Computer Algebra
Independent Integration Tests

Summer 2023 edition

1-Algebraic-functions/1.2-Trinomial-products/1.2.1-Quadratic/38-1.2.1.9-P-x-d+e-x-^m-a+b-x+c-x^2-^p

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Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [400]. This is test number [38].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (400)	0.00 (0)
Mathematica	98.50 (394)	1.50 (6)
Maple	89.25 (357)	10.75 (43)
Fricas	88.00 (352)	12.00 (48)
Giac	87.75 (351)	12.25 (49)
Maxima	72.75 (291)	27.25 (109)
Mupad	48.75 (195)	51.25 (205)
Sympy	47.25 (189)	52.75 (211)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

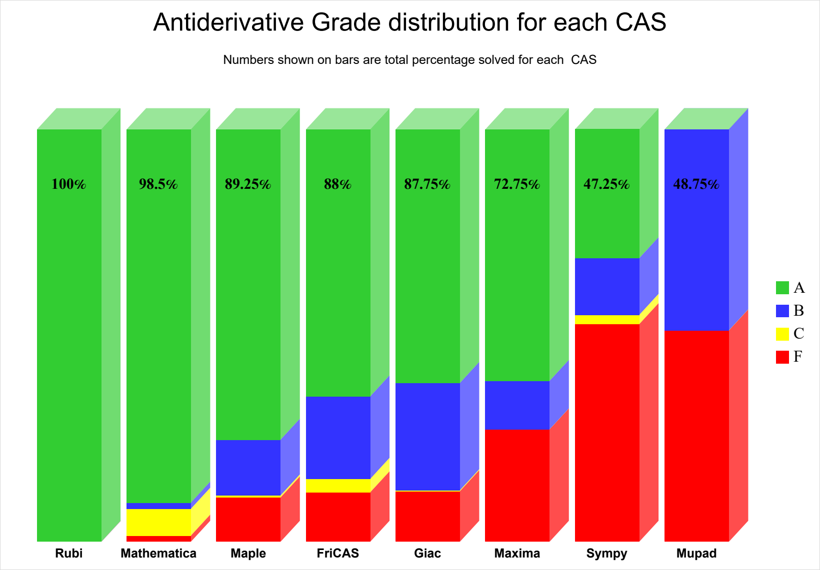
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

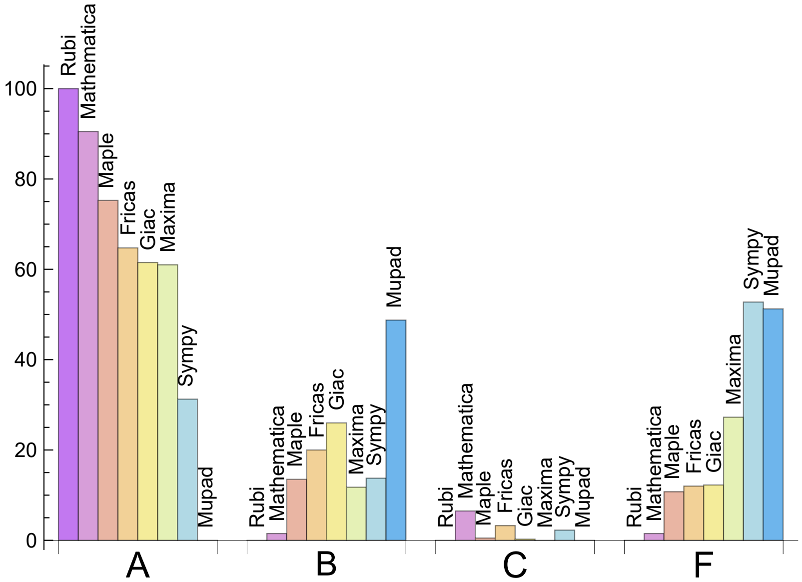
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.000	0.000	0.000	0.000
Mathematica	90.500	1.500	6.500	1.500
Maple	75.250	13.500	0.500	10.750
Fricas	64.750	20.000	3.250	12.000
Giac	61.500	26.000	0.250	12.250
Maxima	61.000	11.750	0.000	27.250
Sympy	31.250	13.750	2.250	52.750
Mupad	0.000	48.750	0.000	51.250

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	6	100.00	0.00	0.00
Maple	43	27.91	72.09	0.00
Fricas	48	25.00	75.00	0.00
Giac	49	67.35	8.16	24.49
Maxima	109	32.11	0.92	66.97
Mupad	205	0.00	100.00	0.00
Sympy	211	72.99	26.54	0.47

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.27
Rubi	0.28
Giac	0.44
Maple	0.78
Mathematica	2.44
Sympy	3.46
Fricas	3.70
Mupad	9.66

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Rubi	238.80	1.00	166.50	1.00
Maxima	419.13	2.05	150.00	1.07
Mathematica	479.92	1.25	136.00	0.95
Fricas	567.76	2.43	190.00	1.39
Maple	770.34	2.12	161.00	0.97
Mupad	773.44	3.16	185.00	1.29
Giac	920.53	2.68	191.00	1.19
Sympy	1766.56	5.34	185.00	1.40

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

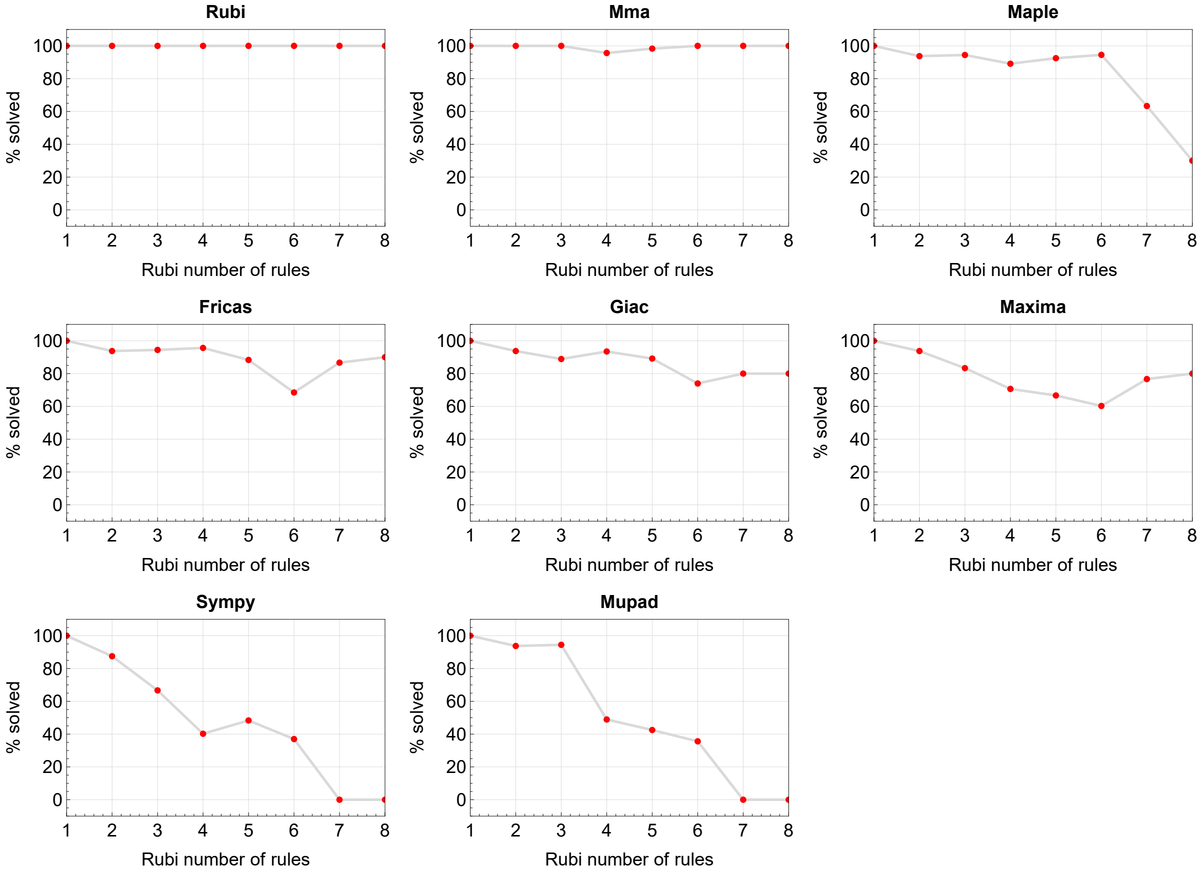


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

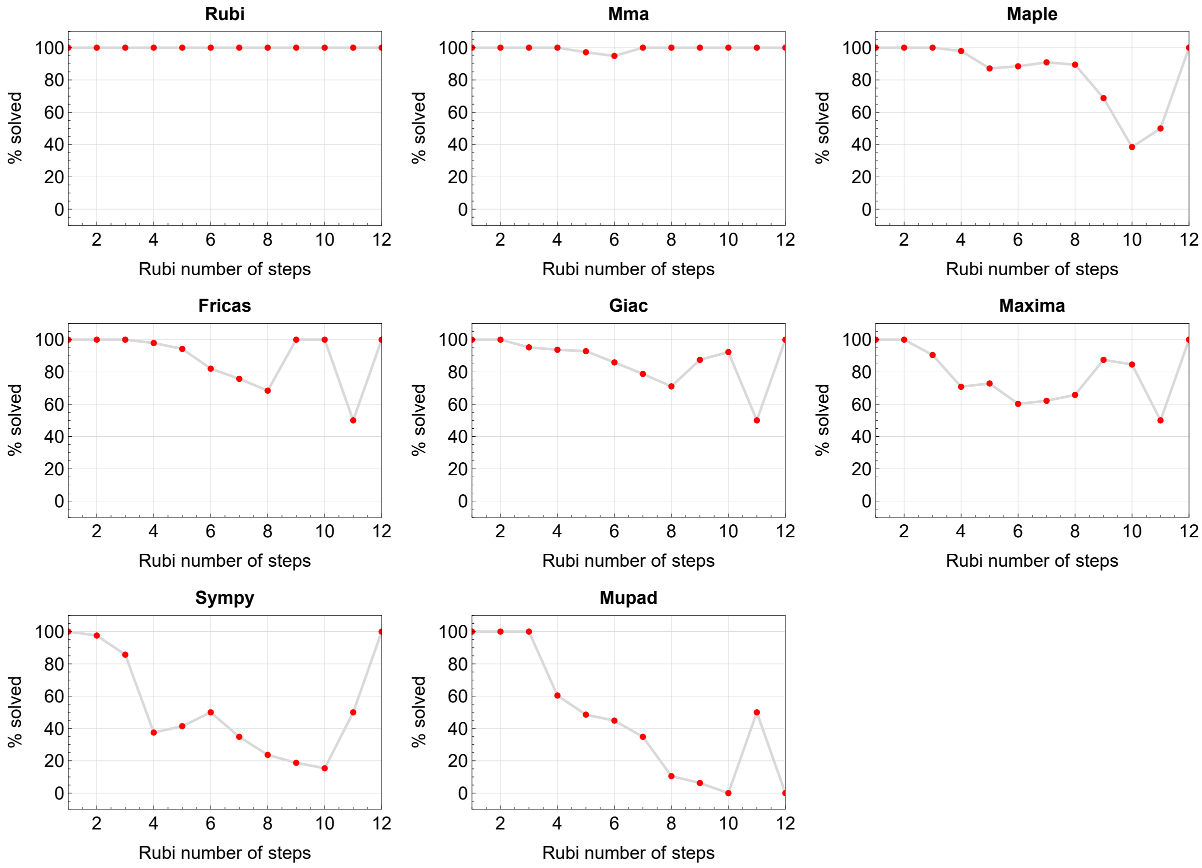


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

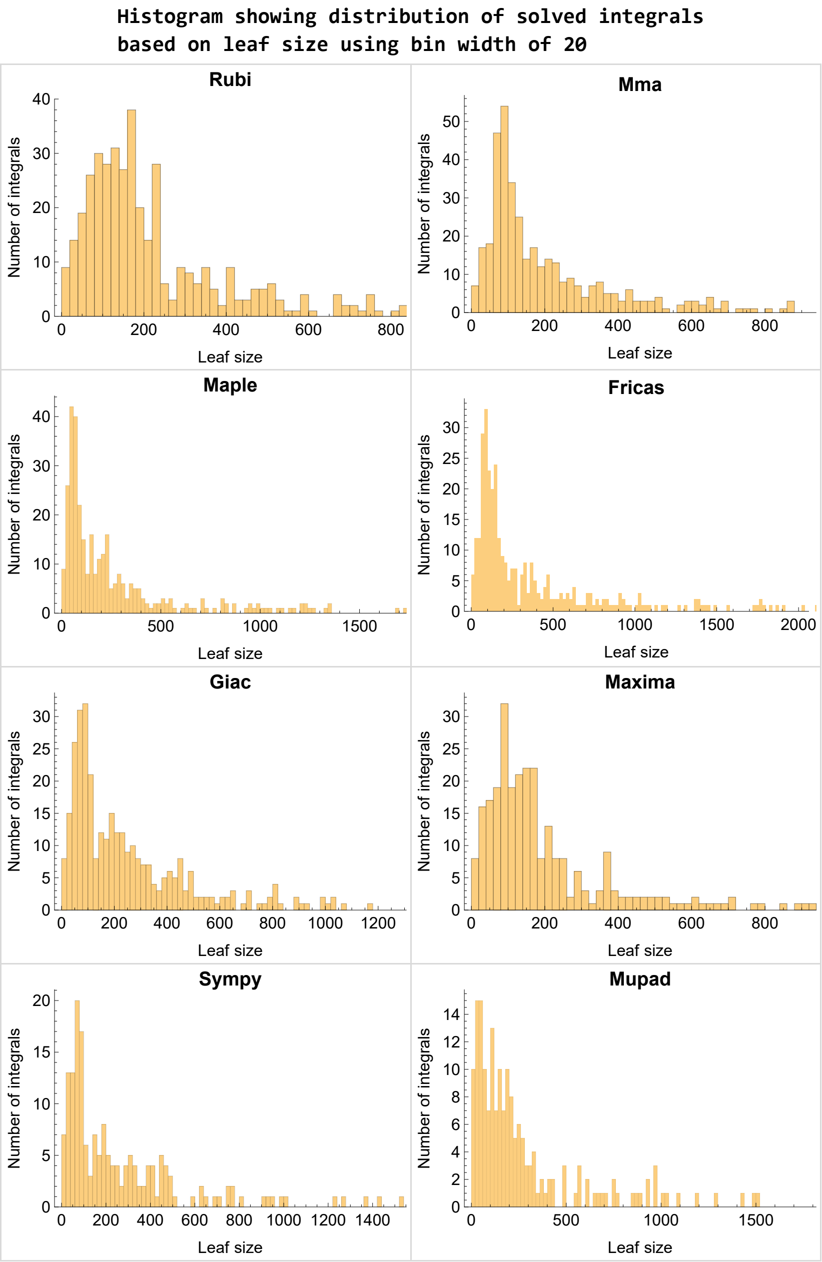


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

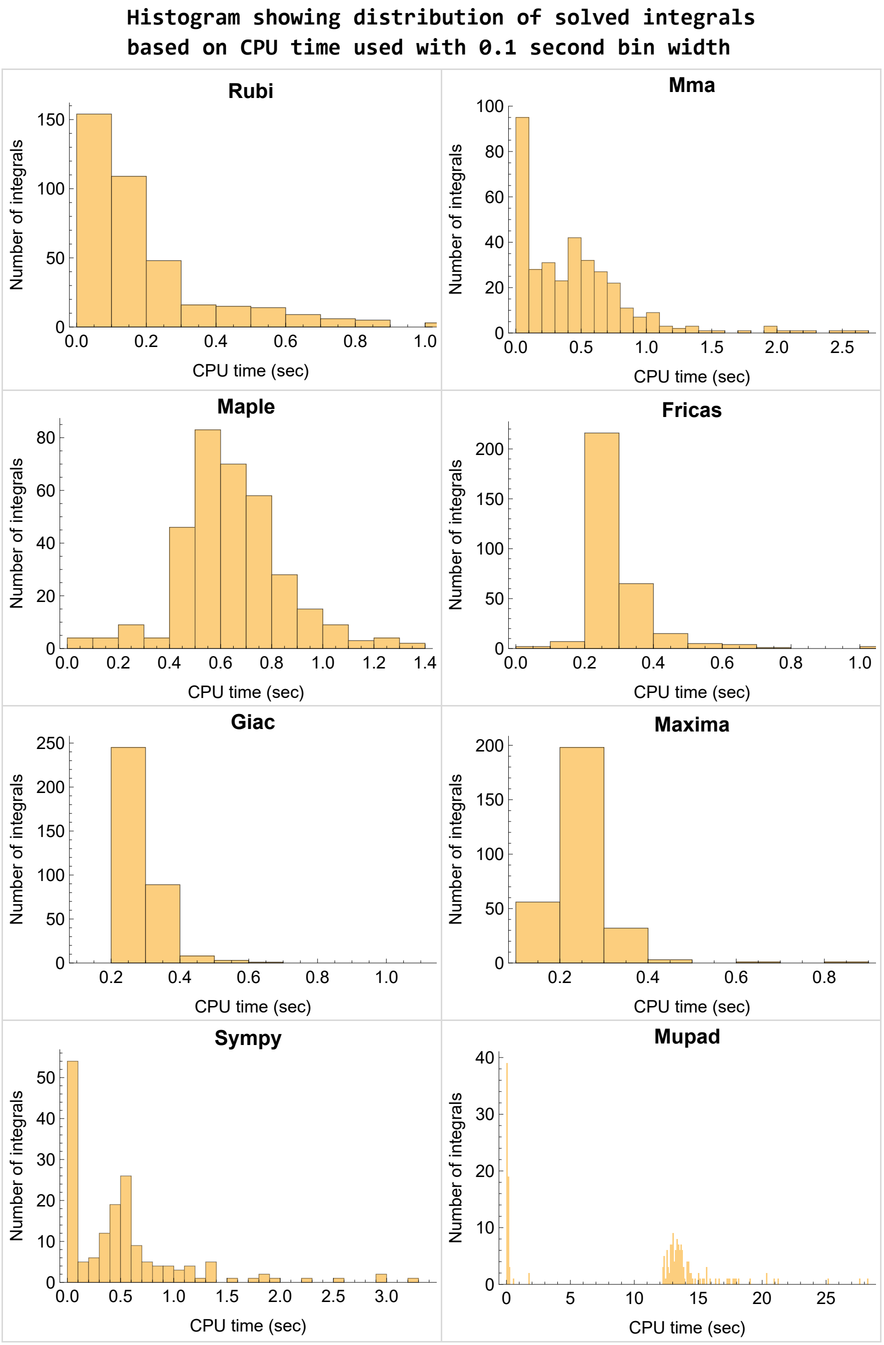


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

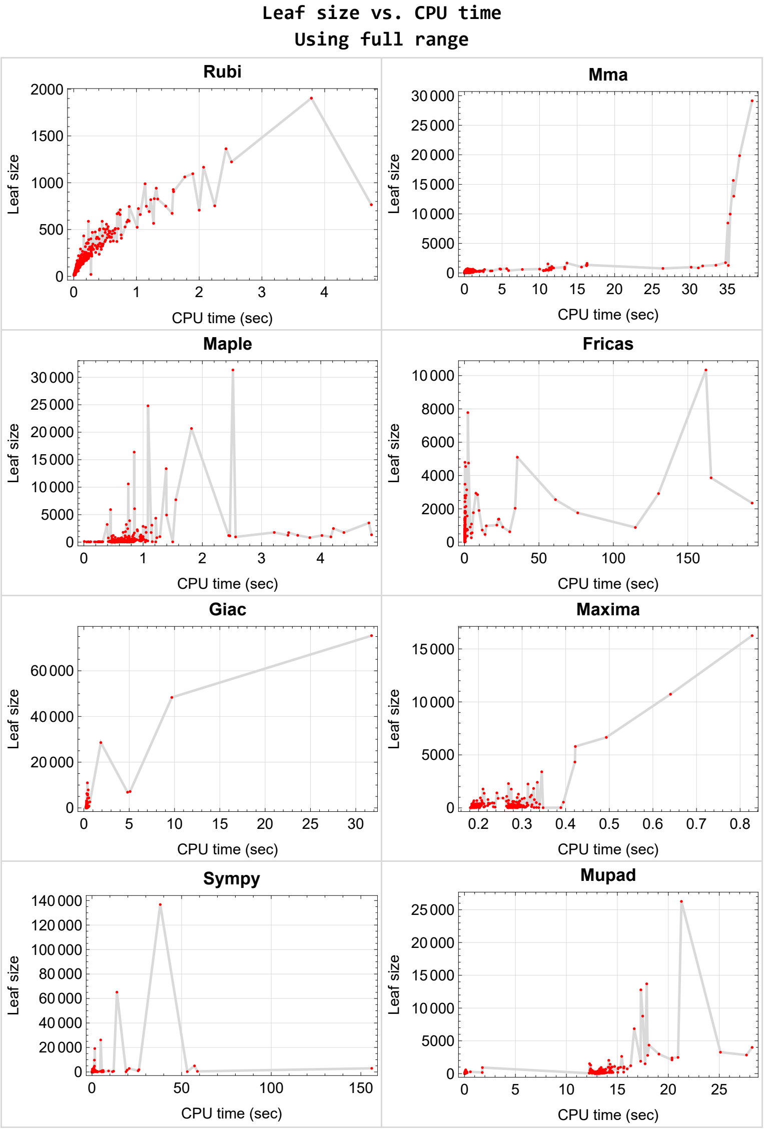


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica $\{264, 265, 398, 399, 400\}$

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
```

```
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

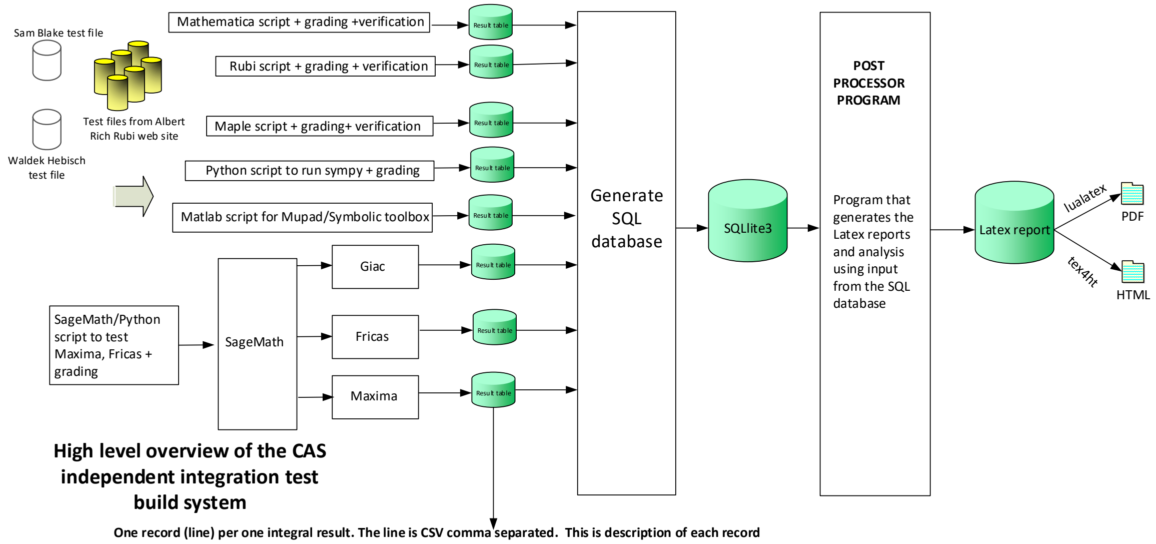
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



High level overview of the CAS independent integration test build system

One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified. (For mma, rubi and maple only)

*The following fields are present only in **Rubi** Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form “{n,n,..}” which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

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June 27, 2023
Design.vsdx

CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 276, 277, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 380, 381, 382, 386, 387, 388, 392, 393, 394, 398, 399, 400 }

B grade { 39, 40, 41, 42, 114, 278 }

C grade { 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 275, 377, 378, 379, 383, 384, 385, 389, 390, 391, 395, 396, 397 }

F normal fail { 136, 137, 138, 272, 273, 274 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 88, 89, 90, 91, 92, 93, 100, 101, 102, 103, 104, 105, 108, 109, 110, 111, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 134, 135, 140, 141, 142, 143, 144, 145, 146, 148, 149, 150, 151, 152, 153, 154, 156, 157, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 179, 180, 181, 182, 183, 184, 186, 187, 188, 189, 190, 191, 199, 200, 201, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 235, 236, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 256, 257, 258, 260, 261, 263, 264, 265, 266, 267, 268, 275, 276, 277, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 332, 333, 334, 335, 344, 345, 352, 353, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397 }

B grade { 41, 42, 64, 84, 85, 86, 87, 94, 95, 96, 97, 98, 99, 106, 107, 112, 113, 114, 147, 155, 158, 159, 178, 185, 192, 193, 194, 195, 196, 197, 198, 202, 203, 204, 205, 206, 207, 231, 232, 233, 234, 237, 238, 239, 259, 262, 269, 270, 271, 278, 366, 367, 368, 369 }

C grade { 133, 255 }

F normal fail { 136, 137, 138, 139, 272, 273, 274, 370, 371, 398, 399, 400 }

F(-1) timedout fail { 327, 328, 329, 330, 331, 336, 337, 338, 339, 340, 341, 342, 343, 346, 347, 348, 349, 350, 351, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365 }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 34, 35, 36, 37, 43, 44, 45, 46, 47, 52, 53, 59, 60, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 88, 89, 90, 91, 100, 101, 102, 103, 104, 105, 108, 109, 110, 111, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 140, 141, 142, 143, 144, 148, 149, 150, 151, 152, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 179, 180, 181, 186, 187, 188, 189, 199, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 254, 255, 256, 257, 275, 276, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 312, 313, 314, 320, 321, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 361, 362, 363, 364, 373, 374, 375, 376, 380, 381, 382, 386, 387, 388, 392, 393, 394 }

B grade { 31, 38, 39, 40, 41, 42, 48, 49, 50, 51, 54, 55, 57, 58, 61, 64, 65, 66, 67, 86, 87, 107, 112, 113, 114, 145, 146, 147, 155, 156, 157, 177, 178, 182, 183, 184, 185, 196, 197, 198, 232, 233, 234, 235, 236, 237, 238, 239, 253, 258, 277, 278, 310, 311, 315, 316, 317, 318, 319, 322, 323, 360, 365, 366, 367, 368, 369, 372, 377, 378, 379, 383, 384, 385, 389, 390, 391, 395, 396, 397 }

C grade { 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271 }

F normal fail { 136, 137, 138, 139, 272, 273, 274, 370, 371, 398, 399, 400 }

F(-1) timedout fail { 56, 62, 63, 82, 83, 84, 85, 92, 93, 94, 95, 96, 97, 98, 99, 106, 153, 154, 158, 159, 190, 191, 192, 193, 194, 195, 200, 201, 202, 203, 204, 205, 206, 207, 230, 231 }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 3, 4, 5, 10, 11, 12, 13, 14, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 57, 58, 59, 60, 61, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 88, 89, 90, 91, 93, 100, 101, 102, 103, 104, 105, 108, 109, 110, 111, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 132, 133, 134, 135, 140, 141, 142, 143, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 254, 255, 256, 257, 275, 276, 277, 279, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 318, 319, 320, 321, 322, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 361, 362, 363, 364, 374, 375, 376, 380, 381, 382, 386, 387, 388, 392, 393, 394 }

B grade { 8, 9, 15, 16, 17, 39, 40, 41, 42, 56, 62, 63, 64, 84, 85, 86, 87, 92, 94, 95, 96, 97, 98, 99, 106, 107, 112, 113, 114, 131, 177, 252, 253, 278, 317, 323, 343, 358, 359, 360, 367, 368, 369, 377, 383, 389, 395 }

C grade { }

F normal fail { 7, 136, 137, 138, 139, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 366, 370, 371, 378, 379, 384, 385, 390, 391, 396, 397, 398, 399, 400 }

F(-1) timedout fail { 6 }

F(-2) exception fail { 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 258, 280, 281, 282, 283, 284, 285, 286, 287, 288, 365, 372, 373 }

Giac

A grade { 1, 2, 3, 4, 6, 10, 11, 12, 13, 14, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 57, 58, 59, 60, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 88, 89, 90, 91, 100, 101, 102, 103, 104, 108, 109, 110, 111, 115, 116, 117, 118, 119, 120, 124, 125, 126, 127, 130, 131, 132, 133, 135, 140, 141, 142, 143, 144, 145, 146, 148, 149, 150, 151, 152, 153, 155, 156, 157, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 179, 180, 181, 182, 183, 186, 187, 188, 189, 199, 208, 209, 210, 211, 214, 215, 216, 217, 220, 221, 222, 223, 226, 227, 228, 229, 235, 236, 240, 241, 242, 243, 246, 247, 248, 249, 252, 253, 254, 255, 279, 280, 281, 282, 284, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 334, 335, 336, 338, 344, 345, 346, 350, 351, 352, 353, 354, 358, 359, 360, 361, 365, 366, 372, 373, 374, 375, 376, 377, 380, 381, 382, 383, 386, 387, 388, 389, 392, 393, 394, 395, 396 }

B grade { 5, 7, 9, 16, 17, 39, 40, 41, 42, 56, 61, 62, 63, 64, 65, 84, 85, 87, 94, 95, 97, 98, 99, 107, 112, 114, 121, 122, 123, 128, 129, 134, 147, 154, 158, 159, 178, 184, 185, 192, 193, 195, 196, 197, 198, 202, 203, 206, 207, 212, 213, 218, 219, 224, 225, 232, 233, 234, 237, 239, 244, 245, 250, 251, 256, 257, 275, 276, 277, 278, 285, 286, 287, 288, 327, 328, 329, 330, 331, 332, 333, 337, 339, 340, 341, 342, 343, 347, 348, 349, 355, 356, 357, 362, 363, 364, 367, 368, 369, 379, 385, 390, 391, 397 }

C grade { 8 }

F normal fail { 83, 86, 93, 96, 113, 136, 137, 138, 139, 194, 238, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 370, 371, 398, 399, 400 }

F(-1) timedout fail { 191, 201, 204, 205 }

F(-2) exception fail { 15, 82, 92, 105, 106, 190, 200, 230, 231, 283, 378, 384 }

Mupad

A grade { }

B grade { 8, 9, 12, 13, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 103, 104, 110, 111, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 180, 182, 183, 184, 185, 186, 187, 188, 189, 208, 209, 210, 236, 254, 258, 275, 276, 277, 278, 279, 284, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 367, 368, 369, 372, 373, 374, 375, 376 }

C grade { }

F normal fail { }

F(-1) timedout fail { 1, 2, 3, 4, 5, 6, 7, 10, 11, 14, 15, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 105, 106, 107, 108, 109, 112, 113, 114, 136, 137, 138, 139, 178, 179, 181, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 255, 256, 257, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 280, 281, 282, 283, 285, 286, 287, 288, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 370, 371, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400 }

F(-2) exception fail { }

Sympy

A grade { 2, 3, 10, 11, 12, 13, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 53, 59, 60, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 79, 80, 81, 91, 101, 102, 103, 104, 110, 111, 115, 118, 119, 120, 140, 141, 142, 143, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 181, 208, 209, 210, 214, 215, 216, 220, 221, 222, 228, 240, 241, 242, 279, 280, 281, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 307, 314, 321, 324, 325, 334, 335, 344, 345, 374, 375, 376, 380, 381, 382, 386, 387, 388 }

B grade { 1, 39, 40, 41, 42, 43, 44, 45, 46, 50, 51, 52, 78, 88, 89, 90, 100, 116, 117, 126, 132, 144, 145, 146, 147, 148, 149, 150, 151, 155, 156, 157, 177, 178, 179, 180, 186, 187, 188, 189, 196, 197, 198, 199, 226, 227, 229, 275, 276, 277, 278, 282, 367, 368, 369 }

C grade { 304, 305, 306, 311, 312, 313, 318, 319, 320 }

F normal fail { 4, 5, 6, 7, 8, 9, 14, 15, 16, 17, 82, 83, 84, 85, 86, 87, 92, 93, 94, 95, 96, 97, 105, 106, 107, 108, 109, 112, 121, 122, 123, 124, 125, 127, 128, 130, 131, 137, 182, 190, 191, 192, 193, 194, 195, 200, 201, 202, 203, 204, 206, 211, 212, 213, 217, 218, 219, 223, 224, 225, 230, 231, 232, 233, 234, 235, 236, 237, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 273, 283, 284, 285, 286, 287, 288, 326, 327, 328, 329, 330, 331, 332, 333, 336, 337, 338, 339, 340, 341, 342, 343, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 377, 378, 379, 383, 384, 385, 389, 390, 391, 392, 393, 394, 395, 396, 397 }

F(-1) timedout fail { 47, 48, 49, 54, 55, 56, 57, 58, 61, 62, 63, 64, 65, 66, 67, 98, 99, 113, 114, 129, 133, 134, 135, 136, 138, 152, 153, 154, 158, 159, 183, 184, 185, 205, 207, 238, 239, 272, 274, 308, 309, 310, 315, 316, 317, 322, 323, 365, 366, 370, 371, 372, 373, 398, 399, 400 }

F(-2) exception fail { 139 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	236	217	218	371	211	466	224	0
N.S.	1	1.00	0.92	0.92	1.57	0.89	1.97	0.95	0.00
time (sec)	N/A	0.286	1.070	0.625	0.277	0.450	0.562	0.296	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	181	180	223	173	328	181	0
N.S.	1	1.00	0.97	0.97	1.20	0.93	1.76	0.97	0.00
time (sec)	N/A	0.138	0.874	0.536	0.280	0.379	0.582	0.312	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	114	107	137	108	165	96	0
N.S.	1	1.00	0.91	0.86	1.10	0.86	1.32	0.77	0.00
time (sec)	N/A	0.042	0.405	0.493	0.285	0.337	0.348	0.297	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	115	111	171	112	0	114	0
N.S.	1	1.00	0.78	0.75	1.16	0.76	0.00	0.77	0.00
time (sec)	N/A	0.114	0.556	0.585	0.311	0.332	0.000	0.294	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	121	196	197	190	0	325	0
N.S.	1	1.00	0.71	1.15	1.16	1.12	0.00	1.91	0.00
time (sec)	N/A	0.130	0.594	0.539	0.293	0.309	0.000	0.312	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	125	238	0	258	0	274	0
N.S.	1	1.00	0.84	1.60	0.00	1.73	0.00	1.84	0.00
time (sec)	N/A	0.114	0.755	0.587	0.000	0.298	0.000	0.299	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	125	293	0	304	0	441	0
N.S.	1	1.00	0.64	1.49	0.00	1.55	0.00	2.25	0.00
time (sec)	N/A	0.123	0.902	0.578	0.000	0.293	0.000	0.296	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	109	116	945	320	0	608	601
N.S.	1	1.00	0.61	0.64	5.25	1.78	0.00	3.38	3.34
time (sec)	N/A	0.134	0.793	0.658	0.204	0.381	0.000	0.309	13.558

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	144	152	1378	399	0	715	960
N.S.	1	1.00	0.62	0.65	5.89	1.71	0.00	3.06	4.10
time (sec)	N/A	0.160	1.029	0.796	0.213	0.350	0.000	0.295	14.181

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	236	185	182	423	178	416	187	0
N.S.	1	1.00	0.78	0.77	1.79	0.75	1.76	0.79	0.00
time (sec)	N/A	0.406	0.837	0.793	0.269	0.292	0.579	0.303	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	150	146	286	145	280	149	0
N.S.	1	1.00	0.79	0.76	1.50	0.76	1.47	0.78	0.00
time (sec)	N/A	0.234	0.704	0.678	0.274	0.303	0.555	0.318	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	114	110	171	109	180	112	270
N.S.	1	1.00	0.80	0.77	1.20	0.76	1.26	0.78	1.89
time (sec)	N/A	0.122	0.497	0.529	0.270	0.311	0.537	0.304	13.763

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	81	72	91	71	124	62	148
N.S.	1	1.00	0.93	0.83	1.05	0.82	1.43	0.71	1.70
time (sec)	N/A	0.032	0.287	0.470	0.272	0.374	0.314	0.292	13.337

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	96	127	138	155	0	109	0
N.S.	1	1.00	0.93	1.23	1.34	1.50	0.00	1.06	0.00
time (sec)	N/A	0.076	0.507	0.531	0.273	0.303	0.000	0.291	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	107	195	317	221	0	0	0
N.S.	1	1.00	0.66	1.20	1.94	1.36	0.00	0.00	0.00
time (sec)	N/A	0.113	0.581	0.526	0.281	0.300	0.000	0.000	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	103	110	608	244	0	350	109
N.S.	1	1.00	0.57	0.61	3.38	1.36	0.00	1.94	0.61
time (sec)	N/A	0.131	0.589	0.566	0.279	0.320	0.000	0.297	12.574

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	139	146	975	320	0	550	204
N.S.	1	1.00	0.59	0.62	4.17	1.37	0.00	2.35	0.87
time (sec)	N/A	0.163	0.750	0.581	0.289	0.332	0.000	0.293	12.543

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	173	208	213	202	202	257	248	206
N.S.	1	0.99	1.19	1.22	1.15	1.15	1.47	1.42	1.18
time (sec)	N/A	0.193	0.054	0.521	0.195	0.306	0.038	0.274	0.090

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	173	150	148	141	141	173	171	143
N.S.	1	0.99	0.86	0.85	0.81	0.81	0.99	0.98	0.82
time (sec)	N/A	0.136	0.037	0.549	0.201	0.293	0.029	0.259	12.366

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	86	79	80	80	97	94	80
N.S.	1	1.00	1.00	0.92	0.93	0.93	1.13	1.09	0.93
time (sec)	N/A	0.075	0.018	0.116	0.187	0.273	0.021	0.288	12.594

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	39	38	38	42	40	39
N.S.	1	1.00	1.00	0.85	0.83	0.83	0.91	0.87	0.85
time (sec)	N/A	0.019	0.009	0.108	0.182	0.262	0.020	0.267	0.025

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	143	136	161	159	161	148	181	175
N.S.	1	0.99	0.94	1.11	1.10	1.11	1.02	1.25	1.21
time (sec)	N/A	0.144	0.048	0.457	0.187	0.267	0.244	0.286	12.647

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	151	142	172	169	250	185	248	192
N.S.	1	0.99	0.93	1.12	1.10	1.63	1.21	1.62	1.25
time (sec)	N/A	0.130	0.090	0.451	0.198	0.290	0.532	0.298	0.092

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	154	176	169	177	273	206	178	185
N.S.	1	0.99	1.13	1.08	1.13	1.75	1.32	1.14	1.19
time (sec)	N/A	0.120	0.060	0.457	0.184	0.277	1.794	0.272	0.094

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	304	301	335	371	360	360	445	432	332
N.S.	1	0.99	1.10	1.22	1.18	1.18	1.46	1.42	1.09
time (sec)	N/A	0.274	0.091	0.501	0.191	0.273	0.042	0.308	0.160

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	216	241	263	257	257	311	302	244
N.S.	1	1.00	1.11	1.21	1.18	1.18	1.43	1.39	1.12
time (sec)	N/A	0.186	0.059	0.485	0.188	0.267	0.035	0.270	0.104

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	144	151	154	154	180	172	140
N.S.	1	1.00	1.12	1.18	1.20	1.20	1.41	1.34	1.09
time (sec)	N/A	0.101	0.034	0.480	0.188	0.278	0.027	0.269	12.837

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	69	75	74	74	83	76	74
N.S.	1	1.00	1.03	1.12	1.10	1.10	1.24	1.13	1.10
time (sec)	N/A	0.027	0.019	0.491	0.186	0.322	0.024	0.265	0.038

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	297	295	285	375	377	379	359	445	422
N.S.	1	0.99	0.96	1.26	1.27	1.28	1.21	1.50	1.42
time (sec)	N/A	0.375	0.106	0.523	0.188	0.303	0.472	0.274	12.505

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	292	289	272	401	392	553	416	520	575
N.S.	1	0.99	0.93	1.37	1.34	1.89	1.42	1.78	1.97
time (sec)	N/A	0.308	0.181	0.486	0.199	0.285	1.088	0.277	0.123

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	295	292	274	392	402	608	474	426	495
N.S.	1	0.99	0.93	1.33	1.36	2.06	1.61	1.44	1.68
time (sec)	N/A	0.305	0.079	0.479	0.203	0.291	4.595	0.271	12.316

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	404	400	459	531	512	512	646	618	490
N.S.	1	0.99	1.14	1.31	1.27	1.27	1.60	1.53	1.21
time (sec)	N/A	0.403	0.134	0.495	0.188	0.257	0.058	0.274	12.549

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	289	288	329	376	367	367	447	432	343
N.S.	1	1.00	1.14	1.30	1.27	1.27	1.55	1.49	1.19
time (sec)	N/A	0.269	0.084	0.496	0.185	0.266	0.046	0.261	12.382

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	196	223	222	222	265	249	187
N.S.	1	1.00	1.16	1.32	1.31	1.31	1.57	1.47	1.11
time (sec)	N/A	0.139	0.046	0.565	0.190	0.271	0.036	0.270	0.119

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	100	109	108	108	122	111	103
N.S.	1	1.00	1.15	1.25	1.24	1.24	1.40	1.28	1.18
time (sec)	N/A	0.046	0.021	0.539	0.187	0.258	0.030	0.268	12.245

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	490	487	498	679	672	674	685	817	741
N.S.	1	0.99	1.02	1.39	1.37	1.38	1.40	1.67	1.51
time (sec)	N/A	0.625	0.296	0.578	0.190	0.275	0.735	0.275	12.377

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	486	483	641	709	691	932	748	882	1511
N.S.	1	0.99	1.32	1.46	1.42	1.92	1.54	1.81	3.11
time (sec)	N/A	0.563	0.227	0.617	0.196	0.288	1.861	0.294	12.272

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	466	463	438	700	701	1025	816	780	1290
N.S.	1	0.99	0.94	1.50	1.50	2.20	1.75	1.67	2.77
time (sec)	N/A	0.567	0.130	0.476	0.208	0.299	9.233	0.286	12.381

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	62	27	82	78	73	111	85
N.S.	1	1.00	3.65	1.59	4.82	4.59	4.29	6.53	5.00
time (sec)	N/A	0.023	0.024	0.483	0.189	0.265	0.181	0.266	0.076

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	62	27	82	78	73	111	85
N.S.	1	1.00	3.65	1.59	4.82	4.59	4.29	6.53	5.00
time (sec)	N/A	0.013	0.008	0.453	0.185	0.276	0.171	0.267	12.481

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	90	38	160	120	153	216	252
N.S.	1	1.00	5.29	2.24	9.41	7.06	9.00	12.71	14.82
time (sec)	N/A	0.027	0.026	0.469	0.194	0.282	0.285	0.281	12.801

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	90	38	160	120	153	216	252
N.S.	1	1.00	5.29	2.24	9.41	7.06	9.00	12.71	14.82
time (sec)	N/A	0.015	0.013	0.569	0.194	0.272	0.267	0.273	0.050

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	237	223	260	244	592	1008	289	277
N.S.	1	0.99	0.93	1.08	1.02	2.47	4.20	1.20	1.15
time (sec)	N/A	0.235	0.133	0.681	0.273	0.278	5.517	0.269	12.874

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	166	155	169	161	404	638	178	181
N.S.	1	0.99	0.92	1.01	0.96	2.40	3.80	1.06	1.08
time (sec)	N/A	0.149	0.099	0.654	0.264	0.284	1.393	0.269	12.922

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	86	84	86	206	337	86	97
N.S.	1	1.00	0.92	0.90	0.92	2.22	3.62	0.92	1.04
time (sec)	N/A	0.072	0.055	0.663	0.275	0.274	0.714	0.392	12.513

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	56	47	48	125	156	48	56
N.S.	1	1.00	1.02	0.85	0.87	2.27	2.84	0.87	1.02
time (sec)	N/A	0.033	0.026	0.504	0.276	0.276	0.235	0.343	12.243

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	120	112	123	262	0	123	840
N.S.	1	1.00	0.90	0.84	0.92	1.97	0.00	0.92	6.32
time (sec)	N/A	0.104	0.062	0.565	0.273	4.613	0.000	0.342	15.629

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	188	204	255	904	0	277	1199
N.S.	1	1.00	0.88	0.95	1.19	4.22	0.00	1.29	5.60
time (sec)	N/A	0.196	0.184	0.624	0.274	25.697	0.000	0.266	16.313

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	305	305	277	317	495	1759	0	516	2980
N.S.	1	1.00	0.91	1.04	1.62	5.77	0.00	1.69	9.77
time (sec)	N/A	0.369	0.185	0.599	0.290	76.070	0.000	0.271	19.082

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	216	216	233	284	287	931	952	299	303
N.S.	1	1.00	1.08	1.31	1.33	4.31	4.41	1.38	1.40
time (sec)	N/A	0.288	0.126	0.649	0.275	0.310	25.718	0.266	13.005

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	175	187	188	631	593	186	195
N.S.	1	1.00	1.20	1.28	1.29	4.32	4.06	1.27	1.34
time (sec)	N/A	0.149	0.086	0.654	0.279	0.301	6.042	0.262	0.228

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	102	108	113	337	318	107	191
N.S.	1	1.00	1.05	1.11	1.16	3.47	3.28	1.10	1.97
time (sec)	N/A	0.049	0.059	0.582	0.272	0.311	2.228	0.275	0.148

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	68	65	62	195	116	60	60
N.S.	1	1.00	0.99	0.94	0.90	2.83	1.68	0.87	0.87
time (sec)	N/A	0.027	0.038	0.599	0.269	0.288	0.331	0.272	0.098

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	226	226	195	293	293	1024	0	364	1493
N.S.	1	1.00	0.86	1.30	1.30	4.53	0.00	1.61	6.61
time (sec)	N/A	0.259	0.136	0.612	0.272	21.753	0.000	0.272	17.721

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	374	371	320	424	604	2916	0	638	2094
N.S.	1	0.99	0.86	1.13	1.61	7.80	0.00	1.71	5.60
time (sec)	N/A	0.584	0.248	0.641	0.280	130.336	0.000	0.280	20.360

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	524	524	466	643	1030	0	0	1013	2828
N.S.	1	1.00	0.89	1.23	1.97	0.00	0.00	1.93	5.40
time (sec)	N/A	1.011	0.373	0.684	0.318	0.000	0.000	0.273	27.671

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	281	333	379	1138	0	358	920
N.S.	1	1.00	1.34	1.59	1.81	5.44	0.00	1.71	4.40
time (sec)	N/A	0.187	0.155	0.573	0.279	0.485	0.000	0.274	1.761

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	175	211	222	253	806	0	256	230
N.S.	1	1.12	1.35	1.42	1.62	5.17	0.00	1.64	1.47
time (sec)	N/A	0.152	0.088	0.589	0.269	0.501	0.000	0.271	12.660

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	137	124	160	470	240	146	128
N.S.	1	1.00	1.05	0.95	1.23	3.62	1.85	1.12	0.98
time (sec)	N/A	0.075	0.065	0.576	0.268	0.356	11.400	0.273	0.148

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	90	83	98	314	156	84	88
N.S.	1	1.00	0.92	0.85	1.00	3.20	1.59	0.86	0.90
time (sec)	N/A	0.038	0.045	0.550	0.279	0.439	0.590	0.266	12.901

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	353	353	321	605	655	2346	0	751	2392
N.S.	1	1.00	0.91	1.71	1.86	6.65	0.00	2.13	6.78
time (sec)	N/A	0.416	0.269	0.811	0.295	193.202	0.000	0.276	20.362

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	571	566	498	833	1196	0	0	1175	6848
N.S.	1	0.99	0.87	1.46	2.09	0.00	0.00	2.06	11.99
time (sec)	N/A	1.273	0.436	0.736	0.321	0.000	0.000	0.296	16.659

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	753	753	672	1055	1835	0	0	1614	8774
N.S.	1	1.00	0.89	1.40	2.44	0.00	0.00	2.14	11.65
time (sec)	N/A	2.248	0.582	0.823	0.326	0.000	0.000	0.280	17.487

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	437	544	599	1864	0	659	669
N.S.	1	1.00	1.87	2.32	2.56	7.97	0.00	2.82	2.86
time (sec)	N/A	0.174	0.182	0.583	0.282	0.373	0.000	0.270	13.859

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	254	288	350	383	457	1378	0	487	402
N.S.	1	1.13	1.38	1.51	1.80	5.43	0.00	1.92	1.58
time (sec)	N/A	0.316	0.181	0.596	0.292	0.474	0.000	0.268	13.678

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	266	268	323	1062	0	331	287
N.S.	1	1.00	1.18	1.19	1.44	4.72	0.00	1.47	1.28
time (sec)	N/A	0.201	0.100	0.602	0.274	0.537	0.000	0.274	0.228

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	171	149	208	636	0	187	164
N.S.	1	1.00	1.04	0.90	1.26	3.85	0.00	1.13	0.99
time (sec)	N/A	0.083	0.084	0.598	0.282	0.397	0.000	0.264	13.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	112	100	133	430	196	109	116
N.S.	1	1.00	0.89	0.79	1.06	3.41	1.56	0.87	0.92
time (sec)	N/A	0.049	0.060	0.517	0.267	0.294	0.901	0.256	12.913

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	29	30	29	46	29	29	30
N.S.	1	1.00	0.67	0.70	0.67	1.07	0.67	0.67	0.70
time (sec)	N/A	0.031	0.015	0.510	0.269	0.275	0.054	0.258	0.034

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	27	24	23	40	20	23	23
N.S.	1	1.00	0.90	0.80	0.77	1.33	0.67	0.77	0.77
time (sec)	N/A	0.025	0.008	0.473	0.278	0.407	0.048	0.271	0.035

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	23	24	23	33	20	23	25
N.S.	1	1.00	0.79	0.83	0.79	1.14	0.69	0.79	0.86
time (sec)	N/A	0.016	0.007	0.493	0.389	0.583	0.057	0.258	0.032

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	20	10	12	14
N.S.	1	1.00	1.00	0.93	0.86	1.43	0.71	0.86	1.00
time (sec)	N/A	0.007	0.005	0.473	0.348	0.261	0.048	0.267	12.938

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	28	26	25	41	24	26	32
N.S.	1	1.00	0.90	0.84	0.81	1.32	0.77	0.84	1.03
time (sec)	N/A	0.027	0.008	0.510	0.338	0.309	0.066	0.282	0.044

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	30	34	49	31	35	38
N.S.	1	1.00	1.00	0.91	1.03	1.48	0.94	1.06	1.15
time (sec)	N/A	0.029	0.012	0.485	0.327	0.350	0.066	0.272	12.851

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	39	38	41	61	42	43	47
N.S.	1	1.00	0.87	0.84	0.91	1.36	0.93	0.96	1.04
time (sec)	N/A	0.041	0.012	0.527	0.333	0.444	0.083	0.257	0.040

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	22	12	13	12	18	8	12	12
N.S.	1	1.83	1.00	1.08	1.00	1.50	0.67	1.00	1.00
time (sec)	N/A	0.005	0.010	0.545	0.290	0.289	0.053	0.267	12.769

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	21	21	25	20	21	21
N.S.	1	1.00	1.00	0.78	0.78	0.93	0.74	0.78	0.78
time (sec)	N/A	0.009	0.009	0.566	0.287	0.273	0.054	0.275	0.035

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	390	387	358	358	436	855	756	463	0
N.S.	1	0.99	0.92	0.92	1.12	2.19	1.94	1.19	0.00
time (sec)	N/A	0.508	1.005	0.813	0.223	0.332	0.580	0.293	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	280	279	245	272	305	595	484	313	0
N.S.	1	1.00	0.88	0.97	1.09	2.12	1.73	1.12	0.00
time (sec)	N/A	0.277	0.721	0.694	0.199	0.461	0.568	0.281	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	145	153	169	329	258	175	0
N.S.	1	1.00	0.83	0.87	0.97	1.88	1.47	1.00	0.00
time (sec)	N/A	0.150	0.486	0.537	0.202	0.453	0.530	0.273	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	87	75	96	190	124	85	0
N.S.	1	1.00	0.82	0.71	0.91	1.79	1.17	0.80	0.00
time (sec)	N/A	0.044	0.214	0.493	0.197	0.274	0.336	0.279	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	206	213	316	362	0	0	0	0
N.S.	1	1.00	1.03	1.53	1.76	0.00	0.00	0.00	0.00
time (sec)	N/A	0.233	0.691	0.550	0.225	0.000	0.000	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	308	303	217	515	478	0	0	0	0
N.S.	1	0.98	0.70	1.67	1.55	0.00	0.00	0.00	0.00
time (sec)	N/A	0.296	0.970	0.565	0.232	0.000	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	296	295	280	946	927	0	0	911	0
N.S.	1	1.00	0.95	3.20	3.13	0.00	0.00	3.08	0.00
time (sec)	N/A	0.302	1.735	0.705	0.256	0.000	0.000	0.325	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	314	314	382	2448	1772	0	0	1701	0
N.S.	1	1.00	1.22	7.80	5.64	0.00	0.00	5.42	0.00
time (sec)	N/A	0.291	10.531	0.740	0.275	0.000	0.000	0.366	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	313	312	439	3903	3404	2552	0	0	0
N.S.	1	1.00	1.40	12.47	10.88	8.15	0.00	0.00	0.00
time (sec)	N/A	0.254	10.746	0.772	0.345	61.128	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	433	432	583	6085	5793	3862	0	4170	0
N.S.	1	1.00	1.35	14.05	13.38	8.92	0.00	9.63	0.00
time (sec)	N/A	0.458	10.881	0.856	0.422	165.638	0.000	0.400	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	462	462	474	406	525	1177	1428	636	0
N.S.	1	1.00	1.03	0.88	1.14	2.55	3.09	1.38	0.00
time (sec)	N/A	0.652	1.331	0.687	0.209	0.332	0.663	0.306	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	346	345	332	320	380	831	933	441	0
N.S.	1	1.00	0.96	0.92	1.10	2.40	2.70	1.27	0.00
time (sec)	N/A	0.307	1.052	0.611	0.195	0.326	0.640	0.302	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	212	197	194	211	477	507	257	0
N.S.	1	1.00	0.92	0.91	0.99	2.24	2.38	1.21	0.00
time (sec)	N/A	0.171	0.718	0.526	0.189	0.286	0.587	0.304	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	117	111	131	262	209	126	0
N.S.	1	1.00	0.85	0.81	0.96	1.91	1.53	0.92	0.00
time (sec)	N/A	0.053	0.366	0.506	0.188	0.281	0.397	0.301	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	326	326	360	563	632	0	0	0	0
N.S.	1	1.00	1.10	1.73	1.94	0.00	0.00	0.00	0.00
time (sec)	N/A	0.462	1.293	0.629	0.264	0.000	0.000	0.000	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	432	428	364	736	708	0	0	0	0
N.S.	1	0.99	0.84	1.70	1.64	0.00	0.00	0.00	0.00
time (sec)	N/A	0.568	1.486	0.740	0.269	0.000	0.000	0.000	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	488	480	361	1167	1299	0	0	1021	0
N.S.	1	0.98	0.74	2.39	2.66	0.00	0.00	2.09	0.00
time (sec)	N/A	0.580	2.013	0.696	0.292	0.000	0.000	0.347	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	475	469	517	1917	2415	0	0	1878	0
N.S.	1	0.99	1.09	4.04	5.08	0.00	0.00	3.95	0.00
time (sec)	N/A	0.502	11.029	0.703	0.335	0.000	0.000	0.385	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	511	511	575	3214	4326	0	0	0	0
N.S.	1	1.00	1.13	6.29	8.47	0.00	0.00	0.00	0.00
time (sec)	N/A	0.663	11.550	0.716	0.421	0.000	0.000	0.000	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	507	507	639	10598	6650	0	0	4363	0
N.S.	1	1.00	1.26	20.90	13.12	0.00	0.00	8.61	0.00
time (sec)	N/A	0.536	11.287	0.750	0.493	0.000	0.000	0.547	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	F(-1)	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	404	403	696	16383	10724	0	0	6061	0
N.S.	1	1.00	1.72	40.55	26.54	0.00	0.00	15.00	0.00
time (sec)	N/A	0.362	11.408	0.850	0.640	0.000	0.000	0.424	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	F(-1)	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	532	531	863	24805	16249	0	0	7857	0
N.S.	1	1.00	1.62	46.63	30.54	0.00	0.00	14.77	0.00
time (sec)	N/A	0.548	11.313	1.083	0.828	0.000	0.000	0.474	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	142	148	166	333	328	168	0
N.S.	1	1.00	0.85	0.88	0.99	1.98	1.95	1.00	0.00
time (sec)	N/A	0.062	0.499	0.596	0.195	0.301	0.483	0.279	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	325	323	244	284	349	559	394	306	0
N.S.	1	0.99	0.75	0.87	1.07	1.72	1.21	0.94	0.00
time (sec)	N/A	0.389	0.737	0.765	0.207	0.319	0.551	0.290	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	222	165	166	230	381	265	201	0
N.S.	1	1.00	0.74	0.74	1.03	1.71	1.19	0.90	0.00
time (sec)	N/A	0.226	0.529	0.598	0.190	0.318	0.511	0.286	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	135	96	87	126	199	150	107	227
N.S.	1	0.99	0.71	0.64	0.93	1.46	1.10	0.79	1.67
time (sec)	N/A	0.112	0.413	0.530	0.196	0.278	0.496	0.296	13.753

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	70	52	61	124	92	57	107
N.S.	1	1.00	0.95	0.70	0.82	1.68	1.24	0.77	1.45
time (sec)	N/A	0.030	0.320	0.482	0.189	0.267	0.305	0.283	13.064

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	137	197	218	881	0	0	0
N.S.	1	1.00	1.05	1.52	1.68	6.78	0.00	0.00	0.00
time (sec)	N/A	0.109	0.457	0.547	0.221	114.782	0.000	0.000	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	177	390	419	0	0	0	0
N.S.	1	1.00	1.05	2.32	2.49	0.00	0.00	0.00	0.00
time (sec)	N/A	0.149	0.853	0.605	0.227	0.000	0.000	0.000	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	224	203	810	896	1088	0	839	0
N.S.	1	1.00	0.90	3.60	3.98	4.84	0.00	3.73	0.00
time (sec)	N/A	0.186	1.094	0.638	0.245	4.715	0.000	0.301	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	228	256	285	346	758	0	331	0
N.S.	1	1.00	1.12	1.24	1.51	3.31	0.00	1.45	0.00
time (sec)	N/A	0.202	0.866	0.848	0.197	0.321	0.000	0.288	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	165	193	227	530	0	214	0
N.S.	1	1.00	1.11	1.30	1.52	3.56	0.00	1.44	0.00
time (sec)	N/A	0.118	0.646	0.668	0.206	0.338	0.000	0.280	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	104	113	126	278	209	113	151
N.S.	1	1.00	1.04	1.13	1.26	2.78	2.09	1.13	1.51
time (sec)	N/A	0.057	0.463	0.514	0.196	0.298	6.334	0.294	13.482

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	62	70	61	181	87	62	68
N.S.	1	1.00	1.02	1.15	1.00	2.97	1.43	1.02	1.11
time (sec)	N/A	0.023	0.352	0.458	0.202	0.284	2.996	0.282	12.750

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	147	386	453	721	0	288	0
N.S.	1	1.00	1.07	2.80	3.28	5.22	0.00	2.09	0.00
time (sec)	N/A	0.102	0.663	0.514	0.236	1.055	0.000	0.288	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	239	248	872	1085	1573	0	0	0
N.S.	1	1.00	1.04	3.65	4.54	6.58	0.00	0.00	0.00
time (sec)	N/A	0.263	1.356	0.529	0.265	1.823	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	374	372	1537	1787	2254	2853	0	1421	0
N.S.	1	0.99	4.11	4.78	6.03	7.63	0.00	3.80	0.00
time (sec)	N/A	0.689	11.111	0.663	0.314	8.792	0.000	0.313	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	50	47	83	68	194	48	59
N.S.	1	1.00	0.75	0.70	1.24	1.01	2.90	0.72	0.88
time (sec)	N/A	0.032	0.454	0.540	0.199	0.273	5.473	0.279	12.969

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	71	72	118	103	638	80	93
N.S.	1	1.00	0.73	0.74	1.22	1.06	6.58	0.82	0.96
time (sec)	N/A	0.040	0.523	0.582	0.204	0.274	12.062	0.274	12.958

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	92	96	153	137	1880	112	115
N.S.	1	1.00	0.72	0.76	1.20	1.08	14.80	0.88	0.91
time (sec)	N/A	0.048	0.636	0.545	0.215	0.274	26.214	0.270	12.990

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	66	45	78	60	94	54	45
N.S.	1	1.00	0.62	0.42	0.74	0.57	0.89	0.51	0.42
time (sec)	N/A	0.057	0.209	0.543	0.285	0.265	0.336	0.264	0.051

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	58	40	64	54	75	48	40
N.S.	1	1.00	0.71	0.49	0.78	0.66	0.91	0.59	0.49
time (sec)	N/A	0.045	0.164	0.500	0.284	0.295	0.226	0.274	12.852

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	56	35	50	49	63	44	35
N.S.	1	1.00	0.90	0.56	0.81	0.79	1.02	0.71	0.56
time (sec)	N/A	0.027	0.126	0.471	0.273	0.255	0.151	0.259	0.034

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	89	55	58	88	0	99	61
N.S.	1	1.00	1.33	0.82	0.87	1.31	0.00	1.48	0.91
time (sec)	N/A	0.044	0.273	0.513	0.270	0.267	0.000	0.291	13.213

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	92	62	65	106	0	191	68
N.S.	1	1.00	1.30	0.87	0.92	1.49	0.00	2.69	0.96
time (sec)	N/A	0.042	0.328	0.524	0.270	0.267	0.000	0.392	13.177

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	71	65	76	89	0	180	77
N.S.	1	1.00	0.92	0.84	0.99	1.16	0.00	2.34	1.00
time (sec)	N/A	0.043	0.386	0.492	0.275	0.263	0.000	0.292	0.112

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	66	45	78	76	0	54	110
N.S.	1	1.00	0.76	0.52	0.90	0.87	0.00	0.62	1.26
time (sec)	N/A	0.065	0.251	0.497	0.277	0.281	0.000	0.273	0.059

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	61	40	64	72	0	49	105
N.S.	1	1.00	0.86	0.56	0.90	1.01	0.00	0.69	1.48
time (sec)	N/A	0.052	0.218	0.514	0.270	0.255	0.000	0.266	13.056

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	56	35	50	67	114	44	100
N.S.	1	1.00	1.02	0.64	0.91	1.22	2.07	0.80	1.82
time (sec)	N/A	0.027	0.181	0.463	0.268	0.255	5.579	0.268	0.038

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	51	48	58	83	0	82	106
N.S.	1	1.00	0.96	0.91	1.09	1.57	0.00	1.55	2.00
time (sec)	N/A	0.035	0.510	0.472	0.270	0.302	0.000	0.280	0.134

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	71	60	84	103	0	168	157
N.S.	1	1.00	0.95	0.80	1.12	1.37	0.00	2.24	2.09
time (sec)	N/A	0.045	0.454	0.484	0.273	0.319	0.000	0.288	13.136

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	81	65	124	119	0	196	180
N.S.	1	1.00	0.84	0.67	1.28	1.23	0.00	2.02	1.86
time (sec)	N/A	0.071	0.485	0.505	0.278	0.268	0.000	0.293	13.088

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	64	45	105	87	0	53	212
N.S.	1	1.00	0.88	0.62	1.44	1.19	0.00	0.73	2.90
time (sec)	N/A	0.054	0.323	0.621	0.268	0.271	0.000	0.264	0.056

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	61	40	91	83	0	48	200
N.S.	1	1.00	1.02	0.67	1.52	1.38	0.00	0.80	3.33
time (sec)	N/A	0.046	0.284	0.591	0.269	0.257	0.000	0.264	0.049

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	30	27	50	40	180	25	185
N.S.	1	1.00	0.73	0.66	1.22	0.98	4.39	0.61	4.51
time (sec)	N/A	0.025	0.223	0.523	0.195	0.260	18.992	0.275	0.049

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	58	74	81	103	0	91	218
N.S.	1	1.00	0.79	1.01	1.11	1.41	0.00	1.25	2.99
time (sec)	N/A	0.049	0.600	0.563	0.275	0.259	0.000	0.281	0.134

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	91	70	107	134	0	233	270
N.S.	1	1.00	0.96	0.74	1.13	1.41	0.00	2.45	2.84
time (sec)	N/A	0.077	0.739	0.512	0.292	0.265	0.000	0.300	13.075

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	91	75	147	149	0	183	301
N.S.	1	1.00	0.78	0.64	1.26	1.27	0.00	1.56	2.57
time (sec)	N/A	0.110	0.622	0.487	0.288	0.283	0.000	0.284	12.666

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	420	417	0	0	0	0	0	0	0
N.S.	1	0.99	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.380	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	403	401	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.274	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	474	474	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.344	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	222	222	165	0	0	0	0	0	0
N.S.	1	1.00	0.74	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.262	0.427	0.000	0.000	0.000	0.000	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	254	254	256	266	263	263	320	308	244
N.S.	1	1.00	1.01	1.05	1.04	1.04	1.26	1.21	0.96
time (sec)	N/A	0.200	0.060	0.701	0.196	0.261	0.042	0.262	0.150

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	163	165	165	165	197	187	149
N.S.	1	1.00	1.01	1.02	1.02	1.02	1.22	1.16	0.93
time (sec)	N/A	0.123	0.030	0.687	0.189	0.250	0.031	0.265	13.679

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	96	90	87	87	102	99	88
N.S.	1	1.00	1.00	0.94	0.91	0.91	1.06	1.03	0.92
time (sec)	N/A	0.069	0.016	0.652	0.189	0.301	0.024	0.269	13.637

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	39	38	38	42	40	39
N.S.	1	1.00	1.00	0.85	0.83	0.83	0.91	0.87	0.85
time (sec)	N/A	0.019	0.006	0.089	0.194	0.296	0.021	0.279	0.024

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	84	82	0	265	413	78	224
N.S.	1	1.00	1.04	1.01	0.00	3.27	5.10	0.96	2.77
time (sec)	N/A	0.067	0.059	0.661	0.000	0.262	0.598	0.261	0.189

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	98	115	0	511	376	108	172
N.S.	1	1.00	0.98	1.15	0.00	5.11	3.76	1.08	1.72
time (sec)	N/A	0.050	0.050	0.603	0.000	0.313	0.587	0.261	13.069

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	160	269	0	1199	774	217	401
N.S.	1	1.00	0.99	1.67	0.00	7.45	4.81	1.35	2.49
time (sec)	N/A	0.077	0.130	0.631	0.000	0.293	1.174	0.264	13.096

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	206	204	510	0	2103	1224	407	698
N.S.	1	1.00	0.99	2.48	0.00	10.21	5.94	1.98	3.39
time (sec)	N/A	0.110	0.224	0.648	0.000	0.319	1.985	0.264	13.249

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	591	591	585	869	0	2150	4972	805	967
N.S.	1	1.00	0.99	1.47	0.00	3.64	8.41	1.36	1.64
time (sec)	N/A	0.883	0.323	0.997	0.000	1.001	57.189	0.268	15.319

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	348	348	345	453	0	1273	2839	433	557
N.S.	1	1.00	0.99	1.30	0.00	3.66	8.16	1.24	1.60
time (sec)	N/A	0.433	0.205	0.944	0.000	0.484	20.767	0.277	13.689

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	173	192	0	654	1265	189	273
N.S.	1	1.00	0.98	1.08	0.00	3.69	7.15	1.07	1.54
time (sec)	N/A	0.181	0.099	0.819	0.000	0.315	6.203	0.267	0.583

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	95	93	0	302	488	89	132
N.S.	1	1.00	1.03	1.01	0.00	3.28	5.30	0.97	1.43
time (sec)	N/A	0.086	0.040	0.795	0.000	0.278	1.100	0.258	0.258

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	193	179	0	625	0	199	2467
N.S.	1	1.00	0.98	0.91	0.00	3.19	0.00	1.02	12.59
time (sec)	N/A	0.209	0.115	0.694	0.000	30.410	0.000	0.272	20.951

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	316	316	281	346	0	0	0	459	3991
N.S.	1	1.00	0.89	1.09	0.00	0.00	0.00	1.45	12.63
time (sec)	N/A	0.432	0.305	0.721	0.000	0.000	0.000	0.266	28.225

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	509	509	504	629	0	0	0	1065	12784
N.S.	1	1.00	0.99	1.24	0.00	0.00	0.00	2.09	25.12
time (sec)	N/A	0.708	0.391	0.993	0.000	0.000	0.000	0.270	17.307

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	288	288	398	628	0	2771	2966	547	742
N.S.	1	1.00	1.38	2.18	0.00	9.62	10.30	1.90	2.58
time (sec)	N/A	0.440	0.482	0.771	0.000	0.548	156.090	0.267	15.068

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	225	308	0	1413	1535	271	376
N.S.	1	1.00	1.26	1.73	0.00	7.94	8.62	1.52	2.11
time (sec)	N/A	0.176	0.225	0.684	0.000	0.370	19.635	0.264	14.451

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	114	133	0	632	459	125	203
N.S.	1	1.00	0.97	1.13	0.00	5.36	3.89	1.06	1.72
time (sec)	N/A	0.064	0.058	0.609	0.000	0.310	0.996	0.259	13.568

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	407	407	405	809	0	0	0	886	13698
N.S.	1	1.00	1.00	1.99	0.00	0.00	0.00	2.18	33.66
time (sec)	N/A	0.760	0.491	0.960	0.000	0.000	0.000	0.271	17.883

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	673	673	650	1345	0	0	0	1506	26278
N.S.	1	1.00	0.97	2.00	0.00	0.00	0.00	2.24	39.05
time (sec)	N/A	1.569	1.133	0.837	0.000	0.000	0.000	0.292	21.285

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	60	53	51	75	60	51	55
N.S.	1	1.00	0.97	0.85	0.82	1.21	0.97	0.82	0.89
time (sec)	N/A	0.048	0.023	0.745	0.267	0.293	0.068	0.264	0.042

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	55	46	46	70	54	46	48
N.S.	1	1.00	1.00	0.84	0.84	1.27	0.98	0.84	0.87
time (sec)	N/A	0.043	0.018	0.776	0.268	0.275	0.067	0.260	0.044

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	52	45	43	60	53	43	59
N.S.	1	1.00	1.00	0.87	0.83	1.15	1.02	0.83	1.13
time (sec)	N/A	0.026	0.014	0.737	0.280	0.267	0.071	0.262	13.204

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	39	34	32	41	41	32	35
N.S.	1	1.00	0.95	0.83	0.78	1.00	1.00	0.78	0.85
time (sec)	N/A	0.016	0.015	0.619	0.267	0.291	0.057	0.256	13.487

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	56	48	47	72	54	48	58
N.S.	1	1.00	1.00	0.86	0.84	1.29	0.96	0.86	1.04
time (sec)	N/A	0.046	0.017	0.621	0.282	0.320	0.079	0.259	0.104

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	61	55	54	85	65	55	68
N.S.	1	1.00	1.00	0.90	0.89	1.39	1.07	0.90	1.11
time (sec)	N/A	0.051	0.017	0.671	0.274	0.302	0.086	0.258	13.745

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	66	60	63	98	71	63	75
N.S.	1	1.00	0.97	0.88	0.93	1.44	1.04	0.93	1.10
time (sec)	N/A	0.058	0.020	0.622	0.279	0.299	0.102	0.259	0.102

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	10	10	7	8	10
N.S.	1	1.00	1.00	1.10	1.00	1.00	0.70	0.80	1.00
time (sec)	N/A	0.005	0.005	0.730	0.182	0.269	0.041	0.263	0.044

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	28	27	27	36	27	29
N.S.	1	1.00	1.00	0.90	0.87	0.87	1.16	0.87	0.94
time (sec)	N/A	0.016	0.007	0.758	0.270	0.302	0.052	0.268	0.032

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	22	21	21	22	21	21
N.S.	1	1.00	1.00	0.96	0.91	0.91	0.96	0.91	0.91
time (sec)	N/A	0.017	0.007	0.766	0.269	0.285	0.046	0.268	0.044

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	19	18	17	25	14	18	17
N.S.	1	1.00	0.90	0.86	0.81	1.19	0.67	0.86	0.81
time (sec)	N/A	0.007	0.007	0.579	0.185	0.280	0.036	0.262	13.130

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	14	14	16	14
N.S.	1	1.00	1.00	0.83	0.78	0.78	0.78	0.89	0.78
time (sec)	N/A	0.010	0.004	0.512	0.190	0.278	0.051	0.255	0.044

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	12	12	14	12
N.S.	1	1.00	1.00	0.93	0.86	0.86	0.86	1.00	0.86
time (sec)	N/A	0.009	0.004	0.524	0.181	0.291	0.058	0.253	13.162

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	31	22	21	21	22	21	17
N.S.	1	1.00	1.15	0.81	0.78	0.78	0.81	0.78	0.63
time (sec)	N/A	0.017	0.006	0.646	0.286	0.262	0.050	0.269	13.248

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	48	30	36	55	46	45	35
N.S.	1	1.00	1.00	0.62	0.75	1.15	0.96	0.94	0.73
time (sec)	N/A	0.027	0.029	0.562	0.267	0.273	0.056	0.257	0.114

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	17	19	19	15	19	17
N.S.	1	1.00	1.00	0.81	0.90	0.90	0.71	0.90	0.81
time (sec)	N/A	0.008	0.007	0.536	0.186	0.264	0.051	0.274	13.020

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	32	30	39	37	30	36
N.S.	1	1.00	1.00	0.82	0.77	1.00	0.95	0.77	0.92
time (sec)	N/A	0.016	0.017	1.037	0.273	0.251	0.063	0.302	12.893

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	33	33	31	11	11
N.S.	1	1.00	1.00	1.09	3.00	3.00	2.82	1.00	1.00
time (sec)	N/A	0.005	0.005	0.763	0.185	0.269	0.050	0.272	12.849

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	267	360	479	0	953	2691	480	0
N.S.	1	1.00	1.35	1.79	0.00	3.57	10.08	1.80	0.00
time (sec)	N/A	0.140	3.664	0.881	0.000	0.361	0.578	0.300	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	230	285	0	605	775	295	0
N.S.	1	1.00	1.08	1.34	0.00	2.85	3.66	1.39	0.00
time (sec)	N/A	0.104	1.986	0.764	0.000	0.312	0.506	0.291	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	145	151	0	355	308	158	240
N.S.	1	1.00	0.92	0.96	0.00	2.26	1.96	1.01	1.53
time (sec)	N/A	0.074	0.781	0.697	0.000	0.290	0.433	0.294	13.748

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	88	76	0	203	189	82	0
N.S.	1	1.00	0.85	0.73	0.00	1.95	1.82	0.79	0.00
time (sec)	N/A	0.047	0.404	0.668	0.000	0.300	0.367	0.293	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	95	145	0	403	0	108	108
N.S.	1	1.00	0.97	1.48	0.00	4.11	0.00	1.10	1.10
time (sec)	N/A	0.044	0.592	0.711	0.000	0.375	0.000	0.287	13.339

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	107	128	0	242	0	193	127
N.S.	1	1.00	0.94	1.12	0.00	2.12	0.00	1.69	1.11
time (sec)	N/A	0.041	0.808	0.679	0.000	0.673	0.000	0.288	13.395

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	234	294	0	563	0	452	578
N.S.	1	1.00	1.40	1.76	0.00	3.37	0.00	2.71	3.46
time (sec)	N/A	0.055	2.519	0.672	0.000	5.164	0.000	0.287	13.679

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	220	410	524	0	978	0	805	1018
N.S.	1	1.00	1.86	2.38	0.00	4.45	0.00	3.66	4.63
time (sec)	N/A	0.077	5.870	0.756	0.000	14.683	0.000	0.293	14.480

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	930	927	1093	1757	0	2817	4820	1657	3262
N.S.	1	1.00	1.18	1.89	0.00	3.03	5.18	1.78	3.51
time (sec)	N/A	1.586	11.638	1.136	0.000	1.118	1.383	0.294	25.114

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	584	581	657	1027	0	1791	2440	983	1881
N.S.	1	0.99	1.12	1.76	0.00	3.07	4.18	1.68	3.22
time (sec)	N/A	0.847	10.008	1.000	0.000	0.690	1.187	0.314	17.289

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	322	322	345	488	0	1009	993	478	877
N.S.	1	1.00	1.07	1.52	0.00	3.13	3.08	1.48	2.72
time (sec)	N/A	0.290	3.465	0.769	0.000	0.465	1.036	0.297	15.104

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	174	197	0	465	384	204	320
N.S.	1	1.00	0.99	1.13	0.00	2.66	2.19	1.17	1.83
time (sec)	N/A	0.102	0.071	0.672	0.000	0.298	0.535	0.302	13.542

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	321	321	324	515	0	0	0	0	0
N.S.	1	1.00	1.01	1.60	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.448	2.212	0.808	0.000	0.000	0.000	0.000	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	459	453	430	704	0	0	0	0	0
N.S.	1	0.99	0.94	1.53	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.610	10.507	0.852	0.000	0.000	0.000	0.000	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	448	446	500	1238	0	0	0	2364	0
N.S.	1	1.00	1.12	2.76	0.00	0.00	0.00	5.28	0.00
time (sec)	N/A	0.540	10.865	0.961	0.000	0.000	0.000	0.467	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	603	601	574	3090	0	0	0	6846	0
N.S.	1	1.00	0.95	5.12	0.00	0.00	0.00	11.35	0.00
time (sec)	N/A	0.865	11.380	1.148	0.000	0.000	0.000	4.818	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	497	499	693	4940	0	0	0	0	0
N.S.	1	1.00	1.39	9.94	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.505	13.347	1.396	0.000	0.000	0.000	0.000	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	824	826	1334	7714	0	0	0	28577	0
N.S.	1	1.00	1.62	9.36	0.00	0.00	0.00	34.68	0.00
time (sec)	N/A	1.339	16.236	1.553	0.000	0.000	0.000	1.870	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1169	1166	1683	2878	0	4751	19122	2902	0
N.S.	1	1.00	1.44	2.46	0.00	4.06	16.36	2.48	0.00
time (sec)	N/A	2.070	13.655	1.000	0.000	2.746	1.543	0.334	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	753	749	877	1680	0	3145	9687	1802	0
N.S.	1	0.99	1.16	2.23	0.00	4.18	12.86	2.39	0.00
time (sec)	N/A	1.158	11.445	0.852	0.000	1.496	1.333	0.323	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	418	418	601	936	0	1833	3990	925	0
N.S.	1	1.00	1.44	2.24	0.00	4.39	9.55	2.21	0.00
time (sec)	N/A	0.369	7.707	0.766	0.000	0.651	1.128	0.309	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	236	293	398	0	839	1360	403	0
N.S.	1	1.00	1.24	1.69	0.00	3.56	5.76	1.71	0.00
time (sec)	N/A	0.135	0.153	0.660	0.000	0.364	0.562	0.287	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	660	660	728	990	0	0	0	0	0
N.S.	1	1.00	1.10	1.50	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.061	5.623	0.947	0.000	0.000	0.000	0.000	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	754	750	641	1324	0	0	0	0	0
N.S.	1	0.99	0.85	1.76	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.467	4.828	0.962	0.000	0.000	0.000	0.000	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	824	819	817	1781	0	0	0	2628	0
N.S.	1	0.99	0.99	2.16	0.00	0.00	0.00	3.19	0.00
time (sec)	N/A	1.228	11.634	1.051	0.000	0.000	0.000	0.665	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	833	829	983	2704	0	0	0	7155	0
N.S.	1	1.00	1.18	3.25	0.00	0.00	0.00	8.59	0.00
time (sec)	N/A	1.285	13.344	1.044	0.000	0.000	0.000	5.099	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1097	1096	1005	4359	0	0	0	0	0
N.S.	1	1.00	0.92	3.97	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.898	15.586	1.214	0.000	0.000	0.000	0.000	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1226	1223	1363	13372	0	0	0	0	0
N.S.	1	1.00	1.11	10.91	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.516	16.342	1.390	0.000	0.000	0.000	0.000	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	657	660	1222	20684	0	0	0	48343	0
N.S.	1	1.00	1.86	31.48	0.00	0.00	0.00	73.58	0.00
time (sec)	N/A	0.738	16.264	1.817	0.000	0.000	0.000	9.703	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1062	1062	1636	31330	0	0	0	75375	0
N.S.	1	1.00	1.54	29.50	0.00	0.00	0.00	70.97	0.00
time (sec)	N/A	1.770	16.323	2.517	0.000	0.000	0.000	31.742	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	80	60	126	83	76	78	170
N.S.	1	1.00	0.56	0.42	0.88	0.58	0.53	0.55	1.19
time (sec)	N/A	0.080	0.422	0.700	0.270	0.337	0.478	0.283	14.570

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	75	55	109	78	70	73	153
N.S.	1	1.00	0.64	0.47	0.92	0.66	0.59	0.62	1.30
time (sec)	N/A	0.063	0.340	0.672	0.272	0.321	0.465	0.293	14.200

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	70	50	92	73	63	68	136
N.S.	1	1.00	0.75	0.54	0.99	0.78	0.68	0.73	1.46
time (sec)	N/A	0.040	0.264	0.672	0.276	0.271	0.492	0.287	13.873

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	104	70	96	115	0	126	0
N.S.	1	1.00	1.03	0.69	0.95	1.14	0.00	1.25	0.00
time (sec)	N/A	0.068	0.242	0.689	0.292	0.306	0.000	0.312	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	110	87	103	133	0	380	0
N.S.	1	1.00	1.02	0.81	0.95	1.23	0.00	3.52	0.00
time (sec)	N/A	0.070	0.419	0.657	0.271	0.350	0.000	0.486	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	111	87	114	149	0	251	0
N.S.	1	1.00	0.97	0.76	0.99	1.30	0.00	2.18	0.00
time (sec)	N/A	0.071	0.451	0.765	0.278	0.280	0.000	0.318	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	90	70	155	93	90	88	0
N.S.	1	1.00	0.57	0.44	0.98	0.59	0.57	0.56	0.00
time (sec)	N/A	0.127	0.646	0.789	0.281	0.268	0.580	0.272	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	85	65	138	88	82	83	0
N.S.	1	1.00	0.60	0.46	0.98	0.62	0.58	0.59	0.00
time (sec)	N/A	0.084	0.544	0.794	0.265	0.270	0.535	0.288	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	80	60	121	83	76	78	0
N.S.	1	1.00	0.69	0.52	1.04	0.72	0.66	0.67	0.00
time (sec)	N/A	0.046	0.402	0.761	0.269	0.272	0.553	0.282	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	114	80	125	125	0	136	0
N.S.	1	1.00	0.92	0.65	1.01	1.01	0.00	1.10	0.00
time (sec)	N/A	0.085	0.382	0.663	0.286	0.267	0.000	0.395	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	121	97	132	143	0	570	0
N.S.	1	1.00	0.92	0.74	1.01	1.09	0.00	4.35	0.00
time (sec)	N/A	0.090	0.450	0.743	0.280	0.262	0.000	0.502	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	121	97	143	159	0	261	0
N.S.	1	1.00	0.88	0.70	1.04	1.15	0.00	1.89	0.00
time (sec)	N/A	0.090	0.589	0.698	0.304	0.267	0.000	0.307	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	100	80	184	103	104	98	0
N.S.	1	1.00	0.53	0.42	0.97	0.54	0.55	0.52	0.00
time (sec)	N/A	0.101	0.838	0.681	0.296	0.257	0.750	0.284	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	95	75	167	98	97	93	0
N.S.	1	1.00	0.58	0.46	1.02	0.60	0.59	0.57	0.00
time (sec)	N/A	0.081	0.764	0.670	0.279	0.259	0.661	0.280	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	90	70	150	93	88	88	0
N.S.	1	1.00	0.65	0.50	1.08	0.67	0.63	0.63	0.00
time (sec)	N/A	0.053	0.638	0.680	0.280	0.248	0.658	0.271	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	124	90	154	135	0	146	0
N.S.	1	1.00	0.84	0.61	1.05	0.92	0.00	0.99	0.00
time (sec)	N/A	0.098	0.621	0.694	0.304	0.275	0.000	0.327	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	131	107	161	153	0	760	0
N.S.	1	1.00	0.85	0.69	1.05	0.99	0.00	4.94	0.00
time (sec)	N/A	0.102	0.657	0.776	0.285	0.273	0.000	0.571	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	131	107	172	169	0	271	0
N.S.	1	1.00	0.81	0.66	1.07	1.05	0.00	1.68	0.00
time (sec)	N/A	0.099	0.784	0.807	0.288	0.267	0.000	0.313	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	693	692	588	810	0	1435	1613	798	0
N.S.	1	1.00	0.85	1.17	0.00	2.07	2.33	1.15	0.00
time (sec)	N/A	1.203	2.670	1.232	0.000	0.495	1.276	0.312	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	420	418	341	437	0	861	910	442	0
N.S.	1	1.00	0.81	1.04	0.00	2.05	2.17	1.05	0.00
time (sec)	N/A	0.578	1.264	0.896	0.000	0.361	1.100	0.307	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	178	192	0	461	437	201	0
N.S.	1	1.00	0.80	0.86	0.00	2.07	1.96	0.90	0.00
time (sec)	N/A	0.173	0.630	0.764	0.000	0.409	0.973	0.362	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	98	86	0	227	226	93	0
N.S.	1	1.00	0.84	0.74	0.00	1.96	1.95	0.80	0.00
time (sec)	N/A	0.062	0.032	0.681	0.000	0.277	0.368	0.298	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	187	251	0	0	0	0	0
N.S.	1	1.00	1.04	1.40	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.174	0.685	0.773	0.000	0.000	0.000	0.000	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	239	236	485	0	0	0	0	0
N.S.	1	0.99	0.98	2.01	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.220	1.045	0.836	0.000	0.000	0.000	0.000	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	336	336	471	1013	0	2034	0	2279	0
N.S.	1	1.00	1.40	3.01	0.00	6.05	0.00	6.78	0.00
time (sec)	N/A	0.391	11.211	0.993	0.000	34.051	0.000	0.331	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	504	502	694	997	0	2937	0	1028	0
N.S.	1	1.00	1.38	1.98	0.00	5.83	0.00	2.04	0.00
time (sec)	N/A	0.658	4.716	1.285	0.000	7.715	0.000	0.300	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	289	288	390	548	0	1769	0	565	0
N.S.	1	1.00	1.35	1.90	0.00	6.12	0.00	1.96	0.00
time (sec)	N/A	0.234	2.452	0.959	0.000	5.980	0.000	0.295	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	194	311	0	905	0	263	0
N.S.	1	1.00	1.04	1.67	0.00	4.87	0.00	1.41	0.00
time (sec)	N/A	0.143	0.907	0.803	0.000	3.865	0.000	0.360	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	106	201	0	429	0	118	143
N.S.	1	1.00	0.95	1.81	0.00	3.86	0.00	1.06	1.29
time (sec)	N/A	0.043	0.145	0.708	0.000	0.474	0.000	0.349	13.602

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	235	547	0	1905	0	708	0
N.S.	1	1.00	1.04	2.43	0.00	8.47	0.00	3.15	0.00
time (sec)	N/A	0.165	1.020	0.764	0.000	9.787	0.000	0.306	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	421	418	518	1115	0	5098	0	0	0
N.S.	1	0.99	1.23	2.65	0.00	12.11	0.00	0.00	0.00
time (sec)	N/A	0.602	10.972	0.760	0.000	35.522	0.000	0.000	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	713	707	847	2268	0	10340	0	5562	0
N.S.	1	0.99	1.19	3.18	0.00	14.50	0.00	7.80	0.00
time (sec)	N/A	2.000	11.870	0.901	0.000	162.168	0.000	0.408	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	70	50	97	73	63	68	0
N.S.	1	1.00	0.58	0.42	0.81	0.61	0.52	0.57	0.00
time (sec)	N/A	0.074	0.341	0.698	0.276	0.280	0.519	0.274	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	65	45	80	68	56	63	0
N.S.	1	1.00	0.68	0.47	0.84	0.72	0.59	0.66	0.00
time (sec)	N/A	0.060	0.285	0.695	0.282	0.253	0.494	0.280	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	60	40	63	63	49	58	0
N.S.	1	1.00	0.86	0.57	0.90	0.90	0.70	0.83	0.00
time (sec)	N/A	0.038	0.211	0.671	0.277	0.259	0.521	0.295	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	93	60	67	105	0	116	0
N.S.	1	1.00	1.19	0.77	0.86	1.35	0.00	1.49	0.00
time (sec)	N/A	0.059	0.233	0.706	0.290	0.270	0.000	0.315	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	99	67	74	123	0	191	0
N.S.	1	1.00	1.19	0.81	0.89	1.48	0.00	2.30	0.00
time (sec)	N/A	0.058	0.290	0.674	0.282	0.285	0.000	0.424	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	77	68	82	96	0	204	0
N.S.	1	1.00	0.87	0.76	0.92	1.08	0.00	2.29	0.00
time (sec)	N/A	0.050	0.341	0.707	0.297	0.269	0.000	0.320	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	70	50	97	97	0	67	0
N.S.	1	1.00	0.68	0.49	0.94	0.94	0.00	0.65	0.00
time (sec)	N/A	0.076	0.479	0.694	0.276	0.262	0.000	0.291	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	65	45	80	92	0	62	0
N.S.	1	1.00	0.79	0.55	0.98	1.12	0.00	0.76	0.00
time (sec)	N/A	0.061	0.438	0.838	0.281	0.256	0.000	0.347	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	60	40	63	87	0	57	0
N.S.	1	1.00	0.95	0.63	1.00	1.38	0.00	0.90	0.00
time (sec)	N/A	0.037	0.357	0.810	0.305	0.278	0.000	0.292	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	70	51	64	96	0	91	0
N.S.	1	1.00	1.13	0.82	1.03	1.55	0.00	1.47	0.00
time (sec)	N/A	0.046	0.368	0.799	0.278	0.274	0.000	0.310	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	90	63	96	106	0	168	0
N.S.	1	1.00	1.03	0.72	1.10	1.22	0.00	1.93	0.00
time (sec)	N/A	0.056	0.407	0.732	0.281	0.270	0.000	0.303	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	87	68	145	126	0	223	0
N.S.	1	1.00	0.78	0.61	1.29	1.12	0.00	1.99	0.00
time (sec)	N/A	0.101	0.493	0.675	0.284	0.259	0.000	0.315	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	70	50	202	117	0	67	0
N.S.	1	1.00	0.81	0.58	2.35	1.36	0.00	0.78	0.00
time (sec)	N/A	0.067	0.651	0.672	0.294	0.259	0.000	0.303	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	65	45	185	112	0	62	0
N.S.	1	1.00	0.96	0.66	2.72	1.65	0.00	0.91	0.00
time (sec)	N/A	0.057	0.563	0.691	0.289	0.248	0.000	0.297	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	33	30	76	51	0	28	49
N.S.	1	1.00	0.70	0.64	1.62	1.09	0.00	0.60	1.04
time (sec)	N/A	0.027	0.405	0.728	0.212	0.260	0.000	0.280	13.336

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	80	80	93	126	0	101	0
N.S.	1	1.00	0.94	0.94	1.09	1.48	0.00	1.19	0.00
time (sec)	N/A	0.055	0.504	0.660	0.294	0.264	0.000	0.292	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	92	73	125	141	0	233	0
N.S.	1	1.00	0.84	0.66	1.14	1.28	0.00	2.12	0.00
time (sec)	N/A	0.090	0.579	0.701	0.282	0.267	0.000	0.318	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	105	78	174	156	0	233	0
N.S.	1	1.00	0.78	0.58	1.29	1.16	0.00	1.73	0.00
time (sec)	N/A	0.127	0.649	0.703	0.290	0.274	0.000	0.308	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	219	324	0	465	0	0	1089
N.S.	1	1.00	1.05	1.56	0.00	2.24	0.00	0.00	5.24
time (sec)	N/A	0.240	0.291	0.934	0.000	13.842	0.000	0.000	14.730

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	906	905	15669	1736	0	1023	0	0	0
N.S.	1	1.00	17.29	1.92	0.00	1.13	0.00	0.00	0.00
time (sec)	N/A	1.592	35.798	3.458	0.000	0.106	0.000	0.000	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	668	668	9965	1202	0	764	0	0	0
N.S.	1	1.00	14.92	1.80	0.00	1.14	0.00	0.00	0.00
time (sec)	N/A	0.692	35.384	2.446	0.000	0.105	0.000	0.000	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	749	746	1276	1215	0	814	0	0	0
N.S.	1	1.00	1.70	1.62	0.00	1.09	0.00	0.00	0.00
time (sec)	N/A	0.885	35.140	4.020	0.000	0.118	0.000	0.000	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	712	711	8456	1340	0	1385	0	0	0
N.S.	1	1.00	11.88	1.88	0.00	1.95	0.00	0.00	0.00
time (sec)	N/A	0.741	35.077	4.858	0.000	0.148	0.000	0.000	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	992	989	12997	1766	0	2430	0	0	0
N.S.	1	1.00	13.10	1.78	0.00	2.45	0.00	0.00	0.00
time (sec)	N/A	1.138	35.861	3.215	0.000	0.308	0.000	0.000	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	1363	1363	19853	2484	0	4543	0	0	0
N.S.	1	1.00	14.57	1.82	0.00	3.33	0.00	0.00	0.00
time (sec)	N/A	2.428	36.637	4.214	0.000	0.779	0.000	0.000	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	1904	1904	29140	3498	0	7780	0	0	0
N.S.	1	1.00	15.30	1.84	0.00	4.09	0.00	0.00	0.00
time (sec)	N/A	3.789	38.306	4.813	0.000	2.280	0.000	0.000	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	724	724	1314	1261	0	761	0	0	0
N.S.	1	1.00	1.81	1.74	0.00	1.05	0.00	0.00	0.00
time (sec)	N/A	1.031	33.480	3.446	0.000	0.107	0.000	0.000	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	557	557	862	955	0	571	0	0	0
N.S.	1	1.00	1.55	1.71	0.00	1.03	0.00	0.00	0.00
time (sec)	N/A	0.522	31.143	2.562	0.000	0.099	0.000	0.000	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	471	470	980	823	0	451	0	0	0
N.S.	1	1.00	2.08	1.75	0.00	0.96	0.00	0.00	0.00
time (sec)	N/A	0.297	30.184	3.816	0.000	0.090	0.000	0.000	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	508	506	772	965	0	708	0	0	0
N.S.	1	1.00	1.52	1.90	0.00	1.39	0.00	0.00	0.00
time (sec)	N/A	0.415	26.426	4.171	0.000	0.107	0.000	0.000	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	684	680	1194	1248	0	1305	0	0	0
N.S.	1	0.99	1.75	1.82	0.00	1.91	0.00	0.00	0.00
time (sec)	N/A	0.716	31.720	3.611	0.000	0.154	0.000	0.000	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	944	942	1746	1757	0	2656	0	0	0
N.S.	1	1.00	1.85	1.86	0.00	2.81	0.00	0.00	0.00
time (sec)	N/A	1.316	34.764	4.391	0.000	0.402	0.000	0.000	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	510	508	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.475	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	496	494	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.409	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	590	588	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.453	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	119	36	59	75	211	141	58
N.S.	1	1.00	2.90	0.88	1.44	1.83	5.15	3.44	1.41
time (sec)	N/A	0.036	0.277	0.496	0.307	0.260	3.225	0.293	13.562

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	34	39	66	83	280	181	78
N.S.	1	1.00	0.74	0.85	1.43	1.80	6.09	3.93	1.70
time (sec)	N/A	0.042	0.433	0.684	0.238	0.266	53.167	0.305	13.489

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	43	51	98	123	483	296	120
N.S.	1	1.00	0.75	0.89	1.72	2.16	8.47	5.19	2.11
time (sec)	N/A	0.073	0.744	0.658	0.234	0.263	58.819	0.304	13.572

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	167	2052	1779	1779	2281	2467	2026
N.S.	1	1.00	8.35	102.60	88.95	88.95	114.05	123.35	101.30
time (sec)	N/A	0.275	0.300	0.906	0.210	0.272	0.162	0.284	14.160

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	19	18	18	20	20	18
N.S.	1	1.00	1.00	0.73	0.69	0.69	0.77	0.77	0.69
time (sec)	N/A	0.015	0.008	0.518	0.203	0.252	0.045	0.281	0.052

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	346	346	282	314	0	701	700	321	0
N.S.	1	1.00	0.82	0.91	0.00	2.03	2.02	0.93	0.00
time (sec)	N/A	0.470	1.097	0.837	0.000	0.366	0.873	0.320	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	245	199	210	0	499	479	221	0
N.S.	1	1.00	0.81	0.86	0.00	2.04	1.96	0.90	0.00
time (sec)	N/A	0.240	0.708	0.755	0.000	0.304	0.840	0.329	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	139	134	0	341	347	144	0
N.S.	1	1.00	0.79	0.76	0.00	1.93	1.96	0.81	0.00
time (sec)	N/A	0.141	0.522	0.727	0.000	0.303	0.476	0.338	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	135	223	0	733	0	0	0
N.S.	1	1.00	0.87	1.44	0.00	4.73	0.00	0.00	0.00
time (sec)	N/A	0.142	0.638	0.717	0.000	2.076	0.000	0.000	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	146	156	0	703	0	170	166
N.S.	1	1.00	1.05	1.12	0.00	5.06	0.00	1.22	1.19
time (sec)	N/A	0.133	0.934	0.775	0.000	1.343	0.000	0.345	13.669

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	170	134	0	783	0	346	0
N.S.	1	1.00	1.07	0.84	0.00	4.92	0.00	2.18	0.00
time (sec)	N/A	0.133	1.131	0.746	0.000	1.745	0.000	0.346	0.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	178	150	0	365	0	680	0
N.S.	1	1.00	0.96	0.81	0.00	1.96	0.00	3.66	0.00
time (sec)	N/A	0.179	0.926	0.818	0.000	1.889	0.000	0.306	0.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	270	237	236	0	525	0	1433	0
N.S.	1	1.00	0.88	0.87	0.00	1.94	0.00	5.31	0.00
time (sec)	N/A	0.278	1.573	0.857	0.000	4.408	0.000	0.328	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	371	371	328	350	0	727	0	2155	0
N.S.	1	1.00	0.88	0.94	0.00	1.96	0.00	5.81	0.00
time (sec)	N/A	0.497	2.137	0.986	0.000	11.939	0.000	0.313	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	258	258	212	200	206	206	230	237	196
N.S.	1	1.00	0.82	0.78	0.80	0.80	0.89	0.92	0.76
time (sec)	N/A	0.176	0.027	0.475	0.200	0.244	0.040	0.275	0.127

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	136	139	145	145	158	160	137
N.S.	1	1.00	0.87	0.89	0.92	0.92	1.01	1.02	0.87
time (sec)	N/A	0.111	0.021	0.541	0.194	0.275	0.029	0.287	13.307

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	93	78	79	79	87	83	77
N.S.	1	1.00	1.00	0.84	0.85	0.85	0.94	0.89	0.83
time (sec)	N/A	0.074	0.009	0.072	0.191	0.245	0.024	0.265	0.049

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	35	34	34	37	34	34
N.S.	1	1.00	1.00	0.83	0.81	0.81	0.88	0.81	0.81
time (sec)	N/A	0.016	0.002	0.047	0.194	0.241	0.023	0.268	0.027

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	228	179	220	228	230	235	249	260
N.S.	1	1.00	0.79	0.96	1.00	1.01	1.03	1.09	1.14
time (sec)	N/A	0.129	0.038	0.576	0.196	0.241	0.244	0.302	0.065

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	228	223	232	234	319	238	325	363
N.S.	1	1.00	0.98	1.02	1.03	1.40	1.04	1.43	1.59
time (sec)	N/A	0.122	0.052	0.463	0.204	0.243	0.448	0.430	13.313

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	204	225	240	360	248	237	297
N.S.	1	1.00	0.88	0.97	1.04	1.56	1.07	1.03	1.29
time (sec)	N/A	0.132	0.039	0.461	0.207	0.248	0.854	0.353	0.095

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	391	391	277	254	263	263	298	305	251
N.S.	1	1.00	0.71	0.65	0.67	0.67	0.76	0.78	0.64
time (sec)	N/A	0.239	0.026	0.546	0.195	0.236	0.045	0.271	13.410

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	201	177	185	185	206	206	175
N.S.	1	1.00	1.00	0.88	0.92	0.92	1.02	1.02	0.87
time (sec)	N/A	0.161	0.022	0.559	0.194	0.251	0.036	0.265	0.132

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	121	100	105	105	112	107	101
N.S.	1	1.00	1.00	0.83	0.87	0.87	0.93	0.88	0.83
time (sec)	N/A	0.108	0.012	0.526	0.194	0.235	0.030	0.277	0.092

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	60	45	44	44	56	44	44
N.S.	1	1.00	1.00	0.75	0.73	0.73	0.93	0.73	0.73
time (sec)	N/A	0.022	0.001	0.529	0.197	0.229	0.024	0.267	0.036

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	352	352	262	354	366	368	372	416	434
N.S.	1	1.00	0.74	1.01	1.04	1.05	1.06	1.18	1.23
time (sec)	N/A	0.203	0.079	0.556	0.189	0.234	0.360	0.264	0.082

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	353	353	342	366	372	490	393	491	939
N.S.	1	1.00	0.97	1.04	1.05	1.39	1.11	1.39	2.66
time (sec)	N/A	0.217	0.088	0.555	0.195	0.241	0.696	0.266	13.408

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	354	354	311	359	378	545	394	392	771
N.S.	1	1.00	0.88	1.01	1.07	1.54	1.11	1.11	2.18
time (sec)	N/A	0.217	0.060	0.529	0.197	0.252	1.357	0.271	13.331

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	360	360	344	361	390	587	401	383	560
N.S.	1	1.00	0.96	1.00	1.08	1.63	1.11	1.06	1.56
time (sec)	N/A	0.220	0.083	0.546	0.212	0.247	2.541	0.260	13.261

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	178	222	206	206	450	221	397
N.S.	1	1.00	0.81	1.00	0.93	0.93	2.04	1.00	1.80
time (sec)	N/A	0.117	0.077	1.079	0.280	0.255	0.750	0.269	13.402

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	130	147	141	141	303	146	223
N.S.	1	1.00	0.83	0.94	0.90	0.90	1.94	0.94	1.43
time (sec)	N/A	0.098	0.053	0.675	0.288	0.249	0.535	0.258	13.737

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	86	83	84	84	163	82	107
N.S.	1	1.00	0.87	0.84	0.85	0.85	1.65	0.83	1.08
time (sec)	N/A	0.064	0.034	0.654	0.281	0.244	0.339	0.281	0.075

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	50	44	43	43	61	43	45
N.S.	1	1.00	0.89	0.79	0.77	0.77	1.09	0.77	0.80
time (sec)	N/A	0.030	0.014	0.629	0.277	0.250	0.062	0.277	0.043

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	146	142	160	171	0	159	713
N.S.	1	1.00	0.87	0.85	0.95	1.02	0.00	0.95	4.24
time (sec)	N/A	0.121	0.076	0.776	0.277	0.281	0.000	0.286	15.969

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	233	233	213	294	416	0	357	312
N.S.	1	1.00	1.00	0.91	1.26	1.79	0.00	1.53	1.34
time (sec)	N/A	0.155	0.102	0.969	0.289	0.322	0.000	0.280	14.221

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	317	317	278	298	498	698	0	438	493
N.S.	1	1.00	0.88	0.94	1.57	2.20	0.00	1.38	1.56
time (sec)	N/A	0.186	0.263	0.843	0.290	0.396	0.000	0.294	13.773

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	209	214	212	350	444	215	333
N.S.	1	1.00	1.11	1.13	1.12	1.85	2.35	1.14	1.76
time (sec)	N/A	0.154	0.099	0.809	0.276	0.263	1.345	0.272	0.155

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	150	145	147	245	298	146	211
N.S.	1	1.00	1.07	1.04	1.05	1.75	2.13	1.04	1.51
time (sec)	N/A	0.125	0.074	0.787	0.279	0.257	0.921	0.271	0.116

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	96	87	90	147	165	88	115
N.S.	1	1.00	0.99	0.90	0.93	1.52	1.70	0.91	1.19
time (sec)	N/A	0.099	0.103	0.758	0.275	0.255	0.617	0.281	13.437

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	59	50	52	78	65	52	52
N.S.	1	1.00	0.94	0.79	0.83	1.24	1.03	0.83	0.83
time (sec)	N/A	0.037	0.023	0.741	0.279	0.243	0.076	0.270	0.053

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	186	214	289	479	0	296	330
N.S.	1	1.00	0.83	0.96	1.29	2.14	0.00	1.32	1.47
time (sec)	N/A	0.213	0.100	0.951	0.283	0.317	0.000	0.280	14.222

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	313	313	270	303	548	910	0	584	601
N.S.	1	1.00	0.86	0.97	1.75	2.91	0.00	1.87	1.92
time (sec)	N/A	0.375	0.152	0.873	0.287	0.350	0.000	0.272	14.283

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	412	412	363	400	851	1499	0	648	887
N.S.	1	1.00	0.88	0.97	2.07	3.64	0.00	1.57	2.15
time (sec)	N/A	0.533	0.218	0.862	0.300	0.447	0.000	0.298	14.361

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	209	209	222	441	469	210	299
N.S.	1	1.00	1.22	1.22	1.30	2.58	2.74	1.23	1.75
time (sec)	N/A	0.204	0.122	0.844	0.282	0.252	2.949	0.261	0.160

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	146	146	155	302	304	145	203
N.S.	1	1.00	1.09	1.09	1.16	2.25	2.27	1.08	1.51
time (sec)	N/A	0.150	0.108	0.852	0.282	0.256	1.882	0.297	13.255

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	107	91	101	172	163	91	125
N.S.	1	1.00	1.04	0.88	0.98	1.67	1.58	0.88	1.21
time (sec)	N/A	0.093	0.055	0.762	0.284	0.246	0.880	0.269	13.555

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	53	47	56	75	61	46	55
N.S.	1	1.00	0.83	0.73	0.88	1.17	0.95	0.72	0.86
time (sec)	N/A	0.032	0.026	0.785	0.281	0.253	0.078	0.265	13.336

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	329	329	282	359	571	1052	0	495	641
N.S.	1	1.00	0.86	1.09	1.74	3.20	0.00	1.50	1.95
time (sec)	N/A	0.344	0.199	1.051	0.293	0.388	0.000	0.281	13.844

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	443	443	389	473	916	1734	0	806	965
N.S.	1	1.00	0.88	1.07	2.07	3.91	0.00	1.82	2.18
time (sec)	N/A	0.756	0.287	1.016	0.305	0.519	0.000	0.318	14.105

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	80	60	126	83	76	78	170
N.S.	1	1.00	0.56	0.42	0.88	0.58	0.53	0.55	1.19
time (sec)	N/A	0.095	0.892	0.795	0.287	0.247	0.629	0.267	1.745

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	75	55	109	78	70	73	153
N.S.	1	1.00	0.60	0.44	0.88	0.63	0.56	0.59	1.23
time (sec)	N/A	0.059	0.372	0.829	0.280	0.246	0.425	0.306	13.963

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	103	127	128	125	0	129	0
N.S.	1	1.00	0.69	0.85	0.86	0.84	0.00	0.87	0.00
time (sec)	N/A	0.138	0.418	0.007	0.284	0.258	0.000	0.288	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	149	149	110	0	132	143	0	531	0
N.S.	1	1.00	0.74	0.00	0.89	0.96	0.00	3.56	0.00
time (sec)	N/A	0.134	0.491	180.000	0.283	0.257	0.000	0.333	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	151	151	110	0	143	159	0	258	0
N.S.	1	1.00	0.73	0.00	0.95	1.05	0.00	1.71	0.00
time (sec)	N/A	0.142	0.593	180.000	0.291	0.262	0.000	0.304	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	158	158	110	0	160	173	0	304	0
N.S.	1	1.00	0.70	0.00	1.01	1.09	0.00	1.92	0.00
time (sec)	N/A	0.139	0.679	180.000	0.294	0.270	0.000	0.298	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	165	165	110	0	181	189	0	327	0
N.S.	1	1.00	0.67	0.00	1.10	1.15	0.00	1.98	0.00
time (sec)	N/A	0.143	0.657	180.000	0.298	0.275	0.000	0.323	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	165	165	110	0	222	203	0	387	0
N.S.	1	1.00	0.67	0.00	1.35	1.23	0.00	2.35	0.00
time (sec)	N/A	0.141	0.755	180.000	0.289	0.285	0.000	0.299	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	86	88	250	156	0	405	0
N.S.	1	1.00	0.51	0.52	1.48	0.92	0.00	2.40	0.00
time (sec)	N/A	0.129	0.777	1.154	0.314	0.259	0.000	0.292	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	91	93	301	171	0	456	0
N.S.	1	1.00	0.47	0.48	1.55	0.88	0.00	2.35	0.00
time (sec)	N/A	0.145	1.019	0.412	0.295	0.263	0.000	0.307	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	90	70	155	93	90	88	0
N.S.	1	1.00	0.54	0.42	0.93	0.56	0.54	0.53	0.00
time (sec)	N/A	0.104	0.679	0.439	0.272	0.270	0.715	0.290	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	85	65	138	88	83	83	0
N.S.	1	1.00	0.58	0.44	0.94	0.60	0.56	0.56	0.00
time (sec)	N/A	0.069	0.567	0.251	0.288	0.253	0.496	0.290	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	172	172	113	0	157	135	0	139	0
N.S.	1	1.00	0.66	0.00	0.91	0.78	0.00	0.81	0.00
time (sec)	N/A	0.161	0.689	180.000	0.290	0.283	0.000	0.310	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	172	172	120	0	161	153	0	707	0
N.S.	1	1.00	0.70	0.00	0.94	0.89	0.00	4.11	0.00
time (sec)	N/A	0.168	0.722	180.000	0.284	0.267	0.000	0.356	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	174	174	120	0	172	169	0	268	0
N.S.	1	1.00	0.69	0.00	0.99	0.97	0.00	1.54	0.00
time (sec)	N/A	0.164	0.780	180.000	0.297	0.271	0.000	0.298	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	181	181	120	0	189	183	0	314	0
N.S.	1	1.00	0.66	0.00	1.04	1.01	0.00	1.73	0.00
time (sec)	N/A	0.158	0.845	180.000	0.297	0.272	0.000	0.288	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	188	188	120	0	210	199	0	503	0
N.S.	1	1.00	0.64	0.00	1.12	1.06	0.00	2.68	0.00
time (sec)	N/A	0.164	0.774	180.000	0.303	0.268	0.000	0.361	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	195	195	120	0	251	213	0	406	0
N.S.	1	1.00	0.62	0.00	1.29	1.09	0.00	2.08	0.00
time (sec)	N/A	0.168	0.892	180.000	0.312	0.284	0.000	0.308	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	195	195	120	0	297	229	0	452	0
N.S.	1	1.00	0.62	0.00	1.52	1.17	0.00	2.32	0.00
time (sec)	N/A	0.166	0.966	180.000	0.323	0.271	0.000	0.297	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	B	A	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	195	195	120	0	348	243	0	489	0
N.S.	1	1.00	0.62	0.00	1.78	1.25	0.00	2.51	0.00
time (sec)	N/A	0.163	1.187	180.000	0.340	0.273	0.000	0.312	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	70	50	96	73	60	68	0
N.S.	1	1.00	0.58	0.42	0.80	0.61	0.50	0.57	0.00
time (sec)	N/A	0.087	0.326	1.500	0.301	0.252	0.639	0.279	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	65	45	80	68	56	63	0
N.S.	1	1.00	0.64	0.45	0.79	0.67	0.55	0.62	0.00
time (sec)	N/A	0.049	0.288	0.235	0.308	0.254	0.433	0.283	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	126	126	93	0	99	115	0	119	0
N.S.	1	1.00	0.74	0.00	0.79	0.91	0.00	0.94	0.00
time (sec)	N/A	0.120	0.333	180.000	0.301	0.278	0.000	0.292	0.000

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	126	126	100	0	103	133	0	339	0
N.S.	1	1.00	0.79	0.00	0.82	1.06	0.00	2.69	0.00
time (sec)	N/A	0.124	0.509	180.000	0.301	0.260	0.000	0.344	0.000

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	128	128	100	0	114	149	0	248	0
N.S.	1	1.00	0.78	0.00	0.89	1.16	0.00	1.94	0.00
time (sec)	N/A	0.129	0.482	180.000	0.322	0.267	0.000	0.299	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	135	135	100	0	131	163	0	285	0
N.S.	1	1.00	0.74	0.00	0.97	1.21	0.00	2.11	0.00
time (sec)	N/A	0.123	0.540	180.000	0.293	0.270	0.000	0.284	0.000

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	139	139	76	0	149	125	0	164	0
N.S.	1	1.00	0.55	0.00	1.07	0.90	0.00	1.18	0.00
time (sec)	N/A	0.117	0.590	180.000	0.298	0.252	0.000	0.313	0.000

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	124	124	75	0	114	102	0	72	0
N.S.	1	1.00	0.60	0.00	0.92	0.82	0.00	0.58	0.00
time (sec)	N/A	0.095	0.506	180.000	0.275	0.256	0.000	0.280	0.000

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	70	50	97	97	0	67	0
N.S.	1	1.00	0.68	0.49	0.94	0.94	0.00	0.65	0.00
time (sec)	N/A	0.066	0.462	1.214	0.286	0.257	0.000	0.291	0.000

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	65	45	80	92	0	62	0
N.S.	1	1.00	0.79	0.55	0.98	1.12	0.00	0.76	0.00
time (sec)	N/A	0.036	0.414	0.235	0.287	0.254	0.000	0.275	0.000

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	101	101	93	0	99	149	0	118	0
N.S.	1	1.00	0.92	0.00	0.98	1.48	0.00	1.17	0.00
time (sec)	N/A	0.094	0.467	180.000	0.287	0.277	0.000	0.286	0.000

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	108	108	110	0	116	157	0	225	0
N.S.	1	1.00	1.02	0.00	1.07	1.45	0.00	2.08	0.00
time (sec)	N/A	0.094	0.526	180.000	0.293	0.266	0.000	0.347	0.000

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	112	112	76	0	149	126	0	220	0
N.S.	1	1.00	0.68	0.00	1.33	1.12	0.00	1.96	0.00
time (sec)	N/A	0.094	0.487	180.000	0.284	0.278	0.000	0.297	0.000

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	137	137	81	0	217	141	0	271	0
N.S.	1	1.00	0.59	0.00	1.58	1.03	0.00	1.98	0.00
time (sec)	N/A	0.125	0.563	180.000	0.296	0.267	0.000	0.284	0.000

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	B	A	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	105	105	75	0	219	122	0	71	0
N.S.	1	1.00	0.71	0.00	2.09	1.16	0.00	0.68	0.00
time (sec)	N/A	0.088	0.537	180.000	0.293	0.273	0.000	0.310	0.000

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	B	A	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	86	86	70	0	202	117	0	66	0
N.S.	1	1.00	0.81	0.00	2.35	1.36	0.00	0.77	0.00
time (sec)	N/A	0.051	0.639	180.000	0.288	0.261	0.000	0.285	0.000

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	B	B	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	68	68	65	0	185	112	0	62	0
N.S.	1	1.00	0.96	0.00	2.72	1.65	0.00	0.91	0.00
time (sec)	N/A	0.031	0.488	180.000	0.285	0.266	0.000	0.282	0.000

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	85	85	69	0	110	126	0	92	0
N.S.	1	1.00	0.81	0.00	1.29	1.48	0.00	1.08	0.00
time (sec)	N/A	0.081	0.524	180.000	0.284	0.260	0.000	0.292	0.000

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	110	110	81	0	127	141	0	206	0
N.S.	1	1.00	0.74	0.00	1.15	1.28	0.00	1.87	0.00
time (sec)	N/A	0.097	0.653	180.000	0.282	0.265	0.000	0.345	0.000

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	135	135	94	0	178	155	0	228	0
N.S.	1	1.00	0.70	0.00	1.32	1.15	0.00	1.69	0.00
time (sec)	N/A	0.129	0.619	180.000	0.297	0.273	0.000	0.290	0.000

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	160	160	91	0	246	170	0	279	0
N.S.	1	1.00	0.57	0.00	1.54	1.06	0.00	1.74	0.00
time (sec)	N/A	0.170	0.706	180.000	0.298	0.261	0.000	0.310	0.000

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	F(-2)	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	354	354	315	0	0	1373	0	463	0
N.S.	1	1.00	0.89	0.00	0.00	3.88	0.00	1.31	0.00
time (sec)	N/A	0.231	1.969	180.000	0.000	22.785	0.000	0.312	0.000

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	353	353	316	1147	0	1385	0	488	0
N.S.	1	1.00	0.90	3.25	0.00	3.92	0.00	1.38	0.00
time (sec)	N/A	0.215	1.973	2.465	0.000	23.128	0.000	0.296	0.000

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	588	588	537	5924	2292	4795	136733	10965	4341
N.S.	1	1.00	0.91	10.07	3.90	8.15	232.54	18.65	7.38
time (sec)	N/A	0.234	0.402	0.451	0.269	0.348	38.078	0.387	18.103

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	432	432	391	3222	1414	2796	65193	6226	2625
N.S.	1	1.00	0.91	7.46	3.27	6.47	150.91	14.41	6.08
time (sec)	N/A	0.160	0.283	0.393	0.242	0.305	13.895	0.325	15.425

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	292	292	261	1220	788	1448	26165	3099	1425
N.S.	1	1.00	0.89	4.18	2.70	4.96	89.61	10.61	4.88
time (sec)	N/A	0.108	0.186	0.495	0.222	0.288	4.847	0.288	14.290

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	255	255	221	0	0	0	0	0	0
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.350	0.589	0.000	0.000	0.000	0.000	0.000	0.000

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	377	377	441	0	0	0	0	0	0
N.S.	1	1.00	1.17	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.631	1.352	0.000	0.000	0.000	0.000	0.000	0.000

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	528	528	488	769	0	3480	0	646	1027
N.S.	1	1.00	0.92	1.46	0.00	6.59	0.00	1.22	1.95
time (sec)	N/A	0.819	0.607	0.413	0.000	0.371	0.000	0.266	15.649

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	765	765	754	1086	0	2643	0	981	2779
N.S.	1	1.00	0.99	1.42	0.00	3.45	0.00	1.28	3.63
time (sec)	N/A	4.744	0.412	0.470	0.000	0.617	0.000	0.258	17.975

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	99	75	177	97	99	92	221
N.S.	1	1.00	0.48	0.36	0.85	0.47	0.48	0.44	1.06
time (sec)	N/A	0.217	0.723	0.263	0.282	0.283	0.482	0.287	15.619

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	89	65	143	87	85	82	187
N.S.	1	1.00	0.54	0.39	0.86	0.52	0.51	0.49	1.13
time (sec)	N/A	0.136	0.480	0.279	0.288	0.274	0.426	0.283	15.043

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	79	55	109	77	71	72	153
N.S.	1	1.00	0.64	0.44	0.88	0.62	0.57	0.58	1.23
time (sec)	N/A	0.075	0.339	0.179	0.301	0.260	0.404	0.275	14.393

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	234	223	500	304	0	144	0
N.S.	1	1.00	1.25	1.19	2.67	1.63	0.00	0.77	0.00
time (sec)	N/A	0.225	0.347	0.618	0.333	0.275	0.000	0.313	0.000

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	427	235	0	378	0	0	0
N.S.	1	1.00	2.15	1.18	0.00	1.90	0.00	0.00	0.00
time (sec)	N/A	0.170	0.534	0.500	0.000	0.263	0.000	0.000	0.000

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	602	231	0	390	0	378	0
N.S.	1	1.00	2.83	1.08	0.00	1.83	0.00	1.77	0.00
time (sec)	N/A	0.162	0.717	0.472	0.000	0.270	0.000	0.299	0.000

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	109	85	206	107	112	102	0
N.S.	1	1.00	0.47	0.37	0.89	0.46	0.48	0.44	0.00
time (sec)	N/A	0.232	0.864	0.245	0.285	0.247	0.630	0.281	0.000

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	99	75	172	97	99	92	0
N.S.	1	1.00	0.52	0.40	0.91	0.51	0.52	0.49	0.00
time (sec)	N/A	0.144	0.714	0.319	0.290	0.253	0.512	0.283	0.000

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	89	65	138	87	85	82	0
N.S.	1	1.00	0.61	0.44	0.94	0.59	0.58	0.56	0.00
time (sec)	N/A	0.080	0.509	0.530	0.284	0.256	0.457	0.276	0.000

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	254	233	535	326	0	154	0
N.S.	1	1.00	1.21	1.11	2.55	1.55	0.00	0.73	0.00
time (sec)	N/A	0.214	0.558	0.506	0.395	0.290	0.000	0.313	0.000

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	222	447	245	0	378	0	0	0
N.S.	1	1.00	2.01	1.10	0.00	1.70	0.00	0.00	0.00
time (sec)	N/A	0.218	0.699	0.513	0.000	0.285	0.000	0.000	0.000

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	636	245	0	447	0	411	0
N.S.	1	1.00	2.72	1.05	0.00	1.91	0.00	1.76	0.00
time (sec)	N/A	0.211	0.809	0.529	0.000	0.292	0.000	0.332	0.000

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	89	65	148	87	85	82	0
N.S.	1	1.00	0.48	0.35	0.80	0.47	0.46	0.44	0.00
time (sec)	N/A	0.207	0.600	0.260	0.288	0.254	0.468	0.282	0.000

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	79	55	114	77	71	72	0
N.S.	1	1.00	0.55	0.38	0.80	0.54	0.50	0.50	0.00
time (sec)	N/A	0.126	0.430	0.304	0.281	0.251	0.453	0.286	0.000

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	69	45	80	67	58	62	0
N.S.	1	1.00	0.68	0.45	0.79	0.66	0.57	0.61	0.00
time (sec)	N/A	0.070	0.294	0.198	0.287	0.249	0.385	0.285	0.000

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	211	204	465	297	0	125	0
N.S.	1	1.00	1.29	1.24	2.84	1.81	0.00	0.76	0.00
time (sec)	N/A	0.146	0.286	0.490	0.311	0.268	0.000	0.330	0.000

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	352	221	0	330	0	276	0
N.S.	1	1.00	1.98	1.24	0.00	1.85	0.00	1.55	0.00
time (sec)	N/A	0.128	0.438	0.452	0.000	0.255	0.000	0.311	0.000

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	433	231	0	390	0	378	0
N.S.	1	1.00	1.91	1.02	0.00	1.72	0.00	1.67	0.00
time (sec)	N/A	0.181	0.696	0.493	0.000	0.291	0.000	0.303	0.000

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	89	65	148	112	0	81	0
N.S.	1	1.00	0.54	0.39	0.89	0.67	0.00	0.49	0.00
time (sec)	N/A	0.154	0.534	0.307	0.278	0.264	0.000	0.280	0.000

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	79	55	114	102	0	71	0
N.S.	1	1.00	0.64	0.44	0.92	0.82	0.00	0.57	0.00
time (sec)	N/A	0.100	0.637	0.244	0.283	0.255	0.000	0.289	0.000

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	69	45	80	92	0	62	0
N.S.	1	1.00	0.84	0.55	0.98	1.12	0.00	0.76	0.00
time (sec)	N/A	0.056	0.412	0.278	0.286	0.258	0.000	0.297	0.000

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	199	209	777	333	0	112	0
N.S.	1	1.00	1.20	1.26	4.68	2.01	0.00	0.67	0.00
time (sec)	N/A	0.127	0.401	0.436	0.328	0.273	0.000	0.284	0.000

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	416	231	0	392	0	295	0
N.S.	1	1.00	1.93	1.07	0.00	1.82	0.00	1.37	0.00
time (sec)	N/A	0.213	0.662	0.475	0.000	0.274	0.000	0.296	0.000

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	250	607	241	0	452	0	397	0
N.S.	1	1.00	2.43	0.96	0.00	1.81	0.00	1.59	0.00
time (sec)	N/A	0.203	0.925	0.523	0.000	0.373	0.000	0.296	0.000

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	166	166	242	0	0	0	0	0	0
N.S.	1	1.00	1.46	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.093	0.320	0.000	0.000	0.000	0.000	0.000	0.000

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	167	167	236	0	0	0	0	0	0
N.S.	1	1.00	1.41	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.106	0.271	0.000	0.000	0.000	0.000	0.000	0.000

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	252	252	302	0	0	0	0	0	0
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.279	0.400	0.000	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [168] had the largest ratio of [.357099999999999973]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	7	5	1.00	34	0.147
2	A	6	5	1.00	32	0.156
3	A	5	5	1.00	27	0.185
4	A	5	5	1.00	34	0.147
5	A	5	5	1.00	34	0.147
6	A	5	5	1.00	34	0.147
7	A	8	6	1.00	34	0.176
8	A	4	4	1.00	34	0.118
9	A	5	4	1.00	34	0.118
10	A	7	4	1.00	34	0.118
11	A	6	4	1.00	34	0.118
12	A	5	4	1.00	32	0.125
13	A	4	4	1.00	27	0.148
14	A	4	4	1.00	34	0.118
15	A	7	5	1.00	34	0.147
16	A	4	4	1.00	34	0.118
17	A	5	4	1.00	34	0.118
18	A	2	1	0.99	25	0.040
19	A	2	1	0.99	25	0.040
20	A	2	1	1.00	23	0.043
21	A	2	1	1.00	18	0.056
22	A	2	1	0.99	25	0.040
23	A	2	1	0.99	25	0.040
24	A	2	1	0.99	25	0.040

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
25	A	3	2	0.99	27	0.074
26	A	3	2	1.00	27	0.074
27	A	3	2	1.00	25	0.080
28	A	3	2	1.00	20	0.100
29	A	2	1	0.99	27	0.037
30	A	2	1	0.99	27	0.037
31	A	2	1	0.99	27	0.037
32	A	3	2	0.99	27	0.074
33	A	3	2	1.00	27	0.074
34	A	3	2	1.00	25	0.080
35	A	3	2	1.00	20	0.100
36	A	2	1	0.99	27	0.037
37	A	2	1	0.99	27	0.037
38	A	2	1	0.99	27	0.037
39	A	1	1	1.00	32	0.031
40	A	1	1	1.00	31	0.032
41	A	1	1	1.00	34	0.029
42	A	1	1	1.00	33	0.030
43	A	5	4	0.99	27	0.148
44	A	5	4	0.99	27	0.148
45	A	5	4	1.00	25	0.160
46	A	5	4	1.00	20	0.200
47	A	5	4	1.00	27	0.148
48	A	5	4	1.00	27	0.148
49	A	5	4	1.00	27	0.148
50	A	6	5	1.00	27	0.185
51	A	5	5	1.00	27	0.185
52	A	4	4	1.00	25	0.160
53	A	3	3	1.00	20	0.150
54	A	6	5	1.00	27	0.185
55	A	6	5	0.99	27	0.185
56	A	6	5	1.00	27	0.185
57	A	5	5	1.00	27	0.185
58	A	3	3	1.12	27	0.111
59	A	3	3	1.00	25	0.120

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
60	A	4	4	1.00	20	0.200
61	A	7	6	1.00	27	0.222
62	A	7	5	0.99	27	0.185
63	A	7	5	1.00	27	0.185
64	A	4	4	1.00	27	0.148
65	A	4	4	1.13	27	0.148
66	A	4	4	1.00	27	0.148
67	A	4	4	1.00	25	0.160
68	A	5	4	1.00	20	0.200
69	A	6	5	1.00	17	0.294
70	A	5	5	1.00	17	0.294
71	A	4	4	1.00	15	0.267
72	A	3	3	1.00	14	0.214
73	A	6	5	1.00	17	0.294
74	A	6	5	1.00	17	0.294
75	A	6	5	1.00	17	0.294
76	A	3	3	1.83	16	0.188
77	A	3	3	1.00	18	0.167
78	A	7	6	0.99	29	0.207
79	A	6	6	1.00	29	0.207
80	A	5	5	1.00	27	0.185
81	A	5	5	1.00	22	0.227
82	A	7	6	1.00	29	0.207
83	A	7	6	0.98	29	0.207
84	A	7	6	1.00	29	0.207
85	A	7	6	1.00	29	0.207
86	A	5	5	1.00	29	0.172
87	A	6	6	1.00	29	0.207
88	A	8	6	1.00	29	0.207
89	A	7	6	1.00	29	0.207
90	A	6	5	1.00	27	0.185
91	A	6	5	1.00	22	0.227
92	A	8	6	1.00	29	0.207
93	A	8	6	0.99	29	0.207
94	A	8	7	0.98	29	0.241

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
95	A	8	6	0.99	29	0.207
96	A	8	7	1.00	29	0.241
97	A	8	6	1.00	29	0.207
98	A	6	5	1.00	29	0.172
99	A	7	6	1.00	29	0.207
100	A	7	5	1.00	22	0.227
101	A	6	5	0.99	29	0.172
102	A	5	5	1.00	29	0.172
103	A	4	4	0.99	27	0.148
104	A	4	4	1.00	22	0.182
105	A	6	5	1.00	29	0.172
106	A	6	5	1.00	29	0.172
107	A	4	4	1.00	29	0.138
108	A	5	5	1.00	29	0.172
109	A	4	4	1.00	29	0.138
110	A	4	4	1.00	27	0.148
111	A	4	4	1.00	22	0.182
112	A	4	4	1.00	29	0.138
113	A	4	4	1.00	29	0.138
114	A	5	5	0.99	29	0.172
115	A	3	3	1.00	22	0.136
116	A	4	4	1.00	22	0.182
117	A	5	4	1.00	22	0.182
118	A	5	4	1.00	29	0.138
119	A	4	4	1.00	29	0.138
120	A	3	3	1.00	27	0.111
121	A	5	5	1.00	29	0.172
122	A	5	5	1.00	29	0.172
123	A	4	4	1.00	29	0.138
124	A	5	4	1.00	29	0.138
125	A	4	4	1.00	29	0.138
126	A	3	3	1.00	27	0.111
127	A	4	4	1.00	29	0.138
128	A	4	4	1.00	29	0.138
129	A	5	5	1.00	29	0.172

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
130	A	4	3	1.00	29	0.103
131	A	4	3	1.00	29	0.103
132	A	2	2	1.00	27	0.074
133	A	5	5	1.00	29	0.172
134	A	5	4	1.00	29	0.138
135	A	6	5	1.00	29	0.172
136	A	6	4	0.99	27	0.148
137	A	6	4	1.00	29	0.138
138	A	5	5	1.00	31	0.161
139	A	6	6	1.00	69	0.087
140	A	2	1	1.00	20	0.050
141	A	2	1	1.00	20	0.050
142	A	2	1	1.00	20	0.050
143	A	2	1	1.00	18	0.056
144	A	6	5	1.00	20	0.250
145	A	4	4	1.00	20	0.200
146	A	5	5	1.00	20	0.250
147	A	6	5	1.00	20	0.250
148	A	6	5	1.00	30	0.167
149	A	6	5	1.00	30	0.167
150	A	6	5	1.00	28	0.179
151	A	6	5	1.00	23	0.217
152	A	6	5	1.00	30	0.167
153	A	6	5	1.00	30	0.167
154	A	6	5	1.00	30	0.167
155	A	6	6	1.00	30	0.200
156	A	5	5	1.00	28	0.179
157	A	4	4	1.00	23	0.174
158	A	7	6	1.00	30	0.200
159	A	7	6	1.00	30	0.200
160	A	7	6	1.00	20	0.300
161	A	7	6	1.00	20	0.300
162	A	5	5	1.00	18	0.278
163	A	4	4	1.00	17	0.235
164	A	7	6	1.00	20	0.300

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
165	A	7	6	1.00	20	0.300
166	A	7	6	1.00	20	0.300
167	A	1	1	1.00	16	0.062
168	A	6	5	1.00	14	0.357
169	A	6	5	1.00	16	0.312
170	A	3	2	1.00	18	0.111
171	A	5	3	1.00	16	0.188
172	A	5	3	1.00	19	0.158
173	A	6	5	1.00	23	0.217
174	A	5	3	1.00	19	0.158
175	A	2	2	1.00	23	0.087
176	A	4	4	1.00	17	0.235
177	A	1	1	1.00	19	0.053
178	A	7	5	1.00	22	0.227
179	A	6	5	1.00	22	0.227
180	A	5	5	1.00	22	0.227
181	A	4	4	1.00	22	0.182
182	A	4	4	1.00	22	0.182
183	A	3	3	1.00	22	0.136
184	A	4	4	1.00	22	0.182
185	A	5	4	1.00	22	0.182
186	A	7	6	1.00	32	0.188
187	A	6	6	0.99	32	0.188
188	A	5	5	1.00	30	0.167
189	A	5	5	1.00	25	0.200
190	A	7	6	1.00	32	0.188
191	A	7	6	0.99	32	0.188
192	A	7	6	1.00	32	0.188
193	A	7	6	1.00	32	0.188
194	A	5	5	1.00	32	0.156
195	A	6	6	1.00	32	0.188
196	A	8	6	1.00	32	0.188
197	A	7	6	0.99	32	0.188
198	A	6	5	1.00	30	0.167
199	A	6	5	1.00	25	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
200	A	8	6	1.00	32	0.188
201	A	8	6	0.99	32	0.188
202	A	8	7	0.99	32	0.219
203	A	8	6	1.00	32	0.188
204	A	8	7	1.00	32	0.219
205	A	8	6	1.00	32	0.188
206	A	6	5	1.00	32	0.156
207	A	7	6	1.00	32	0.188
208	A	7	6	1.00	32	0.188
209	A	6	6	1.00	32	0.188
210	A	5	5	1.00	30	0.167
211	A	7	7	1.00	32	0.219
212	A	7	7	1.00	32	0.219
213	A	7	7	1.00	32	0.219
214	A	9	6	1.00	32	0.188
215	A	7	6	1.00	32	0.188
216	A	6	5	1.00	30	0.167
217	A	8	7	1.00	32	0.219
218	A	8	7	1.00	32	0.219
219	A	8	8	1.00	32	0.250
220	A	9	6	1.00	32	0.188
221	A	8	6	1.00	32	0.188
222	A	7	5	1.00	30	0.167
223	A	9	7	1.00	32	0.219
224	A	9	7	1.00	32	0.219
225	A	9	8	1.00	32	0.250
226	A	6	5	1.00	32	0.156
227	A	5	5	1.00	32	0.156
228	A	4	4	1.00	30	0.133
229	A	4	4	1.00	25	0.160
230	A	6	5	1.00	32	0.156
231	A	6	5	0.99	32	0.156
232	A	4	4	1.00	32	0.125
233	A	5	5	1.00	32	0.156
234	A	4	4	1.00	32	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
235	A	4	4	1.00	30	0.133
236	A	4	4	1.00	25	0.160
237	A	4	4	1.00	32	0.125
238	A	4	4	0.99	32	0.125
239	A	5	5	0.99	32	0.156
240	A	6	5	1.00	32	0.156
241	A	5	5	1.00	32	0.156
242	A	4	4	1.00	30	0.133
243	A	6	6	1.00	32	0.188
244	A	6	6	1.00	32	0.188
245	A	4	4	1.00	32	0.125
246	A	6	5	1.00	32	0.156
247	A	5	5	1.00	32	0.156
248	A	4	4	1.00	30	0.133
249	A	4	4	1.00	32	0.125
250	A	4	4	1.00	32	0.125
251	A	5	5	1.00	32	0.156
252	A	5	4	1.00	32	0.125
253	A	5	4	1.00	32	0.125
254	A	2	2	1.00	30	0.067
255	A	5	5	1.00	32	0.156
256	A	5	4	1.00	32	0.125
257	A	6	5	1.00	32	0.156
258	A	3	3	1.00	47	0.064
259	A	8	7	1.00	34	0.206
260	A	7	6	1.00	34	0.176
261	A	7	6	1.00	34	0.176
262	A	7	6	1.00	34	0.176
263	A	7	6	1.00	34	0.176
264	A	8	7	1.00	34	0.206
265	A	9	7	1.00	34	0.206
266	A	8	6	1.00	34	0.176
267	A	7	6	1.00	34	0.176
268	A	6	5	1.00	34	0.147
269	A	6	5	1.00	34	0.147

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
270	A	7	6	0.99	34	0.176
271	A	8	6	1.00	34	0.176
272	A	6	4	1.00	30	0.133
273	A	6	4	1.00	32	0.125
274	A	5	5	1.00	34	0.147
275	A	3	3	1.00	42	0.071
276	A	2	2	1.00	46	0.043
277	A	2	2	1.00	69	0.029
278	A	2	2	1.00	75	0.027
279	A	6	4	1.00	16	0.250
280	A	6	5	1.00	33	0.152
281	A	5	4	1.00	31	0.129
282	A	5	4	1.00	30	0.133
283	A	7	5	1.00	33	0.152
284	A	7	6	1.00	33	0.182
285	A	7	5	1.00	33	0.152
286	A	5	4	1.00	33	0.121
287	A	6	5	1.00	33	0.152
288	A	7	5	1.00	33	0.152
289	A	2	1	1.00	36	0.028
290	A	2	1	1.00	36	0.028
291	A	2	1	1.00	34	0.029
292	A	2	1	1.00	29	0.034
293	A	2	1	1.00	36	0.028
294	A	2	1	1.00	36	0.028
295	A	2	1	1.00	36	0.028
296	A	2	1	1.00	38	0.026
297	A	2	1	1.00	38	0.026
298	A	2	1	1.00	36	0.028
299	A	2	1	1.00	31	0.032
300	A	2	1	1.00	38	0.026
301	A	2	1	1.00	38	0.026
302	A	2	1	1.00	38	0.026
303	A	2	1	1.00	38	0.026
304	A	6	5	1.00	38	0.132

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
305	A	6	5	1.00	38	0.132
306	A	6	5	1.00	36	0.139
307	A	6	5	1.00	31	0.161
308	A	6	5	1.00	38	0.132
309	A	6	5	1.00	38	0.132
310	A	6	5	1.00	38	0.132
311	A	7	6	1.00	38	0.158
312	A	7	6	1.00	38	0.158
313	A	7	6	1.00	36	0.167
314	A	7	6	1.00	31	0.194
315	A	7	6	1.00	38	0.158
316	A	7	6	1.00	38	0.158
317	A	7	6	1.00	38	0.158
318	A	8	6	1.00	38	0.158
319	A	8	6	1.00	38	0.158
320	A	6	6	1.00	36	0.167
321	A	5	4	1.00	31	0.129
322	A	8	6	1.00	38	0.158
323	A	8	6	1.00	38	0.158
324	A	7	5	1.00	38	0.132
325	A	7	5	1.00	33	0.152
326	A	9	7	1.00	40	0.175
327	A	9	8	1.00	40	0.200
328	A	9	8	1.00	40	0.200
329	A	9	7	1.00	40	0.175
330	A	9	7	1.00	40	0.175
331	A	9	7	1.00	40	0.175
332	A	7	5	1.00	40	0.125
333	A	8	6	1.00	40	0.150
334	A	8	5	1.00	38	0.132
335	A	8	5	1.00	33	0.152
336	A	10	7	1.00	40	0.175
337	A	10	8	1.00	40	0.200
338	A	10	8	1.00	40	0.200
339	A	10	7	1.00	40	0.175

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
340	A	10	8	1.00	40	0.200
341	A	10	7	1.00	40	0.175
342	A	10	8	1.00	40	0.200
343	A	10	7	1.00	40	0.175
344	A	6	4	1.00	38	0.105
345	A	6	4	1.00	33	0.121
346	A	8	6	1.00	40	0.150
347	A	8	7	1.00	40	0.175
348	A	8	7	1.00	40	0.175
349	A	8	6	1.00	40	0.150
350	A	6	4	1.00	40	0.100
351	A	7	5	1.00	40	0.125
352	A	6	5	1.00	38	0.132
353	A	5	5	1.00	33	0.152
354	A	7	7	1.00	40	0.175
355	A	7	7	1.00	40	0.175
356	A	5	5	1.00	40	0.125
357	A	6	5	1.00	40	0.125
358	A	6	5	1.00	40	0.125
359	A	5	4	1.00	38	0.105
360	A	5	4	1.00	33	0.121
361	A	5	4	1.00	40	0.100
362	A	5	4	1.00	40	0.100
363	A	6	5	1.00	40	0.125
364	A	7	5	1.00	40	0.125
365	A	5	4	1.00	35	0.114
366	A	5	4	1.00	36	0.111
367	A	2	1	1.00	38	0.026
368	A	2	1	1.00	38	0.026
369	A	2	1	1.00	36	0.028
370	A	4	2	1.00	38	0.053
371	A	5	3	1.00	38	0.079
372	A	6	5	1.00	38	0.132
373	A	6	5	1.00	53	0.094
374	A	11	5	1.00	35	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
375	A	9	5	1.00	35	0.143
376	A	7	5	1.00	33	0.152
377	A	9	7	1.00	35	0.200
378	A	9	7	1.00	35	0.200
379	A	7	5	1.00	35	0.143
380	A	12	5	1.00	35	0.143
381	A	10	5	1.00	35	0.143
382	A	8	5	1.00	33	0.152
383	A	10	7	1.00	35	0.200
384	A	10	8	1.00	35	0.229
385	A	10	7	1.00	35	0.200
386	A	10	4	1.00	35	0.114
387	A	8	4	1.00	35	0.114
388	A	6	4	1.00	33	0.121
389	A	8	6	1.00	35	0.171
390	A	6	4	1.00	35	0.114
391	A	7	4	1.00	35	0.114
392	A	9	5	1.00	35	0.143
393	A	7	5	1.00	35	0.143
394	A	5	5	1.00	33	0.152
395	A	6	4	1.00	35	0.114
396	A	7	4	1.00	35	0.114
397	A	8	4	1.00	35	0.114
398	A	7	5	1.00	26	0.192
399	A	7	6	1.00	24	0.250
400	A	11	8	1.00	29	0.276

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int (d+ex)^2 \left(A+Bx+Cx^2\right) \sqrt{d^2-e^2x^2} \, dx$

Optimal result

Integrand size = 34, antiderivative size = 236

$$
\begin{aligned}
&\int (d+ex)^2 \left(A+Bx+Cx^2\right) \sqrt{d^2-e^2x^2} \, dx \\
&= \frac{d^2(3Cd^2+4Bde+10Ae^2)\, x\sqrt{d^2-e^2x^2}}{16e^2} - \frac{d(4Cd^2+e(7Bd+10Ae))\left(d^2-e^2x^2\right)^{3/2}}{15e^3} \\
&\quad - \frac{(3Cd^2+2e(2Bd+Ae))\, x(d^2-e^2x^2)^{3/2}}{8e^2} - \frac{(2Cd+Be)x^2(d^2-e^2x^2)^{3/2}}{5e} \\
&\quad - \frac{1}{6}Cx^3\left(d^2-e^2x^2\right)^{3/2} + \frac{d^4(3Cd^2+4Bde+10Ae^2)\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^3}
\end{aligned}
$$

```
[Out] -1/15*d*(4*C*d^2+e*(10*A*e+7*B*d))*(-e^2*x^2+d^2)^(3/2)/e^3-1/8*(3*C*d^2+2*
e*(A*e+2*B*d))*x*(-e^2*x^2+d^2)^(3/2)/e^2-1/5*(B*e+2*C*d)*x^2*(-e^2*x^2+d^2
)^(3/2)/e-1/6*C*x^3*(-e^2*x^2+d^2)^(3/2)+1/16*d^4*(10*A*e^2+4*B*d*e+3*C*d^2
)*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3+1/16*d^2*(10*A*e^2+4*B*d*e+3*C*d^2)*
x*(-e^2*x^2+d^2)^(1/2)/e^2
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used

$= \{1829, 655, 201, 223, 209\}$

$$\int (d+ex)^2 \left(A+Bx+Cx^2\right) \sqrt{d^2-e^2x^2}\,dx = \frac{d^4 \arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) (10Ae^2+4Bde+3Cd^2)}{16e^3} + \frac{d^2x\sqrt{d^2-e^2x^2}(10Ae^2+4Bde+3Cd^2)}{16e^2} - \frac{x(d^2-e^2x^2)^{3/2} \left(2e(Ae+2Bd)+3Cd^2\right)}{8e^2} - \frac{d(d^2-e^2x^2)^{3/2} \left(e(10Ae+7Bd)+4Cd^2\right)}{15e^3} - \frac{x^2(d^2-e^2x^2)^{3/2} (Be+2Cd)}{5e} - \frac{1}{6}Cx^3\left(d^2-e^2x^2\right)^{3/2}$$

```
[In] Int[(d + e*x)^2*(A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (d^2*(3*C*d^2 + 4*B*d*e + 10*A*e^2)*x*Sqrt[d^2 - e^2*x^2])/(16*e^2) - (d*(4
*C*d^2 + e*(7*B*d + 10*A*e))*(d^2 - e^2*x^2)^(3/2))/(15*e^3) - ((3*C*d^2 +
2*e*(2*B*d + A*e))*x*(d^2 - e^2*x^2)^(3/2))/(8*e^2) - ((2*C*d + B*e)*x^2*(d
^2 - e^2*x^2)^(3/2))/(5*e) - (C*x^3*(d^2 - e^2*x^2)^(3/2))/6 + (d^4*(3*C*d^
2 + 4*B*d*e + 10*A*e^2)*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(16*e^3)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
```

```
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1829

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{6}Cx^3\left(d^2-e^2x^2\right)^{3/2} \\
&\quad -\frac{\int \sqrt{d^2-e^2x^2}\left(-6Ad^2e^2-6de^2(Bd+2Ae)x-3e^2\left(3Cd^2+2e(2Bd+Ae)\right)x^2-6e^3(2Cd+Be)x^3\right)\,dx}{6e^2} \\
&= -\frac{(2Cd+Be)x^2\left(d^2-e^2x^2\right)^{3/2}}{5e}-\frac{1}{6}Cx^3\left(d^2-e^2x^2\right)^{3/2} \\
&\quad +\frac{\int \sqrt{d^2-e^2x^2}\left(30Ad^2e^4+6de^3\left(4Cd^2+e(7Bd+10Ae)\right)x+15e^4\left(3Cd^2+2e(2Bd+Ae)\right)x^2\right)\,dx}{30e^4} \\
&= -\frac{\left(3Cd^2+2e(2Bd+Ae)\right)x\left(d^2-e^2x^2\right)^{3/2}}{8e^2} \\
&\quad -\frac{(2Cd+Be)x^2\left(d^2-e^2x^2\right)^{3/2}}{5e}-\frac{1}{6}Cx^3\left(d^2-e^2x^2\right)^{3/2} \\
&\quad -\frac{\int \left(-15d^2e^4\left(3Cd^2+4Bde+10Ae^2\right)-24de^5\left(4Cd^2+e(7Bd+10Ae)\right)x\right)\sqrt{d^2-e^2x^2}\,dx}{120e^6} \\
&= -\frac{d\left(4Cd^2+e(7Bd+10Ae)\right)\left(d^2-e^2x^2\right)^{3/2}}{15e^3} \\
&\quad -\frac{\left(3Cd^2+2e(2Bd+Ae)\right)x\left(d^2-e^2x^2\right)^{3/2}}{8e^2}-\frac{(2Cd+Be)x^2\left(d^2-e^2x^2\right)^{3/2}}{5e} \\
&\quad -\frac{1}{6}Cx^3\left(d^2-e^2x^2\right)^{3/2}+\frac{\left(d^2\left(3Cd^2+4Bde+10Ae^2\right)\right)\int \sqrt{d^2-e^2x^2}\,dx}{8e^2} \\
&= \frac{d^2\left(3Cd^2+4Bde+10Ae^2\right)x\sqrt{d^2-e^2x^2}}{16e^2} \\
&\quad -\frac{d\left(4Cd^2+e(7Bd+10Ae)\right)\left(d^2-e^2x^2\right)^{3/2}}{15e^3} \\
&\quad -\frac{\left(3Cd^2+2e(2Bd+Ae)\right)x\left(d^2-e^2x^2\right)^{3/2}}{8e^2}-\frac{(2Cd+Be)x^2\left(d^2-e^2x^2\right)^{3/2}}{5e} \\
&\quad -\frac{1}{6}Cx^3\left(d^2-e^2x^2\right)^{3/2}+\frac{\left(d^4\left(3Cd^2+4Bde+10Ae^2\right)\right)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{16e^2}
\end{aligned}$$

$$
\begin{aligned}
&=\frac{d^2(3Cd^2+4Bde+10Ae^2)\,x\sqrt{d^2-e^2x^2}}{16e^2}-\frac{d(4Cd^2+e(7Bd+10Ae))\,(d^2-e^2x^2)^{3/2}}{15e^3}\\
&\quad-\frac{(3Cd^2+2e(2Bd+Ae))\,x(d^2-e^2x^2)^{3/2}}{8e^2}-\frac{(2Cd+Be)x^2(d^2-e^2x^2)^{3/2}}{5e}\\
&\quad-\frac{1}{6}Cx^3\left(d^2-e^2x^2\right)^{3/2}+\frac{(d^4(3Cd^2+4Bde+10Ae^2))\,\text{Subst}\left(\int\frac{1}{1+e^2x^2}\,dx,x,\frac{x}{\sqrt{d^2-e^2x^2}}\right)}{16e^2}\\
&=\frac{d^2(3Cd^2+4Bde+10Ae^2)\,x\sqrt{d^2-e^2x^2}}{16e^2}\\
&\quad-\frac{d(4Cd^2+e(7Bd+10Ae))\,(d^2-e^2x^2)^{3/2}}{15e^3}\\
&\quad-\frac{(3Cd^2+2e(2Bd+Ae))\,x(d^2-e^2x^2)^{3/2}}{8e^2}-\frac{(2Cd+Be)x^2(d^2-e^2x^2)^{3/2}}{5e}\\
&\quad-\frac{1}{6}Cx^3\left(d^2-e^2x^2\right)^{3/2}+\frac{d^4(3Cd^2+4Bde+10Ae^2)\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.92

$$\int(d+ex)^2\left(A+Bx+Cx^2\right)\sqrt{d^2-e^2x^2}\,dx$$

$$=\frac{\sqrt{d^2-e^2x^2}(C(-64d^5-45d^4ex-32d^3e^2x^2+50d^2e^3x^3+96de^4x^4+40e^5x^5)+2e(5Ae(-16d^3+9d^2ex+}{}$$

```
[In] Integrate[(d + e*x)^2*(A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(C*(-64*d^5 - 45*d^4*e*x - 32*d^3*e^2*x^2 + 50*d^2*e^3
*x^3 + 96*d*e^4*x^4 + 40*e^5*x^5) + 2*e*(5*A*e*(-16*d^3 + 9*d^2*e*x + 16*d*
e^2*x^2 + 6*e^3*x^3) + B*(-56*d^4 - 30*d^3*e*x + 32*d^2*e^2*x^2 + 60*d*e^3*
x^3 + 24*e^4*x^4))) - 30*d^4*(3*C*d^2 + 2*e*(2*B*d + 5*A*e))*ArcTan[(e*x)/(
Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/(240*e^3)
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.92

method	result
risch	$-\frac{(-40e^5C\,x^5-48B\,x^4e^5-96Cd\,e^4x^4-60A\,e^5x^3-120x^3dB\,e^4-50C\,d^2e^3x^3-160Ad\,e^4x^2-64x^2d^2B\,e^3+32C\,d^3e^2x^2-90A\,d^2e^3x+60}{240e^3}$
default	$A\,d^2\left(\frac{x\sqrt{-e^2x^2+d^2}}{2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}\right)+e^2C\Bigg(-\frac{x^3(-e^2x^2+d^2)^{\frac{3}{2}}}{6e^2}+\frac{d^2\bigg(-\frac{x(-e^2x^2+d^2)^{\frac{3}{2}}}{4e^2}+\frac{d^2\Big(\frac{x\sqrt{-e^2x^2+d^2}}{2}}{}}{2e^2}$

```
[In] int((e*x+d)^2*(C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/240/e^3*(-40*C*e^5*x^5-48*B*e^5*x^4-96*C*d*e^4*x^4-60*A*e^5*x^3-120*B*d*
e^4*x^3-50*C*d^2*e^3*x^3-160*A*d*e^4*x^2-64*B*d^2*e^3*x^2+32*C*d^3*e^2*x^2-
90*A*d^2*e^3*x+60*B*d^3*e^2*x+45*C*d^4*e*x+160*A*d^3*e^2+112*B*d^4*e+64*C*d
^5)*(-e^2*x^2+d^2)^(1/2)+1/16*d^4/e^2*(10*A*e^2+4*B*d*e+3*C*d^2)/(e^2)^(1/2
)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.45 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.89

$$\int (d+ex)^2\left(A+Bx+Cx^2\right)\sqrt{d^2-e^2x^2}\,dx =$$

$$-\frac{30\left(3\,Cd^6+4\,Bd^5e+10\,Ad^4e^2\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-(40\,Ce^5x^5-64\,Cd^5-112\,Bd^4e-160\,Ad^3e^2}{}$$

```
[In] integrate((e*x+d)^2*(C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas
")
```

```
[Out] -1/240*(30*(3*C*d^6 + 4*B*d^5*e + 10*A*d^4*e^2)*arctan(-(d - sqrt(-e^2*x^2
 + d^2))/(e*x)) - (40*C*e^5*x^5 - 64*C*d^5 - 112*B*d^4*e - 160*A*d^3*e^2 + 4
8*(2*C*d*e^4 + B*e^5)*x^4 + 10*(5*C*d^2*e^3 + 12*B*d*e^4 + 6*A*e^5)*x^3 - 3
2*(C*d^3*e^2 - 2*B*d^2*e^3 - 5*A*d*e^4)*x^2 - 15*(3*C*d^4*e + 4*B*d^3*e^2 -
 6*A*d^2*e^3)*x)*sqrt(-e^2*x^2 + d^2))/e^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 466 vs. $2(219) = 438$.

Time = 0.56 (sec) , antiderivative size = 466, normalized size of antiderivative = 1.97

$$\int (d+ex)^2 \left(A+Bx+Cx^2\right)\sqrt{d^2-e^2x^2}\,dx$$
$$= \begin{cases} \sqrt{d^2-e^2x^2}\left(\frac{Ce^2x^5}{6} - \frac{x^4(-Be^4-2Cde^3)}{5e^2} - \frac{x^3\left(-Ae^4-2Bde^3-\frac{5Cd^2e^2}{6}\right)}{4e^2} - \frac{x^2\left(-2Ade^3+2Cd^3e+\frac{4d^2\left(-Be^4-2Cde^3\right)}{5e^2}\right)}{3e^2} - \frac{x\left(2B\right.}{}\right. \\ \left(Ad^2x+\frac{Ce^2x^5}{5}+\frac{x^4(Be^2+2Cde)}{4}+\frac{x^3(Ae^2+2Bde+Cd^2)}{3}+\frac{x^2\cdot(2Ade+Bd^2)}{2}\right)\sqrt{d^2} \end{cases}$$

```
[In] integrate((e*x+d)**2*(C*x**2+B*x+A)*(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Piecewise((sqrt(d**2 - e**2*x**2)*(C*e**2*x**5/6 - x**4*(-B*e**4 - 2*C*d*e*
*3)/(5*e**2) - x**3*(-A*e**4 - 2*B*d*e**3 - 5*C*d**2*e**2/6)/(4*e**2) - x**
2*(-2*A*d*e**3 + 2*C*d**3*e + 4*d**2*(-B*e**4 - 2*C*d*e**3)/(5*e**2))/(3*e*
*2) - x*(2*B*d**3*e + C*d**4 + 3*d**2*(-A*e**4 - 2*B*d*e**3 - 5*C*d**2*e**2
/6)/(4*e**2))/(2*e**2) - (2*A*d**3*e + B*d**4 + 2*d**2*(-2*A*d*e**3 + 2*C*d
**3*e + 4*d**2*(-B*e**4 - 2*C*d*e**3)/(5*e**2))/(3*e**2))/e**2) + (A*d**4 +
 d**2*(2*B*d**3*e + C*d**4 + 3*d**2*(-A*e**4 - 2*B*d*e**3 - 5*C*d**2*e**2/6
)/(4*e**2))/(2*e**2))*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 -
e**2*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True)),
Ne(e**2, 0)), ((A*d**2*x + C*e**2*x**5/5 + x**4*(B*e**2 + 2*C*d*e)/4 + x**3
*(A*e**2 + 2*B*d*e + C*d**2)/3 + x**2*(2*A*d*e + B*d**2)/2)*sqrt(d**2), Tru
e))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.57

$$\begin{aligned}\int (d+ex)^2\left(A+Bx+Cx^2\right)\sqrt{d^2-e^2x^2}\,dx &= -\frac{1}{6}\left(-e^2x^2+d^2\right)^{\frac{3}{2}}Cx^3+\frac{Ad^4\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{2\sqrt{e^2}}\\
&+\frac{Cd^6\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{16\sqrt{e^2}e^2}+\frac{1}{2}\sqrt{-e^2x^2+d^2}Ad^2x\\
&+\frac{\sqrt{-e^2x^2+d^2}Cd^4x}{16\,e^2}-\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}Cd^2x}{8\,e^2}\\
&+\frac{(Cd^2+2\,Bde+Ae^2)d^4\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{8\sqrt{e^2}e^2}\\
&-\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}Bd^2}{3\,e^2}-\frac{2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}Ad}{3\,e}\\
&+\frac{\sqrt{-e^2x^2+d^2}(Cd^2+2\,Bde+Ae^2)d^2x}{8\,e^2}\\
&-\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}(2\,Cde+Be^2)x^2}{5\,e^2}\\
&-\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}(Cd^2+2\,Bde+Ae^2)x}{4\,e^2}\\
&-\frac{2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}(2\,Cde+Be^2)d^2}{15\,e^4}\end{aligned}$$

```
[In] integrate((e*x+d)^2*(C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima
")
```

```
[Out] -1/6*(-e^2*x^2 + d^2)^(3/2)*C*x^3 + 1/2*A*d^4*arcsin(e^2*x/(d*sqrt(e^2)))/s
qrt(e^2) + 1/16*C*d^6*arcsin(e^2*x/(d*sqrt(e^2)))/(sqrt(e^2)*e^2) + 1/2*sqr
t(-e^2*x^2 + d^2)*A*d^2*x + 1/16*sqrt(-e^2*x^2 + d^2)*C*d^4*x/e^2 - 1/8*(-e
^2*x^2 + d^2)^(3/2)*C*d^2*x/e^2 + 1/8*(C*d^2 + 2*B*d*e + A*e^2)*d^4*arcsin(
e^2*x/(d*sqrt(e^2)))/(sqrt(e^2)*e^2) - 1/3*(-e^2*x^2 + d^2)^(3/2)*B*d^2/e^2
 - 2/3*(-e^2*x^2 + d^2)^(3/2)*A*d/e + 1/8*sqrt(-e^2*x^2 + d^2)*(C*d^2 + 2*B
*d*e + A*e^2)*d^2*x/e^2 - 1/5*(-e^2*x^2 + d^2)^(3/2)*(2*C*d*e + B*e^2)*x^2/
e^2 - 1/4*(-e^2*x^2 + d^2)^(3/2)*(C*d^2 + 2*B*d*e + A*e^2)*x/e^2 - 2/15*(-e
^2*x^2 + d^2)^(3/2)*(2*C*d*e + B*e^2)*d^2/e^4
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.95

$$\int (d+ex)^2 \left(A+Bx+Cx^2\right) \sqrt{d^2-e^2x^2}\, dx$$
$$= \frac{1}{240}\sqrt{-e^2x^2+d^2}\Bigg(\Bigg(2\Bigg(\Bigg(4\left(5\,Ce^2x+\frac{6\,(2\,Cde^9+Be^{10})}{e^8}\right)x+\frac{5\,(5\,Cd^2e^8+12\,Bde^9+6\,Ae^{10})}{e^8}\Bigg)x-\frac{16\,(}{}$$
$$+\frac{(3\,Cd^6+4\,Bd^5e+10\,Ad^4e^2)\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{16\,e^2|e|}$$

```
[In] integrate((e*x+d)^2*(C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/240*sqrt(-e^2*x^2 + d^2)*((2*((4*(5*C*e^2*x + 6*(2*C*d*e^9 + B*e^10)/e^8)
*x + 5*(5*C*d^2*e^8 + 12*B*d*e^9 + 6*A*e^10)/e^8)*x - 16*(C*d^3*e^7 - 2*B*d
^2*e^8 - 5*A*d*e^9)/e^8)*x - 15*(3*C*d^4*e^6 + 4*B*d^3*e^7 - 6*A*d^2*e^8)/e
^8)*x - 16*(4*C*d^5*e^5 + 7*B*d^4*e^6 + 10*A*d^3*e^7)/e^8) + 1/16*(3*C*d^6
+ 4*B*d^5*e + 10*A*d^4*e^2)*arcsin(e*x/d)*sgn(d)*sgn(e)/(e^2*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^2 \left(A+Bx+Cx^2\right) \sqrt{d^2-e^2x^2}\, dx = \int \sqrt{d^2-e^2\,x^2}\,{\left(d+e\,x\right)}^2\,\left(C\,x^2+B\,x+A\right) \,d x$$

```
[In] int((d^2 - e^2*x^2)^(1/2)*(d + e*x)^2*(A + B*x + C*x^2),x)
```

```
[Out] int((d^2 - e^2*x^2)^(1/2)*(d + e*x)^2*(A + B*x + C*x^2), x)
```

3.2 $\int (d+ex)\left(A+Bx+Cx^2\right)\sqrt{d^2-e^2x^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 186

$$\begin{aligned}\int (d+ex)\left(A+Bx+Cx^2\right)\sqrt{d^2-e^2x^2}\,dx &= \frac{d(Cd^2+e(Bd+4Ae))\,x\sqrt{d^2-e^2x^2}}{8e^2}\\ &\quad-\frac{\left(2Cd^2+5e(Bd+Ae)\right)\left(d^2-e^2x^2\right)^{3/2}}{15e^3}\\ &\quad-\frac{(Cd+Be)x\left(d^2-e^2x^2\right)^{3/2}}{4e^2}\\ &\quad-\frac{Cx^2\left(d^2-e^2x^2\right)^{3/2}}{5e}\\ &\quad+\frac{d^3(Cd^2+e(Bd+4Ae))\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^3}\end{aligned}$$

```
[Out] -1/15*(2*C*d^2+5*e*(A*e+B*d))*(-e^2*x^2+d^2)^(3/2)/e^3-1/4*(B*e+C*d)*x*(-e^
2*x^2+d^2)^(3/2)/e^2-1/5*C*x^2*(-e^2*x^2+d^2)^(3/2)/e+1/8*d^3*(C*d^2+e*(4*A
*e+B*d))*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3+1/8*d*(C*d^2+e*(4*A*e+B*d))*x
*(-e^2*x^2+d^2)^(1/2)/e^2
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used

$= \{1829, 655, 201, 223, 209\}$

$$\int (d+ex)\left(A+Bx+Cx^2\right)\sqrt{d^2-e^2x^2}\,dx = \frac{d^3\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\left(e(4Ae+Bd)+Cd^2\right)}{8e^3} + \frac{dx\sqrt{d^2-e^2x^2}(e(4Ae+Bd)+Cd^2)}{8e^2} - \frac{\left(d^2-e^2x^2\right)^{3/2}\left(5e(Ae+Bd)+2Cd^2\right)}{15e^3} - \frac{x(d^2-e^2x^2)^{3/2}\left(Be+Cd\right)}{4e^2} - \frac{Cx^2(d^2-e^2x^2)^{3/2}}{5e}$$

```
[In] Int[(d + e*x)*(A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (d*(C*d^2 + e*(B*d + 4*A*e))*x*Sqrt[d^2 - e^2*x^2])/(8*e^2) - ((2*C*d^2 + 5
*e*(B*d + A*e))*(d^2 - e^2*x^2)^(3/2))/(15*e^3) - ((C*d + B*e)*x*(d^2 - e^2
*x^2)^(3/2))/(4*e^2) - (C*x^2*(d^2 - e^2*x^2)^(3/2))/(5*e) + (d^3*(C*d^2 +
e*(B*d + 4*A*e))*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(8*e^3)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1829

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{Cx^2(d^2-e^2x^2)^{3/2}}{5e} \\
&\quad -\frac{\int \sqrt{d^2-e^2x^2}\left(-5Ade^2-e(2Cd^2+5e(Bd+Ae))\,x-5e^2(Cd+Be)x^2\right)\,dx}{5e^2} \\
&= -\frac{(Cd+Be)x(d^2-e^2x^2)^{3/2}}{4e^2}-\frac{Cx^2(d^2-e^2x^2)^{3/2}}{5e} \\
&\quad +\frac{\int \left(5de^2(Cd^2+e(Bd+4Ae))+4e^3(2Cd^2+5e(Bd+Ae))\,x\right)\sqrt{d^2-e^2x^2}\,dx}{20e^4} \\
&= -\frac{(2Cd^2+5e(Bd+Ae))\,(d^2-e^2x^2)^{3/2}}{15e^3}-\frac{(Cd+Be)x(d^2-e^2x^2)^{3/2}}{4e^2} \\
&\quad -\frac{Cx^2(d^2-e^2x^2)^{3/2}}{5e}+\frac{(d(Cd^2+e(Bd+4Ae)))\int \sqrt{d^2-e^2x^2}\,dx}{4e^2} \\
&= \frac{d(Cd^2+e(Bd+4Ae))\,x\sqrt{d^2-e^2x^2}}{8e^2}-\frac{(2Cd^2+5e(Bd+Ae))\,(d^2-e^2x^2)^{3/2}}{15e^3} \\
&\quad -\frac{(Cd+Be)x(d^2-e^2x^2)^{3/2}}{4e^2}-\frac{Cx^2(d^2-e^2x^2)^{3/2}}{5e} \\
&\quad +\frac{(d^3(Cd^2+e(Bd+4Ae)))\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{8e^2} \\
&= \frac{d(Cd^2+e(Bd+4Ae))\,x\sqrt{d^2-e^2x^2}}{8e^2}-\frac{(2Cd^2+5e(Bd+Ae))\,(d^2-e^2x^2)^{3/2}}{15e^3} \\
&\quad -\frac{(Cd+Be)x(d^2-e^2x^2)^{3/2}}{4e^2}-\frac{Cx^2(d^2-e^2x^2)^{3/2}}{5e} \\
&\quad +\frac{(d^3(Cd^2+e(Bd+4Ae)))\,\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx,x,\frac{x}{\sqrt{d^2-e^2x^2}}\right)}{8e^2} \\
&= \frac{d(Cd^2+e(Bd+4Ae))\,x\sqrt{d^2-e^2x^2}}{8e^2}-\frac{(2Cd^2+5e(Bd+Ae))\,(d^2-e^2x^2)^{3/2}}{15e^3} \\
&\quad -\frac{(Cd+Be)x(d^2-e^2x^2)^{3/2}}{4e^2}-\frac{Cx^2(d^2-e^2x^2)^{3/2}}{5e} \\
&\quad +\frac{d^3(Cd^2+e(Bd+4Ae))\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.97

$$\int (d+ex)\left(A+Bx+Cx^2\right)\sqrt{d^2-e^2x^2}\,dx$$

$$= \frac{\sqrt{d^2-e^2x^2}(C(-16d^4-15d^3ex-8d^2e^2x^2+30de^3x^3+24e^4x^4)-5e(-4Ae(-2d^2+3dex+2e^2x^2)+B(}{120e^3}$$

```
[In] Integrate[(d + e*x)*(A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(C*(-16*d^4 - 15*d^3*e*x - 8*d^2*e^2*x^2 + 30*d*e^3*x^
3 + 24*e^4*x^4) - 5*e*(-4*A*e*(-2*d^2 + 3*d*e*x + 2*e^2*x^2) + B*(8*d^3 + 3
*d^2*e*x - 8*d*e^2*x^2 - 6*e^3*x^3))) - 30*d^3*(C*d^2 + e*(B*d + 4*A*e))*Ar
cTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/(120*e^3)
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.97

method	result
risch	$-\frac{(-24e^4C\,x^4-30x^3B\,e^4-30Cd\,e^3x^3-40A\,e^4x^2-40x^2dB\,e^3+8C\,d^2e^2x^2-60Ad\,e^3x+15xB\,d^2e^2+15C\,d^3xe+40A\,d^2e^2+40B\,d^3e+}{120e^3}$
default	$dA\left(\frac{x\sqrt{-e^2x^2+d^2}}{2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}\right)+eC\left(-\frac{x^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}{5e^2}-\frac{2d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}{15e^4}\right)+(Be+Cd)$

```
[In] int((e*x+d)*(C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/120/e^3*(-24*C*e^4*x^4-30*B*e^4*x^3-30*C*d*e^3*x^3-40*A*e^4*x^2-40*B*d*e
^3*x^2+8*C*d^2*e^2*x^2-60*A*d*e^3*x+15*B*d^2*e^2*x+15*C*d^3*e*x+40*A*d^2*e^
2+40*B*d^3*e+16*C*d^4)*(-e^2*x^2+d^2)^(1/2)+1/8*d^3/e^2*(4*A*e^2+B*d*e+C*d^
2)/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.93

$$\int (d+ex)\left(A+Bx+Cx^2\right)\sqrt{d^2-e^2x^2}\,dx =$$

$$-\frac{30\left(Cd^5+Bd^4e+4\,Ad^3e^2\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-\left(24\,Ce^4x^4-16\,Cd^4-40\,Bd^3e-40\,Ad^2e^2+30\,(C}{120\,e^3}$$

```
[In] integrate((e*x+d)*(C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/120*(30*(C*d^5 + B*d^4*e + 4*A*d^3*e^2)*arctan(-(d - sqrt(-e^2*x^2 + d^2
))/(e*x)) - (24*C*e^4*x^4 - 16*C*d^4 - 40*B*d^3*e - 40*A*d^2*e^2 + 30*(C*d*
e^3 + B*e^4)*x^3 - 8*(C*d^2*e^2 - 5*B*d*e^3 - 5*A*e^4)*x^2 - 15*(C*d^3*e +
B*d^2*e^2 - 4*A*d*e^3)*x)*sqrt(-e^2*x^2 + d^2))/e^3
```

Sympy [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.76

$$\int (d+ex)\left(A+Bx+Cx^2\right)\sqrt{d^2-e^2x^2}\,dx$$

$$=\begin{cases}\sqrt{d^2-e^2x^2}\left(\frac{Cex^4}{5}-\frac{x^3\left(-Be^3-Cde^2\right)}{4e^2}-\frac{x^2\left(-Ae^3-Bde^2+\frac{Cd^2e}{5}\right)}{3e^2}-\frac{x\left(-Ade^2+Bd^2e+Cd^3+\frac{3d^2\left(-Be^3-Cde^2\right)}{4e^2}\right)}{2e^2}-\frac{Ad^2e+Bd^3+}{}\right.\\ \left(Adx+\frac{Cex^4}{4}+\frac{x^3(Be+Cd)}{3}+\frac{x^2(Ae+Bd)}{2}\right)\sqrt{d^2}\end{cases}$$

```
[In] integrate((e*x+d)*(C*x**2+B*x+A)*(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Piecewise((sqrt(d**2 - e**2*x**2)*(C*e*x**4/5 - x**3*(-B*e**3 - C*d*e**2)/(
4*e**2) - x**2*(-A*e**3 - B*d*e**2 + C*d**2*e/5)/(3*e**2) - x*(-A*d*e**2 +
B*d**2*e + C*d**3 + 3*d**2*(-B*e**3 - C*d*e**2)/(4*e**2))/(2*e**2) - (A*d**
2*e + B*d**3 + 2*d**2*(-A*e**3 - B*d*e**2 + C*d**2*e/5)/(3*e**2))/e**2) + (
A*d**3 + d**2*(-A*d*e**2 + B*d**2*e + C*d**3 + 3*d**2*(-B*e**3 - C*d*e**2)/
(4*e**2))/(2*e**2))*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e*
*2*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True)), Ne
(e**2, 0)), ((A*d*x + C*e*x**4/4 + x**3*(B*e + C*d)/3 + x**2*(A*e + B*d)/2)
*sqrt(d**2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.20

$$\begin{aligned}
&\int (d+ex)\left(A+Bx+Cx^2\right)\sqrt{d^2-e^2x^2}\,dx \\
&= \frac{Ad^3\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{2\sqrt{e^2}} + \frac{1}{2}\sqrt{-e^2x^2+d^2}Adx - \frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}Cx^2}{5\,e} \\
&\quad + \frac{(Cd+Be)d^4\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{8\sqrt{e^2}e^2} + \frac{\sqrt{-e^2x^2+d^2}(Cd+Be)d^2x}{8\,e^2} - \frac{2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}Cd^2}{15\,e^3} \\
&\quad - \frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}Bd}{3\,e^2} - \frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}A}{3\,e} - \frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}(Cd+Be)x}{4\,e^2}
\end{aligned}$$

```
[In] integrate((e*x+d)*(C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*A*d^3*arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e^2) + 1/2*sqrt(-e^2*x^2 + d^2)*
A*d*x - 1/5*(-e^2*x^2 + d^2)^(3/2)*C*x^2/e + 1/8*(C*d + B*e)*d^4*arcsin(e^2
*x/(d*sqrt(e^2)))/(sqrt(e^2)*e^2) + 1/8*sqrt(-e^2*x^2 + d^2)*(C*d + B*e)*d^
2*x/e^2 - 2/15*(-e^2*x^2 + d^2)^(3/2)*C*d^2/e^3 - 1/3*(-e^2*x^2 + d^2)^(3/2
)*B*d/e^2 - 1/3*(-e^2*x^2 + d^2)^(3/2)*A/e - 1/4*(-e^2*x^2 + d^2)^(3/2)*(C*
d + B*e)*x/e^2
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.97

$$\begin{aligned}
&\int (d+ex)\left(A+Bx+Cx^2\right)\sqrt{d^2-e^2x^2}\,dx \\
&= \frac{1}{120}\sqrt{-e^2x^2+d^2}\left(\left(2\left(3\left(4\,Cex+\frac{5\left(Cde^6+Be^7\right)}{e^6}\right)x-\frac{4\left(Cd^2e^5-5\,Bde^6-5\,Ae^7\right)}{e^6}\right)x-\frac{15\,(Cd^3e^4+}{} \right.\right. \\
&\quad + \frac{\left(Cd^5+Bd^4e+4\,Ad^3e^2\right)\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{8\,e^2|e|}
\end{aligned}$$

```
[In] integrate((e*x+d)*(C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/120*sqrt(-e^2*x^2 + d^2)*((2*(3*(4*C*e*x + 5*(C*d*e^6 + B*e^7)/e^6)*x - 4
*(C*d^2*e^5 - 5*B*d*e^6 - 5*A*e^7)/e^6)*x - 15*(C*d^3*e^4 + B*d^2*e^5 - 4*A
*d*e^6)/e^6)*x - 8*(2*C*d^4*e^3 + 5*B*d^3*e^4 + 5*A*d^2*e^5)/e^6) + 1/8*(C*
d^5 + B*d^4*e + 4*A*d^3*e^2)*arcsin(e*x/d)*sgn(d)*sgn(e)/(e^2*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)\left(A+Bx+Cx^2\right)\sqrt{d^2-e^2x^2}\,dx=\int \sqrt{d^2-e^2\,x^2}\,(d+e\,x)\,\left(C\,x^2+B\,x+A\right)\,dx$$

```
[In] int((d^2 - e^2*x^2)^(1/2)*(d + e*x)*(A + B*x + C*x^2),x)
[Out] int((d^2 - e^2*x^2)^(1/2)*(d + e*x)*(A + B*x + C*x^2), x)
```

3.3 $\int (A+Bx+Cx^2)\sqrt{d^2-e^2x^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 125

$$\int (A+Bx+Cx^2)\sqrt{d^2-e^2x^2}\,dx = \frac{1}{8}\left(4A+\frac{Cd^2}{e^2}\right)x\sqrt{d^2-e^2x^2} - \frac{B(d^2-e^2x^2)^{3/2}}{3e^2} - \frac{Cx(d^2-e^2x^2)^{3/2}}{4e^2} + \frac{d^2(Cd^2+4Ae^2)\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^3}$$

```
[Out] -1/3*B*(-e^2*x^2+d^2)^(3/2)/e^2-1/4*C*x*(-e^2*x^2+d^2)^(3/2)/e^2+1/8*d^2*(4
*A*e^2+C*d^2)*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3+1/8*(4*A+C*d^2/e^2)*x*(-
e^2*x^2+d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1829, 655, 201, 223, 209}

$$\int (A+Bx+Cx^2)\sqrt{d^2-e^2x^2}\,dx = \frac{d^2(4Ae^2+Cd^2)\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^3} + \frac{1}{8}x\sqrt{d^2-e^2x^2}\left(4A+\frac{Cd^2}{e^2}\right) - \frac{B(d^2-e^2x^2)^{3/2}}{3e^2} - \frac{Cx(d^2-e^2x^2)^{3/2}}{4e^2}$$

```
[In] Int[(A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] ((4*A + (C*d^2)/e^2)*x*Sqrt[d^2 - e^2*x^2])/8 - (B*(d^2 - e^2*x^2)^(3/2))/(
3*e^2) - (C*x*(d^2 - e^2*x^2)^(3/2))/(4*e^2) + (d^2*(C*d^2 + 4*A*e^2)*ArcTa
n[(e*x)/Sqrt[d^2 - e^2*x^2]])/(8*e^3)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1829

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{Cx\left(d^2-e^2x^2\right)^{3/2}}{4e^2}-\frac{\int \left(-Cd^2-4Ae^2-4Be^2x\right)\sqrt{d^2-e^2x^2}\,dx}{4e^2}\\
&= -\frac{B\left(d^2-e^2x^2\right)^{3/2}}{3e^2}-\frac{Cx\left(d^2-e^2x^2\right)^{3/2}}{4e^2}-\frac{\left(-Cd^2-4Ae^2\right)\int \sqrt{d^2-e^2x^2}\,dx}{4e^2}\\
&= \frac{\left(Cd^2+4Ae^2\right)x\sqrt{d^2-e^2x^2}}{8e^2}-\frac{B\left(d^2-e^2x^2\right)^{3/2}}{3e^2}\\
&\quad -\frac{Cx\left(d^2-e^2x^2\right)^{3/2}}{4e^2}-\frac{\left(d^2\left(-Cd^2-4Ae^2\right)\right)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{8e^2}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{(Cd^2+4Ae^2)\,x\sqrt{d^2-e^2x^2}}{8e^2} - \frac{B(d^2-e^2x^2)^{3/2}}{3e^2} - \frac{Cx(d^2-e^2x^2)^{3/2}}{4e^2} \\
&\quad - \frac{(d^2(-Cd^2-4Ae^2))\,\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)}{8e^2} \\
&= \frac{(Cd^2+4Ae^2)\,x\sqrt{d^2-e^2x^2}}{8e^2} - \frac{B(d^2-e^2x^2)^{3/2}}{3e^2} \\
&\quad - \frac{Cx(d^2-e^2x^2)^{3/2}}{4e^2} + \frac{d^2(Cd^2+4Ae^2)\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.91

$$
\int \left(A+Bx+Cx^2\right)\sqrt{d^2-e^2x^2}\,dx
$$

$$
= \frac{e\sqrt{d^2-e^2x^2}(-8Bd^2-3Cd^2x+12Ae^2x+8Be^2x^2+6Ce^2x^3)-6d^2(Cd^2+4Ae^2)\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{24e^3}
$$

```
[In] Integrate[(A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (e*Sqrt[d^2 - e^2*x^2]*(-8*B*d^2 - 3*C*d^2*x + 12*A*e^2*x + 8*B*e^2*x^2 + 6
*C*e^2*x^3) - 6*d^2*(C*d^2 + 4*A*e^2)*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 -
e^2*x^2])])/(24*e^3)
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.86

method	result
risch	$\frac{(6C\,e^2x^3+8B\,e^2x^2+12A\,e^2x-3C\,d^2x-8B\,d^2)\sqrt{-e^2x^2+d^2}}{24e^2}+\frac{d^2(4A\,e^2+C\,d^2)\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{8e^2\sqrt{e^2}}$
default	$A\left(\frac{x\sqrt{-e^2x^2+d^2}}{2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}\right)+C\left(-\frac{x(-e^2x^2+d^2)^{\frac{3}{2}}}{4e^2}+\frac{d^2\left(\frac{x\sqrt{-e^2x^2+d^2}}{2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}\right)}{4e^2}\right.$

```
[In] int((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*(6*C*e^2*x^3+8*B*e^2*x^2+12*A*e^2*x-3*C*d^2*x-8*B*d^2)/e^2*(-e^2*x^2+d
^2)^(1/2)+1/8*d^2*(4*A*e^2+C*d^2)/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^
2*x^2+d^2)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.86

$$\int \left(A+Bx+Cx^2\right)\sqrt{d^2-e^2x^2}\,dx = -\frac{6\,(Cd^4+4\,Ad^2e^2)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-(6\,Ce^3x^3+8\,Be^3x^2-8\,Bd^2e-3\,(Cd^2e-4\,Ae^3)x)\sqrt{-e^2x^2}}{24\,e^3}$$

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/24*(6*(C*d^4 + 4*A*d^2*e^2)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) -
(6*C*e^3*x^3 + 8*B*e^3*x^2 - 8*B*d^2*e - 3*(C*d^2*e - 4*A*e^3)*x)*sqrt(-e^2
*x^2 + d^2))/e^3
```

Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.32

$$\int \left(A+Bx+Cx^2\right)\sqrt{d^2-e^2x^2}\,dx = \begin{cases} \sqrt{d^2-e^2x^2}\left(-\frac{Bd^2}{3e^2}+\frac{Bx^2}{3}+\frac{Cx^3}{4}-\frac{x\left(-Ae^2+\frac{Cd^2}{4}\right)}{2e^2}\right)+\left(Ad^2+\frac{d^2\left(-Ae^2+\frac{Cd^2}{4}\right)}{2e^2}\right)\left(\begin{cases}\frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} \\ \frac{x\log\left(x\right)}{\sqrt{-e^2x^2}}\end{cases}\right. \\ \left(Ax+\frac{Bx^2}{2}+\frac{Cx^3}{3}\right)\sqrt{d^2} \end{cases}$$

```
[In] integrate((C*x**2+B*x+A)*(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Piecewise((sqrt(d**2 - e**2*x**2)*(-B*d**2/(3*e**2) + B*x**2/3 + C*x**3/4 -
 x*(-A*e**2 + C*d**2/4)/(2*e**2)) + (A*d**2 + d**2*(-A*e**2 + C*d**2/4)/(2*
e**2))*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e**2*x**2))/sqr
t(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True)), Ne(e**2, 0)), (
(A*x + B*x**2/2 + C*x**3/3)*sqrt(d**2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.10

$$\int \left(A+Bx+Cx^2\right)\sqrt{d^2-e^2x^2}\,dx=\frac{Ad^2\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{2\sqrt{e^2}}+\frac{Cd^4\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{8\sqrt{e^2}e^2}+\frac{1}{2}\sqrt{-e^2x^2+d^2}Ax+\frac{\sqrt{-e^2x^2+d^2}Cd^2x}{8\,e^2}-\frac{(-e^2x^2+d^2)^{\frac{3}{2}}Cx}{4\,e^2}-\frac{(-e^2x^2+d^2)^{\frac{3}{2}}B}{3\,e^2}$$

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*A*d^2*arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e^2) + 1/8*C*d^4*arcsin(e^2*x/(d
*sqrt(e^2)))/(sqrt(e^2)*e^2) + 1/2*sqrt(-e^2*x^2 + d^2)*A*x + 1/8*sqrt(-e^2
*x^2 + d^2)*C*d^2*x/e^2 - 1/4*(-e^2*x^2 + d^2)^(3/2)*C*x/e^2 - 1/3*(-e^2*x^
2 + d^2)^(3/2)*B/e^2
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.77

$$\int \left(A+Bx+Cx^2\right)\sqrt{d^2-e^2x^2}\,dx=\frac{1}{24}\sqrt{-e^2x^2+d^2}\left(\left(2\,(3\,Cx+4\,B)x-\frac{3\,(Cd^2e^2-4\,Ae^4)}{e^4}\right)x-\frac{8\,Bd^2}{e^2}\right)+\frac{(Cd^4+4\,Ad^2e^2)\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{8\,e^2|e|}$$

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*sqrt(-e^2*x^2 + d^2)*((2*(3*C*x + 4*B)*x - 3*(C*d^2*e^2 - 4*A*e^4)/e^4
)*x - 8*B*d^2/e^2) + 1/8*(C*d^4 + 4*A*d^2*e^2)*arcsin(e*x/d)*sgn(d)*sgn(e)/
(e^2*abs(e))
```

Mupad [**F(-1)**]

Timed out.

$$\int \left(A+Bx+Cx^2\right)\sqrt{d^2-e^2x^2}\,dx=\int \sqrt{d^2-e^2\,x^2}\,\left(C\,x^2+B\,x+A\right)\,dx$$

```
[In] int((d^2 - e^2*x^2)^(1/2)*(A + B*x + C*x^2),x)
[Out] int((d^2 - e^2*x^2)^(1/2)*(A + B*x + C*x^2), x)
```

3.4 $\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{d+ex}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 148

$$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{d+ex}\,dx = \frac{(Cd^2-e(Bd-2Ae))\sqrt{d^2-e^2x^2}}{2e^3} - \frac{C(d^2-e^2x^2)^{3/2}}{3e^3} + \frac{(Cd-Be)\,(d^2-e^2x^2)^{3/2}}{2e^3(d+ex)} + \frac{d(Cd^2-e(Bd-2Ae))\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^3}$$

```
[Out] -1/3*C*(-e^2*x^2+d^2)^(3/2)/e^3+1/2*(-B*e+C*d)*(-e^2*x^2+d^2)^(3/2)/e^3/(e*
x+d)+1/2*d*(C*d^2-e*(-2*A*e+B*d))*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3+1/2*
(C*d^2-e*(-2*A*e+B*d))*(-e^2*x^2+d^2)^(1/2)/e^3
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {1653, 809, 679, 223, 209}

$$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{d+ex}\,dx = \frac{d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)(Cd^2-e(Bd-2Ae))}{2e^3} + \frac{\sqrt{d^2-e^2x^2}(Cd^2-e(Bd-2Ae))}{2e^3} + \frac{(d^2-e^2x^2)^{3/2}(Cd-Be)}{2e^3(d+ex)} - \frac{C(d^2-e^2x^2)^{3/2}}{3e^3}$$

```
[In] Int[((A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2])/(d + e*x),x]
```

```
[Out] ((C*d^2 - e*(B*d - 2*A*e))*Sqrt[d^2 - e^2*x^2])/(2*e^3) - (C*(d^2 - e^2*x^2
)^(3/2))/(3*e^3) + ((C*d - B*e)*(d^2 - e^2*x^2)^(3/2))/(2*e^3*(d + e*x)) +
(d*(C*d^2 - e*(B*d - 2*A*e))*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e^3)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 679

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 2*p + 1))), x] - Dist[2*c*d*(p/(e^
2*(m + 2*p + 1))), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 809

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))),
 x] + Dist[(m*(d*g + e*f) + 2*e*f*(p + 1))/(e*(m + 2*p + 2)), Int[(d + e*x)
^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^2
 + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && NeQ[m, 2]
```

Rule 1653

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = -\frac{C(d^2-e^2x^2)^{3/2}}{3e^3} - \frac{\int \frac{(-3Ae^4+3e^3(Cd-Be)x)\sqrt{d^2-e^2x^2}}{d+ex}\,dx}{3e^4}$$

$$= -\frac{C(d^2-e^2x^2)^{3/2}}{3e^3} + \frac{(Cd-Be)\,(d^2-e^2x^2)^{3/2}}{2e^3(d+ex)} + \frac{(Cd^2-e(Bd-2Ae))\int \frac{\sqrt{d^2-e^2x^2}}{d+ex}\,dx}{2e^2}$$

$$= \frac{(Cd^2-e(Bd-2Ae))\sqrt{d^2-e^2x^2}}{2e^3} - \frac{C(d^2-e^2x^2)^{3/2}}{3e^3} + \frac{(Cd-Be)\,(d^2-e^2x^2)^{3/2}}{2e^3(d+ex)} + \frac{(d(Cd^2-e(Bd-2Ae)))\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{2e^2}$$

$$= \frac{(Cd^2-e(Bd-2Ae))\sqrt{d^2-e^2x^2}}{2e^3} - \frac{C(d^2-e^2x^2)^{3/2}}{3e^3} + \frac{(Cd-Be)\,(d^2-e^2x^2)^{3/2}}{2e^3(d+ex)} + \frac{(d(Cd^2-e(Bd-2Ae)))\,\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)}{2e^2}$$

$$= \frac{(Cd^2-e(Bd-2Ae))\sqrt{d^2-e^2x^2}}{2e^3} - \frac{C(d^2-e^2x^2)^{3/2}}{3e^3} + \frac{(Cd-Be)\,(d^2-e^2x^2)^{3/2}}{2e^3(d+ex)} + \frac{d(Cd^2-e(Bd-2Ae))\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^3}$$

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.78

$$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{d+ex}\,dx$$

$$= \frac{\sqrt{d^2-e^2x^2}(3e(-2Bd+2Ae+Bex)+C(4d^2-3dex+2e^2x^2))-6d(Cd^2+e(-Bd+2Ae))\arctan\Bigg(}{6e^3}$$

```
[In] Integrate[((A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2])/(d + e*x),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(3*e*(-2*B*d + 2*A*e + B*e*x) + C*(4*d^2 - 3*d*e*x + 2
*e^2*x^2)) - 6*d*(C*d^2 + e*(-(B*d) + 2*A*e))*ArcTan[(e*x)/(Sqrt[d^2] - Sqr
t[d^2 - e^2*x^2])])/(6*e^3)
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.75

method	result
risch	$\frac{(2C\,e^{2}x^{2}+3xB\,e^{2}-3Cdex+6A\,e^{2}-6Bde+4C\,d^{2})\sqrt{-e^{2}x^{2}+d^{2}}}{6e^{3}}+\frac{d(2A\,e^{2}-Bde+C\,d^{2})\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{2e^{2}\sqrt{e^{2}}}$
default	$\frac{Be\left(\frac{x\sqrt{-e^{2}x^{2}+d^{2}}}{2}+\frac{d^{2}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{2\sqrt{e^{2}}}\right)-\frac{C\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}}{3e}-Cd\left(\frac{x\sqrt{-e^{2}x^{2}+d^{2}}}{2}+\frac{d^{2}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{2\sqrt{e^{2}}}\right)}{e^{2}}+$ $(A\,e^{2}-Bde+C$

```
[In] int((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(2*C*e^2*x^2+3*B*e^2*x-3*C*d*e*x+6*A*e^2-6*B*d*e+4*C*d^2)/e^3*(-e^2*x^2
+d^2)^(1/2)+1/2*d/e^2*(2*A*e^2-B*d*e+C*d^2)/(e^2)^(1/2)*arctan((e^2)^(1/2)*
x/(-e^2*x^2+d^2)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.76

$$\int \frac{(A+Bx+Cx^{2})\sqrt{d^{2}-e^{2}x^{2}}}{d+ex}\,dx =$$
$$-\frac{6\,(Cd^{3}-Bd^{2}e+2\,Ade^{2})\arctan\left(-\frac{d-\sqrt{-e^{2}x^{2}+d^{2}}}{ex}\right)-(2\,Ce^{2}x^{2}+4\,Cd^{2}-6\,Bde+6\,Ae^{2}-3\,(Cde-Be^{2})}{6\,e^{3}}$$

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] -1/6*(6*(C*d^3 - B*d^2*e + 2*A*d*e^2)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e
*x)) - (2*C*e^2*x^2 + 4*C*d^2 - 6*B*d*e + 6*A*e^2 - 3*(C*d*e - B*e^2)*x)*sq
rt(-e^2*x^2 + d^2))/e^3
```

Sympy [F]

$$\int \frac{(A+Bx+Cx^{2})\sqrt{d^{2}-e^{2}x^{2}}}{d+ex}\,dx = \int \frac{\sqrt{-(-d+ex)\,(d+ex)}(A+Bx+Cx^{2})}{d+ex}\,dx$$

```
[In] integrate((C*x**2+B*x+A)*(-e**2*x**2+d**2)**(1/2)/(e*x+d),x)
```

```
[Out] Integral(sqrt(-(-d + e*x)*(d + e*x))*(A + B*x + C*x**2)/(d + e*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.16

$$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{d+ex}\,dx = \frac{Cd^3\arcsin\left(\frac{ex}{d}\right)}{2\,e^3} - \frac{Bd^2\arcsin\left(\frac{ex}{d}\right)}{2\,e^2} + \frac{Ad\arcsin\left(\frac{ex}{d}\right)}{e} - \frac{\sqrt{-e^2x^2+d^2}Cdx}{2\,e^2} + \frac{\sqrt{-e^2x^2+d^2}Bx}{2\,e} + \frac{\sqrt{-e^2x^2+d^2}Cd^2}{e^3} - \frac{\sqrt{-e^2x^2+d^2}Bd}{e^2} + \frac{\sqrt{-e^2x^2+d^2}A}{e} - \frac{(-e^2x^2+d^2)^{\frac{3}{2}}C}{3\,e^3}$$

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/2*C*d^3*arcsin(e*x/d)/e^3 - 1/2*B*d^2*arcsin(e*x/d)/e^2 + A*d*arcsin(e*x/
d)/e - 1/2*sqrt(-e^2*x^2 + d^2)*C*d*x/e^2 + 1/2*sqrt(-e^2*x^2 + d^2)*B*x/e
+ sqrt(-e^2*x^2 + d^2)*C*d^2/e^3 - sqrt(-e^2*x^2 + d^2)*B*d/e^2 + sqrt(-e^2
*x^2 + d^2)*A/e - 1/3*(-e^2*x^2 + d^2)^(3/2)*C/e^3
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.77

$$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{d+ex}\,dx = \frac{1}{6}\sqrt{-e^2x^2+d^2}\left(\left(\frac{2\,Cx}{e}-\frac{3\,(Cde^3-Be^4)}{e^5}\right)x+\frac{2\,(2\,Cd^2e^2-3\,Bde^3+3\,Ae^4)}{e^5}\right) + \frac{(Cd^3-Bd^2e+2\,Ade^2)\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{2\,e^2|e|}$$

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(-e^2*x^2 + d^2)*((2*C*x/e - 3*(C*d*e^3 - B*e^4)/e^5)*x + 2*(2*C*d^
2*e^2 - 3*B*d*e^3 + 3*A*e^4)/e^5) + 1/2*(C*d^3 - B*d^2*e + 2*A*d*e^2)*arcsi
n(e*x/d)*sgn(d)*sgn(e)/(e^2*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{d+ex}\,dx=\int \frac{\sqrt{d^2-e^2\,x^2}\,\left(C\,x^2+B\,x+A\right)}{d+e\,x}\,dx$$

```
[In] int(((d^2 - e^2*x^2)^(1/2)*(A + B*x + C*x^2))/(d + e*x),x)
[Out] int(((d^2 - e^2*x^2)^(1/2)*(A + B*x + C*x^2))/(d + e*x), x)
```

3.5 $\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 170

$$\begin{aligned}\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^2}\,dx = &-\frac{(5Cd^2-2e(2Bd-Ae))\sqrt{d^2-e^2x^2}}{2de^3}\\ &-\frac{(Cd^2-Bde+Ae^2)\left(d^2-e^2x^2\right)^{3/2}}{de^3(d+ex)^2}\\ &-\frac{C\left(d^2-e^2x^2\right)^{3/2}}{2e^3(d+ex)}\\ &-\frac{(5Cd^2-2e(2Bd-Ae))\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^3}\end{aligned}$$

```
[Out] -(A*e^2-B*d*e+C*d^2)*(-e^2*x^2+d^2)^(3/2)/d/e^3/(e*x+d)^2-1/2*C*(-e^2*x^2+d
^2)^(3/2)/e^3/(e*x+d)-1/2*(5*C*d^2-2*e*(-A*e+2*B*d))*arctan(e*x/(-e^2*x^2+d
^2)^(1/2))/e^3-1/2*(5*C*d^2-2*e*(-A*e+2*B*d))*(-e^2*x^2+d^2)^(1/2)/d/e^3
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used

= {1653, 807, 679, 223, 209}

$$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^2}\,dx = -\frac{\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)(5Cd^2-2e(2Bd-Ae))}{2e^3} - \frac{(d^2-e^2x^2)^{3/2}(Ae^2-Bde+Cd^2)}{de^3(d+ex)^2} - \frac{\sqrt{d^2-e^2x^2}(5Cd^2-2e(2Bd-Ae))}{2de^3} - \frac{C(d^2-e^2x^2)^{3/2}}{2e^3(d+ex)}$$

```
[In] Int[((A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2])/(d + e*x)^2,x]
```

```
[Out] -1/2*((5*C*d^2 - 2*e*(2*B*d - A*e))*Sqrt[d^2 - e^2*x^2])/(d*e^3) - ((C*d^2
- B*d*e + A*e^2)*(d^2 - e^2*x^2)^(3/2))/(d*e^3*(d + e*x)^2) - (C*(d^2 - e^2
*x^2)^(3/2))/(2*e^3*(d + e*x)) - ((5*C*d^2 - 2*e*(2*B*d - A*e))*ArcTan[(e*x
)/Sqrt[d^2 - e^2*x^2]])/(2*e^3)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 679

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 2*p + 1))), x] - Dist[2*c*d*(p/(e^
2*(m + 2*p + 1))), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 807

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
```

```
+ 1, 0]
```

Rule 1653

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{C(d^2-e^2x^2)^{3/2}}{2e^3(d+ex)} - \frac{\int \frac{(e^2(Cd^2-2Ae^2)+e^3(3Cd-2Be)x)\sqrt{d^2-e^2x^2}}{(d+ex)^2}\,dx}{2e^4} \\
&= -\frac{(Cd^2-Bde+Ae^2)\,(d^2-e^2x^2)^{3/2}}{de^3(d+ex)^2} - \frac{C(d^2-e^2x^2)^{3/2}}{2e^3(d+ex)} \\
&\quad - \frac{(-3e^5(Cd^2-2Ae^2)-2(-de^5(3Cd-2Be)-e^5(Cd^2-2Ae^2)))\int \frac{\sqrt{d^2-e^2x^2}}{d+ex}\,dx}{2de^7} \\
&= -\frac{(5Cd^2-2e(2Bd-Ae))\sqrt{d^2-e^2x^2}}{2de^3} \\
&\quad - \frac{(Cd^2-Bde+Ae^2)\,(d^2-e^2x^2)^{3/2}}{de^3(d+ex)^2} - \frac{C(d^2-e^2x^2)^{3/2}}{2e^3(d+ex)} \\
&\quad - \frac{(-3e^5(Cd^2-2Ae^2)-2(-de^5(3Cd-2Be)-e^5(Cd^2-2Ae^2)))\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{2e^7} \\
&= -\frac{(5Cd^2-2e(2Bd-Ae))\sqrt{d^2-e^2x^2}}{2de^3} \\
&\quad - \frac{(Cd^2-Bde+Ae^2)\,(d^2-e^2x^2)^{3/2}}{de^3(d+ex)^2} - \frac{C(d^2-e^2x^2)^{3/2}}{2e^3(d+ex)} \\
&\quad - \frac{(-3e^5(Cd^2-2Ae^2)-2(-de^5(3Cd-2Be)-e^5(Cd^2-2Ae^2)))\,\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)}{2e^7} \\
&= -\frac{(5Cd^2-2e(2Bd-Ae))\sqrt{d^2-e^2x^2}}{2de^3} - \frac{(Cd^2-Bde+Ae^2)\,(d^2-e^2x^2)^{3/2}}{de^3(d+ex)^2} \\
&\quad - \frac{C(d^2-e^2x^2)^{3/2}}{2e^3(d+ex)} - \frac{(5Cd^2-2e(2Bd-Ae))\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.71

$$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^2}\,dx = \frac{\frac{\sqrt{d^2-e^2x^2}(2e(3Bd-2Ae+Bex)+C(-8d^2-3dex+e^2x^2))}{d+ex}+2(5Cd^2+2e(-2Bd+Ae))\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{2e^3}$$

```
[In] Integrate[((A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2])/(d + e*x)^2,x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(2*e*(3*B*d - 2*A*e + B*e*x) + C*(-8*d^2 - 3*d*e*x +
e^2*x^2)))/(d + e*x) + 2*(5*C*d^2 + 2*e*(-2*B*d + A*e))*ArcTan[(e*x)/(Sqrt[
d^2] - Sqrt[d^2 - e^2*x^2])])/(2*e^3)
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.15

method	result
risch	$\frac{(Cxe+2Be-4Cd)\sqrt{-e^{2}x^{2}+d^{2}}}{2e^{3}}-\frac{\frac{2A\,e^{2}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{\sqrt{e^{2}}}+\frac{5C\,d^{2}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{\sqrt{e^{2}}}-\frac{4Bde\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{\sqrt{e^{2}}}+4(A\,e^{2}-Bde-}{2e^{2}}$
default	$\frac{C\left(\frac{x\sqrt{-e^{2}x^{2}+d^{2}}}{2}+\frac{d^{2}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{2\sqrt{e^{2}}}\right)}{e^{2}}+\frac{(Be-2Cd)\left(\sqrt{-\left(x+\frac{d}{e}\right)^{2}e^{2}+2de\left(x+\frac{d}{e}\right)}+\frac{de\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-\left(x+\frac{d}{e}\right)^{2}e^{2}+2de\left(x+\frac{d}{e}\right)}}\right)}{\sqrt{e^{2}}}\right)}{e^{3}}$

```
[In] int((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(C*e*x+2*B*e-4*C*d)/e^3*(-e^2*x^2+d^2)^(1/2)-1/2/e^2*(2*A*e^2/(e^2)^(1/
2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))+5*C*d^2/(e^2)^(1/2)*arctan((e
^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))-4*B*d*e/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/
(-e^2*x^2+d^2)^(1/2))+4*(A*e^2-B*d*e+C*d^2)/e^2/(x+d/e)*(-(x+d/e)^2*e^2+2*d
*e*(x+d/e))^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.12

$$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^2}\,dx =$$

$$-\frac{8\,Cd^3 - 6\,Bd^2e + 4\,Ade^2 + 2\,(4\,Cd^2e - 3\,Bde^2 + 2\,Ae^3)x - 2\,(5\,Cd^3 - 4\,Bd^2e + 2\,Ade^2 + (5\,Cd^2e - 4}{2\,(}$$

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^2,x, algorithm="fricas
")
```

```
[Out] -1/2*(8*C*d^3 - 6*B*d^2*e + 4*A*d*e^2 + 2*(4*C*d^2*e - 3*B*d*e^2 + 2*A*e^3)
*x - 2*(5*C*d^3 - 4*B*d^2*e + 2*A*d*e^2 + (5*C*d^2*e - 4*B*d*e^2 + 2*A*e^3)
*x)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (C*e^2*x^2 - 8*C*d^2 + 6*B*
d*e - 4*A*e^2 - (3*C*d*e - 2*B*e^2)*x)*sqrt(-e^2*x^2 + d^2))/(e^4*x + d*e^3
)
```

Sympy [F]

$$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^2}\,dx = \int \frac{\sqrt{-(-d+ex)(d+ex)}(A+Bx+Cx^2)}{(d+ex)^2}\,dx$$

```
[In] integrate((C*x**2+B*x+A)*(-e**2*x**2+d**2)**(1/2)/(e*x+d)**2,x)
```

```
[Out] Integral(sqrt(-(-d + e*x)*(d + e*x))*(A + B*x + C*x**2)/(d + e*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.16

$$\begin{aligned}\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^2}\,dx = &-\frac{2\sqrt{-e^2x^2+d^2}Cd^2}{e^4x+de^3} + \frac{2\sqrt{-e^2x^2+d^2}Bd}{e^3x+de^2} \\ &- \frac{5\,Cd^2\arcsin\left(\frac{ex}{d}\right)}{2\,e^3} + \frac{2\,Bd\arcsin\left(\frac{ex}{d}\right)}{e^2} - \frac{A\arcsin\left(\frac{ex}{d}\right)}{e} \\ &- \frac{2\sqrt{-e^2x^2+d^2}A}{e^2x+de} + \frac{\sqrt{-e^2x^2+d^2}Cx}{2\,e^2} \\ &- \frac{2\sqrt{-e^2x^2+d^2}Cd}{e^3} + \frac{\sqrt{-e^2x^2+d^2}B}{e^2}\end{aligned}$$

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^2,x, algorithm="maxima
")
```

```
[Out] -2*sqrt(-e^2*x^2 + d^2)*C*d^2/(e^4*x + d*e^3) + 2*sqrt(-e^2*x^2 + d^2)*B*d/
(e^3*x + d*e^2) - 5/2*C*d^2*arcsin(e*x/d)/e^3 + 2*B*d*arcsin(e*x/d)/e^2 - A
*arcsin(e*x/d)/e - 2*sqrt(-e^2*x^2 + d^2)*A/(e^2*x + d*e) + 1/2*sqrt(-e^2*x
^2 + d^2)*C*x/e^2 - 2*sqrt(-e^2*x^2 + d^2)*C*d/e^3 + sqrt(-e^2*x^2 + d^2)*B
/e^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 325 vs. $2(156) = 312$.

Time = 0.31 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.91

$$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^2}\,dx =$$

$$-\frac{\left(8\,Cd^3e^3\sqrt{\frac{2\,d}{ex+d}-1}\operatorname{sgn}\left(\frac{1}{ex+d}\right)\operatorname{sgn}(e)-8\,Bd^2e^4\sqrt{\frac{2\,d}{ex+d}-1}\operatorname{sgn}\left(\frac{1}{ex+d}\right)\operatorname{sgn}(e)+8\,Ade^5\sqrt{\frac{2\,d}{ex+d}-1}\operatorname{sgn}\left(\frac{1}{ex+\cdots}\right.\right.}{\cdots}$$

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] -1/4*(8*C*d^3*e^3*sqrt(2*d/(e*x + d) - 1)*sgn(1/(e*x + d))*sgn(e) - 8*B*d^2
*e^4*sqrt(2*d/(e*x + d) - 1)*sgn(1/(e*x + d))*sgn(e) + 8*A*d*e^5*sqrt(2*d/(
e*x + d) - 1)*sgn(1/(e*x + d))*sgn(e) - 4*(5*C*d^3*e^3*sgn(1/(e*x + d))*sgn
(e) - 4*B*d^2*e^4*sgn(1/(e*x + d))*sgn(e) + 2*A*d*e^5*sgn(1/(e*x + d))*sgn(
e))*arctan(sqrt(2*d/(e*x + d) - 1)) + (5*C*d^3*e^3*(2*d/(e*x + d) - 1)^(3/2
)*sgn(1/(e*x + d))*sgn(e) - 2*B*d^2*e^4*(2*d/(e*x + d) - 1)^(3/2)*sgn(1/(e*
x + d))*sgn(e) + 3*C*d^3*e^3*sqrt(2*d/(e*x + d) - 1)*sgn(1/(e*x + d))*sgn(e
) - 2*B*d^2*e^4*sqrt(2*d/(e*x + d) - 1)*sgn(1/(e*x + d))*sgn(e))*(e*x + d)^
2/d^2)*abs(e)/(d*e^7)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^2}\,dx = \int \frac{\sqrt{d^2-e^2\,x^2}\,\left(C\,x^2+B\,x+A\right)}{{\left(d+e\,x\right)}^2}\,dx$$

```
[In] int(((d^2 - e^2*x^2)^(1/2)*(A + B*x + C*x^2))/(d + e*x)^2,x)
```

```
[Out] int(((d^2 - e^2*x^2)^(1/2)*(A + B*x + C*x^2))/(d + e*x)^2, x)
```

3.6 $\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^3}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 149

$$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^3}\,dx = \frac{2(3Cd-Be)\sqrt{d^2-e^2x^2}}{e^3(d+ex)} - \frac{(Cd^2-Bde+Ae^2)\,(d^2-e^2x^2)^{3/2}}{3de^3(d+ex)^3} - \frac{C(d^2-e^2x^2)^{3/2}}{e^3(d+ex)^2} + \frac{(3Cd-Be)\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}$$

```
[Out] -1/3*(A*e^2-B*d*e+C*d^2)*(-e^2*x^2+d^2)^(3/2)/d/e^3/(e*x+d)^3-C*(-e^2*x^2+d
^2)^(3/2)/e^3/(e*x+d)^2+(-B*e+3*C*d)*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3+2
*(-B*e+3*C*d)*(-e^2*x^2+d^2)^(1/2)/e^3/(e*x+d)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {1653, 807, 677, 223, 209}

$$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^3}\,dx = -\frac{(d^2-e^2x^2)^{3/2}\,(Ae^2-Bde+Cd^2)}{3de^3(d+ex)^3} + \frac{\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)(3Cd-Be)}{e^3} + \frac{2\sqrt{d^2-e^2x^2}(3Cd-Be)}{e^3(d+ex)} - \frac{C(d^2-e^2x^2)^{3/2}}{e^3(d+ex)^2}$$

```
[In] Int[((A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2])/(d + e*x)^3,x]
```

```
[Out] (2*(3*C*d - B*e)*Sqrt[d^2 - e^2*x^2])/(e^3*(d + e*x)) - ((C*d^2 - B*d*e + A
*e^2)*(d^2 - e^2*x^2)^(3/2))/(3*d*e^3*(d + e*x)^3) - (C*(d^2 - e^2*x^2)^(3/
2))/(e^3*(d + e*x)^2) + ((3*C*d - B*e)*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e
^3
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 677

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + p + 1))), x] - Dist[c*(p/(e^2*(m +
 p + 1))), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
 2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 807

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 1653

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = -\frac{C\left(d^2-e^2x^2\right)^{3/2}}{e^3(d+ex)^2} - \frac{\int \frac{\left(e^2\left(2Cd^2-Ae^2\right)+e^3(3Cd-Be)x\right)\sqrt{d^2-e^2x^2}}{(d+ex)^3}\,dx}{e^4}$$

$$= -\frac{\left(Cd^2-Bde+Ae^2\right)\left(d^2-e^2x^2\right)^{3/2}}{3de^3(d+ex)^3} - \frac{C\left(d^2-e^2x^2\right)^{3/2}}{e^3(d+ex)^2} - \frac{(3Cd-Be)\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^2}\,dx}{e^2}$$

$$= \frac{2(3Cd-Be)\sqrt{d^2-e^2x^2}}{e^3(d+ex)} - \frac{\left(Cd^2-Bde+Ae^2\right)\left(d^2-e^2x^2\right)^{3/2}}{3de^3(d+ex)^3} - \frac{C\left(d^2-e^2x^2\right)^{3/2}}{e^3(d+ex)^2} + \frac{(3Cd-Be)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e^2}$$

$$= \frac{2(3Cd-Be)\sqrt{d^2-e^2x^2}}{e^3(d+ex)} - \frac{\left(Cd^2-Bde+Ae^2\right)\left(d^2-e^2x^2\right)^{3/2}}{3de^3(d+ex)^3} - \frac{C\left(d^2-e^2x^2\right)^{3/2}}{e^3(d+ex)^2} + \frac{(3Cd-Be)\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)}{e^2}$$

$$= \frac{2(3Cd-Be)\sqrt{d^2-e^2x^2}}{e^3(d+ex)} - \frac{\left(Cd^2-Bde+Ae^2\right)\left(d^2-e^2x^2\right)^{3/2}}{3de^3(d+ex)^3} - \frac{C\left(d^2-e^2x^2\right)^{3/2}}{e^3(d+ex)^2} + \frac{(3Cd-Be)\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}$$

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.84

$$\int \frac{\left(A+Bx+Cx^2\right)\sqrt{d^2-e^2x^2}}{(d+ex)^3}\,dx = \frac{\frac{\sqrt{d^2-e^2x^2}\left(Cd\left(14d^2+19dex+3e^2x^2\right)+e(Ae(-d+ex)-Bd(5d+7ex))\right)}{d(d+ex)^2} + 6(-3Cd+Be)\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{3e^3}$$

```
[In] Integrate[((A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2])/(d + e*x)^3,x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(C*d*(14*d^2 + 19*d*e*x + 3*e^2*x^2) + e*(A*e*(-d + e
*x) - B*d*(5*d + 7*e*x))))/(d*(d + e*x)^2) + 6*(-3*C*d + B*e)*ArcTan[(e*x)/
(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/(3*e^3)
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.60

method	result
risch	$\frac{C\sqrt{-e^2x^2+d^2}}{e^3}-\frac{\frac{(Be-3Cd)\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{e\sqrt{e^2}}-\frac{\left(A\,e^2-3Bde+5C\,d^2\right)\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{e^3d\left(x+\frac{d}{e}\right)}-\frac{2d\left(A\,e^2-Bde+C\,d^2\right)\left(-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^2}}{3de(}\right.}{}}{e}$
default	$\frac{C\left(\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}+\frac{de\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}\right)}{\sqrt{e^2}}\right)}{e^3}+\frac{(Be-2Cd)\left(-\frac{\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{de\left(x+\frac{d}{e}\right)^2}-\frac{e\left(\sqrt{-\left(x+\right.}\right.}{}\right.}{}$

```
[In] int((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] C*(-e^2*x^2+d^2)^(1/2)/e^3-1/e*((B*e-3*C*d)/e/(e^2)^(1/2)*arctan((e^2)^(1/2
)*x/(-e^2*x^2+d^2)^(1/2))-1/e^3*(A*e^2-3*B*d*e+5*C*d^2)/d/(x+d/e)*(-(x+d/e)
^2*e^2+2*d*e*(x+d/e))^(1/2)-2*d*(A*e^2-B*d*e+C*d^2)/e^3*(-1/3/d/e/(x+d/e)^2
*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-1/3/d^2/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e
*(x+d/e))^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.73

$$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^3}\,dx$$

$$=\frac{14\,Cd^4-5\,Bd^3e-Ad^2e^2+(14\,Cd^2e^2-5\,Bde^3-Ae^4)x^2+2\,(14\,Cd^3e-5\,Bd^2e^2-Ade^3)x-6\,(3\,Cd^4-}{}$$

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^3,x, algorithm="fricas
")
```

```
[Out] 1/3*(14*C*d^4 - 5*B*d^3*e - A*d^2*e^2 + (14*C*d^2*e^2 - 5*B*d*e^3 - A*e^4)*
x^2 + 2*(14*C*d^3*e - 5*B*d^2*e^2 - A*d*e^3)*x - 6*(3*C*d^4 - B*d^3*e + (3*
C*d^2*e^2 - B*d*e^3)*x^2 + 2*(3*C*d^3*e - B*d^2*e^2)*x)*arctan(-(d - sqrt(-
e^2*x^2 + d^2))/(e*x)) + (3*C*d*e^2*x^2 + 14*C*d^3 - 5*B*d^2*e - A*d*e^2 +
```

```
(19*C*d^2*e - 7*B*d*e^2 + A*e^3)*x)*sqrt(-e^2*x^2 + d^2))/(d*e^5*x^2 + 2*d^
2*e^4*x + d^3*e^3)
```

Sympy [F]

$$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^3}\,dx = \int \frac{\sqrt{-(-d+ex)(d+ex)}(A+Bx+Cx^2)}{(d+ex)^3}\,dx$$

```
[In] integrate((C*x**2+B*x+A)*(-e**2*x**2+d**2)**(1/2)/(e*x+d)**3,x)
[Out] Integral(sqrt(-(-d + e*x)*(d + e*x))*(A + B*x + C*x**2)/(d + e*x)**3, x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^3}\,dx = \text{Timed out}$$

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^3,x, algorithm="maxima
")
[Out] Timed out
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.84

$$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^3}\,dx = \frac{(3\,Cd-Be)\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{e^2|e|} + \frac{\sqrt{-e^2x^2+d^2}C}{e^3} - \frac{2\left(11\,Cd^2-5\,Bde-Ae^2+\frac{24\left(de+\sqrt{-e^2x^2+d^2}|e|\right)Cd^2}{e^2x}-\frac{12\left(de+\sqrt{-e^2x^2+d^2}|e|\right)Bd}{ex}+\frac{9\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2Cd^2}{e^4x^2}-3\right.}{3\,de^2\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}+1\right)^3|e|}$$

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^3,x, algorithm="giac")
[Out] (3*C*d - B*e)*arcsin(e*x/d)*sgn(d)*sgn(e)/(e^2*abs(e)) + sqrt(-e^2*x^2 + d^
2)*C/e^3 - 2/3*(11*C*d^2 - 5*B*d*e - A*e^2 + 24*(d*e + sqrt(-e^2*x^2 + d^2)
*abs(e))*C*d^2/(e^2*x) - 12*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*B*d/(e*x) +
 9*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*C*d^2/(e^4*x^2) - 3*(d*e + sqrt(-e
^2*x^2 + d^2)*abs(e))^2*B*d/(e^3*x^2) - 3*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e
))^2*A/(e^2*x^2))/(d*e^2*((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) + 1)^
3*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^3}\,dx=\int \frac{\sqrt{d^2-e^2\,x^2}\,\left(C\,x^2+B\,x+A\right)}{{\left(d+e\,x\right)}^3}\,dx$$

```
[In] int(((d^2 - e^2*x^2)^(1/2)*(A + B*x + C*x^2))/(d + e*x)^3,x)
[Out] int(((d^2 - e^2*x^2)^(1/2)*(A + B*x + C*x^2))/(d + e*x)^3, x)
```

3.7 $\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 196

$$\begin{aligned}\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = &-\frac{2C\sqrt{d^2-e^2x^2}}{e^3(d+ex)}-\frac{(Cd^2-Bde+Ae^2)(d^2-e^2x^2)^{3/2}}{5de^3(d+ex)^4}\\&+\frac{(2Cd-Be)(d^2-e^2x^2)^{3/2}}{3de^3(d+ex)^3}\\&-\frac{(Cd^2-Bde+Ae^2)(d^2-e^2x^2)^{3/2}}{15d^2e^3(d+ex)^3}\\&-\frac{C\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}\end{aligned}$$

```
[Out] -1/5*(A*e^2-B*d*e+C*d^2)*(-e^2*x^2+d^2)^(3/2)/d/e^3/(e*x+d)^4+1/3*(-B*e+2*C
*d)*(-e^2*x^2+d^2)^(3/2)/d/e^3/(e*x+d)^3-1/15*(A*e^2-B*d*e+C*d^2)*(-e^2*x^2
+d^2)^(3/2)/d^2/e^3/(e*x+d)^3-C*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3-2*C*(-
e^2*x^2+d^2)^(1/2)/e^3/(e*x+d)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used

$= \{1651, 673, 665, 677, 223, 209\}$

$$\begin{aligned}\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = &-\frac{(d^2-e^2x^2)^{3/2}(Ae^2-Bde+Cd^2)}{15d^2e^3(d+ex)^3}\\ &-\frac{(d^2-e^2x^2)^{3/2}(Ae^2-Bde+Cd^2)}{5de^3(d+ex)^4}\\ &-\frac{C\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}\\ &+\frac{(d^2-e^2x^2)^{3/2}(2Cd-Be)}{3de^3(d+ex)^3}-\frac{2C\sqrt{d^2-e^2x^2}}{e^3(d+ex)}\end{aligned}$$

```
[In] Int[((A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2])/(d + e*x)^4,x]
```

```
[Out] (-2*C*Sqrt[d^2 - e^2*x^2])/(e^3*(d + e*x)) - ((C*d^2 - B*d*e + A*e^2)*(d^2
- e^2*x^2)^(3/2))/(5*d*e^3*(d + e*x)^4) + ((2*C*d - B*e)*(d^2 - e^2*x^2)^(3
/2))/(3*d*e^3*(d + e*x)^3) - ((C*d^2 - B*d*e + A*e^2)*(d^2 - e^2*x^2)^(3/2)
)/(15*d^2*e^3*(d + e*x)^3) - (C*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^3
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 673

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-e)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m + p + 1))), x] + Dist[Simpl
ify[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x]
, x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ
[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 677

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + p + 1))), x] - Dist[c*(p/(e^2*(m +
 p + 1))), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
 2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 Int[ExpandIntegrand[(a + c*x^2)^p, (d + e*x)^m*Pq, x], x] /; FreeQ[{a, c,
d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && EqQ[m + Expon[Pq, x]
+ 2*p + 1, 0] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{\left(Cd^2-Bde+Ae^2\right)\sqrt{d^2-e^2x^2}}{e^2(d+ex)^4}+\frac{(-2Cd+Be)\sqrt{d^2-e^2x^2}}{e^2(d+ex)^3}+\frac{C\sqrt{d^2-e^2x^2}}{e^2(d+ex)^2}\right)dx\\
&= \frac{C\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^2}\,dx}{e^2}-\frac{(2Cd-Be)\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^3}\,dx}{e^2}+\frac{\left(Cd^2-Bde+Ae^2\right)\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx}{e^2}\\
&= -\frac{2C\sqrt{d^2-e^2x^2}}{e^3(d+ex)}-\frac{\left(Cd^2-Bde+Ae^2\right)\left(d^2-e^2x^2\right)^{3/2}}{5de^3(d+ex)^4}+\frac{(2Cd-Be)\left(d^2-e^2x^2\right)^{3/2}}{3de^3(d+ex)^3}\\
&\quad -\frac{C\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e^2}+\frac{\left(Cd^2-Bde+Ae^2\right)\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^3}\,dx}{5de^2}\\
&= -\frac{2C\sqrt{d^2-e^2x^2}}{e^3(d+ex)}-\frac{\left(Cd^2-Bde+Ae^2\right)\left(d^2-e^2x^2\right)^{3/2}}{5de^3(d+ex)^4}+\frac{(2Cd-Be)\left(d^2-e^2x^2\right)^{3/2}}{3de^3(d+ex)^3}\\
&\quad -\frac{\left(Cd^2-Bde+Ae^2\right)\left(d^2-e^2x^2\right)^{3/2}}{15d^2e^3(d+ex)^3}-\frac{C\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx,x,\frac{x}{\sqrt{d^2-e^2x^2}}\right)}{e^2}\\
&= -\frac{2C\sqrt{d^2-e^2x^2}}{e^3(d+ex)}-\frac{\left(Cd^2-Bde+Ae^2\right)\left(d^2-e^2x^2\right)^{3/2}}{5de^3(d+ex)^4}+\frac{(2Cd-Be)\left(d^2-e^2x^2\right)^{3/2}}{3de^3(d+ex)^3}\\
&\quad -\frac{\left(Cd^2-Bde+Ae^2\right)\left(d^2-e^2x^2\right)^{3/2}}{15d^2e^3(d+ex)^3}-\frac{C\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.90 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.64

$$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = \frac{-\frac{\sqrt{d^2-e^2x^2}(3Cd^2(8d^2+19dex+13e^2x^2)+e(d-ex)(Ae(4d+ex)+Bd(d+4ex)))}{d^2(d+ex)^3}+30C\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{15e^3}$$

```
[In] Integrate[((A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2])/(d + e*x)^4,x]
```

```
[Out] (-((Sqrt[d^2 - e^2*x^2]*(3*C*d^2*(8*d^2 + 19*d*e*x + 13*e^2*x^2) + e*(d - e
*x)*(A*e*(4*d + e*x) + B*d*(d + 4*e*x))))/(d^2*(d + e*x)^3)) + 30*C*ArcTan[
(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/(15*e^3)
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.49

method	result
default	$\frac{C\left(-\frac{\left(-\left(x+\frac{d}{e}\right)^2 e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{de\left(x+\frac{d}{e}\right)^2}-\frac{e\left(\sqrt{-\left(x+\frac{d}{e}\right)^2 e^2+2de\left(x+\frac{d}{e}\right)}+\frac{de\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-\left(x+\frac{d}{e}\right)^2 e^2+2de\left(x+\frac{d}{e}\right)}}\right)}{\sqrt{e^2}}\right)}{d}\right)}{e^4}-\frac{(Be-2Cd)(-(x+\frac{d}{e})}{3e^6 d(x}$

```
[In] int((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

```
[Out] C/e^4*(-1/d/e/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)-e/d*((-(x+d/e)
^2*e^2+2*d*e*(x+d/e))^(1/2)+d*e/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(x+d/e)^
2*e^2+2*d*e*(x+d/e))^(1/2))))-1/3*(B*e-2*C*d)/e^6/d/(x+d/e)^3*(-(x+d/e)^2*e
^2+2*d*e*(x+d/e))^(3/2)+(A*e^2-B*d*e+C*d^2)/e^6*(-1/5/d/e/(x+d/e)^4*(-(x+d/
e)^2*e^2+2*d*e*(x+d/e))^(3/2)-1/15/d^2/(x+d/e)^3*(-(x+d/e)^2*e^2+2*d*e*(x+d
/e))^(3/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.55

$$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx =$$

$$-\frac{24\,Cd^5+Bd^4e+4\,Ad^3e^2+(24\,Cd^2e^3+Bde^4+4\,Ae^5)x^3+3\,(24\,Cd^3e^2+Bd^2e^3+4\,Ade^4)x^2+3\,(24}{}$$

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="fricas
")
```

```
[Out] -1/15*(24*C*d^5 + B*d^4*e + 4*A*d^3*e^2 + (24*C*d^2*e^3 + B*d*e^4 + 4*A*e^5
)*x^3 + 3*(24*C*d^3*e^2 + B*d^2*e^3 + 4*A*d*e^4)*x^2 + 3*(24*C*d^4*e + B*d^
3*e^2 + 4*A*d^2*e^3)*x - 30*(C*d^2*e^3*x^3 + 3*C*d^3*e^2*x^2 + 3*C*d^4*e*x
+ C*d^5)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (24*C*d^4 + B*d^3*e +
4*A*d^2*e^2 + (39*C*d^2*e^2 - 4*B*d*e^3 - A*e^4)*x^2 + 3*(19*C*d^3*e + B*d^
2*e^2 - A*d*e^3)*x)*sqrt(-e^2*x^2 + d^2))/(d^2*e^6*x^3 + 3*d^3*e^5*x^2 + 3*
d^4*e^4*x + d^5*e^3)
```

Sympy [F]

$$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = \int \frac{\sqrt{-(-d+ex)\,(d+ex)}(A+Bx+Cx^2)}{(d+ex)^4}\,dx$$

```
[In] integrate((C*x**2+B*x+A)*(-e**2*x**2+d**2)**(1/2)/(e*x+d)**4,x)
```

```
[Out] Integral(sqrt(-(-d + e*x)*(d + e*x))*(A + B*x + C*x**2)/(d + e*x)**4, x)
```

Maxima [F]

$$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = \int \frac{\sqrt{-e^2x^2+d^2}(Cx^2+Bx+A)}{(ex+d)^4}\,dx$$

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="maxima
")
```

```
[Out] integrate(sqrt(-e^2*x^2 + d^2)*(C*x^2 + B*x + A)/(e*x + d)^4, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 441 vs. 2(180) = 360.

Time = 0.30 (sec) , antiderivative size = 441, normalized size of antiderivative = 2.25

$$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = -\frac{C\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{e^2|e|}$$
$$+\frac{2\left(24\,Cd^2+Bde+4\,Ae^2+\frac{5\left(de+\sqrt{-e^2x^2+d^2}|e|\right)A}{x}+\frac{105\left(de+\sqrt{-e^2x^2+d^2}|e|\right)Cd^2}{e^2x}+\frac{5\left(de+\sqrt{-e^2x^2+d^2}|e|\right)Bd}{ex}+\frac{165\left(de\right.}{}\right.}{}$$

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] -C*arcsin(e*x/d)*sgn(d)*sgn(e)/(e^2*abs(e)) + 2/15*(24*C*d^2 + B*d*e + 4*A*
e^2 + 5*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*A/x + 105*(d*e + sqrt(-e^2*x^2
+ d^2)*abs(e))*C*d^2/(e^2*x) + 5*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*B*d/(e
*x) + 165*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*C*d^2/(e^4*x^2) - 5*(d*e +
sqrt(-e^2*x^2 + d^2)*abs(e))^2*B*d/(e^3*x^2) + 25*(d*e + sqrt(-e^2*x^2 + d^
2)*abs(e))^2*A/(e^2*x^2) + 75*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3*C*d^2/(
e^6*x^3) + 15*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3*B*d/(e^5*x^3) + 15*(d*e
 + sqrt(-e^2*x^2 + d^2)*abs(e))^3*A/(e^4*x^3) + 15*(d*e + sqrt(-e^2*x^2 + d
^2)*abs(e))^4*C*d^2/(e^8*x^4) + 15*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4*A/
(e^6*x^4))/(d^2*e^2*((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) + 1)^5*abs
(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = \int \frac{\sqrt{d^2-e^2\,x^2}\,\left(C\,x^2+B\,x+A\right)}{{\left(d+e\,x\right)}^4}\,dx$$

```
[In] int(((d^2 - e^2*x^2)^(1/2)*(A + B*x + C*x^2))/(d + e*x)^4,x)
```

```
[Out] int(((d^2 - e^2*x^2)^(1/2)*(A + B*x + C*x^2))/(d + e*x)^4, x)
```

3.8 $\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^5}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 180

$$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^5}\,dx = -\frac{(Cd^2-Bde+Ae^2)\left(d^2-e^2x^2\right)^{3/2}}{7de^3(d+ex)^5}+\frac{C\left(d^2-e^2x^2\right)^{3/2}}{e^3(d+ex)^4} - \frac{(23Cd^2+e(5Bd+2Ae))\left(d^2-e^2x^2\right)^{3/2}}{35d^2e^3(d+ex)^4} - \frac{(23Cd^2+e(5Bd+2Ae))\left(d^2-e^2x^2\right)^{3/2}}{105d^3e^3(d+ex)^3}$$

```
[Out] -1/7*(A*e^2-B*d*e+C*d^2)*(-e^2*x^2+d^2)^(3/2)/d/e^3/(e*x+d)^5+C*(-e^2*x^2+d
^2)^(3/2)/e^3/(e*x+d)^4-1/35*(23*C*d^2+e*(2*A*e+5*B*d))*(-e^2*x^2+d^2)^(3/2
)/d^2/e^3/(e*x+d)^4-1/105*(23*C*d^2+e*(2*A*e+5*B*d))*(-e^2*x^2+d^2)^(3/2)/d
^3/e^3/(e*x+d)^3
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1653, 807, 673, 665}

$$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^5}\,dx = -\frac{(d^2-e^2x^2)^{3/2}\,(e(2Ae+5Bd)+23Cd^2)}{35d^2e^3(d+ex)^4} - \frac{(d^2-e^2x^2)^{3/2}\,(Ae^2-Bde+Cd^2)}{7de^3(d+ex)^5} - \frac{(d^2-e^2x^2)^{3/2}\,(e(2Ae+5Bd)+23Cd^2)}{105d^3e^3(d+ex)^3} + \frac{C(d^2-e^2x^2)^{3/2}}{e^3(d+ex)^4}$$

```
[In] Int[((A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2])/(d + e*x)^5,x]
```

```
[Out] -1/7*((C*d^2 - B*d*e + A*e^2)*(d^2 - e^2*x^2)^(3/2))/(d*e^3*(d + e*x)^5) +
(C*(d^2 - e^2*x^2)^(3/2))/(e^3*(d + e*x)^4) - ((23*C*d^2 + e*(5*B*d + 2*A*e
))*(d^2 - e^2*x^2)^(3/2))/(35*d^2*e^3*(d + e*x)^4) - ((23*C*d^2 + e*(5*B*d
+ 2*A*e))*(d^2 - e^2*x^2)^(3/2))/(105*d^3*e^3*(d + e*x)^3)
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 673

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-e)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m + p + 1))), x] + Dist[Simpl
ify[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x]
, x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ
[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 807

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
```

```
+ 1, 0]
```

Rule 1653

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{C(d^2-e^2x^2)^{3/2}}{e^3(d+ex)^4} + \frac{\int \frac{(e^2(4Cd^2+Ae^2)+e^3(3Cd+Be)x)\sqrt{d^2-e^2x^2}}{(d+ex)^5}\,dx}{e^4} \\
&= -\frac{(Cd^2-Bde+Ae^2)(d^2-e^2x^2)^{3/2}}{7de^3(d+ex)^5} + \frac{C(d^2-e^2x^2)^{3/2}}{e^3(d+ex)^4} \\
&\quad + \frac{(23Cd^2+e(5Bd+2Ae))\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx}{7de^2} \\
&= -\frac{(Cd^2-Bde+Ae^2)(d^2-e^2x^2)^{3/2}}{7de^3(d+ex)^5} + \frac{C(d^2-e^2x^2)^{3/2}}{e^3(d+ex)^4} \\
&\quad - \frac{(23Cd^2+e(5Bd+2Ae))(d^2-e^2x^2)^{3/2}}{35d^2e^3(d+ex)^4} \\
&\quad + \frac{(23Cd^2+e(5Bd+2Ae))\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^3}\,dx}{35d^2e^2} \\
&= -\frac{(Cd^2-Bde+Ae^2)(d^2-e^2x^2)^{3/2}}{7de^3(d+ex)^5} + \frac{C(d^2-e^2x^2)^{3/2}}{e^3(d+ex)^4} \\
&\quad - \frac{(23Cd^2+e(5Bd+2Ae))(d^2-e^2x^2)^{3/2}}{35d^2e^3(d+ex)^4} \\
&\quad - \frac{(23Cd^2+e(5Bd+2Ae))(d^2-e^2x^2)^{3/2}}{105d^3e^3(d+ex)^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.61

$$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^5}\,dx = -\frac{(d-ex)\sqrt{d^2-e^2x^2}(Cd^2(2d^2+10dex+23e^2x^2)+e(5Bd(d^2+5dex+e^2x^2)+Ae(23d^2+10dex+2e^2x}{105d^3e^3(d+ex)^4}$$

```
[In] Integrate[((A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2])/(d + e*x)^5,x]
```

```
[Out] -1/105*((d - e*x)*Sqrt[d^2 - e^2*x^2]*(C*d^2*(2*d^2 + 10*d*e*x + 23*e^2*x^2
) + e*(5*B*d*(d^2 + 5*d*e*x + e^2*x^2) + A*e*(23*d^2 + 10*d*e*x + 2*e^2*x^2
))))/(d^3*e^3*(d + e*x)^4)
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.64

method	result
gosper	$-\frac{(-ex+d)(2A\,e^4x^2+5x^2dB\,e^3+23C\,d^2e^2x^2+10Ad\,e^3x+25xB\,d^2e^2+10C\,d^3xe+23A\,d^2e^2+5B\,d^3e+2C\,d^4)\sqrt{-e^2x^2+d^2}}{105(ex+d)^4d^3e^3}$
trager	$-\frac{(-2A\,e^5x^3-5x^3dB\,e^4-23C\,d^2e^3x^3-8Ad\,e^4x^2-20x^2d^2B\,e^3+13C\,d^3e^2x^2-13A\,d^2e^3x+20x\,d^3B\,e^2+8C\,d^4ex+23A\,d^3e^2+5B\,d^4e+2}{105d^3(ex+d)^4e^3}$
default	$-\frac{C\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{3e^6d\left(x+\frac{d}{e}\right)^3}+\frac{(Be-2Cd)\left(-\frac{\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{5de\left(x+\frac{d}{e}\right)^4}-\frac{\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{15d^2\left(x+\frac{d}{e}\right)^3}\right)}{e^6}+\frac{(A\,e^2-Bde+}{}$

```
[In] int((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/105*(-e*x+d)*(2*A*e^4*x^2+5*B*d*e^3*x^2+23*C*d^2*e^2*x^2+10*A*d*e^3*x+25
*B*d^2*e^2*x+10*C*d^3*e*x+23*A*d^2*e^2+5*B*d^3*e+2*C*d^4)*(-e^2*x^2+d^2)^(1
/2)/(e*x+d)^4/d^3/e^3
```

Fricas [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.78

$$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^5}\,dx =$$

$$-\frac{2\,Cd^6+5\,Bd^5e+23\,Ad^4e^2+(2\,Cd^2e^4+5\,Bde^5+23\,Ae^6)x^4+4\,(2\,Cd^3e^3+5\,Bd^2e^4+23\,Ade^5)x^3+6}{}$$

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^5,x, algorithm="fricas
")
```

```
[Out] -1/105*(2*C*d^6 + 5*B*d^5*e + 23*A*d^4*e^2 + (2*C*d^2*e^4 + 5*B*d*e^5 + 23*
A*e^6)*x^4 + 4*(2*C*d^3*e^3 + 5*B*d^2*e^4 + 23*A*d*e^5)*x^3 + 6*(2*C*d^4*e^
2 + 5*B*d^3*e^3 + 23*A*d^2*e^4)*x^2 + 4*(2*C*d^5*e + 5*B*d^4*e^2 + 23*A*d^3
*e^3)*x + (2*C*d^5 + 5*B*d^4*e + 23*A*d^3*e^2 - (23*C*d^2*e^3 + 5*B*d*e^4 +
 2*A*e^5)*x^3 + (13*C*d^3*e^2 - 20*B*d^2*e^3 - 8*A*d*e^4)*x^2 + (8*C*d^4*e
+ 20*B*d^3*e^2 - 13*A*d^2*e^3)*x)*sqrt(-e^2*x^2 + d^2))/(d^3*e^7*x^4 + 4*d^
4*e^6*x^3 + 6*d^5*e^5*x^2 + 4*d^6*e^4*x + d^7*e^3)
```

Sympy [F]

$$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^5}\,dx = \int \frac{\sqrt{-(-d+ex)(d+ex)}(A+Bx+Cx^2)}{(d+ex)^5}\,dx$$

```
[In] integrate((C*x**2+B*x+A)*(-e**2*x**2+d**2)**(1/2)/(e*x+d)**5,x)
```

```
[Out] Integral(sqrt(-(-d + e*x)*(d + e*x))*(A + B*x + C*x**2)/(d + e*x)**5, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 945 vs. 2(166) = 332.

Time = 0.20 (sec) , antiderivative size = 945, normalized size of antiderivative = 5.25

$$
\begin{aligned}
\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^5}\,dx = &-\frac{2\sqrt{-e^2x^2+d^2}Cd^2}{7\,(e^7x^4+4\,de^6x^3+6\,d^2e^5x^2+4\,d^3e^4x+d^4e^3)} \\
&+\frac{\sqrt{-e^2x^2+d^2}Cd^2}{35\,(de^6x^3+3\,d^2e^5x^2+3\,d^3e^4x+d^4e^3)} \\
&+\frac{2\sqrt{-e^2x^2+d^2}Cd^2}{105\,(d^2e^5x^2+2\,d^3e^4x+d^4e^3)}+\frac{2\sqrt{-e^2x^2+d^2}Cd^2}{105\,(d^3e^4x+d^4e^3)} \\
&+\frac{2\sqrt{-e^2x^2+d^2}Bd}{7\,(e^6x^4+4\,de^5x^3+6\,d^2e^4x^2+4\,d^3e^3x+d^4e^2)} \\
&-\frac{\sqrt{-e^2x^2+d^2}Bd}{35\,(de^5x^3+3\,d^2e^4x^2+3\,d^3e^3x+d^4e^2)} \\
&-\frac{2\sqrt{-e^2x^2+d^2}Bd}{105\,(d^2e^4x^2+2\,d^3e^3x+d^4e^2)}-\frac{2\sqrt{-e^2x^2+d^2}Bd}{105\,(d^3e^3x+d^4e^2)} \\
&+\frac{4\sqrt{-e^2x^2+d^2}Cd}{5\,(e^6x^3+3\,de^5x^2+3\,d^2e^4x+d^3e^3)} \\
&-\frac{2\sqrt{-e^2x^2+d^2}Cd}{15\,(de^5x^2+2\,d^2e^4x+d^3e^3)}-\frac{2\sqrt{-e^2x^2+d^2}Cd}{15\,(d^2e^4x+d^3e^3)} \\
&-\frac{2\sqrt{-e^2x^2+d^2}A}{7\,(e^5x^4+4\,de^4x^3+6\,d^2e^3x^2+4\,d^3e^2x+d^4e)} \\
&+\frac{\sqrt{-e^2x^2+d^2}A}{35\,(de^4x^3+3\,d^2e^3x^2+3\,d^3e^2x+d^4e)} \\
&+\frac{2\sqrt{-e^2x^2+d^2}A}{105\,(d^2e^3x^2+2\,d^3e^2x+d^4e)}+\frac{2\sqrt{-e^2x^2+d^2}A}{105\,(d^3e^2x+d^4e)} \\
&-\frac{2\sqrt{-e^2x^2+d^2}B}{5\,(e^5x^3+3\,de^4x^2+3\,d^2e^3x+d^3e^2)} \\
&+\frac{\sqrt{-e^2x^2+d^2}B}{15\,(de^4x^2+2\,d^2e^3x+d^3e^2)}+\frac{\sqrt{-e^2x^2+d^2}B}{15\,(d^2e^3x+d^3e^2)} \\
&-\frac{2\sqrt{-e^2x^2+d^2}C}{3\,(e^5x^2+2\,de^4x+d^2e^3)}+\frac{\sqrt{-e^2x^2+d^2}C}{3\,(de^4x+d^2e^3)}
\end{aligned}
$$

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^5,x, algorithm="maxima
")
```

```
[Out] -2/7*sqrt(-e^2*x^2 + d^2)*C*d^2/(e^7*x^4 + 4*d*e^6*x^3 + 6*d^2*e^5*x^2 + 4*
d^3*e^4*x + d^4*e^3) + 1/35*sqrt(-e^2*x^2 + d^2)*C*d^2/(d*e^6*x^3 + 3*d^2*e
^5*x^2 + 3*d^3*e^4*x + d^4*e^3) + 2/105*sqrt(-e^2*x^2 + d^2)*C*d^2/(d^2*e^5
*x^2 + 2*d^3*e^4*x + d^4*e^3) + 2/105*sqrt(-e^2*x^2 + d^2)*C*d^2/(d^3*e^4*x
 + d^4*e^3) + 2/7*sqrt(-e^2*x^2 + d^2)*B*d/(e^6*x^4 + 4*d*e^5*x^3 + 6*d^2*e
^4*x^2 + 4*d^3*e^3*x + d^4*e^2) - 1/35*sqrt(-e^2*x^2 + d^2)*B*d/(d*e^5*x^3
+ 3*d^2*e^4*x^2 + 3*d^3*e^3*x + d^4*e^2) - 2/105*sqrt(-e^2*x^2 + d^2)*B*d/(
```

```
d^2*e^4*x^2 + 2*d^3*e^3*x + d^4*e^2) - 2/105*sqrt(-e^2*x^2 + d^2)*B*d/(d^3*
e^3*x + d^4*e^2) + 4/5*sqrt(-e^2*x^2 + d^2)*C*d/(e^6*x^3 + 3*d*e^5*x^2 + 3*
d^2*e^4*x + d^3*e^3) - 2/15*sqrt(-e^2*x^2 + d^2)*C*d/(d*e^5*x^2 + 2*d^2*e^4
*x + d^3*e^3) - 2/15*sqrt(-e^2*x^2 + d^2)*C*d/(d^2*e^4*x + d^3*e^3) - 2/7*s
qrt(-e^2*x^2 + d^2)*A/(e^5*x^4 + 4*d*e^4*x^3 + 6*d^2*e^3*x^2 + 4*d^3*e^2*x
+ d^4*e) + 1/35*sqrt(-e^2*x^2 + d^2)*A/(d*e^4*x^3 + 3*d^2*e^3*x^2 + 3*d^3*e
^2*x + d^4*e) + 2/105*sqrt(-e^2*x^2 + d^2)*A/(d^2*e^3*x^2 + 2*d^3*e^2*x + d
^4*e) + 2/105*sqrt(-e^2*x^2 + d^2)*A/(d^3*e^2*x + d^4*e) - 2/5*sqrt(-e^2*x^
2 + d^2)*B/(e^5*x^3 + 3*d*e^4*x^2 + 3*d^2*e^3*x + d^3*e^2) + 1/15*sqrt(-e^2
*x^2 + d^2)*B/(d*e^4*x^2 + 2*d^2*e^3*x + d^3*e^2) + 1/15*sqrt(-e^2*x^2 + d^
2)*B/(d^2*e^3*x + d^3*e^2) - 2/3*sqrt(-e^2*x^2 + d^2)*C/(e^5*x^2 + 2*d*e^4*
x + d^2*e^3) + 1/3*sqrt(-e^2*x^2 + d^2)*C/(d*e^4*x + d^2*e^3)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 608, normalized size of antiderivative = 3.38

$$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^5}\,dx =$$

$$-\frac{1}{420}\left(\frac{3\left(5\left(\frac{2d}{ex+d}-1\right)^{\frac{7}{2}}+21\left(\frac{2d}{ex+d}-1\right)^{\frac{5}{2}}+35\left(\frac{2d}{ex+d}-1\right)^{\frac{3}{2}}+35\sqrt{\frac{2d}{ex+d}-1}\right)C\operatorname{sgn}\left(\frac{1}{ex+d}\right)\operatorname{sgn}(e)-35\left(\right.}{}\right.$$

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^5,x, algorithm="giac")
```

```
[Out] -1/420*((3*(5*(2*d/(e*x + d) - 1)^(7/2) + 21*(2*d/(e*x + d) - 1)^(5/2) + 35
*(2*d/(e*x + d) - 1)^(3/2) + 35*sqrt(2*d/(e*x + d) - 1))*C*sgn(1/(e*x + d))
*sgn(e) - 35*(3*(2*d/(e*x + d) - 1)^(5/2) + 10*(2*d/(e*x + d) - 1)^(3/2) +
15*sqrt(2*d/(e*x + d) - 1))*C*sgn(1/(e*x + d))*sgn(e) + 280*((2*d/(e*x + d)
 - 1)^(3/2) + 3*sqrt(2*d/(e*x + d) - 1))*C*sgn(1/(e*x + d))*sgn(e) - 3*(5*(
2*d/(e*x + d) - 1)^(7/2) + 21*(2*d/(e*x + d) - 1)^(5/2) + 35*(2*d/(e*x + d)
 - 1)^(3/2) + 35*sqrt(2*d/(e*x + d) - 1))*B*e*sgn(1/(e*x + d))*sgn(e)/d + 2
1*(3*(2*d/(e*x + d) - 1)^(5/2) + 10*(2*d/(e*x + d) - 1)^(3/2) + 15*sqrt(2*d
/(e*x + d) - 1))*B*e*sgn(1/(e*x + d))*sgn(e)/d - 70*((2*d/(e*x + d) - 1)^(3
/2) + 3*sqrt(2*d/(e*x + d) - 1))*B*e*sgn(1/(e*x + d))*sgn(e)/d + 3*(5*(2*d/
(e*x + d) - 1)^(7/2) + 21*(2*d/(e*x + d) - 1)^(5/2) + 35*(2*d/(e*x + d) - 1
)^(3/2) + 35*sqrt(2*d/(e*x + d) - 1))*A*e^2*sgn(1/(e*x + d))*sgn(e)/d^2 - 7
*(3*(2*d/(e*x + d) - 1)^(5/2) + 10*(2*d/(e*x + d) - 1)^(3/2) + 15*sqrt(2*d/
(e*x + d) - 1))*A*e^2*sgn(1/(e*x + d))*sgn(e)/d^2 - 420*C*sqrt(2*d/(e*x + d
) - 1)*sgn(1/(e*x + d))*sgn(e))/(d*e^4) + 4*(23*I*C*d^2 + 5*I*B*d*e + 2*I*A
*e^2)*sgn(1/(e*x + d))*sgn(e)/(d^3*e^4))*abs(e)
```

Mupad [B] (verification not implemented)

Time = 13.56 (sec) , antiderivative size = 601, normalized size of antiderivative = 3.34

$$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^5}\,dx = \frac{B\sqrt{d^2-e^2\,x^2}}{21\,\left(d^3\,e^2+x\,d^2\,e^3\right)} - \frac{3\,B\sqrt{d^2-e^2\,x^2}}{7\,\left(d^3\,e^2+3\,d^2\,e^3\,x+3\,d\,e^4\,x^2+e^5\,x^3\right)} + \frac{2\,A\sqrt{d^2-e^2\,x^2}}{105\,\left(d^4\,e+2\,d^3\,e^2\,x+d^2\,e^3\,x^2\right)} + \frac{B\sqrt{d^2-e^2\,x^2}}{21\,\left(d^3\,e^2+2\,d^2\,e^3\,x+d\,e^4\,x^2\right)} - \frac{82\,C\sqrt{d^2-e^2\,x^2}}{105\,\left(d^2\,e^3+2\,d\,e^4\,x+e^5\,x^2\right)} + \frac{2\,A\sqrt{d^2-e^2\,x^2}}{105\,\left(d^4\,e+x\,d^3\,e^2\right)} + \frac{23\,C\sqrt{d^2-e^2\,x^2}}{105\,\left(d^2\,e^3+x\,d\,e^4\right)} - \frac{2\,A\sqrt{d^2-e^2\,x^2}}{7\,\left(d^4\,e+4\,d^3\,e^2\,x+6\,d^2\,e^3\,x^2+4\,d\,e^4\,x^3+e^5\,x^4\right)} + \frac{A\sqrt{d^2-e^2\,x^2}}{35\,\left(d^4\,e+3\,d^3\,e^2\,x+3\,d^2\,e^3\,x^2+d\,e^4\,x^3\right)} - \frac{2\,C\,d^2\sqrt{d^2-e^2\,x^2}}{7\,\left(d^4\,e^3+4\,d^3\,e^4\,x+6\,d^2\,e^5\,x^2+4\,d\,e^6\,x^3+e^7\,x^4\right)} + \frac{2\,B\,d\sqrt{d^2-e^2\,x^2}}{7\,\left(d^4\,e^2+4\,d^3\,e^3\,x+6\,d^2\,e^4\,x^2+4\,d\,e^5\,x^3+e^6\,x^4\right)} + \frac{29\,C\,d\sqrt{d^2-e^2\,x^2}}{35\,\left(d^3\,e^3+3\,d^2\,e^4\,x+3\,d\,e^5\,x^2+e^6\,x^3\right)}$$

```
[In] int(((d^2 - e^2*x^2)^(1/2)*(A + B*x + C*x^2))/(d + e*x)^5,x)
```

```
[Out] (B*(d^2 - e^2*x^2)^(1/2))/(21*(d^3*e^2 + d^2*e^3*x)) - (3*B*(d^2 - e^2*x^2)
^(1/2))/(7*(d^3*e^2 + e^5*x^3 + 3*d^2*e^3*x + 3*d*e^4*x^2)) + (2*A*(d^2 - e
^2*x^2)^(1/2))/(105*(d^4*e + 2*d^3*e^2*x + d^2*e^3*x^2)) + (B*(d^2 - e^2*x^
2)^(1/2))/(21*(d^3*e^2 + 2*d^2*e^3*x + d*e^4*x^2)) - (82*C*(d^2 - e^2*x^2)^
(1/2))/(105*(d^2*e^3 + e^5*x^2 + 2*d*e^4*x)) + (2*A*(d^2 - e^2*x^2)^(1/2))/
(105*(d^4*e + d^3*e^2*x)) + (23*C*(d^2 - e^2*x^2)^(1/2))/(105*(d^2*e^3 + d*
e^4*x)) - (2*A*(d^2 - e^2*x^2)^(1/2))/(7*(d^4*e + e^5*x^4 + 4*d^3*e^2*x + 4
*d*e^4*x^3 + 6*d^2*e^3*x^2)) + (A*(d^2 - e^2*x^2)^(1/2))/(35*(d^4*e + 3*d^3
*e^2*x + d*e^4*x^3 + 3*d^2*e^3*x^2)) - (2*C*d^2*(d^2 - e^2*x^2)^(1/2))/(7*(
d^4*e^3 + e^7*x^4 + 4*d^3*e^4*x + 4*d*e^6*x^3 + 6*d^2*e^5*x^2)) + (2*B*d*(d
^2 - e^2*x^2)^(1/2))/(7*(d^4*e^2 + e^6*x^4 + 4*d^3*e^3*x + 4*d*e^5*x^3 + 6*
d^2*e^4*x^2)) + (29*C*d*(d^2 - e^2*x^2)^(1/2))/(35*(d^3*e^3 + e^6*x^3 + 3*d
^2*e^4*x + 3*d*e^5*x^2))
```

3.9 $\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^6}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 234

$$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^6}\,dx=-\frac{(Cd^2-Bde+Ae^2)\,(d^2-e^2x^2)^{3/2}}{9de^3(d+ex)^6}+\frac{C(d^2-e^2x^2)^{3/2}}{2e^3(d+ex)^5}-\frac{(11Cd^2+2e(2Bd+Ae))\,(d^2-e^2x^2)^{3/2}}{42d^2e^3(d+ex)^5}-\frac{(11Cd^2+2e(2Bd+Ae))\,(d^2-e^2x^2)^{3/2}}{105d^3e^3(d+ex)^4}-\frac{(11Cd^2+2e(2Bd+Ae))\,(d^2-e^2x^2)^{3/2}}{315d^4e^3(d+ex)^3}$$

```
[Out] -1/9*(A*e^2-B*d*e+C*d^2)*(-e^2*x^2+d^2)^(3/2)/d/e^3/(e*x+d)^6+1/2*C*(-e^2*x
^2+d^2)^(3/2)/e^3/(e*x+d)^5-1/42*(11*C*d^2+2*e*(A*e+2*B*d))*(-e^2*x^2+d^2)^
(3/2)/d^2/e^3/(e*x+d)^5-1/105*(11*C*d^2+2*e*(A*e+2*B*d))*(-e^2*x^2+d^2)^(3/
2)/d^3/e^3/(e*x+d)^4-1/315*(11*C*d^2+2*e*(A*e+2*B*d))*(-e^2*x^2+d^2)^(3/2)/
d^4/e^3/(e*x+d)^3
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used

$= \{1653, 807, 673, 665\}$

$$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^6}\,dx = -\frac{(d^2-e^2x^2)^{3/2}\,(2e(Ae+2Bd)+11Cd^2)}{42d^2e^3(d+ex)^5} - \frac{(d^2-e^2x^2)^{3/2}\,(Ae^2-Bde+Cd^2)}{9de^3(d+ex)^6} - \frac{(d^2-e^2x^2)^{3/2}\,(2e(Ae+2Bd)+11Cd^2)}{315d^4e^3(d+ex)^3} - \frac{(d^2-e^2x^2)^{3/2}\,(2e(Ae+2Bd)+11Cd^2)}{105d^3e^3(d+ex)^4} + \frac{C(d^2-e^2x^2)^{3/2}}{2e^3(d+ex)^5}$$

```
[In] Int[((A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2])/(d + e*x)^6,x]
```

```
[Out] -1/9*((C*d^2 - B*d*e + A*e^2)*(d^2 - e^2*x^2)^(3/2))/(d*e^3*(d + e*x)^6) +
(C*(d^2 - e^2*x^2)^(3/2))/(2*e^3*(d + e*x)^5) - ((11*C*d^2 + 2*e*(2*B*d + A
*e))*(d^2 - e^2*x^2)^(3/2))/(42*d^2*e^3*(d + e*x)^5) - ((11*C*d^2 + 2*e*(2*
B*d + A*e))*(d^2 - e^2*x^2)^(3/2))/(105*d^3*e^3*(d + e*x)^4) - ((11*C*d^2 +
 2*e*(2*B*d + A*e))*(d^2 - e^2*x^2)^(3/2))/(315*d^4*e^3*(d + e*x)^3)
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 673

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-e)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m + p + 1))), x] + Dist[Simpl
ify[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x]
, x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ
[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 807

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 1653

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{C(d^2-e^2x^2)^{3/2}}{2e^3(d+ex)^5}+\frac{\int \frac{\left(e^2\left(5Cd^2+2Ae^2\right)+e^3(3Cd+2Be)x\right)\sqrt{d^2-e^2x^2}}{(d+ex)^6}\,dx}{2e^4}\\
&= -\frac{\left(Cd^2-Bde+Ae^2\right)\left(d^2-e^2x^2\right)^{3/2}}{9de^3(d+ex)^6}+\frac{C(d^2-e^2x^2)^{3/2}}{2e^3(d+ex)^5}\\
&\quad+\frac{\left(11Cd^2+2e(2Bd+Ae)\right)\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^5}\,dx}{6de^2}\\
&= -\frac{\left(Cd^2-Bde+Ae^2\right)\left(d^2-e^2x^2\right)^{3/2}}{9de^3(d+ex)^6}+\frac{C(d^2-e^2x^2)^{3/2}}{2e^3(d+ex)^5}\\
&\quad-\frac{\left(11Cd^2+2e(2Bd+Ae)\right)\left(d^2-e^2x^2\right)^{3/2}}{42d^2e^3(d+ex)^5}\\
&\quad+\frac{\left(11Cd^2+2e(2Bd+Ae)\right)\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx}{21d^2e^2}\\
&= -\frac{\left(Cd^2-Bde+Ae^2\right)\left(d^2-e^2x^2\right)^{3/2}}{9de^3(d+ex)^6}+\frac{C(d^2-e^2x^2)^{3/2}}{2e^3(d+ex)^5}\\
&\quad-\frac{\left(11Cd^2+2e(2Bd+Ae)\right)\left(d^2-e^2x^2\right)^{3/2}}{42d^2e^3(d+ex)^5}\\
&\quad-\frac{\left(11Cd^2+2e(2Bd+Ae)\right)\left(d^2-e^2x^2\right)^{3/2}}{105d^3e^3(d+ex)^4}\\
&\quad+\frac{\left(11Cd^2+2e(2Bd+Ae)\right)\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^3}\,dx}{105d^3e^2}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{(Cd^2 - Bde + Ae^2)\left(d^2 - e^2x^2\right)^{3/2}}{9de^3(d+ex)^6} + \frac{C(d^2 - e^2x^2)^{3/2}}{2e^3(d+ex)^5} \\
&\quad - \frac{(11Cd^2 + 2e(2Bd + Ae))\left(d^2 - e^2x^2\right)^{3/2}}{42d^2e^3(d+ex)^5} \\
&\quad - \frac{(11Cd^2 + 2e(2Bd + Ae))\left(d^2 - e^2x^2\right)^{3/2}}{105d^3e^3(d+ex)^4} \\
&\quad - \frac{(11Cd^2 + 2e(2Bd + Ae))\left(d^2 - e^2x^2\right)^{3/2}}{315d^4e^3(d+ex)^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.62

$$
\int \frac{(A + Bx + Cx^2)\sqrt{d^2 - e^2x^2}}{(d+ex)^6}\,dx = \\
-\frac{(d-ex)\sqrt{d^2-e^2x^2}(Cd^2(4d^3 + 24d^2ex + 66de^2x^2 + 11e^3x^3) + e(Ae(58d^3 + 33d^2ex + 12de^2x^2 + 2e^3x^3)}{315d^4e^3(d+ex)^5}
$$

```
[In] Integrate[((A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2])/(d + e*x)^6,x]
```

```
[Out] -1/315*((d - e*x)*Sqrt[d^2 - e^2*x^2]*(C*d^2*(4*d^3 + 24*d^2*e*x + 66*d*e^2
*x^2 + 11*e^3*x^3) + e*(A*e*(58*d^3 + 33*d^2*e*x + 12*d*e^2*x^2 + 2*e^3*x^3
) + B*d*(11*d^3 + 66*d^2*e*x + 24*d*e^2*x^2 + 4*e^3*x^3))))/(d^4*e^3*(d + e
*x)^5)
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.65

method	result
gosper	$-\frac{(-ex+d)\left(2A\,e^5x^3+4x^3dB\,e^4+11C\,d^2e^3x^3+12Ad\,e^4x^2+24x^2d^2B\,e^3+66C\,d^3e^2x^2+33A\,d^2e^3x+66x\,d^3B\,e^2+24C\,d^4ex+58A\,d^3e^2\right.}{315(ex+d)^5d^4e^3}$
trager	$-\frac{\left(-2A\,e^6x^4-4Bd\,e^5x^4-11C\,d^2e^4x^4-10Ad\,e^5x^3-20B\,d^2e^4x^3-55C\,d^3e^3x^3-21A\,d^2e^4x^2-42B\,d^3e^3x^2+42C\,d^4e^2x^2-25A\,d^3e^3x+\right.}{315d^4(ex+d)^5e^3}$
default	$\frac{C\left(-\frac{\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{5de\left(x+\frac{d}{e}\right)^4}-\frac{\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{15d^2\left(x+\frac{d}{e}\right)^3}\right)}{e^6}+\frac{(Be-2Cd)\left(-\frac{\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{7de\left(x+\frac{d}{e}\right)^5}+\frac{2e\left(-\frac{\left(-\left(x+\frac{d}{e}\right)^2\right.}{5d}\right.}{}\right.}{e^7}$

```
[In] int((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/315*(-e*x+d)*(2*A*e^5*x^3+4*B*d*e^4*x^3+11*C*d^2*e^3*x^3+12*A*d*e^4*x^2+
24*B*d^2*e^3*x^2+66*C*d^3*e^2*x^2+33*A*d^2*e^3*x+66*B*d^3*e^2*x+24*C*d^4*e*
x+58*A*d^3*e^2+11*B*d^4*e+4*C*d^5)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^5/d^4/e^3
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.71

$$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^6}\,dx = -\frac{4\,Cd^7+11\,Bd^6e+58\,Ad^5e^2+(4\,Cd^2e^5+11\,Bde^6+58\,Ae^7)x^5+5\,(4\,Cd^3e^4+11\,Bd^2e^5+58\,Ade^6)x^4}{}$$

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^6,x, algorithm="fricas
")
```

```
[Out] -1/315*(4*C*d^7 + 11*B*d^6*e + 58*A*d^5*e^2 + (4*C*d^2*e^5 + 11*B*d*e^6 + 5
8*A*e^7)*x^5 + 5*(4*C*d^3*e^4 + 11*B*d^2*e^5 + 58*A*d*e^6)*x^4 + 10*(4*C*d^
4*e^3 + 11*B*d^3*e^4 + 58*A*d^2*e^5)*x^3 + 10*(4*C*d^5*e^2 + 11*B*d^4*e^3 +
 58*A*d^3*e^4)*x^2 + 5*(4*C*d^6*e + 11*B*d^5*e^2 + 58*A*d^4*e^3)*x + (4*C*d
^6 + 11*B*d^5*e + 58*A*d^4*e^2 - (11*C*d^2*e^4 + 4*B*d*e^5 + 2*A*e^6)*x^4 -
```

```
 5*(11*C*d^3*e^3 + 4*B*d^2*e^4 + 2*A*d*e^5)*x^3 + 21*(2*C*d^4*e^2 - 2*B*d^3
*e^3 - A*d^2*e^4)*x^2 + 5*(4*C*d^5*e + 11*B*d^4*e^2 - 5*A*d^3*e^3)*x)*sqrt(
-e^2*x^2 + d^2))/(d^4*e^8*x^5 + 5*d^5*e^7*x^4 + 10*d^6*e^6*x^3 + 10*d^7*e^5
*x^2 + 5*d^8*e^4*x + d^9*e^3)
```

Sympy [F]

$$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^6}\,dx = \int \frac{\sqrt{-(-d+ex)(d+ex)}(A+Bx+Cx^2)}{(d+ex)^6}\,dx$$

```
[In] integrate((C*x**2+B*x+A)*(-e**2*x**2+d**2)**(1/2)/(e*x+d)**6,x)
```

```
[Out] Integral(sqrt(-(-d + e*x)*(d + e*x))*(A + B*x + C*x**2)/(d + e*x)**6, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1378 vs. $2(214) = 428$.

Time = 0.21 (sec) , antiderivative size = 1378, normalized size of antiderivative = 5.89

$$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^6}\,dx = \text{Too large to display}$$

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^6,x, algorithm="maxima
")
```

```
[Out] -2/9*sqrt(-e^2*x^2 + d^2)*C*d^2/(e^8*x^5 + 5*d*e^7*x^4 + 10*d^2*e^6*x^3 + 1
0*d^3*e^5*x^2 + 5*d^4*e^4*x + d^5*e^3) + 1/63*sqrt(-e^2*x^2 + d^2)*C*d^2/(d
*e^7*x^4 + 4*d^2*e^6*x^3 + 6*d^3*e^5*x^2 + 4*d^4*e^4*x + d^5*e^3) + 1/105*s
qrt(-e^2*x^2 + d^2)*C*d^2/(d^2*e^6*x^3 + 3*d^3*e^5*x^2 + 3*d^4*e^4*x + d^5*
e^3) + 2/315*sqrt(-e^2*x^2 + d^2)*C*d^2/(d^3*e^5*x^2 + 2*d^4*e^4*x + d^5*e^
3) + 2/315*sqrt(-e^2*x^2 + d^2)*C*d^2/(d^4*e^4*x + d^5*e^3) + 2/9*sqrt(-e^2
*x^2 + d^2)*B*d/(e^7*x^5 + 5*d*e^6*x^4 + 10*d^2*e^5*x^3 + 10*d^3*e^4*x^2 +
5*d^4*e^3*x + d^5*e^2) - 1/63*sqrt(-e^2*x^2 + d^2)*B*d/(d*e^6*x^4 + 4*d^2*e
^5*x^3 + 6*d^3*e^4*x^2 + 4*d^4*e^3*x + d^5*e^2) - 1/105*sqrt(-e^2*x^2 + d^2
)*B*d/(d^2*e^5*x^3 + 3*d^3*e^4*x^2 + 3*d^4*e^3*x + d^5*e^2) - 2/315*sqrt(-e
^2*x^2 + d^2)*B*d/(d^3*e^4*x^2 + 2*d^4*e^3*x + d^5*e^2) - 2/315*sqrt(-e^2*x
^2 + d^2)*B*d/(d^4*e^3*x + d^5*e^2) + 4/7*sqrt(-e^2*x^2 + d^2)*C*d/(e^7*x^4
 + 4*d*e^6*x^3 + 6*d^2*e^5*x^2 + 4*d^3*e^4*x + d^4*e^3) - 2/35*sqrt(-e^2*x^
2 + d^2)*C*d/(d*e^6*x^3 + 3*d^2*e^5*x^2 + 3*d^3*e^4*x + d^4*e^3) - 4/105*sq
rt(-e^2*x^2 + d^2)*C*d/(d^2*e^5*x^2 + 2*d^3*e^4*x + d^4*e^3) - 4/105*sqrt(-
e^2*x^2 + d^2)*C*d/(d^3*e^4*x + d^4*e^3) - 2/9*sqrt(-e^2*x^2 + d^2)*A/(e^6*
x^5 + 5*d*e^5*x^4 + 10*d^2*e^4*x^3 + 10*d^3*e^3*x^2 + 5*d^4*e^2*x + d^5*e)
+ 1/63*sqrt(-e^2*x^2 + d^2)*A/(d*e^5*x^4 + 4*d^2*e^4*x^3 + 6*d^3*e^3*x^2 +
```

```
4*d^4*e^2*x + d^5*e) + 1/105*sqrt(-e^2*x^2 + d^2)*A/(d^2*e^4*x^3 + 3*d^3*e^
3*x^2 + 3*d^4*e^2*x + d^5*e) + 2/315*sqrt(-e^2*x^2 + d^2)*A/(d^3*e^3*x^2 +
2*d^4*e^2*x + d^5*e) + 2/315*sqrt(-e^2*x^2 + d^2)*A/(d^4*e^2*x + d^5*e) - 2
/7*sqrt(-e^2*x^2 + d^2)*B/(e^6*x^4 + 4*d*e^5*x^3 + 6*d^2*e^4*x^2 + 4*d^3*e^
3*x + d^4*e^2) + 1/35*sqrt(-e^2*x^2 + d^2)*B/(d*e^5*x^3 + 3*d^2*e^4*x^2 + 3
*d^3*e^3*x + d^4*e^2) + 2/105*sqrt(-e^2*x^2 + d^2)*B/(d^2*e^4*x^2 + 2*d^3*e
^3*x + d^4*e^2) + 2/105*sqrt(-e^2*x^2 + d^2)*B/(d^3*e^3*x + d^4*e^2) - 2/5*
sqrt(-e^2*x^2 + d^2)*C/(e^6*x^3 + 3*d*e^5*x^2 + 3*d^2*e^4*x + d^3*e^3) + 1/
15*sqrt(-e^2*x^2 + d^2)*C/(d*e^5*x^2 + 2*d^2*e^4*x + d^3*e^3) + 1/15*sqrt(-
e^2*x^2 + d^2)*C/(d^2*e^4*x + d^3*e^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 715 vs. $2(214) = 428$.

Time = 0.30 (sec) , antiderivative size = 715, normalized size of antiderivative = 3.06

$$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^6}\,dx$$

$$= \frac{2\left(4\,Cd^2+11\,Bde+58\,Ae^2+\frac{207\left(de+\sqrt{-e^2x^2+d^2}|e|\right)A}{x}+\frac{36\left(de+\sqrt{-e^2x^2+d^2}|e|\right)Cd^2}{e^2x}+\frac{99\left(de+\sqrt{-e^2x^2+d^2}|e|\right)Bd}{ex}+14\right.}{}$$

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^6,x, algorithm="giac")
```

```
[Out] 2/315*(4*C*d^2 + 11*B*d*e + 58*A*e^2 + 207*(d*e + sqrt(-e^2*x^2 + d^2)*abs(
e))*A/x + 36*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*C*d^2/(e^2*x) + 99*(d*e +
sqrt(-e^2*x^2 + d^2)*abs(e))*B*d/(e*x) + 144*(d*e + sqrt(-e^2*x^2 + d^2)*ab
s(e))^2*C*d^2/(e^4*x^2) + 81*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*B*d/(e^3
*x^2) + 1143*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*A/(e^2*x^2) - 84*(d*e +
sqrt(-e^2*x^2 + d^2)*abs(e))^3*C*d^2/(e^6*x^3) + 609*(d*e + sqrt(-e^2*x^2 +
 d^2)*abs(e))^3*B*d/(e^5*x^3) + 2247*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3*
A/(e^4*x^3) + 504*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4*C*d^2/(e^8*x^4) + 4
41*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4*B*d/(e^7*x^4) + 3843*(d*e + sqrt(-
e^2*x^2 + d^2)*abs(e))^4*A/(e^6*x^4) + 945*(d*e + sqrt(-e^2*x^2 + d^2)*abs(
e))^5*B*d/(e^9*x^5) + 3465*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^5*A/(e^8*x^5
) + 420*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^6*C*d^2/(e^12*x^6) + 315*(d*e +
 sqrt(-e^2*x^2 + d^2)*abs(e))^6*B*d/(e^11*x^6) + 2625*(d*e + sqrt(-e^2*x^2
+ d^2)*abs(e))^6*A/(e^10*x^6) + 315*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^7*B
*d/(e^13*x^7) + 945*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^7*A/(e^12*x^7) + 31
5*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^8*A/(e^14*x^8))/(d^4*e^2*((d*e + sqrt
(-e^2*x^2 + d^2)*abs(e))/(e^2*x) + 1)^9*abs(e))
```

Mupad [B] (verification not implemented)

Time = 14.18 (sec) , antiderivative size = 960, normalized size of antiderivative = 4.10

$$
\begin{aligned}
&\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^6}\,dx \\
&= \frac{B\sqrt{d^2-e^2\,x^2}}{63\,\left(d^4\,e^2+x\,d^3\,e^3\right)}+\frac{C\sqrt{d^2-e^2\,x^2}}{135\,\left(d^3\,e^3+x\,d^2\,e^4\right)} \\
&\quad-\frac{19\,B\sqrt{d^2-e^2\,x^2}}{63\,\left(d^4\,e^2+4\,d^3\,e^3\,x+6\,d^2\,e^4\,x^2+4\,d\,e^5\,x^3+e^6\,x^4\right)} \\
&\quad+\frac{A\sqrt{d^2-e^2\,x^2}}{105\,\left(d^5\,e+3\,d^4\,e^2\,x+3\,d^3\,e^3\,x^2+d^2\,e^4\,x^3\right)} \\
&\quad+\frac{2\,B\sqrt{d^2-e^2\,x^2}}{105\,\left(d^4\,e^2+3\,d^3\,e^3\,x+3\,d^2\,e^4\,x^2+d\,e^5\,x^3\right)}-\frac{47\,C\sqrt{d^2-e^2\,x^2}}{105\,\left(d^3\,e^3+3\,d^2\,e^4\,x+3\,d\,e^5\,x^2+e^6\,x^3\right)} \\
&\quad+\frac{2\,A\sqrt{d^2-e^2\,x^2}}{315\,\left(d^5\,e+2\,d^4\,e^2\,x+d^3\,e^3\,x^2\right)}+\frac{11\,C\sqrt{d^2-e^2\,x^2}}{315\,\left(d^3\,e^3+2\,d^2\,e^4\,x+d\,e^5\,x^2\right)} \\
&\quad-\frac{2\,A\sqrt{d^2-e^2\,x^2}}{9\,\left(d^5\,e+5\,d^4\,e^2\,x+10\,d^3\,e^3\,x^2+10\,d^2\,e^4\,x^3+5\,d\,e^5\,x^4+e^6\,x^5\right)} \\
&\quad+\frac{A\sqrt{d^2-e^2\,x^2}}{63\,\left(d^5\,e+4\,d^4\,e^2\,x+6\,d^3\,e^3\,x^2+4\,d^2\,e^4\,x^3+d\,e^5\,x^4\right)} \\
&\quad-\frac{2\,A\sqrt{d^2-e^2\,x^2}}{945\,\left(d^5\,e+x\,d^4\,e^2\right)}+\frac{4\,B\sqrt{d^2-e^2\,x^2}}{315\,\left(d^4\,e^2+2\,d^3\,e^3\,x+d^2\,e^4\,x^2\right)} \\
&\quad+\frac{2\,B\,d\sqrt{d^2-e^2\,x^2}}{9\,\left(d^5\,e^2+5\,d^4\,e^3\,x+10\,d^3\,e^4\,x^2+10\,d^2\,e^5\,x^3+5\,d\,e^6\,x^4+e^7\,x^5\right)} \\
&\quad+\frac{37\,C\,d\sqrt{d^2-e^2\,x^2}}{63\,\left(d^4\,e^3+4\,d^3\,e^4\,x+6\,d^2\,e^5\,x^2+4\,d\,e^6\,x^3+e^7\,x^4\right)} \\
&\quad-\frac{2\,C\,d^2\sqrt{d^2-e^2\,x^2}}{9\,\left(d^5\,e^3+5\,d^4\,e^4\,x+10\,d^3\,e^5\,x^2+10\,d^2\,e^6\,x^3+5\,d\,e^7\,x^4+e^8\,x^5\right)} \\
&\quad+\frac{8\,A\,e^2\sqrt{d^2-e^2\,x^2}}{945\,\left(d^5\,e^3+x\,d^4\,e^4\right)}+\frac{26\,C\,d^2\sqrt{d^2-e^2\,x^2}}{945\,\left(d^5\,e^3+x\,d^4\,e^4\right)}-\frac{B\,d\,e\sqrt{d^2-e^2\,x^2}}{315\,\left(d^5\,e^3+x\,d^4\,e^4\right)}
\end{aligned}
$$

```
[In] int(((d^2 - e^2*x^2)^(1/2)*(A + B*x + C*x^2))/(d + e*x)^6,x)
```

```
[Out] (B*(d^2 - e^2*x^2)^(1/2))/(63*(d^4*e^2 + d^3*e^3*x)) + (C*(d^2 - e^2*x^2)^(
1/2))/(135*(d^3*e^3 + d^2*e^4*x)) - (19*B*(d^2 - e^2*x^2)^(1/2))/(63*(d^4*e
^2 + e^6*x^4 + 4*d^3*e^3*x + 4*d*e^5*x^3 + 6*d^2*e^4*x^2)) + (A*(d^2 - e^2*
x^2)^(1/2))/(105*(d^5*e + 3*d^4*e^2*x + 3*d^3*e^3*x^2 + d^2*e^4*x^3)) + (2*
B*(d^2 - e^2*x^2)^(1/2))/(105*(d^4*e^2 + 3*d^3*e^3*x + d*e^5*x^3 + 3*d^2*e^
4*x^2)) - (47*C*(d^2 - e^2*x^2)^(1/2))/(105*(d^3*e^3 + e^6*x^3 + 3*d^2*e^4*
x + 3*d*e^5*x^2)) + (2*A*(d^2 - e^2*x^2)^(1/2))/(315*(d^5*e + 2*d^4*e^2*x +
 d^3*e^3*x^2)) + (11*C*(d^2 - e^2*x^2)^(1/2))/(315*(d^3*e^3 + 2*d^2*e^4*x +
```

```
 d*e^5*x^2)) - (2*A*(d^2 - e^2*x^2)^(1/2))/(9*(d^5*e + e^6*x^5 + 5*d^4*e^2*
x + 5*d*e^5*x^4 + 10*d^3*e^3*x^2 + 10*d^2*e^4*x^3)) + (A*(d^2 - e^2*x^2)^(1
/2))/(63*(d^5*e + 4*d^4*e^2*x + d*e^5*x^4 + 6*d^3*e^3*x^2 + 4*d^2*e^4*x^3))
 - (2*A*(d^2 - e^2*x^2)^(1/2))/(945*(d^5*e + d^4*e^2*x)) + (4*B*(d^2 - e^2*
x^2)^(1/2))/(315*(d^4*e^2 + 2*d^3*e^3*x + d^2*e^4*x^2)) + (2*B*d*(d^2 - e^2
*x^2)^(1/2))/(9*(d^5*e^2 + e^7*x^5 + 5*d^4*e^3*x + 5*d*e^6*x^4 + 10*d^3*e^4
*x^2 + 10*d^2*e^5*x^3)) + (37*C*d*(d^2 - e^2*x^2)^(1/2))/(63*(d^4*e^3 + e^7
*x^4 + 4*d^3*e^4*x + 4*d*e^6*x^3 + 6*d^2*e^5*x^2)) - (2*C*d^2*(d^2 - e^2*x^
2)^(1/2))/(9*(d^5*e^3 + e^8*x^5 + 5*d^4*e^4*x + 5*d*e^7*x^4 + 10*d^3*e^5*x^
2 + 10*d^2*e^6*x^3)) + (8*A*e^2*(d^2 - e^2*x^2)^(1/2))/(945*(d^5*e^3 + d^4*
e^4*x)) + (26*C*d^2*(d^2 - e^2*x^2)^(1/2))/(945*(d^5*e^3 + d^4*e^4*x)) - (B
*d*e*(d^2 - e^2*x^2)^(1/2))/(315*(d^5*e^3 + d^4*e^4*x))
```

3.10 $\int \frac{(d+ex)^3(A+Bx+Cx^2)}{\sqrt{d^2-e^2x^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 236

$$\begin{aligned}\int \frac{(d+ex)^3\,(A+Bx+Cx^2)}{\sqrt{d^2-e^2x^2}}\,dx = & -\frac{d^2(38Cd^2+45Bde+55Ae^2)\sqrt{d^2-e^2x^2}}{15e^3}\\ & -\frac{d(13Cd^2+15Bde+12Ae^2)\,x\sqrt{d^2-e^2x^2}}{8e^2}\\ & -\frac{(19Cd^2+5e(3Bd+Ae))\,x^2\sqrt{d^2-e^2x^2}}{15e}\\ & -\frac{1}{4}(3Cd+Be)x^3\sqrt{d^2-e^2x^2}-\frac{1}{5}Cex^4\sqrt{d^2-e^2x^2}\\ & +\frac{d^3(13Cd^2+15Bde+20Ae^2)\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^3}\end{aligned}$$

```
[Out] 1/8*d^3*(20*A*e^2+15*B*d*e+13*C*d^2)*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3-1
/15*d^2*(55*A*e^2+45*B*d*e+38*C*d^2)*(-e^2*x^2+d^2)^(1/2)/e^3-1/8*d*(12*A*e
^2+15*B*d*e+13*C*d^2)*x*(-e^2*x^2+d^2)^(1/2)/e^2-1/15*(19*C*d^2+5*e*(A*e+3*
B*d))*x^2*(-e^2*x^2+d^2)^(1/2)/e-1/4*(B*e+3*C*d)*x^3*(-e^2*x^2+d^2)^(1/2)-1
/5*C*e*x^4*(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1829, 655, 223, 209}

$$\begin{aligned}\int \frac{(d+ex)^3\left(A+Bx+Cx^2\right)}{\sqrt{d^2-e^2x^2}}\,dx = {} & \frac{d^3\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\left(20Ae^2+15Bde+13Cd^2\right)}{8e^3} \\ & -\frac{x^2\sqrt{d^2-e^2x^2}\left(5e(Ae+3Bd)+19Cd^2\right)}{15e} \\ & -\frac{dx\sqrt{d^2-e^2x^2}\left(12Ae^2+15Bde+13Cd^2\right)}{8e^2} \\ & -\frac{d^2\sqrt{d^2-e^2x^2}\left(55Ae^2+45Bde+38Cd^2\right)}{15e^3} \\ & -\frac{1}{4}x^3\sqrt{d^2-e^2x^2}(Be+3Cd)-\frac{1}{5}Cex^4\sqrt{d^2-e^2x^2}\end{aligned}$$

```
[In] Int[((d + e*x)^3*(A + B*x + C*x^2))/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] -1/15*(d^2*(38*C*d^2 + 45*B*d*e + 55*A*e^2)*Sqrt[d^2 - e^2*x^2])/e^3 - (d*(
13*C*d^2 + 15*B*d*e + 12*A*e^2)*x*Sqrt[d^2 - e^2*x^2])/(8*e^2) - ((19*C*d^2
 + 5*e*(3*B*d + A*e))*x^2*Sqrt[d^2 - e^2*x^2])/(15*e) - ((3*C*d + B*e)*x^3*
Sqrt[d^2 - e^2*x^2])/4 - (C*e*x^4*Sqrt[d^2 - e^2*x^2])/5 + (d^3*(13*C*d^2 +
 15*B*d*e + 20*A*e^2)*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(8*e^3)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1829

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
```

```
q + 2*p + 1))), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
&\text{integral}\\
&=-\frac{1}{5}Cex^4\sqrt{d^2-e^2x^2}\\
&\quad-\frac{\int \frac{-5Ad^3e^2-5d^2e^2(Bd+3Ae)x-5de^2\left(Cd^2+3e(Bd+Ae)\right)x^2-e^3\left(19Cd^2+5e(3Bd+Ae)\right)x^3-5e^4(3Cd+Be)x^4}{\sqrt{d^2-e^2x^2}}\,dx}{5e^2}\\
&=-\frac{1}{4}(3Cd+Be)x^3\sqrt{d^2-e^2x^2}-\frac{1}{5}Cex^4\sqrt{d^2-e^2x^2}\\
&\quad+\frac{\int \frac{20Ad^3e^4+20d^2e^4(Bd+3Ae)x+5de^4\left(13Cd^2+15Bde+12Ae^2\right)x^2+4e^5\left(19Cd^2+5e(3Bd+Ae)\right)x^3}{\sqrt{d^2-e^2x^2}}\,dx}{20e^4}\\
&=-\frac{\left(19Cd^2+5e(3Bd+Ae)\right)x^2\sqrt{d^2-e^2x^2}}{15e}-\frac{1}{4}(3Cd+Be)x^3\sqrt{d^2-e^2x^2}\\
&\quad-\frac{1}{5}Cex^4\sqrt{d^2-e^2x^2}-\frac{\int \frac{-60Ad^3e^6-4d^2e^5\left(38Cd^2+45Bde+55Ae^2\right)x-15de^6\left(13Cd^2+15Bde+12Ae^2\right)x^2}{\sqrt{d^2-e^2x^2}}\,dx}{60e^6}\\
&=-\frac{d\left(13Cd^2+15Bde+12Ae^2\right)x\sqrt{d^2-e^2x^2}}{8e^2}-\frac{\left(19Cd^2+5e(3Bd+Ae)\right)x^2\sqrt{d^2-e^2x^2}}{15e}\\
&\quad-\frac{1}{4}(3Cd+Be)x^3\sqrt{d^2-e^2x^2}-\frac{1}{5}Cex^4\sqrt{d^2-e^2x^2}\\
&\quad+\frac{\int \frac{15d^3e^6\left(13Cd^2+15Bde+20Ae^2\right)+8d^2e^7\left(38Cd^2+45Bde+55Ae^2\right)x}{\sqrt{d^2-e^2x^2}}\,dx}{120e^8}\\
&=-\frac{d^2\left(38Cd^2+45Bde+55Ae^2\right)\sqrt{d^2-e^2x^2}}{15e^3}-\frac{d\left(13Cd^2+15Bde+12Ae^2\right)x\sqrt{d^2-e^2x^2}}{8e^2}\\
&\quad-\frac{\left(19Cd^2+5e(3Bd+Ae)\right)x^2\sqrt{d^2-e^2x^2}}{15e}-\frac{1}{4}(3Cd+Be)x^3\sqrt{d^2-e^2x^2}\\
&\quad-\frac{1}{5}Cex^4\sqrt{d^2-e^2x^2}+\frac{\left(d^3\left(13Cd^2+15Bde+20Ae^2\right)\right)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{8e^2}\\
&=-\frac{d^2\left(38Cd^2+45Bde+55Ae^2\right)\sqrt{d^2-e^2x^2}}{15e^3}-\frac{d\left(13Cd^2+15Bde+12Ae^2\right)x\sqrt{d^2-e^2x^2}}{8e^2}\\
&\quad-\frac{\left(19Cd^2+5e(3Bd+Ae)\right)x^2\sqrt{d^2-e^2x^2}}{15e}-\frac{1}{4}(3Cd+Be)x^3\sqrt{d^2-e^2x^2}\\
&\quad-\frac{1}{5}Cex^4\sqrt{d^2-e^2x^2}+\frac{\left(d^3\left(13Cd^2+15Bde+20Ae^2\right)\right)\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx,x,\frac{x}{\sqrt{d^2-e^2x^2}}\right)}{8e^2}
\end{aligned}
$$

$$= -\frac{d^2(38Cd^2+45Bde+55Ae^2)\sqrt{d^2-e^2x^2}}{15e^3} - \frac{d(13Cd^2+15Bde+12Ae^2)\,x\sqrt{d^2-e^2x^2}}{8e^2} - \frac{(19Cd^2+5e(3Bd+Ae))\,x^2\sqrt{d^2-e^2x^2}}{15e} - \frac{1}{4}(3Cd+Be)x^3\sqrt{d^2-e^2x^2} - \frac{1}{5}Cex^4\sqrt{d^2-e^2x^2} + \frac{d^3(13Cd^2+15Bde+20Ae^2)\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^3}$$

Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.78

$$\int \frac{(d+ex)^3\,(A+Bx+Cx^2)}{\sqrt{d^2-e^2x^2}}\,dx = -\frac{\sqrt{d^2-e^2x^2}(C(304d^4+195d^3ex+152d^2e^2x^2+90de^3x^3+24e^4x^4)+5e(4Ae(22d^2+9dex+2e^2x^2)+}{120e^3}$$

```
[In] Integrate[((d + e*x)^3*(A + B*x + C*x^2))/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] -1/120*(Sqrt[d^2 - e^2*x^2]*(C*(304*d^4 + 195*d^3*e*x + 152*d^2*e^2*x^2 + 9
0*d*e^3*x^3 + 24*e^4*x^4) + 5*e*(4*A*e*(22*d^2 + 9*d*e*x + 2*e^2*x^2) + 3*B
*(24*d^3 + 15*d^2*e*x + 8*d*e^2*x^2 + 2*e^3*x^3))) + 30*d^3*(13*C*d^2 + 5*e
*(3*B*d + 4*A*e))*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/e^3
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.77

method	result
risch	$-\frac{(24e^4C\,x^4+30x^3B\,e^4+90Cd\,e^3x^3+40A\,e^4x^2+120x^2dB\,e^3+152C\,d^2e^2x^2+180Ad\,e^3x+225xB\,d^2e^2+195C\,d^3xe+440A\,d^2e^2+360}{120e^3}$
default	$\frac{A\,d^3\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}}+e^3C\left(-\frac{x^4\sqrt{-e^2x^2+d^2}}{5e^2}+\frac{4d^2\left(-\frac{x^2\sqrt{-e^2x^2+d^2}}{3e^2}-\frac{2d^2\sqrt{-e^2x^2+d^2}}{3e^4}\right)}{5e^2}\right)+(B\,e^3+3d\,e^2C)$

```
[In] int((e*x+d)^3*(C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/120/e^3*(24*C*e^4*x^4+30*B*e^4*x^3+90*C*d*e^3*x^3+40*A*e^4*x^2+120*B*d*e
^3*x^2+152*C*d^2*e^2*x^2+180*A*d*e^3*x+225*B*d^2*e^2*x+195*C*d^3*e*x+440*A*
d^2*e^2+360*B*d^3*e+304*C*d^4)*(-e^2*x^2+d^2)^(1/2)+1/8*d^3/e^2*(20*A*e^2+1
5*B*d*e+13*C*d^2)/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.75

$$\int \frac{(d+ex)^3\left(A+Bx+Cx^2\right)}{\sqrt{d^2-e^2x^2}}\,dx =$$

$$-\frac{30\left(13\,Cd^5+15\,Bd^4e+20\,Ad^3e^2\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(24\,Ce^4x^4+304\,Cd^4+360\,Bd^3e+440\,Ad}{}$$

```
[In] integrate((e*x+d)^3*(C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas
")
```

```
[Out] -1/120*(30*(13*C*d^5 + 15*B*d^4*e + 20*A*d^3*e^2)*arctan(-(d - sqrt(-e^2*x^
2 + d^2))/(e*x)) + (24*C*e^4*x^4 + 304*C*d^4 + 360*B*d^3*e + 440*A*d^2*e^2
+ 30*(3*C*d*e^3 + B*e^4)*x^3 + 8*(19*C*d^2*e^2 + 15*B*d*e^3 + 5*A*e^4)*x^2
+ 15*(13*C*d^3*e + 15*B*d^2*e^2 + 12*A*d*e^3)*x)*sqrt(-e^2*x^2 + d^2))/e^3
```

Sympy [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.76

$$\int \frac{(d+ex)^3\left(A+Bx+Cx^2\right)}{\sqrt{d^2-e^2x^2}}\,dx$$

$$= \begin{cases} \sqrt{d^2-e^2x^2}\left(-\frac{Cex^4}{5}-\frac{x^3\left(Be^3+3Cde^2\right)}{4e^2}-\frac{x^2\left(Ae^3+3Bde^2+\frac{19Cd^2e}{5}\right)}{3e^2}-\frac{x\left(3Ade^2+3Bd^2e+Cd^3+\frac{3d^2\left(Be^3+3Cde^2\right)}{4e^2}\right)}{2e^2}-\frac{3Ad^2e+}{}\right. \\ \frac{Ad^3x+\frac{Ce^3x^6}{6}+\frac{x^5\left(Be^3+3Cde^2\right)}{5}+\frac{x^4\left(Ae^3+3Bde^2+3Cd^2e\right)}{4}+\frac{x^3\cdot\left(3Ade^2+3Bd^2e+Cd^3\right)}{3}+\frac{x^2\cdot\left(3Ad^2e+Bd^3\right)}{2}}{\sqrt{d^2}} \end{cases}$$

```
[In] integrate((e*x+d)**3*(C*x**2+B*x+A)/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Piecewise((sqrt(d**2 - e**2*x**2)*(-C*e*x**4/5 - x**3*(B*e**3 + 3*C*d*e**2)
/(4*e**2) - x**2*(A*e**3 + 3*B*d*e**2 + 19*C*d**2*e/5)/(3*e**2) - x*(3*A*d*
e**2 + 3*B*d**2*e + C*d**3 + 3*d**2*(B*e**3 + 3*C*d*e**2)/(4*e**2))/(2*e**2
) - (3*A*d**2*e + B*d**3 + 2*d**2*(A*e**3 + 3*B*d*e**2 + 19*C*d**2*e/5)/(3*
e**2))/e**2) + (A*d**3 + d**2*(3*A*d*e**2 + 3*B*d**2*e + C*d**3 + 3*d**2*(B
*e**3 + 3*C*d*e**2)/(4*e**2))/(2*e**2))*Piecewise((log(-2*e**2*x + 2*sqrt(-
e**2)*sqrt(d**2 - e**2*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e*
*2*x**2), True)), Ne(e**2, 0)), ((A*d**3*x + C*e**3*x**6/6 + x**5*(B*e**3 +
 3*C*d*e**2)/5 + x**4*(A*e**3 + 3*B*d*e**2 + 3*C*d**2*e)/4 + x**3*(3*A*d*e*
*2 + 3*B*d**2*e + C*d**3)/3 + x**2*(3*A*d**2*e + B*d**3)/2)/sqrt(d**2), Tru
e))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 423, normalized size of antiderivative = 1.79

$$\begin{aligned}\int \frac{(d+ex)^3\,(A+Bx+Cx^2)}{\sqrt{d^2-e^2x^2}}\,dx = &-\frac{1}{5}\,\sqrt{-e^2x^2+d^2}Cex^4-\frac{4\,\sqrt{-e^2x^2+d^2}Cd^2x^2}{15\,e}\\ &+\frac{Ad^3\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{\sqrt{e^2}}-\frac{8\,\sqrt{-e^2x^2+d^2}Cd^4}{15\,e^3}\\ &-\frac{\sqrt{-e^2x^2+d^2}Bd^3}{e^2}-\frac{3\,\sqrt{-e^2x^2+d^2}Ad^2}{e}\\ &-\frac{(3\,Cde^2+Be^3)\sqrt{-e^2x^2+d^2}x^3}{4\,e^2}\\ &-\frac{(3\,Cd^2e+3\,Bde^2+Ae^3)\sqrt{-e^2x^2+d^2}x^2}{3\,e^2}\\ &+\frac{3\,(3\,Cde^2+Be^3)d^4\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{8\,\sqrt{e^2}e^4}\\ &+\frac{(Cd^3+3\,Bd^2e+3\,Ade^2)d^2\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{2\,\sqrt{e^2}e^2}\\ &-\frac{3\,(3\,Cde^2+Be^3)\sqrt{-e^2x^2+d^2}d^2x}{8\,e^4}\\ &-\frac{(Cd^3+3\,Bd^2e+3\,Ade^2)\sqrt{-e^2x^2+d^2}x}{2\,e^2}\\ &-\frac{2\,(3\,Cd^2e+3\,Bde^2+Ae^3)\sqrt{-e^2x^2+d^2}d^2}{3\,e^4}\end{aligned}$$

```
[In] integrate((e*x+d)^3*(C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima
")
```

```
[Out] -1/5*sqrt(-e^2*x^2 + d^2)*C*e*x^4 - 4/15*sqrt(-e^2*x^2 + d^2)*C*d^2*x^2/e +
 A*d^3*arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e^2) - 8/15*sqrt(-e^2*x^2 + d^2)*C*
d^4/e^3 - sqrt(-e^2*x^2 + d^2)*B*d^3/e^2 - 3*sqrt(-e^2*x^2 + d^2)*A*d^2/e -
 1/4*(3*C*d*e^2 + B*e^3)*sqrt(-e^2*x^2 + d^2)*x^3/e^2 - 1/3*(3*C*d^2*e + 3*
B*d*e^2 + A*e^3)*sqrt(-e^2*x^2 + d^2)*x^2/e^2 + 3/8*(3*C*d*e^2 + B*e^3)*d^4
*arcsin(e^2*x/(d*sqrt(e^2)))/(sqrt(e^2)*e^4) + 1/2*(C*d^3 + 3*B*d^2*e + 3*A
*d*e^2)*d^2*arcsin(e^2*x/(d*sqrt(e^2)))/(sqrt(e^2)*e^2) - 3/8*(3*C*d*e^2 +
B*e^3)*sqrt(-e^2*x^2 + d^2)*d^2*x/e^4 - 1/2*(C*d^3 + 3*B*d^2*e + 3*A*d*e^2)
*sqrt(-e^2*x^2 + d^2)*x/e^2 - 2/3*(3*C*d^2*e + 3*B*d*e^2 + A*e^3)*sqrt(-e^2
*x^2 + d^2)*d^2/e^4
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.79

$$\int \frac{(d+ex)^3\left(A+Bx+Cx^2\right)}{\sqrt{d^2-e^2x^2}}\,dx =$$

$$-\frac{1}{120}\sqrt{-e^2x^2+d^2}\bigg(\bigg(2\bigg(3\bigg(4\,Cex+\frac{5\,(3\,Cde^6+Be^7)}{e^6}\bigg)x+\frac{4\,(19\,Cd^2e^5+15\,Bde^6+5\,Ae^7)}{e^6}\bigg)x+15\,(13$$

$$+\frac{(13\,Cd^5+15\,Bd^4e+20\,Ad^3e^2)\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{8\,e^2|e|}$$

```
[In] integrate((e*x+d)^3*(C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/120*sqrt(-e^2*x^2 + d^2)*((2*(3*(4*C*e*x + 5*(3*C*d*e^6 + B*e^7)/e^6)*x
+ 4*(19*C*d^2*e^5 + 15*B*d*e^6 + 5*A*e^7)/e^6)*x + 15*(13*C*d^3*e^4 + 15*B*
d^2*e^5 + 12*A*d*e^6)/e^6)*x + 8*(38*C*d^4*e^3 + 45*B*d^3*e^4 + 55*A*d^2*e^
5)/e^6) + 1/8*(13*C*d^5 + 15*B*d^4*e + 20*A*d^3*e^2)*arcsin(e*x/d)*sgn(d)*s
gn(e)/(e^2*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3\left(A+Bx+Cx^2\right)}{\sqrt{d^2-e^2x^2}}\,dx = \int \frac{{\left(d+e\,x\right)}^3\,\left(C\,x^2+B\,x+A\right)}{\sqrt{d^2-e^2\,x^2}}\,dx$$

```
[In] int(((d + e*x)^3*(A + B*x + C*x^2))/(d^2 - e^2*x^2)^(1/2),x)
```

```
[Out] int(((d + e*x)^3*(A + B*x + C*x^2))/(d^2 - e^2*x^2)^(1/2), x)
```

3.11 $\int \frac{(d+ex)^2(A+Bx+Cx^2)}{\sqrt{d^2-e^2x^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 191

$$\begin{aligned}\int \frac{(d+ex)^2\,(A+Bx+Cx^2)}{\sqrt{d^2-e^2x^2}}\,dx = &-\frac{d(4Cd^2+e(5Bd+6Ae))\sqrt{d^2-e^2x^2}}{3e^3}\\ &-\frac{(7Cd^2+4e(2Bd+Ae))\,x\sqrt{d^2-e^2x^2}}{8e^2}\\ &-\frac{(2Cd+Be)x^2\sqrt{d^2-e^2x^2}}{3e}-\frac{1}{4}Cx^3\sqrt{d^2-e^2x^2}\\ &+\frac{d^2(7Cd^2+8Bde+12Ae^2)\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^3}\end{aligned}$$

```
[Out] 1/8*d^2*(12*A*e^2+8*B*d*e+7*C*d^2)*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3-1/3
*d*(4*C*d^2+e*(6*A*e+5*B*d))*(-e^2*x^2+d^2)^(1/2)/e^3-1/8*(7*C*d^2+4*e*(A*e
+2*B*d))*x*(-e^2*x^2+d^2)^(1/2)/e^2-1/3*(B*e+2*C*d)*x^2*(-e^2*x^2+d^2)^(1/2
)/e-1/4*C*x^3*(-e^2*x^2+d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used

$= \{1829, 655, 223, 209\}$

$$\int \frac{(d+ex)^2 (A+Bx+Cx^2)}{\sqrt{d^2-e^2x^2}}\,dx = \frac{d^2 \arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)(12Ae^2+8Bde+7Cd^2)}{8e^3} - \frac{x\sqrt{d^2-e^2x^2}(4e(Ae+2Bd)+7Cd^2)}{8e^2} - \frac{d\sqrt{d^2-e^2x^2}(e(6Ae+5Bd)+4Cd^2)}{3e^3} - \frac{x^2\sqrt{d^2-e^2x^2}(Be+2Cd)}{3e} - \frac{1}{4}Cx^3\sqrt{d^2-e^2x^2}$$

```
[In] Int[((d + e*x)^2*(A + B*x + C*x^2))/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] -1/3*(d*(4*C*d^2 + e*(5*B*d + 6*A*e))*Sqrt[d^2 - e^2*x^2])/e^3 - ((7*C*d^2
+ 4*e*(2*B*d + A*e))*x*Sqrt[d^2 - e^2*x^2])/(8*e^2) - ((2*C*d + B*e)*x^2*Sq
rt[d^2 - e^2*x^2])/(3*e) - (C*x^3*Sqrt[d^2 - e^2*x^2])/4 + (d^2*(7*C*d^2 +
8*B*d*e + 12*A*e^2)*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(8*e^3)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1829

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{4}Cx^3\sqrt{d^2-e^2x^2}-\frac{\int \frac{-4Ad^2e^2-4de^2(Bd+2Ae)x-e^2\left(7Cd^2+4e(2Bd+Ae)\right)x^2-4e^3(2Cd+Be)x^3}{\sqrt{d^2-e^2x^2}}\,dx}{4e^2}\\
&= -\frac{(2Cd+Be)x^2\sqrt{d^2-e^2x^2}}{3e}-\frac{1}{4}Cx^3\sqrt{d^2-e^2x^2}\\
&\quad+\frac{\int \frac{12Ad^2e^4+4de^3\left(4Cd^2+e(5Bd+6Ae)\right)x+3e^4\left(7Cd^2+4e(2Bd+Ae)\right)x^2}{\sqrt{d^2-e^2x^2}}\,dx}{12e^4}\\
&= -\frac{\left(7Cd^2+4e(2Bd+Ae)\right)x\sqrt{d^2-e^2x^2}}{8e^2}-\frac{(2Cd+Be)x^2\sqrt{d^2-e^2x^2}}{3e}\\
&\quad-\frac{1}{4}Cx^3\sqrt{d^2-e^2x^2}-\frac{\int \frac{-3d^2e^4\left(7Cd^2+8Bde+12Ae^2\right)-8de^5\left(4Cd^2+e(5Bd+6Ae)\right)x}{\sqrt{d^2-e^2x^2}}\,dx}{24e^6}\\
&= -\frac{d\left(4Cd^2+e(5Bd+6Ae)\right)\sqrt{d^2-e^2x^2}}{3e^3}-\frac{\left(7Cd^2+4e(2Bd+Ae)\right)x\sqrt{d^2-e^2x^2}}{8e^2}\\
&\quad-\frac{(2Cd+Be)x^2\sqrt{d^2-e^2x^2}}{3e}-\frac{1}{4}Cx^3\sqrt{d^2-e^2x^2}\\
&\quad+\frac{\left(d^2\left(7Cd^2+8Bde+12Ae^2\right)\right)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{8e^2}\\
&= -\frac{d\left(4Cd^2+e(5Bd+6Ae)\right)\sqrt{d^2-e^2x^2}}{3e^3}-\frac{\left(7Cd^2+4e(2Bd+Ae)\right)x\sqrt{d^2-e^2x^2}}{8e^2}\\
&\quad-\frac{(2Cd+Be)x^2\sqrt{d^2-e^2x^2}}{3e}-\frac{1}{4}Cx^3\sqrt{d^2-e^2x^2}\\
&\quad+\frac{\left(d^2\left(7Cd^2+8Bde+12Ae^2\right)\right)\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx,x,\frac{x}{\sqrt{d^2-e^2x^2}}\right)}{8e^2}\\
&= -\frac{d\left(4Cd^2+e(5Bd+6Ae)\right)\sqrt{d^2-e^2x^2}}{3e^3}-\frac{\left(7Cd^2+4e(2Bd+Ae)\right)x\sqrt{d^2-e^2x^2}}{8e^2}\\
&\quad-\frac{(2Cd+Be)x^2\sqrt{d^2-e^2x^2}}{3e}-\frac{1}{4}Cx^3\sqrt{d^2-e^2x^2}\\
&\quad+\frac{d^2\left(7Cd^2+8Bde+12Ae^2\right)\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.79

$$\int \frac{(d+ex)^2\,(A+Bx+Cx^2)}{\sqrt{d^2-e^2x^2}}\,dx =$$

$$-\frac{\sqrt{d^2-e^2x^2}(C(32d^3+21d^2ex+16de^2x^2+6e^3x^3)+4e(3Ae(4d+ex)+2B(5d^2+3dex+e^2x^2)))+6d^2}{24e^3}$$

```
[In] Integrate[((d + e*x)^2*(A + B*x + C*x^2))/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] -1/24*(Sqrt[d^2 - e^2*x^2]*(C*(32*d^3 + 21*d^2*e*x + 16*d*e^2*x^2 + 6*e^3*x
^3) + 4*e*(3*A*e*(4*d + e*x) + 2*B*(5*d^2 + 3*d*e*x + e^2*x^2))) + 6*d^2*(7
*C*d^2 + 4*e*(2*B*d + 3*A*e))*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2]
)])/e^3
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.76

method	result
risch	$-\frac{(6C\,e^3x^3+8B\,e^3x^2+16Cd\,e^2x^2+12A\,e^3x+24Bd\,e^2x+21C\,d^2ex+48Ad\,e^2+40B\,d^2e+32d^3C)\sqrt{-e^2x^2+d^2}}{24e^3}+\frac{d^2(12A\,e^2+8Bde+7}{}$
default	$\frac{A\,d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}}+e^2C\left(-\frac{x^3\sqrt{-e^2x^2+d^2}}{4e^2}+\frac{3d^2\left(-\frac{x\sqrt{-e^2x^2+d^2}}{2e^2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2e^2\sqrt{e^2}}\right)}{4e^2}\right)+(B\,e^2+2d$

```
[In] int((e*x+d)^2*(C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/24/e^3*(6*C*e^3*x^3+8*B*e^3*x^2+16*C*d*e^2*x^2+12*A*e^3*x+24*B*d*e^2*x+2
1*C*d^2*e*x+48*A*d*e^2+40*B*d^2*e+32*C*d^3)*(-e^2*x^2+d^2)^(1/2)+1/8*d^2/e^
2*(12*A*e^2+8*B*d*e+7*C*d^2)/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2
)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.76

$$\int \frac{(d+ex)^2\,(A+Bx+Cx^2)}{\sqrt{d^2-e^2x^2}}\,dx = \\ -\frac{6\,(7\,Cd^4+8\,Bd^3e+12\,Ad^2e^2)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+(6\,Ce^3x^3+32\,Cd^3+40\,Bd^2e+48\,Ade^2+8\,(}{24\,e^3}$$

```
[In] integrate((e*x+d)^2*(C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas
")
```

```
[Out] -1/24*(6*(7*C*d^4 + 8*B*d^3*e + 12*A*d^2*e^2)*arctan(-(d - sqrt(-e^2*x^2 +
d^2))/(e*x)) + (6*C*e^3*x^3 + 32*C*d^3 + 40*B*d^2*e + 48*A*d*e^2 + 8*(2*C*d
*e^2 + B*e^3)*x^2 + 3*(7*C*d^2*e + 8*B*d*e^2 + 4*A*e^3)*x)*sqrt(-e^2*x^2 +
d^2))/e^3
```

Sympy [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.47

$$\int \frac{(d+ex)^2\,(A+Bx+Cx^2)}{\sqrt{d^2-e^2x^2}}\,dx \\ = \begin{cases} \sqrt{d^2-e^2x^2}\left(-\frac{Cx^3}{4}-\frac{x^2(Be^2+2Cde)}{3e^2}-\frac{x\left(Ae^2+2Bde+\frac{7Cd^2}{4}\right)}{2e^2}-\frac{2Ade+Bd^2+\frac{2d^2\left(Be^2+2Cde\right)}{3e^2}}{e^2}\right)+\left(Ad^2+\frac{d^2\left(Ae^2+2Bde\right.}{2e^2}\right. \\ \frac{Ad^2x+\frac{Ce^2x^5}{5}+\frac{x^4\left(Be^2+2Cde\right)}{4}+\frac{x^3\left(Ae^2+2Bde+Cd^2\right)}{3}+\frac{x^2\cdot\left(2Ade+Bd^2\right)}{2}}{\sqrt{d^2}} \end{cases}$$

```
[In] integrate((e*x+d)**2*(C*x**2+B*x+A)/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Piecewise((sqrt(d**2 - e**2*x**2)*(-C*x**3/4 - x**2*(B*e**2 + 2*C*d*e)/(3*e
**2) - x*(A*e**2 + 2*B*d*e + 7*C*d**2/4)/(2*e**2) - (2*A*d*e + B*d**2 + 2*d
**2*(B*e**2 + 2*C*d*e)/(3*e**2))/e**2) + (A*d**2 + d**2*(A*e**2 + 2*B*d*e +
 7*C*d**2/4)/(2*e**2))*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 -
 e**2*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True)),
 Ne(e**2, 0)), ((A*d**2*x + C*e**2*x**5/5 + x**4*(B*e**2 + 2*C*d*e)/4 + x**
3*(A*e**2 + 2*B*d*e + C*d**2)/3 + x**2*(2*A*d*e + B*d**2)/2)/sqrt(d**2), Tr
ue))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.50

$$\int \frac{(d+ex)^2\,(A+Bx+Cx^2)}{\sqrt{d^2-e^2x^2}}\,dx = -\frac{1}{4}\sqrt{-e^2x^2+d^2}Cx^3 + \frac{Ad^2\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{\sqrt{e^2}} + \frac{3\,Cd^4\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{8\,\sqrt{e^2}e^2} - \frac{3\,\sqrt{-e^2x^2+d^2}Cd^2x}{8\,e^2} - \frac{\sqrt{-e^2x^2+d^2}Bd^2}{e^2} - \frac{2\,\sqrt{-e^2x^2+d^2}Ad}{e} - \frac{\sqrt{-e^2x^2+d^2}(2\,Cde+Be^2)x^2}{3\,e^2} + \frac{(Cd^2+2\,Bde+Ae^2)d^2\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{2\,\sqrt{e^2}e^2} - \frac{\sqrt{-e^2x^2+d^2}(Cd^2+2\,Bde+Ae^2)x}{2\,e^2} - \frac{2\,\sqrt{-e^2x^2+d^2}(2\,Cde+Be^2)d^2}{3\,e^4}$$

```
[In] integrate((e*x+d)^2*(C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima
")
```

```
[Out] -1/4*sqrt(-e^2*x^2 + d^2)*C*x^3 + A*d^2*arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e^
2) + 3/8*C*d^4*arcsin(e^2*x/(d*sqrt(e^2)))/(sqrt(e^2)*e^2) - 3/8*sqrt(-e^2*
x^2 + d^2)*C*d^2*x/e^2 - sqrt(-e^2*x^2 + d^2)*B*d^2/e^2 - 2*sqrt(-e^2*x^2 +
 d^2)*A*d/e - 1/3*sqrt(-e^2*x^2 + d^2)*(2*C*d*e + B*e^2)*x^2/e^2 + 1/2*(C*d
^2 + 2*B*d*e + A*e^2)*d^2*arcsin(e^2*x/(d*sqrt(e^2)))/(sqrt(e^2)*e^2) - 1/2
*sqrt(-e^2*x^2 + d^2)*(C*d^2 + 2*B*d*e + A*e^2)*x/e^2 - 2/3*sqrt(-e^2*x^2 +
 d^2)*(2*C*d*e + B*e^2)*d^2/e^4
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.78

$$\int \frac{(d+ex)^2\,(A+Bx+Cx^2)}{\sqrt{d^2-e^2x^2}}\,dx =$$

$$-\frac{1}{24}\sqrt{-e^2x^2+d^2}\Bigg(\Bigg(2\left(3\,Cx+\frac{4\,(2\,Cde^4+Be^5)}{e^5}\right)x+\frac{3\,(7\,Cd^2e^3+8\,Bde^4+4\,Ae^5)}{e^5}\Bigg)x + 8\,(4\,Cd^3e^2 +$$

$$+ \frac{(7\,Cd^4+8\,Bd^3e+12\,Ad^2e^2)\arcsin\left(\frac{ex}{d}\right)\mathrm{sgn}(d)\,\mathrm{sgn}(e)}{8\,e^2|e|}$$

```
[In] integrate((e*x+d)^2*(C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/24*sqrt(-e^2*x^2 + d^2)*((2*(3*C*x + 4*(2*C*d*e^4 + B*e^5)/e^5)*x + 3*(7
*C*d^2*e^3 + 8*B*d*e^4 + 4*A*e^5)/e^5)*x + 8*(4*C*d^3*e^2 + 5*B*d^2*e^3 + 6
*A*d*e^4)/e^5) + 1/8*(7*C*d^4 + 8*B*d^3*e + 12*A*d^2*e^2)*arcsin(e*x/d)*sgn
(d)*sgn(e)/(e^2*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2\,(A+Bx+Cx^2)}{\sqrt{d^2-e^2x^2}}\,dx=\int \frac{{\left(d+e\,x\right)}^2\,\left(C\,x^2+B\,x+A\right)}{\sqrt{d^2-e^2\,x^2}}\,dx$$

```
[In] int(((d + e*x)^2*(A + B*x + C*x^2))/(d^2 - e^2*x^2)^(1/2),x)
```

```
[Out] int(((d + e*x)^2*(A + B*x + C*x^2))/(d^2 - e^2*x^2)^(1/2), x)
```

3.12 $\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{\sqrt{d^2-e^2x^2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 143

$$\begin{aligned}\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{\sqrt{d^2-e^2x^2}}\,dx = &-\frac{(2Cd^2+3e(Bd+Ae))\sqrt{d^2-e^2x^2}}{3e^3}\\ &-\frac{(Cd+Be)x\sqrt{d^2-e^2x^2}}{2e^2}-\frac{Cx^2\sqrt{d^2-e^2x^2}}{3e}\\ &+\frac{d(Cd^2+e(Bd+2Ae))\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^3}\end{aligned}$$

```
[Out] 1/2*d*(C*d^2+e*(2*A*e+B*d))*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3-1/3*(2*C*d
^2+3*e*(A*e+B*d))*(-e^2*x^2+d^2)^(1/2)/e^3-1/2*(B*e+C*d)*x*(-e^2*x^2+d^2)^(
1/2)/e^2-1/3*C*x^2*(-e^2*x^2+d^2)^(1/2)/e
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1829, 655, 223, 209}

$$\begin{aligned}\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{\sqrt{d^2-e^2x^2}}\,dx = &\frac{d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)(e(2Ae+Bd)+Cd^2)}{2e^3}\\ &-\frac{\sqrt{d^2-e^2x^2}(3e(Ae+Bd)+2Cd^2)}{3e^3}\\ &-\frac{x\sqrt{d^2-e^2x^2}(Be+Cd)}{2e^2}-\frac{Cx^2\sqrt{d^2-e^2x^2}}{3e}\end{aligned}$$

```
[In] Int[((d + e*x)*(A + B*x + C*x^2))/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] -1/3*((2*C*d^2 + 3*e*(B*d + A*e))*Sqrt[d^2 - e^2*x^2])/e^3 - ((C*d + B*e)*x
*Sqrt[d^2 - e^2*x^2])/(2*e^2) - (C*x^2*Sqrt[d^2 - e^2*x^2])/(3*e) + (d*(C*d
^2 + e*(B*d + 2*A*e))*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e^3)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1829

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{Cx^2\sqrt{d^2-e^2x^2}}{3e}-\frac{\int \frac{-3Ade^2-e\left(2Cd^2+3e(Bd+Ae)\right)x-3e^2(Cd+Be)x^2}{\sqrt{d^2-e^2x^2}}\,dx}{3e^2}\\
&= -\frac{(Cd+Be)x\sqrt{d^2-e^2x^2}}{2e^2}-\frac{Cx^2\sqrt{d^2-e^2x^2}}{3e}+\frac{\int \frac{3de^2\left(Cd^2+e(Bd+2Ae)\right)+2e^3\left(2Cd^2+3e(Bd+Ae)\right)x}{\sqrt{d^2-e^2x^2}}\,dx}{6e^4}\\
&= -\frac{\left(2Cd^2+3e(Bd+Ae)\right)\sqrt{d^2-e^2x^2}}{3e^3}-\frac{(Cd+Be)x\sqrt{d^2-e^2x^2}}{2e^2}\\
&\quad -\frac{Cx^2\sqrt{d^2-e^2x^2}}{3e}+\frac{\left(d\left(Cd^2+e(Bd+2Ae)\right)\right)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{2e^2}\\
&= -\frac{\left(2Cd^2+3e(Bd+Ae)\right)\sqrt{d^2-e^2x^2}}{3e^3}-\frac{(Cd+Be)x\sqrt{d^2-e^2x^2}}{2e^2}\\
&\quad -\frac{Cx^2\sqrt{d^2-e^2x^2}}{3e}+\frac{\left(d\left(Cd^2+e(Bd+2Ae)\right)\right)\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx,x,\frac{x}{\sqrt{d^2-e^2x^2}}\right)}{2e^2}
\end{aligned}$$

$$= -\frac{(2Cd^2+3e(Bd+Ae))\sqrt{d^2-e^2x^2}}{3e^3} - \frac{(Cd+Be)x\sqrt{d^2-e^2x^2}}{2e^2} - \frac{Cx^2\sqrt{d^2-e^2x^2}}{3e} + \frac{d(Cd^2+e(Bd+2Ae))\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^3}$$

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.80

$$\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{\sqrt{d^2-e^2x^2}}\,dx = -\frac{\sqrt{d^2-e^2x^2}(3e(2Bd+2Ae+Bex)+C(4d^2+3dex+2e^2x^2))+6d(Cd^2+e(Bd+2Ae))\arctan\left(\frac{e}{\sqrt{d^2}-\sqrt{}}\right.}{6e^3}$$

```
[In] Integrate[((d + e*x)*(A + B*x + C*x^2))/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] -1/6*(Sqrt[d^2 - e^2*x^2]*(3*e*(2*B*d + 2*A*e + B*e*x) + C*(4*d^2 + 3*d*e*x
 + 2*e^2*x^2)) + 6*d*(C*d^2 + e*(B*d + 2*A*e))*ArcTan[(e*x)/(Sqrt[d^2] - Sq
rt[d^2 - e^2*x^2])])/e^3
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.77

method	result
risch	$-\frac{\left(2C\,e^2x^2+3xB\,e^2+3Cdex+6A\,e^2+6Bde+4C\,d^2\right)\sqrt{-e^2x^2+d^2}}{6e^3}+\frac{d\left(2A\,e^2+Bde+C\,d^2\right)\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2e^2\sqrt{e^2}}$
default	$\frac{dA\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}}+eC\left(-\frac{x^2\sqrt{-e^2x^2+d^2}}{3e^2}-\frac{2d^2\sqrt{-e^2x^2+d^2}}{3e^4}\right)+(Be+Cd)\left(-\frac{x\sqrt{-e^2x^2+d^2}}{2e^2}+\frac{d^2\arctan\left(\right.}{2e^2}\right.$

```
[In] int((e*x+d)*(C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(2*C*e^2*x^2+3*B*e^2*x+3*C*d*e*x+6*A*e^2+6*B*d*e+4*C*d^2)/e^3*(-e^2*x^
2+d^2)^(1/2)+1/2*d/e^2*(2*A*e^2+B*d*e+C*d^2)/(e^2)^(1/2)*arctan((e^2)^(1/2)
*x/(-e^2*x^2+d^2)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.76

$$\int \frac{(d+ex)\,(A+Bx+Cx^2)}{\sqrt{d^2-e^2x^2}}\,dx =$$

$$-\frac{6\,(Cd^3+Bd^2e+2\,Ade^2)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+(2\,Ce^2x^2+4\,Cd^2+6\,Bde+6\,Ae^2+3\,(Cde+Be^2}{6\,e^3}$$

```
[In] integrate((e*x+d)*(C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*(6*(C*d^3 + B*d^2*e + 2*A*d*e^2)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e
*x)) + (2*C*e^2*x^2 + 4*C*d^2 + 6*B*d*e + 6*A*e^2 + 3*(C*d*e + B*e^2)*x)*sq
rt(-e^2*x^2 + d^2))/e^3
```

Sympy [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.26

$$\int \frac{(d+ex)\,(A+Bx+Cx^2)}{\sqrt{d^2-e^2x^2}}\,dx$$

$$= \begin{cases} \sqrt{d^2-e^2x^2}\left(-\frac{Cx^2}{3e}-\frac{x(Be+Cd)}{2e^2}-\frac{Ae+Bd+\frac{2Cd^2}{3e}}{e^2}\right)+\left(Ad+\frac{d^2(Be+Cd)}{2e^2}\right)\Bigg(\Bigg(\begin{cases} \frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d \\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{othe} \end{cases} \\ \frac{Adx+\frac{Cex^4}{4}+\frac{x^3(Be+Cd)}{3}+\frac{x^2(Ae+Bd)}{2}}{\sqrt{d^2}} \end{cases}$$

```
[In] integrate((e*x+d)*(C*x**2+B*x+A)/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Piecewise((sqrt(d**2 - e**2*x**2)*(-C*x**2/(3*e) - x*(B*e + C*d)/(2*e**2) -
 (A*e + B*d + 2*C*d**2/(3*e))/e**2) + (A*d + d**2*(B*e + C*d)/(2*e**2))*Pie
cewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e**2*x**2))/sqrt(-e**2),
Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True)), Ne(e**2, 0)), ((A*d*x + C
*e*x**4/4 + x**3*(B*e + C*d)/3 + x**2*(A*e + B*d)/2)/sqrt(d**2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.20

$$\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{\sqrt{d^2-e^2x^2}}\,dx = -\frac{\sqrt{-e^2x^2+d^2}Cx^2}{3\,e} + \frac{Ad\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{\sqrt{e^2}} + \frac{(Cd+Be)d^2\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{2\sqrt{e^2}e^2} - \frac{2\sqrt{-e^2x^2+d^2}Cd^2}{3\,e^3} - \frac{\sqrt{-e^2x^2+d^2}Bd}{e^2} - \frac{\sqrt{-e^2x^2+d^2}A}{e} - \frac{\sqrt{-e^2x^2+d^2}(Cd+Be)x}{2\,e^2}$$

```
[In] integrate((e*x+d)*(C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(-e^2*x^2 + d^2)*C*x^2/e + A*d*arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e^
2) + 1/2*(C*d + B*e)*d^2*arcsin(e^2*x/(d*sqrt(e^2)))/(sqrt(e^2)*e^2) - 2/3*
sqrt(-e^2*x^2 + d^2)*C*d^2/e^3 - sqrt(-e^2*x^2 + d^2)*B*d/e^2 - sqrt(-e^2*x
^2 + d^2)*A/e - 1/2*sqrt(-e^2*x^2 + d^2)*(C*d + B*e)*x/e^2
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.78

$$\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{\sqrt{d^2-e^2x^2}}\,dx = -\frac{1}{6}\sqrt{-e^2x^2+d^2}\left(\left(\frac{2\,Cx}{e}+\frac{3\,(Cde^3+Be^4)}{e^5}\right)x+\frac{2\,(2\,Cd^2e^2+3\,Bde^3+3\,Ae^4)}{e^5}\right) + \frac{(Cd^3+Bd^2e+2\,Ade^2)\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{2\,e^2|e|}$$

```
[In] integrate((e*x+d)*(C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(-e^2*x^2 + d^2)*((2*C*x/e + 3*(C*d*e^3 + B*e^4)/e^5)*x + 2*(2*C*d
^2*e^2 + 3*B*d*e^3 + 3*A*e^4)/e^5) + 1/2*(C*d^3 + B*d^2*e + 2*A*d*e^2)*arcs
in(e*x/d)*sgn(d)*sgn(e)/(e^2*abs(e))
```

Mupad [B] (verification not implemented)

Time = 13.76 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.89

$$\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{\sqrt{d^2-e^2x^2}}\,dx$$

$$= \begin{cases} \frac{2\,C\,d\,x^3+3\,B\,d\,x^2+6\,A\,d\,x}{6\,\sqrt{d^2}} \\ \frac{A\,d\,\ln\left(x\,\sqrt{-e^2}+\sqrt{d^2-e^2\,x^2}\right)}{\sqrt{-e^2}} - \frac{A\,\sqrt{d^2-e^2\,x^2}}{e} - \frac{B\,d\,\sqrt{d^2-e^2\,x^2}}{e^2} - \frac{B\,x\,\sqrt{d^2-e^2\,x^2}}{2\,e} - \frac{C\,\sqrt{d^2-e^2\,x^2}\,\left(2\,d^2+e^2\,x^2\right)}{3\,e^3} - \frac{C\,d^3\,\ln(2\,x\,\sqrt{-}}{2\,(-} \end{cases}$$

```
[In] int(((d + e*x)*(A + B*x + C*x^2))/(d^2 - e^2*x^2)^(1/2),x)
```

```
[Out] piecewise(e == 0, (6*A*d*x + 3*B*d*x^2 + 2*C*d*x^3)/(6*(d^2)^(1/2)), e ~= 0
, - (A*(d^2 - e^2*x^2)^(1/2))/e + (A*d*log(x*(-e^2)^(1/2) + (d^2 - e^2*x^2)
^(1/2)))/(-e^2)^(1/2) - (B*d*(d^2 - e^2*x^2)^(1/2))/e^2 - (B*x*(d^2 - e^2*x
^2)^(1/2))/(2*e) - (C*(d^2 - e^2*x^2)^(1/2)*(2*d^2 + e^2*x^2))/(3*e^3) - (C
*d^3*log(2*x*(-e^2)^(1/2) + 2*(d^2 - e^2*x^2)^(1/2)))/(2*(-e^2)^(3/2)) - (B
*d^2*e*log(2*x*(-e^2)^(1/2) + 2*(d^2 - e^2*x^2)^(1/2)))/(2*(-e^2)^(3/2)) -
(C*d*x*(d^2 - e^2*x^2)^(1/2))/(2*e^2))
```

3.13 $\int \frac{A+Bx+Cx^2}{\sqrt{d^2-e^2x^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 87

$$\int \frac{A+Bx+Cx^2}{\sqrt{d^2-e^2x^2}}\,dx=-\frac{B\sqrt{d^2-e^2x^2}}{e^2}-\frac{Cx\sqrt{d^2-e^2x^2}}{2e^2}+\frac{(Cd^2+2Ae^2)\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^3}$$

```
[Out] 1/2*(2*A*e^2+C*d^2)*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3-B*(-e^2*x^2+d^2)^(
1/2)/e^2-1/2*C*x*(-e^2*x^2+d^2)^(1/2)/e^2
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1829, 655, 223, 209}

$$\int \frac{A+Bx+Cx^2}{\sqrt{d^2-e^2x^2}}\,dx=\frac{(2Ae^2+Cd^2)\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^3}-\frac{B\sqrt{d^2-e^2x^2}}{e^2}-\frac{Cx\sqrt{d^2-e^2x^2}}{2e^2}$$

```
[In] Int[(A + B*x + C*x^2)/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] -((B*Sqrt[d^2 - e^2*x^2])/e^2) - (C*x*Sqrt[d^2 - e^2*x^2])/(2*e^2) + ((C*d^
2 + 2*A*e^2)*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e^3)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1829

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{Cx\sqrt{d^2-e^2x^2}}{2e^2}-\frac{\int \frac{-Cd^2-2Ae^2-2Be^2x}{\sqrt{d^2-e^2x^2}}\,dx}{2e^2}\\
&= -\frac{B\sqrt{d^2-e^2x^2}}{e^2}-\frac{Cx\sqrt{d^2-e^2x^2}}{2e^2}-\frac{(-Cd^2-2Ae^2)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{2e^2}\\
&= -\frac{B\sqrt{d^2-e^2x^2}}{e^2}-\frac{Cx\sqrt{d^2-e^2x^2}}{2e^2}-\frac{(-Cd^2-2Ae^2)\,\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx,x,\frac{x}{\sqrt{d^2-e^2x^2}}\right)}{2e^2}\\
&= -\frac{B\sqrt{d^2-e^2x^2}}{e^2}-\frac{Cx\sqrt{d^2-e^2x^2}}{2e^2}+\frac{(Cd^2+2Ae^2)\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.93

$$\int \frac{A+Bx+Cx^2}{\sqrt{d^2-e^2x^2}}\,dx=\frac{(-2B-Cx)\sqrt{d^2-e^2x^2}}{2e^2}+\frac{(-Cd^2-2Ae^2)\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{e^3}$$

```
[In] Integrate[(A + B*x + C*x^2)/Sqrt[d^2 - e^2*x^2],x]
[Out] ((-2*B - C*x)*Sqrt[d^2 - e^2*x^2])/(2*e^2) + ((-(C*d^2) - 2*A*e^2)*ArcTan[(
e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/e^3
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.83

method	result	size
risch	$-\frac{(Cx+2B)\sqrt{-e^2x^2+d^2}}{2e^2}+\frac{(2A\,e^2+C\,d^2)\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2e^2\sqrt{e^2}}$	72
default	$\frac{A\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}}+C\left(-\frac{x\sqrt{-e^2x^2+d^2}}{2e^2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2e^2\sqrt{e^2}}\right)-\frac{B\sqrt{-e^2x^2+d^2}}{e^2}$	109

```
[In] int((C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/2*(C*x+2*B)/e^2*(-e^2*x^2+d^2)^(1/2)+1/2*(2*A*e^2+C*d^2)/e^2/(e^2)^(1/2)
*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.82

$$\int \frac{A+Bx+Cx^2}{\sqrt{d^2-e^2x^2}}\,dx = -\frac{2\left(Cd^2+2\,Ae^2\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\sqrt{-e^2x^2+d^2}(Cex+2\,Be)}{2\,e^3}$$

```
[In] integrate((C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
[Out] -1/2*(2*(C*d^2 + 2*A*e^2)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + sqrt(
-e^2*x^2 + d^2)*(C*e*x + 2*B*e))/e^3
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.43

$$\int \frac{A+Bx+Cx^2}{\sqrt{d^2-e^2x^2}}\,dx = \begin{cases} \left(A+\frac{Cd^2}{2e^2}\right)\left(\begin{cases} \frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for}\ d^2\neq 0 \\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise} \end{cases}\right)+\sqrt{d^2-e^2x^2}\left(-\frac{B}{e^2}-\frac{Cx}{2e^2}\right) & \text{for}\ e^2\neq 0 \\ \frac{Ax+\frac{Bx^2}{2}+\frac{Cx^3}{3}}{\sqrt{d^2}} & \text{otherwise} \end{cases}$$

```
[In] integrate((C*x**2+B*x+A)/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Piecewise(((A + C*d**2/(2*e**2))*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*s
qrt(d**2 - e**2*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2
), True)) + sqrt(d**2 - e**2*x**2)*(-B/e**2 - C*x/(2*e**2)), Ne(e**2, 0)),
((A*x + B*x**2/2 + C*x**3/3)/sqrt(d**2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.05

$$\int \frac{A+Bx+Cx^2}{\sqrt{d^2-e^2x^2}}\,dx = \frac{A\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{\sqrt{e^2}} + \frac{Cd^2\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{2\sqrt{e^2}e^2} - \frac{\sqrt{-e^2x^2+d^2}Cx}{2e^2} - \frac{\sqrt{-e^2x^2+d^2}B}{e^2}$$

```
[In] integrate((C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] A*arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e^2) + 1/2*C*d^2*arcsin(e^2*x/(d*sqrt(e^
2)))/(sqrt(e^2)*e^2) - 1/2*sqrt(-e^2*x^2 + d^2)*C*x/e^2 - sqrt(-e^2*x^2 + d
^2)*B/e^2
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.71

$$\int \frac{A+Bx+Cx^2}{\sqrt{d^2-e^2x^2}}\,dx = -\frac{1}{2}\sqrt{-e^2x^2+d^2}\left(\frac{Cx}{e^2}+\frac{2B}{e^2}\right) + \frac{(Cd^2+2Ae^2)\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{2e^2|e|}$$

```
[In] integrate((C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(-e^2*x^2 + d^2)*(C*x/e^2 + 2*B/e^2) + 1/2*(C*d^2 + 2*A*e^2)*arcsi
n(e*x/d)*sgn(d)*sgn(e)/(e^2*abs(e))
```

Mupad [B] (verification not implemented)

Time = 13.34 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.70

$$\int \frac{A+Bx+Cx^2}{\sqrt{d^2-e^2x^2}}\,dx$$
$$=\begin{cases} \frac{2\,C\,x^3+3\,B\,x^2+6\,A\,x}{6\,\sqrt{d^2}} & \text{if}\ \ e=0\\ \frac{A\,\ln\left(x\,\sqrt{-e^2}+\sqrt{d^2-e^2\,x^2}\right)}{\sqrt{-e^2}}-\frac{B\,\sqrt{d^2-e^2\,x^2}}{e^2}-\frac{C\,x\,\sqrt{d^2-e^2\,x^2}}{2\,e^2}-\frac{C\,d^2\,\ln\left(2\,x\,\sqrt{-e^2}+2\,\sqrt{d^2-e^2\,x^2}\right)}{2\,{\left(-e^2\right)}^{3/2}} & \text{if}\ \ e\neq 0 \end{cases}$$

```
[In] int((A + B*x + C*x^2)/(d^2 - e^2*x^2)^(1/2),x)
```

```
[Out] piecewise(e == 0, (6*A*x + 3*B*x^2 + 2*C*x^3)/(6*(d^2)^(1/2)), e ~= 0, (A*l
og(x*(-e^2)^(1/2) + (d^2 - e^2*x^2)^(1/2)))/(-e^2)^(1/2) - (B*(d^2 - e^2*x^
2)^(1/2))/e^2 - (C*x*(d^2 - e^2*x^2)^(1/2))/(2*e^2) - (C*d^2*log(2*x*(-e^2)
^(1/2) + 2*(d^2 - e^2*x^2)^(1/2)))/(2*(-e^2)^(3/2)))
```

3.14 $\int \frac{A+Bx+Cx^2}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 103

$$\int \frac{A+Bx+Cx^2}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx=-\frac{C\sqrt{d^2-e^2x^2}}{e^3}-\frac{(Cd^2-Bde+Ae^2)\sqrt{d^2-e^2x^2}}{de^3(d+ex)}-\frac{(Cd-Be)\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}$$

```
[Out] -(-B*e+C*d)*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3-C*(-e^2*x^2+d^2)^(1/2)/e^3
-(A*e^2-B*d*e+C*d^2)*(-e^2*x^2+d^2)^(1/2)/d/e^3/(e*x+d)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1653, 807, 223, 209}

$$\int \frac{A+Bx+Cx^2}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx=-\frac{\sqrt{d^2-e^2x^2}(Ae^2-Bde+Cd^2)}{de^3(d+ex)}-\frac{\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)(Cd-Be)}{e^3}-\frac{C\sqrt{d^2-e^2x^2}}{e^3}$$

```
[In] Int[(A + B*x + C*x^2)/((d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -((C*Sqrt[d^2 - e^2*x^2])/e^3) - ((C*d^2 - B*d*e + A*e^2)*Sqrt[d^2 - e^2*x^
2])/(d*e^3*(d + e*x)) - ((C*d - B*e)*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^3
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 807

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 1653

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{C\sqrt{d^2-e^2x^2}}{e^3}-\frac{\int \frac{-Ae^4+e^3(Cd-Be)x}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx}{e^4}\\
&= -\frac{C\sqrt{d^2-e^2x^2}}{e^3}-\frac{(Cd^2-Bde+Ae^2)\sqrt{d^2-e^2x^2}}{de^3(d+ex)}-\frac{(Cd-Be)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e^2}\\
&= -\frac{C\sqrt{d^2-e^2x^2}}{e^3}-\frac{(Cd^2-Bde+Ae^2)\sqrt{d^2-e^2x^2}}{de^3(d+ex)}\\
&\quad-\frac{(Cd-Be)\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx,x,\frac{x}{\sqrt{d^2-e^2x^2}}\right)}{e^2}\\
&= -\frac{C\sqrt{d^2-e^2x^2}}{e^3}-\frac{(Cd^2-Bde+Ae^2)\sqrt{d^2-e^2x^2}}{de^3(d+ex)}-\frac{(Cd-Be)\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.93

$$\int \frac{A+Bx+Cx^2}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx = \frac{-\frac{\sqrt{d^2-e^2x^2}(e(-Bd+Ae)+Cd(2d+ex))}{d(d+ex)}+2(Cd-Be)\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{e^3}$$

```
[In] Integrate[(A + B*x + C*x^2)/((d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (-((Sqrt[d^2 - e^2*x^2]*(e*(-(B*d) + A*e) + C*d*(2*d + e*x)))/(d*(d + e*x))
) + 2*(C*d - B*e)*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/e^3
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.23

method	result	size
risch	$-\frac{C\sqrt{-e^2x^2+d^2}}{e^3}+\frac{\frac{(Be-Cd)\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{e\sqrt{e^2}}-\frac{\left(A\,e^2-Bde+C\,d^2\right)\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{e^3d\left(x+\frac{d}{e}\right)}}{e}$	127
default	$\frac{\frac{Be\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}}-\frac{C\sqrt{-e^2x^2+d^2}}{e}-\frac{Cd\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}}}{e^2}-\frac{\left(A\,e^2-Bde+C\,d^2\right)\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{e^4d\left(x+\frac{d}{e}\right)}$	149

```
[In] int((C*x^2+B*x+A)/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -C*(-e^2*x^2+d^2)^(1/2)/e^3+1/e*((B*e-C*d)/e/(e^2)^(1/2)*arctan((e^2)^(1/2)
*x/(-e^2*x^2+d^2)^(1/2))-(A*e^2-B*d*e+C*d^2)/e^3/d/(x+d/e)*(-(x+d/e)^2*e^2+
2*d*e*(x+d/e))^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.50

$$\int \frac{A+Bx+Cx^2}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx = -\frac{2\,Cd^3-Bd^2e+Ade^2+(2\,Cd^2e-Bde^2+Ae^3)x-2\,(Cd^3-Bd^2e+(Cd^2e-Bde^2)x)\arctan\Big(-\frac{d-\sqrt{}}{}}{de^4x+d^2e^3}$$

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -(2*C*d^3 - B*d^2*e + A*d*e^2 + (2*C*d^2*e - B*d*e^2 + A*e^3)*x - 2*(C*d^3
- B*d^2*e + (C*d^2*e - B*d*e^2)*x)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)
) + (C*d*e*x + 2*C*d^2 - B*d*e + A*e^2)*sqrt(-e^2*x^2 + d^2))/(d*e^4*x + d^
2*e^3)
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx = \int \frac{A+Bx+Cx^2}{\sqrt{-(-d+ex)(d+ex)}\,(d+ex)}\,dx$$

```
[In] integrate((C*x**2+B*x+A)/(e*x+d)/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral((A + B*x + C*x**2)/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.34

$$\int \frac{A+Bx+Cx^2}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx = -\frac{\sqrt{-e^2x^2+d^2}Cd}{e^4x+de^3} - \frac{\sqrt{-e^2x^2+d^2}A}{de^2x+d^2e} + \frac{\sqrt{-e^2x^2+d^2}B}{e^3x+de^2} - \frac{Cd\arcsin\left(\frac{ex}{d}\right)}{e^3} + \frac{B\arcsin\left(\frac{ex}{d}\right)}{e^2} - \frac{\sqrt{-e^2x^2+d^2}C}{e^3}$$

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(-e^2*x^2 + d^2)*C*d/(e^4*x + d*e^3) - sqrt(-e^2*x^2 + d^2)*A/(d*e^2*x
 + d^2*e) + sqrt(-e^2*x^2 + d^2)*B/(e^3*x + d*e^2) - C*d*arcsin(e*x/d)/e^3
+ B*arcsin(e*x/d)/e^2 - sqrt(-e^2*x^2 + d^2)*C/e^3
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.06

$$\int \frac{A+Bx+Cx^2}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx = -\frac{(Cd-Be)\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{e^2|e|} - \frac{\sqrt{-e^2x^2+d^2}C}{e^3} + \frac{2\,(Cd^2-Bde+Ae^2)}{de^2\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}+1\right)|e|}$$

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] -(C*d - B*e)*arcsin(e*x/d)*sgn(d)*sgn(e)/(e^2*abs(e)) - sqrt(-e^2*x^2 + d^2
)*C/e^3 + 2*(C*d^2 - B*d*e + A*e^2)/(d*e^2*((d*e + sqrt(-e^2*x^2 + d^2)*abs
(e))/(e^2*x) + 1)*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx=\int \frac{C\,x^2+B\,x+A}{\sqrt{d^2-e^2\,x^2}\,\left(d+e\,x\right)}\,dx$$

```
[In] int((A + B*x + C*x^2)/((d^2 - e^2*x^2)^(1/2)*(d + e*x)),x)
[Out] int((A + B*x + C*x^2)/((d^2 - e^2*x^2)^(1/2)*(d + e*x)), x)
```

3.15 $\int \frac{A+Bx+Cx^2}{(d+ex)^2\sqrt{d^2-e^2x^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 163

$$\int \frac{A+Bx+Cx^2}{(d+ex)^2\sqrt{d^2-e^2x^2}}\,dx = -\frac{(Cd^2-Bde+Ae^2)\sqrt{d^2-e^2x^2}}{3de^3(d+ex)^2}+\frac{(2Cd-Be)\sqrt{d^2-e^2x^2}}{de^3(d+ex)} - \frac{(Cd^2-Bde+Ae^2)\sqrt{d^2-e^2x^2}}{3d^2e^3(d+ex)}+\frac{C\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}$$

```
[Out] C*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3-1/3*(A*e^2-B*d*e+C*d^2)*(-e^2*x^2+d^
2)^(1/2)/d/e^3/(e*x+d)^2+(-B*e+2*C*d)*(-e^2*x^2+d^2)^(1/2)/d/e^3/(e*x+d)-1/
3*(A*e^2-B*d*e+C*d^2)*(-e^2*x^2+d^2)^(1/2)/d^2/e^3/(e*x+d)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {1651, 223, 209, 673, 665}

$$\int \frac{A+Bx+Cx^2}{(d+ex)^2\sqrt{d^2-e^2x^2}}\,dx = -\frac{\sqrt{d^2-e^2x^2}(Ae^2-Bde+Cd^2)}{3d^2e^3(d+ex)} - \frac{\sqrt{d^2-e^2x^2}(Ae^2-Bde+Cd^2)}{3de^3(d+ex)^2} + \frac{C\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}+\frac{\sqrt{d^2-e^2x^2}(2Cd-Be)}{de^3(d+ex)}$$

```
[In] Int[(A + B*x + C*x^2)/((d + e*x)^2*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -1/3*((C*d^2 - B*d*e + A*e^2)*Sqrt[d^2 - e^2*x^2])/(d*e^3*(d + e*x)^2) + ((
2*C*d - B*e)*Sqrt[d^2 - e^2*x^2])/(d*e^3*(d + e*x)) - ((C*d^2 - B*d*e + A*e
^2)*Sqrt[d^2 - e^2*x^2])/(3*d^2*e^3*(d + e*x)) + (C*ArcTan[(e*x)/Sqrt[d^2 -
 e^2*x^2]])/e^3
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 673

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-e)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m + p + 1))), x] + Dist[Simpl
ify[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x]
, x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ
[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 Int[ExpandIntegrand[(a + c*x^2)^p, (d + e*x)^m*Pq, x], x] /; FreeQ[{a, c,
d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && EqQ[m + Expon[Pq, x]
+ 2*p + 1, 0] && ILtQ[m, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{C}{e^2\sqrt{d^2-e^2x^2}} + \frac{Cd^2-Bde+Ae^2}{e^2(d+ex)^2\sqrt{d^2-e^2x^2}} + \frac{-2Cd+Be}{e^2(d+ex)\sqrt{d^2-e^2x^2}}\right) dx$$

$$= \frac{C\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e^2} - \frac{(2Cd-Be)\int \frac{1}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx}{e^2} + \frac{(Cd^2-Bde+Ae^2)\int \frac{1}{(d+ex)^2\sqrt{d^2-e^2x^2}}\,dx}{e^2}$$

$$
\begin{aligned}
&= -\frac{(Cd^2 - Bde + Ae^2)\sqrt{d^2 - e^2x^2}}{3de^3(d+ex)^2} + \frac{(2Cd - Be)\sqrt{d^2 - e^2x^2}}{de^3(d+ex)} \\
&\quad + \frac{C\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)}{e^2} + \frac{(Cd^2 - Bde + Ae^2)\int \frac{1}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx}{3de^2} \\
&= -\frac{(Cd^2 - Bde + Ae^2)\sqrt{d^2 - e^2x^2}}{3de^3(d+ex)^2} + \frac{(2Cd - Be)\sqrt{d^2 - e^2x^2}}{de^3(d+ex)} \\
&\quad - \frac{(Cd^2 - Bde + Ae^2)\sqrt{d^2 - e^2x^2}}{3d^2e^3(d+ex)} + \frac{C\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.66

$$
\int \frac{A + Bx + Cx^2}{(d+ex)^2\sqrt{d^2 - e^2x^2}}\,dx = \frac{\frac{\sqrt{d^2-e^2x^2}(Cd^2(4d+5ex)-e(Ae(2d+ex)+Bd(d+2ex)))}{d^2(d+ex)^2} - 6C\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{3e^3}
$$

```
[In] Integrate[(A + B*x + C*x^2)/((d + e*x)^2*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(C*d^2*(4*d + 5*e*x) - e*(A*e*(2*d + e*x) + B*d*(d +
2*e*x))))/(d^2*(d + e*x)^2) - 6*C*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*
x^2])])/(3*e^3)
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.20

method	result
default	$\frac{C\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{e^2\sqrt{e^2}} - \frac{(Be-2Cd)\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{e^4d\left(x+\frac{d}{e}\right)} + \frac{(Ae^2-Bde+Cd^2)\left(-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{3de\left(x+\frac{d}{e}\right)^2} - \frac{\sqrt{-\left(x+\frac{d}{e}\right)}}{3d^2}\right.}{e^4}$

```
[In] int((C*x^2+B*x+A)/(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] C/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))-1/e^4*(B*e-2*C
*d)/d/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+1/e^4*(A*e^2-B*d*e+C*d^2
)*(-1/3/d/e/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-1/3/d^2/(x+d/e)*
(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.36

$$\int \frac{A+Bx+Cx^2}{(d+ex)^2\sqrt{d^2-e^2x^2}}\,dx$$
$$= \frac{4\,Cd^4 - Bd^3e - 2\,Ad^2e^2 + (4\,Cd^2e^2 - Bde^3 - 2\,Ae^4)x^2 + 2\,(4\,Cd^3e - Bd^2e^2 - 2\,Ade^3)x - 6\,(Cd^2e^2x^2 +}{3\,(d^2e^5x^2 +}$$

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/3*(4*C*d^4 - B*d^3*e - 2*A*d^2*e^2 + (4*C*d^2*e^2 - B*d*e^3 - 2*A*e^4)*x^
2 + 2*(4*C*d^3*e - B*d^2*e^2 - 2*A*d*e^3)*x - 6*(C*d^2*e^2*x^2 + 2*C*d^3*e*
x + C*d^4)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (4*C*d^3 - B*d^2*e -
 2*A*d*e^2 + (5*C*d^2*e - 2*B*d*e^2 - A*e^3)*x)*sqrt(-e^2*x^2 + d^2))/(d^2*
e^5*x^2 + 2*d^3*e^4*x + d^4*e^3)
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{(d+ex)^2\sqrt{d^2-e^2x^2}}\,dx = \int \frac{A+Bx+Cx^2}{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}\left(d+ex\right)^2}\,dx$$

```
[In] integrate((C*x**2+B*x+A)/(e*x+d)**2/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral((A + B*x + C*x**2)/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)**2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 317 vs. 2(151) = 302.

Time = 0.28 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.94

$$\begin{aligned}\int \frac{A+Bx+Cx^2}{(d+ex)^2\sqrt{d^2-e^2x^2}}\,dx = &-\frac{\sqrt{-e^2x^2+d^2}\,Cd^2}{3\,(de^5x^2+2\,d^2e^4x+d^3e^3)} - \frac{\sqrt{-e^2x^2+d^2}\,Cd^2}{3\,(d^2e^4x+d^3e^3)}\\ &+\frac{\sqrt{-e^2x^2+d^2}\,Bd}{3\,(de^4x^2+2\,d^2e^3x+d^3e^2)} + \frac{\sqrt{-e^2x^2+d^2}\,Bd}{3\,(d^2e^3x+d^3e^2)}\\ &-\frac{\sqrt{-e^2x^2+d^2}\,A}{3\,(de^3x^2+2\,d^2e^2x+d^3e)} - \frac{\sqrt{-e^2x^2+d^2}\,A}{3\,(d^2e^2x+d^3e)}\\ &-\frac{\sqrt{-e^2x^2+d^2}\,B}{de^3x+d^2e^2} + \frac{2\,\sqrt{-e^2x^2+d^2}\,C}{e^4x+de^3} + \frac{C\arcsin\left(\frac{ex}{d}\right)}{e^3}\end{aligned}$$

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima
")
```

```
[Out] -1/3*sqrt(-e^2*x^2 + d^2)*C*d^2/(d*e^5*x^2 + 2*d^2*e^4*x + d^3*e^3) - 1/3*s
qrt(-e^2*x^2 + d^2)*C*d^2/(d^2*e^4*x + d^3*e^3) + 1/3*sqrt(-e^2*x^2 + d^2)*
B*d/(d*e^4*x^2 + 2*d^2*e^3*x + d^3*e^2) + 1/3*sqrt(-e^2*x^2 + d^2)*B*d/(d^2
*e^3*x + d^3*e^2) - 1/3*sqrt(-e^2*x^2 + d^2)*A/(d*e^3*x^2 + 2*d^2*e^2*x + d
^3*e) - 1/3*sqrt(-e^2*x^2 + d^2)*A/(d^2*e^2*x + d^3*e) - sqrt(-e^2*x^2 + d^
2)*B/(d*e^3*x + d^2*e^2) + 2*sqrt(-e^2*x^2 + d^2)*C/(e^4*x + d*e^3) + C*arc
sin(e*x/d)/e^3
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2}{(d+ex)^2\sqrt{d^2-e^2x^2}}\,dx = \text{Exception raised: NotImplementedError}$$

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> unable to parse Giac output: 1/abs
(sageVARe)*(-((-i)*sageVARA*sageVARe^2+(-2*i)*sageVARB*sageVARd*sageVARe-6*
sageVARC*sageVARd^2*atan(i)+5*i*sageVARC*sageVARd^2)/3/sageVARd^2/sageVARe^
2*sign((sageVARe
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(d+ex)^2\sqrt{d^2-e^2x^2}}\,dx = \int \frac{C\,x^2+B\,x+A}{\sqrt{d^2-e^2\,x^2}\,{\left(d+e\,x\right)}^2}\,dx$$

```
[In] int((A + B*x + C*x^2)/((d^2 - e^2*x^2)^(1/2)*(d + e*x)^2),x)
```

```
[Out] int((A + B*x + C*x^2)/((d^2 - e^2*x^2)^(1/2)*(d + e*x)^2), x)
```

3.16 $\int \frac{A+Bx+Cx^2}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 180

$$\begin{aligned}\int \frac{A+Bx+Cx^2}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = &-\frac{(Cd^2-Bde+Ae^2)\sqrt{d^2-e^2x^2}}{5de^3(d+ex)^3}+\frac{C\sqrt{d^2-e^2x^2}}{e^3(d+ex)^2}\\ &-\frac{(7Cd^2+e(3Bd+2Ae))\sqrt{d^2-e^2x^2}}{15d^2e^3(d+ex)^2}\\ &-\frac{(7Cd^2+e(3Bd+2Ae))\sqrt{d^2-e^2x^2}}{15d^3e^3(d+ex)}\end{aligned}$$

```
[Out] -1/5*(A*e^2-B*d*e+C*d^2)*(-e^2*x^2+d^2)^(1/2)/d/e^3/(e*x+d)^3+C*(-e^2*x^2+d
^2)^(1/2)/e^3/(e*x+d)^2-1/15*(7*C*d^2+e*(2*A*e+3*B*d))*(-e^2*x^2+d^2)^(1/2)
/d^2/e^3/(e*x+d)^2-1/15*(7*C*d^2+e*(2*A*e+3*B*d))*(-e^2*x^2+d^2)^(1/2)/d^3/
e^3/(e*x+d)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1653, 807, 673, 665}

$$\begin{aligned}\int \frac{A+Bx+Cx^2}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = &-\frac{\sqrt{d^2-e^2x^2}(e(2Ae+3Bd)+7Cd^2)}{15d^2e^3(d+ex)^2}\\ &-\frac{\sqrt{d^2-e^2x^2}(Ae^2-Bde+Cd^2)}{5de^3(d+ex)^3}\\ &-\frac{\sqrt{d^2-e^2x^2}(e(2Ae+3Bd)+7Cd^2)}{15d^3e^3(d+ex)}+\frac{C\sqrt{d^2-e^2x^2}}{e^3(d+ex)^2}\end{aligned}$$

```
[In] Int[(A + B*x + C*x^2)/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -1/5*((C*d^2 - B*d*e + A*e^2)*Sqrt[d^2 - e^2*x^2])/(d*e^3*(d + e*x)^3) + (C
*Sqrt[d^2 - e^2*x^2])/(e^3*(d + e*x)^2) - ((7*C*d^2 + e*(3*B*d + 2*A*e))*Sq
rt[d^2 - e^2*x^2])/(15*d^2*e^3*(d + e*x)^2) - ((7*C*d^2 + e*(3*B*d + 2*A*e)
)*Sqrt[d^2 - e^2*x^2])/(15*d^3*e^3*(d + e*x))
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 673

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-e)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m + p + 1))), x] + Dist[Simpl
ify[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x]
, x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ
[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 807

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 1653

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{C\sqrt{d^2-e^2x^2}}{e^3(d+ex)^2} + \frac{\int \frac{e^2(2Cd^2+Ae^2)+e^3(Cd+Be)x}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx}{e^4}$$

$$
\begin{aligned}
&= -\frac{(Cd^2-Bde+Ae^2)\sqrt{d^2-e^2x^2}}{5de^3(d+ex)^3}+\frac{C\sqrt{d^2-e^2x^2}}{e^3(d+ex)^2} \\
&\quad+\frac{(7Cd^2+e(3Bd+2Ae))\int\frac{1}{(d+ex)^2\sqrt{d^2-e^2x^2}}dx}{5de^2} \\
&= -\frac{(Cd^2-Bde+Ae^2)\sqrt{d^2-e^2x^2}}{5de^3(d+ex)^3}+\frac{C\sqrt{d^2-e^2x^2}}{e^3(d+ex)^2} \\
&\quad-\frac{(7Cd^2+e(3Bd+2Ae))\sqrt{d^2-e^2x^2}}{15d^2e^3(d+ex)^2} \\
&\quad+\frac{(7Cd^2+e(3Bd+2Ae))\int\frac{1}{(d+ex)\sqrt{d^2-e^2x^2}}dx}{15d^2e^2} \\
&= -\frac{(Cd^2-Bde+Ae^2)\sqrt{d^2-e^2x^2}}{5de^3(d+ex)^3}+\frac{C\sqrt{d^2-e^2x^2}}{e^3(d+ex)^2} \\
&\quad-\frac{(7Cd^2+e(3Bd+2Ae))\sqrt{d^2-e^2x^2}}{15d^2e^3(d+ex)^2}-\frac{(7Cd^2+e(3Bd+2Ae))\sqrt{d^2-e^2x^2}}{15d^3e^3(d+ex)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.57

$$
\int\frac{A+Bx+Cx^2}{(d+ex)^3\sqrt{d^2-e^2x^2}}dx = -\frac{\sqrt{d^2-e^2x^2}(Cd^2(2d^2+6dex+7e^2x^2)+e(3Bd(d^2+3dex+e^2x^2)+Ae(7d^2+6dex+2e^2x^2)))}{15d^3e^3(d+ex)^3}
$$

```
[In] Integrate[(A + B*x + C*x^2)/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
[Out] -1/15*(Sqrt[d^2 - e^2*x^2]*(C*d^2*(2*d^2 + 6*d*e*x + 7*e^2*x^2) + e*(3*B*d*
(d^2 + 3*d*e*x + e^2*x^2) + A*e*(7*d^2 + 6*d*e*x + 2*e^2*x^2))))/(d^3*e^3*(
d + e*x)^3)
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.61

method	result
trager	$-\frac{\left(2A\,e^{4}x^{2}+3x^{2}dB\,e^{3}+7C\,d^{2}e^{2}x^{2}+6Ad\,e^{3}x+9xB\,d^{2}e^{2}+6C\,d^{3}xe+7A\,d^{2}e^{2}+3B\,d^{3}e+2C\,d^{4}\right)\sqrt{-e^{2}x^{2}+d^{2}}}{15d^{3}e^{3}\left(ex+d\right)^{3}}$
gosper	$-\frac{\left(-ex+d\right)\left(2A\,e^{4}x^{2}+3x^{2}dB\,e^{3}+7C\,d^{2}e^{2}x^{2}+6Ad\,e^{3}x+9xB\,d^{2}e^{2}+6C\,d^{3}xe+7A\,d^{2}e^{2}+3B\,d^{3}e+2C\,d^{4}\right)}{15\left(ex+d\right)^{2}d^{3}e^{3}\sqrt{-e^{2}x^{2}+d^{2}}}$
default	$-\frac{C\sqrt{-\left(x+\frac{d}{e}\right)^{2}e^{2}+2de\left(x+\frac{d}{e}\right)}}{e^{4}d\left(x+\frac{d}{e}\right)}+\frac{\left(Be-2Cd\right)\left(-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^{2}e^{2}+2de\left(x+\frac{d}{e}\right)}}{3de\left(x+\frac{d}{e}\right)^{2}}-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^{2}e^{2}+2de\left(x+\frac{d}{e}\right)}}{3d^{2}\left(x+\frac{d}{e}\right)}\right)}{e^{4}}+\frac{\left(A\,e^{2}-Bde+C\,d^{2}\right)\left(-\right.}{}$

```
[In] int((C*x^2+B*x+A)/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/15*(2*A*e^4*x^2+3*B*d*e^3*x^2+7*C*d^2*e^2*x^2+6*A*d*e^3*x+9*B*d^2*e^2*x+
6*C*d^3*e*x+7*A*d^2*e^2+3*B*d^3*e+2*C*d^4)/d^3/e^3/(e*x+d)^3*(-e^2*x^2+d^2)
^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.36

$$\int \frac{A+Bx+Cx^2}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx =$$
$$-\frac{2\,Cd^5+3\,Bd^4e+7\,Ad^3e^2+(2\,Cd^2e^3+3\,Bde^4+7\,Ae^5)x^3+3\,(2\,Cd^3e^2+3\,Bd^2e^3+7\,Ade^4)x^2+3\,(2\,C}{15}$$

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas
")
```

```
[Out] -1/15*(2*C*d^5 + 3*B*d^4*e + 7*A*d^3*e^2 + (2*C*d^2*e^3 + 3*B*d*e^4 + 7*A*e
^5)*x^3 + 3*(2*C*d^3*e^2 + 3*B*d^2*e^3 + 7*A*d*e^4)*x^2 + 3*(2*C*d^4*e + 3*
B*d^3*e^2 + 7*A*d^2*e^3)*x + (2*C*d^4 + 3*B*d^3*e + 7*A*d^2*e^2 + (7*C*d^2*
e^2 + 3*B*d*e^3 + 2*A*e^4)*x^2 + 3*(2*C*d^3*e + 3*B*d^2*e^2 + 2*A*d*e^3)*x)
*sqrt(-e^2*x^2 + d^2))/(d^3*e^6*x^3 + 3*d^4*e^5*x^2 + 3*d^5*e^4*x + d^6*e^3
)
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx=\int \frac{A+Bx+Cx^2}{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}\left(d+ex\right)^3}\,dx$$

```
[In] integrate((C*x**2+B*x+A)/(e*x+d)**3/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral((A + B*x + C*x**2)/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)**3), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 608 vs. $2(166) = 332$.

Time $= 0.28$ (sec) , antiderivative size $= 608$, normalized size of antiderivative $= 3.38$

$$\begin{aligned}\int \frac{A+Bx+Cx^2}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx &= -\frac{\sqrt{-e^2x^2+d^2}Cd^2}{5\left(de^6x^3+3\,d^2e^5x^2+3\,d^3e^4x+d^4e^3\right)} \\ &\quad -\frac{2\sqrt{-e^2x^2+d^2}Cd^2}{15\left(d^2e^5x^2+2\,d^3e^4x+d^4e^3\right)}-\frac{2\sqrt{-e^2x^2+d^2}Cd^2}{15\left(d^3e^4x+d^4e^3\right)} \\ &\quad +\frac{\sqrt{-e^2x^2+d^2}Bd}{5\left(de^5x^3+3\,d^2e^4x^2+3\,d^3e^3x+d^4e^2\right)} \\ &\quad +\frac{2\sqrt{-e^2x^2+d^2}Bd}{15\left(d^2e^4x^2+2\,d^3e^3x+d^4e^2\right)}+\frac{2\sqrt{-e^2x^2+d^2}Bd}{15\left(d^3e^3x+d^4e^2\right)} \\ &\quad +\frac{2\sqrt{-e^2x^2+d^2}Cd}{3\left(de^5x^2+2\,d^2e^4x+d^3e^3\right)}+\frac{2\sqrt{-e^2x^2+d^2}Cd}{3\left(d^2e^4x+d^3e^3\right)} \\ &\quad -\frac{\sqrt{-e^2x^2+d^2}A}{5\left(de^4x^3+3\,d^2e^3x^2+3\,d^3e^2x+d^4e\right)} \\ &\quad -\frac{2\sqrt{-e^2x^2+d^2}A}{15\left(d^2e^3x^2+2\,d^3e^2x+d^4e\right)} \\ &\quad -\frac{2\sqrt{-e^2x^2+d^2}A}{15\left(d^3e^2x+d^4e\right)}-\frac{\sqrt{-e^2x^2+d^2}B}{3\left(de^4x^2+2\,d^2e^3x+d^3e^2\right)} \\ &\quad -\frac{\sqrt{-e^2x^2+d^2}B}{3\left(d^2e^3x+d^3e^2\right)}-\frac{\sqrt{-e^2x^2+d^2}C}{de^4x+d^2e^3}\end{aligned}$$

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima
")
```

```
[Out] -1/5*sqrt(-e^2*x^2 + d^2)*C*d^2/(d*e^6*x^3 + 3*d^2*e^5*x^2 + 3*d^3*e^4*x +
d^4*e^3) - 2/15*sqrt(-e^2*x^2 + d^2)*C*d^2/(d^2*e^5*x^2 + 2*d^3*e^4*x + d^4
*e^3) - 2/15*sqrt(-e^2*x^2 + d^2)*C*d^2/(d^3*e^4*x + d^4*e^3) + 1/5*sqrt(-e
^2*x^2 + d^2)*B*d/(d*e^5*x^3 + 3*d^2*e^4*x^2 + 3*d^3*e^3*x + d^4*e^2) + 2/1
5*sqrt(-e^2*x^2 + d^2)*B*d/(d^2*e^4*x^2 + 2*d^3*e^3*x + d^4*e^2) + 2/15*sqr
```

```
t(-e^2*x^2 + d^2)*B*d/(d^3*e^3*x + d^4*e^2) + 2/3*sqrt(-e^2*x^2 + d^2)*C*d/
(d*e^5*x^2 + 2*d^2*e^4*x + d^3*e^3) + 2/3*sqrt(-e^2*x^2 + d^2)*C*d/(d^2*e^4
*x + d^3*e^3) - 1/5*sqrt(-e^2*x^2 + d^2)*A/(d*e^4*x^3 + 3*d^2*e^3*x^2 + 3*d
^3*e^2*x + d^4*e) - 2/15*sqrt(-e^2*x^2 + d^2)*A/(d^2*e^3*x^2 + 2*d^3*e^2*x
+ d^4*e) - 2/15*sqrt(-e^2*x^2 + d^2)*A/(d^3*e^2*x + d^4*e) - 1/3*sqrt(-e^2*
x^2 + d^2)*B/(d*e^4*x^2 + 2*d^2*e^3*x + d^3*e^2) - 1/3*sqrt(-e^2*x^2 + d^2)
*B/(d^2*e^3*x + d^3*e^2) - sqrt(-e^2*x^2 + d^2)*C/(d*e^4*x + d^2*e^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 350 vs. $2(166) = 332$.

Time = 0.30 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.94

$$\int \frac{A+Bx+Cx^2}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx$$

$$= \frac{2\left(2\,Cd^2+3\,Bde+7\,Ae^2+\frac{20\left(de+\sqrt{-e^2x^2+d^2}|e|\right)A}{x}+\frac{10\left(de+\sqrt{-e^2x^2+d^2}|e|\right)Cd^2}{e^2x}+\frac{15\left(de+\sqrt{-e^2x^2+d^2}|e|\right)Bd}{ex}+\frac{20\left(de+\right.}{}\right.}{}$$

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*(2*C*d^2 + 3*B*d*e + 7*A*e^2 + 20*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*
A/x + 10*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*C*d^2/(e^2*x) + 15*(d*e + sqrt
(-e^2*x^2 + d^2)*abs(e))*B*d/(e*x) + 20*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))
^2*C*d^2/(e^4*x^2) + 15*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*B*d/(e^3*x^2)
 + 40*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*A/(e^2*x^2) + 15*(d*e + sqrt(-e
^2*x^2 + d^2)*abs(e))^3*B*d/(e^5*x^3) + 30*(d*e + sqrt(-e^2*x^2 + d^2)*abs(
e))^3*A/(e^4*x^3) + 15*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4*A/(e^6*x^4))/(
d^3*e^2*((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) + 1)^5*abs(e))
```

Mupad [B] (verification not implemented)

Time = 12.57 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.61

$$\int \frac{A+Bx+Cx^2}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx =$$

$$-\frac{\sqrt{d^2-e^2\,x^2}\,\left(2\,C\,d^4+6\,C\,d^3\,e\,x+3\,B\,d^3\,e+7\,C\,d^2\,e^2\,x^2+9\,B\,d^2\,e^2\,x+7\,A\,d^2\,e^2+3\,B\,d\,e^3\,x^2+6\,A\,d\,e^3\right.}{15\,d^3\,e^3\,{\left(d+e\,x\right)}^3}$$

```
[In] int((A + B*x + C*x^2)/((d^2 - e^2*x^2)^(1/2)*(d + e*x)^3),x)
```

```
[Out] -((d^2 - e^2*x^2)^(1/2)*(2*C*d^4 + 7*A*d^2*e^2 + 2*A*e^4*x^2 + 3*B*d^3*e +
7*C*d^2*e^2*x^2 + 6*A*d*e^3*x + 6*C*d^3*e*x + 9*B*d^2*e^2*x + 3*B*d*e^3*x^2
))/(15*d^3*e^3*(d + e*x)^3)
```

3.17 $\int \frac{A+Bx+Cx^2}{(d+ex)^4\sqrt{d^2-e^2x^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 234

$$\begin{aligned}\int \frac{A+Bx+Cx^2}{(d+ex)^4\sqrt{d^2-e^2x^2}}\,dx = &-\frac{(Cd^2-Bde+Ae^2)\sqrt{d^2-e^2x^2}}{7de^3(d+ex)^4}+\frac{C\sqrt{d^2-e^2x^2}}{2e^3(d+ex)^3}\\ &-\frac{(13Cd^2+8Bde+6Ae^2)\sqrt{d^2-e^2x^2}}{70d^2e^3(d+ex)^3}\\ &-\frac{(13Cd^2+8Bde+6Ae^2)\sqrt{d^2-e^2x^2}}{105d^3e^3(d+ex)^2}\\ &-\frac{(13Cd^2+8Bde+6Ae^2)\sqrt{d^2-e^2x^2}}{105d^4e^3(d+ex)}\end{aligned}$$

```
[Out] -1/7*(A*e^2-B*d*e+C*d^2)*(-e^2*x^2+d^2)^(1/2)/d/e^3/(e*x+d)^4+1/2*C*(-e^2*x
^2+d^2)^(1/2)/e^3/(e*x+d)^3-1/70*(6*A*e^2+8*B*d*e+13*C*d^2)*(-e^2*x^2+d^2)^
(1/2)/d^2/e^3/(e*x+d)^3-1/105*(6*A*e^2+8*B*d*e+13*C*d^2)*(-e^2*x^2+d^2)^(1/
2)/d^3/e^3/(e*x+d)^2-1/105*(6*A*e^2+8*B*d*e+13*C*d^2)*(-e^2*x^2+d^2)^(1/2)/
d^4/e^3/(e*x+d)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used

$= \{1653, 807, 673, 665\}$

$$\begin{aligned}\int \frac{A+Bx+Cx^2}{(d+ex)^4\sqrt{d^2-e^2x^2}}\,dx = &-\frac{\sqrt{d^2-e^2x^2}(6Ae^2+8Bde+13Cd^2)}{70d^2e^3(d+ex)^3}\\ &-\frac{\sqrt{d^2-e^2x^2}(Ae^2-Bde+Cd^2)}{7de^3(d+ex)^4}\\ &-\frac{\sqrt{d^2-e^2x^2}(6Ae^2+8Bde+13Cd^2)}{105d^4e^3(d+ex)}\\ &-\frac{\sqrt{d^2-e^2x^2}(6Ae^2+8Bde+13Cd^2)}{105d^3e^3(d+ex)^2}+\frac{C\sqrt{d^2-e^2x^2}}{2e^3(d+ex)^3}\end{aligned}$$

```
[In] Int[(A + B*x + C*x^2)/((d + e*x)^4*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -1/7*((C*d^2 - B*d*e + A*e^2)*Sqrt[d^2 - e^2*x^2])/(d*e^3*(d + e*x)^4) + (C
*Sqrt[d^2 - e^2*x^2])/(2*e^3*(d + e*x)^3) - ((13*C*d^2 + 8*B*d*e + 6*A*e^2)
*Sqrt[d^2 - e^2*x^2])/(70*d^2*e^3*(d + e*x)^3) - ((13*C*d^2 + 8*B*d*e + 6*A
*e^2)*Sqrt[d^2 - e^2*x^2])/(105*d^3*e^3*(d + e*x)^2) - ((13*C*d^2 + 8*B*d*e
 + 6*A*e^2)*Sqrt[d^2 - e^2*x^2])/(105*d^4*e^3*(d + e*x))
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 673

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-e)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m + p + 1))), x] + Dist[Simpl
ify[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x]
, x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ
[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 807

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 1653

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{C\sqrt{d^2-e^2x^2}}{2e^3(d+ex)^3} + \frac{\int \frac{e^2(3Cd^2+2Ae^2)+e^3(Cd+2Be)x}{(d+ex)^4\sqrt{d^2-e^2x^2}}\,dx}{2e^4}$$

$$= -\frac{(Cd^2-Bde+Ae^2)\sqrt{d^2-e^2x^2}}{7de^3(d+ex)^4} + \frac{C\sqrt{d^2-e^2x^2}}{2e^3(d+ex)^3} + \frac{(13Cd^2+8Bde+6Ae^2)\int \frac{1}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx}{14de^2}$$

$$= -\frac{(Cd^2-Bde+Ae^2)\sqrt{d^2-e^2x^2}}{7de^3(d+ex)^4} + \frac{C\sqrt{d^2-e^2x^2}}{2e^3(d+ex)^3} - \frac{(13Cd^2+8Bde+6Ae^2)\sqrt{d^2-e^2x^2}}{70d^2e^3(d+ex)^3} + \frac{(13Cd^2+8Bde+6Ae^2)\int \frac{1}{(d+ex)^2\sqrt{d^2-e^2x^2}}\,dx}{35d^2e^2}$$

$$= -\frac{(Cd^2-Bde+Ae^2)\sqrt{d^2-e^2x^2}}{7de^3(d+ex)^4} + \frac{C\sqrt{d^2-e^2x^2}}{2e^3(d+ex)^3} - \frac{(13Cd^2+8Bde+6Ae^2)\sqrt{d^2-e^2x^2}}{70d^2e^3(d+ex)^3} - \frac{(13Cd^2+8Bde+6Ae^2)\sqrt{d^2-e^2x^2}}{105d^3e^3(d+ex)^2} + \frac{(13Cd^2+8Bde+6Ae^2)\int \frac{1}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx}{105d^3e^2}$$

$$= -\frac{(Cd^2-Bde+Ae^2)\sqrt{d^2-e^2x^2}}{7de^3(d+ex)^4} + \frac{C\sqrt{d^2-e^2x^2}}{2e^3(d+ex)^3} - \frac{(13Cd^2+8Bde+6Ae^2)\sqrt{d^2-e^2x^2}}{70d^2e^3(d+ex)^3} - \frac{(13Cd^2+8Bde+6Ae^2)\sqrt{d^2-e^2x^2}}{105d^3e^3(d+ex)^2} - \frac{(13Cd^2+8Bde+6Ae^2)\sqrt{d^2-e^2x^2}}{105d^4e^3(d+ex)}$$

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.59

$$\int \frac{A+Bx+Cx^2}{(d+ex)^4\sqrt{d^2-e^2x^2}}\,dx = \\ -\frac{\sqrt{d^2-e^2x^2}(Cd^2(8d^3+32d^2ex+52de^2x^2+13e^3x^3)+e(3Ae(12d^3+13d^2ex+8de^2x^2+2e^3x^3)+Bd(13}{105d^4e^3(d+ex)^4}$$

```
[In] Integrate[(A + B*x + C*x^2)/((d + e*x)^4*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -1/105*(Sqrt[d^2 - e^2*x^2]*(C*d^2*(8*d^3 + 32*d^2*e*x + 52*d*e^2*x^2 + 13*
e^3*x^3) + e*(3*A*e*(12*d^3 + 13*d^2*e*x + 8*d*e^2*x^2 + 2*e^3*x^3) + B*d*(
13*d^3 + 52*d^2*e*x + 32*d*e^2*x^2 + 8*e^3*x^3))))/(d^4*e^3*(d + e*x)^4)
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.62

method	result
trager	$-\frac{\left(6A\,e^5x^3+8x^3dB\,e^4+13C\,d^2e^3x^3+24Ad\,e^4x^2+32x^2d^2B\,e^3+52C\,d^3e^2x^2+39A\,d^2e^3x+52x\,d^3B\,e^2+32C\,d^4ex+36A\,d^3e^2+13B\,d^4e+\right.}{105d^4\left(ex+d\right)^4e^3}$
gosper	$-\frac{\left(-ex+d\right)\left(6A\,e^5x^3+8x^3dB\,e^4+13C\,d^2e^3x^3+24Ad\,e^4x^2+32x^2d^2B\,e^3+52C\,d^3e^2x^2+39A\,d^2e^3x+52x\,d^3B\,e^2+32C\,d^4ex+36A\,d^3e^2+\right.}{105\left(ex+d\right)^3d^4e^3\sqrt{-e^2x^2+d^2}}$
default	$\frac{C\left(-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{3de\left(x+\frac{d}{e}\right)^2}-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{3d^2\left(x+\frac{d}{e}\right)}\right)}{e^4}+\frac{\left(Be-2Cd\right)\left(-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{5de\left(x+\frac{d}{e}\right)^3}+\frac{2e\left(-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x\right.}}{3de\left(x+\frac{d}{e}\right)^2}\right.}{}\right.}{e^5}$

```
[In] int((C*x^2+B*x+A)/(e*x+d)^4/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/105*(6*A*e^5*x^3+8*B*d*e^4*x^3+13*C*d^2*e^3*x^3+24*A*d*e^4*x^2+32*B*d^2*
e^3*x^2+52*C*d^3*e^2*x^2+39*A*d^2*e^3*x+52*B*d^3*e^2*x+32*C*d^4*e*x+36*A*d^
3*e^2+13*B*d^4*e+8*C*d^5)/d^4/(e*x+d)^4/e^3*(-e^2*x^2+d^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.37

$$\int \frac{A+Bx+Cx^2}{(d+ex)^4\sqrt{d^2-e^2x^2}}\,dx =$$
$$-\frac{8\,Cd^6+13\,Bd^5e+36\,Ad^4e^2+(8\,Cd^2e^4+13\,Bde^5+36\,Ae^6)x^4+4\,(8\,Cd^3e^3+13\,Bd^2e^4+36\,Ade^5)x^3}{}$$

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^4/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas
")
```

```
[Out] -1/105*(8*C*d^6 + 13*B*d^5*e + 36*A*d^4*e^2 + (8*C*d^2*e^4 + 13*B*d*e^5 + 3
6*A*e^6)*x^4 + 4*(8*C*d^3*e^3 + 13*B*d^2*e^4 + 36*A*d*e^5)*x^3 + 6*(8*C*d^4
*e^2 + 13*B*d^3*e^3 + 36*A*d^2*e^4)*x^2 + 4*(8*C*d^5*e + 13*B*d^4*e^2 + 36*
A*d^3*e^3)*x + (8*C*d^5 + 13*B*d^4*e + 36*A*d^3*e^2 + (13*C*d^2*e^3 + 8*B*d
*e^4 + 6*A*e^5)*x^3 + 4*(13*C*d^3*e^2 + 8*B*d^2*e^3 + 6*A*d*e^4)*x^2 + (32*
C*d^4*e + 52*B*d^3*e^2 + 39*A*d^2*e^3)*x)*sqrt(-e^2*x^2 + d^2))/(d^4*e^7*x^
4 + 4*d^5*e^6*x^3 + 6*d^6*e^5*x^2 + 4*d^7*e^4*x + d^8*e^3)
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{(d+ex)^4\sqrt{d^2-e^2x^2}}\,dx = \int \frac{A+Bx+Cx^2}{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}\left(d+ex\right)^4}\,dx$$

```
[In] integrate((C*x**2+B*x+A)/(e*x+d)**4/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral((A + B*x + C*x**2)/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)**4), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 975 vs. 2(214) = 428.

Time = 0.29 (sec) , antiderivative size = 975, normalized size of antiderivative = 4.17

$$
\begin{aligned}
\int \frac{A+Bx+Cx^2}{(d+ex)^4\sqrt{d^2-e^2x^2}}\,dx = {} & -\frac{\sqrt{-e^2x^2+d^2}Cd^2}{7\,(de^7x^4+4\,d^2e^6x^3+6\,d^3e^5x^2+4\,d^4e^4x+d^5e^3)} \\
& -\frac{3\,\sqrt{-e^2x^2+d^2}Cd^2}{35\,(d^2e^6x^3+3\,d^3e^5x^2+3\,d^4e^4x+d^5e^3)} \\
& -\frac{2\,\sqrt{-e^2x^2+d^2}Cd^2}{35\,(d^3e^5x^2+2\,d^4e^4x+d^5e^3)}-\frac{2\,\sqrt{-e^2x^2+d^2}Cd^2}{35\,(d^4e^4x+d^5e^3)} \\
& +\frac{\sqrt{-e^2x^2+d^2}Bd}{7\,(de^6x^4+4\,d^2e^5x^3+6\,d^3e^4x^2+4\,d^4e^3x+d^5e^2)} \\
& +\frac{3\,\sqrt{-e^2x^2+d^2}Bd}{35\,(d^2e^5x^3+3\,d^3e^4x^2+3\,d^4e^3x+d^5e^2)} \\
& +\frac{2\,\sqrt{-e^2x^2+d^2}Bd}{35\,(d^3e^4x^2+2\,d^4e^3x+d^5e^2)}+\frac{2\,\sqrt{-e^2x^2+d^2}Bd}{35\,(d^4e^3x+d^5e^2)} \\
& +\frac{2\,\sqrt{-e^2x^2+d^2}Cd}{5\,(de^6x^3+3\,d^2e^5x^2+3\,d^3e^4x+d^4e^3)} \\
& +\frac{4\,\sqrt{-e^2x^2+d^2}Cd}{15\,(d^2e^5x^2+2\,d^3e^4x+d^4e^3)}+\frac{4\,\sqrt{-e^2x^2+d^2}Cd}{15\,(d^3e^4x+d^4e^3)} \\
& -\frac{\sqrt{-e^2x^2+d^2}A}{7\,(de^5x^4+4\,d^2e^4x^3+6\,d^3e^3x^2+4\,d^4e^2x+d^5e)} \\
& -\frac{3\,\sqrt{-e^2x^2+d^2}A}{35\,(d^2e^4x^3+3\,d^3e^3x^2+3\,d^4e^2x+d^5e)} \\
& -\frac{2\,\sqrt{-e^2x^2+d^2}A}{35\,(d^3e^3x^2+2\,d^4e^2x+d^5e)}-\frac{2\,\sqrt{-e^2x^2+d^2}A}{35\,(d^4e^2x+d^5e)} \\
& -\frac{\sqrt{-e^2x^2+d^2}B}{5\,(de^5x^3+3\,d^2e^4x^2+3\,d^3e^3x+d^4e^2)} \\
& -\frac{2\,\sqrt{-e^2x^2+d^2}B}{15\,(d^2e^4x^2+2\,d^3e^3x+d^4e^2)}-\frac{2\,\sqrt{-e^2x^2+d^2}B}{15\,(d^3e^3x+d^4e^2)} \\
& -\frac{\sqrt{-e^2x^2+d^2}C}{3\,(de^5x^2+2\,d^2e^4x+d^3e^3)}-\frac{\sqrt{-e^2x^2+d^2}C}{3\,(d^2e^4x+d^3e^3)}
\end{aligned}
$$

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^4/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima
")
```

```
[Out] -1/7*sqrt(-e^2*x^2 + d^2)*C*d^2/(d*e^7*x^4 + 4*d^2*e^6*x^3 + 6*d^3*e^5*x^2
+ 4*d^4*e^4*x + d^5*e^3) - 3/35*sqrt(-e^2*x^2 + d^2)*C*d^2/(d^2*e^6*x^3 + 3
*d^3*e^5*x^2 + 3*d^4*e^4*x + d^5*e^3) - 2/35*sqrt(-e^2*x^2 + d^2)*C*d^2/(d^
3*e^5*x^2 + 2*d^4*e^4*x + d^5*e^3) - 2/35*sqrt(-e^2*x^2 + d^2)*C*d^2/(d^4*e
^4*x + d^5*e^3) + 1/7*sqrt(-e^2*x^2 + d^2)*B*d/(d*e^6*x^4 + 4*d^2*e^5*x^3 +
 6*d^3*e^4*x^2 + 4*d^4*e^3*x + d^5*e^2) + 3/35*sqrt(-e^2*x^2 + d^2)*B*d/(d^
2*e^5*x^3 + 3*d^3*e^4*x^2 + 3*d^4*e^3*x + d^5*e^2) + 2/35*sqrt(-e^2*x^2 + d
```

```
^2)*B*d/(d^3*e^4*x^2 + 2*d^4*e^3*x + d^5*e^2) + 2/35*sqrt(-e^2*x^2 + d^2)*B
*d/(d^4*e^3*x + d^5*e^2) + 2/5*sqrt(-e^2*x^2 + d^2)*C*d/(d*e^6*x^3 + 3*d^2*
e^5*x^2 + 3*d^3*e^4*x + d^4*e^3) + 4/15*sqrt(-e^2*x^2 + d^2)*C*d/(d^2*e^5*x
^2 + 2*d^3*e^4*x + d^4*e^3) + 4/15*sqrt(-e^2*x^2 + d^2)*C*d/(d^3*e^4*x + d^
4*e^3) - 1/7*sqrt(-e^2*x^2 + d^2)*A/(d*e^5*x^4 + 4*d^2*e^4*x^3 + 6*d^3*e^3*
x^2 + 4*d^4*e^2*x + d^5*e) - 3/35*sqrt(-e^2*x^2 + d^2)*A/(d^2*e^4*x^3 + 3*d
^3*e^3*x^2 + 3*d^4*e^2*x + d^5*e) - 2/35*sqrt(-e^2*x^2 + d^2)*A/(d^3*e^3*x^
2 + 2*d^4*e^2*x + d^5*e) - 2/35*sqrt(-e^2*x^2 + d^2)*A/(d^4*e^2*x + d^5*e)
- 1/5*sqrt(-e^2*x^2 + d^2)*B/(d*e^5*x^3 + 3*d^2*e^4*x^2 + 3*d^3*e^3*x + d^4
*e^2) - 2/15*sqrt(-e^2*x^2 + d^2)*B/(d^2*e^4*x^2 + 2*d^3*e^3*x + d^4*e^2) -
 2/15*sqrt(-e^2*x^2 + d^2)*B/(d^3*e^3*x + d^4*e^2) - 1/3*sqrt(-e^2*x^2 + d^
2)*C/(d*e^5*x^2 + 2*d^2*e^4*x + d^3*e^3) - 1/3*sqrt(-e^2*x^2 + d^2)*C/(d^2*
e^4*x + d^3*e^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 550 vs. $2(214) = 428$.

Time = 0.29 (sec) , antiderivative size = 550, normalized size of antiderivative = 2.35

$$\int \frac{A+Bx+Cx^2}{(d+ex)^4\sqrt{d^2-e^2x^2}}\,dx$$

$$= \frac{2\left(8\,Cd^2+13\,Bde+36\,Ae^2+\frac{147\left(de+\sqrt{-e^2x^2+d^2}|e|\right)A}{x}+\frac{56\left(de+\sqrt{-e^2x^2+d^2}|e|\right)Cd^2}{e^2x}+\frac{91\left(de+\sqrt{-e^2x^2+d^2}|e|\right)Bd}{ex}+16\right.}{}$$

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^4/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2/105*(8*C*d^2 + 13*B*d*e + 36*A*e^2 + 147*(d*e + sqrt(-e^2*x^2 + d^2)*abs(
e))*A/x + 56*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*C*d^2/(e^2*x) + 91*(d*e +
sqrt(-e^2*x^2 + d^2)*abs(e))*B*d/(e*x) + 168*(d*e + sqrt(-e^2*x^2 + d^2)*ab
s(e))^2*C*d^2/(e^4*x^2) + 168*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*B*d/(e^
3*x^2) + 441*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*A/(e^2*x^2) + 140*(d*e +
 sqrt(-e^2*x^2 + d^2)*abs(e))^3*C*d^2/(e^6*x^3) + 280*(d*e + sqrt(-e^2*x^2
+ d^2)*abs(e))^3*B*d/(e^5*x^3) + 630*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3*
A/(e^4*x^3) + 140*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4*C*d^2/(e^8*x^4) + 1
75*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4*B*d/(e^7*x^4) + 630*(d*e + sqrt(-e
^2*x^2 + d^2)*abs(e))^4*A/(e^6*x^4) + 105*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e
))^5*B*d/(e^9*x^5) + 315*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^5*A/(e^8*x^5)
+ 105*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^6*A/(e^10*x^6))/(d^4*e^2*((d*e +
sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) + 1)^7*abs(e))
```

Mupad [B] (verification not implemented)

Time = 12.54 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.87

$$\int \frac{A+Bx+Cx^2}{(d+ex)^4\sqrt{d^2-e^2x^2}}\,dx = \frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{C}{5\,e^3}-\frac{-4\,C\,d^2+4\,B\,d\,e+3\,A\,e^2}{35\,d^2\,e^3}\right)}{{\left(d+e\,x\right)}^3} - \frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{A}{7\,d\,e}+\frac{d\,\left(\frac{C}{7\,e^2}-\frac{B}{7\,d\,e}\right)}{e}\right)}{{\left(d+e\,x\right)}^4} - \frac{\sqrt{d^2-e^2\,x^2}\,\left(13\,C\,d^2+8\,B\,d\,e+6\,A\,e^2\right)}{105\,d^3\,e^3\,{\left(d+e\,x\right)}^2} - \frac{\sqrt{d^2-e^2\,x^2}\,\left(13\,C\,d^2+8\,B\,d\,e+6\,A\,e^2\right)}{105\,d^4\,e^3\,\left(d+e\,x\right)}$$

```
[In] int((A + B*x + C*x^2)/((d^2 - e^2*x^2)^(1/2)*(d + e*x)^4),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(C/(5*e^3) - (3*A*e^2 - 4*C*d^2 + 4*B*d*e)/(35*d^2*e
^3)))/(d + e*x)^3 - ((d^2 - e^2*x^2)^(1/2)*(A/(7*d*e) + (d*(C/(7*e^2) - B/(
7*d*e)))/e))/(d + e*x)^4 - ((d^2 - e^2*x^2)^(1/2)*(6*A*e^2 + 13*C*d^2 + 8*B
*d*e))/(105*d^3*e^3*(d + e*x)^2) - ((d^2 - e^2*x^2)^(1/2)*(6*A*e^2 + 13*C*d
^2 + 8*B*d*e))/(105*d^4*e^3*(d + e*x))
```

3.18 $\int (d+ex)^3 (a+cx^2) (A+Bx+Cx^2)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 175

$$\begin{aligned}\int (d+ex)^3 (a+cx^2) (A+Bx+Cx^2)\,dx &= \frac{(cd^2+ae^2)(Cd^2-Bde+Ae^2)(d+ex)^4}{4e^5} \\ &- \frac{(ae^2(2Cd-Be)+cd(4Cd^2-e(3Bd-2Ae)))(d+ex)^5}{5e^5} \\ &+ \frac{(aCe^2+c(6Cd^2-e(3Bd-Ae)))(d+ex)^6}{6e^5} - \frac{c(4Cd-Be)(d+ex)^7}{7e^5} + \frac{cC(d+ex)^8}{8e^5}\end{aligned}$$

```
[Out] 1/4*(a*e^2+c*d^2)*(A*e^2-B*d*e+C*d^2)*(e*x+d)^4/e^5-1/5*(a*e^2*(-B*e+2*C*d)
+c*d*(4*C*d^2-e*(-2*A*e+3*B*d)))*(e*x+d)^5/e^5+1/6*(a*C*e^2+c*(6*C*d^2-e*(-
A*e+3*B*d)))*(e*x+d)^6/e^5-1/7*c*(-B*e+4*C*d)*(e*x+d)^7/e^5+1/8*c*C*(e*x+d)
^8/e^5
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {1642}

$$\begin{aligned}\int (d+ex)^3 (a+cx^2) (A+Bx+Cx^2)\,dx &= -\frac{(d+ex)^5 (ae^2(2Cd-Be)-cde(3Bd-2Ae)+4cCd^3)}{5e^5} \\ &+ \frac{(d+ex)^6 (aCe^2-ce(3Bd-Ae)+6cCd^2)}{6e^5} \\ &+ \frac{(d+ex)^4 (ae^2+cd^2)(Ae^2-Bde+Cd^2)}{4e^5} - \frac{c(d+ex)^7(4Cd-Be)}{7e^5} + \frac{cC(d+ex)^8}{8e^5}\end{aligned}$$

```
[In] Int[(d + e*x)^3*(a + c*x^2)*(A + B*x + C*x^2),x]
```

```
[Out] ((c*d^2 + a*e^2)*(C*d^2 - B*d*e + A*e^2)*(d + e*x)^4)/(4*e^5) - ((4*c*C*d^3
 - c*d*e*(3*B*d - 2*A*e) + a*e^2*(2*C*d - B*e))*(d + e*x)^5)/(5*e^5) + ((6*
c*C*d^2 + a*C*e^2 - c*e*(3*B*d - A*e))*(d + e*x)^6)/(6*e^5) - (c*(4*C*d - B
*e)*(d + e*x)^7)/(7*e^5) + (c*C*(d + e*x)^8)/(8*e^5)
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{(cd^2+ae^2)(Cd^2-Bde+Ae^2)(d+ex)^3}{e^4} + \frac{(-4cCd^3+cde(3Bd-2Ae)-ae^2(2Cd-Be))(d+ex)^4}{e^4} + \frac{(6cCd^2+aCe^2-ce(3Bd-Ae))(d+ex)^5}{e^4} + \frac{c(-4Cd+Be)(d+ex)^6}{e^4} + \frac{cC(d+ex)^7}{e^4}\right) dx \\
&= \frac{(cd^2+ae^2)(Cd^2-Bde+Ae^2)(d+ex)^4}{4e^5} - \frac{(4cCd^3-cde(3Bd-2Ae)+ae^2(2Cd-Be))(d+ex)^5}{5e^5} + \frac{(6cCd^2+aCe^2-ce(3Bd-Ae))(d+ex)^6}{6e^5} - \frac{c(4Cd-Be)(d+ex)^7}{7e^5} + \frac{cC(d+ex)^8}{8e^5}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.19

$$\begin{aligned}
&\int (d+ex)^3 (a+cx^2) (A+Bx+Cx^2)\, dx \\
&= aAd^3x + \frac{1}{2}ad^2(Bd+3Ae)x^2 + \frac{1}{3}d\big(ad(Cd+3Be)+A(cd^2+3ae^2)\big)x^3 \\
&\quad + \frac{1}{4}\left(Bcd^3+3Acd^2e+3aCd^2e+3aBde^2+aAe^3\right)x^4 \\
&\quad + \frac{1}{5}\left(cCd^3+3cde(Bd+Ae)+ae^2(3Cd+Be)\right)x^5 \\
&\quad + \frac{1}{6}e\big(3cCd^2+aCe^2+ce(3Bd+Ae)\big)x^6 + \frac{1}{7}ce^2(3Cd+Be)x^7 + \frac{1}{8}cCe^3x^8
\end{aligned}$$

```
[In] Integrate[(d + e*x)^3*(a + c*x^2)*(A + B*x + C*x^2),x]
```

```
[Out] a*A*d^3*x + (a*d^2*(B*d + 3*A*e)*x^2)/2 + (d*(a*d*(C*d + 3*B*e) + A*(c*d^2
+ 3*a*e^2))*x^3)/3 + ((B*c*d^3 + 3*A*c*d^2*e + 3*a*C*d^2*e + 3*a*B*d*e^2 +
a*A*e^3)*x^4)/4 + ((c*C*d^3 + 3*c*d*e*(B*d + A*e) + a*e^2*(3*C*d + B*e))*x^
5)/5 + (e*(3*c*C*d^2 + a*C*e^2 + c*e*(3*B*d + A*e))*x^6)/6 + (c*e^2*(3*C*d
+ B*e)*x^7)/7 + (c*C*e^3*x^8)/8
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.22

method	result
norman	$\frac{c e^3 C x^8}{8}+\left(\frac{1}{7}Bc e^3+\frac{3}{7}cd e^2 C\right)x^7+\left(\frac{1}{6}Ac e^3+\frac{1}{2}Bcd e^2+\frac{1}{6}Ca e^3+\frac{1}{2}Cc d^2 e\right)x^6+\left(\frac{3}{5}Acd e^2+\right.$
default	$\frac{c e^3 C x^8}{8}+\frac{\left(Bc e^3+3cd e^2 C\right)x^7}{7}+\frac{\left(\left(a e^3+3c d^2 e\right)C+3Bcd e^2+Ac e^3\right)x^6}{6}+\frac{\left(\left(3ad e^2+c d^3\right)C+\left(a e^3+3c d^2 e\right)B+3Acd e^2\right)}{5}$
gosper	$\frac{1}{8}c e^3 C x^8+\frac{1}{7}B e^3 c x^7+\frac{3}{7}x^7 cd e^2 C+\frac{1}{6}x^6 Ac e^3+\frac{1}{2}x^6 Bcd e^2+\frac{1}{6}x^6 Ca e^3+\frac{1}{2}x^6 Cc d^2 e+\frac{3}{5}x^5$
risch	$\frac{1}{8}c e^3 C x^8+\frac{1}{7}B e^3 c x^7+\frac{3}{7}x^7 cd e^2 C+\frac{1}{6}x^6 Ac e^3+\frac{1}{2}x^6 Bcd e^2+\frac{1}{6}x^6 Ca e^3+\frac{1}{2}x^6 Cc d^2 e+\frac{3}{5}x^5$
parallelrisch	$\frac{1}{8}c e^3 C x^8+\frac{1}{7}B e^3 c x^7+\frac{3}{7}x^7 cd e^2 C+\frac{1}{6}x^6 Ac e^3+\frac{1}{2}x^6 Bcd e^2+\frac{1}{6}x^6 Ca e^3+\frac{1}{2}x^6 Cc d^2 e+\frac{3}{5}x^5$

```
[In] int((e*x+d)^3*(c*x^2+a)*(C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*c*e^3*C*x^8+(1/7*B*c*e^3+3/7*c*d*e^2*C)*x^7+(1/6*A*c*e^3+1/2*B*c*d*e^2+
1/6*C*a*e^3+1/2*C*c*d^2*e)*x^6+(3/5*A*c*d*e^2+1/5*B*e^3*a+3/5*B*c*d^2*e+3/5
*C*a*d*e^2+1/5*C*c*d^3)*x^5+(1/4*A*a*e^3+3/4*A*c*d^2*e+3/4*B*a*d*e^2+1/4*B*
c*d^3+3/4*a*d^2*e*C)*x^4+(A*a*d*e^2+1/3*A*c*d^3+B*a*d^2*e+1/3*a*d^3*C)*x^3+
(3/2*a*A*d^2*e+1/2*B*a*d^3)*x^2+A*d^3*a*x
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.15

$$\begin{aligned}
&\int (d+ex)^3\left(a+cx^2\right)\left(A+Bx+Cx^2\right)\,dx \\
&=\frac{1}{8}Cce^3x^8+\frac{1}{7}\left(3\,Ccde^2+Bce^3\right)x^7+\frac{1}{6}\left(3\,Ccd^2e+3\,Bcde^2+(Ca+Ac)e^3\right)x^6 \\
&\quad+Aad^3x+\frac{1}{5}\left(Ccd^3+3\,Bcd^2e+Bae^3+3\,(Ca+Ac)de^2\right)x^5 \\
&\quad+\frac{1}{4}\left(Bcd^3+3\,Bade^2+Aae^3+3\,(Ca+Ac)d^2e\right)x^4 \\
&\quad+\frac{1}{3}\left(3\,Bad^2e+3\,Aade^2+(Ca+Ac)d^3\right)x^3+\frac{1}{2}\left(Bad^3+3\,Aad^2e\right)x^2
\end{aligned}$$

```
[In] integrate((e*x+d)^3*(c*x^2+a)*(C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/8*C*c*e^3*x^8 + 1/7*(3*C*c*d*e^2 + B*c*e^3)*x^7 + 1/6*(3*C*c*d^2*e + 3*B*
c*d*e^2 + (C*a + A*c)*e^3)*x^6 + A*a*d^3*x + 1/5*(C*c*d^3 + 3*B*c*d^2*e + B
*a*e^3 + 3*(C*a + A*c)*d*e^2)*x^5 + 1/4*(B*c*d^3 + 3*B*a*d*e^2 + A*a*e^3 +
3*(C*a + A*c)*d^2*e)*x^4 + 1/3*(3*B*a*d^2*e + 3*A*a*d*e^2 + (C*a + A*c)*d^3
)*x^3 + 1/2*(B*a*d^3 + 3*A*a*d^2*e)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.47

$$\begin{aligned}\int (d+ex)^3\left(a+cx^2\right)\left(A+Bx+Cx^2\right)\,dx = {} & Aad^3x+\frac{Cce^3x^8}{8}+x^7\left(\frac{Bce^3}{7}+\frac{3Ccde^2}{7}\right)\\ & +x^6\left(\frac{Ace^3}{6}+\frac{Bcde^2}{2}+\frac{Cae^3}{6}+\frac{Ccd^2e}{2}\right)+x^5\\ & \cdot\left(\frac{3Acde^2}{5}+\frac{Bae^3}{5}+\frac{3Bcd^2e}{5}+\frac{3Cade^2}{5}\right.\\ & \left.+\frac{Ccd^3}{5}\right)+x^4\left(\frac{Aae^3}{4}+\frac{3Acd^2e}{4}+\frac{3Bade^2}{4}\right.\\ & \left.+\frac{Bcd^3}{4}+\frac{3Cad^2e}{4}\right)\\ & +x^3\left(Aade^2+\frac{Acd^3}{3}+Bad^2e+\frac{Cad^3}{3}\right)\\ & +x^2\cdot\left(\frac{3Aad^2e}{2}+\frac{Bad^3}{2}\right)\end{aligned}$$

```
[In] integrate((e*x+d)**3*(c*x**2+a)*(C*x**2+B*x+A),x)
```

```
[Out] A*a*d**3*x + C*c*e**3*x**8/8 + x**7*(B*c*e**3/7 + 3*C*c*d*e**2/7) + x**6*(A
*c*e**3/6 + B*c*d*e**2/2 + C*a*e**3/6 + C*c*d**2*e/2) + x**5*(3*A*c*d*e**2/
5 + B*a*e**3/5 + 3*B*c*d**2*e/5 + 3*C*a*d*e**2/5 + C*c*d**3/5) + x**4*(A*a*
e**3/4 + 3*A*c*d**2*e/4 + 3*B*a*d*e**2/4 + B*c*d**3/4 + 3*C*a*d**2*e/4) + x
**3*(A*a*d*e**2 + A*c*d**3/3 + B*a*d**2*e + C*a*d**3/3) + x**2*(3*A*a*d**2*
e/2 + B*a*d**3/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.15

$$\begin{aligned}\int (d+ex)^3\left(a+cx^2\right)\left(A+Bx+Cx^2\right)\,dx &= \frac{1}{8}Cce^3x^8+\frac{1}{7}\left(3\,Ccde^2+Bce^3\right)x^7+\frac{1}{6}\left(3\,Ccd^2e+3\,Bcde^2+(Ca+Ac)e^3\right)x^6\\&\quad+Aad^3x+\frac{1}{5}\left(Ccd^3+3\,Bcd^2e+Bae^3+3\,(Ca+Ac)de^2\right)x^5\\&\quad+\frac{1}{4}\left(Bcd^3+3\,Bade^2+Aae^3+3\,(Ca+Ac)d^2e\right)x^4\\&\quad+\frac{1}{3}\left(3\,Bad^2e+3\,Aade^2+(Ca+Ac)d^3\right)x^3+\frac{1}{2}\left(Bad^3+3\,Aad^2e\right)x^2\end{aligned}$$

```
[In] integrate((e*x+d)^3*(c*x^2+a)*(C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/8*C*c*e^3*x^8 + 1/7*(3*C*c*d*e^2 + B*c*e^3)*x^7 + 1/6*(3*C*c*d^2*e + 3*B*
c*d*e^2 + (C*a + A*c)*e^3)*x^6 + A*a*d^3*x + 1/5*(C*c*d^3 + 3*B*c*d^2*e + B
*a*e^3 + 3*(C*a + A*c)*d*e^2)*x^5 + 1/4*(B*c*d^3 + 3*B*a*d*e^2 + A*a*e^3 +
3*(C*a + A*c)*d^2*e)*x^4 + 1/3*(3*B*a*d^2*e + 3*A*a*d*e^2 + (C*a + A*c)*d^3
)*x^3 + 1/2*(B*a*d^3 + 3*A*a*d^2*e)*x^2
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.42

$$\begin{aligned}\int (d+ex)^3\left(a+cx^2\right)\left(A+Bx+Cx^2\right)\,dx &= \frac{1}{8}Cce^3x^8+\frac{3}{7}Ccde^2x^7+\frac{1}{7}Bce^3x^7+\frac{1}{2}Ccd^2ex^6\\&\quad+\frac{1}{2}Bcde^2x^6+\frac{1}{6}Cae^3x^6+\frac{1}{6}Ace^3x^6\\&\quad+\frac{1}{5}Ccd^3x^5+\frac{3}{5}Bcd^2ex^5+\frac{3}{5}Cade^2x^5\\&\quad+\frac{3}{5}Acde^2x^5+\frac{1}{5}Bae^3x^5+\frac{1}{4}Bcd^3x^4\\&\quad+\frac{3}{4}Cad^2ex^4+\frac{3}{4}Acd^2ex^4+\frac{3}{4}Bade^2x^4\\&\quad+\frac{1}{4}Aae^3x^4+\frac{1}{3}Cad^3x^3+\frac{1}{3}Acd^3x^3+Bad^2ex^3\\&\quad+Aade^2x^3+\frac{1}{2}Bad^3x^2+\frac{3}{2}Aad^2ex^2+Aad^3x\end{aligned}$$

```
[In] integrate((e*x+d)^3*(c*x^2+a)*(C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/8*C*c*e^3*x^8 + 3/7*C*c*d*e^2*x^7 + 1/7*B*c*e^3*x^7 + 1/2*C*c*d^2*e*x^6 +
 1/2*B*c*d*e^2*x^6 + 1/6*C*a*e^3*x^6 + 1/6*A*c*e^3*x^6 + 1/5*C*c*d^3*x^5 +
3/5*B*c*d^2*e*x^5 + 3/5*C*a*d*e^2*x^5 + 3/5*A*c*d*e^2*x^5 + 1/5*B*a*e^3*x^5
```

```
 + 1/4*B*c*d^3*x^4 + 3/4*C*a*d^2*e*x^4 + 3/4*A*c*d^2*e*x^4 + 3/4*B*a*d*e^2*
x^4 + 1/4*A*a*e^3*x^4 + 1/3*C*a*d^3*x^3 + 1/3*A*c*d^3*x^3 + B*a*d^2*e*x^3 +
 A*a*d*e^2*x^3 + 1/2*B*a*d^3*x^2 + 3/2*A*a*d^2*e*x^2 + A*a*d^3*x
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.18

$$\begin{aligned}\int (d+ex)^3\left(a+cx^2\right)\left(A+Bx+Cx^2\right)\,dx = {} & x^3\left(\frac{A\,c\,d^3}{3}+\frac{C\,a\,d^3}{3}+A\,a\,d\,e^2+B\,a\,d^2\,e\right) \\ & +x^6\left(\frac{A\,c\,e^3}{6}+\frac{C\,a\,e^3}{6}+\frac{B\,c\,d\,e^2}{2}+\frac{C\,c\,d^2\,e}{2}\right) \\ & +x^4\left(\frac{A\,a\,e^3}{4}+\frac{B\,c\,d^3}{4}+\frac{3\,B\,a\,d\,e^2}{4}+\frac{3\,A\,c\,d^2\,e}{4}\right. \\ & \left.+\frac{3\,C\,a\,d^2\,e}{4}\right)+x^5\left(\frac{B\,a\,e^3}{5}+\frac{C\,c\,d^3}{5}+\frac{3\,A\,c\,d\,e^2}{5}\right. \\ & \left.+\frac{3\,C\,a\,d\,e^2}{5}+\frac{3\,B\,c\,d^2\,e}{5}\right)+A\,a\,d^3\,x+\frac{C\,c\,e^3\,x^8}{8} \\ & +\frac{a\,d^2\,x^2\,\left(3\,A\,e+B\,d\right)}{2}+\frac{c\,e^2\,x^7\,\left(B\,e+3\,C\,d\right)}{7}\end{aligned}$$

```
[In] int((a + c*x^2)*(d + e*x)^3*(A + B*x + C*x^2),x)
```

```
[Out] x^3*((A*c*d^3)/3 + (C*a*d^3)/3 + A*a*d*e^2 + B*a*d^2*e) + x^6*((A*c*e^3)/6
+ (C*a*e^3)/6 + (B*c*d*e^2)/2 + (C*c*d^2*e)/2) + x^4*((A*a*e^3)/4 + (B*c*d^
3)/4 + (3*B*a*d*e^2)/4 + (3*A*c*d^2*e)/4 + (3*C*a*d^2*e)/4) + x^5*((B*a*e^3
)/5 + (C*c*d^3)/5 + (3*A*c*d*e^2)/5 + (3*C*a*d*e^2)/5 + (3*B*c*d^2*e)/5) +
A*a*d^3*x + (C*c*e^3*x^8)/8 + (a*d^2*x^2*(3*A*e + B*d))/2 + (c*e^2*x^7*(B*e
 + 3*C*d))/7
```

3.19 $\int (d+ex)^2 (a+cx^2) (A+Bx+Cx^2)\, dx$

Optimal result

Integrand size = 25, antiderivative size = 175

$$\begin{aligned}\int (d+ex)^2 \left(a+cx^2\right) \left(A+Bx+Cx^2\right)\, dx &= \frac{(cd^2+ae^2)\left(Cd^2-Bde+Ae^2\right)(d+ex)^3}{3e^5} \\ &\quad - \frac{(ae^2(2Cd-Be)+cd(4Cd^2-e(3Bd-2Ae)))\,(d+ex)^4}{4e^5} \\ &\quad + \frac{(aCe^2+c(6Cd^2-e(3Bd-Ae)))\,(d+ex)^5}{5e^5} - \frac{c(4Cd-Be)(d+ex)^6}{6e^5} + \frac{cC(d+ex)^7}{7e^5}\end{aligned}$$

```
[Out] 1/3*(a*e^2+c*d^2)*(A*e^2-B*d*e+C*d^2)*(e*x+d)^3/e^5-1/4*(a*e^2*(-B*e+2*C*d)
+c*d*(4*C*d^2-e*(-2*A*e+3*B*d)))*(e*x+d)^4/e^5+1/5*(a*C*e^2+c*(6*C*d^2-e*(-
A*e+3*B*d)))*(e*x+d)^5/e^5-1/6*c*(-B*e+4*C*d)*(e*x+d)^6/e^5+1/7*c*C*(e*x+d)
^7/e^5
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {1642}

$$\begin{aligned}\int (d+ex)^2 \left(a+cx^2\right) \left(A+Bx+Cx^2\right)\, dx &= -\frac{(d+ex)^4 \left(ae^2(2Cd-Be)-cde(3Bd-2Ae)+4cCd^3\right)}{4e^5} \\ &\quad + \frac{(d+ex)^5 \left(aCe^2-ce(3Bd-Ae)+6cCd^2\right)}{5e^5} \\ &\quad + \frac{(d+ex)^3 \left(ae^2+cd^2\right)\left(Ae^2-Bde+Cd^2\right)}{3e^5} - \frac{c(d+ex)^6(4Cd-Be)}{6e^5} + \frac{cC(d+ex)^7}{7e^5}\end{aligned}$$

```
[In] Int[(d + e*x)^2*(a + c*x^2)*(A + B*x + C*x^2),x]
```

```
[Out] ((c*d^2 + a*e^2)*(C*d^2 - B*d*e + A*e^2)*(d + e*x)^3)/(3*e^5) - ((4*c*C*d^3
 - c*d*e*(3*B*d - 2*A*e) + a*e^2*(2*C*d - B*e))*(d + e*x)^4)/(4*e^5) + ((6*
c*C*d^2 + a*C*e^2 - c*e*(3*B*d - A*e))*(d + e*x)^5)/(5*e^5) - (c*(4*C*d - B
*e)*(d + e*x)^6)/(6*e^5) + (c*C*(d + e*x)^7)/(7*e^5)
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{(cd^2+ae^2)(Cd^2-Bde+Ae^2)(d+ex)^2}{e^4} + \frac{(-4cCd^3+cde(3Bd-2Ae)-ae^2(2Cd-Be))(d+ex)^3}{e^4} + \frac{(6cCd^2+aCe^2-ce(3Bd-Ae))(d+ex)^4}{e^4} + \frac{c(-4Cd+Be)(d+ex)^5}{e^4} + \frac{cC(d+ex)^6}{e^4}\right) dx \\ &= \frac{(cd^2+ae^2)(Cd^2-Bde+Ae^2)(d+ex)^3}{3e^5} - \frac{(4cCd^3-cde(3Bd-2Ae)+ae^2(2Cd-Be))(d+ex)^4}{4e^5} + \frac{(6cCd^2+aCe^2-ce(3Bd-Ae))(d+ex)^5}{5e^5} - \frac{c(4Cd-Be)(d+ex)^6}{6e^5} + \frac{cC(d+ex)^7}{7e^5}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.86

$$\int (d+ex)^2 (a+cx^2) (A+Bx+Cx^2) \, dx = aAd^2x + \frac{1}{2}ad(Bd+2Ae)x^2 + \frac{1}{3}(Acd^2+aCd^2+2aBde+aAe^2)x^3 + \frac{1}{4}(Bcd^2+2Acde+2aCde+aBe^2)x^4 + \frac{1}{5}(cCd^2+2Bcde+Ace^2+aCe^2)x^5 + \frac{1}{6}ce(2Cd+Be)x^6 + \frac{1}{7}cCe^2x^7$$

```
[In] Integrate[(d + e*x)^2*(a + c*x^2)*(A + B*x + C*x^2),x]
```

```
[Out] a*A*d^2*x + (a*d*(B*d + 2*A*e)*x^2)/2 + ((A*c*d^2 + a*C*d^2 + 2*a*B*d*e + a
*A*e^2)*x^3)/3 + ((B*c*d^2 + 2*A*c*d*e + 2*a*C*d*e + a*B*e^2)*x^4)/4 + ((c*
C*d^2 + 2*B*c*d*e + A*c*e^2 + a*C*e^2)*x^5)/5 + (c*e*(2*C*d + B*e)*x^6)/6 +
 (c*C*e^2*x^7)/7
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.85

method	result
default	$\frac{c\,e^2C\,x^7}{7}+\frac{(c\,e^2B+2cdeC)x^6}{6}+\frac{((e^2a+c\,d^2)C+2Bcde+Ac\,e^2)x^5}{5}+\frac{(2adeC+(e^2a+c\,d^2)B+2Acde)x^4}{4}+(a\,d^2C+2Bc$
norman	$\frac{c\,e^2C\,x^7}{7}+\left(\frac{1}{6}c\,e^2B+\frac{1}{3}cdeC\right)x^6+\left(\frac{1}{5}Ac\,e^2+\frac{2}{5}Bcde+\frac{1}{5}aC\,e^2+\frac{1}{5}Cc\,d^2\right)x^5+\left(\frac{1}{2}Acde+\frac{1}{4}Bc\right.$
gosper	$\frac{1}{7}c\,e^2C\,x^7+\frac{1}{6}Bc\,e^2x^6+\frac{1}{3}x^6cdeC+\frac{1}{5}x^5Ac\,e^2+\frac{2}{5}x^5Bcde+\frac{1}{5}x^5aC\,e^2+\frac{1}{5}x^5Cc\,d^2+\frac{1}{2}x^4Acd$
risch	$\frac{1}{7}c\,e^2C\,x^7+\frac{1}{6}Bc\,e^2x^6+\frac{1}{3}x^6cdeC+\frac{1}{5}x^5Ac\,e^2+\frac{2}{5}x^5Bcde+\frac{1}{5}x^5aC\,e^2+\frac{1}{5}x^5Cc\,d^2+\frac{1}{2}x^4Acd$
parallelrisch	$\frac{1}{7}c\,e^2C\,x^7+\frac{1}{6}Bc\,e^2x^6+\frac{1}{3}x^6cdeC+\frac{1}{5}x^5Ac\,e^2+\frac{2}{5}x^5Bcde+\frac{1}{5}x^5aC\,e^2+\frac{1}{5}x^5Cc\,d^2+\frac{1}{2}x^4Acd$

```
[In] int((e*x+d)^2*(c*x^2+a)*(C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*c*e^2*C*x^7+1/6*(B*c*e^2+2*C*c*d*e)*x^6+1/5*((a*e^2+c*d^2)*C+2*B*c*d*e+
A*c*e^2)*x^5+1/4*(2*a*d*e*C+(a*e^2+c*d^2)*B+2*A*c*d*e)*x^4+1/3*(a*d^2*C+2*B
*a*d*e+A*(a*e^2+c*d^2))*x^3+1/2*(2*A*a*d*e+B*a*d^2)*x^2+A*d^2*a*x
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.81

$$\begin{aligned}\int (d+ex)^2\left(a+cx^2\right)\left(A+Bx+Cx^2\right)\,dx =& \frac{1}{7}\,Cce^2x^7+\frac{1}{6}\,(2\,Ccde+Bce^2)x^6 \\ &+\frac{1}{5}\,(Ccd^2+2\,Bcde+(Ca+Ac)e^2)x^5 \\ &+Aad^2x \\ &+\frac{1}{4}\,(Bcd^2+Bae^2+2\,(Ca+Ac)de)x^4 \\ &+\frac{1}{3}\,(2\,Bade+Aae^2+(Ca+Ac)d^2)x^3 \\ &+\frac{1}{2}\,(Bad^2+2\,Aade)x^2\end{aligned}$$

```
[In] integrate((e*x+d)^2*(c*x^2+a)*(C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/7*C*c*e^2*x^7 + 1/6*(2*C*c*d*e + B*c*e^2)*x^6 + 1/5*(C*c*d^2 + 2*B*c*d*e
+ (C*a + A*c)*e^2)*x^5 + A*a*d^2*x + 1/4*(B*c*d^2 + B*a*e^2 + 2*(C*a + A*c)
*d*e)*x^4 + 1/3*(2*B*a*d*e + A*a*e^2 + (C*a + A*c)*d^2)*x^3 + 1/2*(B*a*d^2
+ 2*A*a*d*e)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.99

$$\begin{aligned}\int (d+ex)^2\left(a+cx^2\right)\left(A+Bx+Cx^2\right)\,dx = {} & Aad^2x+\frac{Cce^2x^7}{7}+x^6\left(\frac{Bce^2}{6}+\frac{Ccde}{3}\right)\\ & +x^5\left(\frac{Ace^2}{5}+\frac{2Bcde}{5}+\frac{Cae^2}{5}+\frac{Ccd^2}{5}\right)\\ & +x^4\left(\frac{Acde}{2}+\frac{Bae^2}{4}+\frac{Bcd^2}{4}+\frac{Cade}{2}\right)\\ & +x^3\left(\frac{Aae^2}{3}+\frac{Acd^2}{3}+\frac{2Bade}{3}+\frac{Cad^2}{3}\right)\\ & +x^2\left(Aade+\frac{Bad^2}{2}\right)\end{aligned}$$

```
[In] integrate((e*x+d)**2*(c*x**2+a)*(C*x**2+B*x+A),x)
```

```
[Out] A*a*d**2*x + C*c*e**2*x**7/7 + x**6*(B*c*e**2/6 + C*c*d*e/3) + x**5*(A*c*e*
*2/5 + 2*B*c*d*e/5 + C*a*e**2/5 + C*c*d**2/5) + x**4*(A*c*d*e/2 + B*a*e**2/
4 + B*c*d**2/4 + C*a*d*e/2) + x**3*(A*a*e**2/3 + A*c*d**2/3 + 2*B*a*d*e/3 +
 C*a*d**2/3) + x**2*(A*a*d*e + B*a*d**2/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.81

$$\begin{aligned}\int (d+ex)^2\left(a+cx^2\right)\left(A+Bx+Cx^2\right)\,dx = {} & \frac{1}{7}Cce^2x^7+\frac{1}{6}\left(2\,Ccde+Bce^2\right)x^6\\ & +\frac{1}{5}\left(Ccd^2+2\,Bcde+(Ca+Ac)e^2\right)x^5\\ & +Aad^2x\\ & +\frac{1}{4}\left(Bcd^2+Bae^2+2\,(Ca+Ac)de\right)x^4\\ & +\frac{1}{3}\left(2\,Bade+Aae^2+(Ca+Ac)d^2\right)x^3\\ & +\frac{1}{2}\left(Bad^2+2\,Aade\right)x^2\end{aligned}$$

```
[In] integrate((e*x+d)^2*(c*x^2+a)*(C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/7*C*c*e^2*x^7 + 1/6*(2*C*c*d*e + B*c*e^2)*x^6 + 1/5*(C*c*d^2 + 2*B*c*d*e
+ (C*a + A*c)*e^2)*x^5 + A*a*d^2*x + 1/4*(B*c*d^2 + B*a*e^2 + 2*(C*a + A*c)
*d*e)*x^4 + 1/3*(2*B*a*d*e + A*a*e^2 + (C*a + A*c)*d^2)*x^3 + 1/2*(B*a*d^2
+ 2*A*a*d*e)*x^2
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.98

$$
\begin{aligned}
\int (d+ex)^2\left(a+cx^2\right)\left(A+Bx+Cx^2\right)\,dx = {} & \frac{1}{7}Cce^2x^7+\frac{1}{3}Ccdex^6+\frac{1}{6}Bce^2x^6+\frac{1}{5}Ccd^2x^5 \\
& +\frac{2}{5}Bcdex^5+\frac{1}{5}Cae^2x^5+\frac{1}{5}Ace^2x^5+\frac{1}{4}Bcd^2x^4 \\
& +\frac{1}{2}Cadex^4+\frac{1}{2}Acdex^4+\frac{1}{4}Bae^2x^4 \\
& +\frac{1}{3}Cad^2x^3+\frac{1}{3}Acd^2x^3+\frac{2}{3}Badex^3 \\
& +\frac{1}{3}Aae^2x^3+\frac{1}{2}Bad^2x^2+Aadex^2+Aad^2x
\end{aligned}
$$

```
[In] integrate((e*x+d)^2*(c*x^2+a)*(C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/7*C*c*e^2*x^7 + 1/3*C*c*d*e*x^6 + 1/6*B*c*e^2*x^6 + 1/5*C*c*d^2*x^5 + 2/5
*B*c*d*e*x^5 + 1/5*C*a*e^2*x^5 + 1/5*A*c*e^2*x^5 + 1/4*B*c*d^2*x^4 + 1/2*C*
a*d*e*x^4 + 1/2*A*c*d*e*x^4 + 1/4*B*a*e^2*x^4 + 1/3*C*a*d^2*x^3 + 1/3*A*c*d
^2*x^3 + 2/3*B*a*d*e*x^3 + 1/3*A*a*e^2*x^3 + 1/2*B*a*d^2*x^2 + A*a*d*e*x^2
+ A*a*d^2*x
```

Mupad [B] (verification not implemented)

Time = 12.37 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.82

$$
\begin{aligned}
\int (d+ex)^2\left(a+cx^2\right)\left(A+Bx+Cx^2\right)\,dx = {} & x^3\left(\frac{A\,a\,e^2}{3}+\frac{A\,c\,d^2}{3}+\frac{C\,a\,d^2}{3}+\frac{2\,B\,a\,d\,e}{3}\right) \\
& +x^5\left(\frac{A\,c\,e^2}{5}+\frac{C\,a\,e^2}{5}+\frac{C\,c\,d^2}{5}+\frac{2\,B\,c\,d\,e}{5}\right) \\
& +x^4\left(\frac{B\,a\,e^2}{4}+\frac{B\,c\,d^2}{4}+\frac{A\,c\,d\,e}{2}+\frac{C\,a\,d\,e}{2}\right) \\
& +A\,a\,d^2\,x+\frac{a\,d\,x^2\,(2\,A\,e+B\,d)}{2} \\
& +\frac{c\,e\,x^6\,(B\,e+2\,C\,d)}{6}+\frac{C\,c\,e^2\,x^7}{7}
\end{aligned}
$$

```
[In] int((a + c*x^2)*(d + e*x)^2*(A + B*x + C*x^2),x)
```

```
[Out] x^3*((A*a*e^2)/3 + (A*c*d^2)/3 + (C*a*d^2)/3 + (2*B*a*d*e)/3) + x^5*((A*c*e
^2)/5 + (C*a*e^2)/5 + (C*c*d^2)/5 + (2*B*c*d*e)/5) + x^4*((B*a*e^2)/4 + (B*
c*d^2)/4 + (A*c*d*e)/2 + (C*a*d*e)/2) + A*a*d^2*x + (a*d*x^2*(2*A*e + B*d))
/2 + (c*e*x^6*(B*e + 2*C*d))/6 + (C*c*e^2*x^7)/7
```

3.20 $\int (d+ex)\left(a+cx^2\right)\left(A+Bx+Cx^2\right)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 86

$$\int (d+ex)\left(a+cx^2\right)\left(A+Bx+Cx^2\right)\,dx = aAdx+\frac{1}{2}a(Bd+Ae)x^2+\frac{1}{3}(Acd+aCd+aBe)x^3 +\frac{1}{4}(Bcd+Ace+aCe)x^4 +\frac{1}{5}c(Cd+Be)x^5+\frac{1}{6}cCex^6$$

```
[Out] a*A*d*x+1/2*a*(A*e+B*d)*x^2+1/3*(A*c*d+B*a*e+C*a*d)*x^3+1/4*(A*c*e+B*c*d+C*
a*e)*x^4+1/5*c*(B*e+C*d)*x^5+1/6*c*C*e*x^6
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1642}

$$\int (d+ex)\left(a+cx^2\right)\left(A+Bx+Cx^2\right)\,dx = \frac{1}{4}x^4(aCe+Ace+Bcd)+\frac{1}{3}x^3(aBe+aCd+Acd) +\frac{1}{2}ax^2(Ae+Bd)+aAdx +\frac{1}{5}cx^5(Be+Cd)+\frac{1}{6}cCex^6$$

```
[In] Int[(d + e*x)*(a + c*x^2)*(A + B*x + C*x^2),x]
```

```
[Out] a*A*d*x + (a*(B*d + A*e)*x^2)/2 + ((A*c*d + a*C*d + a*B*e)*x^3)/3 + ((B*c*d
 + A*c*e + a*C*e)*x^4)/4 + (c*(C*d + B*e)*x^5)/5 + (c*C*e*x^6)/6
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(aAd + a(Bd+Ae)x + (Acd+aCd+aBe)x^2 + (Bcd+Ace+aCe)x^3 \right.\\ &\qquad \left. + c(Cd+Be)x^4 + cCex^5\right) dx\\ &= aAdx + \frac{1}{2}a(Bd+Ae)x^2 + \frac{1}{3}(Acd+aCd+aBe)x^3\\ &\quad + \frac{1}{4}(Bcd+Ace+aCe)x^4 + \frac{1}{5}c(Cd+Be)x^5 + \frac{1}{6}cCex^6\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00

$$\begin{aligned}\int (d+ex)\left(a+cx^2\right)\left(A+Bx+Cx^2\right)\,dx &= aAdx + \frac{1}{2}a(Bd+Ae)x^2 + \frac{1}{3}(Acd+aCd+aBe)x^3\\ &\quad + \frac{1}{4}(Bcd+Ace+aCe)x^4\\ &\quad + \frac{1}{5}c(Cd+Be)x^5 + \frac{1}{6}cCex^6\end{aligned}$$

```
[In] Integrate[(d + e*x)*(a + c*x^2)*(A + B*x + C*x^2),x]
```

```
[Out] a*A*d*x + (a*(B*d + A*e)*x^2)/2 + ((A*c*d + a*C*d + a*B*e)*x^3)/3 + ((B*c*d
 + A*c*e + a*C*e)*x^4)/4 + (c*(C*d + B*e)*x^5)/5 + (c*C*e*x^6)/6
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.92

method	result
default	$\frac{cCe\,x^6}{6}+\frac{(Bce+cdC)x^5}{5}+\frac{(Ace+Bcd+Cae)x^4}{4}+\frac{(Acd+Bae+Cad)x^3}{3}+\frac{(aAe+Bad)x^2}{2}+aAdx$
norman	$\frac{cCe\,x^6}{6}+\left(\frac{1}{5}Bce+\frac{1}{5}cdC\right)x^5+\left(\frac{1}{4}Ace+\frac{1}{4}Bcd+\frac{1}{4}Cae\right)x^4+\left(\frac{1}{3}Acd+\frac{1}{3}Bae+\frac{1}{3}Cad\right)x^3+\left(\frac{1}{2}\right.$
gosper	$\frac{1}{6}cCe\,x^6+\frac{1}{5}Bce\,x^5+\frac{1}{5}x^5cdC+\frac{1}{4}x^4Ace+\frac{1}{4}x^4Bcd+\frac{1}{4}x^4Cae+\frac{1}{3}x^3Acd+\frac{1}{3}x^3Bae+\frac{1}{3}x^3Ca$
risch	$\frac{1}{6}cCe\,x^6+\frac{1}{5}Bce\,x^5+\frac{1}{5}x^5cdC+\frac{1}{4}x^4Ace+\frac{1}{4}x^4Bcd+\frac{1}{4}x^4Cae+\frac{1}{3}x^3Acd+\frac{1}{3}x^3Bae+\frac{1}{3}x^3Ca$
parallelrisch	$\frac{1}{6}cCe\,x^6+\frac{1}{5}Bce\,x^5+\frac{1}{5}x^5cdC+\frac{1}{4}x^4Ace+\frac{1}{4}x^4Bcd+\frac{1}{4}x^4Cae+\frac{1}{3}x^3Acd+\frac{1}{3}x^3Bae+\frac{1}{3}x^3Ca$

```
[In] int((e*x+d)*(c*x^2+a)*(C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*c*C*e*x^6+1/5*(B*c*e+C*c*d)*x^5+1/4*(A*c*e+B*c*d+C*a*e)*x^4+1/3*(A*c*d+
B*a*e+C*a*d)*x^3+1/2*(A*a*e+B*a*d)*x^2+a*A*d*x
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.93

$$\int (d+ex)\left(a+cx^2\right)\left(A+Bx+Cx^2\right)\,dx=\frac{1}{6}\,Ccex^6+\frac{1}{5}\,(Ccd+Bce)x^5+\frac{1}{4}\,(Bcd+(Ca+Ac)e)x^4+Aadx+\frac{1}{3}\,(Bae+(Ca+Ac)d)x^3+\frac{1}{2}\,(Bad+Aae)x^2$$

```
[In] integrate((e*x+d)*(c*x^2+a)*(C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/6*C*c*e*x^6 + 1/5*(C*c*d + B*c*e)*x^5 + 1/4*(B*c*d + (C*a + A*c)*e)*x^4 +
 A*a*d*x + 1/3*(B*a*e + (C*a + A*c)*d)*x^3 + 1/2*(B*a*d + A*a*e)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.13

$$\int (d+ex)\left(a+cx^2\right)\left(A+Bx+Cx^2\right)\,dx=Aadx+\frac{Ccex^6}{6}+x^5\left(\frac{Bce}{5}+\frac{Ccd}{5}\right)+x^4\left(\frac{Ace}{4}+\frac{Bcd}{4}+\frac{Cae}{4}\right)+x^3\left(\frac{Acd}{3}+\frac{Bae}{3}+\frac{Cad}{3}\right)+x^2\left(\frac{Aae}{2}+\frac{Bad}{2}\right)$$

```
[In] integrate((e*x+d)*(c*x**2+a)*(C*x**2+B*x+A),x)
```

```
[Out] A*a*d*x + C*c*e*x**6/6 + x**5*(B*c*e/5 + C*c*d/5) + x**4*(A*c*e/4 + B*c*d/4
 + C*a*e/4) + x**3*(A*c*d/3 + B*a*e/3 + C*a*d/3) + x**2*(A*a*e/2 + B*a*d/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.93

$$\int (d+ex)\left(a+cx^2\right)\left(A+Bx+Cx^2\right)\,dx = \frac{1}{6}\,Ccex^6 + \frac{1}{5}\,(Ccd+Bce)x^5 + \frac{1}{4}\,(Bcd+(Ca+Ac)e)x^4 + Aadx + \frac{1}{3}\,(Bae+(Ca+Ac)d)x^3 + \frac{1}{2}\,(Bad+Aae)x^2$$

```
[In] integrate((e*x+d)*(c*x^2+a)*(C*x^2+B*x+A),x, algorithm="maxima")
[Out] 1/6*C*c*e*x^6 + 1/5*(C*c*d + B*c*e)*x^5 + 1/4*(B*c*d + (C*a + A*c)*e)*x^4 +
 A*a*d*x + 1/3*(B*a*e + (C*a + A*c)*d)*x^3 + 1/2*(B*a*d + A*a*e)*x^2
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.09

$$\int (d+ex)\left(a+cx^2\right)\left(A+Bx+Cx^2\right)\,dx = \frac{1}{6}\,Ccex^6 + \frac{1}{5}\,Ccdx^5 + \frac{1}{5}\,Bcex^5 + \frac{1}{4}\,Bcdx^4 + \frac{1}{4}\,Caex^4 + \frac{1}{4}\,Acex^4 + \frac{1}{3}\,Cadx^3 + \frac{1}{3}\,Acdx^3 + \frac{1}{3}\,Baex^3 + \frac{1}{2}\,Badx^2 + \frac{1}{2}\,Aaex^2 + Aadx$$

```
[In] integrate((e*x+d)*(c*x^2+a)*(C*x^2+B*x+A),x, algorithm="giac")
[Out] 1/6*C*c*e*x^6 + 1/5*C*c*d*x^5 + 1/5*B*c*e*x^5 + 1/4*B*c*d*x^4 + 1/4*C*a*e*x
^4 + 1/4*A*c*e*x^4 + 1/3*C*a*d*x^3 + 1/3*A*c*d*x^3 + 1/3*B*a*e*x^3 + 1/2*B*
a*d*x^2 + 1/2*A*a*e*x^2 + A*a*d*x
```

Mupad [B] (verification not implemented)

Time = 12.59 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.93

$$\int (d+ex)\left(a+cx^2\right)\left(A+Bx+Cx^2\right)\,dx = \frac{C\,c\,e\,x^6}{6} + \frac{c\,(B\,e+C\,d)\,x^5}{5} + \left(\frac{A\,c\,e}{4}+\frac{B\,c\,d}{4}+\frac{C\,a\,e}{4}\right)\,x^4 + \left(\frac{A\,c\,d}{3}+\frac{B\,a\,e}{3}+\frac{C\,a\,d}{3}\right)\,x^3 + \frac{a\,(A\,e+B\,d)\,x^2}{2} + A\,a\,d\,x$$

```
[In] int((a + c*x^2)*(d + e*x)*(A + B*x + C*x^2),x)
```

```
[Out] x^3*((A*c*d)/3 + (B*a*e)/3 + (C*a*d)/3) + x^4*((A*c*e)/4 + (B*c*d)/4 + (C*a
*e)/4) + (a*x^2*(A*e + B*d))/2 + (c*x^5*(B*e + C*d))/5 + (C*c*e*x^6)/6 + A*
a*d*x
```

3.21 $\int (a+cx^2)(A+Bx+Cx^2)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 46

$$\int (a+cx^2)\left(A+Bx+Cx^2\right)\,dx = aAx+\frac{1}{2}aBx^2+\frac{1}{3}(Ac+aC)x^3+\frac{1}{4}Bcx^4+\frac{1}{5}cCx^5$$

```
[Out] a*A*x+1/2*a*B*x^2+1/3*(A*c+C*a)*x^3+1/4*B*c*x^4+1/5*c*C*x^5
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1671}

$$\int (a+cx^2)\left(A+Bx+Cx^2\right)\,dx = \frac{1}{3}x^3(aC+Ac)+aAx+\frac{1}{2}aBx^2+\frac{1}{4}Bcx^4+\frac{1}{5}cCx^5$$

```
[In] Int[(a + c*x^2)*(A + B*x + C*x^2),x]
```

```
[Out] a*A*x + (a*B*x^2)/2 + ((A*c + a*C)*x^3)/3 + (B*c*x^4)/4 + (c*C*x^5)/5
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(aA+aBx+(Ac+aC)x^2+Bcx^3+cCx^4\right)\,dx\\ &= aAx+\frac{1}{2}aBx^2+\frac{1}{3}(Ac+aC)x^3+\frac{1}{4}Bcx^4+\frac{1}{5}cCx^5\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \left(a+cx^2\right)\left(A+Bx+Cx^2\right)\,dx=aAx+\frac{1}{2}aBx^2+\frac{1}{3}(Ac+aC)x^3+\frac{1}{4}Bcx^4+\frac{1}{5}cCx^5$$

```
[In] Integrate[(a + c*x^2)*(A + B*x + C*x^2),x]
```

```
[Out] a*A*x + (a*B*x^2)/2 + ((A*c + a*C)*x^3)/3 + (B*c*x^4)/4 + (c*C*x^5)/5
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

method	result	size
default	$aAx+\frac{Ba\,x^2}{2}+\frac{(Ac+Ca)x^3}{3}+\frac{Bc\,x^4}{4}+\frac{cC\,x^5}{5}$	39
norman	$\frac{cC\,x^5}{5}+\frac{Bc\,x^4}{4}+\left(\frac{Ac}{3}+\frac{Ca}{3}\right)x^3+\frac{Ba\,x^2}{2}+aAx$	40
gosper	$\frac{1}{5}cC\,x^5+\frac{1}{4}Bc\,x^4+\frac{1}{3}Ac\,x^3+\frac{1}{3}x^3Ca+\frac{1}{2}Ba\,x^2+aAx$	41
risch	$\frac{1}{5}cC\,x^5+\frac{1}{4}Bc\,x^4+\frac{1}{3}Ac\,x^3+\frac{1}{3}x^3Ca+\frac{1}{2}Ba\,x^2+aAx$	41
parallelrisch	$\frac{1}{5}cC\,x^5+\frac{1}{4}Bc\,x^4+\frac{1}{3}Ac\,x^3+\frac{1}{3}x^3Ca+\frac{1}{2}Ba\,x^2+aAx$	41

```
[In] int((c*x^2+a)*(C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

```
[Out] a*A*x+1/2*B*a*x^2+1/3*(A*c+C*a)*x^3+1/4*B*c*x^4+1/5*c*C*x^5
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int \left(a+cx^2\right)\left(A+Bx+Cx^2\right)\,dx=\frac{1}{5}\,Ccx^5+\frac{1}{4}\,Bcx^4+\frac{1}{2}\,Bax^2+\frac{1}{3}\,(Ca+Ac)x^3+Aax$$

```
[In] integrate((c*x^2+a)*(C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/5*C*c*x^5 + 1/4*B*c*x^4 + 1/2*B*a*x^2 + 1/3*(C*a + A*c)*x^3 + A*a*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

$$\int \left(a+cx^2\right)\left(A+Bx+Cx^2\right)\,dx = Aax+\frac{Bax^2}{2}+\frac{Bcx^4}{4}+\frac{Ccx^5}{5}+x^3\left(\frac{Ac}{3}+\frac{Ca}{3}\right)$$

```
[In] integrate((c*x**2+a)*(C*x**2+B*x+A),x)
```

```
[Out] A*a*x + B*a*x**2/2 + B*c*x**4/4 + C*c*x**5/5 + x**3*(A*c/3 + C*a/3)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int \left(a+cx^2\right)\left(A+Bx+Cx^2\right)\,dx = \frac{1}{5}\,Ccx^5+\frac{1}{4}\,Bcx^4+\frac{1}{2}\,Bax^2+\frac{1}{3}\,(Ca+Ac)x^3+Aax$$

```
[In] integrate((c*x^2+a)*(C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/5*C*c*x^5 + 1/4*B*c*x^4 + 1/2*B*a*x^2 + 1/3*(C*a + A*c)*x^3 + A*a*x
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int \left(a+cx^2\right)\left(A+Bx+Cx^2\right)\,dx = \frac{1}{5}\,Ccx^5+\frac{1}{4}\,Bcx^4+\frac{1}{3}\,Cax^3+\frac{1}{3}\,Acx^3+\frac{1}{2}\,Bax^2+Aax$$

```
[In] integrate((c*x^2+a)*(C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/5*C*c*x^5 + 1/4*B*c*x^4 + 1/3*C*a*x^3 + 1/3*A*c*x^3 + 1/2*B*a*x^2 + A*a*x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int \left(a+cx^2\right)\left(A+Bx+Cx^2\right)\,dx = \frac{C\,c\,x^5}{5}+\frac{B\,c\,x^4}{4}+\left(\frac{A\,c}{3}+\frac{C\,a}{3}\right)\,x^3+\frac{B\,a\,x^2}{2}+A\,a\,x$$

```
[In] int((a + c*x^2)*(A + B*x + C*x^2),x)
```

```
[Out] x^3*((A*c)/3 + (C*a)/3) + A*a*x + (B*a*x^2)/2 + (B*c*x^4)/4 + (C*c*x^5)/5
```

3.22 $\int \frac{(a+cx^2)(A+Bx+Cx^2)}{d+ex}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 145

$$\int \frac{(a+cx^2)(A+Bx+Cx^2)}{d+ex}\,dx = -\frac{(ae^2(Cd-Be)+cd(Cd^2-e(Bd-Ae)))\,x}{e^4} + \frac{(aCe^2+c(Cd^2-e(Bd-Ae)))\,x^2}{2e^3} - \frac{c(Cd-Be)x^3}{3e^2} + \frac{cCx^4}{4e} + \frac{(cd^2+ae^2)(Cd^2-Bde+Ae^2)\log(d+ex)}{e^5}$$

```
[Out] -(a*e^2*(-B*e+C*d)+c*d*(C*d^2-e*(-A*e+B*d)))*x/e^4+1/2*(a*C*e^2+c*(C*d^2-e*
(-A*e+B*d)))*x^2/e^3-1/3*c*(-B*e+C*d)*x^3/e^2+1/4*c*C*x^4/e+(a*e^2+c*d^2)*(
A*e^2-B*d*e+C*d^2)*ln(e*x+d)/e^5
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {1642}

$$\int \frac{(a+cx^2)(A+Bx+Cx^2)}{d+ex}\,dx = -\frac{x(ae^2(Cd-Be)-cde(Bd-Ae)+cCd^3)}{e^4} + \frac{(ae^2+cd^2)\log(d+ex)(Ae^2-Bde+Cd^2)}{e^5} + \frac{x^2(aCe^2-ce(Bd-Ae)+cCd^2)}{2e^3} - \frac{cx^3(Cd-Be)}{3e^2} + \frac{cCx^4}{4e}$$

```
[In] Int[((a + c*x^2)*(A + B*x + C*x^2))/(d + e*x),x]
```

```
[Out] -(((c*C*d^3 - c*d*e*(B*d - A*e) + a*e^2*(C*d - B*e))*x)/e^4) + ((c*C*d^2 +
a*C*e^2 - c*e*(B*d - A*e))*x^2)/(2*e^3) - (c*(C*d - B*e)*x^3)/(3*e^2) + (c*
C*x^4)/(4*e) + ((c*d^2 + a*e^2)*(C*d^2 - B*d*e + A*e^2)*Log[d + e*x])/e^5
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{-ae^2(Cd-Be)-c(Cd^3-de(Bd-Ae))}{e^4}+\frac{(cCd^2+aCe^2-ce(Bd-Ae))\,x}{e^3}\right.\\
&\qquad\left.+\frac{c(-Cd+Be)x^2}{e^2}+\frac{cCx^3}{e}+\frac{(cd^2+ae^2)\,(Cd^2-Bde+Ae^2)}{e^4(d+ex)}\right)dx\\
&= -\frac{(cCd^3-cde(Bd-Ae)+ae^2(Cd-Be))\,x}{e^4}+\frac{(cCd^2+aCe^2-ce(Bd-Ae))\,x^2}{2e^3}\\
&\quad-\frac{c(Cd-Be)x^3}{3e^2}+\frac{cCx^4}{4e}+\frac{(cd^2+ae^2)\,(Cd^2-Bde+Ae^2)\log(d+ex)}{e^5}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.94

$$\int \frac{(a+cx^2)\,(A+Bx+Cx^2)}{d+ex}\,dx$$

$$= \frac{ex(6ae^2(-2Cd+2Be+Cex)+cC(-12d^3+6d^2ex-4de^2x^2+3e^3x^3)+2ce(3Ae(-2d+ex)+B(6d^2-3}{12e^5}$$

```
[In] Integrate[((a + c*x^2)*(A + B*x + C*x^2))/(d + e*x),x]
```

```
[Out] (e*x*(6*a*e^2*(-2*C*d + 2*B*e + C*e*x) + c*C*(-12*d^3 + 6*d^2*e*x - 4*d*e^2
*x^2 + 3*e^3*x^3) + 2*c*e*(3*A*e*(-2*d + e*x) + B*(6*d^2 - 3*d*e*x + 2*e^2*
x^2))) + 12*(c*d^2 + a*e^2)*(C*d^2 + e*(-(B*d) + A*e))*Log[d + e*x])/(12*e^
5)
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.11

method	result
norman	$\frac{(Ac\,e^{2}-Bcde+aC\,e^{2}+Cc\,d^{2})x^{2}}{2e^{3}}-\frac{(Acd\,e^{2}-B\,e^{3}a-Bc\,d^{2}e+Cad\,e^{2}+Cc\,d^{3})x}{e^{4}}+\frac{cC\,x^{4}}{4e}+\frac{c(Be-Cd)x^{3}}{3e^{2}}+\frac{(Aa\,e^{4}+Ac\,d}{}$
default	$-\frac{-\frac{1}{4}cC\,x^{4}e^{3}-\frac{1}{3}Bc\,x^{3}e^{3}+\frac{1}{3}Ccd\,e^{2}x^{3}-\frac{1}{2}Ac\,e^{3}x^{2}+\frac{1}{2}B\,x^{2}cd\,e^{2}-\frac{1}{2}Ca\,e^{3}x^{2}-\frac{1}{2}Cc\,d^{2}e\,x^{2}+Acd\,e^{2}x-Bxa\,e^{3}-Bc\,d^{2}ex+Cad\,e^{2}}{e^{4}}$
risch	$\frac{cC\,x^{4}}{4e}+\frac{Bc\,x^{3}}{3e}-\frac{Ccd\,x^{3}}{3e^{2}}+\frac{Ac\,x^{2}}{2e}-\frac{B\,x^{2}cd}{2e^{2}}+\frac{Ca\,x^{2}}{2e}+\frac{Cc\,d^{2}x^{2}}{2e^{3}}-\frac{Acdx}{e^{2}}+\frac{Bxa}{e}+\frac{Bc\,d^{2}x}{e^{3}}-\frac{Cadx}{e^{2}}-\frac{Cc\,d^{3}}{e^{4}}$
parallelrisch	$\frac{3cC\,x^{4}e^{4}+4B\,x^{3}c\,e^{4}-4C\,x^{3}cd\,e^{3}+6A\,x^{2}c\,e^{4}-6B\,x^{2}cd\,e^{3}+6C\,x^{2}a\,e^{4}+6C\,x^{2}c\,d^{2}e^{2}+12A\ln(ex+d)a\,e^{4}+12A\ln(ex+d)c\,d^{2}e^{2}-}{}$

```
[In] int((c*x^2+a)*(C*x^2+B*x+A)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/e^3*(A*c*e^2-B*c*d*e+C*a*e^2+C*c*d^2)*x^2-(A*c*d*e^2-B*a*e^3-B*c*d^2*e+
C*a*d*e^2+C*c*d^3)/e^4*x+1/4*c*C*x^4/e+1/3*c/e^2*(B*e-C*d)*x^3+(A*a*e^4+A*c
*d^2*e^2-B*a*d*e^3-B*c*d^3*e+C*a*d^2*e^2+C*c*d^4)/e^5*ln(e*x+d)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.11

$$\int \frac{(a+cx^{2})(A+Bx+Cx^{2})}{d+ex}\,dx = \frac{3\,Cce^{4}x^{4}-4\,(Ccde^{3}-Bce^{4})x^{3}+6\,(Ccd^{2}e^{2}-Bcde^{3}+(Ca+Ac)e^{4})x^{2}-12\,(Ccd^{3}e-Bcd^{2}e^{2}-Bae^{4}+}{12\,e^{5}}$$

```
[In] integrate((c*x^2+a)*(C*x^2+B*x+A)/(e*x+d),x, algorithm="fricas")
```

```
[Out] 1/12*(3*C*c*e^4*x^4 - 4*(C*c*d*e^3 - B*c*e^4)*x^3 + 6*(C*c*d^2*e^2 - B*c*d*
e^3 + (C*a + A*c)*e^4)*x^2 - 12*(C*c*d^3*e - B*c*d^2*e^2 - B*a*e^4 + (C*a +
 A*c)*d*e^3)*x + 12*(C*c*d^4 - B*c*d^3*e - B*a*d*e^3 + A*a*e^4 + (C*a + A*c
)*d^2*e^2)*log(e*x + d))/e^5
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.02

$$\begin{aligned}\int \frac{(a+cx^{2})(A+Bx+Cx^{2})}{d+ex}\,dx = {} & \frac{Ccx^{4}}{4e}+x^{3}\left(\frac{Bc}{3e}-\frac{Ccd}{3e^{2}}\right)+x^{2}\left(\frac{Ac}{2e}-\frac{Bcd}{2e^{2}}+\frac{Ca}{2e}+\frac{Ccd^{2}}{2e^{3}}\right)\\ & +x\left(-\frac{Acd}{e^{2}}+\frac{Ba}{e}+\frac{Bcd^{2}}{e^{3}}-\frac{Cad}{e^{2}}-\frac{Ccd^{3}}{e^{4}}\right)\\ & +\frac{(ae^{2}+cd^{2})(Ae^{2}-Bde+Cd^{2})\log(d+ex)}{e^{5}}\end{aligned}$$

```
[In] integrate((c*x**2+a)*(C*x**2+B*x+A)/(e*x+d),x)
```

```
[Out] C*c*x**4/(4*e) + x**3*(B*c/(3*e) - C*c*d/(3*e**2)) + x**2*(A*c/(2*e) - B*c*
d/(2*e**2) + C*a/(2*e) + C*c*d**2/(2*e**3)) + x*(-A*c*d/e**2 + B*a/e + B*c*
d**2/e**3 - C*a*d/e**2 - C*c*d**3/e**4) + (a*e**2 + c*d**2)*(A*e**2 - B*d*e
 + C*d**2)*log(d + e*x)/e**5
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.10

$$\int \frac{(a+cx^2)(A+Bx+Cx^2)}{d+ex}\,dx = \frac{3\,Cce^3x^4 - 4\,(Ccde^2 - Bce^3)x^3 + 6\,(Ccd^2e - Bcde^2 + (Ca+Ac)e^3)x^2 - 12\,(Ccd^3 - Bcd^2e - Bae^3 + (C}{12\,e^4} + \frac{(Ccd^4 - Bcd^3e - Bade^3 + Aae^4 + (Ca+Ac)d^2e^2)\log(ex+d)}{e^5}$$

```
[In] integrate((c*x^2+a)*(C*x^2+B*x+A)/(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/12*(3*C*c*e^3*x^4 - 4*(C*c*d*e^2 - B*c*e^3)*x^3 + 6*(C*c*d^2*e - B*c*d*e^
2 + (C*a + A*c)*e^3)*x^2 - 12*(C*c*d^3 - B*c*d^2*e - B*a*e^3 + (C*a + A*c)*
d*e^2)*x)/e^4 + (C*c*d^4 - B*c*d^3*e - B*a*d*e^3 + A*a*e^4 + (C*a + A*c)*d^
2*e^2)*log(e*x + d)/e^5
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.25

$$\int \frac{(a+cx^2)(A+Bx+Cx^2)}{d+ex}\,dx = \frac{3\,Cce^3x^4 - 4\,Ccde^2x^3 + 4\,Bce^3x^3 + 6\,Ccd^2ex^2 - 6\,Bcde^2x^2 + 6\,Cae^3x^2 + 6\,Ace^3x^2 - 12\,Ccd^3x + 12\,Bcd^2}{12\,e^4} + \frac{(Ccd^4 - Bcd^3e + Cad^2e^2 + Acd^2e^2 - Bade^3 + Aae^4)\log(|ex+d|)}{e^5}$$

```
[In] integrate((c*x^2+a)*(C*x^2+B*x+A)/(e*x+d),x, algorithm="giac")
```

```
[Out] 1/12*(3*C*c*e^3*x^4 - 4*C*c*d*e^2*x^3 + 4*B*c*e^3*x^3 + 6*C*c*d^2*e*x^2 - 6
*B*c*d*e^2*x^2 + 6*C*a*e^3*x^2 + 6*A*c*e^3*x^2 - 12*C*c*d^3*x + 12*B*c*d^2*
e*x - 12*C*a*d*e^2*x - 12*A*c*d*e^2*x + 12*B*a*e^3*x)/e^4 + (C*c*d^4 - B*c*
d^3*e + C*a*d^2*e^2 + A*c*d^2*e^2 - B*a*d*e^3 + A*a*e^4)*log(abs(e*x + d))/
e^5
```

Mupad [B] (verification not implemented)

Time = 12.65 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.21

$$\int \frac{(a+cx^2)(A+Bx+Cx^2)}{d+ex}\,dx$$
$$= x^3\left(\frac{B\,c}{3\,e}-\frac{C\,c\,d}{3\,e^2}\right)-x\left(\frac{d\left(\frac{A\,c+C\,a}{e}-\frac{d\left(\frac{B\,c}{e}-\frac{C\,c\,d}{e^2}\right)}{e}\right)}{e}-\frac{B\,a}{e}\right)$$
$$+x^2\left(\frac{A\,c+C\,a}{2\,e}-\frac{d\left(\frac{B\,c}{e}-\frac{C\,c\,d}{e^2}\right)}{2\,e}\right)$$
$$+\frac{\ln(d+e\,x)\,\left(A\,a\,e^4+C\,c\,d^4-B\,a\,d\,e^3-B\,c\,d^3\,e+A\,c\,d^2\,e^2+C\,a\,d^2\,e^2\right)}{e^5}+\frac{C\,c\,x^4}{4\,e}$$

```
[In] int(((a + c*x^2)*(A + B*x + C*x^2))/(d + e*x),x)
```

```
[Out] x^3*((B*c)/(3*e) - (C*c*d)/(3*e^2)) - x*((d*((A*c + C*a)/e - (d*((B*c)/e -
(C*c*d)/e^2))/e))/e - (B*a)/e) + x^2*((A*c + C*a)/(2*e) - (d*((B*c)/e - (C*
c*d)/e^2))/(2*e)) + (log(d + e*x)*(A*a*e^4 + C*c*d^4 - B*a*d*e^3 - B*c*d^3*
e + A*c*d^2*e^2 + C*a*d^2*e^2))/e^5 + (C*c*x^4)/(4*e)
```

3.23 $\int \frac{(a+cx^2)(A+Bx+Cx^2)}{(d+ex)^2} dx$

Optimal result

Integrand size = 25, antiderivative size = 153

$$\begin{aligned}\int \frac{(a+cx^2)(A+Bx+Cx^2)}{(d+ex)^2}\,dx &= \frac{(aCe^2+c(3Cd^2-e(2Bd-Ae)))\,x}{e^4}-\frac{c(2Cd-Be)x^2}{2e^3}\\&\quad+\frac{cCx^3}{3e^2}-\frac{(cd^2+ae^2)(Cd^2-Bde+Ae^2)}{e^5(d+ex)}\\&\quad-\frac{(ae^2(2Cd-Be)+cd(4Cd^2-e(3Bd-2Ae)))\log(d+ex)}{e^5}\end{aligned}$$

```
[Out] (a*C*e^2+c*(3*C*d^2-e*(-A*e+2*B*d)))*x/e^4-1/2*c*(-B*e+2*C*d)*x^2/e^3+1/3*c
*C*x^3/e^2-(a*e^2+c*d^2)*(A*e^2-B*d*e+C*d^2)/e^5/(e*x+d)-(a*e^2*(-B*e+2*C*d
)+c*d*(4*C*d^2-e*(-2*A*e+3*B*d)))*ln(e*x+d)/e^5
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {1642}

$$\begin{aligned}\int \frac{(a+cx^2)(A+Bx+Cx^2)}{(d+ex)^2}\,dx &= -\frac{\log(d+ex)\,(ae^2(2Cd-Be)-cde(3Bd-2Ae)+4cCd^3)}{e^5}\\&\quad-\frac{(ae^2+cd^2)(Ae^2-Bde+Cd^2)}{e^5(d+ex)}\\&\quad+\frac{x(aCe^2-ce(2Bd-Ae)+3cCd^2)}{e^4}-\frac{cx^2(2Cd-Be)}{2e^3}+\frac{cCx^3}{3e^2}\end{aligned}$$

```
[In] Int[((a + c*x^2)*(A + B*x + C*x^2))/(d + e*x)^2,x]
```

```
[Out] ((3*c*C*d^2 + a*C*e^2 - c*e*(2*B*d - A*e))*x)/e^4 - (c*(2*C*d - B*e)*x^2)/(
2*e^3) + (c*C*x^3)/(3*e^2) - ((c*d^2 + a*e^2)*(C*d^2 - B*d*e + A*e^2))/(e^5
*(d + e*x)) - ((4*c*C*d^3 - c*d*e*(3*B*d - 2*A*e) + a*e^2*(2*C*d - B*e))*Lo
g[d + e*x])/e^5
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{3cCd^2+aCe^2-ce(2Bd-Ae)}{e^4}+\frac{c(-2Cd+Be)x}{e^3}+\frac{cCx^2}{e^2}\right.\\
&\qquad+\frac{(cd^2+ae^2)\,(Cd^2-Bde+Ae^2)}{e^4(d+ex)^2}\\
&\qquad\left.+\frac{-4cCd^3+cde(3Bd-2Ae)-ae^2(2Cd-Be)}{e^4(d+ex)}\right)dx\\
&= \frac{(3cCd^2+aCe^2-ce(2Bd-Ae))\,x}{e^4}-\frac{c(2Cd-Be)x^2}{2e^3}\\
&\quad+\frac{cCx^3}{3e^2}-\frac{(cd^2+ae^2)\,(Cd^2-Bde+Ae^2)}{e^5(d+ex)}\\
&\quad-\frac{(4cCd^3-cde(3Bd-2Ae)+ae^2(2Cd-Be))\log(d+ex)}{e^5}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.93

$$\int \frac{(a+cx^2)\,(A+Bx+Cx^2)}{(d+ex)^2}\,dx = \frac{6e(3cCd^2+aCe^2+ce(-2Bd+Ae))\,x+3ce^2(-2Cd+Be)x^2+2cCe^3x^3-\frac{6(cd^2+ae^2)\,(Cd^2+e(-Bd+Ae))}{d+ex}+6}{6e^5}$$

```
[In] Integrate[((a + c*x^2)*(A + B*x + C*x^2))/(d + e*x)^2,x]
```

```
[Out] (6*e*(3*c*C*d^2 + a*C*e^2 + c*e*(-2*B*d + A*e))*x + 3*c*e^2*(-2*C*d + B*e)*
x^2 + 2*c*C*e^3*x^3 - (6*(c*d^2 + a*e^2)*(C*d^2 + e*(-(B*d) + A*e)))/(d + e
*x) + 6*(-4*c*C*d^3 + c*d*e*(3*B*d - 2*A*e) + a*e^2*(-2*C*d + B*e))*Log[d +
 e*x])/(6*e^5)
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.12

method	result
default	$\frac{\frac{1}{3}cC\,x^3e^2+\frac{1}{2}Bc\,e^2x^2-Ccde\,x^2+Ac\,e^2x-2Bcdex+aC\,e^2x+3Cc\,d^2x}{e^4}-\frac{Aa\,e^4+Ac\,d^2e^2-Bad\,e^3-Bc\,d^3e+Ca\,d^2e^2+Cc\,d^4}{e^5(ex+d)}+$
norman	$\frac{\frac{\left(Aa\,e^4+2Ac\,d^2e^2-Bad\,e^3-3Bc\,d^3e+2Ca\,d^2e^2+4Cc\,d^4\right)x}{e^4d}+\frac{\left(2Ac\,e^2-3Bcde+2aC\,e^2+4Cc\,d^2\right)x^2}{2e^3}+\frac{cC\,x^4}{3e}+\frac{c(3Be-4Cd)x^3}{6e^2}}{ex+d}-(2Acd\,e^2-$
risch	$\frac{cC\,x^3}{3e^2}+\frac{Bc\,x^2}{2e^2}-\frac{Ccd\,x^2}{e^3}+\frac{Acx}{e^2}-\frac{2Bcdx}{e^3}+\frac{aCx}{e^2}+\frac{3Cc\,d^2x}{e^4}-\frac{Aa}{e(ex+d)}-\frac{Ac\,d^2}{e^3(ex+d)}+\frac{Bad}{e^2(ex+d)}+\frac{Bc\,d^3}{e^4(ex+d)}$
parallelrisch	$-\frac{6Aa\,e^4+24Cc\,d^4+12A\ln(ex+d)xcd\,e^3+24C\ln(ex+d)xc\,d^3e+24C\ln(ex+d)c\,d^4-3B\,x^3c\,e^4-6A\,x^2c\,e^4-6C\,x^2a\,e^4-2cC\,x^4e^4}{}$

```
[In] int((c*x^2+a)*(C*x^2+B*x+A)/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/e^4*(1/3*c*C*x^3*e^2+1/2*B*c*e^2*x^2-C*c*d*e*x^2+A*c*e^2*x-2*B*c*d*e*x+a*
C*e^2*x+3*C*c*d^2*x)-(A*a*e^4+A*c*d^2*e^2-B*a*d*e^3-B*c*d^3*e+C*a*d^2*e^2+C
*c*d^4)/e^5/(e*x+d)+(-2*A*c*d*e^2+B*a*e^3+3*B*c*d^2*e-2*C*a*d*e^2-4*C*c*d^3
)/e^5*ln(e*x+d)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.63

$$\int \frac{(a+cx^2)\left(A+Bx+Cx^2\right)}{(d+ex)^2}\,dx = \frac{2\,Cce^4x^4-6\,Ccd^4+6\,Bcd^3e+6\,Bade^3-6\,Aae^4-6\,(Ca+Ac)d^2e^2-(4\,Ccde^3-3\,Bce^4)x^3+3\,(4\,Ccd^2e}{}$$

```
[In] integrate((c*x^2+a)*(C*x^2+B*x+A)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] 1/6*(2*C*c*e^4*x^4 - 6*C*c*d^4 + 6*B*c*d^3*e + 6*B*a*d*e^3 - 6*A*a*e^4 - 6*
(C*a + A*c)*d^2*e^2 - (4*C*c*d*e^3 - 3*B*c*e^4)*x^3 + 3*(4*C*c*d^2*e^2 - 3*
B*c*d*e^3 + 2*(C*a + A*c)*e^4)*x^2 + 6*(3*C*c*d^3*e - 2*B*c*d^2*e^2 + (C*a
+ A*c)*d*e^3)*x - 6*(4*C*c*d^4 - 3*B*c*d^3*e - B*a*d*e^3 + 2*(C*a + A*c)*d^
2*e^2 + (4*C*c*d^3*e - 3*B*c*d^2*e^2 - B*a*e^4 + 2*(C*a + A*c)*d*e^3)*x)*lo
g(e*x + d))/(e^6*x + d*e^5)
```

Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.21

$$\begin{aligned}\int \frac{(a+cx^2)(A+Bx+Cx^2)}{(d+ex)^2}\,dx &= \frac{Ccx^3}{3e^2}+x^2\left(\frac{Bc}{2e^2}-\frac{Ccd}{e^3}\right)+x\left(\frac{Ac}{e^2}-\frac{2Bcd}{e^3}+\frac{Ca}{e^2}+\frac{3Ccd^2}{e^4}\right)\\&\quad+\frac{-Aae^4-Acd^2e^2+Bade^3+Bcd^3e-Cad^2e^2-Ccd^4}{de^5+e^6x}\\&\quad-\frac{(2Acde^2-Bae^3-3Bcd^2e+2Cade^2+4Ccd^3)\log{(d+ex)}}{e^5}\end{aligned}$$

```
[In] integrate((c*x**2+a)*(C*x**2+B*x+A)/(e*x+d)**2,x)
```

```
[Out] C*c*x**3/(3*e**2) + x**2*(B*c/(2*e**2) - C*c*d/e**3) + x*(A*c/e**2 - 2*B*c*
d/e**3 + C*a/e**2 + 3*C*c*d**2/e**4) + (-A*a*e**4 - A*c*d**2*e**2 + B*a*d*e
**3 + B*c*d**3*e - C*a*d**2*e**2 - C*c*d**4)/(d*e**5 + e**6*x) - (2*A*c*d*e
**2 - B*a*e**3 - 3*B*c*d**2*e + 2*C*a*d*e**2 + 4*C*c*d**3)*log(d + e*x)/e**
5
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.10

$$\begin{aligned}\int \frac{(a+cx^2)(A+Bx+Cx^2)}{(d+ex)^2}\,dx &= -\frac{Ccd^4-Bcd^3e-Bade^3+Aae^4+(Ca+Ac)d^2e^2}{e^6x+de^5}\\&\quad+\frac{2\,Cce^2x^3-3\,(2\,Ccde-Bce^2)x^2+6\,(3\,Ccd^2-2\,Bcde+(Ca+Ac)e^2)x}{6\,e^4}\\&\quad-\frac{(4\,Ccd^3-3\,Bcd^2e-Bae^3+2\,(Ca+Ac)de^2)\log{(ex+d)}}{e^5}\end{aligned}$$

```
[In] integrate((c*x^2+a)*(C*x^2+B*x+A)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -(C*c*d^4 - B*c*d^3*e - B*a*d*e^3 + A*a*e^4 + (C*a + A*c)*d^2*e^2)/(e^6*x +
 d*e^5) + 1/6*(2*C*c*e^2*x^3 - 3*(2*C*c*d*e - B*c*e^2)*x^2 + 6*(3*C*c*d^2 -
 2*B*c*d*e + (C*a + A*c)*e^2)*x)/e^4 - (4*C*c*d^3 - 3*B*c*d^2*e - B*a*e^3 +
 2*(C*a + A*c)*d*e^2)*log(e*x + d)/e^5
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.62

$$\int \frac{(a+cx^2)(A+Bx+Cx^2)}{(d+ex)^2}\,dx$$
$$= \frac{\left(2\,Cc-\frac{3\,(4\,Ccde-Bce^2)}{(ex+d)e}+\frac{6\,(6\,Ccd^2e^2-3\,Bcde^3+Cae^4+Ace^4)}{(ex+d)^2e^2}\right)(ex+d)^3}{6\,e^5} + \frac{(4\,Ccd^3-3\,Bcd^2e+2\,Cade^2+2\,Acde^2-Bae^3)\log\left(\frac{|ex+d|}{(ex+d)^2|e|}\right)}{e^5} - \frac{\frac{Ccd^4e^3}{ex+d}-\frac{Bcd^3e^4}{ex+d}+\frac{Cad^2e^5}{ex+d}+\frac{Acd^2e^5}{ex+d}-\frac{Bade^6}{ex+d}+\frac{Aae^7}{ex+d}}{e^8}$$

```
[In] integrate((c*x^2+a)*(C*x^2+B*x+A)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] 1/6*(2*C*c - 3*(4*C*c*d*e - B*c*e^2)/((e*x + d)*e) + 6*(6*C*c*d^2*e^2 - 3*B
*c*d*e^3 + C*a*e^4 + A*c*e^4)/((e*x + d)^2*e^2))*(e*x + d)^3/e^5 + (4*C*c*d
^3 - 3*B*c*d^2*e + 2*C*a*d*e^2 + 2*A*c*d*e^2 - B*a*e^3)*log(abs(e*x + d)/((
e*x + d)^2*abs(e)))/e^5 - (C*c*d^4*e^3/(e*x + d) - B*c*d^3*e^4/(e*x + d) +
C*a*d^2*e^5/(e*x + d) + A*c*d^2*e^5/(e*x + d) - B*a*d*e^6/(e*x + d) + A*a*e
^7/(e*x + d))/e^8
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.25

$$\int \frac{(a+cx^2)(A+Bx+Cx^2)}{(d+ex)^2}\,dx$$
$$= x^2\left(\frac{B\,c}{2\,e^2}-\frac{C\,c\,d}{e^3}\right) - x\left(\frac{2\,d\left(\frac{B\,c}{e^2}-\frac{2\,C\,c\,d}{e^3}\right)}{e}-\frac{A\,c+C\,a}{e^2}+\frac{C\,c\,d^2}{e^4}\right) - \frac{\ln(d+e\,x)\,\left(4\,C\,c\,d^3-B\,a\,e^3+2\,A\,c\,d\,e^2+2\,C\,a\,d\,e^2-3\,B\,c\,d^2\,e\right)}{e^5} - \frac{A\,a\,e^4+C\,c\,d^4-B\,a\,d\,e^3-B\,c\,d^3\,e+A\,c\,d^2\,e^2+C\,a\,d^2\,e^2}{e\,\left(x\,e^5+d\,e^4\right)} + \frac{C\,c\,x^3}{3\,e^2}$$

```
[In] int(((a + c*x^2)*(A + B*x + C*x^2))/(d + e*x)^2,x)
```

```
[Out] x^2*((B*c)/(2*e^2) - (C*c*d)/e^3) - x*((2*d*((B*c)/e^2 - (2*C*c*d)/e^3))/e
- (A*c + C*a)/e^2 + (C*c*d^2)/e^4) - (log(d + e*x)*(4*C*c*d^3 - B*a*e^3 + 2
*A*c*d*e^2 + 2*C*a*d*e^2 - 3*B*c*d^2*e))/e^5 - (A*a*e^4 + C*c*d^4 - B*a*d*e
^3 - B*c*d^3*e + A*c*d^2*e^2 + C*a*d^2*e^2)/(e*(d*e^4 + e^5*x)) + (C*c*x^3)
/(3*e^2)
```

3.24 $\int \frac{(a+cx^2)(A+Bx+Cx^2)}{(d+ex)^3} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 156

$$\begin{aligned}\int \frac{(a+cx^2)(A+Bx+Cx^2)}{(d+ex)^3} \, dx = & -\frac{c(3Cd-Be)x}{e^4}+\frac{cCx^2}{2e^3} \\ & -\frac{(cd^2+ae^2)(Cd^2-Bde+Ae^2)}{2e^5(d+ex)^2} \\ & +\frac{ae^2(2Cd-Be)+cd(4Cd^2-e(3Bd-2Ae))}{e^5(d+ex)} \\ & +\frac{(aCe^2+c(6Cd^2-e(3Bd-Ae)))\log(d+ex)}{e^5}\end{aligned}$$

```
[Out] -c*(-B*e+3*C*d)*x/e^4+1/2*c*C*x^2/e^3-1/2*(a*e^2+c*d^2)*(A*e^2-B*d*e+C*d^2)
/e^5/(e*x+d)^2+(a*e^2*(-B*e+2*C*d)+c*d*(4*C*d^2-e*(-2*A*e+3*B*d)))/e^5/(e*x
+d)+(a*C*e^2+c*(6*C*d^2-e*(-A*e+3*B*d)))*ln(e*x+d)/e^5
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {1642}

$$\begin{aligned}\int \frac{(a+cx^2)(A+Bx+Cx^2)}{(d+ex)^3} \, dx = & \frac{ae^2(2Cd-Be)-cde(3Bd-2Ae)+4cCd^3}{e^5(d+ex)} \\ & -\frac{(ae^2+cd^2)(Ae^2-Bde+Cd^2)}{2e^5(d+ex)^2} \\ & +\frac{\log(d+ex)(aCe^2-ce(3Bd-Ae)+6cCd^2)}{e^5} \\ & -\frac{cx(3Cd-Be)}{e^4}+\frac{cCx^2}{2e^3}\end{aligned}$$

```
[In] Int[((a + c*x^2)*(A + B*x + C*x^2))/(d + e*x)^3,x]
```

```
[Out] -((c*(3*C*d - B*e)*x)/e^4) + (c*C*x^2)/(2*e^3) - ((c*d^2 + a*e^2)*(C*d^2 -
B*d*e + A*e^2))/(2*e^5*(d + e*x)^2) + (4*c*C*d^3 - c*d*e*(3*B*d - 2*A*e) +
a*e^2*(2*C*d - B*e))/(e^5*(d + e*x)) + ((6*c*C*d^2 + a*C*e^2 - c*e*(3*B*d -
 A*e))*Log[d + e*x])/e^5
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{c(-3Cd+Be)}{e^4}+\frac{cCx}{e^3}+\frac{(cd^2+ae^2)(Cd^2-Bde+Ae^2)}{e^4(d+ex)^3}\right.\\ &\qquad +\frac{-4cCd^3+cde(3Bd-2Ae)-ae^2(2Cd-Be)}{e^4(d+ex)^2}\\ &\qquad \left.+\frac{6cCd^2+aCe^2-ce(3Bd-Ae)}{e^4(d+ex)}\right)dx\\ &= -\frac{c(3Cd-Be)x}{e^4}+\frac{cCx^2}{2e^3}-\frac{(cd^2+ae^2)(Cd^2-Bde+Ae^2)}{2e^5(d+ex)^2}\\ &\quad +\frac{4cCd^3-cde(3Bd-2Ae)+ae^2(2Cd-Be)}{e^5(d+ex)}\\ &\quad +\frac{(6cCd^2+aCe^2-ce(3Bd-Ae))\log(d+ex)}{e^5}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.13

$$\begin{aligned}\int \frac{(a+cx^2)(A+Bx+Cx^2)}{(d+ex)^3}\,dx &= \frac{c(-3Cd+Be)x}{e^4}+\frac{cCx^2}{2e^3}\\ &\quad +\frac{-cCd^4+Bcd^3e-Acd^2e^2-aCd^2e^2+aBde^3-aAe^4}{2e^5(d+ex)^2}\\ &\quad +\frac{4cCd^3-3Bcd^2e+2Acde^2+2aCde^2-aBe^3}{e^5(d+ex)}\\ &\quad +\frac{(6cCd^2-3Bcde+Ace^2+aCe^2)\log(d+ex)}{e^5}\end{aligned}$$

```
[In] Integrate[((a + c*x^2)*(A + B*x + C*x^2))/(d + e*x)^3,x]
```

```
[Out] (c*(-3*C*d + B*e)*x)/e^4 + (c*C*x^2)/(2*e^3) + (-(c*C*d^4) + B*c*d^3*e - A*
c*d^2*e^2 - a*C*d^2*e^2 + a*B*d*e^3 - a*A*e^4)/(2*e^5*(d + e*x)^2) + (4*c*C
*d^3 - 3*B*c*d^2*e + 2*A*c*d*e^2 + 2*a*C*d*e^2 - a*B*e^3)/(e^5*(d + e*x)) +
 ((6*c*C*d^2 - 3*B*c*d*e + A*c*e^2 + a*C*e^2)*Log[d + e*x])/e^5
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.08

method	result
default	$\frac{c\left(\frac{1}{2}Cx^2e+Bex-3Cdx\right)}{e^4}-\frac{-2Acde^2+Be^3a+3Bcd^2e-2Cade^2-4Ccd^3}{e^5(ex+d)}+\frac{\left(Ace^2-3Bcde+aCe^2+6Ccd^2\right)\ln(ex+d)}{e^5}-$
norman	$\frac{\frac{\left(2Acde^2-Be^3a-6Bcd^2e+2Cade^2+12Ccd^3\right)x}{e^4}+\frac{c(Be-2Cd)x^3}{e^2}-\frac{Aae^4-3Acd^2e^2+Bade^3+9Bcd^3e-3Cad^2e^2-18Ccd^4}{2e^5}+\frac{cCx^4}{2e}}{(ex+d)^2}+$
risch	$\frac{cCx^2}{2e^3}+\frac{Bcx}{e^3}-\frac{3cCdx}{e^4}+\frac{\left(2Acde^2-Be^3a-3Bcd^2e+2Cade^2+4Ccd^3\right)x-\frac{Aae^4-3Acd^2e^2+Bade^3+5Bcd^3e-3Cad^2e^2-7C}{2e}}{e^4(ex+d)^2}$
parallelrisch	$-Aae^4+18Ccd^4+4A\ln(ex+d)xcde^3-6B\ln(ex+d)x^2cde^3+12C\ln(ex+d)x^2cd^2e^2+24C\ln(ex+d)xcd^3e+12C\ln(ex+d)cd^4$

```
[In] int((c*x^2+a)*(C*x^2+B*x+A)/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] c/e^4*(1/2*C*x^2*e+B*e*x-3*C*d*x)-(-2*A*c*d*e^2+B*a*e^3+3*B*c*d^2*e-2*C*a*d
*e^2-4*C*c*d^3)/e^5/(e*x+d)+1/e^5*(A*c*e^2-3*B*c*d*e+C*a*e^2+6*C*c*d^2)*ln(
e*x+d)-1/2*(A*a*e^4+A*c*d^2*e^2-B*a*d*e^3-B*c*d^3*e+C*a*d^2*e^2+C*c*d^4)/e^
5/(e*x+d)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.75

$$\int \frac{(a+cx^2)(A+Bx+Cx^2)}{(d+ex)^3}\,dx$$
$$=\frac{Cce^4x^4+7Ccd^4-5Bcd^3e-Bade^3-Aae^4+3(Ca+Ac)d^2e^2-2(2Ccde^3-Bce^4)x^3-(11Ccd^2e^2-}{}$$

```
[In] integrate((c*x^2+a)*(C*x^2+B*x+A)/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(C*c*e^4*x^4 + 7*C*c*d^4 - 5*B*c*d^3*e - B*a*d*e^3 - A*a*e^4 + 3*(C*a +
 A*c)*d^2*e^2 - 2*(2*C*c*d*e^3 - B*c*e^4)*x^3 - (11*C*c*d^2*e^2 - 4*B*c*d*e
^3)*x^2 + 2*(C*c*d^3*e - 2*B*c*d^2*e^2 - B*a*e^4 + 2*(C*a + A*c)*d*e^3)*x +
 2*(6*C*c*d^4 - 3*B*c*d^3*e + (C*a + A*c)*d^2*e^2 + (6*C*c*d^2*e^2 - 3*B*c*
d*e^3 + (C*a + A*c)*e^4)*x^2 + 2*(6*C*c*d^3*e - 3*B*c*d^2*e^2 + (C*a + A*c)
*d*e^3)*x)*log(e*x + d))/(e^7*x^2 + 2*d*e^6*x + d^2*e^5)
```

Sympy [A] (verification not implemented)

Time = 1.79 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.32

$$\int \frac{(a+cx^2)(A+Bx+Cx^2)}{(d+ex)^3}\,dx = \frac{Ccx^2}{2e^3} + x\left(\frac{Bc}{e^3} - \frac{3Ccd}{e^4}\right) + \frac{-Aae^4 + 3Acd^2e^2 - Bade^3 - 5Bcd^3e + 3Cad^2e^2 + 7Ccd^4 + x(4Acde^3 - 2Bae^4 - 6Bcd^2e^2 + 4Cade^3 +}{2d^2e^5 + 4de^6x + 2e^7x^2} + \frac{(Ace^2 - 3Bcde + Cae^2 + 6Ccd^2)\log(d+ex)}{e^5}$$

```
[In] integrate((c*x**2+a)*(C*x**2+B*x+A)/(e*x+d)**3,x)
```

```
[Out] C*c*x**2/(2*e**3) + x*(B*c/e**3 - 3*C*c*d/e**4) + (-A*a*e**4 + 3*A*c*d**2*e
**2 - B*a*d*e**3 - 5*B*c*d**3*e + 3*C*a*d**2*e**2 + 7*C*c*d**4 + x*(4*A*c*d
*e**3 - 2*B*a*e**4 - 6*B*c*d**2*e**2 + 4*C*a*d*e**3 + 8*C*c*d**3*e))/(2*d**
2*e**5 + 4*d*e**6*x + 2*e**7*x**2) + (A*c*e**2 - 3*B*c*d*e + C*a*e**2 + 6*C
*c*d**2)*log(d + e*x)/e**5
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.13

$$\int \frac{(a+cx^2)(A+Bx+Cx^2)}{(d+ex)^3}\,dx = \frac{7\,Ccd^4 - 5\,Bcd^3e - Bade^3 - Aae^4 + 3\,(Ca+Ac)d^2e^2 + 2\,(4\,Ccd^3e - 3\,Bcd^2e^2 - Bae^4 + 2\,(Ca+Ac)de^3)}{2\,(e^7x^2 + 2\,de^6x + d^2e^5)} + \frac{Ccex^2 - 2\,(3\,Ccd - Bce)x}{2\,e^4} + \frac{(6\,Ccd^2 - 3\,Bcde + (Ca+Ac)e^2)\log(ex+d)}{e^5}$$

```
[In] integrate((c*x^2+a)*(C*x^2+B*x+A)/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(7*C*c*d^4 - 5*B*c*d^3*e - B*a*d*e^3 - A*a*e^4 + 3*(C*a + A*c)*d^2*e^2
+ 2*(4*C*c*d^3*e - 3*B*c*d^2*e^2 - B*a*e^4 + 2*(C*a + A*c)*d*e^3)*x)/(e^7*x
^2 + 2*d*e^6*x + d^2*e^5) + 1/2*(C*c*e*x^2 - 2*(3*C*c*d - B*c*e)*x)/e^4 + (
6*C*c*d^2 - 3*B*c*d*e + (C*a + A*c)*e^2)*log(e*x + d)/e^5
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.14

$$
\begin{aligned}
&\int \frac{(a+cx^2)\,(A+Bx+Cx^2)}{(d+ex)^3}\,dx \\
&= \frac{(6\,Ccd^2-3\,Bcde+Cae^2+Ace^2)\log(|ex+d|)}{e^5}+\frac{Cce^3x^2-6\,Ccde^2x+2\,Bce^3x}{2\,e^6} \\
&\quad+\frac{7\,Ccd^4-5\,Bcd^3e+3\,Cad^2e^2+3\,Acd^2e^2-Bade^3-Aae^4+2\,(4\,Ccd^3e-3\,Bcd^2e^2+2\,Cade^3+2\,Acd}{2\,(ex+d)^2e^5}
\end{aligned}
$$

```
[In] integrate((c*x^2+a)*(C*x^2+B*x+A)/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] (6*C*c*d^2 - 3*B*c*d*e + C*a*e^2 + A*c*e^2)*log(abs(e*x + d))/e^5 + 1/2*(C*
c*e^3*x^2 - 6*C*c*d*e^2*x + 2*B*c*e^3*x)/e^6 + 1/2*(7*C*c*d^4 - 5*B*c*d^3*e
 + 3*C*a*d^2*e^2 + 3*A*c*d^2*e^2 - B*a*d*e^3 - A*a*e^4 + 2*(4*C*c*d^3*e - 3
*B*c*d^2*e^2 + 2*C*a*d*e^3 + 2*A*c*d*e^3 - B*a*e^4)*x)/((e*x + d)^2*e^5)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.19

$$
\begin{aligned}
&\int \frac{(a+cx^2)\,(A+Bx+Cx^2)}{(d+ex)^3}\,dx \\
&= \frac{x\,(4\,C\,c\,d^3-B\,a\,e^3+2\,A\,c\,d\,e^2+2\,C\,a\,d\,e^2-3\,B\,c\,d^2\,e)-\frac{A\,a\,e^4-7\,C\,c\,d^4+B\,a\,d\,e^3+5\,B\,c\,d^3\,e-3\,A\,c\,d^2\,e^2-3\,C\,a\,d^2\,e^2}{2\,e}}{d^2\,e^4+2\,d\,e^5\,x+e^6\,x^2} \\
&\quad+x\left(\frac{B\,c}{e^3}-\frac{3\,C\,c\,d}{e^4}\right)+\frac{\ln(d+e\,x)\,\left(A\,c\,e^2+C\,a\,e^2+6\,C\,c\,d^2-3\,B\,c\,d\,e\right)}{e^5}+\frac{C\,c\,x^2}{2\,e^3}
\end{aligned}
$$

```
[In] int(((a + c*x^2)*(A + B*x + C*x^2))/(d + e*x)^3,x)
```

```
[Out] (x*(4*C*c*d^3 - B*a*e^3 + 2*A*c*d*e^2 + 2*C*a*d*e^2 - 3*B*c*d^2*e) - (A*a*e
^4 - 7*C*c*d^4 + B*a*d*e^3 + 5*B*c*d^3*e - 3*A*c*d^2*e^2 - 3*C*a*d^2*e^2)/(
2*e))/(d^2*e^4 + e^6*x^2 + 2*d*e^5*x) + x*((B*c)/e^3 - (3*C*c*d)/e^4) + (lo
g(d + e*x)*(A*c*e^2 + C*a*e^2 + 6*C*c*d^2 - 3*B*c*d*e))/e^5 + (C*c*x^2)/(2*
e^3)
```

3.25 $\int (d+ex)^3 \left(a+cx^2\right)^2 \left(A+Bx+Cx^2\right) \, dx$

Optimal result

Integrand size = 27, antiderivative size = 304

$$
\begin{aligned}
&\int (d+ex)^3 \left(a+cx^2\right)^2 \left(A+Bx+Cx^2\right) \, dx \\
&= a^2Ad^3x + \frac{1}{3}ad\left(ad(Cd+3Be)+A\left(2cd^2+3ae^2\right)\right)x^3 + \frac{1}{4}a^2e\left(3Cd^2+e(3Bd+Ae)\right)x^4 \\
&\quad + \frac{1}{5}\left(Acd\left(cd^2+6ae^2\right)+a\left(ae^2(3Cd+Be)+2cd^2(Cd+3Be)\right)\right)x^5 \\
&\quad + \frac{1}{6}ae\left(aCe^2+2c\left(3Cd^2+e(3Bd+Ae)\right)\right)x^6 \\
&\quad + \frac{1}{7}c\left(2ae^2(3Cd+Be)+cd\left(Cd^2+3e(Bd+Ae)\right)\right)x^7 \\
&\quad + \frac{1}{8}ce\left(2aCe^2+c\left(3Cd^2+e(3Bd+Ae)\right)\right)x^8 \\
&\quad + \frac{1}{9}c^2e^2(3Cd+Be)x^9 + \frac{1}{10}c^2Ce^3x^{10} + \frac{d^2(Bd+3Ae)\left(a+cx^2\right)^3}{6c}
\end{aligned}
$$

```
[Out] a^2*A*d^3*x+1/3*a*d*(a*d*(3*B*e+C*d)+A*(3*a*e^2+2*c*d^2))*x^3+1/4*a^2*e*(3*
C*d^2+e*(A*e+3*B*d))*x^4+1/5*(A*c*d*(6*a*e^2+c*d^2)+a*(a*e^2*(B*e+3*C*d)+2*
c*d^2*(3*B*e+C*d)))*x^5+1/6*a*e*(a*C*e^2+2*c*(3*C*d^2+e*(A*e+3*B*d)))*x^6+1
/7*c*(2*a*e^2*(B*e+3*C*d)+c*d*(C*d^2+3*e*(A*e+B*d)))*x^7+1/8*c*e*(2*a*C*e^2
+c*(3*C*d^2+e*(A*e+3*B*d)))*x^8+1/9*c^2*e^2*(B*e+3*C*d)*x^9+1/10*c^2*C*e^3*
x^10+1/6*d^2*(3*A*e+B*d)*(c*x^2+a)^3/c
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 301, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {1596, 1824}

$$
\begin{aligned}
&\int (d+ex)^3 \left(a+cx^2\right)^2 \left(A+Bx+Cx^2\right) \, dx \\
&= \frac{1}{4}a^2ex^4\left(e(Ae+3Bd)+3Cd^2\right)+a^2Ad^3x+\frac{1}{7}cx^7\left(2ae^2(Be+3Cd)+3cde(Ae+Bd)+cCd^3\right) \\
&\quad+\frac{1}{8}cex^8\left(2aCe^2+ce(Ae+3Bd)+3cCd^2\right)+\frac{1}{6}aex^6\left(aCe^2+2ce(Ae+3Bd)+6cCd^2\right) \\
&\quad+\frac{1}{5}x^5\left(Acd\left(6ae^2+cd^2\right)+a\left(ae^2(Be+3Cd)+2cd^2(3Be+Cd)\right)\right) \\
&\quad+\frac{1}{3}adx^3\left(A\left(3ae^2+2cd^2\right)+ad(3Be+Cd)\right) \\
&\quad+\frac{d^2\left(a+cx^2\right)^3(3Ae+Bd)}{6c}+\frac{1}{9}c^2e^2x^9(Be+3Cd)+\frac{1}{10}c^2Ce^3x^{10}
\end{aligned}
$$

```
[In] Int[(d + e*x)^3*(a + c*x^2)^2*(A + B*x + C*x^2),x]
```

```
[Out] a^2*A*d^3*x + (a*d*(a*d*(C*d + 3*B*e) + A*(2*c*d^2 + 3*a*e^2))*x^3)/3 + (a^
2*e*(3*C*d^2 + e*(3*B*d + A*e))*x^4)/4 + ((A*c*d*(c*d^2 + 6*a*e^2) + a*(a*e
^2*(3*C*d + B*e) + 2*c*d^2*(C*d + 3*B*e)))*x^5)/5 + (a*e*(6*c*C*d^2 + a*C*e
^2 + 2*c*e*(3*B*d + A*e))*x^6)/6 + (c*(c*C*d^3 + 3*c*d*e*(B*d + A*e) + 2*a*
e^2*(3*C*d + B*e))*x^7)/7 + (c*e*(3*c*C*d^2 + 2*a*C*e^2 + c*e*(3*B*d + A*e)
)*x^8)/8 + (c^2*e^2*(3*C*d + B*e)*x^9)/9 + (c^2*C*e^3*x^10)/10 + (d^2*(B*d
+ 3*A*e)*(a + c*x^2)^3)/(6*c)
```

Rule 1596

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{d^2(Bd+3Ae)\left(a+cx^2\right)^3}{6c} \\
&\quad+\int \left(a+cx^2\right)^2\left(-\left(\left(Bd^3+3Ad^2e\right)x\right)+(d+ex)^3\left(A+Bx+Cx^2\right)\right)\,dx \\
&= \frac{d^2(Bd+3Ae)\left(a+cx^2\right)^3}{6c}+\int \Big(a^2Ad^3+ad\big(ad(Cd+3Be)+A\big(2cd^2+3ae^2\big)\big)x^2 \\
&\quad+a^2e\big(3Cd^2+e(3Bd+Ae)\big)x^3 \\
&\quad+\big(Acd\big(cd^2+6ae^2\big)+a\big(ae^2(3Cd+Be)+2cd^2(Cd+3Be)\big)\big)x^4 \\
&\quad+ae\big(6cCd^2+aCe^2+2ce(3Bd+Ae)\big)x^5 \\
&\quad+c\big(cCd^3+3cde(Bd+Ae)+2ae^2(3Cd+Be)\big)x^6 \\
&\quad+ce\big(3cCd^2+2aCe^2+ce(3Bd+Ae)\big)x^7+c^2e^2(3Cd+Be)x^8+c^2Ce^3x^9\Big)\,dx \\
&= a^2Ad^3x+\frac{1}{3}ad\big(ad(Cd+3Be)+A\big(2cd^2+3ae^2\big)\big)x^3+\frac{1}{4}a^2e\big(3Cd^2+e(3Bd+Ae)\big)x^4 \\
&\quad+\frac{1}{5}\big(Acd\big(cd^2+6ae^2\big)+a\big(ae^2(3Cd+Be)+2cd^2(Cd+3Be)\big)\big)x^5 \\
&\quad+\frac{1}{6}ae\big(6cCd^2+aCe^2+2ce(3Bd+Ae)\big)x^6 \\
&\quad+\frac{1}{7}c\big(cCd^3+3cde(Bd+Ae)+2ae^2(3Cd+Be)\big)x^7 \\
&\quad+\frac{1}{8}ce\big(3cCd^2+2aCe^2+ce(3Bd+Ae)\big)x^8 \\
&\quad+\frac{1}{9}c^2e^2(3Cd+Be)x^9+\frac{1}{10}c^2Ce^3x^{10}+\frac{d^2(Bd+3Ae)\left(a+cx^2\right)^3}{6c}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.10

$$\begin{aligned}\int (d+ex)^3\left(a+cx^2\right)^2\left(A+Bx+Cx^2\right)\,dx&=a^2Ad^3x+\frac{1}{2}a^2d^2(Bd+3Ae)x^2\\&\quad+\frac{1}{3}ad\left(ad(Cd+3Be)+A\left(2cd^2+3ae^2\right)\right)x^3\\&\quad+\frac{1}{4}a\left(2Bcd^3+6Acd^2e+3aCd^2e+3aBde^2\right.\\&\qquad\left.+aAe^3\right)x^4+\frac{1}{5}\left(Acd\left(cd^2+6ae^2\right)\right.\\&\quad\left.+a\left(ae^2(3Cd+Be)+2cd^2(Cd+3Be)\right)\right)x^5\\&\quad+\frac{1}{6}\left(aCe\left(6cd^2+ae^2\right)+Ace\left(3cd^2+2ae^2\right)\right.\\&\qquad\left.+Bcd\left(cd^2+6ae^2\right)\right)x^6+\frac{1}{7}c\left(cCd^3\right.\\&\quad\left.+3cde(Bd+Ae)+2ae^2(3Cd+Be)\right)x^7\\&\quad+\frac{1}{8}ce\left(3cCd^2+2aCe^2+ce(3Bd+Ae)\right)x^8\\&\quad+\frac{1}{9}c^2e^2(3Cd+Be)x^9+\frac{1}{10}c^2Ce^3x^{10}\end{aligned}$$

```
[In] Integrate[(d + e*x)^3*(a + c*x^2)^2*(A + B*x + C*x^2),x]
```

```
[Out] a^2*A*d^3*x + (a^2*d^2*(B*d + 3*A*e)*x^2)/2 + (a*d*(a*d*(C*d + 3*B*e) + A*(
2*c*d^2 + 3*a*e^2))*x^3)/3 + (a*(2*B*c*d^3 + 6*A*c*d^2*e + 3*a*C*d^2*e + 3*
a*B*d*e^2 + a*A*e^3)*x^4)/4 + ((A*c*d*(c*d^2 + 6*a*e^2) + a*(a*e^2*(3*C*d +
 B*e) + 2*c*d^2*(C*d + 3*B*e)))*x^5)/5 + ((a*C*e*(6*c*d^2 + a*e^2) + A*c*e*
(3*c*d^2 + 2*a*e^2) + B*c*d*(c*d^2 + 6*a*e^2))*x^6)/6 + (c*(c*C*d^3 + 3*c*d
*e*(B*d + A*e) + 2*a*e^2*(3*C*d + B*e))*x^7)/7 + (c*e*(3*c*C*d^2 + 2*a*C*e^
2 + c*e*(3*B*d + A*e))*x^8)/8 + (c^2*e^2*(3*C*d + B*e)*x^9)/9 + (c^2*C*e^3*
x^10)/10
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.22

method	result
norman	$\frac{c^2C\,e^3x^{10}}{10}+\left(\frac{1}{9}e^3c^2B+\frac{1}{3}c^2d\,e^2C\right)x^9+\left(\frac{1}{8}e^3c^2A+\frac{3}{8}c^2d\,e^2B+\frac{1}{4}Cac\,e^3+\frac{3}{8}C\,c^2d^2e\right)x^8+(\frac{3}{7}c^2d$
default	$\frac{c^2C\,e^3x^{10}}{10}+\frac{(e^3c^2B+3c^2d\,e^2C)x^9}{9}+\frac{((2c\,e^3a+3c^2d^2e)C+3c^2d\,e^2B+e^3c^2A)x^8}{8}+\frac{((6acd\,e^2+c^2d^3)C+(2c\,e^3a+3c^2d^2e)B}{7}$
gosper	$\frac{6}{7}x^7Cacd\,e^2+x^6Bacd\,e^2+x^6Cac\,d^2e+\frac{6}{5}x^5Aacd\,e^2+\frac{6}{5}x^5Bac\,d^2e+\frac{1}{9}B\,c^2e^3x^9+\frac{1}{8}x^8A\,c^2e^3$
risch	$\frac{6}{7}x^7Cacd\,e^2+x^6Bacd\,e^2+x^6Cac\,d^2e+\frac{6}{5}x^5Aacd\,e^2+\frac{6}{5}x^5Bac\,d^2e+\frac{1}{9}B\,c^2e^3x^9+\frac{1}{8}x^8A\,c^2e^3$
parallelrisch	$\frac{6}{7}x^7Cacd\,e^2+x^6Bacd\,e^2+x^6Cac\,d^2e+\frac{6}{5}x^5Aacd\,e^2+\frac{6}{5}x^5Bac\,d^2e+\frac{1}{9}B\,c^2e^3x^9+\frac{1}{8}x^8A\,c^2e^3$

```
[In] int((e*x+d)^3*(c*x^2+a)^2*(C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*c^2*C*e^3*x^10+(1/9*e^3*c^2*B+1/3*c^2*d*e^2*C)*x^9+(1/8*e^3*c^2*A+3/8*
c^2*d*e^2*B+1/4*C*a*c*e^3+3/8*C*c^2*d^2*e)*x^8+(3/7*c^2*d*e^2*A+2/7*B*e^3*a
*c+3/7*B*c^2*d^2*e+6/7*C*a*c*d*e^2+1/7*C*c^2*d^3)*x^7+(1/3*A*a*c*e^3+1/2*A*
c^2*d^2*e+B*a*c*d*e^2+1/6*B*c^2*d^3+1/6*C*a^2*e^3+C*a*c*d^2*e)*x^6+(6/5*A*a
*c*d*e^2+1/5*A*d^3*c^2+1/5*a^2*B*e^3+6/5*B*a*c*d^2*e+3/5*C*a^2*d*e^2+2/5*C*
a*c*d^3)*x^5+(1/4*A*a^2*e^3+3/2*A*a*c*d^2*e+3/4*B*a^2*d*e^2+1/2*B*a*c*d^3+3
/4*d^2*e*a^2*C)*x^4+(A*a^2*d*e^2+2/3*A*d^3*a*c+B*a^2*d^2*e+1/3*d^3*a^2*C)*x
^3+(3/2*A*a^2*d^2*e+1/2*B*a^2*d^3)*x^2+A*d^3*a^2*x
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.18

$$\begin{aligned}\int (d+ex)^3\left(a+cx^2\right)^2\left(A+Bx+Cx^2\right)\,dx &= \frac{1}{10}Cc^2e^3x^{10}+\frac{1}{9}\left(3\,Cc^2de^2+Bc^2e^3\right)x^9+\frac{1}{8}\left(3\,Cc^2d^2e+3\,Bc^2de^2+\left(2\,Cac+Ac^2\right)e^3\right)x^8\\
&\quad+\frac{1}{7}\left(Cc^2d^3+3\,Bc^2d^2e+2\,Bace^3+3\left(2\,Cac+Ac^2\right)de^2\right)x^7+Aa^2d^3x\\
&\quad+\frac{1}{6}\left(Bc^2d^3+6\,Bacde^2+3\left(2\,Cac+Ac^2\right)d^2e+\left(Ca^2+2\,Aac\right)e^3\right)x^6\\
&\quad+\frac{1}{5}\left(6\,Bacd^2e+Ba^2e^3+\left(2\,Cac+Ac^2\right)d^3+3\left(Ca^2+2\,Aac\right)de^2\right)x^5\\
&\quad+\frac{1}{4}\left(2\,Bacd^3+3\,Ba^2de^2+Aa^2e^3+3\left(Ca^2+2\,Aac\right)d^2e\right)x^4\\
&\quad+\frac{1}{3}\left(3\,Ba^2d^2e+3\,Aa^2de^2+\left(Ca^2+2\,Aac\right)d^3\right)x^3+\frac{1}{2}\left(Ba^2d^3+3\,Aa^2d^2e\right)x^2\end{aligned}$$

```
[In] integrate((e*x+d)^3*(c*x^2+a)^2*(C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/10*C*c^2*e^3*x^10 + 1/9*(3*C*c^2*d*e^2 + B*c^2*e^3)*x^9 + 1/8*(3*C*c^2*d^
2*e + 3*B*c^2*d*e^2 + (2*C*a*c + A*c^2)*e^3)*x^8 + 1/7*(C*c^2*d^3 + 3*B*c^2
*d^2*e + 2*B*a*c*e^3 + 3*(2*C*a*c + A*c^2)*d*e^2)*x^7 + A*a^2*d^3*x + 1/6*(
```

```
B*c^2*d^3 + 6*B*a*c*d*e^2 + 3*(2*C*a*c + A*c^2)*d^2*e + (C*a^2 + 2*A*a*c)*e
^3)*x^6 + 1/5*(6*B*a*c*d^2*e + B*a^2*e^3 + (2*C*a*c + A*c^2)*d^3 + 3*(C*a^2
 + 2*A*a*c)*d*e^2)*x^5 + 1/4*(2*B*a*c*d^3 + 3*B*a^2*d*e^2 + A*a^2*e^3 + 3*(
C*a^2 + 2*A*a*c)*d^2*e)*x^4 + 1/3*(3*B*a^2*d^2*e + 3*A*a^2*d*e^2 + (C*a^2 +
 2*A*a*c)*d^3)*x^3 + 1/2*(B*a^2*d^3 + 3*A*a^2*d^2*e)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 445, normalized size of antiderivative = 1.46

$$
\begin{aligned}
\int (d+ex)^3\left(a+cx^2\right)^2\left(A+Bx+Cx^2\right)\,dx ={}& Aa^2d^3x+\frac{Cc^2e^3x^{10}}{10}+x^9\left(\frac{Bc^2e^3}{9}+\frac{Cc^2de^2}{3}\right)\\
&+x^8\left(\frac{Ac^2e^3}{8}+\frac{3Bc^2de^2}{8}+\frac{Cace^3}{4}+\frac{3Cc^2d^2e}{8}\right)\\
&+x^7\cdot\left(\frac{3Ac^2de^2}{7}+\frac{2Bace^3}{7}+\frac{3Bc^2d^2e}{7}\right.\\
&\left.+\frac{6Cacde^2}{7}+\frac{Cc^2d^3}{7}\right)+x^6\left(\frac{Aace^3}{3}+\frac{Ac^2d^2e}{2}\right.\\
&\left.+Bacde^2+\frac{Bc^2d^3}{6}+\frac{Ca^2e^3}{6}+Cacd^2e\right)+x^5\\
&\cdot\left(\frac{6Aacde^2}{5}+\frac{Ac^2d^3}{5}+\frac{Ba^2e^3}{5}+\frac{6Bacd^2e}{5}\right.\\
&\left.+\frac{3Ca^2de^2}{5}+\frac{2Cacd^3}{5}\right)+x^4\left(\frac{Aa^2e^3}{4}\right.\\
&\left.+\frac{3Aacd^2e}{2}+\frac{3Ba^2de^2}{4}+\frac{Bacd^3}{2}+\frac{3Ca^2d^2e}{4}\right)\\
&+x^3\left(Aa^2de^2+\frac{2Aacd^3}{3}+Ba^2d^2e+\frac{Ca^2d^3}{3}\right)\\
&+x^2\cdot\left(\frac{3Aa^2d^2e}{2}+\frac{Ba^2d^3}{2}\right)
\end{aligned}
$$

```
[In] integrate((e*x+d)**3*(c*x**2+a)**2*(C*x**2+B*x+A),x)
```

```
[Out] A*a**2*d**3*x + C*c**2*e**3*x**10/10 + x**9*(B*c**2*e**3/9 + C*c**2*d*e**2/
3) + x**8*(A*c**2*e**3/8 + 3*B*c**2*d*e**2/8 + C*a*c*e**3/4 + 3*C*c**2*d**2
*e/8) + x**7*(3*A*c**2*d*e**2/7 + 2*B*a*c*e**3/7 + 3*B*c**2*d**2*e/7 + 6*C*
a*c*d*e**2/7 + C*c**2*d**3/7) + x**6*(A*a*c*e**3/3 + A*c**2*d**2*e/2 + B*a*
c*d*e**2 + B*c**2*d**3/6 + C*a**2*e**3/6 + C*a*c*d**2*e) + x**5*(6*A*a*c*d*
e**2/5 + A*c**2*d**3/5 + B*a**2*e**3/5 + 6*B*a*c*d**2*e/5 + 3*C*a**2*d*e**2
/5 + 2*C*a*c*d**3/5) + x**4*(A*a**2*e**3/4 + 3*A*a*c*d**2*e/2 + 3*B*a**2*d*
e**2/4 + B*a*c*d**3/2 + 3*C*a**2*d**2*e/4) + x**3*(A*a**2*d*e**2 + 2*A*a*c*
d**3/3 + B*a**2*d**2*e + C*a**2*d**3/3) + x**2*(3*A*a**2*d**2*e/2 + B*a**2*
d**3/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.18

$$\begin{aligned}\int (d+ex)^3 \left(a+cx^2\right)^2 \left(A+Bx+Cx^2\right) \, dx &= \frac{1}{10} Cc^2e^3x^{10} + \frac{1}{9} \left(3 Cc^2de^2 + Bc^2e^3\right)x^9 + \frac{1}{8} \left(3 Cc^2d^2e + 3 Bc^2de^2 + \left(2 Cac + Ac^2\right)e^3\right)x^8 \\ &\quad + \frac{1}{7} \left(Cc^2d^3 + 3 Bc^2d^2e + 2 Bace^3 + 3 \left(2 Cac + Ac^2\right)de^2\right)x^7 + Aa^2d^3x \\ &\quad + \frac{1}{6} \left(Bc^2d^3 + 6 Bacde^2 + 3 \left(2 Cac + Ac^2\right)d^2e + \left(Ca^2 + 2 Aac\right)e^3\right)x^6 \\ &\quad + \frac{1}{5} \left(6 Bacd^2e + Ba^2e^3 + \left(2 Cac + Ac^2\right)d^3 + 3 \left(Ca^2 + 2 Aac\right)de^2\right)x^5 \\ &\quad + \frac{1}{4} \left(2 Bacd^3 + 3 Ba^2de^2 + Aa^2e^3 + 3 \left(Ca^2 + 2 Aac\right)d^2e\right)x^4 \\ &\quad + \frac{1}{3} \left(3 Ba^2d^2e + 3 Aa^2de^2 + \left(Ca^2 + 2 Aac\right)d^3\right)x^3 + \frac{1}{2} \left(Ba^2d^3 + 3 Aa^2d^2e\right)x^2\end{aligned}$$

```
[In] integrate((e*x+d)^3*(c*x^2+a)^2*(C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/10*C*c^2*e^3*x^10 + 1/9*(3*C*c^2*d*e^2 + B*c^2*e^3)*x^9 + 1/8*(3*C*c^2*d^
2*e + 3*B*c^2*d*e^2 + (2*C*a*c + A*c^2)*e^3)*x^8 + 1/7*(C*c^2*d^3 + 3*B*c^2
*d^2*e + 2*B*a*c*e^3 + 3*(2*C*a*c + A*c^2)*d*e^2)*x^7 + A*a^2*d^3*x + 1/6*(
B*c^2*d^3 + 6*B*a*c*d*e^2 + 3*(2*C*a*c + A*c^2)*d^2*e + (C*a^2 + 2*A*a*c)*e
^3)*x^6 + 1/5*(6*B*a*c*d^2*e + B*a^2*e^3 + (2*C*a*c + A*c^2)*d^3 + 3*(C*a^2
 + 2*A*a*c)*d*e^2)*x^5 + 1/4*(2*B*a*c*d^3 + 3*B*a^2*d*e^2 + A*a^2*e^3 + 3*(
C*a^2 + 2*A*a*c)*d^2*e)*x^4 + 1/3*(3*B*a^2*d^2*e + 3*A*a^2*d*e^2 + (C*a^2 +
 2*A*a*c)*d^3)*x^3 + 1/2*(B*a^2*d^3 + 3*A*a^2*d^2*e)*x^2
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 432, normalized size of antiderivative = 1.42

$$\begin{aligned}\int (d+ex)^3 \left(a+cx^2\right)^2 \left(A+Bx+Cx^2\right)\,dx &= \frac{1}{10}\,Cc^2e^3x^{10} + \frac{1}{3}\,Cc^2de^2x^9 + \frac{1}{9}\,Bc^2e^3x^9 \\ &+ \frac{3}{8}\,Cc^2d^2ex^8 + \frac{3}{8}\,Bc^2de^2x^8 + \frac{1}{4}\,Cace^3x^8 \\ &+ \frac{1}{8}\,Ac^2e^3x^8 + \frac{1}{7}\,Cc^2d^3x^7 + \frac{3}{7}\,Bc^2d^2ex^7 \\ &+ \frac{6}{7}\,Cacde^2x^7 + \frac{3}{7}\,Ac^2de^2x^7 + \frac{2}{7}\,Bace^3x^7 \\ &+ \frac{1}{6}\,Bc^2d^3x^6 + Cacd^2ex^6 + \frac{1}{2}\,Ac^2d^2ex^6 \\ &+ Bacde^2x^6 + \frac{1}{6}\,Ca^2e^3x^6 + \frac{1}{3}\,Aace^3x^6 \\ &+ \frac{2}{5}\,Cacd^3x^5 + \frac{1}{5}\,Ac^2d^3x^5 + \frac{6}{5}\,Bacd^2ex^5 \\ &+ \frac{3}{5}\,Ca^2de^2x^5 + \frac{6}{5}\,Aacde^2x^5 + \frac{1}{5}\,Ba^2e^3x^5 \\ &+ \frac{1}{2}\,Bacd^3x^4 + \frac{3}{4}\,Ca^2d^2ex^4 + \frac{3}{2}\,Aacd^2ex^4 \\ &+ \frac{3}{4}\,Ba^2de^2x^4 + \frac{1}{4}\,Aa^2e^3x^4 + \frac{1}{3}\,Ca^2d^3x^3 \\ &+ \frac{2}{3}\,Aacd^3x^3 + Ba^2d^2ex^3 + Aa^2de^2x^3 \\ &+ \frac{1}{2}\,Ba^2d^3x^2 + \frac{3}{2}\,Aa^2d^2ex^2 + Aa^2d^3x\end{aligned}$$

```
[In] integrate((e*x+d)^3*(c*x^2+a)^2*(C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/10*C*c^2*e^3*x^10 + 1/3*C*c^2*d*e^2*x^9 + 1/9*B*c^2*e^3*x^9 + 3/8*C*c^2*d
^2*e*x^8 + 3/8*B*c^2*d*e^2*x^8 + 1/4*C*a*c*e^3*x^8 + 1/8*A*c^2*e^3*x^8 + 1/
7*C*c^2*d^3*x^7 + 3/7*B*c^2*d^2*e*x^7 + 6/7*C*a*c*d*e^2*x^7 + 3/7*A*c^2*d*e
^2*x^7 + 2/7*B*a*c*e^3*x^7 + 1/6*B*c^2*d^3*x^6 + C*a*c*d^2*e*x^6 + 1/2*A*c^
2*d^2*e*x^6 + B*a*c*d*e^2*x^6 + 1/6*C*a^2*e^3*x^6 + 1/3*A*a*c*e^3*x^6 + 2/5
*C*a*c*d^3*x^5 + 1/5*A*c^2*d^3*x^5 + 6/5*B*a*c*d^2*e*x^5 + 3/5*C*a^2*d*e^2*
x^5 + 6/5*A*a*c*d*e^2*x^5 + 1/5*B*a^2*e^3*x^5 + 1/2*B*a*c*d^3*x^4 + 3/4*C*a
^2*d^2*e*x^4 + 3/2*A*a*c*d^2*e*x^4 + 3/4*B*a^2*d*e^2*x^4 + 1/4*A*a^2*e^3*x^
4 + 1/3*C*a^2*d^3*x^3 + 2/3*A*a*c*d^3*x^3 + B*a^2*d^2*e*x^3 + A*a^2*d*e^2*x
^3 + 1/2*B*a^2*d^3*x^2 + 3/2*A*a^2*d^2*e*x^2 + A*a^2*d^3*x
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.09

$$\begin{aligned}
&\int (d+ex)^3\left(a+cx^2\right)^2\left(A+Bx+Cx^2\right)\,dx \\
&= x^5\left(\frac{3\,C\,a^2\,d\,e^2}{5}+\frac{B\,a^2\,e^3}{5}+\frac{2\,C\,a\,c\,d^3}{5}+\frac{6\,B\,a\,c\,d^2\,e}{5}+\frac{6\,A\,a\,c\,d\,e^2}{5}+\frac{A\,c^2\,d^3}{5}\right) \\
&\quad + x^6\left(\frac{C\,a^2\,e^3}{6}+C\,a\,c\,d^2\,e+B\,a\,c\,d\,e^2+\frac{A\,a\,c\,e^3}{3}+\frac{B\,c^2\,d^3}{6}+\frac{A\,c^2\,d^2\,e}{2}\right) \\
&\quad + \frac{a\,x^4\,\left(A\,a\,e^3+2\,B\,c\,d^3+3\,B\,a\,d\,e^2+6\,A\,c\,d^2\,e+3\,C\,a\,d^2\,e\right)}{4} \\
&\quad + \frac{c\,x^7\,\left(2\,B\,a\,e^3+C\,c\,d^3+3\,A\,c\,d\,e^2+6\,C\,a\,d\,e^2+3\,B\,c\,d^2\,e\right)}{7} \\
&\quad + \frac{C\,c^2\,e^3\,x^{10}}{10}+\frac{a^2\,d^2\,x^2\,\left(3\,A\,e+B\,d\right)}{2}+\frac{c^2\,e^2\,x^9\,\left(B\,e+3\,C\,d\right)}{9} \\
&\quad + \frac{a\,d\,x^3\,\left(3\,A\,a\,e^2+2\,A\,c\,d^2+C\,a\,d^2+3\,B\,a\,d\,e\right)}{3} \\
&\quad + \frac{c\,e\,x^8\,\left(A\,c\,e^2+2\,C\,a\,e^2+3\,C\,c\,d^2+3\,B\,c\,d\,e\right)}{8}+A\,a^2\,d^3\,x
\end{aligned}$$

```
[In] int((a + c*x^2)^2*(d + e*x)^3*(A + B*x + C*x^2),x)
```

```
[Out] x^5*((A*c^2*d^3)/5 + (B*a^2*e^3)/5 + (2*C*a*c*d^3)/5 + (3*C*a^2*d*e^2)/5 +
(6*A*a*c*d*e^2)/5 + (6*B*a*c*d^2*e)/5) + x^6*((B*c^2*d^3)/6 + (C*a^2*e^3)/6
 + (A*a*c*e^3)/3 + (A*c^2*d^2*e)/2 + B*a*c*d*e^2 + C*a*c*d^2*e) + (a*x^4*(A
*a*e^3 + 2*B*c*d^3 + 3*B*a*d*e^2 + 6*A*c*d^2*e + 3*C*a*d^2*e))/4 + (c*x^7*(
2*B*a*e^3 + C*c*d^3 + 3*A*c*d*e^2 + 6*C*a*d*e^2 + 3*B*c*d^2*e))/7 + (C*c^2*
e^3*x^10)/10 + (a^2*d^2*x^2*(3*A*e + B*d))/2 + (c^2*e^2*x^9*(B*e + 3*C*d))/
9 + (a*d*x^3*(3*A*a*e^2 + 2*A*c*d^2 + C*a*d^2 + 3*B*a*d*e))/3 + (c*e*x^8*(A
*c*e^2 + 2*C*a*e^2 + 3*C*c*d^2 + 3*B*c*d*e))/8 + A*a^2*d^3*x
```

3.26 $\int (d+ex)^2 \left(a+cx^2\right)^2 \left(A+Bx+Cx^2\right) \, dx$

Optimal result

Integrand size = 27, antiderivative size = 217

$$
\begin{aligned}
\int (d+ex)^2 \left(a+cx^2\right)^2 \left(A+Bx+Cx^2\right) \, dx = {} & a^2Ad^2x+\frac{1}{3}a\left(ad(Cd+2Be)+A\left(2cd^2+ae^2\right)\right)x^3 \\
& +\frac{1}{4}a^2e(2Cd+Be)x^4+\frac{1}{5}\left(Ac\left(cd^2+2ae^2\right)\right. \\
& \left.+a\left(aCe^2+2cd(Cd+2Be)\right)\right)x^5 \\
& +\frac{1}{3}ace(2Cd+Be)x^6 \\
& +\frac{1}{7}c\left(2aCe^2+c\left(Cd^2+e(2Bd+Ae)\right)\right)x^7 \\
& +\frac{1}{8}c^2e(2Cd+Be)x^8+\frac{1}{9}c^2Ce^2x^9 \\
& +\frac{d(Bd+2Ae)\left(a+cx^2\right)^3}{6c}
\end{aligned}
$$

```
[Out] a^2*A*d^2*x+1/3*a*(a*d*(2*B*e+C*d)+A*(a*e^2+2*c*d^2))*x^3+1/4*a^2*e*(B*e+2*
C*d)*x^4+1/5*(A*c*(2*a*e^2+c*d^2)+a*(a*C*e^2+2*c*d*(2*B*e+C*d)))*x^5+1/3*a*
c*e*(B*e+2*C*d)*x^6+1/7*c*(2*a*C*e^2+c*(C*d^2+e*(A*e+2*B*d)))*x^7+1/8*c^2*e
*(B*e+2*C*d)*x^8+1/9*c^2*C*e^2*x^9+1/6*d*(2*A*e+B*d)*(c*x^2+a)^3/c
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1596, 1824}

$$\begin{aligned}&\int (d+ex)^2 \left(a+cx^2\right)^2 \left(A+Bx+Cx^2\right) \, dx\\ &= a^2Ad^2x+\frac{1}{4}a^2ex^4(Be+2Cd)+\frac{1}{7}cx^7\left(2aCe^2+ce(Ae+2Bd)+cCd^2\right)\\ &\quad +\frac{1}{5}x^5\left(Ac\left(2ae^2+cd^2\right)+a\left(aCe^2+2cd(2Be+Cd)\right)\right)\\ &\quad +\frac{1}{3}ax^3\left(A\left(ae^2+2cd^2\right)+ad(2Be+Cd)\right)+\frac{d\left(a+cx^2\right)^3(2Ae+Bd)}{6c}\\ &\quad +\frac{1}{3}acex^6(Be+2Cd)+\frac{1}{8}c^2ex^8(Be+2Cd)+\frac{1}{9}c^2Ce^2x^9\end{aligned}$$

```
[In] Int[(d + e*x)^2*(a + c*x^2)^2*(A + B*x + C*x^2),x]
```

```
[Out] a^2*A*d^2*x + (a*(a*d*(C*d + 2*B*e) + A*(2*c*d^2 + a*e^2))*x^3)/3 + (a^2*e*
(2*C*d + B*e)*x^4)/4 + ((A*c*(c*d^2 + 2*a*e^2) + a*(a*C*e^2 + 2*c*d*(C*d +
2*B*e)))*x^5)/5 + (a*c*e*(2*C*d + B*e)*x^6)/3 + (c*(c*C*d^2 + 2*a*C*e^2 + c
*e*(2*B*d + A*e))*x^7)/7 + (c^2*e*(2*C*d + B*e)*x^8)/8 + (c^2*C*e^2*x^9)/9
+ (d*(B*d + 2*A*e)*(a + c*x^2)^3)/(6*c)
```

Rule 1596

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{d(Bd+2Ae)\left(a+cx^2\right)^3}{6c}\\ &\quad +\int \left(a+cx^2\right)^2 \left(-\left(\left(Bd^2+2Ade\right)x\right)+(d+ex)^2 \left(A+Bx+Cx^2\right)\right) \, dx\end{aligned}$$

$$= \frac{d(Bd+2Ae)\left(a+cx^2\right)^3}{6c} + \int \left(a^2Ad^2 + a\big(ad(Cd+2Be)+A\big(2cd^2+ae^2\big)\big)\,x^2 + a^2e(2Cd+Be)x^3 + \big(Ac\big(cd^2+2ae^2\big)+a\big(aCe^2+2cd(Cd+2Be)\big)\big)\,x^4 + 2ace(2Cd+Be)x^5 + c\big(cCd^2+2aCe^2+ce(2Bd+Ae)\big)\,x^6 + c^2e(2Cd+Be)x^7 + c^2Ce^2x^8\right)\,dx$$

$$= a^2Ad^2x + \frac{1}{3}a\big(ad(Cd+2Be)+A\big(2cd^2+ae^2\big)\big)\,x^3 + \frac{1}{4}a^2e(2Cd+Be)x^4 + \frac{1}{5}\big(Ac\big(cd^2+2ae^2\big)+a\big(aCe^2+2cd(Cd+2Be)\big)\big)\,x^5 + \frac{1}{3}ace(2Cd+Be)x^6 + \frac{1}{7}c\big(cCd^2+2aCe^2+ce(2Bd+Ae)\big)\,x^7 + \frac{1}{8}c^2e(2Cd+Be)x^8 + \frac{1}{9}c^2Ce^2x^9 + \frac{d(Bd+2Ae)\left(a+cx^2\right)^3}{6c}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.11

$$\int (d+ex)^2\left(a+cx^2\right)^2\left(A+Bx+Cx^2\right)\,dx = a^2Ad^2x + \frac{1}{2}a^2d(Bd+2Ae)x^2 + \frac{1}{3}a\big(ad(Cd+2Be)+A\big(2cd^2+ae^2\big)\big)\,x^3 + \frac{1}{4}a\big(2Bcd^2+4Acde+2aCde+aBe^2\big)\,x^4 + \frac{1}{5}\big(Ac\big(cd^2+2ae^2\big) + a\big(aCe^2+2cd(Cd+2Be)\big)\big)\,x^5 + \frac{1}{6}c\big(Bcd^2+2Acde+4aCde+2aBe^2\big)\,x^6 + \frac{1}{7}c\big(cCd^2+2aCe^2+ce(2Bd+Ae)\big)\,x^7 + \frac{1}{8}c^2e(2Cd+Be)x^8 + \frac{1}{9}c^2Ce^2x^9$$

```
[In] Integrate[(d + e*x)^2*(a + c*x^2)^2*(A + B*x + C*x^2),x]
```

```
[Out] a^2*A*d^2*x + (a^2*d*(B*d + 2*A*e)*x^2)/2 + (a*(a*d*(C*d + 2*B*e) + A*(2*c*
d^2 + a*e^2))*x^3)/3 + (a*(2*B*c*d^2 + 4*A*c*d*e + 2*a*C*d*e + a*B*e^2)*x^4
)/4 + ((A*c*(c*d^2 + 2*a*e^2) + a*(a*C*e^2 + 2*c*d*(C*d + 2*B*e)))*x^5)/5 +
 (c*(B*c*d^2 + 2*A*c*d*e + 4*a*C*d*e + 2*a*B*e^2)*x^6)/6 + (c*(c*C*d^2 + 2*
a*C*e^2 + c*e*(2*B*d + A*e))*x^7)/7 + (c^2*e*(2*C*d + B*e)*x^8)/8 + (c^2*C*
e^2*x^9)/9
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.21

method	result
norman	$\frac{c^2 C e^2 x^9}{9}+\left(\frac{1}{8} B c^2 e^2+\frac{1}{4} c^2 d e C\right) x^8+\left(\frac{1}{7} A c^2 e^2+\frac{2}{7} B c^2 d e+\frac{2}{7} C a c e^2+\frac{1}{7} C c^2 d^2\right) x^7+\left(\frac{1}{3} A c^2 d e\right.$
default	$\frac{c^2 C e^2 x^9}{9}+\frac{\left(B c^2 e^2+2 c^2 d e C\right) x^8}{8}+\frac{\left(\left(2 a c e^2+c^2 d^2\right) C+2 B c^2 d e+A c^2 e^2\right) x^7}{7}+\frac{\left(4 a c d e C+\left(2 a c e^2+c^2 d^2\right) B+2 A c^2 d e\right) x^6}{6}+$
gosper	$\frac{2}{3} x^6 a c d e C+\frac{4}{5} x^5 B a c d e+x^4 A a c d e+\frac{1}{2} x^4 B a c d^2+\frac{1}{8} B c^2 e^2 x^8+\frac{2}{3} x^3 A d^2 a c+\frac{2}{3} x^3 B a^2 d e+x^2$
risch	$\frac{2}{3} x^6 a c d e C+\frac{4}{5} x^5 B a c d e+x^4 A a c d e+\frac{1}{2} x^4 B a c d^2+\frac{1}{8} B c^2 e^2 x^8+\frac{2}{3} x^3 A d^2 a c+\frac{2}{3} x^3 B a^2 d e+x^2$
parallelrisch	$\frac{2}{3} x^6 a c d e C+\frac{4}{5} x^5 B a c d e+x^4 A a c d e+\frac{1}{2} x^4 B a c d^2+\frac{1}{8} B c^2 e^2 x^8+\frac{2}{3} x^3 A d^2 a c+\frac{2}{3} x^3 B a^2 d e+x^2$

```
[In] int((e*x+d)^2*(c*x^2+a)^2*(C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*c^2*C*e^2*x^9+(1/8*B*c^2*e^2+1/4*c^2*d*e*C)*x^8+(1/7*A*c^2*e^2+2/7*B*c^
2*d*e+2/7*C*a*c*e^2+1/7*C*c^2*d^2)*x^7+(1/3*A*c^2*d*e+1/3*B*e^2*a*c+1/6*B*c
^2*d^2+2/3*a*c*d*e*C)*x^6+(2/5*A*a*c*e^2+1/5*A*c^2*d^2+4/5*B*a*c*d*e+1/5*a^
2*C*e^2+2/5*C*a*c*d^2)*x^5+(A*a*c*d*e+1/4*a^2*B*e^2+1/2*B*a*c*d^2+1/2*d*e*a
^2*C)*x^4+(1/3*A*a^2*e^2+2/3*A*d^2*a*c+2/3*B*a^2*d*e+1/3*a^2*d^2*C)*x^3+(d*
e*a^2*A+1/2*a^2*d^2*B)*x^2+A*d^2*a^2*x
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.18

$$\begin{aligned}
\int (d+ex)^2 \left(a+cx^2\right)^2 \left(A+Bx+Cx^2\right)\,dx
&= \frac{1}{9}\,Cc^2e^2x^9 + \frac{1}{8}\,(2\,Cc^2de + Bc^2e^2)x^8 + \frac{1}{7}\,\left(Cc^2d^2 + 2\,Bc^2de + \left(2\,Cac + Ac^2\right)e^2\right)x^7 \\
&\quad + \frac{1}{6}\,\left(Bc^2d^2 + 2\,Bace^2 + 2\,\left(2\,Cac + Ac^2\right)de\right)x^6 + Aa^2d^2x \\
&\quad + \frac{1}{5}\,\left(4\,Bacde + \left(2\,Cac + Ac^2\right)d^2 + \left(Ca^2 + 2\,Aac\right)e^2\right)x^5 \\
&\quad + \frac{1}{4}\,\left(2\,Bacd^2 + Ba^2e^2 + 2\,\left(Ca^2 + 2\,Aac\right)de\right)x^4 \\
&\quad + \frac{1}{3}\,\left(2\,Ba^2de + Aa^2e^2 + \left(Ca^2 + 2\,Aac\right)d^2\right)x^3 + \frac{1}{2}\,\left(Ba^2d^2 + 2\,Aa^2de\right)x^2
\end{aligned}$$

```
[In] integrate((e*x+d)^2*(c*x^2+a)^2*(C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/9*C*c^2*e^2*x^9 + 1/8*(2*C*c^2*d*e + B*c^2*e^2)*x^8 + 1/7*(C*c^2*d^2 + 2*
B*c^2*d*e + (2*C*a*c + A*c^2)*e^2)*x^7 + 1/6*(B*c^2*d^2 + 2*B*a*c*e^2 + 2*(
2*C*a*c + A*c^2)*d*e)*x^6 + A*a^2*d^2*x + 1/5*(4*B*a*c*d*e + (2*C*a*c + A*c
^2)*d^2 + (C*a^2 + 2*A*a*c)*e^2)*x^5 + 1/4*(2*B*a*c*d^2 + B*a^2*e^2 + 2*(C*
```

```
a^2 + 2*A*a*c)*d*e)*x^4 + 1/3*(2*B*a^2*d*e + A*a^2*e^2 + (C*a^2 + 2*A*a*c)*
d^2)*x^3 + 1/2*(B*a^2*d^2 + 2*A*a^2*d*e)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.43

$$\begin{aligned}\int (d+ex)^2\left(a+cx^2\right)^2\left(A+Bx+Cx^2\right)\,dx = {} & Aa^2d^2x+\frac{Cc^2e^2x^9}{9}+x^8\left(\frac{Bc^2e^2}{8}+\frac{Cc^2de}{4}\right)\\ & +x^7\left(\frac{Ac^2e^2}{7}+\frac{2Bc^2de}{7}+\frac{2Cace^2}{7}+\frac{Cc^2d^2}{7}\right)\\ & +x^6\left(\frac{Ac^2de}{3}+\frac{Bace^2}{3}+\frac{Bc^2d^2}{6}+\frac{2Cacde}{3}\right)\\ & +x^5\cdot\left(\frac{2Aace^2}{5}+\frac{Ac^2d^2}{5}+\frac{4Bacde}{5}+\frac{Ca^2e^2}{5}\right.\\ & \left.+\frac{2Cacd^2}{5}\right)\\ & +x^4\left(Aacde+\frac{Ba^2e^2}{4}+\frac{Bacd^2}{2}+\frac{Ca^2de}{2}\right)\\ & +x^3\left(\frac{Aa^2e^2}{3}+\frac{2Aacd^2}{3}+\frac{2Ba^2de}{3}+\frac{Ca^2d^2}{3}\right)\\ & +x^2\left(Aa^2de+\frac{Ba^2d^2}{2}\right)\end{aligned}$$

```
[In] integrate((e*x+d)**2*(c*x**2+a)**2*(C*x**2+B*x+A),x)
```

```
[Out] A*a**2*d**2*x + C*c**2*e**2*x**9/9 + x**8*(B*c**2*e**2/8 + C*c**2*d*e/4) +
x**7*(A*c**2*e**2/7 + 2*B*c**2*d*e/7 + 2*C*a*c*e**2/7 + C*c**2*d**2/7) + x*
*6*(A*c**2*d*e/3 + B*a*c*e**2/3 + B*c**2*d**2/6 + 2*C*a*c*d*e/3) + x**5*(2*
A*a*c*e**2/5 + A*c**2*d**2/5 + 4*B*a*c*d*e/5 + C*a**2*e**2/5 + 2*C*a*c*d**2
/5) + x**4*(A*a*c*d*e + B*a**2*e**2/4 + B*a*c*d**2/2 + C*a**2*d*e/2) + x**3
*(A*a**2*e**2/3 + 2*A*a*c*d**2/3 + 2*B*a**2*d*e/3 + C*a**2*d**2/3) + x**2*(
A*a**2*d*e + B*a**2*d**2/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.18

$$
\begin{aligned}
&\int (d+ex)^2 \left(a+cx^2\right)^2 \left(A+Bx+Cx^2\right)\, dx \\
&= \frac{1}{9}Cc^2e^2x^9 + \frac{1}{8}\left(2\,Cc^2de + Bc^2e^2\right)x^8 + \frac{1}{7}\left(Cc^2d^2 + 2\,Bc^2de + \left(2\,Cac + Ac^2\right)e^2\right)x^7 \\
&\quad + \frac{1}{6}\left(Bc^2d^2 + 2\,Bace^2 + 2\left(2\,Cac + Ac^2\right)de\right)x^6 + Aa^2d^2x \\
&\quad + \frac{1}{5}\left(4\,Bacde + \left(2\,Cac + Ac^2\right)d^2 + \left(Ca^2 + 2\,Aac\right)e^2\right)x^5 \\
&\quad + \frac{1}{4}\left(2\,Bacd^2 + Ba^2e^2 + 2\left(Ca^2 + 2\,Aac\right)de\right)x^4 \\
&\quad + \frac{1}{3}\left(2\,Ba^2de + Aa^2e^2 + \left(Ca^2 + 2\,Aac\right)d^2\right)x^3 + \frac{1}{2}\left(Ba^2d^2 + 2\,Aa^2de\right)x^2
\end{aligned}
$$

```
[In] integrate((e*x+d)^2*(c*x^2+a)^2*(C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/9*C*c^2*e^2*x^9 + 1/8*(2*C*c^2*d*e + B*c^2*e^2)*x^8 + 1/7*(C*c^2*d^2 + 2*
B*c^2*d*e + (2*C*a*c + A*c^2)*e^2)*x^7 + 1/6*(B*c^2*d^2 + 2*B*a*c*e^2 + 2*(
2*C*a*c + A*c^2)*d*e)*x^6 + A*a^2*d^2*x + 1/5*(4*B*a*c*d*e + (2*C*a*c + A*c
^2)*d^2 + (C*a^2 + 2*A*a*c)*e^2)*x^5 + 1/4*(2*B*a*c*d^2 + B*a^2*e^2 + 2*(C*
a^2 + 2*A*a*c)*d*e)*x^4 + 1/3*(2*B*a^2*d*e + A*a^2*e^2 + (C*a^2 + 2*A*a*c)*
d^2)*x^3 + 1/2*(B*a^2*d^2 + 2*A*a^2*d*e)*x^2
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.39

$$\begin{aligned}\int (d+ex)^2\left(a+cx^2\right)^2\left(A+Bx+Cx^2\right)\,dx &= \frac{1}{9}\,Cc^2e^2x^9+\frac{1}{4}\,Cc^2dex^8+\frac{1}{8}\,Bc^2e^2x^8\\ &+\frac{1}{7}\,Cc^2d^2x^7+\frac{2}{7}\,Bc^2dex^7+\frac{2}{7}\,Cace^2x^7\\ &+\frac{1}{7}\,Ac^2e^2x^7+\frac{1}{6}\,Bc^2d^2x^6+\frac{2}{3}\,Cacdex^6\\ &+\frac{1}{3}\,Ac^2dex^6+\frac{1}{3}\,Bace^2x^6+\frac{2}{5}\,Cacd^2x^5\\ &+\frac{1}{5}\,Ac^2d^2x^5+\frac{4}{5}\,Bacdex^5+\frac{1}{5}\,Ca^2e^2x^5\\ &+\frac{2}{5}\,Aace^2x^5+\frac{1}{2}\,Bacd^2x^4+\frac{1}{2}\,Ca^2dex^4\\ &+Aacdex^4+\frac{1}{4}\,Ba^2e^2x^4+\frac{1}{3}\,Ca^2d^2x^3\\ &+\frac{2}{3}\,Aacd^2x^3+\frac{2}{3}\,Ba^2dex^3+\frac{1}{3}\,Aa^2e^2x^3\\ &+\frac{1}{2}\,Ba^2d^2x^2+Aa^2dex^2+Aa^2d^2x\end{aligned}$$

```
[In] integrate((e*x+d)^2*(c*x^2+a)^2*(C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/9*C*c^2*e^2*x^9 + 1/4*C*c^2*d*e*x^8 + 1/8*B*c^2*e^2*x^8 + 1/7*C*c^2*d^2*x
^7 + 2/7*B*c^2*d*e*x^7 + 2/7*C*a*c*e^2*x^7 + 1/7*A*c^2*e^2*x^7 + 1/6*B*c^2*
d^2*x^6 + 2/3*C*a*c*d*e*x^6 + 1/3*A*c^2*d*e*x^6 + 1/3*B*a*c*e^2*x^6 + 2/5*C
*a*c*d^2*x^5 + 1/5*A*c^2*d^2*x^5 + 4/5*B*a*c*d*e*x^5 + 1/5*C*a^2*e^2*x^5 +
2/5*A*a*c*e^2*x^5 + 1/2*B*a*c*d^2*x^4 + 1/2*C*a^2*d*e*x^4 + A*a*c*d*e*x^4 +
 1/4*B*a^2*e^2*x^4 + 1/3*C*a^2*d^2*x^3 + 2/3*A*a*c*d^2*x^3 + 2/3*B*a^2*d*e*
x^3 + 1/3*A*a^2*e^2*x^3 + 1/2*B*a^2*d^2*x^2 + A*a^2*d*e*x^2 + A*a^2*d^2*x
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.12

$$
\begin{aligned}
&\int (d+ex)^2 \left(a+cx^2\right)^2 \left(A+Bx+Cx^2\right)\,dx \\
&= x^3 \left(\frac{C\,a^2\,d^2}{3}+\frac{2\,B\,a^2\,d\,e}{3}+\frac{A\,a^2\,e^2}{3}+\frac{2\,A\,c\,a\,d^2}{3}\right) \\
&\quad + x^7 \left(\frac{C\,c^2\,d^2}{7}+\frac{2\,B\,c^2\,d\,e}{7}+\frac{A\,c^2\,e^2}{7}+\frac{2\,C\,a\,c\,e^2}{7}\right) \\
&\quad + x^5 \left(\frac{C\,a^2\,e^2}{5}+\frac{2\,C\,a\,c\,d^2}{5}+\frac{4\,B\,a\,c\,d\,e}{5}+\frac{2\,A\,a\,c\,e^2}{5}+\frac{A\,c^2\,d^2}{5}\right) \\
&\quad + \frac{a\,x^4\,\left(B\,a\,e^2+2\,B\,c\,d^2+4\,A\,c\,d\,e+2\,C\,a\,d\,e\right)}{4} \\
&\quad + \frac{c\,x^6\,\left(2\,B\,a\,e^2+B\,c\,d^2+2\,A\,c\,d\,e+4\,C\,a\,d\,e\right)}{6}+\frac{C\,c^2\,e^2\,x^9}{9} \\
&\quad + A\,a^2\,d^2\,x+\frac{a^2\,d\,x^2\,\left(2\,A\,e+B\,d\right)}{2}+\frac{c^2\,e\,x^8\,\left(B\,e+2\,C\,d\right)}{8}
\end{aligned}
$$

```
[In] int((a + c*x^2)^2*(d + e*x)^2*(A + B*x + C*x^2),x)
```

```
[Out] x^3*((A*a^2*e^2)/3 + (C*a^2*d^2)/3 + (2*A*a*c*d^2)/3 + (2*B*a^2*d*e)/3) + x
^7*((A*c^2*e^2)/7 + (C*c^2*d^2)/7 + (2*C*a*c*e^2)/7 + (2*B*c^2*d*e)/7) + x^
5*((A*c^2*d^2)/5 + (C*a^2*e^2)/5 + (2*A*a*c*e^2)/5 + (2*C*a*c*d^2)/5 + (4*B
*a*c*d*e)/5) + (a*x^4*(B*a*e^2 + 2*B*c*d^2 + 4*A*c*d*e + 2*C*a*d*e))/4 + (c
*x^6*(2*B*a*e^2 + B*c*d^2 + 2*A*c*d*e + 4*C*a*d*e))/6 + (C*c^2*e^2*x^9)/9 +
 A*a^2*d^2*x + (a^2*d*x^2*(2*A*e + B*d))/2 + (c^2*e*x^8*(B*e + 2*C*d))/8
```

3.27 $\int (d+ex)\left(a+cx^2\right)^2\left(A+Bx+Cx^2\right)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 128

$$\begin{aligned}\int (d+ex)\left(a+cx^2\right)^2\left(A+Bx+Cx^2\right)\,dx &= a^2Adx+\frac{1}{3}a(2Acd+aCd+aBe)x^3\\&\quad+\frac{1}{4}a^2Cex^4+\frac{1}{5}c(Acd+2a(Cd+Be))x^5\\&\quad+\frac{1}{3}acCex^6+\frac{1}{7}c^2(Cd+Be)x^7\\&\quad+\frac{1}{8}c^2Cex^8+\frac{(Bd+Ae)\left(a+cx^2\right)^3}{6c}\end{aligned}$$

```
[Out] a^2*A*d*x+1/3*a*(2*A*c*d+B*a*e+C*a*d)*x^3+1/4*a^2*C*e*x^4+1/5*c*(A*c*d+2*a*
(B*e+C*d))*x^5+1/3*a*c*C*e*x^6+1/7*c^2*(B*e+C*d)*x^7+1/8*c^2*C*e*x^8+1/6*(A
*e+B*d)*(c*x^2+a)^3/c
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {1596, 1824}

$$\begin{aligned}\int (d+ex)\left(a+cx^2\right)^2\left(A+Bx+Cx^2\right)\,dx &= a^2Adx+\frac{1}{4}a^2Cex^4+\frac{1}{5}cx^5(2a(Be+Cd)+Acd)\\&\quad+\frac{1}{3}ax^3(aBe+aCd+2Acd)\\&\quad+\frac{\left(a+cx^2\right)^3(Ae+Bd)}{6c}+\frac{1}{3}acCex^6\\&\quad+\frac{1}{7}c^2x^7(Be+Cd)+\frac{1}{8}c^2Cex^8\end{aligned}$$

```
[In] Int[(d + e*x)*(a + c*x^2)^2*(A + B*x + C*x^2),x]
```

```
[Out] a^2*A*d*x + (a*(2*A*c*d + a*C*d + a*B*e)*x^3)/3 + (a^2*C*e*x^4)/4 + (c*(A*c
*d + 2*a*(C*d + B*e))*x^5)/5 + (a*c*C*e*x^6)/3 + (c^2*(C*d + B*e)*x^7)/7 +
(c^2*C*e*x^8)/8 + ((B*d + A*e)*(a + c*x^2)^3)/(6*c)
```

Rule 1596

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(Bd+Ae)\left(a+cx^2\right)^3}{6c}+\int \left(a+cx^2\right)^2\left(-((Bd+Ae)x)+(d+ex)\left(A+Bx+Cx^2\right)\right)\,dx\\
&= \frac{(Bd+Ae)\left(a+cx^2\right)^3}{6c}+\int \left(a^2Ad+a(2Acd+aCd+aBe)x^2+a^2Cex^3\right.\\
&\qquad \left.+c(Acd+2a(Cd+Be))x^4+2acCex^5+c^2(Cd+Be)x^6+c^2Cex^7\right)\,dx\\
&= a^2Adx+\frac{1}{3}a(2Acd+aCd+aBe)x^3+\frac{1}{4}a^2Cex^4+\frac{1}{5}c(Acd+2a(Cd+Be))x^5\\
&\quad +\frac{1}{3}acCex^6+\frac{1}{7}c^2(Cd+Be)x^7+\frac{1}{8}c^2Cex^8+\frac{(Bd+Ae)\left(a+cx^2\right)^3}{6c}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.12

$$\int (d+ex)\left(a+cx^2\right)^2\left(A+Bx+Cx^2\right)\,dx=a^2Adx+\frac{1}{2}a^2(Bd+Ae)x^2+\frac{1}{3}a(2Acd+aCd+aBe)x^3+\frac{1}{4}a(2Bcd+2Ace+aCe)x^4+\frac{1}{5}c(Acd+2aCd+2aBe)x^5+\frac{1}{6}c(Bcd+Ace+2aCe)x^6+\frac{1}{7}c^2(Cd+Be)x^7+\frac{1}{8}c^2Cex^8$$

```
[In] Integrate[(d + e*x)*(a + c*x^2)^2*(A + B*x + C*x^2),x]
```

```
[Out] a^2*A*d*x + (a^2*(B*d + A*e)*x^2)/2 + (a*(2*A*c*d + a*C*d + a*B*e)*x^3)/3 +
 (a*(2*B*c*d + 2*A*c*e + a*C*e)*x^4)/4 + (c*(A*c*d + 2*a*C*d + 2*a*B*e)*x^5
)/5 + (c*(B*c*d + A*c*e + 2*a*C*e)*x^6)/6 + (c^2*(C*d + B*e)*x^7)/7 + (c^2*
C*e*x^8)/8
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.18

method	result
default	$\frac{c^2Ce\,x^8}{8}+\frac{(c^2eB+c^2dC)x^7}{7}+\frac{(c^2eA+c^2dB+2aceC)x^6}{6}+\frac{(A\,c^2d+2Bace+2acdC)x^5}{5}+\frac{(2aAce+2Bacd+a^2eC)x^4}{4}+$
norman	$\frac{c^2Ce\,x^8}{8}+\left(\frac{1}{7}c^2eB+\frac{1}{7}c^2dC\right)x^7+\left(\frac{1}{6}c^2eA+\frac{1}{6}c^2dB+\frac{1}{3}aceC\right)x^6+\left(\frac{1}{5}A\,c^2d+\frac{2}{5}Bace+\frac{2}{5}acd$
gosper	$\frac{1}{8}c^2Ce\,x^8+\frac{1}{7}B\,c^2e\,x^7+\frac{1}{7}x^7c^2dC+\frac{1}{6}x^6A\,c^2e+\frac{1}{6}x^6B\,c^2d+\frac{1}{3}acCe\,x^6+\frac{1}{5}x^5A\,c^2d+\frac{2}{5}x^5Bac$
risch	$\frac{1}{8}c^2Ce\,x^8+\frac{1}{7}B\,c^2e\,x^7+\frac{1}{7}x^7c^2dC+\frac{1}{6}x^6A\,c^2e+\frac{1}{6}x^6B\,c^2d+\frac{1}{3}acCe\,x^6+\frac{1}{5}x^5A\,c^2d+\frac{2}{5}x^5Bac$
parallelrisch	$\frac{1}{8}c^2Ce\,x^8+\frac{1}{7}B\,c^2e\,x^7+\frac{1}{7}x^7c^2dC+\frac{1}{6}x^6A\,c^2e+\frac{1}{6}x^6B\,c^2d+\frac{1}{3}acCe\,x^6+\frac{1}{5}x^5A\,c^2d+\frac{2}{5}x^5Bac$

```
[In] int((e*x+d)*(c*x^2+a)^2*(C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*c^2*C*e*x^8+1/7*(B*c^2*e+C*c^2*d)*x^7+1/6*(A*c^2*e+B*c^2*d+2*C*a*c*e)*x
^6+1/5*(A*c^2*d+2*B*a*c*e+2*C*a*c*d)*x^5+1/4*(2*A*a*c*e+2*B*a*c*d+C*a^2*e)*
x^4+1/3*(2*A*a*c*d+B*a^2*e+C*a^2*d)*x^3+1/2*(A*a^2*e+B*a^2*d)*x^2+a^2*A*d*x
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.20

$$\int (d+ex)\left(a+cx^2\right)^2\left(A+Bx+Cx^2\right)\,dx = \frac{1}{8}\,Cc^2ex^8+\frac{1}{7}\,\left(Cc^2d+Bc^2e\right)x^7 + \frac{1}{6}\,\left(Bc^2d+\left(2\,Cac+Ac^2\right)e\right)x^6 + \frac{1}{5}\,\left(2\,Bace+\left(2\,Cac+Ac^2\right)d\right)x^5+Aa^2dx + \frac{1}{4}\,\left(2\,Bacd+\left(Ca^2+2\,Aac\right)e\right)x^4 + \frac{1}{3}\,\left(Ba^2e+\left(Ca^2+2\,Aac\right)d\right)x^3 + \frac{1}{2}\,\left(Ba^2d+Aa^2e\right)x^2$$

```
[In] integrate((e*x+d)*(c*x^2+a)^2*(C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/8*C*c^2*e*x^8 + 1/7*(C*c^2*d + B*c^2*e)*x^7 + 1/6*(B*c^2*d + (2*C*a*c + A
*c^2)*e)*x^6 + 1/5*(2*B*a*c*e + (2*C*a*c + A*c^2)*d)*x^5 + A*a^2*d*x + 1/4*
(2*B*a*c*d + (C*a^2 + 2*A*a*c)*e)*x^4 + 1/3*(B*a^2*e + (C*a^2 + 2*A*a*c)*d)
*x^3 + 1/2*(B*a^2*d + A*a^2*e)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.41

$$\int (d+ex)\left(a+cx^2\right)^2\left(A+Bx+Cx^2\right)\,dx = Aa^2dx+\frac{Cc^2ex^8}{8}+x^7\left(\frac{Bc^2e}{7}+\frac{Cc^2d}{7}\right) + x^6\left(\frac{Ac^2e}{6}+\frac{Bc^2d}{6}+\frac{Cace}{3}\right) + x^5\left(\frac{Ac^2d}{5}+\frac{2Bace}{5}+\frac{2Cacd}{5}\right) + x^4\left(\frac{Aace}{2}+\frac{Bacd}{2}+\frac{Ca^2e}{4}\right) + x^3\cdot\left(\frac{2Aacd}{3}+\frac{Ba^2e}{3}+\frac{Ca^2d}{3}\right) + x^2\left(\frac{Aa^2e}{2}+\frac{Ba^2d}{2}\right)$$

```
[In] integrate((e*x+d)*(c*x**2+a)**2*(C*x**2+B*x+A),x)
```

```
[Out] A*a**2*d*x + C*c**2*e*x**8/8 + x**7*(B*c**2*e/7 + C*c**2*d/7) + x**6*(A*c**
2*e/6 + B*c**2*d/6 + C*a*c*e/3) + x**5*(A*c**2*d/5 + 2*B*a*c*e/5 + 2*C*a*c*
d/5) + x**4*(A*a*c*e/2 + B*a*c*d/2 + C*a**2*e/4) + x**3*(2*A*a*c*d/3 + B*a*
*2*e/3 + C*a**2*d/3) + x**2*(A*a**2*e/2 + B*a**2*d/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.20

$$\begin{aligned}\int (d+ex)\left(a+cx^2\right)^2\left(A+Bx+Cx^2\right)\,dx &= \frac{1}{8}\,Cc^2ex^8+\frac{1}{7}\left(Cc^2d+Bc^2e\right)x^7\\&\quad+\frac{1}{6}\left(Bc^2d+\left(2\,Cac+Ac^2\right)e\right)x^6\\&\quad+\frac{1}{5}\left(2\,Bace+\left(2\,Cac+Ac^2\right)d\right)x^5+Aa^2dx\\&\quad+\frac{1}{4}\left(2\,Bacd+\left(Ca^2+2\,Aac\right)e\right)x^4\\&\quad+\frac{1}{3}\left(Ba^2e+\left(Ca^2+2\,Aac\right)d\right)x^3\\&\quad+\frac{1}{2}\left(Ba^2d+Aa^2e\right)x^2\end{aligned}$$

```
[In] integrate((e*x+d)*(c*x^2+a)^2*(C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/8*C*c^2*e*x^8 + 1/7*(C*c^2*d + B*c^2*e)*x^7 + 1/6*(B*c^2*d + (2*C*a*c + A
*c^2)*e)*x^6 + 1/5*(2*B*a*c*e + (2*C*a*c + A*c^2)*d)*x^5 + A*a^2*d*x + 1/4*
(2*B*a*c*d + (C*a^2 + 2*A*a*c)*e)*x^4 + 1/3*(B*a^2*e + (C*a^2 + 2*A*a*c)*d)
*x^3 + 1/2*(B*a^2*d + A*a^2*e)*x^2
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.34

$$\begin{aligned}\int (d+ex)\left(a+cx^2\right)^2\left(A+Bx+Cx^2\right)\,dx &= \frac{1}{8}\,Cc^2ex^8+\frac{1}{7}\,Cc^2dx^7+\frac{1}{7}\,Bc^2ex^7+\frac{1}{6}\,Bc^2dx^6\\&\quad+\frac{1}{3}\,Cacex^6+\frac{1}{6}\,Ac^2ex^6+\frac{2}{5}\,Cacdx^5+\frac{1}{5}\,Ac^2dx^5\\&\quad+\frac{2}{5}\,Bacex^5+\frac{1}{2}\,Bacdx^4+\frac{1}{4}\,Ca^2ex^4\\&\quad+\frac{1}{2}\,Aacex^4+\frac{1}{3}\,Ca^2dx^3+\frac{2}{3}\,Aacdx^3\\&\quad+\frac{1}{3}\,Ba^2ex^3+\frac{1}{2}\,Ba^2dx^2+\frac{1}{2}\,Aa^2ex^2+Aa^2dx\end{aligned}$$

```
[In] integrate((e*x+d)*(c*x^2+a)^2*(C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/8*C*c^2*e*x^8 + 1/7*C*c^2*d*x^7 + 1/7*B*c^2*e*x^7 + 1/6*B*c^2*d*x^6 + 1/3
*C*a*c*e*x^6 + 1/6*A*c^2*e*x^6 + 2/5*C*a*c*d*x^5 + 1/5*A*c^2*d*x^5 + 2/5*B*
a*c*e*x^5 + 1/2*B*a*c*d*x^4 + 1/4*C*a^2*e*x^4 + 1/2*A*a*c*e*x^4 + 1/3*C*a^2
*d*x^3 + 2/3*A*a*c*d*x^3 + 1/3*B*a^2*e*x^3 + 1/2*B*a^2*d*x^2 + 1/2*A*a^2*e*
x^2 + A*a^2*d*x
```

Mupad [B] (verification not implemented)

Time = 12.84 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.09

$$\begin{aligned}\int (d+ex)\left(a+cx^2\right)^2\left(A+Bx+Cx^2\right)\,dx &= x^3\left(\frac{B\,a^2\,e}{3}+\frac{C\,a^2\,d}{3}+\frac{2\,A\,a\,c\,d}{3}\right)\\ &\quad+x^6\left(\frac{A\,c^2\,e}{6}+\frac{B\,c^2\,d}{6}+\frac{C\,a\,c\,e}{3}\right)\\ &\quad+\frac{c\,x^5\,(A\,c\,d+2\,B\,a\,e+2\,C\,a\,d)}{5}\\ &\quad+\frac{a\,x^4\,(2\,A\,c\,e+2\,B\,c\,d+C\,a\,e)}{4}\\ &\quad+\frac{a^2\,x^2\,(A\,e+B\,d)}{2}+\frac{c^2\,x^7\,(B\,e+C\,d)}{7}\\ &\quad+A\,a^2\,d\,x+\frac{C\,c^2\,e\,x^8}{8}\end{aligned}$$

```
[In] int((a + c*x^2)^2*(d + e*x)*(A + B*x + C*x^2),x)
```

```
[Out] x^3*((B*a^2*e)/3 + (C*a^2*d)/3 + (2*A*a*c*d)/3) + x^6*((A*c^2*e)/6 + (B*c^2
*d)/6 + (C*a*c*e)/3) + (c*x^5*(A*c*d + 2*B*a*e + 2*C*a*d))/5 + (a*x^4*(2*A*
c*e + 2*B*c*d + C*a*e))/4 + (a^2*x^2*(A*e + B*d))/2 + (c^2*x^7*(B*e + C*d))
/7 + A*a^2*d*x + (C*c^2*e*x^8)/8
```

3.28 $\int (a+cx^2)^2 (A+Bx+Cx^2)\, dx$

Optimal result

Integrand size = 20, antiderivative size = 67

$$\int (a+cx^2)^2 (A+Bx+Cx^2)\, dx = a^2Ax+\frac{1}{3}a(2Ac+aC)x^3+\frac{1}{5}c(Ac+2aC)x^5 + \frac{1}{7}c^2Cx^7+\frac{B(a+cx^2)^3}{6c}$$

```
[Out] a^2*A*x+1/3*a*(2*A*c+C*a)*x^3+1/5*c*(A*c+2*C*a)*x^5+1/7*c^2*C*x^7+1/6*B*(c*
x^2+a)^3/c
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1596, 380}

$$\int (a+cx^2)^2 (A+Bx+Cx^2)\, dx = a^2Ax+\frac{1}{5}cx^5(2aC+Ac)+\frac{1}{3}ax^3(aC+2Ac) + \frac{B(a+cx^2)^3}{6c}+\frac{1}{7}c^2Cx^7$$

```
[In] Int[(a + c*x^2)^2*(A + B*x + C*x^2),x]
```

```
[Out] a^2*A*x + (a*(2*A*c + a*C)*x^3)/3 + (c*(A*c + 2*a*C)*x^5)/5 + (c^2*C*x^7)/7
 + (B*(a + c*x^2)^3)/(6*c)
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
```

```
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 1596

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{B(a+cx^2)^3}{6c} + \int (a+cx^2)^2 \left(A+Cx^2\right) dx \\
&= \frac{B(a+cx^2)^3}{6c} + \int \left(a^2A + a(2Ac+aC)x^2 + c(Ac+2aC)x^4 + c^2Cx^6\right) dx \\
&= a^2Ax + \frac{1}{3}a(2Ac+aC)x^3 + \frac{1}{5}c(Ac+2aC)x^5 + \frac{1}{7}c^2Cx^7 + \frac{B(a+cx^2)^3}{6c}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.03

$$\begin{aligned}
\int \left(a+cx^2\right)^2 \left(A+Bx+Cx^2\right) dx = \frac{1}{210}x(&35a^2(6A+x(3B+2Cx)) \\
&+ 7acx^2(20A+3x(5B+4Cx)) \\
&+ c^2x^4(42A+5x(7B+6Cx)))
\end{aligned}$$

```
[In] Integrate[(a + c*x^2)^2*(A + B*x + C*x^2),x]
```

```
[Out] (x*(35*a^2*(6*A + x*(3*B + 2*C*x)) + 7*a*c*x^2*(20*A + 3*x*(5*B + 4*C*x)) +
 c^2*x^4*(42*A + 5*x*(7*B + 6*C*x))))/210
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.12

method	result
default	$\frac{c^2C\,x^7}{7}+\frac{B\,c^2x^6}{6}+\frac{(A\,c^2+2acC)x^5}{5}+\frac{Bac\,x^4}{2}+\frac{(2Aac+C\,a^2)x^3}{3}+\frac{B\,a^2x^2}{2}+a^2Ax$
norman	$\frac{c^2C\,x^7}{7}+\frac{B\,c^2x^6}{6}+\left(\frac{1}{5}A\,c^2+\frac{2}{5}acC\right)x^5+\frac{Bac\,x^4}{2}+\left(\frac{2}{3}Aac+\frac{1}{3}C\,a^2\right)x^3+\frac{B\,a^2x^2}{2}+a^2Ax$
gosper	$\frac{1}{7}c^2C\,x^7+\frac{1}{6}B\,c^2x^6+\frac{1}{5}A\,c^2x^5+\frac{2}{5}x^5acC+\frac{1}{2}Bac\,x^4+\frac{2}{3}aAc\,x^3+\frac{1}{3}x^3C\,a^2+\frac{1}{2}B\,a^2x^2+a^2A$
risch	$\frac{1}{7}c^2C\,x^7+\frac{1}{6}B\,c^2x^6+\frac{1}{5}A\,c^2x^5+\frac{2}{5}x^5acC+\frac{1}{2}Bac\,x^4+\frac{2}{3}aAc\,x^3+\frac{1}{3}x^3C\,a^2+\frac{1}{2}B\,a^2x^2+a^2A$
parallelrisch	$\frac{1}{7}c^2C\,x^7+\frac{1}{6}B\,c^2x^6+\frac{1}{5}A\,c^2x^5+\frac{2}{5}x^5acC+\frac{1}{2}Bac\,x^4+\frac{2}{3}aAc\,x^3+\frac{1}{3}x^3C\,a^2+\frac{1}{2}B\,a^2x^2+a^2A$

```
[In] int((c*x^2+a)^2*(C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*c^2*C*x^7+1/6*B*c^2*x^6+1/5*(A*c^2+2*C*a*c)*x^5+1/2*B*a*c*x^4+1/3*(2*A*
a*c+C*a^2)*x^3+1/2*B*a^2*x^2+a^2*A*x
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.10

$$\int \left(a+cx^2\right)^2\left(A+Bx+Cx^2\right)\,dx=\frac{1}{7}\,Cc^2x^7+\frac{1}{6}\,Bc^2x^6+\frac{1}{2}\,Bacx^4+\frac{1}{5}\,(2\,Cac+Ac^2)x^5+\frac{1}{2}\,Ba^2x^2+Aa^2x+\frac{1}{3}\,(Ca^2+2\,Aac)x^3$$

```
[In] integrate((c*x^2+a)^2*(C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/7*C*c^2*x^7 + 1/6*B*c^2*x^6 + 1/2*B*a*c*x^4 + 1/5*(2*C*a*c + A*c^2)*x^5 +
 1/2*B*a^2*x^2 + A*a^2*x + 1/3*(C*a^2 + 2*A*a*c)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.24

$$\int \left(a+cx^2\right)^2\left(A+Bx+Cx^2\right)\,dx=Aa^2x+\frac{Ba^2x^2}{2}+\frac{Bacx^4}{2}+\frac{Bc^2x^6}{6}+\frac{Cc^2x^7}{7}+x^5\left(\frac{Ac^2}{5}+\frac{2Cac}{5}\right)+x^3\cdot\left(\frac{2Aac}{3}+\frac{Ca^2}{3}\right)$$

```
[In] integrate((c*x**2+a)**2*(C*x**2+B*x+A),x)
```

```
[Out] A*a**2*x + B*a**2*x**2/2 + B*a*c*x**4/2 + B*c**2*x**6/6 + C*c**2*x**7/7 + x
**5*(A*c**2/5 + 2*C*a*c/5) + x**3*(2*A*a*c/3 + C*a**2/3)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.10

$$\int \left(a+cx^2\right)^2 \left(A+Bx+Cx^2\right)\,dx = \frac{1}{7}Cc^2x^7+\frac{1}{6}Bc^2x^6+\frac{1}{2}Bacx^4+\frac{1}{5}\left(2\,Cac+Ac^2\right)x^5+\frac{1}{2}Ba^2x^2+Aa^2x+\frac{1}{3}\left(Ca^2+2\,Aac\right)x^3$$

```
[In] integrate((c*x^2+a)^2*(C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/7*C*c^2*x^7 + 1/6*B*c^2*x^6 + 1/2*B*a*c*x^4 + 1/5*(2*C*a*c + A*c^2)*x^5 +
 1/2*B*a^2*x^2 + A*a^2*x + 1/3*(C*a^2 + 2*A*a*c)*x^3
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.13

$$\int \left(a+cx^2\right)^2 \left(A+Bx+Cx^2\right)\,dx = \frac{1}{7}Cc^2x^7+\frac{1}{6}Bc^2x^6+\frac{2}{5}Cacx^5+\frac{1}{5}Ac^2x^5+\frac{1}{2}Bacx^4+\frac{1}{3}Ca^2x^3+\frac{2}{3}Aacx^3+\frac{1}{2}Ba^2x^2+Aa^2x$$

```
[In] integrate((c*x^2+a)^2*(C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/7*C*c^2*x^7 + 1/6*B*c^2*x^6 + 2/5*C*a*c*x^5 + 1/5*A*c^2*x^5 + 1/2*B*a*c*x
^4 + 1/3*C*a^2*x^3 + 2/3*A*a*c*x^3 + 1/2*B*a^2*x^2 + A*a^2*x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.10

$$\int \left(a+cx^2\right)^2 \left(A+Bx+Cx^2\right)\,dx = x^3\left(\frac{C\,a^2}{3}+\frac{2\,A\,c\,a}{3}\right)+x^5\left(\frac{A\,c^2}{5}+\frac{2\,C\,a\,c}{5}\right)+\frac{B\,a^2\,x^2}{2}+\frac{B\,c^2\,x^6}{6}+\frac{C\,c^2\,x^7}{7}+A\,a^2\,x+\frac{B\,a\,c\,x^4}{2}$$

```
[In] int((a + c*x^2)^2*(A + B*x + C*x^2),x)
```

```
[Out] x^3*((C*a^2)/3 + (2*A*a*c)/3) + x^5*((A*c^2)/5 + (2*C*a*c)/5) + (B*a^2*x^2)
/2 + (B*c^2*x^6)/6 + (C*c^2*x^7)/7 + A*a^2*x + (B*a*c*x^4)/2
```

3.29 $\int \frac{(a+cx^2)^2(A+Bx+Cx^2)}{d+ex}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 297

$$\begin{aligned}
&\int \frac{(a+cx^2)^2(A+Bx+Cx^2)}{d+ex}\,dx \\
&= -\frac{(a^2e^4(Cd-Be)+c^2d^3(Cd^2-e(Bd-Ae))+2acde^2(Cd^2-e(Bd-Ae)))\,x}{e^6} \\
&\quad+\frac{(a^2Ce^4+c^2d^2(Cd^2-e(Bd-Ae))+2ace^2(Cd^2-e(Bd-Ae)))\,x^2}{2e^5} \\
&\quad-\frac{c(2ae^2(Cd-Be)+cd(Cd^2-e(Bd-Ae)))\,x^3}{3e^4}+\frac{c(2aCe^2+c(Cd^2-e(Bd-Ae)))\,x^4}{4e^3} \\
&\quad-\frac{c^2(Cd-Be)x^5}{5e^2}+\frac{c^2Cx^6}{6e}+\frac{(cd^2+ae^2)^2\,(Cd^2-Bde+Ae^2)\log(d+ex)}{e^7}
\end{aligned}$$

```
[Out] -(a^2*e^4*(-B*e+C*d)+c^2*d^3*(C*d^2-e*(-A*e+B*d))+2*a*c*d*e^2*(C*d^2-e*(-A*
e+B*d)))*x/e^6+1/2*(a^2*C*e^4+c^2*d^2*(C*d^2-e*(-A*e+B*d))+2*a*c*e^2*(C*d^2
-e*(-A*e+B*d)))*x^2/e^5-1/3*c*(2*a*e^2*(-B*e+C*d)+c*d*(C*d^2-e*(-A*e+B*d)))
*x^3/e^4+1/4*c*(2*a*C*e^2+c*(C*d^2-e*(-A*e+B*d)))*x^4/e^3-1/5*c^2*(-B*e+C*d
)*x^5/e^2+1/6*c^2*C*x^6/e+(a*e^2+c*d^2)^2*(A*e^2-B*d*e+C*d^2)*ln(e*x+d)/e^7
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 295, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used

$= \{1642\}$

$$
\begin{aligned}
&\int \frac{\left(a+cx^2\right)^2\left(A+Bx+Cx^2\right)}{d+ex}\,dx \\
&\quad = \frac{x^2(a^2Ce^4+2ace^2(Cd^2-e(Bd-Ae))+c^2(Cd^4-d^2e(Bd-Ae)))}{2e^5} \\
&\quad - \frac{x(a^2e^4(Cd-Be)+2acde^2(Cd^2-e(Bd-Ae))+c^2(Cd^5-d^3e(Bd-Ae)))}{e^6} \\
&\quad - \frac{cx^3(2ae^2(Cd-Be)-cde(Bd-Ae)+cCd^3)}{3e^4} \\
&\quad + \frac{\left(ae^2+cd^2\right)^2\log(d+ex)\left(Ae^2-Bde+Cd^2\right)}{e^7} \\
&\quad + \frac{cx^4(2aCe^2-ce(Bd-Ae)+cCd^2)}{4e^3} - \frac{c^2x^5(Cd-Be)}{5e^2} + \frac{c^2Cx^6}{6e}
\end{aligned}
$$

```
[In] Int[((a + c*x^2)^2*(A + B*x + C*x^2))/(d + e*x),x]
```

```
[Out] -(((a^2*e^4*(C*d - B*e) + 2*a*c*d*e^2*(C*d^2 - e*(B*d - A*e)) + c^2*(C*d^5
- d^3*e*(B*d - A*e)))*x)/e^6) + ((a^2*C*e^4 + 2*a*c*e^2*(C*d^2 - e*(B*d - A
*e)) + c^2*(C*d^4 - d^2*e*(B*d - A*e)))*x^2)/(2*e^5) - (c*(c*C*d^3 - c*d*e*
(B*d - A*e) + 2*a*e^2*(C*d - B*e))*x^3)/(3*e^4) + (c*(c*C*d^2 + 2*a*C*e^2 -
 c*e*(B*d - A*e))*x^4)/(4*e^3) - (c^2*(C*d - B*e)*x^5)/(5*e^2) + (c^2*C*x^6
)/(6*e) + ((c*d^2 + a*e^2)^2*(C*d^2 - B*d*e + A*e^2)*Log[d + e*x])/e^7
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} = \int \Bigg(&\frac{-a^2e^4(Cd-Be)-2acde^2(Cd^2-e(Bd-Ae))-c^2(Cd^5-d^3e(Bd-Ae))}{e^6} \\
&+ \frac{(a^2Ce^4+2ace^2(Cd^2-e(Bd-Ae))+c^2(Cd^4-d^2e(Bd-Ae)))\,x}{e^5} \\
&+ \frac{c(-cCd^3+cde(Bd-Ae)-2ae^2(Cd-Be))\,x^2}{e^4} \\
&+ \frac{c(cCd^2+2aCe^2-ce(Bd-Ae))\,x^3}{e^3} + \frac{c^2(-Cd+Be)x^4}{e^2} + \frac{c^2Cx^5}{e} \\
&+ \frac{\left(cd^2+ae^2\right)^2\left(Cd^2-Bde+Ae^2\right)}{e^6(d+ex)}\Bigg)\,dx
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{(a^2e^4(Cd-Be)+2acde^2(Cd^2-e(Bd-Ae))+c^2(Cd^5-d^3e(Bd-Ae)))\,x}{e^6} \\
&\quad+\frac{(a^2Ce^4+2ace^2(Cd^2-e(Bd-Ae))+c^2(Cd^4-d^2e(Bd-Ae)))\,x^2}{2e^5} \\
&\quad-\frac{c(cCd^3-cde(Bd-Ae)+2ae^2(Cd-Be))\,x^3}{3e^4} \\
&\quad+\frac{c(cCd^2+2aCe^2-ce(Bd-Ae))\,x^4}{4e^3}-\frac{c^2(Cd-Be)x^5}{5e^2} \\
&\quad+\frac{c^2Cx^6}{6e}+\frac{(cd^2+ae^2)^2\,(Cd^2-Bde+Ae^2)\log(d+ex)}{e^7}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 285, normalized size of antiderivative = 0.96

$$
\int\frac{(a+cx^2)^2\,(A+Bx+Cx^2)}{d+ex}\,dx
$$

$$
=\frac{ex(30a^2e^4(-2Cd+2Be+Cex)+10ace^2(C(-12d^3+6d^2ex-4de^2x^2+3e^3x^3)+2e(3Ae(-2d+ex)+}{}
$$

```
[In] Integrate[((a + c*x^2)^2*(A + B*x + C*x^2))/(d + e*x),x]
```

```
[Out] (e*x*(30*a^2*e^4*(-2*C*d + 2*B*e + C*e*x) + 10*a*c*e^2*(C*(-12*d^3 + 6*d^2*
e*x - 4*d*e^2*x^2 + 3*e^3*x^3) + 2*e*(3*A*e*(-2*d + e*x) + B*(6*d^2 - 3*d*e
*x + 2*e^2*x^2))) + c^2*(C*(-60*d^5 + 30*d^4*e*x - 20*d^3*e^2*x^2 + 15*d^2*
e^3*x^3 - 12*d*e^4*x^4 + 10*e^5*x^5) + e*(5*A*e*(-12*d^3 + 6*d^2*e*x - 4*d*
e^2*x^2 + 3*e^3*x^3) + B*(60*d^4 - 30*d^3*e*x + 20*d^2*e^2*x^2 - 15*d*e^3*x
^3 + 12*e^4*x^4)))) + 60*(c*d^2 + a*e^2)^2*(C*d^2 + e*(-(B*d) + A*e))*Log[d
 + e*x])/(60*e^7)
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.26

method	result
norman	$\frac{(2Aac\,e^4+A\,c^2d^2e^2-2Bacd\,e^3-B\,c^2d^3e+a^2C\,e^4+2Cac\,d^2e^2+C\,c^2d^4)x^2}{2e^5}-\frac{(2Aacd\,e^4+A\,c^2d^3e^2-B\,e^5a^2-2Bac\,d^2e^3-B\,c}{e^6}$
default	$-\frac{2Cac\,d^3e^2x-2Bxac\,d^2e^3+2Axacd\,e^4+B\,x^2acd\,e^4-\frac{1}{2}C\,a^2e^5x^2-\frac{1}{6}c^2C\,x^6e^5-Bx\,a^2e^5-\frac{1}{4}A\,x^4c^2e^5-\frac{1}{5}B\,x^5c^2e^5+C\,c^2d^5x-}{}$
risch	$-\frac{Ax\,c^2d^3}{e^4}+\frac{B\,c^2x^5}{5e}+\frac{\ln(ex+d)A\,a^2}{e}+\frac{Bx\,a^2}{e}+\frac{A\,x^4c^2}{4e}+\frac{C\,a^2x^2}{2e}-\frac{C\,c^2d^5x}{e^6}-\frac{C\,c^2d\,x^5}{5e^2}+\frac{2\ln(ex+d)Aac\,d^2}{e^3}-$
parallelrisch	$\frac{-30B\,x^2c^2d^3e^3-12C\,x^5c^2d\,e^5-15B\,x^4c^2d\,e^5+60C\ln(ex+d)a^2d^2e^4+60A\ln(ex+d)c^2d^4e^2-60B\ln(ex+d)a^2d\,e^5-60B\ln(ex+}{}$

```
[In] int((c*x^2+a)^2*(C*x^2+B*x+A)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/e^5*(2*A*a*c*e^4+A*c^2*d^2*e^2-2*B*a*c*d*e^3-B*c^2*d^3*e+C*a^2*e^4+2*C*
a*c*d^2*e^2+C*c^2*d^4)*x^2-(2*A*a*c*d*e^4+A*c^2*d^3*e^2-B*a^2*e^5-2*B*a*c*d
^2*e^3-B*c^2*d^4*e+C*a^2*d*e^4+2*C*a*c*d^3*e^2+C*c^2*d^5)/e^6*x-1/3*c/e^4*(
A*c*d*e^2-2*B*a*e^3-B*c*d^2*e+2*C*a*d*e^2+C*c*d^3)*x^3+1/4*c/e^3*(A*c*e^2-B
*c*d*e+2*C*a*e^2+C*c*d^2)*x^4+1/6*c^2*C*x^6/e+1/5*c^2/e^2*(B*e-C*d)*x^5+(A*
a^2*e^6+2*A*a*c*d^2*e^4+A*c^2*d^4*e^2-B*a^2*d*e^5-2*B*a*c*d^3*e^3-B*c^2*d^5
*e+C*a^2*d^2*e^4+2*C*a*c*d^4*e^2+C*c^2*d^6)/e^7*ln(e*x+d)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.28

$$\int \frac{(a+cx^2)^2\,(A+Bx+Cx^2)}{d+ex}\,dx$$
$$= \frac{10\,Cc^2e^6x^6 - 12\,(Cc^2de^5 - Bc^2e^6)x^5 + 15\,(Cc^2d^2e^4 - Bc^2de^5 + (2\,Cac + Ac^2)e^6)x^4 - 20\,(Cc^2d^3e^3 - Bc^2d}{}$$

```
[In] integrate((c*x^2+a)^2*(C*x^2+B*x+A)/(e*x+d),x, algorithm="fricas")
```

```
[Out] 1/60*(10*C*c^2*e^6*x^6 - 12*(C*c^2*d*e^5 - B*c^2*e^6)*x^5 + 15*(C*c^2*d^2*e
^4 - B*c^2*d*e^5 + (2*C*a*c + A*c^2)*e^6)*x^4 - 20*(C*c^2*d^3*e^3 - B*c^2*d
^2*e^4 - 2*B*a*c*e^6 + (2*C*a*c + A*c^2)*d*e^5)*x^3 + 30*(C*c^2*d^4*e^2 - B
*c^2*d^3*e^3 - 2*B*a*c*d*e^5 + (2*C*a*c + A*c^2)*d^2*e^4 + (C*a^2 + 2*A*a*c
)*e^6)*x^2 - 60*(C*c^2*d^5*e - B*c^2*d^4*e^2 - 2*B*a*c*d^2*e^4 - B*a^2*e^6
+ (2*C*a*c + A*c^2)*d^3*e^3 + (C*a^2 + 2*A*a*c)*d*e^5)*x + 60*(C*c^2*d^6 -
B*c^2*d^5*e - 2*B*a*c*d^3*e^3 - B*a^2*d*e^5 + A*a^2*e^6 + (2*C*a*c + A*c^2)
*d^4*e^2 + (C*a^2 + 2*A*a*c)*d^2*e^4)*log(e*x + d))/e^7
```

Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.21

$$
\begin{aligned}
\int \frac{(a+cx^2)^2 (A+Bx+Cx^2)}{d+ex}\,dx = {} & \frac{Cc^2x^6}{6e} + x^5\left(\frac{Bc^2}{5e} - \frac{Cc^2d}{5e^2}\right) \\
& + x^4\left(\frac{Ac^2}{4e} - \frac{Bc^2d}{4e^2} + \frac{Cac}{2e} + \frac{Cc^2d^2}{4e^3}\right) \\
& + x^3\left(-\frac{Ac^2d}{3e^2} + \frac{2Bac}{3e} + \frac{Bc^2d^2}{3e^3} - \frac{2Cacd}{3e^2} - \frac{Cc^2d^3}{3e^4}\right) \\
& + x^2\left(\frac{Aac}{e} + \frac{Ac^2d^2}{2e^3} - \frac{Bacd}{e^2} - \frac{Bc^2d^3}{2e^4} + \frac{Ca^2}{2e} + \frac{Cacd^2}{e^3} \right. \\
& \left. + \frac{Cc^2d^4}{2e^5}\right) + x\left(-\frac{2Aacd}{e^2} - \frac{Ac^2d^3}{e^4} + \frac{Ba^2}{e} + \frac{2Bacd^2}{e^3} \right. \\
& \left. + \frac{Bc^2d^4}{e^5} - \frac{Ca^2d}{e^2} - \frac{2Cacd^3}{e^4} - \frac{Cc^2d^5}{e^6}\right) \\
& + \frac{(ae^2+cd^2)^2 (Ae^2 - Bde + Cd^2) \log(d+ex)}{e^7}
\end{aligned}
$$

```
[In] integrate((c*x**2+a)**2*(C*x**2+B*x+A)/(e*x+d),x)
```

```
[Out] C*c**2*x**6/(6*e) + x**5*(B*c**2/(5*e) - C*c**2*d/(5*e**2)) + x**4*(A*c**2/
(4*e) - B*c**2*d/(4*e**2) + C*a*c/(2*e) + C*c**2*d**2/(4*e**3)) + x**3*(-A*
c**2*d/(3*e**2) + 2*B*a*c/(3*e) + B*c**2*d**2/(3*e**3) - 2*C*a*c*d/(3*e**2)
 - C*c**2*d**3/(3*e**4)) + x**2*(A*a*c/e + A*c**2*d**2/(2*e**3) - B*a*c*d/e
**2 - B*c**2*d**3/(2*e**4) + C*a**2/(2*e) + C*a*c*d**2/e**3 + C*c**2*d**4/(
2*e**5)) + x*(-2*A*a*c*d/e**2 - A*c**2*d**3/e**4 + B*a**2/e + 2*B*a*c*d**2/
e**3 + B*c**2*d**4/e**5 - C*a**2*d/e**2 - 2*C*a*c*d**3/e**4 - C*c**2*d**5/e
**6) + (a*e**2 + c*d**2)**2*(A*e**2 - B*d*e + C*d**2)*log(d + e*x)/e**7
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.27

$$
\begin{aligned}
& \int \frac{(a+cx^2)^2 (A+Bx+Cx^2)}{d+ex}\,dx \\
& = \frac{10\,Cc^2e^5x^6 - 12\,(Cc^2de^4 - Bc^2e^5)x^5 + 15\,(Cc^2d^2e^3 - Bc^2de^4 + (2\,Cac + Ac^2)e^5)x^4 - 20\,(Cc^2d^3e^2 - Bc}{} \\
& \quad + \frac{(Cc^2d^6 - Bc^2d^5e - 2\,Bacd^3e^3 - Ba^2de^5 + Aa^2e^6 + (2\,Cac + Ac^2)d^4e^2 + (Ca^2 + 2\,Aac)d^2e^4)\log(ex +}{e^7}
\end{aligned}
$$

```
[In] integrate((c*x^2+a)^2*(C*x^2+B*x+A)/(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/60*(10*C*c^2*e^5*x^6 - 12*(C*c^2*d*e^4 - B*c^2*e^5)*x^5 + 15*(C*c^2*d^2*e
^3 - B*c^2*d*e^4 + (2*C*a*c + A*c^2)*e^5)*x^4 - 20*(C*c^2*d^3*e^2 - B*c^2*d
^2*e^3 - 2*B*a*c*e^5 + (2*C*a*c + A*c^2)*d*e^4)*x^3 + 30*(C*c^2*d^4*e - B*c
^2*d^3*e^2 - 2*B*a*c*d*e^4 + (2*C*a*c + A*c^2)*d^2*e^3 + (C*a^2 + 2*A*a*c)*
e^5)*x^2 - 60*(C*c^2*d^5 - B*c^2*d^4*e - 2*B*a*c*d^2*e^3 - B*a^2*e^5 + (2*C
*a*c + A*c^2)*d^3*e^2 + (C*a^2 + 2*A*a*c)*d*e^4)*x)/e^6 + (C*c^2*d^6 - B*c^
2*d^5*e - 2*B*a*c*d^3*e^3 - B*a^2*d*e^5 + A*a^2*e^6 + (2*C*a*c + A*c^2)*d^4
*e^2 + (C*a^2 + 2*A*a*c)*d^2*e^4)*log(e*x + d)/e^7
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 445, normalized size of antiderivative = 1.50

$$\int \frac{(a+cx^2)^2\,(A+Bx+Cx^2)}{d+ex}\,dx$$

$$= \frac{10\,Cc^2e^5x^6 - 12\,Cc^2de^4x^5 + 12\,Bc^2e^5x^5 + 15\,Cc^2d^2e^3x^4 - 15\,Bc^2de^4x^4 + 30\,Cace^5x^4 + 15\,Ac^2e^5x^4 - 20\,C}{}$$

$$+ \frac{(Cc^2d^6 - Bc^2d^5e + 2\,Cacd^4e^2 + Ac^2d^4e^2 - 2\,Bacd^3e^3 + Ca^2d^2e^4 + 2\,Aacd^2e^4 - Ba^2de^5 + Aa^2e^6)\log{(|e}}{e^7}$$

```
[In] integrate((c*x^2+a)^2*(C*x^2+B*x+A)/(e*x+d),x, algorithm="giac")
```

```
[Out] 1/60*(10*C*c^2*e^5*x^6 - 12*C*c^2*d*e^4*x^5 + 12*B*c^2*e^5*x^5 + 15*C*c^2*d
^2*e^3*x^4 - 15*B*c^2*d*e^4*x^4 + 30*C*a*c*e^5*x^4 + 15*A*c^2*e^5*x^4 - 20*
C*c^2*d^3*e^2*x^3 + 20*B*c^2*d^2*e^3*x^3 - 40*C*a*c*d*e^4*x^3 - 20*A*c^2*d*
e^4*x^3 + 40*B*a*c*e^5*x^3 + 30*C*c^2*d^4*e*x^2 - 30*B*c^2*d^3*e^2*x^2 + 60
*C*a*c*d^2*e^3*x^2 + 30*A*c^2*d^2*e^3*x^2 - 60*B*a*c*d*e^4*x^2 + 30*C*a^2*e
^5*x^2 + 60*A*a*c*e^5*x^2 - 60*C*c^2*d^5*x + 60*B*c^2*d^4*e*x - 120*C*a*c*d
^3*e^2*x - 60*A*c^2*d^3*e^2*x + 120*B*a*c*d^2*e^3*x - 60*C*a^2*d*e^4*x - 12
0*A*a*c*d*e^4*x + 60*B*a^2*e^5*x)/e^6 + (C*c^2*d^6 - B*c^2*d^5*e + 2*C*a*c*
d^4*e^2 + A*c^2*d^4*e^2 - 2*B*a*c*d^3*e^3 + C*a^2*d^2*e^4 + 2*A*a*c*d^2*e^4
 - B*a^2*d*e^5 + A*a^2*e^6)*log(abs(e*x + d))/e^7
```

Mupad [B] (verification not implemented)

Time = 12.50 (sec) , antiderivative size = 422, normalized size of antiderivative = 1.42

$$\int \frac{(a+cx^2)^2\,(A+Bx+Cx^2)}{d+ex}\,dx = x^5\left(\frac{B\,c^2}{5\,e}-\frac{C\,c^2\,d}{5\,e^2}\right)$$

$$-x\left(\frac{d\left(\frac{C\,a^2+2\,A\,c\,a}{e}+\frac{d\left(\frac{d\left(\frac{A\,c^2+2\,C\,a\,c}{e}-\frac{d\left(\frac{B\,c^2}{e}-\frac{C\,c^2\,d}{e^2}\right)}{e}\right)}{e}-\frac{2\,B\,a\,c}{e}\right)}{e}\right)}{e}-\frac{B\,a^2}{e}\right)$$

$$+x^4\left(\frac{A\,c^2+2\,C\,a\,c}{4\,e}-\frac{d\left(\frac{B\,c^2}{e}-\frac{C\,c^2\,d}{e^2}\right)}{4\,e}\right)$$

$$-x^3\left(\frac{d\left(\frac{A\,c^2+2\,C\,a\,c}{e}-\frac{d\left(\frac{B\,c^2}{e}-\frac{C\,c^2\,d}{e^2}\right)}{e}\right)}{3\,e}-\frac{2\,B\,a\,c}{3\,e}\right)$$

$$+x^2\left(\frac{C\,a^2+2\,A\,c\,a}{2\,e}+\frac{d\left(\frac{d\left(\frac{A\,c^2+2\,C\,a\,c}{e}-\frac{d\left(\frac{B\,c^2}{e}-\frac{C\,c^2\,d}{e^2}\right)}{e}\right)}{e}-\frac{2\,B\,a\,c}{e}\right)}{2\,e}\right)$$

$$+\frac{\ln(d+e\,x)\,(C\,a^2\,d^2\,e^4-B\,a^2\,d\,e^5+A\,a^2\,e^6+2\,C\,a\,c\,d^4\,e^2-2\,B\,a\,c\,d^3\,e^3+2\,A\,a\,c\,d^2\,e^4+C\,c^2\,d^6-B\,c}{e^7}$$

$$+\frac{C\,c^2\,x^6}{6\,e}$$

```
[In] int(((a + c*x^2)^2*(A + B*x + C*x^2))/(d + e*x),x)
```

```
[Out] x^5*((B*c^2)/(5*e) - (C*c^2*d)/(5*e^2)) - x*((d*((C*a^2 + 2*A*a*c)/e + (d*(
(d*((A*c^2 + 2*C*a*c)/e - (d*((B*c^2)/e - (C*c^2*d)/e^2))/e))/e - (2*B*a*c)
/e))/e))/e - (B*a^2)/e) + x^4*((A*c^2 + 2*C*a*c)/(4*e) - (d*((B*c^2)/e - (C
*c^2*d)/e^2))/(4*e)) - x^3*((d*((A*c^2 + 2*C*a*c)/e - (d*((B*c^2)/e - (C*c^
2*d)/e^2))/e))/(3*e) - (2*B*a*c)/(3*e)) + x^2*((C*a^2 + 2*A*a*c)/(2*e) + (d
*((d*((A*c^2 + 2*C*a*c)/e - (d*((B*c^2)/e - (C*c^2*d)/e^2))/e))/e - (2*B*a*
c)/e))/(2*e)) + (log(d + e*x)*(A*a^2*e^6 + C*c^2*d^6 - B*a^2*d*e^5 - B*c^2*
d^5*e + A*c^2*d^4*e^2 + C*a^2*d^2*e^4 + 2*A*a*c*d^2*e^4 - 2*B*a*c*d^3*e^3 +
 2*C*a*c*d^4*e^2))/e^7 + (C*c^2*x^6)/(6*e)
```

3.30 $\int \frac{(a+cx^2)^2(A+Bx+Cx^2)}{(d+ex)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 292

$$
\begin{aligned}
&\int \frac{(a+cx^2)^2(A+Bx+Cx^2)}{(d+ex)^2}\,dx \\
&= \frac{(a^2Ce^4+c^2d^2(5Cd^2-e(4Bd-3Ae))+2ace^2(3Cd^2-e(2Bd-Ae)))\,x}{e^6} \\
&\quad - \frac{c(2ae^2(2Cd-Be)+cd(4Cd^2-e(3Bd-2Ae)))\,x^2}{2e^5} \\
&\quad + \frac{c(2aCe^2+c(3Cd^2-e(2Bd-Ae)))\,x^3}{3e^4} - \frac{c^2(2Cd-Be)x^4}{4e^3} \\
&\quad + \frac{c^2Cx^5}{5e^2} - \frac{(cd^2+ae^2)^2\,(Cd^2-Bde+Ae^2)}{e^7(d+ex)} \\
&\quad - \frac{(cd^2+ae^2)\,(ae^2(2Cd-Be)+cd(6Cd^2-e(5Bd-4Ae)))\log(d+ex)}{e^7}
\end{aligned}
$$

```
[Out] (a^2*C*e^4+c^2*d^2*(5*C*d^2-e*(-3*A*e+4*B*d))+2*a*c*e^2*(3*C*d^2-e*(-A*e+2*
B*d)))*x/e^6-1/2*c*(2*a*e^2*(-B*e+2*C*d)+c*d*(4*C*d^2-e*(-2*A*e+3*B*d)))*x^
2/e^5+1/3*c*(2*a*C*e^2+c*(3*C*d^2-e*(-A*e+2*B*d)))*x^3/e^4-1/4*c^2*(-B*e+2*
C*d)*x^4/e^3+1/5*c^2*C*x^5/e^2-(a*e^2+c*d^2)^2*(A*e^2-B*d*e+C*d^2)/e^7/(e*x
+d)-(a*e^2+c*d^2)*(a*e^2*(-B*e+2*C*d)+c*d*(6*C*d^2-e*(-4*A*e+5*B*d)))*ln(e*
x+d)/e^7
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 289, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {1642}

$$\begin{aligned}
&\int \frac{(a+cx^2)^2(A+Bx+Cx^2)}{(d+ex)^2}\,dx \\
&= \frac{x(a^2Ce^4+2ace^2(3Cd^2-e(2Bd-Ae))+c^2(5Cd^4-d^2e(4Bd-3Ae)))}{e^6} \\
&\quad - \frac{cx^2(2ae^2(2Cd-Be)-cde(3Bd-2Ae)+4cCd^3)}{2e^5} \\
&\quad - \frac{(ae^2+cd^2)^2(Ae^2-Bde+Cd^2)}{e^7(d+ex)} + \frac{cx^3(2aCe^2-ce(2Bd-Ae)+3cCd^2)}{3e^4} \\
&\quad - \frac{(ae^2+cd^2)\log(d+ex)(ae^2(2Cd-Be)-cde(5Bd-4Ae)+6cCd^3)}{e^7} \\
&\quad - \frac{c^2x^4(2Cd-Be)}{4e^3} + \frac{c^2Cx^5}{5e^2}
\end{aligned}$$

```
[In] Int[((a + c*x^2)^2*(A + B*x + C*x^2))/(d + e*x)^2,x]
```

```
[Out] ((a^2*C*e^4 + c^2*(5*C*d^4 - d^2*e*(4*B*d - 3*A*e)) + 2*a*c*e^2*(3*C*d^2 -
e*(2*B*d - A*e)))*x)/e^6 - (c*(4*c*C*d^3 - c*d*e*(3*B*d - 2*A*e) + 2*a*e^2*
(2*C*d - B*e))*x^2)/(2*e^5) + (c*(3*c*C*d^2 + 2*a*C*e^2 - c*e*(2*B*d - A*e)
)*x^3)/(3*e^4) - (c^2*(2*C*d - B*e)*x^4)/(4*e^3) + (c^2*C*x^5)/(5*e^2) - ((
c*d^2 + a*e^2)^2*(C*d^2 - B*d*e + A*e^2))/(e^7*(d + e*x)) - ((c*d^2 + a*e^2
)*(6*c*C*d^3 - c*d*e*(5*B*d - 4*A*e) + a*e^2*(2*C*d - B*e))*Log[d + e*x])/e
^7
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{a^2Ce^4 + c^2(5Cd^4 - d^2e(4Bd - 3Ae)) + 2ace^2(3Cd^2 - e(2Bd - Ae))}{e^6}\right. \\
&\quad + \frac{c(-4cCd^3 + cde(3Bd - 2Ae) - 2ae^2(2Cd - Be))\,x}{e^5} \\
&\quad + \frac{c(3cCd^2 + 2aCe^2 - ce(2Bd - Ae))\,x^2}{e^4} + \frac{c^2(-2Cd + Be)x^3}{e^3} + \frac{c^2Cx^4}{e^2} \\
&\quad + \frac{(cd^2 + ae^2)^2\,(Cd^2 - Bde + Ae^2)}{e^6(d + ex)^2} \\
&\quad \left. + \frac{(cd^2 + ae^2)\,(-6cCd^3 + cde(5Bd - 4Ae) - ae^2(2Cd - Be))}{e^6(d + ex)}\right)dx \\
&= \frac{(a^2Ce^4 + c^2(5Cd^4 - d^2e(4Bd - 3Ae)) + 2ace^2(3Cd^2 - e(2Bd - Ae)))\,x}{e^6} \\
&\quad - \frac{c(4cCd^3 - cde(3Bd - 2Ae) + 2ae^2(2Cd - Be))\,x^2}{2e^5} \\
&\quad + \frac{c(3cCd^2 + 2aCe^2 - ce(2Bd - Ae))\,x^3}{3e^4} - \frac{c^2(2Cd - Be)x^4}{4e^3} \\
&\quad + \frac{c^2Cx^5}{5e^2} - \frac{(cd^2 + ae^2)^2\,(Cd^2 - Bde + Ae^2)}{e^7(d + ex)} \\
&\quad - \frac{(cd^2 + ae^2)\,(6cCd^3 - cde(5Bd - 4Ae) + ae^2(2Cd - Be))\log(d + ex)}{e^7}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 272, normalized size of antiderivative = 0.93

$$\int \frac{(a + cx^2)^2\,(A + Bx + Cx^2)}{(d + ex)^2}\,dx$$

$$= \frac{60e(a^2Ce^4 + 2ace^2(3Cd^2 + e(-2Bd + Ae)) + c^2(5Cd^4 + d^2e(-4Bd + 3Ae)))\,x - 30ce^2(4cCd^3 + cde(-3}{}$$

```
[In] Integrate[((a + c*x^2)^2*(A + B*x + C*x^2))/(d + e*x)^2,x]
```

```
[Out] (60*e*(a^2*C*e^4 + 2*a*c*e^2*(3*C*d^2 + e*(-2*B*d + A*e)) + c^2*(5*C*d^4 +
d^2*e*(-4*B*d + 3*A*e)))*x - 30*c*e^2*(4*c*C*d^3 + c*d*e*(-3*B*d + 2*A*e) -
 2*a*e^2*(-2*C*d + B*e))*x^2 + 20*c*e^3*(3*c*C*d^2 + 2*a*C*e^2 + c*e*(-2*B*
d + A*e))*x^3 + 15*c^2*e^4*(-2*C*d + B*e)*x^4 + 12*c^2*C*e^5*x^5 - (60*(c*d
^2 + a*e^2)^2*(C*d^2 + e*(-(B*d) + A*e)))/(d + e*x) - 60*(c*d^2 + a*e^2)*(6
*c*C*d^3 + c*d*e*(-5*B*d + 4*A*e) + a*e^2*(2*C*d - B*e))*Log[d + e*x])/(60*
e^7)
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 401, normalized size of antiderivative = 1.37

method	result
norman	$\frac{\left(A a^2 e^6+4Aac\,d^2e^4+4A\,c^2d^4e^2-B\,a^2d\,e^5-6Bac\,d^3e^3-5B\,c^2d^5e+2C\,a^2d^2e^4+8Cac\,d^4e^2+6C\,c^2d^6\right)x}{e^6 d}+\frac{\left(4Aac\,e^4+4A\,c^2d^2e^2-6Bacd\,e^3-5}{}$
default	$\frac{\frac{1}{5}c^2C\,x^5e^4+\frac{1}{4}B\,c^2e^4x^4-\frac{1}{2}C\,c^2d\,e^3x^4+\frac{1}{3}A\,c^2e^4x^3-\frac{2}{3}B\,c^2d\,e^3x^3+\frac{2}{3}Cac\,e^4x^3+C\,c^2d^2e^2x^3-A\,c^2d\,e^3x^2+Bac\,e^4x^2+\frac{3}{2}B\,c^2d^2e^2x}{e^6}$
risch	$-\frac{C\,c^2d\,x^4}{2e^3}-\frac{2B\,c^2d\,x^3}{3e^3}+\frac{B\,c^2d^5}{e^6(ex+d)}-\frac{C\,a^2d^2}{e^3(ex+d)}-\frac{C\,c^2d^6}{e^7(ex+d)}+\frac{6Cac\,d^2x}{e^4}+\frac{A\,c^2x^3}{3e^2}+\frac{a^2Cx}{e^2}+\frac{B\,c^2x^4}{4e^2}+\frac{\ln(ex+}{e^2}$
parallelrisch	$-\frac{150B\,x^2c^2d^3e^3+18C\,x^5c^2d\,e^5+25B\,x^4c^2d\,e^5+120C\ln(ex+d)a^2d^2e^4+240A\ln(ex+d)c^2d^4e^2-60B\ln(ex+d)a^2d\,e^5-300B\ln(}{}$

```
[In] int((c*x^2+a)^2*(C*x^2+B*x+A)/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] ((A*a^2*e^6+4*A*a*c*d^2*e^4+4*A*c^2*d^4*e^2-B*a^2*d*e^5-6*B*a*c*d^3*e^3-5*B
*c^2*d^5*e+2*C*a^2*d^2*e^4+8*C*a*c*d^4*e^2+6*C*c^2*d^6)/e^6/d*x+1/2*(4*A*a*
c*e^4+4*A*c^2*d^2*e^2-6*B*a*c*d*e^3-5*B*c^2*d^3*e+2*C*a^2*e^4+8*C*a*c*d^2*e
^2+6*C*c^2*d^4)/e^5*x^2+1/12*c*(4*A*c*e^2-5*B*c*d*e+8*C*a*e^2+6*C*c*d^2)/e^
3*x^4-1/6*c*(4*A*c*d*e^2-6*B*a*e^3-5*B*c*d^2*e+8*C*a*d*e^2+6*C*c*d^3)/e^4*x
^3+1/5*c^2*C*x^6/e+1/20*c^2*(5*B*e-6*C*d)/e^2*x^5)/(e*x+d)-(4*A*a*c*d*e^4+4
*A*c^2*d^3*e^2-B*a^2*e^5-6*B*a*c*d^2*e^3-5*B*c^2*d^4*e+2*C*a^2*d*e^4+8*C*a*
c*d^3*e^2+6*C*c^2*d^5)/e^7*ln(e*x+d)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 553, normalized size of antiderivative = 1.89

$$\int \frac{(a+cx^2)^2\,(A+Bx+Cx^2)}{(d+ex)^2}\,dx$$
$$=\frac{12\,Cc^2e^6x^6-60\,Cc^2d^6+60\,Bc^2d^5e+120\,Bacd^3e^3+60\,Ba^2de^5-60\,Aa^2e^6-60\,(2\,Cac+Ac^2)d^4e^2-60\,(}{}$$

```
[In] integrate((c*x^2+a)^2*(C*x^2+B*x+A)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] 1/60*(12*C*c^2*e^6*x^6 - 60*C*c^2*d^6 + 60*B*c^2*d^5*e + 120*B*a*c*d^3*e^3
+ 60*B*a^2*d*e^5 - 60*A*a^2*e^6 - 60*(2*C*a*c + A*c^2)*d^4*e^2 - 60*(C*a^2
+ 2*A*a*c)*d^2*e^4 - 3*(6*C*c^2*d*e^5 - 5*B*c^2*e^6)*x^5 + 5*(6*C*c^2*d^2*e
^4 - 5*B*c^2*d*e^5 + 4*(2*C*a*c + A*c^2)*e^6)*x^4 - 10*(6*C*c^2*d^3*e^3 - 5
*B*c^2*d^2*e^4 - 6*B*a*c*e^6 + 4*(2*C*a*c + A*c^2)*d*e^5)*x^3 + 30*(6*C*c^2
*d^4*e^2 - 5*B*c^2*d^3*e^3 - 6*B*a*c*d*e^5 + 4*(2*C*a*c + A*c^2)*d^2*e^4 +
2*(C*a^2 + 2*A*a*c)*e^6)*x^2 + 60*(5*C*c^2*d^5*e - 4*B*c^2*d^4*e^2 - 4*B*a*
c*d^2*e^4 + 3*(2*C*a*c + A*c^2)*d^3*e^3 + (C*a^2 + 2*A*a*c)*d*e^5)*x - 60*(
6*C*c^2*d^6 - 5*B*c^2*d^5*e - 6*B*a*c*d^3*e^3 - B*a^2*d*e^5 + 4*(2*C*a*c +
```

```
A*c^2)*d^4*e^2 + 2*(C*a^2 + 2*A*a*c)*d^2*e^4 + (6*C*c^2*d^5*e - 5*B*c^2*d^4
*e^2 - 6*B*a*c*d^2*e^4 - B*a^2*e^6 + 4*(2*C*a*c + A*c^2)*d^3*e^3 + 2*(C*a^2
 + 2*A*a*c)*d*e^5)*x)*log(e*x + d))/(e^8*x + d*e^7)
```

Sympy [A] (verification not implemented)

Time = 1.09 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.42

$$\begin{aligned}\int \frac{(a+cx^2)^2\,(A+Bx+Cx^2)}{(d+ex)^2}\,dx &= \frac{Cc^2x^5}{5e^2}+x^4\left(\frac{Bc^2}{4e^2}-\frac{Cc^2d}{2e^3}\right)\\ &\quad +x^3\left(\frac{Ac^2}{3e^2}-\frac{2Bc^2d}{3e^3}+\frac{2Cac}{3e^2}+\frac{Cc^2d^2}{e^4}\right)+x^2\left(-\frac{Ac^2d}{e^3}+\frac{Bac}{e^2}+\frac{3Bc^2d^2}{2e^4}-\frac{2Cacd}{e^3}-\frac{2Cc^2d^3}{e^5}\right)\\ &\quad +x\left(\frac{2Aac}{e^2}+\frac{3Ac^2d^2}{e^4}-\frac{4Bacd}{e^3}-\frac{4Bc^2d^3}{e^5}+\frac{Ca^2}{e^2}+\frac{6Cacd^2}{e^4}+\frac{5Cc^2d^4}{e^6}\right)\\ &\quad +\frac{-Aa^2e^6-2Aacd^2e^4-Ac^2d^4e^2+Ba^2de^5+2Bacd^3e^3+Bc^2d^5e-Ca^2d^2e^4-2Cacd^4e^2-Cc^2d^6}{de^7+e^8x}\\ &\quad -\frac{(ae^2+cd^2)\,(4Acde^2-Bae^3-5Bcd^2e+2Cade^2+6Ccd^3)\log{(d+ex)}}{e^7}\end{aligned}$$

```
[In] integrate((c*x**2+a)**2*(C*x**2+B*x+A)/(e*x+d)**2,x)
```

```
[Out] C*c**2*x**5/(5*e**2) + x**4*(B*c**2/(4*e**2) - C*c**2*d/(2*e**3)) + x**3*(A
*c**2/(3*e**2) - 2*B*c**2*d/(3*e**3) + 2*C*a*c/(3*e**2) + C*c**2*d**2/e**4)
 + x**2*(-A*c**2*d/e**3 + B*a*c/e**2 + 3*B*c**2*d**2/(2*e**4) - 2*C*a*c*d/e
**3 - 2*C*c**2*d**3/e**5) + x*(2*A*a*c/e**2 + 3*A*c**2*d**2/e**4 - 4*B*a*c*
d/e**3 - 4*B*c**2*d**3/e**5 + C*a**2/e**2 + 6*C*a*c*d**2/e**4 + 5*C*c**2*d*
*4/e**6) + (-A*a**2*e**6 - 2*A*a*c*d**2*e**4 - A*c**2*d**4*e**2 + B*a**2*d*
e**5 + 2*B*a*c*d**3*e**3 + B*c**2*d**5*e - C*a**2*d**2*e**4 - 2*C*a*c*d**4*
e**2 - C*c**2*d**6)/(d*e**7 + e**8*x) - (a*e**2 + c*d**2)*(4*A*c*d*e**2 - B
*a*e**3 - 5*B*c*d**2*e + 2*C*a*d*e**2 + 6*C*c*d**3)*log(d + e*x)/e**7
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.34

$$\begin{aligned}\int \frac{(a+cx^2)^2\,(A+Bx+Cx^2)}{(d+ex)^2}\,dx &= \\ &\quad -\frac{Cc^2d^6-Bc^2d^5e-2\,Bacd^3e^3-Ba^2de^5+Aa^2e^6+(2\,Cac+Ac^2)d^4e^2+(Ca^2+2\,Aac)d^2e^4}{e^8x+de^7}\\ &\quad +\frac{12\,Cc^2e^4x^5-15\,(2\,Cc^2de^3-Bc^2e^4)x^4+20\,(3\,Cc^2d^2e^2-2\,Bc^2de^3+(2\,Cac+Ac^2)e^4)x^3-30\,(4\,Cc^2d^3}{}\\ &\quad -\frac{(6\,Cc^2d^5-5\,Bc^2d^4e-6\,Bacd^2e^3-Ba^2e^5+4\,(2\,Cac+Ac^2)d^3e^2+2\,(Ca^2+2\,Aac)de^4)\log{(ex+d)}}{e^7}\end{aligned}$$

```
[In] integrate((c*x^2+a)^2*(C*x^2+B*x+A)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -(C*c^2*d^6 - B*c^2*d^5*e - 2*B*a*c*d^3*e^3 - B*a^2*d*e^5 + A*a^2*e^6 + (2*
C*a*c + A*c^2)*d^4*e^2 + (C*a^2 + 2*A*a*c)*d^2*e^4)/(e^8*x + d*e^7) + 1/60*
(12*C*c^2*e^4*x^5 - 15*(2*C*c^2*d*e^3 - B*c^2*e^4)*x^4 + 20*(3*C*c^2*d^2*e^
2 - 2*B*c^2*d*e^3 + (2*C*a*c + A*c^2)*e^4)*x^3 - 30*(4*C*c^2*d^3*e - 3*B*c^
2*d^2*e^2 - 2*B*a*c*e^4 + 2*(2*C*a*c + A*c^2)*d*e^3)*x^2 + 60*(5*C*c^2*d^4
- 4*B*c^2*d^3*e - 4*B*a*c*d*e^3 + 3*(2*C*a*c + A*c^2)*d^2*e^2 + (C*a^2 + 2*
A*a*c)*e^4)*x)/e^6 - (6*C*c^2*d^5 - 5*B*c^2*d^4*e - 6*B*a*c*d^2*e^3 - B*a^2
*e^5 + 4*(2*C*a*c + A*c^2)*d^3*e^2 + 2*(C*a^2 + 2*A*a*c)*d*e^4)*log(e*x + d
)/e^7
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 520, normalized size of antiderivative = 1.78

$$\int \frac{(a+cx^2)^2\,(A+Bx+Cx^2)}{(d+ex)^2}\,dx$$
$$=\frac{\left(12\,Cc^2-\frac{15\,(6\,Cc^2de-Bc^2e^2)}{(ex+d)e}+\frac{20\,(15\,Cc^2d^2e^2-5\,Bc^2de^3+2\,Cace^4+Ac^2e^4)}{(ex+d)^2e^2}-\frac{60\,\left(10\,Cc^2d^3e^3-5\,Bc^2d^2e^4+4\,Cacde^5+2\,Ac^2de^5-Bace\right.}{(ex+d)^3e^3}\right.}{60\,e^7}$$
$$+\frac{(6\,Cc^2d^5-5\,Bc^2d^4e+8\,Cacd^3e^2+4\,Ac^2d^3e^2-6\,Bacd^2e^3+2\,Ca^2de^4+4\,Aacde^4-Ba^2e^5)\log\left(\frac{|ex+d|}{(ex+d)^2|}\right.}{e^7}$$
$$-\frac{\frac{Cc^2d^6e^5}{ex+d}-\frac{Bc^2d^5e^6}{ex+d}+\frac{2\,Cacd^4e^7}{ex+d}+\frac{Ac^2d^4e^7}{ex+d}-\frac{2\,Bacd^3e^8}{ex+d}+\frac{Ca^2d^2e^9}{ex+d}+\frac{2\,Aacd^2e^9}{ex+d}-\frac{Ba^2de^{10}}{ex+d}+\frac{Aa^2e^{11}}{ex+d}}{e^{12}}$$

```
[In] integrate((c*x^2+a)^2*(C*x^2+B*x+A)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] 1/60*(12*C*c^2 - 15*(6*C*c^2*d*e - B*c^2*e^2)/((e*x + d)*e) + 20*(15*C*c^2*
d^2*e^2 - 5*B*c^2*d*e^3 + 2*C*a*c*e^4 + A*c^2*e^4)/((e*x + d)^2*e^2) - 60*(
10*C*c^2*d^3*e^3 - 5*B*c^2*d^2*e^4 + 4*C*a*c*d*e^5 + 2*A*c^2*d*e^5 - B*a*c*
e^6)/((e*x + d)^3*e^3) + 60*(15*C*c^2*d^4*e^4 - 10*B*c^2*d^3*e^5 + 12*C*a*c
*d^2*e^6 + 6*A*c^2*d^2*e^6 - 6*B*a*c*d*e^7 + C*a^2*e^8 + 2*A*a*c*e^8)/((e*x
 + d)^4*e^4))*(e*x + d)^5/e^7 + (6*C*c^2*d^5 - 5*B*c^2*d^4*e + 8*C*a*c*d^3*
e^2 + 4*A*c^2*d^3*e^2 - 6*B*a*c*d^2*e^3 + 2*C*a^2*d*e^4 + 4*A*a*c*d*e^4 - B
*a^2*e^5)*log(abs(e*x + d)/((e*x + d)^2*abs(e)))/e^7 - (C*c^2*d^6*e^5/(e*x
+ d) - B*c^2*d^5*e^6/(e*x + d) + 2*C*a*c*d^4*e^7/(e*x + d) + A*c^2*d^4*e^7/
(e*x + d) - 2*B*a*c*d^3*e^8/(e*x + d) + C*a^2*d^2*e^9/(e*x + d) + 2*A*a*c*d
^2*e^9/(e*x + d) - B*a^2*d*e^10/(e*x + d) + A*a^2*e^11/(e*x + d))/e^12
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 575, normalized size of antiderivative = 1.97

$$\int \frac{(a+cx^2)^2\,(A+Bx+Cx^2)}{(d+ex)^2}\,dx$$

$$= x^4\left(\frac{B\,c^2}{4\,e^2}-\frac{C\,c^2\,d}{2\,e^3}\right)+x\left(\frac{C\,a^2+2\,A\,c\,a}{e^2}+\frac{d^2\left(\frac{2\,d\left(\frac{B\,c^2}{e^2}-\frac{2\,C\,c^2\,d}{e^3}\right)}{e}-\frac{A\,c^2+2\,C\,a\,c}{e^2}+\frac{C\,c^2\,d^2}{e^4}\right)}{e^2}\right.$$

$$\left.-\frac{2\,d\left(\frac{2\,d\left(\frac{2\,d\left(\frac{B\,c^2}{e^2}-\frac{2\,C\,c^2\,d}{e^3}\right)}{e}-\frac{A\,c^2+2\,C\,a\,c}{e^2}+\frac{C\,c^2\,d^2}{e^4}\right)}{e}-\frac{d^2\left(\frac{B\,c^2}{e^2}-\frac{2\,C\,c^2\,d}{e^3}\right)}{e^2}+\frac{2\,B\,a\,c}{e^2}\right)}{e}\right)$$

$$-x^3\left(\frac{2\,d\left(\frac{B\,c^2}{e^2}-\frac{2\,C\,c^2\,d}{e^3}\right)}{3\,e}-\frac{A\,c^2+2\,C\,a\,c}{3\,e^2}+\frac{C\,c^2\,d^2}{3\,e^4}\right)$$

$$+x^2\left(\frac{d\left(\frac{2\,d\left(\frac{B\,c^2}{e^2}-\frac{2\,C\,c^2\,d}{e^3}\right)}{e}-\frac{A\,c^2+2\,C\,a\,c}{e^2}+\frac{C\,c^2\,d^2}{e^4}\right)}{e}-\frac{d^2\left(\frac{B\,c^2}{e^2}-\frac{2\,C\,c^2\,d}{e^3}\right)}{2\,e^2}+\frac{B\,a\,c}{e^2}\right)$$

$$-\frac{C\,a^2\,d^2\,e^4-B\,a^2\,d\,e^5+A\,a^2\,e^6+2\,C\,a\,c\,d^4\,e^2-2\,B\,a\,c\,d^3\,e^3+2\,A\,a\,c\,d^2\,e^4+C\,c^2\,d^6-B\,c^2\,d^5\,e+A\,c^2}{e\,\left(x\,e^7+d\,e^6\right)}$$

$$-\frac{\ln(d+e\,x)\,\left(2\,C\,a^2\,d\,e^4-B\,a^2\,e^5+8\,C\,a\,c\,d^3\,e^2-6\,B\,a\,c\,d^2\,e^3+4\,A\,a\,c\,d\,e^4+6\,C\,c^2\,d^5-5\,B\,c^2\,d^4\,e+\right.}{e^7}$$

$$+\frac{C\,c^2\,x^5}{5\,e^2}$$

```
[In] int(((a + c*x^2)^2*(A + B*x + C*x^2))/(d + e*x)^2,x)
```

```
[Out] x^4*((B*c^2)/(4*e^2) - (C*c^2*d)/(2*e^3)) + x*((C*a^2 + 2*A*a*c)/e^2 + (d^2
*((2*d*((B*c^2)/e^2 - (2*C*c^2*d)/e^3))/e - (A*c^2 + 2*C*a*c)/e^2 + (C*c^2*
d^2)/e^4))/e^2 - (2*d*((2*d*((2*d*((B*c^2)/e^2 - (2*C*c^2*d)/e^3))/e - (A*c
^2 + 2*C*a*c)/e^2 + (C*c^2*d^2)/e^4))/e - (d^2*((B*c^2)/e^2 - (2*C*c^2*d)/e
^3))/e^2 + (2*B*a*c)/e^2))/e) - x^3*((2*d*((B*c^2)/e^2 - (2*C*c^2*d)/e^3))/
```

```
(3*e) - (A*c^2 + 2*C*a*c)/(3*e^2) + (C*c^2*d^2)/(3*e^4)) + x^2*((d*((2*d*((
B*c^2)/e^2 - (2*C*c^2*d)/e^3))/e - (A*c^2 + 2*C*a*c)/e^2 + (C*c^2*d^2)/e^4)
)/e - (d^2*((B*c^2)/e^2 - (2*C*c^2*d)/e^3))/(2*e^2) + (B*a*c)/e^2) - (A*a^2
*e^6 + C*c^2*d^6 - B*a^2*d*e^5 - B*c^2*d^5*e + A*c^2*d^4*e^2 + C*a^2*d^2*e^
4 + 2*A*a*c*d^2*e^4 - 2*B*a*c*d^3*e^3 + 2*C*a*c*d^4*e^2)/(e*(d*e^6 + e^7*x)
) - (log(d + e*x)*(6*C*c^2*d^5 - B*a^2*e^5 + 2*C*a^2*d*e^4 - 5*B*c^2*d^4*e
+ 4*A*c^2*d^3*e^2 + 4*A*a*c*d*e^4 - 6*B*a*c*d^2*e^3 + 8*C*a*c*d^3*e^2))/e^7
 + (C*c^2*x^5)/(5*e^2)
```

3.31 $\int \frac{(a+cx^2)^2(A+Bx+Cx^2)}{(d+ex)^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 295

$$\begin{aligned}
\int \frac{(a+cx^2)^2(A+Bx+Cx^2)}{(d+ex)^3}\,dx
&= -\frac{c(2ae^2(3Cd-Be)+cd(10Cd^2-3e(2Bd-Ae)))\,x}{e^6} \\
&\quad + \frac{c(2aCe^2+c(6Cd^2-e(3Bd-Ae)))\,x^2}{2e^5} - \frac{c^2(3Cd-Be)x^3}{3e^4} \\
&\quad + \frac{c^2Cx^4}{4e^3} - \frac{(cd^2+ae^2)^2(Cd^2-Bde+Ae^2)}{2e^7(d+ex)^2} \\
&\quad + \frac{(cd^2+ae^2)(ae^2(2Cd-Be)+cd(6Cd^2-e(5Bd-4Ae)))}{e^7(d+ex)} \\
&\quad + \frac{(a^2Ce^4+c^2d^2(15Cd^2-2e(5Bd-3Ae))+2ace^2(6Cd^2-e(3Bd-Ae)))\log(d+ex)}{e^7}
\end{aligned}$$

```
[Out] -c*(2*a*e^2*(-B*e+3*C*d)+c*d*(10*C*d^2-3*e*(-A*e+2*B*d)))*x/e^6+1/2*c*(2*a*
C*e^2+c*(6*C*d^2-e*(-A*e+3*B*d)))*x^2/e^5-1/3*c^2*(-B*e+3*C*d)*x^3/e^4+1/4*
c^2*C*x^4/e^3-1/2*(a*e^2+c*d^2)^2*(A*e^2-B*d*e+C*d^2)/e^7/(e*x+d)^2+(a*e^2+
c*d^2)*(a*e^2*(-B*e+2*C*d)+c*d*(6*C*d^2-e*(-4*A*e+5*B*d)))/e^7/(e*x+d)+(a^2
*C*e^4+c^2*d^2*(15*C*d^2-2*e*(-3*A*e+5*B*d))+2*a*c*e^2*(6*C*d^2-e*(-A*e+3*B
*d)))*ln(e*x+d)/e^7
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 292, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {1642}

$$\begin{aligned}
&\int \frac{(a+cx^2)^2(A+Bx+Cx^2)}{(d+ex)^3}\,dx \\
&= \frac{\log(d+ex)\left(a^2Ce^4+2ace^2(6Cd^2-e(3Bd-Ae))+c^2(15Cd^4-2d^2e(5Bd-3Ae))\right)}{e^7} \\
&\quad -\frac{cx(2ae^2(3Cd-Be)-3cde(2Bd-Ae)+10cCd^3)}{e^6} \\
&\quad -\frac{(ae^2+cd^2)^2(Ae^2-Bde+Cd^2)}{2e^7(d+ex)^2}+\frac{cx^2(2aCe^2-ce(3Bd-Ae)+6cCd^2)}{2e^5} \\
&\quad +\frac{(ae^2+cd^2)(ae^2(2Cd-Be)-cde(5Bd-4Ae)+6cCd^3)}{e^7(d+ex)}-\frac{c^2x^3(3Cd-Be)}{3e^4}+\frac{c^2Cx^4}{4e^3}
\end{aligned}$$

```
[In] Int[((a + c*x^2)^2*(A + B*x + C*x^2))/(d + e*x)^3,x]
```

```
[Out] -((c*(10*c*C*d^3 - 3*c*d*e*(2*B*d - A*e) + 2*a*e^2*(3*C*d - B*e))*x)/e^6) +
 (c*(6*c*C*d^2 + 2*a*C*e^2 - c*e*(3*B*d - A*e))*x^2)/(2*e^5) - (c^2*(3*C*d
- B*e)*x^3)/(3*e^4) + (c^2*C*x^4)/(4*e^3) - ((c*d^2 + a*e^2)^2*(C*d^2 - B*d
*e + A*e^2))/(2*e^7*(d + e*x)^2) + ((c*d^2 + a*e^2)*(6*c*C*d^3 - c*d*e*(5*B
*d - 4*A*e) + a*e^2*(2*C*d - B*e)))/(e^7*(d + e*x)) + ((a^2*C*e^4 + c^2*(15
*C*d^4 - 2*d^2*e*(5*B*d - 3*A*e)) + 2*a*c*e^2*(6*C*d^2 - e*(3*B*d - A*e)))*
Log[d + e*x])/e^7
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} = \int \Bigg(&\frac{c(-10cCd^3+3cde(2Bd-Ae)-2ae^2(3Cd-Be))}{e^6} \\
&+\frac{c(6cCd^2+2aCe^2-ce(3Bd-Ae))\,x}{e^5}+\frac{c^2(-3Cd+Be)x^2}{e^4}+\frac{c^2Cx^3}{e^3} \\
&+\frac{(cd^2+ae^2)^2(Cd^2-Bde+Ae^2)}{e^6(d+ex)^3} \\
&+\frac{(cd^2+ae^2)(-6cCd^3+cde(5Bd-4Ae)-ae^2(2Cd-Be))}{e^6(d+ex)^2} \\
&+\frac{a^2Ce^4+c^2(15Cd^4-2d^2e(5Bd-3Ae))+2ace^2(6Cd^2-e(3Bd-Ae))}{e^6(d+ex)}\Bigg)\,dx
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{c(10cCd^3-3cde(2Bd-Ae)+2ae^2(3Cd-Be))\,x}{e^6} \\
&\quad+\frac{c(6cCd^2+2aCe^2-ce(3Bd-Ae))\,x^2}{2e^5}-\frac{c^2(3Cd-Be)x^3}{3e^4} \\
&\quad+\frac{c^2Cx^4}{4e^3}-\frac{(cd^2+ae^2)^2\,(Cd^2-Bde+Ae^2)}{2e^7(d+ex)^2} \\
&\quad+\frac{(cd^2+ae^2)\,(6cCd^3-cde(5Bd-4Ae)+ae^2(2Cd-Be))}{e^7(d+ex)} \\
&\quad+\frac{(a^2Ce^4+c^2(15Cd^4-2d^2e(5Bd-3Ae))+2ace^2(6Cd^2-e(3Bd-Ae)))\log(d+ex)}{e^7}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.93

$$
\int \frac{(a+cx^2)^2\,(A+Bx+Cx^2)}{(d+ex)^3}\,dx
$$

$$
= \frac{-12ce(10cCd^3+3cde(-2Bd+Ae)-2ae^2(-3Cd+Be))\,x+6ce^2(6cCd^2+2aCe^2+ce(-3Bd+Ae))\,x}{}
$$

```
[In] Integrate[((a + c*x^2)^2*(A + B*x + C*x^2))/(d + e*x)^3,x]
```

```
[Out] (-12*c*e*(10*c*C*d^3 + 3*c*d*e*(-2*B*d + A*e) - 2*a*e^2*(-3*C*d + B*e))*x +
 6*c*e^2*(6*c*C*d^2 + 2*a*C*e^2 + c*e*(-3*B*d + A*e))*x^2 + 4*c^2*e^3*(-3*C
*d + B*e)*x^3 + 3*c^2*C*e^4*x^4 - (6*(c*d^2 + a*e^2)^2*(C*d^2 + e*(-(B*d) +
 A*e)))/(d + e*x)^2 + (12*(c*d^2 + a*e^2)*(6*c*C*d^3 + c*d*e*(-5*B*d + 4*A*
e) + a*e^2*(2*C*d - B*e)))/(d + e*x) + 12*(a^2*C*e^4 + 2*a*c*e^2*(6*C*d^2 +
 e*(-3*B*d + A*e)) + c^2*(15*C*d^4 + 2*d^2*e*(-5*B*d + 3*A*e)))*Log[d + e*x
])/(12*e^7)
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.33

method	result
norman	$\frac{\frac{\left(4Aacde^4+12Ac^2d^3e^2-Be^5a^2-12Bacd^2e^3-20Bc^2d^4e+2Ca^2de^4+24Cacd^3e^2+30Cc^2d^5\right)x}{e^6}-\frac{Aa^2e^6-6Aacd^2e^4-18Ac^2d^4e^2+Ba^2d}{}}{}$
default	$-\frac{c\left(-\frac{1}{4}cCx^4e^3-\frac{1}{3}Bcx^3e^3+Ccde^2x^3-\frac{1}{2}Ace^3x^2+\frac{3}{2}Bx^2cde^2-Cae^3x^2-3Ccd^2ex^2+3Acde^2x-2Bxae^3-6Bcd^2ex+6Cad\right.}{e^6}$
risch	$\frac{c^2Cx^4}{4e^3}+\frac{c^2Bx^3}{3e^3}-\frac{c^2Cdx^3}{e^4}+\frac{c^2Ax^2}{2e^3}-\frac{3c^2Bx^2d}{2e^4}+\frac{cCax^2}{e^3}+\frac{3c^2Cd^2x^2}{e^5}-\frac{3c^2Adx}{e^4}+\frac{2cBxa}{e^3}+\frac{6c^2Bd^2x}{e^5}-$
parallelrisch	$\frac{12C\ln(ex+d)x^2a^2e^6-6Cx^5c^2de^5-10Bx^4c^2de^5+12C\ln(ex+d)a^2d^2e^4+72A\ln(ex+d)c^2d^4e^2-120B\ln(ex+d)c^2d^5e+12Cx^4}{}$

```
[In] int((c*x^2+a)^2*(C*x^2+B*x+A)/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] ((4*A*a*c*d*e^4+12*A*c^2*d^3*e^2-B*a^2*e^5-12*B*a*c*d^2*e^3-20*B*c^2*d^4*e+
2*C*a^2*d*e^4+24*C*a*c*d^3*e^2+30*C*c^2*d^5)/e^6*x-1/2*(A*a^2*e^6-6*A*a*c*d
^2*e^4-18*A*c^2*d^4*e^2+B*a^2*d*e^5+18*B*a*c*d^3*e^3+30*B*c^2*d^5*e-3*C*a^2
*d^2*e^4-36*C*a*c*d^4*e^2-45*C*c^2*d^6)/e^7+1/12*c*(6*A*c*e^2-10*B*c*d*e+12
*C*a*e^2+15*C*c*d^2)/e^3*x^4-1/3*c*(6*A*c*d*e^2-6*B*a*e^3-10*B*c*d^2*e+12*C
*a*d*e^2+15*C*c*d^3)/e^4*x^3+1/4*c^2*C*x^6/e+1/6*c^2*(2*B*e-3*C*d)/e^2*x^5)
/(e*x+d)^2+1/e^7*(2*A*a*c*e^4+6*A*c^2*d^2*e^2-6*B*a*c*d*e^3-10*B*c^2*d^3*e+
C*a^2*e^4+12*C*a*c*d^2*e^2+15*C*c^2*d^4)*ln(e*x+d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 608 vs. 2(287) = 574.

Time = 0.29 (sec) , antiderivative size = 608, normalized size of antiderivative = 2.06

$$\int \frac{(a+cx^2)^2 (A+Bx+Cx^2)}{(d+ex)^3} \, dx$$
$$= \frac{3\,Cc^2e^6x^6 + 66\,Cc^2d^6 - 54\,Bc^2d^5e - 60\,Bacd^3e^3 - 6\,Ba^2de^5 - 6\,Aa^2e^6 + 42\,(2\,Cac + Ac^2)d^4e^2 + 18\,(Ca^2}{}$$

```
[In] integrate((c*x^2+a)^2*(C*x^2+B*x+A)/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] 1/12*(3*C*c^2*e^6*x^6 + 66*C*c^2*d^6 - 54*B*c^2*d^5*e - 60*B*a*c*d^3*e^3 -
6*B*a^2*d*e^5 - 6*A*a^2*e^6 + 42*(2*C*a*c + A*c^2)*d^4*e^2 + 18*(C*a^2 + 2*
A*a*c)*d^2*e^4 - 2*(3*C*c^2*d*e^5 - 2*B*c^2*e^6)*x^5 + (15*C*c^2*d^2*e^4 -
10*B*c^2*d*e^5 + 6*(2*C*a*c + A*c^2)*e^6)*x^4 - 4*(15*C*c^2*d^3*e^3 - 10*B*
c^2*d^2*e^4 - 6*B*a*c*e^6 + 6*(2*C*a*c + A*c^2)*d*e^5)*x^3 - 6*(34*C*c^2*d^
4*e^2 - 21*B*c^2*d^3*e^3 - 8*B*a*c*d*e^5 + 11*(2*C*a*c + A*c^2)*d^2*e^4)*x^
2 - 12*(4*C*c^2*d^5*e - B*c^2*d^4*e^2 + 4*B*a*c*d^2*e^4 + B*a^2*e^6 - (2*C*
a*c + A*c^2)*d^3*e^3 - 2*(C*a^2 + 2*A*a*c)*d*e^5)*x + 12*(15*C*c^2*d^6 - 10
*B*c^2*d^5*e - 6*B*a*c*d^3*e^3 + 6*(2*C*a*c + A*c^2)*d^4*e^2 + (C*a^2 + 2*A
*a*c)*d^2*e^4 + (15*C*c^2*d^4*e^2 - 10*B*c^2*d^3*e^3 - 6*B*a*c*d*e^5 + 6*(2
*C*a*c + A*c^2)*d^2*e^4 + (C*a^2 + 2*A*a*c)*e^6)*x^2 + 2*(15*C*c^2*d^5*e -
10*B*c^2*d^4*e^2 - 6*B*a*c*d^2*e^4 + 6*(2*C*a*c + A*c^2)*d^3*e^3 + (C*a^2 +
 2*A*a*c)*d*e^5)*x)*log(e*x + d))/(e^9*x^2 + 2*d*e^8*x + d^2*e^7)
```

Sympy [A] (verification not implemented)

Time = 4.59 (sec) , antiderivative size = 474, normalized size of antiderivative = 1.61

$$\int \frac{(a+cx^2)^2 (A+Bx+Cx^2)}{(d+ex)^3}\,dx$$
$$= \frac{Cc^2x^4}{4e^3} + x^3\left(\frac{Bc^2}{3e^3} - \frac{Cc^2d}{e^4}\right) + x^2\left(\frac{Ac^2}{2e^3} - \frac{3Bc^2d}{2e^4} + \frac{Cac}{e^3} + \frac{3Cc^2d^2}{e^5}\right)$$
$$+ x\left(-\frac{3Ac^2d}{e^4} + \frac{2Bac}{e^3} + \frac{6Bc^2d^2}{e^5} - \frac{6Cacd}{e^4} - \frac{10Cc^2d^3}{e^6}\right)$$
$$+ \frac{-Aa^2e^6 + 6Aacd^2e^4 + 7Ac^2d^4e^2 - Ba^2de^5 - 10Bacd^3e^3 - 9Bc^2d^5e + 3Ca^2d^2e^4 + 14Cacd^4e^2 + 11Cc^2}{2d^2e^7 + 4d}$$
$$+ \frac{(2Aace^4 + 6Ac^2d^2e^2 - 6Bacde^3 - 10Bc^2d^3e + Ca^2e^4 + 12Cacd^2e^2 + 15Cc^2d^4)\log(d+ex)}{e^7}$$

```
[In] integrate((c*x**2+a)**2*(C*x**2+B*x+A)/(e*x+d)**3,x)
```

```
[Out] C*c**2*x**4/(4*e**3) + x**3*(B*c**2/(3*e**3) - C*c**2*d/e**4) + x**2*(A*c**
2/(2*e**3) - 3*B*c**2*d/(2*e**4) + C*a*c/e**3 + 3*C*c**2*d**2/e**5) + x*(-3
*A*c**2*d/e**4 + 2*B*a*c/e**3 + 6*B*c**2*d**2/e**5 - 6*C*a*c*d/e**4 - 10*C*
c**2*d**3/e**6) + (-A*a**2*e**6 + 6*A*a*c*d**2*e**4 + 7*A*c**2*d**4*e**2 -
B*a**2*d*e**5 - 10*B*a*c*d**3*e**3 - 9*B*c**2*d**5*e + 3*C*a**2*d**2*e**4 +
 14*C*a*c*d**4*e**2 + 11*C*c**2*d**6 + x*(8*A*a*c*d*e**5 + 8*A*c**2*d**3*e*
*3 - 2*B*a**2*e**6 - 12*B*a*c*d**2*e**4 - 10*B*c**2*d**4*e**2 + 4*C*a**2*d*
e**5 + 16*C*a*c*d**3*e**3 + 12*C*c**2*d**5*e))/(2*d**2*e**7 + 4*d*e**8*x +
2*e**9*x**2) + (2*A*a*c*e**4 + 6*A*c**2*d**2*e**2 - 6*B*a*c*d*e**3 - 10*B*c
**2*d**3*e + C*a**2*e**4 + 12*C*a*c*d**2*e**2 + 15*C*c**2*d**4)*log(d + e*x
)/e**7
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.36

$$\int \frac{(a+cx^2)^2 (A+Bx+Cx^2)}{(d+ex)^3}\,dx$$
$$= \frac{11\,Cc^2d^6 - 9\,Bc^2d^5e - 10\,Bacd^3e^3 - Ba^2de^5 - Aa^2e^6 + 7\,(2\,Cac + Ac^2)d^4e^2 + 3\,(Ca^2 + 2\,Aac)d^2e^4 + 2\,(}{2\,(e^9x^2 + 2\,de^8x +}$$
$$+ \frac{3\,Cc^2e^3x^4 - 4\,(3\,Cc^2de^2 - Bc^2e^3)x^3 + 6\,(6\,Cc^2d^2e - 3\,Bc^2de^2 + (2\,Cac + Ac^2)e^3)x^2 - 12\,(10\,Cc^2d^3 -}{12\,e^6}$$
$$+ \frac{(15\,Cc^2d^4 - 10\,Bc^2d^3e - 6\,Bacde^3 + 6\,(2\,Cac + Ac^2)d^2e^2 + (Ca^2 + 2\,Aac)e^4)\log(ex+d)}{e^7}$$

```
[In] integrate((c*x^2+a)^2*(C*x^2+B*x+A)/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(11*C*c^2*d^6 - 9*B*c^2*d^5*e - 10*B*a*c*d^3*e^3 - B*a^2*d*e^5 - A*a^2*
e^6 + 7*(2*C*a*c + A*c^2)*d^4*e^2 + 3*(C*a^2 + 2*A*a*c)*d^2*e^4 + 2*(6*C*c^
2*d^5*e - 5*B*c^2*d^4*e^2 - 6*B*a*c*d^2*e^4 - B*a^2*e^6 + 4*(2*C*a*c + A*c^
2)*d^3*e^3 + 2*(C*a^2 + 2*A*a*c)*d*e^5)*x)/(e^9*x^2 + 2*d*e^8*x + d^2*e^7)
+ 1/12*(3*C*c^2*e^3*x^4 - 4*(3*C*c^2*d*e^2 - B*c^2*e^3)*x^3 + 6*(6*C*c^2*d^
2*e - 3*B*c^2*d*e^2 + (2*C*a*c + A*c^2)*e^3)*x^2 - 12*(10*C*c^2*d^3 - 6*B*c
^2*d^2*e - 2*B*a*c*e^3 + 3*(2*C*a*c + A*c^2)*d*e^2)*x)/e^6 + (15*C*c^2*d^4
- 10*B*c^2*d^3*e - 6*B*a*c*d*e^3 + 6*(2*C*a*c + A*c^2)*d^2*e^2 + (C*a^2 + 2
*A*a*c)*e^4)*log(e*x + d)/e^7
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 426, normalized size of antiderivative = 1.44

$$\int \frac{(a+cx^2)^2(A+Bx+Cx^2)}{(d+ex)^3}\,dx$$

$$= \frac{(15\,Cc^2d^4 - 10\,Bc^2d^3e + 12\,Cacd^2e^2 + 6\,Ac^2d^2e^2 - 6\,Bacde^3 + Ca^2e^4 + 2\,Aace^4)\log(|ex+d|)}{e^7}$$

$$+ \frac{11\,Cc^2d^6 - 9\,Bc^2d^5e + 14\,Cacd^4e^2 + 7\,Ac^2d^4e^2 - 10\,Bacd^3e^3 + 3\,Ca^2d^2e^4 + 6\,Aacd^2e^4 - Ba^2de^5 - Aa^2}{2\,(ex+}$$

$$+ \frac{3\,Cc^2e^9x^4 - 12\,Cc^2de^8x^3 + 4\,Bc^2e^9x^3 + 36\,Cc^2d^2e^7x^2 - 18\,Bc^2de^8x^2 + 12\,Cace^9x^2 + 6\,Ac^2e^9x^2 - 120\,C}{12\,e^{12}}$$

```
[In] integrate((c*x^2+a)^2*(C*x^2+B*x+A)/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] (15*C*c^2*d^4 - 10*B*c^2*d^3*e + 12*C*a*c*d^2*e^2 + 6*A*c^2*d^2*e^2 - 6*B*a
*c*d*e^3 + C*a^2*e^4 + 2*A*a*c*e^4)*log(abs(e*x + d))/e^7 + 1/2*(11*C*c^2*d
^6 - 9*B*c^2*d^5*e + 14*C*a*c*d^4*e^2 + 7*A*c^2*d^4*e^2 - 10*B*a*c*d^3*e^3
+ 3*C*a^2*d^2*e^4 + 6*A*a*c*d^2*e^4 - B*a^2*d*e^5 - A*a^2*e^6 + 2*(6*C*c^2*
d^5*e - 5*B*c^2*d^4*e^2 + 8*C*a*c*d^3*e^3 + 4*A*c^2*d^3*e^3 - 6*B*a*c*d^2*e
^4 + 2*C*a^2*d*e^5 + 4*A*a*c*d*e^5 - B*a^2*e^6)*x)/((e*x + d)^2*e^7) + 1/12
*(3*C*c^2*e^9*x^4 - 12*C*c^2*d*e^8*x^3 + 4*B*c^2*e^9*x^3 + 36*C*c^2*d^2*e^7
*x^2 - 18*B*c^2*d*e^8*x^2 + 12*C*a*c*e^9*x^2 + 6*A*c^2*e^9*x^2 - 120*C*c^2*
d^3*e^6*x + 72*B*c^2*d^2*e^7*x - 72*C*a*c*d*e^8*x - 36*A*c^2*d*e^8*x + 24*B
*a*c*e^9*x)/e^12
```

Mupad [B] (verification not implemented)

Time = 12.32 (sec) , antiderivative size = 495, normalized size of antiderivative = 1.68

$$\int \frac{(a+cx^2)^2\,(A+Bx+Cx^2)}{(d+ex)^3}\,dx$$

$$= x\left(\frac{3\,d\,\left(\frac{3\,d\,\left(\frac{B\,c^2}{e^3}-\frac{3\,C\,c^2\,d}{e^4}\right)}{e}-\frac{A\,c^2+2\,C\,a\,c}{e^3}+\frac{3\,C\,c^2\,d^2}{e^5}\right)}{e}-\frac{3\,d^2\,\left(\frac{B\,c^2}{e^3}-\frac{3\,C\,c^2\,d}{e^4}\right)}{e^2}+\frac{2\,B\,a\,c}{e^3}\right.$$

$$\left.-\frac{C\,c^2\,d^3}{e^6}\right)+x^3\,\left(\frac{B\,c^2}{3\,e^3}-\frac{C\,c^2\,d}{e^4}\right)-x^2\,\left(\frac{3\,d\,\left(\frac{B\,c^2}{e^3}-\frac{3\,C\,c^2\,d}{e^4}\right)}{2\,e}-\frac{A\,c^2+2\,C\,a\,c}{2\,e^3}+\frac{3\,C\,c^2\,d^2}{2\,e^5}\right)$$

$$+\frac{\frac{3\,C\,a^2\,d^2\,e^4-B\,a^2\,d\,e^5-A\,a^2\,e^6+14\,C\,a\,c\,d^4\,e^2-10\,B\,a\,c\,d^3\,e^3+6\,A\,a\,c\,d^2\,e^4+11\,C\,c^2\,d^6-9\,B\,c^2\,d^5\,e+7\,A\,c^2\,d^4\,e^2}{2\,e}+x\,(2\,C\,a^2\,d\,e^4-B}{d^2\,e^6+2\,d\,e^7\,x+e^8}$$

$$+\frac{\ln\left(d+e\,x\right)\,\left(C\,a^2\,e^4+12\,C\,a\,c\,d^2\,e^2-6\,B\,a\,c\,d\,e^3+2\,A\,a\,c\,e^4+15\,C\,c^2\,d^4-10\,B\,c^2\,d^3\,e+6\,A\,c^2\,d^2\,e^2\right)}{e^7}$$

$$+\frac{C\,c^2\,x^4}{4\,e^3}$$

```
[In] int(((a + c*x^2)^2*(A + B*x + C*x^2))/(d + e*x)^3,x)
```

```
[Out] x*((3*d*((3*d*((B*c^2)/e^3 - (3*C*c^2*d)/e^4))/e - (A*c^2 + 2*C*a*c)/e^3 +
(3*C*c^2*d^2)/e^5))/e - (3*d^2*((B*c^2)/e^3 - (3*C*c^2*d)/e^4))/e^2 + (2*B*
a*c)/e^3 - (C*c^2*d^3)/e^6) + x^3*((B*c^2)/(3*e^3) - (C*c^2*d)/e^4) - x^2*(
(3*d*((B*c^2)/e^3 - (3*C*c^2*d)/e^4))/(2*e) - (A*c^2 + 2*C*a*c)/(2*e^3) + (
3*C*c^2*d^2)/(2*e^5)) + ((11*C*c^2*d^6 - A*a^2*e^6 - B*a^2*d*e^5 - 9*B*c^2*
d^5*e + 7*A*c^2*d^4*e^2 + 3*C*a^2*d^2*e^4 + 6*A*a*c*d^2*e^4 - 10*B*a*c*d^3*
e^3 + 14*C*a*c*d^4*e^2)/(2*e) + x*(6*C*c^2*d^5 - B*a^2*e^5 + 2*C*a^2*d*e^4
- 5*B*c^2*d^4*e + 4*A*c^2*d^3*e^2 + 4*A*a*c*d*e^4 - 6*B*a*c*d^2*e^3 + 8*C*a
*c*d^3*e^2))/(d^2*e^6 + e^8*x^2 + 2*d*e^7*x) + (log(d + e*x)*(C*a^2*e^4 + 1
5*C*c^2*d^4 + 2*A*a*c*e^4 - 10*B*c^2*d^3*e + 6*A*c^2*d^2*e^2 - 6*B*a*c*d*e^
3 + 12*C*a*c*d^2*e^2))/e^7 + (C*c^2*x^4)/(4*e^3)
```

3.32 $\int (d+ex)^3 \left(a+cx^2\right)^3 \left(A+Bx+Cx^2\right) \, dx$

Optimal result

Integrand size = 27, antiderivative size = 404

$$
\begin{aligned}
&\int (d+ex)^3 \left(a+cx^2\right)^3 \left(A+Bx+Cx^2\right) \, dx \\
&= a^3Ad^3x + \frac{1}{3}a^2d\left(ad(Cd+3Be)+3A\left(cd^2+ae^2\right)\right)x^3 + \frac{1}{4}a^3e\left(3Cd^2+e(3Bd+Ae)\right)x^4 \\
&\quad + \frac{1}{5}a\left(3Acd\left(cd^2+3ae^2\right)+a\left(ae^2(3Cd+Be)+3cd^2(Cd+3Be)\right)\right)x^5 \\
&\quad + \frac{1}{6}a^2e\left(aCe^2+3c\left(3Cd^2+e(3Bd+Ae)\right)\right)x^6 \\
&\quad + \frac{1}{7}c\left(Acd\left(cd^2+9ae^2\right)+3a\left(ae^2(3Cd+Be)+cd^2(Cd+3Be)\right)\right)x^7 \\
&\quad + \frac{3}{8}ace\left(aCe^2+c\left(3Cd^2+e(3Bd+Ae)\right)\right)x^8 \\
&\quad + \frac{1}{9}c^2\left(3ae^2(3Cd+Be)+cd\left(Cd^2+3e(Bd+Ae)\right)\right)x^9 \\
&\quad + \frac{1}{10}c^2e\left(3aCe^2+c\left(3Cd^2+e(3Bd+Ae)\right)\right)x^{10} \\
&\quad + \frac{1}{11}c^3e^2(3Cd+Be)x^{11} + \frac{1}{12}c^3Ce^3x^{12} + \frac{d^2(Bd+3Ae)\left(a+cx^2\right)^4}{8c}
\end{aligned}
$$

```
[Out] a^3*A*d^3*x+1/3*a^2*d*(a*d*(3*B*e+C*d)+3*A*(a*e^2+c*d^2))*x^3+1/4*a^3*e*(3*
C*d^2+e*(A*e+3*B*d))*x^4+1/5*a*(3*A*c*d*(3*a*e^2+c*d^2)+a*(a*e^2*(B*e+3*C*d
)+3*c*d^2*(3*B*e+C*d)))*x^5+1/6*a^2*e*(a*C*e^2+3*c*(3*C*d^2+e*(A*e+3*B*d)))
*x^6+1/7*c*(A*c*d*(9*a*e^2+c*d^2)+3*a*(a*e^2*(B*e+3*C*d)+c*d^2*(3*B*e+C*d))
)*x^7+3/8*a*c*e*(a*C*e^2+c*(3*C*d^2+e*(A*e+3*B*d)))*x^8+1/9*c^2*(3*a*e^2*(B
*e+3*C*d)+c*d*(C*d^2+3*e*(A*e+B*d)))*x^9+1/10*c^2*e*(3*a*C*e^2+c*(3*C*d^2+e
*(A*e+3*B*d)))*x^10+1/11*c^3*e^2*(B*e+3*C*d)*x^11+1/12*c^3*C*e^3*x^12+1/8*d
^2*(3*A*e+B*d)*(c*x^2+a)^4/c
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 400, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1596, 1824}

$$\begin{aligned}
&\int (d+ex)^3 \left(a+cx^2\right)^3 \left(A+Bx+Cx^2\right) dx \\
&= \frac{1}{4}a^3ex^4\left(e(Ae+3Bd)+3Cd^2\right)+a^3Ad^3x+\frac{1}{6}a^2ex^6\left(aCe^2+3ce(Ae+3Bd)+9cCd^2\right) \\
&\quad+\frac{1}{3}a^2dx^3\left(3A\left(ae^2+cd^2\right)+ad(3Be+Cd)\right) \\
&\quad+\frac{1}{9}c^2x^9\left(3ae^2(Be+3Cd)+3cde(Ae+Bd)+cCd^3\right) \\
&\quad+\frac{1}{10}c^2ex^{10}\left(3aCe^2+ce(Ae+3Bd)+3cCd^2\right)+\frac{3}{8}acex^8\left(aCe^2+ce(Ae+3Bd)+3cCd^2\right) \\
&\quad+\frac{1}{7}cx^7\left(Acd\left(9ae^2+cd^2\right)+3a\left(ae^2(Be+3Cd)+cd^2(3Be+Cd)\right)\right) \\
&\quad+\frac{1}{5}ax^5\left(3Acd\left(3ae^2+cd^2\right)+a\left(ae^2(Be+3Cd)+3cd^2(3Be+Cd)\right)\right) \\
&\quad+\frac{d^2\left(a+cx^2\right)^4(3Ae+Bd)}{8c}+\frac{1}{11}c^3e^2x^{11}(Be+3Cd)+\frac{1}{12}c^3Ce^3x^{12}
\end{aligned}$$

```
[In] Int[(d + e*x)^3*(a + c*x^2)^3*(A + B*x + C*x^2),x]
```

```
[Out] a^3*A*d^3*x + (a^2*d*(a*d*(C*d + 3*B*e) + 3*A*(c*d^2 + a*e^2))*x^3)/3 + (a^
3*e*(3*C*d^2 + e*(3*B*d + A*e))*x^4)/4 + (a*(3*A*c*d*(c*d^2 + 3*a*e^2) + a*
(a*e^2*(3*C*d + B*e) + 3*c*d^2*(C*d + 3*B*e)))*x^5)/5 + (a^2*e*(9*c*C*d^2 +
 a*C*e^2 + 3*c*e*(3*B*d + A*e))*x^6)/6 + (c*(A*c*d*(c*d^2 + 9*a*e^2) + 3*a*
(a*e^2*(3*C*d + B*e) + c*d^2*(C*d + 3*B*e)))*x^7)/7 + (3*a*c*e*(3*c*C*d^2 +
 a*C*e^2 + c*e*(3*B*d + A*e))*x^8)/8 + (c^2*(c*C*d^3 + 3*c*d*e*(B*d + A*e)
+ 3*a*e^2*(3*C*d + B*e))*x^9)/9 + (c^2*e*(3*c*C*d^2 + 3*a*C*e^2 + c*e*(3*B*
d + A*e))*x^10)/10 + (c^3*e^2*(3*C*d + B*e)*x^11)/11 + (c^3*C*e^3*x^12)/12
+ (d^2*(B*d + 3*A*e)*(a + c*x^2)^4)/(8*c)
```

Rule 1596

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{d^2(Bd+3Ae)\left(a+cx^2\right)^4}{8c} \\
&\quad + \int \left(a+cx^2\right)^3 \left(-\left(\left(Bd^3+3Ad^2e\right)x\right)+(d+ex)^3\left(A+Bx+Cx^2\right)\right)dx \\
&= \frac{d^2(Bd+3Ae)\left(a+cx^2\right)^4}{8c} \\
&\quad + \int \Big(a^3Ad^3+a^2d\left(ad(Cd+3Be)+3A\left(cd^2+ae^2\right)\right)x^2+a^3e\left(3Cd^2+e(3Bd+Ae)\right)x^3 \\
&\qquad + a\left(3Acd\left(cd^2+3ae^2\right)+a\left(ae^2(3Cd+Be)+3cd^2(Cd+3Be)\right)\right)x^4 \\
&\qquad + a^2e\left(9cCd^2+aCe^2+3ce(3Bd+Ae)\right)x^5 \\
&\qquad + c\left(Acd\left(cd^2+9ae^2\right)+3a\left(ae^2(3Cd+Be)+cd^2(Cd+3Be)\right)\right)x^6 \\
&\qquad + 3ace\left(3cCd^2+aCe^2+ce(3Bd+Ae)\right)x^7 \\
&\qquad + c^2\left(cCd^3+3cde(Bd+Ae)+3ae^2(3Cd+Be)\right)x^8 \\
&\qquad + c^2e\left(3cCd^2+3aCe^2+ce(3Bd+Ae)\right)x^9+c^3e^2(3Cd+Be)x^{10}+c^3Ce^3x^{11}\Big)\,dx \\
&= a^3Ad^3x+\frac{1}{3}a^2d\left(ad(Cd+3Be)+3A\left(cd^2+ae^2\right)\right)x^3+\frac{1}{4}a^3e\left(3Cd^2+e(3Bd+Ae)\right)x^4 \\
&\quad + \frac{1}{5}a\left(3Acd\left(cd^2+3ae^2\right)+a\left(ae^2(3Cd+Be)+3cd^2(Cd+3Be)\right)\right)x^5 \\
&\quad + \frac{1}{6}a^2e\left(9cCd^2+aCe^2+3ce(3Bd+Ae)\right)x^6 \\
&\quad + \frac{1}{7}c\left(Acd\left(cd^2+9ae^2\right)+3a\left(ae^2(3Cd+Be)+cd^2(Cd+3Be)\right)\right)x^7 \\
&\quad + \frac{3}{8}ace\left(3cCd^2+aCe^2+ce(3Bd+Ae)\right)x^8 \\
&\quad + \frac{1}{9}c^2\left(cCd^3+3cde(Bd+Ae)+3ae^2(3Cd+Be)\right)x^9 \\
&\quad + \frac{1}{10}c^2e\left(3cCd^2+3aCe^2+ce(3Bd+Ae)\right)x^{10} \\
&\quad + \frac{1}{11}c^3e^2(3Cd+Be)x^{11}+\frac{1}{12}c^3Ce^3x^{12}+\frac{d^2(Bd+3Ae)\left(a+cx^2\right)^4}{8c}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.14

$$\begin{aligned}\int (d+ex)^3\left(a+cx^2\right)^3\left(A+Bx+Cx^2\right)\,dx &= a^3Ad^3x+\frac{1}{2}a^3d^2(Bd+3Ae)x^2\\ &\quad+\frac{1}{3}a^2d\left(ad(Cd+3Be)+3A\left(cd^2+ae^2\right)\right)x^3\\ &\quad+\frac{1}{4}a^2\left(3Bcd^3+9Acd^2e+3aCd^2e+3aBde^2\right.\\ &\qquad\left.+aAe^3\right)x^4+\frac{1}{5}a\left(3Acd\left(cd^2+3ae^2\right)\right.\\ &\quad\left.+a\left(ae^2(3Cd+Be)+3cd^2(Cd+3Be)\right)\right)x^5\\ &\quad+\frac{1}{6}a\left(3Ace\left(3cd^2+ae^2\right)+aCe\left(9cd^2+ae^2\right)\right.\\ &\quad\left.+3Bcd\left(cd^2+3ae^2\right)\right)x^6+\frac{1}{7}c\left(Acd\left(cd^2+9ae^2\right)\right.\\ &\quad\left.+3a\left(ae^2(3Cd+Be)+cd^2(Cd+3Be)\right)\right)x^7\\ &\quad+\frac{1}{8}c\left(Bcd\left(cd^2+9ae^2\right)\right.\\ &\quad\left.+3e\left(Ac\left(cd^2+ae^2\right)+aC\left(3cd^2+ae^2\right)\right)\right)x^8\\ &\quad+\frac{1}{9}c^2\left(cCd^3+3cde(Bd+Ae)\right.\\ &\qquad\left.+3ae^2(3Cd+Be)\right)x^9\\ &\quad+\frac{1}{10}c^2e\left(3cCd^2+3aCe^2+ce(3Bd+Ae)\right)x^{10}\\ &\quad+\frac{1}{11}c^3e^2(3Cd+Be)x^{11}+\frac{1}{12}c^3Ce^3x^{12}\end{aligned}$$

```
[In] Integrate[(d + e*x)^3*(a + c*x^2)^3*(A + B*x + C*x^2),x]
```

```
[Out] a^3*A*d^3*x + (a^3*d^2*(B*d + 3*A*e)*x^2)/2 + (a^2*d*(a*d*(C*d + 3*B*e) + 3
*A*(c*d^2 + a*e^2))*x^3)/3 + (a^2*(3*B*c*d^3 + 9*A*c*d^2*e + 3*a*C*d^2*e +
3*a*B*d*e^2 + a*A*e^3)*x^4)/4 + (a*(3*A*c*d*(c*d^2 + 3*a*e^2) + a*(a*e^2*(3
*C*d + B*e) + 3*c*d^2*(C*d + 3*B*e)))*x^5)/5 + (a*(3*A*c*e*(3*c*d^2 + a*e^2
) + a*C*e*(9*c*d^2 + a*e^2) + 3*B*c*d*(c*d^2 + 3*a*e^2))*x^6)/6 + (c*(A*c*d
*(c*d^2 + 9*a*e^2) + 3*a*(a*e^2*(3*C*d + B*e) + c*d^2*(C*d + 3*B*e)))*x^7)/
7 + (c*(B*c*d*(c*d^2 + 9*a*e^2) + 3*e*(A*c*(c*d^2 + a*e^2) + a*C*(3*c*d^2 +
 a*e^2)))*x^8)/8 + (c^2*(c*C*d^3 + 3*c*d*e*(B*d + A*e) + 3*a*e^2*(3*C*d + B
*e))*x^9)/9 + (c^2*e*(3*c*C*d^2 + 3*a*C*e^2 + c*e*(3*B*d + A*e))*x^10)/10 +
 (c^3*e^2*(3*C*d + B*e)*x^11)/11 + (c^3*C*e^3*x^12)/12
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 531, normalized size of antiderivative = 1.31

method	result
norman	$\frac{c^3 C e^3 x^{12}}{12}+\left(\frac{1}{11} B c^3 e^3+\frac{3}{11} c^3 d e^2 C\right) x^{11}+\left(\frac{1}{10} c^3 e^3 A+\frac{3}{10} c^3 d e^2 B+\frac{3}{10} C a c^2 e^3+\frac{3}{10} C c^3 d^2 e\right) x^{10}$
default	$\frac{c^3 C e^3 x^{12}}{12}+\frac{(B c^3 e^3+3 c^3 d e^2 C) x^{11}}{11}+\frac{((3 a c^2 e^3+3 c^3 d^2 e) C+3 c^3 d e^2 B+c^3 e^3 A) x^{10}}{10}+\frac{((9 a c^2 d e^2+c^3 d^3) C+(3 a c^2 e^3+3 c^3}{9}$
gosper	$\frac{1}{11} B c^3 e^3 x^{11}+\frac{3}{8} x^8 C a^2 c e^3+\frac{3}{7} x^7 B e^3 c a^2+\frac{3}{7} x^7 C a c^2 d^3+\frac{3}{8} x^8 A a c^2 e^3+\frac{3}{8} x^8 A c^3 d^2 e+\frac{1}{3} x^9 B c$
risch	$\frac{1}{11} B c^3 e^3 x^{11}+\frac{3}{8} x^8 C a^2 c e^3+\frac{3}{7} x^7 B e^3 c a^2+\frac{3}{7} x^7 C a c^2 d^3+\frac{3}{8} x^8 A a c^2 e^3+\frac{3}{8} x^8 A c^3 d^2 e+\frac{1}{3} x^9 B c$
parallelrisch	$\frac{1}{11} B c^3 e^3 x^{11}+\frac{3}{8} x^8 C a^2 c e^3+\frac{3}{7} x^7 B e^3 c a^2+\frac{3}{7} x^7 C a c^2 d^3+\frac{3}{8} x^8 A a c^2 e^3+\frac{3}{8} x^8 A c^3 d^2 e+\frac{1}{3} x^9 B c$

```
[In] int((e*x+d)^3*(c*x^2+a)^3*(C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*c^3*C*e^3*x^12+(1/11*B*c^3*e^3+3/11*c^3*d*e^2*C)*x^11+(1/10*c^3*e^3*A+
3/10*c^3*d*e^2*B+3/10*C*a*c^2*e^3+3/10*C*c^3*d^2*e)*x^10+(1/3*A*c^3*d*e^2+1
/3*B*e^3*a*c^2+1/3*B*c^3*d^2*e+C*a*c^2*d*e^2+1/9*C*c^3*d^3)*x^9+(3/8*A*a*c^
2*e^3+3/8*A*c^3*d^2*e+9/8*B*a*c^2*d*e^2+1/8*B*c^3*d^3+3/8*C*a^2*c*e^3+9/8*C
*a*c^2*d^2*e)*x^8+(9/7*A*a*c^2*d*e^2+1/7*A*c^3*d^3+3/7*B*e^3*c*a^2+9/7*B*a*
c^2*d^2*e+9/7*C*a^2*c*d*e^2+3/7*C*a*c^2*d^3)*x^7+(1/2*A*a^2*c*e^3+3/2*A*a*c
^2*d^2*e+3/2*B*a^2*c*d*e^2+1/2*B*a*c^2*d^3+1/6*a^3*C*e^3+3/2*C*a^2*c*d^2*e)
*x^6+(9/5*A*a^2*c*d*e^2+3/5*A*d^3*a*c^2+1/5*B*e^3*a^3+9/5*B*a^2*c*d^2*e+3/5
*C*a^3*d*e^2+3/5*C*a^2*c*d^3)*x^5+(1/4*A*a^3*e^3+9/4*A*a^2*c*d^2*e+3/4*B*a^
3*d*e^2+3/4*B*a^2*c*d^3+3/4*a^3*d^2*e*C)*x^4+(A*a^3*d*e^2+A*d^3*c*a^2+a^3*d
^2*e*B+1/3*a^3*d^3*C)*x^3+(3/2*A*a^3*d^2*e+1/2*B*a^3*d^3)*x^2+A*d^3*a^3*x
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 512, normalized size of antiderivative = 1.27

$$
\begin{aligned}
&\int (d+ex)^3\left(a+cx^2\right)^3\left(A+Bx+Cx^2\right)\,dx\\
&=\frac{1}{12}Cc^3e^3x^{12}+\frac{1}{11}\left(3\,Cc^3de^2+Bc^3e^3\right)x^{11}\\
&\quad+\frac{1}{10}\left(3\,Cc^3d^2e+3\,Bc^3de^2+\left(3\,Cac^2+Ac^3\right)e^3\right)x^{10}\\
&\quad+\frac{1}{9}\left(Cc^3d^3+3\,Bc^3d^2e+3\,Bac^2e^3+3\left(3\,Cac^2+Ac^3\right)de^2\right)x^9\\
&\quad+\frac{1}{8}\left(Bc^3d^3+9\,Bac^2de^2+3\left(3\,Cac^2+Ac^3\right)d^2e+3\left(Ca^2c+Aac^2\right)e^3\right)x^8+Aa^3d^3x\\
&\quad+\frac{1}{7}\left(9\,Bac^2d^2e+3\,Ba^2ce^3+\left(3\,Cac^2+Ac^3\right)d^3+9\left(Ca^2c+Aac^2\right)de^2\right)x^7\\
&\quad+\frac{1}{6}\left(3\,Bac^2d^3+9\,Ba^2cde^2+9\left(Ca^2c+Aac^2\right)d^2e+\left(Ca^3+3\,Aa^2c\right)e^3\right)x^6\\
&\quad+\frac{1}{5}\left(9\,Ba^2cd^2e+Ba^3e^3+3\left(Ca^2c+Aac^2\right)d^3+3\left(Ca^3+3\,Aa^2c\right)de^2\right)x^5\\
&\quad+\frac{1}{4}\left(3\,Ba^2cd^3+3\,Ba^3de^2+Aa^3e^3+3\left(Ca^3+3\,Aa^2c\right)d^2e\right)x^4\\
&\quad+\frac{1}{3}\left(3\,Ba^3d^2e+3\,Aa^3de^2+\left(Ca^3+3\,Aa^2c\right)d^3\right)x^3+\frac{1}{2}\left(Ba^3d^3+3\,Aa^3d^2e\right)x^2
\end{aligned}
$$

```
[In] integrate((e*x+d)^3*(c*x^2+a)^3*(C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/12*C*c^3*e^3*x^12 + 1/11*(3*C*c^3*d*e^2 + B*c^3*e^3)*x^11 + 1/10*(3*C*c^3
*d^2*e + 3*B*c^3*d*e^2 + (3*C*a*c^2 + A*c^3)*e^3)*x^10 + 1/9*(C*c^3*d^3 + 3
*B*c^3*d^2*e + 3*B*a*c^2*e^3 + 3*(3*C*a*c^2 + A*c^3)*d*e^2)*x^9 + 1/8*(B*c^
3*d^3 + 9*B*a*c^2*d*e^2 + 3*(3*C*a*c^2 + A*c^3)*d^2*e + 3*(C*a^2*c + A*a*c^
2)*e^3)*x^8 + A*a^3*d^3*x + 1/7*(9*B*a*c^2*d^2*e + 3*B*a^2*c*e^3 + (3*C*a*c
^2 + A*c^3)*d^3 + 9*(C*a^2*c + A*a*c^2)*d*e^2)*x^7 + 1/6*(3*B*a*c^2*d^3 + 9
*B*a^2*c*d*e^2 + 9*(C*a^2*c + A*a*c^2)*d^2*e + (C*a^3 + 3*A*a^2*c)*e^3)*x^6
 + 1/5*(9*B*a^2*c*d^2*e + B*a^3*e^3 + 3*(C*a^2*c + A*a*c^2)*d^3 + 3*(C*a^3
+ 3*A*a^2*c)*d*e^2)*x^5 + 1/4*(3*B*a^2*c*d^3 + 3*B*a^3*d*e^2 + A*a^3*e^3 +
3*(C*a^3 + 3*A*a^2*c)*d^2*e)*x^4 + 1/3*(3*B*a^3*d^2*e + 3*A*a^3*d*e^2 + (C*
a^3 + 3*A*a^2*c)*d^3)*x^3 + 1/2*(B*a^3*d^3 + 3*A*a^3*d^2*e)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 646, normalized size of antiderivative = 1.60

$$
\begin{aligned}
&\int (d+ex)^3\left(a+cx^2\right)^3\left(A+Bx+Cx^2\right)\,dx\\
&= Aa^3d^3x+\frac{Cc^3e^3x^{12}}{12}+x^{11}\left(\frac{Bc^3e^3}{11}+\frac{3Cc^3de^2}{11}\right)\\
&\quad+x^{10}\left(\frac{Ac^3e^3}{10}+\frac{3Bc^3de^2}{10}+\frac{3Cac^2e^3}{10}+\frac{3Cc^3d^2e}{10}\right)\\
&\quad+x^9\left(\frac{Ac^3de^2}{3}+\frac{Bac^2e^3}{3}+\frac{Bc^3d^2e}{3}+Cac^2de^2+\frac{Cc^3d^3}{9}\right)+x^8\\
&\quad\cdot\left(\frac{3Aac^2e^3}{8}+\frac{3Ac^3d^2e}{8}+\frac{9Bac^2de^2}{8}+\frac{Bc^3d^3}{8}+\frac{3Ca^2ce^3}{8}+\frac{9Cac^2d^2e}{8}\right)+x^7\\
&\quad\cdot\left(\frac{9Aac^2de^2}{7}+\frac{Ac^3d^3}{7}+\frac{3Ba^2ce^3}{7}+\frac{9Bac^2d^2e}{7}+\frac{9Ca^2cde^2}{7}+\frac{3Cac^2d^3}{7}\right)\\
&\quad+x^6\left(\frac{Aa^2ce^3}{2}+\frac{3Aac^2d^2e}{2}+\frac{3Ba^2cde^2}{2}+\frac{Bac^2d^3}{2}+\frac{Ca^3e^3}{6}+\frac{3Ca^2cd^2e}{2}\right)\\
&\quad+x^5\cdot\left(\frac{9Aa^2cde^2}{5}+\frac{3Aac^2d^3}{5}+\frac{Ba^3e^3}{5}+\frac{9Ba^2cd^2e}{5}+\frac{3Ca^3de^2}{5}+\frac{3Ca^2cd^3}{5}\right)\\
&\quad+x^4\left(\frac{Aa^3e^3}{4}+\frac{9Aa^2cd^2e}{4}+\frac{3Ba^3de^2}{4}+\frac{3Ba^2cd^3}{4}+\frac{3Ca^3d^2e}{4}\right)\\
&\quad+x^3\left(Aa^3de^2+Aa^2cd^3+Ba^3d^2e+\frac{Ca^3d^3}{3}\right)+x^2\cdot\left(\frac{3Aa^3d^2e}{2}+\frac{Ba^3d^3}{2}\right)
\end{aligned}
$$

```
[In] integrate((e*x+d)**3*(c*x**2+a)**3*(C*x**2+B*x+A),x)
```

```
[Out] A*a**3*d**3*x + C*c**3*e**3*x**12/12 + x**11*(B*c**3*e**3/11 + 3*C*c**3*d*e
**2/11) + x**10*(A*c**3*e**3/10 + 3*B*c**3*d*e**2/10 + 3*C*a*c**2*e**3/10 +
 3*C*c**3*d**2*e/10) + x**9*(A*c**3*d*e**2/3 + B*a*c**2*e**3/3 + B*c**3*d**
2*e/3 + C*a*c**2*d*e**2 + C*c**3*d**3/9) + x**8*(3*A*a*c**2*e**3/8 + 3*A*c*
*3*d**2*e/8 + 9*B*a*c**2*d*e**2/8 + B*c**3*d**3/8 + 3*C*a**2*c*e**3/8 + 9*C
*a*c**2*d**2*e/8) + x**7*(9*A*a*c**2*d*e**2/7 + A*c**3*d**3/7 + 3*B*a**2*c*
e**3/7 + 9*B*a*c**2*d**2*e/7 + 9*C*a**2*c*d*e**2/7 + 3*C*a*c**2*d**3/7) + x
**6*(A*a**2*c*e**3/2 + 3*A*a*c**2*d**2*e/2 + 3*B*a**2*c*d*e**2/2 + B*a*c**2
*d**3/2 + C*a**3*e**3/6 + 3*C*a**2*c*d**2*e/2) + x**5*(9*A*a**2*c*d*e**2/5
+ 3*A*a*c**2*d**3/5 + B*a**3*e**3/5 + 9*B*a**2*c*d**2*e/5 + 3*C*a**3*d*e**2
/5 + 3*C*a**2*c*d**3/5) + x**4*(A*a**3*e**3/4 + 9*A*a**2*c*d**2*e/4 + 3*B*a
**3*d*e**2/4 + 3*B*a**2*c*d**3/4 + 3*C*a**3*d**2*e/4) + x**3*(A*a**3*d*e**2
 + A*a**2*c*d**3 + B*a**3*d**2*e + C*a**3*d**3/3) + x**2*(3*A*a**3*d**2*e/2
 + B*a**3*d**3/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 512, normalized size of antiderivative = 1.27

$$\begin{aligned}\int (d+ex)^3\left(a+cx^2\right)^3\left(A+Bx+Cx^2\right)\,dx &= \frac{1}{12}Cc^3e^3x^{12}+\frac{1}{11}\left(3\,Cc^3de^2+Bc^3e^3\right)x^{11}\\ &+\frac{1}{10}\left(3\,Cc^3d^2e+3\,Bc^3de^2+\left(3\,Cac^2+Ac^3\right)e^3\right)x^{10}\\ &+\frac{1}{9}\left(Cc^3d^3+3\,Bc^3d^2e+3\,Bac^2e^3+3\left(3\,Cac^2+Ac^3\right)de^2\right)x^9\\ &+\frac{1}{8}\left(Bc^3d^3+9\,Bac^2de^2+3\left(3\,Cac^2+Ac^3\right)d^2e+3\left(Ca^2c+Aac^2\right)e^3\right)x^8+Aa^3d^3x\\ &+\frac{1}{7}\left(9\,Bac^2d^2e+3\,Ba^2ce^3+\left(3\,Cac^2+Ac^3\right)d^3+9\left(Ca^2c+Aac^2\right)de^2\right)x^7\\ &+\frac{1}{6}\left(3\,Bac^2d^3+9\,Ba^2cde^2+9\left(Ca^2c+Aac^2\right)d^2e+\left(Ca^3+3\,Aa^2c\right)e^3\right)x^6\\ &+\frac{1}{5}\left(9\,Ba^2cd^2e+Ba^3e^3+3\left(Ca^2c+Aac^2\right)d^3+3\left(Ca^3+3\,Aa^2c\right)de^2\right)x^5\\ &+\frac{1}{4}\left(3\,Ba^2cd^3+3\,Ba^3de^2+Aa^3e^3+3\left(Ca^3+3\,Aa^2c\right)d^2e\right)x^4\\ &+\frac{1}{3}\left(3\,Ba^3d^2e+3\,Aa^3de^2+\left(Ca^3+3\,Aa^2c\right)d^3\right)x^3+\frac{1}{2}\left(Ba^3d^3+3\,Aa^3d^2e\right)x^2\end{aligned}$$

```
[In] integrate((e*x+d)^3*(c*x^2+a)^3*(C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/12*C*c^3*e^3*x^12 + 1/11*(3*C*c^3*d*e^2 + B*c^3*e^3)*x^11 + 1/10*(3*C*c^3
*d^2*e + 3*B*c^3*d*e^2 + (3*C*a*c^2 + A*c^3)*e^3)*x^10 + 1/9*(C*c^3*d^3 + 3
*B*c^3*d^2*e + 3*B*a*c^2*e^3 + 3*(3*C*a*c^2 + A*c^3)*d*e^2)*x^9 + 1/8*(B*c^
3*d^3 + 9*B*a*c^2*d*e^2 + 3*(3*C*a*c^2 + A*c^3)*d^2*e + 3*(C*a^2*c + A*a*c^
2)*e^3)*x^8 + A*a^3*d^3*x + 1/7*(9*B*a*c^2*d^2*e + 3*B*a^2*c*e^3 + (3*C*a*c
^2 + A*c^3)*d^3 + 9*(C*a^2*c + A*a*c^2)*d*e^2)*x^7 + 1/6*(3*B*a*c^2*d^3 + 9
*B*a^2*c*d*e^2 + 9*(C*a^2*c + A*a*c^2)*d^2*e + (C*a^3 + 3*A*a^2*c)*e^3)*x^6
 + 1/5*(9*B*a^2*c*d^2*e + B*a^3*e^3 + 3*(C*a^2*c + A*a*c^2)*d^3 + 3*(C*a^3
+ 3*A*a^2*c)*d*e^2)*x^5 + 1/4*(3*B*a^2*c*d^3 + 3*B*a^3*d*e^2 + A*a^3*e^3 +
3*(C*a^3 + 3*A*a^2*c)*d^2*e)*x^4 + 1/3*(3*B*a^3*d^2*e + 3*A*a^3*d*e^2 + (C*
a^3 + 3*A*a^2*c)*d^3)*x^3 + 1/2*(B*a^3*d^3 + 3*A*a^3*d^2*e)*x^2
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 618, normalized size of antiderivative = 1.53

$$
\begin{aligned}
\int (d+ex)^3 \left(a+cx^2\right)^3 \left(A+Bx+Cx^2\right)\,dx = {} & \frac{1}{12}Cc^3e^3x^{12}+\frac{3}{11}Cc^3de^2x^{11}+\frac{1}{11}Bc^3e^3x^{11} \\
& +\frac{3}{10}Cc^3d^2ex^{10}+\frac{3}{10}Bc^3de^2x^{10} \\
& +\frac{3}{10}Cac^2e^3x^{10}+\frac{1}{10}Ac^3e^3x^{10} \\
& +\frac{1}{9}Cc^3d^3x^9+\frac{1}{3}Bc^3d^2ex^9+Cac^2de^2x^9 \\
& +\frac{1}{3}Ac^3de^2x^9+\frac{1}{3}Bac^2e^3x^9+\frac{1}{8}Bc^3d^3x^8 \\
& +\frac{9}{8}Cac^2d^2ex^8+\frac{3}{8}Ac^3d^2ex^8+\frac{9}{8}Bac^2de^2x^8 \\
& +\frac{3}{8}Ca^2ce^3x^8+\frac{3}{8}Aac^2e^3x^8+\frac{3}{7}Cac^2d^3x^7 \\
& +\frac{1}{7}Ac^3d^3x^7+\frac{9}{7}Bac^2d^2ex^7+\frac{9}{7}Ca^2cde^2x^7 \\
& +\frac{9}{7}Aac^2de^2x^7+\frac{3}{7}Ba^2ce^3x^7+\frac{1}{2}Bac^2d^3x^6 \\
& +\frac{3}{2}Ca^2cd^2ex^6+\frac{3}{2}Aac^2d^2ex^6 \\
& +\frac{3}{2}Ba^2cde^2x^6+\frac{1}{6}Ca^3e^3x^6+\frac{1}{2}Aa^2ce^3x^6 \\
& +\frac{3}{5}Ca^2cd^3x^5+\frac{3}{5}Aac^2d^3x^5+\frac{9}{5}Ba^2cd^2ex^5 \\
& +\frac{3}{5}Ca^3de^2x^5+\frac{9}{5}Aa^2cde^2x^5+\frac{1}{5}Ba^3e^3x^5 \\
& +\frac{3}{4}Ba^2cd^3x^4+\frac{3}{4}Ca^3d^2ex^4+\frac{9}{4}Aa^2cd^2ex^4 \\
& +\frac{3}{4}Ba^3de^2x^4+\frac{1}{4}Aa^3e^3x^4+\frac{1}{3}Ca^3d^3x^3 \\
& +Aa^2cd^3x^3+Ba^3d^2ex^3+Aa^3de^2x^3 \\
& +\frac{1}{2}Ba^3d^3x^2+\frac{3}{2}Aa^3d^2ex^2+Aa^3d^3x
\end{aligned}
$$

```
[In] integrate((e*x+d)^3*(c*x^2+a)^3*(C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/12*C*c^3*e^3*x^12 + 3/11*C*c^3*d*e^2*x^11 + 1/11*B*c^3*e^3*x^11 + 3/10*C*
c^3*d^2*e*x^10 + 3/10*B*c^3*d*e^2*x^10 + 3/10*C*a*c^2*e^3*x^10 + 1/10*A*c^3
*e^3*x^10 + 1/9*C*c^3*d^3*x^9 + 1/3*B*c^3*d^2*e*x^9 + C*a*c^2*d*e^2*x^9 + 1
/3*A*c^3*d*e^2*x^9 + 1/3*B*a*c^2*e^3*x^9 + 1/8*B*c^3*d^3*x^8 + 9/8*C*a*c^2*
d^2*e*x^8 + 3/8*A*c^3*d^2*e*x^8 + 9/8*B*a*c^2*d*e^2*x^8 + 3/8*C*a^2*c*e^3*x
^8 + 3/8*A*a*c^2*e^3*x^8 + 3/7*C*a*c^2*d^3*x^7 + 1/7*A*c^3*d^3*x^7 + 9/7*B*
```

```
a*c^2*d^2*e*x^7 + 9/7*C*a^2*c*d*e^2*x^7 + 9/7*A*a*c^2*d*e^2*x^7 + 3/7*B*a^2
*c*e^3*x^7 + 1/2*B*a*c^2*d^3*x^6 + 3/2*C*a^2*c*d^2*e*x^6 + 3/2*A*a*c^2*d^2*
e*x^6 + 3/2*B*a^2*c*d*e^2*x^6 + 1/6*C*a^3*e^3*x^6 + 1/2*A*a^2*c*e^3*x^6 + 3
/5*C*a^2*c*d^3*x^5 + 3/5*A*a*c^2*d^3*x^5 + 9/5*B*a^2*c*d^2*e*x^5 + 3/5*C*a^
3*d*e^2*x^5 + 9/5*A*a^2*c*d*e^2*x^5 + 1/5*B*a^3*e^3*x^5 + 3/4*B*a^2*c*d^3*x
^4 + 3/4*C*a^3*d^2*e*x^4 + 9/4*A*a^2*c*d^2*e*x^4 + 3/4*B*a^3*d*e^2*x^4 + 1/
4*A*a^3*e^3*x^4 + 1/3*C*a^3*d^3*x^3 + A*a^2*c*d^3*x^3 + B*a^3*d^2*e*x^3 + A
*a^3*d*e^2*x^3 + 1/2*B*a^3*d^3*x^2 + 3/2*A*a^3*d^2*e*x^2 + A*a^3*d^3*x
```

Mupad [B] (verification not implemented)

Time = 12.55 (sec) , antiderivative size = 490, normalized size of antiderivative = 1.21

$$
\begin{aligned}
\int (d+ex)^3\left(a+cx^2\right)^3\left(A+Bx+Cx^2\right)\,dx
&= x^5\left(\frac{3\,C\,a^3\,d\,e^2}{5}+\frac{B\,a^3\,e^3}{5}+\frac{3\,C\,a^2\,c\,d^3}{5}+\frac{9\,B\,a^2\,c\,d^2\,e}{5}+\frac{9\,A\,a^2\,c\,d\,e^2}{5}+\frac{3\,A\,a\,c^2\,d^3}{5}\right)\\
&\quad+x^8\left(\frac{3\,C\,a^2\,c\,e^3}{8}+\frac{9\,C\,a\,c^2\,d^2\,e}{8}+\frac{9\,B\,a\,c^2\,d\,e^2}{8}+\frac{3\,A\,a\,c^2\,e^3}{8}+\frac{B\,c^3\,d^3}{8}+\frac{3\,A\,c^3\,d^2\,e}{8}\right)\\
&\quad+x^6\left(\frac{C\,a^3\,e^3}{6}+\frac{3\,C\,a^2\,c\,d^2\,e}{2}+\frac{3\,B\,a^2\,c\,d\,e^2}{2}+\frac{A\,a^2\,c\,e^3}{2}+\frac{B\,a\,c^2\,d^3}{2}+\frac{3\,A\,a\,c^2\,d^2\,e}{2}\right)\\
&\quad+x^7\left(\frac{9\,C\,a^2\,c\,d\,e^2}{7}+\frac{3\,B\,a^2\,c\,e^3}{7}+\frac{3\,C\,a\,c^2\,d^3}{7}+\frac{9\,B\,a\,c^2\,d^2\,e}{7}+\frac{9\,A\,a\,c^2\,d\,e^2}{7}+\frac{A\,c^3\,d^3}{7}\right)\\
&\quad+\frac{a^2\,x^4\,\left(A\,a\,e^3+3\,B\,c\,d^3+3\,B\,a\,d\,e^2+9\,A\,c\,d^2\,e+3\,C\,a\,d^2\,e\right)}{4}\\
&\quad+\frac{c^2\,x^9\,\left(3\,B\,a\,e^3+C\,c\,d^3+3\,A\,c\,d\,e^2+9\,C\,a\,d\,e^2+3\,B\,c\,d^2\,e\right)}{9}\\
&\quad+\frac{C\,c^3\,e^3\,x^{12}}{12}+\frac{a^3\,d^2\,x^2\,\left(3\,A\,e+B\,d\right)}{2}+\frac{c^3\,e^2\,x^{11}\,\left(B\,e+3\,C\,d\right)}{11}\\
&\quad+A\,a^3\,d^3\,x+\frac{a^2\,d\,x^3\,\left(3\,A\,a\,e^2+3\,A\,c\,d^2+C\,a\,d^2+3\,B\,a\,d\,e\right)}{3}\\
&\quad+\frac{c^2\,e\,x^{10}\,\left(A\,c\,e^2+3\,C\,a\,e^2+3\,C\,c\,d^2+3\,B\,c\,d\,e\right)}{10}
\end{aligned}
$$

```
[In] int((a + c*x^2)^3*(d + e*x)^3*(A + B*x + C*x^2),x)
```

```
[Out] x^5*((B*a^3*e^3)/5 + (3*A*a*c^2*d^3)/5 + (3*C*a^2*c*d^3)/5 + (3*C*a^3*d*e^2
)/5 + (9*A*a^2*c*d*e^2)/5 + (9*B*a^2*c*d^2*e)/5) + x^8*((B*c^3*d^3)/8 + (3*
A*a*c^2*e^3)/8 + (3*C*a^2*c*e^3)/8 + (3*A*c^3*d^2*e)/8 + (9*B*a*c^2*d*e^2)/
8 + (9*C*a*c^2*d^2*e)/8) + x^6*((C*a^3*e^3)/6 + (A*a^2*c*e^3)/2 + (B*a*c^2*
d^3)/2 + (3*A*a*c^2*d^2*e)/2 + (3*B*a^2*c*d*e^2)/2 + (3*C*a^2*c*d^2*e)/2) +
 x^7*((A*c^3*d^3)/7 + (3*B*a^2*c*e^3)/7 + (3*C*a*c^2*d^3)/7 + (9*A*a*c^2*d*
e^2)/7 + (9*B*a*c^2*d^2*e)/7 + (9*C*a^2*c*d*e^2)/7) + (a^2*x^4*(A*a*e^3 + 3
*B*c*d^3 + 3*B*a*d*e^2 + 9*A*c*d^2*e + 3*C*a*d^2*e))/4 + (c^2*x^9*(3*B*a*e^
```

```
3 + C*c*d^3 + 3*A*c*d*e^2 + 9*C*a*d*e^2 + 3*B*c*d^2*e))/9 + (C*c^3*e^3*x^12
)/12 + (a^3*d^2*x^2*(3*A*e + B*d))/2 + (c^3*e^2*x^11*(B*e + 3*C*d))/11 + A*
a^3*d^3*x + (a^2*d*x^3*(3*A*a*e^2 + 3*A*c*d^2 + C*a*d^2 + 3*B*a*d*e))/3 + (
c^2*e*x^10*(A*c*e^2 + 3*C*a*e^2 + 3*C*c*d^2 + 3*B*c*d*e))/10
```

3.33 $\int (d+ex)^2 \left(a+cx^2\right)^3 \left(A+Bx+Cx^2\right) \, dx$

Optimal result

Integrand size = 27, antiderivative size = 289

$$\begin{aligned}\int (d+ex)^2 \left(a+cx^2\right)^3 \left(A+Bx+Cx^2\right) \, dx = {} & a^3Ad^2x+\frac{1}{3}a^2\left(ad(Cd+2Be)+A\left(3cd^2+ae^2\right)\right)x^3 \\ & +\frac{1}{4}a^3e(2Cd+Be)x^4+\frac{1}{5}a\left(3Ac\left(cd^2+ae^2\right)\right. \\ & \left.+a\left(aCe^2+3cd(Cd+2Be)\right)\right)x^5 \\ & +\frac{1}{2}a^2ce(2Cd+Be)x^6+\frac{1}{7}c\left(Ac\left(cd^2+3ae^2\right)\right. \\ & \left.+3a\left(aCe^2+cd(Cd+2Be)\right)\right)x^7 \\ & +\frac{3}{8}ac^2e(2Cd+Be)x^8 \\ & +\frac{1}{9}c^2\left(3aCe^2+c\left(Cd^2+e(2Bd+Ae)\right)\right)x^9 \\ & +\frac{1}{10}c^3e(2Cd+Be)x^{10}+\frac{1}{11}c^3Ce^2x^{11} \\ & +\frac{d(Bd+2Ae)\left(a+cx^2\right)^4}{8c}\end{aligned}$$

```
[Out] a^3*A*d^2*x+1/3*a^2*(a*d*(2*B*e+C*d)+A*(a*e^2+3*c*d^2))*x^3+1/4*a^3*e*(B*e+
2*C*d)*x^4+1/5*a*(3*A*c*(a*e^2+c*d^2)+a*(a*C*e^2+3*c*d*(2*B*e+C*d)))*x^5+1/
2*a^2*c*e*(B*e+2*C*d)*x^6+1/7*c*(A*c*(3*a*e^2+c*d^2)+3*a*(a*C*e^2+c*d*(2*B*
e+C*d)))*x^7+3/8*a*c^2*e*(B*e+2*C*d)*x^8+1/9*c^2*(3*a*C*e^2+c*(C*d^2+e*(A*e
+2*B*d)))*x^9+1/10*c^3*e*(B*e+2*C*d)*x^10+1/11*c^3*C*e^2*x^11+1/8*d*(2*A*e+
B*d)*(c*x^2+a)^4/c
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1596, 1824}

$$\begin{aligned}
&\int (d+ex)^2 \left(a+cx^2\right)^3 \left(A+Bx+Cx^2\right) dx \\
&= a^3Ad^2x+\frac{1}{4}a^3ex^4(Be+2Cd)+\frac{1}{3}a^2x^3\left(A\left(ae^2+3cd^2\right)+ad(2Be+Cd)\right) \\
&\quad +\frac{1}{2}a^2cex^6(Be+2Cd)+\frac{1}{9}c^2x^9\left(3aCe^2+ce(Ae+2Bd)+cCd^2\right) \\
&\quad +\frac{1}{7}cx^7\left(Ac\left(3ae^2+cd^2\right)+3a\left(aCe^2+cd(2Be+Cd)\right)\right) \\
&\quad +\frac{1}{5}ax^5\left(3Ac\left(ae^2+cd^2\right)+a\left(aCe^2+3cd(2Be+Cd)\right)\right)+\frac{d\left(a+cx^2\right)^4(2Ae+Bd)}{8c} \\
&\quad +\frac{3}{8}ac^2ex^8(Be+2Cd)+\frac{1}{10}c^3ex^{10}(Be+2Cd)+\frac{1}{11}c^3Ce^2x^{11}
\end{aligned}$$

```
[In] Int[(d + e*x)^2*(a + c*x^2)^3*(A + B*x + C*x^2),x]
```

```
[Out] a^3*A*d^2*x + (a^2*(a*d*(C*d + 2*B*e) + A*(3*c*d^2 + a*e^2))*x^3)/3 + (a^3*
e*(2*C*d + B*e)*x^4)/4 + (a*(3*A*c*(c*d^2 + a*e^2) + a*(a*C*e^2 + 3*c*d*(C*
d + 2*B*e)))*x^5)/5 + (a^2*c*e*(2*C*d + B*e)*x^6)/2 + (c*(A*c*(c*d^2 + 3*a*
e^2) + 3*a*(a*C*e^2 + c*d*(C*d + 2*B*e)))*x^7)/7 + (3*a*c^2*e*(2*C*d + B*e)
*x^8)/8 + (c^2*(c*C*d^2 + 3*a*C*e^2 + c*e*(2*B*d + A*e))*x^9)/9 + (c^3*e*(2
*C*d + B*e)*x^10)/10 + (c^3*C*e^2*x^11)/11 + (d*(B*d + 2*A*e)*(a + c*x^2)^4
)/(8*c)
```

Rule 1596

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{d(Bd+2Ae)\left(a+cx^2\right)^4}{8c} \\
&\quad + \int \left(a+cx^2\right)^3 \left(-\left(\left(Bd^2+2Ade\right)x\right)+(d+ex)^2\left(A+Bx+Cx^2\right)\right)dx \\
&= \frac{d(Bd+2Ae)\left(a+cx^2\right)^4}{8c} + \int \left(a^3Ad^2+a^2\left(ad(Cd+2Be)+A\left(3cd^2+ae^2\right)\right)x^2\right. \\
&\quad + a^3e(2Cd+Be)x^3+a\left(3Ac\left(cd^2+ae^2\right)+a\left(aCe^2+3cd(Cd+2Be)\right)\right)x^4 \\
&\quad + 3a^2ce(2Cd+Be)x^5+c\left(Ac\left(cd^2+3ae^2\right)+3a\left(aCe^2+cd(Cd+2Be)\right)\right)x^6 \\
&\quad + 3ac^2e(2Cd+Be)x^7+c^2\left(cCd^2+3aCe^2+ce(2Bd+Ae)\right)x^8 \\
&\quad \left. + c^3e(2Cd+Be)x^9+c^3Ce^2x^{10}\right)dx \\
&= a^3Ad^2x+\frac{1}{3}a^2\left(ad(Cd+2Be)+A\left(3cd^2+ae^2\right)\right)x^3+\frac{1}{4}a^3e(2Cd+Be)x^4 \\
&\quad + \frac{1}{5}a\left(3Ac\left(cd^2+ae^2\right)+a\left(aCe^2+3cd(Cd+2Be)\right)\right)x^5 \\
&\quad + \frac{1}{2}a^2ce(2Cd+Be)x^6+\frac{1}{7}c\left(Ac\left(cd^2+3ae^2\right)+3a\left(aCe^2+cd(Cd+2Be)\right)\right)x^7 \\
&\quad + \frac{3}{8}ac^2e(2Cd+Be)x^8+\frac{1}{9}c^2\left(cCd^2+3aCe^2+ce(2Bd+Ae)\right)x^9 \\
&\quad + \frac{1}{10}c^3e(2Cd+Be)x^{10}+\frac{1}{11}c^3Ce^2x^{11}+\frac{d(Bd+2Ae)\left(a+cx^2\right)^4}{8c}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.14

$$\begin{aligned}
&\int (d+ex)^2\left(a+cx^2\right)^3\left(A+Bx+Cx^2\right)dx \\
&= a^3Ad^2x+\frac{1}{2}a^3d(Bd+2Ae)x^2+\frac{1}{3}a^2\left(ad(Cd+2Be)+A\left(3cd^2+ae^2\right)\right)x^3 \\
&\quad + \frac{1}{4}a^2\left(3Bcd^2+6Acde+2aCde+aBe^2\right)x^4 \\
&\quad + \frac{1}{5}a\left(3Ac\left(cd^2+ae^2\right)+a\left(aCe^2+3cd(Cd+2Be)\right)\right)x^5 \\
&\quad + \frac{1}{2}ac\left(2(Ac+aC)de+B\left(cd^2+ae^2\right)\right)x^6 \\
&\quad + \frac{1}{7}c\left(Ac\left(cd^2+3ae^2\right)+3a\left(aCe^2+cd(Cd+2Be)\right)\right)x^7 \\
&\quad + \frac{1}{8}c^2\left(Bcd^2+2Acde+6aCde+3aBe^2\right)x^8 \\
&\quad + \frac{1}{9}c^2\left(cCd^2+3aCe^2+ce(2Bd+Ae)\right)x^9+\frac{1}{10}c^3e(2Cd+Be)x^{10}+\frac{1}{11}c^3Ce^2x^{11}
\end{aligned}$$

```
[In] Integrate[(d + e*x)^2*(a + c*x^2)^3*(A + B*x + C*x^2),x]
```

```
[Out] a^3*A*d^2*x + (a^3*d*(B*d + 2*A*e)*x^2)/2 + (a^2*(a*d*(C*d + 2*B*e) + A*(3*
c*d^2 + a*e^2))*x^3)/3 + (a^2*(3*B*c*d^2 + 6*A*c*d*e + 2*a*C*d*e + a*B*e^2)
*x^4)/4 + (a*(3*A*c*(c*d^2 + a*e^2) + a*(a*C*e^2 + 3*c*d*(C*d + 2*B*e)))*x^
5)/5 + (a*c*(2*(A*c + a*C)*d*e + B*(c*d^2 + a*e^2))*x^6)/2 + (c*(A*c*(c*d^2
 + 3*a*e^2) + 3*a*(a*C*e^2 + c*d*(C*d + 2*B*e)))*x^7)/7 + (c^2*(B*c*d^2 + 2
*A*c*d*e + 6*a*C*d*e + 3*a*B*e^2)*x^8)/8 + (c^2*(c*C*d^2 + 3*a*C*e^2 + c*e*
(2*B*d + A*e))*x^9)/9 + (c^3*e*(2*C*d + B*e)*x^10)/10 + (c^3*C*e^2*x^11)/11
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.30

method	result
norman	$\frac{c^3 C e^2 x^{11}}{11}+\left(\frac{1}{10}c^3e^2B+\frac{1}{5}c^3deC\right)x^{10}+\left(\frac{1}{9}c^3e^2A+\frac{2}{9}c^3deB+\frac{1}{3}Ca\,c^2e^2+\frac{1}{9}C\,c^3d^2\right)x^9+\left(\frac{1}{4}c^3de$
default	$\frac{c^3 C e^2 x^{11}}{11}+\frac{(c^3e^2B+2c^3deC)x^{10}}{10}+\frac{((3a\,c^2e^2+c^3d^2)C+2c^3deB+c^3e^2A)x^9}{9}+\frac{(6a\,c^2deC+(3a\,c^2e^2+c^3d^2)B+2c^3deA)x^8}{8}$
gosper	$\frac{1}{10}B\,c^3e^2x^{10}+\frac{1}{3}x^3d^2a^3C+\frac{1}{2}x^2B\,a^3d^2+\frac{1}{3}x^3A\,a^3e^2+\frac{1}{4}x^4B\,e^2a^3+\frac{1}{8}x^8B\,c^3d^2+\frac{1}{7}x^7A\,c^3d^2+$
risch	$\frac{1}{10}B\,c^3e^2x^{10}+\frac{1}{3}x^3d^2a^3C+\frac{1}{2}x^2B\,a^3d^2+\frac{1}{3}x^3A\,a^3e^2+\frac{1}{4}x^4B\,e^2a^3+\frac{1}{8}x^8B\,c^3d^2+\frac{1}{7}x^7A\,c^3d^2+$
parallelrisch	$\frac{1}{10}B\,c^3e^2x^{10}+\frac{1}{3}x^3d^2a^3C+\frac{1}{2}x^2B\,a^3d^2+\frac{1}{3}x^3A\,a^3e^2+\frac{1}{4}x^4B\,e^2a^3+\frac{1}{8}x^8B\,c^3d^2+\frac{1}{7}x^7A\,c^3d^2+$

```
[In] int((e*x+d)^2*(c*x^2+a)^3*(C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/11*c^3*C*e^2*x^11+(1/10*c^3*e^2*B+1/5*c^3*d*e*C)*x^10+(1/9*c^3*e^2*A+2/9*
c^3*d*e*B+1/3*C*a*c^2*e^2+1/9*C*c^3*d^2)*x^9+(1/4*c^3*d*e*A+3/8*B*e^2*a*c^2
+1/8*B*c^3*d^2+3/4*a*c^2*d*e*C)*x^8+(3/7*A*a*c^2*e^2+1/7*A*c^3*d^2+6/7*B*a*
c^2*d*e+3/7*C*a^2*c*e^2+3/7*C*a*c^2*d^2)*x^7+(A*a*c^2*d*e+1/2*B*e^2*c*a^2+1
/2*B*a*c^2*d^2+a^2*c*d*e*C)*x^6+(3/5*A*a^2*c*e^2+3/5*A*d^2*a*c^2+6/5*a^2*c*
d*e*B+1/5*C*a^3*e^2+3/5*C*a^2*c*d^2)*x^5+(3/2*A*a^2*c*d*e+1/4*B*e^2*a^3+3/4
*B*a^2*c*d^2+1/2*d*e*a^3*C)*x^4+(1/3*A*a^3*e^2+A*d^2*c*a^2+2/3*d*e*a^3*B+1/
3*d^2*a^3*C)*x^3+(A*a^3*d*e+1/2*B*a^3*d^2)*x^2+A*d^2*a^3*x
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.27

$$\begin{aligned}
&\int (d+ex)^2\left(a+cx^2\right)^3\left(A+Bx+Cx^2\right)\,dx \\
&= \frac{1}{11}\,Cc^3e^2x^{11}+\frac{1}{10}\,\left(2\,Cc^3de+Bc^3e^2\right)x^{10}+\frac{1}{9}\,\left(Cc^3d^2+2\,Bc^3de+\left(3\,Cac^2+Ac^3\right)e^2\right)x^9 \\
&\quad+\frac{1}{8}\,\left(Bc^3d^2+3\,Bac^2e^2+2\,\left(3\,Cac^2+Ac^3\right)de\right)x^8 \\
&\quad+\frac{1}{7}\,\left(6\,Bac^2de+\left(3\,Cac^2+Ac^3\right)d^2+3\,\left(Ca^2c+Aac^2\right)e^2\right)x^7 \\
&\quad+Aa^3d^2x+\frac{1}{2}\,\left(Bac^2d^2+Ba^2ce^2+2\,\left(Ca^2c+Aac^2\right)de\right)x^6 \\
&\quad+\frac{1}{5}\,\left(6\,Ba^2cde+3\,\left(Ca^2c+Aac^2\right)d^2+\left(Ca^3+3\,Aa^2c\right)e^2\right)x^5 \\
&\quad+\frac{1}{4}\,\left(3\,Ba^2cd^2+Ba^3e^2+2\,\left(Ca^3+3\,Aa^2c\right)de\right)x^4 \\
&\quad+\frac{1}{3}\,\left(2\,Ba^3de+Aa^3e^2+\left(Ca^3+3\,Aa^2c\right)d^2\right)x^3+\frac{1}{2}\,\left(Ba^3d^2+2\,Aa^3de\right)x^2
\end{aligned}$$

```
[In] integrate((e*x+d)^2*(c*x^2+a)^3*(C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/11*C*c^3*e^2*x^11 + 1/10*(2*C*c^3*d*e + B*c^3*e^2)*x^10 + 1/9*(C*c^3*d^2
+ 2*B*c^3*d*e + (3*C*a*c^2 + A*c^3)*e^2)*x^9 + 1/8*(B*c^3*d^2 + 3*B*a*c^2*e
^2 + 2*(3*C*a*c^2 + A*c^3)*d*e)*x^8 + 1/7*(6*B*a*c^2*d*e + (3*C*a*c^2 + A*c
^3)*d^2 + 3*(C*a^2*c + A*a*c^2)*e^2)*x^7 + A*a^3*d^2*x + 1/2*(B*a*c^2*d^2 +
 B*a^2*c*e^2 + 2*(C*a^2*c + A*a*c^2)*d*e)*x^6 + 1/5*(6*B*a^2*c*d*e + 3*(C*a
^2*c + A*a*c^2)*d^2 + (C*a^3 + 3*A*a^2*c)*e^2)*x^5 + 1/4*(3*B*a^2*c*d^2 + B
*a^3*e^2 + 2*(C*a^3 + 3*A*a^2*c)*d*e)*x^4 + 1/3*(2*B*a^3*d*e + A*a^3*e^2 +
(C*a^3 + 3*A*a^2*c)*d^2)*x^3 + 1/2*(B*a^3*d^2 + 2*A*a^3*d*e)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 447, normalized size of antiderivative = 1.55

$$\begin{aligned}\int (d+ex)^2 \left(a+cx^2\right)^3 \left(A+Bx+Cx^2\right)\, dx = {} & Aa^3d^2x + \frac{Cc^3e^2x^{11}}{11} + x^{10}\left(\frac{Bc^3e^2}{10}+\frac{Cc^3de}{5}\right) \\ & + x^9\left(\frac{Ac^3e^2}{9}+\frac{2Bc^3de}{9}+\frac{Cac^2e^2}{3}+\frac{Cc^3d^2}{9}\right) \\ & + x^8\left(\frac{Ac^3de}{4}+\frac{3Bac^2e^2}{8}+\frac{Bc^3d^2}{8}+\frac{3Cac^2de}{4}\right) \\ & + x^7\cdot\left(\frac{3Aac^2e^2}{7}+\frac{Ac^3d^2}{7}+\frac{6Bac^2de}{7}\right. \\ & \left.+\frac{3Ca^2ce^2}{7}+\frac{3Cac^2d^2}{7}\right) \\ & + x^6\left(Aac^2de+\frac{Ba^2ce^2}{2}+\frac{Bac^2d^2}{2}+Ca^2cde\right) \\ & + x^5\cdot\left(\frac{3Aa^2ce^2}{5}+\frac{3Aac^2d^2}{5}+\frac{6Ba^2cde}{5}\right. \\ & \left.+\frac{Ca^3e^2}{5}+\frac{3Ca^2cd^2}{5}\right) + x^4 \\ & \cdot\left(\frac{3Aa^2cde}{2}+\frac{Ba^3e^2}{4}+\frac{3Ba^2cd^2}{4}+\frac{Ca^3de}{2}\right) \\ & + x^3\left(\frac{Aa^3e^2}{3}+Aa^2cd^2+\frac{2Ba^3de}{3}+\frac{Ca^3d^2}{3}\right) \\ & + x^2\left(Aa^3de+\frac{Ba^3d^2}{2}\right)\end{aligned}$$

```
[In] integrate((e*x+d)**2*(c*x**2+a)**3*(C*x**2+B*x+A),x)
```

```
[Out] A*a**3*d**2*x + C*c**3*e**2*x**11/11 + x**10*(B*c**3*e**2/10 + C*c**3*d*e/5
) + x**9*(A*c**3*e**2/9 + 2*B*c**3*d*e/9 + C*a*c**2*e**2/3 + C*c**3*d**2/9)
 + x**8*(A*c**3*d*e/4 + 3*B*a*c**2*e**2/8 + B*c**3*d**2/8 + 3*C*a*c**2*d*e/
4) + x**7*(3*A*a*c**2*e**2/7 + A*c**3*d**2/7 + 6*B*a*c**2*d*e/7 + 3*C*a**2*
c*e**2/7 + 3*C*a*c**2*d**2/7) + x**6*(A*a*c**2*d*e + B*a**2*c*e**2/2 + B*a*
c**2*d**2/2 + C*a**2*c*d*e) + x**5*(3*A*a**2*c*e**2/5 + 3*A*a*c**2*d**2/5 +
 6*B*a**2*c*d*e/5 + C*a**3*e**2/5 + 3*C*a**2*c*d**2/5) + x**4*(3*A*a**2*c*d
*e/2 + B*a**3*e**2/4 + 3*B*a**2*c*d**2/4 + C*a**3*d*e/2) + x**3*(A*a**3*e**
2/3 + A*a**2*c*d**2 + 2*B*a**3*d*e/3 + C*a**3*d**2/3) + x**2*(A*a**3*d*e +
B*a**3*d**2/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.27

$$\begin{aligned}
&\int (d+ex)^2 \left(a+cx^2\right)^3 \left(A+Bx+Cx^2\right) \, dx \\
&= \frac{1}{11} Cc^3e^2x^{11} + \frac{1}{10} \left(2\,Cc^3de + Bc^3e^2\right)x^{10} + \frac{1}{9} \left(Cc^3d^2 + 2\,Bc^3de + \left(3\,Cac^2 + Ac^3\right)e^2\right)x^9 \\
&\quad + \frac{1}{8} \left(Bc^3d^2 + 3\,Bac^2e^2 + 2\left(3\,Cac^2 + Ac^3\right)de\right)x^8 \\
&\quad + \frac{1}{7} \left(6\,Bac^2de + \left(3\,Cac^2 + Ac^3\right)d^2 + 3\left(Ca^2c + Aac^2\right)e^2\right)x^7 \\
&\quad + Aa^3d^2x + \frac{1}{2} \left(Bac^2d^2 + Ba^2ce^2 + 2\left(Ca^2c + Aac^2\right)de\right)x^6 \\
&\quad + \frac{1}{5} \left(6\,Ba^2cde + 3\left(Ca^2c + Aac^2\right)d^2 + \left(Ca^3 + 3\,Aa^2c\right)e^2\right)x^5 \\
&\quad + \frac{1}{4} \left(3\,Ba^2cd^2 + Ba^3e^2 + 2\left(Ca^3 + 3\,Aa^2c\right)de\right)x^4 \\
&\quad + \frac{1}{3} \left(2\,Ba^3de + Aa^3e^2 + \left(Ca^3 + 3\,Aa^2c\right)d^2\right)x^3 + \frac{1}{2} \left(Ba^3d^2 + 2\,Aa^3de\right)x^2
\end{aligned}$$

```
[In] integrate((e*x+d)^2*(c*x^2+a)^3*(C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/11*C*c^3*e^2*x^11 + 1/10*(2*C*c^3*d*e + B*c^3*e^2)*x^10 + 1/9*(C*c^3*d^2
+ 2*B*c^3*d*e + (3*C*a*c^2 + A*c^3)*e^2)*x^9 + 1/8*(B*c^3*d^2 + 3*B*a*c^2*e
^2 + 2*(3*C*a*c^2 + A*c^3)*d*e)*x^8 + 1/7*(6*B*a*c^2*d*e + (3*C*a*c^2 + A*c
^3)*d^2 + 3*(C*a^2*c + A*a*c^2)*e^2)*x^7 + A*a^3*d^2*x + 1/2*(B*a*c^2*d^2 +
 B*a^2*c*e^2 + 2*(C*a^2*c + A*a*c^2)*d*e)*x^6 + 1/5*(6*B*a^2*c*d*e + 3*(C*a
^2*c + A*a*c^2)*d^2 + (C*a^3 + 3*A*a^2*c)*e^2)*x^5 + 1/4*(3*B*a^2*c*d^2 + B
*a^3*e^2 + 2*(C*a^3 + 3*A*a^2*c)*d*e)*x^4 + 1/3*(2*B*a^3*d*e + A*a^3*e^2 +
(C*a^3 + 3*A*a^2*c)*d^2)*x^3 + 1/2*(B*a^3*d^2 + 2*A*a^3*d*e)*x^2
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 432, normalized size of antiderivative = 1.49

$$\int (d+ex)^2 \left(a+cx^2\right)^3 \left(A+Bx+Cx^2\right)\,dx = \frac{1}{11}Cc^3e^2x^{11}+\frac{1}{5}Cc^3dex^{10}+\frac{1}{10}Bc^3e^2x^{10}+\frac{1}{9}Cc^3d^2x^9+\frac{2}{9}Bc^3dex^9+\frac{1}{3}Cac^2e^2x^9+\frac{1}{9}Ac^3e^2x^9+\frac{1}{8}Bc^3d^2x^8+\frac{3}{4}Cac^2dex^8+\frac{1}{4}Ac^3dex^8+\frac{3}{8}Bac^2e^2x^8+\frac{3}{7}Cac^2d^2x^7+\frac{1}{7}Ac^3d^2x^7+\frac{6}{7}Bac^2dex^7+\frac{3}{7}Ca^2ce^2x^7+\frac{3}{7}Aac^2e^2x^7+\frac{1}{2}Bac^2d^2x^6+Ca^2cdex^6+Aac^2dex^6+\frac{1}{2}Ba^2ce^2x^6+\frac{3}{5}Ca^2cd^2x^5+\frac{3}{5}Aac^2d^2x^5+\frac{6}{5}Ba^2cdex^5+\frac{1}{5}Ca^3e^2x^5+\frac{3}{5}Aa^2ce^2x^5+\frac{3}{4}Ba^2cd^2x^4+\frac{1}{2}Ca^3dex^4+\frac{3}{2}Aa^2cdex^4+\frac{1}{4}Ba^3e^2x^4+\frac{1}{3}Ca^3d^2x^3+Aa^2cd^2x^3+\frac{2}{3}Ba^3dex^3+\frac{1}{3}Aa^3e^2x^3+\frac{1}{2}Ba^3d^2x^2+Aa^3dex^2+Aa^3d^2x$$

```
[In] integrate((e*x+d)^2*(c*x^2+a)^3*(C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/11*C*c^3*e^2*x^11 + 1/5*C*c^3*d*e*x^10 + 1/10*B*c^3*e^2*x^10 + 1/9*C*c^3*
d^2*x^9 + 2/9*B*c^3*d*e*x^9 + 1/3*C*a*c^2*e^2*x^9 + 1/9*A*c^3*e^2*x^9 + 1/8
*B*c^3*d^2*x^8 + 3/4*C*a*c^2*d*e*x^8 + 1/4*A*c^3*d*e*x^8 + 3/8*B*a*c^2*e^2*
x^8 + 3/7*C*a*c^2*d^2*x^7 + 1/7*A*c^3*d^2*x^7 + 6/7*B*a*c^2*d*e*x^7 + 3/7*C
*a^2*c*e^2*x^7 + 3/7*A*a*c^2*e^2*x^7 + 1/2*B*a*c^2*d^2*x^6 + C*a^2*c*d*e*x^
6 + A*a*c^2*d*e*x^6 + 1/2*B*a^2*c*e^2*x^6 + 3/5*C*a^2*c*d^2*x^5 + 3/5*A*a*c
^2*d^2*x^5 + 6/5*B*a^2*c*d*e*x^5 + 1/5*C*a^3*e^2*x^5 + 3/5*A*a^2*c*e^2*x^5
+ 3/4*B*a^2*c*d^2*x^4 + 1/2*C*a^3*d*e*x^4 + 3/2*A*a^2*c*d*e*x^4 + 1/4*B*a^3
*e^2*x^4 + 1/3*C*a^3*d^2*x^3 + A*a^2*c*d^2*x^3 + 2/3*B*a^3*d*e*x^3 + 1/3*A*
a^3*e^2*x^3 + 1/2*B*a^3*d^2*x^2 + A*a^3*d*e*x^2 + A*a^3*d^2*x
```

Mupad [B] (verification not implemented)

Time = 12.38 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.19

$$
\begin{aligned}
&\int (d+ex)^2 \left(a+cx^2\right)^3 \left(A+Bx+Cx^2\right) \,dx \\
&= x^3 \left(\frac{C\,a^3\,d^2}{3}+\frac{2\,B\,a^3\,d\,e}{3}+\frac{A\,a^3\,e^2}{3}+A\,c\,a^2\,d^2\right) \\
&\quad + x^9 \left(\frac{C\,c^3\,d^2}{9}+\frac{2\,B\,c^3\,d\,e}{9}+\frac{A\,c^3\,e^2}{9}+\frac{C\,a\,c^2\,e^2}{3}\right) \\
&\quad + x^5 \left(\frac{C\,a^3\,e^2}{5}+\frac{3\,C\,a^2\,c\,d^2}{5}+\frac{6\,B\,a^2\,c\,d\,e}{5}+\frac{3\,A\,a^2\,c\,e^2}{5}+\frac{3\,A\,a\,c^2\,d^2}{5}\right) \\
&\quad + x^7 \left(\frac{3\,C\,a^2\,c\,e^2}{7}+\frac{3\,C\,a\,c^2\,d^2}{7}+\frac{6\,B\,a\,c^2\,d\,e}{7}+\frac{3\,A\,a\,c^2\,e^2}{7}+\frac{A\,c^3\,d^2}{7}\right) \\
&\quad + \frac{a^2\,x^4\,\left(B\,a\,e^2+3\,B\,c\,d^2+6\,A\,c\,d\,e+2\,C\,a\,d\,e\right)}{4} \\
&\quad + \frac{c^2\,x^8\,\left(3\,B\,a\,e^2+B\,c\,d^2+2\,A\,c\,d\,e+6\,C\,a\,d\,e\right)}{8} \\
&\quad + \frac{C\,c^3\,e^2\,x^{11}}{11}+\frac{a\,c\,x^6\,\left(B\,a\,e^2+B\,c\,d^2+2\,A\,c\,d\,e+2\,C\,a\,d\,e\right)}{2} \\
&\quad + A\,a^3\,d^2\,x+\frac{a^3\,d\,x^2\,\left(2\,A\,e+B\,d\right)}{2}+\frac{c^3\,e\,x^{10}\,\left(B\,e+2\,C\,d\right)}{10}
\end{aligned}
$$

```
[In] int((a + c*x^2)^3*(d + e*x)^2*(A + B*x + C*x^2),x)
```

```
[Out] x^3*((A*a^3*e^2)/3 + (C*a^3*d^2)/3 + (2*B*a^3*d*e)/3 + A*a^2*c*d^2) + x^9*(
(A*c^3*e^2)/9 + (C*c^3*d^2)/9 + (2*B*c^3*d*e)/9 + (C*a*c^2*e^2)/3) + x^5*((
C*a^3*e^2)/5 + (3*A*a*c^2*d^2)/5 + (3*A*a^2*c*e^2)/5 + (3*C*a^2*c*d^2)/5 +
(6*B*a^2*c*d*e)/5) + x^7*((A*c^3*d^2)/7 + (3*A*a*c^2*e^2)/7 + (3*C*a*c^2*d^
2)/7 + (3*C*a^2*c*e^2)/7 + (6*B*a*c^2*d*e)/7) + (a^2*x^4*(B*a*e^2 + 3*B*c*d
^2 + 6*A*c*d*e + 2*C*a*d*e))/4 + (c^2*x^8*(3*B*a*e^2 + B*c*d^2 + 2*A*c*d*e
+ 6*C*a*d*e))/8 + (C*c^3*e^2*x^11)/11 + (a*c*x^6*(B*a*e^2 + B*c*d^2 + 2*A*c
*d*e + 2*C*a*d*e))/2 + A*a^3*d^2*x + (a^3*d*x^2*(2*A*e + B*d))/2 + (c^3*e*x
^10*(B*e + 2*C*d))/10
```

3.34 $\int (d+ex)\left(a+cx^2\right)^3\left(A+Bx+Cx^2\right)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 169

$$\int (d+ex)\left(a+cx^2\right)^3\left(A+Bx+Cx^2\right)\,dx = a^3Adx+\frac{1}{3}a^2(3Acd+aCd+aBe)x^3 +\frac{1}{4}a^3Cex^4+\frac{3}{5}ac(Acd+aCd+aBe)x^5 +\frac{1}{2}a^2cCex^6+\frac{1}{7}c^2(Acd+3a(Cd+Be))x^7 +\frac{3}{8}ac^2Cex^8+\frac{1}{9}c^3(Cd+Be)x^9 +\frac{1}{10}c^3Cex^{10}+\frac{(Bd+Ae)\left(a+cx^2\right)^4}{8c}$$

```
[Out] a^3*A*d*x+1/3*a^2*(3*A*c*d+B*a*e+C*a*d)*x^3+1/4*a^3*C*e*x^4+3/5*a*c*(A*c*d+
B*a*e+C*a*d)*x^5+1/2*a^2*c*C*e*x^6+1/7*c^2*(A*c*d+3*a*(B*e+C*d))*x^7+3/8*a*
c^2*C*e*x^8+1/9*c^3*(B*e+C*d)*x^9+1/10*c^3*C*e*x^10+1/8*(A*e+B*d)*(c*x^2+a)
^4/c
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used

$= \{1596, 1824\}$

$$\int (d+ex)\left(a+cx^2\right)^3\left(A+Bx+Cx^2\right)\,dx = a^3Adx+\frac{1}{4}a^3Cex^4+\frac{1}{3}a^2x^3(aBe+aCd+3Acd)+\frac{1}{2}a^2cCex^6+\frac{1}{7}c^2x^7(3a(Be+Cd)+Acd)+\frac{3}{5}acx^5(aBe+aCd+Acd)+\frac{\left(a+cx^2\right)^4(Ae+Bd)}{8c}+\frac{3}{8}ac^2Cex^8+\frac{1}{9}c^3x^9(Be+Cd)+\frac{1}{10}c^3Cex^{10}$$

```
[In] Int[(d + e*x)*(a + c*x^2)^3*(A + B*x + C*x^2),x]
```

```
[Out] a^3*A*d*x + (a^2*(3*A*c*d + a*C*d + a*B*e)*x^3)/3 + (a^3*C*e*x^4)/4 + (3*a*
c*(A*c*d + a*C*d + a*B*e)*x^5)/5 + (a^2*c*C*e*x^6)/2 + (c^2*(A*c*d + 3*a*(C
*d + B*e))*x^7)/7 + (3*a*c^2*C*e*x^8)/8 + (c^3*(C*d + B*e)*x^9)/9 + (c^3*C*
e*x^10)/10 + ((B*d + A*e)*(a + c*x^2)^4)/(8*c)
```

Rule 1596

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{(Bd+Ae)\left(a+cx^2\right)^4}{8c}+\int \left(a+cx^2\right)^3\left(-((Bd+Ae)x)+(d+ex)\left(A+Bx+Cx^2\right)\right)\,dx\\ &= \frac{(Bd+Ae)\left(a+cx^2\right)^4}{8c}\\ &\quad+\int \left(a^3Ad+a^2(3Acd+aCd+aBe)x^2+a^3Cex^3+3ac(Acd+aCd+aBe)x^4\right.\\ &\quad\left.+3a^2cCex^5+c^2(Acd+3a(Cd+Be))x^6+3ac^2Cex^7+c^3(Cd+Be)x^8+c^3Cex^9\right)\,dx\end{aligned}$$

$$
\begin{aligned}
&= a^3Adx + \frac{1}{3}a^2(3Acd + aCd + aBe)x^3 + \frac{1}{4}a^3Cex^4 + \frac{3}{5}ac(Acd + aCd + aBe)x^5 \\
&\quad + \frac{1}{2}a^2cCex^6 + \frac{1}{7}c^2(Acd + 3a(Cd + Be))x^7 + \frac{3}{8}ac^2Cex^8 \\
&\quad + \frac{1}{9}c^3(Cd + Be)x^9 + \frac{1}{10}c^3Cex^{10} + \frac{(Bd + Ae)\left(a + cx^2\right)^4}{8c}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.16

$$
\begin{aligned}
\int (d + ex)\left(a + cx^2\right)^3\left(A + Bx + Cx^2\right)\,dx = a^3Adx &+ \frac{1}{2}a^3(Bd + Ae)x^2 \\
&+ \frac{1}{3}a^2(3Acd + aCd + aBe)x^3 \\
&+ \frac{1}{4}a^2(3Bcd + 3Ace + aCe)x^4 \\
&+ \frac{3}{5}ac(Acd + aCd + aBe)x^5 \\
&+ \frac{1}{2}ac(Bcd + Ace + aCe)x^6 \\
&+ \frac{1}{7}c^2(Acd + 3aCd + 3aBe)x^7 \\
&+ \frac{1}{8}c^2(Bcd + Ace + 3aCe)x^8 \\
&+ \frac{1}{9}c^3(Cd + Be)x^9 + \frac{1}{10}c^3Cex^{10}
\end{aligned}
$$

```
[In] Integrate[(d + e*x)*(a + c*x^2)^3*(A + B*x + C*x^2),x]
```

```
[Out] a^3*A*d*x + (a^3*(B*d + A*e)*x^2)/2 + (a^2*(3*A*c*d + a*C*d + a*B*e)*x^3)/3
 + (a^2*(3*B*c*d + 3*A*c*e + a*C*e)*x^4)/4 + (3*a*c*(A*c*d + a*C*d + a*B*e)
*x^5)/5 + (a*c*(B*c*d + A*c*e + a*C*e)*x^6)/2 + (c^2*(A*c*d + 3*a*C*d + 3*a
*B*e)*x^7)/7 + (c^2*(B*c*d + A*c*e + 3*a*C*e)*x^8)/8 + (c^3*(C*d + B*e)*x^9
)/9 + (c^3*C*e*x^10)/10
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.32

method	result
default	$\frac{c^3Ce\,x^{10}}{10}+\frac{(e\,c^3B+c^3dC)x^9}{9}+\frac{(e\,c^3A+c^3dB+3a\,c^2eC)x^8}{8}+\frac{(Ad\,c^3+3a\,c^2eB+3a\,c^2dC)x^7}{7}+$
norman	$\frac{c^3Ce\,x^{10}}{10}+\left(\frac{1}{9}e\,c^3B+\frac{1}{9}c^3dC\right)x^9+\left(\frac{1}{8}e\,c^3A+\frac{1}{8}c^3dB+\frac{3}{8}a\,c^2eC\right)x^8+\left(\frac{1}{7}Ad\,c^3+\frac{3}{7}a\,c^2eB+\right.$
gosper	$\frac{1}{10}c^3Ce\,x^{10}+\frac{1}{9}B\,c^3e\,x^9+\frac{1}{9}x^9c^3dC+\frac{1}{8}x^8A\,c^3e+\frac{1}{8}x^8B\,c^3d+\frac{3}{8}a\,c^2Ce\,x^8+\frac{1}{7}x^7Ad\,c^3+$
risch	$\frac{1}{10}c^3Ce\,x^{10}+\frac{1}{9}B\,c^3e\,x^9+\frac{1}{9}x^9c^3dC+\frac{1}{8}x^8A\,c^3e+\frac{1}{8}x^8B\,c^3d+\frac{3}{8}a\,c^2Ce\,x^8+\frac{1}{7}x^7Ad\,c^3+$
parallelrisch	$\frac{1}{10}c^3Ce\,x^{10}+\frac{1}{9}B\,c^3e\,x^9+\frac{1}{9}x^9c^3dC+\frac{1}{8}x^8A\,c^3e+\frac{1}{8}x^8B\,c^3d+\frac{3}{8}a\,c^2Ce\,x^8+\frac{1}{7}x^7Ad\,c^3+$

```
[In] int((e*x+d)*(c*x^2+a)^3*(C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*c^3*C*e*x^10+1/9*(B*c^3*e+C*c^3*d)*x^9+1/8*(A*c^3*e+B*c^3*d+3*C*a*c^2*
e)*x^8+1/7*(A*c^3*d+3*B*a*c^2*e+3*C*a*c^2*d)*x^7+1/6*(3*A*a*c^2*e+3*B*a*c^2
*d+3*C*a^2*c*e)*x^6+1/5*(3*A*a*c^2*d+3*B*a^2*c*e+3*C*a^2*c*d)*x^5+1/4*(3*A*
a^2*c*e+3*B*a^2*c*d+C*a^3*e)*x^4+1/3*(3*A*a^2*c*d+B*a^3*e+C*a^3*d)*x^3+1/2*
(A*a^3*e+B*a^3*d)*x^2+a^3*A*d*x
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.31

$$
\begin{aligned}
&\int (d+ex)\left(a+cx^2\right)^3\left(A+Bx+Cx^2\right)\,dx\\
&=\frac{1}{10}Cc^3ex^{10}+\frac{1}{9}\left(Cc^3d+Bc^3e\right)x^9+\frac{1}{8}\left(Bc^3d+\left(3\,Cac^2+Ac^3\right)e\right)x^8\\
&\quad+\frac{1}{7}\left(3\,Bac^2e+\left(3\,Cac^2+Ac^3\right)d\right)x^7+\frac{1}{2}\left(Bac^2d+\left(Ca^2c+Aac^2\right)e\right)x^6+Aa^3dx\\
&\quad+\frac{3}{5}\left(Ba^2ce+\left(Ca^2c+Aac^2\right)d\right)x^5+\frac{1}{4}\left(3\,Ba^2cd+\left(Ca^3+3\,Aa^2c\right)e\right)x^4\\
&\quad+\frac{1}{3}\left(Ba^3e+\left(Ca^3+3\,Aa^2c\right)d\right)x^3+\frac{1}{2}\left(Ba^3d+Aa^3e\right)x^2
\end{aligned}
$$

```
[In] integrate((e*x+d)*(c*x^2+a)^3*(C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/10*C*c^3*e*x^10 + 1/9*(C*c^3*d + B*c^3*e)*x^9 + 1/8*(B*c^3*d + (3*C*a*c^2
 + A*c^3)*e)*x^8 + 1/7*(3*B*a*c^2*e + (3*C*a*c^2 + A*c^3)*d)*x^7 + 1/2*(B*a
*c^2*d + (C*a^2*c + A*a*c^2)*e)*x^6 + A*a^3*d*x + 3/5*(B*a^2*c*e + (C*a^2*c
 + A*a*c^2)*d)*x^5 + 1/4*(3*B*a^2*c*d + (C*a^3 + 3*A*a^2*c)*e)*x^4 + 1/3*(B
*a^3*e + (C*a^3 + 3*A*a^2*c)*d)*x^3 + 1/2*(B*a^3*d + A*a^3*e)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.57

$$\int (d+ex)\left(a+cx^2\right)^3\left(A+Bx+Cx^2\right)\,dx = Aa^3dx+\frac{Cc^3ex^{10}}{10}+x^9\left(\frac{Bc^3e}{9}+\frac{Cc^3d}{9}\right) + x^8\left(\frac{Ac^3e}{8}+\frac{Bc^3d}{8}+\frac{3Cac^2e}{8}\right) + x^7\left(\frac{Ac^3d}{7}+\frac{3Bac^2e}{7}+\frac{3Cac^2d}{7}\right) + x^6\left(\frac{Aac^2e}{2}+\frac{Bac^2d}{2}+\frac{Ca^2ce}{2}\right)+x^5 \cdot\left(\frac{3Aac^2d}{5}+\frac{3Ba^2ce}{5}+\frac{3Ca^2cd}{5}\right) + x^4\cdot\left(\frac{3Aa^2ce}{4}+\frac{3Ba^2cd}{4}+\frac{Ca^3e}{4}\right) + x^3\left(Aa^2cd+\frac{Ba^3e}{3}+\frac{Ca^3d}{3}\right) + x^2\left(\frac{Aa^3e}{2}+\frac{Ba^3d}{2}\right)$$

```
[In] integrate((e*x+d)*(c*x**2+a)**3*(C*x**2+B*x+A),x)
```

```
[Out] A*a**3*d*x + C*c**3*e*x**10/10 + x**9*(B*c**3*e/9 + C*c**3*d/9) + x**8*(A*c
**3*e/8 + B*c**3*d/8 + 3*C*a*c**2*e/8) + x**7*(A*c**3*d/7 + 3*B*a*c**2*e/7
+ 3*C*a*c**2*d/7) + x**6*(A*a*c**2*e/2 + B*a*c**2*d/2 + C*a**2*c*e/2) + x**
5*(3*A*a*c**2*d/5 + 3*B*a**2*c*e/5 + 3*C*a**2*c*d/5) + x**4*(3*A*a**2*c*e/4
 + 3*B*a**2*c*d/4 + C*a**3*e/4) + x**3*(A*a**2*c*d + B*a**3*e/3 + C*a**3*d/
3) + x**2*(A*a**3*e/2 + B*a**3*d/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.31

$$\int (d+ex)\left(a+cx^2\right)^3\left(A+Bx+Cx^2\right)\,dx = \frac{1}{10}Cc^3ex^{10}+\frac{1}{9}\left(Cc^3d+Bc^3e\right)x^9+\frac{1}{8}\left(Bc^3d+\left(3\,Cac^2+Ac^3\right)e\right)x^8 + \frac{1}{7}\left(3\,Bac^2e+\left(3\,Cac^2+Ac^3\right)d\right)x^7+\frac{1}{2}\left(Bac^2d+\left(Ca^2c+Aac^2\right)e\right)x^6+Aa^3dx + \frac{3}{5}\left(Ba^2ce+\left(Ca^2c+Aac^2\right)d\right)x^5+\frac{1}{4}\left(3\,Ba^2cd+\left(Ca^3+3\,Aa^2c\right)e\right)x^4 + \frac{1}{3}\left(Ba^3e+\left(Ca^3+3\,Aa^2c\right)d\right)x^3+\frac{1}{2}\left(Ba^3d+Aa^3e\right)x^2$$

```
[In] integrate((e*x+d)*(c*x^2+a)^3*(C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/10*C*c^3*e*x^10 + 1/9*(C*c^3*d + B*c^3*e)*x^9 + 1/8*(B*c^3*d + (3*C*a*c^2
 + A*c^3)*e)*x^8 + 1/7*(3*B*a*c^2*e + (3*C*a*c^2 + A*c^3)*d)*x^7 + 1/2*(B*a
*c^2*d + (C*a^2*c + A*a*c^2)*e)*x^6 + A*a^3*d*x + 3/5*(B*a^2*c*e + (C*a^2*c
 + A*a*c^2)*d)*x^5 + 1/4*(3*B*a^2*c*d + (C*a^3 + 3*A*a^2*c)*e)*x^4 + 1/3*(B
*a^3*e + (C*a^3 + 3*A*a^2*c)*d)*x^3 + 1/2*(B*a^3*d + A*a^3*e)*x^2
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.47

$$\begin{aligned}\int (d+ex)\left(a+cx^2\right)^3\left(A+Bx+Cx^2\right)\,dx &= \frac{1}{10}Cc^3ex^{10}+\frac{1}{9}Cc^3dx^9+\frac{1}{9}Bc^3ex^9+\frac{1}{8}Bc^3dx^8\\ &\quad+\frac{3}{8}Cac^2ex^8+\frac{1}{8}Ac^3ex^8+\frac{3}{7}Cac^2dx^7\\ &\quad+\frac{1}{7}Ac^3dx^7+\frac{3}{7}Bac^2ex^7+\frac{1}{2}Bac^2dx^6\\ &\quad+\frac{1}{2}Ca^2cex^6+\frac{1}{2}Aac^2ex^6+\frac{3}{5}Ca^2cdx^5\\ &\quad+\frac{3}{5}Aac^2dx^5+\frac{3}{5}Ba^2cex^5+\frac{3}{4}Ba^2cdx^4\\ &\quad+\frac{1}{4}Ca^3ex^4+\frac{3}{4}Aa^2cex^4+\frac{1}{3}Ca^3dx^3+Aa^2cdx^3\\ &\quad+\frac{1}{3}Ba^3ex^3+\frac{1}{2}Ba^3dx^2+\frac{1}{2}Aa^3ex^2+Aa^3dx\end{aligned}$$

```
[In] integrate((e*x+d)*(c*x^2+a)^3*(C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/10*C*c^3*e*x^10 + 1/9*C*c^3*d*x^9 + 1/9*B*c^3*e*x^9 + 1/8*B*c^3*d*x^8 + 3
/8*C*a*c^2*e*x^8 + 1/8*A*c^3*e*x^8 + 3/7*C*a*c^2*d*x^7 + 1/7*A*c^3*d*x^7 +
3/7*B*a*c^2*e*x^7 + 1/2*B*a*c^2*d*x^6 + 1/2*C*a^2*c*e*x^6 + 1/2*A*a*c^2*e*x
^6 + 3/5*C*a^2*c*d*x^5 + 3/5*A*a*c^2*d*x^5 + 3/5*B*a^2*c*e*x^5 + 3/4*B*a^2*
c*d*x^4 + 1/4*C*a^3*e*x^4 + 3/4*A*a^2*c*e*x^4 + 1/3*C*a^3*d*x^3 + A*a^2*c*d
*x^3 + 1/3*B*a^3*e*x^3 + 1/2*B*a^3*d*x^2 + 1/2*A*a^3*e*x^2 + A*a^3*d*x
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.11

$$\int (d+ex)\left(a+cx^2\right)^3\left(A+Bx+Cx^2\right)\,dx = x^3\left(\frac{B\,a^3\,e}{3}+\frac{C\,a^3\,d}{3}+A\,a^2\,c\,d\right) + x^8\left(\frac{A\,c^3\,e}{8}+\frac{B\,c^3\,d}{8}+\frac{3\,C\,a\,c^2\,e}{8}\right) + \frac{a^3\,x^2\,(A\,e+B\,d)}{2}+\frac{c^3\,x^9\,(B\,e+C\,d)}{9} + \frac{c^2\,x^7\,(A\,c\,d+3\,B\,a\,e+3\,C\,a\,d)}{7} + \frac{a^2\,x^4\,(3\,A\,c\,e+3\,B\,c\,d+C\,a\,e)}{4} + A\,a^3\,d\,x+\frac{3\,a\,c\,x^5\,(A\,c\,d+B\,a\,e+C\,a\,d)}{5} + \frac{a\,c\,x^6\,(A\,c\,e+B\,c\,d+C\,a\,e)}{2}+\frac{C\,c^3\,e\,x^{10}}{10}$$

```
[In] int((a + c*x^2)^3*(d + e*x)*(A + B*x + C*x^2),x)
```

```
[Out] x^3*((B*a^3*e)/3 + (C*a^3*d)/3 + A*a^2*c*d) + x^8*((A*c^3*e)/8 + (B*c^3*d)/
8 + (3*C*a*c^2*e)/8) + (a^3*x^2*(A*e + B*d))/2 + (c^3*x^9*(B*e + C*d))/9 +
(c^2*x^7*(A*c*d + 3*B*a*e + 3*C*a*d))/7 + (a^2*x^4*(3*A*c*e + 3*B*c*d + C*a
*e))/4 + A*a^3*d*x + (3*a*c*x^5*(A*c*d + B*a*e + C*a*d))/5 + (a*c*x^6*(A*c*
e + B*c*d + C*a*e))/2 + (C*c^3*e*x^10)/10
```

3.35 $\int (a+cx^2)^3 (A+Bx+Cx^2)\, dx$

Optimal result

Integrand size = 20, antiderivative size = 87

$$\int (a+cx^2)^3 (A+Bx+Cx^2)\, dx = a^3Ax + \frac{1}{3}a^2(3Ac+aC)x^3 + \frac{3}{5}ac(Ac+aC)x^5 + \frac{1}{7}c^2(Ac+3aC)x^7 + \frac{1}{9}c^3Cx^9 + \frac{B(a+cx^2)^4}{8c}$$

```
[Out] a^3*A*x+1/3*a^2*(3*A*c+C*a)*x^3+3/5*a*c*(A*c+C*a)*x^5+1/7*c^2*(A*c+3*C*a)*x
^7+1/9*c^3*C*x^9+1/8*B*(c*x^2+a)^4/c
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1596, 380}

$$\int (a+cx^2)^3 (A+Bx+Cx^2)\, dx = a^3Ax + \frac{1}{3}a^2x^3(aC+3Ac) + \frac{1}{7}c^2x^7(3aC+Ac) + \frac{3}{5}acx^5(aC+Ac) + \frac{B(a+cx^2)^4}{8c} + \frac{1}{9}c^3Cx^9$$

```
[In] Int[(a + c*x^2)^3*(A + B*x + C*x^2),x]
```

```
[Out] a^3*A*x + (a^2*(3*A*c + a*C)*x^3)/3 + (3*a*c*(A*c + a*C)*x^5)/5 + (c^2*(A*c
 + 3*a*C)*x^7)/7 + (c^3*C*x^9)/9 + (B*(a + c*x^2)^4)/(8*c)
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
```

```
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 1596

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{B(a+cx^2)^4}{8c} + \int (a+cx^2)^3 (A+Cx^2)\, dx \\ &= \frac{B(a+cx^2)^4}{8c} + \int \left(a^3A + a^2(3Ac+aC)x^2 + 3ac(Ac+aC)x^4 + c^2(Ac+3aC)x^6 + c^3Cx^8\right) dx \\ &= a^3Ax + \frac{1}{3}a^2(3Ac+aC)x^3 + \frac{3}{5}ac(Ac+aC)x^5 + \frac{1}{7}c^2(Ac+3aC)x^7 + \frac{1}{9}c^3Cx^9 + \frac{B(a+cx^2)^4}{8c}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.15

$$\begin{aligned}\int (a+cx^2)^3 (A+Bx+Cx^2)\, dx = {} & \frac{1}{6}a^3x(6A+x(3B+2Cx)) + \frac{1}{20}a^2cx^3(20A+3x(5B+4Cx)) \\ & + \frac{1}{70}ac^2x^5(42A+5x(7B+6Cx)) \\ & + \frac{1}{504}c^3x^7(72A+7x(9B+8Cx))\end{aligned}$$

```
[In] Integrate[(a + c*x^2)^3*(A + B*x + C*x^2),x]
```

```
[Out] (a^3*x*(6*A + x*(3*B + 2*C*x)))/6 + (a^2*c*x^3*(20*A + 3*x*(5*B + 4*C*x)))/
20 + (a*c^2*x^5*(42*A + 5*x*(7*B + 6*C*x)))/70 + (c^3*x^7*(72*A + 7*x*(9*B
+ 8*C*x)))/504
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.25

method	result
norman	$\frac{c^3 C x^9}{9}+\frac{B c^3 x^8}{8}+\left(\frac{1}{7}A c^3+\frac{3}{7}a c^2 C\right)x^7+\frac{B a c^2 x^6}{2}+\left(\frac{3}{5}A a c^2+\frac{3}{5}c a^2 C\right)x^5+\frac{3B a^2 c x^4}{4}+(A a^2 c$
default	$\frac{c^3 C x^9}{9}+\frac{B c^3 x^8}{8}+\frac{(A c^3+3a c^2 C)x^7}{7}+\frac{B a c^2 x^6}{2}+\frac{(3A a c^2+3c a^2 C)x^5}{5}+\frac{3B a^2 c x^4}{4}+\frac{(3A a^2 c+a^3 C)x^3}{3}+$
gosper	$\frac{1}{9}c^3 C x^9+\frac{1}{8}B c^3 x^8+\frac{1}{7}A c^3 x^7+\frac{3}{7}x^7 a c^2 C+\frac{1}{2}B a c^2 x^6+\frac{3}{5}a A c^2 x^5+\frac{3}{5}x^5 c a^2 C+\frac{3}{4}B a^2 c x^4$
risch	$\frac{1}{9}c^3 C x^9+\frac{1}{8}B c^3 x^8+\frac{1}{7}A c^3 x^7+\frac{3}{7}x^7 a c^2 C+\frac{1}{2}B a c^2 x^6+\frac{3}{5}a A c^2 x^5+\frac{3}{5}x^5 c a^2 C+\frac{3}{4}B a^2 c x^4$
parallelrisch	$\frac{1}{9}c^3 C x^9+\frac{1}{8}B c^3 x^8+\frac{1}{7}A c^3 x^7+\frac{3}{7}x^7 a c^2 C+\frac{1}{2}B a c^2 x^6+\frac{3}{5}a A c^2 x^5+\frac{3}{5}x^5 c a^2 C+\frac{3}{4}B a^2 c x^4$

```
[In] int((c*x^2+a)^3*(C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*c^3*C*x^9+1/8*B*c^3*x^8+(1/7*A*c^3+3/7*a*c^2*C)*x^7+1/2*B*a*c^2*x^6+(3/
5*A*a*c^2+3/5*c*a^2*C)*x^5+3/4*B*a^2*c*x^4+(A*a^2*c+1/3*a^3*C)*x^3+1/2*B*a^
3*x^2+a^3*A*x
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.24

$$\begin{aligned}\int \left(a+cx^2\right)^3\left(A+Bx+Cx^2\right)\,dx &= \frac{1}{9}\,Cc^3x^9+\frac{1}{8}\,Bc^3x^8+\frac{1}{2}\,Bac^2x^6+\frac{3}{4}\,Ba^2cx^4\\ &\quad+\frac{1}{7}\,(3\,Cac^2+Ac^3)x^7+\frac{1}{2}\,Ba^3x^2\\ &\quad+\frac{3}{5}\,(Ca^2c+Aac^2)x^5+Aa^3x+\frac{1}{3}\,(Ca^3+3\,Aa^2c)x^3\end{aligned}$$

```
[In] integrate((c*x^2+a)^3*(C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/9*C*c^3*x^9 + 1/8*B*c^3*x^8 + 1/2*B*a*c^2*x^6 + 3/4*B*a^2*c*x^4 + 1/7*(3*
C*a*c^2 + A*c^3)*x^7 + 1/2*B*a^3*x^2 + 3/5*(C*a^2*c + A*a*c^2)*x^5 + A*a^3*
x + 1/3*(C*a^3 + 3*A*a^2*c)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.40

$$\int \left(a+cx^2\right)^3 \left(A+Bx+Cx^2\right)\,dx = Aa^3x + \frac{Ba^3x^2}{2} + \frac{3Ba^2cx^4}{4} + \frac{Bac^2x^6}{2} + \frac{Bc^3x^8}{8} + \frac{Cc^3x^9}{9} + x^7\left(\frac{Ac^3}{7} + \frac{3Cac^2}{7}\right) + x^5 \cdot \left(\frac{3Aac^2}{5} + \frac{3Ca^2c}{5}\right) + x^3\left(Aa^2c + \frac{Ca^3}{3}\right)$$

```
[In] integrate((c*x**2+a)**3*(C*x**2+B*x+A),x)
```

```
[Out] A*a**3*x + B*a**3*x**2/2 + 3*B*a**2*c*x**4/4 + B*a*c**2*x**6/2 + B*c**3*x**
8/8 + C*c**3*x**9/9 + x**7*(A*c**3/7 + 3*C*a*c**2/7) + x**5*(3*A*a*c**2/5 +
 3*C*a**2*c/5) + x**3*(A*a**2*c + C*a**3/3)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.24

$$\int \left(a+cx^2\right)^3 \left(A+Bx+Cx^2\right)\,dx = \frac{1}{9}\,Cc^3x^9 + \frac{1}{8}\,Bc^3x^8 + \frac{1}{2}\,Bac^2x^6 + \frac{3}{4}\,Ba^2cx^4 + \frac{1}{7}\,\left(3\,Cac^2 + Ac^3\right)x^7 + \frac{1}{2}\,Ba^3x^2 + \frac{3}{5}\,\left(Ca^2c + Aac^2\right)x^5 + Aa^3x + \frac{1}{3}\,\left(Ca^3 + 3\,Aa^2c\right)x^3$$

```
[In] integrate((c*x^2+a)^3*(C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/9*C*c^3*x^9 + 1/8*B*c^3*x^8 + 1/2*B*a*c^2*x^6 + 3/4*B*a^2*c*x^4 + 1/7*(3*
C*a*c^2 + A*c^3)*x^7 + 1/2*B*a^3*x^2 + 3/5*(C*a^2*c + A*a*c^2)*x^5 + A*a^3*
x + 1/3*(C*a^3 + 3*A*a^2*c)*x^3
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.28

$$\int \left(a+cx^2\right)^3 \left(A+Bx+Cx^2\right)\,dx = \frac{1}{9}\,Cc^3x^9 + \frac{1}{8}\,Bc^3x^8 + \frac{3}{7}\,Cac^2x^7 + \frac{1}{7}\,Ac^3x^7 + \frac{1}{2}\,Bac^2x^6 + \frac{3}{5}\,Ca^2cx^5 + \frac{3}{5}\,Aac^2x^5 + \frac{3}{4}\,Ba^2cx^4 + \frac{1}{3}\,Ca^3x^3 + Aa^2cx^3 + \frac{1}{2}\,Ba^3x^2 + Aa^3x$$

```
[In] integrate((c*x^2+a)^3*(C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/9*C*c^3*x^9 + 1/8*B*c^3*x^8 + 3/7*C*a*c^2*x^7 + 1/7*A*c^3*x^7 + 1/2*B*a*c
^2*x^6 + 3/5*C*a^2*c*x^5 + 3/5*A*a*c^2*x^5 + 3/4*B*a^2*c*x^4 + 1/3*C*a^3*x^
3 + A*a^2*c*x^3 + 1/2*B*a^3*x^2 + A*a^3*x
```

Mupad [B] (verification not implemented)

Time = 12.25 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.18

$$\int \left(a+cx^2\right)^3\left(A+Bx+Cx^2\right)\,dx=x^3\left(\frac{C\,a^3}{3}+A\,c\,a^2\right)+x^7\left(\frac{A\,c^3}{7}+\frac{3\,C\,a\,c^2}{7}\right)+\frac{B\,a^3\,x^2}{2}+\frac{B\,c^3\,x^8}{8}+\frac{C\,c^3\,x^9}{9}+A\,a^3\,x+\frac{3\,a\,c\,x^5\,\left(A\,c+C\,a\right)}{5}+\frac{3\,B\,a^2\,c\,x^4}{4}+\frac{B\,a\,c^2\,x^6}{2}$$

```
[In] int((a + c*x^2)^3*(A + B*x + C*x^2),x)
```

```
[Out] x^3*((C*a^3)/3 + A*a^2*c) + x^7*((A*c^3)/7 + (3*C*a*c^2)/7) + (B*a^3*x^2)/2
 + (B*c^3*x^8)/8 + (C*c^3*x^9)/9 + A*a^3*x + (3*a*c*x^5*(A*c + C*a))/5 + (3
*B*a^2*c*x^4)/4 + (B*a*c^2*x^6)/2
```

3.36 $\int \frac{(a+cx^2)^3(A+Bx+Cx^2)}{d+ex}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 490

$$
\begin{aligned}
&\int \frac{(a+cx^2)^3\,(A+Bx+Cx^2)}{d+ex}\,dx \\
&= -\frac{(cd^2+ae^2)^2\,(ae^2(2Cd-Be)+cd(8Cd^2-e(7Bd-6Ae)))\,x}{e^8} \\
&\quad+\frac{(cd^2+ae^2)\,(a^2Ce^4+c^2d^2(28Cd^2-3e(7Bd-5Ae))+ace^2(17Cd^2-3e(3Bd-Ae)))\,(d+ex)^2}{2e^9} \\
&\quad-\frac{c(3a^2e^4(4Cd-Be)+c^2d^3(56Cd^2-5e(7Bd-4Ae))+6acde^2(10Cd^2-e(5Bd-2Ae)))\,(d+ex)^3}{3e^9} \\
&\quad+\frac{c(3a^2Ce^4+5c^2d^2(14Cd^2-e(7Bd-3Ae))+3ace^2(15Cd^2-e(5Bd-Ae)))\,(d+ex)^4}{4e^9} \\
&\quad-\frac{c^2(3ae^2(6Cd-Be)+cd(56Cd^2-3e(7Bd-2Ae)))\,(d+ex)^5}{5e^9} \\
&\quad+\frac{c^2(3aCe^2+c(28Cd^2-e(7Bd-Ae)))\,(d+ex)^6}{6e^9}-\frac{c^3(8Cd-Be)(d+ex)^7}{7e^9} \\
&\quad+\frac{c^3C(d+ex)^8}{8e^9}+\frac{(cd^2+ae^2)^3\,(Cd^2-Bde+Ae^2)\log(d+ex)}{e^9}
\end{aligned}
$$

```
[Out] -(a*e^2+c*d^2)^2*(a*e^2*(-B*e+2*C*d)+c*d*(8*C*d^2-e*(-6*A*e+7*B*d)))*x/e^8+
1/2*(a*e^2+c*d^2)*(a^2*C*e^4+c^2*d^2*(28*C*d^2-3*e*(-5*A*e+7*B*d))+a*c*e^2*
(17*C*d^2-3*e*(-A*e+3*B*d)))*(e*x+d)^2/e^9-1/3*c*(3*a^2*e^4*(-B*e+4*C*d)+c^
2*d^3*(56*C*d^2-5*e*(-4*A*e+7*B*d))+6*a*c*d*e^2*(10*C*d^2-e*(-2*A*e+5*B*d))
)*(e*x+d)^3/e^9+1/4*c*(3*a^2*C*e^4+5*c^2*d^2*(14*C*d^2-e*(-3*A*e+7*B*d))+3*
a*c*e^2*(15*C*d^2-e*(-A*e+5*B*d)))*(e*x+d)^4/e^9-1/5*c^2*(3*a*e^2*(-B*e+6*C
*d)+c*d*(56*C*d^2-3*e*(-2*A*e+7*B*d)))*(e*x+d)^5/e^9+1/6*c^2*(3*a*C*e^2+c*(
28*C*d^2-e*(-A*e+7*B*d)))*(e*x+d)^6/e^9-1/7*c^3*(-B*e+8*C*d)*(e*x+d)^7/e^9+
1/8*c^3*C*(e*x+d)^8/e^9+(a*e^2+c*d^2)^3*(A*e^2-B*d*e+C*d^2)*ln(e*x+d)/e^9
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 487, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {1642}

$$\begin{aligned}
&\int \frac{(a+cx^2)^3(A+Bx+Cx^2)}{d+ex}\,dx \\
&= \frac{c(d+ex)^4(3a^2Ce^4+3ace^2(15Cd^2-e(5Bd-Ae))+5c^2(14Cd^4-d^2e(7Bd-3Ae)))}{4e^9} \\
&\quad+\frac{(d+ex)^2(ae^2+cd^2)(a^2Ce^4+ace^2(17Cd^2-3e(3Bd-Ae))+c^2(28Cd^4-3d^2e(7Bd-5Ae)))}{2e^9} \\
&\quad-\frac{c(d+ex)^3(3a^2e^4(4Cd-Be)+6acde^2(10Cd^2-e(5Bd-2Ae))+c^2(56Cd^5-5d^3e(7Bd-4Ae)))}{3e^9} \\
&\quad-\frac{c^2(d+ex)^5(3ae^2(6Cd-Be)-3cde(7Bd-2Ae)+56cCd^3)}{5e^9} \\
&\quad+\frac{c^2(d+ex)^6(3aCe^2-ce(7Bd-Ae)+28cCd^2)}{6e^9} \\
&\quad+\frac{(ae^2+cd^2)^3\log(d+ex)(Ae^2-Bde+Cd^2)}{e^9} \\
&\quad-\frac{x(ae^2+cd^2)^2(ae^2(2Cd-Be)-cde(7Bd-6Ae)+8cCd^3)}{e^8} \\
&\quad-\frac{c^3(d+ex)^7(8Cd-Be)}{7e^9}+\frac{c^3C(d+ex)^8}{8e^9}
\end{aligned}$$

```
[In] Int[((a + c*x^2)^3*(A + B*x + C*x^2))/(d + e*x),x]
```

```
[Out] -((((c*d^2 + a*e^2)^2*(8*c*C*d^3 - c*d*e*(7*B*d - 6*A*e) + a*e^2*(2*C*d - B*
e))*x)/e^8) + ((c*d^2 + a*e^2)*(a^2*C*e^4 + c^2*(28*C*d^4 - 3*d^2*e*(7*B*d
- 5*A*e)) + a*c*e^2*(17*C*d^2 - 3*e*(3*B*d - A*e)))*(d + e*x)^2)/(2*e^9) -
(c*(3*a^2*e^4*(4*C*d - B*e) + c^2*(56*C*d^5 - 5*d^3*e*(7*B*d - 4*A*e)) + 6*
a*c*d*e^2*(10*C*d^2 - e*(5*B*d - 2*A*e)))*(d + e*x)^3)/(3*e^9) + (c*(3*a^2*
C*e^4 + 5*c^2*(14*C*d^4 - d^2*e*(7*B*d - 3*A*e)) + 3*a*c*e^2*(15*C*d^2 - e*
(5*B*d - A*e)))*(d + e*x)^4)/(4*e^9) - (c^2*(56*c*C*d^3 - 3*c*d*e*(7*B*d -
2*A*e) + 3*a*e^2*(6*C*d - B*e))*(d + e*x)^5)/(5*e^9) + (c^2*(28*c*C*d^2 + 3
*a*C*e^2 - c*e*(7*B*d - A*e))*(d + e*x)^6)/(6*e^9) - (c^3*(8*C*d - B*e)*(d
+ e*x)^7)/(7*e^9) + (c^3*C*(d + e*x)^8)/(8*e^9) + ((c*d^2 + a*e^2)^3*(C*d^2
 - B*d*e + A*e^2)*Log[d + e*x])/e^9
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} = \int \Bigg(& \frac{\left(cd^2+ae^2\right)^2\left(-8cCd^3+cde(7Bd-6Ae)-ae^2(2Cd-Be)\right)}{e^8} \\
& + \frac{\left(cd^2+ae^2\right)^3\left(Cd^2-Bde+Ae^2\right)}{e^8(d+ex)} \\
& + \frac{\left(cd^2+ae^2\right)\left(a^2Ce^4+c^2(28Cd^4-3d^2e(7Bd-5Ae))+ace^2(17Cd^2-3e(3Bd-Ae))\right)(d+ex)}{e^8} \\
& + \frac{c(-3a^2e^4(4Cd-Be)-c^2(56Cd^5-5d^3e(7Bd-4Ae))-6acde^2(10Cd^2-e(5Bd-2Ae)))(d+ex)^2}{e^8} \\
& + \frac{c(3a^2Ce^4+5c^2(14Cd^4-d^2e(7Bd-3Ae))+3ace^2(15Cd^2-e(5Bd-Ae)))(d+ex)^3}{e^8} \\
& + \frac{c^2(-56cCd^3+3cde(7Bd-2Ae)-3ae^2(6Cd-Be))(d+ex)^4}{e^8} \\
& + \frac{c^2(28cCd^2+3aCe^2-ce(7Bd-Ae))(d+ex)^5}{e^8} + \frac{c^3(-8Cd+Be)(d+ex)^6}{e^8} \\
& + \frac{c^3C(d+ex)^7}{e^8} \Bigg)\, dx
\end{aligned}
$$

$$
\begin{aligned}
= & -\frac{\left(cd^2+ae^2\right)^2\left(8cCd^3-cde(7Bd-6Ae)+ae^2(2Cd-Be)\right)x}{e^8} \\
& + \frac{\left(cd^2+ae^2\right)\left(a^2Ce^4+c^2(28Cd^4-3d^2e(7Bd-5Ae))+ace^2(17Cd^2-3e(3Bd-Ae))\right)(d+ex)^2}{2e^9} \\
& - \frac{c(3a^2e^4(4Cd-Be)+c^2(56Cd^5-5d^3e(7Bd-4Ae))+6acde^2(10Cd^2-e(5Bd-2Ae)))(d+ex}{3e^9} \\
& + \frac{c(3a^2Ce^4+5c^2(14Cd^4-d^2e(7Bd-3Ae))+3ace^2(15Cd^2-e(5Bd-Ae)))(d+ex)^4}{4e^9} \\
& - \frac{c^2(56cCd^3-3cde(7Bd-2Ae)+3ae^2(6Cd-Be))(d+ex)^5}{5e^9} \\
& + \frac{c^2(28cCd^2+3aCe^2-ce(7Bd-Ae))(d+ex)^6}{6e^9} - \frac{c^3(8Cd-Be)(d+ex)^7}{7e^9} \\
& + \frac{c^3C(d+ex)^8}{8e^9} + \frac{\left(cd^2+ae^2\right)^3\left(Cd^2-Bde+Ae^2\right)\log(d+ex)}{e^9}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 498, normalized size of antiderivative = 1.02

$$\int \frac{\left(a+cx^2\right)^3\left(A+Bx+Cx^2\right)}{d+ex}\,dx$$
$$= \frac{x(420a^3e^6(-2Cd+2Be+Cex)+210a^2ce^4(C(-12d^3+6d^2ex-4de^2x^2+3e^3x^3)+2e(3Ae(-2d+ex)+}{}$$
$$+\frac{\left(cd^2+ae^2\right)^3\left(Cd^2+e(-Bd+Ae)\right)\log(d+ex)}{e^9}$$

```
[In] Integrate[((a + c*x^2)^3*(A + B*x + C*x^2))/(d + e*x),x]
```

```
[Out] (x*(420*a^3*e^6*(-2*C*d + 2*B*e + C*e*x) + 210*a^2*c*e^4*(C*(-12*d^3 + 6*d^
2*e*x - 4*d*e^2*x^2 + 3*e^3*x^3) + 2*e*(3*A*e*(-2*d + e*x) + B*(6*d^2 - 3*d
*e*x + 2*e^2*x^2))) + 42*a*c^2*e^2*(C*(-60*d^5 + 30*d^4*e*x - 20*d^3*e^2*x^
2 + 15*d^2*e^3*x^3 - 12*d*e^4*x^4 + 10*e^5*x^5) + e*(5*A*e*(-12*d^3 + 6*d^2
*e*x - 4*d*e^2*x^2 + 3*e^3*x^3) + B*(60*d^4 - 30*d^3*e*x + 20*d^2*e^2*x^2 -
 15*d*e^3*x^3 + 12*e^4*x^4))) + c^3*(C*(-840*d^7 + 420*d^6*e*x - 280*d^5*e^
2*x^2 + 210*d^4*e^3*x^3 - 168*d^3*e^4*x^4 + 140*d^2*e^5*x^5 - 120*d*e^6*x^6
 + 105*e^7*x^7) + 2*e*(7*A*e*(-60*d^5 + 30*d^4*e*x - 20*d^3*e^2*x^2 + 15*d^
2*e^3*x^3 - 12*d*e^4*x^4 + 10*e^5*x^5) + B*(420*d^6 - 210*d^5*e*x + 140*d^4
*e^2*x^2 - 105*d^3*e^3*x^3 + 84*d^2*e^4*x^4 - 70*d*e^5*x^5 + 60*e^6*x^6))))
)/(840*e^8) + ((c*d^2 + a*e^2)^3*(C*d^2 + e*(-(B*d) + A*e))*Log[d + e*x])/e
^9
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 679, normalized size of antiderivative = 1.39

method	result
norman	$\frac{\left(3A\,a^2c\,e^6+3Aa\,c^2d^2e^4+A\,c^3d^4e^2-3B\,a^2cd\,e^5-3Ba\,c^2d^3e^3-B\,c^3d^5e+a^3C\,e^6+3C\,a^2c\,d^2e^4+3Ca\,c^2d^4e^2+C\,c^3d^6\right)x^2}{2e^7}-$
default	$-\frac{C\,a^3d\,e^6x+C\,a^2cd\,e^6x^3+Ca\,c^2d^3e^4x^3-\frac{3}{2}Ca\,c^2d^4e^3x^2-\frac{3}{2}A\,x^2a\,c^2d^2e^5+\frac{3}{2}B\,x^2a^2cd\,e^6+\frac{3}{2}B\,x^2a\,c^2d^3e^4+3Ax\,a^2cd\,e^6+3Ax}{}$
risch	$-\frac{C\,c^3d\,x^7}{7e^2}+\frac{Ca\,c^2x^6}{2e}-\frac{3Ca\,c^2d^5x}{e^6}-\frac{3C\,a^2c\,d^3x}{e^4}-\frac{C\,a^2cd\,x^3}{e^2}+\frac{3A\,x^2a\,c^2d^2}{2e^3}-\frac{3B\,x^2a^2cd}{2e^2}-\frac{3B\,x^2a\,c^2d^3}{2e^4}-$
parallelrisch	$\frac{-280C\,x^3c^3d^5e^3+1260A\,x^2a^2c\,e^8+420A\,x^2c^3d^4e^4-420B\,x^2c^3d^5e^3+420C\,x^2c^3d^6e^2-840Ax\,c^3d^5e^3+840Bx\,c^3d^6e^2-840Cx}{}$

```
[In] int((c*x^2+a)^3*(C*x^2+B*x+A)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/e^7*(3*A*a^2*c*e^6+3*A*a*c^2*d^2*e^4+A*c^3*d^4*e^2-3*B*a^2*c*d*e^5-3*B*
a*c^2*d^3*e^3-B*c^3*d^5*e+C*a^3*e^6+3*C*a^2*c*d^2*e^4+3*C*a*c^2*d^4*e^2+C*c
^3*d^6)*x^2-(3*A*a^2*c*d*e^6+3*A*a*c^2*d^3*e^4+A*c^3*d^5*e^2-B*a^3*e^7-3*B*
a^2*c*d^2*e^5-3*B*a*c^2*d^4*e^3-B*c^3*d^6*e+C*a^3*d*e^6+3*C*a^2*c*d^3*e^4+3
*C*a*c^2*d^5*e^2+C*c^3*d^7)/e^8*x+1/8*C*c^3/e*x^8-1/3*c/e^6*(3*A*a*c*d*e^4+
```

```
A*c^2*d^3*e^2-3*B*a^2*e^5-3*B*a*c*d^2*e^3-B*c^2*d^4*e+3*C*a^2*d*e^4+3*C*a*c
*d^3*e^2+C*c^2*d^5)*x^3+1/4*c/e^5*(3*A*a*c*e^4+A*c^2*d^2*e^2-3*B*a*c*d*e^3-
B*c^2*d^3*e+3*C*a^2*e^4+3*C*a*c*d^2*e^2+C*c^2*d^4)*x^4-1/5*c^2/e^4*(A*c*d*e
^2-3*B*a*e^3-B*c*d^2*e+3*C*a*d*e^2+C*c*d^3)*x^5+1/6*c^2/e^3*(A*c*e^2-B*c*d*
e+3*C*a*e^2+C*c*d^2)*x^6+1/7*c^3/e^2*(B*e-C*d)*x^7+(A*a^3*e^8+3*A*a^2*c*d^2
*e^6+3*A*a*c^2*d^4*e^4+A*c^3*d^6*e^2-B*a^3*d*e^7-3*B*a^2*c*d^3*e^5-3*B*a*c^
2*d^5*e^3-B*c^3*d^7*e+C*a^3*d^2*e^6+3*C*a^2*c*d^4*e^4+3*C*a*c^2*d^6*e^2+C*c
^3*d^8)/e^9*ln(e*x+d)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 674, normalized size of antiderivative = 1.38

$$\int \frac{(a+cx^2)^3\,(A+Bx+Cx^2)}{d+ex}\,dx = \frac{105\,Cc^3e^8x^8 - 120\,(Cc^3de^7 - Bc^3e^8)x^7 + 140\,(Cc^3d^2e^6 - Bc^3de^7 + (3\,Cac^2 + Ac^3)e^8)x^6 - 168\,(Cc^3d^3e^5 -}{}$$

```
[In] integrate((c*x^2+a)^3*(C*x^2+B*x+A)/(e*x+d),x, algorithm="fricas")
```

```
[Out] 1/840*(105*C*c^3*e^8*x^8 - 120*(C*c^3*d*e^7 - B*c^3*e^8)*x^7 + 140*(C*c^3*d
^2*e^6 - B*c^3*d*e^7 + (3*C*a*c^2 + A*c^3)*e^8)*x^6 - 168*(C*c^3*d^3*e^5 -
B*c^3*d^2*e^6 - 3*B*a*c^2*e^8 + (3*C*a*c^2 + A*c^3)*d*e^7)*x^5 + 210*(C*c^3
*d^4*e^4 - B*c^3*d^3*e^5 - 3*B*a*c^2*d*e^7 + (3*C*a*c^2 + A*c^3)*d^2*e^6 +
3*(C*a^2*c + A*a*c^2)*e^8)*x^4 - 280*(C*c^3*d^5*e^3 - B*c^3*d^4*e^4 - 3*B*a
*c^2*d^2*e^6 - 3*B*a^2*c*e^8 + (3*C*a*c^2 + A*c^3)*d^3*e^5 + 3*(C*a^2*c + A
*a*c^2)*d*e^7)*x^3 + 420*(C*c^3*d^6*e^2 - B*c^3*d^5*e^3 - 3*B*a*c^2*d^3*e^5
 - 3*B*a^2*c*d*e^7 + (3*C*a*c^2 + A*c^3)*d^4*e^4 + 3*(C*a^2*c + A*a*c^2)*d^
2*e^6 + (C*a^3 + 3*A*a^2*c)*e^8)*x^2 - 840*(C*c^3*d^7*e - B*c^3*d^6*e^2 - 3
*B*a*c^2*d^4*e^4 - 3*B*a^2*c*d^2*e^6 - B*a^3*e^8 + (3*C*a*c^2 + A*c^3)*d^5*
e^3 + 3*(C*a^2*c + A*a*c^2)*d^3*e^5 + (C*a^3 + 3*A*a^2*c)*d*e^7)*x + 840*(C
*c^3*d^8 - B*c^3*d^7*e - 3*B*a*c^2*d^5*e^3 - 3*B*a^2*c*d^3*e^5 - B*a^3*d*e^
7 + A*a^3*e^8 + (3*C*a*c^2 + A*c^3)*d^6*e^2 + 3*(C*a^2*c + A*a*c^2)*d^4*e^4
 + (C*a^3 + 3*A*a^2*c)*d^2*e^6)*log(e*x + d))/e^9
```

Sympy [A] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 685, normalized size of antiderivative = 1.40

$$
\begin{aligned}
\int \frac{(a+cx^2)^3\,(A+Bx+Cx^2)}{d+ex}\,dx &= \frac{Cc^3x^8}{8e}+x^7\left(\frac{Bc^3}{7e}-\frac{Cc^3d}{7e^2}\right) \\
&\quad +x^6\left(\frac{Ac^3}{6e}-\frac{Bc^3d}{6e^2}+\frac{Cac^2}{2e}+\frac{Cc^3d^2}{6e^3}\right) \\
&\quad +x^5\left(-\frac{Ac^3d}{5e^2}+\frac{3Bac^2}{5e}+\frac{Bc^3d^2}{5e^3}-\frac{3Cac^2d}{5e^2}-\frac{Cc^3d^3}{5e^4}\right) \\
&\quad +x^4\cdot\left(\frac{3Aac^2}{4e}+\frac{Ac^3d^2}{4e^3}-\frac{3Bac^2d}{4e^2}-\frac{Bc^3d^3}{4e^4}+\frac{3Ca^2c}{4e}\right. \\
&\quad \left.+\frac{3Cac^2d^2}{4e^3}+\frac{Cc^3d^4}{4e^5}\right)+x^3\left(-\frac{Aac^2d}{e^2}-\frac{Ac^3d^3}{3e^4}+\frac{Ba^2c}{e}\right. \\
&\quad \left.+\frac{Bac^2d^2}{e^3}+\frac{Bc^3d^4}{3e^5}-\frac{Ca^2cd}{e^2}-\frac{Cac^2d^3}{e^4}-\frac{Cc^3d^5}{3e^6}\right)+x^2 \\
&\quad \cdot\left(\frac{3Aa^2c}{2e}+\frac{3Aac^2d^2}{2e^3}+\frac{Ac^3d^4}{2e^5}-\frac{3Ba^2cd}{2e^2}-\frac{3Bac^2d^3}{2e^4}\right. \\
&\quad \left.-\frac{Bc^3d^5}{2e^6}+\frac{Ca^3}{2e}+\frac{3Ca^2cd^2}{2e^3}+\frac{3Cac^2d^4}{2e^5}+\frac{Cc^3d^6}{2e^7}\right) \\
&\quad +x\left(-\frac{3Aa^2cd}{e^2}-\frac{3Aac^2d^3}{e^4}-\frac{Ac^3d^5}{e^6}+\frac{Ba^3}{e}+\frac{3Ba^2cd^2}{e^3}\right. \\
&\quad +\frac{3Bac^2d^4}{e^5}+\frac{Bc^3d^6}{e^7}-\frac{Ca^3d}{e^2}-\frac{3Ca^2cd^3}{e^4}-\frac{3Cac^2d^5}{e^6} \\
&\quad \left.-\frac{Cc^3d^7}{e^8}\right) \\
&\quad +\frac{(ae^2+cd^2)^3\,(Ae^2-Bde+Cd^2)\log{(d+ex)}}{e^9}
\end{aligned}
$$

```
[In] integrate((c*x**2+a)**3*(C*x**2+B*x+A)/(e*x+d),x)
```

```
[Out] C*c**3*x**8/(8*e) + x**7*(B*c**3/(7*e) - C*c**3*d/(7*e**2)) + x**6*(A*c**3/
(6*e) - B*c**3*d/(6*e**2) + C*a*c**2/(2*e) + C*c**3*d**2/(6*e**3)) + x**5*(
-A*c**3*d/(5*e**2) + 3*B*a*c**2/(5*e) + B*c**3*d**2/(5*e**3) - 3*C*a*c**2*d
/(5*e**2) - C*c**3*d**3/(5*e**4)) + x**4*(3*A*a*c**2/(4*e) + A*c**3*d**2/(4
*e**3) - 3*B*a*c**2*d/(4*e**2) - B*c**3*d**3/(4*e**4) + 3*C*a**2*c/(4*e) +
3*C*a*c**2*d**2/(4*e**3) + C*c**3*d**4/(4*e**5)) + x**3*(-A*a*c**2*d/e**2 -
 A*c**3*d**3/(3*e**4) + B*a**2*c/e + B*a*c**2*d**2/e**3 + B*c**3*d**4/(3*e*
*5) - C*a**2*c*d/e**2 - C*a*c**2*d**3/e**4 - C*c**3*d**5/(3*e**6)) + x**2*(
3*A*a**2*c/(2*e) + 3*A*a*c**2*d**2/(2*e**3) + A*c**3*d**4/(2*e**5) - 3*B*a*
*2*c*d/(2*e**2) - 3*B*a*c**2*d**3/(2*e**4) - B*c**3*d**5/(2*e**6) + C*a**3/
(2*e) + 3*C*a**2*c*d**2/(2*e**3) + 3*C*a*c**2*d**4/(2*e**5) + C*c**3*d**6/(
2*e**7)) + x*(-3*A*a**2*c*d/e**2 - 3*A*a*c**2*d**3/e**4 - A*c**3*d**5/e**6
```

```
+ B*a**3/e + 3*B*a**2*c*d**2/e**3 + 3*B*a*c**2*d**4/e**5 + B*c**3*d**6/e**7
 - C*a**3*d/e**2 - 3*C*a**2*c*d**3/e**4 - 3*C*a*c**2*d**5/e**6 - C*c**3*d**
7/e**8) + (a*e**2 + c*d**2)**3*(A*e**2 - B*d*e + C*d**2)*log(d + e*x)/e**9
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 672, normalized size of antiderivative = 1.37

$$\int \frac{(a+cx^2)^3 (A+Bx+Cx^2)}{d+ex}\,dx = \frac{105\,Cc^3e^7x^8 - 120\,(Cc^3de^6 - Bc^3e^7)x^7 + 140\,(Cc^3d^2e^5 - Bc^3de^6 + (3\,Cac^2 + Ac^3)e^7)x^6 - 168\,(Cc^3d^3e^4 -}{} + \frac{(Cc^3d^8 - Bc^3d^7e - 3\,Bac^2d^5e^3 - 3\,Ba^2cd^3e^5 - Ba^3de^7 + Aa^3e^8 + (3\,Cac^2 + Ac^3)d^6e^2 + 3\,(Ca^2c + Aac}{e^9}$$

```
[In] integrate((c*x^2+a)^3*(C*x^2+B*x+A)/(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/840*(105*C*c^3*e^7*x^8 - 120*(C*c^3*d*e^6 - B*c^3*e^7)*x^7 + 140*(C*c^3*d
^2*e^5 - B*c^3*d*e^6 + (3*C*a*c^2 + A*c^3)*e^7)*x^6 - 168*(C*c^3*d^3*e^4 -
B*c^3*d^2*e^5 - 3*B*a*c^2*e^7 + (3*C*a*c^2 + A*c^3)*d*e^6)*x^5 + 210*(C*c^3
*d^4*e^3 - B*c^3*d^3*e^4 - 3*B*a*c^2*d*e^6 + (3*C*a*c^2 + A*c^3)*d^2*e^5 +
3*(C*a^2*c + A*a*c^2)*e^7)*x^4 - 280*(C*c^3*d^5*e^2 - B*c^3*d^4*e^3 - 3*B*a
*c^2*d^2*e^5 - 3*B*a^2*c*e^7 + (3*C*a*c^2 + A*c^3)*d^3*e^4 + 3*(C*a^2*c + A
*a*c^2)*d*e^6)*x^3 + 420*(C*c^3*d^6*e - B*c^3*d^5*e^2 - 3*B*a*c^2*d^3*e^4 -
 3*B*a^2*c*d*e^6 + (3*C*a*c^2 + A*c^3)*d^4*e^3 + 3*(C*a^2*c + A*a*c^2)*d^2*
e^5 + (C*a^3 + 3*A*a^2*c)*e^7)*x^2 - 840*(C*c^3*d^7 - B*c^3*d^6*e - 3*B*a*c
^2*d^4*e^3 - 3*B*a^2*c*d^2*e^5 - B*a^3*e^7 + (3*C*a*c^2 + A*c^3)*d^5*e^2 +
3*(C*a^2*c + A*a*c^2)*d^3*e^4 + (C*a^3 + 3*A*a^2*c)*d*e^6)*x)/e^8 + (C*c^3*
d^8 - B*c^3*d^7*e - 3*B*a*c^2*d^5*e^3 - 3*B*a^2*c*d^3*e^5 - B*a^3*d*e^7 + A
*a^3*e^8 + (3*C*a*c^2 + A*c^3)*d^6*e^2 + 3*(C*a^2*c + A*a*c^2)*d^4*e^4 + (C
*a^3 + 3*A*a^2*c)*d^2*e^6)*log(e*x + d)/e^9
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 817, normalized size of antiderivative = 1.67

$$\int \frac{(a+cx^2)^3 (A+Bx+Cx^2)}{d+ex}\,dx = \frac{105\,Cc^3e^7x^8 - 120\,Cc^3de^6x^7 + 120\,Bc^3e^7x^7 + 140\,Cc^3d^2e^5x^6 - 140\,Bc^3de^6x^6 + 420\,Cac^2e^7x^6 + 140\,Ac^3e^7}{} + \frac{(Cc^3d^8 - Bc^3d^7e + 3\,Cac^2d^6e^2 + Ac^3d^6e^2 - 3\,Bac^2d^5e^3 + 3\,Ca^2cd^4e^4 + 3\,Aac^2d^4e^4 - 3\,Ba^2cd^3e^5 + Ca^3}{e^9}$$

```
[In] integrate((c*x^2+a)^3*(C*x^2+B*x+A)/(e*x+d),x, algorithm="giac")
```

```
[Out] 1/840*(105*C*c^3*e^7*x^8 - 120*C*c^3*d*e^6*x^7 + 120*B*c^3*e^7*x^7 + 140*C*
c^3*d^2*e^5*x^6 - 140*B*c^3*d*e^6*x^6 + 420*C*a*c^2*e^7*x^6 + 140*A*c^3*e^7
*x^6 - 168*C*c^3*d^3*e^4*x^5 + 168*B*c^3*d^2*e^5*x^5 - 504*C*a*c^2*d*e^6*x^
5 - 168*A*c^3*d*e^6*x^5 + 504*B*a*c^2*e^7*x^5 + 210*C*c^3*d^4*e^3*x^4 - 210
*B*c^3*d^3*e^4*x^4 + 630*C*a*c^2*d^2*e^5*x^4 + 210*A*c^3*d^2*e^5*x^4 - 630*
B*a*c^2*d*e^6*x^4 + 630*C*a^2*c*e^7*x^4 + 630*A*a*c^2*e^7*x^4 - 280*C*c^3*d
^5*e^2*x^3 + 280*B*c^3*d^4*e^3*x^3 - 840*C*a*c^2*d^3*e^4*x^3 - 280*A*c^3*d^
3*e^4*x^3 + 840*B*a*c^2*d^2*e^5*x^3 - 840*C*a^2*c*d*e^6*x^3 - 840*A*a*c^2*d
*e^6*x^3 + 840*B*a^2*c*e^7*x^3 + 420*C*c^3*d^6*e*x^2 - 420*B*c^3*d^5*e^2*x^
2 + 1260*C*a*c^2*d^4*e^3*x^2 + 420*A*c^3*d^4*e^3*x^2 - 1260*B*a*c^2*d^3*e^4
*x^2 + 1260*C*a^2*c*d^2*e^5*x^2 + 1260*A*a*c^2*d^2*e^5*x^2 - 1260*B*a^2*c*d
*e^6*x^2 + 420*C*a^3*e^7*x^2 + 1260*A*a^2*c*e^7*x^2 - 840*C*c^3*d^7*x + 840
*B*c^3*d^6*e*x - 2520*C*a*c^2*d^5*e^2*x - 840*A*c^3*d^5*e^2*x + 2520*B*a*c^
2*d^4*e^3*x - 2520*C*a^2*c*d^3*e^4*x - 2520*A*a*c^2*d^3*e^4*x + 2520*B*a^2*
c*d^2*e^5*x - 840*C*a^3*d*e^6*x - 2520*A*a^2*c*d*e^6*x + 840*B*a^3*e^7*x)/e
^8 + (C*c^3*d^8 - B*c^3*d^7*e + 3*C*a*c^2*d^6*e^2 + A*c^3*d^6*e^2 - 3*B*a*c
^2*d^5*e^3 + 3*C*a^2*c*d^4*e^4 + 3*A*a*c^2*d^4*e^4 - 3*B*a^2*c*d^3*e^5 + C*
a^3*d^2*e^6 + 3*A*a^2*c*d^2*e^6 - B*a^3*d*e^7 + A*a^3*e^8)*log(abs(e*x + d)
)/e^9
```

Mupad [B] (verification not implemented)

Time = 12.38 (sec) , antiderivative size = 741, normalized size of antiderivative = 1.51

$$\int \frac{\left(a+c x^2\right)^3 \left(A+B x+C x^2\right)}{d+e x}\,dx = x\left(\frac{B\,a^3}{e}\right.$$

```
[In] int(((a + c*x^2)^3*(A + B*x + C*x^2))/(d + e*x),x)
```

```
[Out] x*((B*a^3)/e - (d*((C*a^3 + 3*A*a^2*c)/e + (d*((d*((d*((d*((A*c^3 + 3*C*a*c
^2)/e - (d*((B*c^3)/e - (C*c^3*d)/e^2))/e))/e - (3*B*a*c^2)/e))/e + (3*a*c*
(A*c + C*a))/e))/e - (3*B*a^2*c)/e))/e))/e) + x^7*((B*c^3)/(7*e) - (C*c^3*d
)/(7*e^2)) - x^5*((d*((A*c^3 + 3*C*a*c^2)/e - (d*((B*c^3)/e - (C*c^3*d)/e^2
))/e))/(5*e) - (3*B*a*c^2)/(5*e)) + x^4*((d*((d*((A*c^3 + 3*C*a*c^2)/e - (d
*((B*c^3)/e - (C*c^3*d)/e^2))/e))/e - (3*B*a*c^2)/e))/(4*e) + (3*a*c*(A*c +
 C*a))/(4*e)) + x^2*((C*a^3 + 3*A*a^2*c)/(2*e) + (d*((d*((d*((d*((A*c^3 + 3
*C*a*c^2)/e - (d*((B*c^3)/e - (C*c^3*d)/e^2))/e))/e - (3*B*a*c^2)/e))/e + (
3*a*c*(A*c + C*a))/e))/e - (3*B*a^2*c)/e))/(2*e)) + x^6*((A*c^3 + 3*C*a*c^2
)/(6*e) - (d*((B*c^3)/e - (C*c^3*d)/e^2))/(6*e)) - x^3*((d*((d*((d*((A*c^3
+ 3*C*a*c^2)/e - (d*((B*c^3)/e - (C*c^3*d)/e^2))/e))/e - (3*B*a*c^2)/e))/e
+ (3*a*c*(A*c + C*a))/e))/(3*e) - (B*a^2*c)/e) + (log(d + e*x)*(A*a^3*e^8 +
 C*c^3*d^8 - B*a^3*d*e^7 - B*c^3*d^7*e + A*c^3*d^6*e^2 + C*a^3*d^2*e^6 + 3*
A*a*c^2*d^4*e^4 + 3*A*a^2*c*d^2*e^6 - 3*B*a*c^2*d^5*e^3 - 3*B*a^2*c*d^3*e^5
 + 3*C*a*c^2*d^6*e^2 + 3*C*a^2*c*d^4*e^4))/e^9 + (C*c^3*x^8)/(8*e)
```

3.37 $\int \frac{(a+cx^2)^3(A+Bx+Cx^2)}{(d+ex)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 486

$$
\begin{aligned}
&\int \frac{(a+cx^2)^3(A+Bx+Cx^2)}{(d+ex)^2}\,dx \\
&= \frac{(a^3Ce^6+c^3d^4(7Cd^2-e(6Bd-5Ae))+3ac^2d^2e^2(5Cd^2-e(4Bd-3Ae))+3a^2ce^4(3Cd^2-e(2Bd-Ae))}{e^8} \\
&\quad - \frac{c(3a^2e^4(2Cd-Be)+c^2d^3(6Cd^2-e(5Bd-4Ae))+3acde^2(4Cd^2-e(3Bd-2Ae)))\,x^2}{2e^7} \\
&\quad + \frac{c(3a^2Ce^4+c^2d^2(5Cd^2-e(4Bd-3Ae))+3ace^2(3Cd^2-e(2Bd-Ae)))\,x^3}{3e^6} \\
&\quad - \frac{c^2(3ae^2(2Cd-Be)+cd(4Cd^2-e(3Bd-2Ae)))\,x^4}{4e^5} \\
&\quad + \frac{c^2(3aCe^2+c(3Cd^2-e(2Bd-Ae)))\,x^5}{5e^4} - \frac{c^3(2Cd-Be)x^6}{6e^3} \\
&\quad + \frac{c^3Cx^7}{7e^2} - \frac{(cd^2+ae^2)^3(Cd^2-Bde+Ae^2)}{e^9(d+ex)} \\
&\quad - \frac{(cd^2+ae^2)^2(ae^2(2Cd-Be)+cd(8Cd^2-e(7Bd-6Ae)))\log(d+ex)}{e^9}
\end{aligned}
$$

```
[Out] (a^3*C*e^6+c^3*d^4*(7*C*d^2-e*(-5*A*e+6*B*d))+3*a*c^2*d^2*e^2*(5*C*d^2-e*(-
3*A*e+4*B*d))+3*a^2*c*e^4*(3*C*d^2-e*(-A*e+2*B*d)))*x/e^8-1/2*c*(3*a^2*e^4*
(-B*e+2*C*d)+c^2*d^3*(6*C*d^2-e*(-4*A*e+5*B*d))+3*a*c*d*e^2*(4*C*d^2-e*(-2*
A*e+3*B*d)))*x^2/e^7+1/3*c*(3*a^2*C*e^4+c^2*d^2*(5*C*d^2-e*(-3*A*e+4*B*d))+
3*a*c*e^2*(3*C*d^2-e*(-A*e+2*B*d)))*x^3/e^6-1/4*c^2*(3*a*e^2*(-B*e+2*C*d)+c
*d*(4*C*d^2-e*(-2*A*e+3*B*d)))*x^4/e^5+1/5*c^2*(3*a*C*e^2+c*(3*C*d^2-e*(-A*
e+2*B*d)))*x^5/e^4-1/6*c^3*(-B*e+2*C*d)*x^6/e^3+1/7*c^3*C*x^7/e^2-(a*e^2+c*
d^2)^3*(A*e^2-B*d*e+C*d^2)/e^9/(e*x+d)-(a*e^2+c*d^2)^2*(a*e^2*(-B*e+2*C*d)+
c*d*(8*C*d^2-e*(-6*A*e+7*B*d)))*ln(e*x+d)/e^9
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 483, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$, Rules used = {1642}

$$\int \frac{(a+cx^2)^3(A+Bx+Cx^2)}{(d+ex)^2}\,dx$$

$$= \frac{cx^3(3a^2Ce^4+3ace^2(3Cd^2-e(2Bd-Ae))+c^2(5Cd^4-d^2e(4Bd-3Ae)))}{3e^6}$$

$$-\frac{cx^2(3a^2e^4(2Cd-Be)+3acde^2(4Cd^2-e(3Bd-2Ae))+c^2(6Cd^5-d^3e(5Bd-4Ae)))}{2e^7}$$

$$+\frac{x(a^3Ce^6+3a^2ce^4(3Cd^2-e(2Bd-Ae))+3ac^2d^2e^2(5Cd^2-e(4Bd-3Ae))+c^3(7Cd^6-d^4e(6Bd-5}{e^8}$$

$$-\frac{c^2x^4(3ae^2(2Cd-Be)-cde(3Bd-2Ae)+4cCd^3)}{4e^5}$$

$$+\frac{c^2x^5(3aCe^2-ce(2Bd-Ae)+3cCd^2)}{5e^4}-\frac{(ae^2+cd^2)^3(Ae^2-Bde+Cd^2)}{e^9(d+ex)}$$

$$-\frac{(ae^2+cd^2)^2\log(d+ex)(ae^2(2Cd-Be)-cde(7Bd-6Ae)+8cCd^3)}{e^9}$$

$$-\frac{c^3x^6(2Cd-Be)}{6e^3}+\frac{c^3Cx^7}{7e^2}$$

```
[In] Int[((a + c*x^2)^3*(A + B*x + C*x^2))/(d + e*x)^2,x]
```

```
[Out] ((a^3*C*e^6 + c^3*(7*C*d^6 - d^4*e*(6*B*d - 5*A*e)) + 3*a*c^2*d^2*e^2*(5*C*
d^2 - e*(4*B*d - 3*A*e)) + 3*a^2*c*e^4*(3*C*d^2 - e*(2*B*d - A*e)))*x)/e^8
- (c*(3*a^2*e^4*(2*C*d - B*e) + c^2*(6*C*d^5 - d^3*e*(5*B*d - 4*A*e)) + 3*a
*c*d*e^2*(4*C*d^2 - e*(3*B*d - 2*A*e)))*x^2)/(2*e^7) + (c*(3*a^2*C*e^4 + c^
2*(5*C*d^4 - d^2*e*(4*B*d - 3*A*e)) + 3*a*c*e^2*(3*C*d^2 - e*(2*B*d - A*e))
)*x^3)/(3*e^6) - (c^2*(4*c*C*d^3 - c*d*e*(3*B*d - 2*A*e) + 3*a*e^2*(2*C*d -
 B*e))*x^4)/(4*e^5) + (c^2*(3*c*C*d^2 + 3*a*C*e^2 - c*e*(2*B*d - A*e))*x^5)
/(5*e^4) - (c^3*(2*C*d - B*e)*x^6)/(6*e^3) + (c^3*C*x^7)/(7*e^2) - ((c*d^2
+ a*e^2)^3*(C*d^2 - B*d*e + A*e^2))/(e^9*(d + e*x)) - ((c*d^2 + a*e^2)^2*(8
*c*C*d^3 - c*d*e*(7*B*d - 6*A*e) + a*e^2*(2*C*d - B*e))*Log[d + e*x])/e^9
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

integral

$$= \int \left(\frac{a^3Ce^6 + c^3(7Cd^6 - d^4e(6Bd - 5Ae)) + 3ac^2d^2e^2(5Cd^2 - e(4Bd - 3Ae)) + 3a^2ce^4(3Cd^2 - e(2Bd - A}{e^8} \right.$$

$$+ \frac{c(-3a^2e^4(2Cd - Be) - c^2(6Cd^5 - d^3e(5Bd - 4Ae)) - 3acde^2(4Cd^2 - e(3Bd - 2Ae)))\,x}{e^7}$$

$$+ \frac{c(3a^2Ce^4 + c^2(5Cd^4 - d^2e(4Bd - 3Ae)) + 3ace^2(3Cd^2 - e(2Bd - Ae)))\,x^2}{e^6}$$

$$+ \frac{c^2(-4cCd^3 + cde(3Bd - 2Ae) - 3ae^2(2Cd - Be))\,x^3}{e^5}$$

$$+ \frac{c^2(3cCd^2 + 3aCe^2 - ce(2Bd - Ae))\,x^4}{e^4} + \frac{c^3(-2Cd + Be)x^5}{e^3} + \frac{c^3Cx^6}{e^2}$$

$$+ \frac{(cd^2 + ae^2)^3\,(Cd^2 - Bde + Ae^2)}{e^8(d + ex)^2}$$

$$\left. + \frac{(cd^2 + ae^2)^2\,(-8cCd^3 + cde(7Bd - 6Ae) - ae^2(2Cd - Be))}{e^8(d + ex)} \right) dx$$

$$= \frac{(a^3Ce^6 + c^3(7Cd^6 - d^4e(6Bd - 5Ae)) + 3ac^2d^2e^2(5Cd^2 - e(4Bd - 3Ae)) + 3a^2ce^4(3Cd^2 - e(2Bd}{e^8}$$

$$- \frac{c(3a^2e^4(2Cd - Be) + c^2(6Cd^5 - d^3e(5Bd - 4Ae)) + 3acde^2(4Cd^2 - e(3Bd - 2Ae)))\,x^2}{2e^7}$$

$$+ \frac{c(3a^2Ce^4 + c^2(5Cd^4 - d^2e(4Bd - 3Ae)) + 3ace^2(3Cd^2 - e(2Bd - Ae)))\,x^3}{3e^6}$$

$$- \frac{c^2(4cCd^3 - cde(3Bd - 2Ae) + 3ae^2(2Cd - Be))\,x^4}{4e^5}$$

$$+ \frac{c^2(3cCd^2 + 3aCe^2 - ce(2Bd - Ae))\,x^5}{5e^4} - \frac{c^3(2Cd - Be)x^6}{6e^3}$$

$$+ \frac{c^3Cx^7}{7e^2} - \frac{(cd^2 + ae^2)^3\,(Cd^2 - Bde + Ae^2)}{e^9(d + ex)}$$

$$- \frac{(cd^2 + ae^2)^2\,(8cCd^3 - cde(7Bd - 6Ae) + ae^2(2Cd - Be))\log(d + ex)}{e^9}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 641, normalized size of antiderivative = 1.32

$$\int \frac{(a+cx^2)^3 (A+Bx+Cx^2)}{(d+ex)^2}\,dx$$
$$= \frac{420a^3e^6(e(Bd-Ae)+C(-d^2+dex+e^2x^2))+210a^2ce^4(2C(-3d^4+9d^3ex+6d^2e^2x^2-2de^3x^3+e^4x^4)}{}$$

```
[In] Integrate[((a + c*x^2)^3*(A + B*x + C*x^2))/(d + e*x)^2,x]
```

```
[Out] (420*a^3*e^6*(e*(B*d - A*e) + C*(-d^2 + d*e*x + e^2*x^2)) + 210*a^2*c*e^4*(
2*C*(-3*d^4 + 9*d^3*e*x + 6*d^2*e^2*x^2 - 2*d*e^3*x^3 + e^4*x^4) + 3*e*(2*A
*e*(-d^2 + d*e*x + e^2*x^2) + B*(2*d^3 - 4*d^2*e*x - 3*d*e^2*x^2 + e^3*x^3)
)) + 21*a*c^2*e^2*(-6*C*(10*d^6 - 50*d^5*e*x - 30*d^4*e^2*x^2 + 10*d^3*e^3*
x^3 - 5*d^2*e^4*x^4 + 3*d*e^5*x^5 - 2*e^6*x^6) + 5*e*(4*A*e*(-3*d^4 + 9*d^3
*e*x + 6*d^2*e^2*x^2 - 2*d*e^3*x^3 + e^4*x^4) + B*(12*d^5 - 48*d^4*e*x - 30
*d^3*e^2*x^2 + 10*d^2*e^3*x^3 - 5*d*e^4*x^4 + 3*e^5*x^5))) + c^3*(-4*C*(105
*d^8 - 735*d^7*e*x - 420*d^6*e^2*x^2 + 140*d^5*e^3*x^3 - 70*d^4*e^4*x^4 + 4
2*d^3*e^5*x^5 - 28*d^2*e^6*x^6 + 20*d*e^7*x^7 - 15*e^8*x^8) + 7*e*(6*A*e*(-
10*d^6 + 50*d^5*e*x + 30*d^4*e^2*x^2 - 10*d^3*e^3*x^3 + 5*d^2*e^4*x^4 - 3*d
*e^5*x^5 + 2*e^6*x^6) + B*(60*d^7 - 360*d^6*e*x - 210*d^5*e^2*x^2 + 70*d^4*
e^3*x^3 - 35*d^3*e^4*x^4 + 21*d^2*e^5*x^5 - 14*d*e^6*x^6 + 10*e^7*x^7))) -
420*(c*d^2 + a*e^2)^2*(8*c*C*d^3 + c*d*e*(-7*B*d + 6*A*e) + a*e^2*(2*C*d -
B*e))*(d + e*x)*Log[d + e*x])/(420*e^9*(d + e*x))
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 709, normalized size of antiderivative = 1.46

method	result
norman	$\frac{\left(A a^3 e^8+6 A a^2 c d^2 e^6+12 A a c^2 d^4 e^4+6 A c^3 d^6 e^2-B a^3 d e^7-9 B a^2 c d^3 e^5-15 B a c^2 d^5 e^3-7 B c^3 d^7 e+2 C a^3 d^2 e^6+12 C a^2 c d^4 e^4+18 C a c^2 d^6 e\right.}{e^8 d}$
default	$3 C a c^2 d^2 e^4 x^3-3 C c^3 d^5 e x^2+3 A a^2 c e^6 x+a^3 C e^6 x+7 C c^3 d^6 x+\frac{1}{6} B c^3 e^6 x^6+\frac{1}{5} A c^3 e^6 x^5+\frac{1}{7} c^3 C x^7 e^6+A a c^2 e^6 x^3+A c^3 d^2 e^4 x^3+$
risch	$-\frac{A c^3 d^6}{e^7 (ex+d)}+\frac{B a^3 d}{e^2 (ex+d)}+\frac{B c^3 d^7}{e^8 (ex+d)}-\frac{C a^3 d^2}{e^3 (ex+d)}-\frac{C c^3 d^8}{e^9 (ex+d)}-\frac{6 \ln(ex+d) A c^3 d^5}{e^7}+\frac{7 \ln(ex+d) B c^3 d^6}{e^8}-$
parallelrisch	Expression too large to display

```
[In] int((c*x^2+a)^3*(C*x^2+B*x+A)/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] ((A*a^3*e^8+6*A*a^2*c*d^2*e^6+12*A*a*c^2*d^4*e^4+6*A*c^3*d^6*e^2-B*a^3*d*e^
7-9*B*a^2*c*d^3*e^5-15*B*a*c^2*d^5*e^3-7*B*c^3*d^7*e+2*C*a^3*d^2*e^6+12*C*a
^2*c*d^4*e^4+18*C*a*c^2*d^6*e^2+8*C*c^3*d^8)/e^8/d*x+1/2*(6*A*a^2*c*e^6+12*
A*a*c^2*d^2*e^4+6*A*c^3*d^4*e^2-9*B*a^2*c*d*e^5-15*B*a*c^2*d^3*e^3-7*B*c^3*
```

```
d^5*e+2*C*a^3*e^6+12*C*a^2*c*d^2*e^4+18*C*a*c^2*d^4*e^2+8*C*c^3*d^6)/e^7*x^
2+1/7*C*c^3/e*x^8+1/12*c*(12*A*a*c*e^4+6*A*c^2*d^2*e^2-15*B*a*c*d*e^3-7*B*c
^2*d^3*e+12*C*a^2*e^4+18*C*a*c*d^2*e^2+8*C*c^2*d^4)/e^5*x^4-1/6*c*(12*A*a*c
*d*e^4+6*A*c^2*d^3*e^2-9*B*a^2*e^5-15*B*a*c*d^2*e^3-7*B*c^2*d^4*e+12*C*a^2*
d*e^4+18*C*a*c*d^3*e^2+8*C*c^2*d^5)/e^6*x^3+1/30*c^2*(6*A*c*e^2-7*B*c*d*e+1
8*C*a*e^2+8*C*c*d^2)/e^3*x^6-1/20*c^2*(6*A*c*d*e^2-15*B*a*e^3-7*B*c*d^2*e+1
8*C*a*d*e^2+8*C*c*d^3)/e^4*x^5+1/42*c^3*(7*B*e-8*C*d)/e^2*x^7)/(e*x+d)-(6*A
*a^2*c*d*e^6+12*A*a*c^2*d^3*e^4+6*A*c^3*d^5*e^2-B*a^3*e^7-9*B*a^2*c*d^2*e^5
-15*B*a*c^2*d^4*e^3-7*B*c^3*d^6*e+2*C*a^3*d*e^6+12*C*a^2*c*d^3*e^4+18*C*a*c
^2*d^5*e^2+8*C*c^3*d^7)/e^9*ln(e*x+d)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 932, normalized size of antiderivative = 1.92

$$\int \frac{(a+cx^2)^3 (A+Bx+Cx^2)}{(d+ex)^2}\,dx$$

$$= \frac{60\,Cc^3e^8x^8 - 420\,Cc^3d^8 + 420\,Bc^3d^7e + 1260\,Bac^2d^5e^3 + 1260\,Ba^2cd^3e^5 + 420\,Ba^3de^7 - 420\,Aa^3e^8 - 420}{}$$

```
[In] integrate((c*x^2+a)^3*(C*x^2+B*x+A)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] 1/420*(60*C*c^3*e^8*x^8 - 420*C*c^3*d^8 + 420*B*c^3*d^7*e + 1260*B*a*c^2*d^
5*e^3 + 1260*B*a^2*c*d^3*e^5 + 420*B*a^3*d*e^7 - 420*A*a^3*e^8 - 420*(3*C*a
*c^2 + A*c^3)*d^6*e^2 - 1260*(C*a^2*c + A*a*c^2)*d^4*e^4 - 420*(C*a^3 + 3*A
*a^2*c)*d^2*e^6 - 10*(8*C*c^3*d*e^7 - 7*B*c^3*e^8)*x^7 + 14*(8*C*c^3*d^2*e^
6 - 7*B*c^3*d*e^7 + 6*(3*C*a*c^2 + A*c^3)*e^8)*x^6 - 21*(8*C*c^3*d^3*e^5 -
7*B*c^3*d^2*e^6 - 15*B*a*c^2*e^8 + 6*(3*C*a*c^2 + A*c^3)*d*e^7)*x^5 + 35*(8
*C*c^3*d^4*e^4 - 7*B*c^3*d^3*e^5 - 15*B*a*c^2*d*e^7 + 6*(3*C*a*c^2 + A*c^3)
*d^2*e^6 + 12*(C*a^2*c + A*a*c^2)*e^8)*x^4 - 70*(8*C*c^3*d^5*e^3 - 7*B*c^3*
d^4*e^4 - 15*B*a*c^2*d^2*e^6 - 9*B*a^2*c*e^8 + 6*(3*C*a*c^2 + A*c^3)*d^3*e^
5 + 12*(C*a^2*c + A*a*c^2)*d*e^7)*x^3 + 210*(8*C*c^3*d^6*e^2 - 7*B*c^3*d^5*
e^3 - 15*B*a*c^2*d^3*e^5 - 9*B*a^2*c*d*e^7 + 6*(3*C*a*c^2 + A*c^3)*d^4*e^4
+ 12*(C*a^2*c + A*a*c^2)*d^2*e^6 + 2*(C*a^3 + 3*A*a^2*c)*e^8)*x^2 + 420*(7*
C*c^3*d^7*e - 6*B*c^3*d^6*e^2 - 12*B*a*c^2*d^4*e^4 - 6*B*a^2*c*d^2*e^6 + 5*
(3*C*a*c^2 + A*c^3)*d^5*e^3 + 9*(C*a^2*c + A*a*c^2)*d^3*e^5 + (C*a^3 + 3*A*
a^2*c)*d*e^7)*x - 420*(8*C*c^3*d^8 - 7*B*c^3*d^7*e - 15*B*a*c^2*d^5*e^3 - 9
*B*a^2*c*d^3*e^5 - B*a^3*d*e^7 + 6*(3*C*a*c^2 + A*c^3)*d^6*e^2 + 12*(C*a^2*
c + A*a*c^2)*d^4*e^4 + 2*(C*a^3 + 3*A*a^2*c)*d^2*e^6 + (8*C*c^3*d^7*e - 7*B
*c^3*d^6*e^2 - 15*B*a*c^2*d^4*e^4 - 9*B*a^2*c*d^2*e^6 - B*a^3*e^8 + 6*(3*C*
a*c^2 + A*c^3)*d^5*e^3 + 12*(C*a^2*c + A*a*c^2)*d^3*e^5 + 2*(C*a^3 + 3*A*a^
2*c)*d*e^7)*x)*log(e*x + d))/(e^10*x + d*e^9)
```

Sympy [A] (verification not implemented)

Time = 1.86 (sec) , antiderivative size = 748, normalized size of antiderivative = 1.54

$$\int \frac{(a+cx^2)^3 (A+Bx+Cx^2)}{(d+ex)^2}\,dx$$
$$= \frac{Cc^3x^7}{7e^2} + x^6\left(\frac{Bc^3}{6e^2} - \frac{Cc^3d}{3e^3}\right) + x^5\left(\frac{Ac^3}{5e^2} - \frac{2Bc^3d}{5e^3} + \frac{3Cac^2}{5e^2} + \frac{3Cc^3d^2}{5e^4}\right)$$
$$+ x^4\left(-\frac{Ac^3d}{2e^3} + \frac{3Bac^2}{4e^2} + \frac{3Bc^3d^2}{4e^4} - \frac{3Cac^2d}{2e^3} - \frac{Cc^3d^3}{e^5}\right)$$
$$+ x^3\left(\frac{Aac^2}{e^2} + \frac{Ac^3d^2}{e^4} - \frac{2Bac^2d}{e^3} - \frac{4Bc^3d^3}{3e^5} + \frac{Ca^2c}{e^2} + \frac{3Cac^2d^2}{e^4} + \frac{5Cc^3d^4}{3e^6}\right)$$
$$+ x^2\left(-\frac{3Aac^2d}{e^3} - \frac{2Ac^3d^3}{e^5} + \frac{3Ba^2c}{2e^2} + \frac{9Bac^2d^2}{2e^4} + \frac{5Bc^3d^4}{2e^6} - \frac{3Ca^2cd}{e^3} - \frac{6Cac^2d^3}{e^5} - \frac{3Cc^3d^5}{e^7}\right)$$
$$+ x\left(\frac{3Aa^2c}{e^2} + \frac{9Aac^2d^2}{e^4} + \frac{5Ac^3d^4}{e^6} - \frac{6Ba^2cd}{e^3} - \frac{12Bac^2d^3}{e^5} - \frac{6Bc^3d^5}{e^7} + \frac{Ca^3}{e^2} + \frac{9Ca^2cd^2}{e^4} + \frac{15Cac^2d^4}{e^6} + \frac{7Cc^3d^6}{e^8}\right)$$
$$+ \frac{-Aa^3e^8 - 3Aa^2cd^2e^6 - 3Aac^2d^4e^4 - Ac^3d^6e^2 + Ba^3de^7 + 3Ba^2cd^3e^5 + 3Bac^2d^5e^3 + Bc^3d^7e - Ca^3d^2e}{de^9 + e^{10}x}$$
$$- \frac{(ae^2+cd^2)^2 \cdot (6Acde^2 - Bae^3 - 7Bcd^2e + 2Cade^2 + 8Ccd^3)\log(d+ex)}{e^9}$$

```
[In] integrate((c*x**2+a)**3*(C*x**2+B*x+A)/(e*x+d)**2,x)
```

```
[Out] C*c**3*x**7/(7*e**2) + x**6*(B*c**3/(6*e**2) - C*c**3*d/(3*e**3)) + x**5*(A
*c**3/(5*e**2) - 2*B*c**3*d/(5*e**3) + 3*C*a*c**2/(5*e**2) + 3*C*c**3*d**2/
(5*e**4)) + x**4*(-A*c**3*d/(2*e**3) + 3*B*a*c**2/(4*e**2) + 3*B*c**3*d**2/
(4*e**4) - 3*C*a*c**2*d/(2*e**3) - C*c**3*d**3/e**5) + x**3*(A*a*c**2/e**2
+ A*c**3*d**2/e**4 - 2*B*a*c**2*d/e**3 - 4*B*c**3*d**3/(3*e**5) + C*a**2*c/
e**2 + 3*C*a*c**2*d**2/e**4 + 5*C*c**3*d**4/(3*e**6)) + x**2*(-3*A*a*c**2*d
/e**3 - 2*A*c**3*d**3/e**5 + 3*B*a**2*c/(2*e**2) + 9*B*a*c**2*d**2/(2*e**4)
 + 5*B*c**3*d**4/(2*e**6) - 3*C*a**2*c*d/e**3 - 6*C*a*c**2*d**3/e**5 - 3*C*
c**3*d**5/e**7) + x*(3*A*a**2*c/e**2 + 9*A*a*c**2*d**2/e**4 + 5*A*c**3*d**4
/e**6 - 6*B*a**2*c*d/e**3 - 12*B*a*c**2*d**3/e**5 - 6*B*c**3*d**5/e**7 + C*
a**3/e**2 + 9*C*a**2*c*d**2/e**4 + 15*C*a*c**2*d**4/e**6 + 7*C*c**3*d**6/e*
*8) + (-A*a**3*e**8 - 3*A*a**2*c*d**2*e**6 - 3*A*a*c**2*d**4*e**4 - A*c**3*
d**6*e**2 + B*a**3*d*e**7 + 3*B*a**2*c*d**3*e**5 + 3*B*a*c**2*d**5*e**3 + B
*c**3*d**7*e - C*a**3*d**2*e**6 - 3*C*a**2*c*d**4*e**4 - 3*C*a*c**2*d**6*e*
*2 - C*c**3*d**8)/(d*e**9 + e**10*x) - (a*e**2 + c*d**2)**2*(6*A*c*d*e**2 -
 B*a*e**3 - 7*B*c*d**2*e + 2*C*a*d*e**2 + 8*C*c*d**3)*log(d + e*x)/e**9
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 691, normalized size of antiderivative = 1.42

$$\int \frac{(a+cx^2)^3 (A+Bx+Cx^2)}{(d+ex)^2}\,dx = -\frac{Cc^3d^8 - Bc^3d^7e - 3\,Bac^2d^5e^3 - 3\,Ba^2cd^3e^5 - Ba^3de^7 + Aa^3e^8 + (3\,Cac^2 + Ac^3)d^6e^2 + 3\,(Ca^2c + Aac^2}{e^{10}x + de^9} + \frac{60\,Cc^3e^6x^7 - 70\,(2\,Cc^3de^5 - Bc^3e^6)x^6 + 84\,(3\,Cc^3d^2e^4 - 2\,Bc^3de^5 + (3\,Cac^2 + Ac^3)e^6)x^5 - 105\,(4\,Cc^3d}{} - \frac{(8\,Cc^3d^7 - 7\,Bc^3d^6e - 15\,Bac^2d^4e^3 - 9\,Ba^2cd^2e^5 - Ba^3e^7 + 6\,(3\,Cac^2 + Ac^3)d^5e^2 + 12\,(Ca^2c + Aac^2)d}{e^9}$$

```
[In] integrate((c*x^2+a)^3*(C*x^2+B*x+A)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -(C*c^3*d^8 - B*c^3*d^7*e - 3*B*a*c^2*d^5*e^3 - 3*B*a^2*c*d^3*e^5 - B*a^3*d
*e^7 + A*a^3*e^8 + (3*C*a*c^2 + A*c^3)*d^6*e^2 + 3*(C*a^2*c + A*a*c^2)*d^4*
e^4 + (C*a^3 + 3*A*a^2*c)*d^2*e^6)/(e^10*x + d*e^9) + 1/420*(60*C*c^3*e^6*x
^7 - 70*(2*C*c^3*d*e^5 - B*c^3*e^6)*x^6 + 84*(3*C*c^3*d^2*e^4 - 2*B*c^3*d*e
^5 + (3*C*a*c^2 + A*c^3)*e^6)*x^5 - 105*(4*C*c^3*d^3*e^3 - 3*B*c^3*d^2*e^4
- 3*B*a*c^2*e^6 + 2*(3*C*a*c^2 + A*c^3)*d*e^5)*x^4 + 140*(5*C*c^3*d^4*e^2 -
 4*B*c^3*d^3*e^3 - 6*B*a*c^2*d*e^5 + 3*(3*C*a*c^2 + A*c^3)*d^2*e^4 + 3*(C*a
^2*c + A*a*c^2)*e^6)*x^3 - 210*(6*C*c^3*d^5*e - 5*B*c^3*d^4*e^2 - 9*B*a*c^2
*d^2*e^4 - 3*B*a^2*c*e^6 + 4*(3*C*a*c^2 + A*c^3)*d^3*e^3 + 6*(C*a^2*c + A*a
*c^2)*d*e^5)*x^2 + 420*(7*C*c^3*d^6 - 6*B*c^3*d^5*e - 12*B*a*c^2*d^3*e^3 -
6*B*a^2*c*d*e^5 + 5*(3*C*a*c^2 + A*c^3)*d^4*e^2 + 9*(C*a^2*c + A*a*c^2)*d^2
*e^4 + (C*a^3 + 3*A*a^2*c)*e^6)*x)/e^8 - (8*C*c^3*d^7 - 7*B*c^3*d^6*e - 15*
B*a*c^2*d^4*e^3 - 9*B*a^2*c*d^2*e^5 - B*a^3*e^7 + 6*(3*C*a*c^2 + A*c^3)*d^5
*e^2 + 12*(C*a^2*c + A*a*c^2)*d^3*e^4 + 2*(C*a^3 + 3*A*a^2*c)*d*e^6)*log(e*
x + d)/e^9
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 882, normalized size of antiderivative = 1.81

$$\int \frac{(a+cx^2)^3 (A+Bx+Cx^2)}{(d+ex)^2}\,dx = \frac{\Big(60\,Cc^3 - \frac{70\,(8\,Cc^3de - Bc^3e^2)}{(ex+d)e} + \frac{84\,(28\,Cc^3d^2e^2 - 7\,Bc^3de^3 + 3\,Cac^2e^4 + Ac^3e^4)}{(ex+d)^2e^2} - \frac{105\,(56\,Cc^3d^3e^3 - 21\,Bc^3d^2e^4 + 18\,Cac^2de^5 + 6\,Ac^3de^5 -}{(ex+d)^3e^3}}{} + \frac{(8\,Cc^3d^7 - 7\,Bc^3d^6e + 18\,Cac^2d^5e^2 + 6\,Ac^3d^5e^2 - 15\,Bac^2d^4e^3 + 12\,Ca^2cd^3e^4 + 12\,Aac^2d^3e^4 - 9\,Ba^2cd^2}{e^9} - \frac{\frac{Cc^3d^8e^7}{ex+d} - \frac{Bc^3d^7e^8}{ex+d} + \frac{3\,Cac^2d^6e^9}{ex+d} + \frac{Ac^3d^6e^9}{ex+d} - \frac{3\,Bac^2d^5e^{10}}{ex+d} + \frac{3\,Ca^2cd^4e^{11}}{ex+d} + \frac{3\,Aac^2d^4e^{11}}{ex+d} - \frac{3\,Ba^2cd^3e^{12}}{ex+d} + \frac{Ca^3d^2e^{13}}{ex+d} + 3\,A}{e^{16}}$$

```
[In] integrate((c*x^2+a)^3*(C*x^2+B*x+A)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] 1/420*(60*C*c^3 - 70*(8*C*c^3*d*e - B*c^3*e^2)/((e*x + d)*e) + 84*(28*C*c^3
*d^2*e^2 - 7*B*c^3*d*e^3 + 3*C*a*c^2*e^4 + A*c^3*e^4)/((e*x + d)^2*e^2) - 1
05*(56*C*c^3*d^3*e^3 - 21*B*c^3*d^2*e^4 + 18*C*a*c^2*d*e^5 + 6*A*c^3*d*e^5
- 3*B*a*c^2*e^6)/((e*x + d)^3*e^3) + 140*(70*C*c^3*d^4*e^4 - 35*B*c^3*d^3*e
^5 + 45*C*a*c^2*d^2*e^6 + 15*A*c^3*d^2*e^6 - 15*B*a*c^2*d*e^7 + 3*C*a^2*c*e
^8 + 3*A*a*c^2*e^8)/((e*x + d)^4*e^4) - 210*(56*C*c^3*d^5*e^5 - 35*B*c^3*d^
4*e^6 + 60*C*a*c^2*d^3*e^7 + 20*A*c^3*d^3*e^7 - 30*B*a*c^2*d^2*e^8 + 12*C*a
^2*c*d*e^9 + 12*A*a*c^2*d*e^9 - 3*B*a^2*c*e^10)/((e*x + d)^5*e^5) + 420*(28
*C*c^3*d^6*e^6 - 21*B*c^3*d^5*e^7 + 45*C*a*c^2*d^4*e^8 + 15*A*c^3*d^4*e^8 -
 30*B*a*c^2*d^3*e^9 + 18*C*a^2*c*d^2*e^10 + 18*A*a*c^2*d^2*e^10 - 9*B*a^2*c
*d*e^11 + C*a^3*e^12 + 3*A*a^2*c*e^12)/((e*x + d)^6*e^6))*(e*x + d)^7/e^9 +
 (8*C*c^3*d^7 - 7*B*c^3*d^6*e + 18*C*a*c^2*d^5*e^2 + 6*A*c^3*d^5*e^2 - 15*B
*a*c^2*d^4*e^3 + 12*C*a^2*c*d^3*e^4 + 12*A*a*c^2*d^3*e^4 - 9*B*a^2*c*d^2*e^
5 + 2*C*a^3*d*e^6 + 6*A*a^2*c*d*e^6 - B*a^3*e^7)*log(abs(e*x + d)/((e*x + d
)^2*abs(e)))/e^9 - (C*c^3*d^8*e^7/(e*x + d) - B*c^3*d^7*e^8/(e*x + d) + 3*C
*a*c^2*d^6*e^9/(e*x + d) + A*c^3*d^6*e^9/(e*x + d) - 3*B*a*c^2*d^5*e^10/(e*
x + d) + 3*C*a^2*c*d^4*e^11/(e*x + d) + 3*A*a*c^2*d^4*e^11/(e*x + d) - 3*B*
a^2*c*d^3*e^12/(e*x + d) + C*a^3*d^2*e^13/(e*x + d) + 3*A*a^2*c*d^2*e^13/(e
*x + d) - B*a^3*d*e^14/(e*x + d) + A*a^3*e^15/(e*x + d))/e^16
```

Mupad [B] (verification not implemented)

Time = 12.27 (sec) , antiderivative size = 1511, normalized size of antiderivative = 3.11

$$\int \frac{(a+cx^2)^3(A+Bx+Cx^2)}{(d+ex)^2}\,dx = \text{Too large to display}$$

```
[In] int(((a + c*x^2)^3*(A + B*x + C*x^2))/(d + e*x)^2,x)
```

```
[Out] x*((C*a^3 + 3*A*a^2*c)/e^2 + (2*d*((2*d*((d^2*((2*d*((B*c^3)/e^2 - (2*C*c^3
*d)/e^3))/e - (A*c^3 + 3*C*a*c^2)/e^2 + (C*c^3*d^2)/e^4))/e^2 - (2*d*((2*d*
((2*d*((B*c^3)/e^2 - (2*C*c^3*d)/e^3))/e - (A*c^3 + 3*C*a*c^2)/e^2 + (C*c^3
*d^2)/e^4))/e - (d^2*((B*c^3)/e^2 - (2*C*c^3*d)/e^3))/e^2 + (3*B*a*c^2)/e^2
))/e + (3*a*c*(A*c + C*a))/e^2))/e + (d^2*((2*d*((2*d*((B*c^3)/e^2 - (2*C*c
^3*d)/e^3))/e - (A*c^3 + 3*C*a*c^2)/e^2 + (C*c^3*d^2)/e^4))/e - (d^2*((B*c^
3)/e^2 - (2*C*c^3*d)/e^3))/e^2 + (3*B*a*c^2)/e^2))/e^2 - (3*B*a^2*c)/e^2))/
e - (d^2*((d^2*((2*d*((B*c^3)/e^2 - (2*C*c^3*d)/e^3))/e - (A*c^3 + 3*C*a*c^
2)/e^2 + (C*c^3*d^2)/e^4))/e^2 - (2*d*((2*d*((2*d*((B*c^3)/e^2 - (2*C*c^3*d
)/e^3))/e - (A*c^3 + 3*C*a*c^2)/e^2 + (C*c^3*d^2)/e^4))/e - (d^2*((B*c^3)/e
^2 - (2*C*c^3*d)/e^3))/e^2 + (3*B*a*c^2)/e^2))/e + (3*a*c*(A*c + C*a))/e^2)
)/e^2) + x^4*((d*((2*d*((B*c^3)/e^2 - (2*C*c^3*d)/e^3))/e - (A*c^3 + 3*C*a*
c^2)/e^2 + (C*c^3*d^2)/e^4))/(2*e) - (d^2*((B*c^3)/e^2 - (2*C*c^3*d)/e^3))/
(4*e^2) + (3*B*a*c^2)/(4*e^2)) - x^2*((d*((d^2*((2*d*((B*c^3)/e^2 - (2*C*c^
```

```
3*d)/e^3))/e - (A*c^3 + 3*C*a*c^2)/e^2 + (C*c^3*d^2)/e^4))/e^2 - (2*d*((2*d
*((2*d*((B*c^3)/e^2 - (2*C*c^3*d)/e^3))/e - (A*c^3 + 3*C*a*c^2)/e^2 + (C*c^
3*d^2)/e^4))/e - (d^2*((B*c^3)/e^2 - (2*C*c^3*d)/e^3))/e^2 + (3*B*a*c^2)/e^
2))/e + (3*a*c*(A*c + C*a))/e^2))/e + (d^2*((2*d*((2*d*((B*c^3)/e^2 - (2*C*
c^3*d)/e^3))/e - (A*c^3 + 3*C*a*c^2)/e^2 + (C*c^3*d^2)/e^4))/e - (d^2*((B*c
^3)/e^2 - (2*C*c^3*d)/e^3))/e^2 + (3*B*a*c^2)/e^2))/(2*e^2) - (3*B*a^2*c)/(
2*e^2)) + x^6*((B*c^3)/(6*e^2) - (C*c^3*d)/(3*e^3)) - x^5*((2*d*((B*c^3)/e^
2 - (2*C*c^3*d)/e^3))/(5*e) - (A*c^3 + 3*C*a*c^2)/(5*e^2) + (C*c^3*d^2)/(5*
e^4)) + x^3*((d^2*((2*d*((B*c^3)/e^2 - (2*C*c^3*d)/e^3))/e - (A*c^3 + 3*C*a
*c^2)/e^2 + (C*c^3*d^2)/e^4))/(3*e^2) - (2*d*((2*d*((2*d*((B*c^3)/e^2 - (2*
C*c^3*d)/e^3))/e - (A*c^3 + 3*C*a*c^2)/e^2 + (C*c^3*d^2)/e^4))/e - (d^2*((B
*c^3)/e^2 - (2*C*c^3*d)/e^3))/e^2 + (3*B*a*c^2)/e^2))/(3*e) + (a*c*(A*c + C
*a))/e^2) - (A*a^3*e^8 + C*c^3*d^8 - B*a^3*d*e^7 - B*c^3*d^7*e + A*c^3*d^6*
e^2 + C*a^3*d^2*e^6 + 3*A*a*c^2*d^4*e^4 + 3*A*a^2*c*d^2*e^6 - 3*B*a*c^2*d^5
*e^3 - 3*B*a^2*c*d^3*e^5 + 3*C*a*c^2*d^6*e^2 + 3*C*a^2*c*d^4*e^4)/(e*(d*e^8
 + e^9*x)) - (log(d + e*x)*(8*C*c^3*d^7 - B*a^3*e^7 + 2*C*a^3*d*e^6 - 7*B*c
^3*d^6*e + 6*A*c^3*d^5*e^2 + 12*A*a*c^2*d^3*e^4 - 15*B*a*c^2*d^4*e^3 - 9*B*
a^2*c*d^2*e^5 + 18*C*a*c^2*d^5*e^2 + 12*C*a^2*c*d^3*e^4 + 6*A*a^2*c*d*e^6))
/e^9 + (C*c^3*x^7)/(7*e^2)
```

3.38 $\int \frac{(a+cx^2)^3(A+Bx+Cx^2)}{(d+ex)^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 466

$$
\begin{aligned}
&\int \frac{(a+cx^2)^3(A+Bx+Cx^2)}{(d+ex)^3}\,dx = \\
&-\frac{c(3a^2e^4(3Cd-Be)+c^2d^3(21Cd^2-5e(3Bd-2Ae))+3acde^2(10Cd^2-3e(2Bd-Ae)))\,x}{e^8} \\
&+\frac{c(3a^2Ce^4+c^2d^2(15Cd^2-2e(5Bd-3Ae))+3ace^2(6Cd^2-e(3Bd-Ae)))\,x^2}{2e^7} \\
&-\frac{c^2(3ae^2(3Cd-Be)+cd(10Cd^2-3e(2Bd-Ae)))\,x^3}{3e^6} \\
&+\frac{c^2(3aCe^2+c(6Cd^2-e(3Bd-Ae)))\,x^4}{4e^5}-\frac{c^3(3Cd-Be)x^5}{5e^4} \\
&+\frac{c^3Cx^6}{6e^3}-\frac{(cd^2+ae^2)^3\,(Cd^2-Bde+Ae^2)}{2e^9(d+ex)^2} \\
&+\frac{(cd^2+ae^2)^2\,(ae^2(2Cd-Be)+cd(8Cd^2-e(7Bd-6Ae)))}{e^9(d+ex)} \\
&+\frac{(cd^2+ae^2)\,(a^2Ce^4+c^2d^2(28Cd^2-3e(7Bd-5Ae))+ace^2(17Cd^2-3e(3Bd-Ae)))\log(d+ex)}{e^9}
\end{aligned}
$$

```
[Out] -c*(3*a^2*e^4*(-B*e+3*C*d)+c^2*d^3*(21*C*d^2-5*e*(-2*A*e+3*B*d))+3*a*c*d*e^
2*(10*C*d^2-3*e*(-A*e+2*B*d)))*x/e^8+1/2*c*(3*a^2*C*e^4+c^2*d^2*(15*C*d^2-2
*e*(-3*A*e+5*B*d))+3*a*c*e^2*(6*C*d^2-e*(-A*e+3*B*d)))*x^2/e^7-1/3*c^2*(3*a
*e^2*(-B*e+3*C*d)+c*d*(10*C*d^2-3*e*(-A*e+2*B*d)))*x^3/e^6+1/4*c^2*(3*a*C*e
^2+c*(6*C*d^2-e*(-A*e+3*B*d)))*x^4/e^5-1/5*c^3*(-B*e+3*C*d)*x^5/e^4+1/6*c^3
*C*x^6/e^3-1/2*(a*e^2+c*d^2)^3*(A*e^2-B*d*e+C*d^2)/e^9/(e*x+d)^2+(a*e^2+c*d
^2)^2*(a*e^2*(-B*e+2*C*d)+c*d*(8*C*d^2-e*(-6*A*e+7*B*d)))/e^9/(e*x+d)+(a*e^
2+c*d^2)*(a^2*C*e^4+c^2*d^2*(28*C*d^2-3*e*(-5*A*e+7*B*d))+a*c*e^2*(17*C*d^2
-3*e*(-A*e+3*B*d)))*ln(e*x+d)/e^9
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 463, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$, Rules used = {1642}

$$
\begin{aligned}
&\int \frac{(a+cx^2)^3 (A+Bx+Cx^2)}{(d+ex)^3} \, dx \\
&= \frac{(ae^2+cd^2)\log(d+ex)\left(a^2Ce^4+ace^2(17Cd^2-3e(3Bd-Ae))+c^2(28Cd^4-3d^2e(7Bd-5Ae))\right)}{e^9} \\
&\quad + \frac{cx^2(3a^2Ce^4+3ace^2(6Cd^2-e(3Bd-Ae))+c^2(15Cd^4-2d^2e(5Bd-3Ae)))}{2e^7} \\
&\quad - \frac{cx(3a^2e^4(3Cd-Be)+3acde^2(10Cd^2-3e(2Bd-Ae))+c^2(21Cd^5-5d^3e(3Bd-2Ae)))}{e^8} \\
&\quad - \frac{c^2x^3(3ae^2(3Cd-Be)-3cde(2Bd-Ae)+10cCd^3)}{3e^6} \\
&\quad + \frac{c^2x^4(3aCe^2-ce(3Bd-Ae)+6cCd^2)}{4e^5} - \frac{(ae^2+cd^2)^3(Ae^2-Bde+Cd^2)}{2e^9(d+ex)^2} \\
&\quad + \frac{(ae^2+cd^2)^2(ae^2(2Cd-Be)-cde(7Bd-6Ae)+8cCd^3)}{e^9(d+ex)} - \frac{c^3x^5(3Cd-Be)}{5e^4} + \frac{c^3Cx^6}{6e^3}
\end{aligned}
$$

```
[In] Int[((a + c*x^2)^3*(A + B*x + C*x^2))/(d + e*x)^3,x]
```

```
[Out] -((c*(3*a^2*e^4*(3*C*d - B*e) + c^2*(21*C*d^5 - 5*d^3*e*(3*B*d - 2*A*e)) +
3*a*c*d*e^2*(10*C*d^2 - 3*e*(2*B*d - A*e)))*x)/e^8) + (c*(3*a^2*C*e^4 + c^2
*(15*C*d^4 - 2*d^2*e*(5*B*d - 3*A*e)) + 3*a*c*e^2*(6*C*d^2 - e*(3*B*d - A*e
)))*x^2)/(2*e^7) - (c^2*(10*c*C*d^3 - 3*c*d*e*(2*B*d - A*e) + 3*a*e^2*(3*C*
d - B*e))*x^3)/(3*e^6) + (c^2*(6*c*C*d^2 + 3*a*C*e^2 - c*e*(3*B*d - A*e))*x
^4)/(4*e^5) - (c^3*(3*C*d - B*e)*x^5)/(5*e^4) + (c^3*C*x^6)/(6*e^3) - ((c*d
^2 + a*e^2)^3*(C*d^2 - B*d*e + A*e^2))/(2*e^9*(d + e*x)^2) + ((c*d^2 + a*e^
2)^2*(8*c*C*d^3 - c*d*e*(7*B*d - 6*A*e) + a*e^2*(2*C*d - B*e)))/(e^9*(d + e
*x)) + ((c*d^2 + a*e^2)*(a^2*C*e^4 + c^2*(28*C*d^4 - 3*d^2*e*(7*B*d - 5*A*e
)) + a*c*e^2*(17*C*d^2 - 3*e*(3*B*d - A*e)))*Log[d + e*x])/e^9
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

integral

$$= \int \left(\frac{c(-3a^2e^4(3Cd-Be)-c^2(21Cd^5-5d^3e(3Bd-2Ae))-3acde^2(10Cd^2-3e(2Bd-Ae)))}{e^8} + \frac{c(3a^2Ce^4+c^2(15Cd^4-2d^2e(5Bd-3Ae))+3ace^2(6Cd^2-e(3Bd-Ae)))\,x}{e^7} + \frac{c^2(-10cCd^3+3cde(2Bd-Ae)-3ae^2(3Cd-Be))\,x^2}{e^6} + \frac{c^2(6cCd^2+3aCe^2-ce(3Bd-Ae))\,x^3}{e^5} + \frac{c^3(-3Cd+Be)x^4}{e^4} + \frac{c^3Cx^5}{e^3} + \frac{(cd^2+ae^2)^3\,(Cd^2-Bde+Ae^2)}{e^8(d+ex)^3} + \frac{(cd^2+ae^2)^2\,(-8cCd^3+cde(7Bd-6Ae)-ae^2(2Cd-Be))}{e^8(d+ex)^2} + \frac{(cd^2+ae^2)\,(a^2Ce^4+c^2(28Cd^4-3d^2e(7Bd-5Ae))+ace^2(17Cd^2-3e(3Bd-Ae)))}{e^8(d+ex)} \right) dx$$

$$= -\frac{c(3a^2e^4(3Cd-Be)+c^2(21Cd^5-5d^3e(3Bd-2Ae))+3acde^2(10Cd^2-3e(2Bd-Ae)))\,x}{e^8} + \frac{c(3a^2Ce^4+c^2(15Cd^4-2d^2e(5Bd-3Ae))+3ace^2(6Cd^2-e(3Bd-Ae)))\,x^2}{2e^7} - \frac{c^2(10cCd^3-3cde(2Bd-Ae)+3ae^2(3Cd-Be))\,x^3}{3e^6} + \frac{c^2(6cCd^2+3aCe^2-ce(3Bd-Ae))\,x^4}{4e^5} - \frac{c^3(3Cd-Be)x^5}{5e^4} + \frac{c^3Cx^6}{6e^3} - \frac{(cd^2+ae^2)^3\,(Cd^2-Bde+Ae^2)}{2e^9(d+ex)^2} + \frac{(cd^2+ae^2)^2\,(8cCd^3-cde(7Bd-6Ae)+ae^2(2Cd-Be))}{e^9(d+ex)} + \frac{(cd^2+ae^2)\,(a^2Ce^4+c^2(28Cd^4-3d^2e(7Bd-5Ae))+ace^2(17Cd^2-3e(3Bd-Ae)))\log(d+e}{e^9}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 438, normalized size of antiderivative = 0.94

$$\int \frac{(a+cx^2)^3 (A+Bx+Cx^2)}{(d+ex)^3}\,dx$$

$$= \frac{-60ce(-3a^2e^4(-3Cd+Be)+3acde^2(10Cd^2+3e(-2Bd+Ae))+c^2(21Cd^5+5d^3e(-3Bd+2Ae)))\,x+}{}$$

```
[In] Integrate[((a + c*x^2)^3*(A + B*x + C*x^2))/(d + e*x)^3,x]
```

```
[Out] (-60*c*e*(-3*a^2*e^4*(-3*C*d + B*e) + 3*a*c*d*e^2*(10*C*d^2 + 3*e*(-2*B*d +
 A*e)) + c^2*(21*C*d^5 + 5*d^3*e*(-3*B*d + 2*A*e)))*x + 30*c*e^2*(3*a^2*C*e
^4 + 3*a*c*e^2*(6*C*d^2 + e*(-3*B*d + A*e)) + c^2*(15*C*d^4 + 2*d^2*e*(-5*B
*d + 3*A*e)))*x^2 - 20*c^2*e^3*(10*c*C*d^3 + 3*c*d*e*(-2*B*d + A*e) - 3*a*e
^2*(-3*C*d + B*e))*x^3 + 15*c^2*e^4*(6*c*C*d^2 + 3*a*C*e^2 + c*e*(-3*B*d +
A*e))*x^4 + 12*c^3*e^5*(-3*C*d + B*e)*x^5 + 10*c^3*C*e^6*x^6 - (30*(c*d^2 +
 a*e^2)^3*(C*d^2 + e*(-(B*d) + A*e)))/(d + e*x)^2 + (60*(c*d^2 + a*e^2)^2*(
8*c*C*d^3 + c*d*e*(-7*B*d + 6*A*e) + a*e^2*(2*C*d - B*e)))/(d + e*x) + 60*(
c*d^2 + a*e^2)*(a^2*C*e^4 + a*c*e^2*(17*C*d^2 + 3*e*(-3*B*d + A*e)) + c^2*(
28*C*d^4 + 3*d^2*e*(-7*B*d + 5*A*e)))*Log[d + e*x])/(60*e^9)
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 700, normalized size of antiderivative = 1.50

method	result
norman	$\frac{\left(6A\,a^2cd\,e^6+36Aa\,c^2d^3e^4+30A\,c^3d^5e^2-B\,a^3e^7-18B\,a^2c\,d^2e^5-60Ba\,c^2d^4e^3-42B\,c^3d^6e+2C\,a^3d\,e^6+36C\,a^2c\,d^3e^4+90Ca\,c^2d^5e^2+56C\,c^3d\right.}{e^8}$
default	$-\frac{c\left(30Cac\,d^3e^2x-18Bxac\,d^2e^3+9Axacd\,e^4+\frac{9}{2}B\,x^2acd\,e^4-\frac{3}{2}C\,a^2e^5x^2-\frac{1}{6}c^2C\,x^6e^5-3Bx\,a^2e^5-\frac{1}{4}A\,x^4c^2e^5-\frac{1}{5}B\,x^5c^2e^5+21C\right.}{}$
risch	$\frac{3cC\,a^2x^2}{2e^3}+\frac{3cBx\,a^2}{e^3}-\frac{10c^3Ax\,d^3}{e^6}+\frac{15c^3Bx\,d^4}{e^7}-\frac{3c^3B\,x^4d}{4e^4}-\frac{10c^3C\,d^3x^3}{3e^6}+\frac{15c^3C\,d^4x^2}{2e^7}-\frac{3c^2Cad\,x^3}{e^4}-\frac{9cC\,a^2d}{e^4}$
parallelrisch	Expression too large to display

```
[In] int((c*x^2+a)^3*(C*x^2+B*x+A)/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] ((6*A*a^2*c*d*e^6+36*A*a*c^2*d^3*e^4+30*A*c^3*d^5*e^2-B*a^3*e^7-18*B*a^2*c*
d^2*e^5-60*B*a*c^2*d^4*e^3-42*B*c^3*d^6*e+2*C*a^3*d*e^6+36*C*a^2*c*d^3*e^4+
90*C*a*c^2*d^5*e^2+56*C*c^3*d^7)/e^8*x-1/2*(A*a^3*e^8-9*A*a^2*c*d^2*e^6-54*
A*a*c^2*d^4*e^4-45*A*c^3*d^6*e^2+B*a^3*d*e^7+27*B*a^2*c*d^3*e^5+90*B*a*c^2*
d^5*e^3+63*B*c^3*d^7*e-3*C*a^3*d^2*e^6-54*C*a^2*c*d^4*e^4-135*C*a*c^2*d^6*e
^2-84*C*c^3*d^8)/e^9+1/6*C*c^3/e*x^8+1/12*c*(18*A*a*c*e^4+15*A*c^2*d^2*e^2-
30*B*a*c*d*e^3-21*B*c^2*d^3*e+18*C*a^2*e^4+45*C*a*c*d^2*e^2+28*C*c^2*d^4)/e
```

```
^5*x^4-1/3*c*(18*A*a*c*d*e^4+15*A*c^2*d^3*e^2-9*B*a^2*e^5-30*B*a*c*d^2*e^3-
21*B*c^2*d^4*e+18*C*a^2*d*e^4+45*C*a*c*d^3*e^2+28*C*c^2*d^5)/e^6*x^3+1/60*c
^2*(15*A*c*e^2-21*B*c*d*e+45*C*a*e^2+28*C*c*d^2)/e^3*x^6-1/30*c^2*(15*A*c*d
*e^2-30*B*a*e^3-21*B*c*d^2*e+45*C*a*d*e^2+28*C*c*d^3)/e^4*x^5+1/15*c^3*(3*B
*e-4*C*d)/e^2*x^7)/(e*x+d)^2+1/e^9*(3*A*a^2*c*e^6+18*A*a*c^2*d^2*e^4+15*A*c
^3*d^4*e^2-9*B*a^2*c*d*e^5-30*B*a*c^2*d^3*e^3-21*B*c^3*d^5*e+C*a^3*e^6+18*C
*a^2*c*d^2*e^4+45*C*a*c^2*d^4*e^2+28*C*c^3*d^6)*ln(e*x+d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1025 vs. 2(454) = 908.

Time = 0.30 (sec) , antiderivative size = 1025, normalized size of antiderivative = 2.20

$$\int \frac{(a+cx^2)^3\,(A+Bx+Cx^2)}{(d+ex)^3}\,dx$$

$$= \frac{10\,Cc^3e^8x^8 + 450\,Cc^3d^8 - 390\,Bc^3d^7e - 810\,Bac^2d^5e^3 - 450\,Ba^2cd^3e^5 - 30\,Ba^3de^7 - 30\,Aa^3e^8 + 330\,(3}{}$$

```
[In] integrate((c*x^2+a)^3*(C*x^2+B*x+A)/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] 1/60*(10*C*c^3*e^8*x^8 + 450*C*c^3*d^8 - 390*B*c^3*d^7*e - 810*B*a*c^2*d^5*
e^3 - 450*B*a^2*c*d^3*e^5 - 30*B*a^3*d*e^7 - 30*A*a^3*e^8 + 330*(3*C*a*c^2
+ A*c^3)*d^6*e^2 + 630*(C*a^2*c + A*a*c^2)*d^4*e^4 + 90*(C*a^3 + 3*A*a^2*c)
*d^2*e^6 - 4*(4*C*c^3*d*e^7 - 3*B*c^3*e^8)*x^7 + (28*C*c^3*d^2*e^6 - 21*B*c
^3*d*e^7 + 15*(3*C*a*c^2 + A*c^3)*e^8)*x^6 - 2*(28*C*c^3*d^3*e^5 - 21*B*c^3
*d^2*e^6 - 30*B*a*c^2*e^8 + 15*(3*C*a*c^2 + A*c^3)*d*e^7)*x^5 + 5*(28*C*c^3
*d^4*e^4 - 21*B*c^3*d^3*e^5 - 30*B*a*c^2*d*e^7 + 15*(3*C*a*c^2 + A*c^3)*d^2
*e^6 + 18*(C*a^2*c + A*a*c^2)*e^8)*x^4 - 20*(28*C*c^3*d^5*e^3 - 21*B*c^3*d^
4*e^4 - 30*B*a*c^2*d^2*e^6 - 9*B*a^2*c*e^8 + 15*(3*C*a*c^2 + A*c^3)*d^3*e^5
 + 18*(C*a^2*c + A*a*c^2)*d*e^7)*x^3 - 30*(69*C*c^3*d^6*e^2 - 50*B*c^3*d^5*
e^3 - 63*B*a*c^2*d^3*e^5 - 12*B*a^2*c*d*e^7 + 34*(3*C*a*c^2 + A*c^3)*d^4*e^
4 + 33*(C*a^2*c + A*a*c^2)*d^2*e^6)*x^2 - 60*(13*C*c^3*d^7*e - 8*B*c^3*d^6*
e^2 - 3*B*a*c^2*d^4*e^4 + 6*B*a^2*c*d^2*e^6 + B*a^3*e^8 + 4*(3*C*a*c^2 + A*
c^3)*d^5*e^3 - 3*(C*a^2*c + A*a*c^2)*d^3*e^5 - 2*(C*a^3 + 3*A*a^2*c)*d*e^7)
*x + 60*(28*C*c^3*d^8 - 21*B*c^3*d^7*e - 30*B*a*c^2*d^5*e^3 - 9*B*a^2*c*d^3
*e^5 + 15*(3*C*a*c^2 + A*c^3)*d^6*e^2 + 18*(C*a^2*c + A*a*c^2)*d^4*e^4 + (C
*a^3 + 3*A*a^2*c)*d^2*e^6 + (28*C*c^3*d^6*e^2 - 21*B*c^3*d^5*e^3 - 30*B*a*c
^2*d^3*e^5 - 9*B*a^2*c*d*e^7 + 15*(3*C*a*c^2 + A*c^3)*d^4*e^4 + 18*(C*a^2*c
 + A*a*c^2)*d^2*e^6 + (C*a^3 + 3*A*a^2*c)*e^8)*x^2 + 2*(28*C*c^3*d^7*e - 21
*B*c^3*d^6*e^2 - 30*B*a*c^2*d^4*e^4 - 9*B*a^2*c*d^2*e^6 + 15*(3*C*a*c^2 + A
*c^3)*d^5*e^3 + 18*(C*a^2*c + A*a*c^2)*d^3*e^5 + (C*a^3 + 3*A*a^2*c)*d*e^7)
*x)*log(e*x + d))/(e^11*x^2 + 2*d*e^10*x + d^2*e^9)
```

Sympy [A] (verification not implemented)

Time = 9.23 (sec) , antiderivative size = 816, normalized size of antiderivative = 1.75

$$
\begin{aligned}
&\int \frac{(a+cx^2)^3\,(A+Bx+Cx^2)}{(d+ex)^3}\,dx \\
&= \frac{Cc^3x^6}{6e^3} + x^5\left(\frac{Bc^3}{5e^3} - \frac{3Cc^3d}{5e^4}\right) + x^4\left(\frac{Ac^3}{4e^3} - \frac{3Bc^3d}{4e^4} + \frac{3Cac^2}{4e^3} + \frac{3Cc^3d^2}{2e^5}\right) \\
&\quad + x^3\left(-\frac{Ac^3d}{e^4} + \frac{Bac^2}{e^3} + \frac{2Bc^3d^2}{e^5} - \frac{3Cac^2d}{e^4} - \frac{10Cc^3d^3}{3e^6}\right) + x^2 \\
&\quad \cdot\left(\frac{3Aac^2}{2e^3} + \frac{3Ac^3d^2}{e^5} - \frac{9Bac^2d}{2e^4} - \frac{5Bc^3d^3}{e^6} + \frac{3Ca^2c}{2e^3} + \frac{9Cac^2d^2}{e^5} + \frac{15Cc^3d^4}{2e^7}\right) + x\left(-\frac{9Aac^2d}{e^4}\right. \\
&\qquad \left. - \frac{10Ac^3d^3}{e^6} + \frac{3Ba^2c}{e^3} + \frac{18Bac^2d^2}{e^5} + \frac{15Bc^3d^4}{e^7} - \frac{9Ca^2cd}{e^4} - \frac{30Cac^2d^3}{e^6} - \frac{21Cc^3d^5}{e^8}\right) \\
&\quad + \frac{-Aa^3e^8 + 9Aa^2cd^2e^6 + 21Aac^2d^4e^4 + 11Ac^3d^6e^2 - Ba^3de^7 - 15Ba^2cd^3e^5 - 27Bac^2d^5e^3 - 13Bc^3d^7e + 3}{} \\
&\quad + \frac{(ae^2+cd^2)\,(3Aace^4 + 15Ac^2d^2e^2 - 9Bacde^3 - 21Bc^2d^3e + Ca^2e^4 + 17Cacd^2e^2 + 28Cc^2d^4)\log{(d+ex)}}{e^9}
\end{aligned}
$$

```
[In] integrate((c*x**2+a)**3*(C*x**2+B*x+A)/(e*x+d)**3,x)
```

```
[Out] C*c**3*x**6/(6*e**3) + x**5*(B*c**3/(5*e**3) - 3*C*c**3*d/(5*e**4)) + x**4*
(A*c**3/(4*e**3) - 3*B*c**3*d/(4*e**4) + 3*C*a*c**2/(4*e**3) + 3*C*c**3*d**
2/(2*e**5)) + x**3*(-A*c**3*d/e**4 + B*a*c**2/e**3 + 2*B*c**3*d**2/e**5 - 3
*C*a*c**2*d/e**4 - 10*C*c**3*d**3/(3*e**6)) + x**2*(3*A*a*c**2/(2*e**3) + 3
*A*c**3*d**2/e**5 - 9*B*a*c**2*d/(2*e**4) - 5*B*c**3*d**3/e**6 + 3*C*a**2*c
/(2*e**3) + 9*C*a*c**2*d**2/e**5 + 15*C*c**3*d**4/(2*e**7)) + x*(-9*A*a*c**
2*d/e**4 - 10*A*c**3*d**3/e**6 + 3*B*a**2*c/e**3 + 18*B*a*c**2*d**2/e**5 +
15*B*c**3*d**4/e**7 - 9*C*a**2*c*d/e**4 - 30*C*a*c**2*d**3/e**6 - 21*C*c**3
*d**5/e**8) + (-A*a**3*e**8 + 9*A*a**2*c*d**2*e**6 + 21*A*a*c**2*d**4*e**4
+ 11*A*c**3*d**6*e**2 - B*a**3*d*e**7 - 15*B*a**2*c*d**3*e**5 - 27*B*a*c**2
*d**5*e**3 - 13*B*c**3*d**7*e + 3*C*a**3*d**2*e**6 + 21*C*a**2*c*d**4*e**4
+ 33*C*a*c**2*d**6*e**2 + 15*C*c**3*d**8 + x*(12*A*a**2*c*d*e**7 + 24*A*a*c
**2*d**3*e**5 + 12*A*c**3*d**5*e**3 - 2*B*a**3*e**8 - 18*B*a**2*c*d**2*e**6
 - 30*B*a*c**2*d**4*e**4 - 14*B*c**3*d**6*e**2 + 4*C*a**3*d*e**7 + 24*C*a**
2*c*d**3*e**5 + 36*C*a*c**2*d**5*e**3 + 16*C*c**3*d**7*e))/(2*d**2*e**9 + 4
*d*e**10*x + 2*e**11*x**2) + (a*e**2 + c*d**2)*(3*A*a*c*e**4 + 15*A*c**2*d*
*2*e**2 - 9*B*a*c*d*e**3 - 21*B*c**2*d**3*e + C*a**2*e**4 + 17*C*a*c*d**2*e
**2 + 28*C*c**2*d**4)*log(d + e*x)/e**9
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 701, normalized size of antiderivative = 1.50

$$\int \frac{(a+cx^2)^3\,(A+Bx+Cx^2)}{(d+ex)^3}\,dx$$

$$= \frac{15\,Cc^3d^8 - 13\,Bc^3d^7e - 27\,Bac^2d^5e^3 - 15\,Ba^2cd^3e^5 - Ba^3de^7 - Aa^3e^8 + 11\,(3\,Cac^2 + Ac^3)d^6e^2 + 21\,(C}{}$$

$$+ \frac{10\,Cc^3e^5x^6 - 12\,(3\,Cc^3de^4 - Bc^3e^5)x^5 + 15\,(6\,Cc^3d^2e^3 - 3\,Bc^3de^4 + (3\,Cac^2 + Ac^3)e^5)x^4 - 20\,(10\,Cc^3}{}$$

$$+ \frac{(28\,Cc^3d^6 - 21\,Bc^3d^5e - 30\,Bac^2d^3e^3 - 9\,Ba^2cde^5 + 15\,(3\,Cac^2 + Ac^3)d^4e^2 + 18\,(Ca^2c + Aac^2)d^2e^4 +}{e^9}$$

```
[In] integrate((c*x^2+a)^3*(C*x^2+B*x+A)/(e*x+d)^3,x, algorithm="maxima")
[Out] 1/2*(15*C*c^3*d^8 - 13*B*c^3*d^7*e - 27*B*a*c^2*d^5*e^3 - 15*B*a^2*c*d^3*e^
5 - B*a^3*d*e^7 - A*a^3*e^8 + 11*(3*C*a*c^2 + A*c^3)*d^6*e^2 + 21*(C*a^2*c
+ A*a*c^2)*d^4*e^4 + 3*(C*a^3 + 3*A*a^2*c)*d^2*e^6 + 2*(8*C*c^3*d^7*e - 7*B
*c^3*d^6*e^2 - 15*B*a*c^2*d^4*e^4 - 9*B*a^2*c*d^2*e^6 - B*a^3*e^8 + 6*(3*C*
a*c^2 + A*c^3)*d^5*e^3 + 12*(C*a^2*c + A*a*c^2)*d^3*e^5 + 2*(C*a^3 + 3*A*a^
2*c)*d*e^7)*x)/(e^11*x^2 + 2*d*e^10*x + d^2*e^9) + 1/60*(10*C*c^3*e^5*x^6 -
 12*(3*C*c^3*d*e^4 - B*c^3*e^5)*x^5 + 15*(6*C*c^3*d^2*e^3 - 3*B*c^3*d*e^4 +
 (3*C*a*c^2 + A*c^3)*e^5)*x^4 - 20*(10*C*c^3*d^3*e^2 - 6*B*c^3*d^2*e^3 - 3*
B*a*c^2*e^5 + 3*(3*C*a*c^2 + A*c^3)*d*e^4)*x^3 + 30*(15*C*c^3*d^4*e - 10*B*
c^3*d^3*e^2 - 9*B*a*c^2*d*e^4 + 6*(3*C*a*c^2 + A*c^3)*d^2*e^3 + 3*(C*a^2*c
+ A*a*c^2)*e^5)*x^2 - 60*(21*C*c^3*d^5 - 15*B*c^3*d^4*e - 18*B*a*c^2*d^2*e^
3 - 3*B*a^2*c*e^5 + 10*(3*C*a*c^2 + A*c^3)*d^3*e^2 + 9*(C*a^2*c + A*a*c^2)*
d*e^4)*x)/e^8 + (28*C*c^3*d^6 - 21*B*c^3*d^5*e - 30*B*a*c^2*d^3*e^3 - 9*B*a
^2*c*d*e^5 + 15*(3*C*a*c^2 + A*c^3)*d^4*e^2 + 18*(C*a^2*c + A*a*c^2)*d^2*e^
4 + (C*a^3 + 3*A*a^2*c)*e^6)*log(e*x + d)/e^9
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 780, normalized size of antiderivative = 1.67

$$\int \frac{(a+cx^2)^3\,(A+Bx+Cx^2)}{(d+ex)^3}\,dx$$

$$= \frac{(28\,Cc^3d^6 - 21\,Bc^3d^5e + 45\,Cac^2d^4e^2 + 15\,Ac^3d^4e^2 - 30\,Bac^2d^3e^3 + 18\,Ca^2cd^2e^4 + 18\,Aac^2d^2e^4 - 9\,Ba^2}{e^9}$$

$$+ \frac{15\,Cc^3d^8 - 13\,Bc^3d^7e + 33\,Cac^2d^6e^2 + 11\,Ac^3d^6e^2 - 27\,Bac^2d^5e^3 + 21\,Ca^2cd^4e^4 + 21\,Aac^2d^4e^4 - 15\,B}{}$$

$$+ \frac{10\,Cc^3e^{15}x^6 - 36\,Cc^3de^{14}x^5 + 12\,Bc^3e^{15}x^5 + 90\,Cc^3d^2e^{13}x^4 - 45\,Bc^3de^{14}x^4 + 45\,Cac^2e^{15}x^4 + 15\,Ac^3e^{1}}{}$$

```
[In] integrate((c*x^2+a)^3*(C*x^2+B*x+A)/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] (28*C*c^3*d^6 - 21*B*c^3*d^5*e + 45*C*a*c^2*d^4*e^2 + 15*A*c^3*d^4*e^2 - 30
*B*a*c^2*d^3*e^3 + 18*C*a^2*c*d^2*e^4 + 18*A*a*c^2*d^2*e^4 - 9*B*a^2*c*d*e^
5 + C*a^3*e^6 + 3*A*a^2*c*e^6)*log(abs(e*x + d))/e^9 + 1/2*(15*C*c^3*d^8 -
13*B*c^3*d^7*e + 33*C*a*c^2*d^6*e^2 + 11*A*c^3*d^6*e^2 - 27*B*a*c^2*d^5*e^3
 + 21*C*a^2*c*d^4*e^4 + 21*A*a*c^2*d^4*e^4 - 15*B*a^2*c*d^3*e^5 + 3*C*a^3*d
^2*e^6 + 9*A*a^2*c*d^2*e^6 - B*a^3*d*e^7 - A*a^3*e^8 + 2*(8*C*c^3*d^7*e - 7
*B*c^3*d^6*e^2 + 18*C*a*c^2*d^5*e^3 + 6*A*c^3*d^5*e^3 - 15*B*a*c^2*d^4*e^4
+ 12*C*a^2*c*d^3*e^5 + 12*A*a*c^2*d^3*e^5 - 9*B*a^2*c*d^2*e^6 + 2*C*a^3*d*e
^7 + 6*A*a^2*c*d*e^7 - B*a^3*e^8)*x)/((e*x + d)^2*e^9) + 1/60*(10*C*c^3*e^1
5*x^6 - 36*C*c^3*d*e^14*x^5 + 12*B*c^3*e^15*x^5 + 90*C*c^3*d^2*e^13*x^4 - 4
5*B*c^3*d*e^14*x^4 + 45*C*a*c^2*e^15*x^4 + 15*A*c^3*e^15*x^4 - 200*C*c^3*d^
3*e^12*x^3 + 120*B*c^3*d^2*e^13*x^3 - 180*C*a*c^2*d*e^14*x^3 - 60*A*c^3*d*e
^14*x^3 + 60*B*a*c^2*e^15*x^3 + 450*C*c^3*d^4*e^11*x^2 - 300*B*c^3*d^3*e^12
*x^2 + 540*C*a*c^2*d^2*e^13*x^2 + 180*A*c^3*d^2*e^13*x^2 - 270*B*a*c^2*d*e^
14*x^2 + 90*C*a^2*c*e^15*x^2 + 90*A*a*c^2*e^15*x^2 - 1260*C*c^3*d^5*e^10*x
+ 900*B*c^3*d^4*e^11*x - 1800*C*a*c^2*d^3*e^12*x - 600*A*c^3*d^3*e^12*x + 1
080*B*a*c^2*d^2*e^13*x - 540*C*a^2*c*d*e^14*x - 540*A*a*c^2*d*e^14*x + 180*
B*a^2*c*e^15*x)/e^18
```

Mupad [B] (verification not implemented)

Time = 12.38 (sec) , antiderivative size = 1290, normalized size of antiderivative = 2.77

$$\int \frac{(a+cx^2)^3 (A+Bx+Cx^2)}{(d+ex)^3} \, dx = \text{Too large to display}$$

```
[In] int(((a + c*x^2)^3*(A + B*x + C*x^2))/(d + e*x)^3,x)
```

```
[Out] x^3*((d*((3*d*((B*c^3)/e^3 - (3*C*c^3*d)/e^4))/e - (A*c^3 + 3*C*a*c^2)/e^3
+ (3*C*c^3*d^2)/e^5))/e - (d^2*((B*c^3)/e^3 - (3*C*c^3*d)/e^4))/e^2 + (B*a*
c^2)/e^3 - (C*c^3*d^3)/(3*e^6)) + x*((3*d*((3*d*((3*d*((3*d*((B*c^3)/e^3 -
(3*C*c^3*d)/e^4))/e - (A*c^3 + 3*C*a*c^2)/e^3 + (3*C*c^3*d^2)/e^5))/e - (3*
d^2*((B*c^3)/e^3 - (3*C*c^3*d)/e^4))/e^2 + (3*B*a*c^2)/e^3 - (C*c^3*d^3)/e^
6))/e - (3*d^2*((3*d*((B*c^3)/e^3 - (3*C*c^3*d)/e^4))/e - (A*c^3 + 3*C*a*c^
2)/e^3 + (3*C*c^3*d^2)/e^5))/e^2 + (d^3*((B*c^3)/e^3 - (3*C*c^3*d)/e^4))/e^
3 - (3*a*c*(A*c + C*a))/e^3))/e + (d^3*((3*d*((B*c^3)/e^3 - (3*C*c^3*d)/e^4
))/e - (A*c^3 + 3*C*a*c^2)/e^3 + (3*C*c^3*d^2)/e^5))/e^3 - (3*d^2*((3*d*((3
*d*((B*c^3)/e^3 - (3*C*c^3*d)/e^4))/e - (A*c^3 + 3*C*a*c^2)/e^3 + (3*C*c^3*
d^2)/e^5))/e - (3*d^2*((B*c^3)/e^3 - (3*C*c^3*d)/e^4))/e^2 + (3*B*a*c^2)/e^
3 - (C*c^3*d^3)/e^6))/e^2 + (3*B*a^2*c)/e^3) + x^5*((B*c^3)/(5*e^3) - (3*C*
c^3*d)/(5*e^4)) - x^4*((3*d*((B*c^3)/e^3 - (3*C*c^3*d)/e^4))/(4*e) - (A*c^3
 + 3*C*a*c^2)/(4*e^3) + (3*C*c^3*d^2)/(4*e^5)) - x^2*((3*d*((3*d*((3*d*((B*
c^3)/e^3 - (3*C*c^3*d)/e^4))/e - (A*c^3 + 3*C*a*c^2)/e^3 + (3*C*c^3*d^2)/e^
```

```
5))/e - (3*d^2*((B*c^3)/e^3 - (3*C*c^3*d)/e^4))/e^2 + (3*B*a*c^2)/e^3 - (C*
c^3*d^3)/e^6))/(2*e) - (3*d^2*((3*d*((B*c^3)/e^3 - (3*C*c^3*d)/e^4))/e - (A
*c^3 + 3*C*a*c^2)/e^3 + (3*C*c^3*d^2)/e^5))/(2*e^2) + (d^3*((B*c^3)/e^3 - (
3*C*c^3*d)/e^4))/(2*e^3) - (3*a*c*(A*c + C*a))/(2*e^3)) + ((15*C*c^3*d^8 -
A*a^3*e^8 - B*a^3*d*e^7 - 13*B*c^3*d^7*e + 11*A*c^3*d^6*e^2 + 3*C*a^3*d^2*e
^6 + 21*A*a*c^2*d^4*e^4 + 9*A*a^2*c*d^2*e^6 - 27*B*a*c^2*d^5*e^3 - 15*B*a^2
*c*d^3*e^5 + 33*C*a*c^2*d^6*e^2 + 21*C*a^2*c*d^4*e^4)/(2*e) + x*(8*C*c^3*d^
7 - B*a^3*e^7 + 2*C*a^3*d*e^6 - 7*B*c^3*d^6*e + 6*A*c^3*d^5*e^2 + 12*A*a*c^
2*d^3*e^4 - 15*B*a*c^2*d^4*e^3 - 9*B*a^2*c*d^2*e^5 + 18*C*a*c^2*d^5*e^2 + 1
2*C*a^2*c*d^3*e^4 + 6*A*a^2*c*d*e^6))/(d^2*e^8 + e^10*x^2 + 2*d*e^9*x) + (l
og(d + e*x)*(C*a^3*e^6 + 28*C*c^3*d^6 + 3*A*a^2*c*e^6 - 21*B*c^3*d^5*e + 15
*A*c^3*d^4*e^2 + 18*A*a*c^2*d^2*e^4 - 30*B*a*c^2*d^3*e^3 + 45*C*a*c^2*d^4*e
^2 + 18*C*a^2*c*d^2*e^4 - 9*B*a^2*c*d*e^5))/e^9 + (C*c^3*x^6)/(6*e^3)
```

3.39 $\int \frac{(a+bx^2)(-ad+4bcx+3bdx^2)}{(c+dx)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 17

$$\int \frac{(a+bx^2)(-ad+4bcx+3bdx^2)}{(c+dx)^2}\,dx = \frac{(a+bx^2)^2}{c+dx}$$

```
[Out] (b*x^2+a)^2/(d*x+c)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.031, Rules used = {1604}

$$\int \frac{(a+bx^2)(-ad+4bcx+3bdx^2)}{(c+dx)^2}\,dx = \frac{(a+bx^2)^2}{c+dx}$$

```
[In] Int[((a + b*x^2)*(-(a*d) + 4*b*c*x + 3*b*d*x^2))/(c + d*x)^2,x]
```

```
[Out] (a + b*x^2)^2/(c + d*x)
```

Rule 1604

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[Coeff[Pp, x, p]*x^(p - q - r + 1)*Qq
^(m + 1)*(Rr^(n + 1)/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]))
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\text{integral} = \frac{\left(a+bx^2\right)^2}{c+dx}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 62 vs. $2(17) = 34$.

Time = 0.02 (sec) , antiderivative size = 62, normalized size of antiderivative = 3.65

$$\int \frac{(a+bx^2)\left(-ad+4bcx+3bdx^2\right)}{(c+dx)^2}\,dx$$
$$= \frac{a^2d^4+2abd^2(c^2+cdx+d^2x^2)+b^2(c^4+c^3dx+d^4x^4)}{d^4(c+dx)}$$

```
[In] Integrate[((a + b*x^2)*(-(a*d) + 4*b*c*x + 3*b*d*x^2))/(c + d*x)^2,x]
```

```
[Out] (a^2*d^4 + 2*a*b*d^2*(c^2 + c*d*x + d^2*x^2) + b^2*(c^4 + c^3*d*x + d^4*x^4
))/(d^4*(c + d*x))
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.59

method	result	size
gosper	$\frac{b^2x^4+2ab\,x^2+a^2}{dx+c}$	27
norman	$\frac{b^2x^4+2ab\,x^2-\frac{d\,a^2x}{c}}{dx+c}$	34
parallelrisch	$\frac{b^2c\,x^4+2abc\,x^2-a^2dx}{c(dx+c)}$	36
default	$\frac{b\left(b\,x^3d^2-bcd\,x^2+2a\,d^2x+b\,c^2x\right)}{d^3}-\frac{-a^2d^4-2ab\,c^2d^2-b^2c^4}{d^4(dx+c)}$	76
risch	$\frac{b^2x^3}{d}-\frac{b^2c\,x^2}{d^2}+\frac{2bax}{d}+\frac{b^2c^2x}{d^3}+\frac{a^2}{dx+c}+\frac{2ab\,c^2}{d^2(dx+c)}+\frac{b^2c^4}{d^4(dx+c)}$	88

```
[In] int((b*x^2+a)*(3*b*d*x^2+4*b*c*x-a*d)/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (b^2*x^4+2*a*b*x^2+a^2)/(d*x+c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(17) = 34$.

Time $= 0.27$ (sec) , antiderivative size $= 78$, normalized size of antiderivative $= 4.59$

$$\int \frac{(a+bx^2)(-ad+4bcx+3bdx^2)}{(c+dx)^2}\,dx = \frac{b^2d^4x^4+2\,abd^4x^2+b^2c^4+2\,abc^2d^2+a^2d^4+(b^2c^3d+2\,abcd^3)x}{d^5x+cd^4}$$

```
[In] integrate((b*x^2+a)*(3*b*d*x^2+4*b*c*x-a*d)/(d*x+c)^2,x, algorithm="fricas"
)
```

```
[Out] (b^2*d^4*x^4 + 2*a*b*d^4*x^2 + b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4 + (b^2*c^3
*d + 2*a*b*c*d^3)*x)/(d^5*x + c*d^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(12) = 24$.

Time $= 0.18$ (sec) , antiderivative size $= 73$, normalized size of antiderivative $= 4.29$

$$\int \frac{(a+bx^2)(-ad+4bcx+3bdx^2)}{(c+dx)^2}\,dx = -\frac{b^2cx^2}{d^2}+\frac{b^2x^3}{d}+x\left(\frac{2ab}{d}+\frac{b^2c^2}{d^3}\right)+\frac{a^2d^4+2abc^2d^2+b^2c^4}{cd^4+d^5x}$$

```
[In] integrate((b*x**2+a)*(3*b*d*x**2+4*b*c*x-a*d)/(d*x+c)**2,x)
```

```
[Out] -b**2*c*x**2/d**2 + b**2*x**3/d + x*(2*a*b/d + b**2*c**2/d**3) + (a**2*d**4
 + 2*a*b*c**2*d**2 + b**2*c**4)/(c*d**4 + d**5*x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(17) = 34$.

Time $= 0.19$ (sec) , antiderivative size $= 82$, normalized size of antiderivative $= 4.82$

$$\int \frac{(a+bx^2)(-ad+4bcx+3bdx^2)}{(c+dx)^2}\,dx = \frac{b^2c^4+2\,abc^2d^2+a^2d^4}{d^5x+cd^4}+\frac{b^2d^2x^3-b^2cdx^2+(b^2c^2+2\,abd^2)x}{d^3}$$

```
[In] integrate((b*x^2+a)*(3*b*d*x^2+4*b*c*x-a*d)/(d*x+c)^2,x, algorithm="maxima"
)
```

```
[Out] (b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(d^5*x + c*d^4) + (b^2*d^2*x^3 - b^2*c*
d*x^2 + (b^2*c^2 + 2*a*b*d^2)*x)/d^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(17) = 34$.

Time = 0.27 (sec) , antiderivative size = 111, normalized size of antiderivative = 6.53

$$\int \frac{(a+bx^2)(-ad+4bcx+3bdx^2)}{(c+dx)^2}\,dx = \frac{\left(b^2 - \frac{4\,b^2c}{dx+c} + \frac{6\,b^2c^2}{(dx+c)^2} + \frac{2\,abd^2}{(dx+c)^2}\right)(dx+c)^3}{d^4} + \frac{\frac{b^2c^4d^3}{dx+c} + \frac{2\,abc^2d^5}{dx+c} + \frac{a^2d^7}{dx+c}}{d^7}$$

```
[In] integrate((b*x^2+a)*(3*b*d*x^2+4*b*c*x-a*d)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] (b^2 - 4*b^2*c/(d*x + c) + 6*b^2*c^2/(d*x + c)^2 + 2*a*b*d^2/(d*x + c)^2)*(
d*x + c)^3/d^4 + (b^2*c^4*d^3/(d*x + c) + 2*a*b*c^2*d^5/(d*x + c) + a^2*d^7
/(d*x + c))/d^7
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 85, normalized size of antiderivative = 5.00

$$\int \frac{(a+bx^2)(-ad+4bcx+3bdx^2)}{(c+dx)^2}\,dx = x\left(\frac{b^2\,c^2}{d^3} + \frac{2\,a\,b}{d}\right) + \frac{b^2\,x^3}{d} + \frac{a^2\,d^4 + 2\,a\,b\,c^2\,d^2 + b^2\,c^4}{d\,\left(x\,d^4 + c\,d^3\right)} - \frac{b^2\,c\,x^2}{d^2}$$

```
[In] int(((a + b*x^2)*(4*b*c*x - a*d + 3*b*d*x^2))/(c + d*x)^2,x)
```

```
[Out] x*((b^2*c^2)/d^3 + (2*a*b)/d) + (b^2*x^3)/d + (a^2*d^4 + b^2*c^4 + 2*a*b*c^
2*d^2)/(d*(c*d^3 + d^4*x)) - (b^2*c*x^2)/d^2
```

3.40 $\int \frac{(a+bx^2)(-ad+bx(4c+3dx))}{(c+dx)^2} dx$

Optimal result

Integrand size = 31, antiderivative size = 17

$$\int \frac{(a+bx^2)(-ad+bx(4c+3dx))}{(c+dx)^2} dx = \frac{(a+bx^2)^2}{c+dx}$$

```
[Out] (b*x^2+a)^2/(d*x+c)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.032$, Rules used = {1604}

$$\int \frac{(a+bx^2)(-ad+bx(4c+3dx))}{(c+dx)^2} dx = \frac{(a+bx^2)^2}{c+dx}$$

```
[In] Int[((a + b*x^2)*(-(a*d) + b*x*(4*c + 3*d*x)))/(c + d*x)^2,x]
```

```
[Out] (a + b*x^2)^2/(c + d*x)
```

Rule 1604

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[Coeff[Pp, x, p]*x^(p - q - r + 1)*Qq
^(m + 1)*(Rr^(n + 1)/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]))
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\text{integral} = \frac{(a+bx^2)^2}{c+dx}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 62 vs. 2(17) = 34.

Time = 0.01 (sec) , antiderivative size = 62, normalized size of antiderivative = 3.65

$$\int \frac{(a+bx^2)(-ad+bx(4c+3dx))}{(c+dx)^2}\,dx = \frac{a^2d^4+2abd^2(c^2+cdx+d^2x^2)+b^2(c^4+c^3dx+d^4x^4)}{d^4(c+dx)}$$

```
[In] Integrate[((a + b*x^2)*(-(a*d) + b*x*(4*c + 3*d*x)))/(c + d*x)^2,x]
```

```
[Out] (a^2*d^4 + 2*a*b*d^2*(c^2 + c*d*x + d^2*x^2) + b^2*(c^4 + c^3*d*x + d^4*x^4
))/(d^4*(c + d*x))
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.59

method	result	size
gosper	$\frac{b^2x^4+2ab\,x^2+a^2}{dx+c}$	27
norman	$\frac{b^2x^4+2ab\,x^2-\frac{d\,a^2x}{c}}{dx+c}$	34
parallelrisch	$\frac{b^2c\,x^4+2abc\,x^2-a^2dx}{c(dx+c)}$	36
default	$\frac{b\left(b\,x^3d^2-bcd\,x^2+2a\,d^2x+b\,c^2x\right)}{d^3}-\frac{-a^2d^4-2ab\,c^2d^2-b^2c^4}{d^4(dx+c)}$	76
risch	$\frac{b^2x^3}{d}-\frac{b^2c\,x^2}{d^2}+\frac{2bax}{d}+\frac{b^2c^2x}{d^3}+\frac{a^2}{dx+c}+\frac{2ab\,c^2}{d^2(dx+c)}+\frac{b^2c^4}{d^4(dx+c)}$	88

```
[In] int((b*x^2+a)*(-a*d+b*x*(3*d*x+4*c))/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (b^2*x^4+2*a*b*x^2+a^2)/(d*x+c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(17) = 34$.

Time = 0.28 (sec) , antiderivative size = 78, normalized size of antiderivative = 4.59

$$\int \frac{(a+bx^2)(-ad+bx(4c+3dx))}{(c+dx)^2}\,dx = \frac{b^2d^4x^4+2\,abd^4x^2+b^2c^4+2\,abc^2d^2+a^2d^4+(b^2c^3d+2\,abcd^3)x}{d^5x+cd^4}$$

```
[In] integrate((b*x^2+a)*(-a*d+b*x*(3*d*x+4*c))/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] (b^2*d^4*x^4 + 2*a*b*d^4*x^2 + b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4 + (b^2*c^3
*d + 2*a*b*c*d^3)*x)/(d^5*x + c*d^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(12) = 24$.

Time = 0.17 (sec) , antiderivative size = 73, normalized size of antiderivative = 4.29

$$\int \frac{(a+bx^2)(-ad+bx(4c+3dx))}{(c+dx)^2}\,dx = -\frac{b^2cx^2}{d^2}+\frac{b^2x^3}{d}+x\left(\frac{2ab}{d}+\frac{b^2c^2}{d^3}\right) + \frac{a^2d^4+2abc^2d^2+b^2c^4}{cd^4+d^5x}$$

```
[In] integrate((b*x**2+a)*(-a*d+b*x*(3*d*x+4*c))/(d*x+c)**2,x)
```

```
[Out] -b**2*c*x**2/d**2 + b**2*x**3/d + x*(2*a*b/d + b**2*c**2/d**3) + (a**2*d**4
 + 2*a*b*c**2*d**2 + b**2*c**4)/(c*d**4 + d**5*x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(17) = 34$.

Time = 0.19 (sec) , antiderivative size = 82, normalized size of antiderivative = 4.82

$$\int \frac{(a+bx^2)(-ad+bx(4c+3dx))}{(c+dx)^2}\,dx = \frac{b^2c^4+2\,abc^2d^2+a^2d^4}{d^5x+cd^4} + \frac{b^2d^2x^3-b^2cdx^2+(b^2c^2+2\,abd^2)x}{d^3}$$

```
[In] integrate((b*x^2+a)*(-a*d+b*x*(3*d*x+4*c))/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] (b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(d^5*x + c*d^4) + (b^2*d^2*x^3 - b^2*c*
d*x^2 + (b^2*c^2 + 2*a*b*d^2)*x)/d^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(17) = 34$.

Time = 0.27 (sec) , antiderivative size = 111, normalized size of antiderivative = 6.53

$$\int \frac{(a+bx^2)(-ad+bx(4c+3dx))}{(c+dx)^2}\,dx = \frac{\left(b^2 - \frac{4\,b^2c}{dx+c} + \frac{6\,b^2c^2}{(dx+c)^2} + \frac{2\,abd^2}{(dx+c)^2}\right)(dx+c)^3}{d^4} + \frac{\frac{b^2c^4d^3}{dx+c} + \frac{2\,abc^2d^5}{dx+c} + \frac{a^2d^7}{dx+c}}{d^7}$$

```
[In] integrate((b*x^2+a)*(-a*d+b*x*(3*d*x+4*c))/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] (b^2 - 4*b^2*c/(d*x + c) + 6*b^2*c^2/(d*x + c)^2 + 2*a*b*d^2/(d*x + c)^2)*(
d*x + c)^3/d^4 + (b^2*c^4*d^3/(d*x + c) + 2*a*b*c^2*d^5/(d*x + c) + a^2*d^7
/(d*x + c))/d^7
```

Mupad [B] (verification not implemented)

Time = 12.48 (sec) , antiderivative size = 85, normalized size of antiderivative = 5.00

$$\int \frac{(a+bx^2)(-ad+bx(4c+3dx))}{(c+dx)^2}\,dx = x\left(\frac{b^2\,c^2}{d^3} + \frac{2\,a\,b}{d}\right) + \frac{b^2\,x^3}{d} + \frac{a^2\,d^4 + 2\,a\,b\,c^2\,d^2 + b^2\,c^4}{d\,\left(x\,d^4 + c\,d^3\right)} - \frac{b^2\,c\,x^2}{d^2}$$

```
[In] int(-((a*d - b*x*(4*c + 3*d*x))*(a + b*x^2))/(c + d*x)^2,x)
```

```
[Out] x*((b^2*c^2)/d^3 + (2*a*b)/d) + (b^2*x^3)/d + (a^2*d^4 + b^2*c^4 + 2*a*b*c^
2*d^2)/(d*(c*d^3 + d^4*x)) - (b^2*c*x^2)/d^2
```

3.41 $\int \frac{(a+bx^2)^2(-ad+6bcx+5bdx^2)}{(c+dx)^2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 17

$$\int \frac{(a+bx^2)^2(-ad+6bcx+5bdx^2)}{(c+dx)^2}\,dx = \frac{(a+bx^2)^3}{c+dx}$$

```
[Out] (b*x^2+a)^3/(d*x+c)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {1604}

$$\int \frac{(a+bx^2)^2(-ad+6bcx+5bdx^2)}{(c+dx)^2}\,dx = \frac{(a+bx^2)^3}{c+dx}$$

```
[In] Int[((a + b*x^2)^2*(-(a*d) + 6*b*c*x + 5*b*d*x^2))/(c + d*x)^2,x]
```

```
[Out] (a + b*x^2)^3/(c + d*x)
```

Rule 1604

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[Coeff[Pp, x, p]*x^(p - q - r + 1)*Qq
^(m + 1)*(Rr^(n + 1)/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]))
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\text{integral} = \frac{\left(a+bx^2\right)^3}{c+dx}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 90 vs. 2(17) = 34.

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 5.29

$$\int \frac{\left(a+bx^2\right)^2\left(-ad+6bcx+5bdx^2\right)}{(c+dx)^2}\,dx$$
$$= \frac{a^3d^6+3a^2bd^4\left(c^2+cdx+d^2x^2\right)+3ab^2d^2\left(c^4+c^3dx+d^4x^4\right)+b^3\left(c^6+c^5dx+d^6x^6\right)}{d^6(c+dx)}$$

```
[In] Integrate[((a + b*x^2)^2*(-(a*d) + 6*b*c*x + 5*b*d*x^2))/(c + d*x)^2,x]
```

```
[Out] (a^3*d^6 + 3*a^2*b*d^4*(c^2 + c*d*x + d^2*x^2) + 3*a*b^2*d^2*(c^4 + c^3*d*x
 + d^4*x^4) + b^3*(c^6 + c^5*d*x + d^6*x^6))/(d^6*(c + d*x))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. 2(17) = 34.

Time = 0.47 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.24

method	result
gosper	$\frac{b^3x^6+3b^2a\,x^4+3a^2b\,x^2+a^3}{dx+c}$
norman	$\frac{b^3x^6+3a^2b\,x^2+3b^2a\,x^4-\frac{d\,a^3x}{c}}{dx+c}$
parallelrisch	$\frac{b^3d\,x^6+3a\,b^2d\,x^4+3a^2bd\,x^2+d\,a^3}{(dx+c)d}$
default	$\frac{b\left(b^2x^5d^4-x^4b^2d^3c+3ab\,d^4x^3+b^2c^2d^2x^3-3abc\,d^3x^2-b^2c^3d\,x^2+3a^2d^4x+3ab\,c^2d^2x+b^2c^4x\right)}{d^5}-\frac{-a^3d^6-3a^2b\,c^2d^4-3a\,b^2c^4d^2}{d^6(dx+c)}$
risch	$\frac{b^3x^5}{d}-\frac{b^3x^4c}{d^2}+\frac{3b^2a\,x^3}{d}+\frac{b^3c^2x^3}{d^3}-\frac{3b^2ac\,x^2}{d^2}-\frac{b^3c^3x^2}{d^4}+\frac{3b\,a^2x}{d}+\frac{3b^2a\,c^2x}{d^3}+\frac{b^3c^4x}{d^5}+\frac{a^3}{dx+c}+\frac{3a^2b\,c^2}{d^2(dx+c)}+$

```
[In] int((b*x^2+a)^2*(5*b*d*x^2+6*b*c*x-a*d)/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (b^3*x^6+3*a*b^2*x^4+3*a^2*b*x^2+a^3)/(d*x+c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(17) = 34$.

Time = 0.28 (sec) , antiderivative size = 120, normalized size of antiderivative = 7.06

$$\int \frac{(a+bx^2)^2(-ad+6bcx+5bdx^2)}{(c+dx)^2}\,dx$$
$$= \frac{b^3d^6x^6+3\,ab^2d^6x^4+3\,a^2bd^6x^2+b^3c^6+3\,ab^2c^4d^2+3\,a^2bc^2d^4+a^3d^6+(b^3c^5d+3\,ab^2c^3d^3+3\,a^2bcd^5)x}{d^7x+cd^6}$$

```
[In] integrate((b*x^2+a)^2*(5*b*d*x^2+6*b*c*x-a*d)/(d*x+c)^2,x, algorithm="frica
s")
```

```
[Out] (b^3*d^6*x^6 + 3*a*b^2*d^6*x^4 + 3*a^2*b*d^6*x^2 + b^3*c^6 + 3*a*b^2*c^4*d^
2 + 3*a^2*b*c^2*d^4 + a^3*d^6 + (b^3*c^5*d + 3*a*b^2*c^3*d^3 + 3*a^2*b*c*d^
5)*x)/(d^7*x + c*d^6)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(12) = 24$.

Time = 0.28 (sec) , antiderivative size = 153, normalized size of antiderivative = 9.00

$$\int \frac{(a+bx^2)^2(-ad+6bcx+5bdx^2)}{(c+dx)^2}\,dx = -\frac{b^3cx^4}{d^2}+\frac{b^3x^5}{d}+x^3\cdot\left(\frac{3ab^2}{d}+\frac{b^3c^2}{d^3}\right)+x^2\left(-\frac{3ab^2c}{d^2}-\frac{b^3c^3}{d^4}\right)+x\left(\frac{3a^2b}{d}+\frac{3ab^2c^2}{d^3}+\frac{b^3c^4}{d^5}\right)+\frac{a^3d^6+3a^2bc^2d^4+3ab^2c^4d^2+b^3c^6}{cd^6+d^7x}$$

```
[In] integrate((b*x**2+a)**2*(5*b*d*x**2+6*b*c*x-a*d)/(d*x+c)**2,x)
```

```
[Out] -b**3*c*x**4/d**2 + b**3*x**5/d + x**3*(3*a*b**2/d + b**3*c**2/d**3) + x**2
*(-3*a*b**2*c/d**2 - b**3*c**3/d**4) + x*(3*a**2*b/d + 3*a*b**2*c**2/d**3 +
 b**3*c**4/d**5) + (a**3*d**6 + 3*a**2*b*c**2*d**4 + 3*a*b**2*c**4*d**2 + b
**3*c**6)/(c*d**6 + d**7*x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(17) = 34$.

Time = 0.19 (sec) , antiderivative size = 160, normalized size of antiderivative = 9.41

$$\int \frac{(a+bx^2)^2\,(-ad+6bcx+5bdx^2)}{(c+dx)^2}\,dx = \frac{b^3c^6+3\,ab^2c^4d^2+3\,a^2bc^2d^4+a^3d^6}{d^7x+cd^6} + \frac{b^3d^4x^5-b^3cd^3x^4+(b^3c^2d^2+3\,ab^2d^4)x^3-(b^3c^3d+3\,ab^2cd^3)x^2+(b^3c^4+3\,ab^2c^2d^2+3\,a^2bd^4)x}{d^5}$$

```
[In] integrate((b*x^2+a)^2*(5*b*d*x^2+6*b*c*x-a*d)/(d*x+c)^2,x, algorithm="maxim
a")
```

```
[Out] (b^3*c^6 + 3*a*b^2*c^4*d^2 + 3*a^2*b*c^2*d^4 + a^3*d^6)/(d^7*x + c*d^6) + (
b^3*d^4*x^5 - b^3*c*d^3*x^4 + (b^3*c^2*d^2 + 3*a*b^2*d^4)*x^3 - (b^3*c^3*d
+ 3*a*b^2*c*d^3)*x^2 + (b^3*c^4 + 3*a*b^2*c^2*d^2 + 3*a^2*b*d^4)*x)/d^5
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(17) = 34$.

Time = 0.28 (sec) , antiderivative size = 216, normalized size of antiderivative = 12.71

$$\int \frac{(a+bx^2)^2\,(-ad+6bcx+5bdx^2)}{(c+dx)^2}\,dx = \frac{\left(b^3-\frac{6\,b^3c}{dx+c}+\frac{15\,b^3c^2}{(dx+c)^2}-\frac{20\,b^3c^3}{(dx+c)^3}+\frac{15\,b^3c^4}{(dx+c)^4}+\frac{3\,ab^2d^2}{(dx+c)^2}-\frac{12\,ab^2cd^2}{(dx+c)^3}+\frac{18\,ab^2c^2d^2}{(dx+c)^4}+\frac{3\,a^2bd^4}{(dx+c)^4}\right)(dx+c)^5}{d^6} + \frac{\frac{b^3c^6d^5}{dx+c}+\frac{3\,ab^2c^4d^7}{dx+c}+\frac{3\,a^2bc^2d^9}{dx+c}+\frac{a^3d^{11}}{dx+c}}{d^{11}}$$

```
[In] integrate((b*x^2+a)^2*(5*b*d*x^2+6*b*c*x-a*d)/(d*x+c)^2,x, algorithm="giac"
)
```

```
[Out] (b^3 - 6*b^3*c/(d*x + c) + 15*b^3*c^2/(d*x + c)^2 - 20*b^3*c^3/(d*x + c)^3
+ 15*b^3*c^4/(d*x + c)^4 + 3*a*b^2*d^2/(d*x + c)^2 - 12*a*b^2*c*d^2/(d*x +
c)^3 + 18*a*b^2*c^2*d^2/(d*x + c)^4 + 3*a^2*b*d^4/(d*x + c)^4)*(d*x + c)^5/
d^6 + (b^3*c^6*d^5/(d*x + c) + 3*a*b^2*c^4*d^7/(d*x + c) + 3*a^2*b*c^2*d^9/
(d*x + c) + a^3*d^11/(d*x + c))/d^11
```

Mupad [B] (verification not implemented)

Time = 12.80 (sec) , antiderivative size = 252, normalized size of antiderivative = 14.82

$$\int \frac{(a+bx^2)^2\,(-ad+6bcx+5bdx^2)}{(c+dx)^2}\,dx = x^3\left(\frac{3\,a\,b^2}{d}+\frac{b^3\,c^2}{d^3}\right) - x\left(\frac{2\,c\left(\frac{4\,b^3\,c^3}{d^4}-\frac{2\,c\left(\frac{9\,a\,b^2}{d}+\frac{3\,b^3\,c^2}{d^3}\right)}{d}+\frac{12\,a\,b^2\,c}{d^2}\right)}{d} + \frac{c^2\left(\frac{9\,a\,b^2}{d}+\frac{3\,b^3\,c^2}{d^3}\right)}{d^2}-\frac{3\,a^2\,b}{d}\right) + x^2\left(\frac{2\,b^3\,c^3}{d^4}-\frac{c\left(\frac{9\,a\,b^2}{d}+\frac{3\,b^3\,c^2}{d^3}\right)}{d}+\frac{6\,a\,b^2\,c}{d^2}\right) + \frac{a^3\,d^6+3\,a^2\,b\,c^2\,d^4+3\,a\,b^2\,c^4\,d^2+b^3\,c^6}{d\,(x\,d^6+c\,d^5)} + \frac{b^3\,x^5}{d}-\frac{b^3\,c\,x^4}{d^2}$$

```
[In] int(((a + b*x^2)^2*(6*b*c*x - a*d + 5*b*d*x^2))/(c + d*x)^2,x)
```

```
[Out] x^3*((3*a*b^2)/d + (b^3*c^2)/d^3) - x*((2*c*((4*b^3*c^3)/d^4 - (2*c*((9*a*b
^2)/d + (3*b^3*c^2)/d^3))/d + (12*a*b^2*c)/d^2))/d + (c^2*((9*a*b^2)/d + (3
*b^3*c^2)/d^3))/d^2 - (3*a^2*b)/d) + x^2*((2*b^3*c^3)/d^4 - (c*((9*a*b^2)/d
 + (3*b^3*c^2)/d^3))/d + (6*a*b^2*c)/d^2) + (a^3*d^6 + b^3*c^6 + 3*a*b^2*c^
4*d^2 + 3*a^2*b*c^2*d^4)/(d*(c*d^5 + d^6*x)) + (b^3*x^5)/d - (b^3*c*x^4)/d^
2
```

3.42 $\int \frac{(a+bx^2)^2(-ad+bx(6c+5dx))}{(c+dx)^2} dx$

Optimal result

Integrand size = 33, antiderivative size = 17

$$\int \frac{(a+bx^2)^2(-ad+bx(6c+5dx))}{(c+dx)^2} dx = \frac{(a+bx^2)^3}{c+dx}$$

```
[Out] (b*x^2+a)^3/(d*x+c)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.030$, Rules used = $\{1604\}$

$$\int \frac{(a+bx^2)^2(-ad+bx(6c+5dx))}{(c+dx)^2} dx = \frac{(a+bx^2)^3}{c+dx}$$

```
[In] Int[((a + b*x^2)^2*(-(a*d) + b*x*(6*c + 5*d*x)))/(c + d*x)^2,x]
```

```
[Out] (a + b*x^2)^3/(c + d*x)
```

Rule 1604

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[Coeff[Pp, x, p]*x^(p - q - r + 1)*Qq
^(m + 1)*(Rr^(n + 1)/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]))
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\text{integral} = \frac{(a+bx^2)^3}{c+dx}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 90 vs. 2(17) = 34.

Time = 0.01 (sec) , antiderivative size = 90, normalized size of antiderivative = 5.29

$$\int \frac{(a+bx^2)^2(-ad+bx(6c+5dx))}{(c+dx)^2}\,dx$$
$$= \frac{a^3d^6+3a^2bd^4(c^2+cdx+d^2x^2)+3ab^2d^2(c^4+c^3dx+d^4x^4)+b^3(c^6+c^5dx+d^6x^6)}{d^6(c+dx)}$$

```
[In] Integrate[((a + b*x^2)^2*(-(a*d) + b*x*(6*c + 5*d*x)))/(c + d*x)^2,x]
```

```
[Out] (a^3*d^6 + 3*a^2*b*d^4*(c^2 + c*d*x + d^2*x^2) + 3*a*b^2*d^2*(c^4 + c^3*d*x
 + d^4*x^4) + b^3*(c^6 + c^5*d*x + d^6*x^6))/(d^6*(c + d*x))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. 2(17) = 34.

Time = 0.57 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.24

method	result
gosper	$\frac{b^3x^6+3b^2a\,x^4+3a^2b\,x^2+a^3}{dx+c}$
norman	$\frac{b^3x^6+3a^2b\,x^2+3b^2a\,x^4-\frac{d\,a^3x}{c}}{dx+c}$
parallelrisch	$\frac{b^3d\,x^6+3a\,b^2d\,x^4+3a^2bd\,x^2+d\,a^3}{(dx+c)d}$
default	$\frac{b\left(b^2x^5d^4-x^4b^2d^3c+3ab\,d^4x^3+b^2c^2d^2x^3-3abc\,d^3x^2-b^2c^3d\,x^2+3a^2d^4x+3ab\,c^2d^2x+b^2c^4x\right)}{d^5}-\frac{-a^3d^6-3a^2b\,c^2d^4-3a\,b^2c^4d^2-}{d^6(dx+c)}$
risch	$\frac{b^3x^5}{d}-\frac{b^3x^4c}{d^2}+\frac{3b^2a\,x^3}{d}+\frac{b^3c^2x^3}{d^3}-\frac{3b^2ac\,x^2}{d^2}-\frac{b^3c^3x^2}{d^4}+\frac{3b\,a^2x}{d}+\frac{3b^2a\,c^2x}{d^3}+\frac{b^3c^4x}{d^5}+\frac{a^3}{dx+c}+\frac{3a^2b\,c^2}{d^2(dx+c)}+$

```
[In] int((b*x^2+a)^2*(-a*d+b*x*(5*d*x+6*c))/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (b^3*x^6+3*a*b^2*x^4+3*a^2*b*x^2+a^3)/(d*x+c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(17) = 34$.

Time = 0.27 (sec) , antiderivative size = 120, normalized size of antiderivative = 7.06

$$\int \frac{\left(a+bx^2\right)^2 (-ad+bx(6c+5dx))}{(c+dx)^2}\,dx$$
$$=\frac{b^3d^6x^6+3\,ab^2d^6x^4+3\,a^2bd^6x^2+b^3c^6+3\,ab^2c^4d^2+3\,a^2bc^2d^4+a^3d^6+(b^3c^5d+3\,ab^2c^3d^3+3\,a^2bcd^5)x}{d^7x+cd^6}$$

```
[In] integrate((b*x^2+a)^2*(-a*d+b*x*(5*d*x+6*c))/(d*x+c)^2,x, algorithm="fricas
")
```

```
[Out] (b^3*d^6*x^6 + 3*a*b^2*d^6*x^4 + 3*a^2*b*d^6*x^2 + b^3*c^6 + 3*a*b^2*c^4*d^
2 + 3*a^2*b*c^2*d^4 + a^3*d^6 + (b^3*c^5*d + 3*a*b^2*c^3*d^3 + 3*a^2*b*c*d^
5)*x)/(d^7*x + c*d^6)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(12) = 24$.

Time = 0.27 (sec) , antiderivative size = 153, normalized size of antiderivative = 9.00

$$\int \frac{\left(a+bx^2\right)^2 (-ad+bx(6c+5dx))}{(c+dx)^2}\,dx = -\frac{b^3cx^4}{d^2}+\frac{b^3x^5}{d}+x^3\cdot\left(\frac{3ab^2}{d}+\frac{b^3c^2}{d^3}\right)$$
$$+x^2\left(-\frac{3ab^2c}{d^2}-\frac{b^3c^3}{d^4}\right)+x\left(\frac{3a^2b}{d}+\frac{3ab^2c^2}{d^3}+\frac{b^3c^4}{d^5}\right)$$
$$+\frac{a^3d^6+3a^2bc^2d^4+3ab^2c^4d^2+b^3c^6}{cd^6+d^7x}$$

```
[In] integrate((b*x**2+a)**2*(-a*d+b*x*(5*d*x+6*c))/(d*x+c)**2,x)
```

```
[Out] -b**3*c*x**4/d**2 + b**3*x**5/d + x**3*(3*a*b**2/d + b**3*c**2/d**3) + x**2
*(-3*a*b**2*c/d**2 - b**3*c**3/d**4) + x*(3*a**2*b/d + 3*a*b**2*c**2/d**3 +
 b**3*c**4/d**5) + (a**3*d**6 + 3*a**2*b*c**2*d**4 + 3*a*b**2*c**4*d**2 + b
**3*c**6)/(c*d**6 + d**7*x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(17) = 34$.

Time = 0.19 (sec) , antiderivative size = 160, normalized size of antiderivative = 9.41

$$\int \frac{(a+bx^2)^2\,(-ad+bx(6c+5dx))}{(c+dx)^2}\,dx = \frac{b^3c^6+3\,ab^2c^4d^2+3\,a^2bc^2d^4+a^3d^6}{d^7x+cd^6} + \frac{b^3d^4x^5-b^3cd^3x^4+(b^3c^2d^2+3\,ab^2d^4)x^3-(b^3c^3d+3\,ab^2cd^3)x^2+(b^3c^4+3\,ab^2c^2d^2+3\,a^2bd^4)x}{d^5}$$

```
[In] integrate((b*x^2+a)^2*(-a*d+b*x*(5*d*x+6*c))/(d*x+c)^2,x, algorithm="maxima
")
```

```
[Out] (b^3*c^6 + 3*a*b^2*c^4*d^2 + 3*a^2*b*c^2*d^4 + a^3*d^6)/(d^7*x + c*d^6) + (
b^3*d^4*x^5 - b^3*c*d^3*x^4 + (b^3*c^2*d^2 + 3*a*b^2*d^4)*x^3 - (b^3*c^3*d
+ 3*a*b^2*c*d^3)*x^2 + (b^3*c^4 + 3*a*b^2*c^2*d^2 + 3*a^2*b*d^4)*x)/d^5
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(17) = 34$.

Time = 0.27 (sec) , antiderivative size = 216, normalized size of antiderivative = 12.71

$$\int \frac{(a+bx^2)^2\,(-ad+bx(6c+5dx))}{(c+dx)^2}\,dx = \frac{\left(b^3-\frac{6\,b^3c}{dx+c}+\frac{15\,b^3c^2}{(dx+c)^2}-\frac{20\,b^3c^3}{(dx+c)^3}+\frac{15\,b^3c^4}{(dx+c)^4}+\frac{3\,ab^2d^2}{(dx+c)^2}-\frac{12\,ab^2cd^2}{(dx+c)^3}+\frac{18\,ab^2c^2d^2}{(dx+c)^4}+\frac{3\,a^2bd^4}{(dx+c)^4}\right)(dx+c)^5}{d^6} + \frac{\frac{b^3c^6d^5}{dx+c}+\frac{3\,ab^2c^4d^7}{dx+c}+\frac{3\,a^2bc^2d^9}{dx+c}+\frac{a^3d^{11}}{dx+c}}{d^{11}}$$

```
[In] integrate((b*x^2+a)^2*(-a*d+b*x*(5*d*x+6*c))/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] (b^3 - 6*b^3*c/(d*x + c) + 15*b^3*c^2/(d*x + c)^2 - 20*b^3*c^3/(d*x + c)^3
+ 15*b^3*c^4/(d*x + c)^4 + 3*a*b^2*d^2/(d*x + c)^2 - 12*a*b^2*c*d^2/(d*x +
c)^3 + 18*a*b^2*c^2*d^2/(d*x + c)^4 + 3*a^2*b*d^4/(d*x + c)^4)*(d*x + c)^5/
d^6 + (b^3*c^6*d^5/(d*x + c) + 3*a*b^2*c^4*d^7/(d*x + c) + 3*a^2*b*c^2*d^9/
(d*x + c) + a^3*d^11/(d*x + c))/d^11
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 252, normalized size of antiderivative = 14.82

$$\int \frac{(a+bx^2)^2\,(-ad+bx(6c+5dx))}{(c+dx)^2}\,dx = x^3\left(\frac{3\,a\,b^2}{d}+\frac{b^3\,c^2}{d^3}\right) - x\left(\frac{2\,c\left(\frac{4\,b^3\,c^3}{d^4}-\frac{2\,c\left(\frac{9\,a\,b^2}{d}+\frac{3\,b^3\,c^2}{d^3}\right)}{d}+\frac{12\,a\,b^2\,c}{d^2}\right)}{d} + \frac{c^2\left(\frac{9\,a\,b^2}{d}+\frac{3\,b^3\,c^2}{d^3}\right)}{d^2} - \frac{3\,a^2\,b}{d}\right) + x^2\left(\frac{2\,b^3\,c^3}{d^4} - \frac{c\left(\frac{9\,a\,b^2}{d}+\frac{3\,b^3\,c^2}{d^3}\right)}{d} + \frac{6\,a\,b^2\,c}{d^2}\right) + \frac{a^3\,d^6+3\,a^2\,b\,c^2\,d^4+3\,a\,b^2\,c^4\,d^2+b^3\,c^6}{d\,(x\,d^6+c\,d^5)} + \frac{b^3\,x^5}{d} - \frac{b^3\,c\,x^4}{d^2}$$

```
[In] int(-((a*d - b*x*(6*c + 5*d*x))*(a + b*x^2)^2)/(c + d*x)^2,x)
```

```
[Out] x^3*((3*a*b^2)/d + (b^3*c^2)/d^3) - x*((2*c*((4*b^3*c^3)/d^4 - (2*c*((9*a*b
^2)/d + (3*b^3*c^2)/d^3))/d + (12*a*b^2*c)/d^2))/d + (c^2*((9*a*b^2)/d + (3
*b^3*c^2)/d^3))/d^2 - (3*a^2*b)/d) + x^2*((2*b^3*c^3)/d^4 - (c*((9*a*b^2)/d
 + (3*b^3*c^2)/d^3))/d + (6*a*b^2*c)/d^2) + (a^3*d^6 + b^3*c^6 + 3*a*b^2*c^
4*d^2 + 3*a^2*b*c^2*d^4)/(d*(c*d^5 + d^6*x)) + (b^3*x^5)/d - (b^3*c*x^4)/d^
2
```

3.43 $\int \frac{(d+ex)^3(A+Bx+Cx^2)}{a+cx^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 240

$$\begin{aligned}
&\int \frac{(d+ex)^3\,(A+Bx+Cx^2)}{a+cx^2}\,dx \\
&= -\frac{(ae^2(3Cd+Be)-cd(Cd^2+3e(Bd+Ae)))\,x}{c^2} \\
&\quad - \frac{e(aCe^2-c(3Cd^2+e(3Bd+Ae)))\,x^2}{2c^2} + \frac{e^2(3Cd+Be)x^3}{3c} + \frac{Ce^3x^4}{4c} \\
&\quad + \frac{(Acd(cd^2-3ae^2)+a(ae^2(3Cd+Be)-cd^2(Cd+3Be)))\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}c^{5/2}} \\
&\quad + \frac{(Bcd(cd^2-3ae^2)+(Ac-aC)e(3cd^2-ae^2))\log\left(a+cx^2\right)}{2c^3}
\end{aligned}$$

```
[Out] -(a*e^2*(B*e+3*C*d)-c*d*(C*d^2+3*e*(A*e+B*d)))*x/c^2-1/2*e*(a*C*e^2-c*(3*C*
d^2+e*(A*e+3*B*d)))*x^2/c^2+1/3*e^2*(B*e+3*C*d)*x^3/c+1/4*C*e^3*x^4/c+1/2*(
B*c*d*(-3*a*e^2+c*d^2)+(A*c-C*a)*e*(-a*e^2+3*c*d^2))*ln(c*x^2+a)/c^3+(A*c*d
*(-3*a*e^2+c*d^2)+a*(a*e^2*(B*e+3*C*d)-c*d^2*(3*B*e+C*d)))*arctan(x*c^(1/2)
/a^(1/2))/c^(5/2)/a^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1643, 649, 211, 266}

$$\begin{aligned}\int \frac{(d+ex)^3\,(A+Bx+Cx^2)}{a+cx^2}\,dx\\ =\frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(Acd(cd^2-3ae^2)+a(ae^2(Be+3Cd)-cd^2(3Be+Cd))\right)}{\sqrt{a}c^{5/2}}\\ +\frac{\log\left(a+cx^2\right)\left(e(Ac-aC)\left(3cd^2-ae^2\right)+Bcd(cd^2-3ae^2)\right)}{2c^3}\\ +\frac{x(-ae^2(Be+3Cd)+3cde(Ae+Bd)+cCd^3)}{c^2}\\ +\frac{ex^2(-aCe^2+ce(Ae+3Bd)+3cCd^2)}{2c^2}+\frac{e^2x^3(Be+3Cd)}{3c}+\frac{Ce^3x^4}{4c}\end{aligned}$$

```
[In] Int[((d + e*x)^3*(A + B*x + C*x^2))/(a + c*x^2),x]
```

```
[Out] ((c*C*d^3 + 3*c*d*e*(B*d + A*e) - a*e^2*(3*C*d + B*e))*x)/c^2 + (e*(3*c*C*d
^2 - a*C*e^2 + c*e*(3*B*d + A*e))*x^2)/(2*c^2) + (e^2*(3*C*d + B*e)*x^3)/(3
*c) + (C*e^3*x^4)/(4*c) + ((A*c*d*(c*d^2 - 3*a*e^2) + a*(a*e^2*(3*C*d + B*e
) - c*d^2*(C*d + 3*B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*c^(5/2)) +
((B*c*d*(c*d^2 - 3*a*e^2) + (A*c - a*C)*e*(3*c*d^2 - a*e^2))*Log[a + c*x^2]
)/(2*c^3)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
```

```
d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\text{integral} = \int \left(\frac{cCd^3 + 3cde(Bd + Ae) - ae^2(3Cd + Be)}{c^2} + \frac{e(3cCd^2 - aCe^2 + ce(3Bd + Ae))\,x}{c^2} + \frac{e^2(3Cd + Be)x^2}{c} + \frac{Ce^3x^3}{c} + \frac{Acd(cd^2 - 3ae^2) + a(ae^2(3Cd + Be) - cd^2(Cd + 3Be)) + (Bcd(cd^2 - 3ae^2) + (Ac - aC)e(3cd^2 - ae^2))}{c^2\,(a + cx^2)}\right.$$

$$= \frac{(cCd^3 + 3cde(Bd + Ae) - ae^2(3Cd + Be))\,x}{c^2} + \frac{e(3cCd^2 - aCe^2 + ce(3Bd + Ae))\,x^2}{2c^2} + \frac{e^2(3Cd + Be)x^3}{3c} + \frac{Ce^3x^4}{4c} + \frac{\int \frac{Acd(cd^2-3ae^2)+a(ae^2(3Cd+Be)-cd^2(Cd+3Be))+(Bcd(cd^2-3ae^2)+(Ac-aC)e(3cd^2-ae^2))x}{a+cx^2}\,dx}{c^2}$$

$$= \frac{(cCd^3 + 3cde(Bd + Ae) - ae^2(3Cd + Be))\,x}{c^2} + \frac{e(3cCd^2 - aCe^2 + ce(3Bd + Ae))\,x^2}{2c^2} + \frac{e^2(3Cd + Be)x^3}{3c} + \frac{Ce^3x^4}{4c} + \frac{(Bcd(cd^2 - 3ae^2) + (Ac - aC)e(3cd^2 - ae^2))\int \frac{x}{a+cx^2}\,dx}{c^2} + \frac{(Acd(cd^2 - 3ae^2) + a(ae^2(3Cd + Be) - cd^2(Cd + 3Be)))\int \frac{1}{a+cx^2}\,dx}{c^2}$$

$$= \frac{(cCd^3 + 3cde(Bd + Ae) - ae^2(3Cd + Be))\,x}{c^2} + \frac{e(3cCd^2 - aCe^2 + ce(3Bd + Ae))\,x^2}{2c^2} + \frac{e^2(3Cd + Be)x^3}{3c} + \frac{Ce^3x^4}{4c} + \frac{(Acd(cd^2 - 3ae^2) + a(ae^2(3Cd + Be) - cd^2(Cd + 3Be)))\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}c^{5/2}} + \frac{(Bcd(cd^2 - 3ae^2) + (Ac - aC)e(3cd^2 - ae^2))\log\left(a + cx^2\right)}{2c^3}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.93

$$\int \frac{(d+ex)^3\left(A+Bx+Cx^2\right)}{a+cx^2}\,dx$$
$$= \frac{(Acd(cd^2-3ae^2)+a(ae^2(3Cd+Be)-cd^2(Cd+3Be)))\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}c^{5/2}}$$
$$+\frac{cx(-6ae^2(6Cd+2Be+Cex)+3cC(4d^3+6d^2ex+4de^2x^2+e^3x^3)+2ce(3Ae(6d+ex)+B(18d^2+9}{12c^3}$$

```
[In] Integrate[((d + e*x)^3*(A + B*x + C*x^2))/(a + c*x^2),x]
```

```
[Out] ((A*c*d*(c*d^2 - 3*a*e^2) + a*(a*e^2*(3*C*d + B*e) - c*d^2*(C*d + 3*B*e)))*
ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*c^(5/2)) + (c*x*(-6*a*e^2*(6*C*d + 2*
B*e + C*e*x) + 3*c*C*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3) + 2*c*e*(3
*A*e*(6*d + e*x) + B*(18*d^2 + 9*d*e*x + 2*e^2*x^2))) + 6*(B*c*d*(c*d^2 - 3
*a*e^2) + (A*c - a*C)*e*(3*c*d^2 - a*e^2))*Log[a + c*x^2])/(12*c^3)
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.08

method	result
default	$\frac{\frac{1}{4}cC\,x^4e^3+\frac{1}{3}Bc\,x^3e^3+Ccd\,e^2x^3+\frac{1}{2}Ac\,e^3x^2+\frac{3}{2}B\,x^2cd\,e^2-\frac{1}{2}Ca\,e^3x^2+\frac{3}{2}Cc\,d^2e\,x^2+3Acd\,e^2x-Bxa\,e^3+3Bc\,d^2ex-3Cad\,e^2x+Cc\,d^3}{c^2}$
risch	Expression too large to display

```
[In] int((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(1/4*c*C*x^4*e^3+1/3*B*c*x^3*e^3+C*c*d*e^2*x^3+1/2*A*c*e^3*x^2+3/2*B*
x^2*c*d*e^2-1/2*C*a*e^3*x^2+3/2*C*c*d^2*e*x^2+3*A*c*d*e^2*x-B*x*a*e^3+3*B*c
*d^2*e*x-3*C*a*d*e^2*x+C*c*d^3*x)+1/c^2*(1/2*(-A*a*c*e^3+3*A*c^2*d^2*e-3*B*
a*c*d*e^2+B*c^2*d^3+C*a^2*e^3-3*C*a*c*d^2*e)/c*ln(c*x^2+a)+(-3*A*a*c*d*e^2+
A*c^2*d^3+B*a^2*e^3-3*B*a*c*d^2*e+3*C*a^2*d*e^2-C*a*c*d^3)/(a*c)^(1/2)*arct
an(c*x/(a*c)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 592, normalized size of antiderivative = 2.47

$$\int \frac{(d+ex)^3\,(A+Bx+Cx^2)}{a+cx^2}\,dx$$

$$= \left[\frac{3\,Cac^2e^3x^4 + 4\,(3\,Cac^2de^2 + Bac^2e^3)x^3 + 6\,(3\,Cac^2d^2e + 3\,Bac^2de^2 - (Ca^2c - Aac^2)e^3)x^2 + 6\,(3\,Bacd^2e}{} \right.$$

```
[In] integrate((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/12*(3*C*a*c^2*e^3*x^4 + 4*(3*C*a*c^2*d*e^2 + B*a*c^2*e^3)*x^3 + 6*(3*C*a
*c^2*d^2*e + 3*B*a*c^2*d*e^2 - (C*a^2*c - A*a*c^2)*e^3)*x^2 + 6*(3*B*a*c*d^
2*e - B*a^2*e^3 + (C*a*c - A*c^2)*d^3 - 3*(C*a^2 - A*a*c)*d*e^2)*sqrt(-a*c)
*log((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) + 12*(C*a*c^2*d^3 + 3*B*a*c^
2*d^2*e - B*a^2*c*e^3 - 3*(C*a^2*c - A*a*c^2)*d*e^2)*x + 6*(B*a*c^2*d^3 - 3
*B*a^2*c*d*e^2 - 3*(C*a^2*c - A*a*c^2)*d^2*e + (C*a^3 - A*a^2*c)*e^3)*log(c
*x^2 + a))/(a*c^3), 1/12*(3*C*a*c^2*e^3*x^4 + 4*(3*C*a*c^2*d*e^2 + B*a*c^2*
e^3)*x^3 + 6*(3*C*a*c^2*d^2*e + 3*B*a*c^2*d*e^2 - (C*a^2*c - A*a*c^2)*e^3)*
x^2 - 12*(3*B*a*c*d^2*e - B*a^2*e^3 + (C*a*c - A*c^2)*d^3 - 3*(C*a^2 - A*a*
c)*d*e^2)*sqrt(a*c)*arctan(sqrt(a*c)*x/a) + 12*(C*a*c^2*d^3 + 3*B*a*c^2*d^2
*e - B*a^2*c*e^3 - 3*(C*a^2*c - A*a*c^2)*d*e^2)*x + 6*(B*a*c^2*d^3 - 3*B*a^
2*c*d*e^2 - 3*(C*a^2*c - A*a*c^2)*d^2*e + (C*a^3 - A*a^2*c)*e^3)*log(c*x^2
+ a))/(a*c^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1008 vs. 2(224) = 448.

Time = 5.52 (sec) , antiderivative size = 1008, normalized size of antiderivative = 4.20

$$\int \frac{(d+ex)^3\,(A+Bx+Cx^2)}{a+cx^2}\,dx=\frac{Ce^3x^4}{4c}+x^3\left(\frac{Be^3}{3c}+\frac{Cde^2}{c}\right)$$

$$+x^2\left(\frac{Ae^3}{2c}+\frac{3Bde^2}{2c}-\frac{Cae^3}{2c^2}+\frac{3Cd^2e}{2c}\right)+x\left(\frac{3Ade^2}{c}-\frac{Bae^3}{c^2}+\frac{3Bd^2e}{c}-\frac{3Cade^2}{c^2}+\frac{Cd^3}{c}\right)$$

$$+\left(\frac{-Aace^3+3Ac^2d^2e-3Bacde^2+Bc^2d^3+Ca^2e^3-3Cacd^2e}{2c^3}\right.$$

$$\left.-\frac{\sqrt{-ac^7}(-3Aacde^2+Ac^2d^3+Ba^2e^3-3Bacd^2e+3Ca^2de^2-Cacd^3)}{2ac^6}\right)\log\left(x+\frac{Aa^2ce^3-3Aac^2d^2e-}{}\right.$$

$$+\left(\frac{-Aace^3+3Ac^2d^2e-3Bacde^2+Bc^2d^3+Ca^2e^3-3Cacd^2e}{2c^3}\right.$$

$$\left.+\frac{\sqrt{-ac^7}(-3Aacde^2+Ac^2d^3+Ba^2e^3-3Bacd^2e+3Ca^2de^2-Cacd^3)}{2ac^6}\right)\log\left(x+\frac{Aa^2ce^3-3Aac^2d^2e-}{}\right.$$

```
[In] integrate((e*x+d)**3*(C*x**2+B*x+A)/(c*x**2+a),x)
```

```
[Out] C*e**3*x**4/(4*c) + x**3*(B*e**3/(3*c) + C*d*e**2/c) + x**2*(A*e**3/(2*c) +
 3*B*d*e**2/(2*c) - C*a*e**3/(2*c**2) + 3*C*d**2*e/(2*c)) + x*(3*A*d*e**2/c
 - B*a*e**3/c**2 + 3*B*d**2*e/c - 3*C*a*d*e**2/c**2 + C*d**3/c) + ((-A*a*c*
e**3 + 3*A*c**2*d**2*e - 3*B*a*c*d*e**2 + B*c**2*d**3 + C*a**2*e**3 - 3*C*a
*c*d**2*e)/(2*c**3) - sqrt(-a*c**7)*(-3*A*a*c*d*e**2 + A*c**2*d**3 + B*a**2
*e**3 - 3*B*a*c*d**2*e + 3*C*a**2*d*e**2 - C*a*c*d**3)/(2*a*c**6))*log(x +
(A*a**2*c*e**3 - 3*A*a*c**2*d**2*e + 3*B*a**2*c*d*e**2 - B*a*c**2*d**3 - C*
a**3*e**3 + 3*C*a**2*c*d**2*e + 2*a*c**3*((-A*a*c*e**3 + 3*A*c**2*d**2*e -
3*B*a*c*d*e**2 + B*c**2*d**3 + C*a**2*e**3 - 3*C*a*c*d**2*e)/(2*c**3) - sqr
t(-a*c**7)*(-3*A*a*c*d*e**2 + A*c**2*d**3 + B*a**2*e**3 - 3*B*a*c*d**2*e +
3*C*a**2*d*e**2 - C*a*c*d**3)/(2*a*c**6)))/(-3*A*a*c**2*d*e**2 + A*c**3*d**
3 + B*a**2*c*e**3 - 3*B*a*c**2*d**2*e + 3*C*a**2*c*d*e**2 - C*a*c**2*d**3))
 + ((-A*a*c*e**3 + 3*A*c**2*d**2*e - 3*B*a*c*d*e**2 + B*c**2*d**3 + C*a**2*
e**3 - 3*C*a*c*d**2*e)/(2*c**3) + sqrt(-a*c**7)*(-3*A*a*c*d*e**2 + A*c**2*d
**3 + B*a**2*e**3 - 3*B*a*c*d**2*e + 3*C*a**2*d*e**2 - C*a*c*d**3)/(2*a*c**
6))*log(x + (A*a**2*c*e**3 - 3*A*a*c**2*d**2*e + 3*B*a**2*c*d*e**2 - B*a*c*
*2*d**3 - C*a**3*e**3 + 3*C*a**2*c*d**2*e + 2*a*c**3*((-A*a*c*e**3 + 3*A*c*
*2*d**2*e - 3*B*a*c*d*e**2 + B*c**2*d**3 + C*a**2*e**3 - 3*C*a*c*d**2*e)/(2
*c**3) + sqrt(-a*c**7)*(-3*A*a*c*d*e**2 + A*c**2*d**3 + B*a**2*e**3 - 3*B*a
*c*d**2*e + 3*C*a**2*d*e**2 - C*a*c*d**3)/(2*a*c**6)))/(-3*A*a*c**2*d*e**2
+ A*c**3*d**3 + B*a**2*c*e**3 - 3*B*a*c**2*d**2*e + 3*C*a**2*c*d*e**2 - C*a
*c**2*d**3))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.02

$$\int \frac{(d+ex)^3 (A+Bx+Cx^2)}{a+cx^2}\,dx$$
$$= -\frac{(3\,Bacd^2e - Ba^2e^3 + (Cac - Ac^2)d^3 - 3\,(Ca^2 - Aac)de^2)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}c^2}$$
$$+ \frac{3\,Cce^3x^4 + 4\,(3\,Ccde^2 + Bce^3)x^3 + 6\,(3\,Ccd^2e + 3\,Bcde^2 - (Ca - Ac)e^3)x^2 + 12\,(Ccd^3 + 3\,Bcd^2e - Ba}{12\,c^2}$$
$$+ \frac{(Bc^2d^3 - 3\,Bacde^2 - 3\,(Cac - Ac^2)d^2e + (Ca^2 - Aac)e^3)\log\left(cx^2 + a\right)}{2\,c^3}$$

```
[In] integrate((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a),x, algorithm="maxima")
```

```
[Out] -(3*B*a*c*d^2*e - B*a^2*e^3 + (C*a*c - A*c^2)*d^3 - 3*(C*a^2 - A*a*c)*d*e^2
)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*c^2) + 1/12*(3*C*c*e^3*x^4 + 4*(3*C*c*d*
e^2 + B*c*e^3)*x^3 + 6*(3*C*c*d^2*e + 3*B*c*d*e^2 - (C*a - A*c)*e^3)*x^2 +
12*(C*c*d^3 + 3*B*c*d^2*e - B*a*e^3 - 3*(C*a - A*c)*d*e^2)*x)/c^2 + 1/2*(B*
c^2*d^3 - 3*B*a*c*d*e^2 - 3*(C*a*c - A*c^2)*d^2*e + (C*a^2 - A*a*c)*e^3)*lo
g(c*x^2 + a)/c^3
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.20

$$\int \frac{(d+ex)^3 (A+Bx+Cx^2)}{a+cx^2}\,dx$$
$$= -\frac{(Cacd^3 - Ac^2d^3 + 3\,Bacd^2e - 3\,Ca^2de^2 + 3\,Aacde^2 - Ba^2e^3)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}c^2}$$
$$+ \frac{(Bc^2d^3 - 3\,Cacd^2e + 3\,Ac^2d^2e - 3\,Bacde^2 + Ca^2e^3 - Aace^3)\log\left(cx^2 + a\right)}{2\,c^3}$$
$$+ \frac{3\,Cc^3e^3x^4 + 12\,Cc^3de^2x^3 + 4\,Bc^3e^3x^3 + 18\,Cc^3d^2ex^2 + 18\,Bc^3de^2x^2 - 6\,Cac^2e^3x^2 + 6\,Ac^3e^3x^2 + 12\,Cc^3}{12\,c^4}$$

```
[In] integrate((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a),x, algorithm="giac")
```

```
[Out] -(C*a*c*d^3 - A*c^2*d^3 + 3*B*a*c*d^2*e - 3*C*a^2*d*e^2 + 3*A*a*c*d*e^2 - B
*a^2*e^3)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*c^2) + 1/2*(B*c^2*d^3 - 3*C*a*c*
d^2*e + 3*A*c^2*d^2*e - 3*B*a*c*d*e^2 + C*a^2*e^3 - A*a*c*e^3)*log(c*x^2 +
a)/c^3 + 1/12*(3*C*c^3*e^3*x^4 + 12*C*c^3*d*e^2*x^3 + 4*B*c^3*e^3*x^3 + 18*
```

```
C*c^3*d^2*e*x^2 + 18*B*c^3*d*e^2*x^2 - 6*C*a*c^2*e^3*x^2 + 6*A*c^3*e^3*x^2
+ 12*C*c^3*d^3*x + 36*B*c^3*d^2*e*x - 36*C*a*c^2*d*e^2*x + 36*A*c^3*d*e^2*x
 - 12*B*a*c^2*e^3*x)/c^4
```

Mupad [B] (verification not implemented)

Time = 12.87 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.15

$$\begin{aligned}\int \frac{(d+ex)^3\,(A+Bx+Cx^2)}{a+cx^2}\,dx &= x^2\left(\frac{3\,C\,d^2\,e+3\,B\,d\,e^2+A\,e^3}{2\,c}-\frac{C\,a\,e^3}{2\,c^2}\right)\\ &\quad+x\left(\frac{C\,d^3+3\,B\,d^2\,e+3\,A\,d\,e^2}{c}-\frac{a\,(B\,e^3+3\,C\,d\,e^2)}{c^2}\right)+\frac{x^3\,(B\,e^3+3\,C\,d\,e^2)}{3\,c}+\frac{C\,e^3\,x^4}{4\,c}\\ &\quad+\frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\,(3\,C\,a^2\,d\,e^2+B\,a^2\,e^3-C\,a\,c\,d^3-3\,B\,a\,c\,d^2\,e-3\,A\,a\,c\,d\,e^2+A\,c^2\,d^3)}{\sqrt{a}\,c^{5/2}}\\ &\quad+\frac{\ln\left(c\,x^2+a\right)\,(4\,C\,a^3\,c^3\,e^3-12\,C\,a^2\,c^4\,d^2\,e-12\,B\,a^2\,c^4\,d\,e^2-4\,A\,a^2\,c^4\,e^3+4\,B\,a\,c^5\,d^3+12\,A\,a\,c^5\,d^2\,e)}{8\,a\,c^6}\end{aligned}$$

```
[In] int(((d + e*x)^3*(A + B*x + C*x^2))/(a + c*x^2),x)
```

```
[Out] x^2*((A*e^3 + 3*B*d*e^2 + 3*C*d^2*e)/(2*c) - (C*a*e^3)/(2*c^2)) + x*((C*d^3
 + 3*A*d*e^2 + 3*B*d^2*e)/c - (a*(B*e^3 + 3*C*d*e^2))/c^2) + (x^3*(B*e^3 +
3*C*d*e^2))/(3*c) + (C*e^3*x^4)/(4*c) + (atan((c^(1/2)*x)/a^(1/2))*(A*c^2*d
^3 + B*a^2*e^3 - C*a*c*d^3 + 3*C*a^2*d*e^2 - 3*A*a*c*d*e^2 - 3*B*a*c*d^2*e)
)/(a^(1/2)*c^(5/2)) + (log(a + c*x^2)*(4*B*a*c^5*d^3 - 4*A*a^2*c^4*e^3 + 4*
C*a^3*c^3*e^3 - 12*B*a^2*c^4*d*e^2 - 12*C*a^2*c^4*d^2*e + 12*A*a*c^5*d^2*e)
)/(8*a*c^6)
```

3.44 $\int \frac{(d+ex)^2(A+Bx+Cx^2)}{a+cx^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 168

$$\begin{aligned}\int \frac{(d+ex)^2(A+Bx+Cx^2)}{a+cx^2}\,dx &= -\frac{(aCe^2-c(Cd^2+e(2Bd+Ae)))\,x}{c^2}+\frac{e(2Cd+Be)x^2}{2c}+\frac{Ce^2x^3}{3c}\\ &\quad+\frac{(Ac(cd^2-ae^2)+a(aCe^2-cd(Cd+2Be)))\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}c^{5/2}}\\ &\quad+\frac{(Bcd^2+2Acde-2aCde-aBe^2)\log\left(a+cx^2\right)}{2c^2}\end{aligned}$$

```
[Out] -(a*C*e^2-c*(C*d^2+e*(A*e+2*B*d)))*x/c^2+1/2*e*(B*e+2*C*d)*x^2/c+1/3*C*e^2*
x^3/c+1/2*(2*A*c*d*e-B*a*e^2+B*c*d^2-2*C*a*d*e)*ln(c*x^2+a)/c^2+(A*c*(-a*e^
2+c*d^2)+a*(a*C*e^2-c*d*(2*B*e+C*d)))*arctan(x*c^(1/2)/a^(1/2))/c^(5/2)/a^(
1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used

$= \{1643, 649, 211, 266\}$

$$\int \frac{(d+ex)^2\left(A+Bx+Cx^2\right)}{a+cx^2}\,dx$$
$$=\frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(Ac(cd^2-ae^2)+a(aCe^2-cd(2Be+Cd))\right)}{\sqrt{a}c^{5/2}}$$
$$+\frac{\log\left(a+cx^2\right)\left(-aBe^2-2aCde+2Acde+Bcd^2\right)}{2c^2}$$
$$+\frac{x(-aCe^2+ce(Ae+2Bd)+cCd^2)}{c^2}+\frac{ex^2(Be+2Cd)}{2c}+\frac{Ce^2x^3}{3c}$$

```
[In] Int[((d + e*x)^2*(A + B*x + C*x^2))/(a + c*x^2),x]
```

```
[Out] ((c*C*d^2 - a*C*e^2 + c*e*(2*B*d + A*e))*x)/c^2 + (e*(2*C*d + B*e)*x^2)/(2*
c) + (C*e^2*x^3)/(3*c) + ((A*c*(c*d^2 - a*e^2) + a*(a*C*e^2 - c*d*(C*d + 2*
B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*c^(5/2)) + ((B*c*d^2 + 2*A*c*d
*e - 2*a*C*d*e - a*B*e^2)*Log[a + c*x^2])/(2*c^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

integral

$$=\int\left(\frac{cCd^2-aCe^2+ce(2Bd+Ae)}{c^2}+\frac{e(2Cd+Be)x}{c}+\frac{Ce^2x^2}{c}\right.$$
$$\left.+\frac{Ac(cd^2-ae^2)+a(aCe^2-cd(Cd+2Be))+c\left(Bcd^2+2Acde-2aCde-aBe^2\right)x}{c^2\left(a+cx^2\right)}\right)dx$$

$$
\begin{aligned}
&= \frac{(cCd^2-aCe^2+ce(2Bd+Ae))\,x}{c^2}+\frac{e(2Cd+Be)x^2}{2c}+\frac{Ce^2x^3}{3c} \\
&\quad+\frac{\int\frac{Ac(cd^2-ae^2)+a(aCe^2-cd(Cd+2Be))+c(Bcd^2+2Acde-2aCde-aBe^2)x}{a+cx^2}\,dx}{c^2} \\
&= \frac{(cCd^2-aCe^2+ce(2Bd+Ae))\,x}{c^2}+\frac{e(2Cd+Be)x^2}{2c} \\
&\quad+\frac{Ce^2x^3}{3c}+\frac{(Bcd^2+2Acde-2aCde-aBe^2)\int\frac{x}{a+cx^2}\,dx}{c} \\
&\quad+\frac{(Ac(cd^2-ae^2)+a(aCe^2-cd(Cd+2Be)))\int\frac{1}{a+cx^2}\,dx}{c^2} \\
&= \frac{(cCd^2-aCe^2+ce(2Bd+Ae))\,x}{c^2}+\frac{e(2Cd+Be)x^2}{2c}+\frac{Ce^2x^3}{3c} \\
&\quad+\frac{(Ac(cd^2-ae^2)+a(aCe^2-cd(Cd+2Be)))\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}c^{5/2}} \\
&\quad+\frac{(Bcd^2+2Acde-2aCde-aBe^2)\log\left(a+cx^2\right)}{2c^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.92

$$
\begin{aligned}
&\int\frac{(d+ex)^2\left(A+Bx+Cx^2\right)}{a+cx^2}\,dx \\
&= \frac{(Ac(cd^2-ae^2)+a(aCe^2-cd(Cd+2Be)))\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}c^{5/2}} \\
&\quad+\frac{x(-6aCe^2+3ce(4Bd+2Ae+Bex)+2cC(3d^2+3dex+e^2x^2))+3(Bcd^2+2Acde-2aCde-aBe^2)\,\mathrm{l}}{6c^2}
\end{aligned}
$$

```
[In] Integrate[((d + e*x)^2*(A + B*x + C*x^2))/(a + c*x^2),x]
```

```
[Out] ((A*c*(c*d^2 - a*e^2) + a*(a*C*e^2 - c*d*(C*d + 2*B*e)))*ArcTan[(Sqrt[c]*x)
/Sqrt[a]])/(Sqrt[a]*c^(5/2)) + (x*(-6*a*C*e^2 + 3*c*e*(4*B*d + 2*A*e + B*e*
x) + 2*c*C*(3*d^2 + 3*d*e*x + e^2*x^2)) + 3*(B*c*d^2 + 2*A*c*d*e - 2*a*C*d*
e - a*B*e^2)*Log[a + c*x^2])/(6*c^2)
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.01

method	result
default	$\frac{\frac{1}{3}cC\,x^{3}e^{2}+\frac{1}{2}Bc\,e^{2}x^{2}+Ccde\,x^{2}+Ac\,e^{2}x+2Bcdex-aC\,e^{2}x+Cc\,d^{2}x}{c^{2}}+\frac{\frac{\left(2A\,c^{2}de-B\,e^{2}ac+B\,c^{2}d^{2}-2acdeC\right)\ln\left(c\,x^{2}+a\right)}{2c}+\frac{\left(-Aac\,e^{2}+A\,c\right.}{}}{c^{2}}$
risch	Expression too large to display

```
[In] int((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(1/3*c*C*x^3*e^2+1/2*B*c*e^2*x^2+C*c*d*e*x^2+A*c*e^2*x+2*B*c*d*e*x-a*
C*e^2*x+C*c*d^2*x)+1/c^2*(1/2*(2*A*c^2*d*e-B*a*c*e^2+B*c^2*d^2-2*C*a*c*d*e)
/c*ln(c*x^2+a)+(-A*a*c*e^2+A*c^2*d^2-2*B*a*c*d*e+C*a^2*e^2-C*a*c*d^2)/(a*c)
^(1/2)*arctan(c*x/(a*c)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 404, normalized size of antiderivative = 2.40

$$\int \frac{(d+ex)^2\,(A+Bx+Cx^2)}{a+cx^2}\,dx$$

$$=\left[\frac{2\,Cac^2e^2x^3+3\,(2\,Cac^2de+Bac^2e^2)x^2-3\,(2\,Bacde+(Cac-Ac^2)d^2-(Ca^2-Aac)e^2)\sqrt{-ac}\log\left(\frac{cx^2-}{}\right.}{}\right.$$

```
[In] integrate((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/6*(2*C*a*c^2*e^2*x^3 + 3*(2*C*a*c^2*d*e + B*a*c^2*e^2)*x^2 - 3*(2*B*a*c*
d*e + (C*a*c - A*c^2)*d^2 - (C*a^2 - A*a*c)*e^2)*sqrt(-a*c)*log((c*x^2 + 2*
sqrt(-a*c)*x - a)/(c*x^2 + a)) + 6*(C*a*c^2*d^2 + 2*B*a*c^2*d*e - (C*a^2*c
- A*a*c^2)*e^2)*x + 3*(B*a*c^2*d^2 - B*a^2*c*e^2 - 2*(C*a^2*c - A*a*c^2)*d*
e)*log(c*x^2 + a))/(a*c^3), 1/6*(2*C*a*c^2*e^2*x^3 + 3*(2*C*a*c^2*d*e + B*a
*c^2*e^2)*x^2 - 6*(2*B*a*c*d*e + (C*a*c - A*c^2)*d^2 - (C*a^2 - A*a*c)*e^2)
*sqrt(a*c)*arctan(sqrt(a*c)*x/a) + 6*(C*a*c^2*d^2 + 2*B*a*c^2*d*e - (C*a^2*
c - A*a*c^2)*e^2)*x + 3*(B*a*c^2*d^2 - B*a^2*c*e^2 - 2*(C*a^2*c - A*a*c^2)*
d*e)*log(c*x^2 + a))/(a*c^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 638 vs. 2(156) = 312.

Time = 1.39 (sec) , antiderivative size = 638, normalized size of antiderivative = 3.80

$$\int \frac{(d+ex)^2\left(A+Bx+Cx^2\right)}{a+cx^2}\,dx = \frac{Ce^2x^3}{3c} + x^2\left(\frac{Be^2}{2c}+\frac{Cde}{c}\right)$$

$$+ x\left(\frac{Ae^2}{c}+\frac{2Bde}{c}-\frac{Cae^2}{c^2}+\frac{Cd^2}{c}\right) + \left(-\frac{-2Acde+Bae^2-Bcd^2+2Cade}{2c^2}\right.$$

$$\left.-\frac{\sqrt{-ac^5}\left(-Aace^2+Ac^2d^2-2Bacde+Ca^2e^2-Cacd^2\right)}{2ac^5}\right)\log\left(x+\frac{-2Aacde+Ba^2e^2-Bacd^2+2Ca^2}{-}\right.$$

$$+\left(-\frac{-2Acde+Bae^2-Bcd^2+2Cade}{2c^2}\right.$$

$$\left.+\frac{\sqrt{-ac^5}\left(-Aace^2+Ac^2d^2-2Bacde+Ca^2e^2-Cacd^2\right)}{2ac^5}\right)\log\left(x+\frac{-2Aacde+Ba^2e^2-Bacd^2+2Ca^2}{-}\right.$$

```
[In] integrate((e*x+d)**2*(C*x**2+B*x+A)/(c*x**2+a),x)
```

```
[Out] C*e**2*x**3/(3*c) + x**2*(B*e**2/(2*c) + C*d*e/c) + x*(A*e**2/c + 2*B*d*e/c
 - C*a*e**2/c**2 + C*d**2/c) + (-(-2*A*c*d*e + B*a*e**2 - B*c*d**2 + 2*C*a*
d*e)/(2*c**2) - sqrt(-a*c**5)*(-A*a*c*e**2 + A*c**2*d**2 - 2*B*a*c*d*e + C*
a**2*e**2 - C*a*c*d**2)/(2*a*c**5))*log(x + (-2*A*a*c*d*e + B*a**2*e**2 - B
*a*c*d**2 + 2*C*a**2*d*e + 2*a*c**2*(-(-2*A*c*d*e + B*a*e**2 - B*c*d**2 + 2
*C*a*d*e)/(2*c**2) - sqrt(-a*c**5)*(-A*a*c*e**2 + A*c**2*d**2 - 2*B*a*c*d*e
 + C*a**2*e**2 - C*a*c*d**2)/(2*a*c**5)))/(-A*a*c*e**2 + A*c**2*d**2 - 2*B*
a*c*d*e + C*a**2*e**2 - C*a*c*d**2)) + (-(-2*A*c*d*e + B*a*e**2 - B*c*d**2
+ 2*C*a*d*e)/(2*c**2) + sqrt(-a*c**5)*(-A*a*c*e**2 + A*c**2*d**2 - 2*B*a*c*
d*e + C*a**2*e**2 - C*a*c*d**2)/(2*a*c**5))*log(x + (-2*A*a*c*d*e + B*a**2*
e**2 - B*a*c*d**2 + 2*C*a**2*d*e + 2*a*c**2*(-(-2*A*c*d*e + B*a*e**2 - B*c*
d**2 + 2*C*a*d*e)/(2*c**2) + sqrt(-a*c**5)*(-A*a*c*e**2 + A*c**2*d**2 - 2*B
*a*c*d*e + C*a**2*e**2 - C*a*c*d**2)/(2*a*c**5)))/(-A*a*c*e**2 + A*c**2*d**
2 - 2*B*a*c*d*e + C*a**2*e**2 - C*a*c*d**2))
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.96

$$\int \frac{(d+ex)^2 (A+Bx+Cx^2)}{a+cx^2}\,dx = \frac{(Bcd^2 - Bae^2 - 2(Ca - Ac)de)\log(cx^2+a)}{2c^2} - \frac{(2Bacde + (Cac - Ac^2)d^2 - (Ca^2 - Aac)e^2)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}c^2} + \frac{2Cce^2x^3 + 3(2Ccde + Bce^2)x^2 + 6(Ccd^2 + 2Bcde - (Ca - Ac)e^2)x}{6c^2}$$

```
[In] integrate((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*(B*c*d^2 - B*a*e^2 - 2*(C*a - A*c)*d*e)*log(c*x^2 + a)/c^2 - (2*B*a*c*d
*e + (C*a*c - A*c^2)*d^2 - (C*a^2 - A*a*c)*e^2)*arctan(c*x/sqrt(a*c))/(sqrt
(a*c)*c^2) + 1/6*(2*C*c*e^2*x^3 + 3*(2*C*c*d*e + B*c*e^2)*x^2 + 6*(C*c*d^2
+ 2*B*c*d*e - (C*a - A*c)*e^2)*x)/c^2
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.06

$$\int \frac{(d+ex)^2 (A+Bx+Cx^2)}{a+cx^2}\,dx = \frac{(Bcd^2 - 2Cade + 2Acde - Bae^2)\log(cx^2+a)}{2c^2} - \frac{(Cacd^2 - Ac^2d^2 + 2Bacde - Ca^2e^2 + Aace^2)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}c^2} + \frac{2Cc^2e^2x^3 + 6Cc^2dex^2 + 3Bc^2e^2x^2 + 6Cc^2d^2x + 12Bc^2dex - 6Cace^2x + 6Ac^2e^2x}{6c^3}$$

```
[In] integrate((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*(B*c*d^2 - 2*C*a*d*e + 2*A*c*d*e - B*a*e^2)*log(c*x^2 + a)/c^2 - (C*a*c
*d^2 - A*c^2*d^2 + 2*B*a*c*d*e - C*a^2*e^2 + A*a*c*e^2)*arctan(c*x/sqrt(a*c
))/(sqrt(a*c)*c^2) + 1/6*(2*C*c^2*e^2*x^3 + 6*C*c^2*d*e*x^2 + 3*B*c^2*e^2*x
^2 + 6*C*c^2*d^2*x + 12*B*c^2*d*e*x - 6*C*a*c*e^2*x + 6*A*c^2*e^2*x)/c^3
```

Mupad [B] (verification not implemented)

Time = 12.92 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.08

$$
\begin{aligned}
&\int \frac{(d+ex)^2\,(A+Bx+Cx^2)}{a+cx^2}\,dx \\
&= x\left(\frac{C\,d^2+2\,B\,d\,e+A\,e^2}{c}-\frac{C\,a\,e^2}{c^2}\right)+\frac{x^2\,(B\,e^2+2\,C\,d\,e)}{2\,c}+\frac{C\,e^2\,x^3}{3\,c} \\
&\quad-\frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\,(-C\,a^2\,e^2+C\,a\,c\,d^2+2\,B\,a\,c\,d\,e+A\,a\,c\,e^2-A\,c^2\,d^2)}{\sqrt{a}\,c^{5/2}} \\
&\quad+\frac{\ln\left(c\,x^2+a\right)\,(-8\,C\,a^2\,c^3\,d\,e-4\,B\,a^2\,c^3\,e^2+4\,B\,a\,c^4\,d^2+8\,A\,a\,c^4\,d\,e)}{8\,a\,c^5}
\end{aligned}
$$

```
[In] int(((d + e*x)^2*(A + B*x + C*x^2))/(a + c*x^2),x)
```

```
[Out] x*((A*e^2 + C*d^2 + 2*B*d*e)/c - (C*a*e^2)/c^2) + (x^2*(B*e^2 + 2*C*d*e))/(
2*c) + (C*e^2*x^3)/(3*c) - (atan((c^(1/2)*x)/a^(1/2))*(A*a*c*e^2 - C*a^2*e^
2 - A*c^2*d^2 + C*a*c*d^2 + 2*B*a*c*d*e))/(a^(1/2)*c^(5/2)) + (log(a + c*x^
2)*(4*B*a*c^4*d^2 - 4*B*a^2*c^3*e^2 + 8*A*a*c^4*d*e - 8*C*a^2*c^3*d*e))/(8*
a*c^5)
```

3.45 $\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{a+cx^2} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 93

$$\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{a+cx^2} \, dx = \frac{(Cd+Be)x}{c}+\frac{Cex^2}{2c}+\frac{(Acd-a(Cd+Be))\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}c^{3/2}} +\frac{(Bcd+Ace-aCe)\log\left(a+cx^2\right)}{2c^2}$$

```
[Out] (B*e+C*d)*x/c+1/2*C*e*x^2/c+1/2*(A*c*e+B*c*d-C*a*e)*ln(c*x^2+a)/c^2+(A*c*d-
a*(B*e+C*d))*arctan(x*c^(1/2)/a^(1/2))/c^(3/2)/a^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1643, 649, 211, 266}

$$\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{a+cx^2} \, dx = \frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(Acd-a(Be+Cd))}{\sqrt{a}c^{3/2}} +\frac{\log\left(a+cx^2\right)(-aCe+Ace+Bcd)}{2c^2} +\frac{x(Be+Cd)}{c}+\frac{Cex^2}{2c}$$

```
[In] Int[((d + e*x)*(A + B*x + C*x^2))/(a + c*x^2),x]
```

```
[Out] ((C*d + B*e)*x)/c + (C*e*x^2)/(2*c) + ((A*c*d - a*(C*d + B*e))*ArcTan[(Sqrt
[c]*x)/Sqrt[a]])/(Sqrt[a]*c^(3/2)) + ((B*c*d + A*c*e - a*C*e)*Log[a + c*x^2
])/(2*c^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{Cd+Be}{c}+\frac{Cex}{c}+\frac{Acd-a(Cd+Be)+(Bcd+Ace-aCe)x}{c\left(a+cx^2\right)}\right) dx \\
&= \frac{(Cd+Be)x}{c}+\frac{Cex^2}{2c}+\frac{\int \frac{Acd-a(Cd+Be)+(Bcd+Ace-aCe)x}{a+cx^2}\,dx}{c} \\
&= \frac{(Cd+Be)x}{c}+\frac{Cex^2}{2c}+\frac{(Bcd+Ace-aCe)\int \frac{x}{a+cx^2}\,dx}{c}+\frac{(Acd-a(Cd+Be))\int \frac{1}{a+cx^2}\,dx}{c} \\
&= \frac{(Cd+Be)x}{c}+\frac{Cex^2}{2c}+\frac{(Acd-a(Cd+Be))\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}c^{3/2}}+\frac{(Bcd+Ace-aCe)\log\left(a+cx^2\right)}{2c^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.92

$$
\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{a+cx^2}\,dx
$$

$$
=\frac{cx(2Cd+2Be+Cex)-\frac{2\sqrt{c}(-Acd+aCd+aBe)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}}+(Bcd+Ace-aCe)\log\left(a+cx^2\right)}{2c^2}
$$

```
[In] Integrate[((d + e*x)*(A + B*x + C*x^2))/(a + c*x^2),x]
```

```
[Out] (c*x*(2*C*d + 2*B*e + C*e*x) - (2*Sqrt[c]*(-(A*c*d) + a*C*d + a*B*e)*ArcTan
[(Sqrt[c]*x)/Sqrt[a]])/Sqrt[a] + (B*c*d + A*c*e - a*C*e)*Log[a + c*x^2])/(2
*c^2)
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.90

method	result
default	$\frac{\frac{1}{2}C\,x^2e+Bex+Cdx}{c}+\frac{\frac{(Ace+Bcd-Cae)\ln\left(c\,x^2+a\right)}{2c}+\frac{(Acd-Bae-Cad)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}}}{c}$
risch	$\frac{Ce\,x^2}{2c}+\frac{Bex}{c}+\frac{Cdx}{c}+\frac{\ln\left(dAac-Be\,a^2-a^2dC-\sqrt{-ac(Acd-Bae-Cad)^2}\,x\right)Ae}{2c}+\frac{\ln\left(dAac-Be\,a^2-a^2dC-\sqrt{-ac(Acd-B}{2c}$

```
[In] int((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(1/2*C*x^2*e+B*e*x+C*d*x)+1/c*(1/2*(A*c*e+B*c*d-C*a*e)/c*ln(c*x^2+a)+(A
*c*d-B*a*e-C*a*d)/(a*c)^(1/2)*arctan(c*x/(a*c)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 206, normalized size of antiderivative = 2.22

$$\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{a+cx^2}\,dx$$

$$= \left[\frac{Cacex^2-(Bae+(Ca-Ac)d)\sqrt{-ac}\log\left(\frac{cx^2+2\sqrt{-ac}x-a}{cx^2+a}\right)+2\,(Cacd+Bace)x+(Bacd-(Ca^2-Aac)}{2\,ac^2}\right.$$

```
[In] integrate((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/2*(C*a*c*e*x^2 - (B*a*e + (C*a - A*c)*d)*sqrt(-a*c)*log((c*x^2 + 2*sqrt(
-a*c)*x - a)/(c*x^2 + a)) + 2*(C*a*c*d + B*a*c*e)*x + (B*a*c*d - (C*a^2 - A
*a*c)*e)*log(c*x^2 + a))/(a*c^2), 1/2*(C*a*c*e*x^2 - 2*(B*a*e + (C*a - A*c)
*d)*sqrt(a*c)*arctan(sqrt(a*c)*x/a) + 2*(C*a*c*d + B*a*c*e)*x + (B*a*c*d -
(C*a^2 - A*a*c)*e)*log(c*x^2 + a))/(a*c^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 337 vs. 2(88) = 176.

Time = 0.71 (sec) , antiderivative size = 337, normalized size of antiderivative = 3.62

$$\int \frac{(d+ex)\,(A+Bx+Cx^2)}{a+cx^2}\,dx = \frac{Cex^2}{2c} + x\left(\frac{Be}{c} + \frac{Cd}{c}\right) + \left(-\frac{-Ace-Bcd+Cae}{2c^2} - \frac{\sqrt{-ac^5}(-Acd+Bae+Cad)}{2ac^4}\right)\log\left(x + \frac{Aace+Bacd-Ca^2e-2ac^2\left(-\frac{-Ace-Bcd+Cae}{2c^2} - \frac{\sqrt{-ac^5}(-Acd+Ba}{2ac^4}\right.}{-Ac^2d+Bace+Cacd}\right. + \left(-\frac{-Ace-Bcd+Cae}{2c^2} + \frac{\sqrt{-ac^5}(-Acd+Bae+Cad)}{2ac^4}\right)\log\left(x + \frac{Aace+Bacd-Ca^2e-2ac^2\left(-\frac{-Ace-Bcd+Cae}{2c^2} + \frac{\sqrt{-ac^5}(-Acd+Ba}{2ac^4}\right.}{-Ac^2d+Bace+Cacd}\right.$$

```
[In] integrate((e*x+d)*(C*x**2+B*x+A)/(c*x**2+a),x)
```

```
[Out] C*e*x**2/(2*c) + x*(B*e/c + C*d/c) + (-(-A*c*e - B*c*d + C*a*e)/(2*c**2) -
sqrt(-a*c**5)*(-A*c*d + B*a*e + C*a*d)/(2*a*c**4))*log(x + (A*a*c*e + B*a*c
*d - C*a**2*e - 2*a*c**2*(-(-A*c*e - B*c*d + C*a*e)/(2*c**2) - sqrt(-a*c**5
)*(-A*c*d + B*a*e + C*a*d)/(2*a*c**4)))/(-A*c**2*d + B*a*c*e + C*a*c*d)) +
(-(-A*c*e - B*c*d + C*a*e)/(2*c**2) + sqrt(-a*c**5)*(-A*c*d + B*a*e + C*a*d
)/(2*a*c**4))*log(x + (A*a*c*e + B*a*c*d - C*a**2*e - 2*a*c**2*(-(-A*c*e -
B*c*d + C*a*e)/(2*c**2) + sqrt(-a*c**5)*(-A*c*d + B*a*e + C*a*d)/(2*a*c**4)
))/(-A*c**2*d + B*a*c*e + C*a*c*d))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.92

$$\int \frac{(d+ex)\,(A+Bx+Cx^2)}{a+cx^2}\,dx = -\frac{(Bae+(Ca-Ac)d)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}c} + \frac{Cex^2+2\,(Cd+Be)x}{2\,c} + \frac{(Bcd-(Ca-Ac)e)\log\left(cx^2+a\right)}{2\,c^2}$$

```
[In] integrate((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a),x, algorithm="maxima")
```

```
[Out] -(B*a*e + (C*a - A*c)*d)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*c) + 1/2*(C*e*x^2
 + 2*(C*d + B*e)*x)/c + 1/2*(B*c*d - (C*a - A*c)*e)*log(c*x^2 + a)/c^2
```

Giac [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.92

$$\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{a+cx^2}\,dx = -\frac{(Cad-Acd+Bae)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}c} + \frac{(Bcd-Cae+Ace)\log\left(cx^2+a\right)}{2\,c^2} + \frac{Ccex^2+2\,Ccdx+2\,Bcex}{2\,c^2}$$

```
[In] integrate((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a),x, algorithm="giac")
```

```
[Out] -(C*a*d - A*c*d + B*a*e)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*c) + 1/2*(B*c*d -
 C*a*e + A*c*e)*log(c*x^2 + a)/c^2 + 1/2*(C*c*e*x^2 + 2*C*c*d*x + 2*B*c*e*x
)/c^2
```

Mupad [B] (verification not implemented)

Time = 12.51 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.04

$$\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{a+cx^2}\,dx = \frac{x\,(B\,e+C\,d)}{c} - \frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\,\left(B\,a\,e-A\,c\,d+C\,a\,d\right)}{\sqrt{a}\,c^{3/2}} + \frac{C\,e\,x^2}{2\,c} + \frac{\ln\left(c\,x^2+a\right)\,\left(4\,A\,a\,c^3\,e+4\,B\,a\,c^3\,d-4\,C\,a^2\,c^2\,e\right)}{8\,a\,c^4}$$

```
[In] int(((d + e*x)*(A + B*x + C*x^2))/(a + c*x^2),x)
```

```
[Out] (x*(B*e + C*d))/c - (atan((c^(1/2)*x)/a^(1/2))*(B*a*e - A*c*d + C*a*d))/(a^
(1/2)*c^(3/2)) + (C*e*x^2)/(2*c) + (log(a + c*x^2)*(4*A*a*c^3*e + 4*B*a*c^3
*d - 4*C*a^2*c^2*e))/(8*a*c^4)
```

3.46 $\int \frac{A+Bx+Cx^2}{a+cx^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 55

$$\int \frac{A+Bx+Cx^2}{a+cx^2}\,dx=\frac{Cx}{c}+\frac{(Ac-aC)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}c^{3/2}}+\frac{B\log\left(a+cx^2\right)}{2c}$$

```
[Out] C*x/c+1/2*B*ln(c*x^2+a)/c+(A*c-C*a)*arctan(x*c^(1/2)/a^(1/2))/c^(3/2)/a^(1/
2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1824, 649, 211, 266}

$$\int \frac{A+Bx+Cx^2}{a+cx^2}\,dx=\frac{(Ac-aC)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}c^{3/2}}+\frac{B\log\left(a+cx^2\right)}{2c}+\frac{Cx}{c}$$

```
[In] Int[(A + B*x + C*x^2)/(a + c*x^2),x]
```

```
[Out] (C*x)/c + ((A*c - a*C)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*c^(3/2)) + (B*
Log[a + c*x^2])/(2*c)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{C}{c}+\frac{Ac-aC+Bcx}{c\,(a+cx^2)}\right) dx \\
&= \frac{Cx}{c}+\frac{\int \frac{Ac-aC+Bcx}{a+cx^2}\,dx}{c} \\
&= \frac{Cx}{c}+B\int \frac{x}{a+cx^2}\,dx+\frac{(Ac-aC)\int \frac{1}{a+cx^2}\,dx}{c} \\
&= \frac{Cx}{c}+\frac{(Ac-aC)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}c^{3/2}}+\frac{B\log\left(a+cx^2\right)}{2c}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.02

$$\int \frac{A+Bx+Cx^2}{a+cx^2}\,dx=\frac{Cx}{c}-\frac{(-Ac+aC)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}c^{3/2}}+\frac{B\log\left(a+cx^2\right)}{2c}$$

```
[In] Integrate[(A + B*x + C*x^2)/(a + c*x^2),x]
[Out] (C*x)/c - ((-(A*c) + a*C)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*c^(3/2)) +
(B*Log[a + c*x^2])/(2*c)
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.85

method	result
default	$\frac{Cx}{c}+\frac{\frac{B\ln\left(c\,x^{2}+a\right)}{2}+\frac{(Ac-Ca)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}}}{c}$
risch	$\frac{Cx}{c}+\frac{\ln\left(Aac-C\,a^{2}-\sqrt{-ac(Ac-Ca)^{2}}\,x\right)B}{2c}+\frac{\ln\left(Aac-C\,a^{2}-\sqrt{-ac(Ac-Ca)^{2}}\,x\right)\sqrt{-ac(Ac-Ca)^{2}}}{2c^{2}a}+\frac{\ln\left(Aac-C\,a^{2}+\sqrt{-ac(\right.}{2c}$

```
[In] int((C*x^2+B*x+A)/(c*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] C*x/c+1/c*(1/2*B*ln(c*x^2+a)+(A*c-C*a)/(a*c)^(1/2)*arctan(c*x/(a*c)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 125, normalized size of antiderivative = 2.27

$$\int \frac{A+Bx+Cx^2}{a+cx^2}\,dx$$
$$=\left[\frac{2\,Cacx+Bac\log\left(cx^{2}+a\right)+(Ca-Ac)\sqrt{-ac}\log\left(\frac{cx^{2}-2\,\sqrt{-ac}x-a}{cx^{2}+a}\right)}{2\,ac^{2}},\frac{2\,Cacx+Bac\log\left(cx^{2}+a\right)-2\,(Ca}{2\,ac^{2}}\right.$$

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/2*(2*C*a*c*x + B*a*c*log(c*x^2 + a) + (C*a - A*c)*sqrt(-a*c)*log((c*x^2
- 2*sqrt(-a*c)*x - a)/(c*x^2 + a)))/(a*c^2), 1/2*(2*C*a*c*x + B*a*c*log(c*x
^2 + a) - 2*(C*a - A*c)*sqrt(a*c)*arctan(sqrt(a*c)*x/a))/(a*c^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. 2(48) = 96.

Time = 0.23 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.84

$$\int \frac{A+Bx+Cx^2}{a+cx^2}\,dx = \frac{Cx}{c} + \left(\frac{B}{2c} - \frac{\sqrt{-ac^3}(-Ac+Ca)}{2ac^3}\right)\log\left(x + \frac{Ba-2ac\left(\frac{B}{2c}-\frac{\sqrt{-ac^3}(-Ac+Ca)}{2ac^3}\right)}{-Ac+Ca}\right) + \left(\frac{B}{2c} + \frac{\sqrt{-ac^3}(-Ac+Ca)}{2ac^3}\right)\log\left(x + \frac{Ba-2ac\left(\frac{B}{2c}+\frac{\sqrt{-ac^3}(-Ac+Ca)}{2ac^3}\right)}{-Ac+Ca}\right)$$

```
[In] integrate((C*x**2+B*x+A)/(c*x**2+a),x)
```

```
[Out] C*x/c + (B/(2*c) - sqrt(-a*c**3)*(-A*c + C*a)/(2*a*c**3))*log(x + (B*a - 2*
a*c*(B/(2*c) - sqrt(-a*c**3)*(-A*c + C*a)/(2*a*c**3)))/(-A*c + C*a)) + (B/(
2*c) + sqrt(-a*c**3)*(-A*c + C*a)/(2*a*c**3))*log(x + (B*a - 2*a*c*(B/(2*c)
 + sqrt(-a*c**3)*(-A*c + C*a)/(2*a*c**3)))/(-A*c + C*a))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.87

$$\int \frac{A+Bx+Cx^2}{a+cx^2}\,dx = \frac{Cx}{c} + \frac{B\log\left(cx^2+a\right)}{2\,c} - \frac{(Ca-Ac)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}c}$$

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a),x, algorithm="maxima")
```

```
[Out] C*x/c + 1/2*B*log(c*x^2 + a)/c - (C*a - A*c)*arctan(c*x/sqrt(a*c))/(sqrt(a*
c)*c)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.87

$$\int \frac{A+Bx+Cx^2}{a+cx^2}\,dx = \frac{Cx}{c} + \frac{B\log\left(cx^2+a\right)}{2\,c} - \frac{(Ca-Ac)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}c}$$

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a),x, algorithm="giac")
```

```
[Out] C*x/c + 1/2*B*log(c*x^2 + a)/c - (C*a - A*c)*arctan(c*x/sqrt(a*c))/(sqrt(a*
c)*c)
```

Mupad [B] (verification not implemented)

Time = 12.24 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.02

$$\int \frac{A+Bx+Cx^2}{a+cx^2}\,dx=\frac{B\,\ln\left(c\,x^2+a\right)}{2\,c}+\frac{C\,x}{c}+\frac{A\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{c}}-\frac{C\,\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{c^{3/2}}$$

```
[In] int((A + B*x + C*x^2)/(a + c*x^2),x)
```

```
[Out] (B*log(a + c*x^2))/(2*c) + (C*x)/c + (A*atan((c^(1/2)*x)/a^(1/2)))/(a^(1/2)
*c^(1/2)) - (C*a^(1/2)*atan((c^(1/2)*x)/a^(1/2)))/c^(3/2)
```

3.47 $\int \frac{A+Bx+Cx^2}{(d+ex)\left(a+cx^2\right)} \, dx$

Optimal result

Integrand size = 27, antiderivative size = 133

$$\begin{aligned}\int \frac{A+Bx+Cx^2}{(d+ex)\left(a+cx^2\right)}\,dx=&\frac{(Acd-aCd+aBe)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{c}\left(cd^2+ae^2\right)}\\&+\frac{\left(Cd^2-Bde+Ae^2\right)\log(d+ex)}{e\left(cd^2+ae^2\right)}\\&+\frac{(Bcd-Ace+aCe)\log\left(a+cx^2\right)}{2c\left(cd^2+ae^2\right)}\end{aligned}$$

```
[Out] (A*e^2-B*d*e+C*d^2)*ln(e*x+d)/e/(a*e^2+c*d^2)+1/2*(-A*c*e+B*c*d+C*a*e)*ln(c
*x^2+a)/c/(a*e^2+c*d^2)+(A*c*d+B*a*e-C*a*d)*arctan(x*c^(1/2)/a^(1/2))/(a*e^
2+c*d^2)/a^(1/2)/c^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1643, 649, 211, 266}

$$\begin{aligned}\int \frac{A+Bx+Cx^2}{(d+ex)\left(a+cx^2\right)}\,dx=&\frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(aBe-aCd+Acd)}{\sqrt{a}\sqrt{c}\left(ae^2+cd^2\right)}\\&+\frac{\log\left(a+cx^2\right)(aCe-Ace+Bcd)}{2c\left(ae^2+cd^2\right)}\\&+\frac{\log(d+ex)\left(Ae^2-Bde+Cd^2\right)}{e\left(ae^2+cd^2\right)}\end{aligned}$$

```
[In] Int[(A + B*x + C*x^2)/((d + e*x)*(a + c*x^2)),x]
```

```
[Out] ((A*c*d - a*C*d + a*B*e)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[c]*(c*d
^2 + a*e^2)) + ((C*d^2 - B*d*e + A*e^2)*Log[d + e*x])/(e*(c*d^2 + a*e^2)) +
 ((B*c*d - A*c*e + a*C*e)*Log[a + c*x^2])/(2*c*(c*d^2 + a*e^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{Cd^2-Bde+Ae^2}{(cd^2+ae^2)(d+ex)}+\frac{Acd-aCd+aBe+(Bcd-Ace+aCe)x}{(cd^2+ae^2)(a+cx^2)}\right)dx\\
&= \frac{(Cd^2-Bde+Ae^2)\log(d+ex)}{e(cd^2+ae^2)}+\frac{\int \frac{Acd-aCd+aBe+(Bcd-Ace+aCe)x}{a+cx^2}\,dx}{cd^2+ae^2}\\
&= \frac{(Cd^2-Bde+Ae^2)\log(d+ex)}{e(cd^2+ae^2)}+\frac{(Acd-aCd+aBe)\int \frac{1}{a+cx^2}\,dx}{cd^2+ae^2}\\
&\quad+\frac{(Bcd-Ace+aCe)\int \frac{x}{a+cx^2}\,dx}{cd^2+ae^2}\\
&= \frac{(Acd-aCd+aBe)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{c}(cd^2+ae^2)}+\frac{(Cd^2-Bde+Ae^2)\log(d+ex)}{e(cd^2+ae^2)}\\
&\quad+\frac{(Bcd-Ace+aCe)\log(a+cx^2)}{2c(cd^2+ae^2)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.90

$$\int \frac{A+Bx+Cx^2}{(d+ex)\left(a+cx^2\right)}\,dx$$
$$=\frac{2\sqrt{c}e(Acd-aCd+aBe)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)+\sqrt{a}(2c(Cd^2-Bde+Ae^2)\log(d+ex)+e(Bcd-Ace+aCe)}{2\sqrt{a}ce\left(cd^2+ae^2\right)}$$

```
[In] Integrate[(A + B*x + C*x^2)/((d + e*x)*(a + c*x^2)),x]
```

```
[Out] (2*Sqrt[c]*e*(A*c*d - a*C*d + a*B*e)*ArcTan[(Sqrt[c]*x)/Sqrt[a]] + Sqrt[a]*
(2*c*(C*d^2 - B*d*e + A*e^2)*Log[d + e*x] + e*(B*c*d - A*c*e + a*C*e)*Log[a
 + c*x^2]))/(2*Sqrt[a]*c*e*(c*d^2 + a*e^2))
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.84

method	result
default	$\frac{\frac{(-Ace+Bcd+Cae)\ln\left(c\,x^2+a\right)}{2c}+\frac{(Acd+Bae-Cad)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}}}{e^2a+c\,d^2}+\frac{\left(A\,e^2-Bde+C\,d^2\right)\ln(ex+d)}{e\left(e^2a+c\,d^2\right)}$
risch	$\frac{\ln(ex+d)eA}{e^2a+c\,d^2}-\frac{\ln(ex+d)Bd}{e^2a+c\,d^2}+\frac{\ln(ex+d)C\,d^2}{e\left(e^2a+c\,d^2\right)}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\left(a^2c^2e^2+a\,c^3d^2\right)\_Z^2+\left(2Aa\,c^2e-2Ba\,c^2d-2a^2ceC\right)\_Z+A^2c^2-2ACc\right.}\right.}{}$

```
[In] int((C*x^2+B*x+A)/(e*x+d)/(c*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(a*e^2+c*d^2)*(1/2*(-A*c*e+B*c*d+C*a*e)/c*ln(c*x^2+a)+(A*c*d+B*a*e-C*a*d)
/(a*c)^(1/2)*arctan(c*x/(a*c)^(1/2)))+(A*e^2-B*d*e+C*d^2)*ln(e*x+d)/e/(a*e^
2+c*d^2)
```

Fricas [A] (verification not implemented)

none

Time = 4.61 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.97

$$\int \frac{A+Bx+Cx^2}{(d+ex)\left(a+cx^2\right)}\,dx$$
$$=\left[-\frac{(Bae^2-(Ca-Ac)de)\sqrt{-ac}\log\left(\frac{cx^2-2\sqrt{-ac}x-a}{cx^2+a}\right)-(Bacde+(Ca^2-Aac)e^2)\log\left(cx^2+a\right)-2\left(Cac\right.}{2\left(ac^2d^2e+a^2ce^3\right)}\right.$$

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/2*((B*a*e^2 - (C*a - A*c)*d*e)*sqrt(-a*c)*log((c*x^2 - 2*sqrt(-a*c)*x -
 a)/(c*x^2 + a)) - (B*a*c*d*e + (C*a^2 - A*a*c)*e^2)*log(c*x^2 + a) - 2*(C*
a*c*d^2 - B*a*c*d*e + A*a*c*e^2)*log(e*x + d))/(a*c^2*d^2*e + a^2*c*e^3), 1
/2*(2*(B*a*e^2 - (C*a - A*c)*d*e)*sqrt(a*c)*arctan(sqrt(a*c)*x/a) + (B*a*c*
d*e + (C*a^2 - A*a*c)*e^2)*log(c*x^2 + a) + 2*(C*a*c*d^2 - B*a*c*d*e + A*a*
c*e^2)*log(e*x + d))/(a*c^2*d^2*e + a^2*c*e^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(d+ex)(a+cx^2)}\,dx = \text{Timed out}$$

```
[In] integrate((C*x**2+B*x+A)/(e*x+d)/(c*x**2+a),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.92

$$\begin{aligned}\int \frac{A+Bx+Cx^2}{(d+ex)(a+cx^2)}\,dx &= \frac{(Bcd+(Ca-Ac)e)\log\left(cx^2+a\right)}{2\left(c^2d^2+ace^2\right)} \\ &\quad+\frac{(Cd^2-Bde+Ae^2)\log\left(ex+d\right)}{cd^2e+ae^3} \\ &\quad+\frac{(Bae-(Ca-Ac)d)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{(cd^2+ae^2)\sqrt{ac}}\end{aligned}$$

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)/(c*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*(B*c*d + (C*a - A*c)*e)*log(c*x^2 + a)/(c^2*d^2 + a*c*e^2) + (C*d^2 - B
*d*e + A*e^2)*log(e*x + d)/(c*d^2*e + a*e^3) + (B*a*e - (C*a - A*c)*d)*arct
an(c*x/sqrt(a*c))/((c*d^2 + a*e^2)*sqrt(a*c))
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx+Cx^2}{(d+ex)\,(a+cx^2)}\,dx = \frac{(Bcd+Cae-Ace)\log\left(cx^2+a\right)}{2\,(c^2d^2+ace^2)} + \frac{(Cd^2-Bde+Ae^2)\log\left(|ex+d|\right)}{cd^2e+ae^3} - \frac{(Cad-Acd-Bae)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{(cd^2+ae^2)\sqrt{ac}}$$

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)/(c*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*(B*c*d + C*a*e - A*c*e)*log(c*x^2 + a)/(c^2*d^2 + a*c*e^2) + (C*d^2 - B
*d*e + A*e^2)*log(abs(e*x + d))/(c*d^2*e + a*e^3) - (C*a*d - A*c*d - B*a*e)
*arctan(c*x/sqrt(a*c))/((c*d^2 + a*e^2)*sqrt(a*c))
```

Mupad [B] (verification not implemented)

Time = 15.63 (sec) , antiderivative size = 840, normalized size of antiderivative = 6.32

$$\int \frac{A+Bx+Cx^2}{(d+ex)\,(a+cx^2)}\,dx = \frac{\ln\left(d+e\,x\right)\,\left(C\,d^2-B\,d\,e+A\,e^2\right)}{c\,d^2\,e+a\,e^3}$$

$$-\ \ln\left(x\,\left(c\,e\,B^2-c\,d\,B\,C+a\,e\,C^2-A\,c\,e\,C\right)+C^2\,a\,d+\frac{\left(c^2\,\left(\frac{A\,a\,e}{2}-\frac{B\,a\,d}{2}\right)-c\,\left(\frac{C\,a^2\,e}{2}-\frac{A\,d\,\sqrt{-a\,c^3}}{2}\right)+\frac{B\,a\,e\,\sqrt{-a\,c^3}}{2}-C\,a\,d}{}\right.$$

$$-\ \ln\left(x\,\left(c\,e\,B^2-c\,d\,B\,C+a\,e\,C^2-A\,c\,e\,C\right)+C^2\,a\,d+\frac{\left(c^2\,\left(\frac{A\,a\,e}{2}-\frac{B\,a\,d}{2}\right)-c\,\left(\frac{C\,a^2\,e}{2}+\frac{A\,d\,\sqrt{-a\,c^3}}{2}\right)-\frac{B\,a\,e\,\sqrt{-a\,c^3}}{2}+C\,a\,d}{}\right.$$

```
[In] int((A + B*x + C*x^2)/((a + c*x^2)*(d + e*x)),x)
```

```
[Out] (log(d + e*x)*(A*e^2 + C*d^2 - B*d*e))/(a*e^3 + c*d^2*e) - (log(x*(C^2*a*e
+ B^2*c*e - A*C*c*e - B*C*c*d) + C^2*a*d + ((c^2*((A*a*e)/2 - (B*a*d)/2) -
c*((C*a^2*e)/2 - (A*d*(-a*c^3)^(1/2))/2) + (B*a*e*(-a*c^3)^(1/2))/2 - (C*a*
d*(-a*c^3)^(1/2))/2)*(((x*(6*a*c^2*e^3 - 2*c^3*d^2*e) + 8*a*c^2*d*e^2)*(c^2
*((A*a*e)/2 - (B*a*d)/2) - c*((C*a^2*e)/2 - (A*d*(-a*c^3)^(1/2))/2) + (B*a*
e*(-a*c^3)^(1/2))/2 - (C*a*d*(-a*c^3)^(1/2))/2))/(a*c^3*d^2 + a^2*c^2*e^2)
```

```
- x*(3*A*c^2*e^2 + 2*C*c^2*d^2 - 5*C*a*c*e^2 - B*c^2*d*e) + B*a*c*e^2 - A*c
^2*d*e + 5*C*a*c*d*e))/(a*c^3*d^2 + a^2*c^2*e^2) + A*B*c*e - A*C*c*d)*(c^2*
((A*a*e)/2 - (B*a*d)/2) - c*((C*a^2*e)/2 - (A*d*(-a*c^3)^(1/2))/2) + (B*a*e
*(-a*c^3)^(1/2))/2 - (C*a*d*(-a*c^3)^(1/2))/2))/(a*c^3*d^2 + a^2*c^2*e^2) -
 (log(x*(C^2*a*e + B^2*c*e - A*C*c*e - B*C*c*d) + C^2*a*d + ((c^2*((A*a*e)/
2 - (B*a*d)/2) - c*((C*a^2*e)/2 + (A*d*(-a*c^3)^(1/2))/2) - (B*a*e*(-a*c^3)
^(1/2))/2 + (C*a*d*(-a*c^3)^(1/2))/2)*(((x*(6*a*c^2*e^3 - 2*c^3*d^2*e) + 8*
a*c^2*d*e^2)*(c^2*((A*a*e)/2 - (B*a*d)/2) - c*((C*a^2*e)/2 + (A*d*(-a*c^3)^
(1/2))/2) - (B*a*e*(-a*c^3)^(1/2))/2 + (C*a*d*(-a*c^3)^(1/2))/2))/(a*c^3*d^
2 + a^2*c^2*e^2) - x*(3*A*c^2*e^2 + 2*C*c^2*d^2 - 5*C*a*c*e^2 - B*c^2*d*e)
+ B*a*c*e^2 - A*c^2*d*e + 5*C*a*c*d*e))/(a*c^3*d^2 + a^2*c^2*e^2) + A*B*c*e
 - A*C*c*d)*(c^2*((A*a*e)/2 - (B*a*d)/2) - c*((C*a^2*e)/2 + (A*d*(-a*c^3)^(
1/2))/2) - (B*a*e*(-a*c^3)^(1/2))/2 + (C*a*d*(-a*c^3)^(1/2))/2))/(a*c^3*d^2
 + a^2*c^2*e^2)
```

3.48 $\int \frac{A+Bx+Cx^2}{(d+ex)^2(a+cx^2)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 214

$$\begin{aligned}\int \frac{A+Bx+Cx^2}{(d+ex)^2\,(a+cx^2)}\,dx = &-\frac{Cd^2-Bde+Ae^2}{e\,(cd^2+ae^2)\,(d+ex)}\\ &+\frac{(Ac(cd^2-ae^2)+a(aCe^2-cd(Cd-2Be)))\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{c}\,(cd^2+ae^2)^2}\\ &-\frac{(Bcd^2-2Acde+2aCde-aBe^2)\log(d+ex)}{(cd^2+ae^2)^2}\\ &+\frac{(Bcd^2-2Acde+2aCde-aBe^2)\log\left(a+cx^2\right)}{2\,(cd^2+ae^2)^2}\end{aligned}$$

```
[Out] (-A*e^2+B*d*e-C*d^2)/e/(a*e^2+c*d^2)/(e*x+d)-(-2*A*c*d*e-B*a*e^2+B*c*d^2+2*
C*a*d*e)*ln(e*x+d)/(a*e^2+c*d^2)^2+1/2*(-2*A*c*d*e-B*a*e^2+B*c*d^2+2*C*a*d*
e)*ln(c*x^2+a)/(a*e^2+c*d^2)^2+(A*c*(-a*e^2+c*d^2)+a*(a*C*e^2-c*d*(-2*B*e+C
*d)))*arctan(x*c^(1/2)/a^(1/2))/(a*e^2+c*d^2)^2/a^(1/2)/c^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used

$= \{1643, 649, 211, 266\}$

$$\int \frac{A+Bx+Cx^2}{(d+ex)^2\,(a+cx^2)}\,dx = \frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(Ac(cd^2-ae^2)+a(aCe^2-cd(Cd-2Be)))}{\sqrt{a}\sqrt{c}\,(ae^2+cd^2)^2} + \frac{\log(a+cx^2)\,(-aBe^2+2aCde-2Acde+Bcd^2)}{2\,(ae^2+cd^2)^2} - \frac{Ae^2-Bde+Cd^2}{e(d+ex)\,(ae^2+cd^2)} - \frac{\log(d+ex)\,(-aBe^2+2aCde-2Acde+Bcd^2)}{(ae^2+cd^2)^2}$$

```
[In] Int[(A + B*x + C*x^2)/((d + e*x)^2*(a + c*x^2)),x]
```

```
[Out] -((C*d^2 - B*d*e + A*e^2)/(e*(c*d^2 + a*e^2)*(d + e*x))) + ((A*c*(c*d^2 - a
*e^2) + a*(a*C*e^2 - c*d*(C*d - 2*B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt
[a]*Sqrt[c]*(c*d^2 + a*e^2)^2) - ((B*c*d^2 - 2*A*c*d*e + 2*a*C*d*e - a*B*e^
2)*Log[d + e*x])/(c*d^2 + a*e^2)^2 + ((B*c*d^2 - 2*A*c*d*e + 2*a*C*d*e - a*
B*e^2)*Log[a + c*x^2])/(2*(c*d^2 + a*e^2)^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{Cd^2-Bde+Ae^2}{(cd^2+ae^2)\,(d+ex)^2}+\frac{e(-Bcd^2+2Acde-2aCde+aBe^2)}{(cd^2+ae^2)^2\,(d+ex)}\right.\\ &\quad\left.+\frac{Ac(cd^2-ae^2)+a(aCe^2-cd(Cd-2Be))+c(Bcd^2-2Acde+2aCde-aBe^2)\,x}{(cd^2+ae^2)^2\,(a+cx^2)}\right)dx\\
&= -\frac{Cd^2-Bde+Ae^2}{e\,(cd^2+ae^2)\,(d+ex)}-\frac{(Bcd^2-2Acde+2aCde-aBe^2)\log(d+ex)}{(cd^2+ae^2)^2}\\ &\quad+\frac{\int \frac{Ac(cd^2-ae^2)+a(aCe^2-cd(Cd-2Be))+c(Bcd^2-2Acde+2aCde-aBe^2)x}{a+cx^2}\,dx}{(cd^2+ae^2)^2}\\
&= -\frac{Cd^2-Bde+Ae^2}{e\,(cd^2+ae^2)\,(d+ex)}-\frac{(Bcd^2-2Acde+2aCde-aBe^2)\log(d+ex)}{(cd^2+ae^2)^2}\\ &\quad+\frac{(c(Bcd^2-2Acde+2aCde-aBe^2))\int \frac{x}{a+cx^2}\,dx}{(cd^2+ae^2)^2}\\ &\quad+\frac{(Ac(cd^2-ae^2)+a(aCe^2-cd(Cd-2Be)))\int \frac{1}{a+cx^2}\,dx}{(cd^2+ae^2)^2}\\
&= -\frac{Cd^2-Bde+Ae^2}{e\,(cd^2+ae^2)\,(d+ex)}+\frac{(Ac(cd^2-ae^2)+a(aCe^2-cd(Cd-2Be)))\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{c}\,(cd^2+ae^2)^2}\\ &\quad-\frac{(Bcd^2-2Acde+2aCde-aBe^2)\log(d+ex)}{(cd^2+ae^2)^2}\\ &\quad+\frac{(Bcd^2-2Acde+2aCde-aBe^2)\log\left(a+cx^2\right)}{2\,(cd^2+ae^2)^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx+Cx^2}{(d+ex)^2\,(a+cx^2)}\,dx = \frac{-\frac{2(cd^2+ae^2)(Cd^2+e(-Bd+Ae))}{e(d+ex)}+\frac{2\left(Ac(cd^2-ae^2)+a\left(aCe^2+cd(-Cd+2Be)\right)\right)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{c}}+(-2Bcd^2+4Acde-4aCde+}{2\,(cd^2+ae^2)^2}$$

```
[In] Integrate[(A + B*x + C*x^2)/((d + e*x)^2*(a + c*x^2)),x]
```

```
[Out] ((-2*(c*d^2 + a*e^2)*(C*d^2 + e*(-(B*d) + A*e)))/(e*(d + e*x)) + (2*(A*c*(c
*d^2 - a*e^2) + a*(a*C*e^2 + c*d*(-(C*d) + 2*B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt
[a]])/(Sqrt[a]*Sqrt[c]) + (-2*B*c*d^2 + 4*A*c*d*e - 4*a*C*d*e + 2*a*B*e^2)*
Log[d + e*x] + (B*c*d^2 - 2*A*c*d*e + 2*a*C*d*e - a*B*e^2)*Log[a + c*x^2])/
(2*(c*d^2 + a*e^2)^2)
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.95

method	result
default	$\frac{\frac{\left(-2A\,c^2de-B\,e^2ac+B\,c^2d^2+2acdeC\right)\ln\left(c\,x^2+a\right)}{2c}+\frac{\left(-Aac\,e^2+A\,c^2d^2+2Bacde+a^2C\,e^2-Cac\,d^2\right)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}}}{\left(e^2a+c\,d^2\right)^2}-\frac{A\,e^2-Bde+C\,d^2}{\left(e^2a+c\,d^2\right)e(ex+d)}+$ (2
risch	Expression too large to display

```
[In] int((C*x^2+B*x+A)/(e*x+d)^2/(c*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(a*e^2+c*d^2)^2*(1/2*(-2*A*c^2*d*e-B*a*c*e^2+B*c^2*d^2+2*C*a*c*d*e)/c*ln(
c*x^2+a)+(-A*a*c*e^2+A*c^2*d^2+2*B*a*c*d*e+C*a^2*e^2-C*a*c*d^2)/(a*c)^(1/2)
*arctan(c*x/(a*c)^(1/2)))-(A*e^2-B*d*e+C*d^2)/(a*e^2+c*d^2)/e/(e*x+d)+(2*A*
c*d*e+B*a*e^2-B*c*d^2-2*C*a*d*e)/(a*e^2+c*d^2)^2*ln(e*x+d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 442 vs. 2(204) = 408.

Time = 25.70 (sec) , antiderivative size = 904, normalized size of antiderivative = 4.22

$$\int \frac{A+Bx+Cx^2}{(d+ex)^2\,(a+cx^2)}\,dx$$

$$=\left[-\frac{2\,Cac^2d^4-2\,Bac^2d^3e-2\,Ba^2cde^3+2\,Aa^2ce^4+2\,(Ca^2c+Aac^2)d^2e^2-(2\,Bacd^2e^2-(Cac-Ac^2)d^3e}{}\right.$$

$$-\frac{2\,Cac^2d^4-2\,Bac^2d^3e-2\,Ba^2cde^3+2\,Aa^2ce^4+2\,(Ca^2c+Aac^2)d^2e^2-2\,(2\,Bacd^2e^2-(Cac-Ac^2)d^3e}{}$$

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^2/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*C*a*c^2*d^4 - 2*B*a*c^2*d^3*e - 2*B*a^2*c*d*e^3 + 2*A*a^2*c*e^4 +
2*(C*a^2*c + A*a*c^2)*d^2*e^2 - (2*B*a*c*d^2*e^2 - (C*a*c - A*c^2)*d^3*e +
(C*a^2 - A*a*c)*d*e^3 + (2*B*a*c*d*e^3 - (C*a*c - A*c^2)*d^2*e^2 + (C*a^2 -
 A*a*c)*e^4)*x)*sqrt(-a*c)*log((c*x^2 + 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) -
(B*a*c^2*d^3*e - B*a^2*c*d*e^3 + 2*(C*a^2*c - A*a*c^2)*d^2*e^2 + (B*a*c^2*d
^2*e^2 - B*a^2*c*e^4 + 2*(C*a^2*c - A*a*c^2)*d*e^3)*x)*log(c*x^2 + a) + 2*(
B*a*c^2*d^3*e - B*a^2*c*d*e^3 + 2*(C*a^2*c - A*a*c^2)*d^2*e^2 + (B*a*c^2*d^
2*e^2 - B*a^2*c*e^4 + 2*(C*a^2*c - A*a*c^2)*d*e^3)*x)*log(e*x + d))/(a*c^3*
d^5*e + 2*a^2*c^2*d^3*e^3 + a^3*c*d*e^5 + (a*c^3*d^4*e^2 + 2*a^2*c^2*d^2*e^
4 + a^3*c*e^6)*x), -1/2*(2*C*a*c^2*d^4 - 2*B*a*c^2*d^3*e - 2*B*a^2*c*d*e^3
+ 2*A*a^2*c*e^4 + 2*(C*a^2*c + A*a*c^2)*d^2*e^2 - 2*(2*B*a*c*d^2*e^2 - (C*a
*c - A*c^2)*d^3*e + (C*a^2 - A*a*c)*d*e^3 + (2*B*a*c*d*e^3 - (C*a*c - A*c^2
```

```
)*d^2*e^2 + (C*a^2 - A*a*c)*e^4)*x)*sqrt(a*c)*arctan(sqrt(a*c)*x/a) - (B*a*
c^2*d^3*e - B*a^2*c*d*e^3 + 2*(C*a^2*c - A*a*c^2)*d^2*e^2 + (B*a*c^2*d^2*e^
2 - B*a^2*c*e^4 + 2*(C*a^2*c - A*a*c^2)*d*e^3)*x)*log(c*x^2 + a) + 2*(B*a*c
^2*d^3*e - B*a^2*c*d*e^3 + 2*(C*a^2*c - A*a*c^2)*d^2*e^2 + (B*a*c^2*d^2*e^2
 - B*a^2*c*e^4 + 2*(C*a^2*c - A*a*c^2)*d*e^3)*x)*log(e*x + d))/(a*c^3*d^5*e
 + 2*a^2*c^2*d^3*e^3 + a^3*c*d*e^5 + (a*c^3*d^4*e^2 + 2*a^2*c^2*d^2*e^4 + a
^3*c*e^6)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(d+ex)^2\,(a+cx^2)}\,dx = \text{Timed out}$$

```
[In] integrate((C*x**2+B*x+A)/(e*x+d)**2/(c*x**2+a),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.19

$$\begin{aligned}\int \frac{A+Bx+Cx^2}{(d+ex)^2\,(a+cx^2)}\,dx &= \frac{(Bcd^2-Bae^2+2\,(Ca-Ac)de)\log\left(cx^2+a\right)}{2\,(c^2d^4+2\,acd^2e^2+a^2e^4)} \\ &\quad - \frac{(Bcd^2-Bae^2+2\,(Ca-Ac)de)\log\left(ex+d\right)}{c^2d^4+2\,acd^2e^2+a^2e^4} \\ &\quad + \frac{(2\,Bacde-(Cac-Ac^2)d^2+(Ca^2-Aac)e^2)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{(c^2d^4+2\,acd^2e^2+a^2e^4)\sqrt{ac}} \\ &\quad - \frac{Cd^2-Bde+Ae^2}{cd^3e+ade^3+(cd^2e^2+ae^4)x}\end{aligned}$$

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^2/(c*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*(B*c*d^2 - B*a*e^2 + 2*(C*a - A*c)*d*e)*log(c*x^2 + a)/(c^2*d^4 + 2*a*c
*d^2*e^2 + a^2*e^4) - (B*c*d^2 - B*a*e^2 + 2*(C*a - A*c)*d*e)*log(e*x + d)/
(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) + (2*B*a*c*d*e - (C*a*c - A*c^2)*d^2 +
(C*a^2 - A*a*c)*e^2)*arctan(c*x/sqrt(a*c))/((c^2*d^4 + 2*a*c*d^2*e^2 + a^2*
e^4)*sqrt(a*c)) - (C*d^2 - B*d*e + A*e^2)/(c*d^3*e + a*d*e^3 + (c*d^2*e^2 +
 a*e^4)*x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.29

$$\int \frac{A+Bx+Cx^2}{(d+ex)^2\,(a+cx^2)}\,dx$$
$$= \frac{(Bcd^2+2\,Cade-2\,Acde-Bae^2)\log\left(c-\frac{2\,cd}{ex+d}+\frac{cd^2}{(ex+d)^2}+\frac{ae^2}{(ex+d)^2}\right)}{2\,(c^2d^4+2\,acd^2e^2+a^2e^4)} - \frac{\frac{Cd^2e}{ex+d}-\frac{Bde^2}{ex+d}+\frac{Ae^3}{ex+d}}{cd^2e^2+ae^4} - \frac{(Cacd^2e^2-Ac^2d^2e^2-2\,Bacde^3-Ca^2e^4+Aace^4)\arctan\left(\frac{cd-\frac{cd^2}{ex+d}-\frac{ae^2}{ex+d}}{\sqrt{ac}e}\right)}{(c^2d^4+2\,acd^2e^2+a^2e^4)\sqrt{ac}e^2}$$

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^2/(c*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*(B*c*d^2 + 2*C*a*d*e - 2*A*c*d*e - B*a*e^2)*log(c - 2*c*d/(e*x + d) + c
*d^2/(e*x + d)^2 + a*e^2/(e*x + d)^2)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) -
 (C*d^2*e/(e*x + d) - B*d*e^2/(e*x + d) + A*e^3/(e*x + d))/(c*d^2*e^2 + a*e
^4) - (C*a*c*d^2*e^2 - A*c^2*d^2*e^2 - 2*B*a*c*d*e^3 - C*a^2*e^4 + A*a*c*e^
4)*arctan((c*d - c*d^2/(e*x + d) - a*e^2/(e*x + d))/(sqrt(a*c)*e))/((c^2*d^
4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(a*c)*e^2)
```

Mupad [B] (verification not implemented)

Time = 16.31 (sec) , antiderivative size = 1199, normalized size of antiderivative = 5.60

$$\int \frac{A+Bx+Cx^2}{(d+ex)^2\,(a+cx^2)}\,dx$$
$$= \frac{\ln\left(C\,c\,d^4\,{\left(-a\,c\right)}^{3/2}-A\,a\,e^4\,{\left(-a\,c\right)}^{3/2}+3\,B\,a\,c^3\,d^4+3\,B\,a^3\,c\,e^4+A\,c^4\,d^4\,x+A\,c^3\,d^4\,\sqrt{-a\,c}-C\,a^3\,e^4\,\sqrt{-a}\right.}{} - \frac{\ln\left(d+e\,x\right)\,\left(c\,\left(B\,d^2-2\,A\,d\,e\right)-a\,\left(B\,e^2-2\,C\,d\,e\right)\right)}{a^2\,e^4+2\,a\,c\,d^2\,e^2+c^2\,d^4} - \frac{\ln\left(A\,a\,e^4\,{\left(-a\,c\right)}^{3/2}-C\,c\,d^4\,{\left(-a\,c\right)}^{3/2}+3\,B\,a\,c^3\,d^4+3\,B\,a^3\,c\,e^4+A\,c^4\,d^4\,x-A\,c^3\,d^4\,\sqrt{-a\,c}+C\,a^3\,e^4\,\sqrt{-}\right.}{} - \frac{C\,d^2-B\,d\,e+A\,e^2}{e\,\left(c\,d^2+a\,e^2\right)\,\left(d+e\,x\right)}$$

```
[In] int((A + B*x + C*x^2)/((a + c*x^2)*(d + e*x)^2),x)
```

```
[Out] (log(C*c*d^4*(-a*c)^(3/2) - A*a*e^4*(-a*c)^(3/2) + 3*B*a*c^3*d^4 + 3*B*a^3*
c*e^4 + A*c^4*d^4*x + A*c^3*d^4*(-a*c)^(1/2) - C*a^3*e^4*(-a*c)^(1/2) - C*a
*c^3*d^4*x - C*a^3*c*e^4*x + 14*A*c*d^2*e^2*(-a*c)^(3/2) - 14*C*a*d^2*e^2*(
-a*c)^(3/2) - 3*B*c^3*d^4*x*(-a*c)^(1/2) + 8*A*a^2*c^2*d*e^3 + 8*C*a^2*c^2*
d^3*e + A*a^2*c^2*e^4*x - 10*B*a^2*c^2*d^2*e^2 + 8*B*a*d*e^3*(-a*c)^(3/2) -
 8*B*c*d^3*e*(-a*c)^(3/2) + 3*B*a*e^4*x*(-a*c)^(3/2) - 8*A*a*c^3*d^3*e - 8*
C*a^3*c*d*e^3 + 14*C*a^2*c^2*d^2*e^2*x + 8*A*c*d*e^3*x*(-a*c)^(3/2) - 8*C*a
*d*e^3*x*(-a*c)^(3/2) + 8*C*c*d^3*e*x*(-a*c)^(3/2) + 8*B*a*c^3*d^3*e*x + 8*
A*c^3*d^3*e*x*(-a*c)^(1/2) - 10*B*c*d^2*e^2*x*(-a*c)^(3/2) - 14*A*a*c^3*d^2
*e^2*x - 8*B*a^2*c^2*d*e^3*x)*(c^2*(a*((B*d^2)/2 - A*d*e) + (A*d^2*(-a*c)^(
1/2))/2) - c*(a^2*((B*e^2)/2 - C*d*e) + a*((A*e^2*(-a*c)^(1/2))/2 + (C*d^2*
(-a*c)^(1/2))/2 - B*d*e*(-a*c)^(1/2))) + (C*a^2*e^2*(-a*c)^(1/2))/2))/(a*c^
3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2) - (log(d + e*x)*(c*(B*d^2 - 2*A*d*e)
 - a*(B*e^2 - 2*C*d*e)))/(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2) - (log(A*a*e^4
*(-a*c)^(3/2) - C*c*d^4*(-a*c)^(3/2) + 3*B*a*c^3*d^4 + 3*B*a^3*c*e^4 + A*c^
4*d^4*x - A*c^3*d^4*(-a*c)^(1/2) + C*a^3*e^4*(-a*c)^(1/2) - C*a*c^3*d^4*x -
 C*a^3*c*e^4*x - 14*A*c*d^2*e^2*(-a*c)^(3/2) + 14*C*a*d^2*e^2*(-a*c)^(3/2)
+ 3*B*c^3*d^4*x*(-a*c)^(1/2) + 8*A*a^2*c^2*d*e^3 + 8*C*a^2*c^2*d^3*e + A*a^
2*c^2*e^4*x - 10*B*a^2*c^2*d^2*e^2 - 8*B*a*d*e^3*(-a*c)^(3/2) + 8*B*c*d^3*e
*(-a*c)^(3/2) - 3*B*a*e^4*x*(-a*c)^(3/2) - 8*A*a*c^3*d^3*e - 8*C*a^3*c*d*e^
3 + 14*C*a^2*c^2*d^2*e^2*x - 8*A*c*d*e^3*x*(-a*c)^(3/2) + 8*C*a*d*e^3*x*(-a
*c)^(3/2) - 8*C*c*d^3*e*x*(-a*c)^(3/2) + 8*B*a*c^3*d^3*e*x - 8*A*c^3*d^3*e*
x*(-a*c)^(1/2) + 10*B*c*d^2*e^2*x*(-a*c)^(3/2) - 14*A*a*c^3*d^2*e^2*x - 8*B
*a^2*c^2*d*e^3*x)*(c*(a^2*((B*e^2)/2 - C*d*e) - a*((A*e^2*(-a*c)^(1/2))/2 +
 (C*d^2*(-a*c)^(1/2))/2 - B*d*e*(-a*c)^(1/2))) - c^2*(a*((B*d^2)/2 - A*d*e)
 - (A*d^2*(-a*c)^(1/2))/2) + (C*a^2*e^2*(-a*c)^(1/2))/2))/(a*c^3*d^4 + a^3*
c*e^4 + 2*a^2*c^2*d^2*e^2) - (A*e^2 + C*d^2 - B*d*e)/(e*(a*e^2 + c*d^2)*(d
+ e*x))
```

3.49 $\int \frac{A+Bx+Cx^2}{(d+ex)^3(a+cx^2)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 305

$$\begin{aligned}\int \frac{A+Bx+Cx^2}{(d+ex)^3\,(a+cx^2)}\,dx &= -\frac{Cd^2-Bde+Ae^2}{2e\,(cd^2+ae^2)\,(d+ex)^2}+\frac{Bcd^2-2Acde+2aCde-aBe^2}{(cd^2+ae^2)^2\,(d+ex)}\\ &\quad+\frac{\sqrt{c}(Acd(cd^2-3ae^2)-a(cd^2(Cd-3Be)-ae^2(3Cd-Be)))\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}\,(cd^2+ae^2)^3}\\ &\quad-\frac{(Bcd(cd^2-3ae^2)-(Ac-aC)e(3cd^2-ae^2))\log(d+ex)}{(cd^2+ae^2)^3}\\ &\quad+\frac{(Bcd(cd^2-3ae^2)-(Ac-aC)e(3cd^2-ae^2))\log\left(a+cx^2\right)}{2\,(cd^2+ae^2)^3}\end{aligned}$$

```
[Out] 1/2*(-A*e^2+B*d*e-C*d^2)/e/(a*e^2+c*d^2)/(e*x+d)^2+(-2*A*c*d*e-B*a*e^2+B*c*
d^2+2*C*a*d*e)/(a*e^2+c*d^2)^2/(e*x+d)-(B*c*d*(-3*a*e^2+c*d^2)-(A*c-C*a)*e*
(-a*e^2+3*c*d^2))*ln(e*x+d)/(a*e^2+c*d^2)^3+1/2*(B*c*d*(-3*a*e^2+c*d^2)-(A*
c-C*a)*e*(-a*e^2+3*c*d^2))*ln(c*x^2+a)/(a*e^2+c*d^2)^3+(A*c*d*(-3*a*e^2+c*d
^2)-a*(c*d^2*(-3*B*e+C*d)-a*e^2*(-B*e+3*C*d)))*arctan(x*c^(1/2)/a^(1/2))*c^
(1/2)/(a*e^2+c*d^2)^3/a^(1/2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1643, 649, 211, 266}

$$\begin{aligned}
\int \frac{A+Bx+Cx^2}{(d+ex)^3\,(a+cx^2)}\,dx
&= \frac{\sqrt{c}\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(Acd(cd^2-3ae^2)-a(cd^2(Cd-3Be)-ae^2(3Cd-Be))\right)}{\sqrt{a}\,(ae^2+cd^2)^3} \\
&\quad+\frac{\log\left(a+cx^2\right)\left(Bcd(cd^2-3ae^2)-e(Ac-aC)\left(3cd^2-ae^2\right)\right)}{2\left(ae^2+cd^2\right)^3} \\
&\quad-\frac{Ae^2-Bde+Cd^2}{2e(d+ex)^2\,(ae^2+cd^2)}+\frac{-aBe^2+2aCde-2Acde+Bcd^2}{(d+ex)\left(ae^2+cd^2\right)^2} \\
&\quad-\frac{\log(d+ex)\left(Bcd(cd^2-3ae^2)-e(Ac-aC)\left(3cd^2-ae^2\right)\right)}{\left(ae^2+cd^2\right)^3}
\end{aligned}$$

```
[In] Int[(A + B*x + C*x^2)/((d + e*x)^3*(a + c*x^2)),x]
```

```
[Out] -1/2*(C*d^2 - B*d*e + A*e^2)/(e*(c*d^2 + a*e^2)*(d + e*x)^2) + (B*c*d^2 - 2
*A*c*d*e + 2*a*C*d*e - a*B*e^2)/((c*d^2 + a*e^2)^2*(d + e*x)) + (Sqrt[c]*(A
*c*d*(c*d^2 - 3*a*e^2) - a*(c*d^2*(C*d - 3*B*e) - a*e^2*(3*C*d - B*e)))*Arc
Tan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*(c*d^2 + a*e^2)^3) - ((B*c*d*(c*d^2 - 3*
a*e^2) - (A*c - a*C)*e*(3*c*d^2 - a*e^2))*Log[d + e*x])/(c*d^2 + a*e^2)^3 +
 ((B*c*d*(c*d^2 - 3*a*e^2) - (A*c - a*C)*e*(3*c*d^2 - a*e^2))*Log[a + c*x^2
])/(2*(c*d^2 + a*e^2)^3)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\text{integral} = \int \left(\frac{Cd^2-Bde+Ae^2}{(cd^2+ae^2)(d+ex)^3}+\frac{e(-Bcd^2+2Acde-2aCde+aBe^2)}{(cd^2+ae^2)^2(d+ex)^2}+\frac{e(-Bcd(cd^2-3ae^2)+(Ac-aC)e(3cd^2-ae^2))}{(cd^2+ae^2)^3(d+ex)}+\frac{c(Acd(cd^2-3ae^2)-a(cd^2(Cd-3Be)-ae^2(3Cd-Be))+(Bcd(cd^2-3ae^2)-(Ac-aC)e(3cd^2-ae}{(cd^2+ae^2)^3(a+cx^2)}$$

$$= -\frac{Cd^2-Bde+Ae^2}{2e(cd^2+ae^2)(d+ex)^2}+\frac{Bcd^2-2Acde+2aCde-aBe^2}{(cd^2+ae^2)^2(d+ex)} - \frac{(Bcd(cd^2-3ae^2)-(Ac-aC)e(3cd^2-ae^2))\log(d+ex)}{(cd^2+ae^2)^3} + \frac{c\int\frac{Acd(cd^2-3ae^2)-a(cd^2(Cd-3Be)-ae^2(3Cd-Be))+(Bcd(cd^2-3ae^2)-(Ac-aC)e(3cd^2-ae^2))x}{a+cx^2}\,dx}{(cd^2+ae^2)^3}$$

$$= -\frac{Cd^2-Bde+Ae^2}{2e(cd^2+ae^2)(d+ex)^2}+\frac{Bcd^2-2Acde+2aCde-aBe^2}{(cd^2+ae^2)^2(d+ex)} - \frac{(Bcd(cd^2-3ae^2)-(Ac-aC)e(3cd^2-ae^2))\log(d+ex)}{(cd^2+ae^2)^3} + \frac{(c(Bcd(cd^2-3ae^2)-(Ac-aC)e(3cd^2-ae^2)))\int\frac{x}{a+cx^2}\,dx}{(cd^2+ae^2)^3} + \frac{(c(Acd(cd^2-3ae^2)-a(cd^2(Cd-3Be)-ae^2(3Cd-Be))))\int\frac{1}{a+cx^2}\,dx}{(cd^2+ae^2)^3}$$

$$= -\frac{Cd^2-Bde+Ae^2}{2e(cd^2+ae^2)(d+ex)^2}+\frac{Bcd^2-2Acde+2aCde-aBe^2}{(cd^2+ae^2)^2(d+ex)} + \frac{\sqrt{c}(Acd(cd^2-3ae^2)-a(cd^2(Cd-3Be)-ae^2(3Cd-Be)))\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}(cd^2+ae^2)^3} - \frac{(Bcd(cd^2-3ae^2)-(Ac-aC)e(3cd^2-ae^2))\log(d+ex)}{(cd^2+ae^2)^3} + \frac{(Bcd(cd^2-3ae^2)-(Ac-aC)e(3cd^2-ae^2))\log(a+cx^2)}{2(cd^2+ae^2)^3}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.91

$$\int \frac{A+Bx+Cx^2}{(d+ex)^3\,(a+cx^2)}\,dx$$
$$= \frac{-\frac{(cd^2+ae^2)^2(Cd^2+e(-Bd+Ae))}{e(d+ex)^2} + \frac{2(cd^2+ae^2)(Bcd^2-2Acde+2aCde-aBe^2)}{d+ex} + \frac{2\sqrt{c}\left(Acd(cd^2-3ae^2)+a\left(ae^2(3Cd-Be)+cd^2(-Cd+3Be\right.\right.}{\sqrt{a}}}{}$$

```
[In] Integrate[(A + B*x + C*x^2)/((d + e*x)^3*(a + c*x^2)),x]
```

```
[Out] (-(((c*d^2 + a*e^2)^2*(C*d^2 + e*(-(B*d) + A*e)))/(e*(d + e*x)^2)) + (2*(c*
d^2 + a*e^2)*(B*c*d^2 - 2*A*c*d*e + 2*a*C*d*e - a*B*e^2))/(d + e*x) + (2*Sq
rt[c]*(A*c*d*(c*d^2 - 3*a*e^2) + a*(a*e^2*(3*C*d - B*e) + c*d^2*(-(C*d) + 3
*B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/Sqrt[a] - 2*(B*c*d*(c*d^2 - 3*a*e^2) -
 (A*c - a*C)*e*(3*c*d^2 - a*e^2))*Log[d + e*x] + (B*c*d*(c*d^2 - 3*a*e^2) -
 (A*c - a*C)*e*(3*c*d^2 - a*e^2))*Log[a + c*x^2])/(2*(c*d^2 + a*e^2)^3)
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.04

method	result
default	$-\frac{c\left(\frac{\left(-Aac\,e^3+3A\,c^2d^2e+3Bacd\,e^2-B\,c^2d^3+C\,a^2e^3-3Cac\,d^2e\right)\ln\left(c\,x^2+a\right)}{2c}+\frac{\left(3Aacd\,e^2-A\,d^3c^2+a^2B\,e^3-3Bac\,d^2e-3C\,a^2d\,e^2+Cac\,d^3\right)\mathrm{arct}}{\sqrt{ac}}\right.}{\left(e^2a+c\,d^2\right)^3}$
risch	Expression too large to display

```
[In] int((C*x^2+B*x+A)/(e*x+d)^3/(c*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -c/(a*e^2+c*d^2)^3*(1/2*(-A*a*c*e^3+3*A*c^2*d^2*e+3*B*a*c*d*e^2-B*c^2*d^3+C
*a^2*e^3-3*C*a*c*d^2*e)/c*ln(c*x^2+a)+(3*A*a*c*d*e^2-A*c^2*d^3+B*a^2*e^3-3*
B*a*c*d^2*e-3*C*a^2*d*e^2+C*a*c*d^3)/(a*c)^(1/2)*arctan(c*x/(a*c)^(1/2)))-1
/2*(A*e^2-B*d*e+C*d^2)/(a*e^2+c*d^2)/e/(e*x+d)^2-(2*A*c*d*e+B*a*e^2-B*c*d^2
-2*C*a*d*e)/(a*e^2+c*d^2)^2/(e*x+d)-(A*a*c*e^3-3*A*c^2*d^2*e-3*B*a*c*d*e^2+
B*c^2*d^3-C*a^2*e^3+3*C*a*c*d^2*e)/(a*e^2+c*d^2)^3*ln(e*x+d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 868 vs. $2(291) = 582$.

Time = 76.07 (sec) , antiderivative size = 1759, normalized size of antiderivative = 5.77

$$\int \frac{A+Bx+Cx^2}{(d+ex)^3\,(a+cx^2)}\,dx = \text{Too large to display}$$

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^3/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/2*(C*c^2*d^6 - 3*B*c^2*d^5*e - 2*B*a*c*d^3*e^3 + B*a^2*d*e^5 + A*a^2*e^
6 - (2*C*a*c - 5*A*c^2)*d^4*e^2 - 3*(C*a^2 - 2*A*a*c)*d^2*e^4 + (3*B*a*c*d^
4*e^2 - B*a^2*d^2*e^4 - (C*a*c - A*c^2)*d^5*e + 3*(C*a^2 - A*a*c)*d^3*e^3 +
 (3*B*a*c*d^2*e^4 - B*a^2*e^6 - (C*a*c - A*c^2)*d^3*e^3 + 3*(C*a^2 - A*a*c)
*d*e^5)*x^2 + 2*(3*B*a*c*d^3*e^3 - B*a^2*d*e^5 - (C*a*c - A*c^2)*d^4*e^2 +
3*(C*a^2 - A*a*c)*d^2*e^4)*x)*sqrt(-c/a)*log((c*x^2 - 2*a*x*sqrt(-c/a) - a)
/(c*x^2 + a)) - 2*(B*c^2*d^4*e^2 - B*a^2*e^6 + 2*(C*a*c - A*c^2)*d^3*e^3 +
2*(C*a^2 - A*a*c)*d*e^5)*x - (B*c^2*d^5*e - 3*B*a*c*d^3*e^3 + 3*(C*a*c - A*
c^2)*d^4*e^2 - (C*a^2 - A*a*c)*d^2*e^4 + (B*c^2*d^3*e^3 - 3*B*a*c*d*e^5 + 3
*(C*a*c - A*c^2)*d^2*e^4 - (C*a^2 - A*a*c)*e^6)*x^2 + 2*(B*c^2*d^4*e^2 - 3*
B*a*c*d^2*e^4 + 3*(C*a*c - A*c^2)*d^3*e^3 - (C*a^2 - A*a*c)*d*e^5)*x)*log(c
*x^2 + a) + 2*(B*c^2*d^5*e - 3*B*a*c*d^3*e^3 + 3*(C*a*c - A*c^2)*d^4*e^2 -
(C*a^2 - A*a*c)*d^2*e^4 + (B*c^2*d^3*e^3 - 3*B*a*c*d*e^5 + 3*(C*a*c - A*c^2
)*d^2*e^4 - (C*a^2 - A*a*c)*e^6)*x^2 + 2*(B*c^2*d^4*e^2 - 3*B*a*c*d^2*e^4 +
 3*(C*a*c - A*c^2)*d^3*e^3 - (C*a^2 - A*a*c)*d*e^5)*x)*log(e*x + d))/(c^3*d
^8*e + 3*a*c^2*d^6*e^3 + 3*a^2*c*d^4*e^5 + a^3*d^2*e^7 + (c^3*d^6*e^3 + 3*a
*c^2*d^4*e^5 + 3*a^2*c*d^2*e^7 + a^3*e^9)*x^2 + 2*(c^3*d^7*e^2 + 3*a*c^2*d^
5*e^4 + 3*a^2*c*d^3*e^6 + a^3*d*e^8)*x), -1/2*(C*c^2*d^6 - 3*B*c^2*d^5*e -
2*B*a*c*d^3*e^3 + B*a^2*d*e^5 + A*a^2*e^6 - (2*C*a*c - 5*A*c^2)*d^4*e^2 - 3
*(C*a^2 - 2*A*a*c)*d^2*e^4 - 2*(3*B*a*c*d^4*e^2 - B*a^2*d^2*e^4 - (C*a*c -
A*c^2)*d^5*e + 3*(C*a^2 - A*a*c)*d^3*e^3 + (3*B*a*c*d^2*e^4 - B*a^2*e^6 - (
C*a*c - A*c^2)*d^3*e^3 + 3*(C*a^2 - A*a*c)*d*e^5)*x^2 + 2*(3*B*a*c*d^3*e^3
- B*a^2*d*e^5 - (C*a*c - A*c^2)*d^4*e^2 + 3*(C*a^2 - A*a*c)*d^2*e^4)*x)*sqr
t(c/a)*arctan(x*sqrt(c/a)) - 2*(B*c^2*d^4*e^2 - B*a^2*e^6 + 2*(C*a*c - A*c^
2)*d^3*e^3 + 2*(C*a^2 - A*a*c)*d*e^5)*x - (B*c^2*d^5*e - 3*B*a*c*d^3*e^3 +
3*(C*a*c - A*c^2)*d^4*e^2 - (C*a^2 - A*a*c)*d^2*e^4 + (B*c^2*d^3*e^3 - 3*B*
a*c*d*e^5 + 3*(C*a*c - A*c^2)*d^2*e^4 - (C*a^2 - A*a*c)*e^6)*x^2 + 2*(B*c^2
*d^4*e^2 - 3*B*a*c*d^2*e^4 + 3*(C*a*c - A*c^2)*d^3*e^3 - (C*a^2 - A*a*c)*d*
e^5)*x)*log(c*x^2 + a) + 2*(B*c^2*d^5*e - 3*B*a*c*d^3*e^3 + 3*(C*a*c - A*c^
2)*d^4*e^2 - (C*a^2 - A*a*c)*d^2*e^4 + (B*c^2*d^3*e^3 - 3*B*a*c*d*e^5 + 3*(
C*a*c - A*c^2)*d^2*e^4 - (C*a^2 - A*a*c)*e^6)*x^2 + 2*(B*c^2*d^4*e^2 - 3*B*
a*c*d^2*e^4 + 3*(C*a*c - A*c^2)*d^3*e^3 - (C*a^2 - A*a*c)*d*e^5)*x)*log(e*x
 + d))/(c^3*d^8*e + 3*a*c^2*d^6*e^3 + 3*a^2*c*d^4*e^5 + a^3*d^2*e^7 + (c^3*
d^6*e^3 + 3*a*c^2*d^4*e^5 + 3*a^2*c*d^2*e^7 + a^3*e^9)*x^2 + 2*(c^3*d^7*e^2
 + 3*a*c^2*d^5*e^4 + 3*a^2*c*d^3*e^6 + a^3*d*e^8)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(d+ex)^3\,(a+cx^2)}\,dx = \text{Timed out}$$

```
[In] integrate((C*x**2+B*x+A)/(e*x+d)**3/(c*x**2+a),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 495, normalized size of antiderivative = 1.62

$$\begin{aligned}
&\int \frac{A+Bx+Cx^2}{(d+ex)^3\,(a+cx^2)}\,dx \\
&= \frac{(Bc^2d^3 - 3\,Bacde^2 + 3\,(Cac - Ac^2)d^2e - (Ca^2 - Aac)e^3)\log\left(cx^2+a\right)}{2\,(c^3d^6 + 3\,ac^2d^4e^2 + 3\,a^2cd^2e^4 + a^3e^6)} \\
&\quad - \frac{(Bc^2d^3 - 3\,Bacde^2 + 3\,(Cac - Ac^2)d^2e - (Ca^2 - Aac)e^3)\log\left(ex+d\right)}{c^3d^6 + 3\,ac^2d^4e^2 + 3\,a^2cd^2e^4 + a^3e^6} \\
&\quad + \frac{(3\,Bac^2d^2e - Ba^2ce^3 - (Cac^2 - Ac^3)d^3 + 3\,(Ca^2c - Aac^2)de^2)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{(c^3d^6 + 3\,ac^2d^4e^2 + 3\,a^2cd^2e^4 + a^3e^6)\sqrt{ac}} \\
&\quad - \frac{Ccd^4 - 3\,Bcd^3e + Bade^3 + Aae^4 - (3\,Ca - 5\,Ac)d^2e^2 - 2\,(Bcd^2e^2 - Bae^4 + 2\,(Ca - Ac)de^3)x}{2\,(c^2d^6e + 2\,acd^4e^3 + a^2d^2e^5 + (c^2d^4e^3 + 2\,acd^2e^5 + a^2e^7)x^2 + 2\,(c^2d^5e^2 + 2\,acd^3e^4 + a^2de^6)x)}
\end{aligned}$$

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^3/(c*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*(B*c^2*d^3 - 3*B*a*c*d*e^2 + 3*(C*a*c - A*c^2)*d^2*e - (C*a^2 - A*a*c)*
e^3)*log(c*x^2 + a)/(c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + a^3*e^6)
 - (B*c^2*d^3 - 3*B*a*c*d*e^2 + 3*(C*a*c - A*c^2)*d^2*e - (C*a^2 - A*a*c)*e
^3)*log(e*x + d)/(c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + a^3*e^6) +
(3*B*a*c^2*d^2*e - B*a^2*c*e^3 - (C*a*c^2 - A*c^3)*d^3 + 3*(C*a^2*c - A*a*c
^2)*d*e^2)*arctan(c*x/sqrt(a*c))/((c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*
e^4 + a^3*e^6)*sqrt(a*c)) - 1/2*(C*c*d^4 - 3*B*c*d^3*e + B*a*d*e^3 + A*a*e^
4 - (3*C*a - 5*A*c)*d^2*e^2 - 2*(B*c*d^2*e^2 - B*a*e^4 + 2*(C*a - A*c)*d*e^
3)*x)/(c^2*d^6*e + 2*a*c*d^4*e^3 + a^2*d^2*e^5 + (c^2*d^4*e^3 + 2*a*c*d^2*e
^5 + a^2*e^7)*x^2 + 2*(c^2*d^5*e^2 + 2*a*c*d^3*e^4 + a^2*d*e^6)*x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 516, normalized size of antiderivative = 1.69

$$\begin{aligned}&\int \frac{A+Bx+Cx^2}{(d+ex)^3\,(a+cx^2)}\,dx\\ &= \frac{(Bc^2d^3+3\,Cacd^2e-3\,Ac^2d^2e-3\,Bacde^2-Ca^2e^3+Aace^3)\log{(cx^2+a)}}{2\,(c^3d^6+3\,ac^2d^4e^2+3\,a^2cd^2e^4+a^3e^6)}\\ &\quad-\frac{(Bc^2d^3e+3\,Cacd^2e^2-3\,Ac^2d^2e^2-3\,Bacde^3-Ca^2e^4+Aace^4)\log{(|ex+d|)}}{c^3d^6e+3\,ac^2d^4e^3+3\,a^2cd^2e^5+a^3e^7}\\ &\quad-\frac{(Cac^2d^3-Ac^3d^3-3\,Bac^2d^2e-3\,Ca^2cde^2+3\,Aac^2de^2+Ba^2ce^3)\arctan{\left(\frac{cx}{\sqrt{ac}}\right)}}{(c^3d^6+3\,ac^2d^4e^2+3\,a^2cd^2e^4+a^3e^6)\sqrt{ac}}\\ &\quad-\frac{Cc^2d^6-3\,Bc^2d^5e-2\,Cacd^4e^2+5\,Ac^2d^4e^2-2\,Bacd^3e^3-3\,Ca^2d^2e^4+6\,Aacd^2e^4+Ba^2de^5+Aa^2e^6-2}{2\,(cd^2+ae^2)^3(ex+d)^2e}\end{aligned}$$

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^3/(c*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*(B*c^2*d^3 + 3*C*a*c*d^2*e - 3*A*c^2*d^2*e - 3*B*a*c*d*e^2 - C*a^2*e^3
+ A*a*c*e^3)*log(c*x^2 + a)/(c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 +
a^3*e^6) - (B*c^2*d^3*e + 3*C*a*c*d^2*e^2 - 3*A*c^2*d^2*e^2 - 3*B*a*c*d*e^3
 - C*a^2*e^4 + A*a*c*e^4)*log(abs(e*x + d))/(c^3*d^6*e + 3*a*c^2*d^4*e^3 +
3*a^2*c*d^2*e^5 + a^3*e^7) - (C*a*c^2*d^3 - A*c^3*d^3 - 3*B*a*c^2*d^2*e - 3
*C*a^2*c*d*e^2 + 3*A*a*c^2*d*e^2 + B*a^2*c*e^3)*arctan(c*x/sqrt(a*c))/((c^3
*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + a^3*e^6)*sqrt(a*c)) - 1/2*(C*c^2
*d^6 - 3*B*c^2*d^5*e - 2*C*a*c*d^4*e^2 + 5*A*c^2*d^4*e^2 - 2*B*a*c*d^3*e^3
- 3*C*a^2*d^2*e^4 + 6*A*a*c*d^2*e^4 + B*a^2*d*e^5 + A*a^2*e^6 - 2*(B*c^2*d^
4*e^2 + 2*C*a*c*d^3*e^3 - 2*A*c^2*d^3*e^3 + 2*C*a^2*d*e^5 - 2*A*a*c*d*e^5 -
 B*a^2*e^6)*x)/((c*d^2 + a*e^2)^3*(e*x + d)^2*e)
```

Mupad [B] (verification not implemented)

Time = 19.08 (sec) , antiderivative size = 2980, normalized size of antiderivative = 9.77

$$\int \frac{A+Bx+Cx^2}{(d+ex)^3\,(a+cx^2)}\,dx = \text{Too large to display}$$

```
[In] int((A + B*x + C*x^2)/((a + c*x^2)*(d + e*x)^3),x)
```

```
[Out] (log(d + e*x)*(e^3*(C*a^2 - A*a*c) - B*c^2*d^3 + d^2*e*(3*A*c^2 - 3*C*a*c)
+ 3*B*a*c*d*e^2))/(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4) -
 (log(9*A^2*a^5*e^10*(-a*c)^(5/2) + A^2*c^5*d^10*(-a*c)^(5/2) - B^2*a^7*e^1
0*(-a*c)^(3/2) - 9*B^2*c^3*d^10*(-a*c)^(7/2) + 9*C^2*a^9*e^10*(-a*c)^(1/2)
```

```
+ C^2*c*d^10*(-a*c)^(9/2) + 9*C^2*a^9*c*e^10*x - 6*A^2*a*d^4*e^6*(-a*c)^(9/
2) - 6*B^2*a*d^6*e^4*(-a*c)^(9/2) + 106*A^2*c*d^6*e^4*(-a*c)^(9/2) + 77*C^2
*a*d^8*e^2*(-a*c)^(9/2) - 27*B^2*c*d^8*e^2*(-a*c)^(9/2) + A^2*a^2*c^8*d^10*
x + 9*A^2*a^7*c^3*e^10*x + 9*B^2*a^3*c^7*d^10*x + B^2*a^8*c^2*e^10*x + C^2*
a^4*c^6*d^10*x + 27*A^2*a^3*d^2*e^8*(-a*c)^(7/2) - 106*B^2*a^3*d^4*e^6*(-a*
c)^(7/2) + 77*B^2*a^5*d^2*e^8*(-a*c)^(5/2) - 77*A^2*c^3*d^8*e^2*(-a*c)^(7/2
) - 106*C^2*a^3*d^6*e^4*(-a*c)^(7/2) - 6*C^2*a^5*d^4*e^6*(-a*c)^(5/2) + 27*
C^2*a^7*d^2*e^8*(-a*c)^(3/2) + 18*A*C*a^7*e^10*(-a*c)^(3/2) + 2*A*C*c^3*d^1
0*(-a*c)^(7/2) + 224*A*B*a*d^5*e^5*(-a*c)^(9/2) - 48*A*B*a^5*d*e^9*(-a*c)^(
5/2) - 212*A*C*a*d^6*e^4*(-a*c)^(9/2) + 64*A*B*c*d^7*e^3*(-a*c)^(9/2) + 48*
A*B*c^3*d^9*e*(-a*c)^(7/2) - 64*B*C*a*d^7*e^3*(-a*c)^(9/2) - 48*B*C*a^7*d*e
^9*(-a*c)^(3/2) - 154*A*C*c*d^8*e^2*(-a*c)^(9/2) + 77*A^2*a^3*c^7*d^8*e^2*x
 + 106*A^2*a^4*c^6*d^6*e^4*x - 6*A^2*a^5*c^5*d^4*e^6*x - 27*A^2*a^6*c^4*d^2
*e^8*x - 27*B^2*a^4*c^6*d^8*e^2*x - 6*B^2*a^5*c^5*d^6*e^4*x + 106*B^2*a^6*c
^4*d^4*e^6*x + 77*B^2*a^7*c^3*d^2*e^8*x + 77*C^2*a^5*c^5*d^8*e^2*x + 106*C^
2*a^6*c^4*d^6*e^4*x - 6*C^2*a^7*c^3*d^4*e^6*x - 27*C^2*a^8*c^2*d^2*e^8*x -
2*A*C*a^3*c^7*d^10*x - 18*A*C*a^8*c^2*e^10*x - 64*A*B*a^3*d^3*e^7*(-a*c)^(7
/2) - 12*A*C*a^3*d^4*e^6*(-a*c)^(7/2) + 54*A*C*a^5*d^2*e^8*(-a*c)^(5/2) + 2
24*B*C*a^3*d^5*e^5*(-a*c)^(7/2) - 64*B*C*a^5*d^3*e^7*(-a*c)^(5/2) + 48*B*C*
c*d^9*e*(-a*c)^(9/2) - 48*A*B*a^3*c^7*d^9*e*x - 48*A*B*a^7*c^3*d*e^9*x + 48
*B*C*a^4*c^6*d^9*e*x + 48*B*C*a^8*c^2*d*e^9*x + 64*A*B*a^4*c^6*d^7*e^3*x +
224*A*B*a^5*c^5*d^5*e^5*x + 64*A*B*a^6*c^4*d^3*e^7*x - 154*A*C*a^4*c^6*d^8*
e^2*x - 212*A*C*a^5*c^5*d^6*e^4*x + 12*A*C*a^6*c^4*d^4*e^6*x + 54*A*C*a^7*c
^3*d^2*e^8*x - 64*B*C*a^5*c^5*d^7*e^3*x - 224*B*C*a^6*c^4*d^5*e^5*x - 64*B*
C*a^7*c^3*d^3*e^7*x)*(e^2*((3*B*a^2*c*d)/2 - (3*C*a^2*d*(-a*c)^(1/2))/2 + (
3*A*a*c*d*(-a*c)^(1/2))/2) + e^3*((C*a^3)/2 - (A*a^2*c)/2 + (B*a^2*(-a*c)^(
1/2))/2) - e*((3*C*a^2*c*d^2)/2 - (3*A*a*c^2*d^2)/2 + (3*B*a*c*d^2*(-a*c)^(
1/2))/2) - (B*a*c^2*d^3)/2 - (A*c^2*d^3*(-a*c)^(1/2))/2 + (C*a*c*d^3*(-a*c)
^(1/2))/2))/(a^4*e^6 + a*c^3*d^6 + 3*a^3*c*d^2*e^4 + 3*a^2*c^2*d^4*e^2) - (
log(B^2*a^7*e^10*(-a*c)^(3/2) - A^2*c^5*d^10*(-a*c)^(5/2) - 9*A^2*a^5*e^10*
(-a*c)^(5/2) + 9*B^2*c^3*d^10*(-a*c)^(7/2) - 9*C^2*a^9*e^10*(-a*c)^(1/2) -
C^2*c*d^10*(-a*c)^(9/2) + 9*C^2*a^9*c*e^10*x + 6*A^2*a*d^4*e^6*(-a*c)^(9/2)
 + 6*B^2*a*d^6*e^4*(-a*c)^(9/2) - 106*A^2*c*d^6*e^4*(-a*c)^(9/2) - 77*C^2*a
*d^8*e^2*(-a*c)^(9/2) + 27*B^2*c*d^8*e^2*(-a*c)^(9/2) + A^2*a^2*c^8*d^10*x
+ 9*A^2*a^7*c^3*e^10*x + 9*B^2*a^3*c^7*d^10*x + B^2*a^8*c^2*e^10*x + C^2*a^
4*c^6*d^10*x - 27*A^2*a^3*d^2*e^8*(-a*c)^(7/2) + 106*B^2*a^3*d^4*e^6*(-a*c)
^(7/2) - 77*B^2*a^5*d^2*e^8*(-a*c)^(5/2) + 77*A^2*c^3*d^8*e^2*(-a*c)^(7/2)
+ 106*C^2*a^3*d^6*e^4*(-a*c)^(7/2) + 6*C^2*a^5*d^4*e^6*(-a*c)^(5/2) - 27*C^
2*a^7*d^2*e^8*(-a*c)^(3/2) - 18*A*C*a^7*e^10*(-a*c)^(3/2) - 2*A*C*c^3*d^10*
(-a*c)^(7/2) - 224*A*B*a*d^5*e^5*(-a*c)^(9/2) + 48*A*B*a^5*d*e^9*(-a*c)^(5/
2) + 212*A*C*a*d^6*e^4*(-a*c)^(9/2) - 64*A*B*c*d^7*e^3*(-a*c)^(9/2) - 48*A*
B*c^3*d^9*e*(-a*c)^(7/2) + 64*B*C*a*d^7*e^3*(-a*c)^(9/2) + 48*B*C*a^7*d*e^9
*(-a*c)^(3/2) + 154*A*C*c*d^8*e^2*(-a*c)^(9/2) + 77*A^2*a^3*c^7*d^8*e^2*x +
 106*A^2*a^4*c^6*d^6*e^4*x - 6*A^2*a^5*c^5*d^4*e^6*x - 27*A^2*a^6*c^4*d^2*e
^8*x - 27*B^2*a^4*c^6*d^8*e^2*x - 6*B^2*a^5*c^5*d^6*e^4*x + 106*B^2*a^6*c^4
```

```
*d^4*e^6*x + 77*B^2*a^7*c^3*d^2*e^8*x + 77*C^2*a^5*c^5*d^8*e^2*x + 106*C^2*
a^6*c^4*d^6*e^4*x - 6*C^2*a^7*c^3*d^4*e^6*x - 27*C^2*a^8*c^2*d^2*e^8*x - 2*
A*C*a^3*c^7*d^10*x - 18*A*C*a^8*c^2*e^10*x + 64*A*B*a^3*d^3*e^7*(-a*c)^(7/2
) + 12*A*C*a^3*d^4*e^6*(-a*c)^(7/2) - 54*A*C*a^5*d^2*e^8*(-a*c)^(5/2) - 224
*B*C*a^3*d^5*e^5*(-a*c)^(7/2) + 64*B*C*a^5*d^3*e^7*(-a*c)^(5/2) - 48*B*C*c*
d^9*e*(-a*c)^(9/2) - 48*A*B*a^3*c^7*d^9*e*x - 48*A*B*a^7*c^3*d*e^9*x + 48*B
*C*a^4*c^6*d^9*e*x + 48*B*C*a^8*c^2*d*e^9*x + 64*A*B*a^4*c^6*d^7*e^3*x + 22
4*A*B*a^5*c^5*d^5*e^5*x + 64*A*B*a^6*c^4*d^3*e^7*x - 154*A*C*a^4*c^6*d^8*e^
2*x - 212*A*C*a^5*c^5*d^6*e^4*x + 12*A*C*a^6*c^4*d^4*e^6*x + 54*A*C*a^7*c^3
*d^2*e^8*x - 64*B*C*a^5*c^5*d^7*e^3*x - 224*B*C*a^6*c^4*d^5*e^5*x - 64*B*C*
a^7*c^3*d^3*e^7*x)*(e^2*((3*B*a^2*c*d)/2 + (3*C*a^2*d*(-a*c)^(1/2))/2 - (3*
A*a*c*d*(-a*c)^(1/2))/2) - e^3*((A*a^2*c)/2 - (C*a^3)/2 + (B*a^2*(-a*c)^(1/
2))/2) + e*((3*A*a*c^2*d^2)/2 - (3*C*a^2*c*d^2)/2 + (3*B*a*c*d^2*(-a*c)^(1/
2))/2) - (B*a*c^2*d^3)/2 + (A*c^2*d^3*(-a*c)^(1/2))/2 - (C*a*c*d^3*(-a*c)^(
1/2))/2))/(a^4*e^6 + a*c^3*d^6 + 3*a^3*c*d^2*e^4 + 3*a^2*c^2*d^4*e^2) - ((A
*a*e^4 + C*c*d^4 + B*a*d*e^3 - 3*B*c*d^3*e + 5*A*c*d^2*e^2 - 3*C*a*d^2*e^2)
/(2*e*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)) + (x*(B*a*e^3 + 2*A*c*d*e^2 - 2*
C*a*d*e^2 - B*c*d^2*e))/(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2))/(d^2 + e^2*x^2
 + 2*d*e*x)
```

3.50 $\int \frac{(d+ex)^3(A+Bx+Cx^2)}{(a+cx^2)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 216

$$\begin{aligned}&\int \frac{(d+ex)^3\,(A+Bx+Cx^2)}{(a+cx^2)^2}\,dx\\&=-\frac{3e^2(Acd-a(3Cd+Be))x}{2ac^2}-\frac{(Ac-2aC)e^3x^2}{2ac^2}-\frac{(aB-(Ac-aC)x)(d+ex)^3}{2ac\,(a+cx^2)}\\&\quad+\frac{(Acd(cd^2+3ae^2)-a(3ae^2(3Cd+Be)-cd^2(Cd+3Be)))\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}c^{5/2}}\\&\quad-\frac{e(2aCe^2-c(3Cd^2+e(3Bd+Ae)))\log\left(a+cx^2\right)}{2c^3}\end{aligned}$$

```
[Out] -3/2*e^2*(A*c*d-a*(B*e+3*C*d))*x/a/c^2-1/2*(A*c-2*C*a)*e^3*x^2/a/c^2-1/2*(a
*B-(A*c-C*a)*x)*(e*x+d)^3/a/c/(c*x^2+a)+1/2*(A*c*d*(3*a*e^2+c*d^2)-a*(3*a*e
^2*(B*e+3*C*d)-c*d^2*(3*B*e+C*d)))*arctan(x*c^(1/2)/a^(1/2))/a^(3/2)/c^(5/2
)-1/2*e*(2*a*C*e^2-c*(3*C*d^2+e*(A*e+3*B*d)))*ln(c*x^2+a)/c^3
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used

$= \{1659, 815, 649, 211, 266\}$

$$\begin{aligned}\int \frac{(d+ex)^3 (A+Bx+Cx^2)}{(a+cx^2)^2}\,dx \\ = \frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(Acd(3ae^2+cd^2)-a(3ae^2(Be+3Cd)-cd^2(3Be+Cd)))}{2a^{3/2}c^{5/2}} \\ - \frac{e\log(a+cx^2)(2aCe^2-c(e(Ae+3Bd)+3Cd^2))}{2c^3} \\ - \frac{3e^2x(Acd-a(Be+3Cd))}{2ac^2} - \frac{(d+ex)^3(aB-x(Ac-aC))}{2ac(a+cx^2)} - \frac{e^3x^2(Ac-2aC)}{2ac^2}\end{aligned}$$

```
[In] Int[((d + e*x)^3*(A + B*x + C*x^2))/(a + c*x^2)^2,x]
```

```
[Out] (-3*e^2*(A*c*d - a*(3*C*d + B*e))*x)/(2*a*c^2) - ((A*c - 2*a*C)*e^3*x^2)/(2
*a*c^2) - ((a*B - (A*c - a*C)*x)*(d + e*x)^3)/(2*a*c*(a + c*x^2)) + ((A*c*d
*(c*d^2 + 3*a*e^2) - a*(3*a*e^2*(3*C*d + B*e) - c*d^2*(C*d + 3*B*e)))*ArcTa
n[(Sqrt[c]*x)/Sqrt[a]])/(2*a^(3/2)*c^(5/2)) - (e*(2*a*C*e^2 - c*(3*C*d^2 +
e*(3*B*d + A*e)))*Log[a + c*x^2])/(2*c^3)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1659

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
```

```
 x], x, 1]}, Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c*(p
+ 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\text{integral} = -\frac{(aB-(Ac-aC)x)(d+ex)^3}{2ac\,(a+cx^2)} - \frac{\int \frac{(d+ex)^2(-Acd-aCd-3aBe+2(Ac-2aC)ex)}{a+cx^2}\,dx}{2ac}$$

$$= -\frac{(aB-(Ac-aC)x)(d+ex)^3}{2ac\,(a+cx^2)}$$
$$-\frac{\int \Big(\frac{3e^2(Acd-3aCd-aBe)}{c} + \frac{2(Ac-2aC)e^3x}{c} - \frac{Acd(cd^2+3ae^2)-a(3ae^2(3Cd+Be)-cd^2(Cd+3Be))-2ae(2aCe^2-c(3Cd^2+e}{c(a+cx^2)}}{2ac}$$

$$= -\frac{3e^2(Acd-a(3Cd+Be))x}{2ac^2} - \frac{(Ac-2aC)e^3x^2}{2ac^2} - \frac{(aB-(Ac-aC)x)(d+ex)^3}{2ac\,(a+cx^2)}$$
$$+\frac{\int \frac{Acd(cd^2+3ae^2)-a(3ae^2(3Cd+Be)-cd^2(Cd+3Be))-2ae(2aCe^2-c(3Cd^2+e(3Bd+Ae)))x}{a+cx^2}\,dx}{2ac^2}$$

$$= -\frac{3e^2(Acd-a(3Cd+Be))x}{2ac^2} - \frac{(Ac-2aC)e^3x^2}{2ac^2} - \frac{(aB-(Ac-aC)x)(d+ex)^3}{2ac\,(a+cx^2)}$$
$$-\frac{(e(2aCe^2-c(3Cd^2+e(3Bd+Ae))))\int \frac{x}{a+cx^2}\,dx}{c^2}$$
$$+\frac{(Acd(cd^2+3ae^2)-a(3ae^2(3Cd+Be)-cd^2(Cd+3Be)))\int \frac{1}{a+cx^2}\,dx}{2ac^2}$$

$$= -\frac{3e^2(Acd-a(3Cd+Be))x}{2ac^2} - \frac{(Ac-2aC)e^3x^2}{2ac^2} - \frac{(aB-(Ac-aC)x)(d+ex)^3}{2ac\,(a+cx^2)}$$
$$+\frac{(Acd(cd^2+3ae^2)-a(3ae^2(3Cd+Be)-cd^2(Cd+3Be)))\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}c^{5/2}}$$
$$-\frac{e(2aCe^2-c(3Cd^2+e(3Bd+Ae)))\log\left(a+cx^2\right)}{2c^3}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.08

$$\int \frac{(d+ex)^3\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^2}\,dx$$

$$=\frac{2ce^2(3Cd+Be)x+cCe^3x^2+\frac{-a^3Ce^3+Ac^3d^3x-ac^2d\left(Cd^2x+3Ae(d+ex)+Bd(d+3ex)\right)+a^2ce(3Cd(d+ex)+e(3Bd+Ae+Bex))}{a(a+cx^2)}+}{2c}$$

```
[In] Integrate[((d + e*x)^3*(A + B*x + C*x^2))/(a + c*x^2)^2,x]
```

```
[Out] (2*c*e^2*(3*C*d + B*e)*x + c*C*e^3*x^2 + (-(a^3*C*e^3) + A*c^3*d^3*x - a*c^
2*d*(C*d^2*x + 3*A*e*(d + e*x) + B*d*(d + 3*e*x)) + a^2*c*e*(3*C*d*(d + e*x
) + e*(3*B*d + A*e + B*e*x)))/(a*(a + c*x^2)) + (Sqrt[c]*(A*c*d*(c*d^2 + 3*
a*e^2) + a*(-3*a*e^2*(3*C*d + B*e) + c*d^2*(C*d + 3*B*e)))*ArcTan[(Sqrt[c]*
x)/Sqrt[a]])/a^(3/2) + e*(3*c*C*d^2 - 2*a*C*e^2 + c*e*(3*B*d + A*e))*Log[a
+ c*x^2])/(2*c^3)
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.31

method	result
default	$\frac{e^2\left(\frac{1}{2}Cx^2e+Bex+3Cdx\right)}{c^2}+\frac{-\frac{\left(3Aacde^2-Ad^3c^2-a^2Be^3+3Bacd^2e-3Ca^2de^2+Cacd^3\right)x}{2a}+\frac{Aace^3-3Ac^2d^2e+3Bacde^2-Bc^2d^3-Ca^2e^3+3}{2c}}{cx^2+a}$
risch	Expression too large to display

```
[In] int((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] e^2/c^2*(1/2*C*x^2*e+B*e*x+3*C*d*x)+1/c^2*((-1/2*(3*A*a*c*d*e^2-A*c^2*d^3-B
*a^2*e^3+3*B*a*c*d^2*e-3*C*a^2*d*e^2+C*a*c*d^3)/a*x+1/2*(A*a*c*e^3-3*A*c^2*
d^2*e+3*B*a*c*d*e^2-B*c^2*d^3-C*a^2*e^3+3*C*a*c*d^2*e)/c)/(c*x^2+a)+1/2/a*(
1/2*(2*A*a*c*e^3+6*B*a*c*d*e^2-4*C*a^2*e^3+6*C*a*c*d^2*e)/c*ln(c*x^2+a)+(3*
A*a*c*d*e^2+A*c^2*d^3-3*B*a^2*e^3+3*B*a*c*d^2*e-9*C*a^2*d*e^2+C*a*c*d^3)/(a
*c)^(1/2)*arctan(c*x/(a*c)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 454 vs. $2(196) = 392$.

Time = 0.31 (sec) , antiderivative size = 931, normalized size of antiderivative = 4.31

$$\int \frac{(d+ex)^3\,(A+Bx+Cx^2)}{(a+cx^2)^2}\,dx$$

$$= \Bigg[2\,Ca^2c^2e^3x^4 + 2\,Ca^3ce^3x^2 - 2\,Ba^2c^2d^3 + 6\,Ba^3cde^2 + 6\,(Ca^3c - Aa^2c^2)d^2e - 2\,(Ca^4 - Aa^3c)e^3 + 4\,(3$$

```
[In] integrate((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*C*a^2*c^2*e^3*x^4 + 2*C*a^3*c*e^3*x^2 - 2*B*a^2*c^2*d^3 + 6*B*a^3*c
*d*e^2 + 6*(C*a^3*c - A*a^2*c^2)*d^2*e - 2*(C*a^4 - A*a^3*c)*e^3 + 4*(3*C*a
^2*c^2*d*e^2 + B*a^2*c^2*e^3)*x^3 + (3*B*a^2*c*d^2*e - 3*B*a^3*e^3 + (C*a^2
*c + A*a*c^2)*d^3 - 3*(3*C*a^3 - A*a^2*c)*d*e^2 + (3*B*a*c^2*d^2*e - 3*B*a^
2*c*e^3 + (C*a*c^2 + A*c^3)*d^3 - 3*(3*C*a^2*c - A*a*c^2)*d*e^2)*x^2)*sqrt(
-a*c)*log((c*x^2 + 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) - 2*(3*B*a^2*c^2*d^2*e
- 3*B*a^3*c*e^3 + (C*a^2*c^2 - A*a*c^3)*d^3 - 3*(3*C*a^3*c - A*a^2*c^2)*d*e
^2)*x + 2*(3*C*a^3*c*d^2*e + 3*B*a^3*c*d*e^2 - (2*C*a^4 - A*a^3*c)*e^3 + (3
*C*a^2*c^2*d^2*e + 3*B*a^2*c^2*d*e^2 - (2*C*a^3*c - A*a^2*c^2)*e^3)*x^2)*lo
g(c*x^2 + a))/(a^2*c^4*x^2 + a^3*c^3), 1/2*(C*a^2*c^2*e^3*x^4 + C*a^3*c*e^3
*x^2 - B*a^2*c^2*d^3 + 3*B*a^3*c*d*e^2 + 3*(C*a^3*c - A*a^2*c^2)*d^2*e - (C
*a^4 - A*a^3*c)*e^3 + 2*(3*C*a^2*c^2*d*e^2 + B*a^2*c^2*e^3)*x^3 + (3*B*a^2*
c*d^2*e - 3*B*a^3*e^3 + (C*a^2*c + A*a*c^2)*d^3 - 3*(3*C*a^3 - A*a^2*c)*d*e
^2 + (3*B*a*c^2*d^2*e - 3*B*a^2*c*e^3 + (C*a*c^2 + A*c^3)*d^3 - 3*(3*C*a^2*
c - A*a*c^2)*d*e^2)*x^2)*sqrt(a*c)*arctan(sqrt(a*c)*x/a) - (3*B*a^2*c^2*d^2
*e - 3*B*a^3*c*e^3 + (C*a^2*c^2 - A*a*c^3)*d^3 - 3*(3*C*a^3*c - A*a^2*c^2)*
d*e^2)*x + (3*C*a^3*c*d^2*e + 3*B*a^3*c*d*e^2 - (2*C*a^4 - A*a^3*c)*e^3 + (
3*C*a^2*c^2*d^2*e + 3*B*a^2*c^2*d*e^2 - (2*C*a^3*c - A*a^2*c^2)*e^3)*x^2)*l
og(c*x^2 + a))/(a^2*c^4*x^2 + a^3*c^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 952 vs. $2(197) = 394$.

Time = 25.72 (sec) , antiderivative size = 952, normalized size of antiderivative = 4.41

$$\int \frac{(d+ex)^3\,(A+Bx+Cx^2)}{(a+cx^2)^2}\,dx$$

$$= \frac{Ce^3x^2}{2c^2} + x\left(\frac{Be^3}{c^2} + \frac{3Cde^2}{c^2}\right) + \left(-\frac{e(-Ace^2 - 3Bcde + 2Cae^2 - 3Ccd^2)}{2c^3}\right.$$

$$\left.- \frac{\sqrt{-a^3c^7}(-3Aacde^2 - Ac^2d^3 + 3Ba^2e^3 - 3Bacd^2e + 9Ca^2de^2 - Cacd^3)}{4a^3c^6}\right)\log\left(x + \frac{2Aa^2ce^3 + 6Ba^2cde}{}\right.$$

$$+ \left(-\frac{e(-Ace^2 - 3Bcde + 2Cae^2 - 3Ccd^2)}{2c^3}\right.$$

$$\left.+ \frac{\sqrt{-a^3c^7}(-3Aacde^2 - Ac^2d^3 + 3Ba^2e^3 - 3Bacd^2e + 9Ca^2de^2 - Cacd^3)}{4a^3c^6}\right)\log\left(x + \frac{2Aa^2ce^3 + 6Ba^2cde}{}\right.$$

$$+ \frac{Aa^2ce^3 - 3Aac^2d^2e + 3Ba^2cde^2 - Bac^2d^3 - Ca^3e^3 + 3Ca^2cd^2e + x(-3Aac^2de^2 + Ac^3d^3 + Ba^2ce^3 - 3}{2a^2c^3 + 2ac^4x^2}$$

```
[In] integrate((e*x+d)**3*(C*x**2+B*x+A)/(c*x**2+a)**2,x)
```

```
[Out] C*e**3*x**2/(2*c**2) + x*(B*e**3/c**2 + 3*C*d*e**2/c**2) + (-e*(-A*c*e**2 -
 3*B*c*d*e + 2*C*a*e**2 - 3*C*c*d**2)/(2*c**3) - sqrt(-a**3*c**7)*(-3*A*a*c
*d*e**2 - A*c**2*d**3 + 3*B*a**2*e**3 - 3*B*a*c*d**2*e + 9*C*a**2*d*e**2 -
C*a*c*d**3)/(4*a**3*c**6))*log(x + (2*A*a**2*c*e**3 + 6*B*a**2*c*d*e**2 - 4
*C*a**3*e**3 + 6*C*a**2*c*d**2*e - 4*a**2*c**3*(-e*(-A*c*e**2 - 3*B*c*d*e +
 2*C*a*e**2 - 3*C*c*d**2)/(2*c**3) - sqrt(-a**3*c**7)*(-3*A*a*c*d*e**2 - A*
c**2*d**3 + 3*B*a**2*e**3 - 3*B*a*c*d**2*e + 9*C*a**2*d*e**2 - C*a*c*d**3)/
(4*a**3*c**6)))/(-3*A*a*c**2*d*e**2 - A*c**3*d**3 + 3*B*a**2*c*e**3 - 3*B*a
*c**2*d**2*e + 9*C*a**2*c*d*e**2 - C*a*c**2*d**3)) + (-e*(-A*c*e**2 - 3*B*c
*d*e + 2*C*a*e**2 - 3*C*c*d**2)/(2*c**3) + sqrt(-a**3*c**7)*(-3*A*a*c*d*e**
2 - A*c**2*d**3 + 3*B*a**2*e**3 - 3*B*a*c*d**2*e + 9*C*a**2*d*e**2 - C*a*c*
d**3)/(4*a**3*c**6))*log(x + (2*A*a**2*c*e**3 + 6*B*a**2*c*d*e**2 - 4*C*a**
3*e**3 + 6*C*a**2*c*d**2*e - 4*a**2*c**3*(-e*(-A*c*e**2 - 3*B*c*d*e + 2*C*a
*e**2 - 3*C*c*d**2)/(2*c**3) + sqrt(-a**3*c**7)*(-3*A*a*c*d*e**2 - A*c**2*d
**3 + 3*B*a**2*e**3 - 3*B*a*c*d**2*e + 9*C*a**2*d*e**2 - C*a*c*d**3)/(4*a**
3*c**6)))/(-3*A*a*c**2*d*e**2 - A*c**3*d**3 + 3*B*a**2*c*e**3 - 3*B*a*c**2*
d**2*e + 9*C*a**2*c*d*e**2 - C*a*c**2*d**3)) + (A*a**2*c*e**3 - 3*A*a*c**2*
d**2*e + 3*B*a**2*c*d*e**2 - B*a*c**2*d**3 - C*a**3*e**3 + 3*C*a**2*c*d**2*
e + x*(-3*A*a*c**2*d*e**2 + A*c**3*d**3 + B*a**2*c*e**3 - 3*B*a*c**2*d**2*e
 + 3*C*a**2*c*d*e**2 - C*a*c**2*d**3))/(2*a**2*c**3 + 2*a*c**4*x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.33

$$\int \frac{(d+ex)^3\,(A+Bx+Cx^2)}{(a+cx^2)^2}\,dx = $$

$$-\frac{Bac^2d^3 - 3\,Ba^2cde^2 - 3\,(Ca^2c - Aac^2)d^2e + (Ca^3 - Aa^2c)e^3 + (3\,Bac^2d^2e - Ba^2ce^3 + (Cac^2 - Ac^3)}{2\,(ac^4x^2 + a^2c^3)}$$

$$+\frac{Ce^3x^2 + 2\,(3\,Cde^2 + Be^3)x}{2\,c^2} + \frac{(3\,Ccd^2e + 3\,Bcde^2 - (2\,Ca - Ac)e^3)\log{(cx^2+a)}}{2\,c^3}$$

$$+\frac{(3\,Bacd^2e - 3\,Ba^2e^3 + (Cac + Ac^2)d^3 - 3\,(3\,Ca^2 - Aac)de^2)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\,\sqrt{ac}ac^2}$$

```
[In] integrate((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(B*a*c^2*d^3 - 3*B*a^2*c*d*e^2 - 3*(C*a^2*c - A*a*c^2)*d^2*e + (C*a^3
- A*a^2*c)*e^3 + (3*B*a*c^2*d^2*e - B*a^2*c*e^3 + (C*a*c^2 - A*c^3)*d^3 - 3
*(C*a^2*c - A*a*c^2)*d*e^2)*x)/(a*c^4*x^2 + a^2*c^3) + 1/2*(C*e^3*x^2 + 2*(
3*C*d*e^2 + B*e^3)*x)/c^2 + 1/2*(3*C*c*d^2*e + 3*B*c*d*e^2 - (2*C*a - A*c)*
e^3)*log(c*x^2 + a)/c^3 + 1/2*(3*B*a*c*d^2*e - 3*B*a^2*e^3 + (C*a*c + A*c^2
)*d^3 - 3*(3*C*a^2 - A*a*c)*d*e^2)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a*c^2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.38

$$\int \frac{(d+ex)^3\,(A+Bx+Cx^2)}{(a+cx^2)^2}\,dx = \frac{(3\,Ccd^2e + 3\,Bcde^2 - 2\,Cae^3 + Ace^3)\log{(cx^2+a)}}{2\,c^3}$$

$$+\frac{(Cacd^3 + Ac^2d^3 + 3\,Bacd^2e - 9\,Ca^2de^2 + 3\,Aacde^2 - 3\,Ba^2e^3)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\,\sqrt{ac}ac^2}$$

$$+\frac{Cc^2e^3x^2 + 6\,Cc^2de^2x + 2\,Bc^2e^3x}{2\,c^4}$$

$$-\frac{Bac^2d^3 - 3\,Ca^2cd^2e + 3\,Aac^2d^2e - 3\,Ba^2cde^2 + Ca^3e^3 - Aa^2ce^3 + (Cac^2d^3 - Ac^3d^3 + 3\,Bac^2d^2e - 3}{2\,(cx^2+a)ac^3}$$

```
[In] integrate((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(3*C*c*d^2*e + 3*B*c*d*e^2 - 2*C*a*e^3 + A*c*e^3)*log(c*x^2 + a)/c^3 +
1/2*(C*a*c*d^3 + A*c^2*d^3 + 3*B*a*c*d^2*e - 9*C*a^2*d*e^2 + 3*A*a*c*d*e^2
- 3*B*a^2*e^3)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a*c^2) + 1/2*(C*c^2*e^3*x^2
```

```
 + 6*C*c^2*d*e^2*x + 2*B*c^2*e^3*x)/c^4 - 1/2*(B*a*c^2*d^3 - 3*C*a^2*c*d^2*
e + 3*A*a*c^2*d^2*e - 3*B*a^2*c*d*e^2 + C*a^3*e^3 - A*a^2*c*e^3 + (C*a*c^2*
d^3 - A*c^3*d^3 + 3*B*a*c^2*d^2*e - 3*C*a^2*c*d*e^2 + 3*A*a*c^2*d*e^2 - B*a
^2*c*e^3)*x)/((c*x^2 + a)*a*c^3)
```

Mupad [B] (verification not implemented)

Time = 13.00 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.40

$$\int \frac{(d+ex)^3\,(A+Bx+Cx^2)}{(a+cx^2)^2}\,dx = \frac{x\,(B\,e^3+3\,C\,d\,e^2)}{c^2}$$
$$-\frac{\frac{C\,a^2\,e^3-3\,C\,a\,c\,d^2\,e-3\,B\,a\,c\,d\,e^2-A\,a\,c\,e^3+B\,c^2\,d^3+3\,A\,c^2\,d^2\,e}{2\,c}-\frac{x\,(3\,C\,a^2\,d\,e^2+B\,a^2\,e^3-C\,a\,c\,d^3-3\,B\,a\,c\,d^2\,e-3\,A\,a\,c\,d\,e^2+A\,c^2\,d^3)}{2\,a}}{c^3\,x^2+a\,c^2}$$
$$+\frac{C\,e^3\,x^2}{2\,c^2}$$
$$+\frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\,(-9\,C\,a^2\,d\,e^2-3\,B\,a^2\,e^3+C\,a\,c\,d^3+3\,B\,a\,c\,d^2\,e+3\,A\,a\,c\,d\,e^2+A\,c^2\,d^3)}{2\,a^{3/2}\,c^{5/2}}$$
$$+\frac{\ln(c\,x^2+a)\,(-32\,C\,a^4\,c^3\,e^3+48\,C\,a^3\,c^4\,d^2\,e+48\,B\,a^3\,c^4\,d\,e^2+16\,A\,a^3\,c^4\,e^3)}{32\,a^3\,c^6}$$

```
[In] int(((d + e*x)^3*(A + B*x + C*x^2))/(a + c*x^2)^2,x)
```

```
[Out] (x*(B*e^3 + 3*C*d*e^2))/c^2 - ((B*c^2*d^3 + C*a^2*e^3 - A*a*c*e^3 + 3*A*c^2
*d^2*e - 3*B*a*c*d*e^2 - 3*C*a*c*d^2*e)/(2*c) - (x*(A*c^2*d^3 + B*a^2*e^3 -
 C*a*c*d^3 + 3*C*a^2*d*e^2 - 3*A*a*c*d*e^2 - 3*B*a*c*d^2*e))/(2*a))/(a*c^2
+ c^3*x^2) + (C*e^3*x^2)/(2*c^2) + (atan((c^(1/2)*x)/a^(1/2))*(A*c^2*d^3 -
3*B*a^2*e^3 + C*a*c*d^3 - 9*C*a^2*d*e^2 + 3*A*a*c*d*e^2 + 3*B*a*c*d^2*e))/(
2*a^(3/2)*c^(5/2)) + (log(a + c*x^2)*(16*A*a^3*c^4*e^3 - 32*C*a^4*c^3*e^3 +
 48*B*a^3*c^4*d*e^2 + 48*C*a^3*c^4*d^2*e))/(32*a^3*c^6)
```

3.51 $\int \frac{(d+ex)^2\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^2} \, dx$

Optimal result

Integrand size = 27, antiderivative size = 146

$$\begin{aligned}
&\int \frac{(d+ex)^2\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^2} \, dx \\
&= -\frac{(Ac-3aC)e^2x}{2ac^2} - \frac{(aB-(Ac-aC)x)(d+ex)^2}{2ac\left(a+cx^2\right)} \\
&\quad + \frac{\left(a(Ac-3aC)e^2+cd(Acd+aCd+2aBe)\right)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}c^{5/2}} + \frac{e(2Cd+Be)\log\left(a+cx^2\right)}{2c^2}
\end{aligned}$$

```
[Out] -1/2*(A*c-3*C*a)*e^2*x/a/c^2-1/2*(a*B-(A*c-C*a)*x)*(e*x+d)^2/a/c/(c*x^2+a)+
1/2*(a*(A*c-3*C*a)*e^2+c*d*(A*c*d+2*B*a*e+C*a*d))*arctan(x*c^(1/2)/a^(1/2))
/a^(3/2)/c^(5/2)+1/2*e*(B*e+2*C*d)*ln(c*x^2+a)/c^2
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1659, 788, 649, 211, 266}

$$\begin{aligned}
&\int \frac{(d+ex)^2\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^2} \, dx \\
&= \frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(cd(2aBe+aCd+Acd)+ae^2(Ac-3aC)\right)}{2a^{3/2}c^{5/2}} \\
&\quad - \frac{(d+ex)^2(aB-x(Ac-aC))}{2ac\left(a+cx^2\right)} \\
&\quad - \frac{e^2x(Ac-3aC)}{2ac^2} + \frac{e\log\left(a+cx^2\right)(Be+2Cd)}{2c^2}
\end{aligned}$$

```
[In] Int[((d + e*x)^2*(A + B*x + C*x^2))/(a + c*x^2)^2,x]
```

```
[Out] -1/2*((A*c - 3*a*C)*e^2*x)/(a*c^2) - ((a*B - (A*c - a*C)*x)*(d + e*x)^2)/(2
*a*c*(a + c*x^2)) + ((a*(A*c - 3*a*C)*e^2 + c*d*(A*c*d + a*C*d + 2*a*B*e))*
ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(2*a^(3/2)*c^(5/2)) + (e*(2*C*d + B*e)*Log[a +
 c*x^2])/(2*c^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 788

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[e*g*(x/c), x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x
)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

Rule 1659

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c*(p
+ 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\text{integral} = -\frac{(aB-(Ac-aC)x)(d+ex)^2}{2ac\left(a+cx^2\right)} - \frac{\int \frac{(d+ex)(-Acd-aCd-2aBe+(Ac-3aC)ex)}{a+cx^2}\,dx}{2ac}$$

$$= -\frac{(Ac-3aC)e^2x}{2ac^2} - \frac{(aB-(Ac-aC)x)(d+ex)^2}{2ac\,(a+cx^2)} - \frac{\int \frac{-a(Ac-3aC)e^2+cd(-Acd-aCd-2aBe)+c((Ac-3aC)de+e(-Acd-aCd-2aBe))x}{a+cx^2}\,dx}{2ac^2}$$

$$= -\frac{(Ac-3aC)e^2x}{2ac^2} - \frac{(aB-(Ac-aC)x)(d+ex)^2}{2ac\,(a+cx^2)} + \frac{(e(2Cd+Be))\int \frac{x}{a+cx^2}\,dx}{c} + \frac{(a(Ac-3aC)e^2+cd(Acd+aCd+2aBe))\int \frac{1}{a+cx^2}\,dx}{2ac^2}$$

$$= -\frac{(Ac-3aC)e^2x}{2ac^2} - \frac{(aB-(Ac-aC)x)(d+ex)^2}{2ac\,(a+cx^2)} + \frac{(a(Ac-3aC)e^2+cd(Acd+aCd+2aBe))\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}c^{5/2}} + \frac{e(2Cd+Be)\log(a+cx^2)}{2c^2}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.20

$$\int \frac{(d+ex)^2\,(A+Bx+Cx^2)}{(a+cx^2)^2}\,dx$$

$$= \frac{2\sqrt{c}Ce^2x + \frac{\sqrt{c}(Ac^2d^2x+a^2e(2Cd+Be+Cex)-ac(Cd^2x+Ae(2d+ex)+Bd(d+2ex)))}{a(a+cx^2)} + \frac{(Ac(cd^2+ae^2)+a(-3aCe^2+cd(Cd+2Be)))\,\mathrm{arcta}}{a^{3/2}}}{2c^{5/2}}$$

```
[In] Integrate[((d + e*x)^2*(A + B*x + C*x^2))/(a + c*x^2)^2,x]
```

```
[Out] (2*Sqrt[c]*C*e^2*x + (Sqrt[c]*(A*c^2*d^2*x + a^2*e*(2*C*d + B*e + C*e*x) -
a*c*(C*d^2*x + A*e*(2*d + e*x) + B*d*(d + 2*e*x))))/(a*(a + c*x^2)) + ((A*c
*(c*d^2 + a*e^2) + a*(-3*a*C*e^2 + c*d*(C*d + 2*B*e)))*ArcTan[(Sqrt[c]*x)/S
qrt[a]])/a^(3/2) + Sqrt[c]*e*(2*C*d + B*e)*Log[a + c*x^2])/(2*c^(5/2))
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.28

method	result
default	$\frac{e^2Cx}{c^2}+\frac{\frac{-\frac{(Aac\,e^2-A\,c^2d^2+2Bacde-a^2C\,e^2+Cac\,d^2)x}{2a}-Acde+\frac{Ba\,e^2}{2}-\frac{Bc\,d^2}{2}+adeC}{c\,x^2+a}+\frac{(2B\,e^2ac+4acdeC)\ln(c\,x^2+a)}{2c}+\frac{(Aac\,e^2+A\,c^2d^2+2}{2a}}{c^2}$
risch	$\frac{e^2Cx}{c^2}+\frac{-\frac{(Aac\,e^2-A\,c^2d^2+2Bacde-a^2C\,e^2+Cac\,d^2)x}{2a}-Acde+\frac{Ba\,e^2}{2}-\frac{Bc\,d^2}{2}+adeC}{c^2(c\,x^2+a)}+\frac{\ln(A\,a^2c\,e^2+A\,d^2a\,c^2+2a^2cdeB-3C\,a^3e^2+}{}$

```
[In] int((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] e^2*C/c^2*x+1/c^2*((-1/2*(A*a*c*e^2-A*c^2*d^2+2*B*a*c*d*e-C*a^2*e^2+C*a*c*d
^2)/a*x-A*c*d*e+1/2*B*a*e^2-1/2*B*c*d^2+a*d*e*C)/(c*x^2+a)+1/2/a*(1/2*(2*B*
a*c*e^2+4*C*a*c*d*e)/c*ln(c*x^2+a)+(A*a*c*e^2+A*c^2*d^2+2*B*a*c*d*e-3*C*a^2
*e^2+C*a*c*d^2)/(a*c)^(1/2)*arctan(c*x/(a*c)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 304 vs. $2(132) = 264$.

Time = 0.30 (sec) , antiderivative size = 631, normalized size of antiderivative = 4.32

$$\int \frac{(d+ex)^2\,(A+Bx+Cx^2)}{(a+cx^2)^2}\,dx$$

$$= \Bigg[\frac{4\,Ca^2c^2e^2x^3 - 2\,Ba^2c^2d^2 + 2\,Ba^3ce^2 + 4\,(Ca^3c - Aa^2c^2)de - (2\,Ba^2cde + (Ca^2c + Aac^2)d^2 - (3\,Ca^3 -}{}$$

```
[In] integrate((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*C*a^2*c^2*e^2*x^3 - 2*B*a^2*c^2*d^2 + 2*B*a^3*c*e^2 + 4*(C*a^3*c -
A*a^2*c^2)*d*e - (2*B*a^2*c*d*e + (C*a^2*c + A*a*c^2)*d^2 - (3*C*a^3 - A*a^
2*c)*e^2 + (2*B*a*c^2*d*e + (C*a*c^2 + A*c^3)*d^2 - (3*C*a^2*c - A*a*c^2)*e
^2)*x^2)*sqrt(-a*c)*log((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) - 2*(2*B*
a^2*c^2*d*e + (C*a^2*c^2 - A*a*c^3)*d^2 - (3*C*a^3*c - A*a^2*c^2)*e^2)*x +
2*(2*C*a^3*c*d*e + B*a^3*c*e^2 + (2*C*a^2*c^2*d*e + B*a^2*c^2*e^2)*x^2)*log
(c*x^2 + a))/(a^2*c^4*x^2 + a^3*c^3), 1/2*(2*C*a^2*c^2*e^2*x^3 - B*a^2*c^2*
d^2 + B*a^3*c*e^2 + 2*(C*a^3*c - A*a^2*c^2)*d*e + (2*B*a^2*c*d*e + (C*a^2*c
 + A*a*c^2)*d^2 - (3*C*a^3 - A*a^2*c)*e^2 + (2*B*a*c^2*d*e + (C*a*c^2 + A*c
^3)*d^2 - (3*C*a^2*c - A*a*c^2)*e^2)*x^2)*sqrt(a*c)*arctan(sqrt(a*c)*x/a) -
 (2*B*a^2*c^2*d*e + (C*a^2*c^2 - A*a*c^3)*d^2 - (3*C*a^3*c - A*a^2*c^2)*e^2
)*x + (2*C*a^3*c*d*e + B*a^3*c*e^2 + (2*C*a^2*c^2*d*e + B*a^2*c^2*e^2)*x^2)
*log(c*x^2 + a))/(a^2*c^4*x^2 + a^3*c^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 593 vs. $2(138) = 276$.

Time = 6.04 (sec) , antiderivative size = 593, normalized size of antiderivative = 4.06

$$\int \frac{(d+ex)^2\,(A+Bx+Cx^2)}{(a+cx^2)^2}\,dx = \frac{Ce^2x}{c^2} + \left(\frac{e(Be+2Cd)}{2c^2} - \frac{\sqrt{-a^3c^5}(-Aace^2 - Ac^2d^2 - 2Bacde + 3Ca^2e^2 - Cacd^2)}{4a^3c^5}\right)\log\left(x + \frac{2Ba^2e^2 + 4Ca^2de - 4a^2c^2\left(\frac{e(Be+}{2c}\right.}{-Aace^2 - Ac^2d}\right.$$
$$+ \left(\frac{e(Be+2Cd)}{2c^2} + \frac{\sqrt{-a^3c^5}(-Aace^2 - Ac^2d^2 - 2Bacde + 3Ca^2e^2 - Cacd^2)}{4a^3c^5}\right)\log\left(x + \frac{2Ba^2e^2 + 4Ca^2de - 4a^2c^2\left(\frac{e(Be+}{2c}\right.}{-Aace^2 - Ac^2d}\right.$$
$$+ \frac{-2Aacde + Ba^2e^2 - Bacd^2 + 2Ca^2de + x(-Aace^2 + Ac^2d^2 - 2Bacde + Ca^2e^2 - Cacd^2)}{2a^2c^2 + 2ac^3x^2}$$

```
[In] integrate((e*x+d)**2*(C*x**2+B*x+A)/(c*x**2+a)**2,x)
```

```
[Out] C*e**2*x/c**2 + (e*(B*e + 2*C*d)/(2*c**2) - sqrt(-a**3*c**5)*(-A*a*c*e**2 -
 A*c**2*d**2 - 2*B*a*c*d*e + 3*C*a**2*e**2 - C*a*c*d**2)/(4*a**3*c**5))*log
(x + (2*B*a**2*e**2 + 4*C*a**2*d*e - 4*a**2*c**2*(e*(B*e + 2*C*d)/(2*c**2)
- sqrt(-a**3*c**5)*(-A*a*c*e**2 - A*c**2*d**2 - 2*B*a*c*d*e + 3*C*a**2*e**2
 - C*a*c*d**2)/(4*a**3*c**5)))/(-A*a*c*e**2 - A*c**2*d**2 - 2*B*a*c*d*e + 3
*C*a**2*e**2 - C*a*c*d**2)) + (e*(B*e + 2*C*d)/(2*c**2) + sqrt(-a**3*c**5)*
(-A*a*c*e**2 - A*c**2*d**2 - 2*B*a*c*d*e + 3*C*a**2*e**2 - C*a*c*d**2)/(4*a
**3*c**5))*log(x + (2*B*a**2*e**2 + 4*C*a**2*d*e - 4*a**2*c**2*(e*(B*e + 2*
C*d)/(2*c**2) + sqrt(-a**3*c**5)*(-A*a*c*e**2 - A*c**2*d**2 - 2*B*a*c*d*e +
 3*C*a**2*e**2 - C*a*c*d**2)/(4*a**3*c**5)))/(-A*a*c*e**2 - A*c**2*d**2 - 2
*B*a*c*d*e + 3*C*a**2*e**2 - C*a*c*d**2)) + (-2*A*a*c*d*e + B*a**2*e**2 - B
*a*c*d**2 + 2*C*a**2*d*e + x*(-A*a*c*e**2 + A*c**2*d**2 - 2*B*a*c*d*e + C*a
**2*e**2 - C*a*c*d**2))/(2*a**2*c**2 + 2*a*c**3*x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.29

$$\int \frac{(d+ex)^2\,(A+Bx+Cx^2)}{(a+cx^2)^2}\,dx$$
$$= \frac{Ce^2x}{c^2} - \frac{Bacd^2 - Ba^2e^2 - 2\,(Ca^2 - Aac)de + (2\,Bacde + (Cac - Ac^2)d^2 - (Ca^2 - Aac)e^2)x}{2\,(ac^3x^2 + a^2c^2)} + \frac{(2\,Cde + Be^2)\log\left(cx^2 + a\right)}{2\,c^2} + \frac{(2\,Bacde + (Cac + Ac^2)d^2 - (3\,Ca^2 - Aac)e^2)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\,\sqrt{ac}ac^2}$$

```
[In] integrate((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] C*e^2*x/c^2 - 1/2*(B*a*c*d^2 - B*a^2*e^2 - 2*(C*a^2 - A*a*c)*d*e + (2*B*a*c
*d*e + (C*a*c - A*c^2)*d^2 - (C*a^2 - A*a*c)*e^2)*x)/(a*c^3*x^2 + a^2*c^2)
+ 1/2*(2*C*d*e + B*e^2)*log(c*x^2 + a)/c^2 + 1/2*(2*B*a*c*d*e + (C*a*c + A*
c^2)*d^2 - (3*C*a^2 - A*a*c)*e^2)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a*c^2)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.27

$$\int \frac{(d+ex)^2\,(A+Bx+Cx^2)}{(a+cx^2)^2}\,dx = \frac{Ce^2x}{c^2} + \frac{(2\,Cde + Be^2)\log\left(cx^2 + a\right)}{2\,c^2} + \frac{(Cacd^2 + Ac^2d^2 + 2\,Bacde - 3\,Ca^2e^2 + Aace^2)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\,\sqrt{ac}ac^2} - \frac{Bacd^2 - 2\,Ca^2de + 2\,Aacde - Ba^2e^2 + (Cacd^2 - Ac^2d^2 + 2\,Bacde - Ca^2e^2 + Aace^2)x}{2\,(cx^2 + a)ac^2}$$

```
[In] integrate((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a)^2,x, algorithm="giac")
```

```
[Out] C*e^2*x/c^2 + 1/2*(2*C*d*e + B*e^2)*log(c*x^2 + a)/c^2 + 1/2*(C*a*c*d^2 + A
*c^2*d^2 + 2*B*a*c*d*e - 3*C*a^2*e^2 + A*a*c*e^2)*arctan(c*x/sqrt(a*c))/(sq
rt(a*c)*a*c^2) - 1/2*(B*a*c*d^2 - 2*C*a^2*d*e + 2*A*a*c*d*e - B*a^2*e^2 + (
C*a*c*d^2 - A*c^2*d^2 + 2*B*a*c*d*e - C*a^2*e^2 + A*a*c*e^2)*x)/((c*x^2 + a
)*a*c^2)
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.34

$$\int \frac{(d+ex)^2 (A+Bx+Cx^2)}{(a+cx^2)^2}\,dx$$

$$= \frac{C\,e^2\,x}{c^2} - \frac{\frac{x\,(-C\,a^2\,e^2+C\,a\,c\,d^2+2\,B\,a\,c\,d\,e+A\,a\,c\,e^2-A\,c^2\,d^2)}{2\,a} - \frac{B\,a\,e^2}{2} + \frac{B\,c\,d^2}{2} + A\,c\,d\,e - C\,a\,d\,e}{c^3\,x^2 + a\,c^2}$$

$$+ \frac{\operatorname{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\,(-3\,C\,a^2\,e^2 + C\,a\,c\,d^2 + 2\,B\,a\,c\,d\,e + A\,a\,c\,e^2 + A\,c^2\,d^2)}{2\,a^{3/2}\,c^{5/2}}$$

$$+ \frac{\ln(c\,x^2 + a)\,(16\,B\,a^3\,c^3\,e^2 + 32\,C\,d\,a^3\,c^3\,e)}{32\,a^3\,c^5}$$

```
[In] int(((d + e*x)^2*(A + B*x + C*x^2))/(a + c*x^2)^2,x)
```

```
[Out] (C*e^2*x)/c^2 - ((x*(A*a*c*e^2 - C*a^2*e^2 - A*c^2*d^2 + C*a*c*d^2 + 2*B*a*
c*d*e))/(2*a) - (B*a*e^2)/2 + (B*c*d^2)/2 + A*c*d*e - C*a*d*e)/(a*c^2 + c^3
*x^2) + (atan((c^(1/2)*x)/a^(1/2))*(A*c^2*d^2 - 3*C*a^2*e^2 + A*a*c*e^2 + C
*a*c*d^2 + 2*B*a*c*d*e))/(2*a^(3/2)*c^(5/2)) + (log(a + c*x^2)*(16*B*a^3*c^
3*e^2 + 32*C*a^3*c^3*d*e))/(32*a^3*c^5)
```

3.52 $\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 97

$$\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^2}\,dx = -\frac{(aB-(Ac-aC)x)(d+ex)}{2ac\left(a+cx^2\right)} + \frac{(Acd+aCd+aBe)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}c^{3/2}} + \frac{Ce\log\left(a+cx^2\right)}{2c^2}$$

```
[Out] -1/2*(a*B-(A*c-C*a)*x)*(e*x+d)/a/c/(c*x^2+a)+1/2*(A*c*d+B*a*e+C*a*d)*arctan
(x*c^(1/2)/a^(1/2))/a^(3/2)/c^(3/2)+1/2*C*e*ln(c*x^2+a)/c^2
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1659, 649, 211, 266}

$$\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^2}\,dx = \frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(aBe+aCd+Acd)}{2a^{3/2}c^{3/2}} - \frac{(d+ex)(aB-x(Ac-aC))}{2ac\left(a+cx^2\right)} + \frac{Ce\log\left(a+cx^2\right)}{2c^2}$$

```
[In] Int[((d + e*x)*(A + B*x + C*x^2))/(a + c*x^2)^2,x]
```

```
[Out] -1/2*((a*B - (A*c - a*C)*x)*(d + e*x))/(a*c*(a + c*x^2)) + ((A*c*d + a*C*d
+ a*B*e)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(2*a^(3/2)*c^(3/2)) + (C*e*Log[a + c*
x^2])/(2*c^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1659

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c*(p
+ 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(aB-(Ac-aC)x)(d+ex)}{2ac\,(a+cx^2)}-\frac{\int\frac{-Acd-a(Cd+Be)-2aCex}{a+cx^2}\,dx}{2ac}\\
&= -\frac{(aB-(Ac-aC)x)(d+ex)}{2ac\,(a+cx^2)}+\frac{(Ce)\int\frac{x}{a+cx^2}\,dx}{c}+\frac{(Acd+aCd+aBe)\int\frac{1}{a+cx^2}\,dx}{2ac}\\
&= -\frac{(aB-(Ac-aC)x)(d+ex)}{2ac\,(a+cx^2)}+\frac{(Acd+aCd+aBe)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{2a^{3/2}c^{3/2}}+\frac{Ce\log\left(a+cx^2\right)}{2c^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.05

$$\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^2}\,dx$$
$$=\frac{\frac{a^2Ce+Ac^2dx-ac(Ae+Cdx+B(d+ex))}{a(a+cx^2)}+\frac{\sqrt{c}(Acd+aCd+aBe)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{a^{3/2}}+Ce\log\left(a+cx^2\right)}{2c^2}$$

```
[In] Integrate[((d + e*x)*(A + B*x + C*x^2))/(a + c*x^2)^2,x]
```

```
[Out] ((a^2*C*e + A*c^2*d*x - a*c*(A*e + C*d*x + B*(d + e*x)))/(a*(a + c*x^2)) +
(Sqrt[c]*(A*c*d + a*C*d + a*B*e)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/a^(3/2) + C*e
*Log[a + c*x^2])/(2*c^2)
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.11

method	result
default	$\frac{\frac{(Acd-Bae-Cad)x}{2ac}-\frac{Ace+Bcd-Cae}{2c^2}}{c\,x^2+a}+\frac{\frac{Cae\ln\left(c\,x^2+a\right)}{c}+\frac{(Acd+Bae+Cad)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}}}{2ac}$
risch	$\frac{\frac{(Acd-Bae-Cad)x}{2ac}-\frac{Ace+Bcd-Cae}{2c^2}}{c\,x^2+a}+\frac{\ln\left(dAac+Be\,a^2+a^2dC-\sqrt{-ac(Acd+Bae+Cad)^2}\,x\right)eC}{2c^2}+$ $\ln\Bigl(dAac+Be\,a^2+a^2dC-\sqrt{-ac(}$

```
[In] int((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (1/2*(A*c*d-B*a*e-C*a*d)/a/c*x-1/2*(A*c*e+B*c*d-C*a*e)/c^2)/(c*x^2+a)+1/2/a
/c*(C*a*e/c*ln(c*x^2+a)+(A*c*d+B*a*e+C*a*d)/(a*c)^(1/2)*arctan(c*x/(a*c)^(1
/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 337, normalized size of antiderivative = 3.47

$$\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^2}\,dx$$

$$= \left[-\frac{2\,Ba^2cd + (Ba^2e + (Bace + (Cac + Ac^2)d)x^2 + (Ca^2 + Aac)d)\sqrt{-ac}\log\left(\frac{cx^2-2\sqrt{-ac}x-a}{cx^2+a}\right) - 2\,(Ca^3 -}{4\,(a^2c^3x^2 + a^3c^2)}\right.$$

$$-\frac{Ba^2cd - (Ba^2e + (Bace + (Cac + Ac^2)d)x^2 + (Ca^2 + Aac)d)\sqrt{ac}\arctan\left(\frac{\sqrt{ac}x}{a}\right) - (Ca^3 - Aa^2c)e +}{2\,(a^2c^3x^2 + a^3c^2)}$$

```
[In] integrate((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*B*a^2*c*d + (B*a^2*e + (B*a*c*e + (C*a*c + A*c^2)*d)*x^2 + (C*a^2
+ A*a*c)*d)*sqrt(-a*c)*log((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) - 2*(C
*a^3 - A*a^2*c)*e + 2*(B*a^2*c*e + (C*a^2*c - A*a*c^2)*d)*x - 2*(C*a^2*c*e*
x^2 + C*a^3*e)*log(c*x^2 + a))/(a^2*c^3*x^2 + a^3*c^2), -1/2*(B*a^2*c*d - (
B*a^2*e + (B*a*c*e + (C*a*c + A*c^2)*d)*x^2 + (C*a^2 + A*a*c)*d)*sqrt(a*c)*
arctan(sqrt(a*c)*x/a) - (C*a^3 - A*a^2*c)*e + (B*a^2*c*e + (C*a^2*c - A*a*c
^2)*d)*x - (C*a^2*c*e*x^2 + C*a^3*e)*log(c*x^2 + a))/(a^2*c^3*x^2 + a^3*c^2
)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 318 vs. 2(90) = 180.

Time = 2.23 (sec) , antiderivative size = 318, normalized size of antiderivative = 3.28

$$\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^2}\,dx = \left(\frac{Ce}{2c^2} - \frac{\sqrt{-a^3c^5}(Acd+Bae+Cad)}{4a^3c^4}\right)\log\left(x + \frac{-2Ca^2e + 4a^2c^2\left(\frac{Ce}{2c^2} - \frac{\sqrt{-a^3c^5}(Acd+Bae+Cad)}{4a^3c^4}\right)}{Ac^2d + Bace + Cacd}\right) + \left(\frac{Ce}{2c^2} + \frac{\sqrt{-a^3c^5}(Acd+Bae+Cad)}{4a^3c^4}\right)\log\left(x + \frac{-2Ca^2e + 4a^2c^2\left(\frac{Ce}{2c^2} + \frac{\sqrt{-a^3c^5}(Acd+Bae+Cad)}{4a^3c^4}\right)}{Ac^2d + Bace + Cacd}\right) + \frac{-Aace - Bacd + Ca^2e + x(Ac^2d - Bace - Cacd)}{2a^2c^2 + 2ac^3x^2}$$

```
[In] integrate((e*x+d)*(C*x**2+B*x+A)/(c*x**2+a)**2,x)
```

```
[Out] (C*e/(2*c**2) - sqrt(-a**3*c**5)*(A*c*d + B*a*e + C*a*d)/(4*a**3*c**4))*log
(x + (-2*C*a**2*e + 4*a**2*c**2*(C*e/(2*c**2) - sqrt(-a**3*c**5)*(A*c*d + B
*a*e + C*a*d)/(4*a**3*c**4)))/(A*c**2*d + B*a*c*e + C*a*c*d)) + (C*e/(2*c**
2) + sqrt(-a**3*c**5)*(A*c*d + B*a*e + C*a*d)/(4*a**3*c**4))*log(x + (-2*C*
a**2*e + 4*a**2*c**2*(C*e/(2*c**2) + sqrt(-a**3*c**5)*(A*c*d + B*a*e + C*a*
d)/(4*a**3*c**4)))/(A*c**2*d + B*a*c*e + C*a*c*d)) + (-A*a*c*e - B*a*c*d +
C*a**2*e + x*(A*c**2*d - B*a*c*e - C*a*c*d))/(2*a**2*c**2 + 2*a*c**3*x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.16

$$\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^2}\,dx = \frac{Ce\log\left(cx^2+a\right)}{2\,c^2} - \frac{Bacd-(Ca^2-Aac)e+(Bace+(Cac-Ac^2)d)x}{2\left(ac^3x^2+a^2c^2\right)} + \frac{(Bae+(Ca+Ac)d)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\sqrt{ac}ac}$$

```
[In] integrate((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*C*e*log(c*x^2 + a)/c^2 - 1/2*(B*a*c*d - (C*a^2 - A*a*c)*e + (B*a*c*e +
(C*a*c - A*c^2)*d)*x)/(a*c^3*x^2 + a^2*c^2) + 1/2*(B*a*e + (C*a + A*c)*d)*a
rctan(c*x/sqrt(a*c))/(sqrt(a*c)*a*c)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.10

$$\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^2}\,dx = \frac{Ce\log\left(cx^2+a\right)}{2\,c^2} + \frac{(Cad+Acd+Bae)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\sqrt{ac}ac} - \frac{(Cad-Acd+Bae)x+\frac{Bacd-Ca^2e+Aace}{c}}{2\left(cx^2+a\right)ac}$$

```
[In] integrate((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*C*e*log(c*x^2 + a)/c^2 + 1/2*(C*a*d + A*c*d + B*a*e)*arctan(c*x/sqrt(a*
c))/(sqrt(a*c)*a*c) - 1/2*((C*a*d - A*c*d + B*a*e)*x + (B*a*c*d - C*a^2*e +
 A*a*c*e)/c)/((c*x^2 + a)*a*c)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.97

$$\begin{aligned}\int \frac{(d+ex)\,(A+Bx+Cx^2)}{(a+cx^2)^2}\,dx = {} & \frac{C\,e\,\ln\left(c\,x^2+a\right)}{2\,c^2}-\frac{B\,d}{2\,\left(c^2\,x^2+a\,c\right)}-\frac{B\,e\,x}{2\,\left(c^2\,x^2+a\,c\right)}\\ & -\frac{C\,d\,x}{2\,\left(c^2\,x^2+a\,c\right)}-\frac{A\,e}{2\,\left(c^2\,x^2+a\,c\right)}+\frac{C\,a\,e}{2\,\left(c^3\,x^2+a\,c^2\right)}\\ & +\frac{A\,d\,x}{2\,\left(a^2+c\,a\,x^2\right)}+\frac{A\,d\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{2\,a^{3/2}\,\sqrt{c}}\\ & +\frac{B\,e\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{2\,\sqrt{a}\,c^{3/2}}+\frac{C\,d\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{2\,\sqrt{a}\,c^{3/2}}\end{aligned}$$

```
[In] int(((d + e*x)*(A + B*x + C*x^2))/(a + c*x^2)^2,x)
```

```
[Out] (C*e*log(a + c*x^2))/(2*c^2) - (B*d)/(2*(a*c + c^2*x^2)) - (B*e*x)/(2*(a*c
+ c^2*x^2)) - (C*d*x)/(2*(a*c + c^2*x^2)) - (A*e)/(2*(a*c + c^2*x^2)) + (C*
a*e)/(2*(a*c^2 + c^3*x^2)) + (A*d*x)/(2*(a^2 + a*c*x^2)) + (A*d*atan((c^(1/
2)*x)/a^(1/2)))/(2*a^(3/2)*c^(1/2)) + (B*e*atan((c^(1/2)*x)/a^(1/2)))/(2*a^
(1/2)*c^(3/2)) + (C*d*atan((c^(1/2)*x)/a^(1/2)))/(2*a^(1/2)*c^(3/2))
```

3.53 $\int \frac{A+Bx+Cx^2}{\left(a+cx^2\right)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 69

$$\int \frac{A+Bx+Cx^2}{\left(a+cx^2\right)^2}\,dx = -\frac{aB-(Ac-aC)x}{2ac\left(a+cx^2\right)}+\frac{(Ac+aC)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}c^{3/2}}$$

```
[Out] 1/2*(-a*B+(A*c-C*a)*x)/a/c/(c*x^2+a)+1/2*(A*c+C*a)*arctan(x*c^(1/2)/a^(1/2)
)/a^(3/2)/c^(3/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1828, 12, 211}

$$\int \frac{A+Bx+Cx^2}{\left(a+cx^2\right)^2}\,dx = \frac{(aC+Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}c^{3/2}}-\frac{aB-x(Ac-aC)}{2ac\left(a+cx^2\right)}$$

```
[In] Int[(A + B*x + C*x^2)/(a + c*x^2)^2,x]
```

```
[Out] -1/2*(a*B - (A*c - a*C)*x)/(a*c*(a + c*x^2)) + ((A*c + a*C)*ArcTan[(Sqrt[c]
*x)/Sqrt[a]])/(2*a^(3/2)*c^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{aB-(Ac-aC)x}{2ac\,(a+cx^2)}-\frac{\int \frac{-A-\frac{aC}{c}}{a+cx^2}\,dx}{2a}\\
&= -\frac{aB-(Ac-aC)x}{2ac\,(a+cx^2)}+\frac{(Ac+aC)\int \frac{1}{a+cx^2}\,dx}{2ac}\\
&= -\frac{aB-(Ac-aC)x}{2ac\,(a+cx^2)}+\frac{(Ac+aC)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}c^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.99

$$\int \frac{A+Bx+Cx^2}{(a+cx^2)^2}\,dx = \frac{-aB+Acx-aCx}{2ac\,(a+cx^2)}+\frac{(Ac+aC)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}c^{3/2}}$$

```
[In] Integrate[(A + B*x + C*x^2)/(a + c*x^2)^2,x]
[Out] (-(a*B) + A*c*x - a*C*x)/(2*a*c*(a + c*x^2)) + ((A*c + a*C)*ArcTan[(Sqrt[c]
*x)/Sqrt[a]])/(2*a^(3/2)*c^(3/2))
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{\frac{(Ac-Ca)x}{2ac}-\frac{B}{2c}}{c\,x^{2}+a}+\frac{(Ac+Ca)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2ac\sqrt{ac}}$	65
risch	$\frac{\frac{(Ac-Ca)x}{2ac}-\frac{B}{2c}}{c\,x^{2}+a}-\frac{A\ln\left(cx+\sqrt{-ac}\right)}{4\sqrt{-ac}\,a}-\frac{\ln\left(cx+\sqrt{-ac}\right)C}{4\sqrt{-ac}\,c}+\frac{A\ln\left(-cx+\sqrt{-ac}\right)}{4\sqrt{-ac}\,a}+\frac{\ln\left(-cx+\sqrt{-ac}\right)C}{4\sqrt{-ac}\,c}$	130

```
[In] int((C*x^2+B*x+A)/(c*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (1/2*(A*c-C*a)/a/c*x-1/2*B/c)/(c*x^2+a)+1/2*(A*c+C*a)/a/c/(a*c)^(1/2)*arcta
n(c*x/(a*c)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 195, normalized size of antiderivative = 2.83

$$\int \frac{A+Bx+Cx^2}{(a+cx^2)^2}\,dx = \left[-\frac{2\,Ba^2c+(Ca^2+Aac+(Cac+Ac^2)x^2)\sqrt{-ac}\log\left(\frac{cx^2-2\,\sqrt{-ac}x-a}{cx^2+a}\right)+2\,(Ca^2c-Aac^2)x}{4\,(a^2c^3x^2+a^3c^2)},\right.$$
$$\left.-\frac{Ba^2c-(Ca^2+Aac+(Cac+Ac^2)x^2)\sqrt{ac}\arctan\left(\frac{\sqrt{ac}x}{a}\right)+(Ca^2c-Aac^2)x}{2\,(a^2c^3x^2+a^3c^2)}\right]$$

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*B*a^2*c + (C*a^2 + A*a*c + (C*a*c + A*c^2)*x^2)*sqrt(-a*c)*log((c*
x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) + 2*(C*a^2*c - A*a*c^2)*x)/(a^2*c^3*
x^2 + a^3*c^2), -1/2*(B*a^2*c - (C*a^2 + A*a*c + (C*a*c + A*c^2)*x^2)*sqrt(
a*c)*arctan(sqrt(a*c)*x/a) + (C*a^2*c - A*a*c^2)*x)/(a^2*c^3*x^2 + a^3*c^2)
]
```

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.68

$$\int \frac{A+Bx+Cx^2}{(a+cx^2)^2}\,dx = -\frac{\sqrt{-\frac{1}{a^3c^3}}(Ac+Ca)\log\left(-a^2c\sqrt{-\frac{1}{a^3c^3}}+x\right)}{4}$$
$$+\frac{\sqrt{-\frac{1}{a^3c^3}}(Ac+Ca)\log\left(a^2c\sqrt{-\frac{1}{a^3c^3}}+x\right)}{4}+\frac{-Ba+x(Ac-Ca)}{2a^2c+2ac^2x^2}$$

```
[In] integrate((C*x**2+B*x+A)/(c*x**2+a)**2,x)
```

```
[Out] -sqrt(-1/(a**3*c**3))*(A*c + C*a)*log(-a**2*c*sqrt(-1/(a**3*c**3)) + x)/4 +
 sqrt(-1/(a**3*c**3))*(A*c + C*a)*log(a**2*c*sqrt(-1/(a**3*c**3)) + x)/4 +
(-B*a + x*(A*c - C*a))/(2*a**2*c + 2*a*c**2*x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.90

$$\int \frac{A+Bx+Cx^2}{\left(a+cx^2\right)^2}\,dx=-\frac{Ba+(Ca-Ac)x}{2\,(ac^2x^2+a^2c)}+\frac{(Ca+Ac)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\,\sqrt{ac}ac}$$

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(B*a + (C*a - A*c)*x)/(a*c^2*x^2 + a^2*c) + 1/2*(C*a + A*c)*arctan(c*x
/sqrt(a*c))/(sqrt(a*c)*a*c)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.87

$$\int \frac{A+Bx+Cx^2}{\left(a+cx^2\right)^2}\,dx=\frac{(Ca+Ac)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\,\sqrt{ac}ac}-\frac{Cax-Acx+Ba}{2\,(cx^2+a)ac}$$

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(C*a + A*c)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a*c) - 1/2*(C*a*x - A*c*x
+ B*a)/((c*x^2 + a)*a*c)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.87

$$\int \frac{A+Bx+Cx^2}{\left(a+cx^2\right)^2}\,dx=\frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\,\left(A\,c+C\,a\right)}{2\,a^{3/2}\,c^{3/2}}-\frac{\frac{B}{2\,c}-\frac{x\,\left(A\,c-C\,a\right)}{2\,a\,c}}{c\,x^2+a}$$

```
[In] int((A + B*x + C*x^2)/(a + c*x^2)^2,x)
```

```
[Out] (atan((c^(1/2)*x)/a^(1/2))*(A*c + C*a))/(2*a^(3/2)*c^(3/2)) - (B/(2*c) - (x
*(A*c - C*a))/(2*a*c))/(a + c*x^2)
```

3.54 $\int \frac{A+Bx+Cx^2}{(d+ex)\left(a+cx^2\right)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 226

$$
\begin{aligned}
\int \frac{A+Bx+Cx^2}{(d+ex)\left(a+cx^2\right)^2}\,dx = &-\frac{a(Bcd-Ace+aCe)-c(Acd-aCd+aBe)x}{2ac\left(cd^2+ae^2\right)\left(a+cx^2\right)} \\
&+\frac{\left(a(Cd-Be)\left(cd^2-ae^2\right)+Acd\left(cd^2+3ae^2\right)\right)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{c}\left(cd^2+ae^2\right)^2} \\
&+\frac{e\left(Cd^2-Bde+Ae^2\right)\log(d+ex)}{\left(cd^2+ae^2\right)^2} \\
&-\frac{e\left(Cd^2-Bde+Ae^2\right)\log\left(a+cx^2\right)}{2\left(cd^2+ae^2\right)^2}
\end{aligned}
$$

```
[Out] 1/2*(-a*(-A*c*e+B*c*d+C*a*e)+c*(A*c*d+B*a*e-C*a*d)*x)/a/c/(a*e^2+c*d^2)/(c*
x^2+a)+e*(A*e^2-B*d*e+C*d^2)*ln(e*x+d)/(a*e^2+c*d^2)^2-1/2*e*(A*e^2-B*d*e+C
*d^2)*ln(c*x^2+a)/(a*e^2+c*d^2)^2+1/2*(a*(-B*e+C*d)*(-a*e^2+c*d^2)+A*c*d*(3
*a*e^2+c*d^2))*arctan(x*c^(1/2)/a^(1/2))/a^(3/2)/(a*e^2+c*d^2)^2/c^(1/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used

$= \{1661, 815, 649, 211, 266\}$

$$\int \frac{A+Bx+Cx^2}{(d+ex)\left(a+cx^2\right)^2}\,dx = \frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(Acd(3ae^2+cd^2)+a(cd^2-ae^2)\left(Cd-Be\right)\right)}{2a^{3/2}\sqrt{c}\left(ae^2+cd^2\right)^2} - \frac{a(aCe-Ace+Bcd)-cx(aBe-aCd+Acd)}{2ac\left(a+cx^2\right)\left(ae^2+cd^2\right)} - \frac{e\log\left(a+cx^2\right)\left(Ae^2-Bde+Cd^2\right)}{2\left(ae^2+cd^2\right)^2} + \frac{e\log(d+ex)\left(Ae^2-Bde+Cd^2\right)}{\left(ae^2+cd^2\right)^2}$$

```
[In] Int[(A + B*x + C*x^2)/((d + e*x)*(a + c*x^2)^2),x]
```

```
[Out] -1/2*(a*(B*c*d - A*c*e + a*C*e) - c*(A*c*d - a*C*d + a*B*e)*x)/(a*c*(c*d^2
+ a*e^2)*(a + c*x^2)) + ((a*(C*d - B*e)*(c*d^2 - a*e^2) + A*c*d*(c*d^2 + 3*
a*e^2))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(2*a^(3/2)*Sqrt[c]*(c*d^2 + a*e^2)^2)
+ (e*(C*d^2 - B*d*e + A*e^2)*Log[d + e*x])/(c*d^2 + a*e^2)^2 - (e*(C*d^2 -
B*d*e + A*e^2)*Log[a + c*x^2])/(2*(c*d^2 + a*e^2)^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
```

```
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{a(Bcd-Ace+aCe)-c(Acd-aCd+aBe)x}{2ac\left(cd^2+ae^2\right)\left(a+cx^2\right)} - \frac{\int \frac{-\frac{c\left(ad(Cd-Be)+A\left(cd^2+2ae^2\right)\right)}{cd^2+ae^2}-\frac{ce(Acd-aCd+aBe)x}{cd^2+ae^2}}{(d+ex)\left(a+cx^2\right)}\,dx}{2ac}\\
&= -\frac{a(Bcd-Ace+aCe)-c(Acd-aCd+aBe)x}{2ac\left(cd^2+ae^2\right)\left(a+cx^2\right)} - \frac{\int \left(-\frac{2ace^2\left(Cd^2-Bde+Ae^2\right)}{\left(cd^2+ae^2\right)^2(d+ex)}+\frac{c\left(-a(Cd-Be)\left(cd^2-ae^2\right)-Acd\left(cd^2+3ae^2\right)+2ace\left(Cd^2-Bde+Ae^2\right)x\right)}{\left(cd^2+ae^2\right)^2\left(a+cx^2\right)}\right)dx}{2ac}\\
&= -\frac{a(Bcd-Ace+aCe)-c(Acd-aCd+aBe)x}{2ac\left(cd^2+ae^2\right)\left(a+cx^2\right)} + \frac{e\left(Cd^2-Bde+Ae^2\right)\log(d+ex)}{\left(cd^2+ae^2\right)^2} - \frac{\int \frac{-a(Cd-Be)\left(cd^2-ae^2\right)-Acd\left(cd^2+3ae^2\right)+2ace\left(Cd^2-Bde+Ae^2\right)x}{a+cx^2}\,dx}{2a\left(cd^2+ae^2\right)^2}\\
&= -\frac{a(Bcd-Ace+aCe)-c(Acd-aCd+aBe)x}{2ac\left(cd^2+ae^2\right)\left(a+cx^2\right)} + \frac{e\left(Cd^2-Bde+Ae^2\right)\log(d+ex)}{\left(cd^2+ae^2\right)^2} - \frac{\left(ce\left(Cd^2-Bde+Ae^2\right)\right)\int \frac{x}{a+cx^2}\,dx}{\left(cd^2+ae^2\right)^2} + \frac{\left(a(Cd-Be)\left(cd^2-ae^2\right)+Acd\left(cd^2+3ae^2\right)\right)\int \frac{1}{a+cx^2}\,dx}{2a\left(cd^2+ae^2\right)^2}\\
&= -\frac{a(Bcd-Ace+aCe)-c(Acd-aCd+aBe)x}{2ac\left(cd^2+ae^2\right)\left(a+cx^2\right)} + \frac{\left(a(Cd-Be)\left(cd^2-ae^2\right)+Acd\left(cd^2+3ae^2\right)\right)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{c}\left(cd^2+ae^2\right)^2} + \frac{e\left(Cd^2-Bde+Ae^2\right)\log(d+ex)}{\left(cd^2+ae^2\right)^2} - \frac{e\left(Cd^2-Bde+Ae^2\right)\log\left(a+cx^2\right)}{2\left(cd^2+ae^2\right)^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.86

$$\int \frac{A+Bx+Cx^2}{(d+ex)\left(a+cx^2\right)^2}\,dx$$
$$=\frac{\frac{\left(cd^2+ae^2\right)\left(-a^2Ce+Ac^2dx+ac(-Bd+Ae-Cdx+Bex)\right)}{ac(a+cx^2)}+\frac{\left(a(Cd-Be)\left(cd^2-ae^2\right)+Acd\left(cd^2+3ae^2\right)\right)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{c}}+2e(Cd^2+e(-Bc}{2\left(cd^2+ae^2\right)^2}$$

```
[In] Integrate[(A + B*x + C*x^2)/((d + e*x)*(a + c*x^2)^2),x]
```

```
[Out] (((c*d^2 + a*e^2)*(-(a^2*C*e) + A*c^2*d*x + a*c*(-(B*d) + A*e - C*d*x + B*e
*x)))/(a*c*(a + c*x^2)) + ((a*(C*d - B*e)*(c*d^2 - a*e^2) + A*c*d*(c*d^2 +
3*a*e^2))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(a^(3/2)*Sqrt[c]) + 2*e*(C*d^2 + e*(
-(B*d) + A*e))*Log[d + e*x] - e*(C*d^2 + e*(-(B*d) + A*e))*Log[a + c*x^2])/
(2*(c*d^2 + a*e^2)^2)
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.30

method	result
default	$\frac{\frac{\frac{\left(Aacd\,e^2+A\,d^3c^2+a^2B\,e^3+Bac\,d^2e-C\,a^2d\,e^2-Cac\,d^3\right)x}{2a}+\frac{Aac\,e^3+A\,c^2d^2e-Bacd\,e^2-B\,c^2d^3-C\,a^2e^3-Cac\,d^2e}{2c}}{c\,x^2+a}+\frac{\frac{\left(-2Aac\,e^3+2Bacd\,e^2-2Cac}{2c}}{\left(e^2a+c\,d^2\right)^2}$
risch	Expression too large to display

```
[In] int((C*x^2+B*x+A)/(e*x+d)/(c*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/(a*e^2+c*d^2)^2*((1/2*(A*a*c*d*e^2+A*c^2*d^3+B*a^2*e^3+B*a*c*d^2*e-C*a^2*
d*e^2-C*a*c*d^3)/a*x+1/2*(A*a*c*e^3+A*c^2*d^2*e-B*a*c*d*e^2-B*c^2*d^3-C*a^2
*e^3-C*a*c*d^2*e)/c)/(c*x^2+a)+1/2/a*(1/2*(-2*A*a*c*e^3+2*B*a*c*d*e^2-2*C*a
*c*d^2*e)/c*ln(c*x^2+a)+(3*A*a*c*d*e^2+A*c^2*d^3+B*a^2*e^3-B*a*c*d^2*e-C*a^
2*d*e^2+C*a*c*d^3)/(a*c)^(1/2)*arctan(c*x/(a*c)^(1/2))))+e*(A*e^2-B*d*e+C*d
^2)*ln(e*x+d)/(a*e^2+c*d^2)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 499 vs. $2(212) = 424$.

Time = 21.75 (sec) , antiderivative size = 1024, normalized size of antiderivative = 4.53

$$\int \frac{A+Bx+Cx^2}{(d+ex)\left(a+cx^2\right)^2}\,dx$$

$$= \left[-\frac{2\,Ba^2c^2d^3 + 2\,Ba^3cde^2 + 2\,(Ca^3c - Aa^2c^2)d^2e + 2\,(Ca^4 - Aa^3c)e^3 - (Ba^2cd^2e - Ba^3e^3 - (Ca^2c + Aa}{}\right.$$

$$-\frac{Ba^2c^2d^3 + Ba^3cde^2 + (Ca^3c - Aa^2c^2)d^2e + (Ca^4 - Aa^3c)e^3 + (Ba^2cd^2e - Ba^3e^3 - (Ca^2c + Aac^2)d^3 +}{}$$

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)/(c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*B*a^2*c^2*d^3 + 2*B*a^3*c*d*e^2 + 2*(C*a^3*c - A*a^2*c^2)*d^2*e +
2*(C*a^4 - A*a^3*c)*e^3 - (B*a^2*c*d^2*e - B*a^3*e^3 - (C*a^2*c + A*a*c^2)*
d^3 + (C*a^3 - 3*A*a^2*c)*d*e^2 + (B*a*c^2*d^2*e - B*a^2*c*e^3 - (C*a*c^2 +
 A*c^3)*d^3 + (C*a^2*c - 3*A*a*c^2)*d*e^2)*x^2)*sqrt(-a*c)*log((c*x^2 - 2*s
qrt(-a*c)*x - a)/(c*x^2 + a)) - 2*(B*a^2*c^2*d^2*e + B*a^3*c*e^3 - (C*a^2*c
^2 - A*a*c^3)*d^3 - (C*a^3*c - A*a^2*c^2)*d*e^2)*x + 2*(C*a^3*c*d^2*e - B*a
^3*c*d*e^2 + A*a^3*c*e^3 + (C*a^2*c^2*d^2*e - B*a^2*c^2*d*e^2 + A*a^2*c^2*e
^3)*x^2)*log(c*x^2 + a) - 4*(C*a^3*c*d^2*e - B*a^3*c*d*e^2 + A*a^3*c*e^3 +
(C*a^2*c^2*d^2*e - B*a^2*c^2*d*e^2 + A*a^2*c^2*e^3)*x^2)*log(e*x + d))/(a^3
*c^3*d^4 + 2*a^4*c^2*d^2*e^2 + a^5*c*e^4 + (a^2*c^4*d^4 + 2*a^3*c^3*d^2*e^2
 + a^4*c^2*e^4)*x^2), -1/2*(B*a^2*c^2*d^3 + B*a^3*c*d*e^2 + (C*a^3*c - A*a^
2*c^2)*d^2*e + (C*a^4 - A*a^3*c)*e^3 + (B*a^2*c*d^2*e - B*a^3*e^3 - (C*a^2*
c + A*a*c^2)*d^3 + (C*a^3 - 3*A*a^2*c)*d*e^2 + (B*a*c^2*d^2*e - B*a^2*c*e^3
 - (C*a*c^2 + A*c^3)*d^3 + (C*a^2*c - 3*A*a*c^2)*d*e^2)*x^2)*sqrt(a*c)*arct
an(sqrt(a*c)*x/a) - (B*a^2*c^2*d^2*e + B*a^3*c*e^3 - (C*a^2*c^2 - A*a*c^3)*
d^3 - (C*a^3*c - A*a^2*c^2)*d*e^2)*x + (C*a^3*c*d^2*e - B*a^3*c*d*e^2 + A*a
^3*c*e^3 + (C*a^2*c^2*d^2*e - B*a^2*c^2*d*e^2 + A*a^2*c^2*e^3)*x^2)*log(c*x
^2 + a) - 2*(C*a^3*c*d^2*e - B*a^3*c*d*e^2 + A*a^3*c*e^3 + (C*a^2*c^2*d^2*e
 - B*a^2*c^2*d*e^2 + A*a^2*c^2*e^3)*x^2)*log(e*x + d))/(a^3*c^3*d^4 + 2*a^4
*c^2*d^2*e^2 + a^5*c*e^4 + (a^2*c^4*d^4 + 2*a^3*c^3*d^2*e^2 + a^4*c^2*e^4)*
x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(d+ex)\left(a+cx^2\right)^2}\,dx = \text{Timed out}$$

```
[In] integrate((C*x**2+B*x+A)/(e*x+d)/(c*x**2+a)**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.30

$$\begin{aligned}
&\int \frac{A+Bx+Cx^2}{(d+ex)\left(a+cx^2\right)^2}\,dx \\
&= -\frac{(Cd^2e-Bde^2+Ae^3)\log\left(cx^2+a\right)}{2\left(c^2d^4+2\,acd^2e^2+a^2e^4\right)}+\frac{(Cd^2e-Bde^2+Ae^3)\log\left(ex+d\right)}{c^2d^4+2\,acd^2e^2+a^2e^4} \\
&\quad-\frac{\left(Bacd^2e-Ba^2e^3-(Cac+Ac^2)d^3+(Ca^2-3\,Aac)de^2\right)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\left(ac^2d^4+2\,a^2cd^2e^2+a^3e^4\right)\sqrt{ac}} \\
&\quad-\frac{Bacd+(Ca^2-Aac)e-(Bace-(Cac-Ac^2)d)x}{2\left(a^2c^2d^2+a^3ce^2+(ac^3d^2+a^2c^2e^2)x^2\right)}
\end{aligned}$$

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)/(c*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(C*d^2*e - B*d*e^2 + A*e^3)*log(c*x^2 + a)/(c^2*d^4 + 2*a*c*d^2*e^2 +
a^2*e^4) + (C*d^2*e - B*d*e^2 + A*e^3)*log(e*x + d)/(c^2*d^4 + 2*a*c*d^2*e^
2 + a^2*e^4) - 1/2*(B*a*c*d^2*e - B*a^2*e^3 - (C*a*c + A*c^2)*d^3 + (C*a^2
- 3*A*a*c)*d*e^2)*arctan(c*x/sqrt(a*c))/((a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3
*e^4)*sqrt(a*c)) - 1/2*(B*a*c*d + (C*a^2 - A*a*c)*e - (B*a*c*e - (C*a*c - A
*c^2)*d)*x)/(a^2*c^2*d^2 + a^3*c*e^2 + (a*c^3*d^2 + a^2*c^2*e^2)*x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.61

$$\begin{aligned}&\int \frac{A+Bx+Cx^2}{(d+ex)\left(a+cx^2\right)^2}\,dx\\&=-\frac{(Cd^2e-Bde^2+Ae^3)\log\left(cx^2+a\right)}{2\left(c^2d^4+2\,acd^2e^2+a^2e^4\right)}+\frac{(Cd^2e^2-Bde^3+Ae^4)\log\left(|ex+d|\right)}{c^2d^4e+2\,acd^2e^3+a^2e^5}\\&\quad+\frac{(Cacd^3+Ac^2d^3-Bacd^2e-Ca^2de^2+3\,Aacde^2+Ba^2e^3)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\left(ac^2d^4+2\,a^2cd^2e^2+a^3e^4\right)\sqrt{ac}}\\&\quad-\frac{Bac^2d^3+Ca^2cd^2e-Aac^2d^2e+Ba^2cde^2+Ca^3e^3-Aa^2ce^3+(Cac^2d^3-Ac^3d^3-Bac^2d^2e+Ca^2cde^2-}{2\left(cd^2+ae^2\right)^2(cx^2+a)ac}\end{aligned}$$

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)/(c*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(C*d^2*e - B*d*e^2 + A*e^3)*log(c*x^2 + a)/(c^2*d^4 + 2*a*c*d^2*e^2 +
a^2*e^4) + (C*d^2*e^2 - B*d*e^3 + A*e^4)*log(abs(e*x + d))/(c^2*d^4*e + 2*a
*c*d^2*e^3 + a^2*e^5) + 1/2*(C*a*c*d^3 + A*c^2*d^3 - B*a*c*d^2*e - C*a^2*d*
e^2 + 3*A*a*c*d*e^2 + B*a^2*e^3)*arctan(c*x/sqrt(a*c))/((a*c^2*d^4 + 2*a^2*
c*d^2*e^2 + a^3*e^4)*sqrt(a*c)) - 1/2*(B*a*c^2*d^3 + C*a^2*c*d^2*e - A*a*c^
2*d^2*e + B*a^2*c*d*e^2 + C*a^3*e^3 - A*a^2*c*e^3 + (C*a*c^2*d^3 - A*c^3*d^
3 - B*a*c^2*d^2*e + C*a^2*c*d*e^2 - A*a*c^2*d*e^2 - B*a^2*c*e^3)*x)/((c*d^2
 + a*e^2)^2*(c*x^2 + a)*a*c)
```

Mupad [B] (verification not implemented)

Time = 17.72 (sec) , antiderivative size = 1493, normalized size of antiderivative = 6.61

$$\int \frac{A+Bx+Cx^2}{(d+ex)\left(a+cx^2\right)^2}\,dx=\text{Too large to display}$$

```
[In] int((A + B*x + C*x^2)/((a + c*x^2)^2*(d + e*x)),x)
```

```
[Out] (log(A*c^3*d^5*(-a^3*c)^(1/2) - B*a^3*e^5*(-a^3*c)^(1/2) + 6*A*a^4*c*e^5 -
B*a^4*c*e^5*x - 2*A*a^2*c^3*d^4*e - 8*C*a^3*c^2*d^4*e + 8*C*a^4*c*d^2*e^3 +
 C*a^2*c^3*d^5*x + C*a*c^2*d^5*(-a^3*c)^(1/2) + C*a^3*d*e^4*(-a^3*c)^(1/2)
- 12*A*a^3*c^2*d^2*e^3 + 8*B*a^3*c^2*d^3*e^2 - 8*B*a^4*c*d*e^4 + A*a*c^4*d^
5*x + 2*A*a^2*c^3*d^3*e^2*x + 14*B*a^3*c^2*d^2*e^3*x - 14*C*a^3*c^2*d^3*e^2
*x + 2*A*a*c^2*d^3*e^2*(-a^3*c)^(1/2) + 14*B*a^2*c*d^2*e^3*(-a^3*c)^(1/2) -
 14*C*a^2*c*d^3*e^2*(-a^3*c)^(1/2) + C*a^4*c*d*e^4*x - 15*A*a^3*c^2*d*e^4*x
 - B*a^2*c^3*d^4*e*x - 15*A*a^2*c*d*e^4*(-a^3*c)^(1/2) - B*a*c^2*d^4*e*(-a^
3*c)^(1/2) - 6*A*a^2*c*e^5*x*(-a^3*c)^(1/2) + 2*A*c^3*d^4*e*x*(-a^3*c)^(1/2
) + 8*B*a^2*c*d*e^4*x*(-a^3*c)^(1/2) + 8*C*a*c^2*d^4*e*x*(-a^3*c)^(1/2) + 1
2*A*a*c^2*d^2*e^3*x*(-a^3*c)^(1/2) - 8*B*a*c^2*d^3*e^2*x*(-a^3*c)^(1/2) - 8
*C*a^2*c*d^2*e^3*x*(-a^3*c)^(1/2))*(a^2*((B*e^3*(-a^3*c)^(1/2))/4 - (C*d*e^
```

```
2*(-a^3*c)^(1/2))/4) - c*(a^3*((A*e^3)/2 - (B*d*e^2)/2 + (C*d^2*e)/2) - a*(
(C*d^3*(-a^3*c)^(1/2))/4 + (3*A*d*e^2*(-a^3*c)^(1/2))/4 - (B*d^2*e*(-a^3*c)
^(1/2))/4)) + (A*c^2*d^3*(-a^3*c)^(1/2))/4))/(a^5*c*e^4 + a^3*c^3*d^4 + 2*a
^4*c^2*d^2*e^2) - (log(A*c^3*d^5*(-a^3*c)^(1/2) - B*a^3*e^5*(-a^3*c)^(1/2)
- 6*A*a^4*c*e^5 + B*a^4*c*e^5*x + 2*A*a^2*c^3*d^4*e + 8*C*a^3*c^2*d^4*e - 8
*C*a^4*c*d^2*e^3 - C*a^2*c^3*d^5*x + C*a*c^2*d^5*(-a^3*c)^(1/2) + C*a^3*d*e
^4*(-a^3*c)^(1/2) + 12*A*a^3*c^2*d^2*e^3 - 8*B*a^3*c^2*d^3*e^2 + 8*B*a^4*c*
d*e^4 - A*a*c^4*d^5*x - 2*A*a^2*c^3*d^3*e^2*x - 14*B*a^3*c^2*d^2*e^3*x + 14
*C*a^3*c^2*d^3*e^2*x + 2*A*a*c^2*d^3*e^2*(-a^3*c)^(1/2) + 14*B*a^2*c*d^2*e^
3*(-a^3*c)^(1/2) - 14*C*a^2*c*d^3*e^2*(-a^3*c)^(1/2) - C*a^4*c*d*e^4*x + 15
*A*a^3*c^2*d*e^4*x + B*a^2*c^3*d^4*e*x - 15*A*a^2*c*d*e^4*(-a^3*c)^(1/2) -
B*a*c^2*d^4*e*(-a^3*c)^(1/2) - 6*A*a^2*c*e^5*x*(-a^3*c)^(1/2) + 2*A*c^3*d^4
*e*x*(-a^3*c)^(1/2) + 8*B*a^2*c*d*e^4*x*(-a^3*c)^(1/2) + 8*C*a*c^2*d^4*e*x*
(-a^3*c)^(1/2) + 12*A*a*c^2*d^2*e^3*x*(-a^3*c)^(1/2) - 8*B*a*c^2*d^3*e^2*x*
(-a^3*c)^(1/2) - 8*C*a^2*c*d^2*e^3*x*(-a^3*c)^(1/2))*(c*(a^3*((A*e^3)/2 - (
B*d*e^2)/2 + (C*d^2*e)/2) + a*((C*d^3*(-a^3*c)^(1/2))/4 + (3*A*d*e^2*(-a^3*
c)^(1/2))/4 - (B*d^2*e*(-a^3*c)^(1/2))/4)) + a^2*((B*e^3*(-a^3*c)^(1/2))/4
- (C*d*e^2*(-a^3*c)^(1/2))/4) + (A*c^2*d^3*(-a^3*c)^(1/2))/4))/(a^5*c*e^4 +
 a^3*c^3*d^4 + 2*a^4*c^2*d^2*e^2) - ((B*c*d - A*c*e + C*a*e)/(2*c*(a*e^2 +
c*d^2)) - (x*(A*c*d + B*a*e - C*a*d))/(2*a*(a*e^2 + c*d^2)))/(a + c*x^2) +
(e*log(d + e*x)*(A*e^2 + C*d^2 - B*d*e))/(a*e^2 + c*d^2)^2
```

3.55 $\int \frac{A+Bx+Cx^2}{(d+ex)^2\left(a+cx^2\right)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 374

$$
\begin{aligned}
\int \frac{A+Bx+Cx^2}{(d+ex)^2\left(a+cx^2\right)^2}\,dx = {} & -\frac{e(Cd^2-Bde+Ae^2)}{\left(cd^2+ae^2\right)^2(d+ex)} \\
& -\frac{a(Bcd^2-2Acde+2aCde-aBe^2)-(Ac(cd^2-ae^2)+a(aCe^2-cd(Cd-2Be)))\,x}{2a\left(cd^2+ae^2\right)^2(a+cx^2)} \\
& +\frac{(Ac(c^2d^4+6acd^2e^2-3a^2e^4)+a(a^2Ce^4+c^2d^3(Cd-2Be)-6acde^2(Cd-Be)))\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{c}\left(cd^2+ae^2\right)^3} \\
& -\frac{e(ae^2(2Cd-Be)-cd(2Cd^2-e(3Bd-4Ae)))\log(d+ex)}{\left(cd^2+ae^2\right)^3} \\
& +\frac{e(ae^2(2Cd-Be)-cd(2Cd^2-e(3Bd-4Ae)))\log\left(a+cx^2\right)}{2\left(cd^2+ae^2\right)^3}
\end{aligned}
$$

```
[Out] -e*(A*e^2-B*d*e+C*d^2)/(a*e^2+c*d^2)^2/(e*x+d)+1/2*(-a*(-2*A*c*d*e-B*a*e^2+
B*c*d^2+2*C*a*d*e)+(A*c*(-a*e^2+c*d^2)+a*(a*C*e^2-c*d*(-2*B*e+C*d)))*x)/a/(
a*e^2+c*d^2)^2/(c*x^2+a)-e*(a*e^2*(-B*e+2*C*d)-c*d*(2*C*d^2-e*(-4*A*e+3*B*d
)))*ln(e*x+d)/(a*e^2+c*d^2)^3+1/2*e*(a*e^2*(-B*e+2*C*d)-c*d*(2*C*d^2-e*(-4*
A*e+3*B*d)))*ln(c*x^2+a)/(a*e^2+c*d^2)^3+1/2*(A*c*(-3*a^2*e^4+6*a*c*d^2*e^2
+c^2*d^4)+a*(a^2*C*e^4+c^2*d^3*(-2*B*e+C*d)-6*a*c*d*e^2*(-B*e+C*d)))*arctan
(x*c^(1/2)/a^(1/2))/a^(3/2)/(a*e^2+c*d^2)^3/c^(1/2)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 371, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1661, 1643, 649, 211, 266}

$$\begin{aligned}
&\int \frac{A+Bx+Cx^2}{(d+ex)^2\left(a+cx^2\right)^2}\,dx \\
&=\frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(Ac\left(-3a^2e^4+6acd^2e^2+c^2d^4\right)+a\left(a^2Ce^4-6acde^2(Cd-Be)+c^2d^3(Cd-2Be)\right)\right)}{2a^{3/2}\sqrt{c}\left(ae^2+cd^2\right)^3} \\
&\quad-\frac{a\left(-aBe^2+2aCde-2Acde+Bcd^2\right)-x\left(Ac\left(cd^2-ae^2\right)+a\left(aCe^2-cd(Cd-2Be)\right)\right)}{2a\left(a+cx^2\right)\left(ae^2+cd^2\right)^2} \\
&\quad-\frac{e\left(Ae^2-Bde+Cd^2\right)}{(d+ex)\left(ae^2+cd^2\right)^2}-\frac{e\log\left(a+cx^2\right)\left(-ae^2(2Cd-Be)-cde(3Bd-4Ae)+2cCd^3\right)}{2\left(ae^2+cd^2\right)^3} \\
&\quad+\frac{e\log(d+ex)\left(-ae^2(2Cd-Be)-cde(3Bd-4Ae)+2cCd^3\right)}{\left(ae^2+cd^2\right)^3}
\end{aligned}$$

```
[In] Int[(A + B*x + C*x^2)/((d + e*x)^2*(a + c*x^2)^2),x]
```

```
[Out] -((e*(C*d^2 - B*d*e + A*e^2))/((c*d^2 + a*e^2)^2*(d + e*x))) - (a*(B*c*d^2
- 2*A*c*d*e + 2*a*C*d*e - a*B*e^2) - (A*c*(c*d^2 - a*e^2) + a*(a*C*e^2 - c*
d*(C*d - 2*B*e)))*x)/(2*a*(c*d^2 + a*e^2)^2*(a + c*x^2)) + ((A*c*(c^2*d^4 +
 6*a*c*d^2*e^2 - 3*a^2*e^4) + a*(a^2*C*e^4 + c^2*d^3*(C*d - 2*B*e) - 6*a*c*
d*e^2*(C*d - B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(2*a^(3/2)*Sqrt[c]*(c*d^2
+ a*e^2)^3) + (e*(2*c*C*d^3 - c*d*e*(3*B*d - 4*A*e) - a*e^2*(2*C*d - B*e))*
Log[d + e*x])/(c*d^2 + a*e^2)^3 - (e*(2*c*C*d^3 - c*d*e*(3*B*d - 4*A*e) - a
*e^2*(2*C*d - B*e))*Log[a + c*x^2])/(2*(c*d^2 + a*e^2)^3)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

integral

$$= -\frac{a(Bcd^2-2Acde+2aCde-aBe^2)-(Ac(cd^2-ae^2)+a(aCe^2-cd(Cd-2Be)))\,x}{2a\left(cd^2+ae^2\right)^2(a+cx^2)} - \frac{\int \frac{-\frac{c\left(A\left(c^2d^4+5acd^2e^2+2a^2e^4\right)-ad^2\left(aCe^2-cd(Cd-2Be)\right)\right)}{\left(cd^2+ae^2\right)^2}-\frac{2ce(Acd-aCd+aBe)x}{cd^2+ae^2}-\frac{ce^2\left(Ac\left(cd^2-ae^2\right)+a\left(aCe^2-cd(Cd-2Be)\right)\right)x^2}{\left(cd^2+ae^2\right)^2}}{(d+ex)^2(a+cx^2)}\,dx}{2ac}$$

$$= -\frac{a(Bcd^2-2Acde+2aCde-aBe^2)-(Ac(cd^2-ae^2)+a(aCe^2-cd(Cd-2Be)))\,x}{2a\left(cd^2+ae^2\right)^2(a+cx^2)} - \frac{\int \Bigg(-\frac{2ace^2(Cd^2-Bde+Ae^2)}{(cd^2+ae^2)^2(d+ex)^2}+\frac{2ace^2(-2cCd^3+cde(3Bd-4Ae)+ae^2(2Cd-Be))}{(cd^2+ae^2)^3(d+ex)}+\frac{c(-Ac(c^2d^4+6acd^2e^2-3a^2e^4)-a(a^2Ce^4-}{}}{2ac}$$

$$= -\frac{e(Cd^2-Bde+Ae^2)}{\left(cd^2+ae^2\right)^2(d+ex)} - \frac{a(Bcd^2-2Acde+2aCde-aBe^2)-(Ac(cd^2-ae^2)+a(aCe^2-cd(Cd-2Be)))\,x}{2a\left(cd^2+ae^2\right)^2(a+cx^2)} + \frac{e(2cCd^3-cde(3Bd-4Ae)-ae^2(2Cd-Be))\log(d+ex)}{\left(cd^2+ae^2\right)^3} - \frac{\int \frac{-Ac(c^2d^4+6acd^2e^2-3a^2e^4)-a\left(a^2Ce^4+c^2d^3(Cd-2Be)-6acde^2(Cd-Be)\right)+2ace\left(2cCd^3-cde(3Bd-4Ae)-ae^2(2Cd-Be)\right)x}{a+cx^2}\,dx}{2a\left(cd^2+ae^2\right)^3}$$

$$
\begin{aligned}
&= -\frac{e(Cd^2-Bde+Ae^2)}{(cd^2+ae^2)^2(d+ex)} \\
&\quad -\frac{a(Bcd^2-2Acde+2aCde-aBe^2)-(Ac(cd^2-ae^2)+a(aCe^2-cd(Cd-2Be)))\,x}{2a\,(cd^2+ae^2)^2\,(a+cx^2)} \\
&\quad +\frac{e(2cCd^3-cde(3Bd-4Ae)-ae^2(2Cd-Be))\log(d+ex)}{(cd^2+ae^2)^3} \\
&\quad -\frac{(ce(2cCd^3-cde(3Bd-4Ae)-ae^2(2Cd-Be)))\int\frac{x}{a+cx^2}\,dx}{(cd^2+ae^2)^3} \\
&\quad +\frac{(Ac(c^2d^4+6acd^2e^2-3a^2e^4)+a(a^2Ce^4+c^2d^3(Cd-2Be)-6acde^2(Cd-Be)))\int\frac{1}{a+cx^2}\,dx}{2a\,(cd^2+ae^2)^3}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{e(Cd^2-Bde+Ae^2)}{(cd^2+ae^2)^2(d+ex)} \\
&\quad -\frac{a(Bcd^2-2Acde+2aCde-aBe^2)-(Ac(cd^2-ae^2)+a(aCe^2-cd(Cd-2Be)))\,x}{2a\,(cd^2+ae^2)^2\,(a+cx^2)} \\
&\quad +\frac{(Ac(c^2d^4+6acd^2e^2-3a^2e^4)+a(a^2Ce^4+c^2d^3(Cd-2Be)-6acde^2(Cd-Be)))\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{c}\,(cd^2+ae^2)^3} \\
&\quad +\frac{e(2cCd^3-cde(3Bd-4Ae)-ae^2(2Cd-Be))\log(d+ex)}{(cd^2+ae^2)^3} \\
&\quad -\frac{e(2cCd^3-cde(3Bd-4Ae)-ae^2(2Cd-Be))\log\left(a+cx^2\right)}{2\,(cd^2+ae^2)^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 320, normalized size of antiderivative = 0.86

$$
\int\frac{A+Bx+Cx^2}{(d+ex)^2\,(a+cx^2)^2}\,dx
$$

$$
= \frac{-\frac{2e(cd^2+ae^2)(Cd^2+e(-Bd+Ae))}{d+ex}+\frac{(cd^2+ae^2)(Ac^2d^2x+a^2e(-2Cd+Be+Cex)-ac(Cd^2x+Bd(d-2ex)+Ae(-2d+ex)))}{a(a+cx^2)}+\frac{(Ac(c^2d^4+6a}{}}{}
$$

```
[In] Integrate[(A + B*x + C*x^2)/((d + e*x)^2*(a + c*x^2)^2),x]
```

```
[Out] ((-2*e*(c*d^2 + a*e^2)*(C*d^2 + e*(-(B*d) + A*e)))/(d + e*x) + ((c*d^2 + a*
e^2)*(A*c^2*d^2*x + a^2*e*(-2*C*d + B*e + C*e*x) - a*c*(C*d^2*x + B*d*(d -
2*e*x) + A*e*(-2*d + e*x))))/(a*(a + c*x^2)) + ((A*c*(c^2*d^4 + 6*a*c*d^2*e
^2 - 3*a^2*e^4) + a*(a^2*C*e^4 + c^2*d^3*(C*d - 2*B*e) + 6*a*c*d*e^2*(-(C*d
) + B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(a^(3/2)*Sqrt[c]) + 2*e*(2*c*C*d^3
+ c*d*e*(-3*B*d + 4*A*e) + a*e^2*(-2*C*d + B*e))*Log[d + e*x] - e*(2*c*C*d^
3 + c*d*e*(-3*B*d + 4*A*e) + a*e^2*(-2*C*d + B*e))*Log[a + c*x^2])/(2*(c*d^
2 + a*e^2)^3)
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 424, normalized size of antiderivative = 1.13

method	result
default	$-\frac{\frac{\left(A a^{2} c e^{4}-A d^{4} c^{3}-2 B a^{2} c d e^{3}-2 B a c^{2} d^{3} e-C a^{3} e^{4}+C a c^{2} d^{4}\right) x}{2 a}-A a c d e^{3}-A c^{2} d^{3} e-\frac{B e^{4} a^{2}}{2}+\frac{B c^{2} d^{4}}{2}+C a^{2} d e^{3}+C a c d^{3} e}{c x^{2}+a}+\frac{\left(8 A a c^{2} d e^{3}+2 B e}{}$
risch	Expression too large to display

```
[In] int((C*x^2+B*x+A)/(e*x+d)^2/(c*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/(a*e^2+c*d^2)^3*((1/2*(A*a^2*c*e^4-A*c^3*d^4-2*B*a^2*c*d*e^3-2*B*a*c^2*d
^3*e-C*a^3*e^4+C*a*c^2*d^4)/a*x-A*a*c*d*e^3-A*c^2*d^3*e-1/2*B*e^4*a^2+1/2*B
*c^2*d^4+C*a^2*d*e^3+C*a*c*d^3*e)/(c*x^2+a)+1/2/a*(1/2*(8*A*a*c^2*d*e^3+2*B
*a^2*c*e^4-6*B*a*c^2*d^2*e^2-4*C*a^2*c*d*e^3+4*C*a*c^2*d^3*e)/c*ln(c*x^2+a)
+(3*A*a^2*c*e^4-6*A*a*c^2*d^2*e^2-A*c^3*d^4-6*B*a^2*c*d*e^3+2*B*a*c^2*d^3*e
-C*a^3*e^4+6*C*a^2*c*d^2*e^2-C*a*c^2*d^4)/(a*c)^(1/2)*arctan(c*x/(a*c)^(1/2
))))+e*(4*A*c*d*e^2+B*a*e^3-3*B*c*d^2*e-2*C*a*d*e^2+2*C*c*d^3)/(a*e^2+c*d^2
)^3*ln(e*x+d)-e*(A*e^2-B*d*e+C*d^2)/(a*e^2+c*d^2)^2/(e*x+d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1447 vs. 2(360) = 720.

Time = 130.34 (sec) , antiderivative size = 2916, normalized size of antiderivative = 7.80

$$\int \frac{A+Bx+Cx^2}{(d+ex)^2\left(a+cx^2\right)^2}\,dx=\text{Too large to display}$$

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^2/(c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*B*a^2*c^3*d^5 - 4*B*a^3*c^2*d^3*e^2 + 8*C*a^4*c*d^2*e^3 - 6*B*a^4*
c*d*e^4 + 4*A*a^4*c*e^5 + 4*(2*C*a^3*c^2 - A*a^2*c^3)*d^4*e - 2*(4*B*a^2*c^
3*d^3*e^2 + 4*B*a^3*c^2*d*e^4 - (3*C*a^2*c^3 - A*a*c^4)*d^4*e - 2*(C*a^3*c^
2 + A*a^2*c^3)*d^2*e^3 + (C*a^4*c - 3*A*a^3*c^2)*e^5)*x^2 + (2*B*a^2*c^2*d^
4*e - 6*B*a^3*c*d^2*e^3 - (C*a^2*c^2 + A*a*c^3)*d^5 + 6*(C*a^3*c - A*a^2*c^
2)*d^3*e^2 - (C*a^4 - 3*A*a^3*c)*d*e^4 + (2*B*a*c^3*d^3*e^2 - 6*B*a^2*c^2*d
*e^4 - (C*a*c^3 + A*c^4)*d^4*e + 6*(C*a^2*c^2 - A*a*c^3)*d^2*e^3 - (C*a^3*c
 - 3*A*a^2*c^2)*e^5)*x^3 + (2*B*a*c^3*d^4*e - 6*B*a^2*c^2*d^2*e^3 - (C*a*c^
3 + A*c^4)*d^5 + 6*(C*a^2*c^2 - A*a*c^3)*d^3*e^2 - (C*a^3*c - 3*A*a^2*c^2)*
d*e^4)*x^2 + (2*B*a^2*c^2*d^3*e^2 - 6*B*a^3*c*d*e^4 - (C*a^2*c^2 + A*a*c^3)
*d^4*e + 6*(C*a^3*c - A*a^2*c^2)*d^2*e^3 - (C*a^4 - 3*A*a^3*c)*e^5)*x)*sqrt
(-a*c)*log((c*x^2 + 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) - 2*(B*a^2*c^3*d^4*e +
 2*B*a^3*c^2*d^2*e^3 + B*a^4*c*e^5 - (C*a^2*c^3 - A*a*c^4)*d^5 - 2*(C*a^3*c
^2 - A*a^2*c^3)*d^3*e^2 - (C*a^4*c - A*a^3*c^2)*d*e^4)*x + 2*(2*C*a^3*c^2*d
```

```
^4*e - 3*B*a^3*c^2*d^3*e^2 + B*a^4*c*d*e^4 - 2*(C*a^4*c - 2*A*a^3*c^2)*d^2*
e^3 + (2*C*a^2*c^3*d^3*e^2 - 3*B*a^2*c^3*d^2*e^3 + B*a^3*c^2*e^5 - 2*(C*a^3
*c^2 - 2*A*a^2*c^3)*d*e^4)*x^3 + (2*C*a^2*c^3*d^4*e - 3*B*a^2*c^3*d^3*e^2 +
 B*a^3*c^2*d*e^4 - 2*(C*a^3*c^2 - 2*A*a^2*c^3)*d^2*e^3)*x^2 + (2*C*a^3*c^2*
d^3*e^2 - 3*B*a^3*c^2*d^2*e^3 + B*a^4*c*e^5 - 2*(C*a^4*c - 2*A*a^3*c^2)*d*e
^4)*x)*log(c*x^2 + a) - 4*(2*C*a^3*c^2*d^4*e - 3*B*a^3*c^2*d^3*e^2 + B*a^4*
c*d*e^4 - 2*(C*a^4*c - 2*A*a^3*c^2)*d^2*e^3 + (2*C*a^2*c^3*d^3*e^2 - 3*B*a^
2*c^3*d^2*e^3 + B*a^3*c^2*e^5 - 2*(C*a^3*c^2 - 2*A*a^2*c^3)*d*e^4)*x^3 + (2
*C*a^2*c^3*d^4*e - 3*B*a^2*c^3*d^3*e^2 + B*a^3*c^2*d*e^4 - 2*(C*a^3*c^2 - 2
*A*a^2*c^3)*d^2*e^3)*x^2 + (2*C*a^3*c^2*d^3*e^2 - 3*B*a^3*c^2*d^2*e^3 + B*a
^4*c*e^5 - 2*(C*a^4*c - 2*A*a^3*c^2)*d*e^4)*x)*log(e*x + d))/(a^3*c^4*d^7 +
 3*a^4*c^3*d^5*e^2 + 3*a^5*c^2*d^3*e^4 + a^6*c*d*e^6 + (a^2*c^5*d^6*e + 3*a
^3*c^4*d^4*e^3 + 3*a^4*c^3*d^2*e^5 + a^5*c^2*e^7)*x^3 + (a^2*c^5*d^7 + 3*a^
3*c^4*d^5*e^2 + 3*a^4*c^3*d^3*e^4 + a^5*c^2*d*e^6)*x^2 + (a^3*c^4*d^6*e + 3
*a^4*c^3*d^4*e^3 + 3*a^5*c^2*d^2*e^5 + a^6*c*e^7)*x), -1/2*(B*a^2*c^3*d^5 -
 2*B*a^3*c^2*d^3*e^2 + 4*C*a^4*c*d^2*e^3 - 3*B*a^4*c*d*e^4 + 2*A*a^4*c*e^5
+ 2*(2*C*a^3*c^2 - A*a^2*c^3)*d^4*e - (4*B*a^2*c^3*d^3*e^2 + 4*B*a^3*c^2*d*
e^4 - (3*C*a^2*c^3 - A*a*c^4)*d^4*e - 2*(C*a^3*c^2 + A*a^2*c^3)*d^2*e^3 + (
C*a^4*c - 3*A*a^3*c^2)*e^5)*x^2 + (2*B*a^2*c^2*d^4*e - 6*B*a^3*c*d^2*e^3 -
(C*a^2*c^2 + A*a*c^3)*d^5 + 6*(C*a^3*c - A*a^2*c^2)*d^3*e^2 - (C*a^4 - 3*A*
a^3*c)*d*e^4 + (2*B*a*c^3*d^3*e^2 - 6*B*a^2*c^2*d*e^4 - (C*a*c^3 + A*c^4)*d
^4*e + 6*(C*a^2*c^2 - A*a*c^3)*d^2*e^3 - (C*a^3*c - 3*A*a^2*c^2)*e^5)*x^3 +
 (2*B*a*c^3*d^4*e - 6*B*a^2*c^2*d^2*e^3 - (C*a*c^3 + A*c^4)*d^5 + 6*(C*a^2*
c^2 - A*a*c^3)*d^3*e^2 - (C*a^3*c - 3*A*a^2*c^2)*d*e^4)*x^2 + (2*B*a^2*c^2*
d^3*e^2 - 6*B*a^3*c*d*e^4 - (C*a^2*c^2 + A*a*c^3)*d^4*e + 6*(C*a^3*c - A*a^
2*c^2)*d^2*e^3 - (C*a^4 - 3*A*a^3*c)*e^5)*x)*sqrt(a*c)*arctan(sqrt(a*c)*x/a
) - (B*a^2*c^3*d^4*e + 2*B*a^3*c^2*d^2*e^3 + B*a^4*c*e^5 - (C*a^2*c^3 - A*a
*c^4)*d^5 - 2*(C*a^3*c^2 - A*a^2*c^3)*d^3*e^2 - (C*a^4*c - A*a^3*c^2)*d*e^4
)*x + (2*C*a^3*c^2*d^4*e - 3*B*a^3*c^2*d^3*e^2 + B*a^4*c*d*e^4 - 2*(C*a^4*c
 - 2*A*a^3*c^2)*d^2*e^3 + (2*C*a^2*c^3*d^3*e^2 - 3*B*a^2*c^3*d^2*e^3 + B*a^
3*c^2*e^5 - 2*(C*a^3*c^2 - 2*A*a^2*c^3)*d*e^4)*x^3 + (2*C*a^2*c^3*d^4*e - 3
*B*a^2*c^3*d^3*e^2 + B*a^3*c^2*d*e^4 - 2*(C*a^3*c^2 - 2*A*a^2*c^3)*d^2*e^3)
*x^2 + (2*C*a^3*c^2*d^3*e^2 - 3*B*a^3*c^2*d^2*e^3 + B*a^4*c*e^5 - 2*(C*a^4*
c - 2*A*a^3*c^2)*d*e^4)*x)*log(c*x^2 + a) - 2*(2*C*a^3*c^2*d^4*e - 3*B*a^3*
c^2*d^3*e^2 + B*a^4*c*d*e^4 - 2*(C*a^4*c - 2*A*a^3*c^2)*d^2*e^3 + (2*C*a^2*
c^3*d^3*e^2 - 3*B*a^2*c^3*d^2*e^3 + B*a^3*c^2*e^5 - 2*(C*a^3*c^2 - 2*A*a^2*
c^3)*d*e^4)*x^3 + (2*C*a^2*c^3*d^4*e - 3*B*a^2*c^3*d^3*e^2 + B*a^3*c^2*d*e^
4 - 2*(C*a^3*c^2 - 2*A*a^2*c^3)*d^2*e^3)*x^2 + (2*C*a^3*c^2*d^3*e^2 - 3*B*a
^3*c^2*d^2*e^3 + B*a^4*c*e^5 - 2*(C*a^4*c - 2*A*a^3*c^2)*d*e^4)*x)*log(e*x
+ d))/(a^3*c^4*d^7 + 3*a^4*c^3*d^5*e^2 + 3*a^5*c^2*d^3*e^4 + a^6*c*d*e^6 +
(a^2*c^5*d^6*e + 3*a^3*c^4*d^4*e^3 + 3*a^4*c^3*d^2*e^5 + a^5*c^2*e^7)*x^3 +
 (a^2*c^5*d^7 + 3*a^3*c^4*d^5*e^2 + 3*a^4*c^3*d^3*e^4 + a^5*c^2*d*e^6)*x^2
+ (a^3*c^4*d^6*e + 3*a^4*c^3*d^4*e^3 + 3*a^5*c^2*d^2*e^5 + a^6*c*e^7)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(d+ex)^2\left(a+cx^2\right)^2}\,dx=\text{Timed out}$$

```
[In] integrate((C*x**2+B*x+A)/(e*x+d)**2/(c*x**2+a)**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 604, normalized size of antiderivative = 1.61

$$\begin{aligned}&\int \frac{A+Bx+Cx^2}{(d+ex)^2\left(a+cx^2\right)^2}\,dx\\&=-\frac{(2\,Ccd^3e-3\,Bcd^2e^2+Bae^4-2\,(Ca-2\,Ac)de^3)\log\left(cx^2+a\right)}{2\,(c^3d^6+3\,ac^2d^4e^2+3\,a^2cd^2e^4+a^3e^6)}\\&\quad+\frac{(2\,Ccd^3e-3\,Bcd^2e^2+Bae^4-2\,(Ca-2\,Ac)de^3)\log\left(ex+d\right)}{c^3d^6+3\,ac^2d^4e^2+3\,a^2cd^2e^4+a^3e^6}\\&\quad-\frac{(2\,Bac^2d^3e-6\,Ba^2cde^3-(Cac^2+Ac^3)d^4+6\,(Ca^2c-Aac^2)d^2e^2-(Ca^3-3\,Aa^2c)e^4)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\,(ac^3d^6+3\,a^2c^2d^4e^2+3\,a^3cd^2e^4+a^4e^6)\sqrt{ac}}\\&\quad-\frac{Bacd^3-3\,Ba^2de^2+2\,Aa^2e^3+2\,(2\,Ca^2-Aac)d^2e-(4\,Bacde^2-(3\,Cac-Ac^2)d^2e+(Ca^2-3\,Aac)e^3)}{2\,(a^2c^2d^5+2\,a^3cd^3e^2+a^4de^4+(ac^3d^4e+2\,a^2c^2d^2e^3+a^3ce^5)x^3+(ac^3d^5+2\,a^2c^2d^3e^2}\end{aligned}$$

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^2/(c*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*C*c*d^3*e - 3*B*c*d^2*e^2 + B*a*e^4 - 2*(C*a - 2*A*c)*d*e^3)*log(c*
x^2 + a)/(c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + a^3*e^6) + (2*C*c*d
^3*e - 3*B*c*d^2*e^2 + B*a*e^4 - 2*(C*a - 2*A*c)*d*e^3)*log(e*x + d)/(c^3*d
^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + a^3*e^6) - 1/2*(2*B*a*c^2*d^3*e -
6*B*a^2*c*d*e^3 - (C*a*c^2 + A*c^3)*d^4 + 6*(C*a^2*c - A*a*c^2)*d^2*e^2 - (
C*a^3 - 3*A*a^2*c)*e^4)*arctan(c*x/sqrt(a*c))/((a*c^3*d^6 + 3*a^2*c^2*d^4*e
^2 + 3*a^3*c*d^2*e^4 + a^4*e^6)*sqrt(a*c)) - 1/2*(B*a*c*d^3 - 3*B*a^2*d*e^2
 + 2*A*a^2*e^3 + 2*(2*C*a^2 - A*a*c)*d^2*e - (4*B*a*c*d*e^2 - (3*C*a*c - A*
c^2)*d^2*e + (C*a^2 - 3*A*a*c)*e^3)*x^2 - (B*a*c*d^2*e + B*a^2*e^3 - (C*a*c
 - A*c^2)*d^3 - (C*a^2 - A*a*c)*d*e^2)*x)/(a^2*c^2*d^5 + 2*a^3*c*d^3*e^2 +
a^4*d*e^4 + (a*c^3*d^4*e + 2*a^2*c^2*d^2*e^3 + a^3*c*e^5)*x^3 + (a*c^3*d^5
+ 2*a^2*c^2*d^3*e^2 + a^3*c*d*e^4)*x^2 + (a^2*c^2*d^4*e + 2*a^3*c*d^2*e^3 +
 a^4*e^5)*x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 638, normalized size of antiderivative = 1.71

$$
\begin{aligned}
&\int \frac{A+Bx+Cx^2}{(d+ex)^2\left(a+cx^2\right)^2}\,dx \\
&= -\frac{\left(2\,Ccd^3e-3\,Bcd^2e^2-2\,Cade^3+4\,Acde^3+Bae^4\right)\log\left(c-\frac{2\,cd}{ex+d}+\frac{cd^2}{(ex+d)^2}+\frac{ae^2}{(ex+d)^2}\right)}{2\left(c^3d^6+3\,ac^2d^4e^2+3\,a^2cd^2e^4+a^3e^6\right)} \\
&\quad-\frac{\frac{Cd^2e^5}{ex+d}-\frac{Bde^6}{ex+d}+\frac{Ae^7}{ex+d}}{c^2d^4e^4+2\,acd^2e^6+a^2e^8} \\
&\quad+\frac{\left(Cac^2d^4e^2+Ac^3d^4e^2-2\,Bac^2d^3e^3-6\,Ca^2cd^2e^4+6\,Aac^2d^2e^4+6\,Ba^2cde^5+Ca^3e^6-3\,Aa^2ce^6\right)\mathrm{arcta}}{2\left(ac^3d^6+3\,a^2c^2d^4e^2+3\,a^3cd^2e^4+a^4e^6\right)\sqrt{ac}e^2} \\
&\quad-\frac{\frac{Cac^2d^3e-Ac^3d^3e-3\,Bac^2d^2e^2-3\,Ca^2cde^3+3\,Aac^2de^3+Ba^2ce^4}{cd^2+ae^2}-\frac{Cac^2d^4e^2-Ac^3d^4e^2-4\,Bac^2d^3e^3-6\,Ca^2cd^2e^4+6\,Aac^2d^2e^4+4\,Ba^2cde^5}{(cd^2+ae^2)(ex+d)e}}{2\left(cd^2+ae^2\right)^2a\left(c-\frac{2\,cd}{ex+d}+\frac{cd^2}{(ex+d)^2}+\frac{ae^2}{(ex+d)^2}\right)}
\end{aligned}
$$

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^2/(c*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*C*c*d^3*e - 3*B*c*d^2*e^2 - 2*C*a*d*e^3 + 4*A*c*d*e^3 + B*a*e^4)*lo
g(c - 2*c*d/(e*x + d) + c*d^2/(e*x + d)^2 + a*e^2/(e*x + d)^2)/(c^3*d^6 + 3
*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + a^3*e^6) - (C*d^2*e^5/(e*x + d) - B*d*e^
6/(e*x + d) + A*e^7/(e*x + d))/(c^2*d^4*e^4 + 2*a*c*d^2*e^6 + a^2*e^8) + 1/
2*(C*a*c^2*d^4*e^2 + A*c^3*d^4*e^2 - 2*B*a*c^2*d^3*e^3 - 6*C*a^2*c*d^2*e^4
+ 6*A*a*c^2*d^2*e^4 + 6*B*a^2*c*d*e^5 + C*a^3*e^6 - 3*A*a^2*c*e^6)*arctan((
c*d - c*d^2/(e*x + d) - a*e^2/(e*x + d))/(sqrt(a*c)*e))/((a*c^3*d^6 + 3*a^2
*c^2*d^4*e^2 + 3*a^3*c*d^2*e^4 + a^4*e^6)*sqrt(a*c)*e^2) - 1/2*((C*a*c^2*d^
3*e - A*c^3*d^3*e - 3*B*a*c^2*d^2*e^2 - 3*C*a^2*c*d*e^3 + 3*A*a*c^2*d*e^3 +
 B*a^2*c*e^4)/(c*d^2 + a*e^2) - (C*a*c^2*d^4*e^2 - A*c^3*d^4*e^2 - 4*B*a*c^
2*d^3*e^3 - 6*C*a^2*c*d^2*e^4 + 6*A*a*c^2*d^2*e^4 + 4*B*a^2*c*d*e^5 + C*a^3
*e^6 - A*a^2*c*e^6)/((c*d^2 + a*e^2)*(e*x + d)*e))/((c*d^2 + a*e^2)^2*a*(c
- 2*c*d/(e*x + d) + c*d^2/(e*x + d)^2 + a*e^2/(e*x + d)^2))
```

Mupad [B] (verification not implemented)

Time = 20.36 (sec) , antiderivative size = 2094, normalized size of antiderivative = 5.60

$$
\int \frac{A+Bx+Cx^2}{(d+ex)^2\left(a+cx^2\right)^2}\,dx = \text{Too large to display}
$$

```
[In] int((A + B*x + C*x^2)/((a + c*x^2)^2*(d + e*x)^2),x)
```

```
[Out] ((x^2*(C*a^2*e^3 - 3*A*a*c*e^3 + A*c^2*d^2*e + 4*B*a*c*d*e^2 - 3*C*a*c*d^2*
e))/(2*a*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)) - (2*A*a*e^3 + B*c*d^3 - 3*B*
a*d*e^2 - 2*A*c*d^2*e + 4*C*a*d^2*e)/(2*(a*e^2 + c*d^2)^2) + (x*(A*c*d + B*
a*e - C*a*d))/(2*a*(a*e^2 + c*d^2)))/(a*d + a*e*x + c*d*x^2 + c*e*x^3) - (l
og(3*A*e^6*(-a^3*c)^(3/2) - A*c^4*d^6*(-a^3*c)^(1/2) + C*a^4*e^6*(-a^3*c)^(
1/2) + 31*C*d^2*e^4*(-a^3*c)^(3/2) + 6*B*a^5*c*e^6 - 18*B*d*e^5*(-a^3*c)^(3
/2) - 6*B*e^6*x*(-a^3*c)^(3/2) - C*a^5*c*e^6*x + 14*C*d*e^5*x*(-a^3*c)^(3/2
) - 2*A*a^2*c^4*d^5*e + 30*A*a^4*c^2*d*e^5 - 14*C*a^3*c^3*d^5*e + 3*A*a^4*c
^2*e^6*x + C*a^2*c^4*d^6*x - C*a*c^3*d^6*(-a^3*c)^(1/2) - 36*A*a^3*c^3*d^3*
e^3 + 22*B*a^3*c^3*d^4*e^2 - 36*B*a^4*c^2*d^2*e^4 + 36*C*a^4*c^2*d^3*e^3 -
14*C*a^5*c*d*e^5 + A*a*c^5*d^6*x + 5*A*a^2*c^4*d^4*e^2*x - 57*A*a^3*c^3*d^2
*e^4*x + 44*B*a^3*c^3*d^3*e^3*x - 31*C*a^3*c^3*d^4*e^2*x + 31*C*a^4*c^2*d^2
*e^4*x - 5*A*a*c^3*d^4*e^2*(-a^3*c)^(1/2) + 57*A*a^2*c^2*d^2*e^4*(-a^3*c)^(
1/2) - 44*B*a^2*c^2*d^3*e^3*(-a^3*c)^(1/2) + 31*C*a^2*c^2*d^4*e^2*(-a^3*c)^
(1/2) - 2*B*a^2*c^4*d^5*e*x - 18*B*a^4*c^2*d*e^5*x + 2*B*a*c^3*d^5*e*(-a^3*
c)^(1/2) - 2*A*c^4*d^5*e*x*(-a^3*c)^(1/2) - 36*B*a^2*c^2*d^2*e^4*x*(-a^3*c)
^(1/2) + 36*C*a^2*c^2*d^3*e^3*x*(-a^3*c)^(1/2) - 14*C*a*c^3*d^5*e*x*(-a^3*c
)^(1/2) - 36*A*a*c^3*d^3*e^3*x*(-a^3*c)^(1/2) + 30*A*a^2*c^2*d*e^5*x*(-a^3*
c)^(1/2) + 22*B*a*c^3*d^4*e^2*x*(-a^3*c)^(1/2))*(c^2*(a*((C*d^4*(-a^3*c)^(1
/2))/4 + (3*A*d^2*e^2*(-a^3*c)^(1/2))/2 - (B*d^3*e*(-a^3*c)^(1/2))/2) + a^3
*(2*A*d*e^3 - (3*B*d^2*e^2)/2 + C*d^3*e)) - c*(a^2*((3*A*e^4*(-a^3*c)^(1/2)
)/4 + (3*C*d^2*e^2*(-a^3*c)^(1/2))/2 - (3*B*d*e^3*(-a^3*c)^(1/2))/2) - a^4*
((B*e^4)/2 - C*d*e^3)) + (A*c^3*d^4*(-a^3*c)^(1/2))/4 + (C*a^3*e^4*(-a^3*c)
^(1/2))/4))/(a^6*c*e^6 + a^3*c^4*d^6 + 3*a^4*c^3*d^4*e^2 + 3*a^5*c^2*d^2*e^
4) + (log(3*A*e^6*(-a^3*c)^(3/2) - A*c^4*d^6*(-a^3*c)^(1/2) + C*a^4*e^6*(-a
^3*c)^(1/2) + 31*C*d^2*e^4*(-a^3*c)^(3/2) - 6*B*a^5*c*e^6 - 18*B*d*e^5*(-a^
3*c)^(3/2) - 6*B*e^6*x*(-a^3*c)^(3/2) + C*a^5*c*e^6*x + 14*C*d*e^5*x*(-a^3*
c)^(3/2) + 2*A*a^2*c^4*d^5*e - 30*A*a^4*c^2*d*e^5 + 14*C*a^3*c^3*d^5*e - 3*
A*a^4*c^2*e^6*x - C*a^2*c^4*d^6*x - C*a*c^3*d^6*(-a^3*c)^(1/2) + 36*A*a^3*c
^3*d^3*e^3 - 22*B*a^3*c^3*d^4*e^2 + 36*B*a^4*c^2*d^2*e^4 - 36*C*a^4*c^2*d^3
*e^3 + 14*C*a^5*c*d*e^5 - A*a*c^5*d^6*x - 5*A*a^2*c^4*d^4*e^2*x + 57*A*a^3*
c^3*d^2*e^4*x - 44*B*a^3*c^3*d^3*e^3*x + 31*C*a^3*c^3*d^4*e^2*x - 31*C*a^4*
c^2*d^2*e^4*x - 5*A*a*c^3*d^4*e^2*(-a^3*c)^(1/2) + 57*A*a^2*c^2*d^2*e^4*(-a
^3*c)^(1/2) - 44*B*a^2*c^2*d^3*e^3*(-a^3*c)^(1/2) + 31*C*a^2*c^2*d^4*e^2*(-
a^3*c)^(1/2) + 2*B*a^2*c^4*d^5*e*x + 18*B*a^4*c^2*d*e^5*x + 2*B*a*c^3*d^5*e
*(-a^3*c)^(1/2) - 2*A*c^4*d^5*e*x*(-a^3*c)^(1/2) - 36*B*a^2*c^2*d^2*e^4*x*(
-a^3*c)^(1/2) + 36*C*a^2*c^2*d^3*e^3*x*(-a^3*c)^(1/2) - 14*C*a*c^3*d^5*e*x*
(-a^3*c)^(1/2) - 36*A*a*c^3*d^3*e^3*x*(-a^3*c)^(1/2) + 30*A*a^2*c^2*d*e^5*x
*(-a^3*c)^(1/2) + 22*B*a*c^3*d^4*e^2*x*(-a^3*c)^(1/2))*(c^2*(a*((C*d^4*(-a^
3*c)^(1/2))/4 + (3*A*d^2*e^2*(-a^3*c)^(1/2))/2 - (B*d^3*e*(-a^3*c)^(1/2))/2
) - a^3*(2*A*d*e^3 - (3*B*d^2*e^2)/2 + C*d^3*e)) - c*(a^2*((3*A*e^4*(-a^3*c
)^(1/2))/4 + (3*C*d^2*e^2*(-a^3*c)^(1/2))/2 - (3*B*d*e^3*(-a^3*c)^(1/2))/2)
 + a^4*((B*e^4)/2 - C*d*e^3)) + (A*c^3*d^4*(-a^3*c)^(1/2))/4 + (C*a^3*e^4*(
-a^3*c)^(1/2))/4))/(a^6*c*e^6 + a^3*c^4*d^6 + 3*a^4*c^3*d^4*e^2 + 3*a^5*c^2
*d^2*e^4) + (log(d + e*x)*(a*(B*e^4 - 2*C*d*e^3) + c*(4*A*d*e^3 - 3*B*d^2*e
```

```
^2 + 2*C*d^3*e)))/(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)
```

3.56 $\int \frac{A+Bx+Cx^2}{(d+ex)^3\left(a+cx^2\right)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 524

$$\begin{aligned}
&\int \frac{A+Bx+Cx^2}{(d+ex)^3\left(a+cx^2\right)^2}\,dx \\
&= -\frac{e(Cd^2-Bde+Ae^2)}{2\left(cd^2+ae^2\right)^2(d+ex)^2}+\frac{e(ae^2(2Cd-Be)-cd(2Cd^2-e(3Bd-4Ae)))}{\left(cd^2+ae^2\right)^3(d+ex)} \\
&\quad -\frac{a(Bcd(cd^2-3ae^2)-(Ac-aC)e(3cd^2-ae^2))-c(Acd(cd^2-3ae^2)-a(cd^2(Cd-3Be)-ae^2(3Cd-}{2a\left(cd^2+ae^2\right)^3(a+cx^2)} \\
&\quad +\frac{\sqrt{c}(Acd(c^2d^4+10acd^2e^2-15a^2e^4)-a(2acd^2e^2(7Cd-9Be)-c^2d^4(Cd-3Be)-3a^2e^4(3Cd-Be)))\,a}{2a^{3/2}\left(cd^2+ae^2\right)^4} \\
&\quad +\frac{e(a^2Ce^4+c^2d^2(3Cd^2-2e(3Bd-5Ae))-2ace^2(4Cd^2-e(3Bd-Ae)))\log(d+ex)}{\left(cd^2+ae^2\right)^4} \\
&\quad -\frac{e(a^2Ce^4+c^2d^2(3Cd^2-2e(3Bd-5Ae))-2ace^2(4Cd^2-e(3Bd-Ae)))\log\left(a+cx^2\right)}{2\left(cd^2+ae^2\right)^4}
\end{aligned}$$

```
[Out] -1/2*e*(A*e^2-B*d*e+C*d^2)/(a*e^2+c*d^2)^2/(e*x+d)^2+e*(a*e^2*(-B*e+2*C*d)-
c*d*(2*C*d^2-e*(-4*A*e+3*B*d)))/(a*e^2+c*d^2)^3/(e*x+d)+1/2*(-a*(B*c*d*(-3*
a*e^2+c*d^2)-(A*c-C*a)*e*(-a*e^2+3*c*d^2))+c*(A*c*d*(-3*a*e^2+c*d^2)-a*(c*d
^2*(-3*B*e+C*d)-a*e^2*(-B*e+3*C*d)))*x)/a/(a*e^2+c*d^2)^3/(c*x^2+a)+e*(a^2*
C*e^4+c^2*d^2*(3*C*d^2-2*e*(-5*A*e+3*B*d))-2*a*c*e^2*(4*C*d^2-e*(-A*e+3*B*d
)))*ln(e*x+d)/(a*e^2+c*d^2)^4-1/2*e*(a^2*C*e^4+c^2*d^2*(3*C*d^2-2*e*(-5*A*e
+3*B*d))-2*a*c*e^2*(4*C*d^2-e*(-A*e+3*B*d)))*ln(c*x^2+a)/(a*e^2+c*d^2)^4+1/
2*(A*c*d*(-15*a^2*e^4+10*a*c*d^2*e^2+c^2*d^4)-a*(2*a*c*d^2*e^2*(-9*B*e+7*C*
d)-c^2*d^4*(-3*B*e+C*d)-3*a^2*e^4*(-B*e+3*C*d)))*arctan(x*c^(1/2)/a^(1/2))*
c^(1/2)/a^(3/2)/(a*e^2+c*d^2)^4
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 524, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1661, 1643, 649, 211, 266}

$$\int \frac{A+Bx+Cx^2}{(d+ex)^3\left(a+cx^2\right)^2}\,dx$$

$$= -\frac{e\log\left(a+cx^2\right)\left(a^2Ce^4-2ace^2(4Cd^2-e(3Bd-Ae))+c^2(3Cd^4-2d^2e(3Bd-5Ae))\right)}{2\left(ae^2+cd^2\right)^4}$$

$$+\frac{e\log(d+ex)\left(a^2Ce^4-2ace^2(4Cd^2-e(3Bd-Ae))+c^2(3Cd^4-2d^2e(3Bd-5Ae))\right)}{\left(ae^2+cd^2\right)^4}$$

$$+\frac{\sqrt{c}\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(Acd(-15a^2e^4+10acd^2e^2+c^2d^4)-a(-3a^2e^4(3Cd-Be)+2acd^2e^2(7Cd-9Be)-\right.}{2a^{3/2}\left(ae^2+cd^2\right)^4}$$

$$-\frac{a(Bcd(cd^2-3ae^2)-e(Ac-aC)\left(3cd^2-ae^2\right))-cx(Acd(cd^2-3ae^2)-a(cd^2(Cd-3Be)-ae^2(3Cd}{2a\left(a+cx^2\right)\left(ae^2+cd^2\right)^3}$$

$$-\frac{e(Ae^2-Bde+Cd^2)}{2(d+ex)^2\left(ae^2+cd^2\right)^2}-\frac{e(-ae^2(2Cd-Be)-cde(3Bd-4Ae)+2cCd^3)}{(d+ex)\left(ae^2+cd^2\right)^3}$$

```
[In] Int[(A + B*x + C*x^2)/((d + e*x)^3*(a + c*x^2)^2),x]
```

```
[Out] -1/2*(e*(C*d^2 - B*d*e + A*e^2))/((c*d^2 + a*e^2)^2*(d + e*x)^2) - (e*(2*c*
C*d^3 - c*d*e*(3*B*d - 4*A*e) - a*e^2*(2*C*d - B*e)))/((c*d^2 + a*e^2)^3*(d
 + e*x)) - (a*(B*c*d*(c*d^2 - 3*a*e^2) - (A*c - a*C)*e*(3*c*d^2 - a*e^2)) -
 c*(A*c*d*(c*d^2 - 3*a*e^2) - a*(c*d^2*(C*d - 3*B*e) - a*e^2*(3*C*d - B*e))
)*x)/(2*a*(c*d^2 + a*e^2)^3*(a + c*x^2)) + (Sqrt[c]*(A*c*d*(c^2*d^4 + 10*a*
c*d^2*e^2 - 15*a^2*e^4) - a*(2*a*c*d^2*e^2*(7*C*d - 9*B*e) - c^2*d^4*(C*d -
 3*B*e) - 3*a^2*e^4*(3*C*d - B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(2*a^(3/2)
*(c*d^2 + a*e^2)^4) + (e*(a^2*C*e^4 + c^2*(3*C*d^4 - 2*d^2*e*(3*B*d - 5*A*e
)) - 2*a*c*e^2*(4*C*d^2 - e*(3*B*d - A*e)))*Log[d + e*x])/(c*d^2 + a*e^2)^4
 - (e*(a^2*C*e^4 + c^2*(3*C*d^4 - 2*d^2*e*(3*B*d - 5*A*e)) - 2*a*c*e^2*(4*C
*d^2 - e*(3*B*d - A*e)))*Log[a + c*x^2])/(2*(c*d^2 + a*e^2)^4)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

integral =

$$-\frac{a(Bcd(cd^2-3ae^2)-(Ac-aC)e(3cd^2-ae^2))-c(Acd(cd^2-3ae^2)-a(cd^2(Cd-3Be)-ae^2(3Cd-}{2a\left(cd^2+ae^2\right)^3(a+cx^2)}$$

$$-\frac{\int\frac{-\frac{c\left(A\left(c^3d^6+9ac^2d^4e^2+6a^2cd^2e^4+2a^3e^6\right)+acd^3\left(cd^2(Cd-3Be)-ae^2(3Cd-Be)\right)\right)}{\left(cd^2+ae^2\right)^3}-\frac{ce\left(Ac^2d^3\left(3cd^2+7ae^2\right)+a\left(2a^2Be^5-acd^2e^2(7Cd-9Be)-3c^2d^4(Cd-\right.\right.}{\left(cd^2+ae^2\right)^3}}{(d+e}}{}$$

$$=-\frac{a(Bcd(cd^2-3ae^2)-(Ac-aC)e(3cd^2-ae^2))-c(Acd(cd^2-3ae^2)-a(cd^2(Cd-3Be)-ae^2(}{2a\left(cd^2+ae^2\right)^3(a+cx^2)}$$

$$-\frac{\int\left(-\frac{2ace^2\left(Cd^2-Bde+Ae^2\right)}{(cd^2+ae^2)^2(d+ex)^3}+\frac{2ace^2\left(-2cCd^3+cde(3Bd-4Ae)+ae^2(2Cd-Be)\right)}{(cd^2+ae^2)^3(d+ex)^2}+\frac{2ace^2\left(-a^2Ce^4-c^2\left(3Cd^4-2d^2e(3Bd-5A\right.\right.}{(cd^2+ae^2)^4(d}\right.}{}$$

$$=-\frac{e(Cd^2-Bde+Ae^2)}{2\left(cd^2+ae^2\right)^2(d+ex)^2}-\frac{e(2cCd^3-cde(3Bd-4Ae)-ae^2(2Cd-Be))}{\left(cd^2+ae^2\right)^3(d+ex)}$$

$$-\frac{a(Bcd(cd^2-3ae^2)-(Ac-aC)e(3cd^2-ae^2))-c(Acd(cd^2-3ae^2)-a(cd^2(Cd-3Be)-ae^2(}{2a\left(cd^2+ae^2\right)^3(a+cx^2)}$$

$$+\frac{e(a^2Ce^4+c^2(3Cd^4-2d^2e(3Bd-5Ae))-2ace^2(4Cd^2-e(3Bd-Ae)))\log(d+ex)}{\left(cd^2+ae^2\right)^4}$$

$$-\frac{c\int\frac{-Acd\left(c^2d^4+10acd^2e^2-15a^2e^4\right)+a\left(2acd^2e^2(7Cd-9Be)-c^2d^4(Cd-3Be)-3a^2e^4(3Cd-Be)\right)+2ae\left(a^2Ce^4+c^2\left(3Cd^4-2d^2e(3B\right.\right.}{a+cx^2}}{2a\left(cd^2+ae^2\right)^4}$$

$$= -\frac{e(Cd^2-Bde+Ae^2)}{2\left(cd^2+ae^2\right)^2(d+ex)^2} - \frac{e(2cCd^3-cde(3Bd-4Ae)-ae^2(2Cd-Be))}{\left(cd^2+ae^2\right)^3(d+ex)}$$
$$-\frac{a(Bcd(cd^2-3ae^2)-(Ac-aC)e(3cd^2-ae^2))-c(Acd(cd^2-3ae^2)-a(cd^2(Cd-3Be)-ae^2}{2a\left(cd^2+ae^2\right)^3(a+cx^2)}$$
$$+\frac{e(a^2Ce^4+c^2(3Cd^4-2d^2e(3Bd-5Ae))-2ace^2(4Cd^2-e(3Bd-Ae)))\log(d+ex)}{\left(cd^2+ae^2\right)^4}$$
$$-\frac{(ce(a^2Ce^4+c^2(3Cd^4-2d^2e(3Bd-5Ae))-2ace^2(4Cd^2-e(3Bd-Ae))))\int\frac{x}{a+cx^2}\,dx}{\left(cd^2+ae^2\right)^4}$$
$$+\frac{(c(Acd(c^2d^4+10acd^2e^2-15a^2e^4)-a(2acd^2e^2(7Cd-9Be)-c^2d^4(Cd-3Be)-3a^2e^4(3Cd-}{2a\left(cd^2+ae^2\right)^4}$$

$$= -\frac{e(Cd^2-Bde+Ae^2)}{2\left(cd^2+ae^2\right)^2(d+ex)^2} - \frac{e(2cCd^3-cde(3Bd-4Ae)-ae^2(2Cd-Be))}{\left(cd^2+ae^2\right)^3(d+ex)}$$
$$-\frac{a(Bcd(cd^2-3ae^2)-(Ac-aC)e(3cd^2-ae^2))-c(Acd(cd^2-3ae^2)-a(cd^2(Cd-3Be)-ae^2}{2a\left(cd^2+ae^2\right)^3(a+cx^2)}$$
$$+\frac{\sqrt{c}(Acd(c^2d^4+10acd^2e^2-15a^2e^4)-a(2acd^2e^2(7Cd-9Be)-c^2d^4(Cd-3Be)-3a^2e^4(3Cd}{2a^{3/2}\left(cd^2+ae^2\right)^4}$$
$$+\frac{e(a^2Ce^4+c^2(3Cd^4-2d^2e(3Bd-5Ae))-2ace^2(4Cd^2-e(3Bd-Ae)))\log(d+ex)}{\left(cd^2+ae^2\right)^4}$$
$$-\frac{e(a^2Ce^4+c^2(3Cd^4-2d^2e(3Bd-5Ae))-2ace^2(4Cd^2-e(3Bd-Ae)))\log\left(a+cx^2\right)}{2\left(cd^2+ae^2\right)^4}$$

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 466, normalized size of antiderivative = 0.89

$$\int\frac{A+Bx+Cx^2}{(d+ex)^3\left(a+cx^2\right)^2}\,dx$$
$$=\frac{-\frac{e\left(cd^2+ae^2\right)^2\left(Cd^2+e(-Bd+Ae)\right)}{(d+ex)^2}-\frac{2e\left(cd^2+ae^2\right)\left(2cCd^3+cde(-3Bd+4Ae)+ae^2(-2Cd+Be)\right)}{d+ex}+\frac{\left(cd^2+ae^2\right)\left(a^3Ce^3+Ac^3d^3x-ac^2d(Cd^2}{}}{}$$

```
[In] Integrate[(A + B*x + C*x^2)/((d + e*x)^3*(a + c*x^2)^2),x]
```

```
[Out] (-((e*(c*d^2 + a*e^2)^2*(C*d^2 + e*(-(B*d) + A*e)))/(d + e*x)^2) - (2*e*(c*
d^2 + a*e^2)*(2*c*C*d^3 + c*d*e*(-3*B*d + 4*A*e) + a*e^2*(-2*C*d + B*e)))/(
d + e*x) + ((c*d^2 + a*e^2)*(a^3*C*e^3 + A*c^3*d^3*x - a*c^2*d*(C*d^2*x + B
*d*(d - 3*e*x) + 3*A*e*(-d + e*x)) - a^2*c*e*(3*C*d*(d - e*x) + e*(-3*B*d +
 A*e + B*e*x))))/(a*(a + c*x^2)) + (Sqrt[c]*(A*c*d*(c^2*d^4 + 10*a*c*d^2*e^
2 - 15*a^2*e^4) + a*(-2*a*c*d^2*e^2*(7*C*d - 9*B*e) + c^2*d^4*(C*d - 3*B*e)
 - 3*a^2*e^4*(-3*C*d + B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/a^(3/2) + 2*(a^2
*C*e^5 - 2*a*c*e^3*(4*C*d^2 + e*(-3*B*d + A*e)) + c^2*d^2*e*(3*C*d^2 + 2*e*
```

```
(-3*B*d + 5*A*e)))*Log[d + e*x] - (a^2*C*e^5 - 2*a*c*e^3*(4*C*d^2 + e*(-3*B
*d + A*e)) + c^2*d^2*e*(3*C*d^2 + 2*e*(-3*B*d + 5*A*e)))*Log[a + c*x^2])/(2
*(c*d^2 + a*e^2)^4)
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 643, normalized size of antiderivative = 1.23

method	result
default	$-\frac{c\left(\frac{\frac{\left(3A\,a^{2}cd\,e^{4}+2Aa\,c^{2}d^{3}e^{2}-d^{5}A\,c^{3}+B\,e^{5}a^{3}-2B\,a^{2}c\,d^{2}e^{3}-3Ba\,c^{2}d^{4}e-3C\,a^{3}d\,e^{4}-2C\,a^{2}c\,d^{3}e^{2}+C\,c^{2}d^{5}a\right)x}{2a}+\frac{A\,a^{2}c\,e^{5}-2Aa\,c^{2}d^{2}e^{3}-3A\,c^{3}d^{4}}{}}{c\,x^{2}+a}\right.}{}$
risch	Expression too large to display

```
[In] int((C*x^2+B*x+A)/(e*x+d)^3/(c*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -c/(a*e^2+c*d^2)^4*((1/2*(3*A*a^2*c*d*e^4+2*A*a*c^2*d^3*e^2-A*c^3*d^5+B*a^3
*e^5-2*B*a^2*c*d^2*e^3-3*B*a*c^2*d^4*e-3*C*a^3*d*e^4-2*C*a^2*c*d^3*e^2+C*a*
c^2*d^5)/a*x+1/2*(A*a^2*c*e^5-2*A*a*c^2*d^2*e^3-3*A*c^3*d^4*e-3*B*a^2*c*d*e
^4-2*B*a*c^2*d^3*e^2+B*c^3*d^5-C*a^3*e^5+2*C*a^2*c*d^2*e^3+3*C*a*c^2*d^4*e)
/c)/(c*x^2+a)+1/2/a*(1/2*(-4*A*a^2*c*e^5+20*A*a*c^2*d^2*e^3+12*B*a^2*c*d*e^
4-12*B*a*c^2*d^3*e^2+2*C*a^3*e^5-16*C*a^2*c*d^2*e^3+6*C*a*c^2*d^4*e)/c*ln(c
*x^2+a)+(15*A*a^2*c*d*e^4-10*A*a*c^2*d^3*e^2-A*c^3*d^5+3*B*a^3*e^5-18*B*a^2
*c*d^2*e^3+3*B*a*c^2*d^4*e-9*C*a^3*d*e^4+14*C*a^2*c*d^3*e^2-C*a*c^2*d^5)/(a
*c)^(1/2)*arctan(c*x/(a*c)^(1/2))))-e*(4*A*c*d*e^2+B*a*e^3-3*B*c*d^2*e-2*C*
a*d*e^2+2*C*c*d^3)/(a*e^2+c*d^2)^3/(e*x+d)-1/2*e*(A*e^2-B*d*e+C*d^2)/(a*e^2
+c*d^2)^2/(e*x+d)^2-e*(2*A*a*c*e^4-10*A*c^2*d^2*e^2-6*B*a*c*d*e^3+6*B*c^2*d
^3*e-C*a^2*e^4+8*C*a*c*d^2*e^2-3*C*c^2*d^4)/(a*e^2+c*d^2)^4*ln(e*x+d)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(d+ex)^3\left(a+cx^2\right)^2}\,dx=\text{Timed out}$$

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^3/(c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(d+ex)^3\left(a+cx^2\right)^2}\,dx=\text{Timed out}$$

```
[In] integrate((C*x**2+B*x+A)/(e*x+d)**3/(c*x**2+a)**2,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1030 vs. $2(505) = 1010$.

Time = 0.32 (sec) , antiderivative size = 1030, normalized size of antiderivative = 1.97

$$\int \frac{A+Bx+Cx^2}{(d+ex)^3\left(a+cx^2\right)^2}\,dx=$$
$$-\frac{(3\,Cc^2d^4e-6\,Bc^2d^3e^2+6\,Bacde^4-2\,(4\,Cac-5\,Ac^2)d^2e^3+(Ca^2-2\,Aac)e^5)\log\left(cx^2+a\right)}{2\,(c^4d^8+4\,ac^3d^6e^2+6\,a^2c^2d^4e^4+4\,a^3cd^2e^6+a^4e^8)}$$
$$+\frac{(3\,Cc^2d^4e-6\,Bc^2d^3e^2+6\,Bacde^4-2\,(4\,Cac-5\,Ac^2)d^2e^3+(Ca^2-2\,Aac)e^5)\log\left(ex+d\right)}{c^4d^8+4\,ac^3d^6e^2+6\,a^2c^2d^4e^4+4\,a^3cd^2e^6+a^4e^8}$$
$$-\frac{(3\,Bac^3d^4e-18\,Ba^2c^2d^2e^3+3\,Ba^3ce^5-(Cac^3+Ac^4)d^5+2\,(7\,Ca^2c^2-5\,Aac^3)d^3e^2-3\,(3\,Ca^3c-5\,A}{2\,(ac^4d^8+4\,a^2c^3d^6e^2+6\,a^3c^2d^4e^4+4\,a^4cd^2e^6+a^5e^8)\sqrt{ac}}$$
$$-\frac{Bac^2d^5-10\,Ba^2cd^3e^2+Ba^3de^4+Aa^3e^5+(8\,Ca^2c-3\,Aac^2)d^4e-2\,(2\,Ca^3-5\,Aa^2c)d^2e^3-(9\,Bac^2}{2\,(a^2c^3d^8+3\,a^3c^2d^6e^2+3\,a^4cd^4e^4+a^5d^2e^6}$$

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^3/(c*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(3*C*c^2*d^4*e - 6*B*c^2*d^3*e^2 + 6*B*a*c*d*e^4 - 2*(4*C*a*c - 5*A*c^
2)*d^2*e^3 + (C*a^2 - 2*A*a*c)*e^5)*log(c*x^2 + a)/(c^4*d^8 + 4*a*c^3*d^6*e
^2 + 6*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8) + (3*C*c^2*d^4*e - 6*B*
c^2*d^3*e^2 + 6*B*a*c*d*e^4 - 2*(4*C*a*c - 5*A*c^2)*d^2*e^3 + (C*a^2 - 2*A*
a*c)*e^5)*log(e*x + d)/(c^4*d^8 + 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 + 4*a
^3*c*d^2*e^6 + a^4*e^8) - 1/2*(3*B*a*c^3*d^4*e - 18*B*a^2*c^2*d^2*e^3 + 3*B
*a^3*c*e^5 - (C*a*c^3 + A*c^4)*d^5 + 2*(7*C*a^2*c^2 - 5*A*a*c^3)*d^3*e^2 -
3*(3*C*a^3*c - 5*A*a^2*c^2)*d*e^4)*arctan(c*x/sqrt(a*c))/((a*c^4*d^8 + 4*a^
2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4*c*d^2*e^6 + a^5*e^8)*sqrt(a*c)) -
 1/2*(B*a*c^2*d^5 - 10*B*a^2*c*d^3*e^2 + B*a^3*d*e^4 + A*a^3*e^5 + (8*C*a^2
*c - 3*A*a*c^2)*d^4*e - 2*(2*C*a^3 - 5*A*a^2*c)*d^2*e^3 - (9*B*a*c^2*d^2*e^
3 - 3*B*a^2*c*e^5 - (5*C*a*c^2 - A*c^3)*d^3*e^2 + (7*C*a^2*c - 11*A*a*c^2)*
d*e^4)*x^3 - (12*B*a*c^2*d^3*e^2 - (7*C*a*c^2 - 2*A*c^3)*d^4*e + 6*(C*a^2*c
 - 2*A*a*c^2)*d^2*e^3 + (C*a^3 - 2*A*a^2*c)*e^5)*x^2 - (B*a*c^2*d^4*e + 11*
B*a^2*c*d^2*e^3 - 2*B*a^3*e^5 - (C*a*c^2 - A*c^3)*d^5 - (7*C*a^2*c - 3*A*a*
```

```
c^2)*d^3*e^2 + 2*(3*C*a^3 - 5*A*a^2*c)*d*e^4)*x)/(a^2*c^3*d^8 + 3*a^3*c^2*d
^6*e^2 + 3*a^4*c*d^4*e^4 + a^5*d^2*e^6 + (a*c^4*d^6*e^2 + 3*a^2*c^3*d^4*e^4
 + 3*a^3*c^2*d^2*e^6 + a^4*c*e^8)*x^4 + 2*(a*c^4*d^7*e + 3*a^2*c^3*d^5*e^3
+ 3*a^3*c^2*d^3*e^5 + a^4*c*d*e^7)*x^3 + (a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6
*a^3*c^2*d^4*e^4 + 4*a^4*c*d^2*e^6 + a^5*e^8)*x^2 + 2*(a^2*c^3*d^7*e + 3*a^
3*c^2*d^5*e^3 + 3*a^4*c*d^3*e^5 + a^5*d*e^7)*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1013 vs. $2(505) = 1010$.

Time = 0.27 (sec) , antiderivative size = 1013, normalized size of antiderivative = 1.93

$$\int \frac{A+Bx+Cx^2}{(d+ex)^3\left(a+cx^2\right)^2}\,dx = -\frac{(3Cc^2d^4e-6Bc^2d^3e^2-8Cacd^2e^3+10Ac^2d^2e^3+6Bacde^4+Ca^2e^5-2Aace^5)\log\left(cx^2+a\right)}{2\left(c^4d^8+4ac^3d^6e^2+6a^2c^2d^4e^4+4a^3cd^2e^6+a^4e^8\right)} + \frac{(3Cc^2d^4e^2-6Bc^2d^3e^3-8Cacd^2e^4+10Ac^2d^2e^4+6Bacde^5+Ca^2e^6-2Aace^6)\log\left(|ex+d|\right)}{c^4d^8e+4ac^3d^6e^3+6a^2c^2d^4e^5+4a^3cd^2e^7+a^4e^9} + \frac{(Cac^3d^5+Ac^4d^5-3Bac^3d^4e-14Ca^2c^2d^3e^2+10Aac^3d^3e^2+18Ba^2c^2d^2e^3+9Ca^3cde^4-15Aa^2c^2de^4}{2\left(ac^4d^8+4a^2c^3d^6e^2+6a^3c^2d^4e^4+4a^4cd^2e^6+a^5e^8\right)\sqrt{ac}} - \frac{Bac^3d^7+8Ca^2c^2d^6e-3Aac^3d^6e-9Ba^2c^2d^5e^2+4Ca^3cd^4e^3+7Aa^2c^2d^4e^3-9Ba^3cd^3e^4-4Ca^4d^2e^5}{}$$

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^3/(c*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(3*C*c^2*d^4*e - 6*B*c^2*d^3*e^2 - 8*C*a*c*d^2*e^3 + 10*A*c^2*d^2*e^3
+ 6*B*a*c*d*e^4 + C*a^2*e^5 - 2*A*a*c*e^5)*log(c*x^2 + a)/(c^4*d^8 + 4*a*c^
3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8) + (3*C*c^2*d^4*e
^2 - 6*B*c^2*d^3*e^3 - 8*C*a*c*d^2*e^4 + 10*A*c^2*d^2*e^4 + 6*B*a*c*d*e^5 +
 C*a^2*e^6 - 2*A*a*c*e^6)*log(abs(e*x + d))/(c^4*d^8*e + 4*a*c^3*d^6*e^3 +
6*a^2*c^2*d^4*e^5 + 4*a^3*c*d^2*e^7 + a^4*e^9) + 1/2*(C*a*c^3*d^5 + A*c^4*d
^5 - 3*B*a*c^3*d^4*e - 14*C*a^2*c^2*d^3*e^2 + 10*A*a*c^3*d^3*e^2 + 18*B*a^2
*c^2*d^2*e^3 + 9*C*a^3*c*d*e^4 - 15*A*a^2*c^2*d*e^4 - 3*B*a^3*c*e^5)*arctan
(c*x/sqrt(a*c))/((a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4
*c*d^2*e^6 + a^5*e^8)*sqrt(a*c)) - 1/2*(B*a*c^3*d^7 + 8*C*a^2*c^2*d^6*e - 3
*A*a*c^3*d^6*e - 9*B*a^2*c^2*d^5*e^2 + 4*C*a^3*c*d^4*e^3 + 7*A*a^2*c^2*d^4*
e^3 - 9*B*a^3*c*d^3*e^4 - 4*C*a^4*d^2*e^5 + 11*A*a^3*c*d^2*e^5 + B*a^4*d*e^
6 + A*a^4*e^7 + (5*C*a*c^3*d^5*e^2 - A*c^4*d^5*e^2 - 9*B*a*c^3*d^4*e^3 - 2*
C*a^2*c^2*d^3*e^4 + 10*A*a*c^3*d^3*e^4 - 6*B*a^2*c^2*d^2*e^5 - 7*C*a^3*c*d*
e^6 + 11*A*a^2*c^2*d*e^6 + 3*B*a^3*c*e^7)*x^3 + (7*C*a*c^3*d^6*e - 2*A*c^4*
d^6*e - 12*B*a*c^3*d^5*e^2 + C*a^2*c^2*d^4*e^3 + 10*A*a*c^3*d^4*e^3 - 12*B*
a^2*c^2*d^3*e^4 - 7*C*a^3*c*d^2*e^5 + 14*A*a^2*c^2*d^2*e^5 - C*a^4*e^7 + 2*
A*a^3*c*e^7)*x^2 + (C*a*c^3*d^7 - A*c^4*d^7 - B*a*c^3*d^6*e + 8*C*a^2*c^2*d
```

```
^5*e^2 - 4*A*a*c^3*d^5*e^2 - 12*B*a^2*c^2*d^4*e^3 + C*a^3*c*d^3*e^4 + 7*A*a
^2*c^2*d^3*e^4 - 9*B*a^3*c*d^2*e^5 - 6*C*a^4*d*e^6 + 10*A*a^3*c*d*e^6 + 2*B
*a^4*e^7)*x)/((c*d^2 + a*e^2)^4*(c*x^2 + a)*(e*x + d)^2*a)
```

Mupad [B] (verification not implemented)

Time = 27.67 (sec) , antiderivative size = 2828, normalized size of antiderivative = 5.40

$$\int \frac{A+Bx+Cx^2}{(d+ex)^3\,(a+cx^2)^2}\,dx = \text{Too large to display}$$

```
[In] int((A + B*x + C*x^2)/((a + c*x^2)^2*(d + e*x)^3),x)
```

```
[Out] (log(C*c^2*d^7*(-a^3*c)^(3/2) - 3*B*a^6*e^7*(-a^3*c)^(1/2) - 6*C*a^8*e^7 +
12*A*a^7*c*e^7 - 3*B*a^7*c*e^7*x + 2*A*a^4*c^4*d^6*e + 20*C*a^5*c^3*d^6*e +
 72*C*a^7*c*d^2*e^5 - A*a^3*c^5*d^7*x - C*a^4*c^4*d^7*x + 39*A*a^2*d*e^6*(-
a^3*c)^(3/2) + 21*C*a^6*d*e^6*(-a^3*c)^(1/2) - 3*B*c^2*d^6*e*(-a^3*c)^(3/2)
 + 12*A*a^2*e^7*x*(-a^3*c)^(3/2) + 6*C*a^6*e^7*x*(-a^3*c)^(1/2) + 80*A*a^5*
c^3*d^4*e^3 - 102*A*a^6*c^2*d^2*e^5 - 42*B*a^5*c^3*d^5*e^2 + 108*B*a^6*c^2*
d^3*e^4 - 94*C*a^6*c^2*d^4*e^3 - A*a^2*c^4*d^7*(-a^3*c)^(1/2) - 93*B*a^2*d^
2*e^5*(-a^3*c)^(3/2) + 9*A*c^2*d^5*e^2*(-a^3*c)^(3/2) + 119*C*a^2*d^3*e^4*(
-a^3*c)^(3/2) - 42*B*a^7*c*d*e^6 - 9*A*a^4*c^4*d^5*e^2*x + 145*A*a^5*c^3*d^
3*e^4*x - 93*B*a^5*c^3*d^4*e^3*x + 93*B*a^6*c^2*d^2*e^5*x + 51*C*a^5*c^3*d^
5*e^2*x - 119*C*a^6*c^2*d^3*e^4*x + 80*A*c^2*d^4*e^3*x*(-a^3*c)^(3/2) + 72*
C*a^2*d^2*e^5*x*(-a^3*c)^(3/2) - 42*B*c^2*d^5*e^2*x*(-a^3*c)^(3/2) + 21*C*a
^7*c*d*e^6*x - 39*A*a^6*c^2*d*e^6*x + 3*B*a^4*c^4*d^6*e*x - 145*A*a*c*d^3*e
^4*(-a^3*c)^(3/2) + 93*B*a*c*d^4*e^3*(-a^3*c)^(3/2) - 51*C*a*c*d^5*e^2*(-a^
3*c)^(3/2) - 42*B*a^2*d*e^6*x*(-a^3*c)^(3/2) + 20*C*c^2*d^6*e*x*(-a^3*c)^(3
/2) - 102*A*a*c*d^2*e^5*x*(-a^3*c)^(3/2) + 108*B*a*c*d^3*e^4*x*(-a^3*c)^(3/
2) - 94*C*a*c*d^4*e^3*x*(-a^3*c)^(3/2) - 2*A*a^2*c^4*d^6*e*x*(-a^3*c)^(1/2)
)*(e^2*(3*B*a^3*c^2*d^3 + (5*A*a*c^2*d^3*(-a^3*c)^(1/2))/2 - (7*C*a^2*c*d^3
*(-a^3*c)^(1/2))/2) + e^3*(4*C*a^4*c*d^2 - 5*A*a^3*c^2*d^2 + (9*B*a^2*c*d^2
*(-a^3*c)^(1/2))/2) - e^4*(3*B*a^4*c*d - (9*C*a^3*d*(-a^3*c)^(1/2))/4 + (15
*A*a^2*c*d*(-a^3*c)^(1/2))/4) - e*((3*C*a^3*c^2*d^4)/2 + (3*B*a*c^2*d^4*(-a
^3*c)^(1/2))/4) - e^5*((C*a^5)/2 + (3*B*a^3*(-a^3*c)^(1/2))/4 - A*a^4*c) +
(A*c^3*d^5*(-a^3*c)^(1/2))/4 + (C*a*c^2*d^5*(-a^3*c)^(1/2))/4))/(a^7*e^8 +
a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4) - (l
og(3*B*a^6*e^7*(-a^3*c)^(1/2) - 6*C*a^8*e^7 - C*c^2*d^7*(-a^3*c)^(3/2) + 12
*A*a^7*c*e^7 - 3*B*a^7*c*e^7*x + 2*A*a^4*c^4*d^6*e + 20*C*a^5*c^3*d^6*e + 7
2*C*a^7*c*d^2*e^5 - A*a^3*c^5*d^7*x - C*a^4*c^4*d^7*x - 39*A*a^2*d*e^6*(-a^
3*c)^(3/2) - 21*C*a^6*d*e^6*(-a^3*c)^(1/2) + 3*B*c^2*d^6*e*(-a^3*c)^(3/2) -
 12*A*a^2*e^7*x*(-a^3*c)^(3/2) - 6*C*a^6*e^7*x*(-a^3*c)^(1/2) + 80*A*a^5*c^
3*d^4*e^3 - 102*A*a^6*c^2*d^2*e^5 - 42*B*a^5*c^3*d^5*e^2 + 108*B*a^6*c^2*d^
3*e^4 - 94*C*a^6*c^2*d^4*e^3 + A*a^2*c^4*d^7*(-a^3*c)^(1/2) + 93*B*a^2*d^2*
e^5*(-a^3*c)^(3/2) - 9*A*c^2*d^5*e^2*(-a^3*c)^(3/2) - 119*C*a^2*d^3*e^4*(-a
```

```
^3*c)^(3/2) - 42*B*a^7*c*d*e^6 - 9*A*a^4*c^4*d^5*e^2*x + 145*A*a^5*c^3*d^3*
e^4*x - 93*B*a^5*c^3*d^4*e^3*x + 93*B*a^6*c^2*d^2*e^5*x + 51*C*a^5*c^3*d^5*
e^2*x - 119*C*a^6*c^2*d^3*e^4*x - 80*A*c^2*d^4*e^3*x*(-a^3*c)^(3/2) - 72*C*
a^2*d^2*e^5*x*(-a^3*c)^(3/2) + 42*B*c^2*d^5*e^2*x*(-a^3*c)^(3/2) + 21*C*a^7
*c*d*e^6*x - 39*A*a^6*c^2*d*e^6*x + 3*B*a^4*c^4*d^6*e*x + 145*A*a*c*d^3*e^4
*(-a^3*c)^(3/2) - 93*B*a*c*d^4*e^3*(-a^3*c)^(3/2) + 51*C*a*c*d^5*e^2*(-a^3*
c)^(3/2) + 42*B*a^2*d*e^6*x*(-a^3*c)^(3/2) - 20*C*c^2*d^6*e*x*(-a^3*c)^(3/2
) + 102*A*a*c*d^2*e^5*x*(-a^3*c)^(3/2) - 108*B*a*c*d^3*e^4*x*(-a^3*c)^(3/2)
 + 94*C*a*c*d^4*e^3*x*(-a^3*c)^(3/2) + 2*A*a^2*c^4*d^6*e*x*(-a^3*c)^(1/2))*
(e^3*(5*A*a^3*c^2*d^2 - 4*C*a^4*c*d^2 + (9*B*a^2*c*d^2*(-a^3*c)^(1/2))/2) -
 e^2*(3*B*a^3*c^2*d^3 - (5*A*a*c^2*d^3*(-a^3*c)^(1/2))/2 + (7*C*a^2*c*d^3*(
-a^3*c)^(1/2))/2) + e^4*(3*B*a^4*c*d + (9*C*a^3*d*(-a^3*c)^(1/2))/4 - (15*A
*a^2*c*d*(-a^3*c)^(1/2))/4) + e*((3*C*a^3*c^2*d^4)/2 - (3*B*a*c^2*d^4*(-a^3
*c)^(1/2))/4) - e^5*((3*B*a^3*(-a^3*c)^(1/2))/4 - (C*a^5)/2 + A*a^4*c) + (A
*c^3*d^5*(-a^3*c)^(1/2))/4 + (C*a*c^2*d^5*(-a^3*c)^(1/2))/4))/(a^7*e^8 + a^
3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4) - ((A*
a^2*e^5 + B*c^2*d^5 + B*a^2*d*e^4 - 3*A*c^2*d^4*e - 4*C*a^2*d^2*e^3 + 8*C*a
*c*d^4*e + 10*A*a*c*d^2*e^3 - 10*B*a*c*d^3*e^2)/(2*(a*e^2 + c*d^2)*(a^2*e^4
 + c^2*d^4 + 2*a*c*d^2*e^2)) + (x^3*(3*B*a^2*c*e^5 - A*c^3*d^3*e^2 - 9*B*a*
c^2*d^2*e^3 + 5*C*a*c^2*d^3*e^2 + 11*A*a*c^2*d*e^4 - 7*C*a^2*c*d*e^4))/(2*a
*(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)) - (x*(A*c^3*d^5 -
 2*B*a^3*e^5 - C*a*c^2*d^5 + 6*C*a^3*d*e^4 + 3*A*a*c^2*d^3*e^2 + 11*B*a^2*c
*d^2*e^3 - 7*C*a^2*c*d^3*e^2 - 10*A*a^2*c*d*e^4 + B*a*c^2*d^4*e))/(2*a*(a*e
^2 + c*d^2)*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)) - (x^2*(C*a^3*e^5 - 2*A*a^
2*c*e^5 + 2*A*c^3*d^4*e - 12*A*a*c^2*d^2*e^3 + 12*B*a*c^2*d^3*e^2 + 6*C*a^2
*c*d^2*e^3 - 7*C*a*c^2*d^4*e))/(2*a*(a*e^2 + c*d^2)*(a^2*e^4 + c^2*d^4 + 2*
a*c*d^2*e^2)))/(a*d^2 + x^2*(a*e^2 + c*d^2) + c*e^2*x^4 + 2*a*d*e*x + 2*c*d
*e*x^3) + (log(d + e*x)*(c^2*(10*A*d^2*e^3 - 6*B*d^3*e^2 + 3*C*d^4*e) - c*(
2*A*a*e^5 - 6*B*a*d*e^4 + 8*C*a*d^2*e^3) + C*a^2*e^5))/(a^4*e^8 + c^4*d^8 +
 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)
```

3.57 $\int \frac{(d+ex)^3(A+Bx+Cx^2)}{(a+cx^2)^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 209

$$
\begin{aligned}
\int \frac{(d+ex)^3\,(A+Bx+Cx^2)}{(a+cx^2)^3}\,dx
&= -\frac{(aB-(Ac-aC)x)(d+ex)^3}{4ac\,(a+cx^2)^2} \\
&\quad - \frac{(d+ex)\,(ae(3Acd+5aCd+3aBe)-(3Ac^2d^2-a(4aCe^2-cd(Cd+3Be)))\,x)}{8a^2c^2\,(a+cx^2)} \\
&\quad + \frac{(3ae^2(Acd+3aCd+aBe)+cd^2(3Acd+aCd+3aBe))\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8a^{5/2}c^{5/2}} \\
&\quad + \frac{Ce^3\log\left(a+cx^2\right)}{2c^3}
\end{aligned}
$$

```
[Out] -1/4*(a*B-(A*c-C*a)*x)*(e*x+d)^3/a/c/(c*x^2+a)^2-1/8*(e*x+d)*(a*e*(3*A*c*d+
3*B*a*e+5*C*a*d)-(3*A*c^2*d^2-a*(4*a*C*e^2-c*d*(3*B*e+C*d)))*x)/a^2/c^2/(c*
x^2+a)+1/8*(3*A*c*d*(a*e^2+c*d^2)+a*(3*a*e^2*(B*e+3*C*d)+c*d^2*(3*B*e+C*d))
)*arctan(x*c^(1/2)/a^(1/2))/a^(5/2)/c^(5/2)+1/2*C*e^3*ln(c*x^2+a)/c^3
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1659, 833, 649, 211, 266}

$$\begin{aligned}
&\int \frac{(d+ex)^3\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^3}\,dx \\
&\quad= \frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(cd^2(3aBe+aCd+3Acd)+3ae^2(aBe+3aCd+Acd)\right)}{8a^{5/2}c^{5/2}} \\
&\quad-\frac{(d+ex)\left(ae(3aBe+5aCd+3Acd)-x\left(3Ac^2d^2-a\left(4aCe^2-cd(3Be+Cd)\right)\right)\right)}{8a^2c^2\left(a+cx^2\right)} \\
&\quad-\frac{(d+ex)^3(aB-x(Ac-aC))}{4ac\left(a+cx^2\right)^2}+\frac{Ce^3\log\left(a+cx^2\right)}{2c^3}
\end{aligned}$$

```
[In] Int[((d + e*x)^3*(A + B*x + C*x^2))/(a + c*x^2)^3,x]
```

```
[Out] -1/4*((a*B - (A*c - a*C)*x)*(d + e*x)^3)/(a*c*(a + c*x^2)^2) - ((d + e*x)*(
a*e*(3*A*c*d + 5*a*C*d + 3*a*B*e) - (3*A*c^2*d^2 - a*(4*a*C*e^2 - c*d*(C*d
+ 3*B*e)))*x))/(8*a^2*c^2*(a + c*x^2)) + ((3*a*e^2*(A*c*d + 3*a*C*d + a*B*e
) + c*d^2*(3*A*c*d + a*C*d + 3*a*B*e))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(8*a^(5
/2)*c^(5/2)) + (C*e^3*Log[a + c*x^2])/(2*c^3)
```

<u>Rule 211</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

<u>Rule 266</u>

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

<u>Rule 649</u>

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

<u>Rule 833</u>

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
```

```
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 1659

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c*(p
+ 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(aB-(Ac-aC)x)(d+ex)^3}{4ac\left(a+cx^2\right)^2}-\frac{\int \frac{(d+ex)^2(-3Acd-aCd-3aBe-4aCex)}{(a+cx^2)^2}\,dx}{4ac} \\
&= -\frac{(aB-(Ac-aC)x)(d+ex)^3}{4ac\left(a+cx^2\right)^2} \\
&\quad -\frac{(d+ex)\left(ae(3Acd+5aCd+3aBe)-\left(3Ac^2d^2-a(4aCe^2-cd(Cd+3Be))\right)x\right)}{8a^2c^2\left(a+cx^2\right)} \\
&\quad -\frac{\int \frac{-3ae^2(Acd+3aCd+aBe)-cd^2(3Acd+aCd+3aBe)-8a^2Ce^3x}{a+cx^2}\,dx}{8a^2c^2} \\
&= -\frac{(aB-(Ac-aC)x)(d+ex)^3}{4ac\left(a+cx^2\right)^2} \\
&\quad -\frac{(d+ex)\left(ae(3Acd+5aCd+3aBe)-\left(3Ac^2d^2-a(4aCe^2-cd(Cd+3Be))\right)x\right)}{8a^2c^2\left(a+cx^2\right)} \\
&\quad +\frac{\left(Ce^3\right)\int \frac{x}{a+cx^2}\,dx}{c^2} \\
&\quad +\frac{\left(3ae^2(Acd+3aCd+aBe)+cd^2(3Acd+aCd+3aBe)\right)\int \frac{1}{a+cx^2}\,dx}{8a^2c^2}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{(aB-(Ac-aC)x)(d+ex)^3}{4ac\left(a+cx^2\right)^2} \\
&\quad -\frac{(d+ex)\left(ae(3Acd+5aCd+3aBe)-\left(3Ac^2d^2-a(4aCe^2-cd(Cd+3Be))\right)x\right)}{8a^2c^2\left(a+cx^2\right)} \\
&\quad +\frac{\left(3ae^2(Acd+3aCd+aBe)+cd^2(3Acd+aCd+3aBe)\right)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8a^{5/2}c^{5/2}} \\
&\quad +\frac{Ce^3\log\left(a+cx^2\right)}{2c^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.34

$$
\int \frac{(d+ex)^3\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^3}\,dx
$$

$$
= \frac{\frac{-2a^3Ce^3+2Ac^3d^3x-2ac^2d(Cd^2x+3Ae(d+ex)+Bd(d+3ex))+2a^2ce(3Cd(d+ex)+e(3Bd+Ae+Bex))}{a(a+cx^2)^2} + 8a^3Ce^3+3Ac^3d^3x+ac^2d(Cd^2+3e(Bd}{}
$$

```
[In] Integrate[((d + e*x)^3*(A + B*x + C*x^2))/(a + c*x^2)^3,x]
```

```
[Out] ((-2*a^3*C*e^3 + 2*A*c^3*d^3*x - 2*a*c^2*d*(C*d^2*x + 3*A*e*(d + e*x) + B*d
*(d + 3*e*x)) + 2*a^2*c*e*(3*C*d*(d + e*x) + e*(3*B*d + A*e + B*e*x)))/(a*(
a + c*x^2)^2) + (8*a^3*C*e^3 + 3*A*c^3*d^3*x + a*c^2*d*(C*d^2 + 3*e*(B*d +
A*e))*x - a^2*c*e*(3*C*d*(4*d + 5*e*x) + e*(12*B*d + 4*A*e + 5*B*e*x)))/(a^
2*(a + c*x^2)) + (Sqrt[c]*(3*A*c*d*(c*d^2 + a*e^2) + a*(3*a*e^2*(3*C*d + B*
e) + c*d^2*(C*d + 3*B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/a^(5/2) + 4*C*e^3*L
og[a + c*x^2])/(8*c^3)
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.59

method	result
default	$\frac{\frac{\left(3Aacde^2+3Ad^3c^2-5a^2Be^3+3Bacd^2e-15Ca^2de^2+Cacd^3\right)x^3}{8ca^2}-\frac{e\left(Ace^2+3Bcde-2aCe^2+3Ccd^2\right)x^2}{2c^2}-\frac{(3Aacde^2-5Ad^3c^2+3a^2Be^3+3Bacd^2}{8ac^2}}{\left(cx^2+a\right)^2}$
risch	$\frac{\frac{\left(3Aacde^2+3Ad^3c^2-5a^2Be^3+3Bacd^2e-15Ca^2de^2+Cacd^3\right)x^3}{8ca^2}-\frac{e\left(Ace^2+3Bcde-2aCe^2+3Ccd^2\right)x^2}{2c^2}-\frac{(3Aacde^2-5Ad^3c^2+3a^2Be^3+3Bacd^2}{8ac^2}}{\left(cx^2+a\right)^2}$

```
[In] int((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (1/8*(3*A*a*c*d*e^2+3*A*c^2*d^3-5*B*a^2*e^3+3*B*a*c*d^2*e-15*C*a^2*d*e^2+C*
a*c*d^3)/c/a^2*x^3-1/2*e*(A*c*e^2+3*B*c*d*e-2*C*a*e^2+3*C*c*d^2)/c^2*x^2-1/
```

```
8*(3*A*a*c*d*e^2-5*A*c^2*d^3+3*B*a^2*e^3+3*B*a*c*d^2*e+9*C*a^2*d*e^2+C*a*c*
d^3)/a/c^2*x-1/4*(A*a*c*e^3+3*A*c^2*d^2*e+3*B*a*c*d*e^2+B*c^2*d^3-3*C*a^2*e
^3+3*C*a*c*d^2*e)/c^3)/(c*x^2+a)^2+1/8/a^2/c^2*(4*C*a^2*e^3/c*ln(c*x^2+a)+(
3*A*a*c*d*e^2+3*A*c^2*d^3+3*B*a^2*e^3+3*B*a*c*d^2*e+9*C*a^2*d*e^2+C*a*c*d^3
)/(a*c)^(1/2)*arctan(c*x/(a*c)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 559 vs. 2(198) = 396.

Time = 0.48 (sec) , antiderivative size = 1138, normalized size of antiderivative = 5.44

$$\int \frac{(d+ex)^3\,(A+Bx+Cx^2)}{(a+cx^2)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*B*a^3*c^2*d^3 + 12*B*a^4*c*d*e^2 + 12*(C*a^4*c + A*a^3*c^2)*d^2*e
 - 4*(3*C*a^5 - A*a^4*c)*e^3 - 2*(3*B*a^2*c^3*d^2*e - 5*B*a^3*c^2*e^3 + (C*
a^2*c^3 + 3*A*a*c^4)*d^3 - 3*(5*C*a^3*c^2 - A*a^2*c^3)*d*e^2)*x^3 + 8*(3*C*
a^3*c^2*d^2*e + 3*B*a^3*c^2*d*e^2 - (2*C*a^4*c - A*a^3*c^2)*e^3)*x^2 + (3*B
*a^3*c*d^2*e + 3*B*a^4*e^3 + (3*B*a*c^3*d^2*e + 3*B*a^2*c^2*e^3 + (C*a*c^3
+ 3*A*c^4)*d^3 + 3*(3*C*a^2*c^2 + A*a*c^3)*d*e^2)*x^4 + (C*a^3*c + 3*A*a^2*
c^2)*d^3 + 3*(3*C*a^4 + A*a^3*c)*d*e^2 + 2*(3*B*a^2*c^2*d^2*e + 3*B*a^3*c*e
^3 + (C*a^2*c^2 + 3*A*a*c^3)*d^3 + 3*(3*C*a^3*c + A*a^2*c^2)*d*e^2)*x^2)*sq
rt(-a*c)*log((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) + 2*(3*B*a^3*c^2*d^2
*e + 3*B*a^4*c*e^3 + (C*a^3*c^2 - 5*A*a^2*c^3)*d^3 + 3*(3*C*a^4*c + A*a^3*c
^2)*d*e^2)*x - 8*(C*a^3*c^2*e^3*x^4 + 2*C*a^4*c*e^3*x^2 + C*a^5*e^3)*log(c*
x^2 + a))/(a^3*c^5*x^4 + 2*a^4*c^4*x^2 + a^5*c^3), -1/8*(2*B*a^3*c^2*d^3 +
6*B*a^4*c*d*e^2 + 6*(C*a^4*c + A*a^3*c^2)*d^2*e - 2*(3*C*a^5 - A*a^4*c)*e^3
 - (3*B*a^2*c^3*d^2*e - 5*B*a^3*c^2*e^3 + (C*a^2*c^3 + 3*A*a*c^4)*d^3 - 3*(
5*C*a^3*c^2 - A*a^2*c^3)*d*e^2)*x^3 + 4*(3*C*a^3*c^2*d^2*e + 3*B*a^3*c^2*d*
e^2 - (2*C*a^4*c - A*a^3*c^2)*e^3)*x^2 - (3*B*a^3*c*d^2*e + 3*B*a^4*e^3 + (
3*B*a*c^3*d^2*e + 3*B*a^2*c^2*e^3 + (C*a*c^3 + 3*A*c^4)*d^3 + 3*(3*C*a^2*c^
2 + A*a*c^3)*d*e^2)*x^4 + (C*a^3*c + 3*A*a^2*c^2)*d^3 + 3*(3*C*a^4 + A*a^3*
c)*d*e^2 + 2*(3*B*a^2*c^2*d^2*e + 3*B*a^3*c*e^3 + (C*a^2*c^2 + 3*A*a*c^3)*d
^3 + 3*(3*C*a^3*c + A*a^2*c^2)*d*e^2)*x^2)*sqrt(a*c)*arctan(sqrt(a*c)*x/a)
+ (3*B*a^3*c^2*d^2*e + 3*B*a^4*c*e^3 + (C*a^3*c^2 - 5*A*a^2*c^3)*d^3 + 3*(3
*C*a^4*c + A*a^3*c^2)*d*e^2)*x - 4*(C*a^3*c^2*e^3*x^4 + 2*C*a^4*c*e^3*x^2 +
 C*a^5*e^3)*log(c*x^2 + a))/(a^3*c^5*x^4 + 2*a^4*c^4*x^2 + a^5*c^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3 (A+Bx+Cx^2)}{(a+cx^2)^3}\,dx = \text{Timed out}$$

```
[In] integrate((e*x+d)**3*(C*x**2+B*x+A)/(c*x**2+a)**3,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.81

$$\int \frac{(d+ex)^3 (A+Bx+Cx^2)}{(a+cx^2)^3}\,dx = \frac{Ce^3 \log (cx^2+a)}{2\,c^3} - \frac{2\,Ba^2c^2d^3 + 6\,Ba^3cde^2 + 6\,(Ca^3c + Aa^2c^2)d^2e - 2\,(3\,Ca^4 - Aa^3c)e^3 - (3\,Bac^3d^2e - 5\,Ba^2c^2e^3 + (Cac^3}{} + \frac{(3\,Bacd^2e + 3\,Ba^2e^3 + (Cac + 3\,Ac^2)d^3 + 3\,(3\,Ca^2 + Aac)de^2) \arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\,\sqrt{ac}a^2c^2}$$

```
[In] integrate((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*C*e^3*log(c*x^2 + a)/c^3 - 1/8*(2*B*a^2*c^2*d^3 + 6*B*a^3*c*d*e^2 + 6*(
C*a^3*c + A*a^2*c^2)*d^2*e - 2*(3*C*a^4 - A*a^3*c)*e^3 - (3*B*a*c^3*d^2*e -
 5*B*a^2*c^2*e^3 + (C*a*c^3 + 3*A*c^4)*d^3 - 3*(5*C*a^2*c^2 - A*a*c^3)*d*e^
2)*x^3 + 4*(3*C*a^2*c^2*d^2*e + 3*B*a^2*c^2*d*e^2 - (2*C*a^3*c - A*a^2*c^2)
*e^3)*x^2 + (3*B*a^2*c^2*d^2*e + 3*B*a^3*c*e^3 + (C*a^2*c^2 - 5*A*a*c^3)*d^
3 + 3*(3*C*a^3*c + A*a^2*c^2)*d*e^2)*x)/(a^2*c^5*x^4 + 2*a^3*c^4*x^2 + a^4*
c^3) + 1/8*(3*B*a*c*d^2*e + 3*B*a^2*e^3 + (C*a*c + 3*A*c^2)*d^3 + 3*(3*C*a^
2 + A*a*c)*d*e^2)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a^2*c^2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.71

$$\int \frac{(d+ex)^3 (A+Bx+Cx^2)}{(a+cx^2)^3}\,dx = \frac{Ce^3 \log (cx^2+a)}{2\,c^3} + \frac{(Cacd^3 + 3\,Ac^2d^3 + 3\,Bacd^2e + 9\,Ca^2de^2 + 3\,Aacde^2 + 3\,Ba^2e^3) \arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\,\sqrt{ac}a^2c^2} + \frac{(Cac^2d^3 + 3\,Ac^3d^3 + 3\,Bac^2d^2e - 15\,Ca^2cde^2 + 3\,Aac^2de^2 - 5\,Ba^2ce^3)x^3 - 4\,(3\,Ca^2cd^2e + 3\,Ba^2cde^2 -}{}$$

```
[In] integrate((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/2*C*e^3*log(c*x^2 + a)/c^3 + 1/8*(C*a*c*d^3 + 3*A*c^2*d^3 + 3*B*a*c*d^2*e
 + 9*C*a^2*d*e^2 + 3*A*a*c*d*e^2 + 3*B*a^2*e^3)*arctan(c*x/sqrt(a*c))/(sqrt
(a*c)*a^2*c^2) + 1/8*((C*a*c^2*d^3 + 3*A*c^3*d^3 + 3*B*a*c^2*d^2*e - 15*C*a
^2*c*d*e^2 + 3*A*a*c^2*d*e^2 - 5*B*a^2*c*e^3)*x^3 - 4*(3*C*a^2*c*d^2*e + 3*
B*a^2*c*d*e^2 - 2*C*a^3*e^3 + A*a^2*c*e^3)*x^2 - (C*a^2*c*d^3 - 5*A*a*c^2*d
^3 + 3*B*a^2*c*d^2*e + 9*C*a^3*d*e^2 + 3*A*a^2*c*d*e^2 + 3*B*a^3*e^3)*x - 2
*(B*a^2*c^2*d^3 + 3*C*a^3*c*d^2*e + 3*A*a^2*c^2*d^2*e + 3*B*a^3*c*d*e^2 - 3
*C*a^4*e^3 + A*a^3*c*e^3)/c)/((c*x^2 + a)^2*a^2*c^2)
```

Mupad [B] (verification not implemented)

Time = 1.76 (sec) , antiderivative size = 920, normalized size of antiderivative = 4.40

$$\int \frac{(d+ex)^3\,(A+Bx+Cx^2)}{\left(a+cx^2\right)^3}\,dx = \frac{5\,A\,d^3\,x}{8\,\left(a^3+2\,a^2\,c\,x^2+a\,c^2\,x^4\right)}-\frac{B\,d^3}{4\,\left(a^2\,c+2\,a\,c^2\,x^2+c^3\,x^4\right)}+\frac{3\,C\,a^2\,e^3}{4\,\left(a^2\,c^3+2\,a\,c^4\,x^2+c^5\,x^4\right)}-\frac{3\,A\,d^2\,e}{4\,\left(a^2\,c+2\,a\,c^2\,x^2+c^3\,x^4\right)}+\frac{C\,d^3\,x^3}{8\,\left(a^3+2\,a^2\,c\,x^2+a\,c^2\,x^4\right)}-\frac{C\,d^3\,x}{8\,\left(a^2\,c+2\,a\,c^2\,x^2+c^3\,x^4\right)}-\frac{A\,a\,e^3}{4\,\left(a^2\,c^2+2\,a\,c^3\,x^2+c^4\,x^4\right)}-\frac{A\,e^3\,x^2}{2\,\left(a^2\,c+2\,a\,c^2\,x^2+c^3\,x^4\right)}-\frac{5\,B\,e^3\,x^3}{8\,\left(a^2\,c+2\,a\,c^2\,x^2+c^3\,x^4\right)}+\frac{C\,e^3\,\ln\left(c\,x^2+a\right)}{2\,c^3}-\frac{3\,B\,a\,d\,e^2}{4\,\left(a^2\,c^2+2\,a\,c^3\,x^2+c^4\,x^4\right)}-\frac{3\,C\,a\,d^2\,e}{4\,\left(a^2\,c^2+2\,a\,c^3\,x^2+c^4\,x^4\right)}+\frac{3\,A\,c\,d^3\,x^3}{8\,\left(a^4+2\,a^3\,c\,x^2+a^2\,c^2\,x^4\right)}-\frac{3\,B\,a\,e^3\,x}{8\,\left(a^2\,c^2+2\,a\,c^3\,x^2+c^4\,x^4\right)}-\frac{3\,B\,d\,e^2\,x^2}{2\,\left(a^2\,c+2\,a\,c^2\,x^2+c^3\,x^4\right)}-\frac{3\,C\,d^2\,e\,x^2}{2\,\left(a^2\,c+2\,a\,c^2\,x^2+c^3\,x^4\right)}-\frac{15\,C\,d\,e^2\,x^3}{8\,\left(a^2\,c+2\,a\,c^2\,x^2+c^3\,x^4\right)}+\frac{C\,a\,e^3\,x^2}{a^2\,c^2+2\,a\,c^3\,x^2+c^4\,x^4}+\frac{3\,A\,d^3\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{8\,a^{5/2}\,\sqrt{c}}+\frac{3\,B\,e^3\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{8\,\sqrt{a}\,c^{5/2}}+\frac{C\,d^3\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{8\,a^{3/2}\,c^{3/2}}+\frac{3\,A\,d\,e^2\,x^3}{8\,\left(a^3+2\,a^2\,c\,x^2+a\,c^2\,x^4\right)}+\frac{3\,B\,d^2\,e\,x^3}{8\,\left(a^3+2\,a^2\,c\,x^2+a\,c^2\,x^4\right)}-\frac{3\,A\,d\,e^2\,x}{8\,\left(a^2\,c+2\,a\,c^2\,x^2+c^3\,x^4\right)}-\frac{3\,B\,d^2\,e\,x}{8\,\left(a^2\,c+2\,a\,c^2\,x^2+c^3\,x^4\right)}+\frac{3\,A\,d\,e^2\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{8\,a^{3/2}\,c^{3/2}}+\frac{3\,B\,d^2\,e\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{8\,a^{3/2}\,c^{3/2}}+\frac{9\,C\,d\,e^2\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{8\,\sqrt{a}\,c^{5/2}}-\frac{9\,C\,a\,d\,e^2\,x}{8\,\left(a^2\,c^2+2\,a\,c^3\,x^2+c^4\,x^4\right)}$$

```
[In] int(((d + e*x)^3*(A + B*x + C*x^2))/(a + c*x^2)^3,x)
```

```
[Out] (5*A*d^3*x)/(8*(a^3 + 2*a^2*c*x^2 + a*c^2*x^4)) - (B*d^3)/(4*(a^2*c + c^3*x
^4 + 2*a*c^2*x^2)) + (3*C*a^2*e^3)/(4*(a^2*c^3 + c^5*x^4 + 2*a*c^4*x^2)) -
(3*A*d^2*e)/(4*(a^2*c + c^3*x^4 + 2*a*c^2*x^2)) + (C*d^3*x^3)/(8*(a^3 + 2*a
^2*c*x^2 + a*c^2*x^4)) - (C*d^3*x)/(8*(a^2*c + c^3*x^4 + 2*a*c^2*x^2)) - (A
*a*e^3)/(4*(a^2*c^2 + c^4*x^4 + 2*a*c^3*x^2)) - (A*e^3*x^2)/(2*(a^2*c + c^3
*x^4 + 2*a*c^2*x^2)) - (5*B*e^3*x^3)/(8*(a^2*c + c^3*x^4 + 2*a*c^2*x^2)) +
(C*e^3*log(a + c*x^2))/(2*c^3) - (3*B*a*d*e^2)/(4*(a^2*c^2 + c^4*x^4 + 2*a*
c^3*x^2)) - (3*C*a*d^2*e)/(4*(a^2*c^2 + c^4*x^4 + 2*a*c^3*x^2)) + (3*A*c*d^
3*x^3)/(8*(a^4 + 2*a^3*c*x^2 + a^2*c^2*x^4)) - (3*B*a*e^3*x)/(8*(a^2*c^2 +
c^4*x^4 + 2*a*c^3*x^2)) - (3*B*d*e^2*x^2)/(2*(a^2*c + c^3*x^4 + 2*a*c^2*x^2
)) - (3*C*d^2*e*x^2)/(2*(a^2*c + c^3*x^4 + 2*a*c^2*x^2)) - (15*C*d*e^2*x^3)
/(8*(a^2*c + c^3*x^4 + 2*a*c^2*x^2)) + (C*a*e^3*x^2)/(a^2*c^2 + c^4*x^4 + 2
*a*c^3*x^2) + (3*A*d^3*atan((c^(1/2)*x)/a^(1/2)))/(8*a^(5/2)*c^(1/2)) + (3*
B*e^3*atan((c^(1/2)*x)/a^(1/2)))/(8*a^(1/2)*c^(5/2)) + (C*d^3*atan((c^(1/2)
*x)/a^(1/2)))/(8*a^(3/2)*c^(3/2)) + (3*A*d*e^2*x^3)/(8*(a^3 + 2*a^2*c*x^2 +
 a*c^2*x^4)) + (3*B*d^2*e*x^3)/(8*(a^3 + 2*a^2*c*x^2 + a*c^2*x^4)) - (3*A*d
*e^2*x)/(8*(a^2*c + c^3*x^4 + 2*a*c^2*x^2)) - (3*B*d^2*e*x)/(8*(a^2*c + c^3
*x^4 + 2*a*c^2*x^2)) + (3*A*d*e^2*atan((c^(1/2)*x)/a^(1/2)))/(8*a^(3/2)*c^(
3/2)) + (3*B*d^2*e*atan((c^(1/2)*x)/a^(1/2)))/(8*a^(3/2)*c^(3/2)) + (9*C*d*
e^2*atan((c^(1/2)*x)/a^(1/2)))/(8*a^(1/2)*c^(5/2)) - (9*C*a*d*e^2*x)/(8*(a^
2*c^2 + c^4*x^4 + 2*a*c^3*x^2))
```

3.58 $\int \frac{(d+ex)^2(A+Bx+Cx^2)}{(a+cx^2)^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 156

$$\begin{aligned}\int \frac{(d+ex)^2\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^3}\,dx &= -\frac{(aB-(Ac-aC)x)(d+ex)^2}{4ac\left(a+cx^2\right)^2}-\frac{(d+ex)(a(Ac+3aC)e-c(3Acd+aCd+2aBe)x)}{8a^2c^2\left(a+cx^2\right)}\\&\quad+\frac{(a(Ac+3aC)e^2+cd(3Acd+aCd+2aBe))\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8a^{5/2}c^{5/2}}\end{aligned}$$

```
[Out] -1/4*(a*B-(A*c-C*a)*x)*(e*x+d)^2/a/c/(c*x^2+a)^2-1/8*(e*x+d)*(a*(A*c+3*C*a)
*e-c*(3*A*c*d+2*B*a*e+C*a*d)*x)/a^2/c^2/(c*x^2+a)+1/8*(a*(A*c+3*C*a)*e^2+c*
d*(3*A*c*d+2*B*a*e+C*a*d))*arctan(x*c^(1/2)/a^(1/2))/a^(5/2)/c^(5/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.12, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1659, 792, 211}

$$\begin{aligned}\int \frac{(d+ex)^2\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^3}\,dx &= \frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(cd(2aBe+aCd+3Acd)+ae^2(3aC+Ac)\right)}{8a^{5/2}c^{5/2}}\\&\quad-\frac{x\left(ae^2(3aC+Ac)-cd(2aBe+aCd+3Acd)\right)+2ae(aBe+2aCd+2Acd)}{8a^2c^2\left(a+cx^2\right)}\\&\quad-\frac{(d+ex)^2(aB-x(Ac-aC))}{4ac\left(a+cx^2\right)^2}\end{aligned}$$

```
[In] Int[((d + e*x)^2*(A + B*x + C*x^2))/(a + c*x^2)^3,x]
```

```
[Out] -1/4*((a*B - (A*c - a*C)*x)*(d + e*x)^2)/(a*c*(a + c*x^2)^2) - (2*a*e*(2*A*
c*d + 2*a*C*d + a*B*e) + (a*(A*c + 3*a*C)*e^2 - c*d*(3*A*c*d + a*C*d + 2*a*
B*e))*x)/(8*a^2*c^2*(a + c*x^2)) + ((a*(A*c + 3*a*C)*e^2 + c*d*(3*A*c*d + a
*C*d + 2*a*B*e))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(8*a^(5/2)*c^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(a*(e*f + d*g) - (c*d*f - a*e*g)*x)*((a + c*x^2)^(p + 1)/(
2*a*c*(p + 1))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 1659

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c*(p
+ 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(aB-(Ac-aC)x)(d+ex)^2}{4ac\left(a+cx^2\right)^2} - \frac{\int \frac{(d+ex)(-3Acd-aCd-2aBe-(Ac+3aC)ex)}{\left(a+cx^2\right)^2}\,dx}{4ac} \\
&= -\frac{(aB-(Ac-aC)x)(d+ex)^2}{4ac\left(a+cx^2\right)^2} \\
&\quad - \frac{2ae(2Acd+2aCd+aBe)+\left(a(Ac+3aC)e^2-cd(3Acd+aCd+2aBe)\right)x}{8a^2c^2\left(a+cx^2\right)} \\
&\quad + \frac{\left(a(Ac+3aC)e^2+cd(3Acd+aCd+2aBe)\right)\int \frac{1}{a+cx^2}\,dx}{8a^2c^2}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{(aB-(Ac-aC)x)(d+ex)^2}{4ac\left(a+cx^2\right)^2}\\
&\quad -\frac{2ae(2Acd+2aCd+aBe)+(a(Ac+3aC)e^2-cd(3Acd+aCd+2aBe))\,x}{8a^2c^2\left(a+cx^2\right)}\\
&\quad +\frac{(a(Ac+3aC)e^2+cd(3Acd+aCd+2aBe))\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8a^{5/2}c^{5/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.35

$$
\begin{aligned}
&\int \frac{(d+ex)^2\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^3}\,dx\\
&= \frac{3Ac^2d^2x+ac(Cd^2+e(2Bd+Ae))\,x-a^2e(8Cd+4Be+5Cex)}{8a^2c^2\left(a+cx^2\right)}\\
&\quad +\frac{Ac^2d^2x+a^2e(2Cd+Be+Cex)-ac(Cd^2x+Ae(2d+ex)+Bd(d+2ex))}{4ac^2\left(a+cx^2\right)^2}\\
&\quad +\frac{(Ac(3cd^2+ae^2)+a(3aCe^2+cd(Cd+2Be)))\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8a^{5/2}c^{5/2}}
\end{aligned}
$$

```
[In] Integrate[((d + e*x)^2*(A + B*x + C*x^2))/(a + c*x^2)^3,x]
```

```
[Out] (3*A*c^2*d^2*x + a*c*(C*d^2 + e*(2*B*d + A*e))*x - a^2*e*(8*C*d + 4*B*e + 5
*C*e*x))/(8*a^2*c^2*(a + c*x^2)) + (A*c^2*d^2*x + a^2*e*(2*C*d + B*e + C*e*
x) - a*c*(C*d^2*x + A*e*(2*d + e*x) + B*d*(d + 2*e*x)))/(4*a*c^2*(a + c*x^2
)^2) + ((A*c*(3*c*d^2 + a*e^2) + a*(3*a*C*e^2 + c*d*(C*d + 2*B*e)))*ArcTan[
(Sqrt[c]*x)/Sqrt[a]])/(8*a^(5/2)*c^(5/2))
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.42

method	result
default	$\frac{\frac{\left(Aac\,e^{2}+3A\,c^{2}d^{2}+2Bacde-5a^{2}C\,e^{2}+Cac\,d^{2}\right)x^{3}}{8c\,a^{2}}-\frac{e(Be+2Cd)x^{2}}{2c}-\frac{\left(Aac\,e^{2}-5A\,c^{2}d^{2}+2Bacde+3a^{2}C\,e^{2}+Cac\,d^{2}\right)x}{8a\,c^{2}}-\frac{2Acde+Ba\,e^{2}+Bc\,d^{2}+2adeC}{4c^{2}}}{\left(c\,x^{2}+a\right)^{2}}$
risch	$\frac{\frac{\left(Aac\,e^{2}+3A\,c^{2}d^{2}+2Bacde-5a^{2}C\,e^{2}+Cac\,d^{2}\right)x^{3}}{8c\,a^{2}}-\frac{e(Be+2Cd)x^{2}}{2c}-\frac{\left(Aac\,e^{2}-5A\,c^{2}d^{2}+2Bacde+3a^{2}C\,e^{2}+Cac\,d^{2}\right)x}{8a\,c^{2}}-\frac{2Acde+Ba\,e^{2}+Bc\,d^{2}+2adeC}{4c^{2}}}{\left(c\,x^{2}+a\right)^{2}}$

```
[In] int((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (1/8*(A*a*c*e^2+3*A*c^2*d^2+2*B*a*c*d*e-5*C*a^2*e^2+C*a*c*d^2)/c/a^2*x^3-1/
2*e*(B*e+2*C*d)*x^2/c-1/8*(A*a*c*e^2-5*A*c^2*d^2+2*B*a*c*d*e+3*C*a^2*e^2+C*
```

```
a*c*d^2)/a/c^2*x-1/4*(2*A*c*d*e+B*a*e^2+B*c*d^2+2*C*a*d*e)/c^2)/(c*x^2+a)^2
+1/8*(A*a*c*e^2+3*A*c^2*d^2+2*B*a*c*d*e+3*C*a^2*e^2+C*a*c*d^2)/a^2/c^2/(a*c
)^(1/2)*arctan(c*x/(a*c)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 393 vs. 2(141) = 282.

Time = 0.50 (sec) , antiderivative size = 806, normalized size of antiderivative = 5.17

$$\int \frac{(d+ex)^2\,(A+Bx+Cx^2)}{(a+cx^2)^3}\,dx$$

$$= \left[-\frac{4\,Ba^3c^2d^2+4\,Ba^4ce^2-2\,(2\,Ba^2c^3de+(Ca^2c^3+3\,Aac^4)d^2-(5\,Ca^3c^2-Aa^2c^3)e^2)x^3+8\,(Ca^4c+A}{}\right.$$

$$-\frac{2\,Ba^3c^2d^2+2\,Ba^4ce^2-(2\,Ba^2c^3de+(Ca^2c^3+3\,Aac^4)d^2-(5\,Ca^3c^2-Aa^2c^3)e^2)x^3+4\,(Ca^4c+Aa^3}{}$$

```
[In] integrate((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*B*a^3*c^2*d^2 + 4*B*a^4*c*e^2 - 2*(2*B*a^2*c^3*d*e + (C*a^2*c^3 +
 3*A*a*c^4)*d^2 - (5*C*a^3*c^2 - A*a^2*c^3)*e^2)*x^3 + 8*(C*a^4*c + A*a^3*c
^2)*d*e + 8*(2*C*a^3*c^2*d*e + B*a^3*c^2*e^2)*x^2 + (2*B*a^3*c*d*e + (2*B*a
*c^3*d*e + (C*a*c^3 + 3*A*c^4)*d^2 + (3*C*a^2*c^2 + A*a*c^3)*e^2)*x^4 + (C*
a^3*c + 3*A*a^2*c^2)*d^2 + (3*C*a^4 + A*a^3*c)*e^2 + 2*(2*B*a^2*c^2*d*e + (
C*a^2*c^2 + 3*A*a*c^3)*d^2 + (3*C*a^3*c + A*a^2*c^2)*e^2)*x^2)*sqrt(-a*c)*l
og((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) + 2*(2*B*a^3*c^2*d*e + (C*a^3*
c^2 - 5*A*a^2*c^3)*d^2 + (3*C*a^4*c + A*a^3*c^2)*e^2)*x)/(a^3*c^5*x^4 + 2*a
^4*c^4*x^2 + a^5*c^3), -1/8*(2*B*a^3*c^2*d^2 + 2*B*a^4*c*e^2 - (2*B*a^2*c^3
*d*e + (C*a^2*c^3 + 3*A*a*c^4)*d^2 - (5*C*a^3*c^2 - A*a^2*c^3)*e^2)*x^3 + 4
*(C*a^4*c + A*a^3*c^2)*d*e + 4*(2*C*a^3*c^2*d*e + B*a^3*c^2*e^2)*x^2 - (2*B
*a^3*c*d*e + (2*B*a*c^3*d*e + (C*a*c^3 + 3*A*c^4)*d^2 + (3*C*a^2*c^2 + A*a*
c^3)*e^2)*x^4 + (C*a^3*c + 3*A*a^2*c^2)*d^2 + (3*C*a^4 + A*a^3*c)*e^2 + 2*(
2*B*a^2*c^2*d*e + (C*a^2*c^2 + 3*A*a*c^3)*d^2 + (3*C*a^3*c + A*a^2*c^2)*e^2
)*x^2)*sqrt(a*c)*arctan(sqrt(a*c)*x/a) + (2*B*a^3*c^2*d*e + (C*a^3*c^2 - 5*
A*a^2*c^3)*d^2 + (3*C*a^4*c + A*a^3*c^2)*e^2)*x)/(a^3*c^5*x^4 + 2*a^4*c^4*x
^2 + a^5*c^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2 (A+Bx+Cx^2)}{(a+cx^2)^3}\,dx = \text{Timed out}$$

```
[In] integrate((e*x+d)**2*(C*x**2+B*x+A)/(c*x**2+a)**3,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.62

$$\int \frac{(d+ex)^2 (A+Bx+Cx^2)}{(a+cx^2)^3}\,dx = -\frac{2Ba^2cd^2 + 2Ba^3e^2 - (2Bac^2de + (Cac^2 + 3Ac^3)d^2 - (5Ca^2c - Aac^2)e^2)x^3 + 4(Ca^3 + Aa^2c)de + 4(2}{8(a^2c^4x^4 + 2a^3c^3x^2 + a^4c^2} + \frac{(2Bacde + (Cac + 3Ac^2)d^2 + (3Ca^2 + Aac)e^2)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\sqrt{ac}a^2c^2}$$

```
[In] integrate((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(2*B*a^2*c*d^2 + 2*B*a^3*e^2 - (2*B*a*c^2*d*e + (C*a*c^2 + 3*A*c^3)*d^
2 - (5*C*a^2*c - A*a*c^2)*e^2)*x^3 + 4*(C*a^3 + A*a^2*c)*d*e + 4*(2*C*a^2*c
*d*e + B*a^2*c*e^2)*x^2 + (2*B*a^2*c*d*e + (C*a^2*c - 5*A*a*c^2)*d^2 + (3*C
*a^3 + A*a^2*c)*e^2)*x)/(a^2*c^4*x^4 + 2*a^3*c^3*x^2 + a^4*c^2) + 1/8*(2*B*
a*c*d*e + (C*a*c + 3*A*c^2)*d^2 + (3*C*a^2 + A*a*c)*e^2)*arctan(c*x/sqrt(a*
c))/(sqrt(a*c)*a^2*c^2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.64

$$\int \frac{(d+ex)^2 (A+Bx+Cx^2)}{(a+cx^2)^3}\,dx = \frac{(Cacd^2 + 3Ac^2d^2 + 2Bacde + 3Ca^2e^2 + Aace^2)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\sqrt{ac}a^2c^2} + \frac{Cac^2d^2x^3 + 3Ac^3d^2x^3 + 2Bac^2dex^3 - 5Ca^2ce^2x^3 + Aac^2e^2x^3 - 8Ca^2cdex^2 - 4Ba^2ce^2x^2 - Ca^2cd^2x +}{8(cx^2+a)^2a}$$

```
[In] integrate((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(C*a*c*d^2 + 3*A*c^2*d^2 + 2*B*a*c*d*e + 3*C*a^2*e^2 + A*a*c*e^2)*arcta
n(c*x/sqrt(a*c))/(sqrt(a*c)*a^2*c^2) + 1/8*(C*a*c^2*d^2*x^3 + 3*A*c^3*d^2*x
^3 + 2*B*a*c^2*d*e*x^3 - 5*C*a^2*c*e^2*x^3 + A*a*c^2*e^2*x^3 - 8*C*a^2*c*d*
e*x^2 - 4*B*a^2*c*e^2*x^2 - C*a^2*c*d^2*x + 5*A*a*c^2*d^2*x - 2*B*a^2*c*d*e
*x - 3*C*a^3*e^2*x - A*a^2*c*e^2*x - 2*B*a^2*c*d^2 - 4*C*a^3*d*e - 4*A*a^2*
c*d*e - 2*B*a^3*e^2)/((c*x^2 + a)^2*a^2*c^2)
```

Mupad [B] (verification not implemented)

Time = 12.66 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.47

$$\int \frac{(d+ex)^2\,(A+Bx+Cx^2)}{(a+cx^2)^3}\,dx$$

$$= \frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\,\left(3\,C\,a^2\,e^2+C\,a\,c\,d^2+2\,B\,a\,c\,d\,e+A\,a\,c\,e^2+3\,A\,c^2\,d^2\right)}{8\,a^{5/2}\,c^{5/2}}$$

$$- \frac{\frac{B\,a\,e^2+B\,c\,d^2+2\,A\,c\,d\,e+2\,C\,a\,d\,e}{4\,c^2}+\frac{x^2\,\left(B\,e^2+2\,C\,d\,e\right)}{2\,c}+\frac{x\,\left(3\,C\,a^2\,e^2+C\,a\,c\,d^2+2\,B\,a\,c\,d\,e+A\,a\,c\,e^2-5\,A\,c^2\,d^2\right)}{8\,a\,c^2}-\frac{x^3\,\left(-5\,C\,a^2\,e^2+C\,a\,c}{}}{a^2+2\,a\,c\,x^2+c^2\,x^4}$$

```
[In] int(((d + e*x)^2*(A + B*x + C*x^2))/(a + c*x^2)^3,x)
```

```
[Out] (atan((c^(1/2)*x)/a^(1/2))*(3*A*c^2*d^2 + 3*C*a^2*e^2 + A*a*c*e^2 + C*a*c*d
^2 + 2*B*a*c*d*e))/(8*a^(5/2)*c^(5/2)) - ((B*a*e^2 + B*c*d^2 + 2*A*c*d*e +
2*C*a*d*e)/(4*c^2) + (x^2*(B*e^2 + 2*C*d*e))/(2*c) + (x*(3*C*a^2*e^2 - 5*A*
c^2*d^2 + A*a*c*e^2 + C*a*c*d^2 + 2*B*a*c*d*e))/(8*a*c^2) - (x^3*(3*A*c^2*d
^2 - 5*C*a^2*e^2 + A*a*c*e^2 + C*a*c*d^2 + 2*B*a*c*d*e))/(8*a^2*c))/(a^2 +
c^2*x^4 + 2*a*c*x^2)
```

3.59 $\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^3} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 130

$$\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^3} \, dx = -\frac{(aB-(Ac-aC)x)(d+ex)}{4ac\left(a+cx^2\right)^2} - \frac{2a(Ac+aC)e-c(3Acd+aCd+aBe)x}{8a^2c^2\left(a+cx^2\right)} + \frac{(3Acd+aCd+aBe)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8a^{5/2}c^{3/2}}$$

```
[Out] -1/4*(a*B-(A*c-C*a)*x)*(e*x+d)/a/c/(c*x^2+a)^2+1/8*(-2*a*(A*c+C*a)*e+c*(3*A
*c*d+B*a*e+C*a*d)*x)/a^2/c^2/(c*x^2+a)+1/8*(3*A*c*d+B*a*e+C*a*d)*arctan(x*c
^(1/2)/a^(1/2))/a^(5/2)/c^(3/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1659, 653, 211}

$$\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^3} \, dx = \frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(aBe+aCd+3Acd)}{8a^{5/2}c^{3/2}} - \frac{2ae(aC+Ac)-cx(aBe+aCd+3Acd)}{8a^2c^2\left(a+cx^2\right)} - \frac{(d+ex)(aB-x(Ac-aC))}{4ac\left(a+cx^2\right)^2}$$

```
[In] Int[((d + e*x)*(A + B*x + C*x^2))/(a + c*x^2)^3,x]
```

```
[Out] -1/4*((a*B - (A*c - a*C)*x)*(d + e*x))/(a*c*(a + c*x^2)^2) - (2*a*(A*c + a*
C)*e - c*(3*A*c*d + a*C*d + a*B*e)*x)/(8*a^2*c^2*(a + c*x^2)) + ((3*A*c*d +
 a*C*d + a*B*e)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(8*a^(5/2)*c^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)/(2*a*c*(p + 1)))*(a + c*x^2)^(p + 1), x] + Dist[d*((2*p + 3)/(2*a
*(p + 1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 1659

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c*(p
+ 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(aB-(Ac-aC)x)(d+ex)}{4ac\left(a+cx^2\right)^2}-\frac{\int \frac{-3Acd-a(Cd+Be)-2(Ac+aC)ex}{\left(a+cx^2\right)^2}\,dx}{4ac}\\
&= -\frac{(aB-(Ac-aC)x)(d+ex)}{4ac\left(a+cx^2\right)^2}-\frac{2a(Ac+aC)e-c(3Acd+aCd+aBe)x}{8a^2c^2\left(a+cx^2\right)}\\
&\quad+\frac{(3Acd+aCd+aBe)\int \frac{1}{a+cx^2}\,dx}{8a^2c}\\
&= -\frac{(aB-(Ac-aC)x)(d+ex)}{4ac\left(a+cx^2\right)^2}-\frac{2a(Ac+aC)e-c(3Acd+aCd+aBe)x}{8a^2c^2\left(a+cx^2\right)}\\
&\quad+\frac{(3Acd+aCd+aBe)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8a^{5/2}c^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.05

$$\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^3}\,dx = \frac{\frac{\sqrt{a}\left(-4a^2Ce+3Ac^2dx+ac(Cd+Be)x\right)}{a+cx^2}+\frac{2a^{3/2}\left(a^2Ce+Ac^2dx-ac(Ae+Cdx+B(d+ex))\right)}{\left(a+cx^2\right)^2}+\sqrt{c}(3Acd+aCd+aBe)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8a^{5/2}c^2}$$

```
[In] Integrate[((d + e*x)*(A + B*x + C*x^2))/(a + c*x^2)^3,x]
```

```
[Out] ((Sqrt[a]*(-4*a^2*C*e + 3*A*c^2*d*x + a*c*(C*d + B*e)*x))/(a + c*x^2) + (2*
a^(3/2)*(a^2*C*e + A*c^2*d*x - a*c*(A*e + C*d*x + B*(d + e*x))))/(a + c*x^2
)^2 + Sqrt[c]*(3*A*c*d + a*C*d + a*B*e)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(8*a^(
5/2)*c^2)
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.95

method	result
default	$\frac{\frac{(3Acd+Bae+Cad)x^3}{8a^2}-\frac{Ce\,x^2}{2c}+\frac{(5Acd-Bae-Cad)x}{8ac}-\frac{Ace+Bcd+Cae}{4c^2}}{\left(c\,x^2+a\right)^2}+\frac{(3Acd+Bae+Cad)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8a^2c\sqrt{ac}}$
risch	$\frac{\frac{(3Acd+Bae+Cad)x^3}{8a^2}-\frac{Ce\,x^2}{2c}+\frac{(5Acd-Bae-Cad)x}{8ac}-\frac{Ace+Bcd+Cae}{4c^2}}{\left(c\,x^2+a\right)^2}-\frac{3\ln\left(cx+\sqrt{-ac}\right)Ad}{16\sqrt{-ac}\,a^2}-\frac{\ln\left(cx+\sqrt{-ac}\right)Be}{16\sqrt{-ac}\,ca}-\frac{\ln\left(cx+\sqrt{-ac}\right)Cd}{16\sqrt{-ac}\,ca}-$

```
[In] int((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (1/8*(3*A*c*d+B*a*e+C*a*d)/a^2*x^3-1/2*C*e*x^2/c+1/8*(5*A*c*d-B*a*e-C*a*d)/
a/c*x-1/4*(A*c*e+B*c*d+C*a*e)/c^2)/(c*x^2+a)^2+1/8*(3*A*c*d+B*a*e+C*a*d)/a^
2/c/(a*c)^(1/2)*arctan(c*x/(a*c)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 470, normalized size of antiderivative = 3.62

$$\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^3}\,dx$$
$$= \left[-\frac{8\,Ca^3cex^2+4\,Ba^3cd-2\left(Ba^2c^2e+\left(Ca^2c^2+3\,Aac^3\right)d\right)x^3+\left(Ba^3e+\left(Bac^2e+\left(Cac^2+3\,Ac^3\right)d\right)x^4+\right.}{}\right.$$
$$-\frac{4\,Ca^3cex^2+2\,Ba^3cd-\left(Ba^2c^2e+\left(Ca^2c^2+3\,Aac^3\right)d\right)x^3-\left(Ba^3e+\left(Bac^2e+\left(Cac^2+3\,Ac^3\right)d\right)x^4+2\right.}{8\,(}$$

```
[In] integrate((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(8*C*a^3*c*e*x^2 + 4*B*a^3*c*d - 2*(B*a^2*c^2*e + (C*a^2*c^2 + 3*A*a
*c^3)*d)*x^3 + (B*a^3*e + (B*a*c^2*e + (C*a*c^2 + 3*A*c^3)*d)*x^4 + 2*(B*a^
2*c*e + (C*a^2*c + 3*A*a*c^2)*d)*x^2 + (C*a^3 + 3*A*a^2*c)*d)*sqrt(-a*c)*lo
g((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) + 4*(C*a^4 + A*a^3*c)*e + 2*(B*
a^3*c*e + (C*a^3*c - 5*A*a^2*c^2)*d)*x)/(a^3*c^4*x^4 + 2*a^4*c^3*x^2 + a^5*
c^2), -1/8*(4*C*a^3*c*e*x^2 + 2*B*a^3*c*d - (B*a^2*c^2*e + (C*a^2*c^2 + 3*A
*a*c^3)*d)*x^3 - (B*a^3*e + (B*a*c^2*e + (C*a*c^2 + 3*A*c^3)*d)*x^4 + 2*(B*
a^2*c*e + (C*a^2*c + 3*A*a*c^2)*d)*x^2 + (C*a^3 + 3*A*a^2*c)*d)*sqrt(a*c)*a
rctan(sqrt(a*c)*x/a) + 2*(C*a^4 + A*a^3*c)*e + (B*a^3*c*e + (C*a^3*c - 5*A*
a^2*c^2)*d)*x)/(a^3*c^4*x^4 + 2*a^4*c^3*x^2 + a^5*c^2)]
```

Sympy [A] (verification not implemented)

Time = 11.40 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.85

$$\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^3}\,dx = -\frac{\sqrt{-\frac{1}{a^5c^3}}\cdot(3Acd+Bae+Cad)\log\left(-a^3c\sqrt{-\frac{1}{a^5c^3}}+x\right)}{16}$$
$$+\frac{\sqrt{-\frac{1}{a^5c^3}}\cdot(3Acd+Bae+Cad)\log\left(a^3c\sqrt{-\frac{1}{a^5c^3}}+x\right)}{16}$$
$$+\frac{-2Aa^2ce-2Ba^2cd-2Ca^3e-4Ca^2cex^2+x^3\cdot\left(3Ac^3d+Bac^2e+Cac^2d\right)+x(5Aac^2d-Ba^2ce-Ca}{8a^4c^2+16a^3c^3x^2+8a^2c^4x^4}$$

```
[In] integrate((e*x+d)*(C*x**2+B*x+A)/(c*x**2+a)**3,x)
```

```
[Out] -sqrt(-1/(a**5*c**3))*(3*A*c*d + B*a*e + C*a*d)*log(-a**3*c*sqrt(-1/(a**5*c
**3)) + x)/16 + sqrt(-1/(a**5*c**3))*(3*A*c*d + B*a*e + C*a*d)*log(a**3*c*s
qrt(-1/(a**5*c**3)) + x)/16 + (-2*A*a**2*c*e - 2*B*a**2*c*d - 2*C*a**3*e -
4*C*a**2*c*e*x**2 + x**3*(3*A*c**3*d + B*a*c**2*e + C*a*c**2*d) + x*(5*A*a*
c**2*d - B*a**2*c*e - C*a**2*c*d))/(8*a**4*c**2 + 16*a**3*c**3*x**2 + 8*a**
2*c**4*x**4)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.23

$$\int \frac{(d+ex)(A+Bx+Cx^2)}{(a+cx^2)^3}\,dx = -\frac{4\,Ca^2cex^2+2\,Ba^2cd-(Bac^2e+(Cac^2+3\,Ac^3)d)x^3+2\,(Ca^3+Aa^2c)e+(Ba^2ce+(Ca^2c-5\,Aac^2)d)}{8\,(a^2c^4x^4+2\,a^3c^3x^2+a^4c^2)} + \frac{(Bae+(Ca+3\,Ac)d)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\,\sqrt{ac}a^2c}$$

```
[In] integrate((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(4*C*a^2*c*e*x^2 + 2*B*a^2*c*d - (B*a*c^2*e + (C*a*c^2 + 3*A*c^3)*d)*x
^3 + 2*(C*a^3 + A*a^2*c)*e + (B*a^2*c*e + (C*a^2*c - 5*A*a*c^2)*d)*x)/(a^2*
c^4*x^4 + 2*a^3*c^3*x^2 + a^4*c^2) + 1/8*(B*a*e + (C*a + 3*A*c)*d)*arctan(c
*x/sqrt(a*c))/(sqrt(a*c)*a^2*c)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.12

$$\int \frac{(d+ex)(A+Bx+Cx^2)}{(a+cx^2)^3}\,dx = \frac{(Cad+3\,Acd+Bae)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\,\sqrt{ac}a^2c} + \frac{Cac^2dx^3+3\,Ac^3dx^3+Bac^2ex^3-4\,Ca^2cex^2-Ca^2cdx+5\,Aac^2dx-Ba^2cex-2\,Ba^2cd-2\,Ca^3e-2\,A}{8\,(cx^2+a)^2a^2c^2}$$

```
[In] integrate((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(C*a*d + 3*A*c*d + B*a*e)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a^2*c) + 1/8
*(C*a*c^2*d*x^3 + 3*A*c^3*d*x^3 + B*a*c^2*e*x^3 - 4*C*a^2*c*e*x^2 - C*a^2*c
*d*x + 5*A*a*c^2*d*x - B*a^2*c*e*x - 2*B*a^2*c*d - 2*C*a^3*e - 2*A*a^2*c*e)
/((c*x^2 + a)^2*a^2*c^2)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.98

$$\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^3}\,dx$$
$$=\frac{\operatorname{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\,\left(3\,A\,c\,d+B\,a\,e+C\,a\,d\right)}{8\,a^{5/2}\,c^{3/2}}-\frac{\frac{A\,c\,e+B\,c\,d+C\,a\,e}{4\,c^2}-\frac{x^3\,\left(3\,A\,c\,d+B\,a\,e+C\,a\,d\right)}{8\,a^2}+\frac{C\,e\,x^2}{2\,c}+\frac{x\,\left(B\,a\,e-5\,A\,c\,d+C\,a\,d\right)}{8\,a\,c}}{a^2+2\,a\,c\,x^2+c^2\,x^4}$$

```
[In] int(((d + e*x)*(A + B*x + C*x^2))/(a + c*x^2)^3,x)
```

```
[Out] (atan((c^(1/2)*x)/a^(1/2))*(3*A*c*d + B*a*e + C*a*d))/(8*a^(5/2)*c^(3/2)) -
 ((A*c*e + B*c*d + C*a*e)/(4*c^2) - (x^3*(3*A*c*d + B*a*e + C*a*d))/(8*a^2)
 + (C*e*x^2)/(2*c) + (x*(B*a*e - 5*A*c*d + C*a*d))/(8*a*c))/(a^2 + c^2*x^4
+ 2*a*c*x^2)
```

3.60 $\int \frac{A+Bx+Cx^2}{\left(a+cx^2\right)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 98

$$\int \frac{A+Bx+Cx^2}{\left(a+cx^2\right)^3}\,dx=-\frac{aB-(Ac-aC)x}{4ac\left(a+cx^2\right)^2}+\frac{(3Ac+aC)x}{8a^2c\left(a+cx^2\right)}+\frac{(3Ac+aC)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8a^{5/2}c^{3/2}}$$

```
[Out] 1/4*(-a*B+(A*c-C*a)*x)/a/c/(c*x^2+a)^2+1/8*(3*A*c+C*a)*x/a^2/c/(c*x^2+a)+1/
8*(3*A*c+C*a)*arctan(x*c^(1/2)/a^(1/2))/a^(5/2)/c^(3/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1828, 12, 205, 211}

$$\int \frac{A+Bx+Cx^2}{\left(a+cx^2\right)^3}\,dx=\frac{(aC+3Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8a^{5/2}c^{3/2}}+\frac{x(aC+3Ac)}{8a^2c\left(a+cx^2\right)}-\frac{aB-x(Ac-aC)}{4ac\left(a+cx^2\right)^2}$$

```
[In] Int[(A + B*x + C*x^2)/(a + c*x^2)^3,x]
```

```
[Out] -1/4*(a*B - (A*c - a*C)*x)/(a*c*(a + c*x^2)^2) + ((3*A*c + a*C)*x)/(8*a^2*c
*(a + c*x^2)) + ((3*A*c + a*C)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(8*a^(5/2)*c^(3
/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{aB-(Ac-aC)x}{4ac\left(a+cx^2\right)^2}-\frac{\int \frac{-3A-\frac{aC}{c}}{\left(a+cx^2\right)^2}\,dx}{4a}\\
&= -\frac{aB-(Ac-aC)x}{4ac\left(a+cx^2\right)^2}+\frac{(3Ac+aC)\int \frac{1}{\left(a+cx^2\right)^2}\,dx}{4ac}\\
&= -\frac{aB-(Ac-aC)x}{4ac\left(a+cx^2\right)^2}+\frac{(3Ac+aC)x}{8a^2c\left(a+cx^2\right)}+\frac{(3Ac+aC)\int \frac{1}{a+cx^2}\,dx}{8a^2c}\\
&= -\frac{aB-(Ac-aC)x}{4ac\left(a+cx^2\right)^2}+\frac{(3Ac+aC)x}{8a^2c\left(a+cx^2\right)}+\frac{(3Ac+aC)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8a^{5/2}c^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx+Cx^2}{\left(a+cx^2\right)^3}\,dx$$
$$=\frac{3Ac^2x^3-a^2(2B+Cx)+acx\left(5A+Cx^2\right)}{8a^2c\left(a+cx^2\right)^2}+\frac{(3Ac+aC)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8a^{5/2}c^{3/2}}$$

```
[In] Integrate[(A + B*x + C*x^2)/(a + c*x^2)^3,x]
```

```
[Out] (3*A*c^2*x^3 - a^2*(2*B + C*x) + a*c*x*(5*A + C*x^2))/(8*a^2*c*(a + c*x^2)^
2) + ((3*A*c + a*C)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(8*a^(5/2)*c^(3/2))
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{\frac{(3Ac+Ca)x^3}{8a^2}+\frac{(5Ac-Ca)x}{8ac}-\frac{B}{4c}}{\left(c\,x^2+a\right)^2}+\frac{(3Ac+Ca)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8a^2c\sqrt{ac}}$	83
risch	$\frac{\frac{(3Ac+Ca)x^3}{8a^2}+\frac{(5Ac-Ca)x}{8ac}-\frac{B}{4c}}{\left(c\,x^2+a\right)^2}-\frac{3A\ln\left(cx+\sqrt{-ac}\right)}{16\sqrt{-ac}\,a^2}-\frac{\ln\left(cx+\sqrt{-ac}\right)C}{16\sqrt{-ac}\,ca}+\frac{3A\ln\left(-cx+\sqrt{-ac}\right)}{16\sqrt{-ac}\,a^2}+\frac{\ln\left(-cx+\sqrt{-ac}\right)C}{16\sqrt{-ac}\,ca}$	153

```
[In] int((C*x^2+B*x+A)/(c*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (1/8*(3*A*c+C*a)/a^2*x^3+1/8*(5*A*c-C*a)/a/c*x-1/4*B/c)/(c*x^2+a)^2+1/8*(3*
A*c+C*a)/a^2/c/(a*c)^(1/2)*arctan(c*x/(a*c)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.44 (sec) , antiderivative size = 314, normalized size of antiderivative = 3.20

$$\int \frac{A+Bx+Cx^2}{\left(a+cx^2\right)^3}\,dx$$
$$=\left[-\frac{4\,Ba^3c-2\,(Ca^2c^2+3\,Aac^3)x^3+((Cac^2+3\,Ac^3)x^4+Ca^3+3\,Aa^2c+2\,(Ca^2c+3\,Aac^2)x^2)\sqrt{-ac}\log}{16\,(a^3c^4x^4+2\,a^4c^3x^2+a^5c^2)}\right.$$
$$-\frac{2\,Ba^3c-(Ca^2c^2+3\,Aac^3)x^3-((Cac^2+3\,Ac^3)x^4+Ca^3+3\,Aa^2c+2\,(Ca^2c+3\,Aac^2)x^2)\sqrt{ac}\arctan\left(\right.}{8\,(a^3c^4x^4+2\,a^4c^3x^2+a^5c^2)}$$

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*B*a^3*c - 2*(C*a^2*c^2 + 3*A*a*c^3)*x^3 + ((C*a*c^2 + 3*A*c^3)*x^
4 + C*a^3 + 3*A*a^2*c + 2*(C*a^2*c + 3*A*a*c^2)*x^2)*sqrt(-a*c)*log((c*x^2
- 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) + 2*(C*a^3*c - 5*A*a^2*c^2)*x)/(a^3*c^4*
x^4 + 2*a^4*c^3*x^2 + a^5*c^2), -1/8*(2*B*a^3*c - (C*a^2*c^2 + 3*A*a*c^3)*x
^3 - ((C*a*c^2 + 3*A*c^3)*x^4 + C*a^3 + 3*A*a^2*c + 2*(C*a^2*c + 3*A*a*c^2)
*x^2)*sqrt(a*c)*arctan(sqrt(a*c)*x/a) + (C*a^3*c - 5*A*a^2*c^2)*x)/(a^3*c^4
*x^4 + 2*a^4*c^3*x^2 + a^5*c^2)]
```

Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.59

$$\int \frac{A+Bx+Cx^2}{(a+cx^2)^3}\,dx = -\frac{\sqrt{-\frac{1}{a^5c^3}}\cdot(3Ac+Ca)\log\left(-a^3c\sqrt{-\frac{1}{a^5c^3}}+x\right)}{16} + \frac{\sqrt{-\frac{1}{a^5c^3}}\cdot(3Ac+Ca)\log\left(a^3c\sqrt{-\frac{1}{a^5c^3}}+x\right)}{16} + \frac{-2Ba^2+x^3\cdot(3Ac^2+Cac)+x(5Aac-Ca^2)}{8a^4c+16a^3c^2x^2+8a^2c^3x^4}$$

```
[In] integrate((C*x**2+B*x+A)/(c*x**2+a)**3,x)
```

```
[Out] -sqrt(-1/(a**5*c**3))*(3*A*c + C*a)*log(-a**3*c*sqrt(-1/(a**5*c**3)) + x)/1
6 + sqrt(-1/(a**5*c**3))*(3*A*c + C*a)*log(a**3*c*sqrt(-1/(a**5*c**3)) + x)
/16 + (-2*B*a**2 + x**3*(3*A*c**2 + C*a*c) + x*(5*A*a*c - C*a**2))/(8*a**4*
c + 16*a**3*c**2*x**2 + 8*a**2*c**3*x**4)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx+Cx^2}{(a+cx^2)^3}\,dx = \frac{(Cac+3\,Ac^2)x^3-2\,Ba^2-(Ca^2-5\,Aac)x}{8\,(a^2c^3x^4+2\,a^3c^2x^2+a^4c)} + \frac{(Ca+3\,Ac)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\,\sqrt{ac}a^2c}$$

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((C*a*c + 3*A*c^2)*x^3 - 2*B*a^2 - (C*a^2 - 5*A*a*c)*x)/(a^2*c^3*x^4 +
2*a^3*c^2*x^2 + a^4*c) + 1/8*(C*a + 3*A*c)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)
*a^2*c)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.86

$$\int \frac{A+Bx+Cx^2}{(a+cx^2)^3}\,dx = \frac{(Ca+3\,Ac)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\,\sqrt{ac}a^2c} + \frac{Cacx^3+3\,Ac^2x^3-Ca^2x+5\,Aacx-2\,Ba^2}{8\,(cx^2+a)^2a^2c}$$

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(C*a + 3*A*c)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a^2*c) + 1/8*(C*a*c*x^3
+ 3*A*c^2*x^3 - C*a^2*x + 5*A*a*c*x - 2*B*a^2)/((c*x^2 + a)^2*a^2*c)
```

Mupad [B] (verification not implemented)

Time = 12.90 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.90

$$\int \frac{A+Bx+Cx^2}{(a+cx^2)^3}\,dx = \frac{\frac{x^3\,(3\,A\,c+C\,a)}{8\,a^2}-\frac{B}{4\,c}+\frac{x\,(5\,A\,c-C\,a)}{8\,a\,c}}{a^2+2\,a\,c\,x^2+c^2\,x^4}+\frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\,(3\,A\,c+C\,a)}{8\,a^{5/2}\,c^{3/2}}$$

```
[In] int((A + B*x + C*x^2)/(a + c*x^2)^3,x)
```

```
[Out] ((x^3*(3*A*c + C*a))/(8*a^2) - B/(4*c) + (x*(5*A*c - C*a))/(8*a*c))/(a^2 +
c^2*x^4 + 2*a*c*x^2) + (atan((c^(1/2)*x)/a^(1/2))*(3*A*c + C*a))/(8*a^(5/2)
*c^(3/2))
```

3.61 $\int \frac{A+Bx+Cx^2}{(d+ex)\left(a+cx^2\right)^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 353

$$
\begin{aligned}
\int \frac{A+Bx+Cx^2}{(d+ex)\left(a+cx^2\right)^3}\,dx &= -\frac{a(Bcd-Ace+aCe)-c(Acd-aCd+aBe)x}{4ac\left(cd^2+ae^2\right)\left(a+cx^2\right)^2} \\
&+\frac{4a^2e\left(Cd^2-Bde+Ae^2\right)+\left(a(Cd-Be)\left(cd^2-3ae^2\right)+Acd\left(3cd^2+7ae^2\right)\right)x}{8a^2\left(cd^2+ae^2\right)^2\left(a+cx^2\right)} \\
&+\frac{\left(a(Cd-Be)\left(c^2d^4+6acd^2e^2-3a^2e^4\right)+Acd\left(3c^2d^4+10acd^2e^2+15a^2e^4\right)\right)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{c}\left(cd^2+ae^2\right)^3} \\
&+\frac{e^3\left(Cd^2-Bde+Ae^2\right)\log(d+ex)}{\left(cd^2+ae^2\right)^3}-\frac{e^3\left(Cd^2-Bde+Ae^2\right)\log\left(a+cx^2\right)}{2\left(cd^2+ae^2\right)^3}
\end{aligned}
$$

```
[Out] 1/4*(-a*(-A*c*e+B*c*d+C*a*e)+c*(A*c*d+B*a*e-C*a*d)*x)/a/c/(a*e^2+c*d^2)/(c*
x^2+a)^2+1/8*(4*a^2*e*(A*e^2-B*d*e+C*d^2)+(a*(-B*e+C*d)*(-3*a*e^2+c*d^2)+A*
c*d*(7*a*e^2+3*c*d^2))*x)/a^2/(a*e^2+c*d^2)^2/(c*x^2+a)+e^3*(A*e^2-B*d*e+C*
d^2)*ln(e*x+d)/(a*e^2+c*d^2)^3-1/2*e^3*(A*e^2-B*d*e+C*d^2)*ln(c*x^2+a)/(a*e
^2+c*d^2)^3+1/8*(a*(-B*e+C*d)*(-3*a^2*e^4+6*a*c*d^2*e^2+c^2*d^4)+A*c*d*(15*
a^2*e^4+10*a*c*d^2*e^2+3*c^2*d^4))*arctan(x*c^(1/2)/a^(1/2))/a^(5/2)/(a*e^2
+c*d^2)^3/c^(1/2)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1661, 837, 815, 649, 211, 266}

$$\begin{aligned}
&\int \frac{A+Bx+Cx^2}{(d+ex)\left(a+cx^2\right)^3}\,dx \\
&= \frac{4a^2e(Ae^2-Bde+Cd^2)+x(Acd(7ae^2+3cd^2)+a(cd^2-3ae^2)\left(Cd-Be\right))}{8a^2\left(a+cx^2\right)\left(ae^2+cd^2\right)^2} \\
&\quad+\frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(Acd(15a^2e^4+10acd^2e^2+3c^2d^4)+a(-3a^2e^4+6acd^2e^2+c^2d^4)\left(Cd-Be\right)\right)}{8a^{5/2}\sqrt{c}\left(ae^2+cd^2\right)^3} \\
&\quad-\frac{a(aCe-Ace+Bcd)-cx(aBe-aCd+Acd)}{4ac\left(a+cx^2\right)^2\left(ae^2+cd^2\right)} \\
&\quad-\frac{e^3\log\left(a+cx^2\right)\left(Ae^2-Bde+Cd^2\right)}{2\left(ae^2+cd^2\right)^3}+\frac{e^3\log(d+ex)\left(Ae^2-Bde+Cd^2\right)}{\left(ae^2+cd^2\right)^3}
\end{aligned}$$

```
[In] Int[(A + B*x + C*x^2)/((d + e*x)*(a + c*x^2)^3),x]
```

```
[Out] -1/4*(a*(B*c*d - A*c*e + a*C*e) - c*(A*c*d - a*C*d + a*B*e)*x)/(a*c*(c*d^2
+ a*e^2)*(a + c*x^2)^2) + (4*a^2*e*(C*d^2 - B*d*e + A*e^2) + (a*(C*d - B*e)
*(c*d^2 - 3*a*e^2) + A*c*d*(3*c*d^2 + 7*a*e^2))*x)/(8*a^2*(c*d^2 + a*e^2)^2
*(a + c*x^2)) + ((a*(C*d - B*e)*(c^2*d^4 + 6*a*c*d^2*e^2 - 3*a^2*e^4) + A*c
*d*(3*c^2*d^4 + 10*a*c*d^2*e^2 + 15*a^2*e^4))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/
(8*a^(5/2)*Sqrt[c]*(c*d^2 + a*e^2)^3) + (e^3*(C*d^2 - B*d*e + A*e^2)*Log[d
+ e*x])/(c*d^2 + a*e^2)^3 - (e^3*(C*d^2 - B*d*e + A*e^2)*Log[a + c*x^2])/(2
*(c*d^2 + a*e^2)^3)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a(Bcd-Ace+aCe)-c(Acd-aCd+aBe)x}{4ac\left(cd^2+ae^2\right)\left(a+cx^2\right)^2} \\
&\quad -\frac{\int \frac{-\frac{c\left(ad(Cd-Be)+A\left(3cd^2+4ae^2\right)\right)}{cd^2+ae^2}-\frac{3ce(Acd-aCd+aBe)x}{cd^2+ae^2}}{(d+ex)\left(a+cx^2\right)^2}\,dx}{4ac} \\
&= -\frac{a(Bcd-Ace+aCe)-c(Acd-aCd+aBe)x}{4ac\left(cd^2+ae^2\right)\left(a+cx^2\right)^2} \\
&\quad +\frac{4a^2e(Cd^2-Bde+Ae^2)+\left(a(Cd-Be)\left(cd^2-3ae^2\right)+Acd(3cd^2+7ae^2)\right)x}{8a^2\left(cd^2+ae^2\right)^2\left(a+cx^2\right)} \\
&\quad +\frac{\int \frac{\frac{c^2\left(ad(Cd-Be)\left(cd^2+5ae^2\right)+A\left(3c^2d^4+7acd^2e^2+8a^2e^4\right)\right)}{cd^2+ae^2}+\frac{c^2e\left(a(Cd-Be)\left(cd^2-3ae^2\right)+Acd\left(3cd^2+7ae^2\right)\right)x}{cd^2+ae^2}}{(d+ex)\left(a+cx^2\right)}\,dx}{8a^2c^2\left(cd^2+ae^2\right)}
\end{aligned}$$

$$= -\frac{a(Bcd - Ace + aCe) - c(Acd - aCd + aBe)x}{4ac\left(cd^2 + ae^2\right)\left(a + cx^2\right)^2} + \frac{4a^2e(Cd^2 - Bde + Ae^2) + \left(a(Cd - Be)\left(cd^2 - 3ae^2\right) + Acd(3cd^2 + 7ae^2)\right)x}{8a^2\left(cd^2 + ae^2\right)^2\left(a + cx^2\right)} + \frac{\int \Bigg(\frac{8a^2c^2e^4(Cd^2-Bde+Ae^2)}{\left(cd^2+ae^2\right)^2(d+ex)} + \frac{c^2\left(a(Cd-Be)\left(c^2d^4+6acd^2e^2-3a^2e^4\right)+Acd\left(3c^2d^4+10acd^2e^2+15a^2e^4\right)-8a^2ce^3\left(Cd^2-Bde+Ae\right.}{\left(cd^2+ae^2\right)^2\left(a+cx^2\right)}}{8a^2c^2\left(cd^2 + ae^2\right)}$$

$$= -\frac{a(Bcd - Ace + aCe) - c(Acd - aCd + aBe)x}{4ac\left(cd^2 + ae^2\right)\left(a + cx^2\right)^2} + \frac{4a^2e(Cd^2 - Bde + Ae^2) + \left(a(Cd - Be)\left(cd^2 - 3ae^2\right) + Acd(3cd^2 + 7ae^2)\right)x}{8a^2\left(cd^2 + ae^2\right)^2\left(a + cx^2\right)} + \frac{e^3(Cd^2 - Bde + Ae^2)\log(d + ex)}{\left(cd^2 + ae^2\right)^3} + \frac{\int \frac{a(Cd-Be)\left(c^2d^4+6acd^2e^2-3a^2e^4\right)+Acd\left(3c^2d^4+10acd^2e^2+15a^2e^4\right)-8a^2ce^3\left(Cd^2-Bde+Ae^2\right)x}{a+cx^2}\,dx}{8a^2\left(cd^2 + ae^2\right)^3}$$

$$= -\frac{a(Bcd - Ace + aCe) - c(Acd - aCd + aBe)x}{4ac\left(cd^2 + ae^2\right)\left(a + cx^2\right)^2} + \frac{4a^2e(Cd^2 - Bde + Ae^2) + \left(a(Cd - Be)\left(cd^2 - 3ae^2\right) + Acd(3cd^2 + 7ae^2)\right)x}{8a^2\left(cd^2 + ae^2\right)^2\left(a + cx^2\right)} + \frac{e^3(Cd^2 - Bde + Ae^2)\log(d + ex)}{\left(cd^2 + ae^2\right)^3} - \frac{\left(ce^3(Cd^2 - Bde + Ae^2)\right)\int \frac{x}{a+cx^2}\,dx}{\left(cd^2 + ae^2\right)^3} + \frac{\left(a(Cd - Be)\left(c^2d^4 + 6acd^2e^2 - 3a^2e^4\right) + Acd(3c^2d^4 + 10acd^2e^2 + 15a^2e^4)\right)\int \frac{1}{a+cx^2}\,dx}{8a^2\left(cd^2 + ae^2\right)^3}$$

$$= -\frac{a(Bcd - Ace + aCe) - c(Acd - aCd + aBe)x}{4ac\left(cd^2 + ae^2\right)\left(a + cx^2\right)^2} + \frac{4a^2e(Cd^2 - Bde + Ae^2) + \left(a(Cd - Be)\left(cd^2 - 3ae^2\right) + Acd(3cd^2 + 7ae^2)\right)x}{8a^2\left(cd^2 + ae^2\right)^2\left(a + cx^2\right)} + \frac{\left(a(Cd - Be)\left(c^2d^4 + 6acd^2e^2 - 3a^2e^4\right) + Acd(3c^2d^4 + 10acd^2e^2 + 15a^2e^4)\right)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{c}\left(cd^2 + ae^2\right)^3} + \frac{e^3(Cd^2 - Bde + Ae^2)\log(d + ex)}{\left(cd^2 + ae^2\right)^3} - \frac{e^3(Cd^2 - Bde + Ae^2)\log\left(a + cx^2\right)}{2\left(cd^2 + ae^2\right)^3}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 321, normalized size of antiderivative = 0.91

$$\int \frac{A+Bx+Cx^2}{(d+ex)\left(a+cx^2\right)^3}\,dx$$

$$= \frac{\frac{2(cd^2+ae^2)^2(-a^2Ce+Ac^2dx+ac(-Bd+Ae-Cdx+Bex))}{ac(a+cx^2)^2}+\frac{(cd^2+ae^2)(3Ac^2d^3x+acd(Cd^2+e(-Bd+7Ae))x+a^2e(Cd(4d-3ex)+e(-4Bd+4}{a^2(a+cx^2)}}{}$$

```
[In] Integrate[(A + B*x + C*x^2)/((d + e*x)*(a + c*x^2)^3),x]
```

```
[Out] ((2*(c*d^2 + a*e^2)^2*(-(a^2*C*e) + A*c^2*d*x + a*c*(-(B*d) + A*e - C*d*x +
 B*e*x)))/(a*c*(a + c*x^2)^2) + ((c*d^2 + a*e^2)*(3*A*c^2*d^3*x + a*c*d*(C*
d^2 + e*(-(B*d) + 7*A*e))*x + a^2*e*(C*d*(4*d - 3*e*x) + e*(-4*B*d + 4*A*e
+ 3*B*e*x))))/(a^2*(a + c*x^2)) + ((a*(C*d - B*e)*(c^2*d^4 + 6*a*c*d^2*e^2
- 3*a^2*e^4) + A*c*d*(3*c^2*d^4 + 10*a*c*d^2*e^2 + 15*a^2*e^4))*ArcTan[(Sqr
t[c]*x)/Sqrt[a]])/(a^(5/2)*Sqrt[c]) + 8*e^3*(C*d^2 + e*(-(B*d) + A*e))*Log[
d + e*x] - 4*e^3*(C*d^2 + e*(-(B*d) + A*e))*Log[a + c*x^2])/(8*(c*d^2 + a*e
^2)^3)
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 605, normalized size of antiderivative = 1.71

method	result
default	$\frac{\frac{c\left(7A\,a^2cd\,e^4+10Aa\,c^2d^3e^2+3d^5A\,c^3+3B\,e^5a^3+2B\,a^2c\,d^2e^3-Ba\,c^2d^4e-3C\,a^3d\,e^4-2C\,a^2c\,d^3e^2+C\,c^2d^5a\right)x^3}{8a^2}+\left(\frac{1}{2}Aac\,e^5+\frac{1}{2}A\,c^2d^2e^3-\frac{1}{2}Bacd}{}$
risch	Expression too large to display

```
[In] int((C*x^2+B*x+A)/(e*x+d)/(c*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/(a*e^2+c*d^2)^3*((1/8*c*(7*A*a^2*c*d*e^4+10*A*a*c^2*d^3*e^2+3*A*c^3*d^5+3
*B*a^3*e^5+2*B*a^2*c*d^2*e^3-B*a*c^2*d^4*e-3*C*a^3*d*e^4-2*C*a^2*c*d^3*e^2+
C*a*c^2*d^5)/a^2*x^3+(1/2*A*a*c*e^5+1/2*A*c^2*d^2*e^3-1/2*B*a*c*d*e^4-1/2*B
*c^2*d^3*e^2+1/2*C*a*c*d^2*e^3+1/2*C*c^2*d^4*e)*x^2+1/8*(9*A*a^2*c*d*e^4+14
*A*a*c^2*d^3*e^2+5*A*c^3*d^5+5*B*a^3*e^5+6*B*a^2*c*d^2*e^3+B*a*c^2*d^4*e-5*
C*a^3*d*e^4-6*C*a^2*c*d^3*e^2-C*a*c^2*d^5)/a*x+1/4*(3*A*a^2*c*e^5+4*A*a*c^2
*d^2*e^3+A*c^3*d^4*e-3*B*a^2*c*d*e^4-4*B*a*c^2*d^3*e^2-B*c^3*d^5-C*a^3*e^5+
C*a*c^2*d^4*e)/c)/(c*x^2+a)^2+1/8/a^2*(1/2*(-8*A*a^2*c*e^5+8*B*a^2*c*d*e^4-
8*C*a^2*c*d^2*e^3)/c*ln(c*x^2+a)+(15*A*a^2*c*d*e^4+10*A*a*c^2*d^3*e^2+3*A*c
^3*d^5+3*B*a^3*e^5-6*B*a^2*c*d^2*e^3-B*a*c^2*d^4*e-3*C*a^3*d*e^4+6*C*a^2*c*
```

```
d^3*e^2+C*a*c^2*d^5)/(a*c)^(1/2)*arctan(c*x/(a*c)^(1/2))))+e^3*(A*e^2-B*d*e
+C*d^2)*ln(e*x+d)/(a*e^2+c*d^2)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1162 vs. $2(337) = 674$.

Time = 193.20 (sec) , antiderivative size = 2346, normalized size of antiderivative = 6.65

$$\int \frac{A+Bx+Cx^2}{(d+ex)\left(a+cx^2\right)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)/(c*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*B*a^3*c^3*d^5 + 16*B*a^4*c^2*d^3*e^2 - 16*A*a^4*c^2*d^2*e^3 + 12*
B*a^5*c*d*e^4 - 4*(C*a^4*c^2 + A*a^3*c^3)*d^4*e + 4*(C*a^6 - 3*A*a^5*c)*e^5
 + 2*(B*a^2*c^4*d^4*e - 2*B*a^3*c^3*d^2*e^3 - 3*B*a^4*c^2*e^5 - (C*a^2*c^4
+ 3*A*a*c^5)*d^5 + 2*(C*a^3*c^3 - 5*A*a^2*c^4)*d^3*e^2 + (3*C*a^4*c^2 - 7*A
*a^3*c^3)*d*e^4)*x^3 - 8*(C*a^3*c^3*d^4*e - B*a^3*c^3*d^3*e^2 - B*a^4*c^2*d
*e^4 + A*a^4*c^2*e^5 + (C*a^4*c^2 + A*a^3*c^3)*d^2*e^3)*x^2 - (B*a^3*c^2*d^
4*e + 6*B*a^4*c*d^2*e^3 - 3*B*a^5*e^5 - (C*a^3*c^2 + 3*A*a^2*c^3)*d^5 - 2*(
3*C*a^4*c + 5*A*a^3*c^2)*d^3*e^2 + 3*(C*a^5 - 5*A*a^4*c)*d*e^4 + (B*a*c^4*d
^4*e + 6*B*a^2*c^3*d^2*e^3 - 3*B*a^3*c^2*e^5 - (C*a*c^4 + 3*A*c^5)*d^5 - 2*
(3*C*a^2*c^3 + 5*A*a*c^4)*d^3*e^2 + 3*(C*a^3*c^2 - 5*A*a^2*c^3)*d*e^4)*x^4
+ 2*(B*a^2*c^3*d^4*e + 6*B*a^3*c^2*d^2*e^3 - 3*B*a^4*c*e^5 - (C*a^2*c^3 + 3
*A*a*c^4)*d^5 - 2*(3*C*a^3*c^2 + 5*A*a^2*c^3)*d^3*e^2 + 3*(C*a^4*c - 5*A*a^
3*c^2)*d*e^4)*x^2)*sqrt(-a*c)*log((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a))
 - 2*(B*a^3*c^3*d^4*e + 6*B*a^4*c^2*d^2*e^3 + 5*B*a^5*c*e^5 - (C*a^3*c^3 -
5*A*a^2*c^4)*d^5 - 2*(3*C*a^4*c^2 - 7*A*a^3*c^3)*d^3*e^2 - (5*C*a^5*c - 9*A
*a^4*c^2)*d*e^4)*x + 8*(C*a^5*c*d^2*e^3 - B*a^5*c*d*e^4 + A*a^5*c*e^5 + (C*
a^3*c^3*d^2*e^3 - B*a^3*c^3*d*e^4 + A*a^3*c^3*e^5)*x^4 + 2*(C*a^4*c^2*d^2*e
^3 - B*a^4*c^2*d*e^4 + A*a^4*c^2*e^5)*x^2)*log(c*x^2 + a) - 16*(C*a^5*c*d^2
*e^3 - B*a^5*c*d*e^4 + A*a^5*c*e^5 + (C*a^3*c^3*d^2*e^3 - B*a^3*c^3*d*e^4 +
 A*a^3*c^3*e^5)*x^4 + 2*(C*a^4*c^2*d^2*e^3 - B*a^4*c^2*d*e^4 + A*a^4*c^2*e^
5)*x^2)*log(e*x + d))/(a^5*c^4*d^6 + 3*a^6*c^3*d^4*e^2 + 3*a^7*c^2*d^2*e^4
+ a^8*c*e^6 + (a^3*c^6*d^6 + 3*a^4*c^5*d^4*e^2 + 3*a^5*c^4*d^2*e^4 + a^6*c^
3*e^6)*x^4 + 2*(a^4*c^5*d^6 + 3*a^5*c^4*d^4*e^2 + 3*a^6*c^3*d^2*e^4 + a^7*c
^2*e^6)*x^2), -1/8*(2*B*a^3*c^3*d^5 + 8*B*a^4*c^2*d^3*e^2 - 8*A*a^4*c^2*d^2
*e^3 + 6*B*a^5*c*d*e^4 - 2*(C*a^4*c^2 + A*a^3*c^3)*d^4*e + 2*(C*a^6 - 3*A*a
^5*c)*e^5 + (B*a^2*c^4*d^4*e - 2*B*a^3*c^3*d^2*e^3 - 3*B*a^4*c^2*e^5 - (C*a
^2*c^4 + 3*A*a*c^5)*d^5 + 2*(C*a^3*c^3 - 5*A*a^2*c^4)*d^3*e^2 + (3*C*a^4*c^
2 - 7*A*a^3*c^3)*d*e^4)*x^3 - 4*(C*a^3*c^3*d^4*e - B*a^3*c^3*d^3*e^2 - B*a^
4*c^2*d*e^4 + A*a^4*c^2*e^5 + (C*a^4*c^2 + A*a^3*c^3)*d^2*e^3)*x^2 + (B*a^3
*c^2*d^4*e + 6*B*a^4*c*d^2*e^3 - 3*B*a^5*e^5 - (C*a^3*c^2 + 3*A*a^2*c^3)*d^
5 - 2*(3*C*a^4*c + 5*A*a^3*c^2)*d^3*e^2 + 3*(C*a^5 - 5*A*a^4*c)*d*e^4 + (B*
```

```
a*c^4*d^4*e + 6*B*a^2*c^3*d^2*e^3 - 3*B*a^3*c^2*e^5 - (C*a*c^4 + 3*A*c^5)*d
^5 - 2*(3*C*a^2*c^3 + 5*A*a*c^4)*d^3*e^2 + 3*(C*a^3*c^2 - 5*A*a^2*c^3)*d*e^
4)*x^4 + 2*(B*a^2*c^3*d^4*e + 6*B*a^3*c^2*d^2*e^3 - 3*B*a^4*c*e^5 - (C*a^2*
c^3 + 3*A*a*c^4)*d^5 - 2*(3*C*a^3*c^2 + 5*A*a^2*c^3)*d^3*e^2 + 3*(C*a^4*c -
 5*A*a^3*c^2)*d*e^4)*x^2)*sqrt(a*c)*arctan(sqrt(a*c)*x/a) - (B*a^3*c^3*d^4*
e + 6*B*a^4*c^2*d^2*e^3 + 5*B*a^5*c*e^5 - (C*a^3*c^3 - 5*A*a^2*c^4)*d^5 - 2
*(3*C*a^4*c^2 - 7*A*a^3*c^3)*d^3*e^2 - (5*C*a^5*c - 9*A*a^4*c^2)*d*e^4)*x +
 4*(C*a^5*c*d^2*e^3 - B*a^5*c*d*e^4 + A*a^5*c*e^5 + (C*a^3*c^3*d^2*e^3 - B*
a^3*c^3*d*e^4 + A*a^3*c^3*e^5)*x^4 + 2*(C*a^4*c^2*d^2*e^3 - B*a^4*c^2*d*e^4
 + A*a^4*c^2*e^5)*x^2)*log(c*x^2 + a) - 8*(C*a^5*c*d^2*e^3 - B*a^5*c*d*e^4
+ A*a^5*c*e^5 + (C*a^3*c^3*d^2*e^3 - B*a^3*c^3*d*e^4 + A*a^3*c^3*e^5)*x^4 +
 2*(C*a^4*c^2*d^2*e^3 - B*a^4*c^2*d*e^4 + A*a^4*c^2*e^5)*x^2)*log(e*x + d))
/(a^5*c^4*d^6 + 3*a^6*c^3*d^4*e^2 + 3*a^7*c^2*d^2*e^4 + a^8*c*e^6 + (a^3*c^
6*d^6 + 3*a^4*c^5*d^4*e^2 + 3*a^5*c^4*d^2*e^4 + a^6*c^3*e^6)*x^4 + 2*(a^4*c
^5*d^6 + 3*a^5*c^4*d^4*e^2 + 3*a^6*c^3*d^2*e^4 + a^7*c^2*e^6)*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + Bx + Cx^2}{(d + ex)\left(a + cx^2\right)^3}\,dx = \text{Timed out}$$

```
[In] integrate((C*x**2+B*x+A)/(e*x+d)/(c*x**2+a)**3,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 655, normalized size of antiderivative = 1.86

$$\int \frac{A + Bx + Cx^2}{(d + ex)\left(a + cx^2\right)^3}\,dx$$
$$= -\frac{(Cd^2e^3 - Bde^4 + Ae^5)\log\left(cx^2 + a\right)}{2\left(c^3d^6 + 3\,ac^2d^4e^2 + 3\,a^2cd^2e^4 + a^3e^6\right)} + \frac{(Cd^2e^3 - Bde^4 + Ae^5)\log\left(ex + d\right)}{c^3d^6 + 3\,ac^2d^4e^2 + 3\,a^2cd^2e^4 + a^3e^6}$$
$$- \frac{(Bac^2d^4e + 6\,Ba^2cd^2e^3 - 3\,Ba^3e^5 - (Cac^2 + 3\,Ac^3)d^5 - 2\,(3\,Ca^2c + 5\,Aac^2)d^3e^2 + 3\,(Ca^3 - 5\,Aa^2c)de}{8\left(a^2c^3d^6 + 3\,a^3c^2d^4e^2 + 3\,a^4cd^2e^4 + a^5e^6\right)\sqrt{ac}}$$
$$- \frac{2\,Ba^2c^2d^3 + 6\,Ba^3cde^2 - 2\,(Ca^3c + Aa^2c^2)d^2e + 2\,(Ca^4 - 3\,Aa^3c)e^3 + (Bac^3d^2e - 3\,Ba^2c^2e^3 - (Cac^3}{8\,(a^4c^3d^4 + 2\,a^5c^2d^2e^2 + a}$$

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)/(c*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(C*d^2*e^3 - B*d*e^4 + A*e^5)*log(c*x^2 + a)/(c^3*d^6 + 3*a*c^2*d^4*e^
2 + 3*a^2*c*d^2*e^4 + a^3*e^6) + (C*d^2*e^3 - B*d*e^4 + A*e^5)*log(e*x + d)
```

```
/(c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + a^3*e^6) - 1/8*(B*a*c^2*d^4
*e + 6*B*a^2*c*d^2*e^3 - 3*B*a^3*e^5 - (C*a*c^2 + 3*A*c^3)*d^5 - 2*(3*C*a^2
*c + 5*A*a*c^2)*d^3*e^2 + 3*(C*a^3 - 5*A*a^2*c)*d*e^4)*arctan(c*x/sqrt(a*c)
)/((a^2*c^3*d^6 + 3*a^3*c^2*d^4*e^2 + 3*a^4*c*d^2*e^4 + a^5*e^6)*sqrt(a*c))
 - 1/8*(2*B*a^2*c^2*d^3 + 6*B*a^3*c*d*e^2 - 2*(C*a^3*c + A*a^2*c^2)*d^2*e +
 2*(C*a^4 - 3*A*a^3*c)*e^3 + (B*a*c^3*d^2*e - 3*B*a^2*c^2*e^3 - (C*a*c^3 +
3*A*c^4)*d^3 + (3*C*a^2*c^2 - 7*A*a*c^3)*d*e^2)*x^3 - 4*(C*a^2*c^2*d^2*e -
B*a^2*c^2*d*e^2 + A*a^2*c^2*e^3)*x^2 - (B*a^2*c^2*d^2*e + 5*B*a^3*c*e^3 - (
C*a^2*c^2 - 5*A*a*c^3)*d^3 - (5*C*a^3*c - 9*A*a^2*c^2)*d*e^2)*x)/(a^4*c^3*d
^4 + 2*a^5*c^2*d^2*e^2 + a^6*c*e^4 + (a^2*c^5*d^4 + 2*a^3*c^4*d^2*e^2 + a^4
*c^3*e^4)*x^4 + 2*(a^3*c^4*d^4 + 2*a^4*c^3*d^2*e^2 + a^5*c^2*e^4)*x^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 751 vs. $2(337) = 674$.

Time = 0.28 (sec) , antiderivative size = 751, normalized size of antiderivative = 2.13

$$\int \frac{A+Bx+Cx^2}{(d+ex)\left(a+cx^2\right)^3}\,dx$$

$$= -\frac{(Cd^2e^3 - Bde^4 + Ae^5)\log\left(cx^2+a\right)}{2\left(c^3d^6 + 3\,ac^2d^4e^2 + 3\,a^2cd^2e^4 + a^3e^6\right)} + \frac{(Cd^2e^4 - Bde^5 + Ae^6)\log\left(|ex+d|\right)}{c^3d^6e + 3\,ac^2d^4e^3 + 3\,a^2cd^2e^5 + a^3e^7}$$

$$+ \frac{(Cac^2d^5 + 3\,Ac^3d^5 - Bac^2d^4e + 6\,Ca^2cd^3e^2 + 10\,Aac^2d^3e^2 - 6\,Ba^2cd^2e^3 - 3\,Ca^3de^4 + 15\,Aa^2cde^4 + 3\,B}{8\left(a^2c^3d^6 + 3\,a^3c^2d^4e^2 + 3\,a^4cd^2e^4 + a^5e^6\right)\sqrt{ac}}$$

$$- \frac{2\,Ba^2c^3d^5 - 2\,Ca^3c^2d^4e - 2\,Aa^2c^3d^4e + 8\,Ba^3c^2d^3e^2 - 8\,Aa^3c^2d^2e^3 + 6\,Ba^4cde^4 + 2\,Ca^5e^5 - 6\,Aa^4ce^5 -}{}$$

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)/(c*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/2*(C*d^2*e^3 - B*d*e^4 + A*e^5)*log(c*x^2 + a)/(c^3*d^6 + 3*a*c^2*d^4*e^
2 + 3*a^2*c*d^2*e^4 + a^3*e^6) + (C*d^2*e^4 - B*d*e^5 + A*e^6)*log(abs(e*x
+ d))/(c^3*d^6*e + 3*a*c^2*d^4*e^3 + 3*a^2*c*d^2*e^5 + a^3*e^7) + 1/8*(C*a*
c^2*d^5 + 3*A*c^3*d^5 - B*a*c^2*d^4*e + 6*C*a^2*c*d^3*e^2 + 10*A*a*c^2*d^3*
e^2 - 6*B*a^2*c*d^2*e^3 - 3*C*a^3*d*e^4 + 15*A*a^2*c*d*e^4 + 3*B*a^3*e^5)*a
rctan(c*x/sqrt(a*c))/((a^2*c^3*d^6 + 3*a^3*c^2*d^4*e^2 + 3*a^4*c*d^2*e^4 +
a^5*e^6)*sqrt(a*c)) - 1/8*(2*B*a^2*c^3*d^5 - 2*C*a^3*c^2*d^4*e - 2*A*a^2*c^
3*d^4*e + 8*B*a^3*c^2*d^3*e^2 - 8*A*a^3*c^2*d^2*e^3 + 6*B*a^4*c*d*e^4 + 2*C
*a^5*e^5 - 6*A*a^4*c*e^5 - (C*a*c^4*d^5 + 3*A*c^5*d^5 - B*a*c^4*d^4*e - 2*C
*a^2*c^3*d^3*e^2 + 10*A*a*c^4*d^3*e^2 + 2*B*a^2*c^3*d^2*e^3 - 3*C*a^3*c^2*d
*e^4 + 7*A*a^2*c^3*d*e^4 + 3*B*a^3*c^2*e^5)*x^3 - 4*(C*a^2*c^3*d^4*e - B*a^
2*c^3*d^3*e^2 + C*a^3*c^2*d^2*e^3 + A*a^2*c^3*d^2*e^3 - B*a^3*c^2*d*e^4 + A
*a^3*c^2*e^5)*x^2 + (C*a^2*c^3*d^5 - 5*A*a*c^4*d^5 - B*a^2*c^3*d^4*e + 6*C*
a^3*c^2*d^3*e^2 - 14*A*a^2*c^3*d^3*e^2 - 6*B*a^3*c^2*d^2*e^3 + 5*C*a^4*c*d*
e^4 - 9*A*a^3*c^2*d*e^4 - 5*B*a^4*c*e^5)*x)/((c*d^2 + a*e^2)^3*(c*x^2 + a)^
2*a^2*c)
```

Mupad [B] (verification not implemented)

Time = 20.36 (sec) , antiderivative size = 2392, normalized size of antiderivative = 6.78

$$\int \frac{A+Bx+Cx^2}{(d+ex)\left(a+cx^2\right)^3}\,dx = \text{Too large to display}$$

```
[In] int((A + B*x + C*x^2)/((a + c*x^2)^3*(d + e*x)),x)
```

```
[Out] ((x^2*(A*c*e^3 - B*c*d*e^2 + C*c*d^2*e))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*
e^2)) - (B*c^2*d^3 + C*a^2*e^3 - 3*A*a*c*e^3 - A*c^2*d^2*e + 3*B*a*c*d*e^2
- C*a*c*d^2*e)/(4*c*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)) + (x*(5*A*c^2*d^3
+ 5*B*a^2*e^3 - C*a*c*d^3 - 5*C*a^2*d*e^2 + 9*A*a*c*d*e^2 + B*a*c*d^2*e))/(
8*a*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)) + (x^3*(3*A*c^3*d^3 + 3*B*a^2*c*e^
3 + C*a*c^2*d^3 + 7*A*a*c^2*d*e^2 - B*a*c^2*d^2*e - 3*C*a^2*c*d*e^2))/(8*a^
2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)))/(a^2 + c^2*x^4 + 2*a*c*x^2) - (log(
3*A*c^4*d^7*(-a^5*c)^(1/2) - 3*B*a^4*e^7*(-a^5*c)^(1/2) - 24*A*a^6*c*e^7 +
3*B*a^6*c*e^7*x + 6*A*a^3*c^4*d^6*e + 2*C*a^4*c^3*d^6*e - 30*C*a^6*c*d^2*e^
5 - 3*A*a^2*c^5*d^7*x - C*a^3*c^4*d^7*x + C*a*c^3*d^7*(-a^5*c)^(1/2) + 3*C*
a^4*d*e^6*(-a^5*c)^(1/2) + 20*A*a^4*c^3*d^4*e^3 + 54*A*a^5*c^2*d^2*e^5 - 2*
B*a^4*c^3*d^5*e^2 - 36*B*a^5*c^2*d^3*e^4 + 36*C*a^5*c^2*d^4*e^3 + 30*B*a^6*
c*d*e^6 - 7*A*a^3*c^4*d^5*e^2*x - 5*A*a^4*c^3*d^3*e^4*x + 5*B*a^4*c^3*d^4*e
^3*x - 57*B*a^5*c^2*d^2*e^5*x - 5*C*a^4*c^3*d^5*e^2*x + 57*C*a^5*c^2*d^3*e^
4*x + 7*A*a*c^3*d^5*e^2*(-a^5*c)^(1/2) + 57*B*a^3*c*d^2*e^5*(-a^5*c)^(1/2)
- 57*C*a^3*c*d^3*e^4*(-a^5*c)^(1/2) - 3*C*a^6*c*d*e^6*x + 5*A*a^2*c^2*d^3*e
^4*(-a^5*c)^(1/2) - 5*B*a^2*c^2*d^4*e^3*(-a^5*c)^(1/2) + 5*C*a^2*c^2*d^5*e^
2*(-a^5*c)^(1/2) + 63*A*a^5*c^2*d*e^6*x + B*a^3*c^4*d^6*e*x - 63*A*a^3*c*d*
e^6*(-a^5*c)^(1/2) - B*a*c^3*d^6*e*(-a^5*c)^(1/2) - 24*A*a^3*c*e^7*x*(-a^5*
c)^(1/2) + 6*A*c^4*d^6*e*x*(-a^5*c)^(1/2) + 54*A*a^2*c^2*d^2*e^5*x*(-a^5*c)
^(1/2) - 36*B*a^2*c^2*d^3*e^4*x*(-a^5*c)^(1/2) + 36*C*a^2*c^2*d^4*e^3*x*(-a
^5*c)^(1/2) + 30*B*a^3*c*d*e^6*x*(-a^5*c)^(1/2) + 2*C*a*c^3*d^6*e*x*(-a^5*c
)^(1/2) + 20*A*a*c^3*d^4*e^3*x*(-a^5*c)^(1/2) - 2*B*a*c^3*d^5*e^2*x*(-a^5*c
)^(1/2) - 30*C*a^3*c*d^2*e^5*x*(-a^5*c)^(1/2))*(c*(a^2*((3*C*d^3*e^2*(-a^5*
c)^(1/2))/8 - (3*B*d^2*e^3*(-a^5*c)^(1/2))/8 + (15*A*d*e^4*(-a^5*c)^(1/2))/
16) + a^5*((A*e^5)/2 + (C*d^2*e^3)/2 - (B*d*e^4)/2)) + a^3*((3*B*e^5*(-a^5*
c)^(1/2))/16 - (3*C*d*e^4*(-a^5*c)^(1/2))/16) + a*c^2*((C*d^5*(-a^5*c)^(1/2
))/16 + (5*A*d^3*e^2*(-a^5*c)^(1/2))/8 - (B*d^4*e*(-a^5*c)^(1/2))/16) + (3*
A*c^3*d^5*(-a^5*c)^(1/2))/16))/(a^8*c*e^6 + a^5*c^4*d^6 + 3*a^6*c^3*d^4*e^2
 + 3*a^7*c^2*d^2*e^4) + (log(3*A*c^4*d^7*(-a^5*c)^(1/2) - 3*B*a^4*e^7*(-a^5
*c)^(1/2) + 24*A*a^6*c*e^7 - 3*B*a^6*c*e^7*x - 6*A*a^3*c^4*d^6*e - 2*C*a^4*
c^3*d^6*e + 30*C*a^6*c*d^2*e^5 + 3*A*a^2*c^5*d^7*x + C*a^3*c^4*d^7*x + C*a*
c^3*d^7*(-a^5*c)^(1/2) + 3*C*a^4*d*e^6*(-a^5*c)^(1/2) - 20*A*a^4*c^3*d^4*e^
3 - 54*A*a^5*c^2*d^2*e^5 + 2*B*a^4*c^3*d^5*e^2 + 36*B*a^5*c^2*d^3*e^4 - 36*
C*a^5*c^2*d^4*e^3 - 30*B*a^6*c*d*e^6 + 7*A*a^3*c^4*d^5*e^2*x + 5*A*a^4*c^3*
d^3*e^4*x - 5*B*a^4*c^3*d^4*e^3*x + 57*B*a^5*c^2*d^2*e^5*x + 5*C*a^4*c^3*d^
```

```
5*e^2*x - 57*C*a^5*c^2*d^3*e^4*x + 7*A*a*c^3*d^5*e^2*(-a^5*c)^(1/2) + 57*B*
a^3*c*d^2*e^5*(-a^5*c)^(1/2) - 57*C*a^3*c*d^3*e^4*(-a^5*c)^(1/2) + 3*C*a^6*
c*d*e^6*x + 5*A*a^2*c^2*d^3*e^4*(-a^5*c)^(1/2) - 5*B*a^2*c^2*d^4*e^3*(-a^5*
c)^(1/2) + 5*C*a^2*c^2*d^5*e^2*(-a^5*c)^(1/2) - 63*A*a^5*c^2*d*e^6*x - B*a^
3*c^4*d^6*e*x - 63*A*a^3*c*d*e^6*(-a^5*c)^(1/2) - B*a*c^3*d^6*e*(-a^5*c)^(1
/2) - 24*A*a^3*c*e^7*x*(-a^5*c)^(1/2) + 6*A*c^4*d^6*e*x*(-a^5*c)^(1/2) + 54
*A*a^2*c^2*d^2*e^5*x*(-a^5*c)^(1/2) - 36*B*a^2*c^2*d^3*e^4*x*(-a^5*c)^(1/2)
 + 36*C*a^2*c^2*d^4*e^3*x*(-a^5*c)^(1/2) + 30*B*a^3*c*d*e^6*x*(-a^5*c)^(1/2
) + 2*C*a*c^3*d^6*e*x*(-a^5*c)^(1/2) + 20*A*a*c^3*d^4*e^3*x*(-a^5*c)^(1/2)
- 2*B*a*c^3*d^5*e^2*x*(-a^5*c)^(1/2) - 30*C*a^3*c*d^2*e^5*x*(-a^5*c)^(1/2))
*(c*(a^2*((3*C*d^3*e^2*(-a^5*c)^(1/2))/8 - (3*B*d^2*e^3*(-a^5*c)^(1/2))/8 +
 (15*A*d*e^4*(-a^5*c)^(1/2))/16) - a^5*((A*e^5)/2 + (C*d^2*e^3)/2 - (B*d*e^
4)/2)) + a^3*((3*B*e^5*(-a^5*c)^(1/2))/16 - (3*C*d*e^4*(-a^5*c)^(1/2))/16)
+ a*c^2*((C*d^5*(-a^5*c)^(1/2))/16 + (5*A*d^3*e^2*(-a^5*c)^(1/2))/8 - (B*d^
4*e*(-a^5*c)^(1/2))/16) + (3*A*c^3*d^5*(-a^5*c)^(1/2))/16))/(a^8*c*e^6 + a^
5*c^4*d^6 + 3*a^6*c^3*d^4*e^2 + 3*a^7*c^2*d^2*e^4) + (e^3*log(d + e*x)*(A*e
^2 + C*d^2 - B*d*e))/(a*e^2 + c*d^2)^3
```

3.62 $\int \frac{A+Bx+Cx^2}{(d+ex)^2\left(a+cx^2\right)^3} \, dx$

Optimal result

Integrand size = 27, antiderivative size = 571

$$\begin{aligned}\int \frac{A+Bx+Cx^2}{(d+ex)^2\left(a+cx^2\right)^3} \, dx = & -\frac{e^3(Cd^2-Bde+Ae^2)}{\left(cd^2+ae^2\right)^3(d+ex)} \\ & -\frac{a(Bcd^2-2Acde+2aCde-aBe^2)-(Ac(cd^2-ae^2)+a(aCe^2-cd(Cd-2Be)))\,x}{4a\left(cd^2+ae^2\right)^2\left(a+cx^2\right)^2} \\ & -\frac{4a^2e(ae^2(2Cd-Be)-cd(2Cd^2-e(3Bd-4Ae)))-(Ac(3c^2d^4+12acd^2e^2-7a^2e^4)+a(3a^2Ce^4-2a}{8a^2\left(cd^2+ae^2\right)^3\left(a+cx^2\right)} \\ & +\frac{(3Ac(c^3d^6+5ac^2d^4e^2+15a^2cd^2e^4-5a^3e^6)+a(3a^3Ce^6+ac^2d^3e^2(13Cd-20Be)-3a^2cde^4(11Cd-1}{8a^{5/2}\sqrt{c}\left(cd^2+ae^2\right)^4} \\ & -\frac{e^3(ae^2(2Cd-Be)-cd(4Cd^2-e(5Bd-6Ae)))\log(d+ex)}{\left(cd^2+ae^2\right)^4} \\ & +\frac{e^3(ae^2(2Cd-Be)-cd(4Cd^2-e(5Bd-6Ae)))\log\left(a+cx^2\right)}{2\left(cd^2+ae^2\right)^4}\end{aligned}$$

```
[Out] -e^3*(A*e^2-B*d*e+C*d^2)/(a*e^2+c*d^2)^3/(e*x+d)+1/4*(-a*(-2*A*c*d*e-B*a*e^
2+B*c*d^2+2*C*a*d*e)+(A*c*(-a*e^2+c*d^2)+a*(a*C*e^2-c*d*(-2*B*e+C*d)))*x)/a
/(a*e^2+c*d^2)^2/(c*x^2+a)^2+1/8*(-4*a^2*e*(a*e^2*(-B*e+2*C*d)-c*d*(2*C*d^2
-e*(-4*A*e+3*B*d)))+(A*c*(-7*a^2*e^4+12*a*c*d^2*e^2+3*c^2*d^4)+a*(3*a^2*C*e
^4-2*a*c*d*e^2*(-7*B*e+6*C*d)+c^2*d^3*(-2*B*e+C*d)))*x)/a^2/(a*e^2+c*d^2)^3
/(c*x^2+a)-e^3*(a*e^2*(-B*e+2*C*d)-c*d*(4*C*d^2-e*(-6*A*e+5*B*d)))*ln(e*x+d
)/(a*e^2+c*d^2)^4+1/2*e^3*(a*e^2*(-B*e+2*C*d)-c*d*(4*C*d^2-e*(-6*A*e+5*B*d)
))*ln(c*x^2+a)/(a*e^2+c*d^2)^4+1/8*(3*A*c*(-5*a^3*e^6+15*a^2*c*d^2*e^4+5*a*
c^2*d^4*e^2+c^3*d^6)+a*(3*a^3*C*e^6+a*c^2*d^3*e^2*(-20*B*e+13*C*d)-3*a^2*c*
d*e^4*(-10*B*e+11*C*d)+c^3*d^5*(-2*B*e+C*d)))*arctan(x*c^(1/2)/a^(1/2))/a^(
5/2)/(a*e^2+c*d^2)^4/c^(1/2)
```

Rubi [A] (verified)

Time = 1.27 (sec) , antiderivative size = 566, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1661, 1643, 649, 211, 266}

$$\int \frac{A+Bx+Cx^2}{(d+ex)^2\left(a+cx^2\right)^3}\,dx$$
$$=\frac{x(Ac(-7a^2e^4+12acd^2e^2+3c^2d^4)+a(3a^2Ce^4-2acde^2(6Cd-7Be)+c^2d^3(Cd-2Be)))+4a^2e(-ae^2(2}{8a^2\left(a+cx^2\right)\left(ae^2+cd^2\right)^3}$$
$$+\frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(3Ac(-5a^3e^6+15a^2cd^2e^4+5ac^2d^4e^2+c^3d^6)+a(3a^3Ce^6-3a^2cde^4(11Cd-10Be)+ac^2}{8a^{5/2}\sqrt{c}\left(ae^2+cd^2\right)^4}$$
$$-\frac{a(-aBe^2+2aCde-2Acde+Bcd^2)-x(Ac(cd^2-ae^2)+a(aCe^2-cd(Cd-2Be)))}{4a\left(a+cx^2\right)^2\left(ae^2+cd^2\right)^2}$$
$$-\frac{e^3(Ae^2-Bde+Cd^2)}{(d+ex)\left(ae^2+cd^2\right)^3}-\frac{e^3\log\left(a+cx^2\right)(-ae^2(2Cd-Be)-cde(5Bd-6Ae)+4cCd^3)}{2\left(ae^2+cd^2\right)^4}$$
$$+\frac{e^3\log(d+ex)(-ae^2(2Cd-Be)-cde(5Bd-6Ae)+4cCd^3)}{\left(ae^2+cd^2\right)^4}$$

```
[In] Int[(A + B*x + C*x^2)/((d + e*x)^2*(a + c*x^2)^3),x]
```

```
[Out] -((e^3*(C*d^2 - B*d*e + A*e^2))/((c*d^2 + a*e^2)^3*(d + e*x))) - (a*(B*c*d^
2 - 2*A*c*d*e + 2*a*C*d*e - a*B*e^2) - (A*c*(c*d^2 - a*e^2) + a*(a*C*e^2 -
c*d*(C*d - 2*B*e)))*x)/(4*a*(c*d^2 + a*e^2)^2*(a + c*x^2)^2) + (4*a^2*e*(2*
c*C*d^3 - c*d*e*(3*B*d - 4*A*e) - a*e^2*(2*C*d - B*e)) + (A*c*(3*c^2*d^4 +
12*a*c*d^2*e^2 - 7*a^2*e^4) + a*(3*a^2*C*e^4 - 2*a*c*d*e^2*(6*C*d - 7*B*e)
+ c^2*d^3*(C*d - 2*B*e)))*x)/(8*a^2*(c*d^2 + a*e^2)^3*(a + c*x^2)) + ((3*A*
c*(c^3*d^6 + 5*a*c^2*d^4*e^2 + 15*a^2*c*d^2*e^4 - 5*a^3*e^6) + a*(3*a^3*C*e
^6 + a*c^2*d^3*e^2*(13*C*d - 20*B*e) - 3*a^2*c*d*e^4*(11*C*d - 10*B*e) + c^
3*d^5*(C*d - 2*B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(8*a^(5/2)*Sqrt[c]*(c*d^
2 + a*e^2)^4) + (e^3*(4*c*C*d^3 - c*d*e*(5*B*d - 6*A*e) - a*e^2*(2*C*d - B*
e))*Log[d + e*x])/(c*d^2 + a*e^2)^4 - (e^3*(4*c*C*d^3 - c*d*e*(5*B*d - 6*A*
e) - a*e^2*(2*C*d - B*e))*Log[a + c*x^2])/(2*(c*d^2 + a*e^2)^4)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

integral

$$= -\frac{a(Bcd^2-2Acde+2aCde-aBe^2)-(Ac(cd^2-ae^2)+a(aCe^2-cd(Cd-2Be)))\,x}{4a\,(cd^2+ae^2)^2\,(a+cx^2)^2}$$

$$-\frac{\int \frac{-\frac{c(A(3c^2d^4+9acd^2e^2+4a^2e^4)-ad^2(aCe^2-cd(Cd-2Be)))}{(cd^2+ae^2)^2}-\frac{2ce(Acd(3cd^2+ae^2)-a(cd^2(3Cd-4Be)+ae^2(Cd-2Be)))x}{(cd^2+ae^2)^2}-\frac{3ce^2(Ac(cd^2-ae^2)+a(aC}{(cd^2+a}}{(d+ex)^2(a+cx^2)^2}}{4ac}$$

$$= -\frac{a(Bcd^2-2Acde+2aCde-aBe^2)-(Ac(cd^2-ae^2)+a(aCe^2-cd(Cd-2Be)))\,x}{4a\,(cd^2+ae^2)^2\,(a+cx^2)^2}$$

$$+\frac{4a^2e(2cCd^3-cde(3Bd-4Ae)-ae^2(2Cd-Be))+(Ac(3c^2d^4+12acd^2e^2-7a^2e^4)+a(3a^2Ce}{8a^2\,(cd^2+ae^2)^3\,(a+cx^2)}$$

$$+\frac{\int \frac{\frac{c^2(A(3c^3d^6+12ac^2d^4e^2+33a^2cd^2e^4+8a^3e^6)-ad^2(5a^2Ce^4-6acde^2(2Cd-3Be)-c^2d^3(Cd-2Be)))}{(cd^2+ae^2)^3}+\frac{2c^2e(3Acd(cd^2+3ae^2)-a(ae^2(5Cd-}{(cd^2+ae^2)^2}}{(d+ex)^2(a+cx^2)}}{8a^2c^2}$$

$$=-\frac{a(Bcd^2-2Acde+2aCde-aBe^2)-(Ac(cd^2-ae^2)+a(aCe^2-cd(Cd-2Be)))\,x}{4a\left(cd^2+ae^2\right)^2\left(a+cx^2\right)^2}$$

$$+\frac{4a^2e(2cCd^3-cde(3Bd-4Ae)-ae^2(2Cd-Be))+(Ac(3c^2d^4+12acd^2e^2-7a^2e^4)+a(3a^2Ce^4}{8a^2\left(cd^2+ae^2\right)^3\left(a+cx^2\right)}$$

$$+\frac{\int\left(\frac{8a^2c^2e^4\left(Cd^2-Bde+Ae^2\right)}{\left(cd^2+ae^2\right)^3(d+ex)^2}+\frac{8a^2c^2e^4\left(4cCd^3-cde(5Bd-6Ae)-ae^2(2Cd-Be)\right)}{\left(cd^2+ae^2\right)^4(d+ex)}+\frac{c^2\left(3Ac\left(c^3d^6+5ac^2d^4e^2+15a^2cd^2e^4-5a^3e\right.\right.}{}\right.}{}$$

$$=-\frac{e^3(Cd^2-Bde+Ae^2)}{\left(cd^2+ae^2\right)^3(d+ex)}$$

$$-\frac{a(Bcd^2-2Acde+2aCde-aBe^2)-(Ac(cd^2-ae^2)+a(aCe^2-cd(Cd-2Be)))\,x}{4a\left(cd^2+ae^2\right)^2\left(a+cx^2\right)^2}$$

$$+\frac{4a^2e(2cCd^3-cde(3Bd-4Ae)-ae^2(2Cd-Be))+(Ac(3c^2d^4+12acd^2e^2-7a^2e^4)+a(3a^2Ce^4}{8a^2\left(cd^2+ae^2\right)^3\left(a+cx^2\right)}$$

$$+\frac{e^3(4cCd^3-cde(5Bd-6Ae)-ae^2(2Cd-Be))\log(d+ex)}{\left(cd^2+ae^2\right)^4}$$

$$+\frac{\int\frac{3Ac\left(c^3d^6+5ac^2d^4e^2+15a^2cd^2e^4-5a^3e^6\right)+a\left(3a^3Ce^6+ac^2d^3e^2(13Cd-20Be)-3a^2cde^4(11Cd-10Be)+c^3d^5(Cd-2Be)\right)-8a^2ce^3(}{a+cx^2}}{8a^2\left(cd^2+ae^2\right)^4}$$

$$=-\frac{e^3(Cd^2-Bde+Ae^2)}{\left(cd^2+ae^2\right)^3(d+ex)}$$

$$-\frac{a(Bcd^2-2Acde+2aCde-aBe^2)-(Ac(cd^2-ae^2)+a(aCe^2-cd(Cd-2Be)))\,x}{4a\left(cd^2+ae^2\right)^2\left(a+cx^2\right)^2}$$

$$+\frac{4a^2e(2cCd^3-cde(3Bd-4Ae)-ae^2(2Cd-Be))+(Ac(3c^2d^4+12acd^2e^2-7a^2e^4)+a(3a^2Ce^4}{8a^2\left(cd^2+ae^2\right)^3\left(a+cx^2\right)}$$

$$+\frac{e^3(4cCd^3-cde(5Bd-6Ae)-ae^2(2Cd-Be))\log(d+ex)}{\left(cd^2+ae^2\right)^4}$$

$$-\frac{(ce^3(4cCd^3-cde(5Bd-6Ae)-ae^2(2Cd-Be)))\int\frac{x}{a+cx^2}\,dx}{\left(cd^2+ae^2\right)^4}$$

$$+\frac{(3Ac(c^3d^6+5ac^2d^4e^2+15a^2cd^2e^4-5a^3e^6)+a(3a^3Ce^6+ac^2d^3e^2(13Cd-20Be)-3a^2cde^4(11C}{8a^2\left(cd^2+ae^2\right)^4}$$

$$
\begin{aligned}
&= -\frac{e^3(Cd^2-Bde+Ae^2)}{(cd^2+ae^2)^3\,(d+ex)} \\
&\quad -\frac{a(Bcd^2-2Acde+2aCde-aBe^2)-(Ac(cd^2-ae^2)+a(aCe^2-cd(Cd-2Be)))\,x}{4a\,(cd^2+ae^2)^2\,(a+cx^2)^2} \\
&\quad +\frac{4a^2e(2cCd^3-cde(3Bd-4Ae)-ae^2(2Cd-Be))+(Ac(3c^2d^4+12acd^2e^2-7a^2e^4)+a(3a^2Ce}{8a^2\,(cd^2+ae^2)^3\,(a+cx^2)} \\
&\quad +\frac{(3Ac(c^3d^6+5ac^2d^4e^2+15a^2cd^2e^4-5a^3e^6)+a(3a^3Ce^6+ac^2d^3e^2(13Cd-20Be)-3a^2cde^4(11}{8a^{5/2}\sqrt{c}\,(cd^2+ae^2)^4} \\
&\quad +\frac{e^3(4cCd^3-cde(5Bd-6Ae)-ae^2(2Cd-Be))\log(d+ex)}{(cd^2+ae^2)^4} \\
&\quad -\frac{e^3(4cCd^3-cde(5Bd-6Ae)-ae^2(2Cd-Be))\log\left(a+cx^2\right)}{2\,(cd^2+ae^2)^4}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 498, normalized size of antiderivative = 0.87

$$
\int \frac{A+Bx+Cx^2}{(d+ex)^2\,(a+cx^2)^3}\,dx
$$

$$
= \frac{-\frac{8e^3(cd^2+ae^2)(Cd^2+e(-Bd+Ae))}{d+ex}+\frac{2(cd^2+ae^2)^2(Ac^2d^2x+a^2e(-2Cd+Be+Cex)-ac(Cd^2x+Bd(d-2ex)+Ae(-2d+ex)))}{a(a+cx^2)^2}+\frac{(cd^2+ae^2}{}}{}
$$

```
[In] Integrate[(A + B*x + C*x^2)/((d + e*x)^2*(a + c*x^2)^3),x]
```

```
[Out] ((-8*e^3*(c*d^2 + a*e^2)*(C*d^2 + e*(-(B*d) + A*e)))/(d + e*x) + (2*(c*d^2
+ a*e^2)^2*(A*c^2*d^2*x + a^2*e*(-2*C*d + B*e + C*e*x) - a*c*(C*d^2*x + B*d
*(d - 2*e*x) + A*e*(-2*d + e*x))))/(a*(a + c*x^2)^2) + ((c*d^2 + a*e^2)*(3*
A*c^3*d^4*x + a*c^2*d^2*(C*d^2 + 2*e*(-(B*d) + 6*A*e))*x + a^3*e^3*(-8*C*d
+ 4*B*e + 3*C*e*x) + a^2*c*e*(4*C*d^2*(2*d - 3*e*x) + e*(-2*B*d*(6*d - 7*e*
x) + A*e*(16*d - 7*e*x)))))/(a^2*(a + c*x^2)) + ((3*A*c*(c^3*d^6 + 5*a*c^2*
d^4*e^2 + 15*a^2*c*d^2*e^4 - 5*a^3*e^6) + a*(3*a^3*C*e^6 + a*c^2*d^3*e^2*(1
3*C*d - 20*B*e) + c^3*d^5*(C*d - 2*B*e) + 3*a^2*c*d*e^4*(-11*C*d + 10*B*e))
)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(a^(5/2)*Sqrt[c]) + 8*e^3*(4*c*C*d^3 + c*d*e
*(-5*B*d + 6*A*e) + a*e^2*(-2*C*d + B*e))*Log[d + e*x] - 4*e^3*(4*c*C*d^3 +
 c*d*e*(-5*B*d + 6*A*e) + a*e^2*(-2*C*d + B*e))*Log[a + c*x^2])/(8*(c*d^2 +
 a*e^2)^4)
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 833, normalized size of antiderivative = 1.46

method	result
default	$-\frac{\frac{c\left(7A\,a^{3}c\,e^{6}-5A\,a^{2}c^{2}d^{2}e^{4}-15Aa\,c^{3}d^{4}e^{2}-3A\,d^{6}c^{4}-14B\,a^{3}cd\,e^{5}-12B\,a^{2}c^{2}d^{3}e^{3}+2Ba\,c^{3}d^{5}e-3C\,a^{4}e^{6}+9C\,a^{3}c\,d^{2}e^{4}+11C\,a^{2}c^{2}d^{4}e^{2}-Ca\,c^{3}d^{6}\right)x}{8a^{2}}}{}$
risch	Expression too large to display

```
[In] int((C*x^2+B*x+A)/(e*x+d)^2/(c*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/(a*e^2+c*d^2)^4*((1/8*c*(7*A*a^3*c*e^6-5*A*a^2*c^2*d^2*e^4-15*A*a*c^3*d^
4*e^2-3*A*c^4*d^6-14*B*a^3*c*d*e^5-12*B*a^2*c^2*d^3*e^3+2*B*a*c^3*d^5*e-3*C
*a^4*e^6+9*C*a^3*c*d^2*e^4+11*C*a^2*c^2*d^4*e^2-C*a*c^3*d^6)/a^2*x^3+(-2*A*
a*c^2*d*e^5-2*A*c^3*d^3*e^3-1/2*B*a^2*c*e^6+B*a*c^2*d^2*e^4+3/2*B*c^3*d^4*e
^2+C*a^2*c*d*e^5-C*c^3*d^5*e)*x^2+1/8*(9*A*a^3*c*e^6-3*A*a^2*c^2*d^2*e^4-17
*A*a*c^3*d^4*e^2-5*A*c^4*d^6-18*B*a^3*c*d*e^5-20*B*a^2*c^2*d^3*e^3-2*B*a*c^
3*d^5*e-5*C*a^4*e^6+7*C*a^3*c*d^2*e^4+13*C*a^2*c^2*d^4*e^2+C*a*c^3*d^6)/a*x
-5/2*A*a^2*c*d*e^5-3*A*a*c^2*d^3*e^3-1/2*A*c^3*d^5*e-3/4*B*a^3*e^6+3/4*B*a^
2*c*d^2*e^4+7/4*B*a*c^2*d^4*e^2+1/4*B*c^3*d^6+3/2*C*a^3*d*e^5+C*a^2*c*d^3*e
^3-1/2*C*a*c^2*d^5*e)/(c*x^2+a)^2+1/8/a^2*(1/2*(48*A*a^2*c^2*d*e^5+8*B*a^3*
c*e^6-40*B*a^2*c^2*d^2*e^4-16*C*a^3*c*d*e^5+32*C*a^2*c^2*d^3*e^3)/c*ln(c*x^
2+a)+(15*A*a^3*c*e^6-45*A*a^2*c^2*d^2*e^4-15*A*a*c^3*d^4*e^2-3*A*c^4*d^6-30
*B*a^3*c*d*e^5+20*B*a^2*c^2*d^3*e^3+2*B*a*c^3*d^5*e-3*C*a^4*e^6+33*C*a^3*c*
d^2*e^4-13*C*a^2*c^2*d^4*e^2-C*a*c^3*d^6)/(a*c)^(1/2)*arctan(c*x/(a*c)^(1/2
))))+e^3*(6*A*c*d*e^2+B*a*e^3-5*B*c*d^2*e-2*C*a*d*e^2+4*C*c*d^3)/(a*e^2+c*d
^2)^4*ln(e*x+d)-e^3*(A*e^2-B*d*e+C*d^2)/(a*e^2+c*d^2)^3/(e*x+d)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^{2}}{(d+ex)^{2}\left(a+cx^{2}\right)^{3}}\,dx=\text{Timed out}$$

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^2/(c*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(d+ex)^2\left(a+cx^2\right)^3}\,dx=\text{Timed out}$$

```
[In] integrate((C*x**2+B*x+A)/(e*x+d)**2/(c*x**2+a)**3,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1196 vs. $2(555) = 1110$.

Time = 0.32 (sec) , antiderivative size = 1196, normalized size of antiderivative = 2.09

$$\int \frac{A+Bx+Cx^2}{(d+ex)^2\left(a+cx^2\right)^3}\,dx=\text{Too large to display}$$

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^2/(c*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(4*C*c*d^3*e^3 - 5*B*c*d^2*e^4 + B*a*e^6 - 2*(C*a - 3*A*c)*d*e^5)*log(
c*x^2 + a)/(c^4*d^8 + 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6
 + a^4*e^8) + (4*C*c*d^3*e^3 - 5*B*c*d^2*e^4 + B*a*e^6 - 2*(C*a - 3*A*c)*d*
e^5)*log(e*x + d)/(c^4*d^8 + 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 + 4*a^3*c*
d^2*e^6 + a^4*e^8) - 1/8*(2*B*a*c^3*d^5*e + 20*B*a^2*c^2*d^3*e^3 - 30*B*a^3
*c*d*e^5 - (C*a*c^3 + 3*A*c^4)*d^6 - (13*C*a^2*c^2 + 15*A*a*c^3)*d^4*e^2 +
3*(11*C*a^3*c - 15*A*a^2*c^2)*d^2*e^4 - 3*(C*a^4 - 5*A*a^3*c)*e^6)*arctan(c
*x/sqrt(a*c))/((a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5
*c*d^2*e^6 + a^6*e^8)*sqrt(a*c)) - 1/8*(2*B*a^2*c^2*d^5 + 12*B*a^3*c*d^3*e^
2 - 14*B*a^4*d*e^4 + 8*A*a^4*e^5 - 4*(C*a^3*c + A*a^2*c^2)*d^4*e + 20*(C*a^
4 - A*a^3*c)*d^2*e^3 + (2*B*a*c^3*d^3*e^2 - 22*B*a^2*c^2*d*e^4 - (C*a*c^3 +
 3*A*c^4)*d^4*e + 4*(5*C*a^2*c^2 - 3*A*a*c^3)*d^2*e^3 - 3*(C*a^3*c - 5*A*a^
2*c^2)*e^5)*x^4 + (2*B*a*c^3*d^4*e - 2*B*a^2*c^2*d^2*e^3 - 4*B*a^3*c*e^5 -
(C*a*c^3 + 3*A*c^4)*d^5 + 4*(C*a^2*c^2 - 3*A*a*c^3)*d^3*e^2 + (5*C*a^3*c -
9*A*a^2*c^2)*d*e^4)*x^3 + (10*B*a^2*c^2*d^3*e^2 - 38*B*a^3*c*d*e^4 - (7*C*a
^2*c^2 + 5*A*a*c^3)*d^4*e + 4*(9*C*a^3*c - 7*A*a^2*c^2)*d^2*e^3 - 5*(C*a^4
- 5*A*a^3*c)*e^5)*x^2 - (6*B*a^3*c*d^2*e^3 + 6*B*a^4*e^5 - (C*a^2*c^2 - 5*A
*a*c^3)*d^5 - 8*(C*a^3*c - 2*A*a^2*c^2)*d^3*e^2 - (7*C*a^4 - 11*A*a^3*c)*d*
e^4)*x)/(a^4*c^3*d^7 + 3*a^5*c^2*d^5*e^2 + 3*a^6*c*d^3*e^4 + a^7*d*e^6 + (a
^2*c^5*d^6*e + 3*a^3*c^4*d^4*e^3 + 3*a^4*c^3*d^2*e^5 + a^5*c^2*e^7)*x^5 + (
a^2*c^5*d^7 + 3*a^3*c^4*d^5*e^2 + 3*a^4*c^3*d^3*e^4 + a^5*c^2*d*e^6)*x^4 +
2*(a^3*c^4*d^6*e + 3*a^4*c^3*d^4*e^3 + 3*a^5*c^2*d^2*e^5 + a^6*c*e^7)*x^3 +
 2*(a^3*c^4*d^7 + 3*a^4*c^3*d^5*e^2 + 3*a^5*c^2*d^3*e^4 + a^6*c*d*e^6)*x^2
+ (a^4*c^3*d^6*e + 3*a^5*c^2*d^4*e^3 + 3*a^6*c*d^2*e^5 + a^7*e^7)*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1175 vs. $2(555) = 1110$.

Time = 0.30 (sec) , antiderivative size = 1175, normalized size of antiderivative = 2.06

$$\int \frac{A+Bx+Cx^2}{(d+ex)^2\,(a+cx^2)^3}\,dx$$

$$= -\frac{(4\,Ccd^3e^3 - 5\,Bcd^2e^4 - 2\,Cade^5 + 6\,Acde^5 + Bae^6)\log\left(c - \frac{2\,cd}{ex+d} + \frac{cd^2}{(ex+d)^2} + \frac{ae^2}{(ex+d)^2}\right)}{2\,(c^4d^8 + 4\,ac^3d^6e^2 + 6\,a^2c^2d^4e^4 + 4\,a^3cd^2e^6 + a^4e^8)}$$

$$- \frac{\frac{Cd^2e^9}{ex+d} - \frac{Bde^{10}}{ex+d} + \frac{Ae^{11}}{ex+d}}{c^3d^6e^6 + 3\,ac^2d^4e^8 + 3\,a^2cd^2e^{10} + a^3e^{12}}$$

$$+ \frac{(Cac^3d^6e^2 + 3\,Ac^4d^6e^2 - 2\,Bac^3d^5e^3 + 13\,Ca^2c^2d^4e^4 + 15\,Aac^3d^4e^4 - 20\,Ba^2c^2d^3e^5 - 33\,Ca^3cd^2e^6 + 45}{8\,(a^2c^4d^8 + 4\,a^3c^3d^6e^2 + 6\,a^4c^2d^4e^4 + 4\,a^5cd^2e^6}$$

$$+ \frac{Cac^4d^5e + 3\,Ac^5d^5e - 2\,Bac^4d^4e^2 - 22\,Ca^2c^3d^3e^3 + 14\,Aac^4d^3e^3 + 32\,Ba^2c^3d^2e^4 + 17\,Ca^3c^2de^5 - 29\,Aa}{}$$

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^2/(c*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/2*(4*C*c*d^3*e^3 - 5*B*c*d^2*e^4 - 2*C*a*d*e^5 + 6*A*c*d*e^5 + B*a*e^6)*
log(c - 2*c*d/(e*x + d) + c*d^2/(e*x + d)^2 + a*e^2/(e*x + d)^2)/(c^4*d^8 +
 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8) - (C*d^2*
e^9/(e*x + d) - B*d*e^10/(e*x + d) + A*e^11/(e*x + d))/(c^3*d^6*e^6 + 3*a*c
^2*d^4*e^8 + 3*a^2*c*d^2*e^10 + a^3*e^12) + 1/8*(C*a*c^3*d^6*e^2 + 3*A*c^4*
d^6*e^2 - 2*B*a*c^3*d^5*e^3 + 13*C*a^2*c^2*d^4*e^4 + 15*A*a*c^3*d^4*e^4 - 2
0*B*a^2*c^2*d^3*e^5 - 33*C*a^3*c*d^2*e^6 + 45*A*a^2*c^2*d^2*e^6 + 30*B*a^3*
c*d*e^7 + 3*C*a^4*e^8 - 15*A*a^3*c*e^8)*arctan((c*d - c*d^2/(e*x + d) - a*e
^2/(e*x + d))/(sqrt(a*c)*e))/((a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*
d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8)*sqrt(a*c)*e^2) + 1/8*(C*a*c^4*d^5*e +
3*A*c^5*d^5*e - 2*B*a*c^4*d^4*e^2 - 22*C*a^2*c^3*d^3*e^3 + 14*A*a*c^4*d^3*e
^3 + 32*B*a^2*c^3*d^2*e^4 + 17*C*a^3*c^2*d*e^5 - 29*A*a^2*c^3*d*e^5 - 6*B*a
^3*c^2*e^6 - (3*C*a*c^4*d^6*e^2 + 9*A*c^5*d^6*e^2 - 6*B*a*c^4*d^5*e^3 - 77*
C*a^2*c^3*d^4*e^4 + 41*A*a*c^4*d^4*e^4 + 116*B*a^2*c^3*d^3*e^5 + 77*C*a^3*c
^2*d^2*e^6 - 121*A*a^2*c^3*d^2*e^6 - 38*B*a^3*c^2*d*e^7 - 3*C*a^4*c*e^8 + 7
*A*a^3*c^2*e^8)/((e*x + d)*e) + (3*C*a*c^4*d^7*e^3 + 9*A*c^5*d^7*e^3 - 6*B*
a*c^4*d^6*e^4 - 89*C*a^2*c^3*d^5*e^5 + 45*A*a*c^4*d^5*e^5 + 140*B*a^2*c^3*d
^4*e^6 + 85*C*a^3*c^2*d^3*e^7 - 145*A*a^2*c^3*d^3*e^7 - 22*B*a^3*c^2*d^2*e^
8 + 17*C*a^4*c*d*e^9 - 21*A*a^3*c^2*d*e^9 - 8*B*a^4*c*e^10)/((e*x + d)^2*e^
2) - (C*a*c^4*d^8*e^4 + 3*A*c^5*d^8*e^4 - 2*B*a*c^4*d^7*e^5 - 34*C*a^2*c^3*
d^6*e^6 + 18*A*a*c^4*d^6*e^6 + 58*B*a^2*c^3*d^5*e^7 + 20*C*a^3*c^2*d^4*e^8
- 60*A*a^2*c^3*d^4*e^8 + 26*B*a^3*c^2*d^3*e^9 + 50*C*a^4*c*d^2*e^10 - 66*A*
a^3*c^2*d^2*e^10 - 34*B*a^4*c*d*e^11 - 5*C*a^5*e^12 + 9*A*a^4*c*e^12)/((e*x
```

```
 + d)^3*e^3))/((c*d^2 + a*e^2)^4*a^2*(c - 2*c*d/(e*x + d) + c*d^2/(e*x + d)
^2 + a*e^2/(e*x + d)^2)^2)
```

Mupad [B] (verification not implemented)

Time = 16.66 (sec) , antiderivative size = 6848, normalized size of antiderivative = 11.99

$$\int \frac{A+Bx+Cx^2}{(d+ex)^2\,(a+cx^2)^3}\,dx = \text{Too large to display}$$

```
[In] int((A + B*x + C*x^2)/((a + c*x^2)^3*(d + e*x)^2),x)
```

```
[Out] symsum(log(root(17920*a^9*c^5*d^8*e^8*z^3 + 14336*a^10*c^4*d^6*e^10*z^3 + 1
4336*a^8*c^6*d^10*e^6*z^3 + 7168*a^11*c^3*d^4*e^12*z^3 + 7168*a^7*c^7*d^12*
e^4*z^3 + 2048*a^12*c^2*d^2*e^14*z^3 + 2048*a^6*c^8*d^14*e^2*z^3 + 256*a^5*
c^9*d^16*z^3 + 256*a^13*c*e^16*z^3 + 948*B*C*a^7*c*d*e^11*z - 12*A*B*a*c^7*
d^11*e*z + 9768*B*C*a^5*c^3*d^5*e^7*z - 7476*B*C*a^6*c^2*d^3*e^9*z - 328*B*
C*a^4*c^4*d^7*e^5*z - 92*B*C*a^3*c^5*d^9*e^3*z - 12486*A*C*a^5*c^3*d^4*e^8*
z + 5868*A*C*a^6*c^2*d^2*e^10*z + 282*A*C*a^3*c^5*d^8*e^4*z + 168*A*C*a^4*c
^4*d^6*e^6*z + 108*A*C*a^2*c^6*d^10*e^2*z + 14820*A*B*a^5*c^3*d^3*e^9*z - 8
40*A*B*a^4*c^4*d^5*e^7*z - 600*A*B*a^3*c^5*d^7*e^5*z - 180*A*B*a^2*c^6*d^9*
e^3*z - 4*B*C*a^2*c^6*d^11*e*z - 3204*A*B*a^6*c^2*d*e^11*z + 4239*C^2*a^6*c
^2*d^4*e^8*z - 3924*C^2*a^5*c^3*d^6*e^6*z + 103*C^2*a^4*c^4*d^8*e^4*z + 26*
C^2*a^3*c^5*d^10*e^2*z - 6000*B^2*a^5*c^3*d^4*e^8*z + 2820*B^2*a^6*c^2*d^2*
e^10*z + 280*B^2*a^4*c^4*d^6*e^6*z + 80*B^2*a^3*c^5*d^8*e^4*z + 4*B^2*a^2*c
^6*d^10*e^2*z - 8262*A^2*a^5*c^3*d^2*e^10*z + 1575*A^2*a^4*c^4*d^4*e^8*z +
1260*A^2*a^3*c^5*d^6*e^6*z + 495*A^2*a^2*c^6*d^8*e^4*z - 90*A*C*a^7*c*e^12*
z + 6*A*C*a*c^7*d^12*z - 966*C^2*a^7*c*d^2*e^10*z + 90*A^2*a*c^7*d^10*e^2*z
 + C^2*a^2*c^6*d^12*z + 225*A^2*a^6*c^2*e^12*z - 192*B^2*a^7*c*e^12*z + 9*A
^2*c^8*d^12*z + 9*C^2*a^8*e^12*z + 78*A*B*C*a*c^4*d^6*e^4 + 942*A*B*C*a^2*c
^3*d^4*e^6 - 342*A*B*C*a^3*c^2*d^2*e^8 - 129*B*C^2*a^4*c*d^2*e^8 + 990*A^2*
C*a^3*c^2*d*e^9 - 234*A^2*C*a*c^4*d^5*e^5 - 24*A*C^2*a*c^4*d^7*e^3 + 333*A^
2*B*a*c^4*d^4*e^6 - 252*A*B^2*a^3*c^2*d*e^9 - 60*A*B^2*a*c^4*d^5*e^5 + 204*
B^2*C*a^4*c*d*e^9 - 234*A*C^2*a^4*c*d*e^9 - 624*B^2*C*a^3*c^2*d^3*e^7 + 405
*B*C^2*a^3*c^2*d^4*e^6 - 36*B^2*C*a^2*c^3*d^5*e^5 + 21*B*C^2*a^2*c^3*d^6*e^
4 - 1296*A^2*C*a^2*c^3*d^3*e^7 + 396*A*C^2*a^3*c^2*d^3*e^7 - 330*A*C^2*a^2*
c^3*d^5*e^5 + 1863*A^2*B*a^2*c^3*d^2*e^8 - 672*A*B^2*a^2*c^3*d^3*e^7 + 90*A
*B*C*a^4*c*e^10 + 8*C^3*a^4*c*d^3*e^7 - 1350*A^3*a^2*c^3*d*e^9 - 324*A^3*a*
c^4*d^3*e^7 - 36*A^2*C*c^5*d^7*e^3 + 45*A^2*B*c^5*d^6*e^4 - 225*A^2*B*a^3*c
^2*e^10 - 86*C^3*a^3*c^2*d^5*e^5 - 4*C^3*a^2*c^3*d^7*e^3 + 316*B^3*a^3*c^2*
d^2*e^8 + 20*B^3*a^2*c^3*d^4*e^6 + 18*C^3*a^5*d*e^9 - 64*B^3*a^4*c*e^10 - 9
*B*C^2*a^5*e^10 - 54*A^3*c^5*d^5*e^5, z, k)*((120*A*a^8*c^2*e^13 - 24*C*a^9
*c*e^13 + 24*A*a^2*c^8*d^12*e - 112*B*a^8*c^2*d*e^12 + 8*C*a^3*c^7*d^12*e +
 144*A*a^3*c^7*d^10*e^3 + 456*A*a^4*c^6*d^8*e^5 + 864*A*a^5*c^5*d^6*e^7 + 9
36*A*a^6*c^4*d^4*e^9 + 528*A*a^7*c^3*d^2*e^11 - 16*B*a^3*c^7*d^11*e^2 - 176
```

```
*B*a^4*c^6*d^9*e^4 - 544*B*a^5*c^5*d^7*e^6 - 736*B*a^6*c^4*d^5*e^8 - 464*B*
a^7*c^3*d^3*e^10 + 112*C*a^4*c^6*d^10*e^3 + 344*C*a^5*c^5*d^8*e^5 + 416*C*a
^6*c^4*d^6*e^7 + 184*C*a^7*c^3*d^4*e^9 - 16*C*a^8*c^2*d^2*e^11)/(64*(a^10*e
^12 + a^4*c^6*d^12 + 6*a^9*c*d^2*e^10 + 6*a^5*c^5*d^10*e^2 + 15*a^6*c^4*d^8
*e^4 + 20*a^7*c^3*d^6*e^6 + 15*a^8*c^2*d^4*e^8)) + root(17920*a^9*c^5*d^8*e
^8*z^3 + 14336*a^10*c^4*d^6*e^10*z^3 + 14336*a^8*c^6*d^10*e^6*z^3 + 7168*a^
11*c^3*d^4*e^12*z^3 + 7168*a^7*c^7*d^12*e^4*z^3 + 2048*a^12*c^2*d^2*e^14*z^
3 + 2048*a^6*c^8*d^14*e^2*z^3 + 256*a^5*c^9*d^16*z^3 + 256*a^13*c*e^16*z^3
+ 948*B*C*a^7*c*d*e^11*z - 12*A*B*a*c^7*d^11*e*z + 9768*B*C*a^5*c^3*d^5*e^7
*z - 7476*B*C*a^6*c^2*d^3*e^9*z - 328*B*C*a^4*c^4*d^7*e^5*z - 92*B*C*a^3*c^
5*d^9*e^3*z - 12486*A*C*a^5*c^3*d^4*e^8*z + 5868*A*C*a^6*c^2*d^2*e^10*z + 2
82*A*C*a^3*c^5*d^8*e^4*z + 168*A*C*a^4*c^4*d^6*e^6*z + 108*A*C*a^2*c^6*d^10
*e^2*z + 14820*A*B*a^5*c^3*d^3*e^9*z - 840*A*B*a^4*c^4*d^5*e^7*z - 600*A*B*
a^3*c^5*d^7*e^5*z - 180*A*B*a^2*c^6*d^9*e^3*z - 4*B*C*a^2*c^6*d^11*e*z - 32
04*A*B*a^6*c^2*d*e^11*z + 4239*C^2*a^6*c^2*d^4*e^8*z - 3924*C^2*a^5*c^3*d^6
*e^6*z + 103*C^2*a^4*c^4*d^8*e^4*z + 26*C^2*a^3*c^5*d^10*e^2*z - 6000*B^2*a
^5*c^3*d^4*e^8*z + 2820*B^2*a^6*c^2*d^2*e^10*z + 280*B^2*a^4*c^4*d^6*e^6*z
+ 80*B^2*a^3*c^5*d^8*e^4*z + 4*B^2*a^2*c^6*d^10*e^2*z - 8262*A^2*a^5*c^3*d^
2*e^10*z + 1575*A^2*a^4*c^4*d^4*e^8*z + 1260*A^2*a^3*c^5*d^6*e^6*z + 495*A^
2*a^2*c^6*d^8*e^4*z - 90*A*C*a^7*c*e^12*z + 6*A*C*a*c^7*d^12*z - 966*C^2*a^
7*c*d^2*e^10*z + 90*A^2*a*c^7*d^10*e^2*z + C^2*a^2*c^6*d^12*z + 225*A^2*a^6
*c^2*e^12*z - 192*B^2*a^7*c*e^12*z + 9*A^2*c^8*d^12*z + 9*C^2*a^8*e^12*z +
78*A*B*C*a*c^4*d^6*e^4 + 942*A*B*C*a^2*c^3*d^4*e^6 - 342*A*B*C*a^3*c^2*d^2*
e^8 - 129*B*C^2*a^4*c*d^2*e^8 + 990*A^2*C*a^3*c^2*d*e^9 - 234*A^2*C*a*c^4*d
^5*e^5 - 24*A*C^2*a*c^4*d^7*e^3 + 333*A^2*B*a*c^4*d^4*e^6 - 252*A*B^2*a^3*c
^2*d*e^9 - 60*A*B^2*a*c^4*d^5*e^5 + 204*B^2*C*a^4*c*d*e^9 - 234*A*C^2*a^4*c
*d*e^9 - 624*B^2*C*a^3*c^2*d^3*e^7 + 405*B*C^2*a^3*c^2*d^4*e^6 - 36*B^2*C*a
^2*c^3*d^5*e^5 + 21*B*C^2*a^2*c^3*d^6*e^4 - 1296*A^2*C*a^2*c^3*d^3*e^7 + 39
6*A*C^2*a^3*c^2*d^3*e^7 - 330*A*C^2*a^2*c^3*d^5*e^5 + 1863*A^2*B*a^2*c^3*d^
2*e^8 - 672*A*B^2*a^2*c^3*d^3*e^7 + 90*A*B*C*a^4*c*e^10 + 8*C^3*a^4*c*d^3*e
^7 - 1350*A^3*a^2*c^3*d*e^9 - 324*A^3*a*c^4*d^3*e^7 - 36*A^2*C*c^5*d^7*e^3
+ 45*A^2*B*c^5*d^6*e^4 - 225*A^2*B*a^3*c^2*e^10 - 86*C^3*a^3*c^2*d^5*e^5 -
4*C^3*a^2*c^3*d^7*e^3 + 316*B^3*a^3*c^2*d^2*e^8 + 20*B^3*a^2*c^3*d^4*e^6 +
18*C^3*a^5*d*e^9 - 64*B^3*a^4*c*e^10 - 9*B*C^2*a^5*e^10 - 54*A^3*c^5*d^5*e^
5, z, k)*((512*a^11*c^2*d*e^14 + 512*a^5*c^8*d^13*e^2 + 3072*a^6*c^7*d^11*e
^4 + 7680*a^7*c^6*d^9*e^6 + 10240*a^8*c^5*d^7*e^8 + 7680*a^9*c^4*d^5*e^10 +
 3072*a^10*c^3*d^3*e^12)/(64*(a^10*e^12 + a^4*c^6*d^12 + 6*a^9*c*d^2*e^10 +
 6*a^5*c^5*d^10*e^2 + 15*a^6*c^4*d^8*e^4 + 20*a^7*c^3*d^6*e^6 + 15*a^8*c^2*
d^4*e^8)) + (x*(384*a^11*c^2*e^15 - 128*a^4*c^9*d^14*e - 384*a^5*c^8*d^12*e
^3 + 384*a^6*c^7*d^10*e^5 + 3200*a^7*c^6*d^8*e^7 + 5760*a^8*c^5*d^6*e^9 + 4
992*a^9*c^4*d^4*e^11 + 2176*a^10*c^3*d^2*e^13))/(64*(a^10*e^12 + a^4*c^6*d^
12 + 6*a^9*c*d^2*e^10 + 6*a^5*c^5*d^10*e^2 + 15*a^6*c^4*d^8*e^4 + 20*a^7*c^
3*d^6*e^6 + 15*a^8*c^2*d^4*e^8))) + (x*(192*B*a^8*c^2*e^13 + 912*A*a^7*c^3*
d*e^12 - 336*C*a^8*c^2*d*e^12 + 48*A*a^2*c^8*d^11*e^2 + 336*A*a^3*c^7*d^9*e
^4 + 1632*A*a^4*c^6*d^7*e^6 + 3360*A*a^5*c^5*d^5*e^8 + 2928*A*a^6*c^4*d^3*e
```

```
^10 - 32*B*a^3*c^7*d^10*e^3 - 704*B*a^4*c^6*d^8*e^5 - 1728*B*a^5*c^5*d^6*e^
7 - 1280*B*a^6*c^4*d^4*e^9 - 32*B*a^7*c^3*d^2*e^11 + 16*C*a^3*c^7*d^11*e^2
+ 496*C*a^4*c^6*d^9*e^4 + 1056*C*a^5*c^5*d^7*e^6 + 352*C*a^6*c^4*d^5*e^8 -
560*C*a^7*c^3*d^3*e^10))/(64*(a^10*e^12 + a^4*c^6*d^12 + 6*a^9*c*d^2*e^10 +
 6*a^5*c^5*d^10*e^2 + 15*a^6*c^4*d^8*e^4 + 20*a^7*c^3*d^6*e^6 + 15*a^8*c^2*
d^4*e^8))) + (9*A^2*c^7*d^9*e^2 + 198*A^2*a^2*c^5*d^5*e^6 + 216*A^2*a^3*c^4
*d^3*e^8 + 4*B^2*a^2*c^5*d^7*e^4 - 8*B^2*a^3*c^4*d^5*e^6 - 412*B^2*a^4*c^3*
d^3*e^8 + C^2*a^2*c^5*d^9*e^2 - 8*C^2*a^3*c^4*d^7*e^4 - 250*C^2*a^4*c^3*d^5
*e^6 + 296*C^2*a^5*c^2*d^3*e^8 - 120*A*B*a^5*c^2*e^11 - 39*C^2*a^6*c*d*e^10
 + 72*A^2*a*c^6*d^7*e^4 - 495*A^2*a^4*c^3*d*e^10 + 176*B^2*a^5*c^2*d*e^10 +
 24*B*C*a^6*c*e^11 - 12*A*B*a*c^6*d^8*e^3 + 6*A*C*a*c^6*d^9*e^2 + 294*A*C*a
^5*c^2*d*e^10 - 36*A*B*a^2*c^5*d^6*e^5 + 36*A*B*a^3*c^4*d^4*e^7 + 1092*A*B*
a^4*c^3*d^2*e^9 - 108*A*C*a^3*c^4*d^5*e^6 - 960*A*C*a^4*c^3*d^3*e^8 - 4*B*C
*a^2*c^5*d^8*e^3 + 20*B*C*a^3*c^4*d^6*e^5 + 652*B*C*a^4*c^3*d^4*e^7 - 500*B
*C*a^5*c^2*d^2*e^9)/(64*(a^10*e^12 + a^4*c^6*d^12 + 6*a^9*c*d^2*e^10 + 6*a^
5*c^5*d^10*e^2 + 15*a^6*c^4*d^8*e^4 + 20*a^7*c^3*d^6*e^6 + 15*a^8*c^2*d^4*e
^8)) + (x*(225*A^2*a^4*c^3*e^11 + 9*A^2*c^7*d^8*e^3 + 9*C^2*a^6*c*e^11 + 54
*A^2*a^2*c^5*d^4*e^7 - 360*A^2*a^3*c^4*d^2*e^9 + 4*B^2*a^2*c^5*d^6*e^5 - 88
*B^2*a^3*c^4*d^4*e^7 + 484*B^2*a^4*c^3*d^2*e^9 + C^2*a^2*c^5*d^8*e^3 - 40*C
^2*a^3*c^4*d^6*e^5 + 406*C^2*a^4*c^3*d^4*e^7 - 120*C^2*a^5*c^2*d^2*e^9 - 90
*A*C*a^5*c^2*e^11 + 72*A^2*a*c^6*d^6*e^5 - 12*A*B*a*c^6*d^7*e^4 - 660*A*B*a
^4*c^3*d*e^10 + 6*A*C*a*c^6*d^8*e^3 + 132*B*C*a^5*c^2*d*e^10 + 84*A*B*a^2*c
^5*d^5*e^6 + 588*A*B*a^3*c^4*d^3*e^8 - 96*A*C*a^2*c^5*d^6*e^5 - 492*A*C*a^3
*c^4*d^4*e^7 + 672*A*C*a^4*c^3*d^2*e^9 - 4*B*C*a^2*c^5*d^7*e^4 + 124*B*C*a^
3*c^4*d^5*e^6 - 892*B*C*a^4*c^3*d^3*e^8))/(64*(a^10*e^12 + a^4*c^6*d^12 + 6
*a^9*c*d^2*e^10 + 6*a^5*c^5*d^10*e^2 + 15*a^6*c^4*d^8*e^4 + 20*a^7*c^3*d^6*
e^6 + 15*a^8*c^2*d^4*e^8)))*root(17920*a^9*c^5*d^8*e^8*z^3 + 14336*a^10*c^4
*d^6*e^10*z^3 + 14336*a^8*c^6*d^10*e^6*z^3 + 7168*a^11*c^3*d^4*e^12*z^3 + 7
168*a^7*c^7*d^12*e^4*z^3 + 2048*a^12*c^2*d^2*e^14*z^3 + 2048*a^6*c^8*d^14*e
^2*z^3 + 256*a^5*c^9*d^16*z^3 + 256*a^13*c*e^16*z^3 + 948*B*C*a^7*c*d*e^11*
z - 12*A*B*a*c^7*d^11*e*z + 9768*B*C*a^5*c^3*d^5*e^7*z - 7476*B*C*a^6*c^2*d
^3*e^9*z - 328*B*C*a^4*c^4*d^7*e^5*z - 92*B*C*a^3*c^5*d^9*e^3*z - 12486*A*C
*a^5*c^3*d^4*e^8*z + 5868*A*C*a^6*c^2*d^2*e^10*z + 282*A*C*a^3*c^5*d^8*e^4*
z + 168*A*C*a^4*c^4*d^6*e^6*z + 108*A*C*a^2*c^6*d^10*e^2*z + 14820*A*B*a^5*
c^3*d^3*e^9*z - 840*A*B*a^4*c^4*d^5*e^7*z - 600*A*B*a^3*c^5*d^7*e^5*z - 180
*A*B*a^2*c^6*d^9*e^3*z - 4*B*C*a^2*c^6*d^11*e*z - 3204*A*B*a^6*c^2*d*e^11*z
 + 4239*C^2*a^6*c^2*d^4*e^8*z - 3924*C^2*a^5*c^3*d^6*e^6*z + 103*C^2*a^4*c^
4*d^8*e^4*z + 26*C^2*a^3*c^5*d^10*e^2*z - 6000*B^2*a^5*c^3*d^4*e^8*z + 2820
*B^2*a^6*c^2*d^2*e^10*z + 280*B^2*a^4*c^4*d^6*e^6*z + 80*B^2*a^3*c^5*d^8*e^
4*z + 4*B^2*a^2*c^6*d^10*e^2*z - 8262*A^2*a^5*c^3*d^2*e^10*z + 1575*A^2*a^4
*c^4*d^4*e^8*z + 1260*A^2*a^3*c^5*d^6*e^6*z + 495*A^2*a^2*c^6*d^8*e^4*z - 9
0*A*C*a^7*c*e^12*z + 6*A*C*a*c^7*d^12*z - 966*C^2*a^7*c*d^2*e^10*z + 90*A^2
*a*c^7*d^10*e^2*z + C^2*a^2*c^6*d^12*z + 225*A^2*a^6*c^2*e^12*z - 192*B^2*a
^7*c*e^12*z + 9*A^2*c^8*d^12*z + 9*C^2*a^8*e^12*z + 78*A*B*C*a*c^4*d^6*e^4
+ 942*A*B*C*a^2*c^3*d^4*e^6 - 342*A*B*C*a^3*c^2*d^2*e^8 - 129*B*C^2*a^4*c*d
```

```
^2*e^8 + 990*A^2*C*a^3*c^2*d*e^9 - 234*A^2*C*a*c^4*d^5*e^5 - 24*A*C^2*a*c^4
*d^7*e^3 + 333*A^2*B*a*c^4*d^4*e^6 - 252*A*B^2*a^3*c^2*d*e^9 - 60*A*B^2*a*c
^4*d^5*e^5 + 204*B^2*C*a^4*c*d*e^9 - 234*A*C^2*a^4*c*d*e^9 - 624*B^2*C*a^3*
c^2*d^3*e^7 + 405*B*C^2*a^3*c^2*d^4*e^6 - 36*B^2*C*a^2*c^3*d^5*e^5 + 21*B*C
^2*a^2*c^3*d^6*e^4 - 1296*A^2*C*a^2*c^3*d^3*e^7 + 396*A*C^2*a^3*c^2*d^3*e^7
 - 330*A*C^2*a^2*c^3*d^5*e^5 + 1863*A^2*B*a^2*c^3*d^2*e^8 - 672*A*B^2*a^2*c
^3*d^3*e^7 + 90*A*B*C*a^4*c*e^10 + 8*C^3*a^4*c*d^3*e^7 - 1350*A^3*a^2*c^3*d
*e^9 - 324*A^3*a*c^4*d^3*e^7 - 36*A^2*C*c^5*d^7*e^3 + 45*A^2*B*c^5*d^6*e^4
- 225*A^2*B*a^3*c^2*e^10 - 86*C^3*a^3*c^2*d^5*e^5 - 4*C^3*a^2*c^3*d^7*e^3 +
 316*B^3*a^3*c^2*d^2*e^8 + 20*B^3*a^2*c^3*d^4*e^6 + 18*C^3*a^5*d*e^9 - 64*B
^3*a^4*c*e^10 - 9*B*C^2*a^5*e^10 - 54*A^3*c^5*d^5*e^5, z, k), k, 1, 3) + ((
x^4*(3*C*a^3*c*e^5 + 3*A*c^4*d^4*e - 15*A*a^2*c^2*e^5 + 12*A*a*c^3*d^2*e^3
- 2*B*a*c^3*d^3*e^2 + 22*B*a^2*c^2*d*e^4 - 20*C*a^2*c^2*d^2*e^3 + C*a*c^3*d
^4*e))/(8*a^2*(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)) - (4
*A*a^2*e^5 + B*c^2*d^5 - 7*B*a^2*d*e^4 - 2*A*c^2*d^4*e + 10*C*a^2*d^2*e^3 -
 2*C*a*c*d^4*e - 10*A*a*c*d^2*e^3 + 6*B*a*c*d^3*e^2)/(4*(a*e^2 + c*d^2)*(a^
2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)) + (x^3*(3*A*c^3*d^3 + 4*B*a^2*c*e^3 + C*a
*c^2*d^3 + 9*A*a*c^2*d*e^2 - 2*B*a*c^2*d^2*e - 5*C*a^2*c*d*e^2))/(8*a^2*(a^
2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)) + (x*(5*A*c^2*d^3 + 6*B*a^2*e^3 - C*a*c*d
^3 - 7*C*a^2*d*e^2 + 11*A*a*c*d*e^2))/(8*a*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e
^2)) + (x^2*(5*C*a^3*e^5 - 25*A*a^2*c*e^5 + 5*A*c^3*d^4*e + 28*A*a*c^2*d^2*
e^3 - 10*B*a*c^2*d^3*e^2 - 36*C*a^2*c*d^2*e^3 + 38*B*a^2*c*d*e^4 + 7*C*a*c^
2*d^4*e))/(8*a*(a*e^2 + c*d^2)*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)))/(a^2*d
 + c^2*d*x^4 + c^2*e*x^5 + a^2*e*x + 2*a*c*d*x^2 + 2*a*c*e*x^3)
```

3.63 $\int \frac{A+Bx+Cx^2}{(d+ex)^3\left(a+cx^2\right)^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 753

$$
\begin{aligned}
&\int \frac{A+Bx+Cx^2}{(d+ex)^3\left(a+cx^2\right)^3}\,dx \\
&= -\frac{e^3(Cd^2-Bde+Ae^2)}{2\left(cd^2+ae^2\right)^3(d+ex)^2}+\frac{e^3(ae^2(2Cd-Be)-cd(4Cd^2-e(5Bd-6Ae)))}{\left(cd^2+ae^2\right)^4(d+ex)} \\
&\quad -\frac{a(Bcd(cd^2-3ae^2)-(Ac-aC)e(3cd^2-ae^2))-c(Acd(cd^2-3ae^2)-a(cd^2(Cd-3Be)-ae^2(3Cd-}{4a\left(cd^2+ae^2\right)^3\left(a+cx^2\right)^2} \\
&\quad +\frac{4a^2e(a^2Ce^4+c^2d^2(3Cd^2-2e(3Bd-5Ae))-2ace^2(4Cd^2-e(3Bd-Ae)))+c(3Acd(c^2d^4+6acd^2e^2}{8a^2\left(cd^2+ae^2\right)^4(a+cx} \\
&\quad +\frac{\sqrt{c}(3Acd(c^3d^6+7ac^2d^4e^2+35a^2cd^2e^4-35a^3e^6)+a(ac^2d^4e^2(23Cd-45Be)-5a^2cd^2e^4(25Cd-27Be}{8a^{5/2}\left(cd^2+ae^2\right)^5} \\
&\quad +\frac{e^3(a^2Ce^4-ace^2(13Cd^2-9Bde+3Ae^2)+c^2d^2(10Cd^2-3e(5Bd-7Ae)))\log(d+ex)}{\left(cd^2+ae^2\right)^5} \\
&\quad -\frac{e^3(a^2Ce^4-ace^2(13Cd^2-9Bde+3Ae^2)+c^2d^2(10Cd^2-3e(5Bd-7Ae)))\log\left(a+cx^2\right)}{2\left(cd^2+ae^2\right)^5}
\end{aligned}
$$

```
[Out] -1/2*e^3*(A*e^2-B*d*e+C*d^2)/(a*e^2+c*d^2)^3/(e*x+d)^2+e^3*(a*e^2*(-B*e+2*C
*d)-c*d*(4*C*d^2-e*(-6*A*e+5*B*d)))/(a*e^2+c*d^2)^4/(e*x+d)+1/4*(-a*(B*c*d*
(-3*a*e^2+c*d^2)-(A*c-C*a)*e*(-a*e^2+3*c*d^2))+c*(A*c*d*(-3*a*e^2+c*d^2)-a*
(c*d^2*(-3*B*e+C*d)-a*e^2*(-B*e+3*C*d)))*x)/a/(a*e^2+c*d^2)^3/(c*x^2+a)^2+1
/8*(4*a^2*e*(a^2*C*e^4+c^2*d^2*(3*C*d^2-2*e*(-5*A*e+3*B*d))-2*a*c*e^2*(4*C*
d^2-e*(-A*e+3*B*d)))+c*(3*A*c*d*(-11*a^2*e^4+6*a*c*d^2*e^2+c^2*d^4)-a*(2*a*
c*d^2*e^2*(-19*B*e+13*C*d)-c^2*d^4*(-3*B*e+C*d)-7*a^2*e^4*(-B*e+3*C*d)))*x)
/a^2/(a*e^2+c*d^2)^4/(c*x^2+a)+e^3*(a^2*C*e^4-a*c*e^2*(3*A*e^2-9*B*d*e+13*C
*d^2)+c^2*d^2*(10*C*d^2-3*e*(-7*A*e+5*B*d)))*ln(e*x+d)/(a*e^2+c*d^2)^5-1/2*
```

```
e^3*(a^2*C*e^4-a*c*e^2*(3*A*e^2-9*B*d*e+13*C*d^2)+c^2*d^2*(10*C*d^2-3*e*(-7
*A*e+5*B*d)))*ln(c*x^2+a)/(a*e^2+c*d^2)^5+1/8*(3*A*c*d*(-35*a^3*e^6+35*a^2*
c*d^2*e^4+7*a*c^2*d^4*e^2+c^3*d^6)+a*(a*c^2*d^4*e^2*(-45*B*e+23*C*d)-5*a^2*
c*d^2*e^4*(-27*B*e+25*C*d)+c^3*d^6*(-3*B*e+C*d)+15*a^3*e^6*(-B*e+3*C*d)))*a
rctan(x*c^(1/2)/a^(1/2))*c^(1/2)/a^(5/2)/(a*e^2+c*d^2)^5
```

Rubi [A] (verified)

Time = 2.25 (sec) , antiderivative size = 753, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1661, 1643, 649, 211, 266}

$$\int \frac{A+Bx+Cx^2}{(d+ex)^3\left(a+cx^2\right)^3}\,dx$$

$$= \frac{cx(3Acd(-11a^2e^4+6acd^2e^2+c^2d^4)-a(-7a^2e^4(3Cd-Be)+2acd^2e^2(13Cd-19Be)-c^2d^4(Cd-3Be)}{8a^2\left(a+cx^2\right)\left(ae^2+cd^2\right)^4}$$

$$-\frac{e^3\log\left(a+cx^2\right)\left(a^2Ce^4-ace^2(3Ae^2-9Bde+13Cd^2)+c^2(10Cd^4-3d^2e(5Bd-7Ae))\right)}{2\left(ae^2+cd^2\right)^5}$$

$$+\frac{e^3\log(d+ex)\left(a^2Ce^4-ace^2(3Ae^2-9Bde+13Cd^2)+c^2(10Cd^4-3d^2e(5Bd-7Ae))\right)}{\left(ae^2+cd^2\right)^5}$$

$$+\frac{\sqrt{c}\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(3Acd(-35a^3e^6+35a^2cd^2e^4+7ac^2d^4e^2+c^3d^6)+a(15a^3e^6(3Cd-Be)-5a^2cd^2e^4(25}{8a^{5/2}\left(ae^2+cd^2\right)^5}$$

$$-\frac{a(Bcd(cd^2-3ae^2)-e(Ac-aC)\left(3cd^2-ae^2\right))-cx(Acd(cd^2-3ae^2)-a(cd^2(Cd-3Be)-ae^2(3Cd-}{4a\left(a+cx^2\right)^2\left(ae^2+cd^2\right)^3}$$

$$-\frac{e^3(Ae^2-Bde+Cd^2)}{2(d+ex)^2\left(ae^2+cd^2\right)^3}-\frac{e^3(-ae^2(2Cd-Be)-cde(5Bd-6Ae)+4cCd^3)}{(d+ex)\left(ae^2+cd^2\right)^4}$$

```
[In] Int[(A + B*x + C*x^2)/((d + e*x)^3*(a + c*x^2)^3),x]
```

```
[Out] -1/2*(e^3*(C*d^2 - B*d*e + A*e^2))/((c*d^2 + a*e^2)^3*(d + e*x)^2) - (e^3*(
4*c*C*d^3 - c*d*e*(5*B*d - 6*A*e) - a*e^2*(2*C*d - B*e)))/((c*d^2 + a*e^2)^
4*(d + e*x)) - (a*(B*c*d*(c*d^2 - 3*a*e^2) - (A*c - a*C)*e*(3*c*d^2 - a*e^2
)) - c*(A*c*d*(c*d^2 - 3*a*e^2) - a*(c*d^2*(C*d - 3*B*e) - a*e^2*(3*C*d - B
*e)))*x)/(4*a*(c*d^2 + a*e^2)^3*(a + c*x^2)^2) + (4*a^2*e*(a^2*C*e^4 + c^2*
(3*C*d^4 - 2*d^2*e*(3*B*d - 5*A*e)) - 2*a*c*e^2*(4*C*d^2 - e*(3*B*d - A*e))
) + c*(3*A*c*d*(c^2*d^4 + 6*a*c*d^2*e^2 - 11*a^2*e^4) - a*(2*a*c*d^2*e^2*(1
3*C*d - 19*B*e) - c^2*d^4*(C*d - 3*B*e) - 7*a^2*e^4*(3*C*d - B*e)))*x)/(8*a
^2*(c*d^2 + a*e^2)^4*(a + c*x^2)) + (Sqrt[c]*(3*A*c*d*(c^3*d^6 + 7*a*c^2*d^
4*e^2 + 35*a^2*c*d^2*e^4 - 35*a^3*e^6) + a*(a*c^2*d^4*e^2*(23*C*d - 45*B*e)
 - 5*a^2*c*d^2*e^4*(25*C*d - 27*B*e) + c^3*d^6*(C*d - 3*B*e) + 15*a^3*e^6*(
3*C*d - B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(8*a^(5/2)*(c*d^2 + a*e^2)^5) +
 (e^3*(a^2*C*e^4 - a*c*e^2*(13*C*d^2 - 9*B*d*e + 3*A*e^2) + c^2*(10*C*d^4 -
```

```
 3*d^2*e*(5*B*d - 7*A*e)))*Log[d + e*x])/(c*d^2 + a*e^2)^5 - (e^3*(a^2*C*e^
4 - a*c*e^2*(13*C*d^2 - 9*B*d*e + 3*A*e^2) + c^2*(10*C*d^4 - 3*d^2*e*(5*B*d
 - 7*A*e)))*Log[a + c*x^2])/(2*(c*d^2 + a*e^2)^5)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

integral =

$$-\frac{a(Bcd(cd^2-3ae^2)-(Ac-aC)e(3cd^2-ae^2))-c(Acd(cd^2-3ae^2)-a(cd^2(Cd-3Be)-ae^2(3Cd-}{4a\left(cd^2+ae^2\right)^3\left(a+cx^2\right)^2}$$

$$-\frac{\int\frac{-\frac{c\left(A\left(3c^3d^6+15ac^2d^4e^2+12a^2cd^2e^4+4a^3e^6\right)+acd^3\left(cd^2(Cd-3Be)-ae^2(3Cd-Be)\right)\right)}{\left(cd^2+ae^2\right)^3}-\frac{ce\left(Ac^2d^3\left(9cd^2+5ae^2\right)+a\left(4a^2Be^5-3c^2d^4(3Cd-5Be)-5acd^2e\right.\right.}{\left(cd^2+ae^2\right)^3}}{}}{}$$

$$=$$

$$-\frac{a(Bcd(cd^2-3ae^2)-(Ac-aC)e(3cd^2-ae^2))-c(Acd(cd^2-3ae^2)-a(cd^2(Cd-3Be)-ae^2(3}{4a(cd^2+ae^2)^3(a+cx^2)^2}$$

$$+\frac{4a^2e(a^2Ce^4+c^2(3Cd^4-2d^2e(3Bd-5Ae))-2ace^2(4Cd^2-e(3Bd-Ae)))+c(3Acd(c^2d^4+6a}{8a^2(cd^2+ae^2)^4(}$$

$$+\frac{\int \frac{\frac{c^2(A(3c^4d^8+18ac^3d^6e^2+87a^2c^2d^4e^4+32a^3cd^2e^6+8a^4e^8)+acd^3(2acd^2e^2(11Cd-21Be)+c^2d^4(Cd-3Be)-9a^2e^4(3Cd-Be)))}{(cd^2+ae^2)^4}+\frac{c^2e(3Ac^2d^3(}{}}{}}{}$$

$$=$$

$$-\frac{a(Bcd(cd^2-3ae^2)-(Ac-aC)e(3cd^2-ae^2))-c(Acd(cd^2-3ae^2)-a(cd^2(Cd-3Be)-ae^2(3}{4a(cd^2+ae^2)^3(a+cx^2)^2}$$

$$+\frac{4a^2e(a^2Ce^4+c^2(3Cd^4-2d^2e(3Bd-5Ae))-2ace^2(4Cd^2-e(3Bd-Ae)))+c(3Acd(c^2d^4+6a}{8a^2(cd^2+ae^2)^4(}$$

$$+\frac{\int (\frac{8a^2c^2e^4(Cd^2-Bde+Ae^2)}{(cd^2+ae^2)^3(d+ex)^3}+\frac{8a^2c^2e^4(4cCd^3-cde(5Bd-6Ae)-ae^2(2Cd-Be))}{(cd^2+ae^2)^4(d+ex)^2}+\frac{8a^2c^2e^4(a^2Ce^4-ace^2(13Cd^2-9Bde+3Ae}{(cd^2+ae^2)^5(d}}{}$$

$$=-\frac{e^3(Cd^2-Bde+Ae^2)}{2(cd^2+ae^2)^3(d+ex)^2}-\frac{e^3(4cCd^3-cde(5Bd-6Ae)-ae^2(2Cd-Be))}{(cd^2+ae^2)^4(d+ex)}$$

$$-\frac{a(Bcd(cd^2-3ae^2)-(Ac-aC)e(3cd^2-ae^2))-c(Acd(cd^2-3ae^2)-a(cd^2(Cd-3Be)-ae^2(3}{4a(cd^2+ae^2)^3(a+cx^2)^2}$$

$$+\frac{4a^2e(a^2Ce^4+c^2(3Cd^4-2d^2e(3Bd-5Ae))-2ace^2(4Cd^2-e(3Bd-Ae)))+c(3Acd(c^2d^4+6a}{8a^2(cd^2+ae^2)^4(}$$

$$+\frac{e^3(a^2Ce^4-ace^2(13Cd^2-9Bde+3Ae^2)+c^2(10Cd^4-3d^2e(5Bd-7Ae)))\log(d+ex)}{(cd^2+ae^2)^5}$$

$$+\frac{c\int \frac{3Acd(c^3d^6+7ac^2d^4e^2+35a^2cd^2e^4-35a^3e^6)+a(ac^2d^4e^2(23Cd-45Be)-5a^2cd^2e^4(25Cd-27Be)+c^3d^6(Cd-3Be)+15a^3e^6(3Cd-}{a+cx^2}}{8a^2(cd^2+ae^2)^5}$$

$$=-\frac{e^3(Cd^2-Bde+Ae^2)}{2(cd^2+ae^2)^3(d+ex)^2}-\frac{e^3(4cCd^3-cde(5Bd-6Ae)-ae^2(2Cd-Be))}{(cd^2+ae^2)^4(d+ex)}$$

$$-\frac{a(Bcd(cd^2-3ae^2)-(Ac-aC)e(3cd^2-ae^2))-c(Acd(cd^2-3ae^2)-a(cd^2(Cd-3Be)-ae^2(3}{4a(cd^2+ae^2)^3(a+cx^2)^2}$$

$$+\frac{4a^2e(a^2Ce^4+c^2(3Cd^4-2d^2e(3Bd-5Ae))-2ace^2(4Cd^2-e(3Bd-Ae)))+c(3Acd(c^2d^4+6a}{8a^2(cd^2+ae^2)^4(}$$

$$+\frac{e^3(a^2Ce^4-ace^2(13Cd^2-9Bde+3Ae^2)+c^2(10Cd^4-3d^2e(5Bd-7Ae)))\log(d+ex)}{(cd^2+ae^2)^5}$$

$$-\frac{(ce^3(a^2Ce^4-ace^2(13Cd^2-9Bde+3Ae^2)+c^2(10Cd^4-3d^2e(5Bd-7Ae))))\int \frac{x}{a+cx^2}\,dx}{(cd^2+ae^2)^5}$$

$$+\frac{(c(3Acd(c^3d^6+7ac^2d^4e^2+35a^2cd^2e^4-35a^3e^6)+a(ac^2d^4e^2(23Cd-45Be)-5a^2cd^2e^4(25Cd-}{8a^2(cd^2+ae^2)^5}$$

$$= -\frac{e^3(Cd^2 - Bde + Ae^2)}{2\left(cd^2 + ae^2\right)^3 (d + ex)^2} - \frac{e^3(4cCd^3 - cde(5Bd - 6Ae) - ae^2(2Cd - Be))}{\left(cd^2 + ae^2\right)^4 (d + ex)}$$

$$- \frac{a(Bcd(cd^2 - 3ae^2) - (Ac - aC)e(3cd^2 - ae^2)) - c(Acd(cd^2 - 3ae^2) - a(cd^2(Cd - 3Be) - ae^2}{4a\left(cd^2 + ae^2\right)^3 \left(a + cx^2\right)^2}$$

$$+ \frac{4a^2e(a^2Ce^4 + c^2(3Cd^4 - 2d^2e(3Bd - 5Ae)) - 2ace^2(4Cd^2 - e(3Bd - Ae))) + c(3Acd(c^2d^4 + 6}{8a^2\left(cd^2 + ae^2\right)^4}$$

$$+ \frac{\sqrt{c}(3Acd(c^3d^6 + 7ac^2d^4e^2 + 35a^2cd^2e^4 - 35a^3e^6) + a(ac^2d^4e^2(23Cd - 45Be) - 5a^2cd^2e^4(25Cd}{8a^{5/2}\left(cd^2 + ae^2\right)^5}$$

$$+ \frac{e^3(a^2Ce^4 - ace^2(13Cd^2 - 9Bde + 3Ae^2) + c^2(10Cd^4 - 3d^2e(5Bd - 7Ae)))\log(d + ex)}{\left(cd^2 + ae^2\right)^5}$$

$$- \frac{e^3(a^2Ce^4 - ace^2(13Cd^2 - 9Bde + 3Ae^2) + c^2(10Cd^4 - 3d^2e(5Bd - 7Ae)))\log\left(a + cx^2\right)}{2\left(cd^2 + ae^2\right)^5}$$

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 672, normalized size of antiderivative = 0.89

$$\int \frac{A + Bx + Cx^2}{(d + ex)^3 \left(a + cx^2\right)^3}\,dx$$

$$= \frac{-\frac{4e^3\left(cd^2+ae^2\right)^2\left(Cd^2+e(-Bd+Ae)\right)}{(d+ex)^2} - \frac{8e^3\left(cd^2+ae^2\right)\left(4cCd^3+cde(-5Bd+6Ae)+ae^2(-2Cd+Be)\right)}{d+ex} + \frac{2\left(cd^2+ae^2\right)^2\left(a^3Ce^3+Ac^3d^3x-ac^2d}{}}{}$$

```
[In] Integrate[(A + B*x + C*x^2)/((d + e*x)^3*(a + c*x^2)^3),x]
```

```
[Out] ((-4*e^3*(c*d^2 + a*e^2)^2*(C*d^2 + e*(-(B*d) + A*e)))/(d + e*x)^2 - (8*e^3
*(c*d^2 + a*e^2)*(4*c*C*d^3 + c*d*e*(-5*B*d + 6*A*e) + a*e^2*(-2*C*d + B*e)
))/(d + e*x) + (2*(c*d^2 + a*e^2)^2*(a^3*C*e^3 + A*c^3*d^3*x - a*c^2*d*(C*d
^2*x + B*d*(d - 3*e*x) + 3*A*e*(-d + e*x)) - a^2*c*e*(3*C*d*(d - e*x) + e*(
-3*B*d + A*e + B*e*x))))/(a*(a + c*x^2)^2) + ((c*d^2 + a*e^2)*(4*a^4*C*e^5
+ 3*A*c^4*d^5*x + a*c^3*d^3*(C*d^2 + 3*e*(-(B*d) + 6*A*e))*x + a^3*c*e^3*(C
*d*(-32*d + 21*e*x) + e*(24*B*d - 8*A*e - 7*B*e*x)) + a^2*c^2*d*e*(2*C*d^2*
(6*d - 13*e*x) + e*(-24*B*d^2 + 40*A*d*e + 38*B*d*e*x - 33*A*e^2*x))))/(a^2
*(a + c*x^2)) + (Sqrt[c]*(3*A*c*d*(c^3*d^6 + 7*a*c^2*d^4*e^2 + 35*a^2*c*d^2
*e^4 - 35*a^3*e^6) + a*(a*c^2*d^4*e^2*(23*C*d - 45*B*e) - 5*a^2*c*d^2*e^4*(
25*C*d - 27*B*e) + c^3*d^6*(C*d - 3*B*e) - 15*a^3*e^6*(-3*C*d + B*e)))*ArcT
an[(Sqrt[c]*x)/Sqrt[a]])/a^(5/2) + 8*(a^2*C*e^7 + a*c*e^5*(-13*C*d^2 + 9*B*
d*e - 3*A*e^2) + c^2*d^2*e^3*(10*C*d^2 + 3*e*(-5*B*d + 7*A*e)))*Log[d + e*x
] - 4*(a^2*C*e^7 + a*c*e^5*(-13*C*d^2 + 9*B*d*e - 3*A*e^2) + c^2*d^2*e^3*(1
0*C*d^2 + 3*e*(-5*B*d + 7*A*e)))*Log[a + c*x^2])/(8*(c*d^2 + a*e^2)^5)
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 1055, normalized size of antiderivative = 1.40

method	result	size
default	Expression too large to display	1055
risch	Expression too large to display	104197

```
[In] int((C*x^2+B*x+A)/(e*x+d)^3/(c*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -c/(a*e^2+c*d^2)^5*((1/8*c*(33*A*a^3*c*d*e^6+15*A*a^2*c^2*d^3*e^4-21*A*a*c^
3*d^5*e^2-3*A*c^4*d^7+7*B*a^4*e^7-31*B*a^3*c*d^2*e^5-35*B*a^2*c^2*d^4*e^3+3
*B*a*c^3*d^6*e-21*C*a^4*d*e^6+5*C*a^3*c*d^3*e^4+25*C*a^2*c^2*d^5*e^2-C*a*c^
3*d^7)/a^2*x^3+(A*a^2*c*e^7-4*A*a*c^2*d^2*e^5-5*A*c^3*d^4*e^3-3*B*a^2*c*d*e
^6+3*B*c^3*d^5*e^2-1/2*C*a^3*e^7+7/2*C*a^2*c*d^2*e^5+5/2*C*a*c^2*d^4*e^3-3/
2*C*c^3*d^6*e)*x^2+1/8*(39*A*a^3*c*d*e^6+25*A*a^2*c^2*d^3*e^4-19*A*a*c^3*d^
5*e^2-5*A*c^4*d^7+9*B*a^4*e^7-33*B*a^3*c*d^2*e^5-45*B*a^2*c^2*d^4*e^3-3*B*a
*c^3*d^6*e-27*C*a^4*d*e^6-5*C*a^3*c*d^3*e^4+23*C*a^2*c^2*d^5*e^2+C*a*c^3*d^
7)/a*x+1/4*(5*A*a^3*c*e^7-17*A*a^2*c^2*d^2*e^5-25*A*a*c^3*d^4*e^3-3*A*c^4*d
^6*e-15*B*a^3*c*d*e^6-5*B*a^2*c^2*d^3*e^4+11*B*a*c^3*d^5*e^2+B*c^4*d^7-3*C*
a^4*e^7+15*C*a^3*c*d^2*e^5+15*C*a^2*c^2*d^4*e^3-3*C*a*c^3*d^6*e)/c)/(c*x^2+
a)^2+1/8/a^2*(1/2*(-24*A*a^3*c*e^7+168*A*a^2*c^2*d^2*e^5+72*B*a^3*c*d*e^6-1
20*B*a^2*c^2*d^3*e^4+8*C*a^4*e^7-104*C*a^3*c*d^2*e^5+80*C*a^2*c^2*d^4*e^3)/
c*ln(c*x^2+a)+(105*A*a^3*c*d*e^6-105*A*a^2*c^2*d^3*e^4-21*A*a*c^3*d^5*e^2-3
*A*c^4*d^7+15*B*a^4*e^7-135*B*a^3*c*d^2*e^5+45*B*a^2*c^2*d^4*e^3+3*B*a*c^3*
d^6*e-45*C*a^4*d*e^6+125*C*a^3*c*d^3*e^4-23*C*a^2*c^2*d^5*e^2-C*a*c^3*d^7)/
(a*c)^(1/2)*arctan(c*x/(a*c)^(1/2))))-e^3*(6*A*c*d*e^2+B*a*e^3-5*B*c*d^2*e-
2*C*a*d*e^2+4*C*c*d^3)/(a*e^2+c*d^2)^4/(e*x+d)-1/2*e^3*(A*e^2-B*d*e+C*d^2)/
(a*e^2+c*d^2)^3/(e*x+d)^2-e^3*(3*A*a*c*e^4-21*A*c^2*d^2*e^2-9*B*a*c*d*e^3+1
5*B*c^2*d^3*e-C*a^2*e^4+13*C*a*c*d^2*e^2-10*C*c^2*d^4)/(a*e^2+c*d^2)^5*ln(e
*x+d)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(d+ex)^3\,(a+cx^2)^3}\,dx=\text{Timed out}$$

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^3/(c*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(d+ex)^3\,(a+cx^2)^3}\,dx = \text{Timed out}$$

```
[In] integrate((C*x**2+B*x+A)/(e*x+d)**3/(c*x**2+a)**3,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1835 vs. 2(732) = 1464.

Time = 0.33 (sec) , antiderivative size = 1835, normalized size of antiderivative = 2.44

$$\int \frac{A+Bx+Cx^2}{(d+ex)^3\,(a+cx^2)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^3/(c*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(10*C*c^2*d^4*e^3 - 15*B*c^2*d^3*e^4 + 9*B*a*c*d*e^6 - (13*C*a*c - 21*
A*c^2)*d^2*e^5 + (C*a^2 - 3*A*a*c)*e^7)*log(c*x^2 + a)/(c^5*d^10 + 5*a*c^4*
d^8*e^2 + 10*a^2*c^3*d^6*e^4 + 10*a^3*c^2*d^4*e^6 + 5*a^4*c*d^2*e^8 + a^5*e
^10) + (10*C*c^2*d^4*e^3 - 15*B*c^2*d^3*e^4 + 9*B*a*c*d*e^6 - (13*C*a*c - 2
1*A*c^2)*d^2*e^5 + (C*a^2 - 3*A*a*c)*e^7)*log(e*x + d)/(c^5*d^10 + 5*a*c^4*
d^8*e^2 + 10*a^2*c^3*d^6*e^4 + 10*a^3*c^2*d^4*e^6 + 5*a^4*c*d^2*e^8 + a^5*e
^10) - 1/8*(3*B*a*c^4*d^6*e + 45*B*a^2*c^3*d^4*e^3 - 135*B*a^3*c^2*d^2*e^5
+ 15*B*a^4*c*e^7 - (C*a*c^4 + 3*A*c^5)*d^7 - (23*C*a^2*c^3 + 21*A*a*c^4)*d^
5*e^2 + 5*(25*C*a^3*c^2 - 21*A*a^2*c^3)*d^3*e^4 - 15*(3*C*a^4*c - 7*A*a^3*c
^2)*d*e^6)*arctan(c*x/sqrt(a*c))/((a^2*c^5*d^10 + 5*a^3*c^4*d^8*e^2 + 10*a^
4*c^3*d^6*e^4 + 10*a^5*c^2*d^4*e^6 + 5*a^6*c*d^2*e^8 + a^7*e^10)*sqrt(a*c))
 - 1/8*(2*B*a^2*c^3*d^7 + 20*B*a^3*c^2*d^5*e^2 - 74*B*a^4*c*d^3*e^4 + 4*B*a
^5*d*e^6 + 4*A*a^5*e^7 - 6*(C*a^3*c^2 + A*a^2*c^3)*d^6*e + 4*(18*C*a^4*c -
11*A*a^3*c^2)*d^4*e^3 - 2*(9*C*a^5 - 31*A*a^4*c)*d^2*e^5 + (3*B*a*c^4*d^4*e
^3 - 78*B*a^2*c^3*d^2*e^5 + 15*B*a^3*c^2*e^7 - (C*a*c^4 + 3*A*c^5)*d^5*e^2
+ 2*(29*C*a^2*c^3 - 9*A*a*c^4)*d^3*e^4 - (37*C*a^3*c^2 - 81*A*a^2*c^3)*d*e^
6)*x^5 + 2*(3*B*a*c^4*d^5*e^2 - 48*B*a^2*c^3*d^3*e^4 - 3*B*a^3*c^2*d*e^6 -
(C*a*c^4 + 3*A*c^5)*d^6*e + 2*(19*C*a^2*c^3 - 9*A*a*c^4)*d^4*e^3 - (11*C*a^
3*c^2 - 39*A*a^2*c^3)*d^2*e^5 - 2*(C*a^4*c - 3*A*a^3*c^2)*e^7)*x^4 + (3*B*a
*c^4*d^6*e + 7*B*a^2*c^3*d^4*e^3 - 163*B*a^3*c^2*d^2*e^5 + 25*B*a^4*c*e^7 -
 (C*a*c^4 + 3*A*c^5)*d^7 + (3*C*a^2*c^3 - 23*A*a*c^4)*d^5*e^2 + (129*C*a^3*
c^2 - 61*A*a^2*c^3)*d^3*e^4 - (67*C*a^4*c - 151*A*a^3*c^2)*d*e^6)*x^3 + 2*(
10*B*a^2*c^3*d^5*e^2 - 88*B*a^3*c^2*d^3*e^4 - 2*B*a^4*c*d*e^6 - 5*(C*a^2*c^
3 + A*a*c^4)*d^6*e + (71*C*a^3*c^2 - 37*A*a^2*c^3)*d^4*e^3 - (23*C*a^4*c -
73*A*a^3*c^2)*d^2*e^5 - 3*(C*a^5 - 3*A*a^4*c)*e^7)*x^2 + (B*a^2*c^3*d^6*e -
```

```
 2*B*a^3*c^2*d^4*e^3 - 91*B*a^4*c*d^2*e^5 + 8*B*a^5*e^7 + (C*a^2*c^3 - 5*A*
a*c^4)*d^7 + 2*(5*C*a^3*c^2 - 13*A*a^2*c^3)*d^5*e^2 + 7*(11*C*a^4*c - 7*A*a
^3*c^2)*d^3*e^4 - 4*(7*C*a^5 - 17*A*a^4*c)*d*e^6)*x)/(a^4*c^4*d^10 + 4*a^5*
c^3*d^8*e^2 + 6*a^6*c^2*d^6*e^4 + 4*a^7*c*d^4*e^6 + a^8*d^2*e^8 + (a^2*c^6*
d^8*e^2 + 4*a^3*c^5*d^6*e^4 + 6*a^4*c^4*d^4*e^6 + 4*a^5*c^3*d^2*e^8 + a^6*c
^2*e^10)*x^6 + 2*(a^2*c^6*d^9*e + 4*a^3*c^5*d^7*e^3 + 6*a^4*c^4*d^5*e^5 + 4
*a^5*c^3*d^3*e^7 + a^6*c^2*d*e^9)*x^5 + (a^2*c^6*d^10 + 6*a^3*c^5*d^8*e^2 +
 14*a^4*c^4*d^6*e^4 + 16*a^5*c^3*d^4*e^6 + 9*a^6*c^2*d^2*e^8 + 2*a^7*c*e^10
)*x^4 + 4*(a^3*c^5*d^9*e + 4*a^4*c^4*d^7*e^3 + 6*a^5*c^3*d^5*e^5 + 4*a^6*c^
2*d^3*e^7 + a^7*c*d*e^9)*x^3 + (2*a^3*c^5*d^10 + 9*a^4*c^4*d^8*e^2 + 16*a^5
*c^3*d^6*e^4 + 14*a^6*c^2*d^4*e^6 + 6*a^7*c*d^2*e^8 + a^8*e^10)*x^2 + 2*(a^
4*c^4*d^9*e + 4*a^5*c^3*d^7*e^3 + 6*a^6*c^2*d^5*e^5 + 4*a^7*c*d^3*e^7 + a^8
*d*e^9)*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1614 vs. 2(732) = 1464.

Time = 0.28 (sec) , antiderivative size = 1614, normalized size of antiderivative = 2.14

$$\int \frac{A+Bx+Cx^2}{(d+ex)^3\,(a+cx^2)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^3/(c*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/2*(10*C*c^2*d^4*e^3 - 15*B*c^2*d^3*e^4 - 13*C*a*c*d^2*e^5 + 21*A*c^2*d^2
*e^5 + 9*B*a*c*d*e^6 + C*a^2*e^7 - 3*A*a*c*e^7)*log(c*x^2 + a)/(c^5*d^10 +
5*a*c^4*d^8*e^2 + 10*a^2*c^3*d^6*e^4 + 10*a^3*c^2*d^4*e^6 + 5*a^4*c*d^2*e^8
 + a^5*e^10) + (10*C*c^2*d^4*e^4 - 15*B*c^2*d^3*e^5 - 13*C*a*c*d^2*e^6 + 21
*A*c^2*d^2*e^6 + 9*B*a*c*d*e^7 + C*a^2*e^8 - 3*A*a*c*e^8)*log(abs(e*x + d))
/(c^5*d^10*e + 5*a*c^4*d^8*e^3 + 10*a^2*c^3*d^6*e^5 + 10*a^3*c^2*d^4*e^7 +
5*a^4*c*d^2*e^9 + a^5*e^11) + 1/8*(C*a*c^4*d^7 + 3*A*c^5*d^7 - 3*B*a*c^4*d^
6*e + 23*C*a^2*c^3*d^5*e^2 + 21*A*a*c^4*d^5*e^2 - 45*B*a^2*c^3*d^4*e^3 - 12
5*C*a^3*c^2*d^3*e^4 + 105*A*a^2*c^3*d^3*e^4 + 135*B*a^3*c^2*d^2*e^5 + 45*C*
a^4*c*d*e^6 - 105*A*a^3*c^2*d*e^6 - 15*B*a^4*c*e^7)*arctan(c*x/sqrt(a*c))/(
(a^2*c^5*d^10 + 5*a^3*c^4*d^8*e^2 + 10*a^4*c^3*d^6*e^4 + 10*a^5*c^2*d^4*e^6
 + 5*a^6*c*d^2*e^8 + a^7*e^10)*sqrt(a*c)) + 1/8*(C*a*c^4*d^5*e^2*x^5 + 3*A*
c^5*d^5*e^2*x^5 - 3*B*a*c^4*d^4*e^3*x^5 - 58*C*a^2*c^3*d^3*e^4*x^5 + 18*A*a
*c^4*d^3*e^4*x^5 + 78*B*a^2*c^3*d^2*e^5*x^5 + 37*C*a^3*c^2*d*e^6*x^5 - 81*A
*a^2*c^3*d*e^6*x^5 - 15*B*a^3*c^2*e^7*x^5 + 2*C*a*c^4*d^6*e*x^4 + 6*A*c^5*d
^6*e*x^4 - 6*B*a*c^4*d^5*e^2*x^4 - 76*C*a^2*c^3*d^4*e^3*x^4 + 36*A*a*c^4*d^
4*e^3*x^4 + 96*B*a^2*c^3*d^3*e^4*x^4 + 22*C*a^3*c^2*d^2*e^5*x^4 - 78*A*a^2*
c^3*d^2*e^5*x^4 + 6*B*a^3*c^2*d*e^6*x^4 + 4*C*a^4*c*e^7*x^4 - 12*A*a^3*c^2*
e^7*x^4 + C*a*c^4*d^7*x^3 + 3*A*c^5*d^7*x^3 - 3*B*a*c^4*d^6*e*x^3 - 3*C*a^2
*c^3*d^5*e^2*x^3 + 23*A*a*c^4*d^5*e^2*x^3 - 7*B*a^2*c^3*d^4*e^3*x^3 - 129*C
```

```
*a^3*c^2*d^3*e^4*x^3 + 61*A*a^2*c^3*d^3*e^4*x^3 + 163*B*a^3*c^2*d^2*e^5*x^3
 + 67*C*a^4*c*d*e^6*x^3 - 151*A*a^3*c^2*d*e^6*x^3 - 25*B*a^4*c*e^7*x^3 + 10
*C*a^2*c^3*d^6*e*x^2 + 10*A*a*c^4*d^6*e*x^2 - 20*B*a^2*c^3*d^5*e^2*x^2 - 14
2*C*a^3*c^2*d^4*e^3*x^2 + 74*A*a^2*c^3*d^4*e^3*x^2 + 176*B*a^3*c^2*d^3*e^4*
x^2 + 46*C*a^4*c*d^2*e^5*x^2 - 146*A*a^3*c^2*d^2*e^5*x^2 + 4*B*a^4*c*d*e^6*
x^2 + 6*C*a^5*e^7*x^2 - 18*A*a^4*c*e^7*x^2 - C*a^2*c^3*d^7*x + 5*A*a*c^4*d^
7*x - B*a^2*c^3*d^6*e*x - 10*C*a^3*c^2*d^5*e^2*x + 26*A*a^2*c^3*d^5*e^2*x +
 2*B*a^3*c^2*d^4*e^3*x - 77*C*a^4*c*d^3*e^4*x + 49*A*a^3*c^2*d^3*e^4*x + 91
*B*a^4*c*d^2*e^5*x + 28*C*a^5*d*e^6*x - 68*A*a^4*c*d*e^6*x - 8*B*a^5*e^7*x
- 2*B*a^2*c^3*d^7 + 6*C*a^3*c^2*d^6*e + 6*A*a^2*c^3*d^6*e - 20*B*a^3*c^2*d^
5*e^2 - 72*C*a^4*c*d^4*e^3 + 44*A*a^3*c^2*d^4*e^3 + 74*B*a^4*c*d^3*e^4 + 18
*C*a^5*d^2*e^5 - 62*A*a^4*c*d^2*e^5 - 4*B*a^5*d*e^6 - 4*A*a^5*e^7)/((a^2*c^
4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8)*
(c*e*x^3 + c*d*x^2 + a*e*x + a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 17.49 (sec) , antiderivative size = 8774, normalized size of antiderivative = 11.65

$$\int \frac{A+Bx+Cx^2}{(d+ex)^3\,(a+cx^2)^3}\,dx = \text{Too large to display}$$

```
[In] int((A + B*x + C*x^2)/((a + c*x^2)^3*(d + e*x)^3),x)
```

```
[Out] ((x^5*(3*A*c^5*d^5*e^2 - 15*B*a^3*c^2*e^7 + 18*A*a*c^4*d^3*e^4 - 81*A*a^2*c
^3*d*e^6 - 3*B*a*c^4*d^4*e^3 + C*a*c^4*d^5*e^2 + 37*C*a^3*c^2*d*e^6 + 78*B*
a^2*c^3*d^2*e^5 - 58*C*a^2*c^3*d^3*e^4))/(8*a^2*(a^4*e^8 + c^4*d^8 + 4*a*c^
3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)) - (2*A*a^3*e^7 + B*c^3*d^
7 + 2*B*a^3*d*e^6 - 3*A*c^3*d^6*e - 9*C*a^3*d^2*e^5 - 22*A*a*c^2*d^4*e^3 +
31*A*a^2*c*d^2*e^5 + 10*B*a*c^2*d^5*e^2 - 37*B*a^2*c*d^3*e^4 + 36*C*a^2*c*d
^4*e^3 - 3*C*a*c^2*d^6*e)/(4*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c
*d^2*e^6 + 6*a^2*c^2*d^4*e^4)) + (x*(5*A*c^4*d^7 - 8*B*a^4*e^7 - C*a*c^3*d^
7 + 28*C*a^4*d*e^6 + 26*A*a*c^3*d^5*e^2 + 91*B*a^3*c*d^2*e^5 - 77*C*a^3*c*d
^3*e^4 + 49*A*a^2*c^2*d^3*e^4 + 2*B*a^2*c^2*d^4*e^3 - 10*C*a^2*c^2*d^5*e^2
- 68*A*a^3*c*d*e^6 - B*a*c^3*d^6*e))/(8*a*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*
e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)) + (x^2*(3*C*a^4*e^7 - 9*A*a^3*c
*e^7 + 5*A*c^4*d^6*e + 37*A*a*c^3*d^4*e^3 - 10*B*a*c^3*d^5*e^2 + 23*C*a^3*c
*d^2*e^5 - 73*A*a^2*c^2*d^2*e^5 + 88*B*a^2*c^2*d^3*e^4 - 71*C*a^2*c^2*d^4*e
^3 + 2*B*a^3*c*d*e^6 + 5*C*a*c^3*d^6*e))/(4*a*(a^4*e^8 + c^4*d^8 + 4*a*c^3*
d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)) + (x^3*(3*A*c^5*d^7 - 25*B*
a^4*c*e^7 + C*a*c^4*d^7 + 23*A*a*c^4*d^5*e^2 - 151*A*a^3*c^2*d*e^6 + 61*A*a
^2*c^3*d^3*e^4 - 7*B*a^2*c^3*d^4*e^3 + 163*B*a^3*c^2*d^2*e^5 - 3*C*a^2*c^3*
d^5*e^2 - 129*C*a^3*c^2*d^3*e^4 - 3*B*a*c^4*d^6*e + 67*C*a^4*c*d*e^6))/(8*a
^2*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e
^4)) + (x^4*(2*C*a^4*c*e^7 + 3*A*c^5*d^6*e - 6*A*a^3*c^2*e^7 + 18*A*a*c^4*d
```

```
^4*e^3 - 3*B*a*c^4*d^5*e^2 + 3*B*a^3*c^2*d*e^6 - 39*A*a^2*c^3*d^2*e^5 + 48*
B*a^2*c^3*d^3*e^4 - 38*C*a^2*c^3*d^4*e^3 + 11*C*a^3*c^2*d^2*e^5 + C*a*c^4*d
^6*e))/(4*a^2*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^
2*c^2*d^4*e^4)))/(x^2*(a^2*e^2 + 2*a*c*d^2) + x^4*(c^2*d^2 + 2*a*c*e^2) + a
^2*d^2 + c^2*e^2*x^6 + 2*a^2*d*e*x + 2*c^2*d*e*x^5 + 4*a*c*d*e*x^3) + symsu
m(log(root(2560*a^14*c*d^2*e^18*z^3 + 64512*a^10*c^5*d^10*e^10*z^3 + 53760*
a^11*c^4*d^8*e^12*z^3 + 53760*a^9*c^6*d^12*e^8*z^3 + 30720*a^12*c^3*d^6*e^1
4*z^3 + 30720*a^8*c^7*d^14*e^6*z^3 + 11520*a^13*c^2*d^4*e^16*z^3 + 11520*a^
7*c^8*d^16*e^4*z^3 + 2560*a^6*c^9*d^18*e^2*z^3 + 256*a^5*c^10*d^20*z^3 + 25
6*a^15*e^20*z^3 - 4806*B*C*a^8*c*d*e^13*z - 18*A*B*a*c^8*d^13*e*z - 147930*
B*C*a^6*c^3*d^5*e^9*z + 74760*B*C*a^5*c^4*d^7*e^7*z + 66588*B*C*a^7*c^2*d^3
*e^11*z - 1050*B*C*a^4*c^5*d^9*e^5*z - 228*B*C*a^3*c^6*d^11*e^3*z + 152052*
A*C*a^6*c^3*d^4*e^10*z - 109830*A*C*a^5*c^4*d^6*e^8*z - 32490*A*C*a^7*c^2*d
^2*e^12*z + 426*A*C*a^3*c^6*d^10*e^4*z - 360*A*C*a^4*c^5*d^8*e^6*z + 180*A*
C*a^2*c^7*d^12*e^2*z + 158130*A*B*a^5*c^4*d^5*e^9*z - 121356*A*B*a^6*c^3*d^
3*e^11*z - 3240*A*B*a^4*c^5*d^7*e^7*z - 1710*A*B*a^3*c^6*d^9*e^5*z - 396*A*
B*a^2*c^7*d^11*e^3*z - 6*B*C*a^2*c^7*d^13*e*z + 13518*A*B*a^7*c^2*d*e^13*z
 + 67615*C^2*a^6*c^3*d^6*e^8*z - 47538*C^2*a^7*c^2*d^4*e^10*z - 24860*C^2*a^
5*c^4*d^8*e^6*z + 279*C^2*a^4*c^5*d^10*e^4*z + 46*C^2*a^3*c^6*d^12*e^2*z +
71415*B^2*a^6*c^3*d^4*e^10*z - 55260*B^2*a^5*c^4*d^6*e^8*z - 19602*B^2*a^7*
c^2*d^2*e^12*z + 1215*B^2*a^4*c^5*d^8*e^6*z + 270*B^2*a^3*c^6*d^10*e^4*z +
9*B^2*a^2*c^7*d^12*e^2*z - 106722*A^2*a^5*c^4*d^4*e^10*z + 35217*A^2*a^6*c^
3*d^2*e^12*z + 6615*A^2*a^4*c^5*d^6*e^8*z + 3780*A^2*a^3*c^6*d^8*e^6*z + 10
71*A^2*a^2*c^7*d^10*e^4*z + 1152*A*C*a^8*c*e^14*z + 6*A*C*a*c^8*d^14*z + 70
17*C^2*a^8*c*d^2*e^12*z + 126*A^2*a*c^8*d^12*e^2*z + C^2*a^2*c^7*d^14*z - 1
728*A^2*a^7*c^2*e^14*z + 225*B^2*a^8*c*e^14*z + 9*A^2*c^9*d^14*z - 192*C^2*
a^9*e^14*z + 3168*A*B*C*a^4*c^2*d*e^10 + 270*A*B*C*a*c^5*d^7*e^4 - 6930*A*B
*C*a^3*c^3*d^3*e^8 + 5148*A*B*C*a^2*c^4*d^5*e^6 - 819*A^2*C*a*c^5*d^6*e^5 -
 60*A*C^2*a*c^5*d^8*e^3 - 6102*A^2*B*a^3*c^3*d*e^10 + 1512*A^2*B*a*c^5*d^5*
e^6 - 270*A*B^2*a*c^5*d^6*e^5 - 378*B*C^2*a^5*c*d*e^10 - 5049*B^2*C*a^3*c^3
*d^4*e^7 + 4698*B^2*C*a^4*c^2*d^2*e^9 + 2508*B*C^2*a^3*c^3*d^5*e^6 - 1977*B
*C^2*a^4*c^2*d^3*e^8 - 180*B^2*C*a^2*c^4*d^6*e^5 + 75*B*C^2*a^2*c^4*d^7*e^4
 + 15921*A^2*C*a^3*c^3*d^2*e^9 - 7848*A^2*C*a^2*c^4*d^4*e^7 - 6363*A*C^2*a^
4*c^2*d^2*e^9 + 4926*A*C^2*a^3*c^3*d^4*e^7 - 1443*A*C^2*a^2*c^4*d^6*e^5 + 1
4283*A^2*B*a^2*c^4*d^3*e^8 - 4617*A*B^2*a^2*c^4*d^4*e^7 - 1944*A*B^2*a^3*c^
3*d^2*e^9 + 791*C^3*a^5*c*d^2*e^9 - 2025*B^3*a^4*c^2*d*e^10 - 1674*A^3*a*c^
5*d^4*e^7 - 90*A^2*C*c^6*d^8*e^3 + 135*A^2*B*c^6*d^7*e^4 - 1728*A^2*C*a^4*c
^2*e^11 + 675*A*B^2*a^4*c^2*e^11 - 225*B^2*C*a^5*c*e^11 + 576*A*C^2*a^5*c*e
^11 - 397*C^3*a^3*c^3*d^6*e^5 - 108*C^3*a^4*c^2*d^4*e^7 - 10*C^3*a^2*c^4*d^
8*e^3 + 3294*B^3*a^3*c^3*d^3*e^8 + 135*B^3*a^2*c^4*d^5*e^6 - 11853*A^3*a^2*
c^4*d^2*e^9 - 189*A^3*c^6*d^6*e^5 + 1728*A^3*a^3*c^3*e^11 - 64*C^3*a^6*e^11
, z, k)*(root(2560*a^14*c*d^2*e^18*z^3 + 64512*a^10*c^5*d^10*e^10*z^3 + 537
60*a^11*c^4*d^8*e^12*z^3 + 53760*a^9*c^6*d^12*e^8*z^3 + 30720*a^12*c^3*d^6*
e^14*z^3 + 30720*a^8*c^7*d^14*e^6*z^3 + 11520*a^13*c^2*d^4*e^16*z^3 + 11520
*a^7*c^8*d^16*e^4*z^3 + 2560*a^6*c^9*d^18*e^2*z^3 + 256*a^5*c^10*d^20*z^3 +
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 256*a^15*e^20*z^3 - 4806*B*C*a^8*c*d*e^13*z - 18*A*B*a*c^8*d^13*e*z - 1479
30*B*C*a^6*c^3*d^5*e^9*z + 74760*B*C*a^5*c^4*d^7*e^7*z + 66588*B*C*a^7*c^2*
d^3*e^11*z - 1050*B*C*a^4*c^5*d^9*e^5*z - 228*B*C*a^3*c^6*d^11*e^3*z + 1520
52*A*C*a^6*c^3*d^4*e^10*z - 109830*A*C*a^5*c^4*d^6*e^8*z - 32490*A*C*a^7*c^
2*d^2*e^12*z + 426*A*C*a^3*c^6*d^10*e^4*z - 360*A*C*a^4*c^5*d^8*e^6*z + 180
*A*C*a^2*c^7*d^12*e^2*z + 158130*A*B*a^5*c^4*d^5*e^9*z - 121356*A*B*a^6*c^3
*d^3*e^11*z - 3240*A*B*a^4*c^5*d^7*e^7*z - 1710*A*B*a^3*c^6*d^9*e^5*z - 396
*A*B*a^2*c^7*d^11*e^3*z - 6*B*C*a^2*c^7*d^13*e*z + 13518*A*B*a^7*c^2*d*e^13
*z + 67615*C^2*a^6*c^3*d^6*e^8*z - 47538*C^2*a^7*c^2*d^4*e^10*z - 24860*C^2
*a^5*c^4*d^8*e^6*z + 279*C^2*a^4*c^5*d^10*e^4*z + 46*C^2*a^3*c^6*d^12*e^2*z
 + 71415*B^2*a^6*c^3*d^4*e^10*z - 55260*B^2*a^5*c^4*d^6*e^8*z - 19602*B^2*a
^7*c^2*d^2*e^12*z + 1215*B^2*a^4*c^5*d^8*e^6*z + 270*B^2*a^3*c^6*d^10*e^4*z
 + 9*B^2*a^2*c^7*d^12*e^2*z - 106722*A^2*a^5*c^4*d^4*e^10*z + 35217*A^2*a^6
*c^3*d^2*e^12*z + 6615*A^2*a^4*c^5*d^6*e^8*z + 3780*A^2*a^3*c^6*d^8*e^6*z +
 1071*A^2*a^2*c^7*d^10*e^4*z + 1152*A*C*a^8*c*e^14*z + 6*A*C*a*c^8*d^14*z +
 7017*C^2*a^8*c*d^2*e^12*z + 126*A^2*a*c^8*d^12*e^2*z + C^2*a^2*c^7*d^14*z
- 1728*A^2*a^7*c^2*e^14*z + 225*B^2*a^8*c*e^14*z + 9*A^2*c^9*d^14*z - 192*C
^2*a^9*e^14*z + 3168*A*B*C*a^4*c^2*d*e^10 + 270*A*B*C*a*c^5*d^7*e^4 - 6930*
A*B*C*a^3*c^3*d^3*e^8 + 5148*A*B*C*a^2*c^4*d^5*e^6 - 819*A^2*C*a*c^5*d^6*e^
5 - 60*A*C^2*a*c^5*d^8*e^3 - 6102*A^2*B*a^3*c^3*d*e^10 + 1512*A^2*B*a*c^5*d
^5*e^6 - 270*A*B^2*a*c^5*d^6*e^5 - 378*B*C^2*a^5*c*d*e^10 - 5049*B^2*C*a^3*
c^3*d^4*e^7 + 4698*B^2*C*a^4*c^2*d^2*e^9 + 2508*B*C^2*a^3*c^3*d^5*e^6 - 197
7*B*C^2*a^4*c^2*d^3*e^8 - 180*B^2*C*a^2*c^4*d^6*e^5 + 75*B*C^2*a^2*c^4*d^7*
e^4 + 15921*A^2*C*a^3*c^3*d^2*e^9 - 7848*A^2*C*a^2*c^4*d^4*e^7 - 6363*A*C^2
*a^4*c^2*d^2*e^9 + 4926*A*C^2*a^3*c^3*d^4*e^7 - 1443*A*C^2*a^2*c^4*d^6*e^5
+ 14283*A^2*B*a^2*c^4*d^3*e^8 - 4617*A*B^2*a^2*c^4*d^4*e^7 - 1944*A*B^2*a^3
*c^3*d^2*e^9 + 791*C^3*a^5*c*d^2*e^9 - 2025*B^3*a^4*c^2*d*e^10 - 1674*A^3*a
*c^5*d^4*e^7 - 90*A^2*C*c^6*d^8*e^3 + 135*A^2*B*c^6*d^7*e^4 - 1728*A^2*C*a^
4*c^2*e^11 + 675*A*B^2*a^4*c^2*e^11 - 225*B^2*C*a^5*c*e^11 + 576*A*C^2*a^5*
c*e^11 - 397*C^3*a^3*c^3*d^6*e^5 - 108*C^3*a^4*c^2*d^4*e^7 - 10*C^3*a^2*c^4
*d^8*e^3 + 3294*B^3*a^3*c^3*d^3*e^8 + 135*B^3*a^2*c^4*d^5*e^6 - 11853*A^3*a
^2*c^4*d^2*e^9 - 189*A^3*c^6*d^6*e^5 + 1728*A^3*a^3*c^3*e^11 - 64*C^3*a^6*e
^11, z, k)*((512*a^13*c^2*d*e^18 + 512*a^5*c^10*d^17*e^2 + 4096*a^6*c^9*d^1
5*e^4 + 14336*a^7*c^8*d^13*e^6 + 28672*a^8*c^7*d^11*e^8 + 35840*a^9*c^6*d^9
*e^10 + 28672*a^10*c^5*d^7*e^12 + 14336*a^11*c^4*d^5*e^14 + 4096*a^12*c^3*d
^3*e^16)/(64*(a^12*e^16 + a^4*c^8*d^16 + 8*a^11*c*d^2*e^14 + 8*a^5*c^7*d^14
*e^2 + 28*a^6*c^6*d^12*e^4 + 56*a^7*c^5*d^10*e^6 + 70*a^8*c^4*d^8*e^8 + 56*
a^9*c^3*d^6*e^10 + 28*a^10*c^2*d^4*e^12)) + (x*(384*a^13*c^2*e^19 - 128*a^4
*c^11*d^18*e - 640*a^5*c^10*d^16*e^3 - 512*a^6*c^9*d^14*e^5 + 3584*a^7*c^8*
d^12*e^7 + 12544*a^8*c^7*d^10*e^9 + 19712*a^9*c^6*d^8*e^11 + 17920*a^10*c^5
*d^6*e^13 + 9728*a^11*c^4*d^4*e^15 + 2944*a^12*c^3*d^2*e^17))/(64*(a^12*e^1
6 + a^4*c^8*d^16 + 8*a^11*c*d^2*e^14 + 8*a^5*c^7*d^14*e^2 + 28*a^6*c^6*d^12
*e^4 + 56*a^7*c^5*d^10*e^6 + 70*a^8*c^4*d^8*e^8 + 56*a^9*c^3*d^6*e^10 + 28*
a^10*c^2*d^4*e^12))) + (120*B*a^10*c^2*e^16 + 24*A*a^2*c^10*d^15*e + 456*A*
a^9*c^3*d*e^15 + 8*C*a^3*c^9*d^15*e - 232*C*a^10*c^2*d*e^15 + 216*A*a^3*c^9
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*d^13*e^3 + 1176*A*a^4*c^8*d^11*e^5 + 3480*A*a^5*c^7*d^9*e^7 + 5640*A*a^6*c
^6*d^7*e^9 + 5064*A*a^7*c^5*d^5*e^11 + 2376*A*a^8*c^4*d^3*e^13 - 24*B*a^3*c
^9*d^14*e^2 - 408*B*a^4*c^8*d^12*e^4 - 1560*B*a^5*c^7*d^10*e^6 - 2520*B*a^6
*c^6*d^8*e^8 - 1800*B*a^7*c^5*d^6*e^10 - 264*B*a^8*c^4*d^4*e^12 + 312*B*a^9
*c^3*d^2*e^14 + 200*C*a^4*c^8*d^13*e^3 + 648*C*a^5*c^7*d^11*e^5 + 520*C*a^6
*c^6*d^9*e^7 - 680*C*a^7*c^5*d^7*e^9 - 1512*C*a^8*c^4*d^5*e^11 - 1000*C*a^9
*c^3*d^3*e^13)/(64*(a^12*e^16 + a^4*c^8*d^16 + 8*a^11*c*d^2*e^14 + 8*a^5*c^
7*d^14*e^2 + 28*a^6*c^6*d^12*e^4 + 56*a^7*c^5*d^10*e^6 + 70*a^8*c^4*d^8*e^8
 + 56*a^9*c^3*d^6*e^10 + 28*a^10*c^2*d^4*e^12)) + (x*(192*C*a^10*c^2*e^16 -
 576*A*a^9*c^3*e^16 + 1488*B*a^9*c^3*d*e^15 + 48*A*a^2*c^10*d^14*e^2 + 480*
A*a^3*c^9*d^12*e^4 + 4176*A*a^4*c^8*d^10*e^6 + 12288*A*a^5*c^7*d^8*e^8 + 15
312*A*a^6*c^6*d^6*e^10 + 7776*A*a^7*c^5*d^4*e^12 + 432*A*a^8*c^4*d^2*e^14 -
 48*B*a^3*c^9*d^13*e^3 - 1824*B*a^4*c^8*d^11*e^5 - 5328*B*a^5*c^7*d^9*e^7 -
 4032*B*a^6*c^6*d^7*e^9 + 2352*B*a^7*c^5*d^5*e^11 + 4320*B*a^8*c^4*d^3*e^13
 + 16*C*a^3*c^9*d^14*e^2 + 1056*C*a^4*c^8*d^12*e^4 + 2160*C*a^5*c^7*d^10*e^
6 - 1408*C*a^6*c^6*d^8*e^8 - 6672*C*a^7*c^5*d^6*e^10 - 5472*C*a^8*c^4*d^4*e
^12 - 1136*C*a^9*c^3*d^2*e^14))/(64*(a^12*e^16 + a^4*c^8*d^16 + 8*a^11*c*d^
2*e^14 + 8*a^5*c^7*d^14*e^2 + 28*a^6*c^6*d^12*e^4 + 56*a^7*c^5*d^10*e^6 + 7
0*a^8*c^4*d^8*e^8 + 56*a^9*c^3*d^6*e^10 + 28*a^10*c^2*d^4*e^12))) + (9*A^2*
c^9*d^11*e^2 + 342*A^2*a^2*c^7*d^7*e^6 + 36*A^2*a^3*c^6*d^5*e^8 - 7479*A^2*
a^4*c^5*d^3*e^10 + 9*B^2*a^2*c^7*d^9*e^4 - 108*B^2*a^3*c^6*d^7*e^6 - 3402*B
^2*a^4*c^5*d^5*e^8 + 5076*B^2*a^5*c^4*d^3*e^10 + C^2*a^2*c^7*d^11*e^2 - 36*
C^2*a^3*c^6*d^9*e^4 - 1306*C^2*a^4*c^5*d^7*e^6 + 4708*C^2*a^5*c^4*d^5*e^8 -
 2943*C^2*a^6*c^3*d^3*e^10 + 360*A*B*a^6*c^3*e^13 - 120*B*C*a^7*c^2*e^13 +
108*A^2*a*c^8*d^9*e^4 + 1944*A^2*a^5*c^4*d*e^12 - 855*B^2*a^6*c^3*d*e^12 +
296*C^2*a^7*c^2*d*e^12 - 18*A*B*a*c^8*d^10*e^3 + 6*A*C*a*c^8*d^11*e^2 - 153
6*A*C*a^6*c^3*d*e^12 + 756*A*B*a^3*c^6*d^6*e^7 + 11016*A*B*a^4*c^5*d^4*e^9
- 7794*A*B*a^5*c^4*d^2*e^11 - 72*A*C*a^2*c^7*d^9*e^4 - 732*A*C*a^3*c^6*d^7*
e^6 - 7368*A*C*a^4*c^5*d^5*e^8 + 10182*A*C*a^5*c^4*d^3*e^10 - 6*B*C*a^2*c^7
*d^10*e^3 + 144*B*C*a^3*c^6*d^8*e^5 + 4284*B*C*a^4*c^5*d^6*e^7 - 10440*B*C*
a^5*c^4*d^4*e^9 + 3738*B*C*a^6*c^3*d^2*e^11)/(64*(a^12*e^16 + a^4*c^8*d^16
+ 8*a^11*c*d^2*e^14 + 8*a^5*c^7*d^14*e^2 + 28*a^6*c^6*d^12*e^4 + 56*a^7*c^5
*d^10*e^6 + 70*a^8*c^4*d^8*e^8 + 56*a^9*c^3*d^6*e^10 + 28*a^10*c^2*d^4*e^12
)) + (x*(225*B^2*a^6*c^3*e^13 + 9*A^2*c^9*d^10*e^3 - 162*A^2*a^2*c^7*d^6*e^
7 - 2916*A^2*a^3*c^6*d^4*e^9 + 6561*A^2*a^4*c^5*d^2*e^11 + 9*B^2*a^2*c^7*d^
8*e^5 - 468*B^2*a^3*c^6*d^6*e^7 + 6174*B^2*a^4*c^5*d^4*e^9 - 2340*B^2*a^5*c
^4*d^2*e^11 + C^2*a^2*c^7*d^10*e^3 - 116*C^2*a^3*c^6*d^8*e^5 + 3438*C^2*a^4
*c^5*d^6*e^7 - 4292*C^2*a^5*c^4*d^4*e^9 + 1369*C^2*a^6*c^3*d^2*e^11 + 108*A
^2*a*c^8*d^8*e^5 - 18*A*B*a*c^8*d^9*e^4 + 2430*A*B*a^5*c^4*d*e^12 + 6*A*C*a
*c^8*d^10*e^3 - 1110*B*C*a^6*c^3*d*e^12 + 360*A*B*a^2*c^7*d^7*e^6 + 3204*A*
B*a^3*c^6*d^5*e^8 - 13176*A*B*a^4*c^5*d^3*e^10 - 312*A*C*a^2*c^7*d^8*e^5 -
2028*A*C*a^3*c^6*d^6*e^7 + 10728*A*C*a^4*c^5*d^4*e^9 - 5994*A*C*a^5*c^4*d^2
*e^11 - 6*B*C*a^2*c^7*d^9*e^4 + 504*B*C*a^3*c^6*d^7*e^6 - 9300*B*C*a^4*c^5*
d^5*e^8 + 7512*B*C*a^5*c^4*d^3*e^10))/(64*(a^12*e^16 + a^4*c^8*d^16 + 8*a^1
1*c*d^2*e^14 + 8*a^5*c^7*d^14*e^2 + 28*a^6*c^6*d^12*e^4 + 56*a^7*c^5*d^10*e
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^6 + 70*a^8*c^4*d^8*e^8 + 56*a^9*c^3*d^6*e^10 + 28*a^10*c^2*d^4*e^12)))*roo
t(2560*a^14*c*d^2*e^18*z^3 + 64512*a^10*c^5*d^10*e^10*z^3 + 53760*a^11*c^4*
d^8*e^12*z^3 + 53760*a^9*c^6*d^12*e^8*z^3 + 30720*a^12*c^3*d^6*e^14*z^3 + 3
0720*a^8*c^7*d^14*e^6*z^3 + 11520*a^13*c^2*d^4*e^16*z^3 + 11520*a^7*c^8*d^1
6*e^4*z^3 + 2560*a^6*c^9*d^18*e^2*z^3 + 256*a^5*c^10*d^20*z^3 + 256*a^15*e^
20*z^3 - 4806*B*C*a^8*c*d*e^13*z - 18*A*B*a*c^8*d^13*e*z - 147930*B*C*a^6*c
^3*d^5*e^9*z + 74760*B*C*a^5*c^4*d^7*e^7*z + 66588*B*C*a^7*c^2*d^3*e^11*z -
 1050*B*C*a^4*c^5*d^9*e^5*z - 228*B*C*a^3*c^6*d^11*e^3*z + 152052*A*C*a^6*c
^3*d^4*e^10*z - 109830*A*C*a^5*c^4*d^6*e^8*z - 32490*A*C*a^7*c^2*d^2*e^12*z
 + 426*A*C*a^3*c^6*d^10*e^4*z - 360*A*C*a^4*c^5*d^8*e^6*z + 180*A*C*a^2*c^7
*d^12*e^2*z + 158130*A*B*a^5*c^4*d^5*e^9*z - 121356*A*B*a^6*c^3*d^3*e^11*z
- 3240*A*B*a^4*c^5*d^7*e^7*z - 1710*A*B*a^3*c^6*d^9*e^5*z - 396*A*B*a^2*c^7
*d^11*e^3*z - 6*B*C*a^2*c^7*d^13*e*z + 13518*A*B*a^7*c^2*d*e^13*z + 67615*C
^2*a^6*c^3*d^6*e^8*z - 47538*C^2*a^7*c^2*d^4*e^10*z - 24860*C^2*a^5*c^4*d^8
*e^6*z + 279*C^2*a^4*c^5*d^10*e^4*z + 46*C^2*a^3*c^6*d^12*e^2*z + 71415*B^2
*a^6*c^3*d^4*e^10*z - 55260*B^2*a^5*c^4*d^6*e^8*z - 19602*B^2*a^7*c^2*d^2*e
^12*z + 1215*B^2*a^4*c^5*d^8*e^6*z + 270*B^2*a^3*c^6*d^10*e^4*z + 9*B^2*a^2
*c^7*d^12*e^2*z - 106722*A^2*a^5*c^4*d^4*e^10*z + 35217*A^2*a^6*c^3*d^2*e^1
2*z + 6615*A^2*a^4*c^5*d^6*e^8*z + 3780*A^2*a^3*c^6*d^8*e^6*z + 1071*A^2*a^
2*c^7*d^10*e^4*z + 1152*A*C*a^8*c*e^14*z + 6*A*C*a*c^8*d^14*z + 7017*C^2*a^
8*c*d^2*e^12*z + 126*A^2*a*c^8*d^12*e^2*z + C^2*a^2*c^7*d^14*z - 1728*A^2*a
^7*c^2*e^14*z + 225*B^2*a^8*c*e^14*z + 9*A^2*c^9*d^14*z - 192*C^2*a^9*e^14*
z + 3168*A*B*C*a^4*c^2*d*e^10 + 270*A*B*C*a*c^5*d^7*e^4 - 6930*A*B*C*a^3*c^
3*d^3*e^8 + 5148*A*B*C*a^2*c^4*d^5*e^6 - 819*A^2*C*a*c^5*d^6*e^5 - 60*A*C^2
*a*c^5*d^8*e^3 - 6102*A^2*B*a^3*c^3*d*e^10 + 1512*A^2*B*a*c^5*d^5*e^6 - 270
*A*B^2*a*c^5*d^6*e^5 - 378*B*C^2*a^5*c*d*e^10 - 5049*B^2*C*a^3*c^3*d^4*e^7
+ 4698*B^2*C*a^4*c^2*d^2*e^9 + 2508*B*C^2*a^3*c^3*d^5*e^6 - 1977*B*C^2*a^4*
c^2*d^3*e^8 - 180*B^2*C*a^2*c^4*d^6*e^5 + 75*B*C^2*a^2*c^4*d^7*e^4 + 15921*
A^2*C*a^3*c^3*d^2*e^9 - 7848*A^2*C*a^2*c^4*d^4*e^7 - 6363*A*C^2*a^4*c^2*d^2
*e^9 + 4926*A*C^2*a^3*c^3*d^4*e^7 - 1443*A*C^2*a^2*c^4*d^6*e^5 + 14283*A^2*
B*a^2*c^4*d^3*e^8 - 4617*A*B^2*a^2*c^4*d^4*e^7 - 1944*A*B^2*a^3*c^3*d^2*e^9
 + 791*C^3*a^5*c*d^2*e^9 - 2025*B^3*a^4*c^2*d*e^10 - 1674*A^3*a*c^5*d^4*e^7
 - 90*A^2*C*c^6*d^8*e^3 + 135*A^2*B*c^6*d^7*e^4 - 1728*A^2*C*a^4*c^2*e^11 +
 675*A*B^2*a^4*c^2*e^11 - 225*B^2*C*a^5*c*e^11 + 576*A*C^2*a^5*c*e^11 - 397
*C^3*a^3*c^3*d^6*e^5 - 108*C^3*a^4*c^2*d^4*e^7 - 10*C^3*a^2*c^4*d^8*e^3 + 3
294*B^3*a^3*c^3*d^3*e^8 + 135*B^3*a^2*c^4*d^5*e^6 - 11853*A^3*a^2*c^4*d^2*e
^9 - 189*A^3*c^6*d^6*e^5 + 1728*A^3*a^3*c^3*e^11 - 64*C^3*a^6*e^11, z, k),
k, 1, 3)
```

3.64 $\int \frac{(d+ex)^4(A+Bx+Cx^2)}{(a+cx^2)^4}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 234

$$\begin{aligned}
&\int \frac{(d+ex)^4\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^4}\,dx \\
&= -\frac{(aB-(Ac-aC)x)(d+ex)^4}{6ac\left(a+cx^2\right)^3} - \frac{(d+ex)^3(a(Ac+5aC)e-c(5Acd+aCd+4aBe)x)}{24a^2c^2\left(a+cx^2\right)^2} \\
&\quad - \frac{\left(a(Ac+5aC)e^2+cd(5Acd+aCd+4aBe)\right)(ae-cdx)(d+ex)}{16a^3c^3\left(a+cx^2\right)} \\
&\quad + \frac{\left(cd^2+ae^2\right)\left(a(Ac+5aC)e^2+cd(5Acd+aCd+4aBe)\right)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{16a^{7/2}c^{7/2}}
\end{aligned}$$

```
[Out] -1/6*(a*B-(A*c-C*a)*x)*(e*x+d)^4/a/c/(c*x^2+a)^3-1/24*(e*x+d)^3*(a*(A*c+5*C
*a)*e-c*(5*A*c*d+4*B*a*e+C*a*d)*x)/a^2/c^2/(c*x^2+a)^2-1/16*(a*(A*c+5*C*a)*
e^2+c*d*(5*A*c*d+4*B*a*e+C*a*d))*(-c*d*x+a*e)*(e*x+d)/a^3/c^3/(c*x^2+a)+1/1
6*(a*e^2+c*d^2)*(a*(A*c+5*C*a)*e^2+c*d*(5*A*c*d+4*B*a*e+C*a*d))*arctan(x*c^
(1/2)/a^(1/2))/a^(7/2)/c^(7/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used

$= \{1659, 819, 737, 211\}$

$$\begin{aligned}&\int \frac{(d+ex)^4 (A+Bx+Cx^2)}{(a+cx^2)^4}\,dx\\&=\frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(ae^2+cd^2)\left(cd(4aBe+aCd+5Acd)+ae^2(5aC+Ac)\right)}{16a^{7/2}c^{7/2}}\\&\quad-\frac{(d+ex)(ae-cdx)\left(cd(4aBe+aCd+5Acd)+ae^2(5aC+Ac)\right)}{16a^3c^3(a+cx^2)}\\&\quad-\frac{(d+ex)^3(ae(5aC+Ac)-cx(4aBe+aCd+5Acd))}{24a^2c^2(a+cx^2)^2}-\frac{(d+ex)^4(aB-x(Ac-aC))}{6ac(a+cx^2)^3}\end{aligned}$$

```
[In] Int[((d + e*x)^4*(A + B*x + C*x^2))/(a + c*x^2)^4,x]
```

```
[Out] -1/6*((a*B - (A*c - a*C)*x)*(d + e*x)^4)/(a*c*(a + c*x^2)^3) - ((d + e*x)^3
*(a*(A*c + 5*a*C)*e - c*(5*A*c*d + a*C*d + 4*a*B*e)*x))/(24*a^2*c^2*(a + c*
x^2)^2) - ((a*(A*c + 5*a*C)*e^2 + c*d*(5*A*c*d + a*C*d + 4*a*B*e))*(a*e - c
*d*x)*(d + e*x))/(16*a^3*c^3*(a + c*x^2)) + ((c*d^2 + a*e^2)*(a*(A*c + 5*a*
C)*e^2 + c*d*(5*A*c*d + a*C*d + 4*a*B*e))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(16*
a^(7/2)*c^(7/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 737

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m - 1)*(a*e - c*d*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] +
 Dist[(2*p + 3)*((c*d^2 + a*e^2)/(2*a*c*(p + 1))), Int[(d + e*x)^(m - 2)*(a
 + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0
] && EqQ[m + 2*p + 2, 0] && LtQ[p, -1]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c
*(p + 1))), x] - Dist[m*((c*d*f + a*e*g)/(2*a*c*(p + 1))), Int[(d + e*x)^(m
 - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0] && LtQ[p, -1]
```

Rule 1659

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
```

```
 x], x, 1]}, Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c*(p
+ 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(aB-(Ac-aC)x)(d+ex)^4}{6ac\left(a+cx^2\right)^3}-\frac{\int \frac{(d+ex)^3(-5Acd-aCd-4aBe-(Ac+5aC)ex)}{\left(a+cx^2\right)^3}\,dx}{6ac}\\
&= -\frac{(aB-(Ac-aC)x)(d+ex)^4}{6ac\left(a+cx^2\right)^3}\\
&\quad -\frac{(d+ex)^3(a(Ac+5aC)e-c(5Acd+aCd+4aBe)x)}{24a^2c^2\left(a+cx^2\right)^2}\\
&\quad +\frac{\left(a(Ac+5aC)e^2+cd(5Acd+aCd+4aBe)\right)\int \frac{(d+ex)^2}{\left(a+cx^2\right)^2}\,dx}{8a^2c^2}\\
&= -\frac{(aB-(Ac-aC)x)(d+ex)^4}{6ac\left(a+cx^2\right)^3}\\
&\quad -\frac{(d+ex)^3(a(Ac+5aC)e-c(5Acd+aCd+4aBe)x)}{24a^2c^2\left(a+cx^2\right)^2}\\
&\quad -\frac{\left(a(Ac+5aC)e^2+cd(5Acd+aCd+4aBe)\right)(ae-cdx)(d+ex)}{16a^3c^3\left(a+cx^2\right)}\\
&\quad +\frac{\left(\left(cd^2+ae^2\right)\left(a(Ac+5aC)e^2+cd(5Acd+aCd+4aBe)\right)\right)\int \frac{1}{a+cx^2}\,dx}{16a^3c^3}\\
&= -\frac{(aB-(Ac-aC)x)(d+ex)^4}{6ac\left(a+cx^2\right)^3}\\
&\quad -\frac{(d+ex)^3(a(Ac+5aC)e-c(5Acd+aCd+4aBe)x)}{24a^2c^2\left(a+cx^2\right)^2}\\
&\quad -\frac{\left(a(Ac+5aC)e^2+cd(5Acd+aCd+4aBe)\right)(ae-cdx)(d+ex)}{16a^3c^3\left(a+cx^2\right)}\\
&\quad +\frac{\left(cd^2+ae^2\right)\left(a(Ac+5aC)e^2+cd(5Acd+aCd+4aBe)\right)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{16a^{7/2}c^{7/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 437, normalized size of antiderivative = 1.87

$$\int \frac{(d+ex)^4\,(A+Bx+Cx^2)}{(a+cx^2)^4}\,dx$$

$$= \frac{5Ac^3d^4x + ac^2d^2(Cd^2+4Bde+6Ae^2)\,x + a^2ce^2(6Cd^2+e(4Bd+Ae))\,x - a^3e^3(32Cd+8Be+11Cex)}{16a^3c^3\,(a+cx^2)}$$

$$+ \frac{Ac^3d^4x - a^3e^3(4Cd+Be+Cex) - ac^2d^2(4Ade+Cd^2x+6Ae^2x+Bd(d+4ex)) + a^2ce(2Cd^2(2d+3}{6ac^3\,(a+cx^2)^3}$$

$$+ \frac{5Ac^3d^4x + ac^2d^2(Cd^2+4Bde+6Ae^2)\,x + a^3e^3(48Cd+12Be+13Cex) - a^2ce(6Cd^2(4d+7ex)+e(4}{24a^2c^3\,(a+cx^2)^2}$$

$$+ \frac{(cd^2+ae^2)\,(Ac(5cd^2+ae^2)+a(5aCe^2+cd(Cd+4Be)))\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{16a^{7/2}c^{7/2}}$$

```
[In] Integrate[((d + e*x)^4*(A + B*x + C*x^2))/(a + c*x^2)^4,x]
```

```
[Out] (5*A*c^3*d^4*x + a*c^2*d^2*(C*d^2 + 4*B*d*e + 6*A*e^2)*x + a^2*c*e^2*(6*C*d
^2 + e*(4*B*d + A*e))*x - a^3*e^3*(32*C*d + 8*B*e + 11*C*e*x))/(16*a^3*c^3*
(a + c*x^2)) + (A*c^3*d^4*x - a^3*e^3*(4*C*d + B*e + C*e*x) - a*c^2*d^2*(4*
A*d*e + C*d^2*x + 6*A*e^2*x + B*d*(d + 4*e*x)) + a^2*c*e*(2*C*d^2*(2*d + 3*
e*x) + e*(A*e*(4*d + e*x) + 2*B*d*(3*d + 2*e*x))))/(6*a*c^3*(a + c*x^2)^3)
+ (5*A*c^3*d^4*x + a*c^2*d^2*(C*d^2 + 4*B*d*e + 6*A*e^2)*x + a^3*e^3*(48*C*
d + 12*B*e + 13*C*e*x) - a^2*c*e*(6*C*d^2*(4*d + 7*e*x) + e*(4*B*d*(9*d + 7
*e*x) + A*e*(24*d + 7*e*x))))/(24*a^2*c^3*(a + c*x^2)^2) + ((c*d^2 + a*e^2)
*(A*c*(5*c*d^2 + a*e^2) + a*(5*a*C*e^2 + c*d*(C*d + 4*B*e)))*ArcTan[(Sqrt[c
]*x)/Sqrt[a]])/(16*a^(7/2)*c^(7/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 543 vs. 2(218) = 436.

Time = 0.58 (sec) , antiderivative size = 544, normalized size of antiderivative = 2.32

method	result
default	$\frac{\left(A a^2 c e^4+6Aa c^2 d^2 e^2+5A d^4 c^3+4B a^2 cd e^3+4Ba c^2 d^3 e-11C a^3 e^4+6C a^2 c d^2 e^2+Ca c^2 d^4\right)x^5}{16a^3c} - \frac{e^3(Be+4Cd)x^4}{2c} - \left(A a^2 c e^4-6Aa c^2 d^2 e^2-5A d$
risch	$\frac{\left(A a^2 c e^4+6Aa c^2 d^2 e^2+5A d^4 c^3+4B a^2 cd e^3+4Ba c^2 d^3 e-11C a^3 e^4+6C a^2 c d^2 e^2+Ca c^2 d^4\right)x^5}{16a^3c} - \frac{e^3(Be+4Cd)x^4}{2c} - \left(A a^2 c e^4-6Aa c^2 d^2 e^2-5A d$

```
[In] int((e*x+d)^4*(C*x^2+B*x+A)/(c*x^2+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] (1/16*(A*a^2*c*e^4+6*A*a*c^2*d^2*e^2+5*A*c^3*d^4+4*B*a^2*c*d*e^3+4*B*a*c^2*
d^3*e-11*C*a^3*e^4+6*C*a^2*c*d^2*e^2+C*a*c^2*d^4)/a^3/c*x^5-1/2*e^3*(B*e+4*
```

```
C*d)/c*x^4-1/6*(A*a^2*c*e^4-6*A*a*c^2*d^2*e^2-5*A*c^3*d^4+4*B*a^2*c*d*e^3-4
*B*a*c^2*d^3*e+5*C*a^3*e^4+6*C*a^2*c*d^2*e^2-C*a*c^2*d^4)/a^2/c^2*x^3-1/2*e
*(2*A*c*d*e^2+B*a*e^3+3*B*c*d^2*e+4*C*a*d*e^2+2*C*c*d^3)/c^2*x^2-1/16*(A*a^
2*c*e^4+6*A*a*c^2*d^2*e^2-11*A*c^3*d^4+4*B*a^2*c*d*e^3+4*B*a*c^2*d^3*e+5*C*
a^3*e^4+6*C*a^2*c*d^2*e^2+C*a*c^2*d^4)/a/c^3*x-1/6*(2*A*a*c*d*e^3+4*A*c^2*d
^3*e+B*a^2*e^4+3*B*a*c*d^2*e^2+B*c^2*d^4+4*C*a^2*d*e^3+2*C*a*c*d^3*e)/c^3)/
(c*x^2+a)^3+1/16*(A*a^2*c*e^4+6*A*a*c^2*d^2*e^2+5*A*c^3*d^4+4*B*a^2*c*d*e^3
+4*B*a*c^2*d^3*e+5*C*a^3*e^4+6*C*a^2*c*d^2*e^2+C*a*c^2*d^4)/a^3/c^3/(a*c)^(
1/2)*arctan(c*x/(a*c)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 922 vs. 2(217) = 434.

Time = 0.37 (sec) , antiderivative size = 1864, normalized size of antiderivative = 7.97

$$\int \frac{(d+ex)^4 (A+Bx+Cx^2)}{(a+cx^2)^4} \, dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)^4*(C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="fricas")
```

```
[Out] [-1/96*(16*B*a^4*c^3*d^4 + 48*B*a^5*c^2*d^2*e^2 + 16*B*a^6*c*e^4 - 6*(4*B*a
^2*c^5*d^3*e + 4*B*a^3*c^4*d*e^3 + (C*a^2*c^5 + 5*A*a*c^6)*d^4 + 6*(C*a^3*c
^4 + A*a^2*c^5)*d^2*e^2 - (11*C*a^4*c^3 - A*a^3*c^4)*e^4)*x^5 + 32*(C*a^5*c
^2 + 2*A*a^4*c^3)*d^3*e + 32*(2*C*a^6*c + A*a^5*c^2)*d*e^3 + 48*(4*C*a^4*c^
3*d*e^3 + B*a^4*c^3*e^4)*x^4 - 16*(4*B*a^3*c^4*d^3*e - 4*B*a^4*c^3*d*e^3 +
(C*a^3*c^4 + 5*A*a^2*c^5)*d^4 - 6*(C*a^4*c^3 - A*a^3*c^4)*d^2*e^2 - (5*C*a^
5*c^2 + A*a^4*c^3)*e^4)*x^3 + 48*(2*C*a^4*c^3*d^3*e + 3*B*a^4*c^3*d^2*e^2 +
 B*a^5*c^2*e^4 + 2*(2*C*a^5*c^2 + A*a^4*c^3)*d*e^3)*x^2 + 3*(4*B*a^4*c^2*d^
3*e + 4*B*a^5*c*d*e^3 + (4*B*a*c^5*d^3*e + 4*B*a^2*c^4*d*e^3 + (C*a*c^5 + 5
*A*c^6)*d^4 + 6*(C*a^2*c^4 + A*a*c^5)*d^2*e^2 + (5*C*a^3*c^3 + A*a^2*c^4)*e
^4)*x^6 + (C*a^4*c^2 + 5*A*a^3*c^3)*d^4 + 6*(C*a^5*c + A*a^4*c^2)*d^2*e^2 +
 (5*C*a^6 + A*a^5*c)*e^4 + 3*(4*B*a^2*c^4*d^3*e + 4*B*a^3*c^3*d*e^3 + (C*a^
2*c^4 + 5*A*a*c^5)*d^4 + 6*(C*a^3*c^3 + A*a^2*c^4)*d^2*e^2 + (5*C*a^4*c^2 +
 A*a^3*c^3)*e^4)*x^4 + 3*(4*B*a^3*c^3*d^3*e + 4*B*a^4*c^2*d*e^3 + (C*a^3*c^
3 + 5*A*a^2*c^4)*d^4 + 6*(C*a^4*c^2 + A*a^3*c^3)*d^2*e^2 + (5*C*a^5*c + A*a
^4*c^2)*e^4)*x^2)*sqrt(-a*c)*log((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a))
+ 6*(4*B*a^4*c^3*d^3*e + 4*B*a^5*c^2*d*e^3 + (C*a^4*c^3 - 11*A*a^3*c^4)*d^4
 + 6*(C*a^5*c^2 + A*a^4*c^3)*d^2*e^2 + (5*C*a^6*c + A*a^5*c^2)*e^4)*x)/(a^4
*c^7*x^6 + 3*a^5*c^6*x^4 + 3*a^6*c^5*x^2 + a^7*c^4), -1/48*(8*B*a^4*c^3*d^4
 + 24*B*a^5*c^2*d^2*e^2 + 8*B*a^6*c*e^4 - 3*(4*B*a^2*c^5*d^3*e + 4*B*a^3*c^
4*d*e^3 + (C*a^2*c^5 + 5*A*a*c^6)*d^4 + 6*(C*a^3*c^4 + A*a^2*c^5)*d^2*e^2 -
 (11*C*a^4*c^3 - A*a^3*c^4)*e^4)*x^5 + 16*(C*a^5*c^2 + 2*A*a^4*c^3)*d^3*e +
 16*(2*C*a^6*c + A*a^5*c^2)*d*e^3 + 24*(4*C*a^4*c^3*d*e^3 + B*a^4*c^3*e^4)*
x^4 - 8*(4*B*a^3*c^4*d^3*e - 4*B*a^4*c^3*d*e^3 + (C*a^3*c^4 + 5*A*a^2*c^5)*
```

```
d^4 - 6*(C*a^4*c^3 - A*a^3*c^4)*d^2*e^2 - (5*C*a^5*c^2 + A*a^4*c^3)*e^4)*x^
3 + 24*(2*C*a^4*c^3*d^3*e + 3*B*a^4*c^3*d^2*e^2 + B*a^5*c^2*e^4 + 2*(2*C*a^
5*c^2 + A*a^4*c^3)*d*e^3)*x^2 - 3*(4*B*a^4*c^2*d^3*e + 4*B*a^5*c*d*e^3 + (4
*B*a*c^5*d^3*e + 4*B*a^2*c^4*d*e^3 + (C*a*c^5 + 5*A*c^6)*d^4 + 6*(C*a^2*c^4
 + A*a*c^5)*d^2*e^2 + (5*C*a^3*c^3 + A*a^2*c^4)*e^4)*x^6 + (C*a^4*c^2 + 5*A
*a^3*c^3)*d^4 + 6*(C*a^5*c + A*a^4*c^2)*d^2*e^2 + (5*C*a^6 + A*a^5*c)*e^4 +
 3*(4*B*a^2*c^4*d^3*e + 4*B*a^3*c^3*d*e^3 + (C*a^2*c^4 + 5*A*a*c^5)*d^4 + 6
*(C*a^3*c^3 + A*a^2*c^4)*d^2*e^2 + (5*C*a^4*c^2 + A*a^3*c^3)*e^4)*x^4 + 3*(
4*B*a^3*c^3*d^3*e + 4*B*a^4*c^2*d*e^3 + (C*a^3*c^3 + 5*A*a^2*c^4)*d^4 + 6*(
C*a^4*c^2 + A*a^3*c^3)*d^2*e^2 + (5*C*a^5*c + A*a^4*c^2)*e^4)*x^2)*sqrt(a*c
)*arctan(sqrt(a*c)*x/a) + 3*(4*B*a^4*c^3*d^3*e + 4*B*a^5*c^2*d*e^3 + (C*a^4
*c^3 - 11*A*a^3*c^4)*d^4 + 6*(C*a^5*c^2 + A*a^4*c^3)*d^2*e^2 + (5*C*a^6*c +
 A*a^5*c^2)*e^4)*x)/(a^4*c^7*x^6 + 3*a^5*c^6*x^4 + 3*a^6*c^5*x^2 + a^7*c^4)
]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^4\,(A+Bx+Cx^2)}{(a+cx^2)^4}\,dx = \text{Timed out}$$

```
[In] integrate((e*x+d)**4*(C*x**2+B*x+A)/(c*x**2+a)**4,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 599 vs. $2(217) = 434$.

Time = 0.28 (sec) , antiderivative size = 599, normalized size of antiderivative = 2.56

$$\int \frac{(d+ex)^4\,(A+Bx+Cx^2)}{(a+cx^2)^4}\,dx = -\frac{8\,Ba^3c^2d^4 + 24\,Ba^4cd^2e^2 + 8\,Ba^5e^4 - 3\,(4\,Bac^4d^3e + 4\,Ba^2c^3de^3 + (Cac^4 + 5\,Ac^5)d^4 + 6\,(Ca^2c^3 + Aa}{} + \frac{(4\,Bac^2d^3e + 4\,Ba^2cde^3 + (Cac^2 + 5\,Ac^3)d^4 + 6\,(Ca^2c + Aac^2)d^2e^2 + (5\,Ca^3 + Aa^2c)e^4)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\,\sqrt{ac}a^3c^3}$$

```
[In] integrate((e*x+d)^4*(C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="maxima")
```

```
[Out] -1/48*(8*B*a^3*c^2*d^4 + 24*B*a^4*c*d^2*e^2 + 8*B*a^5*e^4 - 3*(4*B*a*c^4*d^
3*e + 4*B*a^2*c^3*d*e^3 + (C*a*c^4 + 5*A*c^5)*d^4 + 6*(C*a^2*c^3 + A*a*c^4)
*d^2*e^2 - (11*C*a^3*c^2 - A*a^2*c^3)*e^4)*x^5 + 16*(C*a^4*c + 2*A*a^3*c^2)
*d^3*e + 16*(2*C*a^5 + A*a^4*c)*d*e^3 + 24*(4*C*a^3*c^2*d*e^3 + B*a^3*c^2*e
```

```
^4)*x^4 - 8*(4*B*a^2*c^3*d^3*e - 4*B*a^3*c^2*d*e^3 + (C*a^2*c^3 + 5*A*a*c^4
)*d^4 - 6*(C*a^3*c^2 - A*a^2*c^3)*d^2*e^2 - (5*C*a^4*c + A*a^3*c^2)*e^4)*x^
3 + 24*(2*C*a^3*c^2*d^3*e + 3*B*a^3*c^2*d^2*e^2 + B*a^4*c*e^4 + 2*(2*C*a^4*
c + A*a^3*c^2)*d*e^3)*x^2 + 3*(4*B*a^3*c^2*d^3*e + 4*B*a^4*c*d*e^3 + (C*a^3
*c^2 - 11*A*a^2*c^3)*d^4 + 6*(C*a^4*c + A*a^3*c^2)*d^2*e^2 + (5*C*a^5 + A*a
^4*c)*e^4)*x)/(a^3*c^6*x^6 + 3*a^4*c^5*x^4 + 3*a^5*c^4*x^2 + a^6*c^3) + 1/1
6*(4*B*a*c^2*d^3*e + 4*B*a^2*c*d*e^3 + (C*a*c^2 + 5*A*c^3)*d^4 + 6*(C*a^2*c
 + A*a*c^2)*d^2*e^2 + (5*C*a^3 + A*a^2*c)*e^4)*arctan(c*x/sqrt(a*c))/(sqrt(
a*c)*a^3*c^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 659 vs. 2(217) = 434.

Time = 0.27 (sec) , antiderivative size = 659, normalized size of antiderivative = 2.82

$$\int \frac{(d+ex)^4\,(A+Bx+Cx^2)}{(a+cx^2)^4}\,dx$$

$$= \frac{(Cac^2d^4 + 5\,Ac^3d^4 + 4\,Bac^2d^3e + 6\,Ca^2cd^2e^2 + 6\,Aac^2d^2e^2 + 4\,Ba^2cde^3 + 5\,Ca^3e^4 + Aa^2ce^4)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\,\sqrt{ac}a^3c^3}$$

$$+ \frac{3\,Cac^4d^4x^5 + 15\,Ac^5d^4x^5 + 12\,Bac^4d^3ex^5 + 18\,Ca^2c^3d^2e^2x^5 + 18\,Aac^4d^2e^2x^5 + 12\,Ba^2c^3de^3x^5 - 33\,Ca^3}{}$$

```
[In] integrate((e*x+d)^4*(C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="giac")
```

```
[Out] 1/16*(C*a*c^2*d^4 + 5*A*c^3*d^4 + 4*B*a*c^2*d^3*e + 6*C*a^2*c*d^2*e^2 + 6*A
*a*c^2*d^2*e^2 + 4*B*a^2*c*d*e^3 + 5*C*a^3*e^4 + A*a^2*c*e^4)*arctan(c*x/sq
rt(a*c))/(sqrt(a*c)*a^3*c^3) + 1/48*(3*C*a*c^4*d^4*x^5 + 15*A*c^5*d^4*x^5 +
 12*B*a*c^4*d^3*e*x^5 + 18*C*a^2*c^3*d^2*e^2*x^5 + 18*A*a*c^4*d^2*e^2*x^5 +
 12*B*a^2*c^3*d*e^3*x^5 - 33*C*a^3*c^2*e^4*x^5 + 3*A*a^2*c^3*e^4*x^5 - 96*C
*a^3*c^2*d*e^3*x^4 - 24*B*a^3*c^2*e^4*x^4 + 8*C*a^2*c^3*d^4*x^3 + 40*A*a*c^
4*d^4*x^3 + 32*B*a^2*c^3*d^3*e*x^3 - 48*C*a^3*c^2*d^2*e^2*x^3 + 48*A*a^2*c^
3*d^2*e^2*x^3 - 32*B*a^3*c^2*d*e^3*x^3 - 40*C*a^4*c*e^4*x^3 - 8*A*a^3*c^2*e
^4*x^3 - 48*C*a^3*c^2*d^3*e*x^2 - 72*B*a^3*c^2*d^2*e^2*x^2 - 96*C*a^4*c*d*e
^3*x^2 - 48*A*a^3*c^2*d*e^3*x^2 - 24*B*a^4*c*e^4*x^2 - 3*C*a^3*c^2*d^4*x +
33*A*a^2*c^3*d^4*x - 12*B*a^3*c^2*d^3*e*x - 18*C*a^4*c*d^2*e^2*x - 18*A*a^3
*c^2*d^2*e^2*x - 12*B*a^4*c*d*e^3*x - 15*C*a^5*e^4*x - 3*A*a^4*c*e^4*x - 8*
B*a^3*c^2*d^4 - 16*C*a^4*c*d^3*e - 32*A*a^3*c^2*d^3*e - 24*B*a^4*c*d^2*e^2
- 32*C*a^5*d*e^3 - 16*A*a^4*c*d*e^3 - 8*B*a^5*e^4)/((c*x^2 + a)^3*a^3*c^3)
```

Mupad [B] (verification not implemented)

Time = 13.86 (sec) , antiderivative size = 669, normalized size of antiderivative = 2.86

$$\int \frac{(d+ex)^4 \left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^4}\,dx$$

$$= \frac{\operatorname{atan}\left(\frac{\sqrt{c}\,x\,\left(c\,d^2+a\,e^2\right)\,\left(5\,C\,a^2\,e^2+C\,a\,c\,d^2+4\,B\,a\,c\,d\,e+A\,a\,c\,e^2+5\,A\,c^2\,d^2\right)}{\sqrt{a}\,\left(5\,C\,a^3\,e^4+6\,C\,a^2\,c\,d^2\,e^2+4\,B\,a^2\,c\,d\,e^3+A\,a^2\,c\,e^4+C\,a\,c^2\,d^4+4\,B\,a\,c^2\,d^3\,e+6\,A\,a\,c^2\,d^2\,e^2+5\,A\,c^3\,d^4\right)}\right)\,\left(c\,d^2+a\,e^2\right)\,\left(5\,C\,a^2\,e\right.}{16\,a^{7/2}\,c^{7/2}}$$

$$- \frac{\frac{4\,C\,a^2\,d\,e^3+B\,a^2\,e^4+2\,C\,a\,c\,d^3\,e+3\,B\,a\,c\,d^2\,e^2+2\,A\,a\,c\,d\,e^3+B\,c^2\,d^4+4\,A\,c^2\,d^3\,e}{6\,c^3}+\frac{x^2\,\left(B\,a\,e^4+2\,A\,c\,d\,e^3+4\,C\,a\,d\,e^3+2\,C\,c\,d^3\,e+3\,B\,c\,d^2\,e^2\right)}{2\,c^2}}{}$$

```
[In] int(((d + e*x)^4*(A + B*x + C*x^2))/(a + c*x^2)^4,x)
```

```
[Out] (atan((c^(1/2)*x*(a*e^2 + c*d^2)*(5*A*c^2*d^2 + 5*C*a^2*e^2 + A*a*c*e^2 + C
*a*c*d^2 + 4*B*a*c*d*e))/(a^(1/2)*(5*A*c^3*d^4 + 5*C*a^3*e^4 + A*a^2*c*e^4
+ C*a*c^2*d^4 + 6*A*a*c^2*d^2*e^2 + 6*C*a^2*c*d^2*e^2 + 4*B*a*c^2*d^3*e + 4
*B*a^2*c*d*e^3)))*(a*e^2 + c*d^2)*(5*A*c^2*d^2 + 5*C*a^2*e^2 + A*a*c*e^2 +
C*a*c*d^2 + 4*B*a*c*d*e))/(16*a^(7/2)*c^(7/2)) - ((B*a^2*e^4 + B*c^2*d^4 +
4*A*c^2*d^3*e + 4*C*a^2*d*e^3 + 2*A*a*c*d*e^3 + 2*C*a*c*d^3*e + 3*B*a*c*d^2
*e^2)/(6*c^3) + (x^2*(B*a*e^4 + 2*A*c*d*e^3 + 4*C*a*d*e^3 + 2*C*c*d^3*e + 3
*B*c*d^2*e^2))/(2*c^2) + (x^4*(B*e^4 + 4*C*d*e^3))/(2*c) + (x*(5*C*a^3*e^4
- 11*A*c^3*d^4 + A*a^2*c*e^4 + C*a*c^2*d^4 + 6*A*a*c^2*d^2*e^2 + 6*C*a^2*c*
d^2*e^2 + 4*B*a*c^2*d^3*e + 4*B*a^2*c*d*e^3))/(16*a*c^3) - (x^3*(5*A*c^3*d^
4 - 5*C*a^3*e^4 - A*a^2*c*e^4 + C*a*c^2*d^4 + 6*A*a*c^2*d^2*e^2 - 6*C*a^2*c
*d^2*e^2 + 4*B*a*c^2*d^3*e - 4*B*a^2*c*d*e^3))/(6*a^2*c^2) - (x^5*(5*A*c^3*
d^4 - 11*C*a^3*e^4 + A*a^2*c*e^4 + C*a*c^2*d^4 + 6*A*a*c^2*d^2*e^2 + 6*C*a^
2*c*d^2*e^2 + 4*B*a*c^2*d^3*e + 4*B*a^2*c*d*e^3))/(16*a^3*c))/(a^3 + c^3*x^
6 + 3*a^2*c*x^2 + 3*a*c^2*x^4)
```

3.65 $\int \frac{(d+ex)^3(A+Bx+Cx^2)}{(a+cx^2)^4}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 254

$$
\begin{aligned}
&\int \frac{(d+ex)^3\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^4}\,dx\\
&=-\frac{(aB-(Ac-aC)x)(d+ex)^3}{6ac\left(a+cx^2\right)^3}-\frac{(d+ex)^2(2a(Ac+2aC)e-c(5Acd+aCd+3aBe)x)}{24a^2c^2\left(a+cx^2\right)^2}\\
&\quad-\frac{(d+ex)\left(ae(5Acd+aCd+3aBe)-\left(4a(Ac+2aC)e^2+3cd(5Acd+aCd+3aBe)\right)x\right)}{48a^3c^2\left(a+cx^2\right)}\\
&\quad+\frac{\left(Acd\left(5cd^2+3ae^2\right)+a\left(ae^2(3Cd+Be)+cd^2(Cd+3Be)\right)\right)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{16a^{7/2}c^{5/2}}
\end{aligned}
$$

```
[Out] -1/6*(a*B-(A*c-C*a)*x)*(e*x+d)^3/a/c/(c*x^2+a)^3-1/24*(e*x+d)^2*(2*a*(A*c+2
*C*a)*e-c*(5*A*c*d+3*B*a*e+C*a*d)*x)/a^2/c^2/(c*x^2+a)^2-1/48*(e*x+d)*(a*e*
(5*A*c*d+3*B*a*e+C*a*d)-(4*a*(A*c+2*C*a)*e^2+3*c*d*(5*A*c*d+3*B*a*e+C*a*d))
*x)/a^3/c^2/(c*x^2+a)+1/16*(A*c*d*(3*a*e^2+5*c*d^2)+a*(a*e^2*(B*e+3*C*d)+c*
d^2*(3*B*e+C*d)))*arctan(x*c^(1/2)/a^(1/2))/a^(7/2)/c^(5/2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used

$= \{1659, 835, 792, 211\}$

$$\int \frac{(d+ex)^3\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^4}\,dx$$

$$= \frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(Acd(3ae^2+5cd^2)+a(ae^2(Be+3Cd)+cd^2(3Be+Cd)))}{16a^{7/2}c^{5/2}} - \frac{4ae(Ac(ae^2+5cd^2)+a(2aCe^2+cd(3Be+Cd)))-cx(Acd(15cd^2-ae^2)+a(ae^2(7Cd-3Be)+3cd^2}{48a^3c^3\left(a+cx^2\right)} - \frac{(d+ex)^2(2ae(2aC+Ac)-cx(3aBe+aCd+5Acd))}{24a^2c^2\left(a+cx^2\right)^2} - \frac{(d+ex)^3(aB-x(Ac-aC))}{6ac\left(a+cx^2\right)^3}$$

```
[In] Int[((d + e*x)^3*(A + B*x + C*x^2))/(a + c*x^2)^4,x]
```

```
[Out] -1/6*((a*B - (A*c - a*C)*x)*(d + e*x)^3)/(a*c*(a + c*x^2)^3) - ((d + e*x)^2
*(2*a*(A*c + 2*a*C)*e - c*(5*A*c*d + a*C*d + 3*a*B*e)*x))/(24*a^2*c^2*(a +
c*x^2)^2) - (4*a*e*(A*c*(5*c*d^2 + a*e^2) + a*(2*a*C*e^2 + c*d*(C*d + 3*B*e
))) - c*(A*c*d*(15*c*d^2 - a*e^2) + a*(a*e^2*(7*C*d - 3*B*e) + 3*c*d^2*(C*d
 + 3*B*e)))*x)/(48*a^3*c^3*(a + c*x^2)) + ((A*c*d*(5*c*d^2 + 3*a*e^2) + a*(
a*e^2*(3*C*d + B*e) + c*d^2*(C*d + 3*B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(1
6*a^(7/2)*c^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(a*(e*f + d*g) - (c*d*f - a*e*g)*x)*((a + c*x^2)^(p + 1)/(
2*a*c*(p + 1))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c
*(p + 1))), x] - Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^
(p + 1)*Simp[a*e*g*m - c*d*f*(2*p + 3) - c*e*f*(m + 2*p + 3)*x, x], x], x]
/; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && G
tQ[m, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1659

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
```

```
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c*(p
+ 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(aB-(Ac-aC)x)(d+ex)^3}{6ac\left(a+cx^2\right)^3}-\frac{\int \frac{(d+ex)^2(-5Acd-aCd-3aBe-2(Ac+2aC)ex)}{\left(a+cx^2\right)^3}\,dx}{6ac}\\
&= -\frac{(aB-(Ac-aC)x)(d+ex)^3}{6ac\left(a+cx^2\right)^3}\\
&\quad -\frac{(d+ex)^2(2a(Ac+2aC)e-c(5Acd+aCd+3aBe)x)}{24a^2c^2\left(a+cx^2\right)^2}\\
&\quad -\frac{\int \frac{(d+ex)\left(-4a(Ac+2aC)e^2-3cd(5Acd+aCd+3aBe)-ce(5Acd+aCd+3aBe)x\right)}{\left(a+cx^2\right)^2}\,dx}{24a^2c^2}\\
&= -\frac{(aB-(Ac-aC)x)(d+ex)^3}{6ac\left(a+cx^2\right)^3}-\frac{(d+ex)^2(2a(Ac+2aC)e-c(5Acd+aCd+3aBe)x)}{24a^2c^2\left(a+cx^2\right)^2}\\
&\quad -\frac{4ae(Ac(5cd^2+ae^2)+a(2aCe^2+cd(Cd+3Be)))-c(Acd(15cd^2-ae^2)+a(ae^2(7Cd-3Be)+}{48a^3c^3\left(a+cx^2\right)}\\
&\quad +\frac{(Acd(5cd^2+3ae^2)+a(ae^2(3Cd+Be)+cd^2(Cd+3Be)))\int \frac{1}{a+cx^2}\,dx}{16a^3c^2}\\
&= -\frac{(aB-(Ac-aC)x)(d+ex)^3}{6ac\left(a+cx^2\right)^3}-\frac{(d+ex)^2(2a(Ac+2aC)e-c(5Acd+aCd+3aBe)x)}{24a^2c^2\left(a+cx^2\right)^2}\\
&\quad -\frac{4ae(Ac(5cd^2+ae^2)+a(2aCe^2+cd(Cd+3Be)))-c(Acd(15cd^2-ae^2)+a(ae^2(7Cd-3Be)+}{48a^3c^3\left(a+cx^2\right)}\\
&\quad +\frac{(Acd(5cd^2+3ae^2)+a(ae^2(3Cd+Be)+cd^2(Cd+3Be)))\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{16a^{7/2}c^{5/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.38

$$\int \frac{(d+ex)^3\,(A+Bx+Cx^2)}{(a+cx^2)^4}\,dx = \frac{-\frac{3\sqrt{a}(8a^3Ce^3-5Ac^3d^3x-a^2ce^2(3Cd+Be)x-ac^2d(Cd^2+3e(Bd+Ae))x)}{a+cx^2} - \frac{8a^{5/2}(a^3Ce^3-Ac^3d^3x+ac^2d(Cd^2x+3Ae(d+ex)+Bd(d+3ex))-}{(a+cx^2)^3}}{}$$

```
[In] Integrate[((d + e*x)^3*(A + B*x + C*x^2))/(a + c*x^2)^4,x]
```

```
[Out] ((-3*Sqrt[a]*(8*a^3*C*e^3 - 5*A*c^3*d^3*x - a^2*c*e^2*(3*C*d + B*e)*x - a*c
^2*d*(C*d^2 + 3*e*(B*d + A*e))*x))/(a + c*x^2) - (8*a^(5/2)*(a^3*C*e^3 - A*
c^3*d^3*x + a*c^2*d*(C*d^2*x + 3*A*e*(d + e*x) + B*d*(d + 3*e*x)) - a^2*c*e
*(3*C*d*(d + e*x) + e*(3*B*d + A*e + B*e*x))))/(a + c*x^2)^3 + (2*a^(3/2)*(
12*a^3*C*e^3 + 5*A*c^3*d^3*x + a*c^2*d*(C*d^2 + 3*e*(B*d + A*e))*x - a^2*c*
e*(3*C*d*(6*d + 7*e*x) + e*(18*B*d + 6*A*e + 7*B*e*x))))/(a + c*x^2)^2 + 3*
Sqrt[c]*(A*c*d*(5*c*d^2 + 3*a*e^2) + a*(a*e^2*(3*C*d + B*e) + c*d^2*(C*d +
3*B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(48*a^(7/2)*c^3)
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.51

method	result
default	$\frac{\frac{(3Aacde^2+5Ad^3c^2+a^2Be^3+3Bacd^2e+3Ca^2de^2+Cacd^3)x^5}{16a^3}-\frac{Ce^3x^4}{2c}+\frac{(3Aacde^2+5Ad^3c^2-a^2Be^3+3Bacd^2e-3Ca^2de^2+Cacd^3)x^3}{6ca^2}-}{}$
risch	$\frac{\frac{(3Aacde^2+5Ad^3c^2+a^2Be^3+3Bacd^2e+3Ca^2de^2+Cacd^3)x^5}{16a^3}-\frac{Ce^3x^4}{2c}+\frac{(3Aacde^2+5Ad^3c^2-a^2Be^3+3Bacd^2e-3Ca^2de^2+Cacd^3)x^3}{6ca^2}-}{}$

```
[In] int((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] (1/16*(3*A*a*c*d*e^2+5*A*c^2*d^3+B*a^2*e^3+3*B*a*c*d^2*e+3*C*a^2*d*e^2+C*a*
c*d^3)/a^3*x^5-1/2*C*e^3*x^4/c+1/6*(3*A*a*c*d*e^2+5*A*c^2*d^3-B*a^2*e^3+3*B
*a*c*d^2*e-3*C*a^2*d*e^2+C*a*c*d^3)/c/a^2*x^3-1/4*e*(A*c*e^2+3*B*c*d*e+2*C*
a*e^2+3*C*c*d^2)/c^2*x^2-1/16*(3*A*a*c*d*e^2-11*A*c^2*d^3+B*a^2*e^3+3*B*a*c
*d^2*e+3*C*a^2*d*e^2+C*a*c*d^3)/a/c^2*x-1/12*(A*a*c*e^3+6*A*c^2*d^2*e+3*B*a
*c*d*e^2+2*B*c^2*d^3+2*C*a^2*e^3+3*C*a*c*d^2*e)/c^3)/(c*x^2+a)^3+1/16*(3*A*
a*c*d*e^2+5*A*c^2*d^3+B*a^2*e^3+3*B*a*c*d^2*e+3*C*a^2*d*e^2+C*a*c*d^3)/a^3/
c^2/(a*c)^(1/2)*arctan(c*x/(a*c)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 679 vs. $2(237) = 474$.

Time = 0.47 (sec) , antiderivative size = 1378, normalized size of antiderivative = 5.43

$$\int \frac{(d+ex)^3 (A+Bx+Cx^2)}{(a+cx^2)^4}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="fricas")
```

```
[Out] [-1/96*(48*C*a^4*c^2*e^3*x^4 + 16*B*a^4*c^2*d^3 + 24*B*a^5*c*d*e^2 - 6*(3*B
*a^2*c^4*d^2*e + B*a^3*c^3*e^3 + (C*a^2*c^4 + 5*A*a*c^5)*d^3 + 3*(C*a^3*c^3
 + A*a^2*c^4)*d*e^2)*x^5 + 24*(C*a^5*c + 2*A*a^4*c^2)*d^2*e + 8*(2*C*a^6 +
A*a^5*c)*e^3 - 16*(3*B*a^3*c^3*d^2*e - B*a^4*c^2*e^3 + (C*a^3*c^3 + 5*A*a^2
*c^4)*d^3 - 3*(C*a^4*c^2 - A*a^3*c^3)*d*e^2)*x^3 + 24*(3*C*a^4*c^2*d^2*e +
3*B*a^4*c^2*d*e^2 + (2*C*a^5*c + A*a^4*c^2)*e^3)*x^2 + 3*(3*B*a^4*c*d^2*e +
 B*a^5*e^3 + (3*B*a*c^4*d^2*e + B*a^2*c^3*e^3 + (C*a*c^4 + 5*A*c^5)*d^3 + 3
*(C*a^2*c^3 + A*a*c^4)*d*e^2)*x^6 + 3*(3*B*a^2*c^3*d^2*e + B*a^3*c^2*e^3 +
(C*a^2*c^3 + 5*A*a*c^4)*d^3 + 3*(C*a^3*c^2 + A*a^2*c^3)*d*e^2)*x^4 + (C*a^4
*c + 5*A*a^3*c^2)*d^3 + 3*(C*a^5 + A*a^4*c)*d*e^2 + 3*(3*B*a^3*c^2*d^2*e +
B*a^4*c*e^3 + (C*a^3*c^2 + 5*A*a^2*c^3)*d^3 + 3*(C*a^4*c + A*a^3*c^2)*d*e^2
)*x^2)*sqrt(-a*c)*log((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) + 6*(3*B*a^
4*c^2*d^2*e + B*a^5*c*e^3 + (C*a^4*c^2 - 11*A*a^3*c^3)*d^3 + 3*(C*a^5*c + A
*a^4*c^2)*d*e^2)*x)/(a^4*c^6*x^6 + 3*a^5*c^5*x^4 + 3*a^6*c^4*x^2 + a^7*c^3)
, -1/48*(24*C*a^4*c^2*e^3*x^4 + 8*B*a^4*c^2*d^3 + 12*B*a^5*c*d*e^2 - 3*(3*B
*a^2*c^4*d^2*e + B*a^3*c^3*e^3 + (C*a^2*c^4 + 5*A*a*c^5)*d^3 + 3*(C*a^3*c^3
 + A*a^2*c^4)*d*e^2)*x^5 + 12*(C*a^5*c + 2*A*a^4*c^2)*d^2*e + 4*(2*C*a^6 +
A*a^5*c)*e^3 - 8*(3*B*a^3*c^3*d^2*e - B*a^4*c^2*e^3 + (C*a^3*c^3 + 5*A*a^2*
c^4)*d^3 - 3*(C*a^4*c^2 - A*a^3*c^3)*d*e^2)*x^3 + 12*(3*C*a^4*c^2*d^2*e + 3
*B*a^4*c^2*d*e^2 + (2*C*a^5*c + A*a^4*c^2)*e^3)*x^2 - 3*(3*B*a^4*c*d^2*e +
B*a^5*e^3 + (3*B*a*c^4*d^2*e + B*a^2*c^3*e^3 + (C*a*c^4 + 5*A*c^5)*d^3 + 3*
(C*a^2*c^3 + A*a*c^4)*d*e^2)*x^6 + 3*(3*B*a^2*c^3*d^2*e + B*a^3*c^2*e^3 + (
C*a^2*c^3 + 5*A*a*c^4)*d^3 + 3*(C*a^3*c^2 + A*a^2*c^3)*d*e^2)*x^4 + (C*a^4*
c + 5*A*a^3*c^2)*d^3 + 3*(C*a^5 + A*a^4*c)*d*e^2 + 3*(3*B*a^3*c^2*d^2*e + B
*a^4*c*e^3 + (C*a^3*c^2 + 5*A*a^2*c^3)*d^3 + 3*(C*a^4*c + A*a^3*c^2)*d*e^2)
*x^2)*sqrt(a*c)*arctan(sqrt(a*c)*x/a) + 3*(3*B*a^4*c^2*d^2*e + B*a^5*c*e^3
+ (C*a^4*c^2 - 11*A*a^3*c^3)*d^3 + 3*(C*a^5*c + A*a^4*c^2)*d*e^2)*x)/(a^4*c
^6*x^6 + 3*a^5*c^5*x^4 + 3*a^6*c^4*x^2 + a^7*c^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3 (A+Bx+Cx^2)}{(a+cx^2)^4}\,dx = \text{Timed out}$$

```
[In] integrate((e*x+d)**3*(C*x**2+B*x+A)/(c*x**2+a)**4,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 457, normalized size of antiderivative = 1.80

$$\int \frac{(d+ex)^3 (A+Bx+Cx^2)}{(a+cx^2)^4}\,dx =$$

$$-\frac{24\,Ca^3c^2e^3x^4 + 8\,Ba^3c^2d^3 + 12\,Ba^4cde^2 - 3\,(3\,Bac^4d^2e + Ba^2c^3e^3 + (Cac^4 + 5\,Ac^5)d^3 + 3\,(Ca^2c^3 + A}{}$$

$$+ \frac{(3\,Bacd^2e + Ba^2e^3 + (Cac + 5\,Ac^2)d^3 + 3\,(Ca^2 + Aac)de^2)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\,\sqrt{ac}a^3c^2}$$

```
[In] integrate((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="maxima")
```

```
[Out] -1/48*(24*C*a^3*c^2*e^3*x^4 + 8*B*a^3*c^2*d^3 + 12*B*a^4*c*d*e^2 - 3*(3*B*a
*c^4*d^2*e + B*a^2*c^3*e^3 + (C*a*c^4 + 5*A*c^5)*d^3 + 3*(C*a^2*c^3 + A*a*c
^4)*d*e^2)*x^5 + 12*(C*a^4*c + 2*A*a^3*c^2)*d^2*e + 4*(2*C*a^5 + A*a^4*c)*e
^3 - 8*(3*B*a^2*c^3*d^2*e - B*a^3*c^2*e^3 + (C*a^2*c^3 + 5*A*a*c^4)*d^3 - 3
*(C*a^3*c^2 - A*a^2*c^3)*d*e^2)*x^3 + 12*(3*C*a^3*c^2*d^2*e + 3*B*a^3*c^2*d
*e^2 + (2*C*a^4*c + A*a^3*c^2)*e^3)*x^2 + 3*(3*B*a^3*c^2*d^2*e + B*a^4*c*e^
3 + (C*a^3*c^2 - 11*A*a^2*c^3)*d^3 + 3*(C*a^4*c + A*a^3*c^2)*d*e^2)*x)/(a^3
*c^6*x^6 + 3*a^4*c^5*x^4 + 3*a^5*c^4*x^2 + a^6*c^3) + 1/16*(3*B*a*c*d^2*e +
 B*a^2*e^3 + (C*a*c + 5*A*c^2)*d^3 + 3*(C*a^2 + A*a*c)*d*e^2)*arctan(c*x/sq
rt(a*c))/(sqrt(a*c)*a^3*c^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 487 vs. $2(237) = 474$.

Time = 0.27 (sec) , antiderivative size = 487, normalized size of antiderivative = 1.92

$$\int \frac{(d+ex)^3\,(A+Bx+Cx^2)}{(a+cx^2)^4}\,dx$$
$$= \frac{(Cacd^3 + 5\,Ac^2d^3 + 3\,Bacd^2e + 3\,Ca^2de^2 + 3\,Aacde^2 + Ba^2e^3)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\,\sqrt{ac}a^3c^2}$$
$$+ \frac{3\,Cac^4d^3x^5 + 15\,Ac^5d^3x^5 + 9\,Bac^4d^2ex^5 + 9\,Ca^2c^3de^2x^5 + 9\,Aac^4de^2x^5 + 3\,Ba^2c^3e^3x^5 - 24\,Ca^3c^2e^3x^4 +}{}$$

```
[In] integrate((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="giac")
```

```
[Out] 1/16*(C*a*c*d^3 + 5*A*c^2*d^3 + 3*B*a*c*d^2*e + 3*C*a^2*d*e^2 + 3*A*a*c*d*e
^2 + B*a^2*e^3)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a^3*c^2) + 1/48*(3*C*a*c^4
*d^3*x^5 + 15*A*c^5*d^3*x^5 + 9*B*a*c^4*d^2*e*x^5 + 9*C*a^2*c^3*d*e^2*x^5 +
 9*A*a*c^4*d*e^2*x^5 + 3*B*a^2*c^3*e^3*x^5 - 24*C*a^3*c^2*e^3*x^4 + 8*C*a^2
*c^3*d^3*x^3 + 40*A*a*c^4*d^3*x^3 + 24*B*a^2*c^3*d^2*e*x^3 - 24*C*a^3*c^2*d
*e^2*x^3 + 24*A*a^2*c^3*d*e^2*x^3 - 8*B*a^3*c^2*e^3*x^3 - 36*C*a^3*c^2*d^2*
e*x^2 - 36*B*a^3*c^2*d*e^2*x^2 - 24*C*a^4*c*e^3*x^2 - 12*A*a^3*c^2*e^3*x^2
- 3*C*a^3*c^2*d^3*x + 33*A*a^2*c^3*d^3*x - 9*B*a^3*c^2*d^2*e*x - 9*C*a^4*c*
d*e^2*x - 9*A*a^3*c^2*d*e^2*x - 3*B*a^4*c*e^3*x - 8*B*a^3*c^2*d^3 - 12*C*a^
4*c*d^2*e - 24*A*a^3*c^2*d^2*e - 12*B*a^4*c*d*e^2 - 8*C*a^5*e^3 - 4*A*a^4*c
*e^3)/((c*x^2 + a)^3*a^3*c^3)
```

Mupad [B] (verification not implemented)

Time = 13.68 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.58

$$\int \frac{(d+ex)^3\,(A+Bx+Cx^2)}{(a+cx^2)^4}\,dx$$
$$= \frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\,(3\,C\,a^2\,d\,e^2 + B\,a^2\,e^3 + C\,a\,c\,d^3 + 3\,B\,a\,c\,d^2\,e + 3\,A\,a\,c\,d\,e^2 + 5\,A\,c^2\,d^3)}{16\,a^{7/2}\,c^{5/2}}$$
$$- \frac{\frac{2\,C\,a^2\,e^3+3\,C\,a\,c\,d^2\,e+3\,B\,a\,c\,d\,e^2+A\,a\,c\,e^3+2\,B\,c^2\,d^3+6\,A\,c^2\,d^2\,e}{12\,c^3} + \frac{x^2\,(A\,c\,e^3+2\,C\,a\,e^3+3\,B\,c\,d\,e^2+3\,C\,c\,d^2\,e)}{4\,c^2} - \frac{x^5\,(3\,C\,a^2\,d\,e^2+B\,a^2\,e^3+}{}}{}$$

```
[In] int(((d + e*x)^3*(A + B*x + C*x^2))/(a + c*x^2)^4,x)
```

```
[Out] (atan((c^(1/2)*x)/a^(1/2))*(5*A*c^2*d^3 + B*a^2*e^3 + C*a*c*d^3 + 3*C*a^2*d
*e^2 + 3*A*a*c*d*e^2 + 3*B*a*c*d^2*e))/(16*a^(7/2)*c^(5/2)) - ((2*B*c^2*d^3
 + 2*C*a^2*e^3 + A*a*c*e^3 + 6*A*c^2*d^2*e + 3*B*a*c*d*e^2 + 3*C*a*c*d^2*e)
```

```
/(12*c^3) + (x^2*(A*c*e^3 + 2*C*a*e^3 + 3*B*c*d*e^2 + 3*C*c*d^2*e))/(4*c^2)
 - (x^5*(5*A*c^2*d^3 + B*a^2*e^3 + C*a*c*d^3 + 3*C*a^2*d*e^2 + 3*A*a*c*d*e^
2 + 3*B*a*c*d^2*e))/(16*a^3) + (C*e^3*x^4)/(2*c) - (x^3*(5*A*c^2*d^3 - B*a^
2*e^3 + C*a*c*d^3 - 3*C*a^2*d*e^2 + 3*A*a*c*d*e^2 + 3*B*a*c*d^2*e))/(6*a^2*
c) + (x*(B*a^2*e^3 - 11*A*c^2*d^3 + C*a*c*d^3 + 3*C*a^2*d*e^2 + 3*A*a*c*d*e
^2 + 3*B*a*c*d^2*e))/(16*a*c^2))/(a^3 + c^3*x^6 + 3*a^2*c*x^2 + 3*a*c^2*x^4
)
```

3.66 $\int \frac{(d+ex)^2(A+Bx+Cx^2)}{(a+cx^2)^4}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 225

$$\begin{aligned}\int \frac{(d+ex)^2(A+Bx+Cx^2)}{(a+cx^2)^4}\,dx &= -\frac{(aB-(Ac-aC)x)(d+ex)^2}{6ac(a+cx^2)^3} \\ &\quad - \frac{2ae(4Acd+2aCd+aBe)+(3a(Ac+aC)e^2-cd(5Acd+aCd+2aBe))x}{24a^2c^2(a+cx^2)^2} \\ &\quad + \frac{(a(Ac+aC)e^2+cd(5Acd+aCd+2aBe))x}{16a^3c^2(a+cx^2)} \\ &\quad + \frac{(a(Ac+aC)e^2+cd(5Acd+aCd+2aBe))\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{16a^{7/2}c^{5/2}}\end{aligned}$$

```
[Out] -1/6*(a*B-(A*c-C*a)*x)*(e*x+d)^2/a/c/(c*x^2+a)^3+1/24*(-2*a*e*(4*A*c*d+B*a*
e+2*C*a*d)-(3*a*(A*c+C*a)*e^2-c*d*(5*A*c*d+2*B*a*e+C*a*d))*x)/a^2/c^2/(c*x^
2+a)^2+1/16*(a*(A*c+C*a)*e^2+c*d*(5*A*c*d+2*B*a*e+C*a*d))*x/a^3/c^2/(c*x^2+
a)+1/16*(a*(A*c+C*a)*e^2+c*d*(5*A*c*d+2*B*a*e+C*a*d))*arctan(x*c^(1/2)/a^(1
/2))/a^(7/2)/c^(5/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1659, 792, 205, 211}

$$\int \frac{(d+ex)^2 (A+Bx+Cx^2)}{(a+cx^2)^4} dx$$
$$= \frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right) (cd(2aBe+aCd+5Acd)+ae^2(aC+Ac))}{16a^{7/2}c^{5/2}} + \frac{x(cd(2aBe+aCd+5Acd)+ae^2(aC+Ac))}{16a^3c^2(a+cx^2)} - \frac{x(3ae^2(aC+Ac)-cd(2aBe+aCd+5Acd))+2ae(aBe+2aCd+4Acd)}{24a^2c^2(a+cx^2)^2} - \frac{(d+ex)^2(aB-x(Ac-aC))}{6ac(a+cx^2)^3}$$

```
[In] Int[((d + e*x)^2*(A + B*x + C*x^2))/(a + c*x^2)^4,x]
```

```
[Out] -1/6*((a*B - (A*c - a*C)*x)*(d + e*x)^2)/(a*c*(a + c*x^2)^3) - (2*a*e*(4*A*
c*d + 2*a*C*d + a*B*e) + (3*a*(A*c + a*C)*e^2 - c*d*(5*A*c*d + a*C*d + 2*a*
B*e))*x)/(24*a^2*c^2*(a + c*x^2)^2) + ((a*(A*c + a*C)*e^2 + c*d*(5*A*c*d +
a*C*d + 2*a*B*e))*x)/(16*a^3*c^2*(a + c*x^2)) + ((a*(A*c + a*C)*e^2 + c*d*(
5*A*c*d + a*C*d + 2*a*B*e))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(16*a^(7/2)*c^(5/2
))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(a*(e*f + d*g) - (c*d*f - a*e*g)*x)*((a + c*x^2)^(p + 1)/(
2*a*c*(p + 1))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 1659

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c*(p
+ 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(aB-(Ac-aC)x)(d+ex)^2}{6ac\left(a+cx^2\right)^3}-\frac{\int\frac{(d+ex)(-5Acd-aCd-2aBe-3(Ac+aC)ex)}{\left(a+cx^2\right)^3}\,dx}{6ac}\\
&= -\frac{(aB-(Ac-aC)x)(d+ex)^2}{6ac\left(a+cx^2\right)^3}\\
&\quad-\frac{2ae(4Acd+2aCd+aBe)+\left(3a(Ac+aC)e^2-cd(5Acd+aCd+2aBe)\right)x}{24a^2c^2\left(a+cx^2\right)^2}\\
&\quad+\frac{\left(a(Ac+aC)e^2+cd(5Acd+aCd+2aBe)\right)\int\frac{1}{\left(a+cx^2\right)^2}\,dx}{8a^2c^2}\\
&= -\frac{(aB-(Ac-aC)x)(d+ex)^2}{6ac\left(a+cx^2\right)^3}\\
&\quad-\frac{2ae(4Acd+2aCd+aBe)+\left(3a(Ac+aC)e^2-cd(5Acd+aCd+2aBe)\right)x}{24a^2c^2\left(a+cx^2\right)^2}\\
&\quad+\frac{\left(a(Ac+aC)e^2+cd(5Acd+aCd+2aBe)\right)x}{16a^3c^2\left(a+cx^2\right)}\\
&\quad+\frac{\left(a(Ac+aC)e^2+cd(5Acd+aCd+2aBe)\right)\int\frac{1}{a+cx^2}\,dx}{16a^3c^2}\\
&= -\frac{(aB-(Ac-aC)x)(d+ex)^2}{6ac\left(a+cx^2\right)^3}\\
&\quad-\frac{2ae(4Acd+2aCd+aBe)+\left(3a(Ac+aC)e^2-cd(5Acd+aCd+2aBe)\right)x}{24a^2c^2\left(a+cx^2\right)^2}\\
&\quad+\frac{\left(a(Ac+aC)e^2+cd(5Acd+aCd+2aBe)\right)x}{16a^3c^2\left(a+cx^2\right)}\\
&\quad+\frac{\left(a(Ac+aC)e^2+cd(5Acd+aCd+2aBe)\right)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{16a^{7/2}c^{5/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.18

$$\begin{aligned}
&\int \frac{(d+ex)^2\,(A+Bx+Cx^2)}{(a+cx^2)^4}\,dx \\
&= \frac{(Ac(5cd^2+ae^2)+a(aCe^2+cd(Cd+2Be)))\,x}{16a^3c^2\,(a+cx^2)} \\
&\quad + \frac{5Ac^2d^2x+ac(Cd^2+e(2Bd+Ae))\,x-a^2e(12Cd+6Be+7Cex)}{24a^2c^2\,(a+cx^2)^2} \\
&\quad + \frac{Ac^2d^2x+a^2e(2Cd+Be+Cex)-ac(Cd^2x+Ae(2d+ex)+Bd(d+2ex))}{6ac^2\,(a+cx^2)^3} \\
&\quad + \frac{(Ac(5cd^2+ae^2)+a(aCe^2+cd(Cd+2Be)))\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{16a^{7/2}c^{5/2}}
\end{aligned}$$

```
[In] Integrate[((d + e*x)^2*(A + B*x + C*x^2))/(a + c*x^2)^4,x]
```

```
[Out] ((A*c*(5*c*d^2 + a*e^2) + a*(a*C*e^2 + c*d*(C*d + 2*B*e)))*x)/(16*a^3*c^2*(
a + c*x^2)) + (5*A*c^2*d^2*x + a*c*(C*d^2 + e*(2*B*d + A*e))*x - a^2*e*(12*
C*d + 6*B*e + 7*C*e*x))/(24*a^2*c^2*(a + c*x^2)^2) + (A*c^2*d^2*x + a^2*e*(
2*C*d + B*e + C*e*x) - a*c*(C*d^2*x + A*e*(2*d + e*x) + B*d*(d + 2*e*x)))/(
6*a*c^2*(a + c*x^2)^3) + ((A*c*(5*c*d^2 + a*e^2) + a*(a*C*e^2 + c*d*(C*d +
2*B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(16*a^(7/2)*c^(5/2))
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.19

method	result
default	$\frac{\frac{\left(Aac\,e^2+5A\,c^2d^2+2Bacde+a^2C\,e^2+Cac\,d^2\right)x^5}{16a^3}+\frac{\left(Aac\,e^2+5A\,c^2d^2+2Bacde-a^2C\,e^2+Cac\,d^2\right)x^3}{6c\,a^2}-\frac{e(Be+2Cd)x^2}{4c}-\frac{(Aac\,e^2-11A\,c^2d^2+2Bacd}{16a\,c^2}}{(c\,x^2+a)^3}$
risch	$\frac{\frac{\left(Aac\,e^2+5A\,c^2d^2+2Bacde+a^2C\,e^2+Cac\,d^2\right)x^5}{16a^3}+\frac{\left(Aac\,e^2+5A\,c^2d^2+2Bacde-a^2C\,e^2+Cac\,d^2\right)x^3}{6c\,a^2}-\frac{e(Be+2Cd)x^2}{4c}-\frac{(Aac\,e^2-11A\,c^2d^2+2Bacd}{16a\,c^2}}{(c\,x^2+a)^3}$

```
[In] int((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] (1/16*(A*a*c*e^2+5*A*c^2*d^2+2*B*a*c*d*e+C*a^2*e^2+C*a*c*d^2)/a^3*x^5+1/6*(
A*a*c*e^2+5*A*c^2*d^2+2*B*a*c*d*e-C*a^2*e^2+C*a*c*d^2)/c/a^2*x^3-1/4*e*(B*e
+2*C*d)*x^2/c-1/16*(A*a*c*e^2-11*A*c^2*d^2+2*B*a*c*d*e+C*a^2*e^2+C*a*c*d^2)
/a/c^2*x-1/12*(4*A*c*d*e+B*a*e^2+2*B*c*d^2+2*C*a*d*e)/c^2)/(c*x^2+a)^3+1/16
*(A*a*c*e^2+5*A*c^2*d^2+2*B*a*c*d*e+C*a^2*e^2+C*a*c*d^2)/a^3/c^2/(a*c)^(1/2
)*arctan(c*x/(a*c)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 521 vs. $2(208) = 416$.

Time = 0.54 (sec) , antiderivative size = 1062, normalized size of antiderivative = 4.72

$$\int \frac{(d+ex)^2 (A+Bx+Cx^2)}{(a+cx^2)^4}\,dx$$

$$= \Bigg[-\frac{16\,Ba^4c^2d^2 + 8\,Ba^5ce^2 - 6\,(2\,Ba^2c^4de + (Ca^2c^4 + 5\,Aac^5)d^2 + (Ca^3c^3 + Aa^2c^4)e^2)x^5 - 16\,(2\,Ba^3c^3de +}{}$$

$$-\frac{8\,Ba^4c^2d^2 + 4\,Ba^5ce^2 - 3\,(2\,Ba^2c^4de + (Ca^2c^4 + 5\,Aac^5)d^2 + (Ca^3c^3 + Aa^2c^4)e^2)x^5 - 8\,(2\,Ba^3c^3de + (}{}$$

```
[In] integrate((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="fricas")
```

```
[Out] [-1/96*(16*B*a^4*c^2*d^2 + 8*B*a^5*c*e^2 - 6*(2*B*a^2*c^4*d*e + (C*a^2*c^4
+ 5*A*a*c^5)*d^2 + (C*a^3*c^3 + A*a^2*c^4)*e^2)*x^5 - 16*(2*B*a^3*c^3*d*e +
 (C*a^3*c^3 + 5*A*a^2*c^4)*d^2 - (C*a^4*c^2 - A*a^3*c^3)*e^2)*x^3 + 16*(C*a
^5*c + 2*A*a^4*c^2)*d*e + 24*(2*C*a^4*c^2*d*e + B*a^4*c^2*e^2)*x^2 + 3*(2*B
*a^4*c*d*e + (2*B*a*c^4*d*e + (C*a*c^4 + 5*A*c^5)*d^2 + (C*a^2*c^3 + A*a*c^
4)*e^2)*x^6 + 3*(2*B*a^2*c^3*d*e + (C*a^2*c^3 + 5*A*a*c^4)*d^2 + (C*a^3*c^2
 + A*a^2*c^3)*e^2)*x^4 + (C*a^4*c + 5*A*a^3*c^2)*d^2 + (C*a^5 + A*a^4*c)*e^
2 + 3*(2*B*a^3*c^2*d*e + (C*a^3*c^2 + 5*A*a^2*c^3)*d^2 + (C*a^4*c + A*a^3*c
^2)*e^2)*x^2)*sqrt(-a*c)*log((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) + 6*
(2*B*a^4*c^2*d*e + (C*a^4*c^2 - 11*A*a^3*c^3)*d^2 + (C*a^5*c + A*a^4*c^2)*e
^2)*x)/(a^4*c^6*x^6 + 3*a^5*c^5*x^4 + 3*a^6*c^4*x^2 + a^7*c^3), -1/48*(8*B*
a^4*c^2*d^2 + 4*B*a^5*c*e^2 - 3*(2*B*a^2*c^4*d*e + (C*a^2*c^4 + 5*A*a*c^5)*
d^2 + (C*a^3*c^3 + A*a^2*c^4)*e^2)*x^5 - 8*(2*B*a^3*c^3*d*e + (C*a^3*c^3 +
5*A*a^2*c^4)*d^2 - (C*a^4*c^2 - A*a^3*c^3)*e^2)*x^3 + 8*(C*a^5*c + 2*A*a^4*
c^2)*d*e + 12*(2*C*a^4*c^2*d*e + B*a^4*c^2*e^2)*x^2 - 3*(2*B*a^4*c*d*e + (2
*B*a*c^4*d*e + (C*a*c^4 + 5*A*c^5)*d^2 + (C*a^2*c^3 + A*a*c^4)*e^2)*x^6 + 3
*(2*B*a^2*c^3*d*e + (C*a^2*c^3 + 5*A*a*c^4)*d^2 + (C*a^3*c^2 + A*a^2*c^3)*e
^2)*x^4 + (C*a^4*c + 5*A*a^3*c^2)*d^2 + (C*a^5 + A*a^4*c)*e^2 + 3*(2*B*a^3*
c^2*d*e + (C*a^3*c^2 + 5*A*a^2*c^3)*d^2 + (C*a^4*c + A*a^3*c^2)*e^2)*x^2)*s
qrt(a*c)*arctan(sqrt(a*c)*x/a) + 3*(2*B*a^4*c^2*d*e + (C*a^4*c^2 - 11*A*a^3
*c^3)*d^2 + (C*a^5*c + A*a^4*c^2)*e^2)*x)/(a^4*c^6*x^6 + 3*a^5*c^5*x^4 + 3*
a^6*c^4*x^2 + a^7*c^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2 \left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^4}\,dx = \text{Timed out}$$

```
[In] integrate((e*x+d)**2*(C*x**2+B*x+A)/(c*x**2+a)**4,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.44

$$\int \frac{(d+ex)^2 \left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^4}\,dx = -\frac{8\,Ba^3cd^2 + 4\,Ba^4e^2 - 3\,(2\,Bac^3de + (Cac^3 + 5\,Ac^4)d^2 + (Ca^2c^2 + Aac^3)e^2)x^5 - 8\,(2\,Ba^2c^2de + (Ca^2c}{} + \frac{(2\,Bacde + (Cac + 5\,Ac^2)d^2 + (Ca^2 + Aac)e^2)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\,\sqrt{ac}a^3c^2}$$

```
[In] integrate((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="maxima")
```

```
[Out] -1/48*(8*B*a^3*c*d^2 + 4*B*a^4*e^2 - 3*(2*B*a*c^3*d*e + (C*a*c^3 + 5*A*c^4)
*d^2 + (C*a^2*c^2 + A*a*c^3)*e^2)*x^5 - 8*(2*B*a^2*c^2*d*e + (C*a^2*c^2 + 5
*A*a*c^3)*d^2 - (C*a^3*c - A*a^2*c^2)*e^2)*x^3 + 8*(C*a^4 + 2*A*a^3*c)*d*e
+ 12*(2*C*a^3*c*d*e + B*a^3*c*e^2)*x^2 + 3*(2*B*a^3*c*d*e + (C*a^3*c - 11*A
*a^2*c^2)*d^2 + (C*a^4 + A*a^3*c)*e^2)*x)/(a^3*c^5*x^6 + 3*a^4*c^4*x^4 + 3*
a^5*c^3*x^2 + a^6*c^2) + 1/16*(2*B*a*c*d*e + (C*a*c + 5*A*c^2)*d^2 + (C*a^2
 + A*a*c)*e^2)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a^3*c^2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.47

$$\int \frac{(d+ex)^2 \left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^4}\,dx = \frac{(Cacd^2 + 5\,Ac^2d^2 + 2\,Bacde + Ca^2e^2 + Aace^2)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\,\sqrt{ac}a^3c^2} + \frac{3\,Cac^3d^2x^5 + 15\,Ac^4d^2x^5 + 6\,Bac^3dex^5 + 3\,Ca^2c^2e^2x^5 + 3\,Aac^3e^2x^5 + 8\,Ca^2c^2d^2x^3 + 40\,Aac^3d^2x^3 + 1}{}$$

```
[In] integrate((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="giac")
```

```
[Out] 1/16*(C*a*c*d^2 + 5*A*c^2*d^2 + 2*B*a*c*d*e + C*a^2*e^2 + A*a*c*e^2)*arctan
(c*x/sqrt(a*c))/(sqrt(a*c)*a^3*c^2) + 1/48*(3*C*a*c^3*d^2*x^5 + 15*A*c^4*d^
2*x^5 + 6*B*a*c^3*d*e*x^5 + 3*C*a^2*c^2*e^2*x^5 + 3*A*a*c^3*e^2*x^5 + 8*C*a
^2*c^2*d^2*x^3 + 40*A*a*c^3*d^2*x^3 + 16*B*a^2*c^2*d*e*x^3 - 8*C*a^3*c*e^2*
x^3 + 8*A*a^2*c^2*e^2*x^3 - 24*C*a^3*c*d*e*x^2 - 12*B*a^3*c*e^2*x^2 - 3*C*a
^3*c*d^2*x + 33*A*a^2*c^2*d^2*x - 6*B*a^3*c*d*e*x - 3*C*a^4*e^2*x - 3*A*a^3
*c*e^2*x - 8*B*a^3*c*d^2 - 8*C*a^4*d*e - 16*A*a^3*c*d*e - 4*B*a^4*e^2)/((c*
x^2 + a)^3*a^3*c^2)
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.28

$$\int \frac{(d+ex)^2\,(A+Bx+Cx^2)}{(a+cx^2)^4}\,dx$$
$$=\frac{\operatorname{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\,(C\,a^2\,e^2+C\,a\,c\,d^2+2\,B\,a\,c\,d\,e+A\,a\,c\,e^2+5\,A\,c^2\,d^2)}{16\,a^{7/2}\,c^{5/2}}$$
$$-\frac{\frac{B\,a\,e^2+2\,B\,c\,d^2+4\,A\,c\,d\,e+2\,C\,a\,d\,e}{12\,c^2}-\frac{x^5\,(C\,a^2\,e^2+C\,a\,c\,d^2+2\,B\,a\,c\,d\,e+A\,a\,c\,e^2+5\,A\,c^2\,d^2)}{16\,a^3}+\frac{x^2\,(B\,e^2+2\,C\,d\,e)}{4\,c}+\frac{x\,(C\,a^2\,e^2+C\,a\,c\,d^2+2}{}}{a^3+3\,a^2\,c\,x^2+3\,a\,c^2\,x^4+c^3\,x^6}$$

```
[In] int(((d + e*x)^2*(A + B*x + C*x^2))/(a + c*x^2)^4,x)
```

```
[Out] (atan((c^(1/2)*x)/a^(1/2))*(5*A*c^2*d^2 + C*a^2*e^2 + A*a*c*e^2 + C*a*c*d^2
 + 2*B*a*c*d*e))/(16*a^(7/2)*c^(5/2)) - ((B*a*e^2 + 2*B*c*d^2 + 4*A*c*d*e +
 2*C*a*d*e)/(12*c^2) - (x^5*(5*A*c^2*d^2 + C*a^2*e^2 + A*a*c*e^2 + C*a*c*d^
2 + 2*B*a*c*d*e))/(16*a^3) + (x^2*(B*e^2 + 2*C*d*e))/(4*c) + (x*(C*a^2*e^2
- 11*A*c^2*d^2 + A*a*c*e^2 + C*a*c*d^2 + 2*B*a*c*d*e))/(16*a*c^2) - (x^3*(5
*A*c^2*d^2 - C*a^2*e^2 + A*a*c*e^2 + C*a*c*d^2 + 2*B*a*c*d*e))/(6*a^2*c))/(
a^3 + c^3*x^6 + 3*a^2*c*x^2 + 3*a*c^2*x^4)
```

3.67 $\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^4} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 165

$$\begin{aligned}\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^4} \, dx = & -\frac{(aB-(Ac-aC)x)(d+ex)}{6ac\left(a+cx^2\right)^3} \\ & -\frac{2a(2Ac+aC)e-c(5Acd+aCd+aBe)x}{24a^2c^2\left(a+cx^2\right)^2} \\ & +\frac{(5Acd+aCd+aBe)x}{16a^3c\left(a+cx^2\right)} \\ & +\frac{(5Acd+aCd+aBe)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{16a^{7/2}c^{3/2}}\end{aligned}$$

```
[Out] -1/6*(a*B-(A*c-C*a)*x)*(e*x+d)/a/c/(c*x^2+a)^3+1/24*(-2*a*(2*A*c+C*a)*e+c*(
5*A*c*d+B*a*e+C*a*d)*x)/a^2/c^2/(c*x^2+a)^2+1/16*(5*A*c*d+B*a*e+C*a*d)*x/a^
3/c/(c*x^2+a)+1/16*(5*A*c*d+B*a*e+C*a*d)*arctan(x*c^(1/2)/a^(1/2))/a^(7/2)/
c^(3/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used

$= \{1659, 653, 205, 211\}$

$$\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^4}\,dx = \frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(aBe+aCd+5Acd)}{16a^{7/2}c^{3/2}} + \frac{x(aBe+aCd+5Acd)}{16a^3c\left(a+cx^2\right)} - \frac{2ae(aC+2Ac)-cx(aBe+aCd+5Acd)}{24a^2c^2\left(a+cx^2\right)^2} - \frac{(d+ex)(aB-x(Ac-aC))}{6ac\left(a+cx^2\right)^3}$$

```
[In] Int[((d + e*x)*(A + B*x + C*x^2))/(a + c*x^2)^4,x]
```

```
[Out] -1/6*((a*B - (A*c - a*C)*x)*(d + e*x))/(a*c*(a + c*x^2)^3) - (2*a*(2*A*c +
a*C)*e - c*(5*A*c*d + a*C*d + a*B*e)*x)/(24*a^2*c^2*(a + c*x^2)^2) + ((5*A*
c*d + a*C*d + a*B*e)*x)/(16*a^3*c*(a + c*x^2)) + ((5*A*c*d + a*C*d + a*B*e)
*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(16*a^(7/2)*c^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)/(2*a*c*(p + 1)))*(a + c*x^2)^(p + 1), x] + Dist[d*((2*p + 3)/(2*a
*(p + 1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 1659

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c*(p
+ 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
```

```
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(aB-(Ac-aC)x)(d+ex)}{6ac\left(a+cx^2\right)^3}-\frac{\int\frac{-5Acd-a(Cd+Be)-2(2Ac+aC)ex}{\left(a+cx^2\right)^3}\,dx}{6ac}\\
&= -\frac{(aB-(Ac-aC)x)(d+ex)}{6ac\left(a+cx^2\right)^3}-\frac{2a(2Ac+aC)e-c(5Acd+aCd+aBe)x}{24a^2c^2\left(a+cx^2\right)^2}\\
&\quad+\frac{(5Acd+aCd+aBe)\int\frac{1}{\left(a+cx^2\right)^2}\,dx}{8a^2c}\\
&= -\frac{(aB-(Ac-aC)x)(d+ex)}{6ac\left(a+cx^2\right)^3}-\frac{2a(2Ac+aC)e-c(5Acd+aCd+aBe)x}{24a^2c^2\left(a+cx^2\right)^2}\\
&\quad+\frac{(5Acd+aCd+aBe)x}{16a^3c\left(a+cx^2\right)}+\frac{(5Acd+aCd+aBe)\int\frac{1}{a+cx^2}\,dx}{16a^3c}\\
&= -\frac{(aB-(Ac-aC)x)(d+ex)}{6ac\left(a+cx^2\right)^3}-\frac{2a(2Ac+aC)e-c(5Acd+aCd+aBe)x}{24a^2c^2\left(a+cx^2\right)^2}\\
&\quad+\frac{(5Acd+aCd+aBe)x}{16a^3c\left(a+cx^2\right)}+\frac{(5Acd+aCd+aBe)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{16a^{7/2}c^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.04

$$\int\frac{(d+ex)\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^4}\,dx$$

$$=\frac{\frac{2a^{3/2}\left(-6a^2Ce+5Ac^2dx+ac(Cd+Be)x\right)}{\left(a+cx^2\right)^2}+\frac{3\sqrt{a}c(5Acd+aCd+aBe)x}{a+cx^2}+\frac{8a^{5/2}\left(a^2Ce+Ac^2dx-ac(Ae+Cdx+B(d+ex))\right)}{\left(a+cx^2\right)^3}+3\sqrt{c}(5Acd+}{48a^{7/2}c^2}$$

```
[In] Integrate[((d + e*x)*(A + B*x + C*x^2))/(a + c*x^2)^4,x]
```

```
[Out] ((2*a^(3/2)*(-6*a^2*C*e + 5*A*c^2*d*x + a*c*(C*d + B*e)*x))/(a + c*x^2)^2 +
 (3*Sqrt[a]*c*(5*A*c*d + a*C*d + a*B*e)*x)/(a + c*x^2) + (8*a^(5/2)*(a^2*C*
e + A*c^2*d*x - a*c*(A*e + C*d*x + B*(d + e*x))))/(a + c*x^2)^3 + 3*Sqrt[c]
*(5*A*c*d + a*C*d + a*B*e)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(48*a^(7/2)*c^2)
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.90

method	result
default	$\frac{\frac{(5Acd+Bae+Cad)c\,x^5}{16a^3}+\frac{(5Acd+Bae+Cad)x^3}{6a^2}-\frac{Ce\,x^2}{4c}+\frac{(11Acd-Bae-Cad)x}{16ac}-\frac{2Ace+2Bcd+Cae}{12c^2}}{(c\,x^2+a)^3}+\frac{(5Acd+Bae+Cad)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16a^3c\sqrt{ac}}$
risch	$\frac{\frac{(5Acd+Bae+Cad)c\,x^5}{16a^3}+\frac{(5Acd+Bae+Cad)x^3}{6a^2}-\frac{Ce\,x^2}{4c}+\frac{(11Acd-Bae-Cad)x}{16ac}-\frac{2Ace+2Bcd+Cae}{12c^2}}{(c\,x^2+a)^3}-\frac{5\ln\left(cx+\sqrt{-ac}\right)Ad}{32\sqrt{-ac}\,a^3}-\frac{\ln\left(cx+\sqrt{-ac}\right)}{32\sqrt{-ac}\,c\,a^2}$

```
[In] int((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] (1/16*(5*A*c*d+B*a*e+C*a*d)/a^3*c*x^5+1/6/a^2*(5*A*c*d+B*a*e+C*a*d)*x^3-1/4
*C*e*x^2/c+1/16*(11*A*c*d-B*a*e-C*a*d)/a/c*x-1/12*(2*A*c*e+2*B*c*d+C*a*e)/c
^2)/(c*x^2+a)^3+1/16*(5*A*c*d+B*a*e+C*a*d)/a^3/c/(a*c)^(1/2)*arctan(c*x/(a*
c)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 308 vs. 2(148) = 296.

Time = 0.40 (sec) , antiderivative size = 636, normalized size of antiderivative = 3.85

$$\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^4}\,dx$$

$$=\left[-\frac{24\,Ca^4cex^2+16\,Ba^4cd-6\,(Ba^2c^3e+(Ca^2c^3+5\,Aac^4)d)x^5-16\,(Ba^3c^2e+(Ca^3c^2+5\,Aa^2c^3)d)x^3+}{}\right.$$

$$-\frac{12\,Ca^4cex^2+8\,Ba^4cd-3\,(Ba^2c^3e+(Ca^2c^3+5\,Aac^4)d)x^5-8\,(Ba^3c^2e+(Ca^3c^2+5\,Aa^2c^3)d)x^3-3\,((}{}$$

```
[In] integrate((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="fricas")
```

```
[Out] [-1/96*(24*C*a^4*c*e*x^2 + 16*B*a^4*c*d - 6*(B*a^2*c^3*e + (C*a^2*c^3 + 5*A
*a*c^4)*d)*x^5 - 16*(B*a^3*c^2*e + (C*a^3*c^2 + 5*A*a^2*c^3)*d)*x^3 + 3*((B
*a*c^3*e + (C*a*c^3 + 5*A*c^4)*d)*x^6 + B*a^4*e + 3*(B*a^2*c^2*e + (C*a^2*c
^2 + 5*A*a*c^3)*d)*x^4 + 3*(B*a^3*c*e + (C*a^3*c + 5*A*a^2*c^2)*d)*x^2 + (C
*a^4 + 5*A*a^3*c)*d)*sqrt(-a*c)*log((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a
)) + 8*(C*a^5 + 2*A*a^4*c)*e + 6*(B*a^4*c*e + (C*a^4*c - 11*A*a^3*c^2)*d)*x
)/(a^4*c^5*x^6 + 3*a^5*c^4*x^4 + 3*a^6*c^3*x^2 + a^7*c^2), -1/48*(12*C*a^4*
c*e*x^2 + 8*B*a^4*c*d - 3*(B*a^2*c^3*e + (C*a^2*c^3 + 5*A*a*c^4)*d)*x^5 - 8
*(B*a^3*c^2*e + (C*a^3*c^2 + 5*A*a^2*c^3)*d)*x^3 - 3*((B*a*c^3*e + (C*a*c^3
 + 5*A*c^4)*d)*x^6 + B*a^4*e + 3*(B*a^2*c^2*e + (C*a^2*c^2 + 5*A*a*c^3)*d)*
x^4 + 3*(B*a^3*c*e + (C*a^3*c + 5*A*a^2*c^2)*d)*x^2 + (C*a^4 + 5*A*a^3*c)*d
```

```
)*sqrt(a*c)*arctan(sqrt(a*c)*x/a) + 4*(C*a^5 + 2*A*a^4*c)*e + 3*(B*a^4*c*e
+ (C*a^4*c - 11*A*a^3*c^2)*d)*x)/(a^4*c^5*x^6 + 3*a^5*c^4*x^4 + 3*a^6*c^3*x
^2 + a^7*c^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^4}\,dx = \text{Timed out}$$

```
[In] integrate((e*x+d)*(C*x**2+B*x+A)/(c*x**2+a)**4,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.26

$$\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^4}\,dx = -\frac{12\,Ca^3cex^2 + 8\,Ba^3cd - 3\,(Bac^3e + (Cac^3 + 5\,Ac^4)d)x^5 - 8\,(Ba^2c^2e + (Ca^2c^2 + 5\,Aac^3)d)x^3 + 4\,(Ca}{48\,(a^3c^5x^6 + 3\,a^4c^4x^4 + 3\,a^5c^3x^2 + a^6c^2)} + \frac{(Bae + (Ca + 5\,Ac)d)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\,\sqrt{ac}a^3c}$$

```
[In] integrate((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="maxima")
```

```
[Out] -1/48*(12*C*a^3*c*e*x^2 + 8*B*a^3*c*d - 3*(B*a*c^3*e + (C*a*c^3 + 5*A*c^4)*
d)*x^5 - 8*(B*a^2*c^2*e + (C*a^2*c^2 + 5*A*a*c^3)*d)*x^3 + 4*(C*a^4 + 2*A*a
^3*c)*e + 3*(B*a^3*c*e + (C*a^3*c - 11*A*a^2*c^2)*d)*x)/(a^3*c^5*x^6 + 3*a^
4*c^4*x^4 + 3*a^5*c^3*x^2 + a^6*c^2) + 1/16*(B*a*e + (C*a + 5*A*c)*d)*arcta
n(c*x/sqrt(a*c))/(sqrt(a*c)*a^3*c)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.13

$$\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^4}\,dx = \frac{(Cad+5\,Acd+Bae)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\,\sqrt{ac}a^3c} + \frac{3\,Cac^3dx^5+15\,Ac^4dx^5+3\,Bac^3ex^5+8\,Ca^2c^2dx^3+40\,Aac^3dx^3+8\,Ba^2c^2ex^3-12\,Ca^3cex^2-3\,Ca^3cdx}{48\left(cx^2+a\right)^3a^3c^2}$$

```
[In] integrate((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="giac")
```

```
[Out] 1/16*(C*a*d + 5*A*c*d + B*a*e)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a^3*c) + 1/
48*(3*C*a*c^3*d*x^5 + 15*A*c^4*d*x^5 + 3*B*a*c^3*e*x^5 + 8*C*a^2*c^2*d*x^3
+ 40*A*a*c^3*d*x^3 + 8*B*a^2*c^2*e*x^3 - 12*C*a^3*c*e*x^2 - 3*C*a^3*c*d*x +
 33*A*a^2*c^2*d*x - 3*B*a^3*c*e*x - 8*B*a^3*c*d - 4*C*a^4*e - 8*A*a^3*c*e)/
((c*x^2 + a)^3*a^3*c^2)
```

Mupad [B] (verification not implemented)

Time = 13.00 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.99

$$\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^4}\,dx = \frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\,\left(5\,A\,c\,d+B\,a\,e+C\,a\,d\right)}{16\,a^{7/2}\,c^{3/2}} - \frac{\frac{2\,A\,c\,e+2\,B\,c\,d+C\,a\,e}{12\,c^2}-\frac{x^3\,\left(5\,A\,c\,d+B\,a\,e+C\,a\,d\right)}{6\,a^2}+\frac{C\,e\,x^2}{4\,c}+\frac{x\,\left(B\,a\,e-11\,A\,c\,d+C\,a\,d\right)}{16\,a\,c}-\frac{c\,x^5\,\left(5\,A\,c\,d+B\,a\,e+C\,a\,d\right)}{16\,a^3}}{a^3+3\,a^2\,c\,x^2+3\,a\,c^2\,x^4+c^3\,x^6}$$

```
[In] int(((d + e*x)*(A + B*x + C*x^2))/(a + c*x^2)^4,x)
```

```
[Out] (atan((c^(1/2)*x)/a^(1/2))*(5*A*c*d + B*a*e + C*a*d))/(16*a^(7/2)*c^(3/2))
- ((2*A*c*e + 2*B*c*d + C*a*e)/(12*c^2) - (x^3*(5*A*c*d + B*a*e + C*a*d))/(
6*a^2) + (C*e*x^2)/(4*c) + (x*(B*a*e - 11*A*c*d + C*a*d))/(16*a*c) - (c*x^5
*(5*A*c*d + B*a*e + C*a*d))/(16*a^3))/(a^3 + c^3*x^6 + 3*a^2*c*x^2 + 3*a*c^
2*x^4)
```

3.68 $\int \frac{A+Bx+Cx^2}{(a+cx^2)^4} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 126

$$\int \frac{A+Bx+Cx^2}{(a+cx^2)^4} \, dx = -\frac{aB-(Ac-aC)x}{6ac(a+cx^2)^3} + \frac{(5Ac+aC)x}{24a^2c(a+cx^2)^2} + \frac{(5Ac+aC)x}{16a^3c(a+cx^2)} + \frac{(5Ac+aC)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{16a^{7/2}c^{3/2}}$$

```
[Out] 1/6*(-a*B+(A*c-C*a)*x)/a/c/(c*x^2+a)^3+1/24*(5*A*c+C*a)*x/a^2/c/(c*x^2+a)^2
+1/16*(5*A*c+C*a)*x/a^3/c/(c*x^2+a)+1/16*(5*A*c+C*a)*arctan(x*c^(1/2)/a^(1/
2))/a^(7/2)/c^(3/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1828, 12, 205, 211}

$$\int \frac{A+Bx+Cx^2}{(a+cx^2)^4} \, dx = \frac{(aC+5Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{16a^{7/2}c^{3/2}} + \frac{x(aC+5Ac)}{16a^3c(a+cx^2)} + \frac{x(aC+5Ac)}{24a^2c(a+cx^2)^2} - \frac{aB-x(Ac-aC)}{6ac(a+cx^2)^3}$$

```
[In] Int[(A + B*x + C*x^2)/(a + c*x^2)^4,x]
```

```
[Out] -1/6*(a*B - (A*c - a*C)*x)/(a*c*(a + c*x^2)^3) + ((5*A*c + a*C)*x)/(24*a^2*
c*(a + c*x^2)^2) + ((5*A*c + a*C)*x)/(16*a^3*c*(a + c*x^2)) + ((5*A*c + a*C
)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(16*a^(7/2)*c^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{aB-(Ac-aC)x}{6ac\left(a+cx^2\right)^3}-\frac{\int \frac{-5A-\frac{aC}{c}}{\left(a+cx^2\right)^3}\,dx}{6a}\\
&= -\frac{aB-(Ac-aC)x}{6ac\left(a+cx^2\right)^3}+\frac{(5Ac+aC)\int \frac{1}{\left(a+cx^2\right)^3}\,dx}{6ac}\\
&= -\frac{aB-(Ac-aC)x}{6ac\left(a+cx^2\right)^3}+\frac{(5Ac+aC)x}{24a^2c\left(a+cx^2\right)^2}+\frac{(5Ac+aC)\int \frac{1}{\left(a+cx^2\right)^2}\,dx}{8a^2c}\\
&= -\frac{aB-(Ac-aC)x}{6ac\left(a+cx^2\right)^3}+\frac{(5Ac+aC)x}{24a^2c\left(a+cx^2\right)^2}+\frac{(5Ac+aC)x}{16a^3c\left(a+cx^2\right)}+\frac{(5Ac+aC)\int \frac{1}{a+cx^2}\,dx}{16a^3c}\\
&= -\frac{aB-(Ac-aC)x}{6ac\left(a+cx^2\right)^3}+\frac{(5Ac+aC)x}{24a^2c\left(a+cx^2\right)^2}+\frac{(5Ac+aC)x}{16a^3c\left(a+cx^2\right)}+\frac{(5Ac+aC)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{16a^{7/2}c^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.89

$$\int \frac{A+Bx+Cx^2}{(a+cx^2)^4}\,dx = \frac{15Ac^3x^5-a^3(8B+3Cx)+ac^2x^3(40A+3Cx^2)+a^2cx(33A+8Cx^2)}{48a^3c\,(a+cx^2)^3} + \frac{(5Ac+aC)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{16a^{7/2}c^{3/2}}$$

```
[In] Integrate[(A + B*x + C*x^2)/(a + c*x^2)^4,x]
```

```
[Out] (15*A*c^3*x^5 - a^3*(8*B + 3*C*x) + a*c^2*x^3*(40*A + 3*C*x^2) + a^2*c*x*(3
3*A + 8*C*x^2))/(48*a^3*c*(a + c*x^2)^3) + ((5*A*c + a*C)*ArcTan[(Sqrt[c]*x
)/Sqrt[a]])/(16*a^(7/2)*c^(3/2))
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.79

method	result
default	$\frac{\frac{(5Ac+Ca)c\,x^5}{16a^3}+\frac{(5Ac+Ca)x^3}{6a^2}+\frac{(11Ac-Ca)x}{16ac}-\frac{B}{6c}}{(c\,x^2+a)^3}+\frac{(5Ac+Ca)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16a^3c\sqrt{ac}}$
risch	$\frac{\frac{(5Ac+Ca)c\,x^5}{16a^3}+\frac{(5Ac+Ca)x^3}{6a^2}+\frac{(11Ac-Ca)x}{16ac}-\frac{B}{6c}}{(c\,x^2+a)^3}-\frac{5\ln\left(cx+\sqrt{-ac}\right)A}{32\sqrt{-ac}\,a^3}-\frac{\ln\left(cx+\sqrt{-ac}\right)C}{32\sqrt{-ac}\,c\,a^2}+\frac{5\ln\left(-cx+\sqrt{-ac}\right)A}{32\sqrt{-ac}\,a^3}+\frac{\ln\left(-cx+\sqrt{-}\right.}{32\sqrt{-ac}\,c}$

```
[In] int((C*x^2+B*x+A)/(c*x^2+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] (1/16*(5*A*c+C*a)/a^3*c*x^5+1/6/a^2*(5*A*c+C*a)*x^3+1/16*(11*A*c-C*a)/a/c*x
-1/6*B/c)/(c*x^2+a)^3+1/16*(5*A*c+C*a)/a^3/c/(a*c)^(1/2)*arctan(c*x/(a*c)^(
1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 430, normalized size of antiderivative = 3.41

$$\int \frac{A+Bx+Cx^2}{\left(a+cx^2\right)^4}\,dx = \left[-\frac{16\,Ba^4c-6\,(Ca^2c^3+5\,Aac^4)x^5-16\,(Ca^3c^2+5\,Aa^2c^3)x^3+3\,((Cac^3+5\,Ac^4)x^6+Ca^4+5\,Aa^3c+3\,(}{96\,(a^4c^5x^6+3\,a^5c^4x^4+3}\right.$$

$$-\frac{8\,Ba^4c-3\,(Ca^2c^3+5\,Aac^4)x^5-8\,(Ca^3c^2+5\,Aa^2c^3)x^3-3\,((Cac^3+5\,Ac^4)x^6+Ca^4+5\,Aa^3c+3\,(Ca}{48\,(a^4c^5x^6+3\,a^5c^4x^4+3\,a^6c^3}$$

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="fricas")
```

```
[Out] [-1/96*(16*B*a^4*c - 6*(C*a^2*c^3 + 5*A*a*c^4)*x^5 - 16*(C*a^3*c^2 + 5*A*a^
2*c^3)*x^3 + 3*((C*a*c^3 + 5*A*c^4)*x^6 + C*a^4 + 5*A*a^3*c + 3*(C*a^2*c^2
+ 5*A*a*c^3)*x^4 + 3*(C*a^3*c + 5*A*a^2*c^2)*x^2)*sqrt(-a*c)*log((c*x^2 - 2
*sqrt(-a*c)*x - a)/(c*x^2 + a)) + 6*(C*a^4*c - 11*A*a^3*c^2)*x)/(a^4*c^5*x^
6 + 3*a^5*c^4*x^4 + 3*a^6*c^3*x^2 + a^7*c^2), -1/48*(8*B*a^4*c - 3*(C*a^2*c
^3 + 5*A*a*c^4)*x^5 - 8*(C*a^3*c^2 + 5*A*a^2*c^3)*x^3 - 3*((C*a*c^3 + 5*A*c
^4)*x^6 + C*a^4 + 5*A*a^3*c + 3*(C*a^2*c^2 + 5*A*a*c^3)*x^4 + 3*(C*a^3*c +
5*A*a^2*c^2)*x^2)*sqrt(a*c)*arctan(sqrt(a*c)*x/a) + 3*(C*a^4*c - 11*A*a^3*c
^2)*x)/(a^4*c^5*x^6 + 3*a^5*c^4*x^4 + 3*a^6*c^3*x^2 + a^7*c^2)]
```

Sympy [A] (verification not implemented)

Time = 0.90 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.56

$$\begin{aligned}\int \frac{A+Bx+Cx^2}{\left(a+cx^2\right)^4}\,dx &= -\frac{\sqrt{-\frac{1}{a^7c^3}}\cdot(5Ac+Ca)\log\left(-a^4c\sqrt{-\frac{1}{a^7c^3}}+x\right)}{32}\\ &\quad+\frac{\sqrt{-\frac{1}{a^7c^3}}\cdot(5Ac+Ca)\log\left(a^4c\sqrt{-\frac{1}{a^7c^3}}+x\right)}{32}\\ &\quad+\frac{-8Ba^3+x^5\cdot(15Ac^3+3Cac^2)+x^3\cdot(40Aac^2+8Ca^2c)+x(33Aa^2c-3Ca^3)}{48a^6c+144a^5c^2x^2+144a^4c^3x^4+48a^3c^4x^6}\end{aligned}$$

```
[In] integrate((C*x**2+B*x+A)/(c*x**2+a)**4,x)
```

```
[Out] -sqrt(-1/(a**7*c**3))*(5*A*c + C*a)*log(-a**4*c*sqrt(-1/(a**7*c**3)) + x)/3
2 + sqrt(-1/(a**7*c**3))*(5*A*c + C*a)*log(a**4*c*sqrt(-1/(a**7*c**3)) + x)
/32 + (-8*B*a**3 + x**5*(15*A*c**3 + 3*C*a*c**2) + x**3*(40*A*a*c**2 + 8*C*
a**2*c) + x*(33*A*a**2*c - 3*C*a**3))/(48*a**6*c + 144*a**5*c**2*x**2 + 144
*a**4*c**3*x**4 + 48*a**3*c**4*x**6)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.06

$$\int \frac{A+Bx+Cx^2}{(a+cx^2)^4}\,dx = \frac{3\,(Cac^2+5\,Ac^3)x^5-8\,Ba^3+8\,(Ca^2c+5\,Aac^2)x^3-3\,(Ca^3-11\,Aa^2c)x}{48\,(a^3c^4x^6+3\,a^4c^3x^4+3\,a^5c^2x^2+a^6c)} + \frac{(Ca+5\,Ac)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\,\sqrt{ac}a^3c}$$

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="maxima")
```

```
[Out] 1/48*(3*(C*a*c^2 + 5*A*c^3)*x^5 - 8*B*a^3 + 8*(C*a^2*c + 5*A*a*c^2)*x^3 - 3
*(C*a^3 - 11*A*a^2*c)*x)/(a^3*c^4*x^6 + 3*a^4*c^3*x^4 + 3*a^5*c^2*x^2 + a^6
*c) + 1/16*(C*a + 5*A*c)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a^3*c)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.87

$$\int \frac{A+Bx+Cx^2}{(a+cx^2)^4}\,dx = \frac{(Ca+5\,Ac)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\,\sqrt{ac}a^3c} + \frac{3\,Cac^2x^5+15\,Ac^3x^5+8\,Ca^2cx^3+40\,Aac^2x^3-3\,Ca^3x+33\,Aa^2cx-8\,Ba^3}{48\,(cx^2+a)^3a^3c}$$

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="giac")
```

```
[Out] 1/16*(C*a + 5*A*c)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a^3*c) + 1/48*(3*C*a*c^
2*x^5 + 15*A*c^3*x^5 + 8*C*a^2*c*x^3 + 40*A*a*c^2*x^3 - 3*C*a^3*x + 33*A*a^
2*c*x - 8*B*a^3)/((c*x^2 + a)^3*a^3*c)
```

Mupad [B] (verification not implemented)

Time = 12.91 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx+Cx^2}{\left(a+cx^2\right)^4}\,dx=\frac{\frac{x^3\,\left(5\,A\,c+C\,a\right)}{6\,a^2}-\frac{B}{6\,c}+\frac{c\,x^5\,\left(5\,A\,c+C\,a\right)}{16\,a^3}+\frac{x\,\left(11\,A\,c-C\,a\right)}{16\,a\,c}}{a^3+3\,a^2\,c\,x^2+3\,a\,c^2\,x^4+c^3\,x^6}+\frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\,\left(5\,A\,c+C\,a\right)}{16\,a^{7/2}\,c^{3/2}}$$

```
[In] int((A + B*x + C*x^2)/(a + c*x^2)^4,x)
```

```
[Out] ((x^3*(5*A*c + C*a))/(6*a^2) - B/(6*c) + (c*x^5*(5*A*c + C*a))/(16*a^3) + (
x*(11*A*c - C*a))/(16*a*c))/(a^3 + c^3*x^6 + 3*a^2*c*x^2 + 3*a*c^2*x^4) + (
atan((c^(1/2)*x)/a^(1/2))*(5*A*c + C*a))/(16*a^(7/2)*c^(3/2))
```

3.69 $\int \frac{x^3(1+x+x^2)}{\left(1+x^2\right)^2} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 43

$$\int \frac{x^3(1+x+x^2)}{(1+x^2)^2} \, dx = \frac{3x}{2}+\frac{x^2}{2}-\frac{x^3}{2\left(1+x^2\right)}-\frac{3\arctan(x)}{2}-\frac{1}{2}\log\left(1+x^2\right)$$

```
[Out] 3/2*x+1/2*x^2-1/2*x^3/(x^2+1)-3/2*arctan(x)-1/2*ln(x^2+1)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {1818, 815, 649, 209, 266}

$$\int \frac{x^3(1+x+x^2)}{(1+x^2)^2} \, dx = -\frac{3\arctan(x)}{2}+\frac{x^2}{2}-\frac{1}{2}\log\left(x^2+1\right)-\frac{x^3}{2\left(x^2+1\right)}+\frac{3x}{2}$$

```
[In] Int[(x^3*(1 + x + x^2))/(1 + x^2)^2,x]
```

```
[Out] (3*x)/2 + x^2/2 - x^3/(2*(1 + x^2)) - (3*ArcTan[x])/2 - Log[1 + x^2]/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1818

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x^3}{2\left(1+x^2\right)}-\frac{1}{2}\int\frac{(-3-2x)x^2}{1+x^2}\,dx\\
&= -\frac{x^3}{2\left(1+x^2\right)}-\frac{1}{2}\int\left(-3-2x+\frac{3+2x}{1+x^2}\right)dx\\
&= \frac{3x}{2}+\frac{x^2}{2}-\frac{x^3}{2\left(1+x^2\right)}-\frac{1}{2}\int\frac{3+2x}{1+x^2}\,dx\\
&= \frac{3x}{2}+\frac{x^2}{2}-\frac{x^3}{2\left(1+x^2\right)}-\frac{3}{2}\int\frac{1}{1+x^2}\,dx-\int\frac{x}{1+x^2}\,dx\\
&= \frac{3x}{2}+\frac{x^2}{2}-\frac{x^3}{2\left(1+x^2\right)}-\frac{3}{2}\tan^{-1}(x)-\frac{1}{2}\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.67

$$\int \frac{x^3(1+x+x^2)}{(1+x^2)^2}\,dx=\frac{1}{2}\left(x\left(2+x+\frac{1}{1+x^2}\right)-3\arctan(x)-\log\left(1+x^2\right)\right)$$

```
[In] Integrate[(x^3*(1 + x + x^2))/(1 + x^2)^2,x]
```

```
[Out] (x*(2 + x + (1 + x^2)^(-1)) - 3*ArcTan[x] - Log[1 + x^2])/2
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.70

method	result	siz
default	$x+\frac{x^2}{2}+\frac{x}{2x^2+2}-\frac{\ln(x^2+1)}{2}-\frac{3\arctan(x)}{2}$	30
risch	$x+\frac{x^2}{2}+\frac{x}{2x^2+2}-\frac{\ln(x^2+1)}{2}-\frac{3\arctan(x)}{2}$	30
meijerg	$\frac{x^2(3x^2+6)}{6x^2+6}-\frac{\ln(x^2+1)}{2}+\frac{x(10x^2+15)}{10x^2+10}-\frac{3\arctan(x)}{2}-\frac{x^2}{2(x^2+1)}$	62
parallelrisch	$\frac{3i\ln(x-i)x^2-3i\ln(x+i)x^2+2x^4-2\ln(x-i)x^2-2\ln(x+i)x^2+4x^3-2+3i\ln(x-i)-3i\ln(x+i)-2\ln(x-i)-2\ln(x+i)+6x}{4x^2+4}$	97

```
[In] int(x^3*(x^2+x+1)/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] x+1/2*x^2+1/2/(x^2+1)*x-1/2*ln(x^2+1)-3/2*arctan(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.07

$$\int \frac{x^3(1+x+x^2)}{(1+x^2)^2}\,dx=\frac{x^4+2\,x^3+x^2-3\,(x^2+1)\arctan\left(x\right)-(x^2+1)\log\left(x^2+1\right)+3\,x}{2\,(x^2+1)}$$

```
[In] integrate(x^3*(x^2+x+1)/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(x^4 + 2*x^3 + x^2 - 3*(x^2 + 1)*arctan(x) - (x^2 + 1)*log(x^2 + 1) + 3
*x)/(x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.67

$$\int \frac{x^3(1+x+x^2)}{\left(1+x^2\right)^2}\,dx = \frac{x^2}{2}+x+\frac{x}{2x^2+2}-\frac{\log\left(x^2+1\right)}{2}-\frac{3\operatorname{atan}(x)}{2}$$

```
[In] integrate(x**3*(x**2+x+1)/(x**2+1)**2,x)
```

```
[Out] x**2/2 + x + x/(2*x**2 + 2) - log(x**2 + 1)/2 - 3*atan(x)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.67

$$\int \frac{x^3(1+x+x^2)}{\left(1+x^2\right)^2}\,dx = \frac{1}{2}x^2+x+\frac{x}{2\left(x^2+1\right)}-\frac{3}{2}\arctan\left(x\right)-\frac{1}{2}\log\left(x^2+1\right)$$

```
[In] integrate(x^3*(x^2+x+1)/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x^2 + x + 1/2*x/(x^2 + 1) - 3/2*arctan(x) - 1/2*log(x^2 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.67

$$\int \frac{x^3(1+x+x^2)}{\left(1+x^2\right)^2}\,dx = \frac{1}{2}x^2+x+\frac{x}{2\left(x^2+1\right)}-\frac{3}{2}\arctan\left(x\right)-\frac{1}{2}\log\left(x^2+1\right)$$

```
[In] integrate(x^3*(x^2+x+1)/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] 1/2*x^2 + x + 1/2*x/(x^2 + 1) - 3/2*arctan(x) - 1/2*log(x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.70

$$\int \frac{x^3(1+x+x^2)}{\left(1+x^2\right)^2}\,dx = x-\frac{\ln\left(x^2+1\right)}{2}-\frac{3\operatorname{atan}(x)}{2}+\frac{x}{2\,\left(x^2+1\right)}+\frac{x^2}{2}$$

```
[In] int((x^3*(x + x^2 + 1))/(x^2 + 1)^2,x)
```

```
[Out] x - log(x^2 + 1)/2 - (3*atan(x))/2 + x/(2*(x^2 + 1)) + x^2/2
```

3.70 $\int \frac{x^2(1+x+x^2)}{(1+x^2)^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 30

$$\int \frac{x^2(1+x+x^2)}{(1+x^2)^2}\,dx = x-\frac{x^2}{2(1+x^2)}-\arctan(x)+\frac{1}{2}\log\left(1+x^2\right)$$

```
[Out] x-1/2*x^2/(x^2+1)-arctan(x)+1/2*ln(x^2+1)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {1818, 788, 649, 209, 266}

$$\int \frac{x^2(1+x+x^2)}{(1+x^2)^2}\,dx = -\arctan(x)-\frac{x^2}{2(x^2+1)}+\frac{1}{2}\log\left(x^2+1\right)+x$$

```
[In] Int[(x^2*(1 + x + x^2))/(1 + x^2)^2,x]
```

```
[Out] x - x^2/(2*(1 + x^2)) - ArcTan[x] + Log[1 + x^2]/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 788

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[e*g*(x/c), x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x
)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

Rule 1818

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x^2}{2(1+x^2)} - \frac{1}{2}\int \frac{(-2-2x)x}{1+x^2}\,dx \\
&= x - \frac{x^2}{2(1+x^2)} - \frac{1}{2}\int \frac{2-2x}{1+x^2}\,dx \\
&= x - \frac{x^2}{2(1+x^2)} - \int \frac{1}{1+x^2}\,dx + \int \frac{x}{1+x^2}\,dx \\
&= x - \frac{x^2}{2(1+x^2)} - \tan^{-1}(x) + \frac{1}{2}\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

$$\int \frac{x^2(1+x+x^2)}{(1+x^2)^2}\,dx = x + \frac{1}{2(1+x^2)} - \arctan(x) + \frac{1}{2}\log\left(1+x^2\right)$$

```
[In] Integrate[(x^2*(1 + x + x^2))/(1 + x^2)^2,x]
[Out] x + 1/(2*(1 + x^2)) - ArcTan[x] + Log[1 + x^2]/2
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

method	result	size
default	$x+\frac{1}{2x^{2}+2}+\frac{\ln\left(x^{2}+1\right)}{2}-\arctan\left(x\right)$	24
risch	$x+\frac{1}{2x^{2}+2}+\frac{\ln\left(x^{2}+1\right)}{2}-\arctan\left(x\right)$	24
meijerg	$\frac{x\left(10x^{2}+15\right)}{10x^{2}+10}-\arctan\left(x\right)-\frac{x^{2}}{2\left(x^{2}+1\right)}+\frac{\ln\left(x^{2}+1\right)}{2}-\frac{x}{2\left(x^{2}+1\right)}$	53
parallelrisch	$\frac{i\ln\left(x-i\right)x^{2}-i\ln\left(x+i\right)x^{2}+\ln\left(x-i\right)x^{2}+\ln\left(x+i\right)x^{2}+2x^{3}+1+i\ln\left(x-i\right)-i\ln\left(x+i\right)+\ln\left(x-i\right)+\ln\left(x+i\right)+2x}{2x^{2}+2}$	86

```
[In] int(x^2*(x^2+x+1)/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] x+1/2/(x^2+1)+1/2*ln(x^2+1)-arctan(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.33

$$\int \frac{x^2(1+x+x^2)}{\left(1+x^2\right)^2}\,dx=\frac{2\,x^{3}-2\,\left(x^{2}+1\right)\arctan\left(x\right)+\left(x^{2}+1\right)\log\left(x^{2}+1\right)+2\,x+1}{2\,\left(x^{2}+1\right)}$$

```
[In] integrate(x^2*(x^2+x+1)/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*x^3 - 2*(x^2 + 1)*arctan(x) + (x^2 + 1)*log(x^2 + 1) + 2*x + 1)/(x^2
 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.67

$$\int \frac{x^2(1+x+x^2)}{\left(1+x^2\right)^2}\,dx=x+\frac{\log\left(x^{2}+1\right)}{2}-\operatorname{atan}\left(x\right)+\frac{1}{2x^{2}+2}$$

```
[In] integrate(x**2*(x**2+x+1)/(x**2+1)**2,x)
```

```
[Out] x + log(x**2 + 1)/2 - atan(x) + 1/(2*x**2 + 2)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.77

$$\int \frac{x^2(1+x+x^2)}{(1+x^2)^2}\,dx = x + \frac{1}{2\,(x^2+1)} - \arctan\left(x\right) + \frac{1}{2}\,\log\left(x^2+1\right)$$

```
[In] integrate(x^2*(x^2+x+1)/(x^2+1)^2,x, algorithm="maxima")
[Out] x + 1/2/(x^2 + 1) - arctan(x) + 1/2*log(x^2 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.77

$$\int \frac{x^2(1+x+x^2)}{(1+x^2)^2}\,dx = x + \frac{1}{2\,(x^2+1)} - \arctan\left(x\right) + \frac{1}{2}\,\log\left(x^2+1\right)$$

```
[In] integrate(x^2*(x^2+x+1)/(x^2+1)^2,x, algorithm="giac")
[Out] x + 1/2/(x^2 + 1) - arctan(x) + 1/2*log(x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.77

$$\int \frac{x^2(1+x+x^2)}{(1+x^2)^2}\,dx = x + \frac{\ln\left(x^2+1\right)}{2} - \mathrm{atan}(x) + \frac{1}{2\,\left(x^2+1\right)}$$

```
[In] int((x^2*(x + x^2 + 1))/(x^2 + 1)^2,x)
[Out] x + log(x^2 + 1)/2 - atan(x) + 1/(2*(x^2 + 1))
```

3.71 $\int \frac{x(1+x+x^2)}{(1+x^2)^2} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 29

$$\int \frac{x(1+x+x^2)}{(1+x^2)^2} \, dx = -\frac{x}{2(1+x^2)} + \frac{\arctan(x)}{2} + \frac{1}{2}\log(1+x^2)$$

```
[Out] -1/2*x/(x^2+1)+1/2*arctan(x)+1/2*ln(x^2+1)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1818, 649, 209, 266}

$$\int \frac{x(1+x+x^2)}{(1+x^2)^2} \, dx = \frac{\arctan(x)}{2} - \frac{x}{2(x^2+1)} + \frac{1}{2}\log(x^2+1)$$

```
[In] Int[(x*(1 + x + x^2))/(1 + x^2)^2,x]
```

```
[Out] -1/2*x/(1 + x^2) + ArcTan[x]/2 + Log[1 + x^2]/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1818

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x}{2(1+x^2)} - \frac{1}{2}\int \frac{-1-2x}{1+x^2}\,dx \\
&= -\frac{x}{2(1+x^2)} + \frac{1}{2}\int \frac{1}{1+x^2}\,dx + \int \frac{x}{1+x^2}\,dx \\
&= -\frac{x}{2(1+x^2)} + \frac{1}{2}\tan^{-1}(x) + \frac{1}{2}\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \frac{x(1+x+x^2)}{(1+x^2)^2}\,dx = \frac{1}{2}\left(-\frac{x}{1+x^2} + \arctan(x) + \log\left(1+x^2\right)\right)$$

```
[In] Integrate[(x*(1 + x + x^2))/(1 + x^2)^2,x]
[Out] (-(x/(1 + x^2)) + ArcTan[x] + Log[1 + x^2])/2
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

method	result	size
default	$-\frac{x}{2(x^2+1)}+\frac{\arctan(x)}{2}+\frac{\ln(x^2+1)}{2}$	24
meijerg	$-\frac{x}{2(x^2+1)}+\frac{\arctan(x)}{2}+\frac{\ln(x^2+1)}{2}$	24
risch	$-\frac{x}{2(x^2+1)}+\frac{\arctan(x)}{2}+\frac{\ln(x^2+1)}{2}$	24
parallelrisch	$-\frac{i\ln(x-i)x^2-i\ln(x+i)x^2-2\ln(x-i)x^2-2\ln(x+i)x^2+i\ln(x-i)-i\ln(x+i)-2\ln(x-i)-2\ln(x+i)+2x}{4(x^2+1)}$	86

```
[In] int(x*(x^2+x+1)/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/(x^2+1)*x+1/2*arctan(x)+1/2*ln(x^2+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.58 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.14

$$\int \frac{x(1+x+x^2)}{(1+x^2)^2}\,dx=\frac{(x^2+1)\arctan(x)+(x^2+1)\log(x^2+1)-x}{2\,(x^2+1)}$$

```
[In] integrate(x*(x^2+x+1)/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] 1/2*((x^2 + 1)*arctan(x) + (x^2 + 1)*log(x^2 + 1) - x)/(x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.69

$$\int \frac{x(1+x+x^2)}{(1+x^2)^2}\,dx=-\frac{x}{2x^2+2}+\frac{\log(x^2+1)}{2}+\frac{\operatorname{atan}(x)}{2}$$

```
[In] integrate(x*(x**2+x+1)/(x**2+1)**2,x)
```

```
[Out] -x/(2*x**2 + 2) + log(x**2 + 1)/2 + atan(x)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \frac{x(1+x+x^2)}{(1+x^2)^2}\,dx = -\frac{x}{2\,(x^2+1)} + \frac{1}{2}\arctan\left(x\right) + \frac{1}{2}\log\left(x^2+1\right)$$

```
[In] integrate(x*(x^2+x+1)/(x^2+1)^2,x, algorithm="maxima")
[Out] -1/2*x/(x^2 + 1) + 1/2*arctan(x) + 1/2*log(x^2 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \frac{x(1+x+x^2)}{(1+x^2)^2}\,dx = -\frac{x}{2\,(x^2+1)} + \frac{1}{2}\arctan\left(x\right) + \frac{1}{2}\log\left(x^2+1\right)$$

```
[In] integrate(x*(x^2+x+1)/(x^2+1)^2,x, algorithm="giac")
[Out] -1/2*x/(x^2 + 1) + 1/2*arctan(x) + 1/2*log(x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{x(1+x+x^2)}{(1+x^2)^2}\,dx = \frac{\ln\left(x^2+1\right)}{2} + \frac{\mathrm{atan}(x)}{2} - \frac{x}{2\,\left(x^2+1\right)}$$

```
[In] int((x*(x + x^2 + 1))/(x^2 + 1)^2,x)
[Out] log(x^2 + 1)/2 + atan(x)/2 - x/(2*(x^2 + 1))
```

3.72 $\int \frac{1+x+x^2}{\left(1+x^2\right)^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{1+x+x^2}{\left(1+x^2\right)^2}\,dx = -\frac{1}{2\left(1+x^2\right)}+\arctan(x)$$

```
[Out] -1/2/(x^2+1)+arctan(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {1828, 12, 209}

$$\int \frac{1+x+x^2}{\left(1+x^2\right)^2}\,dx = \arctan(x)-\frac{1}{2\left(x^2+1\right)}$$

```
[In] Int[(1 + x + x^2)/(1 + x^2)^2,x]
```

```
[Out] -1/2*1/(1 + x^2) + ArcTan[x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{1}{2(1+x^2)} - \frac{1}{2}\int -\frac{2}{1+x^2}\,dx\\ &= -\frac{1}{2(1+x^2)} + \int \frac{1}{1+x^2}\,dx\\ &= -\frac{1}{2(1+x^2)} + \tan^{-1}(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1+x+x^2}{(1+x^2)^2}\,dx = -\frac{1}{2(1+x^2)} + \arctan(x)$$

```
[In] Integrate[(1 + x + x^2)/(1 + x^2)^2,x]
```

```
[Out] -1/2*1/(1 + x^2) + ArcTan[x]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
default	$-\frac{1}{2(x^2+1)} + \arctan(x)$	13
risch	$-\frac{1}{2(x^2+1)} + \arctan(x)$	13
meijerg	$-\frac{x}{2(x^2+1)} + \arctan(x) + \frac{x^2}{2x^2+2} + \frac{x}{2x^2+2}$	37
parallelrisch	$-\frac{i\ln(x-i)x^2 - i\ln(x+i)x^2 + 1 + i\ln(x-i) - i\ln(x+i)}{2(x^2+1)}$	50

```
[In] int((x^2+x+1)/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/(x^2+1)+arctan(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \frac{1+x+x^2}{(1+x^2)^2}\,dx = \frac{2\,(x^2+1)\arctan{(x)}-1}{2\,(x^2+1)}$$

```
[In] integrate((x^2+x+1)/(x^2+1)^2,x, algorithm="fricas")
[Out] 1/2*(2*(x^2 + 1)*arctan(x) - 1)/(x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{1+x+x^2}{(1+x^2)^2}\,dx = \operatorname{atan}{(x)} - \frac{1}{2x^2+2}$$

```
[In] integrate((x**2+x+1)/(x**2+1)**2,x)
[Out] atan(x) - 1/(2*x**2 + 2)
```

Maxima [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1+x+x^2}{(1+x^2)^2}\,dx = -\frac{1}{2\,(x^2+1)} + \arctan{(x)}$$

```
[In] integrate((x^2+x+1)/(x^2+1)^2,x, algorithm="maxima")
[Out] -1/2/(x^2 + 1) + arctan(x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1+x+x^2}{(1+x^2)^2}\,dx = -\frac{1}{2\,(x^2+1)} + \arctan{(x)}$$

```
[In] integrate((x^2+x+1)/(x^2+1)^2,x, algorithm="giac")
[Out] -1/2/(x^2 + 1) + arctan(x)
```

Mupad [B] (verification not implemented)

Time = 12.94 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1+x+x^2}{(1+x^2)^2}\,dx = \operatorname{atan}(x) - \frac{1}{2\,(x^2+1)}$$

```
[In] int((x + x^2 + 1)/(x^2 + 1)^2,x)
[Out] atan(x) - 1/(2*(x^2 + 1))
```

3.73 $\int \frac{1+x+x^2}{x\left(1+x^2\right)^2} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 31

$$\int \frac{1+x+x^2}{x\left(1+x^2\right)^2} \, dx=\frac{x}{2\left(1+x^2\right)}+\frac{\arctan(x)}{2}+\log(x)-\frac{1}{2}\log\left(1+x^2\right)$$

```
[Out] 1/2*x/(x^2+1)+1/2*arctan(x)+ln(x)-1/2*ln(x^2+1)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {1819, 815, 649, 209, 266}

$$\int \frac{1+x+x^2}{x\left(1+x^2\right)^2} \, dx=\frac{\arctan(x)}{2}+\frac{x}{2\left(x^2+1\right)}-\frac{1}{2}\log\left(x^2+1\right)+\log(x)$$

```
[In] Int[(1 + x + x^2)/(x*(1 + x^2)^2),x]
```

```
[Out] x/(2*(1 + x^2)) + ArcTan[x]/2 + Log[x] - Log[1 + x^2]/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{x}{2(1+x^2)} - \frac{1}{2}\int \frac{-2-x}{x(1+x^2)}\,dx \\
&= \frac{x}{2(1+x^2)} - \frac{1}{2}\int \left(-\frac{2}{x} + \frac{-1+2x}{1+x^2}\right)dx \\
&= \frac{x}{2(1+x^2)} + \log(x) - \frac{1}{2}\int \frac{-1+2x}{1+x^2}\,dx \\
&= \frac{x}{2(1+x^2)} + \log(x) + \frac{1}{2}\int \frac{1}{1+x^2}\,dx - \int \frac{x}{1+x^2}\,dx \\
&= \frac{x}{2(1+x^2)} + \frac{1}{2}\tan^{-1}(x) + \log(x) - \frac{1}{2}\log\left(1+x^2\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

$$\int \frac{1+x+x^2}{x\left(1+x^2\right)^2}\,dx=\frac{1}{2}\left(\frac{x}{1+x^2}+\arctan(x)+2\log(x)-\log\left(1+x^2\right)\right)$$

```
[In] Integrate[(1 + x + x^2)/(x*(1 + x^2)^2),x]
```

```
[Out] (x/(1 + x^2) + ArcTan[x] + 2*Log[x] - Log[1 + x^2])/2
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

method	result
default	$\frac{x}{2x^2+2}+\frac{\arctan(x)}{2}+\ln\left(x\right)-\frac{\ln(x^2+1)}{2}$
risch	$\frac{x}{2x^2+2}+\frac{\arctan(x)}{2}+\ln\left(x\right)-\frac{\ln(x^2+1)}{2}$
meijerg	$\frac{x}{2x^2+2}+\frac{\arctan(x)}{2}-\frac{\ln(x^2+1)}{2}+\frac{1}{2}+\ln\left(x\right)$
parallelrisch	$\frac{-i\ln(x-i)x^2+i\ln(x+i)x^2+4\ln(x)x^2-2\ln(x-i)x^2-2\ln(x+i)x^2-i\ln(x-i)+i\ln(x+i)+4\ln(x)-2\ln(x-i)-2\ln(x+i)+2x}{4x^2+4}$

```
[In] int((x^2+x+1)/x/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/(x^2+1)*x+1/2*arctan(x)+ln(x)-1/2*ln(x^2+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.32

$$\int \frac{1+x+x^2}{x\left(1+x^2\right)^2}\,dx=\frac{\left(x^2+1\right)\arctan\left(x\right)-\left(x^2+1\right)\log\left(x^2+1\right)+2\left(x^2+1\right)\log\left(x\right)+x}{2\left(x^2+1\right)}$$

```
[In] integrate((x^2+x+1)/x/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] 1/2*((x^2 + 1)*arctan(x) - (x^2 + 1)*log(x^2 + 1) + 2*(x^2 + 1)*log(x) + x)
/(x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int \frac{1+x+x^2}{x\left(1+x^2\right)^2}\,dx = \frac{x}{2x^2+2} + \log\left(x\right) - \frac{\log\left(x^2+1\right)}{2} + \frac{\operatorname{atan}\left(x\right)}{2}$$

```
[In] integrate((x**2+x+1)/x/(x**2+1)**2,x)
[Out] x/(2*x**2 + 2) + log(x) - log(x**2 + 1)/2 + atan(x)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{1+x+x^2}{x\left(1+x^2\right)^2}\,dx = \frac{x}{2\left(x^2+1\right)} + \frac{1}{2}\arctan\left(x\right) - \frac{1}{2}\log\left(x^2+1\right) + \log\left(x\right)$$

```
[In] integrate((x^2+x+1)/x/(x^2+1)^2,x, algorithm="maxima")
[Out] 1/2*x/(x^2 + 1) + 1/2*arctan(x) - 1/2*log(x^2 + 1) + log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{1+x+x^2}{x\left(1+x^2\right)^2}\,dx = \frac{x}{2\left(x^2+1\right)} + \frac{1}{2}\arctan\left(x\right) - \frac{1}{2}\log\left(x^2+1\right) + \log\left(|x|\right)$$

```
[In] integrate((x^2+x+1)/x/(x^2+1)^2,x, algorithm="giac")
[Out] 1/2*x/(x^2 + 1) + 1/2*arctan(x) - 1/2*log(x^2 + 1) + log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \frac{1+x+x^2}{x\left(1+x^2\right)^2}\,dx = \ln\left(x\right) + \frac{x}{2\left(x^2+1\right)} + \ln\left(x-\mathrm{i}\right)\left(-\frac{1}{2}-\frac{1}{4}\mathrm{i}\right) + \ln\left(x+1\mathrm{i}\right)\left(-\frac{1}{2}+\frac{1}{4}\mathrm{i}\right)$$

```
[In] int((x + x^2 + 1)/(x*(x^2 + 1)^2),x)
[Out] log(x) - log(x + 1i)*(1/2 - 1i/4) - log(x - 1i)*(1/2 + 1i/4) + x/(2*(x^2 +
1))
```

3.74 $\int \frac{1+x+x^2}{x^2\left(1+x^2\right)^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 33

$$\int \frac{1+x+x^2}{x^2\left(1+x^2\right)^2}\,dx=-\frac{1}{x}+\frac{1}{2\left(1+x^2\right)}-\arctan(x)+\log(x)-\frac{1}{2}\log\left(1+x^2\right)$$

```
[Out] -1/x+1/2/(x^2+1)-arctan(x)+ln(x)-1/2*ln(x^2+1)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {1819, 815, 649, 209, 266}

$$\int \frac{1+x+x^2}{x^2\left(1+x^2\right)^2}\,dx=-\arctan(x)+\frac{1}{2\left(x^2+1\right)}-\frac{1}{2}\log\left(x^2+1\right)-\frac{1}{x}+\log(x)$$

```
[In] Int[(1 + x + x^2)/(x^2*(1 + x^2)^2),x]
```

```
[Out] -x^(-1) + 1/(2*(1 + x^2)) - ArcTan[x] + Log[x] - Log[1 + x^2]/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2\left(1+x^2\right)}-\frac{1}{2}\int\frac{-2-2x}{x^2\left(1+x^2\right)}\,dx\\
&= \frac{1}{2\left(1+x^2\right)}-\frac{1}{2}\int\left(-\frac{2}{x^2}-\frac{2}{x}+\frac{2(1+x)}{1+x^2}\right)dx\\
&= -\frac{1}{x}+\frac{1}{2\left(1+x^2\right)}+\log(x)-\int\frac{1+x}{1+x^2}\,dx\\
&= -\frac{1}{x}+\frac{1}{2\left(1+x^2\right)}+\log(x)-\int\frac{1}{1+x^2}\,dx-\int\frac{x}{1+x^2}\,dx\\
&= -\frac{1}{x}+\frac{1}{2\left(1+x^2\right)}-\tan^{-1}(x)+\log(x)-\frac{1}{2}\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{1+x+x^2}{x^2\left(1+x^2\right)^2}\,dx=-\frac{1}{x}+\frac{1}{2\left(1+x^2\right)}-\arctan(x)+\log(x)-\frac{1}{2}\log\left(1+x^2\right)$$

```
[In] Integrate[(1 + x + x^2)/(x^2*(1 + x^2)^2),x]
[Out] -x^(-1) + 1/(2*(1 + x^2)) - ArcTan[x] + Log[x] - Log[1 + x^2]/2
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.91

method	result
default	$-\frac{1}{x}+\frac{1}{2x^2+2}-\arctan\left(x\right)+\ln\left(x\right)-\frac{\ln\left(x^2+1\right)}{2}$
risch	$\frac{-x^2+\frac{1}{2}x-1}{x\left(x^2+1\right)}-\frac{\ln\left(x^2+1\right)}{2}-\arctan\left(x\right)+\ln\left(x\right)$
meijerg	$\frac{x}{2x^2+2}-\arctan\left(x\right)-\frac{x^2}{2x^2+2}-\frac{\ln\left(x^2+1\right)}{2}+\frac{1}{2}+\ln\left(x\right)-\frac{3x^2+2}{x\left(2x^2+2\right)}$
parallelrisch	$\frac{i\ln(x-i)x^3-i\ln(x+i)x^3+2\ln(x)x^3-\ln(x-i)x^3-\ln(x+i)x^3-2+i\ln(x-i)x-i\ln(x+i)x+2\ln(x)x-\ln(x-i)x-\ln(x+i)x-2x^2+}{2\left(x^2+1\right)x}$

```
[In] int((x^2+x+1)/x^2/(x^2+1)^2,x,method=_RETURNVERBOSE)
[Out] -1/x+1/2/(x^2+1)-arctan(x)+ln(x)-1/2*ln(x^2+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.48

$$\int \frac{1+x+x^2}{x^2\left(1+x^2\right)^2}\,dx$$
$$=-\frac{2\,x^2+2\left(x^3+x\right)\arctan\left(x\right)+\left(x^3+x\right)\log\left(x^2+1\right)-2\left(x^3+x\right)\log\left(x\right)-x+2}{2\left(x^3+x\right)}$$

```
[In] integrate((x^2+x+1)/x^2/(x^2+1)^2,x, algorithm="fricas")
[Out] -1/2*(2*x^2 + 2*(x^3 + x)*arctan(x) + (x^3 + x)*log(x^2 + 1) - 2*(x^3 + x)*
log(x) - x + 2)/(x^3 + x)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int \frac{1+x+x^2}{x^2\left(1+x^2\right)^2}\,dx = \log\left(x\right) - \frac{\log\left(x^2+1\right)}{2} - \operatorname{atan}\left(x\right) + \frac{-2x^2+x-2}{2x^3+2x}$$

```
[In] integrate((x**2+x+1)/x**2/(x**2+1)**2,x)
[Out] log(x) - log(x**2 + 1)/2 - atan(x) + (-2*x**2 + x - 2)/(2*x**3 + 2*x)
```

Maxima [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.03

$$\int \frac{1+x+x^2}{x^2\left(1+x^2\right)^2}\,dx = -\frac{2\,x^2-x+2}{2\left(x^3+x\right)} - \arctan\left(x\right) - \frac{1}{2}\,\log\left(x^2+1\right) + \log\left(x\right)$$

```
[In] integrate((x^2+x+1)/x^2/(x^2+1)^2,x, algorithm="maxima")
[Out] -1/2*(2*x^2 - x + 2)/(x^3 + x) - arctan(x) - 1/2*log(x^2 + 1) + log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{1+x+x^2}{x^2\left(1+x^2\right)^2}\,dx = -\frac{2\,x^2-x+2}{2\left(x^3+x\right)} - \arctan\left(x\right) - \frac{1}{2}\,\log\left(x^2+1\right) + \log\left(|x|\right)$$

```
[In] integrate((x^2+x+1)/x^2/(x^2+1)^2,x, algorithm="giac")
[Out] -1/2*(2*x^2 - x + 2)/(x^3 + x) - arctan(x) - 1/2*log(x^2 + 1) + log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 12.85 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.15

$$\int \frac{1+x+x^2}{x^2\left(1+x^2\right)^2}\,dx = \ln\left(x\right) - \frac{x^2-\frac{x}{2}+1}{x^3+x} + \ln\left(x-\mathrm{i}\right)\left(-\frac{1}{2}+\frac{1}{2}\mathrm{i}\right) + \ln\left(x+1\mathrm{i}\right)\left(-\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)$$

```
[In] int((x + x^2 + 1)/(x^2*(x^2 + 1)^2),x)
[Out] log(x) - log(x + 1i)*(1/2 + 1i/2) - log(x - 1i)*(1/2 - 1i/2) - (x^2 - x/2 +
 1)/(x + x^3)
```

3.75 $\int \frac{1+x+x^2}{x^3\left(1+x^2\right)^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 45

$$\int \frac{1+x+x^2}{x^3\left(1+x^2\right)^2}\,dx=-\frac{1}{2x^2}-\frac{1}{x}-\frac{x}{2\left(1+x^2\right)}-\frac{3\arctan(x)}{2}-\log(x)+\frac{1}{2}\log\left(1+x^2\right)$$

```
[Out] -1/2/x^2-1/x-1/2*x/(x^2+1)-3/2*arctan(x)-ln(x)+1/2*ln(x^2+1)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {1819, 1816, 649, 209, 266}

$$\int \frac{1+x+x^2}{x^3\left(1+x^2\right)^2}\,dx=-\frac{3\arctan(x)}{2}-\frac{x}{2\left(x^2+1\right)}-\frac{1}{2x^2}+\frac{1}{2}\log\left(x^2+1\right)-\frac{1}{x}-\log(x)$$

```
[In] Int[(1 + x + x^2)/(x^3*(1 + x^2)^2),x]
```

```
[Out] -1/2*1/x^2 - x^(-1) - x/(2*(1 + x^2)) - (3*ArcTan[x])/2 - Log[x] + Log[1 +
x^2]/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x}{2(1+x^2)} - \frac{1}{2}\int \frac{-2-2x+x^3}{x^3(1+x^2)}\,dx \\
&= -\frac{x}{2(1+x^2)} - \frac{1}{2}\int \left(-\frac{2}{x^3} - \frac{2}{x^2} + \frac{2}{x} + \frac{3-2x}{1+x^2}\right) dx \\
&= -\frac{1}{2x^2} - \frac{1}{x} - \frac{x}{2(1+x^2)} - \log(x) - \frac{1}{2}\int \frac{3-2x}{1+x^2}\,dx \\
&= -\frac{1}{2x^2} - \frac{1}{x} - \frac{x}{2(1+x^2)} - \log(x) - \frac{3}{2}\int \frac{1}{1+x^2}\,dx + \int \frac{x}{1+x^2}\,dx \\
&= -\frac{1}{2x^2} - \frac{1}{x} - \frac{x}{2(1+x^2)} - \frac{3}{2}\tan^{-1}(x) - \log(x) + \frac{1}{2}\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.87

$$\int \frac{1+x+x^2}{x^3\left(1+x^2\right)^2}\,dx=\frac{1}{2}\left(-\frac{1}{x^2}-\frac{2}{x}-\frac{x}{1+x^2}-3\arctan(x)-2\log(x)+\log\left(1+x^2\right)\right)$$

```
[In] Integrate[(1 + x + x^2)/(x^3*(1 + x^2)^2),x]
[Out] (-x^(-2) - 2/x - x/(1 + x^2) - 3*ArcTan[x] - 2*Log[x] + Log[1 + x^2])/2
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.84

method	result
default	$-\frac{1}{2x^2}-\frac{1}{x}-\frac{x}{2(x^2+1)}-\frac{3\arctan(x)}{2}-\ln(x)+\frac{\ln(x^2+1)}{2}$
risch	$\frac{-\frac{3}{2}x^3-\frac{1}{2}x^2-x-\frac{1}{2}}{x^2(x^2+1)}+\frac{\ln(x^2+1)}{2}-\frac{3\arctan(x)}{2}-\ln(x)$
meijerg	$-\frac{x^2}{2x^2+2}+\frac{\ln(x^2+1)}{2}-\ln(x)-\frac{3x^2+2}{x(2x^2+2)}-\frac{3\arctan(x)}{2}+\frac{3x^2}{2(3x^2+3)}-\frac{1}{2x^2}$
parallelrisch	$-\frac{3i\ln(x+i)x^4+3i\ln(x+i)x^2+4\ln(x)x^4-2\ln(x-i)x^4-2\ln(x+i)x^4+2-3i\ln(x-i)x^4-3i\ln(x-i)x^2+4\ln(x)x^2-2\ln(x-i)x^2-}{4x^2(x^2+1)}$

```
[In] int((x^2+x+1)/x^3/(x^2+1)^2,x,method=_RETURNVERBOSE)
[Out] -1/2/x^2-1/x-1/2/(x^2+1)*x-3/2*arctan(x)-ln(x)+1/2*ln(x^2+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.44 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.36

$$\int \frac{1+x+x^2}{x^3\left(1+x^2\right)^2}\,dx = -\frac{3\,x^3+x^2+3\,\left(x^4+x^2\right)\arctan\left(x\right)-\left(x^4+x^2\right)\log\left(x^2+1\right)+2\,\left(x^4+x^2\right)\log\left(x\right)+2\,x+1}{2\,\left(x^4+x^2\right)}$$

```
[In] integrate((x^2+x+1)/x^3/(x^2+1)^2,x, algorithm="fricas")
[Out] -1/2*(3*x^3 + x^2 + 3*(x^4 + x^2)*arctan(x) - (x^4 + x^2)*log(x^2 + 1) + 2*
(x^4 + x^2)*log(x) + 2*x + 1)/(x^4 + x^2)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93

$$\int \frac{1+x+x^2}{x^3(1+x^2)^2}\,dx = -\log{(x)} + \frac{\log{(x^2+1)}}{2} - \frac{3\operatorname{atan}{(x)}}{2} + \frac{-3x^3-x^2-2x-1}{2x^4+2x^2}$$

```
[In] integrate((x**2+x+1)/x**3/(x**2+1)**2,x)
```

```
[Out] -log(x) + log(x**2 + 1)/2 - 3*atan(x)/2 + (-3*x**3 - x**2 - 2*x - 1)/(2*x**
4 + 2*x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91

$$\int \frac{1+x+x^2}{x^3(1+x^2)^2}\,dx = -\frac{3\,x^3+x^2+2\,x+1}{2\,(x^4+x^2)} - \frac{3}{2}\,\arctan\left(x\right) + \frac{1}{2}\,\log\left(x^2+1\right) - \log\left(x\right)$$

```
[In] integrate((x^2+x+1)/x^3/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(3*x^3 + x^2 + 2*x + 1)/(x^4 + x^2) - 3/2*arctan(x) + 1/2*log(x^2 + 1)
 - log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96

$$\int \frac{1+x+x^2}{x^3(1+x^2)^2}\,dx = -\frac{3\,x^3+x^2+2\,x+1}{2\,(x^2+1)x^2} - \frac{3}{2}\,\arctan\left(x\right) + \frac{1}{2}\,\log\left(x^2+1\right) - \log\left(|x|\right)$$

```
[In] integrate((x^2+x+1)/x^3/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] -1/2*(3*x^3 + x^2 + 2*x + 1)/((x^2 + 1)*x^2) - 3/2*arctan(x) + 1/2*log(x^2
+ 1) - log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.04

$$\int \frac{1+x+x^2}{x^3\left(1+x^2\right)^2}\,dx=-\ln\left(x\right)-\frac{\frac{3\,x^3}{2}+\frac{x^2}{2}+x+\frac{1}{2}}{x^4+x^2}+\ln\left(x-\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{3}{4}\mathrm{i}\right)+\ln\left(x+1\mathrm{i}\right)\,\left(\frac{1}{2}-\frac{3}{4}\mathrm{i}\right)$$

```
[In] int((x + x^2 + 1)/(x^3*(x^2 + 1)^2),x)
```

```
[Out] log(x - 1i)*(1/2 + 3i/4) + log(x + 1i)*(1/2 - 3i/4) - log(x) - (x + x^2/2 +
 (3*x^3)/2 + 1/2)/(x^2 + x^4)
```

3.76 $\int \frac{1+2x+x^2}{\left(1+x^2\right)^2} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 12

$$\int \frac{1+2x+x^2}{\left(1+x^2\right)^2} \, dx=-\frac{1}{1+x^2}+\arctan(x)$$

```
[Out] -1/(x^2+1)+arctan(x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {27, 737, 209}

$$\int \frac{1+2x+x^2}{\left(1+x^2\right)^2} \, dx=\arctan(x)-\frac{(1-x)(x+1)}{2\left(x^2+1\right)}$$

```
[In] Int[(1 + 2*x + x^2)/(1 + x^2)^2,x]
```

```
[Out] -1/2*((1 - x)*(1 + x))/(1 + x^2) + ArcTan[x]
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 737

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m - 1)*(a*e - c*d*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] +
 Dist[(2*p + 3)*((c*d^2 + a*e^2)/(2*a*c*(p + 1))), Int[(d + e*x)^(m - 2)*(a
 + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0
] && EqQ[m + 2*p + 2, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{(1+x)^2}{(1+x^2)^2}\,dx\\ &= -\frac{(1-x)(1+x)}{2(1+x^2)} + \int \frac{1}{1+x^2}\,dx\\ &= -\frac{(1-x)(1+x)}{2(1+x^2)} + \tan^{-1}(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1+2x+x^2}{(1+x^2)^2}\,dx = -\frac{1}{1+x^2} + \arctan(x)$$

```
[In] Integrate[(1 + 2*x + x^2)/(1 + x^2)^2,x]
[Out] -(1 + x^2)^(-1) + ArcTan[x]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

method	result	size
default	$-\frac{1}{x^2+1} + \arctan(x)$	13
risch	$-\frac{1}{x^2+1} + \arctan(x)$	13
meijerg	$-\frac{x}{2(x^2+1)} + \arctan(x) + \frac{x^2}{x^2+1} + \frac{x}{2x^2+2}$	36
parallelrisch	$-\frac{i\ln(x-i)x^2 - i\ln(x+i)x^2 + 2 + i\ln(x-i) - i\ln(x+i)}{2(x^2+1)}$	50

```
[In] int((x^2+2*x+1)/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/(x^2+1)+arctan(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.50

$$\int \frac{1+2x+x^2}{(1+x^2)^2}\,dx = \frac{(x^2+1)\arctan(x)-1}{x^2+1}$$

```
[In] integrate((x^2+2*x+1)/(x^2+1)^2,x, algorithm="fricas")
[Out] ((x^2 + 1)*arctan(x) - 1)/(x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1+2x+x^2}{(1+x^2)^2}\,dx = \operatorname{atan}(x) - \frac{1}{x^2+1}$$

```
[In] integrate((x**2+2*x+1)/(x**2+1)**2,x)
[Out] atan(x) - 1/(x**2 + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1+2x+x^2}{(1+x^2)^2}\,dx = -\frac{1}{x^2+1} + \arctan(x)$$

```
[In] integrate((x^2+2*x+1)/(x^2+1)^2,x, algorithm="maxima")
[Out] -1/(x^2 + 1) + arctan(x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1+2x+x^2}{\left(1+x^2\right)^2}\,dx = -\frac{1}{x^2+1} + \arctan\left(x\right)$$

```
[In] integrate((x^2+2*x+1)/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] -1/(x^2 + 1) + arctan(x)
```

Mupad [B] (verification not implemented)

Time = 12.77 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1+2x+x^2}{\left(1+x^2\right)^2}\,dx = \operatorname{atan}(x) - \frac{1}{x^2+1}$$

```
[In] int((2*x + x^2 + 1)/(x^2 + 1)^2,x)
```

```
[Out] atan(x) - 1/(x^2 + 1)
```

3.77 $\int \frac{2+12x+3x^2}{\left(4+x^2\right)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 27

$$\int \frac{2+12x+3x^2}{\left(4+x^2\right)^2}\,dx=-\frac{24+5x}{4\left(4+x^2\right)}+\frac{7}{8}\arctan\left(\frac{x}{2}\right)$$

```
[Out] 1/4*(-24-5*x)/(x^2+4)+7/8*arctan(1/2*x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1828, 12, 209}

$$\int \frac{2+12x+3x^2}{\left(4+x^2\right)^2}\,dx=\frac{7}{8}\arctan\left(\frac{x}{2}\right)-\frac{5x+24}{4\left(x^2+4\right)}$$

```
[In] Int[(2 + 12*x + 3*x^2)/(4 + x^2)^2,x]
```

```
[Out] -1/4*(24 + 5*x)/(4 + x^2) + (7*ArcTan[x/2])/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{24+5x}{4\,(4+x^2)}-\frac{1}{8}\int -\frac{14}{4+x^2}\,dx\\ &= -\frac{24+5x}{4\,(4+x^2)}+\frac{7}{4}\int \frac{1}{4+x^2}\,dx\\ &= -\frac{24+5x}{4\,(4+x^2)}+\frac{7}{8}\tan^{-1}\left(\frac{x}{2}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{2+12x+3x^2}{(4+x^2)^2}\,dx=\frac{-24-5x}{4\,(4+x^2)}+\frac{7}{8}\arctan\left(\frac{x}{2}\right)$$

```
[In] Integrate[(2 + 12*x + 3*x^2)/(4 + x^2)^2,x]
```

```
[Out] (-24 - 5*x)/(4*(4 + x^2)) + (7*ArcTan[x/2])/8
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

method	result	size
default	$\frac{-\frac{5x}{4}-6}{x^2+4}+\frac{7\arctan\left(\frac{x}{2}\right)}{8}$	21
risch	$\frac{-\frac{5x}{4}-6}{x^2+4}+\frac{7\arctan\left(\frac{x}{2}\right)}{8}$	21
meijerg	$\frac{x}{4x^2+16}+\frac{7\arctan\left(\frac{x}{2}\right)}{8}-\frac{3x}{8\left(\frac{x^2}{4}+1\right)}+\frac{3x^2}{8\left(\frac{x^2}{4}+1\right)}$	46
parallelrisch	$-\frac{7i\ln(x-2i)x^2-7i\ln(x+2i)x^2+28i\ln(x-2i)-28i\ln(x+2i)-24x^2+20x}{16(x^2+4)}$	57

```
[In] int((3*x^2+12*x+2)/(x^2+4)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-5/4*x-6)/(x^2+4)+7/8*arctan(1/2*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{2+12x+3x^2}{(4+x^2)^2}\,dx = \frac{7\,(x^2+4)\arctan\left(\frac{1}{2}\,x\right)-10\,x-48}{8\,(x^2+4)}$$

```
[In] integrate((3*x^2+12*x+2)/(x^2+4)^2,x, algorithm="fricas")
[Out] 1/8*(7*(x^2 + 4)*arctan(1/2*x) - 10*x - 48)/(x^2 + 4)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int \frac{2+12x+3x^2}{(4+x^2)^2}\,dx = \frac{-5x-24}{4x^2+16}+\frac{7\operatorname{atan}\left(\frac{x}{2}\right)}{8}$$

```
[In] integrate((3*x**2+12*x+2)/(x**2+4)**2,x)
[Out] (-5*x - 24)/(4*x**2 + 16) + 7*atan(x/2)/8
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{2+12x+3x^2}{(4+x^2)^2}\,dx = -\frac{5\,x+24}{4\,(x^2+4)}+\frac{7}{8}\,\arctan\left(\frac{1}{2}\,x\right)$$

```
[In] integrate((3*x^2+12*x+2)/(x^2+4)^2,x, algorithm="maxima")
[Out] -1/4*(5*x + 24)/(x^2 + 4) + 7/8*arctan(1/2*x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{2+12x+3x^2}{(4+x^2)^2}\,dx = -\frac{5\,x+24}{4\,(x^2+4)} + \frac{7}{8}\,\arctan\left(\frac{1}{2}\,x\right)$$

```
[In] integrate((3*x^2+12*x+2)/(x^2+4)^2,x, algorithm="giac")
[Out] -1/4*(5*x + 24)/(x^2 + 4) + 7/8*arctan(1/2*x)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{2+12x+3x^2}{(4+x^2)^2}\,dx = \frac{7\,\mathrm{atan}\left(\frac{x}{2}\right)}{8} - \frac{\frac{5\,x}{4}+6}{x^2+4}$$

```
[In] int((12*x + 3*x^2 + 2)/(x^2 + 4)^2,x)
[Out] (7*atan(x/2))/8 - ((5*x)/4 + 6)/(x^2 + 4)
```

3.78 $\int (g+hx)^3\sqrt{a+cx^2}\left(d+ex+fx^2\right)\,dx$

Optimal result

Integrand size = 29, antiderivative size = 390

$$\begin{aligned}
&\int (g+hx)^3\sqrt{a+cx^2}\left(d+ex+fx^2\right)\,dx\\
&=\frac{\left(8c^2dg^3+a^2h^2(3fg+eh)-2acg\left(fg^2+3h(eg+dh)\right)\right)x\sqrt{a+cx^2}}{16c^2}\\
&\quad-\frac{\left(8afh^2+c\left(3fg^2-7h(eg+2dh)\right)\right)(g+hx)^2\left(a+cx^2\right)^{3/2}}{70c^2h}\\
&\quad-\frac{(3fg-7eh)(g+hx)^3\left(a+cx^2\right)^{3/2}}{42ch}+\frac{f(g+hx)^4\left(a+cx^2\right)^{3/2}}{7ch}\\
&\quad+\frac{\left(8\left(8a^2fh^4-2ach^2\left(15fg^2+7h(3eg+dh)\right)-c^2g^2\left(3fg^2-7h(eg+12dh)\right)\right)-3ch\left(ah^2(41fg+35eh)+2\right.}{840c^3h}\\
&\quad+\frac{a\left(8c^2dg^3+a^2h^2(3fg+eh)-2acg\left(fg^2+3h(eg+dh)\right)\right)\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{16c^{5/2}}
\end{aligned}$$

```
[Out] -1/70*(8*a*f*h^2+c*(3*f*g^2-7*h*(2*d*h+e*g)))*(h*x+g)^2*(c*x^2+a)^(3/2)/c^2
/h-1/42*(-7*e*h+3*f*g)*(h*x+g)^3*(c*x^2+a)^(3/2)/c/h+1/7*f*(h*x+g)^4*(c*x^2
+a)^(3/2)/c/h+1/840*(64*a^2*f*h^4-16*a*c*h^2*(15*f*g^2+7*h*(d*h+3*e*g))-8*c
^2*g^2*(3*f*g^2-7*h*(12*d*h+e*g))-3*c*h*(a*h^2*(35*e*h+41*f*g)+2*c*g*(3*f*g
^2-7*h*(7*d*h+e*g)))*x)*(c*x^2+a)^(3/2)/c^3/h+1/16*a*(8*c^2*d*g^3+a^2*h^2*(
e*h+3*f*g)-2*a*c*g*(f*g^2+3*h*(d*h+e*g)))*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2)
)/c^(5/2)+1/16*(8*c^2*d*g^3+a^2*h^2*(e*h+3*f*g)-2*a*c*g*(f*g^2+3*h*(d*h+e*g
)))*x*(c*x^2+a)^(1/2)/c^2
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 387, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1668, 847, 794, 201, 223, 212}

$$\int (g+hx)^3\sqrt{a+cx^2}\left(d+ex+fx^2\right)\,dx$$
$$= \frac{a\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(a^2h^2(eh+3fg)-2acg(3h(dh+eg)+fg^2)+8c^2dg^3\right)}{16c^{5/2}}$$
$$+\frac{x\sqrt{a+cx^2}(a^2h^2(eh+3fg)-2acg(3h(dh+eg)+fg^2)+8c^2dg^3)}{16c^2}$$
$$+\frac{(a+cx^2)^{3/2}\,(8(8a^2fh^4-2ach^2(7h(dh+3eg)+15fg^2)-c^2(3fg^4-7g^2h(12dh+eg)))-3chx(ah^2(35}{840c^3h}$$
$$-\frac{(a+cx^2)^{3/2}\,(g+hx)^2\,(8afh^2-7ch(2dh+eg)+3cfg^2)}{70c^2h}$$
$$-\frac{(a+cx^2)^{3/2}\,(g+hx)^3(3fg-7eh)}{42ch}+\frac{f(a+cx^2)^{3/2}\,(g+hx)^4}{7ch}$$

```
[In] Int[(g + h*x)^3*Sqrt[a + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] ((8*c^2*d*g^3 + a^2*h^2*(3*f*g + e*h) - 2*a*c*g*(f*g^2 + 3*h*(e*g + d*h)))*
x*Sqrt[a + c*x^2])/(16*c^2) - ((3*c*f*g^2 + 8*a*f*h^2 - 7*c*h*(e*g + 2*d*h)
)*(g + h*x)^2*(a + c*x^2)^(3/2))/(70*c^2*h) - ((3*f*g - 7*e*h)*(g + h*x)^3*
(a + c*x^2)^(3/2))/(42*c*h) + (f*(g + h*x)^4*(a + c*x^2)^(3/2))/(7*c*h) + (
(8*(8*a^2*f*h^4 - 2*a*c*h^2*(15*f*g^2 + 7*h*(3*e*g + d*h)) - c^2*(3*f*g^4 -
 7*g^2*h*(e*g + 12*d*h))) - 3*c*h*(6*c*f*g^3 - 14*c*g*h*(e*g + 7*d*h) + a*h
^2*(41*f*g + 35*e*h))*x)*(a + c*x^2)^(3/2))/(840*c^3*h) + (a*(8*c^2*d*g^3 +
 a^2*h^2*(3*f*g + e*h) - 2*a*c*g*(f*g^2 + 3*h*(e*g + d*h)))*ArcTanh[(Sqrt[c
]*x)/Sqrt[a + c*x^2]])/(16*c^(5/2))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{f(g+hx)^4\left(a+cx^2\right)^{3/2}}{7ch} \\ &\quad+\frac{\int (g+hx)^3\left((7cd-4af)h^2-ch(3fg-7eh)x\right)\sqrt{a+cx^2}\,dx}{7ch^2} \\ &= -\frac{(3fg-7eh)(g+hx)^3\left(a+cx^2\right)^{3/2}}{42ch}+\frac{f(g+hx)^4\left(a+cx^2\right)^{3/2}}{7ch} \\ &\quad+\frac{\int (g+hx)^2\left(3ch^2(14cdg-5afg-7aeh)-3ch\left(3cfg^2+8afh^2-7ch(eg+2dh)\right)x\right)\sqrt{a+cx^2}\,dx}{42c^2h^2}\end{aligned}$$

$$= -\frac{\left(3cfg^2+8afh^2-7ch(eg+2dh)\right)(g+hx)^2\left(a+cx^2\right)^{3/2}}{70c^2h}$$
$$-\frac{(3fg-7eh)(g+hx)^3\left(a+cx^2\right)^{3/2}}{42ch}+\frac{f(g+hx)^4\left(a+cx^2\right)^{3/2}}{7ch}$$
$$+\frac{\int(g+hx)\,(3ch^2(70c^2dg^2+16a^2fh^2-ac(19fg^2+7h(7eg+4dh)))-3c^2h(6cfg^3-14cgh(eg}{210c^3h^2}$$

$$= -\frac{\left(3cfg^2+8afh^2-7ch(eg+2dh)\right)(g+hx)^2\left(a+cx^2\right)^{3/2}}{70c^2h}$$
$$-\frac{(3fg-7eh)(g+hx)^3\left(a+cx^2\right)^{3/2}}{42ch}+\frac{f(g+hx)^4\left(a+cx^2\right)^{3/2}}{7ch}$$
$$+\frac{(8(8a^2fh^4-2ach^2(15fg^2+7h(3eg+dh))-c^2(3fg^4-7g^2h(eg+12dh)))-3ch(6cfg^3-14c}{840c^3h}$$
$$+\frac{\left(8c^2dg^3+a^2h^2(3fg+eh)-2acg(fg^2+3h(eg+dh))\right)\int\sqrt{a+cx^2}\,dx}{8c^2}$$

$$= \frac{\left(8c^2dg^3+a^2h^2(3fg+eh)-2acg(fg^2+3h(eg+dh))\right)x\sqrt{a+cx^2}}{16c^2}$$
$$-\frac{\left(3cfg^2+8afh^2-7ch(eg+2dh)\right)(g+hx)^2\left(a+cx^2\right)^{3/2}}{70c^2h}$$
$$-\frac{(3fg-7eh)(g+hx)^3\left(a+cx^2\right)^{3/2}}{42ch}+\frac{f(g+hx)^4\left(a+cx^2\right)^{3/2}}{7ch}$$
$$+\frac{(8(8a^2fh^4-2ach^2(15fg^2+7h(3eg+dh))-c^2(3fg^4-7g^2h(eg+12dh)))-3ch(6cfg^3-14c}{840c^3h}$$
$$+\frac{\left(a\left(8c^2dg^3+a^2h^2(3fg+eh)-2acg(fg^2+3h(eg+dh))\right)\right)\int\frac{1}{\sqrt{a+cx^2}}\,dx}{16c^2}$$

$$= \frac{\left(8c^2dg^3+a^2h^2(3fg+eh)-2acg(fg^2+3h(eg+dh))\right)x\sqrt{a+cx^2}}{16c^2}$$
$$-\frac{\left(3cfg^2+8afh^2-7ch(eg+2dh)\right)(g+hx)^2\left(a+cx^2\right)^{3/2}}{70c^2h}$$
$$-\frac{(3fg-7eh)(g+hx)^3\left(a+cx^2\right)^{3/2}}{42ch}+\frac{f(g+hx)^4\left(a+cx^2\right)^{3/2}}{7ch}$$
$$+\frac{(8(8a^2fh^4-2ach^2(15fg^2+7h(3eg+dh))-c^2(3fg^4-7g^2h(eg+12dh)))-3ch(6cfg^3-14c}{840c^3h}$$
$$+\frac{\left(a\left(8c^2dg^3+a^2h^2(3fg+eh)-2acg(fg^2+3h(eg+dh))\right)\right)\text{Subst}\left(\int\frac{1}{1-cx^2}\,dx,x,\frac{x}{\sqrt{a+cx^2}}\right)}{16c^2}$$

$$
\begin{aligned}
&= \frac{\left(8c^2dg^3+a^2h^2(3fg+eh)-2acg(fg^2+3h(eg+dh))\right)x\sqrt{a+cx^2}}{16c^2} \\
&- \frac{\left(3cfg^2+8afh^2-7ch(eg+2dh)\right)(g+hx)^2\left(a+cx^2\right)^{3/2}}{70c^2h} \\
&- \frac{(3fg-7eh)(g+hx)^3\left(a+cx^2\right)^{3/2}}{42ch} + \frac{f(g+hx)^4\left(a+cx^2\right)^{3/2}}{7ch} \\
&+ \frac{(8(8a^2fh^4-2ach^2(15fg^2+7h(3eg+dh))-c^2(3fg^4-7g^2h(eg+12dh)))-3ch(6cfg^3-14cg}{840c^3h} \\
&+ \frac{a\left(8c^2dg^3+a^2h^2(3fg+eh)-2acg(fg^2+3h(eg+dh))\right)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{16c^{5/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 358, normalized size of antiderivative = 0.92

$$
\int (g+hx)^3\sqrt{a+cx^2}\left(d+ex+fx^2\right)\,dx
$$
$$
= \frac{\sqrt{a+cx^2}(128a^3fh^3-a^2ch(7h(96eg+32dh+15ehx)+f(672g^2+315ghx+64h^2x^2))+2ac^2(7dh(120g^2}{}
$$

```
[In] Integrate[(g + h*x)^3*Sqrt[a + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] (Sqrt[a + c*x^2]*(128*a^3*f*h^3 - a^2*c*h*(7*h*(96*e*g + 32*d*h + 15*e*h*x)
 + f*(672*g^2 + 315*g*h*x + 64*h^2*x^2)) + 2*a*c^2*(7*d*h*(120*g^2 + 45*g*h
*x + 8*h^2*x^2) + 7*e*(40*g^3 + 45*g^2*h*x + 24*g*h^2*x^2 + 5*h^3*x^3) + 3*
f*x*(35*g^3 + 56*g^2*h*x + 35*g*h^2*x^2 + 8*h^3*x^3)) + 4*c^3*x*(21*d*(10*g
^3 + 20*g^2*h*x + 15*g*h^2*x^2 + 4*h^3*x^3) + x*(7*e*(20*g^3 + 45*g^2*h*x +
 36*g*h^2*x^2 + 10*h^3*x^3) + 3*f*x*(35*g^3 + 84*g^2*h*x + 70*g*h^2*x^2 + 2
0*h^3*x^3)))) - 105*a*Sqrt[c]*(8*c^2*d*g^3 + a^2*h^2*(3*f*g + e*h) - 2*a*c*
g*(f*g^2 + 3*h*(e*g + d*h)))*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/(1680*c^3
)
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 358, normalized size of antiderivative = 0.92

method	result
default	$d\,g^{3} \left(\frac{x \sqrt{c\,x^{2}+a}}{2}+\frac{a \ln \left(x \sqrt{c}+\sqrt{c\,x^{2}+a}\right)}{2 \sqrt{c}}\right)+f\,h^{3} \left(\frac{x^{4} \left(c\,x^{2}+a\right)^{\frac{3}{2}}}{7 c}-\frac{4 a \left(\frac{x^{2} \left(c\,x^{2}+a\right)^{\frac{3}{2}}}{5 c}-\frac{2 a \left(c\,x^{2}+a\right)^{\frac{3}{2}}}{15 c^{2}}\right)}{7 c}\right)+\left(e\,h^{3}+3\right.$
risch	$\frac{(240 f\,h^{3} c^{3} x^{6}+280 c^{3} e\,h^{3} x^{5}+840 c^{3} f g\,h^{2} x^{5}+48 a\,c^{2} f\,h^{3} x^{4}+336 c^{3} d\,h^{3} x^{4}+1008 c^{3} e g\,h^{2} x^{4}+1008 c^{3} f\,g^{2} h\,x^{4}+70 a\,c^{2} e\,h^{3} x^{3}+210 a\,c^{2} f g}{}$

```
[In] int((h*x+g)^3*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] d*g^3*(1/2*x*(c*x^2+a)^(1/2)+1/2*a/c^(1/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2)))+f
*h^3*(1/7*x^4*(c*x^2+a)^(3/2)/c-4/7*a/c*(1/5*x^2*(c*x^2+a)^(3/2)/c-2/15*a/c
^2*(c*x^2+a)^(3/2)))+(e*h^3+3*f*g*h^2)*(1/6*x^3*(c*x^2+a)^(3/2)/c-1/2*a/c*(
1/4*x*(c*x^2+a)^(3/2)/c-1/4*a/c*(1/2*x*(c*x^2+a)^(1/2)+1/2*a/c^(1/2)*ln(x*c
^(1/2)+(c*x^2+a)^(1/2)))))+1/3*(3*d*g^2*h+e*g^3)*(c*x^2+a)^(3/2)/c+(d*h^3+3
*e*g*h^2+3*f*g^2*h)*(1/5*x^2*(c*x^2+a)^(3/2)/c-2/15*a/c^2*(c*x^2+a)^(3/2))+
(3*d*g*h^2+3*e*g^2*h+f*g^3)*(1/4*x*(c*x^2+a)^(3/2)/c-1/4*a/c*(1/2*x*(c*x^2+
a)^(1/2)+1/2*a/c^(1/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 855, normalized size of antiderivative = 2.19

$$\int (g+hx)^3\sqrt{a+cx^2}\left(d+ex+fx^2\right)\,dx$$

$$=\left[-\frac{105\,\left(6\,a^{2}ceg^{2}h-a^{3}eh^{3}-2\,\left(4\,ac^{2}d-a^{2}cf\right)g^{3}+3\,\left(2\,a^{2}cd-a^{3}f\right)gh^{2}\right)\sqrt{c}\log\left(-2\,cx^{2}-2\,\sqrt{cx^{2}+a}\sqrt{c}x\right.}{}\right.$$

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/3360*(105*(6*a^2*c*e*g^2*h - a^3*e*h^3 - 2*(4*a*c^2*d - a^2*c*f)*g^3 +
3*(2*a^2*c*d - a^3*f)*g*h^2)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(
c)*x - a) - 2*(240*c^3*f*h^3*x^6 + 560*a*c^2*e*g^3 - 672*a^2*c*e*g*h^2 + 28
0*(3*c^3*f*g*h^2 + c^3*e*h^3)*x^5 + 48*(21*c^3*f*g^2*h + 21*c^3*e*g*h^2 + (
7*c^3*d + a*c^2*f)*h^3)*x^4 + 336*(5*a*c^2*d - 2*a^2*c*f)*g^2*h - 32*(7*a^2
*c*d - 4*a^3*f)*h^3 + 70*(6*c^3*f*g^3 + 18*c^3*e*g^2*h + a*c^2*e*h^3 + 3*(6
*c^3*d + a*c^2*f)*g*h^2)*x^3 + 16*(35*c^3*e*g^3 + 21*a*c^2*e*g*h^2 + 21*(5*
c^3*d + a*c^2*f)*g^2*h + (7*a*c^2*d - 4*a^2*c*f)*h^3)*x^2 + 105*(6*a*c^2*e*
g^2*h - a^2*c*e*h^3 + 2*(4*c^3*d + a*c^2*f)*g^3 + 3*(2*a*c^2*d - a^2*c*f)*g
*h^2)*x)*sqrt(c*x^2 + a))/c^3, 1/1680*(105*(6*a^2*c*e*g^2*h - a^3*e*h^3 - 2
```

```
*(4*a*c^2*d - a^2*c*f)*g^3 + 3*(2*a^2*c*d - a^3*f)*g*h^2)*sqrt(-c)*arctan(s
qrt(-c)*x/sqrt(c*x^2 + a)) + (240*c^3*f*h^3*x^6 + 560*a*c^2*e*g^3 - 672*a^2
*c*e*g*h^2 + 280*(3*c^3*f*g*h^2 + c^3*e*h^3)*x^5 + 48*(21*c^3*f*g^2*h + 21*
c^3*e*g*h^2 + (7*c^3*d + a*c^2*f)*h^3)*x^4 + 336*(5*a*c^2*d - 2*a^2*c*f)*g^
2*h - 32*(7*a^2*c*d - 4*a^3*f)*h^3 + 70*(6*c^3*f*g^3 + 18*c^3*e*g^2*h + a*c
^2*e*h^3 + 3*(6*c^3*d + a*c^2*f)*g*h^2)*x^3 + 16*(35*c^3*e*g^3 + 21*a*c^2*e
*g*h^2 + 21*(5*c^3*d + a*c^2*f)*g^2*h + (7*a*c^2*d - 4*a^2*c*f)*h^3)*x^2 +
105*(6*a*c^2*e*g^2*h - a^2*c*e*h^3 + 2*(4*c^3*d + a*c^2*f)*g^3 + 3*(2*a*c^2
*d - a^2*c*f)*g*h^2)*x)*sqrt(c*x^2 + a))/c^3]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 756 vs. $2(377) = 754$.

Time = 0.58 (sec) , antiderivative size = 756, normalized size of antiderivative = 1.94

$$\int (g+hx)^3\sqrt{a+cx^2}\left(d+ex+fx^2\right)\,dx$$

$$= \begin{cases} \sqrt{a+cx^2}\left(\frac{fh^3x^6}{7}+\frac{x^5\left(ceh^3+3cfgh^2\right)}{6c}+\frac{x^4\left(\frac{afh^3}{7}+cdh^3+3cegh^2+3cfg^2h\right)}{5c}+\frac{x^3\left(aeh^3+3afgh^2-\frac{5a\left(ceh^3+3cfgh^2\right)}{6c}+3cdgh^2+3ceg\right.}{4c}\right. \\ \sqrt{a}\left(dg^3x+\frac{fh^3x^6}{6}+\frac{x^5\left(eh^3+3fgh^2\right)}{5}+\frac{x^4\left(dh^3+3egh^2+3fg^2h\right)}{4}+\frac{x^3\cdot\left(3dgh^2+3eg^2h+fg^3\right)}{3}+\frac{x^2\cdot\left(3dg^2h+eg^3\right)}{2}\right) \end{cases}$$

```
[In] integrate((h*x+g)**3*(f*x**2+e*x+d)*(c*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((sqrt(a + c*x**2)*(f*h**3*x**6/7 + x**5*(c*e*h**3 + 3*c*f*g*h**2)
/(6*c) + x**4*(a*f*h**3/7 + c*d*h**3 + 3*c*e*g*h**2 + 3*c*f*g**2*h)/(5*c) +
 x**3*(a*e*h**3 + 3*a*f*g*h**2 - 5*a*(c*e*h**3 + 3*c*f*g*h**2)/(6*c) + 3*c*
d*g*h**2 + 3*c*e*g**2*h + c*f*g**3)/(4*c) + x**2*(a*d*h**3 + 3*a*e*g*h**2 +
 3*a*f*g**2*h - 4*a*(a*f*h**3/7 + c*d*h**3 + 3*c*e*g*h**2 + 3*c*f*g**2*h)/(
5*c) + 3*c*d*g**2*h + c*e*g**3)/(3*c) + x*(3*a*d*g*h**2 + 3*a*e*g**2*h + a*
f*g**3 - 3*a*(a*e*h**3 + 3*a*f*g*h**2 - 5*a*(c*e*h**3 + 3*c*f*g*h**2)/(6*c)
 + 3*c*d*g*h**2 + 3*c*e*g**2*h + c*f*g**3)/(4*c) + c*d*g**3)/(2*c) + (3*a*d
*g**2*h + a*e*g**3 - 2*a*(a*d*h**3 + 3*a*e*g*h**2 + 3*a*f*g**2*h - 4*a*(a*f
*h**3/7 + c*d*h**3 + 3*c*e*g*h**2 + 3*c*f*g**2*h)/(5*c) + 3*c*d*g**2*h + c*
e*g**3)/(3*c))/c) + (a*d*g**3 - a*(3*a*d*g*h**2 + 3*a*e*g**2*h + a*f*g**3 -
 3*a*(a*e*h**3 + 3*a*f*g*h**2 - 5*a*(c*e*h**3 + 3*c*f*g*h**2)/(6*c) + 3*c*d
*g*h**2 + 3*c*e*g**2*h + c*f*g**3)/(4*c) + c*d*g**3)/(2*c))*Piecewise((log(
2*sqrt(c)*sqrt(a + c*x**2) + 2*c*x)/sqrt(c), Ne(a, 0)), (x*log(x)/sqrt(c*x*
*2), True)), Ne(c, 0)), (sqrt(a)*(d*g**3*x + f*h**3*x**6/6 + x**5*(e*h**3 +
 3*f*g*h**2)/5 + x**4*(d*h**3 + 3*e*g*h**2 + 3*f*g**2*h)/4 + x**3*(3*d*g*h*
*2 + 3*e*g**2*h + f*g**3)/3 + x**2*(3*d*g**2*h + e*g**3)/2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 436, normalized size of antiderivative = 1.12

$$
\begin{aligned}
\int (g+hx)^3\sqrt{a+cx^2}\left(d+ex+fx^2\right)\,dx &= \frac{(cx^2+a)^{\frac{3}{2}}fh^3x^4}{7\,c} - \frac{4\,(cx^2+a)^{\frac{3}{2}}afh^3x^2}{35\,c^2} \\
&+ \frac{1}{2}\,\sqrt{cx^2+a}dg^3x + \frac{adg^3\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{2\,\sqrt{c}} \\
&+ \frac{(cx^2+a)^{\frac{3}{2}}eg^3}{3\,c} + \frac{(cx^2+a)^{\frac{3}{2}}dg^2h}{c} \\
&+ \frac{8\,(cx^2+a)^{\frac{3}{2}}a^2fh^3}{105\,c^3} \\
&+ \frac{(3\,fgh^2+eh^3)(cx^2+a)^{\frac{3}{2}}x^3}{6\,c} \\
&+ \frac{(3\,fg^2h+3\,egh^2+dh^3)(cx^2+a)^{\frac{3}{2}}x^2}{5\,c} \\
&- \frac{(3\,fgh^2+eh^3)(cx^2+a)^{\frac{3}{2}}ax}{8\,c^2} \\
&+ \frac{(3\,fgh^2+eh^3)\sqrt{cx^2+a}a^2x}{16\,c^2} \\
&+ \frac{(fg^3+3\,eg^2h+3\,dgh^2)(cx^2+a)^{\frac{3}{2}}x}{4\,c} \\
&- \frac{(fg^3+3\,eg^2h+3\,dgh^2)\sqrt{cx^2+a}ax}{8\,c} \\
&+ \frac{(3\,fgh^2+eh^3)a^3\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{16\,c^{\frac{5}{2}}} \\
&- \frac{(fg^3+3\,eg^2h+3\,dgh^2)a^2\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{8\,c^{\frac{3}{2}}} \\
&- \frac{2\,(3\,fg^2h+3\,egh^2+dh^3)(cx^2+a)^{\frac{3}{2}}a}{15\,c^2}
\end{aligned}
$$

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/7*(c*x^2 + a)^(3/2)*f*h^3*x^4/c - 4/35*(c*x^2 + a)^(3/2)*a*f*h^3*x^2/c^2
+ 1/2*sqrt(c*x^2 + a)*d*g^3*x + 1/2*a*d*g^3*arcsinh(c*x/sqrt(a*c))/sqrt(c)
+ 1/3*(c*x^2 + a)^(3/2)*e*g^3/c + (c*x^2 + a)^(3/2)*d*g^2*h/c + 8/105*(c*x^
2 + a)^(3/2)*a^2*f*h^3/c^3 + 1/6*(3*f*g*h^2 + e*h^3)*(c*x^2 + a)^(3/2)*x^3/
c + 1/5*(3*f*g^2*h + 3*e*g*h^2 + d*h^3)*(c*x^2 + a)^(3/2)*x^2/c - 1/8*(3*f*
g*h^2 + e*h^3)*(c*x^2 + a)^(3/2)*a*x/c^2 + 1/16*(3*f*g*h^2 + e*h^3)*sqrt(c*
```

```
x^2 + a)*a^2*x/c^2 + 1/4*(f*g^3 + 3*e*g^2*h + 3*d*g*h^2)*(c*x^2 + a)^(3/2)*
x/c - 1/8*(f*g^3 + 3*e*g^2*h + 3*d*g*h^2)*sqrt(c*x^2 + a)*a*x/c + 1/16*(3*f
*g*h^2 + e*h^3)*a^3*arcsinh(c*x/sqrt(a*c))/c^(5/2) - 1/8*(f*g^3 + 3*e*g^2*h
 + 3*d*g*h^2)*a^2*arcsinh(c*x/sqrt(a*c))/c^(3/2) - 2/15*(3*f*g^2*h + 3*e*g*
h^2 + d*h^3)*(c*x^2 + a)^(3/2)*a/c^2
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 463, normalized size of antiderivative = 1.19

$$\int (g+hx)^3\sqrt{a+cx^2}\left(d+ex+fx^2\right)\,dx$$
$$=\frac{1}{1680}\sqrt{cx^2+a}\Bigg(\Bigg(2\Bigg(\Bigg(\Bigg(4\Bigg(5\Bigg(6\,fh^3x+\frac{7\,(3\,c^5fgh^2+c^5eh^3)}{c^5}\Bigg)x+\frac{6\,(21\,c^5fg^2h+21\,c^5egh^2+7\,c^5dh^3+ac}{c^5}$$
$$-\frac{(8\,ac^2dg^3-2\,a^2cfg^3-6\,a^2ceg^2h-6\,a^2cdgh^2+3\,a^3fgh^2+a^3eh^3)\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+a}\right|\right)}{16\,c^{\frac{5}{2}}}$$

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/1680*sqrt(c*x^2 + a)*((2*((4*(5*(6*f*h^3*x + 7*(3*c^5*f*g*h^2 + c^5*e*h^3
)/c^5)*x + 6*(21*c^5*f*g^2*h + 21*c^5*e*g*h^2 + 7*c^5*d*h^3 + a*c^4*f*h^3)/
c^5)*x + 35*(6*c^5*f*g^3 + 18*c^5*e*g^2*h + 18*c^5*d*g*h^2 + 3*a*c^4*f*g*h^
2 + a*c^4*e*h^3)/c^5)*x + 8*(35*c^5*e*g^3 + 105*c^5*d*g^2*h + 21*a*c^4*f*g^
2*h + 21*a*c^4*e*g*h^2 + 7*a*c^4*d*h^3 - 4*a^2*c^3*f*h^3)/c^5)*x + 105*(8*c
^5*d*g^3 + 2*a*c^4*f*g^3 + 6*a*c^4*e*g^2*h + 6*a*c^4*d*g*h^2 - 3*a^2*c^3*f*
g*h^2 - a^2*c^3*e*h^3)/c^5)*x + 16*(35*a*c^4*e*g^3 + 105*a*c^4*d*g^2*h - 42
*a^2*c^3*f*g^2*h - 42*a^2*c^3*e*g*h^2 - 14*a^2*c^3*d*h^3 + 8*a^3*c^2*f*h^3)
/c^5) - 1/16*(8*a*c^2*d*g^3 - 2*a^2*c*f*g^3 - 6*a^2*c*e*g^2*h - 6*a^2*c*d*g
*h^2 + 3*a^3*f*g*h^2 + a^3*e*h^3)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c^
(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int (g+hx)^3\sqrt{a+cx^2}\left(d+ex+fx^2\right)\,dx=\int {\left(g+h\,x\right)}^3\,\sqrt{c\,x^2+a}\,\left(f\,x^2+e\,x+d\right)\,dx$$

```
[In] int((g + h*x)^3*(a + c*x^2)^(1/2)*(d + e*x + f*x^2),x)
```

```
[Out] int((g + h*x)^3*(a + c*x^2)^(1/2)*(d + e*x + f*x^2), x)
```

3.79 $\int (g+hx)^2\sqrt{a+cx^2}\left(d+ex+fx^2\right)\,dx$

Optimal result

Integrand size = 29, antiderivative size = 280

$$\begin{aligned}
&\int (g+hx)^2\sqrt{a+cx^2}\left(d+ex+fx^2\right)\,dx \\
&= \frac{(8c^2dg^2+a^2fh^2-2ac(fg^2+h(2eg+dh)))\,x\sqrt{a+cx^2}}{16c^2} \\
&\quad - \frac{(fg-2eh)(g+hx)^2\left(a+cx^2\right)^{3/2}}{10ch} + \frac{f(g+hx)^3\left(a+cx^2\right)^{3/2}}{6ch} \\
&\quad - \frac{(8(2ah^2(2fg+eh)+cg(fg^2-2h(eg+5dh)))-3h(5(2cd-af)h^2-2cg(fg-2eh))\,x)\left(a+cx^2\right)^{3/2}}{120c^2h} \\
&\quad + \frac{a(8c^2dg^2+a^2fh^2-2ac(fg^2+h(2eg+dh)))\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{16c^{5/2}}
\end{aligned}$$

```
[Out] -1/10*(-2*e*h+f*g)*(h*x+g)^2*(c*x^2+a)^(3/2)/c/h+1/6*f*(h*x+g)^3*(c*x^2+a)^
(3/2)/c/h-1/120*(16*a*h^2*(e*h+2*f*g)+8*c*g*(f*g^2-2*h*(5*d*h+e*g))-3*h*(5*
(-a*f+2*c*d)*h^2-2*c*g*(-2*e*h+f*g))*x)*(c*x^2+a)^(3/2)/c^2/h+1/16*a*(8*c^2
*d*g^2+a^2*f*h^2-2*a*c*(f*g^2+h*(d*h+2*e*g)))*arctanh(x*c^(1/2)/(c*x^2+a)^(
1/2))/c^(5/2)+1/16*(8*c^2*d*g^2+a^2*f*h^2-2*a*c*(f*g^2+h*(d*h+2*e*g)))*x*(c
*x^2+a)^(1/2)/c^2
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1668, 847, 794, 201, 223, 212}

$$\begin{aligned}&\int (g+hx)^2\sqrt{a+cx^2}\left(d+ex+fx^2\right)\,dx\\ &=\frac{a\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(a^2fh^2-2ac(h(dh+2eg)+fg^2)+8c^2dg^2\right)}{16c^{5/2}}\\ &\quad+\frac{x\sqrt{a+cx^2}(a^2fh^2-2ac(h(dh+2eg)+fg^2)+8c^2dg^2)}{16c^2}\\ &\quad-\frac{\left(a+cx^2\right)^{3/2}\left(8(2ah^2(eh+2fg)-2cgh(5dh+eg)+cfg^3)-3hx(5h^2(2cd-af)-2cg(fg-2eh))\right)}{120c^2h}\\ &\quad-\frac{\left(a+cx^2\right)^{3/2}(g+hx)^2(fg-2eh)}{10ch}+\frac{f\left(a+cx^2\right)^{3/2}(g+hx)^3}{6ch}\end{aligned}$$

```
[In] Int[(g + h*x)^2*Sqrt[a + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] ((8*c^2*d*g^2 + a^2*f*h^2 - 2*a*c*(f*g^2 + h*(2*e*g + d*h)))*x*Sqrt[a + c*x
^2])/(16*c^2) - ((f*g - 2*e*h)*(g + h*x)^2*(a + c*x^2)^(3/2))/(10*c*h) + (f
*(g + h*x)^3*(a + c*x^2)^(3/2))/(6*c*h) - ((8*(c*f*g^3 - 2*c*g*h*(e*g + 5*d
*h) + 2*a*h^2*(2*f*g + e*h)) - 3*h*(5*(2*c*d - a*f)*h^2 - 2*c*g*(f*g - 2*e*
h))*x)*(a + c*x^2)^(3/2))/(120*c^2*h) + (a*(8*c^2*d*g^2 + a^2*f*h^2 - 2*a*c
*(f*g^2 + h*(2*e*g + d*h)))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(16*c^(5/
2))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{f(g+hx)^3\left(a+cx^2\right)^{3/2}}{6ch}\\ &\quad+\frac{\int (g+hx)^2\left(3(2cd-af)h^2-3ch(fg-2eh)x\right)\sqrt{a+cx^2}\,dx}{6ch^2}\\ &= -\frac{(fg-2eh)(g+hx)^2\left(a+cx^2\right)^{3/2}}{10ch}+\frac{f(g+hx)^3\left(a+cx^2\right)^{3/2}}{6ch}\\ &\quad+\frac{\int (g+hx)\left(3ch^2(10cdg-3afg-4aeh)+3ch\left(5(2cd-af)h^2-2cg(fg-2eh)\right)x\right)\sqrt{a+cx^2}\,dx}{30c^2h^2}\end{aligned}$$

$$= -\frac{(fg-2eh)(g+hx)^2\left(a+cx^2\right)^{3/2}}{10ch} + \frac{f(g+hx)^3\left(a+cx^2\right)^{3/2}}{6ch}$$
$$-\frac{\left(8\left(cfg^3-2cgh(eg+5dh)+2ah^2(2fg+eh)\right)-3h\left(5(2cd-af)h^2-2cg(fg-2eh)\right)x\right)(a+cx^2}{120c^2h}$$
$$+\frac{\left(8c^2dg^2+a^2fh^2-2ac\left(fg^2+h(2eg+dh)\right)\right)\int\sqrt{a+cx^2}\,dx}{8c^2}$$

$$= \frac{\left(8c^2dg^2+a^2fh^2-2ac\left(fg^2+h(2eg+dh)\right)\right)x\sqrt{a+cx^2}}{16c^2}$$
$$-\frac{(fg-2eh)(g+hx)^2\left(a+cx^2\right)^{3/2}}{10ch} + \frac{f(g+hx)^3\left(a+cx^2\right)^{3/2}}{6ch}$$
$$-\frac{\left(8\left(cfg^3-2cgh(eg+5dh)+2ah^2(2fg+eh)\right)-3h\left(5(2cd-af)h^2-2cg(fg-2eh)\right)x\right)(a+cx^2}{120c^2h}$$
$$+\frac{\left(a\left(8c^2dg^2+a^2fh^2-2ac\left(fg^2+h(2eg+dh)\right)\right)\right)\int\frac{1}{\sqrt{a+cx^2}}\,dx}{16c^2}$$

$$= \frac{\left(8c^2dg^2+a^2fh^2-2ac\left(fg^2+h(2eg+dh)\right)\right)x\sqrt{a+cx^2}}{16c^2}$$
$$-\frac{(fg-2eh)(g+hx)^2\left(a+cx^2\right)^{3/2}}{10ch} + \frac{f(g+hx)^3\left(a+cx^2\right)^{3/2}}{6ch}$$
$$-\frac{\left(8\left(cfg^3-2cgh(eg+5dh)+2ah^2(2fg+eh)\right)-3h\left(5(2cd-af)h^2-2cg(fg-2eh)\right)x\right)(a+cx^2}{120c^2h}$$
$$+\frac{\left(a\left(8c^2dg^2+a^2fh^2-2ac\left(fg^2+h(2eg+dh)\right)\right)\right)\text{Subst}\left(\int\frac{1}{1-cx^2}\,dx,x,\frac{x}{\sqrt{a+cx^2}}\right)}{16c^2}$$

$$= \frac{\left(8c^2dg^2+a^2fh^2-2ac\left(fg^2+h(2eg+dh)\right)\right)x\sqrt{a+cx^2}}{16c^2}$$
$$-\frac{(fg-2eh)(g+hx)^2\left(a+cx^2\right)^{3/2}}{10ch} + \frac{f(g+hx)^3\left(a+cx^2\right)^{3/2}}{6ch}$$
$$-\frac{\left(8\left(cfg^3-2cgh(eg+5dh)+2ah^2(2fg+eh)\right)-3h\left(5(2cd-af)h^2-2cg(fg-2eh)\right)x\right)(a+cx^2}{120c^2h}$$
$$+\frac{a\left(8c^2dg^2+a^2fh^2-2ac\left(fg^2+h(2eg+dh)\right)\right)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{16c^{5/2}}$$

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.88

$$\int (g+hx)^2\sqrt{a+cx^2}\left(d+ex+fx^2\right)\,dx$$

$$= \sqrt{a+cx^2}(-a^2h(64fg+32eh+15fhx)+2ac(5dh(16g+3hx)+fx(15g^2+16ghx+5h^2x^2)+e(40g^2+$$

$$-\frac{a(8c^2dg^2+a^2fh^2-2ac(fg^2+h(2eg+dh)))\log\left(-\sqrt{c}x+\sqrt{a+cx^2}\right)}{16c^{5/2}}$$

```
[In] Integrate[(g + h*x)^2*Sqrt[a + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] (Sqrt[a + c*x^2]*(-(a^2*h*(64*f*g + 32*e*h + 15*f*h*x)) + 2*a*c*(5*d*h*(16*
g + 3*h*x) + f*x*(15*g^2 + 16*g*h*x + 5*h^2*x^2) + e*(40*g^2 + 30*g*h*x + 8
*h^2*x^2)) + 4*c^2*x*(5*d*(6*g^2 + 8*g*h*x + 3*h^2*x^2) + x*(2*e*(10*g^2 +
15*g*h*x + 6*h^2*x^2) + f*x*(15*g^2 + 24*g*h*x + 10*h^2*x^2)))))/(240*c^2)
- (a*(8*c^2*d*g^2 + a^2*f*h^2 - 2*a*c*(f*g^2 + h*(2*e*g + d*h)))*Log[-(Sqrt
[c]*x) + Sqrt[a + c*x^2]])/(16*c^(5/2))
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 272, normalized size of antiderivative = 0.97

method	result
default	$d\,g^2\left(\frac{x\sqrt{c\,x^2+a}}{2}+\frac{a\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{2\sqrt{c}}\right)+f\,h^2 (\frac{x^3\left(c\,x^2+a\right)^{\frac{3}{2}}}{6c}-\frac{a (\frac{x\left(c\,x^2+a\right)^{\frac{3}{2}}}{4c}-\frac{a\left(\frac{x\sqrt{c\,x^2+a}}{2}+\frac{a\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{2\sqrt{c}}\right)}{4c}}{2c}$
risch	$-\frac{(-40f\,h^2c^2x^5-48c^2e\,h^2x^4-96c^2fgh\,x^4-10af\,h^2c\,x^3-60c^2d\,h^2x^3-120c^2egh\,x^3-60c^2f\,g^2x^3-16ae\,h^2c\,x^2-32afghc\,x^2-160c^2d}{24}$

```
[In] int((h*x+g)^2*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] d*g^2*(1/2*x*(c*x^2+a)^(1/2)+1/2*a/c^(1/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2)))+f
*h^2*(1/6*x^3*(c*x^2+a)^(3/2)/c-1/2*a/c*(1/4*x*(c*x^2+a)^(3/2)/c-1/4*a/c*(1
/2*x*(c*x^2+a)^(1/2)+1/2*a/c^(1/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2)))))+(e*h^2+
2*f*g*h)*(1/5*x^2*(c*x^2+a)^(3/2)/c-2/15*a/c^2*(c*x^2+a)^(3/2))+1/3*(2*d*g*
h+e*g^2)*(c*x^2+a)^(3/2)/c+(d*h^2+2*e*g*h+f*g^2)*(1/4*x*(c*x^2+a)^(3/2)/c-1
/4*a/c*(1/2*x*(c*x^2+a)^(1/2)+1/2*a/c^(1/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.46 (sec) , antiderivative size = 595, normalized size of antiderivative = 2.12

$$\int (g+hx)^2\sqrt{a+cx^2}\left(d+ex+fx^2\right)\,dx$$

$$=\left[-\frac{15\,(4\,a^2cegh-2\,(4\,ac^2d-a^2cf)g^2+(2\,a^2cd-a^3f)h^2)\sqrt{c}\log\left(-2\,cx^2-2\,\sqrt{cx^2+a}\sqrt{c}x-a\right)-2\,(40\,c^3}{}\right.$$

```
[In] integrate((h*x+g)^2*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/480*(15*(4*a^2*c*e*g*h - 2*(4*a*c^2*d - a^2*c*f)*g^2 + (2*a^2*c*d - a^3
*f)*h^2)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) - 2*(40*c^
3*f*h^2*x^5 + 80*a*c^2*e*g^2 - 32*a^2*c*e*h^2 + 48*(2*c^3*f*g*h + c^3*e*h^2
)*x^4 + 10*(6*c^3*f*g^2 + 12*c^3*e*g*h + (6*c^3*d + a*c^2*f)*h^2)*x^3 + 32*
(5*a*c^2*d - 2*a^2*c*f)*g*h + 16*(5*c^3*e*g^2 + a*c^2*e*h^2 + 2*(5*c^3*d +
a*c^2*f)*g*h)*x^2 + 15*(4*a*c^2*e*g*h + 2*(4*c^3*d + a*c^2*f)*g^2 + (2*a*c^
2*d - a^2*c*f)*h^2)*x)*sqrt(c*x^2 + a))/c^3, 1/240*(15*(4*a^2*c*e*g*h - 2*(
4*a*c^2*d - a^2*c*f)*g^2 + (2*a^2*c*d - a^3*f)*h^2)*sqrt(-c)*arctan(sqrt(-c
)*x/sqrt(c*x^2 + a)) + (40*c^3*f*h^2*x^5 + 80*a*c^2*e*g^2 - 32*a^2*c*e*h^2
+ 48*(2*c^3*f*g*h + c^3*e*h^2)*x^4 + 10*(6*c^3*f*g^2 + 12*c^3*e*g*h + (6*c^
3*d + a*c^2*f)*h^2)*x^3 + 32*(5*a*c^2*d - 2*a^2*c*f)*g*h + 16*(5*c^3*e*g^2
+ a*c^2*e*h^2 + 2*(5*c^3*d + a*c^2*f)*g*h)*x^2 + 15*(4*a*c^2*e*g*h + 2*(4*c
^3*d + a*c^2*f)*g^2 + (2*a*c^2*d - a^2*c*f)*h^2)*x)*sqrt(c*x^2 + a))/c^3]
```

Sympy [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 484, normalized size of antiderivative = 1.73

$$\int (g+hx)^2\sqrt{a+cx^2}\left(d+ex+fx^2\right)\,dx$$

$$=\begin{cases}\sqrt{a+cx^2}\left(\frac{fh^2x^5}{6}+\frac{x^4(ceh^2+2cfgh)}{5c}+\frac{x^3\left(\frac{afh^2}{6}+cdh^2+2cegh+cfg^2\right)}{4c}+\frac{x^2\left(aeh^2+2afgh-\frac{4a\left(ceh^2+2cfgh\right)}{5c}+2cdgh+ceg^2\right)}{3c}+\right. \\ \sqrt{a}\left(dg^2x+\frac{fh^2x^5}{5}+\frac{x^4(eh^2+2fgh)}{4}+\frac{x^3(dh^2+2egh+fg^2)}{3}+\frac{x^2\cdot(2dgh+eg^2)}{2}\right)\end{cases}$$

```
[In] integrate((h*x+g)**2*(f*x**2+e*x+d)*(c*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((sqrt(a + c*x**2)*(f*h**2*x**5/6 + x**4*(c*e*h**2 + 2*c*f*g*h)/(5
*c) + x**3*(a*f*h**2/6 + c*d*h**2 + 2*c*e*g*h + c*f*g**2)/(4*c) + x**2*(a*e
```

```
*h**2 + 2*a*f*g*h - 4*a*(c*e*h**2 + 2*c*f*g*h)/(5*c) + 2*c*d*g*h + c*e*g**2
)/(3*c) + x*(a*d*h**2 + 2*a*e*g*h + a*f*g**2 - 3*a*(a*f*h**2/6 + c*d*h**2 +
 2*c*e*g*h + c*f*g**2)/(4*c) + c*d*g**2)/(2*c) + (2*a*d*g*h + a*e*g**2 - 2*
a*(a*e*h**2 + 2*a*f*g*h - 4*a*(c*e*h**2 + 2*c*f*g*h)/(5*c) + 2*c*d*g*h + c*
e*g**2)/(3*c))/c) + (a*d*g**2 - a*(a*d*h**2 + 2*a*e*g*h + a*f*g**2 - 3*a*(a
*f*h**2/6 + c*d*h**2 + 2*c*e*g*h + c*f*g**2)/(4*c) + c*d*g**2)/(2*c))*Piece
wise((log(2*sqrt(c)*sqrt(a + c*x**2) + 2*c*x)/sqrt(c), Ne(a, 0)), (x*log(x)
/sqrt(c*x**2), True)), Ne(c, 0)), (sqrt(a)*(d*g**2*x + f*h**2*x**5/5 + x**4
*(e*h**2 + 2*f*g*h)/4 + x**3*(d*h**2 + 2*e*g*h + f*g**2)/3 + x**2*(2*d*g*h
+ e*g**2)/2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.09

$$
\begin{aligned}
\int (g+hx)^2\sqrt{a+cx^2}\left(d+ex+fx^2\right)\,dx ={}& \frac{(cx^2+a)^{\frac{3}{2}}fh^2x^3}{6\,c} + \frac{1}{2}\sqrt{cx^2+a}dg^2x \\
&- \frac{(cx^2+a)^{\frac{3}{2}}afh^2x}{8\,c^2} + \frac{\sqrt{cx^2+a}a^2fh^2x}{16\,c^2} \\
&+ \frac{adg^2\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{2\sqrt{c}} + \frac{a^3fh^2\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{16\,c^{\frac{5}{2}}} \\
&+ \frac{(cx^2+a)^{\frac{3}{2}}eg^2}{3\,c} + \frac{2\,(cx^2+a)^{\frac{3}{2}}dgh}{3\,c} \\
&+ \frac{(2\,fgh+eh^2)(cx^2+a)^{\frac{3}{2}}x^2}{5\,c} \\
&+ \frac{(fg^2+2\,egh+dh^2)(cx^2+a)^{\frac{3}{2}}x}{4\,c} \\
&- \frac{(fg^2+2\,egh+dh^2)\sqrt{cx^2+a}ax}{8\,c} \\
&- \frac{(fg^2+2\,egh+dh^2)a^2\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{8\,c^{\frac{3}{2}}} \\
&- \frac{2\,(2\,fgh+eh^2)(cx^2+a)^{\frac{3}{2}}a}{15\,c^2}
\end{aligned}
$$

```
[In] integrate((h*x+g)^2*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*(c*x^2 + a)^(3/2)*f*h^2*x^3/c + 1/2*sqrt(c*x^2 + a)*d*g^2*x - 1/8*(c*x^
2 + a)^(3/2)*a*f*h^2*x/c^2 + 1/16*sqrt(c*x^2 + a)*a^2*f*h^2*x/c^2 + 1/2*a*d
*g^2*arcsinh(c*x/sqrt(a*c))/sqrt(c) + 1/16*a^3*f*h^2*arcsinh(c*x/sqrt(a*c))
/c^(5/2) + 1/3*(c*x^2 + a)^(3/2)*e*g^2/c + 2/3*(c*x^2 + a)^(3/2)*d*g*h/c +
```

```
1/5*(2*f*g*h + e*h^2)*(c*x^2 + a)^(3/2)*x^2/c + 1/4*(f*g^2 + 2*e*g*h + d*h^
2)*(c*x^2 + a)^(3/2)*x/c - 1/8*(f*g^2 + 2*e*g*h + d*h^2)*sqrt(c*x^2 + a)*a*
x/c - 1/8*(f*g^2 + 2*e*g*h + d*h^2)*a^2*arcsinh(c*x/sqrt(a*c))/c^(3/2) - 2/
15*(2*f*g*h + e*h^2)*(c*x^2 + a)^(3/2)*a/c^2
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.12

$$\int (g+hx)^2\sqrt{a+cx^2}\left(d+ex+fx^2\right)\,dx$$
$$=\frac{1}{240}\sqrt{cx^2+a}\Bigg(\Bigg(2\Bigg(\Bigg(\Bigg(4\left(5\,fh^2x+\frac{6\,(2\,c^4fgh+c^4eh^2)}{c^4}\right)x+\frac{5\,(6\,c^4fg^2+12\,c^4egh+6\,c^4dh^2+ac^3fh^2)}{c^4}\Bigg)x$$
$$-\frac{(8\,ac^2dg^2-2\,a^2cfg^2-4\,a^2cegh-2\,a^2cdh^2+a^3fh^2)\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+a}\right|\right)}{16\,c^{\frac{5}{2}}}$$

```
[In] integrate((h*x+g)^2*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/240*sqrt(c*x^2 + a)*((2*((4*(5*f*h^2*x + 6*(2*c^4*f*g*h + c^4*e*h^2)/c^4)
*x + 5*(6*c^4*f*g^2 + 12*c^4*e*g*h + 6*c^4*d*h^2 + a*c^3*f*h^2)/c^4)*x + 8*
(5*c^4*e*g^2 + 10*c^4*d*g*h + 2*a*c^3*f*g*h + a*c^3*e*h^2)/c^4)*x + 15*(8*c
^4*d*g^2 + 2*a*c^3*f*g^2 + 4*a*c^3*e*g*h + 2*a*c^3*d*h^2 - a^2*c^2*f*h^2)/c
^4)*x + 16*(5*a*c^3*e*g^2 + 10*a*c^3*d*g*h - 4*a^2*c^2*f*g*h - 2*a^2*c^2*e*
h^2)/c^4) - 1/16*(8*a*c^2*d*g^2 - 2*a^2*c*f*g^2 - 4*a^2*c*e*g*h - 2*a^2*c*d
*h^2 + a^3*f*h^2)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int (g+hx)^2\sqrt{a+cx^2}\left(d+ex+fx^2\right)\,dx=\int {\left(g+h\,x\right)}^2\,\sqrt{c\,x^2+a}\,\left(f\,x^2+e\,x+d\right)\,dx$$

```
[In] int((g + h*x)^2*(a + c*x^2)^(1/2)*(d + e*x + f*x^2),x)
```

```
[Out] int((g + h*x)^2*(a + c*x^2)^(1/2)*(d + e*x + f*x^2), x)
```

3.80 $\int (g+hx)\sqrt{a+cx^2}\,(d+ex+fx^2)\,dx$

Optimal result

Integrand size = 27, antiderivative size = 175

$$\begin{aligned}\int (g+hx)\sqrt{a+cx^2}\,(d+ex+fx^2)\,dx &= \frac{(4cdg-a(fg+eh))x\sqrt{a+cx^2}}{8c}+\frac{f(g+hx)^2\,(a+cx^2)^{3/2}}{5ch}\\ &\quad-\frac{(4(2afh^2+c(3fg^2-5h(eg+dh)))+3ch(3fg-5eh)x)\,(a+cx^2)^{3/2}}{60c^2h}\\ &\quad+\frac{a(4cdg-afg-aeh)\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{8c^{3/2}}\end{aligned}$$

```
[Out] 1/5*f*(h*x+g)^2*(c*x^2+a)^(3/2)/c/h-1/60*(8*a*f*h^2+4*c*(3*f*g^2-5*h*(d*h+e
*g))+3*c*h*(-5*e*h+3*f*g)*x)*(c*x^2+a)^(3/2)/c^2/h+1/8*a*(-a*e*h-a*f*g+4*c*
d*g)*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(3/2)+1/8*(4*c*d*g-a*(e*h+f*g))*x
*(c*x^2+a)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used

$= \{1668,\ 794,\ 201,\ 223,\ 212\}$

$$\int (g+hx)\sqrt{a+cx^2}\left(d+ex+fx^2\right)\,dx$$

$$= \frac{a\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)(-aeh-afg+4cdg)}{8c^{3/2}}$$

$$- \frac{\left(a+cx^2\right)^{3/2}\left(4\left(2afh^2+c\left(3fg^2-5h(dh+eg)\right)\right)+3chx(3fg-5eh)\right)}{60c^2h}$$

$$+ \frac{x\sqrt{a+cx^2}(4cdg-a(eh+fg))}{8c} + \frac{f\left(a+cx^2\right)^{3/2}(g+hx)^2}{5ch}$$

```
[In] Int[(g + h*x)*Sqrt[a + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] ((4*c*d*g - a*(f*g + e*h))*x*Sqrt[a + c*x^2])/(8*c) + (f*(g + h*x)^2*(a + c
*x^2)^(3/2))/(5*c*h) - ((4*(2*a*f*h^2 + c*(3*f*g^2 - 5*h*(e*g + d*h))) + 3*
c*h*(3*f*g - 5*e*h)*x)*(a + c*x^2)^(3/2))/(60*c^2*h) + (a*(4*c*d*g - a*f*g
- a*e*h)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(8*c^(3/2))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\begin{aligned}
&\text{integral}\\
&=\frac{f(g+hx)^2\left(a+cx^2\right)^{3/2}}{5ch}+\frac{\int (g+hx)\left((5cd-2af)h^2-ch(3fg-5eh)x\right)\sqrt{a+cx^2}\,dx}{5ch^2}\\
&=\frac{f(g+hx)^2\left(a+cx^2\right)^{3/2}}{5ch}\\
&\quad-\frac{\left(4(2afh^2+c(3fg^2-5h(eg+dh)))+3ch(3fg-5eh)x\right)\left(a+cx^2\right)^{3/2}}{60c^2h}\\
&\quad+\frac{(4cdg-afg-aeh)\int \sqrt{a+cx^2}\,dx}{4c}\\
&=\frac{(4cdg-a(fg+eh))x\sqrt{a+cx^2}}{8c}+\frac{f(g+hx)^2\left(a+cx^2\right)^{3/2}}{5ch}\\
&\quad-\frac{\left(4(2afh^2+c(3fg^2-5h(eg+dh)))+3ch(3fg-5eh)x\right)\left(a+cx^2\right)^{3/2}}{60c^2h}\\
&\quad+\frac{(a(4cdg-afg-aeh))\int \frac{1}{\sqrt{a+cx^2}}\,dx}{8c}\\
&=\frac{(4cdg-a(fg+eh))x\sqrt{a+cx^2}}{8c}+\frac{f(g+hx)^2\left(a+cx^2\right)^{3/2}}{5ch}\\
&\quad-\frac{\left(4(2afh^2+c(3fg^2-5h(eg+dh)))+3ch(3fg-5eh)x\right)\left(a+cx^2\right)^{3/2}}{60c^2h}\\
&\quad+\frac{(a(4cdg-afg-aeh))\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x}{\sqrt{a+cx^2}}\right)}{8c}\\
&=\frac{(4cdg-a(fg+eh))x\sqrt{a+cx^2}}{8c}+\frac{f(g+hx)^2\left(a+cx^2\right)^{3/2}}{5ch}\\
&\quad-\frac{\left(4(2afh^2+c(3fg^2-5h(eg+dh)))+3ch(3fg-5eh)x\right)\left(a+cx^2\right)^{3/2}}{60c^2h}\\
&\quad+\frac{a(4cdg-afg-aeh)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{8c^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.83

$$\int (g+hx)\sqrt{a+cx^2}\left(d+ex+fx^2\right)\,dx$$
$$=\frac{\sqrt{a+cx^2}(-16a^2fh+ac(40dh+5e(8g+3hx)+fx(15g+8hx))+2c^2x(10d(3g+2hx)+x(5e(4g+3hx)}{120c^2}$$

```
[In] Integrate[(g + h*x)*Sqrt[a + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] (Sqrt[a + c*x^2]*(-16*a^2*f*h + a*c*(40*d*h + 5*e*(8*g + 3*h*x) + f*x*(15*g
 + 8*h*x)) + 2*c^2*x*(10*d*(3*g + 2*h*x) + x*(5*e*(4*g + 3*h*x) + 3*f*x*(5*
g + 4*h*x)))) + 15*a*Sqrt[c]*(-4*c*d*g + a*f*g + a*e*h)*Log[-(Sqrt[c]*x) +
Sqrt[a + c*x^2]])/(120*c^2)
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.87

method	result
risch	$-\frac{\left(-24hf\,c^2x^4-30c^2eh\,x^3-30c^2fg\,x^3-8acfh\,x^2-40c^2dh\,x^2-40c^2eg\,x^2-15aehxc-15afgxc-60c^2dgx+16a^2fh-40acdh-40aceg\right)\surd}{120c^2}$
default	$dg\left(\frac{x\sqrt{c\,x^2+a}}{2}+\frac{a\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{2\sqrt{c}}\right)+hf\left(\frac{x^2\left(c\,x^2+a\right)^{\frac{3}{2}}}{5c}-\frac{2a\left(c\,x^2+a\right)^{\frac{3}{2}}}{15c^2}\right)+(eh+fg)\left(\frac{x\left(c\,x^2+a\right)^{\frac{3}{2}}}{4c}-\frac{a\left(x\surd\right.}{}\right.$

```
[In] int((h*x+g)*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/120*(-24*c^2*f*h*x^4-30*c^2*e*h*x^3-30*c^2*f*g*x^3-8*a*c*f*h*x^2-40*c^2*
d*h*x^2-40*c^2*e*g*x^2-15*a*c*e*h*x-15*a*c*f*g*x-60*c^2*d*g*x+16*a^2*f*h-40
*a*c*d*h-40*a*c*e*g)/c^2*(c*x^2+a)^(1/2)-1/8*a/c^(3/2)*(a*e*h+a*f*g-4*c*d*g
)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.45 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.88

$$\int (g+hx)\sqrt{a+cx^2}\left(d+ex+fx^2\right)\,dx$$
$$=\left[\frac{15\left(a^2eh-\left(4\,acd-a^2f\right)g\right)\sqrt{c}\log\left(-2\,cx^2+2\,\sqrt{cx^2+a}\sqrt{c}x-a\right)+2\left(24\,c^2fhx^4+40\,aceg+30\left(c^2fg+\right.\right.}{240}\right.$$

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/240*(15*(a^2*e*h - (4*a*c*d - a^2*f)*g)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*
x^2 + a)*sqrt(c)*x - a) + 2*(24*c^2*f*h*x^4 + 40*a*c*e*g + 30*(c^2*f*g + c^
2*e*h)*x^3 + 8*(5*c^2*e*g + (5*c^2*d + a*c*f)*h)*x^2 + 8*(5*a*c*d - 2*a^2*f
)*h + 15*(a*c*e*h + (4*c^2*d + a*c*f)*g)*x)*sqrt(c*x^2 + a))/c^2, 1/120*(15
*(a^2*e*h - (4*a*c*d - a^2*f)*g)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)
) + (24*c^2*f*h*x^4 + 40*a*c*e*g + 30*(c^2*f*g + c^2*e*h)*x^3 + 8*(5*c^2*e*
g + (5*c^2*d + a*c*f)*h)*x^2 + 8*(5*a*c*d - 2*a^2*f)*h + 15*(a*c*e*h + (4*c
^2*d + a*c*f)*g)*x)*sqrt(c*x^2 + a))/c^2]
```

Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.47

$$\int (g+hx)\sqrt{a+cx^2}\left(d+ex+fx^2\right)\,dx$$
$$=\begin{cases}\sqrt{a+cx^2}\left(\frac{fhx^4}{5}+\frac{x^3(ceh+cfg)}{4c}+\frac{x^2\left(\frac{afh}{5}+cdh+ceg\right)}{3c}+\frac{x\left(aeh+afg-\frac{3a(ceh+cfg)}{4c}+cdg\right)}{2c}+\frac{adh+aeg-\frac{2a\left(\frac{afh}{5}+cdh+ceg\right)}{3c}}{c}\right)+ \\ \sqrt{a}\left(dgx+\frac{fhx^4}{4}+\frac{x^3(eh+fg)}{3}+\frac{x^2(dh+eg)}{2}\right)\end{cases}$$

```
[In] integrate((h*x+g)*(f*x**2+e*x+d)*(c*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((sqrt(a + c*x**2)*(f*h*x**4/5 + x**3*(c*e*h + c*f*g)/(4*c) + x**2
*(a*f*h/5 + c*d*h + c*e*g)/(3*c) + x*(a*e*h + a*f*g - 3*a*(c*e*h + c*f*g)/(
4*c) + c*d*g)/(2*c) + (a*d*h + a*e*g - 2*a*(a*f*h/5 + c*d*h + c*e*g)/(3*c))
/c) + (a*d*g - a*(a*e*h + a*f*g - 3*a*(c*e*h + c*f*g)/(4*c) + c*d*g)/(2*c))
*Piecewise((log(2*sqrt(c)*sqrt(a + c*x**2) + 2*c*x)/sqrt(c), Ne(a, 0)), (x*
log(x)/sqrt(c*x**2), True)), Ne(c, 0)), (sqrt(a)*(d*g*x + f*h*x**4/4 + x**3
*(e*h + f*g)/3 + x**2*(d*h + e*g)/2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.97

$$\int (g+hx)\sqrt{a+cx^2}\left(d+ex+fx^2\right)\,dx = \frac{(cx^2+a)^{\frac{3}{2}}fhx^2}{5\,c} + \frac{1}{2}\sqrt{cx^2+a}dgx + \frac{adg\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{2\sqrt{c}} + \frac{(cx^2+a)^{\frac{3}{2}}eg}{3\,c} + \frac{(cx^2+a)^{\frac{3}{2}}dh}{3\,c} - \frac{2\,(cx^2+a)^{\frac{3}{2}}afh}{15\,c^2} + \frac{(cx^2+a)^{\frac{3}{2}}(fg+eh)x}{4\,c} - \frac{\sqrt{cx^2+a}(fg+eh)ax}{8\,c} - \frac{(fg+eh)a^2\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{8\,c^{\frac{3}{2}}}$$

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/5*(c*x^2 + a)^(3/2)*f*h*x^2/c + 1/2*sqrt(c*x^2 + a)*d*g*x + 1/2*a*d*g*arc
sinh(c*x/sqrt(a*c))/sqrt(c) + 1/3*(c*x^2 + a)^(3/2)*e*g/c + 1/3*(c*x^2 + a)
^(3/2)*d*h/c - 2/15*(c*x^2 + a)^(3/2)*a*f*h/c^2 + 1/4*(c*x^2 + a)^(3/2)*(f*
g + e*h)*x/c - 1/8*sqrt(c*x^2 + a)*(f*g + e*h)*a*x/c - 1/8*(f*g + e*h)*a^2*
arcsinh(c*x/sqrt(a*c))/c^(3/2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.00

$$\int (g+hx)\sqrt{a+cx^2}\left(d+ex+fx^2\right)\,dx = \frac{1}{120}\sqrt{cx^2+a}\Bigg(\Bigg(2\Bigg(3\left(4\,fhx+\frac{5\,(c^3fg+c^3eh)}{c^3}\right)x+\frac{4\,(5\,c^3eg+5\,c^3dh+ac^2fh)}{c^3}\Bigg)x+\frac{15\,(4\,c^3dg+ac^2fg}{c^3} - \frac{(4\,acdg-a^2fg-a^2eh)\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+a}\right|\right)}{8\,c^{\frac{3}{2}}}$$

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/120*sqrt(c*x^2 + a)*((2*(3*(4*f*h*x + 5*(c^3*f*g + c^3*e*h)/c^3)*x + 4*(5
*c^3*e*g + 5*c^3*d*h + a*c^2*f*h)/c^3)*x + 15*(4*c^3*d*g + a*c^2*f*g + a*c^
2*e*h)/c^3)*x + 8*(5*a*c^2*e*g + 5*a*c^2*d*h - 2*a^2*c*f*h)/c^3) - 1/8*(4*a
*c*d*g - a^2*f*g - a^2*e*h)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int (g+hx)\sqrt{a+cx^2}\left(d+ex+fx^2\right)\,dx=\int (g+h\,x)\,\sqrt{c\,x^2+a}\,\left(f\,x^2+e\,x+d\right)\,dx$$

```
[In] int((g + h*x)*(a + c*x^2)^(1/2)*(d + e*x + f*x^2),x)
[Out] int((g + h*x)*(a + c*x^2)^(1/2)*(d + e*x + f*x^2), x)
```

3.81 $\int \sqrt{a+cx^2}(d+ex+fx^2)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 106

$$\int \sqrt{a+cx^2}\left(d+ex+fx^2\right)\,dx=\frac{(4cd-af)x\sqrt{a+cx^2}}{8c}+\frac{e\left(a+cx^2\right)^{3/2}}{3c}+\frac{fx\left(a+cx^2\right)^{3/2}}{4c}+\frac{a(4cd-af)\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{8c^{3/2}}$$

```
[Out] 1/3*e*(c*x^2+a)^(3/2)/c+1/4*f*x*(c*x^2+a)^(3/2)/c+1/8*a*(-a*f+4*c*d)*arctan
h(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(3/2)+1/8*(-a*f+4*c*d)*x*(c*x^2+a)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1829, 655, 201, 223, 212}

$$\int \sqrt{a+cx^2}\left(d+ex+fx^2\right)\,dx=\frac{a\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)(4cd-af)}{8c^{3/2}}+\frac{x\sqrt{a+cx^2}(4cd-af)}{8c}+\frac{e\left(a+cx^2\right)^{3/2}}{3c}+\frac{fx\left(a+cx^2\right)^{3/2}}{4c}$$

```
[In] Int[Sqrt[a + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] ((4*c*d - a*f)*x*Sqrt[a + c*x^2])/(8*c) + (e*(a + c*x^2)^(3/2))/(3*c) + (f*
x*(a + c*x^2)^(3/2))/(4*c) + (a*(4*c*d - a*f)*ArcTanh[(Sqrt[c]*x)/Sqrt[a +
c*x^2]])/(8*c^(3/2))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1829

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{fx(a+cx^2)^{3/2}}{4c}+\frac{\int (4cd-af+4cex)\sqrt{a+cx^2}\,dx}{4c}\\
&= \frac{e(a+cx^2)^{3/2}}{3c}+\frac{fx(a+cx^2)^{3/2}}{4c}+\frac{(4cd-af)\int \sqrt{a+cx^2}\,dx}{4c}\\
&= \frac{(4cd-af)x\sqrt{a+cx^2}}{8c}+\frac{e(a+cx^2)^{3/2}}{3c}+\frac{fx(a+cx^2)^{3/2}}{4c}+\frac{(a(4cd-af))\int \frac{1}{\sqrt{a+cx^2}}\,dx}{8c}\\
&= \frac{(4cd-af)x\sqrt{a+cx^2}}{8c}+\frac{e(a+cx^2)^{3/2}}{3c}+\frac{fx(a+cx^2)^{3/2}}{4c}\\
&\quad+\frac{(a(4cd-af))\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x}{\sqrt{a+cx^2}}\right)}{8c}
\end{aligned}
$$

$$=\frac{(4cd-af)x\sqrt{a+cx^2}}{8c}+\frac{e(a+cx^2)^{3/2}}{3c}+\frac{fx(a+cx^2)^{3/2}}{4c}+\frac{a(4cd-af)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{8c^{3/2}}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.82

$$\int\sqrt{a+cx^2}\left(d+ex+fx^2\right)\,dx=\frac{\sqrt{a+cx^2}(8ae+12cdx+3afx+8cex^2+6cfx^3)}{24c}+\frac{a(-4cd+af)\log\left(-\sqrt{c}x+\sqrt{a+cx^2}\right)}{8c^{3/2}}$$

```
[In] Integrate[Sqrt[a + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] (Sqrt[a + c*x^2]*(8*a*e + 12*c*d*x + 3*a*f*x + 8*c*e*x^2 + 6*c*f*x^3))/(24*
c) + (a*(-4*c*d + a*f)*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/(8*c^(3/2))
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.71

method	result	size
risch	$\frac{(6cf\,x^3+8ce\,x^2+3afx+12cdx+8ae)\sqrt{c\,x^2+a}}{24c}-\frac{a(fa-4cd)\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{8c^{\frac{3}{2}}}$	75
default	$d\left(\frac{x\sqrt{c\,x^2+a}}{2}+\frac{a\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{2\sqrt{c}}\right)+f\left(\frac{x(c\,x^2+a)^{\frac{3}{2}}}{4c}-\frac{a\left(\frac{x\sqrt{c\,x^2+a}}{2}+\frac{a\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{2\sqrt{c}}\right)}{4c}\right)+\frac{e(c\,x^2+a)^{\frac{3}{2}}}{3c}$	113

```
[In] int((f*x^2+e*x+d)*(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*(6*c*f*x^3+8*c*e*x^2+3*a*f*x+12*c*d*x+8*a*e)/c*(c*x^2+a)^(1/2)-1/8*a*(
a*f-4*c*d)/c^(3/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.79

$$\int \sqrt{a+cx^2}\left(d+ex+fx^2\right)\,dx$$
$$=\left[-\frac{3\,(4\,acd-a^2f)\sqrt{c}\log\left(-2\,cx^2+2\,\sqrt{cx^2+a}\sqrt{c}x-a\right)-2\,(6\,c^2fx^3+8\,c^2ex^2+8\,ace+3\,(4\,c^2d+acf)}{48\,c^2}\right.$$
$$\left.-\frac{3\,(4\,acd-a^2f)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}x}{\sqrt{cx^2+a}}\right)-(6\,c^2fx^3+8\,c^2ex^2+8\,ace+3\,(4\,c^2d+acf)x)\sqrt{cx^2+a}}{24\,c^2}\right]$$

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/48*(3*(4*a*c*d - a^2*f)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c
)*x - a) - 2*(6*c^2*f*x^3 + 8*c^2*e*x^2 + 8*a*c*e + 3*(4*c^2*d + a*c*f)*x)*
sqrt(c*x^2 + a))/c^2, -1/24*(3*(4*a*c*d - a^2*f)*sqrt(-c)*arctan(sqrt(-c)*x
/sqrt(c*x^2 + a)) - (6*c^2*f*x^3 + 8*c^2*e*x^2 + 8*a*c*e + 3*(4*c^2*d + a*c
*f)*x)*sqrt(c*x^2 + a))/c^2]
```

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.17

$$\int \sqrt{a+cx^2}\left(d+ex+fx^2\right)\,dx$$
$$=\begin{cases}\sqrt{a+cx^2}\left(\frac{ae}{3c}+\frac{ex^2}{3}+\frac{fx^3}{4}+\frac{x\left(\frac{af}{4}+cd\right)}{2c}\right)+\left(ad-\frac{a\left(\frac{af}{4}+cd\right)}{2c}\right)\left(\begin{cases}\frac{\log\left(2\sqrt{c}\sqrt{a+cx^2}+2cx\right)}{\sqrt{c}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{cx^2}} & \text{otherwise}\end{cases}\right) & \text{for } c\\ \sqrt{a}\left(dx+\frac{ex^2}{2}+\frac{fx^3}{3}\right) & \text{othe}\end{cases}$$

```
[In] integrate((f*x**2+e*x+d)*(c*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((sqrt(a + c*x**2)*(a*e/(3*c) + e*x**2/3 + f*x**3/4 + x*(a*f/4 + c
*d)/(2*c)) + (a*d - a*(a*f/4 + c*d)/(2*c))*Piecewise((log(2*sqrt(c)*sqrt(a
+ c*x**2) + 2*c*x)/sqrt(c), Ne(a, 0)), (x*log(x)/sqrt(c*x**2), True)), Ne(c
, 0)), (sqrt(a)*(d*x + e*x**2/2 + f*x**3/3), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.91

$$\int \sqrt{a+cx^2}\left(d+ex+fx^2\right)\,dx = \frac{1}{2}\sqrt{cx^2+a}dx + \frac{(cx^2+a)^{\frac{3}{2}}fx}{4\,c} - \frac{\sqrt{cx^2+a}afx}{8\,c} + \frac{ad\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{2\sqrt{c}} - \frac{a^2f\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{8\,c^{\frac{3}{2}}} + \frac{(cx^2+a)^{\frac{3}{2}}e}{3\,c}$$

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(c*x^2 + a)*d*x + 1/4*(c*x^2 + a)^(3/2)*f*x/c - 1/8*sqrt(c*x^2 + a)
*a*f*x/c + 1/2*a*d*arcsinh(c*x/sqrt(a*c))/sqrt(c) - 1/8*a^2*f*arcsinh(c*x/s
qrt(a*c))/c^(3/2) + 1/3*(c*x^2 + a)^(3/2)*e/c
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.80

$$\int \sqrt{a+cx^2}\left(d+ex+fx^2\right)\,dx = \frac{1}{24}\sqrt{cx^2+a}\left(\left(2\,(3\,fx+4\,e)x + \frac{3\,(4\,c^2d+acf)}{c^2}\right)x + \frac{8\,ae}{c}\right) - \frac{(4\,acd-a^2f)\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+a}\right|\right)}{8\,c^{\frac{3}{2}}}$$

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*sqrt(c*x^2 + a)*((2*(3*f*x + 4*e)*x + 3*(4*c^2*d + a*c*f)/c^2)*x + 8*a
*e/c) - 1/8*(4*a*c*d - a^2*f)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c^(3/2
)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+cx^2}\left(d+ex+fx^2\right)\,dx = \int \sqrt{c\,x^2+a}\,\left(f\,x^2+e\,x+d\right)\,dx$$

```
[In] int((a + c*x^2)^(1/2)*(d + e*x + f*x^2),x)
```

```
[Out] int((a + c*x^2)^(1/2)*(d + e*x + f*x^2), x)
```

3.82 $\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{g+hx}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 206

$$\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{g+hx}\,dx = \frac{(2(fg^2-egh+dh^2)-h(fg-eh)x)\sqrt{a+cx^2}}{2h^3} + \frac{f(a+cx^2)^{3/2}}{3ch} - \frac{(2cdgh^2+(fg-eh)(2cg^2+ah^2))\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2\sqrt{c}h^4} - \frac{\sqrt{cg^2+ah^2}(fg^2-egh+dh^2)\operatorname{arctanh}\left(\frac{ah-cgx}{\sqrt{cg^2+ah^2}\sqrt{a+cx^2}}\right)}{h^4}$$

```
[Out] 1/3*f*(c*x^2+a)^(3/2)/c/h-1/2*(2*c*d*g*h^2+(-e*h+f*g)*(a*h^2+2*c*g^2))*arct
anh(x*c^(1/2)/(c*x^2+a)^(1/2))/h^4/c^(1/2)-(d*h^2-e*g*h+f*g^2)*arctanh((-c*
g*x+a*h)/(a*h^2+c*g^2)^(1/2)/(c*x^2+a)^(1/2))*(a*h^2+c*g^2)^(1/2)/h^4+1/2*(
2*d*h^2-2*e*g*h+2*f*g^2-h*(-e*h+f*g)*x)*(c*x^2+a)^(1/2)/h^3
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used

= {1668, 829, 858, 223, 212, 739}

$$\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{g+hx}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left((ah^2+2cg^2)\,(fg-eh)+2cdgh^2\right)}{2\sqrt{c}h^4} - \frac{\sqrt{ah^2+cg^2}(dh^2-egh+fg^2)\operatorname{arctanh}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)}{h^4} + \frac{\sqrt{a+cx^2}(2(dh^2-egh+fg^2)-hx(fg-eh))}{2h^3} + \frac{f(a+cx^2)^{3/2}}{3ch}$$

```
[In] Int[(Sqrt[a + c*x^2]*(d + e*x + f*x^2))/(g + h*x),x]
```

```
[Out] ((2*(f*g^2 - e*g*h + d*h^2) - h*(f*g - e*h)*x)*Sqrt[a + c*x^2])/(2*h^3) + (
f*(a + c*x^2)^(3/2))/(3*c*h) - ((2*c*d*g*h^2 + (f*g - e*h)*(2*c*g^2 + a*h^2
))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*Sqrt[c]*h^4) - (Sqrt[c*g^2 + a*
h^2]*(f*g^2 - e*g*h + d*h^2)*ArcTanh[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqr
t[a + c*x^2])])/h^4
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
```

```
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{f\left(a+cx^2\right)^{3/2}}{3ch}+\frac{\int \frac{\left(3cdh^2-3ch(fg-eh)x\right)\sqrt{a+cx^2}}{g+hx}\,dx}{3ch^2}\\
&= \frac{\left(2\left(fg^2-egh+dh^2\right)-h(fg-eh)x\right)\sqrt{a+cx^2}}{2h^3}+\frac{f\left(a+cx^2\right)^{3/2}}{3ch}\\
&\quad+\frac{\int \frac{3ac^2h^2\left(fg^2-h(eg-2dh)\right)-3c^2h\left(2cdgh^2+(fg-eh)\left(2cg^2+ah^2\right)\right)x}{(g+hx)\sqrt{a+cx^2}}\,dx}{6c^2h^4}\\
&= \frac{\left(2\left(fg^2-egh+dh^2\right)-h(fg-eh)x\right)\sqrt{a+cx^2}}{2h^3}+\frac{f\left(a+cx^2\right)^{3/2}}{3ch}\\
&\quad+\frac{\left(\left(cg^2+ah^2\right)\left(fg^2-egh+dh^2\right)\right)\int \frac{1}{(g+hx)\sqrt{a+cx^2}}\,dx}{h^4}\\
&\quad-\frac{\left(2cdgh^2+(fg-eh)\left(2cg^2+ah^2\right)\right)\int \frac{1}{\sqrt{a+cx^2}}\,dx}{2h^4}\\
&= \frac{\left(2\left(fg^2-egh+dh^2\right)-h(fg-eh)x\right)\sqrt{a+cx^2}}{2h^3}+\frac{f\left(a+cx^2\right)^{3/2}}{3ch}\\
&\quad-\frac{\left(\left(cg^2+ah^2\right)\left(fg^2-egh+dh^2\right)\right)\operatorname{Subst}\left(\int \frac{1}{cg^2+ah^2-x^2}\,dx,x,\frac{ah-cgx}{\sqrt{a+cx^2}}\right)}{h^4}\\
&\quad-\frac{\left(2cdgh^2+(fg-eh)\left(2cg^2+ah^2\right)\right)\operatorname{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x}{\sqrt{a+cx^2}}\right)}{2h^4}
\end{aligned}
$$

$$= \frac{(2(fg^2-egh+dh^2)-h(fg-eh)x)\sqrt{a+cx^2}}{2h^3}+\frac{f(a+cx^2)^{3/2}}{3ch} - \frac{(2cdgh^2+(fg-eh)(2cg^2+ah^2))\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2\sqrt{c}h^4} - \frac{\sqrt{cg^2+ah^2}(fg^2-egh+dh^2)\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{cg^2+ah^2}\sqrt{a+cx^2}}\right)}{h^4}$$

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.03

$$\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{g+hx}\,dx$$
$$= \frac{\frac{h\sqrt{a+cx^2}(2afh^2+3ch(-2eg+2dh+ehx)+cf(6g^2-3ghx+2h^2x^2))}{c}+12\sqrt{-cg^2-ah^2}(fg^2+h(-eg+dh))\arctan\left(\frac{\sqrt{c}(g+hx)-}{\sqrt{-cg^2}}\right.}{6h^4}$$

```
[In] Integrate[(Sqrt[a + c*x^2]*(d + e*x + f*x^2))/(g + h*x),x]
```

```
[Out] ((h*Sqrt[a + c*x^2]*(2*a*f*h^2 + 3*c*h*(-2*e*g + 2*d*h + e*h*x) + c*f*(6*g^
2 - 3*g*h*x + 2*h^2*x^2)))/c + 12*Sqrt[-(c*g^2) - a*h^2]*(f*g^2 + h*(-(e*g)
 + d*h))*ArcTan[(Sqrt[c]*(g + h*x) - h*Sqrt[a + c*x^2])/Sqrt[-(c*g^2) - a*h
^2]] + (3*(a*h^2*(f*g - e*h) + 2*c*(f*g^3 + g*h*(-(e*g) + d*h)))*Log[-(Sqrt
[c]*x) + Sqrt[a + c*x^2]])/Sqrt[c])/(6*h^4)
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.53

method	result
risch	$\frac{(2f\,h^2c\,x^2+3ce\,h^2x-3cfghx+2af\,h^2+6cd\,h^2-6cegh+6cf\,g^2)\sqrt{c\,x^2+a}}{6c\,h^3}+\frac{\frac{(ae\,h^3-afg\,h^2-2cdg\,h^2+2ce\,g^2h-2cf\,g^3)\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{h\sqrt{c}}}{}$
default	$\frac{eh\left(\frac{x\sqrt{c\,x^2+a}}{2}+\frac{a\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{2\sqrt{c}}\right)+\frac{fh\left(c\,x^2+a\right)^{\frac{3}{2}}}{3c}-fg\left(\frac{x\sqrt{c\,x^2+a}}{2}+\frac{a\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{2\sqrt{c}}\right)}{h^2}+\frac{(d\,h^2-egh+f\,g^2)\left(\sqrt{\left(x+\frac{g}{h}\right)^2c-\frac{2cg}{}}\right.}{}$

```
[In] int((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(2*c*f*h^2*x^2+3*c*e*h^2*x-3*c*f*g*h*x+2*a*f*h^2+6*c*d*h^2-6*c*e*g*h+6*
c*f*g^2)/c*(c*x^2+a)^(1/2)/h^3+1/2/h^3*((a*e*h^3-a*f*g*h^2-2*c*d*g*h^2+2*c*
e*g^2*h-2*c*f*g^3)/h*ln(x*c^(1/2)+(c*x^2+a)^(1/2))/c^(1/2)-(2*a*d*h^4-2*a*e
*g*h^3+2*a*f*g^2*h^2+2*c*d*g^2*h^2-2*c*e*g^3*h+2*c*f*g^4)/h^2/((a*h^2+c*g^2
)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1/h*g)+2*((a*h^2+c*g^2)/h^2
)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+1/h*g
)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{g+hx}\,dx = \text{Timed out}$$

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{g+hx}\,dx = \int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{g+hx}\,dx$$

```
[In] integrate((f*x**2+e*x+d)*(c*x**2+a)**(1/2)/(h*x+g),x)
```

```
[Out] Integral(sqrt(a + c*x**2)*(d + e*x + f*x**2)/(g + h*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.76

$$\begin{aligned}\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{g+hx}\,dx = &-\frac{\sqrt{cx^2+a}fgx}{2\,h^2}+\frac{\sqrt{cx^2+a}ex}{2\,h}-\frac{\sqrt{c}fg^3\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{h^4}\\&+\frac{\sqrt{c}eg^2\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{h^3}-\frac{\sqrt{c}dg\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{h^2}\\&-\frac{afg\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{2\,\sqrt{c}h^2}+\frac{ae\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{2\,\sqrt{c}h}\\&+\frac{\sqrt{a+\frac{cg^2}{h^2}}fg^2\operatorname{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|}-\frac{ah}{\sqrt{ac}|hx+g|}\right)}{h^3}\\&-\frac{\sqrt{a+\frac{cg^2}{h^2}}eg\operatorname{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|}-\frac{ah}{\sqrt{ac}|hx+g|}\right)}{h^2}\\&+\frac{\sqrt{a+\frac{cg^2}{h^2}}d\operatorname{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|}-\frac{ah}{\sqrt{ac}|hx+g|}\right)}{h}\\&+\frac{\sqrt{cx^2+a}fg^2}{h^3}-\frac{\sqrt{cx^2+a}eg}{h^2}+\frac{\sqrt{cx^2+a}d}{h}+\frac{(cx^2+a)^{\frac{3}{2}}f}{3\,ch}\end{aligned}$$

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(c*x^2 + a)*f*g*x/h^2 + 1/2*sqrt(c*x^2 + a)*e*x/h - sqrt(c)*f*g^3*
arcsinh(c*x/sqrt(a*c))/h^4 + sqrt(c)*e*g^2*arcsinh(c*x/sqrt(a*c))/h^3 - sqr
t(c)*d*g*arcsinh(c*x/sqrt(a*c))/h^2 - 1/2*a*f*g*arcsinh(c*x/sqrt(a*c))/(sqr
t(c)*h^2) + 1/2*a*e*arcsinh(c*x/sqrt(a*c))/(sqrt(c)*h) + sqrt(a + c*g^2/h^2
)*f*g^2*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g
)))/h^3 - sqrt(a + c*g^2/h^2)*e*g*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) -
a*h/(sqrt(a*c)*abs(h*x + g)))/h^2 + sqrt(a + c*g^2/h^2)*d*arcsinh(c*g*x/(sq
rt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/h + sqrt(c*x^2 + a)*f
*g^2/h^3 - sqrt(c*x^2 + a)*e*g/h^2 + sqrt(c*x^2 + a)*d/h + 1/3*(c*x^2 + a)^
(3/2)*f/(c*h)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{g+hx}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{g+hx}\,dx = \int \frac{\sqrt{c\,x^2+a}\,\left(f\,x^2+e\,x+d\right)}{g+h\,x}\,dx$$

```
[In] int(((a + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x),x)
```

```
[Out] int(((a + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x), x)
```

3.83 $\int \frac{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}{(g+hx)^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 308

$$\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^2}\,dx = -\frac{(2(ah^2(2fg-eh)+cg(3fg^2-h(2eg-dh)))-h(afh^2+c(3fg^2-2h(eg-dh)))\,x)\sqrt{a+cx^2}}{2h^3\left(cg^2+ah^2\right)} - \frac{\left(fg^2-egh+dh^2\right)\left(a+cx^2\right)^{3/2}}{h\left(cg^2+ah^2\right)(g+hx)} + \frac{(afh^2+2c(3fg^2-h(2eg-dh)))\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2\sqrt{c}h^4} + \frac{(ah^2(2fg-eh)+cg(3fg^2-h(2eg-dh)))\operatorname{arctanh}\left(\frac{ah-cgx}{\sqrt{cg^2+ah^2}\sqrt{a+cx^2}}\right)}{h^4\sqrt{cg^2+ah^2}}$$

```
[Out] -(d*h^2-e*g*h+f*g^2)*(c*x^2+a)^(3/2)/h/(a*h^2+c*g^2)/(h*x+g)+1/2*(a*f*h^2+2
*c*(3*f*g^2-h*(-d*h+2*e*g)))*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/h^4/c^(1/2)
+(a*h^2*(-e*h+2*f*g)+c*g*(3*f*g^2-h*(-d*h+2*e*g)))*arctanh((-c*g*x+a*h)/(a*
h^2+c*g^2)^(1/2)/(c*x^2+a)^(1/2))/h^4/(a*h^2+c*g^2)^(1/2)-1/2*(2*a*h^2*(-e*
h+2*f*g)+2*c*g*(3*f*g^2-h*(-d*h+2*e*g))-h*(a*f*h^2+c*(3*f*g^2-2*h*(-d*h+e*g
)))*x)*(c*x^2+a)^(1/2)/h^3/(a*h^2+c*g^2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1665, 829, 858, 223, 212, 739}

$$\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^2}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)(afh^2-2ch(2eg-dh)+6cfg^2)}{2\sqrt{c}h^4} + \frac{\operatorname{arctanh}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)(ah^2(2fg-eh)-cgh(2eg-dh)+3cfg^3)}{h^4\sqrt{ah^2+cg^2}} - \frac{(a+cx^2)^{3/2}(dh^2-egh+fg^2)}{h(g+hx)(ah^2+cg^2)} - \frac{\sqrt{a+cx^2}(2(ah^2(2fg-eh)-cgh(2eg-dh)+3cfg^3)-hx(afh^2-2ch(eg-dh)+3cfg^2))}{2h^3(ah^2+cg^2)}$$

```
[In] Int[(Sqrt[a + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^2,x]
```

```
[Out] -1/2*((2*(3*c*f*g^3 - c*g*h*(2*e*g - d*h) + a*h^2*(2*f*g - e*h)) - h*(3*c*f
*g^2 + a*f*h^2 - 2*c*h*(e*g - d*h))*x)*Sqrt[a + c*x^2])/(h^3*(c*g^2 + a*h^2
)) - ((f*g^2 - e*g*h + d*h^2)*(a + c*x^2)^(3/2))/(h*(c*g^2 + a*h^2)*(g + h*
x)) + ((6*c*f*g^2 + a*f*h^2 - 2*c*h*(2*e*g - d*h))*ArcTanh[(Sqrt[c]*x)/Sqrt
[a + c*x^2]])/(2*Sqrt[c]*h^4) + ((3*c*f*g^3 - c*g*h*(2*e*g - d*h) + a*h^2*(
2*f*g - e*h))*ArcTanh[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2])])
/(h^4*Sqrt[c*g^2 + a*h^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
```

```
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\left(fg^2-egh+dh^2\right)\left(a+cx^2\right)^{3/2}}{h\left(cg^2+ah^2\right)(g+hx)}-\frac{\int \frac{\left(-cdg+afg-aeh-\left(afh-c\left(2eg-\frac{3fg^2}{h}-2dh\right)\right)x\right)\sqrt{a+cx^2}}{g+hx}\,dx}{cg^2+ah^2}\\
&= -\frac{\left(2\left(3cfg^3-cgh(2eg-dh)+ah^2(2fg-eh)\right)-h\left(3cfg^2+afh^2-2ch(eg-dh)\right)x\right)\sqrt{a+cx^2}}{2h^3\left(cg^2+ah^2\right)}\\
&\quad -\frac{\left(fg^2-egh+dh^2\right)\left(a+cx^2\right)^{3/2}}{h\left(cg^2+ah^2\right)(g+hx)}\\
&\quad -\frac{\int \frac{ac(3fg-2eh)\left(cg^2+ah^2\right)-\frac{c\left(cg^2+ah^2\right)\left(6cfg^2+afh^2-2ch(2eg-dh)\right)x}{h}}{(g+hx)\sqrt{a+cx^2}}\,dx}{2ch^2\left(cg^2+ah^2\right)}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{\left(2(3cfg^3-cgh(2eg-dh)+ah^2(2fg-eh))-h(3cfg^2+afh^2-2ch(eg-dh))\,x\right)\sqrt{a+cx^2}}{2h^3\left(cg^2+ah^2\right)} \\
&\quad -\frac{\left(fg^2-egh+dh^2\right)\left(a+cx^2\right)^{3/2}}{h\left(cg^2+ah^2\right)(g+hx)}+\frac{\left(6cfg^2+afh^2-2ch(2eg-dh)\right)\int\frac{1}{\sqrt{a+cx^2}}\,dx}{2h^4} \\
&\quad -\frac{\left(3cfg^3-cgh(2eg-dh)+ah^2(2fg-eh)\right)\int\frac{1}{(g+hx)\sqrt{a+cx^2}}\,dx}{h^4} \\
&= -\frac{\left(2(3cfg^3-cgh(2eg-dh)+ah^2(2fg-eh))-h(3cfg^2+afh^2-2ch(eg-dh))\,x\right)\sqrt{a+cx^2}}{2h^3\left(cg^2+ah^2\right)} \\
&\quad -\frac{\left(fg^2-egh+dh^2\right)\left(a+cx^2\right)^{3/2}}{h\left(cg^2+ah^2\right)(g+hx)} \\
&\quad +\frac{\left(6cfg^2+afh^2-2ch(2eg-dh)\right)\mathrm{Subst}\left(\int\frac{1}{1-cx^2}\,dx,x,\frac{x}{\sqrt{a+cx^2}}\right)}{2h^4} \\
&\quad +\frac{\left(3cfg^3-cgh(2eg-dh)+ah^2(2fg-eh)\right)\mathrm{Subst}\left(\int\frac{1}{cg^2+ah^2-x^2}\,dx,x,\frac{ah-cgx}{\sqrt{a+cx^2}}\right)}{h^4} \\
&= -\frac{\left(2(3cfg^3-cgh(2eg-dh)+ah^2(2fg-eh))-h(3cfg^2+afh^2-2ch(eg-dh))\,x\right)\sqrt{a+cx^2}}{2h^3\left(cg^2+ah^2\right)} \\
&\quad -\frac{\left(fg^2-egh+dh^2\right)\left(a+cx^2\right)^{3/2}}{h\left(cg^2+ah^2\right)(g+hx)} \\
&\quad +\frac{\left(6cfg^2+afh^2-2ch(2eg-dh)\right)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2\sqrt{c}h^4} \\
&\quad +\frac{\left(3cfg^3-cgh(2eg-dh)+ah^2(2fg-eh)\right)\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{cg^2+ah^2}\sqrt{a+cx^2}}\right)}{h^4\sqrt{cg^2+ah^2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.70

$$
\int\frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^2}\,dx
$$

$$
=\frac{\frac{h\sqrt{a+cx^2}\left(2h(2eg-dh+ehx)+f\left(-6g^2-3ghx+h^2x^2\right)\right)}{g+hx}+\frac{4\left(3cfg^3+cgh(-2eg+dh)+ah^2(2fg-eh)\right)\arctan\left(\frac{\sqrt{c}(g+hx)-h\sqrt{a+cx^2}}{\sqrt{-cg^2-ah^2}}\right)}{\sqrt{-cg^2-ah^2}}-(6cfg^2+}{2h^4}
$$

```
[In] Integrate[(Sqrt[a + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^2,x]
```

```
[Out] ((h*Sqrt[a + c*x^2]*(2*h*(2*e*g - d*h + e*h*x) + f*(-6*g^2 - 3*g*h*x + h^2*
x^2)))/(g + h*x) + (4*(3*c*f*g^3 + c*g*h*(-2*e*g + d*h) + a*h^2*(2*f*g - e*
```

```
h))*ArcTan[(Sqrt[c]*(g + h*x) - h*Sqrt[a + c*x^2])/Sqrt[-(c*g^2) - a*h^2]])
/Sqrt[-(c*g^2) - a*h^2] - ((6*c*f*g^2 + a*f*h^2 + 2*c*h*(-2*e*g + d*h))*Log
[-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/Sqrt[c])/(2*h^4)
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 515, normalized size of antiderivative = 1.67

method	result
risch	$\frac{(fxh+2eh-4fg)\sqrt{c\,x^{2}+a}}{2h^{3}}+\frac{\frac{\left(af\,h^{2}+2cd\,h^{2}-4cegh+6cf\,g^{2}\right)\ln\left(x\sqrt{c}+\sqrt{c\,x^{2}+a}\right)}{h\sqrt{c}}-\frac{\left(2ae\,h^{3}-4afg\,h^{2}-4cdg\,h^{2}+6ce\,g^{2}h-8cf\,g^{3}\right)\ln\left(\frac{2a\,h^{2}+2c}{h^{2}}\right.}{h}}{}$
default	$\frac{f\left(\frac{x\sqrt{c\,x^{2}+a}}{2}+\frac{a\ln\left(x\sqrt{c}+\sqrt{c\,x^{2}+a}\right)}{2\sqrt{c}}\right)}{h^{2}}+\frac{(eh-2fg)\left(\sqrt{\left(x+\frac{g}{h}\right)^{2}c-\frac{2cg\left(x+\frac{g}{h}\right)}{h}+\frac{a\,h^{2}+c\,g^{2}}{h^{2}}}-\frac{\sqrt{c}\,g\ln\left(\frac{-\frac{cg}{h}+c\left(x+\frac{g}{h}\right)}{\sqrt{c}}+\sqrt{\left(x+\frac{g}{h}\right)^{2}c-\frac{2cg\left(x+\right.}{h}}\right.}{h}\right.}{}$

```
[In] int((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(f*h*x+2*e*h-4*f*g)*(c*x^2+a)^(1/2)/h^3+1/2/h^3*((a*f*h^2+2*c*d*h^2-4*c
*e*g*h+6*c*f*g^2)/h*ln(x*c^(1/2)+(c*x^2+a)^(1/2))/c^(1/2)-(2*a*e*h^3-4*a*f*
g*h^2-4*c*d*g*h^2+6*c*e*g^2*h-8*c*f*g^3)/h^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((
2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1/h*g)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+1/h*g
)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+1/h*g))+1/h^3*(2*a*d*h
^4-2*a*e*g*h^3+2*a*f*g^2*h^2+2*c*d*g^2*h^2-2*c*e*g^3*h+2*c*f*g^4)*(-1/(a*h^
2+c*g^2)*h^2/(x+1/h*g)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^
(1/2)-c*g*h/(a*h^2+c*g^2)/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2
-2*c*g/h*(x+1/h*g)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/
h*g)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+1/h*g))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^2}\,dx = \text{Timed out}$$

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^2}\,dx = \int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{\left(g+hx\right)^2}\,dx$$

```
[In] integrate((f*x**2+e*x+d)*(c*x**2+a)**(1/2)/(h*x+g)**2,x)
[Out] Integral(sqrt(a + c*x**2)*(d + e*x + f*x**2)/(g + h*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 478, normalized size of antiderivative = 1.55

$$\begin{aligned}
\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^2}\,dx = &-\frac{\sqrt{cx^2+a}fg^2}{h^4x+gh^3}+\frac{\sqrt{cx^2+a}eg}{h^3x+gh^2}-\frac{\sqrt{cx^2+a}d}{h^2x+gh} \\
&+\frac{\sqrt{cx^2+a}fx}{2\,h^2}+\frac{3\,\sqrt{c}fg^2\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{h^4} \\
&-\frac{2\,\sqrt{c}eg\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{h^3}+\frac{\sqrt{c}d\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{h^2} \\
&+\frac{af\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{2\,\sqrt{c}h^2}-\frac{cfg^3\operatorname{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|}-\frac{ah}{\sqrt{ac}|hx+g|}\right)}{\sqrt{a+\frac{cg^2}{h^2}}h^5} \\
&+\frac{ceg^2\operatorname{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|}-\frac{ah}{\sqrt{ac}|hx+g|}\right)}{\sqrt{a+\frac{cg^2}{h^2}}h^4} \\
&-\frac{cdg\operatorname{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|}-\frac{ah}{\sqrt{ac}|hx+g|}\right)}{\sqrt{a+\frac{cg^2}{h^2}}h^3} \\
&-\frac{2\,\sqrt{a+\frac{cg^2}{h^2}}fg\operatorname{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|}-\frac{ah}{\sqrt{ac}|hx+g|}\right)}{h^3} \\
&+\frac{\sqrt{a+\frac{cg^2}{h^2}}e\operatorname{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|}-\frac{ah}{\sqrt{ac}|hx+g|}\right)}{h^2} \\
&-\frac{2\,\sqrt{cx^2+a}fg}{h^3}+\frac{\sqrt{cx^2+a}e}{h^2}
\end{aligned}$$

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^2,x, algorithm="maxima")
```

```
[Out] -sqrt(c*x^2 + a)*f*g^2/(h^4*x + g*h^3) + sqrt(c*x^2 + a)*e*g/(h^3*x + g*h^2
) - sqrt(c*x^2 + a)*d/(h^2*x + g*h) + 1/2*sqrt(c*x^2 + a)*f*x/h^2 + 3*sqrt(
c)*f*g^2*arcsinh(c*x/sqrt(a*c))/h^4 - 2*sqrt(c)*e*g*arcsinh(c*x/sqrt(a*c))/
h^3 + sqrt(c)*d*arcsinh(c*x/sqrt(a*c))/h^2 + 1/2*a*f*arcsinh(c*x/sqrt(a*c))
/(sqrt(c)*h^2) - c*f*g^3*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt
(a*c)*abs(h*x + g)))/(sqrt(a + c*g^2/h^2)*h^5) + c*e*g^2*arcsinh(c*g*x/(sqr
t(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/(sqrt(a + c*g^2/h^2)*h
^4) - c*d*g*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x
 + g)))/(sqrt(a + c*g^2/h^2)*h^3) - 2*sqrt(a + c*g^2/h^2)*f*g*arcsinh(c*g*x
/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/h^3 + sqrt(a + c*
g^2/h^2)*e*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x
+ g)))/h^2 - 2*sqrt(c*x^2 + a)*f*g/h^3 + sqrt(c*x^2 + a)*e/h^2
```

Giac [F]

$$\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^2}\,dx = \int \frac{\sqrt{cx^2+a}(fx^2+ex+d)}{(hx+g)^2}\,dx$$

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^2,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^2}\,dx = \int \frac{\sqrt{c\,x^2+a}\,(f\,x^2+e\,x+d)}{(g+h\,x)^2}\,dx$$

```
[In] int(((a + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^2,x)
[Out] int(((a + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^2, x)
```

3.84 $\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^3}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 296

$$\begin{aligned}\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^3}\,dx &= \frac{(2(3fg-eh)\left(cg^2+ah^2\right)+h(2afh^2+c(3fg^2-h(eg-dh)))\,x)\sqrt{a+cx^2}}{2h^3\left(cg^2+ah^2\right)(g+hx)} \\ &\quad - \frac{(fg^2-egh+dh^2)\left(a+cx^2\right)^{3/2}}{2h\left(cg^2+ah^2\right)(g+hx)^2} - \frac{\sqrt{c}(3fg-eh)\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{h^4} \\ &\quad - \frac{(2a^2fh^4+2c^2g^3(3fg-eh)+ach^2(9fg^2-h(3eg-dh)))\operatorname{arctanh}\left(\frac{ah-cgx}{\sqrt{cg^2+ah^2}\sqrt{a+cx^2}}\right)}{2h^4\left(cg^2+ah^2\right)^{3/2}}\end{aligned}$$

```
[Out] -1/2*(d*h^2-e*g*h+f*g^2)*(c*x^2+a)^(3/2)/h/(a*h^2+c*g^2)/(h*x+g)^2-1/2*(2*a
^2*f*h^4+2*c^2*g^3*(-e*h+3*f*g)+a*c*h^2*(9*f*g^2-h*(-d*h+3*e*g)))*arctanh((
-c*g*x+a*h)/(a*h^2+c*g^2)^(1/2)/(c*x^2+a)^(1/2))/h^4/(a*h^2+c*g^2)^(3/2)-(-
e*h+3*f*g)*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))*c^(1/2)/h^4+1/2*(2*(-e*h+3*f*
g)*(a*h^2+c*g^2)+h*(2*a*f*h^2+c*(3*f*g^2-h*(-d*h+e*g)))*x)*(c*x^2+a)^(1/2)/
h^3/(a*h^2+c*g^2)/(h*x+g)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1665, 827, 858, 223, 212, 739}

$$\begin{aligned}&\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^3}\,dx\\ &= -\frac{\operatorname{arctanh}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)\left(2a^2fh^4+ach^2(9fg^2-h(3eg-dh))+2c^2g^3(3fg-eh)\right)}{2h^4\left(ah^2+cg^2\right)^{3/2}}\\ &\quad-\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)(3fg-eh)}{h^4}-\frac{\left(a+cx^2\right)^{3/2}\left(dh^2-egh+fg^2\right)}{2h(g+hx)^2\left(ah^2+cg^2\right)}\\ &\quad+\frac{\sqrt{a+cx^2}\left(hx(2afh^2-ch(eg-dh)+3cfg^2)+2\left(ah^2+cg^2\right)(3fg-eh)\right)}{2h^3(g+hx)\left(ah^2+cg^2\right)}\end{aligned}$$

```
[In] Int[(Sqrt[a + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^3,x]
```

```
[Out] ((2*(3*f*g - e*h)*(c*g^2 + a*h^2) + h*(3*c*f*g^2 + 2*a*f*h^2 - c*h*(e*g - d
*h))*x)*Sqrt[a + c*x^2])/(2*h^3*(c*g^2 + a*h^2)*(g + h*x)) - ((f*g^2 - e*g*
h + d*h^2)*(a + c*x^2)^(3/2))/(2*h*(c*g^2 + a*h^2)*(g + h*x)^2) - (Sqrt[c]*
(3*f*g - e*h)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/h^4 - ((2*a^2*f*h^4 + 2
*c^2*g^3*(3*f*g - e*h) + a*c*h^2*(9*f*g^2 - h*(3*e*g - d*h)))*ArcTanh[(a*h
- c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2])])/(2*h^4*(c*g^2 + a*h^2)^(3/
2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 827

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
```

```
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\left(fg^2-egh+dh^2\right)\left(a+cx^2\right)^{3/2}}{2h\left(cg^2+ah^2\right)(g+hx)^2} \\
&\quad -\frac{\int \frac{\left(-2(cdg-afg+aeh)-\left(2afh-c\left(eg-\frac{3fg^2}{h}-dh\right)\right)x\right)\sqrt{a+cx^2}}{(g+hx)^2}\,dx}{2\left(cg^2+ah^2\right)} \\
&= \frac{\left(2(3fg-eh)\left(cg^2+ah^2\right)+h\left(3cfg^2+2afh^2-ch(eg-dh)\right)x\right)\sqrt{a+cx^2}}{2h^3\left(cg^2+ah^2\right)(g+hx)} \\
&\quad -\frac{\left(fg^2-egh+dh^2\right)\left(a+cx^2\right)^{3/2}}{2h\left(cg^2+ah^2\right)(g+hx)^2}+\frac{\int \frac{2a\left(3cfg^2+2afh^2-ch(eg-dh)\right)-\frac{4c(3fg-eh)\left(cg^2+ah^2\right)x}{h}}{(g+hx)\sqrt{a+cx^2}}\,dx}{4h^2\left(cg^2+ah^2\right)} \\
&= \frac{\left(2(3fg-eh)\left(cg^2+ah^2\right)+h\left(3cfg^2+2afh^2-ch(eg-dh)\right)x\right)\sqrt{a+cx^2}}{2h^3\left(cg^2+ah^2\right)(g+hx)} \\
&\quad -\frac{\left(fg^2-egh+dh^2\right)\left(a+cx^2\right)^{3/2}}{2h\left(cg^2+ah^2\right)(g+hx)^2}-\frac{(c(3fg-eh))\int \frac{1}{\sqrt{a+cx^2}}\,dx}{h^4} \\
&\quad +\frac{\left(2a^2fh^4+2c^2g^3(3fg-eh)+ach^2\left(9fg^2-h(3eg-dh)\right)\right)\int \frac{1}{(g+hx)\sqrt{a+cx^2}}\,dx}{2h^4\left(cg^2+ah^2\right)}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{\left(2(3fg-eh)\left(cg^2+ah^2\right)+h\left(3cfg^2+2afh^2-ch(eg-dh)\right)x\right)\sqrt{a+cx^2}}{2h^3\left(cg^2+ah^2\right)(g+hx)} \\
&\quad -\frac{\left(fg^2-egh+dh^2\right)\left(a+cx^2\right)^{3/2}}{2h\left(cg^2+ah^2\right)(g+hx)^2}-\frac{(c(3fg-eh))\text{Subst}\left(\int\frac{1}{1-cx^2}\,dx,x,\frac{x}{\sqrt{a+cx^2}}\right)}{h^4} \\
&\quad -\frac{\left(2a^2fh^4+2c^2g^3(3fg-eh)+ach^2\left(9fg^2-h(3eg-dh)\right)\right)\text{Subst}\left(\int\frac{1}{cg^2+ah^2-x^2}\,dx,x,\frac{ah-cgx}{\sqrt{a+cx^2}}\right)}{2h^4\left(cg^2+ah^2\right)} \\
&= \frac{\left(2(3fg-eh)\left(cg^2+ah^2\right)+h\left(3cfg^2+2afh^2-ch(eg-dh)\right)x\right)\sqrt{a+cx^2}}{2h^3\left(cg^2+ah^2\right)(g+hx)} \\
&\quad -\frac{\left(fg^2-egh+dh^2\right)\left(a+cx^2\right)^{3/2}}{2h\left(cg^2+ah^2\right)(g+hx)^2}-\frac{\sqrt{c}(3fg-eh)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{h^4} \\
&\quad -\frac{\left(2a^2fh^4+2c^2g^3(3fg-eh)+ach^2\left(9fg^2-h(3eg-dh)\right)\right)\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{cg^2+ah^2}\sqrt{a+cx^2}}\right)}{2h^4\left(cg^2+ah^2\right)^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.74 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.95

$$
\int\frac{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}{(g+hx)^3}\,dx
$$

$$
=\frac{\frac{h\sqrt{a+cx^2}\left(ah^2\left(-h(eg+dh+2ehx)+f\left(5g^2+8ghx+2h^2x^2\right)\right)+cg\left(dh^3x-egh(2g+3hx)+fg\left(6g^2+9ghx+2h^2x^2\right)\right)\right)}{(cg^2+ah^2)(g+hx)^2}+\frac{2\left(2a^2fh^4+2c^2g^3(3fg-eh)\right.}{}}{2h^4}
$$

```
[In] Integrate[(Sqrt[a + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^3,x]
```

```
[Out] ((h*Sqrt[a + c*x^2]*(a*h^2*(-(h*(e*g + d*h + 2*e*h*x)) + f*(5*g^2 + 8*g*h*x
 + 2*h^2*x^2)) + c*g*(d*h^3*x - e*g*h*(2*g + 3*h*x) + f*g*(6*g^2 + 9*g*h*x
+ 2*h^2*x^2))))/((c*g^2 + a*h^2)*(g + h*x)^2) + (2*(2*a^2*f*h^4 + 2*c^2*g^3
*(3*f*g - e*h) + a*c*h^2*(9*f*g^2 + h*(-3*e*g + d*h)))*ArcTan[(Sqrt[c]*(g +
 h*x) - h*Sqrt[a + c*x^2])/Sqrt[-(c*g^2) - a*h^2]])/(-(c*g^2) - a*h^2)^(3/2
) + 2*Sqrt[c]*(3*f*g - e*h)*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/(2*h^4)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 945 vs. $2(274) = 548$.

Time = 0.70 (sec) , antiderivative size = 946, normalized size of antiderivative = 3.20

method	result
risch	$\frac{f\sqrt{c x^{2}+a}}{h^{3}}+\frac{\frac{\sqrt{c}\,(eh-3fg)\ln\left(x\sqrt{c}+\sqrt{c x^{2}+a}\right)}{h}-\frac{\left(af h^{2}+cd h^{2}-3cegh+6cf g^{2}\right)\ln\left(\frac{\frac{2a h^{2}+2c g^{2}}{h^{2}}-\frac{2cg\left(x+\frac{g}{h}\right)}{h}+2\sqrt{\frac{a h^{2}+c g^{2}}{h^{2}}}\sqrt{\left(x+\frac{g}{h}\right)^{2}c-\frac{2cg(}{}}}{x+\frac{g}{h}}\right.}{h^{2}\sqrt{\frac{a h^{2}+c g^{2}}{h^{2}}}}}{}$
default	Expression too large to display

```
[In] int((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^3,x,method=_RETURNVERBOSE)
```

```
[Out] f/h^3*(c*x^2+a)^(1/2)+1/h^3*(c^(1/2)*(e*h-3*f*g)/h*ln(x*c^(1/2)+(c*x^2+a)^(
1/2))-1/h^2*(a*f*h^2+c*d*h^2-3*c*e*g*h+6*c*f*g^2)/((a*h^2+c*g^2)/h^2)^(1/2)
*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1/h*g)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+
1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+1/h*g))+1/h^3*(a*
e*h^3-2*a*f*g*h^2-2*c*d*g*h^2+3*c*e*g^2*h-4*c*f*g^3)*(-1/(a*h^2+c*g^2)*h^2/
(x+1/h*g)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)-c*g*h/(
a*h^2+c*g^2)/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1
/h*g)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c
*g^2)/h^2)^(1/2))/(x+1/h*g)))+1/h^4*(a*d*h^4-a*e*g*h^3+a*f*g^2*h^2+c*d*g^2*
h^2-c*e*g^3*h+c*f*g^4)*(-1/2/(a*h^2+c*g^2)*h^2/(x+1/h*g)^2*((x+1/h*g)^2*c-2
*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)+3/2*c*g*h/(a*h^2+c*g^2)*(-1/(a*h^
2+c*g^2)*h^2/(x+1/h*g)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^
(1/2)-c*g*h/(a*h^2+c*g^2)/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2
-2*c*g/h*(x+1/h*g)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/
h*g)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+1/h*g)))+1/2*c/(a*h^2+c*g^2)*h^2/((a*h^2+
c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1/h*g)+2*((a*h^2+c*g^2
)/h^2)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+
1/h*g))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^3}\,dx = \text{Timed out}$$

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^3,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^3}\,dx = \int \frac{\sqrt{a+cx^2}\,(d+ex+fx^2)}{\left(g+hx\right)^3}\,dx$$

```
[In] integrate((f*x**2+e*x+d)*(c*x**2+a)**(1/2)/(h*x+g)**3,x)
[Out] Integral(sqrt(a + c*x**2)*(d + e*x + f*x**2)/(g + h*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 927 vs. $2(275) = 550$.

Time = 0.26 (sec) , antiderivative size = 927, normalized size of antiderivative = 3.13

$$
\begin{aligned}
&\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^3}\,dx \\
&= -\frac{\sqrt{cx^2+a}\,cfg^3}{2\left(cg^2h^4x+ah^6x+cg^3h^3+agh^5\right)}+\frac{\sqrt{cx^2+a}\,ceg^2}{2\left(cg^2h^3x+ah^5x+cg^3h^2+agh^4\right)} \\
&\quad-\frac{(cx^2+a)^{\frac{3}{2}}fg^2}{2\left(cg^2h^3x^2+ah^5x^2+2\,cg^3h^2x+2\,agh^4x+cg^4h+ag^2h^3\right)} \\
&\quad+\frac{\sqrt{cx^2+a}\,cfg^2}{2\left(cg^2h^3+ah^5\right)}-\frac{\sqrt{cx^2+a}\,cdg}{2\left(cg^2h^2x+ah^4x+cg^3h+agh^3\right)} \\
&\quad+\frac{(cx^2+a)^{\frac{3}{2}}eg}{2\left(cg^2h^2x^2+ah^4x^2+2\,cg^3hx+2\,agh^3x+cg^4+ag^2h^2\right)}-\frac{\sqrt{cx^2+a}\,ceg}{2\left(cg^2h^2+ah^4\right)} \\
&\quad-\frac{(cx^2+a)^{\frac{3}{2}}d}{2\left(cg^2hx^2+ah^3x^2+2\,cg^3x+2\,agh^2x+\frac{cg^4}{h}+ag^2h\right)}+\frac{\sqrt{cx^2+a}\,cd}{2\left(cg^2h+ah^3\right)} \\
&\quad+\frac{2\sqrt{cx^2+a}\,fg}{h^4x+gh^3}-\frac{\sqrt{cx^2+a}\,e}{h^3x+gh^2}-\frac{3\sqrt{c}fg\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{h^4}+\frac{\sqrt{c}e\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{h^3} \\
&\quad-\frac{c^2fg^4\operatorname{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|}-\frac{ah}{\sqrt{ac}|hx+g|}\right)}{2\left(a+\frac{cg^2}{h^2}\right)^{\frac{3}{2}}h^7}+\frac{c^2eg^3\operatorname{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|}-\frac{ah}{\sqrt{ac}|hx+g|}\right)}{2\left(a+\frac{cg^2}{h^2}\right)^{\frac{3}{2}}h^6} \\
&\quad-\frac{c^2dg^2\operatorname{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|}-\frac{ah}{\sqrt{ac}|hx+g|}\right)}{2\left(a+\frac{cg^2}{h^2}\right)^{\frac{3}{2}}h^5}+\frac{5\,cfg^2\operatorname{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|}-\frac{ah}{\sqrt{ac}|hx+g|}\right)}{2\sqrt{a+\frac{cg^2}{h^2}}h^5} \\
&\quad-\frac{3\,ceg\operatorname{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|}-\frac{ah}{\sqrt{ac}|hx+g|}\right)}{2\sqrt{a+\frac{cg^2}{h^2}}h^4}+\frac{cd\operatorname{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|}-\frac{ah}{\sqrt{ac}|hx+g|}\right)}{2\sqrt{a+\frac{cg^2}{h^2}}h^3} \\
&\quad+\frac{\sqrt{a+\frac{cg^2}{h^2}}f\operatorname{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|}-\frac{ah}{\sqrt{ac}|hx+g|}\right)}{h^3}+\frac{\sqrt{cx^2+a}\,f}{h^3}
\end{aligned}
$$

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^3,x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(c*x^2 + a)*c*f*g^3/(c*g^2*h^4*x + a*h^6*x + c*g^3*h^3 + a*g*h^5)
+ 1/2*sqrt(c*x^2 + a)*c*e*g^2/(c*g^2*h^3*x + a*h^5*x + c*g^3*h^2 + a*g*h^4)
 - 1/2*(c*x^2 + a)^(3/2)*f*g^2/(c*g^2*h^3*x^2 + a*h^5*x^2 + 2*c*g^3*h^2*x +
 2*a*g*h^4*x + c*g^4*h + a*g^2*h^3) + 1/2*sqrt(c*x^2 + a)*c*f*g^2/(c*g^2*h^
3 + a*h^5) - 1/2*sqrt(c*x^2 + a)*c*d*g/(c*g^2*h^2*x + a*h^4*x + c*g^3*h + a
*g*h^3) + 1/2*(c*x^2 + a)^(3/2)*e*g/(c*g^2*h^2*x^2 + a*h^4*x^2 + 2*c*g^3*h*
x + 2*a*g*h^3*x + c*g^4 + a*g^2*h^2) - 1/2*sqrt(c*x^2 + a)*c*e*g/(c*g^2*h^2
 + a*h^4) - 1/2*(c*x^2 + a)^(3/2)*d/(c*g^2*h*x^2 + a*h^3*x^2 + 2*c*g^3*x +
2*a*g*h^2*x + c*g^4/h + a*g^2*h) + 1/2*sqrt(c*x^2 + a)*c*d/(c*g^2*h + a*h^3
```

```
) + 2*sqrt(c*x^2 + a)*f*g/(h^4*x + g*h^3) - sqrt(c*x^2 + a)*e/(h^3*x + g*h^
2) - 3*sqrt(c)*f*g*arcsinh(c*x/sqrt(a*c))/h^4 + sqrt(c)*e*arcsinh(c*x/sqrt(
a*c))/h^3 - 1/2*c^2*f*g^4*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqr
t(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(3/2)*h^7) + 1/2*c^2*e*g^3*arcsinh(c
*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h
^2)^(3/2)*h^6) - 1/2*c^2*d*g^2*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h
/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(3/2)*h^5) + 5/2*c*f*g^2*arcsin
h(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/(sqrt(a +
c*g^2/h^2)*h^5) - 3/2*c*e*g*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(s
qrt(a*c)*abs(h*x + g)))/(sqrt(a + c*g^2/h^2)*h^4) + 1/2*c*d*arcsinh(c*g*x/(
sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/(sqrt(a + c*g^2/h^2
)*h^3) + sqrt(a + c*g^2/h^2)*f*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h
/(sqrt(a*c)*abs(h*x + g)))/h^3 + sqrt(c*x^2 + a)*f/h^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 911 vs. $2(275) = 550$.

Time = 0.32 (sec) , antiderivative size = 911, normalized size of antiderivative = 3.08

$$\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^3}\,dx$$

$$= \frac{(6\,c^2fg^4 - 2\,c^2eg^3h + 9\,acfg^2h^2 - 3\,acegh^3 + acdh^4 + 2\,a^2fh^4)\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+a}\right)h+\sqrt{c}g}{\sqrt{-cg^2-ah^2}}\right)}{(cg^2h^4+ah^6)\sqrt{-cg^2-ah^2}}$$

$$+ \frac{\sqrt{cx^2+a}f}{h^3}$$

$$+ \frac{6\left(\sqrt{c}x-\sqrt{cx^2+a}\right)^3c^2fg^4h - 4\left(\sqrt{c}x-\sqrt{cx^2+a}\right)^3c^2eg^3h^2 + 2\left(\sqrt{c}x-\sqrt{cx^2+a}\right)^3c^2dg^2h^3 + 5\left(\sqrt{c}x\right.}{}$$

$$+ \frac{\left(3\sqrt{c}fg-\sqrt{c}eh\right)\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+a}\right|\right)}{h^4}$$

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^3,x, algorithm="giac")
```

```
[Out] (6*c^2*f*g^4 - 2*c^2*e*g^3*h + 9*a*c*f*g^2*h^2 - 3*a*c*e*g*h^3 + a*c*d*h^4
+ 2*a^2*f*h^4)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*h + sqrt(c)*g)/sqrt(-
c*g^2 - a*h^2))/((c*g^2*h^4 + a*h^6)*sqrt(-c*g^2 - a*h^2)) + sqrt(c*x^2 + a
)*f/h^3 + (6*(sqrt(c)*x - sqrt(c*x^2 + a))^3*c^2*f*g^4*h - 4*(sqrt(c)*x - s
qrt(c*x^2 + a))^3*c^2*e*g^3*h^2 + 2*(sqrt(c)*x - sqrt(c*x^2 + a))^3*c^2*d*g
^2*h^3 + 5*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a*c*f*g^2*h^3 - 3*(sqrt(c)*x - s
qrt(c*x^2 + a))^3*a*c*e*g*h^4 + (sqrt(c)*x - sqrt(c*x^2 + a))^3*a*c*d*h^5 +
 10*(sqrt(c)*x - sqrt(c*x^2 + a))^2*c^(5/2)*f*g^5 - 6*(sqrt(c)*x - sqrt(c*x
^2 + a))^2*c^(5/2)*e*g^4*h + 2*(sqrt(c)*x - sqrt(c*x^2 + a))^2*c^(5/2)*d*g^
```

```
3*h^2 + 3*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a*c^(3/2)*f*g^3*h^2 - (sqrt(c)*x
- sqrt(c*x^2 + a))^2*a*c^(3/2)*e*g^2*h^3 - (sqrt(c)*x - sqrt(c*x^2 + a))^2*
a*c^(3/2)*d*g*h^4 - 4*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^2*sqrt(c)*f*g*h^4 +
 2*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^2*sqrt(c)*e*h^5 - 14*(sqrt(c)*x - sqrt
(c*x^2 + a))*a*c^2*f*g^4*h + 8*(sqrt(c)*x - sqrt(c*x^2 + a))*a*c^2*e*g^3*h^
2 - 2*(sqrt(c)*x - sqrt(c*x^2 + a))*a*c^2*d*g^2*h^3 - 11*(sqrt(c)*x - sqrt(
c*x^2 + a))*a^2*c*f*g^2*h^3 + 5*(sqrt(c)*x - sqrt(c*x^2 + a))*a^2*c*e*g*h^4
 + (sqrt(c)*x - sqrt(c*x^2 + a))*a^2*c*d*h^5 + 5*a^2*c^(3/2)*f*g^3*h^2 - 3*
a^2*c^(3/2)*e*g^2*h^3 + a^2*c^(3/2)*d*g*h^4 + 4*a^3*sqrt(c)*f*g*h^4 - 2*a^3
*sqrt(c)*e*h^5)/((c*g^2*h^4 + a*h^6)*((sqrt(c)*x - sqrt(c*x^2 + a))^2*h + 2
*(sqrt(c)*x - sqrt(c*x^2 + a))*sqrt(c)*g - a*h)^2) + (3*sqrt(c)*f*g - sqrt(
c)*e*h)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/h^4
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^3}\,dx=\int \frac{\sqrt{c\,x^2+a}\,(f\,x^2+e\,x+d)}{{(g+h\,x)}^3}\,dx$$

```
[In] int(((a + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^3,x)
[Out] int(((a + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^3, x)
```

3.85 $\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^4} dx$

Optimal result

Integrand size = 29, antiderivative size = 314

$$\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^4} dx = -\frac{(2c^2fg^5+a^2eh^5+acgh^2(3fg^2+dh^2)+h(2a^2fh^4+acgh^2(6fg-eh)+c^2(3fg^4-dg^2h^2))x)\sqrt{a+cx}}{2h^3(cg^2+ah^2)^2(g+hx)^2} - \frac{(fg^2-egh+dh^2)(a+cx^2)^{3/2}}{3h(cg^2+ah^2)(g+hx)^3} + \frac{\sqrt{c}f\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{h^4} + \frac{c(2c^2fg^5+a^2h^4(4fg-eh)+acgh^2(5fg^2-dh^2))\operatorname{arctanh}\left(\frac{ah-cgx}{\sqrt{cg^2+ah^2}\sqrt{a+cx^2}}\right)}{2h^4(cg^2+ah^2)^{5/2}}$$

```
[Out] -1/3*(d*h^2-e*g*h+f*g^2)*(c*x^2+a)^(3/2)/h/(a*h^2+c*g^2)/(h*x+g)^3+1/2*c*(2
*c^2*f*g^5+a^2*h^4*(-e*h+4*f*g)+a*c*g*h^2*(-d*h^2+5*f*g^2))*arctanh((-c*g*x
+a*h)/(a*h^2+c*g^2)^(1/2)/(c*x^2+a)^(1/2))/h^4/(a*h^2+c*g^2)^(5/2)+f*arctan
h(x*c^(1/2)/(c*x^2+a)^(1/2))*c^(1/2)/h^4-1/2*(2*c^2*f*g^5+a^2*e*h^5+a*c*g*h
^2*(d*h^2+3*f*g^2)+h*(2*a^2*f*h^4+a*c*g*h^2*(-e*h+6*f*g)+c^2*(-d*g^2*h^2+3*
f*g^4))*x)*(c*x^2+a)^(1/2)/h^3/(a*h^2+c*g^2)^2/(h*x+g)^2
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1665, 825, 858, 223, 212, 739}

$$\begin{aligned}&\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^4}\,dx\\ &= \frac{c\,\mathrm{arctanh}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)(a^2h^4(4fg-eh)+acgh^2(5fg^2-dh^2)+2c^2fg^5)}{2h^4\left(ah^2+cg^2\right)^{5/2}}\\ &\quad-\frac{\sqrt{a+cx^2}(hx(2a^2fh^4+acgh^2(6fg-eh)+c^2(3fg^4-dg^2h^2))+a^2eh^5+acgh^2(dh^2+3fg^2)+2c^2fg^5)}{2h^3(g+hx)^2\left(ah^2+cg^2\right)^2}\\ &\quad+\frac{\sqrt{c}f\,\mathrm{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{h^4}-\frac{\left(a+cx^2\right)^{3/2}\left(dh^2-egh+fg^2\right)}{3h(g+hx)^3\left(ah^2+cg^2\right)}\end{aligned}$$

```
[In] Int[(Sqrt[a + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^4,x]
```

```
[Out] -1/2*((2*c^2*f*g^5 + a^2*e*h^5 + a*c*g*h^2*(3*f*g^2 + d*h^2) + h*(2*a^2*f*h
^4 + a*c*g*h^2*(6*f*g - e*h) + c^2*(3*f*g^4 - d*g^2*h^2))*x)*Sqrt[a + c*x^2
])/(h^3*(c*g^2 + a*h^2)^2*(g + h*x)^2) - ((f*g^2 - e*g*h + d*h^2)*(a + c*x^
2)^(3/2))/(3*h*(c*g^2 + a*h^2)*(g + h*x)^3) + (Sqrt[c]*f*ArcTanh[(Sqrt[c]*x
)/Sqrt[a + c*x^2]])/h^4 + (c*(2*c^2*f*g^5 + a^2*h^4*(4*f*g - e*h) + a*c*g*h
^2*(5*f*g^2 - d*h^2))*ArcTanh[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqrt[a + c
*x^2])])/(2*h^4*(c*g^2 + a*h^2)^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 825

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + c*x^2)^p/(e^2*(m + 1)*(m
```

```
+ 2)*(c*d^2 + a*e^2)))*((d*g - e*f*(m + 2))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*
f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) + 2*c*d*p*(e*f - d*g))*x), x] - Dis
t[p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*
(m + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f,
g}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p,
0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{(fg^2-egh+dh^2)\left(a+cx^2\right)^{3/2}}{3h\left(cg^2+ah^2\right)(g+hx)^3}-\frac{\int \frac{\left(-3(cdg-afg+aeh)-3f\left(\frac{cg^2}{h}+ah\right)x\right)\sqrt{a+cx^2}}{(g+hx)^3}\,dx}{3\left(cg^2+ah^2\right)}$$

$$= -\frac{(2c^2fg^5+a^2eh^5+acgh^2(3fg^2+dh^2)+h(2a^2fh^4+acgh^2(6fg-eh)+c^2(3fg^4-dg^2h^2))\,x)}{2h^3\left(cg^2+ah^2\right)^2(g+hx)^2}$$

$$-\frac{(fg^2-egh+dh^2)\left(a+cx^2\right)^{3/2}}{3h\left(cg^2+ah^2\right)(g+hx)^3}+\frac{\int \frac{-6ac\left(ah^2(2fg-eh)+c\left(fg^3-dgh^2\right)\right)+\frac{12cf\left(cg^2+ah^2\right)^2x}{h}}{(g+hx)\sqrt{a+cx^2}}\,dx}{12h^2\left(cg^2+ah^2\right)^2}$$

$$= -\frac{(2c^2fg^5+a^2eh^5+acgh^2(3fg^2+dh^2)+h(2a^2fh^4+acgh^2(6fg-eh)+c^2(3fg^4-dg^2h^2))\,x)}{2h^3\left(cg^2+ah^2\right)^2(g+hx)^2}$$

$$-\frac{(fg^2-egh+dh^2)\left(a+cx^2\right)^{3/2}}{3h\left(cg^2+ah^2\right)(g+hx)^3}+\frac{(cf)\int \frac{1}{\sqrt{a+cx^2}}\,dx}{h^4}$$

$$-\frac{\left(c(2c^2fg^5+a^2h^4(4fg-eh)+acgh^2(5fg^2-dh^2))\right)\int \frac{1}{(g+hx)\sqrt{a+cx^2}}\,dx}{2h^4\left(cg^2+ah^2\right)^2}$$

$$
\begin{aligned}
=&\\
&-\frac{(2c^2fg^5+a^2eh^5+acgh^2(3fg^2+dh^2)+h(2a^2fh^4+acgh^2(6fg-eh)+c^2(3fg^4-dg^2h^2))\,x)\sqrt{}}{2h^3\left(cg^2+ah^2\right)^2(g+hx)^2}\\
&-\frac{\left(fg^2-egh+dh^2\right)\left(a+cx^2\right)^{3/2}}{3h\left(cg^2+ah^2\right)(g+hx)^3}+\frac{(cf)\mathrm{Subst}\left(\int\frac{1}{1-cx^2}\,dx,x,\frac{x}{\sqrt{a+cx^2}}\right)}{h^4}\\
&+\frac{\left(c(2c^2fg^5+a^2h^4(4fg-eh)+acgh^2(5fg^2-dh^2))\right)\mathrm{Subst}\left(\int\frac{1}{cg^2+ah^2-x^2}\,dx,x,\frac{ah-cgx}{\sqrt{a+cx^2}}\right)}{2h^4\left(cg^2+ah^2\right)^2}\\
=&\\
&-\frac{(2c^2fg^5+a^2eh^5+acgh^2(3fg^2+dh^2)+h(2a^2fh^4+acgh^2(6fg-eh)+c^2(3fg^4-dg^2h^2))\,x)\sqrt{}}{2h^3\left(cg^2+ah^2\right)^2(g+hx)^2}\\
&-\frac{\left(fg^2-egh+dh^2\right)\left(a+cx^2\right)^{3/2}}{3h\left(cg^2+ah^2\right)(g+hx)^3}+\frac{\sqrt{c}f\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{h^4}\\
&+\frac{c(2c^2fg^5+a^2h^4(4fg-eh)+acgh^2(5fg^2-dh^2))\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{cg^2+ah^2}\sqrt{a+cx^2}}\right)}{2h^4\left(cg^2+ah^2\right)^{5/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 10.53 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.22

$$
\int\frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^4}\,dx
$$

$$
=\frac{\dfrac{h\sqrt{a+cx^2}\left(-2\left(fg^2+h(-eg+dh)\right)+\frac{\left(7cfg^3+cgh(-4eg+dh)-3ah^2(-2fg+eh)\right)(g+hx)}{cg^2+ah^2}-\frac{\left(6a^2fh^4+c^2\left(11fg^4-g^2h(2eg+dh)\right)+ach^2\left(20fg^2+h(-5eg+2dh)\right)\right)(}{\left(cg^2+ah^2\right)^2}\right.}{(g+hx)^3}}{}
$$

```
[In] Integrate[(Sqrt[a + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^4,x]
```

```
[Out] ((h*Sqrt[a + c*x^2]*(-2*(f*g^2 + h*(-(e*g) + d*h)) + ((7*c*f*g^3 + c*g*h*(-
4*e*g + d*h) - 3*a*h^2*(-2*f*g + e*h))*(g + h*x))/(c*g^2 + a*h^2) - ((6*a^2
*f*h^4 + c^2*(11*f*g^4 - g^2*h*(2*e*g + d*h)) + a*c*h^2*(20*f*g^2 + h*(-5*e
*g + 2*d*h)))*(g + h*x)^2)/(c*g^2 + a*h^2)^2))/(g + h*x)^3 - (3*c*(2*c^2*f*
g^5 + a^2*h^4*(4*f*g - e*h) + a*c*g*h^2*(5*f*g^2 - d*h^2))*Log[g + h*x])/(c
*g^2 + a*h^2)^(5/2) + 6*Sqrt[c]*f*Log[c*x + Sqrt[c]*Sqrt[a + c*x^2]] + (3*c
*(2*c^2*f*g^5 + a^2*h^4*(4*f*g - e*h) + a*c*g*h^2*(5*f*g^2 - d*h^2))*Log[a*
h - c*g*x + Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2]])/(c*g^2 + a*h^2)^(5/2))/(6
*h^4)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2447 vs. 2(292) = 584.

Time = 0.74 (sec) , antiderivative size = 2448, normalized size of antiderivative = 7.80

method	result	size
default	Expression too large to display	2448

```
[In] int((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^4,x,method=_RETURNVERBOSE)
```

```
[Out] f/h^4*(-1/(a*h^2+c*g^2)*h^2/(x+1/h*g)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h
^2+c*g^2)/h^2)^(3/2)-c*g*h/(a*h^2+c*g^2)*(((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+
(a*h^2+c*g^2)/h^2)^(1/2)-c^(1/2)*g/h*ln((-c*g/h+c*(x+1/h*g))/c^(1/2)+((x+1/
h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))-(a*h^2+c*g^2)/h^2/((a*
h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1/h*g)+2*((a*h^2+c
*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))
/(x+1/h*g)))+2*c/(a*h^2+c*g^2)*h^2*(1/4*(2*c*(x+1/h*g)-2*c*g/h)/c*((x+1/h*g
)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)+1/8*(4*c*(a*h^2+c*g^2)/h^2
-4*c^2*g^2/h^2)/c^(3/2)*ln((-c*g/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g)^2*c-2*c*
g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))))+(e*h-2*f*g)/h^5*(-1/2/(a*h^2+c*g^
2)*h^2/(x+1/h*g)^2*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(3/2
)+1/2*c*g*h/(a*h^2+c*g^2)*(-1/(a*h^2+c*g^2)*h^2/(x+1/h*g)*((x+1/h*g)^2*c-2*
c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(3/2)-c*g*h/(a*h^2+c*g^2)*(((x+1/h*g)^2*
c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)-c^(1/2)*g/h*ln((-c*g/h+c*(x+1/
h*g))/c^(1/2)+((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))-(a
*h^2+c*g^2)/h^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(
x+1/h*g)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^
2+c*g^2)/h^2)^(1/2))/(x+1/h*g)))+2*c/(a*h^2+c*g^2)*h^2*(1/4*(2*c*(x+1/h*g)-
2*c*g/h)/c*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)+1/8*(4
*c*(a*h^2+c*g^2)/h^2-4*c^2*g^2/h^2)/c^(3/2)*ln((-c*g/h+c*(x+1/h*g))/c^(1/2)
+((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))))+1/2*c/(a*h^2+
c*g^2)*h^2*(((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)-c^(1/
2)*g/h*ln((-c*g/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*
h^2+c*g^2)/h^2)^(1/2))-(a*h^2+c*g^2)/h^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a
*h^2+c*g^2)/h^2-2*c*g/h*(x+1/h*g)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*
c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+1/h*g))))+(d*h^2-e*g*h+f*g
^2)/h^6*(-1/3/(a*h^2+c*g^2)*h^2/(x+1/h*g)^3*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g
)+(a*h^2+c*g^2)/h^2)^(3/2)+c*g*h/(a*h^2+c*g^2)*(-1/2/(a*h^2+c*g^2)*h^2/(x+1
/h*g)^2*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(3/2)+1/2*c*g*h
/(a*h^2+c*g^2)*(-1/(a*h^2+c*g^2)*h^2/(x+1/h*g)*((x+1/h*g)^2*c-2*c*g/h*(x+1/
h*g)+(a*h^2+c*g^2)/h^2)^(3/2)-c*g*h/(a*h^2+c*g^2)*(((x+1/h*g)^2*c-2*c*g/h*(
x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)-c^(1/2)*g/h*ln((-c*g/h+c*(x+1/h*g))/c^(1/
2)+((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))-(a*h^2+c*g^2)
/h^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1/h*g)+2*
((a*h^2+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^
```

```
2)^(1/2))/(x+1/h*g))+2*c/(a*h^2+c*g^2)*h^2*(1/4*(2*c*(x+1/h*g)-2*c*g/h)/c*
((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)+1/8*(4*c*(a*h^2+c
*g^2)/h^2-4*c^2*g^2/h^2)/c^(3/2)*ln((-c*g/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g)
^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))))+1/2*c/(a*h^2+c*g^2)*h^2*
(((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)-c^(1/2)*g/h*ln((
-c*g/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/
h^2)^(1/2))-(a*h^2+c*g^2)/h^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)
/h^2-2*c*g/h*(x+1/h*g)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(
x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+1/h*g))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^4}\,dx=\text{Timed out}$$

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^4}\,dx=\int \frac{\sqrt{a+cx^2}\,(d+ex+fx^2)}{(g+hx)^4}\,dx$$

```
[In] integrate((f*x**2+e*x+d)*(c*x**2+a)**(1/2)/(h*x+g)**4,x)
```

```
[Out] Integral(sqrt(a + c*x**2)*(d + e*x + f*x**2)/(g + h*x)**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1772 vs. $2(293) = 586$.

Time $= 0.28$ (sec) , antiderivative size $= 1772$, normalized size of antiderivative $= 5.64$

$$\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^4}\,dx=\text{Too large to display}$$

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^4,x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(c*x^2 + a)*c^2*f*g^4/(c^2*g^4*h^4*x + 2*a*c*g^2*h^6*x + a^2*h^8*x
 + c^2*g^5*h^3 + 2*a*c*g^3*h^5 + a^2*g*h^7) + 1/2*sqrt(c*x^2 + a)*c^2*e*g^3
/(c^2*g^4*h^3*x + 2*a*c*g^2*h^5*x + a^2*h^7*x + c^2*g^5*h^2 + 2*a*c*g^3*h^4
 + a^2*g*h^6) - 1/2*(c*x^2 + a)^(3/2)*c*f*g^3/(c^2*g^4*h^3*x^2 + 2*a*c*g^2*
```

```
h^5*x^2 + a^2*h^7*x^2 + 2*c^2*g^5*h^2*x + 4*a*c*g^3*h^4*x + 2*a^2*g*h^6*x +
 c^2*g^6*h + 2*a*c*g^4*h^3 + a^2*g^2*h^5) + 1/2*sqrt(c*x^2 + a)*c^2*f*g^3/(
c^2*g^4*h^3 + 2*a*c*g^2*h^5 + a^2*h^7) - 1/2*sqrt(c*x^2 + a)*c^2*d*g^2/(c^2
*g^4*h^2*x + 2*a*c*g^2*h^4*x + a^2*h^6*x + c^2*g^5*h + 2*a*c*g^3*h^3 + a^2*
g*h^5) + 1/2*(c*x^2 + a)^(3/2)*c*e*g^2/(c^2*g^4*h^2*x^2 + 2*a*c*g^2*h^4*x^2
 + a^2*h^6*x^2 + 2*c^2*g^5*h*x + 4*a*c*g^3*h^3*x + 2*a^2*g*h^5*x + c^2*g^6
+ 2*a*c*g^4*h^2 + a^2*g^2*h^4) - 1/2*sqrt(c*x^2 + a)*c^2*e*g^2/(c^2*g^4*h^2
 + 2*a*c*g^2*h^4 + a^2*h^6) - 1/2*(c*x^2 + a)^(3/2)*c*d*g/(c^2*g^4*h*x^2 +
2*a*c*g^2*h^3*x^2 + a^2*h^5*x^2 + 2*c^2*g^5*x + 4*a*c*g^3*h^2*x + 2*a^2*g*h
^4*x + c^2*g^6/h + 2*a*c*g^4*h + a^2*g^2*h^3) + 1/2*sqrt(c*x^2 + a)*c^2*d*g
/(c^2*g^4*h + 2*a*c*g^2*h^3 + a^2*h^5) - 1/3*(c*x^2 + a)^(3/2)*f*g^2/(c*g^2
*h^4*x^3 + a*h^6*x^3 + 3*c*g^3*h^3*x^2 + 3*a*g*h^5*x^2 + 3*c*g^4*h^2*x + 3*
a*g^2*h^4*x + c*g^5*h + a*g^3*h^3) + sqrt(c*x^2 + a)*c*f*g^2/(c*g^2*h^4*x +
 a*h^6*x + c*g^3*h^3 + a*g*h^5) + 1/3*(c*x^2 + a)^(3/2)*e*g/(c*g^2*h^3*x^3
+ a*h^5*x^3 + 3*c*g^3*h^2*x^2 + 3*a*g*h^4*x^2 + 3*c*g^4*h*x + 3*a*g^2*h^3*x
 + c*g^5 + a*g^3*h^2) - 1/2*sqrt(c*x^2 + a)*c*e*g/(c*g^2*h^3*x + a*h^5*x +
c*g^3*h^2 + a*g*h^4) + (c*x^2 + a)^(3/2)*f*g/(c*g^2*h^3*x^2 + a*h^5*x^2 + 2
*c*g^3*h^2*x + 2*a*g*h^4*x + c*g^4*h + a*g^2*h^3) - sqrt(c*x^2 + a)*c*f*g/(
c*g^2*h^3 + a*h^5) - 1/3*(c*x^2 + a)^(3/2)*d/(c*g^2*h^2*x^3 + a*h^4*x^3 + 3
*c*g^3*h*x^2 + 3*a*g*h^3*x^2 + 3*c*g^4*x + 3*a*g^2*h^2*x + c*g^5/h + a*g^3*
h) - 1/2*(c*x^2 + a)^(3/2)*e/(c*g^2*h^2*x^2 + a*h^4*x^2 + 2*c*g^3*h*x + 2*a
*g*h^3*x + c*g^4 + a*g^2*h^2) + 1/2*sqrt(c*x^2 + a)*c*e/(c*g^2*h^2 + a*h^4)
 - sqrt(c*x^2 + a)*f/(h^4*x + g*h^3) + sqrt(c)*f*arcsinh(c*x/sqrt(a*c))/h^4
 - 1/2*c^3*f*g^5*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*ab
s(h*x + g)))/((a + c*g^2/h^2)^(5/2)*h^9) + 1/2*c^3*e*g^4*arcsinh(c*g*x/(sqr
t(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(5/2)
*h^8) - 1/2*c^3*d*g^3*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*
c)*abs(h*x + g)))/((a + c*g^2/h^2)^(5/2)*h^7) + 3/2*c^2*f*g^3*arcsinh(c*g*x
/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^
(3/2)*h^7) - c^2*e*g^2*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a
*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(3/2)*h^6) + 1/2*c^2*d*g*arcsinh(c*g*x/
(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(
3/2)*h^5) - 2*c*f*g*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)
*abs(h*x + g)))/(sqrt(a + c*g^2/h^2)*h^5) + 1/2*c*e*arcsinh(c*g*x/(sqrt(a*c
)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/(sqrt(a + c*g^2/h^2)*h^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1701 vs. $2(293) = 586$.

Time = 0.37 (sec) , antiderivative size = 1701, normalized size of antiderivative = 5.42

$$\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^4}\,dx = \text{Too large to display}$$

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^4,x, algorithm="giac")
```

```
[Out] -(2*c^3*f*g^5 + 5*a*c^2*f*g^3*h^2 - a*c^2*d*g*h^4 + 4*a^2*c*f*g*h^4 - a^2*c
*e*h^5)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*h + sqrt(c)*g)/sqrt(-c*g^2 -
 a*h^2))/((c^2*g^4*h^4 + 2*a*c*g^2*h^6 + a^2*h^8)*sqrt(-c*g^2 - a*h^2)) - s
qrt(c)*f*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/h^4 - 1/3*(18*(sqrt(c)*x -
sqrt(c*x^2 + a))^5*c^3*f*g^5*h^2 - 6*(sqrt(c)*x - sqrt(c*x^2 + a))^5*c^3*e*
g^4*h^3 + 33*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a*c^2*f*g^3*h^4 - 12*(sqrt(c)*
x - sqrt(c*x^2 + a))^5*a*c^2*e*g^2*h^5 + 3*(sqrt(c)*x - sqrt(c*x^2 + a))^5*
a*c^2*d*g*h^6 + 12*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^2*c*f*g*h^6 - 3*(sqrt(
c)*x - sqrt(c*x^2 + a))^5*a^2*c*e*h^7 + 54*(sqrt(c)*x - sqrt(c*x^2 + a))^4*
c^(7/2)*f*g^6*h - 12*(sqrt(c)*x - sqrt(c*x^2 + a))^4*c^(7/2)*e*g^5*h^2 - 6*
(sqrt(c)*x - sqrt(c*x^2 + a))^4*c^(7/2)*d*g^4*h^3 + 87*(sqrt(c)*x - sqrt(c*
x^2 + a))^4*a*c^(5/2)*f*g^4*h^3 - 24*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a*c^(5
/2)*e*g^3*h^4 + 3*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a*c^(5/2)*d*g^2*h^5 + 12*
(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^2*c^(3/2)*f*g^2*h^5 + 3*(sqrt(c)*x - sqrt
(c*x^2 + a))^4*a^2*c^(3/2)*e*g*h^6 - 6*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^2*
c^(3/2)*d*h^7 - 6*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^3*sqrt(c)*f*h^7 + 44*(s
qrt(c)*x - sqrt(c*x^2 + a))^3*c^4*f*g^7 - 8*(sqrt(c)*x - sqrt(c*x^2 + a))^3
*c^4*e*g^6*h - 4*(sqrt(c)*x - sqrt(c*x^2 + a))^3*c^4*d*g^5*h^2 + 14*(sqrt(c
)*x - sqrt(c*x^2 + a))^3*a*c^3*f*g^5*h^2 - 8*(sqrt(c)*x - sqrt(c*x^2 + a))^
3*a*c^3*e*g^4*h^3 + 14*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a*c^3*d*g^3*h^4 - 96
*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^2*c^2*f*g^3*h^4 + 30*(sqrt(c)*x - sqrt(c
*x^2 + a))^3*a^2*c^2*e*g^2*h^5 - 12*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^2*c^2
*d*g*h^6 - 36*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^3*c*f*g*h^6 - 78*(sqrt(c)*x
 - sqrt(c*x^2 + a))^2*a*c^(7/2)*f*g^6*h + 12*(sqrt(c)*x - sqrt(c*x^2 + a))^
2*a*c^(7/2)*e*g^5*h^2 + 6*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a*c^(7/2)*d*g^4*h
^3 - 120*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^2*c^(5/2)*f*g^4*h^3 + 30*(sqrt(c
)*x - sqrt(c*x^2 + a))^2*a^2*c^(5/2)*e*g^3*h^4 - 24*(sqrt(c)*x - sqrt(c*x^2
 + a))^2*a^2*c^(5/2)*d*g^2*h^5 - 12*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^3*c^(
3/2)*e*g*h^6 + 12*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^4*sqrt(c)*f*h^7 + 48*(s
qrt(c)*x - sqrt(c*x^2 + a))*a^2*c^3*f*g^5*h^2 - 6*(sqrt(c)*x - sqrt(c*x^2 +
 a))*a^2*c^3*e*g^4*h^3 - 6*(sqrt(c)*x - sqrt(c*x^2 + a))*a^2*c^3*d*g^3*h^4
+ 87*(sqrt(c)*x - sqrt(c*x^2 + a))*a^3*c^2*f*g^3*h^4 - 18*(sqrt(c)*x - sqrt
(c*x^2 + a))*a^3*c^2*e*g^2*h^5 + 9*(sqrt(c)*x - sqrt(c*x^2 + a))*a^3*c^2*d*
g*h^6 + 24*(sqrt(c)*x - sqrt(c*x^2 + a))*a^4*c*f*g*h^6 + 3*(sqrt(c)*x - sqr
t(c*x^2 + a))*a^4*c*e*h^7 - 11*a^3*c^(5/2)*f*g^4*h^3 + 2*a^3*c^(5/2)*e*g^3*
h^4 + a^3*c^(5/2)*d*g^2*h^5 - 20*a^4*c^(3/2)*f*g^2*h^5 + 5*a^4*c^(3/2)*e*g*
h^6 - 2*a^4*c^(3/2)*d*h^7 - 6*a^5*sqrt(c)*f*h^7)/((c^2*g^4*h^4 + 2*a*c*g^2*
h^6 + a^2*h^8)*((sqrt(c)*x - sqrt(c*x^2 + a))^2*h + 2*(sqrt(c)*x - sqrt(c*x
^2 + a))*sqrt(c)*g - a*h)^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^4}\,dx=\int \frac{\sqrt{c\,x^2+a}\,\left(f\,x^2+e\,x+d\right)}{{\left(g+h\,x\right)}^4}\,dx$$

```
[In] int(((a + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^4,x)
[Out] int(((a + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^4, x)
```

3.86 $\int \frac{\sqrt{a+cx^2}\,(d+ex+fx^2)}{(g+hx)^5}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 313

$$\begin{aligned}\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^5}\,dx &= -\frac{(4c^2dg^2+4a^2fh^2-ac(fg^2-h(5eg-dh)))\,(ah-cgx)\sqrt{a+cx^2}}{8\,(cg^2+ah^2)^3\,(g+hx)^2} \\ &\quad -\frac{(fg^2-egh+dh^2)\,(a+cx^2)^{3/2}}{4h\,(cg^2+ah^2)\,(g+hx)^4} \\ &\quad +\frac{(4ah^2(2fg-eh)+cg(3fg^2+h(eg-5dh)))\,(a+cx^2)^{3/2}}{12h\,(cg^2+ah^2)^2\,(g+hx)^3} \\ &\quad -\frac{ac(4c^2dg^2+4a^2fh^2-ac(fg^2-h(5eg-dh)))\,\operatorname{arctanh}\left(\frac{ah-cgx}{\sqrt{cg^2+ah^2}\sqrt{a+cx^2}}\right)}{8\,(cg^2+ah^2)^{7/2}}\end{aligned}$$

```
[Out] -1/4*(d*h^2-e*g*h+f*g^2)*(c*x^2+a)^(3/2)/h/(a*h^2+c*g^2)/(h*x+g)^4+1/12*(4*
a*h^2*(-e*h+2*f*g)+c*g*(3*f*g^2+h*(-5*d*h+e*g)))*(c*x^2+a)^(3/2)/h/(a*h^2+c
*g^2)^2/(h*x+g)^3-1/8*a*c*(4*c^2*d*g^2+4*a^2*f*h^2-a*c*(f*g^2-h*(-d*h+5*e*g
)))*arctanh((-c*g*x+a*h)/(a*h^2+c*g^2)^(1/2)/(c*x^2+a)^(1/2))/(a*h^2+c*g^2)
^(7/2)-1/8*(4*c^2*d*g^2+4*a^2*f*h^2-a*c*(f*g^2-h*(-d*h+5*e*g)))*(-c*g*x+a*h
)*(c*x^2+a)^(1/2)/(a*h^2+c*g^2)^3/(h*x+g)^2
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1665, 821, 735, 739, 212}

$$\begin{aligned}&\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^5}\,dx\\&=-\frac{a\text{carctanh}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)(4a^2fh^2-ac(fg^2-h(5eg-dh))+4c^2dg^2)}{8\left(ah^2+cg^2\right)^{7/2}}\\&\quad-\frac{\sqrt{a+cx^2}(ah-cgx)\left(4a^2fh^2-ac(fg^2-h(5eg-dh))+4c^2dg^2\right)}{8(g+hx)^2\left(ah^2+cg^2\right)^3}\\&\quad-\frac{\left(a+cx^2\right)^{3/2}\left(dh^2-egh+fg^2\right)}{4h(g+hx)^4\left(ah^2+cg^2\right)}\\&\quad+\frac{\left(a+cx^2\right)^{3/2}\left(4ah^2(2fg-eh)+cgh(eg-5dh)+3cfg^3\right)}{12h(g+hx)^3\left(ah^2+cg^2\right)^2}\end{aligned}$$

```
[In] Int[(Sqrt[a + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^5,x]
```

```
[Out] -1/8*((4*c^2*d*g^2 + 4*a^2*f*h^2 - a*c*(f*g^2 - h*(5*e*g - d*h)))*(a*h - c*
g*x)*Sqrt[a + c*x^2])/((c*g^2 + a*h^2)^3*(g + h*x)^2) - ((f*g^2 - e*g*h + d
*h^2)*(a + c*x^2)^(3/2))/(4*h*(c*g^2 + a*h^2)*(g + h*x)^4) + ((3*c*f*g^3 +
c*g*h*(e*g - 5*d*h) + 4*a*h^2*(2*f*g - e*h))*(a + c*x^2)^(3/2))/(12*h*(c*g^
2 + a*h^2)^2*(g + h*x)^3) - (a*c*(4*c^2*d*g^2 + 4*a^2*f*h^2 - a*c*(f*g^2 -
h*(5*e*g - d*h)))*ArcTanh[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2
]])])/(8*(c*g^2 + a*h^2)^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 735

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(-2*a*e + (2*c*d)*x)*((a + c*x^2)^p/(2*(m + 1)*(c*d^2
+ a*e^2))), x] - Dist[4*a*c*(p/(2*(m + 1)*(c*d^2 + a*e^2))), Int[(d + e*x)^
(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2
+ a*e^2, 0] && EqQ[m + 2*p + 2, 0] && GtQ[p, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
```

```
[{a, c, d, e}, x]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(fg^2-egh+dh^2)\,(a+cx^2)^{3/2}}{4h\,(cg^2+ah^2)\,(g+hx)^4} \\
&\quad -\frac{\int \frac{\left(-4(cdg-afg+aeh)-\left(4afh+c\left(eg+\frac{3fg^2}{h}-dh\right)\right)x\right)\sqrt{a+cx^2}}{(g+hx)^4}\,dx}{4\,(cg^2+ah^2)} \\
&= -\frac{(fg^2-egh+dh^2)\,(a+cx^2)^{3/2}}{4h\,(cg^2+ah^2)\,(g+hx)^4} \\
&\quad +\frac{(3cfg^3+cgh(eg-5dh)+4ah^2(2fg-eh))\,(a+cx^2)^{3/2}}{12h\,(cg^2+ah^2)^2\,(g+hx)^3} \\
&\quad +\frac{(4c^2dg^2+4a^2fh^2-ac(fg^2-h(5eg-dh)))\int \frac{\sqrt{a+cx^2}}{(g+hx)^3}\,dx}{4\,(cg^2+ah^2)^2} \\
&= -\frac{(4c^2dg^2+4a^2fh^2-ac(fg^2-h(5eg-dh)))\,(ah-cgx)\sqrt{a+cx^2}}{8\,(cg^2+ah^2)^3\,(g+hx)^2} \\
&\quad -\frac{(fg^2-egh+dh^2)\,(a+cx^2)^{3/2}}{4h\,(cg^2+ah^2)\,(g+hx)^4} \\
&\quad +\frac{(3cfg^3+cgh(eg-5dh)+4ah^2(2fg-eh))\,(a+cx^2)^{3/2}}{12h\,(cg^2+ah^2)^2\,(g+hx)^3} \\
&\quad +\frac{(ac(4c^2dg^2+4a^2fh^2-ac(fg^2-h(5eg-dh))))\int \frac{1}{(g+hx)\sqrt{a+cx^2}}\,dx}{8\,(cg^2+ah^2)^3}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{(4c^2dg^2+4a^2fh^2-ac(fg^2-h(5eg-dh)))\,(ah-cgx)\sqrt{a+cx^2}}{8\left(cg^2+ah^2\right)^3(g+hx)^2} \\
&\quad - \frac{\left(fg^2-egh+dh^2\right)\left(a+cx^2\right)^{3/2}}{4h\left(cg^2+ah^2\right)(g+hx)^4} \\
&\quad + \frac{\left(3cfg^3+cgh(eg-5dh)+4ah^2(2fg-eh)\right)\left(a+cx^2\right)^{3/2}}{12h\left(cg^2+ah^2\right)^2(g+hx)^3} \\
&\quad - \frac{\left(ac(4c^2dg^2+4a^2fh^2-ac(fg^2-h(5eg-dh)))\right)\mathrm{Subst}\left(\int\frac{1}{cg^2+ah^2-x^2}\,dx,x,\frac{ah-cgx}{\sqrt{a+cx^2}}\right)}{8\left(cg^2+ah^2\right)^3} \\
&= -\frac{(4c^2dg^2+4a^2fh^2-ac(fg^2-h(5eg-dh)))\,(ah-cgx)\sqrt{a+cx^2}}{8\left(cg^2+ah^2\right)^3(g+hx)^2} \\
&\quad - \frac{\left(fg^2-egh+dh^2\right)\left(a+cx^2\right)^{3/2}}{4h\left(cg^2+ah^2\right)(g+hx)^4} \\
&\quad + \frac{\left(3cfg^3+cgh(eg-5dh)+4ah^2(2fg-eh)\right)\left(a+cx^2\right)^{3/2}}{12h\left(cg^2+ah^2\right)^2(g+hx)^3} \\
&\quad - \frac{ac(4c^2dg^2+4a^2fh^2-ac(fg^2-h(5eg-dh)))\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{cg^2+ah^2}\sqrt{a+cx^2}}\right)}{8\left(cg^2+ah^2\right)^{7/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 10.75 (sec) , antiderivative size = 439, normalized size of antiderivative = 1.40

$$
\int\frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^5}\,dx
$$

$$
\begin{aligned}
&= \frac{1}{24}\Bigg(-\frac{\sqrt{a+cx^2}\Big(6(cg^2+ah^2)^3\left(fg^2+h(-eg+dh)\right)-2(cg^2+ah^2)^2\left(9cfg^3+cgh(-5eg+dh)-4ah^2(-\right.}{} \\
&\qquad + \frac{3ac(4c^2dg^2+4a^2fh^2-ac(fg^2+h(-5eg+dh)))\log(g+hx)}{\left(cg^2+ah^2\right)^{7/2}} \\
&\quad - \frac{3ac(4c^2dg^2+4a^2fh^2-ac(fg^2+h(-5eg+dh)))\log\left(ah-cgx+\sqrt{cg^2+ah^2}\sqrt{a+cx^2}\right)}{\left(cg^2+ah^2\right)^{7/2}}\Bigg)
\end{aligned}
$$

```
[In] Integrate[(Sqrt[a + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^5,x]
```

```
[Out] (-((Sqrt[a + c*x^2]*(6*(c*g^2 + a*h^2)^3*(f*g^2 + h*(-(e*g) + d*h)) - 2*(c*
g^2 + a*h^2)^2*(9*c*f*g^3 + c*g*h*(-5*e*g + d*h) - 4*a*h^2*(-2*f*g + e*h))*
(g + h*x) + (c*g^2 + a*h^2)*(12*a^2*f*h^4 + 2*c^2*(9*f*g^4 - g^2*h*(e*g + d
*h)) + a*c*h^2*(35*f*g^2 + h*(-7*e*g + 3*d*h)))*(g + h*x)^2 - c*(4*a^2*h^4*
(7*f*g - 2*e*h) + a*c*g*h^2*(19*f*g^2 + h*(9*e*g - 13*d*h)) + 2*c^2*(3*f*g^
```

```
5 + g^3*h*(e*g + d*h)))*(g + h*x)^3))/((c*g^2*h + a*h^3)^3*(g + h*x)^4)) +
(3*a*c*(4*c^2*d*g^2 + 4*a^2*f*h^2 - a*c*(f*g^2 + h*(-5*e*g + d*h)))*Log[g +
 h*x])/(c*g^2 + a*h^2)^(7/2) - (3*a*c*(4*c^2*d*g^2 + 4*a^2*f*h^2 - a*c*(f*g
^2 + h*(-5*e*g + d*h)))*Log[a*h - c*g*x + Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^
2]])/(c*g^2 + a*h^2)^(7/2))/24
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3902 vs. 2(293) = 586.

Time = 0.77 (sec) , antiderivative size = 3903, normalized size of antiderivative = 12.47

method	result	size
default	Expression too large to display	3903

```
[In] int((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^5,x,method=_RETURNVERBOSE)
```

```
[Out] f/h^5*(-1/2/(a*h^2+c*g^2)*h^2/(x+1/h*g)^2*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+
(a*h^2+c*g^2)/h^2)^(3/2)+1/2*c*g*h/(a*h^2+c*g^2)*(-1/(a*h^2+c*g^2)*h^2/(x+1
/h*g)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(3/2)-c*g*h/(a*h^
2+c*g^2)*(((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)-c^(1/2)
*g/h*ln((-c*g/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^
2+c*g^2)/h^2)^(1/2))-(a*h^2+c*g^2)/h^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h
^2+c*g^2)/h^2-2*c*g/h*(x+1/h*g)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c-
2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+1/h*g)))+2*c/(a*h^2+c*g^2)*h
^2*(1/4*(2*c*(x+1/h*g)-2*c*g/h)/c*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c
*g^2)/h^2)^(1/2)+1/8*(4*c*(a*h^2+c*g^2)/h^2-4*c^2*g^2/h^2)/c^(3/2)*ln((-c*g
/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)
^(1/2))))+1/2*c/(a*h^2+c*g^2)*h^2*(((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+
c*g^2)/h^2)^(1/2)-c^(1/2)*g/h*ln((-c*g/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g)^2*
c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))-(a*h^2+c*g^2)/h^2/((a*h^2+c*g
^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1/h*g)+2*((a*h^2+c*g^2)/h
^2)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+1/h
*g))))+(e*h-2*f*g)/h^6*(-1/3/(a*h^2+c*g^2)*h^2/(x+1/h*g)^3*((x+1/h*g)^2*c-2
*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(3/2)+c*g*h/(a*h^2+c*g^2)*(-1/2/(a*h^2+
c*g^2)*h^2/(x+1/h*g)^2*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^
(3/2)+1/2*c*g*h/(a*h^2+c*g^2)*(-1/(a*h^2+c*g^2)*h^2/(x+1/h*g)*((x+1/h*g)^2*
c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(3/2)-c*g*h/(a*h^2+c*g^2)*(((x+1/h*g
)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)-c^(1/2)*g/h*ln((-c*g/h+c*(
x+1/h*g))/c^(1/2)+((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)
)-(a*h^2+c*g^2)/h^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g
/h*(x+1/h*g)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(
a*h^2+c*g^2)/h^2)^(1/2))/(x+1/h*g)))+2*c/(a*h^2+c*g^2)*h^2*(1/4*(2*c*(x+1/h
*g)-2*c*g/h)/c*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)+1/
8*(4*c*(a*h^2+c*g^2)/h^2-4*c^2*g^2/h^2)/c^(3/2)*ln((-c*g/h+c*(x+1/h*g))/c^(
1/2)+((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))))+1/2*c/(a*
```

```
h^2+c*g^2)*h^2*(((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)-c
^(1/2)*g/h*ln((-c*g/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)
+(a*h^2+c*g^2)/h^2)^(1/2))-(a*h^2+c*g^2)/h^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((
2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1/h*g)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+1/h*g
)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+1/h*g)))))+(d*h^2-e*g*
h+f*g^2)/h^7*(-1/4/(a*h^2+c*g^2)*h^2/(x+1/h*g)^4*((x+1/h*g)^2*c-2*c*g/h*(x+
1/h*g)+(a*h^2+c*g^2)/h^2)^(3/2)+5/4*c*g*h/(a*h^2+c*g^2)*(-1/3/(a*h^2+c*g^2)
*h^2/(x+1/h*g)^3*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(3/2)+
c*g*h/(a*h^2+c*g^2)*(-1/2/(a*h^2+c*g^2)*h^2/(x+1/h*g)^2*((x+1/h*g)^2*c-2*c*
g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(3/2)+1/2*c*g*h/(a*h^2+c*g^2)*(-1/(a*h^2+c
*g^2)*h^2/(x+1/h*g)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(3/
2)-c*g*h/(a*h^2+c*g^2)*(((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)
^(1/2)-c^(1/2)*g/h*ln((-c*g/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g)^2*c-2*c*g/h*(
x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))-(a*h^2+c*g^2)/h^2/((a*h^2+c*g^2)/h^2)^(1
/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1/h*g)+2*((a*h^2+c*g^2)/h^2)^(1/2)*(
(x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+1/h*g)))+2*c/(
a*h^2+c*g^2)*h^2*(1/4*(2*c*(x+1/h*g)-2*c*g/h)/c*((x+1/h*g)^2*c-2*c*g/h*(x+1
/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)+1/8*(4*c*(a*h^2+c*g^2)/h^2-4*c^2*g^2/h^2)/c^
(3/2)*ln((-c*g/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h
^2+c*g^2)/h^2)^(1/2))))+1/2*c/(a*h^2+c*g^2)*h^2*(((x+1/h*g)^2*c-2*c*g/h*(x+
1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)-c^(1/2)*g/h*ln((-c*g/h+c*(x+1/h*g))/c^(1/2)
+((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))-(a*h^2+c*g^2)/h
^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1/h*g)+2*((
a*h^2+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)
^(1/2))/(x+1/h*g)))))-1/4*c/(a*h^2+c*g^2)*h^2*(-1/2/(a*h^2+c*g^2)*h^2/(x+1/
h*g)^2*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(3/2)+1/2*c*g*h/
(a*h^2+c*g^2)*(-1/(a*h^2+c*g^2)*h^2/(x+1/h*g)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h
*g)+(a*h^2+c*g^2)/h^2)^(3/2)-c*g*h/(a*h^2+c*g^2)*(((x+1/h*g)^2*c-2*c*g/h*(x
+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)-c^(1/2)*g/h*ln((-c*g/h+c*(x+1/h*g))/c^(1/2
)+((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))-(a*h^2+c*g^2)/
h^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1/h*g)+2*(
(a*h^2+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2
)^(1/2))/(x+1/h*g)))+2*c/(a*h^2+c*g^2)*h^2*(1/4*(2*c*(x+1/h*g)-2*c*g/h)/c*(
(x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)+1/8*(4*c*(a*h^2+c*
g^2)/h^2-4*c^2*g^2/h^2)/c^(3/2)*ln((-c*g/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g)^
2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))))+1/2*c/(a*h^2+c*g^2)*h^2*(
((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)-c^(1/2)*g/h*ln((-
c*g/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h
^2)^(1/2))-(a*h^2+c*g^2)/h^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/
h^2-2*c*g/h*(x+1/h*g)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x
+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+1/h*g)))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1263 vs. $2(294) = 588$.

Time = 61.13 (sec) , antiderivative size = 2552, normalized size of antiderivative = 8.15

$$\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^5}\,dx = \text{Too large to display}$$

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^5,x, algorithm="fricas")
```

```
[Out] [1/48*(3*(5*a^2*c^2*e*g^5*h + (4*a*c^3*d - a^2*c^2*f)*g^6 - (a^2*c^2*d - 4*
a^3*c*f)*g^4*h^2 + (5*a^2*c^2*e*g*h^5 + (4*a*c^3*d - a^2*c^2*f)*g^2*h^4 - (
a^2*c^2*d - 4*a^3*c*f)*h^6)*x^4 + 4*(5*a^2*c^2*e*g^2*h^4 + (4*a*c^3*d - a^2
*c^2*f)*g^3*h^3 - (a^2*c^2*d - 4*a^3*c*f)*g*h^5)*x^3 + 6*(5*a^2*c^2*e*g^3*h
^3 + (4*a*c^3*d - a^2*c^2*f)*g^4*h^2 - (a^2*c^2*d - 4*a^3*c*f)*g^2*h^4)*x^2
 + 4*(5*a^2*c^2*e*g^4*h^2 + (4*a*c^3*d - a^2*c^2*f)*g^5*h - (a^2*c^2*d - 4*
a^3*c*f)*g^3*h^3)*x)*sqrt(c*g^2 + a*h^2)*log((2*a*c*g*h*x - a*c*g^2 - 2*a^2
*h^2 - (2*c^2*g^2 + a*c*h^2)*x^2 - 2*sqrt(c*g^2 + a*h^2)*(c*g*x - a*h)*sqrt
(c*x^2 + a))/(h^2*x^2 + 2*g*h*x + g^2)) + 2*(8*a*c^3*e*g^7 - a^2*c^2*e*g^5*
h^2 - 11*a^3*c*e*g^3*h^4 - 2*a^4*e*g*h^6 - 6*a^4*d*h^7 - (28*a*c^3*d - 13*a
^2*c^2*f)*g^6*h - (47*a^2*c^2*d - 11*a^3*c*f)*g^4*h^3 - (25*a^3*c*d + 2*a^4
*f)*g^2*h^5 + (6*c^4*f*g^7 + 2*c^4*e*g^6*h + 11*a*c^3*e*g^4*h^3 + a^2*c^2*e
*g^2*h^5 - 8*a^3*c*e*h^7 + (2*c^4*d + 25*a*c^3*f)*g^5*h^2 - (11*a*c^3*d - 4
7*a^2*c^2*f)*g^3*h^4 - (13*a^2*c^2*d - 28*a^3*c*f)*g*h^6)*x^3 + (8*c^4*e*g^
7 + 44*a*c^3*e*g^5*h^2 + 19*a^2*c^2*e*g^3*h^4 - 17*a^3*c*e*g*h^6 + 4*(2*c^4
*d + a*c^3*f)*g^6*h - (32*a*c^3*d - 41*a^2*c^2*f)*g^4*h^3 - (43*a^2*c^2*d -
 25*a^3*c*f)*g^2*h^5 - 3*(a^3*c*d + 4*a^4*f)*h^7)*x^2 + (17*a*c^3*e*g^6*h -
 19*a^2*c^2*e*g^4*h^3 - 44*a^3*c*e*g^2*h^5 - 8*a^4*e*h^7 + 3*(4*c^4*d + a*c
^3*f)*g^7 - (25*a*c^3*d - 43*a^2*c^2*f)*g^5*h^2 - (41*a^2*c^2*d - 32*a^3*c*
f)*g^3*h^4 - 4*(a^3*c*d + 2*a^4*f)*g*h^6)*x)*sqrt(c*x^2 + a))/(c^4*g^12 + 4
*a*c^3*g^10*h^2 + 6*a^2*c^2*g^8*h^4 + 4*a^3*c*g^6*h^6 + a^4*g^4*h^8 + (c^4*
g^8*h^4 + 4*a*c^3*g^6*h^6 + 6*a^2*c^2*g^4*h^8 + 4*a^3*c*g^2*h^10 + a^4*h^12
)*x^4 + 4*(c^4*g^9*h^3 + 4*a*c^3*g^7*h^5 + 6*a^2*c^2*g^5*h^7 + 4*a^3*c*g^3*
h^9 + a^4*g*h^11)*x^3 + 6*(c^4*g^10*h^2 + 4*a*c^3*g^8*h^4 + 6*a^2*c^2*g^6*h
^6 + 4*a^3*c*g^4*h^8 + a^4*g^2*h^10)*x^2 + 4*(c^4*g^11*h + 4*a*c^3*g^9*h^3
+ 6*a^2*c^2*g^7*h^5 + 4*a^3*c*g^5*h^7 + a^4*g^3*h^9)*x), -1/24*(3*(5*a^2*c^
2*e*g^5*h + (4*a*c^3*d - a^2*c^2*f)*g^6 - (a^2*c^2*d - 4*a^3*c*f)*g^4*h^2 +
 (5*a^2*c^2*e*g*h^5 + (4*a*c^3*d - a^2*c^2*f)*g^2*h^4 - (a^2*c^2*d - 4*a^3*
c*f)*h^6)*x^4 + 4*(5*a^2*c^2*e*g^2*h^4 + (4*a*c^3*d - a^2*c^2*f)*g^3*h^3 -
(a^2*c^2*d - 4*a^3*c*f)*g*h^5)*x^3 + 6*(5*a^2*c^2*e*g^3*h^3 + (4*a*c^3*d -
a^2*c^2*f)*g^4*h^2 - (a^2*c^2*d - 4*a^3*c*f)*g^2*h^4)*x^2 + 4*(5*a^2*c^2*e*
g^4*h^2 + (4*a*c^3*d - a^2*c^2*f)*g^5*h - (a^2*c^2*d - 4*a^3*c*f)*g^3*h^3)*
x)*sqrt(-c*g^2 - a*h^2)*arctan(sqrt(-c*g^2 - a*h^2)*(c*g*x - a*h)*sqrt(c*x^
2 + a)/(a*c*g^2 + a^2*h^2 + (c^2*g^2 + a*c*h^2)*x^2)) - (8*a*c^3*e*g^7 - a^
2*c^2*e*g^5*h^2 - 11*a^3*c*e*g^3*h^4 - 2*a^4*e*g*h^6 - 6*a^4*d*h^7 - (28*a*
```

```
c^3*d - 13*a^2*c^2*f)*g^6*h - (47*a^2*c^2*d - 11*a^3*c*f)*g^4*h^3 - (25*a^3
*c*d + 2*a^4*f)*g^2*h^5 + (6*c^4*f*g^7 + 2*c^4*e*g^6*h + 11*a*c^3*e*g^4*h^3
 + a^2*c^2*e*g^2*h^5 - 8*a^3*c*e*h^7 + (2*c^4*d + 25*a*c^3*f)*g^5*h^2 - (11
*a*c^3*d - 47*a^2*c^2*f)*g^3*h^4 - (13*a^2*c^2*d - 28*a^3*c*f)*g*h^6)*x^3 +
 (8*c^4*e*g^7 + 44*a*c^3*e*g^5*h^2 + 19*a^2*c^2*e*g^3*h^4 - 17*a^3*c*e*g*h^
6 + 4*(2*c^4*d + a*c^3*f)*g^6*h - (32*a*c^3*d - 41*a^2*c^2*f)*g^4*h^3 - (43
*a^2*c^2*d - 25*a^3*c*f)*g^2*h^5 - 3*(a^3*c*d + 4*a^4*f)*h^7)*x^2 + (17*a*c
^3*e*g^6*h - 19*a^2*c^2*e*g^4*h^3 - 44*a^3*c*e*g^2*h^5 - 8*a^4*e*h^7 + 3*(4
*c^4*d + a*c^3*f)*g^7 - (25*a*c^3*d - 43*a^2*c^2*f)*g^5*h^2 - (41*a^2*c^2*d
 - 32*a^3*c*f)*g^3*h^4 - 4*(a^3*c*d + 2*a^4*f)*g*h^6)*x)*sqrt(c*x^2 + a))/(
c^4*g^12 + 4*a*c^3*g^10*h^2 + 6*a^2*c^2*g^8*h^4 + 4*a^3*c*g^6*h^6 + a^4*g^4
*h^8 + (c^4*g^8*h^4 + 4*a*c^3*g^6*h^6 + 6*a^2*c^2*g^4*h^8 + 4*a^3*c*g^2*h^1
0 + a^4*h^12)*x^4 + 4*(c^4*g^9*h^3 + 4*a*c^3*g^7*h^5 + 6*a^2*c^2*g^5*h^7 +
4*a^3*c*g^3*h^9 + a^4*g*h^11)*x^3 + 6*(c^4*g^10*h^2 + 4*a*c^3*g^8*h^4 + 6*a
^2*c^2*g^6*h^6 + 4*a^3*c*g^4*h^8 + a^4*g^2*h^10)*x^2 + 4*(c^4*g^11*h + 4*a*
c^3*g^9*h^3 + 6*a^2*c^2*g^7*h^5 + 4*a^3*c*g^5*h^7 + a^4*g^3*h^9)*x)]
```

Sympy [F]

$$\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^5}\,dx=\int \frac{\sqrt{a+cx^2}\,(d+ex+fx^2)}{\left(g+hx\right)^5}\,dx$$

```
[In] integrate((f*x**2+e*x+d)*(c*x**2+a)**(1/2)/(h*x+g)**5,x)
```

```
[Out] Integral(sqrt(a + c*x**2)*(d + e*x + f*x**2)/(g + h*x)**5, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3404 vs. $2(294) = 588$.

Time = 0.35 (sec) , antiderivative size = 3404, normalized size of antiderivative = 10.88

$$\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^5}\,dx=\text{Too large to display}$$

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^5,x, algorithm="maxima")
```

```
[Out] -5/8*sqrt(c*x^2 + a)*c^3*f*g^5/(c^3*g^6*h^4*x + 3*a*c^2*g^4*h^6*x + 3*a^2*c
*g^2*h^8*x + a^3*h^10*x + c^3*g^7*h^3 + 3*a*c^2*g^5*h^5 + 3*a^2*c*g^3*h^7 +
 a^3*g*h^9) + 5/8*sqrt(c*x^2 + a)*c^3*e*g^4/(c^3*g^6*h^3*x + 3*a*c^2*g^4*h^
5*x + 3*a^2*c*g^2*h^7*x + a^3*h^9*x + c^3*g^7*h^2 + 3*a*c^2*g^5*h^4 + 3*a^2
*c*g^3*h^6 + a^3*g*h^8) - 5/8*(c*x^2 + a)^(3/2)*c^2*f*g^4/(c^3*g^6*h^3*x^2
+ 3*a*c^2*g^4*h^5*x^2 + 3*a^2*c*g^2*h^7*x^2 + a^3*h^9*x^2 + 2*c^3*g^7*h^2*x
 + 6*a*c^2*g^5*h^4*x + 6*a^2*c*g^3*h^6*x + 2*a^3*g*h^8*x + c^3*g^8*h + 3*a*
```

```
c^2*g^6*h^3 + 3*a^2*c*g^4*h^5 + a^3*g^2*h^7) + 5/8*sqrt(c*x^2 + a)*c^3*f*g^
4/(c^3*g^6*h^3 + 3*a*c^2*g^4*h^5 + 3*a^2*c*g^2*h^7 + a^3*h^9) - 5/8*sqrt(c*
x^2 + a)*c^3*d*g^3/(c^3*g^6*h^2*x + 3*a*c^2*g^4*h^4*x + 3*a^2*c*g^2*h^6*x +
 a^3*h^8*x + c^3*g^7*h + 3*a*c^2*g^5*h^3 + 3*a^2*c*g^3*h^5 + a^3*g*h^7) + 5
/8*(c*x^2 + a)^(3/2)*c^2*e*g^3/(c^3*g^6*h^2*x^2 + 3*a*c^2*g^4*h^4*x^2 + 3*a
^2*c*g^2*h^6*x^2 + a^3*h^8*x^2 + 2*c^3*g^7*h*x + 6*a*c^2*g^5*h^3*x + 6*a^2*
c*g^3*h^5*x + 2*a^3*g*h^7*x + c^3*g^8 + 3*a*c^2*g^6*h^2 + 3*a^2*c*g^4*h^4 +
 a^3*g^2*h^6) - 5/8*sqrt(c*x^2 + a)*c^3*e*g^3/(c^3*g^6*h^2 + 3*a*c^2*g^4*h^
4 + 3*a^2*c*g^2*h^6 + a^3*h^8) - 5/8*(c*x^2 + a)^(3/2)*c^2*d*g^2/(c^3*g^6*h
*x^2 + 3*a*c^2*g^4*h^3*x^2 + 3*a^2*c*g^2*h^5*x^2 + a^3*h^7*x^2 + 2*c^3*g^7*
x + 6*a*c^2*g^5*h^2*x + 6*a^2*c*g^3*h^4*x + 2*a^3*g*h^6*x + c^3*g^8/h + 3*a
*c^2*g^6*h + 3*a^2*c*g^4*h^3 + a^3*g^2*h^5) + 5/8*sqrt(c*x^2 + a)*c^3*d*g^2
/(c^3*g^6*h + 3*a*c^2*g^4*h^3 + 3*a^2*c*g^2*h^5 + a^3*h^7) - 5/12*(c*x^2 +
a)^(3/2)*c*f*g^3/(c^2*g^4*h^4*x^3 + 2*a*c*g^2*h^6*x^3 + a^2*h^8*x^3 + 3*c^2
*g^5*h^3*x^2 + 6*a*c*g^3*h^5*x^2 + 3*a^2*g*h^7*x^2 + 3*c^2*g^6*h^2*x + 6*a*
c*g^4*h^4*x + 3*a^2*g^2*h^6*x + c^2*g^7*h + 2*a*c*g^5*h^3 + a^2*g^3*h^5) +
9/8*sqrt(c*x^2 + a)*c^2*f*g^3/(c^2*g^4*h^4*x + 2*a*c*g^2*h^6*x + a^2*h^8*x
+ c^2*g^5*h^3 + 2*a*c*g^3*h^5 + a^2*g*h^7) + 5/12*(c*x^2 + a)^(3/2)*c*e*g^2
/(c^2*g^4*h^3*x^3 + 2*a*c*g^2*h^5*x^3 + a^2*h^7*x^3 + 3*c^2*g^5*h^2*x^2 + 6
*a*c*g^3*h^4*x^2 + 3*a^2*g*h^6*x^2 + 3*c^2*g^6*h*x + 6*a*c*g^4*h^3*x + 3*a^
2*g^2*h^5*x + c^2*g^7 + 2*a*c*g^5*h^2 + a^2*g^3*h^4) - 5/8*sqrt(c*x^2 + a)*
c^2*e*g^2/(c^2*g^4*h^3*x + 2*a*c*g^2*h^5*x + a^2*h^7*x + c^2*g^5*h^2 + 2*a*
c*g^3*h^4 + a^2*g*h^6) + 9/8*(c*x^2 + a)^(3/2)*c*f*g^2/(c^2*g^4*h^3*x^2 + 2
*a*c*g^2*h^5*x^2 + a^2*h^7*x^2 + 2*c^2*g^5*h^2*x + 4*a*c*g^3*h^4*x + 2*a^2*
g*h^6*x + c^2*g^6*h + 2*a*c*g^4*h^3 + a^2*g^2*h^5) - 9/8*sqrt(c*x^2 + a)*c^
2*f*g^2/(c^2*g^4*h^3 + 2*a*c*g^2*h^5 + a^2*h^7) - 5/12*(c*x^2 + a)^(3/2)*c*
d*g/(c^2*g^4*h^2*x^3 + 2*a*c*g^2*h^4*x^3 + a^2*h^6*x^3 + 3*c^2*g^5*h*x^2 +
6*a*c*g^3*h^3*x^2 + 3*a^2*g*h^5*x^2 + 3*c^2*g^6*x + 6*a*c*g^4*h^2*x + 3*a^2
*g^2*h^4*x + c^2*g^7/h + 2*a*c*g^5*h + a^2*g^3*h^3) + 1/8*sqrt(c*x^2 + a)*c
^2*d*g/(c^2*g^4*h^2*x + 2*a*c*g^2*h^4*x + a^2*h^6*x + c^2*g^5*h + 2*a*c*g^3
*h^3 + a^2*g*h^5) - 5/8*(c*x^2 + a)^(3/2)*c*e*g/(c^2*g^4*h^2*x^2 + 2*a*c*g^
2*h^4*x^2 + a^2*h^6*x^2 + 2*c^2*g^5*h*x + 4*a*c*g^3*h^3*x + 2*a^2*g*h^5*x +
 c^2*g^6 + 2*a*c*g^4*h^2 + a^2*g^2*h^4) + 5/8*sqrt(c*x^2 + a)*c^2*e*g/(c^2*
g^4*h^2 + 2*a*c*g^2*h^4 + a^2*h^6) - 1/4*(c*x^2 + a)^(3/2)*f*g^2/(c*g^2*h^5
*x^4 + a*h^7*x^4 + 4*c*g^3*h^4*x^3 + 4*a*g*h^6*x^3 + 6*c*g^4*h^3*x^2 + 6*a*
g^2*h^5*x^2 + 4*c*g^5*h^2*x + 4*a*g^3*h^4*x + c*g^6*h + a*g^4*h^3) + 1/8*(c
*x^2 + a)^(3/2)*c*d/(c^2*g^4*h*x^2 + 2*a*c*g^2*h^3*x^2 + a^2*h^5*x^2 + 2*c^
2*g^5*x + 4*a*c*g^3*h^2*x + 2*a^2*g*h^4*x + c^2*g^6/h + 2*a*c*g^4*h + a^2*g
^2*h^3) - 1/8*sqrt(c*x^2 + a)*c^2*d/(c^2*g^4*h + 2*a*c*g^2*h^3 + a^2*h^5) +
 1/4*(c*x^2 + a)^(3/2)*e*g/(c*g^2*h^4*x^4 + a*h^6*x^4 + 4*c*g^3*h^3*x^3 + 4
*a*g*h^5*x^3 + 6*c*g^4*h^2*x^2 + 6*a*g^2*h^4*x^2 + 4*c*g^5*h*x + 4*a*g^3*h^
3*x + c*g^6 + a*g^4*h^2) + 2/3*(c*x^2 + a)^(3/2)*f*g/(c*g^2*h^4*x^3 + a*h^6
*x^3 + 3*c*g^3*h^3*x^2 + 3*a*g*h^5*x^2 + 3*c*g^4*h^2*x + 3*a*g^2*h^4*x + c*
g^5*h + a*g^3*h^3) - 1/2*sqrt(c*x^2 + a)*c*f*g/(c*g^2*h^4*x + a*h^6*x + c*g
^3*h^3 + a*g*h^5) - 1/4*(c*x^2 + a)^(3/2)*d/(c*g^2*h^3*x^4 + a*h^5*x^4 + 4*
```

```
c*g^3*h^2*x^3 + 4*a*g*h^4*x^3 + 6*c*g^4*h*x^2 + 6*a*g^2*h^3*x^2 + 4*c*g^5*x
 + 4*a*g^3*h^2*x + c*g^6/h + a*g^4*h) - 1/3*(c*x^2 + a)^(3/2)*e/(c*g^2*h^3*
x^3 + a*h^5*x^3 + 3*c*g^3*h^2*x^2 + 3*a*g*h^4*x^2 + 3*c*g^4*h*x + 3*a*g^2*h
^3*x + c*g^5 + a*g^3*h^2) - 1/2*(c*x^2 + a)^(3/2)*f/(c*g^2*h^3*x^2 + a*h^5*
x^2 + 2*c*g^3*h^2*x + 2*a*g*h^4*x + c*g^4*h + a*g^2*h^3) + 1/2*sqrt(c*x^2 +
 a)*c*f/(c*g^2*h^3 + a*h^5) - 5/8*c^4*f*g^6*arcsinh(c*g*x/(sqrt(a*c)*abs(h*
x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(7/2)*h^11) + 5/8*
c^4*e*g^5*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x +
 g)))/((a + c*g^2/h^2)^(7/2)*h^10) - 5/8*c^4*d*g^4*arcsinh(c*g*x/(sqrt(a*c)
*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(7/2)*h^9)
+ 7/4*c^3*f*g^4*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs
(h*x + g)))/((a + c*g^2/h^2)^(5/2)*h^9) - 5/4*c^3*e*g^3*arcsinh(c*g*x/(sqrt
(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(5/2)*
h^8) + 3/4*c^3*d*g^2*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c
)*abs(h*x + g)))/((a + c*g^2/h^2)^(5/2)*h^7) - 13/8*c^2*f*g^2*arcsinh(c*g*x
/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^
(3/2)*h^7) + 5/8*c^2*e*g*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt
(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(3/2)*h^6) - 1/8*c^2*d*arcsinh(c*g*x/
(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(
3/2)*h^5) + 1/2*c*f*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)
*abs(h*x + g)))/(sqrt(a + c*g^2/h^2)*h^5)
```

Giac [F]

$$\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^5}\,dx=\int \frac{\sqrt{cx^2+a}(fx^2+ex+d)}{(hx+g)^5}\,dx$$

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^5,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^5}\,dx=\int \frac{\sqrt{c\,x^2+a}\,(f\,x^2+e\,x+d)}{(g+h\,x)^5}\,dx$$

```
[In] int(((a + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^5,x)
[Out] int(((a + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^5, x)
```

3.87 $\int \frac{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}{(g+hx)^6} \, dx$

Optimal result

Integrand size = 29, antiderivative size = 433

$$
\begin{aligned}
&\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^6} \, dx \\
&= -\frac{c(4c^2dg^3+a^2h^2(6fg-eh)-acg(fg^2-3h(2eg-dh)))\,(ah-cgx)\sqrt{a+cx^2}}{8\,(cg^2+ah^2)^4\,(g+hx)^2} \\
&\quad -\frac{(fg^2-egh+dh^2)\,(a+cx^2)^{3/2}}{5h\,(cg^2+ah^2)\,(g+hx)^5} \\
&\quad +\frac{(5ah^2(2fg-eh)+cg(3fg^2+h(2eg-7dh)))\,(a+cx^2)^{3/2}}{20h\,(cg^2+ah^2)^2\,(g+hx)^4} \\
&\quad -\frac{(20a^2fh^4-c^2g^2(3fg^2+h(2eg-27dh))-ach^2(18fg^2-h(33eg-8dh)))\,(a+cx^2)^{3/2}}{60h\,(cg^2+ah^2)^3\,(g+hx)^3} \\
&\quad -\frac{ac^2(4c^2dg^3+a^2h^2(6fg-eh)-acg(fg^2-3h(2eg-dh)))\,\operatorname{arctanh}\left(\frac{ah-cgx}{\sqrt{cg^2+ah^2}\sqrt{a+cx^2}}\right)}{8\,(cg^2+ah^2)^{9/2}}
\end{aligned}
$$

```
[Out] -1/5*(d*h^2-e*g*h+f*g^2)*(c*x^2+a)^(3/2)/h/(a*h^2+c*g^2)/(h*x+g)^5+1/20*(5*
a*h^2*(-e*h+2*f*g)+c*g*(3*f*g^2+h*(-7*d*h+2*e*g)))*(c*x^2+a)^(3/2)/h/(a*h^2
+c*g^2)^2/(h*x+g)^4-1/60*(20*a^2*f*h^4-c^2*g^2*(3*f*g^2+h*(-27*d*h+2*e*g))-
a*c*h^2*(18*f*g^2-h*(-8*d*h+33*e*g)))*(c*x^2+a)^(3/2)/h/(a*h^2+c*g^2)^3/(h*
x+g)^3-1/8*a*c^2*(4*c^2*d*g^3+a^2*h^2*(-e*h+6*f*g)-a*c*g*(f*g^2-3*h*(-d*h+2
*e*g)))*arctanh((-c*g*x+a*h)/(a*h^2+c*g^2)^(1/2)/(c*x^2+a)^(1/2))/(a*h^2+c*
g^2)^(9/2)-1/8*c*(4*c^2*d*g^3+a^2*h^2*(-e*h+6*f*g)-a*c*g*(f*g^2-3*h*(-d*h+2
*e*g)))*(-c*g*x+a*h)*(c*x^2+a)^(1/2)/(a*h^2+c*g^2)^4/(h*x+g)^2
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 432, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1665, 849, 821, 735, 739, 212}

$$\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^6}\,dx$$

$$= -\frac{ac^2\text{arctanh}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)(a^2h^2(6fg-eh)-acg(fg^2-3h(2eg-dh))+4c^2dg^3)}{8(ah^2+cg^2)^{9/2}}$$

$$-\frac{(a+cx^2)^{3/2}(20a^2fh^4-ach^2(18fg^2-h(33eg-8dh))-c^2(g^2h(2eg-27dh)+3fg^4))}{60h(g+hx)^3(ah^2+cg^2)^3}$$

$$-\frac{c\sqrt{a+cx^2}(ah-cgx)(a^2h^2(6fg-eh)-acg(fg^2-3h(2eg-dh))+4c^2dg^3)}{8(g+hx)^2(ah^2+cg^2)^4}$$

$$-\frac{(a+cx^2)^{3/2}(dh^2-egh+fg^2)}{5h(g+hx)^5(ah^2+cg^2)}$$

$$+\frac{(a+cx^2)^{3/2}(5ah^2(2fg-eh)+cgh(2eg-7dh)+3cfg^3)}{20h(g+hx)^4(ah^2+cg^2)^2}$$

```
[In] Int[(Sqrt[a + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^6,x]
```

```
[Out] -1/8*(c*(4*c^2*d*g^3 + a^2*h^2*(6*f*g - e*h) - a*c*g*(f*g^2 - 3*h*(2*e*g -
d*h)))*(a*h - c*g*x)*Sqrt[a + c*x^2])/((c*g^2 + a*h^2)^4*(g + h*x)^2) - ((f
*g^2 - e*g*h + d*h^2)*(a + c*x^2)^(3/2))/(5*h*(c*g^2 + a*h^2)*(g + h*x)^5)
+ ((3*c*f*g^3 + c*g*h*(2*e*g - 7*d*h) + 5*a*h^2*(2*f*g - e*h))*(a + c*x^2)^
(3/2))/(20*h*(c*g^2 + a*h^2)^2*(g + h*x)^4) - ((20*a^2*f*h^4 - c^2*(3*f*g^4
 + g^2*h*(2*e*g - 27*d*h)) - a*c*h^2*(18*f*g^2 - h*(33*e*g - 8*d*h)))*(a +
c*x^2)^(3/2))/(60*h*(c*g^2 + a*h^2)^3*(g + h*x)^3) - (a*c^2*(4*c^2*d*g^3 +
a^2*h^2*(6*f*g - e*h) - a*c*g*(f*g^2 - 3*h*(2*e*g - d*h)))*ArcTanh[(a*h - c
*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2])])/(8*(c*g^2 + a*h^2)^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 735

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(-2*a*e + (2*c*d)*x)*((a + c*x^2)^p/(2*(m + 1)*(c*d^2
+ a*e^2))), x] - Dist[4*a*c*(p/(2*(m + 1)*(c*d^2 + a*e^2))), Int[(d + e*x)^
(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2
```

```
+ a*e^2, 0] && EqQ[m + 2*p + 2, 0] && GtQ[p, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(fg^2-egh+dh^2)\,(a+cx^2)^{3/2}}{5h\,(cg^2+ah^2)\,(g+hx)^5} \\
&\quad -\frac{\int \frac{\left(-5(cdg-afg+aeh)-\left(5afh+c\left(2eg+\frac{3fg^2}{h}-2dh\right)\right)x\right)\sqrt{a+cx^2}}{(g+hx)^5}\,dx}{5\,(cg^2+ah^2)}
\end{aligned}
$$

$$= -\frac{(fg^2-egh+dh^2)\left(a+cx^2\right)^{3/2}}{5h\left(cg^2+ah^2\right)(g+hx)^5} + \frac{\left(3cfg^3+cgh(2eg-7dh)+5ah^2(2fg-eh)\right)\left(a+cx^2\right)^{3/2}}{20h\left(cg^2+ah^2\right)^2(g+hx)^4} + \frac{\int \frac{\left(4\left(5c^2dg^2+5a^2fh^2-ac\left(2fg^2-h(7eg-2dh)\right)\right)+\frac{c\left(3cfg^3+cgh(2eg-7dh)+5ah^2(2fg-eh)\right)x}{h}\right)\sqrt{a+cx^2}}{(g+hx)^4}\,dx}{20\left(cg^2+ah^2\right)^2}$$

$$= -\frac{(fg^2-egh+dh^2)\left(a+cx^2\right)^{3/2}}{5h\left(cg^2+ah^2\right)(g+hx)^5} + \frac{\left(3cfg^3+cgh(2eg-7dh)+5ah^2(2fg-eh)\right)\left(a+cx^2\right)^{3/2}}{20h\left(cg^2+ah^2\right)^2(g+hx)^4} - \frac{\left(20a^2fh^4-c^2\left(3fg^4+g^2h(2eg-27dh)\right)-ach^2\left(18fg^2-h(33eg-8dh)\right)\right)\left(a+cx^2\right)^{3/2}}{60h\left(cg^2+ah^2\right)^3(g+hx)^3} + \frac{\left(c\left(4c^2dg^3+a^2h^2(6fg-eh)-acg\left(fg^2-3h(2eg-dh)\right)\right)\right)\int\frac{\sqrt{a+cx^2}}{(g+hx)^3}\,dx}{4\left(cg^2+ah^2\right)^3}$$

$$= -\frac{c\left(4c^2dg^3+a^2h^2(6fg-eh)-acg\left(fg^2-3h(2eg-dh)\right)\right)(ah-cgx)\sqrt{a+cx^2}}{8\left(cg^2+ah^2\right)^4(g+hx)^2} - \frac{(fg^2-egh+dh^2)\left(a+cx^2\right)^{3/2}}{5h\left(cg^2+ah^2\right)(g+hx)^5} + \frac{\left(3cfg^3+cgh(2eg-7dh)+5ah^2(2fg-eh)\right)\left(a+cx^2\right)^{3/2}}{20h\left(cg^2+ah^2\right)^2(g+hx)^4} - \frac{\left(20a^2fh^4-c^2\left(3fg^4+g^2h(2eg-27dh)\right)-ach^2\left(18fg^2-h(33eg-8dh)\right)\right)\left(a+cx^2\right)^{3/2}}{60h\left(cg^2+ah^2\right)^3(g+hx)^3} + \frac{\left(ac^2\left(4c^2dg^3+a^2h^2(6fg-eh)-acg\left(fg^2-3h(2eg-dh)\right)\right)\right)\int\frac{1}{(g+hx)\sqrt{a+cx^2}}\,dx}{8\left(cg^2+ah^2\right)^4}$$

$$= -\frac{c\left(4c^2dg^3+a^2h^2(6fg-eh)-acg\left(fg^2-3h(2eg-dh)\right)\right)(ah-cgx)\sqrt{a+cx^2}}{8\left(cg^2+ah^2\right)^4(g+hx)^2} - \frac{(fg^2-egh+dh^2)\left(a+cx^2\right)^{3/2}}{5h\left(cg^2+ah^2\right)(g+hx)^5} + \frac{\left(3cfg^3+cgh(2eg-7dh)+5ah^2(2fg-eh)\right)\left(a+cx^2\right)^{3/2}}{20h\left(cg^2+ah^2\right)^2(g+hx)^4} - \frac{\left(20a^2fh^4-c^2\left(3fg^4+g^2h(2eg-27dh)\right)-ach^2\left(18fg^2-h(33eg-8dh)\right)\right)\left(a+cx^2\right)^{3/2}}{60h\left(cg^2+ah^2\right)^3(g+hx)^3} - \frac{\left(ac^2\left(4c^2dg^3+a^2h^2(6fg-eh)-acg\left(fg^2-3h(2eg-dh)\right)\right)\right)\operatorname{Subst}\left(\int\frac{1}{cg^2+ah^2-x^2}\,dx, x, \frac{ah-cgx}{\sqrt{a+cx^2}}\right)}{8\left(cg^2+ah^2\right)^4}$$

$$
\begin{aligned}
&= -\frac{c(4c^2dg^3 + a^2h^2(6fg - eh) - acg(fg^2 - 3h(2eg - dh)))\,(ah - cgx)\sqrt{a + cx^2}}{8\,(cg^2 + ah^2)^4\,(g + hx)^2} \\
&\quad - \frac{(fg^2 - egh + dh^2)\,(a + cx^2)^{3/2}}{5h\,(cg^2 + ah^2)\,(g + hx)^5} \\
&\quad + \frac{(3cfg^3 + cgh(2eg - 7dh) + 5ah^2(2fg - eh))\,(a + cx^2)^{3/2}}{20h\,(cg^2 + ah^2)^2\,(g + hx)^4} \\
&\quad - \frac{(20a^2fh^4 - c^2(3fg^4 + g^2h(2eg - 27dh)) - ach^2(18fg^2 - h(33eg - 8dh)))\,(a + cx^2)^{3/2}}{60h\,(cg^2 + ah^2)^3\,(g + hx)^3} \\
&\quad - \frac{ac^2(4c^2dg^3 + a^2h^2(6fg - eh) - acg(fg^2 - 3h(2eg - dh)))\tanh^{-1}\left(\frac{ah - cgx}{\sqrt{cg^2 + ah^2}\sqrt{a + cx^2}}\right)}{8\,(cg^2 + ah^2)^{9/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 10.88 (sec) , antiderivative size = 583, normalized size of antiderivative = 1.35

$$
\int \frac{\sqrt{a + cx^2}(d + ex + fx^2)}{(g + hx)^6}\,dx =
$$

$$
\begin{aligned}
&- \frac{\sqrt{a + cx^2}\Big(24(cg^2 + ah^2)^4\,(fg^2 + h(-eg + dh)) - 6(cg^2 + ah^2)^3\,(11cfg^3 + cgh(-6eg + dh) - 5ah^2(-2f}{} \\
&+ \frac{ac^2(4c^2dg^3 + a^2h^2(6fg - eh) - acg(fg^2 + 3h(-2eg + dh)))\log(g + hx)}{8\,(cg^2 + ah^2)^{9/2}} \\
&- \frac{ac^2(4c^2dg^3 + a^2h^2(6fg - eh) - acg(fg^2 + 3h(-2eg + dh)))\log\left(ah - cgx + \sqrt{cg^2 + ah^2}\sqrt{a + cx^2}\right)}{8\,(cg^2 + ah^2)^{9/2}}
\end{aligned}
$$

```
[In] Integrate[(Sqrt[a + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^6,x]
```

```
[Out] -1/120*(Sqrt[a + c*x^2]*(24*(c*g^2 + a*h^2)^4*(f*g^2 + h*(-(e*g) + d*h)) -
6*(c*g^2 + a*h^2)^3*(11*c*f*g^3 + c*g*h*(-6*e*g + d*h) - 5*a*h^2*(-2*f*g +
e*h))*(g + h*x) + 2*(c*g^2 + a*h^2)^2*(20*a^2*f*h^4 + c^2*(27*f*g^4 - g^2*h
*(2*e*g + 3*d*h)) + a*c*h^2*(54*f*g^2 + h*(-9*e*g + 4*d*h)))*(g + h*x)^2 -
c*(c*g^2 + a*h^2)*(5*a^2*h^4*(10*f*g - 3*e*h) + a*c*g*h^2*(21*f*g^2 + h*(24
*e*g - 29*d*h)) + c^2*(6*f*g^5 + 2*g^3*h*(2*e*g + 3*d*h)))*(g + h*x)^3 - c*
(-40*a^3*f*h^6 + a*c^2*g^2*h^2*(27*f*g^2 + h*(28*e*g - 83*d*h)) + c^3*(6*f*
g^6 + 2*g^4*h*(2*e*g + 3*d*h)) + a^2*c*h^4*(86*f*g^2 + h*(-81*e*g + 16*d*h)
))*(g + h*x)^4))/(h^3*(c*g^2 + a*h^2)^4*(g + h*x)^5) + (a*c^2*(4*c^2*d*g^3
+ a^2*h^2*(6*f*g - e*h) - a*c*g*(f*g^2 + 3*h*(-2*e*g + d*h)))*Log[g + h*x])
/(8*(c*g^2 + a*h^2)^(9/2)) - (a*c^2*(4*c^2*d*g^3 + a^2*h^2*(6*f*g - e*h) -
a*c*g*(f*g^2 + 3*h*(-2*e*g + d*h)))*Log[a*h - c*g*x + Sqrt[c*g^2 + a*h^2]*S
qrt[a + c*x^2]])/(8*(c*g^2 + a*h^2)^(9/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 6084 vs. 2(409) = 818.

Time = 0.86 (sec) , antiderivative size = 6085, normalized size of antiderivative = 14.05

method	result	size
default	Expression too large to display	6085

```
[In] int((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^6,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1918 vs. 2(410) = 820.

Time = 165.64 (sec) , antiderivative size = 3862, normalized size of antiderivative = 8.92

$$\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^6}\,dx = \text{Too large to display}$$

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^6,x, algorithm="fricas")
```

```
[Out] [-1/240*(15*(6*a^2*c^3*e*g^7*h - a^3*c^2*e*g^5*h^3 + (4*a*c^4*d - a^2*c^3*f
)*g^8 - 3*(a^2*c^3*d - 2*a^3*c^2*f)*g^6*h^2 + (6*a^2*c^3*e*g^2*h^6 - a^3*c^
2*e*h^8 + (4*a*c^4*d - a^2*c^3*f)*g^3*h^5 - 3*(a^2*c^3*d - 2*a^3*c^2*f)*g*h
^7)*x^5 + 5*(6*a^2*c^3*e*g^3*h^5 - a^3*c^2*e*g*h^7 + (4*a*c^4*d - a^2*c^3*f
)*g^4*h^4 - 3*(a^2*c^3*d - 2*a^3*c^2*f)*g^2*h^6)*x^4 + 10*(6*a^2*c^3*e*g^4*
h^4 - a^3*c^2*e*g^2*h^6 + (4*a*c^4*d - a^2*c^3*f)*g^5*h^3 - 3*(a^2*c^3*d -
2*a^3*c^2*f)*g^3*h^5)*x^3 + 10*(6*a^2*c^3*e*g^5*h^3 - a^3*c^2*e*g^3*h^5 + (
4*a*c^4*d - a^2*c^3*f)*g^6*h^2 - 3*(a^2*c^3*d - 2*a^3*c^2*f)*g^4*h^4)*x^2 +
 5*(6*a^2*c^3*e*g^6*h^2 - a^3*c^2*e*g^4*h^4 + (4*a*c^4*d - a^2*c^3*f)*g^7*h
 - 3*(a^2*c^3*d - 2*a^3*c^2*f)*g^5*h^3)*x)*sqrt(c*g^2 + a*h^2)*log((2*a*c*g
*h*x - a*c*g^2 - 2*a^2*h^2 - (2*c^2*g^2 + a*c*h^2)*x^2 + 2*sqrt(c*g^2 + a*h
^2)*(c*g*x - a*h)*sqrt(c*x^2 + a))/(h^2*x^2 + 2*g*h*x + g^2)) - 2*(40*a*c^4
*e*g^9 - 46*a^2*c^3*e*g^7*h^2 - 113*a^3*c^2*e*g^5*h^4 - 33*a^4*c*e*g^3*h^6
- 6*a^5*e*g*h^8 - 24*a^5*d*h^9 - 9*(20*a*c^4*d - 9*a^2*c^3*f)*g^8*h - (329*
a^2*c^3*d - 53*a^3*c^2*f)*g^6*h^3 - (247*a^3*c^2*d + 32*a^4*c*f)*g^4*h^5 -
2*(61*a^4*c*d + 2*a^5*f)*g^2*h^7 + (6*c^5*f*g^8*h + 4*c^5*e*g^7*h^2 + 32*a*
c^4*e*g^5*h^4 - 53*a^2*c^3*e*g^3*h^6 - 81*a^3*c^2*e*g*h^8 + 3*(2*c^5*d + 11
*a*c^4*f)*g^6*h^3 - (77*a*c^4*d - 113*a^2*c^3*f)*g^4*h^5 - (67*a^2*c^3*d -
46*a^3*c^2*f)*g^2*h^7 + 8*(2*a^3*c^2*d - 5*a^4*c*f)*h^9)*x^4 + 5*(6*c^5*f*g
^9 + 4*c^5*e*g^8*h + 32*a*c^4*e*g^6*h^3 - 35*a^2*c^3*e*g^4*h^5 - 66*a^3*c^2
*e*g^2*h^7 - 3*a^4*c*e*h^9 + 3*(2*c^5*d + 11*a*c^4*f)*g^7*h^2 - 5*(13*a*c^4
*d - 22*a^2*c^3*f)*g^5*h^4 - (64*a^2*c^3*d - 61*a^3*c^2*f)*g^3*h^6 + (7*a^3
*c^2*d - 22*a^4*c*f)*g*h^8)*x^3 + (40*c^5*e*g^9 + 318*a*c^4*e*g^7*h^2 - 141
```

```
*a^2*c^3*e*g^5*h^4 - 446*a^3*c^2*e*g^3*h^6 - 27*a^4*c*e*g*h^8 + 3*(20*c^5*d
 + 9*a*c^4*f)*g^8*h - (503*a*c^4*d - 446*a^2*c^3*f)*g^6*h^3 - 141*(4*a^2*c^
3*d - a^3*c^2*f)*g^4*h^5 - 3*(3*a^3*c^2*d + 106*a^4*c*f)*g^2*h^7 - 8*(a^4*c
*d + 5*a^5*f)*h^9)*x^2 + 5*(22*a*c^4*e*g^8*h - 61*a^2*c^3*e*g^6*h^3 - 110*a
^3*c^2*e*g^4*h^5 - 33*a^4*c*e*g^2*h^7 - 6*a^5*e*h^9 + 3*(4*c^5*d + a*c^4*f)
*g^9 - 3*(21*a*c^4*d - 22*a^2*c^3*f)*g^7*h^2 - 5*(16*a^2*c^3*d - 7*a^3*c^2*
f)*g^5*h^4 - (7*a^3*c^2*d + 32*a^4*c*f)*g^3*h^6 - 2*(a^4*c*d + 2*a^5*f)*g*h
^8)*x)*sqrt(c*x^2 + a))/(c^5*g^15 + 5*a*c^4*g^13*h^2 + 10*a^2*c^3*g^11*h^4
+ 10*a^3*c^2*g^9*h^6 + 5*a^4*c*g^7*h^8 + a^5*g^5*h^10 + (c^5*g^10*h^5 + 5*a
*c^4*g^8*h^7 + 10*a^2*c^3*g^6*h^9 + 10*a^3*c^2*g^4*h^11 + 5*a^4*c*g^2*h^13
+ a^5*h^15)*x^5 + 5*(c^5*g^11*h^4 + 5*a*c^4*g^9*h^6 + 10*a^2*c^3*g^7*h^8 +
10*a^3*c^2*g^5*h^10 + 5*a^4*c*g^3*h^12 + a^5*g*h^14)*x^4 + 10*(c^5*g^12*h^3
 + 5*a*c^4*g^10*h^5 + 10*a^2*c^3*g^8*h^7 + 10*a^3*c^2*g^6*h^9 + 5*a^4*c*g^4
*h^11 + a^5*g^2*h^13)*x^3 + 10*(c^5*g^13*h^2 + 5*a*c^4*g^11*h^4 + 10*a^2*c^
3*g^9*h^6 + 10*a^3*c^2*g^7*h^8 + 5*a^4*c*g^5*h^10 + a^5*g^3*h^12)*x^2 + 5*(
c^5*g^14*h + 5*a*c^4*g^12*h^3 + 10*a^2*c^3*g^10*h^5 + 10*a^3*c^2*g^8*h^7 +
5*a^4*c*g^6*h^9 + a^5*g^4*h^11)*x), -1/120*(15*(6*a^2*c^3*e*g^7*h - a^3*c^2
*e*g^5*h^3 + (4*a*c^4*d - a^2*c^3*f)*g^8 - 3*(a^2*c^3*d - 2*a^3*c^2*f)*g^6*
h^2 + (6*a^2*c^3*e*g^2*h^6 - a^3*c^2*e*h^8 + (4*a*c^4*d - a^2*c^3*f)*g^3*h^
5 - 3*(a^2*c^3*d - 2*a^3*c^2*f)*g*h^7)*x^5 + 5*(6*a^2*c^3*e*g^3*h^5 - a^3*c
^2*e*g*h^7 + (4*a*c^4*d - a^2*c^3*f)*g^4*h^4 - 3*(a^2*c^3*d - 2*a^3*c^2*f)*
g^2*h^6)*x^4 + 10*(6*a^2*c^3*e*g^4*h^4 - a^3*c^2*e*g^2*h^6 + (4*a*c^4*d - a
^2*c^3*f)*g^5*h^3 - 3*(a^2*c^3*d - 2*a^3*c^2*f)*g^3*h^5)*x^3 + 10*(6*a^2*c^
3*e*g^5*h^3 - a^3*c^2*e*g^3*h^5 + (4*a*c^4*d - a^2*c^3*f)*g^6*h^2 - 3*(a^2*
c^3*d - 2*a^3*c^2*f)*g^4*h^4)*x^2 + 5*(6*a^2*c^3*e*g^6*h^2 - a^3*c^2*e*g^4*
h^4 + (4*a*c^4*d - a^2*c^3*f)*g^7*h - 3*(a^2*c^3*d - 2*a^3*c^2*f)*g^5*h^3)*
x)*sqrt(-c*g^2 - a*h^2)*arctan(sqrt(-c*g^2 - a*h^2)*(c*g*x - a*h)*sqrt(c*x^
2 + a)/(a*c*g^2 + a^2*h^2 + (c^2*g^2 + a*c*h^2)*x^2)) - (40*a*c^4*e*g^9 - 4
6*a^2*c^3*e*g^7*h^2 - 113*a^3*c^2*e*g^5*h^4 - 33*a^4*c*e*g^3*h^6 - 6*a^5*e*
g*h^8 - 24*a^5*d*h^9 - 9*(20*a*c^4*d - 9*a^2*c^3*f)*g^8*h - (329*a^2*c^3*d
 - 53*a^3*c^2*f)*g^6*h^3 - (247*a^3*c^2*d + 32*a^4*c*f)*g^4*h^5 - 2*(61*a^4*
c*d + 2*a^5*f)*g^2*h^7 + (6*c^5*f*g^8*h + 4*c^5*e*g^7*h^2 + 32*a*c^4*e*g^5*
h^4 - 53*a^2*c^3*e*g^3*h^6 - 81*a^3*c^2*e*g*h^8 + 3*(2*c^5*d + 11*a*c^4*f)*
g^6*h^3 - (77*a*c^4*d - 113*a^2*c^3*f)*g^4*h^5 - (67*a^2*c^3*d - 46*a^3*c^2
*f)*g^2*h^7 + 8*(2*a^3*c^2*d - 5*a^4*c*f)*h^9)*x^4 + 5*(6*c^5*f*g^9 + 4*c^5
*e*g^8*h + 32*a*c^4*e*g^6*h^3 - 35*a^2*c^3*e*g^4*h^5 - 66*a^3*c^2*e*g^2*h^7
 - 3*a^4*c*e*h^9 + 3*(2*c^5*d + 11*a*c^4*f)*g^7*h^2 - 5*(13*a*c^4*d - 22*a^
2*c^3*f)*g^5*h^4 - (64*a^2*c^3*d - 61*a^3*c^2*f)*g^3*h^6 + (7*a^3*c^2*d - 2
2*a^4*c*f)*g*h^8)*x^3 + (40*c^5*e*g^9 + 318*a*c^4*e*g^7*h^2 - 141*a^2*c^3*e
*g^5*h^4 - 446*a^3*c^2*e*g^3*h^6 - 27*a^4*c*e*g*h^8 + 3*(20*c^5*d + 9*a*c^4
*f)*g^8*h - (503*a*c^4*d - 446*a^2*c^3*f)*g^6*h^3 - 141*(4*a^2*c^3*d - a^3*
c^2*f)*g^4*h^5 - 3*(3*a^3*c^2*d + 106*a^4*c*f)*g^2*h^7 - 8*(a^4*c*d + 5*a^5
*f)*h^9)*x^2 + 5*(22*a*c^4*e*g^8*h - 61*a^2*c^3*e*g^6*h^3 - 110*a^3*c^2*e*g
^4*h^5 - 33*a^4*c*e*g^2*h^7 - 6*a^5*e*h^9 + 3*(4*c^5*d + a*c^4*f)*g^9 - 3*(
21*a*c^4*d - 22*a^2*c^3*f)*g^7*h^2 - 5*(16*a^2*c^3*d - 7*a^3*c^2*f)*g^5*h^4
```

```
 - (7*a^3*c^2*d + 32*a^4*c*f)*g^3*h^6 - 2*(a^4*c*d + 2*a^5*f)*g*h^8)*x)*sqr
t(c*x^2 + a))/(c^5*g^15 + 5*a*c^4*g^13*h^2 + 10*a^2*c^3*g^11*h^4 + 10*a^3*c
^2*g^9*h^6 + 5*a^4*c*g^7*h^8 + a^5*g^5*h^10 + (c^5*g^10*h^5 + 5*a*c^4*g^8*h
^7 + 10*a^2*c^3*g^6*h^9 + 10*a^3*c^2*g^4*h^11 + 5*a^4*c*g^2*h^13 + a^5*h^15
)*x^5 + 5*(c^5*g^11*h^4 + 5*a*c^4*g^9*h^6 + 10*a^2*c^3*g^7*h^8 + 10*a^3*c^2
*g^5*h^10 + 5*a^4*c*g^3*h^12 + a^5*g*h^14)*x^4 + 10*(c^5*g^12*h^3 + 5*a*c^4
*g^10*h^5 + 10*a^2*c^3*g^8*h^7 + 10*a^3*c^2*g^6*h^9 + 5*a^4*c*g^4*h^11 + a^
5*g^2*h^13)*x^3 + 10*(c^5*g^13*h^2 + 5*a*c^4*g^11*h^4 + 10*a^2*c^3*g^9*h^6
+ 10*a^3*c^2*g^7*h^8 + 5*a^4*c*g^5*h^10 + a^5*g^3*h^12)*x^2 + 5*(c^5*g^14*h
 + 5*a*c^4*g^12*h^3 + 10*a^2*c^3*g^10*h^5 + 10*a^3*c^2*g^8*h^7 + 5*a^4*c*g^
6*h^9 + a^5*g^4*h^11)*x)]
```

Sympy [F]

$$\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^6}\,dx = \int \frac{\sqrt{a+cx^2}\,(d+ex+fx^2)}{(g+hx)^6}\,dx$$

```
[In] integrate((f*x**2+e*x+d)*(c*x**2+a)**(1/2)/(h*x+g)**6,x)
```

```
[Out] Integral(sqrt(a + c*x**2)*(d + e*x + f*x**2)/(g + h*x)**6, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5793 vs. $2(410) = 820$.

Time = 0.42 (sec) , antiderivative size = 5793, normalized size of antiderivative = 13.38

$$\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^6}\,dx = \text{Too large to display}$$

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^6,x, algorithm="maxima")
```

```
[Out] -7/8*sqrt(c*x^2 + a)*c^4*f*g^6/(c^4*g^8*h^4*x + 4*a*c^3*g^6*h^6*x + 6*a^2*c
^2*g^4*h^8*x + 4*a^3*c*g^2*h^10*x + a^4*h^12*x + c^4*g^9*h^3 + 4*a*c^3*g^7*
h^5 + 6*a^2*c^2*g^5*h^7 + 4*a^3*c*g^3*h^9 + a^4*g*h^11) + 7/8*sqrt(c*x^2 +
a)*c^4*e*g^5/(c^4*g^8*h^3*x + 4*a*c^3*g^6*h^5*x + 6*a^2*c^2*g^4*h^7*x + 4*a
^3*c*g^2*h^9*x + a^4*h^11*x + c^4*g^9*h^2 + 4*a*c^3*g^7*h^4 + 6*a^2*c^2*g^5
*h^6 + 4*a^3*c*g^3*h^8 + a^4*g*h^10) - 7/8*(c*x^2 + a)^(3/2)*c^3*f*g^5/(c^4
*g^8*h^3*x^2 + 4*a*c^3*g^6*h^5*x^2 + 6*a^2*c^2*g^4*h^7*x^2 + 4*a^3*c*g^2*h^
9*x^2 + a^4*h^11*x^2 + 2*c^4*g^9*h^2*x + 8*a*c^3*g^7*h^4*x + 12*a^2*c^2*g^5
*h^6*x + 8*a^3*c*g^3*h^8*x + 2*a^4*g*h^10*x + c^4*g^10*h + 4*a*c^3*g^8*h^3
+ 6*a^2*c^2*g^6*h^5 + 4*a^3*c*g^4*h^7 + a^4*g^2*h^9) + 7/8*sqrt(c*x^2 + a)*
c^4*f*g^5/(c^4*g^8*h^3 + 4*a*c^3*g^6*h^5 + 6*a^2*c^2*g^4*h^7 + 4*a^3*c*g^2*
h^9 + a^4*h^11) - 7/8*sqrt(c*x^2 + a)*c^4*d*g^4/(c^4*g^8*h^2*x + 4*a*c^3*g^
```

```
6*h^4*x + 6*a^2*c^2*g^4*h^6*x + 4*a^3*c*g^2*h^8*x + a^4*h^10*x + c^4*g^9*h
+ 4*a*c^3*g^7*h^3 + 6*a^2*c^2*g^5*h^5 + 4*a^3*c*g^3*h^7 + a^4*g*h^9) + 7/8*
(c*x^2 + a)^(3/2)*c^3*e*g^4/(c^4*g^8*h^2*x^2 + 4*a*c^3*g^6*h^4*x^2 + 6*a^2*
c^2*g^4*h^6*x^2 + 4*a^3*c*g^2*h^8*x^2 + a^4*h^10*x^2 + 2*c^4*g^9*h*x + 8*a*
c^3*g^7*h^3*x + 12*a^2*c^2*g^5*h^5*x + 8*a^3*c*g^3*h^7*x + 2*a^4*g*h^9*x +
c^4*g^10 + 4*a*c^3*g^8*h^2 + 6*a^2*c^2*g^6*h^4 + 4*a^3*c*g^4*h^6 + a^4*g^2*
h^8) - 7/8*sqrt(c*x^2 + a)*c^4*e*g^4/(c^4*g^8*h^2 + 4*a*c^3*g^6*h^4 + 6*a^2
*c^2*g^4*h^6 + 4*a^3*c*g^2*h^8 + a^4*h^10) - 7/8*(c*x^2 + a)^(3/2)*c^3*d*g^
3/(c^4*g^8*h*x^2 + 4*a*c^3*g^6*h^3*x^2 + 6*a^2*c^2*g^4*h^5*x^2 + 4*a^3*c*g^
2*h^7*x^2 + a^4*h^9*x^2 + 2*c^4*g^9*x + 8*a*c^3*g^7*h^2*x + 12*a^2*c^2*g^5*
h^4*x + 8*a^3*c*g^3*h^6*x + 2*a^4*g*h^8*x + c^4*g^10/h + 4*a*c^3*g^8*h + 6*
a^2*c^2*g^6*h^3 + 4*a^3*c*g^4*h^5 + a^4*g^2*h^7) + 7/8*sqrt(c*x^2 + a)*c^4*
d*g^3/(c^4*g^8*h + 4*a*c^3*g^6*h^3 + 6*a^2*c^2*g^4*h^5 + 4*a^3*c*g^2*h^7 +
a^4*h^9) - 7/12*(c*x^2 + a)^(3/2)*c^2*f*g^4/(c^3*g^6*h^4*x^3 + 3*a*c^2*g^4*
h^6*x^3 + 3*a^2*c*g^2*h^8*x^3 + a^3*h^10*x^3 + 3*c^3*g^7*h^3*x^2 + 9*a*c^2*
g^5*h^5*x^2 + 9*a^2*c*g^3*h^7*x^2 + 3*a^3*g*h^9*x^2 + 3*c^3*g^8*h^2*x + 9*a
*c^2*g^6*h^4*x + 9*a^2*c*g^4*h^6*x + 3*a^3*g^2*h^8*x + c^3*g^9*h + 3*a*c^2*
g^7*h^3 + 3*a^2*c*g^5*h^5 + a^3*g^3*h^7) + 13/8*sqrt(c*x^2 + a)*c^3*f*g^4/(
c^3*g^6*h^4*x + 3*a*c^2*g^4*h^6*x + 3*a^2*c*g^2*h^8*x + a^3*h^10*x + c^3*g^
7*h^3 + 3*a*c^2*g^5*h^5 + 3*a^2*c*g^3*h^7 + a^3*g*h^9) + 7/12*(c*x^2 + a)^(
3/2)*c^2*e*g^3/(c^3*g^6*h^3*x^3 + 3*a*c^2*g^4*h^5*x^3 + 3*a^2*c*g^2*h^7*x^3
 + a^3*h^9*x^3 + 3*c^3*g^7*h^2*x^2 + 9*a*c^2*g^5*h^4*x^2 + 9*a^2*c*g^3*h^6*
x^2 + 3*a^3*g*h^8*x^2 + 3*c^3*g^8*h*x + 9*a*c^2*g^6*h^3*x + 9*a^2*c*g^4*h^5
*x + 3*a^3*g^2*h^7*x + c^3*g^9 + 3*a*c^2*g^7*h^2 + 3*a^2*c*g^5*h^4 + a^3*g^
3*h^6) - sqrt(c*x^2 + a)*c^3*e*g^3/(c^3*g^6*h^3*x + 3*a*c^2*g^4*h^5*x + 3*a
^2*c*g^2*h^7*x + a^3*h^9*x + c^3*g^7*h^2 + 3*a*c^2*g^5*h^4 + 3*a^2*c*g^3*h^
6 + a^3*g*h^8) + 13/8*(c*x^2 + a)^(3/2)*c^2*f*g^3/(c^3*g^6*h^3*x^2 + 3*a*c^
2*g^4*h^5*x^2 + 3*a^2*c*g^2*h^7*x^2 + a^3*h^9*x^2 + 2*c^3*g^7*h^2*x + 6*a*c
^2*g^5*h^4*x + 6*a^2*c*g^3*h^6*x + 2*a^3*g*h^8*x + c^3*g^8*h + 3*a*c^2*g^6*
h^3 + 3*a^2*c*g^4*h^5 + a^3*g^2*h^7) - 13/8*sqrt(c*x^2 + a)*c^3*f*g^3/(c^3*
g^6*h^3 + 3*a*c^2*g^4*h^5 + 3*a^2*c*g^2*h^7 + a^3*h^9) - 7/12*(c*x^2 + a)^(
3/2)*c^2*d*g^2/(c^3*g^6*h^2*x^3 + 3*a*c^2*g^4*h^4*x^3 + 3*a^2*c*g^2*h^6*x^3
 + a^3*h^8*x^3 + 3*c^3*g^7*h*x^2 + 9*a*c^2*g^5*h^3*x^2 + 9*a^2*c*g^3*h^5*x^
2 + 3*a^3*g*h^7*x^2 + 3*c^3*g^8*x + 9*a*c^2*g^6*h^2*x + 9*a^2*c*g^4*h^4*x +
 3*a^3*g^2*h^6*x + c^3*g^9/h + 3*a*c^2*g^7*h + 3*a^2*c*g^5*h^3 + a^3*g^3*h^
5) + 3/8*sqrt(c*x^2 + a)*c^3*d*g^2/(c^3*g^6*h^2*x + 3*a*c^2*g^4*h^4*x + 3*a
^2*c*g^2*h^6*x + a^3*h^8*x + c^3*g^7*h + 3*a*c^2*g^5*h^3 + 3*a^2*c*g^3*h^5
+ a^3*g*h^7) - (c*x^2 + a)^(3/2)*c^2*e*g^2/(c^3*g^6*h^2*x^2 + 3*a*c^2*g^4*h
^4*x^2 + 3*a^2*c*g^2*h^6*x^2 + a^3*h^8*x^2 + 2*c^3*g^7*h*x + 6*a*c^2*g^5*h^
3*x + 6*a^2*c*g^3*h^5*x + 2*a^3*g*h^7*x + c^3*g^8 + 3*a*c^2*g^6*h^2 + 3*a^2
*c*g^4*h^4 + a^3*g^2*h^6) + sqrt(c*x^2 + a)*c^3*e*g^2/(c^3*g^6*h^2 + 3*a*c^
2*g^4*h^4 + 3*a^2*c*g^2*h^6 + a^3*h^8) - 7/20*(c*x^2 + a)^(3/2)*c*f*g^3/(c^
2*g^4*h^5*x^4 + 2*a*c*g^2*h^7*x^4 + a^2*h^9*x^4 + 4*c^2*g^5*h^4*x^3 + 8*a*c
*g^3*h^6*x^3 + 4*a^2*g*h^8*x^3 + 6*c^2*g^6*h^3*x^2 + 12*a*c*g^4*h^5*x^2 + 6
*a^2*g^2*h^7*x^2 + 4*c^2*g^7*h^2*x + 8*a*c*g^5*h^4*x + 4*a^2*g^3*h^6*x + c^
```

```
2*g^8*h + 2*a*c*g^6*h^3 + a^2*g^4*h^5) + 3/8*(c*x^2 + a)^(3/2)*c^2*d*g/(c^3
*g^6*h*x^2 + 3*a*c^2*g^4*h^3*x^2 + 3*a^2*c*g^2*h^5*x^2 + a^3*h^7*x^2 + 2*c^
3*g^7*x + 6*a*c^2*g^5*h^2*x + 6*a^2*c*g^3*h^4*x + 2*a^3*g*h^6*x + c^3*g^8/h
 + 3*a*c^2*g^6*h + 3*a^2*c*g^4*h^3 + a^3*g^2*h^5) - 3/8*sqrt(c*x^2 + a)*c^3
*d*g/(c^3*g^6*h + 3*a*c^2*g^4*h^3 + 3*a^2*c*g^2*h^5 + a^3*h^7) + 7/20*(c*x^
2 + a)^(3/2)*c*e*g^2/(c^2*g^4*h^4*x^4 + 2*a*c*g^2*h^6*x^4 + a^2*h^8*x^4 + 4
*c^2*g^5*h^3*x^3 + 8*a*c*g^3*h^5*x^3 + 4*a^2*g*h^7*x^3 + 6*c^2*g^6*h^2*x^2
 + 12*a*c*g^4*h^4*x^2 + 6*a^2*g^2*h^6*x^2 + 4*c^2*g^7*h*x + 8*a*c*g^5*h^3*x
 + 4*a^2*g^3*h^5*x + c^2*g^8 + 2*a*c*g^6*h^2 + a^2*g^4*h^4) + 29/30*(c*x^2 +
 a)^(3/2)*c*f*g^2/(c^2*g^4*h^4*x^3 + 2*a*c*g^2*h^6*x^3 + a^2*h^8*x^3 + 3*c^
2*g^5*h^3*x^2 + 6*a*c*g^3*h^5*x^2 + 3*a^2*g*h^7*x^2 + 3*c^2*g^6*h^2*x + 6*a
*c*g^4*h^4*x + 3*a^2*g^2*h^6*x + c^2*g^7*h + 2*a*c*g^5*h^3 + a^2*g^3*h^5) -
 3/4*sqrt(c*x^2 + a)*c^2*f*g^2/(c^2*g^4*h^4*x + 2*a*c*g^2*h^6*x + a^2*h^8*x
 + c^2*g^5*h^3 + 2*a*c*g^3*h^5 + a^2*g*h^7) - 7/20*(c*x^2 + a)^(3/2)*c*d*g/
(c^2*g^4*h^3*x^4 + 2*a*c*g^2*h^5*x^4 + a^2*h^7*x^4 + 4*c^2*g^5*h^2*x^3 + 8*
a*c*g^3*h^4*x^3 + 4*a^2*g*h^6*x^3 + 6*c^2*g^6*h*x^2 + 12*a*c*g^4*h^3*x^2 +
6*a^2*g^2*h^5*x^2 + 4*c^2*g^7*x + 8*a*c*g^5*h^2*x + 4*a^2*g^3*h^4*x + c^2*g
^8/h + 2*a*c*g^6*h + a^2*g^4*h^3) - 11/20*(c*x^2 + a)^(3/2)*c*e*g/(c^2*g^4*
h^3*x^3 + 2*a*c*g^2*h^5*x^3 + a^2*h^7*x^3 + 3*c^2*g^5*h^2*x^2 + 6*a*c*g^3*h
^4*x^2 + 3*a^2*g*h^6*x^2 + 3*c^2*g^6*h*x + 6*a*c*g^4*h^3*x + 3*a^2*g^2*h^5*
x + c^2*g^7 + 2*a*c*g^5*h^2 + a^2*g^3*h^4) + 1/8*sqrt(c*x^2 + a)*c^2*e*g/(c
^2*g^4*h^3*x + 2*a*c*g^2*h^5*x + a^2*h^7*x + c^2*g^5*h^2 + 2*a*c*g^3*h^4 +
a^2*g*h^6) - 3/4*(c*x^2 + a)^(3/2)*c*f*g/(c^2*g^4*h^3*x^2 + 2*a*c*g^2*h^5*x
^2 + a^2*h^7*x^2 + 2*c^2*g^5*h^2*x + 4*a*c*g^3*h^4*x + 2*a^2*g*h^6*x + c^2*
g^6*h + 2*a*c*g^4*h^3 + a^2*g^2*h^5) + 3/4*sqrt(c*x^2 + a)*c^2*f*g/(c^2*g^4
*h^3 + 2*a*c*g^2*h^5 + a^2*h^7) - 1/5*(c*x^2 + a)^(3/2)*f*g^2/(c*g^2*h^6*x^
5 + a*h^8*x^5 + 5*c*g^3*h^5*x^4 + 5*a*g*h^7*x^4 + 10*c*g^4*h^4*x^3 + 10*a*g
^2*h^6*x^3 + 10*c*g^5*h^3*x^2 + 10*a*g^3*h^5*x^2 + 5*c*g^6*h^2*x + 5*a*g^4*
h^4*x + c*g^7*h + a*g^5*h^3) + 2/15*(c*x^2 + a)^(3/2)*c*d/(c^2*g^4*h^2*x^3
 + 2*a*c*g^2*h^4*x^3 + a^2*h^6*x^3 + 3*c^2*g^5*h*x^2 + 6*a*c*g^3*h^3*x^2 + 3
*a^2*g*h^5*x^2 + 3*c^2*g^6*x + 6*a*c*g^4*h^2*x + 3*a^2*g^2*h^4*x + c^2*g^7/
h + 2*a*c*g^5*h + a^2*g^3*h^3) + 1/8*(c*x^2 + a)^(3/2)*c*e/(c^2*g^4*h^2*x^2
 + 2*a*c*g^2*h^4*x^2 + a^2*h^6*x^2 + 2*c^2*g^5*h*x + 4*a*c*g^3*h^3*x + 2*a^
2*g*h^5*x + c^2*g^6 + 2*a*c*g^4*h^2 + a^2*g^2*h^4) - 1/8*sqrt(c*x^2 + a)*c^
2*e/(c^2*g^4*h^2 + 2*a*c*g^2*h^4 + a^2*h^6) + 1/5*(c*x^2 + a)^(3/2)*e*g/(c*
g^2*h^5*x^5 + a*h^7*x^5 + 5*c*g^3*h^4*x^4 + 5*a*g*h^6*x^4 + 10*c*g^4*h^3*x^
3 + 10*a*g^2*h^5*x^3 + 10*c*g^5*h^2*x^2 + 10*a*g^3*h^4*x^2 + 5*c*g^6*h*x +
5*a*g^4*h^3*x + c*g^7 + a*g^5*h^2) + 1/2*(c*x^2 + a)^(3/2)*f*g/(c*g^2*h^5*x
^4 + a*h^7*x^4 + 4*c*g^3*h^4*x^3 + 4*a*g*h^6*x^3 + 6*c*g^4*h^3*x^2 + 6*a*g^
2*h^5*x^2 + 4*c*g^5*h^2*x + 4*a*g^3*h^4*x + c*g^6*h + a*g^4*h^3) - 1/5*(c*x
^2 + a)^(3/2)*d/(c*g^2*h^4*x^5 + a*h^6*x^5 + 5*c*g^3*h^3*x^4 + 5*a*g*h^5*x^
4 + 10*c*g^4*h^2*x^3 + 10*a*g^2*h^4*x^3 + 10*c*g^5*h*x^2 + 10*a*g^3*h^3*x^2
 + 5*c*g^6*x + 5*a*g^4*h^2*x + c*g^7/h + a*g^5*h) - 1/4*(c*x^2 + a)^(3/2)*e
/(c*g^2*h^4*x^4 + a*h^6*x^4 + 4*c*g^3*h^3*x^3 + 4*a*g*h^5*x^3 + 6*c*g^4*h^2
*x^2 + 6*a*g^2*h^4*x^2 + 4*c*g^5*h*x + 4*a*g^3*h^3*x + c*g^6 + a*g^4*h^2) -
```

```
 1/3*(c*x^2 + a)^(3/2)*f/(c*g^2*h^4*x^3 + a*h^6*x^3 + 3*c*g^3*h^3*x^2 + 3*a
*g*h^5*x^2 + 3*c*g^4*h^2*x + 3*a*g^2*h^4*x + c*g^5*h + a*g^3*h^3) - 7/8*c^5
*f*g^7*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)
))/((a + c*g^2/h^2)^(9/2)*h^13) + 7/8*c^5*e*g^6*arcsinh(c*g*x/(sqrt(a*c)*ab
s(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(9/2)*h^12) -
7/8*c^5*d*g^5*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h
*x + g)))/((a + c*g^2/h^2)^(9/2)*h^11) + 5/2*c^4*f*g^5*arcsinh(c*g*x/(sqrt(
a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(7/2)*h
^11) - 15/8*c^4*e*g^4*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*
c)*abs(h*x + g)))/((a + c*g^2/h^2)^(7/2)*h^10) + 5/4*c^4*d*g^3*arcsinh(c*g*
x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)
^(7/2)*h^9) - 19/8*c^3*f*g^3*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(
sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(5/2)*h^9) + 9/8*c^3*e*g^2*arcsin
h(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^
2/h^2)^(5/2)*h^8) - 3/8*c^3*d*g*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*
h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(5/2)*h^7) + 3/4*c^2*f*g*arcsi
nh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g
^2/h^2)^(3/2)*h^7) - 1/8*c^2*e*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h
/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(3/2)*h^6)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4170 vs. $2(410) = 820$.

Time = 0.40 (sec) , antiderivative size = 4170, normalized size of antiderivative = 9.63

$$\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^6}\,dx = \text{Too large to display}$$

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^6,x, algorithm="giac")
```

```
[Out] -1/4*(4*a*c^4*d*g^3 - a^2*c^3*f*g^3 + 6*a^2*c^3*e*g^2*h - 3*a^2*c^3*d*g*h^2
 + 6*a^3*c^2*f*g*h^2 - a^3*c^2*e*h^3)*arctan(((sqrt(c)*x - sqrt(c*x^2 + a))
*h + sqrt(c)*g)/sqrt(-c*g^2 - a*h^2))/((c^4*g^8 + 4*a*c^3*g^6*h^2 + 6*a^2*c
^2*g^4*h^4 + 4*a^3*c*g^2*h^6 + a^4*h^8)*sqrt(-c*g^2 - a*h^2)) - 1/60*(60*(s
qrt(c)*x - sqrt(c*x^2 + a))^9*a*c^4*d*g^3*h^8 - 15*(sqrt(c)*x - sqrt(c*x^2
+ a))^9*a^2*c^3*f*g^3*h^8 + 90*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^2*c^3*e*g^
2*h^9 - 45*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^2*c^3*d*g*h^10 + 90*(sqrt(c)*x
 - sqrt(c*x^2 + a))^9*a^3*c^2*f*g*h^10 - 15*(sqrt(c)*x - sqrt(c*x^2 + a))^9
*a^3*c^2*e*h^11 - 120*(sqrt(c)*x - sqrt(c*x^2 + a))^8*c^(11/2)*f*g^8*h^3 -
480*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a*c^(9/2)*f*g^6*h^5 + 540*(sqrt(c)*x -
sqrt(c*x^2 + a))^8*a*c^(9/2)*d*g^4*h^7 - 855*(sqrt(c)*x - sqrt(c*x^2 + a))^
8*a^2*c^(7/2)*f*g^4*h^7 + 810*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^2*c^(7/2)*e
*g^3*h^8 - 405*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^2*c^(7/2)*d*g^2*h^9 + 330*
(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^3*c^(5/2)*f*g^2*h^9 - 135*(sqrt(c)*x - sq
```

```
rt(c*x^2 + a))^8*a^3*c^(5/2)*e*g*h^10 - 120*(sqrt(c)*x - sqrt(c*x^2 + a))^8
*a^4*c^(3/2)*f*h^11 - 240*(sqrt(c)*x - sqrt(c*x^2 + a))^7*c^6*f*g^9*h^2 - 1
60*(sqrt(c)*x - sqrt(c*x^2 + a))^7*c^6*e*g^8*h^3 - 960*(sqrt(c)*x - sqrt(c*
x^2 + a))^7*a*c^5*f*g^7*h^4 - 640*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a*c^5*e*g
^6*h^5 + 1880*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a*c^5*d*g^5*h^6 - 1910*(sqrt(
c)*x - sqrt(c*x^2 + a))^7*a^2*c^4*f*g^5*h^6 + 1860*(sqrt(c)*x - sqrt(c*x^2
+ a))^7*a^2*c^4*e*g^4*h^7 - 1690*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^2*c^4*d*
g^3*h^8 + 1930*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^3*c^3*f*g^3*h^8 - 1530*(sq
rt(c)*x - sqrt(c*x^2 + a))^7*a^3*c^3*e*g^2*h^9 + 210*(sqrt(c)*x - sqrt(c*x^
2 + a))^7*a^3*c^3*d*g*h^10 - 660*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^4*c^2*f*
g*h^10 - 90*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^4*c^2*e*h^11 - 240*(sqrt(c)*x
 - sqrt(c*x^2 + a))^6*c^(13/2)*f*g^10*h - 160*(sqrt(c)*x - sqrt(c*x^2 + a))
^6*c^(13/2)*e*g^9*h^2 - 240*(sqrt(c)*x - sqrt(c*x^2 + a))^6*c^(13/2)*d*g^8*
h^3 - 720*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a*c^(11/2)*f*g^8*h^3 - 640*(sqrt(
c)*x - sqrt(c*x^2 + a))^6*a*c^(11/2)*e*g^7*h^4 + 2120*(sqrt(c)*x - sqrt(c*x
^2 + a))^6*a*c^(11/2)*d*g^6*h^5 - 1250*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^2*
c^(9/2)*f*g^6*h^5 + 3660*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^2*c^(9/2)*e*g^5*
h^6 - 5710*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^2*c^(9/2)*d*g^4*h^7 + 5590*(sq
rt(c)*x - sqrt(c*x^2 + a))^6*a^3*c^(7/2)*f*g^4*h^7 - 4350*(sqrt(c)*x - sqrt
(c*x^2 + a))^6*a^3*c^(7/2)*e*g^3*h^8 + 510*(sqrt(c)*x - sqrt(c*x^2 + a))^6*
a^3*c^(7/2)*d*g^2*h^9 - 2220*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^4*c^(5/2)*f*
g^2*h^9 + 330*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^4*c^(5/2)*e*g*h^10 - 240*(s
qrt(c)*x - sqrt(c*x^2 + a))^6*a^4*c^(5/2)*d*h^11 + 240*(sqrt(c)*x - sqrt(c*
x^2 + a))^6*a^5*c^(3/2)*f*h^11 - 96*(sqrt(c)*x - sqrt(c*x^2 + a))^5*c^7*f*g
^11 - 64*(sqrt(c)*x - sqrt(c*x^2 + a))^5*c^7*e*g^10*h - 96*(sqrt(c)*x - sqr
t(c*x^2 + a))^5*c^7*d*g^9*h^2 + 48*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a*c^6*f*
g^9*h^2 - 128*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a*c^6*e*g^8*h^3 + 1808*(sqrt(
c)*x - sqrt(c*x^2 + a))^5*a*c^6*d*g^7*h^4 + 604*(sqrt(c)*x - sqrt(c*x^2 + a
))^5*a^2*c^5*f*g^7*h^4 + 3416*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^2*c^5*e*g^6
*h^5 - 7076*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^2*c^5*d*g^5*h^6 + 6710*(sqrt(
c)*x - sqrt(c*x^2 + a))^5*a^3*c^4*f*g^5*h^6 - 7320*(sqrt(c)*x - sqrt(c*x^2
+ a))^5*a^3*c^4*e*g^4*h^7 + 3770*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^3*c^4*d*
g^3*h^8 - 5780*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^4*c^3*f*g^3*h^8 + 2430*(sq
rt(c)*x - sqrt(c*x^2 + a))^5*a^4*c^3*e*g^2*h^9 - 480*(sqrt(c)*x - sqrt(c*x^
2 + a))^5*a^4*c^3*d*g*h^10 + 1200*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^5*c^2*f
*g*h^10 + 240*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a*c^(13/2)*f*g^10*h + 160*(sq
rt(c)*x - sqrt(c*x^2 + a))^4*a*c^(13/2)*e*g^9*h^2 + 240*(sqrt(c)*x - sqrt(c
*x^2 + a))^4*a*c^(13/2)*d*g^8*h^3 + 720*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^2
*c^(11/2)*f*g^8*h^3 + 1120*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^2*c^(11/2)*e*g
^7*h^4 - 5240*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^2*c^(11/2)*d*g^6*h^5 + 2450
*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^3*c^(9/2)*f*g^6*h^5 - 6140*(sqrt(c)*x -
sqrt(c*x^2 + a))^4*a^3*c^(9/2)*e*g^5*h^6 + 5590*(sqrt(c)*x - sqrt(c*x^2 + a
))^4*a^3*c^(9/2)*d*g^4*h^7 - 7660*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^4*c^(7/
2)*f*g^4*h^7 + 5650*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^4*c^(7/2)*e*g^3*h^8 -
 2240*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^4*c^(7/2)*d*g^2*h^9 + 3440*(sqrt(c)
```

```
*x - sqrt(c*x^2 + a))^4*a^5*c^(5/2)*f*g^2*h^9 - 480*(sqrt(c)*x - sqrt(c*x^2
 + a))^4*a^5*c^(5/2)*e*g*h^10 - 80*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^5*c^(5
/2)*d*h^11 - 160*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^6*c^(3/2)*f*h^11 - 240*(
sqrt(c)*x - sqrt(c*x^2 + a))^3*a^2*c^6*f*g^9*h^2 - 160*(sqrt(c)*x - sqrt(c*
x^2 + a))^3*a^2*c^6*e*g^8*h^3 - 480*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^2*c^6
*d*g^7*h^4 - 960*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^3*c^5*f*g^7*h^4 - 1440*(
sqrt(c)*x - sqrt(c*x^2 + a))^3*a^3*c^5*e*g^6*h^5 + 5000*(sqrt(c)*x - sqrt(c
*x^2 + a))^3*a^3*c^5*d*g^5*h^6 - 3890*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^4*c
^4*f*g^5*h^6 + 5740*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^4*c^4*e*g^4*h^7 - 291
0*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^4*c^4*d*g^3*h^8 + 4710*(sqrt(c)*x - sqr
t(c*x^2 + a))^3*a^5*c^3*f*g^3*h^8 - 1710*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^
5*c^3*e*g^2*h^9 + 430*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^5*c^3*d*g*h^10 - 94
0*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^6*c^2*f*g*h^10 + 90*(sqrt(c)*x - sqrt(c
*x^2 + a))^3*a^6*c^2*e*h^11 + 120*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^3*c^(11
/2)*f*g^8*h^3 + 160*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^3*c^(11/2)*e*g^7*h^4
 + 240*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^3*c^(11/2)*d*g^6*h^5 + 570*(sqrt(c)
*x - sqrt(c*x^2 + a))^2*a^4*c^(9/2)*f*g^6*h^5 + 1100*(sqrt(c)*x - sqrt(c*x^
2 + a))^2*a^4*c^(9/2)*e*g^5*h^6 - 2810*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^4*
c^(9/2)*d*g^4*h^7 + 2450*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^5*c^(7/2)*f*g^4*
h^7 - 2570*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^5*c^(7/2)*e*g^3*h^8 + 650*(sqr
t(c)*x - sqrt(c*x^2 + a))^2*a^5*c^(7/2)*d*g^2*h^9 - 1700*(sqrt(c)*x - sqrt(
c*x^2 + a))^2*a^6*c^(5/2)*f*g^2*h^9 + 270*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a
^6*c^(5/2)*e*g*h^10 - 80*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^6*c^(5/2)*d*h^11
 + 80*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^7*c^(3/2)*f*h^11 - 60*(sqrt(c)*x -
sqrt(c*x^2 + a))*a^4*c^5*f*g^7*h^4 - 40*(sqrt(c)*x - sqrt(c*x^2 + a))*a^4*c
^5*e*g^6*h^5 - 60*(sqrt(c)*x - sqrt(c*x^2 + a))*a^4*c^5*d*g^5*h^6 - 270*(sq
rt(c)*x - sqrt(c*x^2 + a))*a^5*c^4*f*g^5*h^6 - 280*(sqrt(c)*x - sqrt(c*x^2
+ a))*a^5*c^4*e*g^4*h^7 + 770*(sqrt(c)*x - sqrt(c*x^2 + a))*a^5*c^4*d*g^3*h
^8 - 845*(sqrt(c)*x - sqrt(c*x^2 + a))*a^6*c^3*f*g^3*h^8 + 720*(sqrt(c)*x -
 sqrt(c*x^2 + a))*a^6*c^3*e*g^2*h^9 - 115*(sqrt(c)*x - sqrt(c*x^2 + a))*a^6
*c^3*d*g*h^10 + 310*(sqrt(c)*x - sqrt(c*x^2 + a))*a^7*c^2*f*g*h^10 + 15*(sq
rt(c)*x - sqrt(c*x^2 + a))*a^7*c^2*e*h^11 + 6*a^5*c^(9/2)*f*g^6*h^5 + 4*a^5
*c^(9/2)*e*g^5*h^6 + 6*a^5*c^(9/2)*d*g^4*h^7 + 27*a^6*c^(7/2)*f*g^4*h^7 + 2
8*a^6*c^(7/2)*e*g^3*h^8 - 83*a^6*c^(7/2)*d*g^2*h^9 + 86*a^7*c^(5/2)*f*g^2*h
^9 - 81*a^7*c^(5/2)*e*g*h^10 + 16*a^7*c^(5/2)*d*h^11 - 40*a^8*c^(3/2)*f*h^1
1)/((c^4*g^8*h^4 + 4*a*c^3*g^6*h^6 + 6*a^2*c^2*g^4*h^8 + 4*a^3*c*g^2*h^10 +
 a^4*h^12)*((sqrt(c)*x - sqrt(c*x^2 + a))^2*h + 2*(sqrt(c)*x - sqrt(c*x^2 +
 a))*sqrt(c)*g - a*h)^5)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^6}\,dx=\int \frac{\sqrt{c\,x^2+a}\,\left(f\,x^2+e\,x+d\right)}{{\left(g+h\,x\right)}^6}\,dx$$

```
[In] int(((a + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^6,x)
[Out] int(((a + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^6, x)
```

3.88 $\int (g+hx)^3 \left(a+cx^2\right)^{3/2} \left(d+ex+fx^2\right) \, dx$

Optimal result

Integrand size = 29, antiderivative size = 462

$$
\begin{aligned}
\int (g+hx)^3 \left(a+cx^2\right)^{3/2} (d+ex \\
+fx^2) \, dx &= \frac{a(48c^2dg^3+3a^2h^2(3fg+eh)-8acg(fg^2+3h(eg+dh)))\,x\sqrt{a+cx^2}}{128c^2} \\
&+ \frac{(48c^2dg^3+3a^2h^2(3fg+eh)-8acg(fg^2+3h(eg+dh)))\,x(a+cx^2)^{3/2}}{192c^2} \\
&+ \frac{(8(9cd-4af)h^2-3cg(5fg-9eh))\,(g+hx)^2\left(a+cx^2\right)^{5/2}}{504c^2h} \\
&- \frac{(5fg-9eh)(g+hx)^3\left(a+cx^2\right)^{5/2}}{72ch} + \frac{f(g+hx)^4\left(a+cx^2\right)^{5/2}}{9ch} \\
&+ \frac{(4(32a^2fh^4-8ach^2(17fg^2+9h(3eg+dh))-3c^2g^2(5fg^2-3h(3eg+64dh)))-5ch(ah^2(61fg+63eh)+}{5040c^3h} \\
&+ \frac{a^2(48c^2dg^3+3a^2h^2(3fg+eh)-8acg(fg^2+3h(eg+dh)))\,\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{128c^{5/2}}
\end{aligned}
$$

```
[Out] 1/192*(48*c^2*d*g^3+3*a^2*h^2*(e*h+3*f*g)-8*a*c*g*(f*g^2+3*h*(d*h+e*g)))*x*
(c*x^2+a)^(3/2)/c^2+1/504*(8*(-4*a*f+9*c*d)*h^2-3*c*g*(-9*e*h+5*f*g))*(h*x+
g)^2*(c*x^2+a)^(5/2)/c^2/h-1/72*(-9*e*h+5*f*g)*(h*x+g)^3*(c*x^2+a)^(5/2)/c/
h+1/9*f*(h*x+g)^4*(c*x^2+a)^(5/2)/c/h+1/5040*(128*a^2*f*h^4-32*a*c*h^2*(17*
f*g^2+9*h*(d*h+3*e*g))-12*c^2*g^2*(5*f*g^2-3*h*(64*d*h+3*e*g))-5*c*h*(a*h^2
*(63*e*h+61*f*g)+2*c*g*(5*f*g^2-9*h*(12*d*h+e*g)))*x)*(c*x^2+a)^(5/2)/c^3/h
+1/128*a^2*(48*c^2*d*g^3+3*a^2*h^2*(e*h+3*f*g)-8*a*c*g*(f*g^2+3*h*(d*h+e*g)
))*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(5/2)+1/128*a*(48*c^2*d*g^3+3*a^2*h
^2*(e*h+3*f*g)-8*a*c*g*(f*g^2+3*h*(d*h+e*g)))*x*(c*x^2+a)^(1/2)/c^2
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 462, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1668, 847, 794, 201, 223, 212}

$$\int (g+hx)^3 \left(a+cx^2\right)^{3/2} \left(d+ex+fx^2\right)\,dx = \frac{a^2 \text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(3a^2h^2(eh+3fg)-8acg(3h(dh+eg)+fg^2)+48c^2dg^3\right)}{128c^{5/2}} + \frac{x\left(a+cx^2\right)^{3/2}\left(3a^2h^2(eh+3fg)-8acg(3h(dh+eg)+fg^2)+48c^2dg^3\right)}{192c^2} + \frac{ax\sqrt{a+cx^2}(3a^2h^2(eh+3fg)-8acg(3h(dh+eg)+fg^2)+48c^2dg^3)}{128c^2} + \frac{\left(a+cx^2\right)^{5/2}(4(32a^2fh^4-8ach^2(9h(dh+3eg)+17fg^2)-c^2(15fg^4-9g^2h(64dh+3eg)))-5chx(ah^2(63}{5040c^3h} + \frac{\left(a+cx^2\right)^{5/2}(g+hx)^2\left(8h^2(9cd-4af)-3cg(5fg-9eh)\right)}{504c^2h} - \frac{\left(a+cx^2\right)^{5/2}(g+hx)^3(5fg-9eh)}{72ch} + \frac{f\left(a+cx^2\right)^{5/2}(g+hx)^4}{9ch}$$

```
[In] Int[(g + h*x)^3*(a + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

```
[Out] (a*(48*c^2*d*g^3 + 3*a^2*h^2*(3*f*g + e*h) - 8*a*c*g*(f*g^2 + 3*h*(e*g + d*
h)))*x*Sqrt[a + c*x^2])/(128*c^2) + ((48*c^2*d*g^3 + 3*a^2*h^2*(3*f*g + e*h
) - 8*a*c*g*(f*g^2 + 3*h*(e*g + d*h)))*x*(a + c*x^2)^(3/2))/(192*c^2) + ((8
*(9*c*d - 4*a*f)*h^2 - 3*c*g*(5*f*g - 9*e*h))*(g + h*x)^2*(a + c*x^2)^(5/2)
)/(504*c^2*h) - ((5*f*g - 9*e*h)*(g + h*x)^3*(a + c*x^2)^(5/2))/(72*c*h) +
(f*(g + h*x)^4*(a + c*x^2)^(5/2))/(9*c*h) + ((4*(32*a^2*f*h^4 - 8*a*c*h^2*(
17*f*g^2 + 9*h*(3*e*g + d*h)) - c^2*(15*f*g^4 - 9*g^2*h*(3*e*g + 64*d*h)))
- 5*c*h*(a*h^2*(61*f*g + 63*e*h) + 2*c*(5*f*g^3 - 9*g*h*(e*g + 12*d*h)))*x)
*(a + c*x^2)^(5/2))/(5040*c^3*h) + (a^2*(48*c^2*d*g^3 + 3*a^2*h^2*(3*f*g +
e*h) - 8*a*c*g*(f*g^2 + 3*h*(e*g + d*h)))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^
2]])/(128*c^(5/2))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{f(g+hx)^4\left(a+cx^2\right)^{5/2}}{9ch}\\ &\quad+\frac{\int(g+hx)^3\left((9cd-4af)h^2-ch(5fg-9eh)x\right)\left(a+cx^2\right)^{3/2}\,dx}{9ch^2}\end{aligned}$$

$$= -\frac{(5fg-9eh)(g+hx)^3\left(a+cx^2\right)^{5/2}}{72ch} + \frac{f(g+hx)^4\left(a+cx^2\right)^{5/2}}{9ch}$$
$$+ \frac{\int (g+hx)^2\left(ch^2(72cdg-17afg-27aeh)+ch(8(9cd-4af)h^2-3cg(5fg-9eh))\,x\right)(a+cx^2)}{72c^2h^2}$$

$$= \frac{\left(8(9cd-4af)h^2-3cg(5fg-9eh)\right)(g+hx)^2\left(a+cx^2\right)^{5/2}}{504c^2h}$$
$$- \frac{(5fg-9eh)(g+hx)^3\left(a+cx^2\right)^{5/2}}{72ch} + \frac{f(g+hx)^4\left(a+cx^2\right)^{5/2}}{9ch}$$
$$+ \frac{\int (g+hx)\left(ch^2(504c^2dg^2+64a^2fh^2-ac(89fg^2+9h(27eg+16dh)))-3c^2h(ah^2(61fg+63eh\right.}{504c^3h^2}$$

$$= \frac{\left(8(9cd-4af)h^2-3cg(5fg-9eh)\right)(g+hx)^2\left(a+cx^2\right)^{5/2}}{504c^2h}$$
$$- \frac{(5fg-9eh)(g+hx)^3\left(a+cx^2\right)^{5/2}}{72ch} + \frac{f(g+hx)^4\left(a+cx^2\right)^{5/2}}{9ch}$$
$$+ \frac{(4(32a^2fh^4-8ach^2(17fg^2+9h(3eg+dh))-c^2(15fg^4-9g^2h(3eg+64dh)))-5ch(ah^2(61fg}{5040c^3h}$$
$$+ \frac{\left(48c^2dg^3+3a^2h^2(3fg+eh)-8acg(fg^2+3h(eg+dh))\right)\int \left(a+cx^2\right)^{3/2}\,dx}{48c^2}$$

$$= \frac{\left(48c^2dg^3+3a^2h^2(3fg+eh)-8acg(fg^2+3h(eg+dh))\right)x\left(a+cx^2\right)^{3/2}}{192c^2}$$
$$+ \frac{\left(8(9cd-4af)h^2-3cg(5fg-9eh)\right)(g+hx)^2\left(a+cx^2\right)^{5/2}}{504c^2h}$$
$$- \frac{(5fg-9eh)(g+hx)^3\left(a+cx^2\right)^{5/2}}{72ch} + \frac{f(g+hx)^4\left(a+cx^2\right)^{5/2}}{9ch}$$
$$+ \frac{(4(32a^2fh^4-8ach^2(17fg^2+9h(3eg+dh))-c^2(15fg^4-9g^2h(3eg+64dh)))-5ch(ah^2(61fg}{5040c^3h}$$
$$+ \frac{\left(a(48c^2dg^3+3a^2h^2(3fg+eh)-8acg(fg^2+3h(eg+dh)))\right)\int \sqrt{a+cx^2}\,dx}{64c^2}$$

$$= \frac{a\left(48c^2dg^3+3a^2h^2(3fg+eh)-8acg(fg^2+3h(eg+dh))\right)x\sqrt{a+cx^2}}{128c^2}$$
$$+ \frac{\left(48c^2dg^3+3a^2h^2(3fg+eh)-8acg(fg^2+3h(eg+dh))\right)x\left(a+cx^2\right)^{3/2}}{192c^2}$$
$$+ \frac{\left(8(9cd-4af)h^2-3cg(5fg-9eh)\right)(g+hx)^2\left(a+cx^2\right)^{5/2}}{504c^2h}$$
$$- \frac{(5fg-9eh)(g+hx)^3\left(a+cx^2\right)^{5/2}}{72ch} + \frac{f(g+hx)^4\left(a+cx^2\right)^{5/2}}{9ch}$$
$$+ \frac{(4(32a^2fh^4-8ach^2(17fg^2+9h(3eg+dh))-c^2(15fg^4-9g^2h(3eg+64dh)))-5ch(ah^2(61fg}{5040c^3h}$$
$$+ \frac{\left(a^2(48c^2dg^3+3a^2h^2(3fg+eh)-8acg(fg^2+3h(eg+dh)))\right)\int \frac{1}{\sqrt{a+cx^2}}\,dx}{128c^2}$$

$$= \frac{a(48c^2dg^3+3a^2h^2(3fg+eh)-8acg(fg^2+3h(eg+dh)))\,x\sqrt{a+cx^2}}{128c^2}$$
$$+\frac{(48c^2dg^3+3a^2h^2(3fg+eh)-8acg(fg^2+3h(eg+dh)))\,x(a+cx^2)^{3/2}}{192c^2}$$
$$+\frac{(8(9cd-4af)h^2-3cg(5fg-9eh))\,(g+hx)^2\,(a+cx^2)^{5/2}}{504c^2h}$$
$$-\frac{(5fg-9eh)(g+hx)^3\,(a+cx^2)^{5/2}}{72ch}+\frac{f(g+hx)^4\,(a+cx^2)^{5/2}}{9ch}$$
$$+\frac{(4(32a^2fh^4-8ach^2(17fg^2+9h(3eg+dh))-c^2(15fg^4-9g^2h(3eg+64dh)))-5ch(ah^2(61fg}{5040c^3h}$$
$$+\frac{(a^2(48c^2dg^3+3a^2h^2(3fg+eh)-8acg(fg^2+3h(eg+dh))))\,\mathrm{Subst}\left(\int\frac{1}{1-cx^2}\,dx,x,\frac{x}{\sqrt{a+cx^2}}\right)}{128c^2}$$
$$= \frac{a(48c^2dg^3+3a^2h^2(3fg+eh)-8acg(fg^2+3h(eg+dh)))\,x\sqrt{a+cx^2}}{128c^2}$$
$$+\frac{(48c^2dg^3+3a^2h^2(3fg+eh)-8acg(fg^2+3h(eg+dh)))\,x(a+cx^2)^{3/2}}{192c^2}$$
$$+\frac{(8(9cd-4af)h^2-3cg(5fg-9eh))\,(g+hx)^2\,(a+cx^2)^{5/2}}{504c^2h}$$
$$-\frac{(5fg-9eh)(g+hx)^3\,(a+cx^2)^{5/2}}{72ch}+\frac{f(g+hx)^4\,(a+cx^2)^{5/2}}{9ch}$$
$$+\frac{(4(32a^2fh^4-8ach^2(17fg^2+9h(3eg+dh))-c^2(15fg^4-9g^2h(3eg+64dh)))-5ch(ah^2(61fg}{5040c^3h}$$
$$+\frac{a^2(48c^2dg^3+3a^2h^2(3fg+eh)-8acg(fg^2+3h(eg+dh)))\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{128c^{5/2}}$$

Mathematica [A] (verified)

Time = 1.33 (sec) , antiderivative size = 474, normalized size of antiderivative = 1.03

$$\int(g+hx)^3\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)\,dx=\frac{\sqrt{a+cx^2}(1024a^4fh^3-a^3ch(9h(768eg+256dh+105ehx)+f(6912g^2+2835ghx+512h^2x^2))}{}$$

```
[In] Integrate[(g + h*x)^3*(a + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

```
[Out] (Sqrt[a + c*x^2]*(1024*a^4*f*h^3 - a^3*c*h*(9*h*(768*e*g + 256*d*h + 105*e*
h*x) + f*(6912*g^2 + 2835*g*h*x + 512*h^2*x^2)) + 6*a^2*c^2*(12*d*h*(336*g^
2 + 105*g*h*x + 16*h^2*x^2) + 3*e*(448*g^3 + 420*g^2*h*x + 192*g*h^2*x^2 +
35*h^3*x^3) + f*x*(420*g^3 + 576*g^2*h*x + 315*g*h^2*x^2 + 64*h^3*x^3)) + 1
6*c^4*x^3*(18*d*(35*g^3 + 84*g^2*h*x + 70*g*h^2*x^2 + 20*h^3*x^3) + x*(9*e*
(56*g^3 + 140*g^2*h*x + 120*g*h^2*x^2 + 35*h^3*x^3) + 5*f*x*(84*g^3 + 216*g
```

```
^2*h*x + 189*g*h^2*x^2 + 56*h^3*x^3))) + 8*a*c^3*x*(18*d*(175*g^3 + 336*g^2
*h*x + 245*g*h^2*x^2 + 64*h^3*x^3) + x*(9*e*(224*g^3 + 490*g^2*h*x + 384*g*
h^2*x^2 + 105*h^3*x^3) + f*x*(1470*g^3 + 3456*g^2*h*x + 2835*g*h^2*x^2 + 80
0*h^3*x^3)))) - 315*a^2*Sqrt[c]*(48*c^2*d*g^3 + 3*a^2*h^2*(3*f*g + e*h) - 8
*a*c*g*(f*g^2 + 3*h*(e*g + d*h)))*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/(403
20*c^3)
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 406, normalized size of antiderivative = 0.88

method	result
default	$d\,g^3\left(\frac{x\left(c\,x^2+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{c\,x^2+a}}{2}+\frac{a\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{2\sqrt{c}}\right)}{4}\right)+f\,h^3\left(\frac{x^4\left(c\,x^2+a\right)^{\frac{5}{2}}}{9c}-\frac{4a\left(\frac{x^2\left(c\,x^2+a\right)^{\frac{5}{2}}}{7c}-\frac{2a\left(c\,x^2+a\right)^{\frac{5}{2}}}{35c^2}\right)}{9c}\right.$
risch	$\left(4480c^4 f\,h^3x^8+5040c^4e\,h^3x^7+15120c^4fg\,h^2x^7+6400a\,c^3f\,h^3x^6+5760c^4d\,h^3x^6+17280c^4eg\,h^2x^6+17280c^4f\,g^2h\,x^6+7560a\,c^3e\,h^3x\right.$

```
[In] int((h*x+g)^3*(c*x^2+a)^(3/2)*(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] d*g^3*(1/4*x*(c*x^2+a)^(3/2)+3/4*a*(1/2*x*(c*x^2+a)^(1/2)+1/2*a/c^(1/2)*ln(
x*c^(1/2)+(c*x^2+a)^(1/2))))+f*h^3*(1/9*x^4*(c*x^2+a)^(5/2)/c-4/9*a/c*(1/7*
x^2*(c*x^2+a)^(5/2)/c-2/35*a/c^2*(c*x^2+a)^(5/2)))+(e*h^3+3*f*g*h^2)*(1/8*x
^3*(c*x^2+a)^(5/2)/c-3/8*a/c*(1/6*x*(c*x^2+a)^(5/2)/c-1/6*a/c*(1/4*x*(c*x^2
+a)^(3/2)+3/4*a*(1/2*x*(c*x^2+a)^(1/2)+1/2*a/c^(1/2)*ln(x*c^(1/2)+(c*x^2+a)
^(1/2))))))+1/5*(3*d*g^2*h+e*g^3)*(c*x^2+a)^(5/2)/c+(d*h^3+3*e*g*h^2+3*f*g^
2*h)*(1/7*x^2*(c*x^2+a)^(5/2)/c-2/35*a/c^2*(c*x^2+a)^(5/2))+(3*d*g*h^2+3*e*
g^2*h+f*g^3)*(1/6*x*(c*x^2+a)^(5/2)/c-1/6*a/c*(1/4*x*(c*x^2+a)^(3/2)+3/4*a*
(1/2*x*(c*x^2+a)^(1/2)+1/2*a/c^(1/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2)))))
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 1177, normalized size of antiderivative = 2.55

$$\int (g+hx)^3 \left(a+cx^2\right)^{3/2} \left(d+ex+fx^2\right)\,dx = \text{Too large to display}$$

```
[In] integrate((h*x+g)^3*(c*x^2+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] [-1/80640*(315*(24*a^3*c*e*g^2*h - 3*a^4*e*h^3 - 8*(6*a^2*c^2*d - a^3*c*f)*
g^3 + 3*(8*a^3*c*d - 3*a^4*f)*g*h^2)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 +
a)*sqrt(c)*x - a) - 2*(4480*c^4*f*h^3*x^8 + 8064*a^2*c^2*e*g^3 - 6912*a^3*c
*e*g*h^2 + 5040*(3*c^4*f*g*h^2 + c^4*e*h^3)*x^7 + 640*(27*c^4*f*g^2*h + 27*
c^4*e*g*h^2 + (9*c^4*d + 10*a*c^3*f)*h^3)*x^6 + 840*(8*c^4*f*g^3 + 24*c^4*e
*g^2*h + 9*a*c^3*e*h^3 + 3*(8*c^4*d + 9*a*c^3*f)*g*h^2)*x^5 + 384*(21*c^4*e
*g^3 + 72*a*c^3*e*g*h^2 + 9*(7*c^4*d + 8*a*c^3*f)*g^2*h + (24*a*c^3*d + a^2
*c^2*f)*h^3)*x^4 + 3456*(7*a^2*c^2*d - 2*a^3*c*f)*g^2*h - 256*(9*a^3*c*d -
4*a^4*f)*h^3 + 210*(168*a*c^3*e*g^2*h + 3*a^2*c^2*e*h^3 + 8*(6*c^4*d + 7*a*
c^3*f)*g^3 + 3*(56*a*c^3*d + 3*a^2*c^2*f)*g*h^2)*x^3 + 128*(126*a*c^3*e*g^3
 + 27*a^2*c^2*e*g*h^2 + 27*(14*a*c^3*d + a^2*c^2*f)*g^2*h + (9*a^2*c^2*d -
4*a^3*c*f)*h^3)*x^2 + 315*(24*a^2*c^2*e*g^2*h - 3*a^3*c*e*h^3 + 8*(10*a*c^3
*d + a^2*c^2*f)*g^3 + 3*(8*a^2*c^2*d - 3*a^3*c*f)*g*h^2)*x)*sqrt(c*x^2 + a)
)/c^3, 1/40320*(315*(24*a^3*c*e*g^2*h - 3*a^4*e*h^3 - 8*(6*a^2*c^2*d - a^3*
c*f)*g^3 + 3*(8*a^3*c*d - 3*a^4*f)*g*h^2)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c
*x^2 + a)) + (4480*c^4*f*h^3*x^8 + 8064*a^2*c^2*e*g^3 - 6912*a^3*c*e*g*h^2
+ 5040*(3*c^4*f*g*h^2 + c^4*e*h^3)*x^7 + 640*(27*c^4*f*g^2*h + 27*c^4*e*g*h
^2 + (9*c^4*d + 10*a*c^3*f)*h^3)*x^6 + 840*(8*c^4*f*g^3 + 24*c^4*e*g^2*h +
9*a*c^3*e*h^3 + 3*(8*c^4*d + 9*a*c^3*f)*g*h^2)*x^5 + 384*(21*c^4*e*g^3 + 72
*a*c^3*e*g*h^2 + 9*(7*c^4*d + 8*a*c^3*f)*g^2*h + (24*a*c^3*d + a^2*c^2*f)*h
^3)*x^4 + 3456*(7*a^2*c^2*d - 2*a^3*c*f)*g^2*h - 256*(9*a^3*c*d - 4*a^4*f)*
h^3 + 210*(168*a*c^3*e*g^2*h + 3*a^2*c^2*e*h^3 + 8*(6*c^4*d + 7*a*c^3*f)*g^
3 + 3*(56*a*c^3*d + 3*a^2*c^2*f)*g*h^2)*x^3 + 128*(126*a*c^3*e*g^3 + 27*a^2
*c^2*e*g*h^2 + 27*(14*a*c^3*d + a^2*c^2*f)*g^2*h + (9*a^2*c^2*d - 4*a^3*c*f
)*h^3)*x^2 + 315*(24*a^2*c^2*e*g^2*h - 3*a^3*c*e*h^3 + 8*(10*a*c^3*d + a^2*
c^2*f)*g^3 + 3*(8*a^2*c^2*d - 3*a^3*c*f)*g*h^2)*x)*sqrt(c*x^2 + a))/c^3]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1428 vs. 2(449) = 898.

Time = 0.66 (sec) , antiderivative size = 1428, normalized size of antiderivative = 3.09

$$\int (g+hx)^3 \left(a+cx^2\right)^{3/2} \left(d+ex+fx^2\right) \, dx = \text{Too large to display}$$

```
[In] integrate((h*x+g)**3*(c*x**2+a)**(3/2)*(f*x**2+e*x+d),x)
```

```
[Out] Piecewise((sqrt(a + c*x**2)*(c*f*h**3*x**8/9 + x**7*(c**2*e*h**3 + 3*c**2*f
*g*h**2)/(8*c) + x**6*(10*a*c*f*h**3/9 + c**2*d*h**3 + 3*c**2*e*g*h**2 + 3*
c**2*f*g**2*h)/(7*c) + x**5*(2*a*c*e*h**3 + 6*a*c*f*g*h**2 - 7*a*(c**2*e*h*
*3 + 3*c**2*f*g*h**2)/(8*c) + 3*c**2*d*g*h**2 + 3*c**2*e*g**2*h + c**2*f*g*
*3)/(6*c) + x**4*(a**2*f*h**3 + 2*a*c*d*h**3 + 6*a*c*e*g*h**2 + 6*a*c*f*g**
2*h - 6*a*(10*a*c*f*h**3/9 + c**2*d*h**3 + 3*c**2*e*g*h**2 + 3*c**2*f*g**2*
h)/(7*c) + 3*c**2*d*g**2*h + c**2*e*g**3)/(5*c) + x**3*(a**2*e*h**3 + 3*a**
2*f*g*h**2 + 6*a*c*d*g*h**2 + 6*a*c*e*g**2*h + 2*a*c*f*g**3 - 5*a*(2*a*c*e*
h**3 + 6*a*c*f*g*h**2 - 7*a*(c**2*e*h**3 + 3*c**2*f*g*h**2)/(8*c) + 3*c**2*
d*g*h**2 + 3*c**2*e*g**2*h + c**2*f*g**3)/(6*c) + c**2*d*g**3)/(4*c) + x**2
*(a**2*d*h**3 + 3*a**2*e*g*h**2 + 3*a**2*f*g**2*h + 6*a*c*d*g**2*h + 2*a*c*
e*g**3 - 4*a*(a**2*f*h**3 + 2*a*c*d*h**3 + 6*a*c*e*g*h**2 + 6*a*c*f*g**2*h
- 6*a*(10*a*c*f*h**3/9 + c**2*d*h**3 + 3*c**2*e*g*h**2 + 3*c**2*f*g**2*h)/(
7*c) + 3*c**2*d*g**2*h + c**2*e*g**3)/(5*c))/(3*c) + x*(3*a**2*d*g*h**2 + 3
*a**2*e*g**2*h + a**2*f*g**3 + 2*a*c*d*g**3 - 3*a*(a**2*e*h**3 + 3*a**2*f*g
*h**2 + 6*a*c*d*g*h**2 + 6*a*c*e*g**2*h + 2*a*c*f*g**3 - 5*a*(2*a*c*e*h**3
+ 6*a*c*f*g*h**2 - 7*a*(c**2*e*h**3 + 3*c**2*f*g*h**2)/(8*c) + 3*c**2*d*g*h
**2 + 3*c**2*e*g**2*h + c**2*f*g**3)/(6*c) + c**2*d*g**3)/(4*c))/(2*c) + (3
*a**2*d*g**2*h + a**2*e*g**3 - 2*a*(a**2*d*h**3 + 3*a**2*e*g*h**2 + 3*a**2*
f*g**2*h + 6*a*c*d*g**2*h + 2*a*c*e*g**3 - 4*a*(a**2*f*h**3 + 2*a*c*d*h**3
+ 6*a*c*e*g*h**2 + 6*a*c*f*g**2*h - 6*a*(10*a*c*f*h**3/9 + c**2*d*h**3 + 3*
c**2*e*g*h**2 + 3*c**2*f*g**2*h)/(7*c) + 3*c**2*d*g**2*h + c**2*e*g**3)/(5*
c))/(3*c))/c) + (a**2*d*g**3 - a*(3*a**2*d*g*h**2 + 3*a**2*e*g**2*h + a**2*
f*g**3 + 2*a*c*d*g**3 - 3*a*(a**2*e*h**3 + 3*a**2*f*g*h**2 + 6*a*c*d*g*h**2
 + 6*a*c*e*g**2*h + 2*a*c*f*g**3 - 5*a*(2*a*c*e*h**3 + 6*a*c*f*g*h**2 - 7*a
*(c**2*e*h**3 + 3*c**2*f*g*h**2)/(8*c) + 3*c**2*d*g*h**2 + 3*c**2*e*g**2*h
+ c**2*f*g**3)/(6*c) + c**2*d*g**3)/(4*c))/(2*c))*Piecewise((log(2*sqrt(c)*
sqrt(a + c*x**2) + 2*c*x)/sqrt(c), Ne(a, 0)), (x*log(x)/sqrt(c*x**2), True)
), Ne(c, 0)), (a**(3/2)*(d*g**3*x + f*h**3*x**6/6 + x**5*(e*h**3 + 3*f*g*h*
*2)/5 + x**4*(d*h**3 + 3*e*g*h**2 + 3*f*g**2*h)/4 + x**3*(3*d*g*h**2 + 3*e*
g**2*h + f*g**3)/3 + x**2*(3*d*g**2*h + e*g**3)/2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 525, normalized size of antiderivative = 1.14

$$
\begin{aligned}
\int (g+hx)^3 \left(a+cx^2\right)^{3/2} \left(d+ex+fx^2\right)\,dx &= \frac{(cx^2+a)^{\frac{5}{2}} fh^3x^4}{9\,c} \\
&- \frac{4\,(cx^2+a)^{\frac{5}{2}} afh^3x^2}{63\,c^2} + \frac{1}{4}\left(cx^2+a\right)^{\frac{3}{2}} dg^3x + \frac{3}{8}\sqrt{cx^2+a}\,adg^3x \\
&+ \frac{3\,a^2dg^3 \operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{8\,\sqrt{c}} + \frac{(cx^2+a)^{\frac{5}{2}} eg^3}{5\,c} + \frac{3\,(cx^2+a)^{\frac{5}{2}} dg^2h}{5\,c} + \frac{8\,(cx^2+a)^{\frac{5}{2}} a^2fh^3}{315\,c^3} \\
&+ \frac{(3\,fgh^2+eh^3)(cx^2+a)^{\frac{5}{2}} x^3}{8\,c} + \frac{(3\,fg^2h+3\,egh^2+dh^3)(cx^2+a)^{\frac{5}{2}} x^2}{7\,c} \\
&- \frac{(3\,fgh^2+eh^3)(cx^2+a)^{\frac{5}{2}} ax}{16\,c^2} + \frac{(3\,fgh^2+eh^3)(cx^2+a)^{\frac{3}{2}} a^2x}{64\,c^2} \\
&+ \frac{3\,(3\,fgh^2+eh^3)\sqrt{cx^2+a}\,a^3x}{128\,c^2} + \frac{(fg^3+3\,eg^2h+3\,dgh^2)(cx^2+a)^{\frac{5}{2}} x}{6\,c} \\
&- \frac{(fg^3+3\,eg^2h+3\,dgh^2)(cx^2+a)^{\frac{3}{2}} ax}{24\,c} - \frac{(fg^3+3\,eg^2h+3\,dgh^2)\sqrt{cx^2+a}\,a^2x}{16\,c} \\
&+ \frac{3\,(3\,fgh^2+eh^3)a^4 \operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{128\,c^{\frac{5}{2}}} - \frac{(fg^3+3\,eg^2h+3\,dgh^2)a^3 \operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{16\,c^{\frac{3}{2}}} \\
&- \frac{2\,(3\,fg^2h+3\,egh^2+dh^3)(cx^2+a)^{\frac{5}{2}} a}{35\,c^2}
\end{aligned}
$$

```
[In] integrate((h*x+g)^3*(c*x^2+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] 1/9*(c*x^2 + a)^(5/2)*f*h^3*x^4/c - 4/63*(c*x^2 + a)^(5/2)*a*f*h^3*x^2/c^2
+ 1/4*(c*x^2 + a)^(3/2)*d*g^3*x + 3/8*sqrt(c*x^2 + a)*a*d*g^3*x + 3/8*a^2*d
*g^3*arcsinh(c*x/sqrt(a*c))/sqrt(c) + 1/5*(c*x^2 + a)^(5/2)*e*g^3/c + 3/5*(
c*x^2 + a)^(5/2)*d*g^2*h/c + 8/315*(c*x^2 + a)^(5/2)*a^2*f*h^3/c^3 + 1/8*(3
*f*g*h^2 + e*h^3)*(c*x^2 + a)^(5/2)*x^3/c + 1/7*(3*f*g^2*h + 3*e*g*h^2 + d*
h^3)*(c*x^2 + a)^(5/2)*x^2/c - 1/16*(3*f*g*h^2 + e*h^3)*(c*x^2 + a)^(5/2)*a
*x/c^2 + 1/64*(3*f*g*h^2 + e*h^3)*(c*x^2 + a)^(3/2)*a^2*x/c^2 + 3/128*(3*f*
g*h^2 + e*h^3)*sqrt(c*x^2 + a)*a^3*x/c^2 + 1/6*(f*g^3 + 3*e*g^2*h + 3*d*g*h
^2)*(c*x^2 + a)^(5/2)*x/c - 1/24*(f*g^3 + 3*e*g^2*h + 3*d*g*h^2)*(c*x^2 + a
)^(3/2)*a*x/c - 1/16*(f*g^3 + 3*e*g^2*h + 3*d*g*h^2)*sqrt(c*x^2 + a)*a^2*x/
c + 3/128*(3*f*g*h^2 + e*h^3)*a^4*arcsinh(c*x/sqrt(a*c))/c^(5/2) - 1/16*(f*
g^3 + 3*e*g^2*h + 3*d*g*h^2)*a^3*arcsinh(c*x/sqrt(a*c))/c^(3/2) - 2/35*(3*f
*g^2*h + 3*e*g*h^2 + d*h^3)*(c*x^2 + a)^(5/2)*a/c^2
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 636, normalized size of antiderivative = 1.38

$$\int (g+hx)^3\left(a+cx^2\right)^{3/2}(d+ex$$
$$+fx^2)\;dx = \frac{1}{40320}\sqrt{cx^2+a}\Bigg(\Bigg(2\Bigg(\Bigg(\Bigg(4\Bigg(5\Bigg(2\Bigg(7\Bigg(8\,cfh^3x+\frac{9\,(3\,c^8fgh^2+c^8eh^3)}{c^7}\Bigg)x+\frac{8\,(27\,c^8fg^2h+27\,c}{}$$
$$-\frac{(48\,a^2c^2dg^3-8\,a^3cfg^3-24\,a^3ceg^2h-24\,a^3cdgh^2+9\,a^4fgh^2+3\,a^4eh^3)\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+a}\right|\right)}{128\,c^{\frac{5}{2}}}$$

```
[In] integrate((h*x+g)^3*(c*x^2+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] 1/40320*sqrt(c*x^2 + a)*((2*((4*(5*(2*(7*(8*c*f*h^3*x + 9*(3*c^8*f*g*h^2 +
c^8*e*h^3)/c^7)*x + 8*(27*c^8*f*g^2*h + 27*c^8*e*g*h^2 + 9*c^8*d*h^3 + 10*a
*c^7*f*h^3)/c^7)*x + 21*(8*c^8*f*g^3 + 24*c^8*e*g^2*h + 24*c^8*d*g*h^2 + 27
*a*c^7*f*g*h^2 + 9*a*c^7*e*h^3)/c^7)*x + 48*(21*c^8*e*g^3 + 63*c^8*d*g^2*h
+ 72*a*c^7*f*g^2*h + 72*a*c^7*e*g*h^2 + 24*a*c^7*d*h^3 + a^2*c^6*f*h^3)/c^7
)*x + 105*(48*c^8*d*g^3 + 56*a*c^7*f*g^3 + 168*a*c^7*e*g^2*h + 168*a*c^7*d*
g*h^2 + 9*a^2*c^6*f*g*h^2 + 3*a^2*c^6*e*h^3)/c^7)*x + 64*(126*a*c^7*e*g^3 +
 378*a*c^7*d*g^2*h + 27*a^2*c^6*f*g^2*h + 27*a^2*c^6*e*g*h^2 + 9*a^2*c^6*d*
h^3 - 4*a^3*c^5*f*h^3)/c^7)*x + 315*(80*a*c^7*d*g^3 + 8*a^2*c^6*f*g^3 + 24*
a^2*c^6*e*g^2*h + 24*a^2*c^6*d*g*h^2 - 9*a^3*c^5*f*g*h^2 - 3*a^3*c^5*e*h^3)
/c^7)*x + 128*(63*a^2*c^6*e*g^3 + 189*a^2*c^6*d*g^2*h - 54*a^3*c^5*f*g^2*h
- 54*a^3*c^5*e*g*h^2 - 18*a^3*c^5*d*h^3 + 8*a^4*c^4*f*h^3)/c^7) - 1/128*(48
*a^2*c^2*d*g^3 - 8*a^3*c*f*g^3 - 24*a^3*c*e*g^2*h - 24*a^3*c*d*g*h^2 + 9*a^
4*f*g*h^2 + 3*a^4*e*h^3)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int (g+hx)^3\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)\,dx=\int (g+h\,x)^3\,\left(c\,x^2+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right)\,dx$$

```
[In] int((g + h*x)^3*(a + c*x^2)^(3/2)*(d + e*x + f*x^2),x)
```

```
[Out] int((g + h*x)^3*(a + c*x^2)^(3/2)*(d + e*x + f*x^2), x)
```

3.89 $\int (g+hx)^2 \left(a+cx^2\right)^{3/2} \left(d+ex+fx^2\right) \, dx$

Optimal result

Integrand size = 29, antiderivative size = 346

$$\begin{aligned}
&\int (g+hx)^2 \left(a+cx^2\right)^{3/2} \left(d+ex+fx^2\right) \, dx = \frac{a\left(48c^2dg^2+3a^2fh^2-8ac(fg^2+h(2eg+dh))\right)x\sqrt{a+cx^2}}{128c^2} \\
&+ \frac{\left(48c^2dg^2+3a^2fh^2-8ac(fg^2+h(2eg+dh))\right)x\left(a+cx^2\right)^{3/2}}{192c^2} \\
&- \frac{(5fg-8eh)(g+hx)^2\left(a+cx^2\right)^{5/2}}{56ch} + \frac{f(g+hx)^3\left(a+cx^2\right)^{5/2}}{8ch} \\
&- \frac{\left(12(8ah^2(2fg+eh)+cg(5fg^2-8h(eg+7dh)))-5h(7(8cd-3af)h^2-2cg(5fg-8eh))x\right)\left(a+cx^2\right)^{5/2}}{1680c^2h} \\
&+ \frac{a^2\left(48c^2dg^2+3a^2fh^2-8ac(fg^2+h(2eg+dh))\right)\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{128c^{5/2}}
\end{aligned}$$

```
[Out] 1/192*(48*c^2*d*g^2+3*a^2*f*h^2-8*a*c*(f*g^2+h*(d*h+2*e*g)))*x*(c*x^2+a)^(3
/2)/c^2-1/56*(-8*e*h+5*f*g)*(h*x+g)^2*(c*x^2+a)^(5/2)/c/h+1/8*f*(h*x+g)^3*(
c*x^2+a)^(5/2)/c/h-1/1680*(96*a*h^2*(e*h+2*f*g)+12*c*g*(5*f*g^2-8*h*(7*d*h+
e*g))-5*h*(7*(-3*a*f+8*c*d)*h^2-2*c*g*(-8*e*h+5*f*g))*x)*(c*x^2+a)^(5/2)/c^
2/h+1/128*a^2*(48*c^2*d*g^2+3*a^2*f*h^2-8*a*c*(f*g^2+h*(d*h+2*e*g)))*arctan
h(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(5/2)+1/128*a*(48*c^2*d*g^2+3*a^2*f*h^2-8*a*
c*(f*g^2+h*(d*h+2*e*g)))*x*(c*x^2+a)^(1/2)/c^2
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1668, 847, 794, 201, 223, 212}

$$\begin{aligned}\int (g+hx)^2\left(a+cx^2\right)^{3/2}(d+ex &+fx^2)\,dx = \frac{a^2\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(3a^2fh^2-8ac(h(dh+2eg)+fg^2)+48c^2dg^2\right)}{128c^{5/2}}\\ &+\frac{x\left(a+cx^2\right)^{3/2}\left(3a^2fh^2-8ac(h(dh+2eg)+fg^2)+48c^2dg^2\right)}{192c^2}\\ &+\frac{ax\sqrt{a+cx^2}\left(3a^2fh^2-8ac(h(dh+2eg)+fg^2)+48c^2dg^2\right)}{128c^2}\\ &-\frac{\left(a+cx^2\right)^{5/2}\left(12(8ah^2(eh+2fg)-8cgh(7dh+eg)+5cfg^3)-5hx(7h^2(8cd-3af)-2cg(5fg-8eh))\right)}{1680c^2h}\\ &-\frac{\left(a+cx^2\right)^{5/2}(g+hx)^2(5fg-8eh)}{56ch}+\frac{f\left(a+cx^2\right)^{5/2}(g+hx)^3}{8ch}\end{aligned}$$

```
[In] Int[(g + h*x)^2*(a + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

```
[Out] (a*(48*c^2*d*g^2 + 3*a^2*f*h^2 - 8*a*c*(f*g^2 + h*(2*e*g + d*h)))*x*Sqrt[a
+ c*x^2])/(128*c^2) + ((48*c^2*d*g^2 + 3*a^2*f*h^2 - 8*a*c*(f*g^2 + h*(2*e*
g + d*h)))*x*(a + c*x^2)^(3/2))/(192*c^2) - ((5*f*g - 8*e*h)*(g + h*x)^2*(a
 + c*x^2)^(5/2))/(56*c*h) + (f*(g + h*x)^3*(a + c*x^2)^(5/2))/(8*c*h) - ((1
2*(5*c*f*g^3 - 8*c*g*h*(e*g + 7*d*h) + 8*a*h^2*(2*f*g + e*h)) - 5*h*(7*(8*c
*d - 3*a*f)*h^2 - 2*c*g*(5*f*g - 8*e*h))*x)*(a + c*x^2)^(5/2))/(1680*c^2*h)
 + (a^2*(48*c^2*d*g^2 + 3*a^2*f*h^2 - 8*a*c*(f*g^2 + h*(2*e*g + d*h)))*ArcT
anh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(128*c^(5/2))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{f(g+hx)^3\left(a+cx^2\right)^{5/2}}{8ch} \\
&\quad+\frac{\int (g+hx)^2\left((8cd-3af)h^2-ch(5fg-8eh)x\right)\left(a+cx^2\right)^{3/2}\,dx}{8ch^2} \\
&= -\frac{(5fg-8eh)(g+hx)^2\left(a+cx^2\right)^{5/2}}{56ch}+\frac{f(g+hx)^3\left(a+cx^2\right)^{5/2}}{8ch} \\
&\quad+\frac{\int (g+hx)\left(ch^2(56cdg-11afg-16aeh)+ch\left(7(8cd-3af)h^2-2cg(5fg-8eh)\right)x\right)\left(a+cx^2\right)^{3/}}{56c^2h^2}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{(5fg-8eh)(g+hx)^2\left(a+cx^2\right)^{5/2}}{56ch}+\frac{f(g+hx)^3\left(a+cx^2\right)^{5/2}}{8ch} \\
&\quad -\frac{\left(12\left(5cfg^3-8cgh(eg+7dh)+8ah^2(2fg+eh)\right)-5h\left(7(8cd-3af)h^2-2cg(5fg-8eh)\right)x\right)(a}{1680c^2h} \\
&\quad +\frac{\left(48c^2dg^2+3a^2fh^2-8ac\left(fg^2+h(2eg+dh)\right)\right)\int\left(a+cx^2\right)^{3/2}\,dx}{48c^2}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{\left(48c^2dg^2+3a^2fh^2-8ac\left(fg^2+h(2eg+dh)\right)\right)x\left(a+cx^2\right)^{3/2}}{192c^2} \\
&\quad -\frac{(5fg-8eh)(g+hx)^2\left(a+cx^2\right)^{5/2}}{56ch}+\frac{f(g+hx)^3\left(a+cx^2\right)^{5/2}}{8ch} \\
&\quad -\frac{\left(12\left(5cfg^3-8cgh(eg+7dh)+8ah^2(2fg+eh)\right)-5h\left(7(8cd-3af)h^2-2cg(5fg-8eh)\right)x\right)(a}{1680c^2h} \\
&\quad +\frac{\left(a\left(48c^2dg^2+3a^2fh^2-8ac\left(fg^2+h(2eg+dh)\right)\right)\right)\int\sqrt{a+cx^2}\,dx}{64c^2}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{a\left(48c^2dg^2+3a^2fh^2-8ac\left(fg^2+h(2eg+dh)\right)\right)x\sqrt{a+cx^2}}{128c^2} \\
&\quad +\frac{\left(48c^2dg^2+3a^2fh^2-8ac\left(fg^2+h(2eg+dh)\right)\right)x\left(a+cx^2\right)^{3/2}}{192c^2} \\
&\quad -\frac{(5fg-8eh)(g+hx)^2\left(a+cx^2\right)^{5/2}}{56ch}+\frac{f(g+hx)^3\left(a+cx^2\right)^{5/2}}{8ch} \\
&\quad -\frac{\left(12\left(5cfg^3-8cgh(eg+7dh)+8ah^2(2fg+eh)\right)-5h\left(7(8cd-3af)h^2-2cg(5fg-8eh)\right)x\right)(a}{1680c^2h} \\
&\quad +\frac{\left(a^2\left(48c^2dg^2+3a^2fh^2-8ac\left(fg^2+h(2eg+dh)\right)\right)\right)\int\frac{1}{\sqrt{a+cx^2}}\,dx}{128c^2}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{a\left(48c^2dg^2+3a^2fh^2-8ac\left(fg^2+h(2eg+dh)\right)\right)x\sqrt{a+cx^2}}{128c^2} \\
&\quad +\frac{\left(48c^2dg^2+3a^2fh^2-8ac\left(fg^2+h(2eg+dh)\right)\right)x\left(a+cx^2\right)^{3/2}}{192c^2} \\
&\quad -\frac{(5fg-8eh)(g+hx)^2\left(a+cx^2\right)^{5/2}}{56ch}+\frac{f(g+hx)^3\left(a+cx^2\right)^{5/2}}{8ch} \\
&\quad -\frac{\left(12\left(5cfg^3-8cgh(eg+7dh)+8ah^2(2fg+eh)\right)-5h\left(7(8cd-3af)h^2-2cg(5fg-8eh)\right)x\right)(a}{1680c^2h} \\
&\quad +\frac{\left(a^2\left(48c^2dg^2+3a^2fh^2-8ac\left(fg^2+h(2eg+dh)\right)\right)\right)\mathrm{Subst}\left(\int\frac{1}{1-cx^2}\,dx,x,\frac{x}{\sqrt{a+cx^2}}\right)}{128c^2}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{a(48c^2dg^2+3a^2fh^2-8ac(fg^2+h(2eg+dh)))\,x\sqrt{a+cx^2}}{128c^2} \\
&\quad + \frac{(48c^2dg^2+3a^2fh^2-8ac(fg^2+h(2eg+dh)))\,x(a+cx^2)^{3/2}}{192c^2} \\
&\quad - \frac{(5fg-8eh)(g+hx)^2\left(a+cx^2\right)^{5/2}}{56ch} + \frac{f(g+hx)^3\left(a+cx^2\right)^{5/2}}{8ch} \\
&\quad - \frac{(12(5cfg^3-8cgh(eg+7dh)+8ah^2(2fg+eh))-5h(7(8cd-3af)h^2-2cg(5fg-8eh))\,x)\,(a+}{1680c^2h} \\
&\quad + \frac{a^2(48c^2dg^2+3a^2fh^2-8ac(fg^2+h(2eg+dh)))\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{128c^{5/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 332, normalized size of antiderivative = 0.96

$$
\int (g+hx)^2\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)dx = \frac{\sqrt{c}\sqrt{a+cx^2}(-3a^3h(512fg+256eh+105fhx)+6a^2c(28dh(32g+5hx)+8e(56g^2+35ghx+8}{}
$$

```
[In] Integrate[(g + h*x)^2*(a + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

```
[Out] (Sqrt[c]*Sqrt[a + c*x^2]*(-3*a^3*h*(512*f*g + 256*e*h + 105*f*h*x) + 6*a^2*
c*(28*d*h*(32*g + 5*h*x) + 8*e*(56*g^2 + 35*g*h*x + 8*h^2*x^2) + f*x*(140*g
^2 + 128*g*h*x + 35*h^2*x^2)) + 16*c^3*x^3*(14*d*(15*g^2 + 24*g*h*x + 10*h^
2*x^2) + x*(8*e*(21*g^2 + 35*g*h*x + 15*h^2*x^2) + 5*f*x*(28*g^2 + 48*g*h*x
 + 21*h^2*x^2))) + 8*a*c^2*x*(14*d*(75*g^2 + 96*g*h*x + 35*h^2*x^2) + x*(4*
e*(168*g^2 + 245*g*h*x + 96*h^2*x^2) + f*x*(490*g^2 + 768*g*h*x + 315*h^2*x
^2)))) - 105*a^2*(48*c^2*d*g^2 + 3*a^2*f*h^2 - 8*a*c*(f*g^2 + h*(2*e*g + d*
h)))*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/(13440*c^(5/2))
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 320, normalized size of antiderivative = 0.92

method	result
default	$d\,g^2\left(\frac{x\left(c\,x^2+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{c\,x^2+a}}{2}+\frac{a\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{2\sqrt{c}}\right)}{4}\right)+f\,h^2\left(\frac{x^3\left(c\,x^2+a\right)^{\frac{5}{2}}}{8c}-\frac{3a\left(\frac{x\left(c\,x^2+a\right)^{\frac{5}{2}}}{6c}-\frac{a\left(\frac{x\left(c\,x^2+a\right)^{\frac{3}{2}}}{4}+\right.}{}\right.}{}\right.$
risch	$-\frac{\left(-1680c^3 f\,h^2 x^7-1920c^3 e\,h^2 x^6-3840c^3 f g h\,x^6-2520a\,c^2 f\,h^2 x^5-2240c^3 d\,h^2 x^5-4480c^3 e g h\,x^5-2240c^3 f\,g^2 x^5-3072a\,c^2 e\,h^2 x^4-\right.}{}$

```
[In] int((h*x+g)^2*(c*x^2+a)^(3/2)*(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] d*g^2*(1/4*x*(c*x^2+a)^(3/2)+3/4*a*(1/2*x*(c*x^2+a)^(1/2)+1/2*a/c^(1/2)*ln(
x*c^(1/2)+(c*x^2+a)^(1/2))))+f*h^2*(1/8*x^3*(c*x^2+a)^(5/2)/c-3/8*a/c*(1/6*
x*(c*x^2+a)^(5/2)/c-1/6*a/c*(1/4*x*(c*x^2+a)^(3/2)+3/4*a*(1/2*x*(c*x^2+a)^(
1/2)+1/2*a/c^(1/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))))))+(e*h^2+2*f*g*h)*(1/7*x
^2*(c*x^2+a)^(5/2)/c-2/35*a/c^2*(c*x^2+a)^(5/2))+1/5*(2*d*g*h+e*g^2)*(c*x^2
+a)^(5/2)/c+(d*h^2+2*e*g*h+f*g^2)*(1/6*x*(c*x^2+a)^(5/2)/c-1/6*a/c*(1/4*x*(
c*x^2+a)^(3/2)+3/4*a*(1/2*x*(c*x^2+a)^(1/2)+1/2*a/c^(1/2)*ln(x*c^(1/2)+(c*x
^2+a)^(1/2)))))
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 831, normalized size of antiderivative = 2.40

$$\int (g+hx)^2\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)dx=\left[-\frac{105\left(16\,a^3cegh-8\left(6\,a^2c^2d-a^3cf\right)g^2+\left(8\,a^3cd-3\,a^4f\right)h^2\right)\sqrt{c}\log\left(-2\,cx^2-2\,\sqrt{cx^2+a}\sqrt{c}\right.}{}\right.$$

```
[In] integrate((h*x+g)^2*(c*x^2+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] [-1/26880*(105*(16*a^3*c*e*g*h - 8*(6*a^2*c^2*d - a^3*c*f)*g^2 + (8*a^3*c*d
 - 3*a^4*f)*h^2)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) -
2*(1680*c^4*f*h^2*x^7 + 2688*a^2*c^2*e*g^2 - 768*a^3*c*e*h^2 + 1920*(2*c^4*
f*g*h + c^4*e*h^2)*x^6 + 280*(8*c^4*f*g^2 + 16*c^4*e*g*h + (8*c^4*d + 9*a*c
```

```
^3*f)*h^2)*x^5 + 384*(7*c^4*e*g^2 + 8*a*c^3*e*h^2 + 2*(7*c^4*d + 8*a*c^3*f)
*g*h)*x^4 + 70*(112*a*c^3*e*g*h + 8*(6*c^4*d + 7*a*c^3*f)*g^2 + (56*a*c^3*d
 + 3*a^2*c^2*f)*h^2)*x^3 + 768*(7*a^2*c^2*d - 2*a^3*c*f)*g*h + 384*(14*a*c^
3*e*g^2 + a^2*c^2*e*h^2 + 2*(14*a*c^3*d + a^2*c^2*f)*g*h)*x^2 + 105*(16*a^2
*c^2*e*g*h + 8*(10*a*c^3*d + a^2*c^2*f)*g^2 + (8*a^2*c^2*d - 3*a^3*c*f)*h^2
)*x)*sqrt(c*x^2 + a))/c^3, 1/13440*(105*(16*a^3*c*e*g*h - 8*(6*a^2*c^2*d -
a^3*c*f)*g^2 + (8*a^3*c*d - 3*a^4*f)*h^2)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c
*x^2 + a)) + (1680*c^4*f*h^2*x^7 + 2688*a^2*c^2*e*g^2 - 768*a^3*c*e*h^2 + 1
920*(2*c^4*f*g*h + c^4*e*h^2)*x^6 + 280*(8*c^4*f*g^2 + 16*c^4*e*g*h + (8*c^
4*d + 9*a*c^3*f)*h^2)*x^5 + 384*(7*c^4*e*g^2 + 8*a*c^3*e*h^2 + 2*(7*c^4*d +
 8*a*c^3*f)*g*h)*x^4 + 70*(112*a*c^3*e*g*h + 8*(6*c^4*d + 7*a*c^3*f)*g^2 +
(56*a*c^3*d + 3*a^2*c^2*f)*h^2)*x^3 + 768*(7*a^2*c^2*d - 2*a^3*c*f)*g*h + 3
84*(14*a*c^3*e*g^2 + a^2*c^2*e*h^2 + 2*(14*a*c^3*d + a^2*c^2*f)*g*h)*x^2 +
105*(16*a^2*c^2*e*g*h + 8*(10*a*c^3*d + a^2*c^2*f)*g^2 + (8*a^2*c^2*d - 3*a
^3*c*f)*h^2)*x)*sqrt(c*x^2 + a))/c^3]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 933 vs. $2(330) = 660$.

Time = 0.64 (sec) , antiderivative size = 933, normalized size of antiderivative = 2.70

$$\int (g+hx)^2\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)\,dx=\begin{cases}\sqrt{a+cx^2}\left(\frac{cfh^2x^7}{8}+\frac{x^6\left(c^2eh^2+2c^2fgh\right)}{7c}+\frac{x^5\cdot\left(\frac{9acfh^2}{8}+c^2dh^2+2c^2egh+c^2fg^2\right)}{6c}+\frac{x^4\cdot\left(2aceh^2+4acfgh-\frac{6a\left(c^2eh^2+2\right.}{7c}\right.}{5c}\right. \\ a^{\frac{3}{2}}\left(dg^2x+\frac{fh^2x^5}{5}+\frac{x^4\left(eh^2+2fgh\right)}{4}+\frac{x^3\left(dh^2+2egh+fg^2\right)}{3}+\frac{x^2\cdot\left(2dgh+eg^2\right)}{2}\right)\end{cases}$$

```
[In] integrate((h*x+g)**2*(c*x**2+a)**(3/2)*(f*x**2+e*x+d),x)
```

```
[Out] Piecewise((sqrt(a + c*x**2)*(c*f*h**2*x**7/8 + x**6*(c**2*e*h**2 + 2*c**2*f
*g*h)/(7*c) + x**5*(9*a*c*f*h**2/8 + c**2*d*h**2 + 2*c**2*e*g*h + c**2*f*g*
*2)/(6*c) + x**4*(2*a*c*e*h**2 + 4*a*c*f*g*h - 6*a*(c**2*e*h**2 + 2*c**2*f*
g*h)/(7*c) + 2*c**2*d*g*h + c**2*e*g**2)/(5*c) + x**3*(a**2*f*h**2 + 2*a*c*
d*h**2 + 4*a*c*e*g*h + 2*a*c*f*g**2 - 5*a*(9*a*c*f*h**2/8 + c**2*d*h**2 + 2
*c**2*e*g*h + c**2*f*g**2)/(6*c) + c**2*d*g**2)/(4*c) + x**2*(a**2*e*h**2 +
 2*a**2*f*g*h + 4*a*c*d*g*h + 2*a*c*e*g**2 - 4*a*(2*a*c*e*h**2 + 4*a*c*f*g*
h - 6*a*(c**2*e*h**2 + 2*c**2*f*g*h)/(7*c) + 2*c**2*d*g*h + c**2*e*g**2)/(5
```

```
*c))/(3*c) + x*(a**2*d*h**2 + 2*a**2*e*g*h + a**2*f*g**2 + 2*a*c*d*g**2 - 3
*a*(a**2*f*h**2 + 2*a*c*d*h**2 + 4*a*c*e*g*h + 2*a*c*f*g**2 - 5*a*(9*a*c*f*
h**2/8 + c**2*d*h**2 + 2*c**2*e*g*h + c**2*f*g**2)/(6*c) + c**2*d*g**2)/(4*
c))/(2*c) + (2*a**2*d*g*h + a**2*e*g**2 - 2*a*(a**2*e*h**2 + 2*a**2*f*g*h +
 4*a*c*d*g*h + 2*a*c*e*g**2 - 4*a*(2*a*c*e*h**2 + 4*a*c*f*g*h - 6*a*(c**2*e
*h**2 + 2*c**2*f*g*h)/(7*c) + 2*c**2*d*g*h + c**2*e*g**2)/(5*c))/(3*c))/c)
+ (a**2*d*g**2 - a*(a**2*d*h**2 + 2*a**2*e*g*h + a**2*f*g**2 + 2*a*c*d*g**2
 - 3*a*(a**2*f*h**2 + 2*a*c*d*h**2 + 4*a*c*e*g*h + 2*a*c*f*g**2 - 5*a*(9*a*
c*f*h**2/8 + c**2*d*h**2 + 2*c**2*e*g*h + c**2*f*g**2)/(6*c) + c**2*d*g**2)
/(4*c))/(2*c))*Piecewise((log(2*sqrt(c)*sqrt(a + c*x**2) + 2*c*x)/sqrt(c),
Ne(a, 0)), (x*log(x)/sqrt(c*x**2), True)), Ne(c, 0)), (a**(3/2)*(d*g**2*x +
 f*h**2*x**5/5 + x**4*(e*h**2 + 2*f*g*h)/4 + x**3*(d*h**2 + 2*e*g*h + f*g**
2)/3 + x**2*(2*d*g*h + e*g**2)/2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 380, normalized size of antiderivative = 1.10

$$
\begin{aligned}
\int (g+hx)^2\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)\,dx &= \frac{\left(cx^2+a\right)^{\frac{5}{2}}fh^2x^3}{8\,c}+\frac{1}{4}\left(cx^2+a\right)^{\frac{3}{2}}dg^2x \\
&+\frac{3}{8}\sqrt{cx^2+a}adg^2x-\frac{\left(cx^2+a\right)^{\frac{5}{2}}afh^2x}{16\,c^2}+\frac{\left(cx^2+a\right)^{\frac{3}{2}}a^2fh^2x}{64\,c^2}+\frac{3\sqrt{cx^2+a}a^3fh^2x}{128\,c^2} \\
&+\frac{3\,a^2dg^2\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{8\sqrt{c}}+\frac{3\,a^4fh^2\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{128\,c^{\frac{5}{2}}}+\frac{\left(cx^2+a\right)^{\frac{5}{2}}eg^2}{5\,c}+\frac{2\left(cx^2+a\right)^{\frac{5}{2}}dgh}{5\,c} \\
&+\frac{(2\,fgh+eh^2)(cx^2+a)^{\frac{5}{2}}x^2}{7\,c}+\frac{(fg^2+2\,egh+dh^2)(cx^2+a)^{\frac{5}{2}}x}{6\,c} \\
&-\frac{(fg^2+2\,egh+dh^2)(cx^2+a)^{\frac{3}{2}}ax}{24\,c}-\frac{(fg^2+2\,egh+dh^2)\sqrt{cx^2+a}a^2x}{16\,c} \\
&-\frac{(fg^2+2\,egh+dh^2)a^3\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{16\,c^{\frac{3}{2}}}-\frac{2\,(2\,fgh+eh^2)(cx^2+a)^{\frac{5}{2}}a}{35\,c^2}
\end{aligned}
$$

```
[In] integrate((h*x+g)^2*(c*x^2+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] 1/8*(c*x^2 + a)^(5/2)*f*h^2*x^3/c + 1/4*(c*x^2 + a)^(3/2)*d*g^2*x + 3/8*sqr
t(c*x^2 + a)*a*d*g^2*x - 1/16*(c*x^2 + a)^(5/2)*a*f*h^2*x/c^2 + 1/64*(c*x^2
 + a)^(3/2)*a^2*f*h^2*x/c^2 + 3/128*sqrt(c*x^2 + a)*a^3*f*h^2*x/c^2 + 3/8*a
^2*d*g^2*arcsinh(c*x/sqrt(a*c))/sqrt(c) + 3/128*a^4*f*h^2*arcsinh(c*x/sqrt(
a*c))/c^(5/2) + 1/5*(c*x^2 + a)^(5/2)*e*g^2/c + 2/5*(c*x^2 + a)^(5/2)*d*g*h
/c + 1/7*(2*f*g*h + e*h^2)*(c*x^2 + a)^(5/2)*x^2/c + 1/6*(f*g^2 + 2*e*g*h +
 d*h^2)*(c*x^2 + a)^(5/2)*x/c - 1/24*(f*g^2 + 2*e*g*h + d*h^2)*(c*x^2 + a)^
(3/2)*a*x/c - 1/16*(f*g^2 + 2*e*g*h + d*h^2)*sqrt(c*x^2 + a)*a^2*x/c - 1/16
```

```
*(f*g^2 + 2*e*g*h + d*h^2)*a^3*arcsinh(c*x/sqrt(a*c))/c^(3/2) - 2/35*(2*f*g
*h + e*h^2)*(c*x^2 + a)^(5/2)*a/c^2
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.27

$$\int (g+hx)^2 \left(a+cx^2\right)^{3/2} (d+ex$$

$$+fx^2) \, dx = \frac{1}{13440} \sqrt{cx^2+a} ((2 (((4 (5 (6 \left(7\,cfh^2x + \frac{8\,(2\,c^7fgh + c^7eh^2)}{c^6} \right) x + \frac{7\,(8\,c^7fg^2 + 16\,c^7egh + 8}{c^6}$$

$$- \frac{(48\,a^2c^2dg^2 - 8\,a^3cfg^2 - 16\,a^3cegh - 8\,a^3cdh^2 + 3\,a^4fh^2) \log\left(\left|-\sqrt{c}x + \sqrt{cx^2+a}\right|\right)}{128\,c^{\frac{5}{2}}}$$

```
[In] integrate((h*x+g)^2*(c*x^2+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] 1/13440*sqrt(c*x^2 + a)*((2*((4*(5*(6*(7*c*f*h^2*x + 8*(2*c^7*f*g*h + c^7*e
*h^2)/c^6)*x + 7*(8*c^7*f*g^2 + 16*c^7*e*g*h + 8*c^7*d*h^2 + 9*a*c^6*f*h^2)
/c^6)*x + 48*(7*c^7*e*g^2 + 14*c^7*d*g*h + 16*a*c^6*f*g*h + 8*a*c^6*e*h^2)/
c^6)*x + 35*(48*c^7*d*g^2 + 56*a*c^6*f*g^2 + 112*a*c^6*e*g*h + 56*a*c^6*d*h
^2 + 3*a^2*c^5*f*h^2)/c^6)*x + 192*(14*a*c^6*e*g^2 + 28*a*c^6*d*g*h + 2*a^2
*c^5*f*g*h + a^2*c^5*e*h^2)/c^6)*x + 105*(80*a*c^6*d*g^2 + 8*a^2*c^5*f*g^2
+ 16*a^2*c^5*e*g*h + 8*a^2*c^5*d*h^2 - 3*a^3*c^4*f*h^2)/c^6)*x + 384*(7*a^2
*c^5*e*g^2 + 14*a^2*c^5*d*g*h - 4*a^3*c^4*f*g*h - 2*a^3*c^4*e*h^2)/c^6) - 1
/128*(48*a^2*c^2*d*g^2 - 8*a^3*c*f*g^2 - 16*a^3*c*e*g*h - 8*a^3*c*d*h^2 + 3
*a^4*f*h^2)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int (g+hx)^2 \left(a+cx^2\right)^{3/2} \left(d+ex+fx^2\right) \, dx = \int (g+h\,x)^2\,\left(c\,x^2+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right) \, dx$$

```
[In] int((g + h*x)^2*(a + c*x^2)^(3/2)*(d + e*x + f*x^2),x)
```

```
[Out] int((g + h*x)^2*(a + c*x^2)^(3/2)*(d + e*x + f*x^2), x)
```

3.90 $\int (g+hx)\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)\,dx$

Optimal result

Integrand size = 27, antiderivative size = 213

$$\begin{aligned}\int (g+hx)\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)\,dx &= \frac{a(6cdg-afg-aeh)x\sqrt{a+cx^2}}{16c}\\ &+\frac{(6cdg-a(fg+eh))x(a+cx^2)^{3/2}}{24c}+\frac{f(g+hx)^2\left(a+cx^2\right)^{5/2}}{7ch}\\ &-\frac{(6(2afh^2+c(5fg^2-7h(eg+dh)))+5ch(5fg-7eh)x)\left(a+cx^2\right)^{5/2}}{210c^2h}\\ &+\frac{a^2(6cdg-afg-aeh)\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{16c^{3/2}}\end{aligned}$$

```
[Out] 1/24*(6*c*d*g-a*(e*h+f*g))*x*(c*x^2+a)^(3/2)/c+1/7*f*(h*x+g)^2*(c*x^2+a)^(5
/2)/c/h-1/210*(12*a*f*h^2+6*c*(5*f*g^2-7*h*(d*h+e*g))+5*c*h*(-7*e*h+5*f*g)*
x)*(c*x^2+a)^(5/2)/c^2/h+1/16*a^2*(-a*e*h-a*f*g+6*c*d*g)*arctanh(x*c^(1/2)/
(c*x^2+a)^(1/2))/c^(3/2)+1/16*a*(-a*e*h-a*f*g+6*c*d*g)*x*(c*x^2+a)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used

= {1668, 794, 201, 223, 212}

$$\int (g+hx)\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)\,dx = \frac{a^2\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)(-aeh-afg+6cdg)}{16c^{3/2}} - \frac{\left(a+cx^2\right)^{5/2}\left(6(2afh^2-7ch(dh+eg)+5cfg^2)+5chx(5fg-7eh)\right)}{210c^2h} + \frac{x\left(a+cx^2\right)^{3/2}(6cdg-a(eh+fg))}{24c} + \frac{ax\sqrt{a+cx^2}(-aeh-afg+6cdg)}{16c} + \frac{f\left(a+cx^2\right)^{5/2}(g+hx)^2}{7ch}$$

```
[In] Int[(g + h*x)*(a + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

```
[Out] (a*(6*c*d*g - a*f*g - a*e*h)*x*Sqrt[a + c*x^2])/(16*c) + ((6*c*d*g - a*(f*g
 + e*h))*x*(a + c*x^2)^(3/2))/(24*c) + (f*(g + h*x)^2*(a + c*x^2)^(5/2))/(7
*c*h) - ((6*(5*c*f*g^2 + 2*a*f*h^2 - 7*c*h*(e*g + d*h)) + 5*c*h*(5*f*g - 7*
e*h)*x)*(a + c*x^2)^(5/2))/(210*c^2*h) + (a^2*(6*c*d*g - a*f*g - a*e*h)*Arc
Tanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(16*c^(3/2))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{f(g+hx)^2\left(a+cx^2\right)^{5/2}}{7ch} \\
&\quad+\frac{\int (g+hx)\left((7cd-2af)h^2-ch(5fg-7eh)x\right)\left(a+cx^2\right)^{3/2}\,dx}{7ch^2} \\
&= \frac{f(g+hx)^2\left(a+cx^2\right)^{5/2}}{7ch} \\
&\quad-\frac{\left(6(5cfg^2+2afh^2-7ch(eg+dh))+5ch(5fg-7eh)x\right)\left(a+cx^2\right)^{5/2}}{210c^2h} \\
&\quad+\frac{(6cdg-afg-aeh)\int \left(a+cx^2\right)^{3/2}\,dx}{6c} \\
&= \frac{(6cdg-a(fg+eh))x\left(a+cx^2\right)^{3/2}}{24c}+\frac{f(g+hx)^2\left(a+cx^2\right)^{5/2}}{7ch} \\
&\quad-\frac{\left(6(5cfg^2+2afh^2-7ch(eg+dh))+5ch(5fg-7eh)x\right)\left(a+cx^2\right)^{5/2}}{210c^2h} \\
&\quad+\frac{(a(6cdg-afg-aeh))\int \sqrt{a+cx^2}\,dx}{8c} \\
&= \frac{a(6cdg-afg-aeh)x\sqrt{a+cx^2}}{16c} \\
&\quad+\frac{(6cdg-a(fg+eh))x\left(a+cx^2\right)^{3/2}}{24c}+\frac{f(g+hx)^2\left(a+cx^2\right)^{5/2}}{7ch} \\
&\quad-\frac{\left(6(5cfg^2+2afh^2-7ch(eg+dh))+5ch(5fg-7eh)x\right)\left(a+cx^2\right)^{5/2}}{210c^2h} \\
&\quad+\frac{\left(a^2(6cdg-afg-aeh)\right)\int \frac{1}{\sqrt{a+cx^2}}\,dx}{16c}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{a(6cdg-afg-aeh)x\sqrt{a+cx^2}}{16c} \\
&\quad + \frac{(6cdg-a(fg+eh))x(a+cx^2)^{3/2}}{24c} + \frac{f(g+hx)^2\left(a+cx^2\right)^{5/2}}{7ch} \\
&\quad - \frac{\left(6(5cfg^2+2afh^2-7ch(eg+dh))+5ch(5fg-7eh)x\right)\left(a+cx^2\right)^{5/2}}{210c^2h} \\
&\quad + \frac{\left(a^2(6cdg-afg-aeh)\right)\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{16c} \\
&= \frac{a(6cdg-afg-aeh)x\sqrt{a+cx^2}}{16c} \\
&\quad + \frac{(6cdg-a(fg+eh))x(a+cx^2)^{3/2}}{24c} + \frac{f(g+hx)^2\left(a+cx^2\right)^{5/2}}{7ch} \\
&\quad - \frac{\left(6(5cfg^2+2afh^2-7ch(eg+dh))+5ch(5fg-7eh)x\right)\left(a+cx^2\right)^{5/2}}{210c^2h} \\
&\quad + \frac{a^2(6cdg-afg-aeh)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{16c^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.92

$$
\int (g+hx)\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)\,dx = \frac{\sqrt{a+cx^2}(-96a^3fh+3a^2c(112dh+7e(16g+5hx)+fx(35g+16hx))+4c^3x^3(21d(5g+4hx)}{}
$$

```
[In] Integrate[(g + h*x)*(a + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

```
[Out] (Sqrt[a + c*x^2]*(-96*a^3*f*h + 3*a^2*c*(112*d*h + 7*e*(16*g + 5*h*x) + f*x
*(35*g + 16*h*x)) + 4*c^3*x^3*(21*d*(5*g + 4*h*x) + 2*x*(7*e*(6*g + 5*h*x)
+ 5*f*x*(7*g + 6*h*x))) + 2*a*c^2*x*(21*d*(25*g + 16*h*x) + x*(7*e*(48*g +
35*h*x) + f*x*(245*g + 192*h*x)))) + 105*a^2*Sqrt[c]*(-6*c*d*g + a*f*g + a*
e*h)*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/(1680*c^2)
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.91

method	result
default	$dg\left(\frac{x(cx^2+a)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{cx^2+a}}{2}+\frac{a\ln\left(x\sqrt{c}+\sqrt{cx^2+a}\right)}{2\sqrt{c}}\right)}{4}\right)+hf\left(\frac{x^2(cx^2+a)^{\frac{5}{2}}}{7c}-\frac{2a(cx^2+a)^{\frac{5}{2}}}{35c^2}\right)+(eh+fg)\Bigg($
risch	$-\frac{(-240c^3fhx^6-280c^3ehx^5-280c^3fgx^5-384ac^2fhx^4-336c^3dhx^4-336c^3egx^4-490ac^2ehx^3-490ac^2fgx^3-420c^3dgx^3-48a^2}{1680c^2}$

```
[In] int((h*x+g)*(c*x^2+a)^(3/2)*(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] d*g*(1/4*x*(c*x^2+a)^(3/2)+3/4*a*(1/2*x*(c*x^2+a)^(1/2)+1/2*a/c^(1/2)*ln(x*
c^(1/2)+(c*x^2+a)^(1/2))))+h*f*(1/7*x^2*(c*x^2+a)^(5/2)/c-2/35*a/c^2*(c*x^2
+a)^(5/2))+(e*h+f*g)*(1/6*x*(c*x^2+a)^(5/2)/c-1/6*a/c*(1/4*x*(c*x^2+a)^(3/2
)+3/4*a*(1/2*x*(c*x^2+a)^(1/2)+1/2*a/c^(1/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2)))
))+1/5*(d*h+e*g)*(c*x^2+a)^(5/2)/c
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 477, normalized size of antiderivative = 2.24

$$\int (g+hx)\left(a+cx^2\right)^{3/2}(d+ex+fx^2)\,dx = \Bigg[\frac{105\,(a^3eh-(6\,a^2cd-a^3f)g)\sqrt{c}\log\left(-2\,cx^2+2\,\sqrt{cx^2+a}\sqrt{c}x-a\right)+2\,(240\,c^3fhx^6+280\,(c^3}{}$$

```
[In] integrate((h*x+g)*(c*x^2+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] [1/3360*(105*(a^3*e*h - (6*a^2*c*d - a^3*f)*g)*sqrt(c)*log(-2*c*x^2 + 2*sqr
t(c*x^2 + a)*sqrt(c)*x - a) + 2*(240*c^3*f*h*x^6 + 280*(c^3*f*g + c^3*e*h)*
x^5 + 336*a^2*c*e*g + 48*(7*c^3*e*g + (7*c^3*d + 8*a*c^2*f)*h)*x^4 + 70*(7*
a*c^2*e*h + (6*c^3*d + 7*a*c^2*f)*g)*x^3 + 48*(14*a*c^2*e*g + (14*a*c^2*d +
 a^2*c*f)*h)*x^2 + 48*(7*a^2*c*d - 2*a^3*f)*h + 105*(a^2*c*e*h + (10*a*c^2*
d + a^2*c*f)*g)*x)*sqrt(c*x^2 + a))/c^2, 1/1680*(105*(a^3*e*h - (6*a^2*c*d
- a^3*f)*g)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) + (240*c^3*f*h*x^6
+ 280*(c^3*f*g + c^3*e*h)*x^5 + 336*a^2*c*e*g + 48*(7*c^3*e*g + (7*c^3*d +
8*a*c^2*f)*h)*x^4 + 70*(7*a*c^2*e*h + (6*c^3*d + 7*a*c^2*f)*g)*x^3 + 48*(14
*a*c^2*e*g + (14*a*c^2*d + a^2*c*f)*h)*x^2 + 48*(7*a^2*c*d - 2*a^3*f)*h + 1
05*(a^2*c*e*h + (10*a*c^2*d + a^2*c*f)*g)*x)*sqrt(c*x^2 + a))/c^2]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 507 vs. $2(196) = 392$.

Time = 0.59 (sec) , antiderivative size = 507, normalized size of antiderivative = 2.38

$$\int (g+hx)\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)\,dx = \begin{cases} \sqrt{a+cx^2}\left(\frac{cfhx^6}{7} + \frac{x^5\left(c^2eh+c^2fg\right)}{6c} + \frac{x^4\cdot\left(\frac{8acfh}{7}+c^2dh+c^2eg\right)}{5c} + \frac{x^3\cdot\left(2aceh+2acfg-\frac{5a\left(c^2eh+c^2fg\right)}{6c}+c^2dg\right)}{4c} + \right. \\ a^{\frac{3}{2}}\left(dgx+\frac{fhx^4}{4}+\frac{x^3(eh+fg)}{3}+\frac{x^2(dh+eg)}{2}\right) \end{cases}$$

```
[In] integrate((h*x+g)*(c*x**2+a)**(3/2)*(f*x**2+e*x+d),x)
```

```
[Out] Piecewise((sqrt(a + c*x**2)*(c*f*h*x**6/7 + x**5*(c**2*e*h + c**2*f*g)/(6*c
) + x**4*(8*a*c*f*h/7 + c**2*d*h + c**2*e*g)/(5*c) + x**3*(2*a*c*e*h + 2*a*
c*f*g - 5*a*(c**2*e*h + c**2*f*g)/(6*c) + c**2*d*g)/(4*c) + x**2*(a**2*f*h
+ 2*a*c*d*h + 2*a*c*e*g - 4*a*(8*a*c*f*h/7 + c**2*d*h + c**2*e*g)/(5*c))/(3
*c) + x*(a**2*e*h + a**2*f*g + 2*a*c*d*g - 3*a*(2*a*c*e*h + 2*a*c*f*g - 5*a
*(c**2*e*h + c**2*f*g)/(6*c) + c**2*d*g)/(4*c))/(2*c) + (a**2*d*h + a**2*e*
g - 2*a*(a**2*f*h + 2*a*c*d*h + 2*a*c*e*g - 4*a*(8*a*c*f*h/7 + c**2*d*h + c
**2*e*g)/(5*c))/(3*c))/c) + (a**2*d*g - a*(a**2*e*h + a**2*f*g + 2*a*c*d*g
- 3*a*(2*a*c*e*h + 2*a*c*f*g - 5*a*(c**2*e*h + c**2*f*g)/(6*c) + c**2*d*g)/
(4*c))/(2*c))*Piecewise((log(2*sqrt(c)*sqrt(a + c*x**2) + 2*c*x)/sqrt(c), N
e(a, 0)), (x*log(x)/sqrt(c*x**2), True)), Ne(c, 0)), (a**(3/2)*(d*g*x + f*h
*x**4/4 + x**3*(e*h + f*g)/3 + x**2*(d*h + e*g)/2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.99

$$\begin{aligned}\int (g+hx)\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)\,dx &= \frac{\left(cx^2+a\right)^{\frac{5}{2}}fhx^2}{7\,c} \\ &+ \frac{1}{4}\left(cx^2+a\right)^{\frac{3}{2}}dgx + \frac{3}{8}\sqrt{cx^2+a}\,adgx + \frac{3\,a^2dg\,\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{8\sqrt{c}} \\ &+ \frac{\left(cx^2+a\right)^{\frac{5}{2}}eg}{5\,c} + \frac{\left(cx^2+a\right)^{\frac{5}{2}}dh}{5\,c} - \frac{2\left(cx^2+a\right)^{\frac{5}{2}}afh}{35\,c^2} + \frac{\left(cx^2+a\right)^{\frac{5}{2}}(fg+eh)x}{6\,c} \\ &- \frac{\left(cx^2+a\right)^{\frac{3}{2}}(fg+eh)ax}{24\,c} - \frac{\sqrt{cx^2+a}(fg+eh)a^2x}{16\,c} - \frac{(fg+eh)a^3\,\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{16\,c^{\frac{3}{2}}}\end{aligned}$$

```
[In] integrate((h*x+g)*(c*x^2+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] 1/7*(c*x^2 + a)^(5/2)*f*h*x^2/c + 1/4*(c*x^2 + a)^(3/2)*d*g*x + 3/8*sqrt(c*
x^2 + a)*a*d*g*x + 3/8*a^2*d*g*arcsinh(c*x/sqrt(a*c))/sqrt(c) + 1/5*(c*x^2
+ a)^(5/2)*e*g/c + 1/5*(c*x^2 + a)^(5/2)*d*h/c - 2/35*(c*x^2 + a)^(5/2)*a*f
*h/c^2 + 1/6*(c*x^2 + a)^(5/2)*(f*g + e*h)*x/c - 1/24*(c*x^2 + a)^(3/2)*(f*
g + e*h)*a*x/c - 1/16*sqrt(c*x^2 + a)*(f*g + e*h)*a^2*x/c - 1/16*(f*g + e*h
)*a^3*arcsinh(c*x/sqrt(a*c))/c^(3/2)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.21

$$\int (g+hx)\left(a+cx^2\right)^{3/2}(d+ex+fx^2)\,dx = \frac{1}{1680}\sqrt{cx^2+a}\Bigg(\Bigg(2\Bigg(\Bigg(4\Bigg(5\Bigg(6\,cfhx+\frac{7\,(c^6fg+c^6eh)}{c^5}\Bigg)x+\frac{6\,(7\,c^6eg+7\,c^6dh+8\,ac^5fh)}{c^5}\Bigg)x$$

$$-\frac{(6\,a^2cdg-a^3fg-a^3eh)\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+a}\right|\right)}{16\,c^{\frac{3}{2}}}$$

```
[In] integrate((h*x+g)*(c*x^2+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] 1/1680*sqrt(c*x^2 + a)*((2*((4*(5*(6*c*f*h*x + 7*(c^6*f*g + c^6*e*h)/c^5)*x
 + 6*(7*c^6*e*g + 7*c^6*d*h + 8*a*c^5*f*h)/c^5)*x + 35*(6*c^6*d*g + 7*a*c^5
*f*g + 7*a*c^5*e*h)/c^5)*x + 24*(14*a*c^5*e*g + 14*a*c^5*d*h + a^2*c^4*f*h)
/c^5)*x + 105*(10*a*c^5*d*g + a^2*c^4*f*g + a^2*c^4*e*h)/c^5)*x + 48*(7*a^2
*c^4*e*g + 7*a^2*c^4*d*h - 2*a^3*c^3*f*h)/c^5) - 1/16*(6*a^2*c*d*g - a^3*f*
g - a^3*e*h)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int (g+hx)\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)\,dx = \int (g+h\,x)\,\left(c\,x^2+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right) \,dx$$

```
[In] int((g + h*x)*(a + c*x^2)^(3/2)*(d + e*x + f*x^2),x)
```

```
[Out] int((g + h*x)*(a + c*x^2)^(3/2)*(d + e*x + f*x^2), x)
```

3.91 $\int \left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 137

$$\int \left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)\,dx=\frac{a(6cd-af)x\sqrt{a+cx^2}}{16c}+\frac{(6cd-af)x\left(a+cx^2\right)^{3/2}}{24c}+\frac{e\left(a+cx^2\right)^{5/2}}{5c}+\frac{fx\left(a+cx^2\right)^{5/2}}{6c}+\frac{a^2(6cd-af)\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{16c^{3/2}}$$

```
[Out] 1/24*(-a*f+6*c*d)*x*(c*x^2+a)^(3/2)/c+1/5*e*(c*x^2+a)^(5/2)/c+1/6*f*x*(c*x^
2+a)^(5/2)/c+1/16*a^2*(-a*f+6*c*d)*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(3/
2)+1/16*a*(-a*f+6*c*d)*x*(c*x^2+a)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1829, 655, 201, 223, 212}

$$\int \left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)\,dx=\frac{a^2\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)(6cd-af)}{16c^{3/2}}+\frac{x\left(a+cx^2\right)^{3/2}(6cd-af)}{24c}+\frac{ax\sqrt{a+cx^2}(6cd-af)}{16c}+\frac{e\left(a+cx^2\right)^{5/2}}{5c}+\frac{fx\left(a+cx^2\right)^{5/2}}{6c}$$

```
[In] Int[(a + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

```
[Out] (a*(6*c*d - a*f)*x*Sqrt[a + c*x^2])/(16*c) + ((6*c*d - a*f)*x*(a + c*x^2)^(
3/2))/(24*c) + (e*(a + c*x^2)^(5/2))/(5*c) + (f*x*(a + c*x^2)^(5/2))/(6*c)
 + (a^2*(6*c*d - a*f)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(16*c^(3/2))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1829

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{fx(a+cx^2)^{5/2}}{6c} + \frac{\int (6cd-af+6cex)\,(a+cx^2)^{3/2}\,dx}{6c} \\
&= \frac{e(a+cx^2)^{5/2}}{5c} + \frac{fx(a+cx^2)^{5/2}}{6c} + \frac{(6cd-af)\int (a+cx^2)^{3/2}\,dx}{6c} \\
&= \frac{(6cd-af)x(a+cx^2)^{3/2}}{24c} + \frac{e(a+cx^2)^{5/2}}{5c} + \frac{fx(a+cx^2)^{5/2}}{6c} + \frac{(a(6cd-af))\int \sqrt{a+cx^2}\,dx}{8c} \\
&= \frac{a(6cd-af)x\sqrt{a+cx^2}}{16c} + \frac{(6cd-af)x(a+cx^2)^{3/2}}{24c} \\
&\quad + \frac{e(a+cx^2)^{5/2}}{5c} + \frac{fx(a+cx^2)^{5/2}}{6c} + \frac{(a^2(6cd-af))\int \frac{1}{\sqrt{a+cx^2}}\,dx}{16c}
\end{aligned}
$$

$$= \frac{a(6cd-af)x\sqrt{a+cx^2}}{16c} + \frac{(6cd-af)x\left(a+cx^2\right)^{3/2}}{24c} + \frac{e\left(a+cx^2\right)^{5/2}}{5c} + \frac{fx\left(a+cx^2\right)^{5/2}}{6c} + \frac{\left(a^2(6cd-af)\right)\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{16c}$$

$$= \frac{a(6cd-af)x\sqrt{a+cx^2}}{16c} + \frac{(6cd-af)x\left(a+cx^2\right)^{3/2}}{24c} + \frac{e\left(a+cx^2\right)^{5/2}}{5c} + \frac{fx\left(a+cx^2\right)^{5/2}}{6c} + \frac{a^2(6cd-af)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{16c^{3/2}}$$

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.85

$$\int \left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)\,dx = \frac{\sqrt{c}\sqrt{a+cx^2}\left(3a^2(16e+5fx)+4c^2x^3(15d+2x(6e+5fx))+2acx(75d+x(48e+35fx))\right)+15}{240c^{3/2}}$$

```
[In] Integrate[(a + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

```
[Out] (Sqrt[c]*Sqrt[a + c*x^2]*(3*a^2*(16*e + 5*f*x) + 4*c^2*x^3*(15*d + 2*x*(6*e
 + 5*f*x)) + 2*a*c*x*(75*d + x*(48*e + 35*f*x))) + 15*a^2*(-6*c*d + a*f)*Lo
g[-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/(240*c^(3/2))
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.81

method	result
risch	$\frac{\left(40c^2fx^5+48ex^4c^2+70acfx^3+60c^2dx^3+96acex^2+15a^2fx+150acdx+48a^2e\right)\sqrt{cx^2+a}}{240c}-\frac{a^2(fa-6cd)\ln\left(x\sqrt{c}+\sqrt{cx^2+a}\right)}{16c^{\frac{3}{2}}}$
default	$d\left(\frac{x\left(cx^2+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{cx^2+a}}{2}+\frac{a\ln\left(x\sqrt{c}+\sqrt{cx^2+a}\right)}{2\sqrt{c}}\right)}{4}\right)+f\left(\frac{x\left(cx^2+a\right)^{\frac{5}{2}}}{6c}-\frac{a\left(\frac{x\left(cx^2+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{cx^2+a}}{2}+\frac{a\ln\left(x\sqrt{c}+\sqrt{}\right.}{2\sqrt{c}}\right.}{4}\right.}{6c}\right.$

```
[In] int((c*x^2+a)^(3/2)*(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/240/c*(40*c^2*f*x^5+48*c^2*e*x^4+70*a*c*f*x^3+60*c^2*d*x^3+96*a*c*e*x^2+1
5*a^2*f*x+150*a*c*d*x+48*a^2*e)*(c*x^2+a)^(1/2)-1/16*a^2*(a*f-6*c*d)/c^(3/2
)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.91

$$\int \left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)\,dx = \left[-\frac{15\left(6a^2cd-a^3f\right)\sqrt{c}\log\left(-2cx^2+2\sqrt{cx^2+a}\sqrt{c}x-a\right)-2\left(40c^3fx^5+48c^3ex^4+96ac^2ex}{480c^2}\right.$$

$$-\frac{15\left(6a^2cd-a^3f\right)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}x}{\sqrt{cx^2+a}}\right)-\left(40c^3fx^5+48c^3ex^4+96ac^2ex^2+48a^2ce+10\left(6c^3d+7ac^2f\right.\right.}{240c^2}$$

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] [-1/480*(15*(6*a^2*c*d - a^3*f)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sq
rt(c)*x - a) - 2*(40*c^3*f*x^5 + 48*c^3*e*x^4 + 96*a*c^2*e*x^2 + 48*a^2*c*e
 + 10*(6*c^3*d + 7*a*c^2*f)*x^3 + 15*(10*a*c^2*d + a^2*c*f)*x)*sqrt(c*x^2 +
 a))/c^2, -1/240*(15*(6*a^2*c*d - a^3*f)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*
x^2 + a)) - (40*c^3*f*x^5 + 48*c^3*e*x^4 + 96*a*c^2*e*x^2 + 48*a^2*c*e + 10
*(6*c^3*d + 7*a*c^2*f)*x^3 + 15*(10*a*c^2*d + a^2*c*f)*x)*sqrt(c*x^2 + a))/
c^2]
```

Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.53

$$\int \left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)\,dx = \begin{cases} \sqrt{a+cx^2}\left(\frac{a^2e}{5c}+\frac{2aex^2}{5}+\frac{cex^4}{5}+\frac{cfx^5}{6}+\frac{x^3\cdot\left(\frac{7acf}{6}+c^2d\right)}{4c}+\frac{x\left(a^2f+2acd-\frac{3a\left(\frac{7acf}{6}+c^2d\right)}{4c}\right)}{2c}\right)+\left(a^2d-\frac{a\left(\right.}{} \right. \\ a^{\frac{3}{2}}\left(dx+\frac{ex^2}{2}+\frac{fx^3}{3}\right) \end{cases}$$

```
[In] integrate((c*x**2+a)**(3/2)*(f*x**2+e*x+d),x)
```

```
[Out] Piecewise((sqrt(a + c*x**2)*(a**2*e/(5*c) + 2*a*e*x**2/5 + c*e*x**4/5 + c*f
*x**5/6 + x**3*(7*a*c*f/6 + c**2*d)/(4*c) + x*(a**2*f + 2*a*c*d - 3*a*(7*a*
c*f/6 + c**2*d)/(4*c))/(2*c)) + (a**2*d - a*(a**2*f + 2*a*c*d - 3*a*(7*a*c*
f/6 + c**2*d)/(4*c))/(2*c))*Piecewise((log(2*sqrt(c)*sqrt(a + c*x**2) + 2*c
*x)/sqrt(c), Ne(a, 0)), (x*log(x)/sqrt(c*x**2), True)), Ne(c, 0)), (a**(3/2
)*(d*x + e*x**2/2 + f*x**3/3), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.96

$$\int \left(a+cx^{2}\right)^{3/2}\left(d+ex+fx^{2}\right)\,dx=\frac{1}{4}\left(cx^{2}+a\right)^{\frac{3}{2}}dx+\frac{3}{8}\sqrt{cx^{2}+a}adx+\frac{\left(cx^{2}+a\right)^{\frac{5}{2}}fx}{6\,c}-\frac{\left(cx^{2}+a\right)^{\frac{3}{2}}afx}{24\,c}-\frac{\sqrt{cx^{2}+a}a^{2}fx}{16\,c}+\frac{3\,a^{2}d\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{8\,\sqrt{c}}-\frac{a^{3}f\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{16\,c^{\frac{3}{2}}}+\frac{\left(cx^{2}+a\right)^{\frac{5}{2}}e}{5\,c}$$

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] 1/4*(c*x^2 + a)^(3/2)*d*x + 3/8*sqrt(c*x^2 + a)*a*d*x + 1/6*(c*x^2 + a)^(5/
2)*f*x/c - 1/24*(c*x^2 + a)^(3/2)*a*f*x/c - 1/16*sqrt(c*x^2 + a)*a^2*f*x/c
+ 3/8*a^2*d*arcsinh(c*x/sqrt(a*c))/sqrt(c) - 1/16*a^3*f*arcsinh(c*x/sqrt(a*
c))/c^(3/2) + 1/5*(c*x^2 + a)^(5/2)*e/c
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.92

$$\int \left(a+cx^{2}\right)^{3/2}\left(d+ex+fx^{2}\right)\,dx=\frac{1}{240}\,\sqrt{cx^{2}+a}\Bigg(\frac{48\,a^{2}e}{c}+\Bigg(2\,\Bigg(48\,ae+\Bigg(4\,(5\,cfx+6\,ce)x+\frac{5\,(6\,c^{5}d+7\,ac^{4}f)}{c^{4}}\Bigg)x\Bigg)x+\frac{15\,(10\,ac^{4}}{c}-\frac{(6\,a^{2}cd-a^{3}f)\log\left(\left|-\sqrt{c}x+\sqrt{cx^{2}+a}\right|\right)}{16\,c^{\frac{3}{2}}}$$

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] 1/240*sqrt(c*x^2 + a)*(48*a^2*e/c + (2*(48*a*e + (4*(5*c*f*x + 6*c*e)*x + 5
*(6*c^5*d + 7*a*c^4*f)/c^4)*x)*x + 15*(10*a*c^4*d + a^2*c^3*f)/c^4)*x) - 1/
16*(6*a^2*c*d - a^3*f)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)\,dx=\int \left(c\,x^2+a\right)^{3/2}\left(f\,x^2+e\,x+d\right)\,dx$$

```
[In] int((a + c*x^2)^(3/2)*(d + e*x + f*x^2),x)
[Out] int((a + c*x^2)^(3/2)*(d + e*x + f*x^2), x)
```

3.92 $\int \frac{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{g+hx}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 326

$$\int \frac{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{g+hx}\,dx=\frac{\left(8\left(cg^2+ah^2\right)\left(fg^2-egh+dh^2\right)-h\left(4cdgh^2+(fg-eh)\left(4cg^2+3ah^2\right)\right)}{8h^5}$$

$$+\frac{\left(4\left(fg^2-egh+dh^2\right)-3h(fg-eh)x\right)\left(a+cx^2\right)^{3/2}}{12h^3}+\frac{f\left(a+cx^2\right)^{5/2}}{5ch}$$

$$-\frac{\left(3a^2h^4(fg-eh)+8c^2g^3\left(fg^2-h(eg-dh)\right)+12acgh^2\left(fg^2-h(eg-dh)\right)\right)\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{8\sqrt{c}h^6}$$

$$-\frac{\left(cg^2+ah^2\right)^{3/2}\left(fg^2-egh+dh^2\right)\operatorname{arctanh}\left(\frac{ah-cgx}{\sqrt{cg^2+ah^2}\sqrt{a+cx^2}}\right)}{h^6}$$

```
[Out] 1/12*(4*d*h^2-4*e*g*h+4*f*g^2-3*h*(-e*h+f*g)*x)*(c*x^2+a)^(3/2)/h^3+1/5*f*(
c*x^2+a)^(5/2)/c/h-(a*h^2+c*g^2)^(3/2)*(d*h^2-e*g*h+f*g^2)*arctanh((-c*g*x+
a*h)/(a*h^2+c*g^2)^(1/2)/(c*x^2+a)^(1/2))/h^6-1/8*(3*a^2*h^4*(-e*h+f*g)+8*c
^2*g^3*(f*g^2-h*(-d*h+e*g))+12*a*c*g*h^2*(f*g^2-h*(-d*h+e*g)))*arctanh(x*c^
(1/2)/(c*x^2+a)^(1/2))/h^6/c^(1/2)+1/8*(8*(a*h^2+c*g^2)*(d*h^2-e*g*h+f*g^2)
-h*(4*c*d*g*h^2+(-e*h+f*g)*(3*a*h^2+4*c*g^2))*x)*(c*x^2+a)^(1/2)/h^5
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1668, 829, 858, 223, 212, 739}

$$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{g+hx}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)(3a^2h^4(fg-eh)+12acgh^2(fg^2-h(eg-dh))+8c^2(fg^5-g^3h(eg-dh)))}{8\sqrt{c}h^6} - \frac{(ah^2+cg^2)^{3/2}(dh^2-egh+fg^2)\operatorname{arctanh}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)}{h^6} + \frac{\sqrt{a+cx^2}(8(ah^2+cg^2)(dh^2-egh+fg^2)-hx((3ah^2+4cg^2)(fg-eh)+4cdgh^2))}{8h^5} + \frac{(a+cx^2)^{3/2}(4(dh^2-egh+fg^2)-3hx(fg-eh))}{12h^3} + \frac{f(a+cx^2)^{5/2}}{5ch}$$

```
[In] Int[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x),x]
```

```
[Out] ((8*(c*g^2 + a*h^2)*(f*g^2 - e*g*h + d*h^2) - h*(4*c*d*g*h^2 + (f*g - e*h)*
(4*c*g^2 + 3*a*h^2))*x)*Sqrt[a + c*x^2])/(8*h^5) + ((4*(f*g^2 - e*g*h + d*h
^2) - 3*h*(f*g - e*h)*x)*(a + c*x^2)^(3/2))/(12*h^3) + (f*(a + c*x^2)^(5/2)
)/(5*c*h) - ((3*a^2*h^4*(f*g - e*h) + 12*a*c*g*h^2*(f*g^2 - h*(e*g - d*h))
+ 8*c^2*(f*g^5 - g^3*h*(e*g - d*h)))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/
(8*Sqrt[c]*h^6) - ((c*g^2 + a*h^2)^(3/2)*(f*g^2 - e*g*h + d*h^2)*ArcTanh[(a
*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2])])/h^6
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{f\left(a+cx^2\right)^{5/2}}{5ch}+\frac{\int \frac{\left(5cdh^2-5ch(fg-eh)x\right)\left(a+cx^2\right)^{3/2}}{g+hx}\,dx}{5ch^2}\\
&= \frac{\left(4\left(fg^2-egh+dh^2\right)-3h(fg-eh)x\right)\left(a+cx^2\right)^{3/2}}{12h^3}+\frac{f\left(a+cx^2\right)^{5/2}}{5ch}\\
&\quad+\frac{\int \frac{\left(5ac^2h^2\left(fg^2-h(eg-4dh)\right)-5c^2h\left(4cdgh^2+(fg-eh)\left(4cg^2+3ah^2\right)\right)x\right)\sqrt{a+cx^2}}{g+hx}\,dx}{20c^2h^4}
\end{aligned}
$$

$$= \frac{\left(8\left(cg^2+ah^2\right)\left(fg^2-egh+dh^2\right)-h\left(4cdgh^2+(fg-eh)\left(4cg^2+3ah^2\right)\right)x\right)\sqrt{a+cx^2}}{8h^5}$$
$$+\frac{\left(4(fg^2-egh+dh^2)-3h(fg-eh)x\right)\left(a+cx^2\right)^{3/2}}{12h^3}+\frac{f\left(a+cx^2\right)^{5/2}}{5ch}$$
$$+\frac{\int \frac{5ac^3h^2(ah^2(5fg^2-h(5eg-8dh))+4c(fg^4-g^2h(eg-dh)))-5c^3h(3a^2h^4(fg-eh)+12acgh^2(fg^2-h(eg-dh))+8c^2(fg^5-g^3h(eg-}{(g+hx)\sqrt{a+cx^2}}}{40c^3h^6}$$

$$= \frac{\left(8\left(cg^2+ah^2\right)\left(fg^2-egh+dh^2\right)-h\left(4cdgh^2+(fg-eh)\left(4cg^2+3ah^2\right)\right)x\right)\sqrt{a+cx^2}}{8h^5}$$
$$+\frac{\left(4(fg^2-egh+dh^2)-3h(fg-eh)x\right)\left(a+cx^2\right)^{3/2}}{12h^3}+\frac{f\left(a+cx^2\right)^{5/2}}{5ch}$$
$$+\frac{\left(\left(cg^2+ah^2\right)^2\left(fg^2-egh+dh^2\right)\right)\int \frac{1}{(g+hx)\sqrt{a+cx^2}}\,dx}{h^6}$$
$$-\frac{\left(3a^2h^4(fg-eh)+12acgh^2(fg^2-h(eg-dh))+8c^2(fg^5-g^3h(eg-dh))\right)\int \frac{1}{\sqrt{a+cx^2}}\,dx}{8h^6}$$

$$= \frac{\left(8\left(cg^2+ah^2\right)\left(fg^2-egh+dh^2\right)-h\left(4cdgh^2+(fg-eh)\left(4cg^2+3ah^2\right)\right)x\right)\sqrt{a+cx^2}}{8h^5}$$
$$+\frac{\left(4(fg^2-egh+dh^2)-3h(fg-eh)x\right)\left(a+cx^2\right)^{3/2}}{12h^3}+\frac{f\left(a+cx^2\right)^{5/2}}{5ch}$$
$$-\frac{\left(\left(cg^2+ah^2\right)^2\left(fg^2-egh+dh^2\right)\right)\text{Subst}\left(\int \frac{1}{cg^2+ah^2-x^2}\,dx,x,\frac{ah-cgx}{\sqrt{a+cx^2}}\right)}{h^6}$$
$$-\frac{\left(3a^2h^4(fg-eh)+12acgh^2(fg^2-h(eg-dh))+8c^2(fg^5-g^3h(eg-dh))\right)\text{Subst}(\int \frac{1}{1-cx^2}\,dx,}{8h^6}$$

$$= \frac{\left(8\left(cg^2+ah^2\right)\left(fg^2-egh+dh^2\right)-h\left(4cdgh^2+(fg-eh)\left(4cg^2+3ah^2\right)\right)x\right)\sqrt{a+cx^2}}{8h^5}$$
$$+\frac{\left(4(fg^2-egh+dh^2)-3h(fg-eh)x\right)\left(a+cx^2\right)^{3/2}}{12h^3}+\frac{f\left(a+cx^2\right)^{5/2}}{5ch}$$
$$-\frac{\left(3a^2h^4(fg-eh)+12acgh^2(fg^2-h(eg-dh))+8c^2(fg^5-g^3h(eg-dh))\right)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{8\sqrt{c}h^6}$$
$$-\frac{\left(cg^2+ah^2\right)^{3/2}\left(fg^2-egh+dh^2\right)\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{cg^2+ah^2}\sqrt{a+cx^2}}\right)}{h^6}$$

Mathematica [A] (verified)

Time = 1.29 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.10

$$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{g+hx}\,dx = \frac{h\sqrt{a+cx^2}(24a^2fh^4+ach^2(5h(-32eg+32dh+15ehx)+f(160g^2-75ghx+48h^2x^2))+2c^2(f(60g^4-30}{}$$

```
[In] Integrate[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x),x]
```

```
[Out] ((h*Sqrt[a + c*x^2]*(24*a^2*f*h^4 + a*c*h^2*(5*h*(-32*e*g + 32*d*h + 15*e*h
*x) + f*(160*g^2 - 75*g*h*x + 48*h^2*x^2)) + 2*c^2*(f*(60*g^4 - 30*g^3*h*x
+ 20*g^2*h^2*x^2 - 15*g*h^3*x^3 + 12*h^4*x^4) + 5*h*(2*d*h*(6*g^2 - 3*g*h*x
 + 2*h^2*x^2) + e*(-12*g^3 + 6*g^2*h*x - 4*g*h^2*x^2 + 3*h^3*x^3)))))/c - 2
40*(-(c*g^2) - a*h^2)^(3/2)*(f*g^2 + h*(-(e*g) + d*h))*ArcTan[(Sqrt[c]*(g +
 h*x) - h*Sqrt[a + c*x^2])/Sqrt[-(c*g^2) - a*h^2]] + (15*(3*a^2*h^4*(f*g -
e*h) + 12*a*c*g*h^2*(f*g^2 + h*(-(e*g) + d*h)) + 8*c^2*(f*g^5 + g^3*h*(-(e*
g) + d*h)))*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/Sqrt[c])/(120*h^6)
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 563, normalized size of antiderivative = 1.73

method	result
risch	$\frac{(24c^2fh^4x^4+30c^2eh^4x^3-30c^2fgh^3x^3+48acfh^4x^2+40c^2dh^4x^2-40c^2egh^3x^2+40c^2fg^2h^2x^2+75aceh^4x-75acfgh^3x-60c^2dgh^3x-}{120ch^5}$
default	$\frac{eh\left(\frac{x(cx^2+a)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{cx^2+a}}{2}+\frac{a\ln\left(x\sqrt{c}+\sqrt{cx^2+a}\right)}{2\sqrt{c}}\right)}{4}\right)+\frac{fh(cx^2+a)^{\frac{5}{2}}}{5c}-fg\left(\frac{x(cx^2+a)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{cx^2+a}}{2}+\frac{a\ln\left(x\sqrt{c}+\sqrt{cx^2+a}\right)}{2\sqrt{c}}\right)}{4}\right)}{h^2}$

```
[In] int((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g),x,method=_RETURNVERBOSE)
```

```
[Out] 1/120/c*(24*c^2*f*h^4*x^4+30*c^2*e*h^4*x^3-30*c^2*f*g*h^3*x^3+48*a*c*f*h^4*
x^2+40*c^2*d*h^4*x^2-40*c^2*e*g*h^3*x^2+40*c^2*f*g^2*h^2*x^2+75*a*c*e*h^4*x
-75*a*c*f*g*h^3*x-60*c^2*d*g*h^3*x+60*c^2*e*g^2*h^2*x-60*c^2*f*g^3*h*x+24*a
```

```
^2*f*h^4+160*a*c*d*h^4-160*a*c*e*g*h^3+160*a*c*f*g^2*h^2+120*c^2*d*g^2*h^2-
120*c^2*e*g^3*h+120*c^2*f*g^4)*(c*x^2+a)^(1/2)/h^5+1/8/h^5*((3*a^2*e*h^5-3*
a^2*f*g*h^4-12*a*c*d*g*h^4+12*a*c*e*g^2*h^3-12*a*c*f*g^3*h^2-8*c^2*d*g^3*h^
2+8*c^2*e*g^4*h-8*c^2*f*g^5)/h*ln(x*c^(1/2)+(c*x^2+a)^(1/2))/c^(1/2)-(8*a^2
*d*h^6-8*a^2*e*g*h^5+8*a^2*f*g^2*h^4+16*a*c*d*g^2*h^4-16*a*c*e*g^3*h^3+16*a
*c*f*g^4*h^2+8*c^2*d*g^4*h^2-8*c^2*e*g^5*h+8*c^2*f*g^6)/h^2/((a*h^2+c*g^2)/
h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1/h*g)+2*((a*h^2+c*g^2)/h^2)^
(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+1/h*g))
)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{g+hx}\,dx = \text{Timed out}$$

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{g+hx}\,dx = \int \frac{(a+cx^2)^{\frac{3}{2}}(d+ex+fx^2)}{g+hx}\,dx$$

```
[In] integrate((c*x**2+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g),x)
```

```
[Out] Integral((a + c*x**2)**(3/2)*(d + e*x + f*x**2)/(g + h*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 632 vs. $2(299) = 598$.

Time = 0.26 (sec) , antiderivative size = 632, normalized size of antiderivative = 1.94

$$
\begin{aligned}
\int \frac{(a+cx^2)^{3/2}\,(d+ex+fx^2)}{g+hx}\,dx = &-\frac{\sqrt{cx^2+a}cfg^3x}{2\,h^4}+\frac{\sqrt{cx^2+a}ceg^2x}{2\,h^3} \\
&-\frac{\sqrt{cx^2+a}cdgx}{2\,h^2}-\frac{(cx^2+a)^{\frac{3}{2}}fgx}{4\,h^2}-\frac{3\,\sqrt{cx^2+a}afgx}{8\,h^2} \\
&+\frac{(cx^2+a)^{\frac{3}{2}}ex}{4\,h}+\frac{3\,\sqrt{cx^2+a}aex}{8\,h}-\frac{c^{\frac{3}{2}}fg^5\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{h^6} \\
&+\frac{c^{\frac{3}{2}}eg^4\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{h^5}-\frac{c^{\frac{3}{2}}dg^3\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{h^4}-\frac{3\,a\sqrt{c}fg^3\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{2\,h^4} \\
&+\frac{3\,a\sqrt{c}eg^2\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{2\,h^3}-\frac{3\,a\sqrt{c}dg\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{2\,h^2}-\frac{3\,a^2fg\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{8\,\sqrt{c}h^2} \\
&+\frac{3\,a^2e\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{8\,\sqrt{c}h}+\frac{\left(a+\frac{cg^2}{h^2}\right)^{\frac{3}{2}}fg^2\,\mathrm{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|}-\frac{ah}{\sqrt{ac}|hx+g|}\right)}{h^3} \\
&-\frac{\left(a+\frac{cg^2}{h^2}\right)^{\frac{3}{2}}eg\,\mathrm{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|}-\frac{ah}{\sqrt{ac}|hx+g|}\right)}{h^2} \\
&+\frac{\left(a+\frac{cg^2}{h^2}\right)^{\frac{3}{2}}d\,\mathrm{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|}-\frac{ah}{\sqrt{ac}|hx+g|}\right)}{h}+\frac{\sqrt{cx^2+a}cfg^4}{h^5} \\
&-\frac{\sqrt{cx^2+a}ceg^3}{h^4}+\frac{\sqrt{cx^2+a}cdg^2}{h^3}+\frac{(cx^2+a)^{\frac{3}{2}}fg^2}{3\,h^3}+\frac{\sqrt{cx^2+a}afg^2}{h^3} \\
&-\frac{(cx^2+a)^{\frac{3}{2}}eg}{3\,h^2}-\frac{\sqrt{cx^2+a}aeg}{h^2}+\frac{(cx^2+a)^{\frac{3}{2}}d}{3\,h}+\frac{\sqrt{cx^2+a}ad}{h}+\frac{(cx^2+a)^{\frac{5}{2}}f}{5\,ch}
\end{aligned}
$$

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(c*x^2 + a)*c*f*g^3*x/h^4 + 1/2*sqrt(c*x^2 + a)*c*e*g^2*x/h^3 - 1/
2*sqrt(c*x^2 + a)*c*d*g*x/h^2 - 1/4*(c*x^2 + a)^(3/2)*f*g*x/h^2 - 3/8*sqrt(
c*x^2 + a)*a*f*g*x/h^2 + 1/4*(c*x^2 + a)^(3/2)*e*x/h + 3/8*sqrt(c*x^2 + a)*
a*e*x/h - c^(3/2)*f*g^5*arcsinh(c*x/sqrt(a*c))/h^6 + c^(3/2)*e*g^4*arcsinh(
c*x/sqrt(a*c))/h^5 - c^(3/2)*d*g^3*arcsinh(c*x/sqrt(a*c))/h^4 - 3/2*a*sqrt(
c)*f*g^3*arcsinh(c*x/sqrt(a*c))/h^4 + 3/2*a*sqrt(c)*e*g^2*arcsinh(c*x/sqrt(
a*c))/h^3 - 3/2*a*sqrt(c)*d*g*arcsinh(c*x/sqrt(a*c))/h^2 - 3/8*a^2*f*g*arcs
inh(c*x/sqrt(a*c))/(sqrt(c)*h^2) + 3/8*a^2*e*arcsinh(c*x/sqrt(a*c))/(sqrt(c
)*h) + (a + c*g^2/h^2)^(3/2)*f*g^2*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) -
 a*h/(sqrt(a*c)*abs(h*x + g)))/h^3 - (a + c*g^2/h^2)^(3/2)*e*g*arcsinh(c*g*
x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/h^2 + (a + c*g^2
/h^2)^(3/2)*d*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h
*x + g)))/h + sqrt(c*x^2 + a)*c*f*g^4/h^5 - sqrt(c*x^2 + a)*c*e*g^3/h^4 + s
qrt(c*x^2 + a)*c*d*g^2/h^3 + 1/3*(c*x^2 + a)^(3/2)*f*g^2/h^3 + sqrt(c*x^2 +
```

```
 a)*a*f*g^2/h^3 - 1/3*(c*x^2 + a)^(3/2)*e*g/h^2 - sqrt(c*x^2 + a)*a*e*g/h^2
 + 1/3*(c*x^2 + a)^(3/2)*d/h + sqrt(c*x^2 + a)*a*d/h + 1/5*(c*x^2 + a)^(5/2
)*f/(c*h)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+cx^2)^{3/2}\,(d+ex+fx^2)}{g+hx}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+cx^2)^{3/2}\,(d+ex+fx^2)}{g+hx}\,dx = \int \frac{(c\,x^2+a)^{3/2}\,(f\,x^2+e\,x+d)}{g+h\,x}\,dx$$

```
[In] int(((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x),x)
```

```
[Out] int(((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x), x)
```

3.93 $\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 432

$$
\begin{aligned}
\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^2}\,dx = & -\frac{\left(8(ah^2(2fg-eh)+cg(5fg^2-h(4eg-3dh)))-h(20cfg^2-16cegh+12cdh^2+3afh^2)\,x\right)\sqrt{a+cx^2}}{8h^5} \\
& -\frac{\left(4(ah^2(2fg-eh)+cg(5fg^2-h(4eg-3dh)))-3h(afh^2+c(5fg^2-4h(eg-dh)))\,x\right)(a+cx^2)^{3/2}}{12h^3\left(cg^2+ah^2\right)} \\
& -\frac{\left(fg^2-egh+dh^2\right)(a+cx^2)^{5/2}}{h\left(cg^2+ah^2\right)(g+hx)} \\
& +\frac{\left(3a^2fh^4+8c^2g^2(5fg^2-h(4eg-3dh))+12ach^2(3fg^2-h(2eg-dh))\right)\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{8\sqrt{c}h^6} \\
& +\frac{\sqrt{cg^2+ah^2}\left(ah^2(2fg-eh)+cg(5fg^2-h(4eg-3dh))\right)\operatorname{arctanh}\left(\frac{ah-cgx}{\sqrt{cg^2+ah^2}\sqrt{a+cx^2}}\right)}{h^6}
\end{aligned}
$$

```
[Out] -1/12*(4*a*h^2*(-e*h+2*f*g)+4*c*g*(5*f*g^2-h*(-3*d*h+4*e*g))-3*h*(a*f*h^2+c
*(5*f*g^2-4*h*(-d*h+e*g)))*x)*(c*x^2+a)^(3/2)/h^3/(a*h^2+c*g^2)-(d*h^2-e*g*
h+f*g^2)*(c*x^2+a)^(5/2)/h/(a*h^2+c*g^2)/(h*x+g)+1/8*(3*a^2*f*h^4+8*c^2*g^2
*(5*f*g^2-h*(-3*d*h+4*e*g))+12*a*c*h^2*(3*f*g^2-h*(-d*h+2*e*g)))*arctanh(x*
c^(1/2)/(c*x^2+a)^(1/2))/h^6/c^(1/2)+(a*h^2*(-e*h+2*f*g)+c*g*(5*f*g^2-h*(-3
*d*h+4*e*g)))*arctanh((-c*g*x+a*h)/(a*h^2+c*g^2)^(1/2)/(c*x^2+a)^(1/2))*(a*
h^2+c*g^2)^(1/2)/h^6-1/8*(8*a*h^2*(-e*h+2*f*g)+8*c*g*(5*f*g^2-h*(-3*d*h+4*e
*g))-h*(3*a*f*h^2+12*c*d*h^2-16*c*e*g*h+20*c*f*g^2)*x)*(c*x^2+a)^(1/2)/h^5
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 428, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1665, 829, 858, 223, 212, 739}

$$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^2}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)(3a^2fh^4+12ach^2(3fg^2-h(2eg-dh))+8c^2(5fg^4 -}{8\sqrt{c}h^6}$$

$$+\frac{\sqrt{ah^2+cg^2}\operatorname{arctanh}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)(ah^2(2fg-eh)-cgh(4eg-3dh)+5cfg^3)}{h^6}$$

$$-\frac{(a+cx^2)^{5/2}(dh^2-egh+fg^2)}{h(g+hx)(ah^2+cg^2)}$$

$$-\frac{\sqrt{a+cx^2}(8(ah^2(2fg-eh)-cgh(4eg-3dh)+5cfg^3)-hx(3afh^2+12cdh^2-16cegh+20cfg^2))}{8h^5}$$

$$-\frac{(a+cx^2)^{3/2}(4(ah^2(2fg-eh)-cgh(4eg-3dh)+5cfg^3)-3hx(afh^2-4ch(eg-dh)+5cfg^2))}{12h^3(ah^2+cg^2)}$$

```
[In] Int[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^2,x]
```

```
[Out] -1/8*((8*(5*c*f*g^3 - c*g*h*(4*e*g - 3*d*h) + a*h^2*(2*f*g - e*h)) - h*(20*
c*f*g^2 - 16*c*e*g*h + 12*c*d*h^2 + 3*a*f*h^2)*x)*Sqrt[a + c*x^2])/h^5 - ((
4*(5*c*f*g^3 - c*g*h*(4*e*g - 3*d*h) + a*h^2*(2*f*g - e*h)) - 3*h*(5*c*f*g^
2 + a*f*h^2 - 4*c*h*(e*g - d*h))*x)*(a + c*x^2)^(3/2))/(12*h^3*(c*g^2 + a*h
^2)) - ((f*g^2 - e*g*h + d*h^2)*(a + c*x^2)^(5/2))/(h*(c*g^2 + a*h^2)*(g +
h*x)) + ((3*a^2*f*h^4 + 8*c^2*(5*f*g^4 - g^2*h*(4*e*g - 3*d*h)) + 12*a*c*h^
2*(3*f*g^2 - h*(2*e*g - d*h)))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(8*Sqr
t[c]*h^6) + (Sqrt[c*g^2 + a*h^2]*(5*c*f*g^3 - c*g*h*(4*e*g - 3*d*h) + a*h^2
*(2*f*g - e*h))*ArcTanh[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2])
])/h^6
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
```

```
[{a, c, d, e}, x]
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{(fg^2-egh+dh^2)\left(a+cx^2\right)^{5/2}}{h\left(cg^2+ah^2\right)(g+hx)} - \frac{\int \frac{\left(-cdg+afg-aeh-\left(afh-c\left(4eg-\frac{5fg^2}{h}-4dh\right)\right)x\right)\left(a+cx^2\right)^{3/2}}{g+hx}\,dx}{cg^2+ah^2}$$

$$= -\frac{(4(5cfg^3 - cgh(4eg-3dh) + ah^2(2fg-eh)) - 3h(5cfg^2 + afh^2 - 4ch(eg-dh))\,x)\,(a+cx^2)}{12h^3\,(cg^2+ah^2)}$$

$$-\frac{(fg^2 - egh + dh^2)\,(a+cx^2)^{5/2}}{h\,(cg^2+ah^2)\,(g+hx)}$$

$$-\frac{\int \frac{\left(ac(5fg-4eh)(cg^2+ah^2) - \frac{c\left(cg^2+ah^2\right)\left(20cfg^2-16cegh+12cdh^2+3afh^2\right)x}{h}\right)\sqrt{a+cx^2}}{g+hx}\,dx}{4ch^2\,(cg^2+ah^2)}$$

$$= -\frac{(8(5cfg^3 - cgh(4eg-3dh) + ah^2(2fg-eh)) - h(20cfg^2 - 16cegh + 12cdh^2 + 3afh^2)\,x)\,\sqrt{a}}{8h^5}$$

$$-\frac{(4(5cfg^3 - cgh(4eg-3dh) + ah^2(2fg-eh)) - 3h(5cfg^2 + afh^2 - 4ch(eg-dh))\,x)\,(a+cx^2)}{12h^3\,(cg^2+ah^2)}$$

$$-\frac{(fg^2 - egh + dh^2)\,(a+cx^2)^{5/2}}{h\,(cg^2+ah^2)\,(g+hx)}$$

$$-\frac{\int \frac{ac^2\left(cg^2+ah^2\right)\left(ah^2(13fg-8eh)+4c\left(5fg^3-gh(4eg-3dh)\right)\right) - \frac{c^2\left(cg^2+ah^2\right)\left(3a^2fh^4+8c^2\left(5fg^4-g^2h(4eg-3dh)\right)+12ach^2\left(3fg^2-h(2eg\right.\right.}{h}}{(g+hx)\sqrt{a+cx^2}}}{8c^2h^4\,(cg^2+ah^2)}$$

$$= -\frac{(8(5cfg^3 - cgh(4eg-3dh) + ah^2(2fg-eh)) - h(20cfg^2 - 16cegh + 12cdh^2 + 3afh^2)\,x)\,\sqrt{a}}{8h^5}$$

$$-\frac{(4(5cfg^3 - cgh(4eg-3dh) + ah^2(2fg-eh)) - 3h(5cfg^2 + afh^2 - 4ch(eg-dh))\,x)\,(a+cx^2)}{12h^3\,(cg^2+ah^2)}$$

$$-\frac{(fg^2 - egh + dh^2)\,(a+cx^2)^{5/2}}{h\,(cg^2+ah^2)\,(g+hx)}$$

$$-\frac{((cg^2+ah^2)\,(5cfg^3 - cgh(4eg-3dh) + ah^2(2fg-eh)))\int \frac{1}{(g+hx)\sqrt{a+cx^2}}\,dx}{h^6}$$

$$+\frac{(3a^2fh^4 + 8c^2(5fg^4 - g^2h(4eg-3dh)) + 12ach^2(3fg^2 - h(2eg-dh)))\int \frac{1}{\sqrt{a+cx^2}}\,dx}{8h^6}$$

$$= -\frac{(8(5cfg^3 - cgh(4eg-3dh) + ah^2(2fg-eh)) - h(20cfg^2 - 16cegh + 12cdh^2 + 3afh^2)\,x)\,\sqrt{a}}{8h^5}$$

$$-\frac{(4(5cfg^3 - cgh(4eg-3dh) + ah^2(2fg-eh)) - 3h(5cfg^2 + afh^2 - 4ch(eg-dh))\,x)\,(a+cx^2)}{12h^3\,(cg^2+ah^2)}$$

$$-\frac{(fg^2 - egh + dh^2)\,(a+cx^2)^{5/2}}{h\,(cg^2+ah^2)\,(g+hx)}$$

$$+\frac{((cg^2+ah^2)\,(5cfg^3 - cgh(4eg-3dh) + ah^2(2fg-eh)))\,\mathrm{Subst}\left(\int \frac{1}{cg^2+ah^2-x^2}\,dx, x, \frac{ah-cgx}{\sqrt{a+cx^2}}\right)}{h^6}$$

$$+\frac{(3a^2fh^4 + 8c^2(5fg^4 - g^2h(4eg-3dh)) + 12ach^2(3fg^2 - h(2eg-dh)))\,\mathrm{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \right.}{8h^6}$$

$$=$$

$$-\frac{\left(8\left(5cfg^3-cgh(4eg-3dh)+ah^2(2fg-eh)\right)-h\left(20cfg^2-16cegh+12cdh^2+3afh^2\right)x\right)\sqrt{a+}}{8h^5}$$

$$-\frac{\left(4\left(5cfg^3-cgh(4eg-3dh)+ah^2(2fg-eh)\right)-3h\left(5cfg^2+afh^2-4ch(eg-dh)\right)x\right)\left(a+cx^2\right)^3}{12h^3\left(cg^2+ah^2\right)}$$

$$-\frac{\left(fg^2-egh+dh^2\right)\left(a+cx^2\right)^{5/2}}{h\left(cg^2+ah^2\right)(g+hx)}$$

$$+\frac{\left(3a^2fh^4+8c^2\left(5fg^4-g^2h(4eg-3dh)\right)+12ach^2\left(3fg^2-h(2eg-dh)\right)\right)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{8\sqrt{c}h^6}$$

$$+\frac{\sqrt{cg^2+ah^2}\left(5cfg^3-cgh(4eg-3dh)+ah^2(2fg-eh)\right)\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{cg^2+ah^2}\sqrt{a+cx^2}}\right)}{h^6}$$

Mathematica [A] (verified)

Time = 1.49 (sec) , antiderivative size = 364, normalized size of antiderivative = 0.84

$$\int\frac{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^2}\,dx=\frac{\frac{h\sqrt{a+cx^2}\left(-2cf\left(60g^4+30g^3hx-10g^2h^2x^2+5gh^3x^3-3h^4x^4\right)+ah^2\left(8h(7eg-3dh+4ehx)+f\left(-88g^2\right.\right.\right.}{g+hx}}{}$$

```
[In] Integrate[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^2,x]
```

```
[Out] ((h*Sqrt[a + c*x^2]*(-2*c*f*(60*g^4 + 30*g^3*h*x - 10*g^2*h^2*x^2 + 5*g*h^3
*x^3 - 3*h^4*x^4) + a*h^2*(8*h*(7*e*g - 3*d*h + 4*e*h*x) + f*(-88*g^2 - 49*
g*h*x + 15*h^2*x^2)) + 4*c*h*(3*d*h*(-6*g^2 - 3*g*h*x + h^2*x^2) + 2*e*(12*
g^3 + 6*g^2*h*x - 2*g*h^2*x^2 + h^3*x^3))))/(g + h*x) - 48*Sqrt[-(c*g^2) -
a*h^2]*(5*c*f*g^3 + c*g*h*(-4*e*g + 3*d*h) + a*h^2*(2*f*g - e*h))*ArcTan[(S
qrt[c]*(g + h*x) - h*Sqrt[a + c*x^2])/Sqrt[-(c*g^2) - a*h^2]] - (3*(3*a^2*f
*h^4 + 12*a*c*h^2*(3*f*g^2 + h*(-2*e*g + d*h)) + 8*c^2*(5*f*g^4 + g^2*h*(-4
*e*g + 3*d*h)))*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/Sqrt[c])/(24*h^6)
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 736, normalized size of antiderivative = 1.70

method	result
risch	$\frac{(6f\,x^3h^3c+8ce\,h^3x^2-16cfg\,h^2x^2+15af\,h^3x+12cd\,h^3x-24ceg\,h^2x+36cf\,g^2hx+32ae\,h^3-64afg\,h^2-48cdg\,h^2+72ce\,g^2h-96cf\,g^3)}{24h^5}$
default	Expression too large to display

```
[In] int((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*(6*c*f*h^3*x^3+8*c*e*h^3*x^2-16*c*f*g*h^2*x^2+15*a*f*h^3*x+12*c*d*h^3*
x-24*c*e*g*h^2*x+36*c*f*g^2*h*x+32*a*e*h^3-64*a*f*g*h^2-48*c*d*g*h^2+72*c*e
*g^2*h-96*c*f*g^3)*(c*x^2+a)^(1/2)/h^5+1/8/h^5*((3*a^2*f*h^4+12*a*c*d*h^4-2
4*a*c*e*g*h^3+36*a*c*f*g^2*h^2+24*c^2*d*g^2*h^2-32*c^2*e*g^3*h+40*c^2*f*g^4
)/h*ln(x*c^(1/2)+(c*x^2+a)^(1/2))/c^(1/2)-(8*a^2*e*h^5-16*a^2*f*g*h^4-32*a*
c*d*g*h^4+48*a*c*e*g^2*h^3-64*a*c*f*g^3*h^2-32*c^2*d*g^3*h^2+40*c^2*e*g^4*h
-48*c^2*f*g^5)/h^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/
h*(x+1/h*g)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a
*h^2+c*g^2)/h^2)^(1/2))/(x+1/h*g))+1/h^3*(8*a^2*d*h^6-8*a^2*e*g*h^5+8*a^2*f
*g^2*h^4+16*a*c*d*g^2*h^4-16*a*c*e*g^3*h^3+16*a*c*f*g^4*h^2+8*c^2*d*g^4*h^2
-8*c^2*e*g^5*h+8*c^2*f*g^6)*(-1/(a*h^2+c*g^2)*h^2/(x+1/h*g)*((x+1/h*g)^2*c-
2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)-c*g*h/(a*h^2+c*g^2)/((a*h^2+c*g^
2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1/h*g)+2*((a*h^2+c*g^2)/h^
2)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+1/h*
g))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+cx^2)^{3/2}\,(d+ex+fx^2)}{(g+hx)^2}\,dx = \text{Timed out}$$

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^2}\,dx=\int \frac{\left(a+cx^2\right)^{\frac{3}{2}}\left(d+ex+fx^2\right)}{\left(g+hx\right)^2}\,dx$$

```
[In] integrate((c*x**2+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**2,x)
```

```
[Out] Integral((a + c*x**2)**(3/2)*(d + e*x + f*x**2)/(g + h*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 708, normalized size of antiderivative = 1.64

$$\begin{aligned}
\int \frac{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^2}\,dx &= -\frac{\left(cx^2+a\right)^{\frac{3}{2}}fg^2}{h^4x+gh^3}+\frac{\left(cx^2+a\right)^{\frac{3}{2}}eg}{h^3x+gh^2} \\
&\quad -\frac{\left(cx^2+a\right)^{\frac{3}{2}}d}{h^2x+gh}+\frac{5\sqrt{cx^2+a}\,cfg^2x}{2\,h^4}-\frac{2\sqrt{cx^2+a}\,cegx}{h^3} \\
&\quad +\frac{3\sqrt{cx^2+a}\,cdx}{2\,h^2}+\frac{\left(cx^2+a\right)^{\frac{3}{2}}fx}{4\,h^2}+\frac{3\sqrt{cx^2+a}\,afx}{8\,h^2} \\
&\quad +\frac{5\,c^{\frac{3}{2}}fg^4\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{h^6}-\frac{4\,c^{\frac{3}{2}}eg^3\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{h^5}+\frac{3\,c^{\frac{3}{2}}dg^2\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{h^4} \\
&\quad +\frac{9\,a\sqrt{c}fg^2\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{2\,h^4}-\frac{3\,a\sqrt{c}eg\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{h^3}+\frac{3\,a\sqrt{c}d\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{2\,h^2} \\
&\quad +\frac{3\,a^2f\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{8\,\sqrt{c}h^2}-\frac{3\,\sqrt{a+\frac{cg^2}{h^2}}cfg^3\operatorname{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|}-\frac{ah}{\sqrt{ac}|hx+g|}\right)}{h^5} \\
&\quad +\frac{3\,\sqrt{a+\frac{cg^2}{h^2}}ceg^2\operatorname{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|}-\frac{ah}{\sqrt{ac}|hx+g|}\right)}{h^4} \\
&\quad -\frac{3\,\sqrt{a+\frac{cg^2}{h^2}}cdg\operatorname{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|}-\frac{ah}{\sqrt{ac}|hx+g|}\right)}{h^3} \\
&\quad -\frac{2\,\left(a+\frac{cg^2}{h^2}\right)^{\frac{3}{2}}fg\operatorname{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|}-\frac{ah}{\sqrt{ac}|hx+g|}\right)}{h^3} \\
&\quad +\frac{\left(a+\frac{cg^2}{h^2}\right)^{\frac{3}{2}}e\operatorname{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|}-\frac{ah}{\sqrt{ac}|hx+g|}\right)}{h^2}-\frac{5\sqrt{cx^2+a}\,cfg^3}{h^5} \\
&\quad +\frac{4\sqrt{cx^2+a}\,ceg^2}{h^4}-\frac{3\sqrt{cx^2+a}\,cdg}{h^3}-\frac{2\left(cx^2+a\right)^{\frac{3}{2}}fg}{3\,h^3} \\
&\quad -\frac{2\sqrt{cx^2+a}\,afg}{h^3}+\frac{\left(cx^2+a\right)^{\frac{3}{2}}e}{3\,h^2}+\frac{\sqrt{cx^2+a}\,ae}{h^2}
\end{aligned}$$

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^2,x, algorithm="maxima")
```

```
[Out] -(c*x^2 + a)^(3/2)*f*g^2/(h^4*x + g*h^3) + (c*x^2 + a)^(3/2)*e*g/(h^3*x + g
*h^2) - (c*x^2 + a)^(3/2)*d/(h^2*x + g*h) + 5/2*sqrt(c*x^2 + a)*c*f*g^2*x/h
^4 - 2*sqrt(c*x^2 + a)*c*e*g*x/h^3 + 3/2*sqrt(c*x^2 + a)*c*d*x/h^2 + 1/4*(c
*x^2 + a)^(3/2)*f*x/h^2 + 3/8*sqrt(c*x^2 + a)*a*f*x/h^2 + 5*c^(3/2)*f*g^4*a
rcsinh(c*x/sqrt(a*c))/h^6 - 4*c^(3/2)*e*g^3*arcsinh(c*x/sqrt(a*c))/h^5 + 3*
c^(3/2)*d*g^2*arcsinh(c*x/sqrt(a*c))/h^4 + 9/2*a*sqrt(c)*f*g^2*arcsinh(c*x/
sqrt(a*c))/h^4 - 3*a*sqrt(c)*e*g*arcsinh(c*x/sqrt(a*c))/h^3 + 3/2*a*sqrt(c)
*d*arcsinh(c*x/sqrt(a*c))/h^2 + 3/8*a^2*f*arcsinh(c*x/sqrt(a*c))/(sqrt(c)*h
^2) - 3*sqrt(a + c*g^2/h^2)*c*f*g^3*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g))
- a*h/(sqrt(a*c)*abs(h*x + g)))/h^5 + 3*sqrt(a + c*g^2/h^2)*c*e*g^2*arcsinh
(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/h^4 - 3*sqr
t(a + c*g^2/h^2)*c*d*g*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a
*c)*abs(h*x + g)))/h^3 - 2*(a + c*g^2/h^2)^(3/2)*f*g*arcsinh(c*g*x/(sqrt(a*
c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/h^3 + (a + c*g^2/h^2)^(3/2
)*e*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/
h^2 - 5*sqrt(c*x^2 + a)*c*f*g^3/h^5 + 4*sqrt(c*x^2 + a)*c*e*g^2/h^4 - 3*sqr
t(c*x^2 + a)*c*d*g/h^3 - 2/3*(c*x^2 + a)^(3/2)*f*g/h^3 - 2*sqrt(c*x^2 + a)*
a*f*g/h^3 + 1/3*(c*x^2 + a)^(3/2)*e/h^2 + sqrt(c*x^2 + a)*a*e/h^2
```

Giac [F]

$$\int \frac{(a+cx^2)^{3/2}\,(d+ex+fx^2)}{(g+hx)^2}\,dx = \int \frac{(cx^2+a)^{\frac{3}{2}}(fx^2+ex+d)}{(hx+g)^2}\,dx$$

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+cx^2)^{3/2}\,(d+ex+fx^2)}{(g+hx)^2}\,dx = \int \frac{(c\,x^2+a)^{3/2}\,(f\,x^2+e\,x+d)}{(g+h\,x)^2}\,dx$$

```
[In] int(((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^2,x)
```

```
[Out] int(((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^2, x)
```

3.94 $\int \frac{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^3}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 488

$$\int \frac{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^3}\,dx = \frac{(2a^2fh^4+2c^2g^2(10fg^2-3h(2eg-dh))+ach^2(19fg^2-3h(3eg-dh))}{2h^5\,(cg^2+a}$$

$$-\frac{\left(2\left(cg\left(6eg-\frac{10fg^2}{h}-3dh\right)-ah(7fg-3eh)\right)-\left(2afh^2+c(5fg^2-3h(eg-dh))\right)x\right)\left(a+cx^2\right)^{3/2}}{6h^2\left(cg^2+ah^2\right)(g+hx)}$$

$$-\frac{\left(fg^2-egh+dh^2\right)\left(a+cx^2\right)^{5/2}}{2h\left(cg^2+ah^2\right)(g+hx)^2}$$

$$-\frac{\sqrt{c}\left(3ah^2(3fg-eh)+2cg(10fg^2-3h(2eg-dh))\right)\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2h^6}$$

$$-\frac{\left(2a^2fh^4+2c^2g^2(10fg^2-3h(2eg-dh))+ach^2(19fg^2-3h(3eg-dh))\right)\operatorname{arctanh}\left(\frac{ah-cgx}{\sqrt{cg^2+ah^2}\sqrt{a+cx^2}}\right)}{2h^6\sqrt{cg^2+ah^2}}$$

```
[Out] -1/6*(2*c*g*(6*e*g-10*f*g^2/h-3*d*h)-2*a*h*(-3*e*h+7*f*g)-(2*a*f*h^2+c*(5*f
*g^2-3*h*(-d*h+e*g)))*x)*(c*x^2+a)^(3/2)/h^2/(a*h^2+c*g^2)/(h*x+g)-1/2*(d*h
^2-e*g*h+f*g^2)*(c*x^2+a)^(5/2)/h/(a*h^2+c*g^2)/(h*x+g)^2-1/2*(3*a*h^2*(-e*
h+3*f*g)+2*c*g*(10*f*g^2-3*h*(-d*h+2*e*g)))*arctanh(x*c^(1/2)/(c*x^2+a)^(1/
2))*c^(1/2)/h^6-1/2*(2*a^2*f*h^4+2*c^2*g^2*(10*f*g^2-3*h*(-d*h+2*e*g))+a*c*
h^2*(19*f*g^2-3*h*(-d*h+3*e*g)))*arctanh((-c*g*x+a*h)/(a*h^2+c*g^2)^(1/2)/(
c*x^2+a)^(1/2))/h^6/(a*h^2+c*g^2)^(1/2)+1/2*(2*a^2*f*h^4+2*c^2*g^2*(10*f*g^
2-3*h*(-d*h+2*e*g))+a*c*h^2*(19*f*g^2-3*h*(-d*h+3*e*g))-c*h*(a*h^2*(-3*e*h+
7*f*g)+c*g*(10*f*g^2-3*h*(-d*h+2*e*g)))*x)*(c*x^2+a)^(1/2)/h^5/(a*h^2+c*g^2
)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 480, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {1665, 827, 829, 858, 223, 212, 739}

$$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^3}\,dx =$$

$$-\frac{\operatorname{arctanh}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)(2a^2fh^4+ach^2(19fg^2-3h(3eg-dh))+2c^2(10fg^4-3g^2h(2eg-dh)))}{2h^6\sqrt{ah^2+cg^2}}$$

$$+\frac{\sqrt{a+cx^2}\left(2a^2fh^3-cx(ah^2(7fg-3eh)-3cgh(2eg-dh)+10cfg^3)+ach(19fg^2-3h(3eg-dh))-2c^2\right.}{2h^4(ah^2+cg^2)}$$

$$-\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)(3ah^2(3fg-eh)-6cgh(2eg-dh)+20cfg^3)}{2h^6}$$

$$-\frac{(a+cx^2)^{5/2}(dh^2-egh+fg^2)}{2h(g+hx)^2(ah^2+cg^2)}$$

$$-\frac{(a+cx^2)^{3/2}\left(2\left(cg\left(-3dh+6eg-\frac{10fg^2}{h}\right)-ah(7fg-3eh)\right)-x(2afh^2-3ch(eg-dh)+5cfg^2)\right)}{6h^2(g+hx)(ah^2+cg^2)}$$

```
[In] Int[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^3,x]
```

```
[Out] ((2*a^2*f*h^3 - 2*c^2*g^2*(6*e*g - (10*f*g^2)/h - 3*d*h) + a*c*h*(19*f*g^2
 - 3*h*(3*e*g - d*h)) - c*(10*c*f*g^3 - 3*c*g*h*(2*e*g - d*h) + a*h^2*(7*f*g
 - 3*e*h))*x)*Sqrt[a + c*x^2])/(2*h^4*(c*g^2 + a*h^2)) - ((2*(c*g*(6*e*g -
(10*f*g^2)/h - 3*d*h) - a*h*(7*f*g - 3*e*h)) - (5*c*f*g^2 + 2*a*f*h^2 - 3*c
*h*(e*g - d*h))*x)*(a + c*x^2)^(3/2))/(6*h^2*(c*g^2 + a*h^2)*(g + h*x)) - (
(f*g^2 - e*g*h + d*h^2)*(a + c*x^2)^(5/2))/(2*h*(c*g^2 + a*h^2)*(g + h*x)^2
) - (Sqrt[c]*(20*c*f*g^3 - 6*c*g*h*(2*e*g - d*h) + 3*a*h^2*(3*f*g - e*h))*A
rcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*h^6) - ((2*a^2*f*h^4 + 2*c^2*(10*f*
g^4 - 3*g^2*h*(2*e*g - d*h)) + a*c*h^2*(19*f*g^2 - 3*h*(3*e*g - d*h)))*ArcT
anh[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2])])/(2*h^6*Sqrt[c*g^2
 + a*h^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 827

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{(fg^2-egh+dh^2)\,(a+cx^2)^{5/2}}{2h\,(cg^2+ah^2)\,(g+hx)^2} - \frac{\int \frac{\left(-2(cdg-afg+aeh)-\left(2afh-c\left(3eg-\frac{5fg^2}{h}-3dh\right)\right)x\right)(a+cx^2)^{3/2}}{(g+hx)^2}\,dx}{2\,(cg^2+ah^2)}$$

$$= -\frac{\Big(2\Big(cg\Big(6eg-\frac{10fg^2}{h}-3dh\Big)-ah(7fg-3eh)\Big)-(5cfg^2+2afh^2-3ch(eg-dh))\,x\Big)\,(a+cx^2)}{6h^2\,(cg^2+ah^2)\,(g+hx)} - \frac{(fg^2-egh+dh^2)\,(a+cx^2)^{5/2}}{2h\,(cg^2+ah^2)\,(g+hx)^2} + \frac{\int \frac{\left(2a\left(5cfg^2+2afh^2-3ch(eg-dh)\right)-\frac{4c\left(10cfg^3-3cgh(2eg-dh)+ah^2(7fg-3eh)\right)x}{h}\right)\sqrt{a+cx^2}}{g+hx}\,dx}{4h^2\,(cg^2+ah^2)}$$

$$= \frac{\Big(2a^2fh^3-2c^2g^2\Big(6eg-\frac{10fg^2}{h}-3dh\Big)+ach(19fg^2-3h(3eg-dh))-c(10cfg^3-3cgh(2eg-d}{2h^4\,(cg^2+ah^2)} - \frac{\Big(2\Big(cg\Big(6eg-\frac{10fg^2}{h}-3dh\Big)-ah(7fg-3eh)\Big)-(5cfg^2+2afh^2-3ch(eg-dh))\,x\Big)\,(a+cx^2)}{6h^2\,(cg^2+ah^2)\,(g+hx)} - \frac{(fg^2-egh+dh^2)\,(a+cx^2)^{5/2}}{2h\,(cg^2+ah^2)\,(g+hx)^2} + \frac{\int \frac{4ac(cg^2+ah^2)\left(10cfg^2+2afh^2-3ch(2eg-dh)\right)-\frac{4c^2\left(cg^2+ah^2\right)\left(20cfg^3-6cgh(2eg-dh)+3ah^2(3fg-eh)\right)x}{h}}{(g+hx)\sqrt{a+cx^2}}\,dx}{8ch^4\,(cg^2+ah^2)}$$

$$= \frac{\Big(2a^2fh^3-2c^2g^2\Big(6eg-\frac{10fg^2}{h}-3dh\Big)+ach(19fg^2-3h(3eg-dh))-c(10cfg^3-3cgh(2eg-d}{2h^4\,(cg^2+ah^2)} - \frac{\Big(2\Big(cg\Big(6eg-\frac{10fg^2}{h}-3dh\Big)-ah(7fg-3eh)\Big)-(5cfg^2+2afh^2-3ch(eg-dh))\,x\Big)\,(a+cx^2)}{6h^2\,(cg^2+ah^2)\,(g+hx)} - \frac{(fg^2-egh+dh^2)\,(a+cx^2)^{5/2}}{2h\,(cg^2+ah^2)\,(g+hx)^2} - \frac{(c(20cfg^3-6cgh(2eg-dh)+3ah^2(3fg-eh)))\int \frac{1}{\sqrt{a+cx^2}}\,dx}{2h^6} + \frac{(2a^2fh^4+2c^2(10fg^4-3g^2h(2eg-dh))+ach^2(19fg^2-3h(3eg-dh)))\int \frac{1}{(g+hx)\sqrt{a+cx^2}}\,dx}{2h^6}$$

$$= \frac{\Big(2a^2fh^3 - 2c^2g^2\Big(6eg - \frac{10fg^2}{h} - 3dh\Big) + ach(19fg^2 - 3h(3eg - dh)) - c(10cfg^3 - 3cgh(2eg - dh)}{2h^4\,(cg^2 + ah^2)}$$
$$- \frac{\Big(2\Big(cg\Big(6eg - \frac{10fg^2}{h} - 3dh\Big) - ah(7fg - 3eh)\Big) - (5cfg^2 + 2afh^2 - 3ch(eg - dh))\,x\Big)\,(a + cx^2)^{3/}}{6h^2\,(cg^2 + ah^2)\,(g + hx)}$$
$$- \frac{(fg^2 - egh + dh^2)\,(a + cx^2)^{5/2}}{2h\,(cg^2 + ah^2)\,(g + hx)^2}$$
$$- \frac{(c(20cfg^3 - 6cgh(2eg - dh) + 3ah^2(3fg - eh)))\,\mathrm{Subst}\Big(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\Big)}{2h^6}$$
$$- \frac{(2a^2fh^4 + 2c^2(10fg^4 - 3g^2h(2eg - dh)) + ach^2(19fg^2 - 3h(3eg - dh)))\,\mathrm{Subst}\Big(\int \frac{1}{cg^2+ah^2-x^2}\,dx,}{2h^6}$$

$$= \frac{\Big(2a^2fh^3 - 2c^2g^2\Big(6eg - \frac{10fg^2}{h} - 3dh\Big) + ach(19fg^2 - 3h(3eg - dh)) - c(10cfg^3 - 3cgh(2eg - dh)}{2h^4\,(cg^2 + ah^2)}$$
$$- \frac{\Big(2\Big(cg\Big(6eg - \frac{10fg^2}{h} - 3dh\Big) - ah(7fg - 3eh)\Big) - (5cfg^2 + 2afh^2 - 3ch(eg - dh))\,x\Big)\,(a + cx^2)^{3/}}{6h^2\,(cg^2 + ah^2)\,(g + hx)}$$
$$- \frac{(fg^2 - egh + dh^2)\,(a + cx^2)^{5/2}}{2h\,(cg^2 + ah^2)\,(g + hx)^2}$$
$$- \frac{\sqrt{c}(20cfg^3 - 6cgh(2eg - dh) + 3ah^2(3fg - eh))\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2h^6}$$
$$- \frac{(2a^2fh^4 + 2c^2(10fg^4 - 3g^2h(2eg - dh)) + ach^2(19fg^2 - 3h(3eg - dh)))\tanh^{-1}\Big(\frac{ah-cgx}{\sqrt{cg^2+ah^2}\sqrt{a+cx}}}{2h^6\sqrt{cg^2 + ah^2}}$$

Mathematica [A] (verified)

Time = 2.01 (sec) , antiderivative size = 361, normalized size of antiderivative = 0.74

$$\int \frac{(a + cx^2)^{3/2}\,(d + ex + fx^2)}{(g + hx)^3}\,dx = \frac{\frac{h\sqrt{a+cx^2}\big(ah^2\big(-3h(eg+dh+2ehx)+f\big(17g^2+28ghx+8h^2x^2\big)\big)+c\big(f\big(60g^4+90g^3hx+20g^2h^2x^2-5g}{(g+hx)^2}}{}$$

```
[In] Integrate[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^3,x]
```

```
[Out] ((h*Sqrt[a + c*x^2]*(a*h^2*(-3*h*(e*g + d*h + 2*e*h*x) + f*(17*g^2 + 28*g*h
*x + 8*h^2*x^2)) + c*(f*(60*g^4 + 90*g^3*h*x + 20*g^2*h^2*x^2 - 5*g*h^3*x^3
 + 2*h^4*x^4) + 3*h*(d*h*(6*g^2 + 9*g*h*x + 2*h^2*x^2) + e*(-12*g^3 - 18*g^
2*h*x - 4*g*h^2*x^2 + h^3*x^3)))))/(g + h*x)^2 - (6*(2*a^2*f*h^4 + a*c*h^2*
(19*f*g^2 + 3*h*(-3*e*g + d*h)) + 2*c^2*(10*f*g^4 + 3*g^2*h*(-2*e*g + d*h))
```

```
)*ArcTan[(Sqrt[c]*(g + h*x) - h*Sqrt[a + c*x^2])/Sqrt[-(c*g^2) - a*h^2]])/S
qrt[-(c*g^2) - a*h^2] + 3*Sqrt[c]*(20*c*f*g^3 + 6*c*g*h*(-2*e*g + d*h) - 3*
a*h^2*(-3*f*g + e*h))*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/(6*h^6)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1166 vs. 2(458) = 916.

Time = 0.70 (sec) , antiderivative size = 1167, normalized size of antiderivative = 2.39

method	result	size
risch	Expression too large to display	1167
default	Expression too large to display	2863

```
[In] int((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(2*c*f*h^2*x^2+3*c*e*h^2*x-9*c*f*g*h*x+8*a*f*h^2+6*c*d*h^2-18*c*e*g*h+3
6*c*f*g^2)*(c*x^2+a)^(1/2)/h^5+1/2/h^5*(c^(1/2)*(3*a*e*h^3-9*a*f*g*h^2-6*c*
d*g*h^2+12*c*e*g^2*h-20*c*f*g^3)/h*ln(x*c^(1/2)+(c*x^2+a)^(1/2))-(2*a^2*f*h
^4+4*a*c*d*h^4-12*a*c*e*g*h^3+24*a*c*f*g^2*h^2+12*c^2*d*g^2*h^2-20*c^2*e*g^
3*h+30*c^2*f*g^4)/h^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c
*g/h*(x+1/h*g)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)
+(a*h^2+c*g^2)/h^2)^(1/2))/(x+1/h*g))+(2*a^2*e*h^5-4*a^2*f*g*h^4-8*a*c*d*g*
h^4+12*a*c*e*g^2*h^3-16*a*c*f*g^3*h^2-8*c^2*d*g^3*h^2+10*c^2*e*g^4*h-12*c^2
*f*g^5)/h^3*(-1/(a*h^2+c*g^2)*h^2/(x+1/h*g)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g
)+(a*h^2+c*g^2)/h^2)^(1/2)-c*g*h/(a*h^2+c*g^2)/((a*h^2+c*g^2)/h^2)^(1/2)*ln
((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1/h*g)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+1/h
*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+1/h*g)))+1/h^4*(2*a^
2*d*h^6-2*a^2*e*g*h^5+2*a^2*f*g^2*h^4+4*a*c*d*g^2*h^4-4*a*c*e*g^3*h^3+4*a*c
*f*g^4*h^2+2*c^2*d*g^4*h^2-2*c^2*e*g^5*h+2*c^2*f*g^6)*(-1/2/(a*h^2+c*g^2)*h
^2/(x+1/h*g)^2*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)+3/
2*c*g*h/(a*h^2+c*g^2)*(-1/(a*h^2+c*g^2)*h^2/(x+1/h*g)*((x+1/h*g)^2*c-2*c*g/
h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)-c*g*h/(a*h^2+c*g^2)/((a*h^2+c*g^2)/h^2
)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1/h*g)+2*((a*h^2+c*g^2)/h^2)^(1/
2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+1/h*g)))+1
/2*c/(a*h^2+c*g^2)*h^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*
c*g/h*(x+1/h*g)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g
)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+1/h*g))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+cx^2)^{3/2}\,(d+ex+fx^2)}{(g+hx)^3}\,dx = \text{Timed out}$$

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{(a+cx^2)^{3/2}\,(d+ex+fx^2)}{(g+hx)^3}\,dx = \int \frac{(a+cx^2)^{\frac{3}{2}}\,(d+ex+fx^2)}{(g+hx)^3}\,dx$$

```
[In] integrate((c*x**2+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**3,x)
```

```
[Out] Integral((a + c*x**2)**(3/2)*(d + e*x + f*x**2)/(g + h*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1299 vs. $2(459) = 918$.

Time $= 0.29$ (sec) , antiderivative size $= 1299$, normalized size of antiderivative $= 2.66$

$$\int \frac{(a+cx^2)^{3/2}\,(d+ex+fx^2)}{(g+hx)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^3,x, algorithm="maxima")
```

```
[Out] 3/2*sqrt(c*x^2 + a)*c^2*f*g^4/(c*g^2*h^5 + a*h^7) - 3/2*sqrt(c*x^2 + a)*c^2
*f*g^3*x/(c*g^2*h^4 + a*h^6) - 3/2*sqrt(c*x^2 + a)*c^2*e*g^3/(c*g^2*h^4 + a
*h^6) + 1/2*(c*x^2 + a)^(3/2)*c*f*g^3/(c*g^2*h^4*x + a*h^6*x + c*g^3*h^3 +
a*g*h^5) + 3/2*sqrt(c*x^2 + a)*c^2*e*g^2*x/(c*g^2*h^3 + a*h^5) + 3/2*sqrt(c
*x^2 + a)*c^2*d*g^2/(c*g^2*h^3 + a*h^5) - 1/2*(c*x^2 + a)^(3/2)*c*e*g^2/(c*
g^2*h^3*x + a*h^5*x + c*g^3*h^2 + a*g*h^4) - 1/2*(c*x^2 + a)^(5/2)*f*g^2/(c
*g^2*h^3*x^2 + a*h^5*x^2 + 2*c*g^3*h^2*x + 2*a*g*h^4*x + c*g^4*h + a*g^2*h^
3) + 1/2*(c*x^2 + a)^(3/2)*c*f*g^2/(c*g^2*h^3 + a*h^5) - 3/2*sqrt(c*x^2 + a
)*c^2*d*g*x/(c*g^2*h^2 + a*h^4) + 1/2*(c*x^2 + a)^(3/2)*c*d*g/(c*g^2*h^2*x
+ a*h^4*x + c*g^3*h + a*g*h^3) + 1/2*(c*x^2 + a)^(5/2)*e*g/(c*g^2*h^2*x^2 +
 a*h^4*x^2 + 2*c*g^3*h*x + 2*a*g*h^3*x + c*g^4 + a*g^2*h^2) - 1/2*(c*x^2 +
a)^(3/2)*c*e*g/(c*g^2*h^2 + a*h^4) - 1/2*(c*x^2 + a)^(5/2)*d/(c*g^2*h*x^2 +
 a*h^3*x^2 + 2*c*g^3*x + 2*a*g*h^2*x + c*g^4/h + a*g^2*h) + 1/2*(c*x^2 + a)
^(3/2)*c*d/(c*g^2*h + a*h^3) + 2*(c*x^2 + a)^(3/2)*f*g/(h^4*x + g*h^3) - (c
```

```
*x^2 + a)^(3/2)*e/(h^3*x + g*h^2) - 7/2*sqrt(c*x^2 + a)*c*f*g*x/h^4 + 3/2*s
qrt(c*x^2 + a)*c*e*x/h^3 - 10*c^(3/2)*f*g^3*arcsinh(c*x/sqrt(a*c))/h^6 + 6*
c^(3/2)*e*g^2*arcsinh(c*x/sqrt(a*c))/h^5 - 3*c^(3/2)*d*g*arcsinh(c*x/sqrt(a
*c))/h^4 - 9/2*a*sqrt(c)*f*g*arcsinh(c*x/sqrt(a*c))/h^4 + 3/2*a*sqrt(c)*e*a
rcsinh(c*x/sqrt(a*c))/h^3 + 3/2*c^2*f*g^4*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x
+ g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/(sqrt(a + c*g^2/h^2)*h^7) - 3/2*c^2*e
*g^3*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))
/(sqrt(a + c*g^2/h^2)*h^6) + 3/2*c^2*d*g^2*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x
 + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/(sqrt(a + c*g^2/h^2)*h^5) + 15/2*sqr
t(a + c*g^2/h^2)*c*f*g^2*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt
(a*c)*abs(h*x + g)))/h^5 - 9/2*sqrt(a + c*g^2/h^2)*c*e*g*arcsinh(c*g*x/(sqr
t(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/h^4 + 3/2*sqrt(a + c*g
^2/h^2)*c*d*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x
 + g)))/h^3 + (a + c*g^2/h^2)^(3/2)*f*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)
) - a*h/(sqrt(a*c)*abs(h*x + g)))/h^3 + 17/2*sqrt(c*x^2 + a)*c*f*g^2/h^5 -
9/2*sqrt(c*x^2 + a)*c*e*g/h^4 + 3/2*sqrt(c*x^2 + a)*c*d/h^3 + 1/3*(c*x^2 +
a)^(3/2)*f/h^3 + sqrt(c*x^2 + a)*a*f/h^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1021 vs. 2(459) = 918.

Time = 0.35 (sec) , antiderivative size = 1021, normalized size of antiderivative = 2.09

$$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^3}\,dx = \frac{1}{6}\sqrt{cx^2+a}\left(x\left(\frac{2cfx}{h^3} - \frac{3(3c^2fgh^{14}-c^2eh^{15})}{ch^{18}}\right) + \frac{2(18c^2fg^2h^{13}-9\right.$$

$$+ \frac{\left(20c^{\frac{3}{2}}fg^3 - 12c^{\frac{3}{2}}eg^2h + 6c^{\frac{3}{2}}dgh^2 + 9a\sqrt{c}fgh^2 - 3a\sqrt{c}eh^3\right)\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+a}\right|\right)}{2h^6}$$

$$+ \frac{(20c^2fg^4 - 12c^2eg^3h + 6c^2dg^2h^2 + 19acfg^2h^2 - 9acegh^3 + 3acdh^4 + 2a^2fh^4)\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+a}\right)h}{\sqrt{-cg^2-ah^2}}\right.}{\sqrt{-cg^2-ah^2}\,h^6}$$

$$+ \frac{10\left(\sqrt{c}x-\sqrt{cx^2+a}\right)^3c^2fg^4h - 8\left(\sqrt{c}x-\sqrt{cx^2+a}\right)^3c^2eg^3h^2 + 6\left(\sqrt{c}x-\sqrt{cx^2+a}\right)^3c^2dg^2h^3 + 5\left(\sqrt{c}x-\right.}{}$$

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^3,x, algorithm="giac")
```

```
[Out] 1/6*sqrt(c*x^2 + a)*(x*(2*c*f*x/h^3 - 3*(3*c^2*f*g*h^14 - c^2*e*h^15)/(c*h^
18)) + 2*(18*c^2*f*g^2*h^13 - 9*c^2*e*g*h^14 + 3*c^2*d*h^15 + 4*a*c*f*h^15)
/(c*h^18)) + 1/2*(20*c^(3/2)*f*g^3 - 12*c^(3/2)*e*g^2*h + 6*c^(3/2)*d*g*h^2
 + 9*a*sqrt(c)*f*g*h^2 - 3*a*sqrt(c)*e*h^3)*log(abs(-sqrt(c)*x + sqrt(c*x^2
 + a)))/h^6 + (20*c^2*f*g^4 - 12*c^2*e*g^3*h + 6*c^2*d*g^2*h^2 + 19*a*c*f*g
^2*h^2 - 9*a*c*e*g*h^3 + 3*a*c*d*h^4 + 2*a^2*f*h^4)*arctan(-((sqrt(c)*x - s
qrt(c*x^2 + a))*h + sqrt(c)*g)/sqrt(-c*g^2 - a*h^2))/(sqrt(-c*g^2 - a*h^2)*
```

```
h^6) + (10*(sqrt(c)*x - sqrt(c*x^2 + a))^3*c^2*f*g^4*h - 8*(sqrt(c)*x - sqr
t(c*x^2 + a))^3*c^2*e*g^3*h^2 + 6*(sqrt(c)*x - sqrt(c*x^2 + a))^3*c^2*d*g^2
*h^3 + 5*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a*c*f*g^2*h^3 - 3*(sqrt(c)*x - sqr
t(c*x^2 + a))^3*a*c*e*g*h^4 + (sqrt(c)*x - sqrt(c*x^2 + a))^3*a*c*d*h^5 + 1
8*(sqrt(c)*x - sqrt(c*x^2 + a))^2*c^(5/2)*f*g^5 - 14*(sqrt(c)*x - sqrt(c*x^
2 + a))^2*c^(5/2)*e*g^4*h + 10*(sqrt(c)*x - sqrt(c*x^2 + a))^2*c^(5/2)*d*g^
3*h^2 - (sqrt(c)*x - sqrt(c*x^2 + a))^2*a*c^(3/2)*f*g^3*h^2 + 3*(sqrt(c)*x
- sqrt(c*x^2 + a))^2*a*c^(3/2)*e*g^2*h^3 - 5*(sqrt(c)*x - sqrt(c*x^2 + a))^
2*a*c^(3/2)*d*g*h^4 - 4*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^2*sqrt(c)*f*g*h^4
 + 2*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^2*sqrt(c)*e*h^5 - 26*(sqrt(c)*x - sq
rt(c*x^2 + a))*a*c^2*f*g^4*h + 20*(sqrt(c)*x - sqrt(c*x^2 + a))*a*c^2*e*g^3
*h^2 - 14*(sqrt(c)*x - sqrt(c*x^2 + a))*a*c^2*d*g^2*h^3 - 11*(sqrt(c)*x - s
qrt(c*x^2 + a))*a^2*c*f*g^2*h^3 + 5*(sqrt(c)*x - sqrt(c*x^2 + a))*a^2*c*e*g
*h^4 + (sqrt(c)*x - sqrt(c*x^2 + a))*a^2*c*d*h^5 + 9*a^2*c^(3/2)*f*g^3*h^2
- 7*a^2*c^(3/2)*e*g^2*h^3 + 5*a^2*c^(3/2)*d*g*h^4 + 4*a^3*sqrt(c)*f*g*h^4 -
 2*a^3*sqrt(c)*e*h^5)/(((sqrt(c)*x - sqrt(c*x^2 + a))^2*h + 2*(sqrt(c)*x -
sqrt(c*x^2 + a))*sqrt(c)*g - a*h)^2*h^6)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^3}\,dx=\int \frac{{\left(c\,x^2+a\right)}^{3/2}\,\left(f\,x^2+e\,x+d\right)}{{\left(g+h\,x\right)}^3}\,dx$$

```
[In] int(((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^3,x)
[Out] int(((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^3, x)
```

3.95 $\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^4}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 475

$$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^4}\,dx =$$

$$-\frac{((cg^2+ah^2)(3afh^2+2c(10fg^2-h(4eg-dh)))+ch(3ah^2(3fg-eh)+cg(10fg^2-h(4eg-dh)))x)\sqrt{}}{2h^5(cg^2+ah^2)(g+hx)}$$

$$-\frac{\left(cg\left(4eg-\frac{10fg^2}{h}-dh\right)-3ah(3fg-eh)-(3afh^2+c(5fg^2-2h(eg-dh)))x\right)(a+cx^2)^{3/2}}{6h^2(cg^2+ah^2)(g+hx)^2}$$

$$-\frac{(fg^2-egh+dh^2)(a+cx^2)^{5/2}}{3h(cg^2+ah^2)(g+hx)^3}+\frac{\sqrt{c}(3afh^2+2c(10fg^2-h(4eg-dh)))\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2h^6}$$

$$+\frac{c(3a^2h^4(4fg-eh)+2c^2g^3(10fg^2-h(4eg-dh))+3acgh^2(11fg^2-h(4eg-dh)))\operatorname{arctanh}\left(\frac{ah-cgx}{\sqrt{cg^2+ah^2}\sqrt{a+}}\right.}{2h^6(cg^2+ah^2)^{3/2}}$$

```
[Out] -1/6*(c*g*(4*e*g-10*f*g^2/h-d*h)-3*a*h*(-e*h+3*f*g)-(3*a*f*h^2+c*(5*f*g^2-2
*h*(-d*h+e*g)))*x)*(c*x^2+a)^(3/2)/h^2/(a*h^2+c*g^2)/(h*x+g)^2-1/3*(d*h^2-e
*g*h+f*g^2)*(c*x^2+a)^(5/2)/h/(a*h^2+c*g^2)/(h*x+g)^3+1/2*c*(3*a^2*h^4*(-e*
h+4*f*g)+2*c^2*g^3*(10*f*g^2-h*(-d*h+4*e*g))+3*a*c*g*h^2*(11*f*g^2-h*(-d*h+
4*e*g)))*arctanh((-c*g*x+a*h)/(a*h^2+c*g^2)^(1/2)/(c*x^2+a)^(1/2))/h^6/(a*h
^2+c*g^2)^(3/2)+1/2*(3*a*f*h^2+2*c*(10*f*g^2-h*(-d*h+4*e*g)))*arctanh(x*c^(
1/2)/(c*x^2+a)^(1/2))*c^(1/2)/h^6-1/2*((a*h^2+c*g^2)*(3*a*f*h^2+2*c*(10*f*g
^2-h*(-d*h+4*e*g)))+c*h*(3*a*h^2*(-e*h+3*f*g)+c*g*(10*f*g^2-h*(-d*h+4*e*g))
)*x)*(c*x^2+a)^(1/2)/h^5/(a*h^2+c*g^2)/(h*x+g)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 469, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1665, 827, 858, 223, 212, 739}

$$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^4}\,dx = \frac{c\operatorname{arctanh}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)\left(3a^2h^4(4fg-eh)+3acgh^2(11fg^2-h(4eg-\right.}{2h^6\left(ah^2+cg^2\right)^{3/2}}$$
$$+\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(3afh^2-2ch(4eg-dh)+20cfg^2\right)}{2h^6}-\frac{(a+cx^2)^{5/2}\left(dh^2-egh+fg^2\right)}{3h(g+hx)^3\left(ah^2+cg^2\right)}-\frac{(a+cx^2)^{3/2}\left(-\right.}{}$$
$$-\frac{\sqrt{a+cx^2}\left(chx(3ah^2(3fg-eh)-cgh(4eg-dh)+10cfg^3)+\left(ah^2+cg^2\right)\left(3afh^2-2ch(4eg-dh)+20cfg\right.\right.}{2h^5(g+hx)\left(ah^2+cg^2\right)}$$

```
[In] Int[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^4,x]
```

```
[Out] -1/2*(((c*g^2 + a*h^2)*(20*c*f*g^2 + 3*a*f*h^2 - 2*c*h*(4*e*g - d*h)) + c*h
*(10*c*f*g^3 - c*g*h*(4*e*g - d*h) + 3*a*h^2*(3*f*g - e*h))*x)*Sqrt[a + c*x
^2])/(h^5*(c*g^2 + a*h^2)*(g + h*x)) - ((c*g*(4*e*g - (10*f*g^2)/h - d*h) -
 3*a*h*(3*f*g - e*h) - (5*c*f*g^2 + 3*a*f*h^2 - 2*c*h*(e*g - d*h))*x)*(a +
c*x^2)^(3/2))/(6*h^2*(c*g^2 + a*h^2)*(g + h*x)^2) - ((f*g^2 - e*g*h + d*h^2
)*(a + c*x^2)^(5/2))/(3*h*(c*g^2 + a*h^2)*(g + h*x)^3) + (Sqrt[c]*(20*c*f*g
^2 + 3*a*f*h^2 - 2*c*h*(4*e*g - d*h))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])
/(2*h^6) + (c*(3*a^2*h^4*(4*f*g - e*h) + 3*a*c*g*h^2*(11*f*g^2 - h*(4*e*g -
 d*h)) + 2*c^2*(10*f*g^5 - g^3*h*(4*e*g - d*h)))*ArcTanh[(a*h - c*g*x)/(Sqr
t[c*g^2 + a*h^2]*Sqrt[a + c*x^2])])/(2*h^6*(c*g^2 + a*h^2)^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 827

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] && !Rational
Q[m])) && NeQ[m, -1] && !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(fg^2-egh+dh^2)\left(a+cx^2\right)^{5/2}}{3h\left(cg^2+ah^2\right)(g+hx)^3} - \frac{\int \frac{\left(-3(cdg-afg+aeh)-\left(3afh-c\left(2eg-\frac{5fg^2}{h}-2dh\right)\right)x\right)\left(a+cx^2\right)^{3/2}}{(g+hx)^3}\,dx}{3\left(cg^2+ah^2\right)} \\
&= -\frac{\left(cg\left(4eg-\frac{10fg^2}{h}-dh\right)-3ah(3fg-eh)-\left(5cfg^2+3afh^2-2ch(eg-dh)\right)x\right)\left(a+cx^2\right)^{3/2}}{6h^2\left(cg^2+ah^2\right)(g+hx)^2} \\
&\quad - \frac{(fg^2-egh+dh^2)\left(a+cx^2\right)^{5/2}}{3h\left(cg^2+ah^2\right)(g+hx)^3} \\
&\quad + \frac{\int \frac{\left(4a\left(5cfg^2+3afh^2-2ch(eg-dh)\right)-\frac{4c\left(10cfg^3-cgh(4eg-dh)+3ah^2(3fg-eh)\right)x}{h}\right)\sqrt{a+cx^2}}{(g+hx)^2}\,dx}{8h^2\left(cg^2+ah^2\right)}
\end{aligned}$$

$$= -\frac{((cg^2+ah^2)(20cfg^2+3afh^2-2ch(4eg-dh))+ch(10cfg^3-cgh(4eg-dh)+3ah^2(3fg-eh}{2h^5(cg^2+ah^2)(g+hx)}$$

$$-\frac{\left(cg\left(4eg-\frac{10fg^2}{h}-dh\right)-3ah(3fg-eh)-(5cfg^2+3afh^2-2ch(eg-dh))x\right)(a+cx^2)^{3/2}}{6h^2(cg^2+ah^2)(g+hx)^2}$$

$$-\frac{(fg^2-egh+dh^2)(a+cx^2)^{5/2}}{3h(cg^2+ah^2)(g+hx)^3}$$

$$-\frac{\int \frac{8ac\left(10cfg^3-cgh(4eg-dh)+3ah^2(3fg-eh)\right)-\frac{8c\left(cg^2+ah^2\right)\left(20cfg^2+3afh^2-2ch(4eg-dh)\right)x}{h}}{(g+hx)\sqrt{a+cx^2}}\,dx}{16h^4(cg^2+ah^2)}$$

$$= -\frac{((cg^2+ah^2)(20cfg^2+3afh^2-2ch(4eg-dh))+ch(10cfg^3-cgh(4eg-dh)+3ah^2(3fg-eh}{2h^5(cg^2+ah^2)(g+hx)}$$

$$-\frac{\left(cg\left(4eg-\frac{10fg^2}{h}-dh\right)-3ah(3fg-eh)-(5cfg^2+3afh^2-2ch(eg-dh))x\right)(a+cx^2)^{3/2}}{6h^2(cg^2+ah^2)(g+hx)^2}$$

$$-\frac{(fg^2-egh+dh^2)(a+cx^2)^{5/2}}{3h(cg^2+ah^2)(g+hx)^3}$$

$$+\frac{(c(20cfg^2+3afh^2-2ch(4eg-dh)))\int \frac{1}{\sqrt{a+cx^2}}\,dx}{2h^6}$$

$$-\frac{(c(3a^2h^4(4fg-eh)+3acgh^2(11fg^2-h(4eg-dh))+2c^2(10fg^5-g^3h(4eg-dh))))\int \frac{1}{(g+hx)\sqrt{a-}}}{2h^6(cg^2+ah^2)}$$

$$= -\frac{((cg^2+ah^2)(20cfg^2+3afh^2-2ch(4eg-dh))+ch(10cfg^3-cgh(4eg-dh)+3ah^2(3fg-eh}{2h^5(cg^2+ah^2)(g+hx)}$$

$$-\frac{\left(cg\left(4eg-\frac{10fg^2}{h}-dh\right)-3ah(3fg-eh)-(5cfg^2+3afh^2-2ch(eg-dh))x\right)(a+cx^2)^{3/2}}{6h^2(cg^2+ah^2)(g+hx)^2}$$

$$-\frac{(fg^2-egh+dh^2)(a+cx^2)^{5/2}}{3h(cg^2+ah^2)(g+hx)^3}$$

$$+\frac{(c(20cfg^2+3afh^2-2ch(4eg-dh)))\,\mathrm{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x}{\sqrt{a+cx^2}}\right)}{2h^6}$$

$$+\frac{(c(3a^2h^4(4fg-eh)+3acgh^2(11fg^2-h(4eg-dh))+2c^2(10fg^5-g^3h(4eg-dh))))\,\mathrm{Subst}\left(\int \frac{}{cg}\right.}{2h^6(cg^2+ah^2)}$$

$$
\begin{aligned}
=& \\
&-\frac{((cg^2+ah^2)\,(20cfg^2+3afh^2-2ch(4eg-dh))+ch(10cfg^3-cgh(4eg-dh)+3ah^2(3fg-e}{2h^5\,(cg^2+ah^2)\,(g+hx)} \\
&-\frac{\left(cg\left(4eg-\frac{10fg^2}{h}-dh\right)-3ah(3fg-eh)-(5cfg^2+3afh^2-2ch(eg-dh))\,x\right)(a+cx^2)^{3/2}}{6h^2\,(cg^2+ah^2)\,(g+hx)^2} \\
&-\frac{(fg^2-egh+dh^2)\,(a+cx^2)^{5/2}}{3h\,(cg^2+ah^2)\,(g+hx)^3} \\
&+\frac{\sqrt{c}(20cfg^2+3afh^2-2ch(4eg-dh))\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2h^6} \\
&+\frac{c(3a^2h^4(4fg-eh)+3acgh^2(11fg^2-h(4eg-dh))+2c^2(10fg^5-g^3h(4eg-dh)))\tanh^{-1}\Big(}{2h^6\,(cg^2+ah^2)^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 11.03 (sec) , antiderivative size = 517, normalized size of antiderivative = 1.09

$$
\int\frac{(a+cx^2)^{3/2}\,(d+ex+fx^2)}{(g+hx)^4}\,dx=\frac{-\frac{h\sqrt{a+cx^2}\big(2(cg^2+ah^2)^2(fg^2+h(-eg+dh))-(cg^2+ah^2)(13cfg^3+cgh(-10eg+7dh)-3ah^2(-2}{}}{}
$$

```
[In] Integrate[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^4,x]
```

```
[Out] (-((h*Sqrt[a + c*x^2]*(2*(c*g^2 + a*h^2)^2*(f*g^2 + h*(-(e*g) + d*h)) - (c*
g^2 + a*h^2)*(13*c*f*g^3 + c*g*h*(-10*e*g + 7*d*h) - 3*a*h^2*(-2*f*g + e*h)
)*(g + h*x) + (6*a^2*f*h^4 + a*c*h^2*(50*f*g^2 + h*(-23*e*g + 8*d*h)) + c^2
*(47*f*g^4 + g^2*h*(-26*e*g + 11*d*h)))*(g + h*x)^2 + 6*c*(4*f*g - e*h)*(c*
g^2 + a*h^2)*(g + h*x)^3 - 3*c*f*h*(c*g^2 + a*h^2)*x*(g + h*x)^3))/((c*g^2
+ a*h^2)*(g + h*x)^3)) - (3*c*(-3*a^2*h^4*(-4*f*g + e*h) + 3*a*c*g*h^2*(11*
f*g^2 + h*(-4*e*g + d*h)) + 2*c^2*(10*f*g^5 + g^3*h*(-4*e*g + d*h)))*Log[g
+ h*x])/(c*g^2 + a*h^2)^(3/2) + 3*Sqrt[c]*(20*c*f*g^2 + 3*a*f*h^2 + 2*c*h*(
-4*e*g + d*h))*Log[c*x + Sqrt[c]*Sqrt[a + c*x^2]] + (3*c*(-3*a^2*h^4*(-4*f*
g + e*h) + 3*a*c*g*h^2*(11*f*g^2 + h*(-4*e*g + d*h)) + 2*c^2*(10*f*g^5 + g^
3*h*(-4*e*g + d*h)))*Log[a*h - c*g*x + Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2]]
)/(c*g^2 + a*h^2)^(3/2))/(6*h^6)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1916 vs. 2(447) = 894.

Time = 0.70 (sec) , antiderivative size = 1917, normalized size of antiderivative = 4.04

method	result	size
risch	Expression too large to display	1917
default	Expression too large to display	4813

```
[In] int((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*c*(f*h*x+2*e*h-8*f*g)*(c*x^2+a)^(1/2)/h^5+1/2/h^5*(c^(1/2)*(3*a*f*h^2+2
*c*d*h^2-8*c*e*g*h+20*c*f*g^2)/h*ln(x*c^(1/2)+(c*x^2+a)^(1/2))-4*c/h^2*(a*e
*h^3-4*a*f*g*h^2-2*c*d*g*h^2+5*c*e*g^2*h-10*c*f*g^3)/((a*h^2+c*g^2)/h^2)^(1
/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1/h*g)+2*((a*h^2+c*g^2)/h^2)^(1/2)*(
(x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+1/h*g))+(2*a^2
*f*h^4+4*a*c*d*h^4-12*a*c*e*g*h^3+24*a*c*f*g^2*h^2+12*c^2*d*g^2*h^2-20*c^2*
e*g^3*h+30*c^2*f*g^4)/h^3*(-1/(a*h^2+c*g^2)*h^2/(x+1/h*g)*((x+1/h*g)^2*c-2*
c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)-c*g*h/(a*h^2+c*g^2)/((a*h^2+c*g^2)
/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1/h*g)+2*((a*h^2+c*g^2)/h^2)
^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+1/h*g)
))+(2*a^2*e*h^5-4*a^2*f*g*h^4-8*a*c*d*g*h^4+12*a*c*e*g^2*h^3-16*a*c*f*g^3*h
^2-8*c^2*d*g^3*h^2+10*c^2*e*g^4*h-12*c^2*f*g^5)/h^4*(-1/2/(a*h^2+c*g^2)*h^2
/(x+1/h*g)^2*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)+3/2*
c*g*h/(a*h^2+c*g^2)*(-1/(a*h^2+c*g^2)*h^2/(x+1/h*g)*((x+1/h*g)^2*c-2*c*g/h*
(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)-c*g*h/(a*h^2+c*g^2)/((a*h^2+c*g^2)/h^2)^
(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1/h*g)+2*((a*h^2+c*g^2)/h^2)^(1/2)
*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+1/h*g)))+1/2
*c/(a*h^2+c*g^2)*h^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*
g/h*(x+1/h*g)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+
(a*h^2+c*g^2)/h^2)^(1/2))/(x+1/h*g)))+1/h^5*(2*a^2*d*h^6-2*a^2*e*g*h^5+2*a^
2*f*g^2*h^4+4*a*c*d*g^2*h^4-4*a*c*e*g^3*h^3+4*a*c*f*g^4*h^2+2*c^2*d*g^4*h^2
-2*c^2*e*g^5*h+2*c^2*f*g^6)*(-1/3/(a*h^2+c*g^2)*h^2/(x+1/h*g)^3*((x+1/h*g)^
2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)+5/3*c*g*h/(a*h^2+c*g^2)*(-1/
2/(a*h^2+c*g^2)*h^2/(x+1/h*g)^2*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g
^2)/h^2)^(1/2)+3/2*c*g*h/(a*h^2+c*g^2)*(-1/(a*h^2+c*g^2)*h^2/(x+1/h*g)*((x+
1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)-c*g*h/(a*h^2+c*g^2)/(
(a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1/h*g)+2*((a*h^
2+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/
2))/(x+1/h*g)))+1/2*c/(a*h^2+c*g^2)*h^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*
h^2+c*g^2)/h^2-2*c*g/h*(x+1/h*g)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c
-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+1/h*g)))-2/3*c/(a*h^2+c*g^2
)*h^2*(-1/(a*h^2+c*g^2)*h^2/(x+1/h*g)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h
^2+c*g^2)/h^2)^(1/2)-c*g*h/(a*h^2+c*g^2)/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a
*h^2+c*g^2)/h^2-2*c*g/h*(x+1/h*g)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*
```

```
c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+1/h*g)))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^4}\,dx=\text{Timed out}$$

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^4}\,dx=\int \frac{\left(a+cx^2\right)^{\frac{3}{2}}\left(d+ex+fx^2\right)}{\left(g+hx\right)^4}\,dx$$

```
[In] integrate((c*x**2+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**4,x)
```

```
[Out] Integral((a + c*x**2)**(3/2)*(d + e*x + f*x**2)/(g + h*x)**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2415 vs. 2(448) = 896.

Time = 0.33 (sec) , antiderivative size = 2415, normalized size of antiderivative = 5.08

$$\int \frac{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^4}\,dx=\text{Too large to display}$$

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^4,x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(c*x^2 + a)*c^3*f*g^5/(c^2*g^4*h^5 + 2*a*c*g^2*h^7 + a^2*h^9) - 1/2
*sqrt(c*x^2 + a)*c^3*f*g^4*x/(c^2*g^4*h^4 + 2*a*c*g^2*h^6 + a^2*h^8) - 1/2*
sqrt(c*x^2 + a)*c^3*e*g^4/(c^2*g^4*h^4 + 2*a*c*g^2*h^6 + a^2*h^8) + 1/6*(c*
x^2 + a)^(3/2)*c^2*f*g^4/(c^2*g^4*h^4*x + 2*a*c*g^2*h^6*x + a^2*h^8*x + c^2
*g^5*h^3 + 2*a*c*g^3*h^5 + a^2*g*h^7) + 1/2*sqrt(c*x^2 + a)*c^3*e*g^3*x/(c^
2*g^4*h^3 + 2*a*c*g^2*h^5 + a^2*h^7) + 1/2*sqrt(c*x^2 + a)*c^3*d*g^3/(c^2*g
^4*h^3 + 2*a*c*g^2*h^5 + a^2*h^7) - 1/6*(c*x^2 + a)^(3/2)*c^2*e*g^3/(c^2*g^
4*h^3*x + 2*a*c*g^2*h^5*x + a^2*h^7*x + c^2*g^5*h^2 + 2*a*c*g^3*h^4 + a^2*g
*h^6) - 1/6*(c*x^2 + a)^(5/2)*c*f*g^3/(c^2*g^4*h^3*x^2 + 2*a*c*g^2*h^5*x^2
+ a^2*h^7*x^2 + 2*c^2*g^5*h^2*x + 4*a*c*g^3*h^4*x + 2*a^2*g*h^6*x + c^2*g^6
*h + 2*a*c*g^4*h^3 + a^2*g^2*h^5) + 1/6*(c*x^2 + a)^(3/2)*c^2*f*g^3/(c^2*g^
```

```
4*h^3 + 2*a*c*g^2*h^5 + a^2*h^7) - 1/2*sqrt(c*x^2 + a)*c^3*d*g^2*x/(c^2*g^4
*h^2 + 2*a*c*g^2*h^4 + a^2*h^6) + 1/6*(c*x^2 + a)^(3/2)*c^2*d*g^2/(c^2*g^4*
h^2*x + 2*a*c*g^2*h^4*x + a^2*h^6*x + c^2*g^5*h + 2*a*c*g^3*h^3 + a^2*g*h^5
) + 1/6*(c*x^2 + a)^(5/2)*c*e*g^2/(c^2*g^4*h^2*x^2 + 2*a*c*g^2*h^4*x^2 + a^
2*h^6*x^2 + 2*c^2*g^5*h*x + 4*a*c*g^3*h^3*x + 2*a^2*g*h^5*x + c^2*g^6 + 2*a
*c*g^4*h^2 + a^2*g^2*h^4) - 1/6*(c*x^2 + a)^(3/2)*c^2*e*g^2/(c^2*g^4*h^2 +
2*a*c*g^2*h^4 + a^2*h^6) - 9/2*sqrt(c*x^2 + a)*c^2*f*g^3/(c*g^2*h^5 + a*h^7
) + 4*sqrt(c*x^2 + a)*c^2*f*g^2*x/(c*g^2*h^4 + a*h^6) - 1/6*(c*x^2 + a)^(5/
2)*c*d*g/(c^2*g^4*h*x^2 + 2*a*c*g^2*h^3*x^2 + a^2*h^5*x^2 + 2*c^2*g^5*x + 4
*a*c*g^3*h^2*x + 2*a^2*g*h^4*x + c^2*g^6/h + 2*a*c*g^4*h + a^2*g^2*h^3) + 1
/6*(c*x^2 + a)^(3/2)*c^2*d*g/(c^2*g^4*h + 2*a*c*g^2*h^3 + a^2*h^5) + 3*sqrt
(c*x^2 + a)*c^2*e*g^2/(c*g^2*h^4 + a*h^6) - 1/3*(c*x^2 + a)^(5/2)*f*g^2/(c*
g^2*h^4*x^3 + a*h^6*x^3 + 3*c*g^3*h^3*x^2 + 3*a*g*h^5*x^2 + 3*c*g^4*h^2*x +
 3*a*g^2*h^4*x + c*g^5*h + a*g^3*h^3) - 5/3*(c*x^2 + a)^(3/2)*c*f*g^2/(c*g^
2*h^4*x + a*h^6*x + c*g^3*h^3 + a*g*h^5) - 5/2*sqrt(c*x^2 + a)*c^2*e*g*x/(c
*g^2*h^3 + a*h^5) - 3/2*sqrt(c*x^2 + a)*c^2*d*g/(c*g^2*h^3 + a*h^5) + 1/3*(
c*x^2 + a)^(5/2)*e*g/(c*g^2*h^3*x^3 + a*h^5*x^3 + 3*c*g^3*h^2*x^2 + 3*a*g*h
^4*x^2 + 3*c*g^4*h*x + 3*a*g^2*h^3*x + c*g^5 + a*g^3*h^2) + 7/6*(c*x^2 + a)
^(3/2)*c*e*g/(c*g^2*h^3*x + a*h^5*x + c*g^3*h^2 + a*g*h^4) + (c*x^2 + a)^(5
/2)*f*g/(c*g^2*h^3*x^2 + a*h^5*x^2 + 2*c*g^3*h^2*x + 2*a*g*h^4*x + c*g^4*h
 + a*g^2*h^3) - (c*x^2 + a)^(3/2)*c*f*g/(c*g^2*h^3 + a*h^5) + sqrt(c*x^2 + a
)*c^2*d*x/(c*g^2*h^2 + a*h^4) - 1/3*(c*x^2 + a)^(5/2)*d/(c*g^2*h^2*x^3 + a*
h^4*x^3 + 3*c*g^3*h*x^2 + 3*a*g*h^3*x^2 + 3*c*g^4*x + 3*a*g^2*h^2*x + c*g^5
/h + a*g^3*h) - 2/3*(c*x^2 + a)^(3/2)*c*d/(c*g^2*h^2*x + a*h^4*x + c*g^3*h
 + a*g*h^3) - 1/2*(c*x^2 + a)^(5/2)*e/(c*g^2*h^2*x^2 + a*h^4*x^2 + 2*c*g^3*h
*x + 2*a*g*h^3*x + c*g^4 + a*g^2*h^2) + 1/2*(c*x^2 + a)^(3/2)*c*e/(c*g^2*h^
2 + a*h^4) - (c*x^2 + a)^(3/2)*f/(h^4*x + g*h^3) + 3/2*sqrt(c*x^2 + a)*c*f*
x/h^4 + 10*c^(3/2)*f*g^2*arcsinh(c*x/sqrt(a*c))/h^6 - 4*c^(3/2)*e*g*arcsinh
(c*x/sqrt(a*c))/h^5 + c^(3/2)*d*arcsinh(c*x/sqrt(a*c))/h^4 + 3/2*a*sqrt(c)*
f*arcsinh(c*x/sqrt(a*c))/h^4 + 1/2*c^3*f*g^5*arcsinh(c*g*x/(sqrt(a*c)*abs(h
*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(3/2)*h^9) - 1/2*
c^3*e*g^4*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x +
 g)))/((a + c*g^2/h^2)^(3/2)*h^8) + 1/2*c^3*d*g^3*arcsinh(c*g*x/(sqrt(a*c)*
abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(3/2)*h^7) -
 9/2*c^2*f*g^3*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(
h*x + g)))/(sqrt(a + c*g^2/h^2)*h^7) + 3*c^2*e*g^2*arcsinh(c*g*x/(sqrt(a*c)
*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/(sqrt(a + c*g^2/h^2)*h^6) -
3/2*c^2*d*g*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x
 + g)))/(sqrt(a + c*g^2/h^2)*h^5) - 6*sqrt(a + c*g^2/h^2)*c*f*g*arcsinh(c*g
*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/h^5 + 3/2*sqrt(
a + c*g^2/h^2)*c*e*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*
abs(h*x + g)))/h^4 - 6*sqrt(c*x^2 + a)*c*f*g/h^5 + 3/2*sqrt(c*x^2 + a)*c*e/
h^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1878 vs. 2(448) = 896.

Time = 0.39 (sec) , antiderivative size = 1878, normalized size of antiderivative = 3.95

$$\int \frac{(a+cx^2)^{3/2}\,(d+ex+fx^2)}{(g+hx)^4}\,dx = \text{Too large to display}$$

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^4,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(c*x^2 + a)*(c*f*x/h^4 - 2*(4*c*f*g*h^10 - c*e*h^11)/h^15) - (20*c^
3*f*g^5 - 8*c^3*e*g^4*h + 2*c^3*d*g^3*h^2 + 33*a*c^2*f*g^3*h^2 - 12*a*c^2*e
*g^2*h^3 + 3*a*c^2*d*g*h^4 + 12*a^2*c*f*g*h^4 - 3*a^2*c*e*h^5)*arctan(-((sq
rt(c)*x - sqrt(c*x^2 + a))*h + sqrt(c)*g)/sqrt(-c*g^2 - a*h^2))/((c*g^2*h^6
 + a*h^8)*sqrt(-c*g^2 - a*h^2)) - 1/3*(60*(sqrt(c)*x - sqrt(c*x^2 + a))^5*c
^3*f*g^5*h^2 - 36*(sqrt(c)*x - sqrt(c*x^2 + a))^5*c^3*e*g^4*h^3 + 18*(sqrt(
c)*x - sqrt(c*x^2 + a))^5*c^3*d*g^3*h^4 + 69*(sqrt(c)*x - sqrt(c*x^2 + a))^
5*a*c^2*f*g^3*h^4 - 36*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a*c^2*e*g^2*h^5 + 15
*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a*c^2*d*g*h^6 + 12*(sqrt(c)*x - sqrt(c*x^2
 + a))^5*a^2*c*f*g*h^6 - 3*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^2*c*e*h^7 + 21
0*(sqrt(c)*x - sqrt(c*x^2 + a))^4*c^(7/2)*f*g^6*h - 120*(sqrt(c)*x - sqrt(c
*x^2 + a))^4*c^(7/2)*e*g^5*h^2 + 54*(sqrt(c)*x - sqrt(c*x^2 + a))^4*c^(7/2)
*d*g^4*h^3 + 183*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a*c^(5/2)*f*g^4*h^3 - 84*(
sqrt(c)*x - sqrt(c*x^2 + a))^4*a*c^(5/2)*e*g^3*h^4 + 27*(sqrt(c)*x - sqrt(c
*x^2 + a))^4*a*c^(5/2)*d*g^2*h^5 - 18*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^2*c
^(3/2)*f*g^2*h^5 + 21*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^2*c^(3/2)*e*g*h^6 -
 12*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^2*c^(3/2)*d*h^7 - 6*(sqrt(c)*x - sqrt
(c*x^2 + a))^4*a^3*sqrt(c)*f*h^7 + 188*(sqrt(c)*x - sqrt(c*x^2 + a))^3*c^4*
f*g^7 - 104*(sqrt(c)*x - sqrt(c*x^2 + a))^3*c^4*e*g^6*h + 44*(sqrt(c)*x - s
qrt(c*x^2 + a))^3*c^4*d*g^5*h^2 - 82*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a*c^3*
f*g^5*h^2 + 64*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a*c^3*e*g^4*h^3 - 34*(sqrt(c
)*x - sqrt(c*x^2 + a))^3*a*c^3*d*g^3*h^4 - 276*(sqrt(c)*x - sqrt(c*x^2 + a)
)^3*a^2*c^2*f*g^3*h^4 + 138*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^2*c^2*e*g^2*h
^5 - 48*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^2*c^2*d*g*h^6 - 36*(sqrt(c)*x - s
qrt(c*x^2 + a))^3*a^3*c*f*g*h^6 - 354*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a*c^(
7/2)*f*g^6*h + 192*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a*c^(7/2)*e*g^5*h^2 - 78
*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a*c^(7/2)*d*g^4*h^3 - 276*(sqrt(c)*x - sqr
t(c*x^2 + a))^2*a^2*c^(5/2)*f*g^4*h^3 + 114*(sqrt(c)*x - sqrt(c*x^2 + a))^2
*a^2*c^(5/2)*e*g^3*h^4 - 36*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^2*c^(5/2)*d*g
^2*h^5 + 60*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^3*c^(3/2)*f*g^2*h^5 - 48*(sqr
t(c)*x - sqrt(c*x^2 + a))^2*a^3*c^(3/2)*e*g*h^6 + 12*(sqrt(c)*x - sqrt(c*x^
2 + a))^2*a^3*c^(3/2)*d*h^7 + 12*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^4*sqrt(c
)*f*h^7 + 222*(sqrt(c)*x - sqrt(c*x^2 + a))*a^2*c^3*f*g^5*h^2 - 120*(sqrt(c
)*x - sqrt(c*x^2 + a))*a^2*c^3*e*g^4*h^3 + 48*(sqrt(c)*x - sqrt(c*x^2 + a))
*a^2*c^3*d*g^3*h^4 + 231*(sqrt(c)*x - sqrt(c*x^2 + a))*a^3*c^2*f*g^3*h^4 -
```

```
102*(sqrt(c)*x - sqrt(c*x^2 + a))*a^3*c^2*e*g^2*h^5 + 33*(sqrt(c)*x - sqrt(
c*x^2 + a))*a^3*c^2*d*g*h^6 + 24*(sqrt(c)*x - sqrt(c*x^2 + a))*a^4*c*f*g*h^
6 + 3*(sqrt(c)*x - sqrt(c*x^2 + a))*a^4*c*e*h^7 - 47*a^3*c^(5/2)*f*g^4*h^3
+ 26*a^3*c^(5/2)*e*g^3*h^4 - 11*a^3*c^(5/2)*d*g^2*h^5 - 50*a^4*c^(3/2)*f*g^
2*h^5 + 23*a^4*c^(3/2)*e*g*h^6 - 8*a^4*c^(3/2)*d*h^7 - 6*a^5*sqrt(c)*f*h^7)
/((c*g^2*h^6 + a*h^8)*((sqrt(c)*x - sqrt(c*x^2 + a))^2*h + 2*(sqrt(c)*x - s
qrt(c*x^2 + a))*sqrt(c)*g - a*h)^3) - 1/2*(20*c^(3/2)*f*g^2 - 8*c^(3/2)*e*g
*h + 2*c^(3/2)*d*h^2 + 3*a*sqrt(c)*f*h^2)*log(abs(-sqrt(c)*x + sqrt(c*x^2 +
 a)))/h^6
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+cx^2)^{3/2}\,(d+ex+fx^2)}{(g+hx)^4}\,dx=\int \frac{{\left(c\,x^2+a\right)}^{3/2}\,\left(f\,x^2+e\,x+d\right)}{{\left(g+h\,x\right)}^4}\,dx$$

```
[In] int(((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^4,x)
```

```
[Out] int(((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^4, x)
```

3.96 $\int \frac{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^5}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 511

$$\int \frac{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^5}\,dx = \frac{c\Big(8(5fg-eh)\left(cg^2+ah^2\right)^2+h(12a^2fh^4+4c^2g^3(5fg-eh)+ach^2(35}{8h^5\left(cg^2+ah^2\right)^2(g+hx)}$$
$$+\frac{(4a^2h^4(fg-2eh)-4c^2g^4(5fg-eh)-acgh^2(25fg^2-h(5eg-9dh))-3h(4a^2fh^4+ach^2(17fg^2-h(5eg}{24h^3\left(cg^2+ah^2\right)^2(g+hx)^3}$$
$$-\frac{\left(fg^2-egh+dh^2\right)\left(a+cx^2\right)^{5/2}}{4h\left(cg^2+ah^2\right)(g+hx)^4}-\frac{c^{3/2}(5fg-eh)\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{h^6}$$
$$-\frac{c(12a^3fh^6+8c^3g^5(5fg-eh)+20ac^2g^3h^2(5fg-eh)+3a^2ch^4(25fg^2-h(5eg-dh)))\,\text{arctanh}\Big(\frac{ah-c}{\sqrt{cg^2+ah^2}}}{8h^6\left(cg^2+ah^2\right)^{5/2}}$$

```
[Out] 1/24*(4*a^2*h^4*(-2*e*h+f*g)-4*c^2*g^4*(-e*h+5*f*g)-a*c*g*h^2*(25*f*g^2-h*(
-9*d*h+5*e*g))-3*h*(4*a^2*f*h^4+a*c*h^2*(17*f*g^2-h*(-d*h+5*e*g))+2*c^2*g^2
*(5*f*g^2-h*(d*h+e*g)))*x)*(c*x^2+a)^(3/2)/h^3/(a*h^2+c*g^2)^2/(h*x+g)^3-1/
4*(d*h^2-e*g*h+f*g^2)*(c*x^2+a)^(5/2)/h/(a*h^2+c*g^2)/(h*x+g)^4-c^(3/2)*(-e
*h+5*f*g)*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/h^6-1/8*c*(12*a^3*f*h^6+8*c^3*
g^5*(-e*h+5*f*g)+20*a*c^2*g^3*h^2*(-e*h+5*f*g)+3*a^2*c*h^4*(25*f*g^2-h*(-d*
h+5*e*g)))*arctanh((-c*g*x+a*h)/(a*h^2+c*g^2)^(1/2)/(c*x^2+a)^(1/2))/h^6/(a
*h^2+c*g^2)^(5/2)+1/8*c*(8*(-e*h+5*f*g)*(a*h^2+c*g^2)^2+h*(12*a^2*f*h^4+4*c
^2*g^3*(-e*h+5*f*g)+a*c*h^2*(35*f*g^2-h*(-3*d*h+7*e*g))))*x)*(c*x^2+a)^(1/2)
/h^5/(a*h^2+c*g^2)^2/(h*x+g)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 511, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {1665, 825, 827, 858, 223, 212, 739}

$$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^5}\,dx = \frac{(a+cx^2)^{3/2}\Big(-3x(4a^2fh^4+ach^2(17fg^2-h(5eg-dh))+2c^2(5fg^4-g}{24h^2(g+}$$

$$+\frac{c\sqrt{a+cx^2}\Big(hx(12a^2fh^4+ach^2(35fg^2-h(7eg-3dh))+4c^2g^3(5fg-eh))+8(ah^2+cg^2)^2(5fg-eh)\Big)}{8h^5(g+hx)(ah^2+cg^2)^2}$$

$$-\frac{c\operatorname{arctanh}\Big(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\Big)(12a^3fh^6+3a^2ch^4(25fg^2-h(5eg-dh))+20ac^2g^3h^2(5fg-eh)+8c^3g^5(5fg}{8h^6(ah^2+cg^2)^{5/2}}$$

$$-\frac{c^{3/2}\operatorname{arctanh}\Big(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\Big)(5fg-eh)}{h^6}-\frac{(a+cx^2)^{5/2}(dh^2-egh+fg^2)}{4h(g+hx)^4(ah^2+cg^2)}$$

```
[In] Int[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^5,x]
```

```
[Out] (c*(8*(5*f*g - e*h)*(c*g^2 + a*h^2)^2 + h*(12*a^2*f*h^4 + 4*c^2*g^3*(5*f*g
- e*h) + a*c*h^2*(35*f*g^2 - h*(7*e*g - 3*d*h)))*x)*Sqrt[a + c*x^2])/(8*h^5
*(c*g^2 + a*h^2)^2*(g + h*x)) + ((4*a^2*h^3*(f*g - 2*e*h) - (4*c^2*g^4*(5*f
*g - e*h))/h - a*c*g*h*(25*f*g^2 - h*(5*e*g - 9*d*h)) - 3*(4*a^2*f*h^4 + a*
c*h^2*(17*f*g^2 - h*(5*e*g - d*h)) + 2*c^2*(5*f*g^4 - g^2*h*(e*g + d*h)))*x
)*(a + c*x^2)^(3/2))/(24*h^2*(c*g^2 + a*h^2)^2*(g + h*x)^3) - ((f*g^2 - e*g
*h + d*h^2)*(a + c*x^2)^(5/2))/(4*h*(c*g^2 + a*h^2)*(g + h*x)^4) - (c^(3/2)
*(5*f*g - e*h)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/h^6 - (c*(12*a^3*f*h^6
 + 8*c^3*g^5*(5*f*g - e*h) + 20*a*c^2*g^3*h^2*(5*f*g - e*h) + 3*a^2*c*h^4*(
25*f*g^2 - h*(5*e*g - d*h)))*ArcTanh[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqr
t[a + c*x^2])])/(8*h^6*(c*g^2 + a*h^2)^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
```

```
[{a, c, d, e}, x]
```

Rule 825

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + c*x^2)^p/(e^2*(m + 1)*(m
+ 2)*(c*d^2 + a*e^2)))*((d*g - e*f*(m + 2))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*
f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) + 2*c*d*p*(e*f - d*g))*x), x] - Dis
t[p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*
(m + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f,
g}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p,
0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 827

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{(fg^2-egh+dh^2)(a+cx^2)^{5/2}}{4h(cg^2+ah^2)(g+hx)^4} - \frac{\int \frac{\left(-4(cdg-afg+aeh)-\left(4afh-c\left(eg-\frac{5fg^2}{h}-dh\right)\right)x\right)(a+cx^2)^{3/2}}{(g+hx)^4}\,dx}{4(cg^2+ah^2)}$$

$$= \frac{\bigl(4a^2h^3(fg-2eh)-\frac{4c^2g^4(5fg-eh)}{h}-acgh(25fg^2-h(5eg-9dh))-3(4a^2fh^4+ach^2(17fg^2-h(5e}{24h^2(cg^2+ah^2)^2(g+hx)^3}$$

$$-\frac{(fg^2-egh+dh^2)(a+cx^2)^{5/2}}{4h(cg^2+ah^2)(g+hx)^4}$$

$$+\frac{\int \frac{\left(-4ac(5cfg^3-cgh(eg+3dh)+4ah^2(2fg-eh))+\frac{2c\left(12a^2fh^4+4c^2g^3(5fg-eh)+ach^2\left(35fg^2-h(7eg-3dh)\right)\right)x}{h}\right)\sqrt{a+cx^2}}{(g+hx)^2}\,dx}{16h^2(cg^2+ah^2)^2}$$

$$= \frac{c\left(8(5fg-eh)(cg^2+ah^2)^2+h(12a^2fh^4+4c^2g^3(5fg-eh)+ach^2(35fg^2-h(7eg-3dh)))x\right)\sqrt{}}{8h^5(cg^2+ah^2)^2(g+hx)}$$

$$+\frac{\bigl(4a^2h^3(fg-2eh)-\frac{4c^2g^4(5fg-eh)}{h}-acgh(25fg^2-h(5eg-9dh))-3(4a^2fh^4+ach^2(17fg^2-h(}{24h^2(cg^2+ah^2)^2(g+hx)^3}$$

$$-\frac{(fg^2-egh+dh^2)(a+cx^2)^{5/2}}{4h(cg^2+ah^2)(g+hx)^4}$$

$$-\frac{\int \frac{-4ac\left(12a^2fh^4+4c^2g^3(5fg-eh)+ach^2\left(35fg^2-h(7eg-3dh)\right)\right)+\frac{32c^2(5fg-eh)\left(cg^2+ah^2\right)^2x}{h}}{(g+hx)\sqrt{a+cx^2}}\,dx}{32h^4(cg^2+ah^2)^2}$$

$$= \frac{c\left(8(5fg-eh)(cg^2+ah^2)^2+h(12a^2fh^4+4c^2g^3(5fg-eh)+ach^2(35fg^2-h(7eg-3dh)))x\right)\sqrt{}}{8h^5(cg^2+ah^2)^2(g+hx)}$$

$$+\frac{\bigl(4a^2h^3(fg-2eh)-\frac{4c^2g^4(5fg-eh)}{h}-acgh(25fg^2-h(5eg-9dh))-3(4a^2fh^4+ach^2(17fg^2-h(}{24h^2(cg^2+ah^2)^2(g+hx)^3}$$

$$-\frac{(fg^2-egh+dh^2)(a+cx^2)^{5/2}}{4h(cg^2+ah^2)(g+hx)^4}-\frac{(c^2(5fg-eh))\int \frac{1}{\sqrt{a+cx^2}}\,dx}{h^6}$$

$$+\frac{(c(12a^3fh^6+8c^3g^5(5fg-eh)+20ac^2g^3h^2(5fg-eh)+3a^2ch^4(25fg^2-h(5eg-dh))))\int \frac{}{(g+hx}}{8h^6(cg^2+ah^2)^2}$$

$$= \frac{c\left(8(5fg-eh)\left(cg^2+ah^2\right)^2+h(12a^2fh^4+4c^2g^3(5fg-eh)+ach^2(35fg^2-h(7eg-3dh)))x\right)}{8h^5\left(cg^2+ah^2\right)^2(g+hx)}$$

$$+\frac{\left(4a^2h^3(fg-2eh)-\frac{4c^2g^4(5fg-eh)}{h}-acgh(25fg^2-h(5eg-9dh))-3(4a^2fh^4+ach^2(17fg^2-h\right.}{24h^2\left(cg^2+ah^2\right)^2(g+hx)^3}$$

$$-\frac{\left(fg^2-egh+dh^2\right)\left(a+cx^2\right)^{5/2}}{4h\left(cg^2+ah^2\right)(g+hx)^4}-\frac{\left(c^2(5fg-eh)\right)\text{Subst}\left(\int\frac{1}{1-cx^2}\,dx,x,\frac{x}{\sqrt{a+cx^2}}\right)}{h^6}$$

$$-\frac{\left(c(12a^3fh^6+8c^3g^5(5fg-eh)+20ac^2g^3h^2(5fg-eh)+3a^2ch^4(25fg^2-h(5eg-dh)))\right)\text{Subst}}{8h^6\left(cg^2+ah^2\right)^2}$$

$$= \frac{c\left(8(5fg-eh)\left(cg^2+ah^2\right)^2+h(12a^2fh^4+4c^2g^3(5fg-eh)+ach^2(35fg^2-h(7eg-3dh)))x\right)}{8h^5\left(cg^2+ah^2\right)^2(g+hx)}$$

$$+\frac{\left(4a^2h^3(fg-2eh)-\frac{4c^2g^4(5fg-eh)}{h}-acgh(25fg^2-h(5eg-9dh))-3(4a^2fh^4+ach^2(17fg^2-h\right.}{24h^2\left(cg^2+ah^2\right)^2(g+hx)^3}$$

$$-\frac{\left(fg^2-egh+dh^2\right)\left(a+cx^2\right)^{5/2}}{4h\left(cg^2+ah^2\right)(g+hx)^4}-\frac{c^{3/2}(5fg-eh)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{h^6}$$

$$-\frac{c(12a^3fh^6+8c^3g^5(5fg-eh)+20ac^2g^3h^2(5fg-eh)+3a^2ch^4(25fg^2-h(5eg-dh)))\tanh^{-1}}{8h^6\left(cg^2+ah^2\right)^{5/2}}$$

Mathematica [A] (verified)

Time = 11.55 (sec) , antiderivative size = 575, normalized size of antiderivative = 1.13

$$\int\frac{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^5}\,dx =$$

$$-\frac{h\sqrt{a+cx^2}\left(6\left(cg^2+ah^2\right)^3\left(fg^2+h(-eg+dh)\right)-2\left(cg^2+ah^2\right)^2\left(17cfg^3+cgh(-13eg+9dh)-4ah^2(-2fg+eh)\right)(g+hx)+\left(cg^2+ah^2\right)\left(12a^2fh^4+2c^2(43\right.\right.}{}$$

```
[In] Integrate[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^5,x]
```

```
[Out] -1/24*((h*Sqrt[a + c*x^2]*(6*(c*g^2 + a*h^2)^3*(f*g^2 + h*(-(e*g) + d*h)) -
 2*(c*g^2 + a*h^2)^2*(17*c*f*g^3 + c*g*h*(-13*e*g + 9*d*h) - 4*a*h^2*(-2*f*
g + e*h))*(g + h*x) + (c*g^2 + a*h^2)*(12*a^2*f*h^4 + 2*c^2*(43*f*g^4 + g^2
*h*(-23*e*g + 9*d*h)) + a*c*h^2*(95*f*g^2 + h*(-43*e*g + 15*d*h)))*(g + h*x
)^2 - c*(4*a^2*h^4*(31*f*g - 8*e*h) + 2*c^2*(77*f*g^5 + g^3*h*(-25*e*g + 3*
d*h)) + a*c*g*h^2*(287*f*g^2 + h*(-91*e*g + 15*d*h)))*(g + h*x)^3 - 24*c*f*
(c*g^2 + a*h^2)^2*(g + h*x)^4))/((c*g^2 + a*h^2)^2*(g + h*x)^4) - (3*c*(12*
```

```
a^3*f*h^6 + 8*c^3*g^5*(5*f*g - e*h) + 20*a*c^2*g^3*h^2*(5*f*g - e*h) + 3*a^
2*c*h^4*(25*f*g^2 + h*(-5*e*g + d*h)))*Log[g + h*x])/(c*g^2 + a*h^2)^(5/2)
+ 24*c^(3/2)*(5*f*g - e*h)*Log[c*x + Sqrt[c]*Sqrt[a + c*x^2]] + (3*c*(12*a^
3*f*h^6 + 8*c^3*g^5*(5*f*g - e*h) + 20*a*c^2*g^3*h^2*(5*f*g - e*h) + 3*a^2*
c*h^4*(25*f*g^2 + h*(-5*e*g + d*h)))*Log[a*h - c*g*x + Sqrt[c*g^2 + a*h^2]*
Sqrt[a + c*x^2]])/(c*g^2 + a*h^2)^(5/2))/h^6
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3213 vs. 2(485) = 970.

Time = 0.72 (sec) , antiderivative size = 3214, normalized size of antiderivative = 6.29

method	result	size
risch	Expression too large to display	3214
default	Expression too large to display	7961

```
[In] int((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^5,x,method=_RETURNVERBOSE)
```

```
[Out] f/h^5*c*(c*x^2+a)^(1/2)+1/h^5*(c^(3/2)*(e*h-5*f*g)/h*ln(x*c^(1/2)+(c*x^2+a)
^(1/2))-c/h^2*(2*a*f*h^2+c*d*h^2-5*c*e*g*h+15*c*f*g^2)/((a*h^2+c*g^2)/h^2)^
(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1/h*g)+2*((a*h^2+c*g^2)/h^2)^(1/2)
*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+1/h*g))+2*c/
h^3*(a*e*h^3-4*a*f*g*h^2-2*c*d*g*h^2+5*c*e*g^2*h-10*c*f*g^3)*(-1/(a*h^2+c*g
^2)*h^2/(x+1/h*g)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)
-c*g*h/(a*h^2+c*g^2)/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*
g/h*(x+1/h*g)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+
(a*h^2+c*g^2)/h^2)^(1/2))/(x+1/h*g)))+1/h^4*(a^2*f*h^4+2*a*c*d*h^4-6*a*c*e*
g*h^3+12*a*c*f*g^2*h^2+6*c^2*d*g^2*h^2-10*c^2*e*g^3*h+15*c^2*f*g^4)*(-1/2/(
a*h^2+c*g^2)*h^2/(x+1/h*g)^2*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)
/h^2)^(1/2)+3/2*c*g*h/(a*h^2+c*g^2)*(-1/(a*h^2+c*g^2)*h^2/(x+1/h*g)*((x+1/h
*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)-c*g*h/(a*h^2+c*g^2)/((a*
h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1/h*g)+2*((a*h^2+c
*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))
/(x+1/h*g)))+1/2*c/(a*h^2+c*g^2)*h^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2
+c*g^2)/h^2-2*c*g/h*(x+1/h*g)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c-2*
c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+1/h*g)))+1/h^5*(a^2*e*h^5-2*a^
2*f*g*h^4-4*a*c*d*g*h^4+6*a*c*e*g^2*h^3-8*a*c*f*g^3*h^2-4*c^2*d*g^3*h^2+5*c
^2*e*g^4*h-6*c^2*f*g^5)*(-1/3/(a*h^2+c*g^2)*h^2/(x+1/h*g)^3*((x+1/h*g)^2*c-
2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)+5/3*c*g*h/(a*h^2+c*g^2)*(-1/2/(a
*h^2+c*g^2)*h^2/(x+1/h*g)^2*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/
h^2)^(1/2)+3/2*c*g*h/(a*h^2+c*g^2)*(-1/(a*h^2+c*g^2)*h^2/(x+1/h*g)*((x+1/h*
g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)-c*g*h/(a*h^2+c*g^2)/((a*h
^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1/h*g)+2*((a*h^2+c*
g^2)/h^2)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))/
```

```
(x+1/h*g)))+1/2*c/(a*h^2+c*g^2)*h^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+
c*g^2)/h^2-2*c*g/h*(x+1/h*g)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c-2*c
*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+1/h*g)))-2/3*c/(a*h^2+c*g^2)*h^
2*(-1/(a*h^2+c*g^2)*h^2/(x+1/h*g)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c
*g^2)/h^2)^(1/2)-c*g*h/(a*h^2+c*g^2)/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2
+c*g^2)/h^2-2*c*g/h*(x+1/h*g)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c-2*
c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+1/h*g))))+1/h^6*(a^2*d*h^6-a^2
*e*g*h^5+a^2*f*g^2*h^4+2*a*c*d*g^2*h^4-2*a*c*e*g^3*h^3+2*a*c*f*g^4*h^2+c^2*
d*g^4*h^2-c^2*e*g^5*h+c^2*f*g^6)*(-1/4/(a*h^2+c*g^2)*h^2/(x+1/h*g)^4*((x+1/
h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)+7/4*c*g*h/(a*h^2+c*g^2)
*(-1/3/(a*h^2+c*g^2)*h^2/(x+1/h*g)^3*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^
2+c*g^2)/h^2)^(1/2)+5/3*c*g*h/(a*h^2+c*g^2)*(-1/2/(a*h^2+c*g^2)*h^2/(x+1/h*
g)^2*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)+3/2*c*g*h/(a
*h^2+c*g^2)*(-1/(a*h^2+c*g^2)*h^2/(x+1/h*g)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g
)+(a*h^2+c*g^2)/h^2)^(1/2)-c*g*h/(a*h^2+c*g^2)/((a*h^2+c*g^2)/h^2)^(1/2)*ln
((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1/h*g)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+1/h
*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+1/h*g)))+1/2*c/(a*h^
2+c*g^2)*h^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1
/h*g)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c
*g^2)/h^2)^(1/2))/(x+1/h*g)))-2/3*c/(a*h^2+c*g^2)*h^2*(-1/(a*h^2+c*g^2)*h^2
/(x+1/h*g)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)-c*g*h/
(a*h^2+c*g^2)/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+
1/h*g)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+
c*g^2)/h^2)^(1/2))/(x+1/h*g))))-3/4*c/(a*h^2+c*g^2)*h^2*(-1/2/(a*h^2+c*g^2)
*h^2/(x+1/h*g)^2*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)+
3/2*c*g*h/(a*h^2+c*g^2)*(-1/(a*h^2+c*g^2)*h^2/(x+1/h*g)*((x+1/h*g)^2*c-2*c*
g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)-c*g*h/(a*h^2+c*g^2)/((a*h^2+c*g^2)/h
^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1/h*g)+2*((a*h^2+c*g^2)/h^2)^(
1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+1/h*g)))
+1/2*c/(a*h^2+c*g^2)*h^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-
2*c*g/h*(x+1/h*g)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h
*g)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+1/h*g)))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^5}\,dx = \text{Timed out}$$

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^5,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^5}\,dx = \int \frac{\left(a+cx^2\right)^{\frac{3}{2}}\left(d+ex+fx^2\right)}{\left(g+hx\right)^5}\,dx$$

```
[In] integrate((c*x**2+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**5,x)
```

```
[Out] Integral((a + c*x**2)**(3/2)*(d + e*x + f*x**2)/(g + h*x)**5, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4326 vs. 2(485) = 970.

Time = 0.42 (sec) , antiderivative size = 4326, normalized size of antiderivative = 8.47

$$\int \frac{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^5}\,dx = \text{Too large to display}$$

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^5,x, algorithm="maxima")
```

```
[Out] 3/8*sqrt(c*x^2 + a)*c^4*f*g^6/(c^3*g^6*h^5 + 3*a*c^2*g^4*h^7 + 3*a^2*c*g^2*
h^9 + a^3*h^11) - 3/8*sqrt(c*x^2 + a)*c^4*f*g^5*x/(c^3*g^6*h^4 + 3*a*c^2*g^
4*h^6 + 3*a^2*c*g^2*h^8 + a^3*h^10) - 3/8*sqrt(c*x^2 + a)*c^4*e*g^5/(c^3*g^
6*h^4 + 3*a*c^2*g^4*h^6 + 3*a^2*c*g^2*h^8 + a^3*h^10) + 1/8*(c*x^2 + a)^(3/
2)*c^3*f*g^5/(c^3*g^6*h^4*x + 3*a*c^2*g^4*h^6*x + 3*a^2*c*g^2*h^8*x + a^3*h
^10*x + c^3*g^7*h^3 + 3*a*c^2*g^5*h^5 + 3*a^2*c*g^3*h^7 + a^3*g*h^9) + 3/8*
sqrt(c*x^2 + a)*c^4*e*g^4*x/(c^3*g^6*h^3 + 3*a*c^2*g^4*h^5 + 3*a^2*c*g^2*h^
7 + a^3*h^9) + 3/8*sqrt(c*x^2 + a)*c^4*d*g^4/(c^3*g^6*h^3 + 3*a*c^2*g^4*h^5
 + 3*a^2*c*g^2*h^7 + a^3*h^9) - 1/8*(c*x^2 + a)^(3/2)*c^3*e*g^4/(c^3*g^6*h^
3*x + 3*a*c^2*g^4*h^5*x + 3*a^2*c*g^2*h^7*x + a^3*h^9*x + c^3*g^7*h^2 + 3*a
*c^2*g^5*h^4 + 3*a^2*c*g^3*h^6 + a^3*g*h^8) - 1/8*(c*x^2 + a)^(5/2)*c^2*f*g
^4/(c^3*g^6*h^3*x^2 + 3*a*c^2*g^4*h^5*x^2 + 3*a^2*c*g^2*h^7*x^2 + a^3*h^9*x
^2 + 2*c^3*g^7*h^2*x + 6*a*c^2*g^5*h^4*x + 6*a^2*c*g^3*h^6*x + 2*a^3*g*h^8*
x + c^3*g^8*h + 3*a*c^2*g^6*h^3 + 3*a^2*c*g^4*h^5 + a^3*g^2*h^7) + 1/8*(c*x
^2 + a)^(3/2)*c^3*f*g^4/(c^3*g^6*h^3 + 3*a*c^2*g^4*h^5 + 3*a^2*c*g^2*h^7 +
a^3*h^9) - 3/8*sqrt(c*x^2 + a)*c^4*d*g^3*x/(c^3*g^6*h^2 + 3*a*c^2*g^4*h^4 +
 3*a^2*c*g^2*h^6 + a^3*h^8) + 1/8*(c*x^2 + a)^(3/2)*c^3*d*g^3/(c^3*g^6*h^2*
x + 3*a*c^2*g^4*h^4*x + 3*a^2*c*g^2*h^6*x + a^3*h^8*x + c^3*g^7*h + 3*a*c^2
*g^5*h^3 + 3*a^2*c*g^3*h^5 + a^3*g*h^7) + 1/8*(c*x^2 + a)^(5/2)*c^2*e*g^3/(
c^3*g^6*h^2*x^2 + 3*a*c^2*g^4*h^4*x^2 + 3*a^2*c*g^2*h^6*x^2 + a^3*h^8*x^2 +
 2*c^3*g^7*h*x + 6*a*c^2*g^5*h^3*x + 6*a^2*c*g^3*h^5*x + 2*a^3*g*h^7*x + c^
3*g^8 + 3*a*c^2*g^6*h^2 + 3*a^2*c*g^4*h^4 + a^3*g^2*h^6) - 1/8*(c*x^2 + a)^
(3/2)*c^3*e*g^3/(c^3*g^6*h^2 + 3*a*c^2*g^4*h^4 + 3*a^2*c*g^2*h^6 + a^3*h^8)
 - 7/4*sqrt(c*x^2 + a)*c^3*f*g^4/(c^2*g^4*h^5 + 2*a*c*g^2*h^7 + a^2*h^9) +
```

```
11/8*sqrt(c*x^2 + a)*c^3*f*g^3*x/(c^2*g^4*h^4 + 2*a*c*g^2*h^6 + a^2*h^8) -
1/8*(c*x^2 + a)^(5/2)*c^2*d*g^2/(c^3*g^6*h*x^2 + 3*a*c^2*g^4*h^3*x^2 + 3*a^
2*c*g^2*h^5*x^2 + a^3*h^7*x^2 + 2*c^3*g^7*x + 6*a*c^2*g^5*h^2*x + 6*a^2*c*g
^3*h^4*x + 2*a^3*g*h^6*x + c^3*g^8/h + 3*a*c^2*g^6*h + 3*a^2*c*g^4*h^3 + a^
3*g^2*h^5) + 1/8*(c*x^2 + a)^(3/2)*c^3*d*g^2/(c^3*g^6*h + 3*a*c^2*g^4*h^3 +
 3*a^2*c*g^2*h^5 + a^3*h^7) + 5/4*sqrt(c*x^2 + a)*c^3*e*g^3/(c^2*g^4*h^4 +
2*a*c*g^2*h^6 + a^2*h^8) - 1/4*(c*x^2 + a)^(5/2)*c*f*g^3/(c^2*g^4*h^4*x^3 +
 2*a*c*g^2*h^6*x^3 + a^2*h^8*x^3 + 3*c^2*g^5*h^3*x^2 + 6*a*c*g^3*h^5*x^2 +
3*a^2*g*h^7*x^2 + 3*c^2*g^6*h^2*x + 6*a*c*g^4*h^4*x + 3*a^2*g^2*h^6*x + c^2
*g^7*h + 2*a*c*g^5*h^3 + a^2*g^3*h^5) - 17/24*(c*x^2 + a)^(3/2)*c^2*f*g^3/(
c^2*g^4*h^4*x + 2*a*c*g^2*h^6*x + a^2*h^8*x + c^2*g^5*h^3 + 2*a*c*g^3*h^5 +
 a^2*g*h^7) - 7/8*sqrt(c*x^2 + a)*c^3*e*g^2*x/(c^2*g^4*h^3 + 2*a*c*g^2*h^5
+ a^2*h^7) - 3/4*sqrt(c*x^2 + a)*c^3*d*g^2/(c^2*g^4*h^3 + 2*a*c*g^2*h^5 + a
^2*h^7) + 1/4*(c*x^2 + a)^(5/2)*c*e*g^2/(c^2*g^4*h^3*x^3 + 2*a*c*g^2*h^5*x^
3 + a^2*h^7*x^3 + 3*c^2*g^5*h^2*x^2 + 6*a*c*g^3*h^4*x^2 + 3*a^2*g*h^6*x^2 +
 3*c^2*g^6*h*x + 6*a*c*g^4*h^3*x + 3*a^2*g^2*h^5*x + c^2*g^7 + 2*a*c*g^5*h^
2 + a^2*g^3*h^4) + 13/24*(c*x^2 + a)^(3/2)*c^2*e*g^2/(c^2*g^4*h^3*x + 2*a*c
*g^2*h^5*x + a^2*h^7*x + c^2*g^5*h^2 + 2*a*c*g^3*h^4 + a^2*g*h^6) + 5/24*(c
*x^2 + a)^(5/2)*c*f*g^2/(c^2*g^4*h^3*x^2 + 2*a*c*g^2*h^5*x^2 + a^2*h^7*x^2
+ 2*c^2*g^5*h^2*x + 4*a*c*g^3*h^4*x + 2*a^2*g*h^6*x + c^2*g^6*h + 2*a*c*g^4
*h^3 + a^2*g^2*h^5) - 5/24*(c*x^2 + a)^(3/2)*c^2*f*g^2/(c^2*g^4*h^3 + 2*a*c
*g^2*h^5 + a^2*h^7) + 3/8*sqrt(c*x^2 + a)*c^3*d*g*x/(c^2*g^4*h^2 + 2*a*c*g^
2*h^4 + a^2*h^6) - 1/4*(c*x^2 + a)^(5/2)*c*d*g/(c^2*g^4*h^2*x^3 + 2*a*c*g^2
*h^4*x^3 + a^2*h^6*x^3 + 3*c^2*g^5*h*x^2 + 6*a*c*g^3*h^3*x^2 + 3*a^2*g*h^5*
x^2 + 3*c^2*g^6*x + 6*a*c*g^4*h^2*x + 3*a^2*g^2*h^4*x + c^2*g^7/h + 2*a*c*g
^5*h + a^2*g^3*h^3) - 3/8*(c*x^2 + a)^(3/2)*c^2*d*g/(c^2*g^4*h^2*x + 2*a*c*
g^2*h^4*x + a^2*h^6*x + c^2*g^5*h + 2*a*c*g^3*h^3 + a^2*g*h^5) - 1/24*(c*x^
2 + a)^(5/2)*c*e*g/(c^2*g^4*h^2*x^2 + 2*a*c*g^2*h^4*x^2 + a^2*h^6*x^2 + 2*c
^2*g^5*h*x + 4*a*c*g^3*h^3*x + 2*a^2*g*h^5*x + c^2*g^6 + 2*a*c*g^4*h^2 + a^
2*g^2*h^4) + 1/24*(c*x^2 + a)^(3/2)*c^2*e*g/(c^2*g^4*h^2 + 2*a*c*g^2*h^4 +
a^2*h^6) - 1/4*(c*x^2 + a)^(5/2)*f*g^2/(c*g^2*h^5*x^4 + a*h^7*x^4 + 4*c*g^3
*h^4*x^3 + 4*a*g*h^6*x^3 + 6*c*g^4*h^3*x^2 + 6*a*g^2*h^5*x^2 + 4*c*g^5*h^2*
x + 4*a*g^3*h^4*x + c*g^6*h + a*g^4*h^3) + 39/8*sqrt(c*x^2 + a)*c^2*f*g^2/(
c*g^2*h^5 + a*h^7) - 7/2*sqrt(c*x^2 + a)*c^2*f*g*x/(c*g^2*h^4 + a*h^6) - 1/
8*(c*x^2 + a)^(5/2)*c*d/(c^2*g^4*h*x^2 + 2*a*c*g^2*h^3*x^2 + a^2*h^5*x^2 +
2*c^2*g^5*x + 4*a*c*g^3*h^2*x + 2*a^2*g*h^4*x + c^2*g^6/h + 2*a*c*g^4*h + a
^2*g^2*h^3) + 1/8*(c*x^2 + a)^(3/2)*c^2*d/(c^2*g^4*h + 2*a*c*g^2*h^3 + a^2*
h^5) + 1/4*(c*x^2 + a)^(5/2)*e*g/(c*g^2*h^4*x^4 + a*h^6*x^4 + 4*c*g^3*h^3*x
^3 + 4*a*g*h^5*x^3 + 6*c*g^4*h^2*x^2 + 6*a*g^2*h^4*x^2 + 4*c*g^5*h*x + 4*a*
g^3*h^3*x + c*g^6 + a*g^4*h^2) - 15/8*sqrt(c*x^2 + a)*c^2*e*g/(c*g^2*h^4 +
a*h^6) + 2/3*(c*x^2 + a)^(5/2)*f*g/(c*g^2*h^4*x^3 + a*h^6*x^3 + 3*c*g^3*h^3
*x^2 + 3*a*g*h^5*x^2 + 3*c*g^4*h^2*x + 3*a*g^2*h^4*x + c*g^5*h + a*g^3*h^3)
 + 11/6*(c*x^2 + a)^(3/2)*c*f*g/(c*g^2*h^4*x + a*h^6*x + c*g^3*h^3 + a*g*h^
5) + sqrt(c*x^2 + a)*c^2*e*x/(c*g^2*h^3 + a*h^5) - 1/4*(c*x^2 + a)^(5/2)*d/
(c*g^2*h^3*x^4 + a*h^5*x^4 + 4*c*g^3*h^2*x^3 + 4*a*g*h^4*x^3 + 6*c*g^4*h*x^
```

```
2 + 6*a*g^2*h^3*x^2 + 4*c*g^5*x + 4*a*g^3*h^2*x + c*g^6/h + a*g^4*h) + 3/8*
sqrt(c*x^2 + a)*c^2*d/(c*g^2*h^3 + a*h^5) - 1/3*(c*x^2 + a)^(5/2)*e/(c*g^2*
h^3*x^3 + a*h^5*x^3 + 3*c*g^3*h^2*x^2 + 3*a*g*h^4*x^2 + 3*c*g^4*h*x + 3*a*g
^2*h^3*x + c*g^5 + a*g^3*h^2) - 2/3*(c*x^2 + a)^(3/2)*c*e/(c*g^2*h^3*x + a*
h^5*x + c*g^3*h^2 + a*g*h^4) - 1/2*(c*x^2 + a)^(5/2)*f/(c*g^2*h^3*x^2 + a*h
^5*x^2 + 2*c*g^3*h^2*x + 2*a*g*h^4*x + c*g^4*h + a*g^2*h^3) + 1/2*(c*x^2 +
a)^(3/2)*c*f/(c*g^2*h^3 + a*h^5) - 5*c^(3/2)*f*g*arcsinh(c*x/sqrt(a*c))/h^6
 + c^(3/2)*e*arcsinh(c*x/sqrt(a*c))/h^5 + 3/8*c^4*f*g^6*arcsinh(c*g*x/(sqrt
(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(5/2)*
h^11) - 3/8*c^4*e*g^5*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*
c)*abs(h*x + g)))/((a + c*g^2/h^2)^(5/2)*h^10) + 3/8*c^4*d*g^4*arcsinh(c*g*
x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)
^(5/2)*h^9) - 7/4*c^3*f*g^4*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(s
qrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(3/2)*h^9) + 5/4*c^3*e*g^3*arcsinh
(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2
/h^2)^(3/2)*h^8) - 3/4*c^3*d*g^2*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a
*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(3/2)*h^7) + 39/8*c^2*f*g^2*a
rcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/(sqrt
(a + c*g^2/h^2)*h^7) - 15/8*c^2*e*g*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g))
- a*h/(sqrt(a*c)*abs(h*x + g)))/(sqrt(a + c*g^2/h^2)*h^6) + 3/8*c^2*d*arcsi
nh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/(sqrt(a +
 c*g^2/h^2)*h^5) + 3/2*sqrt(a + c*g^2/h^2)*c*f*arcsinh(c*g*x/(sqrt(a*c)*abs
(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/h^5 + 3/2*sqrt(c*x^2 + a)*c*f/h^
5
```

Giac [F]

$$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^5}\,dx=\int \frac{(cx^2+a)^{\frac{3}{2}}(fx^2+ex+d)}{(hx+g)^5}\,dx$$

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^5,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^5}\,dx=\int \frac{(c\,x^2+a)^{3/2}\,(f\,x^2+e\,x+d)}{(g+h\,x)^5}\,dx$$

```
[In] int(((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^5,x)
[Out] int(((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^5, x)
```

3.97 $\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^6}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 507

$$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^6}\,dx =$$

$$-\frac{c(8c^3fg^7+20ac^2fg^5h^2-a^3h^6(2fg-3eh)+a^2cgh^4(13fg^2+3dh^2)+h(12c^3fg^6+8a^3fh^6+a^2cgh^4(34f}{8h^5(cg^2+ah^2)^3(g+hx)^2}$$

$$-\frac{(4c^2fg^5-a^2h^4(2fg-3eh)+acgh^2(5fg^2+3dh^2)+h(4a^2fh^4+acgh^2(14fg-3eh)+c^2(7fg^4-3dg^2h^2}{12h^3(cg^2+ah^2)^2(g+hx)^4}$$

$$-\frac{(fg^2-egh+dh^2)(a+cx^2)^{5/2}}{5h(cg^2+ah^2)(g+hx)^5}+\frac{c^{3/2}f\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{h^6}$$

$$+\frac{c^2(8c^3fg^7+28ac^2fg^5h^2+3a^3h^6(6fg-eh)+a^2cgh^4(35fg^2-3dh^2))\operatorname{arctanh}\left(\frac{ah-cgx}{\sqrt{cg^2+ah^2}\sqrt{a+cx^2}}\right)}{8h^6(cg^2+ah^2)^{7/2}}$$

```
[Out] -1/12*(4*c^2*f*g^5-a^2*h^4*(-3*e*h+2*f*g)+a*c*g*h^2*(3*d*h^2+5*f*g^2)+h*(4*
a^2*f*h^4+a*c*g*h^2*(-3*e*h+14*f*g)+c^2*(-3*d*g^2*h^2+7*f*g^4))*x)*(c*x^2+a
)^(3/2)/h^3/(a*h^2+c*g^2)^2/(h*x+g)^4-1/5*(d*h^2-e*g*h+f*g^2)*(c*x^2+a)^(5/
2)/h/(a*h^2+c*g^2)/(h*x+g)^5+c^(3/2)*f*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/h
^6+1/8*c^2*(8*c^3*f*g^7+28*a*c^2*f*g^5*h^2+3*a^3*h^6*(-e*h+6*f*g)+a^2*c*g*h
^4*(-3*d*h^2+35*f*g^2))*arctanh((-c*g*x+a*h)/(a*h^2+c*g^2)^(1/2)/(c*x^2+a)^
(1/2))/h^6/(a*h^2+c*g^2)^(7/2)-1/8*c*(8*c^3*f*g^7+20*a*c^2*f*g^5*h^2-a^3*h^
6*(-3*e*h+2*f*g)+a^2*c*g*h^4*(3*d*h^2+13*f*g^2)+h*(12*c^3*f*g^6+8*a^3*f*h^6
+a^2*c*g*h^4*(-3*e*h+34*f*g)+a*c^2*g^2*h^2*(-3*d*h^2+35*f*g^2))*x)*(c*x^2+a
)^(1/2)/h^5/(a*h^2+c*g^2)^3/(h*x+g)^2
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 507, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1665, 825, 858, 223, 212, 739}

$$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^6}\,dx =$$

$$-\frac{(a+cx^2)^{3/2}(hx(4a^2fh^4+acgh^2(14fg-3eh)+c^2(7fg^4-3dg^2h^2))-a^2h^4(2fg-3eh)+acgh^2(3dh^2+5}{12h^3(g+hx)^4(ah^2+cg^2)^2}$$

$$+\frac{c^2\operatorname{arctanh}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)(3a^3h^6(6fg-eh)+a^2cgh^4(35fg^2-3dh^2)+28ac^2fg^5h^2+8c^3fg^7)}{8h^6(ah^2+cg^2)^{7/2}}$$

$$-\frac{c\sqrt{a+cx^2}(-a^3h^6(2fg-3eh)+a^2cgh^4(3dh^2+13fg^2)+hx(8a^3fh^6+a^2cgh^4(34fg-3eh)+ac^2g^2h^2(35f}{8h^5(g+hx)^2(ah^2+cg^2)^3}$$

$$+\frac{c^{3/2}f\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{h^6}-\frac{(a+cx^2)^{5/2}(dh^2-egh+fg^2)}{5h(g+hx)^5(ah^2+cg^2)}$$

```
[In] Int[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^6,x]
```

```
[Out] -1/8*(c*(8*c^3*f*g^7 + 20*a*c^2*f*g^5*h^2 - a^3*h^6*(2*f*g - 3*e*h) + a^2*c
*g*h^4*(13*f*g^2 + 3*d*h^2) + h*(12*c^3*f*g^6 + 8*a^3*f*h^6 + a^2*c*g*h^4*(
34*f*g - 3*e*h) + a*c^2*g^2*h^2*(35*f*g^2 - 3*d*h^2))*x)*Sqrt[a + c*x^2])/(
h^5*(c*g^2 + a*h^2)^3*(g + h*x)^2) - ((4*c^2*f*g^5 - a^2*h^4*(2*f*g - 3*e*h
) + a*c*g*h^2*(5*f*g^2 + 3*d*h^2) + h*(4*a^2*f*h^4 + a*c*g*h^2*(14*f*g - 3*
e*h) + c^2*(7*f*g^4 - 3*d*g^2*h^2))*x)*(a + c*x^2)^(3/2))/(12*h^3*(c*g^2 +
a*h^2)^2*(g + h*x)^4) - ((f*g^2 - e*g*h + d*h^2)*(a + c*x^2)^(5/2))/(5*h*(c
*g^2 + a*h^2)*(g + h*x)^5) + (c^(3/2)*f*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]
])/h^6 + (c^2*(8*c^3*f*g^7 + 28*a*c^2*f*g^5*h^2 + 3*a^3*h^6*(6*f*g - e*h) +
 a^2*c*g*h^4*(35*f*g^2 - 3*d*h^2))*ArcTanh[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^
2]*Sqrt[a + c*x^2])])/(8*h^6*(c*g^2 + a*h^2)^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 825

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + c*x^2)^p/(e^2*(m + 1)*(m
+ 2)*(c*d^2 + a*e^2)))*((d*g - e*f*(m + 2))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*
f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) + 2*c*d*p*(e*f - d*g))*x), x] - Dis
t[p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*
(m + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f,
g}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p,
0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{(fg^2-egh+dh^2)\,(a+cx^2)^{5/2}}{5h\,(cg^2+ah^2)\,(g+hx)^5} - \frac{\int \frac{\left(-5(cdg-afg+aeh)-5f\left(\frac{cg^2}{h}+ah\right)x\right)(a+cx^2)^{3/2}}{(g+hx)^5}\,dx}{5\,(cg^2+ah^2)}$$

$$=$$

$$-\frac{(4c^2fg^5-a^2h^4(2fg-3eh)+acgh^2(5fg^2+3dh^2)+h(4a^2fh^4+acgh^2(14fg-3eh)+c^2(7fg^4}{12h^3\left(cg^2+ah^2\right)^2(g+hx)^4}$$

$$-\frac{\left(fg^2-egh+dh^2\right)\left(a+cx^2\right)^{5/2}}{5h\left(cg^2+ah^2\right)(g+hx)^5}$$

$$+\frac{\int\frac{\left(-30ac(ah^2(2fg-eh)+c(fg^3-dgh^2))+\frac{40cf\left(cg^2+ah^2\right)^2x}{h}\right)\sqrt{a+cx^2}}{(g+hx)^3}\,dx}{40h^2\left(cg^2+ah^2\right)^2}$$

$$=$$

$$-\frac{c(8c^3fg^7+20ac^2fg^5h^2-a^3h^6(2fg-3eh)+a^2cgh^4(13fg^2+3dh^2)+h(12c^3fg^6+8a^3fh^6+a^2}{8h^5\left(cg^2+ah^2\right)^3(g+hx)^2}$$

$$-\frac{(4c^2fg^5-a^2h^4(2fg-3eh)+acgh^2(5fg^2+3dh^2)+h(4a^2fh^4+acgh^2(14fg-3eh)+c^2(7fg^4}{12h^3\left(cg^2+ah^2\right)^2(g+hx)^4}$$

$$-\frac{\left(fg^2-egh+dh^2\right)\left(a+cx^2\right)^{5/2}}{5h\left(cg^2+ah^2\right)(g+hx)^5}$$

$$-\frac{\int\frac{20ac^2\left(4c^2fg^5+a^2h^4(10fg-3eh)+acgh^2\left(11fg^2-3dh^2\right)\right)-\frac{160c^2f\left(cg^2+ah^2\right)^3x}{h}}{(g+hx)\sqrt{a+cx^2}}\,dx}{160h^4\left(cg^2+ah^2\right)^3}$$

$$=$$

$$-\frac{c(8c^3fg^7+20ac^2fg^5h^2-a^3h^6(2fg-3eh)+a^2cgh^4(13fg^2+3dh^2)+h(12c^3fg^6+8a^3fh^6+a^2}{8h^5\left(cg^2+ah^2\right)^3(g+hx)^2}$$

$$-\frac{(4c^2fg^5-a^2h^4(2fg-3eh)+acgh^2(5fg^2+3dh^2)+h(4a^2fh^4+acgh^2(14fg-3eh)+c^2(7fg^4}{12h^3\left(cg^2+ah^2\right)^2(g+hx)^4}$$

$$-\frac{\left(fg^2-egh+dh^2\right)\left(a+cx^2\right)^{5/2}}{5h\left(cg^2+ah^2\right)(g+hx)^5}+\frac{\left(c^2f\right)\int\frac{1}{\sqrt{a+cx^2}}\,dx}{h^6}$$

$$-\frac{\left(c^2(8c^3fg^7+28ac^2fg^5h^2+3a^3h^6(6fg-eh)+a^2cgh^4(35fg^2-3dh^2))\right)\int\frac{1}{(g+hx)\sqrt{a+cx^2}}\,dx}{8h^6\left(cg^2+ah^2\right)^3}$$

$$=$$

$$-\frac{c(8c^3fg^7+20ac^2fg^5h^2-a^3h^6(2fg-3eh)+a^2cgh^4(13fg^2+3dh^2)+h(12c^3fg^6+8a^3fh^6+a^2}{8h^5\left(cg^2+ah^2\right)^3(g+hx)^2}$$

$$-\frac{(4c^2fg^5-a^2h^4(2fg-3eh)+acgh^2(5fg^2+3dh^2)+h(4a^2fh^4+acgh^2(14fg-3eh)+c^2(7fg^4}{12h^3\left(cg^2+ah^2\right)^2(g+hx)^4}$$

$$-\frac{\left(fg^2-egh+dh^2\right)\left(a+cx^2\right)^{5/2}}{5h\left(cg^2+ah^2\right)(g+hx)^5}+\frac{\left(c^2f\right)\text{Subst}\left(\int\frac{1}{1-cx^2}\,dx,x,\frac{x}{\sqrt{a+cx^2}}\right)}{h^6}$$

$$+\frac{\left(c^2(8c^3fg^7+28ac^2fg^5h^2+3a^3h^6(6fg-eh)+a^2cgh^4(35fg^2-3dh^2))\right)\text{Subst}\left(\int\frac{1}{cg^2+ah^2-x^2}\,dx,x\right.}{8h^6\left(cg^2+ah^2\right)^3}$$

$$
\begin{aligned}
= & \\
& -\frac{c(8c^3fg^7+20ac^2fg^5h^2-a^3h^6(2fg-3eh)+a^2cgh^4(13fg^2+3dh^2)+h(12c^3fg^6+8a^3fh^6+a}{8h^5\left(cg^2+ah^2\right)^3(g+hx)^2} \\
& -\frac{(4c^2fg^5-a^2h^4(2fg-3eh)+acgh^2(5fg^2+3dh^2)+h(4a^2fh^4+acgh^2(14fg-3eh)+c^2(7fg^4}{12h^3\left(cg^2+ah^2\right)^2(g+hx)^4} \\
& -\frac{\left(fg^2-egh+dh^2\right)\left(a+cx^2\right)^{5/2}}{5h\left(cg^2+ah^2\right)(g+hx)^5}+\frac{c^{3/2}f\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{h^6} \\
& +\frac{c^2(8c^3fg^7+28ac^2fg^5h^2+3a^3h^6(6fg-eh)+a^2cgh^4(35fg^2-3dh^2))\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{cg^2+ah^2}\sqrt{a+cx^2}}\right)}{8h^6\left(cg^2+ah^2\right)^{7/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 11.29 (sec) , antiderivative size = 639, normalized size of antiderivative = 1.26

$$
\int\frac{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^6}\,dx=\frac{-\frac{h\sqrt{a+cx^2}\left(24\left(cg^2+ah^2\right)^4\left(fg^2+h(-eg+dh)\right)-6\left(cg^2+ah^2\right)^3\left(21cfg^3+cgh(-16eg+11dh)-5ah\right.}{}}{}
$$

```
[In] Integrate[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^6,x]
```

```
[Out] (-((h*Sqrt[a + c*x^2]*(24*(c*g^2 + a*h^2)^4*(f*g^2 + h*(-(e*g) + d*h)) - 6*
(c*g^2 + a*h^2)^3*(21*c*f*g^3 + c*g*h*(-16*e*g + 11*d*h) - 5*a*h^2*(-2*f*g
+ e*h))*(g + h*x) + 2*(c*g^2 + a*h^2)^2*(20*a^2*f*h^4 + c^2*(137*f*g^4 + 9*
g^2*h*(-8*e*g + 3*d*h)) + a*c*h^2*(154*f*g^2 + 3*h*(-23*e*g + 8*d*h)))*(g +
 h*x)^2 - c*(c*g^2 + a*h^2)*(5*a^2*h^4*(58*f*g - 15*e*h) + c^2*(326*f*g^5 +
 6*g^3*h*(-16*e*g + d*h)) + a*c*g*h^2*(631*f*g^2 + 3*h*(-62*e*g + 7*d*h)))*
(g + h*x)^3 + c*(160*a^3*f*h^6 + c^3*(274*f*g^6 - 6*g^4*h*(4*e*g + d*h)) +
3*a^2*c*h^4*(238*f*g^2 + h*(-33*e*g + 8*d*h)) + 3*a*c^2*g^2*h^2*(261*f*g^2
- h*(26*e*g + 9*d*h)))*(g + h*x)^4))/((c*g^2 + a*h^2)^3*(g + h*x)^5)) - (15
*c^2*(8*c^3*f*g^7 + 28*a*c^2*f*g^5*h^2 - 3*a^3*h^6*(-6*f*g + e*h) + a^2*c*g
*h^4*(35*f*g^2 - 3*d*h^2))*Log[g + h*x])/(c*g^2 + a*h^2)^(7/2) + 120*c^(3/2
)*f*Log[c*x + Sqrt[c]*Sqrt[a + c*x^2]] + (15*c^2*(8*c^3*f*g^7 + 28*a*c^2*f*
g^5*h^2 - 3*a^3*h^6*(-6*f*g + e*h) + a^2*c*g*h^4*(35*f*g^2 - 3*d*h^2))*Log[
a*h - c*g*x + Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2]])/(c*g^2 + a*h^2)^(7/2))/
(120*h^6)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 10597 vs. 2(481) = 962.

Time = 0.75 (sec) , antiderivative size = 10598, normalized size of antiderivative = 20.90

method	result	size
default	Expression too large to display	10598

```
[In] int((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^6,x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^6}\,dx = \text{Timed out}$$

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^6,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^6}\,dx = \int \frac{(a+cx^2)^{\frac{3}{2}}(d+ex+fx^2)}{(g+hx)^6}\,dx$$

```
[In] integrate((c*x**2+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**6,x)
[Out] Integral((a + c*x**2)**(3/2)*(d + e*x + f*x**2)/(g + h*x)**6, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6650 vs. 2(482) = 964.

Time = 0.49 (sec) , antiderivative size = 6650, normalized size of antiderivative = 13.12

$$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^6}\,dx = \text{Too large to display}$$

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^6,x, algorithm="maxima")
[Out] 3/8*sqrt(c*x^2 + a)*c^5*f*g^7/(c^4*g^8*h^5 + 4*a*c^3*g^6*h^7 + 6*a^2*c^2*g^
4*h^9 + 4*a^3*c*g^2*h^11 + a^4*h^13) - 3/8*sqrt(c*x^2 + a)*c^5*f*g^6*x/(c^4
```

```
*g^8*h^4 + 4*a*c^3*g^6*h^6 + 6*a^2*c^2*g^4*h^8 + 4*a^3*c*g^2*h^10 + a^4*h^1
2) - 3/8*sqrt(c*x^2 + a)*c^5*e*g^6/(c^4*g^8*h^4 + 4*a*c^3*g^6*h^6 + 6*a^2*c
^2*g^4*h^8 + 4*a^3*c*g^2*h^10 + a^4*h^12) + 1/8*(c*x^2 + a)^(3/2)*c^4*f*g^6
/(c^4*g^8*h^4*x + 4*a*c^3*g^6*h^6*x + 6*a^2*c^2*g^4*h^8*x + 4*a^3*c*g^2*h^1
0*x + a^4*h^12*x + c^4*g^9*h^3 + 4*a*c^3*g^7*h^5 + 6*a^2*c^2*g^5*h^7 + 4*a^
3*c*g^3*h^9 + a^4*g*h^11) + 3/8*sqrt(c*x^2 + a)*c^5*e*g^5*x/(c^4*g^8*h^3 +
4*a*c^3*g^6*h^5 + 6*a^2*c^2*g^4*h^7 + 4*a^3*c*g^2*h^9 + a^4*h^11) + 3/8*sqr
t(c*x^2 + a)*c^5*d*g^5/(c^4*g^8*h^3 + 4*a*c^3*g^6*h^5 + 6*a^2*c^2*g^4*h^7 +
 4*a^3*c*g^2*h^9 + a^4*h^11) - 1/8*(c*x^2 + a)^(3/2)*c^4*e*g^5/(c^4*g^8*h^3
*x + 4*a*c^3*g^6*h^5*x + 6*a^2*c^2*g^4*h^7*x + 4*a^3*c*g^2*h^9*x + a^4*h^11
*x + c^4*g^9*h^2 + 4*a*c^3*g^7*h^4 + 6*a^2*c^2*g^5*h^6 + 4*a^3*c*g^3*h^8 +
a^4*g*h^10) - 1/8*(c*x^2 + a)^(5/2)*c^3*f*g^5/(c^4*g^8*h^3*x^2 + 4*a*c^3*g^
6*h^5*x^2 + 6*a^2*c^2*g^4*h^7*x^2 + 4*a^3*c*g^2*h^9*x^2 + a^4*h^11*x^2 + 2*
c^4*g^9*h^2*x + 8*a*c^3*g^7*h^4*x + 12*a^2*c^2*g^5*h^6*x + 8*a^3*c*g^3*h^8*
x + 2*a^4*g*h^10*x + c^4*g^10*h + 4*a*c^3*g^8*h^3 + 6*a^2*c^2*g^6*h^5 + 4*a
^3*c*g^4*h^7 + a^4*g^2*h^9) + 1/8*(c*x^2 + a)^(3/2)*c^4*f*g^5/(c^4*g^8*h^3
+ 4*a*c^3*g^6*h^5 + 6*a^2*c^2*g^4*h^7 + 4*a^3*c*g^2*h^9 + a^4*h^11) - 3/8*s
qrt(c*x^2 + a)*c^5*d*g^4*x/(c^4*g^8*h^2 + 4*a*c^3*g^6*h^4 + 6*a^2*c^2*g^4*h
^6 + 4*a^3*c*g^2*h^8 + a^4*h^10) + 1/8*(c*x^2 + a)^(3/2)*c^4*d*g^4/(c^4*g^8
*h^2*x + 4*a*c^3*g^6*h^4*x + 6*a^2*c^2*g^4*h^6*x + 4*a^3*c*g^2*h^8*x + a^4*
h^10*x + c^4*g^9*h + 4*a*c^3*g^7*h^3 + 6*a^2*c^2*g^5*h^5 + 4*a^3*c*g^3*h^7
+ a^4*g*h^9) + 1/8*(c*x^2 + a)^(5/2)*c^3*e*g^4/(c^4*g^8*h^2*x^2 + 4*a*c^3*g
^6*h^4*x^2 + 6*a^2*c^2*g^4*h^6*x^2 + 4*a^3*c*g^2*h^8*x^2 + a^4*h^10*x^2 + 2
*c^4*g^9*h*x + 8*a*c^3*g^7*h^3*x + 12*a^2*c^2*g^5*h^5*x + 8*a^3*c*g^3*h^7*x
 + 2*a^4*g*h^9*x + c^4*g^10 + 4*a*c^3*g^8*h^2 + 6*a^2*c^2*g^6*h^4 + 4*a^3*c
*g^4*h^6 + a^4*g^2*h^8) - 1/8*(c*x^2 + a)^(3/2)*c^4*e*g^4/(c^4*g^8*h^2 + 4*
a*c^3*g^6*h^4 + 6*a^2*c^2*g^4*h^6 + 4*a^3*c*g^2*h^8 + a^4*h^10) - 3/2*sqrt(
c*x^2 + a)*c^4*f*g^5/(c^3*g^6*h^5 + 3*a*c^2*g^4*h^7 + 3*a^2*c*g^2*h^9 + a^3
*h^11) + 9/8*sqrt(c*x^2 + a)*c^4*f*g^4*x/(c^3*g^6*h^4 + 3*a*c^2*g^4*h^6 + 3
*a^2*c*g^2*h^8 + a^3*h^10) - 1/8*(c*x^2 + a)^(5/2)*c^3*d*g^3/(c^4*g^8*h*x^2
 + 4*a*c^3*g^6*h^3*x^2 + 6*a^2*c^2*g^4*h^5*x^2 + 4*a^3*c*g^2*h^7*x^2 + a^4*
h^9*x^2 + 2*c^4*g^9*x + 8*a*c^3*g^7*h^2*x + 12*a^2*c^2*g^5*h^4*x + 8*a^3*c*
g^3*h^6*x + 2*a^4*g*h^8*x + c^4*g^10/h + 4*a*c^3*g^8*h + 6*a^2*c^2*g^6*h^3
 + 4*a^3*c*g^4*h^5 + a^4*g^2*h^7) + 1/8*(c*x^2 + a)^(3/2)*c^4*d*g^3/(c^4*g^8
*h + 4*a*c^3*g^6*h^3 + 6*a^2*c^2*g^4*h^5 + 4*a^3*c*g^2*h^7 + a^4*h^9) + 9/8
*sqrt(c*x^2 + a)*c^4*e*g^4/(c^3*g^6*h^4 + 3*a*c^2*g^4*h^6 + 3*a^2*c*g^2*h^8
 + a^3*h^10) - 1/4*(c*x^2 + a)^(5/2)*c^2*f*g^4/(c^3*g^6*h^4*x^3 + 3*a*c^2*g
^4*h^6*x^3 + 3*a^2*c*g^2*h^8*x^3 + a^3*h^10*x^3 + 3*c^3*g^7*h^3*x^2 + 9*a*c
^2*g^5*h^5*x^2 + 9*a^2*c*g^3*h^7*x^2 + 3*a^3*g*h^9*x^2 + 3*c^3*g^8*h^2*x +
9*a*c^2*g^6*h^4*x + 9*a^2*c*g^4*h^6*x + 3*a^3*g^2*h^8*x + c^3*g^9*h + 3*a*c
^2*g^7*h^3 + 3*a^2*c*g^5*h^5 + a^3*g^3*h^7) - 5/8*(c*x^2 + a)^(3/2)*c^3*f*g
^4/(c^3*g^6*h^4*x + 3*a*c^2*g^4*h^6*x + 3*a^2*c*g^2*h^8*x + a^3*h^10*x + c^
3*g^7*h^3 + 3*a*c^2*g^5*h^5 + 3*a^2*c*g^3*h^7 + a^3*g*h^9) - 3/4*sqrt(c*x^2
 + a)*c^4*e*g^3*x/(c^3*g^6*h^3 + 3*a*c^2*g^4*h^5 + 3*a^2*c*g^2*h^7 + a^3*h^
9) - 3/4*sqrt(c*x^2 + a)*c^4*d*g^3/(c^3*g^6*h^3 + 3*a*c^2*g^4*h^5 + 3*a^2*c
```

```
*g^2*h^7 + a^3*h^9) + 1/4*(c*x^2 + a)^(5/2)*c^2*e*g^3/(c^3*g^6*h^3*x^3 + 3*
a*c^2*g^4*h^5*x^3 + 3*a^2*c*g^2*h^7*x^3 + a^3*h^9*x^3 + 3*c^3*g^7*h^2*x^2 +
 9*a*c^2*g^5*h^4*x^2 + 9*a^2*c*g^3*h^6*x^2 + 3*a^3*g*h^8*x^2 + 3*c^3*g^8*h*
x + 9*a*c^2*g^6*h^3*x + 9*a^2*c*g^4*h^5*x + 3*a^3*g^2*h^7*x + c^3*g^9 + 3*a
*c^2*g^7*h^2 + 3*a^2*c*g^5*h^4 + a^3*g^3*h^6) + 1/2*(c*x^2 + a)^(3/2)*c^3*e
*g^3/(c^3*g^6*h^3*x + 3*a*c^2*g^4*h^5*x + 3*a^2*c*g^2*h^7*x + a^3*h^9*x + c
^3*g^7*h^2 + 3*a*c^2*g^5*h^4 + 3*a^2*c*g^3*h^6 + a^3*g*h^8) + 1/8*(c*x^2 +
a)^(5/2)*c^2*f*g^3/(c^3*g^6*h^3*x^2 + 3*a*c^2*g^4*h^5*x^2 + 3*a^2*c*g^2*h^7
*x^2 + a^3*h^9*x^2 + 2*c^3*g^7*h^2*x + 6*a*c^2*g^5*h^4*x + 6*a^2*c*g^3*h^6*
x + 2*a^3*g*h^8*x + c^3*g^8*h + 3*a*c^2*g^6*h^3 + 3*a^2*c*g^4*h^5 + a^3*g^2
*h^7) - 1/8*(c*x^2 + a)^(3/2)*c^3*f*g^3/(c^3*g^6*h^3 + 3*a*c^2*g^4*h^5 + 3*
a^2*c*g^2*h^7 + a^3*h^9) + 3/8*sqrt(c*x^2 + a)*c^4*d*g^2*x/(c^3*g^6*h^2 + 3
*a*c^2*g^4*h^4 + 3*a^2*c*g^2*h^6 + a^3*h^8) - 1/4*(c*x^2 + a)^(5/2)*c^2*d*g
^2/(c^3*g^6*h^2*x^3 + 3*a*c^2*g^4*h^4*x^3 + 3*a^2*c*g^2*h^6*x^3 + a^3*h^8*x
^3 + 3*c^3*g^7*h*x^2 + 9*a*c^2*g^5*h^3*x^2 + 9*a^2*c*g^3*h^5*x^2 + 3*a^3*g*
h^7*x^2 + 3*c^3*g^8*x + 9*a*c^2*g^6*h^2*x + 9*a^2*c*g^4*h^4*x + 3*a^3*g^2*h
^6*x + c^3*g^9/h + 3*a*c^2*g^7*h + 3*a^2*c*g^5*h^3 + a^3*g^3*h^5) - 3/8*(c*
x^2 + a)^(3/2)*c^3*d*g^2/(c^3*g^6*h^2*x + 3*a*c^2*g^4*h^4*x + 3*a^2*c*g^2*h
^6*x + a^3*h^8*x + c^3*g^7*h + 3*a*c^2*g^5*h^3 + 3*a^2*c*g^3*h^5 + a^3*g*h^
7) - 1/4*(c*x^2 + a)^(5/2)*c*f*g^3/(c^2*g^4*h^5*x^4 + 2*a*c*g^2*h^7*x^4 + a
^2*h^9*x^4 + 4*c^2*g^5*h^4*x^3 + 8*a*c*g^3*h^6*x^3 + 4*a^2*g*h^8*x^3 + 6*c^
2*g^6*h^3*x^2 + 12*a*c*g^4*h^5*x^2 + 6*a^2*g^2*h^7*x^2 + 4*c^2*g^7*h^2*x +
8*a*c*g^5*h^4*x + 4*a^2*g^3*h^6*x + c^2*g^8*h + 2*a*c*g^6*h^3 + a^2*g^4*h^5
) + 19/8*sqrt(c*x^2 + a)*c^3*f*g^3/(c^2*g^4*h^5 + 2*a*c*g^2*h^7 + a^2*h^9)
 - 5/4*sqrt(c*x^2 + a)*c^3*f*g^2*x/(c^2*g^4*h^4 + 2*a*c*g^2*h^6 + a^2*h^8) -
 1/8*(c*x^2 + a)^(5/2)*c^2*d*g/(c^3*g^6*h*x^2 + 3*a*c^2*g^4*h^3*x^2 + 3*a^2
*c*g^2*h^5*x^2 + a^3*h^7*x^2 + 2*c^3*g^7*x + 6*a*c^2*g^5*h^2*x + 6*a^2*c*g^
3*h^4*x + 2*a^3*g*h^6*x + c^3*g^8/h + 3*a*c^2*g^6*h + 3*a^2*c*g^4*h^3 + a^3
*g^2*h^5) + 1/8*(c*x^2 + a)^(3/2)*c^3*d*g/(c^3*g^6*h + 3*a*c^2*g^4*h^3 + 3*
a^2*c*g^2*h^5 + a^3*h^7) + 1/4*(c*x^2 + a)^(5/2)*c*e*g^2/(c^2*g^4*h^4*x^4 +
 2*a*c*g^2*h^6*x^4 + a^2*h^8*x^4 + 4*c^2*g^5*h^3*x^3 + 8*a*c*g^3*h^5*x^3 +
4*a^2*g*h^7*x^3 + 6*c^2*g^6*h^2*x^2 + 12*a*c*g^4*h^4*x^2 + 6*a^2*g^2*h^6*x^
2 + 4*c^2*g^7*h*x + 8*a*c*g^5*h^3*x + 4*a^2*g^3*h^5*x + c^2*g^8 + 2*a*c*g^6
*h^2 + a^2*g^4*h^4) - 9/8*sqrt(c*x^2 + a)*c^3*e*g^2/(c^2*g^4*h^4 + 2*a*c*g^
2*h^6 + a^2*h^8) + 1/2*(c*x^2 + a)^(5/2)*c*f*g^2/(c^2*g^4*h^4*x^3 + 2*a*c*g
^2*h^6*x^3 + a^2*h^8*x^3 + 3*c^2*g^5*h^3*x^2 + 6*a*c*g^3*h^5*x^2 + 3*a^2*g*
h^7*x^2 + 3*c^2*g^6*h^2*x + 6*a*c*g^4*h^4*x + 3*a^2*g^2*h^6*x + c^2*g^7*h +
 2*a*c*g^5*h^3 + a^2*g^3*h^5) + 11/12*(c*x^2 + a)^(3/2)*c^2*f*g^2/(c^2*g^4*
h^4*x + 2*a*c*g^2*h^6*x + a^2*h^8*x + c^2*g^5*h^3 + 2*a*c*g^3*h^5 + a^2*g*h
^7) + 3/8*sqrt(c*x^2 + a)*c^3*e*g*x/(c^2*g^4*h^3 + 2*a*c*g^2*h^5 + a^2*h^7)
 - 1/4*(c*x^2 + a)^(5/2)*c*d*g/(c^2*g^4*h^3*x^4 + 2*a*c*g^2*h^5*x^4 + a^2*h
^7*x^4 + 4*c^2*g^5*h^2*x^3 + 8*a*c*g^3*h^4*x^3 + 4*a^2*g*h^6*x^3 + 6*c^2*g^
6*h*x^2 + 12*a*c*g^4*h^3*x^2 + 6*a^2*g^2*h^5*x^2 + 4*c^2*g^7*x + 8*a*c*g^5*
h^2*x + 4*a^2*g^3*h^4*x + c^2*g^8/h + 2*a*c*g^6*h + a^2*g^4*h^3) + 3/8*sqrt
(c*x^2 + a)*c^3*d*g/(c^2*g^4*h^3 + 2*a*c*g^2*h^5 + a^2*h^7) - 1/4*(c*x^2 +
```

```
a)^(5/2)*c*e*g/(c^2*g^4*h^3*x^3 + 2*a*c*g^2*h^5*x^3 + a^2*h^7*x^3 + 3*c^2*g
^5*h^2*x^2 + 6*a*c*g^3*h^4*x^2 + 3*a^2*g*h^6*x^2 + 3*c^2*g^6*h*x + 6*a*c*g^
4*h^3*x + 3*a^2*g^2*h^5*x + c^2*g^7 + 2*a*c*g^5*h^2 + a^2*g^3*h^4) - 3/8*(c
*x^2 + a)^(3/2)*c^2*e*g/(c^2*g^4*h^3*x + 2*a*c*g^2*h^5*x + a^2*h^7*x + c^2*
g^5*h^2 + 2*a*c*g^3*h^4 + a^2*g*h^6) + 1/12*(c*x^2 + a)^(5/2)*c*f*g/(c^2*g^
4*h^3*x^2 + 2*a*c*g^2*h^5*x^2 + a^2*h^7*x^2 + 2*c^2*g^5*h^2*x + 4*a*c*g^3*h
^4*x + 2*a^2*g*h^6*x + c^2*g^6*h + 2*a*c*g^4*h^3 + a^2*g^2*h^5) - 1/12*(c*x
^2 + a)^(3/2)*c^2*f*g/(c^2*g^4*h^3 + 2*a*c*g^2*h^5 + a^2*h^7) - 1/5*(c*x^2
+ a)^(5/2)*f*g^2/(c*g^2*h^6*x^5 + a*h^8*x^5 + 5*c*g^3*h^5*x^4 + 5*a*g*h^7*x
^4 + 10*c*g^4*h^4*x^3 + 10*a*g^2*h^6*x^3 + 10*c*g^5*h^3*x^2 + 10*a*g^3*h^5*
x^2 + 5*c*g^6*h^2*x + 5*a*g^4*h^4*x + c*g^7*h + a*g^5*h^3) - 1/8*(c*x^2 + a
)^(5/2)*c*e/(c^2*g^4*h^2*x^2 + 2*a*c*g^2*h^4*x^2 + a^2*h^6*x^2 + 2*c^2*g^5*
h*x + 4*a*c*g^3*h^3*x + 2*a^2*g*h^5*x + c^2*g^6 + 2*a*c*g^4*h^2 + a^2*g^2*h
^4) + 1/8*(c*x^2 + a)^(3/2)*c^2*e/(c^2*g^4*h^2 + 2*a*c*g^2*h^4 + a^2*h^6) +
 1/5*(c*x^2 + a)^(5/2)*e*g/(c*g^2*h^5*x^5 + a*h^7*x^5 + 5*c*g^3*h^4*x^4 + 5
*a*g*h^6*x^4 + 10*c*g^4*h^3*x^3 + 10*a*g^2*h^5*x^3 + 10*c*g^5*h^2*x^2 + 10*
a*g^3*h^4*x^2 + 5*c*g^6*h*x + 5*a*g^4*h^3*x + c*g^7 + a*g^5*h^2) + 1/2*(c*x
^2 + a)^(5/2)*f*g/(c*g^2*h^5*x^4 + a*h^7*x^4 + 4*c*g^3*h^4*x^3 + 4*a*g*h^6*
x^3 + 6*c*g^4*h^3*x^2 + 6*a*g^2*h^5*x^2 + 4*c*g^5*h^2*x + 4*a*g^3*h^4*x + c
*g^6*h + a*g^4*h^3) - 9/4*sqrt(c*x^2 + a)*c^2*f*g/(c*g^2*h^5 + a*h^7) + sqr
t(c*x^2 + a)*c^2*f*x/(c*g^2*h^4 + a*h^6) - 1/5*(c*x^2 + a)^(5/2)*d/(c*g^2*h
^4*x^5 + a*h^6*x^5 + 5*c*g^3*h^3*x^4 + 5*a*g*h^5*x^4 + 10*c*g^4*h^2*x^3 + 1
0*a*g^2*h^4*x^3 + 10*c*g^5*h*x^2 + 10*a*g^3*h^3*x^2 + 5*c*g^6*x + 5*a*g^4*h
^2*x + c*g^7/h + a*g^5*h) - 1/4*(c*x^2 + a)^(5/2)*e/(c*g^2*h^4*x^4 + a*h^6*
x^4 + 4*c*g^3*h^3*x^3 + 4*a*g*h^5*x^3 + 6*c*g^4*h^2*x^2 + 6*a*g^2*h^4*x^2 +
 4*c*g^5*h*x + 4*a*g^3*h^3*x + c*g^6 + a*g^4*h^2) + 3/8*sqrt(c*x^2 + a)*c^2
*e/(c*g^2*h^4 + a*h^6) - 1/3*(c*x^2 + a)^(5/2)*f/(c*g^2*h^4*x^3 + a*h^6*x^3
 + 3*c*g^3*h^3*x^2 + 3*a*g*h^5*x^2 + 3*c*g^4*h^2*x + 3*a*g^2*h^4*x + c*g^5*
h + a*g^3*h^3) - 2/3*(c*x^2 + a)^(3/2)*c*f/(c*g^2*h^4*x + a*h^6*x + c*g^3*h
^3 + a*g*h^5) + c^(3/2)*f*arcsinh(c*x/sqrt(a*c))/h^6 + 3/8*c^5*f*g^7*arcsin
h(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^
2/h^2)^(7/2)*h^13) - 3/8*c^5*e*g^6*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) -
 a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(7/2)*h^12) + 3/8*c^5*d*g^5
*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a
 + c*g^2/h^2)^(7/2)*h^11) - 3/2*c^4*f*g^5*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x
+ g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(5/2)*h^11) + 9/8*c^
4*e*g^4*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g
)))/((a + c*g^2/h^2)^(5/2)*h^10) - 3/4*c^4*d*g^3*arcsinh(c*g*x/(sqrt(a*c)*a
bs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(5/2)*h^9) +
19/8*c^3*f*g^3*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(
h*x + g)))/((a + c*g^2/h^2)^(3/2)*h^9) - 9/8*c^3*e*g^2*arcsinh(c*g*x/(sqrt(
a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(3/2)*h
^8) + 3/8*c^3*d*g*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*a
bs(h*x + g)))/((a + c*g^2/h^2)^(3/2)*h^7) - 9/4*c^2*f*g*arcsinh(c*g*x/(sqrt
(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/(sqrt(a + c*g^2/h^2)*h^
```

```
7) + 3/8*c^2*e*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(
h*x + g)))/(sqrt(a + c*g^2/h^2)*h^6)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4363 vs. $2(482) = 964$.

Time = 0.55 (sec) , antiderivative size = 4363, normalized size of antiderivative = 8.61

$$\int \frac{(a+cx^2)^{3/2}\,(d+ex+fx^2)}{(g+hx)^6}\,dx = \text{Too large to display}$$

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^6,x, algorithm="giac")
```

```
[Out] -1/4*(8*c^5*f*g^7 + 28*a*c^4*f*g^5*h^2 + 35*a^2*c^3*f*g^3*h^4 - 3*a^2*c^3*d
*g*h^6 + 18*a^3*c^2*f*g*h^6 - 3*a^3*c^2*e*h^7)*arctan(-((sqrt(c)*x - sqrt(c
*x^2 + a))*h + sqrt(c)*g)/sqrt(-c*g^2 - a*h^2))/((c^3*g^6*h^6 + 3*a*c^2*g^4
*h^8 + 3*a^2*c*g^2*h^10 + a^3*h^12)*sqrt(-c*g^2 - a*h^2)) - c^(3/2)*f*log(a
bs(-sqrt(c)*x + sqrt(c*x^2 + a)))/h^6 - 1/60*(600*(sqrt(c)*x - sqrt(c*x^2 +
 a))^9*c^5*f*g^7*h^4 - 120*(sqrt(c)*x - sqrt(c*x^2 + a))^9*c^5*e*g^6*h^5 +
1740*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a*c^4*f*g^5*h^6 - 360*(sqrt(c)*x - sqr
t(c*x^2 + a))^9*a*c^4*e*g^4*h^7 + 1635*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^2*
c^3*f*g^3*h^8 - 360*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^2*c^3*e*g^2*h^9 + 45*
(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^2*c^3*d*g*h^10 + 450*(sqrt(c)*x - sqrt(c*
x^2 + a))^9*a^3*c^2*f*g*h^10 - 75*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^3*c^2*e
*h^11 + 3600*(sqrt(c)*x - sqrt(c*x^2 + a))^8*c^(11/2)*f*g^8*h^3 - 480*(sqrt
(c)*x - sqrt(c*x^2 + a))^8*c^(11/2)*e*g^7*h^4 - 120*(sqrt(c)*x - sqrt(c*x^2
 + a))^8*c^(11/2)*d*g^6*h^5 + 10020*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a*c^(9/
2)*f*g^6*h^5 - 1440*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a*c^(9/2)*e*g^5*h^6 - 3
60*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a*c^(9/2)*d*g^4*h^7 + 8595*(sqrt(c)*x -
sqrt(c*x^2 + a))^8*a^2*c^(7/2)*f*g^4*h^7 - 1440*(sqrt(c)*x - sqrt(c*x^2 + a
))^8*a^2*c^(7/2)*e*g^3*h^8 + 45*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^2*c^(7/2)
*d*g^2*h^9 + 1530*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^3*c^(5/2)*f*g^2*h^9 - 7
5*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^3*c^(5/2)*e*g*h^10 - 120*(sqrt(c)*x - s
qrt(c*x^2 + a))^8*a^3*c^(5/2)*d*h^11 - 240*(sqrt(c)*x - sqrt(c*x^2 + a))^8*
a^4*c^(3/2)*f*h^11 + 8800*(sqrt(c)*x - sqrt(c*x^2 + a))^7*c^6*f*g^9*h^2 - 9
60*(sqrt(c)*x - sqrt(c*x^2 + a))^7*c^6*e*g^8*h^3 - 240*(sqrt(c)*x - sqrt(c*
x^2 + a))^7*c^6*d*g^7*h^4 + 21240*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a*c^5*f*g
^7*h^4 - 2640*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a*c^5*e*g^6*h^5 - 720*(sqrt(c
)*x - sqrt(c*x^2 + a))^7*a*c^5*d*g^5*h^6 + 11670*(sqrt(c)*x - sqrt(c*x^2 +
a))^7*a^2*c^4*f*g^5*h^6 - 2160*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^2*c^4*e*g^
4*h^7 + 690*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^2*c^4*d*g^3*h^8 - 4970*(sqrt(
c)*x - sqrt(c*x^2 + a))^7*a^3*c^3*f*g^3*h^8 + 1170*(sqrt(c)*x - sqrt(c*x^2
+ a))^7*a^3*c^3*e*g^2*h^9 - 450*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^3*c^3*d*g
*h^10 - 2580*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^4*c^2*f*g*h^10 + 30*(sqrt(c)
```

```
*x - sqrt(c*x^2 + a))^7*a^4*c^2*e*h^11 + 10000*(sqrt(c)*x - sqrt(c*x^2 + a)
)^6*c^(13/2)*f*g^10*h - 960*(sqrt(c)*x - sqrt(c*x^2 + a))^6*c^(13/2)*e*g^9*
h^2 - 240*(sqrt(c)*x - sqrt(c*x^2 + a))^6*c^(13/2)*d*g^8*h^3 + 14040*(sqrt(
c)*x - sqrt(c*x^2 + a))^6*a*c^(11/2)*f*g^8*h^3 - 1680*(sqrt(c)*x - sqrt(c*x
^2 + a))^6*a*c^(11/2)*e*g^7*h^4 - 720*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a*c^(
11/2)*d*g^6*h^5 - 14430*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^2*c^(9/2)*f*g^6*h
^5 + 720*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^2*c^(9/2)*e*g^5*h^6 + 1590*(sqrt
(c)*x - sqrt(c*x^2 + a))^6*a^2*c^(9/2)*d*g^4*h^7 - 28790*(sqrt(c)*x - sqrt(
c*x^2 + a))^6*a^3*c^(7/2)*f*g^4*h^7 + 4950*(sqrt(c)*x - sqrt(c*x^2 + a))^6*
a^3*c^(7/2)*e*g^3*h^8 - 1710*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^3*c^(7/2)*d*
g^2*h^9 - 5820*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^4*c^(5/2)*f*g^2*h^9 - 270*
(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^4*c^(5/2)*e*g*h^10 + 720*(sqrt(c)*x - sqr
t(c*x^2 + a))^6*a^5*c^(3/2)*f*h^11 + 4384*(sqrt(c)*x - sqrt(c*x^2 + a))^5*c
^7*f*g^11 - 384*(sqrt(c)*x - sqrt(c*x^2 + a))^5*c^7*e*g^10*h - 96*(sqrt(c)*
x - sqrt(c*x^2 + a))^5*c^7*d*g^9*h^2 - 9392*(sqrt(c)*x - sqrt(c*x^2 + a))^5
*a*c^6*f*g^9*h^2 + 672*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a*c^6*e*g^8*h^3 + 48
*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a*c^6*d*g^7*h^4 - 42996*(sqrt(c)*x - sqrt(
c*x^2 + a))^5*a^2*c^5*f*g^7*h^4 + 3936*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^2*
c^5*e*g^6*h^5 + 2364*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^2*c^5*d*g^5*h^6 - 31
070*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^3*c^4*f*g^5*h^6 + 5580*(sqrt(c)*x - s
qrt(c*x^2 + a))^5*a^3*c^4*e*g^4*h^7 - 2730*(sqrt(c)*x - sqrt(c*x^2 + a))^5*
a^3*c^4*d*g^3*h^8 + 8620*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^4*c^3*f*g^3*h^8
 - 2970*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^4*c^3*e*g^2*h^9 + 720*(sqrt(c)*x -
 sqrt(c*x^2 + a))^5*a^4*c^3*d*g*h^10 + 4800*(sqrt(c)*x - sqrt(c*x^2 + a))^5
*a^5*c^2*f*g*h^10 - 11920*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a*c^(13/2)*f*g^10
*h + 960*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a*c^(13/2)*e*g^9*h^2 + 240*(sqrt(c
)*x - sqrt(c*x^2 + a))^4*a*c^(13/2)*d*g^8*h^3 - 15720*(sqrt(c)*x - sqrt(c*x
^2 + a))^4*a^2*c^(11/2)*f*g^8*h^3 + 1680*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^
2*c^(11/2)*e*g^7*h^4 + 720*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^2*c^(11/2)*d*g
^6*h^5 + 19670*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^3*c^(9/2)*f*g^6*h^5 - 480*
(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^3*c^(9/2)*e*g^5*h^6 - 3510*(sqrt(c)*x - s
qrt(c*x^2 + a))^4*a^3*c^(9/2)*d*g^4*h^7 + 36260*(sqrt(c)*x - sqrt(c*x^2 + a
))^4*a^4*c^(7/2)*f*g^4*h^7 - 6150*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^4*c^(7/
2)*e*g^3*h^8 + 1440*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^4*c^(7/2)*d*g^2*h^9 +
 6240*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^5*c^(5/2)*f*g^2*h^9 + 720*(sqrt(c)*
x - sqrt(c*x^2 + a))^4*a^5*c^(5/2)*e*g*h^10 - 240*(sqrt(c)*x - sqrt(c*x^2 +
 a))^4*a^5*c^(5/2)*d*h^11 - 880*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^6*c^(3/2)
*f*h^11 + 13120*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^2*c^6*f*g^9*h^2 - 960*(sq
rt(c)*x - sqrt(c*x^2 + a))^3*a^2*c^6*e*g^8*h^3 - 240*(sqrt(c)*x - sqrt(c*x^
2 + a))^3*a^2*c^6*d*g^7*h^4 + 30440*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^3*c^5
*f*g^7*h^4 - 2640*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^3*c^5*e*g^6*h^5 - 1200*
(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^3*c^5*d*g^5*h^6 + 14130*(sqrt(c)*x - sqrt
(c*x^2 + a))^3*a^4*c^4*f*g^5*h^6 - 2640*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^4
*c^4*e*g^4*h^7 + 2310*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^4*c^4*d*g^3*h^8 - 1
0790*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^5*c^3*f*g^3*h^8 + 2790*(sqrt(c)*x -
```

```
sqrt(c*x^2 + a))^3*a^5*c^3*e*g^2*h^9 - 510*(sqrt(c)*x - sqrt(c*x^2 + a))^3*
a^5*c^3*d*g*h^10 - 3820*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^6*c^2*f*g*h^10 -
30*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^6*c^2*e*h^11 - 7360*(sqrt(c)*x - sqrt(
c*x^2 + a))^2*a^3*c^(11/2)*f*g^8*h^3 + 480*(sqrt(c)*x - sqrt(c*x^2 + a))^2*
a^3*c^(11/2)*e*g^7*h^4 + 120*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^3*c^(11/2)*d
*g^6*h^5 - 19930*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^4*c^(9/2)*f*g^6*h^5 + 15
60*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^4*c^(9/2)*e*g^5*h^6 + 690*(sqrt(c)*x -
 sqrt(c*x^2 + a))^2*a^4*c^(9/2)*d*g^4*h^7 - 16050*(sqrt(c)*x - sqrt(c*x^2 +
 a))^2*a^5*c^(7/2)*f*g^4*h^7 + 2130*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^5*c^(
7/2)*e*g^3*h^8 - 1050*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^5*c^(7/2)*d*g^2*h^9
 - 1300*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^6*c^(5/2)*f*g^2*h^9 - 570*(sqrt(c
)*x - sqrt(c*x^2 + a))^2*a^6*c^(5/2)*e*g*h^10 + 560*(sqrt(c)*x - sqrt(c*x^2
 + a))^2*a^7*c^(3/2)*f*h^11 + 2140*(sqrt(c)*x - sqrt(c*x^2 + a))*a^4*c^5*f*
g^7*h^4 - 120*(sqrt(c)*x - sqrt(c*x^2 + a))*a^4*c^5*e*g^6*h^5 - 60*(sqrt(c)
*x - sqrt(c*x^2 + a))*a^4*c^5*d*g^5*h^6 + 6090*(sqrt(c)*x - sqrt(c*x^2 + a)
)*a^5*c^4*f*g^5*h^6 - 420*(sqrt(c)*x - sqrt(c*x^2 + a))*a^5*c^4*e*g^4*h^7 -
 270*(sqrt(c)*x - sqrt(c*x^2 + a))*a^5*c^4*d*g^3*h^8 + 5505*(sqrt(c)*x - sq
rt(c*x^2 + a))*a^6*c^3*f*g^3*h^8 - 630*(sqrt(c)*x - sqrt(c*x^2 + a))*a^6*c^
3*e*g^2*h^9 + 195*(sqrt(c)*x - sqrt(c*x^2 + a))*a^6*c^3*d*g*h^10 + 1150*(sq
rt(c)*x - sqrt(c*x^2 + a))*a^7*c^2*f*g*h^10 + 75*(sqrt(c)*x - sqrt(c*x^2 +
a))*a^7*c^2*e*h^11 - 274*a^5*c^(9/2)*f*g^6*h^5 + 24*a^5*c^(9/2)*e*g^5*h^6 +
 6*a^5*c^(9/2)*d*g^4*h^7 - 783*a^6*c^(7/2)*f*g^4*h^7 + 78*a^6*c^(7/2)*e*g^3
*h^8 + 27*a^6*c^(7/2)*d*g^2*h^9 - 714*a^7*c^(5/2)*f*g^2*h^9 + 99*a^7*c^(5/2
)*e*g*h^10 - 24*a^7*c^(5/2)*d*h^11 - 160*a^8*c^(3/2)*f*h^11)/((c^3*g^6*h^6
+ 3*a*c^2*g^4*h^8 + 3*a^2*c*g^2*h^10 + a^3*h^12)*((sqrt(c)*x - sqrt(c*x^2 +
 a))^2*h + 2*(sqrt(c)*x - sqrt(c*x^2 + a))*sqrt(c)*g - a*h)^5)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^6}\,dx = \int \frac{(c\,x^2+a)^{3/2}\,(f\,x^2+e\,x+d)}{(g+h\,x)^6}\,dx$$

```
[In] int(((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^6,x)
[Out] int(((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^6, x)
```

3.98 $\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^7}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 404

$$
\begin{aligned}
&\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^7}\,dx = \\
&-\frac{ac(6c^2dg^2+6a^2fh^2-ac(fg^2-h(7eg-dh)))(ah-cgx)\sqrt{a+cx^2}}{16(cg^2+ah^2)^4(g+hx)^2} \\
&-\frac{(6c^2dg^2+6a^2fh^2-ac(fg^2-h(7eg-dh)))(ah-cgx)(a+cx^2)^{3/2}}{24(cg^2+ah^2)^3(g+hx)^4} \\
&-\frac{(fg^2-egh+dh^2)(a+cx^2)^{5/2}}{6h(cg^2+ah^2)(g+hx)^6} \\
&+\frac{(6ah^2(2fg-eh)+cg(5fg^2+h(eg-7dh)))(a+cx^2)^{5/2}}{30h(cg^2+ah^2)^2(g+hx)^5} \\
&-\frac{a^2c^2(6c^2dg^2+6a^2fh^2-ac(fg^2-h(7eg-dh)))\operatorname{arctanh}\left(\frac{ah-cgx}{\sqrt{cg^2+ah^2}\sqrt{a+cx^2}}\right)}{16(cg^2+ah^2)^{9/2}}
\end{aligned}
$$

```
[Out] -1/24*(6*c^2*d*g^2+6*a^2*f*h^2-a*c*(f*g^2-h*(-d*h+7*e*g)))*(-c*g*x+a*h)*(c*
x^2+a)^(3/2)/(a*h^2+c*g^2)^3/(h*x+g)^4-1/6*(d*h^2-e*g*h+f*g^2)*(c*x^2+a)^(5
/2)/h/(a*h^2+c*g^2)/(h*x+g)^6+1/30*(6*a*h^2*(-e*h+2*f*g)+c*g*(5*f*g^2+h*(-7
*d*h+e*g)))*(c*x^2+a)^(5/2)/h/(a*h^2+c*g^2)^2/(h*x+g)^5-1/16*a^2*c^2*(6*c^2
*d*g^2+6*a^2*f*h^2-a*c*(f*g^2-h*(-d*h+7*e*g)))*arctanh((-c*g*x+a*h)/(a*h^2+
c*g^2)^(1/2)/(c*x^2+a)^(1/2))/(a*h^2+c*g^2)^(9/2)-1/16*a*c*(6*c^2*d*g^2+6*a
^2*f*h^2-a*c*(f*g^2-h*(-d*h+7*e*g)))*(-c*g*x+a*h)*(c*x^2+a)^(1/2)/(a*h^2+c*
g^2)^4/(h*x+g)^2
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {1665, 821, 735, 739, 212}

$$\begin{aligned}
&\int \frac{(a+cx^2)^{3/2}\,(d+ex+fx^2)}{(g+hx)^7}\,dx = \\
&-\frac{a^2c^2\text{arctanh}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)(6a^2fh^2-ac(fg^2-h(7eg-dh))+6c^2dg^2)}{16\,(ah^2+cg^2)^{9/2}} \\
&-\frac{(a+cx^2)^{3/2}\,(ah-cgx)\,(6a^2fh^2-ac(fg^2-h(7eg-dh))+6c^2dg^2)}{24(g+hx)^4\,(ah^2+cg^2)^3} \\
&-\frac{ac\sqrt{a+cx^2}(ah-cgx)\,(6a^2fh^2-ac(fg^2-h(7eg-dh))+6c^2dg^2)}{16(g+hx)^2\,(ah^2+cg^2)^4} \\
&-\frac{(a+cx^2)^{5/2}\,(dh^2-egh+fg^2)}{6h(g+hx)^6\,(ah^2+cg^2)} \\
&+\frac{(a+cx^2)^{5/2}\,(6ah^2(2fg-eh)+cgh(eg-7dh)+5cfg^3)}{30h(g+hx)^5\,(ah^2+cg^2)^2}
\end{aligned}$$

```
[In] Int[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^7,x]
```

```
[Out] -1/16*(a*c*(6*c^2*d*g^2 + 6*a^2*f*h^2 - a*c*(f*g^2 - h*(7*e*g - d*h)))*(a*h
 - c*g*x)*Sqrt[a + c*x^2])/((c*g^2 + a*h^2)^4*(g + h*x)^2) - ((6*c^2*d*g^2
+ 6*a^2*f*h^2 - a*c*(f*g^2 - h*(7*e*g - d*h)))*(a*h - c*g*x)*(a + c*x^2)^(3
/2))/(24*(c*g^2 + a*h^2)^3*(g + h*x)^4) - ((f*g^2 - e*g*h + d*h^2)*(a + c*x
^2)^(5/2))/(6*h*(c*g^2 + a*h^2)*(g + h*x)^6) + ((5*c*f*g^3 + c*g*h*(e*g - 7
*d*h) + 6*a*h^2*(2*f*g - e*h))*(a + c*x^2)^(5/2))/(30*h*(c*g^2 + a*h^2)^2*(
g + h*x)^5) - (a^2*c^2*(6*c^2*d*g^2 + 6*a^2*f*h^2 - a*c*(f*g^2 - h*(7*e*g -
 d*h)))*ArcTanh[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2])])/(16*(
c*g^2 + a*h^2)^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 735

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(-2*a*e + (2*c*d)*x)*((a + c*x^2)^p/(2*(m + 1)*(c*d^2
+ a*e^2))), x] - Dist[4*a*c*(p/(2*(m + 1)*(c*d^2 + a*e^2))), Int[(d + e*x)^
(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2
```

```
+ a*e^2, 0] && EqQ[m + 2*p + 2, 0] && GtQ[p, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(fg^2-egh+dh^2)\left(a+cx^2\right)^{5/2}}{6h\left(cg^2+ah^2\right)(g+hx)^6} \\
&\quad -\frac{\int \frac{\left(-6(cdg-afg+aeh)-\left(6afh+c\left(eg+\frac{5fg^2}{h}-dh\right)\right)x\right)\left(a+cx^2\right)^{3/2}}{(g+hx)^6}\,dx}{6\left(cg^2+ah^2\right)} \\
&= -\frac{(fg^2-egh+dh^2)\left(a+cx^2\right)^{5/2}}{6h\left(cg^2+ah^2\right)(g+hx)^6} \\
&\quad +\frac{\left(5cfg^3+cgh(eg-7dh)+6ah^2(2fg-eh)\right)\left(a+cx^2\right)^{5/2}}{30h\left(cg^2+ah^2\right)^2(g+hx)^5} \\
&\quad +\frac{\left(6c^2dg^2+6a^2fh^2-ac(fg^2-h(7eg-dh))\right)\int \frac{\left(a+cx^2\right)^{3/2}}{(g+hx)^5}\,dx}{6\left(cg^2+ah^2\right)^2}
\end{aligned}$$

$$= -\frac{\left(6c^2dg^2+6a^2fh^2-ac(fg^2-h(7eg-dh))\right)(ah-cgx)\left(a+cx^2\right)^{3/2}}{24\left(cg^2+ah^2\right)^3(g+hx)^4} - \frac{\left(fg^2-egh+dh^2\right)\left(a+cx^2\right)^{5/2}}{6h\left(cg^2+ah^2\right)(g+hx)^6} + \frac{\left(5cfg^3+cgh(eg-7dh)+6ah^2(2fg-eh)\right)\left(a+cx^2\right)^{5/2}}{30h\left(cg^2+ah^2\right)^2(g+hx)^5} + \frac{\left(ac(6c^2dg^2+6a^2fh^2-ac(fg^2-h(7eg-dh)))\right)\int \frac{\sqrt{a+cx^2}}{(g+hx)^3}\,dx}{8\left(cg^2+ah^2\right)^3}$$

$$= -\frac{ac(6c^2dg^2+6a^2fh^2-ac(fg^2-h(7eg-dh)))(ah-cgx)\sqrt{a+cx^2}}{16\left(cg^2+ah^2\right)^4(g+hx)^2} - \frac{\left(6c^2dg^2+6a^2fh^2-ac(fg^2-h(7eg-dh))\right)(ah-cgx)\left(a+cx^2\right)^{3/2}}{24\left(cg^2+ah^2\right)^3(g+hx)^4} - \frac{\left(fg^2-egh+dh^2\right)\left(a+cx^2\right)^{5/2}}{6h\left(cg^2+ah^2\right)(g+hx)^6} + \frac{\left(5cfg^3+cgh(eg-7dh)+6ah^2(2fg-eh)\right)\left(a+cx^2\right)^{5/2}}{30h\left(cg^2+ah^2\right)^2(g+hx)^5} + \frac{\left(a^2c^2(6c^2dg^2+6a^2fh^2-ac(fg^2-h(7eg-dh)))\right)\int \frac{1}{(g+hx)\sqrt{a+cx^2}}\,dx}{16\left(cg^2+ah^2\right)^4}$$

$$= -\frac{ac(6c^2dg^2+6a^2fh^2-ac(fg^2-h(7eg-dh)))(ah-cgx)\sqrt{a+cx^2}}{16\left(cg^2+ah^2\right)^4(g+hx)^2} - \frac{\left(6c^2dg^2+6a^2fh^2-ac(fg^2-h(7eg-dh))\right)(ah-cgx)\left(a+cx^2\right)^{3/2}}{24\left(cg^2+ah^2\right)^3(g+hx)^4} - \frac{\left(fg^2-egh+dh^2\right)\left(a+cx^2\right)^{5/2}}{6h\left(cg^2+ah^2\right)(g+hx)^6} + \frac{\left(5cfg^3+cgh(eg-7dh)+6ah^2(2fg-eh)\right)\left(a+cx^2\right)^{5/2}}{30h\left(cg^2+ah^2\right)^2(g+hx)^5} - \frac{\left(a^2c^2(6c^2dg^2+6a^2fh^2-ac(fg^2-h(7eg-dh)))\right)\mathrm{Subst}\left(\int \frac{1}{cg^2+ah^2-x^2}\,dx, x, \frac{ah-cgx}{\sqrt{a+cx^2}}\right)}{16\left(cg^2+ah^2\right)^4}$$

$$= -\frac{ac(6c^2dg^2+6a^2fh^2-ac(fg^2-h(7eg-dh)))\,(ah-cgx)\sqrt{a+cx^2}}{16\,(cg^2+ah^2)^4\,(g+hx)^2} - \frac{(6c^2dg^2+6a^2fh^2-ac(fg^2-h(7eg-dh)))\,(ah-cgx)\,(a+cx^2)^{3/2}}{24\,(cg^2+ah^2)^3\,(g+hx)^4} - \frac{(fg^2-egh+dh^2)\,(a+cx^2)^{5/2}}{6h\,(cg^2+ah^2)\,(g+hx)^6} + \frac{(5cfg^3+cgh(eg-7dh)+6ah^2(2fg-eh))\,(a+cx^2)^{5/2}}{30h\,(cg^2+ah^2)^2\,(g+hx)^5} - \frac{a^2c^2(6c^2dg^2+6a^2fh^2-ac(fg^2-h(7eg-dh)))\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{cg^2+ah^2}\sqrt{a+cx^2}}\right)}{16\,(cg^2+ah^2)^{9/2}}$$

Mathematica [A] (verified)

Time = 11.41 (sec) , antiderivative size = 696, normalized size of antiderivative = 1.72

$$\int \frac{(a+cx^2)^{3/2}\,(d+ex+fx^2)}{(g+hx)^7}\,dx = \frac{1}{240}\left(-\frac{\sqrt{a+cx^2}\Big(40(cg^2+ah^2)^5\,(fg^2+h(-eg+dh))-8(cg^2+ah^2)}{} + \frac{15a^2c^2(6c^2dg^2+6a^2fh^2-ac(fg^2+h(-7eg+dh)))\log(g+hx)}{(cg^2+ah^2)^{9/2}} - \frac{15a^2c^2(6c^2dg^2+6a^2fh^2-ac(fg^2+h(-7eg+dh)))\log\left(ah-cgx+\sqrt{cg^2+ah^2}\sqrt{a+cx^2}\right)}{(cg^2+ah^2)^{9/2}}\right)$$

```
[In] Integrate[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^7,x]
```

```
[Out] (-((Sqrt[a + c*x^2]*(40*(c*g^2 + a*h^2)^5*(f*g^2 + h*(-(e*g) + d*h)) - 8*(c
*g^2 + a*h^2)^4*(25*c*f*g^3 + c*g*h*(-19*e*g + 13*d*h) - 6*a*h^2*(-2*f*g +
e*h))*(g + h*x) + 2*(c*g^2 + a*h^2)^3*(30*a^2*f*h^4 + 2*c^2*(100*f*g^4 + g^
2*h*(-52*e*g + 19*d*h)) + a*c*h^2*(227*f*g^2 + h*(-101*e*g + 35*d*h)))*(g +
 h*x)^2 - 2*c*(c*g^2 + a*h^2)^2*(6*a^2*h^4*(31*f*g - 8*e*h) + 2*c^2*(100*f*
g^5 + g^3*h*(-28*e*g + d*h)) + 3*a*c*g*h^2*(131*f*g^2 + h*(-37*e*g + 3*d*h)
))*(g + h*x)^3 + c*(c*g^2 + a*h^2)*(150*a^3*f*h^6 + 4*c^3*(50*f*g^6 - g^4*h
*(2*e*g + d*h)) + 6*a*c^2*g^2*h^2*(99*f*g^2 - h*(5*e*g + 4*d*h)) + 3*a^2*c*
h^4*(193*f*g^2 + h*(-19*e*g + 5*d*h)))*(g + h*x)^4 - c^2*(6*a^3*h^6*(41*f*g
 - 8*e*h) + 3*a^2*c*g*h^4*(89*f*g^2 + h*(29*e*g - 27*d*h)) + 4*c^3*(10*f*g^
7 + g^5*h*(2*e*g + d*h)) + 2*a*c^2*g^3*h^2*(83*f*g^2 + h*(19*e*g + 14*d*h))
)*(g + h*x)^5))/(h^5*(c*g^2 + a*h^2)^4*(g + h*x)^6)) + (15*a^2*c^2*(6*c^2*d
*g^2 + 6*a^2*f*h^2 - a*c*(f*g^2 + h*(-7*e*g + d*h)))*Log[g + h*x])/(c*g^2 +
 a*h^2)^(9/2) - (15*a^2*c^2*(6*c^2*d*g^2 + 6*a^2*f*h^2 - a*c*(f*g^2 + h*(-7
*e*g + d*h)))*Log[a*h - c*g*x + Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2]])/(c*g^
2 + a*h^2)^(9/2))/240
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 16382 vs. $2(380) = 760$.

Time = 0.85 (sec) , antiderivative size = 16383, normalized size of antiderivative = 40.55

method	result	size
default	Expression too large to display	16383

```
[In] int((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^7,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^7}\,dx = \text{Timed out}$$

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^7,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^7}\,dx = \text{Timed out}$$

```
[In] integrate((c*x**2+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**7,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10724 vs. $2(381) = 762$.

Time = 0.64 (sec) , antiderivative size = 10724, normalized size of antiderivative = 26.54

$$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^7}\,dx = \text{Too large to display}$$

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^7,x, algorithm="maxima")
```

```
[Out] 7/16*sqrt(c*x^2 + a)*c^6*f*g^8/(c^5*g^10*h^5 + 5*a*c^4*g^8*h^7 + 10*a^2*c^3
*g^6*h^9 + 10*a^3*c^2*g^4*h^11 + 5*a^4*c*g^2*h^13 + a^5*h^15) - 7/16*sqrt(c
```

```
*x^2 + a)*c^6*f*g^7*x/(c^5*g^10*h^4 + 5*a*c^4*g^8*h^6 + 10*a^2*c^3*g^6*h^8
+ 10*a^3*c^2*g^4*h^10 + 5*a^4*c*g^2*h^12 + a^5*h^14) - 7/16*sqrt(c*x^2 + a)
*c^6*e*g^7/(c^5*g^10*h^4 + 5*a*c^4*g^8*h^6 + 10*a^2*c^3*g^6*h^8 + 10*a^3*c^
2*g^4*h^10 + 5*a^4*c*g^2*h^12 + a^5*h^14) + 7/48*(c*x^2 + a)^(3/2)*c^5*f*g^
7/(c^5*g^10*h^4*x + 5*a*c^4*g^8*h^6*x + 10*a^2*c^3*g^6*h^8*x + 10*a^3*c^2*g
^4*h^10*x + 5*a^4*c*g^2*h^12*x + a^5*h^14*x + c^5*g^11*h^3 + 5*a*c^4*g^9*h^
5 + 10*a^2*c^3*g^7*h^7 + 10*a^3*c^2*g^5*h^9 + 5*a^4*c*g^3*h^11 + a^5*g*h^13
) + 7/16*sqrt(c*x^2 + a)*c^6*e*g^6*x/(c^5*g^10*h^3 + 5*a*c^4*g^8*h^5 + 10*a
^2*c^3*g^6*h^7 + 10*a^3*c^2*g^4*h^9 + 5*a^4*c*g^2*h^11 + a^5*h^13) + 7/16*s
qrt(c*x^2 + a)*c^6*d*g^6/(c^5*g^10*h^3 + 5*a*c^4*g^8*h^5 + 10*a^2*c^3*g^6*h
^7 + 10*a^3*c^2*g^4*h^9 + 5*a^4*c*g^2*h^11 + a^5*h^13) - 7/48*(c*x^2 + a)^(
3/2)*c^5*e*g^6/(c^5*g^10*h^3*x + 5*a*c^4*g^8*h^5*x + 10*a^2*c^3*g^6*h^7*x +
 10*a^3*c^2*g^4*h^9*x + 5*a^4*c*g^2*h^11*x + a^5*h^13*x + c^5*g^11*h^2 + 5*
a*c^4*g^9*h^4 + 10*a^2*c^3*g^7*h^6 + 10*a^3*c^2*g^5*h^8 + 5*a^4*c*g^3*h^10
+ a^5*g*h^12) - 7/48*(c*x^2 + a)^(5/2)*c^4*f*g^6/(c^5*g^10*h^3*x^2 + 5*a*c^
4*g^8*h^5*x^2 + 10*a^2*c^3*g^6*h^7*x^2 + 10*a^3*c^2*g^4*h^9*x^2 + 5*a^4*c*g
^2*h^11*x^2 + a^5*h^13*x^2 + 2*c^5*g^11*h^2*x + 10*a*c^4*g^9*h^4*x + 20*a^2
*c^3*g^7*h^6*x + 20*a^3*c^2*g^5*h^8*x + 10*a^4*c*g^3*h^10*x + 2*a^5*g*h^12*
x + c^5*g^12*h + 5*a*c^4*g^10*h^3 + 10*a^2*c^3*g^8*h^5 + 10*a^3*c^2*g^6*h^7
 + 5*a^4*c*g^4*h^9 + a^5*g^2*h^11) + 7/48*(c*x^2 + a)^(3/2)*c^5*f*g^6/(c^5*
g^10*h^3 + 5*a*c^4*g^8*h^5 + 10*a^2*c^3*g^6*h^7 + 10*a^3*c^2*g^4*h^9 + 5*a^
4*c*g^2*h^11 + a^5*h^13) - 7/16*sqrt(c*x^2 + a)*c^6*d*g^5*x/(c^5*g^10*h^2 +
 5*a*c^4*g^8*h^4 + 10*a^2*c^3*g^6*h^6 + 10*a^3*c^2*g^4*h^8 + 5*a^4*c*g^2*h^
10 + a^5*h^12) + 7/48*(c*x^2 + a)^(3/2)*c^5*d*g^5/(c^5*g^10*h^2*x + 5*a*c^4
*g^8*h^4*x + 10*a^2*c^3*g^6*h^6*x + 10*a^3*c^2*g^4*h^8*x + 5*a^4*c*g^2*h^10
*x + a^5*h^12*x + c^5*g^11*h + 5*a*c^4*g^9*h^3 + 10*a^2*c^3*g^7*h^5 + 10*a^
3*c^2*g^5*h^7 + 5*a^4*c*g^3*h^9 + a^5*g*h^11) + 7/48*(c*x^2 + a)^(5/2)*c^4*
e*g^5/(c^5*g^10*h^2*x^2 + 5*a*c^4*g^8*h^4*x^2 + 10*a^2*c^3*g^6*h^6*x^2 + 10
*a^3*c^2*g^4*h^8*x^2 + 5*a^4*c*g^2*h^10*x^2 + a^5*h^12*x^2 + 2*c^5*g^11*h*x
 + 10*a*c^4*g^9*h^3*x + 20*a^2*c^3*g^7*h^5*x + 20*a^3*c^2*g^5*h^7*x + 10*a^
4*c*g^3*h^9*x + 2*a^5*g*h^11*x + c^5*g^12 + 5*a*c^4*g^10*h^2 + 10*a^2*c^3*g
^8*h^4 + 10*a^3*c^2*g^6*h^6 + 5*a^4*c*g^4*h^8 + a^5*g^2*h^10) - 7/48*(c*x^2
 + a)^(3/2)*c^5*e*g^5/(c^5*g^10*h^2 + 5*a*c^4*g^8*h^4 + 10*a^2*c^3*g^6*h^6
+ 10*a^3*c^2*g^4*h^8 + 5*a^4*c*g^2*h^10 + a^5*h^12) - 27/16*sqrt(c*x^2 + a)
*c^5*f*g^6/(c^4*g^8*h^5 + 4*a*c^3*g^6*h^7 + 6*a^2*c^2*g^4*h^9 + 4*a^3*c*g^2
*h^11 + a^4*h^13) + 5/4*sqrt(c*x^2 + a)*c^5*f*g^5*x/(c^4*g^8*h^4 + 4*a*c^3*
g^6*h^6 + 6*a^2*c^2*g^4*h^8 + 4*a^3*c*g^2*h^10 + a^4*h^12) - 7/48*(c*x^2 +
a)^(5/2)*c^4*d*g^4/(c^5*g^10*h*x^2 + 5*a*c^4*g^8*h^3*x^2 + 10*a^2*c^3*g^6*h
^5*x^2 + 10*a^3*c^2*g^4*h^7*x^2 + 5*a^4*c*g^2*h^9*x^2 + a^5*h^11*x^2 + 2*c^
5*g^11*x + 10*a*c^4*g^9*h^2*x + 20*a^2*c^3*g^7*h^4*x + 20*a^3*c^2*g^5*h^6*x
 + 10*a^4*c*g^3*h^8*x + 2*a^5*g*h^10*x + c^5*g^12/h + 5*a*c^4*g^10*h + 10*a
^2*c^3*g^8*h^3 + 10*a^3*c^2*g^6*h^5 + 5*a^4*c*g^4*h^7 + a^5*g^2*h^9) + 7/48
*(c*x^2 + a)^(3/2)*c^5*d*g^4/(c^5*g^10*h + 5*a*c^4*g^8*h^3 + 10*a^2*c^3*g^6
*h^5 + 10*a^3*c^2*g^4*h^7 + 5*a^4*c*g^2*h^9 + a^5*h^11) + 21/16*sqrt(c*x^2
+ a)*c^5*e*g^5/(c^4*g^8*h^4 + 4*a*c^3*g^6*h^6 + 6*a^2*c^2*g^4*h^8 + 4*a^3*c
```

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*g^2*h^10 + a^4*h^12) - 7/24*(c*x^2 + a)^(5/2)*c^3*f*g^5/(c^4*g^8*h^4*x^3 +
 4*a*c^3*g^6*h^6*x^3 + 6*a^2*c^2*g^4*h^8*x^3 + 4*a^3*c*g^2*h^10*x^3 + a^4*h
^12*x^3 + 3*c^4*g^9*h^3*x^2 + 12*a*c^3*g^7*h^5*x^2 + 18*a^2*c^2*g^5*h^7*x^2
 + 12*a^3*c*g^3*h^9*x^2 + 3*a^4*g*h^11*x^2 + 3*c^4*g^10*h^2*x + 12*a*c^3*g^
8*h^4*x + 18*a^2*c^2*g^6*h^6*x + 12*a^3*c*g^4*h^8*x + 3*a^4*g^2*h^10*x + c^
4*g^11*h + 4*a*c^3*g^9*h^3 + 6*a^2*c^2*g^7*h^5 + 4*a^3*c*g^5*h^7 + a^4*g^3*
h^9) - 17/24*(c*x^2 + a)^(3/2)*c^4*f*g^5/(c^4*g^8*h^4*x + 4*a*c^3*g^6*h^6*x
 + 6*a^2*c^2*g^4*h^8*x + 4*a^3*c*g^2*h^10*x + a^4*h^12*x + c^4*g^9*h^3 + 4*
a*c^3*g^7*h^5 + 6*a^2*c^2*g^5*h^7 + 4*a^3*c*g^3*h^9 + a^4*g*h^11) - 7/8*sqr
t(c*x^2 + a)*c^5*e*g^4*x/(c^4*g^8*h^3 + 4*a*c^3*g^6*h^5 + 6*a^2*c^2*g^4*h^7
 + 4*a^3*c*g^2*h^9 + a^4*h^11) - 15/16*sqrt(c*x^2 + a)*c^5*d*g^4/(c^4*g^8*h
^3 + 4*a*c^3*g^6*h^5 + 6*a^2*c^2*g^4*h^7 + 4*a^3*c*g^2*h^9 + a^4*h^11) + 7/
24*(c*x^2 + a)^(5/2)*c^3*e*g^4/(c^4*g^8*h^3*x^3 + 4*a*c^3*g^6*h^5*x^3 + 6*a
^2*c^2*g^4*h^7*x^3 + 4*a^3*c*g^2*h^9*x^3 + a^4*h^11*x^3 + 3*c^4*g^9*h^2*x^2
 + 12*a*c^3*g^7*h^4*x^2 + 18*a^2*c^2*g^5*h^6*x^2 + 12*a^3*c*g^3*h^8*x^2 + 3
*a^4*g*h^10*x^2 + 3*c^4*g^10*h*x + 12*a*c^3*g^8*h^3*x + 18*a^2*c^2*g^6*h^5*
x + 12*a^3*c*g^4*h^7*x + 3*a^4*g^2*h^9*x + c^4*g^11 + 4*a*c^3*g^9*h^2 + 6*a
^2*c^2*g^7*h^4 + 4*a^3*c*g^5*h^6 + a^4*g^3*h^8) + 7/12*(c*x^2 + a)^(3/2)*c^
4*e*g^4/(c^4*g^8*h^3*x + 4*a*c^3*g^6*h^5*x + 6*a^2*c^2*g^4*h^7*x + 4*a^3*c*
g^2*h^9*x + a^4*h^11*x + c^4*g^9*h^2 + 4*a*c^3*g^7*h^4 + 6*a^2*c^2*g^5*h^6
 + 4*a^3*c*g^3*h^8 + a^4*g*h^10) + 1/8*(c*x^2 + a)^(5/2)*c^3*f*g^4/(c^4*g^8*
h^3*x^2 + 4*a*c^3*g^6*h^5*x^2 + 6*a^2*c^2*g^4*h^7*x^2 + 4*a^3*c*g^2*h^9*x^2
 + a^4*h^11*x^2 + 2*c^4*g^9*h^2*x + 8*a*c^3*g^7*h^4*x + 12*a^2*c^2*g^5*h^6*
x + 8*a^3*c*g^3*h^8*x + 2*a^4*g*h^10*x + c^4*g^10*h + 4*a*c^3*g^8*h^3 + 6*a
^2*c^2*g^6*h^5 + 4*a^3*c*g^4*h^7 + a^4*g^2*h^9) - 1/8*(c*x^2 + a)^(3/2)*c^4
*f*g^4/(c^4*g^8*h^3 + 4*a*c^3*g^6*h^5 + 6*a^2*c^2*g^4*h^7 + 4*a^3*c*g^2*h^9
 + a^4*h^11) + 1/2*sqrt(c*x^2 + a)*c^5*d*g^3*x/(c^4*g^8*h^2 + 4*a*c^3*g^6*h
^4 + 6*a^2*c^2*g^4*h^6 + 4*a^3*c*g^2*h^8 + a^4*h^10) - 7/24*(c*x^2 + a)^(5/
2)*c^3*d*g^3/(c^4*g^8*h^2*x^3 + 4*a*c^3*g^6*h^4*x^3 + 6*a^2*c^2*g^4*h^6*x^3
 + 4*a^3*c*g^2*h^8*x^3 + a^4*h^10*x^3 + 3*c^4*g^9*h*x^2 + 12*a*c^3*g^7*h^3*
x^2 + 18*a^2*c^2*g^5*h^5*x^2 + 12*a^3*c*g^3*h^7*x^2 + 3*a^4*g*h^9*x^2 + 3*c
^4*g^10*x + 12*a*c^3*g^8*h^2*x + 18*a^2*c^2*g^6*h^4*x + 12*a^3*c*g^4*h^6*x
+ 3*a^4*g^2*h^8*x + c^4*g^11/h + 4*a*c^3*g^9*h + 6*a^2*c^2*g^7*h^3 + 4*a^3*
c*g^5*h^5 + a^4*g^3*h^7) - 11/24*(c*x^2 + a)^(3/2)*c^4*d*g^3/(c^4*g^8*h^2*x
 + 4*a*c^3*g^6*h^4*x + 6*a^2*c^2*g^4*h^6*x + 4*a^3*c*g^2*h^8*x + a^4*h^10*x
 + c^4*g^9*h + 4*a*c^3*g^7*h^3 + 6*a^2*c^2*g^5*h^5 + 4*a^3*c*g^3*h^7 + a^4*
g*h^9) - 7/24*(c*x^2 + a)^(5/2)*c^2*f*g^4/(c^3*g^6*h^5*x^4 + 3*a*c^2*g^4*h^
7*x^4 + 3*a^2*c*g^2*h^9*x^4 + a^3*h^11*x^4 + 4*c^3*g^7*h^4*x^3 + 12*a*c^2*g
^5*h^6*x^3 + 12*a^2*c*g^3*h^8*x^3 + 4*a^3*g*h^10*x^3 + 6*c^3*g^8*h^3*x^2 +
18*a*c^2*g^6*h^5*x^2 + 18*a^2*c*g^4*h^7*x^2 + 6*a^3*g^2*h^9*x^2 + 4*c^3*g^9
*h^2*x + 12*a*c^2*g^7*h^4*x + 12*a^2*c*g^5*h^6*x + 4*a^3*g^3*h^8*x + c^3*g^
10*h + 3*a*c^2*g^8*h^3 + 3*a^2*c*g^6*h^5 + a^3*g^4*h^7) + 39/16*sqrt(c*x^2
+ a)*c^4*f*g^4/(c^3*g^6*h^5 + 3*a*c^2*g^4*h^7 + 3*a^2*c*g^2*h^9 + a^3*h^11)
 - 19/16*sqrt(c*x^2 + a)*c^4*f*g^3*x/(c^3*g^6*h^4 + 3*a*c^2*g^4*h^6 + 3*a^2
*c*g^2*h^8 + a^3*h^10) - 1/8*(c*x^2 + a)^(5/2)*c^3*d*g^2/(c^4*g^8*h*x^2 + 4
```

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*a*c^3*g^6*h^3*x^2 + 6*a^2*c^2*g^4*h^5*x^2 + 4*a^3*c*g^2*h^7*x^2 + a^4*h^9*
x^2 + 2*c^4*g^9*x + 8*a*c^3*g^7*h^2*x + 12*a^2*c^2*g^5*h^4*x + 8*a^3*c*g^3*
h^6*x + 2*a^4*g*h^8*x + c^4*g^10/h + 4*a*c^3*g^8*h + 6*a^2*c^2*g^6*h^3 + 4*
a^3*c*g^4*h^5 + a^4*g^2*h^7) + 1/8*(c*x^2 + a)^(3/2)*c^4*d*g^2/(c^4*g^8*h +
 4*a*c^3*g^6*h^3 + 6*a^2*c^2*g^4*h^5 + 4*a^3*c*g^2*h^7 + a^4*h^9) + 7/24*(c
*x^2 + a)^(5/2)*c^2*e*g^3/(c^3*g^6*h^4*x^4 + 3*a*c^2*g^4*h^6*x^4 + 3*a^2*c*
g^2*h^8*x^4 + a^3*h^10*x^4 + 4*c^3*g^7*h^3*x^3 + 12*a*c^2*g^5*h^5*x^3 + 12*
a^2*c*g^3*h^7*x^3 + 4*a^3*g*h^9*x^3 + 6*c^3*g^8*h^2*x^2 + 18*a*c^2*g^6*h^4*
x^2 + 18*a^2*c*g^4*h^6*x^2 + 6*a^3*g^2*h^8*x^2 + 4*c^3*g^9*h*x + 12*a*c^2*g
^7*h^3*x + 12*a^2*c*g^5*h^5*x + 4*a^3*g^3*h^7*x + c^3*g^10 + 3*a*c^2*g^8*h^
2 + 3*a^2*c*g^6*h^4 + a^3*g^4*h^6) - 21/16*sqrt(c*x^2 + a)*c^4*e*g^3/(c^3*g
^6*h^4 + 3*a*c^2*g^4*h^6 + 3*a^2*c*g^2*h^8 + a^3*h^10) + 13/24*(c*x^2 + a)^
(5/2)*c^2*f*g^3/(c^3*g^6*h^4*x^3 + 3*a*c^2*g^4*h^6*x^3 + 3*a^2*c*g^2*h^8*x^
3 + a^3*h^10*x^3 + 3*c^3*g^7*h^3*x^2 + 9*a*c^2*g^5*h^5*x^2 + 9*a^2*c*g^3*h^
7*x^2 + 3*a^3*g*h^9*x^2 + 3*c^3*g^8*h^2*x + 9*a*c^2*g^6*h^4*x + 9*a^2*c*g^4
*h^6*x + 3*a^3*g^2*h^8*x + c^3*g^9*h + 3*a*c^2*g^7*h^3 + 3*a^2*c*g^5*h^5 +
a^3*g^3*h^7) + 15/16*(c*x^2 + a)^(3/2)*c^3*f*g^3/(c^3*g^6*h^4*x + 3*a*c^2*g
^4*h^6*x + 3*a^2*c*g^2*h^8*x + a^3*h^10*x + c^3*g^7*h^3 + 3*a*c^2*g^5*h^5 +
 3*a^2*c*g^3*h^7 + a^3*g*h^9) + 7/16*sqrt(c*x^2 + a)*c^4*e*g^2*x/(c^3*g^6*h
^3 + 3*a*c^2*g^4*h^5 + 3*a^2*c*g^2*h^7 + a^3*h^9) - 7/24*(c*x^2 + a)^(5/2)*
c^2*d*g^2/(c^3*g^6*h^3*x^4 + 3*a*c^2*g^4*h^5*x^4 + 3*a^2*c*g^2*h^7*x^4 + a^
3*h^9*x^4 + 4*c^3*g^7*h^2*x^3 + 12*a*c^2*g^5*h^4*x^3 + 12*a^2*c*g^3*h^6*x^3
 + 4*a^3*g*h^8*x^3 + 6*c^3*g^8*h*x^2 + 18*a*c^2*g^6*h^3*x^2 + 18*a^2*c*g^4*
h^5*x^2 + 6*a^3*g^2*h^7*x^2 + 4*c^3*g^9*x + 12*a*c^2*g^7*h^2*x + 12*a^2*c*g
^5*h^4*x + 4*a^3*g^3*h^6*x + c^3*g^10/h + 3*a*c^2*g^8*h + 3*a^2*c*g^6*h^3 +
 a^3*g^4*h^5) + 9/16*sqrt(c*x^2 + a)*c^4*d*g^2/(c^3*g^6*h^3 + 3*a*c^2*g^4*h
^5 + 3*a^2*c*g^2*h^7 + a^3*h^9) - 7/24*(c*x^2 + a)^(5/2)*c^2*e*g^2/(c^3*g^6
*h^3*x^3 + 3*a*c^2*g^4*h^5*x^3 + 3*a^2*c*g^2*h^7*x^3 + a^3*h^9*x^3 + 3*c^3*
g^7*h^2*x^2 + 9*a*c^2*g^5*h^4*x^2 + 9*a^2*c*g^3*h^6*x^2 + 3*a^3*g*h^8*x^2 +
 3*c^3*g^8*h*x + 9*a*c^2*g^6*h^3*x + 9*a^2*c*g^4*h^5*x + 3*a^3*g^2*h^7*x +
c^3*g^9 + 3*a*c^2*g^7*h^2 + 3*a^2*c*g^5*h^4 + a^3*g^3*h^6) - 7/16*(c*x^2 +
a)^(3/2)*c^3*e*g^2/(c^3*g^6*h^3*x + 3*a*c^2*g^4*h^5*x + 3*a^2*c*g^2*h^7*x +
 a^3*h^9*x + c^3*g^7*h^2 + 3*a*c^2*g^5*h^4 + 3*a^2*c*g^3*h^6 + a^3*g*h^8) +
 7/48*(c*x^2 + a)^(5/2)*c^2*f*g^2/(c^3*g^6*h^3*x^2 + 3*a*c^2*g^4*h^5*x^2 +
3*a^2*c*g^2*h^7*x^2 + a^3*h^9*x^2 + 2*c^3*g^7*h^2*x + 6*a*c^2*g^5*h^4*x + 6
*a^2*c*g^3*h^6*x + 2*a^3*g*h^8*x + c^3*g^8*h + 3*a*c^2*g^6*h^3 + 3*a^2*c*g^
4*h^5 + a^3*g^2*h^7) - 7/48*(c*x^2 + a)^(3/2)*c^3*f*g^2/(c^3*g^6*h^3 + 3*a*
c^2*g^4*h^5 + 3*a^2*c*g^2*h^7 + a^3*h^9) - 7/30*(c*x^2 + a)^(5/2)*c*f*g^3/(
c^2*g^4*h^6*x^5 + 2*a*c*g^2*h^8*x^5 + a^2*h^10*x^5 + 5*c^2*g^5*h^5*x^4 + 10
*a*c*g^3*h^7*x^4 + 5*a^2*g*h^9*x^4 + 10*c^2*g^6*h^4*x^3 + 20*a*c*g^4*h^6*x^
3 + 10*a^2*g^2*h^8*x^3 + 10*c^2*g^7*h^3*x^2 + 20*a*c*g^5*h^5*x^2 + 10*a^2*g
^3*h^7*x^2 + 5*c^2*g^8*h^2*x + 10*a*c*g^6*h^4*x + 5*a^2*g^4*h^6*x + c^2*g^9
*h + 2*a*c*g^7*h^3 + a^2*g^5*h^5) - 1/16*sqrt(c*x^2 + a)*c^4*d*g*x/(c^3*g^6
*h^2 + 3*a*c^2*g^4*h^4 + 3*a^2*c*g^2*h^6 + a^3*h^8) + 1/24*(c*x^2 + a)^(5/2
)*c^2*d*g/(c^3*g^6*h^2*x^3 + 3*a*c^2*g^4*h^4*x^3 + 3*a^2*c*g^2*h^6*x^3 + a^
```

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3*h^8*x^3 + 3*c^3*g^7*h*x^2 + 9*a*c^2*g^5*h^3*x^2 + 9*a^2*c*g^3*h^5*x^2 + 3
*a^3*g*h^7*x^2 + 3*c^3*g^8*x + 9*a*c^2*g^6*h^2*x + 9*a^2*c*g^4*h^4*x + 3*a^
3*g^2*h^6*x + c^3*g^9/h + 3*a*c^2*g^7*h + 3*a^2*c*g^5*h^3 + a^3*g^3*h^5) +
1/16*(c*x^2 + a)^(3/2)*c^3*d*g/(c^3*g^6*h^2*x + 3*a*c^2*g^4*h^4*x + 3*a^2*c
*g^2*h^6*x + a^3*h^8*x + c^3*g^7*h + 3*a*c^2*g^5*h^3 + 3*a^2*c*g^3*h^5 + a^
3*g*h^7) - 7/48*(c*x^2 + a)^(5/2)*c^2*e*g/(c^3*g^6*h^2*x^2 + 3*a*c^2*g^4*h^
4*x^2 + 3*a^2*c*g^2*h^6*x^2 + a^3*h^8*x^2 + 2*c^3*g^7*h*x + 6*a*c^2*g^5*h^3
*x + 6*a^2*c*g^3*h^5*x + 2*a^3*g*h^7*x + c^3*g^8 + 3*a*c^2*g^6*h^2 + 3*a^2*
c*g^4*h^4 + a^3*g^2*h^6) + 7/48*(c*x^2 + a)^(3/2)*c^3*e*g/(c^3*g^6*h^2 + 3*
a*c^2*g^4*h^4 + 3*a^2*c*g^2*h^6 + a^3*h^8) + 7/30*(c*x^2 + a)^(5/2)*c*e*g^2
/(c^2*g^4*h^5*x^5 + 2*a*c*g^2*h^7*x^5 + a^2*h^9*x^5 + 5*c^2*g^5*h^4*x^4 + 1
0*a*c*g^3*h^6*x^4 + 5*a^2*g*h^8*x^4 + 10*c^2*g^6*h^3*x^3 + 20*a*c*g^4*h^5*x
^3 + 10*a^2*g^2*h^7*x^3 + 10*c^2*g^7*h^2*x^2 + 20*a*c*g^5*h^4*x^2 + 10*a^2*
g^3*h^6*x^2 + 5*c^2*g^8*h*x + 10*a*c*g^6*h^3*x + 5*a^2*g^4*h^5*x + c^2*g^9
 + 2*a*c*g^7*h^2 + a^2*g^5*h^4) + 13/24*(c*x^2 + a)^(5/2)*c*f*g^2/(c^2*g^4*h
^5*x^4 + 2*a*c*g^2*h^7*x^4 + a^2*h^9*x^4 + 4*c^2*g^5*h^4*x^3 + 8*a*c*g^3*h^
6*x^3 + 4*a^2*g*h^8*x^3 + 6*c^2*g^6*h^3*x^2 + 12*a*c*g^4*h^5*x^2 + 6*a^2*g^
2*h^7*x^2 + 4*c^2*g^7*h^2*x + 8*a*c*g^5*h^4*x + 4*a^2*g^3*h^6*x + c^2*g^8*h
 + 2*a*c*g^6*h^3 + a^2*g^4*h^5) - 25/16*sqrt(c*x^2 + a)*c^3*f*g^2/(c^2*g^4*
h^5 + 2*a*c*g^2*h^7 + a^2*h^9) + 3/8*sqrt(c*x^2 + a)*c^3*f*g*x/(c^2*g^4*h^4
 + 2*a*c*g^2*h^6 + a^2*h^8) + 1/48*(c*x^2 + a)^(5/2)*c^2*d/(c^3*g^6*h*x^2 +
 3*a*c^2*g^4*h^3*x^2 + 3*a^2*c*g^2*h^5*x^2 + a^3*h^7*x^2 + 2*c^3*g^7*x + 6*
a*c^2*g^5*h^2*x + 6*a^2*c*g^3*h^4*x + 2*a^3*g*h^6*x + c^3*g^8/h + 3*a*c^2*g
^6*h + 3*a^2*c*g^4*h^3 + a^3*g^2*h^5) - 1/48*(c*x^2 + a)^(3/2)*c^3*d/(c^3*g
^6*h + 3*a*c^2*g^4*h^3 + 3*a^2*c*g^2*h^5 + a^3*h^7) - 7/30*(c*x^2 + a)^(5/2
)*c*d*g/(c^2*g^4*h^4*x^5 + 2*a*c*g^2*h^6*x^5 + a^2*h^8*x^5 + 5*c^2*g^5*h^3*
x^4 + 10*a*c*g^3*h^5*x^4 + 5*a^2*g*h^7*x^4 + 10*c^2*g^6*h^2*x^3 + 20*a*c*g^
4*h^4*x^3 + 10*a^2*g^2*h^6*x^3 + 10*c^2*g^7*h*x^2 + 20*a*c*g^5*h^3*x^2 + 10
*a^2*g^3*h^5*x^2 + 5*c^2*g^8*x + 10*a*c*g^6*h^2*x + 5*a^2*g^4*h^4*x + c^2*g
^9/h + 2*a*c*g^7*h + a^2*g^5*h^3) - 7/24*(c*x^2 + a)^(5/2)*c*e*g/(c^2*g^4*h
^4*x^4 + 2*a*c*g^2*h^6*x^4 + a^2*h^8*x^4 + 4*c^2*g^5*h^3*x^3 + 8*a*c*g^3*h^
5*x^3 + 4*a^2*g*h^7*x^3 + 6*c^2*g^6*h^2*x^2 + 12*a*c*g^4*h^4*x^2 + 6*a^2*g^
2*h^6*x^2 + 4*c^2*g^7*h*x + 8*a*c*g^5*h^3*x + 4*a^2*g^3*h^5*x + c^2*g^8 + 2
*a*c*g^6*h^2 + a^2*g^4*h^4) + 7/16*sqrt(c*x^2 + a)*c^3*e*g/(c^2*g^4*h^4 + 2
*a*c*g^2*h^6 + a^2*h^8) - 1/4*(c*x^2 + a)^(5/2)*c*f*g/(c^2*g^4*h^4*x^3 + 2*
a*c*g^2*h^6*x^3 + a^2*h^8*x^3 + 3*c^2*g^5*h^3*x^2 + 6*a*c*g^3*h^5*x^2 + 3*a
^2*g*h^7*x^2 + 3*c^2*g^6*h^2*x + 6*a*c*g^4*h^4*x + 3*a^2*g^2*h^6*x + c^2*g^
7*h + 2*a*c*g^5*h^3 + a^2*g^3*h^5) - 3/8*(c*x^2 + a)^(3/2)*c^2*f*g/(c^2*g^4
*h^4*x + 2*a*c*g^2*h^6*x + a^2*h^8*x + c^2*g^5*h^3 + 2*a*c*g^3*h^5 + a^2*g*
h^7) - 1/6*(c*x^2 + a)^(5/2)*f*g^2/(c*g^2*h^7*x^6 + a*h^9*x^6 + 6*c*g^3*h^6
*x^5 + 6*a*g*h^8*x^5 + 15*c*g^4*h^5*x^4 + 15*a*g^2*h^7*x^4 + 20*c*g^5*h^4*x
^3 + 20*a*g^3*h^6*x^3 + 15*c*g^6*h^3*x^2 + 15*a*g^4*h^5*x^2 + 6*c*g^7*h^2*x
 + 6*a*g^5*h^4*x + c*g^8*h + a*g^6*h^3) + 1/24*(c*x^2 + a)^(5/2)*c*d/(c^2*g
^4*h^3*x^4 + 2*a*c*g^2*h^5*x^4 + a^2*h^7*x^4 + 4*c^2*g^5*h^2*x^3 + 8*a*c*g^
3*h^4*x^3 + 4*a^2*g*h^6*x^3 + 6*c^2*g^6*h*x^2 + 12*a*c*g^4*h^3*x^2 + 6*a^2*
```

```
g^2*h^5*x^2 + 4*c^2*g^7*x + 8*a*c*g^5*h^2*x + 4*a^2*g^3*h^4*x + c^2*g^8/h +
 2*a*c*g^6*h + a^2*g^4*h^3) - 1/16*sqrt(c*x^2 + a)*c^3*d/(c^2*g^4*h^3 + 2*a
*c*g^2*h^5 + a^2*h^7) - 1/8*(c*x^2 + a)^(5/2)*c*f/(c^2*g^4*h^3*x^2 + 2*a*c*
g^2*h^5*x^2 + a^2*h^7*x^2 + 2*c^2*g^5*h^2*x + 4*a*c*g^3*h^4*x + 2*a^2*g*h^6
*x + c^2*g^6*h + 2*a*c*g^4*h^3 + a^2*g^2*h^5) + 1/8*(c*x^2 + a)^(3/2)*c^2*f
/(c^2*g^4*h^3 + 2*a*c*g^2*h^5 + a^2*h^7) + 1/6*(c*x^2 + a)^(5/2)*e*g/(c*g^2
*h^6*x^6 + a*h^8*x^6 + 6*c*g^3*h^5*x^5 + 6*a*g*h^7*x^5 + 15*c*g^4*h^4*x^4 +
 15*a*g^2*h^6*x^4 + 20*c*g^5*h^3*x^3 + 20*a*g^3*h^5*x^3 + 15*c*g^6*h^2*x^2
+ 15*a*g^4*h^4*x^2 + 6*c*g^7*h*x + 6*a*g^5*h^3*x + c*g^8 + a*g^6*h^2) + 2/5
*(c*x^2 + a)^(5/2)*f*g/(c*g^2*h^6*x^5 + a*h^8*x^5 + 5*c*g^3*h^5*x^4 + 5*a*g
*h^7*x^4 + 10*c*g^4*h^4*x^3 + 10*a*g^2*h^6*x^3 + 10*c*g^5*h^3*x^2 + 10*a*g^
3*h^5*x^2 + 5*c*g^6*h^2*x + 5*a*g^4*h^4*x + c*g^7*h + a*g^5*h^3) - 1/6*(c*x
^2 + a)^(5/2)*d/(c*g^2*h^5*x^6 + a*h^7*x^6 + 6*c*g^3*h^4*x^5 + 6*a*g*h^6*x^
5 + 15*c*g^4*h^3*x^4 + 15*a*g^2*h^5*x^4 + 20*c*g^5*h^2*x^3 + 20*a*g^3*h^4*x
^3 + 15*c*g^6*h*x^2 + 15*a*g^4*h^3*x^2 + 6*c*g^7*x + 6*a*g^5*h^2*x + c*g^8/
h + a*g^6*h) - 1/5*(c*x^2 + a)^(5/2)*e/(c*g^2*h^5*x^5 + a*h^7*x^5 + 5*c*g^3
*h^4*x^4 + 5*a*g*h^6*x^4 + 10*c*g^4*h^3*x^3 + 10*a*g^2*h^5*x^3 + 10*c*g^5*h
^2*x^2 + 10*a*g^3*h^4*x^2 + 5*c*g^6*h*x + 5*a*g^4*h^3*x + c*g^7 + a*g^5*h^2
) - 1/4*(c*x^2 + a)^(5/2)*f/(c*g^2*h^5*x^4 + a*h^7*x^4 + 4*c*g^3*h^4*x^3 +
4*a*g*h^6*x^3 + 6*c*g^4*h^3*x^2 + 6*a*g^2*h^5*x^2 + 4*c*g^5*h^2*x + 4*a*g^3
*h^4*x + c*g^6*h + a*g^4*h^3) + 3/8*sqrt(c*x^2 + a)*c^2*f/(c*g^2*h^5 + a*h^
7) + 7/16*c^6*f*g^8*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)
*abs(h*x + g)))/((a + c*g^2/h^2)^(9/2)*h^15) - 7/16*c^6*e*g^7*arcsinh(c*g*x
/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^
(9/2)*h^14) + 7/16*c^6*d*g^6*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(
sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(9/2)*h^13) - 27/16*c^5*f*g^6*arc
sinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c
*g^2/h^2)^(7/2)*h^13) + 21/16*c^5*e*g^5*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x +
g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(7/2)*h^12) - 15/16*c^
5*d*g^4*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g
)))/((a + c*g^2/h^2)^(7/2)*h^11) + 39/16*c^4*f*g^4*arcsinh(c*g*x/(sqrt(a*c)
*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(5/2)*h^11)
 - 21/16*c^4*e*g^3*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*
abs(h*x + g)))/((a + c*g^2/h^2)^(5/2)*h^10) + 9/16*c^4*d*g^2*arcsinh(c*g*x/
(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(
5/2)*h^9) - 25/16*c^3*f*g^2*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(s
qrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(3/2)*h^9) + 7/16*c^3*e*g*arcsinh(
c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/
h^2)^(3/2)*h^8) - 1/16*c^3*d*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(
sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(3/2)*h^7) + 3/8*c^2*f*arcsinh(c*
g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/(sqrt(a + c*g^
2/h^2)*h^7)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6061 vs. 2(381) = 762.

Time = 0.42 (sec) , antiderivative size = 6061, normalized size of antiderivative = 15.00

$$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^7}\,dx = \text{Too large to display}$$

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^7,x, algorithm="giac")
```

```
[Out] 1/8*(6*a^2*c^4*d*g^2 - a^3*c^3*f*g^2 + 7*a^3*c^3*e*g*h - a^3*c^3*d*h^2 + 6*
a^4*c^2*f*h^2)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*h + sqrt(c)*g)/sqrt(-
c*g^2 - a*h^2))/((c^4*g^8 + 4*a*c^3*g^6*h^2 + 6*a^2*c^2*g^4*h^4 + 4*a^3*c*g
^2*h^6 + a^4*h^8)*sqrt(-c*g^2 - a*h^2)) + 1/120*(240*(sqrt(c)*x - sqrt(c*x^
2 + a))^11*c^6*f*g^8*h^5 + 960*(sqrt(c)*x - sqrt(c*x^2 + a))^11*a*c^5*f*g^6
*h^7 + 1440*(sqrt(c)*x - sqrt(c*x^2 + a))^11*a^2*c^4*f*g^4*h^9 - 90*(sqrt(c
)*x - sqrt(c*x^2 + a))^11*a^2*c^4*d*g^2*h^11 + 975*(sqrt(c)*x - sqrt(c*x^2
+ a))^11*a^3*c^3*f*g^2*h^11 - 105*(sqrt(c)*x - sqrt(c*x^2 + a))^11*a^3*c^3*
e*g*h^12 + 15*(sqrt(c)*x - sqrt(c*x^2 + a))^11*a^3*c^3*d*h^13 + 150*(sqrt(c
)*x - sqrt(c*x^2 + a))^11*a^4*c^2*f*h^13 + 1200*(sqrt(c)*x - sqrt(c*x^2 + a
))^10*c^(13/2)*f*g^9*h^4 + 240*(sqrt(c)*x - sqrt(c*x^2 + a))^10*c^(13/2)*e*
g^8*h^5 + 4800*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a*c^(11/2)*f*g^7*h^6 + 960*
(sqrt(c)*x - sqrt(c*x^2 + a))^10*a*c^(11/2)*e*g^6*h^7 + 7200*(sqrt(c)*x - s
qrt(c*x^2 + a))^10*a^2*c^(9/2)*f*g^5*h^8 + 1440*(sqrt(c)*x - sqrt(c*x^2 + a
))^10*a^2*c^(9/2)*e*g^4*h^9 - 990*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a^2*c^(9
/2)*d*g^3*h^10 + 4965*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a^3*c^(7/2)*f*g^3*h^
10 - 195*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a^3*c^(7/2)*e*g^2*h^11 + 165*(sqr
t(c)*x - sqrt(c*x^2 + a))^10*a^3*c^(7/2)*d*g*h^12 + 210*(sqrt(c)*x - sqrt(c
*x^2 + a))^10*a^4*c^(5/2)*f*g*h^12 + 240*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a
^4*c^(5/2)*e*h^13 + 3200*(sqrt(c)*x - sqrt(c*x^2 + a))^9*c^7*f*g^10*h^3 + 6
40*(sqrt(c)*x - sqrt(c*x^2 + a))^9*c^7*e*g^9*h^4 + 320*(sqrt(c)*x - sqrt(c*
x^2 + a))^9*c^7*d*g^8*h^5 + 12080*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a*c^6*f*g
^8*h^5 + 2560*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a*c^6*e*g^7*h^6 + 1280*(sqrt(
c)*x - sqrt(c*x^2 + a))^9*a*c^6*d*g^6*h^7 + 16320*(sqrt(c)*x - sqrt(c*x^2 +
 a))^9*a^2*c^5*f*g^6*h^7 + 3840*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^2*c^5*e*g
^5*h^8 - 2520*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^2*c^5*d*g^4*h^9 + 9220*(sqr
t(c)*x - sqrt(c*x^2 + a))^9*a^3*c^4*f*g^4*h^9 - 2620*(sqrt(c)*x - sqrt(c*x^
2 + a))^9*a^3*c^4*e*g^3*h^10 + 2530*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^3*c^4
*d*g^2*h^11 - 4205*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^4*c^3*f*g^2*h^11 + 123
5*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^4*c^3*e*g*h^12 + 235*(sqrt(c)*x - sqrt(
c*x^2 + a))^9*a^4*c^3*d*h^13 - 210*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^5*c^2*
f*h^13 + 4800*(sqrt(c)*x - sqrt(c*x^2 + a))^8*c^(15/2)*f*g^11*h^2 + 960*(sq
rt(c)*x - sqrt(c*x^2 + a))^8*c^(15/2)*e*g^10*h^3 + 480*(sqrt(c)*x - sqrt(c*
x^2 + a))^8*c^(15/2)*d*g^9*h^4 + 15120*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a*c^
(13/2)*f*g^9*h^4 + 3600*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a*c^(13/2)*e*g^8*h^
```

```
5 + 1920*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a*c^(13/2)*d*g^7*h^6 + 12480*(sqrt
(c)*x - sqrt(c*x^2 + a))^8*a^2*c^(11/2)*f*g^7*h^6 + 4800*(sqrt(c)*x - sqrt(
c*x^2 + a))^8*a^2*c^(11/2)*e*g^6*h^7 - 7380*(sqrt(c)*x - sqrt(c*x^2 + a))^8
*a^2*c^(11/2)*d*g^5*h^8 - 3570*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^3*c^(9/2)*
f*g^5*h^8 - 9570*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^3*c^(9/2)*e*g^4*h^9 + 82
20*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^3*c^(9/2)*d*g^3*h^10 - 22545*(sqrt(c)*
x - sqrt(c*x^2 + a))^8*a^4*c^(7/2)*f*g^3*h^10 + 5355*(sqrt(c)*x - sqrt(c*x^
2 + a))^8*a^4*c^(7/2)*e*g^2*h^11 - 285*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^4*
c^(7/2)*d*g*h^12 + 510*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^5*c^(5/2)*f*g*h^12
 - 240*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^5*c^(5/2)*e*h^13 + 3840*(sqrt(c)*x
 - sqrt(c*x^2 + a))^7*c^8*f*g^12*h + 768*(sqrt(c)*x - sqrt(c*x^2 + a))^7*c^
8*e*g^11*h^2 + 384*(sqrt(c)*x - sqrt(c*x^2 + a))^7*c^8*d*g^10*h^3 + 6336*(s
qrt(c)*x - sqrt(c*x^2 + a))^7*a*c^7*f*g^10*h^3 + 1728*(sqrt(c)*x - sqrt(c*x
^2 + a))^7*a*c^7*e*g^9*h^4 + 1728*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a*c^7*d*g
^8*h^5 - 11808*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^2*c^6*f*g^8*h^5 - 768*(sqr
t(c)*x - sqrt(c*x^2 + a))^7*a^2*c^6*e*g^7*h^6 - 9456*(sqrt(c)*x - sqrt(c*x^
2 + a))^7*a^2*c^6*d*g^6*h^7 - 31704*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^3*c^5
*f*g^6*h^7 - 19608*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^3*c^5*e*g^5*h^8 + 2076
0*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^3*c^5*d*g^4*h^9 - 39960*(sqrt(c)*x - sq
rt(c*x^2 + a))^7*a^4*c^4*f*g^4*h^9 + 14040*(sqrt(c)*x - sqrt(c*x^2 + a))^7*
a^4*c^4*e*g^3*h^10 - 2700*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^4*c^4*d*g^2*h^1
1 + 12150*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^5*c^3*f*g^2*h^11 - 2730*(sqrt(c
)*x - sqrt(c*x^2 + a))^7*a^5*c^3*e*g*h^12 + 390*(sqrt(c)*x - sqrt(c*x^2 + a
))^7*a^5*c^3*d*h^13 + 60*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^6*c^2*f*h^13 + 1
280*(sqrt(c)*x - sqrt(c*x^2 + a))^6*c^(17/2)*f*g^13 + 256*(sqrt(c)*x - sqrt
(c*x^2 + a))^6*c^(17/2)*e*g^12*h + 128*(sqrt(c)*x - sqrt(c*x^2 + a))^6*c^(1
7/2)*d*g^11*h^2 - 4288*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a*c^(15/2)*f*g^11*h^
2 - 704*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a*c^(15/2)*e*g^10*h^3 - 64*(sqrt(c)
*x - sqrt(c*x^2 + a))^6*a*c^(15/2)*d*g^9*h^4 - 24096*(sqrt(c)*x - sqrt(c*x^
2 + a))^6*a^2*c^(13/2)*f*g^9*h^4 - 4896*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^2
*c^(13/2)*e*g^8*h^5 - 8592*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^2*c^(13/2)*d*g
^7*h^6 - 26728*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^3*c^(11/2)*f*g^7*h^6 - 156
56*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^3*c^(11/2)*e*g^6*h^7 + 24440*(sqrt(c)*
x - sqrt(c*x^2 + a))^6*a^3*c^(11/2)*d*g^5*h^8 - 12640*(sqrt(c)*x - sqrt(c*x
^2 + a))^6*a^4*c^(9/2)*f*g^5*h^8 + 26800*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^
4*c^(9/2)*e*g^4*h^9 - 14860*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^4*c^(9/2)*d*g
^3*h^10 + 41610*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^5*c^(7/2)*f*g^3*h^10 - 95
10*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^5*c^(7/2)*e*g^2*h^11 + 810*(sqrt(c)*x
- sqrt(c*x^2 + a))^6*a^5*c^(7/2)*d*g*h^12 - 2460*(sqrt(c)*x - sqrt(c*x^2 +
a))^6*a^6*c^(5/2)*f*g*h^12 + 480*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^6*c^(5/2
)*e*h^13 - 3840*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a*c^8*f*g^12*h - 768*(sqrt(
c)*x - sqrt(c*x^2 + a))^5*a*c^8*e*g^11*h^2 - 384*(sqrt(c)*x - sqrt(c*x^2 +
a))^5*a*c^8*d*g^10*h^3 - 6336*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^2*c^7*f*g^1
0*h^3 - 1728*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^2*c^7*e*g^9*h^4 - 1728*(sqrt
(c)*x - sqrt(c*x^2 + a))^5*a^2*c^7*d*g^8*h^5 + 11808*(sqrt(c)*x - sqrt(c*x^
```

```
2 + a))^5*a^3*c^6*f*g^8*h^5 - 192*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^3*c^6*e
*g^7*h^6 + 19056*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^3*c^6*d*g^6*h^7 + 29304*
(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^4*c^5*f*g^6*h^7 + 26808*(sqrt(c)*x - sqrt
(c*x^2 + a))^5*a^4*c^5*e*g^5*h^8 - 21480*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^
4*c^5*d*g^4*h^9 + 46080*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^5*c^4*f*g^4*h^9 -
 19440*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^5*c^4*e*g^3*h^10 + 7020*(sqrt(c)*x
 - sqrt(c*x^2 + a))^5*a^5*c^4*d*g^2*h^11 - 17370*(sqrt(c)*x - sqrt(c*x^2 +
a))^5*a^6*c^3*f*g^2*h^11 + 3030*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^6*c^3*e*g
*h^12 + 390*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^6*c^3*d*h^13 + 60*(sqrt(c)*x
- sqrt(c*x^2 + a))^5*a^7*c^2*f*h^13 + 4800*(sqrt(c)*x - sqrt(c*x^2 + a))^4*
a^2*c^(15/2)*f*g^11*h^2 + 960*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^2*c^(15/2)*
e*g^10*h^3 + 480*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^2*c^(15/2)*d*g^9*h^4 + 1
5120*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^3*c^(13/2)*f*g^9*h^4 + 3600*(sqrt(c)
*x - sqrt(c*x^2 + a))^4*a^3*c^(13/2)*e*g^8*h^5 + 3840*(sqrt(c)*x - sqrt(c*x
^2 + a))^4*a^3*c^(13/2)*d*g^7*h^6 + 12360*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a
^4*c^(11/2)*f*g^7*h^6 + 7080*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^4*c^(11/2)*e
*g^6*h^7 - 18720*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^4*c^(11/2)*d*g^5*h^8 + 1
020*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^5*c^(9/2)*f*g^5*h^8 - 22260*(sqrt(c)*
x - sqrt(c*x^2 + a))^4*a^5*c^(9/2)*e*g^4*h^9 + 11640*(sqrt(c)*x - sqrt(c*x^
2 + a))^4*a^5*c^(9/2)*d*g^3*h^10 - 32490*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^
6*c^(7/2)*f*g^3*h^10 + 7470*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^6*c^(7/2)*e*g
^2*h^11 - 930*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^6*c^(7/2)*d*g*h^12 + 3180*(
sqrt(c)*x - sqrt(c*x^2 + a))^4*a^7*c^(5/2)*f*g*h^12 - 480*(sqrt(c)*x - sqrt
(c*x^2 + a))^4*a^7*c^(5/2)*e*h^13 - 3200*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^
3*c^7*f*g^10*h^3 - 640*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^3*c^7*e*g^9*h^4 -
320*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^3*c^7*d*g^8*h^5 - 12080*(sqrt(c)*x -
sqrt(c*x^2 + a))^3*a^4*c^6*f*g^8*h^5 - 3040*(sqrt(c)*x - sqrt(c*x^2 + a))^3
*a^4*c^6*e*g^7*h^6 - 2960*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^4*c^6*d*g^6*h^7
 - 16440*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^5*c^5*f*g^6*h^7 - 7800*(sqrt(c)*
x - sqrt(c*x^2 + a))^3*a^5*c^5*e*g^5*h^8 + 12120*(sqrt(c)*x - sqrt(c*x^2 +
a))^3*a^5*c^5*d*g^4*h^9 - 14120*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^6*c^4*f*g
^4*h^9 + 10280*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^6*c^4*e*g^3*h^10 - 2330*(s
qrt(c)*x - sqrt(c*x^2 + a))^3*a^6*c^4*d*g^2*h^11 + 10555*(sqrt(c)*x - sqrt(
c*x^2 + a))^3*a^7*c^3*f*g^2*h^11 - 1645*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^7
*c^3*e*g*h^12 + 235*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^7*c^3*d*h^13 - 210*(s
qrt(c)*x - sqrt(c*x^2 + a))^3*a^8*c^2*f*h^13 + 1200*(sqrt(c)*x - sqrt(c*x^2
 + a))^2*a^4*c^(13/2)*f*g^9*h^4 + 240*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^4*c
^(13/2)*e*g^8*h^5 + 240*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^4*c^(13/2)*d*g^7*
h^6 + 4920*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^5*c^(11/2)*f*g^7*h^6 + 1272*(s
qrt(c)*x - sqrt(c*x^2 + a))^2*a^5*c^(11/2)*e*g^6*h^7 + 1656*(sqrt(c)*x - sq
rt(c*x^2 + a))^2*a^5*c^(11/2)*d*g^5*h^8 + 7824*(sqrt(c)*x - sqrt(c*x^2 + a)
)^2*a^6*c^(9/2)*f*g^5*h^8 + 3552*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^6*c^(9/2
)*e*g^4*h^9 - 4038*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^6*c^(9/2)*d*g^3*h^10 +
 8193*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^7*c^(7/2)*f*g^3*h^10 - 3207*(sqrt(c
)*x - sqrt(c*x^2 + a))^2*a^7*c^(7/2)*e*g^2*h^11 + 321*(sqrt(c)*x - sqrt(c*x
```

```
^2 + a))^2*a^7*c^(7/2)*d*g*h^12 - 1686*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^8*
c^(5/2)*f*g*h^12 + 48*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^8*c^(5/2)*e*h^13 -
240*(sqrt(c)*x - sqrt(c*x^2 + a))*a^5*c^6*f*g^8*h^5 - 96*(sqrt(c)*x - sqrt(
c*x^2 + a))*a^5*c^6*e*g^7*h^6 - 48*(sqrt(c)*x - sqrt(c*x^2 + a))*a^5*c^6*d*
g^6*h^7 - 1032*(sqrt(c)*x - sqrt(c*x^2 + a))*a^6*c^5*f*g^6*h^7 - 456*(sqrt(
c)*x - sqrt(c*x^2 + a))*a^6*c^5*e*g^5*h^8 - 336*(sqrt(c)*x - sqrt(c*x^2 + a
))*a^6*c^5*d*g^4*h^9 - 1764*(sqrt(c)*x - sqrt(c*x^2 + a))*a^7*c^4*f*g^4*h^9
 - 1044*(sqrt(c)*x - sqrt(c*x^2 + a))*a^7*c^4*e*g^3*h^10 + 882*(sqrt(c)*x -
 sqrt(c*x^2 + a))*a^7*c^4*d*g^2*h^11 - 1977*(sqrt(c)*x - sqrt(c*x^2 + a))*a
^8*c^3*f*g^2*h^11 + 471*(sqrt(c)*x - sqrt(c*x^2 + a))*a^8*c^3*e*g*h^12 + 15
*(sqrt(c)*x - sqrt(c*x^2 + a))*a^8*c^3*d*h^13 + 150*(sqrt(c)*x - sqrt(c*x^2
 + a))*a^9*c^2*f*h^13 + 40*a^6*c^(11/2)*f*g^7*h^6 + 8*a^6*c^(11/2)*e*g^6*h^
7 + 4*a^6*c^(11/2)*d*g^5*h^8 + 166*a^7*c^(9/2)*f*g^5*h^8 + 38*a^7*c^(9/2)*e
*g^4*h^9 + 28*a^7*c^(9/2)*d*g^3*h^10 + 267*a^8*c^(7/2)*f*g^3*h^10 + 87*a^8*
c^(7/2)*e*g^2*h^11 - 81*a^8*c^(7/2)*d*g*h^12 + 246*a^9*c^(5/2)*f*g*h^12 - 4
8*a^9*c^(5/2)*e*h^13)/((c^4*g^8*h^6 + 4*a*c^3*g^6*h^8 + 6*a^2*c^2*g^4*h^10
+ 4*a^3*c*g^2*h^12 + a^4*h^14)*((sqrt(c)*x - sqrt(c*x^2 + a))^2*h + 2*(sqrt
(c)*x - sqrt(c*x^2 + a))*sqrt(c)*g - a*h)^6)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+cx^2)^{3/2}\,(d+ex+fx^2)}{(g+hx)^7}\,dx=\int \frac{{\left(c\,x^2+a\right)}^{3/2}\,\left(f\,x^2+e\,x+d\right)}{{\left(g+h\,x\right)}^7}\,dx$$

```
[In] int(((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^7,x)
[Out] int(((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^7, x)
```

3.99 $\int \frac{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^8}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 532

$$\begin{aligned}
&\int \frac{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^8}\,dx = \\
&-\frac{ac^2(6c^2dg^3+a^2h^2(8fg-eh)-acg(fg^2-h(8eg-3dh)))\,(ah-cgx)\sqrt{a+cx^2}}{16\left(cg^2+ah^2\right)^5(g+hx)^2} \\
&-\frac{c(6c^2dg^3+a^2h^2(8fg-eh)-acg(fg^2-h(8eg-3dh)))\,(ah-cgx)\left(a+cx^2\right)^{3/2}}{24\left(cg^2+ah^2\right)^4(g+hx)^4} \\
&-\frac{\left(fg^2-egh+dh^2\right)\left(a+cx^2\right)^{5/2}}{7h\left(cg^2+ah^2\right)(g+hx)^7} \\
&+\frac{\left(7ah^2(2fg-eh)+cg(5fg^2+h(2eg-9dh))\right)\left(a+cx^2\right)^{5/2}}{42h\left(cg^2+ah^2\right)^2(g+hx)^6} \\
&-\frac{\left(42a^2fh^4-c^2g^2(5fg^2+h(2eg-51dh))-ach^2(26fg^2-h(61eg-12dh))\right)\left(a+cx^2\right)^{5/2}}{210h\left(cg^2+ah^2\right)^3(g+hx)^5} \\
&-\frac{a^2c^3(6c^2dg^3+a^2h^2(8fg-eh)-acg(fg^2-h(8eg-3dh)))\,\text{arctanh}\left(\frac{ah-cgx}{\sqrt{cg^2+ah^2}\sqrt{a+cx^2}}\right)}{16\left(cg^2+ah^2\right)^{11/2}}
\end{aligned}$$

```
[Out] -1/24*c*(6*c^2*d*g^3+a^2*h^2*(-e*h+8*f*g)-a*c*g*(f*g^2-h*(-3*d*h+8*e*g)))*(
-c*g*x+a*h)*(c*x^2+a)^(3/2)/(a*h^2+c*g^2)^4/(h*x+g)^4-1/7*(d*h^2-e*g*h+f*g^
2)*(c*x^2+a)^(5/2)/h/(a*h^2+c*g^2)/(h*x+g)^7+1/42*(7*a*h^2*(-e*h+2*f*g)+c*g
*(5*f*g^2+h*(-9*d*h+2*e*g)))*(c*x^2+a)^(5/2)/h/(a*h^2+c*g^2)^2/(h*x+g)^6-1/
210*(42*a^2*f*h^4-c^2*g^2*(5*f*g^2+h*(-51*d*h+2*e*g))-a*c*h^2*(26*f*g^2-h*(
-12*d*h+61*e*g)))*(c*x^2+a)^(5/2)/h/(a*h^2+c*g^2)^3/(h*x+g)^5-1/16*a^2*c^3*
(6*c^2*d*g^3+a^2*h^2*(-e*h+8*f*g)-a*c*g*(f*g^2-h*(-3*d*h+8*e*g)))*arctanh((
-c*g*x+a*h)/(a*h^2+c*g^2)^(1/2)/(c*x^2+a)^(1/2))/(a*h^2+c*g^2)^(11/2)-1/16*
```

```
a*c^2*(6*c^2*d*g^3+a^2*h^2*(-e*h+8*f*g)-a*c*g*(f*g^2-h*(-3*d*h+8*e*g)))*(-c
*g*x+a*h)*(c*x^2+a)^(1/2)/(a*h^2+c*g^2)^5/(h*x+g)^2
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 531, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1665, 849, 821, 735, 739, 212}

$$\begin{aligned}
&\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^8}\,dx = \\
&-\frac{a^2c^3\text{arctanh}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)(a^2h^2(8fg-eh)-acg(fg^2-h(8eg-3dh))+6c^2dg^3)}{16(ah^2+cg^2)^{11/2}} \\
&-\frac{(a+cx^2)^{5/2}(42a^2fh^4-ach^2(26fg^2-h(61eg-12dh))-c^2(g^2h(2eg-51dh)+5fg^4))}{210h(g+hx)^5(ah^2+cg^2)^3} \\
&-\frac{ac^2\sqrt{a+cx^2}(ah-cgx)(a^2h^2(8fg-eh)-acg(fg^2-h(8eg-3dh))+6c^2dg^3)}{16(g+hx)^2(ah^2+cg^2)^5} \\
&-\frac{c(a+cx^2)^{3/2}(ah-cgx)(a^2h^2(8fg-eh)-acg(fg^2-h(8eg-3dh))+6c^2dg^3)}{24(g+hx)^4(ah^2+cg^2)^4} \\
&-\frac{(a+cx^2)^{5/2}(dh^2-egh+fg^2)}{7h(g+hx)^7(ah^2+cg^2)} \\
&+\frac{(a+cx^2)^{5/2}(7ah^2(2fg-eh)+cgh(2eg-9dh)+5cfg^3)}{42h(g+hx)^6(ah^2+cg^2)^2}
\end{aligned}$$

```
[In] Int[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^8,x]
```

```
[Out] -1/16*(a*c^2*(6*c^2*d*g^3 + a^2*h^2*(8*f*g - e*h) - a*c*g*(f*g^2 - h*(8*e*g
 - 3*d*h)))*(a*h - c*g*x)*Sqrt[a + c*x^2])/((c*g^2 + a*h^2)^5*(g + h*x)^2)
- (c*(6*c^2*d*g^3 + a^2*h^2*(8*f*g - e*h) - a*c*g*(f*g^2 - h*(8*e*g - 3*d*h
)))*(a*h - c*g*x)*(a + c*x^2)^(3/2))/(24*(c*g^2 + a*h^2)^4*(g + h*x)^4) - (
(f*g^2 - e*g*h + d*h^2)*(a + c*x^2)^(5/2))/(7*h*(c*g^2 + a*h^2)*(g + h*x)^7
) + ((5*c*f*g^3 + c*g*h*(2*e*g - 9*d*h) + 7*a*h^2*(2*f*g - e*h))*(a + c*x^2
)^(5/2))/(42*h*(c*g^2 + a*h^2)^2*(g + h*x)^6) - ((42*a^2*f*h^4 - c^2*(5*f*g
^4 + g^2*h*(2*e*g - 51*d*h)) - a*c*h^2*(26*f*g^2 - h*(61*e*g - 12*d*h)))*(a
 + c*x^2)^(5/2))/(210*h*(c*g^2 + a*h^2)^3*(g + h*x)^5) - (a^2*c^3*(6*c^2*d*
g^3 + a^2*h^2*(8*f*g - e*h) - a*c*g*(f*g^2 - h*(8*e*g - 3*d*h)))*ArcTanh[(a
*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2])])/(16*(c*g^2 + a*h^2)^(11
/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 735

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(-2*a*e + (2*c*d)*x)*((a + c*x^2)^p/(2*(m + 1)*(c*d^2
+ a*e^2))), x] - Dist[4*a*c*(p/(2*(m + 1)*(c*d^2 + a*e^2))), Int[(d + e*x)^
(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2
+ a*e^2, 0] && EqQ[m + 2*p + 2, 0] && GtQ[p, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(fg^2-egh+dh^2)\left(a+cx^2\right)^{5/2}}{7h\left(cg^2+ah^2\right)(g+hx)^7} \\
&\quad -\frac{\int \frac{\left(-7(cdg-afg+aeh)-\left(7afh+c\left(2eg+\frac{5fg^2}{h}-2dh\right)\right)x\right)\left(a+cx^2\right)^{3/2}}{(g+hx)^7}\,dx}{7\left(cg^2+ah^2\right)} \\
&= -\frac{(fg^2-egh+dh^2)\left(a+cx^2\right)^{5/2}}{7h\left(cg^2+ah^2\right)(g+hx)^7} \\
&\quad +\frac{\left(5cfg^3+cgh(2eg-9dh)+7ah^2(2fg-eh)\right)\left(a+cx^2\right)^{5/2}}{42h\left(cg^2+ah^2\right)^2(g+hx)^6} \\
&\quad +\frac{\int \frac{\left(6\left(7c^2dg^2+7a^2fh^2-ac\left(2fg^2-h(9eg-2dh)\right)\right)+\frac{c\left(5cfg^3+cgh(2eg-9dh)+7ah^2(2fg-eh)\right)x}{h}\right)\left(a+cx^2\right)^{3/2}}{(g+hx)^6}\,dx}{42\left(cg^2+ah^2\right)^2} \\
&= -\frac{(fg^2-egh+dh^2)\left(a+cx^2\right)^{5/2}}{7h\left(cg^2+ah^2\right)(g+hx)^7} \\
&\quad +\frac{\left(5cfg^3+cgh(2eg-9dh)+7ah^2(2fg-eh)\right)\left(a+cx^2\right)^{5/2}}{42h\left(cg^2+ah^2\right)^2(g+hx)^6} \\
&\quad -\frac{\left(42a^2fh^4-c^2\left(5fg^4+g^2h(2eg-51dh)\right)-ach^2\left(26fg^2-h(61eg-12dh)\right)\right)\left(a+cx^2\right)^{5/2}}{210h\left(cg^2+ah^2\right)^3(g+hx)^5} \\
&\quad +\frac{\left(c\left(6c^2dg^3+a^2h^2(8fg-eh)-acg\left(fg^2-h(8eg-3dh)\right)\right)\right)\int \frac{\left(a+cx^2\right)^{3/2}}{(g+hx)^5}\,dx}{6\left(cg^2+ah^2\right)^3} \\
&= -\frac{c\left(6c^2dg^3+a^2h^2(8fg-eh)-acg\left(fg^2-h(8eg-3dh)\right)\right)(ah-cgx)\left(a+cx^2\right)^{3/2}}{24\left(cg^2+ah^2\right)^4(g+hx)^4} \\
&\quad -\frac{(fg^2-egh+dh^2)\left(a+cx^2\right)^{5/2}}{7h\left(cg^2+ah^2\right)(g+hx)^7} \\
&\quad +\frac{\left(5cfg^3+cgh(2eg-9dh)+7ah^2(2fg-eh)\right)\left(a+cx^2\right)^{5/2}}{42h\left(cg^2+ah^2\right)^2(g+hx)^6} \\
&\quad -\frac{\left(42a^2fh^4-c^2\left(5fg^4+g^2h(2eg-51dh)\right)-ach^2\left(26fg^2-h(61eg-12dh)\right)\right)\left(a+cx^2\right)^{5/2}}{210h\left(cg^2+ah^2\right)^3(g+hx)^5} \\
&\quad +\frac{\left(ac^2\left(6c^2dg^3+a^2h^2(8fg-eh)-acg\left(fg^2-h(8eg-3dh)\right)\right)\right)\int \frac{\sqrt{a+cx^2}}{(g+hx)^3}\,dx}{8\left(cg^2+ah^2\right)^4}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{ac^2(6c^2dg^3+a^2h^2(8fg-eh)-acg(fg^2-h(8eg-3dh)))\,(ah-cgx)\sqrt{a+cx^2}}{16\,(cg^2+ah^2)^5\,(g+hx)^2} \\
&\quad - \frac{c(6c^2dg^3+a^2h^2(8fg-eh)-acg(fg^2-h(8eg-3dh)))\,(ah-cgx)\,(a+cx^2)^{3/2}}{24\,(cg^2+ah^2)^4\,(g+hx)^4} \\
&\quad - \frac{(fg^2-egh+dh^2)\,(a+cx^2)^{5/2}}{7h\,(cg^2+ah^2)\,(g+hx)^7} \\
&\quad + \frac{(5cfg^3+cgh(2eg-9dh)+7ah^2(2fg-eh))\,(a+cx^2)^{5/2}}{42h\,(cg^2+ah^2)^2\,(g+hx)^6} \\
&\quad - \frac{(42a^2fh^4-c^2(5fg^4+g^2h(2eg-51dh))-ach^2(26fg^2-h(61eg-12dh)))\,(a+cx^2)^{5/2}}{210h\,(cg^2+ah^2)^3\,(g+hx)^5} \\
&\quad + \frac{(a^2c^3(6c^2dg^3+a^2h^2(8fg-eh)-acg(fg^2-h(8eg-3dh))))\int\frac{1}{(g+hx)\sqrt{a+cx^2}}\,dx}{16\,(cg^2+ah^2)^5}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{ac^2(6c^2dg^3+a^2h^2(8fg-eh)-acg(fg^2-h(8eg-3dh)))\,(ah-cgx)\sqrt{a+cx^2}}{16\,(cg^2+ah^2)^5\,(g+hx)^2} \\
&\quad - \frac{c(6c^2dg^3+a^2h^2(8fg-eh)-acg(fg^2-h(8eg-3dh)))\,(ah-cgx)\,(a+cx^2)^{3/2}}{24\,(cg^2+ah^2)^4\,(g+hx)^4} \\
&\quad - \frac{(fg^2-egh+dh^2)\,(a+cx^2)^{5/2}}{7h\,(cg^2+ah^2)\,(g+hx)^7} \\
&\quad + \frac{(5cfg^3+cgh(2eg-9dh)+7ah^2(2fg-eh))\,(a+cx^2)^{5/2}}{42h\,(cg^2+ah^2)^2\,(g+hx)^6} \\
&\quad - \frac{(42a^2fh^4-c^2(5fg^4+g^2h(2eg-51dh))-ach^2(26fg^2-h(61eg-12dh)))\,(a+cx^2)^{5/2}}{210h\,(cg^2+ah^2)^3\,(g+hx)^5} \\
&\quad - \frac{(a^2c^3(6c^2dg^3+a^2h^2(8fg-eh)-acg(fg^2-h(8eg-3dh))))\,\mathrm{Subst}\left(\int\frac{1}{cg^2+ah^2-x^2}\,dx,x,\frac{ah-cgx}{\sqrt{a+cx^2}}\right)}{16\,(cg^2+ah^2)^5}
\end{aligned}
$$

$$= -\frac{ac^2(6c^2dg^3+a^2h^2(8fg-eh)-acg(fg^2-h(8eg-3dh)))\,(ah-cgx)\sqrt{a+cx^2}}{16\,(cg^2+ah^2)^5\,(g+hx)^2}$$
$$-\frac{c(6c^2dg^3+a^2h^2(8fg-eh)-acg(fg^2-h(8eg-3dh)))\,(ah-cgx)\,(a+cx^2)^{3/2}}{24\,(cg^2+ah^2)^4\,(g+hx)^4}$$
$$-\frac{(fg^2-egh+dh^2)\,(a+cx^2)^{5/2}}{7h\,(cg^2+ah^2)\,(g+hx)^7}$$
$$+\frac{(5cfg^3+cgh(2eg-9dh)+7ah^2(2fg-eh))\,(a+cx^2)^{5/2}}{42h\,(cg^2+ah^2)^2\,(g+hx)^6}$$
$$-\frac{(42a^2fh^4-c^2(5fg^4+g^2h(2eg-51dh))-ach^2(26fg^2-h(61eg-12dh)))\,(a+cx^2)^{5/2}}{210h\,(cg^2+ah^2)^3\,(g+hx)^5}$$
$$-\frac{a^2c^3(6c^2dg^3+a^2h^2(8fg-eh)-acg(fg^2-h(8eg-3dh)))\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{cg^2+ah^2}\sqrt{a+cx^2}}\right)}{16\,(cg^2+ah^2)^{11/2}}$$

Mathematica [A] (verified)

Time = 11.31 (sec) , antiderivative size = 863, normalized size of antiderivative = 1.62

$$\int\frac{(a+cx^2)^{3/2}\,(d+ex+fx^2)}{(g+hx)^8}\,dx =$$
$$-\frac{\sqrt{a+cx^2}\Big(240(cg^2+ah^2)^6\,(fg^2+h(-eg+dh))-40(cg^2+ah^2)^5\,(29cfg^3+cgh(-22eg+15dh)-7ah^2(}{}$$
$$+\frac{a^2c^3(6c^2dg^3+a^2h^2(8fg-eh)-acg(fg^2+h(-8eg+3dh)))\log(g+hx)}{16\,(cg^2+ah^2)^{11/2}}$$
$$-\frac{a^2c^3(6c^2dg^3+a^2h^2(8fg-eh)-acg(fg^2+h(-8eg+3dh)))\log\left(ah-cgx+\sqrt{cg^2+ah^2}\sqrt{a+cx^2}\right)}{16\,(cg^2+ah^2)^{11/2}}$$

```
[In] Integrate[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^8,x]
```

```
[Out] -1/1680*(Sqrt[a + c*x^2]*(240*(c*g^2 + a*h^2)^6*(f*g^2 + h*(-(e*g) + d*h))
- 40*(c*g^2 + a*h^2)^5*(29*c*f*g^3 + c*g*h*(-22*e*g + 15*d*h) - 7*a*h^2*(-2
*f*g + e*h))*(g + h*x) + 8*(c*g^2 + a*h^2)^4*(42*a^2*f*h^4 + a*c*h^2*(314*f
*g^2 + h*(-139*e*g + 48*d*h)) + c^2*(275*f*g^4 + g^2*h*(-142*e*g + 51*d*h))
)*(g + h*x)^2 - 2*c*(c*g^2 + a*h^2)^3*(7*a^2*h^4*(136*f*g - 35*e*h) + 2*c^2
*(500*f*g^5 + g^3*h*(-136*e*g + 3*d*h)) + a*c*g*h^2*(1979*f*g^2 + h*(-544*e
*g + 33*d*h)))*(g + h*x)^3 + 2*c*(c*g^2 + a*h^2)^2*(336*a^3*f*h^6 + c^3*(40
0*f*g^6 - 2*g^4*h*(4*e*g + 3*d*h)) + 3*a^2*c*h^4*(400*f*g^2 + h*(-29*e*g +
8*d*h)) + a*c^2*g^2*h^2*(1201*f*g^2 - h*(32*e*g + 45*d*h)))*(g + h*x)^4 - c
^2*(c*g^2 + a*h^2)*(21*a^3*h^6*(24*f*g - 5*e*h) + 2*a*c^2*g^3*h^2*(89*f*g^2
 + 44*e*g*h + 54*d*h^2) + 3*a^2*c*g*h^4*(109*f*g^2 + h*(94*e*g - 73*d*h)) +
 4*c^3*(10*f*g^7 + g^5*h*(4*e*g + 3*d*h)))*(g + h*x)^5 - c^2*(-336*a^4*f*h^
```

```
8 + 2*a*c^3*g^4*h^2*(109*f*g^2 + 52*e*g*h + 60*d*h^2) + a^2*c^2*g^2*h^4*(50
5*f*g^2 + h*(370*e*g - 741*d*h)) + 4*c^4*(10*f*g^8 + g^6*h*(4*e*g + 3*d*h))
 + 3*a^3*c*h^6*(312*f*g^2 + h*(-221*e*g + 32*d*h)))*(g + h*x)^6))/((c*g^2*h
 + a*h^3)^5*(g + h*x)^7) + (a^2*c^3*(6*c^2*d*g^3 + a^2*h^2*(8*f*g - e*h) -
a*c*g*(f*g^2 + h*(-8*e*g + 3*d*h)))*Log[g + h*x])/(16*(c*g^2 + a*h^2)^(11/2
)) - (a^2*c^3*(6*c^2*d*g^3 + a^2*h^2*(8*f*g - e*h) - a*c*g*(f*g^2 + h*(-8*e
*g + 3*d*h)))*Log[a*h - c*g*x + Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2]])/(16*(
c*g^2 + a*h^2)^(11/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 24804 vs. $2(504) = 1008$.

Time = 1.08 (sec) , antiderivative size = 24805, normalized size of antiderivative = 46.63

method	result	size
default	Expression too large to display	24805

```
[In] int((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^8,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^8}\,dx = \text{Timed out}$$

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^8,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^8}\,dx = \text{Timed out}$$

```
[In] integrate((c*x**2+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**8,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 16249 vs. 2(505) = 1010.

Time = 0.83 (sec) , antiderivative size = 16249, normalized size of antiderivative = 30.54

$$\int \frac{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^8}\,dx = \text{Too large to display}$$

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^8,x, algorithm="maxima")
```

```
[Out] 9/16*sqrt(c*x^2 + a)*c^7*f*g^9/(c^6*g^12*h^5 + 6*a*c^5*g^10*h^7 + 15*a^2*c^
4*g^8*h^9 + 20*a^3*c^3*g^6*h^11 + 15*a^4*c^2*g^4*h^13 + 6*a^5*c*g^2*h^15 +
a^6*h^17) - 9/16*sqrt(c*x^2 + a)*c^7*f*g^8*x/(c^6*g^12*h^4 + 6*a*c^5*g^10*h
^6 + 15*a^2*c^4*g^8*h^8 + 20*a^3*c^3*g^6*h^10 + 15*a^4*c^2*g^4*h^12 + 6*a^5
*c*g^2*h^14 + a^6*h^16) - 9/16*sqrt(c*x^2 + a)*c^7*e*g^8/(c^6*g^12*h^4 + 6*
a*c^5*g^10*h^6 + 15*a^2*c^4*g^8*h^8 + 20*a^3*c^3*g^6*h^10 + 15*a^4*c^2*g^4*
h^12 + 6*a^5*c*g^2*h^14 + a^6*h^16) + 3/16*(c*x^2 + a)^(3/2)*c^6*f*g^8/(c^6
*g^12*h^4*x + 6*a*c^5*g^10*h^6*x + 15*a^2*c^4*g^8*h^8*x + 20*a^3*c^3*g^6*h^
10*x + 15*a^4*c^2*g^4*h^12*x + 6*a^5*c*g^2*h^14*x + a^6*h^16*x + c^6*g^13*h
^3 + 6*a*c^5*g^11*h^5 + 15*a^2*c^4*g^9*h^7 + 20*a^3*c^3*g^7*h^9 + 15*a^4*c^
2*g^5*h^11 + 6*a^5*c*g^3*h^13 + a^6*g*h^15) + 9/16*sqrt(c*x^2 + a)*c^7*e*g^
7*x/(c^6*g^12*h^3 + 6*a*c^5*g^10*h^5 + 15*a^2*c^4*g^8*h^7 + 20*a^3*c^3*g^6*
h^9 + 15*a^4*c^2*g^4*h^11 + 6*a^5*c*g^2*h^13 + a^6*h^15) + 9/16*sqrt(c*x^2
+ a)*c^7*d*g^7/(c^6*g^12*h^3 + 6*a*c^5*g^10*h^5 + 15*a^2*c^4*g^8*h^7 + 20*a
^3*c^3*g^6*h^9 + 15*a^4*c^2*g^4*h^11 + 6*a^5*c*g^2*h^13 + a^6*h^15) - 3/16*
(c*x^2 + a)^(3/2)*c^6*e*g^7/(c^6*g^12*h^3*x + 6*a*c^5*g^10*h^5*x + 15*a^2*c
^4*g^8*h^7*x + 20*a^3*c^3*g^6*h^9*x + 15*a^4*c^2*g^4*h^11*x + 6*a^5*c*g^2*h
^13*x + a^6*h^15*x + c^6*g^13*h^2 + 6*a*c^5*g^11*h^4 + 15*a^2*c^4*g^9*h^6 +
 20*a^3*c^3*g^7*h^8 + 15*a^4*c^2*g^5*h^10 + 6*a^5*c*g^3*h^12 + a^6*g*h^14)
- 3/16*(c*x^2 + a)^(5/2)*c^5*f*g^7/(c^6*g^12*h^3*x^2 + 6*a*c^5*g^10*h^5*x^2
 + 15*a^2*c^4*g^8*h^7*x^2 + 20*a^3*c^3*g^6*h^9*x^2 + 15*a^4*c^2*g^4*h^11*x^
2 + 6*a^5*c*g^2*h^13*x^2 + a^6*h^15*x^2 + 2*c^6*g^13*h^2*x + 12*a*c^5*g^11*
h^4*x + 30*a^2*c^4*g^9*h^6*x + 40*a^3*c^3*g^7*h^8*x + 30*a^4*c^2*g^5*h^10*x
 + 12*a^5*c*g^3*h^12*x + 2*a^6*g*h^14*x + c^6*g^14*h + 6*a*c^5*g^12*h^3 + 1
5*a^2*c^4*g^10*h^5 + 20*a^3*c^3*g^8*h^7 + 15*a^4*c^2*g^6*h^9 + 6*a^5*c*g^4*
h^11 + a^6*g^2*h^13) + 3/16*(c*x^2 + a)^(3/2)*c^6*f*g^7/(c^6*g^12*h^3 + 6*a
*c^5*g^10*h^5 + 15*a^2*c^4*g^8*h^7 + 20*a^3*c^3*g^6*h^9 + 15*a^4*c^2*g^4*h^
11 + 6*a^5*c*g^2*h^13 + a^6*h^15) - 9/16*sqrt(c*x^2 + a)*c^7*d*g^6*x/(c^6*g
^12*h^2 + 6*a*c^5*g^10*h^4 + 15*a^2*c^4*g^8*h^6 + 20*a^3*c^3*g^6*h^8 + 15*a
^4*c^2*g^4*h^10 + 6*a^5*c*g^2*h^12 + a^6*h^14) + 3/16*(c*x^2 + a)^(3/2)*c^6
*d*g^6/(c^6*g^12*h^2*x + 6*a*c^5*g^10*h^4*x + 15*a^2*c^4*g^8*h^6*x + 20*a^3
*c^3*g^6*h^8*x + 15*a^4*c^2*g^4*h^10*x + 6*a^5*c*g^2*h^12*x + a^6*h^14*x +
c^6*g^13*h + 6*a*c^5*g^11*h^3 + 15*a^2*c^4*g^9*h^5 + 20*a^3*c^3*g^7*h^7 + 1
5*a^4*c^2*g^5*h^9 + 6*a^5*c*g^3*h^11 + a^6*g*h^13) + 3/16*(c*x^2 + a)^(5/2)
*c^5*e*g^6/(c^6*g^12*h^2*x^2 + 6*a*c^5*g^10*h^4*x^2 + 15*a^2*c^4*g^8*h^6*x^
```

```
2 + 20*a^3*c^3*g^6*h^8*x^2 + 15*a^4*c^2*g^4*h^10*x^2 + 6*a^5*c*g^2*h^12*x^2
 + a^6*h^14*x^2 + 2*c^6*g^13*h*x + 12*a*c^5*g^11*h^3*x + 30*a^2*c^4*g^9*h^5
*x + 40*a^3*c^3*g^7*h^7*x + 30*a^4*c^2*g^5*h^9*x + 12*a^5*c*g^3*h^11*x + 2*
a^6*g*h^13*x + c^6*g^14 + 6*a*c^5*g^12*h^2 + 15*a^2*c^4*g^10*h^4 + 20*a^3*c
^3*g^8*h^6 + 15*a^4*c^2*g^6*h^8 + 6*a^5*c*g^4*h^10 + a^6*g^2*h^12) - 3/16*(
c*x^2 + a)^(3/2)*c^6*e*g^6/(c^6*g^12*h^2 + 6*a*c^5*g^10*h^4 + 15*a^2*c^4*g^
8*h^6 + 20*a^3*c^3*g^6*h^8 + 15*a^4*c^2*g^4*h^10 + 6*a^5*c*g^2*h^12 + a^6*h
^14) - 35/16*sqrt(c*x^2 + a)*c^6*f*g^7/(c^5*g^10*h^5 + 5*a*c^4*g^8*h^7 + 10
*a^2*c^3*g^6*h^9 + 10*a^3*c^2*g^4*h^11 + 5*a^4*c*g^2*h^13 + a^5*h^15) + 13/
8*sqrt(c*x^2 + a)*c^6*f*g^6*x/(c^5*g^10*h^4 + 5*a*c^4*g^8*h^6 + 10*a^2*c^3*
g^6*h^8 + 10*a^3*c^2*g^4*h^10 + 5*a^4*c*g^2*h^12 + a^5*h^14) - 3/16*(c*x^2
 + a)^(5/2)*c^5*d*g^5/(c^6*g^12*h*x^2 + 6*a*c^5*g^10*h^3*x^2 + 15*a^2*c^4*g^
8*h^5*x^2 + 20*a^3*c^3*g^6*h^7*x^2 + 15*a^4*c^2*g^4*h^9*x^2 + 6*a^5*c*g^2*h
^11*x^2 + a^6*h^13*x^2 + 2*c^6*g^13*x + 12*a*c^5*g^11*h^2*x + 30*a^2*c^4*g^
9*h^4*x + 40*a^3*c^3*g^7*h^6*x + 30*a^4*c^2*g^5*h^8*x + 12*a^5*c*g^3*h^10*x
 + 2*a^6*g*h^12*x + c^6*g^14/h + 6*a*c^5*g^12*h + 15*a^2*c^4*g^10*h^3 + 20*
a^3*c^3*g^8*h^5 + 15*a^4*c^2*g^6*h^7 + 6*a^5*c*g^4*h^9 + a^6*g^2*h^11) + 3/
16*(c*x^2 + a)^(3/2)*c^6*d*g^5/(c^6*g^12*h + 6*a*c^5*g^10*h^3 + 15*a^2*c^4*
g^8*h^5 + 20*a^3*c^3*g^6*h^7 + 15*a^4*c^2*g^4*h^9 + 6*a^5*c*g^2*h^11 + a^6*
h^13) + 7/4*sqrt(c*x^2 + a)*c^6*e*g^6/(c^5*g^10*h^4 + 5*a*c^4*g^8*h^6 + 10*
a^2*c^3*g^6*h^8 + 10*a^3*c^2*g^4*h^10 + 5*a^4*c*g^2*h^12 + a^5*h^14) - 3/8*
(c*x^2 + a)^(5/2)*c^4*f*g^6/(c^5*g^10*h^4*x^3 + 5*a*c^4*g^8*h^6*x^3 + 10*a^
2*c^3*g^6*h^8*x^3 + 10*a^3*c^2*g^4*h^10*x^3 + 5*a^4*c*g^2*h^12*x^3 + a^5*h^
14*x^3 + 3*c^5*g^11*h^3*x^2 + 15*a*c^4*g^9*h^5*x^2 + 30*a^2*c^3*g^7*h^7*x^2
 + 30*a^3*c^2*g^5*h^9*x^2 + 15*a^4*c*g^3*h^11*x^2 + 3*a^5*g*h^13*x^2 + 3*c^
5*g^12*h^2*x + 15*a*c^4*g^10*h^4*x + 30*a^2*c^3*g^8*h^6*x + 30*a^3*c^2*g^6*
h^8*x + 15*a^4*c*g^4*h^10*x + 3*a^5*g^2*h^12*x + c^5*g^13*h + 5*a*c^4*g^11*
h^3 + 10*a^2*c^3*g^9*h^5 + 10*a^3*c^2*g^7*h^7 + 5*a^4*c*g^5*h^9 + a^5*g^3*h
^11) - 11/12*(c*x^2 + a)^(3/2)*c^5*f*g^6/(c^5*g^10*h^4*x + 5*a*c^4*g^8*h^6*
x + 10*a^2*c^3*g^6*h^8*x + 10*a^3*c^2*g^4*h^10*x + 5*a^4*c*g^2*h^12*x + a^5
*h^14*x + c^5*g^11*h^3 + 5*a*c^4*g^9*h^5 + 10*a^2*c^3*g^7*h^7 + 10*a^3*c^2*
g^5*h^9 + 5*a^4*c*g^3*h^11 + a^5*g*h^13) - 19/16*sqrt(c*x^2 + a)*c^6*e*g^5*
x/(c^5*g^10*h^3 + 5*a*c^4*g^8*h^5 + 10*a^2*c^3*g^6*h^7 + 10*a^3*c^2*g^4*h^9
 + 5*a^4*c*g^2*h^11 + a^5*h^13) - 21/16*sqrt(c*x^2 + a)*c^6*d*g^5/(c^5*g^10
*h^3 + 5*a*c^4*g^8*h^5 + 10*a^2*c^3*g^6*h^7 + 10*a^3*c^2*g^4*h^9 + 5*a^4*c*
g^2*h^11 + a^5*h^13) + 3/8*(c*x^2 + a)^(5/2)*c^4*e*g^5/(c^5*g^10*h^3*x^3 +
5*a*c^4*g^8*h^5*x^3 + 10*a^2*c^3*g^6*h^7*x^3 + 10*a^3*c^2*g^4*h^9*x^3 + 5*a
^4*c*g^2*h^11*x^3 + a^5*h^13*x^3 + 3*c^5*g^11*h^2*x^2 + 15*a*c^4*g^9*h^4*x^
2 + 30*a^2*c^3*g^7*h^6*x^2 + 30*a^3*c^2*g^5*h^8*x^2 + 15*a^4*c*g^3*h^10*x^2
 + 3*a^5*g*h^12*x^2 + 3*c^5*g^12*h*x + 15*a*c^4*g^10*h^3*x + 30*a^2*c^3*g^8
*h^5*x + 30*a^3*c^2*g^6*h^7*x + 15*a^4*c*g^4*h^9*x + 3*a^5*g^2*h^11*x + c^5
*g^13 + 5*a*c^4*g^11*h^2 + 10*a^2*c^3*g^9*h^4 + 10*a^3*c^2*g^7*h^6 + 5*a^4*
c*g^5*h^8 + a^5*g^3*h^10) + 37/48*(c*x^2 + a)^(3/2)*c^5*e*g^5/(c^5*g^10*h^3
*x + 5*a*c^4*g^8*h^5*x + 10*a^2*c^3*g^6*h^7*x + 10*a^3*c^2*g^4*h^9*x + 5*a^
4*c*g^2*h^11*x + a^5*h^13*x + c^5*g^11*h^2 + 5*a*c^4*g^9*h^4 + 10*a^2*c^3*g
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^7*h^6 + 10*a^3*c^2*g^5*h^8 + 5*a^4*c*g^3*h^10 + a^5*g*h^12) + 1/6*(c*x^2 +
 a)^(5/2)*c^4*f*g^5/(c^5*g^10*h^3*x^2 + 5*a*c^4*g^8*h^5*x^2 + 10*a^2*c^3*g^
6*h^7*x^2 + 10*a^3*c^2*g^4*h^9*x^2 + 5*a^4*c*g^2*h^11*x^2 + a^5*h^13*x^2 +
2*c^5*g^11*h^2*x + 10*a*c^4*g^9*h^4*x + 20*a^2*c^3*g^7*h^6*x + 20*a^3*c^2*g
^5*h^8*x + 10*a^4*c*g^3*h^10*x + 2*a^5*g*h^12*x + c^5*g^12*h + 5*a*c^4*g^10
*h^3 + 10*a^2*c^3*g^8*h^5 + 10*a^3*c^2*g^6*h^7 + 5*a^4*c*g^4*h^9 + a^5*g^2*
h^11) - 1/6*(c*x^2 + a)^(3/2)*c^5*f*g^5/(c^5*g^10*h^3 + 5*a*c^4*g^8*h^5 + 1
0*a^2*c^3*g^6*h^7 + 10*a^3*c^2*g^4*h^9 + 5*a^4*c*g^2*h^11 + a^5*h^13) + 3/4
*sqrt(c*x^2 + a)*c^6*d*g^4*x/(c^5*g^10*h^2 + 5*a*c^4*g^8*h^4 + 10*a^2*c^3*g
^6*h^6 + 10*a^3*c^2*g^4*h^8 + 5*a^4*c*g^2*h^10 + a^5*h^12) - 3/8*(c*x^2 + a
)^(5/2)*c^4*d*g^4/(c^5*g^10*h^2*x^3 + 5*a*c^4*g^8*h^4*x^3 + 10*a^2*c^3*g^6*
h^6*x^3 + 10*a^3*c^2*g^4*h^8*x^3 + 5*a^4*c*g^2*h^10*x^3 + a^5*h^12*x^3 + 3*
c^5*g^11*h*x^2 + 15*a*c^4*g^9*h^3*x^2 + 30*a^2*c^3*g^7*h^5*x^2 + 30*a^3*c^2
*g^5*h^7*x^2 + 15*a^4*c*g^3*h^9*x^2 + 3*a^5*g*h^11*x^2 + 3*c^5*g^12*x + 15*
a*c^4*g^10*h^2*x + 30*a^2*c^3*g^8*h^4*x + 30*a^3*c^2*g^6*h^6*x + 15*a^4*c*g
^4*h^8*x + 3*a^5*g^2*h^10*x + c^5*g^13/h + 5*a*c^4*g^11*h + 10*a^2*c^3*g^9*
h^3 + 10*a^3*c^2*g^7*h^5 + 5*a^4*c*g^5*h^7 + a^5*g^3*h^9) - 5/8*(c*x^2 + a)
^(3/2)*c^5*d*g^4/(c^5*g^10*h^2*x + 5*a*c^4*g^8*h^4*x + 10*a^2*c^3*g^6*h^6*x
 + 10*a^3*c^2*g^4*h^8*x + 5*a^4*c*g^2*h^10*x + a^5*h^12*x + c^5*g^11*h + 5*
a*c^4*g^9*h^3 + 10*a^2*c^3*g^7*h^5 + 10*a^3*c^2*g^5*h^7 + 5*a^4*c*g^3*h^9 +
 a^5*g*h^11) - 1/48*(c*x^2 + a)^(5/2)*c^4*e*g^4/(c^5*g^10*h^2*x^2 + 5*a*c^4
*g^8*h^4*x^2 + 10*a^2*c^3*g^6*h^6*x^2 + 10*a^3*c^2*g^4*h^8*x^2 + 5*a^4*c*g^
2*h^10*x^2 + a^5*h^12*x^2 + 2*c^5*g^11*h*x + 10*a*c^4*g^9*h^3*x + 20*a^2*c^
3*g^7*h^5*x + 20*a^3*c^2*g^5*h^7*x + 10*a^4*c*g^3*h^9*x + 2*a^5*g*h^11*x +
c^5*g^12 + 5*a*c^4*g^10*h^2 + 10*a^2*c^3*g^8*h^4 + 10*a^3*c^2*g^6*h^6 + 5*a
^4*c*g^4*h^8 + a^5*g^2*h^10) + 1/48*(c*x^2 + a)^(3/2)*c^5*e*g^4/(c^5*g^10*h
^2 + 5*a*c^4*g^8*h^4 + 10*a^2*c^3*g^6*h^6 + 10*a^3*c^2*g^4*h^8 + 5*a^4*c*g^
2*h^10 + a^5*h^12) - 3/8*(c*x^2 + a)^(5/2)*c^3*f*g^5/(c^4*g^8*h^5*x^4 + 4*a
*c^3*g^6*h^7*x^4 + 6*a^2*c^2*g^4*h^9*x^4 + 4*a^3*c*g^2*h^11*x^4 + a^4*h^13*
x^4 + 4*c^4*g^9*h^4*x^3 + 16*a*c^3*g^7*h^6*x^3 + 24*a^2*c^2*g^5*h^8*x^3 + 1
6*a^3*c*g^3*h^10*x^3 + 4*a^4*g*h^12*x^3 + 6*c^4*g^10*h^3*x^2 + 24*a*c^3*g^8
*h^5*x^2 + 36*a^2*c^2*g^6*h^7*x^2 + 24*a^3*c*g^4*h^9*x^2 + 6*a^4*g^2*h^11*x
^2 + 4*c^4*g^11*h^2*x + 16*a*c^3*g^9*h^4*x + 24*a^2*c^2*g^7*h^6*x + 16*a^3*
c*g^5*h^8*x + 4*a^4*g^3*h^10*x + c^4*g^12*h + 4*a*c^3*g^10*h^3 + 6*a^2*c^2*
g^8*h^5 + 4*a^3*c*g^6*h^7 + a^4*g^4*h^9) + 51/16*sqrt(c*x^2 + a)*c^5*f*g^5/
(c^4*g^8*h^5 + 4*a*c^3*g^6*h^7 + 6*a^2*c^2*g^4*h^9 + 4*a^3*c*g^2*h^11 + a^4
*h^13) - 25/16*sqrt(c*x^2 + a)*c^5*f*g^4*x/(c^4*g^8*h^4 + 4*a*c^3*g^6*h^6 +
 6*a^2*c^2*g^4*h^8 + 4*a^3*c*g^2*h^10 + a^4*h^12) - 1/8*(c*x^2 + a)^(5/2)*c
^4*d*g^3/(c^5*g^10*h*x^2 + 5*a*c^4*g^8*h^3*x^2 + 10*a^2*c^3*g^6*h^5*x^2 + 1
0*a^3*c^2*g^4*h^7*x^2 + 5*a^4*c*g^2*h^9*x^2 + a^5*h^11*x^2 + 2*c^5*g^11*x +
 10*a*c^4*g^9*h^2*x + 20*a^2*c^3*g^7*h^4*x + 20*a^3*c^2*g^5*h^6*x + 10*a^4*
c*g^3*h^8*x + 2*a^5*g*h^10*x + c^5*g^12/h + 5*a*c^4*g^10*h + 10*a^2*c^3*g^8
*h^3 + 10*a^3*c^2*g^6*h^5 + 5*a^4*c*g^4*h^7 + a^5*g^2*h^9) + 1/8*(c*x^2 + a
)^(3/2)*c^5*d*g^3/(c^5*g^10*h + 5*a*c^4*g^8*h^3 + 10*a^2*c^3*g^6*h^5 + 10*a
^3*c^2*g^4*h^7 + 5*a^4*c*g^2*h^9 + a^5*h^11) + 3/8*(c*x^2 + a)^(5/2)*c^3*e*
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g^4/(c^4*g^8*h^4*x^4 + 4*a*c^3*g^6*h^6*x^4 + 6*a^2*c^2*g^4*h^8*x^4 + 4*a^3*
c*g^2*h^10*x^4 + a^4*h^12*x^4 + 4*c^4*g^9*h^3*x^3 + 16*a*c^3*g^7*h^5*x^3 +
24*a^2*c^2*g^5*h^7*x^3 + 16*a^3*c*g^3*h^9*x^3 + 4*a^4*g*h^11*x^3 + 6*c^4*g^
10*h^2*x^2 + 24*a*c^3*g^8*h^4*x^2 + 36*a^2*c^2*g^6*h^6*x^2 + 24*a^3*c*g^4*h
^8*x^2 + 6*a^4*g^2*h^10*x^2 + 4*c^4*g^11*h*x + 16*a*c^3*g^9*h^3*x + 24*a^2*
c^2*g^7*h^5*x + 16*a^3*c*g^5*h^7*x + 4*a^4*g^3*h^9*x + c^4*g^12 + 4*a*c^3*g
^10*h^2 + 6*a^2*c^2*g^8*h^4 + 4*a^3*c*g^6*h^6 + a^4*g^4*h^8) - 15/8*sqrt(c*
x^2 + a)*c^5*e*g^4/(c^4*g^8*h^4 + 4*a*c^3*g^6*h^6 + 6*a^2*c^2*g^4*h^8 + 4*a
^3*c*g^2*h^10 + a^4*h^12) + 17/24*(c*x^2 + a)^(5/2)*c^3*f*g^4/(c^4*g^8*h^4*
x^3 + 4*a*c^3*g^6*h^6*x^3 + 6*a^2*c^2*g^4*h^8*x^3 + 4*a^3*c*g^2*h^10*x^3 +
a^4*h^12*x^3 + 3*c^4*g^9*h^3*x^2 + 12*a*c^3*g^7*h^5*x^2 + 18*a^2*c^2*g^5*h^
7*x^2 + 12*a^3*c*g^3*h^9*x^2 + 3*a^4*g*h^11*x^2 + 3*c^4*g^10*h^2*x + 12*a*c
^3*g^8*h^4*x + 18*a^2*c^2*g^6*h^6*x + 12*a^3*c*g^4*h^8*x + 3*a^4*g^2*h^10*x
 + c^4*g^11*h + 4*a*c^3*g^9*h^3 + 6*a^2*c^2*g^7*h^5 + 4*a^3*c*g^5*h^7 + a^4
*g^3*h^9) + 59/48*(c*x^2 + a)^(3/2)*c^4*f*g^4/(c^4*g^8*h^4*x + 4*a*c^3*g^6*
h^6*x + 6*a^2*c^2*g^4*h^8*x + 4*a^3*c*g^2*h^10*x + a^4*h^12*x + c^4*g^9*h^3
 + 4*a*c^3*g^7*h^5 + 6*a^2*c^2*g^5*h^7 + 4*a^3*c*g^3*h^9 + a^4*g*h^11) + 11
/16*sqrt(c*x^2 + a)*c^5*e*g^3*x/(c^4*g^8*h^3 + 4*a*c^3*g^6*h^5 + 6*a^2*c^2*
g^4*h^7 + 4*a^3*c*g^2*h^9 + a^4*h^11) - 3/8*(c*x^2 + a)^(5/2)*c^3*d*g^3/(c^
4*g^8*h^3*x^4 + 4*a*c^3*g^6*h^5*x^4 + 6*a^2*c^2*g^4*h^7*x^4 + 4*a^3*c*g^2*h
^9*x^4 + a^4*h^11*x^4 + 4*c^4*g^9*h^2*x^3 + 16*a*c^3*g^7*h^4*x^3 + 24*a^2*c
^2*g^5*h^6*x^3 + 16*a^3*c*g^3*h^8*x^3 + 4*a^4*g*h^10*x^3 + 6*c^4*g^10*h*x^2
 + 24*a*c^3*g^8*h^3*x^2 + 36*a^2*c^2*g^6*h^5*x^2 + 24*a^3*c*g^4*h^7*x^2 + 6
*a^4*g^2*h^9*x^2 + 4*c^4*g^11*x + 16*a*c^3*g^9*h^2*x + 24*a^2*c^2*g^7*h^4*x
 + 16*a^3*c*g^5*h^6*x + 4*a^4*g^3*h^8*x + c^4*g^12/h + 4*a*c^3*g^10*h + 6*a
^2*c^2*g^8*h^3 + 4*a^3*c*g^6*h^5 + a^4*g^4*h^7) + 15/16*sqrt(c*x^2 + a)*c^5
*d*g^3/(c^4*g^8*h^3 + 4*a*c^3*g^6*h^5 + 6*a^2*c^2*g^4*h^7 + 4*a^3*c*g^2*h^9
 + a^4*h^11) - 5/12*(c*x^2 + a)^(5/2)*c^3*e*g^3/(c^4*g^8*h^3*x^3 + 4*a*c^3*
g^6*h^5*x^3 + 6*a^2*c^2*g^4*h^7*x^3 + 4*a^3*c*g^2*h^9*x^3 + a^4*h^11*x^3 +
3*c^4*g^9*h^2*x^2 + 12*a*c^3*g^7*h^4*x^2 + 18*a^2*c^2*g^5*h^6*x^2 + 12*a^3*
c*g^3*h^8*x^2 + 3*a^4*g*h^10*x^2 + 3*c^4*g^10*h*x + 12*a*c^3*g^8*h^3*x + 18
*a^2*c^2*g^6*h^5*x + 12*a^3*c*g^4*h^7*x + 3*a^4*g^2*h^9*x + c^4*g^11 + 4*a*
c^3*g^9*h^2 + 6*a^2*c^2*g^7*h^4 + 4*a^3*c*g^5*h^6 + a^4*g^3*h^8) - 31/48*(c
*x^2 + a)^(3/2)*c^4*e*g^3/(c^4*g^8*h^3*x + 4*a*c^3*g^6*h^5*x + 6*a^2*c^2*g^
4*h^7*x + 4*a^3*c*g^2*h^9*x + a^4*h^11*x + c^4*g^9*h^2 + 4*a*c^3*g^7*h^4 +
6*a^2*c^2*g^5*h^6 + 4*a^3*c*g^3*h^8 + a^4*g*h^10) + 3/16*(c*x^2 + a)^(5/2)*
c^3*f*g^3/(c^4*g^8*h^3*x^2 + 4*a*c^3*g^6*h^5*x^2 + 6*a^2*c^2*g^4*h^7*x^2 +
4*a^3*c*g^2*h^9*x^2 + a^4*h^11*x^2 + 2*c^4*g^9*h^2*x + 8*a*c^3*g^7*h^4*x +
12*a^2*c^2*g^5*h^6*x + 8*a^3*c*g^3*h^8*x + 2*a^4*g*h^10*x + c^4*g^10*h + 4*
a*c^3*g^8*h^3 + 6*a^2*c^2*g^6*h^5 + 4*a^3*c*g^4*h^7 + a^4*g^2*h^9) - 3/16*(
c*x^2 + a)^(3/2)*c^4*f*g^3/(c^4*g^8*h^3 + 4*a*c^3*g^6*h^5 + 6*a^2*c^2*g^4*h
^7 + 4*a^3*c*g^2*h^9 + a^4*h^11) - 3/10*(c*x^2 + a)^(5/2)*c^2*f*g^4/(c^3*g^
6*h^6*x^5 + 3*a*c^2*g^4*h^8*x^5 + 3*a^2*c*g^2*h^10*x^5 + a^3*h^12*x^5 + 5*c
^3*g^7*h^5*x^4 + 15*a*c^2*g^5*h^7*x^4 + 15*a^2*c*g^3*h^9*x^4 + 5*a^3*g*h^11
*x^4 + 10*c^3*g^8*h^4*x^3 + 30*a*c^2*g^6*h^6*x^3 + 30*a^2*c*g^4*h^8*x^3 + 1
```

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0*a^3*g^2*h^10*x^3 + 10*c^3*g^9*h^3*x^2 + 30*a*c^2*g^7*h^5*x^2 + 30*a^2*c*g
^5*h^7*x^2 + 10*a^3*g^3*h^9*x^2 + 5*c^3*g^10*h^2*x + 15*a*c^2*g^8*h^4*x + 1
5*a^2*c*g^6*h^6*x + 5*a^3*g^4*h^8*x + c^3*g^11*h + 3*a*c^2*g^9*h^3 + 3*a^2*
c*g^7*h^5 + a^3*g^5*h^7) - 3/16*sqrt(c*x^2 + a)*c^5*d*g^2*x/(c^4*g^8*h^2 +
4*a*c^3*g^6*h^4 + 6*a^2*c^2*g^4*h^6 + 4*a^3*c*g^2*h^8 + a^4*h^10) + 1/8*(c*
x^2 + a)^(5/2)*c^3*d*g^2/(c^4*g^8*h^2*x^3 + 4*a*c^3*g^6*h^4*x^3 + 6*a^2*c^2
*g^4*h^6*x^3 + 4*a^3*c*g^2*h^8*x^3 + a^4*h^10*x^3 + 3*c^4*g^9*h*x^2 + 12*a*
c^3*g^7*h^3*x^2 + 18*a^2*c^2*g^5*h^5*x^2 + 12*a^3*c*g^3*h^7*x^2 + 3*a^4*g*h
^9*x^2 + 3*c^4*g^10*x + 12*a*c^3*g^8*h^2*x + 18*a^2*c^2*g^6*h^4*x + 12*a^3*
c*g^4*h^6*x + 3*a^4*g^2*h^8*x + c^4*g^11/h + 4*a*c^3*g^9*h + 6*a^2*c^2*g^7*
h^3 + 4*a^3*c*g^5*h^5 + a^4*g^3*h^7) + 3/16*(c*x^2 + a)^(3/2)*c^4*d*g^2/(c^
4*g^8*h^2*x + 4*a*c^3*g^6*h^4*x + 6*a^2*c^2*g^4*h^6*x + 4*a^3*c*g^2*h^8*x +
 a^4*h^10*x + c^4*g^9*h + 4*a*c^3*g^7*h^3 + 6*a^2*c^2*g^5*h^5 + 4*a^3*c*g^3
*h^7 + a^4*g*h^9) - 3/16*(c*x^2 + a)^(5/2)*c^3*e*g^2/(c^4*g^8*h^2*x^2 + 4*a
*c^3*g^6*h^4*x^2 + 6*a^2*c^2*g^4*h^6*x^2 + 4*a^3*c*g^2*h^8*x^2 + a^4*h^10*x
^2 + 2*c^4*g^9*h*x + 8*a*c^3*g^7*h^3*x + 12*a^2*c^2*g^5*h^5*x + 8*a^3*c*g^3
*h^7*x + 2*a^4*g*h^9*x + c^4*g^10 + 4*a*c^3*g^8*h^2 + 6*a^2*c^2*g^6*h^4 + 4
*a^3*c*g^4*h^6 + a^4*g^2*h^8) + 3/16*(c*x^2 + a)^(3/2)*c^4*e*g^2/(c^4*g^8*h
^2 + 4*a*c^3*g^6*h^4 + 6*a^2*c^2*g^4*h^6 + 4*a^3*c*g^2*h^8 + a^4*h^10) + 3/
10*(c*x^2 + a)^(5/2)*c^2*e*g^3/(c^3*g^6*h^5*x^5 + 3*a*c^2*g^4*h^7*x^5 + 3*a
^2*c*g^2*h^9*x^5 + a^3*h^11*x^5 + 5*c^3*g^7*h^4*x^4 + 15*a*c^2*g^5*h^6*x^4
+ 15*a^2*c*g^3*h^8*x^4 + 5*a^3*g*h^10*x^4 + 10*c^3*g^8*h^3*x^3 + 30*a*c^2*g
^6*h^5*x^3 + 30*a^2*c*g^4*h^7*x^3 + 10*a^3*g^2*h^9*x^3 + 10*c^3*g^9*h^2*x^2
 + 30*a*c^2*g^7*h^4*x^2 + 30*a^2*c*g^5*h^6*x^2 + 10*a^3*g^3*h^8*x^2 + 5*c^3
*g^10*h*x + 15*a*c^2*g^8*h^3*x + 15*a^2*c*g^6*h^5*x + 5*a^3*g^4*h^7*x + c^3
*g^11 + 3*a*c^2*g^9*h^2 + 3*a^2*c*g^7*h^4 + a^3*g^5*h^6) + 17/24*(c*x^2 + a
)^(5/2)*c^2*f*g^3/(c^3*g^6*h^5*x^4 + 3*a*c^2*g^4*h^7*x^4 + 3*a^2*c*g^2*h^9*
x^4 + a^3*h^11*x^4 + 4*c^3*g^7*h^4*x^3 + 12*a*c^2*g^5*h^6*x^3 + 12*a^2*c*g^
3*h^8*x^3 + 4*a^3*g*h^10*x^3 + 6*c^3*g^8*h^3*x^2 + 18*a*c^2*g^6*h^5*x^2 + 1
8*a^2*c*g^4*h^7*x^2 + 6*a^3*g^2*h^9*x^2 + 4*c^3*g^9*h^2*x + 12*a*c^2*g^7*h^
4*x + 12*a^2*c*g^5*h^6*x + 4*a^3*g^3*h^8*x + c^3*g^10*h + 3*a*c^2*g^8*h^3 +
 3*a^2*c*g^6*h^5 + a^3*g^4*h^7) - 33/16*sqrt(c*x^2 + a)*c^4*f*g^3/(c^3*g^6*
h^5 + 3*a*c^2*g^4*h^7 + 3*a^2*c*g^2*h^9 + a^3*h^11) + 1/2*sqrt(c*x^2 + a)*c
^4*f*g^2*x/(c^3*g^6*h^4 + 3*a*c^2*g^4*h^6 + 3*a^2*c*g^2*h^8 + a^3*h^10) + 1
/16*(c*x^2 + a)^(5/2)*c^3*d*g/(c^4*g^8*h*x^2 + 4*a*c^3*g^6*h^3*x^2 + 6*a^2*
c^2*g^4*h^5*x^2 + 4*a^3*c*g^2*h^7*x^2 + a^4*h^9*x^2 + 2*c^4*g^9*x + 8*a*c^3
*g^7*h^2*x + 12*a^2*c^2*g^5*h^4*x + 8*a^3*c*g^3*h^6*x + 2*a^4*g*h^8*x + c^4
*g^10/h + 4*a*c^3*g^8*h + 6*a^2*c^2*g^6*h^3 + 4*a^3*c*g^4*h^5 + a^4*g^2*h^7
) - 1/16*(c*x^2 + a)^(3/2)*c^4*d*g/(c^4*g^8*h + 4*a*c^3*g^6*h^3 + 6*a^2*c^2
*g^4*h^5 + 4*a^3*c*g^2*h^7 + a^4*h^9) - 3/10*(c*x^2 + a)^(5/2)*c^2*d*g^2/(c
^3*g^6*h^4*x^5 + 3*a*c^2*g^4*h^6*x^5 + 3*a^2*c*g^2*h^8*x^5 + a^3*h^10*x^5 +
 5*c^3*g^7*h^3*x^4 + 15*a*c^2*g^5*h^5*x^4 + 15*a^2*c*g^3*h^7*x^4 + 5*a^3*g*
h^9*x^4 + 10*c^3*g^8*h^2*x^3 + 30*a*c^2*g^6*h^4*x^3 + 30*a^2*c*g^4*h^6*x^3
+ 10*a^3*g^2*h^8*x^3 + 10*c^3*g^9*h*x^2 + 30*a*c^2*g^7*h^3*x^2 + 30*a^2*c*g
^5*h^5*x^2 + 10*a^3*g^3*h^7*x^2 + 5*c^3*g^10*x + 15*a*c^2*g^8*h^2*x + 15*a^
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2*c*g^6*h^4*x + 5*a^3*g^4*h^6*x + c^3*g^11/h + 3*a*c^2*g^9*h + 3*a^2*c*g^7*
h^3 + a^3*g^5*h^5) - 5/12*(c*x^2 + a)^(5/2)*c^2*e*g^2/(c^3*g^6*h^4*x^4 + 3*
a*c^2*g^4*h^6*x^4 + 3*a^2*c*g^2*h^8*x^4 + a^3*h^10*x^4 + 4*c^3*g^7*h^3*x^3
+ 12*a*c^2*g^5*h^5*x^3 + 12*a^2*c*g^3*h^7*x^3 + 4*a^3*g*h^9*x^3 + 6*c^3*g^8
*h^2*x^2 + 18*a*c^2*g^6*h^4*x^2 + 18*a^2*c*g^4*h^6*x^2 + 6*a^3*g^2*h^8*x^2
+ 4*c^3*g^9*h*x + 12*a*c^2*g^7*h^3*x + 12*a^2*c*g^5*h^5*x + 4*a^3*g^3*h^7*x
 + c^3*g^10 + 3*a*c^2*g^8*h^2 + 3*a^2*c*g^6*h^4 + a^3*g^4*h^6) + 3/4*sqrt(c
*x^2 + a)*c^4*e*g^2/(c^3*g^6*h^4 + 3*a*c^2*g^4*h^6 + 3*a^2*c*g^2*h^8 + a^3*
h^10) - 1/3*(c*x^2 + a)^(5/2)*c^2*f*g^2/(c^3*g^6*h^4*x^3 + 3*a*c^2*g^4*h^6*
x^3 + 3*a^2*c*g^2*h^8*x^3 + a^3*h^10*x^3 + 3*c^3*g^7*h^3*x^2 + 9*a*c^2*g^5*
h^5*x^2 + 9*a^2*c*g^3*h^7*x^2 + 3*a^3*g*h^9*x^2 + 3*c^3*g^8*h^2*x + 9*a*c^2
*g^6*h^4*x + 9*a^2*c*g^4*h^6*x + 3*a^3*g^2*h^8*x + c^3*g^9*h + 3*a*c^2*g^7*
h^3 + 3*a^2*c*g^5*h^5 + a^3*g^3*h^7) - 1/2*(c*x^2 + a)^(3/2)*c^3*f*g^2/(c^3
*g^6*h^4*x + 3*a*c^2*g^4*h^6*x + 3*a^2*c*g^2*h^8*x + a^3*h^10*x + c^3*g^7*h
^3 + 3*a*c^2*g^5*h^5 + 3*a^2*c*g^3*h^7 + a^3*g*h^9) - 3/14*(c*x^2 + a)^(5/2
)*c*f*g^3/(c^2*g^4*h^7*x^6 + 2*a*c*g^2*h^9*x^6 + a^2*h^11*x^6 + 6*c^2*g^5*h
^6*x^5 + 12*a*c*g^3*h^8*x^5 + 6*a^2*g*h^10*x^5 + 15*c^2*g^6*h^5*x^4 + 30*a*
c*g^4*h^7*x^4 + 15*a^2*g^2*h^9*x^4 + 20*c^2*g^7*h^4*x^3 + 40*a*c*g^5*h^6*x^
3 + 20*a^2*g^3*h^8*x^3 + 15*c^2*g^8*h^3*x^2 + 30*a*c*g^6*h^5*x^2 + 15*a^2*g
^4*h^7*x^2 + 6*c^2*g^9*h^2*x + 12*a*c*g^7*h^4*x + 6*a^2*g^5*h^6*x + c^2*g^1
0*h + 2*a*c*g^8*h^3 + a^2*g^6*h^5) - 1/16*sqrt(c*x^2 + a)*c^4*e*g*x/(c^3*g^
6*h^3 + 3*a*c^2*g^4*h^5 + 3*a^2*c*g^2*h^7 + a^3*h^9) + 1/8*(c*x^2 + a)^(5/2
)*c^2*d*g/(c^3*g^6*h^3*x^4 + 3*a*c^2*g^4*h^5*x^4 + 3*a^2*c*g^2*h^7*x^4 + a^
3*h^9*x^4 + 4*c^3*g^7*h^2*x^3 + 12*a*c^2*g^5*h^4*x^3 + 12*a^2*c*g^3*h^6*x^3
 + 4*a^3*g*h^8*x^3 + 6*c^3*g^8*h*x^2 + 18*a*c^2*g^6*h^3*x^2 + 18*a^2*c*g^4*
h^5*x^2 + 6*a^3*g^2*h^7*x^2 + 4*c^3*g^9*x + 12*a*c^2*g^7*h^2*x + 12*a^2*c*g
^5*h^4*x + 4*a^3*g^3*h^6*x + c^3*g^10/h + 3*a*c^2*g^8*h + 3*a^2*c*g^6*h^3 +
 a^3*g^4*h^5) - 3/16*sqrt(c*x^2 + a)*c^4*d*g/(c^3*g^6*h^3 + 3*a*c^2*g^4*h^5
 + 3*a^2*c*g^2*h^7 + a^3*h^9) + 1/24*(c*x^2 + a)^(5/2)*c^2*e*g/(c^3*g^6*h^3
*x^3 + 3*a*c^2*g^4*h^5*x^3 + 3*a^2*c*g^2*h^7*x^3 + a^3*h^9*x^3 + 3*c^3*g^7*
h^2*x^2 + 9*a*c^2*g^5*h^4*x^2 + 9*a^2*c*g^3*h^6*x^2 + 3*a^3*g*h^8*x^2 + 3*c
^3*g^8*h*x + 9*a*c^2*g^6*h^3*x + 9*a^2*c*g^4*h^5*x + 3*a^3*g^2*h^7*x + c^3*
g^9 + 3*a*c^2*g^7*h^2 + 3*a^2*c*g^5*h^4 + a^3*g^3*h^6) + 1/16*(c*x^2 + a)^(
3/2)*c^3*e*g/(c^3*g^6*h^3*x + 3*a*c^2*g^4*h^5*x + 3*a^2*c*g^2*h^7*x + a^3*h
^9*x + c^3*g^7*h^2 + 3*a*c^2*g^5*h^4 + 3*a^2*c*g^3*h^6 + a^3*g*h^8) - 1/6*(
c*x^2 + a)^(5/2)*c^2*f*g/(c^3*g^6*h^3*x^2 + 3*a*c^2*g^4*h^5*x^2 + 3*a^2*c*g
^2*h^7*x^2 + a^3*h^9*x^2 + 2*c^3*g^7*h^2*x + 6*a*c^2*g^5*h^4*x + 6*a^2*c*g^
3*h^6*x + 2*a^3*g*h^8*x + c^3*g^8*h + 3*a*c^2*g^6*h^3 + 3*a^2*c*g^4*h^5 + a
^3*g^2*h^7) + 1/6*(c*x^2 + a)^(3/2)*c^3*f*g/(c^3*g^6*h^3 + 3*a*c^2*g^4*h^5
+ 3*a^2*c*g^2*h^7 + a^3*h^9) + 3/14*(c*x^2 + a)^(5/2)*c*e*g^2/(c^2*g^4*h^6*
x^6 + 2*a*c*g^2*h^8*x^6 + a^2*h^10*x^6 + 6*c^2*g^5*h^5*x^5 + 12*a*c*g^3*h^7
*x^5 + 6*a^2*g*h^9*x^5 + 15*c^2*g^6*h^4*x^4 + 30*a*c*g^4*h^6*x^4 + 15*a^2*g
^2*h^8*x^4 + 20*c^2*g^7*h^3*x^3 + 40*a*c*g^5*h^5*x^3 + 20*a^2*g^3*h^7*x^3 +
 15*c^2*g^8*h^2*x^2 + 30*a*c*g^6*h^4*x^2 + 15*a^2*g^4*h^6*x^2 + 6*c^2*g^9*h
*x + 12*a*c*g^7*h^3*x + 6*a^2*g^5*h^5*x + c^2*g^10 + 2*a*c*g^8*h^2 + a^2*g^
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6*h^4) + 11/21*(c*x^2 + a)^(5/2)*c*f*g^2/(c^2*g^4*h^6*x^5 + 2*a*c*g^2*h^8*x
^5 + a^2*h^10*x^5 + 5*c^2*g^5*h^5*x^4 + 10*a*c*g^3*h^7*x^4 + 5*a^2*g*h^9*x^
4 + 10*c^2*g^6*h^4*x^3 + 20*a*c*g^4*h^6*x^3 + 10*a^2*g^2*h^8*x^3 + 10*c^2*g
^7*h^3*x^2 + 20*a*c*g^5*h^5*x^2 + 10*a^2*g^3*h^7*x^2 + 5*c^2*g^8*h^2*x + 10
*a*c*g^6*h^4*x + 5*a^2*g^4*h^6*x + c^2*g^9*h + 2*a*c*g^7*h^3 + a^2*g^5*h^5)
 + 1/48*(c*x^2 + a)^(5/2)*c^2*e/(c^3*g^6*h^2*x^2 + 3*a*c^2*g^4*h^4*x^2 + 3*
a^2*c*g^2*h^6*x^2 + a^3*h^8*x^2 + 2*c^3*g^7*h*x + 6*a*c^2*g^5*h^3*x + 6*a^2
*c*g^3*h^5*x + 2*a^3*g*h^7*x + c^3*g^8 + 3*a*c^2*g^6*h^2 + 3*a^2*c*g^4*h^4
+ a^3*g^2*h^6) - 1/48*(c*x^2 + a)^(3/2)*c^3*e/(c^3*g^6*h^2 + 3*a*c^2*g^4*h^
4 + 3*a^2*c*g^2*h^6 + a^3*h^8) - 3/14*(c*x^2 + a)^(5/2)*c*d*g/(c^2*g^4*h^5*
x^6 + 2*a*c*g^2*h^7*x^6 + a^2*h^9*x^6 + 6*c^2*g^5*h^4*x^5 + 12*a*c*g^3*h^6*
x^5 + 6*a^2*g*h^8*x^5 + 15*c^2*g^6*h^3*x^4 + 30*a*c*g^4*h^5*x^4 + 15*a^2*g^
2*h^7*x^4 + 20*c^2*g^7*h^2*x^3 + 40*a*c*g^5*h^4*x^3 + 20*a^2*g^3*h^6*x^3 +
15*c^2*g^8*h*x^2 + 30*a*c*g^6*h^3*x^2 + 15*a^2*g^4*h^5*x^2 + 6*c^2*g^9*x +
12*a*c*g^7*h^2*x + 6*a^2*g^5*h^4*x + c^2*g^10/h + 2*a*c*g^8*h + a^2*g^6*h^3
) - 61/210*(c*x^2 + a)^(5/2)*c*e*g/(c^2*g^4*h^5*x^5 + 2*a*c*g^2*h^7*x^5 + a
^2*h^9*x^5 + 5*c^2*g^5*h^4*x^4 + 10*a*c*g^3*h^6*x^4 + 5*a^2*g*h^8*x^4 + 10*
c^2*g^6*h^3*x^3 + 20*a*c*g^4*h^5*x^3 + 10*a^2*g^2*h^7*x^3 + 10*c^2*g^7*h^2*
x^2 + 20*a*c*g^5*h^4*x^2 + 10*a^2*g^3*h^6*x^2 + 5*c^2*g^8*h*x + 10*a*c*g^6*
h^3*x + 5*a^2*g^4*h^5*x + c^2*g^9 + 2*a*c*g^7*h^2 + a^2*g^5*h^4) - 1/3*(c*x
^2 + a)^(5/2)*c*f*g/(c^2*g^4*h^5*x^4 + 2*a*c*g^2*h^7*x^4 + a^2*h^9*x^4 + 4*
c^2*g^5*h^4*x^3 + 8*a*c*g^3*h^6*x^3 + 4*a^2*g*h^8*x^3 + 6*c^2*g^6*h^3*x^2 +
 12*a*c*g^4*h^5*x^2 + 6*a^2*g^2*h^7*x^2 + 4*c^2*g^7*h^2*x + 8*a*c*g^5*h^4*x
 + 4*a^2*g^3*h^6*x + c^2*g^8*h + 2*a*c*g^6*h^3 + a^2*g^4*h^5) + 1/2*sqrt(c*
x^2 + a)*c^3*f*g/(c^2*g^4*h^5 + 2*a*c*g^2*h^7 + a^2*h^9) - 1/7*(c*x^2 + a)^
(5/2)*f*g^2/(c*g^2*h^8*x^7 + a*h^10*x^7 + 7*c*g^3*h^7*x^6 + 7*a*g*h^9*x^6 +
 21*c*g^4*h^6*x^5 + 21*a*g^2*h^8*x^5 + 35*c*g^5*h^5*x^4 + 35*a*g^3*h^7*x^4
+ 35*c*g^6*h^4*x^3 + 35*a*g^4*h^6*x^3 + 21*c*g^7*h^3*x^2 + 21*a*g^5*h^5*x^2
 + 7*c*g^8*h^2*x + 7*a*g^6*h^4*x + c*g^9*h + a*g^7*h^3) + 2/35*(c*x^2 + a)^
(5/2)*c*d/(c^2*g^4*h^4*x^5 + 2*a*c*g^2*h^6*x^5 + a^2*h^8*x^5 + 5*c^2*g^5*h^
3*x^4 + 10*a*c*g^3*h^5*x^4 + 5*a^2*g*h^7*x^4 + 10*c^2*g^6*h^2*x^3 + 20*a*c*
g^4*h^4*x^3 + 10*a^2*g^2*h^6*x^3 + 10*c^2*g^7*h*x^2 + 20*a*c*g^5*h^3*x^2 +
10*a^2*g^3*h^5*x^2 + 5*c^2*g^8*x + 10*a*c*g^6*h^2*x + 5*a^2*g^4*h^4*x + c^2
*g^9/h + 2*a*c*g^7*h + a^2*g^5*h^3) + 1/24*(c*x^2 + a)^(5/2)*c*e/(c^2*g^4*h
^4*x^4 + 2*a*c*g^2*h^6*x^4 + a^2*h^8*x^4 + 4*c^2*g^5*h^3*x^3 + 8*a*c*g^3*h^
5*x^3 + 4*a^2*g*h^7*x^3 + 6*c^2*g^6*h^2*x^2 + 12*a*c*g^4*h^4*x^2 + 6*a^2*g^
2*h^6*x^2 + 4*c^2*g^7*h*x + 8*a*c*g^5*h^3*x + 4*a^2*g^3*h^5*x + c^2*g^8 + 2
*a*c*g^6*h^2 + a^2*g^4*h^4) - 1/16*sqrt(c*x^2 + a)*c^3*e/(c^2*g^4*h^4 + 2*a
*c*g^2*h^6 + a^2*h^8) + 1/7*(c*x^2 + a)^(5/2)*e*g/(c*g^2*h^7*x^7 + a*h^9*x^
7 + 7*c*g^3*h^6*x^6 + 7*a*g*h^8*x^6 + 21*c*g^4*h^5*x^5 + 21*a*g^2*h^7*x^5 +
 35*c*g^5*h^4*x^4 + 35*a*g^3*h^6*x^4 + 35*c*g^6*h^3*x^3 + 35*a*g^4*h^5*x^3
+ 21*c*g^7*h^2*x^2 + 21*a*g^5*h^4*x^2 + 7*c*g^8*h*x + 7*a*g^6*h^3*x + c*g^9
 + a*g^7*h^2) + 1/3*(c*x^2 + a)^(5/2)*f*g/(c*g^2*h^7*x^6 + a*h^9*x^6 + 6*c*
g^3*h^6*x^5 + 6*a*g*h^8*x^5 + 15*c*g^4*h^5*x^4 + 15*a*g^2*h^7*x^4 + 20*c*g^
5*h^4*x^3 + 20*a*g^3*h^6*x^3 + 15*c*g^6*h^3*x^2 + 15*a*g^4*h^5*x^2 + 6*c*g^
```

```
7*h^2*x + 6*a*g^5*h^4*x + c*g^8*h + a*g^6*h^3) - 1/7*(c*x^2 + a)^(5/2)*d/(c
*g^2*h^6*x^7 + a*h^8*x^7 + 7*c*g^3*h^5*x^6 + 7*a*g*h^7*x^6 + 21*c*g^4*h^4*x
^5 + 21*a*g^2*h^6*x^5 + 35*c*g^5*h^3*x^4 + 35*a*g^3*h^5*x^4 + 35*c*g^6*h^2*
x^3 + 35*a*g^4*h^4*x^3 + 21*c*g^7*h*x^2 + 21*a*g^5*h^3*x^2 + 7*c*g^8*x + 7*
a*g^6*h^2*x + c*g^9/h + a*g^7*h) - 1/6*(c*x^2 + a)^(5/2)*e/(c*g^2*h^6*x^6 +
 a*h^8*x^6 + 6*c*g^3*h^5*x^5 + 6*a*g*h^7*x^5 + 15*c*g^4*h^4*x^4 + 15*a*g^2*
h^6*x^4 + 20*c*g^5*h^3*x^3 + 20*a*g^3*h^5*x^3 + 15*c*g^6*h^2*x^2 + 15*a*g^4
*h^4*x^2 + 6*c*g^7*h*x + 6*a*g^5*h^3*x + c*g^8 + a*g^6*h^2) - 1/5*(c*x^2 +
a)^(5/2)*f/(c*g^2*h^6*x^5 + a*h^8*x^5 + 5*c*g^3*h^5*x^4 + 5*a*g*h^7*x^4 + 1
0*c*g^4*h^4*x^3 + 10*a*g^2*h^6*x^3 + 10*c*g^5*h^3*x^2 + 10*a*g^3*h^5*x^2 +
5*c*g^6*h^2*x + 5*a*g^4*h^4*x + c*g^7*h + a*g^5*h^3) + 9/16*c^7*f*g^9*arcsi
nh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g
^2/h^2)^(11/2)*h^17) - 9/16*c^7*e*g^8*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)
) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(11/2)*h^16) + 9/16*c^7*
d*g^7*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g))
)/((a + c*g^2/h^2)^(11/2)*h^15) - 35/16*c^6*f*g^7*arcsinh(c*g*x/(sqrt(a*c)*
abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(9/2)*h^15)
+ 7/4*c^6*e*g^6*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs
(h*x + g)))/((a + c*g^2/h^2)^(9/2)*h^14) - 21/16*c^6*d*g^5*arcsinh(c*g*x/(s
qrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(9/
2)*h^13) + 51/16*c^5*f*g^5*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sq
rt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(7/2)*h^13) - 15/8*c^5*e*g^4*arcsin
h(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^
2/h^2)^(7/2)*h^12) + 15/16*c^5*d*g^3*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g))
 - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(7/2)*h^11) - 33/16*c^4*f
*g^3*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))
/((a + c*g^2/h^2)^(5/2)*h^11) + 3/4*c^4*e*g^2*arcsinh(c*g*x/(sqrt(a*c)*abs(
h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(5/2)*h^10) - 3/
16*c^4*d*g*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x
+ g)))/((a + c*g^2/h^2)^(5/2)*h^9) + 1/2*c^3*f*g*arcsinh(c*g*x/(sqrt(a*c)*a
bs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(3/2)*h^9) -
1/16*c^3*e*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x
+ g)))/((a + c*g^2/h^2)^(3/2)*h^8)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7857 vs. $2(505) = 1010$.

Time = 0.47 (sec) , antiderivative size = 7857, normalized size of antiderivative = 14.77

$$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^8}\,dx = \text{Too large to display}$$

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^8,x, algorithm="giac")
```

```
[Out] -1/8*(6*a^2*c^5*d*g^3 - a^3*c^4*f*g^3 + 8*a^3*c^4*e*g^2*h - 3*a^3*c^4*d*g*h
^2 + 8*a^4*c^3*f*g*h^2 - a^4*c^3*e*h^3)*arctan(((sqrt(c)*x - sqrt(c*x^2 + a
))*h + sqrt(c)*g)/sqrt(-c*g^2 - a*h^2))/((c^5*g^10 + 5*a*c^4*g^8*h^2 + 10*a
^2*c^3*g^6*h^4 + 10*a^3*c^2*g^4*h^6 + 5*a^4*c*g^2*h^8 + a^5*h^10)*sqrt(-c*g
^2 - a*h^2)) - 1/840*(630*(sqrt(c)*x - sqrt(c*x^2 + a))^13*a^2*c^5*d*g^3*h^
12 - 105*(sqrt(c)*x - sqrt(c*x^2 + a))^13*a^3*c^4*f*g^3*h^12 + 840*(sqrt(c)
*x - sqrt(c*x^2 + a))^13*a^3*c^4*e*g^2*h^13 - 315*(sqrt(c)*x - sqrt(c*x^2 +
 a))^13*a^3*c^4*d*g*h^14 + 840*(sqrt(c)*x - sqrt(c*x^2 + a))^13*a^4*c^3*f*g
*h^14 - 105*(sqrt(c)*x - sqrt(c*x^2 + a))^13*a^4*c^3*e*h^15 - 1680*(sqrt(c)
*x - sqrt(c*x^2 + a))^12*c^(15/2)*f*g^10*h^5 - 8400*(sqrt(c)*x - sqrt(c*x^2
 + a))^12*a*c^(13/2)*f*g^8*h^7 - 16800*(sqrt(c)*x - sqrt(c*x^2 + a))^12*a^2
*c^(11/2)*f*g^6*h^9 + 8190*(sqrt(c)*x - sqrt(c*x^2 + a))^12*a^2*c^(11/2)*d*
g^4*h^11 - 18165*(sqrt(c)*x - sqrt(c*x^2 + a))^12*a^3*c^(9/2)*f*g^4*h^11 +
10920*(sqrt(c)*x - sqrt(c*x^2 + a))^12*a^3*c^(9/2)*e*g^3*h^12 - 4095*(sqrt(
c)*x - sqrt(c*x^2 + a))^12*a^3*c^(9/2)*d*g^2*h^13 + 2520*(sqrt(c)*x - sqrt(
c*x^2 + a))^12*a^4*c^(7/2)*f*g^2*h^13 - 1365*(sqrt(c)*x - sqrt(c*x^2 + a))^
12*a^4*c^(7/2)*e*g*h^14 - 1680*(sqrt(c)*x - sqrt(c*x^2 + a))^12*a^5*c^(5/2)
*f*h^15 - 5600*(sqrt(c)*x - sqrt(c*x^2 + a))^11*c^8*f*g^11*h^4 - 2240*(sqrt
(c)*x - sqrt(c*x^2 + a))^11*c^8*e*g^10*h^5 - 28000*(sqrt(c)*x - sqrt(c*x^2
+ a))^11*a*c^7*f*g^9*h^6 - 11200*(sqrt(c)*x - sqrt(c*x^2 + a))^11*a*c^7*e*g
^8*h^7 - 56000*(sqrt(c)*x - sqrt(c*x^2 + a))^11*a^2*c^6*f*g^7*h^8 - 22400*(
sqrt(c)*x - sqrt(c*x^2 + a))^11*a^2*c^6*e*g^6*h^9 + 44940*(sqrt(c)*x - sqrt
(c*x^2 + a))^11*a^2*c^6*d*g^5*h^10 - 63490*(sqrt(c)*x - sqrt(c*x^2 + a))^11
*a^3*c^5*f*g^5*h^10 + 37520*(sqrt(c)*x - sqrt(c*x^2 + a))^11*a^3*c^5*e*g^4*
h^11 - 26670*(sqrt(c)*x - sqrt(c*x^2 + a))^11*a^3*c^5*d*g^3*h^12 + 32620*(s
qrt(c)*x - sqrt(c*x^2 + a))^11*a^4*c^4*f*g^3*h^12 - 24290*(sqrt(c)*x - sqrt
(c*x^2 + a))^11*a^4*c^4*e*g^2*h^13 + 2100*(sqrt(c)*x - sqrt(c*x^2 + a))^11*
a^4*c^4*d*g*h^14 - 11200*(sqrt(c)*x - sqrt(c*x^2 + a))^11*a^5*c^3*f*g*h^14
- 1540*(sqrt(c)*x - sqrt(c*x^2 + a))^11*a^5*c^3*e*h^15 - 11200*(sqrt(c)*x -
 sqrt(c*x^2 + a))^10*c^(17/2)*f*g^12*h^3 - 4480*(sqrt(c)*x - sqrt(c*x^2 + a
))^10*c^(17/2)*e*g^11*h^4 - 3360*(sqrt(c)*x - sqrt(c*x^2 + a))^10*c^(17/2)*
d*g^10*h^5 - 52640*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a*c^(15/2)*f*g^10*h^5 -
 22400*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a*c^(15/2)*e*g^9*h^6 - 16800*(sqrt(
c)*x - sqrt(c*x^2 + a))^10*a*c^(15/2)*d*g^8*h^7 - 95200*(sqrt(c)*x - sqrt(c
*x^2 + a))^10*a^2*c^(13/2)*f*g^8*h^7 - 44800*(sqrt(c)*x - sqrt(c*x^2 + a))^
10*a^2*c^(13/2)*e*g^7*h^8 + 100380*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a^2*c^(
13/2)*d*g^6*h^9 - 100730*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a^3*c^(11/2)*f*g^
6*h^9 + 133840*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a^3*c^(11/2)*e*g^5*h^10 - 1
46790*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a^3*c^(11/2)*d*g^4*h^11 + 163940*(sq
rt(c)*x - sqrt(c*x^2 + a))^10*a^4*c^(9/2)*f*g^4*h^11 - 106330*(sqrt(c)*x -
sqrt(c*x^2 + a))^10*a^4*c^(9/2)*e*g^3*h^12 + 6300*(sqrt(c)*x - sqrt(c*x^2 +
 a))^10*a^4*c^(9/2)*d*g^2*h^13 - 56000*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a^5
*c^(7/2)*f*g^2*h^13 + 3220*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a^5*c^(7/2)*e*g
*h^14 - 3360*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a^5*c^(7/2)*d*h^15 + 3360*(sq
rt(c)*x - sqrt(c*x^2 + a))^10*a^6*c^(5/2)*f*h^15 - 13440*(sqrt(c)*x - sqrt(
```

```
c*x^2 + a))^9*c^9*f*g^13*h^2 - 5376*(sqrt(c)*x - sqrt(c*x^2 + a))^9*c^9*e*g
^12*h^3 - 4032*(sqrt(c)*x - sqrt(c*x^2 + a))^9*c^9*d*g^11*h^4 - 50848*(sqrt
(c)*x - sqrt(c*x^2 + a))^9*a*c^8*f*g^11*h^4 - 25984*(sqrt(c)*x - sqrt(c*x^2
 + a))^9*a*c^8*e*g^10*h^5 - 20160*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a*c^8*d*g
^9*h^6 - 52640*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^2*c^7*f*g^9*h^6 - 49280*(s
qrt(c)*x - sqrt(c*x^2 + a))^9*a^2*c^7*e*g^8*h^7 + 191016*(sqrt(c)*x - sqrt(
c*x^2 + a))^9*a^2*c^7*d*g^7*h^8 - 9436*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^3*
c^6*f*g^7*h^8 + 263648*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^3*c^6*e*g^6*h^9 -
363216*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^3*c^6*d*g^5*h^10 + 439306*(sqrt(c)
*x - sqrt(c*x^2 + a))^9*a^4*c^5*f*g^5*h^10 - 332780*(sqrt(c)*x - sqrt(c*x^2
 + a))^9*a^4*c^5*e*g^4*h^11 + 95340*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^4*c^5
*d*g^3*h^12 - 209965*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^5*c^4*f*g^3*h^12 + 4
9490*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^5*c^4*e*g^2*h^13 - 9975*(sqrt(c)*x -
 sqrt(c*x^2 + a))^9*a^5*c^4*d*g*h^14 + 32200*(sqrt(c)*x - sqrt(c*x^2 + a))^
9*a^6*c^3*f*g*h^14 - 1085*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^6*c^3*e*h^15 -
8960*(sqrt(c)*x - sqrt(c*x^2 + a))^8*c^(19/2)*f*g^14*h - 3584*(sqrt(c)*x -
sqrt(c*x^2 + a))^8*c^(19/2)*e*g^13*h^2 - 2688*(sqrt(c)*x - sqrt(c*x^2 + a))
^8*c^(19/2)*d*g^12*h^3 - 15232*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a*c^(17/2)*f
*g^12*h^3 - 9856*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a*c^(17/2)*e*g^11*h^4 - 16
800*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a*c^(17/2)*d*g^10*h^5 + 53200*(sqrt(c)*
x - sqrt(c*x^2 + a))^8*a^2*c^(15/2)*f*g^10*h^5 + 4480*(sqrt(c)*x - sqrt(c*x
^2 + a))^8*a^2*c^(15/2)*e*g^9*h^6 + 181104*(sqrt(c)*x - sqrt(c*x^2 + a))^8*
a^2*c^(15/2)*d*g^8*h^7 + 143416*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^3*c^(13/2
)*f*g^8*h^7 + 344512*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^3*c^(13/2)*e*g^7*h^8
 - 651924*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^3*c^(13/2)*d*g^6*h^9 + 580034*(
sqrt(c)*x - sqrt(c*x^2 + a))^8*a^4*c^(11/2)*f*g^6*h^9 - 613480*(sqrt(c)*x -
 sqrt(c*x^2 + a))^8*a^4*c^(11/2)*e*g^5*h^10 + 299460*(sqrt(c)*x - sqrt(c*x^
2 + a))^8*a^4*c^(11/2)*d*g^4*h^11 - 568085*(sqrt(c)*x - sqrt(c*x^2 + a))^8*
a^5*c^(9/2)*f*g^4*h^11 + 259210*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^5*c^(9/2)
*e*g^3*h^12 - 72975*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^5*c^(9/2)*d*g^2*h^13
+ 147000*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^6*c^(7/2)*f*g^2*h^13 - 9765*(sqr
t(c)*x - sqrt(c*x^2 + a))^8*a^6*c^(7/2)*e*g*h^14 - 3360*(sqrt(c)*x - sqrt(c
*x^2 + a))^8*a^6*c^(7/2)*d*h^15 - 5040*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^7*
c^(5/2)*f*h^15 - 2560*(sqrt(c)*x - sqrt(c*x^2 + a))^7*c^10*f*g^15 - 1024*(s
qrt(c)*x - sqrt(c*x^2 + a))^7*c^10*e*g^14*h - 768*(sqrt(c)*x - sqrt(c*x^2 +
 a))^7*c^10*d*g^13*h^2 + 12928*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a*c^9*f*g^13
*h^2 + 4096*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a*c^9*e*g^12*h^3 + 384*(sqrt(c)
*x - sqrt(c*x^2 + a))^7*a*c^9*d*g^11*h^4 + 80576*(sqrt(c)*x - sqrt(c*x^2 +
a))^7*a^2*c^8*f*g^11*h^4 + 32768*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^2*c^8*e*
g^10*h^5 + 117984*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^2*c^8*d*g^9*h^6 + 10193
6*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^3*c^7*f*g^9*h^6 + 205952*(sqrt(c)*x - s
qrt(c*x^2 + a))^7*a^3*c^7*e*g^8*h^7 - 603216*(sqrt(c)*x - sqrt(c*x^2 + a))^
7*a^3*c^7*d*g^7*h^8 + 256816*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^4*c^6*f*g^7*
h^8 - 741776*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^4*c^6*e*g^6*h^9 + 703752*(sq
rt(c)*x - sqrt(c*x^2 + a))^7*a^4*c^6*d*g^5*h^10 - 941332*(sqrt(c)*x - sqrt(
```

```
c*x^2 + a))^7*a^5*c^5*f*g^5*h^10 + 608720*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a
^5*c^5*e*g^4*h^11 - 184380*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^5*c^5*d*g^3*h^
12 + 413280*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^6*c^4*f*g^3*h^12 - 92820*(sqr
t(c)*x - sqrt(c*x^2 + a))^7*a^6*c^4*e*g^2*h^13 + 13440*(sqrt(c)*x - sqrt(c*
x^2 + a))^7*a^6*c^4*d*g*h^14 - 47040*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^7*c^
3*f*g*h^14 + 8960*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a*c^(19/2)*f*g^14*h + 358
4*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a*c^(19/2)*e*g^13*h^2 + 2688*(sqrt(c)*x -
 sqrt(c*x^2 + a))^6*a*c^(19/2)*d*g^12*h^3 + 15232*(sqrt(c)*x - sqrt(c*x^2 +
 a))^6*a^2*c^(17/2)*f*g^12*h^3 + 9856*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^2*c
^(17/2)*e*g^11*h^4 + 16800*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^2*c^(17/2)*d*g
^10*h^5 - 53200*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^3*c^(15/2)*f*g^10*h^5 + 8
960*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^3*c^(15/2)*e*g^9*h^6 - 342384*(sqrt(c
)*x - sqrt(c*x^2 + a))^6*a^3*c^(15/2)*d*g^8*h^7 - 103936*(sqrt(c)*x - sqrt(
c*x^2 + a))^6*a^4*c^(13/2)*f*g^8*h^7 - 487312*(sqrt(c)*x - sqrt(c*x^2 + a))
^6*a^4*c^(13/2)*e*g^7*h^8 + 736344*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^4*c^(1
3/2)*d*g^6*h^9 - 726404*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^5*c^(11/2)*f*g^6*
h^9 + 807520*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^5*c^(11/2)*e*g^5*h^10 - 4884
60*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^5*c^(11/2)*d*g^4*h^11 + 764960*(sqrt(c
)*x - sqrt(c*x^2 + a))^6*a^6*c^(9/2)*f*g^4*h^11 - 310660*(sqrt(c)*x - sqrt(
c*x^2 + a))^6*a^6*c^(9/2)*e*g^3*h^12 + 33600*(sqrt(c)*x - sqrt(c*x^2 + a))^
6*a^6*c^(9/2)*d*g^2*h^13 - 168000*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^7*c^(7/
2)*f*g^2*h^13 + 13440*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^7*c^(7/2)*e*g*h^14
- 6720*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^7*c^(7/2)*d*h^15 + 6720*(sqrt(c)*x
 - sqrt(c*x^2 + a))^6*a^8*c^(5/2)*f*h^15 - 13440*(sqrt(c)*x - sqrt(c*x^2 +
a))^5*a^2*c^9*f*g^13*h^2 - 5376*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^2*c^9*e*g
^12*h^3 - 4032*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^2*c^9*d*g^11*h^4 - 50848*(
sqrt(c)*x - sqrt(c*x^2 + a))^5*a^3*c^8*f*g^11*h^4 - 25984*(sqrt(c)*x - sqrt
(c*x^2 + a))^5*a^3*c^8*e*g^10*h^5 - 47040*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a
^3*c^8*d*g^9*h^6 - 50960*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^4*c^7*f*g^9*h^6
- 86240*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^4*c^7*e*g^8*h^7 + 438816*(sqrt(c)
*x - sqrt(c*x^2 + a))^5*a^4*c^7*d*g^7*h^8 - 99736*(sqrt(c)*x - sqrt(c*x^2 +
 a))^5*a^5*c^6*f*g^7*h^8 + 574448*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^5*c^6*e
*g^6*h^9 - 556416*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^5*c^6*d*g^5*h^10 + 7287
56*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^6*c^5*f*g^5*h^10 - 487480*(sqrt(c)*x -
 sqrt(c*x^2 + a))^5*a^6*c^5*e*g^4*h^11 + 167790*(sqrt(c)*x - sqrt(c*x^2 + a
))^5*a^6*c^5*d*g^3*h^12 - 362915*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^7*c^4*f*
g^3*h^12 + 89740*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^7*c^4*e*g^2*h^13 - 10185
*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^7*c^4*d*g*h^14 + 38360*(sqrt(c)*x - sqrt
(c*x^2 + a))^5*a^8*c^3*f*g*h^14 + 1085*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^8*
c^3*e*h^15 + 11200*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^3*c^(17/2)*f*g^12*h^3
+ 4480*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^3*c^(17/2)*e*g^11*h^4 + 3360*(sqrt
(c)*x - sqrt(c*x^2 + a))^4*a^3*c^(17/2)*d*g^10*h^5 + 52640*(sqrt(c)*x - sqr
t(c*x^2 + a))^4*a^4*c^(15/2)*f*g^10*h^5 + 29120*(sqrt(c)*x - sqrt(c*x^2 + a
))^4*a^4*c^(15/2)*e*g^9*h^6 + 45360*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^4*c^(
15/2)*d*g^8*h^7 + 96880*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^5*c^(13/2)*f*g^8*
```

```
h^7 + 119056*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^5*c^(13/2)*e*g^7*h^8 - 36472
8*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^5*c^(13/2)*d*g^6*h^9 + 215908*(sqrt(c)*
x - sqrt(c*x^2 + a))^4*a^6*c^(11/2)*f*g^6*h^9 - 390656*(sqrt(c)*x - sqrt(c*
x^2 + a))^4*a^6*c^(11/2)*e*g^5*h^10 + 220710*(sqrt(c)*x - sqrt(c*x^2 + a))^
4*a^6*c^(11/2)*d*g^4*h^11 - 406735*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^7*c^(9
/2)*f*g^4*h^11 + 179900*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^7*c^(9/2)*e*g^3*h
^12 - 49581*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^7*c^(9/2)*d*g^2*h^13 + 104776
*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^8*c^(7/2)*f*g^2*h^13 - 10703*(sqrt(c)*x
 - sqrt(c*x^2 + a))^4*a^8*c^(7/2)*e*g*h^14 - 1344*(sqrt(c)*x - sqrt(c*x^2 +
a))^4*a^8*c^(7/2)*d*h^15 - 3696*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^9*c^(5/2)
*f*h^15 - 5600*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^4*c^8*f*g^11*h^4 - 2240*(s
qrt(c)*x - sqrt(c*x^2 + a))^3*a^4*c^8*e*g^10*h^5 - 3360*(sqrt(c)*x - sqrt(c
*x^2 + a))^3*a^4*c^8*d*g^9*h^6 - 29680*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^5*
c^7*f*g^9*h^6 - 16576*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^5*c^7*e*g^8*h^7 - 3
2592*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^5*c^7*d*g^7*h^8 - 67088*(sqrt(c)*x -
 sqrt(c*x^2 + a))^3*a^6*c^6*f*g^7*h^8 - 72464*(sqrt(c)*x - sqrt(c*x^2 + a))
^3*a^6*c^6*e*g^6*h^9 + 172620*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^6*c^6*d*g^5
*h^10 - 156170*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^7*c^5*f*g^5*h^10 + 179200*
(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^7*c^5*e*g^4*h^11 - 62454*(sqrt(c)*x - sqr
t(c*x^2 + a))^3*a^7*c^5*d*g^3*h^12 + 140084*(sqrt(c)*x - sqrt(c*x^2 + a))^3
*a^8*c^4*f*g^3*h^12 - 31402*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^8*c^4*e*g^2*h
^13 + 5964*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^8*c^4*d*g*h^14 - 17024*(sqrt(c
)*x - sqrt(c*x^2 + a))^3*a^9*c^3*f*g*h^14 + 1540*(sqrt(c)*x - sqrt(c*x^2 +
a))^3*a^9*c^3*e*h^15 + 1680*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^5*c^(15/2)*f*
g^10*h^5 + 1344*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^5*c^(15/2)*e*g^9*h^6 + 10
08*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^5*c^(15/2)*d*g^8*h^7 + 9632*(sqrt(c)*x
 - sqrt(c*x^2 + a))^2*a^6*c^(13/2)*f*g^8*h^7 + 8624*(sqrt(c)*x - sqrt(c*x^2
 + a))^2*a^6*c^(13/2)*e*g^7*h^8 + 9996*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^6*
c^(13/2)*d*g^6*h^9 + 24094*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^7*c^(11/2)*f*g
^6*h^9 + 30352*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^7*c^(11/2)*e*g^5*h^10 - 54
894*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^7*c^(11/2)*d*g^4*h^11 + 56924*(sqrt(c
)*x - sqrt(c*x^2 + a))^2*a^8*c^(9/2)*f*g^4*h^11 - 47362*(sqrt(c)*x - sqrt(c
*x^2 + a))^2*a^8*c^(9/2)*e*g^3*h^12 + 9156*(sqrt(c)*x - sqrt(c*x^2 + a))^2*
a^8*c^(9/2)*d*g^2*h^13 - 32256*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^9*c^(7/2)*
f*g^2*h^13 + 3276*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^9*c^(7/2)*e*g*h^14 - 67
2*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^9*c^(7/2)*d*h^15 + 672*(sqrt(c)*x - sqr
t(c*x^2 + a))^2*a^10*c^(5/2)*f*h^15 - 560*(sqrt(c)*x - sqrt(c*x^2 + a))*a^6
*c^7*f*g^9*h^6 - 224*(sqrt(c)*x - sqrt(c*x^2 + a))*a^6*c^7*e*g^8*h^7 - 168*
(sqrt(c)*x - sqrt(c*x^2 + a))*a^6*c^7*d*g^7*h^8 - 3052*(sqrt(c)*x - sqrt(c*
x^2 + a))*a^7*c^6*f*g^7*h^8 - 1456*(sqrt(c)*x - sqrt(c*x^2 + a))*a^7*c^6*e*
g^6*h^9 - 1680*(sqrt(c)*x - sqrt(c*x^2 + a))*a^7*c^6*d*g^5*h^10 - 7070*(sqr
t(c)*x - sqrt(c*x^2 + a))*a^8*c^5*f*g^5*h^10 - 5180*(sqrt(c)*x - sqrt(c*x^2
 + a))*a^8*c^5*e*g^4*h^11 + 9744*(sqrt(c)*x - sqrt(c*x^2 + a))*a^8*c^5*d*g^
3*h^12 - 12999*(sqrt(c)*x - sqrt(c*x^2 + a))*a^9*c^4*f*g^3*h^12 + 8442*(sqr
t(c)*x - sqrt(c*x^2 + a))*a^9*c^4*e*g^2*h^13 - 1029*(sqrt(c)*x - sqrt(c*x^2
```

```
 + a))*a^9*c^4*d*g*h^14 + 3864*(sqrt(c)*x - sqrt(c*x^2 + a))*a^10*c^3*f*g*h
^14 + 105*(sqrt(c)*x - sqrt(c*x^2 + a))*a^10*c^3*e*h^15 + 40*a^7*c^(13/2)*f
*g^8*h^7 + 16*a^7*c^(13/2)*e*g^7*h^8 + 12*a^7*c^(13/2)*d*g^6*h^9 + 218*a^8*
c^(11/2)*f*g^6*h^9 + 104*a^8*c^(11/2)*e*g^5*h^10 + 120*a^8*c^(11/2)*d*g^4*h
^11 + 505*a^9*c^(9/2)*f*g^4*h^11 + 370*a^9*c^(9/2)*e*g^3*h^12 - 741*a^9*c^(
9/2)*d*g^2*h^13 + 936*a^10*c^(7/2)*f*g^2*h^13 - 663*a^10*c^(7/2)*e*g*h^14 +
 96*a^10*c^(7/2)*d*h^15 - 336*a^11*c^(5/2)*f*h^15)/((c^5*g^10*h^6 + 5*a*c^4
*g^8*h^8 + 10*a^2*c^3*g^6*h^10 + 10*a^3*c^2*g^4*h^12 + 5*a^4*c*g^2*h^14 + a
^5*h^16)*((sqrt(c)*x - sqrt(c*x^2 + a))^2*h + 2*(sqrt(c)*x - sqrt(c*x^2 + a
))*sqrt(c)*g - a*h)^7)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^8}\,dx=\int \frac{{\left(c\,x^2+a\right)}^{3/2}\,\left(f\,x^2+e\,x+d\right)}{{\left(g+h\,x\right)}^8}\,dx$$

```
[In] int(((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^8,x)
[Out] int(((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^8, x)
```

3.100 $\int \left(a+cx^2\right)^{5/2}\left(A+Bx+Cx^2\right)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 168

$$\int \left(a+cx^2\right)^{5/2}\left(A+Bx+Cx^2\right)\,dx=\frac{5a^2(8Ac-aC)x\sqrt{a+cx^2}}{128c}+\frac{5a(8Ac-aC)x\left(a+cx^2\right)^{3/2}}{192c}+\frac{(8Ac-aC)x\left(a+cx^2\right)^{5/2}}{48c}+\frac{B\left(a+cx^2\right)^{7/2}}{7c}+\frac{Cx\left(a+cx^2\right)^{7/2}}{8c}+\frac{5a^3(8Ac-aC)\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{128c^{3/2}}$$

```
[Out] 5/192*a*(8*A*c-C*a)*x*(c*x^2+a)^(3/2)/c+1/48*(8*A*c-C*a)*x*(c*x^2+a)^(5/2)/
c+1/7*B*(c*x^2+a)^(7/2)/c+1/8*C*x*(c*x^2+a)^(7/2)/c+5/128*a^3*(8*A*c-C*a)*a
rctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(3/2)+5/128*a^2*(8*A*c-C*a)*x*(c*x^2+a)
^(1/2)/c
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1829, 655, 201, 223, 212}

$$\int \left(a+cx^2\right)^{5/2}\left(A+Bx+Cx^2\right)\,dx=\frac{5a^3(8Ac-aC)\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{128c^{3/2}}+\frac{5a^2x\sqrt{a+cx^2}(8Ac-aC)}{128c}+\frac{x\left(a+cx^2\right)^{5/2}(8Ac-aC)}{48c}+\frac{5ax\left(a+cx^2\right)^{3/2}(8Ac-aC)}{192c}+\frac{B\left(a+cx^2\right)^{7/2}}{7c}+\frac{Cx\left(a+cx^2\right)^{7/2}}{8c}$$

```
[In] Int[(a + c*x^2)^(5/2)*(A + B*x + C*x^2),x]
```

```
[Out] (5*a^2*(8*A*c - a*C)*x*Sqrt[a + c*x^2])/(128*c) + (5*a*(8*A*c - a*C)*x*(a +
 c*x^2)^(3/2))/(192*c) + ((8*A*c - a*C)*x*(a + c*x^2)^(5/2))/(48*c) + (B*(a
 + c*x^2)^(7/2))/(7*c) + (C*x*(a + c*x^2)^(7/2))/(8*c) + (5*a^3*(8*A*c - a*
C)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(128*c^(3/2))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1829

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{Cx(a+cx^2)^{7/2}}{8c}+\frac{\int (8Ac-aC+8Bcx)\left(a+cx^2\right)^{5/2}\,dx}{8c}\\ &= \frac{B(a+cx^2)^{7/2}}{7c}+\frac{Cx(a+cx^2)^{7/2}}{8c}+\frac{(8Ac-aC)\int \left(a+cx^2\right)^{5/2}\,dx}{8c}\end{aligned}$$

$$= \frac{(8Ac-aC)x(a+cx^2)^{5/2}}{48c} + \frac{B(a+cx^2)^{7/2}}{7c} + \frac{Cx(a+cx^2)^{7/2}}{8c} + \frac{(5a(8Ac-aC))\int (a+cx^2)^{3/2}\, dx}{48c}$$

$$= \frac{5a(8Ac-aC)x(a+cx^2)^{3/2}}{192c} + \frac{(8Ac-aC)x(a+cx^2)^{5/2}}{48c} + \frac{B(a+cx^2)^{7/2}}{7c} + \frac{Cx(a+cx^2)^{7/2}}{8c} + \frac{(5a^2(8Ac-aC))\int \sqrt{a+cx^2}\, dx}{64c}$$

$$= \frac{5a^2(8Ac-aC)x\sqrt{a+cx^2}}{128c} + \frac{5a(8Ac-aC)x(a+cx^2)^{3/2}}{192c} + \frac{(8Ac-aC)x(a+cx^2)^{5/2}}{48c} + \frac{B(a+cx^2)^{7/2}}{7c} + \frac{Cx(a+cx^2)^{7/2}}{8c} + \frac{(5a^3(8Ac-aC))\int \frac{1}{\sqrt{a+cx^2}}\, dx}{128c}$$

$$= \frac{5a^2(8Ac-aC)x\sqrt{a+cx^2}}{128c} + \frac{5a(8Ac-aC)x(a+cx^2)^{3/2}}{192c} + \frac{(8Ac-aC)x(a+cx^2)^{5/2}}{48c} + \frac{B(a+cx^2)^{7/2}}{7c} + \frac{Cx(a+cx^2)^{7/2}}{8c} + \frac{(5a^3(8Ac-aC))\,\text{Subst}\left(\int \frac{1}{1-cx^2}\, dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{128c}$$

$$= \frac{5a^2(8Ac-aC)x\sqrt{a+cx^2}}{128c} + \frac{5a(8Ac-aC)x(a+cx^2)^{3/2}}{192c} + \frac{(8Ac-aC)x(a+cx^2)^{5/2}}{48c} + \frac{B(a+cx^2)^{7/2}}{7c} + \frac{Cx(a+cx^2)^{7/2}}{8c} + \frac{5a^3(8Ac-aC)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{128c^{3/2}}$$

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.85

$$\int \left(a+cx^2\right)^{5/2} \left(A+Bx+Cx^2\right)\, dx = \frac{\sqrt{c}\sqrt{a+cx^2}(3a^3(128B+35Cx)+16c^3x^5(28A+3x(8B+7Cx))+8ac^2x^3(182A+x(144B+1}{2688c}$$

```
[In] Integrate[(a + c*x^2)^(5/2)*(A + B*x + C*x^2),x]
```

```
[Out] (Sqrt[c]*Sqrt[a + c*x^2]*(3*a^3*(128*B + 35*C*x) + 16*c^3*x^5*(28*A + 3*x*(
8*B + 7*C*x)) + 8*a*c^2*x^3*(182*A + x*(144*B + 119*C*x)) + 2*a^2*c*x*(924*
A + x*(576*B + 413*C*x))) + 105*a^3*(-8*A*c + a*C)*Log[-(Sqrt[c]*x) + Sqrt[
a + c*x^2]])/(2688*c^(3/2))
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.88

method	result
risch	$\frac{\left(336c^3C\,x^7+384B\,c^3x^6+448A\,c^3x^5+952a\,c^2C\,x^5+1152aB\,c^2x^4+1456aA\,c^2x^3+826C\,a^2c\,x^3+1152a^2Bc\,x^2+1848a^2Acx+105a^3Cx}{2688c}$
default	$A\left(\frac{x\left(c\,x^2+a\right)^{\frac{5}{2}}}{6}+\frac{5a\left(\frac{x\left(c\,x^2+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{c\,x^2+a}}{2}+\frac{a\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{2\sqrt{c}}\right)}{4}\right)}{6}\right)+C\left(\frac{x\left(c\,x^2+a\right)^{\frac{7}{2}}}{8c}-\frac{a\left(\frac{x\left(c\,x^2+a\right)^{\frac{5}{2}}}{6}+\frac{5a}{}\right.}{}\right.$

```
[In] int((c*x^2+a)^(5/2)*(C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2688/c*(336*C*c^3*x^7+384*B*c^3*x^6+448*A*c^3*x^5+952*C*a*c^2*x^5+1152*B*
a*c^2*x^4+1456*A*a*c^2*x^3+826*C*a^2*c*x^3+1152*B*a^2*c*x^2+1848*A*a^2*c*x+
105*C*a^3*x+384*B*a^3)*(c*x^2+a)^(1/2)+5/128*a^3*(8*A*c-C*a)/c^(3/2)*ln(x*c
^(1/2)+(c*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.98

$$\int \left(a+cx^2\right)^{5/2}\left(A+Bx+Cx^2\right)\,dx=\left[-\frac{105\left(Ca^4-8\,Aa^3c\right)\sqrt{c}\log\left(-2\,cx^2-2\,\sqrt{cx^2+a}\sqrt{c}x-a\right)-2\left(336\,Cc^4x^7+384\,Bc^4x^6+11\right.}{}\right.$$

```
[In] integrate((c*x^2+a)^(5/2)*(C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] [-1/5376*(105*(C*a^4 - 8*A*a^3*c)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*
sqrt(c)*x - a) - 2*(336*C*c^4*x^7 + 384*B*c^4*x^6 + 1152*B*a*c^3*x^4 + 1152
*B*a^2*c^2*x^2 + 56*(17*C*a*c^3 + 8*A*c^4)*x^5 + 384*B*a^3*c + 14*(59*C*a^2
*c^2 + 104*A*a*c^3)*x^3 + 21*(5*C*a^3*c + 88*A*a^2*c^2)*x)*sqrt(c*x^2 + a))
```

```
/c^2, 1/2688*(105*(C*a^4 - 8*A*a^3*c)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2
 + a)) + (336*C*c^4*x^7 + 384*B*c^4*x^6 + 1152*B*a*c^3*x^4 + 1152*B*a^2*c^2
*x^2 + 56*(17*C*a*c^3 + 8*A*c^4)*x^5 + 384*B*a^3*c + 14*(59*C*a^2*c^2 + 104
*A*a*c^3)*x^3 + 21*(5*C*a^3*c + 88*A*a^2*c^2)*x)*sqrt(c*x^2 + a))/c^2]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 328 vs. $2(150) = 300$.

Time = 0.48 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.95

$$\int \left(a+cx^2\right)^{5/2}\left(A+Bx+Cx^2\right)\,dx = \begin{cases} \sqrt{a+cx^2}\left(\frac{Ba^3}{7c}+\frac{3Ba^2x^2}{7}+\frac{3Bacx^4}{7}+\frac{Bc^2x^6}{7}+\frac{Cc^2x^7}{8}+\frac{x^5\left(Ac^3+\frac{17Cac^2}{8}\right)}{6c}+\frac{x^3\cdot\left(3Aac^2+3Ca^2c-\frac{5a\left(Ac^3+17C\right.}{6c}\right.}{4c}\right. \\ a^{\frac{5}{2}}\left(Ax+\frac{Bx^2}{2}+\frac{Cx^3}{3}\right) \end{cases}$$

```
[In] integrate((c*x**2+a)**(5/2)*(C*x**2+B*x+A),x)
```

```
[Out] Piecewise((sqrt(a + c*x**2)*(B*a**3/(7*c) + 3*B*a**2*x**2/7 + 3*B*a*c*x**4/
7 + B*c**2*x**6/7 + C*c**2*x**7/8 + x**5*(A*c**3 + 17*C*a*c**2/8)/(6*c) + x
**3*(3*A*a*c**2 + 3*C*a**2*c - 5*a*(A*c**3 + 17*C*a*c**2/8)/(6*c))/(4*c) +
x*(3*A*a**2*c + C*a**3 - 3*a*(3*A*a*c**2 + 3*C*a**2*c - 5*a*(A*c**3 + 17*C*
a*c**2/8)/(6*c))/(4*c))/(2*c)) + (A*a**3 - a*(3*A*a**2*c + C*a**3 - 3*a*(3*
A*a*c**2 + 3*C*a**2*c - 5*a*(A*c**3 + 17*C*a*c**2/8)/(6*c))/(4*c))/(2*c))*P
iecewise((log(2*sqrt(c)*sqrt(a + c*x**2) + 2*c*x)/sqrt(c), Ne(a, 0)), (x*lo
g(x)/sqrt(c*x**2), True)), Ne(c, 0)), (a**(5/2)*(A*x + B*x**2/2 + C*x**3/3)
, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.99

$$\int \left(a+cx^2\right)^{5/2}\left(A+Bx+Cx^2\right)\,dx = \frac{1}{6}\left(cx^2+a\right)^{\frac{5}{2}}Ax+\frac{5}{24}\left(cx^2+a\right)^{\frac{3}{2}}Aax + \frac{5}{16}\sqrt{cx^2+a}Aa^2x+\frac{\left(cx^2+a\right)^{\frac{7}{2}}Cx}{8\,c}-\frac{\left(cx^2+a\right)^{\frac{5}{2}}Cax}{48\,c}-\frac{5\left(cx^2+a\right)^{\frac{3}{2}}Ca^2x}{192\,c} - \frac{5\sqrt{cx^2+a}Ca^3x}{128\,c}-\frac{5\,Ca^4\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{128\,c^{\frac{3}{2}}}+\frac{5\,Aa^3\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{16\sqrt{c}}+\frac{\left(cx^2+a\right)^{\frac{7}{2}}B}{7\,c}$$

```
[In] integrate((c*x^2+a)^(5/2)*(C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/6*(c*x^2 + a)^(5/2)*A*x + 5/24*(c*x^2 + a)^(3/2)*A*a*x + 5/16*sqrt(c*x^2
+ a)*A*a^2*x + 1/8*(c*x^2 + a)^(7/2)*C*x/c - 1/48*(c*x^2 + a)^(5/2)*C*a*x/c
 - 5/192*(c*x^2 + a)^(3/2)*C*a^2*x/c - 5/128*sqrt(c*x^2 + a)*C*a^3*x/c - 5/
128*C*a^4*arcsinh(c*x/sqrt(a*c))/c^(3/2) + 5/16*A*a^3*arcsinh(c*x/sqrt(a*c)
)/sqrt(c) + 1/7*(c*x^2 + a)^(7/2)*B/c
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.00

$$\int \left(a+cx^2\right)^{5/2}(A+Bx +Cx^2)\,dx = \frac{1}{2688}\Bigg(\frac{384\,Ba^3}{c}+\Bigg(2\Bigg(576\,Ba^2+\Bigg(4\Bigg(144\,Bac+\Bigg(6\left(7\,Cc^2x+8\,Bc^2\right)x+\frac{7\,(17\,Cac^7+8\,Ac^8}{c^6} + \frac{5\left(Ca^4-8\,Aa^3c\right)\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+a}\right|\right)}{128\,c^{\frac{3}{2}}}$$

```
[In] integrate((c*x^2+a)^(5/2)*(C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/2688*(384*B*a^3/c + (2*(576*B*a^2 + (4*(144*B*a*c + (6*(7*C*c^2*x + 8*B*c
^2)*x + 7*(17*C*a*c^7 + 8*A*c^8)/c^6)*x)*x + 7*(59*C*a^2*c^6 + 104*A*a*c^7)
/c^6)*x)*x + 21*(5*C*a^3*c^5 + 88*A*a^2*c^6)/c^6)*x)*sqrt(c*x^2 + a) + 5/12
8*(C*a^4 - 8*A*a^3*c)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+cx^2\right)^{5/2}\left(A+Bx+Cx^2\right)\,dx=\int \left(c\,x^2+a\right)^{5/2}\left(C\,x^2+B\,x+A\right)\,dx$$

```
[In] int((a + c*x^2)^(5/2)*(A + B*x + C*x^2),x)
[Out] int((a + c*x^2)^(5/2)*(A + B*x + C*x^2), x)
```

3.101 $\int \frac{(g+hx)^3(d+ex+fx^2)}{\sqrt{a+cx^2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 325

$$\int \frac{(g+hx)^3\left(d+ex+fx^2\right)}{\sqrt{a+cx^2}}\,dx = \frac{\left(4(5cd-4af)h^2-3cg(fg-5eh)\right)(g+hx)^2\sqrt{a+cx^2}}{60c^2h} - \frac{(fg-5eh)(g+hx)^3\sqrt{a+cx^2}}{20ch} + \frac{f(g+hx)^4\sqrt{a+cx^2}}{5ch} + \frac{(4(16a^2fh^4-4ach^2(13fg^2+5h(3eg+dh))-c^2g^2(3fg^2-5h(3eg+16dh)))-ch(ah^2(71fg+45eh)}{120c^3h} + \frac{(8c^2dg^3+3a^2h^2(3fg+eh)-4acg(fg^2+3h(eg+dh)))\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{8c^{5/2}}$$

```
[Out] 1/8*(8*c^2*d*g^3+3*a^2*h^2*(e*h+3*f*g)-4*a*c*g*(f*g^2+3*h*(d*h+e*g)))*arcta
nh(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(5/2)+1/60*(4*(-4*a*f+5*c*d)*h^2-3*c*g*(-5*
e*h+f*g))*(h*x+g)^2*(c*x^2+a)^(1/2)/c^2/h-1/20*(-5*e*h+f*g)*(h*x+g)^3*(c*x^
2+a)^(1/2)/c/h+1/5*f*(h*x+g)^4*(c*x^2+a)^(1/2)/c/h+1/120*(64*a^2*f*h^4-16*a
*c*h^2*(13*f*g^2+5*h*(d*h+3*e*g))-4*c^2*g^2*(3*f*g^2-5*h*(16*d*h+3*e*g))-c*
h*(a*h^2*(45*e*h+71*f*g)+2*c*g*(3*f*g^2-5*h*(10*d*h+3*e*g)))*x)*(c*x^2+a)^(
1/2)/c^3/h
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 323, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1668, 847, 794, 223, 212}

$$\int \frac{(g+hx)^3\,(d+ex+fx^2)}{\sqrt{a+cx^2}}\,dx$$
$$= \frac{\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)(3a^2h^2(eh+3fg)-4acg(3h(dh+eg)+fg^2)+8c^2dg^3)}{8c^{5/2}}$$
$$+\frac{\sqrt{a+cx^2}(4(16a^2fh^4-4ach^2(5h(dh+3eg)+13fg^2)-c^2g^2(3fg^2-5h(16dh+3eg)))-chx(ah^2(45eh}{120c^3h}$$
$$+\frac{\sqrt{a+cx^2}(g+hx)^2\,(4h^2(5cd-4af)-3cg(fg-5eh))}{60c^2h}$$
$$-\frac{\sqrt{a+cx^2}(g+hx)^3(fg-5eh)}{20ch}+\frac{f\sqrt{a+cx^2}(g+hx)^4}{5ch}$$

```
[In] Int[((g + h*x)^3*(d + e*x + f*x^2))/Sqrt[a + c*x^2],x]
```

```
[Out] ((4*(5*c*d - 4*a*f)*h^2 - 3*c*g*(f*g - 5*e*h))*(g + h*x)^2*Sqrt[a + c*x^2])
/(60*c^2*h) - ((f*g - 5*e*h)*(g + h*x)^3*Sqrt[a + c*x^2])/(20*c*h) + (f*(g
+ h*x)^4*Sqrt[a + c*x^2])/(5*c*h) + ((4*(16*a^2*f*h^4 - 4*a*c*h^2*(13*f*g^2
 + 5*h*(3*e*g + d*h)) - c^2*g^2*(3*f*g^2 - 5*h*(3*e*g + 16*d*h))) - c*h*(6*
c*f*g^3 - 10*c*g*h*(3*e*g + 10*d*h) + a*h^2*(71*f*g + 45*e*h))*x)*Sqrt[a +
c*x^2])/(120*c^3*h) + ((8*c^2*d*g^3 + 3*a^2*h^2*(3*f*g + e*h) - 4*a*c*g*(f*
g^2 + 3*h*(e*g + d*h)))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(8*c^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{f(g+hx)^4\sqrt{a+cx^2}}{5ch}+\frac{\int \frac{(g+hx)^3\left((5cd-4af)h^2-ch(fg-5eh)x\right)}{\sqrt{a+cx^2}}\,dx}{5ch^2}\\
&= -\frac{(fg-5eh)(g+hx)^3\sqrt{a+cx^2}}{20ch}+\frac{f(g+hx)^4\sqrt{a+cx^2}}{5ch}\\
&\quad+\frac{\int \frac{(g+hx)^2\left(ch^2(20cdg-13afg-15aeh)+ch\left(4(5cd-4af)h^2-3cg(fg-5eh)\right)x\right)}{\sqrt{a+cx^2}}\,dx}{20c^2h^2}\\
&= \frac{\left(4(5cd-4af)h^2-3cg(fg-5eh)\right)(g+hx)^2\sqrt{a+cx^2}}{60c^2h}\\
&\quad-\frac{(fg-5eh)(g+hx)^3\sqrt{a+cx^2}}{20ch}+\frac{f(g+hx)^4\sqrt{a+cx^2}}{5ch}\\
&\quad+\frac{\int \frac{(g+hx)\left(ch^2\left(60c^2dg^2+32a^2fh^2-ac\left(33fg^2+5h(15eg+8dh)\right)\right)-c^2h\left(6cfg^3-10cgh(3eg+10dh)+ah^2(71fg+45eh)\right)x\right)}{\sqrt{a+cx^2}}\,dx}{60c^3h^2}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{(4(5cd-4af)h^2-3cg(fg-5eh))\,(g+hx)^2\sqrt{a+cx^2}}{60c^2h} \\
&\quad - \frac{(fg-5eh)(g+hx)^3\sqrt{a+cx^2}}{20ch} + \frac{f(g+hx)^4\sqrt{a+cx^2}}{5ch} \\
&\quad + \frac{(4(16a^2fh^4-4ach^2(13fg^2+5h(3eg+dh))-c^2g^2(3fg^2-5h(3eg+16dh)))-ch(6cfg^3-10cg}{120c^3h} \\
&\quad + \frac{(8c^2dg^3+3a^2h^2(3fg+eh)-4acg(fg^2+3h(eg+dh)))\int\frac{1}{\sqrt{a+cx^2}}\,dx}{8c^2}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{(4(5cd-4af)h^2-3cg(fg-5eh))\,(g+hx)^2\sqrt{a+cx^2}}{60c^2h} \\
&\quad - \frac{(fg-5eh)(g+hx)^3\sqrt{a+cx^2}}{20ch} + \frac{f(g+hx)^4\sqrt{a+cx^2}}{5ch} \\
&\quad + \frac{(4(16a^2fh^4-4ach^2(13fg^2+5h(3eg+dh))-c^2g^2(3fg^2-5h(3eg+16dh)))-ch(6cfg^3-10cg}{120c^3h} \\
&\quad + \frac{(8c^2dg^3+3a^2h^2(3fg+eh)-4acg(fg^2+3h(eg+dh)))\,\text{Subst}\left(\int\frac{1}{1-cx^2}\,dx,x,\frac{x}{\sqrt{a+cx^2}}\right)}{8c^2}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{(4(5cd-4af)h^2-3cg(fg-5eh))\,(g+hx)^2\sqrt{a+cx^2}}{60c^2h} \\
&\quad - \frac{(fg-5eh)(g+hx)^3\sqrt{a+cx^2}}{20ch} + \frac{f(g+hx)^4\sqrt{a+cx^2}}{5ch} \\
&\quad + \frac{(4(16a^2fh^4-4ach^2(13fg^2+5h(3eg+dh))-c^2g^2(3fg^2-5h(3eg+16dh)))-ch(6cfg^3-10cg}{120c^3h} \\
&\quad + \frac{(8c^2dg^3+3a^2h^2(3fg+eh)-4acg(fg^2+3h(eg+dh)))\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{8c^{5/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.75

$$
\int\frac{(g+hx)^3\,(d+ex+fx^2)}{\sqrt{a+cx^2}}\,dx
$$

$$
= \sqrt{a+cx^2}(64a^2fh^3-ach(5h(48eg+16dh+9ehx)+f(240g^2+135ghx+32h^2x^2))+2c^2(10dh(18g^2+9g
$$

```
[In] Integrate[((g + h*x)^3*(d + e*x + f*x^2))/Sqrt[a + c*x^2],x]
```

```
[Out] (Sqrt[a + c*x^2]*(64*a^2*f*h^3 - a*c*h*(5*h*(48*e*g + 16*d*h + 9*e*h*x) + f
*(240*g^2 + 135*g*h*x + 32*h^2*x^2)) + 2*c^2*(10*d*h*(18*g^2 + 9*g*h*x + 2*
h^2*x^2) + 15*e*(4*g^3 + 6*g^2*h*x + 4*g*h^2*x^2 + h^3*x^3) + 3*f*x*(10*g^3
 + 20*g^2*h*x + 15*g*h^2*x^2 + 4*h^3*x^3))) - 15*Sqrt[c]*(8*c^2*d*g^3 + 3*a
^2*h^2*(3*f*g + e*h) - 4*a*c*g*(f*g^2 + 3*h*(e*g + d*h)))*Log[-(Sqrt[c]*x)
+ Sqrt[a + c*x^2]])/(120*c^3)
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 284, normalized size of antiderivative = 0.87

method	result
risch	$\frac{(24 f h^3 c^2 x^4+30 c^2 e h^3 x^3+90 c^2 f g h^2 x^3-32 a c f h^3 x^2+40 c^2 d h^3 x^2+120 c^2 e g h^2 x^2+120 c^2 f g^2 h x^2-45 a c e h^3 x-135 a c f g h^2 x+180 c^2 d}{120 c^3}$
default	$\frac{d g^3 \ln(x\sqrt{c}+\sqrt{c x^2+a})}{\sqrt{c}}+f h^3 \Bigg(\frac{x^4\sqrt{c x^2+a}}{5c}-\frac{4a\left(\frac{x^2\sqrt{c x^2+a}}{3c}-\frac{2a\sqrt{c x^2+a}}{3c^2}\right)}{5c}\Bigg)+(e h^3+3 f g h^2)\Bigg(\frac{x^3\sqrt{c x^2+a}}{4c}-$

```
[In] int((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/120*(24*c^2*f*h^3*x^4+30*c^2*e*h^3*x^3+90*c^2*f*g*h^2*x^3-32*a*c*f*h^3*x^
2+40*c^2*d*h^3*x^2+120*c^2*e*g*h^2*x^2+120*c^2*f*g^2*h*x^2-45*a*c*e*h^3*x-1
35*a*c*f*g*h^2*x+180*c^2*d*g*h^2*x+180*c^2*e*g^2*h*x+60*c^2*f*g^3*x+64*a^2*
f*h^3-80*a*c*d*h^3-240*a*c*e*g*h^2-240*a*c*f*g^2*h+360*c^2*d*g^2*h+120*c^2*
e*g^3)/c^3*(c*x^2+a)^(1/2)+1/8/c^(5/2)*(3*a^2*e*h^3+9*a^2*f*g*h^2-12*a*c*d*
g*h^2-12*a*c*e*g^2*h-4*a*c*f*g^3+8*c^2*d*g^3)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 559, normalized size of antiderivative = 1.72

$$\int \frac{(g+hx)^3\left(d+ex+fx^2\right)}{\sqrt{a+cx^2}}\,dx = \Bigg[-\frac{15\left(12\,aceg^2h-3\,a^2eh^3-4\,(2\,c^2d-acf)g^3+3\,(4\,acd-3\,a^2f)gh^2\right)\sqrt{c}\log\big(-2\,cx^2-2\,\sqrt{cx^2+a}\sqrt{c}x}{}$$

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/240*(15*(12*a*c*e*g^2*h - 3*a^2*e*h^3 - 4*(2*c^2*d - a*c*f)*g^3 + 3*(4*
a*c*d - 3*a^2*f)*g*h^2)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x
- a) - 2*(24*c^2*f*h^3*x^4 + 120*c^2*e*g^3 - 240*a*c*e*g*h^2 + 120*(3*c^2*d
 - 2*a*c*f)*g^2*h - 16*(5*a*c*d - 4*a^2*f)*h^3 + 30*(3*c^2*f*g*h^2 + c^2*e*
h^3)*x^3 + 8*(15*c^2*f*g^2*h + 15*c^2*e*g*h^2 + (5*c^2*d - 4*a*c*f)*h^3)*x^
2 + 15*(4*c^2*f*g^3 + 12*c^2*e*g^2*h - 3*a*c*e*h^3 + 3*(4*c^2*d - 3*a*c*f)*
g*h^2)*x)*sqrt(c*x^2 + a))/c^3, 1/120*(15*(12*a*c*e*g^2*h - 3*a^2*e*h^3 - 4
*(2*c^2*d - a*c*f)*g^3 + 3*(4*a*c*d - 3*a^2*f)*g*h^2)*sqrt(-c)*arctan(sqrt(
-c)*x/sqrt(c*x^2 + a)) + (24*c^2*f*h^3*x^4 + 120*c^2*e*g^3 - 240*a*c*e*g*h^
2 + 120*(3*c^2*d - 2*a*c*f)*g^2*h - 16*(5*a*c*d - 4*a^2*f)*h^3 + 30*(3*c^2*
f*g*h^2 + c^2*e*h^3)*x^3 + 8*(15*c^2*f*g^2*h + 15*c^2*e*g*h^2 + (5*c^2*d -
```

```
4*a*c*f)*h^3)*x^2 + 15*(4*c^2*f*g^3 + 12*c^2*e*g^2*h - 3*a*c*e*h^3 + 3*(4*c
^2*d - 3*a*c*f)*g*h^2)*x)*sqrt(c*x^2 + a))/c^3]
```

Sympy [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.21

$$\int \frac{(g+hx)^3\,(d+ex+fx^2)}{\sqrt{a+cx^2}}\,dx$$
$$= \begin{cases} \sqrt{a+cx^2}\left(\frac{fh^3x^4}{5c} + \frac{x^3(eh^3+3fgh^2)}{4c} + \frac{x^2\left(-\frac{4afh^3}{5c}+dh^3+3egh^2+3fg^2h\right)}{3c} + \frac{x\left(-\frac{3a\left(eh^3+3fgh^2\right)}{4c}+3dgh^2+3eg^2h+fg^3\right)}{2c} + \right. \\ \frac{dg^3x+\frac{fh^3x^6}{6}+\frac{x^5\left(eh^3+3fgh^2\right)}{5}+\frac{x^4\left(dh^3+3egh^2+3fg^2h\right)}{4}+\frac{x^3\cdot\left(3dgh^2+3eg^2h+fg^3\right)}{3}+\frac{x^2\cdot\left(3dg^2h+eg^3\right)}{2}}{\sqrt{a}} \end{cases}$$

```
[In] integrate((h*x+g)**3*(f*x**2+e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((sqrt(a + c*x**2)*(f*h**3*x**4/(5*c) + x**3*(e*h**3 + 3*f*g*h**2)
/(4*c) + x**2*(-4*a*f*h**3/(5*c) + d*h**3 + 3*e*g*h**2 + 3*f*g**2*h)/(3*c)
+ x*(-3*a*(e*h**3 + 3*f*g*h**2)/(4*c) + 3*d*g*h**2 + 3*e*g**2*h + f*g**3)/(
2*c) + (-2*a*(-4*a*f*h**3/(5*c) + d*h**3 + 3*e*g*h**2 + 3*f*g**2*h)/(3*c) +
 3*d*g**2*h + e*g**3)/c) + (-a*(-3*a*(e*h**3 + 3*f*g*h**2)/(4*c) + 3*d*g*h*
*2 + 3*e*g**2*h + f*g**3)/(2*c) + d*g**3)*Piecewise((log(2*sqrt(c)*sqrt(a +
 c*x**2) + 2*c*x)/sqrt(c), Ne(a, 0)), (x*log(x)/sqrt(c*x**2), True)), Ne(c,
 0)), ((d*g**3*x + f*h**3*x**6/6 + x**5*(e*h**3 + 3*f*g*h**2)/5 + x**4*(d*h
**3 + 3*e*g*h**2 + 3*f*g**2*h)/4 + x**3*(3*d*g*h**2 + 3*e*g**2*h + f*g**3)/
3 + x**2*(3*d*g**2*h + e*g**3)/2)/sqrt(a), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.07

$$\begin{aligned}\int \frac{(g+hx)^3\,(d+ex+fx^2)}{\sqrt{a+cx^2}}\,dx =\ & \frac{\sqrt{cx^2+a}fh^3x^4}{5\,c}-\frac{4\sqrt{cx^2+a}afh^3x^2}{15\,c^2}\\ & +\frac{dg^3\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{c}}+\frac{\sqrt{cx^2+a}eg^3}{c}+\frac{3\sqrt{cx^2+a}dg^2h}{c}\\ & +\frac{8\sqrt{cx^2+a}a^2fh^3}{15\,c^3}+\frac{(3\,fgh^2+eh^3)\sqrt{cx^2+a}x^3}{4\,c}\\ & +\frac{(3\,fg^2h+3\,egh^2+dh^3)\sqrt{cx^2+a}x^2}{3\,c}\\ & -\frac{3\,(3\,fgh^2+eh^3)\sqrt{cx^2+a}ax}{8\,c^2}\\ & +\frac{(fg^3+3\,eg^2h+3\,dgh^2)\sqrt{cx^2+a}x}{2\,c}\\ & +\frac{3\,(3\,fgh^2+eh^3)a^2\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{8\,c^{\frac{5}{2}}}\\ & -\frac{(fg^3+3\,eg^2h+3\,dgh^2)a\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{2\,c^{\frac{3}{2}}}\\ & -\frac{2\,(3\,fg^2h+3\,egh^2+dh^3)\sqrt{cx^2+a}a}{3\,c^2}\end{aligned}$$

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/5*sqrt(c*x^2 + a)*f*h^3*x^4/c - 4/15*sqrt(c*x^2 + a)*a*f*h^3*x^2/c^2 + d*
g^3*arcsinh(c*x/sqrt(a*c))/sqrt(c) + sqrt(c*x^2 + a)*e*g^3/c + 3*sqrt(c*x^2
 + a)*d*g^2*h/c + 8/15*sqrt(c*x^2 + a)*a^2*f*h^3/c^3 + 1/4*(3*f*g*h^2 + e*h
^3)*sqrt(c*x^2 + a)*x^3/c + 1/3*(3*f*g^2*h + 3*e*g*h^2 + d*h^3)*sqrt(c*x^2
+ a)*x^2/c - 3/8*(3*f*g*h^2 + e*h^3)*sqrt(c*x^2 + a)*a*x/c^2 + 1/2*(f*g^3 +
 3*e*g^2*h + 3*d*g*h^2)*sqrt(c*x^2 + a)*x/c + 3/8*(3*f*g*h^2 + e*h^3)*a^2*a
rcsinh(c*x/sqrt(a*c))/c^(5/2) - 1/2*(f*g^3 + 3*e*g^2*h + 3*d*g*h^2)*a*arcsi
nh(c*x/sqrt(a*c))/c^(3/2) - 2/3*(3*f*g^2*h + 3*e*g*h^2 + d*h^3)*sqrt(c*x^2
+ a)*a/c^2
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 306, normalized size of antiderivative = 0.94

$$\int \frac{(g+hx)^3\,(d+ex+fx^2)}{\sqrt{a+cx^2}}\,dx$$
$$=\frac{1}{120}\sqrt{cx^2+a}\left(\left(2\left(3\left(\frac{4\,fh^3x}{c}+\frac{5\,(3\,c^4fgh^2+c^4eh^3)}{c^5}\right)x+\frac{4\,(15\,c^4fg^2h+15\,c^4egh^2+5\,c^4dh^3-4\,ac^3fh^3}{c^5}\right.\right.\right.$$
$$-\frac{(8\,c^2dg^3-4\,acfg^3-12\,aceg^2h-12\,acdgh^2+9\,a^2fgh^2+3\,a^2eh^3)\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+a}\right|\right)}{8\,c^{\frac{5}{2}}}$$

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/120*sqrt(c*x^2 + a)*((2*(3*(4*f*h^3*x/c + 5*(3*c^4*f*g*h^2 + c^4*e*h^3)/c
^5)*x + 4*(15*c^4*f*g^2*h + 15*c^4*e*g*h^2 + 5*c^4*d*h^3 - 4*a*c^3*f*h^3)/c
^5)*x + 15*(4*c^4*f*g^3 + 12*c^4*e*g^2*h + 12*c^4*d*g*h^2 - 9*a*c^3*f*g*h^2
 - 3*a*c^3*e*h^3)/c^5)*x + 8*(15*c^4*e*g^3 + 45*c^4*d*g^2*h - 30*a*c^3*f*g^
2*h - 30*a*c^3*e*g*h^2 - 10*a*c^3*d*h^3 + 8*a^2*c^2*f*h^3)/c^5) - 1/8*(8*c^
2*d*g^3 - 4*a*c*f*g^3 - 12*a*c*e*g^2*h - 12*a*c*d*g*h^2 + 9*a^2*f*g*h^2 + 3
*a^2*e*h^3)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(g+hx)^3\,(d+ex+fx^2)}{\sqrt{a+cx^2}}\,dx=\int \frac{{\left(g+h\,x\right)}^3\,\left(f\,x^2+e\,x+d\right)}{\sqrt{c\,x^2+a}}\,dx$$

```
[In] int(((g + h*x)^3*(d + e*x + f*x^2))/(a + c*x^2)^(1/2),x)
```

```
[Out] int(((g + h*x)^3*(d + e*x + f*x^2))/(a + c*x^2)^(1/2), x)
```

3.102 $\int \frac{(g+hx)^2(d+ex+fx^2)}{\sqrt{a+cx^2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 223

$$
\begin{aligned}
\int \frac{(g+hx)^2\,(d+ex+fx^2)}{\sqrt{a+cx^2}}\,dx = & -\frac{(fg-4eh)(g+hx)^2\sqrt{a+cx^2}}{12ch}+\frac{f(g+hx)^3\sqrt{a+cx^2}}{4ch} \\
& -\frac{\left(4(4ah^2(2fg+eh)+cg(fg^2-4h(eg+3dh)))-h(3(4cd-3af)h^2-2cg(fg-4eh))\,x\right)\sqrt{a+cx^2}}{24c^2h} \\
& +\frac{(8c^2dg^2+3a^2fh^2-4ac(fg^2+h(2eg+dh)))\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{8c^{5/2}}
\end{aligned}
$$

```
[Out] 1/8*(8*c^2*d*g^2+3*a^2*f*h^2-4*a*c*(f*g^2+h*(d*h+2*e*g)))*arctanh(x*c^(1/2)
/(c*x^2+a)^(1/2))/c^(5/2)-1/12*(-4*e*h+f*g)*(h*x+g)^2*(c*x^2+a)^(1/2)/c/h+1
/4*f*(h*x+g)^3*(c*x^2+a)^(1/2)/c/h-1/24*(16*a*h^2*(e*h+2*f*g)+4*c*g*(f*g^2-
4*h*(3*d*h+e*g))-h*(3*(-3*a*f+4*c*d)*h^2-2*c*g*(-4*e*h+f*g))*x)*(c*x^2+a)^(
1/2)/c^2/h
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {1668, 847, 794, 223, 212}

$$\begin{aligned}
&\int \frac{(g+hx)^2\left(d+ex+fx^2\right)}{\sqrt{a+cx^2}}\,dx \\
&= \frac{\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(3a^2fh^2-4ac\left(h(dh+2eg)+fg^2\right)+8c^2dg^2\right)}{8c^{5/2}} \\
&\quad -\frac{\sqrt{a+cx^2}\left(4\left(4ah^2(eh+2fg)-4cgh(3dh+eg)+cfg^3\right)-hx\left(3h^2(4cd-3af)-2cg(fg-4eh)\right)\right)}{24c^2h} \\
&\quad -\frac{\sqrt{a+cx^2}(g+hx)^2(fg-4eh)}{12ch}+\frac{f\sqrt{a+cx^2}(g+hx)^3}{4ch}
\end{aligned}$$

```
[In] Int[((g + h*x)^2*(d + e*x + f*x^2))/Sqrt[a + c*x^2],x]
```

```
[Out] -1/12*((f*g - 4*e*h)*(g + h*x)^2*Sqrt[a + c*x^2])/(c*h) + (f*(g + h*x)^3*Sq
rt[a + c*x^2])/(4*c*h) - ((4*(c*f*g^3 - 4*c*g*h*(e*g + 3*d*h) + 4*a*h^2*(2*
f*g + e*h)) - h*(3*(4*c*d - 3*a*f)*h^2 - 2*c*g*(f*g - 4*e*h))*x)*Sqrt[a + c
*x^2])/(24*c^2*h) + ((8*c^2*d*g^2 + 3*a^2*f*h^2 - 4*a*c*(f*g^2 + h*(2*e*g +
 d*h)))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(8*c^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
```

```
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) && !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && !(EqQ[d, 0] && T
rue) && !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{f(g+hx)^3\sqrt{a+cx^2}}{4ch} + \frac{\int \frac{(g+hx)^2\left((4cd-3af)h^2-ch(fg-4eh)x\right)}{\sqrt{a+cx^2}}\,dx}{4ch^2}\\
&= -\frac{(fg-4eh)(g+hx)^2\sqrt{a+cx^2}}{12ch} + \frac{f(g+hx)^3\sqrt{a+cx^2}}{4ch}\\
&\quad + \frac{\int \frac{(g+hx)\left(ch^2(12cdg-7afg-8aeh)+ch\left(3(4cd-3af)h^2-2cg(fg-4eh)\right)x\right)}{\sqrt{a+cx^2}}\,dx}{12c^2h^2}\\
&= -\frac{(fg-4eh)(g+hx)^2\sqrt{a+cx^2}}{12ch} + \frac{f(g+hx)^3\sqrt{a+cx^2}}{4ch}\\
&\quad - \frac{\left(4(cfg^3-4cgh(eg+3dh)+4ah^2(2fg+eh))-h(3(4cd-3af)h^2-2cg(fg-4eh))\,x\right)\sqrt{a+c}}{24c^2h}\\
&\quad + \frac{(8c^2dg^2+3a^2fh^2-4ac(fg^2+h(2eg+dh)))\int \frac{1}{\sqrt{a+cx^2}}\,dx}{8c^2}\\
&= -\frac{(fg-4eh)(g+hx)^2\sqrt{a+cx^2}}{12ch} + \frac{f(g+hx)^3\sqrt{a+cx^2}}{4ch}\\
&\quad - \frac{\left(4(cfg^3-4cgh(eg+3dh)+4ah^2(2fg+eh))-h(3(4cd-3af)h^2-2cg(fg-4eh))\,x\right)\sqrt{a+c}}{24c^2h}\\
&\quad + \frac{(8c^2dg^2+3a^2fh^2-4ac(fg^2+h(2eg+dh)))\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{8c^2}
\end{aligned}$$

$$= -\frac{(fg-4eh)(g+hx)^2\sqrt{a+cx^2}}{12ch} + \frac{f(g+hx)^3\sqrt{a+cx^2}}{4ch}$$
$$-\frac{(4(cfg^3-4cgh(eg+3dh)+4ah^2(2fg+eh))-h(3(4cd-3af)h^2-2cg(fg-4eh))\,x)\sqrt{a+cx}}{24c^2h}$$
$$+\frac{(8c^2dg^2+3a^2fh^2-4ac(fg^2+h(2eg+dh)))\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{8c^{5/2}}$$

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.74

$$\int \frac{(g+hx)^2\,(d+ex+fx^2)}{\sqrt{a+cx^2}}\,dx$$
$$= \frac{\sqrt{a+cx^2}(-ah(32fg+16eh+9fhx)+2c(6dh(4g+hx)+4e(3g^2+3ghx+h^2x^2)+fx(6g^2+8ghx+3h}{24c^2}$$
$$-\frac{(8c^2dg^2+3a^2fh^2-4ac(fg^2+h(2eg+dh)))\log\left(-\sqrt{c}x+\sqrt{a+cx^2}\right)}{8c^{5/2}}$$

```
[In] Integrate[((g + h*x)^2*(d + e*x + f*x^2))/Sqrt[a + c*x^2],x]
```

```
[Out] (Sqrt[a + c*x^2]*(-(a*h*(32*f*g + 16*e*h + 9*f*h*x)) + 2*c*(6*d*h*(4*g + h*
x) + 4*e*(3*g^2 + 3*g*h*x + h^2*x^2) + f*x*(6*g^2 + 8*g*h*x + 3*h^2*x^2))))
/(24*c^2) - ((8*c^2*d*g^2 + 3*a^2*f*h^2 - 4*a*c*(f*g^2 + h*(2*e*g + d*h)))*
Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/(8*c^(5/2))
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.74

method	result
risch	$-\frac{\left(-6f\,h^2c\,x^3-8ce\,h^2x^2-16cfgh\,x^2+9xaf\,h^2-12d\,h^2xc-24eghxc-12f\,g^2xc+16ae\,h^2+32afgh-48cdgh-24ce\,g^2\right)\sqrt{c\,x^2+a}}{24c^2}+\frac{(3a}{}$
default	$\frac{d\,g^2\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{\sqrt{c}}+f\,h^2\left(\frac{x^3\sqrt{c\,x^2+a}}{4c}-\frac{3a\left(\frac{x\sqrt{c\,x^2+a}}{2c}-\frac{a\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{2c^{\frac{3}{2}}}\right)}{4c}\right)+(e\,h^2+2fgh)\left(\frac{x^2\sqrt{c\,x^2+a}}{3c}-\right.$

```
[In] int((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*(-6*c*f*h^2*x^3-8*c*e*h^2*x^2-16*c*f*g*h*x^2+9*a*f*h^2*x-12*c*d*h^2*x
-24*c*e*g*h*x-12*c*f*g^2*x+16*a*e*h^2+32*a*f*g*h-48*c*d*g*h-24*c*e*g^2)/c^2
*(c*x^2+a)^(1/2)+1/8*(3*a^2*f*h^2-4*a*c*d*h^2-8*a*c*e*g*h-4*a*c*f*g^2+8*c^2
*d*g^2)/c^(5/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.71

$$\int \frac{(g+hx)^2\left(d+ex+fx^2\right)}{\sqrt{a+cx^2}}\,dx$$

$$= \left[-\frac{3\left(8\,acegh-4\left(2\,c^2d-acf\right)g^2+\left(4\,acd-3\,a^2f\right)h^2\right)\sqrt{c}\log\left(-2\,cx^2-2\,\sqrt{cx^2+a}\sqrt{c}x-a\right)-2\left(6\,c^2fh\right.}{}\right.$$

```
[In] integrate((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/48*(3*(8*a*c*e*g*h - 4*(2*c^2*d - a*c*f)*g^2 + (4*a*c*d - 3*a^2*f)*h^2)
*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) - 2*(6*c^2*f*h^2*x
^3 + 24*c^2*e*g^2 - 16*a*c*e*h^2 + 16*(3*c^2*d - 2*a*c*f)*g*h + 8*(2*c^2*f*
g*h + c^2*e*h^2)*x^2 + 3*(4*c^2*f*g^2 + 8*c^2*e*g*h + (4*c^2*d - 3*a*c*f)*h
^2)*x)*sqrt(c*x^2 + a))/c^3, 1/24*(3*(8*a*c*e*g*h - 4*(2*c^2*d - a*c*f)*g^2
 + (4*a*c*d - 3*a^2*f)*h^2)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) + (
6*c^2*f*h^2*x^3 + 24*c^2*e*g^2 - 16*a*c*e*h^2 + 16*(3*c^2*d - 2*a*c*f)*g*h
+ 8*(2*c^2*f*g*h + c^2*e*h^2)*x^2 + 3*(4*c^2*f*g^2 + 8*c^2*e*g*h + (4*c^2*d
 - 3*a*c*f)*h^2)*x)*sqrt(c*x^2 + a))/c^3]
```

Sympy [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.19

$$\int \frac{(g+hx)^2\left(d+ex+fx^2\right)}{\sqrt{a+cx^2}}\,dx$$

$$= \begin{cases} \sqrt{a+cx^2}\left(\frac{fh^2x^3}{4c}+\frac{x^2\left(eh^2+2fgh\right)}{3c}+\frac{x\left(-\frac{3afh^2}{4c}+dh^2+2egh+fg^2\right)}{2c}+\frac{-\frac{2a\left(eh^2+2fgh\right)}{3c}+2dgh+eg^2}{c}\right)+\left(-\frac{a\left(-\frac{3afh^2}{4c}+dh^2+\right.}{2c}\right. \\ \frac{dg^2x+\frac{fh^2x^5}{5}+\frac{x^4\left(eh^2+2fgh\right)}{4}+\frac{x^3\left(dh^2+2egh+fg^2\right)}{3}+\frac{x^2\cdot\left(2dgh+eg^2\right)}{2}}{\sqrt{a}} \end{cases}$$

```
[In] integrate((h*x+g)**2*(f*x**2+e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((sqrt(a + c*x**2)*(f*h**2*x**3/(4*c) + x**2*(e*h**2 + 2*f*g*h)/(3
*c) + x*(-3*a*f*h**2/(4*c) + d*h**2 + 2*e*g*h + f*g**2)/(2*c) + (-2*a*(e*h*
*2 + 2*f*g*h)/(3*c) + 2*d*g*h + e*g**2)/c) + (-a*(-3*a*f*h**2/(4*c) + d*h**
2 + 2*e*g*h + f*g**2)/(2*c) + d*g**2)*Piecewise((log(2*sqrt(c)*sqrt(a + c*x
**2) + 2*c*x)/sqrt(c), Ne(a, 0)), (x*log(x)/sqrt(c*x**2), True)), Ne(c, 0))
, ((d*g**2*x + f*h**2*x**5/5 + x**4*(e*h**2 + 2*f*g*h)/4 + x**3*(d*h**2 + 2
*e*g*h + f*g**2)/3 + x**2*(2*d*g*h + e*g**2)/2)/sqrt(a), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.03

$$\begin{aligned}\int \frac{(g+hx)^2\,(d+ex+fx^2)}{\sqrt{a+cx^2}}\,dx &= \frac{\sqrt{cx^2+a}fh^2x^3}{4\,c} - \frac{3\,\sqrt{cx^2+a}afh^2x}{8\,c^2} + \frac{dg^2\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{c}} \\ &\quad + \frac{3\,a^2fh^2\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{8\,c^{\frac{5}{2}}} + \frac{\sqrt{cx^2+a}eg^2}{c} \\ &\quad + \frac{2\,\sqrt{cx^2+a}dgh}{c} + \frac{(2\,fgh+eh^2)\sqrt{cx^2+a}x^2}{3\,c} \\ &\quad + \frac{(fg^2+2\,egh+dh^2)\sqrt{cx^2+a}x}{2\,c} \\ &\quad - \frac{(fg^2+2\,egh+dh^2)a\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{2\,c^{\frac{3}{2}}} \\ &\quad - \frac{2\,(2\,fgh+eh^2)\sqrt{cx^2+a}a}{3\,c^2}\end{aligned}$$

```
[In] integrate((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(c*x^2 + a)*f*h^2*x^3/c - 3/8*sqrt(c*x^2 + a)*a*f*h^2*x/c^2 + d*g^2
*arcsinh(c*x/sqrt(a*c))/sqrt(c) + 3/8*a^2*f*h^2*arcsinh(c*x/sqrt(a*c))/c^(5
/2) + sqrt(c*x^2 + a)*e*g^2/c + 2*sqrt(c*x^2 + a)*d*g*h/c + 1/3*(2*f*g*h +
e*h^2)*sqrt(c*x^2 + a)*x^2/c + 1/2*(f*g^2 + 2*e*g*h + d*h^2)*sqrt(c*x^2 + a
)*x/c - 1/2*(f*g^2 + 2*e*g*h + d*h^2)*a*arcsinh(c*x/sqrt(a*c))/c^(3/2) - 2/
3*(2*f*g*h + e*h^2)*sqrt(c*x^2 + a)*a/c^2
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.90

$$\begin{aligned}&\int \frac{(g+hx)^2\,(d+ex+fx^2)}{\sqrt{a+cx^2}}\,dx \\ &= \frac{1}{24}\,\sqrt{cx^2+a}\Bigg(\left(2\left(\frac{3\,fh^2x}{c} + \frac{4\,(2\,c^3fgh+c^3eh^2)}{c^4}\right)x + \frac{3\,(4\,c^3fg^2+8\,c^3egh+4\,c^3dh^2-3\,ac^2fh^2)}{c^4}\right)x + 8\,(\\ &\quad - \frac{(8\,c^2dg^2-4\,acfg^2-8\,acegh-4\,acdh^2+3\,a^2fh^2)\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+a}\right|\right)}{8\,c^{\frac{5}{2}}}\end{aligned}$$

```
[In] integrate((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*sqrt(c*x^2 + a)*((2*(3*f*h^2*x/c + 4*(2*c^3*f*g*h + c^3*e*h^2)/c^4)*x
+ 3*(4*c^3*f*g^2 + 8*c^3*e*g*h + 4*c^3*d*h^2 - 3*a*c^2*f*h^2)/c^4)*x + 8*(3
*c^3*e*g^2 + 6*c^3*d*g*h - 4*a*c^2*f*g*h - 2*a*c^2*e*h^2)/c^4) - 1/8*(8*c^2
*d*g^2 - 4*a*c*f*g^2 - 8*a*c*e*g*h - 4*a*c*d*h^2 + 3*a^2*f*h^2)*log(abs(-sq
rt(c)*x + sqrt(c*x^2 + a)))/c^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(g+hx)^2\,(d+ex+fx^2)}{\sqrt{a+cx^2}}\,dx=\int \frac{{\left(g+h\,x\right)}^2\,\left(f\,x^2+e\,x+d\right)}{\sqrt{c\,x^2+a}}\,dx$$

```
[In] int(((g + h*x)^2*(d + e*x + f*x^2))/(a + c*x^2)^(1/2),x)
```

```
[Out] int(((g + h*x)^2*(d + e*x + f*x^2))/(a + c*x^2)^(1/2), x)
```

3.103 $\int \frac{(g+hx)(d+ex+fx^2)}{\sqrt{a+cx^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 136

$$\begin{aligned}\int \frac{(g+hx)\left(d+ex+fx^2\right)}{\sqrt{a+cx^2}}\,dx &= \frac{f(g+hx)^2\sqrt{a+cx^2}}{3ch} - \frac{(2(2afh^2+c(fg^2-3h(eg+dh)))+ch(fg-3eh)x)\sqrt{a+cx^2}}{6c^2h}\\ &\quad+ \frac{(2cdg-a(fg+eh))\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2c^{3/2}}\end{aligned}$$

```
[Out] 1/2*(2*c*d*g-a*(e*h+f*g))*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(3/2)+1/3*f*
(h*x+g)^2*(c*x^2+a)^(1/2)/c/h-1/6*(4*a*f*h^2+2*c*(f*g^2-3*h*(d*h+e*g))+c*h*
(-3*e*h+f*g)*x)*(c*x^2+a)^(1/2)/c^2/h
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1668, 794, 223, 212}

$$\begin{aligned}\int \frac{(g+hx)\left(d+ex+fx^2\right)}{\sqrt{a+cx^2}}\,dx &= \frac{\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)(2cdg-a(eh+fg))}{2c^{3/2}}\\ &\quad- \frac{\sqrt{a+cx^2}(2(2afh^2-3ch(dh+eg)+cfg^2)+chx(fg-3eh))}{6c^2h} + \frac{f\sqrt{a+cx^2}(g+hx)^2}{3ch}\end{aligned}$$

```
[In] Int[((g + h*x)*(d + e*x + f*x^2))/Sqrt[a + c*x^2],x]
```

```
[Out] (f*(g + h*x)^2*Sqrt[a + c*x^2])/(3*c*h) - ((2*(c*f*g^2 + 2*a*f*h^2 - 3*c*h*
(e*g + d*h)) + c*h*(f*g - 3*e*h)*x)*Sqrt[a + c*x^2])/(6*c^2*h) + ((2*c*d*g
- a*(f*g + e*h))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*c^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{f(g+hx)^2\sqrt{a+cx^2}}{3ch} + \frac{\int \frac{(g+hx)\left((3cd-2af)h^2-ch(fg-3eh)x\right)}{\sqrt{a+cx^2}}\,dx}{3ch^2} \\
&= \frac{f(g+hx)^2\sqrt{a+cx^2}}{3ch} \\
&\quad - \frac{\left(2\left(cfg^2+2afh^2-3ch(eg+dh)\right)+ch(fg-3eh)x\right)\sqrt{a+cx^2}}{6c^2h} \\
&\quad + \frac{(2cdg-afg-aeh)\int \frac{1}{\sqrt{a+cx^2}}\,dx}{2c}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{f(g+hx)^2\sqrt{a+cx^2}}{3ch} \\
&\quad - \frac{\left(2(cfg^2+2afh^2-3ch(eg+dh))+ch(fg-3eh)x\right)\sqrt{a+cx^2}}{6c^2h} \\
&\quad + \frac{(2cdg-afg-aeh)\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{2c} \\
&= \frac{f(g+hx)^2\sqrt{a+cx^2}}{3ch} \\
&\quad - \frac{\left(2(cfg^2+2afh^2-3ch(eg+dh))+ch(fg-3eh)x\right)\sqrt{a+cx^2}}{6c^2h} \\
&\quad + \frac{(2cdg-a(fg+eh))\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2c^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.71

$$
\int \frac{(g+hx)\left(d+ex+fx^2\right)}{\sqrt{a+cx^2}}\,dx
$$
$$
= \frac{\sqrt{a+cx^2}(-4afh+c(6eg+6dh+3fgx+3ehx+2fhx^2))+3\sqrt{c}(-2cdg+afg+aeh)\log\left(-\sqrt{c}x+\sqrt{a\right.}}{6c^2}
$$

```
[In] Integrate[((g + h*x)*(d + e*x + f*x^2))/Sqrt[a + c*x^2],x]
```

```
[Out] (Sqrt[a + c*x^2]*(-4*a*f*h + c*(6*e*g + 6*d*h + 3*f*g*x + 3*e*h*x + 2*f*h*x
^2)) + 3*Sqrt[c]*(-2*c*d*g + a*f*g + a*e*h)*Log[-(Sqrt[c]*x) + Sqrt[a + c*x
^2]])/(6*c^2)
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.64

method	result
risch	$-\frac{\left(-2fhc\,x^2-3ehxc-3cfxg+4afh-6cdh-6ceg\right)\sqrt{c\,x^2+a}}{6c^2}-\frac{(aeh+afg-2cdg)\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{2c^{\frac{3}{2}}}$
default	$\frac{dg\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{\sqrt{c}}+fh\left(\frac{x^2\sqrt{c\,x^2+a}}{3c}-\frac{2a\sqrt{c\,x^2+a}}{3c^2}\right)+(eh+fg)\left(\frac{x\sqrt{c\,x^2+a}}{2c}-\frac{a\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{2c^{\frac{3}{2}}}\right)+ (dh+e$

```
[In] int((h*x+g)*(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(-2*c*f*h*x^2-3*c*e*h*x-3*c*f*g*x+4*a*f*h-6*c*d*h-6*c*e*g)/c^2*(c*x^2+
a)^(1/2)-1/2/c^(3/2)*(a*e*h+a*f*g-2*c*d*g)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.46

$$\int \frac{(g+hx)\left(d+ex+fx^2\right)}{\sqrt{a+cx^2}}\,dx$$
$$= \left[\frac{3\left(aeh-(2\,cd-af)g\right)\sqrt{c}\log\left(-2\,cx^2+2\,\sqrt{cx^2+a}\sqrt{c}x-a\right)+2\left(2\,cfhx^2+6\,ceg+2\,(3\,cd-2\,af)h+\right.}{12\,c^2}\right.$$

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(a*e*h - (2*c*d - a*f)*g)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)
*sqrt(c)*x - a) + 2*(2*c*f*h*x^2 + 6*c*e*g + 2*(3*c*d - 2*a*f)*h + 3*(c*f*g
 + c*e*h)*x)*sqrt(c*x^2 + a))/c^2, 1/6*(3*(a*e*h - (2*c*d - a*f)*g)*sqrt(-c
)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) + (2*c*f*h*x^2 + 6*c*e*g + 2*(3*c*d -
2*a*f)*h + 3*(c*f*g + c*e*h)*x)*sqrt(c*x^2 + a))/c^2]
```

Sympy [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.10

$$\int \frac{(g+hx)\left(d+ex+fx^2\right)}{\sqrt{a+cx^2}}\,dx$$
$$= \begin{cases} \sqrt{a+cx^2}\left(\frac{fhx^2}{3c}+\frac{x(eh+fg)}{2c}+\frac{-\frac{2afh}{3c}+dh+eg}{c}\right)+\left(-\frac{a(eh+fg)}{2c}+dg\right)\left(\begin{cases}\frac{\log\left(2\sqrt{c}\sqrt{a+cx^2}+2cx\right)}{\sqrt{c}} & \text{for}\ a\neq 0\\ \frac{x\log(x)}{\sqrt{cx^2}} & \text{otherwise}\end{cases}\right) & \\ \frac{dgx+\frac{fhx^4}{4}+\frac{x^3(eh+fg)}{3}+\frac{x^2(dh+eg)}{2}}{\sqrt{a}} & \end{cases}$$

```
[In] integrate((h*x+g)*(f*x**2+e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((sqrt(a + c*x**2)*(f*h*x**2/(3*c) + x*(e*h + f*g)/(2*c) + (-2*a*f
*h/(3*c) + d*h + e*g)/c) + (-a*(e*h + f*g)/(2*c) + d*g)*Piecewise((log(2*sq
rt(c)*sqrt(a + c*x**2) + 2*c*x)/sqrt(c), Ne(a, 0)), (x*log(x)/sqrt(c*x**2),
 True)), Ne(c, 0)), ((d*g*x + f*h*x**4/4 + x**3*(e*h + f*g)/3 + x**2*(d*h +
 e*g)/2)/sqrt(a), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.93

$$\int \frac{(g+hx)\left(d+ex+fx^2\right)}{\sqrt{a+cx^2}}\,dx = \frac{\sqrt{cx^2+a}fhx^2}{3\,c}+\frac{dg\,\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{c}} + \frac{\sqrt{cx^2+a}eg}{c}+\frac{\sqrt{cx^2+a}dh}{c}-\frac{2\,\sqrt{cx^2+a}afh}{3\,c^2} + \frac{\sqrt{cx^2+a}(fg+eh)x}{2\,c}-\frac{(fg+eh)a\,\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{2\,c^{\frac{3}{2}}}$$

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(c*x^2 + a)*f*h*x^2/c + d*g*arcsinh(c*x/sqrt(a*c))/sqrt(c) + sqrt(c
*x^2 + a)*e*g/c + sqrt(c*x^2 + a)*d*h/c - 2/3*sqrt(c*x^2 + a)*a*f*h/c^2 + 1
/2*sqrt(c*x^2 + a)*(f*g + e*h)*x/c - 1/2*(f*g + e*h)*a*arcsinh(c*x/sqrt(a*c
))/c^(3/2)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.79

$$\int \frac{(g+hx)\left(d+ex+fx^2\right)}{\sqrt{a+cx^2}}\,dx = \frac{1}{6}\,\sqrt{cx^2+a}\left(\left(\frac{2\,fhx}{c}+\frac{3\,(c^2fg+c^2eh)}{c^3}\right)x+\frac{2\,(3\,c^2eg+3\,c^2dh-2\,acfh)}{c^3}\right) - \frac{(2\,cdg-afg-aeh)\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+a}\right|\right)}{2\,c^{\frac{3}{2}}}$$

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(c*x^2 + a)*((2*f*h*x/c + 3*(c^2*f*g + c^2*e*h)/c^3)*x + 2*(3*c^2*e
*g + 3*c^2*d*h - 2*a*c*f*h)/c^3) - 1/2*(2*c*d*g - a*f*g - a*e*h)*log(abs(-s
qrt(c)*x + sqrt(c*x^2 + a)))/c^(3/2)
```

Mupad [B] (verification not implemented)

Time = 13.75 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.67

$$\int \frac{(g+hx)\left(d+ex+fx^2\right)}{\sqrt{a+cx^2}}\,dx$$
$$=\begin{cases} \frac{2\,f\,g\,x^3+3\,e\,g\,x^2+6\,d\,g\,x}{6\,\sqrt{a}}+\frac{3\,f\,h\,x^4+4\,e\,h\,x^3+6\,d\,h\,x^2}{12\,\sqrt{a}} \\ \frac{d\,g\,\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+a}\right)}{\sqrt{c}}+\frac{d\,h\,\sqrt{c\,x^2+a}}{c}+\frac{e\,g\,\sqrt{c\,x^2+a}}{c}+\frac{e\,h\,x\,\sqrt{c\,x^2+a}}{2\,c}+\frac{f\,g\,x\,\sqrt{c\,x^2+a}}{2\,c}-\frac{f\,h\,\sqrt{c\,x^2+a}\,\left(2\,a-c\,x^2\right)}{3\,c^2}-\frac{a\,e\,h\,\ln\left(2\,\sqrt{c}\right.}{2} \end{cases}$$

```
[In] int(((g + h*x)*(d + e*x + f*x^2))/(a + c*x^2)^(1/2),x)
```

```
[Out] piecewise(c == 0, (3*e*g*x^2 + 2*f*g*x^3 + 6*d*g*x)/(6*a^(1/2)) + (6*d*h*x^
2 + 4*e*h*x^3 + 3*f*h*x^4)/(12*a^(1/2)), c ~= 0, (d*g*log(c^(1/2)*x + (a +
c*x^2)^(1/2)))/c^(1/2) + (d*h*(a + c*x^2)^(1/2))/c + (e*g*(a + c*x^2)^(1/2)
)/c + (e*h*x*(a + c*x^2)^(1/2))/(2*c) + (f*g*x*(a + c*x^2)^(1/2))/(2*c) - (
f*h*(a + c*x^2)^(1/2)*(2*a - c*x^2))/(3*c^2) - (a*e*h*log(2*c^(1/2)*x + 2*(
a + c*x^2)^(1/2)))/(2*c^(3/2)) - (a*f*g*log(2*c^(1/2)*x + 2*(a + c*x^2)^(1/
2)))/(2*c^(3/2)))
```

3.104 $\int \frac{d+ex+fx^2}{\sqrt{a+cx^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 74

$$\int \frac{d+ex+fx^2}{\sqrt{a+cx^2}}\,dx=\frac{e\sqrt{a+cx^2}}{c}+\frac{fx\sqrt{a+cx^2}}{2c}+\frac{(2cd-af)\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2c^{3/2}}$$

```
[Out] 1/2*(-a*f+2*c*d)*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(3/2)+e*(c*x^2+a)^(1/
2)/c+1/2*f*x*(c*x^2+a)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1829, 655, 223, 212}

$$\int \frac{d+ex+fx^2}{\sqrt{a+cx^2}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)(2cd-af)}{2c^{3/2}}+\frac{e\sqrt{a+cx^2}}{c}+\frac{fx\sqrt{a+cx^2}}{2c}$$

```
[In] Int[(d + e*x + f*x^2)/Sqrt[a + c*x^2],x]
```

```
[Out] (e*Sqrt[a + c*x^2])/c + (f*x*Sqrt[a + c*x^2])/(2*c) + ((2*c*d - a*f)*ArcTan
h[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*c^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1829

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{fx\sqrt{a+cx^2}}{2c} + \frac{\int \frac{2cd-af+2cex}{\sqrt{a+cx^2}}\,dx}{2c} \\
&= \frac{e\sqrt{a+cx^2}}{c} + \frac{fx\sqrt{a+cx^2}}{2c} + \frac{(2cd-af)\int \frac{1}{\sqrt{a+cx^2}}\,dx}{2c} \\
&= \frac{e\sqrt{a+cx^2}}{c} + \frac{fx\sqrt{a+cx^2}}{2c} + \frac{(2cd-af)\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{2c} \\
&= \frac{e\sqrt{a+cx^2}}{c} + \frac{fx\sqrt{a+cx^2}}{2c} + \frac{(2cd-af)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2c^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.95

$$\int \frac{d+ex+fx^2}{\sqrt{a+cx^2}}\,dx = \frac{(2e+fx)\sqrt{a+cx^2}}{2c} + \frac{(2cd-af)\text{arctanh}\left(\frac{\sqrt{c}x}{-\sqrt{a}+\sqrt{a+cx^2}}\right)}{c^{3/2}}$$

```
[In] Integrate[(d + e*x + f*x^2)/Sqrt[a + c*x^2],x]
```

```
[Out] ((2*e + f*x)*Sqrt[a + c*x^2])/(2*c) + ((2*c*d - a*f)*ArcTanh[(Sqrt[c]*x)/(-
Sqrt[a] + Sqrt[a + c*x^2])])/c^(3/2)
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.70

method	result	size
risch	$\frac{(fx+2e)\sqrt{c\,x^{2}+a}}{2c}-\frac{(fa-2cd)\ln\left(x\sqrt{c}+\sqrt{c\,x^{2}+a}\right)}{2c^{\frac{3}{2}}}$	52
default	$\frac{d\ln\left(x\sqrt{c}+\sqrt{c\,x^{2}+a}\right)}{\sqrt{c}}+f\left(\frac{x\sqrt{c\,x^{2}+a}}{2c}-\frac{a\ln\left(x\sqrt{c}+\sqrt{c\,x^{2}+a}\right)}{2c^{\frac{3}{2}}}\right)+\frac{e\sqrt{c\,x^{2}+a}}{c}$	77

```
[In] int((f*x^2+e*x+d)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(f*x+2*e)/c*(c*x^2+a)^(1/2)-1/2*(a*f-2*c*d)/c^(3/2)*ln(x*c^(1/2)+(c*x^2
+a)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.68

$$\int \frac{d+ex+fx^2}{\sqrt{a+cx^2}}\,dx = \left[-\frac{(2\,cd-af)\sqrt{c}\log\left(-2\,cx^2+2\,\sqrt{cx^2+a}\sqrt{c}x-a\right)-2\,(cfx+2\,ce)\sqrt{cx^2+a}}{4\,c^2}, -\frac{(2\,cd-af)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}x}{\sqrt{cx^2+a}}\right)-(cfx+2\,ce)\sqrt{cx^2+a}}{2\,c^2}\right]$$

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((2*c*d - a*f)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x - a
) - 2*(c*f*x + 2*c*e)*sqrt(c*x^2 + a))/c^2, -1/2*((2*c*d - a*f)*sqrt(-c)*ar
ctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - (c*f*x + 2*c*e)*sqrt(c*x^2 + a))/c^2]
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.24

$$\int \frac{d+ex+fx^2}{\sqrt{a+cx^2}}\,dx = \begin{cases} \sqrt{a+cx^2}\left(\frac{e}{c}+\frac{fx}{2c}\right)+\left(-\frac{af}{2c}+d\right)\left(\begin{cases}\frac{\log\left(2\sqrt{c}\sqrt{a+cx^2}+2cx\right)}{\sqrt{c}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{cx^2}} & \text{otherwise}\end{cases}\right) & \text{for } c\neq 0\\ \frac{dx+\frac{ex^2}{2}+\frac{fx^3}{3}}{\sqrt{a}} & \text{otherwise}\end{cases}$$

```
[In] integrate((f*x**2+e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((sqrt(a + c*x**2)*(e/c + f*x/(2*c)) + (-a*f/(2*c) + d)*Piecewise(
(log(2*sqrt(c)*sqrt(a + c*x**2) + 2*c*x)/sqrt(c), Ne(a, 0)), (x*log(x)/sqrt
(c*x**2), True)), Ne(c, 0)), ((d*x + e*x**2/2 + f*x**3/3)/sqrt(a), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.82

$$\int \frac{d+ex+fx^2}{\sqrt{a+cx^2}}\,dx = \frac{\sqrt{cx^2+a}fx}{2\,c}+\frac{d\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{c}}-\frac{af\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{2\,c^{\frac{3}{2}}}+\frac{\sqrt{cx^2+a}e}{c}$$

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(c*x^2 + a)*f*x/c + d*arcsinh(c*x/sqrt(a*c))/sqrt(c) - 1/2*a*f*arcs
inh(c*x/sqrt(a*c))/c^(3/2) + sqrt(c*x^2 + a)*e/c
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.77

$$\int \frac{d+ex+fx^2}{\sqrt{a+cx^2}}\,dx = \frac{1}{2}\sqrt{cx^2+a}\left(\frac{fx}{c}+\frac{2\,e}{c}\right)-\frac{(2\,cd-af)\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+a}\right|\right)}{2\,c^{\frac{3}{2}}}$$

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(c*x^2 + a)*(f*x/c + 2*e/c) - 1/2*(2*c*d - a*f)*log(abs(-sqrt(c)*x
+ sqrt(c*x^2 + a)))/c^(3/2)
```

Mupad [B] (verification not implemented)

Time = 13.06 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.45

$$\int \frac{d+ex+fx^2}{\sqrt{a+cx^2}}\,dx$$
$$= \begin{cases} \frac{2\,f\,x^3+3\,e\,x^2+6\,d\,x}{6\,\sqrt{a}} & \text{if}\ \ c=0 \\ \frac{e\,\sqrt{c\,x^2+a}}{c}+\frac{d\,\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+a}\right)}{\sqrt{c}}-\frac{a\,f\,\ln\left(2\,\sqrt{c}\,x+2\,\sqrt{c\,x^2+a}\right)}{2\,c^{3/2}}+\frac{f\,x\,\sqrt{c\,x^2+a}}{2\,c} & \text{if}\ \ c\neq 0 \end{cases}$$

```
[In] int((d + e*x + f*x^2)/(a + c*x^2)^(1/2),x)
```

```
[Out] piecewise(c == 0, (6*d*x + 3*e*x^2 + 2*f*x^3)/(6*a^(1/2)), c ~= 0, (e*(a +
c*x^2)^(1/2))/c + (d*log(c^(1/2)*x + (a + c*x^2)^(1/2)))/c^(1/2) - (a*f*log
(2*c^(1/2)*x + 2*(a + c*x^2)^(1/2)))/(2*c^(3/2)) + (f*x*(a + c*x^2)^(1/2))/
(2*c))
```

3.105 $\int \frac{d+ex+fx^2}{(g+hx)\sqrt{a+cx^2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 130

$$\int \frac{d+ex+fx^2}{(g+hx)\sqrt{a+cx^2}}\,dx=\frac{f\sqrt{a+cx^2}}{ch}-\frac{(fg-eh)\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}h^2}-\frac{(fg^2-egh+dh^2)\,\text{arctanh}\left(\frac{ah-cgx}{\sqrt{cg^2+ah^2}\sqrt{a+cx^2}}\right)}{h^2\sqrt{cg^2+ah^2}}$$

```
[Out] -(-e*h+f*g)*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/h^2/c^(1/2)-(d*h^2-e*g*h+f*g
^2)*arctanh((-c*g*x+a*h)/(a*h^2+c*g^2)^(1/2)/(c*x^2+a)^(1/2))/h^2/(a*h^2+c*
g^2)^(1/2)+f*(c*x^2+a)^(1/2)/c/h
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1668, 858, 223, 212, 739}

$$\int \frac{d+ex+fx^2}{(g+hx)\sqrt{a+cx^2}}\,dx=-\frac{(dh^2-egh+fg^2)\,\text{arctanh}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)}{h^2\sqrt{ah^2+cg^2}}-\frac{\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)(fg-eh)}{\sqrt{c}h^2}+\frac{f\sqrt{a+cx^2}}{ch}$$

```
[In] Int[(d + e*x + f*x^2)/((g + h*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] (f*Sqrt[a + c*x^2])/(c*h) - ((f*g - e*h)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2
]])/(Sqrt[c]*h^2) - ((f*g^2 - e*g*h + d*h^2)*ArcTanh[(a*h - c*g*x)/(Sqrt[c*
g^2 + a*h^2]*Sqrt[a + c*x^2])])/(h^2*Sqrt[c*g^2 + a*h^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\text{integral} = \frac{f\sqrt{a+cx^2}}{ch} + \frac{\int \frac{cdh^2-ch(fg-eh)x}{(g+hx)\sqrt{a+cx^2}}\,dx}{ch^2}$$

$$= \frac{f\sqrt{a+cx^2}}{ch} - \frac{(fg-eh)\int \frac{1}{\sqrt{a+cx^2}}\,dx}{h^2} + \frac{\left(fg^2-egh+dh^2\right)\int \frac{1}{(g+hx)\sqrt{a+cx^2}}\,dx}{h^2}$$

$$
\begin{aligned}
&=\frac{f\sqrt{a+cx^2}}{ch}-\frac{(fg-eh)\text{Subst}\left(\int\frac{1}{1-cx^2}\,dx,x,\frac{x}{\sqrt{a+cx^2}}\right)}{h^2}\\
&\quad-\frac{(fg^2-egh+dh^2)\,\text{Subst}\left(\int\frac{1}{cg^2+ah^2-x^2}\,dx,x,\frac{ah-cgx}{\sqrt{a+cx^2}}\right)}{h^2}\\
&=\frac{f\sqrt{a+cx^2}}{ch}-\frac{(fg-eh)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}h^2}-\frac{(fg^2-egh+dh^2)\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{cg^2+ah^2}\sqrt{a+cx^2}}\right)}{h^2\sqrt{cg^2+ah^2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.05

$$
\int\frac{d+ex+fx^2}{(g+hx)\sqrt{a+cx^2}}\,dx
$$

$$
=\frac{\frac{fh\sqrt{a+cx^2}}{c}-\frac{2\left(fg^2+h(-eg+dh)\right)\arctan\left(\frac{\sqrt{c}(g+hx)-h\sqrt{a+cx^2}}{\sqrt{-cg^2-ah^2}}\right)}{\sqrt{-cg^2-ah^2}}+\frac{(fg-eh)\log\left(-\sqrt{c}x+\sqrt{a+cx^2}\right)}{\sqrt{c}}}{h^2}
$$

```
[In] Integrate[(d + e*x + f*x^2)/((g + h*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] ((f*h*Sqrt[a + c*x^2])/c - (2*(f*g^2 + h*(-(e*g) + d*h))*ArcTan[(Sqrt[c]*(g
 + h*x) - h*Sqrt[a + c*x^2])/Sqrt[-(c*g^2) - a*h^2]])/Sqrt[-(c*g^2) - a*h^2
] + ((f*g - e*h)*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/Sqrt[c])/h^2
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.52

method	result
risch	$\frac{f\sqrt{c\,x^2+a}}{ch}+\frac{\frac{(eh-fg)\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{h\sqrt{c}}-\frac{\left(d\,h^2-egh+f\,g^2\right)\ln\left(\frac{\frac{2a\,h^2+2c\,g^2}{h^2}-\frac{2cg\left(x+\frac{g}{h}\right)}{h}+2\sqrt{\frac{a\,h^2+c\,g^2}{h^2}}\sqrt{\left(x+\frac{g}{h}\right)^2c-\frac{2cg\left(x+\frac{g}{h}\right)}{h}+\frac{a\,h^2+}{h^2}}}{x+\frac{g}{h}}\right.}{h^2\sqrt{\frac{a\,h^2+c\,g^2}{h^2}}}}{h}$
default	$\frac{\frac{eh\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{\sqrt{c}}+\frac{fh\sqrt{c\,x^2+a}}{c}-\frac{fg\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{\sqrt{c}}}{h^2}-\frac{(d\,h^2-egh+f\,g^2)\ln\left(\frac{\frac{2a\,h^2+2c\,g^2}{h^2}-\frac{2cg\left(x+\frac{g}{h}\right)}{h}+2\sqrt{\frac{a\,h^2+c\,g^2}{h^2}}\sqrt{\left(x+\frac{g}{h}\right)^2}}{x+\frac{g}{h}}\right.}{h^3\sqrt{\frac{a\,h^2+c\,g^2}{h^2}}}$

```
[In] int((f*x^2+e*x+d)/(h*x+g)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] f*(c*x^2+a)^(1/2)/c/h+1/h*((e*h-f*g)/h*ln(x*c^(1/2)+(c*x^2+a)^(1/2))/c^(1/2
)-(d*h^2-e*g*h+f*g^2)/h^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2
```

```
-2*c*g/h*(x+1/h*g)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/
h*g)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+1/h*g)))
```

Fricas [A] (verification not implemented)

none

Time = 114.78 (sec) , antiderivative size = 881, normalized size of antiderivative = 6.78

$$\int \frac{d+ex+fx^2}{(g+hx)\sqrt{a+cx^2}}\,dx$$

$$=\Bigg[-\frac{(cfg^3-ceg^2h+afgh^2-aeh^3)\sqrt{c}\log\left(-2\,cx^2-2\,\sqrt{cx^2+a}\sqrt{c}x-a\right)-(cfg^2-cegh+cdh^2)\sqrt{cg^2+}}{2\,(c^2g^2h^2+a}$$

$$-\frac{2\,(cfg^2-cegh+cdh^2)\sqrt{-cg^2-ah^2}\arctan\left(\frac{\sqrt{-cg^2-ah^2}(cgx-ah)\sqrt{cx^2+a}}{acg^2+a^2h^2+(c^2g^2+ach^2)x^2}\right)+(cfg^3-ceg^2h+afgh^2-aeh^3}{2\,(c^2g^2h^2+ach^4)}$$

$$-\frac{(cfg^2-cegh+cdh^2)\sqrt{-cg^2-ah^2}\arctan\left(\frac{\sqrt{-cg^2-ah^2}(cgx-ah)\sqrt{cx^2+a}}{acg^2+a^2h^2+(c^2g^2+ach^2)x^2}\right)-(cfg^3-ceg^2h+afgh^2-aeh^3)}{c^2g^2h^2+ach^4}$$

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)/(c*x^2+a)^(1/2),x, algorithm="fricas")

[Out] [-1/2*((c*f*g^3 - c*e*g^2*h + a*f*g*h^2 - a*e*h^3)*sqrt(c)*log(-2*c*x^2 - 2
*sqrt(c*x^2 + a)*sqrt(c)*x - a) - (c*f*g^2 - c*e*g*h + c*d*h^2)*sqrt(c*g^2
+ a*h^2)*log((2*a*c*g*h*x - a*c*g^2 - 2*a^2*h^2 - (2*c^2*g^2 + a*c*h^2)*x^2
 - 2*sqrt(c*g^2 + a*h^2)*(c*g*x - a*h)*sqrt(c*x^2 + a))/(h^2*x^2 + 2*g*h*x
+ g^2)) - 2*(c*f*g^2*h + a*f*h^3)*sqrt(c*x^2 + a))/(c^2*g^2*h^2 + a*c*h^4),
 -1/2*(2*(c*f*g^2 - c*e*g*h + c*d*h^2)*sqrt(-c*g^2 - a*h^2)*arctan(sqrt(-c*
g^2 - a*h^2)*(c*g*x - a*h)*sqrt(c*x^2 + a)/(a*c*g^2 + a^2*h^2 + (c^2*g^2 +
a*c*h^2)*x^2)) + (c*f*g^3 - c*e*g^2*h + a*f*g*h^2 - a*e*h^3)*sqrt(c)*log(-2
*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) - 2*(c*f*g^2*h + a*f*h^3)*sqrt(c*
x^2 + a))/(c^2*g^2*h^2 + a*c*h^4), 1/2*(2*(c*f*g^3 - c*e*g^2*h + a*f*g*h^2
- a*e*h^3)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) + (c*f*g^2 - c*e*g*h
 + c*d*h^2)*sqrt(c*g^2 + a*h^2)*log((2*a*c*g*h*x - a*c*g^2 - 2*a^2*h^2 - (2
*c^2*g^2 + a*c*h^2)*x^2 - 2*sqrt(c*g^2 + a*h^2)*(c*g*x - a*h)*sqrt(c*x^2 +
a))/(h^2*x^2 + 2*g*h*x + g^2)) + 2*(c*f*g^2*h + a*f*h^3)*sqrt(c*x^2 + a))/(
c^2*g^2*h^2 + a*c*h^4), -((c*f*g^2 - c*e*g*h + c*d*h^2)*sqrt(-c*g^2 - a*h^2
)*arctan(sqrt(-c*g^2 - a*h^2)*(c*g*x - a*h)*sqrt(c*x^2 + a)/(a*c*g^2 + a^2*
h^2 + (c^2*g^2 + a*c*h^2)*x^2)) - (c*f*g^3 - c*e*g^2*h + a*f*g*h^2 - a*e*h^
3)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - (c*f*g^2*h + a*f*h^3)*sqrt
(c*x^2 + a))/(c^2*g^2*h^2 + a*c*h^4)]
```

Sympy [F]

$$\int \frac{d+ex+fx^2}{(g+hx)\sqrt{a+cx^2}}\,dx = \int \frac{d+ex+fx^2}{\sqrt{a+cx^2}\,(g+hx)}\,dx$$

```
[In] integrate((f*x**2+e*x+d)/(h*x+g)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral((d + e*x + f*x**2)/(sqrt(a + c*x**2)*(g + h*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.68

$$\begin{aligned}\int \frac{d+ex+fx^2}{(g+hx)\sqrt{a+cx^2}}\,dx = & -\frac{fg\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{c}h^2} + \frac{e\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{c}h} \\ & + \frac{fg^2\operatorname{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|} - \frac{ah}{\sqrt{ac}|hx+g|}\right)}{\sqrt{a+\frac{cg^2}{h^2}}h^3} \\ & - \frac{eg\operatorname{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|} - \frac{ah}{\sqrt{ac}|hx+g|}\right)}{\sqrt{a+\frac{cg^2}{h^2}}h^2} \\ & + \frac{d\operatorname{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|} - \frac{ah}{\sqrt{ac}|hx+g|}\right)}{\sqrt{a+\frac{cg^2}{h^2}}h} + \frac{\sqrt{cx^2+a}f}{ch}\end{aligned}$$

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -f*g*arcsinh(c*x/sqrt(a*c))/(sqrt(c)*h^2) + e*arcsinh(c*x/sqrt(a*c))/(sqrt(
c)*h) + f*g^2*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h
*x + g)))/(sqrt(a + c*g^2/h^2)*h^3) - e*g*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x
+ g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/(sqrt(a + c*g^2/h^2)*h^2) + d*arcsinh
(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/(sqrt(a + c
*g^2/h^2)*h) + sqrt(c*x^2 + a)*f/(c*h)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{d+ex+fx^2}{(g+hx)\sqrt{a+cx^2}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2}{(g+hx)\sqrt{a+cx^2}}\,dx=\int \frac{f\,x^2+e\,x+d}{(g+h\,x)\,\sqrt{c\,x^2+a}}\,dx$$

```
[In] int((d + e*x + f*x^2)/((g + h*x)*(a + c*x^2)^(1/2)),x)
```

```
[Out] int((d + e*x + f*x^2)/((g + h*x)*(a + c*x^2)^(1/2)), x)
```

3.106 $\int \frac{d+ex+fx^2}{(g+hx)^2\sqrt{a+cx^2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 168

$$\int \frac{d+ex+fx^2}{(g+hx)^2\sqrt{a+cx^2}}\,dx = -\frac{(fg^2-egh+dh^2)\sqrt{a+cx^2}}{h\,(cg^2+ah^2)\,(g+hx)}+\frac{f\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}h^2}+\frac{(ah^2(2fg-eh)+c(fg^3-dgh^2))\operatorname{arctanh}\left(\frac{ah-cgx}{\sqrt{cg^2+ah^2}\sqrt{a+cx^2}}\right)}{h^2\,(cg^2+ah^2)^{3/2}}$$

```
[Out] (a*h^2*(-e*h+2*f*g)+c*(-d*g*h^2+f*g^3))*arctanh((-c*g*x+a*h)/(a*h^2+c*g^2)^
(1/2)/(c*x^2+a)^(1/2))/h^2/(a*h^2+c*g^2)^(3/2)+f*arctanh(x*c^(1/2)/(c*x^2+a
)^(1/2))/h^2/c^(1/2)-(d*h^2-e*g*h+f*g^2)*(c*x^2+a)^(1/2)/h/(a*h^2+c*g^2)/(h
*x+g)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1665, 858, 223, 212, 739}

$$\int \frac{d+ex+fx^2}{(g+hx)^2\sqrt{a+cx^2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)(ah^2(2fg-eh)+c(fg^3-dgh^2))}{h^2\,(ah^2+cg^2)^{3/2}}+\frac{f\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}h^2}-\frac{\sqrt{a+cx^2}(dh^2-egh+fg^2)}{h(g+hx)\,(ah^2+cg^2)}$$

```
[In] Int[(d + e*x + f*x^2)/((g + h*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] -(((f*g^2 - e*g*h + d*h^2)*Sqrt[a + c*x^2])/(h*(c*g^2 + a*h^2)*(g + h*x)))
+ (f*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(Sqrt[c]*h^2) + ((a*h^2*(2*f*g -
 e*h) + c*(f*g^3 - d*g*h^2))*ArcTanh[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqr
t[a + c*x^2])])/(h^2*(c*g^2 + a*h^2)^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{(fg^2-egh+dh^2)\sqrt{a+cx^2}}{h\,(cg^2+ah^2)\,(g+hx)} - \frac{\int \frac{-cdg+afg-aeh-f\left(\frac{cg^2}{h}+ah\right)x}{(g+hx)\sqrt{a+cx^2}}\,dx}{cg^2+ah^2}$$

$$
\begin{aligned}
&= -\frac{(fg^2-egh+dh^2)\sqrt{a+cx^2}}{h\left(cg^2+ah^2\right)(g+hx)} + \frac{f\int \frac{1}{\sqrt{a+cx^2}}\,dx}{h^2} \\
&\quad + \frac{\left(cdg-2afg-\frac{cfg^3}{h^2}+aeh\right)\int \frac{1}{(g+hx)\sqrt{a+cx^2}}\,dx}{cg^2+ah^2}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{(fg^2-egh+dh^2)\sqrt{a+cx^2}}{h\left(cg^2+ah^2\right)(g+hx)} + \frac{f\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x}{\sqrt{a+cx^2}}\right)}{h^2} \\
&\quad - \frac{\left(cdg-2afg-\frac{cfg^3}{h^2}+aeh\right)\text{Subst}\left(\int \frac{1}{cg^2+ah^2-x^2}\,dx,x,\frac{ah-cgx}{\sqrt{a+cx^2}}\right)}{cg^2+ah^2}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{(fg^2-egh+dh^2)\sqrt{a+cx^2}}{h\left(cg^2+ah^2\right)(g+hx)} + \frac{f\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}h^2} \\
&\quad - \frac{\left(cdg-2afg-\frac{cfg^3}{h^2}+aeh\right)\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{cg^2+ah^2}\sqrt{a+cx^2}}\right)}{\left(cg^2+ah^2\right)^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.05

$$
\int \frac{d+ex+fx^2}{(g+hx)^2\sqrt{a+cx^2}}\,dx = -\frac{\frac{h(fg^2+h(-eg+dh))\sqrt{a+cx^2}}{(cg^2+ah^2)(g+hx)} + \frac{2\left(ah^2(2fg-eh)+c\left(fg^3-dgh^2\right)\right)\arctan\left(\frac{\sqrt{c}(g+hx)-h\sqrt{a+cx^2}}{\sqrt{-cg^2-ah^2}}\right)}{\left(-cg^2-ah^2\right)^{3/2}} + \frac{f\log\left(-\sqrt{c}x+\sqrt{a+cx^2}\right)}{\sqrt{c}}}{h^2}
$$

```
[In] Integrate[(d + e*x + f*x^2)/((g + h*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] -(((h*(f*g^2 + h*(-(e*g) + d*h))*Sqrt[a + c*x^2])/((c*g^2 + a*h^2)*(g + h*x
)) + (2*(a*h^2*(2*f*g - e*h) + c*(f*g^3 - d*g*h^2))*ArcTan[(Sqrt[c]*(g + h*
x) - h*Sqrt[a + c*x^2])/Sqrt[-(c*g^2) - a*h^2]])/(-(c*g^2) - a*h^2)^(3/2) +
 (f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/Sqrt[c])/h^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 389 vs. 2(154) = 308.

Time = 0.60 (sec) , antiderivative size = 390, normalized size of antiderivative = 2.32

method	result
default	$\frac{f \ln\left(x\sqrt{c}+\sqrt{c\,x^{2}+a}\right)}{h^{2}\sqrt{c}}-\frac{(eh-2fg)\ln\left(\frac{\frac{2a\,h^{2}+2c\,g^{2}}{h^{2}}-\frac{2cg\left(x+\frac{g}{h}\right)}{h}+2\sqrt{\frac{a\,h^{2}+c\,g^{2}}{h^{2}}}\sqrt{\left(x+\frac{g}{h}\right)^{2}c-\frac{2cg\left(x+\frac{g}{h}\right)}{h}+\frac{a\,h^{2}+c\,g^{2}}{h^{2}}}}{x+\frac{g}{h}}\right)}{h^{3}\sqrt{\frac{a\,h^{2}+c\,g^{2}}{h^{2}}}}+\frac{(d\,h^{2}-egh+f}{}$

```
[In] int((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] f/h^2*ln(x*c^(1/2)+(c*x^2+a)^(1/2))/c^(1/2)-1/h^3*(e*h-2*f*g)/((a*h^2+c*g^2
)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1/h*g)+2*((a*h^2+c*g^2)/h^2
)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+1/h*g
))+1/h^4*(d*h^2-e*g*h+f*g^2)*(-1/(a*h^2+c*g^2)*h^2/(x+1/h*g)*((x+1/h*g)^2*c
-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)-c*g*h/(a*h^2+c*g^2)/((a*h^2+c*g
^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1/h*g)+2*((a*h^2+c*g^2)/h
^2)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+1/h
*g)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2}{(g+hx)^2\sqrt{a+cx^2}}\,dx = \text{Timed out}$$

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{d+ex+fx^2}{(g+hx)^2\sqrt{a+cx^2}}\,dx = \int \frac{d+ex+fx^2}{\sqrt{a+cx^2}\,(g+hx)^2}\,dx$$

```
[In] integrate((f*x**2+e*x+d)/(h*x+g)**2/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral((d + e*x + f*x**2)/(sqrt(a + c*x**2)*(g + h*x)**2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 419 vs. $2(155) = 310$.

Time = 0.23 (sec) , antiderivative size = 419, normalized size of antiderivative = 2.49

$$
\begin{aligned}
\int \frac{d+ex+fx^2}{(g+hx)^2\sqrt{a+cx^2}}\,dx = &-\frac{\sqrt{cx^2+a}fg^2}{cg^2h^2x+ah^4x+cg^3h+agh^3}+\frac{\sqrt{cx^2+a}eg}{cg^2hx+ah^3x+cg^3+agh^2} \\
&-\frac{\sqrt{cx^2+a}d}{cg^2x+ah^2x+\frac{cg^3}{h}+agh}+\frac{f\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{c}h^2} \\
&+\frac{cfg^3\operatorname{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|}-\frac{ah}{\sqrt{ac}|hx+g|}\right)}{\left(a+\frac{cg^2}{h^2}\right)^{\frac{3}{2}}h^5} \\
&-\frac{ceg^2\operatorname{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|}-\frac{ah}{\sqrt{ac}|hx+g|}\right)}{\left(a+\frac{cg^2}{h^2}\right)^{\frac{3}{2}}h^4} \\
&+\frac{cdg\operatorname{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|}-\frac{ah}{\sqrt{ac}|hx+g|}\right)}{\left(a+\frac{cg^2}{h^2}\right)^{\frac{3}{2}}h^3} \\
&-\frac{2\,fg\operatorname{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|}-\frac{ah}{\sqrt{ac}|hx+g|}\right)}{\sqrt{a+\frac{cg^2}{h^2}}h^3} \\
&+\frac{e\operatorname{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|}-\frac{ah}{\sqrt{ac}|hx+g|}\right)}{\sqrt{a+\frac{cg^2}{h^2}}h^2}
\end{aligned}
$$

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(c*x^2 + a)*f*g^2/(c*g^2*h^2*x + a*h^4*x + c*g^3*h + a*g*h^3) + sqrt(c
*x^2 + a)*e*g/(c*g^2*h*x + a*h^3*x + c*g^3 + a*g*h^2) - sqrt(c*x^2 + a)*d/(
c*g^2*x + a*h^2*x + c*g^3/h + a*g*h) + f*arcsinh(c*x/sqrt(a*c))/(sqrt(c)*h^
2) + c*f*g^3*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*
x + g)))/((a + c*g^2/h^2)^(3/2)*h^5) - c*e*g^2*arcsinh(c*g*x/(sqrt(a*c)*abs
(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(3/2)*h^4) + c*
d*g*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/
((a + c*g^2/h^2)^(3/2)*h^3) - 2*f*g*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g))
- a*h/(sqrt(a*c)*abs(h*x + g)))/(sqrt(a + c*g^2/h^2)*h^3) + e*arcsinh(c*g*x
/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/(sqrt(a + c*g^2/h
^2)*h^2)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{d+ex+fx^2}{(g+hx)^2\sqrt{a+cx^2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2}{(g+hx)^2\sqrt{a+cx^2}}\,dx = \int \frac{f\,x^2+e\,x+d}{{\left(g+h\,x\right)}^2\,\sqrt{c\,x^2+a}}\,dx$$

```
[In] int((d + e*x + f*x^2)/((g + h*x)^2*(a + c*x^2)^(1/2)),x)
```

```
[Out] int((d + e*x + f*x^2)/((g + h*x)^2*(a + c*x^2)^(1/2)), x)
```

3.107 $\int \frac{d+ex+fx^2}{(g+hx)^3\sqrt{a+cx^2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 225

$$\begin{aligned}\int \frac{d+ex+fx^2}{(g+hx)^3\sqrt{a+cx^2}}\,dx &= -\frac{(fg^2-egh+dh^2)\sqrt{a+cx^2}}{2h\,(cg^2+ah^2)\,(g+hx)^2}+\frac{(2ah^2(2fg-eh)+cg(fg^2+h(eg-3dh)))\sqrt{a+cx^2}}{2h\,(cg^2+ah^2)^2\,(g+hx)}\\ &\quad-\frac{(2c^2dg^2+2a^2fh^2-ac(fg^2-h(3eg-dh)))\,\text{arctanh}\left(\frac{ah-cgx}{\sqrt{cg^2+ah^2}\sqrt{a+cx^2}}\right)}{2\,(cg^2+ah^2)^{5/2}}\end{aligned}$$

```
[Out] -1/2*(2*c^2*d*g^2+2*a^2*f*h^2-a*c*(f*g^2-h*(-d*h+3*e*g)))*arctanh((-c*g*x+a
*h)/(a*h^2+c*g^2)^(1/2)/(c*x^2+a)^(1/2))/(a*h^2+c*g^2)^(5/2)-1/2*(d*h^2-e*g
*h+f*g^2)*(c*x^2+a)^(1/2)/h/(a*h^2+c*g^2)/(h*x+g)^2+1/2*(2*a*h^2*(-e*h+2*f*
g)+c*g*(f*g^2+h*(-3*d*h+e*g)))*(c*x^2+a)^(1/2)/h/(a*h^2+c*g^2)^2/(h*x+g)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1665, 821, 739, 212}

$$\begin{aligned}\int \frac{d+ex+fx^2}{(g+hx)^3\sqrt{a+cx^2}}\,dx &= -\frac{\text{arctanh}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)(2a^2fh^2-ac(fg^2-h(3eg-dh))+2c^2dg^2)}{2\,(ah^2+cg^2)^{5/2}}\\ &\quad-\frac{\sqrt{a+cx^2}(dh^2-egh+fg^2)}{2h(g+hx)^2\,(ah^2+cg^2)}+\frac{\sqrt{a+cx^2}(2ah^2(2fg-eh)+cgh(eg-3dh)+cfg^3)}{2h(g+hx)\,(ah^2+cg^2)^2}\end{aligned}$$

```
[In] Int[(d + e*x + f*x^2)/((g + h*x)^3*Sqrt[a + c*x^2]),x]
```

```
[Out] -1/2*((f*g^2 - e*g*h + d*h^2)*Sqrt[a + c*x^2])/(h*(c*g^2 + a*h^2)*(g + h*x)
^2) + ((c*f*g^3 + c*g*h*(e*g - 3*d*h) + 2*a*h^2*(2*f*g - e*h))*Sqrt[a + c*x
^2])/(2*h*(c*g^2 + a*h^2)^2*(g + h*x)) - ((2*c^2*d*g^2 + 2*a^2*f*h^2 - a*c*
(f*g^2 - h*(3*e*g - d*h)))*ArcTanh[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqrt[
a + c*x^2])])/(2*(c*g^2 + a*h^2)^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{\left(fg^2-egh+dh^2\right)\sqrt{a+cx^2}}{2h\left(cg^2+ah^2\right)(g+hx)^2}-\frac{\int \frac{-2(cdg-afg+aeh)-\left(2afh+c\left(eg+\frac{fg^2}{h}-dh\right)\right)x}{(g+hx)^2\sqrt{a+cx^2}}\,dx}{2\left(cg^2+ah^2\right)}$$

$$
\begin{aligned}
&= -\frac{(fg^2-egh+dh^2)\sqrt{a+cx^2}}{2h\,(cg^2+ah^2)\,(g+hx)^2} \\
&\quad + \frac{(cfg^3+cgh(eg-3dh)+2ah^2(2fg-eh))\sqrt{a+cx^2}}{2h\,(cg^2+ah^2)^2\,(g+hx)} \\
&\quad + \frac{(2c^2dg^2+2a^2fh^2-ac(fg^2-h(3eg-dh)))\int\frac{1}{(g+hx)\sqrt{a+cx^2}}\,dx}{2\,(cg^2+ah^2)^2}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{(fg^2-egh+dh^2)\sqrt{a+cx^2}}{2h\,(cg^2+ah^2)\,(g+hx)^2} + \frac{(cfg^3+cgh(eg-3dh)+2ah^2(2fg-eh))\sqrt{a+cx^2}}{2h\,(cg^2+ah^2)^2\,(g+hx)} \\
&\quad - \frac{(2c^2dg^2+2a^2fh^2-ac(fg^2-h(3eg-dh)))\,\mathrm{Subst}\left(\int\frac{1}{cg^2+ah^2-x^2}\,dx, x, \frac{ah-cgx}{\sqrt{a+cx^2}}\right)}{2\,(cg^2+ah^2)^2}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{(fg^2-egh+dh^2)\sqrt{a+cx^2}}{2h\,(cg^2+ah^2)\,(g+hx)^2} + \frac{(cfg^3+cgh(eg-3dh)+2ah^2(2fg-eh))\sqrt{a+cx^2}}{2h\,(cg^2+ah^2)^2\,(g+hx)} \\
&\quad - \frac{(2c^2dg^2+2a^2fh^2-ac(fg^2-h(3eg-dh)))\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{cg^2+ah^2}\sqrt{a+cx^2}}\right)}{2\,(cg^2+ah^2)^{5/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.90

$$
\begin{aligned}
&\int\frac{d+ex+fx^2}{(g+hx)^3\sqrt{a+cx^2}}\,dx \\
&= \frac{\sqrt{a+cx^2}(cg(fg^2x+eg(2g+hx)-dh(4g+3hx))-ah(-fg(3g+4hx)+h(dh+e(g+2hx))))}{2\,(cg^2+ah^2)^2\,(g+hx)^2} \\
&\quad - \frac{(2c^2dg^2+2a^2fh^2-ac(fg^2+h(-3eg+dh)))\arctan\left(\frac{\sqrt{c}(g+hx)-h\sqrt{a+cx^2}}{\sqrt{-cg^2-ah^2}}\right)}{(-cg^2-ah^2)^{5/2}}
\end{aligned}
$$

```
[In] Integrate[(d + e*x + f*x^2)/((g + h*x)^3*Sqrt[a + c*x^2]),x]
```

```
[Out] (Sqrt[a + c*x^2]*(c*g*(f*g^2*x + e*g*(2*g + h*x) - d*h*(4*g + 3*h*x)) - a*h
*(-(f*g*(3*g + 4*h*x)) + h*(d*h + e*(g + 2*h*x)))))/(2*(c*g^2 + a*h^2)^2*(g
 + h*x)^2) - ((2*c^2*d*g^2 + 2*a^2*f*h^2 - a*c*(f*g^2 + h*(-3*e*g + d*h)))*
ArcTan[(Sqrt[c]*(g + h*x) - h*Sqrt[a + c*x^2])/Sqrt[-(c*g^2) - a*h^2]])/(-(
c*g^2) - a*h^2)^(5/2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 809 vs. 2(209) = 418.

Time = 0.64 (sec) , antiderivative size = 810, normalized size of antiderivative = 3.60

method	result
default	$-\frac{f\ln\left(\frac{\frac{2a\,h^{2}+2c\,g^{2}}{h^{2}}-\frac{2cg\left(x+\frac{g}{h}\right)}{h}+2\sqrt{\frac{a\,h^{2}+c\,g^{2}}{h^{2}}}\sqrt{\left(x+\frac{g}{h}\right)^{2}c-\frac{2cg\left(x+\frac{g}{h}\right)}{h}+\frac{a\,h^{2}+c\,g^{2}}{h^{2}}}}{x+\frac{g}{h}}\right)}{h^{3}\sqrt{\frac{a\,h^{2}+c\,g^{2}}{h^{2}}}}+\frac{(eh-2fg)\left(-\frac{h^{2}\sqrt{\left(x+\frac{g}{h}\right)^{2}c-\frac{2cg\left(x+\frac{g}{h}\right)}{h}+\frac{a\,h^{2}-}{h}}}{\left(a\,h^{2}+c\,g^{2}\right)\left(x+\frac{g}{h}\right)}\right.}{}$

```
[In] int((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -f/h^3/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1/h*g)+
2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/
h^2)^(1/2))/(x+1/h*g))+(e*h-2*f*g)/h^4*(-1/(a*h^2+c*g^2)*h^2/(x+1/h*g)*((x+
1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)-c*g*h/(a*h^2+c*g^2)/(
(a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1/h*g)+2*((a*h^
2+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/
2))/(x+1/h*g)))+(d*h^2-e*g*h+f*g^2)/h^5*(-1/2/(a*h^2+c*g^2)*h^2/(x+1/h*g)^2
*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)+3/2*c*g*h/(a*h^2
+c*g^2)*(-1/(a*h^2+c*g^2)*h^2/(x+1/h*g)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a
*h^2+c*g^2)/h^2)^(1/2)-c*g*h/(a*h^2+c*g^2)/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*
(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1/h*g)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+1/h*g)^
2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+1/h*g)))+1/2*c/(a*h^2+c*
g^2)*h^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1/h*g
)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2
)/h^2)^(1/2))/(x+1/h*g)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 531 vs. $2(210) = 420$.

Time = 4.71 (sec) , antiderivative size = 1088, normalized size of antiderivative = 4.84

$$\int \frac{d+ex+fx^2}{(g+hx)^3\sqrt{a+cx^2}}\,dx$$

$$= \left[\frac{(3\,aceg^3h + (2\,c^2d - acf)g^4 - (acd - 2\,a^2f)g^2h^2 + (3\,acegh^3 + (2\,c^2d - acf)g^2h^2 - (acd - 2\,a^2f)h^4)x^2}{}\right.$$

$$-\frac{(3\,aceg^3h + (2\,c^2d - acf)g^4 - (acd - 2\,a^2f)g^2h^2 + (3\,acegh^3 + (2\,c^2d - acf)g^2h^2 - (acd - 2\,a^2f)h^4)x}{}$$

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*((3*a*c*e*g^3*h + (2*c^2*d - a*c*f)*g^4 - (a*c*d - 2*a^2*f)*g^2*h^2 +
(3*a*c*e*g*h^3 + (2*c^2*d - a*c*f)*g^2*h^2 - (a*c*d - 2*a^2*f)*h^4)*x^2 + 2
*(3*a*c*e*g^2*h^2 + (2*c^2*d - a*c*f)*g^3*h - (a*c*d - 2*a^2*f)*g*h^3)*x)*s
qrt(c*g^2 + a*h^2)*log((2*a*c*g*h*x - a*c*g^2 - 2*a^2*h^2 - (2*c^2*g^2 + a*
c*h^2)*x^2 - 2*sqrt(c*g^2 + a*h^2)*(c*g*x - a*h)*sqrt(c*x^2 + a))/(h^2*x^2
+ 2*g*h*x + g^2)) + 2*(2*c^2*e*g^5 + a*c*e*g^3*h^2 - a^2*e*g*h^4 - a^2*d*h^
5 - (4*c^2*d - 3*a*c*f)*g^4*h - (5*a*c*d - 3*a^2*f)*g^2*h^3 + (c^2*f*g^5 +
c^2*e*g^4*h - a*c*e*g^2*h^3 - 2*a^2*e*h^5 - (3*c^2*d - 5*a*c*f)*g^3*h^2 - (
3*a*c*d - 4*a^2*f)*g*h^4)*x)*sqrt(c*x^2 + a))/(c^3*g^8 + 3*a*c^2*g^6*h^2 +
3*a^2*c*g^4*h^4 + a^3*g^2*h^6 + (c^3*g^6*h^2 + 3*a*c^2*g^4*h^4 + 3*a^2*c*g^
2*h^6 + a^3*h^8)*x^2 + 2*(c^3*g^7*h + 3*a*c^2*g^5*h^3 + 3*a^2*c*g^3*h^5 + a
^3*g*h^7)*x), -1/2*((3*a*c*e*g^3*h + (2*c^2*d - a*c*f)*g^4 - (a*c*d - 2*a^2
*f)*g^2*h^2 + (3*a*c*e*g*h^3 + (2*c^2*d - a*c*f)*g^2*h^2 - (a*c*d - 2*a^2*f
)*h^4)*x^2 + 2*(3*a*c*e*g^2*h^2 + (2*c^2*d - a*c*f)*g^3*h - (a*c*d - 2*a^2*
f)*g*h^3)*x)*sqrt(-c*g^2 - a*h^2)*arctan(sqrt(-c*g^2 - a*h^2)*(c*g*x - a*h)
*sqrt(c*x^2 + a)/(a*c*g^2 + a^2*h^2 + (c^2*g^2 + a*c*h^2)*x^2)) - (2*c^2*e*
g^5 + a*c*e*g^3*h^2 - a^2*e*g*h^4 - a^2*d*h^5 - (4*c^2*d - 3*a*c*f)*g^4*h -
 (5*a*c*d - 3*a^2*f)*g^2*h^3 + (c^2*f*g^5 + c^2*e*g^4*h - a*c*e*g^2*h^3 - 2
*a^2*e*h^5 - (3*c^2*d - 5*a*c*f)*g^3*h^2 - (3*a*c*d - 4*a^2*f)*g*h^4)*x)*sq
rt(c*x^2 + a))/(c^3*g^8 + 3*a*c^2*g^6*h^2 + 3*a^2*c*g^4*h^4 + a^3*g^2*h^6 +
 (c^3*g^6*h^2 + 3*a*c^2*g^4*h^4 + 3*a^2*c*g^2*h^6 + a^3*h^8)*x^2 + 2*(c^3*g
^7*h + 3*a*c^2*g^5*h^3 + 3*a^2*c*g^3*h^5 + a^3*g*h^7)*x)]
```

Sympy [F]

$$\int \frac{d+ex+fx^2}{(g+hx)^3\sqrt{a+cx^2}}\,dx = \int \frac{d+ex+fx^2}{\sqrt{a+cx^2}\,(g+hx)^3}\,dx$$

```
[In] integrate((f*x**2+e*x+d)/(h*x+g)**3/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral((d + e*x + f*x**2)/(sqrt(a + c*x**2)*(g + h*x)**3), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 896 vs. $2(210) = 420$.

Time = 0.24 (sec) , antiderivative size = 896, normalized size of antiderivative = 3.98

$$
\begin{aligned}
\int \frac{d+ex+fx^2}{(g+hx)^3\sqrt{a+cx^2}}\,dx = & -\frac{3\sqrt{cx^2+a}cfg^3}{2\left(c^2g^4h^2x+2\,acg^2h^4x+a^2h^6x+c^2g^5h+2\,acg^3h^3+a^2gh^5\right)} \\
& +\frac{3\sqrt{cx^2+a}ceg^2}{2\left(c^2g^4hx+2\,acg^2h^3x+a^2h^5x+c^2g^5+2\,acg^3h^2+a^2gh^4\right)} \\
& -\frac{3\sqrt{cx^2+a}cdg}{2\left(c^2g^4x+2\,acg^2h^2x+a^2h^4x+\frac{c^2g^5}{h}+2\,acg^3h+a^2gh^3\right)} \\
& -\frac{\sqrt{cx^2+a}fg^2}{2\left(cg^2h^3x^2+ah^5x^2+2\,cg^3h^2x+2\,agh^4x+cg^4h+ag^2h^3\right)} \\
& +\frac{\sqrt{cx^2+a}eg}{2\left(cg^2h^2x^2+ah^4x^2+2\,cg^3hx+2\,agh^3x+cg^4+ag^2h^2\right)} \\
& +\frac{2\sqrt{cx^2+a}fg}{cg^2h^2x+ah^4x+cg^3h+agh^3} \\
& -\frac{\sqrt{cx^2+a}d}{2\left(cg^2hx^2+ah^3x^2+2\,cg^3x+2\,agh^2x+\frac{cg^4}{h}+ag^2h\right)} \\
& -\frac{\sqrt{cx^2+a}e}{cg^2hx+ah^3x+cg^3+agh^2} \\
& +\frac{3\,c^2fg^4\,\mathrm{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|}-\frac{ah}{\sqrt{ac}|hx+g|}\right)}{2\left(a+\frac{cg^2}{h^2}\right)^{\frac{5}{2}}h^7} \\
& -\frac{3\,c^2eg^3\,\mathrm{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|}-\frac{ah}{\sqrt{ac}|hx+g|}\right)}{2\left(a+\frac{cg^2}{h^2}\right)^{\frac{5}{2}}h^6} \\
& +\frac{3\,c^2dg^2\,\mathrm{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|}-\frac{ah}{\sqrt{ac}|hx+g|}\right)}{2\left(a+\frac{cg^2}{h^2}\right)^{\frac{5}{2}}h^5} \\
& -\frac{5\,cfg^2\,\mathrm{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|}-\frac{ah}{\sqrt{ac}|hx+g|}\right)}{2\left(a+\frac{cg^2}{h^2}\right)^{\frac{3}{2}}h^5} \\
& +\frac{3\,ceg\,\mathrm{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|}-\frac{ah}{\sqrt{ac}|hx+g|}\right)}{2\left(a+\frac{cg^2}{h^2}\right)^{\frac{3}{2}}h^4} \\
& -\frac{cd\,\mathrm{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|}-\frac{ah}{\sqrt{ac}|hx+g|}\right)}{2\left(a+\frac{cg^2}{h^2}\right)^{\frac{3}{2}}h^3} \\
& +\frac{f\,\mathrm{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|}-\frac{ah}{\sqrt{ac}|hx+g|}\right)}{\sqrt{a+\frac{cg^2}{h^2}}h^3}
\end{aligned}
$$

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -3/2*sqrt(c*x^2 + a)*c*f*g^3/(c^2*g^4*h^2*x + 2*a*c*g^2*h^4*x + a^2*h^6*x +
 c^2*g^5*h + 2*a*c*g^3*h^3 + a^2*g*h^5) + 3/2*sqrt(c*x^2 + a)*c*e*g^2/(c^2*
g^4*h*x + 2*a*c*g^2*h^3*x + a^2*h^5*x + c^2*g^5 + 2*a*c*g^3*h^2 + a^2*g*h^4
) - 3/2*sqrt(c*x^2 + a)*c*d*g/(c^2*g^4*x + 2*a*c*g^2*h^2*x + a^2*h^4*x + c^
2*g^5/h + 2*a*c*g^3*h + a^2*g*h^3) - 1/2*sqrt(c*x^2 + a)*f*g^2/(c*g^2*h^3*x
^2 + a*h^5*x^2 + 2*c*g^3*h^2*x + 2*a*g*h^4*x + c*g^4*h + a*g^2*h^3) + 1/2*s
qrt(c*x^2 + a)*e*g/(c*g^2*h^2*x^2 + a*h^4*x^2 + 2*c*g^3*h*x + 2*a*g*h^3*x +
 c*g^4 + a*g^2*h^2) + 2*sqrt(c*x^2 + a)*f*g/(c*g^2*h^2*x + a*h^4*x + c*g^3*
h + a*g*h^3) - 1/2*sqrt(c*x^2 + a)*d/(c*g^2*h*x^2 + a*h^3*x^2 + 2*c*g^3*x +
 2*a*g*h^2*x + c*g^4/h + a*g^2*h) - sqrt(c*x^2 + a)*e/(c*g^2*h*x + a*h^3*x
+ c*g^3 + a*g*h^2) + 3/2*c^2*f*g^4*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) -
 a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(5/2)*h^7) - 3/2*c^2*e*g^3*
arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a
+ c*g^2/h^2)^(5/2)*h^6) + 3/2*c^2*d*g^2*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x +
g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(5/2)*h^5) - 5/2*c*f*g
^2*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/(
(a + c*g^2/h^2)^(3/2)*h^5) + 3/2*c*e*g*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g
)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(3/2)*h^4) - 1/2*c*d*ar
csinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a +
c*g^2/h^2)^(3/2)*h^3) + f*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqr
t(a*c)*abs(h*x + g)))/(sqrt(a + c*g^2/h^2)*h^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 839 vs. 2(210) = 420.

Time = 0.30 (sec) , antiderivative size = 839, normalized size of antiderivative = 3.73

$$\int \frac{d+ex+fx^2}{(g+hx)^3\sqrt{a+cx^2}}\,dx = -\frac{\left(2\,c^2dg^2-acfg^2+3\,acegh-acdh^2+2\,a^2fh^2\right)\arctan\left(\frac{\left(\sqrt{c}x-\sqrt{cx^2+a}\right)h+\sqrt{c}g}{\sqrt{-cg^2-ah^2}}\right)}{\left(c^2g^4+2\,acg^2h^2+a^2h^4\right)\sqrt{-cg^2-ah^2}} + \frac{2\left(\sqrt{c}x-\sqrt{cx^2+a}\right)^3c^2fg^4h-2\left(\sqrt{c}x-\sqrt{cx^2+a}\right)^3c^2dg^2h^3+5\left(\sqrt{c}x-\sqrt{cx^2+a}\right)^3acfg^2h^3-3\left(\sqrt{c}x\right.}{}$$

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -(2*c^2*d*g^2 - a*c*f*g^2 + 3*a*c*e*g*h - a*c*d*h^2 + 2*a^2*f*h^2)*arctan((
(sqrt(c)*x - sqrt(c*x^2 + a))*h + sqrt(c)*g)/sqrt(-c*g^2 - a*h^2))/((c^2*g^
4 + 2*a*c*g^2*h^2 + a^2*h^4)*sqrt(-c*g^2 - a*h^2)) + (2*(sqrt(c)*x - sqrt(c
*x^2 + a))^3*c^2*f*g^4*h - 2*(sqrt(c)*x - sqrt(c*x^2 + a))^3*c^2*d*g^2*h^3
```

```
+ 5*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a*c*f*g^2*h^3 - 3*(sqrt(c)*x - sqrt(c*x
^2 + a))^3*a*c*e*g*h^4 + (sqrt(c)*x - sqrt(c*x^2 + a))^3*a*c*d*h^5 + 2*(sqr
t(c)*x - sqrt(c*x^2 + a))^2*c^(5/2)*f*g^5 + 2*(sqrt(c)*x - sqrt(c*x^2 + a))
^2*c^(5/2)*e*g^4*h - 6*(sqrt(c)*x - sqrt(c*x^2 + a))^2*c^(5/2)*d*g^3*h^2 +
7*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a*c^(3/2)*f*g^3*h^2 - 5*(sqrt(c)*x - sqrt
(c*x^2 + a))^2*a*c^(3/2)*e*g^2*h^3 + 3*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a*c^
(3/2)*d*g*h^4 - 4*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^2*sqrt(c)*f*g*h^4 + 2*(
sqrt(c)*x - sqrt(c*x^2 + a))^2*a^2*sqrt(c)*e*h^5 - 2*(sqrt(c)*x - sqrt(c*x^
2 + a))*a*c^2*f*g^4*h - 4*(sqrt(c)*x - sqrt(c*x^2 + a))*a*c^2*e*g^3*h^2 + 1
0*(sqrt(c)*x - sqrt(c*x^2 + a))*a*c^2*d*g^2*h^3 - 11*(sqrt(c)*x - sqrt(c*x^
2 + a))*a^2*c*f*g^2*h^3 + 5*(sqrt(c)*x - sqrt(c*x^2 + a))*a^2*c*e*g*h^4 + (
sqrt(c)*x - sqrt(c*x^2 + a))*a^2*c*d*h^5 + a^2*c^(3/2)*f*g^3*h^2 + a^2*c^(3
/2)*e*g^2*h^3 - 3*a^2*c^(3/2)*d*g*h^4 + 4*a^3*sqrt(c)*f*g*h^4 - 2*a^3*sqrt(
c)*e*h^5)/((c^2*g^4*h^2 + 2*a*c*g^2*h^4 + a^2*h^6)*((sqrt(c)*x - sqrt(c*x^2
 + a))^2*h + 2*(sqrt(c)*x - sqrt(c*x^2 + a))*sqrt(c)*g - a*h)^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2}{(g+hx)^3\sqrt{a+cx^2}}\,dx = \int \frac{f\,x^2+e\,x+d}{\left(g+h\,x\right)^3\,\sqrt{c\,x^2+a}}\,dx$$

```
[In] int((d + e*x + f*x^2)/((g + h*x)^3*(a + c*x^2)^(1/2)),x)
```

```
[Out] int((d + e*x + f*x^2)/((g + h*x)^3*(a + c*x^2)^(1/2)), x)
```

3.108 $\int \frac{(g+hx)^3(d+ex+fx^2)}{(a+cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 229

$$\int \frac{(g+hx)^3\,(d+ex+fx^2)}{(a+cx^2)^{3/2}}\,dx = -\frac{(ae-(cd-af)x)(g+hx)^3}{ac\sqrt{a+cx^2}}-\frac{(3cd-4af)h(g+hx)^2\sqrt{a+cx^2}}{3ac^2} -\frac{h(4(3c^2dg^2+4a^2fh^2-ac(7fg^2+3h(3eg+dh)))+ch(6cdg-11afg-9aeh)x)\,\sqrt{a+cx^2}}{6ac^3} -\frac{(3ah^2(3fg+eh)-2cg(fg^2+3h(eg+dh)))\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2c^{5/2}}$$

```
[Out] -1/2*(3*a*h^2*(e*h+3*f*g)-2*c*g*(f*g^2+3*h*(d*h+e*g)))*arctanh(x*c^(1/2)/(c
*x^2+a)^(1/2))/c^(5/2)-(a*e-(-a*f+c*d)*x)*(h*x+g)^3/a/c/(c*x^2+a)^(1/2)-1/3
*(-4*a*f+3*c*d)*h*(h*x+g)^2*(c*x^2+a)^(1/2)/a/c^2-1/6*h*(12*c^2*d*g^2+16*a^
2*f*h^2-4*a*c*(7*f*g^2+3*h*(d*h+3*e*g))+c*h*(-9*a*e*h-11*a*f*g+6*c*d*g)*x)*
(c*x^2+a)^(1/2)/a/c^3
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used

$= \{1659, 847, 794, 223, 212\}$

$$\int \frac{(g+hx)^3\left(d+ex+fx^2\right)}{\left(a+cx^2\right)^{3/2}}\,dx = -\frac{h\sqrt{a+cx^2}(4(4a^2fh^2-ac(3h(dh+3eg)+7fg^2)+3c^2dg^2)+chx(-9aeh-11afg+6cdg))}{6ac^3} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(-3ah^2(eh+3fg)+6cgh(dh+eg)+2cfg^3\right)}{2c^{5/2}} - \frac{h\sqrt{a+cx^2}(g+hx)^2(3cd-4af)}{3ac^2} - \frac{(g+hx)^3(ae-x(cd-af))}{ac\sqrt{a+cx^2}}$$

```
[In] Int[((g + h*x)^3*(d + e*x + f*x^2))/(a + c*x^2)^(3/2),x]
```

```
[Out] -(((a*e - (c*d - a*f)*x)*(g + h*x)^3)/(a*c*Sqrt[a + c*x^2])) - ((3*c*d - 4*
a*f)*h*(g + h*x)^2*Sqrt[a + c*x^2])/(3*a*c^2) - (h*(4*(3*c^2*d*g^2 + 4*a^2*
f*h^2 - a*c*(7*f*g^2 + 3*h*(3*e*g + d*h))) + c*h*(6*c*d*g - 11*a*f*g - 9*a*
e*h)*x)*Sqrt[a + c*x^2])/(6*a*c^3) + ((2*c*f*g^3 + 6*c*g*h*(e*g + d*h) - 3*
a*h^2*(3*f*g + e*h))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*c^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
```

```
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1659

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c*(p
+ 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(ae-(cd-af)x)(g+hx)^3}{ac\sqrt{a+cx^2}}-\frac{\int \frac{(g+hx)^2(-a(fg+3eh)+(3cd-4af)hx)}{\sqrt{a+cx^2}}\,dx}{ac}\\
&= -\frac{(ae-(cd-af)x)(g+hx)^3}{ac\sqrt{a+cx^2}}-\frac{(3cd-4af)h(g+hx)^2\sqrt{a+cx^2}}{3ac^2}\\
&\quad -\frac{\int \frac{(g+hx)(-a(2(3cd-4af)h^2+3cg(fg+3eh))+ch(6cdg-11afg-9aeh)x)}{\sqrt{a+cx^2}}\,dx}{3ac^2}\\
&= -\frac{(ae-(cd-af)x)(g+hx)^3}{ac\sqrt{a+cx^2}}-\frac{(3cd-4af)h(g+hx)^2\sqrt{a+cx^2}}{3ac^2}\\
&\quad -\frac{h(4(3c^2dg^2+4a^2fh^2-ac(7fg^2+3h(3eg+dh)))+ch(6cdg-11afg-9aeh)x)\sqrt{a+cx^2}}{6ac^3}\\
&\quad +\frac{(2cfg^3+6cgh(eg+dh)-3ah^2(3fg+eh))\int \frac{1}{\sqrt{a+cx^2}}\,dx}{2c^2}\\
&= -\frac{(ae-(cd-af)x)(g+hx)^3}{ac\sqrt{a+cx^2}}-\frac{(3cd-4af)h(g+hx)^2\sqrt{a+cx^2}}{3ac^2}\\
&\quad -\frac{h(4(3c^2dg^2+4a^2fh^2-ac(7fg^2+3h(3eg+dh)))+ch(6cdg-11afg-9aeh)x)\sqrt{a+cx^2}}{6ac^3}\\
&\quad +\frac{(2cfg^3+6cgh(eg+dh)-3ah^2(3fg+eh))\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x}{\sqrt{a+cx^2}}\right)}{2c^2}\\
&= -\frac{(ae-(cd-af)x)(g+hx)^3}{ac\sqrt{a+cx^2}}-\frac{(3cd-4af)h(g+hx)^2\sqrt{a+cx^2}}{3ac^2}\\
&\quad -\frac{h(4(3c^2dg^2+4a^2fh^2-ac(7fg^2+3h(3eg+dh)))+ch(6cdg-11afg-9aeh)x)\sqrt{a+cx^2}}{6ac^3}\\
&\quad +\frac{(2cfg^3+6cgh(eg+dh)-3ah^2(3fg+eh))\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2c^{5/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.12

$$\int \frac{(g+hx)^3\left(d+ex+fx^2\right)}{\left(a+cx^2\right)^{3/2}}\,dx=\frac{-16a^3fh^3+6c^3dg^3x+ac^2(6dh(-3g^2-3ghx+h^2x^2)-3e(2g^3+6g^2hx}{}$$

```
[In] Integrate[((g + h*x)^3*(d + e*x + f*x^2))/(a + c*x^2)^(3/2),x]
```

```
[Out] (-16*a^3*f*h^3 + 6*c^3*d*g^3*x + a*c^2*(6*d*h*(-3*g^2 - 3*g*h*x + h^2*x^2)
- 3*e*(2*g^3 + 6*g^2*h*x - 6*g*h^2*x^2 - h^3*x^3) + f*x*(-6*g^3 + 18*g^2*h*
x + 9*g*h^2*x^2 + 2*h^3*x^3)) + a^2*c*h*(f*(36*g^2 + 27*g*h*x - 8*h^2*x^2)
+ 3*h*(4*d*h + 3*e*(4*g + h*x))) + 3*a*Sqrt[c]*(3*a*h^2*(3*f*g + e*h) - 2*c
*(f*g^3 + 3*g*h*(e*g + d*h)))*Sqrt[a + c*x^2]*Log[-(Sqrt[c]*x) + Sqrt[a + c
*x^2]])/(6*a*c^3*Sqrt[a + c*x^2])
```

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.24

method	result
risch	$-\frac{h\left(-2f\,h^2c\,x^2-3ce\,h^2x-9cfghx+10af\,h^2-6cd\,h^2-18cegh-18cf\,g^2\right)\sqrt{c\,x^2+a}}{6c^3}-\frac{\frac{ae\,h^3x}{\sqrt{c\,x^2+a}}-\frac{2c^2d\,g^3x}{a\sqrt{c\,x^2+a}}+\frac{3afg\,h^2x}{\sqrt{c\,x^2+a}}+(3ace\,h^3+9a}{}$
default	$\frac{d\,g^3x}{a\sqrt{c\,x^2+a}}+f\,h^3\left(\frac{x^4}{3c\sqrt{c\,x^2+a}}-\frac{4a\left(\frac{x^2}{c\sqrt{c\,x^2+a}}+\frac{2a}{c^2\sqrt{c\,x^2+a}}\right)}{3c}\right)+(e\,h^3+3fg\,h^2)\left(\frac{x^3}{2c\sqrt{c\,x^2+a}}-\frac{3a\left(-\frac{x}{c\sqrt{c\,x^2+a}}+\right.}{}\right.$

```
[In] int((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*h*(-2*c*f*h^2*x^2-3*c*e*h^2*x-9*c*f*g*h*x+10*a*f*h^2-6*c*d*h^2-18*c*e*
g*h-18*c*f*g^2)/c^3*(c*x^2+a)^(1/2)-1/2/c^2*(a*e*h^3*x/(c*x^2+a)^(1/2)-2*c^
2*d*g^3*x/a/(c*x^2+a)^(1/2)+3*a*f*g*h^2*x/(c*x^2+a)^(1/2)+(3*a*c*e*h^3+9*a*
c*f*g*h^2-6*c^2*d*g*h^2-6*c^2*e*g^2*h-2*c^2*f*g^3)*(-x/c/(c*x^2+a)^(1/2)+1/
c^(3/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2)))-(-2*a^2*f*h^3+2*a*c*d*h^3+6*a*c*e*g*
h^2+6*a*c*f*g^2*h-6*c^2*d*g^2*h-2*c^2*e*g^3)/c/(c*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 758, normalized size of antiderivative = 3.31

$$\int \frac{(g+hx)^3\,(d+ex+fx^2)}{(a+cx^2)^{3/2}}\,dx = \left[-\frac{3\,(2\,a^2cfg^3+6\,a^2ceg^2h-3\,a^3eh^3+3\,(2\,a^2cd-3\,a^3f)gh^2+(2\,ac^2fg^3+}{}\right.$$

$$-\frac{3\,(2\,a^2cfg^3+6\,a^2ceg^2h-3\,a^3eh^3+3\,(2\,a^2cd-3\,a^3f)gh^2+(2\,ac^2fg^3+6\,ac^2eg^2h-3\,a^2ceh^3+3\,(2\,ac^2d-}{}$$

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*(2*a^2*c*f*g^3 + 6*a^2*c*e*g^2*h - 3*a^3*e*h^3 + 3*(2*a^2*c*d - 3
*a^3*f)*g*h^2 + (2*a*c^2*f*g^3 + 6*a*c^2*e*g^2*h - 3*a^2*c*e*h^3 + 3*(2*a*c
^2*d - 3*a^2*c*f)*g*h^2)*x^2)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sqrt
(c)*x - a) - 2*(2*a*c^2*f*h^3*x^4 - 6*a*c^2*e*g^3 + 36*a^2*c*e*g*h^2 - 18*(
a*c^2*d - 2*a^2*c*f)*g^2*h + 4*(3*a^2*c*d - 4*a^3*f)*h^3 + 3*(3*a*c^2*f*g*h
^2 + a*c^2*e*h^3)*x^3 + 2*(9*a*c^2*f*g^2*h + 9*a*c^2*e*g*h^2 + (3*a*c^2*d -
 4*a^2*c*f)*h^3)*x^2 - 3*(6*a*c^2*e*g^2*h - 3*a^2*c*e*h^3 - 2*(c^3*d - a*c^
2*f)*g^3 + 3*(2*a*c^2*d - 3*a^2*c*f)*g*h^2)*x)*sqrt(c*x^2 + a))/(a*c^4*x^2
+ a^2*c^3), -1/6*(3*(2*a^2*c*f*g^3 + 6*a^2*c*e*g^2*h - 3*a^3*e*h^3 + 3*(2*a
^2*c*d - 3*a^3*f)*g*h^2 + (2*a*c^2*f*g^3 + 6*a*c^2*e*g^2*h - 3*a^2*c*e*h^3
+ 3*(2*a*c^2*d - 3*a^2*c*f)*g*h^2)*x^2)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x
^2 + a)) - (2*a*c^2*f*h^3*x^4 - 6*a*c^2*e*g^3 + 36*a^2*c*e*g*h^2 - 18*(a*c^
2*d - 2*a^2*c*f)*g^2*h + 4*(3*a^2*c*d - 4*a^3*f)*h^3 + 3*(3*a*c^2*f*g*h^2 +
 a*c^2*e*h^3)*x^3 + 2*(9*a*c^2*f*g^2*h + 9*a*c^2*e*g*h^2 + (3*a*c^2*d - 4*a
^2*c*f)*h^3)*x^2 - 3*(6*a*c^2*e*g^2*h - 3*a^2*c*e*h^3 - 2*(c^3*d - a*c^2*f)
*g^3 + 3*(2*a*c^2*d - 3*a^2*c*f)*g*h^2)*x)*sqrt(c*x^2 + a))/(a*c^4*x^2 + a^
2*c^3)]
```

Sympy [F]

$$\int \frac{(g+hx)^3\,(d+ex+fx^2)}{(a+cx^2)^{3/2}}\,dx = \int \frac{(g+hx)^3\,(d+ex+fx^2)}{(a+cx^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((h*x+g)**3*(f*x**2+e*x+d)/(c*x**2+a)**(3/2),x)
```

```
[Out] Integral((g + h*x)**3*(d + e*x + f*x**2)/(a + c*x**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.51

$$
\begin{aligned}
\int \frac{(g+hx)^3\,(d+ex+fx^2)}{(a+cx^2)^{3/2}}\,dx &= \frac{fh^3x^4}{3\sqrt{cx^2+a}c} - \frac{4\,afh^3x^2}{3\sqrt{cx^2+a}c^2} \\
&+ \frac{dg^3x}{\sqrt{cx^2+a}a} - \frac{eg^3}{\sqrt{cx^2+a}c} - \frac{3\,dg^2h}{\sqrt{cx^2+a}c} - \frac{8\,a^2fh^3}{3\sqrt{cx^2+a}c^3} \\
&+ \frac{(3\,fgh^2+eh^3)x^3}{2\sqrt{cx^2+a}c} + \frac{(3\,fg^2h+3\,egh^2+dh^3)x^2}{\sqrt{cx^2+a}c} + \frac{3\,(3\,fgh^2+eh^3)ax}{2\sqrt{cx^2+a}c^2} \\
&- \frac{(fg^3+3\,eg^2h+3\,dgh^2)x}{\sqrt{cx^2+a}c} - \frac{3\,(3\,fgh^2+eh^3)a\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{2\,c^{\frac{5}{2}}} \\
&+ \frac{(fg^3+3\,eg^2h+3\,dgh^2)\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{c^{\frac{3}{2}}} + \frac{2\,(3\,fg^2h+3\,egh^2+dh^3)a}{\sqrt{cx^2+a}c^2}
\end{aligned}
$$

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/3*f*h^3*x^4/(sqrt(c*x^2 + a)*c) - 4/3*a*f*h^3*x^2/(sqrt(c*x^2 + a)*c^2) +
 d*g^3*x/(sqrt(c*x^2 + a)*a) - e*g^3/(sqrt(c*x^2 + a)*c) - 3*d*g^2*h/(sqrt(
c*x^2 + a)*c) - 8/3*a^2*f*h^3/(sqrt(c*x^2 + a)*c^3) + 1/2*(3*f*g*h^2 + e*h^
3)*x^3/(sqrt(c*x^2 + a)*c) + (3*f*g^2*h + 3*e*g*h^2 + d*h^3)*x^2/(sqrt(c*x^
2 + a)*c) + 3/2*(3*f*g*h^2 + e*h^3)*a*x/(sqrt(c*x^2 + a)*c^2) - (f*g^3 + 3*
e*g^2*h + 3*d*g*h^2)*x/(sqrt(c*x^2 + a)*c) - 3/2*(3*f*g*h^2 + e*h^3)*a*arcs
inh(c*x/sqrt(a*c))/c^(5/2) + (f*g^3 + 3*e*g^2*h + 3*d*g*h^2)*arcsinh(c*x/sq
rt(a*c))/c^(3/2) + 2*(3*f*g^2*h + 3*e*g*h^2 + d*h^3)*a/(sqrt(c*x^2 + a)*c^2
)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.45

$$
\begin{aligned}
\int \frac{(g+hx)^3\,(d+ex+fx^2)}{(a+cx^2)^{3/2}}\,dx &= \frac{\left(\left(\left(\left(\frac{2\,fh^3x}{c} + \frac{3\,(3\,ac^4fgh^2+ac^4eh^3)}{ac^5}\right)x + \frac{2\,(9\,ac^4fg^2h+9\,ac^4egh^2+3\,ac^4dh^3-4\,a^2c^3fh^3)}{ac^5}\right)x\right.\right.}{} \\
&- \frac{(2\,cfg^3+6\,ceg^2h+6\,cdgh^2-9\,afgh^2-3\,aeh^3)\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+a}\right|\right)}{2\,c^{\frac{5}{2}}}
\end{aligned}
$$

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/6*((((2*f*h^3*x/c + 3*(3*a*c^4*f*g*h^2 + a*c^4*e*h^3)/(a*c^5))*x + 2*(9*a
*c^4*f*g^2*h + 9*a*c^4*e*g*h^2 + 3*a*c^4*d*h^3 - 4*a^2*c^3*f*h^3)/(a*c^5))*
```

```
x + 3*(2*c^5*d*g^3 - 2*a*c^4*f*g^3 - 6*a*c^4*e*g^2*h - 6*a*c^4*d*g*h^2 + 9*
a^2*c^3*f*g*h^2 + 3*a^2*c^3*e*h^3)/(a*c^5))*x - 2*(3*a*c^4*e*g^3 + 9*a*c^4*
d*g^2*h - 18*a^2*c^3*f*g^2*h - 18*a^2*c^3*e*g*h^2 - 6*a^2*c^3*d*h^3 + 8*a^3
*c^2*f*h^3)/(a*c^5))/sqrt(c*x^2 + a) - 1/2*(2*c*f*g^3 + 6*c*e*g^2*h + 6*c*d
*g*h^2 - 9*a*f*g*h^2 - 3*a*e*h^3)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c^
(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(g+hx)^3\,(d+ex+fx^2)}{(a+cx^2)^{3/2}}\,dx = \int \frac{(g+h\,x)^3\,(f\,x^2+e\,x+d)}{(c\,x^2+a)^{3/2}}\,dx$$

```
[In] int(((g + h*x)^3*(d + e*x + f*x^2))/(a + c*x^2)^(3/2),x)
[Out] int(((g + h*x)^3*(d + e*x + f*x^2))/(a + c*x^2)^(3/2), x)
```

3.109 $\int \frac{(g+hx)^2(d+ex+fx^2)}{(a+cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 149

$$\begin{aligned}\int \frac{(g+hx)^2(d+ex+fx^2)}{(a+cx^2)^{3/2}}\,dx &= -\frac{(ae-(cd-af)x)(g+hx)^2}{ac\sqrt{a+cx^2}} \\ &\quad - \frac{h(4(cdg-a(2fg+eh))+(2cd-3af)hx)\sqrt{a+cx^2}}{2ac^2} \\ &\quad + \frac{((2cd-3af)h^2+2cg(fg+2eh))\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2c^{5/2}}\end{aligned}$$

```
[Out] 1/2*((-3*a*f+2*c*d)*h^2+2*c*g*(2*e*h+f*g))*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2
))/c^(5/2)-(a*e-(-a*f+c*d)*x)*(h*x+g)^2/a/c/(c*x^2+a)^(1/2)-1/2*h*(4*c*d*g-
4*a*(e*h+2*f*g)+(-3*a*f+2*c*d)*h*x)*(c*x^2+a)^(1/2)/a/c^2
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1659, 794, 223, 212}

$$\begin{aligned}\int \frac{(g+hx)^2(d+ex+fx^2)}{(a+cx^2)^{3/2}}\,dx &= \frac{\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)(h^2(2cd-3af)+2cg(2eh+fg))}{2c^{5/2}} \\ &\quad - \frac{h\sqrt{a+cx^2}(4(cdg-a(eh+2fg))+hx(2cd-3af))}{2ac^2} - \frac{(g+hx)^2(ae-x(cd-af))}{ac\sqrt{a+cx^2}}\end{aligned}$$

```
[In] Int[((g + h*x)^2*(d + e*x + f*x^2))/(a + c*x^2)^(3/2),x]
```

```
[Out] -(((a*e - (c*d - a*f)*x)*(g + h*x)^2)/(a*c*Sqrt[a + c*x^2])) - (h*(4*(c*d*g
 - a*(2*f*g + e*h)) + (2*c*d - 3*a*f)*h*x)*Sqrt[a + c*x^2])/(2*a*c^2) + (((
2*c*d - 3*a*f)*h^2 + 2*c*g*(f*g + 2*e*h))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^
2]])/(2*c^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 1659

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c*(p
+ 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(ae-(cd-af)x)(g+hx)^2}{ac\sqrt{a+cx^2}}-\frac{\int \frac{(g+hx)(-a(fg+2eh)+(2cd-3af)hx)}{\sqrt{a+cx^2}}\,dx}{ac}\\
&= -\frac{(ae-(cd-af)x)(g+hx)^2}{ac\sqrt{a+cx^2}}\\
&\quad -\frac{h(4(cdg-a(2fg+eh))+(2cd-3af)hx)\sqrt{a+cx^2}}{2ac^2}\\
&\quad +\frac{((2cd-3af)h^2+2cg(fg+2eh))\int \frac{1}{\sqrt{a+cx^2}}\,dx}{2c^2}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{(ae-(cd-af)x)(g+hx)^2}{ac\sqrt{a+cx^2}} \\
&\quad - \frac{h(4(cdg-a(2fg+eh))+(2cd-3af)hx)\sqrt{a+cx^2}}{2ac^2} \\
&\quad + \frac{((2cd-3af)h^2+2cg(fg+2eh))\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{2c^2} \\
&= -\frac{(ae-(cd-af)x)(g+hx)^2}{ac\sqrt{a+cx^2}} \\
&\quad - \frac{h(4(cdg-a(2fg+eh))+(2cd-3af)hx)\sqrt{a+cx^2}}{2ac^2} \\
&\quad + \frac{((2cd-3af)h^2+2cg(fg+2eh))\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2c^{5/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.11

$$
\int \frac{(g+hx)^2\,(d+ex+fx^2)}{(a+cx^2)^{3/2}}\,dx = \frac{\frac{\sqrt{c}(2c^2dg^2x+a^2h(8fg+4eh+3fhx)+ac(-2dh(2g+hx)-2e(g^2+2ghx-h^2x^2)+fx(-2g^2+4ghx+h^2}{a\sqrt{a+cx^2}}}{2c^{5/2}}
$$

```
[In] Integrate[((g + h*x)^2*(d + e*x + f*x^2))/(a + c*x^2)^(3/2),x]
```

```
[Out] ((Sqrt[c]*(2*c^2*d*g^2*x + a^2*h*(8*f*g + 4*e*h + 3*f*h*x) + a*c*(-2*d*h*(2
*g + h*x) - 2*e*(g^2 + 2*g*h*x - h^2*x^2) + f*x*(-2*g^2 + 4*g*h*x + h^2*x^2
))))/(a*Sqrt[a + c*x^2]) + (3*a*f*h^2 - 2*c*(f*g^2 + h*(2*e*g + d*h)))*Log[
-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/(2*c^(5/2))
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.30

method	result
risch	$\frac{h(fxh+2eh+4fg)\sqrt{c x^2+a}}{2c^2} - \frac{\frac{afh^2x}{\sqrt{cx^2+a}} - \frac{2c^2dg^2x}{a\sqrt{cx^2+a}} + (3acfh^2-2c^2dh^2-4c^2egh-2c^2fg^2)\left(-\frac{x}{c\sqrt{cx^2+a}}+\frac{\ln\left(x\sqrt{c}+\sqrt{cx^2+a}\right)}{c^{\frac{3}{2}}}\right) - 2e}{2c^2}$
default	$\frac{dg^2x}{a\sqrt{cx^2+a}} + fh^2\left(\frac{x^3}{2c\sqrt{cx^2+a}} - \frac{3a\left(-\frac{x}{c\sqrt{cx^2+a}}+\frac{\ln\left(x\sqrt{c}+\sqrt{cx^2+a}\right)}{c^{\frac{3}{2}}}\right)}{2c}\right) + (eh^2+2fgh)\left(\frac{x^2}{c\sqrt{cx^2+a}} + \frac{2a}{c^2\sqrt{cx^2+a}}\right.$

```
[In] int((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*h*(f*h*x+2*e*h+4*f*g)/c^2*(c*x^2+a)^(1/2)-1/2/c^2*(a*f*h^2*x/(c*x^2+a)^
(1/2)-2*c^2*d*g^2*x/a/(c*x^2+a)^(1/2)+(3*a*c*f*h^2-2*c^2*d*h^2-4*c^2*e*g*h-
2*c^2*f*g^2)*(-x/c/(c*x^2+a)^(1/2)+1/c^(3/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2)))
-(2*a*c*e*h^2+4*a*c*f*g*h-4*c^2*d*g*h-2*c^2*e*g^2)/c/(c*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 530, normalized size of antiderivative = 3.56

$$\int \frac{(g+hx)^2\,(d+ex+fx^2)}{(a+cx^2)^{3/2}}\,dx = \Bigg[-\frac{(2\,a^2cfg^2+4\,a^2cegh+(2\,a^2cd-3\,a^3f)h^2+(2\,ac^2fg^2+4\,ac^2egh+(2}{}$$

$$-\frac{(2\,a^2cfg^2+4\,a^2cegh+(2\,a^2cd-3\,a^3f)h^2+(2\,ac^2fg^2+4\,ac^2egh+(2\,ac^2d-3\,a^2cf)h^2)x^2)\sqrt{-c}\arctan\Big(}{}$$

```
[In] integrate((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((2*a^2*c*f*g^2 + 4*a^2*c*e*g*h + (2*a^2*c*d - 3*a^3*f)*h^2 + (2*a*c^
2*f*g^2 + 4*a*c^2*e*g*h + (2*a*c^2*d - 3*a^2*c*f)*h^2)*x^2)*sqrt(c)*log(-2*
c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) - 2*(a*c^2*f*h^2*x^3 - 2*a*c^2*e*g
^2 + 4*a^2*c*e*h^2 - 4*(a*c^2*d - 2*a^2*c*f)*g*h + 2*(2*a*c^2*f*g*h + a*c^2
*e*h^2)*x^2 - (4*a*c^2*e*g*h - 2*(c^3*d - a*c^2*f)*g^2 + (2*a*c^2*d - 3*a^2
*c*f)*h^2)*x)*sqrt(c*x^2 + a))/(a*c^4*x^2 + a^2*c^3), -1/2*((2*a^2*c*f*g^2
+ 4*a^2*c*e*g*h + (2*a^2*c*d - 3*a^3*f)*h^2 + (2*a*c^2*f*g^2 + 4*a*c^2*e*g*
h + (2*a*c^2*d - 3*a^2*c*f)*h^2)*x^2)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2
 + a)) - (a*c^2*f*h^2*x^3 - 2*a*c^2*e*g^2 + 4*a^2*c*e*h^2 - 4*(a*c^2*d - 2*
a^2*c*f)*g*h + 2*(2*a*c^2*f*g*h + a*c^2*e*h^2)*x^2 - (4*a*c^2*e*g*h - 2*(c^
3*d - a*c^2*f)*g^2 + (2*a*c^2*d - 3*a^2*c*f)*h^2)*x)*sqrt(c*x^2 + a))/(a*c^
4*x^2 + a^2*c^3)]
```

Sympy [F]

$$\int \frac{(g+hx)^2\,(d+ex+fx^2)}{(a+cx^2)^{3/2}}\,dx = \int \frac{(g+hx)^2\,(d+ex+fx^2)}{(a+cx^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((h*x+g)**2*(f*x**2+e*x+d)/(c*x**2+a)**(3/2),x)
```

```
[Out] Integral((g + h*x)**2*(d + e*x + f*x**2)/(a + c*x**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.52

$$\int \frac{(g+hx)^2\left(d+ex+fx^2\right)}{\left(a+cx^2\right)^{3/2}}\,dx = \frac{fh^2x^3}{2\sqrt{cx^2+a}c} + \frac{dg^2x}{\sqrt{cx^2+a}a} + \frac{3\,afh^2x}{2\sqrt{cx^2+a}c^2} - \frac{3\,afh^2\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{2\,c^{\frac{5}{2}}} - \frac{eg^2}{\sqrt{cx^2+a}c} - \frac{2\,dgh}{\sqrt{cx^2+a}c} + \frac{(2\,fgh+eh^2)x^2}{\sqrt{cx^2+a}c} - \frac{(fg^2+2\,egh+dh^2)x}{\sqrt{cx^2+a}c} + \frac{(fg^2+2\,egh+dh^2)\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{c^{\frac{3}{2}}} + \frac{2\,(2\,fgh+eh^2)a}{\sqrt{cx^2+a}c^2}$$

```
[In] integrate((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*f*h^2*x^3/(sqrt(c*x^2 + a)*c) + d*g^2*x/(sqrt(c*x^2 + a)*a) + 3/2*a*f*h
^2*x/(sqrt(c*x^2 + a)*c^2) - 3/2*a*f*h^2*arcsinh(c*x/sqrt(a*c))/c^(5/2) - e
*g^2/(sqrt(c*x^2 + a)*c) - 2*d*g*h/(sqrt(c*x^2 + a)*c) + (2*f*g*h + e*h^2)*
x^2/(sqrt(c*x^2 + a)*c) - (f*g^2 + 2*e*g*h + d*h^2)*x/(sqrt(c*x^2 + a)*c) +
 (f*g^2 + 2*e*g*h + d*h^2)*arcsinh(c*x/sqrt(a*c))/c^(3/2) + 2*(2*f*g*h + e*
h^2)*a/(sqrt(c*x^2 + a)*c^2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.44

$$\int \frac{(g+hx)^2\left(d+ex+fx^2\right)}{\left(a+cx^2\right)^{3/2}}\,dx = \frac{\left(\left(\frac{fh^2x}{c} + \frac{2\,(2\,ac^3fgh+ac^3eh^2)}{ac^4}\right)x + \frac{2\,c^4dg^2-2\,ac^3fg^2-4\,ac^3egh-2\,ac^3dh^2+3\,a^2c^2fh^2}{ac^4}\right)x -}{2\sqrt{cx^2+a}} - \frac{(2\,cfg^2+4\,cegh+2\,cdh^2-3\,afh^2)\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+a}\right|\right)}{2\,c^{\frac{5}{2}}}$$

```
[In] integrate((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(((f*h^2*x/c + 2*(2*a*c^3*f*g*h + a*c^3*e*h^2)/(a*c^4))*x + (2*c^4*d*g^
2 - 2*a*c^3*f*g^2 - 4*a*c^3*e*g*h - 2*a*c^3*d*h^2 + 3*a^2*c^2*f*h^2)/(a*c^4
))*x - 2*(a*c^3*e*g^2 + 2*a*c^3*d*g*h - 4*a^2*c^2*f*g*h - 2*a^2*c^2*e*h^2)/
(a*c^4))/sqrt(c*x^2 + a) - 1/2*(2*c*f*g^2 + 4*c*e*g*h + 2*c*d*h^2 - 3*a*f*h
^2)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(g+hx)^2\,(d+ex+fx^2)}{\left(a+cx^2\right)^{3/2}}\,dx=\int \frac{{\left(g+h\,x\right)}^2\,\left(f\,x^2+e\,x+d\right)}{{\left(c\,x^2+a\right)}^{3/2}}\,dx$$

```
[In] int(((g + h*x)^2*(d + e*x + f*x^2))/(a + c*x^2)^(3/2),x)
[Out] int(((g + h*x)^2*(d + e*x + f*x^2))/(a + c*x^2)^(3/2), x)
```

3.110 $\int \frac{(g+hx)\left(d+ex+fx^2\right)}{\left(a+cx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 100

$$\int \frac{(g+hx)\left(d+ex+fx^2\right)}{\left(a+cx^2\right)^{3/2}}\,dx = -\frac{(ae-(cd-af)x)(g+hx)}{ac\sqrt{a+cx^2}} - \frac{(cd-2af)h\sqrt{a+cx^2}}{ac^2} + \frac{(fg+eh)\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{c^{3/2}}$$

```
[Out] (e*h+f*g)*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(3/2)-(a*e-(-a*f+c*d)*x)*(h*
x+g)/a/c/(c*x^2+a)^(1/2)-(-2*a*f+c*d)*h*(c*x^2+a)^(1/2)/a/c^2
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1659, 655, 223, 212}

$$\int \frac{(g+hx)\left(d+ex+fx^2\right)}{\left(a+cx^2\right)^{3/2}}\,dx = \frac{\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)(eh+fg)}{c^{3/2}} - \frac{h\sqrt{a+cx^2}(cd-2af)}{ac^2} - \frac{(g+hx)(ae-x(cd-af))}{ac\sqrt{a+cx^2}}$$

```
[In] Int[((g + h*x)*(d + e*x + f*x^2))/(a + c*x^2)^(3/2),x]
```

```
[Out] -(((a*e - (c*d - a*f)*x)*(g + h*x))/(a*c*Sqrt[a + c*x^2])) - ((c*d - 2*a*f)
*h*Sqrt[a + c*x^2])/(a*c^2) + ((f*g + e*h)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x
^2]])/c^(3/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1659

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c*(p
+ 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(ae-(cd-af)x)(g+hx)}{ac\sqrt{a+cx^2}}-\frac{\int \frac{-a(fg+eh)+(cd-2af)hx}{\sqrt{a+cx^2}}\,dx}{ac}\\
&= -\frac{(ae-(cd-af)x)(g+hx)}{ac\sqrt{a+cx^2}}-\frac{(cd-2af)h\sqrt{a+cx^2}}{ac^2}+\frac{(fg+eh)\int \frac{1}{\sqrt{a+cx^2}}\,dx}{c}\\
&= -\frac{(ae-(cd-af)x)(g+hx)}{ac\sqrt{a+cx^2}}-\frac{(cd-2af)h\sqrt{a+cx^2}}{ac^2}\\
&\quad+\frac{(fg+eh)\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x}{\sqrt{a+cx^2}}\right)}{c}\\
&= -\frac{(ae-(cd-af)x)(g+hx)}{ac\sqrt{a+cx^2}}-\frac{(cd-2af)h\sqrt{a+cx^2}}{ac^2}+\frac{(fg+eh)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{c^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.04

$$\int \frac{(g+hx)\left(d+ex+fx^2\right)}{\left(a+cx^2\right)^{3/2}}\,dx = \frac{-aceg-acdh+2a^2fh+c^2dgx-acfgx-acehx+acfhx^2}{ac^2\sqrt{a+cx^2}} + \frac{(-fg-eh)\log\left(-\sqrt{c}x+\sqrt{a+cx^2}\right)}{c^{3/2}}$$

```
[In] Integrate[((g + h*x)*(d + e*x + f*x^2))/(a + c*x^2)^(3/2),x]
```

```
[Out] (-(a*c*e*g) - a*c*d*h + 2*a^2*f*h + c^2*d*g*x - a*c*f*g*x - a*c*e*h*x + a*c
*f*h*x^2)/(a*c^2*Sqrt[a + c*x^2]) + ((-(f*g) - e*h)*Log[-(Sqrt[c]*x) + Sqrt
[a + c*x^2]])/c^(3/2)
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.13

method	result	size
risch	$\frac{fh\sqrt{c\,x^2+a}}{c^2}+\frac{\frac{cdgx}{a\sqrt{c\,x^2+a}}+(ehc+cfg)\left(-\frac{x}{c\sqrt{c\,x^2+a}}+\frac{\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{c^{\frac{3}{2}}}\right)-\frac{-afh+cdh+ceg}{c\sqrt{c\,x^2+a}}}{c}$	113
default	$\frac{dgx}{a\sqrt{c\,x^2+a}}+fh\left(\frac{x^2}{c\sqrt{c\,x^2+a}}+\frac{2a}{c^2\sqrt{c\,x^2+a}}\right)+(eh+fg)\left(-\frac{x}{c\sqrt{c\,x^2+a}}+\frac{\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{c^{\frac{3}{2}}}\right)-\frac{dh+eg}{c\sqrt{c\,x^2+a}}$	118

```
[In] int((h*x+g)*(f*x^2+e*x+d)/(c*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*f*h*(c*x^2+a)^(1/2)+1/c*(c*d*g*x/a/(c*x^2+a)^(1/2)+(c*e*h+c*f*g)*(-x/
c/(c*x^2+a)^(1/2)+1/c^(3/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2)))-(-a*f*h+c*d*h+c*
e*g)/c/(c*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 278, normalized size of antiderivative = 2.78

$$\int \frac{(g+hx)\left(d+ex+fx^2\right)}{\left(a+cx^2\right)^{3/2}}\,dx = \left[\frac{\left(a^2fg+a^2eh+(acfg+aceh)x^2\right)\sqrt{c}\log\left(-2\,cx^2-2\,\sqrt{cx^2+a}\sqrt{c}x-a\right)}{2\,(ac^3x}\right.$$

$$-\frac{\left(a^2fg+a^2eh+(acfg+aceh)x^2\right)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}x}{\sqrt{cx^2+a}}\right)-\left(acfhx^2-aceg-\left(acd-2\,a^2f\right)h-(aceh-(c\right.}{ac^3x^2+a^2c^2}$$

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((a^2*f*g + a^2*e*h + (a*c*f*g + a*c*e*h)*x^2)*sqrt(c)*log(-2*c*x^2 -
2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 2*(a*c*f*h*x^2 - a*c*e*g - (a*c*d - 2*a^
2*f)*h - (a*c*e*h - (c^2*d - a*c*f)*g)*x)*sqrt(c*x^2 + a))/(a*c^3*x^2 + a^2
*c^2), -((a^2*f*g + a^2*e*h + (a*c*f*g + a*c*e*h)*x^2)*sqrt(-c)*arctan(sqrt
(-c)*x/sqrt(c*x^2 + a)) - (a*c*f*h*x^2 - a*c*e*g - (a*c*d - 2*a^2*f)*h - (a
*c*e*h - (c^2*d - a*c*f)*g)*x)*sqrt(c*x^2 + a))/(a*c^3*x^2 + a^2*c^2)]
```

Sympy [A] (verification not implemented)

Time = 6.33 (sec) , antiderivative size = 209, normalized size of antiderivative = 2.09

$$
\begin{aligned}
\int \frac{(g+hx)\left(d+ex+fx^2\right)}{\left(a+cx^2\right)^{3/2}}\,dx &= dh\left(\begin{cases} -\frac{1}{c\sqrt{a+cx^2}} & \text{for}\ c \neq 0 \\ \frac{x^2}{2a^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right) \\
&+ eg\left(\begin{cases} -\frac{1}{c\sqrt{a+cx^2}} & \text{for}\ c \neq 0 \\ \frac{x^2}{2a^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right) + eh\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{c^{\frac{3}{2}}} - \frac{x}{\sqrt{a}c\sqrt{1+\frac{cx^2}{a}}}\right) \\
&+ fg\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{c^{\frac{3}{2}}} - \frac{x}{\sqrt{a}c\sqrt{1+\frac{cx^2}{a}}}\right) \\
&+ fh\left(\begin{cases} \frac{2a}{c^2\sqrt{a+cx^2}} + \frac{x^2}{c\sqrt{a+cx^2}} & \text{for}\ c \neq 0 \\ \frac{x^4}{4a^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right) + \frac{dgx}{a^{\frac{3}{2}}\sqrt{1+\frac{cx^2}{a}}}
\end{aligned}
$$

```
[In] integrate((h*x+g)*(f*x**2+e*x+d)/(c*x**2+a)**(3/2),x)
```

```
[Out] d*h*Piecewise((-1/(c*sqrt(a + c*x**2)), Ne(c, 0)), (x**2/(2*a**(3/2)), True
)) + e*g*Piecewise((-1/(c*sqrt(a + c*x**2)), Ne(c, 0)), (x**2/(2*a**(3/2)),
 True)) + e*h*(asinh(sqrt(c)*x/sqrt(a))/c**(3/2) - x/(sqrt(a)*c*sqrt(1 + c*
x**2/a))) + f*g*(asinh(sqrt(c)*x/sqrt(a))/c**(3/2) - x/(sqrt(a)*c*sqrt(1 +
c*x**2/a))) + f*h*Piecewise((2*a/(c**2*sqrt(a + c*x**2)) + x**2/(c*sqrt(a +
 c*x**2)), Ne(c, 0)), (x**4/(4*a**(3/2)), True)) + d*g*x/(a**(3/2)*sqrt(1 +
 c*x**2/a))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.26

$$\int \frac{(g+hx)\left(d+ex+fx^2\right)}{\left(a+cx^2\right)^{3/2}}\,dx = \frac{fhx^2}{\sqrt{cx^2+ac}} + \frac{dgx}{\sqrt{cx^2+aa}} - \frac{eg}{\sqrt{cx^2+ac}} - \frac{dh}{\sqrt{cx^2+ac}} + \frac{2\,afh}{\sqrt{cx^2+ac^2}} - \frac{(fg+eh)x}{\sqrt{cx^2+ac}} + \frac{(fg+eh)\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{c^{\frac{3}{2}}}$$

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] f*h*x^2/(sqrt(c*x^2 + a)*c) + d*g*x/(sqrt(c*x^2 + a)*a) - e*g/(sqrt(c*x^2 +
 a)*c) - d*h/(sqrt(c*x^2 + a)*c) + 2*a*f*h/(sqrt(c*x^2 + a)*c^2) - (f*g + e
*h)*x/(sqrt(c*x^2 + a)*c) + (f*g + e*h)*arcsinh(c*x/sqrt(a*c))/c^(3/2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.13

$$\int \frac{(g+hx)\left(d+ex+fx^2\right)}{\left(a+cx^2\right)^{3/2}}\,dx = \frac{\left(\frac{fhx}{c} + \frac{c^3dg-ac^2fg-ac^2eh}{ac^3}\right)x - \frac{ac^2eg+ac^2dh-2\,a^2cfh}{ac^3}}{\sqrt{cx^2+a}} - \frac{(fg+eh)\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+a}\right|\right)}{c^{\frac{3}{2}}}$$

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)/(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] ((f*h*x/c + (c^3*d*g - a*c^2*f*g - a*c^2*e*h)/(a*c^3))*x - (a*c^2*e*g + a*c
^2*d*h - 2*a^2*c*f*h)/(a*c^3))/sqrt(c*x^2 + a) - (f*g + e*h)*log(abs(-sqrt(
c)*x + sqrt(c*x^2 + a)))/c^(3/2)
```

Mupad [B] (verification not implemented)

Time = 13.48 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.51

$$\int \frac{(g+hx)\left(d+ex+fx^2\right)}{\left(a+cx^2\right)^{3/2}}\,dx = \frac{e\,h\,\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+a}\right)}{c^{3/2}} + \frac{f\,g\,\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+a}\right)}{c^{3/2}} - \frac{d\,h}{c\,\sqrt{c\,x^2+a}} - \frac{e\,g}{c\,\sqrt{c\,x^2+a}} + \frac{d\,g\,x}{a\,\sqrt{c\,x^2+a}} - \frac{e\,h\,x}{c\,\sqrt{c\,x^2+a}} - \frac{f\,g\,x}{c\,\sqrt{c\,x^2+a}} + \frac{f\,h\,\left(c\,x^2+2\,a\right)}{c^2\,\sqrt{c\,x^2+a}}$$

```
[In] int(((g + h*x)*(d + e*x + f*x^2))/(a + c*x^2)^(3/2),x)

[Out] (e*h*log(c^(1/2)*x + (a + c*x^2)^(1/2)))/c^(3/2) + (f*g*log(c^(1/2)*x + (a
+ c*x^2)^(1/2)))/c^(3/2) - (d*h)/(c*(a + c*x^2)^(1/2)) - (e*g)/(c*(a + c*x^
2)^(1/2)) + (d*g*x)/(a*(a + c*x^2)^(1/2)) - (e*h*x)/(c*(a + c*x^2)^(1/2)) -
 (f*g*x)/(c*(a + c*x^2)^(1/2)) + (f*h*(2*a + c*x^2))/(c^2*(a + c*x^2)^(1/2)
)
```

3.111 $\int \frac{d+ex+fx^2}{\left(a+cx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 61

$$\int \frac{d+ex+fx^2}{\left(a+cx^2\right)^{3/2}}\,dx=-\frac{ae-(cd-af)x}{ac\sqrt{a+cx^2}}+\frac{f\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{c^{3/2}}$$

```
[Out] f*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(3/2)+(-a*e+(-a*f+c*d)*x)/a/c/(c*x^2
+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1828, 12, 223, 212}

$$\int \frac{d+ex+fx^2}{\left(a+cx^2\right)^{3/2}}\,dx=\frac{f\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{c^{3/2}}-\frac{ae-x(cd-af)}{ac\sqrt{a+cx^2}}$$

```
[In] Int[(d + e*x + f*x^2)/(a + c*x^2)^(3/2),x]
```

```
[Out] -((a*e - (c*d - a*f)*x)/(a*c*Sqrt[a + c*x^2])) + (f*ArcTanh[(Sqrt[c]*x)/Sqr
t[a + c*x^2]])/c^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{ae-(cd-af)x}{ac\sqrt{a+cx^2}}+\frac{\int \frac{af}{c\sqrt{a+cx^2}}\,dx}{a}\\
&= -\frac{ae-(cd-af)x}{ac\sqrt{a+cx^2}}+\frac{f\int \frac{1}{\sqrt{a+cx^2}}\,dx}{c}\\
&= -\frac{ae-(cd-af)x}{ac\sqrt{a+cx^2}}+\frac{f\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x}{\sqrt{a+cx^2}}\right)}{c}\\
&= -\frac{ae-(cd-af)x}{ac\sqrt{a+cx^2}}+\frac{f\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{c^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.02

$$\int \frac{d+ex+fx^2}{(a+cx^2)^{3/2}}\,dx=\frac{-ae+cdx-afx}{ac\sqrt{a+cx^2}}-\frac{f\log\left(-\sqrt{c}x+\sqrt{a+cx^2}\right)}{c^{3/2}}$$

```
[In] Integrate[(d + e*x + f*x^2)/(a + c*x^2)^(3/2),x]
[Out] (-(a*e) + c*d*x - a*f*x)/(a*c*Sqrt[a + c*x^2]) - (f*Log[-(Sqrt[c]*x) + Sqrt
[a + c*x^2]])/c^(3/2)
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.15

method	result	size
default	$\frac{dx}{a\sqrt{c\,x^{2}+a}}+f\left(-\frac{x}{c\sqrt{c\,x^{2}+a}}+\frac{\ln\left(x\sqrt{c}+\sqrt{c\,x^{2}+a}\right)}{c^{\frac{3}{2}}}\right)-\frac{e}{c\sqrt{c\,x^{2}+a}}$	70

```
[In] int((f*x^2+e*x+d)/(c*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] d*x/a/(c*x^2+a)^(1/2)+f*(-x/c/(c*x^2+a)^(1/2)+1/c^(3/2)*ln(x*c^(1/2)+(c*x^2
+a)^(1/2)))-e/c/(c*x^2+a)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 181, normalized size of antiderivative = 2.97

$$\int \frac{d+ex+fx^2}{\left(a+cx^2\right)^{3/2}}\,dx = \left[\frac{\left(acfx^2+a^2f\right)\sqrt{c}\log\left(-2\,cx^2-2\,\sqrt{cx^2+a}\sqrt{c}x-a\right)-2\,\left(ace-\left(c^2d-acf\right)x\right)\sqrt{cx^2}}{2\,\left(ac^3x^2+a^2c^2\right)}\right.$$
$$\left.-\frac{\left(acfx^2+a^2f\right)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}x}{\sqrt{cx^2+a}}\right)+\left(ace-\left(c^2d-acf\right)x\right)\sqrt{cx^2+a}}{ac^3x^2+a^2c^2}\right]$$

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((a*c*f*x^2 + a^2*f)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*
x - a) - 2*(a*c*e - (c^2*d - a*c*f)*x)*sqrt(c*x^2 + a))/(a*c^3*x^2 + a^2*c^
2), -((a*c*f*x^2 + a^2*f)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) + (a*
c*e - (c^2*d - a*c*f)*x)*sqrt(c*x^2 + a))/(a*c^3*x^2 + a^2*c^2)]
```

Sympy [A] (verification not implemented)

Time = 3.00 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.43

$$\int \frac{d+ex+fx^2}{\left(a+cx^2\right)^{3/2}}\,dx = e\left(\begin{cases} -\frac{1}{c\sqrt{a+cx^2}} & \text{for}\ c\neq 0 \\ \frac{x^2}{2a^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right) + f\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{c^{\frac{3}{2}}}-\frac{x}{\sqrt{a}c\sqrt{1+\frac{cx^2}{a}}}\right)+\frac{dx}{a^{\frac{3}{2}}\sqrt{1+\frac{cx^2}{a}}}$$

```
[In] integrate((f*x**2+e*x+d)/(c*x**2+a)**(3/2),x)
```

```
[Out] e*Piecewise((-1/(c*sqrt(a + c*x**2)), Ne(c, 0)), (x**2/(2*a**(3/2)), True))
 + f*(asinh(sqrt(c)*x/sqrt(a))/c**(3/2) - x/(sqrt(a)*c*sqrt(1 + c*x**2/a)))
 + d*x/(a**(3/2)*sqrt(1 + c*x**2/a))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00

$$\int \frac{d+ex+fx^2}{(a+cx^2)^{3/2}}\,dx=\frac{dx}{\sqrt{cx^2+aa}}-\frac{fx}{\sqrt{cx^2+ac}}+\frac{f\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{c^{\frac{3}{2}}}-\frac{e}{\sqrt{cx^2+ac}}$$

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] d*x/(sqrt(c*x^2 + a)*a) - f*x/(sqrt(c*x^2 + a)*c) + f*arcsinh(c*x/sqrt(a*c)
)/c^(3/2) - e/(sqrt(c*x^2 + a)*c)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.02

$$\int \frac{d+ex+fx^2}{(a+cx^2)^{3/2}}\,dx=-\frac{\frac{e}{c}-\frac{(c^2d-acf)x}{ac^2}}{\sqrt{cx^2+a}}-\frac{f\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+a}\right|\right)}{c^{\frac{3}{2}}}$$

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -(e/c - (c^2*d - a*c*f)*x/(a*c^2))/sqrt(c*x^2 + a) - f*log(abs(-sqrt(c)*x +
 sqrt(c*x^2 + a)))/c^(3/2)
```

Mupad [B] (verification not implemented)

Time = 12.75 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.11

$$\int \frac{d+ex+fx^2}{(a+cx^2)^{3/2}}\,dx=\frac{f\,\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+a}\right)}{c^{3/2}}-\frac{e}{c\,\sqrt{c\,x^2+a}}+\frac{d\,x}{a\,\sqrt{c\,x^2+a}}-\frac{f\,x}{c\,\sqrt{c\,x^2+a}}$$

```
[In] int((d + e*x + f*x^2)/(a + c*x^2)^(3/2),x)
```

```
[Out] (f*log(c^(1/2)*x + (a + c*x^2)^(1/2)))/c^(3/2) - e/(c*(a + c*x^2)^(1/2)) +
(d*x)/(a*(a + c*x^2)^(1/2)) - (f*x)/(c*(a + c*x^2)^(1/2))
```

3.112 $\int \frac{d+ex+fx^2}{(g+hx)\left(a+cx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 138

$$\int \frac{d+ex+fx^2}{(g+hx)\left(a+cx^2\right)^{3/2}}\,dx = -\frac{a(ceg-cdh+afh)-c(cdg-afg+aeh)x}{ac\left(cg^2+ah^2\right)\sqrt{a+cx^2}} - \frac{\left(fg^2-egh+dh^2\right)\operatorname{arctanh}\left(\frac{ah-cgx}{\sqrt{cg^2+ah^2}\sqrt{a+cx^2}}\right)}{\left(cg^2+ah^2\right)^{3/2}}$$

```
[Out] -(d*h^2-e*g*h+f*g^2)*arctanh((-c*g*x+a*h)/(a*h^2+c*g^2)^(1/2)/(c*x^2+a)^(1/
2))/(a*h^2+c*g^2)^(3/2)+(-a*(a*f*h-c*d*h+c*e*g)+c*(a*e*h-a*f*g+c*d*g)*x)/a/
c/(a*h^2+c*g^2)/(c*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1661, 12, 739, 212}

$$\int \frac{d+ex+fx^2}{(g+hx)\left(a+cx^2\right)^{3/2}}\,dx = -\frac{\left(dh^2-egh+fg^2\right)\operatorname{arctanh}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)}{\left(ah^2+cg^2\right)^{3/2}} - \frac{a(afh-cdh+ceg)-cx(aeh-afg+cdg)}{ac\sqrt{a+cx^2}\left(ah^2+cg^2\right)}$$

```
[In] Int[(d + e*x + f*x^2)/((g + h*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] -((a*(c*e*g - c*d*h + a*f*h) - c*(c*d*g - a*f*g + a*e*h)*x)/(a*c*(c*g^2 + a
*h^2)*Sqrt[a + c*x^2])) - ((f*g^2 - e*g*h + d*h^2)*ArcTanh[(a*h - c*g*x)/(S
qrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2])])/(c*g^2 + a*h^2)^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{a(ceg-cdh+afh)-c(cdg-afg+aeh)x}{ac\,(cg^2+ah^2)\sqrt{a+cx^2}}+\frac{\int \frac{ac(fg^2-egh+dh^2)}{(cg^2+ah^2)(g+hx)\sqrt{a+cx^2}}\,dx}{ac}\\
&= -\frac{a(ceg-cdh+afh)-c(cdg-afg+aeh)x}{ac\,(cg^2+ah^2)\sqrt{a+cx^2}}+\frac{(fg^2-egh+dh^2)\int \frac{1}{(g+hx)\sqrt{a+cx^2}}\,dx}{cg^2+ah^2}\\
&= -\frac{a(ceg-cdh+afh)-c(cdg-afg+aeh)x}{ac\,(cg^2+ah^2)\sqrt{a+cx^2}}\\
&\quad -\frac{(fg^2-egh+dh^2)\,\text{Subst}\left(\int \frac{1}{cg^2+ah^2-x^2}\,dx,x,\frac{ah-cgx}{\sqrt{a+cx^2}}\right)}{cg^2+ah^2}\\
&= -\frac{a(ceg-cdh+afh)-c(cdg-afg+aeh)x}{ac\,(cg^2+ah^2)\sqrt{a+cx^2}}-\frac{(fg^2-egh+dh^2)\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{cg^2+ah^2}\sqrt{a+cx^2}}\right)}{(cg^2+ah^2)^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.07

$$\int \frac{d+ex+fx^2}{(g+hx)\left(a+cx^2\right)^{3/2}}\,dx=\frac{-a^2fh+c^2dgx+ac(-eg+dh-fgx+ehx)}{ac\left(cg^2+ah^2\right)\sqrt{a+cx^2}}+\frac{2\left(fg^2+h(-eg+dh)\right)\arctan\left(\frac{\sqrt{c}(g+hx)-h\sqrt{a+cx^2}}{\sqrt{-cg^2-ah^2}}\right)}{\left(-cg^2-ah^2\right)^{3/2}}$$

```
[In] Integrate[(d + e*x + f*x^2)/((g + h*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] (-(a^2*f*h) + c^2*d*g*x + a*c*(-(e*g) + d*h - f*g*x + e*h*x))/(a*c*(c*g^2 +
 a*h^2)*Sqrt[a + c*x^2]) + (2*(f*g^2 + h*(-(e*g) + d*h))*ArcTan[(Sqrt[c]*(g
 + h*x) - h*Sqrt[a + c*x^2])/Sqrt[-(c*g^2) - a*h^2]])/(-(c*g^2) - a*h^2)^(3
/2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 385 vs. 2(129) = 258.

Time = 0.51 (sec) , antiderivative size = 386, normalized size of antiderivative = 2.80

method	result
default	$\frac{\frac{ehx}{a\sqrt{c\,x^2+a}}-\frac{fh}{c\sqrt{c\,x^2+a}}-\frac{fgx}{a\sqrt{c\,x^2+a}}}{h^2}+\frac{\left(d\,h^2-egh+f\,g^2\right)\left(\frac{h^2}{\left(a\,h^2+c\,g^2\right)\sqrt{\left(x+\frac{g}{h}\right)^2c-\frac{2cg\left(x+\frac{g}{h}\right)}{h}+\frac{a\,h^2+c\,g^2}{h^2}}}+\frac{2}{\left(a\,h^2+c\,g^2\right)\left(\frac{4c\left(a\,h^2+c\,g^2\right)}{h^2}\right.}\right.}{}$

```
[In] int((f*x^2+e*x+d)/(h*x+g)/(c*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/h^2*(e*h*x/a/(c*x^2+a)^(1/2)-f*h/c/(c*x^2+a)^(1/2)-f*g*x/a/(c*x^2+a)^(1/2
))+(d*h^2-e*g*h+f*g^2)/h^3*(1/(a*h^2+c*g^2)*h^2/((x+1/h*g)^2*c-2*c*g/h*(x+1
/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)+2*c*g*h/(a*h^2+c*g^2)*(2*c*(x+1/h*g)-2*c*g/h
)/(4*c*(a*h^2+c*g^2)/h^2-4*c^2*g^2/h^2)/((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a
*h^2+c*g^2)/h^2)^(1/2)-1/(a*h^2+c*g^2)*h^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*
(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1/h*g)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+1/h*g)^
2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+1/h*g)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 347 vs. 2(130) = 260.

Time = 1.06 (sec) , antiderivative size = 721, normalized size of antiderivative = 5.22

$$\int \frac{d+ex+fx^2}{(g+hx)\left(a+cx^2\right)^{3/2}}\,dx = \left[\frac{\left(a^2cfg^2 - a^2cegh + a^2cdh^2 + \left(ac^2fg^2 - ac^2egh + ac^2dh^2\right)x^2\right)\sqrt{cg^2+ah^2}\log}{}\right.$$

$$-\frac{\left(a^2cfg^2 - a^2cegh + a^2cdh^2 + \left(ac^2fg^2 - ac^2egh + ac^2dh^2\right)x^2\right)\sqrt{-cg^2-ah^2}\arctan\left(\frac{\sqrt{-cg^2-ah^2}(cgx-ah)\sqrt{cx^2+a}}{acg^2+a^2h^2+(c^2g^2+ach^2)x^2}\right.}{a^2c^3g^4+2\,a^3c^2g^2h}$$

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((a^2*c*f*g^2 - a^2*c*e*g*h + a^2*c*d*h^2 + (a*c^2*f*g^2 - a*c^2*e*g*h
 + a*c^2*d*h^2)*x^2)*sqrt(c*g^2 + a*h^2)*log((2*a*c*g*h*x - a*c*g^2 - 2*a^2
*h^2 - (2*c^2*g^2 + a*c*h^2)*x^2 - 2*sqrt(c*g^2 + a*h^2)*(c*g*x - a*h)*sqrt
(c*x^2 + a))/(h^2*x^2 + 2*g*h*x + g^2)) - 2*(a*c^2*e*g^3 + a^2*c*e*g*h^2 -
(a*c^2*d - a^2*c*f)*g^2*h - (a^2*c*d - a^3*f)*h^3 - (a*c^2*e*g^2*h + a^2*c*
e*h^3 + (c^3*d - a*c^2*f)*g^3 + (a*c^2*d - a^2*c*f)*g*h^2)*x)*sqrt(c*x^2 +
a))/(a^2*c^3*g^4 + 2*a^3*c^2*g^2*h^2 + a^4*c*h^4 + (a*c^4*g^4 + 2*a^2*c^3*g
^2*h^2 + a^3*c^2*h^4)*x^2), -((a^2*c*f*g^2 - a^2*c*e*g*h + a^2*c*d*h^2 + (a
*c^2*f*g^2 - a*c^2*e*g*h + a*c^2*d*h^2)*x^2)*sqrt(-c*g^2 - a*h^2)*arctan(sq
rt(-c*g^2 - a*h^2)*(c*g*x - a*h)*sqrt(c*x^2 + a)/(a*c*g^2 + a^2*h^2 + (c^2*
g^2 + a*c*h^2)*x^2)) + (a*c^2*e*g^3 + a^2*c*e*g*h^2 - (a*c^2*d - a^2*c*f)*g
^2*h - (a^2*c*d - a^3*f)*h^3 - (a*c^2*e*g^2*h + a^2*c*e*h^3 + (c^3*d - a*c^
2*f)*g^3 + (a*c^2*d - a^2*c*f)*g*h^2)*x)*sqrt(c*x^2 + a))/(a^2*c^3*g^4 + 2*
a^3*c^2*g^2*h^2 + a^4*c*h^4 + (a*c^4*g^4 + 2*a^2*c^3*g^2*h^2 + a^3*c^2*h^4)
*x^2)]
```

Sympy [F]

$$\int \frac{d+ex+fx^2}{(g+hx)\left(a+cx^2\right)^{3/2}}\,dx = \int \frac{d+ex+fx^2}{\left(a+cx^2\right)^{\frac{3}{2}}\left(g+hx\right)}\,dx$$

```
[In] integrate((f*x**2+e*x+d)/(h*x+g)/(c*x**2+a)**(3/2),x)
```

```
[Out] Integral((d + e*x + f*x**2)/((a + c*x**2)**(3/2)*(g + h*x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 453 vs. $2(130) = 260$.

Time = 0.24 (sec) , antiderivative size = 453, normalized size of antiderivative = 3.28

$$
\begin{aligned}
\int \frac{d+ex+fx^2}{(g+hx)\left(a+cx^2\right)^{3/2}}\,dx &= \frac{cfg^3x}{\sqrt{cx^2+a}acg^2h^2+\sqrt{cx^2+a}a^2h^4} \\
&- \frac{ceg^2x}{\sqrt{cx^2+a}acg^2h+\sqrt{cx^2+a}a^2h^3} + \frac{cdgx}{\sqrt{cx^2+a}acg^2+\sqrt{cx^2+a}a^2h^2} \\
&+ \frac{fg^2}{\sqrt{cx^2+a}cg^2h+\sqrt{cx^2+a}ah^3} - \frac{eg}{\sqrt{cx^2+a}cg^2+\sqrt{cx^2+a}ah^2} \\
&+ \frac{d}{\frac{\sqrt{cx^2+a}cg^2}{h}+\sqrt{cx^2+a}ah} - \frac{fgx}{\sqrt{cx^2+a}ah^2} + \frac{ex}{\sqrt{cx^2+a}ah} \\
&+ \frac{fg^2\operatorname{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|}-\frac{ah}{\sqrt{ac}|hx+g|}\right)}{\left(a+\frac{cg^2}{h^2}\right)^{\frac{3}{2}}h^3} - \frac{eg\operatorname{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|}-\frac{ah}{\sqrt{ac}|hx+g|}\right)}{\left(a+\frac{cg^2}{h^2}\right)^{\frac{3}{2}}h^2} \\
&+ \frac{d\operatorname{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|}-\frac{ah}{\sqrt{ac}|hx+g|}\right)}{\left(a+\frac{cg^2}{h^2}\right)^{\frac{3}{2}}h} - \frac{f}{\sqrt{cx^2+a}ch}
\end{aligned}
$$

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] c*f*g^3*x/(sqrt(c*x^2 + a)*a*c*g^2*h^2 + sqrt(c*x^2 + a)*a^2*h^4) - c*e*g^2
*x/(sqrt(c*x^2 + a)*a*c*g^2*h + sqrt(c*x^2 + a)*a^2*h^3) + c*d*g*x/(sqrt(c*
x^2 + a)*a*c*g^2 + sqrt(c*x^2 + a)*a^2*h^2) + f*g^2/(sqrt(c*x^2 + a)*c*g^2*
h + sqrt(c*x^2 + a)*a*h^3) - e*g/(sqrt(c*x^2 + a)*c*g^2 + sqrt(c*x^2 + a)*a
*h^2) + d/(sqrt(c*x^2 + a)*c*g^2/h + sqrt(c*x^2 + a)*a*h) - f*g*x/(sqrt(c*x
^2 + a)*a*h^2) + e*x/(sqrt(c*x^2 + a)*a*h) + f*g^2*arcsinh(c*g*x/(sqrt(a*c)
*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(3/2)*h^3)
- e*g*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g))
)/((a + c*g^2/h^2)^(3/2)*h^2) + d*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) -
a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(3/2)*h) - f/(sqrt(c*x^2 + a
)*c*h)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 288 vs. $2(130) = 260$.

Time = 0.29 (sec) , antiderivative size = 288, normalized size of antiderivative = 2.09

$$\int \frac{d+ex+fx^2}{(g+hx)\left(a+cx^2\right)^{3/2}}\,dx = \frac{\frac{\left(c^3dg^3-ac^2fg^3+ac^2eg^2h+ac^2dgh^2-a^2cfgh^2+a^2ceh^3\right)x}{ac^3g^4+2\,a^2c^2g^2h^2+a^3ch^4} - \frac{ac^2eg^3-ac^2dg^2h+a^2cfg^2h+a^2cegh^2-a^2cdh^3}{ac^3g^4+2\,a^2c^2g^2h^2+a^3ch^4}}{\sqrt{cx^2+a}}$$

$$-\frac{2\left(fg^2-egh+dh^2\right)\arctan\left(\frac{\left(\sqrt{c}x-\sqrt{cx^2+a}\right)h+\sqrt{c}g}{\sqrt{-cg^2-ah^2}}\right)}{\left(cg^2+ah^2\right)\sqrt{-cg^2-ah^2}}$$

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)/(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] ((c^3*d*g^3 - a*c^2*f*g^3 + a*c^2*e*g^2*h + a*c^2*d*g*h^2 - a^2*c*f*g*h^2 +
 a^2*c*e*h^3)*x/(a*c^3*g^4 + 2*a^2*c^2*g^2*h^2 + a^3*c*h^4) - (a*c^2*e*g^3
- a*c^2*d*g^2*h + a^2*c*f*g^2*h + a^2*c*e*g*h^2 - a^2*c*d*h^3 + a^3*f*h^3)/
(a*c^3*g^4 + 2*a^2*c^2*g^2*h^2 + a^3*c*h^4))/sqrt(c*x^2 + a) - 2*(f*g^2 - e
*g*h + d*h^2)*arctan(((sqrt(c)*x - sqrt(c*x^2 + a))*h + sqrt(c)*g)/sqrt(-c*
g^2 - a*h^2))/((c*g^2 + a*h^2)*sqrt(-c*g^2 - a*h^2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2}{(g+hx)\left(a+cx^2\right)^{3/2}}\,dx = \int \frac{f\,x^2+e\,x+d}{\left(g+h\,x\right)\,{\left(c\,x^2+a\right)}^{3/2}} \,dx$$

```
[In] int((d + e*x + f*x^2)/((g + h*x)*(a + c*x^2)^(3/2)),x)
```

```
[Out] int((d + e*x + f*x^2)/((g + h*x)*(a + c*x^2)^(3/2)), x)
```

3.113 $\int \frac{d+ex+fx^2}{(g+hx)^2\left(a+cx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 239

$$\begin{aligned}\int \frac{d+ex+fx^2}{(g+hx)^2\left(a+cx^2\right)^{3/2}}\,dx=&\\ &-\frac{a(cg(eg-2dh)+ah(2fg-eh))-\left(c^2dg^2+a^2fh^2-ac(fg^2-h(2eg-dh))\right)x}{a\left(cg^2+ah^2\right)^2\sqrt{a+cx^2}}\\ &-\frac{h\left(fg^2-egh+dh^2\right)\sqrt{a+cx^2}}{\left(cg^2+ah^2\right)^2(g+hx)}\\ &+\frac{\left(ah^2(2fg-eh)-cg(fg^2-h(2eg-3dh))\right)\operatorname{arctanh}\left(\frac{ah-cgx}{\sqrt{cg^2+ah^2}\sqrt{a+cx^2}}\right)}{\left(cg^2+ah^2\right)^{5/2}}\end{aligned}$$

```
[Out] (a*h^2*(-e*h+2*f*g)-c*g*(f*g^2-h*(-3*d*h+2*e*g)))*arctanh((-c*g*x+a*h)/(a*h
^2+c*g^2)^(1/2)/(c*x^2+a)^(1/2))/(a*h^2+c*g^2)^(5/2)+(-a*(c*g*(-2*d*h+e*g)+
a*h*(-e*h+2*f*g))+(c^2*d*g^2+a^2*f*h^2-a*c*(f*g^2-h*(-d*h+2*e*g)))*x)/a/(a*
h^2+c*g^2)^2/(c*x^2+a)^(1/2)-h*(d*h^2-e*g*h+f*g^2)*(c*x^2+a)^(1/2)/(a*h^2+c
*g^2)^2/(h*x+g)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used

= {1661, 821, 739, 212}

$$\int \frac{d+ex+fx^2}{(g+hx)^2\left(a+cx^2\right)^{3/2}}\,dx = -\frac{a(ah(2fg-eh)+cg(eg-2dh))-x(a^2fh^2-ac(fg^2-h(2eg-dh))+c^2dg^2)}{a\sqrt{a+cx^2}\left(ah^2+cg^2\right)^2} - \frac{\operatorname{arctanh}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)\left(-ah^2(2fg-eh)-cgh(2eg-3dh)+cfg^3\right)}{\left(ah^2+cg^2\right)^{5/2}} - \frac{h\sqrt{a+cx^2}(dh^2-egh+fg^2)}{(g+hx)\left(ah^2+cg^2\right)^2}$$

```
[In] Int[(d + e*x + f*x^2)/((g + h*x)^2*(a + c*x^2)^(3/2)),x]
```

```
[Out] -((a*(c*g*(e*g - 2*d*h) + a*h*(2*f*g - e*h)) - (c^2*d*g^2 + a^2*f*h^2 - a*c
*(f*g^2 - h*(2*e*g - d*h)))*x)/(a*(c*g^2 + a*h^2)^2*Sqrt[a + c*x^2])) - (h*
(f*g^2 - e*g*h + d*h^2)*Sqrt[a + c*x^2])/((c*g^2 + a*h^2)^2*(g + h*x)) - ((
c*f*g^3 - c*g*h*(2*e*g - 3*d*h) - a*h^2*(2*f*g - e*h))*ArcTanh[(a*h - c*g*x
)/(Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2])])/(c*g^2 + a*h^2)^(5/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
```

```
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\text{integral} = -\frac{a(cg(eg-2dh)+ah(2fg-eh))-(c^2dg^2+a^2fh^2-ac(fg^2-h(2eg-dh)))x}{a(cg^2+ah^2)^2\sqrt{a+cx^2}} - \frac{\int \frac{\frac{ac\left(ah^2\left(fg^2-dh^2\right)-c\left(fg^4-g^2h(2eg-3dh)\right)\right)}{\left(cg^2+ah^2\right)^2}+\frac{ach^2(cg(eg-2dh)+ah(2fg-eh))x}{\left(cg^2+ah^2\right)^2}}{(g+hx)^2\sqrt{a+cx^2}}\,dx}{ac}$$

$$= -\frac{a(cg(eg-2dh)+ah(2fg-eh))-(c^2dg^2+a^2fh^2-ac(fg^2-h(2eg-dh)))x}{a(cg^2+ah^2)^2\sqrt{a+cx^2}} - \frac{h(fg^2-egh+dh^2)\sqrt{a+cx^2}}{(cg^2+ah^2)^2(g+hx)} - \frac{\left(\frac{a^2ch^3(cg(eg-2dh)+ah(2fg-eh))}{(cg^2+ah^2)^2}+\frac{ac^2g\left(ah^2\left(fg^2-dh^2\right)-c\left(fg^4-g^2h(2eg-3dh)\right)\right)}{(cg^2+ah^2)^2}\right)\int \frac{1}{(g+hx)\sqrt{a+cx^2}}\,dx}{ac(cg^2+ah^2)}$$

$$= -\frac{a(cg(eg-2dh)+ah(2fg-eh))-(c^2dg^2+a^2fh^2-ac(fg^2-h(2eg-dh)))x}{a(cg^2+ah^2)^2\sqrt{a+cx^2}} - \frac{h(fg^2-egh+dh^2)\sqrt{a+cx^2}}{(cg^2+ah^2)^2(g+hx)} + \frac{\left(\frac{a^2ch^3(cg(eg-2dh)+ah(2fg-eh))}{(cg^2+ah^2)^2}+\frac{ac^2g\left(ah^2\left(fg^2-dh^2\right)-c\left(fg^4-g^2h(2eg-3dh)\right)\right)}{(cg^2+ah^2)^2}\right)\text{Subst}\left(\int \frac{1}{cg^2+ah^2-x^2}\,dx, x, \frac{ah-}{\sqrt{a-}}\right.}{ac(cg^2+ah^2)}$$

$$= -\frac{a(cg(eg-2dh)+ah(2fg-eh))-(c^2dg^2+a^2fh^2-ac(fg^2-h(2eg-dh)))x}{a(cg^2+ah^2)^2\sqrt{a+cx^2}} - \frac{h(fg^2-egh+dh^2)\sqrt{a+cx^2}}{(cg^2+ah^2)^2(g+hx)} - \frac{(cfg^3-cgh(2eg-3dh)-ah^2(2fg-eh))\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{cg^2+ah^2}\sqrt{a+cx^2}}\right)}{(cg^2+ah^2)^{5/2}}$$

Mathematica [A] (verified)

Time = 1.36 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.04

$$\int \frac{d+ex+fx^2}{(g+hx)^2\,(a+cx^2)^{3/2}}\,dx = \frac{c^2dg^2x(g+hx)+a^2h(h(2eg-dh+ehx)+f(-3g^2-ghx+h^2x^2))+ac(-f}{a\,(cg^2+ah^2)^2\,(g+hx}$$
$$-\frac{2(cfg^3+cgh(-2eg+3dh)+ah^2(-2fg+eh))\arctan\left(\frac{\sqrt{c}(g+hx)-h\sqrt{a+cx^2}}{\sqrt{-cg^2-ah^2}}\right)}{(-cg^2-ah^2)^{5/2}}$$

```
[In] Integrate[(d + e*x + f*x^2)/((g + h*x)^2*(a + c*x^2)^(3/2)),x]
```

```
[Out] (c^2*d*g^2*x*(g + h*x) + a^2*h*(h*(2*e*g - d*h + e*h*x) + f*(-3*g^2 - g*h*x
 + h^2*x^2)) + a*c*(-(f*g^2*x*(g + 2*h*x)) + d*h*(2*g^2 + g*h*x - 2*h^2*x^2
) + e*g*(-g^2 + g*h*x + 3*h^2*x^2)))/(a*(c*g^2 + a*h^2)^2*(g + h*x)*Sqrt[a
+ c*x^2]) - (2*(c*f*g^3 + c*g*h*(-2*e*g + 3*d*h) + a*h^2*(-2*f*g + e*h))*Ar
cTan[(Sqrt[c]*(g + h*x) - h*Sqrt[a + c*x^2])/Sqrt[-(c*g^2) - a*h^2]])/(-(c*
g^2) - a*h^2)^(5/2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 871 vs. 2(228) = 456.

Time = 0.53 (sec) , antiderivative size = 872, normalized size of antiderivative = 3.65

method	result
default	$\frac{fx}{h^2a\sqrt{cx^2+a}}+\frac{(eh-2fg)\left(\frac{h^2}{(ah^2+cg^2)\sqrt{\left(x+\frac{g}{h}\right)^2c-\frac{2cg\left(x+\frac{g}{h}\right)}{h}+\frac{ah^2+cg^2}{h^2}}}+\frac{2cgh\left(2c\left(x+\frac{g}{h}\right)-\frac{2cg}{h}\right)}{(ah^2+cg^2)\left(\frac{4c(ah^2+cg^2)}{h^2}-\frac{4c^2g^2}{h^2}\right)\sqrt{\left(x+\frac{g}{h}\right)^2c-\frac{2cg(x+}{h}}}\right.}{h^3}$

```
[In] int((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] f/h^2*x/a/(c*x^2+a)^(1/2)+1/h^3*(e*h-2*f*g)*(1/(a*h^2+c*g^2)*h^2/((x+1/h*g)
^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)+2*c*g*h/(a*h^2+c*g^2)*(2*c*
(x+1/h*g)-2*c*g/h)/(4*c*(a*h^2+c*g^2)/h^2-4*c^2*g^2/h^2)/((x+1/h*g)^2*c-2*c
*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)-1/(a*h^2+c*g^2)*h^2/((a*h^2+c*g^2)/
h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1/h*g)+2*((a*h^2+c*g^2)/h^2)^
(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+1/h*g))
```

```
)+1/h^4*(d*h^2-e*g*h+f*g^2)*(-1/(a*h^2+c*g^2)*h^2/(x+1/h*g)/((x+1/h*g)^2*c-
2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)+3*c*g*h/(a*h^2+c*g^2)*(1/(a*h^2+
c*g^2)*h^2/((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)+2*c*g*
h/(a*h^2+c*g^2)*(2*c*(x+1/h*g)-2*c*g/h)/(4*c*(a*h^2+c*g^2)/h^2-4*c^2*g^2/h^
2)/((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)-1/(a*h^2+c*g^2
)*h^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1/h*g)+2
*((a*h^2+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h
^2)^(1/2))/(x+1/h*g)))-4*c/(a*h^2+c*g^2)*h^2*(2*c*(x+1/h*g)-2*c*g/h)/(4*c*(
a*h^2+c*g^2)/h^2-4*c^2*g^2/h^2)/((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g
^2)/h^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 773 vs. $2(230) = 460$.

Time = 1.82 (sec) , antiderivative size = 1573, normalized size of antiderivative = 6.58

$$\int \frac{d+ex+fx^2}{(g+hx)^2\left(a+cx^2\right)^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2*((a^2*c*f*g^4 - 2*a^2*c*e*g^3*h + a^3*e*g*h^3 + (3*a^2*c*d - 2*a^3*f)
*g^2*h^2 + (a*c^2*f*g^3*h - 2*a*c^2*e*g^2*h^2 + a^2*c*e*h^4 + (3*a*c^2*d -
2*a^2*c*f)*g*h^3)*x^3 + (a*c^2*f*g^4 - 2*a*c^2*e*g^3*h + a^2*c*e*g*h^3 + (3
*a*c^2*d - 2*a^2*c*f)*g^2*h^2)*x^2 + (a^2*c*f*g^3*h - 2*a^2*c*e*g^2*h^2 + a
^3*e*h^4 + (3*a^2*c*d - 2*a^3*f)*g*h^3)*x)*sqrt(c*g^2 + a*h^2)*log((2*a*c*g
*h*x - a*c*g^2 - 2*a^2*h^2 - (2*c^2*g^2 + a*c*h^2)*x^2 + 2*sqrt(c*g^2 + a*h
^2)*(c*g*x - a*h)*sqrt(c*x^2 + a))/(h^2*x^2 + 2*g*h*x + g^2)) + 2*(a*c^2*e*
g^5 - a^2*c*e*g^3*h^2 - 2*a^3*e*g*h^4 + a^3*d*h^5 - (2*a*c^2*d - 3*a^2*c*f)
*g^4*h - (a^2*c*d - 3*a^3*f)*g^2*h^3 - (3*a*c^2*e*g^3*h^2 + 3*a^2*c*e*g*h^4
 + (c^3*d - 2*a*c^2*f)*g^4*h - (a*c^2*d + a^2*c*f)*g^2*h^3 - (2*a^2*c*d - a
^3*f)*h^5)*x^2 - (a*c^2*e*g^4*h + 2*a^2*c*e*g^2*h^3 + a^3*e*h^5 + (c^3*d -
a*c^2*f)*g^5 + 2*(a*c^2*d - a^2*c*f)*g^3*h^2 + (a^2*c*d - a^3*f)*g*h^4)*x)*
sqrt(c*x^2 + a))/(a^2*c^3*g^7 + 3*a^3*c^2*g^5*h^2 + 3*a^4*c*g^3*h^4 + a^5*g
*h^6 + (a*c^4*g^6*h + 3*a^2*c^3*g^4*h^3 + 3*a^3*c^2*g^2*h^5 + a^4*c*h^7)*x^
3 + (a*c^4*g^7 + 3*a^2*c^3*g^5*h^2 + 3*a^3*c^2*g^3*h^4 + a^4*c*g*h^6)*x^2 +
 (a^2*c^3*g^6*h + 3*a^3*c^2*g^4*h^3 + 3*a^4*c*g^2*h^5 + a^5*h^7)*x), -((a^2
*c*f*g^4 - 2*a^2*c*e*g^3*h + a^3*e*g*h^3 + (3*a^2*c*d - 2*a^3*f)*g^2*h^2 +
(a*c^2*f*g^3*h - 2*a*c^2*e*g^2*h^2 + a^2*c*e*h^4 + (3*a*c^2*d - 2*a^2*c*f)*
g*h^3)*x^3 + (a*c^2*f*g^4 - 2*a*c^2*e*g^3*h + a^2*c*e*g*h^3 + (3*a*c^2*d -
2*a^2*c*f)*g^2*h^2)*x^2 + (a^2*c*f*g^3*h - 2*a^2*c*e*g^2*h^2 + a^3*e*h^4 +
(3*a^2*c*d - 2*a^3*f)*g*h^3)*x)*sqrt(-c*g^2 - a*h^2)*arctan(sqrt(-c*g^2 - a
*h^2)*(c*g*x - a*h)*sqrt(c*x^2 + a)/(a*c*g^2 + a^2*h^2 + (c^2*g^2 + a*c*h^2
)*x^2)) + (a*c^2*e*g^5 - a^2*c*e*g^3*h^2 - 2*a^3*e*g*h^4 + a^3*d*h^5 - (2*a
```

```
*c^2*d - 3*a^2*c*f)*g^4*h - (a^2*c*d - 3*a^3*f)*g^2*h^3 - (3*a*c^2*e*g^3*h^
2 + 3*a^2*c*e*g*h^4 + (c^3*d - 2*a*c^2*f)*g^4*h - (a*c^2*d + a^2*c*f)*g^2*h
^3 - (2*a^2*c*d - a^3*f)*h^5)*x^2 - (a*c^2*e*g^4*h + 2*a^2*c*e*g^2*h^3 + a^
3*e*h^5 + (c^3*d - a*c^2*f)*g^5 + 2*(a*c^2*d - a^2*c*f)*g^3*h^2 + (a^2*c*d
- a^3*f)*g*h^4)*x)*sqrt(c*x^2 + a))/(a^2*c^3*g^7 + 3*a^3*c^2*g^5*h^2 + 3*a^
4*c*g^3*h^4 + a^5*g*h^6 + (a*c^4*g^6*h + 3*a^2*c^3*g^4*h^3 + 3*a^3*c^2*g^2*
h^5 + a^4*c*h^7)*x^3 + (a*c^4*g^7 + 3*a^2*c^3*g^5*h^2 + 3*a^3*c^2*g^3*h^4 +
 a^4*c*g*h^6)*x^2 + (a^2*c^3*g^6*h + 3*a^3*c^2*g^4*h^3 + 3*a^4*c*g^2*h^5 +
a^5*h^7)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2}{(g+hx)^2\left(a+cx^2\right)^{3/2}}\,dx=\text{Timed out}$$

```
[In] integrate((f*x**2+e*x+d)/(h*x+g)**2/(c*x**2+a)**(3/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1085 vs. $2(230) = 460$.

Time = 0.26 (sec) , antiderivative size = 1085, normalized size of antiderivative = 4.54

$$
\begin{aligned}
\int \frac{d+ex+fx^2}{(g+hx)^2\,(a+cx^2)^{3/2}}\,dx &= \frac{3\,c^2fg^4x}{\sqrt{cx^2+a}ac^2g^4h^2+2\,\sqrt{cx^2+a}a^2cg^2h^4+\sqrt{cx^2+a}a^3h^6} \\
&- \frac{3\,c^2eg^3x}{\sqrt{cx^2+a}ac^2g^4h+2\,\sqrt{cx^2+a}a^2cg^2h^3+\sqrt{cx^2+a}a^3h^5} \\
&+ \frac{3\,c^2dg^2x}{\sqrt{cx^2+a}ac^2g^4+2\,\sqrt{cx^2+a}a^2cg^2h^2+\sqrt{cx^2+a}a^3h^4} \\
&+ \frac{3\,cfg^3}{\sqrt{cx^2+a}c^2g^4h+2\,\sqrt{cx^2+a}acg^2h^3+\sqrt{cx^2+a}a^2h^5} \\
&- \frac{4\,cfg^2x}{\sqrt{cx^2+a}acg^2h^2+\sqrt{cx^2+a}a^2h^4} - \frac{3\,ceg^2}{\sqrt{cx^2+a}c^2g^4+2\,\sqrt{cx^2+a}acg^2h^2+\sqrt{cx^2+a}a^2h^4} \\
&+ \frac{3\,cegx}{\sqrt{cx^2+a}acg^2h+\sqrt{cx^2+a}a^2h^3} + \frac{3\,cdg}{\frac{\sqrt{cx^2+a}c^2g^4}{h}+2\,\sqrt{cx^2+a}acg^2h+\sqrt{cx^2+a}a^2h^3} \\
&- \frac{fg^2}{\sqrt{cx^2+a}cg^2h^2x+\sqrt{cx^2+a}ah^4x+\sqrt{cx^2+a}cg^3h+\sqrt{cx^2+a}agh^3} \\
&- \frac{2\,cdx}{\sqrt{cx^2+a}acg^2+\sqrt{cx^2+a}a^2h^2} \\
&+ \frac{eg}{\sqrt{cx^2+a}cg^2hx+\sqrt{cx^2+a}ah^3x+\sqrt{cx^2+a}cg^3+\sqrt{cx^2+a}agh^2} \\
&- \frac{2\,fg}{\sqrt{cx^2+a}cg^2h+\sqrt{cx^2+a}ah^3} \\
&- \frac{d}{\sqrt{cx^2+a}cg^2x+\sqrt{cx^2+a}ah^2x+\frac{\sqrt{cx^2+a}cg^3}{h}+\sqrt{cx^2+a}agh} \\
&+ \frac{e}{\sqrt{cx^2+a}cg^2+\sqrt{cx^2+a}ah^2} + \frac{fx}{\sqrt{cx^2+a}ah^2} + \frac{3\,cfg^3\operatorname{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|}-\frac{ah}{\sqrt{ac}|hx+g|}\right)}{\left(a+\frac{cg^2}{h^2}\right)^{\frac{5}{2}}h^5} \\
&- \frac{3\,ceg^2\operatorname{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|}-\frac{ah}{\sqrt{ac}|hx+g|}\right)}{\left(a+\frac{cg^2}{h^2}\right)^{\frac{5}{2}}h^4} + \frac{3\,cdg\operatorname{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|}-\frac{ah}{\sqrt{ac}|hx+g|}\right)}{\left(a+\frac{cg^2}{h^2}\right)^{\frac{5}{2}}h^3} \\
&- \frac{2\,fg\operatorname{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|}-\frac{ah}{\sqrt{ac}|hx+g|}\right)}{\left(a+\frac{cg^2}{h^2}\right)^{\frac{3}{2}}h^3} + \frac{e\operatorname{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|}-\frac{ah}{\sqrt{ac}|hx+g|}\right)}{\left(a+\frac{cg^2}{h^2}\right)^{\frac{3}{2}}h^2}
\end{aligned}
$$

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 3*c^2*f*g^4*x/(sqrt(c*x^2 + a)*a*c^2*g^4*h^2 + 2*sqrt(c*x^2 + a)*a^2*c*g^2*
h^4 + sqrt(c*x^2 + a)*a^3*h^6) - 3*c^2*e*g^3*x/(sqrt(c*x^2 + a)*a*c^2*g^4*h
 + 2*sqrt(c*x^2 + a)*a^2*c*g^2*h^3 + sqrt(c*x^2 + a)*a^3*h^5) + 3*c^2*d*g^2
*x/(sqrt(c*x^2 + a)*a*c^2*g^4 + 2*sqrt(c*x^2 + a)*a^2*c*g^2*h^2 + sqrt(c*x^
```

```
2 + a)*a^3*h^4) + 3*c*f*g^3/(sqrt(c*x^2 + a)*c^2*g^4*h + 2*sqrt(c*x^2 + a)*
a*c*g^2*h^3 + sqrt(c*x^2 + a)*a^2*h^5) - 4*c*f*g^2*x/(sqrt(c*x^2 + a)*a*c*g
^2*h^2 + sqrt(c*x^2 + a)*a^2*h^4) - 3*c*e*g^2/(sqrt(c*x^2 + a)*c^2*g^4 + 2*
sqrt(c*x^2 + a)*a*c*g^2*h^2 + sqrt(c*x^2 + a)*a^2*h^4) + 3*c*e*g*x/(sqrt(c*
x^2 + a)*a*c*g^2*h + sqrt(c*x^2 + a)*a^2*h^3) + 3*c*d*g/(sqrt(c*x^2 + a)*c^
2*g^4/h + 2*sqrt(c*x^2 + a)*a*c*g^2*h + sqrt(c*x^2 + a)*a^2*h^3) - f*g^2/(s
qrt(c*x^2 + a)*c*g^2*h^2*x + sqrt(c*x^2 + a)*a*h^4*x + sqrt(c*x^2 + a)*c*g^
3*h + sqrt(c*x^2 + a)*a*g*h^3) - 2*c*d*x/(sqrt(c*x^2 + a)*a*c*g^2 + sqrt(c*
x^2 + a)*a^2*h^2) + e*g/(sqrt(c*x^2 + a)*c*g^2*h*x + sqrt(c*x^2 + a)*a*h^3*
x + sqrt(c*x^2 + a)*c*g^3 + sqrt(c*x^2 + a)*a*g*h^2) - 2*f*g/(sqrt(c*x^2 +
a)*c*g^2*h + sqrt(c*x^2 + a)*a*h^3) - d/(sqrt(c*x^2 + a)*c*g^2*x + sqrt(c*x
^2 + a)*a*h^2*x + sqrt(c*x^2 + a)*c*g^3/h + sqrt(c*x^2 + a)*a*g*h) + e/(sqr
t(c*x^2 + a)*c*g^2 + sqrt(c*x^2 + a)*a*h^2) + f*x/(sqrt(c*x^2 + a)*a*h^2) +
 3*c*f*g^3*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x
+ g)))/((a + c*g^2/h^2)^(5/2)*h^5) - 3*c*e*g^2*arcsinh(c*g*x/(sqrt(a*c)*abs
(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(5/2)*h^4) + 3*
c*d*g*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g))
)/((a + c*g^2/h^2)^(5/2)*h^3) - 2*f*g*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)
) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(3/2)*h^3) + e*arcsinh(c
*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h
^2)^(3/2)*h^2)
```

Giac [F]

$$\int \frac{d+ex+fx^2}{(g+hx)^2\left(a+cx^2\right)^{3/2}}\,dx=\int \frac{fx^2+ex+d}{\left(cx^2+a\right)^{\frac{3}{2}}\left(hx+g\right)^2}\,dx$$

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+a)^(3/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2}{(g+hx)^2\left(a+cx^2\right)^{3/2}}\,dx=\int \frac{f\,x^2+e\,x+d}{\left(g+h\,x\right)^2\left(c\,x^2+a\right)^{3/2}}\,dx$$

```
[In] int((d + e*x + f*x^2)/((g + h*x)^2*(a + c*x^2)^(3/2)),x)
[Out] int((d + e*x + f*x^2)/((g + h*x)^2*(a + c*x^2)^(3/2)), x)
```

3.114 $\int \frac{d+ex+fx^2}{(g+hx)^3\left(a+cx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 374

$$\int \frac{d+ex+fx^2}{(g+hx)^3\left(a+cx^2\right)^{3/2}}\,dx=\frac{a(a^2fh^3-c^2g^2(eg-3dh)-ach(3fg^2-h(3eg-dh)))+c(c^2dg^3+a^2h^2(3f}{a\left(cg^2+ah^2\right)^3\sqrt{a+cx^2}}$$
$$-\frac{h(fg^2-egh+dh^2)\sqrt{a+cx^2}}{2\left(cg^2+ah^2\right)^2(g+hx)^2}+\frac{h(2ah^2(2fg-eh)-cg(3fg^2-h(5eg-7dh)))\sqrt{a+cx^2}}{2\left(cg^2+ah^2\right)^3(g+hx)}$$
$$-\frac{(2a^2fh^4-ach^2(11fg^2-9egh+3dh^2)+2c^2g^2(fg^2-3egh+6dh^2))\,\mathrm{arctanh}\left(\frac{ah-cgx}{\sqrt{cg^2+ah^2}\sqrt{a+cx^2}}\right)}{2\left(cg^2+ah^2\right)^{7/2}}$$

```
[Out] -1/2*(2*a^2*f*h^4-a*c*h^2*(3*d*h^2-9*e*g*h+11*f*g^2)+2*c^2*g^2*(6*d*h^2-3*e
*g*h+f*g^2))*arctanh((-c*g*x+a*h)/(a*h^2+c*g^2)^(1/2)/(c*x^2+a)^(1/2))/(a*h
^2+c*g^2)^(7/2)+(a*(a^2*f*h^3-c^2*g^2*(-3*d*h+e*g)-a*c*h*(3*f*g^2-h*(-d*h+3
*e*g)))+c*(c^2*d*g^3+a^2*h^2*(-e*h+3*f*g)-a*c*g*(f*g^2-3*h*(-d*h+e*g)))*x)/
a/(a*h^2+c*g^2)^3/(c*x^2+a)^(1/2)-1/2*h*(d*h^2-e*g*h+f*g^2)*(c*x^2+a)^(1/2)
/(a*h^2+c*g^2)^2/(h*x+g)^2+1/2*h*(2*a*h^2*(-e*h+2*f*g)-c*g*(3*f*g^2-h*(-7*d
*h+5*e*g)))*(c*x^2+a)^(1/2)/(a*h^2+c*g^2)^3/(h*x+g)
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 372, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used

$= \{1661, 1665, 821, 739, 212\}$

$$\int \frac{d+ex+fx^2}{(g+hx)^3\,(a+cx^2)^{3/2}}\,dx =$$

$$-\frac{\operatorname{arctanh}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\,\sqrt{ah^2+cg^2}}\right)(2a^2fh^4-ach^2(3dh^2-9egh+11fg^2)+2c^2g^2(6dh^2-3egh+fg^2))}{2\,(ah^2+cg^2)^{7/2}}$$

$$+\frac{a(a^2fh^3-ach(3fg^2-h(3eg-dh))-c^2g^2(eg-3dh))+cx(a^2h^2(3fg-eh)-acg(fg^2-3h(eg-dh))+}{a\sqrt{a+cx^2}\,(ah^2+cg^2)^3}$$

$$-\frac{h\sqrt{a+cx^2}(dh^2-egh+fg^2)}{2(g+hx)^2\,(ah^2+cg^2)^2}-\frac{h\sqrt{a+cx^2}(-2ah^2(2fg-eh)-cgh(5eg-7dh)+3cfg^3)}{2(g+hx)\,(ah^2+cg^2)^3}$$

```
[In] Int[(d + e*x + f*x^2)/((g + h*x)^3*(a + c*x^2)^(3/2)),x]
```

```
[Out] (a*(a^2*f*h^3 - c^2*g^2*(e*g - 3*d*h) - a*c*h*(3*f*g^2 - h*(3*e*g - d*h)))
+ c*(c^2*d*g^3 + a^2*h^2*(3*f*g - e*h) - a*c*g*(f*g^2 - 3*h*(e*g - d*h)))*x
)/(a*(c*g^2 + a*h^2)^3*Sqrt[a + c*x^2]) - (h*(f*g^2 - e*g*h + d*h^2)*Sqrt[a
 + c*x^2])/(2*(c*g^2 + a*h^2)^2*(g + h*x)^2) - (h*(3*c*f*g^3 - c*g*h*(5*e*g
 - 7*d*h) - 2*a*h^2*(2*f*g - e*h))*Sqrt[a + c*x^2])/(2*(c*g^2 + a*h^2)^3*(g
 + h*x)) - ((2*a^2*f*h^4 - a*c*h^2*(11*f*g^2 - 9*e*g*h + 3*d*h^2) + 2*c^2*g
^2*(f*g^2 - 3*e*g*h + 6*d*h^2))*ArcTanh[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*
Sqrt[a + c*x^2])])/(2*(c*g^2 + a*h^2)^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
```

```
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

integral

$$= \frac{a(a^2fh^3-c^2g^2(eg-3dh)-ach(3fg^2-h(3eg-dh)))+c(c^2dg^3+a^2h^2(3fg-eh)-acg(fg^2-3h(eg-}{a\,(cg^2+ah^2)^3\sqrt{a+cx^2}}$$

$$- \frac{\int \frac{-\frac{ac(a^2dh^6+c^2g^4(fg^2-3egh+6dh^2)-acg^2h^2(3fg^2-h(eg+3dh)))}{(cg^2+ah^2)^3}-\frac{ach^2(a^2eh^4-c^2g^3(3eg-8dh)-2acg^2h(4fg-3eh))x}{(cg^2+ah^2)^3}-\frac{ach^3(a^2fh^3-c^2g^2(eg-3dh}{(cg^2}}{(g+hx)^3\sqrt{a+cx^2}}}{ac}$$

$$= \frac{a(a^2fh^3-c^2g^2(eg-3dh)-ach(3fg^2-h(3eg-dh)))+c(c^2dg^3+a^2h^2(3fg-eh)-acg(fg^2-}{a\,(cg^2+ah^2)^3\sqrt{a+cx^2}}$$

$$- \frac{h(fg^2-egh+dh^2)\sqrt{a+cx^2}}{2\,(cg^2+ah^2)^2\,(g+hx)^2}$$

$$+ \frac{\int \frac{-\frac{2ac(a^2h^4(fg-eh)-c^2g^3(fg^2-3egh+6dh^2)+2acgh^2(2fg^2-h(eg+dh)))}{(cg^2+ah^2)^2}+\frac{ach(2a^2fh^4-c^2(fg^4+g^2h(eg-5dh))-ach^2(7fg^2-h(7eg-3dh}{(cg^2+ah^2)^2}}{(g+hx)^2\sqrt{a+cx^2}}}{2ac\,(cg^2+ah^2)}$$

$$= \frac{a(a^2fh^3-c^2g^2(eg-3dh)-ach(3fg^2-h(3eg-dh)))+c(c^2dg^3+a^2h^2(3fg-eh)-acg(fg^2-}{a\,(cg^2+ah^2)^3\sqrt{a+cx^2}}$$

$$- \frac{h(fg^2-egh+dh^2)\sqrt{a+cx^2}}{2\,(cg^2+ah^2)^2\,(g+hx)^2}$$

$$- \frac{h(3cfg^3-cgh(5eg-7dh)-2ah^2(2fg-eh))\sqrt{a+cx^2}}{2\,(cg^2+ah^2)^3\,(g+hx)}$$

$$+ \frac{(2a^2fh^4-ach^2(11fg^2-9egh+3dh^2)+2c^2g^2(fg^2-3egh+6dh^2))\int \frac{1}{(g+hx)\sqrt{a+cx^2}}\,dx}{2\,(cg^2+ah^2)^3}$$

$$
\begin{aligned}
&= \frac{a(a^2fh^3 - c^2g^2(eg-3dh) - ach(3fg^2 - h(3eg-dh))) + c(c^2dg^3 + a^2h^2(3fg-eh) - acg(fg^2 - 3}{a\left(cg^2+ah^2\right)^3\sqrt{a+cx^2}} \\
&\quad - \frac{h(fg^2 - egh + dh^2)\sqrt{a+cx^2}}{2\left(cg^2+ah^2\right)^2(g+hx)^2} \\
&\quad - \frac{h(3cfg^3 - cgh(5eg-7dh) - 2ah^2(2fg-eh))\sqrt{a+cx^2}}{2\left(cg^2+ah^2\right)^3(g+hx)} \\
&\quad - \frac{(2a^2fh^4 - ach^2(11fg^2 - 9egh + 3dh^2) + 2c^2g^2(fg^2 - 3egh + 6dh^2))\,\text{Subst}\left(\int \frac{1}{cg^2+ah^2-x^2}\,dx, x,\right.}{2\left(cg^2+ah^2\right)^3}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{a(a^2fh^3 - c^2g^2(eg-3dh) - ach(3fg^2 - h(3eg-dh))) + c(c^2dg^3 + a^2h^2(3fg-eh) - acg(fg^2 - 3}{a\left(cg^2+ah^2\right)^3\sqrt{a+cx^2}} \\
&\quad - \frac{h(fg^2 - egh + dh^2)\sqrt{a+cx^2}}{2\left(cg^2+ah^2\right)^2(g+hx)^2} \\
&\quad - \frac{h(3cfg^3 - cgh(5eg-7dh) - 2ah^2(2fg-eh))\sqrt{a+cx^2}}{2\left(cg^2+ah^2\right)^3(g+hx)} \\
&\quad - \frac{(2a^2fh^4 - ach^2(11fg^2 - 9egh + 3dh^2) + 2c^2g^2(fg^2 - 3egh + 6dh^2))\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{cg^2+ah^2}\sqrt{a+cx^2}}\right)}{2\left(cg^2+ah^2\right)^{7/2}}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1537 vs. 2(374) = 748.

Time = 11.11 (sec) , antiderivative size = 1537, normalized size of antiderivative = 4.11

$$
\begin{aligned}
&\int \frac{d+ex+fx^2}{(g+hx)^3\left(a+cx^2\right)^{3/2}}\,dx = \frac{a^4h^3(f(5g^2+8ghx+2h^2x^2) - h(dh+e(g+2hx))) - 4c^{7/2}g^2x^2\left(-\sqrt{c}x+\sqrt{a}\right.}{} \\
&- \frac{15afh^2\arctan\left(\frac{-\sqrt{c}(g+hx)+h\sqrt{a+cx^2}}{\sqrt{-cg^2-ah^2}}\right)}{\left(-cg^2-ah^2\right)^{5/2}} + \frac{21aeh^3\arctan\left(\frac{-\sqrt{c}(g+hx)+h\sqrt{a+cx^2}}{\sqrt{-cg^2-ah^2}}\right)}{g\left(-cg^2-ah^2\right)^{5/2}} \\
&- \frac{2f\arctan\left(\frac{-\sqrt{c}(g+hx)+h\sqrt{a+cx^2}}{\sqrt{-cg^2-ah^2}}\right)}{\left(-cg^2-ah^2\right)^{3/2}} + \frac{6eh\arctan\left(\frac{-\sqrt{c}(g+hx)+h\sqrt{a+cx^2}}{\sqrt{-cg^2-ah^2}}\right)}{g\left(-cg^2-ah^2\right)^{3/2}} \\
&- \frac{12dh^2\arctan\left(\frac{-\sqrt{c}(g+hx)+h\sqrt{a+cx^2}}{\sqrt{-cg^2-ah^2}}\right)}{g^2\left(-cg^2-ah^2\right)^{3/2}} - \frac{27adh^4\arctan\left(\frac{-\sqrt{c}(g+hx)+h\sqrt{a+cx^2}}{\sqrt{-cg^2-ah^2}}\right)}{\sqrt{-cg^2-ah^2}\left(cg^3+agh^2\right)^2} \\
&- \frac{15a^2h^4(fg^2+h(-eg+dh))\arctan\left(\frac{-\sqrt{c}(g+hx)+h\sqrt{a+cx^2}}{\sqrt{-cg^2-ah^2}}\right)}{g^2\left(-cg^2-ah^2\right)^{7/2}}
\end{aligned}
$$

```
[In] Integrate[(d + e*x + f*x^2)/((g + h*x)^3*(a + c*x^2)^(3/2)),x]
```

```
[Out] (a^4*h^3*(f*(5*g^2 + 8*g*h*x + 2*h^2*x^2) - h*(d*h + e*(g + 2*h*x))) - 4*c^
(7/2)*g^2*x^2*(-(Sqrt[c]*x) + Sqrt[a + c*x^2])*(d*(-g^3 + 4*g^2*h*x + 18*g*
h^2*x^2 + 12*h^3*x^3) + g*x*(f*g*x*(3*g + 2*h*x) - e*(2*g^2 + 9*g*h*x + 6*h
^2*x^2))) - a^3*(Sqrt[c]*h^2*Sqrt[a + c*x^2]*(-3*d*h^3*x + e*h*(4*g^2 + 5*g
*h*x - 2*h^2*x^2) + f*(-10*g^3 - 5*g^2*h*x + 14*g*h^2*x^2 + 6*h^3*x^3)) + c
*h*(f*(10*g^4 + 39*g^3*h*x + 26*g^2*h^2*x^2 - 20*g*h^3*x^3 - 10*h^4*x^4) +
h*(d*h*(10*g^2 + 11*g*h*x + 8*h^2*x^2) - e*(12*g^3 + 27*g^2*h*x + 20*g*h^2*
x^2 - 2*h^3*x^3)))) + a^2*(c^2*(d*h*(6*g^4 + 45*g^3*h*x + 14*g^2*h^2*x^2 -
29*g*h^3*x^3 - 19*h^4*x^4) + e*g*(-2*g^4 - 31*g^3*h*x + 10*g^2*h^2*x^2 + 81
*g*h^3*x^3 + 57*h^4*x^4) + f*x*(13*g^5 - 28*g^4*h*x - 105*g^3*h^2*x^2 - 75*
g^2*h^3*x^3 + 12*g*h^4*x^4 + 8*h^5*x^5)) + c^(3/2)*Sqrt[a + c*x^2]*(f*(-5*g
^5 + 20*g^4*h*x + 72*g^3*h^2*x^2 + 53*g^2*h^3*x^3 - 12*g*h^4*x^4 - 8*h^5*x^
5) + h*(e*g*(11*g^3 - 14*g^2*h*x - 54*g*h^2*x^2 - 39*h^3*x^3) + d*h*(-13*g^
3 + 4*g^2*h*x + 20*g*h^2*x^2 + 13*h^3*x^3)))) + a*(c^(5/2)*Sqrt[a + c*x^2]*
(d*(2*g^5 - 14*g^4*h*x - 81*g^3*h^2*x^2 - 48*g^2*h^3*x^3 + 18*g*h^4*x^4 + 1
2*h^5*x^5) + g*x*(f*g*x*(-19*g^3 + 14*g^2*h*x + 66*g*h^2*x^2 + 44*h^3*x^3)
+ e*(6*g^4 + 49*g^3*h*x + 14*g^2*h^2*x^2 - 54*g*h^3*x^3 - 36*h^4*x^4))) - c
^3*x*(d*(4*g^5 - 22*g^4*h*x - 117*g^3*h^2*x^2 - 72*g^2*h^3*x^3 + 18*g*h^4*x
^4 + 12*h^5*x^5) + g*x*(f*g*x*(-25*g^3 + 10*g^2*h*x + 66*g*h^2*x^2 + 44*h^3
*x^3) + e*(10*g^4 + 67*g^3*h*x + 26*g^2*h^2*x^2 - 54*g*h^3*x^3 - 36*h^4*x^4
)))))/(2*(c*g^2 + a*h^2)^3*(g + h*x)^2*(a + c*x^2)*(a*(-3*Sqrt[c]*x + Sqrt[
a + c*x^2]) + 4*c*x^2*(-(Sqrt[c]*x) + Sqrt[a + c*x^2]))) - (15*a*f*h^2*ArcT
an[(-(Sqrt[c]*(g + h*x)) + h*Sqrt[a + c*x^2])/Sqrt[-(c*g^2) - a*h^2]])/(-(c
*g^2) - a*h^2)^(5/2) + (21*a*e*h^3*ArcTan[(-(Sqrt[c]*(g + h*x)) + h*Sqrt[a
+ c*x^2])/Sqrt[-(c*g^2) - a*h^2]])/(g*(-(c*g^2) - a*h^2)^(5/2)) - (2*f*ArcT
an[(-(Sqrt[c]*(g + h*x)) + h*Sqrt[a + c*x^2])/Sqrt[-(c*g^2) - a*h^2]])/(-(c
*g^2) - a*h^2)^(3/2) + (6*e*h*ArcTan[(-(Sqrt[c]*(g + h*x)) + h*Sqrt[a + c*x
^2])/Sqrt[-(c*g^2) - a*h^2]])/(g*(-(c*g^2) - a*h^2)^(3/2)) - (12*d*h^2*ArcT
an[(-(Sqrt[c]*(g + h*x)) + h*Sqrt[a + c*x^2])/Sqrt[-(c*g^2) - a*h^2]])/(g^2
*(-(c*g^2) - a*h^2)^(3/2)) - (27*a*d*h^4*ArcTan[(-(Sqrt[c]*(g + h*x)) + h*S
qrt[a + c*x^2])/Sqrt[-(c*g^2) - a*h^2]])/(Sqrt[-(c*g^2) - a*h^2]*(c*g^3 + a
*g*h^2)^2) - (15*a^2*h^4*(f*g^2 + h*(-(e*g) + d*h))*ArcTan[(-(Sqrt[c]*(g +
h*x)) + h*Sqrt[a + c*x^2])/Sqrt[-(c*g^2) - a*h^2]])/(g^2*(-(c*g^2) - a*h^2)
^(7/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1786 vs. 2(356) = 712.

Time = 0.66 (sec) , antiderivative size = 1787, normalized size of antiderivative = 4.78

method	result	size
default	Expression too large to display	1787

```
[In] int((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] f/h^3*(1/(a*h^2+c*g^2)*h^2/((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h
^2)^(1/2)+2*c*g*h/(a*h^2+c*g^2)*(2*c*(x+1/h*g)-2*c*g/h)/(4*c*(a*h^2+c*g^2)/
h^2-4*c^2*g^2/h^2)/((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2
)-1/(a*h^2+c*g^2)*h^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c
*g/h*(x+1/h*g)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)
+(a*h^2+c*g^2)/h^2)^(1/2))/(x+1/h*g)))+(e*h-2*f*g)/h^4*(-1/(a*h^2+c*g^2)*h^
2/(x+1/h*g)/((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)+3*c*g
*h/(a*h^2+c*g^2)*(1/(a*h^2+c*g^2)*h^2/((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h
^2+c*g^2)/h^2)^(1/2)+2*c*g*h/(a*h^2+c*g^2)*(2*c*(x+1/h*g)-2*c*g/h)/(4*c*(a*
h^2+c*g^2)/h^2-4*c^2*g^2/h^2)/((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2
)/h^2)^(1/2)-1/(a*h^2+c*g^2)*h^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g
^2)/h^2-2*c*g/h*(x+1/h*g)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c-2*c*g/
h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+1/h*g)))-4*c/(a*h^2+c*g^2)*h^2*(2*
c*(x+1/h*g)-2*c*g/h)/(4*c*(a*h^2+c*g^2)/h^2-4*c^2*g^2/h^2)/((x+1/h*g)^2*c-2
*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))+(d*h^2-e*g*h+f*g^2)/h^5*(-1/2/(a
*h^2+c*g^2)*h^2/(x+1/h*g)^2/((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/
h^2)^(1/2)+5/2*c*g*h/(a*h^2+c*g^2)*(-1/(a*h^2+c*g^2)*h^2/(x+1/h*g)/((x+1/h*
g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)+3*c*g*h/(a*h^2+c*g^2)*(1/
(a*h^2+c*g^2)*h^2/((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)
+2*c*g*h/(a*h^2+c*g^2)*(2*c*(x+1/h*g)-2*c*g/h)/(4*c*(a*h^2+c*g^2)/h^2-4*c^2
*g^2/h^2)/((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)-1/(a*h^
2+c*g^2)*h^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1
/h*g)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c
*g^2)/h^2)^(1/2))/(x+1/h*g)))-4*c/(a*h^2+c*g^2)*h^2*(2*c*(x+1/h*g)-2*c*g/h)
/(4*c*(a*h^2+c*g^2)/h^2-4*c^2*g^2/h^2)/((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*
h^2+c*g^2)/h^2)^(1/2))-3/2*c/(a*h^2+c*g^2)*h^2*(1/(a*h^2+c*g^2)*h^2/((x+1/h
*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)+2*c*g*h/(a*h^2+c*g^2)*(2
*c*(x+1/h*g)-2*c*g/h)/(4*c*(a*h^2+c*g^2)/h^2-4*c^2*g^2/h^2)/((x+1/h*g)^2*c-
2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)-1/(a*h^2+c*g^2)*h^2/((a*h^2+c*g^
2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1/h*g)+2*((a*h^2+c*g^2)/h^
2)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+1/h*
g))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1413 vs. 2(357) = 714.

Time = 8.79 (sec) , antiderivative size = 2853, normalized size of antiderivative = 7.63

$$\int \frac{d+ex+fx^2}{(g+hx)^3\left(a+cx^2\right)^{3/2}}\,dx=\text{Too large to display}$$

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((2*a^2*c^2*f*g^6 - 6*a^2*c^2*e*g^5*h + 9*a^3*c*e*g^3*h^3 + (12*a^2*c^
2*d - 11*a^3*c*f)*g^4*h^2 - (3*a^3*c*d - 2*a^4*f)*g^2*h^4 + (2*a*c^3*f*g^4*
```

```
h^2 - 6*a*c^3*e*g^3*h^3 + 9*a^2*c^2*e*g*h^5 + (12*a*c^3*d - 11*a^2*c^2*f)*g
^2*h^4 - (3*a^2*c^2*d - 2*a^3*c*f)*h^6)*x^4 + 2*(2*a*c^3*f*g^5*h - 6*a*c^3*
e*g^4*h^2 + 9*a^2*c^2*e*g^2*h^4 + (12*a*c^3*d - 11*a^2*c^2*f)*g^3*h^3 - (3*
a^2*c^2*d - 2*a^3*c*f)*g*h^5)*x^3 + (2*a*c^3*f*g^6 - 6*a*c^3*e*g^5*h + 3*a^
2*c^2*e*g^3*h^3 + 9*a^3*c*e*g*h^5 + 3*(4*a*c^3*d - 3*a^2*c^2*f)*g^4*h^2 + 9
*(a^2*c^2*d - a^3*c*f)*g^2*h^4 - (3*a^3*c*d - 2*a^4*f)*h^6)*x^2 + 2*(2*a^2*
c^2*f*g^5*h - 6*a^2*c^2*e*g^4*h^2 + 9*a^3*c*e*g^2*h^4 + (12*a^2*c^2*d - 11*
a^3*c*f)*g^3*h^3 - (3*a^3*c*d - 2*a^4*f)*g*h^5)*x)*sqrt(c*g^2 + a*h^2)*log(
(2*a*c*g*h*x - a*c*g^2 - 2*a^2*h^2 - (2*c^2*g^2 + a*c*h^2)*x^2 - 2*sqrt(c*g
^2 + a*h^2)*(c*g*x - a*h)*sqrt(c*x^2 + a))/(h^2*x^2 + 2*g*h*x + g^2)) - 2*(
2*a*c^3*e*g^7 - 10*a^2*c^2*e*g^5*h^2 - 11*a^3*c*e*g^3*h^4 + a^4*e*g*h^6 + a
^4*d*h^7 - 2*(3*a*c^3*d - 5*a^2*c^2*f)*g^6*h + (4*a^2*c^2*d + 5*a^3*c*f)*g^
4*h^3 + (11*a^3*c*d - 5*a^4*f)*g^2*h^5 - (11*a*c^3*e*g^4*h^3 + 7*a^2*c^2*e*
g^2*h^5 - 4*a^3*c*e*h^7 + (2*c^4*d - 5*a*c^3*f)*g^5*h^2 - (11*a*c^3*d - 5*a
^2*c^2*f)*g^3*h^4 - (13*a^2*c^2*d - 10*a^3*c*f)*g*h^6)*x^3 - (16*a*c^3*e*g^
5*h^2 + 17*a^2*c^2*e*g^3*h^4 + a^3*c*e*g*h^6 + 4*(c^4*d - 2*a*c^3*f)*g^6*h
 - (10*a*c^3*d - a^2*c^2*f)*g^4*h^3 - (17*a^2*c^2*d - 11*a^3*c*f)*g^2*h^5 -
(3*a^3*c*d - 2*a^4*f)*h^7)*x^2 - (2*a*c^3*e*g^6*h + 17*a^2*c^2*e*g^4*h^3 +
13*a^3*c*e*g^2*h^5 - 2*a^4*e*h^7 + 2*(c^4*d - a*c^3*f)*g^7 + (8*a*c^3*d - 1
1*a^2*c^2*f)*g^5*h^2 - (5*a^2*c^2*d + a^3*c*f)*g^3*h^4 - (11*a^3*c*d - 8*a^
4*f)*g*h^6)*x)*sqrt(c*x^2 + a))/(a^2*c^4*g^10 + 4*a^3*c^3*g^8*h^2 + 6*a^4*c
^2*g^6*h^4 + 4*a^5*c*g^4*h^6 + a^6*g^2*h^8 + (a*c^5*g^8*h^2 + 4*a^2*c^4*g^6
*h^4 + 6*a^3*c^3*g^4*h^6 + 4*a^4*c^2*g^2*h^8 + a^5*c*h^10)*x^4 + 2*(a*c^5*g
^9*h + 4*a^2*c^4*g^7*h^3 + 6*a^3*c^3*g^5*h^5 + 4*a^4*c^2*g^3*h^7 + a^5*c*g*
h^9)*x^3 + (a*c^5*g^10 + 5*a^2*c^4*g^8*h^2 + 10*a^3*c^3*g^6*h^4 + 10*a^4*c^
2*g^4*h^6 + 5*a^5*c*g^2*h^8 + a^6*h^10)*x^2 + 2*(a^2*c^4*g^9*h + 4*a^3*c^3*
g^7*h^3 + 6*a^4*c^2*g^5*h^5 + 4*a^5*c*g^3*h^7 + a^6*g*h^9)*x), -1/2*((2*a^2
*c^2*f*g^6 - 6*a^2*c^2*e*g^5*h + 9*a^3*c*e*g^3*h^3 + (12*a^2*c^2*d - 11*a^3
*c*f)*g^4*h^2 - (3*a^3*c*d - 2*a^4*f)*g^2*h^4 + (2*a*c^3*f*g^4*h^2 - 6*a*c^
3*e*g^3*h^3 + 9*a^2*c^2*e*g*h^5 + (12*a*c^3*d - 11*a^2*c^2*f)*g^2*h^4 - (3*
a^2*c^2*d - 2*a^3*c*f)*h^6)*x^4 + 2*(2*a*c^3*f*g^5*h - 6*a*c^3*e*g^4*h^2 +
9*a^2*c^2*e*g^2*h^4 + (12*a*c^3*d - 11*a^2*c^2*f)*g^3*h^3 - (3*a^2*c^2*d -
2*a^3*c*f)*g*h^5)*x^3 + (2*a*c^3*f*g^6 - 6*a*c^3*e*g^5*h + 3*a^2*c^2*e*g^3*
h^3 + 9*a^3*c*e*g*h^5 + 3*(4*a*c^3*d - 3*a^2*c^2*f)*g^4*h^2 + 9*(a^2*c^2*d
 - a^3*c*f)*g^2*h^4 - (3*a^3*c*d - 2*a^4*f)*h^6)*x^2 + 2*(2*a^2*c^2*f*g^5*h
 - 6*a^2*c^2*e*g^4*h^2 + 9*a^3*c*e*g^2*h^4 + (12*a^2*c^2*d - 11*a^3*c*f)*g^3
*h^3 - (3*a^3*c*d - 2*a^4*f)*g*h^5)*x)*sqrt(-c*g^2 - a*h^2)*arctan(sqrt(-c*
g^2 - a*h^2)*(c*g*x - a*h)*sqrt(c*x^2 + a)/(a*c*g^2 + a^2*h^2 + (c^2*g^2 +
a*c*h^2)*x^2)) + (2*a*c^3*e*g^7 - 10*a^2*c^2*e*g^5*h^2 - 11*a^3*c*e*g^3*h^4
 + a^4*e*g*h^6 + a^4*d*h^7 - 2*(3*a*c^3*d - 5*a^2*c^2*f)*g^6*h + (4*a^2*c^2
*d + 5*a^3*c*f)*g^4*h^3 + (11*a^3*c*d - 5*a^4*f)*g^2*h^5 - (11*a*c^3*e*g^4*
h^3 + 7*a^2*c^2*e*g^2*h^5 - 4*a^3*c*e*h^7 + (2*c^4*d - 5*a*c^3*f)*g^5*h^2 -
 (11*a*c^3*d - 5*a^2*c^2*f)*g^3*h^4 - (13*a^2*c^2*d - 10*a^3*c*f)*g*h^6)*x^
3 - (16*a*c^3*e*g^5*h^2 + 17*a^2*c^2*e*g^3*h^4 + a^3*c*e*g*h^6 + 4*(c^4*d -
 2*a*c^3*f)*g^6*h - (10*a*c^3*d - a^2*c^2*f)*g^4*h^3 - (17*a^2*c^2*d - 11*a
```

```
^3*c*f)*g^2*h^5 - (3*a^3*c*d - 2*a^4*f)*h^7)*x^2 - (2*a*c^3*e*g^6*h + 17*a^
2*c^2*e*g^4*h^3 + 13*a^3*c*e*g^2*h^5 - 2*a^4*e*h^7 + 2*(c^4*d - a*c^3*f)*g^
7 + (8*a*c^3*d - 11*a^2*c^2*f)*g^5*h^2 - (5*a^2*c^2*d + a^3*c*f)*g^3*h^4 -
(11*a^3*c*d - 8*a^4*f)*g*h^6)*x)*sqrt(c*x^2 + a))/(a^2*c^4*g^10 + 4*a^3*c^3
*g^8*h^2 + 6*a^4*c^2*g^6*h^4 + 4*a^5*c*g^4*h^6 + a^6*g^2*h^8 + (a*c^5*g^8*h
^2 + 4*a^2*c^4*g^6*h^4 + 6*a^3*c^3*g^4*h^6 + 4*a^4*c^2*g^2*h^8 + a^5*c*h^10
)*x^4 + 2*(a*c^5*g^9*h + 4*a^2*c^4*g^7*h^3 + 6*a^3*c^3*g^5*h^5 + 4*a^4*c^2*
g^3*h^7 + a^5*c*g*h^9)*x^3 + (a*c^5*g^10 + 5*a^2*c^4*g^8*h^2 + 10*a^3*c^3*g
^6*h^4 + 10*a^4*c^2*g^4*h^6 + 5*a^5*c*g^2*h^8 + a^6*h^10)*x^2 + 2*(a^2*c^4*
g^9*h + 4*a^3*c^3*g^7*h^3 + 6*a^4*c^2*g^5*h^5 + 4*a^5*c*g^3*h^7 + a^6*g*h^9
)*x)]
```

Sympy [**F(-1)**]

Timed out.

$$\int \frac{d+ex+fx^2}{(g+hx)^3\left(a+cx^2\right)^{3/2}}\,dx=\text{Timed out}$$

```
[In] integrate((f*x**2+e*x+d)/(h*x+g)**3/(c*x**2+a)**(3/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2254 vs. 2(357) = 714.

Time = 0.31 (sec) , antiderivative size = 2254, normalized size of antiderivative = 6.03

$$\int \frac{d+ex+fx^2}{(g+hx)^3\left(a+cx^2\right)^{3/2}}\,dx=\text{Too large to display}$$

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 15/2*c^3*f*g^5*x/(sqrt(c*x^2 + a)*a*c^3*g^6*h^2 + 3*sqrt(c*x^2 + a)*a^2*c^2
*g^4*h^4 + 3*sqrt(c*x^2 + a)*a^3*c*g^2*h^6 + sqrt(c*x^2 + a)*a^4*h^8) - 15/
2*c^3*e*g^4*x/(sqrt(c*x^2 + a)*a*c^3*g^6*h + 3*sqrt(c*x^2 + a)*a^2*c^2*g^4*
h^3 + 3*sqrt(c*x^2 + a)*a^3*c*g^2*h^5 + sqrt(c*x^2 + a)*a^4*h^7) + 15/2*c^3
*d*g^3*x/(sqrt(c*x^2 + a)*a*c^3*g^6 + 3*sqrt(c*x^2 + a)*a^2*c^2*g^4*h^2 + 3
*sqrt(c*x^2 + a)*a^3*c*g^2*h^4 + sqrt(c*x^2 + a)*a^4*h^6) + 15/2*c^2*f*g^4/
(sqrt(c*x^2 + a)*c^3*g^6*h + 3*sqrt(c*x^2 + a)*a*c^2*g^4*h^3 + 3*sqrt(c*x^2
 + a)*a^2*c*g^2*h^5 + sqrt(c*x^2 + a)*a^3*h^7) - 25/2*c^2*f*g^3*x/(sqrt(c*x
^2 + a)*a*c^2*g^4*h^2 + 2*sqrt(c*x^2 + a)*a^2*c*g^2*h^4 + sqrt(c*x^2 + a)*a
^3*h^6) - 15/2*c^2*e*g^3/(sqrt(c*x^2 + a)*c^3*g^6 + 3*sqrt(c*x^2 + a)*a*c^2
*g^4*h^2 + 3*sqrt(c*x^2 + a)*a^2*c*g^2*h^4 + sqrt(c*x^2 + a)*a^3*h^6) + 19/
2*c^2*e*g^2*x/(sqrt(c*x^2 + a)*a*c^2*g^4*h + 2*sqrt(c*x^2 + a)*a^2*c*g^2*h^
```

```
3 + sqrt(c*x^2 + a)*a^3*h^5) + 15/2*c^2*d*g^2/(sqrt(c*x^2 + a)*c^3*g^6/h +
3*sqrt(c*x^2 + a)*a*c^2*g^4*h + 3*sqrt(c*x^2 + a)*a^2*c*g^2*h^3 + sqrt(c*x^
2 + a)*a^3*h^5) - 5/2*c*f*g^3/(sqrt(c*x^2 + a)*c^2*g^4*h^2*x + 2*sqrt(c*x^2
 + a)*a*c*g^2*h^4*x + sqrt(c*x^2 + a)*a^2*h^6*x + sqrt(c*x^2 + a)*c^2*g^5*h
 + 2*sqrt(c*x^2 + a)*a*c*g^3*h^3 + sqrt(c*x^2 + a)*a^2*g*h^5) - 13/2*c^2*d*
g*x/(sqrt(c*x^2 + a)*a*c^2*g^4 + 2*sqrt(c*x^2 + a)*a^2*c*g^2*h^2 + sqrt(c*x
^2 + a)*a^3*h^4) + 5/2*c*e*g^2/(sqrt(c*x^2 + a)*c^2*g^4*h*x + 2*sqrt(c*x^2
+ a)*a*c*g^2*h^3*x + sqrt(c*x^2 + a)*a^2*h^5*x + sqrt(c*x^2 + a)*c^2*g^5 +
2*sqrt(c*x^2 + a)*a*c*g^3*h^2 + sqrt(c*x^2 + a)*a^2*g*h^4) - 15/2*c*f*g^2/(
sqrt(c*x^2 + a)*c^2*g^4*h + 2*sqrt(c*x^2 + a)*a*c*g^2*h^3 + sqrt(c*x^2 + a)
*a^2*h^5) + 5*c*f*g*x/(sqrt(c*x^2 + a)*a*c*g^2*h^2 + sqrt(c*x^2 + a)*a^2*h^
4) - 5/2*c*d*g/(sqrt(c*x^2 + a)*c^2*g^4*x + 2*sqrt(c*x^2 + a)*a*c*g^2*h^2*x
 + sqrt(c*x^2 + a)*a^2*h^4*x + sqrt(c*x^2 + a)*c^2*g^5/h + 2*sqrt(c*x^2 + a
)*a*c*g^3*h + sqrt(c*x^2 + a)*a^2*g*h^3) + 9/2*c*e*g/(sqrt(c*x^2 + a)*c^2*g
^4 + 2*sqrt(c*x^2 + a)*a*c*g^2*h^2 + sqrt(c*x^2 + a)*a^2*h^4) - 1/2*f*g^2/(
sqrt(c*x^2 + a)*c*g^2*h^3*x^2 + sqrt(c*x^2 + a)*a*h^5*x^2 + 2*sqrt(c*x^2 +
a)*c*g^3*h^2*x + 2*sqrt(c*x^2 + a)*a*g*h^4*x + sqrt(c*x^2 + a)*c*g^4*h + sq
rt(c*x^2 + a)*a*g^2*h^3) - 2*c*e*x/(sqrt(c*x^2 + a)*a*c*g^2*h + sqrt(c*x^2
+ a)*a^2*h^3) - 3/2*c*d/(sqrt(c*x^2 + a)*c^2*g^4/h + 2*sqrt(c*x^2 + a)*a*c*
g^2*h + sqrt(c*x^2 + a)*a^2*h^3) + 1/2*e*g/(sqrt(c*x^2 + a)*c*g^2*h^2*x^2 +
 sqrt(c*x^2 + a)*a*h^4*x^2 + 2*sqrt(c*x^2 + a)*c*g^3*h*x + 2*sqrt(c*x^2 + a
)*a*g*h^3*x + sqrt(c*x^2 + a)*c*g^4 + sqrt(c*x^2 + a)*a*g^2*h^2) + 2*f*g/(s
qrt(c*x^2 + a)*c*g^2*h^2*x + sqrt(c*x^2 + a)*a*h^4*x + sqrt(c*x^2 + a)*c*g^
3*h + sqrt(c*x^2 + a)*a*g*h^3) - 1/2*d/(sqrt(c*x^2 + a)*c*g^2*h*x^2 + sqrt(
c*x^2 + a)*a*h^3*x^2 + 2*sqrt(c*x^2 + a)*c*g^3*x + 2*sqrt(c*x^2 + a)*a*g*h^
2*x + sqrt(c*x^2 + a)*c*g^4/h + sqrt(c*x^2 + a)*a*g^2*h) - e/(sqrt(c*x^2 +
a)*c*g^2*h*x + sqrt(c*x^2 + a)*a*h^3*x + sqrt(c*x^2 + a)*c*g^3 + sqrt(c*x^2
 + a)*a*g*h^2) + f/(sqrt(c*x^2 + a)*c*g^2*h + sqrt(c*x^2 + a)*a*h^3) + 15/2
*c^2*f*g^4*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x
+ g)))/((a + c*g^2/h^2)^(7/2)*h^7) - 15/2*c^2*e*g^3*arcsinh(c*g*x/(sqrt(a*c
)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(7/2)*h^6)
 + 15/2*c^2*d*g^2*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*a
bs(h*x + g)))/((a + c*g^2/h^2)^(7/2)*h^5) - 15/2*c*f*g^2*arcsinh(c*g*x/(sqr
t(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(5/2)
*h^5) + 9/2*c*e*g*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*a
bs(h*x + g)))/((a + c*g^2/h^2)^(5/2)*h^4) - 3/2*c*d*arcsinh(c*g*x/(sqrt(a*c
)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(5/2)*h^3)
 + f*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))
/((a + c*g^2/h^2)^(3/2)*h^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1421 vs. 2(357) = 714.

Time = 0.31 (sec) , antiderivative size = 1421, normalized size of antiderivative = 3.80

$$\int \frac{d + ex + fx^2}{(g + hx)^3 \left(a + cx^2\right)^{3/2}}\, dx = \text{Too large to display}$$

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] ((c^6*d*g^9 - a*c^5*f*g^9 + 3*a*c^5*e*g^8*h + 8*a^2*c^4*e*g^6*h^3 - 6*a^2*c
^4*d*g^5*h^4 + 6*a^3*c^3*f*g^5*h^4 + 6*a^3*c^3*e*g^4*h^5 - 8*a^3*c^3*d*g^3*
h^6 + 8*a^4*c^2*f*g^3*h^6 - 3*a^4*c^2*d*g*h^8 + 3*a^5*c*f*g*h^8 - a^5*c*e*h
^9)*x/(a*c^6*g^12 + 6*a^2*c^5*g^10*h^2 + 15*a^3*c^4*g^8*h^4 + 20*a^4*c^3*g^
6*h^6 + 15*a^5*c^2*g^4*h^8 + 6*a^6*c*g^2*h^10 + a^7*h^12) - (a*c^5*e*g^9 -
3*a*c^5*d*g^8*h + 3*a^2*c^4*f*g^8*h - 8*a^2*c^4*d*g^6*h^3 + 8*a^3*c^3*f*g^6
*h^3 - 6*a^3*c^3*e*g^5*h^4 - 6*a^3*c^3*d*g^4*h^5 + 6*a^4*c^2*f*g^4*h^5 - 8*
a^4*c^2*e*g^3*h^6 - 3*a^5*c*e*g*h^8 + a^5*c*d*h^9 - a^6*f*h^9)/(a*c^6*g^12
+ 6*a^2*c^5*g^10*h^2 + 15*a^3*c^4*g^8*h^4 + 20*a^4*c^3*g^6*h^6 + 15*a^5*c^2
*g^4*h^8 + 6*a^6*c*g^2*h^10 + a^7*h^12))/sqrt(c*x^2 + a) - (2*c^2*f*g^4 - 6
*c^2*e*g^3*h + 12*c^2*d*g^2*h^2 - 11*a*c*f*g^2*h^2 + 9*a*c*e*g*h^3 - 3*a*c*
d*h^4 + 2*a^2*f*h^4)*arctan(((sqrt(c)*x - sqrt(c*x^2 + a))*h + sqrt(c)*g)/s
qrt(-c*g^2 - a*h^2))/((c^3*g^6 + 3*a*c^2*g^4*h^2 + 3*a^2*c*g^2*h^4 + a^3*h^
6)*sqrt(-c*g^2 - a*h^2)) - (2*(sqrt(c)*x - sqrt(c*x^2 + a))^3*c^2*f*g^4*h -
 4*(sqrt(c)*x - sqrt(c*x^2 + a))^3*c^2*e*g^3*h^2 + 6*(sqrt(c)*x - sqrt(c*x^
2 + a))^3*c^2*d*g^2*h^3 - 5*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a*c*f*g^2*h^3 +
 3*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a*c*e*g*h^4 - (sqrt(c)*x - sqrt(c*x^2 +
a))^3*a*c*d*h^5 + 6*(sqrt(c)*x - sqrt(c*x^2 + a))^2*c^(5/2)*f*g^5 - 10*(sqr
t(c)*x - sqrt(c*x^2 + a))^2*c^(5/2)*e*g^4*h + 14*(sqrt(c)*x - sqrt(c*x^2 +
a))^2*c^(5/2)*d*g^3*h^2 - 11*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a*c^(3/2)*f*g^
3*h^2 + 9*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a*c^(3/2)*e*g^2*h^3 - 7*(sqrt(c)*
x - sqrt(c*x^2 + a))^2*a*c^(3/2)*d*g*h^4 + 4*(sqrt(c)*x - sqrt(c*x^2 + a))^
2*a^2*sqrt(c)*f*g*h^4 - 2*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^2*sqrt(c)*e*h^5
 - 10*(sqrt(c)*x - sqrt(c*x^2 + a))*a*c^2*f*g^4*h + 16*(sqrt(c)*x - sqrt(c*
x^2 + a))*a*c^2*e*g^3*h^2 - 22*(sqrt(c)*x - sqrt(c*x^2 + a))*a*c^2*d*g^2*h^
3 + 11*(sqrt(c)*x - sqrt(c*x^2 + a))*a^2*c*f*g^2*h^3 - 5*(sqrt(c)*x - sqrt(
c*x^2 + a))*a^2*c*e*g*h^4 - (sqrt(c)*x - sqrt(c*x^2 + a))*a^2*c*d*h^5 + 3*a
^2*c^(3/2)*f*g^3*h^2 - 5*a^2*c^(3/2)*e*g^2*h^3 + 7*a^2*c^(3/2)*d*g*h^4 - 4*
a^3*sqrt(c)*f*g*h^4 + 2*a^3*sqrt(c)*e*h^5)/((c^3*g^6 + 3*a*c^2*g^4*h^2 + 3*
a^2*c*g^2*h^4 + a^3*h^6)*((sqrt(c)*x - sqrt(c*x^2 + a))^2*h + 2*(sqrt(c)*x
- sqrt(c*x^2 + a))*sqrt(c)*g - a*h)^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2}{(g+hx)^3\left(a+cx^2\right)^{3/2}}\,dx=\int \frac{f\,x^2+e\,x+d}{{\left(g+h\,x\right)}^3\,{\left(c\,x^2+a\right)}^{3/2}} \,dx$$

```
[In] int((d + e*x + f*x^2)/((g + h*x)^3*(a + c*x^2)^(3/2)),x)
[Out] int((d + e*x + f*x^2)/((g + h*x)^3*(a + c*x^2)^(3/2)), x)
```

3.115 $\int \frac{A+Bx+Cx^2}{\left(a+cx^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 67

$$\int \frac{A+Bx+Cx^2}{\left(a+cx^2\right)^{5/2}}\,dx=-\frac{aB-(Ac-aC)x}{3ac\left(a+cx^2\right)^{3/2}}+\frac{(2Ac+aC)x}{3a^2c\sqrt{a+cx^2}}$$

```
[Out] 1/3*(-a*B+(A*c-C*a)*x)/a/c/(c*x^2+a)^(3/2)+1/3*(2*A*c+C*a)*x/a^2/c/(c*x^2+a
)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1828, 12, 197}

$$\int \frac{A+Bx+Cx^2}{\left(a+cx^2\right)^{5/2}}\,dx=\frac{x(aC+2Ac)}{3a^2c\sqrt{a+cx^2}}-\frac{aB-x(Ac-aC)}{3ac\left(a+cx^2\right)^{3/2}}$$

```
[In] Int[(A + B*x + C*x^2)/(a + c*x^2)^(5/2),x]
```

```
[Out] -1/3*(a*B - (A*c - a*C)*x)/(a*c*(a + c*x^2)^(3/2)) + ((2*A*c + a*C)*x)/(3*a
^2*c*Sqrt[a + c*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{aB-(Ac-aC)x}{3ac\left(a+cx^2\right)^{3/2}}-\frac{\int\frac{-2A-\frac{aC}{c}}{\left(a+cx^2\right)^{3/2}}\,dx}{3a}\\ &= -\frac{aB-(Ac-aC)x}{3ac\left(a+cx^2\right)^{3/2}}+\frac{(2Ac+aC)\int\frac{1}{\left(a+cx^2\right)^{3/2}}\,dx}{3ac}\\ &= -\frac{aB-(Ac-aC)x}{3ac\left(a+cx^2\right)^{3/2}}+\frac{(2Ac+aC)x}{3a^2c\sqrt{a+cx^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.75

$$\int\frac{A+Bx+Cx^2}{\left(a+cx^2\right)^{5/2}}\,dx=\frac{-a^2B+3aAcx+2Ac^2x^3+acCx^3}{3a^2c\left(a+cx^2\right)^{3/2}}$$

```
[In] Integrate[(A + B*x + C*x^2)/(a + c*x^2)^(5/2),x]
[Out] (-(a^2*B) + 3*a*A*c*x + 2*A*c^2*x^3 + a*c*C*x^3)/(3*a^2*c*(a + c*x^2)^(3/2)
)
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.70

method	result	size
gosper	$\frac{2A\,c^{2}x^{3}+Cac\,x^{3}+3aAcx-B\,a^{2}}{3\left(c\,x^{2}+a\right)^{\frac{3}{2}}c\,a^{2}}$	47
trager	$\frac{2A\,c^{2}x^{3}+Cac\,x^{3}+3aAcx-B\,a^{2}}{3\left(c\,x^{2}+a\right)^{\frac{3}{2}}c\,a^{2}}$	47
default	$A\left(\frac{x}{3a\left(c\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{2x}{3a^{2}\sqrt{c\,x^{2}+a}}\right)+C\left(-\frac{x}{2c\left(c\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{a\left(\frac{x}{3a\left(c\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{2x}{3a^{2}\sqrt{c\,x^{2}+a}}\right)}{2c}\right)-\frac{B}{3c\left(c\,x^{2}+a\right)^{\frac{3}{2}}}$	105

```
[In] int((C*x^2+B*x+A)/(c*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(2*A*c^2*x^3+C*a*c*x^3+3*A*a*c*x-B*a^2)/(c*x^2+a)^(3/2)/c/a^2
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.01

$$\int \frac{A+Bx+Cx^2}{\left(a+cx^2\right)^{5/2}}\,dx=\frac{\left(3\,Aacx+\left(Cac+2\,Ac^{2}\right)x^{3}-Ba^{2}\right)\sqrt{cx^{2}+a}}{3\,\left(a^{2}c^{3}x^{4}+2\,a^{3}c^{2}x^{2}+a^{4}c\right)}$$

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(3*A*a*c*x + (C*a*c + 2*A*c^2)*x^3 - B*a^2)*sqrt(c*x^2 + a)/(a^2*c^3*x^
4 + 2*a^3*c^2*x^2 + a^4*c)
```

Sympy [A] (verification not implemented)

Time = 5.47 (sec) , antiderivative size = 194, normalized size of antiderivative = 2.90

$$\begin{aligned}\int \frac{A+Bx+Cx^2}{\left(a+cx^2\right)^{5/2}}\,dx=\;&A\left(\frac{3ax}{3a^{\frac{7}{2}}\sqrt{1+\frac{cx^{2}}{a}}+3a^{\frac{5}{2}}cx^{2}\sqrt{1+\frac{cx^{2}}{a}}}\right.\\&\left.+\frac{2cx^{3}}{3a^{\frac{7}{2}}\sqrt{1+\frac{cx^{2}}{a}}+3a^{\frac{5}{2}}cx^{2}\sqrt{1+\frac{cx^{2}}{a}}}\right)\\&+B\left(\begin{cases}-\frac{1}{3ac\sqrt{a+cx^{2}}+3c^{2}x^{2}\sqrt{a+cx^{2}}} & \text{for}\ c\neq 0\\ \frac{x^{2}}{2a^{\frac{5}{2}}} & \text{otherwise}\end{cases}\right)+\frac{Cx^{3}}{3a^{\frac{5}{2}}\sqrt{1+\frac{cx^{2}}{a}}+3a^{\frac{3}{2}}cx^{2}\sqrt{1+\frac{cx^{2}}{a}}}\end{aligned}$$

```
[In] integrate((C*x**2+B*x+A)/(c*x**2+a)**(5/2),x)
```

```
[Out] A*(3*a*x/(3*a**(7/2)*sqrt(1 + c*x**2/a) + 3*a**(5/2)*c*x**2*sqrt(1 + c*x**2
/a)) + 2*c*x**3/(3*a**(7/2)*sqrt(1 + c*x**2/a) + 3*a**(5/2)*c*x**2*sqrt(1 +
 c*x**2/a))) + B*Piecewise((-1/(3*a*c*sqrt(a + c*x**2) + 3*c**2*x**2*sqrt(a
 + c*x**2)), Ne(c, 0)), (x**2/(2*a**(5/2)), True)) + C*x**3/(3*a**(5/2)*sqr
t(1 + c*x**2/a) + 3*a**(3/2)*c*x**2*sqrt(1 + c*x**2/a))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.24

$$\int \frac{A+Bx+Cx^2}{(a+cx^2)^{5/2}}\,dx = \frac{2\,Ax}{3\,\sqrt{cx^2+a}a^2} + \frac{Ax}{3\,(cx^2+a)^{\frac{3}{2}}a} - \frac{Cx}{3\,(cx^2+a)^{\frac{3}{2}}c} + \frac{Cx}{3\,\sqrt{cx^2+a}ac} - \frac{B}{3\,(cx^2+a)^{\frac{3}{2}}c}$$

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*A*x/(sqrt(c*x^2 + a)*a^2) + 1/3*A*x/((c*x^2 + a)^(3/2)*a) - 1/3*C*x/((c
*x^2 + a)^(3/2)*c) + 1/3*C*x/(sqrt(c*x^2 + a)*a*c) - 1/3*B/((c*x^2 + a)^(3/
2)*c)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.72

$$\int \frac{A+Bx+Cx^2}{(a+cx^2)^{5/2}}\,dx = \frac{x\left(\frac{3\,A}{a} + \frac{(Cac+2\,Ac^2)x^2}{a^2c}\right) - \frac{B}{c}}{3\,(cx^2+a)^{\frac{3}{2}}}$$

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(x*(3*A/a + (C*a*c + 2*A*c^2)*x^2/(a^2*c)) - B/c)/(c*x^2 + a)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 12.97 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx+Cx^2}{(a+cx^2)^{5/2}}\,dx = \frac{2\,A\,c\,x\,(c\,x^2+a)-C\,a^2\,x-B\,a^2+C\,a\,x\,(c\,x^2+a)+A\,a\,c\,x}{3\,a^2\,c\,(c\,x^2+a)^{3/2}}$$

```
[In] int((A + B*x + C*x^2)/(a + c*x^2)^(5/2),x)
```

```
[Out] (2*A*c*x*(a + c*x^2) - C*a^2*x - B*a^2 + C*a*x*(a + c*x^2) + A*a*c*x)/(3*a^
2*c*(a + c*x^2)^(3/2))
```

3.116 $\int \frac{A+Bx+Cx^2}{\left(a+cx^2\right)^{7/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 97

$$\int \frac{A+Bx+Cx^2}{\left(a+cx^2\right)^{7/2}}\,dx=-\frac{aB-(Ac-aC)x}{5ac\left(a+cx^2\right)^{5/2}}+\frac{(4Ac+aC)x}{15a^2c\left(a+cx^2\right)^{3/2}}+\frac{2(4Ac+aC)x}{15a^3c\sqrt{a+cx^2}}$$

```
[Out] 1/5*(-a*B+(A*c-C*a)*x)/a/c/(c*x^2+a)^(5/2)+1/15*(4*A*c+C*a)*x/a^2/c/(c*x^2+
a)^(3/2)+2/15*(4*A*c+C*a)*x/a^3/c/(c*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1828, 12, 198, 197}

$$\int \frac{A+Bx+Cx^2}{\left(a+cx^2\right)^{7/2}}\,dx=\frac{2x(aC+4Ac)}{15a^3c\sqrt{a+cx^2}}+\frac{x(aC+4Ac)}{15a^2c\left(a+cx^2\right)^{3/2}}-\frac{aB-x(Ac-aC)}{5ac\left(a+cx^2\right)^{5/2}}$$

```
[In] Int[(A + B*x + C*x^2)/(a + c*x^2)^(7/2),x]
```

```
[Out] -1/5*(a*B - (A*c - a*C)*x)/(a*c*(a + c*x^2)^(5/2)) + ((4*A*c + a*C)*x)/(15*
a^2*c*(a + c*x^2)^(3/2)) + (2*(4*A*c + a*C)*x)/(15*a^3*c*Sqrt[a + c*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{aB-(Ac-aC)x}{5ac\,(a+cx^2)^{5/2}}-\frac{\int \frac{-4A-\frac{aC}{c}}{(a+cx^2)^{5/2}}\,dx}{5a}\\
&= -\frac{aB-(Ac-aC)x}{5ac\,(a+cx^2)^{5/2}}+\frac{(4Ac+aC)\int \frac{1}{(a+cx^2)^{5/2}}\,dx}{5ac}\\
&= -\frac{aB-(Ac-aC)x}{5ac\,(a+cx^2)^{5/2}}+\frac{(4Ac+aC)x}{15a^2c\,(a+cx^2)^{3/2}}+\frac{(2(4Ac+aC))\int \frac{1}{(a+cx^2)^{3/2}}\,dx}{15a^2c}\\
&= -\frac{aB-(Ac-aC)x}{5ac\,(a+cx^2)^{5/2}}+\frac{(4Ac+aC)x}{15a^2c\,(a+cx^2)^{3/2}}+\frac{2(4Ac+aC)x}{15a^3c\sqrt{a+cx^2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.73

$$
\int \frac{A+Bx+Cx^2}{(a+cx^2)^{7/2}}\,dx=\frac{-3a^3B+8Ac^3x^5+5a^2cx(3A+Cx^2)+2ac^2x^3(10A+Cx^2)}{15a^3c\,(a+cx^2)^{5/2}}
$$

```
[In] Integrate[(A + B*x + C*x^2)/(a + c*x^2)^(7/2),x]
```

```
[Out] (-3*a^3*B + 8*A*c^3*x^5 + 5*a^2*c*x*(3*A + C*x^2) + 2*a*c^2*x^3*(10*A + C*x
^2))/(15*a^3*c*(a + c*x^2)^(5/2))
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.74

method	result
gosper	$\frac{8A\,c^3x^5+2a\,c^2C\,x^5+20aA\,c^2x^3+5C\,a^2c\,x^3+15a^2Acx-3B\,a^3}{15(c\,x^2+a)^{\frac{5}{2}}\,a^3c}$
trager	$\frac{8A\,c^3x^5+2a\,c^2C\,x^5+20aA\,c^2x^3+5C\,a^2c\,x^3+15a^2Acx-3B\,a^3}{15(c\,x^2+a)^{\frac{5}{2}}\,a^3c}$
default	$A\left(\frac{x}{5a(c\,x^2+a)^{\frac{5}{2}}}+\frac{\frac{4x}{15a\left(c\,x^2+a\right)^{\frac{3}{2}}}+\frac{8x}{15a^2\sqrt{c\,x^2+a}}}{a}\right)+C\left(-\frac{x}{4c(c\,x^2+a)^{\frac{5}{2}}}+\frac{a\left(\frac{x}{5a\left(c\,x^2+a\right)^{\frac{5}{2}}}+\frac{\frac{4x}{15a\left(c\,x^2+a\right)^{\frac{3}{2}}}+\frac{8x}{15a^2\sqrt{c\,x^2+}}}{a}\right.}{4c}\right.$

```
[In] int((C*x^2+B*x+A)/(c*x^2+a)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*(8*A*c^3*x^5+2*C*a*c^2*x^5+20*A*a*c^2*x^3+5*C*a^2*c*x^3+15*A*a^2*c*x-3
*B*a^3)/(c*x^2+a)^(5/2)/a^3/c
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.06

$$\int \frac{A+Bx+Cx^2}{(a+cx^2)^{7/2}}\,dx=\frac{(2\,(Cac^2+4\,Ac^3)x^5+15\,Aa^2cx-3\,Ba^3+5\,(Ca^2c+4\,Aac^2)x^3)\sqrt{cx^2+a}}{15\,(a^3c^4x^6+3\,a^4c^3x^4+3\,a^5c^2x^2+a^6c)}$$

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(2*(C*a*c^2 + 4*A*c^3)*x^5 + 15*A*a^2*c*x - 3*B*a^3 + 5*(C*a^2*c + 4*A
*a*c^2)*x^3)*sqrt(c*x^2 + a)/(a^3*c^4*x^6 + 3*a^4*c^3*x^4 + 3*a^5*c^2*x^2 +
 a^6*c)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 415 vs. 2(87) = 174.

Time = 12.06 (sec) , antiderivative size = 638, normalized size of antiderivative = 6.58

$$\int \frac{A+Bx+Cx^2}{(a+cx^2)^{7/2}}\,dx = A\left(\frac{15a^5x}{15a^{\frac{17}{2}}\sqrt{1+\frac{cx^2}{a}}+45a^{\frac{15}{2}}cx^2\sqrt{1+\frac{cx^2}{a}}+45a^{\frac{13}{2}}c^2x^4\sqrt{1+\frac{cx^2}{a}}+15a^{\frac{11}{2}}c^3x^6\sqrt{1+\frac{cx^2}{a}}}\right.$$

$$+\frac{35a^4cx^3}{15a^{\frac{17}{2}}\sqrt{1+\frac{cx^2}{a}}+45a^{\frac{15}{2}}cx^2\sqrt{1+\frac{cx^2}{a}}+45a^{\frac{13}{2}}c^2x^4\sqrt{1+\frac{cx^2}{a}}+15a^{\frac{11}{2}}c^3x^6\sqrt{1+\frac{cx^2}{a}}}$$

$$+\frac{28a^3c^2x^5}{15a^{\frac{17}{2}}\sqrt{1+\frac{cx^2}{a}}+45a^{\frac{15}{2}}cx^2\sqrt{1+\frac{cx^2}{a}}+45a^{\frac{13}{2}}c^2x^4\sqrt{1+\frac{cx^2}{a}}+15a^{\frac{11}{2}}c^3x^6\sqrt{1+\frac{cx^2}{a}}}$$

$$\left.+\frac{8a^2c^3x^7}{15a^{\frac{17}{2}}\sqrt{1+\frac{cx^2}{a}}+45a^{\frac{15}{2}}cx^2\sqrt{1+\frac{cx^2}{a}}+45a^{\frac{13}{2}}c^2x^4\sqrt{1+\frac{cx^2}{a}}+15a^{\frac{11}{2}}c^3x^6\sqrt{1+\frac{cx^2}{a}}}\right)$$

$$+B\left(\begin{cases}-\frac{1}{5a^2c\sqrt{a+cx^2}+10ac^2x^2\sqrt{a+cx^2}+5c^3x^4\sqrt{a+cx^2}} & \text{for } c\neq 0\\ \frac{x^2}{2a^{\frac{7}{2}}} & \text{otherwise}\end{cases}\right)$$

$$+C\left(\frac{5ax^3}{15a^{\frac{9}{2}}\sqrt{1+\frac{cx^2}{a}}+30a^{\frac{7}{2}}cx^2\sqrt{1+\frac{cx^2}{a}}+15a^{\frac{5}{2}}c^2x^4\sqrt{1+\frac{cx^2}{a}}}\right.$$

$$\left.+\frac{2cx^5}{15a^{\frac{9}{2}}\sqrt{1+\frac{cx^2}{a}}+30a^{\frac{7}{2}}cx^2\sqrt{1+\frac{cx^2}{a}}+15a^{\frac{5}{2}}c^2x^4\sqrt{1+\frac{cx^2}{a}}}\right)$$

```
[In] integrate((C*x**2+B*x+A)/(c*x**2+a)**(7/2),x)
```

```
[Out] A*(15*a**5*x/(15*a**(17/2)*sqrt(1 + c*x**2/a) + 45*a**(15/2)*c*x**2*sqrt(1
+ c*x**2/a) + 45*a**(13/2)*c**2*x**4*sqrt(1 + c*x**2/a) + 15*a**(11/2)*c**3
*x**6*sqrt(1 + c*x**2/a)) + 35*a**4*c*x**3/(15*a**(17/2)*sqrt(1 + c*x**2/a)
 + 45*a**(15/2)*c*x**2*sqrt(1 + c*x**2/a) + 45*a**(13/2)*c**2*x**4*sqrt(1 +
 c*x**2/a) + 15*a**(11/2)*c**3*x**6*sqrt(1 + c*x**2/a)) + 28*a**3*c**2*x**5
/(15*a**(17/2)*sqrt(1 + c*x**2/a) + 45*a**(15/2)*c*x**2*sqrt(1 + c*x**2/a)
+ 45*a**(13/2)*c**2*x**4*sqrt(1 + c*x**2/a) + 15*a**(11/2)*c**3*x**6*sqrt(1
 + c*x**2/a)) + 8*a**2*c**3*x**7/(15*a**(17/2)*sqrt(1 + c*x**2/a) + 45*a**(
15/2)*c*x**2*sqrt(1 + c*x**2/a) + 45*a**(13/2)*c**2*x**4*sqrt(1 + c*x**2/a)
 + 15*a**(11/2)*c**3*x**6*sqrt(1 + c*x**2/a))) + B*Piecewise((-1/(5*a**2*c*
sqrt(a + c*x**2) + 10*a*c**2*x**2*sqrt(a + c*x**2) + 5*c**3*x**4*sqrt(a + c
*x**2)), Ne(c, 0)), (x**2/(2*a**(7/2)), True)) + C*(5*a*x**3/(15*a**(9/2)*s
qrt(1 + c*x**2/a) + 30*a**(7/2)*c*x**2*sqrt(1 + c*x**2/a) + 15*a**(5/2)*c**
2*x**4*sqrt(1 + c*x**2/a)) + 2*c*x**5/(15*a**(9/2)*sqrt(1 + c*x**2/a) + 30*
a**(7/2)*c*x**2*sqrt(1 + c*x**2/a) + 15*a**(5/2)*c**2*x**4*sqrt(1 + c*x**2/
a)))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.22

$$\int \frac{A+Bx+Cx^2}{(a+cx^2)^{7/2}}\,dx = \frac{8\,Ax}{15\,\sqrt{cx^2+a}a^3}+\frac{4\,Ax}{15\,(cx^2+a)^{\frac{3}{2}}a^2}+\frac{Ax}{5\,(cx^2+a)^{\frac{5}{2}}a} - \frac{Cx}{5\,(cx^2+a)^{\frac{5}{2}}c}+\frac{2\,Cx}{15\,\sqrt{cx^2+a}a^2c}+\frac{Cx}{15\,(cx^2+a)^{\frac{3}{2}}ac}-\frac{B}{5\,(cx^2+a)^{\frac{5}{2}}c}$$

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a)^(7/2),x, algorithm="maxima")
```

```
[Out] 8/15*A*x/(sqrt(c*x^2 + a)*a^3) + 4/15*A*x/((c*x^2 + a)^(3/2)*a^2) + 1/5*A*x
/((c*x^2 + a)^(5/2)*a) - 1/5*C*x/((c*x^2 + a)^(5/2)*c) + 2/15*C*x/(sqrt(c*x
^2 + a)*a^2*c) + 1/15*C*x/((c*x^2 + a)^(3/2)*a*c) - 1/5*B/((c*x^2 + a)^(5/2
)*c)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.82

$$\int \frac{A+Bx+Cx^2}{(a+cx^2)^{7/2}}\,dx = \frac{\left(x^2\left(\frac{2\,(Cac^3+4\,Ac^4)x^2}{a^3c^2}+\frac{5\,(Ca^2c^2+4\,Aac^3)}{a^3c^2}\right)+\frac{15\,A}{a}\right)x-\frac{3\,B}{c}}{15\,(cx^2+a)^{\frac{5}{2}}}$$

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a)^(7/2),x, algorithm="giac")
```

```
[Out] 1/15*((x^2*(2*(C*a*c^3 + 4*A*c^4)*x^2/(a^3*c^2) + 5*(C*a^2*c^2 + 4*A*a*c^3)
/(a^3*c^2)) + 15*A/a)*x - 3*B/c)/(c*x^2 + a)^(5/2)
```

Mupad [B] (verification not implemented)

Time = 12.96 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.96

$$\int \frac{A+Bx+Cx^2}{(a+cx^2)^{7/2}}\,dx = \frac{8\,A\,c\,x\,(c\,x^2+a)^2-3\,C\,a^3\,x-3\,B\,a^3+2\,C\,a\,x\,(c\,x^2+a)^2+C\,a^2\,x\,(c\,x^2+a)+3\,A}{15\,a^3\,c\,(c\,x^2+a)^{5/2}}$$

```
[In] int((A + B*x + C*x^2)/(a + c*x^2)^(7/2),x)
```

```
[Out] (8*A*c*x*(a + c*x^2)^2 - 3*C*a^3*x - 3*B*a^3 + 2*C*a*x*(a + c*x^2)^2 + C*a^
2*x*(a + c*x^2) + 3*A*a^2*c*x + 4*A*a*c*x*(a + c*x^2))/(15*a^3*c*(a + c*x^2
)^(5/2))
```

3.117 $\int \frac{A+Bx+Cx^2}{\left(a+cx^2\right)^{9/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 127

$$\int \frac{A+Bx+Cx^2}{\left(a+cx^2\right)^{9/2}}\,dx = -\frac{aB-(Ac-aC)x}{7ac\left(a+cx^2\right)^{7/2}} + \frac{(6Ac+aC)x}{35a^2c\left(a+cx^2\right)^{5/2}} + \frac{4(6Ac+aC)x}{105a^3c\left(a+cx^2\right)^{3/2}} + \frac{8(6Ac+aC)x}{105a^4c\sqrt{a+cx^2}}$$

```
[Out] 1/7*(-a*B+(A*c-C*a)*x)/a/c/(c*x^2+a)^(7/2)+1/35*(6*A*c+C*a)*x/a^2/c/(c*x^2+
a)^(5/2)+4/105*(6*A*c+C*a)*x/a^3/c/(c*x^2+a)^(3/2)+8/105*(6*A*c+C*a)*x/a^4/
c/(c*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1828, 12, 198, 197}

$$\int \frac{A+Bx+Cx^2}{\left(a+cx^2\right)^{9/2}}\,dx = \frac{8x(aC+6Ac)}{105a^4c\sqrt{a+cx^2}} + \frac{4x(aC+6Ac)}{105a^3c\left(a+cx^2\right)^{3/2}} + \frac{x(aC+6Ac)}{35a^2c\left(a+cx^2\right)^{5/2}} - \frac{aB-x(Ac-aC)}{7ac\left(a+cx^2\right)^{7/2}}$$

```
[In] Int[(A + B*x + C*x^2)/(a + c*x^2)^(9/2),x]
```

```
[Out] -1/7*(a*B - (A*c - a*C)*x)/(a*c*(a + c*x^2)^(7/2)) + ((6*A*c + a*C)*x)/(35*
a^2*c*(a + c*x^2)^(5/2)) + (4*(6*A*c + a*C)*x)/(105*a^3*c*(a + c*x^2)^(3/2)
) + (8*(6*A*c + a*C)*x)/(105*a^4*c*Sqrt[a + c*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{aB-(Ac-aC)x}{7ac\left(a+cx^2\right)^{7/2}}-\frac{\int\frac{-6A-\frac{aC}{c}}{\left(a+cx^2\right)^{7/2}}\,dx}{7a}\\
&= -\frac{aB-(Ac-aC)x}{7ac\left(a+cx^2\right)^{7/2}}+\frac{(6Ac+aC)\int\frac{1}{\left(a+cx^2\right)^{7/2}}\,dx}{7ac}\\
&= -\frac{aB-(Ac-aC)x}{7ac\left(a+cx^2\right)^{7/2}}+\frac{(6Ac+aC)x}{35a^2c\left(a+cx^2\right)^{5/2}}+\frac{(4(6Ac+aC))\int\frac{1}{\left(a+cx^2\right)^{5/2}}\,dx}{35a^2c}\\
&= -\frac{aB-(Ac-aC)x}{7ac\left(a+cx^2\right)^{7/2}}+\frac{(6Ac+aC)x}{35a^2c\left(a+cx^2\right)^{5/2}}\\
&\quad+\frac{4(6Ac+aC)x}{105a^3c\left(a+cx^2\right)^{3/2}}+\frac{(8(6Ac+aC))\int\frac{1}{\left(a+cx^2\right)^{3/2}}\,dx}{105a^3c}\\
&= -\frac{aB-(Ac-aC)x}{7ac\left(a+cx^2\right)^{7/2}}+\frac{(6Ac+aC)x}{35a^2c\left(a+cx^2\right)^{5/2}}+\frac{4(6Ac+aC)x}{105a^3c\left(a+cx^2\right)^{3/2}}+\frac{8(6Ac+aC)x}{105a^4c\sqrt{a+cx^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.72

$$\int \frac{A+Bx+Cx^2}{\left(a+cx^2\right)^{9/2}}\,dx=\frac{-15a^4B+48Ac^4x^7+35a^3cx(3A+Cx^2)+8ac^3x^5(21A+Cx^2)+14a^2c^2x^3(15A+2}{105a^4c\left(a+cx^2\right)^{7/2}}$$

```
[In] Integrate[(A + B*x + C*x^2)/(a + c*x^2)^(9/2),x]
```

```
[Out] (-15*a^4*B + 48*A*c^4*x^7 + 35*a^3*c*x*(3*A + C*x^2) + 8*a*c^3*x^5*(21*A +
C*x^2) + 14*a^2*c^2*x^3*(15*A + 2*C*x^2))/(105*a^4*c*(a + c*x^2)^(7/2))
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.76

method	result
gosper	$\frac{48A\,c^4x^7+8Ca\,c^3x^7+168aA\,c^3x^5+28C\,a^2c^2x^5+210a^2A\,c^2x^3+35C\,a^3c\,x^3+105a^3Acx-15B\,a^4}{105(c\,x^2+a)^{\frac{7}{2}}a^4c}$
trager	$\frac{48A\,c^4x^7+8Ca\,c^3x^7+168aA\,c^3x^5+28C\,a^2c^2x^5+210a^2A\,c^2x^3+35C\,a^3c\,x^3+105a^3Acx-15B\,a^4}{105(c\,x^2+a)^{\frac{7}{2}}a^4c}$
default	$A\left(\frac{x}{7a(c\,x^2+a)^{\frac{7}{2}}}+\frac{\frac{6x}{35a\left(c\,x^2+a\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15a\left(c\,x^2+a\right)^{\frac{3}{2}}}+\frac{8x}{15a^2\sqrt{c\,x^2+a}}\right)}{7a}}{a}\right)+C\left(-\frac{x}{6c(c\,x^2+a)^{\frac{7}{2}}}+\frac{a\left(\frac{x}{7a\left(c\,x^2+a\right)^{\frac{7}{2}}}+\frac{6x}{35a\left(c\,x^2+\right.}\right.}{}\right.$

```
[In] int((C*x^2+B*x+A)/(c*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/105*(48*A*c^4*x^7+8*C*a*c^3*x^7+168*A*a*c^3*x^5+28*C*a^2*c^2*x^5+210*A*a^
2*c^2*x^3+35*C*a^3*c*x^3+105*A*a^3*c*x-15*B*a^4)/(c*x^2+a)^(7/2)/a^4/c
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.08

$$\int \frac{A+Bx+Cx^2}{\left(a+cx^2\right)^{9/2}}\,dx = \frac{(8\,(Cac^3+6\,Ac^4)x^7+105\,Aa^3cx+28\,(Ca^2c^2+6\,Aac^3)x^5-15\,Ba^4+35\,(Ca^3c+6}{105\,(a^4c^5x^8+4\,a^5c^4x^6+6\,a^6c^3x^4+4\,a^7c^2x^2+a^8c)}$$

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/105*(8*(C*a*c^3 + 6*A*c^4)*x^7 + 105*A*a^3*c*x + 28*(C*a^2*c^2 + 6*A*a*c^
3)*x^5 - 15*B*a^4 + 35*(C*a^3*c + 6*A*a^2*c^2)*x^3)*sqrt(c*x^2 + a)/(a^4*c^
5*x^8 + 4*a^5*c^4*x^6 + 6*a^6*c^3*x^4 + 4*a^7*c^2*x^2 + a^8*c)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1266 vs. 2(117) = 234.

Time = 26.21 (sec) , antiderivative size = 1880, normalized size of antiderivative = 14.80

$$\int \frac{A+Bx+Cx^2}{\left(a+cx^2\right)^{9/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((C*x**2+B*x+A)/(c*x**2+a)**(9/2),x)
```

```
[Out] A*(35*a**14*x/(35*a**(37/2)*sqrt(1 + c*x**2/a) + 210*a**(35/2)*c*x**2*sqrt(
1 + c*x**2/a) + 525*a**(33/2)*c**2*x**4*sqrt(1 + c*x**2/a) + 700*a**(31/2)*
c**3*x**6*sqrt(1 + c*x**2/a) + 525*a**(29/2)*c**4*x**8*sqrt(1 + c*x**2/a) +
 210*a**(27/2)*c**5*x**10*sqrt(1 + c*x**2/a) + 35*a**(25/2)*c**6*x**12*sqrt
(1 + c*x**2/a)) + 175*a**13*c*x**3/(35*a**(37/2)*sqrt(1 + c*x**2/a) + 210*a
**(35/2)*c*x**2*sqrt(1 + c*x**2/a) + 525*a**(33/2)*c**2*x**4*sqrt(1 + c*x**
2/a) + 700*a**(31/2)*c**3*x**6*sqrt(1 + c*x**2/a) + 525*a**(29/2)*c**4*x**8
*sqrt(1 + c*x**2/a) + 210*a**(27/2)*c**5*x**10*sqrt(1 + c*x**2/a) + 35*a**(
25/2)*c**6*x**12*sqrt(1 + c*x**2/a)) + 371*a**12*c**2*x**5/(35*a**(37/2)*sq
rt(1 + c*x**2/a) + 210*a**(35/2)*c*x**2*sqrt(1 + c*x**2/a) + 525*a**(33/2)*
c**2*x**4*sqrt(1 + c*x**2/a) + 700*a**(31/2)*c**3*x**6*sqrt(1 + c*x**2/a) +
 525*a**(29/2)*c**4*x**8*sqrt(1 + c*x**2/a) + 210*a**(27/2)*c**5*x**10*sqrt
(1 + c*x**2/a) + 35*a**(25/2)*c**6*x**12*sqrt(1 + c*x**2/a)) + 429*a**11*c*
*3*x**7/(35*a**(37/2)*sqrt(1 + c*x**2/a) + 210*a**(35/2)*c*x**2*sqrt(1 + c*
x**2/a) + 525*a**(33/2)*c**2*x**4*sqrt(1 + c*x**2/a) + 700*a**(31/2)*c**3*x
**6*sqrt(1 + c*x**2/a) + 525*a**(29/2)*c**4*x**8*sqrt(1 + c*x**2/a) + 210*a
**(27/2)*c**5*x**10*sqrt(1 + c*x**2/a) + 35*a**(25/2)*c**6*x**12*sqrt(1 + c
*x**2/a)) + 286*a**10*c**4*x**9/(35*a**(37/2)*sqrt(1 + c*x**2/a) + 210*a**(
35/2)*c*x**2*sqrt(1 + c*x**2/a) + 525*a**(33/2)*c**2*x**4*sqrt(1 + c*x**2/a
) + 700*a**(31/2)*c**3*x**6*sqrt(1 + c*x**2/a) + 525*a**(29/2)*c**4*x**8*sq
```

```
rt(1 + c*x**2/a) + 210*a**(27/2)*c**5*x**10*sqrt(1 + c*x**2/a) + 35*a**(25/
2)*c**6*x**12*sqrt(1 + c*x**2/a)) + 104*a**9*c**5*x**11/(35*a**(37/2)*sqrt(
1 + c*x**2/a) + 210*a**(35/2)*c*x**2*sqrt(1 + c*x**2/a) + 525*a**(33/2)*c**
2*x**4*sqrt(1 + c*x**2/a) + 700*a**(31/2)*c**3*x**6*sqrt(1 + c*x**2/a) + 52
5*a**(29/2)*c**4*x**8*sqrt(1 + c*x**2/a) + 210*a**(27/2)*c**5*x**10*sqrt(1
+ c*x**2/a) + 35*a**(25/2)*c**6*x**12*sqrt(1 + c*x**2/a)) + 16*a**8*c**6*x*
*13/(35*a**(37/2)*sqrt(1 + c*x**2/a) + 210*a**(35/2)*c*x**2*sqrt(1 + c*x**2
/a) + 525*a**(33/2)*c**2*x**4*sqrt(1 + c*x**2/a) + 700*a**(31/2)*c**3*x**6*
sqrt(1 + c*x**2/a) + 525*a**(29/2)*c**4*x**8*sqrt(1 + c*x**2/a) + 210*a**(2
7/2)*c**5*x**10*sqrt(1 + c*x**2/a) + 35*a**(25/2)*c**6*x**12*sqrt(1 + c*x**
2/a))) + B*Piecewise((-1/(7*a**3*c*sqrt(a + c*x**2) + 21*a**2*c**2*x**2*sqr
t(a + c*x**2) + 21*a*c**3*x**4*sqrt(a + c*x**2) + 7*c**4*x**6*sqrt(a + c*x*
*2)), Ne(c, 0)), (x**2/(2*a**(9/2)), True)) + C*(35*a**5*x**3/(105*a**(19/2
)*sqrt(1 + c*x**2/a) + 420*a**(17/2)*c*x**2*sqrt(1 + c*x**2/a) + 630*a**(15
/2)*c**2*x**4*sqrt(1 + c*x**2/a) + 420*a**(13/2)*c**3*x**6*sqrt(1 + c*x**2/
a) + 105*a**(11/2)*c**4*x**8*sqrt(1 + c*x**2/a)) + 63*a**4*c*x**5/(105*a**(
19/2)*sqrt(1 + c*x**2/a) + 420*a**(17/2)*c*x**2*sqrt(1 + c*x**2/a) + 630*a*
*(15/2)*c**2*x**4*sqrt(1 + c*x**2/a) + 420*a**(13/2)*c**3*x**6*sqrt(1 + c*x
**2/a) + 105*a**(11/2)*c**4*x**8*sqrt(1 + c*x**2/a)) + 36*a**3*c**2*x**7/(1
05*a**(19/2)*sqrt(1 + c*x**2/a) + 420*a**(17/2)*c*x**2*sqrt(1 + c*x**2/a) +
 630*a**(15/2)*c**2*x**4*sqrt(1 + c*x**2/a) + 420*a**(13/2)*c**3*x**6*sqrt(
1 + c*x**2/a) + 105*a**(11/2)*c**4*x**8*sqrt(1 + c*x**2/a)) + 8*a**2*c**3*x
**9/(105*a**(19/2)*sqrt(1 + c*x**2/a) + 420*a**(17/2)*c*x**2*sqrt(1 + c*x**
2/a) + 630*a**(15/2)*c**2*x**4*sqrt(1 + c*x**2/a) + 420*a**(13/2)*c**3*x**6
*sqrt(1 + c*x**2/a) + 105*a**(11/2)*c**4*x**8*sqrt(1 + c*x**2/a)))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.20

$$
\begin{aligned}
\int \frac{A+Bx+Cx^2}{\left(a+cx^2\right)^{9/2}}\,dx &= \frac{16\,Ax}{35\,\sqrt{cx^2+a}a^4}+\frac{8\,Ax}{35\,\left(cx^2+a\right)^{\frac{3}{2}}a^3} \\
&\quad+\frac{6\,Ax}{35\,\left(cx^2+a\right)^{\frac{5}{2}}a^2}+\frac{Ax}{7\,\left(cx^2+a\right)^{\frac{7}{2}}a}-\frac{Cx}{7\,\left(cx^2+a\right)^{\frac{7}{2}}c}+\frac{8\,Cx}{105\,\sqrt{cx^2+a}a^3c} \\
&\quad+\frac{4\,Cx}{105\,\left(cx^2+a\right)^{\frac{3}{2}}a^2c}+\frac{Cx}{35\,\left(cx^2+a\right)^{\frac{5}{2}}ac}-\frac{B}{7\,\left(cx^2+a\right)^{\frac{7}{2}}c}
\end{aligned}
$$

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 16/35*A*x/(sqrt(c*x^2 + a)*a^4) + 8/35*A*x/((c*x^2 + a)^(3/2)*a^3) + 6/35*A
*x/((c*x^2 + a)^(5/2)*a^2) + 1/7*A*x/((c*x^2 + a)^(7/2)*a) - 1/7*C*x/((c*x^
2 + a)^(7/2)*c) + 8/105*C*x/(sqrt(c*x^2 + a)*a^3*c) + 4/105*C*x/((c*x^2 + a
)^(3/2)*a^2*c) + 1/35*C*x/((c*x^2 + a)^(5/2)*a*c) - 1/7*B/((c*x^2 + a)^(7/2
)*c)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx+Cx^2}{\left(a+cx^2\right)^{9/2}}\,dx = \frac{\left(\left(4\,x^2\left(\frac{2\,\left(Cac^5+6\,Ac^6\right)x^2}{a^4c^3}+\frac{7\,\left(Ca^2c^4+6\,Aac^5\right)}{a^4c^3}\right)+\frac{35\,\left(Ca^3c^3+6\,Aa^2c^4\right)}{a^4c^3}\right)x^2+\frac{105\,A}{a}\right)x-\frac{15\,B}{c}}{105\,\left(cx^2+a\right)^{\frac{7}{2}}}$$

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*(((4*x^2*(2*(C*a*c^5 + 6*A*c^6)*x^2/(a^4*c^3) + 7*(C*a^2*c^4 + 6*A*a*
c^5)/(a^4*c^3)) + 35*(C*a^3*c^3 + 6*A*a^2*c^4)/(a^4*c^3))*x^2 + 105*A/a)*x
- 15*B/c)/(c*x^2 + a)^(7/2)
```

Mupad [B] (verification not implemented)

Time = 12.99 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.91

$$\int \frac{A+Bx+Cx^2}{\left(a+cx^2\right)^{9/2}}\,dx = \frac{x\,\left(6\,A\,c+C\,a\right)}{35\,a^2\,c\,{\left(c\,x^2+a\right)}^{5/2}}-\frac{\frac{B}{7\,c}-x\,\left(\frac{A}{7\,a}-\frac{C}{7\,c}\right)}{{\left(c\,x^2+a\right)}^{7/2}}+\frac{x\,\left(24\,A\,c+4\,C\,a\right)}{105\,a^3\,c\,{\left(c\,x^2+a\right)}^{3/2}}+\frac{x\,\left(48\,A\,c+8\,C\,a\right)}{105\,a^4\,c\,\sqrt{c\,x^2+a}}$$

```
[In] int((A + B*x + C*x^2)/(a + c*x^2)^(9/2),x)
```

```
[Out] (x*(6*A*c + C*a))/(35*a^2*c*(a + c*x^2)^(5/2)) - (B/(7*c) - x*(A/(7*a) - C/
(7*c)))/(a + c*x^2)^(7/2) + (x*(24*A*c + 4*C*a))/(105*a^3*c*(a + c*x^2)^(3/
2)) + (x*(48*A*c + 8*C*a))/(105*a^4*c*(a + c*x^2)^(1/2))
```

3.118 $\int \frac{(1+2x)^3(1+3x+4x^2)}{\sqrt{2+3x^2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 106

$$
\begin{aligned}
\int \frac{(1+2x)^3(1+3x+4x^2)}{\sqrt{2+3x^2}}\,dx = {} & -\frac{19}{540}(1+2x)^2\sqrt{2+3x^2} \\
& +\frac{13}{60}(1+2x)^3\sqrt{2+3x^2}+\frac{2}{15}(1+2x)^4\sqrt{2+3x^2} \\
& -\frac{1}{810}(3937+2073x)\sqrt{2+3x^2}+\frac{5\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{3\sqrt{3}}
\end{aligned}
$$

```
[Out] 5/9*arcsinh(1/2*x*6^(1/2))*3^(1/2)-19/540*(1+2*x)^2*(3*x^2+2)^(1/2)+13/60*(
1+2*x)^3*(3*x^2+2)^(1/2)+2/15*(1+2*x)^4*(3*x^2+2)^(1/2)-1/810*(3937+2073*x)
*(3*x^2+2)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1668, 847, 794, 221}

$$
\begin{aligned}
\int \frac{(1+2x)^3(1+3x+4x^2)}{\sqrt{2+3x^2}}\,dx = {} & \frac{5\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{3\sqrt{3}}+\frac{2}{15}\sqrt{3x^2+2}(2x+1)^4 \\
& +\frac{13}{60}\sqrt{3x^2+2}(2x+1)^3-\frac{19}{540}\sqrt{3x^2+2}(2x+1)^2 \\
& -\frac{1}{810}(2073x+3937)\sqrt{3x^2+2}
\end{aligned}
$$

```
[In] Int[((1 + 2*x)^3*(1 + 3*x + 4*x^2))/Sqrt[2 + 3*x^2],x]
```

```
[Out] (-19*(1 + 2*x)^2*Sqrt[2 + 3*x^2])/540 + (13*(1 + 2*x)^3*Sqrt[2 + 3*x^2])/60
 + (2*(1 + 2*x)^4*Sqrt[2 + 3*x^2])/15 - ((3937 + 2073*x)*Sqrt[2 + 3*x^2])/8
10 + (5*ArcSinh[Sqrt[3/2]*x])/(3*Sqrt[3])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2}{15}(1+2x)^4\sqrt{2+3x^2}+\frac{1}{60}\int\frac{(1+2x)^3(-68+156x)}{\sqrt{2+3x^2}}\,dx\\ &= \frac{13}{60}(1+2x)^3\sqrt{2+3x^2}+\frac{2}{15}(1+2x)^4\sqrt{2+3x^2}+\frac{1}{720}\int\frac{(-2688-228x)(1+2x)^2}{\sqrt{2+3x^2}}\,dx\end{aligned}$$

$$
\begin{aligned}
&= -\frac{19}{540}(1+2x)^2\sqrt{2+3x^2}+\frac{13}{60}(1+2x)^3\sqrt{2+3x^2} \\
&\quad +\frac{2}{15}(1+2x)^4\sqrt{2+3x^2}+\frac{\int\frac{(-22368-49752x)(1+2x)}{\sqrt{2+3x^2}}\,dx}{6480} \\
&= -\frac{19}{540}(1+2x)^2\sqrt{2+3x^2}+\frac{13}{60}(1+2x)^3\sqrt{2+3x^2}+\frac{2}{15}(1+2x)^4\sqrt{2+3x^2} \\
&\quad -\frac{1}{810}(3937+2073x)\sqrt{2+3x^2}+\frac{5}{3}\int\frac{1}{\sqrt{2+3x^2}}\,dx \\
&= -\frac{19}{540}(1+2x)^2\sqrt{2+3x^2}+\frac{13}{60}(1+2x)^3\sqrt{2+3x^2}+\frac{2}{15}(1+2x)^4\sqrt{2+3x^2} \\
&\quad -\frac{1}{810}(3937+2073x)\sqrt{2+3x^2}+\frac{5\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{3\sqrt{3}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.62

$$
\begin{aligned}
\int\frac{(1+2x)^3\left(1+3x+4x^2\right)}{\sqrt{2+3x^2}}\,dx &= \frac{1}{405}\sqrt{2+3x^2}\left(-1841-135x+2292x^2+2430x^3+864x^4\right) \\
&\quad -\frac{5\log\left(-\sqrt{3}x+\sqrt{2+3x^2}\right)}{3\sqrt{3}}
\end{aligned}
$$

```
[In] Integrate[((1 + 2*x)^3*(1 + 3*x + 4*x^2))/Sqrt[2 + 3*x^2],x]
```

```
[Out] (Sqrt[2 + 3*x^2]*(-1841 - 135*x + 2292*x^2 + 2430*x^3 + 864*x^4))/405 - (5*
Log[-(Sqrt[3]*x) + Sqrt[2 + 3*x^2]])/(3*Sqrt[3])
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.42

method	result
risch	$\frac{\left(864x^4+2430x^3+2292x^2-135x-1841\right)\sqrt{3x^2+2}}{405}+\frac{5\operatorname{arcsinh}\left(\frac{x\sqrt{6}}{2}\right)\sqrt{3}}{9}$
trager	$\left(\frac{32}{15}x^4+6x^3+\frac{764}{135}x^2-\frac{1}{3}x-\frac{1841}{405}\right)\sqrt{3x^2+2}+\frac{5\operatorname{RootOf}\left(\_Z^2-3\right)\ln\left(\operatorname{RootOf}\left(\_Z^2-3\right)\sqrt{3x^2+2}+3x\right)}{9}$
default	$\frac{5\operatorname{arcsinh}\left(\frac{x\sqrt{6}}{2}\right)\sqrt{3}}{9}-\frac{1841\sqrt{3x^2+2}}{405}+\frac{32x^4\sqrt{3x^2+2}}{15}+\frac{764x^2\sqrt{3x^2+2}}{135}+6x^3\sqrt{3x^2+2}-\frac{x\sqrt{3x^2+2}}{3}$
meijerg	$\frac{\sqrt{3}\operatorname{arcsinh}\left(\frac{x\sqrt{3}\sqrt{2}}{2}\right)}{3}+\frac{34\sqrt{3}\left(\frac{\sqrt{\pi}\,x\sqrt{3}\sqrt{2}\sqrt{\frac{3x^2}{2}+1}}{2}-\sqrt{\pi}\operatorname{arcsinh}\left(\frac{x\sqrt{3}\sqrt{2}}{2}\right)\right)}{9\sqrt{\pi}}+\frac{3\sqrt{2}\left(-2\sqrt{\pi}+2\sqrt{\pi}\sqrt{\frac{3x^2}{2}+1}\right)}{2\sqrt{\pi}}+68\sqrt{2}\left(\frac{4\sqrt{\pi}}{3}-\right.$

```
[In] int((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/405*(864*x^4+2430*x^3+2292*x^2-135*x-1841)*(3*x^2+2)^(1/2)+5/9*arcsinh(1/
2*x*6^(1/2))*3^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.57

$$\int \frac{(1+2x)^3\left(1+3x+4x^2\right)}{\sqrt{2+3x^2}}\,dx = \frac{1}{405}\left(864\,x^4+2430\,x^3+2292\,x^2-135\,x-1841\right)\sqrt{3\,x^2+2} + \frac{5}{18}\,\sqrt{3}\log\left(-\sqrt{3}\sqrt{3\,x^2+2}x-3\,x^2-1\right)$$

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/405*(864*x^4 + 2430*x^3 + 2292*x^2 - 135*x - 1841)*sqrt(3*x^2 + 2) + 5/18
*sqrt(3)*log(-sqrt(3)*sqrt(3*x^2 + 2)*x - 3*x^2 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.89

$$\int \frac{(1+2x)^3\left(1+3x+4x^2\right)}{\sqrt{2+3x^2}}\,dx = \frac{32x^4\sqrt{3x^2+2}}{15}+6x^3\sqrt{3x^2+2}+\frac{764x^2\sqrt{3x^2+2}}{135} - \frac{x\sqrt{3x^2+2}}{3}-\frac{1841\sqrt{3x^2+2}}{405}+\frac{5\sqrt{3}\operatorname{asinh}\left(\frac{\sqrt{6}x}{2}\right)}{9}$$

```
[In] integrate((1+2*x)**3*(4*x**2+3*x+1)/(3*x**2+2)**(1/2),x)
```

```
[Out] 32*x**4*sqrt(3*x**2 + 2)/15 + 6*x**3*sqrt(3*x**2 + 2) + 764*x**2*sqrt(3*x**
2 + 2)/135 - x*sqrt(3*x**2 + 2)/3 - 1841*sqrt(3*x**2 + 2)/405 + 5*sqrt(3)*a
sinh(sqrt(6)*x/2)/9
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.74

$$\int \frac{(1+2x)^3\,(1+3x+4x^2)}{\sqrt{2+3x^2}}\,dx = \frac{32}{15}\,\sqrt{3\,x^2+2}x^4+6\,\sqrt{3\,x^2+2}x^3+\frac{764}{135}\,\sqrt{3\,x^2+2}x^2 - \frac{1}{3}\,\sqrt{3\,x^2+2}x+\frac{5}{9}\,\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{2}\,\sqrt{6}x\right)-\frac{1841}{405}\,\sqrt{3\,x^2+2}$$

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 32/15*sqrt(3*x^2 + 2)*x^4 + 6*sqrt(3*x^2 + 2)*x^3 + 764/135*sqrt(3*x^2 + 2)
*x^2 - 1/3*sqrt(3*x^2 + 2)*x + 5/9*sqrt(3)*arcsinh(1/2*sqrt(6)*x) - 1841/40
5*sqrt(3*x^2 + 2)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.51

$$\int \frac{(1+2x)^3\,(1+3x+4x^2)}{\sqrt{2+3x^2}}\,dx = \frac{1}{405}\,(3\,(2\,(9\,(16\,x+45)x+382)x-45)x-1841)\sqrt{3\,x^2+2} - \frac{5}{9}\,\sqrt{3}\log\left(-\sqrt{3}x+\sqrt{3\,x^2+2}\right)$$

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/405*(3*(2*(9*(16*x + 45)*x + 382)*x - 45)*x - 1841)*sqrt(3*x^2 + 2) - 5/9
*sqrt(3)*log(-sqrt(3)*x + sqrt(3*x^2 + 2))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.42

$$\int \frac{(1+2x)^3\,(1+3x+4x^2)}{\sqrt{2+3x^2}}\,dx = \frac{5\,\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{6}\,x}{2}\right)}{9} + \frac{\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}\,\left(\frac{32\,x^4}{5}+18\,x^3+\frac{764\,x^2}{45}-x-\frac{1841}{135}\right)}{3}$$

```
[In] int(((2*x + 1)^3*(3*x + 4*x^2 + 1))/(3*x^2 + 2)^(1/2),x)
```

```
[Out] (5*3^(1/2)*asinh((6^(1/2)*x)/2))/9 + (3^(1/2)*(x^2 + 2/3)^(1/2)*((764*x^2)/
45 - x + 18*x^3 + (32*x^4)/5 - 1841/135))/3
```

3.119 $\int \frac{(1+2x)^2(1+3x+4x^2)}{\sqrt{2+3x^2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 82

$$\int \frac{(1+2x)^2(1+3x+4x^2)}{\sqrt{2+3x^2}}\,dx = \frac{5}{18}(1+2x)^2\sqrt{2+3x^2}+\frac{1}{6}(1+2x)^3\sqrt{2+3x^2} -\frac{1}{27}(61+3x)\sqrt{2+3x^2}-\sqrt{3}\operatorname{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)$$

```
[Out] -arcsinh(1/2*x*6^(1/2))*3^(1/2)+5/18*(1+2*x)^2*(3*x^2+2)^(1/2)+1/6*(1+2*x)^
3*(3*x^2+2)^(1/2)-1/27*(61+3*x)*(3*x^2+2)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1668, 847, 794, 221}

$$\int \frac{(1+2x)^2(1+3x+4x^2)}{\sqrt{2+3x^2}}\,dx = -\sqrt{3}\operatorname{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)+\frac{1}{6}\sqrt{3x^2+2}(2x+1)^3 +\frac{5}{18}\sqrt{3x^2+2}(2x+1)^2-\frac{1}{27}(3x+61)\sqrt{3x^2+2}$$

```
[In] Int[((1 + 2*x)^2*(1 + 3*x + 4*x^2))/Sqrt[2 + 3*x^2],x]
```

```
[Out] (5*(1 + 2*x)^2*Sqrt[2 + 3*x^2])/18 + ((1 + 2*x)^3*Sqrt[2 + 3*x^2])/6 - ((61
 + 3*x)*Sqrt[2 + 3*x^2])/27 - Sqrt[3]*ArcSinh[Sqrt[3/2]*x]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{6}(1+2x)^3\sqrt{2+3x^2}+\frac{1}{48}\int \frac{(1+2x)^2(-48+120x)}{\sqrt{2+3x^2}}\,dx \\
&= \frac{5}{18}(1+2x)^2\sqrt{2+3x^2}+\frac{1}{6}(1+2x)^3\sqrt{2+3x^2}+\frac{1}{432}\int \frac{(-1392-144x)(1+2x)}{\sqrt{2+3x^2}}\,dx \\
&= \frac{5}{18}(1+2x)^2\sqrt{2+3x^2}+\frac{1}{6}(1+2x)^3\sqrt{2+3x^2}-\frac{1}{27}(61+3x)\sqrt{2+3x^2}-3\int \frac{1}{\sqrt{2+3x^2}}\,dx \\
&= \frac{5}{18}(1+2x)^2\sqrt{2+3x^2}+\frac{1}{6}(1+2x)^3\sqrt{2+3x^2}-\frac{1}{27}(61+3x)\sqrt{2+3x^2}-\sqrt{3}\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.71

$$\int \frac{(1+2x)^2\left(1+3x+4x^2\right)}{\sqrt{2+3x^2}}\,dx = \frac{1}{27}\sqrt{2+3x^2}\left(-49+54x+84x^2+36x^3\right) + \sqrt{3}\log\left(-\sqrt{3}x+\sqrt{2+3x^2}\right)$$

```
[In] Integrate[((1 + 2*x)^2*(1 + 3*x + 4*x^2))/Sqrt[2 + 3*x^2],x]
```

```
[Out] (Sqrt[2 + 3*x^2]*(-49 + 54*x + 84*x^2 + 36*x^3))/27 + Sqrt[3]*Log[-(Sqrt[3]
*x) + Sqrt[2 + 3*x^2]]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.49

method	result
risch	$\frac{\left(36x^3+84x^2+54x-49\right)\sqrt{3x^2+2}}{27}-\operatorname{arcsinh}\left(\frac{x\sqrt{6}}{2}\right)\sqrt{3}$
trager	$\left(\frac{4}{3}x^3+\frac{28}{9}x^2+2x-\frac{49}{27}\right)\sqrt{3x^2+2}+\operatorname{RootOf}\left(\_Z^2-3\right)\ln\left(-\operatorname{RootOf}\left(\_Z^2-3\right)\sqrt{3x^2+2}+3x\right.$
default	$-\operatorname{arcsinh}\left(\frac{x\sqrt{6}}{2}\right)\sqrt{3}-\frac{49\sqrt{3x^2+2}}{27}+\frac{4x^3\sqrt{3x^2+2}}{3}+2x\sqrt{3x^2+2}+\frac{28x^2\sqrt{3x^2+2}}{9}$
meijerg	$\frac{\sqrt{3}\operatorname{arcsinh}\left(\frac{x\sqrt{3}\sqrt{2}}{2}\right)}{3}+\frac{20\sqrt{3}\left(\frac{\sqrt{\pi}\,x\sqrt{3}\sqrt{2}\sqrt{\frac{3x^2}{2}+1}}{2}-\sqrt{\pi}\operatorname{arcsinh}\left(\frac{x\sqrt{3}\sqrt{2}}{2}\right)\right)}{9\sqrt{\pi}}+\frac{7\sqrt{2}\left(-2\sqrt{\pi}+2\sqrt{\pi}\sqrt{\frac{3x^2}{2}+1}\right)}{6\sqrt{\pi}}+$

```
[In] int((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/27*(36*x^3+84*x^2+54*x-49)*(3*x^2+2)^(1/2)-arcsinh(1/2*x*6^(1/2))*3^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.66

$$\int \frac{(1+2x)^2\left(1+3x+4x^2\right)}{\sqrt{2+3x^2}}\,dx = \frac{1}{27}\left(36\,x^3+84\,x^2+54\,x-49\right)\sqrt{3\,x^2+2} + \frac{1}{2}\sqrt{3}\log\left(\sqrt{3}\sqrt{3\,x^2+2}x-3\,x^2-1\right)$$

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/27*(36*x^3 + 84*x^2 + 54*x - 49)*sqrt(3*x^2 + 2) + 1/2*sqrt(3)*log(sqrt(3
)*sqrt(3*x^2 + 2)*x - 3*x^2 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.91

$$\int \frac{(1+2x)^2\,(1+3x+4x^2)}{\sqrt{2+3x^2}}\,dx = \frac{4x^3\sqrt{3x^2+2}}{3} + \frac{28x^2\sqrt{3x^2+2}}{9} + 2x\sqrt{3x^2+2} - \frac{49\sqrt{3x^2+2}}{27} - \sqrt{3}\operatorname{asinh}\left(\frac{\sqrt{6}x}{2}\right)$$

```
[In] integrate((1+2*x)**2*(4*x**2+3*x+1)/(3*x**2+2)**(1/2),x)
```

```
[Out] 4*x**3*sqrt(3*x**2 + 2)/3 + 28*x**2*sqrt(3*x**2 + 2)/9 + 2*x*sqrt(3*x**2 +
2) - 49*sqrt(3*x**2 + 2)/27 - sqrt(3)*asinh(sqrt(6)*x/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.78

$$\int \frac{(1+2x)^2\,(1+3x+4x^2)}{\sqrt{2+3x^2}}\,dx = \frac{4}{3}\,\sqrt{3\,x^2+2}x^3 + \frac{28}{9}\,\sqrt{3\,x^2+2}x^2 + 2\,\sqrt{3\,x^2+2}x - \sqrt{3}\operatorname{arsinh}\left(\frac{1}{2}\,\sqrt{6}x\right) - \frac{49}{27}\,\sqrt{3\,x^2+2}$$

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 4/3*sqrt(3*x^2 + 2)*x^3 + 28/9*sqrt(3*x^2 + 2)*x^2 + 2*sqrt(3*x^2 + 2)*x -
sqrt(3)*arcsinh(1/2*sqrt(6)*x) - 49/27*sqrt(3*x^2 + 2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.59

$$\int \frac{(1+2x)^2\,(1+3x+4x^2)}{\sqrt{2+3x^2}}\,dx = \frac{1}{27}\,(6\,(2\,(3\,x+7)x+9)x-49)\sqrt{3\,x^2+2} + \sqrt{3}\log\left(-\sqrt{3}x+\sqrt{3\,x^2+2}\right)$$

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/27*(6*(2*(3*x + 7)*x + 9)*x - 49)*sqrt(3*x^2 + 2) + sqrt(3)*log(-sqrt(3)*
x + sqrt(3*x^2 + 2))
```

Mupad [B] (verification not implemented)

Time = 12.85 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.49

$$\int \frac{(1+2x)^2\left(1+3x+4x^2\right)}{\sqrt{2+3x^2}}\,dx=\frac{\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}\,\left(4\,x^3+\frac{28\,x^2}{3}+6\,x-\frac{49}{9}\right)}{3}-\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{6}\,x}{2}\right)$$

```
[In] int(((2*x + 1)^2*(3*x + 4*x^2 + 1))/(3*x^2 + 2)^(1/2),x)
```

```
[Out] (3^(1/2)*(x^2 + 2/3)^(1/2)*(6*x + (28*x^2)/3 + 4*x^3 - 49/9))/3 - 3^(1/2)*a
sinh((6^(1/2)*x)/2)
```

3.120 $\int \frac{(1+2x)(1+3x+4x^2)}{\sqrt{2+3x^2}} \, dx$

Optimal result

Integrand size = 27, antiderivative size = 62

$$\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\sqrt{2+3x^2}} \, dx = \frac{2}{9}(1+2x)^2\sqrt{2+3x^2}+\frac{7}{27}(1+3x)\sqrt{2+3x^2}-\frac{7\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{3\sqrt{3}}$$

```
[Out] -7/9*arcsinh(1/2*x*6^(1/2))*3^(1/2)+2/9*(1+2*x)^2*(3*x^2+2)^(1/2)+7/27*(1+3
*x)*(3*x^2+2)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1668, 794, 221}

$$\begin{aligned}\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\sqrt{2+3x^2}} \, dx &= -\frac{7\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{3\sqrt{3}}+\frac{2}{9}\sqrt{3x^2+2}(2x+1)^2 \\ &\quad +\frac{7}{27}(3x+1)\sqrt{3x^2+2}\end{aligned}$$

```
[In] Int[((1 + 2*x)*(1 + 3*x + 4*x^2))/Sqrt[2 + 3*x^2],x]
```

```
[Out] (2*(1 + 2*x)^2*Sqrt[2 + 3*x^2])/9 + (7*(1 + 3*x)*Sqrt[2 + 3*x^2])/27 - (7*A
rcSinh[Sqrt[3/2]*x])/(3*Sqrt[3])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2}{9}(1+2x)^2\sqrt{2+3x^2}+\frac{1}{36}\int\frac{(1+2x)(-28+84x)}{\sqrt{2+3x^2}}\,dx\\
&= \frac{2}{9}(1+2x)^2\sqrt{2+3x^2}+\frac{7}{27}(1+3x)\sqrt{2+3x^2}-\frac{7}{3}\int\frac{1}{\sqrt{2+3x^2}}\,dx\\
&= \frac{2}{9}(1+2x)^2\sqrt{2+3x^2}+\frac{7}{27}(1+3x)\sqrt{2+3x^2}-\frac{7\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{3\sqrt{3}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.90

$$\int\frac{(1+2x)\left(1+3x+4x^2\right)}{\sqrt{2+3x^2}}\,dx=\frac{1}{27}\sqrt{2+3x^2}\left(13+45x+24x^2\right)+\frac{7\log\left(-\sqrt{3}x+\sqrt{2+3x^2}\right)}{3\sqrt{3}}$$

```
[In] Integrate[((1 + 2*x)*(1 + 3*x + 4*x^2))/Sqrt[2 + 3*x^2],x]
[Out] (Sqrt[2 + 3*x^2]*(13 + 45*x + 24*x^2))/27 + (7*Log[-(Sqrt[3]*x) + Sqrt[2 +
3*x^2]])/(3*Sqrt[3])
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.56

method	result
risch	$\frac{(24x^2+45x+13)\sqrt{3x^2+2}}{27}-\frac{7\ \operatorname{arcsinh}\left(\frac{x\sqrt{6}}{2}\right)\sqrt{3}}{9}$
default	$-\frac{7\ \operatorname{arcsinh}\left(\frac{x\sqrt{6}}{2}\right)\sqrt{3}}{9}+\frac{13\sqrt{3x^2+2}}{27}+\frac{8x^2\sqrt{3x^2+2}}{9}+\frac{5x\sqrt{3x^2+2}}{3}$
trager	$\left(\frac{8}{9}x^2+\frac{5}{3}x+\frac{13}{27}\right)\sqrt{3x^2+2}+\frac{7\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(-\mathrm{RootOf}\left(\_Z^2-3\right)\sqrt{3x^2+2}+3x\right)}{9}$
meijerg	$\frac{\sqrt{3}\ \operatorname{arcsinh}\left(\frac{x\sqrt{3}\,\sqrt{2}}{2}\right)}{3}+\frac{10\sqrt{3}\left(\frac{\sqrt{\pi}\,x\sqrt{3}\,\sqrt{2}\,\sqrt{\frac{3x^2}{2}+1}}{2}-\sqrt{\pi}\ \operatorname{arcsinh}\left(\frac{x\sqrt{3}\,\sqrt{2}}{2}\right)\right)}{9\sqrt{\pi}}+\frac{5\sqrt{2}\left(-2\sqrt{\pi}+2\sqrt{\pi}\,\sqrt{\frac{3x^2}{2}+1}\right)}{6\sqrt{\pi}}+\frac{8\sqrt{2}\left(\frac{4\sqrt{\pi}}{3}-\right.}{}$

```
[In] int((1+2*x)*(4*x^2+3*x+1)/(3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/27*(24*x^2+45*x+13)*(3*x^2+2)^(1/2)-7/9*arcsinh(1/2*x*6^(1/2))*3^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.79

$$\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\sqrt{2+3x^2}}\,dx=\frac{1}{27}\left(24\,x^2+45\,x+13\right)\sqrt{3\,x^2+2}+\frac{7}{18}\,\sqrt{3}\log\left(\sqrt{3}\sqrt{3\,x^2+2}x-3\,x^2-1\right)$$

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2+2)^(1/2),x, algorithm="fricas")
[Out] 1/27*(24*x^2 + 45*x + 13)*sqrt(3*x^2 + 2) + 7/18*sqrt(3)*log(sqrt(3)*sqrt(3
*x^2 + 2)*x - 3*x^2 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.02

$$\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\sqrt{2+3x^2}}\,dx=\frac{8x^2\sqrt{3x^2+2}}{9}+\frac{5x\sqrt{3x^2+2}}{3}+\frac{13\sqrt{3x^2+2}}{27}-\frac{7\sqrt{3}\operatorname{asinh}\left(\frac{\sqrt{6}x}{2}\right)}{9}$$

```
[In] integrate((1+2*x)*(4*x**2+3*x+1)/(3*x**2+2)**(1/2),x)
[Out] 8*x**2*sqrt(3*x**2 + 2)/9 + 5*x*sqrt(3*x**2 + 2)/3 + 13*sqrt(3*x**2 + 2)/27
 - 7*sqrt(3)*asinh(sqrt(6)*x/2)/9
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.81

$$\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\sqrt{2+3x^2}}\,dx = \frac{8}{9}\,\sqrt{3\,x^2+2}x^2+\frac{5}{3}\,\sqrt{3\,x^2+2}x - \frac{7}{9}\,\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{2}\,\sqrt{6}x\right)+\frac{13}{27}\,\sqrt{3\,x^2+2}$$

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 8/9*sqrt(3*x^2 + 2)*x^2 + 5/3*sqrt(3*x^2 + 2)*x - 7/9*sqrt(3)*arcsinh(1/2*s
qrt(6)*x) + 13/27*sqrt(3*x^2 + 2)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.71

$$\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\sqrt{2+3x^2}}\,dx = \frac{1}{27}\,(3\,(8\,x+15)x+13)\sqrt{3\,x^2+2} + \frac{7}{9}\,\sqrt{3}\log\left(-\sqrt{3}x+\sqrt{3\,x^2+2}\right)$$

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/27*(3*(8*x + 15)*x + 13)*sqrt(3*x^2 + 2) + 7/9*sqrt(3)*log(-sqrt(3)*x + s
qrt(3*x^2 + 2))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.56

$$\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\sqrt{2+3x^2}}\,dx = \frac{\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}\,\left(\frac{8\,x^2}{3}+5\,x+\frac{13}{9}\right)}{3}-\frac{7\,\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{6}\,x}{2}\right)}{9}$$

```
[In] int(((2*x + 1)*(3*x + 4*x^2 + 1))/(3*x^2 + 2)^(1/2),x)
```

```
[Out] (3^(1/2)*(x^2 + 2/3)^(1/2)*(5*x + (8*x^2)/3 + 13/9))/3 - (7*3^(1/2)*asinh((
6^(1/2)*x)/2))/9
```

3.121 $\int \frac{1+3x+4x^2}{(1+2x)\sqrt{2+3x^2}} \, dx$

Optimal result

Integrand size = 29, antiderivative size = 67

$$\int \frac{1+3x+4x^2}{(1+2x)\sqrt{2+3x^2}} \, dx = \frac{2}{3}\sqrt{2+3x^2}+\frac{\operatorname{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{2\sqrt{3}}-\frac{\operatorname{arctanh}\left(\frac{4-3x}{\sqrt{11}\sqrt{2+3x^2}}\right)}{2\sqrt{11}}$$

```
[Out] 1/6*arcsinh(1/2*x*6^(1/2))*3^(1/2)-1/22*arctanh(1/11*(4-3*x)*11^(1/2)/(3*x^
2+2)^(1/2))*11^(1/2)+2/3*(3*x^2+2)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1668, 858, 221, 739, 212}

$$\int \frac{1+3x+4x^2}{(1+2x)\sqrt{2+3x^2}} \, dx = \frac{\operatorname{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{2\sqrt{3}}-\frac{\operatorname{arctanh}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{2\sqrt{11}}+\frac{2}{3}\sqrt{3x^2+2}$$

```
[In] Int[(1 + 3*x + 4*x^2)/((1 + 2*x)*Sqrt[2 + 3*x^2]),x]
```

```
[Out] (2*Sqrt[2 + 3*x^2])/3 + ArcSinh[Sqrt[3/2]*x]/(2*Sqrt[3]) - ArcTanh[(4 - 3*x
)/(Sqrt[11]*Sqrt[2 + 3*x^2])]/(2*Sqrt[11])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2}{3}\sqrt{2+3x^2}+\frac{1}{12}\int \frac{12+12x}{(1+2x)\sqrt{2+3x^2}}\,dx\\
&= \frac{2}{3}\sqrt{2+3x^2}+\frac{1}{2}\int \frac{1}{\sqrt{2+3x^2}}\,dx+\frac{1}{2}\int \frac{1}{(1+2x)\sqrt{2+3x^2}}\,dx\\
&= \frac{2}{3}\sqrt{2+3x^2}+\frac{\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{2\sqrt{3}}-\frac{1}{2}\text{Subst}\left(\int \frac{1}{11-x^2}\,dx,x,\frac{4-3x}{\sqrt{2+3x^2}}\right)\\
&= \frac{2}{3}\sqrt{2+3x^2}+\frac{\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{2\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{2+3x^2}}\right)}{2\sqrt{11}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.33

$$\int \frac{1+3x+4x^2}{(1+2x)\sqrt{2+3x^2}}\,dx = \frac{2}{3}\sqrt{2+3x^2}+\frac{\operatorname{arctanh}\left(\sqrt{\frac{3}{11}}+2\sqrt{\frac{3}{11}}x-\frac{2\sqrt{2+3x^2}}{\sqrt{11}}\right)}{\sqrt{11}} - \frac{\log\left(-\sqrt{3}x+\sqrt{2+3x^2}\right)}{2\sqrt{3}}$$

```
[In] Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)*Sqrt[2 + 3*x^2]),x]
```

```
[Out] (2*Sqrt[2 + 3*x^2])/3 + ArcTanh[Sqrt[3/11] + 2*Sqrt[3/11]*x - (2*Sqrt[2 + 3
*x^2])/Sqrt[11]]/Sqrt[11] - Log[-(Sqrt[3]*x) + Sqrt[2 + 3*x^2]]/(2*Sqrt[3])
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.82

method	result
default	$\frac{\operatorname{arcsinh}\left(\frac{x\sqrt{6}}{2}\right)\sqrt{3}}{6}+\frac{2\sqrt{3x^2+2}}{3}-\frac{\sqrt{11}\,\operatorname{arctanh}\left(\frac{2(4-3x)\sqrt{11}}{11\sqrt{12\left(x+\frac{1}{2}\right)^2-12x+5}}\right)}{22}$
risch	$\frac{\operatorname{arcsinh}\left(\frac{x\sqrt{6}}{2}\right)\sqrt{3}}{6}+\frac{2\sqrt{3x^2+2}}{3}-\frac{\sqrt{11}\,\operatorname{arctanh}\left(\frac{2(4-3x)\sqrt{11}}{11\sqrt{12\left(x+\frac{1}{2}\right)^2-12x+5}}\right)}{22}$
trager	$\frac{2\sqrt{3x^2+2}}{3}+\frac{\operatorname{RootOf}\left(\_Z^2-3\right)\ln\left(\operatorname{RootOf}\left(\_Z^2-3\right)\sqrt{3x^2+2}+3x\right)}{6}+\frac{\operatorname{RootOf}\left(\_Z^2-11\right)\ln\left(\frac{3\operatorname{RootOf}\left(\_Z^2-11\right)x+11\sqrt{3x^2+2}}{1+2x}\right.}{22}$

```
[In] int((4*x^2+3*x+1)/(1+2*x)/(3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*arcsinh(1/2*x*6^(1/2))*3^(1/2)+2/3*(3*x^2+2)^(1/2)-1/22*11^(1/2)*arctan
h(2/11*(4-3*x)*11^(1/2)/(12*(x+1/2)^2-12*x+5)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.31

$$\int \frac{1+3x+4x^2}{(1+2x)\sqrt{2+3x^2}}\,dx = \frac{1}{12}\,\sqrt{3}\log\left(-\sqrt{3}\sqrt{3\,x^2+2}x-3\,x^2-1\right) + \frac{1}{44}\,\sqrt{11}\log\left(-\frac{\sqrt{11}\sqrt{3\,x^2+2}(3\,x-4)+21\,x^2-12\,x+19}{4\,x^2+4\,x+1}\right) + \frac{2}{3}\,\sqrt{3\,x^2+2}$$

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*log(-sqrt(3)*sqrt(3*x^2 + 2)*x - 3*x^2 - 1) + 1/44*sqrt(11)*lo
g(-(sqrt(11)*sqrt(3*x^2 + 2)*(3*x - 4) + 21*x^2 - 12*x + 19)/(4*x^2 + 4*x +
 1)) + 2/3*sqrt(3*x^2 + 2)
```

Sympy [F]

$$\int \frac{1+3x+4x^2}{(1+2x)\sqrt{2+3x^2}}\,dx = \int \frac{4x^2+3x+1}{(2x+1)\sqrt{3x^2+2}}\,dx$$

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)/(3*x**2+2)**(1/2),x)
```

```
[Out] Integral((4*x**2 + 3*x + 1)/((2*x + 1)*sqrt(3*x**2 + 2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.87

$$\int \frac{1+3x+4x^2}{(1+2x)\sqrt{2+3x^2}}\,dx = \frac{1}{6}\,\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{2}\,\sqrt{6}x\right) + \frac{1}{22}\,\sqrt{11}\,\mathrm{arsinh}\left(\frac{\sqrt{6}x}{2\,|2\,x+1|}-\frac{2\,\sqrt{6}}{3\,|2\,x+1|}\right) + \frac{2}{3}\,\sqrt{3\,x^2+2}$$

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*arcsinh(1/2*sqrt(6)*x) + 1/22*sqrt(11)*arcsinh(1/2*sqrt(6)*x/ab
s(2*x + 1) - 2/3*sqrt(6)/abs(2*x + 1)) + 2/3*sqrt(3*x^2 + 2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(49) = 98$.

Time = 0.29 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.48

$$\int \frac{1+3x+4x^2}{(1+2x)\sqrt{2+3x^2}}\,dx = -\frac{1}{6}\sqrt{3}\log\left(-\sqrt{3}x+\sqrt{3\,x^2+2}\right) + \frac{1}{22}\sqrt{11}\log\left(-\frac{\left|-2\sqrt{3}x-\sqrt{11}-\sqrt{3}+2\sqrt{3\,x^2+2}\right|}{2\sqrt{3}x-\sqrt{11}+\sqrt{3}-2\sqrt{3\,x^2+2}}\right) + \frac{2}{3}\sqrt{3\,x^2+2}$$

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(3)*log(-sqrt(3)*x + sqrt(3*x^2 + 2)) + 1/22*sqrt(11)*log(-abs(-2*
sqrt(3)*x - sqrt(11) - sqrt(3) + 2*sqrt(3*x^2 + 2))/(2*sqrt(3)*x - sqrt(11)
 + sqrt(3) - 2*sqrt(3*x^2 + 2))) + 2/3*sqrt(3*x^2 + 2)
```

Mupad [B] (verification not implemented)

Time = 13.21 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.91

$$\int \frac{1+3x+4x^2}{(1+2x)\sqrt{2+3x^2}}\,dx = \frac{\sqrt{11}\left(2\,\ln\left(x+\frac{1}{2}\right)-2\,\ln\left(x-\frac{\sqrt{3}\sqrt{11}\sqrt{x^2+\frac{2}{3}}}{3}-\frac{4}{3}\right)\right)}{44} + \frac{2\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{3} + \frac{\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{2}\sqrt{3}\,x}{2}\right)}{6}$$

```
[In] int((3*x + 4*x^2 + 1)/((2*x + 1)*(3*x^2 + 2)^(1/2)),x)
```

```
[Out] (11^(1/2)*(2*log(x + 1/2) - 2*log(x - (3^(1/2)*11^(1/2)*(x^2 + 2/3)^(1/2))/
3 - 4/3)))/44 + (2*3^(1/2)*(x^2 + 2/3)^(1/2))/3 + (3^(1/2)*asinh((2^(1/2)*3
^(1/2)*x)/2))/6
```

3.122 $\int \frac{1+3x+4x^2}{(1+2x)^2\sqrt{2+3x^2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 71

$$\int \frac{1+3x+4x^2}{(1+2x)^2\sqrt{2+3x^2}}\,dx=-\frac{\sqrt{2+3x^2}}{11(1+2x)}+\frac{\operatorname{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{3}}+\frac{4\operatorname{arctanh}\left(\frac{4-3x}{\sqrt{11}\sqrt{2+3x^2}}\right)}{11\sqrt{11}}$$

```
[Out] 1/3*arcsinh(1/2*x*6^(1/2))*3^(1/2)+4/121*arctanh(1/11*(4-3*x)*11^(1/2)/(3*x
^2+2)^(1/2))*11^(1/2)-1/11*(3*x^2+2)^(1/2)/(1+2*x)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1665, 858, 221, 739, 212}

$$\int \frac{1+3x+4x^2}{(1+2x)^2\sqrt{2+3x^2}}\,dx=\frac{\operatorname{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{3}}+\frac{4\operatorname{arctanh}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{11\sqrt{11}}-\frac{\sqrt{3x^2+2}}{11(2x+1)}$$

```
[In] Int[(1 + 3*x + 4*x^2)/((1 + 2*x)^2*Sqrt[2 + 3*x^2]),x]
```

```
[Out] -1/11*Sqrt[2 + 3*x^2]/(1 + 2*x) + ArcSinh[Sqrt[3/2]*x]/Sqrt[3] + (4*ArcTanh
[(4 - 3*x)/(Sqrt[11]*Sqrt[2 + 3*x^2])])/(11*Sqrt[11])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sqrt{2+3x^2}}{11(1+2x)}-\frac{1}{11}\int\frac{-7-22x}{(1+2x)\sqrt{2+3x^2}}\,dx\\
&= -\frac{\sqrt{2+3x^2}}{11(1+2x)}-\frac{4}{11}\int\frac{1}{(1+2x)\sqrt{2+3x^2}}\,dx+\int\frac{1}{\sqrt{2+3x^2}}\,dx\\
&= -\frac{\sqrt{2+3x^2}}{11(1+2x)}+\frac{\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{3}}+\frac{4}{11}\text{Subst}\left(\int\frac{1}{11-x^2}\,dx,x,\frac{4-3x}{\sqrt{2+3x^2}}\right)\\
&= -\frac{\sqrt{2+3x^2}}{11(1+2x)}+\frac{\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{3}}+\frac{4\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{2+3x^2}}\right)}{11\sqrt{11}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.30

$$\int \frac{1+3x+4x^2}{(1+2x)^2\sqrt{2+3x^2}}\,dx = -\frac{\sqrt{2+3x^2}}{11+22x} - \frac{8\text{arctanh}\left(\frac{\sqrt{3}+2\sqrt{3}x-2\sqrt{2+3x^2}}{\sqrt{11}}\right)}{11\sqrt{11}} - \frac{\log\left(-\sqrt{3}x+\sqrt{2+3x^2}\right)}{\sqrt{3}}$$

```
[In] Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)^2*Sqrt[2 + 3*x^2]),x]
```

```
[Out] -(Sqrt[2 + 3*x^2]/(11 + 22*x)) - (8*ArcTanh[(Sqrt[3] + 2*Sqrt[3]*x - 2*Sqrt
[2 + 3*x^2])/Sqrt[11]])/(11*Sqrt[11]) - Log[-(Sqrt[3]*x) + Sqrt[2 + 3*x^2]]
/Sqrt[3]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.87

method	result
risch	$-\frac{\sqrt{3x^2+2}}{11(1+2x)}+\frac{\text{arcsinh}\left(\frac{x\sqrt{6}}{2}\right)\sqrt{3}}{3}+\frac{4\sqrt{11}\,\text{arctanh}\left(\frac{2(4-3x)\sqrt{11}}{11\sqrt{12\left(x+\frac{1}{2}\right)^2-12x+5}}\right)}{121}$
default	$\frac{\text{arcsinh}\left(\frac{x\sqrt{6}}{2}\right)\sqrt{3}}{3}-\frac{\sqrt{3\left(x+\frac{1}{2}\right)^2-3x+\frac{5}{4}}}{22\left(x+\frac{1}{2}\right)}+\frac{4\sqrt{11}\,\text{arctanh}\left(\frac{2(4-3x)\sqrt{11}}{11\sqrt{12\left(x+\frac{1}{2}\right)^2-12x+5}}\right)}{121}$
trager	$-\frac{\sqrt{3x^2+2}}{11(1+2x)}+\frac{\text{RootOf}\left(\_Z^2-3\right)\ln\left(\text{RootOf}\left(\_Z^2-3\right)\sqrt{3x^2+2}+3x\right)}{3}-\frac{4\,\text{RootOf}\left(\_Z^2-11\right)\ln\left(\frac{3\,\text{RootOf}\left(\_Z^2-11\right)x+11\sqrt{}}{1+}\right.}{121}$

```
[In] int((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/11*(3*x^2+2)^(1/2)/(1+2*x)+1/3*arcsinh(1/2*x*6^(1/2))*3^(1/2)+4/121*11^(
1/2)*arctanh(2/11*(4-3*x)*11^(1/2)/(12*(x+1/2)^2-12*x+5)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.49

$$\int \frac{1+3x+4x^2}{(1+2x)^2\sqrt{2+3x^2}}\,dx = \frac{121\sqrt{3}(2x+1)\log\left(-\sqrt{3}\sqrt{3x^2+2}x-3x^2-1\right)+12\sqrt{11}(2x+1)\log\left(\frac{\sqrt{11}\sqrt{3x^2+2}(3x-4)-21x^2+12x-19}{4x^2+4x+1}\right)-}{726\,(2x+1)}$$

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/726*(121*sqrt(3)*(2*x + 1)*log(-sqrt(3)*sqrt(3*x^2 + 2)*x - 3*x^2 - 1) +
12*sqrt(11)*(2*x + 1)*log((sqrt(11)*sqrt(3*x^2 + 2)*(3*x - 4) - 21*x^2 + 12
*x - 19)/(4*x^2 + 4*x + 1)) - 66*sqrt(3*x^2 + 2))/(2*x + 1)
```

Sympy [F]

$$\int \frac{1+3x+4x^2}{(1+2x)^2\sqrt{2+3x^2}}\,dx = \int \frac{4x^2+3x+1}{(2x+1)^2\sqrt{3x^2+2}}\,dx$$

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)**2/(3*x**2+2)**(1/2),x)
```

```
[Out] Integral((4*x**2 + 3*x + 1)/((2*x + 1)**2*sqrt(3*x**2 + 2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.92

$$\int \frac{1+3x+4x^2}{(1+2x)^2\sqrt{2+3x^2}}\,dx = \frac{1}{3}\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{2}\sqrt{6}x\right) - \frac{4}{121}\sqrt{11}\,\mathrm{arsinh}\left(\frac{\sqrt{6}x}{2\,|2x+1|}-\frac{2\sqrt{6}}{3\,|2x+1|}\right)-\frac{\sqrt{3x^2+2}}{11\,(2x+1)}$$

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arcsinh(1/2*sqrt(6)*x) - 4/121*sqrt(11)*arcsinh(1/2*sqrt(6)*x/a
bs(2*x + 1) - 2/3*sqrt(6)/abs(2*x + 1)) - 1/11*sqrt(3*x^2 + 2)/(2*x + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 191 vs. $2(56) = 112$.

Time = 0.39 (sec) , antiderivative size = 191, normalized size of antiderivative = 2.69

$$\int \frac{1+3x+4x^2}{(1+2x)^2\sqrt{2+3x^2}}\,dx = \frac{4\sqrt{11}\log\left(\sqrt{11}\left(\sqrt{-\frac{6}{2x+1}+\frac{11}{(2x+1)^2}+3}+\frac{\sqrt{11}}{2x+1}\right)-3\right)}{121\,\mathrm{sgn}\left(\frac{1}{2x+1}\right)} - \frac{\sqrt{3}\log\left(\frac{\left|-2\sqrt{3}+2\sqrt{-\frac{6}{2x+1}+\frac{11}{(2x+1)^2}+3}+\frac{2\sqrt{11}}{2x+1}\right|}{2\left(\sqrt{3}+\sqrt{-\frac{6}{2x+1}+\frac{11}{(2x+1)^2}+3}+\frac{\sqrt{11}}{2x+1}\right)}\right)}{3\,\mathrm{sgn}\left(\frac{1}{2x+1}\right)} - \frac{\sqrt{-\frac{6}{2x+1}+\frac{11}{(2x+1)^2}+3}}{22\,\mathrm{sgn}\left(\frac{1}{2x+1}\right)}$$

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] 4/121*sqrt(11)*log(sqrt(11)*(sqrt(-6/(2*x + 1) + 11/(2*x + 1)^2 + 3) + sqrt
(11)/(2*x + 1)) - 3)/sgn(1/(2*x + 1)) - 1/3*sqrt(3)*log(1/2*abs(-2*sqrt(3)
+ 2*sqrt(-6/(2*x + 1) + 11/(2*x + 1)^2 + 3) + 2*sqrt(11)/(2*x + 1))/(sqrt(3
) + sqrt(-6/(2*x + 1) + 11/(2*x + 1)^2 + 3) + sqrt(11)/(2*x + 1)))/sgn(1/(2
*x + 1)) - 1/22*sqrt(-6/(2*x + 1) + 11/(2*x + 1)^2 + 3)/sgn(1/(2*x + 1))
```

Mupad [B] (verification not implemented)

Time = 13.18 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.96

$$\int \frac{1+3x+4x^2}{(1+2x)^2\sqrt{2+3x^2}}\,dx = \frac{\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{2}\sqrt{3}\,x}{2}\right)}{3} - \frac{4\sqrt{11}\,\ln\left(x+\frac{1}{2}\right)}{121} + \frac{4\sqrt{11}\,\ln\left(x-\frac{\sqrt{3}\sqrt{11}\sqrt{x^2+\frac{2}{3}}}{3}-\frac{4}{3}\right)}{121} - \frac{\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{22\left(x+\frac{1}{2}\right)}$$

```
[In] int((3*x + 4*x^2 + 1)/((2*x + 1)^2*(3*x^2 + 2)^(1/2)),x)
```

```
[Out] (3^(1/2)*asinh((2^(1/2)*3^(1/2)*x)/2))/3 - (4*11^(1/2)*log(x + 1/2))/121 +
(4*11^(1/2)*log(x - (3^(1/2)*11^(1/2)*(x^2 + 2/3)^(1/2))/3 - 4/3))/121 - (3
^(1/2)*(x^2 + 2/3)^(1/2))/(22*(x + 1/2))
```

3.123 $\int \frac{1+3x+4x^2}{(1+2x)^3\sqrt{2+3x^2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 77

$$\int \frac{1+3x+4x^2}{(1+2x)^3\sqrt{2+3x^2}}\,dx=-\frac{\sqrt{2+3x^2}}{22(1+2x)^2}+\frac{13\sqrt{2+3x^2}}{242(1+2x)}-\frac{103\text{arctanh}\left(\frac{4-3x}{\sqrt{11}\sqrt{2+3x^2}}\right)}{121\sqrt{11}}$$

```
[Out] -103/1331*arctanh(1/11*(4-3*x)*11^(1/2)/(3*x^2+2)^(1/2))*11^(1/2)-1/22*(3*x
^2+2)^(1/2)/(1+2*x)^2+13/242*(3*x^2+2)^(1/2)/(1+2*x)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1665, 821, 739, 212}

$$\int \frac{1+3x+4x^2}{(1+2x)^3\sqrt{2+3x^2}}\,dx=-\frac{103\text{arctanh}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{121\sqrt{11}}+\frac{13\sqrt{3x^2+2}}{242(2x+1)}-\frac{\sqrt{3x^2+2}}{22(2x+1)^2}$$

```
[In] Int[(1 + 3*x + 4*x^2)/((1 + 2*x)^3*Sqrt[2 + 3*x^2]),x]
```

```
[Out] -1/22*Sqrt[2 + 3*x^2]/(1 + 2*x)^2 + (13*Sqrt[2 + 3*x^2])/(242*(1 + 2*x)) -
(103*ArcTanh[(4 - 3*x)/(Sqrt[11]*Sqrt[2 + 3*x^2])])/(121*Sqrt[11])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sqrt{2+3x^2}}{22(1+2x)^2} - \frac{1}{22}\int \frac{-14-41x}{(1+2x)^2\sqrt{2+3x^2}}\,dx \\
&= -\frac{\sqrt{2+3x^2}}{22(1+2x)^2} + \frac{13\sqrt{2+3x^2}}{242(1+2x)} + \frac{103}{121}\int \frac{1}{(1+2x)\sqrt{2+3x^2}}\,dx \\
&= -\frac{\sqrt{2+3x^2}}{22(1+2x)^2} + \frac{13\sqrt{2+3x^2}}{242(1+2x)} - \frac{103}{121}\text{Subst}\left(\int \frac{1}{11-x^2}\,dx, x, \frac{4-3x}{\sqrt{2+3x^2}}\right) \\
&= -\frac{\sqrt{2+3x^2}}{22(1+2x)^2} + \frac{13\sqrt{2+3x^2}}{242(1+2x)} - \frac{103\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{2+3x^2}}\right)}{121\sqrt{11}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.92

$$\int \frac{1+3x+4x^2}{(1+2x)^3\sqrt{2+3x^2}}\,dx = \frac{\frac{11(1+13x)\sqrt{2+3x^2}}{(1+2x)^2}+206\sqrt{11}\text{arctanh}\left(\frac{\sqrt{3}+2\sqrt{3}x-2\sqrt{2+3x^2}}{\sqrt{11}}\right)}{1331}$$

```
[In] Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)^3*Sqrt[2 + 3*x^2]),x]
```

```
[Out] ((11*(1 + 13*x)*Sqrt[2 + 3*x^2])/(1 + 2*x)^2 + 206*Sqrt[11]*ArcTanh[(Sqrt[3
] + 2*Sqrt[3]*x - 2*Sqrt[2 + 3*x^2])/Sqrt[11]])/1331
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.84

method	result	size
risch	$\frac{39x^3+3x^2+26x+2}{121(1+2x)^2\sqrt{3x^2+2}}-\frac{103\sqrt{11}\ \text{arctanh}\left(\frac{2(4-3x)\sqrt{11}}{11\sqrt{12\left(x+\frac{1}{2}\right)^2-12x+5}}\right)}{1331}$	65
trager	$\frac{(13x+1)\sqrt{3x^2+2}}{121(1+2x)^2}-\frac{103\,\text{RootOf}\left(\_Z^2-11\right)\ln\left(\frac{-3\,\text{RootOf}\left(\_Z^2-11\right)x+11\sqrt{3x^2+2}+4\,\text{RootOf}\left(\_Z^2-11\right)}{1+2x}\right)}{1331}$	71
default	$-\frac{103\sqrt{11}\ \text{arctanh}\left(\frac{2(4-3x)\sqrt{11}}{11\sqrt{12\left(x+\frac{1}{2}\right)^2-12x+5}}\right)}{1331}+\frac{13\sqrt{3\left(x+\frac{1}{2}\right)^2-3x+\frac{5}{4}}}{484\left(x+\frac{1}{2}\right)}-\frac{\sqrt{3\left(x+\frac{1}{2}\right)^2-3x+\frac{5}{4}}}{88\left(x+\frac{1}{2}\right)^2}$	74

```
[In] int((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/121*(39*x^3+3*x^2+26*x+2)/(1+2*x)^2/(3*x^2+2)^(1/2)-103/1331*11^(1/2)*arc
tanh(2/11*(4-3*x)*11^(1/2)/(12*(x+1/2)^2-12*x+5)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.16

$$\int \frac{1+3x+4x^2}{(1+2x)^3\sqrt{2+3x^2}}\,dx = \frac{103\,\sqrt{11}(4\,x^2+4\,x+1)\log\left(-\frac{\sqrt{11}\sqrt{3\,x^2+2}(3\,x-4)+21\,x^2-12\,x+19}{4\,x^2+4\,x+1}\right)+22\,\sqrt{3\,x^2+2}(13\,x+1)}{2662\,(4\,x^2+4\,x+1)}$$

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2662*(103*sqrt(11)*(4*x^2 + 4*x + 1)*log(-(sqrt(11)*sqrt(3*x^2 + 2)*(3*x
- 4) + 21*x^2 - 12*x + 19)/(4*x^2 + 4*x + 1)) + 22*sqrt(3*x^2 + 2)*(13*x +
1))/(4*x^2 + 4*x + 1)
```

Sympy [F]

$$\int \frac{1+3x+4x^2}{(1+2x)^3\sqrt{2+3x^2}}\,dx=\int \frac{4x^2+3x+1}{(2x+1)^3\sqrt{3x^2+2}}\,dx$$

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)**3/(3*x**2+2)**(1/2),x)
```

```
[Out] Integral((4*x**2 + 3*x + 1)/((2*x + 1)**3*sqrt(3*x**2 + 2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.99

$$\int \frac{1+3x+4x^2}{(1+2x)^3\sqrt{2+3x^2}}\,dx=\frac{103}{1331}\sqrt{11}\,\operatorname{arsinh}\left(\frac{\sqrt{6}x}{2\,|2\,x+1|}-\frac{2\,\sqrt{6}}{3\,|2\,x+1|}\right)-\frac{\sqrt{3\,x^2+2}}{22\,(4\,x^2+4\,x+1)}+\frac{13\,\sqrt{3\,x^2+2}}{242\,(2\,x+1)}$$

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 103/1331*sqrt(11)*arcsinh(1/2*sqrt(6)*x/abs(2*x + 1) - 2/3*sqrt(6)/abs(2*x
+ 1)) - 1/22*sqrt(3*x^2 + 2)/(4*x^2 + 4*x + 1) + 13/242*sqrt(3*x^2 + 2)/(2*
x + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. 2(62) = 124.

Time = 0.29 (sec) , antiderivative size = 180, normalized size of antiderivative = 2.34

$$\int \frac{1+3x+4x^2}{(1+2x)^3\sqrt{2+3x^2}}\,dx=\frac{103}{1331}\sqrt{11}\log\left(-\frac{\left|-2\,\sqrt{3}x-\sqrt{11}-\sqrt{3}+2\,\sqrt{3\,x^2+2}\right|}{2\,\sqrt{3}x-\sqrt{11}+\sqrt{3}-2\,\sqrt{3\,x^2+2}}\right)+\frac{72\,\left(\sqrt{3}x-\sqrt{3\,x^2+2}\right)^3-13\,\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2+2}\right)^2-168\,\sqrt{3}x+104\,\sqrt{3}+168\,\sqrt{3\,x^2+2}}{484\,\left(\left(\sqrt{3}x-\sqrt{3\,x^2+2}\right)^2+\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2+2}\right)-2\right)^2}$$

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] 103/1331*sqrt(11)*log(-abs(-2*sqrt(3)*x - sqrt(11) - sqrt(3) + 2*sqrt(3*x^2
 + 2))/(2*sqrt(3)*x - sqrt(11) + sqrt(3) - 2*sqrt(3*x^2 + 2))) + 1/484*(72*
(sqrt(3)*x - sqrt(3*x^2 + 2))^3 - 13*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 2))^
2 - 168*sqrt(3)*x + 104*sqrt(3) + 168*sqrt(3*x^2 + 2))/((sqrt(3)*x - sqrt(3
*x^2 + 2))^2 + sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 2)) - 2)^2
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00

$$\int \frac{1+3x+4x^2}{(1+2x)^3\sqrt{2+3x^2}}\,dx = \frac{103\sqrt{11}\,\ln\left(x+\frac{1}{2}\right)}{1331} - \frac{103\sqrt{11}\,\ln\left(x-\frac{\sqrt{3}\,\sqrt{11}\,\sqrt{x^2+\frac{2}{3}}}{3}-\frac{4}{3}\right)}{1331} - \frac{\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}}{88\,\left(x^2+x+\frac{1}{4}\right)} + \frac{13\,\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}}{484\,\left(x+\frac{1}{2}\right)}$$

```
[In] int((3*x + 4*x^2 + 1)/((2*x + 1)^3*(3*x^2 + 2)^(1/2)),x)
```

```
[Out] (103*11^(1/2)*log(x + 1/2))/1331 - (103*11^(1/2)*log(x - (3^(1/2)*11^(1/2)*
(x^2 + 2/3)^(1/2))/3 - 4/3))/1331 - (3^(1/2)*(x^2 + 2/3)^(1/2))/(88*(x + x^
2 + 1/4)) + (13*3^(1/2)*(x^2 + 2/3)^(1/2))/(484*(x + 1/2))
```

3.124 $\int \frac{(1+2x)^3(1+3x+4x^2)}{(2+3x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 87

$$\int \frac{(1+2x)^3(1+3x+4x^2)}{(2+3x^2)^{3/2}}\,dx = \frac{398+279x}{54\sqrt{2+3x^2}}+\frac{292}{81}\sqrt{2+3x^2} + 4x\sqrt{2+3x^2}+\frac{32}{27}x^2\sqrt{2+3x^2}-\frac{38\operatorname{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{3\sqrt{3}}$$

```
[Out] -38/9*arcsinh(1/2*x*6^(1/2))*3^(1/2)+1/54*(398+279*x)/(3*x^2+2)^(1/2)+292/8
1*(3*x^2+2)^(1/2)+4*x*(3*x^2+2)^(1/2)+32/27*x^2*(3*x^2+2)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1828, 1829, 655, 221}

$$\int \frac{(1+2x)^3(1+3x+4x^2)}{(2+3x^2)^{3/2}}\,dx = -\frac{38\operatorname{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{3\sqrt{3}} + \frac{32}{27}\sqrt{3x^2+2}x^2+4\sqrt{3x^2+2}x+\frac{292}{81}\sqrt{3x^2+2}+\frac{279x+398}{54\sqrt{3x^2+2}}$$

```
[In] Int[((1 + 2*x)^3*(1 + 3*x + 4*x^2))/(2 + 3*x^2)^(3/2),x]
```

```
[Out] (398 + 279*x)/(54*Sqrt[2 + 3*x^2]) + (292*Sqrt[2 + 3*x^2])/81 + 4*x*Sqrt[2
+ 3*x^2] + (32*x^2*Sqrt[2 + 3*x^2])/27 - (38*ArcSinh[Sqrt[3/2]*x])/(3*Sqrt[
3])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 1829

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{398+279x}{54\sqrt{2+3x^2}} - \frac{1}{2}\int \frac{\frac{28}{3}-\frac{280x}{9}-48x^2-\frac{64x^3}{3}}{\sqrt{2+3x^2}}\,dx \\
&= \frac{398+279x}{54\sqrt{2+3x^2}} + \frac{32}{27}x^2\sqrt{2+3x^2} - \frac{1}{18}\int \frac{84-\frac{584x}{3}-432x^2}{\sqrt{2+3x^2}}\,dx \\
&= \frac{398+279x}{54\sqrt{2+3x^2}} + 4x\sqrt{2+3x^2} + \frac{32}{27}x^2\sqrt{2+3x^2} - \frac{1}{108}\int \frac{1368-1168x}{\sqrt{2+3x^2}}\,dx \\
&= \frac{398+279x}{54\sqrt{2+3x^2}} + \frac{292}{81}\sqrt{2+3x^2} + 4x\sqrt{2+3x^2} + \frac{32}{27}x^2\sqrt{2+3x^2} - \frac{38}{3}\int \frac{1}{\sqrt{2+3x^2}}\,dx \\
&= \frac{398+279x}{54\sqrt{2+3x^2}} + \frac{292}{81}\sqrt{2+3x^2} + 4x\sqrt{2+3x^2} + \frac{32}{27}x^2\sqrt{2+3x^2} - \frac{38\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{3\sqrt{3}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.76

$$\int \frac{(1+2x)^3\,(1+3x+4x^2)}{(2+3x^2)^{3/2}}\,dx = \frac{2362+2133x+2136x^2+1944x^3+576x^4}{162\sqrt{2+3x^2}} + \frac{38\log\left(-\sqrt{3}x+\sqrt{2+3x^2}\right)}{3\sqrt{3}}$$

```
[In] Integrate[((1 + 2*x)^3*(1 + 3*x + 4*x^2))/(2 + 3*x^2)^(3/2),x]
```

```
[Out] (2362 + 2133*x + 2136*x^2 + 1944*x^3 + 576*x^4)/(162*Sqrt[2 + 3*x^2]) + (38
*Log[-(Sqrt[3]*x) + Sqrt[2 + 3*x^2]])/(3*Sqrt[3])
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.52

method	result
risch	$\frac{576x^4+1944x^3+2136x^2+2133x+2362}{162\sqrt{3x^2+2}} - \frac{38\ \mathrm{arcsinh}\left(\frac{x\sqrt{6}}{2}\right)\sqrt{3}}{9}$
trager	$\frac{576x^4+1944x^3+2136x^2+2133x+2362}{162\sqrt{3x^2+2}} - \frac{38\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(\mathrm{RootOf}\left(\_Z^2-3\right)\sqrt{3x^2+2}+3x\right)}{9}$
default	$\frac{79x}{6\sqrt{3x^2+2}} + \frac{1181}{81\sqrt{3x^2+2}} + \frac{32x^4}{9\sqrt{3x^2+2}} + \frac{356x^2}{27\sqrt{3x^2+2}} + \frac{12x^3}{\sqrt{3x^2+2}} - \frac{38\ \mathrm{arcsinh}\left(\frac{x\sqrt{6}}{2}\right)\sqrt{3}}{9}$
meijerg	$\frac{\sqrt{2}\,x}{4\sqrt{\frac{3x^2}{2}+1}} + \frac{34\sqrt{3}\left(-\frac{\sqrt{\pi}\,x\sqrt{3}\,\sqrt{2}}{2\sqrt{\frac{3x^2}{2}+1}}+\sqrt{\pi}\ \mathrm{arcsinh}\left(\frac{x\sqrt{3}\,\sqrt{2}}{2}\right)\right)}{9\sqrt{\pi}} + \frac{3\sqrt{2}\left(\sqrt{\pi}-\frac{\sqrt{\pi}}{\sqrt{\frac{3x^2}{2}+1}}\right)}{2\sqrt{\pi}} + \frac{68\sqrt{2}\left(-2\sqrt{\pi}+\frac{\sqrt{\pi}\left(6x^2+8\right)}{4\sqrt{\frac{3x^2}{2}+1}}\right)}{9\sqrt{\pi}} + \frac{16}{}$

```
[In] int((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/162*(576*x^4+1944*x^3+2136*x^2+2133*x+2362)/(3*x^2+2)^(1/2)-38/9*arcsinh(
1/2*x*6^(1/2))*3^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.87

$$\int \frac{(1+2x)^3\,(1+3x+4x^2)}{(2+3x^2)^{3/2}}\,dx = \frac{342\,\sqrt{3}(3\,x^2+2)\log\left(\sqrt{3}\sqrt{3\,x^2+2}x-3\,x^2-1\right)+(576\,x^4+1944\,x^3+213}{162\,(3\,x^2+2)}$$

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/162*(342*sqrt(3)*(3*x^2 + 2)*log(sqrt(3)*sqrt(3*x^2 + 2)*x - 3*x^2 - 1) +
 (576*x^4 + 1944*x^3 + 2136*x^2 + 2133*x + 2362)*sqrt(3*x^2 + 2))/(3*x^2 +
2)
```

Sympy [F]

$$\int \frac{(1+2x)^3\,(1+3x+4x^2)}{(2+3x^2)^{3/2}}\,dx = \int \frac{(2x+1)^3\cdot(4x^2+3x+1)}{(3x^2+2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((1+2*x)**3*(4*x**2+3*x+1)/(3*x**2+2)**(3/2),x)
```

```
[Out] Integral((2*x + 1)**3*(4*x**2 + 3*x + 1)/(3*x**2 + 2)**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.90

$$\int \frac{(1+2x)^3\,(1+3x+4x^2)}{(2+3x^2)^{3/2}}\,dx = \frac{32\,x^4}{9\,\sqrt{3\,x^2+2}} + \frac{12\,x^3}{\sqrt{3\,x^2+2}} + \frac{356\,x^2}{27\,\sqrt{3\,x^2+2}} - \frac{38}{9}\,\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{2}\,\sqrt{6}x\right) + \frac{79\,x}{6\,\sqrt{3\,x^2+2}} + \frac{1181}{81\,\sqrt{3\,x^2+2}}$$

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 32/9*x^4/sqrt(3*x^2 + 2) + 12*x^3/sqrt(3*x^2 + 2) + 356/27*x^2/sqrt(3*x^2 +
 2) - 38/9*sqrt(3)*arcsinh(1/2*sqrt(6)*x) + 79/6*x/sqrt(3*x^2 + 2) + 1181/8
1/sqrt(3*x^2 + 2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.62

$$\int \frac{(1+2x)^3\left(1+3x+4x^2\right)}{\left(2+3x^2\right)^{3/2}}\,dx = \frac{38}{9}\sqrt{3}\log\left(-\sqrt{3}x+\sqrt{3\,x^2+2}\right) + \frac{3\left(8\left(3\left(8\,x+27\right)x+89\right)x+711\right)x+2362}{162\sqrt{3\,x^2+2}}$$

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] 38/9*sqrt(3)*log(-sqrt(3)*x + sqrt(3*x^2 + 2)) + 1/162*(3*(8*(3*(8*x + 27)*
x + 89)*x + 711)*x + 2362)/sqrt(3*x^2 + 2)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.26

$$\int \frac{(1+2x)^3\left(1+3x+4x^2\right)}{\left(2+3x^2\right)^{3/2}}\,dx = \frac{\sqrt{3}\sqrt{x^2+\frac{2}{3}}\left(\frac{32\,x^2}{9}+12\,x+\frac{292}{27}\right)}{3} - \frac{38\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{2}\sqrt{3}\,x}{2}\right)}{9} - \frac{\sqrt{3}\sqrt{6}\left(-1194+\sqrt{6}\,279\mathrm{i}\right)\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{1944\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)} - \frac{\sqrt{3}\sqrt{6}\left(1194+\sqrt{6}\,279\mathrm{i}\right)\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{1944\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}$$

```
[In] int(((2*x + 1)^3*(3*x + 4*x^2 + 1))/(3*x^2 + 2)^(3/2),x)
```

```
[Out] (3^(1/2)*(x^2 + 2/3)^(1/2)*(12*x + (32*x^2)/9 + 292/27))/3 - (38*3^(1/2)*as
inh((2^(1/2)*3^(1/2)*x)/2))/9 - (3^(1/2)*6^(1/2)*(6^(1/2)*279i - 1194)*(x^2
 + 2/3)^(1/2)*1i)/(1944*(x + (6^(1/2)*1i)/3)) - (3^(1/2)*6^(1/2)*(6^(1/2)*2
79i + 1194)*(x^2 + 2/3)^(1/2)*1i)/(1944*(x - (6^(1/2)*1i)/3))
```

3.125 $\int \frac{(1+2x)^2(1+3x+4x^2)}{(2+3x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 71

$$\int \frac{(1+2x)^2\left(1+3x+4x^2\right)}{\left(2+3x^2\right)^{3/2}}\,dx=\frac{70-47x}{18\sqrt{2+3x^2}}+\frac{28}{9}\sqrt{2+3x^2}+\frac{8}{9}x\sqrt{2+3x^2}+\frac{4\operatorname{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{3\sqrt{3}}$$

```
[Out] 4/9*arcsinh(1/2*x*6^(1/2))*3^(1/2)+1/18*(70-47*x)/(3*x^2+2)^(1/2)+28/9*(3*x
^2+2)^(1/2)+8/9*x*(3*x^2+2)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1828, 1829, 655, 221}

$$\int \frac{(1+2x)^2\left(1+3x+4x^2\right)}{\left(2+3x^2\right)^{3/2}}\,dx=\frac{4\operatorname{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{3\sqrt{3}}+\frac{70-47x}{18\sqrt{3x^2+2}}+\frac{8}{9}x\sqrt{3x^2+2}+\frac{28}{9}\sqrt{3x^2+2}$$

```
[In] Int[((1 + 2*x)^2*(1 + 3*x + 4*x^2))/(2 + 3*x^2)^(3/2),x]
```

```
[Out] (70 - 47*x)/(18*Sqrt[2 + 3*x^2]) + (28*Sqrt[2 + 3*x^2])/9 + (8*x*Sqrt[2 + 3
*x^2])/9 + (4*ArcSinh[Sqrt[3/2]*x])/(3*Sqrt[3])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 1829

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{70-47x}{18\sqrt{2+3x^2}} - \frac{1}{2}\int \frac{-\frac{56}{9}-\frac{56x}{3}-\frac{32x^2}{3}}{\sqrt{2+3x^2}}\,dx \\
&= \frac{70-47x}{18\sqrt{2+3x^2}} + \frac{8}{9}x\sqrt{2+3x^2} - \frac{1}{12}\int \frac{-16-112x}{\sqrt{2+3x^2}}\,dx \\
&= \frac{70-47x}{18\sqrt{2+3x^2}} + \frac{28}{9}\sqrt{2+3x^2} + \frac{8}{9}x\sqrt{2+3x^2} + \frac{4}{3}\int \frac{1}{\sqrt{2+3x^2}}\,dx \\
&= \frac{70-47x}{18\sqrt{2+3x^2}} + \frac{28}{9}\sqrt{2+3x^2} + \frac{8}{9}x\sqrt{2+3x^2} + \frac{4\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{3\sqrt{3}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.86

$$\int \frac{(1+2x)^2\left(1+3x+4x^2\right)}{\left(2+3x^2\right)^{3/2}}\,dx = \frac{182-15x+168x^2+48x^3}{18\sqrt{2+3x^2}} - \frac{4\log\left(-\sqrt{3}x+\sqrt{2+3x^2}\right)}{3\sqrt{3}}$$

```
[In] Integrate[((1 + 2*x)^2*(1 + 3*x + 4*x^2))/(2 + 3*x^2)^(3/2),x]
```

```
[Out] (182 - 15*x + 168*x^2 + 48*x^3)/(18*Sqrt[2 + 3*x^2]) - (4*Log[-(Sqrt[3]*x)
+ Sqrt[2 + 3*x^2]])/(3*Sqrt[3])
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.56

method	result
risch	$\frac{48x^3+168x^2-15x+182}{18\sqrt{3x^2+2}}+\frac{4\ \mathrm{arcsinh}\left(\frac{x\sqrt{6}}{2}\right)\sqrt{3}}{9}$
trager	$\frac{48x^3+168x^2-15x+182}{18\sqrt{3x^2+2}}+\frac{4\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(\mathrm{RootOf}\left(\_Z^2-3\right)\sqrt{3x^2+2}+3x\right)}{9}$
default	$-\frac{5x}{6\sqrt{3x^2+2}}+\frac{91}{9\sqrt{3x^2+2}}+\frac{8x^3}{3\sqrt{3x^2+2}}+\frac{4\ \mathrm{arcsinh}\left(\frac{x\sqrt{6}}{2}\right)\sqrt{3}}{9}+\frac{28x^2}{3\sqrt{3x^2+2}}$
meijerg	$\frac{\sqrt{2}\,x}{4\sqrt{\frac{3x^2}{2}+1}}+\frac{20\sqrt{3}\left(-\frac{\sqrt{\pi}\,x\sqrt{3}\,\sqrt{2}}{2\sqrt{\frac{3x^2}{2}+1}}+\sqrt{\pi}\ \mathrm{arcsinh}\left(\frac{x\sqrt{3}\,\sqrt{2}}{2}\right)\right)}{9\sqrt{\pi}}+\frac{7\sqrt{2}\left(\sqrt{\pi}-\frac{\sqrt{\pi}}{\sqrt{\frac{3x^2}{2}+1}}\right)}{6\sqrt{\pi}}+\frac{28\sqrt{2}\left(-2\sqrt{\pi}+\frac{\sqrt{\pi}\left(6x^2+8\right)}{4\sqrt{\frac{3x^2}{2}+1}}\right)}{9\sqrt{\pi}}+$ 32√3

```
[In] int((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/18*(48*x^3+168*x^2-15*x+182)/(3*x^2+2)^(1/2)+4/9*arcsinh(1/2*x*6^(1/2))*3
^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.01

$$\int\frac{(1+2x)^2\,(1+3x+4x^2)}{(2+3x^2)^{3/2}}\,dx=\frac{4\sqrt{3}(3\,x^2+2)\log\left(-\sqrt{3}\sqrt{3\,x^2+2}x-3\,x^2-1\right)+(48\,x^3+168\,x^2-15\,x+}{18\,(3\,x^2+2)}$$

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/18*(4*sqrt(3)*(3*x^2 + 2)*log(-sqrt(3)*sqrt(3*x^2 + 2)*x - 3*x^2 - 1) + (
48*x^3 + 168*x^2 - 15*x + 182)*sqrt(3*x^2 + 2))/(3*x^2 + 2)
```

Sympy [F]

$$\int\frac{(1+2x)^2\,(1+3x+4x^2)}{(2+3x^2)^{3/2}}\,dx=\int\frac{(2x+1)^2\cdot(4x^2+3x+1)}{(3x^2+2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((1+2*x)**2*(4*x**2+3*x+1)/(3*x**2+2)**(3/2),x)
```

```
[Out] Integral((2*x + 1)**2*(4*x**2 + 3*x + 1)/(3*x**2 + 2)**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.90

$$\int \frac{(1+2x)^2\,(1+3x+4x^2)}{(2+3x^2)^{3/2}}\,dx = \frac{8\,x^3}{3\,\sqrt{3\,x^2+2}} + \frac{28\,x^2}{3\,\sqrt{3\,x^2+2}} + \frac{4}{9}\,\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{2}\,\sqrt{6}x\right) - \frac{5\,x}{6\,\sqrt{3\,x^2+2}} + \frac{91}{9\,\sqrt{3\,x^2+2}}$$

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x, algorithm="maxima")
[Out] 8/3*x^3/sqrt(3*x^2 + 2) + 28/3*x^2/sqrt(3*x^2 + 2) + 4/9*sqrt(3)*arcsinh(1/
2*sqrt(6)*x) - 5/6*x/sqrt(3*x^2 + 2) + 91/9/sqrt(3*x^2 + 2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.69

$$\int \frac{(1+2x)^2\,(1+3x+4x^2)}{(2+3x^2)^{3/2}}\,dx = -\frac{4}{9}\,\sqrt{3}\log\left(-\sqrt{3}x+\sqrt{3\,x^2+2}\right) + \frac{3\,(8\,(2\,x+7)x-5)x+182}{18\,\sqrt{3\,x^2+2}}$$

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x, algorithm="giac")
[Out] -4/9*sqrt(3)*log(-sqrt(3)*x + sqrt(3*x^2 + 2)) + 1/18*(3*(8*(2*x + 7)*x - 5
)*x + 182)/sqrt(3*x^2 + 2)
```

Mupad [B] (verification not implemented)

Time = 13.06 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.48

$$\int \frac{(1+2x)^2\,(1+3x+4x^2)}{(2+3x^2)^{3/2}}\,dx = \frac{4\,\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{2}\,\sqrt{3}\,x}{2}\right)}{9} + \frac{\sqrt{3}\,\left(\frac{8\,x}{3}+\frac{28}{3}\right)\,\sqrt{x^2+\frac{2}{3}}}{3} + \frac{\sqrt{3}\,\sqrt{6}\,\left(-630+\sqrt{6}\,141\mathrm{i}\right)\,\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{1944\,\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)} + \frac{\sqrt{3}\,\sqrt{6}\,\left(630+\sqrt{6}\,141\mathrm{i}\right)\,\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{1944\,\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}$$

```
[In] int(((2*x + 1)^2*(3*x + 4*x^2 + 1))/(3*x^2 + 2)^(3/2),x)
[Out] (4*3^(1/2)*asinh((2^(1/2)*3^(1/2)*x)/2))/9 + (3^(1/2)*((8*x)/3 + 28/3)*(x^2
 + 2/3)^(1/2))/3 + (3^(1/2)*6^(1/2)*(6^(1/2)*141i - 630)*(x^2 + 2/3)^(1/2)*
1i)/(1944*(x - (6^(1/2)*1i)/3)) + (3^(1/2)*6^(1/2)*(6^(1/2)*141i + 630)*(x^
2 + 2/3)^(1/2)*1i)/(1944*(x + (6^(1/2)*1i)/3))
```

3.126 $\int \frac{(1+2x)(1+3x+4x^2)}{(2+3x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 55

$$\int \frac{(1+2x)\,(1+3x+4x^2)}{(2+3x^2)^{3/2}}\,dx=\frac{2-51x}{18\sqrt{2+3x^2}}+\frac{8}{9}\sqrt{2+3x^2}+\frac{10\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{3\sqrt{3}}$$

```
[Out] 10/9*arcsinh(1/2*x*6^(1/2))*3^(1/2)+1/18*(2-51*x)/(3*x^2+2)^(1/2)+8/9*(3*x^
2+2)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1828, 655, 221}

$$\int \frac{(1+2x)\,(1+3x+4x^2)}{(2+3x^2)^{3/2}}\,dx=\frac{10\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{3\sqrt{3}}+\frac{2-51x}{18\sqrt{3x^2+2}}+\frac{8}{9}\sqrt{3x^2+2}$$

```
[In] Int[((1 + 2*x)*(1 + 3*x + 4*x^2))/(2 + 3*x^2)^(3/2),x]
```

```
[Out] (2 - 51*x)/(18*Sqrt[2 + 3*x^2]) + (8*Sqrt[2 + 3*x^2])/9 + (10*ArcSinh[Sqrt[
3/2]*x])/(3*Sqrt[3])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2-51x}{18\sqrt{2+3x^2}} - \frac{1}{2}\int \frac{-\frac{20}{3}-\frac{16x}{3}}{\sqrt{2+3x^2}}\,dx \\
&= \frac{2-51x}{18\sqrt{2+3x^2}} + \frac{8}{9}\sqrt{2+3x^2} + \frac{10}{3}\int \frac{1}{\sqrt{2+3x^2}}\,dx \\
&= \frac{2-51x}{18\sqrt{2+3x^2}} + \frac{8}{9}\sqrt{2+3x^2} + \frac{10\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{3\sqrt{3}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.02

$$\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\left(2+3x^2\right)^{3/2}}\,dx = \frac{34-51x+48x^2}{18\sqrt{2+3x^2}} - \frac{10\log\left(-\sqrt{3}x+\sqrt{2+3x^2}\right)}{3\sqrt{3}}$$

```
[In] Integrate[((1 + 2*x)*(1 + 3*x + 4*x^2))/(2 + 3*x^2)^(3/2),x]
[Out] (34 - 51*x + 48*x^2)/(18*Sqrt[2 + 3*x^2]) - (10*Log[-(Sqrt[3]*x) + Sqrt[2 +
 3*x^2]])/(3*Sqrt[3])
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.64

method	result	size
risch	$\frac{48x^2-51x+34}{18\sqrt{3x^2+2}}+\frac{10\,\operatorname{arcsinh}\left(\frac{x\sqrt{6}}{2}\right)\sqrt{3}}{9}$	35
default	$-\frac{17x}{6\sqrt{3x^2+2}}+\frac{17}{9\sqrt{3x^2+2}}+\frac{8x^2}{3\sqrt{3x^2+2}}+\frac{10\,\operatorname{arcsinh}\left(\frac{x\sqrt{6}}{2}\right)\sqrt{3}}{9}$	51
trager	$\frac{48x^2-51x+34}{18\sqrt{3x^2+2}}-\frac{10\,\operatorname{RootOf}\left(\_Z^2-3\right)\ln\left(-\operatorname{RootOf}\left(\_Z^2-3\right)\sqrt{3x^2+2}+3x\right)}{9}$	53
meijerg	$\frac{\sqrt{2}\,x}{4\sqrt{\frac{3x^2}{2}+1}}+\frac{10\sqrt{3}\left(-\frac{\sqrt{\pi}\,x\sqrt{3}\,\sqrt{2}}{2\sqrt{\frac{3x^2}{2}+1}}+\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{x\sqrt{3}\,\sqrt{2}}{2}\right)\right)}{9\sqrt{\pi}}+\frac{5\sqrt{2}\left(\sqrt{\pi}-\frac{\sqrt{\pi}}{\sqrt{\frac{3x^2}{2}+1}}\right)}{6\sqrt{\pi}}+\frac{8\sqrt{2}\left(-2\sqrt{\pi}+\frac{\sqrt{\pi}\,\left(6x^2+8\right)}{4\sqrt{\frac{3x^2}{2}+1}}\right)}{9\sqrt{\pi}}$	122

```
[In] int((1+2*x)*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/18*(48*x^2-51*x+34)/(3*x^2+2)^(1/2)+10/9*arcsinh(1/2*x*6^(1/2))*3^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.22

$$\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\left(2+3x^2\right)^{3/2}}\,dx=\frac{10\,\sqrt{3}(3\,x^2+2)\log\left(-\sqrt{3}\sqrt{3\,x^2+2}x-3\,x^2-1\right)+(48\,x^2-51\,x+34)\sqrt{3\,x}}{18\,(3\,x^2+2)}$$

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/18*(10*sqrt(3)*(3*x^2 + 2)*log(-sqrt(3)*sqrt(3*x^2 + 2)*x - 3*x^2 - 1) +
(48*x^2 - 51*x + 34)*sqrt(3*x^2 + 2))/(3*x^2 + 2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(49) = 98$.

Time = 5.58 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.07

$$\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\left(2+3x^2\right)^{3/2}}\,dx=\frac{30\sqrt{3}x^2\operatorname{asinh}\left(\frac{\sqrt{6}x}{2}\right)}{27x^2+18}+\frac{8x^2}{3\sqrt{3x^2+2}}-\frac{30x\sqrt{3x^2+2}}{27x^2+18}+\frac{x}{2\sqrt{3x^2+2}}+\frac{20\sqrt{3}\operatorname{asinh}\left(\frac{\sqrt{6}x}{2}\right)}{27x^2+18}+\frac{17}{9\sqrt{3x^2+2}}$$

```
[In] integrate((1+2*x)*(4*x**2+3*x+1)/(3*x**2+2)**(3/2),x)
```

```
[Out] 30*sqrt(3)*x**2*asinh(sqrt(6)*x/2)/(27*x**2 + 18) + 8*x**2/(3*sqrt(3*x**2 +
 2)) - 30*x*sqrt(3*x**2 + 2)/(27*x**2 + 18) + x/(2*sqrt(3*x**2 + 2)) + 20*s
qrt(3)*asinh(sqrt(6)*x/2)/(27*x**2 + 18) + 17/(9*sqrt(3*x**2 + 2))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91

$$\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\left(2+3x^2\right)^{3/2}}\,dx = \frac{8\,x^2}{3\,\sqrt{3\,x^2+2}} + \frac{10}{9}\,\sqrt{3}\,\operatorname{arsinh}\left(\frac{1}{2}\,\sqrt{6}x\right) - \frac{17\,x}{6\,\sqrt{3\,x^2+2}} + \frac{17}{9\,\sqrt{3\,x^2+2}}$$

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 8/3*x^2/sqrt(3*x^2 + 2) + 10/9*sqrt(3)*arcsinh(1/2*sqrt(6)*x) - 17/6*x/sqrt
(3*x^2 + 2) + 17/9/sqrt(3*x^2 + 2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.80

$$\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\left(2+3x^2\right)^{3/2}}\,dx = -\frac{10}{9}\,\sqrt{3}\log\left(-\sqrt{3}x+\sqrt{3\,x^2+2}\right) + \frac{3\,(16\,x-17)x+34}{18\,\sqrt{3\,x^2+2}}$$

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] -10/9*sqrt(3)*log(-sqrt(3)*x + sqrt(3*x^2 + 2)) + 1/18*(3*(16*x - 17)*x + 3
4)/sqrt(3*x^2 + 2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.82

$$\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\left(2+3x^2\right)^{3/2}}\,dx = \frac{8\,\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}}{9} + \frac{10\,\sqrt{3}\,\operatorname{asinh}\left(\frac{\sqrt{2}\,\sqrt{3}\,x}{2}\right)}{9} + \frac{\sqrt{3}\,\sqrt{6}\,\left(-6+\sqrt{6}\,51\mathrm{i}\right)\,\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{648\,\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)} + \frac{\sqrt{3}\,\sqrt{6}\,\left(6+\sqrt{6}\,51\mathrm{i}\right)\,\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{648\,\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}$$

```
[In] int(((2*x + 1)*(3*x + 4*x^2 + 1))/(3*x^2 + 2)^(3/2),x)
```

```
[Out] (8*3^(1/2)*(x^2 + 2/3)^(1/2))/9 + (10*3^(1/2)*asinh((2^(1/2)*3^(1/2)*x)/2))
/9 + (3^(1/2)*6^(1/2)*(6^(1/2)*51i - 6)*(x^2 + 2/3)^(1/2)*1i)/(648*(x - (6^
(1/2)*1i)/3)) + (3^(1/2)*6^(1/2)*(6^(1/2)*51i + 6)*(x^2 + 2/3)^(1/2)*1i)/(6
48*(x + (6^(1/2)*1i)/3))
```

3.127 $\int \frac{1+3x+4x^2}{(1+2x)\left(2+3x^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 53

$$\int \frac{1+3x+4x^2}{(1+2x)\left(2+3x^2\right)^{3/2}}\,dx=\frac{-38+21x}{66\sqrt{2+3x^2}}-\frac{2\text{arctanh}\left(\frac{4-3x}{\sqrt{11}\sqrt{2+3x^2}}\right)}{11\sqrt{11}}$$

```
[Out] -2/121*arctanh(1/11*(4-3*x)*11^(1/2)/(3*x^2+2)^(1/2))*11^(1/2)+1/66*(-38+21
*x)/(3*x^2+2)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1661, 12, 739, 212}

$$\int \frac{1+3x+4x^2}{(1+2x)\left(2+3x^2\right)^{3/2}}\,dx=-\frac{2\text{arctanh}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{11\sqrt{11}}-\frac{38-21x}{66\sqrt{3x^2+2}}$$

```
[In] Int[(1 + 3*x + 4*x^2)/((1 + 2*x)*(2 + 3*x^2)^(3/2)),x]
```

```
[Out] -1/66*(38 - 21*x)/Sqrt[2 + 3*x^2] - (2*ArcTanh[(4 - 3*x)/(Sqrt[11]*Sqrt[2 +
 3*x^2])])/(11*Sqrt[11])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{38-21x}{66\sqrt{2+3x^2}}-\frac{1}{6}\int -\frac{12}{11(1+2x)\sqrt{2+3x^2}}\,dx\\
&= -\frac{38-21x}{66\sqrt{2+3x^2}}+\frac{2}{11}\int \frac{1}{(1+2x)\sqrt{2+3x^2}}\,dx\\
&= -\frac{38-21x}{66\sqrt{2+3x^2}}-\frac{2}{11}\text{Subst}\left(\int \frac{1}{11-x^2}\,dx,x,\frac{4-3x}{\sqrt{2+3x^2}}\right)\\
&= -\frac{38-21x}{66\sqrt{2+3x^2}}-\frac{2\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{2+3x^2}}\right)}{11\sqrt{11}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96

$$\int \frac{1+3x+4x^2}{(1+2x)\left(2+3x^2\right)^{3/2}}\,dx=\frac{-418+231x-12\sqrt{22+33x^2}\operatorname{arctanh}\left(\frac{4-3x}{\sqrt{22+33x^2}}\right)}{726\sqrt{2+3x^2}}$$

```
[In] Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)*(2 + 3*x^2)^(3/2)),x]
[Out] (-418 + 231*x - 12*Sqrt[22 + 33*x^2]*ArcTanh[(4 - 3*x)/Sqrt[22 + 33*x^2]])/
(726*Sqrt[2 + 3*x^2])
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.91

method	result	size
risch	$\frac{-38+21x}{66\sqrt{3x^2+2}}-\frac{2\sqrt{11}\,\operatorname{arctanh}\left(\frac{2(4-3x)\sqrt{11}}{11\sqrt{12\left(x+\frac{1}{2}\right)^2-12x+5}}\right)}{121}$	48
trager	$\frac{-38+21x}{66\sqrt{3x^2+2}}-\frac{2\,\mathrm{RootOf}\left(\_Z^2-11\right)\ln\left(\frac{-3\,\mathrm{RootOf}\left(\_Z^2-11\right)x+11\sqrt{3x^2+2}+4\,\mathrm{RootOf}\left(\_Z^2-11\right)}{1+2x}\right)}{121}$	64
default	$\frac{x}{4\sqrt{3x^2+2}}-\frac{2}{3\sqrt{3x^2+2}}+\frac{1}{11\sqrt{3\left(x+\frac{1}{2}\right)^2-3x+\frac{5}{4}}}+\frac{3x}{44\sqrt{3\left(x+\frac{1}{2}\right)^2-3x+\frac{5}{4}}}-\frac{2\sqrt{11}\,\operatorname{arctanh}\left(\frac{2(4-3x)\sqrt{11}}{11\sqrt{12\left(x+\frac{1}{2}\right)^2-12x+5}}\right)}{121}$	88

```
[In] int((4*x^2+3*x+1)/(1+2*x)/(3*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/66*(-38+21*x)/(3*x^2+2)^(1/2)-2/121*11^(1/2)*arctanh(2/11*(4-3*x)*11^(1/2
)/(12*(x+1/2)^2-12*x+5)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.57

$$\int \frac{1+3x+4x^2}{(1+2x)\left(2+3x^2\right)^{3/2}}\,dx=\frac{6\sqrt{11}(3\,x^2+2)\log\left(-\frac{\sqrt{11}\sqrt{3\,x^2+2}(3\,x-4)+21\,x^2-12\,x+19}{4\,x^2+4\,x+1}\right)+11\sqrt{3\,x^2+2}(21\,x-38}{726\,(3\,x^2+2)}$$

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/726*(6*sqrt(11)*(3*x^2 + 2)*log(-(sqrt(11)*sqrt(3*x^2 + 2)*(3*x - 4) + 21
*x^2 - 12*x + 19)/(4*x^2 + 4*x + 1)) + 11*sqrt(3*x^2 + 2)*(21*x - 38))/(3*x
^2 + 2)
```

Sympy [F]

$$\int \frac{1+3x+4x^2}{(1+2x)\left(2+3x^2\right)^{3/2}}\,dx=\int \frac{4x^2+3x+1}{(2x+1)\left(3x^2+2\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)/(3*x**2+2)**(3/2),x)
```

```
[Out] Integral((4*x**2 + 3*x + 1)/((2*x + 1)*(3*x**2 + 2)**(3/2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.09

$$\int \frac{1+3x+4x^2}{(1+2x)\left(2+3x^2\right)^{3/2}}\,dx = \frac{2}{121}\sqrt{11}\,\text{arsinh}\left(\frac{\sqrt{6}x}{2\,|2\,x+1|} - \frac{2\,\sqrt{6}}{3\,|2\,x+1|}\right) + \frac{7\,x}{22\,\sqrt{3\,x^2+2}} - \frac{19}{33\,\sqrt{3\,x^2+2}}$$

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/121*sqrt(11)*arcsinh(1/2*sqrt(6)*x/abs(2*x + 1) - 2/3*sqrt(6)/abs(2*x + 1
)) + 7/22*x/sqrt(3*x^2 + 2) - 19/33/sqrt(3*x^2 + 2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.55

$$\int \frac{1+3x+4x^2}{(1+2x)\left(2+3x^2\right)^{3/2}}\,dx = \frac{2}{121}\sqrt{11}\log\left(-\frac{\left|-2\,\sqrt{3}x-\sqrt{11}-\sqrt{3}+2\,\sqrt{3\,x^2+2}\right|}{2\,\sqrt{3}x-\sqrt{11}+\sqrt{3}-2\,\sqrt{3\,x^2+2}}\right) + \frac{21\,x-38}{66\,\sqrt{3\,x^2+2}}$$

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] 2/121*sqrt(11)*log(-abs(-2*sqrt(3)*x - sqrt(11) - sqrt(3) + 2*sqrt(3*x^2 +
2))/(2*sqrt(3)*x - sqrt(11) + sqrt(3) - 2*sqrt(3*x^2 + 2))) + 1/66*(21*x -
38)/sqrt(3*x^2 + 2)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.00

$$\int \frac{1+3x+4x^2}{(1+2x)\left(2+3x^2\right)^{3/2}}\,dx = \frac{\sqrt{11}\left(2\,\ln\left(x+\frac{1}{2}\right)-2\,\ln\left(x-\frac{\sqrt{3}\,\sqrt{11}\,\sqrt{x^2+\frac{2}{3}}}{3}-\frac{4}{3}\right)\right)}{121} - \frac{\sqrt{3}\,\sqrt{6}\,\left(-114+\sqrt{6}\,21\mathrm{i}\right)\,\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{2376\,\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)} - \frac{\sqrt{3}\,\sqrt{6}\,\left(114+\sqrt{6}\,21\mathrm{i}\right)\,\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{2376\,\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}$$

```
[In] int((3*x + 4*x^2 + 1)/((2*x + 1)*(3*x^2 + 2)^(3/2)),x)
```

```
[Out] (11^(1/2)*(2*log(x + 1/2) - 2*log(x - (3^(1/2)*11^(1/2)*(x^2 + 2/3)^(1/2))/
3 - 4/3)))/121 - (3^(1/2)*6^(1/2)*(6^(1/2)*21i - 114)*(x^2 + 2/3)^(1/2)*1i)
/(2376*(x - (6^(1/2)*1i)/3)) - (3^(1/2)*6^(1/2)*(6^(1/2)*21i + 114)*(x^2 +
2/3)^(1/2)*1i)/(2376*(x + (6^(1/2)*1i)/3))
```

3.128 $\int \frac{1+3x+4x^2}{(1+2x)^2\left(2+3x^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 75

$$\int \frac{1+3x+4x^2}{(1+2x)^2\left(2+3x^2\right)^{3/2}}\,dx=\frac{-10+97x}{242\sqrt{2+3x^2}}-\frac{4\sqrt{2+3x^2}}{121(1+2x)}+\frac{4\text{arctanh}\left(\frac{4-3x}{\sqrt{11}\sqrt{2+3x^2}}\right)}{121\sqrt{11}}$$

```
[Out] 4/1331*arctanh(1/11*(4-3*x)*11^(1/2)/(3*x^2+2)^(1/2))*11^(1/2)+1/242*(-10+9
7*x)/(3*x^2+2)^(1/2)-4/121*(3*x^2+2)^(1/2)/(1+2*x)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1661, 821, 739, 212}

$$\int \frac{1+3x+4x^2}{(1+2x)^2\left(2+3x^2\right)^{3/2}}\,dx=\frac{4\text{arctanh}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{121\sqrt{11}}-\frac{10-97x}{242\sqrt{3x^2+2}}-\frac{4\sqrt{3x^2+2}}{121(2x+1)}$$

```
[In] Int[(1 + 3*x + 4*x^2)/((1 + 2*x)^2*(2 + 3*x^2)^(3/2)),x]
```

```
[Out] -1/242*(10 - 97*x)/Sqrt[2 + 3*x^2] - (4*Sqrt[2 + 3*x^2])/(121*(1 + 2*x)) +
(4*ArcTanh[(4 - 3*x)/(Sqrt[11]*Sqrt[2 + 3*x^2])])/(121*Sqrt[11])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{10-97x}{242\sqrt{2+3x^2}}-\frac{1}{6}\int\frac{-\frac{72}{121}+\frac{120x}{121}}{(1+2x)^2\sqrt{2+3x^2}}\,dx\\
&= -\frac{10-97x}{242\sqrt{2+3x^2}}-\frac{4\sqrt{2+3x^2}}{121(1+2x)}-\frac{4}{121}\int\frac{1}{(1+2x)\sqrt{2+3x^2}}\,dx\\
&= -\frac{10-97x}{242\sqrt{2+3x^2}}-\frac{4\sqrt{2+3x^2}}{121(1+2x)}+\frac{4}{121}\text{Subst}\left(\int\frac{1}{11-x^2}\,dx,x,\frac{4-3x}{\sqrt{2+3x^2}}\right)\\
&= -\frac{10-97x}{242\sqrt{2+3x^2}}-\frac{4\sqrt{2+3x^2}}{121(1+2x)}+\frac{4\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{2+3x^2}}\right)}{121\sqrt{11}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.95

$$\int \frac{1+3x+4x^2}{(1+2x)^2\left(2+3x^2\right)^{3/2}}\,dx=\frac{11\left(-26+77x+170x^2\right)+8(1+2x)\sqrt{22+33x^2}\,\mathrm{arctanh}\left(\frac{4-3x}{\sqrt{22+33x^2}}\right)}{2662(1+2x)\sqrt{2+3x^2}}$$

```
[In] Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)^2*(2 + 3*x^2)^(3/2)),x]
```

```
[Out] (11*(-26 + 77*x + 170*x^2) + 8*(1 + 2*x)*Sqrt[22 + 33*x^2]*ArcTanh[(4 - 3*x
)/Sqrt[22 + 33*x^2]])/(2662*(1 + 2*x)*Sqrt[2 + 3*x^2])
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.80

method	result
risch	$\frac{170x^2+77x-26}{242(1+2x)\sqrt{3x^2+2}}+\frac{4\sqrt{11}\,\mathrm{arctanh}\left(\frac{2(4-3x)\sqrt{11}}{11\sqrt{12\left(x+\frac{1}{2}\right)^2-12x+5}}\right)}{1331}$
trager	$\frac{\left(170x^2+77x-26\right)\sqrt{3x^2+2}}{1452x^3+726x^2+968x+484}-\frac{4\,\mathrm{RootOf}\left(\_Z^2-11\right)\ln\left(\frac{3\,\mathrm{RootOf}\left(\_Z^2-11\right)x+11\sqrt{3x^2+2}-4\,\mathrm{RootOf}\left(\_Z^2-11\right)}{1+2x}\right)}{1331}$
default	$\frac{x}{2\sqrt{3x^2+2}}-\frac{1}{22\left(x+\frac{1}{2}\right)\sqrt{3\left(x+\frac{1}{2}\right)^2-3x+\frac{5}{4}}}-\frac{2}{121\sqrt{3\left(x+\frac{1}{2}\right)^2-3x+\frac{5}{4}}}-\frac{18x}{121\sqrt{3\left(x+\frac{1}{2}\right)^2-3x+\frac{5}{4}}}+\frac{4\sqrt{11}\,\mathrm{arctanh}\left(\frac{2(4-3x}{11\sqrt{12\left(x+\frac{1}{2}\right.}}\right.}{1331}$

```
[In] int((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/242*(170*x^2+77*x-26)/(1+2*x)/(3*x^2+2)^(1/2)+4/1331*11^(1/2)*arctanh(2/1
1*(4-3*x)*11^(1/2)/(12*(x+1/2)^2-12*x+5)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.37

$$\int \frac{1+3x+4x^2}{(1+2x)^2\left(2+3x^2\right)^{3/2}}\,dx=\frac{4\sqrt{11}\left(6x^3+3x^2+4x+2\right)\log\left(\frac{\sqrt{11}\sqrt{3x^2+2}(3x-4)-21x^2+12x-19}{4x^2+4x+1}\right)+11\left(170x^2\right.}{2662\left(6x^3+3x^2+4x+2\right)}$$

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2662*(4*sqrt(11)*(6*x^3 + 3*x^2 + 4*x + 2)*log((sqrt(11)*sqrt(3*x^2 + 2)*
(3*x - 4) - 21*x^2 + 12*x - 19)/(4*x^2 + 4*x + 1)) + 11*(170*x^2 + 77*x - 2
6)*sqrt(3*x^2 + 2))/(6*x^3 + 3*x^2 + 4*x + 2)
```

Sympy [F]

$$\int \frac{1+3x+4x^2}{(1+2x)^2\left(2+3x^2\right)^{3/2}}\,dx=\int \frac{4x^2+3x+1}{\left(2x+1\right)^2\left(3x^2+2\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)**2/(3*x**2+2)**(3/2),x)
[Out] Integral((4*x**2 + 3*x + 1)/((2*x + 1)**2*(3*x**2 + 2)**(3/2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.12

$$\int \frac{1+3x+4x^2}{(1+2x)^2\left(2+3x^2\right)^{3/2}}\,dx=-\frac{4}{1331}\sqrt{11}\,\operatorname{arsinh}\left(\frac{\sqrt{6}x}{2\,|2\,x+1|}-\frac{2\,\sqrt{6}}{3\,|2\,x+1|}\right)+\frac{85\,x}{242\,\sqrt{3\,x^2+2}}-\frac{2}{121\,\sqrt{3\,x^2+2}}-\frac{1}{11\,\left(2\,\sqrt{3\,x^2+2}x+\sqrt{3\,x^2+2}\right)}$$

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2+2)^(3/2),x, algorithm="maxima")
[Out] -4/1331*sqrt(11)*arcsinh(1/2*sqrt(6)*x/abs(2*x + 1) - 2/3*sqrt(6)/abs(2*x +
 1)) + 85/242*x/sqrt(3*x^2 + 2) - 2/121/sqrt(3*x^2 + 2) - 1/11/(2*sqrt(3*x^
2 + 2)*x + sqrt(3*x^2 + 2))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. 2(60) = 120.

Time = 0.29 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.24

$$\int \frac{1+3x+4x^2}{(1+2x)^2\left(2+3x^2\right)^{3/2}}\,dx=-\frac{1}{7986}\,\sqrt{11}\left(85\,\sqrt{11}\sqrt{3}+24\,\log\left(\sqrt{11}\sqrt{3}-3\right)\right)\operatorname{sgn}\left(\frac{1}{2\,x+1}\right)-\frac{\frac{\frac{93}{\operatorname{sgn}\left(\frac{1}{2\,x+1}\right)}+\frac{44}{(2\,x+1)\operatorname{sgn}\left(\frac{1}{2\,x+1}\right)}}{2\,x+1}-\frac{85}{\operatorname{sgn}\left(\frac{1}{2\,x+1}\right)}}{242\,\sqrt{-\frac{6}{2\,x+1}+\frac{11}{(2\,x+1)^2}+3}}+\frac{4\,\sqrt{11}\log\left(\sqrt{11}\left(\sqrt{-\frac{6}{2\,x+1}+\frac{11}{(2\,x+1)^2}+3}+\frac{\sqrt{11}}{2\,x+1}\right)-3\right)}{1331\,\operatorname{sgn}\left(\frac{1}{2\,x+1}\right)}$$

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/7986*sqrt(11)*(85*sqrt(11)*sqrt(3) + 24*log(sqrt(11)*sqrt(3) - 3))*sgn(1
/(2*x + 1)) - 1/242*((93/sgn(1/(2*x + 1)) + 44/((2*x + 1)*sgn(1/(2*x + 1)))
)/(2*x + 1) - 85/sgn(1/(2*x + 1)))/sqrt(-6/(2*x + 1) + 11/(2*x + 1)^2 + 3)
+ 4/1331*sqrt(11)*log(sqrt(11)*(sqrt(-6/(2*x + 1) + 11/(2*x + 1)^2 + 3) + s
qrt(11)/(2*x + 1)) - 3)/sgn(1/(2*x + 1))
```

Mupad [B] (verification not implemented)

Time = 13.14 (sec) , antiderivative size = 157, normalized size of antiderivative = 2.09

$$\int \frac{1+3x+4x^2}{(1+2x)^2\,(2+3x^2)^{3/2}}\,dx = \frac{4\sqrt{11}\,\ln\left(x-\frac{\sqrt{3}\,\sqrt{11}\,\sqrt{x^2+\frac{2}{3}}}{3}-\frac{4}{3}\right)}{1331} - \frac{4\sqrt{11}\,\ln\left(x+\frac{1}{2}\right)}{1331} + \frac{97\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}}{1452\,\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)} + \frac{97\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}}{1452\,\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)} - \frac{2\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}}{121\,\left(x+\frac{1}{2}\right)} + \frac{\sqrt{3}\,\sqrt{6}\,\sqrt{x^2+\frac{2}{3}}\,5\mathrm{i}}{1452\,\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)} - \frac{\sqrt{3}\,\sqrt{6}\,\sqrt{x^2+\frac{2}{3}}\,5\mathrm{i}}{1452\,\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}$$

```
[In] int((3*x + 4*x^2 + 1)/((2*x + 1)^2*(3*x^2 + 2)^(3/2)),x)
```

```
[Out] (4*11^(1/2)*log(x - (3^(1/2)*11^(1/2)*(x^2 + 2/3)^(1/2))/3 - 4/3))/1331 - (
4*11^(1/2)*log(x + 1/2))/1331 + (97*3^(1/2)*(x^2 + 2/3)^(1/2))/(1452*(x - (
6^(1/2)*1i)/3)) + (97*3^(1/2)*(x^2 + 2/3)^(1/2))/(1452*(x + (6^(1/2)*1i)/3)
) - (2*3^(1/2)*(x^2 + 2/3)^(1/2))/(121*(x + 1/2)) + (3^(1/2)*6^(1/2)*(x^2 +
 2/3)^(1/2)*5i)/(1452*(x - (6^(1/2)*1i)/3)) - (3^(1/2)*6^(1/2)*(x^2 + 2/3)^
(1/2)*5i)/(1452*(x + (6^(1/2)*1i)/3))
```

3.129 $\int \frac{1+3x+4x^2}{(1+2x)^3\left(2+3x^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 97

$$\int \frac{1+3x+4x^2}{(1+2x)^3\left(2+3x^2\right)^{3/2}}\,dx=\frac{358+351x}{2662\sqrt{2+3x^2}}-\frac{2\sqrt{2+3x^2}}{121(1+2x)^2}+\frac{2\sqrt{2+3x^2}}{1331(1+2x)}-\frac{322\operatorname{arctanh}\left(\frac{4-3x}{\sqrt{11}\sqrt{2+3x^2}}\right)}{1331\sqrt{11}}$$

```
[Out] -322/14641*arctanh(1/11*(4-3*x)*11^(1/2)/(3*x^2+2)^(1/2))*11^(1/2)+1/2662*(
358+351*x)/(3*x^2+2)^(1/2)-2/121*(3*x^2+2)^(1/2)/(1+2*x)^2+2/1331*(3*x^2+2)
^(1/2)/(1+2*x)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1661, 1665, 821, 739, 212}

$$\int \frac{1+3x+4x^2}{(1+2x)^3\left(2+3x^2\right)^{3/2}}\,dx=-\frac{322\operatorname{arctanh}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{1331\sqrt{11}}+\frac{351x+358}{2662\sqrt{3x^2+2}}+\frac{2\sqrt{3x^2+2}}{1331(2x+1)}-\frac{2\sqrt{3x^2+2}}{121(2x+1)^2}$$

```
[In] Int[(1 + 3*x + 4*x^2)/((1 + 2*x)^3*(2 + 3*x^2)^(3/2)),x]
```

```
[Out] (358 + 351*x)/(2662*Sqrt[2 + 3*x^2]) - (2*Sqrt[2 + 3*x^2])/(121*(1 + 2*x)^2
) + (2*Sqrt[2 + 3*x^2])/(1331*(1 + 2*x)) - (322*ArcTanh[(4 - 3*x)/(Sqrt[11]
*Sqrt[2 + 3*x^2])])/(1331*Sqrt[11])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{358+351x}{2662\sqrt{2+3x^2}} - \frac{1}{6}\int \frac{-\frac{2940}{1331}-\frac{7272x}{1331}-\frac{8592x^2}{1331}}{(1+2x)^3\sqrt{2+3x^2}}\,dx$$

$$
\begin{aligned}
&= \frac{358+351x}{2662\sqrt{2+3x^2}} - \frac{2\sqrt{2+3x^2}}{121(1+2x)^2} + \frac{1}{132}\int \frac{\frac{3768}{121}+\frac{7800x}{121}}{(1+2x)^2\sqrt{2+3x^2}}\,dx \\
&= \frac{358+351x}{2662\sqrt{2+3x^2}} - \frac{2\sqrt{2+3x^2}}{121(1+2x)^2} + \frac{2\sqrt{2+3x^2}}{1331(1+2x)} + \frac{322\int \frac{1}{(1+2x)\sqrt{2+3x^2}}\,dx}{1331} \\
&= \frac{358+351x}{2662\sqrt{2+3x^2}} - \frac{2\sqrt{2+3x^2}}{121(1+2x)^2} + \frac{2\sqrt{2+3x^2}}{1331(1+2x)} - \frac{322\text{Subst}\left(\int \frac{1}{11-x^2}\,dx, x, \frac{4-3x}{\sqrt{2+3x^2}}\right)}{1331} \\
&= \frac{358+351x}{2662\sqrt{2+3x^2}} - \frac{2\sqrt{2+3x^2}}{121(1+2x)^2} + \frac{2\sqrt{2+3x^2}}{1331(1+2x)} - \frac{322\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{2+3x^2}}\right)}{1331\sqrt{11}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.84

$$
\int \frac{1+3x+4x^2}{(1+2x)^3\left(2+3x^2\right)^{3/2}}\,dx = \frac{\frac{11\left(278+1799x+2716x^2+1428x^3\right)}{(1+2x)^2\sqrt{2+3x^2}} + 1288\sqrt{11}\text{arctanh}\left(\frac{\sqrt{3}+2\sqrt{3}x-2\sqrt{2+3x^2}}{\sqrt{11}}\right)}{29282}
$$

```
[In] Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)^3*(2 + 3*x^2)^(3/2)),x]
[Out] ((11*(278 + 1799*x + 2716*x^2 + 1428*x^3))/((1 + 2*x)^2*Sqrt[2 + 3*x^2]) +
1288*Sqrt[11]*ArcTanh[(Sqrt[3] + 2*Sqrt[3]*x - 2*Sqrt[2 + 3*x^2])/Sqrt[11]]
)/29282
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.67

method	result
risch	$\frac{1428x^3+2716x^2+1799x+278}{2662(1+2x)^2\sqrt{3x^2+2}} - \frac{322\sqrt{11}\,\text{arctanh}\left(\frac{2(4-3x)\sqrt{11}}{11\sqrt{12\left(x+\frac{1}{2}\right)^2-12x+5}}\right)}{14641}$
trager	$\frac{1428x^3+2716x^2+1799x+278}{2662(1+2x)^2\sqrt{3x^2+2}} - \frac{322\,\text{RootOf}\left(\_Z^2-11\right)\ln\left(\frac{-3\,\text{RootOf}\left(\_Z^2-11\right)x+11\sqrt{3x^2+2}+4\,\text{RootOf}\left(\_Z^2-11\right)}{1+2x}\right)}{14641}$
default	$\frac{161}{1331\sqrt{3\left(x+\frac{1}{2}\right)^2-3x+\frac{5}{4}}} + \frac{357x}{2662\sqrt{3\left(x+\frac{1}{2}\right)^2-3x+\frac{5}{4}}} - \frac{322\sqrt{11}\,\text{arctanh}\left(\frac{2(4-3x)\sqrt{11}}{11\sqrt{12\left(x+\frac{1}{2}\right)^2-12x+5}}\right)}{14641} + \frac{7}{484\left(x+\frac{1}{2}\right)\sqrt{3\left(x+\frac{1}{2}\right)^2-3x+}}$

```
[In] int((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2662*(1428*x^3+2716*x^2+1799*x+278)/(1+2*x)^2/(3*x^2+2)^(1/2)-322/14641*1
1^(1/2)*arctanh(2/11*(4-3*x)*11^(1/2)/(12*(x+1/2)^2-12*x+5)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.23

$$\int \frac{1+3x+4x^2}{(1+2x)^3\left(2+3x^2\right)^{3/2}}\,dx = \frac{322\,\sqrt{11}(12\,x^4+12\,x^3+11\,x^2+8\,x+2)\log\Big(-\frac{\sqrt{11}\sqrt{3\,x^2+2}(3\,x-4)+21\,x^2-12\,x+19}{4\,x^2+4\,x+1}}{29282\,(12\,x^4+12\,x^3+11\,x^2+}$$

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/29282*(322*sqrt(11)*(12*x^4 + 12*x^3 + 11*x^2 + 8*x + 2)*log(-(sqrt(11)*s
qrt(3*x^2 + 2)*(3*x - 4) + 21*x^2 - 12*x + 19)/(4*x^2 + 4*x + 1)) + 11*(142
8*x^3 + 2716*x^2 + 1799*x + 278)*sqrt(3*x^2 + 2))/(12*x^4 + 12*x^3 + 11*x^2
 + 8*x + 2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1+3x+4x^2}{(1+2x)^3\left(2+3x^2\right)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)**3/(3*x**2+2)**(3/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.28

$$\begin{aligned}\int \frac{1+3x+4x^2}{(1+2x)^3\left(2+3x^2\right)^{3/2}}\,dx &= \frac{322}{14641}\,\sqrt{11}\,\operatorname{arsinh}\left(\frac{\sqrt{6}x}{2\,|2\,x+1|}-\frac{2\,\sqrt{6}}{3\,|2\,x+1|}\right)\\&+\frac{357\,x}{2662\,\sqrt{3\,x^2+2}}+\frac{161}{1331\,\sqrt{3\,x^2+2}}\\&-\frac{1}{22\,\left(4\,\sqrt{3\,x^2+2}x^2+4\,\sqrt{3\,x^2+2}x+\sqrt{3\,x^2+2}\right)}+\frac{7}{242\,\left(2\,\sqrt{3\,x^2+2}x+\sqrt{3\,x^2+2}\right)}\end{aligned}$$

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 322/14641*sqrt(11)*arcsinh(1/2*sqrt(6)*x/abs(2*x + 1) - 2/3*sqrt(6)/abs(2*x
 + 1)) + 357/2662*x/sqrt(3*x^2 + 2) + 161/1331/sqrt(3*x^2 + 2) - 1/22/(4*sq
rt(3*x^2 + 2)*x^2 + 4*sqrt(3*x^2 + 2)*x + sqrt(3*x^2 + 2)) + 7/242/(2*sqrt(
3*x^2 + 2)*x + sqrt(3*x^2 + 2))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(78) = 156$.

Time = 0.29 (sec) , antiderivative size = 196, normalized size of antiderivative = 2.02

$$\int \frac{1+3x+4x^2}{(1+2x)^3\left(2+3x^2\right)^{3/2}}\,dx = \frac{322}{14641}\sqrt{11}\log\left(-\frac{\left|-2\sqrt{3}x-\sqrt{11}-\sqrt{3}+2\sqrt{3x^2+2}\right|}{2\sqrt{3}x-\sqrt{11}+\sqrt{3}-2\sqrt{3x^2+2}}\right) + \frac{351x+358}{2662\sqrt{3x^2+2}} + \frac{36\left(\sqrt{3}x-\sqrt{3x^2+2}\right)^3-\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2+2}\right)^2+48\sqrt{3}x+8\sqrt{3}-48\sqrt{3x^2+2}}{1331\left(\left(\sqrt{3}x-\sqrt{3x^2+2}\right)^2+\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2+2}\right)-2\right)^2}$$

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] 322/14641*sqrt(11)*log(-abs(-2*sqrt(3)*x - sqrt(11) - sqrt(3) + 2*sqrt(3*x^
2 + 2))/(2*sqrt(3)*x - sqrt(11) + sqrt(3) - 2*sqrt(3*x^2 + 2))) + 1/2662*(3
51*x + 358)/sqrt(3*x^2 + 2) + 1/1331*(36*(sqrt(3)*x - sqrt(3*x^2 + 2))^3 -
sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 2))^2 + 48*sqrt(3)*x + 8*sqrt(3) - 48*sqr
t(3*x^2 + 2))/((sqrt(3)*x - sqrt(3*x^2 + 2))^2 + sqrt(3)*(sqrt(3)*x - sqrt(
3*x^2 + 2)) - 2)^2
```

Mupad [B] (verification not implemented)

Time = 13.09 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.86

$$\int \frac{1+3x+4x^2}{(1+2x)^3\left(2+3x^2\right)^{3/2}}\,dx = \frac{322\sqrt{11}\,\ln\left(x+\frac{1}{2}\right)}{14641} - \frac{322\sqrt{11}\,\ln\left(x-\frac{\sqrt{3}\sqrt{11}\sqrt{x^2+\frac{2}{3}}}{3}-\frac{4}{3}\right)}{14641} + \frac{117\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{5324\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)} + \frac{117\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{5324\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)} - \frac{\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{242\left(x^2+x+\frac{1}{4}\right)} + \frac{\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{1331\left(x+\frac{1}{2}\right)} - \frac{\sqrt{3}\sqrt{6}\sqrt{x^2+\frac{2}{3}}\,179\mathrm{i}}{15972\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)} + \frac{\sqrt{3}\sqrt{6}\sqrt{x^2+\frac{2}{3}}\,179\mathrm{i}}{15972\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}$$

```
[In] int((3*x + 4*x^2 + 1)/((2*x + 1)^3*(3*x^2 + 2)^(3/2)),x)
```

```
[Out] (322*11^(1/2)*log(x + 1/2))/14641 - (322*11^(1/2)*log(x - (3^(1/2)*11^(1/2)
*(x^2 + 2/3)^(1/2))/3 - 4/3))/14641 + (117*3^(1/2)*(x^2 + 2/3)^(1/2))/(5324
*(x - (6^(1/2)*1i)/3)) + (117*3^(1/2)*(x^2 + 2/3)^(1/2))/(5324*(x + (6^(1/2
)*1i)/3)) - (3^(1/2)*(x^2 + 2/3)^(1/2))/(242*(x + x^2 + 1/4)) + (3^(1/2)*(x
^2 + 2/3)^(1/2))/(1331*(x + 1/2)) - (3^(1/2)*6^(1/2)*(x^2 + 2/3)^(1/2)*179i
)/(15972*(x - (6^(1/2)*1i)/3)) + (3^(1/2)*6^(1/2)*(x^2 + 2/3)^(1/2)*179i)/(
15972*(x + (6^(1/2)*1i)/3))
```

3.130 $\int \frac{(1+2x)^3\left(1+3x+4x^2\right)}{\left(2+3x^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 73

$$\int \frac{(1+2x)^3\left(1+3x+4x^2\right)}{\left(2+3x^2\right)^{5/2}}\,dx = \frac{398+279x}{162\left(2+3x^2\right)^{3/2}} - \frac{152+465x}{54\sqrt{2+3x^2}} + \frac{32}{27}\sqrt{2+3x^2} + \frac{8\operatorname{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{3}}$$

```
[Out] 1/162*(398+279*x)/(3*x^2+2)^(3/2)+8/3*arcsinh(1/2*x*6^(1/2))*3^(1/2)+1/54*(
-152-465*x)/(3*x^2+2)^(1/2)+32/27*(3*x^2+2)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1828, 655, 221}

$$\int \frac{(1+2x)^3\left(1+3x+4x^2\right)}{\left(2+3x^2\right)^{5/2}}\,dx = \frac{8\operatorname{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{3}} + \frac{279x+398}{162\left(3x^2+2\right)^{3/2}} + \frac{32}{27}\sqrt{3x^2+2} - \frac{465x+152}{54\sqrt{3x^2+2}}$$

```
[In] Int[((1 + 2*x)^3*(1 + 3*x + 4*x^2))/(2 + 3*x^2)^(5/2),x]
```

```
[Out] (398 + 279*x)/(162*(2 + 3*x^2)^(3/2)) - (152 + 465*x)/(54*Sqrt[2 + 3*x^2])
+ (32*Sqrt[2 + 3*x^2])/27 + (8*ArcSinh[Sqrt[3/2]*x])/Sqrt[3]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{398+279x}{162\left(2+3x^2\right)^{3/2}}-\frac{1}{6}\int\frac{\frac{22}{3}-\frac{280x}{3}-144x^2-64x^3}{\left(2+3x^2\right)^{3/2}}\,dx\\
&= \frac{398+279x}{162\left(2+3x^2\right)^{3/2}}-\frac{152+465x}{54\sqrt{2+3x^2}}+\frac{1}{12}\int\frac{96+\frac{128x}{3}}{\sqrt{2+3x^2}}\,dx\\
&= \frac{398+279x}{162\left(2+3x^2\right)^{3/2}}-\frac{152+465x}{54\sqrt{2+3x^2}}+\frac{32}{27}\sqrt{2+3x^2}+8\int\frac{1}{\sqrt{2+3x^2}}\,dx\\
&= \frac{398+279x}{162\left(2+3x^2\right)^{3/2}}-\frac{152+465x}{54\sqrt{2+3x^2}}+\frac{32}{27}\sqrt{2+3x^2}+\frac{8\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{3}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.88

$$\int\frac{(1+2x)^3\left(1+3x+4x^2\right)}{\left(2+3x^2\right)^{5/2}}\,dx=\frac{254-2511x+936x^2-4185x^3+1728x^4}{162\left(2+3x^2\right)^{3/2}}-\frac{8\log\left(-\sqrt{3}x+\sqrt{2+3x^2}\right)}{\sqrt{3}}$$

```
[In] Integrate[((1 + 2*x)^3*(1 + 3*x + 4*x^2))/(2 + 3*x^2)^(5/2),x]
```

```
[Out] (254 - 2511*x + 936*x^2 - 4185*x^3 + 1728*x^4)/(162*(2 + 3*x^2)^(3/2)) - (8
*Log[-(Sqrt[3]*x) + Sqrt[2 + 3*x^2]])/Sqrt[3]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.62

method	result
risch	$\frac{1728x^4-4185x^3+936x^2-2511x+254}{162(3x^2+2)^{\frac{3}{2}}}+\frac{8\ \operatorname{arcsinh}\left(\frac{x\sqrt{6}}{2}\right)\sqrt{3}}{3}$
trager	$\frac{1728x^4-4185x^3+936x^2-2511x+254}{162(3x^2+2)^{\frac{3}{2}}}+\frac{8\operatorname{RootOf}\left(\_Z^2-3\right)\ln\left(\operatorname{RootOf}\left(\_Z^2-3\right)\sqrt{3x^2+2}+3x\right)}{3}$
default	$-\frac{65x}{18(3x^2+2)^{\frac{3}{2}}}-\frac{107x}{18\sqrt{3x^2+2}}+\frac{127}{81(3x^2+2)^{\frac{3}{2}}}+\frac{32x^4}{3(3x^2+2)^{\frac{3}{2}}}+\frac{52x^2}{9(3x^2+2)^{\frac{3}{2}}}-\frac{8x^3}{(3x^2+2)^{\frac{3}{2}}}+\frac{8\ \operatorname{arcsinh}\left(\frac{x\sqrt{6}}{2}\right)\sqrt{3}}{3}$
meijerg	$\frac{\sqrt{2}\,x(3x^2+3)}{24\left(\frac{3x^2}{2}+1\right)^{\frac{3}{2}}}+\frac{17\sqrt{2}\,x^3}{12\left(\frac{3x^2}{2}+1\right)^{\frac{3}{2}}}+\frac{\sqrt{2}\left(\frac{\sqrt{\pi}}{2}-\frac{\sqrt{\pi}}{2\left(\frac{3x^2}{2}+1\right)^{\frac{3}{2}}}\right)}{2\sqrt{\pi}}+\frac{68\sqrt{2}\left(\sqrt{\pi}-\frac{\sqrt{\pi}\left(18x^2+8\right)}{8\left(\frac{3x^2}{2}+1\right)^{\frac{3}{2}}}\right)}{27\sqrt{\pi}}+\frac{16\sqrt{3}\Bigg(-\frac{\sqrt{\pi}\,x\sqrt{2}\,\sqrt{3}\left(30x^2+15\right)}{20\left(\frac{3x^2}{2}+1\right)^{\frac{3}{2}}}}{9\sqrt{\pi}}$

```
[In] int((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2+2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/162*(1728*x^4-4185*x^3+936*x^2-2511*x+254)/(3*x^2+2)^(3/2)+8/3*arcsinh(1/
2*x*6^(1/2))*3^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.19

$$\int \frac{(1+2x)^3\,(1+3x+4x^2)}{(2+3x^2)^{5/2}}\,dx=\frac{216\,\sqrt{3}(9\,x^4+12\,x^2+4)\log\left(-\sqrt{3}\sqrt{3\,x^2+2}x-3\,x^2-1\right)+(1728\,x^4-4}{162\,(9\,x^4+12\,x^2+4)}$$

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2+2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/162*(216*sqrt(3)*(9*x^4 + 12*x^2 + 4)*log(-sqrt(3)*sqrt(3*x^2 + 2)*x - 3*
x^2 - 1) + (1728*x^4 - 4185*x^3 + 936*x^2 - 2511*x + 254)*sqrt(3*x^2 + 2))/
(9*x^4 + 12*x^2 + 4)
```

Sympy [F]

$$\int \frac{(1+2x)^3 (1+3x+4x^2)}{(2+3x^2)^{5/2}}\,dx = \int \frac{(2x+1)^3 \cdot (4x^2+3x+1)}{(3x^2+2)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((1+2*x)**3*(4*x**2+3*x+1)/(3*x**2+2)**(5/2),x)
```

```
[Out] Integral((2*x + 1)**3*(4*x**2 + 3*x + 1)/(3*x**2 + 2)**(5/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.44

$$\int \frac{(1+2x)^3 (1+3x+4x^2)}{(2+3x^2)^{5/2}}\,dx = \frac{32\,x^4}{3\,(3\,x^2+2)^{\frac{3}{2}}} - \frac{8}{3}\,x\left(\frac{9\,x^2}{(3\,x^2+2)^{\frac{3}{2}}} + \frac{4}{(3\,x^2+2)^{\frac{3}{2}}}\right) + \frac{8}{3}\,\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{2}\,\sqrt{6}x\right) - \frac{11\,x}{18\,\sqrt{3\,x^2+2}} + \frac{52\,x^2}{9\,(3\,x^2+2)^{\frac{3}{2}}} - \frac{65\,x}{18\,(3\,x^2+2)^{\frac{3}{2}}} + \frac{127}{81\,(3\,x^2+2)^{\frac{3}{2}}}$$

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 32/3*x^4/(3*x^2 + 2)^(3/2) - 8/3*x*(9*x^2/(3*x^2 + 2)^(3/2) + 4/(3*x^2 + 2)
^(3/2)) + 8/3*sqrt(3)*arcsinh(1/2*sqrt(6)*x) - 11/18*x/sqrt(3*x^2 + 2) + 52
/9*x^2/(3*x^2 + 2)^(3/2) - 65/18*x/(3*x^2 + 2)^(3/2) + 127/81/(3*x^2 + 2)^(
3/2)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.73

$$\int \frac{(1+2x)^3 (1+3x+4x^2)}{(2+3x^2)^{5/2}}\,dx = -\frac{8}{3}\,\sqrt{3}\log\left(-\sqrt{3}x + \sqrt{3\,x^2+2}\right) + \frac{9\,((3\,(64\,x-155)x+104)x-279)x+254}{162\,(3\,x^2+2)^{\frac{3}{2}}}$$

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2+2)^(5/2),x, algorithm="giac")
```

```
[Out] -8/3*sqrt(3)*log(-sqrt(3)*x + sqrt(3*x^2 + 2)) + 1/162*(9*((3*(64*x - 155)*
x + 104)*x - 279)*x + 254)/(3*x^2 + 2)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 212, normalized size of antiderivative = 2.90

$$\int \frac{(1+2x)^3\left(1+3x+4x^2\right)}{\left(2+3x^2\right)^{5/2}}\,dx = \frac{32\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}}{27} + \frac{8\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{2}\sqrt{3}\,x}{2}\right)}{3} - \frac{\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}\left(\frac{-\frac{31}{16}+\frac{\sqrt{6}\,199\mathrm{i}}{144}}{x-\frac{\sqrt{6}\,1\mathrm{i}}{3}} - \frac{\sqrt{6}\left(-\frac{31}{24}+\frac{\sqrt{6}\,199\mathrm{i}}{216}\right)1\mathrm{i}}{2\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)^2}\right)}{27} + \frac{\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}\left(\frac{\frac{31}{16}+\frac{\sqrt{6}\,199\mathrm{i}}{144}}{x+\frac{\sqrt{6}\,1\mathrm{i}}{3}} + \frac{\sqrt{6}\left(\frac{31}{24}+\frac{\sqrt{6}\,199\mathrm{i}}{216}\right)1\mathrm{i}}{2\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)^2}\right)}{27} + \frac{\sqrt{3}\,\sqrt{6}\left(-1824+\sqrt{6}\,1953\mathrm{i}\right)\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{7776\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)} + \frac{\sqrt{3}\,\sqrt{6}\left(1824+\sqrt{6}\,1953\mathrm{i}\right)\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{7776\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}$$

```
[In] int(((2*x + 1)^3*(3*x + 4*x^2 + 1))/(3*x^2 + 2)^(5/2),x)
```

```
[Out] (32*3^(1/2)*(x^2 + 2/3)^(1/2))/27 + (8*3^(1/2)*asinh((2^(1/2)*3^(1/2)*x)/2)
)/3 - (3^(1/2)*(x^2 + 2/3)^(1/2)*(((6^(1/2)*199i)/144 - 31/16)/(x - (6^(1/2
)*1i)/3) - (6^(1/2)*((6^(1/2)*199i)/216 - 31/24)*1i)/(2*(x - (6^(1/2)*1i)/3
)^2)))/27 + (3^(1/2)*(x^2 + 2/3)^(1/2)*(((6^(1/2)*199i)/144 + 31/16)/(x + (
6^(1/2)*1i)/3) + (6^(1/2)*((6^(1/2)*199i)/216 + 31/24)*1i)/(2*(x + (6^(1/2)
*1i)/3)^2)))/27 + (3^(1/2)*6^(1/2)*(6^(1/2)*1953i - 1824)*(x^2 + 2/3)^(1/2)
*1i)/(7776*(x + (6^(1/2)*1i)/3)) + (3^(1/2)*6^(1/2)*(6^(1/2)*1953i + 1824)*
(x^2 + 2/3)^(1/2)*1i)/(7776*(x - (6^(1/2)*1i)/3))
```

3.131 $\int \frac{(1+2x)^2(1+3x+4x^2)}{(2+3x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 60

$$\int \frac{(1+2x)^2(1+3x+4x^2)}{(2+3x^2)^{5/2}}\,dx=\frac{70-47x}{54(2+3x^2)^{3/2}}-\frac{168+59x}{54\sqrt{2+3x^2}}+\frac{16\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{9\sqrt{3}}$$

```
[Out] 1/54*(70-47*x)/(3*x^2+2)^(3/2)+16/27*arcsinh(1/2*x*6^(1/2))*3^(1/2)+1/54*(-
168-59*x)/(3*x^2+2)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1828, 12, 221}

$$\int \frac{(1+2x)^2(1+3x+4x^2)}{(2+3x^2)^{5/2}}\,dx=\frac{16\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{9\sqrt{3}}+\frac{70-47x}{54(3x^2+2)^{3/2}}-\frac{59x+168}{54\sqrt{3x^2+2}}$$

```
[In] Int[((1 + 2*x)^2*(1 + 3*x + 4*x^2))/(2 + 3*x^2)^(5/2),x]
```

```
[Out] (70 - 47*x)/(54*(2 + 3*x^2)^(3/2)) - (168 + 59*x)/(54*Sqrt[2 + 3*x^2]) + (1
6*ArcSinh[Sqrt[3/2]*x])/(9*Sqrt[3])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{70-47x}{54\left(2+3x^2\right)^{3/2}}-\frac{1}{6}\int \frac{-\frac{74}{9}-56x-32x^2}{\left(2+3x^2\right)^{3/2}}\,dx\\
&= \frac{70-47x}{54\left(2+3x^2\right)^{3/2}}-\frac{168+59x}{54\sqrt{2+3x^2}}+\frac{1}{12}\int \frac{64}{3\sqrt{2+3x^2}}\,dx\\
&= \frac{70-47x}{54\left(2+3x^2\right)^{3/2}}-\frac{168+59x}{54\sqrt{2+3x^2}}+\frac{16}{9}\int \frac{1}{\sqrt{2+3x^2}}\,dx\\
&= \frac{70-47x}{54\left(2+3x^2\right)^{3/2}}-\frac{168+59x}{54\sqrt{2+3x^2}}+\frac{16\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{9\sqrt{3}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.02

$$\int \frac{(1+2x)^2\left(1+3x+4x^2\right)}{\left(2+3x^2\right)^{5/2}}\,dx=\frac{-266-165x-504x^2-177x^3}{54\left(2+3x^2\right)^{3/2}}-\frac{16\log\left(-\sqrt{3}x+\sqrt{2+3x^2}\right)}{9\sqrt{3}}$$

```
[In] Integrate[((1 + 2*x)^2*(1 + 3*x + 4*x^2))/(2 + 3*x^2)^(5/2),x]
[Out] (-266 - 165*x - 504*x^2 - 177*x^3)/(54*(2 + 3*x^2)^(3/2)) - (16*Log[-(Sqrt[
3]*x) + Sqrt[2 + 3*x^2]])/(9*Sqrt[3])
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.67

method	result
risch	$-\frac{177x^3+504x^2+165x+266}{54(3x^2+2)^{\frac{3}{2}}}+\frac{16\ \mathrm{arcsinh}\left(\frac{x\sqrt{6}}{2}\right)\sqrt{3}}{27}$
trager	$-\frac{177x^3+504x^2+165x+266}{54(3x^2+2)^{\frac{3}{2}}}+\frac{16\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(\mathrm{RootOf}\left(\_Z^2-3\right)\sqrt{3x^2+2}+3x\right)}{27}$
default	$-\frac{37x}{18(3x^2+2)^{\frac{3}{2}}}-\frac{x}{2\sqrt{3x^2+2}}-\frac{133}{27(3x^2+2)^{\frac{3}{2}}}-\frac{16x^3}{9(3x^2+2)^{\frac{3}{2}}}+\frac{16\ \mathrm{arcsinh}\left(\frac{x\sqrt{6}}{2}\right)\sqrt{3}}{27}-\frac{28x^2}{3(3x^2+2)^{\frac{3}{2}}}$
meijerg	$\frac{\sqrt{2}\,x(3x^2+3)}{24\left(\frac{3x^2}{2}+1\right)^{\frac{3}{2}}}+\frac{5\sqrt{2}\,x^3}{6\left(\frac{3x^2}{2}+1\right)^{\frac{3}{2}}}+\frac{7\sqrt{2}\left(\frac{\sqrt{\pi}}{2}-\frac{\sqrt{\pi}}{2\left(\frac{3x^2}{2}+1\right)^{\frac{3}{2}}}\right)}{18\sqrt{\pi}}+\frac{28\sqrt{2}\left(\sqrt{\pi}-\frac{\sqrt{\pi}\left(18x^2+8\right)}{8\left(\frac{3x^2}{2}+1\right)^{\frac{3}{2}}}\right)}{27\sqrt{\pi}}+\frac{32\sqrt{3}\left(-\frac{\sqrt{\pi}\,x\sqrt{2}\,\sqrt{3}\left(30x^2+15\right)}{20\left(\frac{3x^2}{2}+1\right)^{\frac{3}{2}}}+\right.}{81\sqrt{\pi}}$

```
[In] int((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2+2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/54*(177*x^3+504*x^2+165*x+266)/(3*x^2+2)^(3/2)+16/27*arcsinh(1/2*x*6^(1/
2))*3^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.38

$$\int \frac{(1+2x)^2\left(1+3x+4x^2\right)}{\left(2+3x^2\right)^{5/2}}\,dx=\frac{16\,\sqrt{3}(9\,x^4+12\,x^2+4)\log\left(-\sqrt{3}\sqrt{3\,x^2+2}x-3\,x^2-1\right)-(177\,x^3+504\,x}{54\,(9\,x^4+12\,x^2+4)}$$

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2+2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/54*(16*sqrt(3)*(9*x^4 + 12*x^2 + 4)*log(-sqrt(3)*sqrt(3*x^2 + 2)*x - 3*x^
2 - 1) - (177*x^3 + 504*x^2 + 165*x + 266)*sqrt(3*x^2 + 2))/(9*x^4 + 12*x^2
 + 4)
```

Sympy [F]

$$\int \frac{(1+2x)^2\left(1+3x+4x^2\right)}{\left(2+3x^2\right)^{5/2}}\,dx=\int \frac{\left(2x+1\right)^{2}\cdot\left(4x^{2}+3x+1\right)}{\left(3x^{2}+2\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((1+2*x)**2*(4*x**2+3*x+1)/(3*x**2+2)**(5/2),x)
[Out] Integral((2*x + 1)**2*(4*x**2 + 3*x + 1)/(3*x**2 + 2)**(5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 91 vs. $2(45) = 90$.

Time = 0.27 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.52

$$\int \frac{(1+2x)^2\left(1+3x+4x^2\right)}{\left(2+3x^2\right)^{5/2}}\,dx=-\frac{16}{27}\,x\left(\frac{9\,x^{2}}{\left(3\,x^{2}+2\right)^{\frac{3}{2}}}+\frac{4}{\left(3\,x^{2}+2\right)^{\frac{3}{2}}}\right)+\frac{16}{27}\,\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{2}\,\sqrt{6}x\right)+\frac{37\,x}{54\,\sqrt{3\,x^{2}+2}}-\frac{28\,x^{2}}{3\,\left(3\,x^{2}+2\right)^{\frac{3}{2}}}-\frac{37\,x}{18\,\left(3\,x^{2}+2\right)^{\frac{3}{2}}}-\frac{133}{27\,\left(3\,x^{2}+2\right)^{\frac{3}{2}}}$$

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2+2)^(5/2),x, algorithm="maxima")
[Out] -16/27*x*(9*x^2/(3*x^2 + 2)^(3/2) + 4/(3*x^2 + 2)^(3/2)) + 16/27*sqrt(3)*ar
csinh(1/2*sqrt(6)*x) + 37/54*x/sqrt(3*x^2 + 2) - 28/3*x^2/(3*x^2 + 2)^(3/2)
 - 37/18*x/(3*x^2 + 2)^(3/2) - 133/27/(3*x^2 + 2)^(3/2)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.80

$$\int \frac{(1+2x)^2\left(1+3x+4x^2\right)}{\left(2+3x^2\right)^{5/2}}\,dx=-\frac{16}{27}\,\sqrt{3}\log\left(-\sqrt{3}x+\sqrt{3\,x^{2}+2}\right)-\frac{3\,\left(\left(59\,x+168\right)x+55\right)x+266}{54\,\left(3\,x^{2}+2\right)^{\frac{3}{2}}}$$

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2+2)^(5/2),x, algorithm="giac")
[Out] -16/27*sqrt(3)*log(-sqrt(3)*x + sqrt(3*x^2 + 2)) - 1/54*(3*((59*x + 168)*x
+ 55)*x + 266)/(3*x^2 + 2)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 200, normalized size of antiderivative = 3.33

$$\int \frac{(1+2x)^2\,(1+3x+4x^2)}{(2+3x^2)^{5/2}}\,dx = \frac{16\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{2}\sqrt{3}\,x}{2}\right)}{27} + \frac{\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}\left(\frac{-\frac{47}{48}+\frac{\sqrt{6}\,35\mathrm{i}}{48}}{x+\frac{\sqrt{6}\,1\mathrm{i}}{3}} + \frac{\sqrt{6}\left(-\frac{47}{72}+\frac{\sqrt{6}\,35\mathrm{i}}{72}\right)1\mathrm{i}}{2\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)^2}\right)}{27} - \frac{\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}\left(\frac{\frac{47}{48}+\frac{\sqrt{6}\,35\mathrm{i}}{48}}{x-\frac{\sqrt{6}\,1\mathrm{i}}{3}} - \frac{\sqrt{6}\left(\frac{47}{72}+\frac{\sqrt{6}\,35\mathrm{i}}{72}\right)1\mathrm{i}}{2\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)^2}\right)}{27} + \frac{\sqrt{3}\,\sqrt{6}\left(-672+\sqrt{6}\,63\mathrm{i}\right)\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{2592\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)} + \frac{\sqrt{3}\,\sqrt{6}\left(672+\sqrt{6}\,63\mathrm{i}\right)\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{2592\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}$$

```
[In] int(((2*x + 1)^2*(3*x + 4*x^2 + 1))/(3*x^2 + 2)^(5/2),x)
```

```
[Out] (16*3^(1/2)*asinh((2^(1/2)*3^(1/2)*x)/2))/27 + (3^(1/2)*(x^2 + 2/3)^(1/2)*(
((6^(1/2)*35i)/48 - 47/48)/(x + (6^(1/2)*1i)/3) + (6^(1/2)*((6^(1/2)*35i)/7
2 - 47/72)*1i)/(2*(x + (6^(1/2)*1i)/3)^2)))/27 - (3^(1/2)*(x^2 + 2/3)^(1/2)
*(((6^(1/2)*35i)/48 + 47/48)/(x - (6^(1/2)*1i)/3) - (6^(1/2)*((6^(1/2)*35i)
/72 + 47/72)*1i)/(2*(x - (6^(1/2)*1i)/3)^2)))/27 + (3^(1/2)*6^(1/2)*(6^(1/2
)*63i - 672)*(x^2 + 2/3)^(1/2)*1i)/(2592*(x + (6^(1/2)*1i)/3)) + (3^(1/2)*6
^(1/2)*(6^(1/2)*63i + 672)*(x^2 + 2/3)^(1/2)*1i)/(2592*(x - (6^(1/2)*1i)/3)
)
```

3.132 $\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\left(2+3x^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 41

$$\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\left(2+3x^2\right)^{5/2}}\,dx=\frac{2-51x}{54\left(2+3x^2\right)^{3/2}}-\frac{16-13x}{18\sqrt{2+3x^2}}$$

```
[Out] 1/54*(2-51*x)/(3*x^2+2)^(3/2)+1/18*(-16+13*x)/(3*x^2+2)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1828, 651}

$$\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\left(2+3x^2\right)^{5/2}}\,dx=\frac{2-51x}{54\left(3x^2+2\right)^{3/2}}-\frac{16-13x}{18\sqrt{3x^2+2}}$$

```
[In] Int[((1 + 2*x)*(1 + 3*x + 4*x^2))/(2 + 3*x^2)^(5/2),x]
```

```
[Out] (2 - 51*x)/(54*(2 + 3*x^2)^(3/2)) - (16 - 13*x)/(18*Sqrt[2 + 3*x^2])
```

Rule 651

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[((-a
)*e + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
```

```
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2-51x}{54\left(2+3x^2\right)^{3/2}}-\frac{1}{6}\int\frac{-\frac{26}{3}-16x}{\left(2+3x^2\right)^{3/2}}\,dx\\ &= \frac{2-51x}{54\left(2+3x^2\right)^{3/2}}-\frac{16-13x}{18\sqrt{2+3x^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.73

$$\int\frac{(1+2x)\left(1+3x+4x^2\right)}{\left(2+3x^2\right)^{5/2}}\,dx=\frac{-94+27x-144x^2+117x^3}{54\left(2+3x^2\right)^{3/2}}$$

```
[In] Integrate[((1 + 2*x)*(1 + 3*x + 4*x^2))/(2 + 3*x^2)^(5/2),x]
[Out] (-94 + 27*x - 144*x^2 + 117*x^3)/(54*(2 + 3*x^2)^(3/2))
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.66

method	result	size
gosper	$\frac{117x^3-144x^2+27x-94}{54(3x^2+2)^{\frac{3}{2}}}$	27
trager	$\frac{117x^3-144x^2+27x-94}{54(3x^2+2)^{\frac{3}{2}}}$	27
risch	$\frac{117x^3-144x^2+27x-94}{54(3x^2+2)^{\frac{3}{2}}}$	27
default	$-\frac{17x}{18(3x^2+2)^{\frac{3}{2}}}+\frac{13x}{18\sqrt{3x^2+2}}-\frac{47}{27(3x^2+2)^{\frac{3}{2}}}-\frac{8x^2}{3(3x^2+2)^{\frac{3}{2}}}$	51
meijerg	$\frac{\sqrt{2}\,x(3x^2+3)}{24\left(\frac{3x^2}{2}+1\right)^{\frac{3}{2}}}+\frac{5\sqrt{2}\,x^3}{12\left(\frac{3x^2}{2}+1\right)^{\frac{3}{2}}}+\frac{5\sqrt{2}\left(\frac{\sqrt{\pi}}{2}-\frac{\sqrt{\pi}}{2\left(\frac{3x^2}{2}+1\right)^{\frac{3}{2}}}\right)}{18\sqrt{\pi}}+\frac{8\sqrt{2}\left(\sqrt{\pi}-\frac{\sqrt{\pi}\left(18x^2+8\right)}{8\left(\frac{3x^2}{2}+1\right)^{\frac{3}{2}}}\right)}{27\sqrt{\pi}}$	102

```
[In] int((1+2*x)*(4*x^2+3*x+1)/(3*x^2+2)^(5/2),x,method=_RETURNVERBOSE)
[Out] 1/54*(117*x^3-144*x^2+27*x-94)/(3*x^2+2)^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.98

$$\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\left(2+3x^2\right)^{5/2}}\,dx = \frac{\left(117\,x^3-144\,x^2+27\,x-94\right)\sqrt{3\,x^2+2}}{54\,\left(9\,x^4+12\,x^2+4\right)}$$

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2+2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/54*(117*x^3 - 144*x^2 + 27*x - 94)*sqrt(3*x^2 + 2)/(9*x^4 + 12*x^2 + 4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. 2(36) = 72.

Time = 18.99 (sec) , antiderivative size = 180, normalized size of antiderivative = 4.39

$$\begin{aligned}\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\left(2+3x^2\right)^{5/2}}\,dx &= \frac{10x^3}{18x^2\sqrt{3x^2+2}+12\sqrt{3x^2+2}} \\ &+ \frac{x^3}{6x^2\sqrt{3x^2+2}+4\sqrt{3x^2+2}} - \frac{72x^2}{81x^2\sqrt{3x^2+2}+54\sqrt{3x^2+2}} \\ &+ \frac{x}{6x^2\sqrt{3x^2+2}+4\sqrt{3x^2+2}} - \frac{32}{81x^2\sqrt{3x^2+2}+54\sqrt{3x^2+2}} \\ &- \frac{5}{27x^2\sqrt{3x^2+2}+18\sqrt{3x^2+2}}\end{aligned}$$

```
[In] integrate((1+2*x)*(4*x**2+3*x+1)/(3*x**2+2)**(5/2),x)
```

```
[Out] 10*x**3/(18*x**2*sqrt(3*x**2 + 2) + 12*sqrt(3*x**2 + 2)) + x**3/(6*x**2*sqr
t(3*x**2 + 2) + 4*sqrt(3*x**2 + 2)) - 72*x**2/(81*x**2*sqrt(3*x**2 + 2) + 5
4*sqrt(3*x**2 + 2)) + x/(6*x**2*sqrt(3*x**2 + 2) + 4*sqrt(3*x**2 + 2)) - 32
/(81*x**2*sqrt(3*x**2 + 2) + 54*sqrt(3*x**2 + 2)) - 5/(27*x**2*sqrt(3*x**2
+ 2) + 18*sqrt(3*x**2 + 2))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.22

$$\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\left(2+3x^2\right)^{5/2}}\,dx = \frac{13\,x}{18\,\sqrt{3\,x^2+2}} - \frac{8\,x^2}{3\,\left(3\,x^2+2\right)^{\frac{3}{2}}} - \frac{17\,x}{18\,\left(3\,x^2+2\right)^{\frac{3}{2}}} - \frac{47}{27\,\left(3\,x^2+2\right)^{\frac{3}{2}}}$$

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 13/18*x/sqrt(3*x^2 + 2) - 8/3*x^2/(3*x^2 + 2)^(3/2) - 17/18*x/(3*x^2 + 2)^(
3/2) - 47/27/(3*x^2 + 2)^(3/2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.61

$$\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\left(2+3x^2\right)^{5/2}}\,dx=\frac{9\left((13\,x-16)x+3\right)x-94}{54\left(3\,x^2+2\right)^{\frac{3}{2}}}$$

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2+2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/54*(9*((13*x - 16)*x + 3)*x - 94)/(3*x^2 + 2)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 185, normalized size of antiderivative = 4.51

$$\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\left(2+3x^2\right)^{5/2}}\,dx=\frac{\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}\left(\frac{-\frac{17}{16}+\frac{\sqrt{6}\,1\mathrm{i}}{48}}{x+\frac{\sqrt{6}\,1\mathrm{i}}{3}}+\frac{\sqrt{6}\left(-\frac{17}{24}+\frac{\sqrt{6}\,1\mathrm{i}}{72}\right)1\mathrm{i}}{2\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)^2}\right)}{27}$$

$$-\frac{\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}\left(\frac{\frac{17}{16}+\frac{\sqrt{6}\,1\mathrm{i}}{48}}{x-\frac{\sqrt{6}\,1\mathrm{i}}{3}}-\frac{\sqrt{6}\left(\frac{17}{24}+\frac{\sqrt{6}\,1\mathrm{i}}{72}\right)1\mathrm{i}}{2\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)^2}\right)}{27}$$

$$-\frac{\sqrt{3}\,\sqrt{6}\left(-192+\sqrt{6}\,69\mathrm{i}\right)\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{2592\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}-\frac{\sqrt{3}\,\sqrt{6}\left(192+\sqrt{6}\,69\mathrm{i}\right)\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{2592\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}$$

```
[In] int(((2*x + 1)*(3*x + 4*x^2 + 1))/(3*x^2 + 2)^(5/2),x)
```

```
[Out] (3^(1/2)*(x^2 + 2/3)^(1/2)*(((6^(1/2)*1i)/48 - 17/16)/(x + (6^(1/2)*1i)/3)
+ (6^(1/2)*((6^(1/2)*1i)/72 - 17/24)*1i)/(2*(x + (6^(1/2)*1i)/3)^2)))/27 -
(3^(1/2)*(x^2 + 2/3)^(1/2)*(((6^(1/2)*1i)/48 + 17/16)/(x - (6^(1/2)*1i)/3)
- (6^(1/2)*((6^(1/2)*1i)/72 + 17/24)*1i)/(2*(x - (6^(1/2)*1i)/3)^2)))/27 -
(3^(1/2)*6^(1/2)*(6^(1/2)*69i - 192)*(x^2 + 2/3)^(1/2)*1i)/(2592*(x - (6^(1
/2)*1i)/3)) - (3^(1/2)*6^(1/2)*(6^(1/2)*69i + 192)*(x^2 + 2/3)^(1/2)*1i)/(2
592*(x + (6^(1/2)*1i)/3))
```

3.133 $\int \frac{1+3x+4x^2}{(1+2x)\left(2+3x^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 73

$$\int \frac{1+3x+4x^2}{(1+2x)\left(2+3x^2\right)^{5/2}}\,dx=\frac{-38+21x}{198\left(2+3x^2\right)^{3/2}}+\frac{24+95x}{726\sqrt{2+3x^2}}-\frac{8\operatorname{arctanh}\left(\frac{4-3x}{\sqrt{11}\sqrt{2+3x^2}}\right)}{121\sqrt{11}}$$

```
[Out] 1/198*(-38+21*x)/(3*x^2+2)^(3/2)-8/1331*arctanh(1/11*(4-3*x)*11^(1/2)/(3*x^
2+2)^(1/2))*11^(1/2)+1/726*(24+95*x)/(3*x^2+2)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1661, 837, 12, 739, 212}

$$\int \frac{1+3x+4x^2}{(1+2x)\left(2+3x^2\right)^{5/2}}\,dx=-\frac{8\operatorname{arctanh}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{121\sqrt{11}}-\frac{38-21x}{198\left(3x^2+2\right)^{3/2}}+\frac{95x+24}{726\sqrt{3x^2+2}}$$

```
[In] Int[(1 + 3*x + 4*x^2)/((1 + 2*x)*(2 + 3*x^2)^(5/2)),x]
```

```
[Out] -1/198*(38 - 21*x)/(2 + 3*x^2)^(3/2) + (24 + 95*x)/(726*Sqrt[2 + 3*x^2]) -
(8*ArcTanh[(4 - 3*x)/(Sqrt[11]*Sqrt[2 + 3*x^2])])/(121*Sqrt[11])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{38-21x}{198\left(2+3x^2\right)^{3/2}}-\frac{1}{18}\int \frac{-\frac{78}{11}-\frac{84x}{11}}{(1+2x)\left(2+3x^2\right)^{3/2}}\,dx \\
&= -\frac{38-21x}{198\left(2+3x^2\right)^{3/2}}+\frac{24+95x}{726\sqrt{2+3x^2}}+\frac{\int \frac{864}{11(1+2x)\sqrt{2+3x^2}}\,dx}{1188} \\
&= -\frac{38-21x}{198\left(2+3x^2\right)^{3/2}}+\frac{24+95x}{726\sqrt{2+3x^2}}+\frac{8}{121}\int \frac{1}{(1+2x)\sqrt{2+3x^2}}\,dx \\
&= -\frac{38-21x}{198\left(2+3x^2\right)^{3/2}}+\frac{24+95x}{726\sqrt{2+3x^2}}-\frac{8}{121}\text{Subst}\left(\int \frac{1}{11-x^2}\,dx,x,\frac{4-3x}{\sqrt{2+3x^2}}\right)
\end{aligned}$$

$$= -\frac{38-21x}{198\left(2+3x^2\right)^{3/2}}+\frac{24+95x}{726\sqrt{2+3x^2}}-\frac{8\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{2+3x^2}}\right)}{121\sqrt{11}}$$

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.79

$$\int\frac{1+3x+4x^2}{(1+2x)\left(2+3x^2\right)^{5/2}}\,dx=\frac{-274+801x+216x^2+855x^3}{2178\left(2+3x^2\right)^{3/2}}-\frac{8\text{arctanh}\left(\frac{4-3x}{\sqrt{22+33x^2}}\right)}{121\sqrt{11}}$$

```
[In] Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)*(2 + 3*x^2)^(5/2)),x]
```

```
[Out] (-274 + 801*x + 216*x^2 + 855*x^3)/(2178*(2 + 3*x^2)^(3/2)) - (8*ArcTanh[(4
 - 3*x)/Sqrt[22 + 33*x^2]])/(121*Sqrt[11])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.56 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.01

method	result
trager	$\frac{855x^3+216x^2+801x-274}{2178(3x^2+2)^{\frac{3}{2}}}+\frac{8\,\mathrm{RootOf}\left(\_Z^2-11\right)\ln\left(\frac{3\,\mathrm{RootOf}\left(\_Z^2-11\right)x+11\sqrt{3x^2+2}-4\,\mathrm{RootOf}\left(\_Z^2-11\right)}{1+2x}\right)}{1331}$
default	$\frac{x}{12(3x^2+2)^{\frac{3}{2}}}+\frac{x}{12\sqrt{3x^2+2}}-\frac{2}{9(3x^2+2)^{\frac{3}{2}}}+\frac{1}{33\left(3\left(x+\frac{1}{2}\right)^2-3x+\frac{5}{4}\right)^{\frac{3}{2}}}+\frac{x}{44\left(3\left(x+\frac{1}{2}\right)^2-3x+\frac{5}{4}\right)^{\frac{3}{2}}}+\frac{23x}{484\sqrt{3\left(x+\frac{1}{2}\right)^2-3x+\frac{5}{4}}}$

```
[In] int((4*x^2+3*x+1)/(1+2*x)/(3*x^2+2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2178*(855*x^3+216*x^2+801*x-274)/(3*x^2+2)^(3/2)+8/1331*RootOf(_Z^2-11)*l
n((3*RootOf(_Z^2-11)*x+11*(3*x^2+2)^(1/2)-4*RootOf(_Z^2-11))/(1+2*x))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.41

$$\int \frac{1+3x+4x^2}{(1+2x)\left(2+3x^2\right)^{5/2}}\,dx = \frac{72\sqrt{11}(9\,x^4+12\,x^2+4)\log\left(-\frac{\sqrt{11}\sqrt{3\,x^2+2}(3\,x-4)+21\,x^2-12\,x+19}{4\,x^2+4\,x+1}\right)+11\,(855\,x^3+2}{23958\,(9\,x^4+12\,x^2+4)}$$

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2+2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/23958*(72*sqrt(11)*(9*x^4 + 12*x^2 + 4)*log(-(sqrt(11)*sqrt(3*x^2 + 2)*(3
*x - 4) + 21*x^2 - 12*x + 19)/(4*x^2 + 4*x + 1)) + 11*(855*x^3 + 216*x^2 +
801*x - 274)*sqrt(3*x^2 + 2))/(9*x^4 + 12*x^2 + 4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1+3x+4x^2}{(1+2x)\left(2+3x^2\right)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)/(3*x**2+2)**(5/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.11

$$\begin{aligned}\int \frac{1+3x+4x^2}{(1+2x)\left(2+3x^2\right)^{5/2}}\,dx &= \frac{8}{1331}\sqrt{11}\,\operatorname{arsinh}\left(\frac{\sqrt{6}x}{2\,|2\,x+1|}-\frac{2\sqrt{6}}{3\,|2\,x+1|}\right)\\ &\quad+\frac{95\,x}{726\sqrt{3\,x^2+2}}+\frac{4}{121\sqrt{3\,x^2+2}}+\frac{7\,x}{66\,(3\,x^2+2)^{\frac{3}{2}}}-\frac{19}{99\,(3\,x^2+2)^{\frac{3}{2}}}\end{aligned}$$

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 8/1331*sqrt(11)*arcsinh(1/2*sqrt(6)*x/abs(2*x + 1) - 2/3*sqrt(6)/abs(2*x +
1)) + 95/726*x/sqrt(3*x^2 + 2) + 4/121/sqrt(3*x^2 + 2) + 7/66*x/(3*x^2 + 2)
^(3/2) - 19/99/(3*x^2 + 2)^(3/2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.25

$$\int \frac{1+3x+4x^2}{(1+2x)\left(2+3x^2\right)^{5/2}}\,dx = \frac{8}{1331}\sqrt{11}\log\left(-\frac{\left|-2\sqrt{3}x-\sqrt{11}-\sqrt{3}+2\sqrt{3x^2+2}\right|}{2\sqrt{3}x-\sqrt{11}+\sqrt{3}-2\sqrt{3x^2+2}}\right) + \frac{9\left((95x+24)x+89\right)x-274}{2178\left(3x^2+2\right)^{\frac{3}{2}}}$$

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2+2)^(5/2),x, algorithm="giac")
```

```
[Out] 8/1331*sqrt(11)*log(-abs(-2*sqrt(3)*x - sqrt(11) - sqrt(3) + 2*sqrt(3*x^2 +
 2))/(2*sqrt(3)*x - sqrt(11) + sqrt(3) - 2*sqrt(3*x^2 + 2))) + 1/2178*(9*((
95*x + 24)*x + 89)*x - 274)/(3*x^2 + 2)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 218, normalized size of antiderivative = 2.99

$$\int \frac{1+3x+4x^2}{(1+2x)\left(2+3x^2\right)^{5/2}}\,dx = \frac{\sqrt{11}\left(8\ln\left(x+\frac{1}{2}\right)-8\ln\left(x-\frac{\sqrt{3}\sqrt{11}\sqrt{x^2+\frac{2}{3}}}{3}-\frac{4}{3}\right)\right)}{1331} - \frac{\sqrt{3}\sqrt{x^2+\frac{2}{3}}\left(\frac{-\frac{21}{176}+\frac{\sqrt{6}\,19\mathrm{i}}{176}}{x+\frac{\sqrt{6}\,1\mathrm{i}}{3}}+\frac{\sqrt{6}\left(-\frac{7}{88}+\frac{\sqrt{6}\,19\mathrm{i}}{264}\right)1\mathrm{i}}{2\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)^2}\right)}{27} + \frac{\sqrt{3}\sqrt{x^2+\frac{2}{3}}\left(\frac{\frac{21}{176}+\frac{\sqrt{6}\,19\mathrm{i}}{176}}{x-\frac{\sqrt{6}\,1\mathrm{i}}{3}}-\frac{\sqrt{6}\left(\frac{7}{88}+\frac{\sqrt{6}\,19\mathrm{i}}{264}\right)1\mathrm{i}}{2\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)^2}\right)}{27} - \frac{\sqrt{3}\sqrt{6}\left(-288+\sqrt{6}\,303\mathrm{i}\right)\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{104544\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)} - \frac{\sqrt{3}\sqrt{6}\left(288+\sqrt{6}\,303\mathrm{i}\right)\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{104544\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}$$

```
[In] int((3*x + 4*x^2 + 1)/((2*x + 1)*(3*x^2 + 2)^(5/2)),x)
```

```
[Out] (11^(1/2)*(8*log(x + 1/2) - 8*log(x - (3^(1/2)*11^(1/2)*(x^2 + 2/3)^(1/2))/
3 - 4/3)))/1331 - (3^(1/2)*(x^2 + 2/3)^(1/2)*(((6^(1/2)*19i)/176 - 21/176)/
(x + (6^(1/2)*1i)/3) + (6^(1/2)*((6^(1/2)*19i)/264 - 7/88)*1i)/(2*(x + (6^(
1/2)*1i)/3)^2)))/27 + (3^(1/2)*(x^2 + 2/3)^(1/2)*(((6^(1/2)*19i)/176 + 21/1
76)/(x - (6^(1/2)*1i)/3) - (6^(1/2)*((6^(1/2)*19i)/264 + 7/88)*1i)/(2*(x -
(6^(1/2)*1i)/3)^2)))/27 - (3^(1/2)*6^(1/2)*(6^(1/2)*303i - 288)*(x^2 + 2/3)
^(1/2)*1i)/(104544*(x + (6^(1/2)*1i)/3)) - (3^(1/2)*6^(1/2)*(6^(1/2)*303i +
 288)*(x^2 + 2/3)^(1/2)*1i)/(104544*(x - (6^(1/2)*1i)/3))
```

3.134 $\int \frac{1+3x+4x^2}{(1+2x)^2\left(2+3x^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 95

$$\int \frac{1+3x+4x^2}{(1+2x)^2\left(2+3x^2\right)^{5/2}}\,dx = \frac{-10+97x}{726\left(2+3x^2\right)^{3/2}} + \frac{24+887x}{7986\sqrt{2+3x^2}} - \frac{16\sqrt{2+3x^2}}{1331(1+2x)} - \frac{32\text{arctanh}\left(\frac{4-3x}{\sqrt{11}\sqrt{2+3x^2}}\right)}{1331\sqrt{11}}$$

```
[Out] 1/726*(-10+97*x)/(3*x^2+2)^(3/2)-32/14641*arctanh(1/11*(4-3*x)*11^(1/2)/(3*
x^2+2)^(1/2))*11^(1/2)+1/7986*(24+887*x)/(3*x^2+2)^(1/2)-16/1331*(3*x^2+2)^
(1/2)/(1+2*x)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1661, 821, 739, 212}

$$\int \frac{1+3x+4x^2}{(1+2x)^2\left(2+3x^2\right)^{5/2}}\,dx = -\frac{32\text{arctanh}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{1331\sqrt{11}} - \frac{10-97x}{726\left(3x^2+2\right)^{3/2}} - \frac{16\sqrt{3x^2+2}}{1331(2x+1)} + \frac{887x+24}{7986\sqrt{3x^2+2}}$$

```
[In] Int[(1 + 3*x + 4*x^2)/((1 + 2*x)^2*(2 + 3*x^2)^(5/2)),x]
```

```
[Out] -1/726*(10 - 97*x)/(2 + 3*x^2)^(3/2) + (24 + 887*x)/(7986*Sqrt[2 + 3*x^2])
- (16*Sqrt[2 + 3*x^2])/(1331*(1 + 2*x)) - (32*ArcTanh[(4 - 3*x)/(Sqrt[11]*S
qrt[2 + 3*x^2])])/(1331*Sqrt[11])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{10-97x}{726\left(2+3x^2\right)^{3/2}}-\frac{1}{18}\int\frac{-\frac{798}{121}-\frac{1968x}{121}-\frac{2328x^2}{121}}{(1+2x)^2\left(2+3x^2\right)^{3/2}}\,dx\\
&= -\frac{10-97x}{726\left(2+3x^2\right)^{3/2}}+\frac{24+887x}{7986\sqrt{2+3x^2}}+\frac{1}{108}\int\frac{\frac{10368}{1331}+\frac{1728x}{1331}}{(1+2x)^2\sqrt{2+3x^2}}\,dx\\
&= -\frac{10-97x}{726\left(2+3x^2\right)^{3/2}}+\frac{24+887x}{7986\sqrt{2+3x^2}}-\frac{16\sqrt{2+3x^2}}{1331(1+2x)}+\frac{32\int\frac{1}{(1+2x)\sqrt{2+3x^2}}\,dx}{1331}\\
&= -\frac{10-97x}{726\left(2+3x^2\right)^{3/2}}+\frac{24+887x}{7986\sqrt{2+3x^2}}-\frac{16\sqrt{2+3x^2}}{1331(1+2x)}-\frac{32\text{Subst}\left(\int\frac{1}{11-x^2}\,dx,x,\frac{4-3x}{\sqrt{2+3x^2}}\right)}{1331}
\end{aligned}$$

$$=-\frac{10-97x}{726\left(2+3x^2\right)^{3/2}}+\frac{24+887x}{7986\sqrt{2+3x^2}}-\frac{16\sqrt{2+3x^2}}{1331(1+2x)}-\frac{32\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{2+3x^2}}\right)}{1331\sqrt{11}}$$

Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.96

$$\int\frac{1+3x+4x^2}{(1+2x)^2\left(2+3x^2\right)^{5/2}}\,dx=\frac{11(-446+2717x+4602x^2+2805x^3+4458x^4)-192\sqrt{22+33x^2}(2+4x+3}{87846(1+2x)\left(2+3x^2\right)^{3/2}}$$

```
[In] Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)^2*(2 + 3*x^2)^(5/2)),x]
```

```
[Out] (11*(-446 + 2717*x + 4602*x^2 + 2805*x^3 + 4458*x^4) - 192*Sqrt[22 + 33*x^2
]*(2 + 4*x + 3*x^2 + 6*x^3)*ArcTanh[(4 - 3*x)/Sqrt[22 + 33*x^2]])/(87846*(1
 + 2*x)*(2 + 3*x^2)^(3/2))
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.74

method	result
risch	$\frac{4458x^4+2805x^3+4602x^2+2717x-446}{7986(3x^2+2)^{\frac{3}{2}}(1+2x)}-\frac{32\sqrt{11}\,\operatorname{arctanh}\left(\frac{2(4-3x)\sqrt{11}}{11\sqrt{12\left(x+\frac{1}{2}\right)^2-12x+5}}\right)}{14641}$
trager	$\frac{4458x^4+2805x^3+4602x^2+2717x-446}{7986(3x^2+2)^{\frac{3}{2}}(1+2x)}+\frac{32\operatorname{RootOf}\left(\_Z^2-11\right)\ln\left(\frac{3\operatorname{RootOf}\left(\_Z^2-11\right)x+11\sqrt{3x^2+2}-4\operatorname{RootOf}\left(\_Z^2-11\right)}{1+2x}\right)}{14641}$
default	$\frac{x}{6(3x^2+2)^{\frac{3}{2}}}+\frac{x}{6\sqrt{3x^2+2}}-\frac{1}{22\left(x+\frac{1}{2}\right)\left(3\left(x+\frac{1}{2}\right)^2-3x+\frac{5}{4}\right)^{\frac{3}{2}}}+\frac{4}{363\left(3\left(x+\frac{1}{2}\right)^2-3x+\frac{5}{4}\right)^{\frac{3}{2}}}-\frac{10x}{121\left(3\left(x+\frac{1}{2}\right)^2-3x+\frac{5}{4}\right)^{\frac{3}{2}}}-\frac{}{1331\sqrt{}}$

```
[In] int((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2+2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/7986*(4458*x^4+2805*x^3+4602*x^2+2717*x-446)/(3*x^2+2)^(3/2)/(1+2*x)-32/1
4641*11^(1/2)*arctanh(2/11*(4-3*x)*11^(1/2)/(12*(x+1/2)^2-12*x+5)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.41

$$\int \frac{1+3x+4x^2}{(1+2x)^2\left(2+3x^2\right)^{5/2}}\,dx=\frac{96\sqrt{11}(18\,x^5+9\,x^4+24\,x^3+12\,x^2+8\,x+4)\log\left(-\frac{\sqrt{11}\sqrt{3\,x^2+2}(3\,x-4)+21\,x^2-1}{4\,x^2+4\,x+1}\right.}{87846\left(18\,x^5+9\,x^4+24\,x^3\right.}$$

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2+2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/87846*(96*sqrt(11)*(18*x^5 + 9*x^4 + 24*x^3 + 12*x^2 + 8*x + 4)*log(-(sqr
t(11)*sqrt(3*x^2 + 2)*(3*x - 4) + 21*x^2 - 12*x + 19)/(4*x^2 + 4*x + 1)) +
11*(4458*x^4 + 2805*x^3 + 4602*x^2 + 2717*x - 446)*sqrt(3*x^2 + 2))/(18*x^5
 + 9*x^4 + 24*x^3 + 12*x^2 + 8*x + 4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1+3x+4x^2}{(1+2x)^2\left(2+3x^2\right)^{5/2}}\,dx=\text{Timed out}$$

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)**2/(3*x**2+2)**(5/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.13

$$\begin{aligned}\int \frac{1+3x+4x^2}{(1+2x)^2\left(2+3x^2\right)^{5/2}}\,dx&=\frac{32}{14641}\sqrt{11}\,\mathrm{arsinh}\left(\frac{\sqrt{6}x}{2\,|2\,x+1|}-\frac{2\,\sqrt{6}}{3\,|2\,x+1|}\right)\\&\quad+\frac{743\,x}{7986\,\sqrt{3\,x^2+2}}+\frac{16}{1331\,\sqrt{3\,x^2+2}}+\frac{61\,x}{726\,(3\,x^2+2)^{\frac{3}{2}}}\\&\quad-\frac{1}{11\,\left(2\,(3\,x^2+2)^{\frac{3}{2}}x+(3\,x^2+2)^{\frac{3}{2}}\right)}+\frac{4}{363\,(3\,x^2+2)^{\frac{3}{2}}}\end{aligned}$$

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 32/14641*sqrt(11)*arcsinh(1/2*sqrt(6)*x/abs(2*x + 1) - 2/3*sqrt(6)/abs(2*x
+ 1)) + 743/7986*x/sqrt(3*x^2 + 2) + 16/1331/sqrt(3*x^2 + 2) + 61/726*x/(3*
x^2 + 2)^(3/2) - 1/11/(2*(3*x^2 + 2)^(3/2)*x + (3*x^2 + 2)^(3/2)) + 4/363/(
3*x^2 + 2)^(3/2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(76) = 152$.

Time = 0.30 (sec) , antiderivative size = 233, normalized size of antiderivative = 2.45

$$\int \frac{1+3x+4x^2}{(1+2x)^2\left(2+3x^2\right)^{5/2}}\,dx =$$

$$-\frac{1}{263538}\sqrt{11}\left(743\sqrt{11}\sqrt{3}-576\log\left(\sqrt{11}\sqrt{3}-3\right)\right)\operatorname{sgn}\left(\frac{1}{2x+1}\right)$$

$$-\frac{32\sqrt{11}\log\left(\sqrt{11}\left(\sqrt{-\frac{6}{2x+1}+\frac{11}{(2x+1)^2}+3}+\frac{\sqrt{11}}{2x+1}\right)-3\right)}{14641\operatorname{sgn}\left(\frac{1}{2x+1}\right)}$$

$$+\frac{\frac{\frac{\frac{11\left(\frac{731}{\operatorname{sgn}\left(\frac{1}{2x+1}\right)}+\frac{528}{(2x+1)\operatorname{sgn}\left(\frac{1}{2x+1}\right)}\right)}{2x+1}-\frac{14163}{\operatorname{sgn}\left(\frac{1}{2x+1}\right)}}{2x+1}+\frac{6111}{\operatorname{sgn}\left(\frac{1}{2x+1}\right)}}{2x+1}-\frac{2229}{\operatorname{sgn}\left(\frac{1}{2x+1}\right)}}{7986\left(\frac{6}{2x+1}-\frac{11}{(2x+1)^2}-3\right)\sqrt{-\frac{6}{2x+1}+\frac{11}{(2x+1)^2}+3}}$$

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2+2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/263538*sqrt(11)*(743*sqrt(11)*sqrt(3) - 576*log(sqrt(11)*sqrt(3) - 3))*s
gn(1/(2*x + 1)) - 32/14641*sqrt(11)*log(sqrt(11)*(sqrt(-6/(2*x + 1) + 11/(2
*x + 1)^2 + 3) + sqrt(11)/(2*x + 1)) - 3)/sgn(1/(2*x + 1)) + 1/7986*(((11*(
731/sgn(1/(2*x + 1)) + 528/((2*x + 1)*sgn(1/(2*x + 1))))/(2*x + 1) - 14163/
sgn(1/(2*x + 1)))/(2*x + 1) + 6111/sgn(1/(2*x + 1)))/(2*x + 1) - 2229/sgn(1
/(2*x + 1)))/((6/(2*x + 1) - 11/(2*x + 1)^2 - 3)*sqrt(-6/(2*x + 1) + 11/(2*
x + 1)^2 + 3))
```

Mupad [B] (verification not implemented)

Time = 13.07 (sec) , antiderivative size = 270, normalized size of antiderivative = 2.84

$$\int \frac{1+3x+4x^2}{(1+2x)^2\left(2+3x^2\right)^{5/2}}\,dx = \frac{\sqrt{11}\left(8\,\ln\left(x+\frac{1}{2}\right)-8\,\ln\left(x-\frac{\sqrt{3}\,\sqrt{11}\,\sqrt{x^2+\frac{2}{3}}}{3}-\frac{4}{3}\right)\right)}{14641}$$

$$+\frac{\sqrt{11}\left(\frac{48\,\ln\left(x+\frac{1}{2}\right)}{1331}-\frac{48\,\ln\left(x-\frac{\sqrt{3}\,\sqrt{11}\,\sqrt{x^2+\frac{2}{3}}}{3}-\frac{4}{3}\right)}{1331}\right)}{22}-\frac{8\,\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}}{1331\,\left(x+\frac{1}{2}\right)}$$

$$-\frac{\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}\left(\frac{-\frac{291}{1936}+\frac{\sqrt{6}\,15\mathrm{i}}{1936}}{x+\frac{\sqrt{6}\,1\mathrm{i}}{3}}+\frac{\sqrt{6}\,\left(-\frac{97}{968}+\frac{\sqrt{6}\,5\mathrm{i}}{968}\right)\,1\mathrm{i}}{2\,\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)^2}\right)}{27}$$

$$+\frac{\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}\left(\frac{\frac{291}{1936}+\frac{\sqrt{6}\,15\mathrm{i}}{1936}}{x-\frac{\sqrt{6}\,1\mathrm{i}}{3}}-\frac{\sqrt{6}\,\left(\frac{97}{968}+\frac{\sqrt{6}\,5\mathrm{i}}{968}\right)\,1\mathrm{i}}{2\,\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)^2}\right)}{27}$$

$$-\frac{\sqrt{3}\,\sqrt{6}\,\left(-288+\sqrt{6}\,2481\mathrm{i}\right)\,\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{1149984\,\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}-\frac{\sqrt{3}\,\sqrt{6}\,\left(288+\sqrt{6}\,2481\mathrm{i}\right)\,\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{1149984\,\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}$$

```
[In] int((3*x + 4*x^2 + 1)/((2*x + 1)^2*(3*x^2 + 2)^(5/2)),x)
```

```
[Out] (11^(1/2)*(8*log(x + 1/2) - 8*log(x - (3^(1/2)*11^(1/2)*(x^2 + 2/3)^(1/2))/
3 - 4/3)))/14641 + (11^(1/2)*((48*log(x + 1/2))/1331 - (48*log(x - (3^(1/2)
*11^(1/2)*(x^2 + 2/3)^(1/2))/3 - 4/3))/1331))/22 - (8*3^(1/2)*(x^2 + 2/3)^(
1/2))/(1331*(x + 1/2)) - (3^(1/2)*(x^2 + 2/3)^(1/2)*(((6^(1/2)*15i)/1936 -
291/1936)/(x + (6^(1/2)*1i)/3) + (6^(1/2)*((6^(1/2)*5i)/968 - 97/968)*1i)/(
2*(x + (6^(1/2)*1i)/3)^2)))/27 + (3^(1/2)*(x^2 + 2/3)^(1/2)*(((6^(1/2)*15i)
/1936 + 291/1936)/(x - (6^(1/2)*1i)/3) - (6^(1/2)*((6^(1/2)*5i)/968 + 97/96
8)*1i)/(2*(x - (6^(1/2)*1i)/3)^2)))/27 - (3^(1/2)*6^(1/2)*(6^(1/2)*2481i -
288)*(x^2 + 2/3)^(1/2)*1i)/(1149984*(x + (6^(1/2)*1i)/3)) - (3^(1/2)*6^(1/2
)*(6^(1/2)*2481i + 288)*(x^2 + 2/3)^(1/2)*1i)/(1149984*(x - (6^(1/2)*1i)/3)
)
```

3.135 $\int \frac{1+3x+4x^2}{(1+2x)^3\left(2+3x^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 117

$$\int \frac{1+3x+4x^2}{(1+2x)^3\left(2+3x^2\right)^{5/2}}\,dx = \frac{358+351x}{7986\left(2+3x^2\right)^{3/2}}+\frac{1216+2133x}{29282\sqrt{2+3x^2}} - \frac{8\sqrt{2+3x^2}}{1331(1+2x)^2}-\frac{8\sqrt{2+3x^2}}{1331(1+2x)}-\frac{1216\operatorname{arctanh}\left(\frac{4-3x}{\sqrt{11}\sqrt{2+3x^2}}\right)}{14641\sqrt{11}}$$

```
[Out] 1/7986*(358+351*x)/(3*x^2+2)^(3/2)-1216/161051*arctanh(1/11*(4-3*x)*11^(1/2
)/(3*x^2+2)^(1/2))*11^(1/2)+1/29282*(1216+2133*x)/(3*x^2+2)^(1/2)-8/1331*(3
*x^2+2)^(1/2)/(1+2*x)^2-8/1331*(3*x^2+2)^(1/2)/(1+2*x)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1661, 1665, 821, 739, 212}

$$\int \frac{1+3x+4x^2}{(1+2x)^3\left(2+3x^2\right)^{5/2}}\,dx = -\frac{1216\operatorname{arctanh}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{14641\sqrt{11}} + \frac{351x+358}{7986\left(3x^2+2\right)^{3/2}}-\frac{8\sqrt{3x^2+2}}{1331(2x+1)}-\frac{8\sqrt{3x^2+2}}{1331(2x+1)^2}+\frac{2133x+1216}{29282\sqrt{3x^2+2}}$$

```
[In] Int[(1 + 3*x + 4*x^2)/((1 + 2*x)^3*(2 + 3*x^2)^(5/2)),x]
```

```
[Out] (358 + 351*x)/(7986*(2 + 3*x^2)^(3/2)) + (1216 + 2133*x)/(29282*Sqrt[2 + 3*
x^2]) - (8*Sqrt[2 + 3*x^2])/(1331*(1 + 2*x)^2) - (8*Sqrt[2 + 3*x^2])/(1331*
```

```
(1 + 2*x)) - (1216*ArcTanh[(4 - 3*x)/(Sqrt[11]*Sqrt[2 + 3*x^2])])/(14641*Sq
rt[11])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{358+351x}{7986\left(2+3x^2\right)^{3/2}} - \frac{1}{18}\int \frac{-\frac{10926}{1331}-\frac{3132x}{121}-\frac{51048x^2}{1331}-\frac{16848x^3}{1331}}{(1+2x)^3\left(2+3x^2\right)^{3/2}}\,dx \\
&= \frac{358+351x}{7986\left(2+3x^2\right)^{3/2}} + \frac{1216+2133x}{29282\sqrt{2+3x^2}} + \frac{1}{108}\int \frac{\frac{245376}{14641}+\frac{544320x}{14641}+\frac{525312x^2}{14641}}{(1+2x)^3\sqrt{2+3x^2}}\,dx \\
&= \frac{358+351x}{7986\left(2+3x^2\right)^{3/2}} + \frac{1216+2133x}{29282\sqrt{2+3x^2}} - \frac{8\sqrt{2+3x^2}}{1331(1+2x)^2} - \frac{\int \frac{-\frac{338688}{1331}-\frac{468288x}{1331}}{(1+2x)^2\sqrt{2+3x^2}}\,dx}{2376} \\
&= \frac{358+351x}{7986\left(2+3x^2\right)^{3/2}} + \frac{1216+2133x}{29282\sqrt{2+3x^2}} - \frac{8\sqrt{2+3x^2}}{1331(1+2x)^2} \\
&\quad - \frac{8\sqrt{2+3x^2}}{1331(1+2x)} + \frac{1216\int \frac{1}{(1+2x)\sqrt{2+3x^2}}\,dx}{14641} \\
&= \frac{358+351x}{7986\left(2+3x^2\right)^{3/2}} + \frac{1216+2133x}{29282\sqrt{2+3x^2}} - \frac{8\sqrt{2+3x^2}}{1331(1+2x)^2} \\
&\quad - \frac{8\sqrt{2+3x^2}}{1331(1+2x)} - \frac{1216\text{Subst}\left(\int \frac{1}{11-x^2}\,dx, x, \frac{4-3x}{\sqrt{2+3x^2}}\right)}{14641} \\
&= \frac{358+351x}{7986\left(2+3x^2\right)^{3/2}} + \frac{1216+2133x}{29282\sqrt{2+3x^2}} - \frac{8\sqrt{2+3x^2}}{1331(1+2x)^2} \\
&\quad - \frac{8\sqrt{2+3x^2}}{1331(1+2x)} - \frac{1216\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{2+3x^2}}\right)}{14641\sqrt{11}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.78

$$\int \frac{1+3x+4x^2}{(1+2x)^3\left(2+3x^2\right)^{5/2}}\,dx = \frac{\frac{11\left(7010+57371x+109844x^2+116937x^3+111060x^4+67284x^5\right)}{(1+2x)^2\left(2+3x^2\right)^{3/2}} + 14592\sqrt{11}\operatorname{arctanh}\left(\frac{\sqrt{3}+2\sqrt{3}x-}{\sqrt{11}}\right.}{966306}$$

```
[In] Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)^3*(2 + 3*x^2)^(5/2)),x]
```

```
[Out] ((11*(7010 + 57371*x + 109844*x^2 + 116937*x^3 + 111060*x^4 + 67284*x^5))/(
(1 + 2*x)^2*(2 + 3*x^2)^(3/2)) + 14592*Sqrt[11]*ArcTanh[(Sqrt[3] + 2*Sqrt[3
]*x - 2*Sqrt[2 + 3*x^2])/Sqrt[11]])/966306
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.64

method	result
risch	$\frac{67284x^5+111060x^4+116937x^3+109844x^2+57371x+7010}{87846(3x^2+2)^{\frac{3}{2}}(1+2x)^2}-\frac{1216\sqrt{11}\,\operatorname{arctanh}\left(\frac{2(4-3x)\sqrt{11}}{11\sqrt{12\left(x+\frac{1}{2}\right)^2-12x+5}}\right)}{161051}$
trager	$\frac{(67284x^5+111060x^4+116937x^3+109844x^2+57371x+7010)\sqrt{3x^2+2}}{87846(6x^3+3x^2+4x+2)^2}+\frac{1216\,\mathrm{RootOf}\left(\_Z^2-11\right)\ln\left(\frac{3\,\mathrm{RootOf}\left(\_Z^2-11\right)x+11\sqrt{3}}{1+2}\right.}{161051}$
default	$\frac{152}{3993\left(3\left(x+\frac{1}{2}\right)^2-3x+\frac{5}{4}\right)^{\frac{3}{2}}}+\frac{87x}{2662\left(3\left(x+\frac{1}{2}\right)^2-3x+\frac{5}{4}\right)^{\frac{3}{2}}}+\frac{1869x}{29282\sqrt{3\left(x+\frac{1}{2}\right)^2-3x+\frac{5}{4}}}+\frac{608}{14641\sqrt{3\left(x+\frac{1}{2}\right)^2-3x+\frac{5}{4}}}-\frac{1216\sqrt{11}}{}$

```
[In] int((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2+2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/87846*(67284*x^5+111060*x^4+116937*x^3+109844*x^2+57371*x+7010)/(3*x^2+2)
^(3/2)/(1+2*x)^2-1216/161051*11^(1/2)*arctanh(2/11*(4-3*x)*11^(1/2)/(12*(x+
1/2)^2-12*x+5)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.27

$$\int \frac{1+3x+4x^2}{(1+2x)^3\,(2+3x^2)^{5/2}}\,dx=\frac{3648\sqrt{11}(36\,x^6+36\,x^5+57\,x^4+48\,x^3+28\,x^2+16\,x+4)\log\left(-\frac{\sqrt{11}\sqrt{3\,x^2+2}}{}\right.}{966306\,(36\,x^6+}$$

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2+2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/966306*(3648*sqrt(11)*(36*x^6 + 36*x^5 + 57*x^4 + 48*x^3 + 28*x^2 + 16*x
+ 4)*log(-(sqrt(11)*sqrt(3*x^2 + 2)*(3*x - 4) + 21*x^2 - 12*x + 19)/(4*x^2
+ 4*x + 1)) + 11*(67284*x^5 + 111060*x^4 + 116937*x^3 + 109844*x^2 + 57371*
x + 7010)*sqrt(3*x^2 + 2))/(36*x^6 + 36*x^5 + 57*x^4 + 48*x^3 + 28*x^2 + 16
*x + 4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1+3x+4x^2}{(1+2x)^3 \left(2+3x^2\right)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)**3/(3*x**2+2)**(5/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.26

$$\begin{aligned}\int \frac{1+3x+4x^2}{(1+2x)^3 \left(2+3x^2\right)^{5/2}}\,dx &= \frac{1216}{161051}\,\sqrt{11}\,\mathrm{arsinh}\left(\frac{\sqrt{6}x}{2\,|2\,x+1|}-\frac{2\,\sqrt{6}}{3\,|2\,x+1|}\right) \\ &\quad + \frac{1869\,x}{29282\,\sqrt{3\,x^2+2}}+\frac{608}{14641\,\sqrt{3\,x^2+2}}+\frac{87\,x}{2662\,\left(3\,x^2+2\right)^{\frac{3}{2}}} \\ &\quad - \frac{1}{22\,\left(4\,\left(3\,x^2+2\right)^{\frac{3}{2}}x^2+4\,\left(3\,x^2+2\right)^{\frac{3}{2}}x+\left(3\,x^2+2\right)^{\frac{3}{2}}\right)} \\ &\quad + \frac{1}{242\,\left(2\,\left(3\,x^2+2\right)^{\frac{3}{2}}x+\left(3\,x^2+2\right)^{\frac{3}{2}}\right)}+\frac{152}{3993\,\left(3\,x^2+2\right)^{\frac{3}{2}}}\end{aligned}$$

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1216/161051*sqrt(11)*arcsinh(1/2*sqrt(6)*x/abs(2*x + 1) - 2/3*sqrt(6)/abs(2
*x + 1)) + 1869/29282*x/sqrt(3*x^2 + 2) + 608/14641/sqrt(3*x^2 + 2) + 87/26
62*x/(3*x^2 + 2)^(3/2) - 1/22/(4*(3*x^2 + 2)^(3/2)*x^2 + 4*(3*x^2 + 2)^(3/2
)*x + (3*x^2 + 2)^(3/2)) + 1/242/(2*(3*x^2 + 2)^(3/2)*x + (3*x^2 + 2)^(3/2)
) + 152/3993/(3*x^2 + 2)^(3/2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.56

$$\int \frac{1+3x+4x^2}{(1+2x)^3\left(2+3x^2\right)^{5/2}}\,dx = \frac{1216}{161051}\sqrt{11}\log\left(-\frac{\left|-2\sqrt{3}x-\sqrt{11}-\sqrt{3}+2\sqrt{3\,x^2+2}\right|}{2\sqrt{3}x-\sqrt{11}+\sqrt{3}-2\sqrt{3\,x^2+2}}\right)$$
$$+\frac{9\left((2133\,x+1216)x+1851\right)x+11234}{87846\left(3\,x^2+2\right)^{\frac{3}{2}}}$$
$$+\frac{4\left(\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2+2}\right)^2+24\sqrt{3}x-8\sqrt{3}-24\sqrt{3\,x^2+2}\right)}{1331\left(\left(\sqrt{3}x-\sqrt{3\,x^2+2}\right)^2+\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2+2}\right)-2\right)^2}$$

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2+2)^(5/2),x, algorithm="giac")
```

```
[Out] 1216/161051*sqrt(11)*log(-abs(-2*sqrt(3)*x - sqrt(11) - sqrt(3) + 2*sqrt(3*
x^2 + 2))/(2*sqrt(3)*x - sqrt(11) + sqrt(3) - 2*sqrt(3*x^2 + 2))) + 1/87846
*(9*((2133*x + 1216)*x + 1851)*x + 11234)/(3*x^2 + 2)^(3/2) + 4/1331*(sqrt(
3)*(sqrt(3)*x - sqrt(3*x^2 + 2))^2 + 24*sqrt(3)*x - 8*sqrt(3) - 24*sqrt(3*x
^2 + 2))/((sqrt(3)*x - sqrt(3*x^2 + 2))^2 + sqrt(3)*(sqrt(3)*x - sqrt(3*x^2
 + 2)) - 2)^2
```

Mupad [B] (verification not implemented)

Time = 12.67 (sec) , antiderivative size = 301, normalized size of antiderivative = 2.57

$$\int \frac{1+3x+4x^2}{(1+2x)^3\left(2+3x^2\right)^{5/2}}\,dx = \frac{1216\sqrt{11}\,\ln\left(x+\frac{1}{2}\right)}{161051}$$
$$-\frac{1216\sqrt{11}\,\ln\left(x-\frac{\sqrt{3}\sqrt{11}\sqrt{x^2+\frac{2}{3}}}{3}-\frac{4}{3}\right)}{161051}-\frac{179\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{95832\left(x^2+\frac{2\mathrm{i}\sqrt{6}\,x}{3}-\frac{2}{3}\right)}$$
$$+\frac{711\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{58564\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}+\frac{711\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{58564\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}-\frac{2\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{1331\left(x^2+x+\frac{1}{4}\right)}$$
$$+\frac{179\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{95832\left(-x^2+\frac{2\mathrm{i}\sqrt{6}\,x}{3}+\frac{2}{3}\right)}-\frac{4\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{1331\left(x+\frac{1}{2}\right)}+\frac{\sqrt{3}\sqrt{6}\sqrt{x^2+\frac{2}{3}}\,13\mathrm{i}}{21296\left(x^2+\frac{2\mathrm{i}\sqrt{6}\,x}{3}-\frac{2}{3}\right)}$$
$$-\frac{\sqrt{3}\sqrt{6}\sqrt{x^2+\frac{2}{3}}\,9265\mathrm{i}}{2108304\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}+\frac{\sqrt{3}\sqrt{6}\sqrt{x^2+\frac{2}{3}}\,9265\mathrm{i}}{2108304\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}+\frac{\sqrt{3}\sqrt{6}\sqrt{x^2+\frac{2}{3}}\,13\mathrm{i}}{21296\left(-x^2+\frac{2\mathrm{i}\sqrt{6}\,x}{3}+\frac{2}{3}\right)}$$

```
[In] int((3*x + 4*x^2 + 1)/((2*x + 1)^3*(3*x^2 + 2)^(5/2)),x)
```

```
[Out] (1216*11^(1/2)*log(x + 1/2))/161051 - (1216*11^(1/2)*log(x - (3^(1/2)*11^(1
/2)*(x^2 + 2/3)^(1/2))/3 - 4/3))/161051 - (179*3^(1/2)*(x^2 + 2/3)^(1/2))/(
95832*((6^(1/2)*x*2i)/3 + x^2 - 2/3)) + (711*3^(1/2)*(x^2 + 2/3)^(1/2))/(58
564*(x - (6^(1/2)*1i)/3)) + (711*3^(1/2)*(x^2 + 2/3)^(1/2))/(58564*(x + (6^
(1/2)*1i)/3)) - (2*3^(1/2)*(x^2 + 2/3)^(1/2))/(1331*(x + x^2 + 1/4)) + (179
*3^(1/2)*(x^2 + 2/3)^(1/2))/(95832*((6^(1/2)*x*2i)/3 - x^2 + 2/3)) - (4*3^(
1/2)*(x^2 + 2/3)^(1/2))/(1331*(x + 1/2)) + (3^(1/2)*6^(1/2)*(x^2 + 2/3)^(1/
2)*13i)/(21296*((6^(1/2)*x*2i)/3 + x^2 - 2/3)) - (3^(1/2)*6^(1/2)*(x^2 + 2/
3)^(1/2)*9265i)/(2108304*(x - (6^(1/2)*1i)/3)) + (3^(1/2)*6^(1/2)*(x^2 + 2/
3)^(1/2)*9265i)/(2108304*(x + (6^(1/2)*1i)/3)) + (3^(1/2)*6^(1/2)*(x^2 + 2/
3)^(1/2)*13i)/(21296*((6^(1/2)*x*2i)/3 - x^2 + 2/3))
```

3.136 $\int (g+hx)^m \left(a+cx^2\right)^p \left(d+ex+fx^2\right) dx$

Optimal result

Integrand size = 27, antiderivative size = 420

$$\int (g+hx)^m \left(a+cx^2\right)^p \left(d+ex+fx^2\right) dx = \frac{f(g+hx)^{1+m}\left(a+cx^2\right)^{1+p}}{ch(3+m+2p)}$$

$$- \frac{(afh^2(1+m)-c(2fg^2(1+p)-h(eg-dh)(3+m+2p)))\,(g+hx)^{1+m}\left(a+cx^2\right)^p\left(1-\frac{g+hx}{g-\frac{\sqrt{-a}h}{\sqrt{c}}}\right)^{-p}\Bigg(}{ch^3(1+m)(3+m+2p)}$$

$$- \frac{(2fg(1+p)-eh(3+m+2p))(g+hx)^{2+m}\left(a+cx^2\right)^p\left(1-\frac{g+hx}{g-\frac{\sqrt{-a}h}{\sqrt{c}}}\right)^{-p}\left(1-\frac{g+hx}{g+\frac{\sqrt{-a}h}{\sqrt{c}}}\right)^{-p}\operatorname{AppellF1}\Bigg(2}{h^3(2+m)(3+m+2p)}$$

```
[Out] f*(h*x+g)^(1+m)*(c*x^2+a)^(p+1)/c/h/(3+m+2*p)-(a*f*h^2*(1+m)-c*(2*f*g^2*(p+
1)-h*(-d*h+e*g)*(3+m+2*p)))*(h*x+g)^(1+m)*(c*x^2+a)^p*AppellF1(1+m,-p,-p,2+
m,(h*x+g)/(g-h*(-a)^(1/2)/c^(1/2)),(h*x+g)/(g+h*(-a)^(1/2)/c^(1/2)))/c/h^3/
(1+m)/(3+m+2*p)/((1+(-h*x-g)/(g-h*(-a)^(1/2)/c^(1/2)))^p)/((1+(-h*x-g)/(g+h
*(-a)^(1/2)/c^(1/2)))^p)-(2*f*g*(p+1)-e*h*(3+m+2*p))*(h*x+g)^(2+m)*(c*x^2+a
)^p*AppellF1(2+m,-p,-p,3+m,(h*x+g)/(g-h*(-a)^(1/2)/c^(1/2)),(h*x+g)/(g+h*(-
a)^(1/2)/c^(1/2)))/h^3/(2+m)/(3+m+2*p)/((1+(-h*x-g)/(g-h*(-a)^(1/2)/c^(1/2)
))^p)/((1+(-h*x-g)/(g+h*(-a)^(1/2)/c^(1/2)))^p)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 417, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1668, 858, 774, 138}

$$\int (g+hx)^m \left(a+cx^2\right)^p \left(d+ex+fx^2\right)\,dx =$$

$$-\frac{\left(a+cx^2\right)^p (g+hx)^{m+1} \left(1-\frac{g+hx}{g-\frac{\sqrt{-a}h}{\sqrt{c}}}\right)^{-p} \left(1-\frac{g+hx}{\frac{\sqrt{-a}h}{\sqrt{c}}+g}\right)^{-p} \text{AppellF1}\Big(m+1,-p,-p,m+2,\frac{g+hx}{g-\frac{\sqrt{-a}h}{\sqrt{c}}},}{ch^3(m+1)(m+2p+3)}$$

$$-\frac{\left(a+cx^2\right)^p (g+hx)^{m+2} \left(1-\frac{g+hx}{g-\frac{\sqrt{-a}h}{\sqrt{c}}}\right)^{-p} \left(1-\frac{g+hx}{\frac{\sqrt{-a}h}{\sqrt{c}}+g}\right)^{-p} (2fg(p+1)-eh(m+2p+3))\,\text{AppellF1}\Big(m}{h^3(m+2)(m+2p+3)}$$

$$+\frac{f\left(a+cx^2\right)^{p+1} (g+hx)^{m+1}}{ch(m+2p+3)}$$

```
[In] Int[(g + h*x)^m*(a + c*x^2)^p*(d + e*x + f*x^2),x]
```

```
[Out] (f*(g + h*x)^(1 + m)*(a + c*x^2)^(1 + p))/(c*h*(3 + m + 2*p)) - ((a*f*h^2*(
1 + m) - 2*c*f*g^2*(1 + p) + c*h*(e*g - d*h)*(3 + m + 2*p))*(g + h*x)^(1 +
m)*(a + c*x^2)^p*AppellF1[1 + m, -p, -p, 2 + m, (g + h*x)/(g - (Sqrt[-a]*h)
/Sqrt[c]), (g + h*x)/(g + (Sqrt[-a]*h)/Sqrt[c])])/(c*h^3*(1 + m)*(3 + m + 2
*p)*(1 - (g + h*x)/(g - (Sqrt[-a]*h)/Sqrt[c]))^p*(1 - (g + h*x)/(g + (Sqrt[
-a]*h)/Sqrt[c]))^p) - ((2*f*g*(1 + p) - e*h*(3 + m + 2*p))*(g + h*x)^(2 + m
)*(a + c*x^2)^p*AppellF1[2 + m, -p, -p, 3 + m, (g + h*x)/(g - (Sqrt[-a]*h)/
Sqrt[c]), (g + h*x)/(g + (Sqrt[-a]*h)/Sqrt[c])])/(h^3*(2 + m)*(3 + m + 2*p)
*(1 - (g + h*x)/(g - (Sqrt[-a]*h)/Sqrt[c]))^p*(1 - (g + h*x)/(g + (Sqrt[-a]
*h)/Sqrt[c]))^p)
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 774

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Dist[(a + c*x^2)^p/(e*(1 - (d + e*x)/(d + e*(q/c)))^p*
(1 - (d + e*x)/(d - e*(q/c)))^p), Subst[Int[x^m*Simp[1 - x/(d + e*(q/c)), x
]^p*Simp[1 - x/(d - e*(q/c)), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, c, d,
 e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\text{integral} = \frac{f(g+hx)^{1+m}\left(a+cx^2\right)^{1+p}}{ch(3+m+2p)} + \frac{\int (g+hx)^m\left(-h^2(af(1+m)-cd(3+m+2p))-ch(2fg(1+p)-eh(3+m+2p))x\right)\left(a+cx^2\right)^p\,dx}{ch^2(3+m+2p)}$$

$$= \frac{f(g+hx)^{1+m}\left(a+cx^2\right)^{1+p}}{ch(3+m+2p)} + \frac{\left(eh-\frac{2fg(1+p)}{3+m+2p}\right)\int (g+hx)^{1+m}\left(a+cx^2\right)^p\,dx}{h^2} - \frac{\left(afh^2(1+m)-2cfg^2(1+p)+ch(eg-dh)(3+m+2p)\right)\int (g+hx)^m\left(a+cx^2\right)^p\,dx}{ch^2(3+m+2p)}$$

$$= \frac{f(g+hx)^{1+m}\left(a+cx^2\right)^{1+p}}{ch(3+m+2p)} + \frac{\Bigg(\left(eh-\frac{2fg(1+p)}{3+m+2p}\right)\left(a+cx^2\right)^p\left(1-\frac{g+hx}{g-\frac{\sqrt{-a}h}{\sqrt{c}}}\right)^{-p}\left(1-\frac{g+hx}{g+\frac{\sqrt{-a}h}{\sqrt{c}}}\right)^{-p}\Bigg)\operatorname{Subst}\Bigg(\int x^{1+m}\Bigg(1-\frac{x}{g-\frac{\sqrt{-a}h}{\sqrt{c}}}}{h^3} - \frac{\Bigg(\left(afh^2(1+m)-2cfg^2(1+p)+ch(eg-dh)(3+m+2p)\right)\left(a+cx^2\right)^p\left(1-\frac{g+hx}{g-\frac{\sqrt{-a}h}{\sqrt{c}}}\right)^{-p}\Bigg(1-}{ch^3(3+m+2p)}$$

$$\begin{aligned}&=\frac{f(g+hx)^{1+m}\left(a+cx^2\right)^{1+p}}{ch(3+m+2p)}\\&-\frac{\left(afh^2(1+m)-2cfg^2(1+p)+ch(eg-dh)(3+m+2p)\right)(g+hx)^{1+m}\left(a+cx^2\right)^p\left(1-\frac{g+hx}{g-\frac{\sqrt{-a}h}{\sqrt{c}}}\right.}{ch^3(1+m)(3+m+2p)}\\&+\frac{\left(eh-\frac{2fg(1+p)}{3+m+2p}\right)(g+hx)^{2+m}\left(a+cx^2\right)^p\left(1-\frac{g+hx}{g-\frac{\sqrt{-a}h}{\sqrt{c}}}\right)^{-p}\left(1-\frac{g+hx}{g+\frac{\sqrt{-a}h}{\sqrt{c}}}\right)^{-p}F_1\left(2+m;-p,-p;3\right.}{h^3(2+m)}\end{aligned}$$

Mathematica [F]

$$\int (g+hx)^m\left(a+cx^2\right)^p\left(d+ex+fx^2\right)\,dx=\int (g+hx)^m\left(a+cx^2\right)^p\left(d+ex+fx^2\right)\,dx$$

```
[In] Integrate[(g + h*x)^m*(a + c*x^2)^p*(d + e*x + f*x^2),x]
[Out] Integrate[(g + h*x)^m*(a + c*x^2)^p*(d + e*x + f*x^2), x]
```

Maple [F]

$$\int (hx+g)^m\left(c\,x^2+a\right)^p\left(f\,x^2+ex+d\right)dx$$

```
[In] int((h*x+g)^m*(c*x^2+a)^p*(f*x^2+e*x+d),x)
[Out] int((h*x+g)^m*(c*x^2+a)^p*(f*x^2+e*x+d),x)
```

Fricas [F]

$$\int (g+hx)^m\left(a+cx^2\right)^p\left(d+ex+fx^2\right)\,dx=\int\left(fx^2+ex+d\right)\left(cx^2+a\right)^p(hx+g)^m\,dx$$

```
[In] integrate((h*x+g)^m*(c*x^2+a)^p*(f*x^2+e*x+d),x, algorithm="fricas")
[Out] integral((f*x^2 + e*x + d)*(c*x^2 + a)^p*(h*x + g)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (g+hx)^m \left(a+cx^2\right)^p \left(d+ex+fx^2\right)\,dx=\text{Timed out}$$

```
[In] integrate((h*x+g)**m*(c*x**2+a)**p*(f*x**2+e*x+d),x)
[Out] Timed out
```

Maxima [F]

$$\int (g+hx)^m \left(a+cx^2\right)^p \left(d+ex+fx^2\right)\,dx=\int \left(fx^2+ex+d\right)\left(cx^2+a\right)^p(hx+g)^m\,dx$$

```
[In] integrate((h*x+g)^m*(c*x^2+a)^p*(f*x^2+e*x+d),x, algorithm="maxima")
[Out] integrate((f*x^2 + e*x + d)*(c*x^2 + a)^p*(h*x + g)^m, x)
```

Giac [F]

$$\int (g+hx)^m \left(a+cx^2\right)^p \left(d+ex+fx^2\right)\,dx=\int \left(fx^2+ex+d\right)\left(cx^2+a\right)^p(hx+g)^m\,dx$$

```
[In] integrate((h*x+g)^m*(c*x^2+a)^p*(f*x^2+e*x+d),x, algorithm="giac")
[Out] integrate((f*x^2 + e*x + d)*(c*x^2 + a)^p*(h*x + g)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (g+hx)^m \left(a+cx^2\right)^p \left(d+ex+fx^2\right)\,dx=\int {\left(g+h\,x\right)}^m\,{\left(c\,x^2+a\right)}^p\,\left(f\,x^2+e\,x+d\right)\,dx$$

```
[In] int((g + h*x)^m*(a + c*x^2)^p*(d + e*x + f*x^2),x)
[Out] int((g + h*x)^m*(a + c*x^2)^p*(d + e*x + f*x^2), x)
```

3.137 $\int (g+hx)^m\sqrt{a+cx^2}(d+ex+fx^2)\,dx$

Optimal result

Integrand size = 29, antiderivative size = 403

$$\int (g+hx)^m\sqrt{a+cx^2}\left(d+ex+fx^2\right)\,dx=\frac{f(g+hx)^{1+m}\left(a+cx^2\right)^{3/2}}{ch(4+m)}$$

$$-\frac{(afh^2(1+m)-c(3fg^2-h(eg-dh)(4+m)))\,(g+hx)^{1+m}\sqrt{a+cx^2}\,\text{AppellF1}\left(1+m,-\frac{1}{2},-\frac{1}{2},2+m,\right.}{ch^3(1+m)(4+m)\sqrt{1-\frac{g+hx}{g-\frac{\sqrt{-a}h}{\sqrt{c}}}}\sqrt{1-\frac{g+hx}{g+\frac{\sqrt{-a}h}{\sqrt{c}}}}}$$

$$-\frac{(3fg-eh(4+m))(g+hx)^{2+m}\sqrt{a+cx^2}\,\text{AppellF1}\left(2+m,-\frac{1}{2},-\frac{1}{2},3+m,\frac{g+hx}{g-\frac{\sqrt{-a}h}{\sqrt{c}}},\frac{g+hx}{g+\frac{\sqrt{-a}h}{\sqrt{c}}}\right)}{h^3(2+m)(4+m)\sqrt{1-\frac{g+hx}{g-\frac{\sqrt{-a}h}{\sqrt{c}}}}\sqrt{1-\frac{g+hx}{g+\frac{\sqrt{-a}h}{\sqrt{c}}}}}$$

```
[Out] f*(h*x+g)^(1+m)*(c*x^2+a)^(3/2)/c/h/(4+m)-(a*f*h^2*(1+m)-c*(3*f*g^2-h*(-d*h
+e*g)*(4+m)))*(h*x+g)^(1+m)*AppellF1(1+m,-1/2,-1/2,2+m,(h*x+g)/(g-h*(-a)^(1
/2)/c^(1/2)),(h*x+g)/(g+h*(-a)^(1/2)/c^(1/2)))*(c*x^2+a)^(1/2)/c/h^3/(1+m)/
(4+m)/(1+(-h*x-g)/(g-h*(-a)^(1/2)/c^(1/2)))^(1/2)/(1+(-h*x-g)/(g+h*(-a)^(1/
2)/c^(1/2)))^(1/2)-(3*f*g-e*h*(4+m))*(h*x+g)^(2+m)*AppellF1(2+m,-1/2,-1/2,3
+m,(h*x+g)/(g-h*(-a)^(1/2)/c^(1/2)),(h*x+g)/(g+h*(-a)^(1/2)/c^(1/2)))*(c*x^
2+a)^(1/2)/h^3/(2+m)/(4+m)/(1+(-h*x-g)/(g-h*(-a)^(1/2)/c^(1/2)))^(1/2)/(1+(
-h*x-g)/(g+h*(-a)^(1/2)/c^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 401, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1668, 858, 774, 138}

$$\int (g+hx)^m\sqrt{a+cx^2}\left(d+ex+fx^2\right)\,dx$$

$$= \frac{\sqrt{a+cx^2}(g+hx)^{m+1}\operatorname{AppellF1}\left(m+1,-\frac{1}{2},-\frac{1}{2},m+2,\frac{g+hx}{g-\frac{\sqrt{-a}h}{\sqrt{c}}},\frac{g+hx}{g+\frac{\sqrt{-a}h}{\sqrt{c}}}\right)\left(-afh^2(m+1)-ch(m+4)(\right.}{ch^3(m+1)(m+4)\sqrt{1-\frac{g+hx}{g-\frac{\sqrt{-a}h}{\sqrt{c}}}}\sqrt{1-\frac{g+hx}{\frac{\sqrt{-a}h}{\sqrt{c}}+g}}}$$

$$-\frac{\sqrt{a+cx^2}(g+hx)^{m+2}(3fg-eh(m+4))\operatorname{AppellF1}\left(m+2,-\frac{1}{2},-\frac{1}{2},m+3,\frac{g+hx}{g-\frac{\sqrt{-a}h}{\sqrt{c}}},\frac{g+hx}{g+\frac{\sqrt{-a}h}{\sqrt{c}}}\right)}{h^3(m+2)(m+4)\sqrt{1-\frac{g+hx}{g-\frac{\sqrt{-a}h}{\sqrt{c}}}}\sqrt{1-\frac{g+hx}{\frac{\sqrt{-a}h}{\sqrt{c}}+g}}}$$

$$+\frac{f\left(a+cx^2\right)^{3/2}(g+hx)^{m+1}}{ch(m+4)}$$

```
[In] Int[(g + h*x)^m*Sqrt[a + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] (f*(g + h*x)^(1 + m)*(a + c*x^2)^(3/2))/(c*h*(4 + m)) + ((3*c*f*g^2 - a*f*h
^2*(1 + m) - c*h*(e*g - d*h)*(4 + m))*(g + h*x)^(1 + m)*Sqrt[a + c*x^2]*App
ellF1[1 + m, -1/2, -1/2, 2 + m, (g + h*x)/(g - (Sqrt[-a]*h)/Sqrt[c]), (g +
h*x)/(g + (Sqrt[-a]*h)/Sqrt[c])])/(c*h^3*(1 + m)*(4 + m)*Sqrt[1 - (g + h*x)
/(g - (Sqrt[-a]*h)/Sqrt[c])]*Sqrt[1 - (g + h*x)/(g + (Sqrt[-a]*h)/Sqrt[c])]
) - ((3*f*g - e*h*(4 + m))*(g + h*x)^(2 + m)*Sqrt[a + c*x^2]*AppellF1[2 + m
, -1/2, -1/2, 3 + m, (g + h*x)/(g - (Sqrt[-a]*h)/Sqrt[c]), (g + h*x)/(g + (
Sqrt[-a]*h)/Sqrt[c])])/(h^3*(2 + m)*(4 + m)*Sqrt[1 - (g + h*x)/(g - (Sqrt[-
a]*h)/Sqrt[c])]*Sqrt[1 - (g + h*x)/(g + (Sqrt[-a]*h)/Sqrt[c])])
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 774

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Dist[(a + c*x^2)^p/(e*(1 - (d + e*x)/(d + e*(q/c)))^p*
(1 - (d + e*x)/(d - e*(q/c)))^p), Subst[Int[x^m*Simp[1 - x/(d + e*(q/c)), x
]^p*Simp[1 - x/(d - e*(q/c)), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, c, d,
 e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

integral

$$= \frac{f(g+hx)^{1+m}\left(a+cx^2\right)^{3/2}}{ch(4+m)} + \frac{\int (g+hx)^m\left(-h^2(af(1+m)-cd(4+m))-ch(3fg-eh(4+m))x\right)\sqrt{a+cx^2}\,dx}{ch^2(4+m)}$$

$$= \frac{f(g+hx)^{1+m}\left(a+cx^2\right)^{3/2}}{ch(4+m)} - \frac{(3fg-eh(4+m))\int (g+hx)^{1+m}\sqrt{a+cx^2}\,dx}{h^2(4+m)} + \frac{\left(3cfg^2-afh^2(1+m)-ch(eg-dh)(4+m)\right)\int (g+hx)^m\sqrt{a+cx^2}\,dx}{ch^2(4+m)}$$

$$= \frac{f(g+hx)^{1+m}\left(a+cx^2\right)^{3/2}}{ch(4+m)} - \frac{\left((3fg-eh(4+m))\sqrt{a+cx^2}\right)\text{Subst}\left(\int x^{1+m}\sqrt{1-\frac{x}{g-\frac{\sqrt{-a}h}{\sqrt{c}}}}\sqrt{1-\frac{x}{g+\frac{\sqrt{-a}h}{\sqrt{c}}}}\,dx,x,g+hx\right)}{h^3(4+m)\sqrt{1-\frac{g+hx}{g-\frac{\sqrt{-a}h}{\sqrt{c}}}}\sqrt{1-\frac{g+hx}{g+\frac{\sqrt{-a}h}{\sqrt{c}}}}} + \frac{\left(\left(3cfg^2-afh^2(1+m)-ch(eg-dh)(4+m)\right)\sqrt{a+cx^2}\right)\text{Subst}\left(\int x^m\sqrt{1-\frac{x}{g-\frac{\sqrt{-a}h}{\sqrt{c}}}}\sqrt{1-\frac{x}{g+\frac{\sqrt{-a}h}{\sqrt{c}}}}\,dx,x\right.}{ch^3(4+m)\sqrt{1-\frac{g+hx}{g-\frac{\sqrt{-a}h}{\sqrt{c}}}}\sqrt{1-\frac{g+hx}{g+\frac{\sqrt{-a}h}{\sqrt{c}}}}}$$

$$= \frac{f(g+hx)^{1+m}\left(a+cx^2\right)^{3/2}}{ch(4+m)} + \frac{\left(3cfg^2-afh^2(1+m)-ch(eg-dh)(4+m)\right)(g+hx)^{1+m}\sqrt{a+cx^2}F_1\left(1+m;-\frac{1}{2},-\frac{1}{2};2+m;\frac{g+hx}{g-\frac{\sqrt{-a}h}{\sqrt{c}}}\right.}{ch^3(1+m)(4+m)\sqrt{1-\frac{g+hx}{g-\frac{\sqrt{-a}h}{\sqrt{c}}}}\sqrt{1-\frac{g+hx}{g+\frac{\sqrt{-a}h}{\sqrt{c}}}}} - \frac{(3fg-eh(4+m))(g+hx)^{2+m}\sqrt{a+cx^2}F_1\left(2+m;-\frac{1}{2},-\frac{1}{2};3+m;\frac{g+hx}{g-\frac{\sqrt{-a}h}{\sqrt{c}}},\frac{g+hx}{g+\frac{\sqrt{-a}h}{\sqrt{c}}}\right)}{h^3(2+m)(4+m)\sqrt{1-\frac{g+hx}{g-\frac{\sqrt{-a}h}{\sqrt{c}}}}\sqrt{1-\frac{g+hx}{g+\frac{\sqrt{-a}h}{\sqrt{c}}}}}$$

Mathematica [F]

$$\int (g+hx)^m\sqrt{a+cx^2}\left(d+ex+fx^2\right)\,dx = \int (g+hx)^m\sqrt{a+cx^2}\left(d+ex+fx^2\right)\,dx$$

```
[In] Integrate[(g + h*x)^m*Sqrt[a + c*x^2]*(d + e*x + f*x^2),x]
[Out] Integrate[(g + h*x)^m*Sqrt[a + c*x^2]*(d + e*x + f*x^2), x]
```

Maple [F]

$$\int (hx+g)^m \left(f\,x^2+ex+d\right)\sqrt{c\,x^2+a}dx$$

```
[In] int((h*x+g)^m*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x)
[Out] int((h*x+g)^m*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x)
```

Fricas [F]

$$\int (g+hx)^m\sqrt{a+cx^2}\left(d+ex+fx^2\right)\,dx = \int \sqrt{cx^2+a}\left(fx^2+ex+d\right)(hx+g)^m\,dx$$

```
[In] integrate((h*x+g)^m*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(c*x^2 + a)*(f*x^2 + e*x + d)*(h*x + g)^m, x)
```

Sympy [F]

$$\int (g+hx)^m\sqrt{a+cx^2}\left(d+ex+fx^2\right)\,dx=\int \sqrt{a+cx^2}(g+hx)^m\left(d+ex+fx^2\right)\,dx$$

```
[In] integrate((h*x+g)**m*(f*x**2+e*x+d)*(c*x**2+a)**(1/2),x)
[Out] Integral(sqrt(a + c*x**2)*(g + h*x)**m*(d + e*x + f*x**2), x)
```

Maxima [F]

$$\int (g+hx)^m\sqrt{a+cx^2}\left(d+ex+fx^2\right)\,dx=\int \sqrt{cx^2+a}\left(fx^2+ex+d\right)(hx+g)^m\,dx$$

```
[In] integrate((h*x+g)^m*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^2 + a)*(f*x^2 + e*x + d)*(h*x + g)^m, x)
```

Giac [F]

$$\int (g+hx)^m\sqrt{a+cx^2}\left(d+ex+fx^2\right)\,dx=\int \sqrt{cx^2+a}\left(fx^2+ex+d\right)(hx+g)^m\,dx$$

```
[In] integrate((h*x+g)^m*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^2 + a)*(f*x^2 + e*x + d)*(h*x + g)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (g+hx)^m\sqrt{a+cx^2}\left(d+ex+fx^2\right)\,dx=\int (g+h\,x)^m\,\sqrt{c\,x^2+a}\,\left(f\,x^2+e\,x+d\right)\,dx$$

```
[In] int((g + h*x)^m*(a + c*x^2)^(1/2)*(d + e*x + f*x^2),x)
[Out] int((g + h*x)^m*(a + c*x^2)^(1/2)*(d + e*x + f*x^2), x)
```

3.138 $\int (g+hx)^{-3-2p}\left(a+cx^2\right)^p\left(d+ex+fx^2\right)\,dx$

Optimal result

Integrand size = 31, antiderivative size = 474

$$\int (g+hx)^{-3-2p}\left(a+cx^2\right)^p\left(d+ex+fx^2\right)\,dx$$

$$= -\frac{\left(fg^2-egh+dh^2\right)(g+hx)^{-2(1+p)}\left(a+cx^2\right)^{1+p}}{2h\left(cg^2+ah^2\right)(1+p)}$$

$$-\frac{f(g+hx)^{-2p}\left(a+cx^2\right)^p\left(1-\frac{g+hx}{g-\frac{\sqrt{-a}h}{\sqrt{c}}}\right)^{-p}\left(1-\frac{g+hx}{g+\frac{\sqrt{-a}h}{\sqrt{c}}}\right)^{-p}\operatorname{AppellF1}\Bigg(-2p,-p,-p,1-2p,\frac{g+hx}{g-\frac{\sqrt{-a}h}{\sqrt{c}}},}{2h^3p}$$

$$+\frac{\left(ah^2(2fg-eh)+c\left(fg^3-dgh^2\right)\right)\left(\sqrt{-a}-\sqrt{c}x\right)\left(-\frac{\left(\sqrt{c}g+\sqrt{-a}h\right)\left(\sqrt{-a}+\sqrt{c}x\right)}{\left(\sqrt{c}g-\sqrt{-a}h\right)\left(\sqrt{-a}-\sqrt{c}x\right)}\right)^{-p}(g+hx)^{-1-2p}\left(a+cx^2\right)^p}{h^2\left(\sqrt{c}g+\sqrt{-a}h\right)\left(cg^2+ah^2\right)(1+2p)}$$

```
[Out] -1/2*(d*h^2-e*g*h+f*g^2)*(c*x^2+a)^(p+1)/h/(a*h^2+c*g^2)/(p+1)/((h*x+g)^(2+
2*p))-1/2*f*(c*x^2+a)^p*AppellF1(-2*p,-p,-p,1-2*p,(h*x+g)/(g-h*(-a)^(1/2)/c
^(1/2)),(h*x+g)/(g+h*(-a)^(1/2)/c^(1/2)))/h^3/p/((h*x+g)^(2*p))/((1+(-h*x-g
)/(g-h*(-a)^(1/2)/c^(1/2)))^p)/((1+(-h*x-g)/(g+h*(-a)^(1/2)/c^(1/2)))^p)+(a
*h^2*(-e*h+2*f*g)+c*(-d*g*h^2+f*g^3))*(h*x+g)^(-1-2*p)*(c*x^2+a)^p*hypergeo
m([-p, -1-2*p],[-2*p],2*(h*x+g)*(-a)^(1/2)*c^(1/2)/(-h*(-a)^(1/2)+g*c^(1/2)
)/((-a)^(1/2)-x*c^(1/2)))*((-a)^(1/2)-x*c^(1/2))/h^2/(a*h^2+c*g^2)/(1+2*p)/
(h*(-a)^(1/2)+g*c^(1/2))/((-(h*(-a)^(1/2)+g*c^(1/2))*((-a)^(1/2)+x*c^(1/2))
/(-h*(-a)^(1/2)+g*c^(1/2))/((-a)^(1/2)-x*c^(1/2)))^p)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 474, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {1670, 774, 138, 821, 741}

$$\int (g+hx)^{-3-2p}\left(a+cx^2\right)^p\left(d+ex+fx^2\right)\,dx =$$

$$-\frac{f(a+cx^2)^p(g+hx)^{-2p}\left(1-\frac{g+hx}{g-\frac{\sqrt{-a}h}{\sqrt{c}}}\right)^{-p}\left(1-\frac{g+hx}{\frac{\sqrt{-a}h}{\sqrt{c}}+g}\right)^{-p}\text{AppellF1}\left(-2p,-p,-p,1-2p,\frac{g+hx}{g-\frac{\sqrt{-a}h}{\sqrt{c}}},\frac{g+}{g+}\right.}{2h^3p}$$

$$-\frac{(a+cx^2)^{p+1}(g+hx)^{-2(p+1)}(dh^2-egh+fg^2)}{2h(p+1)(ah^2+cg^2)}$$

$$+\frac{\left(\sqrt{-a}-\sqrt{c}x\right)(a+cx^2)^p(g+hx)^{-2p-1}\left(-\frac{(\sqrt{-a}+\sqrt{c}x)(\sqrt{-a}h+\sqrt{c}g)}{(\sqrt{-a}-\sqrt{c}x)(\sqrt{c}g-\sqrt{-a}h)}\right)^{-p}(ah^2(2fg-eh)+c(fg^3-dgh^2))\,\text{H}}{h^2(2p+1)\left(\sqrt{-a}h+\sqrt{c}g\right)(ah^2+cg^2)}$$

```
[In] Int[(g + h*x)^(-3 - 2*p)*(a + c*x^2)^p*(d + e*x + f*x^2),x]
```

```
[Out] -1/2*((f*g^2 - e*g*h + d*h^2)*(a + c*x^2)^(1 + p))/(h*(c*g^2 + a*h^2)*(1 +
p)*(g + h*x)^(2*(1 + p))) - (f*(a + c*x^2)^p*AppellF1[-2*p, -p, -p, 1 - 2*p
, (g + h*x)/(g - (Sqrt[-a]*h)/Sqrt[c]), (g + h*x)/(g + (Sqrt[-a]*h)/Sqrt[c]
)])/(2*h^3*p*(g + h*x)^(2*p)*(1 - (g + h*x)/(g - (Sqrt[-a]*h)/Sqrt[c]))^p*(
1 - (g + h*x)/(g + (Sqrt[-a]*h)/Sqrt[c]))^p) + ((a*h^2*(2*f*g - e*h) + c*(f
*g^3 - d*g*h^2))*(Sqrt[-a] - Sqrt[c]*x)*(g + h*x)^(-1 - 2*p)*(a + c*x^2)^p*
Hypergeometric2F1[-1 - 2*p, -p, -2*p, (2*Sqrt[-a]*Sqrt[c]*(g + h*x))/((Sqrt
[c]*g - Sqrt[-a]*h)*(Sqrt[-a] - Sqrt[c]*x))]])/(h^2*(Sqrt[c]*g + Sqrt[-a]*h)
*(c*g^2 + a*h^2)*(1 + 2*p)*(-(((Sqrt[c]*g + Sqrt[-a]*h)*(Sqrt[-a] + Sqrt[c]
*x))/((Sqrt[c]*g - Sqrt[-a]*h)*(Sqrt[-a] - Sqrt[c]*x))))^p)
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(Rt[(-a)*c, 2] - c*x)*(d + e*x)^(m + 1)*((a + c*x^2)^p/((m + 1)*(c*d + e*Rt
[(-a)*c, 2])*((c*d + e*Rt[(-a)*c, 2])*((Rt[(-a)*c, 2] + c*x)/((c*d - e*Rt[(
-a)*c, 2])*(-Rt[(-a)*c, 2] + c*x))))^p))*Hypergeometric2F1[m + 1, -p, m + 2
, 2*c*Rt[(-a)*c, 2]*((d + e*x)/((c*d - e*Rt[(-a)*c, 2])*(Rt[(-a)*c, 2] - c*
x)))], x] /; FreeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Int
egerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 774

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Dist[(a + c*x^2)^p/(e*(1 - (d + e*x)/(d + e*(q/c)))^p*
(1 - (d + e*x)/(d - e*(q/c)))^p), Subst[Int[x^m*Simp[1 - x/(d + e*(q/c)), x
]^p*Simp[1 - x/(d - e*(q/c)), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, c, d,
 e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1670

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> With[{q = Expon[Pq, x]}, Dist[Coeff[Pq, x, q]/e^q, Int[(d + e*x)^(m + q)
*(a + c*x^2)^p, x], x] + Dist[1/e^q, Int[(d + e*x)^m*(a + c*x^2)^p*ExpandTo
Sum[e^q*Pq - Coeff[Pq, x, q]*(d + e*x)^q, x], x], x]] /; FreeQ[{a, c, d, e,
 m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(IGtQ[m, 0] && Rat
ionalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int (g+hx)^{-3-2p}\left(-fg^2+dh^2-h(2fg-eh)x\right)\left(a+cx^2\right)^p\,dx}{h^2} \\
&\quad+\frac{f\int (g+hx)^{-1-2p}\left(a+cx^2\right)^p\,dx}{h^2} \\
&= -\frac{\left(fg^2-egh+dh^2\right)(g+hx)^{-2(1+p)}\left(a+cx^2\right)^{1+p}}{2h\left(cg^2+ah^2\right)(1+p)} \\
&\quad-\frac{\left(ah^2(2fg-eh)+c\left(fg^3-dgh^2\right)\right)\int (g+hx)^{-2-2p}\left(a+cx^2\right)^p\,dx}{h^2\left(cg^2+ah^2\right)} \\
&\quad+\frac{\left(f\left(a+cx^2\right)^p\left(1-\frac{g+hx}{g-\frac{\sqrt{-a}h}{\sqrt{c}}}\right)^{-p}\left(1-\frac{g+hx}{g+\frac{\sqrt{-a}h}{\sqrt{c}}}\right)^{-p}\right)\text{Subst}\left(\int x^{-1-2p}\left(1-\frac{x}{g-\frac{\sqrt{-a}h}{\sqrt{c}}}\right)^p\left(1-\frac{x}{g+\frac{\sqrt{-}}{\sqrt{}}}\right.\right.}{h^3}
\end{aligned}$$

$$= -\frac{(fg^2 - egh + dh^2)\,(g+hx)^{-2(1+p)}\,(a+cx^2)^{1+p}}{2h\,(cg^2+ah^2)\,(1+p)}$$

$$- \frac{f(g+hx)^{-2p}\,(a+cx^2)^p \left(1 - \frac{g+hx}{g-\frac{\sqrt{-a}h}{\sqrt{c}}}\right)^{-p} \left(1 - \frac{g+hx}{g+\frac{\sqrt{-a}h}{\sqrt{c}}}\right)^{-p} F_1\Big(-2p; -p, -p; 1-2p; \frac{g+hx}{g-\frac{\sqrt{-a}h}{\sqrt{c}}}, \frac{g+}{g+}}{2h^3p}$$

$$+ \frac{(ah^2(2fg-eh) + c(fg^3 - dgh^2))\,(\sqrt{-a}-\sqrt{c}x) \left(-\frac{(\sqrt{c}g+\sqrt{-a}h)\,(\sqrt{-a}+\sqrt{c}x)}{(\sqrt{c}g-\sqrt{-a}h)\,(\sqrt{-a}-\sqrt{c}x)}\right)^{-p} (g+hx)^{-1-2p}\,(a+}{h^2\,(\sqrt{c}g+\sqrt{-a}h)\,(cg^2+ah^2)\,(1+2p)}$$

Mathematica [F]

$$\int (g+hx)^{-3-2p}\left(a+cx^2\right)^p\left(d+ex+fx^2\right)\,dx = \int (g+hx)^{-3-2p}\left(a+cx^2\right)^p\left(d+ex+fx^2\right)\,dx$$

```
[In] Integrate[(g + h*x)^(-3 - 2*p)*(a + c*x^2)^p*(d + e*x + f*x^2),x]
[Out] Integrate[(g + h*x)^(-3 - 2*p)*(a + c*x^2)^p*(d + e*x + f*x^2), x]
```

Maple [F]

$$\int (hx+g)^{-3-2p}\left(c\,x^2+a\right)^p\left(f\,x^2+ex+d\right)dx$$

```
[In] int((h*x+g)^(-3-2*p)*(c*x^2+a)^p*(f*x^2+e*x+d),x)
[Out] int((h*x+g)^(-3-2*p)*(c*x^2+a)^p*(f*x^2+e*x+d),x)
```

Fricas [F]

$$\int (g+hx)^{-3-2p}\left(a+cx^2\right)^p\left(d+ex+fx^2\right)\,dx = \int (fx^2+ex+d)\,(cx^2+a)^p(hx+g)^{-2\,p-3}\,dx$$

```
[In] integrate((h*x+g)^(-3-2*p)*(c*x^2+a)^p*(f*x^2+e*x+d),x, algorithm="fricas")
[Out] integral((f*x^2 + e*x + d)*(c*x^2 + a)^p*(h*x + g)^(-2*p - 3), x)
```

Sympy [F(-1)]

Timed out.

$$\int (g+hx)^{-3-2p}\left(a+cx^2\right)^p\left(d+ex+fx^2\right)\,dx=\text{Timed out}$$

```
[In] integrate((h*x+g)**(-3-2*p)*(c*x**2+a)**p*(f*x**2+e*x+d),x)
[Out] Timed out
```

Maxima [F]

$$\int (g+hx)^{-3-2p}\left(a+cx^2\right)^p\left(d+ex+fx^2\right)\,dx=\int \left(fx^2+ex+d\right)\left(cx^2+a\right)^p(hx+g)^{-2\,p-3}\,dx$$

```
[In] integrate((h*x+g)^(-3-2*p)*(c*x^2+a)^p*(f*x^2+e*x+d),x, algorithm="maxima")
[Out] integrate((f*x^2 + e*x + d)*(c*x^2 + a)^p*(h*x + g)^(-2*p - 3), x)
```

Giac [F]

$$\int (g+hx)^{-3-2p}\left(a+cx^2\right)^p\left(d+ex+fx^2\right)\,dx=\int \left(fx^2+ex+d\right)\left(cx^2+a\right)^p(hx+g)^{-2\,p-3}\,dx$$

```
[In] integrate((h*x+g)^(-3-2*p)*(c*x^2+a)^p*(f*x^2+e*x+d),x, algorithm="giac")
[Out] integrate((f*x^2 + e*x + d)*(c*x^2 + a)^p*(h*x + g)^(-2*p - 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int (g+hx)^{-3-2p}\left(a+cx^2\right)^p\left(d+ex+fx^2\right)\,dx=\int \frac{\left(c\,x^2+a\right)^p\,\left(f\,x^2+e\,x+d\right)}{{\left(g+h\,x\right)}^{2\,p+3}}\,dx$$

```
[In] int(((a + c*x^2)^p*(d + e*x + f*x^2))/(g + h*x)^(2*p + 3),x)
[Out] int(((a + c*x^2)^p*(d + e*x + f*x^2))/(g + h*x)^(2*p + 3), x)
```

3.139 $\int (d+ex)^m \left(-cd^2+bde+be^2x+ce^2x^2\right)^p ((-cd+be)$

Optimal result

Integrand size = 69, antiderivative size = 222

$$\int (d+ex)^m \left(-cd^2+bde+be^2x+ce^2x^2\right)^p \left((-cd+be)f+(cef-cdg+beg)x+cegx^2\right)\,dx$$

$$=\frac{g(d+ex)^{-1+m}\left(-d(cd-be)+be^2x+ce^2x^2\right)^{2+p}}{ce^2(3+m+2p)}$$

$$-\frac{(beg(1+m+p)+c(dg(1-m)-ef(3+m+2p)))(d+ex)^m\left(\frac{c(d+ex)}{2cd-be}\right)^{-m-p}(cd-be-cex)^2\,(-d(cd-}{c^2e^2(2+p)(3+m+2p)}$$

```
[Out] g*(e*x+d)^(-1+m)*(-d*(-b*e+c*d)+b*e^2*x+c*e^2*x^2)^(2+p)/c/e^2/(3+m+2*p)-(b
*e*g*(1+m+p)+c*(d*g*(1-m)-e*f*(3+m+2*p)))*(e*x+d)^m*(c*(e*x+d)/(-b*e+2*c*d)
)^(-m-p)*(-c*e*x-b*e+c*d)^2*(-d*(-b*e+c*d)+b*e^2*x+c*e^2*x^2)^p*hypergeom([
-m-p, 2+p],[3+p],(-c*e*x-b*e+c*d)/(-b*e+2*c*d))/c^2/e^2/(2+p)/(3+m+2*p)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1646, 808, 693, 691, 72, 71}

$$\int (d+ex)^m \left(-cd^2+bde+be^2x+ce^2x^2\right)^p \left((-cd+be)f+(cef-cdg+beg)x+cegx^2\right)\,dx$$

$$=\frac{g(d+ex)^{m-1}\left(-d(cd-be)+be^2x+ce^2x^2\right)^{p+2}}{ce^2(m+2p+3)}$$

$$-\frac{(d+ex)^m(-be+cd-cex)^2\left(-d(cd-be)+be^2x+ce^2x^2\right)^p\left(\frac{c(d+ex)}{2cd-be}\right)^{-m-p}(beg(m+p+1)+c(dg(1-}{c^2e^2(p+2)(m+2p+3)}$$

```
[In] Int[(d + e*x)^m*(-(c*d^2) + b*d*e + b*e^2*x + c*e^2*x^2)^p*((-(c*d) + b*e)*
f + (c*e*f - c*d*g + b*e*g)*x + c*e*g*x^2),x]
```

```
[Out] (g*(d + e*x)^(-1 + m)*(-(d*(c*d - b*e)) + b*e^2*x + c*e^2*x^2)^(2 + p))/(c*
e^2*(3 + m + 2*p)) - ((b*e*g*(1 + m + p) + c*(d*g*(1 - m) - e*f*(3 + m + 2*
p)))*(d + e*x)^m*((c*(d + e*x))/(2*c*d - b*e))^(-m - p)*(c*d - b*e - c*e*x)
^2*(-(d*(c*d - b*e)) + b*e^2*x + c*e^2*x^2)^p*Hypergeometric2F1[-m - p, 2 +
 p, 3 + p, (c*d - b*e - c*e*x)/(2*c*d - b*e)])/(c^2*e^2*(2 + p)*(3 + m + 2*
p))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 691

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[d^m*((a + b*x + c*x^2)^FracPart[p]/((1 + e*(x/d))^FracPart[p]
*(a/d + (c*x)/e)^FracPart[p])), Int[(1 + e*(x/d))^(m + p)*(a/d + (c/e)*x)^p
, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || GtQ[d, 0]) &&  !(
IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 693

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[d^IntPart[m]*((d + e*x)^FracPart[m]/(1 + e*(x/d))^FracPart[m]
), Int[(1 + e*(x/d))^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !Intege
rQ[p] &&  !(IntegerQ[m] || GtQ[d, 0])
```

Rule 808

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
```

```
)/(c*(m + 2*p + 2))), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 1646

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
.), x_Symbol] :> Dist[d*e, Int[(d + e*x)^(m - 1)*PolynomialQuotient[Pq, a*e
 + c*d*x, x]*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2,
 0] && EqQ[PolynomialRemainder[Pq, a*e + c*d*x, x], 0]
```

Rubi steps

$$\text{integral} = (de)\int (d+ex)^{-1+m}\left(\frac{f}{de}+\frac{gx}{de}\right)\left(-cd^2+bde+be^2x+ce^2x^2\right)^{1+p}\,dx$$

$$= \frac{g(d+ex)^{-1+m}\left(-d(cd-be)+be^2x+ce^2x^2\right)^{2+p}}{ce^2(3+m+2p)} + \frac{\left(d\left(\left(\frac{ce^2f}{d}+\frac{(cde^2-be^3)g}{de}\right)(-1+m)+e\left(\frac{2cef}{d}-\frac{beg}{d}\right)(2+p)\right)\right)\int (d+ex)^{-1+m}\left(-cd^2+bde+be^2x\right.}{ce^2(1+m+2(1+p))}$$

$$= \frac{g(d+ex)^{-1+m}\left(-d(cd-be)+be^2x+ce^2x^2\right)^{2+p}}{ce^2(3+m+2p)} + \frac{\left(\left(\left(\frac{ce^2f}{d}+\frac{(cde^2-be^3)g}{de}\right)(-1+m)+e\left(\frac{2cef}{d}-\frac{beg}{d}\right)(2+p)\right)(d+ex)^m\left(1+\frac{ex}{d}\right)^{-m}\right)\int\left(1+\frac{ex}{d}\right)^{-1+m}}{ce^2(1+m+2(1+p))}$$

$$= \frac{g(d+ex)^{-1+m}\left(-d(cd-be)+be^2x+ce^2x^2\right)^{2+p}}{ce^2(3+m+2p)} + \frac{\left(\left(\left(\frac{ce^2f}{d}+\frac{(cde^2-be^3)g}{de}\right)(-1+m)+e\left(\frac{2cef}{d}-\frac{beg}{d}\right)(2+p)\right)(d+ex)^m\left(1+\frac{ex}{d}\right)^{-m-p}\left(-cd^2+bde+\right.\right.}{ce^2(1+m+2(1+}$$

$$= \frac{g(d+ex)^{-1+m}\left(-d(cd-be)+be^2x+ce^2x^2\right)^{2+p}}{ce^2(3+m+2p)} + \frac{\left(\left(\left(\frac{ce^2f}{d}+\frac{(cde^2-be^3)g}{de}\right)(-1+m)+e\left(\frac{2cef}{d}-\frac{beg}{d}\right)(2+p)\right)(d+ex)^m\left(\frac{cde\left(1+\frac{ex}{d}\right)}{cde-\frac{e\left(-cd^2+bde\right)}{d}}\right)^{-m-p}\left(-c\right.\right.}{ce^2(1+}$$

$$= \frac{g(d+ex)^{-1+m}\left(-d(cd-be)+be^2x+ce^2x^2\right)^{2+p}}{ce^2(3+m+2p)} - \frac{(cdg(1-m)+beg(1+m+p)-cef(3+m+2p))(d+ex)^m\left(\frac{c(d+ex)}{2cd-be}\right)^{-m-p}(cd-be-cex)^2\left(-\right.}{c^2e^2(2+p)(3+m+2p)}$$

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.74

$$\int (d+ex)^m \left(-cd^2+bde+be^2x+ce^2x^2\right)^p \left((-cd+be)f+(cef-cdg+beg)x+cegx^2\right) \, dx$$

$$= \frac{(d+ex)^m(-cd+be+cex)^2(-((d+ex)(-be+c(d-ex))))^p \left(ceg(d+ex)+\frac{e(cdg(-1+m)-beg(1+m+p)+cef(3}{}\right.}{c^2e^3(3+m+2p)}$$

```
[In] Integrate[(d + e*x)^m*(-(c*d^2) + b*d*e + b*e^2*x + c*e^2*x^2)^p*((-(c*d) +
 b*e)*f + (c*e*f - c*d*g + b*e*g)*x + c*e*g*x^2),x]
```

```
[Out] ((d + e*x)^m*(-(c*d) + b*e + c*e*x)^2*(-((d + e*x)*(-(b*e) + c*(d - e*x))))
^p*(c*e*g*(d + e*x) + (e*(c*d*g*(-1 + m) - b*e*g*(1 + m + p) + c*e*f*(3 + m
 + 2*p))*((c*(d + e*x))/(2*c*d - b*e))^(-m - p)*Hypergeometric2F1[-m - p, 2
 + p, 3 + p, (-(c*d) + b*e + c*e*x)/(-2*c*d + b*e)])/(2 + p)))/(c^2*e^3*(3
+ m + 2*p))
```

Maple [F]

$$\int (ex+d)^m \left(c\,e^2x^2+b\,e^2x+bde-c\,d^2\right)^p \left(-(-be+cd)\,f+(beg-cdg+cef)\,x+ceg\,x^2\right) dx$$

```
[In] int((e*x+d)^m*(c*e^2*x^2+b*e^2*x+b*d*e-c*d^2)^p*(-(-b*e+c*d)*f+(b*e*g-c*d*g
+c*e*f)*x+c*e*g*x^2),x)
```

```
[Out] int((e*x+d)^m*(c*e^2*x^2+b*e^2*x+b*d*e-c*d^2)^p*(-(-b*e+c*d)*f+(b*e*g-c*d*g
+c*e*f)*x+c*e*g*x^2),x)
```

Fricas [F]

$$\int (d+ex)^m \left(-cd^2+bde+be^2x+ce^2x^2\right)^p \left((-cd+be)f+(cef-cdg+beg)x+cegx^2\right) \, dx$$
$$= \int \left(cegx^2-(cd-be)f+(cef-cdg+beg)x\right)\left(ce^2x^2+be^2x-cd^2+bde\right)^p(ex+d)^m \, dx$$

```
[In] integrate((e*x+d)^m*(c*e^2*x^2+b*e^2*x+b*d*e-c*d^2)^p*(-(-b*e+c*d)*f+(b*e*g
-c*d*g+c*e*f)*x+c*e*g*x^2),x, algorithm="fricas")
```

```
[Out] integral((c*e*g*x^2 - (c*d - b*e)*f + (c*e*f - (c*d - b*e)*g)*x)*(c*e^2*x^2
 + b*e^2*x - c*d^2 + b*d*e)^p*(e*x + d)^m, x)
```

Sympy [F(-2)]

Exception generated.

$$\int (d+ex)^m \left(-cd^2+bde+be^2x+ce^2x^2\right)^p \left((-cd+be)f+(cef-cdg+beg)x+cegx^2\right)\,dx$$
$$= \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((e*x+d)**m*(c*e**2*x**2+b*e**2*x+b*d*e-c*d**2)**p*(-(-b*e+c*d)*f+
(b*e*g-c*d*g+c*e*f)*x+c*e*g*x**2),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int (d+ex)^m \left(-cd^2+bde+be^2x+ce^2x^2\right)^p \left((-cd+be)f+(cef-cdg+beg)x+cegx^2\right)\,dx$$
$$= \int \left(cegx^2-(cd-be)f+(cef-cdg+beg)x\right)\left(ce^2x^2+be^2x-cd^2+bde\right)^p(ex+d)^m\,dx$$

```
[In] integrate((e*x+d)^m*(c*e^2*x^2+b*e^2*x+b*d*e-c*d^2)^p*(-(-b*e+c*d)*f+(b*e*g
-c*d*g+c*e*f)*x+c*e*g*x^2),x, algorithm="maxima")
```

```
[Out] integrate((c*e*g*x^2 - (c*d - b*e)*f + (c*e*f - c*d*g + b*e*g)*x)*(c*e^2*x^
2 + b*e^2*x - c*d^2 + b*d*e)^p*(e*x + d)^m, x)
```

Giac [F]

$$\int (d+ex)^m \left(-cd^2+bde+be^2x+ce^2x^2\right)^p \left((-cd+be)f+(cef-cdg+beg)x+cegx^2\right)\,dx$$
$$= \int \left(cegx^2-(cd-be)f+(cef-cdg+beg)x\right)\left(ce^2x^2+be^2x-cd^2+bde\right)^p(ex+d)^m\,dx$$

```
[In] integrate((e*x+d)^m*(c*e^2*x^2+b*e^2*x+b*d*e-c*d^2)^p*(-(-b*e+c*d)*f+(b*e*g
-c*d*g+c*e*f)*x+c*e*g*x^2),x, algorithm="giac")
```

```
[Out] integrate((c*e*g*x^2 - (c*d - b*e)*f + (c*e*f - c*d*g + b*e*g)*x)*(c*e^2*x^
2 + b*e^2*x - c*d^2 + b*d*e)^p*(e*x + d)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^m \left(-cd^2+bde+be^2x+ce^2x^2\right)^p \left((-cd+be)f+(cef-cdg+beg)x+cegx^2\right) \, dx$$
$$= \int (d+e\,x)^m \left(c\,e\,g\,x^2+(b\,e\,g-c\,d\,g+c\,e\,f)\;x + f\,(b\,e-c\,d)\right) \left(-c\,d^2+b\,d\,e+c\,e^2\,x^2+b\,e^2\,x\right)^p dx$$

```
[In] int((d + e*x)^m*(f*(b*e - c*d) + x*(b*e*g - c*d*g + c*e*f) + c*e*g*x^2)*(c*
e^2*x^2 - c*d^2 + b*d*e + b*e^2*x)^p,x)
```

```
[Out] int((d + e*x)^m*(f*(b*e - c*d) + x*(b*e*g - c*d*g + c*e*f) + c*e*g*x^2)*(c*
e^2*x^2 - c*d^2 + b*d*e + b*e^2*x)^p, x)
```

3.140 $\int \left(a+bx+cx^2\right)^4 \left(A+Cx^2\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 254

$$\begin{aligned}\int \left(a+bx+cx^2\right)^4 \left(A+Cx^2\right)\,dx = {} & a^4Ax+2a^3Abx^2+\frac{1}{3}a^2\left(6Ab^2+4aAc+a^2C\right)x^3 \\ & +ab\left(A\left(b^2+3ac\right)+a^2C\right)x^4 \\ & +\frac{1}{5}\left(A\left(b^4+12ab^2c+6a^2c^2\right)+2a^2\left(3b^2+2ac\right)C\right)x^5 \\ & +\frac{2}{3}b\left(b^2+3ac\right)(Ac+aC)x^6 \\ & +\frac{1}{7}\left(2Ac^2\left(3b^2+2ac\right)+\left(b^4+12ab^2c+6a^2c^2\right)C\right)x^7 \\ & +\frac{1}{2}bc\left(Ac^2+\left(b^2+3ac\right)C\right)x^8 \\ & +\frac{1}{9}c^2\left(Ac^2+6b^2C+4acC\right)x^9+\frac{2}{5}bc^3Cx^{10}+\frac{1}{11}c^4Cx^{11}\end{aligned}$$

```
[Out] a^4*A*x+2*a^3*A*b*x^2+1/3*a^2*(4*A*a*c+6*A*b^2+C*a^2)*x^3+a*b*(A*(3*a*c+b^2
)+C*a^2)*x^4+1/5*(A*(6*a^2*c^2+12*a*b^2*c+b^4)+2*a^2*(2*a*c+3*b^2)*C)*x^5+2
/3*b*(3*a*c+b^2)*(A*c+C*a)*x^6+1/7*(2*A*c^2*(2*a*c+3*b^2)+(6*a^2*c^2+12*a*b
^2*c+b^4)*C)*x^7+1/2*b*c*(A*c^2+(3*a*c+b^2)*C)*x^8+1/9*c^2*(A*c^2+4*C*a*c+6
*C*b^2)*x^9+2/5*b*c^3*C*x^10+1/11*c^4*C*x^11
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1671}

$$\begin{aligned}\int \left(a+bx+cx^2\right)^4 \left(A+Cx^2\right)\,dx &= a^4Ax+2a^3Abx^2+abx^4\left(a^2C+A\left(3ac+b^2\right)\right)\\ &\quad+\frac{1}{3}a^2x^3\left(a^2C+4aAc+6Ab^2\right)\\ &\quad+\frac{1}{7}x^7\left(C\left(6a^2c^2+12ab^2c+b^4\right)+2Ac^2\left(2ac+3b^2\right)\right)\\ &\quad+\frac{1}{5}x^5\left(A\left(6a^2c^2+12ab^2c+b^4\right)+2a^2C\left(2ac+3b^2\right)\right)\\ &\quad+\frac{1}{9}c^2x^9\left(4acC+Ac^2+6b^2C\right)+\frac{1}{2}bcx^8\left(C\left(3ac+b^2\right)+Ac^2\right)\\ &\quad+\frac{2}{3}bx^6\left(3ac+b^2\right)(aC+Ac)+\frac{2}{5}bc^3Cx^{10}+\frac{1}{11}c^4Cx^{11}\end{aligned}$$

```
[In] Int[(a + b*x + c*x^2)^4*(A + C*x^2),x]
```

```
[Out] a^4*A*x + 2*a^3*A*b*x^2 + (a^2*(6*A*b^2 + 4*a*A*c + a^2*C)*x^3)/3 + a*b*(A*
(b^2 + 3*a*c) + a^2*C)*x^4 + ((A*(b^4 + 12*a*b^2*c + 6*a^2*c^2) + 2*a^2*(3*
b^2 + 2*a*c)*C)*x^5)/5 + (2*b*(b^2 + 3*a*c)*(A*c + a*C)*x^6)/3 + ((2*A*c^2*
(3*b^2 + 2*a*c) + (b^4 + 12*a*b^2*c + 6*a^2*c^2)*C)*x^7)/7 + (b*c*(A*c^2 +
(b^2 + 3*a*c)*C)*x^8)/2 + (c^2*(A*c^2 + 6*b^2*C + 4*a*c*C)*x^9)/9 + (2*b*c^
3*C*x^10)/5 + (c^4*C*x^11)/11
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \Big(a^4A+4a^3Abx+a^2\left(6Ab^2+4aAc+a^2C\right)x^2+4ab\left(A\left(b^2+3ac\right)+a^2C\right)x^3\\ &\quad+\left(A\left(b^4+12ab^2c+6a^2c^2\right)+2a^2\left(3b^2+2ac\right)C\right)x^4+4b\left(b^2+3ac\right)(Ac+aC)x^5\\ &\quad+\left(2Ac^2\left(3b^2+2ac\right)+\left(b^4+12ab^2c+6a^2c^2\right)C\right)x^6+4bc\left(Ac^2+\left(b^2+3ac\right)C\right)x^7\\ &\quad+c^2\left(Ac^2+6b^2C+4acC\right)x^8+4bc^3Cx^9+c^4Cx^{10}\Big)\,dx\\ &= a^4Ax+2a^3Abx^2+\frac{1}{3}a^2\left(6Ab^2+4aAc+a^2C\right)x^3+ab\left(A\left(b^2+3ac\right)+a^2C\right)x^4\\ &\quad+\frac{1}{5}\left(A\left(b^4+12ab^2c+6a^2c^2\right)+2a^2\left(3b^2+2ac\right)C\right)x^5+\frac{2}{3}b\left(b^2+3ac\right)(Ac+aC)x^6\\ &\quad+\frac{1}{7}\left(2Ac^2\left(3b^2+2ac\right)+\left(b^4+12ab^2c+6a^2c^2\right)C\right)x^7+\frac{1}{2}bc\left(Ac^2+\left(b^2+3ac\right)C\right)x^8\\ &\quad+\frac{1}{9}c^2\left(Ac^2+6b^2C+4acC\right)x^9+\frac{2}{5}bc^3Cx^{10}+\frac{1}{11}c^4Cx^{11}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.01

$$\begin{aligned}\int \left(a+bx+cx^2\right)^4\left(A+Cx^2\right)\,dx &= a^4Ax+2a^3Abx^2+\frac{1}{3}a^2\left(6Ab^2+4aAc+a^2C\right)x^3\\ &\quad+ab\left(Ab^2+3aAc+a^2C\right)x^4\\ &\quad+\frac{1}{5}\left(Ab^4+12aAb^2c+6a^2Ac^2+6a^2b^2C+4a^3cC\right)x^5\\ &\quad+\frac{2}{3}b\left(b^2+3ac\right)(Ac+aC)x^6\\ &\quad+\frac{1}{7}\left(6Ab^2c^2+4aAc^3+b^4C+12ab^2cC+6a^2c^2C\right)x^7\\ &\quad+\frac{1}{2}bc\left(Ac^2+b^2C+3acC\right)x^8\\ &\quad+\frac{1}{9}c^2\left(Ac^2+6b^2C+4acC\right)x^9+\frac{2}{5}bc^3Cx^{10}+\frac{1}{11}c^4Cx^{11}\end{aligned}$$

```
[In] Integrate[(a + b*x + c*x^2)^4*(A + C*x^2),x]
```

```
[Out] a^4*A*x + 2*a^3*A*b*x^2 + (a^2*(6*A*b^2 + 4*a*A*c + a^2*C)*x^3)/3 + a*b*(A*
b^2 + 3*a*A*c + a^2*C)*x^4 + ((A*b^4 + 12*a*A*b^2*c + 6*a^2*A*c^2 + 6*a^2*b
^2*C + 4*a^3*c*C)*x^5)/5 + (2*b*(b^2 + 3*a*c)*(A*c + a*C)*x^6)/3 + ((6*A*b^
2*c^2 + 4*a*A*c^3 + b^4*C + 12*a*b^2*c*C + 6*a^2*c^2*C)*x^7)/7 + (b*c*(A*c^
2 + b^2*C + 3*a*c*C)*x^8)/2 + (c^2*(A*c^2 + 6*b^2*C + 4*a*c*C)*x^9)/9 + (2*
b*c^3*C*x^10)/5 + (c^4*C*x^11)/11
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.05

method	result
norman	$\frac{c^4C\,x^{11}}{11}+\frac{2b\,c^3C\,x^{10}}{5}+\left(\frac{1}{9}c^4A+\frac{4}{9}Ca\,c^3+\frac{2}{3}C\,b^2c^2\right)x^9+\left(\frac{1}{2}Ab\,c^3+\frac{3}{2}Cab\,c^2+\frac{1}{2}C\,b^3c\right)x^8+\Big(\frac{4}{7}Aa$
gosper	$\frac{1}{3}x^3a^4C+\frac{1}{7}x^7C\,b^4+\frac{4}{3}a^3Ac\,x^3+\frac{6}{5}a^2A\,c^2x^5+\frac{4}{7}aA\,c^3x^7+x^4A\,b^3a+2x^3A\,a^2b^2+2x^2A\,a^3b+$
risch	$\frac{1}{3}x^3a^4C+\frac{1}{7}x^7C\,b^4+\frac{4}{3}a^3Ac\,x^3+\frac{6}{5}a^2A\,c^2x^5+\frac{4}{7}aA\,c^3x^7+x^4A\,b^3a+2x^3A\,a^2b^2+2x^2A\,a^3b+$
parallelrisch	$\frac{1}{3}x^3a^4C+\frac{1}{7}x^7C\,b^4+\frac{4}{3}a^3Ac\,x^3+\frac{6}{5}a^2A\,c^2x^5+\frac{4}{7}aA\,c^3x^7+x^4A\,b^3a+2x^3A\,a^2b^2+2x^2A\,a^3b+$
default	$\frac{c^4C\,x^{11}}{11}+\frac{2b\,c^3C\,x^{10}}{5}+\frac{\left(\left(2\left(2ac+b^2\right)c^2+4b^2c^2\right)C+c^4A\right)x^9}{9}+\frac{\left(\left(4ba\,c^2+4\left(2ac+b^2\right)bc\right)C+4Ab\,c^3\right)x^8}{8}+\Big(\Big(2a^2c^2+8a\,b^2c+$

```
[In] int((c*x^2+b*x+a)^4*(C*x^2+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/11*c^4*C*x^11+2/5*b*c^3*C*x^10+(1/9*c^4*A+4/9*C*a*c^3+2/3*C*b^2*c^2)*x^9+
(1/2*A*b*c^3+3/2*C*a*b*c^2+1/2*C*b^3*c)*x^8+(4/7*A*a*c^3+6/7*A*b^2*c^2+6/7*
C*a^2*c^2+12/7*C*a*b^2*c+1/7*C*b^4)*x^7+(2*a*A*b*c^2+2/3*A*b^3*c+2*C*a^2*b*
```

```
c+2/3*C*a*b^3)*x^6+(6/5*A*a^2*c^2+12/5*A*a*b^2*c+1/5*A*b^4+4/5*C*a^3*c+6/5*
C*a^2*b^2)*x^5+(3*A*a^2*b*c+A*a*b^3+C*a^3*b)*x^4+(4/3*A*a^3*c+2*A*a^2*b^2+1
/3*a^4*C)*x^3+2*x^2*A*a^3*b+a^4*A*x
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.04

$$\begin{aligned}\int \left(a+bx+cx^2\right)^4 \left(A+Cx^2\right)\,dx = {} & \frac{1}{11}\,Cc^4x^{11} + \frac{2}{5}\,Cbc^3x^{10} + \frac{1}{9}\left(6\,Cb^2c^2 + 4\,Cac^3 + Ac^4\right)x^9 \\ & + \frac{1}{2}\left(Cb^3c + 3\,Cabc^2 + Abc^3\right)x^8 \\ & + \frac{1}{7}\left(Cb^4 + 12\,Cab^2c + 4\,Aac^3 + 6\left(Ca^2 + Ab^2\right)c^2\right)x^7 \\ & + 2\,Aa^3bx^2 \\ & + \frac{2}{3}\left(Cab^3 + 3\,Aabc^2 + \left(3\,Ca^2b + Ab^3\right)c\right)x^6 + Aa^4x \\ & + \frac{1}{5}\left(6\,Ca^2b^2 + Ab^4 + 6\,Aa^2c^2 + 4\left(Ca^3 + 3\,Aab^2\right)c\right)x^5 \\ & + \left(Ca^3b + Aab^3 + 3\,Aa^2bc\right)x^4 \\ & + \frac{1}{3}\left(Ca^4 + 6\,Aa^2b^2 + 4\,Aa^3c\right)x^3\end{aligned}$$

```
[In] integrate((c*x^2+b*x+a)^4*(C*x^2+A),x, algorithm="fricas")
```

```
[Out] 1/11*C*c^4*x^11 + 2/5*C*b*c^3*x^10 + 1/9*(6*C*b^2*c^2 + 4*C*a*c^3 + A*c^4)*
x^9 + 1/2*(C*b^3*c + 3*C*a*b*c^2 + A*b*c^3)*x^8 + 1/7*(C*b^4 + 12*C*a*b^2*c
 + 4*A*a*c^3 + 6*(C*a^2 + A*b^2)*c^2)*x^7 + 2*A*a^3*b*x^2 + 2/3*(C*a*b^3 +
3*A*a*b*c^2 + (3*C*a^2*b + A*b^3)*c)*x^6 + A*a^4*x + 1/5*(6*C*a^2*b^2 + A*b
^4 + 6*A*a^2*c^2 + 4*(C*a^3 + 3*A*a*b^2)*c)*x^5 + (C*a^3*b + A*a*b^3 + 3*A*
a^2*b*c)*x^4 + 1/3*(C*a^4 + 6*A*a^2*b^2 + 4*A*a^3*c)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.26

$$\int \left(a+bx+cx^2\right)^4\left(A+Cx^2\right)\,dx = Aa^4x+2Aa^3bx^2+\frac{2Cbc^3x^{10}}{5}+\frac{Cc^4x^{11}}{11} + x^9\left(\frac{Ac^4}{9}+\frac{4Cac^3}{9}+\frac{2Cb^2c^2}{3}\right) + x^8\left(\frac{Abc^3}{2}+\frac{3Cabc^2}{2}+\frac{Cb^3c}{2}\right)+x^7 \cdot\left(\frac{4Aac^3}{7}+\frac{6Ab^2c^2}{7}+\frac{6Ca^2c^2}{7}+\frac{12Cab^2c}{7}+\frac{Cb^4}{7}\right) + x^6\cdot\left(2Aabc^2+\frac{2Ab^3c}{3}+2Ca^2bc+\frac{2Cab^3}{3}\right)+x^5 \cdot\left(\frac{6Aa^2c^2}{5}+\frac{12Aab^2c}{5}+\frac{Ab^4}{5}+\frac{4Ca^3c}{5}+\frac{6Ca^2b^2}{5}\right)+x^4 \cdot\left(3Aa^2bc+Aab^3+Ca^3b\right)+x^3\cdot\left(\frac{4Aa^3c}{3}+2Aa^2b^2+\frac{Ca^4}{3}\right)$$

```
[In] integrate((c*x**2+b*x+a)**4*(C*x**2+A),x)
```

```
[Out] A*a**4*x + 2*A*a**3*b*x**2 + 2*C*b*c**3*x**10/5 + C*c**4*x**11/11 + x**9*(A
*c**4/9 + 4*C*a*c**3/9 + 2*C*b**2*c**2/3) + x**8*(A*b*c**3/2 + 3*C*a*b*c**2
/2 + C*b**3*c/2) + x**7*(4*A*a*c**3/7 + 6*A*b**2*c**2/7 + 6*C*a**2*c**2/7 +
 12*C*a*b**2*c/7 + C*b**4/7) + x**6*(2*A*a*b*c**2 + 2*A*b**3*c/3 + 2*C*a**2
*b*c + 2*C*a*b**3/3) + x**5*(6*A*a**2*c**2/5 + 12*A*a*b**2*c/5 + A*b**4/5 +
 4*C*a**3*c/5 + 6*C*a**2*b**2/5) + x**4*(3*A*a**2*b*c + A*a*b**3 + C*a**3*b
) + x**3*(4*A*a**3*c/3 + 2*A*a**2*b**2 + C*a**4/3)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.04

$$
\begin{aligned}
\int \left(a+bx+cx^2\right)^4 \left(A+Cx^2\right)\,dx = {} & \frac{1}{11}\,Cc^4x^{11}+\frac{2}{5}\,Cbc^3x^{10}+\frac{1}{9}\left(6\,Cb^2c^2+4\,Cac^3+Ac^4\right)x^9\\
&+\frac{1}{2}\left(Cb^3c+3\,Cabc^2+Abc^3\right)x^8\\
&+\frac{1}{7}\left(Cb^4+12\,Cab^2c+4\,Aac^3+6\left(Ca^2+Ab^2\right)c^2\right)x^7\\
&+2\,Aa^3bx^2\\
&+\frac{2}{3}\left(Cab^3+3\,Aabc^2+\left(3\,Ca^2b+Ab^3\right)c\right)x^6+Aa^4x\\
&+\frac{1}{5}\left(6\,Ca^2b^2+Ab^4+6\,Aa^2c^2+4\left(Ca^3+3\,Aab^2\right)c\right)x^5\\
&+\left(Ca^3b+Aab^3+3\,Aa^2bc\right)x^4\\
&+\frac{1}{3}\left(Ca^4+6\,Aa^2b^2+4\,Aa^3c\right)x^3
\end{aligned}
$$

```
[In] integrate((c*x^2+b*x+a)^4*(C*x^2+A),x, algorithm="maxima")
```

```
[Out] 1/11*C*c^4*x^11 + 2/5*C*b*c^3*x^10 + 1/9*(6*C*b^2*c^2 + 4*C*a*c^3 + A*c^4)*
x^9 + 1/2*(C*b^3*c + 3*C*a*b*c^2 + A*b*c^3)*x^8 + 1/7*(C*b^4 + 12*C*a*b^2*c
 + 4*A*a*c^3 + 6*(C*a^2 + A*b^2)*c^2)*x^7 + 2*A*a^3*b*x^2 + 2/3*(C*a*b^3 +
3*A*a*b*c^2 + (3*C*a^2*b + A*b^3)*c)*x^6 + A*a^4*x + 1/5*(6*C*a^2*b^2 + A*b
^4 + 6*A*a^2*c^2 + 4*(C*a^3 + 3*A*a*b^2)*c)*x^5 + (C*a^3*b + A*a*b^3 + 3*A*
a^2*b*c)*x^4 + 1/3*(C*a^4 + 6*A*a^2*b^2 + 4*A*a^3*c)*x^3
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.21

$$
\begin{aligned}
\int \left(a+bx+cx^2\right)^4 \left(A+Cx^2\right)\,dx = {} & \frac{1}{11}\,Cc^4x^{11}+\frac{2}{5}\,Cbc^3x^{10}+\frac{2}{3}\,Cb^2c^2x^9+\frac{4}{9}\,Cac^3x^9+\frac{1}{9}\,Ac^4x^9\\
&+\frac{1}{2}\,Cb^3cx^8+\frac{3}{2}\,Cabc^2x^8+\frac{1}{2}\,Abc^3x^8+\frac{1}{7}\,Cb^4x^7\\
&+\frac{12}{7}\,Cab^2cx^7+\frac{6}{7}\,Ca^2c^2x^7+\frac{6}{7}\,Ab^2c^2x^7+\frac{4}{7}\,Aac^3x^7\\
&+\frac{2}{3}\,Cab^3x^6+2\,Ca^2bcx^6+\frac{2}{3}\,Ab^3cx^6+2\,Aabc^2x^6\\
&+\frac{6}{5}\,Ca^2b^2x^5+\frac{1}{5}\,Ab^4x^5+\frac{4}{5}\,Ca^3cx^5+\frac{12}{5}\,Aab^2cx^5\\
&+\frac{6}{5}\,Aa^2c^2x^5+Ca^3bx^4+Aab^3x^4+3\,Aa^2bcx^4\\
&+\frac{1}{3}\,Ca^4x^3+2\,Aa^2b^2x^3+\frac{4}{3}\,Aa^3cx^3+2\,Aa^3bx^2+Aa^4x
\end{aligned}
$$

```
[In] integrate((c*x^2+b*x+a)^4*(C*x^2+A),x, algorithm="giac")
```

```
[Out] 1/11*C*c^4*x^11 + 2/5*C*b*c^3*x^10 + 2/3*C*b^2*c^2*x^9 + 4/9*C*a*c^3*x^9 +
1/9*A*c^4*x^9 + 1/2*C*b^3*c*x^8 + 3/2*C*a*b*c^2*x^8 + 1/2*A*b*c^3*x^8 + 1/7
*C*b^4*x^7 + 12/7*C*a*b^2*c*x^7 + 6/7*C*a^2*c^2*x^7 + 6/7*A*b^2*c^2*x^7 + 4
/7*A*a*c^3*x^7 + 2/3*C*a*b^3*x^6 + 2*C*a^2*b*c*x^6 + 2/3*A*b^3*c*x^6 + 2*A*
a*b*c^2*x^6 + 6/5*C*a^2*b^2*x^5 + 1/5*A*b^4*x^5 + 4/5*C*a^3*c*x^5 + 12/5*A*
a*b^2*c*x^5 + 6/5*A*a^2*c^2*x^5 + C*a^3*b*x^4 + A*a*b^3*x^4 + 3*A*a^2*b*c*x
^4 + 1/3*C*a^4*x^3 + 2*A*a^2*b^2*x^3 + 4/3*A*a^3*c*x^3 + 2*A*a^3*b*x^2 + A*
a^4*x
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.96

$$
\begin{aligned}
\int \left(a+bx+cx^2\right)^4 \left(A+Cx^2\right)\,dx &= x^5\left(\frac{4\,C\,a^3\,c}{5}+\frac{6\,C\,a^2\,b^2}{5}+\frac{6\,A\,a^2\,c^2}{5}+\frac{12\,A\,a\,b^2\,c}{5}+\frac{A\,b^4}{5}\right) \\
&\quad + x^7\left(\frac{6\,C\,a^2\,c^2}{7}+\frac{12\,C\,a\,b^2\,c}{7}+\frac{4\,A\,a\,c^3}{7}+\frac{C\,b^4}{7}\right. \\
&\qquad \left.+\frac{6\,A\,b^2\,c^2}{7}\right)+x^3\left(\frac{C\,a^4}{3}+\frac{4\,A\,c\,a^3}{3}+2\,A\,a^2\,b^2\right) \\
&\quad + x^9\left(\frac{2\,C\,b^2\,c^2}{3}+\frac{A\,c^4}{9}+\frac{4\,C\,a\,c^3}{9}\right)+\frac{C\,c^4\,x^{11}}{11} \\
&\quad + A\,a^4\,x+\frac{2\,b\,x^6\,\left(b^2+3\,a\,c\right)\,\left(A\,c+C\,a\right)}{3} \\
&\quad + a\,b\,x^4\,\left(C\,a^2+3\,A\,c\,a+A\,b^2\right) \\
&\quad + \frac{b\,c\,x^8\,\left(C\,b^2+A\,c^2+3\,C\,a\,c\right)}{2}+2\,A\,a^3\,b\,x^2+\frac{2\,C\,b\,c^3\,x^{10}}{5}
\end{aligned}
$$

```
[In] int((A + C*x^2)*(a + b*x + c*x^2)^4,x)
```

```
[Out] x^5*((A*b^4)/5 + (6*A*a^2*c^2)/5 + (6*C*a^2*b^2)/5 + (4*C*a^3*c)/5 + (12*A*
a*b^2*c)/5) + x^7*((C*b^4)/7 + (6*A*b^2*c^2)/7 + (6*C*a^2*c^2)/7 + (4*A*a*c
^3)/7 + (12*C*a*b^2*c)/7) + x^3*((C*a^4)/3 + 2*A*a^2*b^2 + (4*A*a^3*c)/3) +
 x^9*((A*c^4)/9 + (2*C*b^2*c^2)/3 + (4*C*a*c^3)/9) + (C*c^4*x^11)/11 + A*a^
4*x + (2*b*x^6*(3*a*c + b^2)*(A*c + C*a))/3 + a*b*x^4*(A*b^2 + C*a^2 + 3*A*
a*c) + (b*c*x^8*(A*c^2 + C*b^2 + 3*C*a*c))/2 + 2*A*a^3*b*x^2 + (2*C*b*c^3*x
^10)/5
```

3.141 $\int (a+bx+cx^2)^3 (A+Cx^2)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 161

$$\begin{aligned}\int (a+bx+cx^2)^3 (A+Cx^2)\,dx &= a^3Ax+\frac{3}{2}a^2Abx^2+\frac{1}{3}a\left(3A\left(b^2+ac\right)+a^2C\right)x^3\\ &\quad+\frac{1}{4}b\left(A\left(b^2+6ac\right)+3a^2C\right)x^4+\frac{3}{5}\left(b^2+ac\right)(Ac+aC)x^5\\ &\quad+\frac{1}{6}b\left(3Ac^2+\left(b^2+6ac\right)C\right)x^6\\ &\quad+\frac{1}{7}c\left(Ac^2+3\left(b^2+ac\right)C\right)x^7+\frac{3}{8}bc^2Cx^8+\frac{1}{9}c^3Cx^9\end{aligned}$$

```
[Out] a^3*A*x+3/2*a^2*A*b*x^2+1/3*a*(3*A*(a*c+b^2)+C*a^2)*x^3+1/4*b*(A*(6*a*c+b^2
)+3*C*a^2)*x^4+3/5*(a*c+b^2)*(A*c+C*a)*x^5+1/6*b*(3*A*c^2+(6*a*c+b^2)*C)*x^
6+1/7*c*(A*c^2+3*(a*c+b^2)*C)*x^7+3/8*b*c^2*C*x^8+1/9*c^3*C*x^9
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1671}

$$\begin{aligned}\int (a+bx+cx^2)^3 (A+Cx^2)\,dx &= a^3Ax+\frac{1}{4}bx^4\left(3a^2C+A\left(6ac+b^2\right)\right)\\ &\quad+\frac{1}{3}ax^3\left(a^2C+3A\left(ac+b^2\right)\right)+\frac{3}{2}a^2Abx^2\\ &\quad+\frac{1}{7}cx^7\left(3C\left(ac+b^2\right)+Ac^2\right)+\frac{1}{6}bx^6\left(C\left(6ac+b^2\right)+3Ac^2\right)\\ &\quad+\frac{3}{5}x^5\left(ac+b^2\right)(aC+Ac)+\frac{3}{8}bc^2Cx^8+\frac{1}{9}c^3Cx^9\end{aligned}$$

```
[In] Int[(a + b*x + c*x^2)^3*(A + C*x^2),x]
```

```
[Out] a^3*A*x + (3*a^2*A*b*x^2)/2 + (a*(3*A*(b^2 + a*c) + a^2*C)*x^3)/3 + (b*(A*(
b^2 + 6*a*c) + 3*a^2*C)*x^4)/4 + (3*(b^2 + a*c)*(A*c + a*C)*x^5)/5 + (b*(3*
A*c^2 + (b^2 + 6*a*c)*C)*x^6)/6 + (c*(A*c^2 + 3*(b^2 + a*c)*C)*x^7)/7 + (3*
b*c^2*C*x^8)/8 + (c^3*C*x^9)/9
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(a^3A+3a^2Abx+a\left(3A\left(b^2+ac\right)+a^2C\right)x^2+b\left(A\left(b^2+6ac\right)+3a^2C\right)x^3\right.\\ &\quad +3\left(b^2+ac\right)(Ac+aC)x^4+b\left(3Ac^2+\left(b^2+6ac\right)C\right)x^5\\ &\quad \left.+c\left(Ac^2+3\left(b^2+ac\right)C\right)x^6+3bc^2Cx^7+c^3Cx^8\right)\,dx\\ &= a^3Ax+\frac{3}{2}a^2Abx^2+\frac{1}{3}a\left(3A\left(b^2+ac\right)+a^2C\right)x^3+\frac{1}{4}b\left(A\left(b^2+6ac\right)+3a^2C\right)x^4\\ &\quad +\frac{3}{5}\left(b^2+ac\right)(Ac+aC)x^5+\frac{1}{6}b\left(3Ac^2+\left(b^2+6ac\right)C\right)x^6\\ &\quad +\frac{1}{7}c\left(Ac^2+3\left(b^2+ac\right)C\right)x^7+\frac{3}{8}bc^2Cx^8+\frac{1}{9}c^3Cx^9\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.01

$$\begin{aligned}\int \left(a+bx+cx^2\right)^3\left(A+Cx^2\right)\,dx &= a^3Ax+\frac{3}{2}a^2Abx^2+\frac{1}{3}a\left(3Ab^2+3aAc+a^2C\right)x^3\\ &\quad +\frac{1}{4}b\left(Ab^2+6aAc+3a^2C\right)x^4\\ &\quad +\frac{3}{5}\left(b^2+ac\right)(Ac+aC)x^5+\frac{1}{6}b\left(3Ac^2+b^2C+6acC\right)x^6\\ &\quad +\frac{1}{7}c\left(Ac^2+3b^2C+3acC\right)x^7+\frac{3}{8}bc^2Cx^8+\frac{1}{9}c^3Cx^9\end{aligned}$$

```
[In] Integrate[(a + b*x + c*x^2)^3*(A + C*x^2),x]
```

```
[Out] a^3*A*x + (3*a^2*A*b*x^2)/2 + (a*(3*A*b^2 + 3*a*A*c + a^2*C)*x^3)/3 + (b*(A
*b^2 + 6*a*A*c + 3*a^2*C)*x^4)/4 + (3*(b^2 + a*c)*(A*c + a*C)*x^5)/5 + (b*(
3*A*c^2 + b^2*C + 6*a*c*C)*x^6)/6 + (c*(A*c^2 + 3*b^2*C + 3*a*c*C)*x^7)/7 +
 (3*b*c^2*C*x^8)/8 + (c^3*C*x^9)/9
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.02

method	result
norman	$\frac{c^3 C x^9}{9}+\frac{3b c^2 C x^8}{8}+\left(\frac{1}{7}A c^3+\frac{3}{7}a c^2 C+\frac{3}{7}C b^2 c\right)x^7+\left(\frac{1}{2}Ab c^2+Cabc+\frac{1}{6}C b^3\right)x^6+\left(\frac{3}{5}Aa c^2+\right.$
gosper	$\frac{1}{9}c^3 C x^9+\frac{3}{8}b c^2 C x^8+\frac{1}{7}A c^3 x^7+\frac{3}{7}x^7 a c^2 C+\frac{3}{7}x^7 C b^2 c+\frac{1}{2}x^6 Ab c^2+x^6 Cabc+\frac{1}{6}x^6 C b^3+$
risch	$\frac{1}{9}c^3 C x^9+\frac{3}{8}b c^2 C x^8+\frac{1}{7}A c^3 x^7+\frac{3}{7}x^7 a c^2 C+\frac{3}{7}x^7 C b^2 c+\frac{1}{2}x^6 Ab c^2+x^6 Cabc+\frac{1}{6}x^6 C b^3+$
parallelrisch	$\frac{1}{9}c^3 C x^9+\frac{3}{8}b c^2 C x^8+\frac{1}{7}A c^3 x^7+\frac{3}{7}x^7 a c^2 C+\frac{3}{7}x^7 C b^2 c+\frac{1}{2}x^6 Ab c^2+x^6 Cabc+\frac{1}{6}x^6 C b^3+$
default	$\frac{c^3 C x^9}{9}+\frac{3b c^2 C x^8}{8}+\frac{\left(\left(a c^2+2b^2 c+c\left(2ac+b^2\right)\right)C+A c^3\right)x^7}{7}+\frac{\left(\left(4abc+b\left(2ac+b^2\right)\right)C+3Ab c^2\right)x^6}{6}+$

```
[In] int((c*x^2+b*x+a)^3*(C*x^2+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*c^3*C*x^9+3/8*b*c^2*C*x^8+(1/7*A*c^3+3/7*a*c^2*C+3/7*C*b^2*c)*x^7+(1/2*
A*b*c^2+C*a*b*c+1/6*C*b^3)*x^6+(3/5*A*a*c^2+3/5*A*b^2*c+3/5*c*a^2*C+3/5*C*a
*b^2)*x^5+(3/2*A*a*b*c+1/4*A*b^3+3/4*b*a^2*C)*x^4+(A*a^2*c+A*a*b^2+1/3*a^3*
C)*x^3+3/2*a^2*A*b*x^2+a^3*A*x
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.02

$$\begin{aligned}\int \left(a+bx+cx^2\right)^3\left(A+Cx^2\right)\,dx &= \frac{1}{9}\,Cc^3x^9+\frac{3}{8}\,Cbc^2x^8+\frac{1}{7}\,\left(3\,Cb^2c+3\,Cac^2+Ac^3\right)x^7\\ &\quad+\frac{1}{6}\,\left(Cb^3+6\,Cabc+3\,Abc^2\right)x^6+\frac{3}{2}\,Aa^2bx^2\\ &\quad+\frac{3}{5}\,\left(Cab^2+Aac^2+\left(Ca^2+Ab^2\right)c\right)x^5\\ &\quad+Aa^3x+\frac{1}{4}\,\left(3\,Ca^2b+Ab^3+6\,Aabc\right)x^4\\ &\quad+\frac{1}{3}\,\left(Ca^3+3\,Aab^2+3\,Aa^2c\right)x^3\end{aligned}$$

```
[In] integrate((c*x^2+b*x+a)^3*(C*x^2+A),x, algorithm="fricas")
```

```
[Out] 1/9*C*c^3*x^9 + 3/8*C*b*c^2*x^8 + 1/7*(3*C*b^2*c + 3*C*a*c^2 + A*c^3)*x^7 +
 1/6*(C*b^3 + 6*C*a*b*c + 3*A*b*c^2)*x^6 + 3/2*A*a^2*b*x^2 + 3/5*(C*a*b^2 +
 A*a*c^2 + (C*a^2 + A*b^2)*c)*x^5 + A*a^3*x + 1/4*(3*C*a^2*b + A*b^3 + 6*A*
a*b*c)*x^4 + 1/3*(C*a^3 + 3*A*a*b^2 + 3*A*a^2*c)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.22

$$\int \left(a+bx+cx^2\right)^3\left(A+Cx^2\right)\,dx = Aa^3x+\frac{3Aa^2bx^2}{2}+\frac{3Cbc^2x^8}{8}+\frac{Cc^3x^9}{9}+x^7\left(\frac{Ac^3}{7}+\frac{3Cac^2}{7}+\frac{3Cb^2c}{7}\right)+x^6\left(\frac{Abc^2}{2}+Cabc+\frac{Cb^3}{6}\right)+x^5\cdot\left(\frac{3Aac^2}{5}+\frac{3Ab^2c}{5}+\frac{3Ca^2c}{5}+\frac{3Cab^2}{5}\right)+x^4\cdot\left(\frac{3Aabc}{2}+\frac{Ab^3}{4}+\frac{3Ca^2b}{4}\right)+x^3\left(Aa^2c+Aab^2+\frac{Ca^3}{3}\right)$$

```
[In] integrate((c*x**2+b*x+a)**3*(C*x**2+A),x)
```

```
[Out] A*a**3*x + 3*A*a**2*b*x**2/2 + 3*C*b*c**2*x**8/8 + C*c**3*x**9/9 + x**7*(A*
c**3/7 + 3*C*a*c**2/7 + 3*C*b**2*c/7) + x**6*(A*b*c**2/2 + C*a*b*c + C*b**3
/6) + x**5*(3*A*a*c**2/5 + 3*A*b**2*c/5 + 3*C*a**2*c/5 + 3*C*a*b**2/5) + x*
*4*(3*A*a*b*c/2 + A*b**3/4 + 3*C*a**2*b/4) + x**3*(A*a**2*c + A*a*b**2 + C*
a**3/3)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.02

$$\int \left(a+bx+cx^2\right)^3\left(A+Cx^2\right)\,dx = \frac{1}{9}\,Cc^3x^9+\frac{3}{8}\,Cbc^2x^8+\frac{1}{7}\,\left(3\,Cb^2c+3\,Cac^2+Ac^3\right)x^7+\frac{1}{6}\,\left(Cb^3+6\,Cabc+3\,Abc^2\right)x^6+\frac{3}{2}\,Aa^2bx^2+\frac{3}{5}\,\left(Cab^2+Aac^2+\left(Ca^2+Ab^2\right)c\right)x^5+Aa^3x+\frac{1}{4}\,\left(3\,Ca^2b+Ab^3+6\,Aabc\right)x^4+\frac{1}{3}\,\left(Ca^3+3\,Aab^2+3\,Aa^2c\right)x^3$$

```
[In] integrate((c*x^2+b*x+a)^3*(C*x^2+A),x, algorithm="maxima")
```

```
[Out] 1/9*C*c^3*x^9 + 3/8*C*b*c^2*x^8 + 1/7*(3*C*b^2*c + 3*C*a*c^2 + A*c^3)*x^7 +
 1/6*(C*b^3 + 6*C*a*b*c + 3*A*b*c^2)*x^6 + 3/2*A*a^2*b*x^2 + 3/5*(C*a*b^2 +
 A*a*c^2 + (C*a^2 + A*b^2)*c)*x^5 + A*a^3*x + 1/4*(3*C*a^2*b + A*b^3 + 6*A*
a*b*c)*x^4 + 1/3*(C*a^3 + 3*A*a*b^2 + 3*A*a^2*c)*x^3
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.16

$$\int (a+bx+cx^2)^3 (A+Cx^2)\, dx = \frac{1}{9}Cc^3x^9 + \frac{3}{8}Cbc^2x^8 + \frac{3}{7}Cb^2cx^7 + \frac{3}{7}Cac^2x^7 + \frac{1}{7}Ac^3x^7 + \frac{1}{6}Cb^3x^6 + Cabcx^6 + \frac{1}{2}Abc^2x^6 + \frac{3}{5}Cab^2x^5 + \frac{3}{5}Ca^2cx^5 + \frac{3}{5}Ab^2cx^5 + \frac{3}{5}Aac^2x^5 + \frac{3}{4}Ca^2bx^4 + \frac{1}{4}Ab^3x^4 + \frac{3}{2}Aabcx^4 + \frac{1}{3}Ca^3x^3 + Aab^2x^3 + Aa^2cx^3 + \frac{3}{2}Aa^2bx^2 + Aa^3x$$

```
[In] integrate((c*x^2+b*x+a)^3*(C*x^2+A),x, algorithm="giac")
```

```
[Out] 1/9*C*c^3*x^9 + 3/8*C*b*c^2*x^8 + 3/7*C*b^2*c*x^7 + 3/7*C*a*c^2*x^7 + 1/7*A
*c^3*x^7 + 1/6*C*b^3*x^6 + C*a*b*c*x^6 + 1/2*A*b*c^2*x^6 + 3/5*C*a*b^2*x^5
+ 3/5*C*a^2*c*x^5 + 3/5*A*b^2*c*x^5 + 3/5*A*a*c^2*x^5 + 3/4*C*a^2*b*x^4 + 1
/4*A*b^3*x^4 + 3/2*A*a*b*c*x^4 + 1/3*C*a^3*x^3 + A*a*b^2*x^3 + A*a^2*c*x^3
+ 3/2*A*a^2*b*x^2 + A*a^3*x
```

Mupad [B] (verification not implemented)

Time = 13.68 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.93

$$\int (a+bx+cx^2)^3 (A+Cx^2)\, dx = x^3 \left(\frac{C\,a^3}{3} + A\,c\,a^2 + A\,a\,b^2\right) + x^7 \left(\frac{3\,C\,b^2\,c}{7} + \frac{A\,c^3}{7} + \frac{3\,C\,a\,c^2}{7}\right) + \frac{b\,x^4\,(3\,C\,a^2 + 6\,A\,c\,a + A\,b^2)}{4} + \frac{b\,x^6\,(C\,b^2 + 3\,A\,c^2 + 6\,C\,a\,c)}{6} + \frac{C\,c^3\,x^9}{9} + A\,a^3\,x + \frac{3\,x^5\,(b^2 + a\,c)\,(A\,c + C\,a)}{5} + \frac{3\,A\,a^2\,b\,x^2}{2} + \frac{3\,C\,b\,c^2\,x^8}{8}$$

```
[In] int((A + C*x^2)*(a + b*x + c*x^2)^3,x)
```

```
[Out] x^3*((C*a^3)/3 + A*a*b^2 + A*a^2*c) + x^7*((A*c^3)/7 + (3*C*a*c^2)/7 + (3*C
*b^2*c)/7) + (b*x^4*(A*b^2 + 3*C*a^2 + 6*A*a*c))/4 + (b*x^6*(3*A*c^2 + C*b^
2 + 6*C*a*c))/6 + (C*c^3*x^9)/9 + A*a^3*x + (3*x^5*(a*c + b^2)*(A*c + C*a))
/5 + (3*A*a^2*b*x^2)/2 + (3*C*b*c^2*x^8)/8
```

3.142 $\int (a+bx+cx^2)^2 (A+Cx^2)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 96

$$\int (a+bx+cx^2)^2 (A+Cx^2)\,dx = a^2Ax+aAbx^2+\frac{1}{3}\left(A\left(b^2+2ac\right)+a^2C\right)x^3+\frac{1}{2}b(Ac+aC)x^4 + \frac{1}{5}\left(Ac^2+\left(b^2+2ac\right)C\right)x^5+\frac{1}{3}bcCx^6+\frac{1}{7}c^2Cx^7$$

```
[Out] a^2*A*x+a*A*b*x^2+1/3*(A*(2*a*c+b^2)+C*a^2)*x^3+1/2*b*(A*c+C*a)*x^4+1/5*(A*
c^2+(2*a*c+b^2)*C)*x^5+1/3*b*c*C*x^6+1/7*c^2*C*x^7
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1671}

$$\int (a+bx+cx^2)^2 (A+Cx^2)\,dx = \frac{1}{3}x^3\left(a^2C+A\left(2ac+b^2\right)\right)+a^2Ax+\frac{1}{5}x^5\left(C\left(2ac+b^2\right)+Ac^2\right) + \frac{1}{2}bx^4(aC+Ac)+aAbx^2+\frac{1}{3}bcCx^6+\frac{1}{7}c^2Cx^7$$

```
[In] Int[(a + b*x + c*x^2)^2*(A + C*x^2),x]
```

```
[Out] a^2*A*x + a*A*b*x^2 + ((A*(b^2 + 2*a*c) + a^2*C)*x^3)/3 + (b*(A*c + a*C)*x^
4)/2 + ((A*c^2 + (b^2 + 2*a*c)*C)*x^5)/5 + (b*c*C*x^6)/3 + (c^2*C*x^7)/7
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(a^2A+2aAbx+\left(A\left(b^2+2ac\right)+a^2C\right)x^2+2b(Ac+aC)x^3\right.\\&\qquad\left.+\left(Ac^2+\left(b^2+2ac\right)C\right)x^4+2bcCx^5+c^2Cx^6\right)\,dx\\&=a^2Ax+aAbx^2+\frac{1}{3}\left(A\left(b^2+2ac\right)+a^2C\right)x^3+\frac{1}{2}b(Ac+aC)x^4\\&\quad+\frac{1}{5}\left(Ac^2+\left(b^2+2ac\right)C\right)x^5+\frac{1}{3}bcCx^6+\frac{1}{7}c^2Cx^7\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00

$$\begin{aligned}\int \left(a+bx+cx^2\right)^2\left(A+Cx^2\right)\,dx &= a^2Ax+aAbx^2+\frac{1}{3}\left(Ab^2+2aAc+a^2C\right)x^3+\frac{1}{2}b(Ac+aC)x^4\\&\quad+\frac{1}{5}\left(Ac^2+b^2C+2acC\right)x^5+\frac{1}{3}bcCx^6+\frac{1}{7}c^2Cx^7\end{aligned}$$

```
[In] Integrate[(a + b*x + c*x^2)^2*(A + C*x^2),x]
```

```
[Out] a^2*A*x + a*A*b*x^2 + ((A*b^2 + 2*a*A*c + a^2*C)*x^3)/3 + (b*(A*c + a*C)*x^
4)/2 + ((A*c^2 + b^2*C + 2*a*c*C)*x^5)/5 + (b*c*C*x^6)/3 + (c^2*C*x^7)/7
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.94

method	result
default	$\frac{c^{2} C \,x^{7}}{7}+\frac{b c C \,x^{6}}{3}+\frac{\left(A \,c^{2}+\left(2 a c +b^{2}\right) C \right) x^{5}}{5}+\frac{\left(2 A b c +2 b a C \right) x^{4}}{4}+\frac{\left(A \left(2 a c +b^{2}\right)+C \,a^{2}\right) x^{3}}{3}+x^{2} A b a +a^{2} A x$
norman	$\frac{c^{2} C \,x^{7}}{7}+\frac{b c C \,x^{6}}{3}+\left(\frac{1}{5} A \,c^{2}+\frac{2}{5} C a c +\frac{1}{5} C \,b^{2}\right) x^{5}+\left(\frac{1}{2} A b c +\frac{1}{2} b a C \right) x^{4}+\left(\frac{2}{3} A a c +\frac{1}{3} A \,b^{2}+\frac{1}{3} C \,a^{2}$
gosper	$\frac{1}{7} c^{2} C \,x^{7}+\frac{1}{3} b c C \,x^{6}+\frac{1}{5} A \,c^{2} x^{5}+\frac{2}{5} x^{5} C a c +\frac{1}{5} x^{5} C \,b^{2}+\frac{1}{2} x^{4} A b c +\frac{1}{2} x^{4} b a C +\frac{2}{3} a A c \,x^{3}+\frac{1}{3} A \,b^{2}$
risch	$\frac{1}{7} c^{2} C \,x^{7}+\frac{1}{3} b c C \,x^{6}+\frac{1}{5} A \,c^{2} x^{5}+\frac{2}{5} x^{5} C a c +\frac{1}{5} x^{5} C \,b^{2}+\frac{1}{2} x^{4} A b c +\frac{1}{2} x^{4} b a C +\frac{2}{3} a A c \,x^{3}+\frac{1}{3} A \,b^{2}$
parallelrisch	$\frac{1}{7} c^{2} C \,x^{7}+\frac{1}{3} b c C \,x^{6}+\frac{1}{5} A \,c^{2} x^{5}+\frac{2}{5} x^{5} C a c +\frac{1}{5} x^{5} C \,b^{2}+\frac{1}{2} x^{4} A b c +\frac{1}{2} x^{4} b a C +\frac{2}{3} a A c \,x^{3}+\frac{1}{3} A \,b^{2}$

```
[In] int((c*x^2+b*x+a)^2*(C*x^2+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*c^2*C*x^7+1/3*b*c*C*x^6+1/5*(A*c^2+(2*a*c+b^2)*C)*x^5+1/4*(2*A*b*c+2*C*
a*b)*x^4+1/3*(A*(2*a*c+b^2)+C*a^2)*x^3+x^2*A*b*a+a^2*A*x
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.91

$$\int \left(a+bx+cx^2\right)^2 \left(A+Cx^2\right) \, dx = \frac{1}{7} \, Cc^2 x^7 + \frac{1}{3} \, Cbcx^6 + \frac{1}{5} \, (Cb^2 + 2 \, Cac + Ac^2)x^5 + Aabx^2 + \frac{1}{2} \, (Cab + Abc)x^4 + Aa^2 x + \frac{1}{3} \, (Ca^2 + Ab^2 + 2 \, Aac)x^3$$

```
[In] integrate((c*x^2+b*x+a)^2*(C*x^2+A),x, algorithm="fricas")
```

```
[Out] 1/7*C*c^2*x^7 + 1/3*C*b*c*x^6 + 1/5*(C*b^2 + 2*C*a*c + A*c^2)*x^5 + A*a*b*x
^2 + 1/2*(C*a*b + A*b*c)*x^4 + A*a^2*x + 1/3*(C*a^2 + A*b^2 + 2*A*a*c)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.06

$$\int \left(a+bx+cx^2\right)^2 \left(A+Cx^2\right) \, dx = Aa^2 x + Aabx^2 + \frac{Cbcx^6}{3} + \frac{Cc^2 x^7}{7} + x^5 \left(\frac{Ac^2}{5} + \frac{2Cac}{5} + \frac{Cb^2}{5}\right) + x^4 \left(\frac{Abc}{2} + \frac{Cab}{2}\right) + x^3 \cdot \left(\frac{2Aac}{3} + \frac{Ab^2}{3} + \frac{Ca^2}{3}\right)$$

```
[In] integrate((c*x**2+b*x+a)**2*(C*x**2+A),x)
```

```
[Out] A*a**2*x + A*a*b*x**2 + C*b*c*x**6/3 + C*c**2*x**7/7 + x**5*(A*c**2/5 + 2*C
*a*c/5 + C*b**2/5) + x**4*(A*b*c/2 + C*a*b/2) + x**3*(2*A*a*c/3 + A*b**2/3
+ C*a**2/3)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.91

$$\int \left(a+bx+cx^2\right)^2 \left(A+Cx^2\right) \, dx = \frac{1}{7} \, Cc^2 x^7 + \frac{1}{3} \, Cbcx^6 + \frac{1}{5} \, (Cb^2 + 2 \, Cac + Ac^2)x^5 + Aabx^2 + \frac{1}{2} \, (Cab + Abc)x^4 + Aa^2 x + \frac{1}{3} \, (Ca^2 + Ab^2 + 2 \, Aac)x^3$$

```
[In] integrate((c*x^2+b*x+a)^2*(C*x^2+A),x, algorithm="maxima")
```

```
[Out] 1/7*C*c^2*x^7 + 1/3*C*b*c*x^6 + 1/5*(C*b^2 + 2*C*a*c + A*c^2)*x^5 + A*a*b*x
^2 + 1/2*(C*a*b + A*b*c)*x^4 + A*a^2*x + 1/3*(C*a^2 + A*b^2 + 2*A*a*c)*x^3
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.03

$$\int \left(a+bx+cx^2\right)^2 \left(A+Cx^2\right) \, dx = \frac{1}{7}\,Cc^2x^7+\frac{1}{3}\,Cbcx^6+\frac{1}{5}\,Cb^2x^5+\frac{2}{5}\,Cacx^5+\frac{1}{5}\,Ac^2x^5+\frac{1}{2}\,Cabx^4+\frac{1}{2}\,Abcx^4+\frac{1}{3}\,Ca^2x^3+\frac{1}{3}\,Ab^2x^3+\frac{2}{3}\,Aacx^3+Aabx^2+Aa^2x$$

```
[In] integrate((c*x^2+b*x+a)^2*(C*x^2+A),x, algorithm="giac")
```

```
[Out] 1/7*C*c^2*x^7 + 1/3*C*b*c*x^6 + 1/5*C*b^2*x^5 + 2/5*C*a*c*x^5 + 1/5*A*c^2*x
^5 + 1/2*C*a*b*x^4 + 1/2*A*b*c*x^4 + 1/3*C*a^2*x^3 + 1/3*A*b^2*x^3 + 2/3*A*
a*c*x^3 + A*a*b*x^2 + A*a^2*x
```

Mupad [B] (verification not implemented)

Time = 13.64 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.92

$$\int \left(a+bx+cx^2\right)^2 \left(A+Cx^2\right) \, dx = x^3\left(\frac{C\,a^2}{3}+\frac{2\,A\,c\,a}{3}+\frac{A\,b^2}{3}\right)+x^5\left(\frac{C\,b^2}{5}+\frac{A\,c^2}{5}+\frac{2\,C\,a\,c}{5}\right)+\frac{C\,c^2\,x^7}{7}+A\,a^2\,x+\frac{b\,x^4\,\left(A\,c+C\,a\right)}{2}+A\,a\,b\,x^2+\frac{C\,b\,c\,x^6}{3}$$

```
[In] int((A + C*x^2)*(a + b*x + c*x^2)^2,x)
```

```
[Out] x^3*((A*b^2)/3 + (C*a^2)/3 + (2*A*a*c)/3) + x^5*((A*c^2)/5 + (C*b^2)/5 + (2
*C*a*c)/5) + (C*c^2*x^7)/7 + A*a^2*x + (b*x^4*(A*c + C*a))/2 + A*a*b*x^2 +
(C*b*c*x^6)/3
```

3.143 $\int \left(a+bx+cx^2\right)\left(A+Cx^2\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 46

$$\int \left(a+bx+cx^2\right)\left(A+Cx^2\right)\,dx=aAx+\frac{1}{2}Abx^2+\frac{1}{3}(Ac+aC)x^3+\frac{1}{4}bCx^4+\frac{1}{5}cCx^5$$

```
[Out] a*A*x+1/2*A*b*x^2+1/3*(A*c+C*a)*x^3+1/4*b*C*x^4+1/5*c*C*x^5
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1671}

$$\int \left(a+bx+cx^2\right)\left(A+Cx^2\right)\,dx=\frac{1}{3}x^3(aC+Ac)+aAx+\frac{1}{2}Abx^2+\frac{1}{4}bCx^4+\frac{1}{5}cCx^5$$

```
[In] Int[(a + b*x + c*x^2)*(A + C*x^2),x]
```

```
[Out] a*A*x + (A*b*x^2)/2 + ((A*c + a*C)*x^3)/3 + (b*C*x^4)/4 + (c*C*x^5)/5
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\int \left(aA+Abx+(Ac+aC)x^2+bCx^3+cCx^4\right)\,dx\\&=aAx+\frac{1}{2}Abx^2+\frac{1}{3}(Ac+aC)x^3+\frac{1}{4}bCx^4+\frac{1}{5}cCx^5\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \left(a+bx+cx^2\right)\left(A+Cx^2\right)\,dx=aAx+\frac{1}{2}Abx^2+\frac{1}{3}(Ac+aC)x^3+\frac{1}{4}bCx^4+\frac{1}{5}cCx^5$$

```
[In] Integrate[(a + b*x + c*x^2)*(A + C*x^2),x]
[Out] a*A*x + (A*b*x^2)/2 + ((A*c + a*C)*x^3)/3 + (b*C*x^4)/4 + (c*C*x^5)/5
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

method	result	size
default	$aAx+\frac{Ab\,x^2}{2}+\frac{(Ac+Ca)x^3}{3}+\frac{bC\,x^4}{4}+\frac{cC\,x^5}{5}$	39
norman	$\frac{cC\,x^5}{5}+\frac{bC\,x^4}{4}+\left(\frac{Ac}{3}+\frac{Ca}{3}\right)x^3+\frac{Ab\,x^2}{2}+aAx$	40
gosper	$\frac{1}{5}cC\,x^5+\frac{1}{4}bC\,x^4+\frac{1}{3}Ac\,x^3+\frac{1}{3}x^3Ca+\frac{1}{2}Ab\,x^2+aAx$	41
risch	$\frac{1}{5}cC\,x^5+\frac{1}{4}bC\,x^4+\frac{1}{3}Ac\,x^3+\frac{1}{3}x^3Ca+\frac{1}{2}Ab\,x^2+aAx$	41
parallelrisch	$\frac{1}{5}cC\,x^5+\frac{1}{4}bC\,x^4+\frac{1}{3}Ac\,x^3+\frac{1}{3}x^3Ca+\frac{1}{2}Ab\,x^2+aAx$	41

```
[In] int((c*x^2+b*x+a)*(C*x^2+A),x,method=_RETURNVERBOSE)
[Out] a*A*x+1/2*A*b*x^2+1/3*(A*c+C*a)*x^3+1/4*b*C*x^4+1/5*c*C*x^5
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int \left(a+bx+cx^2\right)\left(A+Cx^2\right)\,dx=\frac{1}{5}\,Ccx^5+\frac{1}{4}\,Cbx^4+\frac{1}{2}\,Abx^2+\frac{1}{3}\,(Ca+Ac)x^3+Aax$$

```
[In] integrate((c*x^2+b*x+a)*(C*x^2+A),x, algorithm="fricas")
[Out] 1/5*C*c*x^5 + 1/4*C*b*x^4 + 1/2*A*b*x^2 + 1/3*(C*a + A*c)*x^3 + A*a*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

$$\int \left(a+bx+cx^2\right)\left(A+Cx^2\right)\,dx=Aax+\frac{Abx^2}{2}+\frac{Cbx^4}{4}+\frac{Ccx^5}{5}+x^3\left(\frac{Ac}{3}+\frac{Ca}{3}\right)$$

```
[In] integrate((c*x**2+b*x+a)*(C*x**2+A),x)
[Out] A*a*x + A*b*x**2/2 + C*b*x**4/4 + C*c*x**5/5 + x**3*(A*c/3 + C*a/3)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int \left(a+bx+cx^2\right)\left(A+Cx^2\right)\,dx=\frac{1}{5}\,Ccx^5+\frac{1}{4}\,Cbx^4+\frac{1}{2}\,Abx^2+\frac{1}{3}\,(Ca+Ac)x^3+Aax$$

```
[In] integrate((c*x^2+b*x+a)*(C*x^2+A),x, algorithm="maxima")
[Out] 1/5*C*c*x^5 + 1/4*C*b*x^4 + 1/2*A*b*x^2 + 1/3*(C*a + A*c)*x^3 + A*a*x
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int \left(a+bx+cx^2\right)\left(A+Cx^2\right)\,dx=\frac{1}{5}\,Ccx^5+\frac{1}{4}\,Cbx^4+\frac{1}{3}\,Cax^3+\frac{1}{3}\,Acx^3+\frac{1}{2}\,Abx^2+Aax$$

```
[In] integrate((c*x^2+b*x+a)*(C*x^2+A),x, algorithm="giac")
[Out] 1/5*C*c*x^5 + 1/4*C*b*x^4 + 1/3*C*a*x^3 + 1/3*A*c*x^3 + 1/2*A*b*x^2 + A*a*x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int \left(a+bx+cx^2\right)\left(A+Cx^2\right)\,dx=\frac{C\,c\,x^5}{5}+\frac{C\,b\,x^4}{4}+\left(\frac{A\,c}{3}+\frac{C\,a}{3}\right)\,x^3+\frac{A\,b\,x^2}{2}+A\,a\,x$$

```
[In] int((A + C*x^2)*(a + b*x + c*x^2),x)
[Out] x^3*((A*c)/3 + (C*a)/3) + A*a*x + (A*b*x^2)/2 + (C*b*x^4)/4 + (C*c*x^5)/5
```

3.144 $\int \frac{A+Cx^2}{a+bx+cx^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 81

$$\int \frac{A+Cx^2}{a+bx+cx^2}\,dx=\frac{Cx}{c}-\frac{(2Ac^2+(b^2-2ac)\,C)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}-\frac{bC\log\left(a+bx+cx^2\right)}{2c^2}$$

```
[Out] C*x/c-1/2*b*C*ln(c*x^2+b*x+a)/c^2-(2*A*c^2+(-2*a*c+b^2)*C)*arctanh((2*c*x+b
)/(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1671, 648, 632, 212, 642}

$$\int \frac{A+Cx^2}{a+bx+cx^2}\,dx=-\frac{(C(b^2-2ac)+2Ac^2)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}-\frac{bC\log\left(a+bx+cx^2\right)}{2c^2}+\frac{Cx}{c}$$

```
[In] Int[(A + C*x^2)/(a + b*x + c*x^2),x]
```

```
[Out] (C*x)/c - ((2*A*c^2 + (b^2 - 2*a*c)*C)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c
]])/(c^2*Sqrt[b^2 - 4*a*c]) - (b*C*Log[a + b*x + c*x^2])/(2*c^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{C}{c}+\frac{Ac-aC-bCx}{c\,(a+bx+cx^2)}\right)dx\\
&= \frac{Cx}{c}+\frac{\int \frac{Ac-aC-bCx}{a+bx+cx^2}\,dx}{c}\\
&= \frac{Cx}{c}-\frac{(bC)\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2c^2}+\frac{1}{2}\left(2A+\frac{(b^2-2ac)\,C}{c^2}\right)\int \frac{1}{a+bx+cx^2}\,dx\\
&= \frac{Cx}{c}-\frac{bC\log\left(a+bx+cx^2\right)}{2c^2}\\
&\quad+\left(-2A-\frac{(b^2-2ac)\,C}{c^2}\right)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx\right)\\
&= \frac{Cx}{c}-\frac{\left(2A+\frac{(b^2-2ac)C}{c^2}\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}-\frac{bC\log\left(a+bx+cx^2\right)}{2c^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.04

$$\int \frac{A+Cx^2}{a+bx+cx^2}\,dx=\frac{Cx}{c}+\frac{(2Ac^2+b^2C-2acC)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{c^2\sqrt{-b^2+4ac}}-\frac{bC\log\left(a+bx+cx^2\right)}{2c^2}$$

```
[In] Integrate[(A + C*x^2)/(a + b*x + c*x^2),x]
```

```
[Out] (C*x)/c + ((2*A*c^2 + b^2*C - 2*a*c*C)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c
]])/(c^2*Sqrt[-b^2 + 4*a*c]) - (b*C*Log[a + b*x + c*x^2])/(2*c^2)
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.01

method	result
default	$\frac{Cx}{c}+\frac{-\frac{bC\ln\left(c\,x^2+bx+a\right)}{2c}+\frac{2\left(Ac-Ca+\frac{b^2C}{2c}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{c}$
risch	$\frac{Cx}{c}-\frac{2\ln\bigg(8Aa\,c^3-2A\,b^2c^2-8C\,a^2c^2+6Ca\,b^2c-C\,b^4-2\sqrt{-(4ac-b^2)(2A\,c^2-2Cac+C\,b^2)^2}\,cx-\sqrt{-(4ac-b^2)(2A\,c^2-2Cac+C\,b^2)}}{c(4ac-b^2)}$

```
[In] int((C*x^2+A)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] C*x/c+1/c*(-1/2*b*C/c*ln(c*x^2+b*x+a)+2*(A*c-C*a+1/2*b^2*C/c)/(4*a*c-b^2)^(
1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 265, normalized size of antiderivative = 3.27

$$\int \frac{A+Cx^2}{a+bx+cx^2}\,dx$$

$$=\Bigg[\frac{(Cb^2-2\,Cac+2\,Ac^2)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)+2\,(Cb^2c-4\,Cac^2)x-(Cb^3-}{2\,(b^2c^2-4\,ac^3)}$$

$$-\frac{2\,(Cb^2-2\,Cac+2\,Ac^2)\sqrt{-b^2+4\,ac}\arctan\left(-\frac{\sqrt{-b^2+4\,ac}(2\,cx+b)}{b^2-4\,ac}\right)-2\,(Cb^2c-4\,Cac^2)x+(Cb^3-4\,Cab}{2\,(b^2c^2-4\,ac^3)}$$

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] [1/2*((C*b^2 - 2*C*a*c + 2*A*c^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*
x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*(C*
b^2*c - 4*C*a*c^2)*x - (C*b^3 - 4*C*a*b*c)*log(c*x^2 + b*x + a))/(b^2*c^2 -
 4*a*c^3), -1/2*(2*(C*b^2 - 2*C*a*c + 2*A*c^2)*sqrt(-b^2 + 4*a*c)*arctan(-s
qrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - 2*(C*b^2*c - 4*C*a*c^2)*x +
(C*b^3 - 4*C*a*b*c)*log(c*x^2 + b*x + a))/(b^2*c^2 - 4*a*c^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 413 vs. $2(76) = 152$.

Time = 0.60 (sec) , antiderivative size = 413, normalized size of antiderivative = 5.10

$$\int \frac{A+Cx^2}{a+bx+cx^2}\,dx = \frac{Cx}{c} + \left(-\frac{Cb}{2c^2} - \frac{\sqrt{-4ac+b^2}\,(-2Ac^2+2Cac-Cb^2)}{2c^2\cdot(4ac-b^2)}\right)\log\left(x + \frac{-Abc-Cab-4ac^2\left(-\frac{Cb}{2c^2}-\frac{\sqrt{-4ac+b^2}(-2Ac^2+2Cac-Cb^2)}{2c^2\cdot(4ac-b^2)}\right)}{-2Ac^2+2Cac-}\right. + \left(-\frac{Cb}{2c^2} + \frac{\sqrt{-4ac+b^2}\,(-2Ac^2+2Cac-Cb^2)}{2c^2\cdot(4ac-b^2)}\right)\log\left(x + \frac{-Abc-Cab-4ac^2\left(-\frac{Cb}{2c^2}+\frac{\sqrt{-4ac+b^2}(-2Ac^2+2Cac-Cb^2)}{2c^2\cdot(4ac-b^2)}\right)}{-2Ac^2+2Cac-}\right.$$

```
[In] integrate((C*x**2+A)/(c*x**2+b*x+a),x)
```

```
[Out] C*x/c + (-C*b/(2*c**2) - sqrt(-4*a*c + b**2)*(-2*A*c**2 + 2*C*a*c - C*b**2)
/(2*c**2*(4*a*c - b**2)))*log(x + (-A*b*c - C*a*b - 4*a*c**2*(-C*b/(2*c**2)
 - sqrt(-4*a*c + b**2)*(-2*A*c**2 + 2*C*a*c - C*b**2)/(2*c**2*(4*a*c - b**2
))) + b**2*c*(-C*b/(2*c**2) - sqrt(-4*a*c + b**2)*(-2*A*c**2 + 2*C*a*c - C*
b**2)/(2*c**2*(4*a*c - b**2))))/(-2*A*c**2 + 2*C*a*c - C*b**2)) + (-C*b/(2*
c**2) + sqrt(-4*a*c + b**2)*(-2*A*c**2 + 2*C*a*c - C*b**2)/(2*c**2*(4*a*c -
 b**2)))*log(x + (-A*b*c - C*a*b - 4*a*c**2*(-C*b/(2*c**2) + sqrt(-4*a*c +
b**2)*(-2*A*c**2 + 2*C*a*c - C*b**2)/(2*c**2*(4*a*c - b**2))) + b**2*c*(-C*
b/(2*c**2) + sqrt(-4*a*c + b**2)*(-2*A*c**2 + 2*C*a*c - C*b**2)/(2*c**2*(4*
a*c - b**2))))/(-2*A*c**2 + 2*C*a*c - C*b**2))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Cx^2}{a+bx+cx^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.96

$$\int \frac{A+Cx^2}{a+bx+cx^2}\,dx = \frac{Cx}{c} - \frac{Cb\log\left(cx^2+bx+a\right)}{2\,c^2} + \frac{\left(Cb^2-2\,Cac+2\,Ac^2\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}c^2}$$

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] C*x/c - 1/2*C*b*log(c*x^2 + b*x + a)/c^2 + (C*b^2 - 2*C*a*c + 2*A*c^2)*arct
an((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2)
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 224, normalized size of antiderivative = 2.77

$$\begin{aligned}\int \frac{A+Cx^2}{a+bx+cx^2}\,dx &= \frac{2\,A\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)}{\sqrt{4\,a\,c-b^2}} + \frac{C\,x}{c} \\ &\quad + \frac{C\,b^3\,\ln\left(c\,x^2+b\,x+a\right)}{2\,\left(4\,a\,c^3-b^2\,c^2\right)} - \frac{2\,C\,a\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)}{c\,\sqrt{4\,a\,c-b^2}} \\ &\quad + \frac{C\,b^2\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)}{c^2\,\sqrt{4\,a\,c-b^2}} - \frac{2\,C\,a\,b\,c\,\ln\left(c\,x^2+b\,x+a\right)}{4\,a\,c^3-b^2\,c^2}\end{aligned}$$

```
[In] int((A + C*x^2)/(a + b*x + c*x^2),x)
```

```
[Out] (2*A*atan(b/(4*a*c - b^2)^(1/2) + (2*c*x)/(4*a*c - b^2)^(1/2)))/(4*a*c - b^
2)^(1/2) + (C*x)/c + (C*b^3*log(a + b*x + c*x^2))/(2*(4*a*c^3 - b^2*c^2)) -
 (2*C*a*atan(b/(4*a*c - b^2)^(1/2) + (2*c*x)/(4*a*c - b^2)^(1/2)))/(c*(4*a*
c - b^2)^(1/2)) + (C*b^2*atan(b/(4*a*c - b^2)^(1/2) + (2*c*x)/(4*a*c - b^2)
^(1/2)))/(c^2*(4*a*c - b^2)^(1/2)) - (2*C*a*b*c*log(a + b*x + c*x^2))/(4*a*
c^3 - b^2*c^2)
```

3.145 $\int \frac{A+Cx^2}{(a+bx+cx^2)^2} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 100

$$\int \frac{A+Cx^2}{(a+bx+cx^2)^2} \, dx = -\frac{bc\left(A+\frac{aC}{c}\right)+(2Ac^2+(b^2-2ac)\,C)\,x}{c\,(b^2-4ac)\,(a+bx+cx^2)} + \frac{4(Ac+aC)\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

```
[Out] (-b*c*(A+a*C/c)-(2*A*c^2+(-2*a*c+b^2)*C)*x)/c/(-4*a*c+b^2)/(c*x^2+b*x+a)+4*
(A*c+C*a)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1674, 12, 632, 212}

$$\int \frac{A+Cx^2}{(a+bx+cx^2)^2} \, dx = \frac{4(aC+Ac)\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}} - \frac{x(C(b^2-2ac)+2Ac^2)+bc\left(\frac{aC}{c}+A\right)}{c\,(b^2-4ac)\,(a+bx+cx^2)}$$

```
[In] Int[(A + C*x^2)/(a + b*x + c*x^2)^2,x]
```

```
[Out] -((b*c*(A + (a*C)/c) + (2*A*c^2 + (b^2 - 2*a*c)*C)*x)/(c*(b^2 - 4*a*c)*(a +
 b*x + c*x^2))) + (4*(A*c + a*C)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(b
^2 - 4*a*c)^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{\int \frac{2(Ac+aC)}{a+bx+cx^2}\,dx}{-b^2+4ac}\\
&= -\frac{bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{\left(2(Ac+aC)\right)\int \frac{1}{a+bx+cx^2}\,dx}{b^2-4ac}\\
&= -\frac{bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{(4(Ac+aC))\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx\right)}{b^2-4ac}\\
&= -\frac{bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{4(Ac+aC)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.98

$$\int \frac{A+Cx^2}{\left(a+bx+cx^2\right)^2}\,dx = \frac{b^2Cx+aC(b-2cx)+Ac(b+2cx)}{c\left(-b^2+4ac\right)\left(a+x(b+cx)\right)} + \frac{4(Ac+aC)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\left(-b^2+4ac\right)^{3/2}}$$

```
[In] Integrate[(A + C*x^2)/(a + b*x + c*x^2)^2,x]
```

```
[Out] (b^2*C*x + a*C*(b - 2*c*x) + A*c*(b + 2*c*x))/(c*(-b^2 + 4*a*c)*(a + x*(b +
 c*x))) + (4*(A*c + a*C)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*
a*c)^(3/2)
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.15

method	result
default	$\frac{\frac{\left(2A\,c^2-2Cac+C\,b^2\right)x}{c\left(4ac-b^2\right)}+\frac{b(Ac+Ca)}{c\left(4ac-b^2\right)}}{c\,x^2+bx+a}+\frac{4(Ac+Ca)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{\frac{3}{2}}}$
risch	$\frac{\frac{\left(2A\,c^2-2Cac+C\,b^2\right)x}{c\left(4ac-b^2\right)}+\frac{b(Ac+Ca)}{c\left(4ac-b^2\right)}}{c\,x^2+bx+a}+\frac{2\ln\left(\left(-8a\,c^2+2b^2c\right)x+\left(-4ac+b^2\right)^{\frac{3}{2}}-4abc+b^3\right)Ac}{(-4ac+b^2)^{\frac{3}{2}}}+\frac{2\ln\left(\left(-8a\,c^2+2b^2c\right)x+\left(-4ac+b^2\right)^{\frac{3}{2}}-4\right.}{(-4ac+b^2)^{\frac{3}{2}}}$

```
[In] int((C*x^2+A)/(c*x^2+b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] ((2*A*c^2-2*C*a*c+C*b^2)/c/(4*a*c-b^2)*x+b/c*(A*c+C*a)/(4*a*c-b^2))/(c*x^2+
b*x+a)+4*(A*c+C*a)/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. 2(96) = 192.

Time = 0.31 (sec) , antiderivative size = 511, normalized size of antiderivative = 5.11

$$\int \frac{A+Cx^2}{\left(a+bx+cx^2\right)^2}\,dx$$

$$= \left[-\frac{Cab^3-4\,Aabc^2+2\,(Ca^2c+Aac^2+(Cac^2+Ac^3)x^2+(Cabc+Abc^2)x)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2}{cx^2}\right.}{ab^4c-8\,a^2b^2c^2+16\,a^3c^3+(b^4c^2-8\,ab^2c^3+16\,a^2}\right.$$

$$-\frac{Cab^3-4\,Aabc^2-4\,(Ca^2c+Aac^2+(Cac^2+Ac^3)x^2+(Cabc+Abc^2)x)\sqrt{-b^2+4\,ac}\arctan\left(-\frac{\sqrt{-b^2+4\,a}}{b^2-}\right.}{ab^4c-8\,a^2b^2c^2+16\,a^3c^3+(b^4c^2-8\,ab^2c^3+16\,a^2c^4)x^2}$$

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [-(C*a*b^3 - 4*A*a*b*c^2 + 2*(C*a^2*c + A*a*c^2 + (C*a*c^2 + A*c^3)*x^2 + (
C*a*b*c + A*b*c^2)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*
a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - (4*C*a^2*b - A*b^
3)*c + (C*b^4 - 6*C*a*b^2*c - 8*A*a*c^3 + 2*(4*C*a^2 + A*b^2)*c^2)*x)/(a*b^
4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^2
 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x), -(C*a*b^3 - 4*A*a*b*c^2 - 4*(C*
a^2*c + A*a*c^2 + (C*a*c^2 + A*c^3)*x^2 + (C*a*b*c + A*b*c^2)*x)*sqrt(-b^2
+ 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - (4*C*a^2*b
 - A*b^3)*c + (C*b^4 - 6*C*a*b^2*c - 8*A*a*c^3 + 2*(4*C*a^2 + A*b^2)*c^2)*x
)/(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c
^4)*x^2 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 376 vs. $2(92) = 184$.

Time = 0.59 (sec) , antiderivative size = 376, normalized size of antiderivative = 3.76

$$\int \frac{A+Cx^2}{(a+bx+cx^2)^2}\,dx = -2\sqrt{-\frac{1}{(4ac-b^2)^3}}(Ac$$

$$+Ca)\log\left(x+\frac{2Abc+2Cab-32a^2c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}(Ac+Ca)+16ab^2c\sqrt{-\frac{1}{(4ac-b^2)^3}}(Ac+Ca)-2b^4\sqrt{-}}{4Ac^2+4Cac}\right.$$

$$+2\sqrt{-\frac{1}{(4ac-b^2)^3}}(Ac$$

$$+Ca)\log\left(x+\frac{2Abc+2Cab+32a^2c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}(Ac+Ca)-16ab^2c\sqrt{-\frac{1}{(4ac-b^2)^3}}(Ac+Ca)+2b^4\sqrt{-}}{4Ac^2+4Cac}\right.$$

$$+\frac{Abc+Cab+x(2Ac^2-2Cac+Cb^2)}{4a^2c^2-ab^2c+x^2\cdot(4ac^3-b^2c^2)+x\,(4abc^2-b^3c)}$$

```
[In] integrate((C*x**2+A)/(c*x**2+b*x+a)**2,x)
```

```
[Out] -2*sqrt(-1/(4*a*c - b**2)**3)*(A*c + C*a)*log(x + (2*A*b*c + 2*C*a*b - 32*a
**2*c**2*sqrt(-1/(4*a*c - b**2)**3)*(A*c + C*a) + 16*a*b**2*c*sqrt(-1/(4*a*
c - b**2)**3)*(A*c + C*a) - 2*b**4*sqrt(-1/(4*a*c - b**2)**3)*(A*c + C*a))/
(4*A*c**2 + 4*C*a*c)) + 2*sqrt(-1/(4*a*c - b**2)**3)*(A*c + C*a)*log(x + (2
*A*b*c + 2*C*a*b + 32*a**2*c**2*sqrt(-1/(4*a*c - b**2)**3)*(A*c + C*a) - 16
*a*b**2*c*sqrt(-1/(4*a*c - b**2)**3)*(A*c + C*a) + 2*b**4*sqrt(-1/(4*a*c -
b**2)**3)*(A*c + C*a))/(4*A*c**2 + 4*C*a*c)) + (A*b*c + C*a*b + x*(2*A*c**2
 - 2*C*a*c + C*b**2))/(4*a**2*c**2 - a*b**2*c + x**2*(4*a*c**3 - b**2*c**2)
 + x*(4*a*b*c**2 - b**3*c))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Cx^2}{\left(a+bx+cx^2\right)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.08

$$\int \frac{A+Cx^2}{\left(a+bx+cx^2\right)^2}\,dx = -\frac{4\,(Ca+Ac)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2-4\,ac)\sqrt{-b^2+4\,ac}} - \frac{Cb^2x-2\,Cacx+2\,Ac^2x+Cab+Abc}{(b^2c-4\,ac^2)(cx^2+bx+a)}$$

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] -4*(C*a + A*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-
b^2 + 4*a*c)) - (C*b^2*x - 2*C*a*c*x + 2*A*c^2*x + C*a*b + A*b*c)/((b^2*c -
 4*a*c^2)*(c*x^2 + b*x + a))
```

Mupad [B] (verification not implemented)

Time = 13.07 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.72

$$\int \frac{A+Cx^2}{\left(a+bx+cx^2\right)^2}\,dx = \frac{\frac{A\,b\,c+C\,a\,b}{c\,(4\,a\,c-b^2)}+\frac{x\,(C\,b^2+2\,A\,c^2-2\,C\,a\,c)}{c\,(4\,a\,c-b^2)}}{c\,x^2+b\,x+a} - \frac{4\,\mathrm{atan}\left(\frac{\left(\frac{2\,(A\,c+C\,a)\,\left(b^3-4\,a\,b\,c\right)}{\left(4\,a\,c-b^2\right)^{5/2}}-\frac{4\,c\,x\,(A\,c+C\,a)}{\left(4\,a\,c-b^2\right)^{3/2}}\right)\,(4\,a\,c-b^2)}{2\,A\,c+2\,C\,a}\right)\,(A\,c+C\,a)}{\left(4\,a\,c-b^2\right)^{3/2}}$$

```
[In] int((A + C*x^2)/(a + b*x + c*x^2)^2,x)
```

```
[Out] ((A*b*c + C*a*b)/(c*(4*a*c - b^2)) + (x*(2*A*c^2 + C*b^2 - 2*C*a*c))/(c*(4*
a*c - b^2)))/(a + b*x + c*x^2) - (4*atan((((2*(A*c + C*a)*(b^3 - 4*a*b*c))/
(4*a*c - b^2)^(5/2) - (4*c*x*(A*c + C*a))/(4*a*c - b^2)^(3/2))*(4*a*c - b^2
))/(2*A*c + 2*C*a))*(A*c + C*a))/(4*a*c - b^2)^(3/2)
```

3.146 $\int \frac{A+Cx^2}{(a+bx+cx^2)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 161

$$\int \frac{A+Cx^2}{(a+bx+cx^2)^3}\,dx = -\frac{bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x}{2c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{\left(6Ac+2aC+\frac{b^2C}{c}\right)(b+2cx)}{2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} - \frac{2\left(6Ac^2+\left(b^2+2ac\right)C\right)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$

```
[Out] 1/2*(-b*c*(A+a*C/c)-(2*A*c^2+(-2*a*c+b^2)*C)*x)/c/(-4*a*c+b^2)/(c*x^2+b*x+a
)^2+1/2*(6*A*c+2*C*a+b^2*C/c)*(2*c*x+b)/(-4*a*c+b^2)^2/(c*x^2+b*x+a)-2*(6*A
*c^2+(2*a*c+b^2)*C)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(5/2
)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1674, 12, 628, 632, 212}

$$\int \frac{A+Cx^2}{(a+bx+cx^2)^3}\,dx = -\frac{2\left(C\left(2ac+b^2\right)+6Ac^2\right)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}} - \frac{x\left(C\left(b^2-2ac\right)+2Ac^2\right)+bc\left(\frac{aC}{c}+A\right)}{2c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{(b+2cx)\left(2aC+6Ac+\frac{b^2C}{c}\right)}{2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}$$

```
[In] Int[(A + C*x^2)/(a + b*x + c*x^2)^3,x]
```

```
[Out] -1/2*(b*c*(A + (a*C)/c) + (2*A*c^2 + (b^2 - 2*a*c)*C)*x)/(c*(b^2 - 4*a*c)*(
a + b*x + c*x^2)^2) + ((6*A*c + 2*a*C + (b^2*C)/c)*(b + 2*c*x))/(2*(b^2 - 4
*a*c)^2*(a + b*x + c*x^2)) - (2*(6*A*c^2 + (b^2 + 2*a*c)*C)*ArcTanh[(b + 2*
c*x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\text{integral} = -\frac{bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x}{2c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}-\frac{\int\frac{6Ac+2aC+\frac{b^2C}{c}}{\left(a+bx+cx^2\right)^2}\,dx}{2\left(b^2-4ac\right)}$$

$$= -\frac{bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x}{2c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} - \frac{\left(6Ac+2aC+\frac{b^2C}{c}\right)\int\frac{1}{\left(a+bx+cx^2\right)^2}\,dx}{2\left(b^2-4ac\right)}$$

$$= -\frac{bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x}{2c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{\left(6Ac+2aC+\frac{b^2C}{c}\right)(b+2cx)}{2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} + \frac{\left(6Ac^2+\left(b^2+2ac\right)C\right)\int\frac{1}{a+bx+cx^2}\,dx}{\left(b^2-4ac\right)^2}$$

$$= -\frac{bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x}{2c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{\left(6Ac+2aC+\frac{b^2C}{c}\right)(b+2cx)}{2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} - \frac{\left(2\left(6Ac^2+\left(b^2+2ac\right)C\right)\right)\operatorname{Subst}\left(\int\frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx\right)}{\left(b^2-4ac\right)^2}$$

$$= -\frac{bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x}{2c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{\left(6Ac+2aC+\frac{b^2C}{c}\right)(b+2cx)}{2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} - \frac{2\left(6Ac^2+\left(b^2+2ac\right)C\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.99

$$\int\frac{A+Cx^2}{\left(a+bx+cx^2\right)^3}\,dx = \frac{1}{2}\left(\frac{\left(6Ac^2+\left(b^2+2ac\right)C\right)(b+2cx)}{c\left(b^2-4ac\right)^2\left(a+x(b+cx)\right)} + \frac{b^2Cx+aC(b-2cx)+Ac(b+2cx)}{c\left(-b^2+4ac\right)\left(a+x(b+cx)\right)^2} + \frac{4\left(6Ac^2+\left(b^2+2ac\right)C\right)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\left(-b^2+4ac\right)^{5/2}}\right)$$

```
[In] Integrate[(A + C*x^2)/(a + b*x + c*x^2)^3,x]
```

```
[Out] (((6*A*c^2 + (b^2 + 2*a*c)*C)*(b + 2*c*x))/(c*(b^2 - 4*a*c)^2*(a + x*(b + c
*x))) + (b^2*C*x + a*C*(b - 2*c*x) + A*c*(b + 2*c*x))/(c*(-b^2 + 4*a*c)*(a
+ x*(b + c*x))^2) + (4*(6*A*c^2 + (b^2 + 2*a*c)*C)*ArcTan[(b + 2*c*x)/Sqrt[
-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2))/2
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.67

method	result
default	$\frac{\frac{c\left(6A\,c^2+2Cac+C\,b^2\right)x^3}{16a^2c^2-8a\,b^2c+b^4}+\frac{3b\left(6A\,c^2+2Cac+C\,b^2\right)x^2}{2\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{\left(10Aa\,c^2+2A\,b^2c-2c\,a^2C+5Ca\,b^2\right)x}{16a^2c^2-8a\,b^2c+b^4}+\frac{b\left(10Aac-A\,b^2+6C\,a^2\right)}{32a^2c^2-16a\,b^2c+2b^4}}{\left(c\,x^2+bx+a\right)^2}+\frac{2(6A\,c^2+2Cac+C}{(16a^2c^2-8a}$
risch	$\frac{\frac{c\left(6A\,c^2+2Cac+C\,b^2\right)x^3}{16a^2c^2-8a\,b^2c+b^4}+\frac{3b\left(6A\,c^2+2Cac+C\,b^2\right)x^2}{2\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{\left(10Aa\,c^2+2A\,b^2c-2c\,a^2C+5Ca\,b^2\right)x}{16a^2c^2-8a\,b^2c+b^4}+\frac{b\left(10Aac-A\,b^2+6C\,a^2\right)}{32a^2c^2-16a\,b^2c+2b^4}}{\left(c\,x^2+bx+a\right)^2}-\frac{6\ln\Big((32a^2c^3-16a}{}$

```
[In] int((C*x^2+A)/(c*x^2+b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (c*(6*A*c^2+2*C*a*c+C*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3+3/2*b*(6*A*c^2+2*
C*a*c+C*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2+(10*A*a*c^2+2*A*b^2*c-2*C*a^2*c
+5*C*a*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x+1/2*b*(10*A*a*c-A*b^2+6*C*a^2)/(16
*a^2*c^2-8*a*b^2*c+b^4))/(c*x^2+b*x+a)^2+2*(6*A*c^2+2*C*a*c+C*b^2)/(16*a^2*
c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 589 vs. 2(153) = 306.

Time = 0.29 (sec) , antiderivative size = 1199, normalized size of antiderivative = 7.45

$$\int \frac{A+Cx^2}{\left(a+bx+cx^2\right)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [1/2*(6*C*a^2*b^3 - A*b^5 - 40*A*a^2*b*c^2 + 2*(C*b^4*c - 2*C*a*b^2*c^2 - 2
4*A*a*c^4 - 2*(4*C*a^2 - 3*A*b^2)*c^3)*x^3 + 3*(C*b^5 - 2*C*a*b^3*c - 24*A*
a*b*c^3 - 2*(4*C*a^2*b - 3*A*b^3)*c^2)*x^2 + 2*(C*a^2*b^2 + 2*C*a^3*c + 6*A
*a^2*c^2 + (C*b^2*c^2 + 2*C*a*c^3 + 6*A*c^4)*x^4 + 2*(C*b^3*c + 2*C*a*b*c^2
 + 6*A*b*c^3)*x^3 + (C*b^4 + 4*C*a*b^2*c + 12*A*a*c^3 + 2*(2*C*a^2 + 3*A*b^
2)*c^2)*x^2 + 2*(C*a*b^3 + 2*C*a^2*b*c + 6*A*a*b*c^2)*x)*sqrt(b^2 - 4*a*c)*
log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*
x^2 + b*x + a)) - 2*(12*C*a^3*b - 7*A*a*b^3)*c + 2*(5*C*a*b^4 - 40*A*a^2*c^
3 + 2*(4*C*a^3 + A*a*b^2)*c^2 - 2*(11*C*a^2*b^2 - A*b^4)*c)*x)/(a^2*b^6 - 1
2*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^
2*b^2*c^4 - 64*a^3*c^5)*x^4 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64
*a^3*b*c^4)*x^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128
*a^4*c^4)*x^2 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x)
, 1/2*(6*C*a^2*b^3 - A*b^5 - 40*A*a^2*b*c^2 + 2*(C*b^4*c - 2*C*a*b^2*c^2 -
24*A*a*c^4 - 2*(4*C*a^2 - 3*A*b^2)*c^3)*x^3 + 3*(C*b^5 - 2*C*a*b^3*c - 24*A
*a*b*c^3 - 2*(4*C*a^2*b - 3*A*b^3)*c^2)*x^2 - 4*(C*a^2*b^2 + 2*C*a^3*c + 6*
```

```
A*a^2*c^2 + (C*b^2*c^2 + 2*C*a*c^3 + 6*A*c^4)*x^4 + 2*(C*b^3*c + 2*C*a*b*c^
2 + 6*A*b*c^3)*x^3 + (C*b^4 + 4*C*a*b^2*c + 12*A*a*c^3 + 2*(2*C*a^2 + 3*A*b
^2)*c^2)*x^2 + 2*(C*a*b^3 + 2*C*a^2*b*c + 6*A*a*b*c^2)*x)*sqrt(-b^2 + 4*a*c
)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - 2*(12*C*a^3*b - 7
*A*a*b^3)*c + 2*(5*C*a*b^4 - 40*A*a^2*c^3 + 2*(4*C*a^3 + A*a*b^2)*c^2 - 2*(
11*C*a^2*b^2 - A*b^4)*c)*x)/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a
^5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4 + 2*(b^
7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^3 + (b^8 - 10*a*b^6*c
 + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + 2*(a*b^7 - 12*a^2*b
^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 774 vs. $2(150) = 300$.

Time = 1.17 (sec) , antiderivative size = 774, normalized size of antiderivative = 4.81

$$\int \frac{A+Cx^2}{(a+bx+cx^2)^3}\,dx = -\sqrt{-\frac{1}{(4ac-b^2)^5}}\cdot\big(6Ac^2+2Cac$$

$$+Cb^2\big)\log\left(x+\frac{6Abc^2+2Cabc+Cb^3-64a^3c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}\cdot(6Ac^2+2Cac+Cb^2)+48a^2b^2c^2\sqrt{-\frac{1}{(4ac-b^2}}}{1}\right.$$

$$+\sqrt{-\frac{1}{(4ac-b^2)^5}}\cdot\big(6Ac^2+2Cac$$

$$+Cb^2\big)\log\left(x+\frac{6Abc^2+2Cabc+Cb^3+64a^3c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}\cdot(6Ac^2+2Cac+Cb^2)-48a^2b^2c^2\sqrt{-\frac{1}{(4ac-b^2}}}{1}\right.$$

$$+\frac{10Aabc-Ab^3+6Ca^2b+x^3\cdot(12Ac^3+4Cac^2+2Cb^2c)+x^2\cdot(18Abc^2+6Cabc+3Cb^3)+}{32a^4c^2-16a^3b^2c+2a^2b^4+x^4\cdot(32a^2c^4-16ab^2c^3+2b^4c^2)+x^3\cdot(64a^2bc^3-32ab^3c^2+4b^5c)+x^2\cdot(64a}$$

```
[In] integrate((C*x**2+A)/(c*x**2+b*x+a)**3,x)
```

```
[Out] -sqrt(-1/(4*a*c - b**2)**5)*(6*A*c**2 + 2*C*a*c + C*b**2)*log(x + (6*A*b*c*
*2 + 2*C*a*b*c + C*b**3 - 64*a**3*c**3*sqrt(-1/(4*a*c - b**2)**5)*(6*A*c**2
 + 2*C*a*c + C*b**2) + 48*a**2*b**2*c**2*sqrt(-1/(4*a*c - b**2)**5)*(6*A*c*
*2 + 2*C*a*c + C*b**2) - 12*a*b**4*c*sqrt(-1/(4*a*c - b**2)**5)*(6*A*c**2 +
 2*C*a*c + C*b**2) + b**6*sqrt(-1/(4*a*c - b**2)**5)*(6*A*c**2 + 2*C*a*c +
C*b**2))/(12*A*c**3 + 4*C*a*c**2 + 2*C*b**2*c)) + sqrt(-1/(4*a*c - b**2)**5
)*(6*A*c**2 + 2*C*a*c + C*b**2)*log(x + (6*A*b*c**2 + 2*C*a*b*c + C*b**3 +
64*a**3*c**3*sqrt(-1/(4*a*c - b**2)**5)*(6*A*c**2 + 2*C*a*c + C*b**2) - 48*
a**2*b**2*c**2*sqrt(-1/(4*a*c - b**2)**5)*(6*A*c**2 + 2*C*a*c + C*b**2) + 1
2*a*b**4*c*sqrt(-1/(4*a*c - b**2)**5)*(6*A*c**2 + 2*C*a*c + C*b**2) - b**6*
sqrt(-1/(4*a*c - b**2)**5)*(6*A*c**2 + 2*C*a*c + C*b**2))/(12*A*c**3 + 4*C*
```

```
a*c**2 + 2*C*b**2*c)) + (10*A*a*b*c - A*b**3 + 6*C*a**2*b + x**3*(12*A*c**3
 + 4*C*a*c**2 + 2*C*b**2*c) + x**2*(18*A*b*c**2 + 6*C*a*b*c + 3*C*b**3) + x
*(20*A*a*c**2 + 4*A*b**2*c - 4*C*a**2*c + 10*C*a*b**2))/(32*a**4*c**2 - 16*
a**3*b**2*c + 2*a**2*b**4 + x**4*(32*a**2*c**4 - 16*a*b**2*c**3 + 2*b**4*c*
*2) + x**3*(64*a**2*b*c**3 - 32*a*b**3*c**2 + 4*b**5*c) + x**2*(64*a**3*c**
3 - 12*a*b**4*c + 2*b**6) + x*(64*a**3*b*c**2 - 32*a**2*b**3*c + 4*a*b**5))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A + Cx^2}{\left(a + bx + cx^2\right)^3}\, dx = \text{Exception raised: ValueError}$$

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.35

$$\int \frac{A + Cx^2}{\left(a + bx + cx^2\right)^3}\, dx = \frac{2\,\left(Cb^2 + 2\,Cac + 6\,Ac^2\right)\arctan\left(\frac{2\,cx + b}{\sqrt{-b^2 + 4\,ac}}\right)}{\left(b^4 - 8\,ab^2c + 16\,a^2c^2\right)\sqrt{-b^2 + 4\,ac}} + \frac{2\,Cb^2cx^3 + 4\,Cac^2x^3 + 12\,Ac^3x^3 + 3\,Cb^3x^2 + 6\,Cabcx^2 + 18\,Abc^2x^2 + 10\,Cab^2x - 4\,Ca^2cx + 4\,Ab^2cx +}{2\,\left(b^4 - 8\,ab^2c + 16\,a^2c^2\right)\left(cx^2 + bx + a\right)^2}$$

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^3,x, algorithm="giac")
```

```
[Out] 2*(C*b^2 + 2*C*a*c + 6*A*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^4
- 8*a*b^2*c + 16*a^2*c^2)*sqrt(-b^2 + 4*a*c)) + 1/2*(2*C*b^2*c*x^3 + 4*C*a*
c^2*x^3 + 12*A*c^3*x^3 + 3*C*b^3*x^2 + 6*C*a*b*c*x^2 + 18*A*b*c^2*x^2 + 10*
C*a*b^2*x - 4*C*a^2*c*x + 4*A*b^2*c*x + 20*A*a*c^2*x + 6*C*a^2*b - A*b^3 +
10*A*a*b*c)/((b^4 - 8*a*b^2*c + 16*a^2*c^2)*(c*x^2 + b*x + a)^2)
```

Mupad [B] (verification not implemented)

Time = 13.10 (sec) , antiderivative size = 401, normalized size of antiderivative = 2.49

$$\int \frac{A+Cx^2}{\left(a+bx+cx^2\right)^3}\,dx$$
$$=\frac{\frac{6\,C\,a^2\,b+10\,A\,c\,a\,b-A\,b^3}{2\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{x\,\left(-2\,C\,a^2\,c+5\,C\,a\,b^2+10\,A\,a\,c^2+2\,A\,b^2\,c\right)}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}+\frac{3\,b\,x^2\,\left(C\,b^2+6\,A\,c^2+2\,C\,a\,c\right)}{2\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{c\,x^3\,\left(C\,b^2+6\,A\,c^2+2\,C\,a\,c\right)}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}}{x^2\,\left(b^2+2\,a\,c\right)+a^2+c^2\,x^4+2\,a\,b\,x+2\,b\,c\,x^3}$$
$$+\frac{2\,\mathrm{atan}\left(\frac{\left(\frac{\left(16\,a^2\,b\,c^2-8\,a\,b^3\,c+b^5\right)\,\left(C\,b^2+6\,A\,c^2+2\,C\,a\,c\right)}{{\left(4\,a\,c-b^2\right)}^{5/2}\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{2\,c\,x\,\left(C\,b^2+6\,A\,c^2+2\,C\,a\,c\right)}{{\left(4\,a\,c-b^2\right)}^{5/2}}\right)\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}{C\,b^2+6\,A\,c^2+2\,C\,a\,c}\right)\,(C\,b^2+6\,A\,c^2+2\,C\,a}{{\left(4\,a\,c-b^2\right)}^{5/2}}$$

```
[In] int((A + C*x^2)/(a + b*x + c*x^2)^3,x)
```

```
[Out] ((6*C*a^2*b - A*b^3 + 10*A*a*b*c)/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x*(
10*A*a*c^2 + 2*A*b^2*c + 5*C*a*b^2 - 2*C*a^2*c))/(b^4 + 16*a^2*c^2 - 8*a*b^
2*c) + (3*b*x^2*(6*A*c^2 + C*b^2 + 2*C*a*c))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2
*c)) + (c*x^3*(6*A*c^2 + C*b^2 + 2*C*a*c))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))/
(x^2*(2*a*c + b^2) + a^2 + c^2*x^4 + 2*a*b*x + 2*b*c*x^3) + (2*atan(((((b^5
 + 16*a^2*b*c^2 - 8*a*b^3*c)*(6*A*c^2 + C*b^2 + 2*C*a*c))/((4*a*c - b^2)^(5
/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (2*c*x*(6*A*c^2 + C*b^2 + 2*C*a*c))/(
4*a*c - b^2)^(5/2))*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(6*A*c^2 + C*b^2 + 2*C*
a*c))*(6*A*c^2 + C*b^2 + 2*C*a*c))/(4*a*c - b^2)^(5/2)
```

3.147 $\int \frac{A+Cx^2}{\left(a+bx+cx^2\right)^4}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 206

$$\begin{aligned}\int \frac{A+Cx^2}{\left(a+bx+cx^2\right)^4}\,dx= & -\frac{bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3} \\ & +\frac{\left(5Ac+\left(a+\frac{b^2}{c}\right)C\right)(b+2cx)}{3\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^2}-\frac{2\left(5Ac^2+\left(b^2+ac\right)C\right)(b+2cx)}{\left(b^2-4ac\right)^3\left(a+bx+cx^2\right)} \\ & +\frac{8c\left(5Ac^2+\left(b^2+ac\right)C\right)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{7/2}}\end{aligned}$$

```
[Out] 1/3*(-b*c*(A+a*C/c)-(2*A*c^2+(-2*a*c+b^2)*C)*x)/c/(-4*a*c+b^2)/(c*x^2+b*x+a
)^3+1/3*(5*A*c+(a+b^2/c)*C)*(2*c*x+b)/(-4*a*c+b^2)^2/(c*x^2+b*x+a)^2-2*(5*A
*c^2+(a*c+b^2)*C)*(2*c*x+b)/(-4*a*c+b^2)^3/(c*x^2+b*x+a)+8*c*(5*A*c^2+(a*c+
b^2)*C)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(7/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1674, 12, 628, 632, 212}

$$\int \frac{A+Cx^2}{(a+bx+cx^2)^4}\,dx = \frac{8c(C(ac+b^2)+5Ac^2)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{7/2}} - \frac{2(b+2cx)\,(C(ac+b^2)+5Ac^2)}{(b^2-4ac)^3\,(a+bx+cx^2)} - \frac{x(C(b^2-2ac)+2Ac^2)+bc\left(\frac{aC}{c}+A\right)}{3c\,(b^2-4ac)\,(a+bx+cx^2)^3} + \frac{(b+2cx)\left(C\left(a+\frac{b^2}{c}\right)+5Ac\right)}{3\,(b^2-4ac)^2\,(a+bx+cx^2)^2}$$

```
[In] Int[(A + C*x^2)/(a + b*x + c*x^2)^4,x]
```

```
[Out] -1/3*(b*c*(A + (a*C)/c) + (2*A*c^2 + (b^2 - 2*a*c)*C)*x)/(c*(b^2 - 4*a*c)*(
a + b*x + c*x^2)^3) + ((5*A*c + (a + b^2/c)*C)*(b + 2*c*x))/(3*(b^2 - 4*a*c
)^2*(a + b*x + c*x^2)^2) - (2*(5*A*c^2 + (b^2 + a*c)*C)*(b + 2*c*x))/((b^2
- 4*a*c)^3*(a + b*x + c*x^2)) + (8*c*(5*A*c^2 + (b^2 + a*c)*C)*ArcTanh[(b +
 2*c*x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(7/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3}-\frac{\int \frac{2\left(5Ac+\left(a+\frac{b^2}{c}\right)C\right)}{\left(a+bx+cx^2\right)^3}\,dx}{3\left(b^2-4ac\right)}\\
&= -\frac{bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3}-\frac{\left(2\left(5Ac+\left(a+\frac{b^2}{c}\right)C\right)\right)\int \frac{1}{\left(a+bx+cx^2\right)^3}\,dx}{3\left(b^2-4ac\right)}\\
&= -\frac{bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3}+\frac{\left(5Ac+\left(a+\frac{b^2}{c}\right)C\right)(b+2cx)}{3\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^2}\\
&\quad+\frac{\left(2\left(5Ac^2+\left(b^2+ac\right)C\right)\right)\int \frac{1}{\left(a+bx+cx^2\right)^2}\,dx}{\left(b^2-4ac\right)^2}\\
&= -\frac{bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3}+\frac{\left(5Ac+\left(a+\frac{b^2}{c}\right)C\right)(b+2cx)}{3\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^2}\\
&\quad-\frac{2\left(5Ac^2+\left(b^2+ac\right)C\right)(b+2cx)}{\left(b^2-4ac\right)^3\left(a+bx+cx^2\right)}-\frac{\left(4c\left(5Ac^2+\left(b^2+ac\right)C\right)\right)\int \frac{1}{a+bx+cx^2}\,dx}{\left(b^2-4ac\right)^3}\\
&= -\frac{bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3}\\
&\quad+\frac{\left(5Ac+\left(a+\frac{b^2}{c}\right)C\right)(b+2cx)}{3\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^2}-\frac{2\left(5Ac^2+\left(b^2+ac\right)C\right)(b+2cx)}{\left(b^2-4ac\right)^3\left(a+bx+cx^2\right)}\\
&\quad+\frac{\left(8c\left(5Ac^2+\left(b^2+ac\right)C\right)\right)\operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx\right)}{\left(b^2-4ac\right)^3}
\end{aligned}
$$

$$= -\frac{bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3}+\frac{\left(5Ac+\left(a+\frac{b^2}{c}\right)C\right)(b+2cx)}{3\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^2}$$

$$-\frac{2\left(5Ac^2+\left(b^2+ac\right)C\right)(b+2cx)}{\left(b^2-4ac\right)^3\left(a+bx+cx^2\right)}+\frac{8c\left(5Ac^2+\left(b^2+ac\right)C\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{7/2}}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.99

$$\int \frac{A+Cx^2}{\left(a+bx+cx^2\right)^4}\,dx = \frac{1}{3}\left(\frac{\left(5Ac^2+\left(b^2+ac\right)C\right)(b+2cx)}{c\left(b^2-4ac\right)^2(a+x(b+cx))^2}-\frac{6\left(5Ac^2+\left(b^2+ac\right)C\right)(b+2cx)}{\left(b^2-4ac\right)^3(a+x(b+cx))}+\frac{b^2Cx+aC(b-2cx)+Ac(b+2cx)}{c\left(-b^2+4ac\right)(a+x(b+cx))^3}+\frac{24c\left(5Ac^2+\left(b^2+ac\right)C\right)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\left(-b^2+4ac\right)^{7/2}}\right)$$

```
[In] Integrate[(A + C*x^2)/(a + b*x + c*x^2)^4,x]
```

```
[Out] (((5*A*c^2 + (b^2 + a*c)*C)*(b + 2*c*x))/(c*(b^2 - 4*a*c)^2*(a + x*(b + c*x
))^2) - (6*(5*A*c^2 + (b^2 + a*c)*C)*(b + 2*c*x))/((b^2 - 4*a*c)^3*(a + x*(
b + c*x))) + (b^2*C*x + a*C*(b - 2*c*x) + A*c*(b + 2*c*x))/(c*(-b^2 + 4*a*c
)*(a + x*(b + c*x))^3) + (24*c*(5*A*c^2 + (b^2 + a*c)*C)*ArcTan[(b + 2*c*x)
/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(7/2))/3
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 509 vs. 2(200) = 400.

Time = 0.65 (sec) , antiderivative size = 510, normalized size of antiderivative = 2.48

method	result
default	$\frac{\frac{4c^3\left(5A\,c^2+Cac+C\,b^2\right)x^5}{64a^3c^3-48a^2b^2c^2+12a\,b^4c-b^6}+\frac{10c^2\left(5A\,c^2+Cac+C\,b^2\right)b\,x^4}{64a^3c^3-48a^2b^2c^2+12a\,b^4c-b^6}+\frac{2\left(16ac+11b^2\right)c\left(5A\,c^2+Cac+C\,b^2\right)x^3}{3\left(64a^3c^3-48a^2b^2c^2+12a\,b^4c-b^6\right)}+\frac{b\left(16ac+b^2\right)\left(5A\,c^2+Cac+C\,b^2\right)x^2}{64a^3c^3-48a^2b^2c^2+12a\,b^4c-b^6}+}{\left(c\,x^2+bx+a\right)^3}$
risch	Expression too large to display

```
[In] int((C*x^2+A)/(c*x^2+b*x+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] (4*c^3*(5*A*c^2+C*a*c+C*b^2)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)*x^5
+10*c^2*(5*A*c^2+C*a*c+C*b^2)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)*b*
x^4+2/3*(16*a*c+11*b^2)*c*(5*A*c^2+C*a*c+C*b^2)/(64*a^3*c^3-48*a^2*b^2*c^2+
12*a*b^4*c-b^6)*x^3+b*(16*a*c+b^2)*(5*A*c^2+C*a*c+C*b^2)/(64*a^3*c^3-48*a^2
*b^2*c^2+12*a*b^4*c-b^6)*x^2+(44*A*a^2*c^3+18*A*a*b^2*c^2-A*b^4*c-4*C*a^3*c
^2+22*C*a^2*b^2*c+C*a*b^4)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)*x+1/3
*(66*A*a^2*c^2-13*A*a*b^2*c+A*b^4+26*C*a^3*c+C*a^2*b^2)*b/(64*a^3*c^3-48*a^
2*b^2*c^2+12*a*b^4*c-b^6))/(c*x^2+b*x+a)^3+8*c*(5*A*c^2+C*a*c+C*b^2)/(64*a^
3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*
a*c-b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1041 vs. 2(198) = 396.

Time = 0.32 (sec) , antiderivative size = 2103, normalized size of antiderivative = 10.21

$$\int \frac{A+Cx^2}{(a+bx+cx^2)^4}\,dx = \text{Too large to display}$$

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^4,x, algorithm="fricas")
```

```
[Out] [-1/3*(C*a^2*b^5 + A*b^7 - 264*A*a^3*b*c^3 + 12*(C*b^4*c^3 - 3*C*a*b^2*c^4
- 20*A*a*c^6 - (4*C*a^2 - 5*A*b^2)*c^5)*x^5 + 30*(C*b^5*c^2 - 3*C*a*b^3*c^3
 - 20*A*a*b*c^5 - (4*C*a^2*b - 5*A*b^3)*c^4)*x^4 + 2*(11*C*b^6*c - 17*C*a*b
^4*c^2 - 320*A*a^2*c^5 - 4*(16*C*a^3 + 35*A*a*b^2)*c^4 - (92*C*a^2*b^2 - 55
*A*b^4)*c^3)*x^3 - 2*(52*C*a^4*b - 59*A*a^2*b^3)*c^2 + 3*(C*b^7 + 13*C*a*b^
5*c - 320*A*a^2*b*c^4 - 4*(16*C*a^3*b - 15*A*a*b^3)*c^3 - (52*C*a^2*b^3 - 5
*A*b^5)*c^2)*x^2 + 12*(C*a^3*b^2*c + C*a^4*c^2 + 5*A*a^3*c^3 + (C*b^2*c^4 +
 C*a*c^5 + 5*A*c^6)*x^6 + 3*(C*b^3*c^3 + C*a*b*c^4 + 5*A*b*c^5)*x^5 + 3*(C*
b^4*c^2 + 2*C*a*b^2*c^3 + 5*A*a*c^5 + (C*a^2 + 5*A*b^2)*c^4)*x^4 + (C*b^5*c
 + 7*C*a*b^3*c^2 + 30*A*a*b*c^4 + (6*C*a^2*b + 5*A*b^3)*c^3)*x^3 + 3*(C*a*b
^4*c + 2*C*a^2*b^2*c^2 + 5*A*a^2*c^4 + (C*a^3 + 5*A*a*b^2)*c^3)*x^2 + 3*(C*
a^2*b^3*c + C*a^3*b*c^2 + 5*A*a^2*b*c^3)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^
2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a
)) + (22*C*a^3*b^3 - 17*A*a*b^5)*c + 3*(C*a*b^6 - 176*A*a^3*c^4 + 4*(4*C*a^
4 - 7*A*a^2*b^2)*c^3 - 2*(46*C*a^3*b^2 - 11*A*a*b^4)*c^2 + (18*C*a^2*b^4 -
A*b^6)*c)*x)/(a^3*b^8 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3 + 2
56*a^7*c^4 + (b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6 + 2
56*a^4*c^7)*x^6 + 3*(b^9*c^2 - 16*a*b^7*c^3 + 96*a^2*b^5*c^4 - 256*a^3*b^3*
c^5 + 256*a^4*b*c^6)*x^5 + 3*(b^10*c - 15*a*b^8*c^2 + 80*a^2*b^6*c^3 - 160*
a^3*b^4*c^4 + 256*a^5*c^6)*x^4 + (b^11 - 10*a*b^9*c + 320*a^3*b^5*c^3 - 128
0*a^4*b^3*c^4 + 1536*a^5*b*c^5)*x^3 + 3*(a*b^10 - 15*a^2*b^8*c + 80*a^3*b^6
*c^2 - 160*a^4*b^4*c^3 + 256*a^6*c^5)*x^2 + 3*(a^2*b^9 - 16*a^3*b^7*c + 96*
a^4*b^5*c^2 - 256*a^5*b^3*c^3 + 256*a^6*b*c^4)*x), -1/3*(C*a^2*b^5 + A*b^7
```

```
- 264*A*a^3*b*c^3 + 12*(C*b^4*c^3 - 3*C*a*b^2*c^4 - 20*A*a*c^6 - (4*C*a^2 -
 5*A*b^2)*c^5)*x^5 + 30*(C*b^5*c^2 - 3*C*a*b^3*c^3 - 20*A*a*b*c^5 - (4*C*a^
2*b - 5*A*b^3)*c^4)*x^4 + 2*(11*C*b^6*c - 17*C*a*b^4*c^2 - 320*A*a^2*c^5 -
4*(16*C*a^3 + 35*A*a*b^2)*c^4 - (92*C*a^2*b^2 - 55*A*b^4)*c^3)*x^3 - 2*(52*
C*a^4*b - 59*A*a^2*b^3)*c^2 + 3*(C*b^7 + 13*C*a*b^5*c - 320*A*a^2*b*c^4 - 4
*(16*C*a^3*b - 15*A*a*b^3)*c^3 - (52*C*a^2*b^3 - 5*A*b^5)*c^2)*x^2 - 24*(C*
a^3*b^2*c + C*a^4*c^2 + 5*A*a^3*c^3 + (C*b^2*c^4 + C*a*c^5 + 5*A*c^6)*x^6 +
 3*(C*b^3*c^3 + C*a*b*c^4 + 5*A*b*c^5)*x^5 + 3*(C*b^4*c^2 + 2*C*a*b^2*c^3 +
 5*A*a*c^5 + (C*a^2 + 5*A*b^2)*c^4)*x^4 + (C*b^5*c + 7*C*a*b^3*c^2 + 30*A*a
*b*c^4 + (6*C*a^2*b + 5*A*b^3)*c^3)*x^3 + 3*(C*a*b^4*c + 2*C*a^2*b^2*c^2 +
5*A*a^2*c^4 + (C*a^3 + 5*A*a*b^2)*c^3)*x^2 + 3*(C*a^2*b^3*c + C*a^3*b*c^2 +
 5*A*a^2*b*c^3)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b
)/(b^2 - 4*a*c)) + (22*C*a^3*b^3 - 17*A*a*b^5)*c + 3*(C*a*b^6 - 176*A*a^3*c
^4 + 4*(4*C*a^4 - 7*A*a^2*b^2)*c^3 - 2*(46*C*a^3*b^2 - 11*A*a*b^4)*c^2 + (1
8*C*a^2*b^4 - A*b^6)*c)*x)/(a^3*b^8 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a
^6*b^2*c^3 + 256*a^7*c^4 + (b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a
^3*b^2*c^6 + 256*a^4*c^7)*x^6 + 3*(b^9*c^2 - 16*a*b^7*c^3 + 96*a^2*b^5*c^4
- 256*a^3*b^3*c^5 + 256*a^4*b*c^6)*x^5 + 3*(b^10*c - 15*a*b^8*c^2 + 80*a^2*
b^6*c^3 - 160*a^3*b^4*c^4 + 256*a^5*c^6)*x^4 + (b^11 - 10*a*b^9*c + 320*a^3
*b^5*c^3 - 1280*a^4*b^3*c^4 + 1536*a^5*b*c^5)*x^3 + 3*(a*b^10 - 15*a^2*b^8*
c + 80*a^3*b^6*c^2 - 160*a^4*b^4*c^3 + 256*a^6*c^5)*x^2 + 3*(a^2*b^9 - 16*a
^3*b^7*c + 96*a^4*b^5*c^2 - 256*a^5*b^3*c^3 + 256*a^6*b*c^4)*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1224 vs. $2(196) = 392$.

Time = 1.99 (sec) , antiderivative size = 1224, normalized size of antiderivative = 5.94

$$\int \frac{A + Cx^2}{\left(a + bx + cx^2\right)^4}\, dx = -4c\sqrt{-\frac{1}{\left(4ac - b^2\right)^7}} \cdot (5Ac^2 + Cac$$

$$+Cb^2) \log\left(x + \frac{20Abc^3 + 4Cabc^2 + 4Cb^3c - 1024a^4c^5\sqrt{-\frac{1}{(4ac-b^2)^7}} \cdot (5Ac^2 + Cac + Cb^2) + 1024a^3b^2c^4\sqrt{}}{}\right.$$

$$+ 4c\sqrt{-\frac{1}{\left(4ac - b^2\right)^7}} \cdot (5Ac^2 + Cac$$

$$+Cb^2) \log\left(x + \frac{20Abc^3 + 4Cabc^2 + 4Cb^3c + 1024a^4c^5\sqrt{-\frac{1}{(4ac-b^2)^7}} \cdot (5Ac^2 + Cac + Cb^2) - 1024a^3b^2c^4\sqrt{}}{}\right.$$

$$+ \frac{66Aa^2bc^2 - 13Aab^3c + Ab^5 + 26Ca^3bc + Ca^2b^3 + x^5 \cdot (60Ac^5 + 12Cac^4 + 12Cb^2c}{192a^6c^3 - 144a^5b^2c^2 + 36a^4b^4c - 3a^3b^6 + x^6 \cdot (192a^3c^6 - 144a^2b^2c^5 + 36ab^4c^4 - 3b^6c^3) + x^5 \cdot (576a^3bc^5}$$

```
[In] integrate((C*x**2+A)/(c*x**2+b*x+a)**4,x)
```

```
[Out] -4*c*sqrt(-1/(4*a*c - b**2)**7)*(5*A*c**2 + C*a*c + C*b**2)*log(x + (20*A*b
*c**3 + 4*C*a*b*c**2 + 4*C*b**3*c - 1024*a**4*c**5*sqrt(-1/(4*a*c - b**2)**
7)*(5*A*c**2 + C*a*c + C*b**2) + 1024*a**3*b**2*c**4*sqrt(-1/(4*a*c - b**2)
**7)*(5*A*c**2 + C*a*c + C*b**2) - 384*a**2*b**4*c**3*sqrt(-1/(4*a*c - b**2
)**7)*(5*A*c**2 + C*a*c + C*b**2) + 64*a*b**6*c**2*sqrt(-1/(4*a*c - b**2)**
7)*(5*A*c**2 + C*a*c + C*b**2) - 4*b**8*c*sqrt(-1/(4*a*c - b**2)**7)*(5*A*c
**2 + C*a*c + C*b**2))/(40*A*c**4 + 8*C*a*c**3 + 8*C*b**2*c**2)) + 4*c*sqrt
(-1/(4*a*c - b**2)**7)*(5*A*c**2 + C*a*c + C*b**2)*log(x + (20*A*b*c**3 + 4
*C*a*b*c**2 + 4*C*b**3*c + 1024*a**4*c**5*sqrt(-1/(4*a*c - b**2)**7)*(5*A*c
**2 + C*a*c + C*b**2) - 1024*a**3*b**2*c**4*sqrt(-1/(4*a*c - b**2)**7)*(5*A
*c**2 + C*a*c + C*b**2) + 384*a**2*b**4*c**3*sqrt(-1/(4*a*c - b**2)**7)*(5*
A*c**2 + C*a*c + C*b**2) - 64*a*b**6*c**2*sqrt(-1/(4*a*c - b**2)**7)*(5*A*c
**2 + C*a*c + C*b**2) + 4*b**8*c*sqrt(-1/(4*a*c - b**2)**7)*(5*A*c**2 + C*a
*c + C*b**2))/(40*A*c**4 + 8*C*a*c**3 + 8*C*b**2*c**2)) + (66*A*a**2*b*c**2
 - 13*A*a*b**3*c + A*b**5 + 26*C*a**3*b*c + C*a**2*b**3 + x**5*(60*A*c**5 +
 12*C*a*c**4 + 12*C*b**2*c**3) + x**4*(150*A*b*c**4 + 30*C*a*b*c**3 + 30*C*
b**3*c**2) + x**3*(160*A*a*c**4 + 110*A*b**2*c**3 + 32*C*a**2*c**3 + 54*C*a
*b**2*c**2 + 22*C*b**4*c) + x**2*(240*A*a*b*c**3 + 15*A*b**3*c**2 + 48*C*a*
*2*b*c**2 + 51*C*a*b**3*c + 3*C*b**5) + x*(132*A*a**2*c**3 + 54*A*a*b**2*c*
*2 - 3*A*b**4*c - 12*C*a**3*c**2 + 66*C*a**2*b**2*c + 3*C*a*b**4))/(192*a**
6*c**3 - 144*a**5*b**2*c**2 + 36*a**4*b**4*c - 3*a**3*b**6 + x**6*(192*a**3
*c**6 - 144*a**2*b**2*c**5 + 36*a*b**4*c**4 - 3*b**6*c**3) + x**5*(576*a**3
*b*c**5 - 432*a**2*b**3*c**4 + 108*a*b**5*c**3 - 9*b**7*c**2) + x**4*(576*a
**4*c**5 + 144*a**3*b**2*c**4 - 324*a**2*b**4*c**3 + 99*a*b**6*c**2 - 9*b**
8*c) + x**3*(1152*a**4*b*c**4 - 672*a**3*b**3*c**3 + 72*a**2*b**5*c**2 + 18
*a*b**7*c - 3*b**9) + x**2*(576*a**5*c**4 + 144*a**4*b**2*c**3 - 324*a**3*b
**4*c**2 + 99*a**2*b**6*c - 9*a*b**8) + x*(576*a**5*b*c**3 - 432*a**4*b**3*
c**2 + 108*a**3*b**5*c - 9*a**2*b**7))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Cx^2}{(a+bx+cx^2)^4}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 407 vs. $2(198) = 396$.

Time = 0.26 (sec) , antiderivative size = 407, normalized size of antiderivative = 1.98

$$\int \frac{A+Cx^2}{(a+bx+cx^2)^4}\,dx = -\frac{8\left(Cb^2c+Cac^2+5\,Ac^3\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^6-12\,ab^4c+48\,a^2b^2c^2-64\,a^3c^3\right)\sqrt{-b^2+4\,ac}} - \frac{12\,Cb^2c^3x^5+12\,Cac^4x^5+60\,Ac^5x^5+30\,Cb^3c^2x^4+30\,Cabc^3x^4+150\,Abc^4x^4+22\,Cb^4cx^3+54\,Cab^2c^2x}{}$$

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^4,x, algorithm="giac")
```

```
[Out] -8*(C*b^2*c + C*a*c^2 + 5*A*c^3)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b
^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3)*sqrt(-b^2 + 4*a*c)) - 1/3*(1
2*C*b^2*c^3*x^5 + 12*C*a*c^4*x^5 + 60*A*c^5*x^5 + 30*C*b^3*c^2*x^4 + 30*C*a
*b*c^3*x^4 + 150*A*b*c^4*x^4 + 22*C*b^4*c*x^3 + 54*C*a*b^2*c^2*x^3 + 32*C*a
^2*c^3*x^3 + 110*A*b^2*c^3*x^3 + 160*A*a*c^4*x^3 + 3*C*b^5*x^2 + 51*C*a*b^3
*c*x^2 + 48*C*a^2*b*c^2*x^2 + 15*A*b^3*c^2*x^2 + 240*A*a*b*c^3*x^2 + 3*C*a*
b^4*x + 66*C*a^2*b^2*c*x - 3*A*b^4*c*x - 12*C*a^3*c^2*x + 54*A*a*b^2*c^2*x
+ 132*A*a^2*c^3*x + C*a^2*b^3 + A*b^5 + 26*C*a^3*b*c - 13*A*a*b^3*c + 66*A*
a^2*b*c^2)/((b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3)*(c*x^2 + b*x +
 a)^3)
```

Mupad [B] (verification not implemented)

Time = 13.25 (sec) , antiderivative size = 698, normalized size of antiderivative = 3.39

$$\int \frac{A+Cx^2}{(a+bx+cx^2)^4}\,dx = -\frac{\frac{26\,C\,a^3\,b\,c+C\,a^2\,b^3+66\,A\,a^2\,b\,c^2-13\,A\,a\,b^3\,c+A\,b^5}{3\,\left(-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6\right)}+\frac{x\,\left(-4\,C\,a^3\,c^2+22\,C\,a^2\,b^2\,c+44\,A\,a^2\,c^3+C\,a\,b^4+18\,A\,a\,b^2\,c^2-A\,b^4\,c\right)}{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}+\frac{2\,x^3\,\left(11\,b^2\,c+16}{3\,\left(-64\,a^3\,c^3+}}{x^2\,\left(3\,c\,a^2+3\,a\,b^2\right)+x^4\,\left(3\,b^2\,c+3\,a\,c^2\right)+a} - \frac{8\,c\,\mathrm{atan}\left(\frac{\left(\frac{8\,c^2\,x\,\left(C\,b^2+5\,A\,c^2+C\,a\,c\right)}{{\left(4\,a\,c-b^2\right)}^{7/2}}+\frac{4\,c\,\left(C\,b^2+5\,A\,c^2+C\,a\,c\right)\,\left(-64\,a^3\,b\,c^3+48\,a^2\,b^3\,c^2-12\,a\,b^5\,c+b^7\right)}{{\left(4\,a\,c-b^2\right)}^{7/2}\,\left(-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6\right)}\right)\,\left(-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6\right)}{4\,C\,b^2\,c+20\,A\,c^3+4\,C\,a\,c^2}\right)}{{\left(4\,a\,c-b^2\right)}^{7/2}}$$

```
[In] int((A + C*x^2)/(a + b*x + c*x^2)^4,x)
```

```
[Out] - ((A*b^5 + C*a^2*b^3 - 13*A*a*b^3*c + 26*C*a^3*b*c + 66*A*a^2*b*c^2)/(3*(b
^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(44*A*a^2*c^3 - 4*C*a^
3*c^2 - A*b^4*c + C*a*b^4 + 18*A*a*b^2*c^2 + 22*C*a^2*b^2*c))/(b^6 - 64*a^3
```

```
*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c) + (2*x^3*(16*a*c^2 + 11*b^2*c)*(5*A*c^2
 + C*b^2 + C*a*c))/(3*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (
x^2*(b^3 + 16*a*b*c)*(5*A*c^2 + C*b^2 + C*a*c))/(b^6 - 64*a^3*c^3 + 48*a^2*
b^2*c^2 - 12*a*b^4*c) + (4*c^3*x^5*(5*A*c^2 + C*b^2 + C*a*c))/(b^6 - 64*a^3
*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c) + (10*b*c^2*x^4*(5*A*c^2 + C*b^2 + C*a*
c))/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c))/(x^2*(3*a*b^2 + 3*a^2
*c) + x^4*(3*a*c^2 + 3*b^2*c) + a^3 + x^3*(b^3 + 6*a*b*c) + c^3*x^6 + 3*b*c
^2*x^5 + 3*a^2*b*x) - (8*c*atan((((8*c^2*x*(5*A*c^2 + C*b^2 + C*a*c))/(4*a*
c - b^2)^(7/2) + (4*c*(5*A*c^2 + C*b^2 + C*a*c)*(b^7 - 64*a^3*b*c^3 + 48*a^
2*b^3*c^2 - 12*a*b^5*c))/((4*a*c - b^2)^(7/2)*(b^6 - 64*a^3*c^3 + 48*a^2*b^
2*c^2 - 12*a*b^4*c)))*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c))/(20
*A*c^3 + 4*C*a*c^2 + 4*C*b^2*c))*(5*A*c^2 + C*b^2 + C*a*c))/(4*a*c - b^2)^(
7/2)
```

3.148 $\int \frac{(d+ex)^3\left(f+gx+hx^2\right)}{a+bx+cx^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 591

$$\int \frac{(d+ex)^3\left(f+gx+hx^2\right)}{a+bx+cx^2}\,dx =$$

$$-\frac{(b^3e^3h-c^3d(3e^2f+3deg+d^2h)-bce^2(beg+3bdh+2aeh)+c^2e(ae(eg+3dh)+b(e^2f+3deg+3d^2h}{c^4}$$

$$+\frac{e(b^2e^2h+c^2(e^2f+3deg+3d^2h)-ce(beg+3bdh+aeh))\,x^2}{2c^3}$$

$$+\frac{e^2(ceg+3cdh-beh)x^3}{3c^2}+\frac{e^3hx^4}{4c}$$

$$-\frac{(2c^5d^3f-b^5e^3h+b^3ce^2(beg+3bdh+5aeh)-c^4d(bd(3ef+dg)+2a(3e^2f+3deg+d^2h))-bc^2e(5a^2e}{}$$

$$+\frac{(c^4d^2(3ef+dg)+b^4e^3h-b^2ce^2(beg+3bdh+3aeh)+c^2e(a^2e^2h+2abe(eg+3dh)+b^2(e^2f+3deg+3}{2c^5}$$

```
[Out] -(b^3*e^3*h-c^3*d*(d^2*h+3*d*e*g+3*e^2*f)-b*c*e^2*(2*a*e*h+3*b*d*h+b*e*g)+c
^2*e*(a*e*(3*d*h+e*g)+b*(3*d^2*h+3*d*e*g+e^2*f)))*x/c^4+1/2*e*(b^2*e^2*h+c^
2*(3*d^2*h+3*d*e*g+e^2*f)-c*e*(a*e*h+3*b*d*h+b*e*g))*x^2/c^3+1/3*e^2*(-b*e*
h+3*c*d*h+c*e*g)*x^3/c^2+1/4*e^3*h*x^4/c+1/2*(c^4*d^2*(d*g+3*e*f)+b^4*e^3*h
-b^2*c*e^2*(3*a*e*h+3*b*d*h+b*e*g)+c^2*e*(a^2*e^2*h+2*a*b*e*(3*d*h+e*g)+b^2
*(3*d^2*h+3*d*e*g+e^2*f))-c^3*(b*d*(d^2*h+3*d*e*g+3*e^2*f)+a*e*(3*d^2*h+3*d
*e*g+e^2*f)))*ln(c*x^2+b*x+a)/c^5-(2*c^5*d^3*f-b^5*e^3*h+b^3*c*e^2*(5*a*e*h
+3*b*d*h+b*e*g)-c^4*d*(b*d*(d*g+3*e*f)+2*a*(d^2*h+3*d*e*g+3*e^2*f))-b*c^2*e
*(5*a^2*e^2*h+4*a*b*e*(3*d*h+e*g)+b^2*(3*d^2*h+3*d*e*g+e^2*f))+c^3*(2*a^2*e
^2*(3*d*h+e*g)+b^2*d*(d^2*h+3*d*e*g+3*e^2*f)+3*a*b*e*(3*d^2*h+3*d*e*g+e^2*f
)))*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c^5/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 591, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1642, 648, 632, 212, 642}

$$\int \frac{(d+ex)^3\,(f+gx+hx^2)}{a+bx+cx^2}\,dx =$$

$$-\frac{\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)(c^3(2a^2e^2(3dh+eg)+3abe(3d^2h+3deg+e^2f)+b^2d(d^2h+3deg+3e^2f))-bc^2e(5}{}$$

$$+\frac{\log(a+bx+cx^2)\,(c^2e(a^2e^2h+2abe(3dh+eg)+b^2(3d^2h+3deg+e^2f))-b^2ce^2(3aeh+3bdh+beg)}{2c^5}$$

$$-\frac{x(c^2e(ae(3dh+eg)+b(3d^2h+3deg+e^2f))-bce^2(2aeh+3bdh+beg)+b^3e^3h+c^3(-d)\,(d^2h+3deg}{c^4}$$

$$+\frac{ex^2(-ce(aeh+3bdh+beg)+b^2e^2h+c^2(3d^2h+3deg+e^2f))}{2c^3}$$

$$+\frac{e^2x^3(-beh+3cdh+ceg)}{3c^2}+\frac{e^3hx^4}{4c}$$

```
[In] Int[((d + e*x)^3*(f + g*x + h*x^2))/(a + b*x + c*x^2),x]
```

```
[Out] -(((b^3*e^3*h - c^3*d*(3*e^2*f + 3*d*e*g + d^2*h) - b*c*e^2*(b*e*g + 3*b*d*
h + 2*a*e*h) + c^2*e*(a*e*(e*g + 3*d*h) + b*(e^2*f + 3*d*e*g + 3*d^2*h)))*x
)/c^4) + (e*(b^2*e^2*h + c^2*(e^2*f + 3*d*e*g + 3*d^2*h) - c*e*(b*e*g + 3*b
*d*h + a*e*h))*x^2)/(2*c^3) + (e^2*(c*e*g + 3*c*d*h - b*e*h)*x^3)/(3*c^2) +
 (e^3*h*x^4)/(4*c) - ((2*c^5*d^3*f - b^5*e^3*h + b^3*c*e^2*(b*e*g + 3*b*d*h
 + 5*a*e*h) - c^4*d*(b*d*(3*e*f + d*g) + 2*a*(3*e^2*f + 3*d*e*g + d^2*h)) -
 b*c^2*e*(5*a^2*e^2*h + 4*a*b*e*(e*g + 3*d*h) + b^2*(e^2*f + 3*d*e*g + 3*d^
2*h)) + c^3*(2*a^2*e^2*(e*g + 3*d*h) + b^2*d*(3*e^2*f + 3*d*e*g + d^2*h) +
3*a*b*e*(e^2*f + 3*d*e*g + 3*d^2*h)))*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]
])/(c^5*Sqrt[b^2 - 4*a*c]) + ((c^4*d^2*(3*e*f + d*g) + b^4*e^3*h - b^2*c*e^
2*(b*e*g + 3*b*d*h + 3*a*e*h) + c^2*e*(a^2*e^2*h + 2*a*b*e*(e*g + 3*d*h) +
b^2*(e^2*f + 3*d*e*g + 3*d^2*h)) - c^3*(b*d*(3*e^2*f + 3*d*e*g + d^2*h) + a
*e*(e^2*f + 3*d*e*g + 3*d^2*h)))*Log[a + b*x + c*x^2])/(2*c^5)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

integral

$$= \int \Bigg(-\frac{b^3e^3h - c^3d(3e^2f + 3deg + d^2h) - bce^2(beg + 3bdh + 2aeh) + c^2e(ae(eg + 3dh) + b(e^2f + 3deg + 3}{c^4} + \frac{e(b^2e^2h + c^2(e^2f + 3deg + 3d^2h) - ce(beg + 3bdh + aeh))\,x}{c^3} + \frac{e^2(ceg + 3cdh - beh)x^2}{c^2} + \frac{e^3hx^3}{c} + \frac{c^4d^3f + ab^3e^3h - ac^3d(3e^2f + 3deg + d^2h) - abce^2(beg + 3bdh + 2aeh) + ac^2e(ae(eg + 3dh) + b(e^2f +}{}$$

$$= -\frac{(b^3e^3h - c^3d(3e^2f + 3deg + d^2h) - bce^2(beg + 3bdh + 2aeh) + c^2e(ae(eg + 3dh) + b(e^2f + 3deg}{c^4} + \frac{e(b^2e^2h + c^2(e^2f + 3deg + 3d^2h) - ce(beg + 3bdh + aeh))\,x^2}{2c^3} + \frac{e^2(ceg + 3cdh - beh)x^3}{3c^2} + \frac{e^3hx^4}{4c} + \int \frac{c^4d^3f + ab^3e^3h - ac^3d(3e^2f + 3deg + d^2h) - abce^2(beg + 3bdh + 2aeh) + ac^2e(ae(eg + 3dh) + b(e^2f + 3deg + 3d^2h)) + (c^4d^2(3ef + dg) + b^4}{a+}$$

$$
\begin{aligned}
= & \\
& -\frac{(b^3e^3h-c^3d(3e^2f+3deg+d^2h)-bce^2(beg+3bdh+2aeh)+c^2e(ae(eg+3dh)+b(e^2f+3de}{c^4} \\
& +\frac{e(b^2e^2h+c^2(e^2f+3deg+3d^2h)-ce(beg+3bdh+aeh))\,x^2}{2c^3} \\
& +\frac{e^2(ceg+3cdh-beh)x^3}{3c^2}+\frac{e^3hx^4}{4c} \\
& +\frac{(c^4d^2(3ef+dg)+b^4e^3h-b^2ce^2(beg+3bdh+3aeh)+c^2e(a^2e^2h+2abe(eg+3dh)+b^2(e^2f+}{2c^5} \\
& +\frac{(2c^5d^3f-b^5e^3h+b^3ce^2(beg+3bdh+5aeh)-c^4d(bd(3ef+dg)+2a(3e^2f+3deg+d^2h))-}{}
\end{aligned}
$$

$$
\begin{aligned}
= & \\
& -\frac{(b^3e^3h-c^3d(3e^2f+3deg+d^2h)-bce^2(beg+3bdh+2aeh)+c^2e(ae(eg+3dh)+b(e^2f+3de}{c^4} \\
& +\frac{e(b^2e^2h+c^2(e^2f+3deg+3d^2h)-ce(beg+3bdh+aeh))\,x^2}{2c^3} \\
& +\frac{e^2(ceg+3cdh-beh)x^3}{3c^2}+\frac{e^3hx^4}{4c} \\
& +\frac{(c^4d^2(3ef+dg)+b^4e^3h-b^2ce^2(beg+3bdh+3aeh)+c^2e(a^2e^2h+2abe(eg+3dh)+b^2(e^2f+}{2c^5} \\
& -\frac{(2c^5d^3f-b^5e^3h+b^3ce^2(beg+3bdh+5aeh)-c^4d(bd(3ef+dg)+2a(3e^2f+3deg+d^2h))-}{}
\end{aligned}
$$

$$
\begin{aligned}
= & \\
& -\frac{(b^3e^3h-c^3d(3e^2f+3deg+d^2h)-bce^2(beg+3bdh+2aeh)+c^2e(ae(eg+3dh)+b(e^2f+3de}{c^4} \\
& +\frac{e(b^2e^2h+c^2(e^2f+3deg+3d^2h)-ce(beg+3bdh+aeh))\,x^2}{2c^3} \\
& +\frac{e^2(ceg+3cdh-beh)x^3}{3c^2}+\frac{e^3hx^4}{4c} \\
& -\frac{(2c^5d^3f-b^5e^3h+b^3ce^2(beg+3bdh+5aeh)-c^4d(bd(3ef+dg)+2a(3e^2f+3deg+d^2h))-}{} \\
& +\frac{(c^4d^2(3ef+dg)+b^4e^3h-b^2ce^2(beg+3bdh+3aeh)+c^2e(a^2e^2h+2abe(eg+3dh)+b^2(e^2f+}{2c^5}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 585, normalized size of antiderivative = 0.99

$$\int \frac{(d+ex)^3\left(f+gx+hx^2\right)}{a+bx+cx^2}\,dx$$

$$= \frac{12c(-b^3e^3h+c^3d(3e^2f+3deg+d^2h)+bce^2(beg+3bdh+2aeh)-c^2e(ae(eg+3dh)+b(e^2f+3deg+3}{}$$

```
[In] Integrate[((d + e*x)^3*(f + g*x + h*x^2))/(a + b*x + c*x^2),x]
```

```
[Out] (12*c*(-(b^3*e^3*h) + c^3*d*(3*e^2*f + 3*d*e*g + d^2*h) + b*c*e^2*(b*e*g +
3*b*d*h + 2*a*e*h) - c^2*e*(a*e*(e*g + 3*d*h) + b*(e^2*f + 3*d*e*g + 3*d^2*
h)))*x + 6*c^2*e*(b^2*e^2*h + c^2*(e^2*f + 3*d*e*g + 3*d^2*h) - c*e*(b*e*g
+ 3*b*d*h + a*e*h))*x^2 + 4*c^3*e^2*(c*e*g + 3*c*d*h - b*e*h)*x^3 + 3*c^4*e
^3*h*x^4 + (12*(2*c^5*d^3*f - b^5*e^3*h + b^3*c*e^2*(b*e*g + 3*b*d*h + 5*a*
e*h) - c^4*d*(b*d*(3*e*f + d*g) + 2*a*(3*e^2*f + 3*d*e*g + d^2*h)) - b*c^2*
e*(5*a^2*e^2*h + 4*a*b*e*(e*g + 3*d*h) + b^2*(e^2*f + 3*d*e*g + 3*d^2*h)) +
 c^3*(2*a^2*e^2*(e*g + 3*d*h) + b^2*d*(3*e^2*f + 3*d*e*g + d^2*h) + 3*a*b*e
*(e^2*f + 3*d*e*g + 3*d^2*h)))*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt
[-b^2 + 4*a*c] + 6*(c^4*d^2*(3*e*f + d*g) + b^4*e^3*h - b^2*c*e^2*(b*e*g +
3*b*d*h + 3*a*e*h) + c^2*e*(a^2*e^2*h + 2*a*b*e*(e*g + 3*d*h) + b^2*(e^2*f
+ 3*d*e*g + 3*d^2*h)) - c^3*(b*d*(3*e^2*f + 3*d*e*g + d^2*h) + a*e*(e^2*f +
 3*d*e*g + 3*d^2*h)))*Log[a + x*(b + c*x)])/(12*c^5)
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 869, normalized size of antiderivative = 1.47

method	result
default	$\frac{\frac{1}{4}h\,e^3x^4c^3-\frac{1}{3}b\,c^2e^3h\,x^3+c^3d\,e^2h\,x^3+\frac{1}{3}g\,e^3x^3c^3-\frac{1}{2}a\,c^2e^3h\,x^2+\frac{1}{2}b^2c\,e^3h\,x^2-\frac{3}{2}b\,c^2d\,e^2h\,x^2-\frac{1}{2}b\,c^2e^3g\,x^2+\frac{3}{2}c^3d^2eh\,x^2+\frac{3}{2}c^3d\,e^2g\,x^2+}{}$
risch	Expression too large to display

```
[In] int((e*x+d)^3*(h*x^2+g*x+f)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(1/4*h*e^3*x^4*c^3-1/3*b*c^2*e^3*h*x^3+c^3*d*e^2*h*x^3+1/3*g*e^3*x^3*
c^3-1/2*a*c^2*e^3*h*x^2+1/2*b^2*c*e^3*h*x^2-3/2*b*c^2*d*e^2*h*x^2-1/2*b*c^2
*e^3*g*x^2+3/2*c^3*d^2*e*h*x^2+3/2*c^3*d*e^2*g*x^2+1/2*c^3*e^3*f*x^2+2*a*b*
c*e^3*h*x-3*a*c^2*d*e^2*h*x-a*c^2*e^3*g*x-b^3*e^3*h*x+3*b^2*c*d*e^2*h*x+b^2
*c*e^3*g*x-3*b*c^2*d^2*e*h*x-3*b*c^2*d*e^2*g*x-b*c^2*e^3*f*x+c^3*d^3*h*x+3*
c^3*d^2*e*g*x+3*c^3*d*e^2*f*x)+1/c^4*(1/2*(a^2*c^2*e^3*h-3*a*b^2*c*e^3*h+6*
a*b*c^2*d*e^2*h+2*a*b*c^2*e^3*g-3*a*c^3*d^2*e*h-3*a*c^3*d*e^2*g-a*c^3*e^3*f
```

```
+b^4*e^3*h-3*b^3*c*d*e^2*h-b^3*c*e^3*g+3*b^2*c^2*d^2*e*h+3*b^2*c^2*d*e^2*g+
b^2*c^2*e^3*f-b*c^3*d^3*h-3*b*c^3*d^2*e*g-3*b*c^3*d*e^2*f+c^4*d^3*g+3*c^4*d
^2*e*f)/c*ln(c*x^2+b*x+a)+2*(-2*a^2*b*c*e^3*h+3*a^2*c^2*d*e^2*h+a^2*c^2*e^3
*g+a*b^3*e^3*h-3*a*b^2*c*d*e^2*h-a*b^2*c*e^3*g+3*a*b*c^2*d^2*e*h+3*a*b*c^2*
d*e^2*g+a*b*c^2*e^3*f-a*c^3*d^3*h-3*a*c^3*d^2*e*g-3*a*c^3*d*e^2*f+c^4*d^3*f
-1/2*(a^2*c^2*e^3*h-3*a*b^2*c*e^3*h+6*a*b*c^2*d*e^2*h+2*a*b*c^2*e^3*g-3*a*c
^3*d^2*e*h-3*a*c^3*d*e^2*g-a*c^3*e^3*f+b^4*e^3*h-3*b^3*c*d*e^2*h-b^3*c*e^3*
g+3*b^2*c^2*d^2*e*h+3*b^2*c^2*d*e^2*g+b^2*c^2*e^3*f-b*c^3*d^3*h-3*b*c^3*d^2
*e*g-3*b*c^3*d*e^2*f+c^4*d^3*g+3*c^4*d^2*e*f)*b/c)/(4*a*c-b^2)^(1/2)*arctan
((2*c*x+b)/(4*a*c-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 1.00 (sec) , antiderivative size = 2150, normalized size of antiderivative = 3.64

$$\int \frac{(d+ex)^3 \left(f+gx+hx^2\right)}{a+bx+cx^2}\,dx=\text{Too large to display}$$

```
[In] integrate((e*x+d)^3*(h*x^2+g*x+f)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(b^2*c^4 - 4*a*c^5)*e^3*h*x^4 + 4*((b^2*c^4 - 4*a*c^5)*e^3*g + (3*
(b^2*c^4 - 4*a*c^5)*d*e^2 - (b^3*c^3 - 4*a*b*c^4)*e^3)*h)*x^3 + 6*((b^2*c^4
 - 4*a*c^5)*e^3*f + (3*(b^2*c^4 - 4*a*c^5)*d*e^2 - (b^3*c^3 - 4*a*b*c^4)*e^
3)*g + (3*(b^2*c^4 - 4*a*c^5)*d^2*e - 3*(b^3*c^3 - 4*a*b*c^4)*d*e^2 + (b^4*
c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*e^3)*h)*x^2 - 6*sqrt(b^2 - 4*a*c)*((2*c^5*d^
3 - 3*b*c^4*d^2*e + 3*(b^2*c^3 - 2*a*c^4)*d*e^2 - (b^3*c^2 - 3*a*b*c^3)*e^3
)*f - (b*c^4*d^3 - 3*(b^2*c^3 - 2*a*c^4)*d^2*e + 3*(b^3*c^2 - 3*a*b*c^3)*d*
e^2 - (b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*e^3)*g + ((b^2*c^3 - 2*a*c^4)*d^3 -
 3*(b^3*c^2 - 3*a*b*c^3)*d^2*e + 3*(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d*e^2
- (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e^3)*h)*log((2*c^2*x^2 + 2*b*c*x + b^2 -
2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 12*((3*(b^2*c^4
 - 4*a*c^5)*d*e^2 - (b^3*c^3 - 4*a*b*c^4)*e^3)*f + (3*(b^2*c^4 - 4*a*c^5)*d
^2*e - 3*(b^3*c^3 - 4*a*b*c^4)*d*e^2 + (b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*
e^3)*g + ((b^2*c^4 - 4*a*c^5)*d^3 - 3*(b^3*c^3 - 4*a*b*c^4)*d^2*e + 3*(b^4*
c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*d*e^2 - (b^5*c - 6*a*b^3*c^2 + 8*a^2*b*c^3)*
e^3)*h)*x + 6*((3*(b^2*c^4 - 4*a*c^5)*d^2*e - 3*(b^3*c^3 - 4*a*b*c^4)*d*e^2
 + (b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*e^3)*f + ((b^2*c^4 - 4*a*c^5)*d^3 -
3*(b^3*c^3 - 4*a*b*c^4)*d^2*e + 3*(b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*d*e^2
 - (b^5*c - 6*a*b^3*c^2 + 8*a^2*b*c^3)*e^3)*g - ((b^3*c^3 - 4*a*b*c^4)*d^3
- 3*(b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*d^2*e + 3*(b^5*c - 6*a*b^3*c^2 + 8*
a^2*b*c^3)*d*e^2 - (b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*e^3)*h)*l
og(c*x^2 + b*x + a))/(b^2*c^5 - 4*a*c^6), 1/12*(3*(b^2*c^4 - 4*a*c^5)*e^3*h
*x^4 + 4*((b^2*c^4 - 4*a*c^5)*e^3*g + (3*(b^2*c^4 - 4*a*c^5)*d*e^2 - (b^3*c
^3 - 4*a*b*c^4)*e^3)*h)*x^3 + 6*((b^2*c^4 - 4*a*c^5)*e^3*f + (3*(b^2*c^4 -
```

```
4*a*c^5)*d*e^2 - (b^3*c^3 - 4*a*b*c^4)*e^3)*g + (3*(b^2*c^4 - 4*a*c^5)*d^2*
e - 3*(b^3*c^3 - 4*a*b*c^4)*d*e^2 + (b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*e^3
)*h)*x^2 - 12*sqrt(-b^2 + 4*a*c)*((2*c^5*d^3 - 3*b*c^4*d^2*e + 3*(b^2*c^3 -
 2*a*c^4)*d*e^2 - (b^3*c^2 - 3*a*b*c^3)*e^3)*f - (b*c^4*d^3 - 3*(b^2*c^3 -
2*a*c^4)*d^2*e + 3*(b^3*c^2 - 3*a*b*c^3)*d*e^2 - (b^4*c - 4*a*b^2*c^2 + 2*a
^2*c^3)*e^3)*g + ((b^2*c^3 - 2*a*c^4)*d^3 - 3*(b^3*c^2 - 3*a*b*c^3)*d^2*e +
 3*(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d*e^2 - (b^5 - 5*a*b^3*c + 5*a^2*b*c^2
)*e^3)*h)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 12*((3*(b
^2*c^4 - 4*a*c^5)*d*e^2 - (b^3*c^3 - 4*a*b*c^4)*e^3)*f + (3*(b^2*c^4 - 4*a*
c^5)*d^2*e - 3*(b^3*c^3 - 4*a*b*c^4)*d*e^2 + (b^4*c^2 - 5*a*b^2*c^3 + 4*a^2
*c^4)*e^3)*g + ((b^2*c^4 - 4*a*c^5)*d^3 - 3*(b^3*c^3 - 4*a*b*c^4)*d^2*e + 3
*(b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*d*e^2 - (b^5*c - 6*a*b^3*c^2 + 8*a^2*b
*c^3)*e^3)*h)*x + 6*((3*(b^2*c^4 - 4*a*c^5)*d^2*e - 3*(b^3*c^3 - 4*a*b*c^4)
*d*e^2 + (b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*e^3)*f + ((b^2*c^4 - 4*a*c^5)*
d^3 - 3*(b^3*c^3 - 4*a*b*c^4)*d^2*e + 3*(b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)
*d*e^2 - (b^5*c - 6*a*b^3*c^2 + 8*a^2*b*c^3)*e^3)*g - ((b^3*c^3 - 4*a*b*c^4
)*d^3 - 3*(b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*d^2*e + 3*(b^5*c - 6*a*b^3*c^
2 + 8*a^2*b*c^3)*d*e^2 - (b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*e^3
)*h)*log(c*x^2 + b*x + a))/(b^2*c^5 - 4*a*c^6)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4972 vs. $2(619) = 1238$.

Time = 57.19 (sec) , antiderivative size = 4972, normalized size of antiderivative = 8.41

$$\int \frac{(d+ex)^3\,(f+gx+hx^2)}{a+bx+cx^2}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)**3*(h*x**2+g*x+f)/(c*x**2+b*x+a),x)
```

```
[Out] x**3*(-b*e**3*h/(3*c**2) + d*e**2*h/c + e**3*g/(3*c)) + x**2*(-a*e**3*h/(2*
c**2) + b**2*e**3*h/(2*c**3) - 3*b*d*e**2*h/(2*c**2) - b*e**3*g/(2*c**2) +
3*d**2*e*h/(2*c) + 3*d*e**2*g/(2*c) + e**3*f/(2*c)) + x*(2*a*b*e**3*h/c**3
- 3*a*d*e**2*h/c**2 - a*e**3*g/c**2 - b**3*e**3*h/c**4 + 3*b**2*d*e**2*h/c*
*3 + b**2*e**3*g/c**3 - 3*b*d**2*e*h/c**2 - 3*b*d*e**2*g/c**2 - b*e**3*f/c*
*2 + d**3*h/c + 3*d**2*e*g/c + 3*d*e**2*f/c) + (-sqrt(-4*a*c + b**2)*(5*a**
2*b*c**2*e**3*h - 6*a**2*c**3*d*e**2*h - 2*a**2*c**3*e**3*g - 5*a*b**3*c*e*
*3*h + 12*a*b**2*c**2*d*e**2*h + 4*a*b**2*c**2*e**3*g - 9*a*b*c**3*d**2*e*h
 - 9*a*b*c**3*d*e**2*g - 3*a*b*c**3*e**3*f + 2*a*c**4*d**3*h + 6*a*c**4*d**
2*e*g + 6*a*c**4*d*e**2*f + b**5*e**3*h - 3*b**4*c*d*e**2*h - b**4*c*e**3*g
 + 3*b**3*c**2*d**2*e*h + 3*b**3*c**2*d*e**2*g + b**3*c**2*e**3*f - b**2*c*
*3*d**3*h - 3*b**2*c**3*d**2*e*g - 3*b**2*c**3*d*e**2*f + b*c**4*d**3*g + 3
*b*c**4*d**2*e*f - 2*c**5*d**3*f)/(2*c**5*(4*a*c - b**2)) + (a**2*c**2*e**3
*h - 3*a*b**2*c*e**3*h + 6*a*b*c**2*d*e**2*h + 2*a*b*c**2*e**3*g - 3*a*c**3
*d**2*e*h - 3*a*c**3*d*e**2*g - a*c**3*e**3*f + b**4*e**3*h - 3*b**3*c*d*e*
```

```
*2*h - b**3*c*e**3*g + 3*b**2*c**2*d**2*e*h + 3*b**2*c**2*d*e**2*g + b**2*c
**2*e**3*f - b*c**3*d**3*h - 3*b*c**3*d**2*e*g - 3*b*c**3*d*e**2*f + c**4*d
**3*g + 3*c**4*d**2*e*f)/(2*c**5))*log(x + (2*a**3*c**2*e**3*h - 4*a**2*b**
2*c*e**3*h + 9*a**2*b*c**2*d*e**2*h + 3*a**2*b*c**2*e**3*g - 6*a**2*c**3*d*
*2*e*h - 6*a**2*c**3*d*e**2*g - 2*a**2*c**3*e**3*f + a*b**4*e**3*h - 3*a*b*
*3*c*d*e**2*h - a*b**3*c*e**3*g + 3*a*b**2*c**2*d**2*e*h + 3*a*b**2*c**2*d*
e**2*g + a*b**2*c**2*e**3*f - a*b*c**3*d**3*h - 3*a*b*c**3*d**2*e*g - 3*a*b
*c**3*d*e**2*f - 4*a*c**5*(-sqrt(-4*a*c + b**2)*(5*a**2*b*c**2*e**3*h - 6*a
**2*c**3*d*e**2*h - 2*a**2*c**3*e**3*g - 5*a*b**3*c*e**3*h + 12*a*b**2*c**2
*d*e**2*h + 4*a*b**2*c**2*e**3*g - 9*a*b*c**3*d**2*e*h - 9*a*b*c**3*d*e**2*
g - 3*a*b*c**3*e**3*f + 2*a*c**4*d**3*h + 6*a*c**4*d**2*e*g + 6*a*c**4*d*e*
*2*f + b**5*e**3*h - 3*b**4*c*d*e**2*h - b**4*c*e**3*g + 3*b**3*c**2*d**2*e
*h + 3*b**3*c**2*d*e**2*g + b**3*c**2*e**3*f - b**2*c**3*d**3*h - 3*b**2*c*
*3*d**2*e*g - 3*b**2*c**3*d*e**2*f + b*c**4*d**3*g + 3*b*c**4*d**2*e*f - 2*
c**5*d**3*f)/(2*c**5*(4*a*c - b**2)) + (a**2*c**2*e**3*h - 3*a*b**2*c*e**3*
h + 6*a*b*c**2*d*e**2*h + 2*a*b*c**2*e**3*g - 3*a*c**3*d**2*e*h - 3*a*c**3*
d*e**2*g - a*c**3*e**3*f + b**4*e**3*h - 3*b**3*c*d*e**2*h - b**3*c*e**3*g
 + 3*b**2*c**2*d**2*e*h + 3*b**2*c**2*d*e**2*g + b**2*c**2*e**3*f - b*c**3*d
**3*h - 3*b*c**3*d**2*e*g - 3*b*c**3*d*e**2*f + c**4*d**3*g + 3*c**4*d**2*e
*f)/(2*c**5)) + 2*a*c**4*d**3*g + 6*a*c**4*d**2*e*f + b**2*c**4*(-sqrt(-4*a
*c + b**2)*(5*a**2*b*c**2*e**3*h - 6*a**2*c**3*d*e**2*h - 2*a**2*c**3*e**3*
g - 5*a*b**3*c*e**3*h + 12*a*b**2*c**2*d*e**2*h + 4*a*b**2*c**2*e**3*g - 9*
a*b*c**3*d**2*e*h - 9*a*b*c**3*d*e**2*g - 3*a*b*c**3*e**3*f + 2*a*c**4*d**3
*h + 6*a*c**4*d**2*e*g + 6*a*c**4*d*e**2*f + b**5*e**3*h - 3*b**4*c*d*e**2*
h - b**4*c*e**3*g + 3*b**3*c**2*d**2*e*h + 3*b**3*c**2*d*e**2*g + b**3*c**2
*e**3*f - b**2*c**3*d**3*h - 3*b**2*c**3*d**2*e*g - 3*b**2*c**3*d*e**2*f +
b*c**4*d**3*g + 3*b*c**4*d**2*e*f - 2*c**5*d**3*f)/(2*c**5*(4*a*c - b**2))
+ (a**2*c**2*e**3*h - 3*a*b**2*c*e**3*h + 6*a*b*c**2*d*e**2*h + 2*a*b*c**2*
e**3*g - 3*a*c**3*d**2*e*h - 3*a*c**3*d*e**2*g - a*c**3*e**3*f + b**4*e**3*
h - 3*b**3*c*d*e**2*h - b**3*c*e**3*g + 3*b**2*c**2*d**2*e*h + 3*b**2*c**2*
d*e**2*g + b**2*c**2*e**3*f - b*c**3*d**3*h - 3*b*c**3*d**2*e*g - 3*b*c**3*
d*e**2*f + c**4*d**3*g + 3*c**4*d**2*e*f)/(2*c**5)) - b*c**4*d**3*f)/(5*a**
2*b*c**2*e**3*h - 6*a**2*c**3*d*e**2*h - 2*a**2*c**3*e**3*g - 5*a*b**3*c*e*
*3*h + 12*a*b**2*c**2*d*e**2*h + 4*a*b**2*c**2*e**3*g - 9*a*b*c**3*d**2*e*h
 - 9*a*b*c**3*d*e**2*g - 3*a*b*c**3*e**3*f + 2*a*c**4*d**3*h + 6*a*c**4*d**
2*e*g + 6*a*c**4*d*e**2*f + b**5*e**3*h - 3*b**4*c*d*e**2*h - b**4*c*e**3*g
 + 3*b**3*c**2*d**2*e*h + 3*b**3*c**2*d*e**2*g + b**3*c**2*e**3*f - b**2*c*
*3*d**3*h - 3*b**2*c**3*d**2*e*g - 3*b**2*c**3*d*e**2*f + b*c**4*d**3*g + 3
*b*c**4*d**2*e*f - 2*c**5*d**3*f)) + (sqrt(-4*a*c + b**2)*(5*a**2*b*c**2*e*
*3*h - 6*a**2*c**3*d*e**2*h - 2*a**2*c**3*e**3*g - 5*a*b**3*c*e**3*h + 12*a
*b**2*c**2*d*e**2*h + 4*a*b**2*c**2*e**3*g - 9*a*b*c**3*d**2*e*h - 9*a*b*c*
*3*d*e**2*g - 3*a*b*c**3*e**3*f + 2*a*c**4*d**3*h + 6*a*c**4*d**2*e*g + 6*a
*c**4*d*e**2*f + b**5*e**3*h - 3*b**4*c*d*e**2*h - b**4*c*e**3*g + 3*b**3*c
**2*d**2*e*h + 3*b**3*c**2*d*e**2*g + b**3*c**2*e**3*f - b**2*c**3*d**3*h -
 3*b**2*c**3*d**2*e*g - 3*b**2*c**3*d*e**2*f + b*c**4*d**3*g + 3*b*c**4*d**
```

```
2*e*f - 2*c**5*d**3*f)/(2*c**5*(4*a*c - b**2)) + (a**2*c**2*e**3*h - 3*a*b*
*2*c*e**3*h + 6*a*b*c**2*d*e**2*h + 2*a*b*c**2*e**3*g - 3*a*c**3*d**2*e*h -
 3*a*c**3*d*e**2*g - a*c**3*e**3*f + b**4*e**3*h - 3*b**3*c*d*e**2*h - b**3
*c*e**3*g + 3*b**2*c**2*d**2*e*h + 3*b**2*c**2*d*e**2*g + b**2*c**2*e**3*f
 - b*c**3*d**3*h - 3*b*c**3*d**2*e*g - 3*b*c**3*d*e**2*f + c**4*d**3*g + 3*c
**4*d**2*e*f)/(2*c**5))*log(x + (2*a**3*c**2*e**3*h - 4*a**2*b**2*c*e**3*h
+ 9*a**2*b*c**2*d*e**2*h + 3*a**2*b*c**2*e**3*g - 6*a**2*c**3*d**2*e*h - 6*
a**2*c**3*d*e**2*g - 2*a**2*c**3*e**3*f + a*b**4*e**3*h - 3*a*b**3*c*d*e**2
*h - a*b**3*c*e**3*g + 3*a*b**2*c**2*d**2*e*h + 3*a*b**2*c**2*d*e**2*g + a*
b**2*c**2*e**3*f - a*b*c**3*d**3*h - 3*a*b*c**3*d**2*e*g - 3*a*b*c**3*d*e**
2*f - 4*a*c**5*(sqrt(-4*a*c + b**2)*(5*a**2*b*c**2*e**3*h - 6*a**2*c**3*d*e
**2*h - 2*a**2*c**3*e**3*g - 5*a*b**3*c*e**3*h + 12*a*b**2*c**2*d*e**2*h +
4*a*b**2*c**2*e**3*g - 9*a*b*c**3*d**2*e*h - 9*a*b*c**3*d*e**2*g - 3*a*b*c*
*3*e**3*f + 2*a*c**4*d**3*h + 6*a*c**4*d**2*e*g + 6*a*c**4*d*e**2*f + b**5*
e**3*h - 3*b**4*c*d*e**2*h - b**4*c*e**3*g + 3*b**3*c**2*d**2*e*h + 3*b**3*
c**2*d*e**2*g + b**3*c**2*e**3*f - b**2*c**3*d**3*h - 3*b**2*c**3*d**2*e*g
- 3*b**2*c**3*d*e**2*f + b*c**4*d**3*g + 3*b*c**4*d**2*e*f - 2*c**5*d**3*f)
/(2*c**5*(4*a*c - b**2)) + (a**2*c**2*e**3*h - 3*a*b**2*c*e**3*h + 6*a*b*c*
*2*d*e**2*h + 2*a*b*c**2*e**3*g - 3*a*c**3*d**2*e*h - 3*a*c**3*d*e**2*g - a
*c**3*e**3*f + b**4*e**3*h - 3*b**3*c*d*e**2*h - b**3*c*e**3*g + 3*b**2*c**
2*d**2*e*h + 3*b**2*c**2*d*e**2*g + b**2*c**2*e**3*f - b*c**3*d**3*h - 3*b*
c**3*d**2*e*g - 3*b*c**3*d*e**2*f + c**4*d**3*g + 3*c**4*d**2*e*f)/(2*c**5)
) + 2*a*c**4*d**3*g + 6*a*c**4*d**2*e*f + b**2*c**4*(sqrt(-4*a*c + b**2)*(5
*a**2*b*c**2*e**3*h - 6*a**2*c**3*d*e**2*h - 2*a**2*c**3*e**3*g - 5*a*b**3*
c*e**3*h + 12*a*b**2*c**2*d*e**2*h + 4*a*b**2*c**2*e**3*g - 9*a*b*c**3*d**2
*e*h - 9*a*b*c**3*d*e**2*g - 3*a*b*c**3*e**3*f + 2*a*c**4*d**3*h + 6*a*c**4
*d**2*e*g + 6*a*c**4*d*e**2*f + b**5*e**3*h - 3*b**4*c*d*e**2*h - b**4*c*e*
*3*g + 3*b**3*c**2*d**2*e*h + 3*b**3*c**2*d*e**2*g + b**3*c**2*e**3*f - b**
2*c**3*d**3*h - 3*b**2*c**3*d**2*e*g - 3*b**2*c**3*d*e**2*f + b*c**4*d**3*g
 + 3*b*c**4*d**2*e*f - 2*c**5*d**3*f)/(2*c**5*(4*a*c - b**2)) + (a**2*c**2*
e**3*h - 3*a*b**2*c*e**3*h + 6*a*b*c**2*d*e**2*h + 2*a*b*c**2*e**3*g - 3*a*
c**3*d**2*e*h - 3*a*c**3*d*e**2*g - a*c**3*e**3*f + b**4*e**3*h - 3*b**3*c*
d*e**2*h - b**3*c*e**3*g + 3*b**2*c**2*d**2*e*h + 3*b**2*c**2*d*e**2*g + b*
*2*c**2*e**3*f - b*c**3*d**3*h - 3*b*c**3*d**2*e*g - 3*b*c**3*d*e**2*f + c*
*4*d**3*g + 3*c**4*d**2*e*f)/(2*c**5)) - b*c**4*d**3*f)/(5*a**2*b*c**2*e**3
*h - 6*a**2*c**3*d*e**2*h - 2*a**2*c**3*e**3*g - 5*a*b**3*c*e**3*h + 12*a*b
**2*c**2*d*e**2*h + 4*a*b**2*c**2*e**3*g - 9*a*b*c**3*d**2*e*h - 9*a*b*c**3
*d*e**2*g - 3*a*b*c**3*e**3*f + 2*a*c**4*d**3*h + 6*a*c**4*d**2*e*g + 6*a*c
**4*d*e**2*f + b**5*e**3*h - 3*b**4*c*d*e**2*h - b**4*c*e**3*g + 3*b**3*c**
2*d**2*e*h + 3*b**3*c**2*d*e**2*g + b**3*c**2*e**3*f - b**2*c**3*d**3*h - 3
*b**2*c**3*d**2*e*g - 3*b**2*c**3*d*e**2*f + b*c**4*d**3*g + 3*b*c**4*d**2*
e*f - 2*c**5*d**3*f)) + e**3*h*x**4/(4*c)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^3\,(f+gx+hx^2)}{a+bx+cx^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*x+d)^3*(h*x^2+g*x+f)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 805, normalized size of antiderivative = 1.36

$$\int \frac{(d+ex)^3\,(f+gx+hx^2)}{a+bx+cx^2}\,dx$$
$$= \frac{3\,c^3e^3hx^4+4\,c^3e^3gx^3+12\,c^3de^2hx^3-4\,bc^2e^3hx^3+6\,c^3e^3fx^2+18\,c^3de^2gx^2-6\,bc^2e^3gx^2+18\,c^3d^2ehx^2-}{}$$
$$+\frac{(3\,c^4d^2ef-3\,bc^3de^2f+b^2c^2e^3f-ac^3e^3f+c^4d^3g-3\,bc^3d^2eg+3\,b^2c^2de^2g-3\,ac^3de^2g-b^3ce^3g+2\,ab}{}$$
$$+\frac{(2\,c^5d^3f-3\,bc^4d^2ef+3\,b^2c^3de^2f-6\,ac^4de^2f-b^3c^2e^3f+3\,abc^3e^3f-bc^4d^3g+3\,b^2c^3d^2eg-6\,ac^4d^2eg}{}$$

```
[In] integrate((e*x+d)^3*(h*x^2+g*x+f)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] 1/12*(3*c^3*e^3*h*x^4 + 4*c^3*e^3*g*x^3 + 12*c^3*d*e^2*h*x^3 - 4*b*c^2*e^3*
h*x^3 + 6*c^3*e^3*f*x^2 + 18*c^3*d*e^2*g*x^2 - 6*b*c^2*e^3*g*x^2 + 18*c^3*d
^2*e*h*x^2 - 18*b*c^2*d*e^2*h*x^2 + 6*b^2*c*e^3*h*x^2 - 6*a*c^2*e^3*h*x^2 +
 36*c^3*d*e^2*f*x - 12*b*c^2*e^3*f*x + 36*c^3*d^2*e*g*x - 36*b*c^2*d*e^2*g*
x + 12*b^2*c*e^3*g*x - 12*a*c^2*e^3*g*x + 12*c^3*d^3*h*x - 36*b*c^2*d^2*e*h
*x + 36*b^2*c*d*e^2*h*x - 36*a*c^2*d*e^2*h*x - 12*b^3*e^3*h*x + 24*a*b*c*e^
3*h*x)/c^4 + 1/2*(3*c^4*d^2*e*f - 3*b*c^3*d*e^2*f + b^2*c^2*e^3*f - a*c^3*e
^3*f + c^4*d^3*g - 3*b*c^3*d^2*e*g + 3*b^2*c^2*d*e^2*g - 3*a*c^3*d*e^2*g -
b^3*c*e^3*g + 2*a*b*c^2*e^3*g - b*c^3*d^3*h + 3*b^2*c^2*d^2*e*h - 3*a*c^3*d
^2*e*h - 3*b^3*c*d*e^2*h + 6*a*b*c^2*d*e^2*h + b^4*e^3*h - 3*a*b^2*c*e^3*h
+ a^2*c^2*e^3*h)*log(c*x^2 + b*x + a)/c^5 + (2*c^5*d^3*f - 3*b*c^4*d^2*e*f
+ 3*b^2*c^3*d*e^2*f - 6*a*c^4*d*e^2*f - b^3*c^2*e^3*f + 3*a*b*c^3*e^3*f - b
*c^4*d^3*g + 3*b^2*c^3*d^2*e*g - 6*a*c^4*d^2*e*g - 3*b^3*c^2*d*e^2*g + 9*a*
b*c^3*d*e^2*g + b^4*c*e^3*g - 4*a*b^2*c^2*e^3*g + 2*a^2*c^3*e^3*g + b^2*c^3
```

```
*d^3*h - 2*a*c^4*d^3*h - 3*b^3*c^2*d^2*e*h + 9*a*b*c^3*d^2*e*h + 3*b^4*c*d*
e^2*h - 12*a*b^2*c^2*d*e^2*h + 6*a^2*c^3*d*e^2*h - b^5*e^3*h + 5*a*b^3*c*e^
3*h - 5*a^2*b*c^2*e^3*h)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2
+ 4*a*c)*c^5)
```

Mupad [B] (verification not implemented)

Time = 15.32 (sec) , antiderivative size = 967, normalized size of antiderivative = 1.64

$$\int \frac{(d+ex)^3 (f+gx+hx^2)}{a+bx+cx^2} \, dx = x^3 \left(\frac{g\,e^3+3\,d\,h\,e^2}{3\,c}-\frac{b\,e^3\,h}{3\,c^2}\right)+x\left(\frac{h\,d^3+3\,g\,d^2\,e+3\,f\,d\,e^2}{c}+\frac{b\left(\frac{b\left(\frac{g\,e^3+3\,d\,h\,e^2}{c}-\frac{b\,e^3\,h}{c^2}\right)}{c}-\frac{3\,h\,d^2\,e+3\,g\,d\,e^2+f\,e^3}{c}+\frac{a\,e^3\,h}{c^2}\right)}{c}-\frac{a\left(\frac{g\,e^3+3\,d\,h\,e^2}{c}-\frac{b\,e^3\,h}{c^2}\right)}{c}\right)$$

$$-x^2\left(\frac{b\left(\frac{g\,e^3+3\,d\,h\,e^2}{c}-\frac{b\,e^3\,h}{c^2}\right)}{2\,c}-\frac{3\,h\,d^2\,e+3\,g\,d\,e^2+f\,e^3}{2\,c}+\frac{a\,e^3\,h}{2\,c^2}\right)$$

$$-\frac{\ln\left(c\,x^2+b\,x+a\right)\,\left(-4\,h\,a^3\,c^3\,e^3+13\,h\,a^2\,b^2\,c^2\,e^3-24\,h\,a^2\,b\,c^3\,d\,e^2-8\,g\,a^2\,b\,c^3\,e^3+12\,h\,a^2\,c^4\,d^2\,e+12\,g}{}$$

$$+\frac{e^3\,h\,x^4}{4\,c}$$

$$+\frac{\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)\,\left(-5\,h\,a^2\,b\,c^2\,e^3+6\,h\,a^2\,c^3\,d\,e^2+2\,g\,a^2\,c^3\,e^3+5\,h\,a\,b^3\,c\,e^3-12\,h\,a\,b^2\,c^2\,d\,e^2-}{}$$

```
[In] int(((d + e*x)^3*(f + g*x + h*x^2))/(a + b*x + c*x^2),x)
```

```
[Out] x^3*((e^3*g + 3*d*e^2*h)/(3*c) - (b*e^3*h)/(3*c^2)) + x*((d^3*h + 3*d*e^2*f
 + 3*d^2*e*g)/c + (b*((b*((e^3*g + 3*d*e^2*h)/c - (b*e^3*h)/c^2))/c - (e^3*
f + 3*d*e^2*g + 3*d^2*e*h)/c + (a*e^3*h)/c^2))/c - (a*((e^3*g + 3*d*e^2*h)/
c - (b*e^3*h)/c^2))/c) - x^2*((b*((e^3*g + 3*d*e^2*h)/c - (b*e^3*h)/c^2))/(
2*c) - (e^3*f + 3*d*e^2*g + 3*d^2*e*h)/(2*c) + (a*e^3*h)/(2*c^2)) - (log(a
+ b*x + c*x^2)*(b^6*e^3*h + 4*a^2*c^4*e^3*f + b^2*c^4*d^3*g + b^4*c^2*e^3*f
 - 4*a^3*c^3*e^3*h - b^3*c^3*d^3*h - 4*a*c^5*d^3*g - b^5*c*e^3*g + 4*a*b*c^
4*d^3*h - 7*a*b^4*c*e^3*h - 12*a*c^5*d^2*e*f - 3*b^5*c*d*e^2*h - 5*a*b^2*c^
3*e^3*f + 6*a*b^3*c^2*e^3*g - 8*a^2*b*c^3*e^3*g + 12*a^2*c^4*d*e^2*g + 3*b^
2*c^4*d^2*e*f - 3*b^3*c^3*d*e^2*f + 12*a^2*c^4*d^2*e*h - 3*b^3*c^3*d^2*e*g
+ 3*b^4*c^2*d*e^2*g + 3*b^4*c^2*d^2*e*h + 13*a^2*b^2*c^2*e^3*h + 12*a*b*c^4
```

```
*d*e^2*f + 12*a*b*c^4*d^2*e*g - 15*a*b^2*c^3*d*e^2*g - 15*a*b^2*c^3*d^2*e*h
 + 18*a*b^3*c^2*d*e^2*h - 24*a^2*b*c^3*d*e^2*h))/(2*(4*a*c^6 - b^2*c^5)) +
(e^3*h*x^4)/(4*c) + (atan(b/(4*a*c - b^2)^(1/2) + (2*c*x)/(4*a*c - b^2)^(1/
2))*(2*c^5*d^3*f - b^5*e^3*h + 2*a^2*c^3*e^3*g - b^3*c^2*e^3*f + b^2*c^3*d^
3*h - 2*a*c^4*d^3*h - b*c^4*d^3*g + b^4*c*e^3*g + 3*a*b*c^3*e^3*f + 5*a*b^3
*c*e^3*h - 6*a*c^4*d*e^2*f - 6*a*c^4*d^2*e*g - 3*b*c^4*d^2*e*f + 3*b^4*c*d*
e^2*h - 4*a*b^2*c^2*e^3*g - 5*a^2*b*c^2*e^3*h + 3*b^2*c^3*d*e^2*f + 6*a^2*c
^3*d*e^2*h + 3*b^2*c^3*d^2*e*g - 3*b^3*c^2*d*e^2*g - 3*b^3*c^2*d^2*e*h + 9*
a*b*c^3*d*e^2*g + 9*a*b*c^3*d^2*e*h - 12*a*b^2*c^2*d*e^2*h))/(c^5*(4*a*c -
b^2)^(1/2))
```

3.149 $\int \frac{(d+ex)^2(f+gx+hx^2)}{a+bx+cx^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 348

$$\int \frac{(d+ex)^2\,(f+gx+hx^2)}{a+bx+cx^2}\,dx = \frac{(b^2e^2h+c^2(e^2f+2deg+d^2h)-ce(beg+2bdh+aeh))\,x}{c^3}$$
$$+\frac{e(ceg+2cdh-beh)x^2}{2c^2}+\frac{e^2hx^3}{3c}$$
$$-\frac{(2c^4d^2f+b^4e^2h-b^2ce(beg+2bdh+4aeh)-c^3(bd(2ef+dg)+2a(e^2f+2deg+d^2h))+c^2(2a^2e^2h+3}{c^4\sqrt{b^2-4ac}}$$
$$+\frac{(c^3d(2ef+dg)-b^3e^2h+bce(beg+2bdh+2aeh)-c^2(ae(eg+2dh)+b(e^2f+2deg+d^2h)))\log{(a+b}}{2c^4}$$

```
[Out] (b^2*e^2*h+c^2*(d^2*h+2*d*e*g+e^2*f)-c*e*(a*e*h+2*b*d*h+b*e*g))*x/c^3+1/2*e
*(-b*e*h+2*c*d*h+c*e*g)*x^2/c^2+1/3*e^2*h*x^3/c+1/2*(c^3*d*(d*g+2*e*f)-b^3*
e^2*h+b*c*e*(2*a*e*h+2*b*d*h+b*e*g)-c^2*(a*e*(2*d*h+e*g)+b*(d^2*h+2*d*e*g+e
^2*f)))*ln(c*x^2+b*x+a)/c^4-(2*c^4*d^2*f+b^4*e^2*h-b^2*c*e*(4*a*e*h+2*b*d*h
+b*e*g)-c^3*(b*d*(d*g+2*e*f)+2*a*(d^2*h+2*d*e*g+e^2*f))+c^2*(2*a^2*e^2*h+3*
a*b*e*(2*d*h+e*g)+b^2*(d^2*h+2*d*e*g+e^2*f)))*arctanh((2*c*x+b)/(-4*a*c+b^2
)^(1/2))/c^4/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1642, 648, 632, 212, 642}

$$\int \frac{(d+ex)^2 (f+gx+hx^2)}{a+bx+cx^2} \, dx =$$

$$-\frac{\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right) (c^2(2a^2e^2h+3abe(2dh+eg)+b^2(d^2h+2deg+e^2f))-b^2ce(4aeh+2bdh+beg)-}{c^4\sqrt{b^2-4ac}}$$

$$+\frac{\log (a+bx+cx^2) (-c^2(ae(2dh+eg)+b(d^2h+2deg+e^2f))+bce(2aeh+2bdh+beg)+b^3(-e^2)\,h+}{2c^4}$$

$$+\frac{x(-ce(aeh+2bdh+beg)+b^2e^2h+c^2(d^2h+2deg+e^2f))}{c^3}$$

$$+\frac{ex^2(-beh+2cdh+ceg)}{2c^2}+\frac{e^2hx^3}{3c}$$

```
[In] Int[((d + e*x)^2*(f + g*x + h*x^2))/(a + b*x + c*x^2),x]
```

```
[Out] ((b^2*e^2*h + c^2*(e^2*f + 2*d*e*g + d^2*h) - c*e*(b*e*g + 2*b*d*h + a*e*h)
)*x)/c^3 + (e*(c*e*g + 2*c*d*h - b*e*h)*x^2)/(2*c^2) + (e^2*h*x^3)/(3*c) -
((2*c^4*d^2*f + b^4*e^2*h - b^2*c*e*(b*e*g + 2*b*d*h + 4*a*e*h) - c^3*(b*d*
(2*e*f + d*g) + 2*a*(e^2*f + 2*d*e*g + d^2*h)) + c^2*(2*a^2*e^2*h + 3*a*b*e
*(e*g + 2*d*h) + b^2*(e^2*f + 2*d*e*g + d^2*h)))*ArcTanh[(b + 2*c*x)/Sqrt[b
^2 - 4*a*c]])/(c^4*Sqrt[b^2 - 4*a*c]) + ((c^3*d*(2*e*f + d*g) - b^3*e^2*h +
 b*c*e*(b*e*g + 2*b*d*h + 2*a*e*h) - c^2*(a*e*(e*g + 2*d*h) + b*(e^2*f + 2*
d*e*g + d^2*h)))*Log[a + b*x + c*x^2])/(2*c^4)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\text{integral} = \int \Bigg(\frac{b^2e^2h + c^2(e^2f + 2deg + d^2h) - ce(beg + 2bdh + aeh)}{c^3}$$

$$+ \frac{e(ceg + 2cdh - beh)x}{c^2} + \frac{e^2hx^2}{c}$$

$$+ \frac{c^3d^2f - ab^2e^2h - ac^2(e^2f + 2deg + d^2h) + ace(beg + 2bdh + aeh) + (c^3d(2ef + dg) - b^3e^2h + bce(beg +}{c^3(a + bx + cx^2)}$$

$$= \frac{(b^2e^2h + c^2(e^2f + 2deg + d^2h) - ce(beg + 2bdh + aeh))\,x}{c^3} + \frac{e(ceg + 2cdh - beh)x^2}{2c^2}$$

$$+ \frac{e^2hx^3}{3c} + \frac{\int \frac{c^3d^2f - ab^2e^2h - ac^2(e^2f + 2deg + d^2h) + ace(beg + 2bdh + aeh) + (c^3d(2ef + dg) - b^3e^2h + bce(beg + 2bdh + 2aeh) - c^2(ae(eg}{a + bx + cx^2}}{c^3}$$

$$= \frac{(b^2e^2h + c^2(e^2f + 2deg + d^2h) - ce(beg + 2bdh + aeh))\,x}{c^3} + \frac{e(ceg + 2cdh - beh)x^2}{2c^2}$$

$$+ \frac{e^2hx^3}{3c} + \frac{(c^3d(2ef + dg) - b^3e^2h + bce(beg + 2bdh + 2aeh) - c^2(ae(eg + 2dh) + b(e^2f + 2deg +}{2c^4}$$

$$+ \frac{(2c^4d^2f + b^4e^2h - b^2ce(beg + 2bdh + 4aeh) - c^3(bd(2ef + dg) + 2a(e^2f + 2deg + d^2h)) + c^2(2a}{2c^4}$$

$$= \frac{(b^2e^2h + c^2(e^2f + 2deg + d^2h) - ce(beg + 2bdh + aeh))\,x}{c^3} + \frac{e(ceg + 2cdh - beh)x^2}{2c^2}$$

$$+ \frac{e^2hx^3}{3c} + \frac{(c^3d(2ef + dg) - b^3e^2h + bce(beg + 2bdh + 2aeh) - c^2(ae(eg + 2dh) + b(e^2f + 2deg +}{2c^4}$$

$$- \frac{(2c^4d^2f + b^4e^2h - b^2ce(beg + 2bdh + 4aeh) - c^3(bd(2ef + dg) + 2a(e^2f + 2deg + d^2h)) + c^2(2a}{c^4}$$

$$= \frac{(b^2e^2h + c^2(e^2f + 2deg + d^2h) - ce(beg + 2bdh + aeh))\,x}{c^3} + \frac{e(ceg + 2cdh - beh)x^2}{2c^2}$$

$$+ \frac{e^2hx^3}{3c} - \frac{(2c^4d^2f + b^4e^2h - b^2ce(beg + 2bdh + 4aeh) - c^3(bd(2ef + dg) + 2a(e^2f + 2deg + d^2h))}{c^4\sqrt{b^2 - 4ac}}$$

$$+ \frac{(c^3d(2ef + dg) - b^3e^2h + bce(beg + 2bdh + 2aeh) - c^2(ae(eg + 2dh) + b(e^2f + 2deg + d^2h)))\,\mathrm{lo}}{2c^4}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 345, normalized size of antiderivative = 0.99

$$\int \frac{(d+ex)^2\,(f+gx+hx^2)}{a+bx+cx^2}\,dx$$

$$= 6c(b^2e^2h+c^2(e^2f+2deg+d^2h)-ce(beg+2bdh+aeh))\,x+3c^2e(ceg+2cdh-beh)x^2+2c^3e^2hx^3+$$

```
[In] Integrate[((d + e*x)^2*(f + g*x + h*x^2))/(a + b*x + c*x^2),x]
```

```
[Out] (6*c*(b^2*e^2*h + c^2*(e^2*f + 2*d*e*g + d^2*h) - c*e*(b*e*g + 2*b*d*h + a*
e*h))*x + 3*c^2*e*(c*e*g + 2*c*d*h - b*e*h)*x^2 + 2*c^3*e^2*h*x^3 + (6*(2*c
^4*d^2*f + b^4*e^2*h - b^2*c*e*(b*e*g + 2*b*d*h + 4*a*e*h) - c^3*(b*d*(2*e*
f + d*g) + 2*a*(e^2*f + 2*d*e*g + d^2*h)) + c^2*(2*a^2*e^2*h + 3*a*b*e*(e*g
 + 2*d*h) + b^2*(e^2*f + 2*d*e*g + d^2*h)))*ArcTan[(b + 2*c*x)/Sqrt[-b^2 +
4*a*c]])/Sqrt[-b^2 + 4*a*c] + 3*(c^3*d*(2*e*f + d*g) - b^3*e^2*h + b*c*e*(b
*e*g + 2*b*d*h + 2*a*e*h) - c^2*(a*e*(e*g + 2*d*h) + b*(e^2*f + 2*d*e*g + d
^2*h)))*Log[a + x*(b + c*x)])/(6*c^4)
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 453, normalized size of antiderivative = 1.30

method	result
default	$-\frac{-\frac{1}{3}he^2x^3c^2+\frac{1}{2}bce^2hx^2-c^2dehx^2-\frac{1}{2}gx^2c^2e^2+ace^2hx-b^2e^2hx+2bcdehx+bce^2gx-c^2d^2hx-2c^2degx-c^2e^2fx}{c^3}+$
risch	Expression too large to display

```
[In] int((e*x+d)^2*(h*x^2+g*x+f)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/c^3*(-1/3*h*e^2*x^3*c^2+1/2*b*c*e^2*h*x^2-c^2*d*e*h*x^2-1/2*g*x^2*c^2*e^
2+a*c*e^2*h*x-b^2*e^2*h*x+2*b*c*d*e*h*x+b*c*e^2*g*x-c^2*d^2*h*x-2*c^2*d*e*g
*x-c^2*e^2*f*x)+1/c^3*(1/2*(2*a*b*c*e^2*h-2*a*c^2*d*e*h-a*c^2*e^2*g-b^3*e^2
*h+2*b^2*c*d*e*h+b^2*c*e^2*g-b*c^2*d^2*h-2*b*c^2*d*e*g-b*c^2*e^2*f+c^3*d^2*
g+2*c^3*d*e*f)/c*ln(c*x^2+b*x+a)+2*(a^2*c*e^2*h-a*b^2*e^2*h+2*a*b*c*d*e*h+a
*b*c*e^2*g-a*c^2*d^2*h-2*a*c^2*d*e*g-a*c^2*e^2*f+c^3*d^2*f-1/2*(2*a*b*c*e^2
*h-2*a*c^2*d*e*h-a*c^2*e^2*g-b^3*e^2*h+2*b^2*c*d*e*h+b^2*c*e^2*g-b*c^2*d^2*
h-2*b*c^2*d*e*g-b*c^2*e^2*f+c^3*d^2*g+2*c^3*d*e*f)*b/c)/(4*a*c-b^2)^(1/2)*a
rctan((2*c*x+b)/(4*a*c-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.48 (sec) , antiderivative size = 1273, normalized size of antiderivative = 3.66

$$\int \frac{(d+ex)^2 \left(f+gx+hx^2\right)}{a+bx+cx^2}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)^2*(h*x^2+g*x+f)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] [1/6*(2*(b^2*c^3 - 4*a*c^4)*e^2*h*x^3 + 3*((b^2*c^3 - 4*a*c^4)*e^2*g + (2*(
b^2*c^3 - 4*a*c^4)*d*e - (b^3*c^2 - 4*a*b*c^3)*e^2)*h)*x^2 + 3*sqrt(b^2 - 4
*a*c)*((2*c^4*d^2 - 2*b*c^3*d*e + (b^2*c^2 - 2*a*c^3)*e^2)*f - (b*c^3*d^2 -
 2*(b^2*c^2 - 2*a*c^3)*d*e + (b^3*c - 3*a*b*c^2)*e^2)*g + ((b^2*c^2 - 2*a*c
^3)*d^2 - 2*(b^3*c - 3*a*b*c^2)*d*e + (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e^2)*h)
*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c
*x^2 + b*x + a)) + 6*((b^2*c^3 - 4*a*c^4)*e^2*f + (2*(b^2*c^3 - 4*a*c^4)*d*
e - (b^3*c^2 - 4*a*b*c^3)*e^2)*g + ((b^2*c^3 - 4*a*c^4)*d^2 - 2*(b^3*c^2 -
4*a*b*c^3)*d*e + (b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*e^2)*h)*x + 3*((2*(b^2*c
^3 - 4*a*c^4)*d*e - (b^3*c^2 - 4*a*b*c^3)*e^2)*f + ((b^2*c^3 - 4*a*c^4)*d^2
 - 2*(b^3*c^2 - 4*a*b*c^3)*d*e + (b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*e^2)*g -
 ((b^3*c^2 - 4*a*b*c^3)*d^2 - 2*(b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*d*e + (b^
5 - 6*a*b^3*c + 8*a^2*b*c^2)*e^2)*h)*log(c*x^2 + b*x + a))/(b^2*c^4 - 4*a*c
^5), 1/6*(2*(b^2*c^3 - 4*a*c^4)*e^2*h*x^3 + 3*((b^2*c^3 - 4*a*c^4)*e^2*g +
(2*(b^2*c^3 - 4*a*c^4)*d*e - (b^3*c^2 - 4*a*b*c^3)*e^2)*h)*x^2 - 6*sqrt(-b^
2 + 4*a*c)*((2*c^4*d^2 - 2*b*c^3*d*e + (b^2*c^2 - 2*a*c^3)*e^2)*f - (b*c^3*
d^2 - 2*(b^2*c^2 - 2*a*c^3)*d*e + (b^3*c - 3*a*b*c^2)*e^2)*g + ((b^2*c^2 -
2*a*c^3)*d^2 - 2*(b^3*c - 3*a*b*c^2)*d*e + (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e^
2)*h)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 6*((b^2*c^3 -
 4*a*c^4)*e^2*f + (2*(b^2*c^3 - 4*a*c^4)*d*e - (b^3*c^2 - 4*a*b*c^3)*e^2)*g
 + ((b^2*c^3 - 4*a*c^4)*d^2 - 2*(b^3*c^2 - 4*a*b*c^3)*d*e + (b^4*c - 5*a*b^
2*c^2 + 4*a^2*c^3)*e^2)*h)*x + 3*((2*(b^2*c^3 - 4*a*c^4)*d*e - (b^3*c^2 - 4
*a*b*c^3)*e^2)*f + ((b^2*c^3 - 4*a*c^4)*d^2 - 2*(b^3*c^2 - 4*a*b*c^3)*d*e +
 (b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*e^2)*g - ((b^3*c^2 - 4*a*b*c^3)*d^2 - 2*
(b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*d*e + (b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*e^2
)*h)*log(c*x^2 + b*x + a))/(b^2*c^4 - 4*a*c^5)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2839 vs. $2(359) = 718$.

Time = 20.77 (sec) , antiderivative size = 2839, normalized size of antiderivative = 8.16

$$\int \frac{(d+ex)^2\,(f+gx+hx^2)}{a+bx+cx^2}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)**2*(h*x**2+g*x+f)/(c*x**2+b*x+a),x)
```

```
[Out] x**2*(-b*e**2*h/(2*c**2) + d*e*h/c + e**2*g/(2*c)) + x*(-a*e**2*h/c**2 + b*
*2*e**2*h/c**3 - 2*b*d*e*h/c**2 - b*e**2*g/c**2 + d**2*h/c + 2*d*e*g/c + e*
*2*f/c) + (-sqrt(-4*a*c + b**2)*(2*a**2*c**2*e**2*h - 4*a*b**2*c*e**2*h + 6
*a*b*c**2*d*e*h + 3*a*b*c**2*e**2*g - 2*a*c**3*d**2*h - 4*a*c**3*d*e*g - 2*
a*c**3*e**2*f + b**4*e**2*h - 2*b**3*c*d*e*h - b**3*c*e**2*g + b**2*c**2*d*
*2*h + 2*b**2*c**2*d*e*g + b**2*c**2*e**2*f - b*c**3*d**2*g - 2*b*c**3*d*e*
f + 2*c**4*d**2*f)/(2*c**4*(4*a*c - b**2)) + (2*a*b*c*e**2*h - 2*a*c**2*d*e
*h - a*c**2*e**2*g - b**3*e**2*h + 2*b**2*c*d*e*h + b**2*c*e**2*g - b*c**2*
d**2*h - 2*b*c**2*d*e*g - b*c**2*e**2*f + c**3*d**2*g + 2*c**3*d*e*f)/(2*c*
*4))*log(x + (-3*a**2*b*c*e**2*h + 4*a**2*c**2*d*e*h + 2*a**2*c**2*e**2*g +
 a*b**3*e**2*h - 2*a*b**2*c*d*e*h - a*b**2*c*e**2*g + a*b*c**2*d**2*h + 2*a
*b*c**2*d*e*g + a*b*c**2*e**2*f + 4*a*c**4*(-sqrt(-4*a*c + b**2)*(2*a**2*c*
*2*e**2*h - 4*a*b**2*c*e**2*h + 6*a*b*c**2*d*e*h + 3*a*b*c**2*e**2*g - 2*a*
c**3*d**2*h - 4*a*c**3*d*e*g - 2*a*c**3*e**2*f + b**4*e**2*h - 2*b**3*c*d*e
*h - b**3*c*e**2*g + b**2*c**2*d**2*h + 2*b**2*c**2*d*e*g + b**2*c**2*e**2*
f - b*c**3*d**2*g - 2*b*c**3*d*e*f + 2*c**4*d**2*f)/(2*c**4*(4*a*c - b**2))
 + (2*a*b*c*e**2*h - 2*a*c**2*d*e*h - a*c**2*e**2*g - b**3*e**2*h + 2*b**2*
c*d*e*h + b**2*c*e**2*g - b*c**2*d**2*h - 2*b*c**2*d*e*g - b*c**2*e**2*f +
c**3*d**2*g + 2*c**3*d*e*f)/(2*c**4)) - 2*a*c**3*d**2*g - 4*a*c**3*d*e*f -
b**2*c**3*(-sqrt(-4*a*c + b**2)*(2*a**2*c**2*e**2*h - 4*a*b**2*c*e**2*h + 6
*a*b*c**2*d*e*h + 3*a*b*c**2*e**2*g - 2*a*c**3*d**2*h - 4*a*c**3*d*e*g - 2*
a*c**3*e**2*f + b**4*e**2*h - 2*b**3*c*d*e*h - b**3*c*e**2*g + b**2*c**2*d*
*2*h + 2*b**2*c**2*d*e*g + b**2*c**2*e**2*f - b*c**3*d**2*g - 2*b*c**3*d*e*
f + 2*c**4*d**2*f)/(2*c**4*(4*a*c - b**2)) + (2*a*b*c*e**2*h - 2*a*c**2*d*e
*h - a*c**2*e**2*g - b**3*e**2*h + 2*b**2*c*d*e*h + b**2*c*e**2*g - b*c**2*
d**2*h - 2*b*c**2*d*e*g - b*c**2*e**2*f + c**3*d**2*g + 2*c**3*d*e*f)/(2*c*
*4)) + b*c**3*d**2*f)/(2*a**2*c**2*e**2*h - 4*a*b**2*c*e**2*h + 6*a*b*c**2*
d*e*h + 3*a*b*c**2*e**2*g - 2*a*c**3*d**2*h - 4*a*c**3*d*e*g - 2*a*c**3*e**
2*f + b**4*e**2*h - 2*b**3*c*d*e*h - b**3*c*e**2*g + b**2*c**2*d**2*h + 2*b
**2*c**2*d*e*g + b**2*c**2*e**2*f - b*c**3*d**2*g - 2*b*c**3*d*e*f + 2*c**4
*d**2*f)) + (sqrt(-4*a*c + b**2)*(2*a**2*c**2*e**2*h - 4*a*b**2*c*e**2*h +
6*a*b*c**2*d*e*h + 3*a*b*c**2*e**2*g - 2*a*c**3*d**2*h - 4*a*c**3*d*e*g - 2
*a*c**3*e**2*f + b**4*e**2*h - 2*b**3*c*d*e*h - b**3*c*e**2*g + b**2*c**2*d
**2*h + 2*b**2*c**2*d*e*g + b**2*c**2*e**2*f - b*c**3*d**2*g - 2*b*c**3*d*e
*f + 2*c**4*d**2*f)/(2*c**4*(4*a*c - b**2)) + (2*a*b*c*e**2*h - 2*a*c**2*d*
```

```
e*h - a*c**2*e**2*g - b**3*e**2*h + 2*b**2*c*d*e*h + b**2*c*e**2*g - b*c**2
*d**2*h - 2*b*c**2*d*e*g - b*c**2*e**2*f + c**3*d**2*g + 2*c**3*d*e*f)/(2*c
**4))*log(x + (-3*a**2*b*c*e**2*h + 4*a**2*c**2*d*e*h + 2*a**2*c**2*e**2*g
+ a*b**3*e**2*h - 2*a*b**2*c*d*e*h - a*b**2*c*e**2*g + a*b*c**2*d**2*h + 2*
a*b*c**2*d*e*g + a*b*c**2*e**2*f + 4*a*c**4*(sqrt(-4*a*c + b**2)*(2*a**2*c*
*2*e**2*h - 4*a*b**2*c*e**2*h + 6*a*b*c**2*d*e*h + 3*a*b*c**2*e**2*g - 2*a*
c**3*d**2*h - 4*a*c**3*d*e*g - 2*a*c**3*e**2*f + b**4*e**2*h - 2*b**3*c*d*e
*h - b**3*c*e**2*g + b**2*c**2*d**2*h + 2*b**2*c**2*d*e*g + b**2*c**2*e**2*
f - b*c**3*d**2*g - 2*b*c**3*d*e*f + 2*c**4*d**2*f)/(2*c**4*(4*a*c - b**2))
 + (2*a*b*c*e**2*h - 2*a*c**2*d*e*h - a*c**2*e**2*g - b**3*e**2*h + 2*b**2*
c*d*e*h + b**2*c*e**2*g - b*c**2*d**2*h - 2*b*c**2*d*e*g - b*c**2*e**2*f +
c**3*d**2*g + 2*c**3*d*e*f)/(2*c**4)) - 2*a*c**3*d**2*g - 4*a*c**3*d*e*f -
b**2*c**3*(sqrt(-4*a*c + b**2)*(2*a**2*c**2*e**2*h - 4*a*b**2*c*e**2*h + 6*
a*b*c**2*d*e*h + 3*a*b*c**2*e**2*g - 2*a*c**3*d**2*h - 4*a*c**3*d*e*g - 2*a
*c**3*e**2*f + b**4*e**2*h - 2*b**3*c*d*e*h - b**3*c*e**2*g + b**2*c**2*d**
2*h + 2*b**2*c**2*d*e*g + b**2*c**2*e**2*f - b*c**3*d**2*g - 2*b*c**3*d*e*f
 + 2*c**4*d**2*f)/(2*c**4*(4*a*c - b**2)) + (2*a*b*c*e**2*h - 2*a*c**2*d*e*
h - a*c**2*e**2*g - b**3*e**2*h + 2*b**2*c*d*e*h + b**2*c*e**2*g - b*c**2*d
**2*h - 2*b*c**2*d*e*g - b*c**2*e**2*f + c**3*d**2*g + 2*c**3*d*e*f)/(2*c**
4)) + b*c**3*d**2*f)/(2*a**2*c**2*e**2*h - 4*a*b**2*c*e**2*h + 6*a*b*c**2*d
*e*h + 3*a*b*c**2*e**2*g - 2*a*c**3*d**2*h - 4*a*c**3*d*e*g - 2*a*c**3*e**2
*f + b**4*e**2*h - 2*b**3*c*d*e*h - b**3*c*e**2*g + b**2*c**2*d**2*h + 2*b*
*2*c**2*d*e*g + b**2*c**2*e**2*f - b*c**3*d**2*g - 2*b*c**3*d*e*f + 2*c**4*
d**2*f)) + e**2*h*x**3/(3*c)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^2\,(f+gx+hx^2)}{a+bx+cx^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*x+d)^2*(h*x^2+g*x+f)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 433, normalized size of antiderivative = 1.24

$$\int \frac{(d+ex)^2\,(f+gx+hx^2)}{a+bx+cx^2}\,dx$$
$$= \frac{2\,c^2e^2hx^3 + 3\,c^2e^2gx^2 + 6\,c^2dehx^2 - 3\,bce^2hx^2 + 6\,c^2e^2fx + 12\,c^2degx - 6\,bce^2gx + 6\,c^2d^2hx - 12\,bcdehx}{6\,c^3}$$
$$+ \frac{(2\,c^3def - bc^2e^2f + c^3d^2g - 2\,bc^2deg + b^2ce^2g - ac^2e^2g - bc^2d^2h + 2\,b^2cdeh - 2\,ac^2deh - b^3e^2h + 2\,ab}{2\,c^4}$$
$$+ \frac{(2\,c^4d^2f - 2\,bc^3def + b^2c^2e^2f - 2\,ac^3e^2f - bc^3d^2g + 2\,b^2c^2deg - 4\,ac^3deg - b^3ce^2g + 3\,abc^2e^2g + b^2c^2d}{\sqrt{-b^2+4\,ac}c}$$

```
[In] integrate((e*x+d)^2*(h*x^2+g*x+f)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] 1/6*(2*c^2*e^2*h*x^3 + 3*c^2*e^2*g*x^2 + 6*c^2*d*e*h*x^2 - 3*b*c*e^2*h*x^2
+ 6*c^2*e^2*f*x + 12*c^2*d*e*g*x - 6*b*c*e^2*g*x + 6*c^2*d^2*h*x - 12*b*c*d
*e*h*x + 6*b^2*e^2*h*x - 6*a*c*e^2*h*x)/c^3 + 1/2*(2*c^3*d*e*f - b*c^2*e^2*
f + c^3*d^2*g - 2*b*c^2*d*e*g + b^2*c*e^2*g - a*c^2*e^2*g - b*c^2*d^2*h + 2
*b^2*c*d*e*h - 2*a*c^2*d*e*h - b^3*e^2*h + 2*a*b*c*e^2*h)*log(c*x^2 + b*x +
 a)/c^4 + (2*c^4*d^2*f - 2*b*c^3*d*e*f + b^2*c^2*e^2*f - 2*a*c^3*e^2*f - b*
c^3*d^2*g + 2*b^2*c^2*d*e*g - 4*a*c^3*d*e*g - b^3*c*e^2*g + 3*a*b*c^2*e^2*g
 + b^2*c^2*d^2*h - 2*a*c^3*d^2*h - 2*b^3*c*d*e*h + 6*a*b*c^2*d*e*h + b^4*e^
2*h - 4*a*b^2*c*e^2*h + 2*a^2*c^2*e^2*h)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a
*c))/(sqrt(-b^2 + 4*a*c)*c^4)
```

Mupad [B] (verification not implemented)

Time = 13.69 (sec) , antiderivative size = 557, normalized size of antiderivative = 1.60

$$\int \frac{(d+ex)^2\,(f+gx+hx^2)}{a+bx+cx^2}\,dx$$
$$= x^2\left(\frac{g\,e^2+2\,d\,h\,e}{2\,c} - \frac{b\,e^2\,h}{2\,c^2}\right) - x\left(\frac{b\left(\frac{g\,e^2+2\,d\,h\,e}{c} - \frac{b\,e^2\,h}{c^2}\right)}{c} - \frac{h\,d^2+2\,g\,d\,e+f\,e^2}{c} + \frac{a\,e^2\,h}{c^2}\right)$$
$$- \frac{\ln\left(c\,x^2+b\,x+a\right)\,\left(-8\,h\,a^2\,b\,c^2\,e^2 + 8\,h\,a^2\,c^3\,d\,e + 4\,g\,a^2\,c^3\,e^2 + 6\,h\,a\,b^3\,c\,e^2 - 10\,h\,a\,b^2\,c^2\,d\,e - 5\,g\,a\,b^2\,c^2}{}$$
$$+ \frac{e^2\,h\,x^3}{3\,c}$$
$$+ \frac{\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}} + \frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)\,\left(2\,h\,a^2\,c^2\,e^2 - 4\,h\,a\,b^2\,c\,e^2 + 6\,h\,a\,b\,c^2\,d\,e + 3\,g\,a\,b\,c^2\,e^2 - 2\,h\,a\,c^3\,d^2 - 4\,g\,a\,c^3}{}$$

```
[In] int(((d + e*x)^2*(f + g*x + h*x^2))/(a + b*x + c*x^2),x)
```

```
[Out] x^2*((e^2*g + 2*d*e*h)/(2*c) - (b*e^2*h)/(2*c^2)) - x*((b*((e^2*g + 2*d*e*h
)/c - (b*e^2*h)/c^2))/c - (e^2*f + d^2*h + 2*d*e*g)/c + (a*e^2*h)/c^2) - (l
og(a + b*x + c*x^2)*(4*a^2*c^3*e^2*g - b^5*e^2*h + b^2*c^3*d^2*g - b^3*c^2*
e^2*f - b^3*c^2*d^2*h - 4*a*c^4*d^2*g + b^4*c*e^2*g + 4*a*b*c^3*e^2*f + 4*a
*b*c^3*d^2*h + 6*a*b^3*c*e^2*h + 2*b^2*c^3*d*e*f + 8*a^2*c^3*d*e*h - 2*b^3*
c^2*d*e*g - 5*a*b^2*c^2*e^2*g - 8*a^2*b*c^2*e^2*h - 8*a*c^4*d*e*f + 2*b^4*c
*d*e*h + 8*a*b*c^3*d*e*g - 10*a*b^2*c^2*d*e*h))/(2*(4*a*c^5 - b^2*c^4)) + (
e^2*h*x^3)/(3*c) + (atan(b/(4*a*c - b^2)^(1/2) + (2*c*x)/(4*a*c - b^2)^(1/2
))*(2*c^4*d^2*f + b^4*e^2*h + b^2*c^2*e^2*f + 2*a^2*c^2*e^2*h + b^2*c^2*d^2
*h - 2*a*c^3*e^2*f - 2*a*c^3*d^2*h - b*c^3*d^2*g - b^3*c*e^2*g + 3*a*b*c^2*
e^2*g - 4*a*b^2*c*e^2*h + 2*b^2*c^2*d*e*g - 4*a*c^3*d*e*g - 2*b*c^3*d*e*f -
 2*b^3*c*d*e*h + 6*a*b*c^2*d*e*h))/(c^4*(4*a*c - b^2)^(1/2))
```

3.150 $\int \frac{(d+ex)\left(f+gx+hx^2\right)}{a+bx+cx^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 177

$$\begin{aligned}\int \frac{(d+ex)\left(f+gx+hx^2\right)}{a+bx+cx^2}\,dx &= \frac{(ceg+cdh-beh)x}{c^2}+\frac{ehx^2}{2c}\\ &\quad-\frac{\left(2c^3df-b^3eh-c^2(bef+bdg+2aeg+2adh)+bc(beg+bdh+3aeh)\right)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}}\\ &\quad+\frac{\left(c^2(ef+dg)+b^2eh-c(beg+bdh+aeh)\right)\log\left(a+bx+cx^2\right)}{2c^3}\end{aligned}$$

```
[Out] (-b*e*h+c*d*h+c*e*g)*x/c^2+1/2*e*h*x^2/c+1/2*(c^2*(d*g+e*f)+b^2*e*h-c*(a*e*
h+b*d*h+b*e*g))*ln(c*x^2+b*x+a)/c^3-(2*c^3*d*f-b^3*e*h-c^2*(2*a*d*h+2*a*e*g
+b*d*g+b*e*f)+b*c*(3*a*e*h+b*d*h+b*e*g))*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/
2))/c^3/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1642, 648, 632, 212, 642}

$$\begin{aligned}\int \frac{(d+ex)\left(f+gx+hx^2\right)}{a+bx+cx^2}\,dx &=\\ &-\frac{\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(-c^2(2adh+2aeg+bdg+bef)+bc(3aeh+bdh+beg)+b^3(-e)h+2c^3df\right)}{c^3\sqrt{b^2-4ac}}\\ &+\frac{\log\left(a+bx+cx^2\right)\left(-c(aeh+bdh+beg)+b^2eh+c^2(dg+ef)\right)}{2c^3}\\ &+\frac{x(-beh+cdh+ceg)}{c^2}+\frac{ehx^2}{2c}\end{aligned}$$

```
[In] Int[((d + e*x)*(f + g*x + h*x^2))/(a + b*x + c*x^2),x]
```

```
[Out] ((c*e*g + c*d*h - b*e*h)*x)/c^2 + (e*h*x^2)/(2*c) - ((2*c^3*d*f - b^3*e*h -
 c^2*(b*e*f + b*d*g + 2*a*e*g + 2*a*d*h) + b*c*(b*e*g + b*d*h + 3*a*e*h))*A
rcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^3*Sqrt[b^2 - 4*a*c]) + ((c^2*(e*f
 + d*g) + b^2*e*h - c*(b*e*g + b*d*h + a*e*h))*Log[a + b*x + c*x^2])/(2*c^3
)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{ceg+cdh-beh}{c^2}+\frac{ehx}{c}\right.\\&\qquad\left.+\frac{c^2df+abeh-ac(eg+dh)+\left(c^2(ef+dg)+b^2eh-c(beg+bdh+aeh)\right)x}{c^2\left(a+bx+cx^2\right)}\right)dx\\&=\frac{(ceg+cdh-beh)x}{c^2}+\frac{ehx^2}{2c}+\frac{\int\frac{c^2df+abeh-ac(eg+dh)+\left(c^2(ef+dg)+b^2eh-c(beg+bdh+aeh)\right)x}{a+bx+cx^2}\,dx}{c^2}\end{aligned}$$

$$= \frac{(ceg+cdh-beh)x}{c^2}+\frac{ehx^2}{2c}+\frac{(c^2(ef+dg)+b^2eh-c(beg+bdh+aeh))\int\frac{b+2cx}{a+bx+cx^2}\,dx}{2c^3}$$
$$+\frac{(2c^3df-b^3eh-c^2(bef+bdg+2aeg+2adh)+bc(beg+bdh+3aeh))\int\frac{1}{a+bx+cx^2}\,dx}{2c^3}$$

$$= \frac{(ceg+cdh-beh)x}{c^2}+\frac{ehx^2}{2c}+\frac{(c^2(ef+dg)+b^2eh-c(beg+bdh+aeh))\log\left(a+bx+cx^2\right)}{2c^3}$$
$$-\frac{(2c^3df-b^3eh-c^2(bef+bdg+2aeg+2adh)+bc(beg+bdh+3aeh))\operatorname{Subst}\left(\int\frac{1}{b^2-4ac-x^2}\,dx,x\right.}{c^3}$$

$$= \frac{(ceg+cdh-beh)x}{c^2}+\frac{ehx^2}{2c}$$
$$-\frac{(2c^3df-b^3eh-c^2(bef+bdg+2aeg+2adh)+bc(beg+bdh+3aeh))\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}}$$
$$+\frac{(c^2(ef+dg)+b^2eh-c(beg+bdh+aeh))\log\left(a+bx+cx^2\right)}{2c^3}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.98

$$\int\frac{(d+ex)\left(f+gx+hx^2\right)}{a+bx+cx^2}\,dx$$
$$=\frac{2c(ceg+cdh-beh)x+c^2ehx^2-\frac{2\left(-2c^3df+b^3eh+c^2(bef+bdg+2aeg+2adh)-bc(beg+bdh+3aeh)\right)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+(c^2(}{2c^3}$$

```
[In] Integrate[((d + e*x)*(f + g*x + h*x^2))/(a + b*x + c*x^2),x]
```

```
[Out] (2*c*(c*e*g + c*d*h - b*e*h)*x + c^2*e*h*x^2 - (2*(-2*c^3*d*f + b^3*e*h + c
^2*(b*e*f + b*d*g + 2*a*e*g + 2*a*d*h) - b*c*(b*e*g + b*d*h + 3*a*e*h))*Arc
Tan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + (c^2*(e*f + d*g)
+ b^2*e*h - c*(b*e*g + b*d*h + a*e*h))*Log[a + x*(b + c*x)])/(2*c^3)
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.08

method	result
default	$-\frac{-\frac{1}{2}ceh\,x^2+behx-cdhx-cegx}{c^2}+\frac{\frac{\left(-aehc+b^2eh-bcdh-bceg+c^2dg+c^2ef\right)\ln\left(c\,x^2+bx+a\right)}{2c}+\frac{2\left(baeh-acdh-aceg+c^2df-\frac{(-aehc+b^2eh-b}{}\right.}{\sqrt{4c}}}{c^2}$
risch	Expression too large to display

```
[In] int((e*x+d)*(h*x^2+g*x+f)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/c^2*(-1/2*c*e*h*x^2+b*e*h*x-c*d*h*x-c*e*g*x)+1/c^2*(1/2*(-a*c*e*h+b^2*e*
h-b*c*d*h-b*c*e*g+c^2*d*g+c^2*e*f)/c*ln(c*x^2+b*x+a)+2*(b*a*e*h-a*c*d*h-a*c
*e*g+c^2*d*f-1/2*(-a*c*e*h+b^2*e*h-b*c*d*h-b*c*e*g+c^2*d*g+c^2*e*f)*b/c)/(4
*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 654, normalized size of antiderivative = 3.69

$$\int \frac{(d+ex)\left(f+gx+hx^2\right)}{a+bx+cx^2}\,dx$$

$$= \Bigg[\frac{(b^2c^2-4\,ac^3)ehx^2+\sqrt{b^2-4\,ac}((2\,c^3d-bc^2e)f-(bc^2d-(b^2c-2\,ac^2)e)g+((b^2c-2\,ac^2)d-(b^3-3\,ab}{}$$

```
[In] integrate((e*x+d)*(h*x^2+g*x+f)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] [1/2*((b^2*c^2 - 4*a*c^3)*e*h*x^2 + sqrt(b^2 - 4*a*c)*((2*c^3*d - b*c^2*e)*
f - (b*c^2*d - (b^2*c - 2*a*c^2)*e)*g + ((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b
*c)*e)*h)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x
 + b))/(c*x^2 + b*x + a)) + 2*((b^2*c^2 - 4*a*c^3)*e*g + ((b^2*c^2 - 4*a*c^
3)*d - (b^3*c - 4*a*b*c^2)*e)*h)*x + ((b^2*c^2 - 4*a*c^3)*e*f + ((b^2*c^2 -
 4*a*c^3)*d - (b^3*c - 4*a*b*c^2)*e)*g - ((b^3*c - 4*a*b*c^2)*d - (b^4 - 5*
a*b^2*c + 4*a^2*c^2)*e)*h)*log(c*x^2 + b*x + a))/(b^2*c^3 - 4*a*c^4), 1/2*(
(b^2*c^2 - 4*a*c^3)*e*h*x^2 - 2*sqrt(-b^2 + 4*a*c)*((2*c^3*d - b*c^2*e)*f -
 (b*c^2*d - (b^2*c - 2*a*c^2)*e)*g + ((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)
*e)*h)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*((b^2*c^2
- 4*a*c^3)*e*g + ((b^2*c^2 - 4*a*c^3)*d - (b^3*c - 4*a*b*c^2)*e)*h)*x + ((b
^2*c^2 - 4*a*c^3)*e*f + ((b^2*c^2 - 4*a*c^3)*d - (b^3*c - 4*a*b*c^2)*e)*g -
 ((b^3*c - 4*a*b*c^2)*d - (b^4 - 5*a*b^2*c + 4*a^2*c^2)*e)*h)*log(c*x^2 + b
*x + a))/(b^2*c^3 - 4*a*c^4)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1265 vs. 2(182) = 364.

Time = 6.20 (sec) , antiderivative size = 1265, normalized size of antiderivative = 7.15

$$\int \frac{(d+ex)\,(f+gx+hx^2)}{a+bx+cx^2}\,dx = x\left(-\frac{beh}{c^2}+\frac{dh}{c}+\frac{eg}{c}\right)$$

$$+\left(-\frac{\sqrt{-4ac+b^2}\cdot(3abceh-2ac^2dh-2ac^2eg-b^3eh+b^2cdh+b^2ceg-bc^2dg-bc^2ef+2c^3df)}{2c^3\cdot(4ac-b^2)}\right.$$

$$\left.-\frac{aceh-b^2eh+bcdh+bceg-c^2dg-c^2ef}{2c^3}\right)\log\left(x+\frac{2a^2ceh-ab^2eh+abcdh+abceg+4ac^3\left(-\frac{\sqrt{-4a}}{}\right.}{}\right.$$

$$+\left(\frac{\sqrt{-4ac+b^2}\cdot(3abceh-2ac^2dh-2ac^2eg-b^3eh+b^2cdh+b^2ceg-bc^2dg-bc^2ef+2c^3df)}{2c^3\cdot(4ac-b^2)}\right.$$

$$\left.-\frac{aceh-b^2eh+bcdh+bceg-c^2dg-c^2ef}{2c^3}\right)\log\left(x+\frac{2a^2ceh-ab^2eh+abcdh+abceg+4ac^3\left(\frac{\sqrt{-4ac+}}{}\right.}{}\right.$$

$$+\frac{ehx^2}{2c}$$

```
[In] integrate((e*x+d)*(h*x**2+g*x+f)/(c*x**2+b*x+a),x)
```

```
[Out] x*(-b*e*h/c**2 + d*h/c + e*g/c) + (-sqrt(-4*a*c + b**2)*(3*a*b*c*e*h - 2*a*
c**2*d*h - 2*a*c**2*e*g - b**3*e*h + b**2*c*d*h + b**2*c*e*g - b*c**2*d*g -
 b*c**2*e*f + 2*c**3*d*f)/(2*c**3*(4*a*c - b**2)) - (a*c*e*h - b**2*e*h + b
*c*d*h + b*c*e*g - c**2*d*g - c**2*e*f)/(2*c**3))*log(x + (2*a**2*c*e*h - a
*b**2*e*h + a*b*c*d*h + a*b*c*e*g + 4*a*c**3*(-sqrt(-4*a*c + b**2)*(3*a*b*c
*e*h - 2*a*c**2*d*h - 2*a*c**2*e*g - b**3*e*h + b**2*c*d*h + b**2*c*e*g - b
*c**2*d*g - b*c**2*e*f + 2*c**3*d*f)/(2*c**3*(4*a*c - b**2)) - (a*c*e*h - b
**2*e*h + b*c*d*h + b*c*e*g - c**2*d*g - c**2*e*f)/(2*c**3)) - 2*a*c**2*d*g
 - 2*a*c**2*e*f - b**2*c**2*(-sqrt(-4*a*c + b**2)*(3*a*b*c*e*h - 2*a*c**2*d
*h - 2*a*c**2*e*g - b**3*e*h + b**2*c*d*h + b**2*c*e*g - b*c**2*d*g - b*c**
2*e*f + 2*c**3*d*f)/(2*c**3*(4*a*c - b**2)) - (a*c*e*h - b**2*e*h + b*c*d*h
 + b*c*e*g - c**2*d*g - c**2*e*f)/(2*c**3)) + b*c**2*d*f)/(3*a*b*c*e*h - 2*
a*c**2*d*h - 2*a*c**2*e*g - b**3*e*h + b**2*c*d*h + b**2*c*e*g - b*c**2*d*g
 - b*c**2*e*f + 2*c**3*d*f)) + (sqrt(-4*a*c + b**2)*(3*a*b*c*e*h - 2*a*c**2
*d*h - 2*a*c**2*e*g - b**3*e*h + b**2*c*d*h + b**2*c*e*g - b*c**2*d*g - b*c
**2*e*f + 2*c**3*d*f)/(2*c**3*(4*a*c - b**2)) - (a*c*e*h - b**2*e*h + b*c*d
*h + b*c*e*g - c**2*d*g - c**2*e*f)/(2*c**3))*log(x + (2*a**2*c*e*h - a*b**
2*e*h + a*b*c*d*h + a*b*c*e*g + 4*a*c**3*(sqrt(-4*a*c + b**2)*(3*a*b*c*e*h
- 2*a*c**2*d*h - 2*a*c**2*e*g - b**3*e*h + b**2*c*d*h + b**2*c*e*g - b*c**2
*d*g - b*c**2*e*f + 2*c**3*d*f)/(2*c**3*(4*a*c - b**2)) - (a*c*e*h - b**2*e
*h + b*c*d*h + b*c*e*g - c**2*d*g - c**2*e*f)/(2*c**3)) - 2*a*c**2*d*g - 2*
a*c**2*e*f - b**2*c**2*(sqrt(-4*a*c + b**2)*(3*a*b*c*e*h - 2*a*c**2*d*h - 2
*a*c**2*e*g - b**3*e*h + b**2*c*d*h + b**2*c*e*g - b*c**2*d*g - b*c**2*e*f
+ 2*c**3*d*f)/(2*c**3*(4*a*c - b**2)) - (a*c*e*h - b**2*e*h + b*c*d*h + b*c
```

```
*e*g - c**2*d*g - c**2*e*f)/(2*c**3)) + b*c**2*d*f)/(3*a*b*c*e*h - 2*a*c**2
*d*h - 2*a*c**2*e*g - b**3*e*h + b**2*c*d*h + b**2*c*e*g - b*c**2*d*g - b*c
**2*e*f + 2*c**3*d*f)) + e*h*x**2/(2*c)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)\,(f+gx+hx^2)}{a+bx+cx^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*x+d)*(h*x^2+g*x+f)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.07

$$\int \frac{(d+ex)\,(f+gx+hx^2)}{a+bx+cx^2}\,dx = \frac{cehx^2+2\,cegx+2\,cdhx-2\,behx}{2\,c^2} + \frac{(c^2ef+c^2dg-bceg-bcdh+b^2eh-aceh)\log(cx^2+bx+a)}{2\,c^3} + \frac{(2\,c^3df-bc^2ef-bc^2dg+b^2ceg-2\,ac^2eg+b^2cdh-2\,ac^2dh-b^3eh+3\,abceh)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}c^3}$$

```
[In] integrate((e*x+d)*(h*x^2+g*x+f)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(c*e*h*x^2 + 2*c*e*g*x + 2*c*d*h*x - 2*b*e*h*x)/c^2 + 1/2*(c^2*e*f + c^
2*d*g - b*c*e*g - b*c*d*h + b^2*e*h - a*c*e*h)*log(c*x^2 + b*x + a)/c^3 + (
2*c^3*d*f - b*c^2*e*f - b*c^2*d*g + b^2*c*e*g - 2*a*c^2*e*g + b^2*c*d*h - 2
*a*c^2*d*h - b^3*e*h + 3*a*b*c*e*h)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/
(sqrt(-b^2 + 4*a*c)*c^3)
```

Mupad [B] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.54

$$\int \frac{(d+ex)\left(f+gx+hx^2\right)}{a+bx+cx^2}\,dx = x\left(\frac{d\,h+e\,g}{c}-\frac{b\,e\,h}{c^2}\right)$$

$$-\frac{\ln\left(c\,x^2+b\,x+a\right)\,\left(b^4\,e\,h-4\,a\,c^3\,d\,g-4\,a\,c^3\,e\,f-b^3\,c\,d\,h-b^3\,c\,e\,g+b^2\,c^2\,d\,g+b^2\,c^2\,e\,f+4\,a^2\,c^2\,e\,h+\right.}{2\,\left(4\,a\,c^4-b^2\,c^3\right)}$$

$$-\frac{\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)\,\left(b^3\,e\,h-2\,c^3\,d\,f+2\,a\,c^2\,d\,h+2\,a\,c^2\,e\,g+b\,c^2\,d\,g+b\,c^2\,e\,f-b^2\,c\,d\,h-b^2\,c\,e\right.}{c^3\,\sqrt{4\,a\,c-b^2}}$$

$$+\frac{e\,h\,x^2}{2\,c}$$

```
[In] int(((d + e*x)*(f + g*x + h*x^2))/(a + b*x + c*x^2),x)
```

```
[Out] x*((d*h + e*g)/c - (b*e*h)/c^2) - (log(a + b*x + c*x^2)*(b^4*e*h - 4*a*c^3*
d*g - 4*a*c^3*e*f - b^3*c*d*h - b^3*c*e*g + b^2*c^2*d*g + b^2*c^2*e*f + 4*a
^2*c^2*e*h + 4*a*b*c^2*d*h + 4*a*b*c^2*e*g - 5*a*b^2*c*e*h))/(2*(4*a*c^4 -
b^2*c^3)) - (atan(b/(4*a*c - b^2)^(1/2) + (2*c*x)/(4*a*c - b^2)^(1/2))*(b^3
*e*h - 2*c^3*d*f + 2*a*c^2*d*h + 2*a*c^2*e*g + b*c^2*d*g + b*c^2*e*f - b^2*
c*d*h - b^2*c*e*g - 3*a*b*c*e*h))/(c^3*(4*a*c - b^2)^(1/2)) + (e*h*x^2)/(2*
c)
```

3.151 $\int \frac{f+gx+hx^2}{a+bx+cx^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 92

$$\int \frac{f+gx+hx^2}{a+bx+cx^2}\,dx = \frac{hx}{c} - \frac{\left(2c^2f-bcg+b^2h-2ach\right)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}} + \frac{(cg-bh)\log\left(a+bx+cx^2\right)}{2c^2}$$

```
[Out] h*x/c+1/2*(-b*h+c*g)*ln(c*x^2+b*x+a)/c^2-(-2*a*c*h+b^2*h-b*c*g+2*c^2*f)*arc
tanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1671, 648, 632, 212, 642}

$$\int \frac{f+gx+hx^2}{a+bx+cx^2}\,dx = -\frac{\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(-2ach+b^2h-bcg+2c^2f\right)}{c^2\sqrt{b^2-4ac}} + \frac{(cg-bh)\log\left(a+bx+cx^2\right)}{2c^2} + \frac{hx}{c}$$

```
[In] Int[(f + g*x + h*x^2)/(a + b*x + c*x^2),x]
```

```
[Out] (h*x)/c - ((2*c^2*f - b*c*g + b^2*h - 2*a*c*h)*ArcTanh[(b + 2*c*x)/Sqrt[b^2
 - 4*a*c]])/(c^2*Sqrt[b^2 - 4*a*c]) + ((c*g - b*h)*Log[a + b*x + c*x^2])/(2
*c^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{h}{c}+\frac{cf-ah+(cg-bh)x}{c\,(a+bx+cx^2)}\right)dx\\
&= \frac{hx}{c}+\frac{\int \frac{cf-ah+(cg-bh)x}{a+bx+cx^2}\,dx}{c}\\
&= \frac{hx}{c}+\frac{(cg-bh)\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2c^2}+\frac{(2c^2f-bcg+b^2h-2ach)\int \frac{1}{a+bx+cx^2}\,dx}{2c^2}\\
&= \frac{hx}{c}+\frac{(cg-bh)\log\left(a+bx+cx^2\right)}{2c^2}\\
&\quad-\frac{(2c^2f-bcg+b^2h-2ach)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx\right)}{c^2}\\
&= \frac{hx}{c}-\frac{(2c^2f-bcg+b^2h-2ach)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}+\frac{(cg-bh)\log\left(a+bx+cx^2\right)}{2c^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.03

$$\int \frac{f+gx+hx^2}{a+bx+cx^2}\,dx = \frac{hx}{c}+\frac{(2c^2f-bcg+b^2h-2ach)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{c^2\sqrt{-b^2+4ac}} + \frac{(cg-bh)\log(a+bx+cx^2)}{2c^2}$$

```
[In] Integrate[(f + g*x + h*x^2)/(a + b*x + c*x^2),x]
```

```
[Out] (h*x)/c + ((2*c^2*f - b*c*g + b^2*h - 2*a*c*h)*ArcTan[(b + 2*c*x)/Sqrt[-b^2
 + 4*a*c]])/(c^2*Sqrt[-b^2 + 4*a*c]) + ((c*g - b*h)*Log[a + b*x + c*x^2])/(
2*c^2)
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.01

method	result	size
default	$\frac{hx}{c}+\frac{\frac{(-bh+cg)\ln\left(c\,x^2+bx+a\right)}{2c}+\frac{2\left(-ah+cf-\frac{(-bh+cg)b}{2c}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{c}$	93
risch	Expression too large to display	1649

```
[In] int((h*x^2+g*x+f)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] h*x/c+1/c*(1/2*(-b*h+c*g)/c*ln(c*x^2+b*x+a)+2*(-a*h+c*f-1/2*(-b*h+c*g)*b/c)
/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 302, normalized size of antiderivative = 3.28

$$\int \frac{f+gx+hx^2}{a+bx+cx^2}\,dx = \left[\frac{2\,(b^2c-4\,ac^2)hx-(2\,c^2f-bcg+(b^2-2\,ac)h)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)+((b^2c}{2\,(b^2c^2-4\,ac^3)}\right.$$

```
[In] integrate((h*x^2+g*x+f)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] [1/2*(2*(b^2*c - 4*a*c^2)*h*x - (2*c^2*f - b*c*g + (b^2 - 2*a*c)*h)*sqrt(b^
2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*
```

```
x + b))/(c*x^2 + b*x + a)) + ((b^2*c - 4*a*c^2)*g - (b^3 - 4*a*b*c)*h)*log(
c*x^2 + b*x + a))/(b^2*c^2 - 4*a*c^3), 1/2*(2*(b^2*c - 4*a*c^2)*h*x - 2*(2*
c^2*f - b*c*g + (b^2 - 2*a*c)*h)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a
*c)*(2*c*x + b)/(b^2 - 4*a*c)) + ((b^2*c - 4*a*c^2)*g - (b^3 - 4*a*b*c)*h)*
log(c*x^2 + b*x + a))/(b^2*c^2 - 4*a*c^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 488 vs. $2(88) = 176$.

Time = 1.10 (sec) , antiderivative size = 488, normalized size of antiderivative = 5.30

$$\int \frac{f+gx+hx^2}{a+bx+cx^2}\,dx = \left(-\frac{\sqrt{-4ac+b^2}\cdot(2ach-b^2h+bcg-2c^2f)}{2c^2\cdot(4ac-b^2)}\right.$$

$$\left.-\frac{bh-cg}{2c^2}\right)\log\left(x+\frac{-abh-4ac^2\left(-\frac{\sqrt{-4ac+b^2}\cdot(2ach-b^2h+bcg-2c^2f)}{2c^2\cdot(4ac-b^2)}-\frac{bh-cg}{2c^2}\right)+2acg+b^2c\left(-\frac{\sqrt{-4ac+b^2}\cdot(2ach}{2c^2\cdot(4a}\right.}{2ach-b^2h+bcg-2c^2f}\right.$$

$$+\left(\frac{\sqrt{-4ac+b^2}\cdot(2ach-b^2h+bcg-2c^2f)}{2c^2\cdot(4ac-b^2)}\right.$$

$$\left.-\frac{bh-cg}{2c^2}\right)\log\left(x+\frac{-abh-4ac^2\left(\frac{\sqrt{-4ac+b^2}\cdot(2ach-b^2h+bcg-2c^2f)}{2c^2\cdot(4ac-b^2)}-\frac{bh-cg}{2c^2}\right)+2acg+b^2c\left(\frac{\sqrt{-4ac+b^2}\cdot(2ach-b^2h}{2c^2\cdot(4ac-b^2}\right.}{2ach-b^2h+bcg-2c^2f}\right.$$

$$+\frac{hx}{c}$$

```
[In] integrate((h*x**2+g*x+f)/(c*x**2+b*x+a),x)
```

```
[Out] (-sqrt(-4*a*c + b**2)*(2*a*c*h - b**2*h + b*c*g - 2*c**2*f)/(2*c**2*(4*a*c
- b**2)) - (b*h - c*g)/(2*c**2))*log(x + (-a*b*h - 4*a*c**2*(-sqrt(-4*a*c +
 b**2)*(2*a*c*h - b**2*h + b*c*g - 2*c**2*f)/(2*c**2*(4*a*c - b**2)) - (b*h
 - c*g)/(2*c**2)) + 2*a*c*g + b**2*c*(-sqrt(-4*a*c + b**2)*(2*a*c*h - b**2*
h + b*c*g - 2*c**2*f)/(2*c**2*(4*a*c - b**2)) - (b*h - c*g)/(2*c**2)) - b*c
*f)/(2*a*c*h - b**2*h + b*c*g - 2*c**2*f)) + (sqrt(-4*a*c + b**2)*(2*a*c*h
- b**2*h + b*c*g - 2*c**2*f)/(2*c**2*(4*a*c - b**2)) - (b*h - c*g)/(2*c**2)
)*log(x + (-a*b*h - 4*a*c**2*(sqrt(-4*a*c + b**2)*(2*a*c*h - b**2*h + b*c*g
 - 2*c**2*f)/(2*c**2*(4*a*c - b**2)) - (b*h - c*g)/(2*c**2)) + 2*a*c*g + b*
*2*c*(sqrt(-4*a*c + b**2)*(2*a*c*h - b**2*h + b*c*g - 2*c**2*f)/(2*c**2*(4*
a*c - b**2)) - (b*h - c*g)/(2*c**2)) - b*c*f)/(2*a*c*h - b**2*h + b*c*g - 2
*c**2*f)) + h*x/c
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{f+gx+hx^2}{a+bx+cx^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((h*x^2+g*x+f)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.97

$$\int \frac{f+gx+hx^2}{a+bx+cx^2}\,dx = \frac{hx}{c} + \frac{(cg-bh)\log\left(cx^2+bx+a\right)}{2\,c^2} + \frac{\left(2\,c^2f-bcg+b^2h-2\,ach\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}c^2}$$

```
[In] integrate((h*x^2+g*x+f)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] h*x/c + 1/2*(c*g - b*h)*log(c*x^2 + b*x + a)/c^2 + (2*c^2*f - b*c*g + b^2*h
 - 2*a*c*h)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2)
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.43

$$\int \frac{f+gx+hx^2}{a+bx+cx^2}\,dx = \frac{h\,x}{c} + \frac{\ln\left(c\,x^2+b\,x+a\right)\,\left(h\,b^3-g\,b^2\,c-4\,a\,h\,b\,c+4\,a\,g\,c^2\right)}{2\,\left(4\,a\,c^3-b^2\,c^2\right)} + \frac{\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)\,\left(h\,b^2-g\,b\,c+2\,f\,c^2-2\,a\,h\,c\right)}{c^2\,\sqrt{4\,a\,c-b^2}}$$

```
[In] int((f + g*x + h*x^2)/(a + b*x + c*x^2),x)
```

```
[Out] (h*x)/c + (log(a + b*x + c*x^2)*(b^3*h + 4*a*c^2*g - b^2*c*g - 4*a*b*c*h))/
(2*(4*a*c^3 - b^2*c^2)) + (atan(b/(4*a*c - b^2)^(1/2) + (2*c*x)/(4*a*c - b^
2)^(1/2))*(2*c^2*f + b^2*h - 2*a*c*h - b*c*g))/(c^2*(4*a*c - b^2)^(1/2))
```

3.152 $\int \frac{f+gx+hx^2}{(d+ex)(a+bx+cx^2)} \, dx$

Optimal result

Integrand size = 30, antiderivative size = 196

$$\begin{aligned}\int \frac{f+gx+hx^2}{(d+ex)\,(a+bx+cx^2)}\,dx &= -\frac{(2c^2df+b(bd-ae)h-c(bef+bdg-2aeg+2adh))\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}\,(cd^2-bde+ae^2)} \\ &\quad+\frac{(e^2f-deg+d^2h)\log(d+ex)}{e\,(cd^2-bde+ae^2)}-\frac{(cef-cdg+bdh-aeh)\log\left(a+bx+cx^2\right)}{2c\,(cd^2-bde+ae^2)}\end{aligned}$$

```
[Out] (d^2*h-d*e*g+e^2*f)*ln(e*x+d)/e/(a*e^2-b*d*e+c*d^2)-1/2*(-a*e*h+b*d*h-c*d*g
+c*e*f)*ln(c*x^2+b*x+a)/c/(a*e^2-b*d*e+c*d^2)-(2*c^2*d*f+b*(-a*e+b*d)*h-c*(
2*a*d*h-2*a*e*g+b*d*g+b*e*f))*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c/(a*e^
2-b*d*e+c*d^2)/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1642, 648, 632, 212, 642}

$$\begin{aligned}\int \frac{f+gx+hx^2}{(d+ex)\,(a+bx+cx^2)}\,dx &= -\frac{\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(-c(2adh-2aeg+bdg+bef)+bh(bd-ae)+2c^2df\right)}{c\sqrt{b^2-4ac}\,(ae^2-bde+cd^2)} \\ &\quad-\frac{\log\left(a+bx+cx^2\right)(-aeh+bdh-cdg+cef)}{2c\,(ae^2-bde+cd^2)}+\frac{\log(d+ex)\,(d^2h-deg+e^2f)}{e\,(ae^2-bde+cd^2)}\end{aligned}$$

```
[In] Int[(f + g*x + h*x^2)/((d + e*x)*(a + b*x + c*x^2)),x]
```

```
[Out] -(((2*c^2*d*f + b*(b*d - a*e)*h - c*(b*e*f + b*d*g - 2*a*e*g + 2*a*d*h))*Ar
cTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e +
 a*e^2))) + ((e^2*f - d*e*g + d^2*h)*Log[d + e*x])/(e*(c*d^2 - b*d*e + a*e^
2)) - ((c*e*f - c*d*g + b*d*h - a*e*h)*Log[a + b*x + c*x^2])/(2*c*(c*d^2 -
b*d*e + a*e^2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\text{integral} = \int \left(\frac{e^2f-deg+d^2h}{(cd^2-bde+ae^2)(d+ex)} + \frac{cdf-bef+aeg-adh-(cef-cdg+bdh-aeh)x}{(cd^2-bde+ae^2)(a+bx+cx^2)}\right)dx$$

$$= \frac{(e^2f-deg+d^2h)\log(d+ex)}{e(cd^2-bde+ae^2)} + \frac{\int \frac{cdf-bef+aeg-adh-(cef-cdg+bdh-aeh)x}{a+bx+cx^2}\,dx}{cd^2-bde+ae^2}$$

$$
\begin{aligned}
&= \frac{(e^2f-deg+d^2h)\log(d+ex)}{e\,(cd^2-bde+ae^2)} - \frac{(cef-cdg+bdh-aeh)\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2c\,(cd^2-bde+ae^2)} \\
&\quad + \frac{(2c^2df+b(bd-ae)h-c(bef+bdg-2aeg+2adh))\int \frac{1}{a+bx+cx^2}\,dx}{2c\,(cd^2-bde+ae^2)} \\
&= \frac{(e^2f-deg+d^2h)\log(d+ex)}{e\,(cd^2-bde+ae^2)} - \frac{(cef-cdg+bdh-aeh)\log(a+bx+cx^2)}{2c\,(cd^2-bde+ae^2)} \\
&\quad - \frac{(2c^2df+b(bd-ae)h-c(bef+bdg-2aeg+2adh))\,\mathrm{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx\right)}{c\,(cd^2-bde+ae^2)} \\
&= -\frac{(2c^2df+b(bd-ae)h-c(bef+bdg-2aeg+2adh))\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}\,(cd^2-bde+ae^2)} \\
&\quad + \frac{(e^2f-deg+d^2h)\log(d+ex)}{e\,(cd^2-bde+ae^2)} - \frac{(cef-cdg+bdh-aeh)\log(a+bx+cx^2)}{2c\,(cd^2-bde+ae^2)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.98

$$
\int \frac{f+gx+hx^2}{(d+ex)\,(a+bx+cx^2)}\,dx = \frac{-2e(-2c^2df+b(-bd+ae)h+c(bef+bdg-2aeg+2adh))\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)+2c\sqrt{-b^2+4ac}(e^2f-}{2c\sqrt{-b^2+4ac}e\,(cd^2+e(-bd}
$$

```
[In] Integrate[(f + g*x + h*x^2)/((d + e*x)*(a + b*x + c*x^2)),x]
```

```
[Out] (-2*e*(-2*c^2*d*f + b*(-(b*d) + a*e)*h + c*(b*e*f + b*d*g - 2*a*e*g + 2*a*d
*h))*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]] + 2*c*Sqrt[-b^2 + 4*a*c]*(e^2*f
 - d*e*g + d^2*h)*Log[d + e*x] - Sqrt[-b^2 + 4*a*c]*e*(c*e*f - c*d*g + b*d*
h - a*e*h)*Log[a + x*(b + c*x)])/(2*c*Sqrt[-b^2 + 4*a*c]*e*(c*d^2 + e*(-(b*
d) + a*e)))
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.91

method	result
default	$\frac{(d^2h-deg+e^2f)\ln(ex+d)}{e(e^2a-bde+cd^2)}+\frac{\frac{(aeh-bdh+cdg-cef)\ln\left(cx^2+bx+a\right)}{2c}+\frac{2\left(-adh+aeg-bef+cdf-\frac{(aeh-bdh+cdg-cef)b}{2c}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{e^2a-bde+cd^2}$
risch	$\frac{\ln(ex+d)d^2h}{e(e^2a-bde+cd^2)}-\frac{\ln(ex+d)dg}{e^2a-bde+cd^2}+\frac{\ln(ex+d)ef}{e^2a-bde+cd^2}+\Bigg(\_R=\mathrm{RootOf}\big((4a^2c^2e^2-ab^2ce^2-4abc^2de+4ac^3d^2+b^3cde-b^2c^2d^2)\_$

```
[In] int((h*x^2+g*x+f)/(e*x+d)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] (d^2*h-d*e*g+e^2*f)*ln(e*x+d)/e/(a*e^2-b*d*e+c*d^2)+1/(a*e^2-b*d*e+c*d^2)*(
1/2*(a*e*h-b*d*h+c*d*g-c*e*f)/c*ln(c*x^2+b*x+a)+2*(-a*d*h+a*e*g-b*e*f+c*d*f
-1/2*(a*e*h-b*d*h+c*d*g-c*e*f)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a
*c-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 30.41 (sec) , antiderivative size = 625, normalized size of antiderivative = 3.19

$$\int \frac{f+gx+hx^2}{(d+ex)\,(a+bx+cx^2)}\,dx$$

$$= \Bigg[-\frac{\sqrt{b^2-4\,ac}((2\,c^2de-bce^2)f-(bcde-2\,ace^2)g-(abe^2-(b^2-2\,ac)de)h)\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4}}{cx^2+bx+a}\right.}{}$$

$$-\frac{2\,\sqrt{-b^2+4\,ac}((2\,c^2de-bce^2)f-(bcde-2\,ace^2)g-(abe^2-(b^2-2\,ac)de)h)\arctan\left(-\frac{\sqrt{-b^2+4\,ac}(2\,cx+b)}{b^2-4\,ac}\right)}{}$$

```
[In] integrate((h*x^2+g*x+f)/(e*x+d)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(b^2 - 4*a*c)*((2*c^2*d*e - b*c*e^2)*f - (b*c*d*e - 2*a*c*e^2)*g
 - (a*b*e^2 - (b^2 - 2*a*c)*d*e)*h)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c
+ sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + ((b^2*c - 4*a*c^2)*e^
2*f - (b^2*c - 4*a*c^2)*d*e*g + ((b^3 - 4*a*b*c)*d*e - (a*b^2 - 4*a^2*c)*e^
2)*h)*log(c*x^2 + b*x + a) - 2*((b^2*c - 4*a*c^2)*e^2*f - (b^2*c - 4*a*c^2)
*d*e*g + (b^2*c - 4*a*c^2)*d^2*h)*log(e*x + d))/((b^2*c^2 - 4*a*c^3)*d^2*e
- (b^3*c - 4*a*b*c^2)*d*e^2 + (a*b^2*c - 4*a^2*c^2)*e^3), -1/2*(2*sqrt(-b^2
 + 4*a*c)*((2*c^2*d*e - b*c*e^2)*f - (b*c*d*e - 2*a*c*e^2)*g - (a*b*e^2 - (
b^2 - 2*a*c)*d*e)*h)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c))
+ ((b^2*c - 4*a*c^2)*e^2*f - (b^2*c - 4*a*c^2)*d*e*g + ((b^3 - 4*a*b*c)*d*e
 - (a*b^2 - 4*a^2*c)*e^2)*h)*log(c*x^2 + b*x + a) - 2*((b^2*c - 4*a*c^2)*e^
2*f - (b^2*c - 4*a*c^2)*d*e*g + (b^2*c - 4*a*c^2)*d^2*h)*log(e*x + d))/((b^
2*c^2 - 4*a*c^3)*d^2*e - (b^3*c - 4*a*b*c^2)*d*e^2 + (a*b^2*c - 4*a^2*c^2)*
e^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{f+gx+hx^2}{(d+ex)\left(a+bx+cx^2\right)}\,dx=\text{Timed out}$$

```
[In] integrate((h*x**2+g*x+f)/(e*x+d)/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{f+gx+hx^2}{(d+ex)\left(a+bx+cx^2\right)}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((h*x^2+g*x+f)/(e*x+d)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.02

$$\begin{aligned}\int \frac{f+gx+hx^2}{(d+ex)\left(a+bx+cx^2\right)}\,dx &= -\frac{(cef-cdg+bdh-aeh)\log\left(cx^2+bx+a\right)}{2\left(c^2d^2-bcde+ace^2\right)}+\frac{\left(e^2f-deg+d^2h\right)\log\left(|ex+d|\right)}{cd^2e-bde^2+ae^3}\\&\quad+\frac{\left(2\,c^2df-bcef-bcdg+2\,aceg+b^2dh-2\,acdh-abeh\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(c^2d^2-bcde+ace^2\right)\sqrt{-b^2+4\,ac}}\end{aligned}$$

```
[In] integrate((h*x^2+g*x+f)/(e*x+d)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] -1/2*(c*e*f - c*d*g + b*d*h - a*e*h)*log(c*x^2 + b*x + a)/(c^2*d^2 - b*c*d*
e + a*c*e^2) + (e^2*f - d*e*g + d^2*h)*log(abs(e*x + d))/(c*d^2*e - b*d*e^2
 + a*e^3) + (2*c^2*d*f - b*c*e*f - b*c*d*g + 2*a*c*e*g + b^2*d*h - 2*a*c*d*
h - a*b*e*h)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((c^2*d^2 - b*c*d*e + a
*c*e^2)*sqrt(-b^2 + 4*a*c))
```

Mupad [B] (verification not implemented)

Time = 20.95 (sec) , antiderivative size = 2467, normalized size of antiderivative = 12.59

$$\int \frac{f+gx+hx^2}{(d+ex)\,(a+bx+cx^2)}\,dx=\text{Too large to display}$$

```
[In] int((f + g*x + h*x^2)/((d + e*x)*(a + b*x + c*x^2)),x)
```

```
[Out] (log(a^2*b*e^4*g - 2*a*b^2*e^4*f - 2*a^3*e^4*h + 6*a^2*c*e^4*f - 4*a*c^2*d^
4*h + b^2*c*d^4*h + b^3*d^3*e*h - 2*b^3*e^4*f*x + a^2*e^4*g*(b^2 - 4*a*c)^(
1/2) + a*b^2*d*e^3*g + 6*a*c^2*d^3*e*g + b*c^2*d^3*e*f + 3*a^2*b*d*e^3*h -
10*a^2*c*d*e^3*g - 2*b^2*c*d^3*e*g + a*b^2*e^4*g*x - a^2*b*e^4*h*x - 2*a^2*
c*e^4*g*x + b^3*d*e^3*g*x + 2*c^3*d^3*e*f*x - 3*a^2*d*e^3*h*(b^2 - 4*a*c)^(
1/2) - c^2*d^3*e*f*(b^2 - 4*a*c)^(1/2) - b^2*d^3*e*h*(b^2 - 4*a*c)^(1/2) -
2*b^2*e^4*f*x*(b^2 - 4*a*c)^(1/2) - a^2*e^4*h*x*(b^2 - 4*a*c)^(1/2) - 2*c^2
*d^4*h*x*(b^2 - 4*a*c)^(1/2) - 10*a*c^2*d^2*e^2*f - 4*a*b^2*d^2*e^2*h + b^2
*c*d^2*e^2*f + 10*a^2*c*d^2*e^2*h - b^3*d^2*e^2*h*x - 2*a*b*e^4*f*(b^2 - 4*
a*c)^(1/2) - b*c*d^4*h*(b^2 - 4*a*c)^(1/2) + 3*a*b*c*d*e^3*f - 3*a*b*c*d^3*
e*h + 7*a*b*c*e^4*f*x - 5*c^2*d^2*e^2*f*x*(b^2 - 4*a*c)^(1/2) - b^2*d^2*e^2
*h*x*(b^2 - 4*a*c)^(1/2) + a*b*d*e^3*g*(b^2 - 4*a*c)^(1/2) + 7*a*c*d*e^3*f*
(b^2 - 4*a*c)^(1/2) + 5*a*c*d^3*e*h*(b^2 - 4*a*c)^(1/2) + 2*b*c*d^3*e*g*(b^
2 - 4*a*c)^(1/2) + a*b*e^4*g*x*(b^2 - 4*a*c)^(1/2) + 3*a*c*e^4*f*x*(b^2 - 4
*a*c)^(1/2) + 3*a*b*c*d^2*e^2*g - 14*a*c^2*d*e^3*f*x + 5*b^2*c*d*e^3*f*x -
10*a*c^2*d^3*e*h*x - b*c^2*d^3*e*g*x + 6*a^2*c*d*e^3*h*x + 3*b^2*c*d^3*e*h*
x + 2*a*b*d^2*e^2*h*(b^2 - 4*a*c)^(1/2) - 7*a*c*d^2*e^2*g*(b^2 - 4*a*c)^(1/
2) - b*c*d^2*e^2*f*(b^2 - 4*a*c)^(1/2) + b^2*d*e^3*g*x*(b^2 - 4*a*c)^(1/2)
+ 3*c^2*d^3*e*g*x*(b^2 - 4*a*c)^(1/2) + 14*a*c^2*d^2*e^2*g*x - 3*b*c^2*d^2*
e^2*f*x - 2*b^2*c*d^2*e^2*g*x + 5*a*c*d^2*e^2*h*x*(b^2 - 4*a*c)^(1/2) - 2*b
*c*d^2*e^2*g*x*(b^2 - 4*a*c)^(1/2) - 7*a*b*c*d*e^3*g*x - 5*a*c*d*e^3*g*x*(b
^2 - 4*a*c)^(1/2) + 5*b*c*d*e^3*f*x*(b^2 - 4*a*c)^(1/2) + b*c*d^3*e*h*x*(b^
2 - 4*a*c)^(1/2) + a*b*c*d^2*e^2*h*x)*(b^3*d*h + 4*a*c^2*d*g - 4*a*c^2*e*f
- a*b^2*e*h - b^2*c*d*g + b^2*c*e*f + 4*a^2*c*e*h - 2*c^2*d*f*(b^2 - 4*a*c)
^(1/2) - b^2*d*h*(b^2 - 4*a*c)^(1/2) - 4*a*b*c*d*h + a*b*e*h*(b^2 - 4*a*c)^
(1/2) + 2*a*c*d*h*(b^2 - 4*a*c)^(1/2) - 2*a*c*e*g*(b^2 - 4*a*c)^(1/2) + b*c
*d*g*(b^2 - 4*a*c)^(1/2) + b*c*e*f*(b^2 - 4*a*c)^(1/2)))/(2*(4*a*c^3*d^2 +
4*a^2*c^2*e^2 - b^2*c^2*d^2 + b^3*c*d*e - a*b^2*c*e^2 - 4*a*b*c^2*d*e)) - (
log(a^2*b*e^4*g - 2*a*b^2*e^4*f - 2*a^3*e^4*h + 6*a^2*c*e^4*f - 4*a*c^2*d^4
*h + b^2*c*d^4*h + b^3*d^3*e*h - 2*b^3*e^4*f*x - a^2*e^4*g*(b^2 - 4*a*c)^(1
/2) + a*b^2*d*e^3*g + 6*a*c^2*d^3*e*g + b*c^2*d^3*e*f + 3*a^2*b*d*e^3*h - 1
0*a^2*c*d*e^3*g - 2*b^2*c*d^3*e*g + a*b^2*e^4*g*x - a^2*b*e^4*h*x - 2*a^2*c
*e^4*g*x + b^3*d*e^3*g*x + 2*c^3*d^3*e*f*x + 3*a^2*d*e^3*h*(b^2 - 4*a*c)^(1
/2) + c^2*d^3*e*f*(b^2 - 4*a*c)^(1/2) + b^2*d^3*e*h*(b^2 - 4*a*c)^(1/2) + 2
*b^2*e^4*f*x*(b^2 - 4*a*c)^(1/2) + a^2*e^4*h*x*(b^2 - 4*a*c)^(1/2) + 2*c^2*
d^4*h*x*(b^2 - 4*a*c)^(1/2) - 10*a*c^2*d^2*e^2*f - 4*a*b^2*d^2*e^2*h + b^2*
```

```
c*d^2*e^2*f + 10*a^2*c*d^2*e^2*h - b^3*d^2*e^2*h*x + 2*a*b*e^4*f*(b^2 - 4*a
*c)^(1/2) + b*c*d^4*h*(b^2 - 4*a*c)^(1/2) + 3*a*b*c*d*e^3*f - 3*a*b*c*d^3*e
*h + 7*a*b*c*e^4*f*x + 5*c^2*d^2*e^2*f*x*(b^2 - 4*a*c)^(1/2) + b^2*d^2*e^2*
h*x*(b^2 - 4*a*c)^(1/2) - a*b*d*e^3*g*(b^2 - 4*a*c)^(1/2) - 7*a*c*d*e^3*f*(
b^2 - 4*a*c)^(1/2) - 5*a*c*d^3*e*h*(b^2 - 4*a*c)^(1/2) - 2*b*c*d^3*e*g*(b^2
 - 4*a*c)^(1/2) - a*b*e^4*g*x*(b^2 - 4*a*c)^(1/2) - 3*a*c*e^4*f*x*(b^2 - 4*
a*c)^(1/2) + 3*a*b*c*d^2*e^2*g - 14*a*c^2*d*e^3*f*x + 5*b^2*c*d*e^3*f*x - 1
0*a*c^2*d^3*e*h*x - b*c^2*d^3*e*g*x + 6*a^2*c*d*e^3*h*x + 3*b^2*c*d^3*e*h*x
 - 2*a*b*d^2*e^2*h*(b^2 - 4*a*c)^(1/2) + 7*a*c*d^2*e^2*g*(b^2 - 4*a*c)^(1/2
) + b*c*d^2*e^2*f*(b^2 - 4*a*c)^(1/2) - b^2*d*e^3*g*x*(b^2 - 4*a*c)^(1/2) -
 3*c^2*d^3*e*g*x*(b^2 - 4*a*c)^(1/2) + 14*a*c^2*d^2*e^2*g*x - 3*b*c^2*d^2*e
^2*f*x - 2*b^2*c*d^2*e^2*g*x - 5*a*c*d^2*e^2*h*x*(b^2 - 4*a*c)^(1/2) + 2*b*
c*d^2*e^2*g*x*(b^2 - 4*a*c)^(1/2) - 7*a*b*c*d*e^3*g*x + 5*a*c*d*e^3*g*x*(b^
2 - 4*a*c)^(1/2) - 5*b*c*d*e^3*f*x*(b^2 - 4*a*c)^(1/2) - b*c*d^3*e*h*x*(b^2
 - 4*a*c)^(1/2) + a*b*c*d^2*e^2*h*x)*(4*a*c^2*e*f - 4*a*c^2*d*g - b^3*d*h +
 a*b^2*e*h + b^2*c*d*g - b^2*c*e*f - 4*a^2*c*e*h - 2*c^2*d*f*(b^2 - 4*a*c)^
(1/2) - b^2*d*h*(b^2 - 4*a*c)^(1/2) + 4*a*b*c*d*h + a*b*e*h*(b^2 - 4*a*c)^(
1/2) + 2*a*c*d*h*(b^2 - 4*a*c)^(1/2) - 2*a*c*e*g*(b^2 - 4*a*c)^(1/2) + b*c*
d*g*(b^2 - 4*a*c)^(1/2) + b*c*e*f*(b^2 - 4*a*c)^(1/2)))/(2*(4*a*c^3*d^2 + 4
*a^2*c^2*e^2 - b^2*c^2*d^2 + b^3*c*d*e - a*b^2*c*e^2 - 4*a*b*c^2*d*e)) + (l
og(d + e*x)*(e^2*f + d^2*h - d*e*g))/(a*e^3 - b*d*e^2 + c*d^2*e)
```

3.153 $\int \frac{f+gx+hx^2}{(d+ex)^2(a+bx+cx^2)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 316

$$\int \frac{f+gx+hx^2}{(d+ex)^2\left(a+bx+cx^2\right)}\,dx = -\frac{e^2f-deg+d^2h}{e\left(cd^2-bde+ae^2\right)(d+ex)}$$
$$-\frac{(2c^2d^2f+2a^2e^2h-abe(eg+2dh)+b^2(e^2f+d^2h)-c(bd(2ef+dg)+2a(e^2f-2deg+d^2h)))\operatorname{arctanh}}{\sqrt{b^2-4ac}\left(cd^2-bde+ae^2\right)^2}$$
$$+\frac{(cd(2ef-dg)+ae(eg-2dh)-b(e^2f-d^2h))\log(d+ex)}{\left(cd^2-bde+ae^2\right)^2}$$
$$-\frac{(cd(2ef-dg)+ae(eg-2dh)-b(e^2f-d^2h))\log\left(a+bx+cx^2\right)}{2\left(cd^2-bde+ae^2\right)^2}$$

```
[Out] (-d^2*h+d*e*g-e^2*f)/e/(a*e^2-b*d*e+c*d^2)/(e*x+d)+(c*d*(-d*g+2*e*f)+a*e*(-
2*d*h+e*g)-b*(-d^2*h+e^2*f))*ln(e*x+d)/(a*e^2-b*d*e+c*d^2)^2-1/2*(c*d*(-d*g
+2*e*f)+a*e*(-2*d*h+e*g)-b*(-d^2*h+e^2*f))*ln(c*x^2+b*x+a)/(a*e^2-b*d*e+c*d
^2)^2-(2*c^2*d^2*f+2*a^2*e^2*h-a*b*e*(2*d*h+e*g)+b^2*(d^2*h+e^2*f)-c*(b*d*(
d*g+2*e*f)+2*a*(d^2*h-2*d*e*g+e^2*f)))*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2)
)/(a*e^2-b*d*e+c*d^2)^2/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used

$= \{1642, 648, 632, 212, 642\}$

$$\int \frac{f+gx+hx^2}{(d+ex)^2\,(a+bx+cx^2)}\,dx =$$

$$-\frac{\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)(2a^2e^2h-c(2a(d^2h-2deg+e^2f)+bd(dg+2ef))-abe(2dh+eg)+b^2(d^2h+e^2f}{\sqrt{b^2-4ac}\,(ae^2-bde+cd^2)^2}$$

$$-\frac{\log(a+bx+cx^2)\,(ae(eg-2dh)-b(e^2f-d^2h)+cd(2ef-dg))}{2\,(ae^2-bde+cd^2)^2}$$

$$-\frac{d^2h-deg+e^2f}{e(d+ex)\,(ae^2-bde+cd^2)}$$

$$+\frac{\log(d+ex)\,(ae(eg-2dh)-b(e^2f-d^2h)+cd(2ef-dg))}{(ae^2-bde+cd^2)^2}$$

```
[In] Int[(f + g*x + h*x^2)/((d + e*x)^2*(a + b*x + c*x^2)),x]
```

```
[Out] -((e^2*f - d*e*g + d^2*h)/(e*(c*d^2 - b*d*e + a*e^2)*(d + e*x))) - ((2*c^2*
d^2*f + 2*a^2*e^2*h - a*b*e*(e*g + 2*d*h) + b^2*(e^2*f + d^2*h) - c*(b*d*(2
*e*f + d*g) + 2*a*(e^2*f - 2*d*e*g + d^2*h)))*ArcTanh[(b + 2*c*x)/Sqrt[b^2
- 4*a*c]])/(Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)^2) + ((c*d*(2*e*f - d
*g) + a*e*(e*g - 2*d*h) - b*(e^2*f - d^2*h))*Log[d + e*x])/(c*d^2 - b*d*e +
 a*e^2)^2 - ((c*d*(2*e*f - d*g) + a*e*(e*g - 2*d*h) - b*(e^2*f - d^2*h))*Lo
g[a + b*x + c*x^2])/(2*(c*d^2 - b*d*e + a*e^2)^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\text{integral} = \int \left(\frac{e^2f-deg+d^2h}{(cd^2-bde+ae^2)\,(d+ex)^2}+\frac{e(cd(2ef-dg)+ae(eg-2dh)-b(e^2f-d^2h))}{(cd^2-bde+ae^2)^2\,(d+ex)}\right.$$
$$+\frac{c^2d^2f+e^2(b^2f-abg+a^2h)-c(2bdef+a(e^2f-2deg+d^2h))-c(cd(2ef-dg)+ae(eg-2dh)-b(e}{(cd^2-bde+ae^2)^2\,(a+bx+cx^2)}$$

$$=-\frac{e^2f-deg+d^2h}{e\,(cd^2-bde+ae^2)\,(d+ex)}+\frac{(cd(2ef-dg)+ae(eg-2dh)-b(e^2f-d^2h))\log(d+ex)}{(cd^2-bde+ae^2)^2}$$
$$+\frac{\int\frac{c^2d^2f+e^2(b^2f-abg+a^2h)-c(2bdef+a(e^2f-2deg+d^2h))-c(cd(2ef-dg)+ae(eg-2dh)-b(e^2f-d^2h))x}{a+bx+cx^2}\,dx}{(cd^2-bde+ae^2)^2}$$

$$=-\frac{e^2f-deg+d^2h}{e\,(cd^2-bde+ae^2)\,(d+ex)}$$
$$+\frac{(cd(2ef-dg)+ae(eg-2dh)-b(e^2f-d^2h))\log(d+ex)}{(cd^2-bde+ae^2)^2}$$
$$-\frac{(cd(2ef-dg)+ae(eg-2dh)-b(e^2f-d^2h))\int\frac{b+2cx}{a+bx+cx^2}\,dx}{2\,(cd^2-bde+ae^2)^2}$$
$$+\frac{(2c^2d^2f+2a^2e^2h-abe(eg+2dh)+b^2(e^2f+d^2h)-c(bd(2ef+dg)+2a(e^2f-2deg+d^2h)))}{2\,(cd^2-bde+ae^2)^2}$$

$$=-\frac{e^2f-deg+d^2h}{e\,(cd^2-bde+ae^2)\,(d+ex)}$$
$$+\frac{(cd(2ef-dg)+ae(eg-2dh)-b(e^2f-d^2h))\log(d+ex)}{(cd^2-bde+ae^2)^2}$$
$$-\frac{(cd(2ef-dg)+ae(eg-2dh)-b(e^2f-d^2h))\log\left(a+bx+cx^2\right)}{2\,(cd^2-bde+ae^2)^2}$$
$$-\frac{(2c^2d^2f+2a^2e^2h-abe(eg+2dh)+b^2(e^2f+d^2h)-c(bd(2ef+dg)+2a(e^2f-2deg+d^2h)))}{(cd^2-bde+ae^2)^2}$$

$$= -\frac{e^2f-deg+d^2h}{e\left(cd^2-bde+ae^2\right)(d+ex)}$$
$$-\frac{(2c^2d^2f+2a^2e^2h-abe(eg+2dh)+b^2(e^2f+d^2h)-c(bd(2ef+dg)+2a(e^2f-2deg+d^2h))}{\sqrt{b^2-4ac}\left(cd^2-bde+ae^2\right)^2}$$
$$+\frac{(cd(2ef-dg)+ae(eg-2dh)-b(e^2f-d^2h))\log(d+ex)}{\left(cd^2-bde+ae^2\right)^2}$$
$$-\frac{(cd(2ef-dg)+ae(eg-2dh)-b(e^2f-d^2h))\log\left(a+bx+cx^2\right)}{2\left(cd^2-bde+ae^2\right)^2}$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.89

$$\int \frac{f+gx+hx^2}{(d+ex)^2\left(a+bx+cx^2\right)}\,dx$$
$$= \frac{-\frac{2\left(cd^2+e(-bd+ae)\right)\left(e^2f-deg+d^2h\right)}{e(d+ex)}+\frac{2\left(2c^2d^2f+2a^2e^2h-abe(eg+2dh)+b^2\left(e^2f+d^2h\right)-c\left(bd(2ef+dg)+2a\left(e^2f-2deg+d^2h\right)\right)\right)\arctan(}{\sqrt{-b^2+4ac}}}{}$$

```
[In] Integrate[(f + g*x + h*x^2)/((d + e*x)^2*(a + b*x + c*x^2)),x]
```

```
[Out] ((-2*(c*d^2 + e*(-(b*d) + a*e))*(e^2*f - d*e*g + d^2*h))/(e*(d + e*x)) + (2
*(2*c^2*d^2*f + 2*a^2*e^2*h - a*b*e*(e*g + 2*d*h) + b^2*(e^2*f + d^2*h) - c
*(b*d*(2*e*f + d*g) + 2*a*(e^2*f - 2*d*e*g + d^2*h)))*ArcTan[(b + 2*c*x)/Sq
rt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + 2*(c*d*(2*e*f - d*g) + a*e*(e*g - 2
*d*h) + b*(-(e^2*f) + d^2*h))*Log[d + e*x] + (c*d*(-2*e*f + d*g) + a*e*(-(e
*g) + 2*d*h) + b*(e^2*f - d^2*h))*Log[a + x*(b + c*x)])/(2*(c*d^2 + e*(-(b*
d) + a*e))^2)
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.09

method	result
default	$-\frac{d^2h-deg+e^2f}{(e^2a-bde+c\,d^2)e(ex+d)}-\frac{\left(2adeh-a\,e^2g-b\,d^2h+b\,e^2f+c\,d^2g-2cdef\right)\ln(ex+d)}{(e^2a-bde+c\,d^2)^2}+\frac{\frac{(2acdhe-ac\,e^2g-bc\,d^2h+bc\,e^2f+c^2d^2g-2c^2d}{2c}}{}$
risch	Expression too large to display

```
[In] int((h*x^2+g*x+f)/(e*x+d)^2/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -(d^2*h-d*e*g+e^2*f)/(a*e^2-b*d*e+c*d^2)/e/(e*x+d)-(2*a*d*e*h-a*e^2*g-b*d^2
*h+b*e^2*f+c*d^2*g-2*c*d*e*f)/(a*e^2-b*d*e+c*d^2)^2*ln(e*x+d)+1/(a*e^2-b*d*
```

```
e+c*d^2)^2*(1/2*(2*a*c*d*e*h-a*c*e^2*g-b*c*d^2*h+b*c*e^2*f+c^2*d^2*g-2*c^2*
d*e*f)/c*ln(c*x^2+b*x+a)+2*(a^2*e^2*h-a*b*e^2*g-a*c*d^2*h+2*a*c*d*e*g-a*c*e
^2*f+b^2*e^2*f-2*b*c*d*e*f+c^2*d^2*f-1/2*(2*a*c*d*e*h-a*c*e^2*g-b*c*d^2*h+b
*c*e^2*f+c^2*d^2*g-2*c^2*d*e*f)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*
a*c-b^2)^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{f + gx + hx^2}{(d + ex)^2 (a + bx + cx^2)} \, dx = \text{Timed out}$$

```
[In] integrate((h*x^2+g*x+f)/(e*x+d)^2/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{f + gx + hx^2}{(d + ex)^2 (a + bx + cx^2)} \, dx = \text{Timed out}$$

```
[In] integrate((h*x**2+g*x+f)/(e*x+d)**2/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{f + gx + hx^2}{(d + ex)^2 (a + bx + cx^2)} \, dx = \text{Exception raised: ValueError}$$

```
[In] integrate((h*x^2+g*x+f)/(e*x+d)^2/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.45

$$\int \frac{f+gx+hx^2}{(d+ex)^2\,(a+bx+cx^2)}\,dx =$$

$$-\frac{(2\,cdef-be^2f-cd^2g+ae^2g+bd^2h-2\,adeh)\log\left(c-\frac{2\,cd}{ex+d}+\frac{cd^2}{(ex+d)^2}+\frac{be}{ex+d}-\frac{bde}{(ex+d)^2}+\frac{ae^2}{(ex+d)^2}\right)}{2\,(c^2d^4-2\,bcd^3e+b^2d^2e^2+2\,acd^2e^2-2\,abde^3+a^2e^4)}$$

$$-\frac{\frac{e^3f}{ex+d}-\frac{de^2g}{ex+d}+\frac{d^2eh}{ex+d}}{cd^2e^2-bde^3+ae^4}$$

$$+\frac{(2\,c^2d^2e^2f-2\,bcde^3f+b^2e^4f-2\,ace^4f-bcd^2e^2g+4\,acde^3g-abe^4g+b^2d^2e^2h-2\,acd^2e^2h-2\,abde^3}{(c^2d^4-2\,bcd^3e+b^2d^2e^2+2\,acd^2e^2-2\,abde^3+a^2e^4)\sqrt{-b^2+4}}$$

```
[In] integrate((h*x^2+g*x+f)/(e*x+d)^2/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] -1/2*(2*c*d*e*f - b*e^2*f - c*d^2*g + a*e^2*g + b*d^2*h - 2*a*d*e*h)*log(c
- 2*c*d/(e*x + d) + c*d^2/(e*x + d)^2 + b*e/(e*x + d) - b*d*e/(e*x + d)^2 +
 a*e^2/(e*x + d)^2)/(c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2 + 2*a*c*d^2*e^2 -
2*a*b*d*e^3 + a^2*e^4) - (e^3*f/(e*x + d) - d*e^2*g/(e*x + d) + d^2*e*h/(e*
x + d))/(c*d^2*e^2 - b*d*e^3 + a*e^4) + (2*c^2*d^2*e^2*f - 2*b*c*d*e^3*f +
b^2*e^4*f - 2*a*c*e^4*f - b*c*d^2*e^2*g + 4*a*c*d*e^3*g - a*b*e^4*g + b^2*d
^2*e^2*h - 2*a*c*d^2*e^2*h - 2*a*b*d*e^3*h + 2*a^2*e^4*h)*arctan((2*c*d - 2
*c*d^2/(e*x + d) - b*e + 2*b*d*e/(e*x + d) - 2*a*e^2/(e*x + d))/(sqrt(-b^2
+ 4*a*c)*e))/((c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2 + 2*a*c*d^2*e^2 - 2*a*b*
d*e^3 + a^2*e^4)*sqrt(-b^2 + 4*a*c)*e^2)
```

Mupad [B] (verification not implemented)

Time = 28.23 (sec) , antiderivative size = 3991, normalized size of antiderivative = 12.63

$$\int \frac{f+gx+hx^2}{(d+ex)^2\,(a+bx+cx^2)}\,dx = \text{Too large to display}$$

```
[In] int((f + g*x + h*x^2)/((d + e*x)^2*(a + b*x + c*x^2)),x)
```

```
[Out] (log(d + e*x)*(e^2*(a*g - b*f) + d^2*(b*h - c*g) - d*e*(2*a*h - 2*c*f)))/(a
^2*e^4 + c^2*d^4 + b^2*d^2*e^2 - 2*a*b*d*e^3 - 2*b*c*d^3*e + 2*a*c*d^2*e^2)
 + (log(2*a*b^3*e^4*f - 2*b^2*c^2*d^4*g - 2*a^2*b^2*e^4*g + 6*a*c^3*d^4*g +
 b*c^3*d^4*f + a^3*b*e^4*h + 6*a^3*c*e^4*g + 2*b^3*c*d^4*h + 2*b^4*e^4*f*x
+ 2*c^4*d^4*f*x - c^3*d^4*f*(b^2 - 4*a*c)^(1/2) + a^3*e^4*h*(b^2 - 4*a*c)^(
1/2) - 7*a^2*b*c*e^4*f - 7*a*b*c^2*d^4*h - 16*a*c^3*d^3*e*f - 16*a^3*c*d*e^
```

```
3*h - 2*a*b^3*e^4*g*x - 2*a*c^3*d^4*h*x - b*c^3*d^4*g*x - 2*a^3*c*e^4*h*x +
 2*a*b^2*e^4*f*(b^2 - 4*a*c)^(1/2) - 2*a^2*b*e^4*g*(b^2 - 4*a*c)^(1/2) - a^
2*c*e^4*f*(b^2 - 4*a*c)^(1/2) + a*c^2*d^4*h*(b^2 - 4*a*c)^(1/2) + 2*b*c^2*d
^4*g*(b^2 - 4*a*c)^(1/2) - 2*b^2*c*d^4*h*(b^2 - 4*a*c)^(1/2) + 2*b^3*e^4*f*
x*(b^2 - 4*a*c)^(1/2) + 3*c^3*d^4*g*x*(b^2 - 4*a*c)^(1/2) + 16*a^2*c^2*d*e^
3*f - a*b^3*d^2*e^2*h + 2*a^2*b^2*d*e^3*h + 2*b^2*c^2*d^3*e*f - b^3*c*d^2*e
^2*f + 16*a^2*c^2*d^3*e*h + 2*a^2*c^2*e^4*f*x + a^2*b^2*e^4*h*x + b^2*c^2*d
^4*h*x - b^4*d^2*e^2*h*x - 20*a^2*c^2*d^2*e^2*g + 14*a*c^2*d^2*e^2*f*(b^2 -
 4*a*c)^(1/2) - a*b^2*d^2*e^2*h*(b^2 - 4*a*c)^(1/2) + b^2*c*d^2*e^2*f*(b^2
- 4*a*c)^(1/2) - 14*a^2*c*d^2*e^2*h*(b^2 - 4*a*c)^(1/2) - b^3*d^2*e^2*h*x*(
b^2 - 4*a*c)^(1/2) + 10*b^2*c^2*d^2*e^2*f*x + 28*a^2*c^2*d^2*e^2*h*x - 6*a*
b^2*c*d*e^3*f + 4*a*b*c^2*d^3*e*g + 4*a^2*b*c*d*e^3*g - 6*a*b^2*c*d^3*e*h -
 8*a*b^2*c*e^4*f*x + 7*a^2*b*c*e^4*g*x + 2*a*b^3*d*e^3*h*x + 16*a*c^3*d^3*e
*g*x - 4*b*c^3*d^3*e*f*x - 8*b^3*c*d*e^3*f*x + 2*b^3*c*d^3*e*h*x - 8*a*c^2*
d^3*e*g*(b^2 - 4*a*c)^(1/2) - 2*b*c^2*d^3*e*f*(b^2 - 4*a*c)^(1/2) + 2*a^2*b
*d*e^3*h*(b^2 - 4*a*c)^(1/2) + 8*a^2*c*d*e^3*g*(b^2 - 4*a*c)^(1/2) - 2*a*b^
2*e^4*g*x*(b^2 - 4*a*c)^(1/2) + a^2*b*e^4*h*x*(b^2 - 4*a*c)^(1/2) + 3*a^2*c
*e^4*g*x*(b^2 - 4*a*c)^(1/2) - 3*b*c^2*d^4*h*x*(b^2 - 4*a*c)^(1/2) - 8*c^3*
d^3*e*f*x*(b^2 - 4*a*c)^(1/2) + 10*a*b*c^2*d^2*e^2*f + 2*a*b^2*c*d^2*e^2*g
+ 10*a^2*b*c*d^2*e^2*h - 28*a*c^3*d^2*e^2*f*x - 16*a^2*c^2*d*e^3*g*x - 2*b^
2*c^2*d^3*e*g*x + b^3*c*d^2*e^2*g*x + 8*a*c^2*d*e^3*f*x*(b^2 - 4*a*c)^(1/2)
 + 2*a*b^2*d*e^3*h*x*(b^2 - 4*a*c)^(1/2) - 8*b^2*c*d*e^3*f*x*(b^2 - 4*a*c)^
(1/2) + 8*a*c^2*d^3*e*h*x*(b^2 - 4*a*c)^(1/2) - 2*b*c^2*d^3*e*g*x*(b^2 - 4*
a*c)^(1/2) - 8*a^2*c*d*e^3*h*x*(b^2 - 4*a*c)^(1/2) + 2*b^2*c*d^3*e*h*x*(b^2
 - 4*a*c)^(1/2) - 10*a*b*c^2*d^2*e^2*g*x - 10*a*c^2*d^2*e^2*g*x*(b^2 - 4*a*
c)^(1/2) + 12*b*c^2*d^2*e^2*f*x*(b^2 - 4*a*c)^(1/2) + b^2*c*d^2*e^2*g*x*(b^
2 - 4*a*c)^(1/2) - 10*a*b*c*d*e^3*f*(b^2 - 4*a*c)^(1/2) + 10*a*b*c*d^3*e*h*
(b^2 - 4*a*c)^(1/2) - 4*a*b*c*e^4*f*x*(b^2 - 4*a*c)^(1/2) + 28*a*b*c^2*d*e^
3*f*x + 6*a*b^2*c*d*e^3*g*x - 12*a*b*c^2*d^3*e*h*x - 12*a^2*b*c*d*e^3*h*x +
 6*a*b*c*d*e^3*g*x*(b^2 - 4*a*c)^(1/2) - 2*a*b*c*d^2*e^2*h*x*(b^2 - 4*a*c)^
(1/2))*(b^3*d^2*h - b^3*e^2*f + a*b^2*e^2*g + 4*a*c^2*d^2*g - 4*a^2*c*e^2*g
 - b^2*c*d^2*g - b^2*e^2*f*(b^2 - 4*a*c)^(1/2) - 2*c^2*d^2*f*(b^2 - 4*a*c)^
(1/2) - 2*a^2*e^2*h*(b^2 - 4*a*c)^(1/2) - b^2*d^2*h*(b^2 - 4*a*c)^(1/2) + 4
*a*b*c*e^2*f - 4*a*b*c*d^2*h - 8*a*c^2*d*e*f - 2*a*b^2*d*e*h + 2*b^2*c*d*e*
f + 8*a^2*c*d*e*h + a*b*e^2*g*(b^2 - 4*a*c)^(1/2) + 2*a*c*e^2*f*(b^2 - 4*a*
c)^(1/2) + 2*a*c*d^2*h*(b^2 - 4*a*c)^(1/2) + b*c*d^2*g*(b^2 - 4*a*c)^(1/2)
+ 2*a*b*d*e*h*(b^2 - 4*a*c)^(1/2) - 4*a*c*d*e*g*(b^2 - 4*a*c)^(1/2) + 2*b*c
*d*e*f*(b^2 - 4*a*c)^(1/2)))/(2*(4*a*c^3*d^4 + 4*a^3*c*e^4 - a^2*b^2*e^4 -
b^2*c^2*d^4 - b^4*d^2*e^2 + 8*a^2*c^2*d^2*e^2 + 2*a*b^3*d*e^3 + 2*b^3*c*d^3
*e - 8*a*b*c^2*d^3*e - 8*a^2*b*c*d*e^3 + 2*a*b^2*c*d^2*e^2)) - (log(2*a*b^3
*e^4*f - 2*b^2*c^2*d^4*g - 2*a^2*b^2*e^4*g + 6*a*c^3*d^4*g + b*c^3*d^4*f +
a^3*b*e^4*h + 6*a^3*c*e^4*g + 2*b^3*c*d^4*h + 2*b^4*e^4*f*x + 2*c^4*d^4*f*x
 + c^3*d^4*f*(b^2 - 4*a*c)^(1/2) - a^3*e^4*h*(b^2 - 4*a*c)^(1/2) - 7*a^2*b*
c*e^4*f - 7*a*b*c^2*d^4*h - 16*a*c^3*d^3*e*f - 16*a^3*c*d*e^3*h - 2*a*b^3*e
^4*g*x - 2*a*c^3*d^4*h*x - b*c^3*d^4*g*x - 2*a^3*c*e^4*h*x - 2*a*b^2*e^4*f*
```

```
(b^2 - 4*a*c)^(1/2) + 2*a^2*b*e^4*g*(b^2 - 4*a*c)^(1/2) + a^2*c*e^4*f*(b^2
- 4*a*c)^(1/2) - a*c^2*d^4*h*(b^2 - 4*a*c)^(1/2) - 2*b*c^2*d^4*g*(b^2 - 4*a
*c)^(1/2) + 2*b^2*c*d^4*h*(b^2 - 4*a*c)^(1/2) - 2*b^3*e^4*f*x*(b^2 - 4*a*c)
^(1/2) - 3*c^3*d^4*g*x*(b^2 - 4*a*c)^(1/2) + 16*a^2*c^2*d*e^3*f - a*b^3*d^2
*e^2*h + 2*a^2*b^2*d*e^3*h + 2*b^2*c^2*d^3*e*f - b^3*c*d^2*e^2*f + 16*a^2*c
^2*d^3*e*h + 2*a^2*c^2*e^4*f*x + a^2*b^2*e^4*h*x + b^2*c^2*d^4*h*x - b^4*d^
2*e^2*h*x - 20*a^2*c^2*d^2*e^2*g - 14*a*c^2*d^2*e^2*f*(b^2 - 4*a*c)^(1/2) +
 a*b^2*d^2*e^2*h*(b^2 - 4*a*c)^(1/2) - b^2*c*d^2*e^2*f*(b^2 - 4*a*c)^(1/2)
+ 14*a^2*c*d^2*e^2*h*(b^2 - 4*a*c)^(1/2) + b^3*d^2*e^2*h*x*(b^2 - 4*a*c)^(1
/2) + 10*b^2*c^2*d^2*e^2*f*x + 28*a^2*c^2*d^2*e^2*h*x - 6*a*b^2*c*d*e^3*f +
 4*a*b*c^2*d^3*e*g + 4*a^2*b*c*d*e^3*g - 6*a*b^2*c*d^3*e*h - 8*a*b^2*c*e^4*
f*x + 7*a^2*b*c*e^4*g*x + 2*a*b^3*d*e^3*h*x + 16*a*c^3*d^3*e*g*x - 4*b*c^3*
d^3*e*f*x - 8*b^3*c*d*e^3*f*x + 2*b^3*c*d^3*e*h*x + 8*a*c^2*d^3*e*g*(b^2 -
4*a*c)^(1/2) + 2*b*c^2*d^3*e*f*(b^2 - 4*a*c)^(1/2) - 2*a^2*b*d*e^3*h*(b^2 -
 4*a*c)^(1/2) - 8*a^2*c*d*e^3*g*(b^2 - 4*a*c)^(1/2) + 2*a*b^2*e^4*g*x*(b^2
- 4*a*c)^(1/2) - a^2*b*e^4*h*x*(b^2 - 4*a*c)^(1/2) - 3*a^2*c*e^4*g*x*(b^2 -
 4*a*c)^(1/2) + 3*b*c^2*d^4*h*x*(b^2 - 4*a*c)^(1/2) + 8*c^3*d^3*e*f*x*(b^2
- 4*a*c)^(1/2) + 10*a*b*c^2*d^2*e^2*f + 2*a*b^2*c*d^2*e^2*g + 10*a^2*b*c*d^
2*e^2*h - 28*a*c^3*d^2*e^2*f*x - 16*a^2*c^2*d*e^3*g*x - 2*b^2*c^2*d^3*e*g*x
 + b^3*c*d^2*e^2*g*x - 8*a*c^2*d*e^3*f*x*(b^2 - 4*a*c)^(1/2) - 2*a*b^2*d*e^
3*h*x*(b^2 - 4*a*c)^(1/2) + 8*b^2*c*d*e^3*f*x*(b^2 - 4*a*c)^(1/2) - 8*a*c^2
*d^3*e*h*x*(b^2 - 4*a*c)^(1/2) + 2*b*c^2*d^3*e*g*x*(b^2 - 4*a*c)^(1/2) + 8*
a^2*c*d*e^3*h*x*(b^2 - 4*a*c)^(1/2) - 2*b^2*c*d^3*e*h*x*(b^2 - 4*a*c)^(1/2)
 - 10*a*b*c^2*d^2*e^2*g*x + 10*a*c^2*d^2*e^2*g*x*(b^2 - 4*a*c)^(1/2) - 12*b
*c^2*d^2*e^2*f*x*(b^2 - 4*a*c)^(1/2) - b^2*c*d^2*e^2*g*x*(b^2 - 4*a*c)^(1/2
) + 10*a*b*c*d*e^3*f*(b^2 - 4*a*c)^(1/2) - 10*a*b*c*d^3*e*h*(b^2 - 4*a*c)^(
1/2) + 4*a*b*c*e^4*f*x*(b^2 - 4*a*c)^(1/2) + 28*a*b*c^2*d*e^3*f*x + 6*a*b^2
*c*d*e^3*g*x - 12*a*b*c^2*d^3*e*h*x - 12*a^2*b*c*d*e^3*h*x - 6*a*b*c*d*e^3*
g*x*(b^2 - 4*a*c)^(1/2) + 2*a*b*c*d^2*e^2*h*x*(b^2 - 4*a*c)^(1/2))*(b^3*e^2
*f - b^3*d^2*h - a*b^2*e^2*g - 4*a*c^2*d^2*g + 4*a^2*c*e^2*g + b^2*c*d^2*g
- b^2*e^2*f*(b^2 - 4*a*c)^(1/2) - 2*c^2*d^2*f*(b^2 - 4*a*c)^(1/2) - 2*a^2*e
^2*h*(b^2 - 4*a*c)^(1/2) - b^2*d^2*h*(b^2 - 4*a*c)^(1/2) - 4*a*b*c*e^2*f +
4*a*b*c*d^2*h + 8*a*c^2*d*e*f + 2*a*b^2*d*e*h - 2*b^2*c*d*e*f - 8*a^2*c*d*e
*h + a*b*e^2*g*(b^2 - 4*a*c)^(1/2) + 2*a*c*e^2*f*(b^2 - 4*a*c)^(1/2) + 2*a*
c*d^2*h*(b^2 - 4*a*c)^(1/2) + b*c*d^2*g*(b^2 - 4*a*c)^(1/2) + 2*a*b*d*e*h*(
b^2 - 4*a*c)^(1/2) - 4*a*c*d*e*g*(b^2 - 4*a*c)^(1/2) + 2*b*c*d*e*f*(b^2 - 4
*a*c)^(1/2)))/(2*(4*a*c^3*d^4 + 4*a^3*c*e^4 - a^2*b^2*e^4 - b^2*c^2*d^4 - b
^4*d^2*e^2 + 8*a^2*c^2*d^2*e^2 + 2*a*b^3*d*e^3 + 2*b^3*c*d^3*e - 8*a*b*c^2*
d^3*e - 8*a^2*b*c*d*e^3 + 2*a*b^2*c*d^2*e^2)) - (e^2*f + d^2*h - d*e*g)/(e*
(d + e*x)*(a*e^2 + c*d^2 - b*d*e))
```

3.154 $\int \frac{f+gx+hx^2}{(d+ex)^3(a+bx+cx^2)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 509

$$\int \frac{f+gx+hx^2}{(d+ex)^3\left(a+bx+cx^2\right)}\,dx$$

$$= -\frac{e^2f-deg+d^2h}{2e\left(cd^2-bde+ae^2\right)(d+ex)^2} - \frac{cd(2ef-dg)+ae(eg-2dh)-b(e^2f-d^2h)}{\left(cd^2-bde+ae^2\right)^2(d+ex)}$$

$$- \frac{(2c^3d^3f-be^3(b^2f-abg+a^2h)-c^2d(bd(3ef+dg)+2a(3e^2f-3deg+d^2h))-c(2a^2e^2(eg-3dh)-3}{\sqrt{b^2-4ac}\left(cd^2-bde+ae^2\right)^3}$$

$$+ \frac{(c^2d^2(3ef-dg)+e^3(b^2f-abg+a^2h)-c(ae(e^2f-3deg+3d^2h)+b(3de^2f-d^3h)))\log(d+ex)}{\left(cd^2-bde+ae^2\right)^3}$$

$$- \frac{(c^2d^2(3ef-dg)+e^3(b^2f-abg+a^2h)-c(ae(e^2f-3deg+3d^2h)+b(3de^2f-d^3h)))\log(a+bx+cx^2}{2\left(cd^2-bde+ae^2\right)^3}$$

```
[Out] 1/2*(-d^2*h+d*e*g-e^2*f)/e/(a*e^2-b*d*e+c*d^2)/(e*x+d)^2+(-c*d*(-d*g+2*e*f)
-a*e*(-2*d*h+e*g)+b*(-d^2*h+e^2*f))/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)+(c^2*d^2*
(-d*g+3*e*f)+e^3*(a^2*h-a*b*g+b^2*f)-c*(a*e*(3*d^2*h-3*d*e*g+e^2*f)+b*(-d^3
*h+3*d*e^2*f)))*ln(e*x+d)/(a*e^2-b*d*e+c*d^2)^3-1/2*(c^2*d^2*(-d*g+3*e*f)+e
^3*(a^2*h-a*b*g+b^2*f)-c*(a*e*(3*d^2*h-3*d*e*g+e^2*f)+b*(-d^3*h+3*d*e^2*f))
)*ln(c*x^2+b*x+a)/(a*e^2-b*d*e+c*d^2)^3-(2*c^3*d^3*f-b*e^3*(a^2*h-a*b*g+b^2
*f)-c^2*d*(b*d*(d*g+3*e*f)+2*a*(d^2*h-3*d*e*g+3*e^2*f))-c*(2*a^2*e^2*(-3*d*
h+e*g)-3*a*b*e*(-d^2*h-d*e*g+e^2*f)-b^2*(d^3*h+3*d*e^2*f)))*arctanh((2*c*x+
b)/(-4*a*c+b^2)^(1/2))/(a*e^2-b*d*e+c*d^2)^3/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 509, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1642, 648, 632, 212, 642}

$$\int \frac{f+gx+hx^2}{(d+ex)^3\,(a+bx+cx^2)}\,dx =$$

$$-\frac{\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)(-c(2a^2e^2(eg-3dh)-3abe(d^2(-h)-deg+e^2f)-(b^2(d^3h+3de^2f)))-be^3(a^2h-}{\sqrt{b^2-4ac}\,(ae^2-bde+cd^2)^3}$$

$$-\frac{\log(a+bx+cx^2)\,(e^3(a^2h-abg+b^2f)-ace(3d^2h-3deg+e^2f)-bc(3de^2f-d^3h)+c^2d^2(3ef-dg}{2\,(ae^2-bde+cd^2)^3}$$

$$+\frac{\log(d+ex)\,(e^3(a^2h-abg+b^2f)-ace(3d^2h-3deg+e^2f)-bc(3de^2f-d^3h)+c^2d^2(3ef-dg))}{(ae^2-bde+cd^2)^3}$$

$$-\frac{d^2h-deg+e^2f}{2e(d+ex)^2\,(ae^2-bde+cd^2)}-\frac{ae(eg-2dh)-b(e^2f-d^2h)+cd(2ef-dg)}{(d+ex)\,(ae^2-bde+cd^2)^2}$$

```
[In] Int[(f + g*x + h*x^2)/((d + e*x)^3*(a + b*x + c*x^2)),x]
```

```
[Out] -1/2*(e^2*f - d*e*g + d^2*h)/(e*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^2) - (c*d
*(2*e*f - d*g) + a*e*(e*g - 2*d*h) - b*(e^2*f - d^2*h))/((c*d^2 - b*d*e + a
*e^2)^2*(d + e*x)) - ((2*c^3*d^3*f - b*e^3*(b^2*f - a*b*g + a^2*h) - c^2*d*
(b*d*(3*e*f + d*g) + 2*a*(3*e^2*f - 3*d*e*g + d^2*h)) - c*(2*a^2*e^2*(e*g -
 3*d*h) - 3*a*b*e*(e^2*f - d*e*g - d^2*h) - b^2*(3*d*e^2*f + d^3*h)))*ArcTa
nh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^
2)^3) + ((c^2*d^2*(3*e*f - d*g) + e^3*(b^2*f - a*b*g + a^2*h) - a*c*e*(e^2*
f - 3*d*e*g + 3*d^2*h) - b*c*(3*d*e^2*f - d^3*h))*Log[d + e*x])/(c*d^2 - b*
d*e + a*e^2)^3 - ((c^2*d^2*(3*e*f - d*g) + e^3*(b^2*f - a*b*g + a^2*h) - a*
c*e*(e^2*f - 3*d*e*g + 3*d^2*h) - b*c*(3*d*e^2*f - d^3*h))*Log[a + b*x + c*
x^2])/(2*(c*d^2 - b*d*e + a*e^2)^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\text{integral} = \int \Bigg(\frac{e^2f-deg+d^2h}{(cd^2-bde+ae^2)(d+ex)^3} + \frac{e(cd(2ef-dg)+ae(eg-2dh)-b(e^2f-d^2h))}{(cd^2-bde+ae^2)^2(d+ex)^2}$$

$$+\frac{e(c^2d^2(3ef-dg)+e^3(b^2f-abg+a^2h)-ace(e^2f-3deg+3d^2h)-bc(3de^2f-d^3h))}{(cd^2-bde+ae^2)^3(d+ex)}$$

$$+\frac{c^3d^3f-be^3(b^2f-abg+a^2h)+ce^2(3b^2df+ab(2ef-3dg)-a^2(eg-3dh))-c^2d(3bdef+a(3e^2f-3d}{(cd^2-bde+ae^2)}$$

$$=-\frac{e^2f-deg+d^2h}{2e(cd^2-bde+ae^2)(d+ex)^2}-\frac{cd(2ef-dg)+ae(eg-2dh)-b(e^2f-d^2h)}{(cd^2-bde+ae^2)^2(d+ex)}$$

$$+\frac{(c^2d^2(3ef-dg)+e^3(b^2f-abg+a^2h)-ace(e^2f-3deg+3d^2h)-bc(3de^2f-d^3h))\log(d+ex}{(cd^2-bde+ae^2)^3}$$

$$+\frac{\int \frac{c^3d^3f-be^3(b^2f-abg+a^2h)+ce^2(3b^2df+ab(2ef-3dg)-a^2(eg-3dh))-c^2d(3bdef+a(3e^2f-3deg+d^2h))-c(c^2d^2(3ef-dg)+e^3(b^2}{a+bx+cx^2}}{(cd^2-bde+ae^2)^3}$$

$$=-\frac{e^2f-deg+d^2h}{2e(cd^2-bde+ae^2)(d+ex)^2}-\frac{cd(2ef-dg)+ae(eg-2dh)-b(e^2f-d^2h)}{(cd^2-bde+ae^2)^2(d+ex)}$$

$$+\frac{(c^2d^2(3ef-dg)+e^3(b^2f-abg+a^2h)-ace(e^2f-3deg+3d^2h)-bc(3de^2f-d^3h))\log(d+ex}{(cd^2-bde+ae^2)^3}$$

$$-\frac{(c^2d^2(3ef-dg)+e^3(b^2f-abg+a^2h)-ace(e^2f-3deg+3d^2h)-bc(3de^2f-d^3h))\int \frac{b+2cx}{a+bx+cx^2}}{2(cd^2-bde+ae^2)^3}$$

$$+\frac{(2c^3d^3f-be^3(b^2f-abg+a^2h)-c^2d(bd(3ef+dg)+2a(3e^2f-3deg+d^2h))-c(2a^2e^2(eg-3}{2(cd^2-bde+ae^2)^3}$$

$$
\begin{aligned}
&= -\frac{e^2f-deg+d^2h}{2e\left(cd^2-bde+ae^2\right)(d+ex)^2} - \frac{cd(2ef-dg)+ae(eg-2dh)-b(e^2f-d^2h)}{\left(cd^2-bde+ae^2\right)^2(d+ex)} \\
&\quad + \frac{(c^2d^2(3ef-dg)+e^3(b^2f-abg+a^2h)-ace(e^2f-3deg+3d^2h)-bc(3de^2f-d^3h))\log(d+}{\left(cd^2-bde+ae^2\right)^3} \\
&\quad - \frac{(c^2d^2(3ef-dg)+e^3(b^2f-abg+a^2h)-ace(e^2f-3deg+3d^2h)-bc(3de^2f-d^3h))\log\left(a+\right.}{2\left(cd^2-bde+ae^2\right)^3} \\
&\quad - \frac{(2c^3d^3f-be^3(b^2f-abg+a^2h)-c^2d(bd(3ef+dg)+2a(3e^2f-3deg+d^2h))-c(2a^2e^2(eg-}{(cd^2-bde+a}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{e^2f-deg+d^2h}{2e\left(cd^2-bde+ae^2\right)(d+ex)^2} - \frac{cd(2ef-dg)+ae(eg-2dh)-b(e^2f-d^2h)}{\left(cd^2-bde+ae^2\right)^2(d+ex)} \\
&\quad - \frac{(2c^3d^3f-be^3(b^2f-abg+a^2h)-c^2d(bd(3ef+dg)+2a(3e^2f-3deg+d^2h))-c(2a^2e^2(eg-}{\sqrt{b^2-4ac}\,(cd^2-bde+ae^2} \\
&\quad + \frac{(c^2d^2(3ef-dg)+e^3(b^2f-abg+a^2h)-ace(e^2f-3deg+3d^2h)-bc(3de^2f-d^3h))\log(d+}{\left(cd^2-bde+ae^2\right)^3} \\
&\quad - \frac{(c^2d^2(3ef-dg)+e^3(b^2f-abg+a^2h)-ace(e^2f-3deg+3d^2h)-bc(3de^2f-d^3h))\log\left(a+\right.}{2\left(cd^2-bde+ae^2\right)^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 504, normalized size of antiderivative = 0.99

$$
\begin{aligned}
&\int \frac{f+gx+hx^2}{(d+ex)^3\left(a+bx+cx^2\right)}\,dx \\
&= -\frac{e^2f-deg+d^2h}{2e\left(cd^2+e(-bd+ae)\right)(d+ex)^2} + \frac{cd(-2ef+dg)+ae(-eg+2dh)+b(e^2f-d^2h)}{\left(cd^2+e(-bd+ae)\right)^2(d+ex)} \\
&\quad + \frac{(-2c^3d^3f+be^3(b^2f-abg+a^2h)+c^2d(bd(3ef+dg)+2a(3e^2f-3deg+d^2h))-c(-2a^2e^2(eg-3dh}{\sqrt{-b^2+4ac}\left(-cd^2+e(bd-ae)\right)^3} \\
&\quad - \frac{(c^2d^2(-3ef+dg)-e^3(b^2f-abg+a^2h)+ace(e^2f-3deg+3d^2h)+bc(3de^2f-d^3h))\log(d+ex)}{\left(cd^2+e(-bd+ae)\right)^3} \\
&\quad + \frac{(c^2d^2(-3ef+dg)-e^3(b^2f-abg+a^2h)+ace(e^2f-3deg+3d^2h)+bc(3de^2f-d^3h))\log(a+x(b+}{2\left(cd^2+e(-bd+ae)\right)^3}
\end{aligned}
$$

```
[In] Integrate[(f + g*x + h*x^2)/((d + e*x)^3*(a + b*x + c*x^2)),x]
```

```
[Out] -1/2*(e^2*f - d*e*g + d^2*h)/(e*(c*d^2 + e*(-(b*d) + a*e))*(d + e*x)^2) + (
c*d*(-2*e*f + d*g) + a*e*(-(e*g) + 2*d*h) + b*(e^2*f - d^2*h))/((c*d^2 + e*
(-(b*d) + a*e))^2*(d + e*x)) + ((-2*c^3*d^3*f + b*e^3*(b^2*f - a*b*g + a^2*
h) + c^2*d*(b*d*(3*e*f + d*g) + 2*a*(3*e^2*f - 3*d*e*g + d^2*h)) - c*(-2*a^
2*e^2*(e*g - 3*d*h) + 3*a*b*e*(e^2*f - d*e*g - d^2*h) + b^2*(3*d*e^2*f + d^
3*h)))*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(Sqrt[-b^2 + 4*a*c]*(-(c*d^2
```

```
) + e*(b*d - a*e))^3) - ((c^2*d^2*(-3*e*f + d*g) - e^3*(b^2*f - a*b*g + a^2
*h) + a*c*e*(e^2*f - 3*d*e*g + 3*d^2*h) + b*c*(3*d*e^2*f - d^3*h))*Log[d +
e*x])/(c*d^2 + e*(-(b*d) + a*e))^3 + ((c^2*d^2*(-3*e*f + d*g) - e^3*(b^2*f
- a*b*g + a^2*h) + a*c*e*(e^2*f - 3*d*e*g + 3*d^2*h) + b*c*(3*d*e^2*f - d^3
*h))*Log[a + x*(b + c*x)])/(2*(c*d^2 + e*(-(b*d) + a*e))^3)
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 629, normalized size of antiderivative = 1.24

method	result
default	$-\frac{d^2h-deg+e^2f}{2(e^2a-bde+c\,d^2)e(ex+d)^2}+\frac{(a^2e^3h-ab\,e^3g-3ac\,d^2eh+3acd\,e^2g-ac\,e^3f+b^2e^3f+bc\,d^3h-3bcd\,e^2f-c^2d^3g+3c^2d^2ef)\ln(ex+d)}{(e^2a-bde+c\,d^2)^3}$
risch	Expression too large to display

```
[In] int((h*x^2+g*x+f)/(e*x+d)^3/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(d^2*h-d*e*g+e^2*f)/(a*e^2-b*d*e+c*d^2)/e/(e*x+d)^2+(a^2*e^3*h-a*b*e^3
*g-3*a*c*d^2*e*h+3*a*c*d*e^2*g-a*c*e^3*f+b^2*e^3*f+b*c*d^3*h-3*b*c*d*e^2*f-
c^2*d^3*g+3*c^2*d^2*e*f)/(a*e^2-b*d*e+c*d^2)^3*ln(e*x+d)+(2*a*d*e*h-a*e^2*g
-b*d^2*h+b*e^2*f+c*d^2*g-2*c*d*e*f)/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)+1/(a*e^2-
b*d*e+c*d^2)^3*(1/2*(-a^2*c*e^3*h+a*b*c*e^3*g+3*a*c^2*d^2*e*h-3*a*c^2*d*e^2
*g+a*c^2*e^3*f-b^2*c*e^3*f-b*c^2*d^3*h+3*b*c^2*d*e^2*f+c^3*d^3*g-3*c^3*d^2*
e*f)/c*ln(c*x^2+b*x+a)+2*(-a^2*b*e^3*h+3*a^2*c*d*e^2*h-a^2*c*e^3*g+a*b^2*e^
3*g-3*a*b*c*d*e^2*g+2*a*b*c*e^3*f-a*c^2*d^3*h+3*a*c^2*d^2*e*g-3*a*c^2*d*e^2
*f-b^3*e^3*f+3*b^2*c*d*e^2*f-3*b*c^2*d^2*e*f+c^3*d^3*f-1/2*(-a^2*c*e^3*h+a*
b*c*e^3*g+3*a*c^2*d^2*e*h-3*a*c^2*d*e^2*g+a*c^2*e^3*f-b^2*c*e^3*f-b*c^2*d^3
*h+3*b*c^2*d*e^2*f+c^3*d^3*g-3*c^3*d^2*e*f)*b/c)/(4*a*c-b^2)^(1/2)*arctan((
2*c*x+b)/(4*a*c-b^2)^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{f+gx+hx^2}{(d+ex)^3\,(a+bx+cx^2)}\,dx=\text{Timed out}$$

```
[In] integrate((h*x^2+g*x+f)/(e*x+d)^3/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{f+gx+hx^2}{(d+ex)^3\,(a+bx+cx^2)}\,dx = \text{Timed out}$$

```
[In] integrate((h*x**2+g*x+f)/(e*x+d)**3/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{f+gx+hx^2}{(d+ex)^3\,(a+bx+cx^2)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((h*x^2+g*x+f)/(e*x+d)^3/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1065 vs. 2(501) = 1002.

Time = 0.27 (sec) , antiderivative size = 1065, normalized size of antiderivative = 2.09

$$\begin{aligned}
&\int \frac{f+gx+hx^2}{(d+ex)^3\,(a+bx+cx^2)}\,dx = \\
&-\frac{(3\,c^2d^2ef-3\,bcde^2f+b^2e^3f-ace^3f-c^2d^3g+3\,acde^2g-abe^3g+bcd^3h-3\,acd^2eh+a^2e^3h)\log(cx^2}{2\,(c^3d^6-3\,bc^2d^5e+3\,b^2cd^4e^2+3\,ac^2d^4e^2-b^3d^3e^3-6\,abcd^3e^3+3\,ab^2d^2e^4+3\,a^2cd^2e^4-3\,a^2bde^5+} \\
&+\frac{(3\,c^2d^2e^2f-3\,bcde^3f+b^2e^4f-ace^4f-c^2d^3eg+3\,acde^3g-abe^4g+bcd^3eh-3\,acd^2e^2h+a^2e^4h)\log}{c^3d^6e-3\,bc^2d^5e^2+3\,b^2cd^4e^3+3\,ac^2d^4e^3-b^3d^3e^4-6\,abcd^3e^4+3\,ab^2d^2e^5+3\,a^2cd^2e^5-3\,a^2bde^6+} \\
&+\frac{(2\,c^3d^3f-3\,bc^2d^2ef+3\,b^2cde^2f-6\,ac^2de^2f-b^3e^3f+3\,abce^3f-bc^2d^3g+6\,ac^2d^2eg-3\,abcde^2g+a}{(c^3d^6-3\,bc^2d^5e+3\,b^2cd^4e^2+3\,ac^2d^4e^2-b^3d^3e^3-6\,abcd^3e^3+} \\
&-\frac{5\,c^2d^4e^2f-8\,bcd^3e^3f+3\,b^2d^2e^4f+6\,acd^2e^4f-4\,abde^5f+a^2e^6f-3\,c^2d^5eg+4\,bcd^4e^2g-b^2d^3e^3g-}{}
\end{aligned}$$

```
[In] integrate((h*x^2+g*x+f)/(e*x+d)^3/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] -1/2*(3*c^2*d^2*e*f - 3*b*c*d*e^2*f + b^2*e^3*f - a*c*e^3*f - c^2*d^3*g + 3
*a*c*d*e^2*g - a*b*e^3*g + b*c*d^3*h - 3*a*c*d^2*e*h + a^2*e^3*h)*log(c*x^2
```

```
 + b*x + a)/(c^3*d^6 - 3*b*c^2*d^5*e + 3*b^2*c*d^4*e^2 + 3*a*c^2*d^4*e^2 -
b^3*d^3*e^3 - 6*a*b*c*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a^2*c*d^2*e^4 - 3*a^2*b
*d*e^5 + a^3*e^6) + (3*c^2*d^2*e^2*f - 3*b*c*d*e^3*f + b^2*e^4*f - a*c*e^4*
f - c^2*d^3*e*g + 3*a*c*d*e^3*g - a*b*e^4*g + b*c*d^3*e*h - 3*a*c*d^2*e^2*h
 + a^2*e^4*h)*log(abs(e*x + d))/(c^3*d^6*e - 3*b*c^2*d^5*e^2 + 3*b^2*c*d^4*
e^3 + 3*a*c^2*d^4*e^3 - b^3*d^3*e^4 - 6*a*b*c*d^3*e^4 + 3*a*b^2*d^2*e^5 + 3
*a^2*c*d^2*e^5 - 3*a^2*b*d*e^6 + a^3*e^7) + (2*c^3*d^3*f - 3*b*c^2*d^2*e*f
 + 3*b^2*c*d*e^2*f - 6*a*c^2*d*e^2*f - b^3*e^3*f + 3*a*b*c*e^3*f - b*c^2*d^3
*g + 6*a*c^2*d^2*e*g - 3*a*b*c*d*e^2*g + a*b^2*e^3*g - 2*a^2*c*e^3*g + b^2*
c*d^3*h - 2*a*c^2*d^3*h - 3*a*b*c*d^2*e*h + 6*a^2*c*d*e^2*h - a^2*b*e^3*h)*
arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((c^3*d^6 - 3*b*c^2*d^5*e + 3*b^2*c*
d^4*e^2 + 3*a*c^2*d^4*e^2 - b^3*d^3*e^3 - 6*a*b*c*d^3*e^3 + 3*a*b^2*d^2*e^4
 + 3*a^2*c*d^2*e^4 - 3*a^2*b*d*e^5 + a^3*e^6)*sqrt(-b^2 + 4*a*c)) - 1/2*(5*
c^2*d^4*e^2*f - 8*b*c*d^3*e^3*f + 3*b^2*d^2*e^4*f + 6*a*c*d^2*e^4*f - 4*a*b
*d*e^5*f + a^2*e^6*f - 3*c^2*d^5*e*g + 4*b*c*d^4*e^2*g - b^2*d^3*e^3*g - 2*
a*c*d^3*e^3*g + a^2*d*e^5*g + c^2*d^6*h - b^2*d^4*e^2*h - 2*a*c*d^4*e^2*h +
 4*a*b*d^3*e^3*h - 3*a^2*d^2*e^4*h + 2*(2*c^2*d^3*e^3*f - 3*b*c*d^2*e^4*f +
 b^2*d*e^5*f + 2*a*c*d*e^5*f - a*b*e^6*f - c^2*d^4*e^2*g + b*c*d^3*e^3*g -
a*b*d*e^5*g + a^2*e^6*g + b*c*d^4*e^2*h - b^2*d^3*e^3*h - 2*a*c*d^3*e^3*h +
 3*a*b*d^2*e^4*h - 2*a^2*d*e^5*h)*x)/((c*d^2 - b*d*e + a*e^2)^3*(e*x + d)^2
*e)
```

Mupad [B] (verification not implemented)

Time = 17.31 (sec) , antiderivative size = 12784, normalized size of antiderivative = 25.12

$$\int \frac{f+gx+hx^2}{(d+ex)^3\,(a+bx+cx^2)}\,dx = \text{Too large to display}$$

```
[In] int((f + g*x + h*x^2)/((d + e*x)^3*(a + b*x + c*x^2)),x)
```

```
[Out] symsum(log(root(24*a^6*b*c*d*e^11*z^3 + 24*a*b*c^6*d^11*e*z^3 + 240*a^4*b*c
^3*d^5*e^7*z^3 + 240*a^3*b*c^4*d^7*e^5*z^3 + 120*a^5*b*c^2*d^3*e^9*z^3 + 12
0*a^2*b*c^5*d^9*e^3*z^3 - 54*a^5*b^2*c*d^2*e^10*z^3 - 54*a*b^2*c^5*d^10*e^2
*z^3 + 50*a^4*b^3*c*d^3*e^9*z^3 + 50*a*b^3*c^4*d^9*e^3*z^3 - 36*a^2*b^5*c*d
^5*e^7*z^3 - 36*a*b^5*c^2*d^7*e^5*z^3 + 26*a*b^6*c*d^6*e^6*z^3 - 340*a^3*b^
2*c^3*d^6*e^6*z^3 - 225*a^4*b^2*c^2*d^4*e^8*z^3 - 225*a^2*b^2*c^4*d^8*e^4*z
^3 + 180*a^3*b^3*c^2*d^5*e^7*z^3 + 180*a^2*b^3*c^3*d^7*e^5*z^3 - 30*a^2*b^4
*c^2*d^6*e^6*z^3 - 6*b^7*c*d^7*e^5*z^3 - 6*b^3*c^5*d^11*e*z^3 - 6*a^5*b^3*d
*e^11*z^3 - 6*a*b^7*d^5*e^7*z^3 - 20*b^5*c^3*d^9*e^3*z^3 + 15*b^6*c^2*d^8*e
^4*z^3 + 15*b^4*c^4*d^10*e^2*z^3 - 80*a^4*c^4*d^6*e^6*z^3 - 60*a^5*c^3*d^4*
e^8*z^3 - 60*a^3*c^5*d^8*e^4*z^3 - 24*a^6*c^2*d^2*e^10*z^3 - 24*a^2*c^6*d^1
0*e^2*z^3 - 20*a^3*b^5*d^3*e^9*z^3 + 15*a^4*b^4*d^2*e^10*z^3 + 15*a^2*b^6*d
^4*e^8*z^3 - 4*a^7*c*e^12*z^3 - 4*a*c^7*d^12*z^3 + b^8*d^6*e^6*z^3 + b^2*c^
6*d^12*z^3 + a^6*b^2*e^12*z^3 - 9*a^3*b^2*c*d*e^5*g*h*z - 9*a*b^2*c^3*d^5*e
```

```
*g*h*z - 30*a^3*b*c^2*d*e^5*f*h*z + 9*a^2*b^3*c*d*e^5*f*h*z + 3*a*b^4*c*d^2
*e^4*f*h*z + 27*a*b*c^4*d^4*e^2*f*g*z + 6*a^2*b^2*c^2*d^3*e^3*g*h*z - 33*a^
2*b^2*c^2*d^2*e^4*f*h*z + 18*a*b*c^4*d^5*e*f*h*z - 12*a*b^4*c*d*e^5*f*g*z +
 27*a^3*b*c^2*d^2*e^4*g*h*z + 27*a^2*b*c^3*d^4*e^2*g*h*z - 3*a^2*b^3*c*d^2*
e^4*g*h*z - 3*a*b^3*c^2*d^4*e^2*g*h*z + 52*a^2*b*c^3*d^3*e^3*f*h*z - 4*a*b^
3*c^2*d^3*e^3*f*h*z - 3*a*b^2*c^3*d^4*e^2*f*h*z - 93*a^2*b*c^3*d^2*e^4*f*g*
z + 51*a^2*b^2*c^2*d*e^5*f*g*z - 34*a*b^2*c^3*d^3*e^3*f*g*z + 27*a*b^3*c^2*
d^2*e^4*f*g*z - 24*a*c^5*d^5*e*f*g*z - 7*a^4*b*c*e^6*g*h*z - 7*a*b*c^4*d^6*
g*h*z + a*b^4*c*d^3*e^3*g*h*z - 80*a^3*c^3*d^3*e^3*g*h*z + 3*b^4*c^2*d^4*e^
2*f*h*z - 66*a^2*c^4*d^4*e^2*f*h*z + 54*a^3*c^3*d^2*e^4*f*h*z - 3*b^3*c^3*d
^4*e^2*f*g*z + 80*a^2*c^4*d^3*e^3*f*g*z - 21*a^2*b*c^3*d^5*e*h^2*z + 6*a*b^
3*c^2*d^5*e*h^2*z - 21*a^3*b*c^2*d*e^5*g^2*z + 6*a^2*b^3*c*d*e^5*g^2*z - 66
*a*b*c^4*d^3*e^3*f^2*z - 30*a*b^3*c^2*d*e^5*f^2*z + 27*a^2*b*c^3*d*e^5*f^2*
z - 12*a^2*b^2*c^2*d^4*e^2*h^2*z - 12*a^2*b^2*c^2*d^2*e^4*g^2*z + 24*a^4*c^
2*d*e^5*g*h*z + 24*a^2*c^4*d^5*e*g*h*z - 3*b^3*c^3*d^5*e*f*h*z - b^5*c*d^3*
e^3*f*h*z + 3*b^2*c^4*d^5*e*f*g*z - 24*a^3*c^3*d*e^5*f*g*z + 9*a^3*b^2*c*e^
6*f*h*z - 10*a^2*b^3*c*e^6*f*g*z + 9*a^3*b*c^2*e^6*f*g*z + 3*a^4*b*c*d*e^5*
h^2*z + 3*a*b*c^4*d^5*e*g^2*z + 14*a^3*b*c^2*d^3*e^3*h^2*z + 3*a^3*b^2*c*d^
2*e^4*h^2*z - a^2*b^3*c*d^3*e^3*h^2*z + 14*a^2*b*c^3*d^3*e^3*g^2*z + 3*a*b^
2*c^3*d^4*e^2*g^2*z - a*b^3*c^2*d^3*e^3*g^2*z + 63*a*b^2*c^3*d^2*e^4*f^2*z
+ 2*b^3*c^3*d^6*g*h*z - 6*a^4*c^2*e^6*f*h*z + 2*a^3*b^3*e^6*g*h*z - b^2*c^4
*d^6*f*h*z - 2*a^2*b^4*e^6*f*h*z + 6*b^5*c*d*e^5*f^2*z + 3*b*c^5*d^5*e*f^2*
z + 6*a*b^4*c*e^6*f^2*z + b^4*c^2*d^3*e^3*f*g*z + 33*a^3*c^3*d^4*e^2*h^2*z
- 27*a^4*c^2*d^2*e^4*h^2*z + 33*a^3*c^3*d^2*e^4*g^2*z - 27*a^2*c^4*d^4*e^2*
g^2*z + 19*b^3*c^3*d^3*e^3*f^2*z - 15*b^4*c^2*d^2*e^4*f^2*z - 12*b^2*c^4*d^
4*e^2*f^2*z - 27*a^2*c^4*d^2*e^4*f^2*z - 9*a^2*b^2*c^2*e^6*f^2*z + 2*a*c^5*
d^6*f*h*z + 2*a*b^5*e^6*f*g*z + 33*a*c^5*d^4*e^2*f^2*z + 4*a^3*b^2*c*e^6*g^
2*z + 4*a*b^2*c^3*d^6*h^2*z - b^4*c^2*d^6*h^2*z - b^2*c^4*d^6*g^2*z - a^4*c
^2*e^6*g^2*z - a^4*b^2*e^6*h^2*z - a^2*c^4*d^6*h^2*z + 3*a^3*c^3*e^6*f^2*z
- a^2*b^4*e^6*g^2*z + b*c^5*d^6*f*g*z + 3*a^5*c*e^6*h^2*z + 3*a*c^5*d^6*g^2
*z - c^6*d^6*f^2*z - b^6*e^6*f^2*z + 6*a*b^2*c^2*d*e^2*f*g*h - 2*a*b^3*c*e^
3*f*g*h + 3*a^2*b*c^2*d^2*e*g*h^2 - 3*a^2*b*c^2*d*e^2*g^2*h - 3*a^2*b*c^2*d
*e^2*f*h^2 - 3*a*b^2*c^2*d^2*e*f*h^2 - 6*a^2*c^3*d*e^2*f*g*h + 2*a^2*b*c^2*
e^3*f*g*h + 6*a*b*c^3*d*e^2*f^2*h - 6*a*b*c^3*d*e^2*f*g^2 - 2*b^2*c^3*d^3*f
*g*h - 9*a*c^4*d^2*e*f^2*h - 3*b*c^4*d^2*e*f^2*g + 3*a*c^4*d^2*e*f*g^2 + 3*
a*c^4*d*e^2*f^2*g - 2*a^3*b*c*e^3*g*h^2 + 2*a*b*c^3*d^3*g^2*h - 2*a*b*c^3*d
^3*f*h^2 + 2*a*c^4*d^3*f*g*h - 3*b^3*c^2*d*e^2*f^2*h + 3*b^2*c^3*d^2*e*f^2*
h + 3*a^3*c^2*d*e^2*g*h^2 - 3*a^2*c^3*d^2*e*g^2*h + 9*a^2*c^3*d^2*e*f*h^2 +
 3*b^2*c^3*d*e^2*f^2*g - 3*a*b^2*c^2*e^3*f^2*h + 2*a^2*b^2*c*e^3*f*h^2 - a*
b^2*c^2*d^3*g*h^2 + 2*a*b^2*c^2*e^3*f*g^2 - 3*a^3*c^2*e^3*f*h^2 + 3*a^2*c^3
*e^3*f^2*h - b^3*c^2*e^3*f^2*g - a^2*c^3*d^3*g*h^2 - a^2*c^3*e^3*f*g^2 - 3*
a^3*c^2*d^2*e*h^3 + 3*a^2*c^3*d*e^2*g^3 - a^2*b*c^2*e^3*g^3 - 3*b*c^4*d*e^2
*f^3 + a^2*b^2*c*e^3*g^2*h + a^3*c^2*e^3*g^2*h + b^3*c^2*d^3*f*h^2 + a^2*b*
c^2*d^3*h^3 + b^4*c*e^3*f^2*h + b*c^4*d^3*f^2*h + b*c^4*d^3*f*g^2 - c^5*d^3
*f^2*g + 3*c^5*d^2*e*f^3 - a*c^4*e^3*f^3 - a*c^4*d^3*g^3 + b^2*c^3*e^3*f^3
```

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+ a^4*c*e^3*h^3, z, k)*(root(24*a^6*b*c*d*e^11*z^3 + 24*a*b*c^6*d^11*e*z^3
+ 240*a^4*b*c^3*d^5*e^7*z^3 + 240*a^3*b*c^4*d^7*e^5*z^3 + 120*a^5*b*c^2*d^3
*e^9*z^3 + 120*a^2*b*c^5*d^9*e^3*z^3 - 54*a^5*b^2*c*d^2*e^10*z^3 - 54*a*b^2
*c^5*d^10*e^2*z^3 + 50*a^4*b^3*c*d^3*e^9*z^3 + 50*a*b^3*c^4*d^9*e^3*z^3 - 3
6*a^2*b^5*c*d^5*e^7*z^3 - 36*a*b^5*c^2*d^7*e^5*z^3 + 26*a*b^6*c*d^6*e^6*z^3
 - 340*a^3*b^2*c^3*d^6*e^6*z^3 - 225*a^4*b^2*c^2*d^4*e^8*z^3 - 225*a^2*b^2*
c^4*d^8*e^4*z^3 + 180*a^3*b^3*c^2*d^5*e^7*z^3 + 180*a^2*b^3*c^3*d^7*e^5*z^3
 - 30*a^2*b^4*c^2*d^6*e^6*z^3 - 6*b^7*c*d^7*e^5*z^3 - 6*b^3*c^5*d^11*e*z^3
- 6*a^5*b^3*d*e^11*z^3 - 6*a*b^7*d^5*e^7*z^3 - 20*b^5*c^3*d^9*e^3*z^3 + 15*
b^6*c^2*d^8*e^4*z^3 + 15*b^4*c^4*d^10*e^2*z^3 - 80*a^4*c^4*d^6*e^6*z^3 - 60
*a^5*c^3*d^4*e^8*z^3 - 60*a^3*c^5*d^8*e^4*z^3 - 24*a^6*c^2*d^2*e^10*z^3 - 2
4*a^2*c^6*d^10*e^2*z^3 - 20*a^3*b^5*d^3*e^9*z^3 + 15*a^4*b^4*d^2*e^10*z^3 +
 15*a^2*b^6*d^4*e^8*z^3 - 4*a^7*c*e^12*z^3 - 4*a*c^7*d^12*z^3 + b^8*d^6*e^6
*z^3 + b^2*c^6*d^12*z^3 + a^6*b^2*e^12*z^3 - 9*a^3*b^2*c*d*e^5*g*h*z - 9*a*
b^2*c^3*d^5*e*g*h*z - 30*a^3*b*c^2*d*e^5*f*h*z + 9*a^2*b^3*c*d*e^5*f*h*z +
3*a*b^4*c*d^2*e^4*f*h*z + 27*a*b*c^4*d^4*e^2*f*g*z + 6*a^2*b^2*c^2*d^3*e^3*
g*h*z - 33*a^2*b^2*c^2*d^2*e^4*f*h*z + 18*a*b*c^4*d^5*e*f*h*z - 12*a*b^4*c*
d*e^5*f*g*z + 27*a^3*b*c^2*d^2*e^4*g*h*z + 27*a^2*b*c^3*d^4*e^2*g*h*z - 3*a
^2*b^3*c*d^2*e^4*g*h*z - 3*a*b^3*c^2*d^4*e^2*g*h*z + 52*a^2*b*c^3*d^3*e^3*f
*h*z - 4*a*b^3*c^2*d^3*e^3*f*h*z - 3*a*b^2*c^3*d^4*e^2*f*h*z - 93*a^2*b*c^3
*d^2*e^4*f*g*z + 51*a^2*b^2*c^2*d*e^5*f*g*z - 34*a*b^2*c^3*d^3*e^3*f*g*z +
27*a*b^3*c^2*d^2*e^4*f*g*z - 24*a*c^5*d^5*e*f*g*z - 7*a^4*b*c*e^6*g*h*z - 7
*a*b*c^4*d^6*g*h*z + a*b^4*c*d^3*e^3*g*h*z - 80*a^3*c^3*d^3*e^3*g*h*z + 3*b
^4*c^2*d^4*e^2*f*h*z - 66*a^2*c^4*d^4*e^2*f*h*z + 54*a^3*c^3*d^2*e^4*f*h*z
- 3*b^3*c^3*d^4*e^2*f*g*z + 80*a^2*c^4*d^3*e^3*f*g*z - 21*a^2*b*c^3*d^5*e*h
^2*z + 6*a*b^3*c^2*d^5*e*h^2*z - 21*a^3*b*c^2*d*e^5*g^2*z + 6*a^2*b^3*c*d*e
^5*g^2*z - 66*a*b*c^4*d^3*e^3*f^2*z - 30*a*b^3*c^2*d*e^5*f^2*z + 27*a^2*b*c
^3*d*e^5*f^2*z - 12*a^2*b^2*c^2*d^4*e^2*h^2*z - 12*a^2*b^2*c^2*d^2*e^4*g^2*
z + 24*a^4*c^2*d*e^5*g*h*z + 24*a^2*c^4*d^5*e*g*h*z - 3*b^3*c^3*d^5*e*f*h*z
 - b^5*c*d^3*e^3*f*h*z + 3*b^2*c^4*d^5*e*f*g*z - 24*a^3*c^3*d*e^5*f*g*z + 9
*a^3*b^2*c*e^6*f*h*z - 10*a^2*b^3*c*e^6*f*g*z + 9*a^3*b*c^2*e^6*f*g*z + 3*a
^4*b*c*d*e^5*h^2*z + 3*a*b*c^4*d^5*e*g^2*z + 14*a^3*b*c^2*d^3*e^3*h^2*z + 3
*a^3*b^2*c*d^2*e^4*h^2*z - a^2*b^3*c*d^3*e^3*h^2*z + 14*a^2*b*c^3*d^3*e^3*g
^2*z + 3*a*b^2*c^3*d^4*e^2*g^2*z - a*b^3*c^2*d^3*e^3*g^2*z + 63*a*b^2*c^3*d
^2*e^4*f^2*z + 2*b^3*c^3*d^6*g*h*z - 6*a^4*c^2*e^6*f*h*z + 2*a^3*b^3*e^6*g*
h*z - b^2*c^4*d^6*f*h*z - 2*a^2*b^4*e^6*f*h*z + 6*b^5*c*d*e^5*f^2*z + 3*b*c
^5*d^5*e*f^2*z + 6*a*b^4*c*e^6*f^2*z + b^4*c^2*d^3*e^3*f*g*z + 33*a^3*c^3*d
^4*e^2*h^2*z - 27*a^4*c^2*d^2*e^4*h^2*z + 33*a^3*c^3*d^2*e^4*g^2*z - 27*a^2
*c^4*d^4*e^2*g^2*z + 19*b^3*c^3*d^3*e^3*f^2*z - 15*b^4*c^2*d^2*e^4*f^2*z -
12*b^2*c^4*d^4*e^2*f^2*z - 27*a^2*c^4*d^2*e^4*f^2*z - 9*a^2*b^2*c^2*e^6*f^2
*z + 2*a*c^5*d^6*f*h*z + 2*a*b^5*e^6*f*g*z + 33*a*c^5*d^4*e^2*f^2*z + 4*a^3
*b^2*c*e^6*g^2*z + 4*a*b^2*c^3*d^6*h^2*z - b^4*c^2*d^6*h^2*z - b^2*c^4*d^6*
g^2*z - a^4*c^2*e^6*g^2*z - a^4*b^2*e^6*h^2*z - a^2*c^4*d^6*h^2*z + 3*a^3*c
^3*e^6*f^2*z - a^2*b^4*e^6*g^2*z + b*c^5*d^6*f*g*z + 3*a^5*c*e^6*h^2*z + 3*
a*c^5*d^6*g^2*z - c^6*d^6*f^2*z - b^6*e^6*f^2*z + 6*a*b^2*c^2*d*e^2*f*g*h -
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 2*a*b^3*c*e^3*f*g*h + 3*a^2*b*c^2*d^2*e*g*h^2 - 3*a^2*b*c^2*d*e^2*g^2*h -
3*a^2*b*c^2*d*e^2*f*h^2 - 3*a*b^2*c^2*d^2*e*f*h^2 - 6*a^2*c^3*d*e^2*f*g*h +
 2*a^2*b*c^2*e^3*f*g*h + 6*a*b*c^3*d*e^2*f^2*h - 6*a*b*c^3*d*e^2*f*g^2 - 2*
b^2*c^3*d^3*f*g*h - 9*a*c^4*d^2*e*f^2*h - 3*b*c^4*d^2*e*f^2*g + 3*a*c^4*d^2
*e*f*g^2 + 3*a*c^4*d*e^2*f^2*g - 2*a^3*b*c*e^3*g*h^2 + 2*a*b*c^3*d^3*g^2*h
- 2*a*b*c^3*d^3*f*h^2 + 2*a*c^4*d^3*f*g*h - 3*b^3*c^2*d*e^2*f^2*h + 3*b^2*c
^3*d^2*e*f^2*h + 3*a^3*c^2*d*e^2*g*h^2 - 3*a^2*c^3*d^2*e*g^2*h + 9*a^2*c^3*
d^2*e*f*h^2 + 3*b^2*c^3*d*e^2*f^2*g - 3*a*b^2*c^2*e^3*f^2*h + 2*a^2*b^2*c*e
^3*f*h^2 - a*b^2*c^2*d^3*g*h^2 + 2*a*b^2*c^2*e^3*f*g^2 - 3*a^3*c^2*e^3*f*h^
2 + 3*a^2*c^3*e^3*f^2*h - b^3*c^2*e^3*f^2*g - a^2*c^3*d^3*g*h^2 - a^2*c^3*e
^3*f*g^2 - 3*a^3*c^2*d^2*e*h^3 + 3*a^2*c^3*d*e^2*g^3 - a^2*b*c^2*e^3*g^3 -
3*b*c^4*d*e^2*f^3 + a^2*b^2*c*e^3*g^2*h + a^3*c^2*e^3*g^2*h + b^3*c^2*d^3*f
*h^2 + a^2*b*c^2*d^3*h^3 + b^4*c*e^3*f^2*h + b*c^4*d^3*f^2*h + b*c^4*d^3*f*
g^2 - c^5*d^3*f^2*g + 3*c^5*d^2*e*f^3 - a*c^4*e^3*f^3 - a*c^4*d^3*g^3 + b^2
*c^3*e^3*f^3 + a^4*c*e^3*h^3, z, k)*((8*a*c^6*d^9*e^2 + 8*a^5*c^2*d*e^10 -
b^6*c*d^5*e^6 + 32*a^2*c^5*d^7*e^4 + 48*a^3*c^4*d^5*e^6 + 32*a^4*c^3*d^3*e^
8 + 3*b^2*c^5*d^9*e^2 - 2*b^3*c^4*d^8*e^3 - 2*b^4*c^3*d^7*e^4 + 3*b^5*c^2*d
^6*e^5 - a^5*b*c*e^11 - b*c^6*d^10*e + 114*a^2*b^2*c^3*d^5*e^6 - 38*a^2*b^3
*c^2*d^4*e^7 + 60*a^3*b^2*c^2*d^3*e^8 - 37*a*b*c^5*d^8*e^3 + 3*a*b^5*c*d^4*
e^7 + 3*a^4*b^2*c*d*e^10 + 60*a*b^2*c^4*d^7*e^4 - 38*a*b^3*c^3*d^6*e^5 + 4*
a*b^4*c^2*d^5*e^6 - 106*a^2*b*c^4*d^6*e^5 - 2*a^2*b^4*c*d^3*e^8 - 106*a^3*b
*c^3*d^4*e^7 - 2*a^3*b^3*c*d^2*e^9 - 37*a^4*b*c^2*d^2*e^9)/(a^4*e^8 + c^4*d
^8 + b^4*d^4*e^4 - 4*a*b^3*d^3*e^5 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 - 4*
b^3*c*d^5*e^3 + 6*a^2*b^2*d^2*e^6 + 6*a^2*c^2*d^4*e^4 + 6*b^2*c^2*d^6*e^2 -
 4*a^3*b*d*e^7 - 4*b*c^3*d^7*e - 12*a*b*c^2*d^5*e^3 + 12*a*b^2*c*d^4*e^4 -
12*a^2*b*c*d^3*e^5) + (x*(6*a^5*c^2*e^11 - 2*c^7*d^10*e - 2*a^4*b^2*c*e^11
- 2*a*c^6*d^8*e^3 + 10*b*c^6*d^9*e^2 - 2*b^6*c*d^4*e^7 + 12*a^2*c^5*d^6*e^5
 + 28*a^3*c^4*d^4*e^7 + 22*a^4*c^3*d^2*e^9 - 22*b^2*c^5*d^8*e^3 + 28*b^3*c^
4*d^7*e^4 - 22*b^4*c^3*d^6*e^5 + 10*b^5*c^2*d^5*e^6 + 24*a^2*b^2*c^3*d^4*e^
7 + 12*a^2*b^3*c^2*d^3*e^8 + 20*a^3*b^2*c^2*d^2*e^9 + 8*a*b*c^5*d^7*e^4 + 8
*a*b^5*c*d^3*e^8 + 8*a^3*b^3*c*d*e^10 - 22*a^4*b*c^2*d*e^10 - 20*a*b^2*c^4*
d^6*e^5 + 32*a*b^3*c^3*d^5*e^6 - 26*a*b^4*c^2*d^4*e^7 - 36*a^2*b*c^4*d^5*e^
6 - 12*a^2*b^4*c*d^2*e^9 - 56*a^3*b*c^3*d^3*e^8))/(a^4*e^8 + c^4*d^8 + b^4*
d^4*e^4 - 4*a*b^3*d^3*e^5 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 - 4*b^3*c*d^5
*e^3 + 6*a^2*b^2*d^2*e^6 + 6*a^2*c^2*d^4*e^4 + 6*b^2*c^2*d^6*e^2 - 4*a^3*b*
d*e^7 - 4*b*c^3*d^7*e - 12*a*b*c^2*d^5*e^3 + 12*a*b^2*c*d^4*e^4 - 12*a^2*b*
c*d^3*e^5)) + (a^4*c^2*e^8*g + c^6*d^7*e*f + a^4*b*c*e^8*h - a*c^5*d^7*e*h
- b*c^5*d^7*e*g + a^2*b^3*c*e^8*f - 2*a^3*b*c^2*e^8*f - a^3*b^2*c*e^8*g + 3
*a*c^5*d^5*e^3*f + a^3*c^3*d*e^7*f + a*c^5*d^6*e^2*g - b*c^5*d^6*e^2*f + b^
5*c*d^2*e^6*f - a^4*c^2*d*e^7*h + b^2*c^4*d^7*e*h + 3*a^2*c^4*d^3*e^5*f + 3
*a^2*c^4*d^4*e^4*g + 3*a^3*c^3*d^2*e^6*g - 3*b^2*c^4*d^5*e^3*f + 6*b^3*c^3*
d^4*e^4*f - 4*b^4*c^2*d^3*e^5*f - 3*a^2*c^4*d^5*e^3*h - 3*a^3*c^3*d^3*e^5*h
 + 2*b^2*c^4*d^6*e^2*g - b^3*c^3*d^5*e^3*g - 2*b^3*c^3*d^6*e^2*h + b^4*c^2*
d^5*e^3*h - a*b^2*c^3*d^3*e^5*f + 4*a*b^3*c^2*d^2*e^6*f - 5*a^2*b*c^3*d^2*e
^6*f + 2*a^2*b^2*c^2*d*e^7*f - 2*a*b^2*c^3*d^4*e^4*g + 4*a*b^3*c^2*d^3*e^5*
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g - a^2*b*c^3*d^3*e^5*g + 5*a*b^2*c^3*d^5*e^3*h - 4*a*b^3*c^2*d^4*e^4*h + a
^2*b*c^3*d^4*e^4*h + a^2*b^3*c*d^2*e^6*h + 2*a^3*b*c^2*d^2*e^6*h - 2*a*b^4*
c*d*e^7*f - 5*a^2*b^2*c^2*d^2*e^6*g + 2*a^2*b^2*c^2*d^3*e^5*h - 4*a*b*c^4*d
^4*e^4*f - 2*a*b*c^4*d^5*e^3*g - a*b^4*c*d^2*e^6*g + 2*a^2*b^3*c*d*e^7*g -
2*a^3*b^2*c*d*e^7*h)/(a^4*e^8 + c^4*d^8 + b^4*d^4*e^4 - 4*a*b^3*d^3*e^5 + 4
*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 - 4*b^3*c*d^5*e^3 + 6*a^2*b^2*d^2*e^6 + 6*
a^2*c^2*d^4*e^4 + 6*b^2*c^2*d^6*e^2 - 4*a^3*b*d*e^7 - 4*b*c^3*d^7*e - 12*a*
b*c^2*d^5*e^3 + 12*a*b^2*c*d^4*e^4 - 12*a^2*b*c*d^3*e^5) + (x*(3*a^4*c^2*e^
8*h - 3*a^3*c^3*e^8*f + 5*c^6*d^6*e^2*f - c^6*d^7*e*g + b*c^5*d^7*e*h - 2*a
^3*b*c^2*e^8*g + 7*a*c^5*d^4*e^4*f + 5*a*c^5*d^5*e^3*g - 15*b*c^5*d^5*e^3*f
 + 7*a^3*c^3*d*e^7*g - 5*a*c^5*d^6*e^2*h + b*c^5*d^6*e^2*g + 2*a^2*b^2*c^2*
e^8*f - a^2*c^4*d^2*e^6*f + 13*a^2*c^4*d^3*e^5*g + 17*b^2*c^4*d^4*e^4*f - 9
*b^3*c^3*d^3*e^5*f + 2*b^4*c^2*d^2*e^6*f - 7*a^2*c^4*d^4*e^4*h + a^3*c^3*d^
2*e^6*h + b^2*c^4*d^5*e^3*g - b^3*c^3*d^4*e^4*g - b^2*c^4*d^6*e^2*h - b^3*c
^3*d^5*e^3*h + b^4*c^2*d^4*e^4*h + 11*a*b^2*c^3*d^2*e^6*f + 13*a*b^2*c^3*d^
3*e^5*g - 2*a*b^3*c^2*d^2*e^6*g - 19*a^2*b*c^3*d^2*e^6*g + 4*a^2*b^2*c^2*d*
e^7*g - a*b^2*c^3*d^4*e^4*h - 4*a*b^3*c^2*d^3*e^5*h + a^2*b*c^3*d^3*e^5*h +
 8*a^2*b^2*c^2*d^2*e^6*h - 14*a*b*c^4*d^3*e^5*f - 4*a*b^3*c^2*d*e^7*f + a^2
*b*c^3*d*e^7*f - 16*a*b*c^4*d^4*e^4*g + 10*a*b*c^4*d^5*e^3*h - 8*a^3*b*c^2*
d*e^7*h))/(a^4*e^8 + c^4*d^8 + b^4*d^4*e^4 - 4*a*b^3*d^3*e^5 + 4*a*c^3*d^6*
e^2 + 4*a^3*c*d^2*e^6 - 4*b^3*c*d^5*e^3 + 6*a^2*b^2*d^2*e^6 + 6*a^2*c^2*d^4
*e^4 + 6*b^2*c^2*d^6*e^2 - 4*a^3*b*d*e^7 - 4*b*c^3*d^7*e - 12*a*b*c^2*d^5*e
^3 + 12*a*b^2*c*d^4*e^4 - 12*a^2*b*c*d^3*e^5)) - (2*c^5*d^3*e^2*f^2 - b^3*c
^2*e^5*f^2 - c^5*d^4*e*f*g + 2*a^2*c^3*d^3*e^2*h^2 + a*b*c^3*e^5*f^2 - 2*a*
c^4*d*e^4*f^2 - a^2*c^3*e^5*f*g + a^3*c^2*e^5*g*h - a^2*b*c^2*e^5*g^2 - 2*a
*c^4*d^3*e^2*g^2 - 5*b*c^4*d^2*e^3*f^2 + 2*a^2*c^3*d*e^4*g^2 + 4*b^2*c^3*d*
e^4*f^2 - 2*a^3*c^2*d*e^4*h^2 + a*b*c^3*d^2*e^3*g^2 - b^2*c^3*d^2*e^3*f*g -
 6*a^2*c^3*d^2*e^3*g*h - 2*b^2*c^3*d^3*e^2*f*h + b^3*c^2*d^2*e^3*f*h + a*c^
4*d^4*e*g*h + b*c^4*d^4*e*f*h + a^2*b*c^2*d^2*e^3*h^2 - a*b*c^3*d^4*e*h^2 +
 2*a*b^2*c^2*e^5*f*g - a^2*b*c^2*e^5*f*h + 6*a*c^4*d^2*e^3*f*g - 4*a*c^4*d^
3*e^2*f*h + 2*b*c^4*d^3*e^2*f*g + 4*a^2*c^3*d*e^4*f*h + 4*a*b*c^3*d^2*e^3*f
*h - 2*a*b^2*c^2*d*e^4*f*h + 2*a*b*c^3*d^3*e^2*g*h + 2*a^2*b*c^2*d*e^4*g*h
- a*b^2*c^2*d^2*e^3*g*h - 6*a*b*c^3*d*e^4*f*g)/(a^4*e^8 + c^4*d^8 + b^4*d^4
*e^4 - 4*a*b^3*d^3*e^5 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 - 4*b^3*c*d^5*e^
3 + 6*a^2*b^2*d^2*e^6 + 6*a^2*c^2*d^4*e^4 + 6*b^2*c^2*d^6*e^2 - 4*a^3*b*d*e
^7 - 4*b*c^3*d^7*e - 12*a*b*c^2*d^5*e^3 + 12*a*b^2*c*d^4*e^4 - 12*a^2*b*c*d
^3*e^5) + (x*(c^5*d^4*e*g^2 + a^2*c^3*e^5*g^2 + b^2*c^3*e^5*f^2 + 4*c^5*d^2
*e^3*f^2 + 4*a^2*c^3*d^2*e^3*h^2 - 4*b*c^4*d*e^4*f^2 - 4*c^5*d^3*e^2*f*g -
2*a*c^4*d^2*e^3*g^2 + b^2*c^3*d^4*e*h^2 - 4*a*b*c^3*d^3*e^2*h^2 - 2*b^2*c^3
*d^2*e^3*f*h - 2*a*b*c^3*e^5*f*g + 4*a*c^4*d*e^4*f*g - 2*b*c^4*d^4*e*g*h -
8*a*c^4*d^2*e^3*f*h + 2*b*c^4*d^2*e^3*f*g + 4*a*c^4*d^3*e^2*g*h + 4*b*c^4*d
^3*e^2*f*h - 4*a^2*c^3*d*e^4*g*h + 2*a*b*c^3*d^2*e^3*g*h + 4*a*b*c^3*d*e^4*
f*h))/(a^4*e^8 + c^4*d^8 + b^4*d^4*e^4 - 4*a*b^3*d^3*e^5 + 4*a*c^3*d^6*e^2
+ 4*a^3*c*d^2*e^6 - 4*b^3*c*d^5*e^3 + 6*a^2*b^2*d^2*e^6 + 6*a^2*c^2*d^4*e^4
 + 6*b^2*c^2*d^6*e^2 - 4*a^3*b*d*e^7 - 4*b*c^3*d^7*e - 12*a*b*c^2*d^5*e^3 +
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 12*a*b^2*c*d^4*e^4 - 12*a^2*b*c*d^3*e^5))*root(24*a^6*b*c*d*e^11*z^3 + 24*
a*b*c^6*d^11*e*z^3 + 240*a^4*b*c^3*d^5*e^7*z^3 + 240*a^3*b*c^4*d^7*e^5*z^3
+ 120*a^5*b*c^2*d^3*e^9*z^3 + 120*a^2*b*c^5*d^9*e^3*z^3 - 54*a^5*b^2*c*d^2*
e^10*z^3 - 54*a*b^2*c^5*d^10*e^2*z^3 + 50*a^4*b^3*c*d^3*e^9*z^3 + 50*a*b^3*
c^4*d^9*e^3*z^3 - 36*a^2*b^5*c*d^5*e^7*z^3 - 36*a*b^5*c^2*d^7*e^5*z^3 + 26*
a*b^6*c*d^6*e^6*z^3 - 340*a^3*b^2*c^3*d^6*e^6*z^3 - 225*a^4*b^2*c^2*d^4*e^8
*z^3 - 225*a^2*b^2*c^4*d^8*e^4*z^3 + 180*a^3*b^3*c^2*d^5*e^7*z^3 + 180*a^2*
b^3*c^3*d^7*e^5*z^3 - 30*a^2*b^4*c^2*d^6*e^6*z^3 - 6*b^7*c*d^7*e^5*z^3 - 6*
b^3*c^5*d^11*e*z^3 - 6*a^5*b^3*d*e^11*z^3 - 6*a*b^7*d^5*e^7*z^3 - 20*b^5*c^
3*d^9*e^3*z^3 + 15*b^6*c^2*d^8*e^4*z^3 + 15*b^4*c^4*d^10*e^2*z^3 - 80*a^4*c
^4*d^6*e^6*z^3 - 60*a^5*c^3*d^4*e^8*z^3 - 60*a^3*c^5*d^8*e^4*z^3 - 24*a^6*c
^2*d^2*e^10*z^3 - 24*a^2*c^6*d^10*e^2*z^3 - 20*a^3*b^5*d^3*e^9*z^3 + 15*a^4
*b^4*d^2*e^10*z^3 + 15*a^2*b^6*d^4*e^8*z^3 - 4*a^7*c*e^12*z^3 - 4*a*c^7*d^1
2*z^3 + b^8*d^6*e^6*z^3 + b^2*c^6*d^12*z^3 + a^6*b^2*e^12*z^3 - 9*a^3*b^2*c
*d*e^5*g*h*z - 9*a*b^2*c^3*d^5*e*g*h*z - 30*a^3*b*c^2*d*e^5*f*h*z + 9*a^2*b
^3*c*d*e^5*f*h*z + 3*a*b^4*c*d^2*e^4*f*h*z + 27*a*b*c^4*d^4*e^2*f*g*z + 6*a
^2*b^2*c^2*d^3*e^3*g*h*z - 33*a^2*b^2*c^2*d^2*e^4*f*h*z + 18*a*b*c^4*d^5*e*
f*h*z - 12*a*b^4*c*d*e^5*f*g*z + 27*a^3*b*c^2*d^2*e^4*g*h*z + 27*a^2*b*c^3*
d^4*e^2*g*h*z - 3*a^2*b^3*c*d^2*e^4*g*h*z - 3*a*b^3*c^2*d^4*e^2*g*h*z + 52*
a^2*b*c^3*d^3*e^3*f*h*z - 4*a*b^3*c^2*d^3*e^3*f*h*z - 3*a*b^2*c^3*d^4*e^2*f
*h*z - 93*a^2*b*c^3*d^2*e^4*f*g*z + 51*a^2*b^2*c^2*d*e^5*f*g*z - 34*a*b^2*c
^3*d^3*e^3*f*g*z + 27*a*b^3*c^2*d^2*e^4*f*g*z - 24*a*c^5*d^5*e*f*g*z - 7*a^
4*b*c*e^6*g*h*z - 7*a*b*c^4*d^6*g*h*z + a*b^4*c*d^3*e^3*g*h*z - 80*a^3*c^3*
d^3*e^3*g*h*z + 3*b^4*c^2*d^4*e^2*f*h*z - 66*a^2*c^4*d^4*e^2*f*h*z + 54*a^3
*c^3*d^2*e^4*f*h*z - 3*b^3*c^3*d^4*e^2*f*g*z + 80*a^2*c^4*d^3*e^3*f*g*z - 2
1*a^2*b*c^3*d^5*e*h^2*z + 6*a*b^3*c^2*d^5*e*h^2*z - 21*a^3*b*c^2*d*e^5*g^2*
z + 6*a^2*b^3*c*d*e^5*g^2*z - 66*a*b*c^4*d^3*e^3*f^2*z - 30*a*b^3*c^2*d*e^5
*f^2*z + 27*a^2*b*c^3*d*e^5*f^2*z - 12*a^2*b^2*c^2*d^4*e^2*h^2*z - 12*a^2*b
^2*c^2*d^2*e^4*g^2*z + 24*a^4*c^2*d*e^5*g*h*z + 24*a^2*c^4*d^5*e*g*h*z - 3*
b^3*c^3*d^5*e*f*h*z - b^5*c*d^3*e^3*f*h*z + 3*b^2*c^4*d^5*e*f*g*z - 24*a^3*
c^3*d*e^5*f*g*z + 9*a^3*b^2*c*e^6*f*h*z - 10*a^2*b^3*c*e^6*f*g*z + 9*a^3*b*
c^2*e^6*f*g*z + 3*a^4*b*c*d*e^5*h^2*z + 3*a*b*c^4*d^5*e*g^2*z + 14*a^3*b*c^
2*d^3*e^3*h^2*z + 3*a^3*b^2*c*d^2*e^4*h^2*z - a^2*b^3*c*d^3*e^3*h^2*z + 14*
a^2*b*c^3*d^3*e^3*g^2*z + 3*a*b^2*c^3*d^4*e^2*g^2*z - a*b^3*c^2*d^3*e^3*g^2
*z + 63*a*b^2*c^3*d^2*e^4*f^2*z + 2*b^3*c^3*d^6*g*h*z - 6*a^4*c^2*e^6*f*h*z
 + 2*a^3*b^3*e^6*g*h*z - b^2*c^4*d^6*f*h*z - 2*a^2*b^4*e^6*f*h*z + 6*b^5*c*
d*e^5*f^2*z + 3*b*c^5*d^5*e*f^2*z + 6*a*b^4*c*e^6*f^2*z + b^4*c^2*d^3*e^3*f
*g*z + 33*a^3*c^3*d^4*e^2*h^2*z - 27*a^4*c^2*d^2*e^4*h^2*z + 33*a^3*c^3*d^2
*e^4*g^2*z - 27*a^2*c^4*d^4*e^2*g^2*z + 19*b^3*c^3*d^3*e^3*f^2*z - 15*b^4*c
^2*d^2*e^4*f^2*z - 12*b^2*c^4*d^4*e^2*f^2*z - 27*a^2*c^4*d^2*e^4*f^2*z - 9*
a^2*b^2*c^2*e^6*f^2*z + 2*a*c^5*d^6*f*h*z + 2*a*b^5*e^6*f*g*z + 33*a*c^5*d^
4*e^2*f^2*z + 4*a^3*b^2*c*e^6*g^2*z + 4*a*b^2*c^3*d^6*h^2*z - b^4*c^2*d^6*h
^2*z - b^2*c^4*d^6*g^2*z - a^4*c^2*e^6*g^2*z - a^4*b^2*e^6*h^2*z - a^2*c^4*
d^6*h^2*z + 3*a^3*c^3*e^6*f^2*z - a^2*b^4*e^6*g^2*z + b*c^5*d^6*f*g*z + 3*a
^5*c*e^6*h^2*z + 3*a*c^5*d^6*g^2*z - c^6*d^6*f^2*z - b^6*e^6*f^2*z + 6*a*b^
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2*c^2*d*e^2*f*g*h - 2*a*b^3*c*e^3*f*g*h + 3*a^2*b*c^2*d^2*e*g*h^2 - 3*a^2*b
*c^2*d*e^2*g^2*h - 3*a^2*b*c^2*d*e^2*f*h^2 - 3*a*b^2*c^2*d^2*e*f*h^2 - 6*a^
2*c^3*d*e^2*f*g*h + 2*a^2*b*c^2*e^3*f*g*h + 6*a*b*c^3*d*e^2*f^2*h - 6*a*b*c
^3*d*e^2*f*g^2 - 2*b^2*c^3*d^3*f*g*h - 9*a*c^4*d^2*e*f^2*h - 3*b*c^4*d^2*e*
f^2*g + 3*a*c^4*d^2*e*f*g^2 + 3*a*c^4*d*e^2*f^2*g - 2*a^3*b*c*e^3*g*h^2 + 2
*a*b*c^3*d^3*g^2*h - 2*a*b*c^3*d^3*f*h^2 + 2*a*c^4*d^3*f*g*h - 3*b^3*c^2*d*
e^2*f^2*h + 3*b^2*c^3*d^2*e*f^2*h + 3*a^3*c^2*d*e^2*g*h^2 - 3*a^2*c^3*d^2*e
*g^2*h + 9*a^2*c^3*d^2*e*f*h^2 + 3*b^2*c^3*d*e^2*f^2*g - 3*a*b^2*c^2*e^3*f^
2*h + 2*a^2*b^2*c*e^3*f*h^2 - a*b^2*c^2*d^3*g*h^2 + 2*a*b^2*c^2*e^3*f*g^2 -
 3*a^3*c^2*e^3*f*h^2 + 3*a^2*c^3*e^3*f^2*h - b^3*c^2*e^3*f^2*g - a^2*c^3*d^
3*g*h^2 - a^2*c^3*e^3*f*g^2 - 3*a^3*c^2*d^2*e*h^3 + 3*a^2*c^3*d*e^2*g^3 - a
^2*b*c^2*e^3*g^3 - 3*b*c^4*d*e^2*f^3 + a^2*b^2*c*e^3*g^2*h + a^3*c^2*e^3*g^
2*h + b^3*c^2*d^3*f*h^2 + a^2*b*c^2*d^3*h^3 + b^4*c*e^3*f^2*h + b*c^4*d^3*f
^2*h + b*c^4*d^3*f*g^2 - c^5*d^3*f^2*g + 3*c^5*d^2*e*f^3 - a*c^4*e^3*f^3 -
a*c^4*d^3*g^3 + b^2*c^3*e^3*f^3 + a^4*c*e^3*h^3, z, k), k, 1, 3) - ((a*e^4*
f + c*d^4*h + a*d*e^3*g - 3*b*d*e^3*f + b*d^3*e*h - 3*c*d^3*e*g - 3*a*d^2*e
^2*h + b*d^2*e^2*g + 5*c*d^2*e^2*f)/(2*e*(a^2*e^4 + c^2*d^4 + b^2*d^2*e^2 -
 2*a*b*d*e^3 - 2*b*c*d^3*e + 2*a*c*d^2*e^2)) + (x*(a*e^3*g - b*e^3*f - 2*a*
d*e^2*h + 2*c*d*e^2*f + b*d^2*e*h - c*d^2*e*g))/(a^2*e^4 + c^2*d^4 + b^2*d^
2*e^2 - 2*a*b*d*e^3 - 2*b*c*d^3*e + 2*a*c*d^2*e^2))/(d^2 + e^2*x^2 + 2*d*e*
x)
```

3.155 $\int \frac{(d+ex)^2(f+gx+hx^2)}{(a+bx+cx^2)^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 288

$$\int \frac{(d+ex)^2\left(f+gx+hx^2\right)}{\left(a+bx+cx^2\right)^2}\,dx=\frac{e^2(2c^2f-bcg+2b^2h-6ach)\,x}{c^2\left(b^2-4ac\right)}$$
$$+\frac{(d+ex)^2\left(c\left(2ag-b\left(f+\frac{ah}{c}\right)\right)-\left(2c^2f-bcg+b^2h-2ach\right)x\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$
$$+\frac{(4c^4d^2f-2b^4e^2h-6ac^2e(beg+2bdh+2aeh)+b^2ce(beg+2bdh+12aeh)-c^3(2bd(2ef+dg)-4a(e}{c^3\left(b^2-4ac\right)^{3/2}}$$
$$+\frac{e(ceg+2cdh-2beh)\log\left(a+bx+cx^2\right)}{2c^3}$$

```
[Out] e^2*(2*c^2*f+2*b^2*h-c*(6*a*h+b*g))*x/c^2/(-4*a*c+b^2)+(e*x+d)^2*(c*(2*a*g-
b*(f+a*h/c))-(-2*a*c*h+b^2*h-b*c*g+2*c^2*f)*x)/c/(-4*a*c+b^2)/(c*x^2+b*x+a)
+(4*c^4*d^2*f-2*b^4*e^2*h-6*a*c^2*e*(2*a*e*h+2*b*d*h+b*e*g)+b^2*c*e*(12*a*e
*h+2*b*d*h+b*e*g)-c^3*(2*b*d*(d*g+2*e*f)-4*a*(d^2*h+2*d*e*g+e^2*f)))*arctan
h((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c^3/(-4*a*c+b^2)^(3/2)+1/2*e*(-2*b*e*h+2*c*
d*h+c*e*g)*ln(c*x^2+b*x+a)/c^3
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1658, 787, 648, 632, 212, 642}

$$\int \frac{(d+ex)^2 (f+gx+hx^2)}{(a+bx+cx^2)^2}\,dx$$

$$= \frac{\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)(b^2ce(12aeh+2bdh+beg)-c^3(2bd(dg+2ef)-4a(d^2h+2deg+e^2f))-6ac^2e(2aeh -}{c^3(b^2-4ac)^{3/2}}$$

$$+\frac{(d+ex)^2\left(c\left(2ag-b\left(\frac{ah}{c}+f\right)\right)-x(-2ach+b^2h-bcg+2c^2f)\right)}{c(b^2-4ac)(a+bx+cx^2)}$$

$$+\frac{e^2x(-6ach+2b^2h-bcg+2c^2f)}{c^2(b^2-4ac)}+\frac{e\log(a+bx+cx^2)(-2beh+2cdh+ceg)}{2c^3}$$

```
[In] Int[((d + e*x)^2*(f + g*x + h*x^2))/(a + b*x + c*x^2)^2,x]
```

```
[Out] (e^2*(2*c^2*f - b*c*g + 2*b^2*h - 6*a*c*h)*x)/(c^2*(b^2 - 4*a*c)) + ((d + e
*x)^2*(c*(2*a*g - b*(f + (a*h)/c)) - (2*c^2*f - b*c*g + b^2*h - 2*a*c*h)*x)
)/(c*(b^2 - 4*a*c)*(a + b*x + c*x^2)) + ((4*c^4*d^2*f - 2*b^4*e^2*h - 6*a*c
^2*e*(b*e*g + 2*b*d*h + 2*a*e*h) + b^2*c*e*(b*e*g + 2*b*d*h + 12*a*e*h) - c
^3*(2*b*d*(2*e*f + d*g) - 4*a*(e^2*f + 2*d*e*g + d^2*h)))*ArcTanh[(b + 2*c*
x)/Sqrt[b^2 - 4*a*c]])/(c^3*(b^2 - 4*a*c)^(3/2)) + (e*(c*e*g + 2*c*d*h - 2*
b*e*h)*Log[a + b*x + c*x^2])/(2*c^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 787

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[e*g*(x/c), x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1658

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f =
Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[Polynom
ialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x + c
*x^2)^(p + 1)*((f*b - 2*a*g + (2*c*f - b*g)*x)/((p + 1)*(b^2 - 4*a*c))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Q + g*(2*a*e*m + b*d*(2
*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(m + 2*p + 3)*x, x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Integer
Q[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] && Ra
tionalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\text{integral} = \frac{(d+ex)^2 \left(c\left(2ag-b\left(f+\frac{ah}{c}\right)\right)-\left(2c^2f-bcg+b^2h-2ach\right)x\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \frac{\int \frac{(d+ex)\left(2cdf-2bef-bdg+4aeg+2adh-\frac{2abeh}{c}-e\left(2cf-bg-6ah+\frac{2b^2h}{c}\right)x\right)}{a+bx+cx^2}\,dx}{-b^2+4ac}$$

$$= \frac{e^2\left(2c^2f-bcg+2b^2h-6ach\right)x}{c^2\left(b^2-4ac\right)} + \frac{(d+ex)^2 \left(c\left(2ag-b\left(f+\frac{ah}{c}\right)\right)-\left(2c^2f-bcg+b^2h-2ach\right)x\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)} - \frac{\int \frac{ae^2 (2cf-bg-6ah+\frac{2b^2h}{c})+cd (2cdf-2bef-bdg+4aeg+2adh-\frac{2abeh}{c})+ (-cde (2cf-bg-6ah+\frac{2b^2h}{c})+be^2 (2cf-bg-6ah+\frac{2b^2h}{c}}{a+bx+cx^2}}{c\left(b^2-4ac\right)}$$

$$= \frac{e^2(2c^2f - bcg + 2b^2h - 6ach)\,x}{c^2\,(b^2-4ac)} + \frac{(d+ex)^2\,\left(c\left(2ag - b\left(f + \frac{ah}{c}\right)\right) - (2c^2f - bcg + b^2h - 2ach)\,x\right)}{c\,(b^2-4ac)\,(a+bx+cx^2)} + \frac{(e(ceg + 2cdh - 2beh))\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2c^3} - \frac{(4c^4d^2f - 2b^4e^2h - 6ac^2e(beg + 2bdh + 2aeh) + b^2ce(beg + 2bdh + 12aeh) - c^3(2bd(2ef + dg) -}{2c^3\,(b^2-4ac)}$$

$$= \frac{e^2(2c^2f - bcg + 2b^2h - 6ach)\,x}{c^2\,(b^2-4ac)} + \frac{(d+ex)^2\,\left(c\left(2ag - b\left(f + \frac{ah}{c}\right)\right) - (2c^2f - bcg + b^2h - 2ach)\,x\right)}{c\,(b^2-4ac)\,(a+bx+cx^2)} + \frac{e(ceg + 2cdh - 2beh)\log\left(a+bx+cx^2\right)}{2c^3} + \frac{(4c^4d^2f - 2b^4e^2h - 6ac^2e(beg + 2bdh + 2aeh) + b^2ce(beg + 2bdh + 12aeh) - c^3(2bd(2ef + dg) -}{c^3\,(b^2-4ac)}$$

$$= \frac{e^2(2c^2f - bcg + 2b^2h - 6ach)\,x}{c^2\,(b^2-4ac)} + \frac{(d+ex)^2\,\left(c\left(2ag - b\left(f + \frac{ah}{c}\right)\right) - (2c^2f - bcg + b^2h - 2ach)\,x\right)}{c\,(b^2-4ac)\,(a+bx+cx^2)} + \frac{(4c^4d^2f - 2b^4e^2h - 6ac^2e(beg + 2bdh + 2aeh) + b^2ce(beg + 2bdh + 12aeh) - c^3(2bd(2ef + dg) -}{c^3\,(b^2-4ac)^{3/2}} + \frac{e(ceg + 2cdh - 2beh)\log\left(a+bx+cx^2\right)}{2c^3}$$

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.38

$$\int \frac{(d+ex)^2\,(f+gx+hx^2)}{(a+bx+cx^2)^2}\,dx$$

$$= \frac{2ce^2hx - \frac{2(b^4e^2hx + b^3e(aeh - c(eg+2dh)x) + b^2c(c(e^2f + 2deg + d^2h)x - ae(eg + 2dh + 4ehx)) + 2c^2(c^2d^2fx - ac(e^2fx + 2de(f+gx) + d^2(g+hx)) +}{(b^2-4ac)(a+x(b+cx))}}{}$$

```
[In] Integrate[((d + e*x)^2*(f + g*x + h*x^2))/(a + b*x + c*x^2)^2,x]
```

```
[Out] (2*c*e^2*h*x - (2*(b^4*e^2*h*x + b^3*e*(a*e*h - c*(e*g + 2*d*h)*x) + b^2*c*
(c*(e^2*f + 2*d*e*g + d^2*h)*x - a*e*(e*g + 2*d*h + 4*e*h*x)) + 2*c^2*(c^2*
d^2*f*x - a*c*(e^2*f*x + 2*d*e*(f + g*x) + d^2*(g + h*x)) + a^2*e*(2*d*h +
e*(g + h*x))) + b*c*(-3*a^2*e^2*h + c^2*d*(-2*e*f*x + d*(f - g*x)) + a*c*(d
```

```
^2*h + e^2*(f + 3*g*x) + 2*d*e*(g + 3*h*x)))))/((b^2 - 4*a*c)*(a + x*(b + c
*x))) + (2*(4*c^4*d^2*f - 2*b^4*e^2*h - 6*a*c^2*e*(b*e*g + 2*b*d*h + 2*a*e*
h) + b^2*c*e*(b*e*g + 2*b*d*h + 12*a*e*h) + c^3*(-2*b*d*(2*e*f + d*g) + 4*a
*(e^2*f + 2*d*e*g + d^2*h)))*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2
+ 4*a*c)^(3/2) + e*(c*e*g + 2*c*d*h - 2*b*e*h)*Log[a + x*(b + c*x)])/(2*c^3
)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 627 vs. $2(283) = 566$.

Time = 0.77 (sec) , antiderivative size = 628, normalized size of antiderivative = 2.18

method	result
default	$\frac{h\,e^{2}x}{c^{2}}-\frac{-\frac{\left(2a^{2}c^{2}e^{2}h-4a\,b^{2}c\,e^{2}h+6ab\,c^{2}deh+3ab\,c^{2}e^{2}g-2a\,c^{3}d^{2}h-4a\,c^{3}deg-2a\,c^{3}e^{2}f+b^{4}e^{2}h-2b^{3}cdeh-b^{3}c\,e^{2}g+b^{2}c^{2}d^{2}h+2b^{2}c^{2}deg+b^{2}c^{2}\right.}{c\left(4ac-b^{2}\right)}}{}$
risch	Expression too large to display

```
[In] int((e*x+d)^2*(h*x^2+g*x+f)/(c*x^2+b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] h*e^2/c^2*x-1/c^2*((-(2*a^2*c^2*e^2*h-4*a*b^2*c*e^2*h+6*a*b*c^2*d*e*h+3*a*b
*c^2*e^2*g-2*a*c^3*d^2*h-4*a*c^3*d*e*g-2*a*c^3*e^2*f+b^4*e^2*h-2*b^3*c*d*e*
h-b^3*c*e^2*g+b^2*c^2*d^2*h+2*b^2*c^2*d*e*g+b^2*c^2*e^2*f-b*c^3*d^2*g-2*b*c
^3*d*e*f+2*c^4*d^2*f)/c/(4*a*c-b^2)*x+(3*a^2*b*c*e^2*h-4*a^2*c^2*d*e*h-2*a^
2*c^2*e^2*g-a*b^3*e^2*h+2*a*b^2*c*d*e*h+a*b^2*c*e^2*g-a*b*c^2*d^2*h-2*a*b*c
^2*d*e*g-a*b*c^2*e^2*f+2*a*c^3*d^2*g+4*a*c^3*d*e*f-b*c^3*d^2*f)/c/(4*a*c-b^
2))/(c*x^2+b*x+a)+1/(4*a*c-b^2)*(1/2*(8*a*b*c*e^2*h-8*a*c^2*d*e*h-4*a*c^2*e
^2*g-2*b^3*e^2*h+2*b^2*c*d*e*h+b^2*c*e^2*g)/c*ln(c*x^2+b*x+a)+2*(6*a^2*c*e^
2*h-2*a*b^2*e^2*h+2*a*b*c*d*e*h+a*b*c*e^2*g-2*a*c^2*d^2*h-4*a*c^2*d*e*g-2*a
*c^2*e^2*f+b*c^2*d^2*g+2*b*c^2*d*e*f-2*c^3*d^2*f-1/2*(8*a*b*c*e^2*h-8*a*c^2
*d*e*h-4*a*c^2*e^2*g-2*b^3*e^2*h+2*b^2*c*d*e*h+b^2*c*e^2*g)*b/c)/(4*a*c-b^2
)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1376 vs. $2(281) = 562$.

Time = 0.55 (sec) , antiderivative size = 2771, normalized size of antiderivative = 9.62

$$\int \frac{(d+ex)^2\left(f+gx+hx^2\right)}{\left(a+bx+cx^2\right)^2}\,dx=\text{Too large to display}$$

```
[In] integrate((e*x+d)^2*(h*x^2+g*x+f)/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*e^2*h*x^3 + 2*(b^5*c - 8*a*b^3
*c^2 + 16*a^2*b*c^3)*e^2*h*x^2 + ((4*(c^5*d^2 - b*c^4*d*e + a*c^4*e^2)*f -
(2*b*c^4*d^2 - 8*a*c^4*d*e - (b^3*c^2 - 6*a*b*c^3)*e^2)*g + 2*(2*a*c^4*d^2
+ (b^3*c^2 - 6*a*b*c^3)*d*e - (b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*e^2)*h)*x^2
 + 4*(a*c^4*d^2 - a*b*c^3*d*e + a^2*c^3*e^2)*f - (2*a*b*c^3*d^2 - 8*a^2*c^3
*d*e - (a*b^3*c - 6*a^2*b*c^2)*e^2)*g + 2*(2*a^2*c^3*d^2 + (a*b^3*c - 6*a^2
*b*c^2)*d*e - (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*e^2)*h + (4*(b*c^4*d^2 - b^
2*c^3*d*e + a*b*c^3*e^2)*f - (2*b^2*c^3*d^2 - 8*a*b*c^3*d*e - (b^4*c - 6*a*
b^2*c^2)*e^2)*g + 2*(2*a*b*c^3*d^2 + (b^4*c - 6*a*b^2*c^2)*d*e - (b^5 - 6*a
*b^3*c + 6*a^2*b*c^2)*e^2)*h)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x
 + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - 2*((b^
3*c^3 - 4*a*b*c^4)*d^2 - 4*(a*b^2*c^3 - 4*a^2*c^4)*d*e + (a*b^3*c^2 - 4*a^2
*b*c^3)*e^2)*f + 2*(2*(a*b^2*c^3 - 4*a^2*c^4)*d^2 - 2*(a*b^3*c^2 - 4*a^2*b*
c^3)*d*e + (a*b^4*c - 6*a^2*b^2*c^2 + 8*a^3*c^3)*e^2)*g - 2*((a*b^3*c^2 - 4
*a^2*b*c^3)*d^2 - 2*(a*b^4*c - 6*a^2*b^2*c^2 + 8*a^3*c^3)*d*e + (a*b^5 - 7*
a^2*b^3*c + 12*a^3*b*c^2)*e^2)*h - 2*((2*(b^2*c^4 - 4*a*c^5)*d^2 - 2*(b^3*c
^3 - 4*a*b*c^4)*d*e + (b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*e^2)*f - ((b^3*c^
3 - 4*a*b*c^4)*d^2 - 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d*e + (b^5*c - 7
*a*b^3*c^2 + 12*a^2*b*c^3)*e^2)*g + ((b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^
2 - 2*(b^5*c - 7*a*b^3*c^2 + 12*a^2*b*c^3)*d*e + (b^6 - 9*a*b^4*c + 26*a^2*
b^2*c^2 - 24*a^3*c^3)*e^2)*h)*x + ((a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)*e
^2*g + ((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*e^2*g + 2*((b^4*c^2 - 8*a*b^2*
c^3 + 16*a^2*c^4)*d*e - (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*e^2)*h)*x^2 +
2*((a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)*d*e - (a*b^5 - 8*a^2*b^3*c + 16*a
^3*b*c^2)*e^2)*h + ((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*e^2*g + 2*((b^5*c
- 8*a*b^3*c^2 + 16*a^2*b*c^3)*d*e - (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*e^2)
*h)*x)*log(c*x^2 + b*x + a))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5 + (b^4
*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^2 + (b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^
5)*x), 1/2*(2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*e^2*h*x^3 + 2*(b^5*c - 8
*a*b^3*c^2 + 16*a^2*b*c^3)*e^2*h*x^2 + 2*((4*(c^5*d^2 - b*c^4*d*e + a*c^4*e
^2)*f - (2*b*c^4*d^2 - 8*a*c^4*d*e - (b^3*c^2 - 6*a*b*c^3)*e^2)*g + 2*(2*a*
c^4*d^2 + (b^3*c^2 - 6*a*b*c^3)*d*e - (b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*e^2
)*h)*x^2 + 4*(a*c^4*d^2 - a*b*c^3*d*e + a^2*c^3*e^2)*f - (2*a*b*c^3*d^2 - 8
*a^2*c^3*d*e - (a*b^3*c - 6*a^2*b*c^2)*e^2)*g + 2*(2*a^2*c^3*d^2 + (a*b^3*c
 - 6*a^2*b*c^2)*d*e - (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*e^2)*h + (4*(b*c^4*
d^2 - b^2*c^3*d*e + a*b*c^3*e^2)*f - (2*b^2*c^3*d^2 - 8*a*b*c^3*d*e - (b^4*
c - 6*a*b^2*c^2)*e^2)*g + 2*(2*a*b*c^3*d^2 + (b^4*c - 6*a*b^2*c^2)*d*e - (b
^5 - 6*a*b^3*c + 6*a^2*b*c^2)*e^2)*h)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b
^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - 2*((b^3*c^3 - 4*a*b*c^4)*d^2 - 4*(
a*b^2*c^3 - 4*a^2*c^4)*d*e + (a*b^3*c^2 - 4*a^2*b*c^3)*e^2)*f + 2*(2*(a*b^2
*c^3 - 4*a^2*c^4)*d^2 - 2*(a*b^3*c^2 - 4*a^2*b*c^3)*d*e + (a*b^4*c - 6*a^2*
b^2*c^2 + 8*a^3*c^3)*e^2)*g - 2*((a*b^3*c^2 - 4*a^2*b*c^3)*d^2 - 2*(a*b^4*c
 - 6*a^2*b^2*c^2 + 8*a^3*c^3)*d*e + (a*b^5 - 7*a^2*b^3*c + 12*a^3*b*c^2)*e^
2)*h - 2*((2*(b^2*c^4 - 4*a*c^5)*d^2 - 2*(b^3*c^3 - 4*a*b*c^4)*d*e + (b^4*c
^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*e^2)*f - ((b^3*c^3 - 4*a*b*c^4)*d^2 - 2*(b^4*
```

```
c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d*e + (b^5*c - 7*a*b^3*c^2 + 12*a^2*b*c^3)*e
^2)*g + ((b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^2 - 2*(b^5*c - 7*a*b^3*c^2 +
 12*a^2*b*c^3)*d*e + (b^6 - 9*a*b^4*c + 26*a^2*b^2*c^2 - 24*a^3*c^3)*e^2)*h
)*x + ((a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)*e^2*g + ((b^4*c^2 - 8*a*b^2*c
^3 + 16*a^2*c^4)*e^2*g + 2*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d*e - (b^5
*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*e^2)*h)*x^2 + 2*((a*b^4*c - 8*a^2*b^2*c^2
+ 16*a^3*c^3)*d*e - (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*e^2)*h + ((b^5*c -
 8*a*b^3*c^2 + 16*a^2*b*c^3)*e^2*g + 2*((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3
)*d*e - (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*e^2)*h)*x)*log(c*x^2 + b*x + a))
/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5 + (b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*
c^6)*x^2 + (b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2966 vs. 2(291) = 582.

Time = 156.09 (sec) , antiderivative size = 2966, normalized size of antiderivative = 10.30

$$\int \frac{(d+ex)^2\,(f+gx+hx^2)}{(a+bx+cx^2)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)**2*(h*x**2+g*x+f)/(c*x**2+b*x+a)**2,x)
```

```
[Out] (-e*(2*b*e*h - 2*c*d*h - c*e*g)/(2*c**3) - sqrt(-(4*a*c - b**2)**3)*(12*a**
2*c**2*e**2*h - 12*a*b**2*c*e**2*h + 12*a*b*c**2*d*e*h + 6*a*b*c**2*e**2*g
- 4*a*c**3*d**2*h - 8*a*c**3*d*e*g - 4*a*c**3*e**2*f + 2*b**4*e**2*h - 2*b*
*3*c*d*e*h - b**3*c*e**2*g + 2*b*c**3*d**2*g + 4*b*c**3*d*e*f - 4*c**4*d**2
*f)/(2*c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))*log(x
 + (-10*a**2*b*c*e**2*h - 16*a**2*c**4*(-e*(2*b*e*h - 2*c*d*h - c*e*g)/(2*c
**3) - sqrt(-(4*a*c - b**2)**3)*(12*a**2*c**2*e**2*h - 12*a*b**2*c*e**2*h +
 12*a*b*c**2*d*e*h + 6*a*b*c**2*e**2*g - 4*a*c**3*d**2*h - 8*a*c**3*d*e*g -
 4*a*c**3*e**2*f + 2*b**4*e**2*h - 2*b**3*c*d*e*h - b**3*c*e**2*g + 2*b*c**
3*d**2*g + 4*b*c**3*d*e*f - 4*c**4*d**2*f)/(2*c**3*(64*a**3*c**3 - 48*a**2*
b**2*c**2 + 12*a*b**4*c - b**6))) + 16*a**2*c**2*d*e*h + 8*a**2*c**2*e**2*g
 + 2*a*b**3*e**2*h + 8*a*b**2*c**3*(-e*(2*b*e*h - 2*c*d*h - c*e*g)/(2*c**3)
 - sqrt(-(4*a*c - b**2)**3)*(12*a**2*c**2*e**2*h - 12*a*b**2*c*e**2*h + 12*
a*b*c**2*d*e*h + 6*a*b*c**2*e**2*g - 4*a*c**3*d**2*h - 8*a*c**3*d*e*g - 4*a
*c**3*e**2*f + 2*b**4*e**2*h - 2*b**3*c*d*e*h - b**3*c*e**2*g + 2*b*c**3*d*
*2*g + 4*b*c**3*d*e*f - 4*c**4*d**2*f)/(2*c**3*(64*a**3*c**3 - 48*a**2*b**2
*c**2 + 12*a*b**4*c - b**6))) - 2*a*b**2*c*d*e*h - a*b**2*c*e**2*g - 2*a*b*
c**2*d**2*h - 4*a*b*c**2*d*e*g - 2*a*b*c**2*e**2*f - b**4*c**2*(-e*(2*b*e*h
 - 2*c*d*h - c*e*g)/(2*c**3) - sqrt(-(4*a*c - b**2)**3)*(12*a**2*c**2*e**2*
h - 12*a*b**2*c*e**2*h + 12*a*b*c**2*d*e*h + 6*a*b*c**2*e**2*g - 4*a*c**3*d
**2*h - 8*a*c**3*d*e*g - 4*a*c**3*e**2*f + 2*b**4*e**2*h - 2*b**3*c*d*e*h -
 b**3*c*e**2*g + 2*b*c**3*d**2*g + 4*b*c**3*d*e*f - 4*c**4*d**2*f)/(2*c**3*
```

```
(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) + b**2*c**2*d**2*
g + 2*b**2*c**2*d*e*f - 2*b*c**3*d**2*f)/(12*a**2*c**2*e**2*h - 12*a*b**2*c
*e**2*h + 12*a*b*c**2*d*e*h + 6*a*b*c**2*e**2*g - 4*a*c**3*d**2*h - 8*a*c**
3*d*e*g - 4*a*c**3*e**2*f + 2*b**4*e**2*h - 2*b**3*c*d*e*h - b**3*c*e**2*g
+ 2*b*c**3*d**2*g + 4*b*c**3*d*e*f - 4*c**4*d**2*f)) + (-e*(2*b*e*h - 2*c*d
*h - c*e*g)/(2*c**3) + sqrt(-(4*a*c - b**2)**3)*(12*a**2*c**2*e**2*h - 12*a
*b**2*c*e**2*h + 12*a*b*c**2*d*e*h + 6*a*b*c**2*e**2*g - 4*a*c**3*d**2*h -
8*a*c**3*d*e*g - 4*a*c**3*e**2*f + 2*b**4*e**2*h - 2*b**3*c*d*e*h - b**3*c*
e**2*g + 2*b*c**3*d**2*g + 4*b*c**3*d*e*f - 4*c**4*d**2*f)/(2*c**3*(64*a**3
*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))*log(x + (-10*a**2*b*c*e**
2*h - 16*a**2*c**4*(-e*(2*b*e*h - 2*c*d*h - c*e*g)/(2*c**3) + sqrt(-(4*a*c
- b**2)**3)*(12*a**2*c**2*e**2*h - 12*a*b**2*c*e**2*h + 12*a*b*c**2*d*e*h +
 6*a*b*c**2*e**2*g - 4*a*c**3*d**2*h - 8*a*c**3*d*e*g - 4*a*c**3*e**2*f + 2
*b**4*e**2*h - 2*b**3*c*d*e*h - b**3*c*e**2*g + 2*b*c**3*d**2*g + 4*b*c**3*
d*e*f - 4*c**4*d**2*f)/(2*c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**
4*c - b**6))) + 16*a**2*c**2*d*e*h + 8*a**2*c**2*e**2*g + 2*a*b**3*e**2*h +
 8*a*b**2*c**3*(-e*(2*b*e*h - 2*c*d*h - c*e*g)/(2*c**3) + sqrt(-(4*a*c - b*
*2)**3)*(12*a**2*c**2*e**2*h - 12*a*b**2*c*e**2*h + 12*a*b*c**2*d*e*h + 6*a
*b*c**2*e**2*g - 4*a*c**3*d**2*h - 8*a*c**3*d*e*g - 4*a*c**3*e**2*f + 2*b**
4*e**2*h - 2*b**3*c*d*e*h - b**3*c*e**2*g + 2*b*c**3*d**2*g + 4*b*c**3*d*e*
f - 4*c**4*d**2*f)/(2*c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c
- b**6))) - 2*a*b**2*c*d*e*h - a*b**2*c*e**2*g - 2*a*b*c**2*d**2*h - 4*a*b*
c**2*d*e*g - 2*a*b*c**2*e**2*f - b**4*c**2*(-e*(2*b*e*h - 2*c*d*h - c*e*g)/
(2*c**3) + sqrt(-(4*a*c - b**2)**3)*(12*a**2*c**2*e**2*h - 12*a*b**2*c*e**2
*h + 12*a*b*c**2*d*e*h + 6*a*b*c**2*e**2*g - 4*a*c**3*d**2*h - 8*a*c**3*d*e
*g - 4*a*c**3*e**2*f + 2*b**4*e**2*h - 2*b**3*c*d*e*h - b**3*c*e**2*g + 2*b
*c**3*d**2*g + 4*b*c**3*d*e*f - 4*c**4*d**2*f)/(2*c**3*(64*a**3*c**3 - 48*a
**2*b**2*c**2 + 12*a*b**4*c - b**6))) + b**2*c**2*d**2*g + 2*b**2*c**2*d*e*
f - 2*b*c**3*d**2*f)/(12*a**2*c**2*e**2*h - 12*a*b**2*c*e**2*h + 12*a*b*c**
2*d*e*h + 6*a*b*c**2*e**2*g - 4*a*c**3*d**2*h - 8*a*c**3*d*e*g - 4*a*c**3*e
**2*f + 2*b**4*e**2*h - 2*b**3*c*d*e*h - b**3*c*e**2*g + 2*b*c**3*d**2*g +
4*b*c**3*d*e*f - 4*c**4*d**2*f)) + (-3*a**2*b*c*e**2*h + 4*a**2*c**2*d*e*h
+ 2*a**2*c**2*e**2*g + a*b**3*e**2*h - 2*a*b**2*c*d*e*h - a*b**2*c*e**2*g +
 a*b*c**2*d**2*h + 2*a*b*c**2*d*e*g + a*b*c**2*e**2*f - 2*a*c**3*d**2*g - 4
*a*c**3*d*e*f + b*c**3*d**2*f + x*(2*a**2*c**2*e**2*h - 4*a*b**2*c*e**2*h +
 6*a*b*c**2*d*e*h + 3*a*b*c**2*e**2*g - 2*a*c**3*d**2*h - 4*a*c**3*d*e*g -
2*a*c**3*e**2*f + b**4*e**2*h - 2*b**3*c*d*e*h - b**3*c*e**2*g + b**2*c**2*
d**2*h + 2*b**2*c**2*d*e*g + b**2*c**2*e**2*f - b*c**3*d**2*g - 2*b*c**3*d*
e*f + 2*c**4*d**2*f))/(4*a**2*c**4 - a*b**2*c**3 + x**2*(4*a*c**5 - b**2*c*
*4) + x*(4*a*b*c**4 - b**3*c**3)) + e**2*h*x/c**2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^2\,(f+gx+hx^2)}{(a+bx+cx^2)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*x+d)^2*(h*x^2+g*x+f)/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 547, normalized size of antiderivative = 1.90

$$\int \frac{(d+ex)^2\,(f+gx+hx^2)}{(a+bx+cx^2)^2}\,dx = \frac{e^2hx}{c^2} - \frac{(4\,c^4d^2f - 4\,bc^3def + 4\,ac^3e^2f - 2\,bc^3d^2g + 8\,ac^3deg + b^3ce^2g - 6\,abc^2e^2g + 4\,ac^3d^2h + 2\,b^3cdeh - 12\,a}{(b^2c^3 - 4\,ac^4)\sqrt{-b^2+4\,ac}} + \frac{(ce^2g + 2\,cdeh - 2\,be^2h)\log\left(cx^2+bx+a\right)}{2\,c^3} - \frac{\frac{(2\,c^4d^2f-2\,bc^3def+b^2c^2e^2f-2\,ac^3e^2f-bc^3d^2g+2\,b^2c^2deg-4\,ac^3deg-b^3ce^2g+3\,abc^2e^2g+b^2c^2d^2h-2\,ac^3d^2h-2\,b^3cdeh+6\,abc^2deh+b^4e^2h-4}{c}}{(cx^2+bx}$$

```
[In] integrate((e*x+d)^2*(h*x^2+g*x+f)/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] e^2*h*x/c^2 - (4*c^4*d^2*f - 4*b*c^3*d*e*f + 4*a*c^3*e^2*f - 2*b*c^3*d^2*g
+ 8*a*c^3*d*e*g + b^3*c*e^2*g - 6*a*b*c^2*e^2*g + 4*a*c^3*d^2*h + 2*b^3*c*d
*e*h - 12*a*b*c^2*d*e*h - 2*b^4*e^2*h + 12*a*b^2*c*e^2*h - 12*a^2*c^2*e^2*h
)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2*c^3 - 4*a*c^4)*sqrt(-b^2 + 4
*a*c)) + 1/2*(c*e^2*g + 2*c*d*e*h - 2*b*e^2*h)*log(c*x^2 + b*x + a)/c^3 - (
(2*c^4*d^2*f - 2*b*c^3*d*e*f + b^2*c^2*e^2*f - 2*a*c^3*e^2*f - b*c^3*d^2*g
+ 2*b^2*c^2*d*e*g - 4*a*c^3*d*e*g - b^3*c*e^2*g + 3*a*b*c^2*e^2*g + b^2*c^2
*d^2*h - 2*a*c^3*d^2*h - 2*b^3*c*d*e*h + 6*a*b*c^2*d*e*h + b^4*e^2*h - 4*a*
b^2*c*e^2*h + 2*a^2*c^2*e^2*h)*x/c + (b*c^3*d^2*f - 4*a*c^3*d*e*f + a*b*c^2
*e^2*f - 2*a*c^3*d^2*g + 2*a*b*c^2*d*e*g - a*b^2*c*e^2*g + 2*a^2*c^2*e^2*g
+ a*b*c^2*d^2*h - 2*a*b^2*c*d*e*h + 4*a^2*c^2*d*e*h + a*b^3*e^2*h - 3*a^2*b
*c*e^2*h)/c)/((c*x^2 + b*x + a)*(b^2 - 4*a*c)*c^2)
```

Mupad [B] (verification not implemented)

Time = 15.07 (sec) , antiderivative size = 742, normalized size of antiderivative = 2.58

$$\int \frac{(d+ex)^2\left(f+gx+hx^2\right)}{\left(a+bx+cx^2\right)^2}\,dx$$

$$= \frac{\frac{-3\,h\,a^2\,b\,c\,e^2+4\,h\,a^2\,c^2\,d\,e+2\,g\,a^2\,c^2\,e^2+h\,a\,b^3\,e^2-2\,h\,a\,b^2\,c\,d\,e-g\,a\,b^2\,c\,e^2+h\,a\,b\,c^2\,d^2+2\,g\,a\,b\,c^2\,d\,e+f\,a\,b\,c^2\,e^2-2\,g\,a\,c^3\,d^2-4\,f\,a\,c^3\,d\,e+f\,b\,c^3\,d^2}{c\,(4\,a\,c-b^2)} +}{}$$

$$+ \frac{\ln\left(c\,x^2+b\,x+a\right)\,\left(-128\,h\,a^3\,b\,c^3\,e^2+64\,g\,a^3\,c^4\,e^2+128\,d\,h\,a^3\,c^4\,e+96\,h\,a^2\,b^3\,c^2\,e^2-48\,g\,a^2\,b^2\,c^3\,e^2-96\right.}{2\,\left(64\,a^3\,c^6-48\,a^2\,b^2\,c^5+\right.}$$

$$+ \frac{\mathrm{atan}\left(\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}-\frac{b^3\,c^2-4\,a\,b\,c^3}{c^2\,\left(4\,a\,c-b^2\right)^{3/2}}\right)\,\left(-12\,h\,a^2\,c^2\,e^2+12\,h\,a\,b^2\,c\,e^2-12\,h\,a\,b\,c^2\,d\,e-6\,g\,a\,b\,c^2\,e^2+4\,h\,a\,c^3\,d^2+\right.}{c^3\,\left(4\,a\,c-b^2\right)}$$

$$+ \frac{e^2\,h\,x}{c^2}$$

```
[In] int(((d + e*x)^2*(f + g*x + h*x^2))/(a + b*x + c*x^2)^2,x)
```

```
[Out] ((2*a^2*c^2*e^2*g - 2*a*c^3*d^2*g + b*c^3*d^2*f + a*b^3*e^2*h + a*b*c^2*e^2
*f + a*b*c^2*d^2*h - a*b^2*c*e^2*g - 3*a^2*b*c*e^2*h + 4*a^2*c^2*d*e*h - 4*
a*c^3*d*e*f + 2*a*b*c^2*d*e*g - 2*a*b^2*c*d*e*h)/(c*(4*a*c - b^2)) + (x*(2*
c^4*d^2*f + b^4*e^2*h + b^2*c^2*e^2*f + 2*a^2*c^2*e^2*h + b^2*c^2*d^2*h - 2
*a*c^3*e^2*f - 2*a*c^3*d^2*h - b*c^3*d^2*g - b^3*c*e^2*g + 3*a*b*c^2*e^2*g
- 4*a*b^2*c*e^2*h + 2*b^2*c^2*d*e*g - 4*a*c^3*d*e*g - 2*b*c^3*d*e*f - 2*b^3
*c*d*e*h + 6*a*b*c^2*d*e*h))/(c*(4*a*c - b^2)))/(a*c^2 + c^3*x^2 + b*c^2*x)
 + (log(a + b*x + c*x^2)*(2*b^7*e^2*h + 64*a^3*c^4*e^2*g - b^6*c*e^2*g - 24
*a*b^5*c*e^2*h + 128*a^3*c^4*d*e*h + 12*a*b^4*c^2*e^2*g - 128*a^3*b*c^3*e^2
*h - 2*b^6*c*d*e*h - 48*a^2*b^2*c^3*e^2*g + 96*a^2*b^3*c^2*e^2*h + 24*a*b^4
*c^2*d*e*h - 96*a^2*b^2*c^3*d*e*h))/(2*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4
 - 48*a^2*b^2*c^5)) + (atan((2*c*x)/(4*a*c - b^2)^(1/2) - (b^3*c^2 - 4*a*b*
c^3)/(c^2*(4*a*c - b^2)^(3/2)))*(4*c^4*d^2*f - 2*b^4*e^2*h - 12*a^2*c^2*e^2
*h + 4*a*c^3*e^2*f + 4*a*c^3*d^2*h - 2*b*c^3*d^2*g + b^3*c*e^2*g - 6*a*b*c^
2*e^2*g + 12*a*b^2*c*e^2*h + 8*a*c^3*d*e*g - 4*b*c^3*d*e*f + 2*b^3*c*d*e*h
- 12*a*b*c^2*d*e*h))/(c^3*(4*a*c - b^2)^(3/2)) + (e^2*h*x)/c^2
```

3.156 $\int \frac{(d+ex)\left(f+gx+hx^2\right)}{\left(a+bx+cx^2\right)^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 178

$$\begin{aligned}\int \frac{(d+ex)\left(f+gx+hx^2\right)}{\left(a+bx+cx^2\right)^2}\,dx &= \frac{(d+ex)\left(c\left(2ag-b\left(f+\frac{ah}{c}\right)\right)-\left(2c^2f-bcg+b^2h-2ach\right)x\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}\\ &\quad+\frac{\left(4c^3df+b^3eh-6abceh-2c^2(b(ef+dg)-2a(eg+dh))\right)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\left(b^2-4ac\right)^{3/2}}\\ &\quad+\frac{eh\log\left(a+bx+cx^2\right)}{2c^2}\end{aligned}$$

```
[Out] (e*x+d)*(c*(2*a*g-b*(f+a*h/c))-(-2*a*c*h+b^2*h-b*c*g+2*c^2*f)*x)/c/(-4*a*c+
b^2)/(c*x^2+b*x+a)+(4*c^3*d*f+b^3*e*h-6*a*b*c*e*h-2*c^2*(b*(d*g+e*f)-2*a*(d
*h+e*g)))*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b^2)^(3/2)+1/2*
e*h*ln(c*x^2+b*x+a)/c^2
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used

$= \{1658, 648, 632, 212, 642\}$

$$\int \frac{(d+ex)\left(f+gx+hx^2\right)}{\left(a+bx+cx^2\right)^2}\,dx$$

$$= \frac{\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(-2c^2(b(dg+ef)-2a(dh+eg))-6abceh+b^3eh+4c^3df\right)}{c^2\left(b^2-4ac\right)^{3/2}}$$

$$+ \frac{(d+ex)\left(c\left(2ag-b\left(\frac{ah}{c}+f\right)\right)-x\left(-2ach+b^2h-bcg+2c^2f\right)\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

$$+ \frac{eh\log\left(a+bx+cx^2\right)}{2c^2}$$

```
[In] Int[((d + e*x)*(f + g*x + h*x^2))/(a + b*x + c*x^2)^2,x]
```

```
[Out] ((d + e*x)*(c*(2*a*g - b*(f + (a*h)/c)) - (2*c^2*f - b*c*g + b^2*h - 2*a*c*
h)*x))/(c*(b^2 - 4*a*c)*(a + b*x + c*x^2)) + ((4*c^3*d*f + b^3*e*h - 6*a*b*
c*e*h - 2*c^2*(b*(e*f + d*g) - 2*a*(e*g + d*h)))*ArcTanh[(b + 2*c*x)/Sqrt[b
^2 - 4*a*c]])/(c^2*(b^2 - 4*a*c)^(3/2)) + (e*h*Log[a + b*x + c*x^2])/(2*c^2
)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1658

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f =
Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[Polynom
ialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x + c
*x^2)^(p + 1)*((f*b - 2*a*g + (2*c*f - b*g)*x)/((p + 1)*(b^2 - 4*a*c))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Q + g*(2*a*e*m + b*d*(2
*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(m + 2*p + 3)*x, x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Integer
Q[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] && Ra
tionalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(d+ex)\left(c\left(2ag-b\left(f+\frac{ah}{c}\right)\right)-\left(2c^2f-bcg+b^2h-2ach\right)x\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)} \\
&\quad+\frac{\int\frac{2cdf-b(ef+dg)-\frac{abeh}{c}+2a(eg+dh)+\left(4a-\frac{b^2}{c}\right)ehx}{a+bx+cx^2}\,dx}{-b^2+4ac} \\
&= \frac{(d+ex)\left(c\left(2ag-b\left(f+\frac{ah}{c}\right)\right)-\left(2c^2f-bcg+b^2h-2ach\right)x\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{(eh)\int\frac{b+2cx}{a+bx+cx^2}\,dx}{2c^2} \\
&\quad-\frac{\left(4c^3df+b^3eh-6abceh-2c^2(b(ef+dg)-2a(eg+dh))\right)\int\frac{1}{a+bx+cx^2}\,dx}{2c^2\left(b^2-4ac\right)} \\
&= \frac{(d+ex)\left(c\left(2ag-b\left(f+\frac{ah}{c}\right)\right)-\left(2c^2f-bcg+b^2h-2ach\right)x\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{eh\log\left(a+bx+cx^2\right)}{2c^2} \\
&\quad+\frac{\left(4c^3df+b^3eh-6abceh-2c^2(b(ef+dg)-2a(eg+dh))\right)\text{Subst}\left(\int\frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx\right)}{c^2\left(b^2-4ac\right)} \\
&= \frac{(d+ex)\left(c\left(2ag-b\left(f+\frac{ah}{c}\right)\right)-\left(2c^2f-bcg+b^2h-2ach\right)x\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)} \\
&\quad+\frac{\left(4c^3df+b^3eh-6abceh-2c^2(b(ef+dg)-2a(eg+dh))\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\left(b^2-4ac\right)^{3/2}} \\
&\quad+\frac{eh\log\left(a+bx+cx^2\right)}{2c^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.26

$$\int \frac{(d+ex)\left(f+gx+hx^2\right)}{\left(a+bx+cx^2\right)^2}\,dx$$

$$= \frac{-\frac{2\left(-b^3ehx+b^2(-aeh+c(eg+dh)x)+bc(adh-cefx+cd(f-gx)+ae(g+3hx))+2c\left(a^2eh+c^2dfx-ac(e(f+gx)+d(g+hx))\right)\right)}{(b^2-4ac)(a+x(b+cx))}+\frac{2\left(4c^3df+b^3eh-6ab\right.}{}}{2c^2}$$

```
[In] Integrate[((d + e*x)*(f + g*x + h*x^2))/(a + b*x + c*x^2)^2,x]
```

```
[Out] ((-2*(-(b^3*e*h*x) + b^2*(-(a*e*h) + c*(e*g + d*h)*x) + b*c*(a*d*h - c*e*f*
x + c*d*(f - g*x) + a*e*(g + 3*h*x)) + 2*c*(a^2*e*h + c^2*d*f*x - a*c*(e*(f
 + g*x) + d*(g + h*x)))))/((b^2 - 4*a*c)*(a + x*(b + c*x))) + (2*(4*c^3*d*f
 + b^3*e*h - 6*a*b*c*e*h - 2*c^2*(b*(e*f + d*g) - 2*a*(e*g + d*h)))*ArcTan[
(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + e*h*Log[a + x*(b +
c*x)])/(2*c^2)
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.73

method	result
default	$\frac{\frac{\left(3abceh-2a\,c^2dh-2a\,c^2eg-b^3eh+b^2cdh+b^2ceg-b\,c^2dg-b\,c^2ef+2c^3df\right)x}{c^2\left(4ac-b^2\right)}+\frac{2a^2ceh-a\,b^2eh+abcdh+abceg-2a\,c^2dg-2a\,c^2ef+b\,c^2df}{\left(4ac-b^2\right)c^2}}{c\,x^2+bx+a}+\frac{\left(4aehc-b\right.}{}$
risch	Expression too large to display

```
[In] int((e*x+d)*(h*x^2+g*x+f)/(c*x^2+b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] ((3*a*b*c*e*h-2*a*c^2*d*h-2*a*c^2*e*g-b^3*e*h+b^2*c*d*h+b^2*c*e*g-b*c^2*d*g
-b*c^2*e*f+2*c^3*d*f)/c^2/(4*a*c-b^2)*x+(2*a^2*c*e*h-a*b^2*e*h+a*b*c*d*h+a*
b*c*e*g-2*a*c^2*d*g-2*a*c^2*e*f+b*c^2*d*f)/(4*a*c-b^2)/c^2)/(c*x^2+b*x+a)+1
/c/(4*a*c-b^2)*(1/2*(4*a*c*e*h-b^2*e*h)/c*ln(c*x^2+b*x+a)+2*(-b*a*e*h+2*a*c
*d*h+2*a*c*e*g-b*c*d*g-b*c*e*f+2*c^2*d*f-1/2*(4*a*c*e*h-b^2*e*h)*b/c)/(4*a*
c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 697 vs. $2(172) = 344$.

Time = 0.37 (sec) , antiderivative size = 1413, normalized size of antiderivative = 7.94

$$\int \frac{(d+ex)\left(f+gx+hx^2\right)}{\left(a+bx+cx^2\right)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)*(h*x^2+g*x+f)/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [1/2*(((2*(2*c^4*d - b*c^3*e)*f - 2*(b*c^3*d - 2*a*c^3*e)*g + (4*a*c^3*d +
(b^3*c - 6*a*b*c^2)*e)*h)*x^2 + 2*(2*a*c^3*d - a*b*c^2*e)*f - 2*(a*b*c^2*d
- 2*a^2*c^2*e)*g + (4*a^2*c^2*d + (a*b^3 - 6*a^2*b*c)*e)*h + (2*(2*b*c^3*d
- b^2*c^2*e)*f - 2*(b^2*c^2*d - 2*a*b*c^2*e)*g + (4*a*b*c^2*d + (b^4 - 6*a*
b^2*c)*e)*h)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c +
sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - 2*((b^3*c^2 - 4*a*b*c^3
)*d - 2*(a*b^2*c^2 - 4*a^2*c^3)*e)*f + 2*(2*(a*b^2*c^2 - 4*a^2*c^3)*d - (a*
b^3*c - 4*a^2*b*c^2)*e)*g - 2*((a*b^3*c - 4*a^2*b*c^2)*d - (a*b^4 - 6*a^2*b
^2*c + 8*a^3*c^2)*e)*h - 2*((2*(b^2*c^3 - 4*a*c^4)*d - (b^3*c^2 - 4*a*b*c^3
)*e)*f - ((b^3*c^2 - 4*a*b*c^3)*d - (b^4*c - 6*a*b^2*c^2 + 8*a^2*c^3)*e)*g
+ ((b^4*c - 6*a*b^2*c^2 + 8*a^2*c^3)*d - (b^5 - 7*a*b^3*c + 12*a^2*b*c^2)*e
)*h)*x + ((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*e*h*x^2 + (b^5 - 8*a*b^3*c + 1
6*a^2*b*c^2)*e*h*x + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*e*h)*log(c*x^2 + b*
x + a))/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4 + (b^4*c^3 - 8*a*b^2*c^4 +
16*a^2*c^5)*x^2 + (b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x), 1/2*(2*((2*(2*
c^4*d - b*c^3*e)*f - 2*(b*c^3*d - 2*a*c^3*e)*g + (4*a*c^3*d + (b^3*c - 6*a*
b*c^2)*e)*h)*x^2 + 2*(2*a*c^3*d - a*b*c^2*e)*f - 2*(a*b*c^2*d - 2*a^2*c^2*e
)*g + (4*a^2*c^2*d + (a*b^3 - 6*a^2*b*c)*e)*h + (2*(2*b*c^3*d - b^2*c^2*e)*
f - 2*(b^2*c^2*d - 2*a*b*c^2*e)*g + (4*a*b*c^2*d + (b^4 - 6*a*b^2*c)*e)*h)*
x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c))
 - 2*((b^3*c^2 - 4*a*b*c^3)*d - 2*(a*b^2*c^2 - 4*a^2*c^3)*e)*f + 2*(2*(a*b^
2*c^2 - 4*a^2*c^3)*d - (a*b^3*c - 4*a^2*b*c^2)*e)*g - 2*((a*b^3*c - 4*a^2*b
*c^2)*d - (a*b^4 - 6*a^2*b^2*c + 8*a^3*c^2)*e)*h - 2*((2*(b^2*c^3 - 4*a*c^4
)*d - (b^3*c^2 - 4*a*b*c^3)*e)*f - ((b^3*c^2 - 4*a*b*c^3)*d - (b^4*c - 6*a*
b^2*c^2 + 8*a^2*c^3)*e)*g + ((b^4*c - 6*a*b^2*c^2 + 8*a^2*c^3)*d - (b^5 - 7
*a*b^3*c + 12*a^2*b*c^2)*e)*h)*x + ((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*e*h*
x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*e*h*x + (a*b^4 - 8*a^2*b^2*c + 16*a^
3*c^2)*e*h)*log(c*x^2 + b*x + a))/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4 +
 (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^2 + (b^5*c^2 - 8*a*b^3*c^3 + 16*a^2
*b*c^4)*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1535 vs. $2(170) = 340$.

Time = 19.64 (sec) , antiderivative size = 1535, normalized size of antiderivative = 8.62

$$\int \frac{(d+ex)\left(f+gx+hx^2\right)}{\left(a+bx+cx^2\right)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)*(h*x**2+g*x+f)/(c*x**2+b*x+a)**2,x)
```

```
[Out] (e*h/(2*c**2) - sqrt(-(4*a*c - b**2)**3)*(6*a*b*c*e*h - 4*a*c**2*d*h - 4*a*
c**2*e*g - b**3*e*h + 2*b*c**2*d*g + 2*b*c**2*e*f - 4*c**3*d*f)/(2*c**2*(64
*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))*log(x + (-16*a**2*c*
*3*(e*h/(2*c**2) - sqrt(-(4*a*c - b**2)**3)*(6*a*b*c*e*h - 4*a*c**2*d*h - 4
*a*c**2*e*g - b**3*e*h + 2*b*c**2*d*g + 2*b*c**2*e*f - 4*c**3*d*f)/(2*c**2*
(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) + 8*a**2*c*e*h +
8*a*b**2*c**2*(e*h/(2*c**2) - sqrt(-(4*a*c - b**2)**3)*(6*a*b*c*e*h - 4*a*c
**2*d*h - 4*a*c**2*e*g - b**3*e*h + 2*b*c**2*d*g + 2*b*c**2*e*f - 4*c**3*d*
f)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) - a*b*
*2*e*h - 2*a*b*c*d*h - 2*a*b*c*e*g - b**4*c*(e*h/(2*c**2) - sqrt(-(4*a*c -
b**2)**3)*(6*a*b*c*e*h - 4*a*c**2*d*h - 4*a*c**2*e*g - b**3*e*h + 2*b*c**2*
d*g + 2*b*c**2*e*f - 4*c**3*d*f)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2
+ 12*a*b**4*c - b**6))) + b**2*c*d*g + b**2*c*e*f - 2*b*c**2*d*f)/(6*a*b*c*
e*h - 4*a*c**2*d*h - 4*a*c**2*e*g - b**3*e*h + 2*b*c**2*d*g + 2*b*c**2*e*f
- 4*c**3*d*f)) + (e*h/(2*c**2) + sqrt(-(4*a*c - b**2)**3)*(6*a*b*c*e*h - 4*
a*c**2*d*h - 4*a*c**2*e*g - b**3*e*h + 2*b*c**2*d*g + 2*b*c**2*e*f - 4*c**3
*d*f)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))*log
(x + (-16*a**2*c**3*(e*h/(2*c**2) + sqrt(-(4*a*c - b**2)**3)*(6*a*b*c*e*h -
 4*a*c**2*d*h - 4*a*c**2*e*g - b**3*e*h + 2*b*c**2*d*g + 2*b*c**2*e*f - 4*c
**3*d*f)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))
+ 8*a**2*c*e*h + 8*a*b**2*c**2*(e*h/(2*c**2) + sqrt(-(4*a*c - b**2)**3)*(6*
a*b*c*e*h - 4*a*c**2*d*h - 4*a*c**2*e*g - b**3*e*h + 2*b*c**2*d*g + 2*b*c**
2*e*f - 4*c**3*d*f)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c
 - b**6))) - a*b**2*e*h - 2*a*b*c*d*h - 2*a*b*c*e*g - b**4*c*(e*h/(2*c**2)
+ sqrt(-(4*a*c - b**2)**3)*(6*a*b*c*e*h - 4*a*c**2*d*h - 4*a*c**2*e*g - b**
3*e*h + 2*b*c**2*d*g + 2*b*c**2*e*f - 4*c**3*d*f)/(2*c**2*(64*a**3*c**3 - 4
8*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) + b**2*c*d*g + b**2*c*e*f - 2*b*c*
*2*d*f)/(6*a*b*c*e*h - 4*a*c**2*d*h - 4*a*c**2*e*g - b**3*e*h + 2*b*c**2*d*
g + 2*b*c**2*e*f - 4*c**3*d*f)) + (2*a**2*c*e*h - a*b**2*e*h + a*b*c*d*h +
a*b*c*e*g - 2*a*c**2*d*g - 2*a*c**2*e*f + b*c**2*d*f + x*(3*a*b*c*e*h - 2*a
*c**2*d*h - 2*a*c**2*e*g - b**3*e*h + b**2*c*d*h + b**2*c*e*g - b*c**2*d*g
- b*c**2*e*f + 2*c**3*d*f))/(4*a**2*c**3 - a*b**2*c**2 + x**2*(4*a*c**4 - b
**2*c**3) + x*(4*a*b*c**3 - b**3*c**2))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)\left(f+gx+hx^2\right)}{\left(a+bx+cx^2\right)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*x+d)*(h*x^2+g*x+f)/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.52

$$\int \frac{(d+ex)\left(f+gx+hx^2\right)}{\left(a+bx+cx^2\right)^2}\,dx = \frac{eh\log\left(cx^2+bx+a\right)}{2\,c^2} - \frac{\left(4\,c^3df - 2\,bc^2ef - 2\,bc^2dg + 4\,ac^2eg + 4\,ac^2dh + b^3eh - 6\,abceh\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^2c^2 - 4\,ac^3\right)\sqrt{-b^2+4\,ac}} - \frac{bc^2df - 2\,ac^2ef - 2\,ac^2dg + abceg + abcdh - ab^2eh + 2\,a^2ceh + \left(2\,c^3df - bc^2ef - bc^2dg + b^2ceg - 2\,ac}{(cx^2+bx+a)(b^2-4\,ac)c^2}$$

```
[In] integrate((e*x+d)*(h*x^2+g*x+f)/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*e*h*log(c*x^2 + b*x + a)/c^2 - (4*c^3*d*f - 2*b*c^2*e*f - 2*b*c^2*d*g +
 4*a*c^2*e*g + 4*a*c^2*d*h + b^3*e*h - 6*a*b*c*e*h)*arctan((2*c*x + b)/sqrt
(-b^2 + 4*a*c))/((b^2*c^2 - 4*a*c^3)*sqrt(-b^2 + 4*a*c)) - (b*c^2*d*f - 2*a
*c^2*e*f - 2*a*c^2*d*g + a*b*c*e*g + a*b*c*d*h - a*b^2*e*h + 2*a^2*c*e*h +
(2*c^3*d*f - b*c^2*e*f - b*c^2*d*g + b^2*c*e*g - 2*a*c^2*e*g + b^2*c*d*h -
2*a*c^2*d*h - b^3*e*h + 3*a*b*c*e*h)*x)/((c*x^2 + b*x + a)*(b^2 - 4*a*c)*c^
2)
```

Mupad [B] (verification not implemented)

Time = 14.45 (sec) , antiderivative size = 376, normalized size of antiderivative = 2.11

$$\int \frac{(d+ex)\left(f+gx+hx^2\right)}{\left(a+bx+cx^2\right)^2}\,dx$$
$$=\frac{\frac{b\,c^2\,d\,f-2\,a\,c^2\,e\,f-2\,a\,c^2\,d\,g-a\,b^2\,e\,h+2\,a^2\,c\,e\,h+a\,b\,c\,d\,h+a\,b\,c\,e\,g}{c^2\,\left(4\,a\,c-b^2\right)}-\frac{x\,\left(b^3\,e\,h-2\,c^3\,d\,f+2\,a\,c^2\,d\,h+2\,a\,c^2\,e\,g+b\,c^2\,d\,g+b\,c^2\,e\,f-b^2\,c\,d\,h-b^2\,c\,e\,g-3\right.}{c^2\,\left(4\,a\,c-b^2\right)}}{c\,x^2+b\,x+a}$$
$$-\frac{\ln\left(c\,x^2+b\,x+a\right)\,\left(-64\,e\,h\,a^3\,c^3+48\,e\,h\,a^2\,b^2\,c^2-12\,e\,h\,a\,b^4\,c+e\,h\,b^6\right)}{2\,\left(64\,a^3\,c^5-48\,a^2\,b^2\,c^4+12\,a\,b^4\,c^3-b^6\,c^2\right)}$$
$$+\frac{\mathrm{atan}\left(\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}-\frac{b^3\,c-4\,a\,b\,c^2}{c\,{\left(4\,a\,c-b^2\right)}^{3/2}}\right)\,\left(4\,c^3\,d\,f+b^3\,e\,h+4\,a\,c^2\,d\,h+4\,a\,c^2\,e\,g-2\,b\,c^2\,d\,g-2\,b\,c^2\,e\,f-6\,a\,b\,c\,e\,h\right)}{c^2\,{\left(4\,a\,c-b^2\right)}^{3/2}}$$

```
[In] int(((d + e*x)*(f + g*x + h*x^2))/(a + b*x + c*x^2)^2,x)
```

```
[Out] ((b*c^2*d*f - 2*a*c^2*e*f - 2*a*c^2*d*g - a*b^2*e*h + 2*a^2*c*e*h + a*b*c*d
*h + a*b*c*e*g)/(c^2*(4*a*c - b^2)) - (x*(b^3*e*h - 2*c^3*d*f + 2*a*c^2*d*h
 + 2*a*c^2*e*g + b*c^2*d*g + b*c^2*e*f - b^2*c*d*h - b^2*c*e*g - 3*a*b*c*e*
h))/(c^2*(4*a*c - b^2)))/(a + b*x + c*x^2) - (log(a + b*x + c*x^2)*(b^6*e*h
 - 64*a^3*c^3*e*h + 48*a^2*b^2*c^2*e*h - 12*a*b^4*c*e*h))/(2*(64*a^3*c^5 -
b^6*c^2 + 12*a*b^4*c^3 - 48*a^2*b^2*c^4)) + (atan((2*c*x)/(4*a*c - b^2)^(1/
2) - (b^3*c - 4*a*b*c^2)/(c*(4*a*c - b^2)^(3/2)))*(4*c^3*d*f + b^3*e*h + 4*
a*c^2*d*h + 4*a*c^2*e*g - 2*b*c^2*d*g - 2*b*c^2*e*f - 6*a*b*c*e*h))/(c^2*(4
*a*c - b^2)^(3/2))
```

3.157 $\int \frac{f+gx+hx^2}{\left(a+bx+cx^2\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 118

$$\int \frac{f+gx+hx^2}{\left(a+bx+cx^2\right)^2}\,dx=\frac{c\left(2ag-b\left(f+\frac{ah}{c}\right)\right)-\left(2c^2f-bcg+b^2h-2ach\right)x}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{2(2cf-bg+2ah)\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}$$

```
[Out] (c*(2*a*g-b*(f+a*h/c))-(-2*a*c*h+b^2*h-b*c*g+2*c^2*f)*x)/c/(-4*a*c+b^2)/(c*
x^2+b*x+a)+2*(2*a*h-b*g+2*c*f)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/(-4*a*
c+b^2)^(3/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {1674, 12, 632, 212}

$$\int \frac{f+gx+hx^2}{\left(a+bx+cx^2\right)^2}\,dx=\frac{2\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)(2ah-bg+2cf)}{\left(b^2-4ac\right)^{3/2}}+\frac{c\left(2ag-b\left(\frac{ah}{c}+f\right)\right)-x\left(-2ach+b^2h-bcg+2c^2f\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

```
[In] Int[(f + g*x + h*x^2)/(a + b*x + c*x^2)^2,x]
```

```
[Out] (c*(2*a*g - b*(f + (a*h)/c)) - (2*c^2*f - b*c*g + b^2*h - 2*a*c*h)*x)/(c*(b
^2 - 4*a*c)*(a + b*x + c*x^2)) + (2*(2*c*f - b*g + 2*a*h)*ArcTanh[(b + 2*c*
x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{c\left(2ag-b\left(f+\frac{ah}{c}\right)\right)-\left(2c^2f-bcg+b^2h-2ach\right)x}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{\int \frac{2cf-bg+2ah}{a+bx+cx^2}\,dx}{-b^2+4ac}\\
&= \frac{c\left(2ag-b\left(f+\frac{ah}{c}\right)\right)-\left(2c^2f-bcg+b^2h-2ach\right)x}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{(2cf-bg+2ah)\int \frac{1}{a+bx+cx^2}\,dx}{b^2-4ac}\\
&= \frac{c\left(2ag-b\left(f+\frac{ah}{c}\right)\right)-\left(2c^2f-bcg+b^2h-2ach\right)x}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}\\
&\quad+\frac{(2(2cf-bg+2ah))\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx\right)}{b^2-4ac}\\
&= \frac{c\left(2ag-b\left(f+\frac{ah}{c}\right)\right)-\left(2c^2f-bcg+b^2h-2ach\right)x}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{2(2cf-bg+2ah)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.97

$$\int \frac{f+gx+hx^2}{(a+bx+cx^2)^2}\,dx = \frac{abh+2c^2fx+b^2hx+bc(f-gx)-2ac(g+hx)}{c\left(-b^2+4ac\right)\left(a+x(b+cx)\right)} - \frac{2(-2cf+bg-2ah)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\left(-b^2+4ac\right)^{3/2}}$$

```
[In] Integrate[(f + g*x + h*x^2)/(a + b*x + c*x^2)^2,x]
```

```
[Out] (a*b*h + 2*c^2*f*x + b^2*h*x + b*c*(f - g*x) - 2*a*c*(g + h*x))/(c*(-b^2 +
4*a*c)*(a + x*(b + c*x))) - (2*(-2*c*f + b*g - 2*a*h)*ArcTan[(b + 2*c*x)/Sq
rt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2)
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.13

method	result
default	$\frac{-\frac{\left(2ach-b^2h+bcg-2c^2f\right)x}{c\left(4ac-b^2\right)}+\frac{abh-2acg+bcf}{c\left(4ac-b^2\right)}}{c\,x^2+bx+a}+\frac{2(2ah-bg+2cf)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}}$
risch	$\frac{-\frac{\left(2ach-b^2h+bcg-2c^2f\right)x}{c\left(4ac-b^2\right)}+\frac{abh-2acg+bcf}{c\left(4ac-b^2\right)}}{c\,x^2+bx+a}+\frac{2\ln\left(\left(-8a\,c^2+2b^2c\right)x+\left(-4ac+b^2\right)^{\frac{3}{2}}-4abc+b^3\right)ah}{\left(-4ac+b^2\right)^{\frac{3}{2}}}-\frac{\ln\left(\left(-8a\,c^2+2b^2c\right)x+\left(-4ac+\right.\right.}{\left(-4ac+b^2\right)}$

```
[In] int((h*x^2+g*x+f)/(c*x^2+b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-(2*a*c*h-b^2*h+b*c*g-2*c^2*f)/c/(4*a*c-b^2)*x+1/c*(a*b*h-2*a*c*g+b*c*f)/(
4*a*c-b^2))/(c*x^2+b*x+a)+2*(2*a*h-b*g+2*c*f)/(4*a*c-b^2)^(3/2)*arctan((2*c
*x+b)/(4*a*c-b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 307 vs. 2(113) = 226.

Time = 0.31 (sec) , antiderivative size = 632, normalized size of antiderivative = 5.36

$$\int \frac{f+gx+hx^2}{(a+bx+cx^2)^2}\,dx$$

$$= \left[-\frac{\left(2\,ac^2f-abcg+2\,a^2ch+\left(2\,c^3f-bc^2g+2\,ac^2h\right)x^2+\left(2\,bc^2f-b^2cg+2\,abch\right)x\right)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^2}{}\right.}{ab^4c-8\,a^2b^2c^2+}\right.$$

```
[In] integrate((h*x^2+g*x+f)/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [-((2*a*c^2*f - a*b*c*g + 2*a^2*c*h + (2*c^3*f - b*c^2*g + 2*a*c^2*h)*x^2 +
 (2*b*c^2*f - b^2*c*g + 2*a*b*c*h)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*
b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + (
b^3*c - 4*a*b*c^2)*f - 2*(a*b^2*c - 4*a^2*c^2)*g + (a*b^3 - 4*a^2*b*c)*h +
(2*(b^2*c^2 - 4*a*c^3)*f - (b^3*c - 4*a*b*c^2)*g + (b^4 - 6*a*b^2*c + 8*a^2
*c^2)*h)*x)/(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3 + (b^4*c^2 - 8*a*b^2*c^3
+ 16*a^2*c^4)*x^2 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x), (2*(2*a*c^2*f
- a*b*c*g + 2*a^2*c*h + (2*c^3*f - b*c^2*g + 2*a*c^2*h)*x^2 + (2*b*c^2*f -
b^2*c*g + 2*a*b*c*h)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*
x + b)/(b^2 - 4*a*c)) - (b^3*c - 4*a*b*c^2)*f + 2*(a*b^2*c - 4*a^2*c^2)*g -
 (a*b^3 - 4*a^2*b*c)*h - (2*(b^2*c^2 - 4*a*c^3)*f - (b^3*c - 4*a*b*c^2)*g +
 (b^4 - 6*a*b^2*c + 8*a^2*c^2)*h)*x)/(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3
+ (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^2 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*
b*c^3)*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 459 vs. $2(107) = 214$.

Time $= 1.00$ (sec) , antiderivative size $= 459$, normalized size of antiderivative $= 3.89$

$$\int \frac{f+gx+hx^2}{\left(a+bx+cx^2\right)^2}\,dx = -\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\cdot(2ah-bg$$
$$+2cf)\log\left(x+\frac{-16a^2c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}\cdot(2ah-bg+2cf)+8ab^2c\sqrt{-\frac{1}{(4ac-b^2)^3}}\cdot(2ah-bg+2cf)+2abh-}{4ach-2bcg+4c^2f}\right.$$
$$+\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\cdot(2ah-bg$$
$$+2cf)\log\left(x+\frac{16a^2c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}\cdot(2ah-bg+2cf)-8ab^2c\sqrt{-\frac{1}{(4ac-b^2)^3}}\cdot(2ah-bg+2cf)+2abh+b}{4ach-2bcg+4c^2f}\right.$$
$$+\frac{abh-2acg+bcf+x(-2ach+b^2h-bcg+2c^2f)}{4a^2c^2-ab^2c+x^2\cdot(4ac^3-b^2c^2)+x\left(4abc^2-b^3c\right)}$$

```
[In] integrate((h*x**2+g*x+f)/(c*x**2+b*x+a)**2,x)
```

```
[Out] -sqrt(-1/(4*a*c - b**2)**3)*(2*a*h - b*g + 2*c*f)*log(x + (-16*a**2*c**2*sq
rt(-1/(4*a*c - b**2)**3)*(2*a*h - b*g + 2*c*f) + 8*a*b**2*c*sqrt(-1/(4*a*c
- b**2)**3)*(2*a*h - b*g + 2*c*f) + 2*a*b*h - b**4*sqrt(-1/(4*a*c - b**2)**
3)*(2*a*h - b*g + 2*c*f) - b**2*g + 2*b*c*f)/(4*a*c*h - 2*b*c*g + 4*c**2*f)
) + sqrt(-1/(4*a*c - b**2)**3)*(2*a*h - b*g + 2*c*f)*log(x + (16*a**2*c**2*
sqrt(-1/(4*a*c - b**2)**3)*(2*a*h - b*g + 2*c*f) - 8*a*b**2*c*sqrt(-1/(4*a*
```

```
c - b**2)**3)*(2*a*h - b*g + 2*c*f) + 2*a*b*h + b**4*sqrt(-1/(4*a*c - b**2)
**3)*(2*a*h - b*g + 2*c*f) - b**2*g + 2*b*c*f)/(4*a*c*h - 2*b*c*g + 4*c**2*
f)) + (a*b*h - 2*a*c*g + b*c*f + x*(-2*a*c*h + b**2*h - b*c*g + 2*c**2*f))/
(4*a**2*c**2 - a*b**2*c + x**2*(4*a*c**3 - b**2*c**2) + x*(4*a*b*c**2 - b**
3*c))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{f+gx+hx^2}{\left(a+bx+cx^2\right)^2}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((h*x^2+g*x+f)/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.06

$$\int \frac{f+gx+hx^2}{\left(a+bx+cx^2\right)^2}\,dx=-\frac{2\,(2\,cf-bg+2\,ah)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2-4\,ac)\sqrt{-b^2+4\,ac}}-\frac{2\,c^2fx-bcgx+b^2hx-2\,achx+bcf-2\,acg+abh}{(b^2c-4\,ac^2)(cx^2+bx+a)}$$

```
[In] integrate((h*x^2+g*x+f)/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] -2*(2*c*f - b*g + 2*a*h)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a
*c)*sqrt(-b^2 + 4*a*c)) - (2*c^2*f*x - b*c*g*x + b^2*h*x - 2*a*c*h*x + b*c*
f - 2*a*c*g + a*b*h)/((b^2*c - 4*a*c^2)*(c*x^2 + b*x + a))
```

Mupad [B] (verification not implemented)

Time = 13.57 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.72

$$\int \frac{f+gx+hx^2}{\left(a+bx+cx^2\right)^2}\,dx$$
$$=\frac{\frac{a\,b\,h-2\,a\,c\,g+b\,c\,f}{c\,\left(4\,a\,c-b^2\right)}+\frac{x\,\left(h\,b^2-g\,b\,c+2\,f\,c^2-2\,a\,h\,c\right)}{c\,\left(4\,a\,c-b^2\right)}}{c\,x^2+b\,x+a}$$
$$-\frac{2\,\mathrm{atan}\left(\frac{\left(\frac{\left(b^3-4\,a\,b\,c\right)\,\left(2\,a\,h-b\,g+2\,c\,f\right)}{{\left(4\,a\,c-b^2\right)}^{5/2}}-\frac{2\,c\,x\,\left(2\,a\,h-b\,g+2\,c\,f\right)}{{\left(4\,a\,c-b^2\right)}^{3/2}}\right)\,\left(4\,a\,c-b^2\right)}{2\,a\,h-b\,g+2\,c\,f}\right)\,\left(2\,a\,h-b\,g+2\,c\,f\right)}{{\left(4\,a\,c-b^2\right)}^{3/2}}$$

```
[In] int((f + g*x + h*x^2)/(a + b*x + c*x^2)^2,x)
```

```
[Out] ((a*b*h - 2*a*c*g + b*c*f)/(c*(4*a*c - b^2)) + (x*(2*c^2*f + b^2*h - 2*a*c*
h - b*c*g))/(c*(4*a*c - b^2)))/(a + b*x + c*x^2) - (2*atan(((((b^3 - 4*a*b*
c)*(2*a*h - b*g + 2*c*f))/(4*a*c - b^2)^(5/2) - (2*c*x*(2*a*h - b*g + 2*c*f
))/(4*a*c - b^2)^(3/2))*(4*a*c - b^2))/(2*a*h - b*g + 2*c*f))*(2*a*h - b*g
+ 2*c*f))/(4*a*c - b^2)^(3/2)
```

3.158 $\int \frac{f+gx+hx^2}{(d+ex)\left(a+bx+cx^2\right)^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 407

$$\int \frac{f+gx+hx^2}{(d+ex)\left(a+bx+cx^2\right)^2}\,dx$$

$$= \frac{b^2ef-b(cdf+aeg+adh)-2a(cef-cdg-aeh)-(2c^2df+b(bd-ae)h-c(bef+bdg-2aeg+2adh}{(b^2-4ac)\,(cd^2-bde+ae^2)\,(a+bx+cx^2)}$$

$$+\frac{(4c^3d^3f+be(4abdeh-2a^2e^2h+b^2(e^2f-deg-d^2h))-2c^2d(bd(3ef+dg)-2a(3e^2f-deg+d^2h))}{(b^2-4ac)^{3/2}\,(cd^2-bde+ae^2}$$

$$+\frac{e(e^2f-deg+d^2h)\log(d+ex)}{(cd^2-bde+ae^2)^2}-\frac{e(e^2f-deg+d^2h)\log\left(a+bx+cx^2\right)}{2\left(cd^2-bde+ae^2\right)^2}$$

```
[Out] (b^2*e*f-b*(a*d*h+a*e*g+c*d*f)-2*a*(-a*e*h-c*d*g+c*e*f)-(2*c^2*d*f+b*(-a*e+
b*d)*h-c*(2*a*d*h-2*a*e*g+b*d*g+b*e*f))*x)/(-4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)
/(c*x^2+b*x+a)+(4*c^3*d^3*f+b*e*(4*a*b*d*e*h-2*a^2*e^2*h+b^2*(-d^2*h-d*e*g+
e^2*f))-2*c^2*d*(b*d*(d*g+3*e*f)-2*a*(d^2*h-d*e*g+3*e^2*f))+2*c*e*(2*b^2*d^
2*g+2*a^2*e*(-d*h+e*g)-a*b*(d^2*h+d*e*g+3*e^2*f)))*arctanh((2*c*x+b)/(-4*a*
c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)/(a*e^2-b*d*e+c*d^2)^2+e*(d^2*h-d*e*g+e^2*f
)*ln(e*x+d)/(a*e^2-b*d*e+c*d^2)^2-1/2*e*(d^2*h-d*e*g+e^2*f)*ln(c*x^2+b*x+a)
/(a*e^2-b*d*e+c*d^2)^2
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 407, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1660, 814, 648, 632, 212, 642}

$$\int \frac{f+gx+hx^2}{(d+ex)\left(a+bx+cx^2\right)^2}\,dx$$
$$= \frac{\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(2ce(2a^2e(eg-dh)-ab(d^2h+deg+3e^2f)+2b^2d^2g)+be(-2a^2e^2h+4abdeh+b^2(d^2(}{\left(b^2-4ac\right)^{3/2}\left(ae^2-bde+cd^2\right)}$$
$$+\frac{-x(-c(2adh-2aeg+bdg+bef)+bh(bd-ae)+2c^2df)-b(adh+aeg+cdf)-2a(-aeh-cdg+ce}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)\left(ae^2-bde+cd^2\right)}$$
$$-\frac{e\log\left(a+bx+cx^2\right)\left(d^2h-deg+e^2f\right)}{2\left(ae^2-bde+cd^2\right)^2}+\frac{e\log(d+ex)\left(d^2h-deg+e^2f\right)}{\left(ae^2-bde+cd^2\right)^2}$$

```
[In] Int[(f + g*x + h*x^2)/((d + e*x)*(a + b*x + c*x^2)^2),x]
```

```
[Out] (b^2*e*f - b*(c*d*f + a*e*g + a*d*h) - 2*a*(c*e*f - c*d*g - a*e*h) - (2*c^2
*d*f + b*(b*d - a*e)*h - c*(b*e*f + b*d*g - 2*a*e*g + 2*a*d*h))*x)/((b^2 -
4*a*c)*(c*d^2 - b*d*e + a*e^2)*(a + b*x + c*x^2)) + ((4*c^3*d^3*f + b*e*(4*
a*b*d*e*h - 2*a^2*e^2*h + b^2*(e^2*f - d*e*g - d^2*h)) - 2*c^2*d*(b*d*(3*e*
f + d*g) - 2*a*(3*e^2*f - d*e*g + d^2*h)) + 2*c*e*(2*b^2*d^2*g + 2*a^2*e*(e
*g - d*h) - a*b*(3*e^2*f + d*e*g + d^2*h)))*ArcTanh[(b + 2*c*x)/Sqrt[b^2 -
4*a*c]])/((b^2 - 4*a*c)^(3/2)*(c*d^2 - b*d*e + a*e^2)^2) + (e*(e^2*f - d*e*
g + d^2*h)*Log[d + e*x])/(c*d^2 - b*d*e + a*e^2)^2 - (e*(e^2*f - d*e*g + d^
2*h)*Log[a + b*x + c*x^2])/(2*(c*d^2 - b*d*e + a*e^2)^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1660

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^(p + 1)/((p
+ 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

integral

$$= \frac{b^2ef - b(cdf + aeg + adh) - 2a(cef - cdg - aeh) - (2c^2df + b(bd - ae)h - c(bef + bdg - 2aeg + 2adh)}{(b^2 - 4ac)\left(cd^2 - bde + ae^2\right)\left(a + bx + cx^2\right)} + \frac{\int \frac{\frac{2c^2d^2f - bcd(ef + dg) - be(bef - bdg + adh) + 2ac\left(2e^2f - deg + d^2h\right)}{cd^2 - bde + ae^2} + \frac{e\left(2c^2df + b(bd - ae)h - c(bef + bdg - 2aeg + 2adh)\right)x}{cd^2 - bde + ae^2}}{(d + ex)(a + bx + cx^2)}\,dx}{-b^2 + 4ac}$$

$$= \frac{b^2ef - b(cdf + aeg + adh) - 2a(cef - cdg - aeh) - (2c^2df + b(bd - ae)h - c(bef + bdg - 2aeg}{(b^2 - 4ac)\left(cd^2 - bde + ae^2\right)\left(a + bx + cx^2\right)} + \frac{\int \left(-\frac{(b^2 - 4ac)e^2\left(e^2f - deg + d^2h\right)}{\left(cd^2 - bde + ae^2\right)^2(d + ex)} + \frac{2c^3d^3f + be^2\left(b^2(ef - dg) + 2abdh - a^2eh\right) - c^2d\left(bd(3ef + dg) - 2a\left(3e^2f - deg + d^2h\right)\right) + ce(2b^2c}{\left(cd^2 - bde + ae^2\right)^2(a + bx}\right.}{-b^2 + 4ac}$$

$$= \frac{b^2ef - b(cdf + aeg + adh) - 2a(cef - cdg - aeh) - (2c^2df + b(bd - ae)h - c(bef + bdg - 2aeg +}{(b^2 - 4ac)(cd^2 - bde + ae^2)(a + bx + cx^2)}$$
$$+ \frac{e(e^2f - deg + d^2h)\log(d + ex)}{(cd^2 - bde + ae^2)^2}$$
$$- \frac{\int \frac{2c^3d^3f + be^2(b^2(ef - dg) + 2abdh - a^2eh) - c^2d(bd(3ef + dg) - 2a(3e^2f - deg + d^2h)) + ce(2b^2d^2g + 2a^2e(eg - dh) - ab(5e^2f - deg + 3d^2h}{a + bx + cx^2}}{(b^2 - 4ac)(cd^2 - bde + ae^2)^2}$$

$$= \frac{b^2ef - b(cdf + aeg + adh) - 2a(cef - cdg - aeh) - (2c^2df + b(bd - ae)h - c(bef + bdg - 2aeg +}{(b^2 - 4ac)(cd^2 - bde + ae^2)(a + bx + cx^2)}$$
$$+ \frac{e(e^2f - deg + d^2h)\log(d + ex)}{(cd^2 - bde + ae^2)^2} - \frac{(e(e^2f - deg + d^2h)) \int \frac{b + 2cx}{a + bx + cx^2}\,dx}{2(cd^2 - bde + ae^2)^2}$$
$$- \frac{(4c^3d^3f + be(4abdeh - 2a^2e^2h + b^2(e^2f - deg - d^2h)) - 2c^2d(bd(3ef + dg) - 2a(3e^2f - deg +}{2(b^2 - 4ac)(cd^2 - bde +}$$

$$= \frac{b^2ef - b(cdf + aeg + adh) - 2a(cef - cdg - aeh) - (2c^2df + b(bd - ae)h - c(bef + bdg - 2aeg +}{(b^2 - 4ac)(cd^2 - bde + ae^2)(a + bx + cx^2)}$$
$$+ \frac{e(e^2f - deg + d^2h)\log(d + ex)}{(cd^2 - bde + ae^2)^2} - \frac{e(e^2f - deg + d^2h)\log(a + bx + cx^2)}{2(cd^2 - bde + ae^2)^2}$$
$$+ \frac{(4c^3d^3f + be(4abdeh - 2a^2e^2h + b^2(e^2f - deg - d^2h)) - 2c^2d(bd(3ef + dg) - 2a(3e^2f - deg +}{(b^2 - 4ac)(cd^2}$$

$$= \frac{b^2ef - b(cdf + aeg + adh) - 2a(cef - cdg - aeh) - (2c^2df + b(bd - ae)h - c(bef + bdg - 2aeg +}{(b^2 - 4ac)(cd^2 - bde + ae^2)(a + bx + cx^2)}$$
$$+ \frac{(4c^3d^3f + be(4abdeh - 2a^2e^2h + b^2(e^2f - deg - d^2h)) - 2c^2d(bd(3ef + dg) - 2a(3e^2f - deg +}{(b^2 - 4ac)^{3/2}(cd^2 - bde}$$
$$+ \frac{e(e^2f - deg + d^2h)\log(d + ex)}{(cd^2 - bde + ae^2)^2} - \frac{e(e^2f - deg + d^2h)\log(a + bx + cx^2)}{2(cd^2 - bde + ae^2)^2}$$

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.00

$$\int \frac{f + gx + hx^2}{(d + ex)(a + bx + cx^2)^2}\,dx$$
$$= \frac{-2a^2eh + 2c^2dfx + b^2(-ef + dhx) + bc(-efx + d(f - gx)) + ab(dh + e(g - hx)) + 2ac(e(f + gx) - d(g}{(b^2 - 4ac)(-cd^2 + e(bd - ae))(a + x(b + cx))}$$
$$- \frac{(-4c^3d^3f + 2c^2d(bd(3ef + dg) - 2a(3e^2f - deg + d^2h)) + be(-4abdeh + 2a^2e^2h + b^2(-e^2f + deg + d^2}{(-b^2 + 4ac)^{3/2}(cd^2 + e(-bd}$$
$$+ \frac{e(e^2f - deg + d^2h)\log(d + ex)}{(cd^2 + e(-bd + ae))^2} - \frac{e(e^2f - deg + d^2h)\log(a + x(b + cx))}{2(cd^2 + e(-bd + ae))^2}$$

```
[In] Integrate[(f + g*x + h*x^2)/((d + e*x)*(a + b*x + c*x^2)^2),x]
```

```
[Out] (-2*a^2*e*h + 2*c^2*d*f*x + b^2*(-(e*f) + d*h*x) + b*c*(-(e*f*x) + d*(f - g
*x)) + a*b*(d*h + e*(g - h*x)) + 2*a*c*(e*(f + g*x) - d*(g + h*x)))/((b^2 -
 4*a*c)*(-(c*d^2) + e*(b*d - a*e))*(a + x*(b + c*x))) - ((-4*c^3*d^3*f + 2*
c^2*d*(b*d*(3*e*f + d*g) - 2*a*(3*e^2*f - d*e*g + d^2*h)) + b*e*(-4*a*b*d*e
*h + 2*a^2*e^2*h + b^2*(-(e^2*f) + d*e*g + d^2*h)) + 2*c*e*(-2*b^2*d^2*g +
2*a^2*e*(-(e*g) + d*h) + a*b*(3*e^2*f + d*e*g + d^2*h)))*ArcTan[(b + 2*c*x)
/Sqrt[-b^2 + 4*a*c]])/((-b^2 + 4*a*c)^(3/2)*(c*d^2 + e*(-(b*d) + a*e))^2) +
 (e*(e^2*f - d*e*g + d^2*h)*Log[d + e*x])/(c*d^2 + e*(-(b*d) + a*e))^2 - (e
*(e^2*f - d*e*g + d^2*h)*Log[a + x*(b + c*x)])/(2*(c*d^2 + e*(-(b*d) + a*e)
)^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 808 vs. $2(401) = 802$.

Time = 0.96 (sec) , antiderivative size = 809, normalized size of antiderivative = 1.99

method	result
default	$\frac{e\left(d^2h-deg+e^2f\right)\ln(ex+d)}{\left(e^2a-bde+c\,d^2\right)^2}-\frac{\frac{\left(a^2b\,e^3h+2a^2cd\,e^2h-2a^2c\,e^3g-2a\,b^2d\,e^2h-abc\,d^2eh+3abcd\,e^2g+abc\,e^3f+2a\,c^2d^3h-2a\,c^2d^2eg-2a\,c^2d\,e^2f+\right.}{4ac-b^2}}{}$
risch	Expression too large to display

```
[In] int((h*x^2+g*x+f)/(e*x+d)/(c*x^2+b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] e*(d^2*h-d*e*g+e^2*f)*ln(e*x+d)/(a*e^2-b*d*e+c*d^2)^2-1/(a*e^2-b*d*e+c*d^2)
^2*(((a^2*b*e^3*h+2*a^2*c*d*e^2*h-2*a^2*c*e^3*g-2*a*b^2*d*e^2*h-a*b*c*d^2*e
*h+3*a*b*c*d*e^2*g+a*b*c*e^3*f+2*a*c^2*d^3*h-2*a*c^2*d^2*e*g-2*a*c^2*d*e^2*
f+b^3*d^2*e*h-b^2*c*d^3*h-b^2*c*d^2*e*g-b^2*c*d*e^2*f+b*c^2*d^3*g+3*b*c^2*d
^2*e*f-2*c^3*d^3*f)/(4*a*c-b^2)*x+(2*a^3*e^3*h-3*a^2*b*d*e^2*h-a^2*b*e^3*g+
2*a^2*c*d^2*e*h+2*a^2*c*d*e^2*g-2*a^2*c*e^3*f+a*b^2*d^2*e*h+a*b^2*d*e^2*g+a
*b^2*e^3*f-a*b*c*d^3*h-3*a*b*c*d^2*e*g+a*b*c*d*e^2*f+2*a*c^2*d^3*g-2*a*c^2*
d^2*e*f-b^3*d*e^2*f+2*b^2*c*d^2*e*f-b*c^2*d^3*f)/(4*a*c-b^2))/(c*x^2+b*x+a)
+1/(4*a*c-b^2)*(1/2*(4*a*c^2*d^2*e*h-4*a*c^2*d*e^2*g+4*a*c^2*e^3*f-b^2*c*d^
2*e*h+b^2*c*d*e^2*g-b^2*c*e^3*f)/c*ln(c*x^2+b*x+a)+2*(a^2*b*e^3*h+2*a^2*c*d
*e^2*h-2*a^2*c*e^3*g-2*a*b^2*d*e^2*h+3*a*b*c*d^2*e*h-a*b*c*d*e^2*g+5*a*b*c*
e^3*f-2*a*c^2*d^3*h+2*a*c^2*d^2*e*g-6*a*c^2*d*e^2*f+b^3*d*e^2*g-b^3*e^3*f-2
*b^2*c*d^2*e*g+b*c^2*d^3*g+3*b*c^2*d^2*e*f-2*c^3*d^3*f-1/2*(4*a*c^2*d^2*e*h
-4*a*c^2*d*e^2*g+4*a*c^2*e^3*f-b^2*c*d^2*e*h+b^2*c*d*e^2*g-b^2*c*e^3*f)*b/c
)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{f+gx+hx^2}{(d+ex)\left(a+bx+cx^2\right)^2}\,dx = \text{Timed out}$$

```
[In] integrate((h*x^2+g*x+f)/(e*x+d)/(c*x^2+b*x+a)^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{f+gx+hx^2}{(d+ex)\left(a+bx+cx^2\right)^2}\,dx = \text{Timed out}$$

```
[In] integrate((h*x**2+g*x+f)/(e*x+d)/(c*x**2+b*x+a)**2,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{f+gx+hx^2}{(d+ex)\left(a+bx+cx^2\right)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((h*x^2+g*x+f)/(e*x+d)/(c*x^2+b*x+a)^2,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 886 vs. $2(401) = 802$.

Time = 0.27 (sec) , antiderivative size = 886, normalized size of antiderivative = 2.18

$$\int \frac{f+gx+hx^2}{(d+ex)\left(a+bx+cx^2\right)^2}\,dx = -\frac{\left(e^3f-de^2g+d^2eh\right)\log\left(cx^2+bx+a\right)}{2\left(c^2d^4-2\,bcd^3e+b^2d^2e^2+2\,acd^2e^2-2\,abde^3+a^2e^4\right)} + \frac{\left(e^4f-de^3g+d^2e^2h\right)\log\left(|ex+d|\right)}{c^2d^4e-2\,bcd^3e^2+b^2d^2e^3+2\,acd^2e^3-2\,abde^4+a^2e^5} - \frac{(4\,c^3d^3f-6\,bc^2d^2ef+12\,ac^2de^2f+b^3e^3f-6\,abce^3f-2\,bc^2d^3g+4\,b^2cd^2eg-4\,ac^2d^2eg-b^3de^2g-2\,a}{(b^2c^2d^4-4\,ac^3d^4-2\,b^3cd^3e+8\,abc^2d^3e+b^4d^2e^2-2\,ab^2cd^2e^2-} - \frac{bc^2d^3f-2\,b^2cd^2ef+2\,ac^2d^2ef+b^3de^2f-abcde^2f-ab^2e^3f+2\,a^2ce^3f-2\,ac^2d^3g+3\,abcd^2eg-ab^2de}{}$$

```
[In] integrate((h*x^2+g*x+f)/(e*x+d)/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(e^3*f - d*e^2*g + d^2*e*h)*log(c*x^2 + b*x + a)/(c^2*d^4 - 2*b*c*d^3*
e + b^2*d^2*e^2 + 2*a*c*d^2*e^2 - 2*a*b*d*e^3 + a^2*e^4) + (e^4*f - d*e^3*g
 + d^2*e^2*h)*log(abs(e*x + d))/(c^2*d^4*e - 2*b*c*d^3*e^2 + b^2*d^2*e^3 +
2*a*c*d^2*e^3 - 2*a*b*d*e^4 + a^2*e^5) - (4*c^3*d^3*f - 6*b*c^2*d^2*e*f + 1
2*a*c^2*d*e^2*f + b^3*e^3*f - 6*a*b*c*e^3*f - 2*b*c^2*d^3*g + 4*b^2*c*d^2*e
*g - 4*a*c^2*d^2*e*g - b^3*d*e^2*g - 2*a*b*c*d*e^2*g + 4*a^2*c*e^3*g + 4*a*
c^2*d^3*h - b^3*d^2*e*h - 2*a*b*c*d^2*e*h + 4*a*b^2*d*e^2*h - 4*a^2*c*d*e^2
*h - 2*a^2*b*e^3*h)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2*c^2*d^4 -
4*a*c^3*d^4 - 2*b^3*c*d^3*e + 8*a*b*c^2*d^3*e + b^4*d^2*e^2 - 2*a*b^2*c*d^2
*e^2 - 8*a^2*c^2*d^2*e^2 - 2*a*b^3*d*e^3 + 8*a^2*b*c*d*e^3 + a^2*b^2*e^4 -
4*a^3*c*e^4)*sqrt(-b^2 + 4*a*c)) - (b*c^2*d^3*f - 2*b^2*c*d^2*e*f + 2*a*c^2
*d^2*e*f + b^3*d*e^2*f - a*b*c*d*e^2*f - a*b^2*e^3*f + 2*a^2*c*e^3*f - 2*a*
c^2*d^3*g + 3*a*b*c*d^2*e*g - a*b^2*d*e^2*g - 2*a^2*c*d*e^2*g + a^2*b*e^3*g
 + a*b*c*d^3*h - a*b^2*d^2*e*h - 2*a^2*c*d^2*e*h + 3*a^2*b*d*e^2*h - 2*a^3*
e^3*h + (2*c^3*d^3*f - 3*b*c^2*d^2*e*f + b^2*c*d*e^2*f + 2*a*c^2*d*e^2*f -
a*b*c*e^3*f - b*c^2*d^3*g + b^2*c*d^2*e*g + 2*a*c^2*d^2*e*g - 3*a*b*c*d*e^2
*g + 2*a^2*c*e^3*g + b^2*c*d^3*h - 2*a*c^2*d^3*h - b^3*d^2*e*h + a*b*c*d^2*
e*h + 2*a*b^2*d*e^2*h - 2*a^2*c*d*e^2*h - a^2*b*e^3*h)*x)/((c*d^2 - b*d*e +
 a*e^2)^2*(c*x^2 + b*x + a)*(b^2 - 4*a*c))
```

Mupad [B] (verification not implemented)

Time = 17.88 (sec) , antiderivative size = 13698, normalized size of antiderivative = 33.66

$$\int \frac{f+gx+hx^2}{(d+ex)\left(a+bx+cx^2\right)^2}\,dx = \text{Too large to display}$$

```
[In] int((f + g*x + h*x^2)/((d + e*x)*(a + b*x + c*x^2)^2),x)
```

```
[Out] symsum(log(root(768*a^5*b*c^4*d^3*e^5*z^3 + 768*a^4*b*c^5*d^5*e^3*z^3 - 192
*a^5*b^3*c^2*d*e^7*z^3 - 192*a^2*b^3*c^5*d^7*e*z^3 - 68*a^3*b^6*c*d^2*e^6*z
^3 - 68*a*b^6*c^3*d^6*e^2*z^3 + 36*a^2*b^7*c*d^3*e^5*z^3 + 36*a*b^7*c^2*d^5
*e^3*z^3 + 256*a^6*b*c^3*d*e^7*z^3 + 256*a^3*b*c^6*d^7*e*z^3 + 48*a^4*b^5*c
*d*e^7*z^3 + 48*a*b^5*c^4*d^7*e*z^3 - 480*a^4*b^2*c^4*d^4*e^4*z^3 + 440*a^3
*b^4*c^3*d^4*e^4*z^3 - 320*a^4*b^3*c^3*d^3*e^5*z^3 - 320*a^3*b^3*c^4*d^5*e^
3*z^3 + 240*a^4*b^4*c^2*d^2*e^6*z^3 + 240*a^2*b^4*c^4*d^6*e^2*z^3 - 192*a^5
*b^2*c^3*d^2*e^6*z^3 - 192*a^3*b^2*c^5*d^6*e^2*z^3 - 90*a^2*b^6*c^2*d^4*e^4
*z^3 - 48*a^3*b^5*c^2*d^3*e^5*z^3 - 48*a^2*b^5*c^3*d^5*e^3*z^3 - 4*b^9*c*d^
5*e^3*z^3 - 4*b^7*c^3*d^7*e*z^3 - 4*a^3*b^7*d*e^7*z^3 - 4*a*b^9*d^3*e^5*z^3
 - 12*a^5*b^4*c*e^8*z^3 - 12*a*b^4*c^5*d^8*z^3 + 6*b^8*c^2*d^6*e^2*z^3 - 38
4*a^5*c^5*d^4*e^4*z^3 - 256*a^6*c^4*d^2*e^6*z^3 - 256*a^4*c^6*d^6*e^2*z^3 +
 6*a^2*b^8*d^2*e^6*z^3 + 48*a^6*b^2*c^2*e^8*z^3 + 48*a^2*b^2*c^6*d^8*z^3 -
64*a^7*c^3*e^8*z^3 - 64*a^3*c^7*d^8*z^3 + b^10*d^4*e^4*z^3 + b^6*c^4*d^8*z^
```

```
3 + a^4*b^6*e^8*z^3 - 28*a*b^4*c*d^3*e^3*g*h*z - 10*a^3*b^2*c*d*e^5*g*h*z -
 10*a*b^2*c^3*d^5*e*g*h*z + 16*a*b^4*c*d^2*e^4*f*h*z + 14*a^2*b^3*c*d*e^5*f
*h*z + 4*a*b*c^4*d^4*e^2*f*g*z + 84*a^2*b^2*c^2*d^3*e^3*g*h*z - 108*a^2*b^2
*c^2*d^2*e^4*f*h*z + 16*a*b*c^4*d^5*e*f*h*z - 20*a*b^4*c*d*e^5*f*g*z + 8*a^
2*b^3*c*d^2*e^4*g*h*z + 8*a*b^3*c^2*d^4*e^2*g*h*z - 4*a^3*b*c^2*d^2*e^4*g*h
*z - 4*a^2*b*c^3*d^4*e^2*g*h*z + 16*a^2*b*c^3*d^3*e^3*f*h*z + 16*a*b^3*c^2*
d^3*e^3*f*h*z - 14*a*b^2*c^3*d^4*e^2*f*h*z + 66*a^2*b^2*c^2*d*e^5*f*g*z - 3
6*a*b^2*c^3*d^3*e^3*f*g*z + 20*a*b^3*c^2*d^2*e^4*f*g*z + 12*a^2*b*c^3*d^2*e
^4*f*g*z + 8*a*c^5*d^5*e*f*g*z + 4*a^4*b*c*e^6*g*h*z - 2*a*b^5*d*e^5*f*h*z
+ 4*a*b*c^4*d^6*g*h*z - 112*a^3*c^3*d^3*e^3*g*h*z - 3*b^4*c^2*d^4*e^2*f*h*z
 + 120*a^3*c^3*d^2*e^4*f*h*z - 16*a^2*c^4*d^4*e^2*f*h*z + 14*b^3*c^3*d^4*e^
2*f*g*z - 2*b^4*c^2*d^3*e^3*f*g*z + 16*a^2*c^4*d^3*e^3*f*g*z + 8*a*b^4*c*d^
4*e^2*h^2*z + 4*a^2*b*c^3*d^5*e*h^2*z + 2*a*b^3*c^2*d^5*e*h^2*z + 8*a*b^4*c
*d^2*e^4*g^2*z + 4*a^3*b*c^2*d*e^5*g^2*z + 2*a^2*b^3*c*d*e^5*g^2*z + 48*a*b
*c^4*d^3*e^3*f^2*z + 36*a^2*b*c^3*d*e^5*f^2*z - 6*a*b^3*c^2*d*e^5*f^2*z - 4
5*a^2*b^2*c^2*d^4*e^2*h^2*z - 45*a^2*b^2*c^2*d^2*e^4*g^2*z + 2*b^5*c*d^4*e^
2*g*h*z - b^4*c^2*d^5*e*g*h*z + 8*a^4*c^2*d*e^5*g*h*z + 8*a^2*c^4*d^5*e*g*h
*z + 2*b^3*c^3*d^5*e*f*h*z - 14*b^2*c^4*d^5*e*f*g*z - 2*b^5*c*d^2*e^4*f*g*z
 + 2*a*b^5*d^2*e^4*g*h*z - a^2*b^4*d*e^5*g*h*z - 120*a^3*c^3*d*e^5*f*g*z -
6*a^3*b^2*c*e^6*f*h*z + 12*a^3*b*c^2*e^6*f*g*z - 2*a^2*b^3*c*e^6*f*g*z - 4*
a^4*b*c*d*e^5*h^2*z - 4*a*b*c^4*d^5*e*g^2*z + 6*a^3*b^2*c*d^2*e^4*h^2*z + 2
*a^2*b^3*c*d^3*e^3*h^2*z + 6*a*b^2*c^3*d^4*e^2*g^2*z + 2*a*b^3*c^2*d^3*e^3*
g^2*z - 18*a*b^2*c^3*d^2*e^4*f^2*z - b^6*d^2*e^4*f*h*z + 12*b*c^5*d^5*e*f^2
*z + 12*a*b^4*c*e^6*f^2*z + 56*a^3*c^3*d^4*e^2*h^2*z - 5*b^4*c^2*d^4*e^2*g^
2*z - 4*a^4*c^2*d^2*e^4*h^2*z + 56*a^3*c^3*d^2*e^4*g^2*z - 9*b^2*c^4*d^4*e^
2*f^2*z - 5*a^2*b^4*d^2*e^4*h^2*z - 4*a^2*c^4*d^4*e^2*g^2*z + 3*b^4*c^2*d^2
*e^4*f^2*z - 2*b^3*c^3*d^3*e^3*f^2*z - 36*a^2*c^4*d^2*e^4*f^2*z - 45*a^2*b^
2*c^2*e^6*f^2*z + 2*b^6*d*e^5*f*g*z - 8*a*c^5*d^6*f*h*z + 4*b*c^5*d^6*f*g*z
 + 4*b^3*c^3*d^5*e*g^2*z + 2*b^5*c*d^3*e^3*g^2*z + 4*a^3*b^3*d*e^5*h^2*z +
2*a*b^5*d^3*e^3*h^2*z - 24*a*c^5*d^4*e^2*f^2*z + b^6*d^3*e^3*g*h*z + a^2*b^
4*e^6*f*h*z - b^6*d^4*e^2*h^2*z - b^6*d^2*e^4*g^2*z - 4*a^4*c^2*e^6*g^2*z -
 4*a^2*c^4*d^6*h^2*z - b^2*c^4*d^6*g^2*z - a^4*b^2*e^6*h^2*z + 48*a^3*c^3*e
^6*f^2*z - 4*c^6*d^6*f^2*z - b^6*e^6*f^2*z - 16*a*b*c^2*d^2*e^3*f*g*h - 4*a
*b^2*c*d*e^4*f*g*h - 4*b*c^3*d^4*e*f*g*h - 4*a^2*b*c*e^5*f*g*h + 6*b^2*c^2*
d^3*e^2*f*g*h - 8*a^2*b*c*d^2*e^3*g*h^2 + 8*a*b*c^2*d^3*e^2*g^2*h + 2*a*b^2
*c*d^3*e^2*g*h^2 - 2*a*b^2*c*d^2*e^3*g^2*h + 6*a*b^2*c*d^2*e^3*f*h^2 + 4*b^
3*c*d^2*e^3*f*g*h - 16*a*c^3*d^3*e^2*f*g*h - 8*a^2*c^2*d*e^4*f*g*h + 4*a^2*
b*c*d*e^4*g^2*h - 4*a*b*c^2*d^4*e*g*h^2 + 4*a^2*b*c*d*e^4*f*h^2 + 16*a*b*c^
2*d*e^4*f*g^2 - 2*b^3*c*d*e^4*f^2*h + 8*a*c^3*d^4*e*f*h^2 - 4*b^3*c*d*e^4*f
*g^2 - 24*a*c^3*d*e^4*f^2*g - 2*a*b^3*d*e^4*f*h^2 + 6*a*b^2*c*e^5*f^2*h - 1
2*a*b*c^2*e^5*f^2*g - 12*a^2*c^2*d^3*e^2*g*h^2 + 12*a^2*c^2*d^2*e^3*g^2*h -
 3*b^2*c^2*d^2*e^3*f^2*h - 5*b^2*c^2*d^2*e^3*f*g^2 + 4*a^2*c^2*d^2*e^3*f*h^
2 + 2*b^4*d*e^4*f*g*h - 2*b^3*c*d^3*e^2*g^2*h - 4*b*c^3*d^3*e^2*f^2*h - 2*b
^3*c*d^3*e^2*f*h^2 + 24*a*c^3*d^2*e^3*f^2*h + 9*b^2*c^2*d*e^4*f^2*g + 4*b*c
^3*d^3*e^2*f*g^2 + 2*a*b^3*d^2*e^3*g*h^2 - a^2*b^2*d*e^4*g*h^2 + 8*a*c^3*d^
```

```
2*e^3*f*g^2 + 4*a^2*b*c*d^3*e^2*h^3 - 4*a*b*c^2*d^2*e^3*g^3 - b^4*d^2*e^3*g
^2*h - 4*c^4*d^3*e^2*f^2*g - b^4*d^2*e^3*f*h^2 + 4*a^2*c^2*e^5*f*g^2 + 4*a^
2*c^2*d^4*e*h^3 + 2*b^3*c*d^2*e^3*g^3 - 4*a^2*c^2*d*e^4*g^3 - 2*a*b^3*d^3*e
^2*h^3 + 4*c^4*d^4*e*f^2*h + 2*b^3*c*e^5*f^2*g - 4*b*c^3*d*e^4*f^3 + b^2*c^
2*d^4*e*g^2*h - b^2*c^2*d^3*e^2*g^3 + b^4*d^3*e^2*g*h^2 + a^2*b^2*e^5*f*h^2
 + 4*c^4*d^2*e^3*f^3 - 3*b^2*c^2*e^5*f^3 + a^2*b^2*d^2*e^3*h^3 - b^4*e^5*f^
2*h + 16*a*c^3*e^5*f^3, z, k)*((a*b^5*c*e^6*f - 8*a^4*c^3*e^6*g + 8*a*c^6*d
^5*e*f - b^6*c*d*e^5*f + 20*a^3*b*c^3*e^6*f - a^3*b^3*c*e^6*h + 8*a^3*c^4*d
*e^5*f + 4*a^4*b*c^2*e^6*h - 2*b^2*c^5*d^5*e*f + 8*a^2*c^5*d^5*e*h + 8*a^4*
c^3*d*e^5*h + b^3*c^4*d^5*e*g + b^6*c*d^2*e^4*g - b^6*c*d^3*e^3*h - 9*a^2*b
^3*c^2*e^6*f + 2*a^3*b^2*c^2*e^6*g + 16*a^2*c^5*d^3*e^3*f - 8*a^2*c^5*d^4*e
^2*g - 16*a^3*c^4*d^2*e^4*g + 3*b^3*c^4*d^4*e^2*f + 16*a^3*c^4*d^3*e^3*h -
2*b^4*c^3*d^4*e^2*g + b^5*c^2*d^4*e^2*h - 4*a*b^2*c^4*d^3*e^3*f - 2*a*b^3*c
^3*d^2*e^4*f + 8*a^2*b*c^4*d^2*e^4*f - 26*a^2*b^2*c^3*d*e^5*f + 10*a*b^2*c^
4*d^4*e^2*g + 2*a*b^3*c^3*d^3*e^3*g - 8*a*b^4*c^2*d^2*e^4*g - 8*a^2*b*c^4*d
^3*e^3*g + 5*a^2*b^3*c^2*d*e^5*g - 5*a*b^3*c^3*d^4*e^2*h + 8*a*b^4*c^2*d^3*
e^3*h + 4*a^2*b*c^4*d^4*e^2*h + 8*a^3*b*c^3*d^2*e^4*h - 10*a^3*b^2*c^2*d*e^
5*h - 4*a*b*c^5*d^5*e*g - a*b^5*c*d*e^5*g + 20*a^2*b^2*c^3*d^2*e^4*g - 20*a
^2*b^2*c^3*d^3*e^3*h - 2*a^2*b^3*c^2*d^2*e^4*h - 12*a*b*c^5*d^4*e^2*f + 10*
a*b^4*c^2*d*e^5*f - 4*a^3*b*c^3*d*e^5*g - 2*a*b^2*c^4*d^5*e*h + 2*a^2*b^4*c
*d*e^5*h)/(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^
6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^
5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b
*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3) + root(768*a^5*b*c^4*
d^3*e^5*z^3 + 768*a^4*b*c^5*d^5*e^3*z^3 - 192*a^5*b^3*c^2*d*e^7*z^3 - 192*a
^2*b^3*c^5*d^7*e*z^3 - 68*a^3*b^6*c*d^2*e^6*z^3 - 68*a*b^6*c^3*d^6*e^2*z^3
 + 36*a^2*b^7*c*d^3*e^5*z^3 + 36*a*b^7*c^2*d^5*e^3*z^3 + 256*a^6*b*c^3*d*e^7
*z^3 + 256*a^3*b*c^6*d^7*e*z^3 + 48*a^4*b^5*c*d*e^7*z^3 + 48*a*b^5*c^4*d^7*
e*z^3 - 480*a^4*b^2*c^4*d^4*e^4*z^3 + 440*a^3*b^4*c^3*d^4*e^4*z^3 - 320*a^4
*b^3*c^3*d^3*e^5*z^3 - 320*a^3*b^3*c^4*d^5*e^3*z^3 + 240*a^4*b^4*c^2*d^2*e^
6*z^3 + 240*a^2*b^4*c^4*d^6*e^2*z^3 - 192*a^5*b^2*c^3*d^2*e^6*z^3 - 192*a^3
*b^2*c^5*d^6*e^2*z^3 - 90*a^2*b^6*c^2*d^4*e^4*z^3 - 48*a^3*b^5*c^2*d^3*e^5*
z^3 - 48*a^2*b^5*c^3*d^5*e^3*z^3 - 4*b^9*c*d^5*e^3*z^3 - 4*b^7*c^3*d^7*e*z^
3 - 4*a^3*b^7*d*e^7*z^3 - 4*a*b^9*d^3*e^5*z^3 - 12*a^5*b^4*c*e^8*z^3 - 12*a
*b^4*c^5*d^8*z^3 + 6*b^8*c^2*d^6*e^2*z^3 - 384*a^5*c^5*d^4*e^4*z^3 - 256*a^
6*c^4*d^2*e^6*z^3 - 256*a^4*c^6*d^6*e^2*z^3 + 6*a^2*b^8*d^2*e^6*z^3 + 48*a^
6*b^2*c^2*e^8*z^3 + 48*a^2*b^2*c^6*d^8*z^3 - 64*a^7*c^3*e^8*z^3 - 64*a^3*c^
7*d^8*z^3 + b^10*d^4*e^4*z^3 + b^6*c^4*d^8*z^3 + a^4*b^6*e^8*z^3 - 28*a*b^4
*c*d^3*e^3*g*h*z - 10*a^3*b^2*c*d*e^5*g*h*z - 10*a*b^2*c^3*d^5*e*g*h*z + 16
*a*b^4*c*d^2*e^4*f*h*z + 14*a^2*b^3*c*d*e^5*f*h*z + 4*a*b*c^4*d^4*e^2*f*g*z
 + 84*a^2*b^2*c^2*d^3*e^3*g*h*z - 108*a^2*b^2*c^2*d^2*e^4*f*h*z + 16*a*b*c^
4*d^5*e*f*h*z - 20*a*b^4*c*d*e^5*f*g*z + 8*a^2*b^3*c*d^2*e^4*g*h*z + 8*a*b^
3*c^2*d^4*e^2*g*h*z - 4*a^3*b*c^2*d^2*e^4*g*h*z - 4*a^2*b*c^3*d^4*e^2*g*h*z
 + 16*a^2*b*c^3*d^3*e^3*f*h*z + 16*a*b^3*c^2*d^3*e^3*f*h*z - 14*a*b^2*c^3*d
^4*e^2*f*h*z + 66*a^2*b^2*c^2*d*e^5*f*g*z - 36*a*b^2*c^3*d^3*e^3*f*g*z + 20
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*a*b^3*c^2*d^2*e^4*f*g*z + 12*a^2*b*c^3*d^2*e^4*f*g*z + 8*a*c^5*d^5*e*f*g*z
 + 4*a^4*b*c*e^6*g*h*z - 2*a*b^5*d*e^5*f*h*z + 4*a*b*c^4*d^6*g*h*z - 112*a^
3*c^3*d^3*e^3*g*h*z - 3*b^4*c^2*d^4*e^2*f*h*z + 120*a^3*c^3*d^2*e^4*f*h*z -
 16*a^2*c^4*d^4*e^2*f*h*z + 14*b^3*c^3*d^4*e^2*f*g*z - 2*b^4*c^2*d^3*e^3*f*
g*z + 16*a^2*c^4*d^3*e^3*f*g*z + 8*a*b^4*c*d^4*e^2*h^2*z + 4*a^2*b*c^3*d^5*
e*h^2*z + 2*a*b^3*c^2*d^5*e*h^2*z + 8*a*b^4*c*d^2*e^4*g^2*z + 4*a^3*b*c^2*d
*e^5*g^2*z + 2*a^2*b^3*c*d*e^5*g^2*z + 48*a*b*c^4*d^3*e^3*f^2*z + 36*a^2*b*
c^3*d*e^5*f^2*z - 6*a*b^3*c^2*d*e^5*f^2*z - 45*a^2*b^2*c^2*d^4*e^2*h^2*z -
45*a^2*b^2*c^2*d^2*e^4*g^2*z + 2*b^5*c*d^4*e^2*g*h*z - b^4*c^2*d^5*e*g*h*z
+ 8*a^4*c^2*d*e^5*g*h*z + 8*a^2*c^4*d^5*e*g*h*z + 2*b^3*c^3*d^5*e*f*h*z - 1
4*b^2*c^4*d^5*e*f*g*z - 2*b^5*c*d^2*e^4*f*g*z + 2*a*b^5*d^2*e^4*g*h*z - a^2
*b^4*d*e^5*g*h*z - 120*a^3*c^3*d*e^5*f*g*z - 6*a^3*b^2*c*e^6*f*h*z + 12*a^3
*b*c^2*e^6*f*g*z - 2*a^2*b^3*c*e^6*f*g*z - 4*a^4*b*c*d*e^5*h^2*z - 4*a*b*c^
4*d^5*e*g^2*z + 6*a^3*b^2*c*d^2*e^4*h^2*z + 2*a^2*b^3*c*d^3*e^3*h^2*z + 6*a
*b^2*c^3*d^4*e^2*g^2*z + 2*a*b^3*c^2*d^3*e^3*g^2*z - 18*a*b^2*c^3*d^2*e^4*f
^2*z - b^6*d^2*e^4*f*h*z + 12*b*c^5*d^5*e*f^2*z + 12*a*b^4*c*e^6*f^2*z + 56
*a^3*c^3*d^4*e^2*h^2*z - 5*b^4*c^2*d^4*e^2*g^2*z - 4*a^4*c^2*d^2*e^4*h^2*z
+ 56*a^3*c^3*d^2*e^4*g^2*z - 9*b^2*c^4*d^4*e^2*f^2*z - 5*a^2*b^4*d^2*e^4*h^
2*z - 4*a^2*c^4*d^4*e^2*g^2*z + 3*b^4*c^2*d^2*e^4*f^2*z - 2*b^3*c^3*d^3*e^3
*f^2*z - 36*a^2*c^4*d^2*e^4*f^2*z - 45*a^2*b^2*c^2*e^6*f^2*z + 2*b^6*d*e^5*
f*g*z - 8*a*c^5*d^6*f*h*z + 4*b*c^5*d^6*f*g*z + 4*b^3*c^3*d^5*e*g^2*z + 2*b
^5*c*d^3*e^3*g^2*z + 4*a^3*b^3*d*e^5*h^2*z + 2*a*b^5*d^3*e^3*h^2*z - 24*a*c
^5*d^4*e^2*f^2*z + b^6*d^3*e^3*g*h*z + a^2*b^4*e^6*f*h*z - b^6*d^4*e^2*h^2*
z - b^6*d^2*e^4*g^2*z - 4*a^4*c^2*e^6*g^2*z - 4*a^2*c^4*d^6*h^2*z - b^2*c^4
*d^6*g^2*z - a^4*b^2*e^6*h^2*z + 48*a^3*c^3*e^6*f^2*z - 4*c^6*d^6*f^2*z - b
^6*e^6*f^2*z - 16*a*b*c^2*d^2*e^3*f*g*h - 4*a*b^2*c*d*e^4*f*g*h - 4*b*c^3*d
^4*e*f*g*h - 4*a^2*b*c*e^5*f*g*h + 6*b^2*c^2*d^3*e^2*f*g*h - 8*a^2*b*c*d^2*
e^3*g*h^2 + 8*a*b*c^2*d^3*e^2*g^2*h + 2*a*b^2*c*d^3*e^2*g*h^2 - 2*a*b^2*c*d
^2*e^3*g^2*h + 6*a*b^2*c*d^2*e^3*f*h^2 + 4*b^3*c*d^2*e^3*f*g*h - 16*a*c^3*d
^3*e^2*f*g*h - 8*a^2*c^2*d*e^4*f*g*h + 4*a^2*b*c*d*e^4*g^2*h - 4*a*b*c^2*d^
4*e*g*h^2 + 4*a^2*b*c*d*e^4*f*h^2 + 16*a*b*c^2*d*e^4*f*g^2 - 2*b^3*c*d*e^4*
f^2*h + 8*a*c^3*d^4*e*f*h^2 - 4*b^3*c*d*e^4*f*g^2 - 24*a*c^3*d*e^4*f^2*g -
2*a*b^3*d*e^4*f*h^2 + 6*a*b^2*c*e^5*f^2*h - 12*a*b*c^2*e^5*f^2*g - 12*a^2*c
^2*d^3*e^2*g*h^2 + 12*a^2*c^2*d^2*e^3*g^2*h - 3*b^2*c^2*d^2*e^3*f^2*h - 5*b
^2*c^2*d^2*e^3*f*g^2 + 4*a^2*c^2*d^2*e^3*f*h^2 + 2*b^4*d*e^4*f*g*h - 2*b^3*
c*d^3*e^2*g^2*h - 4*b*c^3*d^3*e^2*f^2*h - 2*b^3*c*d^3*e^2*f*h^2 + 24*a*c^3*
d^2*e^3*f^2*h + 9*b^2*c^2*d*e^4*f^2*g + 4*b*c^3*d^3*e^2*f*g^2 + 2*a*b^3*d^2
*e^3*g*h^2 - a^2*b^2*d*e^4*g*h^2 + 8*a*c^3*d^2*e^3*f*g^2 + 4*a^2*b*c*d^3*e^
2*h^3 - 4*a*b*c^2*d^2*e^3*g^3 - b^4*d^2*e^3*g^2*h - 4*c^4*d^3*e^2*f^2*g - b
^4*d^2*e^3*f*h^2 + 4*a^2*c^2*e^5*f*g^2 + 4*a^2*c^2*d^4*e*h^3 + 2*b^3*c*d^2*
e^3*g^3 - 4*a^2*c^2*d*e^4*g^3 - 2*a*b^3*d^3*e^2*h^3 + 4*c^4*d^4*e*f^2*h + 2
*b^3*c*e^5*f^2*g - 4*b*c^3*d*e^4*f^3 + b^2*c^2*d^4*e*g^2*h - b^2*c^2*d^3*e^
2*g^3 + b^4*d^3*e^2*g*h^2 + a^2*b^2*e^5*f*h^2 + 4*c^4*d^2*e^3*f^3 - 3*b^2*c
^2*e^5*f^3 + a^2*b^2*d^2*e^3*h^3 - b^4*e^5*f^2*h + 16*a*c^3*e^5*f^3, z, k)*
((128*a^5*c^4*d*e^6 - 16*a^5*b*c^3*e^7 - a^3*b^5*c*e^7 - b^5*c^4*d^6*e - b^
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8*c*d^3*e^4 + 8*a^4*b^3*c^2*e^7 + 128*a^3*c^6*d^5*e^2 + 256*a^4*c^5*d^3*e^4
 + b^6*c^3*d^5*e^2 + b^7*c^2*d^4*e^3 - 48*a^2*b^2*c^5*d^5*e^2 + 168*a^2*b^3
*c^4*d^4*e^3 - 80*a^2*b^4*c^3*d^3*e^4 - 27*a^2*b^5*c^2*d^2*e^5 + 32*a^3*b^2
*c^4*d^3*e^4 + 168*a^3*b^3*c^3*d^2*e^5 + 8*a*b^3*c^5*d^6*e + a*b^7*c*d^2*e^
5 - 16*a^2*b*c^6*d^6*e + a^2*b^6*c*d*e^6 - 27*a*b^5*c^3*d^4*e^3 + 18*a*b^6*
c^2*d^3*e^4 - 304*a^3*b*c^5*d^4*e^3 - 304*a^4*b*c^4*d^2*e^5 - 48*a^4*b^2*c^
3*d*e^6)/(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6
*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5
*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*
c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3) - (x*(2*a^2*b^6*c*e^7
 - 96*a^5*c^4*e^7 + 32*a^2*c^7*d^6*e + 2*b^4*c^5*d^6*e + 2*b^8*c*d^2*e^5 - 2
2*a^3*b^4*c^2*e^7 + 80*a^4*b^2*c^3*e^7 - 32*a^3*c^6*d^4*e^3 - 160*a^4*c^5*d
^2*e^5 - 6*b^5*c^4*d^5*e^2 + 8*b^6*c^3*d^4*e^3 - 6*b^7*c^2*d^3*e^4 - 4*a*b^
7*c*d*e^6 + 144*a^2*b^2*c^5*d^4*e^3 - 128*a^2*b^3*c^4*d^3*e^4 + 6*a^2*b^4*c
^3*d^2*e^5 + 112*a^3*b^2*c^4*d^2*e^5 - 16*a*b^2*c^6*d^6*e + 160*a^4*b*c^4*d
*e^6 + 48*a*b^3*c^5*d^5*e^2 - 66*a*b^4*c^4*d^4*e^3 + 52*a*b^5*c^3*d^3*e^4 -
 14*a*b^6*c^2*d^2*e^5 - 96*a^2*b*c^6*d^5*e^2 + 42*a^2*b^5*c^2*d*e^6 + 64*a^
3*b*c^5*d^3*e^4 - 144*a^3*b^3*c^3*d*e^6))/(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 1
6*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e
^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*
e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*
c^2*d*e^3)) - (x*(8*a^3*b*c^3*e^6*g - 2*a*b^4*c^2*e^6*f - 48*a^3*c^4*e^6*f
 - 16*a*c^6*d^4*e^2*f + a^2*b^4*c*e^6*h + 32*a^3*c^4*d*e^5*g + 2*b^5*c^2*d*e
^5*f + b^6*c*d^2*e^4*h + 20*a^2*b^2*c^3*e^6*f - 2*a^2*b^3*c^2*e^6*g - 64*a^
2*c^5*d^2*e^4*f - 4*a^3*b^2*c^2*e^6*h + 32*a^2*c^5*d^3*e^3*g + 4*b^2*c^5*d^
4*e^2*f - 8*b^3*c^4*d^3*e^3*f + 2*b^4*c^3*d^2*e^4*f - 32*a^2*c^5*d^4*e^2*h
 - 32*a^3*c^4*d^2*e^4*h - 2*b^3*c^4*d^4*e^2*g + 6*b^4*c^3*d^3*e^3*g - 4*b^5*
c^2*d^2*e^4*g - b^4*c^3*d^4*e^2*h + 8*a*b^2*c^4*d^2*e^4*f - 32*a*b^2*c^4*d^
3*e^3*g + 20*a*b^3*c^3*d^2*e^4*g - 16*a^2*b*c^4*d^2*e^4*g - 32*a^2*b^2*c^3*
d*e^5*g + 12*a*b^2*c^4*d^4*e^2*h - 8*a*b^3*c^3*d^3*e^3*h - 4*a*b^4*c^2*d^2*
e^4*h + 32*a^2*b*c^4*d^3*e^3*h + 8*a^2*b^3*c^2*d*e^5*h - 2*a*b^5*c*d*e^5*h
 + 8*a^2*b^2*c^3*d^2*e^4*h + 32*a*b*c^5*d^3*e^3*f - 24*a*b^3*c^3*d*e^5*f + 6
4*a^2*b*c^4*d*e^5*f + 8*a*b*c^5*d^4*e^2*g + 6*a*b^4*c^2*d*e^5*g))/(a^2*b^4*
e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2
*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d
^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2
*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)) - (4*a^2*c^3*d^3*e^2*h^2 - 4*c^5*d^3*e^
2*f^2 - b^3*c^2*e^5*f^2 - b^2*c^3*d^3*e^2*g^2 + b^3*c^2*d^2*e^3*g^2 + 4*a*b
*c^3*e^5*f^2 - 8*a*c^4*d*e^4*f^2 - 8*a^2*c^3*e^5*f*g + 4*b*c^4*d^2*e^3*f^2
 + 4*a^2*c^3*d*e^4*g^2 + b^2*c^3*d*e^4*f^2 - 2*a*b^2*c^2*d*e^4*g^2 + a*b^3*c
*d^2*e^3*h^2 - a^2*b^2*c*d*e^4*h^2 - 4*b^2*c^3*d^2*e^3*f*g - 8*a^2*c^3*d^2*
e^3*g*h - 2*b^2*c^3*d^3*e^2*f*h + b^3*c^2*d^2*e^3*f*h + b^3*c^2*d^3*e^2*g*h
 - a*b^3*c*e^5*f*h + b^4*c*d*e^4*f*h - 2*a*b^2*c^2*d^3*e^2*h^2 + 2*a*b^2*c^
2*e^5*f*g + 4*a^2*b*c^2*e^5*f*h + 4*b*c^4*d^3*e^2*f*g + 8*a^2*c^3*d*e^4*f*h
 - b^4*c*d^2*e^3*g*h + 4*a*b*c^3*d^2*e^3*f*h - 8*a*b^2*c^2*d*e^4*f*h + 2*a*
```

```
b^2*c^2*d^2*e^3*g*h + 4*a*b*c^3*d*e^4*f*g + a*b^3*c*d*e^4*g*h)/(a^2*b^4*e^4
 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^
3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*
e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^
3*c*d*e^3 - 32*a^3*b*c^2*d*e^3) + (x*(4*a^2*c^3*e^5*g^2 + b^2*c^3*e^5*f^2 +
 4*c^5*d^2*e^3*f^2 + 4*a^2*c^3*d^2*e^3*h^2 + b^2*c^3*d^2*e^3*g^2 - 4*b*c^4*
d*e^4*f^2 + a^2*b^2*c*e^5*h^2 + b^4*c*d^2*e^3*h^2 + 4*a^2*b*c^2*d*e^4*h^2 +
 4*b^2*c^3*d^2*e^3*f*h - 2*b^3*c^2*d^2*e^3*g*h - 4*a*b*c^3*e^5*f*g + 8*a*c^
4*d*e^4*f*g - 4*a*b^2*c^2*d^2*e^3*h^2 - 4*a*b*c^3*d*e^4*g^2 - 2*a*b^3*c*d*e
^4*h^2 + 2*a*b^2*c^2*e^5*f*h - 4*a^2*b*c^2*e^5*g*h - 8*a*c^4*d^2*e^3*f*h -
4*b*c^4*d^2*e^3*f*g + 2*b^2*c^3*d*e^4*f*g - 8*a^2*c^3*d*e^4*g*h - 2*b^3*c^2
*d*e^4*f*h + 4*a*b*c^3*d^2*e^3*g*h + 6*a*b^2*c^2*d*e^4*g*h))/(a^2*b^4*e^4 +
 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*
d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e
+ 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*
c*d*e^3 - 32*a^3*b*c^2*d*e^3))*root(768*a^5*b*c^4*d^3*e^5*z^3 + 768*a^4*b*c
^5*d^5*e^3*z^3 - 192*a^5*b^3*c^2*d*e^7*z^3 - 192*a^2*b^3*c^5*d^7*e*z^3 - 68
*a^3*b^6*c*d^2*e^6*z^3 - 68*a*b^6*c^3*d^6*e^2*z^3 + 36*a^2*b^7*c*d^3*e^5*z^
3 + 36*a*b^7*c^2*d^5*e^3*z^3 + 256*a^6*b*c^3*d*e^7*z^3 + 256*a^3*b*c^6*d^7*
e*z^3 + 48*a^4*b^5*c*d*e^7*z^3 + 48*a*b^5*c^4*d^7*e*z^3 - 480*a^4*b^2*c^4*d
^4*e^4*z^3 + 440*a^3*b^4*c^3*d^4*e^4*z^3 - 320*a^4*b^3*c^3*d^3*e^5*z^3 - 32
0*a^3*b^3*c^4*d^5*e^3*z^3 + 240*a^4*b^4*c^2*d^2*e^6*z^3 + 240*a^2*b^4*c^4*d
^6*e^2*z^3 - 192*a^5*b^2*c^3*d^2*e^6*z^3 - 192*a^3*b^2*c^5*d^6*e^2*z^3 - 90
*a^2*b^6*c^2*d^4*e^4*z^3 - 48*a^3*b^5*c^2*d^3*e^5*z^3 - 48*a^2*b^5*c^3*d^5*
e^3*z^3 - 4*b^9*c*d^5*e^3*z^3 - 4*b^7*c^3*d^7*e*z^3 - 4*a^3*b^7*d*e^7*z^3 -
 4*a*b^9*d^3*e^5*z^3 - 12*a^5*b^4*c*e^8*z^3 - 12*a*b^4*c^5*d^8*z^3 + 6*b^8*
c^2*d^6*e^2*z^3 - 384*a^5*c^5*d^4*e^4*z^3 - 256*a^6*c^4*d^2*e^6*z^3 - 256*a
^4*c^6*d^6*e^2*z^3 + 6*a^2*b^8*d^2*e^6*z^3 + 48*a^6*b^2*c^2*e^8*z^3 + 48*a^
2*b^2*c^6*d^8*z^3 - 64*a^7*c^3*e^8*z^3 - 64*a^3*c^7*d^8*z^3 + b^10*d^4*e^4*
z^3 + b^6*c^4*d^8*z^3 + a^4*b^6*e^8*z^3 - 28*a*b^4*c*d^3*e^3*g*h*z - 10*a^3
*b^2*c*d*e^5*g*h*z - 10*a*b^2*c^3*d^5*e*g*h*z + 16*a*b^4*c*d^2*e^4*f*h*z +
14*a^2*b^3*c*d*e^5*f*h*z + 4*a*b*c^4*d^4*e^2*f*g*z + 84*a^2*b^2*c^2*d^3*e^3
*g*h*z - 108*a^2*b^2*c^2*d^2*e^4*f*h*z + 16*a*b*c^4*d^5*e*f*h*z - 20*a*b^4*
c*d*e^5*f*g*z + 8*a^2*b^3*c*d^2*e^4*g*h*z + 8*a*b^3*c^2*d^4*e^2*g*h*z - 4*a
^3*b*c^2*d^2*e^4*g*h*z - 4*a^2*b*c^3*d^4*e^2*g*h*z + 16*a^2*b*c^3*d^3*e^3*f
*h*z + 16*a*b^3*c^2*d^3*e^3*f*h*z - 14*a*b^2*c^3*d^4*e^2*f*h*z + 66*a^2*b^2
*c^2*d*e^5*f*g*z - 36*a*b^2*c^3*d^3*e^3*f*g*z + 20*a*b^3*c^2*d^2*e^4*f*g*z
+ 12*a^2*b*c^3*d^2*e^4*f*g*z + 8*a*c^5*d^5*e*f*g*z + 4*a^4*b*c*e^6*g*h*z -
2*a*b^5*d*e^5*f*h*z + 4*a*b*c^4*d^6*g*h*z - 112*a^3*c^3*d^3*e^3*g*h*z - 3*b
^4*c^2*d^4*e^2*f*h*z + 120*a^3*c^3*d^2*e^4*f*h*z - 16*a^2*c^4*d^4*e^2*f*h*z
 + 14*b^3*c^3*d^4*e^2*f*g*z - 2*b^4*c^2*d^3*e^3*f*g*z + 16*a^2*c^4*d^3*e^3*
f*g*z + 8*a*b^4*c*d^4*e^2*h^2*z + 4*a^2*b*c^3*d^5*e*h^2*z + 2*a*b^3*c^2*d^5
*e*h^2*z + 8*a*b^4*c*d^2*e^4*g^2*z + 4*a^3*b*c^2*d*e^5*g^2*z + 2*a^2*b^3*c*
d*e^5*g^2*z + 48*a*b*c^4*d^3*e^3*f^2*z + 36*a^2*b*c^3*d*e^5*f^2*z - 6*a*b^3
*c^2*d*e^5*f^2*z - 45*a^2*b^2*c^2*d^4*e^2*h^2*z - 45*a^2*b^2*c^2*d^2*e^4*g^
```

```
2*z + 2*b^5*c*d^4*e^2*g*h*z - b^4*c^2*d^5*e*g*h*z + 8*a^4*c^2*d*e^5*g*h*z +
 8*a^2*c^4*d^5*e*g*h*z + 2*b^3*c^3*d^5*e*f*h*z - 14*b^2*c^4*d^5*e*f*g*z - 2
*b^5*c*d^2*e^4*f*g*z + 2*a*b^5*d^2*e^4*g*h*z - a^2*b^4*d*e^5*g*h*z - 120*a^
3*c^3*d*e^5*f*g*z - 6*a^3*b^2*c*e^6*f*h*z + 12*a^3*b*c^2*e^6*f*g*z - 2*a^2*
b^3*c*e^6*f*g*z - 4*a^4*b*c*d*e^5*h^2*z - 4*a*b*c^4*d^5*e*g^2*z + 6*a^3*b^2
*c*d^2*e^4*h^2*z + 2*a^2*b^3*c*d^3*e^3*h^2*z + 6*a*b^2*c^3*d^4*e^2*g^2*z +
2*a*b^3*c^2*d^3*e^3*g^2*z - 18*a*b^2*c^3*d^2*e^4*f^2*z - b^6*d^2*e^4*f*h*z
+ 12*b*c^5*d^5*e*f^2*z + 12*a*b^4*c*e^6*f^2*z + 56*a^3*c^3*d^4*e^2*h^2*z -
5*b^4*c^2*d^4*e^2*g^2*z - 4*a^4*c^2*d^2*e^4*h^2*z + 56*a^3*c^3*d^2*e^4*g^2*
z - 9*b^2*c^4*d^4*e^2*f^2*z - 5*a^2*b^4*d^2*e^4*h^2*z - 4*a^2*c^4*d^4*e^2*g
^2*z + 3*b^4*c^2*d^2*e^4*f^2*z - 2*b^3*c^3*d^3*e^3*f^2*z - 36*a^2*c^4*d^2*e
^4*f^2*z - 45*a^2*b^2*c^2*e^6*f^2*z + 2*b^6*d*e^5*f*g*z - 8*a*c^5*d^6*f*h*z
 + 4*b*c^5*d^6*f*g*z + 4*b^3*c^3*d^5*e*g^2*z + 2*b^5*c*d^3*e^3*g^2*z + 4*a^
3*b^3*d*e^5*h^2*z + 2*a*b^5*d^3*e^3*h^2*z - 24*a*c^5*d^4*e^2*f^2*z + b^6*d^
3*e^3*g*h*z + a^2*b^4*e^6*f*h*z - b^6*d^4*e^2*h^2*z - b^6*d^2*e^4*g^2*z - 4
*a^4*c^2*e^6*g^2*z - 4*a^2*c^4*d^6*h^2*z - b^2*c^4*d^6*g^2*z - a^4*b^2*e^6*
h^2*z + 48*a^3*c^3*e^6*f^2*z - 4*c^6*d^6*f^2*z - b^6*e^6*f^2*z - 16*a*b*c^2
*d^2*e^3*f*g*h - 4*a*b^2*c*d*e^4*f*g*h - 4*b*c^3*d^4*e*f*g*h - 4*a^2*b*c*e^
5*f*g*h + 6*b^2*c^2*d^3*e^2*f*g*h - 8*a^2*b*c*d^2*e^3*g*h^2 + 8*a*b*c^2*d^3
*e^2*g^2*h + 2*a*b^2*c*d^3*e^2*g*h^2 - 2*a*b^2*c*d^2*e^3*g^2*h + 6*a*b^2*c*
d^2*e^3*f*h^2 + 4*b^3*c*d^2*e^3*f*g*h - 16*a*c^3*d^3*e^2*f*g*h - 8*a^2*c^2*
d*e^4*f*g*h + 4*a^2*b*c*d*e^4*g^2*h - 4*a*b*c^2*d^4*e*g*h^2 + 4*a^2*b*c*d*e
^4*f*h^2 + 16*a*b*c^2*d*e^4*f*g^2 - 2*b^3*c*d*e^4*f^2*h + 8*a*c^3*d^4*e*f*h
^2 - 4*b^3*c*d*e^4*f*g^2 - 24*a*c^3*d*e^4*f^2*g - 2*a*b^3*d*e^4*f*h^2 + 6*a
*b^2*c*e^5*f^2*h - 12*a*b*c^2*e^5*f^2*g - 12*a^2*c^2*d^3*e^2*g*h^2 + 12*a^2
*c^2*d^2*e^3*g^2*h - 3*b^2*c^2*d^2*e^3*f^2*h - 5*b^2*c^2*d^2*e^3*f*g^2 + 4*
a^2*c^2*d^2*e^3*f*h^2 + 2*b^4*d*e^4*f*g*h - 2*b^3*c*d^3*e^2*g^2*h - 4*b*c^3
*d^3*e^2*f^2*h - 2*b^3*c*d^3*e^2*f*h^2 + 24*a*c^3*d^2*e^3*f^2*h + 9*b^2*c^2
*d*e^4*f^2*g + 4*b*c^3*d^3*e^2*f*g^2 + 2*a*b^3*d^2*e^3*g*h^2 - a^2*b^2*d*e^
4*g*h^2 + 8*a*c^3*d^2*e^3*f*g^2 + 4*a^2*b*c*d^3*e^2*h^3 - 4*a*b*c^2*d^2*e^3
*g^3 - b^4*d^2*e^3*g^2*h - 4*c^4*d^3*e^2*f^2*g - b^4*d^2*e^3*f*h^2 + 4*a^2*
c^2*e^5*f*g^2 + 4*a^2*c^2*d^4*e*h^3 + 2*b^3*c*d^2*e^3*g^3 - 4*a^2*c^2*d*e^4
*g^3 - 2*a*b^3*d^3*e^2*h^3 + 4*c^4*d^4*e*f^2*h + 2*b^3*c*e^5*f^2*g - 4*b*c^
3*d*e^4*f^3 + b^2*c^2*d^4*e*g^2*h - b^2*c^2*d^3*e^2*g^3 + b^4*d^3*e^2*g*h^2
 + a^2*b^2*e^5*f*h^2 + 4*c^4*d^2*e^3*f^3 - 3*b^2*c^2*e^5*f^3 + a^2*b^2*d^2*
e^3*h^3 - b^4*e^5*f^2*h + 16*a*c^3*e^5*f^3, z, k), k, 1, 3) - ((a*b*d*h - 2
*a^2*e*h - b^2*e*f + a*b*e*g - 2*a*c*d*g + 2*a*c*e*f + b*c*d*f)/(a*b^2*e^2
- 4*a*c^2*d^2 - 4*a^2*c*e^2 + b^2*c*d^2 - b^3*d*e + 4*a*b*c*d*e) - (x*(a*b*
e*h - b^2*d*h - 2*c^2*d*f + 2*a*c*d*h - 2*a*c*e*g + b*c*d*g + b*c*e*f))/(a*
b^2*e^2 - 4*a*c^2*d^2 - 4*a^2*c*e^2 + b^2*c*d^2 - b^3*d*e + 4*a*b*c*d*e))/(
a + b*x + c*x^2)
```

3.159 $\int \frac{f+gx+hx^2}{(d+ex)^2\left(a+bx+cx^2\right)^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 673

$$\begin{aligned}\int \frac{f+gx+hx^2}{(d+ex)^2\left(a+bx+cx^2\right)^2}\,dx = &-\frac{e(e^2f-deg+d^2h)}{\left(cd^2-bde+ae^2\right)^2(d+ex)}\\ &-\frac{b^3e^2f-b^2e(2cdf+aeg)+2ac(cd(2ef-dg)+ae(eg-2dh))+b(c^2d^2f+a^2e^2h-ac(3e^2f-2deg-d^2}{(b^2-4ac)\left(cd^2-bde\right.}\\ &+\frac{(4c^4d^4f-b^3e^3(2bef-bdg-aeg+2adh)-2c^3d^2(bd(4ef+dg)-2a(6e^2f-2deg+d^2h))-6c^2e(4abd}{}\\ &-\frac{e(e^2(2bef-bdg-aeg+2adh)-cd(4e^2f-3deg+2d^2h))\log(d+ex)}{\left(cd^2-bde+ae^2\right)^3}\\ &+\frac{e(e^2(2bef-bdg-aeg+2adh)-cd(4e^2f-3deg+2d^2h))\log\left(a+bx+cx^2\right)}{2\left(cd^2-bde+ae^2\right)^3}\end{aligned}$$

```
[Out] -e*(d^2*h-d*e*g+e^2*f)/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)+(-b^3*e^2*f+b^2*e*(a*e
*g+2*c*d*f)-2*a*c*(c*d*(-d*g+2*e*f)+a*e*(-2*d*h+e*g))-b*(c^2*d^2*f+a^2*e^2*
h-a*c*(-d^2*h-2*d*e*g+3*e^2*f))-c*(2*c^2*d^2*f+2*a^2*e^2*h-a*b*e*(2*d*h+e*g
)+b^2*(d^2*h+e^2*f)-c*(b*d*(d*g+2*e*f)+2*a*(d^2*h-2*d*e*g+e^2*f)))*x)/(-4*a
*c+b^2)/(a*e^2-b*d*e+c*d^2)^2/(c*x^2+b*x+a)+(4*c^4*d^4*f-b^3*e^3*(2*a*d*h-a
*e*g-b*d*g+2*b*e*f)-2*c^3*d^2*(b*d*(d*g+4*e*f)-2*a*(d^2*h-2*d*e*g+6*e^2*f))
-6*c^2*e*(4*a*b*d*e^2*f-b^2*d^3*g+2*a^2*e*(2*d^2*h-2*d*e*g+e^2*f))-c*e*(6*a
^2*b*e^3*g-4*a^3*e^3*h-b^3*d*(-2*d^2*h-3*d*e*g+4*e^2*f)-6*a*b^2*e*(2*d^2*h-
d*e*g+2*e^2*f)))*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)/(
a*e^2-b*d*e+c*d^2)^3-e*(e^2*(2*a*d*h-a*e*g-b*d*g+2*b*e*f)-c*d*(2*d^2*h-3*d*
e*g+4*e^2*f))*ln(e*x+d)/(a*e^2-b*d*e+c*d^2)^3+1/2*e*(e^2*(2*a*d*h-a*e*g-b*d
*g+2*b*e*f)-c*d*(2*d^2*h-3*d*e*g+4*e^2*f))*ln(c*x^2+b*x+a)/(a*e^2-b*d*e+c*d
^2)^3
```

Rubi [A] (verified)

Time = 1.57 (sec) , antiderivative size = 673, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1660, 1642, 648, 632, 212, 642}

$$
\begin{aligned}
\int \frac{f+gx+hx^2}{(d+ex)^2\left(a+bx+cx^2\right)^2}\,dx = \\
& -\frac{cx(2a^2e^2h-c(2a(d^2h-2deg+e^2f)+bd(dg+2ef))-abe(2dh+eg)+b^2(d^2h+e^2f)+2c^2d^2f)+b(}{(b^2-4ac)\,(a+bx} \\
& +\frac{\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)(-6c^2e(2a^2e(2d^2h-2deg+e^2f)+4abde^2f-b^2d^3g)-ce(-4a^3e^3h+6a^2be^3g-6ab}{} \\
& +\frac{e\log\left(a+bx+cx^2\right)\left(e^2(2adh-aeg-bdg+2bef)-cd(2d^2h-3deg+4e^2f)\right)}{2\left(ae^2-bde+cd^2\right)^3} \\
& -\frac{e(d^2h-deg+e^2f)}{(d+ex)\left(ae^2-bde+cd^2\right)^2} \\
& -\frac{e\log(d+ex)\left(e^2(2adh-aeg-bdg+2bef)-cd(2d^2h-3deg+4e^2f)\right)}{\left(ae^2-bde+cd^2\right)^3}
\end{aligned}
$$

```
[In] Int[(f + g*x + h*x^2)/((d + e*x)^2*(a + b*x + c*x^2)^2),x]
```

```
[Out] -((e*(e^2*f - d*e*g + d^2*h))/((c*d^2 - b*d*e + a*e^2)^2*(d + e*x))) - (b^3
*e^2*f - b^2*e*(2*c*d*f + a*e*g) + 2*a*c*(c*d*(2*e*f - d*g) + a*e*(e*g - 2*
d*h)) + b*(c^2*d^2*f + a^2*e^2*h - a*c*(3*e^2*f - 2*d*e*g - d^2*h)) + c*(2*
c^2*d^2*f + 2*a^2*e^2*h - a*b*e*(e*g + 2*d*h) + b^2*(e^2*f + d^2*h) - c*(b*
d*(2*e*f + d*g) + 2*a*(e^2*f - 2*d*e*g + d^2*h)))*x)/((b^2 - 4*a*c)*(c*d^2
- b*d*e + a*e^2)^2*(a + b*x + c*x^2)) + ((4*c^4*d^4*f - b^3*e^3*(2*b*e*f -
b*d*g - a*e*g + 2*a*d*h) - 2*c^3*d^2*(b*d*(4*e*f + d*g) - 2*a*(6*e^2*f - 2*
d*e*g + d^2*h)) - 6*c^2*e*(4*a*b*d*e^2*f - b^2*d^3*g + 2*a^2*e*(e^2*f - 2*d
*e*g + 2*d^2*h)) - c*e*(6*a^2*b*e^3*g - 4*a^3*e^3*h - b^3*d*(4*e^2*f - 3*d*
e*g - 2*d^2*h) - 6*a*b^2*e*(2*e^2*f - d*e*g + 2*d^2*h)))*ArcTanh[(b + 2*c*x
)/Sqrt[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(3/2)*(c*d^2 - b*d*e + a*e^2)^3) - (e*
(e^2*(2*b*e*f - b*d*g - a*e*g + 2*a*d*h) - c*d*(4*e^2*f - 3*d*e*g + 2*d^2*h
))*Log[d + e*x])/(c*d^2 - b*d*e + a*e^2)^3 + (e*(e^2*(2*b*e*f - b*d*g - a*e
*g + 2*a*d*h) - c*d*(4*e^2*f - 3*d*e*g + 2*d^2*h))*Log[a + b*x + c*x^2])/(2
*(c*d^2 - b*d*e + a*e^2)^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1660

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^(p + 1)/((p
+ 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\text{integral} = -\frac{b^3e^2f-b^2e(2cdf+aeg)+2ac(cd(2ef-dg)+ae(eg-2dh))+b(c^2d^2f+a^2e^2h-ac(3e^2f-2deg-d^2}{(b^2-4ac)\left(cd^2-bde\right.}$$

$$+\frac{\int\frac{\frac{2c^3d^4f-b^2e^2\left(ae^2f-bd(2ef-dg)-ad^2h\right)-c^2d^2\left(bd(2ef+dg)-2a\left(5e^2f-2deg+d^2h\right)\right)-ce\left(2b^2d^2(ef-dg)-2a^2e\left(2e^2f-d^2h\right)+abd\left(8e^2f-3deg+2d^2h\right)\right)}{\left(cd^2-bde+ae^2\right)^2}+\frac{e}{}}{}}{}$$

$$= -\frac{b^3e^2f - b^2e(2cdf + aeg) + 2ac(cd(2ef - dg) + ae(eg - 2dh)) + b(c^2d^2f + a^2e^2h - ac(3e^2f - 2}{(b^2 - 4ac)\,(c}$$

$$+\frac{\int \left(-\frac{(b^2-4ac)e^2(e^2f-deg+d^2h)}{(cd^2-bde+ae^2)^2(d+ex)^2} + \frac{(b^2-4ac)e^2(e^2(2bef-bdg-aeg+2adh)-cd(4e^2f-3deg+2d^2h))}{(cd^2-bde+ae^2)^3(d+ex)} + \frac{2c^4d^4f-b^3e^3(2bef-}{}}{}$$

$$= -\frac{e(e^2f - deg + d^2h)}{(cd^2 - bde + ae^2)^2\,(d + ex)}$$
$$-\frac{b^3e^2f - b^2e(2cdf + aeg) + 2ac(cd(2ef - dg) + ae(eg - 2dh)) + b(c^2d^2f + a^2e^2h - ac(3e^2f - 2}{(b^2 - 4ac)\,(c}$$
$$-\frac{e(e^2(2bef - bdg - aeg + 2adh) - cd(4e^2f - 3deg + 2d^2h))\log(d + ex)}{(cd^2 - bde + ae^2)^3}$$
$$-\frac{\int \frac{2c^4d^4f-b^3e^3(2bef-bdg-aeg+2adh)-c^3d^2(bd(4ef+dg)-2a(6e^2f-2deg+d^2h))+c^2e(3b^2d^3g-2abd(10e^2f-3deg+2d^2h)-6a^2}{}}{(b}$$

$$= -\frac{e(e^2f - deg + d^2h)}{(cd^2 - bde + ae^2)^2\,(d + ex)}$$
$$-\frac{b^3e^2f - b^2e(2cdf + aeg) + 2ac(cd(2ef - dg) + ae(eg - 2dh)) + b(c^2d^2f + a^2e^2h - ac(3e^2f - 2}{(b^2 - 4ac)\,(c}$$
$$-\frac{e(e^2(2bef - bdg - aeg + 2adh) - cd(4e^2f - 3deg + 2d^2h))\log(d + ex)}{(cd^2 - bde + ae^2)^3}$$
$$+\frac{(e(e^2(2bef - bdg - aeg + 2adh) - cd(4e^2f - 3deg + 2d^2h)))\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2\,(cd^2 - bde + ae^2)^3}$$
$$-\frac{(4c^4d^4f - b^3e^3(2bef - bdg - aeg + 2adh) - 2c^3d^2(bd(4ef + dg) - 2a(6e^2f - 2deg + d^2h)) - 6}{}$$

$$= -\frac{e(e^2f - deg + d^2h)}{(cd^2 - bde + ae^2)^2\,(d + ex)}$$
$$-\frac{b^3e^2f - b^2e(2cdf + aeg) + 2ac(cd(2ef - dg) + ae(eg - 2dh)) + b(c^2d^2f + a^2e^2h - ac(3e^2f - 2}{(b^2 - 4ac)\,(c}$$
$$-\frac{e(e^2(2bef - bdg - aeg + 2adh) - cd(4e^2f - 3deg + 2d^2h))\log(d + ex)}{(cd^2 - bde + ae^2)^3}$$
$$+\frac{e(e^2(2bef - bdg - aeg + 2adh) - cd(4e^2f - 3deg + 2d^2h))\log(a + bx + cx^2)}{2\,(cd^2 - bde + ae^2)^3}$$
$$+\frac{(4c^4d^4f - b^3e^3(2bef - bdg - aeg + 2adh) - 2c^3d^2(bd(4ef + dg) - 2a(6e^2f - 2deg + d^2h)) - 6}{}$$

$$= -\frac{e(e^2f - deg + d^2h)}{(cd^2 - bde + ae^2)^2 (d + ex)}$$

$$- \frac{b^3e^2f - b^2e(2cdf + aeg) + 2ac(cd(2ef - dg) + ae(eg - 2dh)) + b(c^2d^2f + a^2e^2h - ac(3e^2f - 2a}{(b^2 - 4ac)(cd}$$

$$+ \frac{(4c^4d^4f - b^3e^3(2bef - bdg - aeg + 2adh) - 2c^3d^2(bd(4ef + dg) - 2a(6e^2f - 2deg + d^2h)) - 6c}{}$$

$$- \frac{e(e^2(2bef - bdg - aeg + 2adh) - cd(4e^2f - 3deg + 2d^2h))\log(d + ex)}{(cd^2 - bde + ae^2)^3}$$

$$+ \frac{e(e^2(2bef - bdg - aeg + 2adh) - cd(4e^2f - 3deg + 2d^2h))\log(a + bx + cx^2)}{2(cd^2 - bde + ae^2)^3}$$

Mathematica [A] (verified)

Time = 1.13 (sec) , antiderivative size = 650, normalized size of antiderivative = 0.97

$$\int \frac{f + gx + hx^2}{(d + ex)^2 (a + bx + cx^2)^2} \, dx = -\frac{e(e^2f - deg + d^2h)}{(cd^2 + e(-bd + ae))^2 (d + ex)}$$

$$+ \frac{-b^3e^2f + b^2(ae^2g - c(-2def + e^2fx + d^2hx)) + b(-a^2e^2h + c^2d(-df + 2efx + dgx) + ac(-d^2h + e^2(3}{(b^2 - 4ac)(cd^2 + e(-b}$$

$$- \frac{(4c^4d^4f + b^3e^3(-2bef + bdg + aeg - 2adh) - 2c^3d^2(bd(4ef + dg) - 2a(6e^2f - 2deg + d^2h)) - 6c^2e(4ab}{}$$

$$+ \frac{(e^3(-2bef + bdg + aeg - 2adh) + cde(4e^2f - 3deg + 2d^2h))\log(d + ex)}{(cd^2 + e(-bd + ae))^3}$$

$$- \frac{(e^3(-2bef + bdg + aeg - 2adh) + cde(4e^2f - 3deg + 2d^2h))\log(a + x(b + cx))}{2(cd^2 + e(-bd + ae))^3}$$

```
[In] Integrate[(f + g*x + h*x^2)/((d + e*x)^2*(a + b*x + c*x^2)^2),x]
```

```
[Out] -((e*(e^2*f - d*e*g + d^2*h))/((c*d^2 + e*(-(b*d) + a*e))^2*(d + e*x))) + (
-(b^3*e^2*f) + b^2*(a*e^2*g - c*(-2*d*e*f + e^2*f*x + d^2*h*x)) + b*(-(a^2*
e^2*h) + c^2*d*(-(d*f) + 2*e*f*x + d*g*x) + a*c*(-(d^2*h) + e^2*(3*f + g*x)
 - 2*d*e*(g - h*x))) + 2*c*(-(c^2*d^2*f*x) + a*c*(e^2*f*x - 2*d*e*(f + g*x)
 + d^2*(g + h*x)) - a^2*e*(-2*d*h + e*(g + h*x))))/((b^2 - 4*a*c)*(c*d^2 +
e*(-(b*d) + a*e))^2*(a + x*(b + c*x))) - ((4*c^4*d^4*f + b^3*e^3*(-2*b*e*f
+ b*d*g + a*e*g - 2*a*d*h) - 2*c^3*d^2*(b*d*(4*e*f + d*g) - 2*a*(6*e^2*f -
2*d*e*g + d^2*h)) - 6*c^2*e*(4*a*b*d*e^2*f - b^2*d^3*g + 2*a^2*e*(e^2*f - 2
*d*e*g + 2*d^2*h)) + c*e*(-6*a^2*b*e^3*g + 4*a^3*e^3*h + b^3*d*(4*e^2*f - 3
*d*e*g - 2*d^2*h) + 6*a*b^2*e*(2*e^2*f - d*e*g + 2*d^2*h)))*ArcTan[(b + 2*c
*x)/Sqrt[-b^2 + 4*a*c]])/((-b^2 + 4*a*c)^(3/2)*(-(c*d^2) + e*(b*d - a*e))^3
) + ((e^3*(-2*b*e*f + b*d*g + a*e*g - 2*a*d*h) + c*d*e*(4*e^2*f - 3*d*e*g +
 2*d^2*h))*Log[d + e*x])/(c*d^2 + e*(-(b*d) + a*e))^3 - ((e^3*(-2*b*e*f + b
*d*g + a*e*g - 2*a*d*h) + c*d*e*(4*e^2*f - 3*d*e*g + 2*d^2*h))*Log[a + x*(b
 + c*x)])/(2*(c*d^2 + e*(-(b*d) + a*e))^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1344 vs. 2(668) = 1336.

Time = 0.84 (sec) , antiderivative size = 1345, normalized size of antiderivative = 2.00

method	result	size
default	Expression too large to display	1345
risch	Expression too large to display	8771

```
[In] int((h*x^2+g*x+f)/(e*x+d)^2/(c*x^2+b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -e*(d^2*h-d*e*g+e^2*f)/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)-e*(2*a*d*e^2*h-a*e^3*g
-b*d*e^2*g+2*b*e^3*f-2*c*d^3*h+3*c*d^2*e*g-4*c*d*e^2*f)/(a*e^2-b*d*e+c*d^2)
^3*ln(e*x+d)+1/(a*e^2-b*d*e+c*d^2)^3*((c*(2*a^3*e^4*h-4*a^2*b*d*e^3*h-a^2*b
*e^4*g+4*a^2*c*d*e^3*g-2*a^2*c*e^4*f+3*a*b^2*d^2*e^2*h+a*b^2*d*e^3*g+a*b^2*
e^4*f-6*a*b*c*d^2*e^2*g-2*a*c^2*d^4*h+4*a*c^2*d^3*e*g-b^3*d^3*e*h-b^3*d*e^3
*f+b^2*c*d^4*h+b^2*c*d^3*e*g+3*b^2*c*d^2*e^2*f-b*c^2*d^4*g-4*b*c^2*d^3*e*f+
2*c^3*d^4*f)/(4*a*c-b^2)*x+(a^3*b*e^4*h-4*a^3*c*d*e^3*h+2*a^3*c*e^4*g-a^2*b
^2*d*e^3*h-a^2*b^2*e^4*g+6*a^2*b*c*d^2*e^2*h-3*a^2*b*c*e^4*f-4*a^2*c^2*d^3*
e*h+4*a^2*c^2*d*e^3*f+a*b^3*d*e^3*g+a*b^3*e^4*f-a*b^2*c*d^3*e*h-3*a*b^2*c*d
^2*e^2*g+a*b^2*c*d*e^3*f+a*b*c^2*d^4*h+4*a*b*c^2*d^3*e*g-6*a*b*c^2*d^2*e^2*
f-2*a*c^3*d^4*g+4*a*c^3*d^3*e*f-b^4*d*e^3*f+3*b^3*c*d^2*e^2*f-3*b^2*c^2*d^3
*e*f+b*c^3*d^4*f)/(4*a*c-b^2))/(c*x^2+b*x+a)+1/(4*a*c-b^2)*(1/2*(8*a^2*c^2*
d*e^3*h-4*a^2*c^2*e^4*g-2*a*b^2*c*d*e^3*h+a*b^2*c*e^4*g-4*a*b*c^2*d*e^3*g+8
*a*b*c^2*e^4*f-8*a*c^3*d^3*e*h+12*a*c^3*d^2*e^2*g-16*a*c^3*d*e^3*f+b^3*c*d*
e^3*g-2*b^3*c*e^4*f+2*b^2*c^2*d^3*e*h-3*b^2*c^2*d^2*e^2*g+4*b^2*c^2*d*e^3*f
)/c*ln(c*x^2+b*x+a)+2*(a*b^3*e^4*g+b^4*d*e^3*g-b*c^3*d^4*g+2*a^3*c*e^4*h-6*
a^2*c^2*e^4*f+2*a*c^3*d^4*h-4*a*c^3*d^3*e*g+4*b^3*c*d*e^3*f-4*b*c^3*d^3*e*f
-5*a^2*b*c*e^4*g+12*a^2*c^2*d*e^3*g+10*a*b^2*c*e^4*f-1/2*(8*a^2*c^2*d*e^3*h
-4*a^2*c^2*e^4*g-2*a*b^2*c*d*e^3*h+a*b^2*c*e^4*g-4*a*b*c^2*d*e^3*g+8*a*b*c^
2*e^4*f-8*a*c^3*d^3*e*h+12*a*c^3*d^2*e^2*g-16*a*c^3*d*e^3*f+b^3*c*d*e^3*g-2
*b^3*c*e^4*f+2*b^2*c^2*d^3*e*h-3*b^2*c^2*d^2*e^2*g+4*b^2*c^2*d*e^3*f)*b/c+4
*a^2*b*c*d*e^3*h-5*a*b^2*c*d*e^3*g-4*a*b*c^2*d^3*e*h+6*a*b*c^2*d^2*e^2*g-20
*a*b*c^2*d*e^3*f+6*a*b^2*c*d^2*e^2*h+12*a*c^3*d^2*e^2*f-3*b^3*c*d^2*e^2*g+3
*b^2*c^2*d^3*e*g-12*a^2*c^2*d^2*e^2*h-2*a*b^3*d*e^3*h-2*b^4*e^4*f+2*c^4*d^4
*f)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{f+gx+hx^2}{(d+ex)^2\left(a+bx+cx^2\right)^2}\,dx=\text{Timed out}$$

```
[In] integrate((h*x^2+g*x+f)/(e*x+d)^2/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{f+gx+hx^2}{(d+ex)^2\left(a+bx+cx^2\right)^2}\,dx=\text{Timed out}$$

```
[In] integrate((h*x**2+g*x+f)/(e*x+d)**2/(c*x**2+b*x+a)**2,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{f+gx+hx^2}{(d+ex)^2\left(a+bx+cx^2\right)^2}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((h*x^2+g*x+f)/(e*x+d)^2/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1506 vs. $2(666) = 1332$.

Time = 0.29 (sec) , antiderivative size = 1506, normalized size of antiderivative = 2.24

$$\int \frac{f+gx+hx^2}{(d+ex)^2\left(a+bx+cx^2\right)^2}\,dx=\text{Too large to display}$$

```
[In] integrate((h*x^2+g*x+f)/(e*x+d)^2/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(4*c*d*e^3*f - 2*b*e^4*f - 3*c*d^2*e^2*g + b*d*e^3*g + a*e^4*g + 2*c*d
^3*e*h - 2*a*d*e^3*h)*log(c - 2*c*d/(e*x + d) + c*d^2/(e*x + d)^2 + b*e/(e*
x + d) - b*d*e/(e*x + d)^2 + a*e^2/(e*x + d)^2)/(c^3*d^6 - 3*b*c^2*d^5*e +
3*b^2*c*d^4*e^2 + 3*a*c^2*d^4*e^2 - b^3*d^3*e^3 - 6*a*b*c*d^3*e^3 + 3*a*b^2
*d^2*e^4 + 3*a^2*c*d^2*e^4 - 3*a^2*b*d*e^5 + a^3*e^6) - (e^7*f/(e*x + d) -
d*e^6*g/(e*x + d) + d^2*e^5*h/(e*x + d))/(c^2*d^4*e^4 - 2*b*c*d^3*e^5 + b^2
*d^2*e^6 + 2*a*c*d^2*e^6 - 2*a*b*d*e^7 + a^2*e^8) - (4*c^4*d^4*e^2*f - 8*b*
c^3*d^3*e^3*f + 24*a*c^3*d^2*e^4*f + 4*b^3*c*d*e^5*f - 24*a*b*c^2*d*e^5*f -
 2*b^4*e^6*f + 12*a*b^2*c*e^6*f - 12*a^2*c^2*e^6*f - 2*b*c^3*d^4*e^2*g + 6*
b^2*c^2*d^3*e^3*g - 8*a*c^3*d^3*e^3*g - 3*b^3*c*d^2*e^4*g + b^4*d*e^5*g - 6
*a*b^2*c*d*e^5*g + 24*a^2*c^2*d*e^5*g + a*b^3*e^6*g - 6*a^2*b*c*e^6*g + 4*a
*c^3*d^4*e^2*h - 2*b^3*c*d^3*e^3*h + 12*a*b^2*c*d^2*e^4*h - 24*a^2*c^2*d^2*
e^4*h - 2*a*b^3*d*e^5*h + 4*a^3*c*e^6*h)*arctan((2*c*d - 2*c*d^2/(e*x + d)
- b*e + 2*b*d*e/(e*x + d) - 2*a*e^2/(e*x + d))/(sqrt(-b^2 + 4*a*c)*e))/((b^
2*c^3*d^6 - 4*a*c^4*d^6 - 3*b^3*c^2*d^5*e + 12*a*b*c^3*d^5*e + 3*b^4*c*d^4*
e^2 - 9*a*b^2*c^2*d^4*e^2 - 12*a^2*c^3*d^4*e^2 - b^5*d^3*e^3 - 2*a*b^3*c*d^
3*e^3 + 24*a^2*b*c^2*d^3*e^3 + 3*a*b^4*d^2*e^4 - 9*a^2*b^2*c*d^2*e^4 - 12*a
^3*c^2*d^2*e^4 - 3*a^2*b^3*d*e^5 + 12*a^3*b*c*d*e^5 + a^3*b^2*e^6 - 4*a^4*c
*e^6)*sqrt(-b^2 + 4*a*c)*e^2) - ((2*c^4*d^3*e*f - 3*b*c^3*d^2*e^2*f + 3*b^2
*c^2*d*e^3*f - 6*a*c^3*d*e^3*f - b^3*c*e^4*f + 3*a*b*c^2*e^4*f - b*c^3*d^3*
e*g + 6*a*c^3*d^2*e^2*g - 3*a*b*c^2*d*e^3*g + a*b^2*c*e^4*g - 2*a^2*c^2*e^4
*g + b^2*c^2*d^3*e*h - 2*a*c^3*d^3*e*h - 3*a*b*c^2*d^2*e^2*h + 6*a^2*c^2*d*
e^3*h - a^2*b*c*e^4*h)/(c*d^2 - b*d*e + a*e^2) - (2*c^4*d^4*e^2*f - 4*b*c^3
*d^3*e^3*f + 6*b^2*c^2*d^2*e^4*f - 12*a*c^3*d^2*e^4*f - 4*b^3*c*d*e^5*f + 1
2*a*b*c^2*d*e^5*f + b^4*e^6*f - 4*a*b^2*c*e^6*f + 2*a^2*c^2*e^6*f - b*c^3*d
^4*e^2*g + 8*a*c^3*d^3*e^3*g - 6*a*b*c^2*d^2*e^4*g + 4*a*b^2*c*d*e^5*g - 8*
a^2*c^2*d*e^5*g - a*b^3*e^6*g + 3*a^2*b*c*e^6*g + b^2*c^2*d^4*e^2*h - 2*a*c
^3*d^4*e^2*h - 4*a*b*c^2*d^3*e^3*h + 12*a^2*c^2*d^2*e^4*h - 4*a^2*b*c*d*e^5
*h + a^2*b^2*e^6*h - 2*a^3*c*e^6*h)/((c*d^2 - b*d*e + a*e^2)*(e*x + d)*e))/
((c*d^2 - b*d*e + a*e^2)^2*(b^2 - 4*a*c)*(c - 2*c*d/(e*x + d) + c*d^2/(e*x
+ d)^2 + b*e/(e*x + d) - b*d*e/(e*x + d)^2 + a*e^2/(e*x + d)^2))
```

Mupad [B] (verification not implemented)

Time = 21.29 (sec) , antiderivative size = 26278, normalized size of antiderivative = 39.05

$$\int \frac{f+gx+hx^2}{(d+ex)^2\,(a+bx+cx^2)^2}\,dx = \text{Too large to display}$$

```
[In] int((f + g*x + h*x^2)/((d + e*x)^2*(a + b*x + c*x^2)^2),x)
```

```
[Out] ((a*b^2*e^3*f - 2*a*c^2*d^3*g + b*c^2*d^3*f - 4*a^2*c*e^3*f + b^3*d*e^2*f -
 2*a*b^2*d*e^2*g + 4*a*c^2*d^2*e*f + a*b^2*d^2*e*h + a^2*b*d*e^2*h + 6*a^2*
c*d*e^2*g - 2*b^2*c*d^2*e*f - 8*a^2*c*d^2*e*h + a*b*c*d^3*h - 3*a*b*c*d*e^2
*f + 2*a*b*c*d^2*e*g)/(4*a*c^3*d^4 + 4*a^3*c*e^4 - a^2*b^2*e^4 - b^2*c^2*d^
```

```
4 - b^4*d^2*e^2 + 8*a^2*c^2*d^2*e^2 + 2*a*b^3*d*e^3 + 2*b^3*c*d^3*e - 8*a*b
*c^2*d^3*e - 8*a^2*b*c*d*e^3 + 2*a*b^2*c*d^2*e^2) + (x*(2*b^3*e^3*f + 2*c^3
*d^3*f - a*b^2*e^3*g - 2*a*c^2*d^3*h - b*c^2*d^3*g + a^2*b*e^3*h + 2*a^2*c*
e^3*g + b^2*c*d^3*h - b^3*d*e^2*g + b^3*d^2*e*h + 2*a*c^2*d*e^2*f + 2*a*c^2
*d^2*e*g - b*c^2*d^2*e*f - b^2*c*d*e^2*f - 2*a^2*c*d*e^2*h - 7*a*b*c*e^3*f
 + 5*a*b*c*d*e^2*g - 5*a*b*c*d^2*e*h))/(4*a*c^3*d^4 + 4*a^3*c*e^4 - a^2*b^2*
e^4 - b^2*c^2*d^4 - b^4*d^2*e^2 + 8*a^2*c^2*d^2*e^2 + 2*a*b^3*d*e^3 + 2*b^3
*c*d^3*e - 8*a*b*c^2*d^3*e - 8*a^2*b*c*d*e^3 + 2*a*b^2*c*d^2*e^2) - (x^2*(6
*a*c^2*e^3*f - 2*b^2*c*e^3*f - 2*a^2*c*e^3*h - 2*c^3*d^2*e*f - 8*a*c^2*d*e^
2*g + 2*b*c^2*d*e^2*f + 6*a*c^2*d^2*e*h + b*c^2*d^2*e*g + b^2*c*d*e^2*g - 2
*b^2*c*d^2*e*h + a*b*c*e^3*g + 2*a*b*c*d*e^2*h))/(4*a*c^3*d^4 + 4*a^3*c*e^4
 - a^2*b^2*e^4 - b^2*c^2*d^4 - b^4*d^2*e^2 + 8*a^2*c^2*d^2*e^2 + 2*a*b^3*d*
e^3 + 2*b^3*c*d^3*e - 8*a*b*c^2*d^3*e - 8*a^2*b*c*d*e^3 + 2*a*b^2*c*d^2*e^2
))/(a*d + x*(a*e + b*d) + x^2*(b*e + c*d) + c*e*x^3) + symsum(log((x*(36*a^
2*c^5*e^7*f^2 + 4*b^4*c^3*e^7*f^2 + 4*a^4*c^3*e^7*h^2 + 4*c^7*d^4*e^3*f^2 +
 a^2*b^2*c^3*e^7*g^2 + 64*a^2*c^5*d^2*e^5*g^2 + 12*b^2*c^5*d^2*e^5*f^2 + 36
*a^2*c^5*d^4*e^3*h^2 - 24*a^3*c^4*d^2*e^5*h^2 + b^2*c^5*d^4*e^3*g^2 + 2*b^3
*c^4*d^3*e^4*g^2 + b^4*c^3*d^2*e^5*g^2 + 4*b^4*c^3*d^4*e^3*h^2 - 24*a^3*c^4
*e^7*f*h - 24*a*b^2*c^4*e^7*f^2 - 24*a*c^6*d^2*e^5*f^2 - 8*b*c^6*d^3*e^4*f^
2 - 8*b^3*c^4*d*e^6*f^2 - 16*a*b*c^5*d^3*e^4*g^2 + 2*a*b^3*c^3*d*e^6*g^2 -
16*a^2*b*c^4*d*e^6*g^2 - 8*a^3*b*c^3*d*e^6*h^2 + 8*a^2*b^2*c^3*e^7*f*h + 80
*a^2*c^5*d^2*e^5*f*h - 96*a^2*c^5*d^3*e^4*g*h + 8*b^2*c^5*d^4*e^3*f*h - 8*b
^3*c^4*d^3*e^4*f*h + 8*b^4*c^3*d^2*e^5*f*h - 4*b^3*c^4*d^4*e^3*g*h - 4*b^4*
c^3*d^3*e^4*g*h - 14*a*b^2*c^4*d^2*e^5*g^2 - 24*a*b^2*c^4*d^4*e^3*h^2 - 8*a
*b^3*c^3*d^3*e^4*h^2 + 24*a^2*b*c^4*d^3*e^4*h^2 + 24*a*b*c^5*d*e^6*f^2 - 4*
a*b^3*c^3*e^7*f*g + 12*a^2*b*c^4*e^7*f*g + 32*a*c^6*d^3*e^4*f*g - 96*a^2*c^
5*d*e^6*f*g - 4*a^3*b*c^3*e^7*g*h - 24*a*c^6*d^4*e^3*f*h - 4*b*c^6*d^4*e^3*
f*g - 4*b^4*c^3*d*e^6*f*g + 32*a^3*c^4*d*e^6*g*h + 12*a^2*b^2*c^3*d^2*e^5*h
^2 - 24*a*b*c^5*d^2*e^5*f*g + 48*a*b^2*c^4*d*e^6*f*g + 16*a*b*c^5*d^3*e^4*f
*h - 8*a*b^3*c^3*d*e^6*f*h + 16*a^2*b*c^4*d*e^6*f*h + 12*a*b*c^5*d^4*e^3*g*
h - 40*a*b^2*c^4*d^2*e^5*f*h + 48*a*b^2*c^4*d^3*e^4*g*h - 24*a^2*b*c^4*d^2*
e^5*g*h))/(16*a^2*c^6*d^8 + a^4*b^4*e^8 + 16*a^6*c^2*e^8 + b^4*c^4*d^8 + b^
8*d^4*e^4 - 8*a*b^2*c^5*d^8 - 8*a^5*b^2*c*e^8 - 4*a*b^7*d^3*e^5 - 4*a^3*b^5
*d*e^7 - 4*b^5*c^3*d^7*e - 4*b^7*c*d^5*e^3 + 6*a^2*b^6*d^2*e^6 + 64*a^3*c^5
*d^6*e^2 + 96*a^4*c^4*d^4*e^4 + 64*a^5*c^3*d^2*e^6 + 6*b^6*c^2*d^6*e^2 + 64
*a^2*b^2*c^4*d^6*e^2 + 32*a^2*b^3*c^3*d^5*e^3 - 74*a^2*b^4*c^2*d^4*e^4 + 14
4*a^3*b^2*c^3*d^4*e^4 + 32*a^3*b^3*c^2*d^3*e^5 + 64*a^4*b^2*c^2*d^2*e^6 + 3
2*a*b^3*c^4*d^7*e + 4*a*b^6*c*d^4*e^4 - 64*a^2*b*c^5*d^7*e + 32*a^4*b^3*c*d
*e^7 - 64*a^5*b*c^2*d*e^7 - 44*a*b^4*c^3*d^6*e^2 + 20*a*b^5*c^2*d^5*e^3 + 2
0*a^2*b^5*c*d^3*e^5 - 192*a^3*b*c^4*d^5*e^3 - 44*a^3*b^4*c*d^2*e^6 - 192*a^
4*b*c^3*d^3*e^5) - root(3840*a^6*b*c^5*d^5*e^7*z^3 + 3840*a^5*b*c^6*d^7*e^5
*z^3 + 1920*a^7*b*c^4*d^3*e^9*z^3 + 1920*a^4*b*c^7*d^9*e^3*z^3 - 288*a^7*b^
3*c^2*d*e^11*z^3 - 288*a^2*b^3*c^7*d^11*e*z^3 + 210*a^4*b^7*c*d^3*e^9*z^3 +
 210*a*b^7*c^4*d^9*e^3*z^3 - 174*a^5*b^6*c*d^2*e^10*z^3 - 174*a*b^6*c^5*d^1
0*e^2*z^3 - 120*a^3*b^8*c*d^4*e^8*z^3 - 120*a*b^8*c^3*d^8*e^4*z^3 + 12*a^2*
```

```
b^9*c*d^5*e^7*z^3 + 12*a*b^9*c^2*d^7*e^5*z^3 + 384*a^8*b*c^3*d*e^11*z^3 + 3
84*a^3*b*c^8*d^11*e*z^3 + 72*a^6*b^5*c*d*e^11*z^3 + 72*a*b^5*c^6*d^11*e*z^3
 + 18*a*b^10*c*d^6*e^6*z^3 - 4800*a^5*b^2*c^5*d^6*e^6*z^3 - 3120*a^6*b^2*c^
4*d^4*e^8*z^3 - 3120*a^4*b^2*c^6*d^8*e^4*z^3 + 2160*a^4*b^4*c^4*d^6*e^6*z^3
 - 1776*a^4*b^5*c^3*d^5*e^7*z^3 - 1776*a^3*b^5*c^4*d^7*e^5*z^3 + 1740*a^5*b
^4*c^3*d^4*e^8*z^3 + 1740*a^3*b^4*c^5*d^8*e^4*z^3 + 960*a^5*b^3*c^4*d^5*e^7
*z^3 + 960*a^4*b^3*c^5*d^7*e^5*z^3 - 672*a^7*b^2*c^3*d^2*e^10*z^3 - 672*a^3
*b^2*c^7*d^10*e^2*z^3 + 648*a^6*b^4*c^2*d^2*e^10*z^3 + 648*a^2*b^4*c^6*d^10
*e^2*z^3 - 600*a^5*b^5*c^2*d^3*e^9*z^3 - 600*a^2*b^5*c^5*d^9*e^3*z^3 + 372*
a^3*b^7*c^2*d^5*e^7*z^3 + 372*a^2*b^7*c^3*d^7*e^5*z^3 + 316*a^3*b^6*c^3*d^6
*e^6*z^3 - 222*a^2*b^8*c^2*d^6*e^6*z^3 - 160*a^6*b^3*c^3*d^3*e^9*z^3 - 160*
a^3*b^3*c^6*d^9*e^3*z^3 + 15*a^4*b^6*c^2*d^4*e^8*z^3 + 15*a^2*b^6*c^4*d^8*e
^4*z^3 - 6*b^11*c*d^7*e^5*z^3 - 6*b^7*c^5*d^11*e*z^3 - 6*a^5*b^7*d*e^11*z^3
 - 6*a*b^11*d^5*e^7*z^3 - 12*a^7*b^4*c*e^12*z^3 - 12*a*b^4*c^7*d^12*z^3 - 2
0*b^9*c^3*d^9*e^3*z^3 + 15*b^10*c^2*d^8*e^4*z^3 + 15*b^8*c^4*d^10*e^2*z^3 -
 1280*a^6*c^6*d^6*e^6*z^3 - 960*a^7*c^5*d^4*e^8*z^3 - 960*a^5*c^7*d^8*e^4*z
^3 - 384*a^8*c^4*d^2*e^10*z^3 - 384*a^4*c^8*d^10*e^2*z^3 - 20*a^3*b^9*d^3*e
^9*z^3 + 15*a^4*b^8*d^2*e^10*z^3 + 15*a^2*b^10*d^4*e^8*z^3 + 48*a^8*b^2*c^2
*e^12*z^3 + 48*a^2*b^2*c^8*d^12*z^3 - 64*a^9*c^3*e^12*z^3 - 64*a^3*c^9*d^12
*z^3 + b^12*d^6*e^6*z^3 + b^6*c^6*d^12*z^3 + a^6*b^6*e^12*z^3 - 44*a^3*b^4*
c*d*e^7*g*h*z - 20*a*b^6*c*d^3*e^5*g*h*z - 12*a*b^2*c^5*d^7*e*g*h*z + 432*a
^4*b*c^3*d*e^7*f*h*z + 84*a^2*b^5*c*d*e^7*f*h*z + 28*a*b^6*c*d^2*e^6*f*h*z
- 8*a*b*c^6*d^6*e^2*f*g*z - 804*a^3*b^2*c^3*d^3*e^5*g*h*z + 564*a^2*b^2*c^4
*d^5*e^3*g*h*z + 222*a^3*b^3*c^2*d^2*e^6*g*h*z + 186*a^2*b^4*c^2*d^3*e^5*g*
h*z - 166*a^2*b^3*c^3*d^4*e^4*g*h*z + 792*a^3*b^2*c^3*d^2*e^6*f*h*z - 744*a
^2*b^2*c^4*d^4*e^4*f*h*z + 492*a^2*b^3*c^3*d^3*e^5*f*h*z - 264*a^2*b^4*c^2*
d^2*e^6*f*h*z + 996*a^2*b^2*c^4*d^3*e^5*f*g*z - 870*a^2*b^3*c^3*d^2*e^6*f*g
*z + 16*a*b*c^6*d^7*e*f*h*z - 56*a*b^6*c*d*e^7*f*g*z - 264*a^4*b*c^3*d^2*e^
6*g*h*z + 208*a^3*b*c^4*d^4*e^4*g*h*z + 156*a^4*b^2*c^2*d*e^7*g*h*z - 148*a
*b^4*c^3*d^5*e^3*g*h*z + 54*a*b^5*c^2*d^4*e^4*g*h*z - 48*a^2*b^5*c*d^2*e^6*
g*h*z - 24*a^2*b*c^5*d^6*e^2*g*h*z + 10*a*b^3*c^4*d^6*e^2*g*h*z - 656*a^3*b
*c^4*d^3*e^5*f*h*z - 308*a^3*b^3*c^2*d*e^7*f*h*z + 116*a*b^4*c^3*d^4*e^4*f*
h*z - 84*a*b^5*c^2*d^3*e^5*f*h*z + 68*a*b^3*c^4*d^5*e^3*f*h*z - 48*a^2*b*c^
5*d^5*e^3*f*h*z - 24*a*b^2*c^5*d^6*e^2*f*h*z + 1320*a^3*b*c^4*d^2*e^6*f*g*z
 - 732*a^3*b^2*c^3*d*e^7*f*g*z + 306*a^2*b^4*c^2*d*e^7*f*g*z - 304*a*b^4*c^
3*d^3*e^5*f*g*z + 222*a*b^5*c^2*d^2*e^6*f*g*z + 110*a*b^3*c^4*d^4*e^4*f*g*z
 - 84*a*b^2*c^5*d^5*e^3*f*g*z + 16*a*c^7*d^7*e*f*g*z - 8*a*b^7*d*e^7*f*h*z
+ 4*a*b*c^6*d^8*g*h*z + 6*b^6*c^2*d^5*e^3*g*h*z + 6*b^5*c^3*d^6*e^2*g*h*z +
 1072*a^4*c^4*d^3*e^5*g*h*z - 720*a^3*c^5*d^5*e^3*g*h*z - 8*b^6*c^2*d^4*e^4
*f*h*z - 8*b^4*c^4*d^6*e^2*f*h*z + 1072*a^3*c^5*d^4*e^4*f*h*z - 960*a^4*c^4
*d^2*e^6*f*h*z + 30*b^6*c^2*d^3*e^5*f*g*z + 30*b^3*c^5*d^6*e^2*f*g*z - 10*b
^5*c^3*d^4*e^4*f*g*z - 10*b^4*c^4*d^5*e^3*f*g*z - 1488*a^3*c^5*d^3*e^5*f*g*
z + 48*a^2*c^6*d^5*e^3*f*g*z - 24*a^4*b^2*c^2*e^8*f*h*z + 186*a^3*b^3*c^2*e
^8*f*g*z + 4*a^4*b^3*c*d*e^7*h^2*z + 4*a*b^6*c*d^4*e^4*h^2*z + 4*a*b^3*c^4*
d^7*e*h^2*z + 168*a^4*b*c^3*d*e^7*g^2*z + 24*a^2*b^5*c*d*e^7*g^2*z + 18*a*b
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^6*c*d^2*e^6*g^2*z - 912*a^3*b*c^4*d*e^7*f^2*z - 192*a*b^5*c^2*d*e^7*f^2*z
+ 144*a*b*c^6*d^5*e^3*f^2*z + 432*a^3*b^2*c^3*d^4*e^4*h^2*z - 168*a^4*b^2*c
^2*d^2*e^6*h^2*z - 168*a^2*b^2*c^4*d^6*e^2*h^2*z - 108*a^2*b^4*c^2*d^4*e^4*
h^2*z - 20*a^3*b^3*c^2*d^3*e^5*h^2*z - 20*a^2*b^3*c^3*d^5*e^3*h^2*z - 426*a
^2*b^2*c^4*d^4*e^4*g^2*z + 336*a^3*b^2*c^3*d^2*e^6*g^2*z + 274*a^2*b^3*c^3*
d^3*e^5*g^2*z - 120*a^2*b^4*c^2*d^2*e^6*g^2*z - 864*a^2*b^2*c^4*d^2*e^6*f^2
*z - 2*b^7*c*d^4*e^4*g*h*z - 2*b^4*c^4*d^7*e*g*h*z - 240*a^5*c^3*d*e^7*g*h*
z + 16*a^2*c^6*d^7*e*g*h*z + 4*b^7*c*d^3*e^5*f*h*z + 4*b^3*c^5*d^7*e*f*h*z
- 20*b^7*c*d^2*e^6*f*g*z - 20*b^2*c^6*d^7*e*f*g*z + 4*a^2*b^6*d*e^7*g*h*z +
 4*a*b^7*d^2*e^6*g*h*z + 528*a^4*c^4*d*e^7*f*g*z + 12*a^5*b*c^2*e^8*g*h*z -
 2*a^4*b^3*c*e^8*g*h*z + 4*a^3*b^4*c*e^8*f*h*z - 228*a^4*b*c^3*e^8*f*g*z -
48*a^2*b^5*c*e^8*f*g*z - 8*a*b*c^6*d^7*e*g^2*z + 36*a^3*b^4*c*d^2*e^6*h^2*z
 + 36*a*b^4*c^3*d^6*e^2*h^2*z + 12*a^2*b^5*c*d^3*e^5*h^2*z + 12*a*b^5*c^2*d
^5*e^3*h^2*z - 312*a^3*b*c^4*d^3*e^5*g^2*z + 104*a*b^4*c^3*d^4*e^4*g^2*z -
102*a^3*b^3*c^2*d*e^7*g^2*z - 66*a*b^5*c^2*d^3*e^5*g^2*z + 24*a^2*b*c^5*d^5
*e^3*g^2*z + 24*a*b^2*c^5*d^6*e^2*g^2*z - 18*a*b^3*c^4*d^5*e^3*g^2*z + 744*
a^2*b^3*c^3*d*e^7*f^2*z + 240*a^2*b*c^5*d^3*e^5*f^2*z + 216*a*b^4*c^3*d^2*e
^6*f^2*z - 120*a*b^2*c^5*d^4*e^4*f^2*z + 24*a^5*c^3*e^8*f*h*z + 16*b^7*c*d*
e^7*f^2*z + 16*b*c^7*d^7*e*f^2*z - 2*a*b^7*d*e^7*g^2*z + 48*a*b^6*c*e^8*f^2
*z - 4*b^6*c^2*d^6*e^2*h^2*z - 536*a^4*c^4*d^4*e^4*h^2*z + 240*a^5*c^3*d^2*
e^6*h^2*z + 240*a^3*c^5*d^6*e^2*h^2*z - 12*b^6*c^2*d^4*e^4*g^2*z - 12*b^4*c
^4*d^6*e^2*g^2*z + 10*b^5*c^3*d^5*e^3*g^2*z + 528*a^3*c^5*d^4*e^4*g^2*z - 4
32*a^4*c^4*d^2*e^6*g^2*z + 20*b^4*c^4*d^4*e^4*f^2*z - 16*b^6*c^2*d^2*e^6*f^
2*z - 16*b^2*c^6*d^6*e^2*f^2*z - 16*a^2*c^6*d^6*e^2*g^2*z - 8*b^5*c^3*d^3*e
^5*f^2*z - 8*b^3*c^5*d^5*e^3*f^2*z - 4*a^2*b^6*d^2*e^6*h^2*z + 912*a^3*c^5*
d^2*e^6*f^2*z - 120*a^2*c^6*d^4*e^4*f^2*z - 45*a^4*b^2*c^2*e^8*g^2*z + 264*
a^3*b^2*c^3*e^8*f^2*z - 192*a^2*b^4*c^2*e^8*f^2*z + 4*b^8*d*e^7*f*g*z - 8*a
*c^7*d^8*f*h*z + 4*b*c^7*d^8*f*g*z + 4*a*b^7*e^8*f*g*z + 6*b^7*c*d^3*e^5*g^
2*z + 6*b^3*c^5*d^7*e*g^2*z - 48*a*c^7*d^6*e^2*f^2*z + 12*a^3*b^4*c*e^8*g^2
*z - b^8*d^2*e^6*g^2*z - 4*a^6*c^2*e^8*h^2*z + 48*a^5*c^3*e^8*g^2*z - 4*a^2
*c^6*d^8*h^2*z - b^2*c^6*d^8*g^2*z - 36*a^4*c^4*e^8*f^2*z - a^2*b^6*e^8*g^2
*z - 4*c^8*d^8*f^2*z - 4*b^8*e^8*f^2*z - 80*a*b*c^4*d^3*e^3*f*g*h + 24*a^2*
b*c^3*d*e^5*f*g*h + 16*a*b^3*c^2*d*e^5*f*g*h - 72*a*b^2*c^3*d^2*e^4*f*g*h -
 48*a^2*b*c^3*d^3*e^3*g*h^2 + 16*a*b^3*c^2*d^3*e^3*g*h^2 - 12*a*b^2*c^3*d^3
*e^3*g^2*h - 6*a^2*b^2*c^2*d*e^5*g^2*h - 72*a^2*b^2*c^2*d*e^5*f*h^2 + 48*a*
b^2*c^3*d^3*e^3*f*h^2 + 24*a^2*b*c^3*d^2*e^4*f*h^2 - 8*a*b^3*c^2*d^2*e^4*f*
h^2 - 8*b^5*c*d*e^5*f*g*h - 8*b*c^5*d^5*e*f*g*h - 8*a*b^4*c*e^6*f*g*h + 24*
b^3*c^3*d^3*e^3*f*g*h + 16*b^4*c^2*d^2*e^4*f*g*h + 16*b^2*c^4*d^4*e^2*f*g*h
 + 48*a^2*c^4*d^2*e^4*f*g*h + 48*a^2*b^2*c^2*e^6*f*g*h + 40*a^3*b*c^2*d*e^5
*g*h^2 + 28*a*b*c^4*d^4*e^2*g^2*h - 8*a^2*b^3*c*d*e^5*g*h^2 - 8*a*b^4*c*d^2
*e^4*g*h^2 + 96*a*b^2*c^3*d*e^5*f^2*h + 24*a*b*c^4*d^2*e^4*f^2*h + 16*a*b*c
^4*d^4*e^2*f*h^2 + 96*a*b*c^4*d^2*e^4*f*g^2 - 48*a*b^2*c^3*d*e^5*f*g^2 + 12
*a^2*b^2*c^2*d^2*e^4*g*h^2 - 56*a*c^5*d^4*e^2*f*g*h - 8*a*b*c^4*d^5*e*g*h^2
 + 4*a*b^4*c*d*e^5*g^2*h + 16*a*b^4*c*d*e^5*f*h^2 - 48*a*b*c^4*d*e^5*f^2*g
- 24*a^3*c^3*e^6*f*g*h + 16*a*c^5*d^5*e*f*h^2 - 6*b^4*c^2*d^3*e^3*g^2*h - 6
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*b^3*c^3*d^4*e^2*g^2*h + 4*b^4*c^2*d^4*e^2*g*h^2 + 80*a^2*c^4*d^3*e^3*g^2*h
 - 44*a^2*c^4*d^4*e^2*g*h^2 + 24*a^3*c^3*d^2*e^4*g*h^2 - 16*b^3*c^3*d^2*e^4
*f^2*h - 16*b^2*c^4*d^3*e^3*f^2*h - 8*b^4*c^2*d^3*e^3*f*h^2 - 8*b^3*c^3*d^4
*e^2*f*h^2 + 60*b^2*c^4*d^2*e^4*f^2*g - 48*a^2*c^4*d^3*e^3*f*h^2 - 24*b^3*c
^3*d^2*e^4*f*g^2 - 24*b^2*c^4*d^3*e^3*f*g^2 - 24*a^3*b*c^2*d^2*e^4*h^3 + 24
*a^2*b*c^3*d^4*e^2*h^3 + 8*a^2*b^3*c*d^2*e^4*h^3 - 8*a*b^3*c^2*d^4*e^2*h^3
+ 18*a*b^2*c^3*d^2*e^4*g^3 + 2*b^5*c*d^2*e^4*g^2*h + 2*b^2*c^4*d^5*e*g^2*h
- 48*a^3*c^3*d*e^5*g^2*h - 8*b^4*c^2*d*e^5*f^2*h - 8*b*c^5*d^4*e^2*f^2*h -
168*a^2*c^4*d*e^5*f^2*h + 96*a*c^5*d^3*e^3*f^2*h + 64*a^3*c^3*d*e^5*f*h^2 +
 12*b^4*c^2*d*e^5*f*g^2 + 12*b*c^5*d^4*e^2*f*g^2 - 168*a*c^5*d^2*e^4*f^2*g
+ 48*a^2*c^4*d*e^5*f*g^2 + 48*a*c^5*d^3*e^3*f*g^2 - 12*a^3*b*c^2*e^6*g^2*h
+ 2*a^2*b^3*c*e^6*g^2*h + 48*a^2*b*c^3*e^6*f^2*h - 48*a*b^3*c^2*e^6*f^2*h -
 8*a^3*b*c^2*e^6*f*h^2 - 60*a^2*b*c^3*e^6*f*g^2 + 48*a*b^2*c^3*e^6*f^2*g +
12*a*b^3*c^2*e^6*f*g^2 + 24*a^2*b*c^3*d*e^5*g^3 - 24*a*b*c^4*d^3*e^3*g^3 -
6*a*b^3*c^2*d*e^5*g^3 - 12*c^6*d^4*e^2*f^2*g + 4*a^4*c^2*e^6*g*h^2 - 12*b^4
*c^2*e^6*f^2*g + 36*a^2*c^4*e^6*f^2*g - 8*a^4*c^2*d*e^5*h^3 + 8*a^2*c^4*d^5
*e*h^3 - 24*b^2*c^4*d*e^5*f^3 - 24*b*c^5*d^2*e^4*f^3 + 8*c^6*d^5*e*f^2*h +
8*b^5*c*e^6*f^2*h + 144*a*c^5*d*e^5*f^3 - 72*a*b*c^4*e^6*f^3 + 10*b^3*c^3*d
^3*e^3*g^3 - 3*b^4*c^2*d^2*e^4*g^3 - 3*b^2*c^4*d^4*e^2*g^3 - 48*a^2*c^4*d^2
*e^4*g^3 - 3*a^2*b^2*c^2*e^6*g^3 + 16*c^6*d^3*e^3*f^3 + 16*b^3*c^3*e^6*f^3
+ 16*a^3*c^3*e^6*g^3, z, k)*((8*a^6*c^3*e^9*h - 24*a^5*c^4*e^9*f - 8*a*c^8*
d^8*e*f + 2*a^2*b^6*c*e^9*f - a^3*b^5*c*e^9*g - 20*a^5*b*c^3*e^9*g + 16*a^5
*c^4*d*e^8*g + 2*b^2*c^7*d^8*e*f + 2*b^8*c*d^2*e^7*f - 8*a^2*c^7*d^8*e*h -
b^3*c^6*d^8*e*g - b^8*c*d^3*e^6*g - 18*a^3*b^4*c^2*e^9*f + 46*a^4*b^2*c^3*e
^9*f + 9*a^4*b^3*c^2*e^9*g - 48*a^2*c^7*d^6*e^3*f - 96*a^3*c^6*d^4*e^5*f -
80*a^4*c^5*d^2*e^7*f - 2*a^5*b^2*c^2*e^9*h + 16*a^2*c^7*d^7*e^2*g + 48*a^3*
c^6*d^5*e^4*g + 48*a^4*c^5*d^3*e^6*g - 6*b^3*c^6*d^7*e^2*f + 4*b^4*c^5*d^6*
e^3*f + 4*b^6*c^3*d^4*e^5*f - 6*b^7*c^2*d^3*e^6*f - 16*a^3*c^6*d^6*e^3*h +
16*a^5*c^4*d^2*e^7*h + 4*b^4*c^5*d^7*e^2*g - 3*b^5*c^4*d^6*e^3*g - 3*b^6*c^
3*d^5*e^4*g + 4*b^7*c^2*d^4*e^5*g - 2*b^5*c^4*d^7*e^2*h + 4*b^6*c^3*d^6*e^3
*h - 2*b^7*c^2*d^5*e^4*h - 4*a*b^2*c^6*d^6*e^3*f - 14*a*b^3*c^5*d^5*e^4*f -
 38*a*b^4*c^4*d^4*e^5*f + 54*a*b^5*c^3*d^3*e^6*f - 10*a*b^6*c^2*d^2*e^7*f +
 56*a^2*b*c^6*d^5*e^4*f + 34*a^2*b^5*c^2*d*e^8*f + 40*a^3*b*c^5*d^3*e^6*f -
 74*a^3*b^3*c^3*d*e^8*f - 20*a*b^2*c^6*d^7*e^2*g + 10*a*b^3*c^5*d^6*e^3*g +
 34*a*b^4*c^4*d^5*e^4*g - 33*a*b^5*c^3*d^4*e^5*g + 4*a*b^6*c^2*d^3*e^6*g +
8*a^2*b*c^6*d^6*e^3*g - 16*a^3*b*c^5*d^4*e^5*g - 10*a^3*b^4*c^2*d*e^8*g - 4
0*a^4*b*c^4*d^2*e^7*g + 20*a^4*b^2*c^3*d*e^8*g + 10*a*b^3*c^5*d^7*e^2*h - 2
6*a*b^4*c^4*d^6*e^3*h + 12*a*b^5*c^3*d^5*e^4*h - 8*a^2*b*c^6*d^7*e^2*h - 4*
a^2*b^6*c*d^2*e^7*h - 8*a^3*b*c^5*d^5*e^4*h + 8*a^4*b*c^4*d^3*e^6*h - 10*a^
4*b^3*c^2*d*e^8*h - 4*a*b^7*c*d*e^8*f + 4*a*b*c^7*d^8*e*g + 112*a^2*b^2*c^5
*d^4*e^5*f - 130*a^2*b^3*c^4*d^3*e^6*f - 28*a^2*b^4*c^3*d^2*e^7*f + 164*a^3
*b^2*c^4*d^2*e^7*f - 100*a^2*b^2*c^5*d^5*e^4*g + 72*a^2*b^3*c^4*d^4*e^5*g +
 12*a^2*b^4*c^3*d^3*e^6*g - 7*a^2*b^5*c^2*d^2*e^7*g - 60*a^3*b^2*c^4*d^3*e^
6*g + 22*a^3*b^3*c^3*d^2*e^7*g + 44*a^2*b^2*c^5*d^6*e^3*h - 14*a^2*b^3*c^4*
d^5*e^4*h - 12*a^2*b^5*c^2*d^3*e^6*h + 14*a^3*b^3*c^3*d^3*e^6*h + 26*a^3*b^
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4*c^2*d^2*e^7*h - 44*a^4*b^2*c^3*d^2*e^7*h + 24*a*b*c^7*d^7*e^2*f + 8*a^4*b
*c^4*d*e^8*f + a*b^7*c*d^2*e^7*g + a^2*b^6*c*d*e^8*g + 2*a*b^2*c^6*d^8*e*h
+ 2*a*b^7*c*d^3*e^6*h + 2*a^3*b^5*c*d*e^8*h + 8*a^5*b*c^3*d*e^8*h)/(16*a^2*
c^6*d^8 + a^4*b^4*e^8 + 16*a^6*c^2*e^8 + b^4*c^4*d^8 + b^8*d^4*e^4 - 8*a*b^
2*c^5*d^8 - 8*a^5*b^2*c*e^8 - 4*a*b^7*d^3*e^5 - 4*a^3*b^5*d*e^7 - 4*b^5*c^3
*d^7*e - 4*b^7*c*d^5*e^3 + 6*a^2*b^6*d^2*e^6 + 64*a^3*c^5*d^6*e^2 + 96*a^4*
c^4*d^4*e^4 + 64*a^5*c^3*d^2*e^6 + 6*b^6*c^2*d^6*e^2 + 64*a^2*b^2*c^4*d^6*e
^2 + 32*a^2*b^3*c^3*d^5*e^3 - 74*a^2*b^4*c^2*d^4*e^4 + 144*a^3*b^2*c^3*d^4*
e^4 + 32*a^3*b^3*c^2*d^3*e^5 + 64*a^4*b^2*c^2*d^2*e^6 + 32*a*b^3*c^4*d^7*e
+ 4*a*b^6*c*d^4*e^4 - 64*a^2*b*c^5*d^7*e + 32*a^4*b^3*c*d*e^7 - 64*a^5*b*c^
2*d*e^7 - 44*a*b^4*c^3*d^6*e^2 + 20*a*b^5*c^2*d^5*e^3 + 20*a^2*b^5*c*d^3*e^
5 - 192*a^3*b*c^4*d^5*e^3 - 44*a^3*b^4*c*d^2*e^6 - 192*a^4*b*c^3*d^3*e^5) +
 root(3840*a^6*b*c^5*d^5*e^7*z^3 + 3840*a^5*b*c^6*d^7*e^5*z^3 + 1920*a^7*b*
c^4*d^3*e^9*z^3 + 1920*a^4*b*c^7*d^9*e^3*z^3 - 288*a^7*b^3*c^2*d*e^11*z^3 -
 288*a^2*b^3*c^7*d^11*e*z^3 + 210*a^4*b^7*c*d^3*e^9*z^3 + 210*a*b^7*c^4*d^9
*e^3*z^3 - 174*a^5*b^6*c*d^2*e^10*z^3 - 174*a*b^6*c^5*d^10*e^2*z^3 - 120*a^
3*b^8*c*d^4*e^8*z^3 - 120*a*b^8*c^3*d^8*e^4*z^3 + 12*a^2*b^9*c*d^5*e^7*z^3
+ 12*a*b^9*c^2*d^7*e^5*z^3 + 384*a^8*b*c^3*d*e^11*z^3 + 384*a^3*b*c^8*d^11*
e*z^3 + 72*a^6*b^5*c*d*e^11*z^3 + 72*a*b^5*c^6*d^11*e*z^3 + 18*a*b^10*c*d^6
*e^6*z^3 - 4800*a^5*b^2*c^5*d^6*e^6*z^3 - 3120*a^6*b^2*c^4*d^4*e^8*z^3 - 31
20*a^4*b^2*c^6*d^8*e^4*z^3 + 2160*a^4*b^4*c^4*d^6*e^6*z^3 - 1776*a^4*b^5*c^
3*d^5*e^7*z^3 - 1776*a^3*b^5*c^4*d^7*e^5*z^3 + 1740*a^5*b^4*c^3*d^4*e^8*z^3
 + 1740*a^3*b^4*c^5*d^8*e^4*z^3 + 960*a^5*b^3*c^4*d^5*e^7*z^3 + 960*a^4*b^3
*c^5*d^7*e^5*z^3 - 672*a^7*b^2*c^3*d^2*e^10*z^3 - 672*a^3*b^2*c^7*d^10*e^2*
z^3 + 648*a^6*b^4*c^2*d^2*e^10*z^3 + 648*a^2*b^4*c^6*d^10*e^2*z^3 - 600*a^5
*b^5*c^2*d^3*e^9*z^3 - 600*a^2*b^5*c^5*d^9*e^3*z^3 + 372*a^3*b^7*c^2*d^5*e^
7*z^3 + 372*a^2*b^7*c^3*d^7*e^5*z^3 + 316*a^3*b^6*c^3*d^6*e^6*z^3 - 222*a^2
*b^8*c^2*d^6*e^6*z^3 - 160*a^6*b^3*c^3*d^3*e^9*z^3 - 160*a^3*b^3*c^6*d^9*e^
3*z^3 + 15*a^4*b^6*c^2*d^4*e^8*z^3 + 15*a^2*b^6*c^4*d^8*e^4*z^3 - 6*b^11*c*
d^7*e^5*z^3 - 6*b^7*c^5*d^11*e*z^3 - 6*a^5*b^7*d*e^11*z^3 - 6*a*b^11*d^5*e^
7*z^3 - 12*a^7*b^4*c*e^12*z^3 - 12*a*b^4*c^7*d^12*z^3 - 20*b^9*c^3*d^9*e^3*
z^3 + 15*b^10*c^2*d^8*e^4*z^3 + 15*b^8*c^4*d^10*e^2*z^3 - 1280*a^6*c^6*d^6*
e^6*z^3 - 960*a^7*c^5*d^4*e^8*z^3 - 960*a^5*c^7*d^8*e^4*z^3 - 384*a^8*c^4*d
^2*e^10*z^3 - 384*a^4*c^8*d^10*e^2*z^3 - 20*a^3*b^9*d^3*e^9*z^3 + 15*a^4*b^
8*d^2*e^10*z^3 + 15*a^2*b^10*d^4*e^8*z^3 + 48*a^8*b^2*c^2*e^12*z^3 + 48*a^2
*b^2*c^8*d^12*z^3 - 64*a^9*c^3*e^12*z^3 - 64*a^3*c^9*d^12*z^3 + b^12*d^6*e^
6*z^3 + b^6*c^6*d^12*z^3 + a^6*b^6*e^12*z^3 - 44*a^3*b^4*c*d*e^7*g*h*z - 20
*a*b^6*c*d^3*e^5*g*h*z - 12*a*b^2*c^5*d^7*e*g*h*z + 432*a^4*b*c^3*d*e^7*f*h
*z + 84*a^2*b^5*c*d*e^7*f*h*z + 28*a*b^6*c*d^2*e^6*f*h*z - 8*a*b*c^6*d^6*e^
2*f*g*z - 804*a^3*b^2*c^3*d^3*e^5*g*h*z + 564*a^2*b^2*c^4*d^5*e^3*g*h*z + 2
22*a^3*b^3*c^2*d^2*e^6*g*h*z + 186*a^2*b^4*c^2*d^3*e^5*g*h*z - 166*a^2*b^3*
c^3*d^4*e^4*g*h*z + 792*a^3*b^2*c^3*d^2*e^6*f*h*z - 744*a^2*b^2*c^4*d^4*e^4
*f*h*z + 492*a^2*b^3*c^3*d^3*e^5*f*h*z - 264*a^2*b^4*c^2*d^2*e^6*f*h*z + 99
6*a^2*b^2*c^4*d^3*e^5*f*g*z - 870*a^2*b^3*c^3*d^2*e^6*f*g*z + 16*a*b*c^6*d^
7*e*f*h*z - 56*a*b^6*c*d*e^7*f*g*z - 264*a^4*b*c^3*d^2*e^6*g*h*z + 208*a^3*
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b*c^4*d^4*e^4*g*h*z + 156*a^4*b^2*c^2*d*e^7*g*h*z - 148*a*b^4*c^3*d^5*e^3*g
*h*z + 54*a*b^5*c^2*d^4*e^4*g*h*z - 48*a^2*b^5*c*d^2*e^6*g*h*z - 24*a^2*b*c
^5*d^6*e^2*g*h*z + 10*a*b^3*c^4*d^6*e^2*g*h*z - 656*a^3*b*c^4*d^3*e^5*f*h*z
 - 308*a^3*b^3*c^2*d*e^7*f*h*z + 116*a*b^4*c^3*d^4*e^4*f*h*z - 84*a*b^5*c^2
*d^3*e^5*f*h*z + 68*a*b^3*c^4*d^5*e^3*f*h*z - 48*a^2*b*c^5*d^5*e^3*f*h*z -
24*a*b^2*c^5*d^6*e^2*f*h*z + 1320*a^3*b*c^4*d^2*e^6*f*g*z - 732*a^3*b^2*c^3
*d*e^7*f*g*z + 306*a^2*b^4*c^2*d*e^7*f*g*z - 304*a*b^4*c^3*d^3*e^5*f*g*z +
222*a*b^5*c^2*d^2*e^6*f*g*z + 110*a*b^3*c^4*d^4*e^4*f*g*z - 84*a*b^2*c^5*d^
5*e^3*f*g*z + 16*a*c^7*d^7*e*f*g*z - 8*a*b^7*d*e^7*f*h*z + 4*a*b*c^6*d^8*g*
h*z + 6*b^6*c^2*d^5*e^3*g*h*z + 6*b^5*c^3*d^6*e^2*g*h*z + 1072*a^4*c^4*d^3*
e^5*g*h*z - 720*a^3*c^5*d^5*e^3*g*h*z - 8*b^6*c^2*d^4*e^4*f*h*z - 8*b^4*c^4
*d^6*e^2*f*h*z + 1072*a^3*c^5*d^4*e^4*f*h*z - 960*a^4*c^4*d^2*e^6*f*h*z + 3
0*b^6*c^2*d^3*e^5*f*g*z + 30*b^3*c^5*d^6*e^2*f*g*z - 10*b^5*c^3*d^4*e^4*f*g
*z - 10*b^4*c^4*d^5*e^3*f*g*z - 1488*a^3*c^5*d^3*e^5*f*g*z + 48*a^2*c^6*d^5
*e^3*f*g*z - 24*a^4*b^2*c^2*e^8*f*h*z + 186*a^3*b^3*c^2*e^8*f*g*z + 4*a^4*b
^3*c*d*e^7*h^2*z + 4*a*b^6*c*d^4*e^4*h^2*z + 4*a*b^3*c^4*d^7*e*h^2*z + 168*
a^4*b*c^3*d*e^7*g^2*z + 24*a^2*b^5*c*d*e^7*g^2*z + 18*a*b^6*c*d^2*e^6*g^2*z
 - 912*a^3*b*c^4*d*e^7*f^2*z - 192*a*b^5*c^2*d*e^7*f^2*z + 144*a*b*c^6*d^5*
e^3*f^2*z + 432*a^3*b^2*c^3*d^4*e^4*h^2*z - 168*a^4*b^2*c^2*d^2*e^6*h^2*z -
 168*a^2*b^2*c^4*d^6*e^2*h^2*z - 108*a^2*b^4*c^2*d^4*e^4*h^2*z - 20*a^3*b^3
*c^2*d^3*e^5*h^2*z - 20*a^2*b^3*c^3*d^5*e^3*h^2*z - 426*a^2*b^2*c^4*d^4*e^4
*g^2*z + 336*a^3*b^2*c^3*d^2*e^6*g^2*z + 274*a^2*b^3*c^3*d^3*e^5*g^2*z - 12
0*a^2*b^4*c^2*d^2*e^6*g^2*z - 864*a^2*b^2*c^4*d^2*e^6*f^2*z - 2*b^7*c*d^4*e
^4*g*h*z - 2*b^4*c^4*d^7*e*g*h*z - 240*a^5*c^3*d*e^7*g*h*z + 16*a^2*c^6*d^7
*e*g*h*z + 4*b^7*c*d^3*e^5*f*h*z + 4*b^3*c^5*d^7*e*f*h*z - 20*b^7*c*d^2*e^6
*f*g*z - 20*b^2*c^6*d^7*e*f*g*z + 4*a^2*b^6*d*e^7*g*h*z + 4*a*b^7*d^2*e^6*g
*h*z + 528*a^4*c^4*d*e^7*f*g*z + 12*a^5*b*c^2*e^8*g*h*z - 2*a^4*b^3*c*e^8*g
*h*z + 4*a^3*b^4*c*e^8*f*h*z - 228*a^4*b*c^3*e^8*f*g*z - 48*a^2*b^5*c*e^8*f
*g*z - 8*a*b*c^6*d^7*e*g^2*z + 36*a^3*b^4*c*d^2*e^6*h^2*z + 36*a*b^4*c^3*d^
6*e^2*h^2*z + 12*a^2*b^5*c*d^3*e^5*h^2*z + 12*a*b^5*c^2*d^5*e^3*h^2*z - 312
*a^3*b*c^4*d^3*e^5*g^2*z + 104*a*b^4*c^3*d^4*e^4*g^2*z - 102*a^3*b^3*c^2*d*
e^7*g^2*z - 66*a*b^5*c^2*d^3*e^5*g^2*z + 24*a^2*b*c^5*d^5*e^3*g^2*z + 24*a*
b^2*c^5*d^6*e^2*g^2*z - 18*a*b^3*c^4*d^5*e^3*g^2*z + 744*a^2*b^3*c^3*d*e^7*
f^2*z + 240*a^2*b*c^5*d^3*e^5*f^2*z + 216*a*b^4*c^3*d^2*e^6*f^2*z - 120*a*b
^2*c^5*d^4*e^4*f^2*z + 24*a^5*c^3*e^8*f*h*z + 16*b^7*c*d*e^7*f^2*z + 16*b*c
^7*d^7*e*f^2*z - 2*a*b^7*d*e^7*g^2*z + 48*a*b^6*c*e^8*f^2*z - 4*b^6*c^2*d^6
*e^2*h^2*z - 536*a^4*c^4*d^4*e^4*h^2*z + 240*a^5*c^3*d^2*e^6*h^2*z + 240*a^
3*c^5*d^6*e^2*h^2*z - 12*b^6*c^2*d^4*e^4*g^2*z - 12*b^4*c^4*d^6*e^2*g^2*z +
 10*b^5*c^3*d^5*e^3*g^2*z + 528*a^3*c^5*d^4*e^4*g^2*z - 432*a^4*c^4*d^2*e^6
*g^2*z + 20*b^4*c^4*d^4*e^4*f^2*z - 16*b^6*c^2*d^2*e^6*f^2*z - 16*b^2*c^6*d
^6*e^2*f^2*z - 16*a^2*c^6*d^6*e^2*g^2*z - 8*b^5*c^3*d^3*e^5*f^2*z - 8*b^3*c
^5*d^5*e^3*f^2*z - 4*a^2*b^6*d^2*e^6*h^2*z + 912*a^3*c^5*d^2*e^6*f^2*z - 12
0*a^2*c^6*d^4*e^4*f^2*z - 45*a^4*b^2*c^2*e^8*g^2*z + 264*a^3*b^2*c^3*e^8*f^
2*z - 192*a^2*b^4*c^2*e^8*f^2*z + 4*b^8*d*e^7*f*g*z - 8*a*c^7*d^8*f*h*z + 4
*b*c^7*d^8*f*g*z + 4*a*b^7*e^8*f*g*z + 6*b^7*c*d^3*e^5*g^2*z + 6*b^3*c^5*d^
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7*e*g^2*z - 48*a*c^7*d^6*e^2*f^2*z + 12*a^3*b^4*c*e^8*g^2*z - b^8*d^2*e^6*g
^2*z - 4*a^6*c^2*e^8*h^2*z + 48*a^5*c^3*e^8*g^2*z - 4*a^2*c^6*d^8*h^2*z - b
^2*c^6*d^8*g^2*z - 36*a^4*c^4*e^8*f^2*z - a^2*b^6*e^8*g^2*z - 4*c^8*d^8*f^2
*z - 4*b^8*e^8*f^2*z - 80*a*b*c^4*d^3*e^3*f*g*h + 24*a^2*b*c^3*d*e^5*f*g*h
+ 16*a*b^3*c^2*d*e^5*f*g*h - 72*a*b^2*c^3*d^2*e^4*f*g*h - 48*a^2*b*c^3*d^3*
e^3*g*h^2 + 16*a*b^3*c^2*d^3*e^3*g*h^2 - 12*a*b^2*c^3*d^3*e^3*g^2*h - 6*a^2
*b^2*c^2*d*e^5*g^2*h - 72*a^2*b^2*c^2*d*e^5*f*h^2 + 48*a*b^2*c^3*d^3*e^3*f*
h^2 + 24*a^2*b*c^3*d^2*e^4*f*h^2 - 8*a*b^3*c^2*d^2*e^4*f*h^2 - 8*b^5*c*d*e^
5*f*g*h - 8*b*c^5*d^5*e*f*g*h - 8*a*b^4*c*e^6*f*g*h + 24*b^3*c^3*d^3*e^3*f*
g*h + 16*b^4*c^2*d^2*e^4*f*g*h + 16*b^2*c^4*d^4*e^2*f*g*h + 48*a^2*c^4*d^2*
e^4*f*g*h + 48*a^2*b^2*c^2*e^6*f*g*h + 40*a^3*b*c^2*d*e^5*g*h^2 + 28*a*b*c^
4*d^4*e^2*g^2*h - 8*a^2*b^3*c*d*e^5*g*h^2 - 8*a*b^4*c*d^2*e^4*g*h^2 + 96*a*
b^2*c^3*d*e^5*f^2*h + 24*a*b*c^4*d^2*e^4*f^2*h + 16*a*b*c^4*d^4*e^2*f*h^2 +
 96*a*b*c^4*d^2*e^4*f*g^2 - 48*a*b^2*c^3*d*e^5*f*g^2 + 12*a^2*b^2*c^2*d^2*e
^4*g*h^2 - 56*a*c^5*d^4*e^2*f*g*h - 8*a*b*c^4*d^5*e*g*h^2 + 4*a*b^4*c*d*e^5
*g^2*h + 16*a*b^4*c*d*e^5*f*h^2 - 48*a*b*c^4*d*e^5*f^2*g - 24*a^3*c^3*e^6*f
*g*h + 16*a*c^5*d^5*e*f*h^2 - 6*b^4*c^2*d^3*e^3*g^2*h - 6*b^3*c^3*d^4*e^2*g
^2*h + 4*b^4*c^2*d^4*e^2*g*h^2 + 80*a^2*c^4*d^3*e^3*g^2*h - 44*a^2*c^4*d^4*
e^2*g*h^2 + 24*a^3*c^3*d^2*e^4*g*h^2 - 16*b^3*c^3*d^2*e^4*f^2*h - 16*b^2*c^
4*d^3*e^3*f^2*h - 8*b^4*c^2*d^3*e^3*f*h^2 - 8*b^3*c^3*d^4*e^2*f*h^2 + 60*b^
2*c^4*d^2*e^4*f^2*g - 48*a^2*c^4*d^3*e^3*f*h^2 - 24*b^3*c^3*d^2*e^4*f*g^2 -
 24*b^2*c^4*d^3*e^3*f*g^2 - 24*a^3*b*c^2*d^2*e^4*h^3 + 24*a^2*b*c^3*d^4*e^2
*h^3 + 8*a^2*b^3*c*d^2*e^4*h^3 - 8*a*b^3*c^2*d^4*e^2*h^3 + 18*a*b^2*c^3*d^2
*e^4*g^3 + 2*b^5*c*d^2*e^4*g^2*h + 2*b^2*c^4*d^5*e*g^2*h - 48*a^3*c^3*d*e^5
*g^2*h - 8*b^4*c^2*d*e^5*f^2*h - 8*b*c^5*d^4*e^2*f^2*h - 168*a^2*c^4*d*e^5*
f^2*h + 96*a*c^5*d^3*e^3*f^2*h + 64*a^3*c^3*d*e^5*f*h^2 + 12*b^4*c^2*d*e^5*
f*g^2 + 12*b*c^5*d^4*e^2*f*g^2 - 168*a*c^5*d^2*e^4*f^2*g + 48*a^2*c^4*d*e^5
*f*g^2 + 48*a*c^5*d^3*e^3*f*g^2 - 12*a^3*b*c^2*e^6*g^2*h + 2*a^2*b^3*c*e^6*
g^2*h + 48*a^2*b*c^3*e^6*f^2*h - 48*a*b^3*c^2*e^6*f^2*h - 8*a^3*b*c^2*e^6*f
*h^2 - 60*a^2*b*c^3*e^6*f*g^2 + 48*a*b^2*c^3*e^6*f^2*g + 12*a*b^3*c^2*e^6*f
*g^2 + 24*a^2*b*c^3*d*e^5*g^3 - 24*a*b*c^4*d^3*e^3*g^3 - 6*a*b^3*c^2*d*e^5*
g^3 - 12*c^6*d^4*e^2*f^2*g + 4*a^4*c^2*e^6*g*h^2 - 12*b^4*c^2*e^6*f^2*g + 3
6*a^2*c^4*e^6*f^2*g - 8*a^4*c^2*d*e^5*h^3 + 8*a^2*c^4*d^5*e*h^3 - 24*b^2*c^
4*d*e^5*f^3 - 24*b*c^5*d^2*e^4*f^3 + 8*c^6*d^5*e*f^2*h + 8*b^5*c*e^6*f^2*h
+ 144*a*c^5*d*e^5*f^3 - 72*a*b*c^4*e^6*f^3 + 10*b^3*c^3*d^3*e^3*g^3 - 3*b^4
*c^2*d^2*e^4*g^3 - 3*b^2*c^4*d^4*e^2*g^3 - 48*a^2*c^4*d^2*e^4*g^3 - 3*a^2*b
^2*c^2*e^6*g^3 + 16*c^6*d^3*e^3*f^3 + 16*b^3*c^3*e^6*f^3 + 16*a^3*c^3*e^6*g
^3, z, k)*((a^5*b^5*c*e^11 + 16*a^7*b*c^3*e^11 - 128*a^7*c^4*d*e^10 + b^5*c
^6*d^10*e + b^10*c*d^5*e^6 - 8*a^6*b^3*c^2*e^11 - 128*a^3*c^8*d^9*e^2 - 512
*a^4*c^7*d^7*e^4 - 768*a^5*c^6*d^5*e^6 - 512*a^6*c^5*d^3*e^8 - 3*b^6*c^5*d^
9*e^2 + 2*b^7*c^4*d^8*e^3 + 2*b^8*c^3*d^7*e^4 - 3*b^9*c^2*d^6*e^5 + 16*a^2*
b^2*c^7*d^9*e^2 - 264*a^2*b^3*c^6*d^8*e^3 + 480*a^2*b^4*c^5*d^7*e^4 - 246*a
^2*b^5*c^4*d^6*e^5 - 66*a^2*b^6*c^3*d^5*e^6 + 62*a^2*b^7*c^2*d^4*e^7 - 704*
a^3*b^2*c^6*d^7*e^4 - 240*a^3*b^3*c^5*d^6*e^5 + 800*a^3*b^4*c^4*d^5*e^6 - 2
46*a^3*b^5*c^3*d^4*e^7 - 76*a^3*b^6*c^2*d^3*e^8 - 1440*a^4*b^2*c^5*d^5*e^6
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- 240*a^4*b^3*c^4*d^4*e^7 + 480*a^4*b^4*c^3*d^3*e^8 + 21*a^4*b^5*c^2*d^2*e^
9 - 704*a^5*b^2*c^4*d^3*e^8 - 264*a^5*b^3*c^3*d^2*e^9 - 8*a*b^3*c^7*d^10*e
- 3*a*b^9*c*d^4*e^7 + 16*a^2*b*c^8*d^10*e - 3*a^4*b^6*c*d*e^10 + 16*a*b^4*c
^6*d^9*e^2 + 21*a*b^5*c^5*d^8*e^3 - 76*a*b^6*c^4*d^7*e^4 + 62*a*b^7*c^3*d^6
*e^5 - 12*a*b^8*c^2*d^5*e^6 + 2*a^2*b^8*c*d^3*e^8 + 592*a^3*b*c^7*d^8*e^3 +
 2*a^3*b^7*c*d^2*e^9 + 1696*a^4*b*c^6*d^6*e^5 + 1696*a^5*b*c^5*d^4*e^7 + 16
*a^5*b^4*c^2*d*e^10 + 592*a^6*b*c^4*d^2*e^9 + 16*a^6*b^2*c^3*d*e^10)/(16*a^
2*c^6*d^8 + a^4*b^4*e^8 + 16*a^6*c^2*e^8 + b^4*c^4*d^8 + b^8*d^4*e^4 - 8*a*
b^2*c^5*d^8 - 8*a^5*b^2*c*e^8 - 4*a*b^7*d^3*e^5 - 4*a^3*b^5*d*e^7 - 4*b^5*c
^3*d^7*e - 4*b^7*c*d^5*e^3 + 6*a^2*b^6*d^2*e^6 + 64*a^3*c^5*d^6*e^2 + 96*a^
4*c^4*d^4*e^4 + 64*a^5*c^3*d^2*e^6 + 6*b^6*c^2*d^6*e^2 + 64*a^2*b^2*c^4*d^6
*e^2 + 32*a^2*b^3*c^3*d^5*e^3 - 74*a^2*b^4*c^2*d^4*e^4 + 144*a^3*b^2*c^3*d^
4*e^4 + 32*a^3*b^3*c^2*d^3*e^5 + 64*a^4*b^2*c^2*d^2*e^6 + 32*a*b^3*c^4*d^7*
e + 4*a*b^6*c*d^4*e^4 - 64*a^2*b*c^5*d^7*e + 32*a^4*b^3*c*d*e^7 - 64*a^5*b*
c^2*d*e^7 - 44*a*b^4*c^3*d^6*e^2 + 20*a*b^5*c^2*d^5*e^3 + 20*a^2*b^5*c*d^3*
e^5 - 192*a^3*b*c^4*d^5*e^3 - 44*a^3*b^4*c*d^2*e^6 - 192*a^4*b*c^3*d^3*e^5)
 + (x*(2*a^4*b^6*c*e^11 - 96*a^7*c^4*e^11 + 32*a^2*c^9*d^10*e + 2*b^4*c^7*d
^10*e + 2*b^10*c*d^4*e^7 - 22*a^5*b^4*c^2*e^11 + 80*a^6*b^2*c^3*e^11 + 32*a
^3*c^8*d^8*e^3 - 192*a^4*c^7*d^6*e^5 - 448*a^5*c^6*d^4*e^7 - 352*a^6*c^5*d^
2*e^9 - 10*b^5*c^6*d^9*e^2 + 22*b^6*c^5*d^8*e^3 - 28*b^7*c^4*d^7*e^4 + 22*b
^8*c^3*d^6*e^5 - 10*b^9*c^2*d^5*e^6 + 336*a^2*b^2*c^7*d^8*e^3 - 384*a^2*b^3
*c^6*d^7*e^4 + 180*a^2*b^4*c^5*d^6*e^5 + 132*a^2*b^5*c^4*d^5*e^6 - 200*a^2*
b^6*c^3*d^4*e^7 + 52*a^2*b^7*c^2*d^3*e^8 + 416*a^3*b^2*c^6*d^6*e^5 - 800*a^
3*b^3*c^5*d^5*e^6 + 580*a^3*b^4*c^4*d^4*e^7 + 24*a^3*b^5*c^3*d^3*e^8 - 116*
a^3*b^6*c^2*d^2*e^9 - 160*a^4*b^2*c^5*d^4*e^7 - 640*a^4*b^3*c^4*d^3*e^8 + 3
30*a^4*b^4*c^3*d^2*e^9 - 144*a^5*b^2*c^4*d^2*e^9 - 16*a*b^2*c^8*d^10*e - 8*
a*b^9*c*d^3*e^8 - 8*a^3*b^7*c*d*e^10 + 352*a^6*b*c^4*d*e^10 + 80*a*b^3*c^7*
d^9*e^2 - 174*a*b^4*c^6*d^8*e^3 + 216*a*b^5*c^5*d^7*e^4 - 156*a*b^6*c^4*d^6
*e^5 + 48*a*b^7*c^3*d^5*e^6 + 10*a*b^8*c^2*d^4*e^7 - 160*a^2*b*c^8*d^9*e^2
+ 12*a^2*b^8*c*d^2*e^9 - 128*a^3*b*c^7*d^7*e^4 + 576*a^4*b*c^6*d^5*e^6 + 86
*a^4*b^5*c^2*d*e^10 + 896*a^5*b*c^5*d^3*e^8 - 304*a^5*b^3*c^3*d*e^10))/(16*
a^2*c^6*d^8 + a^4*b^4*e^8 + 16*a^6*c^2*e^8 + b^4*c^4*d^8 + b^8*d^4*e^4 - 8*
a*b^2*c^5*d^8 - 8*a^5*b^2*c*e^8 - 4*a*b^7*d^3*e^5 - 4*a^3*b^5*d*e^7 - 4*b^5
*c^3*d^7*e - 4*b^7*c*d^5*e^3 + 6*a^2*b^6*d^2*e^6 + 64*a^3*c^5*d^6*e^2 + 96*
a^4*c^4*d^4*e^4 + 64*a^5*c^3*d^2*e^6 + 6*b^6*c^2*d^6*e^2 + 64*a^2*b^2*c^4*d
^6*e^2 + 32*a^2*b^3*c^3*d^5*e^3 - 74*a^2*b^4*c^2*d^4*e^4 + 144*a^3*b^2*c^3*
d^4*e^4 + 32*a^3*b^3*c^2*d^3*e^5 + 64*a^4*b^2*c^2*d^2*e^6 + 32*a*b^3*c^4*d^
7*e + 4*a*b^6*c*d^4*e^4 - 64*a^2*b*c^5*d^7*e + 32*a^4*b^3*c*d*e^7 - 64*a^5*
b*c^2*d*e^7 - 44*a*b^4*c^3*d^6*e^2 + 20*a*b^5*c^2*d^5*e^3 + 20*a^2*b^5*c*d^
3*e^5 - 192*a^3*b*c^4*d^5*e^3 - 44*a^3*b^4*c*d^2*e^6 - 192*a^4*b*c^3*d^3*e^
5)) - (x*(48*a^5*c^4*e^9*g - 72*a^4*b*c^4*e^9*f + 16*a*c^8*d^7*e^2*f + 144*
a^4*c^5*d*e^8*f - 8*a^5*b*c^3*e^9*h - 80*a^5*c^4*d*e^8*h - 4*a^2*b^5*c^2*e^
9*f + 34*a^3*b^3*c^3*e^9*f + 2*a^3*b^4*c^2*e^9*g - 20*a^4*b^2*c^3*e^9*g + 1
76*a^2*c^7*d^5*e^4*f + 304*a^3*c^6*d^3*e^6*f + 2*a^4*b^3*c^2*e^9*h - 80*a^2
*c^7*d^6*e^3*g - 112*a^3*c^6*d^4*e^5*g + 16*a^4*c^5*d^2*e^7*g - 4*b^2*c^7*d
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^7*e^2*f + 14*b^3*c^6*d^6*e^3*f - 10*b^4*c^5*d^5*e^4*f - 10*b^5*c^4*d^4*e^5
*f + 14*b^6*c^3*d^3*e^6*f - 4*b^7*c^2*d^2*e^7*f + 48*a^2*c^7*d^7*e^2*h + 16
*a^3*c^6*d^5*e^4*h - 112*a^4*c^5*d^3*e^6*h + 2*b^3*c^6*d^7*e^2*g - 12*b^4*c
^5*d^6*e^3*g + 20*b^5*c^4*d^5*e^4*g - 12*b^6*c^3*d^4*e^5*g + 2*b^7*c^2*d^3*
e^6*g + 2*b^4*c^5*d^7*e^2*h - 2*b^5*c^4*d^6*e^3*h - 2*b^6*c^3*d^5*e^4*h + 2
*b^7*c^2*d^4*e^5*h - 4*a*b^2*c^6*d^5*e^4*f + 150*a*b^3*c^5*d^4*e^5*f - 128*
a*b^4*c^4*d^3*e^6*f + 14*a*b^5*c^3*d^2*e^7*f - 440*a^2*b*c^6*d^4*e^5*f - 62
*a^2*b^4*c^3*d*e^8*f - 456*a^3*b*c^5*d^2*e^7*f + 84*a^3*b^2*c^4*d*e^8*f + 6
8*a*b^2*c^6*d^6*e^3*g - 118*a*b^3*c^5*d^5*e^4*g + 54*a*b^4*c^4*d^4*e^5*g +
6*a*b^5*c^3*d^3*e^6*g - 2*a*b^6*c^2*d^2*e^7*g + 152*a^2*b*c^6*d^5*e^4*g - 2
*a^2*b^5*c^2*d*e^8*g + 72*a^3*b*c^5*d^3*e^6*g + 30*a^3*b^3*c^3*d*e^8*g - 20
*a*b^2*c^6*d^7*e^2*h + 30*a*b^3*c^5*d^6*e^3*h - 4*a*b^4*c^4*d^5*e^4*h + 6*a
*b^5*c^3*d^4*e^5*h - 12*a*b^6*c^2*d^3*e^6*h - 88*a^2*b*c^6*d^6*e^3*h + 72*a
^3*b*c^5*d^4*e^5*h - 12*a^3*b^4*c^2*d*e^8*h + 152*a^4*b*c^4*d^2*e^7*h + 68*
a^4*b^2*c^3*d*e^8*h + 212*a^2*b^2*c^5*d^3*e^6*f + 122*a^2*b^3*c^4*d^2*e^7*f
 + 4*a^2*b^2*c^5*d^4*e^5*g - 74*a^2*b^3*c^4*d^3*e^6*g - 4*a^2*b^4*c^3*d^2*e
^7*g + 44*a^3*b^2*c^4*d^2*e^7*g + 44*a^2*b^2*c^5*d^5*e^4*h - 74*a^2*b^3*c^4
*d^4*e^5*h + 54*a^2*b^4*c^3*d^3*e^6*h + 20*a^2*b^5*c^2*d^2*e^7*h + 4*a^3*b^
2*c^4*d^3*e^6*h - 118*a^3*b^3*c^3*d^2*e^7*h - 56*a*b*c^7*d^6*e^3*f + 8*a*b^
6*c^2*d*e^8*f - 8*a*b*c^7*d^7*e^2*g - 88*a^4*b*c^4*d*e^8*g))/(16*a^2*c^6*d^
8 + a^4*b^4*e^8 + 16*a^6*c^2*e^8 + b^4*c^4*d^8 + b^8*d^4*e^4 - 8*a*b^2*c^5*
d^8 - 8*a^5*b^2*c*e^8 - 4*a*b^7*d^3*e^5 - 4*a^3*b^5*d*e^7 - 4*b^5*c^3*d^7*e
 - 4*b^7*c*d^5*e^3 + 6*a^2*b^6*d^2*e^6 + 64*a^3*c^5*d^6*e^2 + 96*a^4*c^4*d^
4*e^4 + 64*a^5*c^3*d^2*e^6 + 6*b^6*c^2*d^6*e^2 + 64*a^2*b^2*c^4*d^6*e^2 + 3
2*a^2*b^3*c^3*d^5*e^3 - 74*a^2*b^4*c^2*d^4*e^4 + 144*a^3*b^2*c^3*d^4*e^4 +
32*a^3*b^3*c^2*d^3*e^5 + 64*a^4*b^2*c^2*d^2*e^6 + 32*a*b^3*c^4*d^7*e + 4*a*
b^6*c*d^4*e^4 - 64*a^2*b*c^5*d^7*e + 32*a^4*b^3*c*d*e^7 - 64*a^5*b*c^2*d*e^
7 - 44*a*b^4*c^3*d^6*e^2 + 20*a*b^5*c^2*d^5*e^3 + 20*a^2*b^5*c*d^3*e^5 - 19
2*a^3*b*c^4*d^5*e^3 - 44*a^3*b^4*c*d^2*e^6 - 192*a^4*b*c^3*d^3*e^5)) - (32*
a^2*c^5*d^3*e^4*g^2 - 4*c^7*d^5*e^2*f^2 - a^2*b^3*c^2*e^7*g^2 - 4*b^5*c^2*e
^7*f^2 - 4*b^2*c^5*d^3*e^4*f^2 - 4*b^3*c^4*d^2*e^5*f^2 + 12*a^2*c^5*d^5*e^2
*h^2 - 40*a^3*c^4*d^3*e^4*h^2 - b^2*c^5*d^5*e^2*g^2 + b^3*c^4*d^4*e^3*g^2 +
 b^4*c^3*d^3*e^4*g^2 - b^5*c^2*d^2*e^5*g^2 + 24*a^3*c^4*e^7*f*g - 8*a^4*c^3
*e^7*g*h + 28*a*b^3*c^3*e^7*f^2 - 48*a^2*b*c^4*e^7*f^2 + 4*a^3*b*c^3*e^7*g^
2 - 8*a*c^6*d^3*e^4*f^2 + 60*a^2*c^5*d*e^6*f^2 + 8*b*c^6*d^4*e^3*f^2 - 32*a
^3*c^4*d*e^6*g^2 + 8*b^4*c^3*d*e^6*f^2 + 12*a^4*c^3*d*e^6*h^2 + 24*a*b*c^5*
d^2*e^5*f^2 - 48*a*b^2*c^4*d*e^6*f^2 + 4*a*b*c^5*d^4*e^3*g^2 - 2*a*b^4*c^2*
d*e^6*g^2 - 22*a^2*b^2*c^3*e^7*f*g - 4*a^2*b^3*c^2*e^7*f*h - 112*a^2*c^5*d^
2*e^5*f*g + 2*a^3*b^2*c^2*e^7*g*h + 80*a^2*c^5*d^3*e^4*f*h - 6*b^2*c^5*d^4*
e^3*f*g + 4*b^3*c^4*d^3*e^4*f*g - 6*b^4*c^3*d^2*e^5*f*g - 40*a^2*c^5*d^4*e^
3*g*h + 80*a^3*c^4*d^2*e^5*g*h - 4*b^2*c^5*d^5*e^2*f*h + 4*b^3*c^4*d^4*e^3*
f*h + 4*b^4*c^3*d^3*e^4*f*h - 4*b^5*c^2*d^2*e^5*f*h + 2*b^3*c^4*d^5*e^2*g*h
 - 4*b^4*c^3*d^4*e^3*g*h + 2*b^5*c^2*d^3*e^4*g*h - 18*a*b^2*c^4*d^3*e^4*g^2
 + 12*a*b^3*c^3*d^2*e^5*g^2 - 24*a^2*b*c^4*d^2*e^5*g^2 + 15*a^2*b^2*c^3*d*e
^6*g^2 - 4*a*b^2*c^4*d^5*e^2*h^2 + 4*a*b^3*c^3*d^4*e^3*h^2 - 4*a*b^4*c^2*d^
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3*e^4*h^2 - 8*a^2*b*c^4*d^4*e^3*h^2 - 8*a^3*b*c^3*d^2*e^5*h^2 - 4*a^3*b^2*c
^2*d*e^6*h^2 + 4*a*b^4*c^2*e^7*f*g + 16*a^3*b*c^3*e^7*f*h - 8*a*c^6*d^4*e^3
*f*g + 8*a*c^6*d^5*e^2*f*h + 4*b*c^6*d^5*e^2*f*g - 56*a^3*c^4*d*e^6*f*h + 4
*b^5*c^2*d*e^6*f*g + 20*a^2*b^2*c^3*d^3*e^4*h^2 + 4*a^2*b^3*c^2*d^2*e^5*h^2
 + 8*a*b*c^5*d^3*e^4*f*g - 40*a*b^3*c^3*d*e^6*f*g + 100*a^2*b*c^4*d*e^6*f*g
 - 4*a*b*c^5*d^5*e^2*g*h - 4*a^3*b*c^3*d*e^6*g*h + 44*a*b^2*c^4*d^2*e^5*f*g
 - 48*a*b^2*c^4*d^3*e^4*f*h + 32*a*b^3*c^3*d^2*e^5*f*h - 48*a^2*b*c^4*d^2*e
^5*f*h + 12*a^2*b^2*c^3*d*e^6*f*h + 18*a*b^2*c^4*d^4*e^3*g*h - 8*a*b^3*c^3*
d^3*e^4*g*h + 2*a*b^4*c^2*d^2*e^5*g*h + 24*a^2*b*c^4*d^3*e^4*g*h + 2*a^2*b^
3*c^2*d*e^6*g*h - 36*a^2*b^2*c^3*d^2*e^5*g*h)/(16*a^2*c^6*d^8 + a^4*b^4*e^8
 + 16*a^6*c^2*e^8 + b^4*c^4*d^8 + b^8*d^4*e^4 - 8*a*b^2*c^5*d^8 - 8*a^5*b^2
*c*e^8 - 4*a*b^7*d^3*e^5 - 4*a^3*b^5*d*e^7 - 4*b^5*c^3*d^7*e - 4*b^7*c*d^5*
e^3 + 6*a^2*b^6*d^2*e^6 + 64*a^3*c^5*d^6*e^2 + 96*a^4*c^4*d^4*e^4 + 64*a^5*
c^3*d^2*e^6 + 6*b^6*c^2*d^6*e^2 + 64*a^2*b^2*c^4*d^6*e^2 + 32*a^2*b^3*c^3*d
^5*e^3 - 74*a^2*b^4*c^2*d^4*e^4 + 144*a^3*b^2*c^3*d^4*e^4 + 32*a^3*b^3*c^2*
d^3*e^5 + 64*a^4*b^2*c^2*d^2*e^6 + 32*a*b^3*c^4*d^7*e + 4*a*b^6*c*d^4*e^4 -
 64*a^2*b*c^5*d^7*e + 32*a^4*b^3*c*d*e^7 - 64*a^5*b*c^2*d*e^7 - 44*a*b^4*c^
3*d^6*e^2 + 20*a*b^5*c^2*d^5*e^3 + 20*a^2*b^5*c*d^3*e^5 - 192*a^3*b*c^4*d^5
*e^3 - 44*a^3*b^4*c*d^2*e^6 - 192*a^4*b*c^3*d^3*e^5))*root(3840*a^6*b*c^5*d
^5*e^7*z^3 + 3840*a^5*b*c^6*d^7*e^5*z^3 + 1920*a^7*b*c^4*d^3*e^9*z^3 + 1920
*a^4*b*c^7*d^9*e^3*z^3 - 288*a^7*b^3*c^2*d*e^11*z^3 - 288*a^2*b^3*c^7*d^11*
e*z^3 + 210*a^4*b^7*c*d^3*e^9*z^3 + 210*a*b^7*c^4*d^9*e^3*z^3 - 174*a^5*b^6
*c*d^2*e^10*z^3 - 174*a*b^6*c^5*d^10*e^2*z^3 - 120*a^3*b^8*c*d^4*e^8*z^3 -
120*a*b^8*c^3*d^8*e^4*z^3 + 12*a^2*b^9*c*d^5*e^7*z^3 + 12*a*b^9*c^2*d^7*e^5
*z^3 + 384*a^8*b*c^3*d*e^11*z^3 + 384*a^3*b*c^8*d^11*e*z^3 + 72*a^6*b^5*c*d
*e^11*z^3 + 72*a*b^5*c^6*d^11*e*z^3 + 18*a*b^10*c*d^6*e^6*z^3 - 4800*a^5*b^
2*c^5*d^6*e^6*z^3 - 3120*a^6*b^2*c^4*d^4*e^8*z^3 - 3120*a^4*b^2*c^6*d^8*e^4
*z^3 + 2160*a^4*b^4*c^4*d^6*e^6*z^3 - 1776*a^4*b^5*c^3*d^5*e^7*z^3 - 1776*a
^3*b^5*c^4*d^7*e^5*z^3 + 1740*a^5*b^4*c^3*d^4*e^8*z^3 + 1740*a^3*b^4*c^5*d^
8*e^4*z^3 + 960*a^5*b^3*c^4*d^5*e^7*z^3 + 960*a^4*b^3*c^5*d^7*e^5*z^3 - 672
*a^7*b^2*c^3*d^2*e^10*z^3 - 672*a^3*b^2*c^7*d^10*e^2*z^3 + 648*a^6*b^4*c^2*
d^2*e^10*z^3 + 648*a^2*b^4*c^6*d^10*e^2*z^3 - 600*a^5*b^5*c^2*d^3*e^9*z^3 -
 600*a^2*b^5*c^5*d^9*e^3*z^3 + 372*a^3*b^7*c^2*d^5*e^7*z^3 + 372*a^2*b^7*c^
3*d^7*e^5*z^3 + 316*a^3*b^6*c^3*d^6*e^6*z^3 - 222*a^2*b^8*c^2*d^6*e^6*z^3 -
 160*a^6*b^3*c^3*d^3*e^9*z^3 - 160*a^3*b^3*c^6*d^9*e^3*z^3 + 15*a^4*b^6*c^2
*d^4*e^8*z^3 + 15*a^2*b^6*c^4*d^8*e^4*z^3 - 6*b^11*c*d^7*e^5*z^3 - 6*b^7*c^
5*d^11*e*z^3 - 6*a^5*b^7*d*e^11*z^3 - 6*a*b^11*d^5*e^7*z^3 - 12*a^7*b^4*c*e
^12*z^3 - 12*a*b^4*c^7*d^12*z^3 - 20*b^9*c^3*d^9*e^3*z^3 + 15*b^10*c^2*d^8*
e^4*z^3 + 15*b^8*c^4*d^10*e^2*z^3 - 1280*a^6*c^6*d^6*e^6*z^3 - 960*a^7*c^5*
d^4*e^8*z^3 - 960*a^5*c^7*d^8*e^4*z^3 - 384*a^8*c^4*d^2*e^10*z^3 - 384*a^4*
c^8*d^10*e^2*z^3 - 20*a^3*b^9*d^3*e^9*z^3 + 15*a^4*b^8*d^2*e^10*z^3 + 15*a^
2*b^10*d^4*e^8*z^3 + 48*a^8*b^2*c^2*e^12*z^3 + 48*a^2*b^2*c^8*d^12*z^3 - 64
*a^9*c^3*e^12*z^3 - 64*a^3*c^9*d^12*z^3 + b^12*d^6*e^6*z^3 + b^6*c^6*d^12*z
^3 + a^6*b^6*e^12*z^3 - 44*a^3*b^4*c*d*e^7*g*h*z - 20*a*b^6*c*d^3*e^5*g*h*z
 - 12*a*b^2*c^5*d^7*e*g*h*z + 432*a^4*b*c^3*d*e^7*f*h*z + 84*a^2*b^5*c*d*e^
```

```
7*f*h*z + 28*a*b^6*c*d^2*e^6*f*h*z - 8*a*b*c^6*d^6*e^2*f*g*z - 804*a^3*b^2*
c^3*d^3*e^5*g*h*z + 564*a^2*b^2*c^4*d^5*e^3*g*h*z + 222*a^3*b^3*c^2*d^2*e^6
*g*h*z + 186*a^2*b^4*c^2*d^3*e^5*g*h*z - 166*a^2*b^3*c^3*d^4*e^4*g*h*z + 79
2*a^3*b^2*c^3*d^2*e^6*f*h*z - 744*a^2*b^2*c^4*d^4*e^4*f*h*z + 492*a^2*b^3*c
^3*d^3*e^5*f*h*z - 264*a^2*b^4*c^2*d^2*e^6*f*h*z + 996*a^2*b^2*c^4*d^3*e^5*
f*g*z - 870*a^2*b^3*c^3*d^2*e^6*f*g*z + 16*a*b*c^6*d^7*e*f*h*z - 56*a*b^6*c
*d*e^7*f*g*z - 264*a^4*b*c^3*d^2*e^6*g*h*z + 208*a^3*b*c^4*d^4*e^4*g*h*z +
156*a^4*b^2*c^2*d*e^7*g*h*z - 148*a*b^4*c^3*d^5*e^3*g*h*z + 54*a*b^5*c^2*d^
4*e^4*g*h*z - 48*a^2*b^5*c*d^2*e^6*g*h*z - 24*a^2*b*c^5*d^6*e^2*g*h*z + 10*
a*b^3*c^4*d^6*e^2*g*h*z - 656*a^3*b*c^4*d^3*e^5*f*h*z - 308*a^3*b^3*c^2*d*e
^7*f*h*z + 116*a*b^4*c^3*d^4*e^4*f*h*z - 84*a*b^5*c^2*d^3*e^5*f*h*z + 68*a*
b^3*c^4*d^5*e^3*f*h*z - 48*a^2*b*c^5*d^5*e^3*f*h*z - 24*a*b^2*c^5*d^6*e^2*f
*h*z + 1320*a^3*b*c^4*d^2*e^6*f*g*z - 732*a^3*b^2*c^3*d*e^7*f*g*z + 306*a^2
*b^4*c^2*d*e^7*f*g*z - 304*a*b^4*c^3*d^3*e^5*f*g*z + 222*a*b^5*c^2*d^2*e^6*
f*g*z + 110*a*b^3*c^4*d^4*e^4*f*g*z - 84*a*b^2*c^5*d^5*e^3*f*g*z + 16*a*c^7
*d^7*e*f*g*z - 8*a*b^7*d*e^7*f*h*z + 4*a*b*c^6*d^8*g*h*z + 6*b^6*c^2*d^5*e^
3*g*h*z + 6*b^5*c^3*d^6*e^2*g*h*z + 1072*a^4*c^4*d^3*e^5*g*h*z - 720*a^3*c^
5*d^5*e^3*g*h*z - 8*b^6*c^2*d^4*e^4*f*h*z - 8*b^4*c^4*d^6*e^2*f*h*z + 1072*
a^3*c^5*d^4*e^4*f*h*z - 960*a^4*c^4*d^2*e^6*f*h*z + 30*b^6*c^2*d^3*e^5*f*g*
z + 30*b^3*c^5*d^6*e^2*f*g*z - 10*b^5*c^3*d^4*e^4*f*g*z - 10*b^4*c^4*d^5*e^
3*f*g*z - 1488*a^3*c^5*d^3*e^5*f*g*z + 48*a^2*c^6*d^5*e^3*f*g*z - 24*a^4*b^
2*c^2*e^8*f*h*z + 186*a^3*b^3*c^2*e^8*f*g*z + 4*a^4*b^3*c*d*e^7*h^2*z + 4*a
*b^6*c*d^4*e^4*h^2*z + 4*a*b^3*c^4*d^7*e*h^2*z + 168*a^4*b*c^3*d*e^7*g^2*z
+ 24*a^2*b^5*c*d*e^7*g^2*z + 18*a*b^6*c*d^2*e^6*g^2*z - 912*a^3*b*c^4*d*e^7
*f^2*z - 192*a*b^5*c^2*d*e^7*f^2*z + 144*a*b*c^6*d^5*e^3*f^2*z + 432*a^3*b^
2*c^3*d^4*e^4*h^2*z - 168*a^4*b^2*c^2*d^2*e^6*h^2*z - 168*a^2*b^2*c^4*d^6*e
^2*h^2*z - 108*a^2*b^4*c^2*d^4*e^4*h^2*z - 20*a^3*b^3*c^2*d^3*e^5*h^2*z - 2
0*a^2*b^3*c^3*d^5*e^3*h^2*z - 426*a^2*b^2*c^4*d^4*e^4*g^2*z + 336*a^3*b^2*c
^3*d^2*e^6*g^2*z + 274*a^2*b^3*c^3*d^3*e^5*g^2*z - 120*a^2*b^4*c^2*d^2*e^6*
g^2*z - 864*a^2*b^2*c^4*d^2*e^6*f^2*z - 2*b^7*c*d^4*e^4*g*h*z - 2*b^4*c^4*d
^7*e*g*h*z - 240*a^5*c^3*d*e^7*g*h*z + 16*a^2*c^6*d^7*e*g*h*z + 4*b^7*c*d^3
*e^5*f*h*z + 4*b^3*c^5*d^7*e*f*h*z - 20*b^7*c*d^2*e^6*f*g*z - 20*b^2*c^6*d^
7*e*f*g*z + 4*a^2*b^6*d*e^7*g*h*z + 4*a*b^7*d^2*e^6*g*h*z + 528*a^4*c^4*d*e
^7*f*g*z + 12*a^5*b*c^2*e^8*g*h*z - 2*a^4*b^3*c*e^8*g*h*z + 4*a^3*b^4*c*e^8
*f*h*z - 228*a^4*b*c^3*e^8*f*g*z - 48*a^2*b^5*c*e^8*f*g*z - 8*a*b*c^6*d^7*e
*g^2*z + 36*a^3*b^4*c*d^2*e^6*h^2*z + 36*a*b^4*c^3*d^6*e^2*h^2*z + 12*a^2*b
^5*c*d^3*e^5*h^2*z + 12*a*b^5*c^2*d^5*e^3*h^2*z - 312*a^3*b*c^4*d^3*e^5*g^2
*z + 104*a*b^4*c^3*d^4*e^4*g^2*z - 102*a^3*b^3*c^2*d*e^7*g^2*z - 66*a*b^5*c
^2*d^3*e^5*g^2*z + 24*a^2*b*c^5*d^5*e^3*g^2*z + 24*a*b^2*c^5*d^6*e^2*g^2*z
- 18*a*b^3*c^4*d^5*e^3*g^2*z + 744*a^2*b^3*c^3*d*e^7*f^2*z + 240*a^2*b*c^5*
d^3*e^5*f^2*z + 216*a*b^4*c^3*d^2*e^6*f^2*z - 120*a*b^2*c^5*d^4*e^4*f^2*z +
 24*a^5*c^3*e^8*f*h*z + 16*b^7*c*d*e^7*f^2*z + 16*b*c^7*d^7*e*f^2*z - 2*a*b
^7*d*e^7*g^2*z + 48*a*b^6*c*e^8*f^2*z - 4*b^6*c^2*d^6*e^2*h^2*z - 536*a^4*c
^4*d^4*e^4*h^2*z + 240*a^5*c^3*d^2*e^6*h^2*z + 240*a^3*c^5*d^6*e^2*h^2*z -
12*b^6*c^2*d^4*e^4*g^2*z - 12*b^4*c^4*d^6*e^2*g^2*z + 10*b^5*c^3*d^5*e^3*g^
```

```
2*z + 528*a^3*c^5*d^4*e^4*g^2*z - 432*a^4*c^4*d^2*e^6*g^2*z + 20*b^4*c^4*d^
4*e^4*f^2*z - 16*b^6*c^2*d^2*e^6*f^2*z - 16*b^2*c^6*d^6*e^2*f^2*z - 16*a^2*
c^6*d^6*e^2*g^2*z - 8*b^5*c^3*d^3*e^5*f^2*z - 8*b^3*c^5*d^5*e^3*f^2*z - 4*a
^2*b^6*d^2*e^6*h^2*z + 912*a^3*c^5*d^2*e^6*f^2*z - 120*a^2*c^6*d^4*e^4*f^2*
z - 45*a^4*b^2*c^2*e^8*g^2*z + 264*a^3*b^2*c^3*e^8*f^2*z - 192*a^2*b^4*c^2*
e^8*f^2*z + 4*b^8*d*e^7*f*g*z - 8*a*c^7*d^8*f*h*z + 4*b*c^7*d^8*f*g*z + 4*a
*b^7*e^8*f*g*z + 6*b^7*c*d^3*e^5*g^2*z + 6*b^3*c^5*d^7*e*g^2*z - 48*a*c^7*d
^6*e^2*f^2*z + 12*a^3*b^4*c*e^8*g^2*z - b^8*d^2*e^6*g^2*z - 4*a^6*c^2*e^8*h
^2*z + 48*a^5*c^3*e^8*g^2*z - 4*a^2*c^6*d^8*h^2*z - b^2*c^6*d^8*g^2*z - 36*
a^4*c^4*e^8*f^2*z - a^2*b^6*e^8*g^2*z - 4*c^8*d^8*f^2*z - 4*b^8*e^8*f^2*z -
 80*a*b*c^4*d^3*e^3*f*g*h + 24*a^2*b*c^3*d*e^5*f*g*h + 16*a*b^3*c^2*d*e^5*f
*g*h - 72*a*b^2*c^3*d^2*e^4*f*g*h - 48*a^2*b*c^3*d^3*e^3*g*h^2 + 16*a*b^3*c
^2*d^3*e^3*g*h^2 - 12*a*b^2*c^3*d^3*e^3*g^2*h - 6*a^2*b^2*c^2*d*e^5*g^2*h -
 72*a^2*b^2*c^2*d*e^5*f*h^2 + 48*a*b^2*c^3*d^3*e^3*f*h^2 + 24*a^2*b*c^3*d^2
*e^4*f*h^2 - 8*a*b^3*c^2*d^2*e^4*f*h^2 - 8*b^5*c*d*e^5*f*g*h - 8*b*c^5*d^5*
e*f*g*h - 8*a*b^4*c*e^6*f*g*h + 24*b^3*c^3*d^3*e^3*f*g*h + 16*b^4*c^2*d^2*e
^4*f*g*h + 16*b^2*c^4*d^4*e^2*f*g*h + 48*a^2*c^4*d^2*e^4*f*g*h + 48*a^2*b^2
*c^2*e^6*f*g*h + 40*a^3*b*c^2*d*e^5*g*h^2 + 28*a*b*c^4*d^4*e^2*g^2*h - 8*a^
2*b^3*c*d*e^5*g*h^2 - 8*a*b^4*c*d^2*e^4*g*h^2 + 96*a*b^2*c^3*d*e^5*f^2*h +
24*a*b*c^4*d^2*e^4*f^2*h + 16*a*b*c^4*d^4*e^2*f*h^2 + 96*a*b*c^4*d^2*e^4*f*
g^2 - 48*a*b^2*c^3*d*e^5*f*g^2 + 12*a^2*b^2*c^2*d^2*e^4*g*h^2 - 56*a*c^5*d^
4*e^2*f*g*h - 8*a*b*c^4*d^5*e*g*h^2 + 4*a*b^4*c*d*e^5*g^2*h + 16*a*b^4*c*d*
e^5*f*h^2 - 48*a*b*c^4*d*e^5*f^2*g - 24*a^3*c^3*e^6*f*g*h + 16*a*c^5*d^5*e*
f*h^2 - 6*b^4*c^2*d^3*e^3*g^2*h - 6*b^3*c^3*d^4*e^2*g^2*h + 4*b^4*c^2*d^4*e
^2*g*h^2 + 80*a^2*c^4*d^3*e^3*g^2*h - 44*a^2*c^4*d^4*e^2*g*h^2 + 24*a^3*c^3
*d^2*e^4*g*h^2 - 16*b^3*c^3*d^2*e^4*f^2*h - 16*b^2*c^4*d^3*e^3*f^2*h - 8*b^
4*c^2*d^3*e^3*f*h^2 - 8*b^3*c^3*d^4*e^2*f*h^2 + 60*b^2*c^4*d^2*e^4*f^2*g -
48*a^2*c^4*d^3*e^3*f*h^2 - 24*b^3*c^3*d^2*e^4*f*g^2 - 24*b^2*c^4*d^3*e^3*f*
g^2 - 24*a^3*b*c^2*d^2*e^4*h^3 + 24*a^2*b*c^3*d^4*e^2*h^3 + 8*a^2*b^3*c*d^2
*e^4*h^3 - 8*a*b^3*c^2*d^4*e^2*h^3 + 18*a*b^2*c^3*d^2*e^4*g^3 + 2*b^5*c*d^2
*e^4*g^2*h + 2*b^2*c^4*d^5*e*g^2*h - 48*a^3*c^3*d*e^5*g^2*h - 8*b^4*c^2*d*e
^5*f^2*h - 8*b*c^5*d^4*e^2*f^2*h - 168*a^2*c^4*d*e^5*f^2*h + 96*a*c^5*d^3*e
^3*f^2*h + 64*a^3*c^3*d*e^5*f*h^2 + 12*b^4*c^2*d*e^5*f*g^2 + 12*b*c^5*d^4*e
^2*f*g^2 - 168*a*c^5*d^2*e^4*f^2*g + 48*a^2*c^4*d*e^5*f*g^2 + 48*a*c^5*d^3*
e^3*f*g^2 - 12*a^3*b*c^2*e^6*g^2*h + 2*a^2*b^3*c*e^6*g^2*h + 48*a^2*b*c^3*e
^6*f^2*h - 48*a*b^3*c^2*e^6*f^2*h - 8*a^3*b*c^2*e^6*f*h^2 - 60*a^2*b*c^3*e^
6*f*g^2 + 48*a*b^2*c^3*e^6*f^2*g + 12*a*b^3*c^2*e^6*f*g^2 + 24*a^2*b*c^3*d*
e^5*g^3 - 24*a*b*c^4*d^3*e^3*g^3 - 6*a*b^3*c^2*d*e^5*g^3 - 12*c^6*d^4*e^2*f
^2*g + 4*a^4*c^2*e^6*g*h^2 - 12*b^4*c^2*e^6*f^2*g + 36*a^2*c^4*e^6*f^2*g -
8*a^4*c^2*d*e^5*h^3 + 8*a^2*c^4*d^5*e*h^3 - 24*b^2*c^4*d*e^5*f^3 - 24*b*c^5
*d^2*e^4*f^3 + 8*c^6*d^5*e*f^2*h + 8*b^5*c*e^6*f^2*h + 144*a*c^5*d*e^5*f^3
- 72*a*b*c^4*e^6*f^3 + 10*b^3*c^3*d^3*e^3*g^3 - 3*b^4*c^2*d^2*e^4*g^3 - 3*b
^2*c^4*d^4*e^2*g^3 - 48*a^2*c^4*d^2*e^4*g^3 - 3*a^2*b^2*c^2*e^6*g^3 + 16*c^
6*d^3*e^3*f^3 + 16*b^3*c^3*e^6*f^3 + 16*a^3*c^3*e^6*g^3, z, k), k, 1, 3)
```

3.160 $\int \frac{x^3(1+x+x^2)}{(1-x+x^2)^2} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 62

$$\int \frac{x^3(1+x+x^2)}{(1-x+x^2)^2} \, dx = 3x + \frac{x^2}{2} + \frac{2(2-x)}{3(1-x+x^2)} + \frac{10 \arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}} + 2\log\left(1-x+x^2\right)$$

```
[Out] 3*x+1/2*x^2+2/3*(2-x)/(x^2-x+1)+2*ln(x^2-x+1)+10/9*arctan(1/3*(1-2*x)*3^(1/
2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1674, 1671, 648, 632, 210, 642}

$$\int \frac{x^3(1+x+x^2)}{(1-x+x^2)^2} \, dx = \frac{10 \arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{x^2}{2} + \frac{2(2-x)}{3(x^2-x+1)} + 2\log\left(x^2-x+1\right) + 3x$$

```
[In] Int[(x^3*(1 + x + x^2))/(1 - x + x^2)^2,x]
```

```
[Out] 3*x + x^2/2 + (2*(2 - x))/(3*(1 - x + x^2)) + (10*ArcTan[(1 - 2*x)/Sqrt[3]]
)/(3*Sqrt[3]) + 2*Log[1 - x + x^2]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2(2-x)}{3\left(1-x+x^2\right)}+\frac{1}{3}\int \frac{-2+6x+6x^2+3x^3}{1-x+x^2}\,dx \\
&= \frac{2(2-x)}{3\left(1-x+x^2\right)}+\frac{1}{3}\int \left(9+3x-\frac{11-12x}{1-x+x^2}\right)dx \\
&= 3x+\frac{x^2}{2}+\frac{2(2-x)}{3\left(1-x+x^2\right)}-\frac{1}{3}\int \frac{11-12x}{1-x+x^2}\,dx
\end{aligned}$$

$$= 3x + \frac{x^2}{2} + \frac{2(2-x)}{3\left(1-x+x^2\right)} - \frac{5}{3}\int \frac{1}{1-x+x^2}\,dx + 2\int \frac{-1+2x}{1-x+x^2}\,dx$$

$$= 3x + \frac{x^2}{2} + \frac{2(2-x)}{3\left(1-x+x^2\right)} + 2\log\left(1-x+x^2\right) + \frac{10}{3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x\right)$$

$$= 3x + \frac{x^2}{2} + \frac{2(2-x)}{3\left(1-x+x^2\right)} + \frac{10\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}} + 2\log\left(1-x+x^2\right)$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.97

$$\int \frac{x^3(1+x+x^2)}{\left(1-x+x^2\right)^2}\,dx = 3x + \frac{x^2}{2} - \frac{2(-2+x)}{3\left(1-x+x^2\right)} - \frac{10\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)}{3\sqrt{3}} + 2\log\left(1-x+x^2\right)$$

```
[In] Integrate[(x^3*(1 + x + x^2))/(1 - x + x^2)^2,x]
```

```
[Out] 3*x + x^2/2 - (2*(-2 + x))/(3*(1 - x + x^2)) - (10*ArcTan[(-1 + 2*x)/Sqrt[3
]])/(3*Sqrt[3]) + 2*Log[1 - x + x^2]
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.85

method	result	size
default	$3x+\frac{x^2}{2}+\frac{-\frac{2x}{3}+\frac{4}{3}}{x^2-x+1}+2\ln\left(x^2-x+1\right)-\frac{10\sqrt{3}\,\arctan\left(\frac{(-1+2x)\sqrt{3}}{3}\right)}{9}$	53
risch	$3x+\frac{x^2}{2}+\frac{-\frac{2x}{3}+\frac{4}{3}}{x^2-x+1}+2\ln\left(4x^2-4x+4\right)-\frac{10\sqrt{3}\,\arctan\left(\frac{(-1+2x)\sqrt{3}}{3}\right)}{9}$	55

```
[In] int(x^3*(x^2+x+1)/(x^2-x+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 3*x+1/2*x^2+(-2/3*x+4/3)/(x^2-x+1)+2*ln(x^2-x+1)-10/9*3^(1/2)*arctan(1/3*(-
1+2*x)*3^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.21

$$\int \frac{x^3(1+x+x^2)}{(1-x+x^2)^2}\,dx$$
$$=\frac{9\,x^4+45\,x^3-20\,\sqrt{3}(x^2-x+1)\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right)-45\,x^2+36\,(x^2-x+1)\log\left(x^2-x+1\right)+42}{18\,(x^2-x+1)}$$

```
[In] integrate(x^3*(x^2+x+1)/(x^2-x+1)^2,x, algorithm="fricas")
```

```
[Out] 1/18*(9*x^4 + 45*x^3 - 20*sqrt(3)*(x^2 - x + 1)*arctan(1/3*sqrt(3)*(2*x - 1
)) - 45*x^2 + 36*(x^2 - x + 1)*log(x^2 - x + 1) + 42*x + 24)/(x^2 - x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.97

$$\int \frac{x^3(1+x+x^2)}{(1-x+x^2)^2}\,dx=\frac{x^2}{2}+3x+\frac{4-2x}{3x^2-3x+3}+2\log\left(x^2-x+1\right)-\frac{10\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

```
[In] integrate(x**3*(x**2+x+1)/(x**2-x+1)**2,x)
```

```
[Out] x**2/2 + 3*x + (4 - 2*x)/(3*x**2 - 3*x + 3) + 2*log(x**2 - x + 1) - 10*sqrt
(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/9
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.82

$$\begin{aligned}\int \frac{x^3(1+x+x^2)}{(1-x+x^2)^2}\,dx&=\frac{1}{2}\,x^2-\frac{10}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right)\\&\quad+3\,x-\frac{2\,(x-2)}{3\,(x^2-x+1)}+2\,\log\left(x^2-x+1\right)\end{aligned}$$

```
[In] integrate(x^3*(x^2+x+1)/(x^2-x+1)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - 10/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 3*x - 2/3*(x - 2)/(x
^2 - x + 1) + 2*log(x^2 - x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.82

$$\int \frac{x^3(1+x+x^2)}{(1-x+x^2)^2}\,dx = \frac{1}{2}\,x^2 - \frac{10}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right) + 3\,x - \frac{2\,(x-2)}{3\,(x^2-x+1)} + 2\,\log\left(x^2-x+1\right)$$

```
[In] integrate(x^3*(x^2+x+1)/(x^2-x+1)^2,x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 10/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 3*x - 2/3*(x - 2)/(x
^2 - x + 1) + 2*log(x^2 - x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.89

$$\int \frac{x^3(1+x+x^2)}{(1-x+x^2)^2}\,dx = 3\,x + 2\,\ln\left(x^2-x+1\right) - \frac{\frac{2\,x}{3}-\frac{4}{3}}{x^2-x+1} - \frac{10\,\sqrt{3}\,\mathrm{atan}\left(\frac{2\,\sqrt{3}\,x}{3}-\frac{\sqrt{3}}{3}\right)}{9} + \frac{x^2}{2}$$

```
[In] int((x^3*(x + x^2 + 1))/(x^2 - x + 1)^2,x)
```

```
[Out] 3*x + 2*log(x^2 - x + 1) - ((2*x)/3 - 4/3)/(x^2 - x + 1) - (10*3^(1/2)*atan
((2*3^(1/2)*x)/3 - 3^(1/2)/3))/9 + x^2/2
```

3.161 $\int \frac{x^2(1+x+x^2)}{(1-x+x^2)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 55

$$\int \frac{x^2(1+x+x^2)}{(1-x+x^2)^2}\,dx = x+\frac{2(1-2x)}{3(1-x+x^2)}-\frac{7\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{3}{2}\log\left(1-x+x^2\right)$$

```
[Out] x+2/3*(1-2*x)/(x^2-x+1)+3/2*ln(x^2-x+1)-7/9*arctan(1/3*(1-2*x)*3^(1/2))*3^(
1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1674, 1671, 648, 632, 210, 642}

$$\int \frac{x^2(1+x+x^2)}{(1-x+x^2)^2}\,dx = -\frac{7\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{2(1-2x)}{3(x^2-x+1)}+\frac{3}{2}\log\left(x^2-x+1\right)+x$$

```
[In] Int[(x^2*(1 + x + x^2))/(1 - x + x^2)^2,x]
```

```
[Out] x + (2*(1 - 2*x))/(3*(1 - x + x^2)) - (7*ArcTan[(1 - 2*x)/Sqrt[3]])/(3*Sqrt
[3]) + (3*Log[1 - x + x^2])/2
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2(1-2x)}{3(1-x+x^2)} + \frac{1}{3}\int \frac{2+6x+3x^2}{1-x+x^2}\,dx \\
&= \frac{2(1-2x)}{3(1-x+x^2)} + \frac{1}{3}\int \left(3 - \frac{1-9x}{1-x+x^2}\right)dx \\
&= x + \frac{2(1-2x)}{3(1-x+x^2)} - \frac{1}{3}\int \frac{1-9x}{1-x+x^2}\,dx
\end{aligned}$$

$$
\begin{aligned}
&= x + \frac{2(1-2x)}{3(1-x+x^2)} + \frac{7}{6}\int \frac{1}{1-x+x^2}\,dx + \frac{3}{2}\int \frac{-1+2x}{1-x+x^2}\,dx \\
&= x + \frac{2(1-2x)}{3(1-x+x^2)} + \frac{3}{2}\log\left(1-x+x^2\right) - \frac{7}{3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x\right) \\
&= x + \frac{2(1-2x)}{3(1-x+x^2)} - \frac{7\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{3}{2}\log\left(1-x+x^2\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$
\int \frac{x^2(1+x+x^2)}{(1-x+x^2)^2}\,dx = x - \frac{2(-1+2x)}{3(1-x+x^2)} + \frac{7\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{3}{2}\log\left(1-x+x^2\right)
$$

```
[In] Integrate[(x^2*(1 + x + x^2))/(1 - x + x^2)^2,x]
[Out] x - (2*(-1 + 2*x))/(3*(1 - x + x^2)) + (7*ArcTan[(-1 + 2*x)/Sqrt[3]])/(3*Sq
rt[3]) + (3*Log[1 - x + x^2])/2
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.84

method	result	size
default	$x+\frac{-\frac{4x}{3}+\frac{2}{3}}{x^2-x+1}+\frac{3\ln(x^2-x+1)}{2}+\frac{7\sqrt{3}\,\arctan\left(\frac{(-1+2x)\sqrt{3}}{3}\right)}{9}$	46
risch	$x+\frac{-\frac{4x}{3}+\frac{2}{3}}{x^2-x+1}+\frac{3\ln(4x^2-4x+4)}{2}+\frac{7\sqrt{3}\,\arctan\left(\frac{(-1+2x)\sqrt{3}}{3}\right)}{9}$	48

```
[In] int(x^2*(x^2+x+1)/(x^2-x+1)^2,x,method=_RETURNVERBOSE)
[Out] x+(-4/3*x+2/3)/(x^2-x+1)+3/2*ln(x^2-x+1)+7/9*3^(1/2)*arctan(1/3*(-1+2*x)*3^
(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.27

$$\int \frac{x^2(1+x+x^2)}{(1-x+x^2)^2}\,dx = \frac{18\,x^3+14\,\sqrt{3}(x^2-x+1)\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right)-18\,x^2+27\,(x^2-x+1)\log\left(x^2-x+1\right)-6\,x+12}{18\,(x^2-x+1)}$$

```
[In] integrate(x^2*(x^2+x+1)/(x^2-x+1)^2,x, algorithm="fricas")
[Out] 1/18*(18*x^3 + 14*sqrt(3)*(x^2 - x + 1)*arctan(1/3*sqrt(3)*(2*x - 1)) - 18*
x^2 + 27*(x^2 - x + 1)*log(x^2 - x + 1) - 6*x + 12)/(x^2 - x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.98

$$\int \frac{x^2(1+x+x^2)}{(1-x+x^2)^2}\,dx = x+\frac{2-4x}{3x^2-3x+3}+\frac{3\log\left(x^2-x+1\right)}{2}+\frac{7\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

```
[In] integrate(x**2*(x**2+x+1)/(x**2-x+1)**2,x)
[Out] x + (2 - 4*x)/(3*x**2 - 3*x + 3) + 3*log(x**2 - x + 1)/2 + 7*sqrt(3)*atan(2
*sqrt(3)*x/3 - sqrt(3)/3)/9
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.84

$$\int \frac{x^2(1+x+x^2)}{(1-x+x^2)^2}\,dx = \frac{7}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right)+x-\frac{2\,(2\,x-1)}{3\,(x^2-x+1)}+\frac{3}{2}\,\log\left(x^2-x+1\right)$$

```
[In] integrate(x^2*(x^2+x+1)/(x^2-x+1)^2,x, algorithm="maxima")
[Out] 7/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + x - 2/3*(2*x - 1)/(x^2 - x + 1)
 + 3/2*log(x^2 - x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.84

$$\int \frac{x^2(1+x+x^2)}{(1-x+x^2)^2}\,dx = \frac{7}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+x-\frac{2(2x-1)}{3(x^2-x+1)}+\frac{3}{2}\log\left(x^2-x+1\right)$$

```
[In] integrate(x^2*(x^2+x+1)/(x^2-x+1)^2,x, algorithm="giac")
```

```
[Out] 7/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + x - 2/3*(2*x - 1)/(x^2 - x + 1)
 + 3/2*log(x^2 - x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.87

$$\int \frac{x^2(1+x+x^2)}{(1-x+x^2)^2}\,dx = x+\frac{3\ln\left(x^2-x+1\right)}{2}-\frac{\frac{4x}{3}-\frac{2}{3}}{x^2-x+1}+\frac{7\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\,x}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

```
[In] int((x^2*(x + x^2 + 1))/(x^2 - x + 1)^2,x)
```

```
[Out] x + (3*log(x^2 - x + 1))/2 - ((4*x)/3 - 2/3)/(x^2 - x + 1) + (7*3^(1/2)*ata
n((2*3^(1/2)*x)/3 - 3^(1/2)/3))/9
```

3.162 $\int \frac{x(1+x+x^2)}{(1-x+x^2)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 52

$$\int \frac{x(1+x+x^2)}{(1-x+x^2)^2}\,dx = -\frac{2(1+x)}{3(1-x+x^2)} - \frac{11\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{1}{2}\log\left(1-x+x^2\right)$$

```
[Out] -2/3*(1+x)/(x^2-x+1)+1/2*ln(x^2-x+1)-11/9*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/
2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1674, 648, 632, 210, 642}

$$\int \frac{x(1+x+x^2)}{(1-x+x^2)^2}\,dx = -\frac{11\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{2(x+1)}{3(x^2-x+1)} + \frac{1}{2}\log\left(x^2-x+1\right)$$

```
[In] Int[(x*(1 + x + x^2))/(1 - x + x^2)^2,x]
```

```
[Out] (-2*(1 + x))/(3*(1 - x + x^2)) - (11*ArcTan[(1 - 2*x)/Sqrt[3]])/(3*Sqrt[3])
 + Log[1 - x + x^2]/2
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2(1+x)}{3\left(1-x+x^2\right)}+\frac{1}{3}\int\frac{4+3x}{1-x+x^2}\,dx\\
&= -\frac{2(1+x)}{3\left(1-x+x^2\right)}+\frac{1}{2}\int\frac{-1+2x}{1-x+x^2}\,dx+\frac{11}{6}\int\frac{1}{1-x+x^2}\,dx\\
&= -\frac{2(1+x)}{3\left(1-x+x^2\right)}+\frac{1}{2}\log\left(1-x+x^2\right)-\frac{11}{3}\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,-1+2x\right)\\
&= -\frac{2(1+x)}{3\left(1-x+x^2\right)}-\frac{11\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{2}\log\left(1-x+x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00

$$\int \frac{x(1+x+x^2)}{(1-x+x^2)^2}\,dx = -\frac{2(1+x)}{3(1-x+x^2)} + \frac{11\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{1}{2}\log\left(1-x+x^2\right)$$

```
[In] Integrate[(x*(1 + x + x^2))/(1 - x + x^2)^2,x]
```

```
[Out] (-2*(1 + x))/(3*(1 - x + x^2)) + (11*ArcTan[(-1 + 2*x)/Sqrt[3]])/(3*Sqrt[3]
) + Log[1 - x + x^2]/2
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{-\frac{2x}{3}-\frac{2}{3}}{x^2-x+1}+\frac{\ln\left(x^2-x+1\right)}{2}+\frac{11\sqrt{3}\,\arctan\left(\frac{(-1+2x)\sqrt{3}}{3}\right)}{9}$	45
risch	$\frac{-\frac{2x}{3}-\frac{2}{3}}{x^2-x+1}+\frac{\ln\left(4x^2-4x+4\right)}{2}+\frac{11\sqrt{3}\,\arctan\left(\frac{(-1+2x)\sqrt{3}}{3}\right)}{9}$	47

```
[In] int(x*(x^2+x+1)/(x^2-x+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-2/3*x-2/3)/(x^2-x+1)+1/2*ln(x^2-x+1)+11/9*3^(1/2)*arctan(1/3*(-1+2*x)*3^(
1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.15

$$\int \frac{x(1+x+x^2)}{(1-x+x^2)^2}\,dx = \frac{22\sqrt{3}(x^2-x+1)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+9(x^2-x+1)\log\left(x^2-x+1\right)-12x-12}{18(x^2-x+1)}$$

```
[In] integrate(x*(x^2+x+1)/(x^2-x+1)^2,x, algorithm="fricas")
```

```
[Out] 1/18*(22*sqrt(3)*(x^2 - x + 1)*arctan(1/3*sqrt(3)*(2*x - 1)) + 9*(x^2 - x +
 1)*log(x^2 - x + 1) - 12*x - 12)/(x^2 - x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.02

$$\int \frac{x(1+x+x^2)}{(1-x+x^2)^2}\,dx = \frac{-2x-2}{3x^2-3x+3} + \frac{\log\left(x^2-x+1\right)}{2} + \frac{11\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

```
[In] integrate(x*(x**2+x+1)/(x**2-x+1)**2,x)
```

```
[Out] (-2*x - 2)/(3*x**2 - 3*x + 3) + log(x**2 - x + 1)/2 + 11*sqrt(3)*atan(2*sqr
t(3)*x/3 - sqrt(3)/3)/9
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.83

$$\int \frac{x(1+x+x^2)}{(1-x+x^2)^2}\,dx = \frac{11}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right) - \frac{2\,(x+1)}{3\,(x^2-x+1)} + \frac{1}{2}\,\log\left(x^2-x+1\right)$$

```
[In] integrate(x*(x^2+x+1)/(x^2-x+1)^2,x, algorithm="maxima")
```

```
[Out] 11/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 2/3*(x + 1)/(x^2 - x + 1) + 1/
2*log(x^2 - x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.83

$$\int \frac{x(1+x+x^2)}{(1-x+x^2)^2}\,dx = \frac{11}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right) - \frac{2\,(x+1)}{3\,(x^2-x+1)} + \frac{1}{2}\,\log\left(x^2-x+1\right)$$

```
[In] integrate(x*(x^2+x+1)/(x^2-x+1)^2,x, algorithm="giac")
```

```
[Out] 11/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 2/3*(x + 1)/(x^2 - x + 1) + 1/
2*log(x^2 - x + 1)
```

Mupad [B] (verification not implemented)

Time = 13.20 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.13

$$\int \frac{x(1+x+x^2)}{(1-x+x^2)^2}\,dx = \frac{\ln\left(x^2-x+1\right)}{2} - \frac{2\,x}{3\,\left(x^2-x+1\right)} - \frac{2}{3\,\left(x^2-x+1\right)} + \frac{11\,\sqrt{3}\,\mathrm{atan}\left(\frac{2\,\sqrt{3}\,x}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

```
[In] int((x*(x + x^2 + 1))/(x^2 - x + 1)^2,x)
```

```
[Out] log(x^2 - x + 1)/2 - (2*x)/(3*(x^2 - x + 1)) - 2/(3*(x^2 - x + 1)) + (11*3^
(1/2)*atan((2*3^(1/2)*x)/3 - 3^(1/2)/3))/9
```

3.163 $\int \frac{1+x+x^2}{\left(1-x+x^2\right)^2} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 41

$$\int \frac{1+x+x^2}{\left(1-x+x^2\right)^2} \, dx=-\frac{2(2-x)}{3\left(1-x+x^2\right)}-\frac{10\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] -2/3*(2-x)/(x^2-x+1)-10/9*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1674, 12, 632, 210}

$$\int \frac{1+x+x^2}{\left(1-x+x^2\right)^2} \, dx=-\frac{10\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{2(2-x)}{3\left(x^2-x+1\right)}$$

```
[In] Int[(1 + x + x^2)/(1 - x + x^2)^2,x]
```

```
[Out] (-2*(2 - x))/(3*(1 - x + x^2)) - (10*ArcTan[(1 - 2*x)/Sqrt[3]])/(3*Sqrt[3])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2(2-x)}{3\left(1-x+x^2\right)}+\frac{1}{3}\int\frac{5}{1-x+x^2}\,dx\\
&= -\frac{2(2-x)}{3\left(1-x+x^2\right)}+\frac{5}{3}\int\frac{1}{1-x+x^2}\,dx\\
&= -\frac{2(2-x)}{3\left(1-x+x^2\right)}-\frac{10}{3}\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,-1+2x\right)\\
&= -\frac{2(2-x)}{3\left(1-x+x^2\right)}-\frac{10\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95

$$\int\frac{1+x+x^2}{\left(1-x+x^2\right)^2}\,dx=\frac{2(-2+x)}{3\left(1-x+x^2\right)}+\frac{10\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[In] Integrate[(1 + x + x^2)/(1 - x + x^2)^2,x]
[Out] (2*(-2 + x))/(3*(1 - x + x^2)) + (10*ArcTan[(-1 + 2*x)/Sqrt[3]])/(3*Sqrt[3]
)
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{\frac{2x}{3}-\frac{4}{3}}{x^2-x+1}+\frac{10\sqrt{3}\,\arctan\left(\frac{(-1+2x)\sqrt{3}}{3}\right)}{9}$	34
risch	$\frac{\frac{2x}{3}-\frac{4}{3}}{x^2-x+1}+\frac{10\sqrt{3}\,\arctan\left(\frac{(-1+2x)\sqrt{3}}{3}\right)}{9}$	34

```
[In] int((x^2+x+1)/(x^2-x+1)^2,x,method=_RETURNVERBOSE)
[Out] (2/3*x-4/3)/(x^2-x+1)+10/9*3^(1/2)*arctan(1/3*(-1+2*x)*3^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{1+x+x^2}{\left(1-x+x^2\right)^2}\,dx=\frac{2\left(5\sqrt{3}(x^2-x+1)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+3x-6\right)}{9\left(x^2-x+1\right)}$$

```
[In] integrate((x^2+x+1)/(x^2-x+1)^2,x, algorithm="fricas")
[Out] 2/9*(5*sqrt(3)*(x^2 - x + 1)*arctan(1/3*sqrt(3)*(2*x - 1)) + 3*x - 6)/(x^2
- x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{1+x+x^2}{\left(1-x+x^2\right)^2}\,dx=\frac{2x-4}{3x^2-3x+3}+\frac{10\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

```
[In] integrate((x**2+x+1)/(x**2-x+1)**2,x)
[Out] (2*x - 4)/(3*x**2 - 3*x + 3) + 10*sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/9
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.78

$$\int \frac{1+x+x^2}{\left(1-x+x^2\right)^2}\,dx = \frac{10}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right)+\frac{2\,(x-2)}{3\,(x^2-x+1)}$$

```
[In] integrate((x^2+x+1)/(x^2-x+1)^2,x, algorithm="maxima")
[Out] 10/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 2/3*(x - 2)/(x^2 - x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.78

$$\int \frac{1+x+x^2}{\left(1-x+x^2\right)^2}\,dx = \frac{10}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right)+\frac{2\,(x-2)}{3\,(x^2-x+1)}$$

```
[In] integrate((x^2+x+1)/(x^2-x+1)^2,x, algorithm="giac")
[Out] 10/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 2/3*(x - 2)/(x^2 - x + 1)
```

Mupad [B] (verification not implemented)

Time = 13.49 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

$$\int \frac{1+x+x^2}{\left(1-x+x^2\right)^2}\,dx = \frac{\frac{2\,x}{3}-\frac{4}{3}}{x^2-x+1}+\frac{10\,\sqrt{3}\,\mathrm{atan}\left(\frac{2\,\sqrt{3}\,x}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

```
[In] int((x + x^2 + 1)/(x^2 - x + 1)^2,x)
[Out] ((2*x)/3 - 4/3)/(x^2 - x + 1) + (10*3^(1/2)*atan((2*3^(1/2)*x)/3 - 3^(1/2)/
3))/9
```

3.164 $\int \frac{1+x+x^2}{x\left(1-x+x^2\right)^2} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 56

$$\int \frac{1+x+x^2}{x \left(1-x+x^2\right)^2} \, dx=-\frac{2(1-2 x)}{3 \left(1-x+x^2\right)}-\frac{11 \arctan \left(\frac{1-2 x}{\sqrt{3}}\right)}{3 \sqrt{3}}+\log (x)-\frac{1}{2} \log \left(1-x+x^2\right)$$

```
[Out] -2/3*(1-2*x)/(x^2-x+1)+ln(x)-1/2*ln(x^2-x+1)-11/9*arctan(1/3*(1-2*x)*3^(1/2
))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1660, 814, 648, 632, 210, 642}

$$\int \frac{1+x+x^2}{x \left(1-x+x^2\right)^2} \, dx=-\frac{11 \arctan \left(\frac{1-2 x}{\sqrt{3}}\right)}{3 \sqrt{3}}-\frac{2(1-2 x)}{3 \left(x^2-x+1\right)}-\frac{1}{2} \log \left(x^2-x+1\right)+\log (x)$$

```
[In] Int[(1 + x + x^2)/(x*(1 - x + x^2)^2),x]
```

```
[Out] (-2*(1 - 2*x))/(3*(1 - x + x^2)) - (11*ArcTan[(1 - 2*x)/Sqrt[3]])/(3*Sqrt[3
]) + Log[x] - Log[1 - x + x^2]/2
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1660

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^(p + 1)/((p
+ 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2(1-2x)}{3\left(1-x+x^2\right)}+\frac{1}{3}\int \frac{3+4x}{x\left(1-x+x^2\right)}\,dx\\ &= -\frac{2(1-2x)}{3\left(1-x+x^2\right)}+\frac{1}{3}\int \left(\frac{3}{x}+\frac{7-3x}{1-x+x^2}\right)\,dx\end{aligned}$$

$$
\begin{aligned}
&= -\frac{2(1-2x)}{3\left(1-x+x^2\right)} + \log(x) + \frac{1}{3}\int \frac{7-3x}{1-x+x^2}\,dx \\
&= -\frac{2(1-2x)}{3\left(1-x+x^2\right)} + \log(x) - \frac{1}{2}\int \frac{-1+2x}{1-x+x^2}\,dx + \frac{11}{6}\int \frac{1}{1-x+x^2}\,dx \\
&= -\frac{2(1-2x)}{3\left(1-x+x^2\right)} + \log(x) - \frac{1}{2}\log\left(1-x+x^2\right) - \frac{11}{3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x\right) \\
&= -\frac{2(1-2x)}{3\left(1-x+x^2\right)} - \frac{11\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}} + \log(x) - \frac{1}{2}\log\left(1-x+x^2\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$
\int \frac{1+x+x^2}{x\left(1-x+x^2\right)^2}\,dx = \frac{2(-1+2x)}{3\left(1-x+x^2\right)} + \frac{11\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)}{3\sqrt{3}} + \log(x) - \frac{1}{2}\log\left(1-x+x^2\right)
$$

```
[In] Integrate[(1 + x + x^2)/(x*(1 - x + x^2)^2),x]
[Out] (2*(-1 + 2*x))/(3*(1 - x + x^2)) + (11*ArcTan[(-1 + 2*x)/Sqrt[3]])/(3*Sqrt[
3]) + Log[x] - Log[1 - x + x^2]/2
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

method	result	size
default	$-\frac{-\frac{4x}{3}+\frac{2}{3}}{x^2-x+1} - \frac{\ln\left(x^2-x+1\right)}{2} + \frac{11\sqrt{3}\,\arctan\left(\frac{(-1+2x)\sqrt{3}}{3}\right)}{9} + \ln\left(x\right)$	48
risch	$\frac{\frac{4x}{3}-\frac{2}{3}}{x^2-x+1} + \frac{11\sqrt{3}\,\arctan\left(\frac{(-1+2x)\sqrt{3}}{3}\right)}{9} - \frac{\ln\left(4x^2-4x+4\right)}{2} + \ln\left(x\right)$	49

```
[In] int((x^2+x+1)/x/(x^2-x+1)^2,x,method=_RETURNVERBOSE)
[Out] -(-4/3*x+2/3)/(x^2-x+1)-1/2*ln(x^2-x+1)+11/9*3^(1/2)*arctan(1/3*(-1+2*x)*3^
(1/2))+ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.29

$$\int \frac{1+x+x^2}{x\left(1-x+x^2\right)^2}\,dx$$
$$=\frac{22\sqrt{3}(x^2-x+1)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-9\left(x^2-x+1\right)\log\left(x^2-x+1\right)+18\left(x^2-x+1\right)\log\left(x\right)+24}{18\left(x^2-x+1\right)}$$

```
[In] integrate((x^2+x+1)/x/(x^2-x+1)^2,x, algorithm="fricas")
```

```
[Out] 1/18*(22*sqrt(3)*(x^2 - x + 1)*arctan(1/3*sqrt(3)*(2*x - 1)) - 9*(x^2 - x +
 1)*log(x^2 - x + 1) + 18*(x^2 - x + 1)*log(x) + 24*x - 12)/(x^2 - x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96

$$\int \frac{1+x+x^2}{x\left(1-x+x^2\right)^2}\,dx=\frac{4x-2}{3x^2-3x+3}+\log\left(x\right)-\frac{\log\left(x^2-x+1\right)}{2}+\frac{11\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

```
[In] integrate((x**2+x+1)/x/(x**2-x+1)**2,x)
```

```
[Out] (4*x - 2)/(3*x**2 - 3*x + 3) + log(x) - log(x**2 - x + 1)/2 + 11*sqrt(3)*at
an(2*sqrt(3)*x/3 - sqrt(3)/3)/9
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.84

$$\begin{aligned}\int \frac{1+x+x^2}{x\left(1-x+x^2\right)^2}\,dx=&\ \frac{11}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)\\&+\frac{2\left(2x-1\right)}{3\left(x^2-x+1\right)}-\frac{1}{2}\log\left(x^2-x+1\right)+\log\left(x\right)\end{aligned}$$

```
[In] integrate((x^2+x+1)/x/(x^2-x+1)^2,x, algorithm="maxima")
```

```
[Out] 11/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 2/3*(2*x - 1)/(x^2 - x + 1) -
1/2*log(x^2 - x + 1) + log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int \frac{1+x+x^2}{x\left(1-x+x^2\right)^2}\,dx = \frac{11}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{2\left(2x-1\right)}{3\left(x^2-x+1\right)} -\frac{1}{2}\log\left(x^2-x+1\right)+\log\left(|x|\right)$$

```
[In] integrate((x^2+x+1)/x/(x^2-x+1)^2,x, algorithm="giac")
```

```
[Out] 11/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 2/3*(2*x - 1)/(x^2 - x + 1) -
1/2*log(x^2 - x + 1) + log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.04

$$\int \frac{1+x+x^2}{x\left(1-x+x^2\right)^2}\,dx = \ln\left(x\right)+\frac{\frac{4x}{3}-\frac{2}{3}}{x^2-x+1}-\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,11\mathrm{i}}{18}\right) +\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,11\mathrm{i}}{18}\right)$$

```
[In] int((x + x^2 + 1)/(x*(x^2 - x + 1)^2),x)
```

```
[Out] log(x) + ((4*x)/3 - 2/3)/(x^2 - x + 1) - log(x - (3^(1/2)*1i)/2 - 1/2)*((3^
(1/2)*11i)/18 + 1/2) + log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*11i)/18 - 1/
2)
```

3.165 $\int \frac{1+x+x^2}{x^2\left(1-x+x^2\right)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 61

$$\int \frac{1+x+x^2}{x^2\left(1-x+x^2\right)^2}\,dx=-\frac{1}{x}+\frac{2(1+x)}{3\left(1-x+x^2\right)}-\frac{7\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}+3\log(x)-\frac{3}{2}\log\left(1-x+x^2\right)$$

```
[Out] -1/x+2/3*(1+x)/(x^2-x+1)+3*ln(x)-3/2*ln(x^2-x+1)-7/9*arctan(1/3*(1-2*x)*3^(
1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1660, 1642, 648, 632, 210, 642}

$$\int \frac{1+x+x^2}{x^2\left(1-x+x^2\right)^2}\,dx=-\frac{7\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{2(x+1)}{3\left(x^2-x+1\right)}-\frac{3}{2}\log\left(x^2-x+1\right)-\frac{1}{x}+3\log(x)$$

```
[In] Int[(1 + x + x^2)/(x^2*(1 - x + x^2)^2),x]
```

```
[Out] -x^(-1) + (2*(1 + x))/(3*(1 - x + x^2)) - (7*ArcTan[(1 - 2*x)/Sqrt[3]])/(3*
Sqrt[3]) + 3*Log[x] - (3*Log[1 - x + x^2])/2
```

<u>Rule 210</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1660

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^(p + 1)/((p
+ 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2(1+x)}{3\left(1-x+x^2\right)}+\frac{1}{3}\int \frac{3+6x+2x^2}{x^2\left(1-x+x^2\right)}\,dx \\
&= \frac{2(1+x)}{3\left(1-x+x^2\right)}+\frac{1}{3}\int \left(\frac{3}{x^2}+\frac{9}{x}+\frac{8-9x}{1-x+x^2}\right)dx
\end{aligned}
$$

$$
\begin{aligned}
&=-\frac{1}{x}+\frac{2(1+x)}{3\left(1-x+x^2\right)}+3\log(x)+\frac{1}{3}\int\frac{8-9x}{1-x+x^2}\,dx\\
&=-\frac{1}{x}+\frac{2(1+x)}{3\left(1-x+x^2\right)}+3\log(x)+\frac{7}{6}\int\frac{1}{1-x+x^2}\,dx-\frac{3}{2}\int\frac{-1+2x}{1-x+x^2}\,dx\\
&=-\frac{1}{x}+\frac{2(1+x)}{3\left(1-x+x^2\right)}+3\log(x)-\frac{3}{2}\log\left(1-x+x^2\right)-\frac{7}{3}\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,\right.\\
&\qquad\left.-1+2x\right)\\
&=-\frac{1}{x}+\frac{2(1+x)}{3\left(1-x+x^2\right)}-\frac{7\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}+3\log(x)-\frac{3}{2}\log\left(1-x+x^2\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00

$$
\int\frac{1+x+x^2}{x^2\left(1-x+x^2\right)^2}\,dx=-\frac{1}{x}+\frac{2(1+x)}{3\left(1-x+x^2\right)}+\frac{7\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)}{3\sqrt{3}}+3\log(x)-\frac{3}{2}\log\left(1-x+x^2\right)
$$

```
[In] Integrate[(1 + x + x^2)/(x^2*(1 - x + x^2)^2),x]
```

```
[Out] -x^(-1) + (2*(1 + x))/(3*(1 - x + x^2)) + (7*ArcTan[(-1 + 2*x)/Sqrt[3]])/(3
*Sqrt[3]) + 3*Log[x] - (3*Log[1 - x + x^2])/2
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.90

method	result	size
default	$-\frac{-\frac{2x}{3}-\frac{2}{3}}{x^2-x+1}-\frac{3\ln\left(x^2-x+1\right)}{2}+\frac{7\sqrt{3}\,\arctan\left(\frac{(-1+2x)\sqrt{3}}{3}\right)}{9}-\frac{1}{x}+3\ln\left(x\right)$	55
risch	$\frac{-\frac{1}{3}x^2+\frac{5}{3}x-1}{x\left(x^2-x+1\right)}+\frac{7\sqrt{3}\,\arctan\left(\frac{(-1+2x)\sqrt{3}}{3}\right)}{9}-\frac{3\ln\left(4x^2-4x+4\right)}{2}+3\ln\left(x\right)$	59

```
[In] int((x^2+x+1)/x^2/(x^2-x+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -(-2/3*x-2/3)/(x^2-x+1)-3/2*ln(x^2-x+1)+7/9*3^(1/2)*arctan(1/3*(-1+2*x)*3^(
1/2))-1/x+3*ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.39

$$\int \frac{1+x+x^2}{x^2\left(1-x+x^2\right)^2}\,dx$$
$$= \frac{14\sqrt{3}(x^3-x^2+x)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-6x^2-27\left(x^3-x^2+x\right)\log\left(x^2-x+1\right)+54\left(x^3-x^2+x\right)}{18\left(x^3-x^2+x\right)}$$

```
[In] integrate((x^2+x+1)/x^2/(x^2-x+1)^2,x, algorithm="fricas")
```

```
[Out] 1/18*(14*sqrt(3)*(x^3 - x^2 + x)*arctan(1/3*sqrt(3)*(2*x - 1)) - 6*x^2 - 27
*(x^3 - x^2 + x)*log(x^2 - x + 1) + 54*(x^3 - x^2 + x)*log(x) + 30*x - 18)/
(x^3 - x^2 + x)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.07

$$\int \frac{1+x+x^2}{x^2\left(1-x+x^2\right)^2}\,dx = \frac{-x^2+5x-3}{3x^3-3x^2+3x}+3\log(x)-\frac{3\log\left(x^2-x+1\right)}{2}+\frac{7\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

```
[In] integrate((x**2+x+1)/x**2/(x**2-x+1)**2,x)
```

```
[Out] (-x**2 + 5*x - 3)/(3*x**3 - 3*x**2 + 3*x) + 3*log(x) - 3*log(x**2 - x + 1)/
2 + 7*sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/9
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.89

$$\int \frac{1+x+x^2}{x^2\left(1-x+x^2\right)^2}\,dx = \frac{7}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{x^2-5x+3}{3\left(x^3-x^2+x\right)}-\frac{3}{2}\log\left(x^2-x+1\right)+3\log(x)$$

```
[In] integrate((x^2+x+1)/x^2/(x^2-x+1)^2,x, algorithm="maxima")
```

```
[Out] 7/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/3*(x^2 - 5*x + 3)/(x^3 - x^2
+ x) - 3/2*log(x^2 - x + 1) + 3*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.90

$$\int \frac{1+x+x^2}{x^2\left(1-x+x^2\right)^2}\,dx = \frac{7}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right) - \frac{x^2-5\,x+3}{3\,(x^3-x^2+x)} - \frac{3}{2}\,\log\left(x^2-x+1\right) + 3\,\log\left(|x|\right)$$

```
[In] integrate((x^2+x+1)/x^2/(x^2-x+1)^2,x, algorithm="giac")
```

```
[Out] 7/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/3*(x^2 - 5*x + 3)/(x^3 - x^2
+ x) - 3/2*log(x^2 - x + 1) + 3*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 13.75 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.11

$$\int \frac{1+x+x^2}{x^2\left(1-x+x^2\right)^2}\,dx = 3\,\ln\left(x\right) - \frac{\frac{x^2}{3}-\frac{5\,x}{3}+1}{x^3-x^2+x} - \ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{3}{2}+\frac{\sqrt{3}\,7\mathrm{i}}{18}\right) + \ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-\frac{3}{2}+\frac{\sqrt{3}\,7\mathrm{i}}{18}\right)$$

```
[In] int((x + x^2 + 1)/(x^2*(x^2 - x + 1)^2),x)
```

```
[Out] 3*log(x) - (x^2/3 - (5*x)/3 + 1)/(x - x^2 + x^3) - log(x - (3^(1/2)*1i)/2 -
 1/2)*((3^(1/2)*7i)/18 + 3/2) + log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*7i)
/18 - 3/2)
```

3.166 $\int \frac{1+x+x^2}{x^3\left(1-x+x^2\right)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 68

$$\int \frac{1+x+x^2}{x^3\left(1-x+x^2\right)^2}\,dx=-\frac{1}{2x^2}-\frac{3}{x}+\frac{2(2-x)}{3\left(1-x+x^2\right)}+\frac{10\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}+4\log(x)-2\log\left(1-x+x^2\right)$$

```
[Out] -1/2/x^2-3/x+2/3*(2-x)/(x^2-x+1)+4*ln(x)-2*ln(x^2-x+1)+10/9*arctan(1/3*(1-2
*x)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1660, 1642, 648, 632, 210, 642}

$$\int \frac{1+x+x^2}{x^3\left(1-x+x^2\right)^2}\,dx=\frac{10\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{2(2-x)}{3\left(x^2-x+1\right)}-\frac{1}{2x^2}-2\log\left(x^2-x+1\right)-\frac{3}{x}+4\log(x)$$

```
[In] Int[(1 + x + x^2)/(x^3*(1 - x + x^2)^2),x]
```

```
[Out] -1/2*1/x^2 - 3/x + (2*(2 - x))/(3*(1 - x + x^2)) + (10*ArcTan[(1 - 2*x)/Sqr
t[3]])/(3*Sqrt[3]) + 4*Log[x] - 2*Log[1 - x + x^2]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1660

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^(p + 1)/((p
+ 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2(2-x)}{3\left(1-x+x^2\right)}+\frac{1}{3}\int\frac{3+6x+6x^2-2x^3}{x^3\left(1-x+x^2\right)}\,dx\\
&= \frac{2(2-x)}{3\left(1-x+x^2\right)}+\frac{1}{3}\int\left(\frac{3}{x^3}+\frac{9}{x^2}+\frac{12}{x}+\frac{1-12x}{1-x+x^2}\right)dx
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{1}{2x^2} - \frac{3}{x} + \frac{2(2-x)}{3\left(1-x+x^2\right)} + 4\log(x) + \frac{1}{3}\int \frac{1-12x}{1-x+x^2}\,dx \\
&= -\frac{1}{2x^2} - \frac{3}{x} + \frac{2(2-x)}{3\left(1-x+x^2\right)} + 4\log(x) - \frac{5}{3}\int \frac{1}{1-x+x^2}\,dx - 2\int \frac{-1+2x}{1-x+x^2}\,dx \\
&= -\frac{1}{2x^2} - \frac{3}{x} + \frac{2(2-x)}{3\left(1-x+x^2\right)} + 4\log(x) - 2\log\left(1-x+x^2\right) \\
&\quad + \frac{10}{3}\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x\right) \\
&= -\frac{1}{2x^2} - \frac{3}{x} + \frac{2(2-x)}{3\left(1-x+x^2\right)} + \frac{10\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}} + 4\log(x) - 2\log\left(1-x+x^2\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.97

$$
\begin{aligned}
\int \frac{1+x+x^2}{x^3\left(1-x+x^2\right)^2}\,dx = {} & -\frac{1}{2x^2} - \frac{3}{x} - \frac{2(-2+x)}{3\left(1-x+x^2\right)} - \frac{10\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)}{3\sqrt{3}} \\
& + 4\log(x) - 2\log\left(1-x+x^2\right)
\end{aligned}
$$

```
[In] Integrate[(1 + x + x^2)/(x^3*(1 - x + x^2)^2),x]
```

```
[Out] -1/2*1/x^2 - 3/x - (2*(-2 + x))/(3*(1 - x + x^2)) - (10*ArcTan[(-1 + 2*x)/S
qrt[3]])/(3*Sqrt[3]) + 4*Log[x] - 2*Log[1 - x + x^2]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{\frac{2x}{3}-\frac{4}{3}}{x^2-x+1}-2\ln\left(x^2-x+1\right)-\frac{10\sqrt{3}\,\arctan\left(\frac{(-1+2x)\sqrt{3}}{3}\right)}{9}-\frac{1}{2x^2}-\frac{3}{x}+4\ln\left(x\right)$	60
risch	$\frac{-\frac{11}{3}x^3+\frac{23}{6}x^2-\frac{5}{2}x-\frac{1}{2}}{x^2\left(x^2-x+1\right)}-2\ln\left(4x^2-4x+4\right)-\frac{10\sqrt{3}\,\arctan\left(\frac{(-1+2x)\sqrt{3}}{3}\right)}{9}+4\ln\left(x\right)$	64

```
[In] int((x^2+x+1)/x^3/(x^2-x+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -(2/3*x-4/3)/(x^2-x+1)-2*ln(x^2-x+1)-10/9*3^(1/2)*arctan(1/3*(-1+2*x)*3^(1/
2))-1/2/x^2-3/x+4*ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.44

$$\int \frac{1+x+x^2}{x^3\left(1-x+x^2\right)^2}\,dx = -\frac{66\,x^3+20\,\sqrt{3}(x^4-x^3+x^2)\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right)-69\,x^2+36\,(x^4-x^3+x^2)\log\left(x^2-x+1\right)-72\,(x}{18\,(x^4-x^3+x^2)}$$

```
[In] integrate((x^2+x+1)/x^3/(x^2-x+1)^2,x, algorithm="fricas")
```

```
[Out] -1/18*(66*x^3 + 20*sqrt(3)*(x^4 - x^3 + x^2)*arctan(1/3*sqrt(3)*(2*x - 1))
- 69*x^2 + 36*(x^4 - x^3 + x^2)*log(x^2 - x + 1) - 72*(x^4 - x^3 + x^2)*log
(x) + 45*x + 9)/(x^4 - x^3 + x^2)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.04

$$\int \frac{1+x+x^2}{x^3\left(1-x+x^2\right)^2}\,dx = 4\log\left(x\right)-2\log\left(x^2-x+1\right) - \frac{10\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{9}+\frac{-22x^3+23x^2-15x-3}{6x^4-6x^3+6x^2}$$

```
[In] integrate((x**2+x+1)/x**3/(x**2-x+1)**2,x)
```

```
[Out] 4*log(x) - 2*log(x**2 - x + 1) - 10*sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)
/9 + (-22*x**3 + 23*x**2 - 15*x - 3)/(6*x**4 - 6*x**3 + 6*x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.93

$$\int \frac{1+x+x^2}{x^3\left(1-x+x^2\right)^2}\,dx = -\frac{10}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right) - \frac{22\,x^3-23\,x^2+15\,x+3}{6\,(x^4-x^3+x^2)}-2\,\log\left(x^2-x+1\right)+4\,\log\left(x\right)$$

```
[In] integrate((x^2+x+1)/x^3/(x^2-x+1)^2,x, algorithm="maxima")
```

```
[Out] -10/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/6*(22*x^3 - 23*x^2 + 15*x +
 3)/(x^4 - x^3 + x^2) - 2*log(x^2 - x + 1) + 4*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.93

$$\int \frac{1+x+x^2}{x^3\left(1-x+x^2\right)^2}\,dx = -\frac{10}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right) - \frac{22\,x^3-23\,x^2+15\,x+3}{6\,(x^2-x+1)x^2} - 2\,\log\left(x^2-x+1\right) + 4\,\log\left(|x|\right)$$

```
[In] integrate((x^2+x+1)/x^3/(x^2-x+1)^2,x, algorithm="giac")
```

```
[Out] -10/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/6*(22*x^3 - 23*x^2 + 15*x +
 3)/((x^2 - x + 1)*x^2) - 2*log(x^2 - x + 1) + 4*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.10

$$\int \frac{1+x+x^2}{x^3\left(1-x+x^2\right)^2}\,dx = 4\,\ln\left(x\right) + \ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-2+\frac{\sqrt{3}\,5\mathrm{i}}{9}\right) - \ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(2+\frac{\sqrt{3}\,5\mathrm{i}}{9}\right) - \frac{\frac{11\,x^3}{3}-\frac{23\,x^2}{6}+\frac{5\,x}{2}+\frac{1}{2}}{x^4-x^3+x^2}$$

```
[In] int((x + x^2 + 1)/(x^3*(x^2 - x + 1)^2),x)
```

```
[Out] 4*log(x) + log(x - (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*5i)/9 - 2) - log(x + (3^
(1/2)*1i)/2 - 1/2)*((3^(1/2)*5i)/9 + 2) - ((5*x)/2 - (23*x^2)/6 + (11*x^3)/
3 + 1/2)/(x^2 - x^3 + x^4)
```

3.167 $\int \frac{1-x^2}{(1+x+x^2)^2} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 10

$$\int \frac{1-x^2}{(1+x+x^2)^2} \, dx = \frac{x}{1+x+x^2}$$

```
[Out] x/(x^2+x+1)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1602}

$$\int \frac{1-x^2}{(1+x+x^2)^2} \, dx = \frac{x}{x^2+x+1}$$

```
[In] Int[(1 - x^2)/(1 + x + x^2)^2,x]
```

```
[Out] x/(1 + x + x^2)
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{x}{1+x+x^2}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1-x^2}{\left(1+x+x^2\right)^2}\,dx=\frac{x}{1+x+x^2}$$

```
[In] Integrate[(1 - x^2)/(1 + x + x^2)^2,x]
[Out] x/(1 + x + x^2)
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

method	result	size
gosper	$\frac{x}{x^{2}+x+1}$	11
default	$\frac{x}{x^{2}+x+1}$	11
norman	$\frac{x}{x^{2}+x+1}$	11
risch	$\frac{x}{x^{2}+x+1}$	11
parallelrisch	$\frac{x}{x^{2}+x+1}$	11

```
[In] int((-x^2+1)/(x^2+x+1)^2,x,method=_RETURNVERBOSE)
[Out] x/(x^2+x+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1-x^2}{\left(1+x+x^2\right)^2}\,dx=\frac{x}{x^2+x+1}$$

```
[In] integrate((-x^2+1)/(x^2+x+1)^2,x, algorithm="fricas")
[Out] x/(x^2 + x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int \frac{1-x^2}{\left(1+x+x^2\right)^2}\,dx=\frac{x}{x^2+x+1}$$

```
[In] integrate((-x**2+1)/(x**2+x+1)**2,x)
[Out] x/(x**2 + x + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1-x^2}{\left(1+x+x^2\right)^2}\,dx=\frac{x}{x^2+x+1}$$

```
[In] integrate((-x^2+1)/(x^2+x+1)^2,x, algorithm="maxima")
[Out] x/(x^2 + x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{1-x^2}{\left(1+x+x^2\right)^2}\,dx=\frac{1}{x+\frac{1}{x}+1}$$

```
[In] integrate((-x^2+1)/(x^2+x+1)^2,x, algorithm="giac")
[Out] 1/(x + 1/x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1-x^2}{\left(1+x+x^2\right)^2}\,dx=\frac{x}{x^2+x+1}$$

```
[In] int(-(x^2 - 1)/(x + x^2 + 1)^2,x)
[Out] x/(x + x^2 + 1)
```

3.168 $\int \frac{1+x^2}{1+x+x^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 31

$$\int \frac{1+x^2}{1+x+x^2}\,dx = x+\frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{2}\log\left(1+x+x^2\right)$$

```
[Out] x-1/2*ln(x^2+x+1)+1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {1671, 648, 632, 210, 642}

$$\int \frac{1+x^2}{1+x+x^2}\,dx = \frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{2}\log\left(x^2+x+1\right)+x$$

```
[In] Int[(1 + x^2)/(1 + x + x^2),x]
```

```
[Out] x + ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] - Log[1 + x + x^2]/2
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(1-\frac{x}{1+x+x^2}\right) dx \\
&= x-\int \frac{x}{1+x+x^2}\,dx \\
&= x+\frac{1}{2}\int \frac{1}{1+x+x^2}\,dx-\frac{1}{2}\int \frac{1+2x}{1+x+x^2}\,dx \\
&= x-\frac{1}{2}\log \left(1+x+x^2\right)-\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2x\right) \\
&= x+\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{2}\log \left(1+x+x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{1+x^2}{1+x+x^2}\,dx = x+\frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{2}\log\left(1+x+x^2\right)$$

```
[In] Integrate[(1 + x^2)/(1 + x + x^2),x]
[Out] x + ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] - Log[1 + x + x^2]/2
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

method	result	size
default	$x-\frac{\ln(x^2+x+1)}{2}+\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{3}$	28
risch	$x-\frac{\ln(4x^2+4x+4)}{2}+\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{3}$	32

```
[In] int((x^2+1)/(x^2+x+1),x,method=_RETURNVERBOSE)
[Out] x-1/2*ln(x^2+x+1)+1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{1+x^2}{1+x+x^2}\,dx = \frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)+x-\frac{1}{2}\,\log\left(x^2+x+1\right)$$

```
[In] integrate((x^2+1)/(x^2+x+1),x, algorithm="fricas")
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + x - 1/2*log(x^2 + x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.16

$$\int \frac{1+x^2}{1+x+x^2}\,dx = x - \frac{\log\left(x^2+x+1\right)}{2} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

```
[In] integrate((x**2+1)/(x**2+x+1),x)
[Out] x - log(x**2 + x + 1)/2 + sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{1+x^2}{1+x+x^2}\,dx = \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + x - \frac{1}{2}\log\left(x^2+x+1\right)$$

```
[In] integrate((x^2+1)/(x^2+x+1),x, algorithm="maxima")
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + x - 1/2*log(x^2 + x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{1+x^2}{1+x+x^2}\,dx = \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + x - \frac{1}{2}\log\left(x^2+x+1\right)$$

```
[In] integrate((x^2+1)/(x^2+x+1),x, algorithm="giac")
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + x - 1/2*log(x^2 + x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{1+x^2}{1+x+x^2}\,dx = x - \frac{\ln\left(x^2+x+1\right)}{2} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\,x}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

```
[In] int((x^2 + 1)/(x + x^2 + 1),x)
[Out] x - log(x + x^2 + 1)/2 + (3^(1/2)*atan((2*3^(1/2)*x)/3 + 3^(1/2)/3))/3
```

3.169 $\int \frac{-1+x^2}{25-6x+x^2} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 23

$$\int \frac{-1+x^2}{25-6x+x^2} \, dx = x-2\arctan\left(\frac{1}{4}(-3+x)\right)+3\log\left(25-6x+x^2\right)$$

```
[Out] x-2*arctan(-3/4+1/4*x)+3*ln(x^2-6*x+25)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1671, 648, 632, 210, 642}

$$\int \frac{-1+x^2}{25-6x+x^2} \, dx = -2\arctan\left(\frac{x-3}{4}\right)+3\log\left(x^2-6x+25\right)+x$$

```
[In] Int[(-1 + x^2)/(25 - 6*x + x^2),x]
```

```
[Out] x - 2*ArcTan[(-3 + x)/4] + 3*Log[25 - 6*x + x^2]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(1-\frac{2(13-3x)}{25-6x+x^2}\right)\,dx\\
&= x-2\int \frac{13-3x}{25-6x+x^2}\,dx\\
&= x+3\int \frac{-6+2x}{25-6x+x^2}\,dx-8\int \frac{1}{25-6x+x^2}\,dx\\
&= x+3\log\left(25-6x+x^2\right)+16\text{Subst}\left(\int \frac{1}{-64-x^2}\,dx,x,-6+2x\right)\\
&= x-2\tan^{-1}\left(\frac{1}{4}(-3+x)\right)+3\log\left(25-6x+x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{-1+x^2}{25-6x+x^2}\,dx = x-2\arctan\left(\frac{1}{4}(-3+x)\right)+3\log\left(25-6x+x^2\right)$$

```
[In] Integrate[(-1 + x^2)/(25 - 6*x + x^2),x]
[Out] x - 2*ArcTan[(-3 + x)/4] + 3*Log[25 - 6*x + x^2]
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

method	result	size
default	$x-2\arctan\left(-\frac{3}{4}+\frac{x}{4}\right)+3\ln\left(x^{2}-6x+25\right)$	22
risch	$x-2\arctan\left(-\frac{3}{4}+\frac{x}{4}\right)+3\ln\left(x^{2}-6x+25\right)$	22
parallelrisch	$i\ln\left(x-3-4i\right)-i\ln\left(x-3+4i\right)+3\ln\left(x-3-4i\right)+3\ln\left(x-3+4i\right)+x$	37

```
[In] int((x^2-1)/(x^2-6*x+25),x,method=_RETURNVERBOSE)
[Out] x-2*arctan(-3/4+1/4*x)+3*ln(x^2-6*x+25)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{-1+x^2}{25-6x+x^2}\,dx = x-2\arctan\left(\frac{1}{4}\,x-\frac{3}{4}\right)+3\log\left(x^{2}-6\,x+25\right)$$

```
[In] integrate((x^2-1)/(x^2-6*x+25),x, algorithm="fricas")
[Out] x - 2*arctan(1/4*x - 3/4) + 3*log(x^2 - 6*x + 25)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{-1+x^2}{25-6x+x^2}\,dx = x + 3\log\left(x^2-6x+25\right) - 2\,\mathrm{atan}\left(\frac{x}{4}-\frac{3}{4}\right)$$

```
[In] integrate((x**2-1)/(x**2-6*x+25),x)
[Out] x + 3*log(x**2 - 6*x + 25) - 2*atan(x/4 - 3/4)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{-1+x^2}{25-6x+x^2}\,dx = x - 2\,\arctan\left(\frac{1}{4}\,x-\frac{3}{4}\right) + 3\,\log\left(x^2-6\,x+25\right)$$

```
[In] integrate((x^2-1)/(x^2-6*x+25),x, algorithm="maxima")
[Out] x - 2*arctan(1/4*x - 3/4) + 3*log(x^2 - 6*x + 25)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{-1+x^2}{25-6x+x^2}\,dx = x - 2\,\arctan\left(\frac{1}{4}\,x-\frac{3}{4}\right) + 3\,\log\left(x^2-6\,x+25\right)$$

```
[In] integrate((x^2-1)/(x^2-6*x+25),x, algorithm="giac")
[Out] x - 2*arctan(1/4*x - 3/4) + 3*log(x^2 - 6*x + 25)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{-1+x^2}{25-6x+x^2}\,dx = x + 3\,\ln\left(x^2-6\,x+25\right) - 2\,\mathrm{atan}\left(\frac{x}{4}-\frac{3}{4}\right)$$

```
[In] int((x^2 - 1)/(x^2 - 6*x + 25),x)
[Out] x + 3*log(x^2 - 6*x + 25) - 2*atan(x/4 - 3/4)
```

3.170 $\int \frac{-10+3x^2}{4-4x+x^2} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 21

$$\int \frac{-10+3x^2}{4-4x+x^2} \, dx = \frac{2}{2-x} + 3x + 12\log(2-x)$$

```
[Out] 2/(2-x)+3*x+12*ln(2-x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {27, 711}

$$\int \frac{-10+3x^2}{4-4x+x^2} \, dx = 3x + \frac{2}{2-x} + 12\log(2-x)$$

```
[In] Int[(-10 + 3*x^2)/(4 - 4*x + x^2),x]
```

```
[Out] 2/(2 - x) + 3*x + 12*Log[2 - x]
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{-10+3x^2}{(-2+x)^2}\,dx \\
&= \int \left(3+\frac{2}{(-2+x)^2}+\frac{12}{-2+x}\right) dx \\
&= \frac{2}{2-x}+3x+12\log(2-x)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{-10+3x^2}{4-4x+x^2}\,dx = -\frac{2}{-2+x}+3(-2+x)+12\log(-2+x)$$

```
[In] Integrate[(-10 + 3*x^2)/(4 - 4*x + x^2),x]
[Out] -2/(-2 + x) + 3*(-2 + x) + 12*Log[-2 + x]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

method	result	size
default	$3x+12\ln\left(-2+x\right)-\frac{2}{-2+x}$	18
risch	$3x+12\ln\left(-2+x\right)-\frac{2}{-2+x}$	18
norman	$\frac{3x^2-14}{-2+x}+12\ln\left(-2+x\right)$	21
parallelrisch	$\frac{12\ln(-2+x)x+3x^2-14-24\ln(-2+x)}{-2+x}$	27
meijerg	$-\frac{5x}{2\left(1-\frac{x}{2}\right)}+\frac{x\left(-\frac{3x}{2}+6\right)}{1-\frac{x}{2}}+12\ln\left(1-\frac{x}{2}\right)$	34

```
[In] int((3*x^2-10)/(x^2-4*x+4),x,method=_RETURNVERBOSE)
[Out] 3*x+12*ln(-2+x)-2/(-2+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \frac{-10+3x^2}{4-4x+x^2}\,dx = \frac{3\,x^2+12\,(x-2)\log{(x-2)}-6\,x-2}{x-2}$$

```
[In] integrate((3*x^2-10)/(x^2-4*x+4),x, algorithm="fricas")
[Out] (3*x^2 + 12*(x - 2)*log(x - 2) - 6*x - 2)/(x - 2)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

$$\int \frac{-10+3x^2}{4-4x+x^2}\,dx = 3x+12\log{(x-2)}-\frac{2}{x-2}$$

```
[In] integrate((3*x**2-10)/(x**2-4*x+4),x)
[Out] 3*x + 12*log(x - 2) - 2/(x - 2)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{-10+3x^2}{4-4x+x^2}\,dx = 3\,x-\frac{2}{x-2}+12\,\log{(x-2)}$$

```
[In] integrate((3*x^2-10)/(x^2-4*x+4),x, algorithm="maxima")
[Out] 3*x - 2/(x - 2) + 12*log(x - 2)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

$$\int \frac{-10+3x^2}{4-4x+x^2}\,dx = 3\,x-\frac{2}{x-2}+12\,\log{(|x-2|)}$$

```
[In] integrate((3*x^2-10)/(x^2-4*x+4),x, algorithm="giac")
[Out] 3*x - 2/(x - 2) + 12*log(abs(x - 2))
```

Mupad [B] (verification not implemented)

Time = 13.13 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{-10+3x^2}{4-4x+x^2}\,dx = 3\,x+12\,\ln\left(x-2\right)-\frac{2}{x-2}$$

```
[In] int((3*x^2 - 10)/(x^2 - 4*x + 4),x)
[Out] 3*x + 12*log(x - 2) - 2/(x - 2)
```

3.171 $\int \frac{8+x^2}{6-5x+x^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 18

$$\int \frac{8+x^2}{6-5x+x^2}\,dx = x-12\log(2-x)+17\log(3-x)$$

```
[Out] x-12*ln(2-x)+17*ln(3-x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1671, 646, 31}

$$\int \frac{8+x^2}{6-5x+x^2}\,dx = x-12\log(2-x)+17\log(3-x)$$

```
[In] Int[(8 + x^2)/(6 - 5*x + x^2),x]
```

```
[Out] x - 12*Log[2 - x] + 17*Log[3 - x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
```

```
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(1+\frac{2+5x}{6-5x+x^2}\right) dx \\ &= x+\int \frac{2+5x}{6-5x+x^2}\,dx \\ &= x-12\int \frac{1}{-2+x}\,dx+17\int \frac{1}{-3+x}\,dx \\ &= x-12\log(2-x)+17\log(3-x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{8+x^2}{6-5x+x^2}\,dx = x-12\log(2-x)+17\log(3-x)$$

```
[In] Integrate[(8 + x^2)/(6 - 5*x + x^2),x]
[Out] x - 12*Log[2 - x] + 17*Log[3 - x]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
default	$x-12\ln\left(-2+x\right)+17\ln\left(-3+x\right)$	15
norman	$x-12\ln\left(-2+x\right)+17\ln\left(-3+x\right)$	15
risch	$x-12\ln\left(-2+x\right)+17\ln\left(-3+x\right)$	15
parallelrisch	$x-12\ln\left(-2+x\right)+17\ln\left(-3+x\right)$	15

```
[In] int((x^2+8)/(x^2-5*x+6),x,method=_RETURNVERBOSE)
[Out] x-12*ln(-2+x)+17*ln(-3+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{8+x^2}{6-5x+x^2}\,dx = x - 12\,\log\left(x-2\right) + 17\,\log\left(x-3\right)$$

```
[In] integrate((x^2+8)/(x^2-5*x+6),x, algorithm="fricas")
[Out] x - 12*log(x - 2) + 17*log(x - 3)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{8+x^2}{6-5x+x^2}\,dx = x + 17\log\left(x-3\right) - 12\log\left(x-2\right)$$

```
[In] integrate((x**2+8)/(x**2-5*x+6),x)
[Out] x + 17*log(x - 3) - 12*log(x - 2)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{8+x^2}{6-5x+x^2}\,dx = x - 12\,\log\left(x-2\right) + 17\,\log\left(x-3\right)$$

```
[In] integrate((x^2+8)/(x^2-5*x+6),x, algorithm="maxima")
[Out] x - 12*log(x - 2) + 17*log(x - 3)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{8+x^2}{6-5x+x^2}\,dx = x - 12\,\log\left(|x-2|\right) + 17\,\log\left(|x-3|\right)$$

```
[In] integrate((x^2+8)/(x^2-5*x+6),x, algorithm="giac")
[Out] x - 12*log(abs(x - 2)) + 17*log(abs(x - 3))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{8+x^2}{6-5x+x^2}\,dx = x-12\,\ln\left(x-2\right)+17\,\ln\left(x-3\right)$$

```
[In] int((x^2 + 8)/(x^2 - 5*x + 6),x)
[Out] x - 12*log(x - 2) + 17*log(x - 3)
```

3.172 $\int \frac{-4+3x+x^2}{-8-2x+x^2} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 14

$$\int \frac{-4+3x+x^2}{-8-2x+x^2} \, dx = x+4\log(4-x)+\log(2+x)$$

```
[Out] x+4*ln(4-x)+ln(2+x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1671, 646, 31}

$$\int \frac{-4+3x+x^2}{-8-2x+x^2} \, dx = x+4\log(4-x)+\log(x+2)$$

```
[In] Int[(-4 + 3*x + x^2)/(-8 - 2*x + x^2),x]
```

```
[Out] x + 4*Log[4 - x] + Log[2 + x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
```

```
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(1+\frac{4+5x}{-8-2x+x^2}\right) dx \\
&= x+\int \frac{4+5x}{-8-2x+x^2}\,dx \\
&= x+4\int \frac{1}{-4+x}\,dx+\int \frac{1}{2+x}\,dx \\
&= x+4\log(4-x)+\log(2+x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{-4+3x+x^2}{-8-2x+x^2}\,dx = x+4\log(4-x)+\log(2+x)$$

```
[In] Integrate[(-4 + 3*x + x^2)/(-8 - 2*x + x^2),x]
[Out] x + 4*Log[4 - x] + Log[2 + x]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
default	$x+4\ln(x-4)+\ln(2+x)$	13
norman	$x+4\ln(x-4)+\ln(2+x)$	13
risch	$x+4\ln(x-4)+\ln(2+x)$	13
parallelrisch	$x+4\ln(x-4)+\ln(2+x)$	13

```
[In] int((x^2+3*x-4)/(x^2-2*x-8),x,method=_RETURNVERBOSE)
[Out] x+4*ln(x-4)+ln(2+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{-4+3x+x^2}{-8-2x+x^2}\,dx = x+\log\left(x+2\right)+4\,\log\left(x-4\right)$$

```
[In] integrate((x^2+3*x-4)/(x^2-2*x-8),x, algorithm="fricas")
[Out] x + log(x + 2) + 4*log(x - 4)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{-4+3x+x^2}{-8-2x+x^2}\,dx = x+4\log\left(x-4\right)+\log\left(x+2\right)$$

```
[In] integrate((x**2+3*x-4)/(x**2-2*x-8),x)
[Out] x + 4*log(x - 4) + log(x + 2)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{-4+3x+x^2}{-8-2x+x^2}\,dx = x+\log\left(x+2\right)+4\,\log\left(x-4\right)$$

```
[In] integrate((x^2+3*x-4)/(x^2-2*x-8),x, algorithm="maxima")
[Out] x + log(x + 2) + 4*log(x - 4)
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{-4+3x+x^2}{-8-2x+x^2}\,dx = x+\log\left(|x+2|\right)+4\,\log\left(|x-4|\right)$$

```
[In] integrate((x^2+3*x-4)/(x^2-2*x-8),x, algorithm="giac")
[Out] x + log(abs(x + 2)) + 4*log(abs(x - 4))
```

Mupad [B] (verification not implemented)

Time = 13.16 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{-4+3x+x^2}{-8-2x+x^2}\,dx = x+\ln{(x+2)}+4\,\ln{(x-4)}$$

```
[In] int(-(3*x + x^2 - 4)/(2*x - x^2 + 8),x)
[Out] x + log(x + 2) + 4*log(x - 4)
```

3.173 $\int \frac{7+5x+4x^2}{5+4x+4x^2} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 27

$$\int \frac{7+5x+4x^2}{5+4x+4x^2} \, dx = x+\frac{3}{8}\arctan\left(\frac{1}{2}+x\right)+\frac{1}{8}\log\left(5+4x+4x^2\right)$$

```
[Out] x+3/8*arctan(1/2+x)+1/8*ln(4*x^2+4*x+5)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1671, 648, 632, 210, 642}

$$\int \frac{7+5x+4x^2}{5+4x+4x^2} \, dx = \frac{3}{8}\arctan\left(x+\frac{1}{2}\right)+\frac{1}{8}\log\left(4x^2+4x+5\right)+x$$

```
[In] Int[(7 + 5*x + 4*x^2)/(5 + 4*x + 4*x^2),x]
```

```
[Out] x + (3*ArcTan[1/2 + x])/8 + Log[5 + 4*x + 4*x^2]/8
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(1+\frac{2+x}{5+4x+4x^2}\right) dx \\
&= x+\int \frac{2+x}{5+4x+4x^2}\, dx \\
&= x+\frac{1}{8}\int \frac{4+8x}{5+4x+4x^2}\, dx+\frac{3}{2}\int \frac{1}{5+4x+4x^2}\, dx \\
&= x+\frac{1}{8}\log\left(5+4x+4x^2\right)-3\text{Subst}\left(\int \frac{1}{-64-x^2}\, dx,x,4+8x\right) \\
&= x+\frac{3}{8}\tan^{-1}\left(\frac{1}{2}+x\right)+\frac{1}{8}\log\left(5+4x+4x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15

$$\int \frac{7+5x+4x^2}{5+4x+4x^2}\,dx = x+\frac{3}{8}\arctan\left(\frac{1}{2}(1+2x)\right)+\frac{1}{8}\log\left(5+4x+4x^2\right)$$

```
[In] Integrate[(7 + 5*x + 4*x^2)/(5 + 4*x + 4*x^2),x]
[Out] x + (3*ArcTan[(1 + 2*x)/2])/8 + Log[5 + 4*x + 4*x^2]/8
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

method	result	size
default	$x+\frac{3\arctan\left(x+\frac{1}{2}\right)}{8}+\frac{\ln\left(4x^2+4x+5\right)}{8}$	22
risch	$x+\frac{3\arctan\left(x+\frac{1}{2}\right)}{8}+\frac{\ln\left(4x^2+4x+5\right)}{8}$	22
parallelrisch	$x+\frac{\ln\left(x+\frac{1}{2}-i\right)}{8}-\frac{3i\ln\left(x+\frac{1}{2}-i\right)}{16}+\frac{\ln\left(x+\frac{1}{2}+i\right)}{8}+\frac{3i\ln\left(x+\frac{1}{2}+i\right)}{16}$	37

```
[In] int((4*x^2+5*x+7)/(4*x^2+4*x+5),x,method=_RETURNVERBOSE)
[Out] x+3/8*arctan(x+1/2)+1/8*ln(4*x^2+4*x+5)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{7+5x+4x^2}{5+4x+4x^2}\,dx = x+\frac{3}{8}\,\arctan\left(x+\frac{1}{2}\right)+\frac{1}{8}\,\log\left(4\,x^2+4\,x+5\right)$$

```
[In] integrate((4*x^2+5*x+7)/(4*x^2+4*x+5),x, algorithm="fricas")
[Out] x + 3/8*arctan(x + 1/2) + 1/8*log(4*x^2 + 4*x + 5)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{7+5x+4x^2}{5+4x+4x^2}\,dx = x+\frac{\log\left(x^2+x+\frac{5}{4}\right)}{8}+\frac{3\,\mathrm{atan}\left(x+\frac{1}{2}\right)}{8}$$

```
[In] integrate((4*x**2+5*x+7)/(4*x**2+4*x+5),x)
```

```
[Out] x + log(x**2 + x + 5/4)/8 + 3*atan(x + 1/2)/8
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{7+5x+4x^2}{5+4x+4x^2}\,dx = x+\frac{3}{8}\,\arctan\left(x+\frac{1}{2}\right)+\frac{1}{8}\,\log\left(4\,x^2+4\,x+5\right)$$

```
[In] integrate((4*x^2+5*x+7)/(4*x^2+4*x+5),x, algorithm="maxima")
```

```
[Out] x + 3/8*arctan(x + 1/2) + 1/8*log(4*x^2 + 4*x + 5)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{7+5x+4x^2}{5+4x+4x^2}\,dx = x+\frac{3}{8}\,\arctan\left(x+\frac{1}{2}\right)+\frac{1}{8}\,\log\left(4\,x^2+4\,x+5\right)$$

```
[In] integrate((4*x^2+5*x+7)/(4*x^2+4*x+5),x, algorithm="giac")
```

```
[Out] x + 3/8*arctan(x + 1/2) + 1/8*log(4*x^2 + 4*x + 5)
```

Mupad [B] (verification not implemented)

Time = 13.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.63

$$\int \frac{7+5x+4x^2}{5+4x+4x^2}\,dx = x+\frac{\ln\left(x^2+x+\frac{5}{4}\right)}{8}+\frac{3\,\mathrm{atan}\left(x+\frac{1}{2}\right)}{8}$$

```
[In] int((5*x + 4*x^2 + 7)/(4*x + 4*x^2 + 5),x)
```

```
[Out] x + log(x + x^2 + 5/4)/8 + (3*atan(x + 1/2))/8
```

3.174 $\int \frac{2-x+x^2}{-5+2x+x^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 48

$$\int \frac{2-x+x^2}{-5+2x+x^2}\,dx = x-\frac{1}{6}\left(9-5\sqrt{6}\right)\log\left(1-\sqrt{6}+x\right)-\frac{1}{6}\left(9+5\sqrt{6}\right)\log\left(1+\sqrt{6}+x\right)$$

```
[Out] x-1/6*ln(1+x-6^(1/2))*(9-5*6^(1/2))-1/6*ln(1+x+6^(1/2))*(9+5*6^(1/2))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1671, 646, 31}

$$\int \frac{2-x+x^2}{-5+2x+x^2}\,dx = x-\frac{1}{6}\left(9-5\sqrt{6}\right)\log\left(x-\sqrt{6}+1\right)-\frac{1}{6}\left(9+5\sqrt{6}\right)\log\left(x+\sqrt{6}+1\right)$$

```
[In] Int[(2 - x + x^2)/(-5 + 2*x + x^2),x]
```

```
[Out] x - ((9 - 5*Sqrt[6])*Log[1 - Sqrt[6] + x])/6 - ((9 + 5*Sqrt[6])*Log[1 + Sqr
t[6] + x])/6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
```

```
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(1+\frac{7-3x}{-5+2x+x^2}\right) dx\\ &= x+\int \frac{7-3x}{-5+2x+x^2}\,dx\\ &= x+\frac{1}{6}\left(-9+5\sqrt{6}\right)\int \frac{1}{1-\sqrt{6}+x}\,dx-\frac{1}{6}\left(9+5\sqrt{6}\right)\int \frac{1}{1+\sqrt{6}+x}\,dx\\ &= x-\frac{1}{6}\left(9-5\sqrt{6}\right)\log\left(1-\sqrt{6}+x\right)-\frac{1}{6}\left(9+5\sqrt{6}\right)\log\left(1+\sqrt{6}+x\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{2-x+x^2}{-5+2x+x^2}\,dx = x+\frac{1}{6}\left(-9+5\sqrt{6}\right)\log\left(-1+\sqrt{6}-x\right)+\frac{1}{6}\left(-9-5\sqrt{6}\right)\log\left(1+\sqrt{6}+x\right)$$

```
[In] Integrate[(2 - x + x^2)/(-5 + 2*x + x^2),x]
[Out] x + ((-9 + 5*Sqrt[6])*Log[-1 + Sqrt[6] - x])/6 + ((-9 - 5*Sqrt[6])*Log[1 +
Sqrt[6] + x])/6
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.62

method	result	size
default	$x-\frac{3\ln\left(x^2+2x-5\right)}{2}-\frac{5\sqrt{6}\,\operatorname{arctanh}\left(\frac{(2x+2)\sqrt{6}}{12}\right)}{3}$	30
risch	$x-\frac{3\ln\left(1+x-\sqrt{6}\right)}{2}+\frac{5\ln\left(1+x-\sqrt{6}\right)\sqrt{6}}{6}-\frac{3\ln\left(1+x+\sqrt{6}\right)}{2}-\frac{5\ln\left(1+x+\sqrt{6}\right)\sqrt{6}}{6}$	49

```
[In] int((x^2-x+2)/(x^2+2*x-5),x,method=_RETURNVERBOSE)
```

```
[Out] x-3/2*ln(x^2+2*x-5)-5/3*6^(1/2)*arctanh(1/12*(2*x+2)*6^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.15

$$\int \frac{2-x+x^2}{-5+2x+x^2}\,dx = \frac{5}{6}\,\sqrt{3}\sqrt{2}\log\left(-\frac{2\,\sqrt{3}\sqrt{2}(x+1)-x^2-2\,x-7}{x^2+2\,x-5}\right) + x - \frac{3}{2}\,\log\left(x^2+2\,x-5\right)$$

```
[In] integrate((x^2-x+2)/(x^2+2*x-5),x, algorithm="fricas")
```

```
[Out] 5/6*sqrt(3)*sqrt(2)*log(-(2*sqrt(3)*sqrt(2)*(x + 1) - x^2 - 2*x - 7)/(x^2 +
 2*x - 5)) + x - 3/2*log(x^2 + 2*x - 5)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96

$$\int \frac{2-x+x^2}{-5+2x+x^2}\,dx = x + \left(-\frac{5\sqrt{6}}{6}-\frac{3}{2}\right)\log\left(x+1+\sqrt{6}\right) + \left(-\frac{3}{2}+\frac{5\sqrt{6}}{6}\right)\log\left(x-\sqrt{6}+1\right)$$

```
[In] integrate((x**2-x+2)/(x**2+2*x-5),x)
```

```
[Out] x + (-5*sqrt(6)/6 - 3/2)*log(x + 1 + sqrt(6)) + (-3/2 + 5*sqrt(6)/6)*log(x
- sqrt(6) + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.75

$$\int \frac{2-x+x^2}{-5+2x+x^2}\,dx = \frac{5}{6}\,\sqrt{6}\log\left(\frac{x-\sqrt{6}+1}{x+\sqrt{6}+1}\right) + x - \frac{3}{2}\,\log\left(x^2+2\,x-5\right)$$

```
[In] integrate((x^2-x+2)/(x^2+2*x-5),x, algorithm="maxima")
```

```
[Out] 5/6*sqrt(6)*log((x - sqrt(6) + 1)/(x + sqrt(6) + 1)) + x - 3/2*log(x^2 + 2*
x - 5)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.94

$$\int \frac{2-x+x^2}{-5+2x+x^2}\,dx = \frac{5}{6}\sqrt{6}\log\left(\frac{\left|2\,x-2\,\sqrt{6}+2\right|}{\left|2\,x+2\,\sqrt{6}+2\right|}\right)+x-\frac{3}{2}\,\log\left(\left|x^2+2\,x-5\right|\right)$$

```
[In] integrate((x^2-x+2)/(x^2+2*x-5),x, algorithm="giac")
```

```
[Out] 5/6*sqrt(6)*log(abs(2*x - 2*sqrt(6) + 2)/abs(2*x + 2*sqrt(6) + 2)) + x - 3/
2*log(abs(x^2 + 2*x - 5))
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.73

$$\int \frac{2-x+x^2}{-5+2x+x^2}\,dx = x-\ln\left(x+\sqrt{6}+1\right)\,\left(\frac{5\,\sqrt{6}}{6}+\frac{3}{2}\right)+\ln\left(x-\sqrt{6}+1\right)\,\left(\frac{5\,\sqrt{6}}{6}-\frac{3}{2}\right)$$

```
[In] int((x^2 - x + 2)/(2*x + x^2 - 5),x)
```

```
[Out] x - log(x + 6^(1/2) + 1)*((5*6^(1/2))/6 + 3/2) + log(x - 6^(1/2) + 1)*((5*6
^(1/2))/6 - 3/2)
```

3.175 $\int \frac{1+4x+3x^2}{\left(4+7x+2x^2\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 21

$$\int \frac{1+4x+3x^2}{\left(4+7x+2x^2\right)^2}\,dx=-\frac{2+3x}{2\left(4+7x+2x^2\right)}$$

```
[Out] 1/2*(-2-3*x)/(2*x^2+7*x+4)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1674, 8}

$$\int \frac{1+4x+3x^2}{\left(4+7x+2x^2\right)^2}\,dx=-\frac{3x+2}{2\left(2x^2+7x+4\right)}$$

```
[In] Int[(1 + 4*x + 3*x^2)/(4 + 7*x + 2*x^2)^2,x]
```

```
[Out] -1/2*(2 + 3*x)/(4 + 7*x + 2*x^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
```

```
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2+3x}{2\left(4+7x+2x^2\right)}-\frac{\int 0\,dx}{17}\\ &= -\frac{2+3x}{2\left(4+7x+2x^2\right)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1+4x+3x^2}{\left(4+7x+2x^2\right)^2}\,dx=\frac{-2-3x}{2\left(4+7x+2x^2\right)}$$

```
[In] Integrate[(1 + 4*x + 3*x^2)/(4 + 7*x + 2*x^2)^2,x]
[Out] (-2 - 3*x)/(2*(4 + 7*x + 2*x^2))
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

method	result	size
default	$\frac{-\frac{3x}{4}-\frac{1}{2}}{x^2+\frac{7}{2}x+2}$	17
risch	$\frac{-\frac{3x}{4}-\frac{1}{2}}{x^2+\frac{7}{2}x+2}$	17
norman	$\frac{-\frac{3x}{2}-1}{2x^2+7x+4}$	19
gosper	$-\frac{2+3x}{2(2x^2+7x+4)}$	20
parallelrisch	$\frac{-2-3x}{4x^2+14x+8}$	20

```
[In] int((3*x^2+4*x+1)/(2*x^2+7*x+4)^2,x,method=_RETURNVERBOSE)
[Out] (-3/4*x-1/2)/(x^2+7/2*x+2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{1+4x+3x^2}{(4+7x+2x^2)^2}\,dx = -\frac{3\,x+2}{2\,(2\,x^2+7\,x+4)}$$

```
[In] integrate((3*x^2+4*x+1)/(2*x^2+7*x+4)^2,x, algorithm="fricas")
[Out] -1/2*(3*x + 2)/(2*x^2 + 7*x + 4)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{1+4x+3x^2}{(4+7x+2x^2)^2}\,dx = \frac{-3x-2}{4x^2+14x+8}$$

```
[In] integrate((3*x**2+4*x+1)/(2*x**2+7*x+4)**2,x)
[Out] (-3*x - 2)/(4*x**2 + 14*x + 8)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{1+4x+3x^2}{(4+7x+2x^2)^2}\,dx = -\frac{3\,x+2}{2\,(2\,x^2+7\,x+4)}$$

```
[In] integrate((3*x^2+4*x+1)/(2*x^2+7*x+4)^2,x, algorithm="maxima")
[Out] -1/2*(3*x + 2)/(2*x^2 + 7*x + 4)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{1+4x+3x^2}{(4+7x+2x^2)^2}\,dx = -\frac{3\,x+2}{2\,(2\,x^2+7\,x+4)}$$

```
[In] integrate((3*x^2+4*x+1)/(2*x^2+7*x+4)^2,x, algorithm="giac")
[Out] -1/2*(3*x + 2)/(2*x^2 + 7*x + 4)
```

Mupad [B] (verification not implemented)

Time = 13.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1+4x+3x^2}{\left(4+7x+2x^2\right)^2}\,dx = -\frac{\frac{3\,x}{4}+\frac{1}{2}}{x^2+\frac{7\,x}{2}+2}$$

```
[In] int((4*x + 3*x^2 + 1)/(7*x + 2*x^2 + 4)^2,x)
[Out] -((3*x)/4 + 1/2)/((7*x)/2 + x^2 + 2)
```

3.176 $\int \frac{1+x+x^2}{\left(3+2x+x^2\right)^2} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 39

$$\int \frac{1+x+x^2}{\left(3+2x+x^2\right)^2} \, dx=\frac{1-x}{4\left(3+2x+x^2\right)}+\frac{3\arctan\left(\frac{1+x}{\sqrt{2}}\right)}{4\sqrt{2}}$$

```
[Out] 1/4*(1-x)/(x^2+2*x+3)+3/8*arctan(1/2*(1+x)*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1674, 12, 632, 210}

$$\int \frac{1+x+x^2}{\left(3+2x+x^2\right)^2} \, dx=\frac{3\arctan\left(\frac{x+1}{\sqrt{2}}\right)}{4\sqrt{2}}+\frac{1-x}{4\left(x^2+2x+3\right)}$$

```
[In] Int[(1 + x + x^2)/(3 + 2*x + x^2)^2,x]
```

```
[Out] (1 - x)/(4*(3 + 2*x + x^2)) + (3*ArcTan[(1 + x)/Sqrt[2]])/(4*Sqrt[2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1-x}{4\left(3+2x+x^2\right)}+\frac{1}{8}\int \frac{6}{3+2x+x^2}\,dx\\
&= \frac{1-x}{4\left(3+2x+x^2\right)}+\frac{3}{4}\int \frac{1}{3+2x+x^2}\,dx\\
&= \frac{1-x}{4\left(3+2x+x^2\right)}-\frac{3}{2}\text{Subst}\left(\int \frac{1}{-8-x^2}\,dx,x,2+2x\right)\\
&= \frac{1-x}{4\left(3+2x+x^2\right)}+\frac{3\tan^{-1}\left(\frac{1+x}{\sqrt{2}}\right)}{4\sqrt{2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{1+x+x^2}{\left(3+2x+x^2\right)^2}\,dx=\frac{1-x}{4\left(3+2x+x^2\right)}+\frac{3\arctan\left(\frac{1+x}{\sqrt{2}}\right)}{4\sqrt{2}}$$

```
[In] Integrate[(1 + x + x^2)/(3 + 2*x + x^2)^2,x]
[Out] (1 - x)/(4*(3 + 2*x + x^2)) + (3*ArcTan[(1 + x)/Sqrt[2]])/(4*Sqrt[2])
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

method	result	size
risch	$\frac{-\frac{x}{4}+\frac{1}{4}}{x^2+2x+3}+\frac{3\arctan\left(\frac{(1+x)\sqrt{2}}{2}\right)\sqrt{2}}{8}$	32
default	$\frac{-\frac{x}{4}+\frac{1}{4}}{x^2+2x+3}+\frac{3\sqrt{2}\,\arctan\left(\frac{(2x+2)\sqrt{2}}{4}\right)}{8}$	34

```
[In] int((x^2+x+1)/(x^2+2*x+3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-1/4*x+1/4)/(x^2+2*x+3)+3/8*arctan(1/2*(1+x)*2^(1/2))*2^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{1+x+x^2}{(3+2x+x^2)^2}\,dx = \frac{3\,\sqrt{2}(x^2+2\,x+3)\arctan\left(\frac{1}{2}\,\sqrt{2}(x+1)\right)-2\,x+2}{8\,(x^2+2\,x+3)}$$

```
[In] integrate((x^2+x+1)/(x^2+2*x+3)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(3*sqrt(2)*(x^2 + 2*x + 3)*arctan(1/2*sqrt(2)*(x + 1)) - 2*x + 2)/(x^2
+ 2*x + 3)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \frac{1+x+x^2}{(3+2x+x^2)^2}\,dx = \frac{1-x}{4x^2+8x+12}+\frac{3\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}+\frac{\sqrt{2}}{2}\right)}{8}$$

```
[In] integrate((x**2+x+1)/(x**2+2*x+3)**2,x)
```

```
[Out] (1 - x)/(4*x**2 + 8*x + 12) + 3*sqrt(2)*atan(sqrt(2)*x/2 + sqrt(2)/2)/8
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.77

$$\int \frac{1+x+x^2}{\left(3+2x+x^2\right)^2}\,dx = \frac{3}{8}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}(x+1)\right) - \frac{x-1}{4\,(x^2+2\,x+3)}$$

```
[In] integrate((x^2+x+1)/(x^2+2*x+3)^2,x, algorithm="maxima")
[Out] 3/8*sqrt(2)*arctan(1/2*sqrt(2)*(x + 1)) - 1/4*(x - 1)/(x^2 + 2*x + 3)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.77

$$\int \frac{1+x+x^2}{\left(3+2x+x^2\right)^2}\,dx = \frac{3}{8}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}(x+1)\right) - \frac{x-1}{4\,(x^2+2\,x+3)}$$

```
[In] integrate((x^2+x+1)/(x^2+2*x+3)^2,x, algorithm="giac")
[Out] 3/8*sqrt(2)*arctan(1/2*sqrt(2)*(x + 1)) - 1/4*(x - 1)/(x^2 + 2*x + 3)
```

Mupad [B] (verification not implemented)

Time = 12.89 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int \frac{1+x+x^2}{\left(3+2x+x^2\right)^2}\,dx = \frac{3\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}+\frac{\sqrt{2}}{2}\right)}{8} - \frac{\frac{x}{4}-\frac{1}{4}}{x^2+2\,x+3}$$

```
[In] int((x + x^2 + 1)/(2*x + x^2 + 3)^2,x)
[Out] (3*2^(1/2)*atan((2^(1/2)*x)/2 + 2^(1/2)/2))/8 - (x/4 - 1/4)/(2*x + x^2 + 3)
```

3.177 $\int \frac{-1+2x+5x^2}{(1+x+x^2)^4}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 11

$$\int \frac{-1+2x+5x^2}{(1+x+x^2)^4}\,dx = -\frac{x}{(1+x+x^2)^3}$$

```
[Out] -x/(x^2+x+1)^3
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1602}

$$\int \frac{-1+2x+5x^2}{(1+x+x^2)^4}\,dx = -\frac{x}{(x^2+x+1)^3}$$

```
[In] Int[(-1 + 2*x + 5*x^2)/(1 + x + x^2)^4,x]
```

```
[Out] -(x/(1 + x + x^2)^3)
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{x}{(1+x+x^2)^3}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{-1+2x+5x^2}{(1+x+x^2)^4}\,dx=-\frac{x}{(1+x+x^2)^3}$$

```
[In] Integrate[(-1 + 2*x + 5*x^2)/(1 + x + x^2)^4,x]
[Out] -(x/(1 + x + x^2)^3)
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
gosper	$-\frac{x}{(x^2+x+1)^3}$	12
default	$-\frac{x}{(x^2+x+1)^3}$	12
norman	$-\frac{x}{(x^2+x+1)^3}$	12
risch	$-\frac{x}{(x^2+x+1)^3}$	12
parallelrisch	$-\frac{x}{(x^2+x+1)^3}$	12

```
[In] int((5*x^2+2*x-1)/(x^2+x+1)^4,x,method=_RETURNVERBOSE)
[Out] -x/(x^2+x+1)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. 2(11) = 22.

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 3.00

$$\int \frac{-1+2x+5x^2}{(1+x+x^2)^4}\,dx=-\frac{x}{x^6+3\,x^5+6\,x^4+7\,x^3+6\,x^2+3\,x+1}$$

```
[In] integrate((5*x^2+2*x-1)/(x^2+x+1)^4,x, algorithm="fricas")
[Out] -x/(x^6 + 3*x^5 + 6*x^4 + 7*x^3 + 6*x^2 + 3*x + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(10) = 20$.

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.82

$$\int \frac{-1+2x+5x^2}{\left(1+x+x^2\right)^4}\,dx = -\frac{x}{x^6+3x^5+6x^4+7x^3+6x^2+3x+1}$$

```
[In] integrate((5*x**2+2*x-1)/(x**2+x+1)**4,x)
[Out] -x/(x**6 + 3*x**5 + 6*x**4 + 7*x**3 + 6*x**2 + 3*x + 1)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(11) = 22$.

Time = 0.18 (sec) , antiderivative size = 33, normalized size of antiderivative = 3.00

$$\int \frac{-1+2x+5x^2}{\left(1+x+x^2\right)^4}\,dx = -\frac{x}{x^6+3\,x^5+6\,x^4+7\,x^3+6\,x^2+3\,x+1}$$

```
[In] integrate((5*x^2+2*x-1)/(x^2+x+1)^4,x, algorithm="maxima")
[Out] -x/(x^6 + 3*x^5 + 6*x^4 + 7*x^3 + 6*x^2 + 3*x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{-1+2x+5x^2}{\left(1+x+x^2\right)^4}\,dx = -\frac{x}{\left(x^2+x+1\right)^3}$$

```
[In] integrate((5*x^2+2*x-1)/(x^2+x+1)^4,x, algorithm="giac")
[Out] -x/(x^2 + x + 1)^3
```

Mupad [B] (verification not implemented)

Time = 12.85 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{-1+2x+5x^2}{(1+x+x^2)^4}\,dx=-\frac{x}{(x^2+x+1)^3}$$

```
[In] int((2*x + 5*x^2 - 1)/(x + x^2 + 1)^4,x)
```

```
[Out] -x/(x + x^2 + 1)^3
```

3.178 $\int \left(a+bx+cx^2\right)^{5/2}\left(A+Cx^2\right)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 267

$$\begin{aligned}\int (a+bx &+cx^2)^{5/2}\left(A+Cx^2\right)\,dx=\frac{5(b^2-4ac)^2\left(32Ac^2+9b^2C-4acC\right)(b+2cx)\sqrt{a+bx+cx^2}}{16384c^5}\\ &-\frac{5(b^2-4ac)\left(32Ac^2+9b^2C-4acC\right)(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{6144c^4}\\ &+\frac{\left(32Ac^2+9b^2C-4acC\right)(b+2cx)\left(a+bx+cx^2\right)^{5/2}}{384c^3}-\frac{9bC\left(a+bx+cx^2\right)^{7/2}}{112c^2}\\ &+\frac{Cx\left(a+bx+cx^2\right)^{7/2}}{8c}-\frac{5(b^2-4ac)^3\left(32Ac^2+9b^2C-4acC\right)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{32768c^{11/2}}\end{aligned}$$

```
[Out] -5/6144*(-4*a*c+b^2)*(32*A*c^2-4*C*a*c+9*C*b^2)*(2*c*x+b)*(c*x^2+b*x+a)^(3/
2)/c^4+1/384*(32*A*c^2-4*C*a*c+9*C*b^2)*(2*c*x+b)*(c*x^2+b*x+a)^(5/2)/c^3-9
/112*b*C*(c*x^2+b*x+a)^(7/2)/c^2+1/8*C*x*(c*x^2+b*x+a)^(7/2)/c-5/32768*(-4*
a*c+b^2)^3*(32*A*c^2-4*C*a*c+9*C*b^2)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+
b*x+a)^(1/2))/c^(11/2)+5/16384*(-4*a*c+b^2)^2*(32*A*c^2-4*C*a*c+9*C*b^2)*(2
*c*x+b)*(c*x^2+b*x+a)^(1/2)/c^5
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1675, 654, 626, 635, 212}

$$
\begin{aligned}
\int \left(a+bx+cx^2\right)^{5/2}\left(A+Cx^2\right)\,dx = \\
&-\frac{5\left(b^2-4ac\right)^3\left(-4acC+32Ac^2+9b^2C\right)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{32768c^{11/2}} \\
&+\frac{5\left(b^2-4ac\right)^2(b+2cx)\sqrt{a+bx+cx^2}\left(-4acC+32Ac^2+9b^2C\right)}{16384c^5} \\
&-\frac{5\left(b^2-4ac\right)(b+2cx)\left(a+bx+cx^2\right)^{3/2}\left(-4acC+32Ac^2+9b^2C\right)}{6144c^4} \\
&+\frac{(b+2cx)\left(a+bx+cx^2\right)^{5/2}\left(-4acC+32Ac^2+9b^2C\right)}{384c^3} \\
&-\frac{9bC\left(a+bx+cx^2\right)^{7/2}}{112c^2}+\frac{Cx\left(a+bx+cx^2\right)^{7/2}}{8c}
\end{aligned}
$$

```
[In] Int[(a + b*x + c*x^2)^(5/2)*(A + C*x^2),x]
```

```
[Out] (5*(b^2 - 4*a*c)^2*(32*A*c^2 + 9*b^2*C - 4*a*c*C)*(b + 2*c*x)*Sqrt[a + b*x
+ c*x^2])/(16384*c^5) - (5*(b^2 - 4*a*c)*(32*A*c^2 + 9*b^2*C - 4*a*c*C)*(b
+ 2*c*x)*(a + b*x + c*x^2)^(3/2))/(6144*c^4) + ((32*A*c^2 + 9*b^2*C - 4*a*c
*C)*(b + 2*c*x)*(a + b*x + c*x^2)^(5/2))/(384*c^3) - (9*b*C*(a + b*x + c*x^
2)^(7/2))/(112*c^2) + (C*x*(a + b*x + c*x^2)^(7/2))/(8*c) - (5*(b^2 - 4*a*c
)^3*(32*A*c^2 + 9*b^2*C - 4*a*c*C)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a +
b*x + c*x^2])])/(32768*c^(11/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
```

```
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{Cx(a+bx+cx^2)^{7/2}}{8c} + \frac{\int \left(8Ac - aC - \frac{9bCx}{2}\right)(a+bx+cx^2)^{5/2}\,dx}{8c} \\
&= -\frac{9bC(a+bx+cx^2)^{7/2}}{112c^2} + \frac{Cx(a+bx+cx^2)^{7/2}}{8c} \\
&\quad + \frac{\left(\frac{9b^2C}{2} + 2c(8Ac - aC)\right)\int (a+bx+cx^2)^{5/2}\,dx}{16c^2} \\
&= \frac{(32Ac^2 + 9b^2C - 4acC)(b+2cx)(a+bx+cx^2)^{5/2}}{384c^3} - \frac{9bC(a+bx+cx^2)^{7/2}}{112c^2} \\
&\quad + \frac{Cx(a+bx+cx^2)^{7/2}}{8c} - \frac{(5(b^2-4ac)(32Ac^2 + 9b^2C - 4acC))\int (a+bx+cx^2)^{3/2}\,dx}{768c^3} \\
&= -\frac{5(b^2-4ac)(32Ac^2 + 9b^2C - 4acC)(b+2cx)(a+bx+cx^2)^{3/2}}{6144c^4} \\
&\quad + \frac{(32Ac^2 + 9b^2C - 4acC)(b+2cx)(a+bx+cx^2)^{5/2}}{384c^3} \\
&\quad - \frac{9bC(a+bx+cx^2)^{7/2}}{112c^2} + \frac{Cx(a+bx+cx^2)^{7/2}}{8c} \\
&\quad + \frac{\left(5(b^2-4ac)^2(32Ac^2 + 9b^2C - 4acC)\right)\int \sqrt{a+bx+cx^2}\,dx}{4096c^4}
\end{aligned}
$$

$$= \frac{5(b^2-4ac)^2(32Ac^2+9b^2C-4acC)(b+2cx)\sqrt{a+bx+cx^2}}{16384c^5} - \frac{5(b^2-4ac)(32Ac^2+9b^2C-4acC)(b+2cx)(a+bx+cx^2)^{3/2}}{6144c^4} + \frac{(32Ac^2+9b^2C-4acC)(b+2cx)(a+bx+cx^2)^{5/2}}{384c^3} - \frac{9bC(a+bx+cx^2)^{7/2}}{112c^2} + \frac{Cx(a+bx+cx^2)^{7/2}}{8c} - \frac{\left(5(b^2-4ac)^3(32Ac^2+9b^2C-4acC)\right)\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{32768c^5}$$

$$= \frac{5(b^2-4ac)^2(32Ac^2+9b^2C-4acC)(b+2cx)\sqrt{a+bx+cx^2}}{16384c^5} - \frac{5(b^2-4ac)(32Ac^2+9b^2C-4acC)(b+2cx)(a+bx+cx^2)^{3/2}}{6144c^4} + \frac{(32Ac^2+9b^2C-4acC)(b+2cx)(a+bx+cx^2)^{5/2}}{384c^3} - \frac{9bC(a+bx+cx^2)^{7/2}}{112c^2} + \frac{Cx(a+bx+cx^2)^{7/2}}{8c} - \frac{\left(5(b^2-4ac)^3(32Ac^2+9b^2C-4acC)\right)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{16384c^5}$$

$$= \frac{5(b^2-4ac)^2(32Ac^2+9b^2C-4acC)(b+2cx)\sqrt{a+bx+cx^2}}{16384c^5} - \frac{5(b^2-4ac)(32Ac^2+9b^2C-4acC)(b+2cx)(a+bx+cx^2)^{3/2}}{6144c^4} + \frac{(32Ac^2+9b^2C-4acC)(b+2cx)(a+bx+cx^2)^{5/2}}{384c^3} - \frac{9bC(a+bx+cx^2)^{7/2}}{112c^2} + \frac{Cx(a+bx+cx^2)^{7/2}}{8c} - \frac{5(b^2-4ac)^3(32Ac^2+9b^2C-4acC)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{32768c^{11/2}}$$

Mathematica [A] (verified)

Time = 3.66 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.35

$$\int (a+bx+cx^2)^{5/2}(A+Cx^2)\,dx = \frac{\sqrt{c}\sqrt{a+x(b+cx)}(224Ac^2(b+2cx)(15b^4-40b^3cx+32bc^2x(13a+8cx^2)+8b^2c(-20a+11cx^2}{}$$

```
[In] Integrate[(a + b*x + c*x^2)^(5/2)*(A + C*x^2),x]
```

```
[Out] (Sqrt[c]*Sqrt[a + x*(b + c*x)]*(224*A*c^2*(b + 2*c*x)*(15*b^4 - 40*b^3*c*x
+ 32*b*c^2*x*(13*a + 8*c*x^2) + 8*b^2*c*(-20*a + 11*c*x^2) + 16*c^2*(33*a^2
 + 26*a*c*x^2 + 8*c^2*x^4)) + C*(945*b^7 - 630*b^6*c*x + 8*b^4*c^2*x*(791*a
 - 54*c*x^2) + 84*b^5*c*(-125*a + 6*c*x^2) + 16*b^3*c^2*(2359*a^2 - 284*a*c
*x^2 + 24*c^2*x^4) + 96*b^2*c^3*x*(-199*a^2 + 36*a*c*x^2 + 648*c^2*x^4) + 8
96*c^4*x*(15*a^3 + 118*a^2*c*x^2 + 136*a*c^2*x^4 + 48*c^3*x^6) + 64*b*c^3*(
-663*a^3 + 174*a^2*c*x^2 + 2456*a*c^2*x^4 + 1584*c^3*x^6))) - 105*(b^2 - 4*
a*c)^3*(32*A*c^2 + 9*b^2*C - 4*a*c*C)*ArcTanh[(Sqrt[c]*x)/(-Sqrt[a] + Sqrt[
a + x*(b + c*x)])])/(344064*c^(11/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 478 vs. $2(237) = 474$.

Time = 0.88 (sec) , antiderivative size = 479, normalized size of antiderivative = 1.79

method	result
risch	$(43008C\,c^7x^7+101376Cb\,c^6x^6+57344A\,c^7x^5+121856Ca\,c^6x^5+62208C\,b^2c^5x^5+143360Ab\,c^6x^4+157184Cab\,c^5x^4+384C\,b^3c^4x^4+1863$
default	$A\left(\frac{(2cx+b)\left(c\,x^2+bx+a\right)^{\frac{5}{2}}}{12c}+\frac{5\left(4ac-b^2\right)\left(\frac{(2cx+b)\left(c\,x^2+bx+a\right)^{\frac{3}{2}}}{8c}+\frac{3\left(4ac-b^2\right)\left(\frac{(2cx+b)\sqrt{c\,x^2+bx+a}}{4c}+\frac{\left(4ac-b^2\right)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+}\right.}{8c^{\frac{3}{2}}}\right.}{16c}\right.}{24c}\right.$

```
[In] int((c*x^2+b*x+a)^(5/2)*(C*x^2+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/344064/c^5*(43008*C*c^7*x^7+101376*C*b*c^6*x^6+57344*A*c^7*x^5+121856*C*a
*c^6*x^5+62208*C*b^2*c^5*x^5+143360*A*b*c^6*x^4+157184*C*a*b*c^5*x^4+384*C*
b^3*c^4*x^4+186368*A*a*c^6*x^3+96768*A*b^2*c^5*x^3+105728*C*a^2*c^5*x^3+345
6*C*a*b^2*c^4*x^3-432*C*b^4*c^3*x^3+279552*A*a*b*c^5*x^2+1792*A*b^3*c^4*x^2
+11136*C*a^2*b*c^4*x^2-4544*C*a*b^3*c^3*x^2+504*C*b^5*c^2*x^2+236544*A*a^2*
c^5*x+21504*A*a*b^2*c^4*x-2240*A*b^4*c^3*x+13440*C*a^3*c^4*x-19104*C*a^2*b^
2*c^3*x+6328*C*a*b^4*c^2*x-630*C*b^6*c*x+118272*A*a^2*b*c^4-35840*A*a*b^3*c
^3+3360*A*b^5*c^2-42432*C*a^3*b*c^3+37744*C*a^2*b^3*c^2-10500*C*a*b^5*c+945
*C*b^7)*(c*x^2+b*x+a)^(1/2)+5/32768*(2048*A*a^3*c^5-1536*A*a^2*b^2*c^4+384*
A*a*b^4*c^3-32*A*b^6*c^2-256*C*a^4*c^4+768*C*a^3*b^2*c^3-480*C*a^2*b^4*c^2+
112*C*a*b^6*c-9*C*b^8)/c^(11/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 475 vs. 2(237) = 474.

Time = 0.36 (sec) , antiderivative size = 953, normalized size of antiderivative = 3.57

$$\int (a+bx+cx^2)^{5/2} (A+Cx^2)\, dx = \left[\frac{105\,(9\,Cb^8 - 112\,Cab^6c - 2048\,Aa^3c^5 + 256\,(Ca^4 + 6\,Aa^2b^2)c^4 - 384\,(2\,Ca^3b^2 + Aab^4)c^3 + 3}{} \right.$$

```
[In] integrate((c*x^2+b*x+a)^(5/2)*(C*x^2+A),x, algorithm="fricas")
```

```
[Out] [1/1376256*(105*(9*C*b^8 - 112*C*a*b^6*c - 2048*A*a^3*c^5 + 256*(C*a^4 + 6*
A*a^2*b^2)*c^4 - 384*(2*C*a^3*b^2 + A*a*b^4)*c^3 + 32*(15*C*a^2*b^4 + A*b^6
)*c^2)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*
c*x + b)*sqrt(c) - 4*a*c) + 4*(43008*C*c^8*x^7 + 101376*C*b*c^7*x^6 + 945*C
*b^7*c - 10500*C*a*b^5*c^2 + 118272*A*a^2*b*c^5 + 256*(243*C*b^2*c^6 + 476*
C*a*c^7 + 224*A*c^8)*x^5 - 64*(663*C*a^3*b + 560*A*a*b^3)*c^4 + 128*(3*C*b^
3*c^5 + 1228*C*a*b*c^6 + 1120*A*b*c^7)*x^4 + 112*(337*C*a^2*b^3 + 30*A*b^5)
*c^3 - 16*(27*C*b^4*c^4 - 216*C*a*b^2*c^5 - 11648*A*a*c^7 - 112*(59*C*a^2 +
 54*A*b^2)*c^6)*x^3 + 8*(63*C*b^5*c^3 - 568*C*a*b^3*c^4 + 34944*A*a*b*c^6 +
 16*(87*C*a^2*b + 14*A*b^3)*c^5)*x^2 - 2*(315*C*b^6*c^2 - 3164*C*a*b^4*c^3
- 118272*A*a^2*c^6 - 1344*(5*C*a^3 + 8*A*a*b^2)*c^5 + 16*(597*C*a^2*b^2 + 7
0*A*b^4)*c^4)*x)*sqrt(c*x^2 + b*x + a))/c^6, 1/688128*(105*(9*C*b^8 - 112*C
*a*b^6*c - 2048*A*a^3*c^5 + 256*(C*a^4 + 6*A*a^2*b^2)*c^4 - 384*(2*C*a^3*b^
2 + A*a*b^4)*c^3 + 32*(15*C*a^2*b^4 + A*b^6)*c^2)*sqrt(-c)*arctan(1/2*sqrt(
c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(43008*C
*c^8*x^7 + 101376*C*b*c^7*x^6 + 945*C*b^7*c - 10500*C*a*b^5*c^2 + 118272*A*
a^2*b*c^5 + 256*(243*C*b^2*c^6 + 476*C*a*c^7 + 224*A*c^8)*x^5 - 64*(663*C*a
^3*b + 560*A*a*b^3)*c^4 + 128*(3*C*b^3*c^5 + 1228*C*a*b*c^6 + 1120*A*b*c^7)
```

```
*x^4 + 112*(337*C*a^2*b^3 + 30*A*b^5)*c^3 - 16*(27*C*b^4*c^4 - 216*C*a*b^2*
c^5 - 11648*A*a*c^7 - 112*(59*C*a^2 + 54*A*b^2)*c^6)*x^3 + 8*(63*C*b^5*c^3
- 568*C*a*b^3*c^4 + 34944*A*a*b*c^6 + 16*(87*C*a^2*b + 14*A*b^3)*c^5)*x^2 -
 2*(315*C*b^6*c^2 - 3164*C*a*b^4*c^3 - 118272*A*a^2*c^6 - 1344*(5*C*a^3 + 8
*A*a*b^2)*c^5 + 16*(597*C*a^2*b^2 + 70*A*b^4)*c^4)*x)*sqrt(c*x^2 + b*x + a)
)/c^6]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2691 vs. $2(270) = 540$.

Time = 0.58 (sec) , antiderivative size = 2691, normalized size of antiderivative = 10.08

$$\int \left(a+bx+cx^2\right)^{5/2} \left(A+Cx^2\right)\,dx = \text{Too large to display}$$

```
[In] integrate((c*x**2+b*x+a)**(5/2)*(C*x**2+A),x)
```

```
[Out] Piecewise((sqrt(a + b*x + c*x**2)*(33*C*b*c*x**6/112 + C*c**2*x**7/8 + x**5
*(A*c**3 + 17*C*a*c**2/8 + 243*C*b**2*c/224)/(6*c) + x**4*(3*A*b*c**2 + 237
*C*a*b*c/56 + C*b**3 - 11*b*(A*c**3 + 17*C*a*c**2/8 + 243*C*b**2*c/224)/(12
*c))/(5*c) + x**3*(3*A*a*c**2 + 3*A*b**2*c + 3*C*a**2*c + 3*C*a*b**2 - 5*a*
(A*c**3 + 17*C*a*c**2/8 + 243*C*b**2*c/224)/(6*c) - 9*b*(3*A*b*c**2 + 237*C
*a*b*c/56 + C*b**3 - 11*b*(A*c**3 + 17*C*a*c**2/8 + 243*C*b**2*c/224)/(12*c
))/(10*c))/(4*c) + x**2*(6*A*a*b*c + A*b**3 + 3*C*a**2*b - 4*a*(3*A*b*c**2
+ 237*C*a*b*c/56 + C*b**3 - 11*b*(A*c**3 + 17*C*a*c**2/8 + 243*C*b**2*c/224
)/(12*c))/(5*c) - 7*b*(3*A*a*c**2 + 3*A*b**2*c + 3*C*a**2*c + 3*C*a*b**2 -
5*a*(A*c**3 + 17*C*a*c**2/8 + 243*C*b**2*c/224)/(6*c) - 9*b*(3*A*b*c**2 + 2
37*C*a*b*c/56 + C*b**3 - 11*b*(A*c**3 + 17*C*a*c**2/8 + 243*C*b**2*c/224)/(
12*c))/(10*c))/(8*c))/(3*c) + x*(3*A*a**2*c + 3*A*a*b**2 + C*a**3 - 3*a*(3*
A*a*c**2 + 3*A*b**2*c + 3*C*a**2*c + 3*C*a*b**2 - 5*a*(A*c**3 + 17*C*a*c**2
/8 + 243*C*b**2*c/224)/(6*c) - 9*b*(3*A*b*c**2 + 237*C*a*b*c/56 + C*b**3 -
11*b*(A*c**3 + 17*C*a*c**2/8 + 243*C*b**2*c/224)/(12*c))/(10*c))/(4*c) - 5*
b*(6*A*a*b*c + A*b**3 + 3*C*a**2*b - 4*a*(3*A*b*c**2 + 237*C*a*b*c/56 + C*b
**3 - 11*b*(A*c**3 + 17*C*a*c**2/8 + 243*C*b**2*c/224)/(12*c))/(5*c) - 7*b*
(3*A*a*c**2 + 3*A*b**2*c + 3*C*a**2*c + 3*C*a*b**2 - 5*a*(A*c**3 + 17*C*a*c
**2/8 + 243*C*b**2*c/224)/(6*c) - 9*b*(3*A*b*c**2 + 237*C*a*b*c/56 + C*b**3
 - 11*b*(A*c**3 + 17*C*a*c**2/8 + 243*C*b**2*c/224)/(12*c))/(10*c))/(8*c))/
(6*c))/(2*c) + (3*A*a**2*b - 2*a*(6*A*a*b*c + A*b**3 + 3*C*a**2*b - 4*a*(3*
A*b*c**2 + 237*C*a*b*c/56 + C*b**3 - 11*b*(A*c**3 + 17*C*a*c**2/8 + 243*C*b
**2*c/224)/(12*c))/(5*c) - 7*b*(3*A*a*c**2 + 3*A*b**2*c + 3*C*a**2*c + 3*C*
a*b**2 - 5*a*(A*c**3 + 17*C*a*c**2/8 + 243*C*b**2*c/224)/(6*c) - 9*b*(3*A*b
*c**2 + 237*C*a*b*c/56 + C*b**3 - 11*b*(A*c**3 + 17*C*a*c**2/8 + 243*C*b**2
*c/224)/(12*c))/(10*c))/(8*c))/(3*c) - 3*b*(3*A*a**2*c + 3*A*a*b**2 + C*a**
3 - 3*a*(3*A*a*c**2 + 3*A*b**2*c + 3*C*a**2*c + 3*C*a*b**2 - 5*a*(A*c**3 +
17*C*a*c**2/8 + 243*C*b**2*c/224)/(6*c) - 9*b*(3*A*b*c**2 + 237*C*a*b*c/56
```

```
+ C*b**3 - 11*b*(A*c**3 + 17*C*a*c**2/8 + 243*C*b**2*c/224)/(12*c))/(10*c))
/(4*c) - 5*b*(6*A*a*b*c + A*b**3 + 3*C*a**2*b - 4*a*(3*A*b*c**2 + 237*C*a*b
*c/56 + C*b**3 - 11*b*(A*c**3 + 17*C*a*c**2/8 + 243*C*b**2*c/224)/(12*c))/(
5*c) - 7*b*(3*A*a*c**2 + 3*A*b**2*c + 3*C*a**2*c + 3*C*a*b**2 - 5*a*(A*c**3
 + 17*C*a*c**2/8 + 243*C*b**2*c/224)/(6*c) - 9*b*(3*A*b*c**2 + 237*C*a*b*c/
56 + C*b**3 - 11*b*(A*c**3 + 17*C*a*c**2/8 + 243*C*b**2*c/224)/(12*c))/(10*
c))/(8*c))/(6*c))/(4*c))/c) + (A*a**3 - a*(3*A*a**2*c + 3*A*a*b**2 + C*a**3
 - 3*a*(3*A*a*c**2 + 3*A*b**2*c + 3*C*a**2*c + 3*C*a*b**2 - 5*a*(A*c**3 + 1
7*C*a*c**2/8 + 243*C*b**2*c/224)/(6*c) - 9*b*(3*A*b*c**2 + 237*C*a*b*c/56 +
 C*b**3 - 11*b*(A*c**3 + 17*C*a*c**2/8 + 243*C*b**2*c/224)/(12*c))/(10*c))/
(4*c) - 5*b*(6*A*a*b*c + A*b**3 + 3*C*a**2*b - 4*a*(3*A*b*c**2 + 237*C*a*b*
c/56 + C*b**3 - 11*b*(A*c**3 + 17*C*a*c**2/8 + 243*C*b**2*c/224)/(12*c))/(5
*c) - 7*b*(3*A*a*c**2 + 3*A*b**2*c + 3*C*a**2*c + 3*C*a*b**2 - 5*a*(A*c**3
+ 17*C*a*c**2/8 + 243*C*b**2*c/224)/(6*c) - 9*b*(3*A*b*c**2 + 237*C*a*b*c/5
6 + C*b**3 - 11*b*(A*c**3 + 17*C*a*c**2/8 + 243*C*b**2*c/224)/(12*c))/(10*c
))/(8*c))/(6*c))/(2*c) - b*(3*A*a**2*b - 2*a*(6*A*a*b*c + A*b**3 + 3*C*a**2
*b - 4*a*(3*A*b*c**2 + 237*C*a*b*c/56 + C*b**3 - 11*b*(A*c**3 + 17*C*a*c**2
/8 + 243*C*b**2*c/224)/(12*c))/(5*c) - 7*b*(3*A*a*c**2 + 3*A*b**2*c + 3*C*a
**2*c + 3*C*a*b**2 - 5*a*(A*c**3 + 17*C*a*c**2/8 + 243*C*b**2*c/224)/(6*c)
- 9*b*(3*A*b*c**2 + 237*C*a*b*c/56 + C*b**3 - 11*b*(A*c**3 + 17*C*a*c**2/8
+ 243*C*b**2*c/224)/(12*c))/(10*c))/(8*c))/(3*c) - 3*b*(3*A*a**2*c + 3*A*a*
b**2 + C*a**3 - 3*a*(3*A*a*c**2 + 3*A*b**2*c + 3*C*a**2*c + 3*C*a*b**2 - 5*
a*(A*c**3 + 17*C*a*c**2/8 + 243*C*b**2*c/224)/(6*c) - 9*b*(3*A*b*c**2 + 237
*C*a*b*c/56 + C*b**3 - 11*b*(A*c**3 + 17*C*a*c**2/8 + 243*C*b**2*c/224)/(12
*c))/(10*c))/(4*c) - 5*b*(6*A*a*b*c + A*b**3 + 3*C*a**2*b - 4*a*(3*A*b*c**2
 + 237*C*a*b*c/56 + C*b**3 - 11*b*(A*c**3 + 17*C*a*c**2/8 + 243*C*b**2*c/22
4)/(12*c))/(5*c) - 7*b*(3*A*a*c**2 + 3*A*b**2*c + 3*C*a**2*c + 3*C*a*b**2 -
 5*a*(A*c**3 + 17*C*a*c**2/8 + 243*C*b**2*c/224)/(6*c) - 9*b*(3*A*b*c**2 +
237*C*a*b*c/56 + C*b**3 - 11*b*(A*c**3 + 17*C*a*c**2/8 + 243*C*b**2*c/224)/
(12*c))/(10*c))/(8*c))/(6*c))/(4*c))/(2*c))*Piecewise((log(b + 2*sqrt(c)*sq
rt(a + b*x + c*x**2) + 2*c*x)/sqrt(c), Ne(a - b**2/(4*c), 0)), ((b/(2*c) +
x)*log(b/(2*c) + x)/sqrt(c*(b/(2*c) + x)**2), True)), Ne(c, 0)), (2*(-2*C*a
*(a + b*x)**(9/2)/(9*b**2) + C*(a + b*x)**(11/2)/(11*b**2) + (a + b*x)**(7/
2)*(A*b**2 + C*a**2)/(7*b**2))/b, Ne(b, 0)), (a**(5/2)*(A*x + C*x**3/3), Tr
ue))
```

Maxima [F(-2)]

Exception generated.

$$\int \left(a+bx+cx^2\right)^{5/2}\left(A+Cx^2\right)\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((c*x^2+b*x+a)^(5/2)*(C*x^2+A),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 480 vs. 2(237) = 474.

Time = 0.30 (sec) , antiderivative size = 480, normalized size of antiderivative = 1.80

$$\int \left(a+bx+cx^2\right)^{5/2} (A$$

$$+Cx^2)\,dx = \frac{1}{344064}\sqrt{cx^2+bx+a}\,(2\,(4\,(2\,(8\,(2\,(12\,(14\,Cc^2x+33\,Cbc)x+\frac{243\,Cb^2c^7+476\,Cac^8+224}{c^7}$$

$$+\frac{5\,(9\,Cb^8-112\,Cab^6c+480\,Ca^2b^4c^2+32\,Ab^6c^2-768\,Ca^3b^2c^3-384\,Aab^4c^3+256\,Ca^4c^4+1536\,Aa^2b^2c^4-}{32768\,c^{\frac{11}{2}}}$$

```
[In] integrate((c*x^2+b*x+a)^(5/2)*(C*x^2+A),x, algorithm="giac")
```

```
[Out] 1/344064*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(2*(12*(14*C*c^2*x + 33*C*b*c)*x
 + (243*C*b^2*c^7 + 476*C*a*c^8 + 224*A*c^9)/c^7)*x + (3*C*b^3*c^6 + 1228*C
*a*b*c^7 + 1120*A*b*c^8)/c^7)*x - (27*C*b^4*c^5 - 216*C*a*b^2*c^6 - 6608*C*
a^2*c^7 - 6048*A*b^2*c^7 - 11648*A*a*c^8)/c^7)*x + (63*C*b^5*c^4 - 568*C*a*
b^3*c^5 + 1392*C*a^2*b*c^6 + 224*A*b^3*c^6 + 34944*A*a*b*c^7)/c^7)*x - (315
*C*b^6*c^3 - 3164*C*a*b^4*c^4 + 9552*C*a^2*b^2*c^5 + 1120*A*b^4*c^5 - 6720*
C*a^3*c^6 - 10752*A*a*b^2*c^6 - 118272*A*a^2*c^7)/c^7)*x + (945*C*b^7*c^2 -
 10500*C*a*b^5*c^3 + 37744*C*a^2*b^3*c^4 + 3360*A*b^5*c^4 - 42432*C*a^3*b*c
^5 - 35840*A*a*b^3*c^5 + 118272*A*a^2*b*c^6)/c^7) + 5/32768*(9*C*b^8 - 112*
C*a*b^6*c + 480*C*a^2*b^4*c^2 + 32*A*b^6*c^2 - 768*C*a^3*b^2*c^3 - 384*A*a*
b^4*c^3 + 256*C*a^4*c^4 + 1536*A*a^2*b^2*c^4 - 2048*A*a^3*c^5)*log(abs(2*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(11/2)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx+cx^2\right)^{5/2}\left(A+Cx^2\right)\,dx=\int \left(C\,x^2+A\right)\,\left(c\,x^2+b\,x+a\right)^{5/2}\,dx$$

```
[In] int((A + C*x^2)*(a + b*x + c*x^2)^(5/2),x)
```

```
[Out] int((A + C*x^2)*(a + b*x + c*x^2)^(5/2), x)
```

3.179 $\int (a+bx+cx^2)^{3/2} (A+Cx^2)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 212

$$\begin{aligned}\int (a+bx+cx^2)^{3/2} (A+Cx^2)\,dx =& -\frac{(b^2-4ac)(24Ac^2+7b^2C-4acC)(b+2cx)\sqrt{a+bx+cx^2}}{512c^4} \\ &+\frac{(24Ac^2+7b^2C-4acC)(b+2cx)(a+bx+cx^2)^{3/2}}{192c^3}-\frac{7bC(a+bx+cx^2)^{5/2}}{60c^2} \\ &+\frac{Cx(a+bx+cx^2)^{5/2}}{6c}+\frac{(b^2-4ac)^2(24Ac^2+7b^2C-4acC)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{1024c^{9/2}}\end{aligned}$$

```
[Out] 1/192*(24*A*c^2-4*C*a*c+7*C*b^2)*(2*c*x+b)*(c*x^2+b*x+a)^(3/2)/c^3-7/60*b*C
*(c*x^2+b*x+a)^(5/2)/c^2+1/6*C*x*(c*x^2+b*x+a)^(5/2)/c+1/1024*(-4*a*c+b^2)^
2*(24*A*c^2-4*C*a*c+7*C*b^2)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1
/2))/c^(9/2)-1/512*(-4*a*c+b^2)*(24*A*c^2-4*C*a*c+7*C*b^2)*(2*c*x+b)*(c*x^2
+b*x+a)^(1/2)/c^4
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used

$= \{1675, 654, 626, 635, 212\}$

$$\begin{aligned}\int \left(a+bx+cx^2\right)^{3/2} \big(A & \\ +Cx^2\big)\,dx &= \frac{\left(b^2-4ac\right)^2 \left(-4acC+24Ac^2+7b^2C\right) \operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{1024c^{9/2}} \\ &- \frac{\left(b^2-4ac\right)(b+2cx)\sqrt{a+bx+cx^2}\left(-4acC+24Ac^2+7b^2C\right)}{512c^4} \\ &+ \frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}\left(-4acC+24Ac^2+7b^2C\right)}{192c^3} \\ &- \frac{7bC\left(a+bx+cx^2\right)^{5/2}}{60c^2} + \frac{Cx\left(a+bx+cx^2\right)^{5/2}}{6c}\end{aligned}$$

```
[In] Int[(a + b*x + c*x^2)^(3/2)*(A + C*x^2),x]
```

```
[Out] -1/512*((b^2 - 4*a*c)*(24*A*c^2 + 7*b^2*C - 4*a*c*C)*(b + 2*c*x)*Sqrt[a + b
*x + c*x^2])/c^4 + ((24*A*c^2 + 7*b^2*C - 4*a*c*C)*(b + 2*c*x)*(a + b*x + c
*x^2)^(3/2))/(192*c^3) - (7*b*C*(a + b*x + c*x^2)^(5/2))/(60*c^2) + (C*x*(a
 + b*x + c*x^2)^(5/2))/(6*c) + ((b^2 - 4*a*c)^2*(24*A*c^2 + 7*b^2*C - 4*a*c
*C)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(1024*c^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{Cx(a+bx+cx^2)^{5/2}}{6c}+\frac{\int\left(6Ac-aC-\frac{7bCx}{2}\right)(a+bx+cx^2)^{3/2}\,dx}{6c}\\
&=-\frac{7bC(a+bx+cx^2)^{5/2}}{60c^2}+\frac{Cx(a+bx+cx^2)^{5/2}}{6c}\\
&\quad+\frac{\left(\frac{7b^2C}{2}+2c(6Ac-aC)\right)\int(a+bx+cx^2)^{3/2}\,dx}{12c^2}\\
&=\frac{(24Ac^2+7b^2C-4acC)(b+2cx)(a+bx+cx^2)^{3/2}}{192c^3}-\frac{7bC(a+bx+cx^2)^{5/2}}{60c^2}\\
&\quad+\frac{Cx(a+bx+cx^2)^{5/2}}{6c}-\frac{((b^2-4ac)(24Ac^2+7b^2C-4acC))\int\sqrt{a+bx+cx^2}\,dx}{128c^3}\\
&=-\frac{(b^2-4ac)(24Ac^2+7b^2C-4acC)(b+2cx)\sqrt{a+bx+cx^2}}{512c^4}\\
&\quad+\frac{(24Ac^2+7b^2C-4acC)(b+2cx)(a+bx+cx^2)^{3/2}}{192c^3}-\frac{7bC(a+bx+cx^2)^{5/2}}{60c^2}\\
&\quad+\frac{Cx(a+bx+cx^2)^{5/2}}{6c}+\frac{\left((b^2-4ac)^2(24Ac^2+7b^2C-4acC)\right)\int\frac{1}{\sqrt{a+bx+cx^2}}\,dx}{1024c^4}\\
&=-\frac{(b^2-4ac)(24Ac^2+7b^2C-4acC)(b+2cx)\sqrt{a+bx+cx^2}}{512c^4}\\
&\quad+\frac{(24Ac^2+7b^2C-4acC)(b+2cx)(a+bx+cx^2)^{3/2}}{192c^3}\\
&\quad-\frac{7bC(a+bx+cx^2)^{5/2}}{60c^2}+\frac{Cx(a+bx+cx^2)^{5/2}}{6c}\\
&\quad+\frac{\left((b^2-4ac)^2(24Ac^2+7b^2C-4acC)\right)\text{Subst}\left(\int\frac{1}{4c-x^2}\,dx,x,\frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{512c^4}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{\left(b^2-4ac\right)\left(24Ac^2+7b^2C-4acC\right)(b+2cx)\sqrt{a+bx+cx^2}}{512c^4} \\
&\quad + \frac{\left(24Ac^2+7b^2C-4acC\right)(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{192c^3} \\
&\quad - \frac{7bC\left(a+bx+cx^2\right)^{5/2}}{60c^2} + \frac{Cx\left(a+bx+cx^2\right)^{5/2}}{6c} \\
&\quad + \frac{\left(b^2-4ac\right)^2\left(24Ac^2+7b^2C-4acC\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{1024c^{9/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.99 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.08

$$
\int \left(a+bx+cx^2\right)^{3/2} \left(A +Cx^2\right) dx = \frac{\sqrt{c}\sqrt{a+x(b+cx)}(120Ac^2(b+2cx)\left(-3b^2+8bcx+4c(5a+2cx^2)\right)+C(-105b^5+70b^4cx+8b}{}
$$

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)*(A + C*x^2),x]
```

```
[Out] (Sqrt[c]*Sqrt[a + x*(b + c*x)]*(120*A*c^2*(b + 2*c*x)*(-3*b^2 + 8*b*c*x + 4
*c*(5*a + 2*c*x^2)) + C*(-105*b^5 + 70*b^4*c*x + 8*b^3*c*(95*a - 7*c*x^2) +
 48*b^2*c^2*x*(-9*a + c*x^2) + 160*c^3*x*(3*a^2 + 14*a*c*x^2 + 8*c^2*x^4) +
 16*b*c^2*(-81*a^2 + 18*a*c*x^2 + 104*c^2*x^4))) + 15*(b^2 - 4*a*c)^2*(24*A
*c^2 + 7*b^2*C - 4*a*c*C)*ArcTanh[(Sqrt[c]*x)/(-Sqrt[a] + Sqrt[a + x*(b + c
*x)])])/(7680*c^(9/2))
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.34

method	result
risch	$\frac{(1280c^5 C x^5+1664bC c^4x^4+1920A c^5x^3+2240Ca c^4x^3+48C b^2c^3x^3+2880Ab c^4x^2+288Cab c^3x^2-56C b^3c^2x^2+4800Aa c^4x+240A}{7680c^4}$
default	$A\left(\frac{(2cx+b)(c x^2+bx+a)^{\frac{3}{2}}}{8c}+\frac{3(4ac-b^2)\left(\frac{(2cx+b)\sqrt{c x^2+bx+a}}{4c}+\frac{(4ac-b^2)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c x^2+bx+a}\right)}{8c^{\frac{3}{2}}}\right)}{16c}\right)+C\left(\frac{x(c x^2+bx+a)}{6c}\right.$

```
[In] int((c*x^2+b*x+a)^(3/2)*(C*x^2+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/7680*(1280*C*c^5*x^5+1664*C*b*c^4*x^4+1920*A*c^5*x^3+2240*C*a*c^4*x^3+48*
C*b^2*c^3*x^3+2880*A*b*c^4*x^2+288*C*a*b*c^3*x^2-56*C*b^3*c^2*x^2+4800*A*a*
c^4*x+240*A*b^2*c^3*x+480*C*a^2*c^3*x-432*C*a*b^2*c^2*x+70*C*b^4*c*x+2400*A
*a*b*c^3-360*A*b^3*c^2-1296*C*a^2*b*c^2+760*C*a*b^3*c-105*C*b^5)/c^4*(c*x^2
+b*x+a)^(1/2)+1/1024*(384*A*a^2*c^4-192*A*a*b^2*c^3+24*A*b^4*c^2-64*C*a^3*c
^3+144*C*a^2*b^2*c^2-60*C*a*b^4*c+7*C*b^6)/c^(9/2)*ln((1/2*b+c*x)/c^(1/2)+(
c*x^2+b*x+a)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 605, normalized size of antiderivative = 2.85

$$\int \left(a+bx+cx^2\right)^{3/2} (A + Cx^2)\, dx = \left[\frac{15\,(7\,Cb^6-60\,Cab^4c+384\,Aa^2c^4-64\,(Ca^3+3\,Aab^2)c^3+24\,(6\,Ca^2b^2+Ab^4)c^2)\sqrt{c}\log\left(-8\,c^2\right.}{}\right.$$

$$-\frac{15\,(7\,Cb^6-60\,Cab^4c+384\,Aa^2c^4-64\,(Ca^3+3\,Aab^2)c^3+24\,(6\,Ca^2b^2+Ab^4)c^2)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^2+bx+a}(2\,c}{2\,(c^2x^2+bcx}\right.}{}$$

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(C*x^2+A),x, algorithm="fricas")
```

```
[Out] [1/30720*(15*(7*C*b^6 - 60*C*a*b^4*c + 384*A*a^2*c^4 - 64*(C*a^3 + 3*A*a*b^
2)*c^3 + 24*(6*C*a^2*b^2 + A*b^4)*c^2)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b
^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(1280*C*c^6*x
^5 + 1664*C*b*c^5*x^4 - 105*C*b^5*c + 760*C*a*b^3*c^2 + 2400*A*a*b*c^4 - 72
*(18*C*a^2*b + 5*A*b^3)*c^3 + 16*(3*C*b^2*c^4 + 140*C*a*c^5 + 120*A*c^6)*x^
3 - 8*(7*C*b^3*c^3 - 36*C*a*b*c^4 - 360*A*b*c^5)*x^2 + 2*(35*C*b^4*c^2 - 21
6*C*a*b^2*c^3 + 2400*A*a*c^5 + 120*(2*C*a^2 + A*b^2)*c^4)*x)*sqrt(c*x^2 + b
*x + a))/c^5, -1/15360*(15*(7*C*b^6 - 60*C*a*b^4*c + 384*A*a^2*c^4 - 64*(C*
a^3 + 3*A*a*b^2)*c^3 + 24*(6*C*a^2*b^2 + A*b^4)*c^2)*sqrt(-c)*arctan(1/2*sq
rt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(1280
*C*c^6*x^5 + 1664*C*b*c^5*x^4 - 105*C*b^5*c + 760*C*a*b^3*c^2 + 2400*A*a*b*
c^4 - 72*(18*C*a^2*b + 5*A*b^3)*c^3 + 16*(3*C*b^2*c^4 + 140*C*a*c^5 + 120*A
*c^6)*x^3 - 8*(7*C*b^3*c^3 - 36*C*a*b*c^4 - 360*A*b*c^5)*x^2 + 2*(35*C*b^4*
c^2 - 216*C*a*b^2*c^3 + 2400*A*a*c^5 + 120*(2*C*a^2 + A*b^2)*c^4)*x)*sqrt(c
*x^2 + b*x + a))/c^5]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 775 vs. $2(207) = 414$.

Time = 0.51 (sec) , antiderivative size = 775, normalized size of antiderivative = 3.66

$$\int \left(a+bx+cx^2\right)^{3/2} \left(A+Cx^2\right) dx = \begin{cases} \sqrt{a+bx+cx^2}\cdot\left(\frac{13Cbx^4}{60} + \frac{Ccx^5}{6} + \frac{x^3\left(Ac^2+\frac{7Cac}{6}+\frac{Cb^2}{40}\right)}{4c} + \frac{x^2\cdot\left(2Abc+\frac{17Cab}{15}-\frac{7b\left(Ac^2+\frac{7Cac}{6}+\frac{Cb^2}{40}\right)}{8c}\right)}{3c} + \right. \\ \frac{2\left(-\frac{2Ca(a+bx)^{\frac{7}{2}}}{7b^2}+\frac{C(a+bx)^{\frac{9}{2}}}{9b^2}+\frac{(a+bx)^{\frac{5}{2}}\left(Ab^2+Ca^2\right)}{5b^2}\right)}{b} \\ a^{\frac{3}{2}}\left(Ax+\frac{Cx^3}{3}\right) \end{cases}$$

```
[In] integrate((c*x**2+b*x+a)**(3/2)*(C*x**2+A),x)
```

```
[Out] Piecewise((sqrt(a + b*x + c*x**2)*(13*C*b*x**4/60 + C*c*x**5/6 + x**3*(A*c*
*2 + 7*C*a*c/6 + C*b**2/40)/(4*c) + x**2*(2*A*b*c + 17*C*a*b/15 - 7*b*(A*c*
*2 + 7*C*a*c/6 + C*b**2/40)/(8*c))/(3*c) + x*(2*A*a*c + A*b**2 + C*a**2 - 3
*a*(A*c**2 + 7*C*a*c/6 + C*b**2/40)/(4*c) - 5*b*(2*A*b*c + 17*C*a*b/15 - 7*
b*(A*c**2 + 7*C*a*c/6 + C*b**2/40)/(8*c))/(6*c))/(2*c) + (2*A*a*b - 2*a*(2*
A*b*c + 17*C*a*b/15 - 7*b*(A*c**2 + 7*C*a*c/6 + C*b**2/40)/(8*c))/(3*c) - 3
*b*(2*A*a*c + A*b**2 + C*a**2 - 3*a*(A*c**2 + 7*C*a*c/6 + C*b**2/40)/(4*c)
- 5*b*(2*A*b*c + 17*C*a*b/15 - 7*b*(A*c**2 + 7*C*a*c/6 + C*b**2/40)/(8*c))/
(6*c))/(4*c))/c) + (A*a**2 - a*(2*A*a*c + A*b**2 + C*a**2 - 3*a*(A*c**2 + 7
*C*a*c/6 + C*b**2/40)/(4*c) - 5*b*(2*A*b*c + 17*C*a*b/15 - 7*b*(A*c**2 + 7*
C*a*c/6 + C*b**2/40)/(8*c))/(6*c))/(2*c) - b*(2*A*a*b - 2*a*(2*A*b*c + 17*C
*a*b/15 - 7*b*(A*c**2 + 7*C*a*c/6 + C*b**2/40)/(8*c))/(3*c) - 3*b*(2*A*a*c
+ A*b**2 + C*a**2 - 3*a*(A*c**2 + 7*C*a*c/6 + C*b**2/40)/(4*c) - 5*b*(2*A*b
```

```
*c + 17*C*a*b/15 - 7*b*(A*c**2 + 7*C*a*c/6 + C*b**2/40)/(8*c))/(6*c))/(4*c)
)/(2*c))*Piecewise((log(b + 2*sqrt(c)*sqrt(a + b*x + c*x**2) + 2*c*x)/sqrt(
c), Ne(a - b**2/(4*c), 0)), ((b/(2*c) + x)*log(b/(2*c) + x)/sqrt(c*(b/(2*c)
 + x)**2), True)), Ne(c, 0)), (2*(-2*C*a*(a + b*x)**(7/2)/(7*b**2) + C*(a +
 b*x)**(9/2)/(9*b**2) + (a + b*x)**(5/2)*(A*b**2 + C*a**2)/(5*b**2))/b, Ne(
b, 0)), (a**(3/2)*(A*x + C*x**3/3), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \left(a+bx+cx^2\right)^{3/2}\left(A+Cx^2\right)\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(C*x^2+A),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.39

$$\int \left(a+bx+cx^2\right)^{3/2}\left(A\right.$$

$$\left.+Cx^2\right)\,dx = \frac{1}{7680}\sqrt{cx^2+bx+a}\left(2\left(4\left(2\left(8\,(10\,Ccx+13\,Cb)x+\frac{3\,Cb^2c^4+140\,Cac^5+120\,Ac^6}{c^5}\right)x-7\,Cb\right.\right.\right.$$

$$-\frac{\left(7\,Cb^6-60\,Cab^4c+144\,Ca^2b^2c^2+24\,Ab^4c^2-64\,Ca^3c^3-192\,Aab^2c^3+384\,Aa^2c^4\right)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+}\right.\right.\right.}{1024\,c^{\frac{9}{2}}}$$

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(C*x^2+A),x, algorithm="giac")
```

```
[Out] 1/7680*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*C*c*x + 13*C*b)*x + (3*C*b^2*c
^4 + 140*C*a*c^5 + 120*A*c^6)/c^5)*x - (7*C*b^3*c^3 - 36*C*a*b*c^4 - 360*A*
b*c^5)/c^5)*x + (35*C*b^4*c^2 - 216*C*a*b^2*c^3 + 240*C*a^2*c^4 + 120*A*b^2
*c^4 + 2400*A*a*c^5)/c^5)*x - (105*C*b^5*c - 760*C*a*b^3*c^2 + 1296*C*a^2*b
*c^3 + 360*A*b^3*c^3 - 2400*A*a*b*c^4)/c^5) - 1/1024*(7*C*b^6 - 60*C*a*b^4*
c + 144*C*a^2*b^2*c^2 + 24*A*b^4*c^2 - 64*C*a^3*c^3 - 192*A*a*b^2*c^3 + 384
*A*a^2*c^4)*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(
9/2)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx+cx^2\right)^{3/2}\left(A+Cx^2\right)\,dx=\int \left(C\,x^2+A\right)\,\left(c\,x^2+b\,x+a\right)^{3/2}\,dx$$

```
[In] int((A + C*x^2)*(a + b*x + c*x^2)^(3/2),x)
[Out] int((A + C*x^2)*(a + b*x + c*x^2)^(3/2), x)
```

3.180 $\int \sqrt{a+bx+cx^2}(A+Cx^2)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 157

$$\begin{aligned}\int \sqrt{a+bx+cx^2}\left(A+Cx^2\right)\,dx &= \frac{(16Ac^2+5b^2C-4acC)\,(b+2cx)\sqrt{a+bx+cx^2}}{64c^3}-\frac{5bC(a+bx+cx^2)^{3/2}}{24c^2}\\&\quad+\frac{Cx(a+bx+cx^2)^{3/2}}{4c}-\frac{(b^2-4ac)\,(16Ac^2+5b^2C-4acC)\,\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{128c^{7/2}}\end{aligned}$$

```
[Out] -5/24*b*C*(c*x^2+b*x+a)^(3/2)/c^2+1/4*C*x*(c*x^2+b*x+a)^(3/2)/c-1/128*(-4*a
*c+b^2)*(16*A*c^2-4*C*a*c+5*C*b^2)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x
+a)^(1/2))/c^(7/2)+1/64*(16*A*c^2-4*C*a*c+5*C*b^2)*(2*c*x+b)*(c*x^2+b*x+a)^
(1/2)/c^3
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1675, 654, 626, 635, 212}

$$\begin{aligned}\int \sqrt{a+bx+cx^2}\left(A+Cx^2\right)\,dx &= -\frac{(b^2-4ac)\,(-4acC+16Ac^2+5b^2C)\,\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{128c^{7/2}}\\&\quad+\frac{(b+2cx)\sqrt{a+bx+cx^2}(-4acC+16Ac^2+5b^2C)}{64c^3}\\&\quad-\frac{5bC(a+bx+cx^2)^{3/2}}{24c^2}+\frac{Cx(a+bx+cx^2)^{3/2}}{4c}\end{aligned}$$

```
[In] Int[Sqrt[a + b*x + c*x^2]*(A + C*x^2),x]
```

```
[Out] ((16*A*c^2 + 5*b^2*C - 4*a*c*C)*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(64*c^3)
 - (5*b*C*(a + b*x + c*x^2)^(3/2))/(24*c^2) + (C*x*(a + b*x + c*x^2)^(3/2))
/(4*c) - ((b^2 - 4*a*c)*(16*A*c^2 + 5*b^2*C - 4*a*c*C)*ArcTanh[(b + 2*c*x)/
(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(128*c^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{Cx\left(a+bx+cx^2\right)^{3/2}}{4c} + \frac{\int \left(4Ac-aC-\frac{5bCx}{2}\right)\sqrt{a+bx+cx^2}\,dx}{4c}$$

$$= -\frac{5bC(a+bx+cx^2)^{3/2}}{24c^2} + \frac{Cx(a+bx+cx^2)^{3/2}}{4c} + \frac{\left(\frac{5b^2C}{2}+2c(4Ac-aC)\right)\int \sqrt{a+bx+cx^2}\,dx}{8c^2}$$

$$= \frac{(16Ac^2+5b^2C-4acC)\,(b+2cx)\sqrt{a+bx+cx^2}}{64c^3} - \frac{5bC(a+bx+cx^2)^{3/2}}{24c^2} + \frac{Cx(a+bx+cx^2)^{3/2}}{4c} - \frac{((b^2-4ac)\,(16Ac^2+5b^2C-4acC))\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{128c^3}$$

$$= \frac{(16Ac^2+5b^2C-4acC)\,(b+2cx)\sqrt{a+bx+cx^2}}{64c^3} - \frac{5bC(a+bx+cx^2)^{3/2}}{24c^2} + \frac{Cx(a+bx+cx^2)^{3/2}}{4c} - \frac{((b^2-4ac)\,(16Ac^2+5b^2C-4acC))\,\mathrm{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{64c^3}$$

$$= \frac{(16Ac^2+5b^2C-4acC)\,(b+2cx)\sqrt{a+bx+cx^2}}{64c^3} - \frac{5bC(a+bx+cx^2)^{3/2}}{24c^2} + \frac{Cx(a+bx+cx^2)^{3/2}}{4c} - \frac{(b^2-4ac)\,(16Ac^2+5b^2C-4acC)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{128c^{7/2}}$$

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.92

$$\int \sqrt{a+bx+cx^2}\left(A+Cx^2\right)\,dx$$

$$= \frac{\sqrt{c}\sqrt{a+x(b+cx)}(48Ac^2(b+2cx)+C(15b^3-10b^2cx+24c^2x(a+2cx^2)+b(-52ac+8c^2x^2)))-3(b^2-}{192c^{7/2}}$$

```
[In] Integrate[Sqrt[a + b*x + c*x^2]*(A + C*x^2),x]
```

```
[Out] (Sqrt[c]*Sqrt[a + x*(b + c*x)]*(48*A*c^2*(b + 2*c*x) + C*(15*b^3 - 10*b^2*c
*x + 24*c^2*x*(a + 2*c*x^2) + b*(-52*a*c + 8*c^2*x^2))) - 3*(b^2 - 4*a*c)*(
16*A*c^2 + 5*b^2*C - 4*a*c*C)*ArcTanh[(Sqrt[c]*x)/(-Sqrt[a] + Sqrt[a + x*(b
 + c*x)])])/(192*c^(7/2))
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.96

method	result
risch	$\frac{\left(48c^{3}C\,x^{3}+8b\,c^{2}C\,x^{2}+96A\,c^{3}x+24a\,c^{2}Cx-10C\,b^{2}cx+48Ab\,c^{2}-52Cabc+15C\,b^{3}\right)\sqrt{c\,x^{2}+bx+a}}{192c^{3}}+\frac{\left(64Aa\,c^{3}-16A\,b^{2}c^{2}-16C\,a^{2}c^{2}+\right.}{}$
default	$A\left(\frac{(2cx+b)\sqrt{c\,x^{2}+bx+a}}{4c}+\frac{\left(4ac-b^{2}\right)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx+a}\right)}{8c^{\frac{3}{2}}}\right)+C\left(\frac{x\left(c\,x^{2}+bx+a\right)^{\frac{3}{2}}}{4c}-\frac{5b\left(\frac{\left(c\,x^{2}+bx+a\right)^{\frac{3}{2}}}{3c}-\frac{b\left(\frac{(2cx+b}{}\right.}{}\right.}{}\right.$

```
[In] int((c*x^2+b*x+a)^(1/2)*(C*x^2+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/192*(48*C*c^3*x^3+8*C*b*c^2*x^2+96*A*c^3*x+24*C*a*c^2*x-10*C*b^2*c*x+48*A
*b*c^2-52*C*a*b*c+15*C*b^3)/c^3*(c*x^2+b*x+a)^(1/2)+1/128*(64*A*a*c^3-16*A*
b^2*c^2-16*C*a^2*c^2+24*C*a*b^2*c-5*C*b^4)/c^(7/2)*ln((1/2*b+c*x)/c^(1/2)+(
c*x^2+b*x+a)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 355, normalized size of antiderivative = 2.26

$$\int \sqrt{a+bx+cx^2}\left(A+Cx^2\right)\,dx$$

$$=\left[-\frac{3\left(5\,Cb^4-24\,Cab^2c-64\,Aac^3+16\,(Ca^2+Ab^2)c^2\right)\sqrt{c}\log\left(-8\,c^2x^2-8\,bcx-b^2-4\,\sqrt{cx^2+bx+a}(2\right.}{}\right.$$

```
[In] integrate((c*x^2+b*x+a)^(1/2)*(C*x^2+A),x, algorithm="fricas")
```

```
[Out] [-1/768*(3*(5*C*b^4 - 24*C*a*b^2*c - 64*A*a*c^3 + 16*(C*a^2 + A*b^2)*c^2)*s
qrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)
*sqrt(c) - 4*a*c) - 4*(48*C*c^4*x^3 + 8*C*b*c^3*x^2 + 15*C*b^3*c - 52*C*a*b
*c^2 + 48*A*b*c^3 - 2*(5*C*b^2*c^2 - 12*C*a*c^3 - 48*A*c^4)*x)*sqrt(c*x^2 +
 b*x + a))/c^4, 1/384*(3*(5*C*b^4 - 24*C*a*b^2*c - 64*A*a*c^3 + 16*(C*a^2 +
 A*b^2)*c^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)
```

```
/(c^2*x^2 + b*c*x + a*c)) + 2*(48*C*c^4*x^3 + 8*C*b*c^3*x^2 + 15*C*b^3*c -
52*C*a*b*c^2 + 48*A*b*c^3 - 2*(5*C*b^2*c^2 - 12*C*a*c^3 - 48*A*c^4)*x)*sqrt
(c*x^2 + b*x + a))/c^4]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 308 vs. 2(151) = 302.

Time = 0.43 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.96

$$\int \sqrt{a+bx+cx^2}\left(A+Cx^2\right)\,dx$$

$$= \begin{cases} \sqrt{a+bx+cx^2}\left(\frac{Cbx^2}{24c}+\frac{Cx^3}{4}+\frac{x\left(Ac+\frac{Ca}{4}-\frac{5Cb^2}{48c}\right)}{2c}+\frac{Ab-\frac{Cab}{12c}-\frac{3b\left(Ac+\frac{Ca}{4}-\frac{5Cb^2}{48c}\right)}{4c}}{c}\right)+\left(Aa-\frac{a\left(Ac+\frac{Ca}{4}-\frac{5Cb^2}{48c}\right)}{2c}-\right. \\ \frac{2\left(-\frac{2Ca(a+bx)^{\frac{5}{2}}}{5b^2}+\frac{C(a+bx)^{\frac{7}{2}}}{7b^2}+\frac{(a+bx)^{\frac{3}{2}}\left(Ab^2+Ca^2\right)}{3b^2}\right)}{b} \\ \sqrt{a}\left(Ax+\frac{Cx^3}{3}\right) \end{cases}$$

```
[In] integrate((c*x**2+b*x+a)**(1/2)*(C*x**2+A),x)
```

```
[Out] Piecewise((sqrt(a + b*x + c*x**2)*(C*b*x**2/(24*c) + C*x**3/4 + x*(A*c + C*
a/4 - 5*C*b**2/(48*c))/(2*c) + (A*b - C*a*b/(12*c) - 3*b*(A*c + C*a/4 - 5*C
*b**2/(48*c))/(4*c))/c) + (A*a - a*(A*c + C*a/4 - 5*C*b**2/(48*c))/(2*c) -
b*(A*b - C*a*b/(12*c) - 3*b*(A*c + C*a/4 - 5*C*b**2/(48*c))/(4*c))/(2*c))*P
iecewise((log(b + 2*sqrt(c)*sqrt(a + b*x + c*x**2) + 2*c*x)/sqrt(c), Ne(a -
 b**2/(4*c), 0)), ((b/(2*c) + x)*log(b/(2*c) + x)/sqrt(c*(b/(2*c) + x)**2),
 True)), Ne(c, 0)), (2*(-2*C*a*(a + b*x)**(5/2)/(5*b**2) + C*(a + b*x)**(7/
2)/(7*b**2) + (a + b*x)**(3/2)*(A*b**2 + C*a**2)/(3*b**2))/b, Ne(b, 0)), (s
qrt(a)*(A*x + C*x**3/3), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{a+bx+cx^2}\left(A+Cx^2\right)\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((c*x^2+b*x+a)^(1/2)*(C*x^2+A),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.01

$$\int \sqrt{a+bx+cx^2}\left(A+Cx^2\right)\,dx$$
$$=\frac{1}{192}\sqrt{cx^2+bx+a}\left(2\left(4\left(6\,Cx+\frac{Cb}{c}\right)x-\frac{5\,Cb^2c-12\,Cac^2-48\,Ac^3}{c^3}\right)x+\frac{15\,Cb^3-52\,Cabc+48\,Abc}{c^3}\right.$$
$$+\frac{\left(5\,Cb^4-24\,Cab^2c+16\,Ca^2c^2+16\,Ab^2c^2-64\,Aac^3\right)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}+b\right|\right)}{128\,c^{\frac{7}{2}}}$$

```
[In] integrate((c*x^2+b*x+a)^(1/2)*(C*x^2+A),x, algorithm="giac")
```

```
[Out] 1/192*sqrt(c*x^2 + b*x + a)*(2*(4*(6*C*x + C*b/c)*x - (5*C*b^2*c - 12*C*a*c
^2 - 48*A*c^3)/c^3)*x + (15*C*b^3 - 52*C*a*b*c + 48*A*b*c^2)/c^3) + 1/128*(
5*C*b^4 - 24*C*a*b^2*c + 16*C*a^2*c^2 + 16*A*b^2*c^2 - 64*A*a*c^3)*log(abs(
2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(7/2)
```

Mupad [B] (verification not implemented)

Time = 13.75 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.53

$$\int \sqrt{a+bx+cx^2}\left(A+Cx^2\right)\,dx$$
$$=A\left(\frac{x}{2}+\frac{b}{4\,c}\right)\,\sqrt{c\,x^2+b\,x+a}$$
$$-\frac{C\,a\left(\left(\frac{x}{2}+\frac{b}{4\,c}\right)\,\sqrt{c\,x^2+b\,x+a}+\frac{\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x+a}\right)\,\left(a\,c-\frac{b^2}{4}\right)}{2\,c^{3/2}}\right)}{4\,c}$$
$$+\frac{A\,\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x+a}\right)\,\left(a\,c-\frac{b^2}{4}\right)}{2\,c^{3/2}}$$
$$-\frac{5\,C\,b\left(\frac{\ln\left(\frac{b+2\,c\,x}{\sqrt{c}}+2\,\sqrt{c\,x^2+b\,x+a}\right)\,\left(b^3-4\,a\,b\,c\right)}{16\,c^{5/2}}+\frac{\left(-3\,b^2+2\,c\,x\,b+8\,c\,\left(c\,x^2+a\right)\right)\,\sqrt{c\,x^2+b\,x+a}}{24\,c^2}\right)}{8\,c}$$
$$+\frac{C\,x\,{\left(c\,x^2+b\,x+a\right)}^{3/2}}{4\,c}$$

```
[In] int((A + C*x^2)*(a + b*x + c*x^2)^(1/2),x)
```

```
[Out] A*(x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) - (C*a*((x/2 + b/(4*c))*(a + b*x
+ c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c -
```

```
 b^2/4))/(2*c^(3/2))))/(4*c) + (A*log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^
2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2)) - (5*C*b*((log((b + 2*c*x)/c^(1/2) + 2
*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2)
 - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) + (C*x*(a + b
*x + c*x^2)^(3/2))/(4*c)
```

3.181 $\int \frac{A+Cx^2}{\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 104

$$\int \frac{A+Cx^2}{\sqrt{a+bx+cx^2}}\,dx = -\frac{3bC\sqrt{a+bx+cx^2}}{4c^2}+\frac{Cx\sqrt{a+bx+cx^2}}{2c}+\frac{(8Ac^2+3b^2C-4acC)\,\mathrm{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{5/2}}$$

```
[Out] 1/8*(8*A*c^2-4*C*a*c+3*C*b^2)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(
1/2))/c^(5/2)-3/4*b*C*(c*x^2+b*x+a)^(1/2)/c^2+1/2*C*x*(c*x^2+b*x+a)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1675, 654, 635, 212}

$$\int \frac{A+Cx^2}{\sqrt{a+bx+cx^2}}\,dx = \frac{(-4acC+8Ac^2+3b^2C)\,\mathrm{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{5/2}}-\frac{3bC\sqrt{a+bx+cx^2}}{4c^2}+\frac{Cx\sqrt{a+bx+cx^2}}{2c}$$

```
[In] Int[(A + C*x^2)/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (-3*b*C*Sqrt[a + b*x + c*x^2])/(4*c^2) + (C*x*Sqrt[a + b*x + c*x^2])/(2*c)
+ ((8*A*c^2 + 3*b^2*C - 4*a*c*C)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*
x + c*x^2])])/(8*c^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{Cx\sqrt{a+bx+cx^2}}{2c}+\frac{\int \frac{2Ac-aC-\frac{3bCx}{2}}{\sqrt{a+bx+cx^2}}\,dx}{2c}\\
&= -\frac{3bC\sqrt{a+bx+cx^2}}{4c^2}+\frac{Cx\sqrt{a+bx+cx^2}}{2c}+\frac{\left(\frac{3b^2C}{2}+2c(2Ac-aC)\right)\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{4c^2}\\
&= -\frac{3bC\sqrt{a+bx+cx^2}}{4c^2}+\frac{Cx\sqrt{a+bx+cx^2}}{2c}\\
&\quad+\frac{\left(\frac{3b^2C}{2}+2c(2Ac-aC)\right)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx,x,\frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{2c^2}\\
&= -\frac{3bC\sqrt{a+bx+cx^2}}{4c^2}+\frac{Cx\sqrt{a+bx+cx^2}}{2c}+\frac{\left(8Ac^2+3b^2C-4acC\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{5/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.85

$$\int \frac{A+Cx^2}{\sqrt{a+bx+cx^2}}\,dx = \frac{C(-3b+2cx)\sqrt{a+x(b+cx)}}{4c^2} + \frac{(8Ac^2+3b^2C-4acC)\operatorname{arctanh}\left(\frac{\sqrt{c}x}{-\sqrt{a}+\sqrt{a+x(b+cx)}}\right)}{4c^{5/2}}$$

```
[In] Integrate[(A + C*x^2)/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (C*(-3*b + 2*c*x)*Sqrt[a + x*(b + c*x)])/(4*c^2) + ((8*A*c^2 + 3*b^2*C - 4*
a*c*C)*ArcTanh[(Sqrt[c]*x)/(-Sqrt[a] + Sqrt[a + x*(b + c*x)])])/(4*c^(5/2))
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.73

method	result
risch	$-\frac{C(-2cx+3b)\sqrt{c\,x^2+bx+a}}{4c^2}+\frac{(8A\,c^2-4Cac+3C\,b^2)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{8c^{\frac{5}{2}}}$
default	$\frac{A\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{\sqrt{c}}+C\left(\frac{x\sqrt{c\,x^2+bx+a}}{2c}-\frac{3b\left(\frac{\sqrt{c\,x^2+bx+a}}{c}-\frac{b\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{2c^{\frac{3}{2}}}\right)}{4c}-\frac{a\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx}\right.}{2c^{\frac{3}{2}}}\right.$

```
[In] int((C*x^2+A)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*C*(-2*c*x+3*b)/c^2*(c*x^2+b*x+a)^(1/2)+1/8*(8*A*c^2-4*C*a*c+3*C*b^2)/c
^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.95

$$\int \frac{A+Cx^2}{\sqrt{a+bx+cx^2}}\,dx$$
$$= \left[\frac{(3\,Cb^2-4\,Cac+8\,Ac^2)\sqrt{c}\log\left(-8\,c^2x^2-8\,bcx-b^2-4\,\sqrt{cx^2+bx+a}(2\,cx+b)\sqrt{c}-4\,ac\right)+4\,(2\,Cc^2x}{16\,c^3}\right.$$
$$\left. -\frac{(3\,Cb^2-4\,Cac+8\,Ac^2)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^2+bx+a}(2\,cx+b)\sqrt{-c}}{2\,(c^2x^2+bcx+ac)}\right)-2\,(2\,Cc^2x-3\,Cbc)\sqrt{cx^2+bx+a}}{8\,c^3}\right]$$

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*((3*C*b^2 - 4*C*a*c + 8*A*c^2)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2
 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(2*C*c^2*x - 3*
C*b*c)*sqrt(c*x^2 + b*x + a))/c^3, -1/8*((3*C*b^2 - 4*C*a*c + 8*A*c^2)*sqrt
(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x
 + a*c)) - 2*(2*C*c^2*x - 3*C*b*c)*sqrt(c*x^2 + b*x + a))/c^3]
```

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.82

$$\int \frac{A+Cx^2}{\sqrt{a+bx+cx^2}}\,dx$$
$$= \begin{cases} \left(-\frac{3Cb}{4c^2}+\frac{Cx}{2c}\right)\sqrt{a+bx+cx^2}+\left(A-\frac{Ca}{2c}+\frac{3Cb^2}{8c^2}\right)\left(\begin{cases}\frac{\log\left(b+2\sqrt{c}\sqrt{a+bx+cx^2}+2cx\right)}{\sqrt{c}} & \text{for}\ a-\frac{b^2}{4c}\neq 0\\ \frac{\left(\frac{b}{2c}+x\right)\log\left(\frac{b}{2c}+x\right)}{\sqrt{c\left(\frac{b}{2c}+x\right)^2}} & \text{otherwise}\end{cases}\right) & \text{for}\ c\neq \\ \frac{2A\sqrt{a+bx}+\frac{2C\left(a^2\sqrt{a+bx}-\frac{2a(a+bx)^{\frac{3}{2}}}{3}+\frac{(a+bx)^{\frac{5}{2}}}{5}\right)}{b^2}}{b} & \text{for}\ b\neq \\ \frac{Ax+\frac{Cx^3}{3}}{\sqrt{a}} & \text{otherw} \end{cases}$$

```
[In] integrate((C*x**2+A)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Piecewise(((-3*C*b/(4*c**2) + C*x/(2*c))*sqrt(a + b*x + c*x**2) + (A - C*a/
(2*c) + 3*C*b**2/(8*c**2))*Piecewise((log(b + 2*sqrt(c)*sqrt(a + b*x + c*x*
*2) + 2*c*x)/sqrt(c), Ne(a - b**2/(4*c), 0)), ((b/(2*c) + x)*log(b/(2*c) +
x)/sqrt(c*(b/(2*c) + x)**2), True)), Ne(c, 0)), ((2*A*sqrt(a + b*x) + 2*C*(
a**2*sqrt(a + b*x) - 2*a*(a + b*x)**(3/2)/3 + (a + b*x)**(5/2)/5)/b**2)/b,
Ne(b, 0)), ((A*x + C*x**3/3)/sqrt(a), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Cx^2}{\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.79

$$\int \frac{A+Cx^2}{\sqrt{a+bx+cx^2}}\,dx = \frac{1}{4}\sqrt{cx^2+bx+a}\left(\frac{2\,Cx}{c}-\frac{3\,Cb}{c^2}\right) - \frac{(3\,Cb^2-4\,Cac+8\,Ac^2)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}+b\right|\right)}{8\,c^{\frac{5}{2}}}$$

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(c*x^2 + b*x + a)*(2*C*x/c - 3*C*b/c^2) - 1/8*(3*C*b^2 - 4*C*a*c +
8*A*c^2)*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(5/2
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Cx^2}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{C\,x^2+A}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

```
[In] int((A + C*x^2)/(a + b*x + c*x^2)^(1/2),x)
```

```
[Out] int((A + C*x^2)/(a + b*x + c*x^2)^(1/2), x)
```

3.182 $\int \frac{A+Cx^2}{\left(a+bx+cx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 98

$$\int \frac{A+Cx^2}{\left(a+bx+cx^2\right)^{3/2}}\,dx = -\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} + \frac{C\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}}$$

```
[Out] C*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(3/2)-2*(b*c*(A+a*C/
c)+(2*A*c^2+(-2*a*c+b^2)*C)*x)/c/(-4*a*c+b^2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1674, 12, 635, 212}

$$\int \frac{A+Cx^2}{\left(a+bx+cx^2\right)^{3/2}}\,dx = \frac{C\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}} - \frac{2\left(x\left(C\left(b^2-2ac\right)+2Ac^2\right)+bc\left(\frac{aC}{c}+A\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

```
[In] Int[(A + C*x^2)/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (-2*(b*c*(A + (a*C)/c) + (2*A*c^2 + (b^2 - 2*a*c)*C)*x))/(c*(b^2 - 4*a*c)*S
qrt[a + b*x + c*x^2]) + (C*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*
x^2])])/c^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}-\frac{2\int -\frac{\left(b^2-4ac\right)C}{2c\sqrt{a+bx+cx^2}}\,dx}{b^2-4ac}\\
&= -\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}+\frac{C\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{c}\\
&= -\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}+\frac{(2C)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx,x,\frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{c}\\
&= -\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}+\frac{C\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.97

$$\int \frac{A+Cx^2}{(a+bx+cx^2)^{3/2}}\,dx = \frac{2(b^2Cx+aC(b-2cx)+Ac(b+2cx))}{c\left(-b^2+4ac\right)\sqrt{a+x(b+cx)}} + \frac{2C\text{arctanh}\left(\frac{\sqrt{c}x}{-\sqrt{a}+\sqrt{a+x(b+cx)}}\right)}{c^{3/2}}$$

```
[In] Integrate[(A + C*x^2)/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (2*(b^2*C*x + a*C*(b - 2*c*x) + A*c*(b + 2*c*x)))/(c*(-b^2 + 4*a*c)*Sqrt[a
+ x*(b + c*x)]) + (2*C*ArcTanh[(Sqrt[c]*x)/(-Sqrt[a] + Sqrt[a + x*(b + c*x)
])])/c^(3/2)
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.48

method	result
default	$\frac{2A(2cx+b)}{(4ac-b^2)\sqrt{c\,x^2+bx+a}}+C\left(-\frac{x}{c\sqrt{c\,x^2+bx+a}}-\frac{b\left(-\frac{1}{c\sqrt{c\,x^2+bx+a}}-\frac{b(2cx+b)}{c\left(4ac-b^2\right)\sqrt{c\,x^2+bx+a}}\right)}{2c}+\frac{\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{c^{\frac{3}{2}}}\right)$

```
[In] int((C*x^2+A)/(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*A*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+C*(-x/c/(c*x^2+b*x+a)^(1/2)-1
/2*b/c*(-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1
/2))+1/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 200 vs. 2(88) = 176.

Time = 0.37 (sec) , antiderivative size = 403, normalized size of antiderivative = 4.11

$$\int \frac{A+Cx^2}{(a+bx+cx^2)^{3/2}}\,dx = \Bigg[\frac{(Cab^2-4\,Ca^2c+(Cb^2c-4\,Cac^2)x^2+(Cb^3-4\,Cabc)x)\sqrt{c}\log\left(-8\,c^2x^2-8\,bcx\right.}{2\left(ab^2c^2-4\,a^2\right.}$$

$$-\frac{(Cab^2-4\,Ca^2c+(Cb^2c-4\,Cac^2)x^2+(Cb^3-4\,Cabc)x)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^2+bx+a}(2\,cx+b)\sqrt{-c}}{2\,(c^2x^2+bcx+ac)}\right)+2\,(Cabc+A}{ab^2c^2-4\,a^2c^3+(b^2c^3-4\,ac^4)x^2+(b^3c^2-4\,abc^3)x}$$

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((C*a*b^2 - 4*C*a^2*c + (C*b^2*c - 4*C*a*c^2)*x^2 + (C*b^3 - 4*C*a*b*c
)*x)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*
x + b)*sqrt(c) - 4*a*c) - 4*(C*a*b*c + A*b*c^2 + (C*b^2*c - 2*C*a*c^2 + 2*A
*c^3)*x)*sqrt(c*x^2 + b*x + a))/(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4
)*x^2 + (b^3*c^2 - 4*a*b*c^3)*x), -((C*a*b^2 - 4*C*a^2*c + (C*b^2*c - 4*C*a
*c^2)*x^2 + (C*b^3 - 4*C*a*b*c)*x)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a
)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(C*a*b*c + A*b*c^2 + (C
*b^2*c - 2*C*a*c^2 + 2*A*c^3)*x)*sqrt(c*x^2 + b*x + a))/(a*b^2*c^2 - 4*a^2*
c^3 + (b^2*c^3 - 4*a*c^4)*x^2 + (b^3*c^2 - 4*a*b*c^3)*x)]
```

Sympy [F]

$$\int \frac{A+Cx^2}{(a+bx+cx^2)^{3/2}}\,dx=\int \frac{A+Cx^2}{(a+bx+cx^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((C*x**2+A)/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral((A + C*x**2)/(a + b*x + c*x**2)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Cx^2}{(a+bx+cx^2)^{3/2}}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.10

$$\int \frac{A+Cx^2}{(a+bx+cx^2)^{3/2}}\,dx=-\frac{2\left(\frac{(Cb^2-2\,Cac+2\,Ac^2)x}{b^2c-4\,ac^2}+\frac{Cab+Abc}{b^2c-4\,ac^2}\right)}{\sqrt{cx^2+bx+a}}-\frac{C\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}+b\right|\right)}{c^{\frac{3}{2}}}$$

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -2*((C*b^2 - 2*C*a*c + 2*A*c^2)*x/(b^2*c - 4*a*c^2) + (C*a*b + A*b*c)/(b^2*
c - 4*a*c^2))/sqrt(c*x^2 + b*x + a) - C*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))*sqrt(c) + b))/c^(3/2)
```

Mupad [B] (verification not implemented)

Time = 13.34 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.10

$$\int \frac{A+Cx^2}{\left(a+bx+cx^2\right)^{3/2}}\,dx = \frac{C\,\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x+a}\right)}{c^{3/2}} + \frac{A\,\left(\frac{b}{2}+c\,x\right)}{\left(a\,c-\frac{b^2}{4}\right)\,\sqrt{c\,x^2+b\,x+a}} + \frac{C\,\left(\frac{a\,b}{2}-x\,\left(a\,c-\frac{b^2}{2}\right)\right)}{c\,\left(a\,c-\frac{b^2}{4}\right)\,\sqrt{c\,x^2+b\,x+a}}$$

```
[In] int((A + C*x^2)/(a + b*x + c*x^2)^(3/2),x)
```

```
[Out] (C*log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2)))/c^(3/2) + (A*(b/2 +
c*x))/((a*c - b^2/4)*(a + b*x + c*x^2)^(1/2)) + (C*((a*b)/2 - x*(a*c - b^2/
2)))/(c*(a*c - b^2/4)*(a + b*x + c*x^2)^(1/2))
```

3.183 $\int \frac{A+Cx^2}{\left(a+bx+cx^2\right)^{5/2}} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 114

$$\int \frac{A+Cx^2}{\left(a+bx+cx^2\right)^{5/2}} \, dx = -\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x\right)}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}} + \frac{2\left(8Ac+4aC+\frac{b^2C}{c}\right)(b+2cx)}{3\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}}$$

```
[Out] -2/3*(b*c*(A+a*C/c)+(2*A*c^2+(-2*a*c+b^2)*C)*x)/c/(-4*a*c+b^2)/(c*x^2+b*x+a
)^(3/2)+2/3*(8*A*c+4*C*a+b^2*C/c)*(2*c*x+b)/(-4*a*c+b^2)^2/(c*x^2+b*x+a)^(1
/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1674, 12, 627}

$$\int \frac{A+Cx^2}{\left(a+bx+cx^2\right)^{5/2}} \, dx = \frac{2(b+2cx)\left(4aC+8Ac+\frac{b^2C}{c}\right)}{3\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}} - \frac{2\left(x\left(C\left(b^2-2ac\right)+2Ac^2\right)+bc\left(\frac{aC}{c}+A\right)\right)}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}$$

```
[In] Int[(A + C*x^2)/(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (-2*(b*c*(A + (a*C)/c) + (2*A*c^2 + (b^2 - 2*a*c)*C)*x))/(3*c*(b^2 - 4*a*c)
*(a + b*x + c*x^2)^(3/2)) + (2*(8*A*c + 4*a*C + (b^2*C)/c)*(b + 2*c*x))/(3*
(b^2 - 4*a*c)^2*Sqrt[a + b*x + c*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 627

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x\right)}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}-\frac{2\int\frac{8Ac+4aC+\frac{b^2C}{c}}{2\left(a+bx+cx^2\right)^{3/2}}\,dx}{3\left(b^2-4ac\right)}\\
&= -\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x\right)}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}-\frac{\left(8Ac+4aC+\frac{b^2C}{c}\right)\int\frac{1}{\left(a+bx+cx^2\right)^{3/2}}\,dx}{3\left(b^2-4ac\right)}\\
&= -\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x\right)}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}+\frac{2\left(8Ac+4aC+\frac{b^2C}{c}\right)(b+2cx)}{3\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.94

$$\int\frac{A+Cx^2}{\left(a+bx+cx^2\right)^{5/2}}\,dx=\frac{-2A(b+2cx)\left(b^2-8bcx-4c(3a+2cx^2)\right)+2C(8a^2b+b^2x^2(3b+2cx)+4ax(3b^2}{3\left(b^2-4ac\right)^2(a+x(b+cx))^{3/2}}$$

```
[In] Integrate[(A + C*x^2)/(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (-2*A*(b + 2*c*x)*(b^2 - 8*b*c*x - 4*c*(3*a + 2*c*x^2)) + 2*C*(8*a^2*b + b^
2*x^2*(3*b + 2*c*x) + 4*a*x*(3*b^2 + 3*b*c*x + 2*c^2*x^2)))/(3*(b^2 - 4*a*c
)^2*(a + x*(b + c*x))^(3/2))
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.12

method	result
trager	$\frac{\frac{32}{3}Ac^3x^3+\frac{16}{3}Cac^2x^3+\frac{4}{3}Cb^2cx^3+16Abc^2x^2+8Cabcx^2+2Cb^3x^2+16aAc^2x+4Ab^2cx+8Cab^2x+8Aabc-\frac{2}{3}Ab^3+\frac{16}{3}ba^2C}{(4ac-b^2)^2(cx^2+bx+a)^{\frac{3}{2}}}$
gosper	$\frac{\frac{32}{3}Ac^3x^3+\frac{16}{3}Cac^2x^3+\frac{4}{3}Cb^2cx^3+16Abc^2x^2+8Cabcx^2+2Cb^3x^2+16aAc^2x+4Ab^2cx+8Cab^2x+8Aabc-\frac{2}{3}Ab^3+\frac{16}{3}ba^2C}{(cx^2+bx+a)^{\frac{3}{2}}(16a^2c^2-8ab^2c+b^4)}$
default	$A\left(\frac{\frac{4cx}{3}+\frac{2b}{3}}{(4ac-b^2)(cx^2+bx+a)^{\frac{3}{2}}}+\frac{16c(2cx+b)}{3(4ac-b^2)^2\sqrt{cx^2+bx+a}}\right)+C\left(-\frac{x}{2c(cx^2+bx+a)^{\frac{3}{2}}}-\frac{b\left(-\frac{1}{3c\left(cx^2+bx+a\right)^{\frac{3}{2}}}-\frac{b\left(\frac{\frac{4c}{3}}{\left(4ac-b^2\right)}\right)}{}\right)}{}\right)$

```
[In] int((C*x^2+A)/(c*x^2+b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(16*A*c^3*x^3+8*C*a*c^2*x^3+2*C*b^2*c*x^3+24*A*b*c^2*x^2+12*C*a*b*c*x^2
+3*C*b^3*x^2+24*A*a*c^2*x+6*A*b^2*c*x+12*C*a*b^2*x+12*A*a*b*c-A*b^3+8*C*a^2
*b)/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 242 vs. 2(106) = 212.

Time = 0.67 (sec) , antiderivative size = 242, normalized size of antiderivative = 2.12

$$\int \frac{A+Cx^2}{(a+bx+cx^2)^{5/2}}\,dx = \frac{2\,(8\,Ca^2b-Ab^3+12\,Aabc+2\,(Cb^2c+4\,Cac^2+8\,Ac^3)x^3+3\,(Cb^3+4\,Ca}{3\,(a^2b^4-8\,a^3b^2c+16\,a^4c^2+(b^4c^2-8\,ab^2c^3+16\,a^2c^4)x^4+2\,(b^5c-8\,ab^3c^2+16\,a}$$

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(8*C*a^2*b - A*b^3 + 12*A*a*b*c + 2*(C*b^2*c + 4*C*a*c^2 + 8*A*c^3)*x^3
 + 3*(C*b^3 + 4*C*a*b*c + 8*A*b*c^2)*x^2 + 6*(2*C*a*b^2 + A*b^2*c + 4*A*a*c
^2)*x)*sqrt(c*x^2 + b*x + a)/(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^4*c^2
 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^4 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x
^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^2 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b
*c^2)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Cx^2}{(a+bx+cx^2)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((C*x**2+A)/(c*x**2+b*x+a)**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Cx^2}{(a+bx+cx^2)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.69

$$\int \frac{A+Cx^2}{(a+bx+cx^2)^{5/2}}\,dx = \frac{2\left(\left(\left(\frac{2\,(Cb^2c+4\,Cac^2+8\,Ac^3)x}{b^4-8\,ab^2c+16\,a^2c^2}+\frac{3\,(Cb^3+4\,Cabc+8\,Abc^2)}{b^4-8\,ab^2c+16\,a^2c^2}\right)x+\frac{6\,(2\,Cab^2+Ab^2c+4\,Aac^2)}{b^4-8\,ab^2c+16\,a^2c^2}\right)x+\frac{8\,Ca^2b}{b^4-8}\right.}{3\,(cx^2+bx+a)^{\frac{3}{2}}}$$

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*(((2*(C*b^2*c + 4*C*a*c^2 + 8*A*c^3)*x/(b^4 - 8*a*b^2*c + 16*a^2*c^2) +
 3*(C*b^3 + 4*C*a*b*c + 8*A*b*c^2)/(b^4 - 8*a*b^2*c + 16*a^2*c^2))*x + 6*(2
*C*a*b^2 + A*b^2*c + 4*A*a*c^2)/(b^4 - 8*a*b^2*c + 16*a^2*c^2))*x + (8*C*a^
2*b - A*b^3 + 12*A*a*b*c)/(b^4 - 8*a*b^2*c + 16*a^2*c^2))/(c*x^2 + b*x + a)
^(3/2)
```

Mupad [B] (verification not implemented)

Time = 13.39 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.11

$$\int \frac{A+Cx^2}{(a+bx+cx^2)^{5/2}}\,dx=\frac{2\,(8\,C\,a^2\,b+12\,C\,a\,b^2\,x+12\,C\,a\,b\,c\,x^2+12\,A\,a\,b\,c+8\,C\,a\,c^2\,x^3+24\,A\,a\,c^2\,x+3}{3\,(4\,a\,c-b^2)^2\,(c\,x^2+b\,x}$$

```
[In] int((A + C*x^2)/(a + b*x + c*x^2)^(5/2),x)
```

```
[Out] (2*(16*A*c^3*x^3 - A*b^3 + 3*C*b^3*x^2 + 8*C*a^2*b + 24*A*a*c^2*x + 6*A*b^2
*c*x + 12*C*a*b^2*x + 24*A*b*c^2*x^2 + 8*C*a*c^2*x^3 + 2*C*b^2*c*x^3 + 12*A
*a*b*c + 12*C*a*b*c*x^2))/(3*(4*a*c - b^2)^2*(a + b*x + c*x^2)^(3/2))
```

3.184 $\int \frac{A+Cx^2}{(a+bx+cx^2)^{7/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 167

$$\int \frac{A+Cx^2}{(a+bx+cx^2)^{7/2}}\,dx = -\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x\right)}{5c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{5/2}} + \frac{2\left(16Ac+4aC+\frac{3b^2C}{c}\right)(b+2cx)}{15\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^{3/2}} - \frac{16(16Ac^2+3b^2C+4acC)\,(b+2cx)}{15\left(b^2-4ac\right)^3\sqrt{a+bx+cx^2}}$$

```
[Out] -2/5*(b*c*(A+a*C/c)+(2*A*c^2+(-2*a*c+b^2)*C)*x)/c/(-4*a*c+b^2)/(c*x^2+b*x+a
)^(5/2)+2/15*(16*A*c+4*C*a+3*b^2*C/c)*(2*c*x+b)/(-4*a*c+b^2)^2/(c*x^2+b*x+a
)^(3/2)-16/15*(16*A*c^2+4*C*a*c+3*C*b^2)*(2*c*x+b)/(-4*a*c+b^2)^3/(c*x^2+b*
x+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1674, 12, 628, 627}

$$\int \frac{A+Cx^2}{(a+bx+cx^2)^{7/2}}\,dx = -\frac{16(b+2cx)\left(4acC+16Ac^2+3b^2C\right)}{15\left(b^2-4ac\right)^3\sqrt{a+bx+cx^2}} - \frac{2\left(x\left(C\left(b^2-2ac\right)+2Ac^2\right)+bc\left(\frac{aC}{c}+A\right)\right)}{5c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{5/2}} + \frac{2(b+2cx)\left(4aC+16Ac+\frac{3b^2C}{c}\right)}{15\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^{3/2}}$$

```
[In] Int[(A + C*x^2)/(a + b*x + c*x^2)^(7/2),x]
```

```
[Out] (-2*(b*c*(A + (a*C)/c) + (2*A*c^2 + (b^2 - 2*a*c)*C)*x))/(5*c*(b^2 - 4*a*c)
*(a + b*x + c*x^2)^(5/2)) + (2*(16*A*c + 4*a*C + (3*b^2*C)/c)*(b + 2*c*x))/
```

```
(15*(b^2 - 4*a*c)^2*(a + b*x + c*x^2)^(3/2)) - (16*(16*A*c^2 + 3*b^2*C + 4*
a*c*C)*(b + 2*c*x))/(15*(b^2 - 4*a*c)^3*Sqrt[a + b*x + c*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 627

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x\right)}{5c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{5/2}}-\frac{2\int\frac{16Ac+4aC+\frac{3b^2C}{c}}{2\left(a+bx+cx^2\right)^{5/2}}\,dx}{5\left(b^2-4ac\right)}\\
&= -\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x\right)}{5c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{5/2}}-\frac{\left(16Ac+4aC+\frac{3b^2C}{c}\right)\int\frac{1}{\left(a+bx+cx^2\right)^{5/2}}\,dx}{5\left(b^2-4ac\right)}\\
&= -\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x\right)}{5c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{5/2}}+\frac{2\left(16Ac+4aC+\frac{3b^2C}{c}\right)(b+2cx)}{15\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^{3/2}}\\
&\quad+\frac{\left(8\left(16Ac^2+3b^2C+4acC\right)\right)\int\frac{1}{\left(a+bx+cx^2\right)^{3/2}}\,dx}{15\left(b^2-4ac\right)^2}
\end{aligned}$$

$$=-\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x\right)}{5c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{5/2}}+\frac{2\left(16Ac+4aC+\frac{3b^2C}{c}\right)(b+2cx)}{15\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^{3/2}}-\frac{16(16Ac^2+3b^2C+4acC)(b+2cx)}{15\left(b^2-4ac\right)^3\sqrt{a+bx+cx^2}}$$

Mathematica [A] (verified)

Time = 2.52 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.40

$$\int\frac{A+Cx^2}{\left(a+bx+cx^2\right)^{7/2}}\,dx=$$

$$-\frac{2(A(b+2cx)\left(3b^4-16b^3cx+64bc^2x(5a+4cx^2)+8b^2c(-5a+14cx^2)+16c^2(15a^2+20acx^2+8c^2x^4)\right)+}{}$$

```
[In] Integrate[(A + C*x^2)/(a + b*x + c*x^2)^(7/2),x]
```

```
[Out] (-2*(A*(b + 2*c*x)*(3*b^4 - 16*b^3*c*x + 64*b*c^2*x*(5*a + 4*c*x^2) + 8*b^2
*c*(-5*a + 14*c*x^2) + 16*c^2*(15*a^2 + 20*a*c*x^2 + 8*c^2*x^4)) + C*(96*a^
3*b*c + 3*b^2*x^2*(5*b^3 + 30*b^2*c*x + 40*b*c^2*x^2 + 16*c^3*x^3) + 8*a^2*
(b^3 + 30*b^2*c*x + 30*b*c^2*x^2 + 20*c^3*x^3) + 4*a*x*(5*b^4 + 50*b^3*c*x
+ 60*b^2*c^2*x^2 + 40*b*c^3*x^3 + 16*c^4*x^4))))/(15*(b^2 - 4*a*c)^3*(a + x
*(b + c*x))^(5/2))
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.76

method	result
trager	$\frac{512}{15} A c^5 x^5 + 128 A a b c^3 x^2 + 32 C a^2 b c^2 x^2 + 32 A a^2 b c^2 - \frac{16}{3} A a b^3 c + \frac{2}{5} A b^5 + \frac{64}{5} C a^3 b c + 12 C b^4 c x^3 + \frac{32}{3} A b^3 c^2 x^2 + 64 A a^2 c^3 x - \frac{4}{3} A b^4 c x +$
gosper	$\frac{512}{15} A c^5 x^5 + 128 A a b c^3 x^2 + 32 C a^2 b c^2 x^2 + 32 A a^2 b c^2 - \frac{16}{3} A a b^3 c + \frac{2}{5} A b^5 + \frac{64}{5} C a^3 b c + 12 C b^4 c x^3 + \frac{32}{3} A b^3 c^2 x^2 + 64 A a^2 c^3 x - \frac{4}{3} A b^4 c x +$
default	$A \left(\frac{\frac{4cx}{5} + \frac{2b}{5}}{(4ac - b^2)(c x^2 + bx + a)^{\frac{5}{2}}} + \frac{16c \left(\frac{\frac{4cx}{3} + \frac{2b}{3}}{\left(4ac - b^2\right)\left(c x^2 + bx + a\right)^{\frac{3}{2}}} + \frac{16c(2cx + b)}{3\left(4ac - b^2\right)^2 \sqrt{c x^2 + bx + a}} \right)}{5(4ac - b^2)} \right) + C \left(- \frac{x}{4c(c x^2 + bx + a)^{\frac{5}{2}}} - 3b$

```
[In] int((C*x^2+A)/(c*x^2+b*x+a)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15*(256*A*c^5*x^5+64*C*a*c^4*x^5+48*C*b^2*c^3*x^5+640*A*b*c^4*x^4+160*C*a
*b*c^3*x^4+120*C*b^3*c^2*x^4+640*A*a*c^4*x^3+480*A*b^2*c^3*x^3+160*C*a^2*c^
3*x^3+240*C*a*b^2*c^2*x^3+90*C*b^4*c*x^3+960*A*a*b*c^3*x^2+80*A*b^3*c^2*x^2
+240*C*a^2*b*c^2*x^2+200*C*a*b^3*c*x^2+15*C*b^5*x^2+480*A*a^2*c^3*x+240*A*a
*b^2*c^2*x-10*A*b^4*c*x+240*C*a^2*b^2*c*x+20*C*a*b^4*x+240*A*a^2*b*c^2-40*A
*a*b^3*c+3*A*b^5+96*C*a^3*b*c+8*C*a^2*b^3)/(4*a*c-b^2)^3/(c*x^2+b*x+a)^(5/2
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 563 vs. $2(155) = 310$.

Time = 5.16 (sec) , antiderivative size = 563, normalized size of antiderivative = 3.37

$$\int \frac{A + Cx^2}{(a + bx + cx^2)^{7/2}}\, dx =$$

$$-\frac{2\,(8\,Ca^2b^3 + 3\,Ab^5 + 240\,Aa^2bc^2 + 16\,(3\,Cb^2c^3 + 4\,Cac^4 + 16\,Ac^5)x^5 + 40\,(3\,Cb^3c^2 + 4\,Cabc^3 + 16}{15\,(a^3b^6 - 12\,a^4b^4c + 48\,a^5b^2c^2 - 64\,a^6c^3 + (b^6c^3 - 12\,ab^4c^4 + 48\,a^2b^2c^5 - 64\,a^3c^6)x^6 + 3\,(b^7c^2 - 12\,ab^5c^3}$$

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^(7/2),x, algorithm="fricas")
```

```
[Out] -2/15*(8*C*a^2*b^3 + 3*A*b^5 + 240*A*a^2*b*c^2 + 16*(3*C*b^2*c^3 + 4*C*a*c^
4 + 16*A*c^5)*x^5 + 40*(3*C*b^3*c^2 + 4*C*a*b*c^3 + 16*A*b*c^4)*x^4 + 10*(9
*C*b^4*c + 24*C*a*b^2*c^2 + 64*A*a*c^4 + 16*(C*a^2 + 3*A*b^2)*c^3)*x^3 + 5*
(3*C*b^5 + 40*C*a*b^3*c + 192*A*a*b*c^3 + 16*(3*C*a^2*b + A*b^3)*c^2)*x^2 +
 8*(12*C*a^3*b - 5*A*a*b^3)*c + 10*(2*C*a*b^4 + 24*A*a*b^2*c^2 + 48*A*a^2*c
^3 + (24*C*a^2*b^2 - A*b^4)*c)*x)*sqrt(c*x^2 + b*x + a)/(a^3*b^6 - 12*a^4*b
^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3 + (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c
^5 - 64*a^3*c^6)*x^6 + 3*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*
b*c^5)*x^5 + 3*(b^8*c - 11*a*b^6*c^2 + 36*a^2*b^4*c^3 - 16*a^3*b^2*c^4 - 64
*a^4*c^5)*x^4 + (b^9 - 6*a*b^7*c - 24*a^2*b^5*c^2 + 224*a^3*b^3*c^3 - 384*a
^4*b*c^4)*x^3 + 3*(a*b^8 - 11*a^2*b^6*c + 36*a^3*b^4*c^2 - 16*a^4*b^2*c^3 -
 64*a^5*c^4)*x^2 + 3*(a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^
3)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Cx^2}{\left(a+bx+cx^2\right)^{7/2}}\,dx=\text{Timed out}$$

```
[In] integrate((C*x**2+A)/(c*x**2+b*x+a)**(7/2),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Cx^2}{\left(a+bx+cx^2\right)^{7/2}}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 452 vs. $2(155) = 310$.

Time = 0.29 (sec) , antiderivative size = 452, normalized size of antiderivative = 2.71

$$\int \frac{A+Cx^2}{(a+bx+cx^2)^{7/2}}\,dx =$$

$$-\frac{2\left(\left(\left(2\left(4\left(\frac{2\,(3\,Cb^2c^3+4\,Cac^4+16\,Ac^5)x}{b^6-12\,ab^4c+48\,a^2b^2c^2-64\,a^3c^3}+\frac{5\,(3\,Cb^3c^2+4\,Cabc^3+16\,Abc^4)}{b^6-12\,ab^4c+48\,a^2b^2c^2-64\,a^3c^3}\right)x+\frac{5\,(9\,Cb^4c+24\,Cab^2c^2+16\,Ca^2c^3+48\,Ab^2c^3+64\,Aac}{b^6-12\,ab^4c+48\,a^2b^2c^2-64\,a^3c^3}\right.\right.\right.\right.}{}$$

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^(7/2),x, algorithm="giac")
```

```
[Out] -2/15*((((2*(4*(2*(3*C*b^2*c^3 + 4*C*a*c^4 + 16*A*c^5)*x/(b^6 - 12*a*b^4*c +
 48*a^2*b^2*c^2 - 64*a^3*c^3) + 5*(3*C*b^3*c^2 + 4*C*a*b*c^3 + 16*A*b*c^4)/
(b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3))*x + 5*(9*C*b^4*c + 24*C*a
*b^2*c^2 + 16*C*a^2*c^3 + 48*A*b^2*c^3 + 64*A*a*c^4)/(b^6 - 12*a*b^4*c + 48
*a^2*b^2*c^2 - 64*a^3*c^3))*x + 5*(3*C*b^5 + 40*C*a*b^3*c + 48*C*a^2*b*c^2
+ 16*A*b^3*c^2 + 192*A*a*b*c^3)/(b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3
*c^3))*x + 10*(2*C*a*b^4 + 24*C*a^2*b^2*c - A*b^4*c + 24*A*a*b^2*c^2 + 48*A
*a^2*c^3)/(b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3))*x + (8*C*a^2*b^
3 + 3*A*b^5 + 96*C*a^3*b*c - 40*A*a*b^3*c + 240*A*a^2*b*c^2)/(b^6 - 12*a*b^
4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3))/(c*x^2 + b*x + a)^(5/2)
```

Mupad [B] (verification not implemented)

Time = 13.68 (sec) , antiderivative size = 578, normalized size of antiderivative = 3.46

$$\int \frac{A+Cx^2}{(a+bx+cx^2)^{7/2}}\,dx = \frac{\frac{b\,c\,(56\,C\,b^2+256\,A\,c^2+32\,C\,a\,c)}{15\,(4\,a\,c^2-b^2\,c)\,(4\,a\,c-b^2)^2}+\frac{2\,c^2\,x\,(56\,C\,b^2+256\,A\,c^2+32\,C\,a\,c)}{15\,(4\,a\,c^2-b^2\,c)\,(4\,a\,c-b^2)^2}}{\sqrt{c\,x^2+b\,x+a}}$$

$$+\frac{\frac{8\,C\,b\,c}{15\,(4\,a\,c^2-b^2\,c)\,(4\,a\,c-b^2)}+\frac{16\,C\,c^2\,x}{15\,(4\,a\,c^2-b^2\,c)\,(4\,a\,c-b^2)}}{\sqrt{c\,x^2+b\,x+a}}-\frac{\frac{4\,C\,x}{15\,(4\,a\,c-b^2)}-\frac{2\,C\,b}{15\,c\,(4\,a\,c-b^2)}}{(c\,x^2+b\,x+a)^{3/2}}$$

$$+\frac{x\left(\frac{4\,A\,c^2}{5\,(4\,a\,c^2-b^2\,c)}+\frac{2\,C\,b^2}{5\,(4\,a\,c^2-b^2\,c)}-\frac{4\,C\,a\,c}{5\,(4\,a\,c^2-b^2\,c)}\right)+\frac{2\,A\,b\,c}{5\,(4\,a\,c^2-b^2\,c)}+\frac{2\,C\,a\,b}{5\,(4\,a\,c^2-b^2\,c)}}{(c\,x^2+b\,x+a)^{5/2}}$$

$$+\frac{x\left(\frac{2\,c\,(8\,C\,b^2+32\,A\,c^2+8\,C\,a\,c)}{15\,(4\,a\,c^2-b^2\,c)\,(4\,a\,c-b^2)}+\frac{16\,C\,a\,c^2}{15\,(4\,a\,c^2-b^2\,c)\,(4\,a\,c-b^2)}-\frac{8\,C\,b^2\,c}{15\,(4\,a\,c^2-b^2\,c)\,(4\,a\,c-b^2)}\right)+\frac{b\,(8\,C\,b^2+32\,A\,c^2+8\,C\,a\,c)}{15\,(4\,a\,c^2-b^2\,c)\,(4\,a\,c-b^2)}-\frac{8\,C\,a}{15\,(4\,a\,c^2-b^2}}{(c\,x^2+b\,x+a)^{3/2}}$$

```
[In] int((A + C*x^2)/(a + b*x + c*x^2)^(7/2),x)
```

```
[Out] ((b*c*(256*A*c^2 + 56*C*b^2 + 32*C*a*c))/(15*(4*a*c^2 - b^2*c)*(4*a*c - b^2
)^2) + (2*c^2*x*(256*A*c^2 + 56*C*b^2 + 32*C*a*c))/(15*(4*a*c^2 - b^2*c)*(4
```

```
*a*c - b^2)^2))/(a + b*x + c*x^2)^(1/2) + ((8*C*b*c)/(15*(4*a*c^2 - b^2*c)*
(4*a*c - b^2)) + (16*C*c^2*x)/(15*(4*a*c^2 - b^2*c)*(4*a*c - b^2)))/(a + b*
x + c*x^2)^(1/2) - ((4*C*x)/(15*(4*a*c - b^2)) - (2*C*b)/(15*c*(4*a*c - b^2
)))/(a + b*x + c*x^2)^(3/2) + (x*((4*A*c^2)/(5*(4*a*c^2 - b^2*c)) + (2*C*b^
2)/(5*(4*a*c^2 - b^2*c)) - (4*C*a*c)/(5*(4*a*c^2 - b^2*c))) + (2*A*b*c)/(5*
(4*a*c^2 - b^2*c)) + (2*C*a*b)/(5*(4*a*c^2 - b^2*c)))/(a + b*x + c*x^2)^(5/
2) + (x*((2*c*(32*A*c^2 + 8*C*b^2 + 8*C*a*c))/(15*(4*a*c^2 - b^2*c)*(4*a*c
- b^2)) + (16*C*a*c^2)/(15*(4*a*c^2 - b^2*c)*(4*a*c - b^2)) - (8*C*b^2*c)/(
15*(4*a*c^2 - b^2*c)*(4*a*c - b^2))) + (b*(32*A*c^2 + 8*C*b^2 + 8*C*a*c))/(
15*(4*a*c^2 - b^2*c)*(4*a*c - b^2)) - (8*C*a*b*c)/(15*(4*a*c^2 - b^2*c)*(4*
a*c - b^2)))/(a + b*x + c*x^2)^(3/2)
```

3.185 $\int \frac{A+Cx^2}{(a+bx+cx^2)^{9/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 220

$$\begin{aligned}\int \frac{A+Cx^2}{(a+bx+cx^2)^{9/2}}\,dx = &-\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x\right)}{7c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{7/2}}\\ &+\frac{2\left(24Ac+4aC+\frac{5b^2C}{c}\right)(b+2cx)}{35\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^{5/2}}-\frac{32(24Ac^2+5b^2C+4acC)\,(b+2cx)}{105\left(b^2-4ac\right)^3\left(a+bx+cx^2\right)^{3/2}}\\ &+\frac{256c(24Ac^2+5b^2C+4acC)\,(b+2cx)}{105\left(b^2-4ac\right)^4\sqrt{a+bx+cx^2}}\end{aligned}$$

```
[Out] -2/7*(b*c*(A+a*C/c)+(2*A*c^2+(-2*a*c+b^2)*C)*x)/c/(-4*a*c+b^2)/(c*x^2+b*x+a
)^(7/2)+2/35*(24*A*c+4*C*a+5*b^2*C/c)*(2*c*x+b)/(-4*a*c+b^2)^2/(c*x^2+b*x+a
)^(5/2)-32/105*(24*A*c^2+4*C*a*c+5*C*b^2)*(2*c*x+b)/(-4*a*c+b^2)^3/(c*x^2+b
*x+a)^(3/2)+256/105*c*(24*A*c^2+4*C*a*c+5*C*b^2)*(2*c*x+b)/(-4*a*c+b^2)^4/(
c*x^2+b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1674, 12, 628, 627}

$$\begin{aligned}\int \frac{A+Cx^2}{(a+bx+cx^2)^{9/2}}\,dx = &\frac{256c(b+2cx)\left(4acC+24Ac^2+5b^2C\right)}{105\left(b^2-4ac\right)^4\sqrt{a+bx+cx^2}}\\ &-\frac{32(b+2cx)\left(4acC+24Ac^2+5b^2C\right)}{105\left(b^2-4ac\right)^3\left(a+bx+cx^2\right)^{3/2}}\\ &-\frac{2\left(x\left(C\left(b^2-2ac\right)+2Ac^2\right)+bc\left(\frac{aC}{c}+A\right)\right)}{7c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{7/2}}+\frac{2(b+2cx)\left(4aC+24Ac+\frac{5b^2C}{c}\right)}{35\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^{5/2}}\end{aligned}$$

```
[In] Int[(A + C*x^2)/(a + b*x + c*x^2)^(9/2),x]
```

```
[Out] (-2*(b*c*(A + (a*C)/c) + (2*A*c^2 + (b^2 - 2*a*c)*C)*x))/(7*c*(b^2 - 4*a*c)
*(a + b*x + c*x^2)^(7/2)) + (2*(24*A*c + 4*a*C + (5*b^2*C)/c)*(b + 2*c*x))/
(35*(b^2 - 4*a*c)^2*(a + b*x + c*x^2)^(5/2)) - (32*(24*A*c^2 + 5*b^2*C + 4*
a*c*C)*(b + 2*c*x))/(105*(b^2 - 4*a*c)^3*(a + b*x + c*x^2)^(3/2)) + (256*c*
(24*A*c^2 + 5*b^2*C + 4*a*c*C)*(b + 2*c*x))/(105*(b^2 - 4*a*c)^4*Sqrt[a + b
*x + c*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 627

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\text{integral} = -\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x\right)}{7c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{7/2}}-\frac{2\int\frac{24Ac+4aC+\frac{5b^2C}{c}}{2\left(a+bx+cx^2\right)^{7/2}}\,dx}{7\left(b^2-4ac\right)}$$

$$=-\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x\right)}{7c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{7/2}}-\frac{\left(24Ac+4aC+\frac{5b^2C}{c}\right)\int\frac{1}{\left(a+bx+cx^2\right)^{7/2}}\,dx}{7\left(b^2-4ac\right)}$$

$$=-\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x\right)}{7c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{7/2}}+\frac{2\left(24Ac+4aC+\frac{5b^2C}{c}\right)(b+2cx)}{35\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^{5/2}}+\frac{\left(16(24Ac^2+5b^2C+4acC)\right)\int\frac{1}{(a+bx+cx^2)^{5/2}}\,dx}{35\left(b^2-4ac\right)^2}$$

$$=-\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x\right)}{7c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{7/2}}+\frac{2\left(24Ac+4aC+\frac{5b^2C}{c}\right)(b+2cx)}{35\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^{5/2}}-\frac{32(24Ac^2+5b^2C+4acC)(b+2cx)}{105\left(b^2-4ac\right)^3\left(a+bx+cx^2\right)^{3/2}}-\frac{\left(128c(24Ac^2+5b^2C+4acC)\right)\int\frac{1}{(a+bx+cx^2)^{3/2}}\,dx}{105\left(b^2-4ac\right)^3}$$

$$=-\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x\right)}{7c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{7/2}}+\frac{2\left(24Ac+4aC+\frac{5b^2C}{c}\right)(b+2cx)}{35\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^{5/2}}-\frac{32(24Ac^2+5b^2C+4acC)(b+2cx)}{105\left(b^2-4ac\right)^3\left(a+bx+cx^2\right)^{3/2}}+\frac{256c(24Ac^2+5b^2C+4acC)(b+2cx)}{105\left(b^2-4ac\right)^4\sqrt{a+bx+cx^2}}$$

Mathematica [A] (verified)

Time = 5.87 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.86

$$\int\frac{A+Cx^2}{\left(a+bx+cx^2\right)^{9/2}}\,dx=\frac{-6A(b+2cx)\left(5b^6-24b^5cx+64b^3c^2x(7a-12cx^2)+4b^4c(-21a+26cx^2)-128\right.}{}$$

```
[In] Integrate[(A + C*x^2)/(a + b*x + c*x^2)^(9/2),x]
```

```
[Out] (-6*A*(b + 2*c*x)*(5*b^6 - 24*b^5*c*x + 64*b^3*c^2*x*(7*a - 12*c*x^2) + 4*b
^4*c*(-21*a + 26*c*x^2) - 128*b*c^3*x*(35*a^2 + 56*a*c*x^2 + 24*c^2*x^4) -
16*b^2*c^2*(-35*a^2 + 196*a*c*x^2 + 184*c^2*x^4) - 64*c^3*(35*a^3 + 70*a^2*
c*x^2 + 56*a*c^2*x^4 + 16*c^3*x^6)) + 2*C*(1920*a^4*b*c^2 + 320*a^3*c*(b^3
+ 21*b^2*c*x + 21*b*c^2*x^2 + 14*c^3*x^3) + 5*b^2*x^2*(-7*b^5 + 70*b^4*c*x
+ 560*b^3*c^2*x^2 + 1120*b^2*c^3*x^3 + 896*b*c^4*x^4 + 256*c^5*x^5) + 8*a^2
*(-b^5 + 140*b^4*c*x + 1190*b^3*c^2*x^2 + 1540*b^2*c^3*x^3 + 1120*b*c^4*x^4
 + 448*c^5*x^5) + 4*a*x*(-7*b^6 + 343*b^5*c*x + 2170*b^4*c^2*x^2 + 3360*b^3
*c^3*x^3 + 2240*b^2*c^4*x^4 + 896*b*c^5*x^5 + 256*c^6*x^6)))/(105*(b^2 - 4*
a*c)^4*(a + x*(b + c*x))^(7/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 523 vs. 2(204) = 408.

Time = 0.76 (sec) , antiderivative size = 524, normalized size of antiderivative = 2.38

method	result
trager	$\frac{-\frac{2}{7}A\,b^{7}+\frac{128}{21}C\,a^{3}b^{3}c+\frac{256}{7}C\,a^{4}b\,c^{2}+\frac{4}{5}A\,b^{6}cx-\frac{8}{15}Ca\,b^{6}x+256A\,a^{3}c^{4}x+\frac{160}{3}C\,b^{5}c^{2}x^{4}+512A\,a^{2}c^{5}x^{3}+32A\,b^{4}c^{3}x^{3}+\frac{256}{3}C\,a^{3}c^{4}x^{3}+\frac{20}{3}C}{}$
default	$A\left(\frac{\frac{4cx}{7}+\frac{2b}{7}}{(4ac-b^{2})(c\,x^{2}+bx+a)^{\frac{7}{2}}}+\frac{24c\left(\frac{\frac{4cx}{5}+\frac{2b}{5}}{\left(4ac-b^{2}\right)\left(c\,x^{2}+bx+a\right)^{\frac{5}{2}}}+\frac{16c\left(\frac{\frac{4cx}{3}+\frac{2b}{3}}{\left(4ac-b^{2}\right)\left(c\,x^{2}+bx+a\right)^{\frac{3}{2}}}+\frac{16c(2cx+b)}{3\left(4ac-b^{2}\right)^{2}\sqrt{c\,x^{2}+bx+a}}\right)}{5\left(4ac-b^{2}\right)}\right)}{7(4ac-b^{2})}\right)+C$
gosper	$\frac{-\frac{2}{7}A\,b^{7}+\frac{128}{21}C\,a^{3}b^{3}c+\frac{256}{7}C\,a^{4}b\,c^{2}+\frac{4}{5}A\,b^{6}cx-\frac{8}{15}Ca\,b^{6}x+256A\,a^{3}c^{4}x+\frac{160}{3}C\,b^{5}c^{2}x^{4}+512A\,a^{2}c^{5}x^{3}+32A\,b^{4}c^{3}x^{3}+\frac{256}{3}C\,a^{3}c^{4}x^{3}+\frac{20}{3}C}{}$

```
[In] int((C*x^2+A)/(c*x^2+b*x+a)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/105*(6144*A*c^7*x^7+1024*C*a*c^6*x^7+1280*C*b^2*c^5*x^7+21504*A*b*c^6*x^6
+3584*C*a*b*c^5*x^6+4480*C*b^3*c^4*x^6+21504*A*a*c^6*x^5+26880*A*b^2*c^5*x^
5+3584*C*a^2*c^5*x^5+8960*C*a*b^2*c^4*x^5+5600*C*b^4*c^3*x^5+53760*A*a*b*c^
5*x^4+13440*A*b^3*c^4*x^4+8960*C*a^2*b*c^4*x^4+13440*C*a*b^3*c^3*x^4+2800*C
*b^5*c^2*x^4+26880*A*a^2*c^5*x^3+40320*A*a*b^2*c^4*x^3+1680*A*b^4*c^3*x^3+4
480*C*a^3*c^4*x^3+12320*C*a^2*b^2*c^3*x^3+8680*C*a*b^4*c^2*x^3+350*C*b^6*c*
x^3+40320*A*a^2*b*c^4*x^2+6720*A*a*b^3*c^3*x^2-168*A*b^5*c^2*x^2+6720*C*a^3
*b*c^3*x^2+9520*C*a^2*b^3*c^2*x^2+1372*C*a*b^5*c*x^2-35*C*b^7*x^2+13440*A*a
^3*c^4*x+10080*A*a^2*b^2*c^3*x-840*A*a*b^4*c^2*x+42*A*b^6*c*x+6720*C*a^3*b^
2*c^2*x+1120*C*a^2*b^4*c*x-28*C*a*b^6*x+6720*A*a^3*b*c^3-1680*A*a^2*b^3*c^2
+252*A*a*b^5*c-15*A*b^7+1920*C*a^4*b*c^2+320*C*a^3*b^3*c-8*C*a^2*b^5)/(4*a*
c-b^2)^4/(c*x^2+b*x+a)^(7/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 978 vs. $2(204) = 408$.

Time = 14.68 (sec) , antiderivative size = 978, normalized size of antiderivative = 4.45

$$\int \frac{A+Cx^2}{(a+bx+cx^2)^{9/2}}\,dx =$$

$$-\frac{2\left(8\,Ca^2b^5+15\,Ab^7-6720\,Aa^3bc^3-256\,(5\,Cb^2c^5+4\,Cac^6+24\,Ac^7)x^7-896\,(5\,Cb^3c^4+4\right.}{105\left(a^4b^8-16\,a^5b^6c+96\,a^6b^4c^2-256\,a^7b^2c^3+256\,a^8c^4+(b^8c^4-16\,ab^6c^5+96\,a^2b^4c^6-256\,a^3b^2c^7+256\right.}$$

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^(9/2),x, algorithm="fricas")
```

```
[Out] -2/105*(8*C*a^2*b^5 + 15*A*b^7 - 6720*A*a^3*b*c^3 - 256*(5*C*b^2*c^5 + 4*C*
a*c^6 + 24*A*c^7)*x^7 - 896*(5*C*b^3*c^4 + 4*C*a*b*c^5 + 24*A*b*c^6)*x^6 -
224*(25*C*b^4*c^3 + 40*C*a*b^2*c^4 + 96*A*a*c^6 + 8*(2*C*a^2 + 15*A*b^2)*c^
5)*x^5 - 560*(5*C*b^5*c^2 + 24*C*a*b^3*c^3 + 96*A*a*b*c^5 + 8*(2*C*a^2*b +
3*A*b^3)*c^4)*x^4 - 70*(5*C*b^6*c + 124*C*a*b^4*c^2 + 384*A*a^2*c^5 + 64*(C
*a^3 + 9*A*a*b^2)*c^4 + 8*(22*C*a^2*b^2 + 3*A*b^4)*c^3)*x^3 - 240*(8*C*a^4*
b - 7*A*a^2*b^3)*c^2 + 7*(5*C*b^7 - 196*C*a*b^5*c - 5760*A*a^2*b*c^4 - 960*
(C*a^3*b + A*a*b^3)*c^3 - 8*(170*C*a^2*b^3 - 3*A*b^5)*c^2)*x^2 - 4*(80*C*a^
3*b^3 + 63*A*a*b^5)*c + 14*(2*C*a*b^6 - 720*A*a^2*b^2*c^3 - 960*A*a^3*c^4 -
 60*(8*C*a^3*b^2 - A*a*b^4)*c^2 - (80*C*a^2*b^4 + 3*A*b^6)*c)*x)*sqrt(c*x^2
 + b*x + a)/(a^4*b^8 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3 + 25
6*a^8*c^4 + (b^8*c^4 - 16*a*b^6*c^5 + 96*a^2*b^4*c^6 - 256*a^3*b^2*c^7 + 25
6*a^4*c^8)*x^8 + 4*(b^9*c^3 - 16*a*b^7*c^4 + 96*a^2*b^5*c^5 - 256*a^3*b^3*c
^6 + 256*a^4*b*c^7)*x^7 + 2*(3*b^10*c^2 - 46*a*b^8*c^3 + 256*a^2*b^6*c^4 -
576*a^3*b^4*c^5 + 256*a^4*b^2*c^6 + 512*a^5*c^7)*x^6 + 4*(b^11*c - 13*a*b^9
*c^2 + 48*a^2*b^7*c^3 + 32*a^3*b^5*c^4 - 512*a^4*b^3*c^5 + 768*a^5*b*c^6)*x
^5 + (b^12 - 4*a*b^10*c - 90*a^2*b^8*c^2 + 800*a^3*b^6*c^3 - 2240*a^4*b^4*c
^4 + 1536*a^5*b^2*c^5 + 1536*a^6*c^6)*x^4 + 4*(a*b^11 - 13*a^2*b^9*c + 48*a
^3*b^7*c^2 + 32*a^4*b^5*c^3 - 512*a^5*b^3*c^4 + 768*a^6*b*c^5)*x^3 + 2*(3*a
```

```
^2*b^10 - 46*a^3*b^8*c + 256*a^4*b^6*c^2 - 576*a^5*b^4*c^3 + 256*a^6*b^2*c^
4 + 512*a^7*c^5)*x^2 + 4*(a^3*b^9 - 16*a^4*b^7*c + 96*a^5*b^5*c^2 - 256*a^6
*b^3*c^3 + 256*a^7*b*c^4)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Cx^2}{(a+bx+cx^2)^{9/2}}\,dx = \text{Timed out}$$

```
[In] integrate((C*x**2+A)/(c*x**2+b*x+a)**(9/2),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Cx^2}{(a+bx+cx^2)^{9/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^(9/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 805 vs. $2(204) = 408$.

Time = 0.29 (sec) , antiderivative size = 805, normalized size of antiderivative = 3.66

$$\int \frac{A+Cx^2}{(a+bx+cx^2)^{9/2}}\,dx = \frac{2 \left(\left(\left(2 \left(8 \left(2 \left(4 \left(\frac{2\,(5\,Cb^2c^5+4\,Cac^6+24\,Ac^7)\,x}{b^8-16\,ab^6c+96\,a^2b^4c^2-256\,a^3b^2c^3+256\,a^4c^4} + \frac{7\,(5\,Cb^3c^4+4\,Cabc^5+24\,Abc^6)}{b^8-16\,ab^6c+96\,a^2b^4c^2-256\,a^3b^2c^3+25} \right.\right.\right.\right.\right.\right.\right.}{}$$

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^(9/2),x, algorithm="giac")
```

```
[Out] 2/105*(((2*(8*(2*(4*(2*(5*C*b^2*c^5 + 4*C*a*c^6 + 24*A*c^7)*x/(b^8 - 16*a*b
^6*c + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 + 256*a^4*c^4) + 7*(5*C*b^3*c^4 + 4
*C*a*b*c^5 + 24*A*b*c^6)/(b^8 - 16*a*b^6*c + 96*a^2*b^4*c^2 - 256*a^3*b^2*c
^3 + 256*a^4*c^4))*x + 7*(25*C*b^4*c^3 + 40*C*a*b^2*c^4 + 16*C*a^2*c^5 + 12
0*A*b^2*c^5 + 96*A*a*c^6)/(b^8 - 16*a*b^6*c + 96*a^2*b^4*c^2 - 256*a^3*b^2*
```

```
c^3 + 256*a^4*c^4))*x + 35*(5*C*b^5*c^2 + 24*C*a*b^3*c^3 + 16*C*a^2*b*c^4 +
 24*A*b^3*c^4 + 96*A*a*b*c^5)/(b^8 - 16*a*b^6*c + 96*a^2*b^4*c^2 - 256*a^3*
b^2*c^3 + 256*a^4*c^4))*x + 35*(5*C*b^6*c + 124*C*a*b^4*c^2 + 176*C*a^2*b^2
*c^3 + 24*A*b^4*c^3 + 64*C*a^3*c^4 + 576*A*a*b^2*c^4 + 384*A*a^2*c^5)/(b^8
- 16*a*b^6*c + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 + 256*a^4*c^4))*x - 7*(5*C*
b^7 - 196*C*a*b^5*c - 1360*C*a^2*b^3*c^2 + 24*A*b^5*c^2 - 960*C*a^3*b*c^3 -
 960*A*a*b^3*c^3 - 5760*A*a^2*b*c^4)/(b^8 - 16*a*b^6*c + 96*a^2*b^4*c^2 - 2
56*a^3*b^2*c^3 + 256*a^4*c^4))*x - 14*(2*C*a*b^6 - 80*C*a^2*b^4*c - 3*A*b^6
*c - 480*C*a^3*b^2*c^2 + 60*A*a*b^4*c^2 - 720*A*a^2*b^2*c^3 - 960*A*a^3*c^4
)/(b^8 - 16*a*b^6*c + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 + 256*a^4*c^4))*x -
(8*C*a^2*b^5 + 15*A*b^7 - 320*C*a^3*b^3*c - 252*A*a*b^5*c - 1920*C*a^4*b*c^
2 + 1680*A*a^2*b^3*c^2 - 6720*A*a^3*b*c^3)/(b^8 - 16*a*b^6*c + 96*a^2*b^4*c
^2 - 256*a^3*b^2*c^3 + 256*a^4*c^4))/(c*x^2 + b*x + a)^(7/2)
```

Mupad [B] (verification not implemented)

Time = 14.48 (sec) , antiderivative size = 1018, normalized size of antiderivative = 4.63

$$\int \frac{A+Cx^2}{(a+bx+cx^2)^{9/2}}\,dx=\frac{x\left(\frac{2\,c^2\,\left(160\,C\,b^2+768\,A\,c^2+96\,C\,a\,c\right)}{105\,\left(4\,a\,c^2-b^2\,c\right)\,{\left(4\,a\,c-b^2\right)}^2}-\frac{64\,C\,a\,c^3}{105\,\left(4\,a\,c^2-b^2\,c\right)\,{\left(4\,a\,c-b^2\right)}^2}+\frac{32\,C\,b^2\,c^2}{105\,\left(4\,a\,c^2-b^2\,c\right)\,{\left(4\,a\,c-b^2\right)}^2}\right)+}{{\left(c\,x^2+b\,x+a\right)}^{3/2}}$$

$$-\frac{\frac{8\,C\,b}{105\,{\left(4\,a\,c-b^2\right)}^2}-\frac{16\,C\,c\,x}{105\,{\left(4\,a\,c-b^2\right)}^2}}{{\left(c\,x^2+b\,x+a\right)}^{3/2}}+\frac{\frac{8\,C\,b\,c}{105\,\left(4\,a\,c^2-b^2\,c\right)\,\left(4\,a\,c-b^2\right)}+\frac{16\,C\,c^2\,x}{105\,\left(4\,a\,c^2-b^2\,c\right)\,\left(4\,a\,c-b^2\right)}}{{\left(c\,x^2+b\,x+a\right)}^{3/2}}$$

$$-\frac{\frac{4\,C\,x}{35\,\left(4\,a\,c-b^2\right)}-\frac{2\,C\,b}{35\,c\,\left(4\,a\,c-b^2\right)}}{{\left(c\,x^2+b\,x+a\right)}^{5/2}}+\frac{\frac{b\,c\,\left(1312\,C\,b^2\,c+6144\,A\,c^3+896\,C\,a\,c^2\right)}{105\,\left(4\,a\,c^2-b^2\,c\right)\,{\left(4\,a\,c-b^2\right)}^3}+\frac{2\,c^2\,x\,\left(1312\,C\,b^2\,c+6144\,A\,c^3+896\,C\,a\,c^2\right)}{105\,\left(4\,a\,c^2-b^2\,c\right)\,{\left(4\,a\,c-b^2\right)}^3}}{\sqrt{c\,x^2+b\,x+a}}$$

$$+\frac{x\left(\frac{4\,A\,c^2}{7\,\left(4\,a\,c^2-b^2\,c\right)}+\frac{2\,C\,b^2}{7\,\left(4\,a\,c^2-b^2\,c\right)}-\frac{4\,C\,a\,c}{7\,\left(4\,a\,c^2-b^2\,c\right)}\right)+\frac{2\,A\,b\,c}{7\,\left(4\,a\,c^2-b^2\,c\right)}+\frac{2\,C\,a\,b}{7\,\left(4\,a\,c^2-b^2\,c\right)}}{{\left(c\,x^2+b\,x+a\right)}^{7/2}}$$

$$+\frac{x\left(\frac{2\,c\,\left(12\,C\,b^2+48\,A\,c^2+8\,C\,a\,c\right)}{35\,\left(4\,a\,c^2-b^2\,c\right)\,\left(4\,a\,c-b^2\right)}+\frac{16\,C\,a\,c^2}{35\,\left(4\,a\,c^2-b^2\,c\right)\,\left(4\,a\,c-b^2\right)}-\frac{8\,C\,b^2\,c}{35\,\left(4\,a\,c^2-b^2\,c\right)\,\left(4\,a\,c-b^2\right)}\right)+\frac{b\,\left(12\,C\,b^2+48\,A\,c^2+8\,C\,a\,c\right)}{35\,\left(4\,a\,c^2-b^2\,c\right)\,\left(4\,a\,c-b^2\right)}-}{{\left(c\,x^2+b\,x+a\right)}^{5/2}}$$

$$-\frac{\frac{32\,C\,b\,c^2}{105\,\left(4\,a\,c^2-b^2\,c\right)\,{\left(4\,a\,c-b^2\right)}^2}+\frac{64\,C\,c^3\,x}{105\,\left(4\,a\,c^2-b^2\,c\right)\,{\left(4\,a\,c-b^2\right)}^2}}{\sqrt{c\,x^2+b\,x+a}}$$

$$+\frac{\frac{64\,C\,b\,c^2}{105\,\left(4\,a\,c^2-b^2\,c\right)\,{\left(4\,a\,c-b^2\right)}^2}+\frac{128\,C\,c^3\,x}{105\,\left(4\,a\,c^2-b^2\,c\right)\,{\left(4\,a\,c-b^2\right)}^2}}{\sqrt{c\,x^2+b\,x+a}}$$

```
[In] int((A + C*x^2)/(a + b*x + c*x^2)^(9/2),x)
```

```
[Out] (x*((2*c^2*(768*A*c^2 + 160*C*b^2 + 96*C*a*c))/(105*(4*a*c^2 - b^2*c)*(4*a*
c - b^2)^2) - (64*C*a*c^3)/(105*(4*a*c^2 - b^2*c)*(4*a*c - b^2)^2) + (32*C*
b^2*c^2)/(105*(4*a*c^2 - b^2*c)*(4*a*c - b^2)^2)) + (b*c*(768*A*c^2 + 160*C
*b^2 + 96*C*a*c))/(105*(4*a*c^2 - b^2*c)*(4*a*c - b^2)^2) + (32*C*a*b*c^2)/
```

```
(105*(4*a*c^2 - b^2*c)*(4*a*c - b^2)^2))/(a + b*x + c*x^2)^(3/2) - ((8*C*b)
/(105*(4*a*c - b^2)^2) - (16*C*c*x)/(105*(4*a*c - b^2)^2))/(a + b*x + c*x^2
)^(3/2) + ((8*C*b*c)/(105*(4*a*c^2 - b^2*c)*(4*a*c - b^2)) + (16*C*c^2*x)/(
105*(4*a*c^2 - b^2*c)*(4*a*c - b^2)))/(a + b*x + c*x^2)^(3/2) - ((4*C*x)/(3
5*(4*a*c - b^2)) - (2*C*b)/(35*c*(4*a*c - b^2)))/(a + b*x + c*x^2)^(5/2) +
((b*c*(6144*A*c^3 + 896*C*a*c^2 + 1312*C*b^2*c))/(105*(4*a*c^2 - b^2*c)*(4*
a*c - b^2)^3) + (2*c^2*x*(6144*A*c^3 + 896*C*a*c^2 + 1312*C*b^2*c))/(105*(4
*a*c^2 - b^2*c)*(4*a*c - b^2)^3))/(a + b*x + c*x^2)^(1/2) + (x*((4*A*c^2)/(
7*(4*a*c^2 - b^2*c)) + (2*C*b^2)/(7*(4*a*c^2 - b^2*c)) - (4*C*a*c)/(7*(4*a*
c^2 - b^2*c))) + (2*A*b*c)/(7*(4*a*c^2 - b^2*c)) + (2*C*a*b)/(7*(4*a*c^2 -
b^2*c)))/(a + b*x + c*x^2)^(7/2) + (x*((2*c*(48*A*c^2 + 12*C*b^2 + 8*C*a*c)
)/(35*(4*a*c^2 - b^2*c)*(4*a*c - b^2)) + (16*C*a*c^2)/(35*(4*a*c^2 - b^2*c)
*(4*a*c - b^2)) - (8*C*b^2*c)/(35*(4*a*c^2 - b^2*c)*(4*a*c - b^2))) + (b*(4
8*A*c^2 + 12*C*b^2 + 8*C*a*c))/(35*(4*a*c^2 - b^2*c)*(4*a*c - b^2)) - (8*C*
a*b*c)/(35*(4*a*c^2 - b^2*c)*(4*a*c - b^2)))/(a + b*x + c*x^2)^(5/2) - ((32
*C*b*c^2)/(105*(4*a*c^2 - b^2*c)*(4*a*c - b^2)^2) + (64*C*c^3*x)/(105*(4*a*
c^2 - b^2*c)*(4*a*c - b^2)^2))/(a + b*x + c*x^2)^(1/2) + ((64*C*b*c^2)/(105
*(4*a*c^2 - b^2*c)*(4*a*c - b^2)^2) + (128*C*c^3*x)/(105*(4*a*c^2 - b^2*c)*
(4*a*c - b^2)^2))/(a + b*x + c*x^2)^(1/2)
```

3.186 $\int (g+hx)^3\sqrt{a+bx+cx^2}(d+ex+fx^2)\,dx$

Optimal result

Integrand size = 32, antiderivative size = 930

$$\int (g+hx)^3\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx$$

$$= \frac{(256c^5dg^3-33b^5fh^3+6b^3ch^2(20afh+7b(3fg+eh))-8bc^2h(10a^2fh^2+14abh(3fg+eh)+7b^2(3fg^2 -}{}$$

$$+\frac{(33b^2fh^2-2ch(8bfg+21beh+16afh)-4c^2(3fg^2-7h(eg+2dh)))\,(g+hx)^2\left(a+bx+cx^2\right)^{3/2}}{280c^3h}$$

$$-\frac{(6cfg-14ceh+11bfh)(g+hx)^3\left(a+bx+cx^2\right)^{3/2}}{84c^2h}+\frac{f(g+hx)^4\left(a+bx+cx^2\right)^{3/2}}{7ch}$$

$$+\frac{(1155b^4fh^4-128c^4g^2(3fg^2-7h(eg+12dh))-42b^2ch^3(78afh+35b(3fg+eh))+8c^2h^2(128a^2fh^2 +}{}$$

$$-\frac{(b^2-4ac)\,(256c^5dg^3-33b^5fh^3+6b^3ch^2(20afh+7b(3fg+eh))-8bc^2h(10a^2fh^2+14abh(3fg+eh)}{}$$

```
[Out] 1/280*(33*b^2*f*h^2-2*c*h*(16*a*f*h+21*b*e*h+8*b*f*g)-4*c^2*(3*f*g^2-7*h*(2
*d*h+e*g)))*(h*x+g)^2*(c*x^2+b*x+a)^(3/2)/c^3/h-1/84*(11*b*f*h-14*c*e*h+6*c
*f*g)*(h*x+g)^3*(c*x^2+b*x+a)^(3/2)/c^2/h+1/7*f*(h*x+g)^4*(c*x^2+b*x+a)^(3/
2)/c/h+1/13440*(1155*b^4*f*h^4-128*c^4*g^2*(3*f*g^2-7*h*(12*d*h+e*g))-42*b^
2*c*h^3*(78*a*f*h+35*b*(e*h+3*f*g))+8*c^2*h^2*(128*a^2*f*h^2+343*a*b*h*(e*h
+3*f*g)+b^2*(537*f*g^2+245*h*(d*h+3*e*g)))-16*c^3*h*(16*a*h*(15*f*g^2+7*h*(
d*h+3*e*g))+b*g*(17*f*g^2+21*h*(25*d*h+19*e*g)))-6*c*h*(231*b^3*f*h^3-6*b*c
*h^2*(74*a*f*h+49*b*e*h+59*b*f*g)+16*c^3*g*(3*f*g^2-7*h*(7*d*h+e*g))+8*c^2*
h*(a*h*(35*e*h+41*f*g)+b*(5*f*g^2+7*h*(7*d*h+9*e*g))))*x)*(c*x^2+b*x+a)^(3/
2)/c^5/h-1/2048*(-4*a*c+b^2)*(256*c^5*d*g^3-33*b^5*f*h^3+6*b^3*c*h^2*(20*a*
f*h+7*b*(e*h+3*f*g))-8*b*c^2*h*(10*a^2*f*h^2+14*a*b*h*(e*h+3*f*g)+7*b^2*(d*
h^2+3*e*g*h+3*f*g^2))-64*c^4*g*(2*b*g*(3*d*h+e*g)+a*(f*g^2+3*h*(d*h+e*g)))+
16*c^3*(2*a^2*h^2*(e*h+3*f*g)+5*b^2*g*(f*g^2+3*h*(d*h+e*g))+6*a*b*h*(3*f*g^
```

```
2+h*(d*h+3*e*g))))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(13
/2)+1/1024*(256*c^5*d*g^3-33*b^5*f*h^3+6*b^3*c*h^2*(20*a*f*h+7*b*(e*h+3*f*g
))-8*b*c^2*h*(10*a^2*f*h^2+14*a*b*h*(e*h+3*f*g)+7*b^2*(d*h^2+3*e*g*h+3*f*g^
2))-64*c^4*g*(2*b*g*(3*d*h+e*g)+a*(f*g^2+3*h*(d*h+e*g)))+16*c^3*(2*a^2*h^2*
(e*h+3*f*g)+5*b^2*g*(f*g^2+3*h*(d*h+e*g))+6*a*b*h*(3*f*g^2+h*(d*h+3*e*g))))
*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/c^6
```

Rubi [A] (verified)

Time = 1.59 (sec) , antiderivative size = 927, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1667, 846, 793, 626, 635, 212}

$$\int (g+hx)^3\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx$$

$$= \frac{f\left(cx^2+bx+a\right)^{3/2}(g+hx)^4}{7ch} - \frac{(6cfg-14ceh+11bfh)\left(cx^2+bx+a\right)^{3/2}(g+hx)^3}{84c^2h}$$

$$+ \frac{\left(-4(3fg^2-7h(eg+2dh))\,c^2-2h(8bfg+21beh+16afh)c+33b^2fh^2\right)\left(cx^2+bx+a\right)^{3/2}(g+hx)^2}{280c^3h}$$

$$+ \frac{(-128(3fg^4-7g^2h(eg+12dh))\,c^4-16h(16ah(15fg^2+7h(3eg+dh))+bg(17fg^2+21h(19eg+25dh)}{}$$

$$- \frac{(b^2-4ac)\left(-33fh^3b^5+6ch^2(20afh+7b(3fg+eh))b^3-8c^2h(7(3fg^2+3ehg+dh^2)\,b^2+14ah(3fg+e}{}$$

$$+ \frac{(-33fh^3b^5+6ch^2(20afh+7b(3fg+eh))b^3-8c^2h(7(3fg^2+3ehg+dh^2)\,b^2+14ah(3fg+eh)b+10a^2}{}$$

```
[In] Int[(g + h*x)^3*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] ((256*c^5*d*g^3 - 33*b^5*f*h^3 - 64*c^4*g*(a*f*g^2 + 3*a*h*(e*g + d*h) + 2*
b*g*(e*g + 3*d*h)) + 6*b^3*c*h^2*(20*a*f*h + 7*b*(3*f*g + e*h)) - 8*b*c^2*h
*(10*a^2*f*h^2 + 14*a*b*h*(3*f*g + e*h) + 7*b^2*(3*f*g^2 + 3*e*g*h + d*h^2)
) + 16*c^3*(2*a^2*h^2*(3*f*g + e*h) + 5*b^2*g*(f*g^2 + 3*h*(e*g + d*h)) + 6
*a*b*h*(3*f*g^2 + h*(3*e*g + d*h))))*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(10
24*c^6) + ((33*b^2*f*h^2 - 2*c*h*(8*b*f*g + 21*b*e*h + 16*a*f*h) - 4*c^2*(3
*f*g^2 - 7*h*(e*g + 2*d*h)))*(g + h*x)^2*(a + b*x + c*x^2)^(3/2))/(280*c^3*
h) - ((6*c*f*g - 14*c*e*h + 11*b*f*h)*(g + h*x)^3*(a + b*x + c*x^2)^(3/2))/
(84*c^2*h) + (f*(g + h*x)^4*(a + b*x + c*x^2)^(3/2))/(7*c*h) + ((1155*b^4*f
*h^4 - 128*c^4*(3*f*g^4 - 7*g^2*h*(e*g + 12*d*h)) - 42*b^2*c*h^3*(78*a*f*h
+ 35*b*(3*f*g + e*h)) + 8*c^2*h^2*(128*a^2*f*h^2 + 343*a*b*h*(3*f*g + e*h)
+ b^2*(537*f*g^2 + 245*h*(3*e*g + d*h))) - 16*c^3*h*(16*a*h*(15*f*g^2 + 7*h
*(3*e*g + d*h)) + b*g*(17*f*g^2 + 21*h*(19*e*g + 25*d*h))) - 6*c*h*(231*b^3
*f*h^3 - 6*b*c*h^2*(59*b*f*g + 49*b*e*h + 74*a*f*h) + 16*c^3*(3*f*g^3 - 7*g
*h*(e*g + 7*d*h)) + 8*c^2*h*(5*b*f*g^2 + 7*b*h*(9*e*g + 7*d*h) + a*h*(41*f*
g + 35*e*h)))*x)*(a + b*x + c*x^2)^(3/2))/(13440*c^5*h) - ((b^2 - 4*a*c)*(2
```

```
56*c^5*d*g^3 - 33*b^5*f*h^3 - 64*c^4*g*(a*f*g^2 + 3*a*h*(e*g + d*h) + 2*b*g
*(e*g + 3*d*h)) + 6*b^3*c*h^2*(20*a*f*h + 7*b*(3*f*g + e*h)) - 8*b*c^2*h*(1
0*a^2*f*h^2 + 14*a*b*h*(3*f*g + e*h) + 7*b^2*(3*f*g^2 + 3*e*g*h + d*h^2)) +
 16*c^3*(2*a^2*h^2*(3*f*g + e*h) + 5*b^2*g*(f*g^2 + 3*h*(e*g + d*h)) + 6*a*
b*h*(3*f*g^2 + h*(3*e*g + d*h))))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b
*x + c*x^2])])/(2048*c^(13/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 846

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\text{integral} = \frac{f(g+hx)^4\left(a+bx+cx^2\right)^{3/2}}{7ch} + \frac{\int (g+hx)^3\left(-\frac{1}{2}h(3bfg-14cdh+8afh)-\frac{1}{2}h(6cfg-14ceh+11bfh)x\right)\sqrt{a+bx+cx^2}\,dx}{7ch^2}$$

$$= -\frac{(6cfg-14ceh+11bfh)(g+hx)^3\left(a+bx+cx^2\right)^{3/2}}{84c^2h} + \frac{f(g+hx)^4\left(a+bx+cx^2\right)^{3/2}}{7ch} + \frac{\int (g+hx)^2 \left(\frac{3}{4}h(11b^2fgh+22abfh^2-2bcg(3fg+7eh)+4ch(14cdg-5afg-7aeh))+\frac{3}{4}h(33b}{42c^2h^2}$$

$$= \frac{\left(33b^2fh^2-2ch(8bfg+21beh+16afh)-4c^2(3fg^2-7h(eg+2dh))\right)(g+hx)^2\left(a+bx+cx^2\right)^{3/2}}{280c^3h} - \frac{(6cfg-14ceh+11bfh)(g+hx)^3\left(a+bx+cx^2\right)^{3/2}}{84c^2h} + \frac{f(g+hx)^4\left(a+bx+cx^2\right)^{3/2}}{7ch} + \frac{\int (g+hx)\left(-\frac{3}{8}h(99b^3fgh^2+4bc(6cfg^3+14cgh(4eg+3dh)-ah^2(95fg+42eh))+2b^2(66afh^3}{}$$

$$= \frac{\left(33b^2fh^2-2ch(8bfg+21beh+16afh)-4c^2(3fg^2-7h(eg+2dh))\right)(g+hx)^2\left(a+bx+cx^2\right)^{3/2}}{280c^3h} - \frac{(6cfg-14ceh+11bfh)(g+hx)^3\left(a+bx+cx^2\right)^{3/2}}{84c^2h} + \frac{f(g+hx)^4\left(a+bx+cx^2\right)^{3/2}}{7ch} + \frac{(1155b^4fh^4-128c^4(3fg^4-7g^2h(eg+12dh))-42b^2ch^3(78afh+35b(3fg+eh))+8c^2h^2(128a}{} + \frac{(256c^5dg^3-33b^5fh^3-64c^4g(afg^2+3ah(eg+dh)+2bg(eg+3dh))+6b^3ch^2(20afh+7b(3fg}{}$$

$$= (256c^5dg^3 - 33b^5fh^3 - 64c^4g(afg^2 + 3ah(eg + dh) + 2bg(eg + 3dh)) + 6b^3ch^2(20afh + 7b(3fg -$$

$$+ \frac{(33b^2fh^2 - 2ch(8bfg + 21beh + 16afh) - 4c^2(3fg^2 - 7h(eg + 2dh)))(g + hx)^2(a + bx + cx^2)}{280c^3h}$$

$$- \frac{(6cfg - 14ceh + 11bfh)(g + hx)^3(a + bx + cx^2)^{3/2}}{84c^2h}$$

$$+ \frac{f(g + hx)^4(a + bx + cx^2)^{3/2}}{7ch}$$

$$+ (1155b^4fh^4 - 128c^4(3fg^4 - 7g^2h(eg + 12dh)) - 42b^2ch^3(78afh + 35b(3fg + eh)) + 8c^2h^2(128$$

$$- ((b^2 - 4ac)(256c^5dg^3 - 33b^5fh^3 - 64c^4g(afg^2 + 3ah(eg + dh) + 2bg(eg + 3dh)) + 6b^3ch^2(20a$$

$$= (256c^5dg^3 - 33b^5fh^3 - 64c^4g(afg^2 + 3ah(eg + dh) + 2bg(eg + 3dh)) + 6b^3ch^2(20afh + 7b(3fg -$$

$$+ \frac{(33b^2fh^2 - 2ch(8bfg + 21beh + 16afh) - 4c^2(3fg^2 - 7h(eg + 2dh)))(g + hx)^2(a + bx + cx^2)}{280c^3h}$$

$$- \frac{(6cfg - 14ceh + 11bfh)(g + hx)^3(a + bx + cx^2)^{3/2}}{84c^2h}$$

$$+ \frac{f(g + hx)^4(a + bx + cx^2)^{3/2}}{7ch}$$

$$+ (1155b^4fh^4 - 128c^4(3fg^4 - 7g^2h(eg + 12dh)) - 42b^2ch^3(78afh + 35b(3fg + eh)) + 8c^2h^2(128$$

$$- ((b^2 - 4ac)(256c^5dg^3 - 33b^5fh^3 - 64c^4g(afg^2 + 3ah(eg + dh) + 2bg(eg + 3dh)) + 6b^3ch^2(20a$$

$$= (256c^5dg^3 - 33b^5fh^3 - 64c^4g(afg^2 + 3ah(eg + dh) + 2bg(eg + 3dh)) + 6b^3ch^2(20afh + 7b(3fg -$$

$$+ \frac{(33b^2fh^2 - 2ch(8bfg + 21beh + 16afh) - 4c^2(3fg^2 - 7h(eg + 2dh)))(g + hx)^2(a + bx + cx^2)}{280c^3h}$$

$$- \frac{(6cfg - 14ceh + 11bfh)(g + hx)^3(a + bx + cx^2)^{3/2}}{84c^2h}$$

$$+ \frac{f(g + hx)^4(a + bx + cx^2)^{3/2}}{7ch}$$

$$+ (1155b^4fh^4 - 128c^4(3fg^4 - 7g^2h(eg + 12dh)) - 42b^2ch^3(78afh + 35b(3fg + eh)) + 8c^2h^2(128$$

$$- (b^2 - 4ac)(256c^5dg^3 - 33b^5fh^3 - 64c^4g(afg^2 + 3ah(eg + dh) + 2bg(eg + 3dh)) + 6b^3ch^2(20af$$

Mathematica [A] (verified)

Time = 11.64 (sec) , antiderivative size = 1093, normalized size of antiderivative = 1.18

$$\int (g+hx)^3\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx$$

$$=\frac{2\sqrt{c}\sqrt{a+x(b+cx)}(-3465b^6fh^3+210b^5ch^2(63fg+21eh+11fhx)-84b^4ch(-260afh^2+35ch(6eg+2}{}$$

```
[In] Integrate[(g + h*x)^3*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(-3465*b^6*f*h^3 + 210*b^5*c*h^2*(63*f*g +
 21*e*h + 11*f*h*x) - 84*b^4*c*h*(-260*a*f*h^2 + 35*c*h*(6*e*g + 2*d*h + e*
h*x) + c*f*(210*g^2 + 105*g*h*x + 22*h^2*x^2)) - 16*b^2*c^2*(2163*a^2*f*h^3
 - 2*a*c*h*(7*h*(345*e*g + 115*d*h + 56*e*h*x) + 3*f*(805*g^2 + 392*g*h*x +
 81*h^2*x^2)) + 2*c^2*(7*d*h*(180*g^2 + 75*g*h*x + 14*h^2*x^2) + 21*e*(20*g
^3 + 25*g^2*h*x + 14*g*h^2*x^2 + 3*h^3*x^3) + f*x*(175*g^3 + 294*g^2*h*x +
189*g*h^2*x^2 + 44*h^3*x^3))) + 16*b^3*c^2*(-42*a*h^2*(35*e*h + 3*f*(35*g +
 6*h*x)) + c*(f*(525*g^3 + 735*g^2*h*x + 441*g*h^2*x^2 + 99*h^3*x^3) + 7*h*
(5*d*h*(45*g + 7*h*x) + 3*e*(75*g^2 + 35*g*h*x + 7*h^2*x^2)))) + 32*b*c^3*(
a^2*h^2*(2373*f*g + 791*e*h + 397*f*h*x) - 2*a*c*(f*(455*g^3 + 609*g^2*h*x
+ 357*g*h^2*x^2 + 79*h^3*x^3) + 7*h*(d*h*(195*g + 29*h*x) + e*(195*g^2 + 87
*g*h*x + 17*h^2*x^2))) + 4*c^2*(21*d*(10*g^3 + 10*g^2*h*x + 5*g*h^2*x^2 + h
^3*x^3) + x*(7*e*(10*g^3 + 15*g^2*h*x + 9*g*h^2*x^2 + 2*h^3*x^3) + f*x*(35*
g^3 + 63*g^2*h*x + 42*g*h^2*x^2 + 10*h^3*x^3)))) + 64*c^3*(128*a^3*f*h^3 -
a^2*c*h*(7*h*(96*e*g + 32*d*h + 15*e*h*x) + f*(672*g^2 + 315*g*h*x + 64*h^2
*x^2)) + 2*a*c^2*(7*d*h*(120*g^2 + 45*g*h*x + 8*h^2*x^2) + 7*e*(40*g^3 + 45
*g^2*h*x + 24*g*h^2*x^2 + 5*h^3*x^3) + 3*f*x*(35*g^3 + 56*g^2*h*x + 35*g*h^
2*x^2 + 8*h^3*x^3)) + 4*c^3*x*(21*d*(10*g^3 + 20*g^2*h*x + 15*g*h^2*x^2 + 4
*h^3*x^3) + x*(7*e*(20*g^3 + 45*g^2*h*x + 36*g*h^2*x^2 + 10*h^3*x^3) + 3*f*
x*(35*g^3 + 84*g^2*h*x + 70*g*h^2*x^2 + 20*h^3*x^3))))) + 105*(b^2 - 4*a*c)
*(-256*c^5*d*g^3 + 33*b^5*f*h^3 + 64*c^4*g*(a*f*g^2 + 3*a*h*(e*g + d*h) + 2
*b*g*(e*g + 3*d*h)) - 6*b^3*c*h^2*(20*a*f*h + 7*b*(3*f*g + e*h)) + 8*b*c^2*
h*(10*a^2*f*h^2 + 14*a*b*h*(3*f*g + e*h) + 7*b^2*(3*f*g^2 + 3*e*g*h + d*h^2
)) - 16*c^3*(2*a^2*h^2*(3*f*g + e*h) + 5*b^2*g*(f*g^2 + 3*h*(e*g + d*h)) +
6*a*b*h*(3*f*g^2 + h*(3*e*g + d*h))))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a
 + x*(b + c*x)])])/(215040*c^(13/2))
```

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 1757, normalized size of antiderivative = 1.89

method	result	size
risch	Expression too large to display	1757
default	Expression too large to display	2098

```
[In] int((h*x+g)^3*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/107520*(15360*c^6*f*h^3*x^6+1280*b*c^5*f*h^3*x^5+17920*c^6*e*h^3*x^5+5376
0*c^6*f*g*h^2*x^5+3072*a*c^5*f*h^3*x^4-1408*b^2*c^4*f*h^3*x^4+1792*b*c^5*e*
h^3*x^4+5376*b*c^5*f*g*h^2*x^4+21504*c^6*d*h^3*x^4+64512*c^6*e*g*h^2*x^4+64
512*c^6*f*g^2*h*x^4-5056*a*b*c^4*f*h^3*x^3+4480*a*c^5*e*h^3*x^3+13440*a*c^5
*f*g*h^2*x^3+1584*b^3*c^3*f*h^3*x^3-2016*b^2*c^4*e*h^3*x^3-6048*b^2*c^4*f*g
*h^2*x^3+2688*b*c^5*d*h^3*x^3+8064*b*c^5*e*g*h^2*x^3+8064*b*c^5*f*g^2*h*x^3
+80640*c^6*d*g*h^2*x^3+80640*c^6*e*g^2*h*x^3+26880*c^6*f*g^3*x^3-4096*a^2*c
^4*f*h^3*x^2+7776*a*b^2*c^3*f*h^3*x^2-7616*a*b*c^4*e*h^3*x^2-22848*a*b*c^4*
f*g*h^2*x^2+7168*a*c^5*d*h^3*x^2+21504*a*c^5*e*g*h^2*x^2+21504*a*c^5*f*g^2*
h*x^2-1848*b^4*c^2*f*h^3*x^2+2352*b^3*c^3*e*h^3*x^2+7056*b^3*c^3*f*g*h^2*x^
2-3136*b^2*c^4*d*h^3*x^2-9408*b^2*c^4*e*g*h^2*x^2-9408*b^2*c^4*f*g^2*h*x^2+
13440*b*c^5*d*g*h^2*x^2+13440*b*c^5*e*g^2*h*x^2+4480*b*c^5*f*g^3*x^2+107520
*c^6*d*g^2*h*x^2+35840*c^6*e*g^3*x^2+12704*a^2*b*c^3*f*h^3*x-6720*a^2*c^4*e
*h^3*x-20160*a^2*c^4*f*g*h^2*x-12096*a*b^3*c^2*f*h^3*x+12544*a*b^2*c^3*e*h^
3*x+37632*a*b^2*c^3*f*g*h^2*x-12992*a*b*c^4*d*h^3*x-38976*a*b*c^4*e*g*h^2*x
-38976*a*b*c^4*f*g^2*h*x+40320*a*c^5*d*g*h^2*x+40320*a*c^5*e*g^2*h*x+13440*
a*c^5*f*g^3*x+2310*b^5*c*f*h^3*x-2940*b^4*c^2*e*h^3*x-8820*b^4*c^2*f*g*h^2*
x+3920*b^3*c^3*d*h^3*x+11760*b^3*c^3*e*g*h^2*x+11760*b^3*c^3*f*g^2*h*x-1680
0*b^2*c^4*d*g*h^2*x-16800*b^2*c^4*e*g^2*h*x-5600*b^2*c^4*f*g^3*x+26880*b*c^
5*d*g^2*h*x+8960*b*c^5*e*g^3*x+53760*c^6*d*g^3*x+8192*a^3*c^3*f*h^3-34608*a
^2*b^2*c^2*f*h^3+25312*a^2*b*c^3*e*h^3+75936*a^2*b*c^3*f*g*h^2-14336*a^2*c^
4*d*h^3-43008*a^2*c^4*e*g*h^2-43008*a^2*c^4*f*g^2*h+21840*a*b^4*c*f*h^3-235
20*a*b^3*c^2*e*h^3-70560*a*b^3*c^2*f*g*h^2+25760*a*b^2*c^3*d*h^3+77280*a*b^
2*c^3*e*g*h^2+77280*a*b^2*c^3*f*g^2*h-87360*a*b*c^4*d*g*h^2-87360*a*b*c^4*e
*g^2*h-29120*a*b*c^4*f*g^3+107520*a*c^5*d*g^2*h+35840*a*c^5*e*g^3-3465*b^6*
f*h^3+4410*b^5*c*e*h^3+13230*b^5*c*f*g*h^2-5880*b^4*c^2*d*h^3-17640*b^4*c^2
*e*g*h^2-17640*b^4*c^2*f*g^2*h+25200*b^3*c^3*d*g*h^2+25200*b^3*c^3*e*g^2*h+
8400*b^3*c^3*f*g^3-40320*b^2*c^4*d*g^2*h-13440*b^2*c^4*e*g^3+26880*b*c^5*d*
g^3)/c^6*(c*x^2+b*x+a)^(1/2)-1/2048*(320*a^3*b*c^3*f*h^3-128*a^3*c^4*e*h^3-
384*a^3*c^4*f*g*h^2-560*a^2*b^3*c^2*f*h^3+480*a^2*b^2*c^3*e*h^3+1440*a^2*b^
2*c^3*f*g*h^2-384*a^2*b*c^4*d*h^3-1152*a^2*b*c^4*e*g*h^2-1152*a^2*b*c^4*f*g
^2*h+768*a^2*c^5*d*g*h^2+768*a^2*c^5*e*g^2*h+256*a^2*c^5*f*g^3+252*a*b^5*c*
f*h^3-280*a*b^4*c^2*e*h^3-840*a*b^4*c^2*f*g*h^2+320*a*b^3*c^3*d*h^3+960*a*b
^3*c^3*e*g*h^2+960*a*b^3*c^3*f*g^2*h-1152*a*b^2*c^4*d*g*h^2-1152*a*b^2*c^4*
e*g^2*h-384*a*b^2*c^4*f*g^3+1536*a*b*c^5*d*g^2*h+512*a*b*c^5*e*g^3-1024*a*c
```

```
^6*d*g^3-33*b^7*f*h^3+42*b^6*c*e*h^3+126*b^6*c*f*g*h^2-56*b^5*c^2*d*h^3-168
*b^5*c^2*e*g*h^2-168*b^5*c^2*f*g^2*h+240*b^4*c^3*d*g*h^2+240*b^4*c^3*e*g^2*
h+80*b^4*c^3*f*g^3-384*b^3*c^4*d*g^2*h-128*b^3*c^4*e*g^3+256*b^2*c^5*d*g^3)
/c^(13/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 1.12 (sec) , antiderivative size = 2817, normalized size of antiderivative = 3.03

$$\int (g+hx)^3\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx=\text{Too large to display}$$

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [1/430080*(105*(16*(16*(b^2*c^5 - 4*a*c^6)*d - 8*(b^3*c^4 - 4*a*b*c^5)*e +
(5*b^4*c^3 - 24*a*b^2*c^4 + 16*a^2*c^5)*f)*g^3 - 24*(16*(b^3*c^4 - 4*a*b*c^
5)*d - 2*(5*b^4*c^3 - 24*a*b^2*c^4 + 16*a^2*c^5)*e + (7*b^5*c^2 - 40*a*b^3*
c^3 + 48*a^2*b*c^4)*f)*g^2*h + 6*(8*(5*b^4*c^3 - 24*a*b^2*c^4 + 16*a^2*c^5)
*d - 4*(7*b^5*c^2 - 40*a*b^3*c^3 + 48*a^2*b*c^4)*e + (21*b^6*c - 140*a*b^4*
c^2 + 240*a^2*b^2*c^3 - 64*a^3*c^4)*f)*g*h^2 - (8*(7*b^5*c^2 - 40*a*b^3*c^3
 + 48*a^2*b*c^4)*d - 2*(21*b^6*c - 140*a*b^4*c^2 + 240*a^2*b^2*c^3 - 64*a^3
*c^4)*e + (33*b^7 - 252*a*b^5*c + 560*a^2*b^3*c^2 - 320*a^3*b*c^3)*f)*h^3)*
sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b
)*sqrt(c) - 4*a*c) + 4*(15360*c^7*f*h^3*x^6 + 1280*(42*c^7*f*g*h^2 + (14*c^
7*e + b*c^6*f)*h^3)*x^5 + 128*(504*c^7*f*g^2*h + 42*(12*c^7*e + b*c^6*f)*g*
h^2 + (168*c^7*d + 14*b*c^6*e - (11*b^2*c^5 - 24*a*c^6)*f)*h^3)*x^4 + 560*(
48*b*c^6*d - 8*(3*b^2*c^5 - 8*a*c^6)*e + (15*b^3*c^4 - 52*a*b*c^5)*f)*g^3 -
 168*(80*(3*b^2*c^5 - 8*a*c^6)*d - 10*(15*b^3*c^4 - 52*a*b*c^5)*e + (105*b^
4*c^3 - 460*a*b^2*c^4 + 256*a^2*c^5)*f)*g^2*h + 42*(40*(15*b^3*c^4 - 52*a*b
*c^5)*d - 4*(105*b^4*c^3 - 460*a*b^2*c^4 + 256*a^2*c^5)*e + (315*b^5*c^2 -
1680*a*b^3*c^3 + 1808*a^2*b*c^4)*f)*g*h^2 - (56*(105*b^4*c^3 - 460*a*b^2*c^
4 + 256*a^2*c^5)*d - 14*(315*b^5*c^2 - 1680*a*b^3*c^3 + 1808*a^2*b*c^4)*e +
 (3465*b^6*c - 21840*a*b^4*c^2 + 34608*a^2*b^2*c^3 - 8192*a^3*c^4)*f)*h^3 +
 16*(1680*c^7*f*g^3 + 504*(10*c^7*e + b*c^6*f)*g^2*h + 42*(120*c^7*d + 12*b
*c^6*e - (9*b^2*c^5 - 20*a*c^6)*f)*g*h^2 + (168*b*c^6*d - 14*(9*b^2*c^5 - 2
0*a*c^6)*e + (99*b^3*c^4 - 316*a*b*c^5)*f)*h^3)*x^3 + 8*(560*(8*c^7*e + b*c
^6*f)*g^3 + 168*(80*c^7*d + 10*b*c^6*e - (7*b^2*c^5 - 16*a*c^6)*f)*g^2*h +
42*(40*b*c^6*d - 4*(7*b^2*c^5 - 16*a*c^6)*e + (21*b^3*c^4 - 68*a*b*c^5)*f)*
g*h^2 - (56*(7*b^2*c^5 - 16*a*c^6)*d - 14*(21*b^3*c^4 - 68*a*b*c^5)*e + (23
1*b^4*c^3 - 972*a*b^2*c^4 + 512*a^2*c^5)*f)*h^3)*x^2 + 2*(560*(48*c^7*d + 8
*b*c^6*e - (5*b^2*c^5 - 12*a*c^6)*f)*g^3 + 168*(80*b*c^6*d - 10*(5*b^2*c^5
- 12*a*c^6)*e + (35*b^3*c^4 - 116*a*b*c^5)*f)*g^2*h - 42*(40*(5*b^2*c^5 - 1
2*a*c^6)*d - 4*(35*b^3*c^4 - 116*a*b*c^5)*e + (105*b^4*c^3 - 448*a*b^2*c^4
```

```
+ 240*a^2*c^5)*f)*g*h^2 + (56*(35*b^3*c^4 - 116*a*b*c^5)*d - 14*(105*b^4*c^
3 - 448*a*b^2*c^4 + 240*a^2*c^5)*e + (1155*b^5*c^2 - 6048*a*b^3*c^3 + 6352*
a^2*b*c^4)*f)*h^3)*x)*sqrt(c*x^2 + b*x + a))/c^7, 1/215040*(105*(16*(16*(b^
2*c^5 - 4*a*c^6)*d - 8*(b^3*c^4 - 4*a*b*c^5)*e + (5*b^4*c^3 - 24*a*b^2*c^4
+ 16*a^2*c^5)*f)*g^3 - 24*(16*(b^3*c^4 - 4*a*b*c^5)*d - 2*(5*b^4*c^3 - 24*a
*b^2*c^4 + 16*a^2*c^5)*e + (7*b^5*c^2 - 40*a*b^3*c^3 + 48*a^2*b*c^4)*f)*g^2
*h + 6*(8*(5*b^4*c^3 - 24*a*b^2*c^4 + 16*a^2*c^5)*d - 4*(7*b^5*c^2 - 40*a*b
^3*c^3 + 48*a^2*b*c^4)*e + (21*b^6*c - 140*a*b^4*c^2 + 240*a^2*b^2*c^3 - 64
*a^3*c^4)*f)*g*h^2 - (8*(7*b^5*c^2 - 40*a*b^3*c^3 + 48*a^2*b*c^4)*d - 2*(21
*b^6*c - 140*a*b^4*c^2 + 240*a^2*b^2*c^3 - 64*a^3*c^4)*e + (33*b^7 - 252*a*
b^5*c + 560*a^2*b^3*c^2 - 320*a^3*b*c^3)*f)*h^3)*sqrt(-c)*arctan(1/2*sqrt(c
*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(15360*c^
7*f*h^3*x^6 + 1280*(42*c^7*f*g*h^2 + (14*c^7*e + b*c^6*f)*h^3)*x^5 + 128*(5
04*c^7*f*g^2*h + 42*(12*c^7*e + b*c^6*f)*g*h^2 + (168*c^7*d + 14*b*c^6*e -
(11*b^2*c^5 - 24*a*c^6)*f)*h^3)*x^4 + 560*(48*b*c^6*d - 8*(3*b^2*c^5 - 8*a*
c^6)*e + (15*b^3*c^4 - 52*a*b*c^5)*f)*g^3 - 168*(80*(3*b^2*c^5 - 8*a*c^6)*d
 - 10*(15*b^3*c^4 - 52*a*b*c^5)*e + (105*b^4*c^3 - 460*a*b^2*c^4 + 256*a^2*
c^5)*f)*g^2*h + 42*(40*(15*b^3*c^4 - 52*a*b*c^5)*d - 4*(105*b^4*c^3 - 460*a
*b^2*c^4 + 256*a^2*c^5)*e + (315*b^5*c^2 - 1680*a*b^3*c^3 + 1808*a^2*b*c^4)
*f)*g*h^2 - (56*(105*b^4*c^3 - 460*a*b^2*c^4 + 256*a^2*c^5)*d - 14*(315*b^5
*c^2 - 1680*a*b^3*c^3 + 1808*a^2*b*c^4)*e + (3465*b^6*c - 21840*a*b^4*c^2 +
 34608*a^2*b^2*c^3 - 8192*a^3*c^4)*f)*h^3 + 16*(1680*c^7*f*g^3 + 504*(10*c^
7*e + b*c^6*f)*g^2*h + 42*(120*c^7*d + 12*b*c^6*e - (9*b^2*c^5 - 20*a*c^6)*
f)*g*h^2 + (168*b*c^6*d - 14*(9*b^2*c^5 - 20*a*c^6)*e + (99*b^3*c^4 - 316*a
*b*c^5)*f)*h^3)*x^3 + 8*(560*(8*c^7*e + b*c^6*f)*g^3 + 168*(80*c^7*d + 10*b
*c^6*e - (7*b^2*c^5 - 16*a*c^6)*f)*g^2*h + 42*(40*b*c^6*d - 4*(7*b^2*c^5 -
16*a*c^6)*e + (21*b^3*c^4 - 68*a*b*c^5)*f)*g*h^2 - (56*(7*b^2*c^5 - 16*a*c^
6)*d - 14*(21*b^3*c^4 - 68*a*b*c^5)*e + (231*b^4*c^3 - 972*a*b^2*c^4 + 512*
a^2*c^5)*f)*h^3)*x^2 + 2*(560*(48*c^7*d + 8*b*c^6*e - (5*b^2*c^5 - 12*a*c^6
)*f)*g^3 + 168*(80*b*c^6*d - 10*(5*b^2*c^5 - 12*a*c^6)*e + (35*b^3*c^4 - 11
6*a*b*c^5)*f)*g^2*h - 42*(40*(5*b^2*c^5 - 12*a*c^6)*d - 4*(35*b^3*c^4 - 116
*a*b*c^5)*e + (105*b^4*c^3 - 448*a*b^2*c^4 + 240*a^2*c^5)*f)*g*h^2 + (56*(3
5*b^3*c^4 - 116*a*b*c^5)*d - 14*(105*b^4*c^3 - 448*a*b^2*c^4 + 240*a^2*c^5)
*e + (1155*b^5*c^2 - 6048*a*b^3*c^3 + 6352*a^2*b*c^4)*f)*h^3)*x)*sqrt(c*x^2
 + b*x + a))/c^7]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4820 vs. $2(966) = 1932$.

Time = 1.38 (sec) , antiderivative size = 4820, normalized size of antiderivative = 5.18

$$\int (g+hx)^3\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx = \text{Too large to display}$$

```
[In] integrate((h*x+g)**3*(f*x**2+e*x+d)*(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Piecewise((sqrt(a + b*x + c*x**2)*(f*h**3*x**6/7 + x**5*(b*f*h**3/14 + c*e*
h**3 + 3*c*f*g*h**2)/(6*c) + x**4*(a*f*h**3/7 + b*e*h**3 + 3*b*f*g*h**2 - 1
1*b*(b*f*h**3/14 + c*e*h**3 + 3*c*f*g*h**2)/(12*c) + c*d*h**3 + 3*c*e*g*h**
2 + 3*c*f*g**2*h)/(5*c) + x**3*(a*e*h**3 + 3*a*f*g*h**2 - 5*a*(b*f*h**3/14
+ c*e*h**3 + 3*c*f*g*h**2)/(6*c) + b*d*h**3 + 3*b*e*g*h**2 + 3*b*f*g**2*h -
 9*b*(a*f*h**3/7 + b*e*h**3 + 3*b*f*g*h**2 - 11*b*(b*f*h**3/14 + c*e*h**3 +
 3*c*f*g*h**2)/(12*c) + c*d*h**3 + 3*c*e*g*h**2 + 3*c*f*g**2*h)/(10*c) + 3*
c*d*g*h**2 + 3*c*e*g**2*h + c*f*g**3)/(4*c) + x**2*(a*d*h**3 + 3*a*e*g*h**2
 + 3*a*f*g**2*h - 4*a*(a*f*h**3/7 + b*e*h**3 + 3*b*f*g*h**2 - 11*b*(b*f*h**
3/14 + c*e*h**3 + 3*c*f*g*h**2)/(12*c) + c*d*h**3 + 3*c*e*g*h**2 + 3*c*f*g*
*2*h)/(5*c) + 3*b*d*g*h**2 + 3*b*e*g**2*h + b*f*g**3 - 7*b*(a*e*h**3 + 3*a*
f*g*h**2 - 5*a*(b*f*h**3/14 + c*e*h**3 + 3*c*f*g*h**2)/(6*c) + b*d*h**3 + 3
*b*e*g*h**2 + 3*b*f*g**2*h - 9*b*(a*f*h**3/7 + b*e*h**3 + 3*b*f*g*h**2 - 11
*b*(b*f*h**3/14 + c*e*h**3 + 3*c*f*g*h**2)/(12*c) + c*d*h**3 + 3*c*e*g*h**2
 + 3*c*f*g**2*h)/(10*c) + 3*c*d*g*h**2 + 3*c*e*g**2*h + c*f*g**3)/(8*c) + 3
*c*d*g**2*h + c*e*g**3)/(3*c) + x*(3*a*d*g*h**2 + 3*a*e*g**2*h + a*f*g**3 -
 3*a*(a*e*h**3 + 3*a*f*g*h**2 - 5*a*(b*f*h**3/14 + c*e*h**3 + 3*c*f*g*h**2)
/(6*c) + b*d*h**3 + 3*b*e*g*h**2 + 3*b*f*g**2*h - 9*b*(a*f*h**3/7 + b*e*h**
3 + 3*b*f*g*h**2 - 11*b*(b*f*h**3/14 + c*e*h**3 + 3*c*f*g*h**2)/(12*c) + c*
d*h**3 + 3*c*e*g*h**2 + 3*c*f*g**2*h)/(10*c) + 3*c*d*g*h**2 + 3*c*e*g**2*h
+ c*f*g**3)/(4*c) + 3*b*d*g**2*h + b*e*g**3 - 5*b*(a*d*h**3 + 3*a*e*g*h**2
+ 3*a*f*g**2*h - 4*a*(a*f*h**3/7 + b*e*h**3 + 3*b*f*g*h**2 - 11*b*(b*f*h**3
/14 + c*e*h**3 + 3*c*f*g*h**2)/(12*c) + c*d*h**3 + 3*c*e*g*h**2 + 3*c*f*g**
2*h)/(5*c) + 3*b*d*g*h**2 + 3*b*e*g**2*h + b*f*g**3 - 7*b*(a*e*h**3 + 3*a*f
*g*h**2 - 5*a*(b*f*h**3/14 + c*e*h**3 + 3*c*f*g*h**2)/(6*c) + b*d*h**3 + 3*
b*e*g*h**2 + 3*b*f*g**2*h - 9*b*(a*f*h**3/7 + b*e*h**3 + 3*b*f*g*h**2 - 11*
b*(b*f*h**3/14 + c*e*h**3 + 3*c*f*g*h**2)/(12*c) + c*d*h**3 + 3*c*e*g*h**2
+ 3*c*f*g**2*h)/(10*c) + 3*c*d*g*h**2 + 3*c*e*g**2*h + c*f*g**3)/(8*c) + 3*
c*d*g**2*h + c*e*g**3)/(6*c) + c*d*g**3)/(2*c) + (3*a*d*g**2*h + a*e*g**3 -
 2*a*(a*d*h**3 + 3*a*e*g*h**2 + 3*a*f*g**2*h - 4*a*(a*f*h**3/7 + b*e*h**3 +
 3*b*f*g*h**2 - 11*b*(b*f*h**3/14 + c*e*h**3 + 3*c*f*g*h**2)/(12*c) + c*d*h
**3 + 3*c*e*g*h**2 + 3*c*f*g**2*h)/(5*c) + 3*b*d*g*h**2 + 3*b*e*g**2*h + b*
f*g**3 - 7*b*(a*e*h**3 + 3*a*f*g*h**2 - 5*a*(b*f*h**3/14 + c*e*h**3 + 3*c*f
*g*h**2)/(6*c) + b*d*h**3 + 3*b*e*g*h**2 + 3*b*f*g**2*h - 9*b*(a*f*h**3/7 +
 b*e*h**3 + 3*b*f*g*h**2 - 11*b*(b*f*h**3/14 + c*e*h**3 + 3*c*f*g*h**2)/(12
*c) + c*d*h**3 + 3*c*e*g*h**2 + 3*c*f*g**2*h)/(10*c) + 3*c*d*g*h**2 + 3*c*e
*g**2*h + c*f*g**3)/(8*c) + 3*c*d*g**2*h + c*e*g**3)/(3*c) + b*d*g**3 - 3*b
*(3*a*d*g*h**2 + 3*a*e*g**2*h + a*f*g**3 - 3*a*(a*e*h**3 + 3*a*f*g*h**2 - 5
*a*(b*f*h**3/14 + c*e*h**3 + 3*c*f*g*h**2)/(6*c) + b*d*h**3 + 3*b*e*g*h**2
+ 3*b*f*g**2*h - 9*b*(a*f*h**3/7 + b*e*h**3 + 3*b*f*g*h**2 - 11*b*(b*f*h**3
/14 + c*e*h**3 + 3*c*f*g*h**2)/(12*c) + c*d*h**3 + 3*c*e*g*h**2 + 3*c*f*g**
2*h)/(10*c) + 3*c*d*g*h**2 + 3*c*e*g**2*h + c*f*g**3)/(4*c) + 3*b*d*g**2*h
+ b*e*g**3 - 5*b*(a*d*h**3 + 3*a*e*g*h**2 + 3*a*f*g**2*h - 4*a*(a*f*h**3/7
+ b*e*h**3 + 3*b*f*g*h**2 - 11*b*(b*f*h**3/14 + c*e*h**3 + 3*c*f*g*h**2)/(1
2*c) + c*d*h**3 + 3*c*e*g*h**2 + 3*c*f*g**2*h)/(5*c) + 3*b*d*g*h**2 + 3*b*e
```

```
*g**2*h + b*f*g**3 - 7*b*(a*e*h**3 + 3*a*f*g*h**2 - 5*a*(b*f*h**3/14 + c*e*
h**3 + 3*c*f*g*h**2)/(6*c) + b*d*h**3 + 3*b*e*g*h**2 + 3*b*f*g**2*h - 9*b*(
a*f*h**3/7 + b*e*h**3 + 3*b*f*g*h**2 - 11*b*(b*f*h**3/14 + c*e*h**3 + 3*c*f
*g*h**2)/(12*c) + c*d*h**3 + 3*c*e*g*h**2 + 3*c*f*g**2*h)/(10*c) + 3*c*d*g*
h**2 + 3*c*e*g**2*h + c*f*g**3)/(8*c) + 3*c*d*g**2*h + c*e*g**3)/(6*c) + c*
d*g**3)/(4*c))/c) + (a*d*g**3 - a*(3*a*d*g*h**2 + 3*a*e*g**2*h + a*f*g**3 -
 3*a*(a*e*h**3 + 3*a*f*g*h**2 - 5*a*(b*f*h**3/14 + c*e*h**3 + 3*c*f*g*h**2)
/(6*c) + b*d*h**3 + 3*b*e*g*h**2 + 3*b*f*g**2*h - 9*b*(a*f*h**3/7 + b*e*h**
3 + 3*b*f*g*h**2 - 11*b*(b*f*h**3/14 + c*e*h**3 + 3*c*f*g*h**2)/(12*c) + c*
d*h**3 + 3*c*e*g*h**2 + 3*c*f*g**2*h)/(10*c) + 3*c*d*g*h**2 + 3*c*e*g**2*h
+ c*f*g**3)/(4*c) + 3*b*d*g**2*h + b*e*g**3 - 5*b*(a*d*h**3 + 3*a*e*g*h**2
+ 3*a*f*g**2*h - 4*a*(a*f*h**3/7 + b*e*h**3 + 3*b*f*g*h**2 - 11*b*(b*f*h**3
/14 + c*e*h**3 + 3*c*f*g*h**2)/(12*c) + c*d*h**3 + 3*c*e*g*h**2 + 3*c*f*g**
2*h)/(5*c) + 3*b*d*g*h**2 + 3*b*e*g**2*h + b*f*g**3 - 7*b*(a*e*h**3 + 3*a*f
*g*h**2 - 5*a*(b*f*h**3/14 + c*e*h**3 + 3*c*f*g*h**2)/(6*c) + b*d*h**3 + 3*
b*e*g*h**2 + 3*b*f*g**2*h - 9*b*(a*f*h**3/7 + b*e*h**3 + 3*b*f*g*h**2 - 11*
b*(b*f*h**3/14 + c*e*h**3 + 3*c*f*g*h**2)/(12*c) + c*d*h**3 + 3*c*e*g*h**2
+ 3*c*f*g**2*h)/(10*c) + 3*c*d*g*h**2 + 3*c*e*g**2*h + c*f*g**3)/(8*c) + 3*
c*d*g**2*h + c*e*g**3)/(6*c) + c*d*g**3)/(2*c) - b*(3*a*d*g**2*h + a*e*g**3
 - 2*a*(a*d*h**3 + 3*a*e*g*h**2 + 3*a*f*g**2*h - 4*a*(a*f*h**3/7 + b*e*h**3
 + 3*b*f*g*h**2 - 11*b*(b*f*h**3/14 + c*e*h**3 + 3*c*f*g*h**2)/(12*c) + c*d
*h**3 + 3*c*e*g*h**2 + 3*c*f*g**2*h)/(5*c) + 3*b*d*g*h**2 + 3*b*e*g**2*h +
b*f*g**3 - 7*b*(a*e*h**3 + 3*a*f*g*h**2 - 5*a*(b*f*h**3/14 + c*e*h**3 + 3*c
*f*g*h**2)/(6*c) + b*d*h**3 + 3*b*e*g*h**2 + 3*b*f*g**2*h - 9*b*(a*f*h**3/7
 + b*e*h**3 + 3*b*f*g*h**2 - 11*b*(b*f*h**3/14 + c*e*h**3 + 3*c*f*g*h**2)/(
12*c) + c*d*h**3 + 3*c*e*g*h**2 + 3*c*f*g**2*h)/(10*c) + 3*c*d*g*h**2 + 3*c
*e*g**2*h + c*f*g**3)/(8*c) + 3*c*d*g**2*h + c*e*g**3)/(3*c) + b*d*g**3 - 3
*b*(3*a*d*g*h**2 + 3*a*e*g**2*h + a*f*g**3 - 3*a*(a*e*h**3 + 3*a*f*g*h**2 -
 5*a*(b*f*h**3/14 + c*e*h**3 + 3*c*f*g*h**2)/(6*c) + b*d*h**3 + 3*b*e*g*h**
2 + 3*b*f*g**2*h - 9*b*(a*f*h**3/7 + b*e*h**3 + 3*b*f*g*h**2 - 11*b*(b*f*h*
*3/14 + c*e*h**3 + 3*c*f*g*h**2)/(12*c) + c*d*h**3 + 3*c*e*g*h**2 + 3*c*f*g
**2*h)/(10*c) + 3*c*d*g*h**2 + 3*c*e*g**2*h + c*f*g**3)/(4*c) + 3*b*d*g**2*
h + b*e*g**3 - 5*b*(a*d*h**3 + 3*a*e*g*h**2 + 3*a*f*g**2*h - 4*a*(a*f*h**3/
7 + b*e*h**3 + 3*b*f*g*h**2 - 11*b*(b*f*h**3/14 + c*e*h**3 + 3*c*f*g*h**2)/
(12*c) + c*d*h**3 + 3*c*e*g*h**2 + 3*c*f*g**2*h)/(5*c) + 3*b*d*g*h**2 + 3*b
*e*g**2*h + b*f*g**3 - 7*b*(a*e*h**3 + 3*a*f*g*h**2 - 5*a*(b*f*h**3/14 + c*
e*h**3 + 3*c*f*g*h**2)/(6*c) + b*d*h**3 + 3*b*e*g*h**2 + 3*b*f*g**2*h - 9*b
*(a*f*h**3/7 + b*e*h**3 + 3*b*f*g*h**2 - 11*b*(b*f*h**3/14 + c*e*h**3 + 3*c
*f*g*h**2)/(12*c) + c*d*h**3 + 3*c*e*g*h**2 + 3*c*f*g**2*h)/(10*c) + 3*c*d*
g*h**2 + 3*c*e*g**2*h + c*f*g**3)/(8*c) + 3*c*d*g**2*h + c*e*g**3)/(6*c) +
c*d*g**3)/(4*c))/(2*c))*Piecewise((log(b + 2*sqrt(c)*sqrt(a + b*x + c*x**2)
 + 2*c*x)/sqrt(c), Ne(a - b**2/(4*c), 0)), ((b/(2*c) + x)*log(b/(2*c) + x)/
sqrt(c*(b/(2*c) + x)**2), True)), Ne(c, 0)), (2*(f*h**3*(a + b*x)**(13/2)/(
13*b**5) + (a + b*x)**(11/2)*(-5*a*f*h**3 + b*e*h**3 + 3*b*f*g*h**2)/(11*b*
*5) + (a + b*x)**(9/2)*(10*a**2*f*h**3 - 4*a*b*e*h**3 - 12*a*b*f*g*h**2 + b
```

```
**2*d*h**3 + 3*b**2*e*g*h**2 + 3*b**2*f*g**2*h)/(9*b**5) + (a + b*x)**(7/2)
*(-10*a**3*f*h**3 + 6*a**2*b*e*h**3 + 18*a**2*b*f*g*h**2 - 3*a*b**2*d*h**3
 - 9*a*b**2*e*g*h**2 - 9*a*b**2*f*g**2*h + 3*b**3*d*g*h**2 + 3*b**3*e*g**2*h
 + b**3*f*g**3)/(7*b**5) + (a + b*x)**(5/2)*(5*a**4*f*h**3 - 4*a**3*b*e*h**
3 - 12*a**3*b*f*g*h**2 + 3*a**2*b**2*d*h**3 + 9*a**2*b**2*e*g*h**2 + 9*a**2
*b**2*f*g**2*h - 6*a*b**3*d*g*h**2 - 6*a*b**3*e*g**2*h - 2*a*b**3*f*g**3 +
3*b**4*d*g**2*h + b**4*e*g**3)/(5*b**5) + (a + b*x)**(3/2)*(-a**5*f*h**3 +
a**4*b*e*h**3 + 3*a**4*b*f*g*h**2 - a**3*b**2*d*h**3 - 3*a**3*b**2*e*g*h**2
 - 3*a**3*b**2*f*g**2*h + 3*a**2*b**3*d*g*h**2 + 3*a**2*b**3*e*g**2*h + a**
2*b**3*f*g**3 - 3*a*b**4*d*g**2*h - a*b**4*e*g**3 + b**5*d*g**3)/(3*b**5))/
b, Ne(b, 0)), (sqrt(a)*(d*g**3*x + f*h**3*x**6/6 + x**5*(e*h**3 + 3*f*g*h**
2)/5 + x**4*(d*h**3 + 3*e*g*h**2 + 3*f*g**2*h)/4 + x**3*(3*d*g*h**2 + 3*e*g
**2*h + f*g**3)/3 + x**2*(3*d*g**2*h + e*g**3)/2), True))
```

Maxima [F(-2)]

Exception generated.

$$\int (g+hx)^3\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 1657, normalized size of antiderivative = 1.78

$$\int (g+hx)^3\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx=\text{Too large to display}$$

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/107520*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*(12*f*h^3*x + (42*c^6*f*g*h^
2 + 14*c^6*e*h^3 + b*c^5*f*h^3)/c^6)*x + (504*c^6*f*g^2*h + 504*c^6*e*g*h^2
 + 42*b*c^5*f*g*h^2 + 168*c^6*d*h^3 + 14*b*c^5*e*h^3 - 11*b^2*c^4*f*h^3 + 2
4*a*c^5*f*h^3)/c^6)*x + (1680*c^6*f*g^3 + 5040*c^6*e*g^2*h + 504*b*c^5*f*g^
2*h + 5040*c^6*d*g*h^2 + 504*b*c^5*e*g*h^2 - 378*b^2*c^4*f*g*h^2 + 840*a*c^
5*f*g*h^2 + 168*b*c^5*d*h^3 - 126*b^2*c^4*e*h^3 + 280*a*c^5*e*h^3 + 99*b^3*
```

```
c^3*f*h^3 - 316*a*b*c^4*f*h^3)/c^6)*x + (4480*c^6*e*g^3 + 560*b*c^5*f*g^3 +
 13440*c^6*d*g^2*h + 1680*b*c^5*e*g^2*h - 1176*b^2*c^4*f*g^2*h + 2688*a*c^5
*f*g^2*h + 1680*b*c^5*d*g*h^2 - 1176*b^2*c^4*e*g*h^2 + 2688*a*c^5*e*g*h^2 +
 882*b^3*c^3*f*g*h^2 - 2856*a*b*c^4*f*g*h^2 - 392*b^2*c^4*d*h^3 + 896*a*c^5
*d*h^3 + 294*b^3*c^3*e*h^3 - 952*a*b*c^4*e*h^3 - 231*b^4*c^2*f*h^3 + 972*a*
b^2*c^3*f*h^3 - 512*a^2*c^4*f*h^3)/c^6)*x + (26880*c^6*d*g^3 + 4480*b*c^5*e
*g^3 - 2800*b^2*c^4*f*g^3 + 6720*a*c^5*f*g^3 + 13440*b*c^5*d*g^2*h - 8400*b
^2*c^4*e*g^2*h + 20160*a*c^5*e*g^2*h + 5880*b^3*c^3*f*g^2*h - 19488*a*b*c^4
*f*g^2*h - 8400*b^2*c^4*d*g*h^2 + 20160*a*c^5*d*g*h^2 + 5880*b^3*c^3*e*g*h^
2 - 19488*a*b*c^4*e*g*h^2 - 4410*b^4*c^2*f*g*h^2 + 18816*a*b^2*c^3*f*g*h^2
 - 10080*a^2*c^4*f*g*h^2 + 1960*b^3*c^3*d*h^3 - 6496*a*b*c^4*d*h^3 - 1470*b^
4*c^2*e*h^3 + 6272*a*b^2*c^3*e*h^3 - 3360*a^2*c^4*e*h^3 + 1155*b^5*c*f*h^3
 - 6048*a*b^3*c^2*f*h^3 + 6352*a^2*b*c^3*f*h^3)/c^6)*x + (26880*b*c^5*d*g^3
 - 13440*b^2*c^4*e*g^3 + 35840*a*c^5*e*g^3 + 8400*b^3*c^3*f*g^3 - 29120*a*b*
c^4*f*g^3 - 40320*b^2*c^4*d*g^2*h + 107520*a*c^5*d*g^2*h + 25200*b^3*c^3*e*
g^2*h - 87360*a*b*c^4*e*g^2*h - 17640*b^4*c^2*f*g^2*h + 77280*a*b^2*c^3*f*g
^2*h - 43008*a^2*c^4*f*g^2*h + 25200*b^3*c^3*d*g*h^2 - 87360*a*b*c^4*d*g*h^
2 - 17640*b^4*c^2*e*g*h^2 + 77280*a*b^2*c^3*e*g*h^2 - 43008*a^2*c^4*e*g*h^2
 + 13230*b^5*c*f*g*h^2 - 70560*a*b^3*c^2*f*g*h^2 + 75936*a^2*b*c^3*f*g*h^2
 - 5880*b^4*c^2*d*h^3 + 25760*a*b^2*c^3*d*h^3 - 14336*a^2*c^4*d*h^3 + 4410*b
^5*c*e*h^3 - 23520*a*b^3*c^2*e*h^3 + 25312*a^2*b*c^3*e*h^3 - 3465*b^6*f*h^3
 + 21840*a*b^4*c*f*h^3 - 34608*a^2*b^2*c^2*f*h^3 + 8192*a^3*c^3*f*h^3)/c^6)
 + 1/2048*(256*b^2*c^5*d*g^3 - 1024*a*c^6*d*g^3 - 128*b^3*c^4*e*g^3 + 512*a
*b*c^5*e*g^3 + 80*b^4*c^3*f*g^3 - 384*a*b^2*c^4*f*g^3 + 256*a^2*c^5*f*g^3 -
 384*b^3*c^4*d*g^2*h + 1536*a*b*c^5*d*g^2*h + 240*b^4*c^3*e*g^2*h - 1152*a*
b^2*c^4*e*g^2*h + 768*a^2*c^5*e*g^2*h - 168*b^5*c^2*f*g^2*h + 960*a*b^3*c^3
*f*g^2*h - 1152*a^2*b*c^4*f*g^2*h + 240*b^4*c^3*d*g*h^2 - 1152*a*b^2*c^4*d*
g*h^2 + 768*a^2*c^5*d*g*h^2 - 168*b^5*c^2*e*g*h^2 + 960*a*b^3*c^3*e*g*h^2 -
 1152*a^2*b*c^4*e*g*h^2 + 126*b^6*c*f*g*h^2 - 840*a*b^4*c^2*f*g*h^2 + 1440*
a^2*b^2*c^3*f*g*h^2 - 384*a^3*c^4*f*g*h^2 - 56*b^5*c^2*d*h^3 + 320*a*b^3*c^
3*d*h^3 - 384*a^2*b*c^4*d*h^3 + 42*b^6*c*e*h^3 - 280*a*b^4*c^2*e*h^3 + 480*
a^2*b^2*c^3*e*h^3 - 128*a^3*c^4*e*h^3 - 33*b^7*f*h^3 + 252*a*b^5*c*f*h^3 -
560*a^2*b^3*c^2*f*h^3 + 320*a^3*b*c^3*f*h^3)*log(abs(2*(sqrt(c)*x - sqrt(c*
x^2 + b*x + a))*sqrt(c) + b))/c^(13/2)
```

Mupad [B] (verification not implemented)

Time = 25.11 (sec) , antiderivative size = 3262, normalized size of antiderivative = 3.51

$$\int (g+hx)^3\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx=\text{Too large to display}$$

```
[In] int((g + h*x)^3*(a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2),x)
```

```
[Out] d*g^3*(x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (8*a^3*f*h^3*(a + b*x + c*x
^2)^(1/2))/(105*c^3) - (33*b^6*f*h^3*(a + b*x + c*x^2)^(1/2))/(1024*c^6) +
```

```
(d*h^3*x^2*(a + b*x + c*x^2)^(3/2))/(5*c) + (e*h^3*x^3*(a + b*x + c*x^2)^(3
/2))/(6*c) + (f*h^3*x^4*(a + b*x + c*x^2)^(3/2))/(7*c) - (a*f*g^3*((x/2 + b
/(4*c))*(a + b*x + c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x
^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c) + (d*g^3*log((b/2 + c*x)/c^(1
/2) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2)) + (e*g^3*log((b +
 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2))
- (2*a*d*h^3*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 -
4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*
x^2)^(1/2))/(24*c^2)))/(5*c) - (5*b*f*g^3*((log((b + 2*c*x)/c^(1/2) + 2*(a
+ b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3
*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) + (e*g^3*(8*c*(a
+ c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2) + (33*b^7*f*h
^3*log(b + 2*c^(1/2)*(a + b*x + c*x^2)^(1/2) + 2*c*x))/(2048*c^(13/2)) + (f
*g^3*x*(a + b*x + c*x^2)^(3/2))/(4*c) + (a*e*h^3*((5*b*((log((b + 2*c*x)/c^
(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a
 + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) - (x
*(a + b*x + c*x^2)^(3/2))/(4*c) + (a*((x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/
2) + (log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*
c^(3/2))))/(4*c)))/(2*c) + (7*b*d*h^3*((5*b*((log((b + 2*c*x)/c^(1/2) + 2*(
a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) -
 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) - (x*(a + b*x +
 c*x^2)^(3/2))/(4*c) + (a*((x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (log((
b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/
(4*c)))/(10*c) - (3*b*e*h^3*((7*b*((5*b*((log((b + 2*c*x)/c^(1/2) + 2*(a +
b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b
^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) - (x*(a + b*x + c*x
^2)^(3/2))/(4*c) + (a*((x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (log((b/2
+ c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c
)))/(10*c) - (2*a*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b
^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x
 + c*x^2)^(1/2))/(24*c^2)))/(5*c) + (x^2*(a + b*x + c*x^2)^(3/2))/(5*c)))/(
4*c) + (3*d*g*h^2*x*(a + b*x + c*x^2)^(3/2))/(4*c) + (3*e*g^2*h*x*(a + b*x
+ c*x^2)^(3/2))/(4*c) + (3*a*f*g*h^2*((5*b*((log((b + 2*c*x)/c^(1/2) + 2*(a
 + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) -
3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) - (x*(a + b*x +
c*x^2)^(3/2))/(4*c) + (a*((x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (log((b
/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(
4*c)))/(2*c) + (21*b*e*g*h^2*((5*b*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x +
 c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 +
2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) - (x*(a + b*x + c*x^2)^(
3/2))/(4*c) + (a*((x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (log((b/2 + c*x
)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c)))/(
10*c) + (21*b*f*g^2*h*((5*b*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)
^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x
)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) - (x*(a + b*x + c*x^2)^(3/2))/(
```

```
4*c) + (a*((x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/
2) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2)))))/(4*c)))/(10*c) -
 (9*b*f*g*h^2*((7*b*((5*b*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(
1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*
(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) - (x*(a + b*x + c*x^2)^(3/2))/(4*
c) + (a*((x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2)
 + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2)))))/(4*c)))/(10*c) - (
2*a*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))
/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2
))/(24*c^2)))/(5*c) + (x^2*(a + b*x + c*x^2)^(3/2))/(5*c)))/(4*c) + (35*a^2
*b^3*f*h^3*log(b + 2*c^(1/2)*(a + b*x + c*x^2)^(1/2) + 2*c*x))/(128*c^(9/2)
) + (13*a*b^4*f*h^3*(a + b*x + c*x^2)^(1/2))/(64*c^5) - (4*a*f*h^3*x^2*(a +
 b*x + c*x^2)^(3/2))/(35*c^2) - (11*b*f*h^3*x^3*(a + b*x + c*x^2)^(3/2))/(8
4*c^2) - (33*b^3*f*h^3*x*(a + b*x + c*x^2)^(3/2))/(320*c^4) + (11*b^5*f*h^3
*x*(a + b*x + c*x^2)^(1/2))/(512*c^5) + (3*e*g*h^2*x^2*(a + b*x + c*x^2)^(3
/2))/(5*c) + (3*f*g^2*h*x^2*(a + b*x + c*x^2)^(3/2))/(5*c) + (f*g*h^2*x^3*(
a + b*x + c*x^2)^(3/2))/(2*c) - (3*a*d*g*h^2*((x/2 + b/(4*c))*(a + b*x + c*
x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2
/4))/(2*c^(3/2))))/(4*c) - (3*a*e*g^2*h*((x/2 + b/(4*c))*(a + b*x + c*x^2)^
(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/
(2*c^(3/2))))/(4*c) + (3*d*g^2*h*log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x
^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) - (103*a^2*b^2*f*h^3*(a + b*x + c*
x^2)^(1/2))/(320*c^4) - (6*a*e*g*h^2*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x
 + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2
+ 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(5*c) - (15*b*d*g*h^2*((log(
(b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/
2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2
)))/(8*c) - (6*a*f*g^2*h*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1
/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(
a + b*x + c*x^2)^(1/2))/(24*c^2)))/(5*c) - (15*b*e*g^2*h*((log((b + 2*c*x)/
c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*
(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) +
(8*a^2*f*h^3*x^2*(a + b*x + c*x^2)^(1/2))/(105*c^2) + (33*b^2*f*h^3*x^2*(a
+ b*x + c*x^2)^(3/2))/(280*c^3) + (11*b^4*f*h^3*x^2*(a + b*x + c*x^2)^(1/2)
)/(128*c^4) + (d*g^2*h*(8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2
)^(1/2))/(8*c^2) - (5*a^3*b*f*h^3*log(b + 2*c^(1/2)*(a + b*x + c*x^2)^(1/2)
 + 2*c*x))/(32*c^(7/2)) - (63*a*b^5*f*h^3*log(b + 2*c^(1/2)*(a + b*x + c*x^
2)^(1/2) + 2*c*x))/(512*c^(11/2)) - (39*a*b^2*f*h^3*x^2*(a + b*x + c*x^2)^(
1/2))/(160*c^3) + (111*a*b*f*h^3*x*(a + b*x + c*x^2)^(3/2))/(560*c^3) - (26
9*a^2*b*f*h^3*x*(a + b*x + c*x^2)^(1/2))/(3360*c^3) - (3*a*b^3*f*h^3*x*(a +
 b*x + c*x^2)^(1/2))/(320*c^4)
```

3.187 $\int (g+hx)^2\sqrt{a+bx+cx^2}(d+ex+fx^2)\,dx$

Optimal result

Integrand size = 32, antiderivative size = 584

$$\int (g+hx)^2\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx$$

$$= \frac{(128c^4dg^2+21b^4fh^2-28b^2ch(2bfg+beh+2afh)-32c^3(2bg(eg+2dh)+a(fg^2+2egh+dh^2))+8c^2(2}{512c^5}$$

$$-\frac{(2cfg-4ceh+3bfh)(g+hx)^2\left(a+bx+cx^2\right)^{3/2}}{20c^2h}+\frac{f(g+hx)^3\left(a+bx+cx^2\right)^{3/2}}{6ch}$$

$$-\frac{(105b^3fh^3+64c^3g(fg^2-2h(eg+5dh))-28bch^2(7afh+5b(2fg+eh))+8c^2h(16ah(2fg+eh)+b(7}{}$$

$$-\frac{\left(b^2-4ac\right)(128c^4dg^2+21b^4fh^2-28b^2ch(2bfg+beh+2afh)-32c^3(2bg(eg+2dh)+a(fg^2+2egh+}{1024c^{11/2}}$$

```
[Out] -1/20*(3*b*f*h-4*c*e*h+2*c*f*g)*(h*x+g)^2*(c*x^2+b*x+a)^(3/2)/c^2/h+1/6*f*(
h*x+g)^3*(c*x^2+b*x+a)^(3/2)/c/h-1/960*(105*b^3*f*h^3+64*c^3*g*(f*g^2-2*h*(
5*d*h+e*g))-28*b*c*h^2*(7*a*f*h+5*b*(e*h+2*f*g))+8*c^2*h*(16*a*h*(e*h+2*f*g
)+b*(7*f*g^2+25*h*(d*h+2*e*g)))-6*c*h*(21*b^2*f*h^2-4*c*h*(5*a*f*h+7*b*e*h+
2*b*f*g)-8*c^2*(f*g^2-h*(5*d*h+2*e*g)))*x)*(c*x^2+b*x+a)^(3/2)/c^4/h-1/1024
*(-4*a*c+b^2)*(128*c^4*d*g^2+21*b^4*f*h^2-28*b^2*c*h*(2*a*f*h+b*e*h+2*b*f*g
)-32*c^3*(2*b*g*(2*d*h+e*g)+a*(d*h^2+2*e*g*h+f*g^2))+8*c^2*(2*a^2*f*h^2+6*a
*b*h*(e*h+2*f*g)+5*b^2*(d*h^2+2*e*g*h+f*g^2)))*arctanh(1/2*(2*c*x+b)/c^(1/2
)/(c*x^2+b*x+a)^(1/2))/c^(11/2)+1/512*(128*c^4*d*g^2+21*b^4*f*h^2-28*b^2*c*
h*(2*a*f*h+b*e*h+2*b*f*g)-32*c^3*(2*b*g*(2*d*h+e*g)+a*(d*h^2+2*e*g*h+f*g^2)
)+8*c^2*(2*a^2*f*h^2+6*a*b*h*(e*h+2*f*g)+5*b^2*(d*h^2+2*e*g*h+f*g^2)))*(2*c
*x+b)*(c*x^2+b*x+a)^(1/2)/c^5
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 581, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1667, 846, 793, 626, 635, 212}

$$\int (g+hx)^2\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx =$$

$$-\frac{(b^2-4ac)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(8c^2(2a^2fh^2+6abh(eh+2fg)+5b^2(h(dh+2eg)+fg^2))-28b^2ch\right.}{1024c^{11/2}}$$

$$+\frac{(b+2cx)\sqrt{a+bx+cx^2}(8c^2(2a^2fh^2+6abh(eh+2fg)+5b^2(h(dh+2eg)+fg^2))-28b^2ch(2afh+be}{512c^5}$$

$$-\frac{\left(a+bx+cx^2\right)^{3/2}\left(-6chx(-4ch(5afh+7beh+2bfg)+21b^2fh^2-8c^2(fg^2-h(5dh+2eg)))+8c^2h(1\right.}{}$$

$$-\frac{(g+hx)^2\left(a+bx+cx^2\right)^{3/2}(3bfh-4ceh+2cfg)}{20c^2h}+\frac{f(g+hx)^3\left(a+bx+cx^2\right)^{3/2}}{6ch}$$

```
[In] Int[(g + h*x)^2*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] ((128*c^4*d*g^2 + 21*b^4*f*h^2 - 28*b^2*c*h*(2*b*f*g + b*e*h + 2*a*f*h) - 3
2*c^3*(a*f*g^2 + a*h*(2*e*g + d*h) + 2*b*g*(e*g + 2*d*h)) + 8*c^2*(2*a^2*f*
h^2 + 6*a*b*h*(2*f*g + e*h) + 5*b^2*(f*g^2 + h*(2*e*g + d*h))))*(b + 2*c*x)
*Sqrt[a + b*x + c*x^2])/(512*c^5) - ((2*c*f*g - 4*c*e*h + 3*b*f*h)*(g + h*x
)^2*(a + b*x + c*x^2)^(3/2))/(20*c^2*h) + (f*(g + h*x)^3*(a + b*x + c*x^2)^
(3/2))/(6*c*h) - ((105*b^3*f*h^3 + 64*c^3*(f*g^3 - 2*g*h*(e*g + 5*d*h)) - 2
8*b*c*h^2*(7*a*f*h + 5*b*(2*f*g + e*h)) + 8*c^2*h*(7*b*f*g^2 + 25*b*h*(2*e*
g + d*h) + 16*a*h*(2*f*g + e*h)) - 6*c*h*(21*b^2*f*h^2 - 4*c*h*(2*b*f*g + 7
*b*e*h + 5*a*f*h) - 8*c^2*(f*g^2 - h*(2*e*g + 5*d*h)))*x)*(a + b*x + c*x^2)
^(3/2))/(960*c^4*h) - ((b^2 - 4*a*c)*(128*c^4*d*g^2 + 21*b^4*f*h^2 - 28*b^2
*c*h*(2*b*f*g + b*e*h + 2*a*f*h) - 32*c^3*(a*f*g^2 + a*h*(2*e*g + d*h) + 2*
b*g*(e*g + 2*d*h)) + 8*c^2*(2*a^2*f*h^2 + 6*a*b*h*(2*f*g + e*h) + 5*b^2*(f*
g^2 + h*(2*e*g + d*h))))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^
2])])/(1024*c^(11/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
```

```
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 846

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

integral

$$=\frac{f(g+hx)^3\left(a+bx+cx^2\right)^{3/2}}{6ch}$$

$$+\frac{\int(g+hx)^2\left(-\frac{3}{2}h(bfg-4cdh+2afh)-\frac{3}{2}h(2cfg-4ceh+3bfh)x\right)\sqrt{a+bx+cx^2}\,dx}{6ch^2}$$

$$=-\frac{(2cfg-4ceh+3bfh)(g+hx)^2\left(a+bx+cx^2\right)^{3/2}}{20c^2h}+\frac{f(g+hx)^3\left(a+bx+cx^2\right)^{3/2}}{6ch}$$

$$+\frac{\int(g+hx)\left(\frac{3}{4}h(9b^2fgh+12abfh^2-4bcg(fg+3eh)+4ch(10cdg-3afg-4aeh))+\frac{3}{4}h(21b^2fh^2-4c}{30c^2h^2}$$

$$=-\frac{(2cfg-4ceh+3bfh)(g+hx)^2\left(a+bx+cx^2\right)^{3/2}}{20c^2h}+\frac{f(g+hx)^3\left(a+bx+cx^2\right)^{3/2}}{6ch}$$

$$-\frac{(105b^3fh^3+64c^3(fg^3-2gh(eg+5dh))-28bch^2(7afh+5b(2fg+eh))+8c^2h(7bfg^2+25bh(2eg+}{}$$

$$+\frac{(128c^4dg^2+21b^4fh^2-28b^2ch(2bfg+beh+2afh)-32c^3(afg^2+ah(2eg+dh)+2bg(eg+2dh))+8}{128c^4}$$

$$=\frac{(128c^4dg^2+21b^4fh^2-28b^2ch(2bfg+beh+2afh)-32c^3(afg^2+ah(2eg+dh)+2bg(eg+2dh))+8c^2}{512c^5}$$

$$-\frac{(2cfg-4ceh+3bfh)(g+hx)^2\left(a+bx+cx^2\right)^{3/2}}{20c^2h}+\frac{f(g+hx)^3\left(a+bx+cx^2\right)^{3/2}}{6ch}$$

$$-\frac{(105b^3fh^3+64c^3(fg^3-2gh(eg+5dh))-28bch^2(7afh+5b(2fg+eh))+8c^2h(7bfg^2+25bh(2eg+}{}$$

$$-\frac{((b^2-4ac)\left(128c^4dg^2+21b^4fh^2-28b^2ch(2bfg+beh+2afh)-32c^3(afg^2+ah(2eg+dh)+2bg(eg\right.}{1024c^5}$$

$$=\frac{(128c^4dg^2+21b^4fh^2-28b^2ch(2bfg+beh+2afh)-32c^3(afg^2+ah(2eg+dh)+2bg(eg+2dh))+8c^2}{512c^5}$$

$$-\frac{(2cfg-4ceh+3bfh)(g+hx)^2\left(a+bx+cx^2\right)^{3/2}}{20c^2h}+\frac{f(g+hx)^3\left(a+bx+cx^2\right)^{3/2}}{6ch}$$

$$-\frac{(105b^3fh^3+64c^3(fg^3-2gh(eg+5dh))-28bch^2(7afh+5b(2fg+eh))+8c^2h(7bfg^2+25bh(2eg+}{}$$

$$-\frac{((b^2-4ac)\left(128c^4dg^2+21b^4fh^2-28b^2ch(2bfg+beh+2afh)-32c^3(afg^2+ah(2eg+dh)+2bg(eg\right.}{512}$$

$$= \frac{(128c^4dg^2 + 21b^4fh^2 - 28b^2ch(2bfg + beh + 2afh) - 32c^3(afg^2 + ah(2eg + dh) + 2bg(eg + 2dh)) + 8c^2(}{512c^5}$$

$$- \frac{(2cfg - 4ceh + 3bfh)(g + hx)^2 (a + bx + cx^2)^{3/2}}{20c^2h} + \frac{f(g + hx)^3 (a + bx + cx^2)^{3/2}}{6ch}$$

$$- \frac{(105b^3fh^3 + 64c^3(fg^3 - 2gh(eg + 5dh)) - 28bch^2(7afh + 5b(2fg + eh)) + 8c^2h(7bfg^2 + 25bh(2eg + d}{}$$

$$- \frac{(b^2 - 4ac)(128c^4dg^2 + 21b^4fh^2 - 28b^2ch(2bfg + beh + 2afh) - 32c^3(afg^2 + ah(2eg + dh) + 2bg(eg +}{1024c^{11/2}}$$

Mathematica [A] (verified)

Time = 10.01 (sec) , antiderivative size = 657, normalized size of antiderivative = 1.12

$$\int (g + hx)^2\sqrt{a + bx + cx^2}\left(d + ex + fx^2\right)\,dx$$

$$= \frac{\sqrt{c}\sqrt{a + x(b + cx)}(315b^5fh^2 - 210b^4ch(4fg + 2eh + fhx) + 8b^3c(-210afh^2 + 5ch(30eg + 15dh + 7ehx)}{}$$

```
[In] Integrate[(g + h*x)^2*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] (Sqrt[c]*Sqrt[a + x*(b + c*x)]*(315*b^5*f*h^2 - 210*b^4*c*h*(4*f*g + 2*e*h
+ f*h*x) + 8*b^3*c*(-210*a*f*h^2 + 5*c*h*(30*e*g + 15*d*h + 7*e*h*x) + c*f*
(75*g^2 + 70*g*h*x + 21*h^2*x^2)) - 16*b^2*c^2*(-(a*h*(230*f*g + 115*e*h +
56*f*h*x)) + c*(5*d*h*(24*g + 5*h*x) + 2*e*(30*g^2 + 25*g*h*x + 7*h^2*x^2)
+ f*x*(25*g^2 + 28*g*h*x + 9*h^2*x^2))) + 16*b*c^2*(113*a^2*f*h^2 - 2*a*c*(
h*(130*e*g + 65*d*h + 29*e*h*x) + f*(65*g^2 + 58*g*h*x + 17*h^2*x^2)) + 4*c
^2*(5*d*(6*g^2 + 4*g*h*x + h^2*x^2) + x*(f*x*(5*g^2 + 6*g*h*x + 2*h^2*x^2)
+ e*(10*g^2 + 10*g*h*x + 3*h^2*x^2)))) - 32*c^3*(a^2*h*(64*f*g + 32*e*h + 1
5*f*h*x) - 2*a*c*(5*d*h*(16*g + 3*h*x) + f*x*(15*g^2 + 16*g*h*x + 5*h^2*x^2
) + e*(40*g^2 + 30*g*h*x + 8*h^2*x^2)) - 4*c^2*x*(5*d*(6*g^2 + 8*g*h*x + 3*
h^2*x^2) + x*(2*e*(10*g^2 + 15*g*h*x + 6*h^2*x^2) + f*x*(15*g^2 + 24*g*h*x
+ 10*h^2*x^2))))) - 15*(b^2 - 4*a*c)*(128*c^4*d*g^2 + 21*b^4*f*h^2 - 28*b^2
*c*h*(2*b*f*g + b*e*h + 2*a*f*h) - 32*c^3*(a*f*g^2 + a*h*(2*e*g + d*h) + 2*
b*g*(e*g + 2*d*h)) + 8*c^2*(2*a^2*f*h^2 + 6*a*b*h*(2*f*g + e*h) + 5*b^2*(f*
g^2 + h*(2*e*g + d*h))))*ArcTanh[(Sqrt[c]*x)/(-Sqrt[a] + Sqrt[a + x*(b + c*
x)])])/(7680*c^(11/2))
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 1027, normalized size of antiderivative = 1.76

method	result	size
risch	Expression too large to display	1027
default	Expression too large to display	1212

```
[In] int((h*x+g)^2*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/7680*(1280*c^5*f*h^2*x^5+128*b*c^4*f*h^2*x^4+1536*c^5*e*h^2*x^4+3072*c^5*
f*g*h*x^4+320*a*c^4*f*h^2*x^3-144*b^2*c^3*f*h^2*x^3+192*b*c^4*e*h^2*x^3+384
*b*c^4*f*g*h*x^3+1920*c^5*d*h^2*x^3+3840*c^5*e*g*h*x^3+1920*c^5*f*g^2*x^3-5
44*a*b*c^3*f*h^2*x^2+512*a*c^4*e*h^2*x^2+1024*a*c^4*f*g*h*x^2+168*b^3*c^2*f
*h^2*x^2-224*b^2*c^3*e*h^2*x^2-448*b^2*c^3*f*g*h*x^2+320*b*c^4*d*h^2*x^2+64
0*b*c^4*e*g*h*x^2+320*b*c^4*f*g^2*x^2+5120*c^5*d*g*h*x^2+2560*c^5*e*g^2*x^2
-480*a^2*c^3*f*h^2*x+896*a*b^2*c^2*f*h^2*x-928*a*b*c^3*e*h^2*x-1856*a*b*c^3
*f*g*h*x+960*a*c^4*d*h^2*x+1920*a*c^4*e*g*h*x+960*a*c^4*f*g^2*x-210*b^4*c*f
*h^2*x+280*b^3*c^2*e*h^2*x+560*b^3*c^2*f*g*h*x-400*b^2*c^3*d*h^2*x-800*b^2*
c^3*e*g*h*x-400*b^2*c^3*f*g^2*x+1280*b*c^4*d*g*h*x+640*b*c^4*e*g^2*x+3840*c
^5*d*g^2*x+1808*a^2*b*c^2*f*h^2-1024*a^2*c^3*e*h^2-2048*a^2*c^3*f*g*h-1680*
a*b^3*c*f*h^2+1840*a*b^2*c^2*e*h^2+3680*a*b^2*c^2*f*g*h-2080*a*b*c^3*d*h^2-
4160*a*b*c^3*e*g*h-2080*a*b*c^3*f*g^2+5120*a*c^4*d*g*h+2560*a*c^4*e*g^2+315
*b^5*f*h^2-420*b^4*c*e*h^2-840*b^4*c*f*g*h+600*b^3*c^2*d*h^2+1200*b^3*c^2*e
*g*h+600*b^3*c^2*f*g^2-1920*b^2*c^3*d*g*h-960*b^2*c^3*e*g^2+1920*b*c^4*d*g^
2)/c^5*(c*x^2+b*x+a)^(1/2)+1/1024*(64*a^3*c^3*f*h^2-240*a^2*b^2*c^2*f*h^2+1
92*a^2*b*c^3*e*h^2+384*a^2*b*c^3*f*g*h-128*a^2*c^4*d*h^2-256*a^2*c^4*e*g*h-
128*a^2*c^4*f*g^2+140*a*b^4*c*f*h^2-160*a*b^3*c^2*e*h^2-320*a*b^3*c^2*f*g*h
+192*a*b^2*c^3*d*h^2+384*a*b^2*c^3*e*g*h+192*a*b^2*c^3*f*g^2-512*a*b*c^4*d*
g*h-256*a*b*c^4*e*g^2+512*a*c^5*d*g^2-21*b^6*f*h^2+28*b^5*c*e*h^2+56*b^5*c*
f*g*h-40*b^4*c^2*d*h^2-80*b^4*c^2*e*g*h-40*b^4*c^2*f*g^2+128*b^3*c^3*d*g*h+
64*b^3*c^3*e*g^2-128*b^2*c^4*d*g^2)/c^(11/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+
b*x+a)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.69 (sec) , antiderivative size = 1791, normalized size of antiderivative = 3.07

$$\int (g+hx)^2\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx=\text{Too large to display}$$

```
[In] integrate((h*x+g)^2*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [-1/30720*(15*(8*(16*(b^2*c^4 - 4*a*c^5)*d - 8*(b^3*c^3 - 4*a*b*c^4)*e + (5
*b^4*c^2 - 24*a*b^2*c^3 + 16*a^2*c^4)*f)*g^2 - 8*(16*(b^3*c^3 - 4*a*b*c^4)*
d - 2*(5*b^4*c^2 - 24*a*b^2*c^3 + 16*a^2*c^4)*e + (7*b^5*c - 40*a*b^3*c^2 +
 48*a^2*b*c^3)*f)*g*h + (8*(5*b^4*c^2 - 24*a*b^2*c^3 + 16*a^2*c^4)*d - 4*(7
*b^5*c - 40*a*b^3*c^2 + 48*a^2*b*c^3)*e + (21*b^6 - 140*a*b^4*c + 240*a^2*b
^2*c^2 - 64*a^3*c^3)*f)*h^2)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqr
t(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(1280*c^6*f*h^2*x^5 + 1
28*(24*c^6*f*g*h + (12*c^6*e + b*c^5*f)*h^2)*x^4 + 16*(120*c^6*f*g^2 + 24*(
10*c^6*e + b*c^5*f)*g*h + (120*c^6*d + 12*b*c^5*e - (9*b^2*c^4 - 20*a*c^5)*
f)*h^2)*x^3 + 40*(48*b*c^5*d - 8*(3*b^2*c^4 - 8*a*c^5)*e + (15*b^3*c^3 - 52
*a*b*c^4)*f)*g^2 - 8*(80*(3*b^2*c^4 - 8*a*c^5)*d - 10*(15*b^3*c^3 - 52*a*b*
c^4)*e + (105*b^4*c^2 - 460*a*b^2*c^3 + 256*a^2*c^4)*f)*g*h + (40*(15*b^3*c
^3 - 52*a*b*c^4)*d - 4*(105*b^4*c^2 - 460*a*b^2*c^3 + 256*a^2*c^4)*e + (315
*b^5*c - 1680*a*b^3*c^2 + 1808*a^2*b*c^3)*f)*h^2 + 8*(40*(8*c^6*e + b*c^5*f
)*g^2 + 8*(80*c^6*d + 10*b*c^5*e - (7*b^2*c^4 - 16*a*c^5)*f)*g*h + (40*b*c^
5*d - 4*(7*b^2*c^4 - 16*a*c^5)*e + (21*b^3*c^3 - 68*a*b*c^4)*f)*h^2)*x^2 +
2*(40*(48*c^6*d + 8*b*c^5*e - (5*b^2*c^4 - 12*a*c^5)*f)*g^2 + 8*(80*b*c^5*d
 - 10*(5*b^2*c^4 - 12*a*c^5)*e + (35*b^3*c^3 - 116*a*b*c^4)*f)*g*h - (40*(5
*b^2*c^4 - 12*a*c^5)*d - 4*(35*b^3*c^3 - 116*a*b*c^4)*e + (105*b^4*c^2 - 44
8*a*b^2*c^3 + 240*a^2*c^4)*f)*h^2)*x)*sqrt(c*x^2 + b*x + a))/c^6, 1/15360*(
15*(8*(16*(b^2*c^4 - 4*a*c^5)*d - 8*(b^3*c^3 - 4*a*b*c^4)*e + (5*b^4*c^2 -
24*a*b^2*c^3 + 16*a^2*c^4)*f)*g^2 - 8*(16*(b^3*c^3 - 4*a*b*c^4)*d - 2*(5*b^
4*c^2 - 24*a*b^2*c^3 + 16*a^2*c^4)*e + (7*b^5*c - 40*a*b^3*c^2 + 48*a^2*b*c
^3)*f)*g*h + (8*(5*b^4*c^2 - 24*a*b^2*c^3 + 16*a^2*c^4)*d - 4*(7*b^5*c - 40
*a*b^3*c^2 + 48*a^2*b*c^3)*e + (21*b^6 - 140*a*b^4*c + 240*a^2*b^2*c^2 - 64
*a^3*c^3)*f)*h^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqr
t(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(1280*c^6*f*h^2*x^5 + 128*(24*c^6*f*g*h
+ (12*c^6*e + b*c^5*f)*h^2)*x^4 + 16*(120*c^6*f*g^2 + 24*(10*c^6*e + b*c^5*
f)*g*h + (120*c^6*d + 12*b*c^5*e - (9*b^2*c^4 - 20*a*c^5)*f)*h^2)*x^3 + 40*
(48*b*c^5*d - 8*(3*b^2*c^4 - 8*a*c^5)*e + (15*b^3*c^3 - 52*a*b*c^4)*f)*g^2
- 8*(80*(3*b^2*c^4 - 8*a*c^5)*d - 10*(15*b^3*c^3 - 52*a*b*c^4)*e + (105*b^4
*c^2 - 460*a*b^2*c^3 + 256*a^2*c^4)*f)*g*h + (40*(15*b^3*c^3 - 52*a*b*c^4)*
d - 4*(105*b^4*c^2 - 460*a*b^2*c^3 + 256*a^2*c^4)*e + (315*b^5*c - 1680*a*b
^3*c^2 + 1808*a^2*b*c^3)*f)*h^2 + 8*(40*(8*c^6*e + b*c^5*f)*g^2 + 8*(80*c^6
*d + 10*b*c^5*e - (7*b^2*c^4 - 16*a*c^5)*f)*g*h + (40*b*c^5*d - 4*(7*b^2*c^
4 - 16*a*c^5)*e + (21*b^3*c^3 - 68*a*b*c^4)*f)*h^2)*x^2 + 2*(40*(48*c^6*d +
 8*b*c^5*e - (5*b^2*c^4 - 12*a*c^5)*f)*g^2 + 8*(80*b*c^5*d - 10*(5*b^2*c^4
- 12*a*c^5)*e + (35*b^3*c^3 - 116*a*b*c^4)*f)*g*h - (40*(5*b^2*c^4 - 12*a*c
^5)*d - 4*(35*b^3*c^3 - 116*a*b*c^4)*e + (105*b^4*c^2 - 448*a*b^2*c^3 + 240
*a^2*c^4)*f)*h^2)*x)*sqrt(c*x^2 + b*x + a))/c^6]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2440 vs. $2(607) = 1214$.

Time = 1.19 (sec) , antiderivative size = 2440, normalized size of antiderivative = 4.18

$$\int (g+hx)^2\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx = \text{Too large to display}$$

```
[In] integrate((h*x+g)**2*(f*x**2+e*x+d)*(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Piecewise((sqrt(a + b*x + c*x**2)*(f*h**2*x**5/6 + x**4*(b*f*h**2/12 + c*e*
h**2 + 2*c*f*g*h)/(5*c) + x**3*(a*f*h**2/6 + b*e*h**2 + 2*b*f*g*h - 9*b*(b*
f*h**2/12 + c*e*h**2 + 2*c*f*g*h)/(10*c) + c*d*h**2 + 2*c*e*g*h + c*f*g**2)
/(4*c) + x**2*(a*e*h**2 + 2*a*f*g*h - 4*a*(b*f*h**2/12 + c*e*h**2 + 2*c*f*g
*h)/(5*c) + b*d*h**2 + 2*b*e*g*h + b*f*g**2 - 7*b*(a*f*h**2/6 + b*e*h**2 +
2*b*f*g*h - 9*b*(b*f*h**2/12 + c*e*h**2 + 2*c*f*g*h)/(10*c) + c*d*h**2 + 2*
c*e*g*h + c*f*g**2)/(8*c) + 2*c*d*g*h + c*e*g**2)/(3*c) + x*(a*d*h**2 + 2*a
*e*g*h + a*f*g**2 - 3*a*(a*f*h**2/6 + b*e*h**2 + 2*b*f*g*h - 9*b*(b*f*h**2/
12 + c*e*h**2 + 2*c*f*g*h)/(10*c) + c*d*h**2 + 2*c*e*g*h + c*f*g**2)/(4*c)
+ 2*b*d*g*h + b*e*g**2 - 5*b*(a*e*h**2 + 2*a*f*g*h - 4*a*(b*f*h**2/12 + c*e
*h**2 + 2*c*f*g*h)/(5*c) + b*d*h**2 + 2*b*e*g*h + b*f*g**2 - 7*b*(a*f*h**2/
6 + b*e*h**2 + 2*b*f*g*h - 9*b*(b*f*h**2/12 + c*e*h**2 + 2*c*f*g*h)/(10*c)
+ c*d*h**2 + 2*c*e*g*h + c*f*g**2)/(8*c) + 2*c*d*g*h + c*e*g**2)/(6*c) + c*
d*g**2)/(2*c) + (2*a*d*g*h + a*e*g**2 - 2*a*(a*e*h**2 + 2*a*f*g*h - 4*a*(b*
f*h**2/12 + c*e*h**2 + 2*c*f*g*h)/(5*c) + b*d*h**2 + 2*b*e*g*h + b*f*g**2 -
 7*b*(a*f*h**2/6 + b*e*h**2 + 2*b*f*g*h - 9*b*(b*f*h**2/12 + c*e*h**2 + 2*c
*f*g*h)/(10*c) + c*d*h**2 + 2*c*e*g*h + c*f*g**2)/(8*c) + 2*c*d*g*h + c*e*g
**2)/(3*c) + b*d*g**2 - 3*b*(a*d*h**2 + 2*a*e*g*h + a*f*g**2 - 3*a*(a*f*h**
2/6 + b*e*h**2 + 2*b*f*g*h - 9*b*(b*f*h**2/12 + c*e*h**2 + 2*c*f*g*h)/(10*c
) + c*d*h**2 + 2*c*e*g*h + c*f*g**2)/(4*c) + 2*b*d*g*h + b*e*g**2 - 5*b*(a*
e*h**2 + 2*a*f*g*h - 4*a*(b*f*h**2/12 + c*e*h**2 + 2*c*f*g*h)/(5*c) + b*d*h
**2 + 2*b*e*g*h + b*f*g**2 - 7*b*(a*f*h**2/6 + b*e*h**2 + 2*b*f*g*h - 9*b*(
b*f*h**2/12 + c*e*h**2 + 2*c*f*g*h)/(10*c) + c*d*h**2 + 2*c*e*g*h + c*f*g**
2)/(8*c) + 2*c*d*g*h + c*e*g**2)/(6*c) + c*d*g**2)/(4*c))/c) + (a*d*g**2 -
a*(a*d*h**2 + 2*a*e*g*h + a*f*g**2 - 3*a*(a*f*h**2/6 + b*e*h**2 + 2*b*f*g*h
 - 9*b*(b*f*h**2/12 + c*e*h**2 + 2*c*f*g*h)/(10*c) + c*d*h**2 + 2*c*e*g*h +
 c*f*g**2)/(4*c) + 2*b*d*g*h + b*e*g**2 - 5*b*(a*e*h**2 + 2*a*f*g*h - 4*a*(
b*f*h**2/12 + c*e*h**2 + 2*c*f*g*h)/(5*c) + b*d*h**2 + 2*b*e*g*h + b*f*g**2
 - 7*b*(a*f*h**2/6 + b*e*h**2 + 2*b*f*g*h - 9*b*(b*f*h**2/12 + c*e*h**2 + 2
*c*f*g*h)/(10*c) + c*d*h**2 + 2*c*e*g*h + c*f*g**2)/(8*c) + 2*c*d*g*h + c*e
*g**2)/(6*c) + c*d*g**2)/(2*c) - b*(2*a*d*g*h + a*e*g**2 - 2*a*(a*e*h**2 +
2*a*f*g*h - 4*a*(b*f*h**2/12 + c*e*h**2 + 2*c*f*g*h)/(5*c) + b*d*h**2 + 2*b
*e*g*h + b*f*g**2 - 7*b*(a*f*h**2/6 + b*e*h**2 + 2*b*f*g*h - 9*b*(b*f*h**2/
12 + c*e*h**2 + 2*c*f*g*h)/(10*c) + c*d*h**2 + 2*c*e*g*h + c*f*g**2)/(8*c)
+ 2*c*d*g*h + c*e*g**2)/(3*c) + b*d*g**2 - 3*b*(a*d*h**2 + 2*a*e*g*h + a*f*
```

```
g**2 - 3*a*(a*f*h**2/6 + b*e*h**2 + 2*b*f*g*h - 9*b*(b*f*h**2/12 + c*e*h**2
 + 2*c*f*g*h)/(10*c) + c*d*h**2 + 2*c*e*g*h + c*f*g**2)/(4*c) + 2*b*d*g*h +
 b*e*g**2 - 5*b*(a*e*h**2 + 2*a*f*g*h - 4*a*(b*f*h**2/12 + c*e*h**2 + 2*c*f
*g*h)/(5*c) + b*d*h**2 + 2*b*e*g*h + b*f*g**2 - 7*b*(a*f*h**2/6 + b*e*h**2
+ 2*b*f*g*h - 9*b*(b*f*h**2/12 + c*e*h**2 + 2*c*f*g*h)/(10*c) + c*d*h**2 +
2*c*e*g*h + c*f*g**2)/(8*c) + 2*c*d*g*h + c*e*g**2)/(6*c) + c*d*g**2)/(4*c)
)/(2*c))*Piecewise((log(b + 2*sqrt(c)*sqrt(a + b*x + c*x**2) + 2*c*x)/sqrt(
c), Ne(a - b**2/(4*c), 0)), ((b/(2*c) + x)*log(b/(2*c) + x)/sqrt(c*(b/(2*c)
 + x)**2), True)), Ne(c, 0)), (2*(f*h**2*(a + b*x)**(11/2)/(11*b**4) + (a +
 b*x)**(9/2)*(-4*a*f*h**2 + b*e*h**2 + 2*b*f*g*h)/(9*b**4) + (a + b*x)**(7/
2)*(6*a**2*f*h**2 - 3*a*b*e*h**2 - 6*a*b*f*g*h + b**2*d*h**2 + 2*b**2*e*g*h
 + b**2*f*g**2)/(7*b**4) + (a + b*x)**(5/2)*(-4*a**3*f*h**2 + 3*a**2*b*e*h*
*2 + 6*a**2*b*f*g*h - 2*a*b**2*d*h**2 - 4*a*b**2*e*g*h - 2*a*b**2*f*g**2 +
2*b**3*d*g*h + b**3*e*g**2)/(5*b**4) + (a + b*x)**(3/2)*(a**4*f*h**2 - a**3
*b*e*h**2 - 2*a**3*b*f*g*h + a**2*b**2*d*h**2 + 2*a**2*b**2*e*g*h + a**2*b*
*2*f*g**2 - 2*a*b**3*d*g*h - a*b**3*e*g**2 + b**4*d*g**2)/(3*b**4))/b, Ne(b
, 0)), (sqrt(a)*(d*g**2*x + f*h**2*x**5/5 + x**4*(e*h**2 + 2*f*g*h)/4 + x**
3*(d*h**2 + 2*e*g*h + f*g**2)/3 + x**2*(2*d*g*h + e*g**2)/2), True))
```

Maxima [F(-2)]

Exception generated.

$$\int (g+hx)^2\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((h*x+g)^2*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 983, normalized size of antiderivative = 1.68

$$\int (g+hx)^2\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx$$

$$= \frac{1}{7680}\sqrt{cx^2+bx+a}\left(2\left(4\left(2\left(8\left(10\,fh^2x+\frac{24\,c^5fgh+12\,c^5eh^2+bc^4fh^2}{c^5}\right)x+\frac{120\,c^5fg^2+240\,c^5egh+}{}\right.\right.\right.\right.$$

$$+\frac{(128\,b^2c^4dg^2-512\,ac^5dg^2-64\,b^3c^3eg^2+256\,abc^4eg^2+40\,b^4c^2fg^2-192\,ab^2c^3fg^2+128\,a^2c^4fg^2-128\,b}{}$$

```
[In] integrate((h*x+g)^2*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/7680*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*f*h^2*x + (24*c^5*f*g*h + 12*c
^5*e*h^2 + b*c^4*f*h^2)/c^5)*x + (120*c^5*f*g^2 + 240*c^5*e*g*h + 24*b*c^4*
f*g*h + 120*c^5*d*h^2 + 12*b*c^4*e*h^2 - 9*b^2*c^3*f*h^2 + 20*a*c^4*f*h^2)/
c^5)*x + (320*c^5*e*g^2 + 40*b*c^4*f*g^2 + 640*c^5*d*g*h + 80*b*c^4*e*g*h -
 56*b^2*c^3*f*g*h + 128*a*c^4*f*g*h + 40*b*c^4*d*h^2 - 28*b^2*c^3*e*h^2 + 6
4*a*c^4*e*h^2 + 21*b^3*c^2*f*h^2 - 68*a*b*c^3*f*h^2)/c^5)*x + (1920*c^5*d*g
^2 + 320*b*c^4*e*g^2 - 200*b^2*c^3*f*g^2 + 480*a*c^4*f*g^2 + 640*b*c^4*d*g*
h - 400*b^2*c^3*e*g*h + 960*a*c^4*e*g*h + 280*b^3*c^2*f*g*h - 928*a*b*c^3*f
*g*h - 200*b^2*c^3*d*h^2 + 480*a*c^4*d*h^2 + 140*b^3*c^2*e*h^2 - 464*a*b*c^
3*e*h^2 - 105*b^4*c*f*h^2 + 448*a*b^2*c^2*f*h^2 - 240*a^2*c^3*f*h^2)/c^5)*x
 + (1920*b*c^4*d*g^2 - 960*b^2*c^3*e*g^2 + 2560*a*c^4*e*g^2 + 600*b^3*c^2*f
*g^2 - 2080*a*b*c^3*f*g^2 - 1920*b^2*c^3*d*g*h + 5120*a*c^4*d*g*h + 1200*b^
3*c^2*e*g*h - 4160*a*b*c^3*e*g*h - 840*b^4*c*f*g*h + 3680*a*b^2*c^2*f*g*h -
 2048*a^2*c^3*f*g*h + 600*b^3*c^2*d*h^2 - 2080*a*b*c^3*d*h^2 - 420*b^4*c*e*
h^2 + 1840*a*b^2*c^2*e*h^2 - 1024*a^2*c^3*e*h^2 + 315*b^5*f*h^2 - 1680*a*b^
3*c*f*h^2 + 1808*a^2*b*c^2*f*h^2)/c^5) + 1/1024*(128*b^2*c^4*d*g^2 - 512*a*
c^5*d*g^2 - 64*b^3*c^3*e*g^2 + 256*a*b*c^4*e*g^2 + 40*b^4*c^2*f*g^2 - 192*a
*b^2*c^3*f*g^2 + 128*a^2*c^4*f*g^2 - 128*b^3*c^3*d*g*h + 512*a*b*c^4*d*g*h
+ 80*b^4*c^2*e*g*h - 384*a*b^2*c^3*e*g*h + 256*a^2*c^4*e*g*h - 56*b^5*c*f*g
*h + 320*a*b^3*c^2*f*g*h - 384*a^2*b*c^3*f*g*h + 40*b^4*c^2*d*h^2 - 192*a*b
^2*c^3*d*h^2 + 128*a^2*c^4*d*h^2 - 28*b^5*c*e*h^2 + 160*a*b^3*c^2*e*h^2 - 1
92*a^2*b*c^3*e*h^2 + 21*b^6*f*h^2 - 140*a*b^4*c*f*h^2 + 240*a^2*b^2*c^2*f*h
^2 - 64*a^3*c^3*f*h^2)*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c
) + b))/c^(11/2)
```

Mupad [B] (verification not implemented)

Time = 17.29 (sec) , antiderivative size = 1881, normalized size of antiderivative = 3.22

$$\int (g+hx)^2\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx = \text{Too large to display}$$

```
[In] int((g + h*x)^2*(a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2),x)
```

```
[Out] d*g^2*(x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (e*h^2*x^2*(a + b*x + c*x^2
)^(3/2))/(5*c) + (f*h^2*x^3*(a + b*x + c*x^2)^(3/2))/(6*c) - (a*d*h^2*((x/2
 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b*x +
 c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c) - (a*f*g^2*((x/2 + b/(4*c
))*(a + b*x + c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(
1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c) + (d*g^2*log((b/2 + c*x)/c^(1/2) +
 (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2)) + (e*g^2*log((b + 2*c*
x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) - (2*
a*e*h^2*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b
*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^
```

```
(1/2))/(24*c^2)))/(5*c) - (5*b*d*h^2*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x
 + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2
+ 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) - (5*b*f*g^2*((log((b
+ 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2))
 + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))
/(8*c) + (e*g^2*(8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2)
)/(24*c^2) + (d*h^2*x*(a + b*x + c*x^2)^(3/2))/(4*c) + (f*g^2*x*(a + b*x +
c*x^2)^(3/2))/(4*c) + (a*f*h^2*((5*b*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x
 + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2
+ 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) - (x*(a + b*x + c*x^2)
^(3/2))/(4*c) + (a*((x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (log((b/2 + c
*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c)))
/(2*c) + (7*b*e*h^2*((5*b*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(
1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*
(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) - (x*(a + b*x + c*x^2)^(3/2))/(4*
c) + (a*((x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2)
 + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c)))/(10*c) - (
3*b*f*h^2*((7*b*((5*b*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2)
)*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a +
 b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) - (x*(a + b*x + c*x^2)^(3/2))/(4*c) +
 (a*((x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) + (
a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c)))/(10*c) - (2*a*
((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16
*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(
24*c^2)))/(5*c) + (x^2*(a + b*x + c*x^2)^(3/2))/(5*c)))/(4*c) + (2*f*g*h*x^
2*(a + b*x + c*x^2)^(3/2))/(5*c) - (a*e*g*h*((x/2 + b/(4*c))*(a + b*x + c*x
^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/
4))/(2*c^(3/2))))/(2*c) + (d*g*h*log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x
^2)^(1/2))*(b^3 - 4*a*b*c))/(8*c^(5/2)) - (4*a*f*g*h*((log((b + 2*c*x)/c^(1
/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a +
 c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(5*c) - (5*b
*e*g*h*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*
c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(
1/2))/(24*c^2)))/(4*c) + (d*g*h*(8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*
x + c*x^2)^(1/2))/(12*c^2) + (e*g*h*x*(a + b*x + c*x^2)^(3/2))/(2*c) + (7*b
*f*g*h*((5*b*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 -
4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*
x^2)^(1/2))/(24*c^2)))/(8*c) - (x*(a + b*x + c*x^2)^(3/2))/(4*c) + (a*((x/2
 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b*x +
 c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c)))/(5*c)
```

3.188 $\int (g+hx)\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx$

Optimal result

Integrand size = 30, antiderivative size = 322

$$\begin{aligned}&\int (g+hx)\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx\\&=\frac{\left(32c^3dg-7b^3fh-8c^2(2beg+afg+2bdh+aeh)+2bc(6afh+5b(fg+eh))\right)(b+2cx)\sqrt{a+bx+cx^2}}{128c^4}\\&\quad+\frac{f(g+hx)^2\left(a+bx+cx^2\right)^{3/2}}{5ch}\\&\quad+\frac{\left(35b^2fh^2-16c^2(3fg^2-5h(eg+dh))-2ch(16afh+25b(fg+eh))-6ch(6cfg-10ceh+7bfh)x\right)}{240c^3h}\\&\quad-\frac{\left(b^2-4ac\right)\left(32c^3dg-7b^3fh-8c^2(2beg+afg+2bdh+aeh)+2bc(6afh+5b(fg+eh))\right)\operatorname{arctanh}\left(\right.}{256c^{9/2}}\end{aligned}$$

```
[Out] 1/5*f*(h*x+g)^2*(c*x^2+b*x+a)^(3/2)/c/h+1/240*(35*b^2*f*h^2-16*c^2*(3*f*g^2
-5*h*(d*h+e*g))-2*c*h*(16*a*f*h+25*b*(e*h+f*g))-6*c*h*(7*b*f*h-10*c*e*h+6*c
*f*g)*x)*(c*x^2+b*x+a)^(3/2)/c^3/h-1/256*(-4*a*c+b^2)*(32*c^3*d*g-7*b^3*f*h
-8*c^2*(a*e*h+a*f*g+2*b*d*h+2*b*e*g)+2*b*c*(6*a*f*h+5*b*(e*h+f*g)))*arctanh
(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(9/2)+1/128*(32*c^3*d*g-7*b^3
*f*h-8*c^2*(a*e*h+a*f*g+2*b*d*h+2*b*e*g)+2*b*c*(6*a*f*h+5*b*(e*h+f*g)))*(2*
c*x+b)*(c*x^2+b*x+a)^(1/2)/c^4
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1667, 793, 626, 635, 212}

$$\int (g+hx)\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx =$$

$$-\frac{(b^2-4ac)\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-8c^2(aeh+afg+2bdh+2beg)+2bc(6afh+5b(eh+fg))-7b^3f\right.}{256c^{9/2}}$$

$$+\frac{(b+2cx)\sqrt{a+bx+cx^2}(-8c^2(aeh+afg+2bdh+2beg)+2bc(6afh+5b(eh+fg))-7b^3fh+32c^3dg}{128c^4}$$

$$+\frac{(a+bx+cx^2)^{3/2}\left(-2ch(16afh+25b(eh+fg))+35b^2fh^2-6chx(7bfh-10ceh+6cfg)-(c^2(48fg^2\right.}{240c^3h}$$

$$+\frac{f(g+hx)^2\left(a+bx+cx^2\right)^{3/2}}{5ch}$$

```
[In] Int[(g + h*x)*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] ((32*c^3*d*g - 7*b^3*f*h - 8*c^2*(2*b*e*g + a*f*g + 2*b*d*h + a*e*h) + 2*b*
c*(6*a*f*h + 5*b*(f*g + e*h)))*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(128*c^4)
 + (f*(g + h*x)^2*(a + b*x + c*x^2)^(3/2))/(5*c*h) + ((35*b^2*f*h^2 - c^2*(
48*f*g^2 - 80*h*(e*g + d*h)) - 2*c*h*(16*a*f*h + 25*b*(f*g + e*h)) - 6*c*h*
(6*c*f*g - 10*c*e*h + 7*b*f*h)*x)*(a + b*x + c*x^2)^(3/2))/(240*c^3*h) - ((
b^2 - 4*a*c)*(32*c^3*d*g - 7*b^3*f*h - 8*c^2*(2*b*e*g + a*f*g + 2*b*d*h + a
*e*h) + 2*b*c*(6*a*f*h + 5*b*(f*g + e*h)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*S
qrt[a + b*x + c*x^2])])/(256*c^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
```

```
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\text{integral} = \frac{f(g+hx)^2\left(a+bx+cx^2\right)^{3/2}}{5ch} + \frac{\int (g+hx)\left(-\frac{1}{2}h(3bfg-10cdh+4afh)-\frac{1}{2}h(6cfg-10ceh+7bfh)x\right)\sqrt{a+bx+cx^2}\,dx}{5ch^2}$$

$$= \frac{f(g+hx)^2\left(a+bx+cx^2\right)^{3/2}}{5ch} + \frac{(35b^2fh^2-c^2(48fg^2-80h(eg+dh))-2ch(16afh+25b(fg+eh))-6ch(6cfg-10ceh+7b}{240c^3h} + \frac{(32c^3dg-7b^3fh-8c^2(2beg+afg+2bdh+aeh)+2bc(6afh+5b(fg+eh)))\int\sqrt{a+bx+cx}}{32c^3}$$

$$= \frac{(32c^3dg-7b^3fh-8c^2(2beg+afg+2bdh+aeh)+2bc(6afh+5b(fg+eh)))(b+2cx)\sqrt{a+bx}}{128c^4} + \frac{f(g+hx)^2\left(a+bx+cx^2\right)^{3/2}}{5ch} + \frac{(35b^2fh^2-c^2(48fg^2-80h(eg+dh))-2ch(16afh+25b(fg+eh))-6ch(6cfg-10ceh+7b}{240c^3h} - \frac{((b^2-4ac)(32c^3dg-7b^3fh-8c^2(2beg+afg+2bdh+aeh)+2bc(6afh+5b(fg+eh))))\int}{256c^4}$$

$$= \frac{(32c^3dg - 7b^3fh - 8c^2(2beg + afg + 2bdh + aeh) + 2bc(6afh + 5b(fg + eh)))\,(b + 2cx)\sqrt{a + bx}}{128c^4}$$

$$+ \frac{f(g + hx)^2\,(a + bx + cx^2)^{3/2}}{5ch}$$

$$+ \frac{(35b^2fh^2 - c^2(48fg^2 - 80h(eg + dh)) - 2ch(16afh + 25b(fg + eh)) - 6ch(6cfg - 10ceh + 7bf}{240c^3h}$$

$$- \frac{((b^2 - 4ac)\,(32c^3dg - 7b^3fh - 8c^2(2beg + afg + 2bdh + aeh) + 2bc(6afh + 5b(fg + eh))))\,\text{Subs}}{128c^4}$$

$$= \frac{(32c^3dg - 7b^3fh - 8c^2(2beg + afg + 2bdh + aeh) + 2bc(6afh + 5b(fg + eh)))\,(b + 2cx)\sqrt{a + bx}}{128c^4}$$

$$+ \frac{f(g + hx)^2\,(a + bx + cx^2)^{3/2}}{5ch}$$

$$+ \frac{(35b^2fh^2 - c^2(48fg^2 - 80h(eg + dh)) - 2ch(16afh + 25b(fg + eh)) - 6ch(6cfg - 10ceh + 7bf}{240c^3h}$$

$$- \frac{(b^2 - 4ac)\,(32c^3dg - 7b^3fh - 8c^2(2beg + afg + 2bdh + aeh) + 2bc(6afh + 5b(fg + eh)))\tanh^{-}}{256c^{9/2}}$$

Mathematica [A] (verified)

Time = 3.46 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.07

$$\int (g + hx)\sqrt{a + bx + cx^2}\left(d + ex + fx^2\right)\,dx$$

$$= \frac{\sqrt{c}\sqrt{a + x(b + cx)}(-105b^4fh + 10b^3c(15fg + 15eh + 7fhx) - 4b^2c(-115afh + c(60eg + 60dh + 25fgx}{}$$

```
[In] Integrate[(g + h*x)*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] (Sqrt[c]*Sqrt[a + x*(b + c*x)]*(-105*b^4*f*h + 10*b^3*c*(15*f*g + 15*e*h +
7*f*h*x) - 4*b^2*c*(-115*a*f*h + c*(60*e*g + 60*d*h + 25*f*g*x + 25*e*h*x +
 14*f*h*x^2)) + 8*b*c^2*(20*c*d*(3*g + h*x) - a*(65*f*g + 65*e*h + 29*f*h*x
) + 2*c*x*(5*e*(2*g + h*x) + f*x*(5*g + 3*h*x))) + 16*c^2*(-16*a^2*f*h + a*
c*(40*d*h + 5*e*(8*g + 3*h*x) + f*x*(15*g + 8*h*x)) + 2*c^2*x*(10*d*(3*g +
2*h*x) + x*(5*e*(4*g + 3*h*x) + 3*f*x*(5*g + 4*h*x))))) + 15*(b^2 - 4*a*c)*
(-32*c^3*d*g + 7*b^3*f*h + 8*c^2*(2*b*e*g + a*f*g + 2*b*d*h + a*e*h) - 2*b*
c*(6*a*f*h + 5*b*(f*g + e*h)))*ArcTanh[(Sqrt[c]*x)/(-Sqrt[a] + Sqrt[a + x*(
b + c*x)])])/(1920*c^(9/2))
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 488, normalized size of antiderivative = 1.52

method	result
risch	$-\frac{\left(-384hf\,c^4x^4-48b\,c^3fh\,x^3-480c^4eh\,x^3-480c^4fg\,x^3-128a\,c^3fh\,x^2+56b^2c^2fh\,x^2-80b\,c^3eh\,x^2-80b\,c^3fg\,x^2-640c^4dh\,x^2-640c^4\right.}{}$
default	$dg\left(\frac{(2cx+b)\sqrt{c\,x^2+bx+a}}{4c}+\frac{(4ac-b^2)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{8c^{\frac{3}{2}}}\right)+hf\left(\frac{x^2(c\,x^2+bx+a)^{\frac{3}{2}}}{5c}-\frac{7b\left(\frac{x(c\,x^2+bx+a)^{\frac{3}{2}}}{4c}-\frac{5b\left(\right.}{}\right)}{}\right.$

```
[In] int((h*x+g)*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/1920*(-384*c^4*f*h*x^4-48*b*c^3*f*h*x^3-480*c^4*e*h*x^3-480*c^4*f*g*x^3-
128*a*c^3*f*h*x^2+56*b^2*c^2*f*h*x^2-80*b*c^3*e*h*x^2-80*b*c^3*f*g*x^2-640*
c^4*d*h*x^2-640*c^4*e*g*x^2+232*a*b*c^2*f*h*x-240*a*c^3*e*h*x-240*a*c^3*f*g
*x-70*b^3*c*f*h*x+100*b^2*c^2*e*h*x+100*b^2*c^2*f*g*x-160*b*c^3*d*h*x-160*b
*c^3*e*g*x-960*c^4*d*g*x+256*a^2*c^2*f*h-460*a*b^2*c*f*h+520*a*b*c^2*e*h+52
0*a*b*c^2*f*g-640*a*c^3*d*h-640*a*c^3*e*g+105*b^4*f*h-150*b^3*c*e*h-150*b^3
*c*f*g+240*b^2*c^2*d*h+240*b^2*c^2*e*g-480*b*c^3*d*g)/c^4*(c*x^2+b*x+a)^(1/
2)+1/256*(48*a^2*b*c^2*f*h-32*a^2*c^3*e*h-32*a^2*c^3*f*g-40*a*b^3*c*f*h+48*
a*b^2*c^2*e*h+48*a*b^2*c^2*f*g-64*a*b*c^3*d*h-64*a*b*c^3*e*g+128*a*c^4*d*g+
7*b^5*f*h-10*b^4*c*e*h-10*b^4*c*f*g+16*b^3*c^2*d*h+16*b^3*c^2*e*g-32*b^2*c^
3*d*g)/c^(9/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.46 (sec) , antiderivative size = 1009, normalized size of antiderivative = 3.13

$$\int (g+hx)\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx=\text{Too large to display}$$

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/7680*(15*(2*(16*(b^2*c^3 - 4*a*c^4)*d - 8*(b^3*c^2 - 4*a*b*c^3)*e + (5*
b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*f)*g - (16*(b^3*c^2 - 4*a*b*c^3)*d - 2*(
5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*e + (7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2
)*f)*h)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2
*c*x + b)*sqrt(c) - 4*a*c) - 4*(384*c^5*f*h*x^4 + 48*(10*c^5*f*g + (10*c^5*
e + b*c^4*f)*h)*x^3 + 8*(10*(8*c^5*e + b*c^4*f)*g + (80*c^5*d + 10*b*c^4*e
- (7*b^2*c^3 - 16*a*c^4)*f)*h)*x^2 + 10*(48*b*c^4*d - 8*(3*b^2*c^3 - 8*a*c^
4)*e + (15*b^3*c^2 - 52*a*b*c^3)*f)*g - (80*(3*b^2*c^3 - 8*a*c^4)*d - 10*(1
5*b^3*c^2 - 52*a*b*c^3)*e + (105*b^4*c - 460*a*b^2*c^2 + 256*a^2*c^3)*f)*h
+ 2*(10*(48*c^5*d + 8*b*c^4*e - (5*b^2*c^3 - 12*a*c^4)*f)*g + (80*b*c^4*d -
 10*(5*b^2*c^3 - 12*a*c^4)*e + (35*b^3*c^2 - 116*a*b*c^3)*f)*h)*x)*sqrt(c*x
^2 + b*x + a))/c^5, 1/3840*(15*(2*(16*(b^2*c^3 - 4*a*c^4)*d - 8*(b^3*c^2 -
4*a*b*c^3)*e + (5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*f)*g - (16*(b^3*c^2 -
4*a*b*c^3)*d - 2*(5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*e + (7*b^5 - 40*a*b^
3*c + 48*a^2*b*c^2)*f)*h)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x
+ b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(384*c^5*f*h*x^4 + 48*(10*c^5*f*
g + (10*c^5*e + b*c^4*f)*h)*x^3 + 8*(10*(8*c^5*e + b*c^4*f)*g + (80*c^5*d +
 10*b*c^4*e - (7*b^2*c^3 - 16*a*c^4)*f)*h)*x^2 + 10*(48*b*c^4*d - 8*(3*b^2*
c^3 - 8*a*c^4)*e + (15*b^3*c^2 - 52*a*b*c^3)*f)*g - (80*(3*b^2*c^3 - 8*a*c^
4)*d - 10*(15*b^3*c^2 - 52*a*b*c^3)*e + (105*b^4*c - 460*a*b^2*c^2 + 256*a^
2*c^3)*f)*h + 2*(10*(48*c^5*d + 8*b*c^4*e - (5*b^2*c^3 - 12*a*c^4)*f)*g + (
80*b*c^4*d - 10*(5*b^2*c^3 - 12*a*c^4)*e + (35*b^3*c^2 - 116*a*b*c^3)*f)*h)
*x)*sqrt(c*x^2 + b*x + a))/c^5]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 993 vs. $2(332) = 664$.

Time = 1.04 (sec) , antiderivative size = 993, normalized size of antiderivative = 3.08

$$\int (g+hx)\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx$$

$$= \begin{cases} \sqrt{a+bx+cx^2}\left(\frac{fhx^4}{5} + \frac{x^3\left(\frac{bfh}{10}+ceh+cfg\right)}{4c} + \frac{x^2\left(\frac{afh}{5}+beh+bfg-\frac{7b\left(\frac{bfh}{10}+ceh+cfg\right)}{8c}+cdh+ceg\right)}{3c} + \frac{x\left(aeh+afg-\frac{3a\left(\frac{bfh}{10}+ceh+\right.}{4c}\right.}{}\right. \\ \frac{2\left(\frac{fh(a+bx)^{\frac{9}{2}}}{9b^3}+\frac{(a+bx)^{\frac{7}{2}}(-3afh+beh+bfg)}{7b^3}+\frac{(a+bx)^{\frac{5}{2}}\cdot\left(3a^2fh-2abeh-2abfg+b^2dh+b^2eg\right)}{5b^3}+\frac{(a+bx)^{\frac{3}{2}}\left(-a^3fh+a^2beh+a^2bfg-ab^2dh-ab^2eg+b^3dg\right)}{3b^3}\right)}{b} \\ \sqrt{a}\left(dgx+\frac{fhx^4}{4}+\frac{x^3(eh+fg)}{3}+\frac{x^2(dh+eg)}{2}\right) \end{cases}$$

```
[In] integrate((h*x+g)*(f*x**2+e*x+d)*(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Piecewise((sqrt(a + b*x + c*x**2)*(f*h*x**4/5 + x**3*(b*f*h/10 + c*e*h + c*
f*g)/(4*c) + x**2*(a*f*h/5 + b*e*h + b*f*g - 7*b*(b*f*h/10 + c*e*h + c*f*g)
/(8*c) + c*d*h + c*e*g)/(3*c) + x*(a*e*h + a*f*g - 3*a*(b*f*h/10 + c*e*h +
c*f*g)/(4*c) + b*d*h + b*e*g - 5*b*(a*f*h/5 + b*e*h + b*f*g - 7*b*(b*f*h/10
 + c*e*h + c*f*g)/(8*c) + c*d*h + c*e*g)/(6*c) + c*d*g)/(2*c) + (a*d*h + a*
e*g - 2*a*(a*f*h/5 + b*e*h + b*f*g - 7*b*(b*f*h/10 + c*e*h + c*f*g)/(8*c) +
 c*d*h + c*e*g)/(3*c) + b*d*g - 3*b*(a*e*h + a*f*g - 3*a*(b*f*h/10 + c*e*h
+ c*f*g)/(4*c) + b*d*h + b*e*g - 5*b*(a*f*h/5 + b*e*h + b*f*g - 7*b*(b*f*h/
10 + c*e*h + c*f*g)/(8*c) + c*d*h + c*e*g)/(6*c) + c*d*g)/(4*c))/c) + (a*d*
g - a*(a*e*h + a*f*g - 3*a*(b*f*h/10 + c*e*h + c*f*g)/(4*c) + b*d*h + b*e*g
 - 5*b*(a*f*h/5 + b*e*h + b*f*g - 7*b*(b*f*h/10 + c*e*h + c*f*g)/(8*c) + c*
d*h + c*e*g)/(6*c) + c*d*g)/(2*c) - b*(a*d*h + a*e*g - 2*a*(a*f*h/5 + b*e*h
 + b*f*g - 7*b*(b*f*h/10 + c*e*h + c*f*g)/(8*c) + c*d*h + c*e*g)/(3*c) + b*
d*g - 3*b*(a*e*h + a*f*g - 3*a*(b*f*h/10 + c*e*h + c*f*g)/(4*c) + b*d*h + b
*e*g - 5*b*(a*f*h/5 + b*e*h + b*f*g - 7*b*(b*f*h/10 + c*e*h + c*f*g)/(8*c)
+ c*d*h + c*e*g)/(6*c) + c*d*g)/(4*c))/(2*c))*Piecewise((log(b + 2*sqrt(c)*
sqrt(a + b*x + c*x**2) + 2*c*x)/sqrt(c), Ne(a - b**2/(4*c), 0)), ((b/(2*c)
```

```
+ x)*log(b/(2*c) + x)/sqrt(c*(b/(2*c) + x)**2), True)), Ne(c, 0)), (2*(f*h*
(a + b*x)**(9/2)/(9*b**3) + (a + b*x)**(7/2)*(-3*a*f*h + b*e*h + b*f*g)/(7*
b**3) + (a + b*x)**(5/2)*(3*a**2*f*h - 2*a*b*e*h - 2*a*b*f*g + b**2*d*h + b
**2*e*g)/(5*b**3) + (a + b*x)**(3/2)*(-a**3*f*h + a**2*b*e*h + a**2*b*f*g -
 a*b**2*d*h - a*b**2*e*g + b**3*d*g)/(3*b**3))/b, Ne(b, 0)), (sqrt(a)*(d*g*
x + f*h*x**4/4 + x**3*(e*h + f*g)/3 + x**2*(d*h + e*g)/2), True))
```

Maxima [F(-2)]

Exception generated.

$$\int (g+hx)\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 478, normalized size of antiderivative = 1.48

$$\int (g+hx)\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx$$
$$= \frac{1}{1920}\sqrt{cx^2+bx+a}\left(2\left(4\left(6\left(8\,fhx+\frac{10\,c^4fg+10\,c^4eh+bc^3fh}{c^4}\right)x+\frac{80\,c^4eg+10\,bc^3fg+80\,c^4dh+1}{c^4}\right.\right.\right.$$
$$+\frac{(32\,b^2c^3dg-128\,ac^4dg-16\,b^3c^2eg+64\,abc^3eg+10\,b^4cfg-48\,ab^2c^2fg+32\,a^2c^3fg-16\,b^3c^2dh+64\,ab}{}$$

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/1920*sqrt(c*x^2 + b*x + a)*(2*(4*(6*(8*f*h*x + (10*c^4*f*g + 10*c^4*e*h +
 b*c^3*f*h)/c^4)*x + (80*c^4*e*g + 10*b*c^3*f*g + 80*c^4*d*h + 10*b*c^3*e*h
 - 7*b^2*c^2*f*h + 16*a*c^3*f*h)/c^4)*x + (480*c^4*d*g + 80*b*c^3*e*g - 50*
b^2*c^2*f*g + 120*a*c^3*f*g + 80*b*c^3*d*h - 50*b^2*c^2*e*h + 120*a*c^3*e*h
 + 35*b^3*c*f*h - 116*a*b*c^2*f*h)/c^4)*x + (480*b*c^3*d*g - 240*b^2*c^2*e*
g + 640*a*c^3*e*g + 150*b^3*c*f*g - 520*a*b*c^2*f*g - 240*b^2*c^2*d*h + 640
*a*c^3*d*h + 150*b^3*c*e*h - 520*a*b*c^2*e*h - 105*b^4*f*h + 460*a*b^2*c*f*
h - 256*a^2*c^2*f*h)/c^4) + 1/256*(32*b^2*c^3*d*g - 128*a*c^4*d*g - 16*b^3*
c^2*e*g + 64*a*b*c^3*e*g + 10*b^4*c*f*g - 48*a*b^2*c^2*f*g + 32*a^2*c^3*f*g
 - 16*b^3*c^2*d*h + 64*a*b*c^3*d*h + 10*b^4*c*e*h - 48*a*b^2*c^2*e*h + 32*a
^2*c^3*e*h - 7*b^5*f*h + 40*a*b^3*c*f*h - 48*a^2*b*c^2*f*h)*log(abs(2*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(9/2)
```

Mupad [B] (verification not implemented)

Time = 15.10 (sec) , antiderivative size = 877, normalized size of antiderivative = 2.72

$$\int (g+hx)\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx=d\,g\,\left(\frac{x}{2}+\frac{b}{4\,c}\right)\,\sqrt{c\,x^2+b\,x+a}$$

$$-\frac{2\,a\,f\,h\,\left(\frac{\ln\left(\frac{b+2\,c\,x}{\sqrt{c}}+2\,\sqrt{c\,x^2+b\,x+a}\right)\,\left(b^3-4\,a\,b\,c\right)}{16\,c^{5/2}}+\frac{\left(-3\,b^2+2\,c\,x\,b+8\,c\,\left(c\,x^2+a\right)\right)\,\sqrt{c\,x^2+b\,x+a}}{24\,c^2}\right)}{5\,c}$$

$$-\frac{5\,b\,e\,h\,\left(\frac{\ln\left(\frac{b+2\,c\,x}{\sqrt{c}}+2\,\sqrt{c\,x^2+b\,x+a}\right)\,\left(b^3-4\,a\,b\,c\right)}{16\,c^{5/2}}+\frac{\left(-3\,b^2+2\,c\,x\,b+8\,c\,\left(c\,x^2+a\right)\right)\,\sqrt{c\,x^2+b\,x+a}}{24\,c^2}\right)}{8\,c}$$

$$-\frac{5\,b\,f\,g\,\left(\frac{\ln\left(\frac{b+2\,c\,x}{\sqrt{c}}+2\,\sqrt{c\,x^2+b\,x+a}\right)\,\left(b^3-4\,a\,b\,c\right)}{16\,c^{5/2}}+\frac{\left(-3\,b^2+2\,c\,x\,b+8\,c\,\left(c\,x^2+a\right)\right)\,\sqrt{c\,x^2+b\,x+a}}{24\,c^2}\right)}{8\,c}$$

$$+\frac{d\,h\,\left(-3\,b^2+2\,c\,x\,b+8\,c\,\left(c\,x^2+a\right)\right)\,\sqrt{c\,x^2+b\,x+a}}{24\,c^2}$$

$$+\frac{e\,g\,\left(-3\,b^2+2\,c\,x\,b+8\,c\,\left(c\,x^2+a\right)\right)\,\sqrt{c\,x^2+b\,x+a}}{24\,c^2}$$

$$+\frac{e\,h\,x\,{\left(c\,x^2+b\,x+a\right)}^{3/2}}{4\,c}+\frac{f\,g\,x\,{\left(c\,x^2+b\,x+a\right)}^{3/2}}{4\,c}$$

$$+\frac{7\,b\,f\,h\,\left(\frac{5\,b\,\left(\frac{\ln\left(\frac{b+2\,c\,x}{\sqrt{c}}+2\,\sqrt{c\,x^2+b\,x+a}\right)\,\left(b^3-4\,a\,b\,c\right)}{16\,c^{5/2}}+\frac{\left(-3\,b^2+2\,c\,x\,b+8\,c\,\left(c\,x^2+a\right)\right)\,\sqrt{c\,x^2+b\,x+a}}{24\,c^2}\right)}{8\,c}-\frac{x\,{\left(c\,x^2+b\,x+a\right)}^{3/2}}{4\,c}+\frac{a\,\left(\left(\frac{x}{2}+\frac{b}{4\,c}\right)\right.}{}\right.}{10\,c}$$

$$+\frac{f\,h\,x^2\,{\left(c\,x^2+b\,x+a\right)}^{3/2}}{5\,c}$$

$$-\frac{a\,e\,h\,\left(\left(\frac{x}{2}+\frac{b}{4\,c}\right)\,\sqrt{c\,x^2+b\,x+a}+\frac{\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x+a}\right)\,\left(a\,c-\frac{b^2}{4}\right)}{2\,c^{3/2}}\right)}{4\,c}$$

$$-\frac{a\,f\,g\,\left(\left(\frac{x}{2}+\frac{b}{4\,c}\right)\,\sqrt{c\,x^2+b\,x+a}+\frac{\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x+a}\right)\,\left(a\,c-\frac{b^2}{4}\right)}{2\,c^{3/2}}\right)}{4\,c}$$

$$+\frac{d\,g\,\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x+a}\right)\,\left(a\,c-\frac{b^2}{4}\right)}{2\,c^{3/2}}$$

$$+\frac{d\,h\,\ln\left(\frac{b+2\,c\,x}{\sqrt{c}}+2\,\sqrt{c\,x^2+b\,x+a}\right)\,\left(b^3-4\,a\,b\,c\right)}{16\,c^{5/2}}$$

$$+\frac{e\,g\,\ln\left(\frac{b+2\,c\,x}{\sqrt{c}}+2\,\sqrt{c\,x^2+b\,x+a}\right)\,\left(b^3-4\,a\,b\,c\right)}{16\,c^{5/2}}$$

```
[In] int((g + h*x)*(a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2),x)
```

```
[Out] d*g*(x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) - (2*a*f*h*((log((b + 2*c*x)/c^
(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a
 + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(5*c) - (5
*b*e*h*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*
c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(
1/2))/(24*c^2)))/(8*c) - (5*b*f*g*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x +
c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2
*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) + (d*h*(8*c*(a + c*x^2) -
 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2) + (e*g*(8*c*(a + c*x^2)
 - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2) + (e*h*x*(a + b*x + c
*x^2)^(3/2))/(4*c) + (f*g*x*(a + b*x + c*x^2)^(3/2))/(4*c) + (7*b*f*h*((5*b
*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(1
6*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/
(24*c^2)))/(8*c) - (x*(a + b*x + c*x^2)^(3/2))/(4*c) + (a*((x/2 + b/(4*c))*
(a + b*x + c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2
))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c)))/(10*c) + (f*h*x^2*(a + b*x + c*x^2)
^(3/2))/(5*c) - (a*e*h*((x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (log((b/2
 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*
c) - (a*f*g*((x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (log((b/2 + c*x)/c^(
1/2) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c) + (d*g*l
og((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2)
) + (d*h*log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*
c))/(16*c^(5/2)) + (e*g*log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2)
)*(b^3 - 4*a*b*c))/(16*c^(5/2))
```

3.189 $\int \sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 175

$$\begin{aligned}\int \sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx &= \frac{\left(16c^2d-8bce+5b^2f-4acf\right)(b+2cx)\sqrt{a+bx+cx^2}}{64c^3}\\ &\quad+\frac{(8ce-5bf)\left(a+bx+cx^2\right)^{3/2}}{24c^2}+\frac{fx\left(a+bx+cx^2\right)^{3/2}}{4c}\\ &\quad-\frac{\left(b^2-4ac\right)\left(16c^2d+5b^2f-4c(2be+af)\right)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{128c^{7/2}}\end{aligned}$$

```
[Out] 1/24*(-5*b*f+8*c*e)*(c*x^2+b*x+a)^(3/2)/c^2+1/4*f*x*(c*x^2+b*x+a)^(3/2)/c-1
/128*(-4*a*c+b^2)*(16*c^2*d+5*b^2*f-4*c*(a*f+2*b*e))*arctanh(1/2*(2*c*x+b)/
c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(7/2)+1/64*(-4*a*c*f+5*b^2*f-8*b*c*e+16*c^2*
d)*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/c^3
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used

$= \{1675, 654, 626, 635, 212\}$

$$\begin{aligned}
&\int \sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx \\
&= -\frac{(b^2-4ac)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-4c(af+2be)+5b^2f+16c^2d\right)}{128c^{7/2}} \\
&\quad+\frac{(b+2cx)\sqrt{a+bx+cx^2}(-4acf+5b^2f-8bce+16c^2d)}{64c^3} \\
&\quad+\frac{\left(a+bx+cx^2\right)^{3/2}(8ce-5bf)}{24c^2}+\frac{fx\left(a+bx+cx^2\right)^{3/2}}{4c}
\end{aligned}$$

```
[In] Int[Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] ((16*c^2*d - 8*b*c*e + 5*b^2*f - 4*a*c*f)*(b + 2*c*x)*Sqrt[a + b*x + c*x^2]
)/(64*c^3) + ((8*c*e - 5*b*f)*(a + b*x + c*x^2)^(3/2))/(24*c^2) + (f*x*(a +
 b*x + c*x^2)^(3/2))/(4*c) - ((b^2 - 4*a*c)*(16*c^2*d + 5*b^2*f - 4*c*(2*b*
e + a*f))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(128*c^(7
/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{fx(a+bx+cx^2)^{3/2}}{4c}+\frac{\int \left(4cd-af+\frac{1}{2}(8ce-5bf)x\right)\sqrt{a+bx+cx^2}\,dx}{4c}\\
&= \frac{(8ce-5bf)\left(a+bx+cx^2\right)^{3/2}}{24c^2}+\frac{fx(a+bx+cx^2)^{3/2}}{4c}\\
&\quad+\frac{(16c^2d-8bce+5b^2f-4acf)\int\sqrt{a+bx+cx^2}\,dx}{16c^2}\\
&= \frac{(16c^2d-8bce+5b^2f-4acf)\,(b+2cx)\sqrt{a+bx+cx^2}}{64c^3}+\frac{(8ce-5bf)\left(a+bx+cx^2\right)^{3/2}}{24c^2}\\
&\quad+\frac{fx(a+bx+cx^2)^{3/2}}{4c}-\frac{\left((b^2-4ac)\left(16c^2d+5b^2f-4c(2be+af)\right)\right)\int\frac{1}{\sqrt{a+bx+cx^2}}\,dx}{128c^3}\\
&= \frac{(16c^2d-8bce+5b^2f-4acf)\,(b+2cx)\sqrt{a+bx+cx^2}}{64c^3}\\
&\quad+\frac{(8ce-5bf)\left(a+bx+cx^2\right)^{3/2}}{24c^2}+\frac{fx(a+bx+cx^2)^{3/2}}{4c}\\
&\quad-\frac{\left((b^2-4ac)\left(16c^2d+5b^2f-4c(2be+af)\right)\right)\text{Subst}\left(\int\frac{1}{4c-x^2}\,dx,x,\frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{64c^3}\\
&= \frac{(16c^2d-8bce+5b^2f-4acf)\,(b+2cx)\sqrt{a+bx+cx^2}}{64c^3}+\frac{(8ce-5bf)\left(a+bx+cx^2\right)^{3/2}}{24c^2}\\
&\quad+\frac{fx(a+bx+cx^2)^{3/2}}{4c}-\frac{(b^2-4ac)\left(16c^2d+5b^2f-4c(2be+af)\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{128c^{7/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.99

$$\int\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx$$

$$=\frac{\sqrt{c}\sqrt{a+x(b+cx)}(15b^3f-2b^2c(12e+5fx)+4bc(-13af+2c(6d+2ex+fx^2))+8c^2(a(8e+3fx)+}{192c^{7/2}}$$

```
[In] Integrate[Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] (Sqrt[c]*Sqrt[a + x*(b + c*x)]*(15*b^3*f - 2*b^2*c*(12*e + 5*f*x) + 4*b*c*(
-13*a*f + 2*c*(6*d + 2*e*x + f*x^2)) + 8*c^2*(a*(8*e + 3*f*x) + 2*c*x*(6*d
+ 4*e*x + 3*f*x^2))) - 3*(b^2 - 4*a*c)*(16*c^2*d + 5*b^2*f - 4*c*(2*b*e + a
*f))*ArcTanh[(Sqrt[c]*x)/(-Sqrt[a] + Sqrt[a + x*(b + c*x)])])/(192*c^(7/2))
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.13

method	result
risch	$-\frac{\left(-48f\,c^3x^3-8b\,c^2f\,x^2-64c^3e\,x^2-24a\,c^2fx+10b^2cfx-16b\,c^2ex-96c^3dx+52abcf-64a\,c^2e-15b^3f+24b^2ce-48bd\,c^2\right)\sqrt{c\,x^2+bx+a}}{192c^3}$
default	$d\left(\frac{(2cx+b)\sqrt{c\,x^2+bx+a}}{4c}+\frac{\left(4ac-b^2\right)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{8c^{\frac{3}{2}}}\right)+f\left(\frac{x\left(c\,x^2+bx+a\right)^{\frac{3}{2}}}{4c}-\frac{5b\left(\frac{\left(c\,x^2+bx+a\right)^{\frac{3}{2}}}{3c}-\frac{b\left(\frac{(2cx+b)\sqrt{}}{4}\right.}{}\right.}{}\right.$

```
[In] int((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/192*(-48*c^3*f*x^3-8*b*c^2*f*x^2-64*c^3*e*x^2-24*a*c^2*f*x+10*b^2*c*f*x-
16*b*c^2*e*x-96*c^3*d*x+52*a*b*c*f-64*a*c^2*e-15*b^3*f+24*b^2*c*e-48*b*c^2*
d)*(c*x^2+b*x+a)^(1/2)/c^3-1/128*(16*a^2*c^2*f-24*a*b^2*c*f+32*a*b*c^2*e-64
*a*c^3*d+5*b^4*f-8*b^3*c*e+16*b^2*c^2*d)/c^(7/2)*ln((1/2*b+c*x)/c^(1/2)+(c*
x^2+b*x+a)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 465, normalized size of antiderivative = 2.66

$$\int \sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx = \left[\frac{3\left(16\left(b^2c^2-4\,ac^3\right)d-8\left(b^3c-4\,abc^2\right)e+\left(5\,b^4-24\,ab^2c+16\,a^2c^2\right)f\right)\sqrt{c}\log\left(-8\,c^2x^2-8\,bcx-b^2+4\,\sqrt{}\right.}{}\right.$$

```
[In] integrate((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] [1/768*(3*(16*(b^2*c^2 - 4*a*c^3)*d - 8*(b^3*c - 4*a*b*c^2)*e + (5*b^4 - 24
*a*b^2*c + 16*a^2*c^2)*f)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c
*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(48*c^4*f*x^3 + 48*b*c^3*d
 + 8*(8*c^4*e + b*c^3*f)*x^2 - 8*(3*b^2*c^2 - 8*a*c^3)*e + (15*b^3*c - 52*a
*b*c^2)*f + 2*(48*c^4*d + 8*b*c^3*e - (5*b^2*c^2 - 12*a*c^3)*f)*x)*sqrt(c*x
^2 + b*x + a))/c^4, 1/384*(3*(16*(b^2*c^2 - 4*a*c^3)*d - 8*(b^3*c - 4*a*b*c
^2)*e + (5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*f)*sqrt(-c)*arctan(1/2*sqrt(c*x^2
 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(48*c^4*f*x^3
 + 48*b*c^3*d + 8*(8*c^4*e + b*c^3*f)*x^2 - 8*(3*b^2*c^2 - 8*a*c^3)*e + (15
*b^3*c - 52*a*b*c^2)*f + 2*(48*c^4*d + 8*b*c^3*e - (5*b^2*c^2 - 12*a*c^3)*f
)*x)*sqrt(c*x^2 + b*x + a))/c^4]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 384 vs. $2(168) = 336$.

Time = 0.54 (sec) , antiderivative size = 384, normalized size of antiderivative = 2.19

$$\int \sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx$$

$$= \begin{cases} \sqrt{a+bx+cx^2}\left(\frac{fx^3}{4}+\frac{x^2\left(\frac{bf}{8}+ce\right)}{3c}+\frac{x\left(\frac{af}{4}+be-\frac{5b\left(\frac{bf}{8}+ce\right)}{6c}+cd\right)}{2c}+\frac{ae-\frac{2a\left(\frac{bf}{8}+ce\right)}{3c}+bd-\frac{3b\left(\frac{af}{4}+be-\frac{5b\left(\frac{bf}{8}+ce\right)}{6c}+cd\right)}{4c}}{c}\right)+\Bigg(\\ \frac{2\left(\frac{f(a+bx)^{\frac{7}{2}}}{7b^2}+\frac{(a+bx)^{\frac{5}{2}}(-2af+be)}{5b^2}+\frac{(a+bx)^{\frac{3}{2}}\left(a^2f-abe+b^2d\right)}{3b^2}\right)}{b} \\ \sqrt{a}\left(dx+\frac{ex^2}{2}+\frac{fx^3}{3}\right) \end{cases}$$

```
[In] integrate((c*x**2+b*x+a)**(1/2)*(f*x**2+e*x+d),x)
```

```
[Out] Piecewise((sqrt(a + b*x + c*x**2)*(f*x**3/4 + x**2*(b*f/8 + c*e)/(3*c) + x*
(a*f/4 + b*e - 5*b*(b*f/8 + c*e)/(6*c) + c*d)/(2*c) + (a*e - 2*a*(b*f/8 + c
*e)/(3*c) + b*d - 3*b*(a*f/4 + b*e - 5*b*(b*f/8 + c*e)/(6*c) + c*d)/(4*c))/
c) + (a*d - a*(a*f/4 + b*e - 5*b*(b*f/8 + c*e)/(6*c) + c*d)/(2*c) - b*(a*e
- 2*a*(b*f/8 + c*e)/(3*c) + b*d - 3*b*(a*f/4 + b*e - 5*b*(b*f/8 + c*e)/(6*c
) + c*d)/(4*c))/(2*c))*Piecewise((log(b + 2*sqrt(c)*sqrt(a + b*x + c*x**2)
+ 2*c*x)/sqrt(c), Ne(a - b**2/(4*c), 0)), ((b/(2*c) + x)*log(b/(2*c) + x)/s
qrt(c*(b/(2*c) + x)**2), True)), Ne(c, 0)), (2*(f*(a + b*x)**(7/2)/(7*b**2)
 + (a + b*x)**(5/2)*(-2*a*f + b*e)/(5*b**2) + (a + b*x)**(3/2)*(a**2*f - a*
b*e + b**2*d)/(3*b**2))/b, Ne(b, 0)), (sqrt(a)*(d*x + e*x**2/2 + f*x**3/3),
 True))
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.17

$$\int \sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx$$
$$=\frac{1}{192}\sqrt{cx^2+bx+a}\left(2\left(4\left(6\,fx+\frac{8\,c^3e+bc^2f}{c^3}\right)x+\frac{48\,c^3d+8\,bc^2e-5\,b^2cf+12\,ac^2f}{c^3}\right)x+\frac{48\,bc^2d-24}{}\right.$$
$$+\frac{\left(16\,b^2c^2d-64\,ac^3d-8\,b^3ce+32\,abc^2e+5\,b^4f-24\,ab^2cf+16\,a^2c^2f\right)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{}\right.\right.}{128\,c^{\frac{7}{2}}}$$

```
[In] integrate((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] 1/192*sqrt(c*x^2 + b*x + a)*(2*(4*(6*f*x + (8*c^3*e + b*c^2*f)/c^3)*x + (48
*c^3*d + 8*b*c^2*e - 5*b^2*c*f + 12*a*c^2*f)/c^3)*x + (48*b*c^2*d - 24*b^2*
c*e + 64*a*c^2*e + 15*b^3*f - 52*a*b*c*f)/c^3) + 1/128*(16*b^2*c^2*d - 64*a
*c^3*d - 8*b^3*c*e + 32*a*b*c^2*e + 5*b^4*f - 24*a*b^2*c*f + 16*a^2*c^2*f)*
log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(7/2)
```

Mupad [B] (verification not implemented)

Time = 13.54 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.83

$$
\begin{aligned}
&\int \sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx \\
&= d\left(\frac{x}{2}+\frac{b}{4c}\right)\sqrt{c\,x^2+b\,x+a} \\
&\quad-\frac{a\,f\left(\left(\frac{x}{2}+\frac{b}{4\,c}\right)\sqrt{c\,x^2+b\,x+a}+\frac{\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x+a}\right)\left(a\,c-\frac{b^2}{4}\right)}{2\,c^{3/2}}\right)}{4\,c} \\
&\quad+\frac{d\,\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x+a}\right)\left(a\,c-\frac{b^2}{4}\right)}{2\,c^{3/2}} \\
&\quad+\frac{e\,\ln\left(\frac{b+2\,c\,x}{\sqrt{c}}+2\,\sqrt{c\,x^2+b\,x+a}\right)\left(b^3-4\,a\,b\,c\right)}{16\,c^{5/2}} \\
&\quad-\frac{5\,b\,f\left(\frac{\ln\left(\frac{b+2\,c\,x}{\sqrt{c}}+2\,\sqrt{c\,x^2+b\,x+a}\right)\left(b^3-4\,a\,b\,c\right)}{16\,c^{5/2}}+\frac{\left(-3\,b^2+2\,c\,x\,b+8\,c\,\left(c\,x^2+a\right)\right)\sqrt{c\,x^2+b\,x+a}}{24\,c^2}\right)}{8\,c} \\
&\quad+\frac{e\left(-3\,b^2+2\,c\,x\,b+8\,c\left(c\,x^2+a\right)\right)\sqrt{c\,x^2+b\,x+a}}{24\,c^2}+\frac{f\,x\left(c\,x^2+b\,x+a\right)^{3/2}}{4\,c}
\end{aligned}
$$

```
[In] int((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2),x)
```

```
[Out] d*(x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) - (a*f*((x/2 + b/(4*c))*(a + b*x
+ c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c -
 b^2/4))/(2*c^(3/2))))/(4*c) + (d*log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^
2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2)) + (e*log((b + 2*c*x)/c^(1/2) + 2*(a +
b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) - (5*b*f*((log((b + 2*c*x
)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*
c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c)
+ (e*(8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)
+ (f*x*(a + b*x + c*x^2)^(3/2))/(4*c)
```

3.190 $\int \frac{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}{g+hx}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 321

$$\int \frac{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}{g+hx}\,dx = -\frac{(4ch(bfg-2cdh)-(4cg-bh)(2cfg-2ceh+bfh)+2ch(2cfg-2ceh+bfh)x)\sqrt{a+bx+cx^2}}{8c^2h^3} + \frac{f\left(a+bx+cx^2\right)^{3/2}}{3ch} + \frac{(4ch(2cg-bh)(bfg-2cdh)-(2cfg-2ceh+bfh)\left(8c^2g^2-b^2h^2-4ch(bg-ah)\right))\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+c}}\right.}{16c^{5/2}h^4} + \frac{\sqrt{cg^2-bgh+ah^2}\left(fg^2-egh+dh^2\right)\operatorname{arctanh}\left(\frac{bg-2ah+(2cg-bh)x}{2\sqrt{cg^2-bgh+ah^2}\sqrt{a+bx+cx^2}}\right)}{h^4}$$

```
[Out] 1/3*f*(c*x^2+b*x+a)^(3/2)/c/h+1/16*(4*c*h*(-b*h+2*c*g)*(b*f*g-2*c*d*h)-(b*f
*h-2*c*e*h+2*c*f*g)*(8*c^2*g^2-b^2*h^2-4*c*h*(-a*h+b*g)))*arctanh(1/2*(2*c*
x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(5/2)/h^4+(d*h^2-e*g*h+f*g^2)*arctanh(1
/2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^(1/2)/(c*x^2+b*x+a)^(1/2)
)*(a*h^2-b*g*h+c*g^2)^(1/2)/h^4-1/8*(4*c*h*(b*f*g-2*c*d*h)-(-b*h+4*c*g)*(b*
f*h-2*c*e*h+2*c*f*g)+2*c*h*(b*f*h-2*c*e*h+2*c*f*g)*x)*(c*x^2+b*x+a)^(1/2)/c
^2/h^3
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1667, 828, 857, 635, 212, 738}

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{g+hx}\,dx$$

$$= \frac{\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(4ch(2cg-bh)(bfg-2cdh)-\left(-4ch(bg-ah)-b^2h^2+8c^2g^2\right)(bfh-2ceh+2c\right.}{16c^{5/2}h^4}$$

$$+\frac{\sqrt{ah^2-bgh+cg^2}(dh^2-egh+fg^2)\operatorname{arctanh}\left(\frac{-2ah+x(2cg-bh)+bg}{2\sqrt{a+bx+cx^2}\sqrt{ah^2-bgh+cg^2}}\right)}{h^4}$$

$$-\frac{\sqrt{a+bx+cx^2}(4ch(bfg-2cdh)+2chx(bfh-2ceh+2cfg)-(4cg-bh)(bfh-2ceh+2cfg))}{8c^2h^3}$$

$$+\frac{f(a+bx+cx^2)^{3/2}}{3ch}$$

```
[In] Int[(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))/(g + h*x),x]
```

```
[Out] -1/8*((4*c*h*(b*f*g - 2*c*d*h) - (4*c*g - b*h)*(2*c*f*g - 2*c*e*h + b*f*h)
+ 2*c*h*(2*c*f*g - 2*c*e*h + b*f*h)*x)*Sqrt[a + b*x + c*x^2])/(c^2*h^3) + (
f*(a + b*x + c*x^2)^(3/2))/(3*c*h) + ((4*c*h*(2*c*g - b*h)*(b*f*g - 2*c*d*h
) - (2*c*f*g - 2*c*e*h + b*f*h)*(8*c^2*g^2 - b^2*h^2 - 4*c*h*(b*g - a*h)))*
ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(5/2)*h^4) +
(Sqrt[c*g^2 - b*g*h + a*h^2]*(f*g^2 - e*g*h + d*h^2)*ArcTanh[(b*g - 2*a*h +
 (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[a + b*x + c*x^2])])/h
^4
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\text{integral} = \frac{f(a+bx+cx^2)^{3/2}}{3ch} + \frac{\int \frac{\left(-\frac{3}{2}h(bfg-2cdh)-\frac{3}{2}h(2cfg-2ceh+bfh)x\right)\sqrt{a+bx+cx^2}}{g+hx}\,dx}{3ch^2}$$

$$=$$

$$-\frac{(4ch(bfg-2cdh)-(4cg-bh)(2cfg-2ceh+bfh)+2ch(2cfg-2ceh+bfh)x)\sqrt{a+bx+c}}{8c^2h^3}$$

$$+\frac{f\left(a+bx+cx^2\right)^{3/2}}{3ch}$$

$$-\frac{\int\frac{-\frac{3}{4}h\left(4ch(bg-2ah)(bfg-2cdh)-g\left(4bcg-b^2h-4ach\right)(2cfg-2ceh+bfh)\right)-\frac{3}{4}h\left(4ch(2cg-bh)(bfg-2cdh)-2(2cfg-2ceh+bfh)\left(4\right.\right.}{(g+hx)\sqrt{a+bx+cx^2}}}{12c^2h^4}$$

$$=$$

$$-\frac{(4ch(bfg-2cdh)-(4cg-bh)(2cfg-2ceh+bfh)+2ch(2cfg-2ceh+bfh)x)\sqrt{a+bx+c}}{8c^2h^3}$$

$$+\frac{f\left(a+bx+cx^2\right)^{3/2}}{3ch}$$

$$+\frac{\left(\left(cg^2-bgh+ah^2\right)\left(fg^2-h(eg-dh)\right)\right)\int\frac{1}{(g+hx)\sqrt{a+bx+cx^2}}\,dx}{h^4}$$

$$+\frac{\left(4ch(2cg-bh)(bfg-2cdh)-(2cfg-2ceh+bfh)\left(8c^2g^2-b^2h^2-4ch(bg-ah)\right)\right)\int\frac{1}{\sqrt{a+bx+cx}}}{16c^2h^4}$$

$$=$$

$$-\frac{(4ch(bfg-2cdh)-(4cg-bh)(2cfg-2ceh+bfh)+2ch(2cfg-2ceh+bfh)x)\sqrt{a+bx+c}}{8c^2h^3}$$

$$+\frac{f\left(a+bx+cx^2\right)^{3/2}}{3ch}$$

$$-\frac{\left(2\left(cg^2-bgh+ah^2\right)\left(fg^2-h(eg-dh)\right)\right)\mathrm{Subst}\left(\int\frac{1}{4cg^2-4bgh+4ah^2-x^2}\,dx,x,\frac{-bg+2ah-(2cg-bh)x}{\sqrt{a+bx+cx^2}}\right)}{h^4}$$

$$+\frac{\left(4ch(2cg-bh)(bfg-2cdh)-(2cfg-2ceh+bfh)\left(8c^2g^2-b^2h^2-4ch(bg-ah)\right)\right)\mathrm{Subst}\left(\int\frac{}{4}\right.}{8c^2h^4}$$

$$=$$

$$-\frac{(4ch(bfg-2cdh)-(4cg-bh)(2cfg-2ceh+bfh)+2ch(2cfg-2ceh+bfh)x)\sqrt{a+bx+c}}{8c^2h^3}$$

$$+\frac{f\left(a+bx+cx^2\right)^{3/2}}{3ch}$$

$$+\frac{\left(4ch(2cg-bh)(bfg-2cdh)-(2cfg-2ceh+bfh)\left(8c^2g^2-b^2h^2-4ch(bg-ah)\right)\right)\tanh^{-1}\left(\frac{}{2\sqrt{}}\right.}{16c^{5/2}h^4}$$

$$+\frac{\sqrt{cg^2-bgh+ah^2}\left(fg^2-h(eg-dh)\right)\tanh^{-1}\left(\frac{bg-2ah+(2cg-bh)x}{2\sqrt{cg^2-bgh+ah^2}\sqrt{a+bx+cx^2}}\right)}{h^4}$$

Mathematica [A] (verified)

Time = 2.21 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.01

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{g+hx}\,dx$$

$$= \frac{\frac{h\sqrt{a+x(b+cx)}\left(-3b^2fh^2+2ch(4afh+b(-3fg+3eh+fhx))+4c^2\left(3h(-2eg+2dh+ehx)+f\left(6g^2-3ghx+2h^2x^2\right)\right)\right)}{c^2}+48\sqrt{-cg^2+h(bg-a}}{}$$

```
[In] Integrate[(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))/(g + h*x),x]
```

```
[Out] ((h*Sqrt[a + x*(b + c*x)]*(-3*b^2*f*h^2 + 2*c*h*(4*a*f*h + b*(-3*f*g + 3*e*
h + f*h*x)) + 4*c^2*(3*h*(-2*e*g + 2*d*h + e*h*x) + f*(6*g^2 - 3*g*h*x + 2*
h^2*x^2))))/c^2 + 48*Sqrt[-(c*g^2) + h*(b*g - a*h)]*(f*g^2 + h*(-(e*g) + d*
h))*ArcTan[(Sqrt[-(c*g^2) + h*(b*g - a*h)]*x)/(Sqrt[a]*(g + h*x) - g*Sqrt[a
 + x*(b + c*x)])] - (3*(-(b^3*f*h^3) + 2*b*c*h^2*(-(b*f*g) + b*e*h + 2*a*f*
h) + 16*c^3*(f*g^3 + g*h*(-(e*g) + d*h)) - 8*c^2*h*(b*f*g^2 + b*h*(-(e*g) +
 d*h) + a*h*(-(f*g) + e*h)))*ArcTanh[(Sqrt[c]*x)/(-Sqrt[a] + Sqrt[a + x*(b
+ c*x)])])/c^(5/2))/(24*h^4)
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 515, normalized size of antiderivative = 1.60

method	result
risch	$\frac{\left(8f\,h^2c^2x^2+2bcf\,h^2x+12c^2e\,h^2x-12c^2fghx+8acf\,h^2-3b^2f\,h^2+6bce\,h^2-6bcfgh+24c^2d\,h^2-24c^2egh+24c^2f\,g^2\right)\sqrt{c\,x^2+bx+a}}{24c^2h^3}-$
default	$\frac{eh\left(\frac{(2cx+b)\sqrt{c\,x^2+bx+a}}{4c}+\frac{\left(4ac-b^2\right)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{8c^{\frac{3}{2}}}\right)+fh\left(\frac{\left(c\,x^2+bx+a\right)^{\frac{3}{2}}}{3c}-\frac{b\left(\frac{(2cx+b)\sqrt{c\,x^2+bx+a}}{4c}+\frac{\left(4ac-b^2\right)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x\right.}\right.}{8c^{\frac{3}{2}}}\right.}{2c}\right.}{h^2}$

```
[In] int((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*(8*c^2*f*h^2*x^2+2*b*c*f*h^2*x+12*c^2*e*h^2*x-12*c^2*f*g*h*x+8*a*c*f*h
^2-3*b^2*f*h^2+6*b*c*e*h^2-6*b*c*f*g*h+24*c^2*d*h^2-24*c^2*e*g*h+24*c^2*f*g
^2)/c^2*(c*x^2+b*x+a)^(1/2)/h^3-1/16/c^2/h^3*(16*(a*d*h^4-a*e*g*h^3+a*f*g^2
*h^2-b*d*g*h^3+b*e*g^2*h^2-b*f*g^3*h+c*d*g^2*h^2-c*e*g^3*h+c*f*g^4)*c^2/h^2
/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/
```

```
h*(x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*
(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g))+(4*a*b*c*f*h^3-8*a*c^2
*e*h^3+8*a*c^2*f*g*h^2-b^3*f*h^3+2*b^2*c*e*h^3-2*b^2*c*f*g*h^2-8*b*c^2*d*h^
3+8*b*c^2*e*g*h^2-8*b*c^2*f*g^2*h+16*c^3*d*g*h^2-16*c^3*e*g^2*h+16*c^3*f*g^
3)/h*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{g+hx}\,dx=\text{Timed out}$$

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{g+hx}\,dx=\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{g+hx}\,dx$$

```
[In] integrate((f*x**2+e*x+d)*(c*x**2+b*x+a)**(1/2)/(h*x+g),x)
[Out] Integral(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)/(g + h*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{g+hx}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b*h-2*c*g>0)', see `assume?` for mo
re deta
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{g+hx}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{g+hx}\,dx=\int \frac{\sqrt{c\,x^2+b\,x+a}\,\left(f\,x^2+e\,x+d\right)}{g+h\,x}\,dx$$

```
[In] int(((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x),x)
```

```
[Out] int(((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x), x)
```

3.191 $\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 459

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^2}\,dx =$$

$$-\frac{\Big(bfh^2(bg-ah)+4c^2g(3fg^2-h(2eg-dh))+ch(4ah(2fg-eh)-b(13fg^2-8egh+4dh^2))+2ch^2\Big(}{4ch^3\left(cg^2-bgh+ah^2\right)}$$

$$-\frac{(fg^2-h(eg-dh))\left(a+bx+cx^2\right)^{3/2}}{h\left(cg^2-bgh+ah^2\right)(g+hx)}$$

$$-\frac{(b^2fh^2+4ch(2bfg-beh-afh)-8c^2(3fg^2-h(2eg-dh)))\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}h^4}$$

$$-\frac{(2cg(3fg^2-h(2eg-dh))+h(2ah(2fg-eh)-b(5fg^2-3egh+dh^2)))\,\text{arctanh}\Big(\frac{bg-2ah+(2cg-bh)x}{2\sqrt{cg^2-bgh+ah^2}\sqrt{a+bx+c}}}{2h^4\sqrt{cg^2-bgh+ah^2}}$$

```
[Out] -(f*g^2-h*(-d*h+e*g))*(c*x^2+b*x+a)^(3/2)/h/(a*h^2-b*g*h+c*g^2)/(h*x+g)-1/8
*(b^2*f*h^2+4*c*h*(-a*f*h-b*e*h+2*b*f*g)-8*c^2*(3*f*g^2-h*(-d*h+2*e*g)))*ar
ctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(3/2)/h^4-1/2*(2*c*g*(3*
f*g^2-h*(-d*h+2*e*g))+h*(2*a*h*(-e*h+2*f*g)-b*(d*h^2-3*e*g*h+5*f*g^2)))*arc
tanh(1/2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^(1/2)/(c*x^2+b*x+a)
^(1/2))/h^4/(a*h^2-b*g*h+c*g^2)^(1/2)-1/4*(b*f*h^2*(-a*h+b*g)+4*c^2*g*(3*f*
g^2-h*(-d*h+2*e*g))+c*h*(4*a*h*(-e*h+2*f*g)-b*(4*d*h^2-8*e*g*h+13*f*g^2))+2
*c*h^2*(2*c*e*g+b*f*g-3*c*f*g^2/h-2*c*d*h-a*f*h)*x)*(c*x^2+b*x+a)^(1/2)/c/h
^3/(a*h^2-b*g*h+c*g^2)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 453, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1664, 828, 857, 635, 212, 738}

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^2}\,dx$$

$$= -\frac{\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(4ch(-afh-beh+2bfg)+b^2fh^2-8c^2(3fg^2-h(2eg-dh)))}{8c^{3/2}h^4}$$

$$-\frac{\operatorname{arctanh}\left(\frac{-2ah+x(2cg-bh)+bg}{2\sqrt{a+bx+cx^2}\sqrt{ah^2-bgh+cg^2}}\right)(2c(3fg^3-gh(2eg-dh))-h(-2ah(2fg-eh)-bh(3eg-dh)+5bf}{2h^4\sqrt{ah^2-bgh+cg^2}}$$

$$-\frac{\sqrt{a+bx+cx^2}\left(2chx\left(-afh+bfg-2cdh+2ceg-\frac{3cfg^2}{h}\right)+bfh(bg-ah)+4ach(2fg-eh)-bc(4dh}{4ch^2(ah^2-bgh+cg^2)}$$

$$-\frac{(a+bx+cx^2)^{3/2}(fg^2-h(eg-dh))}{h(g+hx)(ah^2-bgh+cg^2)}$$

```
[In] Int[(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^2,x]
```

```
[Out] -1/4*((b*f*h*(b*g - a*h) - 4*c^2*g*(2*e*g - (3*f*g^2)/h - d*h) + 4*a*c*h*(2
*f*g - e*h) - b*c*(13*f*g^2 - 8*e*g*h + 4*d*h^2) + 2*c*h*(2*c*e*g + b*f*g -
 (3*c*f*g^2)/h - 2*c*d*h - a*f*h)*x)*Sqrt[a + b*x + c*x^2])/(c*h^2*(c*g^2 -
 b*g*h + a*h^2)) - ((f*g^2 - h*(e*g - d*h))*(a + b*x + c*x^2)^(3/2))/(h*(c*
g^2 - b*g*h + a*h^2)*(g + h*x)) - ((b^2*f*h^2 + 4*c*h*(2*b*f*g - b*e*h - a*
f*h) - 8*c^2*(3*f*g^2 - h*(2*e*g - d*h)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sq
rt[a + b*x + c*x^2])])/(8*c^(3/2)*h^4) - ((2*c*(3*f*g^3 - g*h*(2*e*g - d*h)
) - h*(5*b*f*g^2 - b*h*(3*e*g - d*h) - 2*a*h*(2*f*g - e*h)))*ArcTanh[(b*g -
 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[a + b*x + c*x
^2])])/(2*h^4*Sqrt[c*g^2 - b*g*h + a*h^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{\left(fg^2-h(eg-dh)\right)\left(a+bx+cx^2\right)^{3/2}}{h\left(cg^2-bgh+ah^2\right)(g+hx)} - \frac{\int \frac{\left(\frac{1}{2}\left(-2cdg+3beg+2afg-\frac{3bfg^2}{h}-bdh-2aeh\right)+\left(2ceg+bfg-\frac{3cfg^2}{h}-2cdh-afh\right)x\right)\sqrt{a+bx+cx^2}}{g+hx}\,dx}{cg^2-bgh+ah^2}$$

$$= -\frac{\Big(bfh(bg-ah)-4c^2g\Big(2eg-\frac{3fg^2}{h}-dh\Big)+4ach(2fg-eh)-bc(13fg^2-8egh+4dh^2)+2ch\Big(}{4ch^2\left(cg^2-bgh+ah^2\right)}$$

$$-\frac{\left(fg^2-h(eg-dh)\right)\left(a+bx+cx^2\right)^{3/2}}{h\left(cg^2-bgh+ah^2\right)(g+hx)}$$

$$+\frac{\int\frac{-\frac{\left(cg^2-bgh+ah^2\right)\left(b^2fgh+4ach(3fg-2eh)-4bc\left(3fg^2-h(2eg-dh)\right)\right)}{2h}-\frac{\left(cg^2-bgh+ah^2\right)\Big(b^2fh^2+4ch(2bfg-beh-afh)-8c^2\Big(3fg^2-h(2eg-dh)}{2h}}{(g+hx)\sqrt{a+bx+cx^2}}}{4ch^2\left(cg^2-bgh+ah^2\right)}$$

$$= -\frac{\Big(bfh(bg-ah)-4c^2g\Big(2eg-\frac{3fg^2}{h}-dh\Big)+4ach(2fg-eh)-bc(13fg^2-8egh+4dh^2)+2ch\Big(}{4ch^2\left(cg^2-bgh+ah^2\right)}$$

$$-\frac{\left(fg^2-h(eg-dh)\right)\left(a+bx+cx^2\right)^{3/2}}{h\left(cg^2-bgh+ah^2\right)(g+hx)}$$

$$-\frac{\left(b^2fh^2+4ch(2bfg-beh-afh)-8c^2(3fg^2-h(2eg-dh))\right)\int\frac{1}{\sqrt{a+bx+cx^2}}\,dx}{8ch^4}$$

$$-\frac{\left(2c(3fg^3-gh(2eg-dh))-h(5bfg^2-bh(3eg-dh)-2ah(2fg-eh))\right)\int\frac{1}{(g+hx)\sqrt{a+bx+cx^2}}\,dx}{2h^4}$$

$$= -\frac{\Big(bfh(bg-ah)-4c^2g\Big(2eg-\frac{3fg^2}{h}-dh\Big)+4ach(2fg-eh)-bc(13fg^2-8egh+4dh^2)+2ch\Big(}{4ch^2\left(cg^2-bgh+ah^2\right)}$$

$$-\frac{\left(fg^2-h(eg-dh)\right)\left(a+bx+cx^2\right)^{3/2}}{h\left(cg^2-bgh+ah^2\right)(g+hx)}$$

$$-\frac{\left(b^2fh^2+4ch(2bfg-beh-afh)-8c^2(3fg^2-h(2eg-dh))\right)\text{Subst}\left(\int\frac{1}{4c-x^2}\,dx,x,\frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{4ch^4}$$

$$+\frac{\left(2c(3fg^3-gh(2eg-dh))-h(5bfg^2-bh(3eg-dh)-2ah(2fg-eh))\right)\text{Subst}\Big(\int\frac{1}{4cg^2-4bgh+4ah^2-}}{h^4}$$

$$= -\frac{\Big(bfh(bg-ah)-4c^2g\Big(2eg-\frac{3fg^2}{h}-dh\Big)+4ach(2fg-eh)-bc(13fg^2-8egh+4dh^2)+2ch\Big(}{4ch^2\left(cg^2-bgh+ah^2\right)}$$

$$-\frac{\left(fg^2-h(eg-dh)\right)\left(a+bx+cx^2\right)^{3/2}}{h\left(cg^2-bgh+ah^2\right)(g+hx)}$$

$$-\frac{\left(b^2fh^2+4ch(2bfg-beh-afh)-8c^2(3fg^2-h(2eg-dh))\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}h^4}$$

$$-\frac{\left(2c(3fg^3-gh(2eg-dh))-h(5bfg^2-bh(3eg-dh)-2ah(2fg-eh))\right)\tanh^{-1}\Big(\frac{bg-2ah+(2cg}{2\sqrt{cg^2-bgh+ah^2}}}{2h^4\sqrt{cg^2-bgh+ah^2}}$$

Mathematica [A] (verified)

Time = 10.51 (sec) , antiderivative size = 430, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^2}\,dx$$

$$=\frac{-8(2fg-eh)\sqrt{a+x(b+cx)}+\frac{2fh(b+2cx)\sqrt{a+x(b+cx)}}{c}-\frac{8(fg^2+h(-eg+dh))\sqrt{a+x(b+cx)}}{g+hx}+\frac{(-b^2+4ac)fh\operatorname{arctanh}\left(\frac{}{2\sqrt{}}\right.}{c^{3/2}}}{}$$

```
[In] Integrate[(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^2,x]
```

```
[Out] (-8*(2*f*g - e*h)*Sqrt[a + x*(b + c*x)] + (2*f*h*(b + 2*c*x)*Sqrt[a + x*(b
+ c*x)])/c - (8*(f*g^2 + h*(-(e*g) + d*h))*Sqrt[a + x*(b + c*x)])/(g + h*x)
 + ((-b^2 + 4*a*c)*f*h*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)]
)])/c^(3/2) + (4*(f*g^2 + h*(-(e*g) + d*h))*(2*Sqrt[c]*ArcTanh[(b + 2*c*x)/
(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] - ((2*c*g - b*h)*ArcTanh[(-2*a*h + 2*c*g
*x + b*(g - h*x))/(2*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*Sqrt[a + x*(b + c*x)])]
)/Sqrt[c*g^2 + h*(-(b*g) + a*h)]))/h + (4*(2*f*g - e*h)*((2*c*g - b*h)*ArcT
anh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] - 2*Sqrt[c]*Sqrt[c*g^2 +
 h*(-(b*g) + a*h)]*ArcTanh[(-2*a*h + 2*c*g*x + b*(g - h*x))/(2*Sqrt[c*g^2 +
 h*(-(b*g) + a*h)]*Sqrt[a + x*(b + c*x)])]))/(Sqrt[c]*h))/(8*h^3)
```

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 704, normalized size of antiderivative = 1.53

method	result
risch	$\frac{(2cfhx+bfh+4ehc-8cfg)\sqrt{c\,x^2+bx+a}}{4c\,h^3}+\frac{\frac{\left(4acf\,h^2-b^2f\,h^2+4bce\,h^2-8bcfgh+8c^2d\,h^2-16c^2egh+24c^2f\,g^2\right)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{h\sqrt{c}}-}{}$
default	Expression too large to display

```
[In] int((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(2*c*f*h*x+b*f*h+4*c*e*h-8*c*f*g)/c*(c*x^2+b*x+a)^(1/2)/h^3+1/8/c/h^3*(
(4*a*c*f*h^2-b^2*f*h^2+4*b*c*e*h^2-8*b*c*f*g*h+8*c^2*d*h^2-16*c^2*e*g*h+24*
c^2*f*g^2)/h*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-8*c/h^2*(a
*e*h^3-2*a*f*g*h^2+b*d*h^3-2*b*e*g*h^2+3*b*f*g^2*h-2*c*d*g*h^2+3*c*e*g^2*h-
```

```
4*c*f*g^3)/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b
*h-2*c*g)/h*(x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c+(b*h
-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g))+8*c*(a*d*h^4
-a*e*g*h^3+a*f*g^2*h^2-b*d*g*h^3+b*e*g^2*h^2-b*f*g^3*h+c*d*g^2*h^2-c*e*g^3*
h+c*f*g^4)/h^3*(-1/(a*h^2-b*g*h+c*g^2)*h^2/(x+1/h*g)*((x+1/h*g)^2*c+(b*h-2*
c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)+1/2*(b*h-2*c*g)*h/(a*h^2-b*
g*h+c*g^2)/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b
*h-2*c*g)/h*(x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c+(b*h
-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^2}\,dx=\text{Timed out}$$

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^2,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^2}\,dx=\int \frac{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}{\left(g+hx\right)^2}\,dx$$

```
[In] integrate((f*x**2+e*x+d)*(c*x**2+b*x+a)**(1/2)/(h*x+g)**2,x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)/(g + h*x)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^2}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^2,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b*h-2*c*g>0)', see `assume?` for mo
re deta
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^2}\,dx = \text{Timed out}$$

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^2,x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^2}\,dx = \int \frac{\sqrt{c\,x^2+b\,x+a}\,\left(f\,x^2+e\,x+d\right)}{{\left(g+h\,x\right)}^2}\,dx$$

```
[In] int(((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^2,x)
[Out] int(((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^2, x)
```

3.192 $\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^3}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 448

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^3}\,dx$$
$$= \frac{\left(\frac{4cg^2(3fg-eh)}{h}+4ah(3fg-eh)-b(11fg^2-3egh-dh^2)-2h\left(ceg+2bfg-\frac{3cfg^2}{h}-cdh-2afh\right)x\right)\sqrt{a+}}{4h^2\left(cg^2-bgh+ah^2\right)(g+hx)}$$
$$-\frac{\left(fg^2-h(eg-dh)\right)\left(a+bx+cx^2\right)^{3/2}}{2h\left(cg^2-bgh+ah^2\right)(g+hx)^2}-\frac{(6cfg-2ceh-bfh)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}h^4}$$
$$+\frac{(8c^2g^3(3fg-eh)-4ch(bg^2(10fg-3eh)-ah(9fg^2-3egh+dh^2))+h^2(8a^2fh^2-4abh(6fg-eh)+b}{8h^4\left(cg^2-bgh+ah^2\right)^{3/2}}$$

```
[Out] -1/2*(f*g^2-h*(-d*h+e*g))*(c*x^2+b*x+a)^(3/2)/h/(a*h^2-b*g*h+c*g^2)/(h*x+g)
^2+1/8*(8*c^2*g^3*(-e*h+3*f*g)-4*c*h*(b*g^2*(-3*e*h+10*f*g)-a*h*(d*h^2-3*e*
g*h+9*f*g^2))+h^2*(8*a^2*f*h^2-4*a*b*h*(-e*h+6*f*g)+b^2*(15*f*g^2-h*(d*h+3*
e*g))))*arctanh(1/2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^(1/2)/(c
*x^2+b*x+a)^(1/2))/h^4/(a*h^2-b*g*h+c*g^2)^(3/2)-1/2*(-b*f*h-2*c*e*h+6*c*f*
g)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/h^4/c^(1/2)+1/4*(4*c*
g^2*(-e*h+3*f*g)/h+4*a*h*(-e*h+3*f*g)-b*(-d*h^2-3*e*g*h+11*f*g^2)-2*h*(c*e*
g+2*b*f*g-3*c*f*g^2/h-c*d*h-2*a*f*h)*x)*(c*x^2+b*x+a)^(1/2)/h^2/(a*h^2-b*g*
h+c*g^2)/(h*x+g)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 446, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1664, 826, 857, 635, 212, 738}

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^3}\,dx$$

$$= \frac{\operatorname{arctanh}\left(\frac{-2ah+x(2cg-bh)+bg}{2\sqrt{a+bx+cx^2}\sqrt{ah^2-bgh+cg^2}}\right)(h^2(8a^2fh^2-4abh(6fg-eh)+b^2(15fg^2-h(dh+3eg)))-4ch(bg^2(1}{8h^4\,(ah^2-bgh+cg^2)^{3/2}}$$

$$-\frac{\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(-bfh-2ceh+6cfg)}{2\sqrt{c}h^4}-\frac{(a+bx+cx^2)^{3/2}\,(fg^2-h(eg-dh))}{2h(g+hx)^2\,(ah^2-bgh+cg^2)}$$

$$-\frac{\sqrt{a+bx+cx^2}\left(2hx\left(-2afh+2bfg-cdh+ceg-\frac{3cfg^2}{h}\right)-4ah(3fg-eh)-bh(dh+3eg)+11bfg^2\right.}{4h^2(g+hx)\,(ah^2-bgh+cg^2)}$$

```
[In] Int[(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^3,x]
```

```
[Out] -1/4*((11*b*f*g^2 - b*h*(3*e*g + d*h) - (4*c*g^2*(3*f*g - e*h))/h - 4*a*h*(
3*f*g - e*h) + 2*h*(c*e*g + 2*b*f*g - (3*c*f*g^2)/h - c*d*h - 2*a*f*h)*x)*S
qrt[a + b*x + c*x^2])/(h^2*(c*g^2 - b*g*h + a*h^2)*(g + h*x)) - ((f*g^2 - h
*(e*g - d*h))*(a + b*x + c*x^2)^(3/2))/(2*h*(c*g^2 - b*g*h + a*h^2)*(g + h*
x)^2) - ((6*c*f*g - 2*c*e*h - b*f*h)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a
+ b*x + c*x^2])])/(2*Sqrt[c]*h^4) + ((8*c^2*g^3*(3*f*g - e*h) - 4*c*h*(b*g^
2*(10*f*g - 3*e*h) - a*h*(9*f*g^2 - 3*e*g*h + d*h^2)) + h^2*(8*a^2*f*h^2 -
4*a*b*h*(6*f*g - e*h) + b^2*(15*f*g^2 - h*(3*e*g + d*h))))*ArcTanh[(b*g - 2
*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[a + b*x + c*x^2
]])])/(8*h^4*(c*g^2 - b*g*h + a*h^2)^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 826

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{(fg^2-h(eg-dh))\left(a+bx+cx^2\right)^{3/2}}{2h\left(cg^2-bgh+ah^2\right)(g+hx)^2} - \frac{\int \frac{\left(\frac{1}{2}\left(-4cdg+3beg+4afg-\frac{3bfg^2}{h}+bdh-4aeh\right)+\left(ceg+2bfg-\frac{3cfg^2}{h}-cdh-2afh\right)x\right)\sqrt{a+bx+cx^2}}{(g+hx)^2}\,dx}{2\left(cg^2-bgh+ah^2\right)}$$

$$= -\frac{\Big(11bfg^2 - bh(3eg+dh) - \frac{4cg^2(3fg-eh)}{h} - 4ah(3fg-eh) + 2h\Big(ceg + 2bfg - \frac{3cfg^2}{h} - cdh - 2af}{4h^2\left(cg^2-bgh+ah^2\right)(g+hx)} - \frac{\left(fg^2-h(eg-dh)\right)\left(a+bx+cx^2\right)^{3/2}}{2h\left(cg^2-bgh+ah^2\right)(g+hx)^2} + \frac{\int \frac{\frac{1}{2}\Big(2(2bg-2ah)\Big(ceg+2bfg-\frac{3cfg^2}{h}-cdh-2afh\Big)+b\big(3bfg^2-bh(3eg+dh)+4h(cdg-afg+aeh)\big)\Big)-\frac{2(6cfg-2ceh-bfh)\big(cg^2-bgh+a}{h}}{(g+hx)\sqrt{a+bx+cx^2}}}{4h^2\left(cg^2-bgh+ah^2\right)}$$

$$= -\frac{\Big(11bfg^2 - bh(3eg+dh) - \frac{4cg^2(3fg-eh)}{h} - 4ah(3fg-eh) + 2h\Big(ceg + 2bfg - \frac{3cfg^2}{h} - cdh - 2af}{4h^2\left(cg^2-bgh+ah^2\right)(g+hx)} - \frac{\left(fg^2-h(eg-dh)\right)\left(a+bx+cx^2\right)^{3/2}}{2h\left(cg^2-bgh+ah^2\right)(g+hx)^2} - \frac{(6cfg-2ceh-bfh)\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{2h^4} + \frac{(8c^2g^3(3fg-eh) - 4ch(bg^2(10fg-3eh) - ah(9fg^2-3egh+dh^2)) + h^2(8a^2fh^2 - 4abh(6fg}{8h^4\left(cg^2-bgh+ah^2\right)}$$

$$= -\frac{\Big(11bfg^2 - bh(3eg+dh) - \frac{4cg^2(3fg-eh)}{h} - 4ah(3fg-eh) + 2h\Big(ceg + 2bfg - \frac{3cfg^2}{h} - cdh - 2af}{4h^2\left(cg^2-bgh+ah^2\right)(g+hx)} - \frac{\left(fg^2-h(eg-dh)\right)\left(a+bx+cx^2\right)^{3/2}}{2h\left(cg^2-bgh+ah^2\right)(g+hx)^2} - \frac{(6cfg-2ceh-bfh)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{h^4} - \frac{(8c^2g^3(3fg-eh) - 4ch(bg^2(10fg-3eh) - ah(9fg^2-3egh+dh^2)) + h^2(8a^2fh^2 - 4abh(6fg}{4h^4\,(cg^2-bgh}$$

$$= -\frac{\Big(11bfg^2 - bh(3eg+dh) - \frac{4cg^2(3fg-eh)}{h} - 4ah(3fg-eh) + 2h\Big(ceg + 2bfg - \frac{3cfg^2}{h} - cdh - 2af}{4h^2\left(cg^2-bgh+ah^2\right)(g+hx)} - \frac{\left(fg^2-h(eg-dh)\right)\left(a+bx+cx^2\right)^{3/2}}{2h\left(cg^2-bgh+ah^2\right)(g+hx)^2} - \frac{(6cfg-2ceh-bfh)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}h^4} + \frac{(8c^2g^3(3fg-eh) - 4ch(bg^2(10fg-3eh) - ah(9fg^2-3egh+dh^2)) + h^2(8a^2fh^2 - 4abh(6fg}{8h^4\left(cg^2-bgh+ah^2\right)^{3/2}}$$

Mathematica [A] (verified)

Time = 10.87 (sec) , antiderivative size = 500, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^3}\,dx$$

$$= \frac{8f\sqrt{a+x(b+cx)}+\frac{8(2fg-eh)\sqrt{a+x(b+cx)}}{g+hx}+\frac{2h\left(fg^2+h(-eg+dh)\right)\sqrt{a+x(b+cx)}(-2ah+2cgx+b(g-hx))}{(cg^2+h(-bg+ah))(g+hx)^2}+\frac{(-b^2+4ac)h\left(fg^2+h(-\right.}{}}{}$$

```
[In] Integrate[(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^3,x]
```

```
[Out] (8*f*Sqrt[a + x*(b + c*x)] + (8*(2*f*g - e*h)*Sqrt[a + x*(b + c*x)])/(g + h
*x) + (2*h*(f*g^2 + h*(-(e*g) + d*h))*Sqrt[a + x*(b + c*x)]*(-2*a*h + 2*c*g
*x + b*(g - h*x)))/((c*g^2 + h*(-(b*g) + a*h))*(g + h*x)^2) + ((-b^2 + 4*a*
c)*h*(f*g^2 + h*(-(e*g) + d*h))*ArcTanh[(-2*a*h + 2*c*g*x + b*(g - h*x))/(2
*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*Sqrt[a + x*(b + c*x)])])/(c*g^2 + h*(-(b*g)
 + a*h))^(3/2) - (4*(2*f*g - e*h)*(2*Sqrt[c]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]
*Sqrt[a + x*(b + c*x)])] - ((2*c*g - b*h)*ArcTanh[(-2*a*h + 2*c*g*x + b*(g
- h*x))/(2*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*Sqrt[a + x*(b + c*x)])])/Sqrt[c*g
^2 + h*(-(b*g) + a*h)]))/h + (4*f*(((-2*c*g + b*h)*ArcTanh[(b + 2*c*x)/(2*S
qrt[c]*Sqrt[a + x*(b + c*x)])])/Sqrt[c] + 2*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*
ArcTanh[(-2*a*h + 2*c*g*x + b*(g - h*x))/(2*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*
Sqrt[a + x*(b + c*x)])]))/h)/(8*h^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1237 vs. 2(420) = 840.

Time = 0.96 (sec) , antiderivative size = 1238, normalized size of antiderivative = 2.76

method	result	size
risch	Expression too large to display	1238
default	Expression too large to display	2166

```
[In] int((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^3,x,method=_RETURNVERBOSE)
```

```
[Out] f/h^3*(c*x^2+b*x+a)^(1/2)+1/2/h^3*((b*f*h+2*c*e*h-6*c*f*g)/h*ln((1/2*b+c*x)
/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-(2*a*f*h^2+2*b*e*h^2-6*b*f*g*h+2*c*d*
h^2-6*c*e*g*h+12*c*f*g^2)/h^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-
b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*
((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1
/h*g))+(2*a*e*h^3-4*a*f*g*h^2+2*b*d*h^3-4*b*e*g*h^2+6*b*f*g^2*h-4*c*d*g*h^2
+6*c*e*g^2*h-8*c*f*g^3)/h^3*(-1/(a*h^2-b*g*h+c*g^2)*h^2/(x+1/h*g)*((x+1/h*g
```

```
)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)+1/2*(b*h-2*c*g
)*h/(a*h^2-b*g*h+c*g^2)/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+
c*g^2)/h^2+(b*h-2*c*g)/h*(x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/
h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g))
)+1/h^4*(2*a*d*h^4-2*a*e*g*h^3+2*a*f*g^2*h^2-2*b*d*g*h^3+2*b*e*g^2*h^2-2*b*
f*g^3*h+2*c*d*g^2*h^2-2*c*e*g^3*h+2*c*f*g^4)*(-1/2/(a*h^2-b*g*h+c*g^2)*h^2/
(x+1/h*g)^2*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)
^(1/2)-3/4*(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)*(-1/(a*h^2-b*g*h+c*g^2)*h^2/(x
+1/h*g)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/
2)+1/2*(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln
((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/
h^2)^(1/2)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^
(1/2))/(x+1/h*g)))+1/2*c/(a*h^2-b*g*h+c*g^2)*h^2/((a*h^2-b*g*h+c*g^2)/h^2)^
(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+1/h*g)+2*((a*h^2-b*g*h
+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^
2)/h^2)^(1/2))/(x+1/h*g))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^3}\,dx=\text{Timed out}$$

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^3,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^3}\,dx=\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^3}\,dx$$

```
[In] integrate((f*x**2+e*x+d)*(c*x**2+b*x+a)**(1/2)/(h*x+g)**3,x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)/(g + h*x)**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^3,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*h^2-b*g*h>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2364 vs. 2(420) = 840.

Time = 0.47 (sec) , antiderivative size = 2364, normalized size of antiderivative = 5.28

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^3,x, algorithm="giac")
```

```
[Out] 1/4*(24*c^2*f*g^4 - 8*c^2*e*g^3*h - 40*b*c*f*g^3*h + 12*b*c*e*g^2*h^2 + 15*
b^2*f*g^2*h^2 + 36*a*c*f*g^2*h^2 - 3*b^2*e*g*h^3 - 12*a*c*e*g*h^3 - 24*a*b*
f*g*h^3 - b^2*d*h^4 + 4*a*c*d*h^4 + 4*a*b*e*h^4 + 8*a^2*f*h^4)*arctan(-((sq
rt(c)*x - sqrt(c*x^2 + b*x + a))*h + sqrt(c)*g)/sqrt(-c*g^2 + b*g*h - a*h^2
))/((c*g^2*h^4 - b*g*h^5 + a*h^6)*sqrt(-c*g^2 + b*g*h - a*h^2)) + sqrt(c*x^
2 + b*x + a)*f/h^3 + 1/4*(24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*c^2*f*g^
4*h - 16*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*c^2*e*g^3*h^2 - 32*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^3*b*c*f*g^3*h^2 + 8*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^3*c^2*d*g^2*h^3 + 20*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b*c*e*g^
2*h^3 + 9*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^2*f*g^2*h^3 + 20*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^3*a*c*f*g^2*h^3 - 8*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^3*b*c*d*g*h^4 - 5*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^2*e*g*h^
4 - 12*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*c*e*g*h^4 - 8*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^3*a*b*f*g*h^4 + (sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3
*b^2*d*h^5 + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*c*d*h^5 + 4*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^3*a*b*e*h^5 + 40*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^2*c^(5/2)*f*g^5 - 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c^(5/2)*e*
g^4*h - 40*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b*c^(3/2)*f*g^4*h + 8*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^2*c^(5/2)*d*g^3*h^2 + 20*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^2*b*c^(3/2)*e*g^3*h^2 + 3*(sqrt(c)*x - sqrt(c*x^2 + b*x +
```

```
 a))^2*b^2*sqrt(c)*f*g^3*h^2 + 12*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*c
^(3/2)*f*g^3*h^2 + (sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2*sqrt(c)*e*g^2*
h^3 - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*c^(3/2)*e*g^2*h^3 + 16*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b*sqrt(c)*f*g^2*h^3 - 5*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))^2*b^2*sqrt(c)*d*g*h^4 - 4*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^2*a*c^(3/2)*d*g*h^4 - 12*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b*
sqrt(c)*e*g*h^4 - 16*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*sqrt(c)*f*g*
h^4 + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b*sqrt(c)*d*h^5 + 8*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^2*a^2*sqrt(c)*e*h^5 + 40*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))*b*c^2*f*g^5 - 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b*c^2*e
*g^4*h - 44*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^2*c*f*g^4*h - 56*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))*a*c^2*f*g^4*h + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))*b*c^2*d*g^3*h^2 + 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^2*c*e*g^3
*h^2 + 32*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*c^2*e*g^3*h^2 + 7*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))*b^3*f*g^3*h^2 + 92*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))*a*b*c*f*g^3*h^2 - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^2*c*d*g^2*
h^3 - 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*c^2*d*g^2*h^3 - 3*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))*b^3*e*g^2*h^3 - 48*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))*a*b*c*e*g^2*h^3 - 15*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^2*f*g^2*h
^3 - 44*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*c*f*g^2*h^3 - (sqrt(c)*x -
sqrt(c*x^2 + b*x + a))*b^3*d*g*h^4 + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*
a*b*c*d*g*h^4 + 7*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^2*e*g*h^4 + 20*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*c*e*g*h^4 + 8*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))*a^2*b*f*g*h^4 + (sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^2*d*h^
5 + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*c*d*h^5 - 4*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))*a^2*b*e*h^5 + 10*b^2*c^(3/2)*f*g^5 - 6*b^2*c^(3/2)*e*g^4
*h - 9*b^3*sqrt(c)*f*g^4*h - 28*a*b*c^(3/2)*f*g^4*h + 2*b^2*c^(3/2)*d*g^3*h
^2 + 5*b^3*sqrt(c)*e*g^3*h^2 + 16*a*b*c^(3/2)*e*g^3*h^2 + 33*a*b^2*sqrt(c)*
f*g^3*h^2 + 20*a^2*c^(3/2)*f*g^3*h^2 - b^3*sqrt(c)*d*g^2*h^3 - 4*a*b*c^(3/2
)*d*g^2*h^3 - 17*a*b^2*sqrt(c)*e*g^2*h^3 - 12*a^2*c^(3/2)*e*g^2*h^3 - 40*a^
2*b*sqrt(c)*f*g^2*h^3 + a*b^2*sqrt(c)*d*g*h^4 + 4*a^2*c^(3/2)*d*g*h^4 + 20*
a^2*b*sqrt(c)*e*g*h^4 + 16*a^3*sqrt(c)*f*g*h^4 - 8*a^3*sqrt(c)*e*h^5)/((c*g
^2*h^4 - b*g*h^5 + a*h^6)*((sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*h + 2*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c)*g + b*g - a*h)^2) + 1/2*(6*c*f*g -
2*c*e*h - b*f*h)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b
))/(sqrt(c)*h^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^3}\,dx=\int \frac{\sqrt{c\,x^2+b\,x+a}\,\left(f\,x^2+e\,x+d\right)}{{\left(g+h\,x\right)}^3}\,dx$$

```
[In] int(((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^3,x)
[Out] int(((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^3, x)
```

3.193 $\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^4}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 603

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^4}\,dx =$$

$$-\frac{(8c^2fg^5-2cgh(7bfg^3-6afg^2h+bdgh^2-2adh^3)+h^2(4a^2eh^3+b^2g(5fg^2+egh+dh^2)-2abh(3fg}{}$$

$$-\frac{(fg^2-h(eg-dh))\,(a+bx+cx^2)^{3/2}}{3h\,(cg^2-bgh+ah^2)\,(g+hx)^3}+\frac{\sqrt{c}f\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{h^4}$$

$$-\frac{(16c^3fg^5-8c^2gh(5bfg^3-5afg^2h+adh^3)-bh^3(8a^2fh^2-2abh(6fg+eh)+b^2(5fg^2+egh+dh^2))}{16h^4\,(cg^2}$$

```
[Out] -1/3*(f*g^2-h*(-d*h+e*g))*(c*x^2+b*x+a)^(3/2)/h/(a*h^2-b*g*h+c*g^2)/(h*x+g)
^3-1/16*(16*c^3*f*g^5-8*c^2*g*h*(a*d*h^3-5*a*f*g^2*h+5*b*f*g^3)-b*h^3*(8*a^
2*f*h^2-2*a*b*h*(e*h+6*f*g)+b^2*(d*h^2+e*g*h+5*f*g^2))+2*c*h^2*(4*a^2*h^2*(
-e*h+4*f*g)-2*a*b*h*(-d*h^2-e*g*h+15*f*g^2)+b^2*(d*g*h^2+15*f*g^3)))*arctan
h(1/2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^(1/2)/(c*x^2+b*x+a)^(1
/2))/h^4/(a*h^2-b*g*h+c*g^2)^(5/2)+f*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b
*x+a)^(1/2))*c^(1/2)/h^4-1/8*(8*c^2*f*g^5-2*c*g*h*(-2*a*d*h^3-6*a*f*g^2*h+b
*d*g*h^2+7*b*f*g^3)+h^2*(4*a^2*e*h^3+b^2*g*(d*h^2+e*g*h+5*f*g^2)-2*a*b*h*(d
*h^2+2*e*g*h+3*f*g^2))+h*(4*c^2*(-d*g^2*h^2+3*f*g^4)+h^2*(8*a^2*f*h^2-2*a*b
*h*(-e*h+10*f*g)+b^2*(11*f*g^2-h*(d*h+e*g)))+2*c*g*h*(2*a*h*(-e*h+6*f*g)-b*
(12*f*g^2-h*(2*d*h+e*g))))*x)*(c*x^2+b*x+a)^(1/2)/h^3/(a*h^2-b*g*h+c*g^2)^2
/(h*x+g)^2
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 601, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1664, 824, 857, 635, 212, 738}

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^4}\,dx =$$

$$-\frac{\operatorname{arctanh}\left(\frac{-2ah+x(2cg-bh)+bg}{2\sqrt{a+bx+cx^2}\sqrt{ah^2-bgh+cg^2}}\right)\left(2ch^2(4a^2h^2(4fg-eh)-2abh(-dh^2-egh+15fg^2)+b^2(dgh^2+15f\right.}{16h^4\left(ah^2\right.}$$

$$-\frac{\sqrt{a+bx+cx^2}\left(hx\left(8a^2fh^3-2b(ah^2(10fg-eh)-cgh(2dh+eg)+12cfg^3)+4acgh(6fg-eh)+b^2h(\right.\right.}{}$$

$$+\frac{\sqrt{c}f\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{h^4}-\frac{(a+bx+cx^2)^{3/2}\left(fg^2-h(eg-dh)\right)}{3h(g+hx)^3\left(ah^2-bgh+cg^2\right)}$$

```
[In] Int[(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^4,x]
```

```
[Out] -1/8*(((8*c^2*f*g^5)/h + 4*a^2*e*h^4 + 4*a*c*g*h*(3*f*g^2 + d*h^2) + b^2*g*
h*(5*f*g^2 + h*(e*g + d*h)) - 2*b*(a*h^2*(3*f*g^2 + 2*e*g*h + d*h^2) + c*(7
*f*g^4 + d*g^2*h^2)) + h*(8*a^2*f*h^3 + 4*a*c*g*h*(6*f*g - e*h) + c^2*((12*
f*g^4)/h - 4*d*g^2*h) + b^2*h*(11*f*g^2 - h*(e*g + d*h)) - 2*b*(12*c*f*g^3
- c*g*h*(e*g + 2*d*h) + a*h^2*(10*f*g - e*h)))*x)*Sqrt[a + b*x + c*x^2])/(h
^2*(c*g^2 - b*g*h + a*h^2)^2*(g + h*x)^2) - ((f*g^2 - h*(e*g - d*h))*(a + b
*x + c*x^2)^(3/2))/(3*h*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^3) + (Sqrt[c]*f*A
rcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/h^4 - ((16*c^3*f*g^5
 - 8*c^2*g*h*(5*b*f*g^3 - 5*a*f*g^2*h + a*d*h^3) - b*h^3*(8*a^2*f*h^2 - 2*a
*b*h*(6*f*g + e*h) + b^2*(5*f*g^2 + e*g*h + d*h^2)) + 2*c*h^2*(4*a^2*h^2*(4
*f*g - e*h) - 2*a*b*h*(15*f*g^2 - e*g*h - d*h^2) + b^2*(15*f*g^3 + d*g*h^2)
))*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*S
qrt[a + b*x + c*x^2])])/(16*h^4*(c*g^2 - b*g*h + a*h^2)^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 824

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Dist[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)
^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p
 + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*
(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m
 - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &
& LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{\left(fg^2-h(eg-dh)\right)\left(a+bx+cx^2\right)^{3/2}}{3h\left(cg^2-bgh+ah^2\right)(g+hx)^3}$$
$$-\frac{\int\frac{\left(-\frac{3}{2}\left(2cdg-beg-2afg+\frac{bfg^2}{h}-bdh+2aeh\right)+3f\left(bg-\frac{cg^2}{h}-ah\right)x\right)\sqrt{a+bx+cx^2}}{(g+hx)^3}\,dx}{3\left(cg^2-bgh+ah^2\right)}$$

$$=$$

$$-\frac{\Big(\frac{8c^2fg^5}{h}+4a^2eh^4+4acgh(3fg^2+dh^2)+b^2gh(5fg^2+h(eg+dh))-2b(ah^2(3fg^2+2egh+dh^2}{}$$

$$-\frac{(fg^2-h(eg-dh))\left(a+bx+cx^2\right)^{3/2}}{3h\left(cg^2-bgh+ah^2\right)(g+hx)^3}$$

$$+\frac{\int\frac{\frac{3\big(b^3h^2\left(5fg^2+h(eg+dh)\right)+4b\left(2c^2fg^4+2a^2fh^4+ach^2\left(7fg^2-egh-dh^2\right)\right)-8ach\left(ah^2(2fg-eh)+c\left(fg^3-dgh^2\right)\right)-2b^2\big(ah^3(6fg+eh)+c\big(7fg^3h+}{4h}}{(g+hx)\sqrt{a+bx+cx^2}}}{12h^2\left(cg^2-bgh+ah^2\right)^2}$$

$$=$$

$$-\frac{\Big(\frac{8c^2fg^5}{h}+4a^2eh^4+4acgh(3fg^2+dh^2)+b^2gh(5fg^2+h(eg+dh))-2b(ah^2(3fg^2+2egh+dh^2}{}$$

$$-\frac{(fg^2-h(eg-dh))\left(a+bx+cx^2\right)^{3/2}}{3h\left(cg^2-bgh+ah^2\right)(g+hx)^3}+\frac{(cf)\int\frac{1}{\sqrt{a+bx+cx^2}}\,dx}{h^4}$$

$$-\frac{(16c^3fg^5-8c^2gh(5bfg^3-5afg^2h+adh^3)-bh^3(8a^2fh^2-2abh(6fg+eh)+b^2(5fg^2+egh+}{16h^4\left(cg^2\right.}$$

$$=$$

$$-\frac{\Big(\frac{8c^2fg^5}{h}+4a^2eh^4+4acgh(3fg^2+dh^2)+b^2gh(5fg^2+h(eg+dh))-2b(ah^2(3fg^2+2egh+dh^2}{}$$

$$-\frac{(fg^2-h(eg-dh))\left(a+bx+cx^2\right)^{3/2}}{3h\left(cg^2-bgh+ah^2\right)(g+hx)^3}+\frac{(2cf)\text{Subst}\left(\int\frac{1}{4c-x^2}\,dx,x,\frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{h^4}$$

$$+\frac{(16c^3fg^5-8c^2gh(5bfg^3-5afg^2h+adh^3)-bh^3(8a^2fh^2-2abh(6fg+eh)+b^2(5fg^2+egh+}{}$$

$$=$$

$$-\frac{\Big(\frac{8c^2fg^5}{h}+4a^2eh^4+4acgh(3fg^2+dh^2)+b^2gh(5fg^2+h(eg+dh))-2b(ah^2(3fg^2+2egh+dh^2}{}$$

$$-\frac{(fg^2-h(eg-dh))\left(a+bx+cx^2\right)^{3/2}}{3h\left(cg^2-bgh+ah^2\right)(g+hx)^3}+\frac{\sqrt{c}f\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{h^4}$$

$$-\frac{(16c^3fg^5-8c^2gh(5bfg^3-5afg^2h+adh^3)-bh^3(8a^2fh^2-2abh(6fg+eh)+b^2(5fg^2+egh+}{16h}$$

Mathematica [A] (verified)

Time = 11.38 (sec) , antiderivative size = 574, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^4}\,dx$$

$$= \frac{-\frac{48f\sqrt{a+x(b+cx)}}{g+hx} - \frac{16h^2(fg^2+h(-eg+dh))(a+x(b+cx))^{3/2}}{(cg^2+h(-bg+ah))(g+hx)^3} + \frac{12h(-2fg+eh)\sqrt{a+x(b+cx)}(-2ah+2cgx+b(g-hx))}{(cg^2+h(-bg+ah))(g+hx)^2} - \frac{6(b^2-4ac)h(-2}{}}{}$$

```
[In] Integrate[(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^4,x]
```

```
[Out] ((-48*f*Sqrt[a + x*(b + c*x)])/(g + h*x) - (16*h^2*(f*g^2 + h*(-(e*g) + d*h
))*(a + x*(b + c*x))^(3/2))/((c*g^2 + h*(-(b*g) + a*h))*(g + h*x)^3) + (12*
h*(-2*f*g + e*h)*Sqrt[a + x*(b + c*x)]*(-2*a*h + 2*c*g*x + b*(g - h*x)))/((
c*g^2 + h*(-(b*g) + a*h))*(g + h*x)^2) - (6*(b^2 - 4*a*c)*h*(-2*f*g + e*h)*
ArcTanh[(-2*a*h + 2*c*g*x + b*(g - h*x))/(2*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*
Sqrt[a + x*(b + c*x)])])/(c*g^2 + h*(-(b*g) + a*h))^(3/2) + (3*h*(2*c*g - b
*h)*(f*g^2 + h*(-(e*g) + d*h))*((2*Sqrt[a + x*(b + c*x)]*(-2*a*h + 2*c*g*x
+ b*(g - h*x)))/((c*g^2 + h*(-(b*g) + a*h))*(g + h*x)^2) + ((-b^2 + 4*a*c)*
ArcTanh[(-2*a*h + 2*c*g*x + b*(g - h*x))/(2*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*
Sqrt[a + x*(b + c*x)])])/(c*g^2 + h*(-(b*g) + a*h))^(3/2)))/(c*g^2 + h*(-(b
*g) + a*h)) + (24*f*(2*Sqrt[c]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b
 + c*x)])] - ((2*c*g - b*h)*ArcTanh[(-2*a*h + 2*c*g*x + b*(g - h*x))/(2*Sqr
t[c*g^2 + h*(-(b*g) + a*h)]*Sqrt[a + x*(b + c*x)])])/Sqrt[c*g^2 + h*(-(b*g)
 + a*h)]))/h)/(48*h^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3089 vs. 2(577) = 1154.

Time = 1.15 (sec) , antiderivative size = 3090, normalized size of antiderivative = 5.12

method	result	size
default	Expression too large to display	3090

```
[In] int((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^4,x,method=_RETURNVERBOSE)
```

```
[Out] f/h^4*(-1/(a*h^2-b*g*h+c*g^2)*h^2/(x+1/h*g)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x
+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(3/2)+1/2*(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2
)*(((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)+1/
2*(b*h-2*c*g)/h*ln((1/2*(b*h-2*c*g)/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g)^2*c+(
b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/c^(1/2)-(a*h^2-b*g*h
+c*g^2)/h^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(
```

```
b*h-2*c*g)/h*(x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c+(b*
h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g)))+2*c/(a*h^2
-b*g*h+c*g^2)*h^2*(1/4*(2*c*(x+1/h*g)+(b*h-2*c*g)/h)/c*((x+1/h*g)^2*c+(b*h-
2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)+1/8*(4*c*(a*h^2-b*g*h+c*g
^2)/h^2-(b*h-2*c*g)^2/h^2)/c^(3/2)*ln((1/2*(b*h-2*c*g)/h+c*(x+1/h*g))/c^(1/
2)+((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))))
+(e*h-2*f*g)/h^5*(-1/2/(a*h^2-b*g*h+c*g^2)*h^2/(x+1/h*g)^2*((x+1/h*g)^2*c+(
b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(3/2)-1/4*(b*h-2*c*g)*h/(a*
h^2-b*g*h+c*g^2)*(-1/(a*h^2-b*g*h+c*g^2)*h^2/(x+1/h*g)*((x+1/h*g)^2*c+(b*h-
2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(3/2)+1/2*(b*h-2*c*g)*h/(a*h^2-
b*g*h+c*g^2)*(((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^
2)^(1/2)+1/2*(b*h-2*c*g)/h*ln((1/2*(b*h-2*c*g)/h+c*(x+1/h*g))/c^(1/2)+((x+1
/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/c^(1/2)-(
a*h^2-b*g*h+c*g^2)/h^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c
*g^2)/h^2+(b*h-2*c*g)/h*(x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h
*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g)))
+2*c/(a*h^2-b*g*h+c*g^2)*h^2*(1/4*(2*c*(x+1/h*g)+(b*h-2*c*g)/h)/c*((x+1/h*g
)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)+1/8*(4*c*(a*h^
2-b*g*h+c*g^2)/h^2-(b*h-2*c*g)^2/h^2)/c^(3/2)*ln((1/2*(b*h-2*c*g)/h+c*(x+1/
h*g))/c^(1/2)+((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^
2)^(1/2))))+1/2*c/(a*h^2-b*g*h+c*g^2)*h^2*(((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+
1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)+1/2*(b*h-2*c*g)/h*ln((1/2*(b*h-2*c*g)
/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h
+c*g^2)/h^2)^(1/2))/c^(1/2)-(a*h^2-b*g*h+c*g^2)/h^2/((a*h^2-b*g*h+c*g^2)/h^
2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+1/h*g)+2*((a*h^2-b*
g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c
*g^2)/h^2)^(1/2))/(x+1/h*g))))+(d*h^2-e*g*h+f*g^2)/h^6*(-1/3/(a*h^2-b*g*h+c
*g^2)*h^2/(x+1/h*g)^3*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c
*g^2)/h^2)^(3/2)-1/2*(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)*(-1/2/(a*h^2-b*g*h+c
*g^2)*h^2/(x+1/h*g)^2*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c
*g^2)/h^2)^(3/2)-1/4*(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)*(-1/(a*h^2-b*g*h+c*g
^2)*h^2/(x+1/h*g)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2
)/h^2)^(3/2)+1/2*(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)*(((x+1/h*g)^2*c+(b*h-2*c
*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)+1/2*(b*h-2*c*g)/h*ln((1/2*(b
*h-2*c*g)/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*
h^2-b*g*h+c*g^2)/h^2)^(1/2))/c^(1/2)-(a*h^2-b*g*h+c*g^2)/h^2/((a*h^2-b*g*h+
c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+1/h*g)+2*(
(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^
2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g)))+2*c/(a*h^2-b*g*h+c*g^2)*h^2*(1/4*(2*
c*(x+1/h*g)+(b*h-2*c*g)/h)/c*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-
b*g*h+c*g^2)/h^2)^(1/2)+1/8*(4*c*(a*h^2-b*g*h+c*g^2)/h^2-(b*h-2*c*g)^2/h^2)
/c^(3/2)*ln((1/2*(b*h-2*c*g)/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g)^2*c+(b*h-2*c
*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))))+1/2*c/(a*h^2-b*g*h+c*g^2)
*h^2*(((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)
+1/2*(b*h-2*c*g)/h*ln((1/2*(b*h-2*c*g)/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g)^2*
```

```
c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/c^(1/2)-(a*h^2-b*
g*h+c*g^2)/h^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^
2+(b*h-2*c*g)/h*(x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c+
(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^4}\,dx = \text{Timed out}$$

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^4,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^4}\,dx = \int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^4}\,dx$$

```
[In] integrate((f*x**2+e*x+d)*(c*x**2+b*x+a)**(1/2)/(h*x+g)**4,x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)/(g + h*x)**4, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^4}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^4,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*h^2-b*g*h>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6846 vs. 2(577) = 1154.

Time = 4.82 (sec) , antiderivative size = 6846, normalized size of antiderivative = 11.35

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^4}\,dx = \text{Too large to display}$$

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^4,x, algorithm="giac")
[Out] -1/8*(16*c^3*f*g^5 - 40*b*c^2*f*g^4*h + 30*b^2*c*f*g^3*h^2 + 40*a*c^2*f*g^3
*h^2 - 5*b^3*f*g^2*h^3 - 60*a*b*c*f*g^2*h^3 + 2*b^2*c*d*g*h^4 - 8*a*c^2*d*g
*h^4 - b^3*e*g*h^4 + 4*a*b*c*e*g*h^4 + 12*a*b^2*f*g*h^4 + 32*a^2*c*f*g*h^4
- b^3*d*h^5 + 4*a*b*c*d*h^5 + 2*a*b^2*e*h^5 - 8*a^2*c*e*h^5 - 8*a^2*b*f*h^5
)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*h + sqrt(c)*g)/sqrt(-c*g^2 +
 b*g*h - a*h^2))/((c^2*g^4*h^4 - 2*b*c*g^3*h^5 + b^2*g^2*h^6 + 2*a*c*g^2*h^
6 - 2*a*b*g*h^7 + a^2*h^8)*sqrt(-c*g^2 + b*g*h - a*h^2)) - sqrt(c)*f*log(ab
s(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/h^4 - 1/24*(144*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^5*c^3*f*g^5*h^2 - 48*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^5*c^3*e*g^4*h^3 - 312*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b
*c^2*f*g^4*h^3 + 96*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b*c^2*e*g^3*h^4 +
 198*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^2*c*f*g^3*h^4 + 264*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))^5*a*c^2*f*g^3*h^4 - 48*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))^5*b^2*c*e*g^2*h^5 - 96*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*c^
2*e*g^2*h^5 - 33*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^3*f*g^2*h^5 - 300*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b*c*f*g^2*h^5 - 6*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^5*b^2*c*d*g*h^6 + 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
^5*a*c^2*d*g*h^6 + 3*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^3*e*g*h^6 + 84
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b*c*e*g*h^6 + 60*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^5*a*b^2*f*g*h^6 + 96*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
^5*a^2*c*f*g*h^6 + 3*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^3*d*h^7 - 12*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b*c*d*h^7 - 6*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^5*a*b^2*e*h^7 - 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*
c*e*h^7 - 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*b*f*h^7 + 432*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^4*c^(7/2)*f*g^6*h - 96*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^4*c^(7/2)*e*g^5*h^2 - 840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
^4*b*c^(5/2)*f*g^5*h^2 - 48*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*c^(7/2)*d
*g^4*h^3 + 144*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b*c^(5/2)*e*g^4*h^3 +
414*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^2*c^(3/2)*f*g^4*h^3 + 696*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^4*a*c^(5/2)*f*g^4*h^3 + 96*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^4*b*c^(5/2)*d*g^3*h^4 - 192*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^4*a*c^(5/2)*e*g^3*h^4 - 21*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^
3*sqrt(c)*f*g^3*h^4 - 540*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b*c^(3/2)
*f*g^3*h^4 - 78*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^2*c^(3/2)*d*g^2*h^5
 + 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*c^(5/2)*d*g^2*h^5 - 33*(sqrt(
```

```
c)*x - sqrt(c*x^2 + b*x + a))^4*b^3*sqrt(c)*e*g^2*h^5 + 36*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^4*a*b*c^(3/2)*e*g^2*h^5 - 36*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))^4*a*b^2*sqrt(c)*f*g^2*h^5 + 96*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)^4*a^2*c^(3/2)*f*g^2*h^5 + 15*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^3*sq
rt(c)*d*g*h^6 + 36*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b*c^(3/2)*d*g*h^
6 + 66*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b^2*sqrt(c)*e*g*h^6 + 24*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*c^(3/2)*e*g*h^6 + 120*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^4*a^2*b*sqrt(c)*f*g*h^6 - 48*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^4*a^2*c^(3/2)*d*h^7 - 48*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*
a^2*b*sqrt(c)*e*h^7 - 48*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^3*sqrt(c)*
f*h^7 + 352*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*c^4*f*g^7 - 64*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))^3*c^4*e*g^6*h - 304*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))^3*b*c^3*f*g^6*h - 32*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*c^4*d*g^5
*h^2 + 16*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b*c^3*e*g^5*h^2 - 420*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^3*b^2*c^2*f*g^5*h^2 + 112*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^3*a*c^3*f*g^5*h^2 - 16*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)^3*b*c^3*d*g^4*h^3 + 144*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^2*c^2*e*g
^4*h^3 - 64*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*c^3*e*g^4*h^3 + 382*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^3*c*f*g^4*h^3 + 1176*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^3*a*b*c^2*f*g^4*h^3 + 84*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))^3*b^2*c^2*d*g^3*h^4 + 112*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*c^3*
d*g^3*h^4 - 58*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^3*c*e*g^3*h^4 - 312*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b*c^2*e*g^3*h^4 - 40*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^3*b^4*f*g^3*h^4 - 1176*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^3*a*b^2*c*f*g^3*h^4 - 768*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*c
^2*f*g^3*h^4 - 74*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^3*c*d*g^2*h^5 - 1
20*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b*c^2*d*g^2*h^5 - 8*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))^3*b^4*e*g^2*h^5 + 180*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))^3*a*b^2*c*e*g^2*h^5 + 240*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*
c^2*e*g^2*h^5 + 136*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^3*f*g^2*h^5 +
 1104*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b*c*f*g^2*h^5 + 8*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^3*b^4*d*g*h^6 + 144*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^3*a*b^2*c*d*g*h^6 - 96*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*c^
2*d*g*h^6 + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^3*e*g*h^6 - 144*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b*c*e*g*h^6 - 144*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^3*a^2*b^2*f*g*h^6 - 288*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))^3*a^3*c*f*g*h^6 - 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^3*d*h^7 -
48*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b*c*d*h^7 + 48*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))^3*a^3*b*f*h^7 + 528*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)^2*b*c^(7/2)*f*g^7 - 96*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b*c^(7/2)*e*
g^6*h - 948*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2*c^(5/2)*f*g^6*h - 624
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*c^(7/2)*f*g^6*h - 48*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^2*b*c^(7/2)*d*g^5*h^2 + 144*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^2*b^2*c^(5/2)*e*g^5*h^2 + 96*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)^2*a*c^(7/2)*e*g^5*h^2 + 366*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^3*c^(
```

```
3/2)*f*g^5*h^2 + 1944*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b*c^(5/2)*f*g
^5*h^2 + 36*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2*c^(5/2)*d*g^4*h^3 + 4
8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*c^(7/2)*d*g^4*h^3 + 6*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))^2*b^3*c^(3/2)*e*g^4*h^3 - 360*(sqrt(c)*x - sqrt(c*
x^2 + b*x + a))^2*a*b*c^(5/2)*e*g^4*h^3 + 24*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^2*b^4*sqrt(c)*f*g^4*h^3 - 888*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a
*b^2*c^(3/2)*f*g^4*h^3 - 960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*c^(5
/2)*f*g^4*h^3 + 6*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^3*c^(3/2)*d*g^3*h
^4 + 72*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b*c^(5/2)*d*g^3*h^4 - 24*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^4*sqrt(c)*e*g^3*h^4 - 12*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))^2*a*b^2*c^(3/2)*e*g^3*h^4 + 240*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))^2*a^2*c^(5/2)*e*g^3*h^4 - 168*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^2*a*b^3*sqrt(c)*f*g^3*h^4 + 528*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
^2*a^2*b*c^(3/2)*f*g^3*h^4 - 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^4*s
qrt(c)*d*g^2*h^5 - 48*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b^2*c^(3/2)*d
*g^2*h^5 - 192*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*c^(5/2)*d*g^2*h^5
+ 72*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b^3*sqrt(c)*e*g^2*h^5 + 96*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*b*c^(3/2)*e*g^2*h^5 + 384*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))^2*a^2*b^2*sqrt(c)*f*g^2*h^5 + 72*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^2*a*b^3*sqrt(c)*d*g*h^6 + 48*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))^2*a^2*b*c^(3/2)*d*g*h^6 - 96*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^
2*a^2*b^2*sqrt(c)*e*g*h^6 - 96*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*c^
(3/2)*e*g*h^6 - 336*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*b*sqrt(c)*f*g
*h^6 - 48*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*b^2*sqrt(c)*d*h^7 + 48*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*b*sqrt(c)*e*h^7 + 96*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))^2*a^4*sqrt(c)*f*h^7 + 264*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))*b^2*c^3*f*g^7 - 48*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^2*c^3*e
*g^6*h - 504*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*c^2*f*g^6*h - 624*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))*a*b*c^3*f*g^6*h - 24*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))*b^2*c^3*d*g^5*h^2 + 84*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^
3*c^2*e*g^5*h^2 + 96*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b*c^3*e*g^5*h^2
+ 240*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^4*c*f*g^5*h^2 + 1668*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))*a*b^2*c^2*f*g^5*h^2 + 384*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))*a^2*c^3*f*g^5*h^2 + 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3
*c^2*d*g^4*h^3 + 48*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b*c^3*d*g^4*h^3 -
 18*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^4*c*e*g^4*h^3 - 288*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))*a*b^2*c^2*e*g^4*h^3 - 48*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))*a^2*c^3*e*g^4*h^3 - 15*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^5*f*g
^4*h^3 - 966*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^3*c*f*g^4*h^3 - 1872*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b*c^2*f*g^4*h^3 - 12*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))*b^4*c*d*g^3*h^4 + 12*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))*a*b^2*c^2*d*g^3*h^4 - 48*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*c^3*d*g
^3*h^4 - 3*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^5*e*g^3*h^4 + 66*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))*a*b^3*c*e*g^3*h^4 + 360*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))*a^2*b*c^2*e*g^3*h^4 + 66*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*
```

```
b^4*f*g^3*h^4 + 1434*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^2*c*f*g^3*h^
4 + 696*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*c^2*f*g^3*h^4 - 3*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))*b^5*d*g^2*h^5 + 18*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))*a*b^3*c*d*g^2*h^5 - 120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b*c^
2*d*g^2*h^5 + 12*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^4*e*g^2*h^5 - 84*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^2*c*e*g^2*h^5 - 144*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))*a^3*c^2*e*g^2*h^5 - 111*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))*a^2*b^3*f*g^2*h^5 - 900*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*b*c*
f*g^2*h^5 + 6*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^4*d*g*h^6 + 30*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^2*c*d*g*h^6 + 72*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))*a^3*c^2*d*g*h^6 - 15*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*
b^3*e*g*h^6 + 12*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*b*c*e*g*h^6 + 84*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*b^2*f*g*h^6 + 192*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))*a^4*c*f*g*h^6 - 3*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2
*b^3*d*h^7 - 36*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*b*c*d*h^7 + 6*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))*a^3*b^2*e*h^7 + 24*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))*a^4*c*e*h^7 - 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^4*b*f*h^7
 + 44*b^3*c^(5/2)*f*g^7 - 8*b^3*c^(5/2)*e*g^6*h - 80*b^4*c^(3/2)*f*g^6*h -
156*a*b^2*c^(5/2)*f*g^6*h - 4*b^3*c^(5/2)*d*g^5*h^2 + 14*b^4*c^(3/2)*e*g^5*
h^2 + 24*a*b^2*c^(5/2)*e*g^5*h^2 + 33*b^5*sqrt(c)*f*g^5*h^2 + 362*a*b^3*c^(
3/2)*f*g^5*h^2 + 192*a^2*b*c^(5/2)*f*g^5*h^2 + 4*b^4*c^(3/2)*d*g^4*h^3 + 12
*a*b^2*c^(5/2)*d*g^4*h^3 - 3*b^5*sqrt(c)*e*g^4*h^3 - 62*a*b^3*c^(3/2)*e*g^4
*h^3 - 24*a^2*b*c^(5/2)*e*g^4*h^3 - 174*a*b^4*sqrt(c)*f*g^4*h^3 - 630*a^2*b
^2*c^(3/2)*f*g^4*h^3 - 88*a^3*c^(5/2)*f*g^4*h^3 - 3*b^5*sqrt(c)*d*g^3*h^4 +
 2*a*b^3*c^(3/2)*d*g^3*h^4 - 24*a^2*b*c^(5/2)*d*g^3*h^4 + 12*a*b^4*sqrt(c)*
e*g^3*h^4 + 108*a^2*b^2*c^(3/2)*e*g^3*h^4 + 16*a^3*c^(5/2)*e*g^3*h^4 + 369*
a^2*b^3*sqrt(c)*f*g^3*h^4 + 508*a^3*b*c^(3/2)*f*g^3*h^4 + 6*a*b^4*sqrt(c)*d
*g^2*h^5 - 18*a^2*b^2*c^(3/2)*d*g^2*h^5 + 8*a^3*c^(5/2)*d*g^2*h^5 - 15*a^2*
b^3*sqrt(c)*e*g^2*h^5 - 100*a^3*b*c^(3/2)*e*g^2*h^5 - 396*a^3*b^2*sqrt(c)*f
*g^2*h^5 - 160*a^4*c^(3/2)*f*g^2*h^5 - 3*a^2*b^3*sqrt(c)*d*g*h^6 + 28*a^3*b
*c^(3/2)*d*g*h^6 + 6*a^3*b^2*sqrt(c)*e*g*h^6 + 40*a^4*c^(3/2)*e*g*h^6 + 216
*a^4*b*sqrt(c)*f*g*h^6 - 16*a^4*c^(3/2)*d*h^7 - 48*a^5*sqrt(c)*f*h^7)/((c^2
*g^4*h^4 - 2*b*c*g^3*h^5 + b^2*g^2*h^6 + 2*a*c*g^2*h^6 - 2*a*b*g*h^7 + a^2*
h^8)*((sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*h + 2*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))*sqrt(c)*g + b*g - a*h)^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^4}\,dx=\int \frac{\sqrt{c\,x^2+b\,x+a}\,\left(f\,x^2+e\,x+d\right)}{{\left(g+h\,x\right)}^4}\,dx$$

```
[In] int(((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^4,x)
[Out] int(((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^4, x)
```

3.194 $\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^5}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 497

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^5}\,dx$$

$$= \frac{(16c^2dg^2+16a^2fh^2-8abh(2fg+eh)+b^2(5fg^2+3egh+5dh^2)-4c(2bg(eg+2dh)+a(fg^2-5egh+d}{64\left(cg^2-bgh+ah^2\right)^3(g+hx)^2}$$

$$-\frac{(fg^2-h(eg-dh))\left(a+bx+cx^2\right)^{3/2}}{4h\left(cg^2-bgh+ah^2\right)(g+hx)^4}$$

$$+\frac{(2cg(3fg^2+h(eg-5dh))+h(8ah(2fg-eh)-b(11fg^2-3egh-5dh^2)))\left(a+bx+cx^2\right)^{3/2}}{24h\left(cg^2-bgh+ah^2\right)^2(g+hx)^3}$$

$$-\frac{(b^2-4ac)\left(16c^2dg^2+16a^2fh^2-8abh(2fg+eh)+b^2(5fg^2+3egh+5dh^2)-4c(2bg(eg+2dh)+a(fg\right.}{128\left(cg^2-bgh+ah^2\right)^{7/2}}$$

```
[Out] -1/4*(f*g^2-h*(-d*h+e*g))*(c*x^2+b*x+a)^(3/2)/h/(a*h^2-b*g*h+c*g^2)/(h*x+g)
^4+1/24*(2*c*g*(3*f*g^2+h*(-5*d*h+e*g))+h*(8*a*h*(-e*h+2*f*g)-b*(-5*d*h^2-3
*e*g*h+11*f*g^2)))*(c*x^2+b*x+a)^(3/2)/h/(a*h^2-b*g*h+c*g^2)^2/(h*x+g)^3-1/
128*(-4*a*c+b^2)*(16*c^2*d*g^2+16*a^2*f*h^2-8*a*b*h*(e*h+2*f*g)+b^2*(5*d*h^
2+3*e*g*h+5*f*g^2)-4*c*(2*b*g*(2*d*h+e*g)+a*(d*h^2-5*e*g*h+f*g^2)))*arctanh
(1/2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^(1/2)/(c*x^2+b*x+a)^(1/
2))/(a*h^2-b*g*h+c*g^2)^(7/2)+1/64*(16*c^2*d*g^2+16*a^2*f*h^2-8*a*b*h*(e*h+
2*f*g)+b^2*(5*d*h^2+3*e*g*h+5*f*g^2)-4*c*(2*b*g*(2*d*h+e*g)+a*(d*h^2-5*e*g*
h+f*g^2)))*(b*g-2*a*h+(-b*h+2*c*g)*x)*(c*x^2+b*x+a)^(1/2)/(a*h^2-b*g*h+c*g^
2)^3/(h*x+g)^2
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 499, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {1664, 820, 734, 738, 212}

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^5}\,dx =$$

$$-\frac{(b^2-4ac)\operatorname{arctanh}\left(\frac{-2ah+x(2cg-bh)+bg}{2\sqrt{a+bx+cx^2}\sqrt{ah^2-bgh+cg^2}}\right)\left(16a^2fh^2-4c(-ah(5eg-dh)+afg^2+2bg(2dh+eg))-\right.}{128\left(ah^2-bgh+cg^2\right)^{7/2}}$$

$$+\frac{\sqrt{a+bx+cx^2}(-2ah+x(2cg-bh)+bg)\left(16a^2fh^2-4c(-ah(5eg-dh)+afg^2+2bg(2dh+eg))-8\right.}{64(g+hx)^2\left(ah^2-bgh+cg^2\right)^3}$$

$$-\frac{\left(a+bx+cx^2\right)^{3/2}\left(fg^2-h(eg-dh)\right)}{4h(g+hx)^4\left(ah^2-bgh+cg^2\right)}$$

$$+\frac{\left(a+bx+cx^2\right)^{3/2}\left(8ah^2(2fg-eh)-bh(11fg^2-h(5dh+3eg))+2cgh(eg-5dh)+6cfg^3\right)}{24h(g+hx)^3\left(ah^2-bgh+cg^2\right)^2}$$

```
[In] Int[(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^5,x]
```

```
[Out] ((16*c^2*d*g^2 + 16*a^2*f*h^2 - 8*a*b*h*(2*f*g + e*h) - 4*c*(a*f*g^2 - a*h*
(5*e*g - d*h) + 2*b*g*(e*g + 2*d*h)) + b^2*(5*f*g^2 + h*(3*e*g + 5*d*h)))*(
b*g - 2*a*h + (2*c*g - b*h)*x)*Sqrt[a + b*x + c*x^2])/(64*(c*g^2 - b*g*h +
a*h^2)^3*(g + h*x)^2) - ((f*g^2 - h*(e*g - d*h))*(a + b*x + c*x^2)^(3/2))/(
4*h*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^4) + ((6*c*f*g^3 + 2*c*g*h*(e*g - 5*d
*h) + 8*a*h^2*(2*f*g - e*h) - b*h*(11*f*g^2 - h*(3*e*g + 5*d*h)))*(a + b*x
+ c*x^2)^(3/2))/(24*h*(c*g^2 - b*g*h + a*h^2)^2*(g + h*x)^3) - ((b^2 - 4*a*
c)*(16*c^2*d*g^2 + 16*a^2*f*h^2 - 8*a*b*h*(2*f*g + e*h) - 4*c*(a*f*g^2 - a*
h*(5*e*g - d*h) + 2*b*g*(e*g + 2*d*h)) + b^2*(5*f*g^2 + h*(3*e*g + 5*d*h)))
*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqr
t[a + b*x + c*x^2])])/(128*(c*g^2 - b*g*h + a*h^2)^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), Int[(d + e*x)^(m + 2)*(a + b*x +
c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
```

```
&& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
 0] && GtQ[p, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\left(fg^2-h(eg-dh)\right)\left(a+bx+cx^2\right)^{3/2}}{4h\left(cg^2-bgh+ah^2\right)(g+hx)^4} \\
&\quad -\frac{\int \frac{\left(\frac{1}{2}\left(-8cdg+3beg+8afg-\frac{3bfg^2}{h}+5bdh-8aeh\right)-\left(ceg-4bfg+\frac{3cfg^2}{h}-cdh+4afh\right)x\right)\sqrt{a+bx+cx^2}}{(g+hx)^4}\,dx}{4\left(cg^2-bgh+ah^2\right)} \\
&= -\frac{\left(fg^2-h(eg-dh)\right)\left(a+bx+cx^2\right)^{3/2}}{4h\left(cg^2-bgh+ah^2\right)(g+hx)^4} \\
&\quad +\frac{\left(6cfg^3+2cgh(eg-5dh)+8ah^2(2fg-eh)-bh(11fg^2-h(3eg+5dh))\right)\left(a+bx+cx^2\right)^{3/2}}{24h\left(cg^2-bgh+ah^2\right)^2(g+hx)^3} \\
&\quad +\frac{(16c^2dg^2+16a^2fh^2-8abh(2fg+eh)-4c(afg^2-ah(5eg-dh)+2bg(eg+2dh))+b^2(5fg^2+}{16\left(cg^2-bgh+ah^2\right)^2}
\end{aligned}
$$

$$= \frac{(16c^2dg^2 + 16a^2fh^2 - 8abh(2fg+eh) - 4c(afg^2 - ah(5eg-dh) + 2bg(eg+2dh)) + b^2(5fg^2 +}{64\left(cg^2-bgh+ah^2\right)^3(g+hx)^2}$$
$$- \frac{\left(fg^2-h(eg-dh)\right)\left(a+bx+cx^2\right)^{3/2}}{4h\left(cg^2-bgh+ah^2\right)(g+hx)^4}$$
$$+ \frac{\left(6cfg^3+2cgh(eg-5dh)+8ah^2(2fg-eh)-bh(11fg^2-h(3eg+5dh))\right)\left(a+bx+cx^2\right)^{3/2}}{24h\left(cg^2-bgh+ah^2\right)^2(g+hx)^3}$$
$$- \frac{((b^2-4ac)\left(16c^2dg^2+16a^2fh^2-8abh(2fg+eh)-4c(afg^2-ah(5eg-dh)+2bg(eg+2dh)\right)}{128\left(cg^2-bgh+ah^2\right)^3}$$

$$= \frac{(16c^2dg^2 + 16a^2fh^2 - 8abh(2fg+eh) - 4c(afg^2 - ah(5eg-dh) + 2bg(eg+2dh)) + b^2(5fg^2 +}{64\left(cg^2-bgh+ah^2\right)^3(g+hx)^2}$$
$$- \frac{\left(fg^2-h(eg-dh)\right)\left(a+bx+cx^2\right)^{3/2}}{4h\left(cg^2-bgh+ah^2\right)(g+hx)^4}$$
$$+ \frac{\left(6cfg^3+2cgh(eg-5dh)+8ah^2(2fg-eh)-bh(11fg^2-h(3eg+5dh))\right)\left(a+bx+cx^2\right)^{3/2}}{24h\left(cg^2-bgh+ah^2\right)^2(g+hx)^3}$$
$$+ \frac{((b^2-4ac)\left(16c^2dg^2+16a^2fh^2-8abh(2fg+eh)-4c(afg^2-ah(5eg-dh)+2bg(eg+2dh)\right)}{64\left(cg^2-bgh+a\right.}$$

$$= \frac{(16c^2dg^2 + 16a^2fh^2 - 8abh(2fg+eh) - 4c(afg^2 - ah(5eg-dh) + 2bg(eg+2dh)) + b^2(5fg^2 +}{64\left(cg^2-bgh+ah^2\right)^3(g+hx)^2}$$
$$- \frac{\left(fg^2-h(eg-dh)\right)\left(a+bx+cx^2\right)^{3/2}}{4h\left(cg^2-bgh+ah^2\right)(g+hx)^4}$$
$$+ \frac{\left(6cfg^3+2cgh(eg-5dh)+8ah^2(2fg-eh)-bh(11fg^2-h(3eg+5dh))\right)\left(a+bx+cx^2\right)^{3/2}}{24h\left(cg^2-bgh+ah^2\right)^2(g+hx)^3}$$
$$- \frac{(b^2-4ac)\left(16c^2dg^2+16a^2fh^2-8abh(2fg+eh)-4c(afg^2-ah(5eg-dh)+2bg(eg+2dh))\right.}{128\left(cg^2-bgh+ah^2\right)^{7/2}}$$

Mathematica [A] (verified)

Time = 13.35 (sec) , antiderivative size = 693, normalized size of antiderivative = 1.39

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^5}\,dx$$
$$= \frac{-48h(cg^2+h(-bg+ah))^{5/2}\left(fg^2+h(-eg+dh)\right)(a+x(b+cx))^{3/2}+64h(2fg-eh)\left(cg^2+h(-bg+a\right.}{}$$

```
[In] Integrate[(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^5,x]
```

```
[Out] (-48*h*(c*g^2 + h*(-(b*g) + a*h))^(5/2)*(f*g^2 + h*(-(e*g) + d*h))*(a + x*(
b + c*x))^(3/2) + 64*h*(2*f*g - e*h)*(c*g^2 + h*(-(b*g) + a*h))^(5/2)*(g +
h*x)*(a + x*(b + c*x))^(3/2) + 48*f*(c*g^2 + h*(-(b*g) + a*h))^(5/2)*(g + h
*x)^2*Sqrt[a + x*(b + c*x)]*(-2*a*h + 2*c*g*x + b*(g - h*x)) - (f*g^2 + h*(
-(e*g) + d*h))*(g + h*x)*(40*h*(2*c*g - b*h)*(c*g^2 + h*(-(b*g) + a*h))^(3/
2)*(a + x*(b + c*x))^(3/2) + 3*(8*c^2*g^2 + (5*b^2*h^2)/2 - 2*c*h*(4*b*g +
a*h))*(g + h*x)*(-2*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*Sqrt[a + x*(b + c*x)]*(-
2*a*h + 2*c*g*x + b*(g - h*x)) - (b^2 - 4*a*c)*(g + h*x)^2*ArcTanh[(-(b*g)
+ 2*a*h - 2*c*g*x + b*h*x)/(2*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*Sqrt[a + x*(b
+ c*x)])])) - 24*(b^2 - 4*a*c)*f*(c*g^2 + h*(-(b*g) + a*h))^2*(g + h*x)^4*A
rcTanh[(-2*a*h + 2*c*g*x + b*(g - h*x))/(2*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*S
qrt[a + x*(b + c*x)])] + 12*(2*c*g - b*h)*(2*f*g - e*h)*(c*g^2 + h*(-(b*g)
+ a*h))*(g + h*x)^2*(-2*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*Sqrt[a + x*(b + c*x)
]*(-2*a*h + 2*c*g*x + b*(g - h*x)) + (b^2 - 4*a*c)*(g + h*x)^2*ArcTanh[(-2*
a*h + 2*c*g*x + b*(g - h*x))/(2*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*Sqrt[a + x*(
b + c*x)])]))/(192*h^2*(c*g^2 + h*(-(b*g) + a*h))^(7/2)*(g + h*x)^4)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 4939 vs. 2(475) = 950.

Time = 1.40 (sec) , antiderivative size = 4940, normalized size of antiderivative = 9.94

method	result	size
default	Expression too large to display	4940

```
[In] int((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^5,x,method=_RETURNVERBOSE)
```

```
[Out] f/h^5*(-1/2/(a*h^2-b*g*h+c*g^2)*h^2/(x+1/h*g)^2*((x+1/h*g)^2*c+(b*h-2*c*g)/
h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(3/2)-1/4*(b*h-2*c*g)*h/(a*h^2-b*g*h+c
*g^2)*(-1/(a*h^2-b*g*h+c*g^2)*h^2/(x+1/h*g)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x
+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(3/2)+1/2*(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2
)*(((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)+1/
2*(b*h-2*c*g)/h*ln((1/2*(b*h-2*c*g)/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g)^2*c+(
b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/c^(1/2)-(a*h^2-b*g*h
+c*g^2)/h^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(
b*h-2*c*g)/h*(x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c+(b*
h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g)))+2*c/(a*h^2
-b*g*h+c*g^2)*h^2*(1/4*(2*c*(x+1/h*g)+(b*h-2*c*g)/h)/c*((x+1/h*g)^2*c+(b*h-
2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)+1/8*(4*c*(a*h^2-b*g*h+c*g
^2)/h^2-(b*h-2*c*g)^2/h^2)/c^(3/2)*ln((1/2*(b*h-2*c*g)/h+c*(x+1/h*g))/c^(1/
2)+((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)))
+1/2*c/(a*h^2-b*g*h+c*g^2)*h^2*(((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h
^2-b*g*h+c*g^2)/h^2)^(1/2)+1/2*(b*h-2*c*g)/h*ln((1/2*(b*h-2*c*g)/h+c*(x+1/h
*g))/c^(1/2)+((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2
)^(1/2))/c^(1/2)-(a*h^2-b*g*h+c*g^2)/h^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln
```

```
((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/
h^2)^(1/2)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^
(1/2))/(x+1/h*g))))+(e*h-2*f*g)/h^6*(-1/3/(a*h^2-b*g*h+c*g^2)*h^2/(x+1/h*g)
^3*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(3/2)-1/
2*(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)*(-1/2/(a*h^2-b*g*h+c*g^2)*h^2/(x+1/h*g)
^2*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(3/2)-1/
4*(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)*(-1/(a*h^2-b*g*h+c*g^2)*h^2/(x+1/h*g)*(
(x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(3/2)+1/2*(b
*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)*(((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*
h^2-b*g*h+c*g^2)/h^2)^(1/2)+1/2*(b*h-2*c*g)/h*ln((1/2*(b*h-2*c*g)/h+c*(x+1/
h*g))/c^(1/2)+((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^
2)^(1/2))/c^(1/2)-(a*h^2-b*g*h+c*g^2)/h^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*l
n((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)
/h^2)^(1/2)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)
^(1/2))/(x+1/h*g)))+2*c/(a*h^2-b*g*h+c*g^2)*h^2*(1/4*(2*c*(x+1/h*g)+(b*h-2*
c*g)/h)/c*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(
1/2)+1/8*(4*c*(a*h^2-b*g*h+c*g^2)/h^2-(b*h-2*c*g)^2/h^2)/c^(3/2)*ln((1/2*(b
*h-2*c*g)/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*
h^2-b*g*h+c*g^2)/h^2)^(1/2))))+1/2*c/(a*h^2-b*g*h+c*g^2)*h^2*(((x+1/h*g)^2*
c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)+1/2*(b*h-2*c*g)/h*
ln((1/2*(b*h-2*c*g)/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+
1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/c^(1/2)-(a*h^2-b*g*h+c*g^2)/h^2/((a*
h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+
1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/
h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g)))))+(d*h^2-e*g*h+f*g^2)/h^7*
(-1/4/(a*h^2-b*g*h+c*g^2)*h^2/(x+1/h*g)^4*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1
/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(3/2)-5/8*(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)*
(-1/3/(a*h^2-b*g*h+c*g^2)*h^2/(x+1/h*g)^3*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1
/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(3/2)-1/2*(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)*
(-1/2/(a*h^2-b*g*h+c*g^2)*h^2/(x+1/h*g)^2*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1
/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(3/2)-1/4*(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)*
(-1/(a*h^2-b*g*h+c*g^2)*h^2/(x+1/h*g)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g
)+(a*h^2-b*g*h+c*g^2)/h^2)^(3/2)+1/2*(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)*(((x
+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)+1/2*(b*h
-2*c*g)/h*ln((1/2*(b*h-2*c*g)/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g)^2*c+(b*h-2*
c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/c^(1/2)-(a*h^2-b*g*h+c*g^2
)/h^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*
c*g)/h*(x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c+(b*h-2*c*
g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g)))+2*c/(a*h^2-b*g*h
+c*g^2)*h^2*(1/4*(2*c*(x+1/h*g)+(b*h-2*c*g)/h)/c*((x+1/h*g)^2*c+(b*h-2*c*g)
/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)+1/8*(4*c*(a*h^2-b*g*h+c*g^2)/h^
2-(b*h-2*c*g)^2/h^2)/c^(3/2)*ln((1/2*(b*h-2*c*g)/h+c*(x+1/h*g))/c^(1/2)+((x
+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))))+1/2*c
/(a*h^2-b*g*h+c*g^2)*h^2*(((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g
*h+c*g^2)/h^2)^(1/2)+1/2*(b*h-2*c*g)/h*ln((1/2*(b*h-2*c*g)/h+c*(x+1/h*g))/c
```

```
^(1/2)+((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2
))/c^(1/2)-(a*h^2-b*g*h+c*g^2)/h^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a
*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(
1/2)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))
/(x+1/h*g)))))-1/4*c/(a*h^2-b*g*h+c*g^2)*h^2*(-1/2/(a*h^2-b*g*h+c*g^2)*h^2/
(x+1/h*g)^2*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)
^(3/2)-1/4*(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)*(-1/(a*h^2-b*g*h+c*g^2)*h^2/(x
+1/h*g)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(3/
2)+1/2*(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)*(((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1
/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)+1/2*(b*h-2*c*g)/h*ln((1/2*(b*h-2*c*g)/
h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+
c*g^2)/h^2)^(1/2))/c^(1/2)-(a*h^2-b*g*h+c*g^2)/h^2/((a*h^2-b*g*h+c*g^2)/h^2
)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+1/h*g)+2*((a*h^2-b*g
*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*
g^2)/h^2)^(1/2))/(x+1/h*g)))+2*c/(a*h^2-b*g*h+c*g^2)*h^2*(1/4*(2*c*(x+1/h*g
)+(b*h-2*c*g)/h)/c*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^
2)/h^2)^(1/2)+1/8*(4*c*(a*h^2-b*g*h+c*g^2)/h^2-(b*h-2*c*g)^2/h^2)/c^(3/2)*l
n((1/2*(b*h-2*c*g)/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1
/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))))+1/2*c/(a*h^2-b*g*h+c*g^2)*h^2*(((x+
1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)+1/2*(b*h-
2*c*g)/h*ln((1/2*(b*h-2*c*g)/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g)^2*c+(b*h-2*c
*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/c^(1/2)-(a*h^2-b*g*h+c*g^2)
/h^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c
*g)/h*(x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c+(b*h-2*c*g
)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g)))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^5}\,dx=\text{Timed out}$$

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^5,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^5}\,dx = \int \frac{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}{(g+hx)^5}\,dx$$

```
[In] integrate((f*x**2+e*x+d)*(c*x**2+b*x+a)**(1/2)/(h*x+g)**5,x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)/(g + h*x)**5, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^5}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^5,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*h^2-b*g*h>0)', see `assume?` for
more de
```

Giac [F]

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^5}\,dx = \int \frac{\sqrt{cx^2+bx+a}\,(fx^2+ex+d)}{(hx+g)^5}\,dx$$

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^5,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^5}\,dx = \int \frac{\sqrt{c\,x^2+b\,x+a}\,\left(f\,x^2+e\,x+d\right)}{(g+h\,x)^5}\,dx$$

```
[In] int(((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^5,x)
```

```
[Out] int(((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^5, x)
```

3.195 $\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^6} \, dx$

Optimal result

Integrand size = 32, antiderivative size = 824

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^6} \, dx$$

$$= \frac{(32c^3dg^3 - 8c^2g(2bg(eg+3dh) + a(fg^2 - 6egh + 3dh^2)) - bh(16a^2fh^2 - 2abh(6fg+5eh) + b^2(3fg^2 + 3}{12}$$

$$- \frac{(fg^2 - h(eg-dh))\,(a+bx+cx^2)^{3/2}}{5h\,(cg^2 - bgh + ah^2)\,(g+hx)^5}$$

$$+ \frac{(2cg(3fg^2 + h(2eg-7dh)) + h(10ah(2fg-eh) - b(13fg^2 - 3egh - 7dh^2)))\,(a+bx+cx^2)^{3/2}}{40h\,(cg^2 - bgh + ah^2)^2\,(g+hx)^4}$$

$$+ \frac{(4c^2g^2(3fg^2 + h(2eg-27dh)) - 5h^2(16a^2fh^2 - 2abh(6fg+5eh) + b^2(3fg^2 + 3egh + 7dh^2)) - 2ch(bg(}{240h\,(cg^2 - bgh + ah^2)^3\,(g+h}$$

$$- \frac{(b^2 - 4ac)\,(32c^3dg^3 - 8c^2g(2bg(eg+3dh) + a(fg^2 - 6egh + 3dh^2)) - bh(16a^2fh^2 - 2abh(6fg+5eh) +}{}$$

```
[Out] -1/5*(f*g^2-h*(-d*h+e*g))*(c*x^2+b*x+a)^(3/2)/h/(a*h^2-b*g*h+c*g^2)/(h*x+g)
^5+1/40*(2*c*g*(3*f*g^2+h*(-7*d*h+2*e*g))+h*(10*a*h*(-e*h+2*f*g)-b*(-7*d*h^
2-3*e*g*h+13*f*g^2)))*(c*x^2+b*x+a)^(3/2)/h/(a*h^2-b*g*h+c*g^2)^2/(h*x+g)^4
+1/240*(4*c^2*g^2*(3*f*g^2+h*(-27*d*h+2*e*g))-5*h^2*(16*a^2*f*h^2-2*a*b*h*(
5*e*h+6*f*g)+b^2*(7*d*h^2+3*e*g*h+3*f*g^2))-2*c*h*(b*g*(-54*d*h^2-21*e*g*h+
16*f*g^2)-2*a*h*(8*d*h^2-33*e*g*h+18*f*g^2)))*(c*x^2+b*x+a)^(3/2)/h/(a*h^2-
b*g*h+c*g^2)^3/(h*x+g)^3-1/256*(-4*a*c+b^2)*(32*c^3*d*g^3-8*c^2*g*(2*b*g*(3
*d*h+e*g)+a*(3*d*h^2-6*e*g*h+f*g^2))-b*h*(16*a^2*f*h^2-2*a*b*h*(5*e*h+6*f*g
)+b^2*(7*d*h^2+3*e*g*h+3*f*g^2))+2*c*(4*a^2*h^2*(-e*h+6*f*g)-6*a*b*h*(-d*h^
2+3*e*g*h+3*f*g^2)+b^2*g*(15*d*h^2+6*e*g*h+5*f*g^2)))*arctanh(1/2*(b*g-2*a*
h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/(a*h^2-b*g
```

```
*h+c*g^2)^(9/2)+1/128*(32*c^3*d*g^3-8*c^2*g*(2*b*g*(3*d*h+e*g)+a*(3*d*h^2-6
*e*g*h+f*g^2))-b*h*(16*a^2*f*h^2-2*a*b*h*(5*e*h+6*f*g)+b^2*(7*d*h^2+3*e*g*h
+3*f*g^2))+2*c*(4*a^2*h^2*(-e*h+6*f*g)-6*a*b*h*(-d*h^2+3*e*g*h+3*f*g^2)+b^2
*g*(15*d*h^2+6*e*g*h+5*f*g^2)))*(b*g-2*a*h+(-b*h+2*c*g)*x)*(c*x^2+b*x+a)^(1
/2)/(a*h^2-b*g*h+c*g^2)^4/(h*x+g)^2
```

Rubi [A] (verified)

Time = 1.34 (sec) , antiderivative size = 826, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1664, 848, 820, 734, 738, 212}

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^6}\,dx$$

$$= \frac{\left(4\left(3fg^4+h(2eg-27dh)g^2\right)c^2-2h\left(bg\left(16fg^2-21ehg-54dh^2\right)-2ah\left(18fg^2-33ehg+8dh^2\right)\right)c-5h^2}{240h\left(cg^2-bhg+ah^2\right)^3(g+h}$$

$$+\frac{\left(6cfg^3+2ch(2eg-7dh)g+10ah^2(2fg-eh)-bh\left(13fg^2-h(3eg+7dh)\right)\right)\left(cx^2+bx+a\right)^{3/2}}{40h\left(cg^2-bhg+ah^2\right)^2(g+hx)^4}$$

$$-\frac{\left(fg^2-h(eg-dh)\right)\left(cx^2+bx+a\right)^{3/2}}{5h\left(cg^2-bhg+ah^2\right)(g+hx)^5}$$

$$+\frac{\left(32c^3dg^3-8c^2\left(afg^2+2b(eg+3dh)g-3ah(2eg-dh)\right)g+2c\left(\left(5fg^3+3h(2eg+5dh)g\right)b^2-6ah(3fg\right.\right.}{}$$

$$-\frac{\left(b^2-4ac\right)\left(32c^3dg^3-8c^2\left(afg^2+2b(eg+3dh)g-3ah(2eg-dh)\right)g+2c\left(\left(5fg^3+3h(2eg+5dh)g\right)b^2\right.\right.}{}$$

```
[In] Int[(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^6,x]
```

```
[Out] ((32*c^3*d*g^3 - 8*c^2*g*(a*f*g^2 - 3*a*h*(2*e*g - d*h) + 2*b*g*(e*g + 3*d*
h)) + 2*c*(4*a^2*h^2*(6*f*g - e*h) - 6*a*b*h*(3*f*g^2 + h*(3*e*g - d*h)) +
b^2*(5*f*g^3 + 3*g*h*(2*e*g + 5*d*h))) - b*h*(16*a^2*f*h^2 - 2*a*b*h*(6*f*g
 + 5*e*h) + b^2*(3*f*g^2 + h*(3*e*g + 7*d*h))))*(b*g - 2*a*h + (2*c*g - b*h
)*x)*Sqrt[a + b*x + c*x^2])/(128*(c*g^2 - b*g*h + a*h^2)^4*(g + h*x)^2) - (
(f*g^2 - h*(e*g - d*h))*(a + b*x + c*x^2)^(3/2))/(5*h*(c*g^2 - b*g*h + a*h^
2)*(g + h*x)^5) + ((6*c*f*g^3 + 2*c*g*h*(2*e*g - 7*d*h) + 10*a*h^2*(2*f*g -
 e*h) - b*h*(13*f*g^2 - h*(3*e*g + 7*d*h)))*(a + b*x + c*x^2)^(3/2))/(40*h*
(c*g^2 - b*g*h + a*h^2)^2*(g + h*x)^4) + ((4*c^2*(3*f*g^4 + g^2*h*(2*e*g -
27*d*h)) - 5*h^2*(16*a^2*f*h^2 - 2*a*b*h*(6*f*g + 5*e*h) + b^2*(3*f*g^2 + 3
*e*g*h + 7*d*h^2)) - 2*c*h*(b*g*(16*f*g^2 - 21*e*g*h - 54*d*h^2) - 2*a*h*(1
8*f*g^2 - 33*e*g*h + 8*d*h^2)))*(a + b*x + c*x^2)^(3/2))/(240*h*(c*g^2 - b*
g*h + a*h^2)^3*(g + h*x)^3) - ((b^2 - 4*a*c)*(32*c^3*d*g^3 - 8*c^2*g*(a*f*g
^2 - 3*a*h*(2*e*g - d*h) + 2*b*g*(e*g + 3*d*h)) + 2*c*(4*a^2*h^2*(6*f*g - e
*h) - 6*a*b*h*(3*f*g^2 + h*(3*e*g - d*h)) + b^2*(5*f*g^3 + 3*g*h*(2*e*g + 5
```

```
*d*h))) - b*h*(16*a^2*f*h^2 - 2*a*b*h*(6*f*g + 5*e*h) + b^2*(3*f*g^2 + h*(3
*e*g + 7*d*h))))*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*
g*h + a*h^2]*Sqrt[a + b*x + c*x^2])])/(256*(c*g^2 - b*g*h + a*h^2)^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), Int[(d + e*x)^(m + 2)*(a + b*x +
c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
 0] && GtQ[p, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 848

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
```

```
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

integral

$$= -\frac{(fg^2-h(eg-dh))\left(a+bx+cx^2\right)^{3/2}}{5h\left(cg^2-bgh+ah^2\right)(g+hx)^5} - \frac{\int \frac{\left(\frac{1}{2}\left(-10cdg+3beg+10afg-\frac{3bfg^2}{h}+7bdh-10aeh\right)-\left(2ceg-5bfg+\frac{3cfg^2}{h}-2cdh+5afh\right)x\right)\sqrt{a+bx+cx^2}}{(g+hx)^5}\,dx}{5\left(cg^2-bgh+ah^2\right)}$$

$$= -\frac{(fg^2-h(eg-dh))\left(a+bx+cx^2\right)^{3/2}}{5h\left(cg^2-bgh+ah^2\right)(g+hx)^5} + \frac{(6cfg^3+2cgh(2eg-7dh)+10ah^2(2fg-eh)-bh(13fg^2-h(3eg+7dh)))\left(a+bx+cx^2\right)^{3/2}}{40h\left(cg^2-bgh+ah^2\right)^2(g+hx)^4} + \frac{\int \frac{\left(\frac{1}{4}\left(80c^2dg^2+80a^2fh^2-2bcg\left(18eg-\frac{3fg^2}{h}+47dh\right)-10abh(6fg+5eh)-16ac\left(2fg^2-h(7eg-2dh)\right)+5b^2\left(3fg^2+h(3eg+7dh)\right)\right)+\frac{c\left(6cfg^3+2\right.}{}}{(g+hx)^4}}{20\left(cg^2-bgh+ah^2\right)^2}$$

$$= -\frac{(fg^2-h(eg-dh))\left(a+bx+cx^2\right)^{3/2}}{5h\left(cg^2-bgh+ah^2\right)(g+hx)^5} + \frac{(6cfg^3+2cgh(2eg-7dh)+10ah^2(2fg-eh)-bh(13fg^2-h(3eg+7dh)))\left(a+bx+cx^2\right)^{3/2}}{40h\left(cg^2-bgh+ah^2\right)^2(g+hx)^4} + \frac{(4c^2(3fg^4+g^2h(2eg-27dh))-5h^2(16a^2fh^2-2abh(6fg+5eh)+b^2(3fg^2+3egh+7dh^2))-2ch(bg}{240h\left(cg^2-bgh+ah^2\right)^3(g+} + \frac{(32c^3dg^3-8c^2g(afg^2-3ah(2eg-dh)+2bg(eg+3dh))+2c(4a^2h^2(6fg-eh)-6abh(3fg^2+h(3eg}{32(}$$

$$=\frac{(32c^3dg^3-8c^2g(afg^2-3ah(2eg-dh)+2bg(eg+3dh))+2c(4a^2h^2(6fg-eh)-6abh(3fg^2+h(3eg-d}{}$$

$$-\frac{(fg^2-h(eg-dh))\left(a+bx+cx^2\right)^{3/2}}{5h\left(cg^2-bgh+ah^2\right)(g+hx)^5}$$

$$+\frac{(6cfg^3+2cgh(2eg-7dh)+10ah^2(2fg-eh)-bh(13fg^2-h(3eg+7dh)))\left(a+bx+cx^2\right)^{3/2}}{40h\left(cg^2-bgh+ah^2\right)^2(g+hx)^4}$$

$$+\frac{(4c^2(3fg^4+g^2h(2eg-27dh))-5h^2(16a^2fh^2-2abh(6fg+5eh)+b^2(3fg^2+3egh+7dh^2))-2ch(bg(}{240h\left(cg^2-bgh+ah^2\right)^3(g+h}$$

$$-\frac{((b^2-4ac)\,(32c^3dg^3-8c^2g(afg^2-3ah(2eg-dh)+2bg(eg+3dh))+2c(4a^2h^2(6fg-eh)-6abh(3fg}{}$$

$$=\frac{(32c^3dg^3-8c^2g(afg^2-3ah(2eg-dh)+2bg(eg+3dh))+2c(4a^2h^2(6fg-eh)-6abh(3fg^2+h(3eg-d}{}$$

$$-\frac{(fg^2-h(eg-dh))\left(a+bx+cx^2\right)^{3/2}}{5h\left(cg^2-bgh+ah^2\right)(g+hx)^5}$$

$$+\frac{(6cfg^3+2cgh(2eg-7dh)+10ah^2(2fg-eh)-bh(13fg^2-h(3eg+7dh)))\left(a+bx+cx^2\right)^{3/2}}{40h\left(cg^2-bgh+ah^2\right)^2(g+hx)^4}$$

$$+\frac{(4c^2(3fg^4+g^2h(2eg-27dh))-5h^2(16a^2fh^2-2abh(6fg+5eh)+b^2(3fg^2+3egh+7dh^2))-2ch(bg(}{240h\left(cg^2-bgh+ah^2\right)^3(g+h}$$

$$+\frac{((b^2-4ac)\,(32c^3dg^3-8c^2g(afg^2-3ah(2eg-dh)+2bg(eg+3dh))+2c(4a^2h^2(6fg-eh)-6abh(3fg}{}$$

$$=\frac{(32c^3dg^3-8c^2g(afg^2-3ah(2eg-dh)+2bg(eg+3dh))+2c(4a^2h^2(6fg-eh)-6abh(3fg^2+h(3eg-d}{}$$

$$-\frac{(fg^2-h(eg-dh))\left(a+bx+cx^2\right)^{3/2}}{5h\left(cg^2-bgh+ah^2\right)(g+hx)^5}$$

$$+\frac{(6cfg^3+2cgh(2eg-7dh)+10ah^2(2fg-eh)-bh(13fg^2-h(3eg+7dh)))\left(a+bx+cx^2\right)^{3/2}}{40h\left(cg^2-bgh+ah^2\right)^2(g+hx)^4}$$

$$+\frac{(4c^2(3fg^4+g^2h(2eg-27dh))-5h^2(16a^2fh^2-2abh(6fg+5eh)+b^2(3fg^2+3egh+7dh^2))-2ch(bg(}{240h\left(cg^2-bgh+ah^2\right)^3(g+h}$$

$$-\frac{(b^2-4ac)\,(32c^3dg^3-8c^2g(afg^2-3ah(2eg-dh)+2bg(eg+3dh))+2c(4a^2h^2(6fg-eh)-6abh(3fg^2}{}$$

Mathematica [A] (verified)

Time = 16.24 (sec) , antiderivative size = 1334, normalized size of antiderivative = 1.62

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^6}\,dx = -\frac{(fg^2-h(eg-dh))\,(a+bx+cx^2)\,\sqrt{a+x(b+cx)}}{5h\,(cg^2-h(bg-ah))\,(g+hx)^5}$$

$$+\frac{(2fg-eh)\,(a+bx+cx^2)\,\sqrt{a+x(b+cx)}}{4h\,(cg^2-h(bg-ah))\,(g+hx)^4}-\frac{f(a+bx+cx^2)\,\sqrt{a+x(b+cx)}}{3h\,(cg^2-bgh+ah^2)\,(g+hx)^3}$$

$$-\frac{(-2fg+eh)\sqrt{a+x(b+cx)}\Bigg(\frac{\left(cgh-\frac{1}{2}h(-8cg+5bh)\right)\left(a+bx+cx^2\right)^{3/2}}{3(cg^2-bgh+ah^2)(g+hx)^3}-\frac{\big(-2\big(ach^2+\frac{1}{2}cg(-8cg+5bh)\big)+b\big(cgh+\frac{1}{2}h(-8cg+5bh)\big)}{}}{4h^2\,(cg^2-bgh+ah^2)\,\sqrt{a+bx+cx}}$$

$$-\frac{(fg^2-egh+dh^2)\,\sqrt{a+x(b+cx)}\Bigg(-\frac{\left(-2cgh+\frac{1}{2}h(-10cg+7bh)\right)\left(a+bx+cx^2\right)^{3/2}}{4(cg^2-bgh+ah^2)(g+hx)^4}-\frac{\frac{\big(-\frac{7}{2}cgh(2cg-bh)-\frac{1}{4}h\big(80c^2g^2+35b^2h^2-2c}{3\big(cg^2-bgh+ah^2\big)}}{}}{}$$

$$+\frac{f(2cg-bh)\sqrt{a+x(b+cx)}\left(\frac{2(bg-2ah+(2cg-bh)x)\sqrt{a+bx+cx^2}}{(cg^2-bgh+ah^2)(g+hx)^2}-\frac{(b^2-4ac)\operatorname{arctanh}\left(\frac{bg-2ah+(2cg-bh)x}{2\sqrt{cg^2-h(bg-ah)}\sqrt{a+bx+cx^2}}\right)}{(cg^2-h(bg-ah))^{3/2}}\right)}{16h^2\,(cg^2-h(bg-ah))\,\sqrt{a+bx+cx^2}}$$

```
[In] Integrate[(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^6,x]
```

```
[Out] -1/5*((f*g^2 - h*(e*g - d*h))*(a + b*x + c*x^2)*Sqrt[a + x*(b + c*x)])/(h*(
c*g^2 - h*(b*g - a*h))*(g + h*x)^5) + ((2*f*g - e*h)*(a + b*x + c*x^2)*Sqrt
[a + x*(b + c*x)])/(4*h*(c*g^2 - h*(b*g - a*h))*(g + h*x)^4) - (f*(a + b*x
+ c*x^2)*Sqrt[a + x*(b + c*x)])/(3*h*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^3) -
 ((-2*f*g + e*h)*Sqrt[a + x*(b + c*x)]*(((c*g*h - (h*(-8*c*g + 5*b*h))/2)*(
a + b*x + c*x^2)^(3/2))/(3*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^3) - ((-2*(a*c
*h^2 + (c*g*(-8*c*g + 5*b*h))/2) + b*(c*g*h + (h*(-8*c*g + 5*b*h))/2))*(((b
*g - 2*a*h + (2*c*g - b*h)*x)*Sqrt[a + b*x + c*x^2])/(4*(c*g^2 - b*g*h + a*
h^2)*(g + h*x)^2) + ((b^2 - 4*a*c)*ArcTanh[(-(b*g) + 2*a*h - (2*c*g - b*h)*
x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c*g^2 -
b*g*h + a*h^2]*(4*c*g^2 - 4*b*g*h + 4*a*h^2))))/(2*(c*g^2 - b*g*h + a*h^2))
))/(4*h^2*(c*g^2 - b*g*h + a*h^2)*Sqrt[a + b*x + c*x^2]) - ((f*g^2 - e*g*h
```

```
+ d*h^2)*Sqrt[a + x*(b + c*x)]*(-1/4*((-2*c*g*h + (h*(-10*c*g + 7*b*h))/2)*
(a + b*x + c*x^2)^(3/2))/((c*g^2 - b*g*h + a*h^2)*(g + h*x)^4) - ((((-7*c*g
*h*(2*c*g - b*h))/2 - (h*(80*c^2*g^2 + 35*b^2*h^2 - 2*c*h*(47*b*g + 16*a*h)
))/4)*(a + b*x + c*x^2)^(3/2))/(3*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^3) - ((
-2*((-7*a*c*h^2*(2*c*g - b*h))/2 + (c*g*(80*c^2*g^2 + 35*b^2*h^2 - 2*c*h*(4
7*b*g + 16*a*h)))/4) + b*((-7*c*g*h*(2*c*g - b*h))/2 + (h*(80*c^2*g^2 + 35*
b^2*h^2 - 2*c*h*(47*b*g + 16*a*h)))/4))*(((b*g - 2*a*h + (2*c*g - b*h)*x)*S
qrt[a + b*x + c*x^2])/(4*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^2) + ((b^2 - 4*a
*c)*ArcTanh[(-(b*g) + 2*a*h - (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^
2]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*(4*c*g^2 - 4*b*g
*h + 4*a*h^2))))/(2*(c*g^2 - b*g*h + a*h^2)))/(4*(c*g^2 - b*g*h + a*h^2)))
/(5*h^2*(c*g^2 - b*g*h + a*h^2)*Sqrt[a + b*x + c*x^2]) + (f*(2*c*g - b*h)*S
qrt[a + x*(b + c*x)]*((2*(b*g - 2*a*h + (2*c*g - b*h)*x)*Sqrt[a + b*x + c*x
^2])/((c*g^2 - b*g*h + a*h^2)*(g + h*x)^2) - ((b^2 - 4*a*c)*ArcTanh[(b*g -
2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - h*(b*g - a*h)]*Sqrt[a + b*x + c*x^
2])])/(c*g^2 - h*(b*g - a*h))^(3/2)))/(16*h^2*(c*g^2 - h*(b*g - a*h))*Sqrt[
a + b*x + c*x^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 7713 vs. $2(798) = 1596$.

Time = 1.55 (sec) , antiderivative size = 7714, normalized size of antiderivative = 9.36

method	result	size
default	Expression too large to display	7714

```
[In] int((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^6,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^6}\,dx = \text{Timed out}$$

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^6,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^6}\,dx=\int \frac{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}{(g+hx)^6}\,dx$$

```
[In] integrate((f*x**2+e*x+d)*(c*x**2+b*x+a)**(1/2)/(h*x+g)**6,x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)/(g + h*x)**6, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^6}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^6,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*h^2-b*g*h>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 28577 vs. $2(798) = 1596$.

Time = 1.87 (sec) , antiderivative size = 28577, normalized size of antiderivative = 34.68

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^6}\,dx=\text{Too large to display}$$

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^6,x, algorithm="giac")
```

```
[Out] -1/128*(32*b^2*c^3*d*g^3 - 128*a*c^4*d*g^3 - 16*b^3*c^2*e*g^3 + 64*a*b*c^3*
e*g^3 + 10*b^4*c*f*g^3 - 48*a*b^2*c^2*f*g^3 + 32*a^2*c^3*f*g^3 - 48*b^3*c^2
*d*g^2*h + 192*a*b*c^3*d*g^2*h + 12*b^4*c*e*g^2*h - 192*a^2*c^3*e*g^2*h - 3
*b^5*f*g^2*h - 24*a*b^3*c*f*g^2*h + 144*a^2*b*c^2*f*g^2*h + 30*b^4*c*d*g*h^
2 - 144*a*b^2*c^2*d*g*h^2 + 96*a^2*c^3*d*g*h^2 - 3*b^5*e*g*h^2 - 24*a*b^3*c
*e*g*h^2 + 144*a^2*b*c^2*e*g*h^2 + 12*a*b^4*f*g*h^2 - 192*a^3*c^2*f*g*h^2 -
 7*b^5*d*h^3 + 40*a*b^3*c*d*h^3 - 48*a^2*b*c^2*d*h^3 + 10*a*b^4*e*h^3 - 48*
a^2*b^2*c*e*h^3 + 32*a^3*c^2*e*h^3 - 16*a^2*b^3*f*h^3 + 64*a^3*b*c*f*h^3)*a
rctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*h + sqrt(c)*g)/sqrt(-c*g^2 + b*
```

```
g*h - a*h^2))/((c^4*g^8 - 4*b*c^3*g^7*h + 6*b^2*c^2*g^6*h^2 + 4*a*c^3*g^6*h
^2 - 4*b^3*c*g^5*h^3 - 12*a*b*c^2*g^5*h^3 + b^4*g^4*h^4 + 12*a*b^2*c*g^4*h^
4 + 6*a^2*c^2*g^4*h^4 - 4*a*b^3*g^3*h^5 - 12*a^2*b*c*g^3*h^5 + 6*a^2*b^2*g^
2*h^6 + 4*a^3*c*g^2*h^6 - 4*a^3*b*g*h^7 + a^4*h^8)*sqrt(-c*g^2 + b*g*h - a*
h^2)) + 1/1920*(480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*b^2*c^3*d*g^3*h^8
 - 1920*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a*c^4*d*g^3*h^8 - 240*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^9*b^3*c^2*e*g^3*h^8 + 960*(sqrt(c)*x - sqrt(c*
x^2 + b*x + a))^9*a*b*c^3*e*g^3*h^8 + 150*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))^9*b^4*c*f*g^3*h^8 - 720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a*b^2*c^2*
f*g^3*h^8 + 480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^2*c^3*f*g^3*h^8 - 7
20*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*b^3*c^2*d*g^2*h^9 + 2880*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^9*a*b*c^3*d*g^2*h^9 + 180*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^9*b^4*c*e*g^2*h^9 - 2880*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^
9*a^2*c^3*e*g^2*h^9 - 45*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*b^5*f*g^2*h^
9 - 360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a*b^3*c*f*g^2*h^9 + 2160*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^9*a^2*b*c^2*f*g^2*h^9 + 450*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))^9*b^4*c*d*g*h^10 - 2160*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^9*a*b^2*c^2*d*g*h^10 + 1440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^2
*c^3*d*g*h^10 - 45*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*b^5*e*g*h^10 - 360
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a*b^3*c*e*g*h^10 + 2160*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))^9*a^2*b*c^2*e*g*h^10 + 180*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^9*a*b^4*f*g*h^10 - 2880*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a
^3*c^2*f*g*h^10 - 105*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*b^5*d*h^11 + 60
0*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a*b^3*c*d*h^11 - 720*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^9*a^2*b*c^2*d*h^11 + 150*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^9*a*b^4*e*h^11 - 720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^2*b^2*
c*e*h^11 + 480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^3*c^2*e*h^11 - 240*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^2*b^3*f*h^11 + 960*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^9*a^3*b*c*f*h^11 + 3840*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))^8*c^(11/2)*f*g^8*h^3 - 15360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*b*c^
(9/2)*f*g^7*h^4 + 23040*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*b^2*c^(7/2)*f
*g^6*h^5 + 15360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a*c^(9/2)*f*g^6*h^5
 - 15360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*b^3*c^(5/2)*f*g^5*h^6 - 46080
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a*b*c^(7/2)*f*g^5*h^6 + 4320*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^8*b^2*c^(7/2)*d*g^4*h^7 - 17280*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^8*a*c^(9/2)*d*g^4*h^7 - 2160*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^8*b^3*c^(5/2)*e*g^4*h^7 + 8640*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))^8*a*b*c^(7/2)*e*g^4*h^7 + 5190*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*b^
4*c^(3/2)*f*g^4*h^7 + 39600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a*b^2*c^(
5/2)*f*g^4*h^7 + 27360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^2*c^(7/2)*f*
g^4*h^7 - 6480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*b^3*c^(5/2)*d*g^3*h^8
+ 25920*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a*b*c^(7/2)*d*g^3*h^8 + 1620*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*b^4*c^(3/2)*e*g^3*h^8 - 25920*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^8*a^2*c^(7/2)*e*g^3*h^8 - 405*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^8*b^5*sqrt(c)*f*g^3*h^8 - 18600*(sqrt(c)*x - sqrt(c*x^2
```

```
 + b*x + a))^8*a*b^3*c^(3/2)*f*g^3*h^8 - 26640*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^8*a^2*b*c^(5/2)*f*g^3*h^8 + 4050*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)^8*b^4*c^(3/2)*d*g^2*h^9 - 19440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a*b
^2*c^(5/2)*d*g^2*h^9 + 12960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^2*c^(7
/2)*d*g^2*h^9 - 405*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*b^5*sqrt(c)*e*g^2
*h^9 - 3240*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a*b^3*c^(3/2)*e*g^2*h^9 +
 19440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^2*b*c^(5/2)*e*g^2*h^9 + 1620
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a*b^4*sqrt(c)*f*g^2*h^9 + 23040*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^8*a^2*b^2*c^(3/2)*f*g^2*h^9 - 10560*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^8*a^3*c^(5/2)*f*g^2*h^9 - 945*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^8*b^5*sqrt(c)*d*g*h^10 + 5400*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))^8*a*b^3*c^(3/2)*d*g*h^10 - 6480*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))^8*a^2*b*c^(5/2)*d*g*h^10 + 1350*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a
*b^4*sqrt(c)*e*g*h^10 - 6480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^2*b^2*
c^(3/2)*e*g*h^10 + 4320*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^3*c^(5/2)*e
*g*h^10 - 2160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^2*b^3*sqrt(c)*f*g*h^
10 - 6720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^3*b*c^(3/2)*f*g*h^10 + 38
40*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^4*c^(3/2)*f*h^11 + 7680*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^7*c^6*f*g^9*h^2 + 5120*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^7*c^6*e*g^8*h^3 - 24320*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*b
*c^5*f*g^8*h^3 - 20480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*b*c^5*e*g^7*h^
4 + 20480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*b^2*c^4*f*g^7*h^4 + 30720*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a*c^5*f*g^7*h^4 + 30720*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^7*b^2*c^4*e*g^6*h^5 + 20480*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))^7*a*c^5*e*g^6*h^5 + 7680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*b
^3*c^3*f*g^6*h^5 - 66560*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a*b*c^4*f*g^
6*h^5 + 15040*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*b^2*c^4*d*g^5*h^6 - 601
60*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a*c^5*d*g^5*h^6 - 28000*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))^7*b^3*c^3*e*g^5*h^6 - 31360*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))^7*a*b*c^4*e*g^5*h^6 - 13220*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))^7*b^4*c^2*f*g^5*h^6 - 7200*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a*b^2*
c^3*f*g^5*h^6 + 61120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^2*c^4*f*g^5*h
^6 - 20320*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*b^3*c^3*d*g^4*h^7 + 81280*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a*b*c^4*d*g^4*h^7 + 9640*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))^7*b^4*c^2*e*g^4*h^7 + 65920*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^7*a*b^2*c^3*e*g^4*h^7 - 59520*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))^7*a^2*c^4*e*g^4*h^7 + 5690*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*b^5*c*
f*g^4*h^7 + 31440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a*b^3*c^2*f*g^4*h^7
 + 16160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^2*b*c^3*f*g^4*h^7 + 10740*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*b^4*c^2*d*g^3*h^8 - 56480*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))^7*a*b^2*c^3*d*g^3*h^8 + 54080*(sqrt(c)*x - sqrt(c*
x^2 + b*x + a))^7*a^2*c^4*d*g^3*h^8 - 570*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))^7*b^5*c*e*g^3*h^8 - 30640*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a*b^3*c^
2*e*g^3*h^8 - 11680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^2*b*c^3*e*g^3*h
^8 - 210*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*b^6*f*g^3*h^8 - 22340*(sqrt(
```

```
c)*x - sqrt(c*x^2 + b*x + a))^7*a*b^4*c*f*g^3*h^8 - 17280*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^7*a^2*b^2*c^2*f*g^3*h^8 - 61760*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^7*a^3*c^3*f*g^3*h^8 - 1190*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^
7*b^5*c*d*g^2*h^9 + 12080*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a*b^3*c^2*d
*g^2*h^9 - 29280*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^2*b*c^3*d*g^2*h^9
- 210*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*b^6*e*g^2*h^9 + 2180*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))^7*a*b^4*c*e*g^2*h^9 + 18240*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))^7*a^2*b^2*c^2*e*g^2*h^9 + 48960*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^7*a^3*c^3*e*g^2*h^9 + 1050*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a*
b^5*f*g^2*h^9 + 32560*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^2*b^3*c*f*g^2
*h^9 + 1440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^3*b*c^2*f*g^2*h^9 - 490
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*b^6*d*g*h^10 + 700*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^7*a*b^4*c*d*g*h^10 + 6720*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^7*a^2*b^2*c^2*d*g*h^10 - 6720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a
^3*c^3*d*g*h^10 + 910*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a*b^5*e*g*h^10
- 1680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^2*b^3*c*e*g*h^10 - 28320*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^3*b*c^2*e*g*h^10 - 1960*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^7*a^2*b^4*f*g*h^10 - 21120*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))^7*a^3*b^2*c*f*g*h^10 + 21120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^
7*a^4*c^2*f*g*h^10 + 490*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a*b^5*d*h^11
 - 2800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^2*b^3*c*d*h^11 + 3360*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^7*a^3*b*c^2*d*h^11 - 700*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))^7*a^2*b^4*e*h^11 + 3360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)^7*a^3*b^2*c*e*h^11 + 2880*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^4*c^2*e
*h^11 + 1120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^3*b^3*f*h^11 + 1920*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^4*b*c*f*h^11 + 7680*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^6*c^(13/2)*f*g^10*h + 5120*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))^6*c^(13/2)*e*g^9*h^2 - 16640*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*b
*c^(11/2)*f*g^9*h^2 + 7680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*c^(13/2)*d
*g^8*h^3 - 8960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*b*c^(11/2)*e*g^8*h^3
- 6400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*b^2*c^(9/2)*f*g^8*h^3 + 23040*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a*c^(11/2)*f*g^8*h^3 - 30720*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^6*b*c^(11/2)*d*g^7*h^4 - 15360*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^6*b^2*c^(9/2)*e*g^7*h^4 + 20480*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^6*a*c^(11/2)*e*g^7*h^4 + 38400*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))^6*b^3*c^(7/2)*f*g^7*h^4 - 5120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a
*b*c^(9/2)*f*g^7*h^4 + 70720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*b^2*c^(9
/2)*d*g^6*h^5 - 67840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a*c^(11/2)*d*g^
6*h^5 + 36320*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*b^3*c^(7/2)*e*g^6*h^5 +
 33920*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a*b*c^(9/2)*e*g^6*h^5 - 17900*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*b^4*c^(5/2)*f*g^6*h^5 - 144480*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^6*a*b^2*c^(7/2)*f*g^6*h^5 + 40000*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))^6*a^2*c^(9/2)*f*g^6*h^5 - 52000*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^6*b^3*c^(7/2)*d*g^5*h^6 - 7040*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))^6*a*b*c^(9/2)*d*g^5*h^6 - 39560*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
```

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))^6*b^4*c^(5/2)*e*g^5*h^6 - 45440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a*
b^2*c^(7/2)*e*g^5*h^6 - 117120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^2*c^
(9/2)*e*g^5*h^6 + 1310*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*b^5*c^(3/2)*f*
g^5*h^6 + 80880*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a*b^3*c^(5/2)*f*g^5*h
^6 + 211040*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^2*b*c^(7/2)*f*g^5*h^6 +
 7260*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*b^4*c^(5/2)*d*g^4*h^7 + 59680*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a*b^2*c^(7/2)*d*g^4*h^7 + 182720*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^6*a^2*c^(9/2)*d*g^4*h^7 + 15090*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))^6*b^5*c^(3/2)*e*g^4*h^7 + 107120*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^6*a*b^3*c^(5/2)*e*g^4*h^7 - 6880*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^6*a^2*b*c^(7/2)*e*g^4*h^7 + 2370*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^6*b^6*sqrt(c)*f*g^4*h^7 - 25340*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6
*a*b^4*c^(3/2)*f*g^4*h^7 - 97920*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^2*
b^2*c^(5/2)*f*g^4*h^7 - 178880*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^3*c^
(7/2)*f*g^4*h^7 + 9310*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*b^5*c^(3/2)*d*
g^3*h^8 - 46960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a*b^3*c^(5/2)*d*g^3*h
^8 - 176160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^2*b*c^(7/2)*d*g^3*h^8 -
 1470*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*b^6*sqrt(c)*e*g^3*h^8 - 56020*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a*b^4*c^(3/2)*e*g^3*h^8 - 73920*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^6*a^2*b^2*c^(5/2)*e*g^3*h^8 + 139200*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^6*a^3*c^(7/2)*e*g^3*h^8 - 8010*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^6*a*b^5*sqrt(c)*f*g^3*h^8 + 68560*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))^6*a^2*b^3*c^(3/2)*f*g^3*h^8 + 2400*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))^6*a^3*b*c^(5/2)*f*g^3*h^8 - 3430*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))^6*b^6*sqrt(c)*d*g^2*h^9 + 4900*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a
*b^4*c^(3/2)*d*g^2*h^9 + 93120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^2*b^
2*c^(5/2)*d*g^2*h^9 - 16320*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^3*c^(7/
2)*d*g^2*h^9 + 6370*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a*b^5*sqrt(c)*e*g
^2*h^9 + 57360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^2*b^3*c^(3/2)*e*g^2*
h^9 - 29280*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^3*b*c^(5/2)*e*g^2*h^9 +
 9320*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^2*b^4*sqrt(c)*f*g^2*h^9 - 556
80*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^3*b^2*c^(3/2)*f*g^2*h^9 + 71040*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^4*c^(5/2)*f*g^2*h^9 + 3430*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^6*a*b^5*sqrt(c)*d*g*h^10 - 19600*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^6*a^2*b^3*c^(3/2)*d*g*h^10 - 7200*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))^6*a^3*b*c^(5/2)*d*g*h^10 - 4900*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))^6*a^2*b^4*sqrt(c)*e*g*h^10 - 22560*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^6*a^3*b^2*c^(3/2)*e*g*h^10 - 10560*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)^6*a^4*c^(5/2)*e*g*h^10 - 7520*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^3*b
^3*sqrt(c)*f*g*h^10 + 13440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^4*b*c^(
3/2)*f*g*h^10 + 7680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^4*c^(5/2)*d*h^
11 + 11520*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^4*b*c^(3/2)*e*h^11 + 384
0*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^4*b^2*sqrt(c)*f*h^11 - 7680*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^6*a^5*c^(3/2)*f*h^11 + 3072*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^5*c^7*f*g^11 + 2048*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
```

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^5*c^7*e*g^10*h + 4096*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b*c^6*f*g^10*h
 + 3072*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*c^7*d*g^9*h^2 + 3584*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^5*b*c^6*e*g^9*h^2 - 36608*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))^5*b^2*c^5*f*g^9*h^2 - 1536*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))^5*a*c^6*f*g^9*h^2 + 9216*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b*c^6*d*g
^8*h^3 - 24832*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^2*c^5*e*g^8*h^3 + 40
96*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*c^6*e*g^8*h^3 + 44800*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))^5*b^3*c^4*f*g^8*h^3 + 71936*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))^5*a*b*c^5*f*g^8*h^3 - 50048*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))^5*b^2*c^5*d*g^7*h^4 - 57856*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*c^6
*d*g^7*h^4 + 13760*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^3*c^4*e*g^7*h^4
+ 73984*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b*c^5*e*g^7*h^4 + 2920*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^5*b^4*c^3*f*g^7*h^4 - 178880*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^5*a*b^2*c^4*f*g^7*h^4 - 19328*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^5*a^2*c^5*f*g^7*h^4 + 129280*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))^5*b^3*c^4*d*g^6*h^5 - 1024*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b*c^5
*d*g^6*h^5 + 2000*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^4*c^3*e*g^6*h^5 -
 23680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b^2*c^4*e*g^6*h^5 - 109312*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*c^5*e*g^6*h^5 - 10496*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))^5*b^5*c^2*f*g^6*h^5 - 26880*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^5*a*b^3*c^3*f*g^6*h^5 + 296960*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))^5*a^2*b*c^4*f*g^6*h^5 - 120680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b
^4*c^3*d*g^5*h^6 - 122240*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b^2*c^4*d
*g^5*h^6 + 226432*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*c^5*d*g^5*h^6 -
 17284*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^5*c^2*e*g^5*h^6 + 44160*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b^3*c^3*e*g^5*h^6 - 136640*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))^5*a^2*b*c^4*e*g^5*h^6 + 5062*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^5*b^6*c*f*g^5*h^6 + 44660*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
^5*a*b^4*c^2*f*g^5*h^6 + 100480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*b
^2*c^3*f*g^5*h^6 - 214720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^3*c^4*f*g
^5*h^6 + 47944*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^5*c^2*d*g^4*h^7 + 16
7520*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b^3*c^3*d*g^4*h^7 - 17920*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*b*c^4*d*g^4*h^7 + 7878*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^5*b^6*c*e*g^4*h^7 + 34480*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^5*a*b^4*c^2*e*g^4*h^7 - 10720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^
5*a^2*b^2*c^3*e*g^4*h^7 + 234240*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^3*
c^4*e*g^4*h^7 + 384*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^7*f*g^4*h^7 - 3
2222*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b^5*c*f*g^4*h^7 - 61040*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*b^3*c^2*f*g^4*h^7 - 234080*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))^5*a^3*b*c^3*f*g^4*h^7 - 4658*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^5*b^6*c*d*g^3*h^8 - 80020*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
^5*a*b^4*c^2*d*g^3*h^8 - 155840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*b
^2*c^3*d*g^3*h^8 - 120640*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^3*c^4*d*g
^3*h^8 - 384*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^7*e*g^3*h^8 - 30298*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b^5*c*e*g^3*h^8 - 45520*(sqrt(c)*x -
```

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sqrt(c*x^2 + b*x + a))^5*a^2*b^3*c^2*e*g^3*h^8 + 9440*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))^5*a^3*b*c^3*e*g^3*h^8 - 768*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))^5*a*b^6*f*g^3*h^8 + 75320*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*b^4
*c*f*g^3*h^8 + 48000*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^3*b^2*c^2*f*g^
3*h^8 + 184960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^4*c^3*f*g^3*h^8 - 89
6*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^7*d*g^2*h^9 + 5938*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^5*a*b^5*c*d*g^2*h^9 + 105040*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^5*a^2*b^3*c^2*d*g^2*h^9 + 132000*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^5*a^3*b*c^3*d*g^2*h^9 + 2048*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*
b^6*e*g^2*h^9 + 49300*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*b^4*c*e*g^2
*h^9 - 6240*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^3*b^2*c^2*e*g^2*h^9 - 7
7760*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^4*c^3*e*g^2*h^9 - 896*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^5*a^2*b^5*f*g^2*h^9 - 86560*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))^5*a^3*b^3*c*f*g^2*h^9 + 13440*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^5*a^4*b*c^2*f*g^2*h^9 + 1792*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*
a*b^6*d*g*h^10 - 6400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*b^4*c*d*g*h
^10 - 92160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^3*b^2*c^2*d*g*h^10 + 15
360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^4*c^3*d*g*h^10 - 2944*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^5*a^2*b^5*e*g*h^10 - 30720*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))^5*a^3*b^3*c*e*g*h^10 + 38400*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))^5*a^4*b*c^2*e*g*h^10 + 2560*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^3*
b^4*f*g*h^10 + 42240*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^4*b^2*c*f*g*h^
10 - 38400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^5*c^2*f*g*h^10 - 896*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*b^5*d*h^11 + 5120*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^5*a^3*b^3*c*d*h^11 + 15360*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^5*a^4*b*c^2*d*h^11 + 1280*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^3*b
^4*e*h^11 + 3840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^4*b^2*c*e*h^11 - 1
280*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^4*b^3*f*h^11 - 3840*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))^5*a^5*b*c*f*h^11 + 7680*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))^4*b*c^(13/2)*f*g^11 + 5120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*
b*c^(13/2)*e*g^10*h - 16640*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^2*c^(11
/2)*f*g^10*h - 7680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*c^(13/2)*f*g^10
*h + 7680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b*c^(13/2)*d*g^9*h^2 - 8960
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^2*c^(11/2)*e*g^9*h^2 - 5120*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^4*a*c^(13/2)*e*g^9*h^2 - 6400*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))^4*b^3*c^(9/2)*f*g^9*h^2 + 39680*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^4*a*b*c^(11/2)*f*g^9*h^2 - 3840*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))^4*b^2*c^(11/2)*d*g^8*h^3 - 7680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^
4*a*c^(13/2)*d*g^8*h^3 - 12800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^3*c^
(9/2)*e*g^8*h^3 + 37120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b*c^(11/2)*
e*g^8*h^3 + 39200*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^4*c^(7/2)*f*g^8*h
^3 - 6400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b^2*c^(9/2)*f*g^8*h^3 - 2
3040*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*c^(11/2)*f*g^8*h^3 - 17600*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^3*c^(9/2)*d*g^7*h^4 - 113920*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^4*a*b*c^(11/2)*d*g^7*h^4 + 15520*(sqrt(c)*x -
```

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sqrt(c*x^2 + b*x + a))^4*b^4*c^(7/2)*e*g^7*h^4 + 64640*(sqrt(c)*x - sqrt(c*
x^2 + b*x + a))^4*a*b^2*c^(9/2)*e*g^7*h^4 - 35840*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))^4*a^2*c^(11/2)*e*g^7*h^4 - 14140*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))^4*b^5*c^(5/2)*f*g^7*h^4 - 188640*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4
*a*b^3*c^(7/2)*f*g^7*h^4 + 79680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*
b*c^(9/2)*f*g^7*h^4 + 81920*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^4*c^(7/
2)*d*g^6*h^5 + 114240*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b^2*c^(9/2)*d
*g^6*h^5 + 167680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*c^(11/2)*d*g^6*
h^5 - 14360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^5*c^(5/2)*e*g^6*h^5 - 3
6960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b^3*c^(7/2)*e*g^6*h^5 - 231680
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*b*c^(9/2)*e*g^6*h^5 + 1610*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^4*b^6*c^(3/2)*f*g^6*h^5 + 55900*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))^4*a*b^4*c^(5/2)*f*g^6*h^5 + 406400*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^4*a^2*b^2*c^(7/2)*f*g^6*h^5 - 78400*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))^4*a^3*c^(9/2)*f*g^6*h^5 - 85780*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))^4*b^5*c^(5/2)*d*g^5*h^6 - 237120*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))^4*a*b^3*c^(7/2)*d*g^5*h^6 + 63040*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4
*a^2*b*c^(9/2)*d*g^5*h^6 + 330*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^6*c^
(3/2)*e*g^5*h^6 + 81560*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b^4*c^(5/2)
*e*g^5*h^6 + 13600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*b^2*c^(7/2)*e*
g^5*h^6 + 196480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^3*c^(9/2)*e*g^5*h^
6 + 1920*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^7*sqrt(c)*f*g^5*h^6 - 1189
0*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b^5*c^(3/2)*f*g^5*h^6 - 78160*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*b^3*c^(5/2)*f*g^5*h^6 - 498080*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^4*a^3*b*c^(7/2)*f*g^5*h^6 + 35330*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))^4*b^6*c^(3/2)*d*g^4*h^7 + 244660*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^4*a*b^4*c^(5/2)*d*g^4*h^7 + 32960*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))^4*a^2*b^2*c^(7/2)*d*g^4*h^7 - 178880*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^4*a^3*c^(9/2)*d*g^4*h^7 + 1920*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))^4*b^7*sqrt(c)*e*g^4*h^7 - 24470*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*
a*b^5*c^(3/2)*e*g^4*h^7 - 117680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*
b^3*c^(5/2)*e*g^4*h^7 + 183840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^3*b*
c^(7/2)*e*g^4*h^7 - 11520*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b^6*sqrt(
c)*f*g^4*h^7 + 42760*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*b^4*c^(3/2)*
f*g^4*h^7 - 7040*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^3*b^2*c^(5/2)*f*g^
4*h^7 + 245120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^4*c^(7/2)*f*g^4*h^7
- 4480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^7*sqrt(c)*d*g^3*h^8 - 101890
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b^5*c^(3/2)*d*g^3*h^8 - 179920*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*b^3*c^(5/2)*d*g^3*h^8 + 56160*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^4*a^3*b*c^(7/2)*d*g^3*h^8 - 5120*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))^4*a*b^6*sqrt(c)*e*g^3*h^8 + 54860*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^4*a^2*b^4*c^(3/2)*e*g^3*h^8 - 5280*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))^4*a^3*b^2*c^(5/2)*e*g^3*h^8 - 180800*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^4*a^4*c^(7/2)*e*g^3*h^8 + 26240*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^4*a^2*b^5*sqrt(c)*f*g^3*h^8 - 55520*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
```

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))^4*a^3*b^3*c^(3/2)*f*g^3*h^8 + 152960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
^4*a^4*b*c^(5/2)*f*g^3*h^8 + 8960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b
^6*sqrt(c)*d*g^2*h^9 + 113920*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*b^4
*c^(3/2)*d*g^2*h^9 + 51200*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^3*b^2*c^
(5/2)*d*g^2*h^9 + 71680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^4*c^(7/2)*d
*g^2*h^9 + 8320*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*b^5*sqrt(c)*e*g^2
*h^9 - 38400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^3*b^3*c^(3/2)*e*g^2*h^
9 + 40960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^4*b*c^(5/2)*e*g^2*h^9 - 3
3280*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^3*b^4*sqrt(c)*f*g^2*h^9 + 1920
0*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^4*b^2*c^(3/2)*f*g^2*h^9 - 110080*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^5*c^(5/2)*f*g^2*h^9 - 4480*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^4*a^2*b^5*sqrt(c)*d*g*h^10 - 71680*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))^4*a^3*b^3*c^(3/2)*d*g*h^10 - 33280*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^4*a^4*b*c^(5/2)*d*g*h^10 - 8960*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^4*a^3*b^4*sqrt(c)*e*g*h^10 + 16640*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^4*a^4*b^2*c^(3/2)*e*g*h^10 + 15360*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))^4*a^5*c^(5/2)*e*g*h^10 + 24320*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a
^4*b^3*sqrt(c)*f*g*h^10 - 1280*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^5*b*
c^(3/2)*f*g*h^10 + 24320*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^4*b^2*c^(3
/2)*d*h^11 + 2560*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^5*c^(5/2)*d*h^11
+ 3840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^4*b^3*sqrt(c)*e*h^11 - 8960*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^5*b*c^(3/2)*e*h^11 - 7680*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^4*a^5*b^2*sqrt(c)*f*h^11 + 5120*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^4*a^6*c^(3/2)*f*h^11 + 7680*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^3*b^2*c^6*f*g^11 + 5120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^2*
c^6*e*g^10*h - 24320*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^3*c^5*f*g^10*h
 - 15360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b*c^6*f*g^10*h + 7680*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^3*b^2*c^6*d*g^9*h^2 - 14080*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))^3*b^3*c^5*e*g^9*h^2 - 10240*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))^3*a*b*c^6*e*g^9*h^2 + 20800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3
*b^4*c^4*f*g^9*h^2 + 79360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^2*c^5*
f*g^9*h^2 + 7680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*c^6*f*g^9*h^2 -
11520*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^3*c^5*d*g^8*h^3 - 15360*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b*c^6*d*g^8*h^3 + 3200*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^3*b^4*c^4*e*g^8*h^3 + 61440*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^3*a*b^2*c^5*e*g^8*h^3 + 5120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*
a^2*c^6*e*g^8*h^3 + 7840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^5*c^3*f*g^
8*h^3 - 115200*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^3*c^4*f*g^8*h^3 -
85760*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b*c^5*f*g^8*h^3 + 14080*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^4*c^4*d*g^7*h^4 - 90880*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^3*a*b^2*c^5*d*g^7*h^4 + 15360*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^3*a^2*c^6*d*g^7*h^4 + 1280*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
^3*b^5*c^3*e*g^7*h^4 + 9600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^3*c^4
*e*g^7*h^4 - 99840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b*c^5*e*g^7*h^
4 - 5900*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^6*c^2*f*g^7*h^4 - 51040*(s
```

```
qrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^4*c^3*f*g^7*h^4 + 278080*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))^3*a^2*b^2*c^4*f*g^7*h^4 + 30720*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^3*a^3*c^5*f*g^7*h^4 + 14080*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^3*b^5*c^3*d*g^6*h^5 + 88320*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a
*b^3*c^4*d*g^6*h^5 + 281600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b*c^5
*d*g^6*h^5 - 6920*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^6*c^2*e*g^6*h^5 +
 2560*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^4*c^3*e*g^6*h^5 - 161920*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b^2*c^4*e*g^6*h^5 + 46080*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^3*a^3*c^5*e*g^6*h^5 + 2510*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))^3*b^7*c*f*g^6*h^5 + 26440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)^3*a*b^5*c^2*f*g^6*h^5 + 155680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*
b^3*c^3*f*g^6*h^5 - 310400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^3*b*c^4*
f*g^6*h^5 - 25220*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^6*c^2*d*g^5*h^6 -
 124960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^4*c^3*d*g^5*h^6 - 239360*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b^2*c^4*d*g^5*h^6 - 160000*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^3*a^3*c^5*d*g^5*h^6 + 2370*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^3*b^7*c*e*g^5*h^6 + 34880*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))^3*a*b^5*c^2*e*g^5*h^6 + 43520*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^
2*b^3*c^3*e*g^5*h^6 + 334720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^3*b*c^
4*e*g^5*h^6 + 210*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^8*f*g^5*h^6 - 154
80*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^6*c*f*g^5*h^6 - 45100*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^3*a^2*b^4*c^2*f*g^5*h^6 - 324320*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^3*a^3*b^2*c^3*f*g^5*h^6 + 124480*(sqrt(c)*x - sqrt(c*
x^2 + b*x + a))^3*a^4*c^4*f*g^5*h^6 + 10510*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))^3*b^7*c*d*g^4*h^7 + 120280*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^5
*c^2*d*g^4*h^7 + 200320*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b^3*c^3*d
*g^4*h^7 + 42240*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^3*b*c^4*d*g^4*h^7
+ 210*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^8*e*g^4*h^7 - 18040*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^3*a*b^6*c*e*g^4*h^7 - 88040*(sqrt(c)*x - sqrt(c*
x^2 + b*x + a))^3*a^2*b^4*c^2*e*g^4*h^7 - 61440*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^3*a^3*b^2*c^3*e*g^4*h^7 - 183680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))^3*a^4*c^4*e*g^4*h^7 - 1470*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^7*f
*g^4*h^7 + 43990*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b^5*c*f*g^4*h^7
+ 44240*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^3*b^3*c^2*f*g^4*h^7 + 36272
0*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^4*b*c^3*f*g^4*h^7 - 790*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^3*b^8*d*g^3*h^8 - 47320*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^3*a*b^6*c*d*g^3*h^8 - 160420*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)^3*a^2*b^4*c^2*d*g^3*h^8 - 79200*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^3
*b^2*c^3*d*g^3*h^8 + 93120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^4*c^4*d*
g^3*h^8 - 50*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^7*e*g^3*h^8 + 49850*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b^5*c*e*g^3*h^8 + 77840*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^3*a^3*b^3*c^2*e*g^3*h^8 - 40800*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^3*a^4*b*c^3*e*g^3*h^8 + 2990*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))^3*a^2*b^6*f*g^3*h^8 - 68860*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3
*a^3*b^4*c*f*g^3*h^8 - 960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^4*b^2*c^
```

```
2*f*g^3*h^8 - 150720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^5*c^3*f*g^3*h^
8 + 2370*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^7*d*g^2*h^9 + 72310*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b^5*c*d*g^2*h^9 + 71280*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^3*a^3*b^3*c^2*d*g^2*h^9 + 3680*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))^3*a^4*b*c^3*d*g^2*h^9 - 1110*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))^3*a^2*b^6*e*g^2*h^9 - 58420*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^3*
b^4*c*e*g^2*h^9 + 2240*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^4*b^2*c^2*e*
g^2*h^9 + 54720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^5*c^3*e*g^2*h^9 - 2
250*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^3*b^5*f*g^2*h^9 + 53200*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^3*a^4*b^3*c*f*g^2*h^9 - 48800*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^3*a^5*b*c^2*f*g^2*h^9 - 2370*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))^3*a^2*b^6*d*g*h^10 - 44700*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*
a^3*b^4*c*d*g*h^10 - 17920*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^4*b^2*c^
2*d*g*h^10 - 13760*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^5*c^3*d*g*h^10 +
 1530*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^3*b^5*e*g*h^10 + 27600*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^3*a^4*b^3*c*e*g*h^10 - 17120*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^3*a^5*b*c^2*e*g*h^10 + 360*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^3*a^4*b^4*f*g*h^10 - 13440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^
5*b^2*c*f*g*h^10 + 30080*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^6*c^2*f*g*
h^10 + 790*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^3*b^5*d*h^11 + 9200*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^3*a^4*b^3*c*d*h^11 + 12000*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^3*a^5*b*c^2*d*h^11 - 580*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^3*a^4*b^4*e*h^11 - 3360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^5*b^2
*c*e*h^11 - 2880*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^6*c^2*e*h^11 + 160
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^5*b^3*f*h^11 - 1920*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^3*a^6*b*c*f*h^11 + 3840*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))^2*b^3*c^(11/2)*f*g^11 + 2560*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b
^3*c^(11/2)*e*g^10*h - 13600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^4*c^(9
/2)*f*g^10*h - 11520*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b^2*c^(11/2)*f
*g^10*h + 3840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^3*c^(11/2)*d*g^9*h^2
 - 8000*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^4*c^(9/2)*e*g^9*h^2 - 7680*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b^2*c^(11/2)*e*g^9*h^2 + 15680*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^2*b^5*c^(7/2)*f*g^9*h^2 + 57600*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))^2*a*b^3*c^(9/2)*f*g^9*h^2 + 11520*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^2*a^2*b*c^(11/2)*f*g^9*h^2 - 7200*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))^2*b^4*c^(9/2)*d*g^8*h^3 - 11520*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^2*a*b^2*c^(11/2)*d*g^8*h^3 + 4720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))^2*b^5*c^(7/2)*e*g^8*h^3 + 41600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*
b^3*c^(9/2)*e*g^8*h^3 + 7680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*b*c^
(11/2)*e*g^8*h^3 - 2620*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^6*c^(5/2)*f
*g^8*h^3 - 95600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b^4*c^(7/2)*f*g^8*
h^3 - 91200*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*b^2*c^(9/2)*f*g^8*h^3
 - 3840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*c^(11/2)*f*g^8*h^3 + 1376
0*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^5*c^(7/2)*d*g^7*h^4 - 37760*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b^3*c^(9/2)*d*g^7*h^4 + 23040*(sqrt(c)*x
```

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 - sqrt(c*x^2 + b*x + a))^2*a^2*b*c^(11/2)*d*g^7*h^4 - 3120*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))^2*b^6*c^(5/2)*e*g^7*h^4 - 10240*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^2*a*b^4*c^(7/2)*e*g^7*h^4 - 92160*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^2*a^2*b^2*c^(9/2)*e*g^7*h^4 - 5120*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))^2*a^3*c^(11/2)*e*g^7*h^4 - 150*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b
^7*c^(3/2)*f*g^7*h^4 + 11600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b^5*c^
(5/2)*f*g^7*h^4 + 258080*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*b^3*c^(7
/2)*f*g^7*h^4 + 66560*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*b*c^(9/2)*f
*g^7*h^4 - 6340*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^6*c^(5/2)*d*g^6*h^5
 + 22000*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b^4*c^(7/2)*d*g^6*h^5 + 17
6640*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*b^2*c^(9/2)*d*g^6*h^5 - 7680
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*c^(11/2)*d*g^6*h^5 - 570*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^2*b^7*c^(3/2)*e*g^6*h^5 + 15400*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^2*a*b^5*c^(5/2)*e*g^6*h^5 - 48320*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))^2*a^2*b^3*c^(7/2)*e*g^6*h^5 + 90880*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^2*a^3*b*c^(9/2)*e*g^6*h^5 + 630*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))^2*b^8*sqrt(c)*f*g^6*h^5 - 360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*
a*b^6*c^(3/2)*f*g^6*h^5 - 14180*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*b
^4*c^(5/2)*f*g^6*h^5 - 376480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*b^2
*c^(7/2)*f*g^6*h^5 - 18240*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^4*c^(9/2
)*f*g^6*h^5 - 1750*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^7*c^(3/2)*d*g^5*
h^6 - 13120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b^5*c^(5/2)*d*g^5*h^6 -
 187840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*b^3*c^(7/2)*d*g^5*h^6 - 2
16960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*b*c^(9/2)*d*g^5*h^6 + 630*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^8*sqrt(c)*e*g^5*h^6 + 2200*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^2*a*b^6*c^(3/2)*e*g^5*h^6 - 520*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^2*a^2*b^4*c^(5/2)*e*g^5*h^6 + 204160*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^2*a^3*b^2*c^(7/2)*e*g^5*h^6 - 35200*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))^2*a^4*c^(9/2)*e*g^5*h^6 - 4410*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))^2*a*b^7*sqrt(c)*f*g^5*h^6 + 6530*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
^2*a^2*b^5*c^(3/2)*f*g^5*h^6 - 38160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*
a^3*b^3*c^(5/2)*f*g^5*h^6 + 276640*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^
4*b*c^(7/2)*f*g^5*h^6 + 1470*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^8*sqrt
(c)*d*g^4*h^7 + 16760*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b^6*c^(3/2)*d
*g^4*h^7 + 112660*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*b^4*c^(5/2)*d*g
^4*h^7 + 212960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*b^2*c^(7/2)*d*g^4
*h^7 + 89920*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^4*c^(9/2)*d*g^4*h^7 -
3990*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b^7*sqrt(c)*e*g^4*h^7 - 13010*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*b^5*c^(3/2)*e*g^4*h^7 - 81360*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*b^3*c^(5/2)*e*g^4*h^7 - 232480*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^2*a^4*b*c^(7/2)*e*g^4*h^7 + 12810*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))^2*a^2*b^6*sqrt(c)*f*g^4*h^7 - 9460*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^2*a^3*b^4*c^(3/2)*f*g^4*h^7 + 140480*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^2*a^4*b^2*c^(5/2)*f*g^4*h^7 - 78400*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))^2*a^5*c^(7/2)*f*g^4*h^7 - 8250*(sqrt(c)*x - sqrt(c*x^2 + b
```

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*x + a))^2*a*b^7*sqrt(c)*d*g^3*h^8 - 46750*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))^2*a^2*b^5*c^(3/2)*d*g^3*h^8 - 154800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)^2*a^3*b^3*c^(5/2)*d*g^3*h^8 - 40160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2
*a^4*b*c^(7/2)*d*g^3*h^8 + 12030*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*
b^6*sqrt(c)*e*g^3*h^8 + 34660*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*b^4
*c^(3/2)*e*g^3*h^8 + 109120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^4*b^2*c
^(5/2)*e*g^3*h^8 + 82240*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^5*c^(7/2)*
e*g^3*h^8 - 22110*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*b^5*sqrt(c)*f*g
^3*h^8 - 4240*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^4*b^3*c^(3/2)*f*g^3*h
^8 - 151520*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^5*b*c^(5/2)*f*g^3*h^8 +
 15930*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*b^6*sqrt(c)*d*g^2*h^9 + 55
340*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*b^4*c^(3/2)*d*g^2*h^9 + 58880
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^4*b^2*c^(5/2)*d*g^2*h^9 - 20800*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^5*c^(7/2)*d*g^2*h^9 - 18450*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^2*a^3*b^5*sqrt(c)*e*g^2*h^9 - 37520*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))^2*a^4*b^3*c^(3/2)*e*g^2*h^9 - 30880*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))^2*a^5*b*c^(5/2)*e*g^2*h^9 + 24120*(sqrt(c)*x - sqrt(c*
x^2 + b*x + a))^2*a^4*b^4*sqrt(c)*f*g^2*h^9 + 10880*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^2*a^5*b^2*c^(3/2)*f*g^2*h^9 + 54400*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))^2*a^6*c^(5/2)*f*g^2*h^9 - 12990*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))^2*a^3*b^5*sqrt(c)*d*g*h^10 - 28720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2
*a^4*b^3*c^(3/2)*d*g*h^10 + 160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^5*b
*c^(5/2)*d*g*h^10 + 13620*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^4*b^4*sqr
t(c)*e*g*h^10 + 15520*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^5*b^2*c^(3/2)
*e*g*h^10 - 8640*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^6*c^(5/2)*e*g*h^10
 - 14880*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^5*b^3*sqrt(c)*f*g*h^10 - 6
40*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^6*b*c^(3/2)*f*g*h^10 + 3840*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^2*a^4*b^4*sqrt(c)*d*h^11 + 5120*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))^2*a^5*b^2*c^(3/2)*d*h^11 + 2560*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^2*a^6*c^(5/2)*d*h^11 - 3840*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))^2*a^5*b^3*sqrt(c)*e*h^11 - 1280*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^
2*a^6*b*c^(3/2)*e*h^11 + 3840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^6*b^2
*sqrt(c)*f*h^11 - 2560*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^7*c^(3/2)*f*
h^11 + 960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^4*c^5*f*g^11 + 640*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))*b^4*c^5*e*g^10*h - 3520*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))*b^5*c^4*f*g^10*h - 3840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a
*b^3*c^5*f*g^10*h + 960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^4*c^5*d*g^9*h
^2 - 2080*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^5*c^4*e*g^9*h^2 - 2560*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))*a*b^3*c^5*e*g^9*h^2 + 4360*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))*b^6*c^3*f*g^9*h^2 + 18400*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))*a*b^4*c^4*f*g^9*h^2 + 5760*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b
^2*c^5*f*g^9*h^2 - 1920*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^5*c^4*d*g^8*h
^3 - 3840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^3*c^5*d*g^8*h^3 + 1440*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))*b^6*c^3*e*g^8*h^3 + 12800*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))*a*b^4*c^4*e*g^8*h^3 + 3840*(sqrt(c)*x - sqrt(c*x^2 + b
```

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*x + a))*a^2*b^2*c^5*e*g^8*h^3 - 1200*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b
^7*c^2*f*g^8*h^3 - 30560*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^5*c^3*f*g^
8*h^3 - 38400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^3*c^4*f*g^8*h^3 - 3
840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*b*c^5*f*g^8*h^3 + 4280*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))*b^6*c^3*d*g^7*h^4 - 8480*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))*a*b^4*c^4*d*g^7*h^4 + 11520*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))*a^2*b^2*c^5*d*g^7*h^4 - 1260*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^7*c^2
*e*g^7*h^4 - 4400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^5*c^3*e*g^7*h^4 -
 35200*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^3*c^4*e*g^7*h^4 - 5120*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*b*c^5*e*g^7*h^4 + 300*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))*b^8*c*f*g^7*h^4 + 7020*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))*a*b^6*c^2*f*g^7*h^4 + 96120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^4
*c^3*f*g^7*h^4 + 41600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*b^2*c^4*f*g^
7*h^4 + 1920*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^4*c^5*f*g^7*h^4 - 3080*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^7*c^2*d*g^6*h^5 + 480*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))*a*b^5*c^3*d*g^6*h^5 + 49280*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))*a^2*b^3*c^4*d*g^6*h^5 - 7680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a
^3*b*c^5*d*g^6*h^5 + 270*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^8*c*e*g^6*h^
5 + 6440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^6*c^2*e*g^6*h^5 - 8080*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^4*c^3*e*g^6*h^5 + 55040*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))*a^3*b^2*c^4*e*g^6*h^5 + 1280*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))*a^4*c^5*e*g^6*h^5 + 45*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^
9*f*g^6*h^5 - 2190*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^7*c*f*g^6*h^5 -
16040*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^5*c^2*f*g^6*h^5 - 174880*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*b^3*c^3*f*g^6*h^5 - 26240*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))*a^4*b*c^4*f*g^6*h^5 + 600*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))*b^8*c*d*g^5*h^6 + 5380*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^
6*c^2*d*g^5*h^6 - 46840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^4*c^3*d*g
^5*h^6 - 98880*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*b^2*c^4*d*g^5*h^6 +
1920*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^4*c^5*d*g^5*h^6 + 45*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))*b^9*e*g^5*h^6 - 1770*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))*a*b^7*c*e*g^5*h^6 - 7420*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^5
*c^2*e*g^5*h^6 + 56160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*b^3*c^3*e*g^
5*h^6 - 40000*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^4*b*c^4*e*g^5*h^6 - 360
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^8*f*g^5*h^6 + 6810*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))*a^2*b^6*c*f*g^5*h^6 + 14140*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))*a^3*b^4*c^2*f*g^5*h^6 + 186880*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)*a^4*b^2*c^3*f*g^5*h^6 + 8640*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^5*c^4*
f*g^5*h^6 + 105*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^9*d*g^4*h^7 - 1950*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^7*c*d*g^4*h^7 + 16960*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))*a^2*b^5*c^2*d*g^4*h^7 + 96640*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))*a^3*b^3*c^3*d*g^4*h^7 + 85120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))*a^4*b*c^4*d*g^4*h^7 - 330*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^8*e*g^
4*h^7 + 4050*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^6*c*e*g^4*h^7 - 1816
0*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*b^4*c^2*e*g^4*h^7 - 98720*(sqrt(c
```

```
)*x - sqrt(c*x^2 + b*x + a))*a^4*b^2*c^3*e*g^4*h^7 + 8960*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))*a^5*c^4*e*g^4*h^7 + 1230*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))*a^2*b^7*f*g^4*h^7 - 11250*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*b^5*
c*f*g^4*h^7 + 14800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^4*b^3*c^2*f*g^4*h
^7 - 108960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^5*b*c^3*f*g^4*h^7 - 420*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^8*d*g^3*h^8 - 990*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))*a^2*b^6*c*d*g^3*h^8 - 49820*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))*a^3*b^4*c^2*d*g^3*h^8 - 74400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^
4*b^2*c^3*d*g^3*h^8 - 24640*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^5*c^4*d*g
^3*h^8 + 870*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^7*e*g^3*h^8 - 1110*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*b^5*c*e*g^3*h^8 + 50720*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))*a^4*b^3*c^2*e*g^3*h^8 + 76640*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))*a^5*b*c^3*e*g^3*h^8 - 2220*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))*a^3*b^6*f*g^3*h^8 + 7410*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^4*b^4*c*f
*g^3*h^8 - 45680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^5*b^2*c^2*f*g^3*h^8
 + 27040*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^6*c^3*f*g^3*h^8 + 630*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))*a^2*b^7*d*g^2*h^9 + 8670*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))*a^3*b^5*c*d*g^2*h^9 + 43360*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))*a^4*b^3*c^2*d*g^2*h^9 + 16160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^5*b
*c^3*d*g^2*h^9 - 1080*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*b^6*e*g^2*h^9
 - 7320*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^4*b^4*c*e*g^2*h^9 - 42400*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))*a^5*b^2*c^2*e*g^2*h^9 - 23040*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))*a^6*c^3*e*g^2*h^9 + 2205*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))*a^4*b^5*f*g^2*h^9 + 3720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^5*
b^3*c*f*g^2*h^9 + 36880*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^6*b*c^2*f*g^2
*h^9 - 420*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*b^6*d*g*h^10 - 9570*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))*a^4*b^4*c*d*g*h^10 - 13520*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))*a^5*b^2*c^2*d*g*h^10 + 3680*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))*a^6*c^3*d*g*h^10 + 645*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^4*b^5
*e*g*h^10 + 9000*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^5*b^3*c*e*g*h^10 + 1
2560*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^6*b*c^2*e*g*h^10 - 1140*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))*a^5*b^4*f*g*h^10 - 7680*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))*a^6*b^2*c*f*g*h^10 - 9920*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*
a^7*c^2*f*g*h^10 + 105*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^4*b^5*d*h^11 +
 3240*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^5*b^3*c*d*h^11 + 720*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))*a^6*b*c^2*d*h^11 - 150*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))*a^5*b^4*e*h^11 - 3120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^6*b^2
*c*e*h^11 - 480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^7*c^2*e*h^11 + 240*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))*a^6*b^3*f*h^11 + 2880*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))*a^7*b*c*f*h^11 + 96*b^5*c^(9/2)*f*g^11 + 64*b^5*c^(9/2)*e*
g^10*h - 352*b^6*c^(7/2)*f*g^10*h - 480*a*b^4*c^(9/2)*f*g^10*h + 96*b^5*c^(
9/2)*d*g^9*h^2 - 208*b^6*c^(7/2)*e*g^9*h^2 - 320*a*b^4*c^(9/2)*e*g^9*h^2 +
436*b^7*c^(5/2)*f*g^9*h^2 + 2192*a*b^5*c^(7/2)*f*g^9*h^2 + 960*a^2*b^3*c^(9
/2)*f*g^9*h^2 - 192*b^6*c^(7/2)*d*g^8*h^3 - 480*a*b^4*c^(9/2)*d*g^8*h^3 + 1
44*b^7*c^(5/2)*e*g^8*h^3 + 1488*a*b^5*c^(7/2)*e*g^8*h^3 + 640*a^2*b^3*c^(9/
```

```
2)*e*g^8*h^3 - 120*b^8*c^(3/2)*f*g^8*h^3 - 3492*a*b^6*c^(5/2)*f*g^8*h^3 - 5
680*a^2*b^4*c^(7/2)*f*g^8*h^3 - 960*a^3*b^2*c^(9/2)*f*g^8*h^3 + 476*b^7*c^(
5/2)*d*g^7*h^4 - 848*a*b^5*c^(7/2)*d*g^7*h^4 + 1920*a^2*b^3*c^(9/2)*d*g^7*h
^4 - 150*b^8*c^(3/2)*e*g^7*h^4 - 488*a*b^6*c^(5/2)*e*g^7*h^4 - 4800*a^2*b^4
*c^(7/2)*e*g^7*h^4 - 1280*a^3*b^2*c^(9/2)*e*g^7*h^4 + 45*b^9*sqrt(c)*f*g^7*
h^4 + 750*a*b^7*c^(3/2)*f*g^7*h^4 + 12716*a^2*b^5*c^(5/2)*f*g^7*h^4 + 8000*
a^3*b^3*c^(7/2)*f*g^7*h^4 + 960*a^4*b*c^(9/2)*f*g^7*h^4 - 380*b^8*c^(3/2)*d
*g^6*h^5 - 332*a*b^6*c^(5/2)*d*g^6*h^5 + 5200*a^2*b^4*c^(7/2)*d*g^6*h^5 - 1
920*a^3*b^2*c^(9/2)*d*g^6*h^5 + 45*b^9*sqrt(c)*e*g^6*h^5 + 890*a*b^7*c^(3/2
)*e*g^6*h^5 - 1136*a^2*b^5*c^(5/2)*e*g^6*h^5 + 10560*a^3*b^3*c^(7/2)*e*g^6*
h^5 + 640*a^4*b*c^(9/2)*e*g^6*h^5 - 360*a*b^8*sqrt(c)*f*g^6*h^5 - 1730*a^2*
b^6*c^(3/2)*f*g^6*h^5 - 27340*a^3*b^4*c^(5/2)*f*g^6*h^5 - 8320*a^4*b^2*c^(7
/2)*f*g^6*h^5 - 192*a^5*c^(9/2)*f*g^6*h^5 + 105*b^9*sqrt(c)*d*g^5*h^6 + 990
*a*b^7*c^(3/2)*d*g^5*h^6 - 3244*a^2*b^5*c^(5/2)*d*g^5*h^6 - 15200*a^3*b^3*c
^(7/2)*d*g^5*h^6 + 960*a^4*b*c^(9/2)*d*g^5*h^6 - 330*a*b^8*sqrt(c)*e*g^5*h^
6 - 1410*a^2*b^6*c^(3/2)*e*g^5*h^6 + 6520*a^3*b^4*c^(5/2)*e*g^5*h^6 - 11040
*a^4*b^2*c^(7/2)*e*g^5*h^6 - 128*a^5*c^(9/2)*e*g^5*h^6 + 1230*a^2*b^7*sqrt(
c)*f*g^5*h^6 + 930*a^3*b^5*c^(3/2)*f*g^5*h^6 + 38960*a^4*b^3*c^(5/2)*f*g^5*
h^6 + 5024*a^5*b*c^(7/2)*f*g^5*h^6 - 420*a*b^8*sqrt(c)*d*g^4*h^7 - 90*a^2*b
^6*c^(3/2)*d*g^4*h^7 + 11420*a^3*b^4*c^(5/2)*d*g^4*h^7 + 20320*a^4*b^2*c^(7
/2)*d*g^4*h^7 - 192*a^5*c^(9/2)*d*g^4*h^7 + 870*a^2*b^7*sqrt(c)*e*g^4*h^7 -
 330*a^3*b^5*c^(3/2)*e*g^4*h^7 - 14720*a^4*b^3*c^(5/2)*e*g^4*h^7 + 4896*a^5
*b*c^(7/2)*e*g^4*h^7 - 2220*a^3*b^6*sqrt(c)*f*g^4*h^7 + 2850*a^4*b^4*c^(3/2
)*f*g^4*h^7 - 34672*a^5*b^2*c^(5/2)*f*g^4*h^7 - 864*a^6*c^(7/2)*f*g^4*h^7 +
 630*a^2*b^7*sqrt(c)*d*g^3*h^8 - 3150*a^3*b^5*c^(3/2)*d*g^3*h^8 - 16000*a^4
*b^3*c^(5/2)*d*g^3*h^8 - 11936*a^5*b*c^(7/2)*d*g^3*h^8 - 1080*a^3*b^6*sqrt(
c)*e*g^3*h^8 + 4080*a^4*b^4*c^(3/2)*e*g^3*h^8 + 18592*a^5*b^2*c^(5/2)*e*g^3
*h^8 - 896*a^6*c^(7/2)*e*g^3*h^8 + 2205*a^4*b^5*sqrt(c)*f*g^3*h^8 - 6840*a^
5*b^3*c^(3/2)*f*g^3*h^8 + 16144*a^6*b*c^(5/2)*f*g^3*h^8 - 420*a^3*b^6*sqrt(
c)*d*g^2*h^9 + 5790*a^4*b^4*c^(3/2)*d*g^2*h^9 + 9008*a^5*b^2*c^(5/2)*d*g^2*
h^9 + 2656*a^6*c^(7/2)*d*g^2*h^9 + 645*a^4*b^5*sqrt(c)*e*g^2*h^9 - 6360*a^5
*b^3*c^(3/2)*e*g^2*h^9 - 11504*a^6*b*c^(5/2)*e*g^2*h^9 - 1140*a^5*b^4*sqrt(
c)*f*g^2*h^9 + 7680*a^6*b^2*c^(3/2)*f*g^2*h^9 - 2752*a^7*c^(5/2)*f*g^2*h^9
+ 105*a^4*b^5*sqrt(c)*d*g*h^10 - 4440*a^5*b^3*c^(3/2)*d*g*h^10 - 816*a^6*b*
c^(5/2)*d*g*h^10 - 150*a^5*b^4*sqrt(c)*e*g*h^10 + 4560*a^6*b^2*c^(3/2)*e*g*
h^10 + 2592*a^7*c^(5/2)*e*g*h^10 + 240*a^6*b^3*sqrt(c)*f*g*h^10 - 4800*a^7*
b*c^(3/2)*f*g*h^10 + 1280*a^6*b^2*c^(3/2)*d*h^11 - 512*a^7*c^(5/2)*d*h^11 -
 1280*a^7*b*c^(3/2)*e*h^11 + 1280*a^8*c^(3/2)*f*h^11)/((c^4*g^8*h^4 - 4*b*c
^3*g^7*h^5 + 6*b^2*c^2*g^6*h^6 + 4*a*c^3*g^6*h^6 - 4*b^3*c*g^5*h^7 - 12*a*b
*c^2*g^5*h^7 + b^4*g^4*h^8 + 12*a*b^2*c*g^4*h^8 + 6*a^2*c^2*g^4*h^8 - 4*a*b
^3*g^3*h^9 - 12*a^2*b*c*g^3*h^9 + 6*a^2*b^2*g^2*h^10 + 4*a^3*c*g^2*h^10 - 4
*a^3*b*g*h^11 + a^4*h^12)*((sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*h + 2*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c)*g + b*g - a*h)^5)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^6}\,dx=\int \frac{\sqrt{c\,x^2+b\,x+a}\,\left(f\,x^2+e\,x+d\right)}{{\left(g+h\,x\right)}^6}\,dx$$

```
[In] int(((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^6,x)
[Out] int(((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^6, x)
```

3.196 $\int (g+hx)^3 \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right) dx$

Optimal result

Integrand size = 32, antiderivative size = 1169

$$\int (g+hx)^3 \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right) dx =$$

$$-\frac{\left(b^2-4ac\right)\left(1536c^5dg^3-143b^5fh^3+22b^3ch^2(20afh+9b(3fg+eh))-48bc^2h(5a^2fh^2+9abh(3fg+eh)+\right.}{}$$

$$+\frac{\left(1536c^5dg^3-143b^5fh^3+22b^3ch^2(20afh+9b(3fg+eh))-48bc^2h(5a^2fh^2+9abh(3fg+eh)+6b^2(3fg^2\right.}{}$$

$$+\frac{\left(143b^2fh^2-2ch(24bfg+99beh+64afh)-12c^2(5fg^2-3h(3eg+8dh))\right)(g+hx)^2\left(a+bx+cx^2\right)^{5/2}}{2016c^3h}$$

$$-\frac{(10cfg-18ceh+13bfh)(g+hx)^3\left(a+bx+cx^2\right)^{5/2}}{144c^2h}+\frac{f(g+hx)^4\left(a+bx+cx^2\right)^{5/2}}{9ch}$$

$$+\frac{\left(3003b^4fh^4-192c^4g^2(5fg^2-3h(3eg+64dh))-198b^2ch^3(38afh+21b(3fg+eh))+8c^2h^2(256a^2fh^2+8\right.}{}$$

$$+\frac{\left(b^2-4ac\right)^2\left(1536c^5dg^3-143b^5fh^3+22b^3ch^2(20afh+9b(3fg+eh))-48bc^2h(5a^2fh^2+9abh(3fg+eh)\right.}{}$$

```
[Out] 1/12288*(1536*c^5*d*g^3-143*b^5*f*h^3+22*b^3*c*h^2*(20*a*f*h+9*b*(e*h+3*f*g
))-48*b*c^2*h*(5*a^2*f*h^2+9*a*b*h*(e*h+3*f*g)+6*b^2*(d*h^2+3*e*g*h+3*f*g^2
))-256*c^4*g*(3*b*g*(3*d*h+e*g)+a*(f*g^2+3*h*(d*h+e*g)))+32*c^3*(3*a^2*h^2*
(e*h+3*f*g)+14*b^2*g*(f*g^2+3*h*(d*h+e*g))+12*a*b*h*(3*f*g^2+h*(d*h+3*e*g))
))*(2*c*x+b)*(c*x^2+b*x+a)^(3/2)/c^6+1/2016*(143*b^2*f*h^2-2*c*h*(64*a*f*h+
99*b*e*h+24*b*f*g)-12*c^2*(5*f*g^2-3*h*(8*d*h+3*e*g)))*(h*x+g)^2*(c*x^2+b*x
+a)^(5/2)/c^3/h-1/144*(13*b*f*h-18*c*e*h+10*c*f*g)*(h*x+g)^3*(c*x^2+b*x+a)^
(5/2)/c^2/h+1/9*f*(h*x+g)^4*(c*x^2+b*x+a)^(5/2)/c/h+1/80640*(3003*b^4*f*h^4
-192*c^4*g^2*(5*f*g^2-3*h*(64*d*h+3*e*g))-198*b^2*c*h^3*(38*a*f*h+21*b*(e*h
+3*f*g))+8*c^2*h^2*(256*a^2*f*h^2+837*a*b*h*(e*h+3*f*g)+b^2*(1553*f*g^2+756
*h*(d*h+3*e*g)))-16*c^3*h*(32*a*h*(17*f*g^2+9*h*(d*h+3*e*g))+b*g*(13*f*g^2+
```

```
9*h*(196*d*h+141*e*g)))-10*c*h*(429*b^3*f*h^3-22*b*c*h^2*(34*a*f*h+27*b*e*h
+29*b*f*g)+16*c^3*g*(5*f*g^2-9*h*(12*d*h+e*g))+8*c^2*h*(a*h*(63*e*h+61*f*g)
+3*b*(f*g^2+6*h*(6*d*h+7*e*g)))))*x)*(c*x^2+b*x+a)^(5/2)/c^5/h+1/65536*(-4*a
*c+b^2)^2*(1536*c^5*d*g^3-143*b^5*f*h^3+22*b^3*c*h^2*(20*a*f*h+9*b*(e*h+3*f
*g))-48*b*c^2*h*(5*a^2*f*h^2+9*a*b*h*(e*h+3*f*g)+6*b^2*(d*h^2+3*e*g*h+3*f*g
^2))-256*c^4*g*(3*b*g*(3*d*h+e*g)+a*(f*g^2+3*h*(d*h+e*g)))+32*c^3*(3*a^2*h^
2*(e*h+3*f*g)+14*b^2*g*(f*g^2+3*h*(d*h+e*g))+12*a*b*h*(3*f*g^2+h*(d*h+3*e*g
))))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(15/2)-1/32768*(-
4*a*c+b^2)*(1536*c^5*d*g^3-143*b^5*f*h^3+22*b^3*c*h^2*(20*a*f*h+9*b*(e*h+3*
f*g))-48*b*c^2*h*(5*a^2*f*h^2+9*a*b*h*(e*h+3*f*g)+6*b^2*(d*h^2+3*e*g*h+3*f*
g^2))-256*c^4*g*(3*b*g*(3*d*h+e*g)+a*(f*g^2+3*h*(d*h+e*g)))+32*c^3*(3*a^2*h
^2*(e*h+3*f*g)+14*b^2*g*(f*g^2+3*h*(d*h+e*g))+12*a*b*h*(3*f*g^2+h*(d*h+3*e*
g))))*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/c^7
```

Rubi [A] (verified)

Time = 2.07 (sec) , antiderivative size = 1166, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1667, 846, 793, 626, 635, 212}

$$\int (g+hx)^3 \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right) \, dx = \frac{f\left(cx^2+bx+a\right)^{5/2}(g+hx)^4}{9ch}$$

$$-\frac{(10cfg-18ceh+13bfh)\left(cx^2+bx+a\right)^{5/2}(g+hx)^3}{144c^2h}$$

$$+\frac{\left(-12(5fg^2-3h(3eg+8dh))\,c^2-2h(24bfg+99beh+64afh)c+143b^2fh^2\right)\left(cx^2+bx+a\right)^{5/2}(g+hx)^2}{2016c^3h}$$

$$+\frac{\left(-192(5fg^4-3g^2h(3eg+64dh))\,c^4-16h(32ah(17fg^2+9h(3eg+dh))+bg(13fg^2+9h(141eg+196dh}{}$$

$$+\frac{\left(-143fh^3b^5+22ch^2(20afh+9b(3fg+eh))b^3-48c^2h(6(3fg^2+3ehg+dh^2)\,b^2+9ah(3fg+eh)b+5a^2}{}$$

$$+\frac{\left(b^2-4ac\right)^2\left(-143fh^3b^5+22ch^2(20afh+9b(3fg+eh))b^3-48c^2h(6(3fg^2+3ehg+dh^2)\,b^2+9ah(3fg+}{}$$

$$-\frac{\left(b^2-4ac\right)\left(-143fh^3b^5+22ch^2(20afh+9b(3fg+eh))b^3-48c^2h(6(3fg^2+3ehg+dh^2)\,b^2+9ah(3fg+}{}$$

```
[In] Int[(g + h*x)^3*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

```
[Out] -1/32768*((b^2 - 4*a*c)*(1536*c^5*d*g^3 - 143*b^5*f*h^3 - 256*c^4*g*(a*f*g^
2 + 3*a*h*(e*g + d*h) + 3*b*g*(e*g + 3*d*h)) + 22*b^3*c*h^2*(20*a*f*h + 9*b
*(3*f*g + e*h)) - 48*b*c^2*h*(5*a^2*f*h^2 + 9*a*b*h*(3*f*g + e*h) + 6*b^2*(
3*f*g^2 + 3*e*g*h + d*h^2)) + 32*c^3*(3*a^2*h^2*(3*f*g + e*h) + 14*b^2*g*(f
*g^2 + 3*h*(e*g + d*h)) + 12*a*b*h*(3*f*g^2 + h*(3*e*g + d*h))))*(b + 2*c*x
)*Sqrt[a + b*x + c*x^2])/c^7 + ((1536*c^5*d*g^3 - 143*b^5*f*h^3 - 256*c^4*g
```

```
*(a*f*g^2 + 3*a*h*(e*g + d*h) + 3*b*g*(e*g + 3*d*h)) + 22*b^3*c*h^2*(20*a*f
*h + 9*b*(3*f*g + e*h)) - 48*b*c^2*h*(5*a^2*f*h^2 + 9*a*b*h*(3*f*g + e*h) +
 6*b^2*(3*f*g^2 + 3*e*g*h + d*h^2)) + 32*c^3*(3*a^2*h^2*(3*f*g + e*h) + 14*
b^2*g*(f*g^2 + 3*h*(e*g + d*h)) + 12*a*b*h*(3*f*g^2 + h*(3*e*g + d*h))))*(b
 + 2*c*x)*(a + b*x + c*x^2)^(3/2))/(12288*c^6) + ((143*b^2*f*h^2 - 2*c*h*(2
4*b*f*g + 99*b*e*h + 64*a*f*h) - 12*c^2*(5*f*g^2 - 3*h*(3*e*g + 8*d*h)))*(g
 + h*x)^2*(a + b*x + c*x^2)^(5/2))/(2016*c^3*h) - ((10*c*f*g - 18*c*e*h + 1
3*b*f*h)*(g + h*x)^3*(a + b*x + c*x^2)^(5/2))/(144*c^2*h) + (f*(g + h*x)^4*
(a + b*x + c*x^2)^(5/2))/(9*c*h) + ((3003*b^4*f*h^4 - 192*c^4*(5*f*g^4 - 3*
g^2*h*(3*e*g + 64*d*h)) - 198*b^2*c*h^3*(38*a*f*h + 21*b*(3*f*g + e*h)) + 8
*c^2*h^2*(256*a^2*f*h^2 + 837*a*b*h*(3*f*g + e*h) + b^2*(1553*f*g^2 + 756*h
*(3*e*g + d*h))) - 16*c^3*h*(32*a*h*(17*f*g^2 + 9*h*(3*e*g + d*h)) + b*g*(1
3*f*g^2 + 9*h*(141*e*g + 196*d*h))) - 10*c*h*(429*b^3*f*h^3 - 22*b*c*h^2*(2
9*b*f*g + 27*b*e*h + 34*a*f*h) + 16*c^3*(5*f*g^3 - 9*g*h*(e*g + 12*d*h)) +
8*c^2*h*(a*h*(61*f*g + 63*e*h) + 3*b*(f*g^2 + 6*h*(7*e*g + 6*d*h)))))*x)*(a
+ b*x + c*x^2)^(5/2))/(80640*c^5*h) + ((b^2 - 4*a*c)^2*(1536*c^5*d*g^3 - 14
3*b^5*f*h^3 - 256*c^4*g*(a*f*g^2 + 3*a*h*(e*g + d*h) + 3*b*g*(e*g + 3*d*h))
 + 22*b^3*c*h^2*(20*a*f*h + 9*b*(3*f*g + e*h)) - 48*b*c^2*h*(5*a^2*f*h^2 +
9*a*b*h*(3*f*g + e*h) + 6*b^2*(3*f*g^2 + 3*e*g*h + d*h^2)) + 32*c^3*(3*a^2*
h^2*(3*f*g + e*h) + 14*b^2*g*(f*g^2 + 3*h*(e*g + d*h)) + 12*a*b*h*(3*f*g^2
+ h*(3*e*g + d*h))))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])]
)/(65536*c^(15/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
```

```
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 846

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{f(g+hx)^4\left(a+bx+cx^2\right)^{5/2}}{9ch} \\
&\quad+\frac{\int (g+hx)^3\left(-\frac{1}{2}h(5bfg-18cdh+8afh)-\frac{1}{2}h(10cfg-18ceh+13bfh)x\right)\left(a+bx+cx^2\right)^{3/2}\,dx}{9ch^2} \\
&= -\frac{(10cfg-18ceh+13bfh)(g+hx)^3\left(a+bx+cx^2\right)^{5/2}}{144c^2h}+\frac{f(g+hx)^4\left(a+bx+cx^2\right)^{5/2}}{9ch} \\
&\quad+\frac{\int (g+hx)^2\left(\frac{1}{4}h(65b^2fgh+78abfh^2-30bcg(fg+3eh)+4ch(72cdg-17afg-27aeh))+\frac{1}{4}h(}{72}
\end{aligned}
$$

$$= \frac{(143b^2fh^2 - 2ch(24bfg + 99beh + 64afh) - 12c^2(5fg^2 - 3h(3eg + 8dh)))\,(g + hx)^2\,(a + bx + cx^2}{2016c^3h}$$

$$- \frac{(10cfg - 18ceh + 13bfh)(g + hx)^3\,(a + bx + cx^2)^{5/2}}{144c^2h}$$

$$+ \frac{f(g + hx)^4\,(a + bx + cx^2)^{5/2}}{9ch}$$

$$+ \frac{\int (g + hx)\,\left(-\frac{1}{8}h(715b^3fgh^2 + 2b^2(286afh^3 - 5cgh(115fg + 99eh)) - 4bc(ah^2(481fg + 198eh) -}{}$$

$$= \frac{(143b^2fh^2 - 2ch(24bfg + 99beh + 64afh) - 12c^2(5fg^2 - 3h(3eg + 8dh)))\,(g + hx)^2\,(a + bx + cx^2}{2016c^3h}$$

$$- \frac{(10cfg - 18ceh + 13bfh)(g + hx)^3\,(a + bx + cx^2)^{5/2}}{144c^2h}$$

$$+ \frac{f(g + hx)^4\,(a + bx + cx^2)^{5/2}}{9ch}$$

$$+ \frac{(3003b^4fh^4 - 192c^4(5fg^4 - 3g^2h(3eg + 64dh)) - 198b^2ch^3(38afh + 21b(3fg + eh)) + 8c^2h^2(25}{}$$

$$+ \frac{(1536c^5dg^3 - 143b^5fh^3 - 256c^4g(afg^2 + 3ah(eg + dh) + 3bg(eg + 3dh)) + 22b^3ch^2(20afh + 9b(}{}$$

$$= \frac{(1536c^5dg^3 - 143b^5fh^3 - 256c^4g(afg^2 + 3ah(eg + dh) + 3bg(eg + 3dh)) + 22b^3ch^2(20afh + 9b(3}{}$$

$$+ \frac{(143b^2fh^2 - 2ch(24bfg + 99beh + 64afh) - 12c^2(5fg^2 - 3h(3eg + 8dh)))\,(g + hx)^2\,(a + bx +}{2016c^3h}$$

$$- \frac{(10cfg - 18ceh + 13bfh)(g + hx)^3\,(a + bx + cx^2)^{5/2}}{144c^2h}$$

$$+ \frac{f(g + hx)^4\,(a + bx + cx^2)^{5/2}}{9ch}$$

$$+ \frac{(3003b^4fh^4 - 192c^4(5fg^4 - 3g^2h(3eg + 64dh)) - 198b^2ch^3(38afh + 21b(3fg + eh)) + 8c^2h^2(25}{}$$

$$- \frac{((b^2 - 4ac)\,(1536c^5dg^3 - 143b^5fh^3 - 256c^4g(afg^2 + 3ah(eg + dh) + 3bg(eg + 3dh)) + 22b^3ch^2(}{}$$

$$=$$

$$-\frac{(b^2-4ac)\left(1536c^5dg^3-143b^5fh^3-256c^4g(afg^2+3ah(eg+dh)+3bg(eg+3dh))+22b^3ch^2(}{}$$

$$+\frac{(1536c^5dg^3-143b^5fh^3-256c^4g(afg^2+3ah(eg+dh)+3bg(eg+3dh))+22b^3ch^2(20afh+9}{}$$

$$+\frac{(143b^2fh^2-2ch(24bfg+99beh+64afh)-12c^2(5fg^2-3h(3eg+8dh)))\,(g+hx)^2\,(a+bx+}{2016c^3h}$$

$$-\frac{(10cfg-18ceh+13bfh)(g+hx)^3\left(a+bx+cx^2\right)^{5/2}}{144c^2h}$$

$$+\frac{f(g+hx)^4\left(a+bx+cx^2\right)^{5/2}}{9ch}$$

$$+\frac{(3003b^4fh^4-192c^4(5fg^4-3g^2h(3eg+64dh))-198b^2ch^3(38afh+21b(3fg+eh))+8c^2h^2(2}{}$$

$$+\frac{\Big(\left(b^2-4ac\right)^2\left(1536c^5dg^3-143b^5fh^3-256c^4g(afg^2+3ah(eg+dh)+3bg(eg+3dh))+22b^3ch}{}$$

$$=$$

$$-\frac{(b^2-4ac)\left(1536c^5dg^3-143b^5fh^3-256c^4g(afg^2+3ah(eg+dh)+3bg(eg+3dh))+22b^3ch^2(}{}$$

$$+\frac{(1536c^5dg^3-143b^5fh^3-256c^4g(afg^2+3ah(eg+dh)+3bg(eg+3dh))+22b^3ch^2(20afh+9}{}$$

$$+\frac{(143b^2fh^2-2ch(24bfg+99beh+64afh)-12c^2(5fg^2-3h(3eg+8dh)))\,(g+hx)^2\,(a+bx+}{2016c^3h}$$

$$-\frac{(10cfg-18ceh+13bfh)(g+hx)^3\left(a+bx+cx^2\right)^{5/2}}{144c^2h}$$

$$+\frac{f(g+hx)^4\left(a+bx+cx^2\right)^{5/2}}{9ch}$$

$$+\frac{(3003b^4fh^4-192c^4(5fg^4-3g^2h(3eg+64dh))-198b^2ch^3(38afh+21b(3fg+eh))+8c^2h^2(2}{}$$

$$+\frac{\Big(\left(b^2-4ac\right)^2\left(1536c^5dg^3-143b^5fh^3-256c^4g(afg^2+3ah(eg+dh)+3bg(eg+3dh))+22b^3ch}{}$$

$$=$$

$$-\frac{(b^2-4ac)\left(1536c^5dg^3-143b^5fh^3-256c^4g(afg^2+3ah(eg+dh)+3bg(eg+3dh))+22b^3ch^2(2}{}$$

$$+\frac{(1536c^5dg^3-143b^5fh^3-256c^4g(afg^2+3ah(eg+dh)+3bg(eg+3dh))+22b^3ch^2(20afh+9b(}{}$$

$$+\frac{(143b^2fh^2-2ch(24bfg+99beh+64afh)-12c^2(5fg^2-3h(3eg+8dh)))\,(g+hx)^2\,(a+bx+c}{2016c^3h}$$

$$-\frac{(10cfg-18ceh+13bfh)(g+hx)^3\left(a+bx+cx^2\right)^{5/2}}{144c^2h}$$

$$+\frac{f(g+hx)^4\left(a+bx+cx^2\right)^{5/2}}{9ch}$$

$$+\frac{(3003b^4fh^4-192c^4(5fg^4-3g^2h(3eg+64dh))-198b^2ch^3(38afh+21b(3fg+eh))+8c^2h^2(25}{}$$

$$+\frac{(b^2-4ac)^2\left(1536c^5dg^3-143b^5fh^3-256c^4g(afg^2+3ah(eg+dh)+3bg(eg+3dh))+22b^3ch^2(2}{}$$

Mathematica [A] (verified)

Time = 13.65 (sec) , antiderivative size = 1683, normalized size of antiderivative = 1.44

$$\int (g+hx)^3\left(a+bx+cx^2\right)^{3/2}\left(d+ex\right.$$

$$\left.+fx^2\right)\,dx=\frac{72h(fg^2+h(-eg+dh))\,(g+hx)^2(a+x(b+cx))^{5/2}+63h(-2fg+eh)(g+hx)^3(a+x(b+cx)}{}$$

```
[In] Integrate[(g + h*x)^3*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

```
[Out] (72*h*(f*g^2 + h*(-(e*g) + d*h))*(g + h*x)^2*(a + x*(b + c*x))^(5/2) + 63*h
*(-2*f*g + e*h)*(g + h*x)^3*(a + x*(b + c*x))^(5/2) + 56*f*h*(g + h*x)^4*(a
 + x*(b + c*x))^(5/2) - (f*(2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(-45045*b^8*h^5
 + 2310*b^7*c*h^4*(135*g + 13*h*x) - 84*b^6*c*h^3*(-5225*a*h^2 + c*(10800*g
^2 + 2475*g*h*x + 286*h^2*x^2)) + 72*b^5*c^2*h^2*(-7*a*h^2*(5475*g + 517*h*
x) + 2*c*(9800*g^3 + 4200*g^2*h*x + 1155*g*h^2*x^2 + 143*h^3*x^3)) - 16*b^4
*c^2*h*(86499*a^2*h^4 - 9*a*c*h^2*(50400*g^2 + 11235*g*h*x + 1276*h^2*x^2)
+ 2*c^2*(37800*g^4 + 29400*g^3*h*x + 15120*g^2*h^2*x^2 + 4455*g*h^3*x^3 + 5
72*h^4*x^4)) - 128*b*c^4*(a^3*h^4*(41355*g + 3701*h*x) - 6*a^2*c*h^2*(22680
*g^3 + 8760*g^2*h*x + 2265*g*h^2*x^2 + 269*h^3*x^3) + 40*a*c^2*(630*g^5 + 4
34*g^4*h*x - 1036*g^3*h^2*x^2 - 2292*g^2*h^3*x^3 - 1675*g*h^4*x^4 - 433*h^5
*x^5) + 80*c^3*x^2*(378*g^5 + 1162*g^4*h*x + 1288*g^3*h^2*x^2 + 456*g^2*h^3
*x^3 - 131*g*h^4*x^4 - 91*h^5*x^5)) - 256*c^4*(1024*a^4*h^5 - a^3*c*h^3*(23
040*g^2 + 4725*g*h*x + 512*h^2*x^2) + 80*c^4*g*x^3*(126*g^4 + 448*g^3*h*x +
 616*g^2*h^2*x^2 + 384*g*h^3*x^3 + 91*h^4*x^4) - 2*a^2*c^2*h*(-17920*g^4 -
3640*g^3*h*x + 7680*g^2*h^2*x^2 + 7385*g*h^3*x^3 + 2048*h^4*x^4) + 40*a*c^3
```

```
*x*(630*g^5 + 1792*g^4*h*x + 2044*g^3*h^2*x^2 + 960*g^2*h^3*x^3 + 49*g*h^4*
x^4 - 64*h^5*x^5)) + 32*b^3*c^3*(9*a^2*h^4*(25515*g + 2353*h*x) - 4*a*c*h^2
*(79800*g^3 + 32760*g^2*h*x + 8775*g*h^2*x^2 + 1067*h^3*x^3) + 40*c^2*(378*
g^5 + 630*g^4*h*x + 588*g^3*h^2*x^2 + 324*g^2*h^3*x^3 + 99*g*h^4*x^4 + 13*h
^5*x^5)) + 64*b^2*c^3*(22923*a^3*h^5 - 15*a^2*c*h^3*(16464*g^2 + 3543*g*h*x
 + 394*h^2*x^2) + 12*a*c^2*h*(10500*g^4 + 7560*g^3*h*x + 3720*g^2*h^2*x^2 +
 1065*g*h^3*x^3 + 134*h^4*x^4) + 40*c^3*x*(-126*g^5 + 196*g^4*h*x + 1540*g^
3*h^2*x^2 + 2544*g^2*h^3*x^3 + 1747*g*h^4*x^4 + 442*h^5*x^5))) + 315*(b^2 -
 4*a*c)^2*(-2*c*g + b*h)*(768*c^4*g^4 + 143*b^4*h^4 - 256*c^3*g^2*h*(6*b*g
+ 5*a*h) - 88*b^2*c*h^3*(8*b*g + 5*a*h) + 16*c^2*h^2*(92*b^2*g^2 + 80*a*b*g
*h + 15*a^2*h^2))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))/
(40960*c^(13/2)) - (3*(f*g^2 + h*(-(e*g) + d*h))*(-192*c^(5/2)*h*(a + x*(b
+ c*x))^(5/2)*(21*b^2*h^2 + 4*c^2*g*(32*g + 15*h*x) - 2*c*h*(49*b*g + 8*a*h
 + 15*b*h*x)) - (105*(2*c*g - b*h)*(8*c^2*g^2 + 3*b^2*h^2 - 4*c*h*(2*b*g +
a*h))*(2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c*x)]*(-3*b^2 + 8*b*c*x + 4*c*
(5*a + 2*c*x^2)) + 3*(b^2 - 4*a*c)^2*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a
+ x*(b + c*x)])]))/4))/(320*c^(9/2)) - (3*(2*f*g - e*h)*(42240*c^(9/2)*h*(2
*c*g - b*h)*(g + h*x)^2*(a + x*(b + c*x))^(5/2) + 96*c^(5/2)*h*(a + x*(b +
c*x))^(5/2)*(-231*b^3*h^3 + 16*c^3*g^2*(134*g + 65*h*x) + 6*b*c*h^2*(224*b*
g + 62*a*h + 55*b*h*x) - 8*c^2*h*(a*h*(128*g + 35*h*x) + b*g*(337*g + 130*h
*x))) + (105*(256*c^4*g^4 + 33*b^4*h^4 - 256*c^3*g^2*h*(2*b*g + a*h) - 24*b
^2*c*h^3*(8*b*g + 3*a*h) + 16*c^2*h^2*(28*b^2*g^2 + 16*a*b*g*h + a^2*h^2))*
(2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c*x)]*(-3*b^2 + 8*b*c*x + 4*c*(5*a +
 2*c*x^2)) + 3*(b^2 - 4*a*c)^2*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b
 + c*x)])]))/8))/(2560*c^(11/2)))/(504*c*h^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2877 vs. 2(1135) = 2270.

Time = 1.00 (sec) , antiderivative size = 2878, normalized size of antiderivative = 2.46

method	result	size
default	Expression too large to display	2878
risch	Expression too large to display	3146

```
[In] int((h*x+g)^3*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] d*g^3*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b
)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x
^2+b*x+a)^(1/2))))+f*h^3*(1/9*x^4*(c*x^2+b*x+a)^(5/2)/c-13/18*b/c*(1/8*x^3*
(c*x^2+b*x+a)^(5/2)/c-11/16*b/c*(1/7*x^2*(c*x^2+b*x+a)^(5/2)/c-9/14*b/c*(1/
6*x*(c*x^2+b*x+a)^(5/2)/c-7/12*b/c*(1/5*(c*x^2+b*x+a)^(5/2)/c-1/2*b/c*(1/8*
(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+
b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(
```

```
1/2)))))-1/6*a/c*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1
/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c
^(1/2)+(c*x^2+b*x+a)^(1/2)))))-2/7*a/c*(1/5*(c*x^2+b*x+a)^(5/2)/c-1/2*b/c*(
1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*
x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+
a)^(1/2))))))-3/8*a/c*(1/6*x*(c*x^2+b*x+a)^(5/2)/c-7/12*b/c*(1/5*(c*x^2+b*x
+a)^(5/2)/c-1/2*b/c*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c
*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x
)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))))-1/6*a/c*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3
/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)
/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))))-4/9*a/c*(1/7*x^2*(
c*x^2+b*x+a)^(5/2)/c-9/14*b/c*(1/6*x*(c*x^2+b*x+a)^(5/2)/c-7/12*b/c*(1/5*(c
*x^2+b*x+a)^(5/2)/c-1/2*b/c*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*
c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1
/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))))-1/6*a/c*(1/8*(2*c*x+b)/c*(c*x^2+b
*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*
a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))))-2/7*a/c*(1/
5*(c*x^2+b*x+a)^(5/2)/c-1/2*b/c*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(
4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*l
n((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))))))+(e*h^3+3*f*g*h^2)*(1/8*x^3*
(c*x^2+b*x+a)^(5/2)/c-11/16*b/c*(1/7*x^2*(c*x^2+b*x+a)^(5/2)/c-9/14*b/c*(1/
6*x*(c*x^2+b*x+a)^(5/2)/c-7/12*b/c*(1/5*(c*x^2+b*x+a)^(5/2)/c-1/2*b/c*(1/8*
(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+
b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(
1/2)))))-1/6*a/c*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1
/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c
^(1/2)+(c*x^2+b*x+a)^(1/2)))))-2/7*a/c*(1/5*(c*x^2+b*x+a)^(5/2)/c-1/2*b/c*(
1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*
x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+
a)^(1/2))))))-3/8*a/c*(1/6*x*(c*x^2+b*x+a)^(5/2)/c-7/12*b/c*(1/5*(c*x^2+b*x
+a)^(5/2)/c-1/2*b/c*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c
*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x
)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))))-1/6*a/c*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3
/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)
/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))))+(3*d*g^2*h+e*g^3)*
(1/5*(c*x^2+b*x+a)^(5/2)/c-1/2*b/c*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/1
6*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2
)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))))+(d*h^3+3*e*g*h^2+3*f*g^2*h
)*(1/7*x^2*(c*x^2+b*x+a)^(5/2)/c-9/14*b/c*(1/6*x*(c*x^2+b*x+a)^(5/2)/c-7/12
*b/c*(1/5*(c*x^2+b*x+a)^(5/2)/c-1/2*b/c*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2
)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c
^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))))-1/6*a/c*(1/8*(2*c*x+b
)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(
1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))))
-2/7*a/c*(1/5*(c*x^2+b*x+a)^(5/2)/c-1/2*b/c*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^
```

```
(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^
2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))))+(3*d*g*h^2+3*e*g
^2*h+f*g^3)*(1/6*x*(c*x^2+b*x+a)^(5/2)/c-7/12*b/c*(1/5*(c*x^2+b*x+a)^(5/2)/
c-1/2*b/c*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c
*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+
(c*x^2+b*x+a)^(1/2)))))-1/6*a/c*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(
4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*l
n((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2374 vs. $2(1135) = 2270$.

Time = 2.75 (sec) , antiderivative size = 4751, normalized size of antiderivative = 4.06

$$\int (g+hx)^3\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)\,dx = \text{Too large to display}$$

```
[In] integrate((h*x+g)^3*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="fricas"
)
```

```
[Out] [-1/41287680*(315*(64*(24*(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*d - 12*(b^5*
c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)*e + (7*b^6*c^3 - 60*a*b^4*c^4 + 144*a^2*b
^2*c^5 - 64*a^3*c^6)*f)*g^3 - 96*(24*(b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)
*d - 2*(7*b^6*c^3 - 60*a*b^4*c^4 + 144*a^2*b^2*c^5 - 64*a^3*c^6)*e + 3*(3*b
^7*c^2 - 28*a*b^5*c^3 + 80*a^2*b^3*c^4 - 64*a^3*b*c^5)*f)*g^2*h + 6*(32*(7*
b^6*c^3 - 60*a*b^4*c^4 + 144*a^2*b^2*c^5 - 64*a^3*c^6)*d - 48*(3*b^7*c^2 -
28*a*b^5*c^3 + 80*a^2*b^3*c^4 - 64*a^3*b*c^5)*e + 3*(33*b^8*c - 336*a*b^6*c
^2 + 1120*a^2*b^4*c^3 - 1280*a^3*b^2*c^4 + 256*a^4*c^5)*f)*g*h^2 - (96*(3*b
^7*c^2 - 28*a*b^5*c^3 + 80*a^2*b^3*c^4 - 64*a^3*b*c^5)*d - 6*(33*b^8*c - 33
6*a*b^6*c^2 + 1120*a^2*b^4*c^3 - 1280*a^3*b^2*c^4 + 256*a^4*c^5)*e + (143*b
^9 - 1584*a*b^7*c + 6048*a^2*b^5*c^2 - 8960*a^3*b^3*c^3 + 3840*a^4*b*c^4)*f
)*h^3)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*
c*x + b)*sqrt(c) - 4*a*c) - 4*(1146880*c^9*f*h^3*x^8 + 71680*(54*c^9*f*g*h^
2 + (18*c^9*e + 19*b*c^8*f)*h^3)*x^7 + 5120*(864*c^9*f*g^2*h + 54*(16*c^9*e
 + 17*b*c^8*f)*g*h^2 + (288*c^9*d + 306*b*c^8*e + (3*b^2*c^7 + 320*a*c^8)*f
)*h^3)*x^6 + 1280*(1344*c^9*f*g^3 + 288*(14*c^9*e + 15*b*c^8*f)*g^2*h + 18*
(224*c^9*d + 240*b*c^8*e + 3*(b^2*c^7 + 84*a*c^8)*f)*g*h^2 + (1440*b*c^8*d
+ 18*(b^2*c^7 + 84*a*c^8)*e - (13*b^3*c^6 - 60*a*b*c^7)*f)*h^3)*x^5 + 128*(
1344*(12*c^9*e + 13*b*c^8*f)*g^3 + 288*(168*c^9*d + 182*b*c^8*e + 3*(b^2*c^
7 + 64*a*c^8)*f)*g^2*h + 18*(2912*b*c^8*d + 48*(b^2*c^7 + 64*a*c^8)*e - 3*(
11*b^3*c^6 - 52*a*b*c^7)*f)*g*h^2 + (288*(b^2*c^7 + 64*a*c^8)*d - 18*(11*b^
3*c^6 - 52*a*b*c^7)*e + (143*b^4*c^5 - 804*a*b^2*c^6 + 768*a^2*c^7)*f)*h^3)
*x^4 - 1344*(120*(3*b^3*c^6 - 20*a*b*c^7)*d - 12*(15*b^4*c^5 - 100*a*b^2*c^
6 + 128*a^2*c^7)*e + (105*b^5*c^4 - 760*a*b^3*c^5 + 1296*a^2*b*c^6)*f)*g^3
+ 288*(168*(15*b^4*c^5 - 100*a*b^2*c^6 + 128*a^2*c^7)*d - 14*(105*b^5*c^4 -
```

```
 760*a*b^3*c^5 + 1296*a^2*b*c^6)*e + 3*(315*b^6*c^3 - 2520*a*b^4*c^4 + 5488
*a^2*b^2*c^5 - 2048*a^3*c^6)*f)*g^2*h - 18*(224*(105*b^5*c^4 - 760*a*b^3*c^
5 + 1296*a^2*b*c^6)*d - 48*(315*b^6*c^3 - 2520*a*b^4*c^4 + 5488*a^2*b^2*c^5
 - 2048*a^3*c^6)*e + 3*(3465*b^7*c^2 - 30660*a*b^5*c^3 + 81648*a^2*b^3*c^4
- 58816*a^3*b*c^5)*f)*g*h^2 + (288*(315*b^6*c^3 - 2520*a*b^4*c^4 + 5488*a^2
*b^2*c^5 - 2048*a^3*c^6)*d - 18*(3465*b^7*c^2 - 30660*a*b^5*c^3 + 81648*a^2
*b^3*c^4 - 58816*a^3*b*c^5)*e + (45045*b^8*c - 438900*a*b^6*c^2 + 1383984*a
^2*b^4*c^3 - 1467072*a^3*b^2*c^4 + 262144*a^4*c^5)*f)*h^3 + 16*(1344*(120*c
^9*d + 132*b*c^8*e + (3*b^2*c^7 + 140*a*c^8)*f)*g^3 + 288*(1848*b*c^8*d + 1
4*(3*b^2*c^7 + 140*a*c^8)*e - 3*(9*b^3*c^6 - 44*a*b*c^7)*f)*g^2*h + 18*(224
*(3*b^2*c^7 + 140*a*c^8)*d - 48*(9*b^3*c^6 - 44*a*b*c^7)*e + 3*(99*b^4*c^5
- 568*a*b^2*c^6 + 560*a^2*c^7)*f)*g*h^2 - (288*(9*b^3*c^6 - 44*a*b*c^7)*d -
 18*(99*b^4*c^5 - 568*a*b^2*c^6 + 560*a^2*c^7)*e + (1287*b^5*c^4 - 8536*a*b
^3*c^5 + 12912*a^2*b*c^6)*f)*h^3)*x^3 + 8*(1344*(360*b*c^8*d + 12*(b^2*c^7
+ 32*a*c^8)*e - (7*b^3*c^6 - 36*a*b*c^7)*f)*g^3 + 288*(168*(b^2*c^7 + 32*a*
c^8)*d - 14*(7*b^3*c^6 - 36*a*b*c^7)*e + 3*(21*b^4*c^5 - 124*a*b^2*c^6 + 12
8*a^2*c^7)*f)*g^2*h - 18*(224*(7*b^3*c^6 - 36*a*b*c^7)*d - 48*(21*b^4*c^5 -
 124*a*b^2*c^6 + 128*a^2*c^7)*e + 3*(231*b^5*c^4 - 1560*a*b^3*c^5 + 2416*a^
2*b*c^6)*f)*g*h^2 + (288*(21*b^4*c^5 - 124*a*b^2*c^6 + 128*a^2*c^7)*d - 18*
(231*b^5*c^4 - 1560*a*b^3*c^5 + 2416*a^2*b*c^6)*e + (3003*b^6*c^3 - 22968*a
*b^4*c^4 + 47280*a^2*b^2*c^5 - 16384*a^3*c^6)*f)*h^3)*x^2 + 2*(1344*(120*(b
^2*c^7 + 20*a*c^8)*d - 12*(5*b^3*c^6 - 28*a*b*c^7)*e + (35*b^4*c^5 - 216*a*
b^2*c^6 + 240*a^2*c^7)*f)*g^3 - 288*(168*(5*b^3*c^6 - 28*a*b*c^7)*d - 14*(3
5*b^4*c^5 - 216*a*b^2*c^6 + 240*a^2*c^7)*e + 3*(105*b^5*c^4 - 728*a*b^3*c^5
 + 1168*a^2*b*c^6)*f)*g^2*h + 18*(224*(35*b^4*c^5 - 216*a*b^2*c^6 + 240*a^2
*c^7)*d - 48*(105*b^5*c^4 - 728*a*b^3*c^5 + 1168*a^2*b*c^6)*e + 3*(1155*b^6
*c^3 - 8988*a*b^4*c^4 + 18896*a^2*b^2*c^5 - 6720*a^3*c^6)*f)*g*h^2 - (288*(
105*b^5*c^4 - 728*a*b^3*c^5 + 1168*a^2*b*c^6)*d - 18*(1155*b^6*c^3 - 8988*a
*b^4*c^4 + 18896*a^2*b^2*c^5 - 6720*a^3*c^6)*e + (15015*b^7*c^2 - 130284*a*
b^5*c^3 + 338832*a^2*b^3*c^4 - 236864*a^3*b*c^5)*f)*h^3)*x)*sqrt(c*x^2 + b*
x + a))/c^8, -1/20643840*(315*(64*(24*(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*
d - 12*(b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)*e + (7*b^6*c^3 - 60*a*b^4*c^4
 + 144*a^2*b^2*c^5 - 64*a^3*c^6)*f)*g^3 - 96*(24*(b^5*c^4 - 8*a*b^3*c^5 + 1
6*a^2*b*c^6)*d - 2*(7*b^6*c^3 - 60*a*b^4*c^4 + 144*a^2*b^2*c^5 - 64*a^3*c^6
)*e + 3*(3*b^7*c^2 - 28*a*b^5*c^3 + 80*a^2*b^3*c^4 - 64*a^3*b*c^5)*f)*g^2*h
 + 6*(32*(7*b^6*c^3 - 60*a*b^4*c^4 + 144*a^2*b^2*c^5 - 64*a^3*c^6)*d - 48*(
3*b^7*c^2 - 28*a*b^5*c^3 + 80*a^2*b^3*c^4 - 64*a^3*b*c^5)*e + 3*(33*b^8*c -
 336*a*b^6*c^2 + 1120*a^2*b^4*c^3 - 1280*a^3*b^2*c^4 + 256*a^4*c^5)*f)*g*h^
2 - (96*(3*b^7*c^2 - 28*a*b^5*c^3 + 80*a^2*b^3*c^4 - 64*a^3*b*c^5)*d - 6*(3
3*b^8*c - 336*a*b^6*c^2 + 1120*a^2*b^4*c^3 - 1280*a^3*b^2*c^4 + 256*a^4*c^5
)*e + (143*b^9 - 1584*a*b^7*c + 6048*a^2*b^5*c^2 - 8960*a^3*b^3*c^3 + 3840*
a^4*b*c^4)*f)*h^3)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sq
rt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(1146880*c^9*f*h^3*x^8 + 71680*(54*c^9*
f*g*h^2 + (18*c^9*e + 19*b*c^8*f)*h^3)*x^7 + 5120*(864*c^9*f*g^2*h + 54*(16
*c^9*e + 17*b*c^8*f)*g*h^2 + (288*c^9*d + 306*b*c^8*e + (3*b^2*c^7 + 320*a*
```

```
c^8)*f)*h^3)*x^6 + 1280*(1344*c^9*f*g^3 + 288*(14*c^9*e + 15*b*c^8*f)*g^2*h
 + 18*(224*c^9*d + 240*b*c^8*e + 3*(b^2*c^7 + 84*a*c^8)*f)*g*h^2 + (1440*b*
c^8*d + 18*(b^2*c^7 + 84*a*c^8)*e - (13*b^3*c^6 - 60*a*b*c^7)*f)*h^3)*x^5 +
 128*(1344*(12*c^9*e + 13*b*c^8*f)*g^3 + 288*(168*c^9*d + 182*b*c^8*e + 3*(
b^2*c^7 + 64*a*c^8)*f)*g^2*h + 18*(2912*b*c^8*d + 48*(b^2*c^7 + 64*a*c^8)*e
 - 3*(11*b^3*c^6 - 52*a*b*c^7)*f)*g*h^2 + (288*(b^2*c^7 + 64*a*c^8)*d - 18*
(11*b^3*c^6 - 52*a*b*c^7)*e + (143*b^4*c^5 - 804*a*b^2*c^6 + 768*a^2*c^7)*f
)*h^3)*x^4 - 1344*(120*(3*b^3*c^6 - 20*a*b*c^7)*d - 12*(15*b^4*c^5 - 100*a*
b^2*c^6 + 128*a^2*c^7)*e + (105*b^5*c^4 - 760*a*b^3*c^5 + 1296*a^2*b*c^6)*f
)*g^3 + 288*(168*(15*b^4*c^5 - 100*a*b^2*c^6 + 128*a^2*c^7)*d - 14*(105*b^5
*c^4 - 760*a*b^3*c^5 + 1296*a^2*b*c^6)*e + 3*(315*b^6*c^3 - 2520*a*b^4*c^4
+ 5488*a^2*b^2*c^5 - 2048*a^3*c^6)*f)*g^2*h - 18*(224*(105*b^5*c^4 - 760*a*
b^3*c^5 + 1296*a^2*b*c^6)*d - 48*(315*b^6*c^3 - 2520*a*b^4*c^4 + 5488*a^2*b
^2*c^5 - 2048*a^3*c^6)*e + 3*(3465*b^7*c^2 - 30660*a*b^5*c^3 + 81648*a^2*b^
3*c^4 - 58816*a^3*b*c^5)*f)*g*h^2 + (288*(315*b^6*c^3 - 2520*a*b^4*c^4 + 54
88*a^2*b^2*c^5 - 2048*a^3*c^6)*d - 18*(3465*b^7*c^2 - 30660*a*b^5*c^3 + 816
48*a^2*b^3*c^4 - 58816*a^3*b*c^5)*e + (45045*b^8*c - 438900*a*b^6*c^2 + 138
3984*a^2*b^4*c^3 - 1467072*a^3*b^2*c^4 + 262144*a^4*c^5)*f)*h^3 + 16*(1344*
(120*c^9*d + 132*b*c^8*e + (3*b^2*c^7 + 140*a*c^8)*f)*g^3 + 288*(1848*b*c^8
*d + 14*(3*b^2*c^7 + 140*a*c^8)*e - 3*(9*b^3*c^6 - 44*a*b*c^7)*f)*g^2*h + 1
8*(224*(3*b^2*c^7 + 140*a*c^8)*d - 48*(9*b^3*c^6 - 44*a*b*c^7)*e + 3*(99*b^
4*c^5 - 568*a*b^2*c^6 + 560*a^2*c^7)*f)*g*h^2 - (288*(9*b^3*c^6 - 44*a*b*c^
7)*d - 18*(99*b^4*c^5 - 568*a*b^2*c^6 + 560*a^2*c^7)*e + (1287*b^5*c^4 - 85
36*a*b^3*c^5 + 12912*a^2*b*c^6)*f)*h^3)*x^3 + 8*(1344*(360*b*c^8*d + 12*(b^
2*c^7 + 32*a*c^8)*e - (7*b^3*c^6 - 36*a*b*c^7)*f)*g^3 + 288*(168*(b^2*c^7 +
 32*a*c^8)*d - 14*(7*b^3*c^6 - 36*a*b*c^7)*e + 3*(21*b^4*c^5 - 124*a*b^2*c^
6 + 128*a^2*c^7)*f)*g^2*h - 18*(224*(7*b^3*c^6 - 36*a*b*c^7)*d - 48*(21*b^4
*c^5 - 124*a*b^2*c^6 + 128*a^2*c^7)*e + 3*(231*b^5*c^4 - 1560*a*b^3*c^5 + 2
416*a^2*b*c^6)*f)*g*h^2 + (288*(21*b^4*c^5 - 124*a*b^2*c^6 + 128*a^2*c^7)*d
 - 18*(231*b^5*c^4 - 1560*a*b^3*c^5 + 2416*a^2*b*c^6)*e + (3003*b^6*c^3 - 2
2968*a*b^4*c^4 + 47280*a^2*b^2*c^5 - 16384*a^3*c^6)*f)*h^3)*x^2 + 2*(1344*(
120*(b^2*c^7 + 20*a*c^8)*d - 12*(5*b^3*c^6 - 28*a*b*c^7)*e + (35*b^4*c^5 -
216*a*b^2*c^6 + 240*a^2*c^7)*f)*g^3 - 288*(168*(5*b^3*c^6 - 28*a*b*c^7)*d -
 14*(35*b^4*c^5 - 216*a*b^2*c^6 + 240*a^2*c^7)*e + 3*(105*b^5*c^4 - 728*a*b
^3*c^5 + 1168*a^2*b*c^6)*f)*g^2*h + 18*(224*(35*b^4*c^5 - 216*a*b^2*c^6 + 2
40*a^2*c^7)*d - 48*(105*b^5*c^4 - 728*a*b^3*c^5 + 1168*a^2*b*c^6)*e + 3*(11
55*b^6*c^3 - 8988*a*b^4*c^4 + 18896*a^2*b^2*c^5 - 6720*a^3*c^6)*f)*g*h^2 -
(288*(105*b^5*c^4 - 728*a*b^3*c^5 + 1168*a^2*b*c^6)*d - 18*(1155*b^6*c^3 -
8988*a*b^4*c^4 + 18896*a^2*b^2*c^5 - 6720*a^3*c^6)*e + (15015*b^7*c^2 - 130
284*a*b^5*c^3 + 338832*a^2*b^3*c^4 - 236864*a^3*b*c^5)*f)*h^3)*x)*sqrt(c*x^
2 + b*x + a))/c^8]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19122 vs. $2(1221) = 2442$.

Time = 1.54 (sec) , antiderivative size = 19122, normalized size of antiderivative = 16.36

$$\int (g+hx)^3 \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right)\,dx = \text{Too large to display}$$

```
[In] integrate((h*x+g)**3*(c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d),x)
```

```
[Out] Piecewise((sqrt(a + b*x + c*x**2)*(c*f*h**3*x**8/9 + x**7*(19*b*c*f*h**3/18
 + c**2*e*h**3 + 3*c**2*f*g*h**2)/(8*c) + x**6*(10*a*c*f*h**3/9 + b**2*f*h*
*3 + 2*b*c*e*h**3 + 6*b*c*f*g*h**2 - 15*b*(19*b*c*f*h**3/18 + c**2*e*h**3 +
 3*c**2*f*g*h**2)/(16*c) + c**2*d*h**3 + 3*c**2*e*g*h**2 + 3*c**2*f*g**2*h)
/(7*c) + x**5*(2*a*b*f*h**3 + 2*a*c*e*h**3 + 6*a*c*f*g*h**2 - 7*a*(19*b*c*f
*h**3/18 + c**2*e*h**3 + 3*c**2*f*g*h**2)/(8*c) + b**2*e*h**3 + 3*b**2*f*g*
h**2 + 2*b*c*d*h**3 + 6*b*c*e*g*h**2 + 6*b*c*f*g**2*h - 13*b*(10*a*c*f*h**3
/9 + b**2*f*h**3 + 2*b*c*e*h**3 + 6*b*c*f*g*h**2 - 15*b*(19*b*c*f*h**3/18 +
 c**2*e*h**3 + 3*c**2*f*g*h**2)/(16*c) + c**2*d*h**3 + 3*c**2*e*g*h**2 + 3*
c**2*f*g**2*h)/(14*c) + 3*c**2*d*g*h**2 + 3*c**2*e*g**2*h + c**2*f*g**3)/(6
*c) + x**4*(a**2*f*h**3 + 2*a*b*e*h**3 + 6*a*b*f*g*h**2 + 2*a*c*d*h**3 + 6*
a*c*e*g*h**2 + 6*a*c*f*g**2*h - 6*a*(10*a*c*f*h**3/9 + b**2*f*h**3 + 2*b*c*
e*h**3 + 6*b*c*f*g*h**2 - 15*b*(19*b*c*f*h**3/18 + c**2*e*h**3 + 3*c**2*f*g
*h**2)/(16*c) + c**2*d*h**3 + 3*c**2*e*g*h**2 + 3*c**2*f*g**2*h)/(7*c) + b*
*2*d*h**3 + 3*b**2*e*g*h**2 + 3*b**2*f*g**2*h + 6*b*c*d*g*h**2 + 6*b*c*e*g*
*2*h + 2*b*c*f*g**3 - 11*b*(2*a*b*f*h**3 + 2*a*c*e*h**3 + 6*a*c*f*g*h**2 -
7*a*(19*b*c*f*h**3/18 + c**2*e*h**3 + 3*c**2*f*g*h**2)/(8*c) + b**2*e*h**3
+ 3*b**2*f*g*h**2 + 2*b*c*d*h**3 + 6*b*c*e*g*h**2 + 6*b*c*f*g**2*h - 13*b*(
10*a*c*f*h**3/9 + b**2*f*h**3 + 2*b*c*e*h**3 + 6*b*c*f*g*h**2 - 15*b*(19*b*
c*f*h**3/18 + c**2*e*h**3 + 3*c**2*f*g*h**2)/(16*c) + c**2*d*h**3 + 3*c**2*
e*g*h**2 + 3*c**2*f*g**2*h)/(14*c) + 3*c**2*d*g*h**2 + 3*c**2*e*g**2*h + c*
*2*f*g**3)/(12*c) + 3*c**2*d*g**2*h + c**2*e*g**3)/(5*c) + x**3*(a**2*e*h**
3 + 3*a**2*f*g*h**2 + 2*a*b*d*h**3 + 6*a*b*e*g*h**2 + 6*a*b*f*g**2*h + 6*a*
c*d*g*h**2 + 6*a*c*e*g**2*h + 2*a*c*f*g**3 - 5*a*(2*a*b*f*h**3 + 2*a*c*e*h*
*3 + 6*a*c*f*g*h**2 - 7*a*(19*b*c*f*h**3/18 + c**2*e*h**3 + 3*c**2*f*g*h**2
)/(8*c) + b**2*e*h**3 + 3*b**2*f*g*h**2 + 2*b*c*d*h**3 + 6*b*c*e*g*h**2 + 6
*b*c*f*g**2*h - 13*b*(10*a*c*f*h**3/9 + b**2*f*h**3 + 2*b*c*e*h**3 + 6*b*c*
f*g*h**2 - 15*b*(19*b*c*f*h**3/18 + c**2*e*h**3 + 3*c**2*f*g*h**2)/(16*c) +
 c**2*d*h**3 + 3*c**2*e*g*h**2 + 3*c**2*f*g**2*h)/(14*c) + 3*c**2*d*g*h**2
+ 3*c**2*e*g**2*h + c**2*f*g**3)/(6*c) + 3*b**2*d*g*h**2 + 3*b**2*e*g**2*h
+ b**2*f*g**3 + 6*b*c*d*g**2*h + 2*b*c*e*g**3 - 9*b*(a**2*f*h**3 + 2*a*b*e*
h**3 + 6*a*b*f*g*h**2 + 2*a*c*d*h**3 + 6*a*c*e*g*h**2 + 6*a*c*f*g**2*h - 6*
a*(10*a*c*f*h**3/9 + b**2*f*h**3 + 2*b*c*e*h**3 + 6*b*c*f*g*h**2 - 15*b*(19
*b*c*f*h**3/18 + c**2*e*h**3 + 3*c**2*f*g*h**2)/(16*c) + c**2*d*h**3 + 3*c*
*2*e*g*h**2 + 3*c**2*f*g**2*h)/(7*c) + b**2*d*h**3 + 3*b**2*e*g*h**2 + 3*b*
```

```
*2*f*g**2*h + 6*b*c*d*g*h**2 + 6*b*c*e*g**2*h + 2*b*c*f*g**3 - 11*b*(2*a*b*
f*h**3 + 2*a*c*e*h**3 + 6*a*c*f*g*h**2 - 7*a*(19*b*c*f*h**3/18 + c**2*e*h**
3 + 3*c**2*f*g*h**2)/(8*c) + b**2*e*h**3 + 3*b**2*f*g*h**2 + 2*b*c*d*h**3 +
 6*b*c*e*g*h**2 + 6*b*c*f*g**2*h - 13*b*(10*a*c*f*h**3/9 + b**2*f*h**3 + 2*
b*c*e*h**3 + 6*b*c*f*g*h**2 - 15*b*(19*b*c*f*h**3/18 + c**2*e*h**3 + 3*c**2
*f*g*h**2)/(16*c) + c**2*d*h**3 + 3*c**2*e*g*h**2 + 3*c**2*f*g**2*h)/(14*c)
 + 3*c**2*d*g*h**2 + 3*c**2*e*g**2*h + c**2*f*g**3)/(12*c) + 3*c**2*d*g**2*
h + c**2*e*g**3)/(10*c) + c**2*d*g**3)/(4*c) + x**2*(a**2*d*h**3 + 3*a**2*e
*g*h**2 + 3*a**2*f*g**2*h + 6*a*b*d*g*h**2 + 6*a*b*e*g**2*h + 2*a*b*f*g**3
+ 6*a*c*d*g**2*h + 2*a*c*e*g**3 - 4*a*(a**2*f*h**3 + 2*a*b*e*h**3 + 6*a*b*f
*g*h**2 + 2*a*c*d*h**3 + 6*a*c*e*g*h**2 + 6*a*c*f*g**2*h - 6*a*(10*a*c*f*h*
*3/9 + b**2*f*h**3 + 2*b*c*e*h**3 + 6*b*c*f*g*h**2 - 15*b*(19*b*c*f*h**3/18
 + c**2*e*h**3 + 3*c**2*f*g*h**2)/(16*c) + c**2*d*h**3 + 3*c**2*e*g*h**2 +
3*c**2*f*g**2*h)/(7*c) + b**2*d*h**3 + 3*b**2*e*g*h**2 + 3*b**2*f*g**2*h +
6*b*c*d*g*h**2 + 6*b*c*e*g**2*h + 2*b*c*f*g**3 - 11*b*(2*a*b*f*h**3 + 2*a*c
*e*h**3 + 6*a*c*f*g*h**2 - 7*a*(19*b*c*f*h**3/18 + c**2*e*h**3 + 3*c**2*f*g
*h**2)/(8*c) + b**2*e*h**3 + 3*b**2*f*g*h**2 + 2*b*c*d*h**3 + 6*b*c*e*g*h**
2 + 6*b*c*f*g**2*h - 13*b*(10*a*c*f*h**3/9 + b**2*f*h**3 + 2*b*c*e*h**3 + 6
*b*c*f*g*h**2 - 15*b*(19*b*c*f*h**3/18 + c**2*e*h**3 + 3*c**2*f*g*h**2)/(16
*c) + c**2*d*h**3 + 3*c**2*e*g*h**2 + 3*c**2*f*g**2*h)/(14*c) + 3*c**2*d*g*
h**2 + 3*c**2*e*g**2*h + c**2*f*g**3)/(12*c) + 3*c**2*d*g**2*h + c**2*e*g**
3)/(5*c) + 3*b**2*d*g**2*h + b**2*e*g**3 + 2*b*c*d*g**3 - 7*b*(a**2*e*h**3
+ 3*a**2*f*g*h**2 + 2*a*b*d*h**3 + 6*a*b*e*g*h**2 + 6*a*b*f*g**2*h + 6*a*c*
d*g*h**2 + 6*a*c*e*g**2*h + 2*a*c*f*g**3 - 5*a*(2*a*b*f*h**3 + 2*a*c*e*h**3
 + 6*a*c*f*g*h**2 - 7*a*(19*b*c*f*h**3/18 + c**2*e*h**3 + 3*c**2*f*g*h**2)/
(8*c) + b**2*e*h**3 + 3*b**2*f*g*h**2 + 2*b*c*d*h**3 + 6*b*c*e*g*h**2 + 6*b
*c*f*g**2*h - 13*b*(10*a*c*f*h**3/9 + b**2*f*h**3 + 2*b*c*e*h**3 + 6*b*c*f*
g*h**2 - 15*b*(19*b*c*f*h**3/18 + c**2*e*h**3 + 3*c**2*f*g*h**2)/(16*c) + c
**2*d*h**3 + 3*c**2*e*g*h**2 + 3*c**2*f*g**2*h)/(14*c) + 3*c**2*d*g*h**2 +
3*c**2*e*g**2*h + c**2*f*g**3)/(6*c) + 3*b**2*d*g*h**2 + 3*b**2*e*g**2*h +
b**2*f*g**3 + 6*b*c*d*g**2*h + 2*b*c*e*g**3 - 9*b*(a**2*f*h**3 + 2*a*b*e*h*
*3 + 6*a*b*f*g*h**2 + 2*a*c*d*h**3 + 6*a*c*e*g*h**2 + 6*a*c*f*g**2*h - 6*a*
(10*a*c*f*h**3/9 + b**2*f*h**3 + 2*b*c*e*h**3 + 6*b*c*f*g*h**2 - 15*b*(19*b
*c*f*h**3/18 + c**2*e*h**3 + 3*c**2*f*g*h**2)/(16*c) + c**2*d*h**3 + 3*c**2
*e*g*h**2 + 3*c**2*f*g**2*h)/(7*c) + b**2*d*h**3 + 3*b**2*e*g*h**2 + 3*b**2
*f*g**2*h + 6*b*c*d*g*h**2 + 6*b*c*e*g**2*h + 2*b*c*f*g**3 - 11*b*(2*a*b*f*
h**3 + 2*a*c*e*h**3 + 6*a*c*f*g*h**2 - 7*a*(19*b*c*f*h**3/18 + c**2*e*h**3
+ 3*c**2*f*g*h**2)/(8*c) + b**2*e*h**3 + 3*b**2*f*g*h**2 + 2*b*c*d*h**3 + 6
*b*c*e*g*h**2 + 6*b*c*f*g**2*h - 13*b*(10*a*c*f*h**3/9 + b**2*f*h**3 + 2*b*
c*e*h**3 + 6*b*c*f*g*h**2 - 15*b*(19*b*c*f*h**3/18 + c**2*e*h**3 + 3*c**2*f
*g*h**2)/(16*c) + c**2*d*h**3 + 3*c**2*e*g*h**2 + 3*c**2*f*g**2*h)/(14*c) +
 3*c**2*d*g*h**2 + 3*c**2*e*g**2*h + c**2*f*g**3)/(12*c) + 3*c**2*d*g**2*h
+ c**2*e*g**3)/(10*c) + c**2*d*g**3)/(8*c))/(3*c) + x*(3*a**2*d*g*h**2 + 3*
a**2*e*g**2*h + a**2*f*g**3 + 6*a*b*d*g**2*h + 2*a*b*e*g**3 + 2*a*c*d*g**3
- 3*a*(a**2*e*h**3 + 3*a**2*f*g*h**2 + 2*a*b*d*h**3 + 6*a*b*e*g*h**2 + 6*a*
```

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b*f*g**2*h + 6*a*c*d*g*h**2 + 6*a*c*e*g**2*h + 2*a*c*f*g**3 - 5*a*(2*a*b*f*
h**3 + 2*a*c*e*h**3 + 6*a*c*f*g*h**2 - 7*a*(19*b*c*f*h**3/18 + c**2*e*h**3
+ 3*c**2*f*g*h**2)/(8*c) + b**2*e*h**3 + 3*b**2*f*g*h**2 + 2*b*c*d*h**3 + 6
*b*c*e*g*h**2 + 6*b*c*f*g**2*h - 13*b*(10*a*c*f*h**3/9 + b**2*f*h**3 + 2*b*
c*e*h**3 + 6*b*c*f*g*h**2 - 15*b*(19*b*c*f*h**3/18 + c**2*e*h**3 + 3*c**2*f
*g*h**2)/(16*c) + c**2*d*h**3 + 3*c**2*e*g*h**2 + 3*c**2*f*g**2*h)/(14*c) +
 3*c**2*d*g*h**2 + 3*c**2*e*g**2*h + c**2*f*g**3)/(6*c) + 3*b**2*d*g*h**2 +
 3*b**2*e*g**2*h + b**2*f*g**3 + 6*b*c*d*g**2*h + 2*b*c*e*g**3 - 9*b*(a**2*
f*h**3 + 2*a*b*e*h**3 + 6*a*b*f*g*h**2 + 2*a*c*d*h**3 + 6*a*c*e*g*h**2 + 6*
a*c*f*g**2*h - 6*a*(10*a*c*f*h**3/9 + b**2*f*h**3 + 2*b*c*e*h**3 + 6*b*c*f*
g*h**2 - 15*b*(19*b*c*f*h**3/18 + c**2*e*h**3 + 3*c**2*f*g*h**2)/(16*c) + c
**2*d*h**3 + 3*c**2*e*g*h**2 + 3*c**2*f*g**2*h)/(7*c) + b**2*d*h**3 + 3*b**
2*e*g*h**2 + 3*b**2*f*g**2*h + 6*b*c*d*g*h**2 + 6*b*c*e*g**2*h + 2*b*c*f*g*
*3 - 11*b*(2*a*b*f*h**3 + 2*a*c*e*h**3 + 6*a*c*f*g*h**2 - 7*a*(19*b*c*f*h**
3/18 + c**2*e*h**3 + 3*c**2*f*g*h**2)/(8*c) + b**2*e*h**3 + 3*b**2*f*g*h**2
 + 2*b*c*d*h**3 + 6*b*c*e*g*h**2 + 6*b*c*f*g**2*h - 13*b*(10*a*c*f*h**3/9 +
 b**2*f*h**3 + 2*b*c*e*h**3 + 6*b*c*f*g*h**2 - 15*b*(19*b*c*f*h**3/18 + c**
2*e*h**3 + 3*c**2*f*g*h**2)/(16*c) + c**2*d*h**3 + 3*c**2*e*g*h**2 + 3*c**2
*f*g**2*h)/(14*c) + 3*c**2*d*g*h**2 + 3*c**2*e*g**2*h + c**2*f*g**3)/(12*c)
 + 3*c**2*d*g**2*h + c**2*e*g**3)/(10*c) + c**2*d*g**3)/(4*c) + b**2*d*g**3
 - 5*b*(a**2*d*h**3 + 3*a**2*e*g*h**2 + 3*a**2*f*g**2*h + 6*a*b*d*g*h**2 +
6*a*b*e*g**2*h + 2*a*b*f*g**3 + 6*a*c*d*g**2*h + 2*a*c*e*g**3 - 4*a*(a**2*f
*h**3 + 2*a*b*e*h**3 + 6*a*b*f*g*h**2 + 2*a*c*d*h**3 + 6*a*c*e*g*h**2 + 6*a
*c*f*g**2*h - 6*a*(10*a*c*f*h**3/9 + b**2*f*h**3 + 2*b*c*e*h**3 + 6*b*c*f*g
*h**2 - 15*b*(19*b*c*f*h**3/18 + c**2*e*h**3 + 3*c**2*f*g*h**2)/(16*c) + c*
*2*d*h**3 + 3*c**2*e*g*h**2 + 3*c**2*f*g**2*h)/(7*c) + b**2*d*h**3 + 3*b**2
*e*g*h**2 + 3*b**2*f*g**2*h + 6*b*c*d*g*h**2 + 6*b*c*e*g**2*h + 2*b*c*f*g**
3 - 11*b*(2*a*b*f*h**3 + 2*a*c*e*h**3 + 6*a*c*f*g*h**2 - 7*a*(19*b*c*f*h**3
/18 + c**2*e*h**3 + 3*c**2*f*g*h**2)/(8*c) + b**2*e*h**3 + 3*b**2*f*g*h**2
+ 2*b*c*d*h**3 + 6*b*c*e*g*h**2 + 6*b*c*f*g**2*h - 13*b*(10*a*c*f*h**3/9 +
b**2*f*h**3 + 2*b*c*e*h**3 + 6*b*c*f*g*h**2 - 15*b*(19*b*c*f*h**3/18 + c**2
*e*h**3 + 3*c**2*f*g*h**2)/(16*c) + c**2*d*h**3 + 3*c**2*e*g*h**2 + 3*c**2*
f*g**2*h)/(14*c) + 3*c**2*d*g*h**2 + 3*c**2*e*g**2*h + c**2*f*g**3)/(12*c)
+ 3*c**2*d*g**2*h + c**2*e*g**3)/(5*c) + 3*b**2*d*g**2*h + b**2*e*g**3 + 2*
b*c*d*g**3 - 7*b*(a**2*e*h**3 + 3*a**2*f*g*h**2 + 2*a*b*d*h**3 + 6*a*b*e*g*
h**2 + 6*a*b*f*g**2*h + 6*a*c*d*g*h**2 + 6*a*c*e*g**2*h + 2*a*c*f*g**3 - 5*
a*(2*a*b*f*h**3 + 2*a*c*e*h**3 + 6*a*c*f*g*h**2 - 7*a*(19*b*c*f*h**3/18 + c
**2*e*h**3 + 3*c**2*f*g*h**2)/(8*c) + b**2*e*h**3 + 3*b**2*f*g*h**2 + 2*b*c
*d*h**3 + 6*b*c*e*g*h**2 + 6*b*c*f*g**2*h - 13*b*(10*a*c*f*h**3/9 + b**2*f*
h**3 + 2*b*c*e*h**3 + 6*b*c*f*g*h**2 - 15*b*(19*b*c*f*h**3/18 + c**2*e*h**3
 + 3*c**2*f*g*h**2)/(16*c) + c**2*d*h**3 + 3*c**2*e*g*h**2 + 3*c**2*f*g**2*
h)/(14*c) + 3*c**2*d*g*h**2 + 3*c**2*e*g**2*h + c**2*f*g**3)/(6*c) + 3*b**2
*d*g*h**2 + 3*b**2*e*g**2*h + b**2*f*g**3 + 6*b*c*d*g**2*h + 2*b*c*e*g**3 -
 9*b*(a**2*f*h**3 + 2*a*b*e*h**3 + 6*a*b*f*g*h**2 + 2*a*c*d*h**3 + 6*a*c*e*
g*h**2 + 6*a*c*f*g**2*h - 6*a*(10*a*c*f*h**3/9 + b**2*f*h**3 + 2*b*c*e*h**3
```

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 + 6*b*c*f*g*h**2 - 15*b*(19*b*c*f*h**3/18 + c**2*e*h**3 + 3*c**2*f*g*h**2)
/(16*c) + c**2*d*h**3 + 3*c**2*e*g*h**2 + 3*c**2*f*g**2*h)/(7*c) + b**2*d*h
**3 + 3*b**2*e*g*h**2 + 3*b**2*f*g**2*h + 6*b*c*d*g*h**2 + 6*b*c*e*g**2*h +
 2*b*c*f*g**3 - 11*b*(2*a*b*f*h**3 + 2*a*c*e*h**3 + 6*a*c*f*g*h**2 - 7*a*(1
9*b*c*f*h**3/18 + c**2*e*h**3 + 3*c**2*f*g*h**2)/(8*c) + b**2*e*h**3 + 3*b*
*2*f*g*h**2 + 2*b*c*d*h**3 + 6*b*c*e*g*h**2 + 6*b*c*f*g**2*h - 13*b*(10*a*c
*f*h**3/9 + b**2*f*h**3 + 2*b*c*e*h**3 + 6*b*c*f*g*h**2 - 15*b*(19*b*c*f*h*
*3/18 + c**2*e*h**3 + 3*c**2*f*g*h**2)/(16*c) + c**2*d*h**3 + 3*c**2*e*g*h*
*2 + 3*c**2*f*g**2*h)/(14*c) + 3*c**2*d*g*h**2 + 3*c**2*e*g**2*h + c**2*f*g
**3)/(12*c) + 3*c**2*d*g**2*h + c**2*e*g**3)/(10*c) + c**2*d*g**3)/(8*c))/(
6*c))/(2*c) + (3*a**2*d*g**2*h + a**2*e*g**3 + 2*a*b*d*g**3 - 2*a*(a**2*d*h
**3 + 3*a**2*e*g*h**2 + 3*a**2*f*g**2*h + 6*a*b*d*g*h**2 + 6*a*b*e*g**2*h +
 2*a*b*f*g**3 + 6*a*c*d*g**2*h + 2*a*c*e*g**3 - 4*a*(a**2*f*h**3 + 2*a*b*e*
h**3 + 6*a*b*f*g*h**2 + 2*a*c*d*h**3 + 6*a*c*e*g*h**2 + 6*a*c*f*g**2*h - 6*
a*(10*a*c*f*h**3/9 + b**2*f*h**3 + 2*b*c*e*h**3 + 6*b*c*f*g*h**2 - 15*b*(19
*b*c*f*h**3/18 + c**2*e*h**3 + 3*c**2*f*g*h**2)/(16*c) + c**2*d*h**3 + 3*c*
*2*e*g*h**2 + 3*c**2*f*g**2*h)/(7*c) + b**2*d*h**3 + 3*b**2*e*g*h**2 + 3*b*
*2*f*g**2*h + 6*b*c*d*g*h**2 + 6*b*c*e*g**2*h + 2*b*c*f*g**3 - 11*b*(2*a*b*
f*h**3 + 2*a*c*e*h**3 + 6*a*c*f*g*h**2 - 7*a*(19*b*c*f*h**3/18 + c**2*e*h**
3 + 3*c**2*f*g*h**2)/(8*c) + b**2*e*h**3 + 3*b**2*f*g*h**2 + 2*b*c*d*h**3 +
 6*b*c*e*g*h**2 + 6*b*c*f*g**2*h - 13*b*(10*a*c*f*h**3/9 + b**2*f*h**3 + 2*
b*c*e*h**3 + 6*b*c*f*g*h**2 - 15*b*(19*b*c*f*h**3/18 + c**2*e*h**3 + 3*c**2
*f*g*h**2)/(16*c) + c**2*d*h**3 + 3*c**2*e*g*h**2 + 3*c**2*f*g**2*h)/(14*c)
 + 3*c**2*d*g*h**2 + 3*c**2*e*g**2*h + c**2*f*g**3)/(12*c) + 3*c**2*d*g**2*
h + c**2*e*g**3)/(5*c) + 3*b**2*d*g**2*h + b**2*e*g**3 + 2*b*c*d*g**3 - 7*b
*(a**2*e*h**3 + 3*a**2*f*g*h**2 + 2*a*b*d*h**3 + 6*a*b*e*g*h**2 + 6*a*b*f*g
**2*h + 6*a*c*d*g*h**2 + 6*a*c*e*g**2*h + 2*a*c*f*g**3 - 5*a*(2*a*b*f*h**3
+ 2*a*c*e*h**3 + 6*a*c*f*g*h**2 - 7*a*(19*b*c*f*h**3/18 + c**2*e*h**3 + 3*c
**2*f*g*h**2)/(8*c) + b**2*e*h**3 + 3*b**2*f*g*h**2 + 2*b*c*d*h**3 + 6*b*c*
e*g*h**2 + 6*b*c*f*g**2*h - 13*b*(10*a*c*f*h**3/9 + b**2*f*h**3 + 2*b*c*e*h
**3 + 6*b*c*f*g*h**2 - 15*b*(19*b*c*f*h**3/18 + c**2*e*h**3 + 3*c**2*f*g*h*
*2)/(16*c) + c**2*d*h**3 + 3*c**2*e*g*h**2 + 3*c**2*f*g**2*h)/(14*c) + 3*c*
*2*d*g*h**2 + 3*c**2*e*g**2*h + c**2*f*g**3)/(6*c) + 3*b**2*d*g*h**2 + 3*b*
*2*e*g**2*h + b**2*f*g**3 + 6*b*c*d*g**2*h + 2*b*c*e*g**3 - 9*b*(a**2*f*h**
3 + 2*a*b*e*h**3 + 6*a*b*f*g*h**2 + 2*a*c*d*h**3 + 6*a*c*e*g*h**2 + 6*a*c*f
*g**2*h - 6*a*(10*a*c*f*h**3/9 + b**2*f*h**3 + 2*b*c*e*h**3 + 6*b*c*f*g*h**
2 - 15*b*(19*b*c*f*h**3/18 + c**2*e*h**3 + 3*c**2*f*g*h**2)/(16*c) + c**2*d
*h**3 + 3*c**2*e*g*h**2 + 3*c**2*f*g**2*h)/(7*c) + b**2*d*h**3 + 3*b**2*e*g
*h**2 + 3*b**2*f*g**2*h + 6*b*c*d*g*h**2 + 6*b*c*e*g**2*h + 2*b*c*f*g**3 -
11*b*(2*a*b*f*h**3 + 2*a*c*e*h**3 + 6*a*c*f*g*h**2 - 7*a*(19*b*c*f*h**3/18
+ c**2*e*h**3 + 3*c**2*f*g*h**2)/(8*c) + b**2*e*h**3 + 3*b**2*f*g*h**2 + 2*
b*c*d*h**3 + 6*b*c*e*g*h**2 + 6*b*c*f*g**2*h - 13*b*(10*a*c*f*h**3/9 + b**2
*f*h**3 + 2*b*c*e*h**3 + 6*b*c*f*g*h**2 - 15*b*(19*b*c*f*h**3/18 + c**2*e*h
**3 + 3*c**2*f*g*h**2)/(16*c) + c**2*d*h**3 + 3*c**2*e*g*h**2 + 3*c**2*f*g*
*2*h)/(14*c) + 3*c**2*d*g*h**2 + 3*c**2*e*g**2*h + c**2*f*g**3)/(12*c) + 3*
```

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c**2*d*g**2*h + c**2*e*g**3)/(10*c) + c**2*d*g**3)/(8*c))/(3*c) - 3*b*(3*a*
*2*d*g*h**2 + 3*a**2*e*g**2*h + a**2*f*g**3 + 6*a*b*d*g**2*h + 2*a*b*e*g**3
 + 2*a*c*d*g**3 - 3*a*(a**2*e*h**3 + 3*a**2*f*g*h**2 + 2*a*b*d*h**3 + 6*a*b
*e*g*h**2 + 6*a*b*f*g**2*h + 6*a*c*d*g*h**2 + 6*a*c*e*g**2*h + 2*a*c*f*g**3
 - 5*a*(2*a*b*f*h**3 + 2*a*c*e*h**3 + 6*a*c*f*g*h**2 - 7*a*(19*b*c*f*h**3/1
8 + c**2*e*h**3 + 3*c**2*f*g*h**2)/(8*c) + b**2*e*h**3 + 3*b**2*f*g*h**2 +
2*b*c*d*h**3 + 6*b*c*e*g*h**2 + 6*b*c*f*g**2*h - 13*b*(10*a*c*f*h**3/9 + b*
*2*f*h**3 + 2*b*c*e*h**3 + 6*b*c*f*g*h**2 - 15*b*(19*b*c*f*h**3/18 + c**2*e
*h**3 + 3*c**2*f*g*h**2)/(16*c) + c**2*d*h**3 + 3*c**2*e*g*h**2 + 3*c**2*f*
g**2*h)/(14*c) + 3*c**2*d*g*h**2 + 3*c**2*e*g**2*h + c**2*f*g**3)/(6*c) + 3
*b**2*d*g*h**2 + 3*b**2*e*g**2*h + b**2*f*g**3 + 6*b*c*d*g**2*h + 2*b*c*e*g
**3 - 9*b*(a**2*f*h**3 + 2*a*b*e*h**3 + 6*a*b*f*g*h**2 + 2*a*c*d*h**3 + 6*a
*c*e*g*h**2 + 6*a*c*f*g**2*h - 6*a*(10*a*c*f*h**3/9 + b**2*f*h**3 + 2*b*c*e
*h**3 + 6*b*c*f*g*h**2 - 15*b*(19*b*c*f*h**3/18 + c**2*e*h**3 + 3*c**2*f*g*
h**2)/(16*c) + c**2*d*h**3 + 3*c**2*e*g*h**2 + 3*c**2*f*g**2*h)/(7*c) + b**
2*d*h**3 + 3*b**2*e*g*h**2 + 3*b**2*f*g**2*h + 6*b*c*d*g*h**2 + 6*b*c*e*g**
2*h + 2*b*c*f*g**3 - 11*b*(2*a*b*f*h**3 + 2*a*c*e*h**3 + 6*a*c*f*g*h**2 - 7
*a*(19*b*c*f*h**3/18 + c**2*e*h**3 + 3*c**2*f*g*h**2)/(8*c) + b**2*e*h**3 +
 3*b**2*f*g*h**2 + 2*b*c*d*h**3 + 6*b*c*e*g*h**2 + 6*b*c*f*g**2*h - 13*b*(1
0*a*c*f*h**3/9 + b**2*f*h**3 + 2*b*c*e*h**3 + 6*b*c*f*g*h**2 - 15*b*(19*b*c
*f*h**3/18 + c**2*e*h**3 + 3*c**2*f*g*h**2)/(16*c) + c**2*d*h**3 + 3*c**2*e
*g*h**2 + 3*c**2*f*g**2*h)/(14*c) + 3*c**2*d*g*h**2 + 3*c**2*e*g**2*h + c**
2*f*g**3)/(12*c) + 3*c**2*d*g**2*h + c**2*e*g**3)/(10*c) + c**2*d*g**3)/(4*
c) + b**2*d*g**3 - 5*b*(a**2*d*h**3 + 3*a**2*e*g*h**2 + 3*a**2*f*g**2*h + 6
*a*b*d*g*h**2 + 6*a*b*e*g**2*h + 2*a*b*f*g**3 + 6*a*c*d*g**2*h + 2*a*c*e*g*
*3 - 4*a*(a**2*f*h**3 + 2*a*b*e*h**3 + 6*a*b*f*g*h**2 + 2*a*c*d*h**3 + 6*a*
c*e*g*h**2 + 6*a*c*f*g**2*h - 6*a*(10*a*c*f*h**3/9 + b**2*f*h**3 + 2*b*c*e*
h**3 + 6*b*c*f*g*h**2 - 15*b*(19*b*c*f*h**3/18 + c**2*e*h**3 + 3*c**2*f*g*h
**2)/(16*c) + c**2*d*h**3 + 3*c**2*e*g*h**2 + 3*c**2*f*g**2*h)/(7*c) + b**2
*d*h**3 + 3*b**2*e*g*h**2 + 3*b**2*f*g**2*h + 6*b*c*d*g*h**2 + 6*b*c*e*g**2
*h + 2*b*c*f*g**3 - 11*b*(2*a*b*f*h**3 + 2*a*c*e*h**3 + 6*a*c*f*g*h**2 - 7*
a*(19*b*c*f*h**3/18 + c**2*e*h**3 + 3*c**2*f*g*h**2)/(8*c) + b**2*e*h**3 +
3*b**2*f*g*h**2 + 2*b*c*d*h**3 + 6*b*c*e*g*h**2 + 6*b*c*f*g**2*h - 13*b*(10
*a*c*f*h**3/9 + b**2*f*h**3 + 2*b*c*e*h**3 + 6*b*c*f*g*h**2 - 15*b*(19*b*c*
f*h**3/18 + c**2*e*h**3 + 3*c**2*f*g*h**2)/(16*c) + c**2*d*h**3 + 3*c**2*e*
g*h**2 + 3*c**2*f*g**2*h)/(14*c) + 3*c**2*d*g*h**2 + 3*c**2*e*g**2*h + c**2
*f*g**3)/(12*c) + 3*c**2*d*g**2*h + c**2*e*g**3)/(5*c) + 3*b**2*d*g**2*h +
b**2*e*g**3 + 2*b*c*d*g**3 - 7*b*(a**2*e*h**3 + 3*a**2*f*g*h**2 + 2*a*b*d*h
**3 + 6*a*b*e*g*h**2 + 6*a*b*f*g**2*h + 6*a*c*d*g*h**2 + 6*a*c*e*g**2*h + 2
*a*c*f*g**3 - 5*a*(2*a*b*f*h**3 + 2*a*c*e*h**3 + 6*a*c*f*g*h**2 - 7*a*(19*b
*c*f*h**3/18 + c**2*e*h**3 + 3*c**2*f*g*h**2)/(8*c) + b**2*e*h**3 + 3*b**2*
f*g*h**2 + 2*b*c*d*h**3 + 6*b*c*e*g*h**2 + 6*b*c*f*g**2*h - 13*b*(10*a*c*f*
h**3/9 + b**2*f*h**3 + 2*b*c*e*h**3 + 6*b*c*f*g*h**2 - 15*b*(19*b*c*f*h**3/
18 + c**2*e*h**3 + 3*c**2*f*g*h**2)/(16*c) + c**2*d*h**3 + 3*c**2*e*g*h**2
+ 3*c**2*f*g**2*h)/(14*c) + 3*c**2*d*g*h**2 + 3*c**2*e*g**2*h + c**2*f*g**3
```

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)/(6*c) + 3*b**2*d*g*h**2 + 3*b**2*e*g**2*h + b**2*f*g**3 + 6*b*c*d*g**2*h
+ 2*b*c*e*g**3 - 9*b*(a**2*f*h**3 + 2*a*b*e*h**3 + 6*a*b*f*g*h**2 + 2*a*c*d
*h**3 + 6*a*c*e*g*h**2 + 6*a*c*f*g**2*h - 6*a*(10*a*c*f*h**3/9 + b**2*f*h**
3 + 2*b*c*e*h**3 + 6*b*c*f*g*h**2 - 15*b*(19*b*c*f*h**3/18 + c**2*e*h**3 +
3*c**2*f*g*h**2)/(16*c) + c**2*d*h**3 + 3*c**2*e*g*h**2 + 3*c**2*f*g**2*h)/
(7*c) + b**2*d*h**3 + 3*b**2*e*g*h**2 + 3*b**2*f*g**2*h + 6*b*c*d*g*h**2 +
6*b*c*e*g**2*h + 2*b*c*f*g**3 - 11*b*(2*a*b*f*h**3 + 2*a*c*e*h**3 + 6*a*c*f
*g*h**2 - 7*a*(19*b*c*f*h**3/18 + c**2*e*h**3 + 3*c**2*f*g*h**2)/(8*c) + b*
*2*e*h**3 + 3*b**2*f*g*h**2 + 2*b*c*d*h**3 + 6*b*c*e*g*h**2 + 6*b*c*f*g**2*
h - 13*b*(10*a*c*f*h**3/9 + b**2*f*h**3 + 2*b*c*e*h**3 + 6*b*c*f*g*h**2 - 1
5*b*(19*b*c*f*h**3/18 + c**2*e*h**3 + 3*c**2*f*g*h**2)/(16*c) + c**2*d*h**3
 + 3*c**2*e*g*h**2 + 3*c**2*f*g**2*h)/(14*c) + 3*c**2*d*g*h**2 + 3*c**2*e*g
**2*h + c**2*f*g**3)/(12*c) + 3*c**2*d*g**2*h + c**2*e*g**3)/(10*c) + c**2*
d*g**3)/(8*c))/(6*c))/(4*c))/c) + (a**2*d*g**3 - a*(3*a**2*d*g*h**2 + 3*a**
2*e*g**2*h + a**2*f*g**3 + 6*a*b*d*g**2*h + 2*a*b*e*g**3 + 2*a*c*d*g**3 - 3
*a*(a**2*e*h**3 + 3*a**2*f*g*h**2 + 2*a*b*d*h**3 + 6*a*b*e*g*h**2 + 6*a*b*f
*g**2*h + 6*a*c*d*g*h**2 + 6*a*c*e*g**2*h + 2*a*c*f*g**3 - 5*a*(2*a*b*f*h**
3 + 2*a*c*e*h**3 + 6*a*c*f*g*h**2 - 7*a*(19*b*c*f*h**3/18 + c**2*e*h**3 + 3
*c**2*f*g*h**2)/(8*c) + b**2*e*h**3 + 3*b**2*f*g*h**2 + 2*b*c*d*h**3 + 6*b*
c*e*g*h**2 + 6*b*c*f*g**2*h - 13*b*(10*a*c*f*h**3/9 + b**2*f*h**3 + 2*b*c*e
*h**3 + 6*b*c*f*g*h**2 - 15*b*(19*b*c*f*h**3/18 + c**2*e*h**3 + 3*c**2*f*g*
h**2)/(16*c) + c**2*d*h**3 + 3*c**2*e*g*h**2 + 3*c**2*f*g**2*h)/(14*c) + 3*
c**2*d*g*h**2 + 3*c**2*e*g**2*h + c**2*f*g**3)/(6*c) + 3*b**2*d*g*h**2 + 3*
b**2*e*g**2*h + b**2*f*g**3 + 6*b*c*d*g**2*h + 2*b*c*e*g**3 - 9*b*(a**2*f*h
**3 + 2*a*b*e*h**3 + 6*a*b*f*g*h**2 + 2*a*c*d*h**3 + 6*a*c*e*g*h**2 + 6*a*c
*f*g**2*h - 6*a*(10*a*c*f*h**3/9 + b**2*f*h**3 + 2*b*c*e*h**3 + 6*b*c*f*g*h
**2 - 15*b*(19*b*c*f*h**3/18 + c**2*e*h**3 + 3*c**2*f*g*h**2)/(16*c) + c**2
*d*h**3 + 3*c**2*e*g*h**2 + 3*c**2*f*g**2*h)/(7*c) + b**2*d*h**3 + 3*b**2*e
*g*h**2 + 3*b**2*f*g**2*h + 6*b*c*d*g*h**2 + 6*b*c*e*g**2*h + 2*b*c*f*g**3
- 11*b*(2*a*b*f*h**3 + 2*a*c*e*h**3 + 6*a*c*f*g*h**2 - 7*a*(19*b*c*f*h**3/1
8 + c**2*e*h**3 + 3*c**2*f*g*h**2)/(8*c) + b**2*e*h**3 + 3*b**2*f*g*h**2 +
2*b*c*d*h**3 + 6*b*c*e*g*h**2 + 6*b*c*f*g**2*h - 13*b*(10*a*c*f*h**3/9 + b*
*2*f*h**3 + 2*b*c*e*h**3 + 6*b*c*f*g*h**2 - 15*b*(19*b*c*f*h**3/18 + c**2*e
*h**3 + 3*c**2*f*g*h**2)/(16*c) + c**2*d*h**3 + 3*c**2*e*g*h**2 + 3*c**2*f*
g**2*h)/(14*c) + 3*c**2*d*g*h**2 + 3*c**2*e*g**2*h + c**2*f*g**3)/(12*c) +
3*c**2*d*g**2*h + c**2*e*g**3)/(10*c) + c**2*d*g**3)/(4*c) + b**2*d*g**3 -
5*b*(a**2*d*h**3 + 3*a**2*e*g*h**2 + 3*a**2*f*g**2*h + 6*a*b*d*g*h**2 + 6*a
*b*e*g**2*h + 2*a*b*f*g**3 + 6*a*c*d*g**2*h + 2*a*c*e*g**3 - 4*a*(a**2*f*h*
*3 + 2*a*b*e*h**3 + 6*a*b*f*g*h**2 + 2*a*c*d*h**3 + 6*a*c*e*g*h**2 + 6*a*c*
f*g**2*h - 6*a*(10*a*c*f*h**3/9 + b**2*f*h**3 + 2*b*c*e*h**3 + 6*b*c*f*g*h*
*2 - 15*b*(19*b*c*f*h**3/18 + c**2*e*h**3 + 3*c**2*f*g*h**2)/(16*c) + c**2*
d*h**3 + 3*c**2*e*g*h**2 + 3*c**2*f*g**2*h)/(7*c) + b**2*d*h**3 + 3*b**2*e*
g*h**2 + 3*b**2*f*g**2*h + 6*b*c*d*g*h**2 + 6*b*c*e*g**2*h + 2*b*c*f*g**3 -
 11*b*(2*a*b*f*h**3 + 2*a*c*e*h**3 + 6*a*c*f*g*h**2 - 7*a*(19*b*c*f*h**3/18
 + c**2*e*h**3 + 3*c**2*f*g*h**2)/(8*c) + b**2*e*h**3 + 3*b**2*f*g*h**2 + 2
```

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*b*c*d*h**3 + 6*b*c*e*g*h**2 + 6*b*c*f*g**2*h - 13*b*(10*a*c*f*h**3/9 + b**
2*f*h**3 + 2*b*c*e*h**3 + 6*b*c*f*g*h**2 - 15*b*(19*b*c*f*h**3/18 + c**2*e*
h**3 + 3*c**2*f*g*h**2)/(16*c) + c**2*d*h**3 + 3*c**2*e*g*h**2 + 3*c**2*f*g
**2*h)/(14*c) + 3*c**2*d*g*h**2 + 3*c**2*e*g**2*h + c**2*f*g**3)/(12*c) + 3
*c**2*d*g**2*h + c**2*e*g**3)/(5*c) + 3*b**2*d*g**2*h + b**2*e*g**3 + 2*b*c
*d*g**3 - 7*b*(a**2*e*h**3 + 3*a**2*f*g*h**2 + 2*a*b*d*h**3 + 6*a*b*e*g*h**
2 + 6*a*b*f*g**2*h + 6*a*c*d*g*h**2 + 6*a*c*e*g**2*h + 2*a*c*f*g**3 - 5*a*(
2*a*b*f*h**3 + 2*a*c*e*h**3 + 6*a*c*f*g*h**2 - 7*a*(19*b*c*f*h**3/18 + c**2
*e*h**3 + 3*c**2*f*g*h**2)/(8*c) + b**2*e*h**3 + 3*b**2*f*g*h**2 + 2*b*c*d*
h**3 + 6*b*c*e*g*h**2 + 6*b*c*f*g**2*h - 13*b*(10*a*c*f*h**3/9 + b**2*f*h**
3 + 2*b*c*e*h**3 + 6*b*c*f*g*h**2 - 15*b*(19*b*c*f*h**3/18 + c**2*e*h**3 +
3*c**2*f*g*h**2)/(16*c) + c**2*d*h**3 + 3*c**2*e*g*h**2 + 3*c**2*f*g**2*h)/
(14*c) + 3*c**2*d*g*h**2 + 3*c**2*e*g**2*h + c**2*f*g**3)/(6*c) + 3*b**2*d*
g*h**2 + 3*b**2*e*g**2*h + b**2*f*g**3 + 6*b*c*d*g**2*h + 2*b*c*e*g**3 - 9*
b*(a**2*f*h**3 + 2*a*b*e*h**3 + 6*a*b*f*g*h**2 + 2*a*c*d*h**3 + 6*a*c*e*g*h
**2 + 6*a*c*f*g**2*h - 6*a*(10*a*c*f*h**3/9 + b**2*f*h**3 + 2*b*c*e*h**3 +
6*b*c*f*g*h**2 - 15*b*(19*b*c*f*h**3/18 + c**2*e*h**3 + 3*c**2*f*g*h**2)/(1
6*c) + c**2*d*h**3 + 3*c**2*e*g*h**2 + 3*c**2*f*g**2*h)/(7*c) + b**2*d*h**3
 + 3*b**2*e*g*h**2 + 3*b**2*f*g**2*h + 6*b*c*d*g*h**2 + 6*b*c*e*g**2*h + 2*
b*c*f*g**3 - 11*b*(2*a*b*f*h**3 + 2*a*c*e*h**3 + 6*a*c*f*g*h**2 - 7*a*(19*b
*c*f*h**3/18 + c**2*e*h**3 + 3*c**2*f*g*h**2)/(8*c) + b**2*e*h**3 + 3*b**2*
f*g*h**2 + 2*b*c*d*h**3 + 6*b*c*e*g*h**2 + 6*b*c*f*g**2*h - 13*b*(10*a*c*f*
h**3/9 + b**2*f*h**3 + 2*b*c*e*h**3 + 6*b*c*f*g*h**2 - 15*b*(19*b*c*f*h**3/
18 + c**2*e*h**3 + 3*c**2*f*g*h**2)/(16*c) + c**2*d*h**3 + 3*c**2*e*g*h**2
+ 3*c**2*f*g**2*h)/(14*c) + 3*c**2*d*g*h**2 + 3*c**2*e*g**2*h + c**2*f*g**3
)/(12*c) + 3*c**2*d*g**2*h + c**2*e*g**3)/(10*c) + c**2*d*g**3)/(8*c))/(6*c
))/(2*c) - b*(3*a**2*d*g**2*h + a**2*e*g**3 + 2*a*b*d*g**3 - 2*a*(a**2*d*h*
*3 + 3*a**2*e*g*h**2 + 3*a**2*f*g**2*h + 6*a*b*d*g*h**2 + 6*a*b*e*g**2*h +
2*a*b*f*g**3 + 6*a*c*d*g**2*h + 2*a*c*e*g**3 - 4*a*(a**2*f*h**3 + 2*a*b*e*h
**3 + 6*a*b*f*g*h**2 + 2*a*c*d*h**3 + 6*a*c*e*g*h**2 + 6*a*c*f*g**2*h - 6*a
*(10*a*c*f*h**3/9 + b**2*f*h**3 + 2*b*c*e*h**3 + 6*b*c*f*g*h**2 - 15*b*(19*
b*c*f*h**3/18 + c**2*e*h**3 + 3*c**2*f*g*h**2)/(16*c) + c**2*d*h**3 + 3*c**
2*e*g*h**2 + 3*c**2*f*g**2*h)/(7*c) + b**2*d*h**3 + 3*b**2*e*g*h**2 + 3*b**
2*f*g**2*h + 6*b*c*d*g*h**2 + 6*b*c*e*g**2*h + 2*b*c*f*g**3 - 11*b*(2*a*b*f
*h**3 + 2*a*c*e*h**3 + 6*a*c*f*g*h**2 - 7*a*(19*b*c*f*h**3/18 + c**2*e*h**3
 + 3*c**2*f*g*h**2)/(8*c) + b**2*e*h**3 + 3*b**2*f*g*h**2 + 2*b*c*d*h**3 +
6*b*c*e*g*h**2 + 6*b*c*f*g**2*h - 13*b*(10*a*c*f*h**3/9 + b**2*f*h**3 + 2*b
*c*e*h**3 + 6*b*c*f*g*h**2 - 15*b*(19*b*c*f*h**3/18 + c**2*e*h**3 + 3*c**2*
f*g*h**2)/(16*c) + c**2*d*h**3 + 3*c**2*e*g*h**2 + 3*c**2*f*g**2*h)/(14*c)
+ 3*c**2*d*g*h**2 + 3*c**2*e*g**2*h + c**2*f*g**3)/(12*c) + 3*c**2*d*g**2*h
 + c**2*e*g**3)/(5*c) + 3*b**2*d*g**2*h + b**2*e*g**3 + 2*b*c*d*g**3 - 7*b*
(a**2*e*h**3 + 3*a**2*f*g*h**2 + 2*a*b*d*h**3 + 6*a*b*e*g*h**2 + 6*a*b*f*g*
*2*h + 6*a*c*d*g*h**2 + 6*a*c*e*g**2*h + 2*a*c*f*g**3 - 5*a*(2*a*b*f*h**3 +
 2*a*c*e*h**3 + 6*a*c*f*g*h**2 - 7*a*(19*b*c*f*h**3/18 + c**2*e*h**3 + 3*c*
*2*f*g*h**2)/(8*c) + b**2*e*h**3 + 3*b**2*f*g*h**2 + 2*b*c*d*h**3 + 6*b*c*e
```

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*g*h**2 + 6*b*c*f*g**2*h - 13*b*(10*a*c*f*h**3/9 + b**2*f*h**3 + 2*b*c*e*h*
*3 + 6*b*c*f*g*h**2 - 15*b*(19*b*c*f*h**3/18 + c**2*e*h**3 + 3*c**2*f*g*h**
2)/(16*c) + c**2*d*h**3 + 3*c**2*e*g*h**2 + 3*c**2*f*g**2*h)/(14*c) + 3*c**
2*d*g*h**2 + 3*c**2*e*g**2*h + c**2*f*g**3)/(6*c) + 3*b**2*d*g*h**2 + 3*b**
2*e*g**2*h + b**2*f*g**3 + 6*b*c*d*g**2*h + 2*b*c*e*g**3 - 9*b*(a**2*f*h**3
 + 2*a*b*e*h**3 + 6*a*b*f*g*h**2 + 2*a*c*d*h**3 + 6*a*c*e*g*h**2 + 6*a*c*f*
g**2*h - 6*a*(10*a*c*f*h**3/9 + b**2*f*h**3 + 2*b*c*e*h**3 + 6*b*c*f*g*h**2
 - 15*b*(19*b*c*f*h**3/18 + c**2*e*h**3 + 3*c**2*f*g*h**2)/(16*c) + c**2*d*
h**3 + 3*c**2*e*g*h**2 + 3*c**2*f*g**2*h)/(7*c) + b**2*d*h**3 + 3*b**2*e*g*
h**2 + 3*b**2*f*g**2*h + 6*b*c*d*g*h**2 + 6*b*c*e*g**2*h + 2*b*c*f*g**3 - 1
1*b*(2*a*b*f*h**3 + 2*a*c*e*h**3 + 6*a*c*f*g*h**2 - 7*a*(19*b*c*f*h**3/18 +
 c**2*e*h**3 + 3*c**2*f*g*h**2)/(8*c) + b**2*e*h**3 + 3*b**2*f*g*h**2 + 2*b
*c*d*h**3 + 6*b*c*e*g*h**2 + 6*b*c*f*g**2*h - 13*b*(10*a*c*f*h**3/9 + b**2*
f*h**3 + 2*b*c*e*h**3 + 6*b*c*f*g*h**2 - 15*b*(19*b*c*f*h**3/18 + c**2*e*h*
*3 + 3*c**2*f*g*h**2)/(16*c) + c**2*d*h**3 + 3*c**2*e*g*h**2 + 3*c**2*f*g**
2*h)/(14*c) + 3*c**2*d*g*h**2 + 3*c**2*e*g**2*h + c**2*f*g**3)/(12*c) + 3*c
**2*d*g**2*h + c**2*e*g**3)/(10*c) + c**2*d*g**3)/(8*c))/(3*c) - 3*b*(3*a**
2*d*g*h**2 + 3*a**2*e*g**2*h + a**2*f*g**3 + 6*a*b*d*g**2*h + 2*a*b*e*g**3
+ 2*a*c*d*g**3 - 3*a*(a**2*e*h**3 + 3*a**2*f*g*h**2 + 2*a*b*d*h**3 + 6*a*b*
e*g*h**2 + 6*a*b*f*g**2*h + 6*a*c*d*g*h**2 + 6*a*c*e*g**2*h + 2*a*c*f*g**3
- 5*a*(2*a*b*f*h**3 + 2*a*c*e*h**3 + 6*a*c*f*g*h**2 - 7*a*(19*b*c*f*h**3/18
 + c**2*e*h**3 + 3*c**2*f*g*h**2)/(8*c) + b**2*e*h**3 + 3*b**2*f*g*h**2 + 2
*b*c*d*h**3 + 6*b*c*e*g*h**2 + 6*b*c*f*g**2*h - 13*b*(10*a*c*f*h**3/9 + b**
2*f*h**3 + 2*b*c*e*h**3 + 6*b*c*f*g*h**2 - 15*b*(19*b*c*f*h**3/18 + c**2*e*
h**3 + 3*c**2*f*g*h**2)/(16*c) + c**2*d*h**3 + 3*c**2*e*g*h**2 + 3*c**2*f*g
**2*h)/(14*c) + 3*c**2*d*g*h**2 + 3*c**2*e*g**2*h + c**2*f*g**3)/(6*c) + 3*
b**2*d*g*h**2 + 3*b**2*e*g**2*h + b**2*f*g**3 + 6*b*c*d*g**2*h + 2*b*c*e*g*
*3 - 9*b*(a**2*f*h**3 + 2*a*b*e*h**3 + 6*a*b*f*g*h**2 + 2*a*c*d*h**3 + 6*a*
c*e*g*h**2 + 6*a*c*f*g**2*h - 6*a*(10*a*c*f*h**3/9 + b**2*f*h**3 + 2*b*c*e*
h**3 + 6*b*c*f*g*h**2 - 15*b*(19*b*c*f*h**3/18 + c**2*e*h**3 + 3*c**2*f*g*h
**2)/(16*c) + c**2*d*h**3 + 3*c**2*e*g*h**2 + 3*c**2*f*g**2*h)/(7*c) + b**2
*d*h**3 + 3*b**2*e*g*h**2 + 3*b**2*f*g**2*h + 6*b*c*d*g*h**2 + 6*b*c*e*g**2
*h + 2*b*c*f*g**3 - 11*b*(2*a*b*f*h**3 + 2*a*c*e*h**3 + 6*a*c*f*g*h**2 - 7*
a*(19*b*c*f*h**3/18 + c**2*e*h**3 + 3*c**2*f*g*h**2)/(8*c) + b**2*e*h**3 +
3*b**2*f*g*h**2 + 2*b*c*d*h**3 + 6*b*c*e*g*h**2 + 6*b*c*f*g**2*h - 13*b*(10
*a*c*f*h**3/9 + b**2*f*h**3 + 2*b*c*e*h**3 + 6*b*c*f*g*h**2 - 15*b*(19*b*c*
f*h**3/18 + c**2*e*h**3 + 3*c**2*f*g*h**2)/(16*c) + c**2*d*h**3 + 3*c**2*e*
g*h**2 + 3*c**2*f*g**2*h)/(14*c) + 3*c**2*d*g*h**2 + 3*c**2*e*g**2*h + c**2
*f*g**3)/(12*c) + 3*c**2*d*g**2*h + c**2*e*g**3)/(10*c) + c**2*d*g**3)/(4*c
) + b**2*d*g**3 - 5*b*(a**2*d*h**3 + 3*a**2*e*g*h**2 + 3*a**2*f*g**2*h + 6*
a*b*d*g*h**2 + 6*a*b*e*g**2*h + 2*a*b*f*g**3 + 6*a*c*d*g**2*h + 2*a*c*e*g**
3 - 4*a*(a**2*f*h**3 + 2*a*b*e*h**3 + 6*a*b*f*g*h**2 + 2*a*c*d*h**3 + 6*a*c
*e*g*h**2 + 6*a*c*f*g**2*h - 6*a*(10*a*c*f*h**3/9 + b**2*f*h**3 + 2*b*c*e*h
**3 + 6*b*c*f*g*h**2 - 15*b*(19*b*c*f*h**3/18 + c**2*e*h**3 + 3*c**2*f*g*h*
*2)/(16*c) + c**2*d*h**3 + 3*c**2*e*g*h**2 + 3*c**2*f*g**2*h)/(7*c) + b**2*
```

```
d*h**3 + 3*b**2*e*g*h**2 + 3*b**2*f*g**2*h + 6*b*c*d*g*h**2 + 6*b*c*e*g**2*
h + 2*b*c*f*g**3 - 11*b*(2*a*b*f*h**3 + 2*a*c*e*h**3 + 6*a*c*f*g*h**2 - 7*a
*(19*b*c*f*h**3/18 + c**2*e*h**3 + 3*c**2*f*g*h**2)/(8*c) + b**2*e*h**3 + 3
*b**2*f*g*h**2 + 2*b*c*d*h**3 + 6*b*c*e*g*h**2 + 6*b*c*f*g**2*h - 13*b*(10*
a*c*f*h**3/9 + b**2*f*h**3 + 2*b*c*e*h**3 + 6*b*c*f*g*h**2 - 15*b*(19*b*c*f
*h**3/18 + c**2*e*h**3 + 3*c**2*f*g*h**2)/(16*c) + c**2*d*h**3 + 3*c**2*e*g
*h**2 + 3*c**2*f*g**2*h)/(14*c) + 3*c**2*d*g*h**2 + 3*c**2*e*g**2*h + c**2*
f*g**3)/(12*c) + 3*c**2*d*g**2*h + c**2*e*g**3)/(5*c) + 3*b**2*d*g**2*h + b
**2*e*g**3 + 2*b*c*d*g**3 - 7*b*(a**2*e*h**3 + 3*a**2*f*g*h**2 + 2*a*b*d*h*
*3 + 6*a*b*e*g*h**2 + 6*a*b*f*g**2*h + 6*a*c*d*g*h**2 + 6*a*c*e*g**2*h + 2*
a*c*f*g**3 - 5*a*(2*a*b*f*h**3 + 2*a*c*e*h**3 + 6*a*c*f*g*h**2 - 7*a*(19*b*
c*f*h**3/18 + c**2*e*h**3 + 3*c**2*f*g*h**2)/(8*c) + b**2*e*h**3 + 3*b**2*f
*g*h**2 + 2*b*c*d*h**3 + 6*b*c*e*g*h**2 + 6*b*c*f*g**2*h - 13*b*(10*a*c*f*h
**3/9 + b**2*f*h**3 + 2*b*c*e*h**3 + 6*b*c*f*g*h**2 - 15*b*(19*b*c*f*h**3/1
8 + c**2*e*h**3 + 3*c**2*f*g*h**2)/(16*c) + c**2*d*h**3 + 3*c**2*e*g*h**2 +
 3*c**2*f*g**2*h)/(14*c) + 3*c**2*d*g*h**2 + 3*c**2*e*g**2*h + c**2*f*g**3)
/(6*c) + 3*b**2*d*g*h**2 + 3*b**2*e*g**2*h + b**2*f*g**3 + 6*b*c*d*g**2*h +
 2*b*c*e*g**3 - 9*b*(a**2*f*h**3 + 2*a*b*e*h**3 + 6*a*b*f*g*h**2 + 2*a*c*d*
h**3 + 6*a*c*e*g*h**2 + 6*a*c*f*g**2*h - 6*a*(10*a*c*f*h**3/9 + b**2*f*h**3
 + 2*b*c*e*h**3 + 6*b*c*f*g*h**2 - 15*b*(19*b*c*f*h**3/18 + c**2*e*h**3 + 3
*c**2*f*g*h**2)/(16*c) + c**2*d*h**3 + 3*c**2*e*g*h**2 + 3*c**2*f*g**2*h)/(
7*c) + b**2*d*h**3 + 3*b**2*e*g*h**2 + 3*b**2*f*g**2*h + 6*b*c*d*g*h**2 + 6
*b*c*e*g**2*h + 2*b*c*f*g**3 - 11*b*(2*a*b*f*h**3 + 2*a*c*e*h**3 + 6*a*c*f*
g*h**2 - 7*a*(19*b*c*f*h**3/18 + c**2*e*h**3 + 3*c**2*f*g*h**2)/(8*c) + b**
2*e*h**3 + 3*b**2*f*g*h**2 + 2*b*c*d*h**3 + 6*b*c*e*g*h**2 + 6*b*c*f*g**2*h
 - 13*b*(10*a*c*f*h**3/9 + b**2*f*h**3 + 2*b*c*e*h**3 + 6*b*c*f*g*h**2 - 15
*b*(19*b*c*f*h**3/18 + c**2*e*h**3 + 3*c**2*f*g*h**2)/(16*c) + c**2*d*h**3
+ 3*c**2*e*g*h**2 + 3*c**2*f*g**2*h)/(14*c) + 3*c**2*d*g*h**2 + 3*c**2*e*g*
*2*h + c**2*f*g**3)/(12*c) + 3*c**2*d*g**2*h + c**2*e*g**3)/(10*c) + c**2*d
*g**3)/(8*c))/(6*c))/(4*c))/(2*c))*Piecewise((log(b + 2*sqrt(c)*sqrt(a + b*
x + c*x**2) + 2*c*x)/sqrt(c), Ne(a - b**2/(4*c), 0)), ((b/(2*c) + x)*log(b/
(2*c) + x)/sqrt(c*(b/(2*c) + x)**2), True)), Ne(c, 0)), (2*(f*h**3*(a + b*x
)**(15/2)/(15*b**5) + (a + b*x)**(13/2)*(-5*a*f*h**3 + b*e*h**3 + 3*b*f*g*h
**2)/(13*b**5) + (a + b*x)**(11/2)*(10*a**2*f*h**3 - 4*a*b*e*h**3 - 12*a*b*
f*g*h**2 + b**2*d*h**3 + 3*b**2*e*g*h**2 + 3*b**2*f*g**2*h)/(11*b**5) + (a
+ b*x)**(9/2)*(-10*a**3*f*h**3 + 6*a**2*b*e*h**3 + 18*a**2*b*f*g*h**2 - 3*a
*b**2*d*h**3 - 9*a*b**2*e*g*h**2 - 9*a*b**2*f*g**2*h + 3*b**3*d*g*h**2 + 3*
b**3*e*g**2*h + b**3*f*g**3)/(9*b**5) + (a + b*x)**(7/2)*(5*a**4*f*h**3 - 4
*a**3*b*e*h**3 - 12*a**3*b*f*g*h**2 + 3*a**2*b**2*d*h**3 + 9*a**2*b**2*e*g*
h**2 + 9*a**2*b**2*f*g**2*h - 6*a*b**3*d*g*h**2 - 6*a*b**3*e*g**2*h - 2*a*b
**3*f*g**3 + 3*b**4*d*g**2*h + b**4*e*g**3)/(7*b**5) + (a + b*x)**(5/2)*(-a
**5*f*h**3 + a**4*b*e*h**3 + 3*a**4*b*f*g*h**2 - a**3*b**2*d*h**3 - 3*a**3*
b**2*e*g*h**2 - 3*a**3*b**2*f*g**2*h + 3*a**2*b**3*d*g*h**2 + 3*a**2*b**3*e
*g**2*h + a**2*b**3*f*g**3 - 3*a*b**4*d*g**2*h - a*b**4*e*g**3 + b**5*d*g**
3)/(5*b**5))/b, Ne(b, 0)), (a**(3/2)*(d*g**3*x + f*h**3*x**6/6 + x**5*(e*h*
```

```
*3 + 3*f*g*h**2)/5 + x**4*(d*h**3 + 3*e*g*h**2 + 3*f*g**2*h)/4 + x**3*(3*d*
g*h**2 + 3*e*g**2*h + f*g**3)/3 + x**2*(3*d*g**2*h + e*g**3)/2), True))
```

Maxima [F(-2)]

Exception generated.

$$\int (g+hx)^3 \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right) \, dx = \text{Exception raised: ValueError}$$

```
[In] integrate((h*x+g)^3*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2902 vs. $2(1135) = 2270$.

Time = 0.33 (sec) , antiderivative size = 2902, normalized size of antiderivative = 2.48

$$\int (g+hx)^3 \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right) \, dx = \text{Too large to display}$$

```
[In] integrate((h*x+g)^3*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] 1/10321920*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*(4*(14*(16*c*f*h^3*x + (54
*c^9*f*g*h^2 + 18*c^9*e*h^3 + 19*b*c^8*f*h^3)/c^8)*x + (864*c^9*f*g^2*h + 8
64*c^9*e*g*h^2 + 918*b*c^8*f*g*h^2 + 288*c^9*d*h^3 + 306*b*c^8*e*h^3 + 3*b^
2*c^7*f*h^3 + 320*a*c^8*f*h^3)/c^8)*x + (1344*c^9*f*g^3 + 4032*c^9*e*g^2*h
+ 4320*b*c^8*f*g^2*h + 4032*c^9*d*g*h^2 + 4320*b*c^8*e*g*h^2 + 54*b^2*c^7*f
*g*h^2 + 4536*a*c^8*f*g*h^2 + 1440*b*c^8*d*h^3 + 18*b^2*c^7*e*h^3 + 1512*a*
c^8*e*h^3 - 13*b^3*c^6*f*h^3 + 60*a*b*c^7*f*h^3)/c^8)*x + (16128*c^9*e*g^3
+ 17472*b*c^8*f*g^3 + 48384*c^9*d*g^2*h + 52416*b*c^8*e*g^2*h + 864*b^2*c^7
*f*g^2*h + 55296*a*c^8*f*g^2*h + 52416*b*c^8*d*g*h^2 + 864*b^2*c^7*e*g*h^2
+ 55296*a*c^8*e*g*h^2 - 594*b^3*c^6*f*g*h^2 + 2808*a*b*c^7*f*g*h^2 + 288*b^
2*c^7*d*h^3 + 18432*a*c^8*d*h^3 - 198*b^3*c^6*e*h^3 + 936*a*b*c^7*e*h^3 + 1
43*b^4*c^5*f*h^3 - 804*a*b^2*c^6*f*h^3 + 768*a^2*c^7*f*h^3)/c^8)*x + (16128
0*c^9*d*g^3 + 177408*b*c^8*e*g^3 + 4032*b^2*c^7*f*g^3 + 188160*a*c^8*f*g^3
+ 532224*b*c^8*d*g^2*h + 12096*b^2*c^7*e*g^2*h + 564480*a*c^8*e*g^2*h - 777
6*b^3*c^6*f*g^2*h + 38016*a*b*c^7*f*g^2*h + 12096*b^2*c^7*d*g*h^2 + 564480*
a*c^8*d*g*h^2 - 7776*b^3*c^6*e*g*h^2 + 38016*a*b*c^7*e*g*h^2 + 5346*b^4*c^5
*f*g*h^2 - 30672*a*b^2*c^6*f*g*h^2 + 30240*a^2*c^7*f*g*h^2 - 2592*b^3*c^6*d
```

```
*h^3 + 12672*a*b*c^7*d*h^3 + 1782*b^4*c^5*e*h^3 - 10224*a*b^2*c^6*e*h^3 + 1
0080*a^2*c^7*e*h^3 - 1287*b^5*c^4*f*h^3 + 8536*a*b^3*c^5*f*h^3 - 12912*a^2*
b*c^6*f*h^3)/c^8)*x + (483840*b*c^8*d*g^3 + 16128*b^2*c^7*e*g^3 + 516096*a*
c^8*e*g^3 - 9408*b^3*c^6*f*g^3 + 48384*a*b*c^7*f*g^3 + 48384*b^2*c^7*d*g^2*
h + 1548288*a*c^8*d*g^2*h - 28224*b^3*c^6*e*g^2*h + 145152*a*b*c^7*e*g^2*h
+ 18144*b^4*c^5*f*g^2*h - 107136*a*b^2*c^6*f*g^2*h + 110592*a^2*c^7*f*g^2*h
 - 28224*b^3*c^6*d*g*h^2 + 145152*a*b*c^7*d*g*h^2 + 18144*b^4*c^5*e*g*h^2 -
 107136*a*b^2*c^6*e*g*h^2 + 110592*a^2*c^7*e*g*h^2 - 12474*b^5*c^4*f*g*h^2
+ 84240*a*b^3*c^5*f*g*h^2 - 130464*a^2*b*c^6*f*g*h^2 + 6048*b^4*c^5*d*h^3 -
 35712*a*b^2*c^6*d*h^3 + 36864*a^2*c^7*d*h^3 - 4158*b^5*c^4*e*h^3 + 28080*a
*b^3*c^5*e*h^3 - 43488*a^2*b*c^6*e*h^3 + 3003*b^6*c^3*f*h^3 - 22968*a*b^4*c
^4*f*h^3 + 47280*a^2*b^2*c^5*f*h^3 - 16384*a^3*c^6*f*h^3)/c^8)*x + (161280*
b^2*c^7*d*g^3 + 3225600*a*c^8*d*g^3 - 80640*b^3*c^6*e*g^3 + 451584*a*b*c^7*
e*g^3 + 47040*b^4*c^5*f*g^3 - 290304*a*b^2*c^6*f*g^3 + 322560*a^2*c^7*f*g^3
 - 241920*b^3*c^6*d*g^2*h + 1354752*a*b*c^7*d*g^2*h + 141120*b^4*c^5*e*g^2*
h - 870912*a*b^2*c^6*e*g^2*h + 967680*a^2*c^7*e*g^2*h - 90720*b^5*c^4*f*g^2
*h + 628992*a*b^3*c^5*f*g^2*h - 1009152*a^2*b*c^6*f*g^2*h + 141120*b^4*c^5*
d*g*h^2 - 870912*a*b^2*c^6*d*g*h^2 + 967680*a^2*c^7*d*g*h^2 - 90720*b^5*c^4
*e*g*h^2 + 628992*a*b^3*c^5*e*g*h^2 - 1009152*a^2*b*c^6*e*g*h^2 + 62370*b^6
*c^3*f*g*h^2 - 485352*a*b^4*c^4*f*g*h^2 + 1020384*a^2*b^2*c^5*f*g*h^2 - 362
880*a^3*c^6*f*g*h^2 - 30240*b^5*c^4*d*h^3 + 209664*a*b^3*c^5*d*h^3 - 336384
*a^2*b*c^6*d*h^3 + 20790*b^6*c^3*e*h^3 - 161784*a*b^4*c^4*e*h^3 + 340128*a^
2*b^2*c^5*e*h^3 - 120960*a^3*c^6*e*h^3 - 15015*b^7*c^2*f*h^3 + 130284*a*b^5
*c^3*f*h^3 - 338832*a^2*b^3*c^4*f*h^3 + 236864*a^3*b*c^5*f*h^3)/c^8)*x - (4
83840*b^3*c^6*d*g^3 - 3225600*a*b*c^7*d*g^3 - 241920*b^4*c^5*e*g^3 + 161280
0*a*b^2*c^6*e*g^3 - 2064384*a^2*c^7*e*g^3 + 141120*b^5*c^4*f*g^3 - 1021440*
a*b^3*c^5*f*g^3 + 1741824*a^2*b*c^6*f*g^3 - 725760*b^4*c^5*d*g^2*h + 483840
0*a*b^2*c^6*d*g^2*h - 6193152*a^2*c^7*d*g^2*h + 423360*b^5*c^4*e*g^2*h - 30
64320*a*b^3*c^5*e*g^2*h + 5225472*a^2*b*c^6*e*g^2*h - 272160*b^6*c^3*f*g^2*
h + 2177280*a*b^4*c^4*f*g^2*h - 4741632*a^2*b^2*c^5*f*g^2*h + 1769472*a^3*c
^6*f*g^2*h + 423360*b^5*c^4*d*g*h^2 - 3064320*a*b^3*c^5*d*g*h^2 + 5225472*a
^2*b*c^6*d*g*h^2 - 272160*b^6*c^3*e*g*h^2 + 2177280*a*b^4*c^4*e*g*h^2 - 474
1632*a^2*b^2*c^5*e*g*h^2 + 1769472*a^3*c^6*e*g*h^2 + 187110*b^7*c^2*f*g*h^2
 - 1655640*a*b^5*c^3*f*g*h^2 + 4408992*a^2*b^3*c^4*f*g*h^2 - 3176064*a^3*b*
c^5*f*g*h^2 - 90720*b^6*c^3*d*h^3 + 725760*a*b^4*c^4*d*h^3 - 1580544*a^2*b^
2*c^5*d*h^3 + 589824*a^3*c^6*d*h^3 + 62370*b^7*c^2*e*h^3 - 551880*a*b^5*c^3
*e*h^3 + 1469664*a^2*b^3*c^4*e*h^3 - 1058688*a^3*b*c^5*e*h^3 - 45045*b^8*c*
f*h^3 + 438900*a*b^6*c^2*f*h^3 - 1383984*a^2*b^4*c^3*f*h^3 + 1467072*a^3*b^
2*c^4*f*h^3 - 262144*a^4*c^5*f*h^3)/c^8) - 1/65536*(1536*b^4*c^5*d*g^3 - 12
288*a*b^2*c^6*d*g^3 + 24576*a^2*c^7*d*g^3 - 768*b^5*c^4*e*g^3 + 6144*a*b^3*
c^5*e*g^3 - 12288*a^2*b*c^6*e*g^3 + 448*b^6*c^3*f*g^3 - 3840*a*b^4*c^4*f*g^
3 + 9216*a^2*b^2*c^5*f*g^3 - 4096*a^3*c^6*f*g^3 - 2304*b^5*c^4*d*g^2*h + 18
432*a*b^3*c^5*d*g^2*h - 36864*a^2*b*c^6*d*g^2*h + 1344*b^6*c^3*e*g^2*h - 11
520*a*b^4*c^4*e*g^2*h + 27648*a^2*b^2*c^5*e*g^2*h - 12288*a^3*c^6*e*g^2*h -
 864*b^7*c^2*f*g^2*h + 8064*a*b^5*c^3*f*g^2*h - 23040*a^2*b^3*c^4*f*g^2*h +
```

```
 18432*a^3*b*c^5*f*g^2*h + 1344*b^6*c^3*d*g*h^2 - 11520*a*b^4*c^4*d*g*h^2 +
 27648*a^2*b^2*c^5*d*g*h^2 - 12288*a^3*c^6*d*g*h^2 - 864*b^7*c^2*e*g*h^2 +
8064*a*b^5*c^3*e*g*h^2 - 23040*a^2*b^3*c^4*e*g*h^2 + 18432*a^3*b*c^5*e*g*h^
2 + 594*b^8*c*f*g*h^2 - 6048*a*b^6*c^2*f*g*h^2 + 20160*a^2*b^4*c^3*f*g*h^2
- 23040*a^3*b^2*c^4*f*g*h^2 + 4608*a^4*c^5*f*g*h^2 - 288*b^7*c^2*d*h^3 + 26
88*a*b^5*c^3*d*h^3 - 7680*a^2*b^3*c^4*d*h^3 + 6144*a^3*b*c^5*d*h^3 + 198*b^
8*c*e*h^3 - 2016*a*b^6*c^2*e*h^3 + 6720*a^2*b^4*c^3*e*h^3 - 7680*a^3*b^2*c^
4*e*h^3 + 1536*a^4*c^5*e*h^3 - 143*b^9*f*h^3 + 1584*a*b^7*c*f*h^3 - 6048*a^
2*b^5*c^2*f*h^3 + 8960*a^3*b^3*c^3*f*h^3 - 3840*a^4*b*c^4*f*h^3)*log(abs(2*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(15/2)
```

Mupad [F(-1)]

Timed out.

$$\int (g+hx)^3 \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right)\, dx = \int (g+h\,x)^3\, \left(c\,x^2+b\,x+a\right)^{3/2}\, \left(f\,x^2+e\,x+d\right)\, dx$$

```
[In] int((g + h*x)^3*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x)
[Out] int((g + h*x)^3*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2), x)
```

3.197 $\int (g+hx)^2 \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right) dx$

Optimal result

Integrand size = 32, antiderivative size = 753

$$\int (g+hx)^2 \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right) dx =$$

$$-\frac{(b^2-4ac)\left(768c^4dg^2+99b^4fh^2-72b^2ch(4bfg+2beh+3afh)-128c^3(3bg(eg+2dh)+a(fg^2+2egh+\right.}{16384c^6}$$

$$+\frac{(768c^4dg^2+99b^4fh^2-72b^2ch(4bfg+2beh+3afh)-128c^3(3bg(eg+2dh)+a(fg^2+2egh+dh^2))+16c}{6144c^5}$$

$$-\frac{(10cfg-16ceh+11bfh)(g+hx)^2\left(a+bx+cx^2\right)^{5/2}}{112c^2h}+\frac{f(g+hx)^3\left(a+bx+cx^2\right)^{5/2}}{8ch}$$

$$-\frac{(693b^3fh^3+96c^3g(5fg^2-8h(eg+7dh))-36bch^2(31afh+28b(2fg+eh))+8c^2h(96ah(2fg+eh)+b(3}{}$$

$$+\frac{\left(b^2-4ac\right)^2\left(768c^4dg^2+99b^4fh^2-72b^2ch(4bfg+2beh+3afh)-128c^3(3bg(eg+2dh)+a(fg^2+2egh+\right.}{32768c^{13/2}}$$

```
[Out] 1/6144*(768*c^4*d*g^2+99*b^4*f*h^2-72*b^2*c*h*(3*a*f*h+2*b*e*h+4*b*f*g)-128
*c^3*(3*b*g*(2*d*h+e*g)+a*(d*h^2+2*e*g*h+f*g^2))+16*c^2*(3*a^2*f*h^2+12*a*b
*h*(e*h+2*f*g)+14*b^2*(d*h^2+2*e*g*h+f*g^2)))*(2*c*x+b)*(c*x^2+b*x+a)^(3/2)
/c^5-1/112*(11*b*f*h-16*c*e*h+10*c*f*g)*(h*x+g)^2*(c*x^2+b*x+a)^(5/2)/c^2/h
+1/8*f*(h*x+g)^3*(c*x^2+b*x+a)^(5/2)/c/h-1/13440*(693*b^3*f*h^3+96*c^3*g*(5
*f*g^2-8*h*(7*d*h+e*g))-36*b*c*h^2*(31*a*f*h+28*b*(e*h+2*f*g))+8*c^2*h*(96*
a*h*(e*h+2*f*g)+b*(31*f*g^2+196*h*(d*h+2*e*g)))-10*c*h*(99*b^2*f*h^2-8*c^2*
(5*f*g^2-4*h*(7*d*h+2*e*g))-12*c*h*(7*a*f*h+2*b*(6*e*h+f*g)))*x)*(c*x^2+b*x
+a)^(5/2)/c^4/h+1/32768*(-4*a*c+b^2)^2*(768*c^4*d*g^2+99*b^4*f*h^2-72*b^2*c
*h*(3*a*f*h+2*b*e*h+4*b*f*g)-128*c^3*(3*b*g*(2*d*h+e*g)+a*(d*h^2+2*e*g*h+f*
g^2))+16*c^2*(3*a^2*f*h^2+12*a*b*h*(e*h+2*f*g)+14*b^2*(d*h^2+2*e*g*h+f*g^2)
))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(13/2)-1/16384*(-4*
a*c+b^2)*(768*c^4*d*g^2+99*b^4*f*h^2-72*b^2*c*h*(3*a*f*h+2*b*e*h+4*b*f*g)-1
```

```
28*c^3*(3*b*g*(2*d*h+e*g)+a*(d*h^2+2*e*g*h+f*g^2))+16*c^2*(3*a^2*f*h^2+12*a
*b*h*(e*h+2*f*g)+14*b^2*(d*h^2+2*e*g*h+f*g^2)))*(2*c*x+b)*(c*x^2+b*x+a)^(1/
2)/c^6
```

Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 749, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1667, 846, 793, 626, 635, 212}

$$\int (g+hx)^2 \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right) dx = \frac{\left(b^2-4ac\right)^2 \operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right) \left(16c^2(3a^2fh^2+12abh(eh+2fg)+14b^2(h(dh+2eg)+fg^2\right.}{}$$

$$-\frac{\left(b^2-4ac\right)(b+2cx)\sqrt{a+bx+cx^2}(16c^2(3a^2fh^2+12abh(eh+2fg)+14b^2(h(dh+2eg)+fg^2))-72b^2ch}{16384c^6}$$

$$+\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}(16c^2(3a^2fh^2+12abh(eh+2fg)+14b^2(h(dh+2eg)+fg^2))-72b^2ch(3afh}{6144c^5}$$

$$-\frac{\left(a+bx+cx^2\right)^{5/2}(-10chx(-12ch(7afh+2b(6eh+fg))+99b^2fh^2-8c^2(5fg^2-4h(7dh+2eg)))+8c^2}{}$$

$$-\frac{(g+hx)^2\left(a+bx+cx^2\right)^{5/2}(11bfh-16ceh+10cfg)}{112c^2h}+\frac{f(g+hx)^3\left(a+bx+cx^2\right)^{5/2}}{8ch}$$

```
[In] Int[(g + h*x)^2*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

```
[Out] -1/16384*((b^2 - 4*a*c)*(768*c^4*d*g^2 + 99*b^4*f*h^2 - 72*b^2*c*h*(4*b*f*g
 + 2*b*e*h + 3*a*f*h) - 128*c^3*(a*f*g^2 + a*h*(2*e*g + d*h) + 3*b*g*(e*g +
 2*d*h)) + 16*c^2*(3*a^2*f*h^2 + 12*a*b*h*(2*f*g + e*h) + 14*b^2*(f*g^2 + h
*(2*e*g + d*h))))*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/c^6 + ((768*c^4*d*g^2
+ 99*b^4*f*h^2 - 72*b^2*c*h*(4*b*f*g + 2*b*e*h + 3*a*f*h) - 128*c^3*(a*f*g^
2 + a*h*(2*e*g + d*h) + 3*b*g*(e*g + 2*d*h)) + 16*c^2*(3*a^2*f*h^2 + 12*a*b
*h*(2*f*g + e*h) + 14*b^2*(f*g^2 + h*(2*e*g + d*h))))*(b + 2*c*x)*(a + b*x
+ c*x^2)^(3/2))/(6144*c^5) - ((10*c*f*g - 16*c*e*h + 11*b*f*h)*(g + h*x)^2*
(a + b*x + c*x^2)^(5/2))/(112*c^2*h) + (f*(g + h*x)^3*(a + b*x + c*x^2)^(5/
2))/(8*c*h) - ((693*b^3*f*h^3 + 96*c^3*(5*f*g^3 - 8*g*h*(e*g + 7*d*h)) - 36
*b*c*h^2*(31*a*f*h + 28*b*(2*f*g + e*h)) + 8*c^2*h*(31*b*f*g^2 + 196*b*h*(2
*e*g + d*h) + 96*a*h*(2*f*g + e*h)) - 10*c*h*(99*b^2*f*h^2 - 8*c^2*(5*f*g^2
 - 4*h*(2*e*g + 7*d*h)) - 12*c*h*(7*a*f*h + 2*b*(f*g + 6*e*h)))*x)*(a + b*x
 + c*x^2)^(5/2))/(13440*c^4*h) + ((b^2 - 4*a*c)^2*(768*c^4*d*g^2 + 99*b^4*f
*h^2 - 72*b^2*c*h*(4*b*f*g + 2*b*e*h + 3*a*f*h) - 128*c^3*(a*f*g^2 + a*h*(2
*e*g + d*h) + 3*b*g*(e*g + 2*d*h)) + 16*c^2*(3*a^2*f*h^2 + 12*a*b*h*(2*f*g
+ e*h) + 14*b^2*(f*g^2 + h*(2*e*g + d*h))))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*
Sqrt[a + b*x + c*x^2])])/(32768*c^(13/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 846

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
```

```
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\text{integral} = \frac{f(g+hx)^3\left(a+bx+cx^2\right)^{5/2}}{8ch} + \frac{\int (g+hx)^2\left(-\frac{1}{2}h(5bfg-16cdh+6afh)-\frac{1}{2}h(10cfg-16ceh+11bfh)x\right)\left(a+bx+cx^2\right)^{3/2}\,dx}{8ch^2}$$

$$= -\frac{(10cfg-16ceh+11bfh)(g+hx)^2\left(a+bx+cx^2\right)^{5/2}}{112c^2h} + \frac{f(g+hx)^3\left(a+bx+cx^2\right)^{5/2}}{8ch} + \frac{\int (g+hx)\left(\frac{1}{4}h(55b^2fgh+44abfh^2-20bcg(fg+4eh)+4ch(56cdg-11afg-16aeh))+\frac{1}{4}h(9}{56c^2}$$

$$= -\frac{(10cfg-16ceh+11bfh)(g+hx)^2\left(a+bx+cx^2\right)^{5/2}}{112c^2h} + \frac{f(g+hx)^3\left(a+bx+cx^2\right)^{5/2}}{8ch} - \frac{(693b^3fh^3+96c^3(5fg^3-8gh(eg+7dh))-36bch^2(31afh+28b(2fg+eh))+8c^2h(31bfg^2+}{} + \frac{(768c^4dg^2+99b^4fh^2-72b^2ch(4bfg+2beh+3afh)-128c^3(afg^2+ah(2eg+dh)+3bg(eg+}{768}$$

$$= \frac{(768c^4dg^2+99b^4fh^2-72b^2ch(4bfg+2beh+3afh)-128c^3(afg^2+ah(2eg+dh)+3bg(eg+2}{614} - \frac{(10cfg-16ceh+11bfh)(g+hx)^2\left(a+bx+cx^2\right)^{5/2}}{112c^2h} + \frac{f(g+hx)^3\left(a+bx+cx^2\right)^{5/2}}{8ch} - \frac{(693b^3fh^3+96c^3(5fg^3-8gh(eg+7dh))-36bch^2(31afh+28b(2fg+eh))+8c^2h(31bfg^2+}{} - \frac{((b^2-4ac)(768c^4dg^2+99b^4fh^2-72b^2ch(4bfg+2beh+3afh)-128c^3(afg^2+ah(2eg+dh)}{}$$

$$=$$

$$-\frac{(b^2-4ac)\left(768c^4dg^2+99b^4fh^2-72b^2ch(4bfg+2beh+3afh)-128c^3(afg^2+ah(2eg+dh)+\right.}{}$$

$$+\frac{\left(768c^4dg^2+99b^4fh^2-72b^2ch(4bfg+2beh+3afh)-128c^3(afg^2+ah(2eg+dh)+3bg(eg+2\right.}{614}$$

$$-\frac{(10cfg-16ceh+11bfh)(g+hx)^2\left(a+bx+cx^2\right)^{5/2}}{112c^2h}$$

$$+\frac{f(g+hx)^3\left(a+bx+cx^2\right)^{5/2}}{8ch}$$

$$-\frac{\left(693b^3fh^3+96c^3(5fg^3-8gh(eg+7dh))-36bch^2(31afh+28b(2fg+eh))+8c^2h(31bfg^2+1\right.}{}$$

$$+\frac{\left(\left(b^2-4ac\right)^2\left(768c^4dg^2+99b^4fh^2-72b^2ch(4bfg+2beh+3afh)-128c^3(afg^2+ah(2eg+dh)\right.\right.}{32}$$

$$=$$

$$-\frac{(b^2-4ac)\left(768c^4dg^2+99b^4fh^2-72b^2ch(4bfg+2beh+3afh)-128c^3(afg^2+ah(2eg+dh)+\right.}{}$$

$$+\frac{\left(768c^4dg^2+99b^4fh^2-72b^2ch(4bfg+2beh+3afh)-128c^3(afg^2+ah(2eg+dh)+3bg(eg+2\right.}{614}$$

$$-\frac{(10cfg-16ceh+11bfh)(g+hx)^2\left(a+bx+cx^2\right)^{5/2}}{112c^2h}$$

$$+\frac{f(g+hx)^3\left(a+bx+cx^2\right)^{5/2}}{8ch}$$

$$-\frac{\left(693b^3fh^3+96c^3(5fg^3-8gh(eg+7dh))-36bch^2(31afh+28b(2fg+eh))+8c^2h(31bfg^2+1\right.}{}$$

$$+\frac{\left(\left(b^2-4ac\right)^2\left(768c^4dg^2+99b^4fh^2-72b^2ch(4bfg+2beh+3afh)-128c^3(afg^2+ah(2eg+dh)\right.\right.}{}$$

$$=$$

$$-\frac{(b^2-4ac)\left(768c^4dg^2+99b^4fh^2-72b^2ch(4bfg+2beh+3afh)-128c^3(afg^2+ah(2eg+dh)+\right.}{}$$

$$+\frac{\left(768c^4dg^2+99b^4fh^2-72b^2ch(4bfg+2beh+3afh)-128c^3(afg^2+ah(2eg+dh)+3bg(eg+2\right.}{614}$$

$$-\frac{(10cfg-16ceh+11bfh)(g+hx)^2\left(a+bx+cx^2\right)^{5/2}}{112c^2h}$$

$$+\frac{f(g+hx)^3\left(a+bx+cx^2\right)^{5/2}}{8ch}$$

$$-\frac{\left(693b^3fh^3+96c^3(5fg^3-8gh(eg+7dh))-36bch^2(31afh+28b(2fg+eh))+8c^2h(31bfg^2+1\right.}{}$$

$$+\frac{\left(b^2-4ac\right)^2\left(768c^4dg^2+99b^4fh^2-72b^2ch(4bfg+2beh+3afh)-128c^3(afg^2+ah(2eg+dh)+\right.}{3}$$

Mathematica [A] (verified)

Time = 11.44 (sec) , antiderivative size = 877, normalized size of antiderivative = 1.16

$$\int (g+hx)^2 \left(a+bx+cx^2\right)^{3/2} (d+ex +fx^2) \, dx = \frac{430080dg^2(b+2cx)(a+x(b+cx))^{3/2}+688128g(eg+2dh)(a+x(b+cx))^{5/2}+573440(fg^2+}{}$$

```
[In] Integrate[(g + h*x)^2*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

```
[Out] (430080*d*g^2*(b + 2*c*x)*(a + x*(b + c*x))^(3/2) + 688128*g*(e*g + 2*d*h)*
(a + x*(b + c*x))^(5/2) + 573440*(f*g^2 + h*(2*e*g + d*h))*x*(a + x*(b + c*
x))^(5/2) + 491520*h*(2*f*g + e*h)*x^2*(a + x*(b + c*x))^(5/2) + 430080*f*h
^2*x^3*(a + x*(b + c*x))^(5/2) + (80640*(b^2 - 4*a*c)*d*g^2*(-2*Sqrt[c]*(b
+ 2*c*x)*Sqrt[a + x*(b + c*x)] + (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[
c]*Sqrt[a + x*(b + c*x)])]))/c^(3/2) - (13440*b*g*(e*g + 2*d*h)*(16*c^(3/2)
*(b + 2*c*x)*(a + x*(b + c*x))^(3/2) - 3*(b^2 - 4*a*c)*(2*Sqrt[c]*(b + 2*c*
x)*Sqrt[a + x*(b + c*x)] - (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqr
t[a + x*(b + c*x)])])))/c^(5/2) + (48*h*(2*f*g + e*h)*(-256*c^(5/2)*(-21*b^
2 + 16*a*c + 30*b*c*x)*(a + x*(b + c*x))^(5/2) - 35*b*(3*b^2 - 4*a*c)*(16*c
^(3/2)*(b + 2*c*x)*(a + x*(b + c*x))^(3/2) - 3*(b^2 - 4*a*c)*(2*Sqrt[c]*(b
+ 2*c*x)*Sqrt[a + x*(b + c*x)] - (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[
c]*Sqrt[a + x*(b + c*x)])]))))/c^(9/2) - (224*(f*g^2 + h*(2*e*g + d*h))*(17
92*b*c^(5/2)*(a + x*(b + c*x))^(5/2) - 5*(7*b^2 - 4*a*c)*(16*c^(3/2)*(b + 2
*c*x)*(a + x*(b + c*x))^(3/2) - 3*(b^2 - 4*a*c)*(2*Sqrt[c]*(b + 2*c*x)*Sqrt
[a + x*(b + c*x)] - (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x
*(b + c*x)])]))))/c^(7/2) - (3*f*h^2*(112640*b*c^(9/2)*x^2*(a + x*(b + c*x)
)^(5/2) + 256*c^(5/2)*(231*b^3 - 372*a*b*c - 330*b^2*c*x + 280*a*c^2*x)*(a
+ x*(b + c*x))^(5/2) - 35*(33*b^4 - 72*a*b^2*c + 16*a^2*c^2)*(16*c^(3/2)*(b
 + 2*c*x)*(a + x*(b + c*x))^(3/2) - 3*(b^2 - 4*a*c)*(2*Sqrt[c]*(b + 2*c*x)*
Sqrt[a + x*(b + c*x)] - (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a
 + x*(b + c*x)])]))))/c^(11/2))/(3440640*c)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1679 vs. 2(723) = 1446.

Time = 0.85 (sec) , antiderivative size = 1680, normalized size of antiderivative = 2.23

method	result	size
default	Expression too large to display	1680
risch	Expression too large to display	1933

```
[In] int((h*x+g)^2*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x,method=_RETURNVERBOSE)

[Out] d*g^2*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b
)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x
^2+b*x+a)^(1/2))))+f*h^2*(1/8*x^3*(c*x^2+b*x+a)^(5/2)/c-11/16*b/c*(1/7*x^2*
(c*x^2+b*x+a)^(5/2)/c-9/14*b/c*(1/6*x*(c*x^2+b*x+a)^(5/2)/c-7/12*b/c*(1/5*(
c*x^2+b*x+a)^(5/2)/c-1/2*b/c*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a
*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((
1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))))-1/6*a/c*(1/8*(2*c*x+b)/c*(c*x^2+
b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4
*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))))-2/7*a/c*(1
/5*(c*x^2+b*x+a)^(5/2)/c-1/2*b/c*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*
(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*
ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))))-3/8*a/c*(1/6*x*(c*x^2+b*x+a
)^(5/2)/c-7/12*b/c*(1/5*(c*x^2+b*x+a)^(5/2)/c-1/2*b/c*(1/8*(2*c*x+b)/c*(c*x
^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8
*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))))-1/6*a/c
*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(
c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*
x+a)^(1/2))))))+(e*h^2+2*f*g*h)*(1/7*x^2*(c*x^2+b*x+a)^(5/2)/c-9/14*b/c*(1/
6*x*(c*x^2+b*x+a)^(5/2)/c-7/12*b/c*(1/5*(c*x^2+b*x+a)^(5/2)/c-1/2*b/c*(1/8*
(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+
b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(
1/2)))))-1/6*a/c*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1
/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c
^(1/2)+(c*x^2+b*x+a)^(1/2)))))-2/7*a/c*(1/5*(c*x^2+b*x+a)^(5/2)/c-1/2*b/c*(
1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*
x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+
a)^(1/2))))))+(2*d*g*h+e*g^2)*(1/5*(c*x^2+b*x+a)^(5/2)/c-1/2*b/c*(1/8*(2*c*
x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a
)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
)))+(d*h^2+2*e*g*h+f*g^2)*(1/6*x*(c*x^2+b*x+a)^(5/2)/c-7/12*b/c*(1/5*(c*x^2
+b*x+a)^(5/2)/c-1/2*b/c*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^
2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b
+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))))-1/6*a/c*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a
)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-
b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1571 vs. 2(723) = 1446.

Time = 1.50 (sec) , antiderivative size = 3145, normalized size of antiderivative = 4.18

$$\int (g+hx)^2 \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right)\,dx = \text{Too large to display}$$

```
[In] integrate((h*x+g)^2*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="fricas"
)
```

```
[Out] [1/6881280*(105*(32*(24*(b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*d - 12*(b^5*c^
3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*e + (7*b^6*c^2 - 60*a*b^4*c^3 + 144*a^2*b^2
*c^4 - 64*a^3*c^5)*f)*g^2 - 32*(24*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*d
 - 2*(7*b^6*c^2 - 60*a*b^4*c^3 + 144*a^2*b^2*c^4 - 64*a^3*c^5)*e + 3*(3*b^7
*c - 28*a*b^5*c^2 + 80*a^2*b^3*c^3 - 64*a^3*b*c^4)*f)*g*h + (32*(7*b^6*c^2
- 60*a*b^4*c^3 + 144*a^2*b^2*c^4 - 64*a^3*c^5)*d - 48*(3*b^7*c - 28*a*b^5*c
^2 + 80*a^2*b^3*c^3 - 64*a^3*b*c^4)*e + 3*(33*b^8 - 336*a*b^6*c + 1120*a^2*
b^4*c^2 - 1280*a^3*b^2*c^3 + 256*a^4*c^4)*f)*h^2)*sqrt(c)*log(-8*c^2*x^2 -
8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(2
15040*c^8*f*h^2*x^7 + 15360*(32*c^8*f*g*h + (16*c^8*e + 17*b*c^7*f)*h^2)*x^
6 + 1280*(224*c^8*f*g^2 + 32*(14*c^8*e + 15*b*c^7*f)*g*h + (224*c^8*d + 240
*b*c^7*e + 3*(b^2*c^6 + 84*a*c^7)*f)*h^2)*x^5 + 128*(224*(12*c^8*e + 13*b*c
^7*f)*g^2 + 32*(168*c^8*d + 182*b*c^7*e + 3*(b^2*c^6 + 64*a*c^7)*f)*g*h + (
2912*b*c^7*d + 48*(b^2*c^6 + 64*a*c^7)*e - 3*(11*b^3*c^5 - 52*a*b*c^6)*f)*h
^2)*x^4 + 16*(224*(120*c^8*d + 132*b*c^7*e + (3*b^2*c^6 + 140*a*c^7)*f)*g^2
 + 32*(1848*b*c^7*d + 14*(3*b^2*c^6 + 140*a*c^7)*e - 3*(9*b^3*c^5 - 44*a*b*
c^6)*f)*g*h + (224*(3*b^2*c^6 + 140*a*c^7)*d - 48*(9*b^3*c^5 - 44*a*b*c^6)*
e + 3*(99*b^4*c^4 - 568*a*b^2*c^5 + 560*a^2*c^6)*f)*h^2)*x^3 - 224*(120*(3*
b^3*c^5 - 20*a*b*c^6)*d - 12*(15*b^4*c^4 - 100*a*b^2*c^5 + 128*a^2*c^6)*e +
 (105*b^5*c^3 - 760*a*b^3*c^4 + 1296*a^2*b*c^5)*f)*g^2 + 32*(168*(15*b^4*c^
4 - 100*a*b^2*c^5 + 128*a^2*c^6)*d - 14*(105*b^5*c^3 - 760*a*b^3*c^4 + 1296
*a^2*b*c^5)*e + 3*(315*b^6*c^2 - 2520*a*b^4*c^3 + 5488*a^2*b^2*c^4 - 2048*a
^3*c^5)*f)*g*h - (224*(105*b^5*c^3 - 760*a*b^3*c^4 + 1296*a^2*b*c^5)*d - 48
*(315*b^6*c^2 - 2520*a*b^4*c^3 + 5488*a^2*b^2*c^4 - 2048*a^3*c^5)*e + 3*(34
65*b^7*c - 30660*a*b^5*c^2 + 81648*a^2*b^3*c^3 - 58816*a^3*b*c^4)*f)*h^2 +
8*(224*(360*b*c^7*d + 12*(b^2*c^6 + 32*a*c^7)*e - (7*b^3*c^5 - 36*a*b*c^6)*
f)*g^2 + 32*(168*(b^2*c^6 + 32*a*c^7)*d - 14*(7*b^3*c^5 - 36*a*b*c^6)*e + 3
*(21*b^4*c^4 - 124*a*b^2*c^5 + 128*a^2*c^6)*f)*g*h - (224*(7*b^3*c^5 - 36*a
*b*c^6)*d - 48*(21*b^4*c^4 - 124*a*b^2*c^5 + 128*a^2*c^6)*e + 3*(231*b^5*c^
3 - 1560*a*b^3*c^4 + 2416*a^2*b*c^5)*f)*h^2)*x^2 + 2*(224*(120*(b^2*c^6 + 2
0*a*c^7)*d - 12*(5*b^3*c^5 - 28*a*b*c^6)*e + (35*b^4*c^4 - 216*a*b^2*c^5 +
240*a^2*c^6)*f)*g^2 - 32*(168*(5*b^3*c^5 - 28*a*b*c^6)*d - 14*(35*b^4*c^4 -
 216*a*b^2*c^5 + 240*a^2*c^6)*e + 3*(105*b^5*c^3 - 728*a*b^3*c^4 + 1168*a^2
*b*c^5)*f)*g*h + (224*(35*b^4*c^4 - 216*a*b^2*c^5 + 240*a^2*c^6)*d - 48*(10
```

```
5*b^5*c^3 - 728*a*b^3*c^4 + 1168*a^2*b*c^5)*e + 3*(1155*b^6*c^2 - 8988*a*b^
4*c^3 + 18896*a^2*b^2*c^4 - 6720*a^3*c^5)*f)*h^2)*x)*sqrt(c*x^2 + b*x + a))
/c^7, -1/3440640*(105*(32*(24*(b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*d - 12*(
b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*e + (7*b^6*c^2 - 60*a*b^4*c^3 + 144*a
^2*b^2*c^4 - 64*a^3*c^5)*f)*g^2 - 32*(24*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*
c^5)*d - 2*(7*b^6*c^2 - 60*a*b^4*c^3 + 144*a^2*b^2*c^4 - 64*a^3*c^5)*e + 3*
(3*b^7*c - 28*a*b^5*c^2 + 80*a^2*b^3*c^3 - 64*a^3*b*c^4)*f)*g*h + (32*(7*b^
6*c^2 - 60*a*b^4*c^3 + 144*a^2*b^2*c^4 - 64*a^3*c^5)*d - 48*(3*b^7*c - 28*a
*b^5*c^2 + 80*a^2*b^3*c^3 - 64*a^3*b*c^4)*e + 3*(33*b^8 - 336*a*b^6*c + 112
0*a^2*b^4*c^2 - 1280*a^3*b^2*c^3 + 256*a^4*c^4)*f)*h^2)*sqrt(-c)*arctan(1/2
*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(2
15040*c^8*f*h^2*x^7 + 15360*(32*c^8*f*g*h + (16*c^8*e + 17*b*c^7*f)*h^2)*x^
6 + 1280*(224*c^8*f*g^2 + 32*(14*c^8*e + 15*b*c^7*f)*g*h + (224*c^8*d + 240
*b*c^7*e + 3*(b^2*c^6 + 84*a*c^7)*f)*h^2)*x^5 + 128*(224*(12*c^8*e + 13*b*c
^7*f)*g^2 + 32*(168*c^8*d + 182*b*c^7*e + 3*(b^2*c^6 + 64*a*c^7)*f)*g*h + (
2912*b*c^7*d + 48*(b^2*c^6 + 64*a*c^7)*e - 3*(11*b^3*c^5 - 52*a*b*c^6)*f)*h
^2)*x^4 + 16*(224*(120*c^8*d + 132*b*c^7*e + (3*b^2*c^6 + 140*a*c^7)*f)*g^2
 + 32*(1848*b*c^7*d + 14*(3*b^2*c^6 + 140*a*c^7)*e - 3*(9*b^3*c^5 - 44*a*b*
c^6)*f)*g*h + (224*(3*b^2*c^6 + 140*a*c^7)*d - 48*(9*b^3*c^5 - 44*a*b*c^6)*
e + 3*(99*b^4*c^4 - 568*a*b^2*c^5 + 560*a^2*c^6)*f)*h^2)*x^3 - 224*(120*(3*
b^3*c^5 - 20*a*b*c^6)*d - 12*(15*b^4*c^4 - 100*a*b^2*c^5 + 128*a^2*c^6)*e +
 (105*b^5*c^3 - 760*a*b^3*c^4 + 1296*a^2*b*c^5)*f)*g^2 + 32*(168*(15*b^4*c^
4 - 100*a*b^2*c^5 + 128*a^2*c^6)*d - 14*(105*b^5*c^3 - 760*a*b^3*c^4 + 1296
*a^2*b*c^5)*e + 3*(315*b^6*c^2 - 2520*a*b^4*c^3 + 5488*a^2*b^2*c^4 - 2048*a
^3*c^5)*f)*g*h - (224*(105*b^5*c^3 - 760*a*b^3*c^4 + 1296*a^2*b*c^5)*d - 48
*(315*b^6*c^2 - 2520*a*b^4*c^3 + 5488*a^2*b^2*c^4 - 2048*a^3*c^5)*e + 3*(34
65*b^7*c - 30660*a*b^5*c^2 + 81648*a^2*b^3*c^3 - 58816*a^3*b*c^4)*f)*h^2 +
8*(224*(360*b*c^7*d + 12*(b^2*c^6 + 32*a*c^7)*e - (7*b^3*c^5 - 36*a*b*c^6)*
f)*g^2 + 32*(168*(b^2*c^6 + 32*a*c^7)*d - 14*(7*b^3*c^5 - 36*a*b*c^6)*e + 3
*(21*b^4*c^4 - 124*a*b^2*c^5 + 128*a^2*c^6)*f)*g*h - (224*(7*b^3*c^5 - 36*a
*b*c^6)*d - 48*(21*b^4*c^4 - 124*a*b^2*c^5 + 128*a^2*c^6)*e + 3*(231*b^5*c^
3 - 1560*a*b^3*c^4 + 2416*a^2*b*c^5)*f)*h^2)*x^2 + 2*(224*(120*(b^2*c^6 + 2
0*a*c^7)*d - 12*(5*b^3*c^5 - 28*a*b*c^6)*e + (35*b^4*c^4 - 216*a*b^2*c^5 +
240*a^2*c^6)*f)*g^2 - 32*(168*(5*b^3*c^5 - 28*a*b*c^6)*d - 14*(35*b^4*c^4 -
 216*a*b^2*c^5 + 240*a^2*c^6)*e + 3*(105*b^5*c^3 - 728*a*b^3*c^4 + 1168*a^2
*b*c^5)*f)*g*h + (224*(35*b^4*c^4 - 216*a*b^2*c^5 + 240*a^2*c^6)*d - 48*(10
5*b^5*c^3 - 728*a*b^3*c^4 + 1168*a^2*b*c^5)*e + 3*(1155*b^6*c^2 - 8988*a*b^
4*c^3 + 18896*a^2*b^2*c^4 - 6720*a^3*c^5)*f)*h^2)*x)*sqrt(c*x^2 + b*x + a))
/c^7]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9687 vs. 2(790) = 1580.

Time = 1.33 (sec) , antiderivative size = 9687, normalized size of antiderivative = 12.86

$$\int (g+hx)^2 \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right) \, dx = \text{Too large to display}$$

```
[In] integrate((h*x+g)**2*(c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d),x)

[Out] Piecewise((sqrt(a + b*x + c*x**2)*(c*f*h**2*x**7/8 + x**6*(17*b*c*f*h**2/16
 + c**2*e*h**2 + 2*c**2*f*g*h)/(7*c) + x**5*(9*a*c*f*h**2/8 + b**2*f*h**2 +
 2*b*c*e*h**2 + 4*b*c*f*g*h - 13*b*(17*b*c*f*h**2/16 + c**2*e*h**2 + 2*c**2
*f*g*h)/(14*c) + c**2*d*h**2 + 2*c**2*e*g*h + c**2*f*g**2)/(6*c) + x**4*(2*
a*b*f*h**2 + 2*a*c*e*h**2 + 4*a*c*f*g*h - 6*a*(17*b*c*f*h**2/16 + c**2*e*h*
*2 + 2*c**2*f*g*h)/(7*c) + b**2*e*h**2 + 2*b**2*f*g*h + 2*b*c*d*h**2 + 4*b*
c*e*g*h + 2*b*c*f*g**2 - 11*b*(9*a*c*f*h**2/8 + b**2*f*h**2 + 2*b*c*e*h**2
+ 4*b*c*f*g*h - 13*b*(17*b*c*f*h**2/16 + c**2*e*h**2 + 2*c**2*f*g*h)/(14*c)
 + c**2*d*h**2 + 2*c**2*e*g*h + c**2*f*g**2)/(12*c) + 2*c**2*d*g*h + c**2*e
*g**2)/(5*c) + x**3*(a**2*f*h**2 + 2*a*b*e*h**2 + 4*a*b*f*g*h + 2*a*c*d*h**
2 + 4*a*c*e*g*h + 2*a*c*f*g**2 - 5*a*(9*a*c*f*h**2/8 + b**2*f*h**2 + 2*b*c*
e*h**2 + 4*b*c*f*g*h - 13*b*(17*b*c*f*h**2/16 + c**2*e*h**2 + 2*c**2*f*g*h)
/(14*c) + c**2*d*h**2 + 2*c**2*e*g*h + c**2*f*g**2)/(6*c) + b**2*d*h**2 + 2
*b**2*e*g*h + b**2*f*g**2 + 4*b*c*d*g*h + 2*b*c*e*g**2 - 9*b*(2*a*b*f*h**2
+ 2*a*c*e*h**2 + 4*a*c*f*g*h - 6*a*(17*b*c*f*h**2/16 + c**2*e*h**2 + 2*c**2
*f*g*h)/(7*c) + b**2*e*h**2 + 2*b**2*f*g*h + 2*b*c*d*h**2 + 4*b*c*e*g*h + 2
*b*c*f*g**2 - 11*b*(9*a*c*f*h**2/8 + b**2*f*h**2 + 2*b*c*e*h**2 + 4*b*c*f*g
*h - 13*b*(17*b*c*f*h**2/16 + c**2*e*h**2 + 2*c**2*f*g*h)/(14*c) + c**2*d*h
**2 + 2*c**2*e*g*h + c**2*f*g**2)/(12*c) + 2*c**2*d*g*h + c**2*e*g**2)/(10*
c) + c**2*d*g**2)/(4*c) + x**2*(a**2*e*h**2 + 2*a**2*f*g*h + 2*a*b*d*h**2 +
 4*a*b*e*g*h + 2*a*b*f*g**2 + 4*a*c*d*g*h + 2*a*c*e*g**2 - 4*a*(2*a*b*f*h**
2 + 2*a*c*e*h**2 + 4*a*c*f*g*h - 6*a*(17*b*c*f*h**2/16 + c**2*e*h**2 + 2*c*
*2*f*g*h)/(7*c) + b**2*e*h**2 + 2*b**2*f*g*h + 2*b*c*d*h**2 + 4*b*c*e*g*h +
 2*b*c*f*g**2 - 11*b*(9*a*c*f*h**2/8 + b**2*f*h**2 + 2*b*c*e*h**2 + 4*b*c*f
*g*h - 13*b*(17*b*c*f*h**2/16 + c**2*e*h**2 + 2*c**2*f*g*h)/(14*c) + c**2*d
*h**2 + 2*c**2*e*g*h + c**2*f*g**2)/(12*c) + 2*c**2*d*g*h + c**2*e*g**2)/(5
*c) + 2*b**2*d*g*h + b**2*e*g**2 + 2*b*c*d*g**2 - 7*b*(a**2*f*h**2 + 2*a*b*
e*h**2 + 4*a*b*f*g*h + 2*a*c*d*h**2 + 4*a*c*e*g*h + 2*a*c*f*g**2 - 5*a*(9*a
*c*f*h**2/8 + b**2*f*h**2 + 2*b*c*e*h**2 + 4*b*c*f*g*h - 13*b*(17*b*c*f*h**
2/16 + c**2*e*h**2 + 2*c**2*f*g*h)/(14*c) + c**2*d*h**2 + 2*c**2*e*g*h + c*
*2*f*g**2)/(6*c) + b**2*d*h**2 + 2*b**2*e*g*h + b**2*f*g**2 + 4*b*c*d*g*h +
 2*b*c*e*g**2 - 9*b*(2*a*b*f*h**2 + 2*a*c*e*h**2 + 4*a*c*f*g*h - 6*a*(17*b*
c*f*h**2/16 + c**2*e*h**2 + 2*c**2*f*g*h)/(7*c) + b**2*e*h**2 + 2*b**2*f*g*
h + 2*b*c*d*h**2 + 4*b*c*e*g*h + 2*b*c*f*g**2 - 11*b*(9*a*c*f*h**2/8 + b**2
*f*h**2 + 2*b*c*e*h**2 + 4*b*c*f*g*h - 13*b*(17*b*c*f*h**2/16 + c**2*e*h**2
```

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 + 2*c**2*f*g*h)/(14*c) + c**2*d*h**2 + 2*c**2*e*g*h + c**2*f*g**2)/(12*c)
+ 2*c**2*d*g*h + c**2*e*g**2)/(10*c) + c**2*d*g**2)/(8*c))/(3*c) + x*(a**2*
d*h**2 + 2*a**2*e*g*h + a**2*f*g**2 + 4*a*b*d*g*h + 2*a*b*e*g**2 + 2*a*c*d*
g**2 - 3*a*(a**2*f*h**2 + 2*a*b*e*h**2 + 4*a*b*f*g*h + 2*a*c*d*h**2 + 4*a*c
*e*g*h + 2*a*c*f*g**2 - 5*a*(9*a*c*f*h**2/8 + b**2*f*h**2 + 2*b*c*e*h**2 +
4*b*c*f*g*h - 13*b*(17*b*c*f*h**2/16 + c**2*e*h**2 + 2*c**2*f*g*h)/(14*c) +
 c**2*d*h**2 + 2*c**2*e*g*h + c**2*f*g**2)/(6*c) + b**2*d*h**2 + 2*b**2*e*g
*h + b**2*f*g**2 + 4*b*c*d*g*h + 2*b*c*e*g**2 - 9*b*(2*a*b*f*h**2 + 2*a*c*e
*h**2 + 4*a*c*f*g*h - 6*a*(17*b*c*f*h**2/16 + c**2*e*h**2 + 2*c**2*f*g*h)/(
7*c) + b**2*e*h**2 + 2*b**2*f*g*h + 2*b*c*d*h**2 + 4*b*c*e*g*h + 2*b*c*f*g*
*2 - 11*b*(9*a*c*f*h**2/8 + b**2*f*h**2 + 2*b*c*e*h**2 + 4*b*c*f*g*h - 13*b
*(17*b*c*f*h**2/16 + c**2*e*h**2 + 2*c**2*f*g*h)/(14*c) + c**2*d*h**2 + 2*c
**2*e*g*h + c**2*f*g**2)/(12*c) + 2*c**2*d*g*h + c**2*e*g**2)/(10*c) + c**2
*d*g**2)/(4*c) + b**2*d*g**2 - 5*b*(a**2*e*h**2 + 2*a**2*f*g*h + 2*a*b*d*h*
*2 + 4*a*b*e*g*h + 2*a*b*f*g**2 + 4*a*c*d*g*h + 2*a*c*e*g**2 - 4*a*(2*a*b*f
*h**2 + 2*a*c*e*h**2 + 4*a*c*f*g*h - 6*a*(17*b*c*f*h**2/16 + c**2*e*h**2 +
2*c**2*f*g*h)/(7*c) + b**2*e*h**2 + 2*b**2*f*g*h + 2*b*c*d*h**2 + 4*b*c*e*g
*h + 2*b*c*f*g**2 - 11*b*(9*a*c*f*h**2/8 + b**2*f*h**2 + 2*b*c*e*h**2 + 4*b
*c*f*g*h - 13*b*(17*b*c*f*h**2/16 + c**2*e*h**2 + 2*c**2*f*g*h)/(14*c) + c*
*2*d*h**2 + 2*c**2*e*g*h + c**2*f*g**2)/(12*c) + 2*c**2*d*g*h + c**2*e*g**2
)/(5*c) + 2*b**2*d*g*h + b**2*e*g**2 + 2*b*c*d*g**2 - 7*b*(a**2*f*h**2 + 2*
a*b*e*h**2 + 4*a*b*f*g*h + 2*a*c*d*h**2 + 4*a*c*e*g*h + 2*a*c*f*g**2 - 5*a*
(9*a*c*f*h**2/8 + b**2*f*h**2 + 2*b*c*e*h**2 + 4*b*c*f*g*h - 13*b*(17*b*c*f
*h**2/16 + c**2*e*h**2 + 2*c**2*f*g*h)/(14*c) + c**2*d*h**2 + 2*c**2*e*g*h
+ c**2*f*g**2)/(6*c) + b**2*d*h**2 + 2*b**2*e*g*h + b**2*f*g**2 + 4*b*c*d*g
*h + 2*b*c*e*g**2 - 9*b*(2*a*b*f*h**2 + 2*a*c*e*h**2 + 4*a*c*f*g*h - 6*a*(1
7*b*c*f*h**2/16 + c**2*e*h**2 + 2*c**2*f*g*h)/(7*c) + b**2*e*h**2 + 2*b**2*
f*g*h + 2*b*c*d*h**2 + 4*b*c*e*g*h + 2*b*c*f*g**2 - 11*b*(9*a*c*f*h**2/8 +
b**2*f*h**2 + 2*b*c*e*h**2 + 4*b*c*f*g*h - 13*b*(17*b*c*f*h**2/16 + c**2*e*
h**2 + 2*c**2*f*g*h)/(14*c) + c**2*d*h**2 + 2*c**2*e*g*h + c**2*f*g**2)/(12
*c) + 2*c**2*d*g*h + c**2*e*g**2)/(10*c) + c**2*d*g**2)/(8*c))/(6*c))/(2*c)
 + (2*a**2*d*g*h + a**2*e*g**2 + 2*a*b*d*g**2 - 2*a*(a**2*e*h**2 + 2*a**2*f
*g*h + 2*a*b*d*h**2 + 4*a*b*e*g*h + 2*a*b*f*g**2 + 4*a*c*d*g*h + 2*a*c*e*g*
*2 - 4*a*(2*a*b*f*h**2 + 2*a*c*e*h**2 + 4*a*c*f*g*h - 6*a*(17*b*c*f*h**2/16
 + c**2*e*h**2 + 2*c**2*f*g*h)/(7*c) + b**2*e*h**2 + 2*b**2*f*g*h + 2*b*c*d
*h**2 + 4*b*c*e*g*h + 2*b*c*f*g**2 - 11*b*(9*a*c*f*h**2/8 + b**2*f*h**2 + 2
*b*c*e*h**2 + 4*b*c*f*g*h - 13*b*(17*b*c*f*h**2/16 + c**2*e*h**2 + 2*c**2*f
*g*h)/(14*c) + c**2*d*h**2 + 2*c**2*e*g*h + c**2*f*g**2)/(12*c) + 2*c**2*d*
g*h + c**2*e*g**2)/(5*c) + 2*b**2*d*g*h + b**2*e*g**2 + 2*b*c*d*g**2 - 7*b*
(a**2*f*h**2 + 2*a*b*e*h**2 + 4*a*b*f*g*h + 2*a*c*d*h**2 + 4*a*c*e*g*h + 2*
a*c*f*g**2 - 5*a*(9*a*c*f*h**2/8 + b**2*f*h**2 + 2*b*c*e*h**2 + 4*b*c*f*g*h
 - 13*b*(17*b*c*f*h**2/16 + c**2*e*h**2 + 2*c**2*f*g*h)/(14*c) + c**2*d*h**
2 + 2*c**2*e*g*h + c**2*f*g**2)/(6*c) + b**2*d*h**2 + 2*b**2*e*g*h + b**2*f
*g**2 + 4*b*c*d*g*h + 2*b*c*e*g**2 - 9*b*(2*a*b*f*h**2 + 2*a*c*e*h**2 + 4*a
*c*f*g*h - 6*a*(17*b*c*f*h**2/16 + c**2*e*h**2 + 2*c**2*f*g*h)/(7*c) + b**2
```

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*e*h**2 + 2*b**2*f*g*h + 2*b*c*d*h**2 + 4*b*c*e*g*h + 2*b*c*f*g**2 - 11*b*(
9*a*c*f*h**2/8 + b**2*f*h**2 + 2*b*c*e*h**2 + 4*b*c*f*g*h - 13*b*(17*b*c*f*
h**2/16 + c**2*e*h**2 + 2*c**2*f*g*h)/(14*c) + c**2*d*h**2 + 2*c**2*e*g*h +
 c**2*f*g**2)/(12*c) + 2*c**2*d*g*h + c**2*e*g**2)/(10*c) + c**2*d*g**2)/(8
*c))/(3*c) - 3*b*(a**2*d*h**2 + 2*a**2*e*g*h + a**2*f*g**2 + 4*a*b*d*g*h +
2*a*b*e*g**2 + 2*a*c*d*g**2 - 3*a*(a**2*f*h**2 + 2*a*b*e*h**2 + 4*a*b*f*g*h
 + 2*a*c*d*h**2 + 4*a*c*e*g*h + 2*a*c*f*g**2 - 5*a*(9*a*c*f*h**2/8 + b**2*f
*h**2 + 2*b*c*e*h**2 + 4*b*c*f*g*h - 13*b*(17*b*c*f*h**2/16 + c**2*e*h**2 +
 2*c**2*f*g*h)/(14*c) + c**2*d*h**2 + 2*c**2*e*g*h + c**2*f*g**2)/(6*c) + b
**2*d*h**2 + 2*b**2*e*g*h + b**2*f*g**2 + 4*b*c*d*g*h + 2*b*c*e*g**2 - 9*b*
(2*a*b*f*h**2 + 2*a*c*e*h**2 + 4*a*c*f*g*h - 6*a*(17*b*c*f*h**2/16 + c**2*e
*h**2 + 2*c**2*f*g*h)/(7*c) + b**2*e*h**2 + 2*b**2*f*g*h + 2*b*c*d*h**2 + 4
*b*c*e*g*h + 2*b*c*f*g**2 - 11*b*(9*a*c*f*h**2/8 + b**2*f*h**2 + 2*b*c*e*h*
*2 + 4*b*c*f*g*h - 13*b*(17*b*c*f*h**2/16 + c**2*e*h**2 + 2*c**2*f*g*h)/(14
*c) + c**2*d*h**2 + 2*c**2*e*g*h + c**2*f*g**2)/(12*c) + 2*c**2*d*g*h + c**
2*e*g**2)/(10*c) + c**2*d*g**2)/(4*c) + b**2*d*g**2 - 5*b*(a**2*e*h**2 + 2*
a**2*f*g*h + 2*a*b*d*h**2 + 4*a*b*e*g*h + 2*a*b*f*g**2 + 4*a*c*d*g*h + 2*a*
c*e*g**2 - 4*a*(2*a*b*f*h**2 + 2*a*c*e*h**2 + 4*a*c*f*g*h - 6*a*(17*b*c*f*h
**2/16 + c**2*e*h**2 + 2*c**2*f*g*h)/(7*c) + b**2*e*h**2 + 2*b**2*f*g*h + 2
*b*c*d*h**2 + 4*b*c*e*g*h + 2*b*c*f*g**2 - 11*b*(9*a*c*f*h**2/8 + b**2*f*h*
*2 + 2*b*c*e*h**2 + 4*b*c*f*g*h - 13*b*(17*b*c*f*h**2/16 + c**2*e*h**2 + 2*
c**2*f*g*h)/(14*c) + c**2*d*h**2 + 2*c**2*e*g*h + c**2*f*g**2)/(12*c) + 2*c
**2*d*g*h + c**2*e*g**2)/(5*c) + 2*b**2*d*g*h + b**2*e*g**2 + 2*b*c*d*g**2
- 7*b*(a**2*f*h**2 + 2*a*b*e*h**2 + 4*a*b*f*g*h + 2*a*c*d*h**2 + 4*a*c*e*g*
h + 2*a*c*f*g**2 - 5*a*(9*a*c*f*h**2/8 + b**2*f*h**2 + 2*b*c*e*h**2 + 4*b*c
*f*g*h - 13*b*(17*b*c*f*h**2/16 + c**2*e*h**2 + 2*c**2*f*g*h)/(14*c) + c**2
*d*h**2 + 2*c**2*e*g*h + c**2*f*g**2)/(6*c) + b**2*d*h**2 + 2*b**2*e*g*h +
b**2*f*g**2 + 4*b*c*d*g*h + 2*b*c*e*g**2 - 9*b*(2*a*b*f*h**2 + 2*a*c*e*h**2
 + 4*a*c*f*g*h - 6*a*(17*b*c*f*h**2/16 + c**2*e*h**2 + 2*c**2*f*g*h)/(7*c)
+ b**2*e*h**2 + 2*b**2*f*g*h + 2*b*c*d*h**2 + 4*b*c*e*g*h + 2*b*c*f*g**2 -
11*b*(9*a*c*f*h**2/8 + b**2*f*h**2 + 2*b*c*e*h**2 + 4*b*c*f*g*h - 13*b*(17*
b*c*f*h**2/16 + c**2*e*h**2 + 2*c**2*f*g*h)/(14*c) + c**2*d*h**2 + 2*c**2*e
*g*h + c**2*f*g**2)/(12*c) + 2*c**2*d*g*h + c**2*e*g**2)/(10*c) + c**2*d*g*
*2)/(8*c))/(6*c))/(4*c))/c) + (a**2*d*g**2 - a*(a**2*d*h**2 + 2*a**2*e*g*h
+ a**2*f*g**2 + 4*a*b*d*g*h + 2*a*b*e*g**2 + 2*a*c*d*g**2 - 3*a*(a**2*f*h**
2 + 2*a*b*e*h**2 + 4*a*b*f*g*h + 2*a*c*d*h**2 + 4*a*c*e*g*h + 2*a*c*f*g**2
- 5*a*(9*a*c*f*h**2/8 + b**2*f*h**2 + 2*b*c*e*h**2 + 4*b*c*f*g*h - 13*b*(17
*b*c*f*h**2/16 + c**2*e*h**2 + 2*c**2*f*g*h)/(14*c) + c**2*d*h**2 + 2*c**2*
e*g*h + c**2*f*g**2)/(6*c) + b**2*d*h**2 + 2*b**2*e*g*h + b**2*f*g**2 + 4*b
*c*d*g*h + 2*b*c*e*g**2 - 9*b*(2*a*b*f*h**2 + 2*a*c*e*h**2 + 4*a*c*f*g*h -
6*a*(17*b*c*f*h**2/16 + c**2*e*h**2 + 2*c**2*f*g*h)/(7*c) + b**2*e*h**2 + 2
*b**2*f*g*h + 2*b*c*d*h**2 + 4*b*c*e*g*h + 2*b*c*f*g**2 - 11*b*(9*a*c*f*h**
2/8 + b**2*f*h**2 + 2*b*c*e*h**2 + 4*b*c*f*g*h - 13*b*(17*b*c*f*h**2/16 + c
**2*e*h**2 + 2*c**2*f*g*h)/(14*c) + c**2*d*h**2 + 2*c**2*e*g*h + c**2*f*g**
2)/(12*c) + 2*c**2*d*g*h + c**2*e*g**2)/(10*c) + c**2*d*g**2)/(4*c) + b**2*
```

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d*g**2 - 5*b*(a**2*e*h**2 + 2*a**2*f*g*h + 2*a*b*d*h**2 + 4*a*b*e*g*h + 2*a
*b*f*g**2 + 4*a*c*d*g*h + 2*a*c*e*g**2 - 4*a*(2*a*b*f*h**2 + 2*a*c*e*h**2 +
 4*a*c*f*g*h - 6*a*(17*b*c*f*h**2/16 + c**2*e*h**2 + 2*c**2*f*g*h)/(7*c) +
b**2*e*h**2 + 2*b**2*f*g*h + 2*b*c*d*h**2 + 4*b*c*e*g*h + 2*b*c*f*g**2 - 11
*b*(9*a*c*f*h**2/8 + b**2*f*h**2 + 2*b*c*e*h**2 + 4*b*c*f*g*h - 13*b*(17*b*
c*f*h**2/16 + c**2*e*h**2 + 2*c**2*f*g*h)/(14*c) + c**2*d*h**2 + 2*c**2*e*g
*h + c**2*f*g**2)/(12*c) + 2*c**2*d*g*h + c**2*e*g**2)/(5*c) + 2*b**2*d*g*h
 + b**2*e*g**2 + 2*b*c*d*g**2 - 7*b*(a**2*f*h**2 + 2*a*b*e*h**2 + 4*a*b*f*g
*h + 2*a*c*d*h**2 + 4*a*c*e*g*h + 2*a*c*f*g**2 - 5*a*(9*a*c*f*h**2/8 + b**2
*f*h**2 + 2*b*c*e*h**2 + 4*b*c*f*g*h - 13*b*(17*b*c*f*h**2/16 + c**2*e*h**2
 + 2*c**2*f*g*h)/(14*c) + c**2*d*h**2 + 2*c**2*e*g*h + c**2*f*g**2)/(6*c) +
 b**2*d*h**2 + 2*b**2*e*g*h + b**2*f*g**2 + 4*b*c*d*g*h + 2*b*c*e*g**2 - 9*
b*(2*a*b*f*h**2 + 2*a*c*e*h**2 + 4*a*c*f*g*h - 6*a*(17*b*c*f*h**2/16 + c**2
*e*h**2 + 2*c**2*f*g*h)/(7*c) + b**2*e*h**2 + 2*b**2*f*g*h + 2*b*c*d*h**2 +
 4*b*c*e*g*h + 2*b*c*f*g**2 - 11*b*(9*a*c*f*h**2/8 + b**2*f*h**2 + 2*b*c*e*
h**2 + 4*b*c*f*g*h - 13*b*(17*b*c*f*h**2/16 + c**2*e*h**2 + 2*c**2*f*g*h)/(
14*c) + c**2*d*h**2 + 2*c**2*e*g*h + c**2*f*g**2)/(12*c) + 2*c**2*d*g*h + c
**2*e*g**2)/(10*c) + c**2*d*g**2)/(8*c))/(6*c))/(2*c) - b*(2*a**2*d*g*h + a
**2*e*g**2 + 2*a*b*d*g**2 - 2*a*(a**2*e*h**2 + 2*a**2*f*g*h + 2*a*b*d*h**2
+ 4*a*b*e*g*h + 2*a*b*f*g**2 + 4*a*c*d*g*h + 2*a*c*e*g**2 - 4*a*(2*a*b*f*h*
*2 + 2*a*c*e*h**2 + 4*a*c*f*g*h - 6*a*(17*b*c*f*h**2/16 + c**2*e*h**2 + 2*c
**2*f*g*h)/(7*c) + b**2*e*h**2 + 2*b**2*f*g*h + 2*b*c*d*h**2 + 4*b*c*e*g*h
+ 2*b*c*f*g**2 - 11*b*(9*a*c*f*h**2/8 + b**2*f*h**2 + 2*b*c*e*h**2 + 4*b*c*
f*g*h - 13*b*(17*b*c*f*h**2/16 + c**2*e*h**2 + 2*c**2*f*g*h)/(14*c) + c**2*
d*h**2 + 2*c**2*e*g*h + c**2*f*g**2)/(12*c) + 2*c**2*d*g*h + c**2*e*g**2)/(
5*c) + 2*b**2*d*g*h + b**2*e*g**2 + 2*b*c*d*g**2 - 7*b*(a**2*f*h**2 + 2*a*b
*e*h**2 + 4*a*b*f*g*h + 2*a*c*d*h**2 + 4*a*c*e*g*h + 2*a*c*f*g**2 - 5*a*(9*
a*c*f*h**2/8 + b**2*f*h**2 + 2*b*c*e*h**2 + 4*b*c*f*g*h - 13*b*(17*b*c*f*h*
*2/16 + c**2*e*h**2 + 2*c**2*f*g*h)/(14*c) + c**2*d*h**2 + 2*c**2*e*g*h + c
**2*f*g**2)/(6*c) + b**2*d*h**2 + 2*b**2*e*g*h + b**2*f*g**2 + 4*b*c*d*g*h
+ 2*b*c*e*g**2 - 9*b*(2*a*b*f*h**2 + 2*a*c*e*h**2 + 4*a*c*f*g*h - 6*a*(17*b
*c*f*h**2/16 + c**2*e*h**2 + 2*c**2*f*g*h)/(7*c) + b**2*e*h**2 + 2*b**2*f*g
*h + 2*b*c*d*h**2 + 4*b*c*e*g*h + 2*b*c*f*g**2 - 11*b*(9*a*c*f*h**2/8 + b**
2*f*h**2 + 2*b*c*e*h**2 + 4*b*c*f*g*h - 13*b*(17*b*c*f*h**2/16 + c**2*e*h**
2 + 2*c**2*f*g*h)/(14*c) + c**2*d*h**2 + 2*c**2*e*g*h + c**2*f*g**2)/(12*c)
 + 2*c**2*d*g*h + c**2*e*g**2)/(10*c) + c**2*d*g**2)/(8*c))/(3*c) - 3*b*(a*
*2*d*h**2 + 2*a**2*e*g*h + a**2*f*g**2 + 4*a*b*d*g*h + 2*a*b*e*g**2 + 2*a*c
*d*g**2 - 3*a*(a**2*f*h**2 + 2*a*b*e*h**2 + 4*a*b*f*g*h + 2*a*c*d*h**2 + 4*
a*c*e*g*h + 2*a*c*f*g**2 - 5*a*(9*a*c*f*h**2/8 + b**2*f*h**2 + 2*b*c*e*h**2
 + 4*b*c*f*g*h - 13*b*(17*b*c*f*h**2/16 + c**2*e*h**2 + 2*c**2*f*g*h)/(14*c
) + c**2*d*h**2 + 2*c**2*e*g*h + c**2*f*g**2)/(6*c) + b**2*d*h**2 + 2*b**2*
e*g*h + b**2*f*g**2 + 4*b*c*d*g*h + 2*b*c*e*g**2 - 9*b*(2*a*b*f*h**2 + 2*a*
c*e*h**2 + 4*a*c*f*g*h - 6*a*(17*b*c*f*h**2/16 + c**2*e*h**2 + 2*c**2*f*g*h
)/(7*c) + b**2*e*h**2 + 2*b**2*f*g*h + 2*b*c*d*h**2 + 4*b*c*e*g*h + 2*b*c*f
*g**2 - 11*b*(9*a*c*f*h**2/8 + b**2*f*h**2 + 2*b*c*e*h**2 + 4*b*c*f*g*h - 1
```

```
3*b*(17*b*c*f*h**2/16 + c**2*e*h**2 + 2*c**2*f*g*h)/(14*c) + c**2*d*h**2 +
2*c**2*e*g*h + c**2*f*g**2)/(12*c) + 2*c**2*d*g*h + c**2*e*g**2)/(10*c) + c
**2*d*g**2)/(4*c) + b**2*d*g**2 - 5*b*(a**2*e*h**2 + 2*a**2*f*g*h + 2*a*b*d
*h**2 + 4*a*b*e*g*h + 2*a*b*f*g**2 + 4*a*c*d*g*h + 2*a*c*e*g**2 - 4*a*(2*a*
b*f*h**2 + 2*a*c*e*h**2 + 4*a*c*f*g*h - 6*a*(17*b*c*f*h**2/16 + c**2*e*h**2
 + 2*c**2*f*g*h)/(7*c) + b**2*e*h**2 + 2*b**2*f*g*h + 2*b*c*d*h**2 + 4*b*c*
e*g*h + 2*b*c*f*g**2 - 11*b*(9*a*c*f*h**2/8 + b**2*f*h**2 + 2*b*c*e*h**2 +
4*b*c*f*g*h - 13*b*(17*b*c*f*h**2/16 + c**2*e*h**2 + 2*c**2*f*g*h)/(14*c) +
 c**2*d*h**2 + 2*c**2*e*g*h + c**2*f*g**2)/(12*c) + 2*c**2*d*g*h + c**2*e*g
**2)/(5*c) + 2*b**2*d*g*h + b**2*e*g**2 + 2*b*c*d*g**2 - 7*b*(a**2*f*h**2 +
 2*a*b*e*h**2 + 4*a*b*f*g*h + 2*a*c*d*h**2 + 4*a*c*e*g*h + 2*a*c*f*g**2 - 5
*a*(9*a*c*f*h**2/8 + b**2*f*h**2 + 2*b*c*e*h**2 + 4*b*c*f*g*h - 13*b*(17*b*
c*f*h**2/16 + c**2*e*h**2 + 2*c**2*f*g*h)/(14*c) + c**2*d*h**2 + 2*c**2*e*g
*h + c**2*f*g**2)/(6*c) + b**2*d*h**2 + 2*b**2*e*g*h + b**2*f*g**2 + 4*b*c*
d*g*h + 2*b*c*e*g**2 - 9*b*(2*a*b*f*h**2 + 2*a*c*e*h**2 + 4*a*c*f*g*h - 6*a
*(17*b*c*f*h**2/16 + c**2*e*h**2 + 2*c**2*f*g*h)/(7*c) + b**2*e*h**2 + 2*b*
*2*f*g*h + 2*b*c*d*h**2 + 4*b*c*e*g*h + 2*b*c*f*g**2 - 11*b*(9*a*c*f*h**2/8
 + b**2*f*h**2 + 2*b*c*e*h**2 + 4*b*c*f*g*h - 13*b*(17*b*c*f*h**2/16 + c**2
*e*h**2 + 2*c**2*f*g*h)/(14*c) + c**2*d*h**2 + 2*c**2*e*g*h + c**2*f*g**2)/
(12*c) + 2*c**2*d*g*h + c**2*e*g**2)/(10*c) + c**2*d*g**2)/(8*c))/(6*c))/(4
*c))/(2*c))*Piecewise((log(b + 2*sqrt(c)*sqrt(a + b*x + c*x**2) + 2*c*x)/sq
rt(c), Ne(a - b**2/(4*c), 0)), ((b/(2*c) + x)*log(b/(2*c) + x)/sqrt(c*(b/(2
*c) + x)**2), True)), Ne(c, 0)), (2*(f*h**2*(a + b*x)**(13/2)/(13*b**4) + (
a + b*x)**(11/2)*(-4*a*f*h**2 + b*e*h**2 + 2*b*f*g*h)/(11*b**4) + (a + b*x)
**(9/2)*(6*a**2*f*h**2 - 3*a*b*e*h**2 - 6*a*b*f*g*h + b**2*d*h**2 + 2*b**2*
e*g*h + b**2*f*g**2)/(9*b**4) + (a + b*x)**(7/2)*(-4*a**3*f*h**2 + 3*a**2*b
*e*h**2 + 6*a**2*b*f*g*h - 2*a*b**2*d*h**2 - 4*a*b**2*e*g*h - 2*a*b**2*f*g*
*2 + 2*b**3*d*g*h + b**3*e*g**2)/(7*b**4) + (a + b*x)**(5/2)*(a**4*f*h**2 -
 a**3*b*e*h**2 - 2*a**3*b*f*g*h + a**2*b**2*d*h**2 + 2*a**2*b**2*e*g*h + a*
*2*b**2*f*g**2 - 2*a*b**3*d*g*h - a*b**3*e*g**2 + b**4*d*g**2)/(5*b**4))/b,
 Ne(b, 0)), (a**(3/2)*(d*g**2*x + f*h**2*x**5/5 + x**4*(e*h**2 + 2*f*g*h)/4
 + x**3*(d*h**2 + 2*e*g*h + f*g**2)/3 + x**2*(2*d*g*h + e*g**2)/2), True))
```

Maxima [F(-2)]

Exception generated.

$$\int (g+hx)^2 \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right) \, dx = \text{Exception raised: ValueError}$$

```
[In] integrate((h*x+g)^2*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1802 vs. 2(723) = 1446.

Time = 0.32 (sec) , antiderivative size = 1802, normalized size of antiderivative = 2.39

$$\int (g+hx)^2 \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right)\, dx = \text{Too large to display}$$

```
[In] integrate((h*x+g)^2*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] 1/1720320*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*(12*(14*c*f*h^2*x + (32*c^8
*f*g*h + 16*c^8*e*h^2 + 17*b*c^7*f*h^2)/c^7)*x + (224*c^8*f*g^2 + 448*c^8*e
*g*h + 480*b*c^7*f*g*h + 224*c^8*d*h^2 + 240*b*c^7*e*h^2 + 3*b^2*c^6*f*h^2
+ 252*a*c^7*f*h^2)/c^7)*x + (2688*c^8*e*g^2 + 2912*b*c^7*f*g^2 + 5376*c^8*d
*g*h + 5824*b*c^7*e*g*h + 96*b^2*c^6*f*g*h + 6144*a*c^7*f*g*h + 2912*b*c^7*
d*h^2 + 48*b^2*c^6*e*h^2 + 3072*a*c^7*e*h^2 - 33*b^3*c^5*f*h^2 + 156*a*b*c^
6*f*h^2)/c^7)*x + (26880*c^8*d*g^2 + 29568*b*c^7*e*g^2 + 672*b^2*c^6*f*g^2
+ 31360*a*c^7*f*g^2 + 59136*b*c^7*d*g*h + 1344*b^2*c^6*e*g*h + 62720*a*c^7*
e*g*h - 864*b^3*c^5*f*g*h + 4224*a*b*c^6*f*g*h + 672*b^2*c^6*d*h^2 + 31360*
a*c^7*d*h^2 - 432*b^3*c^5*e*h^2 + 2112*a*b*c^6*e*h^2 + 297*b^4*c^4*f*h^2 -
1704*a*b^2*c^5*f*h^2 + 1680*a^2*c^6*f*h^2)/c^7)*x + (80640*b*c^7*d*g^2 + 26
88*b^2*c^6*e*g^2 + 86016*a*c^7*e*g^2 - 1568*b^3*c^5*f*g^2 + 8064*a*b*c^6*f*
g^2 + 5376*b^2*c^6*d*g*h + 172032*a*c^7*d*g*h - 3136*b^3*c^5*e*g*h + 16128*
a*b*c^6*e*g*h + 2016*b^4*c^4*f*g*h - 11904*a*b^2*c^5*f*g*h + 12288*a^2*c^6*
f*g*h - 1568*b^3*c^5*d*h^2 + 8064*a*b*c^6*d*h^2 + 1008*b^4*c^4*e*h^2 - 5952
*a*b^2*c^5*e*h^2 + 6144*a^2*c^6*e*h^2 - 693*b^5*c^3*f*h^2 + 4680*a*b^3*c^4*
f*h^2 - 7248*a^2*b*c^5*f*h^2)/c^7)*x + (26880*b^2*c^6*d*g^2 + 537600*a*c^7*
d*g^2 - 13440*b^3*c^5*e*g^2 + 75264*a*b*c^6*e*g^2 + 7840*b^4*c^4*f*g^2 - 48
384*a*b^2*c^5*f*g^2 + 53760*a^2*c^6*f*g^2 - 26880*b^3*c^5*d*g*h + 150528*a*
b*c^6*d*g*h + 15680*b^4*c^4*e*g*h - 96768*a*b^2*c^5*e*g*h + 107520*a^2*c^6*
e*g*h - 10080*b^5*c^3*f*g*h + 69888*a*b^3*c^4*f*g*h - 112128*a^2*b*c^5*f*g*
h + 7840*b^4*c^4*d*h^2 - 48384*a*b^2*c^5*d*h^2 + 53760*a^2*c^6*d*h^2 - 5040
*b^5*c^3*e*h^2 + 34944*a*b^3*c^4*e*h^2 - 56064*a^2*b*c^5*e*h^2 + 3465*b^6*c
^2*f*h^2 - 26964*a*b^4*c^3*f*h^2 + 56688*a^2*b^2*c^4*f*h^2 - 20160*a^3*c^5*
f*h^2)/c^7)*x - (80640*b^3*c^5*d*g^2 - 537600*a*b*c^6*d*g^2 - 40320*b^4*c^4
*e*g^2 + 268800*a*b^2*c^5*e*g^2 - 344064*a^2*c^6*e*g^2 + 23520*b^5*c^3*f*g^
2 - 170240*a*b^3*c^4*f*g^2 + 290304*a^2*b*c^5*f*g^2 - 80640*b^4*c^4*d*g*h +
 537600*a*b^2*c^5*d*g*h - 688128*a^2*c^6*d*g*h + 47040*b^5*c^3*e*g*h - 3404
80*a*b^3*c^4*e*g*h + 580608*a^2*b*c^5*e*g*h - 30240*b^6*c^2*f*g*h + 241920*
a*b^4*c^3*f*g*h - 526848*a^2*b^2*c^4*f*g*h + 196608*a^3*c^5*f*g*h + 23520*b
^5*c^3*d*h^2 - 170240*a*b^3*c^4*d*h^2 + 290304*a^2*b*c^5*d*h^2 - 15120*b^6*
c^2*e*h^2 + 120960*a*b^4*c^3*e*h^2 - 263424*a^2*b^2*c^4*e*h^2 + 98304*a^3*c
```

```
^5*e*h^2 + 10395*b^7*c*f*h^2 - 91980*a*b^5*c^2*f*h^2 + 244944*a^2*b^3*c^3*f
*h^2 - 176448*a^3*b*c^4*f*h^2)/c^7) - 1/32768*(768*b^4*c^4*d*g^2 - 6144*a*b
^2*c^5*d*g^2 + 12288*a^2*c^6*d*g^2 - 384*b^5*c^3*e*g^2 + 3072*a*b^3*c^4*e*g
^2 - 6144*a^2*b*c^5*e*g^2 + 224*b^6*c^2*f*g^2 - 1920*a*b^4*c^3*f*g^2 + 4608
*a^2*b^2*c^4*f*g^2 - 2048*a^3*c^5*f*g^2 - 768*b^5*c^3*d*g*h + 6144*a*b^3*c^
4*d*g*h - 12288*a^2*b*c^5*d*g*h + 448*b^6*c^2*e*g*h - 3840*a*b^4*c^3*e*g*h
+ 9216*a^2*b^2*c^4*e*g*h - 4096*a^3*c^5*e*g*h - 288*b^7*c*f*g*h + 2688*a*b^
5*c^2*f*g*h - 7680*a^2*b^3*c^3*f*g*h + 6144*a^3*b*c^4*f*g*h + 224*b^6*c^2*d
*h^2 - 1920*a*b^4*c^3*d*h^2 + 4608*a^2*b^2*c^4*d*h^2 - 2048*a^3*c^5*d*h^2 -
 144*b^7*c*e*h^2 + 1344*a*b^5*c^2*e*h^2 - 3840*a^2*b^3*c^3*e*h^2 + 3072*a^3
*b*c^4*e*h^2 + 99*b^8*f*h^2 - 1008*a*b^6*c*f*h^2 + 3360*a^2*b^4*c^2*f*h^2 -
 3840*a^3*b^2*c^3*f*h^2 + 768*a^4*c^4*f*h^2)*log(abs(2*(sqrt(c)*x - sqrt(c*
x^2 + b*x + a))*sqrt(c) + b))/c^(13/2)
```

Mupad [F(-1)]

Timed out.

$$\int (g+hx)^2 \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right)\,dx=\int (g+h\,x)^2\,\left(c\,x^2+b\,x+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right)\,dx$$

```
[In] int((g + h*x)^2*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x)
[Out] int((g + h*x)^2*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2), x)
```

3.198 $\int (g+hx) \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right) \, dx$

Optimal result

Integrand size = 30, antiderivative size = 418

$$\begin{aligned} &\int (g+hx) \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right) \, dx = \\ &-\frac{\left(b^2-4ac\right) \left(48c^3dg-9b^3fh-8c^2(3beg+afg+3bdh+aeh)+2bc(6afh+7b(fg+eh))\right) (b+2cx)\sqrt{a+b}}{1024c^5} \\ &+\frac{\left(48c^3dg-9b^3fh-8c^2(3beg+afg+3bdh+aeh)+2bc(6afh+7b(fg+eh))\right) (b+2cx) \left(a+bx+cx^2\right)^{3/2}}{384c^4} \\ &+\frac{f(g+hx)^2 \left(a+bx+cx^2\right)^{5/2}}{7ch} \\ &+\frac{\left(63b^2fh^2-24c^2(5fg^2-7h(eg+dh))-2ch(24afh+49b(fg+eh))-10ch(10cfg-14ceh+9bfh)x\right) (a+}{840c^3h} \\ &+\frac{\left(b^2-4ac\right)^2 \left(48c^3dg-9b^3fh-8c^2(3beg+afg+3bdh+aeh)+2bc(6afh+7b(fg+eh))\right) \operatorname{arctanh}\left(\frac{b+2}{2\sqrt{c}\sqrt{a+}}\right.}{2048c^{11/2}} \end{aligned}$$

```
[Out] 1/384*(48*c^3*d*g-9*b^3*f*h-8*c^2*(a*e*h+a*f*g+3*b*d*h+3*b*e*g)+2*b*c*(6*a*
f*h+7*b*(e*h+f*g)))*(2*c*x+b)*(c*x^2+b*x+a)^(3/2)/c^4+1/7*f*(h*x+g)^2*(c*x^
2+b*x+a)^(5/2)/c/h+1/840*(63*b^2*f*h^2-24*c^2*(5*f*g^2-7*h*(d*h+e*g))-2*c*h
*(24*a*f*h+49*b*(e*h+f*g))-10*c*h*(9*b*f*h-14*c*e*h+10*c*f*g)*x)*(c*x^2+b*x
+a)^(5/2)/c^3/h+1/2048*(-4*a*c+b^2)^2*(48*c^3*d*g-9*b^3*f*h-8*c^2*(a*e*h+a*
f*g+3*b*d*h+3*b*e*g)+2*b*c*(6*a*f*h+7*b*(e*h+f*g)))*arctanh(1/2*(2*c*x+b)/c
^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(11/2)-1/1024*(-4*a*c+b^2)*(48*c^3*d*g-9*b^3*
f*h-8*c^2*(a*e*h+a*f*g+3*b*d*h+3*b*e*g)+2*b*c*(6*a*f*h+7*b*(e*h+f*g)))*(2*c
*x+b)*(c*x^2+b*x+a)^(1/2)/c^5
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1667, 793, 626, 635, 212}

$$\int (g+hx)\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)dx = \frac{(b^2-4ac)^2\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(-8c^2(aeh+afg+3bdh+3beg)+2bc(6afh+7b(eh+fg}{2048c^{11/2}} + \frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}(-8c^2(aeh+afg+3bdh+3beg)+2bc(6afh+7b(eh+fg))-9b^3fh+48c^3d}{384c^4} + \frac{\left(a+bx+cx^2\right)^{5/2}(-2ch(24afh+49b(eh+fg))+63b^2fh^2-10chx(9bfh-14ceh+10cfg)-24c^2(5fg^2}{840c^3h} - \frac{(b^2-4ac)(b+2cx)\sqrt{a+bx+cx^2}(-8c^2(aeh+afg+3bdh+3beg)+2bc(6afh+7b(eh+fg))-9b^3fh}{1024c^5} + \frac{f(g+hx)^2\left(a+bx+cx^2\right)^{5/2}}{7ch}$$

```
[In] Int[(g + h*x)*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

```
[Out] -1/1024*((b^2 - 4*a*c)*(48*c^3*d*g - 9*b^3*f*h - 8*c^2*(3*b*e*g + a*f*g + 3
*b*d*h + a*e*h) + 2*b*c*(6*a*f*h + 7*b*(f*g + e*h)))*(b + 2*c*x)*Sqrt[a + b
*x + c*x^2])/c^5 + ((48*c^3*d*g - 9*b^3*f*h - 8*c^2*(3*b*e*g + a*f*g + 3*b*
d*h + a*e*h) + 2*b*c*(6*a*f*h + 7*b*(f*g + e*h)))*(b + 2*c*x)*(a + b*x + c*
x^2)^(3/2))/(384*c^4) + (f*(g + h*x)^2*(a + b*x + c*x^2)^(5/2))/(7*c*h) + (
(63*b^2*f*h^2 - 24*c^2*(5*f*g^2 - 7*h*(e*g + d*h)) - 2*c*h*(24*a*f*h + 49*b
*(f*g + e*h)) - 10*c*h*(10*c*f*g - 14*c*e*h + 9*b*f*h)*x)*(a + b*x + c*x^2)
^(5/2))/(840*c^3*h) + ((b^2 - 4*a*c)^2*(48*c^3*d*g - 9*b^3*f*h - 8*c^2*(3*b
*e*g + a*f*g + 3*b*d*h + a*e*h) + 2*b*c*(6*a*f*h + 7*b*(f*g + e*h)))*ArcTan
h[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2048*c^(11/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\text{integral} = \frac{f(g+hx)^2\left(a+bx+cx^2\right)^{5/2}}{7ch}$$
$$+\frac{\int (g+hx)\left(-\frac{1}{2}h(5bfg-14cdh+4afh)-\frac{1}{2}h(10cfg-14ceh+9bfh)x\right)\left(a+bx+cx^2\right)^{3/2}\,dx}{7ch^2}$$

$$= \frac{f(g+hx)^2\left(a+bx+cx^2\right)^{5/2}}{7ch}$$
$$+\frac{(63b^2fh^2-24c^2(5fg^2-7h(eg+dh))-2ch(24afh+49b(fg+eh))-10ch(10cfg-14ceh+9}{840c^3h}$$
$$+\frac{(48c^3dg-9b^3fh-8c^2(3beg+afg+3bdh+aeh)+2bc(6afh+7b(fg+eh)))\int (a+bx+cx^2)}{48c^3}$$

$$= \frac{(48c^3dg-9b^3fh-8c^2(3beg+afg+3bdh+aeh)+2bc(6afh+7b(fg+eh)))\,(b+2cx)\,(a+bx}{384c^4}$$

$$+\frac{f(g+hx)^2\,(a+bx+cx^2)^{5/2}}{7ch}$$

$$+\frac{(63b^2fh^2-24c^2(5fg^2-7h(eg+dh))-2ch(24afh+49b(fg+eh))-10ch(10cfg-14ceh+}{840c^3h}$$

$$-\frac{((b^2-4ac)\,(48c^3dg-9b^3fh-8c^2(3beg+afg+3bdh+aeh)+2bc(6afh+7b(fg+eh))))\int}{256c^4}$$

$$=$$

$$-\frac{(b^2-4ac)\,(48c^3dg-9b^3fh-8c^2(3beg+afg+3bdh+aeh)+2bc(6afh+7b(fg+eh)))\,(b+}{1024c^5}$$

$$+\frac{(48c^3dg-9b^3fh-8c^2(3beg+afg+3bdh+aeh)+2bc(6afh+7b(fg+eh)))\,(b+2cx)\,(a+b}{384c^4}$$

$$+\frac{f(g+hx)^2\,(a+bx+cx^2)^{5/2}}{7ch}$$

$$+\frac{(63b^2fh^2-24c^2(5fg^2-7h(eg+dh))-2ch(24afh+49b(fg+eh))-10ch(10cfg-14ceh+}{840c^3h}$$

$$+\frac{\left((b^2-4ac)^2\,(48c^3dg-9b^3fh-8c^2(3beg+afg+3bdh+aeh)+2bc(6afh+7b(fg+eh)))\right)\int}{2048c^5}$$

$$=$$

$$-\frac{(b^2-4ac)\,(48c^3dg-9b^3fh-8c^2(3beg+afg+3bdh+aeh)+2bc(6afh+7b(fg+eh)))\,(b+}{1024c^5}$$

$$+\frac{(48c^3dg-9b^3fh-8c^2(3beg+afg+3bdh+aeh)+2bc(6afh+7b(fg+eh)))\,(b+2cx)\,(a+b}{384c^4}$$

$$+\frac{f(g+hx)^2\,(a+bx+cx^2)^{5/2}}{7ch}$$

$$+\frac{(63b^2fh^2-24c^2(5fg^2-7h(eg+dh))-2ch(24afh+49b(fg+eh))-10ch(10cfg-14ceh+}{840c^3h}$$

$$+\frac{\left((b^2-4ac)^2\,(48c^3dg-9b^3fh-8c^2(3beg+afg+3bdh+aeh)+2bc(6afh+7b(fg+eh)))\right)\text{S}}{1024c^5}$$

$$
\begin{aligned}
= & \\
& -\frac{(b^2-4ac)\left(48c^3dg-9b^3fh-8c^2(3beg+afg+3bdh+aeh)+2bc(6afh+7b(fg+eh))\right)(b+2}{1024c^5} \\
& +\frac{\left(48c^3dg-9b^3fh-8c^2(3beg+afg+3bdh+aeh)+2bc(6afh+7b(fg+eh))\right)(b+2cx)\,(a+bx}{384c^4} \\
& +\frac{f(g+hx)^2\left(a+bx+cx^2\right)^{5/2}}{7ch} \\
& +\frac{(63b^2fh^2-24c^2(5fg^2-7h(eg+dh))-2ch(24afh+49b(fg+eh))-10ch(10cfg-14ceh+9}{840c^3h} \\
& +\frac{(b^2-4ac)^2\left(48c^3dg-9b^3fh-8c^2(3beg+afg+3bdh+aeh)+2bc(6afh+7b(fg+eh))\right)\tanh}{2048c^{11/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 7.71 (sec) , antiderivative size = 601, normalized size of antiderivative = 1.44

$$
\int (g+hx)\left(a+bx+cx^2\right)^{3/2}\left(d+ex\right.
$$

$$
\left.+fx^2\right)\,dx=\frac{\sqrt{c}\sqrt{a+x(b+cx)}(945b^6fh-210b^5c(7fg+7eh+3fhx)+28b^4c(-270afh+c(90eg+90dh+}{}
$$

```
[In] Integrate[(g + h*x)*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

```
[Out] (Sqrt[c]*Sqrt[a + x*(b + c*x)]*(945*b^6*f*h - 210*b^5*c*(7*f*g + 7*e*h + 3*
f*h*x) + 28*b^4*c*(-270*a*f*h + c*(90*e*g + 90*d*h + 35*f*g*x + 35*e*h*x +
18*f*h*x^2)) - 16*b^3*c^2*(105*c*d*(3*g + h*x) - 7*a*(95*f*g + 95*e*h + 39*
f*h*x) + c*x*(7*e*(15*g + 7*h*x) + f*x*(49*g + 27*h*x))) + 48*b^2*c^2*(343*
a^2*f*h - 2*a*c*(175*d*h + 7*e*(25*g + 9*h*x) + f*x*(63*g + 31*h*x)) + 2*c^
2*x*(7*d*(5*g + 2*h*x) + x*(7*e*(2*g + h*x) + f*x*(7*g + 4*h*x)))) + 32*b*c
^3*(-3*a^2*(189*f*g + 189*e*h + 73*f*h*x) + 6*a*c*(7*d*(25*g + 7*h*x) + x*(
7*e*(7*g + 3*h*x) + f*x*(21*g + 11*h*x))) + 4*c^2*x^2*(21*d*(15*g + 11*h*x)
 + x*(7*e*(33*g + 26*h*x) + 2*f*x*(91*g + 75*h*x)))) + 64*c^3*(-96*a^3*f*h
+ 3*a^2*c*(112*d*h + 7*e*(16*g + 5*h*x) + f*x*(35*g + 16*h*x)) + 4*c^3*x^3*
(21*d*(5*g + 4*h*x) + 2*x*(7*e*(6*g + 5*h*x) + 5*f*x*(7*g + 6*h*x))) + 2*a*
c^2*x*(21*d*(25*g + 16*h*x) + x*(7*e*(48*g + 35*h*x) + f*x*(245*g + 192*h*x
))))) - 105*(b^2 - 4*a*c)^2*(-48*c^3*d*g + 9*b^3*f*h + 8*c^2*(3*b*e*g + a*f
*g + 3*b*d*h + a*e*h) - 2*b*c*(6*a*f*h + 7*b*(f*g + e*h)))*ArcTanh[(Sqrt[c]
*x)/(-Sqrt[a] + Sqrt[a + x*(b + c*x)])])/(107520*c^(11/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 935 vs. 2(392) = 784.

Time = 0.77 (sec) , antiderivative size = 936, normalized size of antiderivative = 2.24

method	result
default	Expression too large to display
risch	$-\frac{\left(-15360 c^{6} f h \,x^{6}-19200 b \,c^{5} f h \,x^{5}-17920 c^{6} e h \,x^{5}-17920 c^{6} f g \,x^{5}-24576 a \,c^{5} f h \,x^{4}-384 b^{2} c^{4} f h \,x^{4}-23296 b \,c^{5} e h \,x^{4}-23296 b \,c^{5} f g \,x^{4}}{}$

```
[In] int((h*x+g)*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] d*g*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/
c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2
+b*x+a)^(1/2))))+f*h*(1/7*x^2*(c*x^2+b*x+a)^(5/2)/c-9/14*b/c*(1/6*x*(c*x^2+
b*x+a)^(5/2)/c-7/12*b/c*(1/5*(c*x^2+b*x+a)^(5/2)/c-1/2*b/c*(1/8*(2*c*x+b)/c
*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2
)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))))-1/
6*a/c*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b
)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x
^2+b*x+a)^(1/2)))))-2/7*a/c*(1/5*(c*x^2+b*x+a)^(5/2)/c-1/2*b/c*(1/8*(2*c*x+
b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^
(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))
))+(e*h+f*g)*(1/6*x*(c*x^2+b*x+a)^(5/2)/c-7/12*b/c*(1/5*(c*x^2+b*x+a)^(5/2)
/c-1/2*b/c*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*
c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)
+(c*x^2+b*x+a)^(1/2)))))-1/6*a/c*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*
(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*
ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))))+(d*h+e*g)*(1/5*(c*x^2+b*x+a)
^(5/2)/c-1/2*b/c*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1
/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c
^(1/2)+(c*x^2+b*x+a)^(1/2)))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 915 vs. 2(392) = 784.

Time = 0.65 (sec) , antiderivative size = 1833, normalized size of antiderivative = 4.39

$$\int (g+hx)\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)\,dx=\text{Too large to display}$$

```
[In] integrate((h*x+g)*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] [1/430080*(105*(2*(24*(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*d - 12*(b^5*c^2
- 8*a*b^3*c^3 + 16*a^2*b*c^4)*e + (7*b^6*c - 60*a*b^4*c^2 + 144*a^2*b^2*c^3
```

```
 - 64*a^3*c^4)*f)*g - (24*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*d - 2*(7*b
^6*c - 60*a*b^4*c^2 + 144*a^2*b^2*c^3 - 64*a^3*c^4)*e + 3*(3*b^7 - 28*a*b^5
*c + 80*a^2*b^3*c^2 - 64*a^3*b*c^3)*f)*h)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x
- b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(15360*c^7
*f*h*x^6 + 1280*(14*c^7*f*g + (14*c^7*e + 15*b*c^6*f)*h)*x^5 + 128*(14*(12*
c^7*e + 13*b*c^6*f)*g + (168*c^7*d + 182*b*c^6*e + 3*(b^2*c^5 + 64*a*c^6)*f
)*h)*x^4 + 16*(14*(120*c^7*d + 132*b*c^6*e + (3*b^2*c^5 + 140*a*c^6)*f)*g +
 (1848*b*c^6*d + 14*(3*b^2*c^5 + 140*a*c^6)*e - 3*(9*b^3*c^4 - 44*a*b*c^5)*
f)*h)*x^3 + 8*(14*(360*b*c^6*d + 12*(b^2*c^5 + 32*a*c^6)*e - (7*b^3*c^4 - 3
6*a*b*c^5)*f)*g + (168*(b^2*c^5 + 32*a*c^6)*d - 14*(7*b^3*c^4 - 36*a*b*c^5)
*e + 3*(21*b^4*c^3 - 124*a*b^2*c^4 + 128*a^2*c^5)*f)*h)*x^2 - 14*(120*(3*b^
3*c^4 - 20*a*b*c^5)*d - 12*(15*b^4*c^3 - 100*a*b^2*c^4 + 128*a^2*c^5)*e + (
105*b^5*c^2 - 760*a*b^3*c^3 + 1296*a^2*b*c^4)*f)*g + (168*(15*b^4*c^3 - 100
*a*b^2*c^4 + 128*a^2*c^5)*d - 14*(105*b^5*c^2 - 760*a*b^3*c^3 + 1296*a^2*b*
c^4)*e + 3*(315*b^6*c - 2520*a*b^4*c^2 + 5488*a^2*b^2*c^3 - 2048*a^3*c^4)*f
)*h + 2*(14*(120*(b^2*c^5 + 20*a*c^6)*d - 12*(5*b^3*c^4 - 28*a*b*c^5)*e + (
35*b^4*c^3 - 216*a*b^2*c^4 + 240*a^2*c^5)*f)*g - (168*(5*b^3*c^4 - 28*a*b*c
^5)*d - 14*(35*b^4*c^3 - 216*a*b^2*c^4 + 240*a^2*c^5)*e + 3*(105*b^5*c^2 -
728*a*b^3*c^3 + 1168*a^2*b*c^4)*f)*h)*x)*sqrt(c*x^2 + b*x + a))/c^6, -1/215
040*(105*(2*(24*(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*d - 12*(b^5*c^2 - 8*a*
b^3*c^3 + 16*a^2*b*c^4)*e + (7*b^6*c - 60*a*b^4*c^2 + 144*a^2*b^2*c^3 - 64*
a^3*c^4)*f)*g - (24*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*d - 2*(7*b^6*c -
 60*a*b^4*c^2 + 144*a^2*b^2*c^3 - 64*a^3*c^4)*e + 3*(3*b^7 - 28*a*b^5*c + 8
0*a^2*b^3*c^2 - 64*a^3*b*c^3)*f)*h)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x +
a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(15360*c^7*f*h*x^6 + 1
280*(14*c^7*f*g + (14*c^7*e + 15*b*c^6*f)*h)*x^5 + 128*(14*(12*c^7*e + 13*b
*c^6*f)*g + (168*c^7*d + 182*b*c^6*e + 3*(b^2*c^5 + 64*a*c^6)*f)*h)*x^4 + 1
6*(14*(120*c^7*d + 132*b*c^6*e + (3*b^2*c^5 + 140*a*c^6)*f)*g + (1848*b*c^6
*d + 14*(3*b^2*c^5 + 140*a*c^6)*e - 3*(9*b^3*c^4 - 44*a*b*c^5)*f)*h)*x^3 +
8*(14*(360*b*c^6*d + 12*(b^2*c^5 + 32*a*c^6)*e - (7*b^3*c^4 - 36*a*b*c^5)*f
)*g + (168*(b^2*c^5 + 32*a*c^6)*d - 14*(7*b^3*c^4 - 36*a*b*c^5)*e + 3*(21*b
^4*c^3 - 124*a*b^2*c^4 + 128*a^2*c^5)*f)*h)*x^2 - 14*(120*(3*b^3*c^4 - 20*a
*b*c^5)*d - 12*(15*b^4*c^3 - 100*a*b^2*c^4 + 128*a^2*c^5)*e + (105*b^5*c^2
- 760*a*b^3*c^3 + 1296*a^2*b*c^4)*f)*g + (168*(15*b^4*c^3 - 100*a*b^2*c^4 +
 128*a^2*c^5)*d - 14*(105*b^5*c^2 - 760*a*b^3*c^3 + 1296*a^2*b*c^4)*e + 3*(
315*b^6*c - 2520*a*b^4*c^2 + 5488*a^2*b^2*c^3 - 2048*a^3*c^4)*f)*h + 2*(14*
(120*(b^2*c^5 + 20*a*c^6)*d - 12*(5*b^3*c^4 - 28*a*b*c^5)*e + (35*b^4*c^3 -
 216*a*b^2*c^4 + 240*a^2*c^5)*f)*g - (168*(5*b^3*c^4 - 28*a*b*c^5)*d - 14*(
35*b^4*c^3 - 216*a*b^2*c^4 + 240*a^2*c^5)*e + 3*(105*b^5*c^2 - 728*a*b^3*c^
3 + 1168*a^2*b*c^4)*f)*h)*x)*sqrt(c*x^2 + b*x + a))/c^6]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3990 vs. 2(435) = 870.

Time = 1.13 (sec) , antiderivative size = 3990, normalized size of antiderivative = 9.55

$$\int (g+hx)\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)\,dx=\text{Too large to display}$$

```
[In] integrate((h*x+g)*(c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d),x)
```

```
[Out] Piecewise((sqrt(a + b*x + c*x**2)*(c*f*h*x**6/7 + x**5*(15*b*c*f*h/14 + c**
2*e*h + c**2*f*g)/(6*c) + x**4*(8*a*c*f*h/7 + b**2*f*h + 2*b*c*e*h + 2*b*c*
f*g - 11*b*(15*b*c*f*h/14 + c**2*e*h + c**2*f*g)/(12*c) + c**2*d*h + c**2*e
*g)/(5*c) + x**3*(2*a*b*f*h + 2*a*c*e*h + 2*a*c*f*g - 5*a*(15*b*c*f*h/14 +
c**2*e*h + c**2*f*g)/(6*c) + b**2*e*h + b**2*f*g + 2*b*c*d*h + 2*b*c*e*g -
9*b*(8*a*c*f*h/7 + b**2*f*h + 2*b*c*e*h + 2*b*c*f*g - 11*b*(15*b*c*f*h/14 +
 c**2*e*h + c**2*f*g)/(12*c) + c**2*d*h + c**2*e*g)/(10*c) + c**2*d*g)/(4*c
) + x**2*(a**2*f*h + 2*a*b*e*h + 2*a*b*f*g + 2*a*c*d*h + 2*a*c*e*g - 4*a*(8
*a*c*f*h/7 + b**2*f*h + 2*b*c*e*h + 2*b*c*f*g - 11*b*(15*b*c*f*h/14 + c**2*
e*h + c**2*f*g)/(12*c) + c**2*d*h + c**2*e*g)/(5*c) + b**2*d*h + b**2*e*g +
 2*b*c*d*g - 7*b*(2*a*b*f*h + 2*a*c*e*h + 2*a*c*f*g - 5*a*(15*b*c*f*h/14 +
c**2*e*h + c**2*f*g)/(6*c) + b**2*e*h + b**2*f*g + 2*b*c*d*h + 2*b*c*e*g -
9*b*(8*a*c*f*h/7 + b**2*f*h + 2*b*c*e*h + 2*b*c*f*g - 11*b*(15*b*c*f*h/14 +
 c**2*e*h + c**2*f*g)/(12*c) + c**2*d*h + c**2*e*g)/(10*c) + c**2*d*g)/(8*c
))/(3*c) + x*(a**2*e*h + a**2*f*g + 2*a*b*d*h + 2*a*b*e*g + 2*a*c*d*g - 3*a
*(2*a*b*f*h + 2*a*c*e*h + 2*a*c*f*g - 5*a*(15*b*c*f*h/14 + c**2*e*h + c**2*
f*g)/(6*c) + b**2*e*h + b**2*f*g + 2*b*c*d*h + 2*b*c*e*g - 9*b*(8*a*c*f*h/7
 + b**2*f*h + 2*b*c*e*h + 2*b*c*f*g - 11*b*(15*b*c*f*h/14 + c**2*e*h + c**2
*f*g)/(12*c) + c**2*d*h + c**2*e*g)/(10*c) + c**2*d*g)/(4*c) + b**2*d*g - 5
*b*(a**2*f*h + 2*a*b*e*h + 2*a*b*f*g + 2*a*c*d*h + 2*a*c*e*g - 4*a*(8*a*c*f
*h/7 + b**2*f*h + 2*b*c*e*h + 2*b*c*f*g - 11*b*(15*b*c*f*h/14 + c**2*e*h +
c**2*f*g)/(12*c) + c**2*d*h + c**2*e*g)/(5*c) + b**2*d*h + b**2*e*g + 2*b*c
*d*g - 7*b*(2*a*b*f*h + 2*a*c*e*h + 2*a*c*f*g - 5*a*(15*b*c*f*h/14 + c**2*e
*h + c**2*f*g)/(6*c) + b**2*e*h + b**2*f*g + 2*b*c*d*h + 2*b*c*e*g - 9*b*(8
*a*c*f*h/7 + b**2*f*h + 2*b*c*e*h + 2*b*c*f*g - 11*b*(15*b*c*f*h/14 + c**2*
e*h + c**2*f*g)/(12*c) + c**2*d*h + c**2*e*g)/(10*c) + c**2*d*g)/(8*c))/(6*
c))/(2*c) + (a**2*d*h + a**2*e*g + 2*a*b*d*g - 2*a*(a**2*f*h + 2*a*b*e*h +
2*a*b*f*g + 2*a*c*d*h + 2*a*c*e*g - 4*a*(8*a*c*f*h/7 + b**2*f*h + 2*b*c*e*h
 + 2*b*c*f*g - 11*b*(15*b*c*f*h/14 + c**2*e*h + c**2*f*g)/(12*c) + c**2*d*h
 + c**2*e*g)/(5*c) + b**2*d*h + b**2*e*g + 2*b*c*d*g - 7*b*(2*a*b*f*h + 2*a
*c*e*h + 2*a*c*f*g - 5*a*(15*b*c*f*h/14 + c**2*e*h + c**2*f*g)/(6*c) + b**2
*e*h + b**2*f*g + 2*b*c*d*h + 2*b*c*e*g - 9*b*(8*a*c*f*h/7 + b**2*f*h + 2*b
*c*e*h + 2*b*c*f*g - 11*b*(15*b*c*f*h/14 + c**2*e*h + c**2*f*g)/(12*c) + c*
*2*d*h + c**2*e*g)/(10*c) + c**2*d*g)/(8*c))/(3*c) - 3*b*(a**2*e*h + a**2*f
*g + 2*a*b*d*h + 2*a*b*e*g + 2*a*c*d*g - 3*a*(2*a*b*f*h + 2*a*c*e*h + 2*a*c
```

```
*f*g - 5*a*(15*b*c*f*h/14 + c**2*e*h + c**2*f*g)/(6*c) + b**2*e*h + b**2*f*
g + 2*b*c*d*h + 2*b*c*e*g - 9*b*(8*a*c*f*h/7 + b**2*f*h + 2*b*c*e*h + 2*b*c
*f*g - 11*b*(15*b*c*f*h/14 + c**2*e*h + c**2*f*g)/(12*c) + c**2*d*h + c**2*
e*g)/(10*c) + c**2*d*g)/(4*c) + b**2*d*g - 5*b*(a**2*f*h + 2*a*b*e*h + 2*a*
b*f*g + 2*a*c*d*h + 2*a*c*e*g - 4*a*(8*a*c*f*h/7 + b**2*f*h + 2*b*c*e*h + 2
*b*c*f*g - 11*b*(15*b*c*f*h/14 + c**2*e*h + c**2*f*g)/(12*c) + c**2*d*h + c
**2*e*g)/(5*c) + b**2*d*h + b**2*e*g + 2*b*c*d*g - 7*b*(2*a*b*f*h + 2*a*c*e
*h + 2*a*c*f*g - 5*a*(15*b*c*f*h/14 + c**2*e*h + c**2*f*g)/(6*c) + b**2*e*h
 + b**2*f*g + 2*b*c*d*h + 2*b*c*e*g - 9*b*(8*a*c*f*h/7 + b**2*f*h + 2*b*c*e
*h + 2*b*c*f*g - 11*b*(15*b*c*f*h/14 + c**2*e*h + c**2*f*g)/(12*c) + c**2*d
*h + c**2*e*g)/(10*c) + c**2*d*g)/(8*c))/(6*c))/(4*c))/c) + (a**2*d*g - a*(
a**2*e*h + a**2*f*g + 2*a*b*d*h + 2*a*b*e*g + 2*a*c*d*g - 3*a*(2*a*b*f*h +
2*a*c*e*h + 2*a*c*f*g - 5*a*(15*b*c*f*h/14 + c**2*e*h + c**2*f*g)/(6*c) + b
**2*e*h + b**2*f*g + 2*b*c*d*h + 2*b*c*e*g - 9*b*(8*a*c*f*h/7 + b**2*f*h +
2*b*c*e*h + 2*b*c*f*g - 11*b*(15*b*c*f*h/14 + c**2*e*h + c**2*f*g)/(12*c) +
 c**2*d*h + c**2*e*g)/(10*c) + c**2*d*g)/(4*c) + b**2*d*g - 5*b*(a**2*f*h +
 2*a*b*e*h + 2*a*b*f*g + 2*a*c*d*h + 2*a*c*e*g - 4*a*(8*a*c*f*h/7 + b**2*f*
h + 2*b*c*e*h + 2*b*c*f*g - 11*b*(15*b*c*f*h/14 + c**2*e*h + c**2*f*g)/(12*
c) + c**2*d*h + c**2*e*g)/(5*c) + b**2*d*h + b**2*e*g + 2*b*c*d*g - 7*b*(2*
a*b*f*h + 2*a*c*e*h + 2*a*c*f*g - 5*a*(15*b*c*f*h/14 + c**2*e*h + c**2*f*g)
/(6*c) + b**2*e*h + b**2*f*g + 2*b*c*d*h + 2*b*c*e*g - 9*b*(8*a*c*f*h/7 + b
**2*f*h + 2*b*c*e*h + 2*b*c*f*g - 11*b*(15*b*c*f*h/14 + c**2*e*h + c**2*f*g
)/(12*c) + c**2*d*h + c**2*e*g)/(10*c) + c**2*d*g)/(8*c))/(6*c))/(2*c) - b*
(a**2*d*h + a**2*e*g + 2*a*b*d*g - 2*a*(a**2*f*h + 2*a*b*e*h + 2*a*b*f*g +
2*a*c*d*h + 2*a*c*e*g - 4*a*(8*a*c*f*h/7 + b**2*f*h + 2*b*c*e*h + 2*b*c*f*g
 - 11*b*(15*b*c*f*h/14 + c**2*e*h + c**2*f*g)/(12*c) + c**2*d*h + c**2*e*g)
/(5*c) + b**2*d*h + b**2*e*g + 2*b*c*d*g - 7*b*(2*a*b*f*h + 2*a*c*e*h + 2*a
*c*f*g - 5*a*(15*b*c*f*h/14 + c**2*e*h + c**2*f*g)/(6*c) + b**2*e*h + b**2*
f*g + 2*b*c*d*h + 2*b*c*e*g - 9*b*(8*a*c*f*h/7 + b**2*f*h + 2*b*c*e*h + 2*b
*c*f*g - 11*b*(15*b*c*f*h/14 + c**2*e*h + c**2*f*g)/(12*c) + c**2*d*h + c**
2*e*g)/(10*c) + c**2*d*g)/(8*c))/(3*c) - 3*b*(a**2*e*h + a**2*f*g + 2*a*b*d
*h + 2*a*b*e*g + 2*a*c*d*g - 3*a*(2*a*b*f*h + 2*a*c*e*h + 2*a*c*f*g - 5*a*(
15*b*c*f*h/14 + c**2*e*h + c**2*f*g)/(6*c) + b**2*e*h + b**2*f*g + 2*b*c*d*
h + 2*b*c*e*g - 9*b*(8*a*c*f*h/7 + b**2*f*h + 2*b*c*e*h + 2*b*c*f*g - 11*b*
(15*b*c*f*h/14 + c**2*e*h + c**2*f*g)/(12*c) + c**2*d*h + c**2*e*g)/(10*c)
+ c**2*d*g)/(4*c) + b**2*d*g - 5*b*(a**2*f*h + 2*a*b*e*h + 2*a*b*f*g + 2*a*
c*d*h + 2*a*c*e*g - 4*a*(8*a*c*f*h/7 + b**2*f*h + 2*b*c*e*h + 2*b*c*f*g - 1
1*b*(15*b*c*f*h/14 + c**2*e*h + c**2*f*g)/(12*c) + c**2*d*h + c**2*e*g)/(5*
c) + b**2*d*h + b**2*e*g + 2*b*c*d*g - 7*b*(2*a*b*f*h + 2*a*c*e*h + 2*a*c*f
*g - 5*a*(15*b*c*f*h/14 + c**2*e*h + c**2*f*g)/(6*c) + b**2*e*h + b**2*f*g
+ 2*b*c*d*h + 2*b*c*e*g - 9*b*(8*a*c*f*h/7 + b**2*f*h + 2*b*c*e*h + 2*b*c*f
*g - 11*b*(15*b*c*f*h/14 + c**2*e*h + c**2*f*g)/(12*c) + c**2*d*h + c**2*e*
g)/(10*c) + c**2*d*g)/(8*c))/(6*c))/(4*c))/(2*c))*Piecewise((log(b + 2*sqrt
(c)*sqrt(a + b*x + c*x**2) + 2*c*x)/sqrt(c), Ne(a - b**2/(4*c), 0)), ((b/(2
*c) + x)*log(b/(2*c) + x)/sqrt(c*(b/(2*c) + x)**2), True)), Ne(c, 0)), (2*(
```

```
f*h*(a + b*x)**(11/2)/(11*b**3) + (a + b*x)**(9/2)*(-3*a*f*h + b*e*h + b*f*
g)/(9*b**3) + (a + b*x)**(7/2)*(3*a**2*f*h - 2*a*b*e*h - 2*a*b*f*g + b**2*d
*h + b**2*e*g)/(7*b**3) + (a + b*x)**(5/2)*(-a**3*f*h + a**2*b*e*h + a**2*b
*f*g - a*b**2*d*h - a*b**2*e*g + b**3*d*g)/(5*b**3))/b, Ne(b, 0)), (a**(3/2
)*(d*g*x + f*h*x**4/4 + x**3*(e*h + f*g)/3 + x**2*(d*h + e*g)/2), True))
```

Maxima [F(-2)]

Exception generated.

$$\int (g+hx)\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((h*x+g)*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 925 vs. 2(392) = 784.

Time = 0.31 (sec) , antiderivative size = 925, normalized size of antiderivative = 2.21

$$\int (g+hx)\left(a+bx+cx^2\right)^{3/2}(d+ex+fx^2)\,dx = \frac{1}{107520}\sqrt{cx^2+bx+a}\,(2\,(4\,(2\,(8\,(10\,(12\,cfhx+\frac{14\,c^7fg+14\,c^7eh+15\,bc^6fh}{c^6})\,x+\frac{168\,c^7}{}$$

$$-\frac{(48\,b^4c^3dg-384\,ab^2c^4dg+768\,a^2c^5dg-24\,b^5c^2eg+192\,ab^3c^3eg-384\,a^2bc^4eg+14\,b^6cfg-120\,ab^4c^2fg}{}$$

```
[In] integrate((h*x+g)*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] 1/107520*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*(12*c*f*h*x + (14*c^7*f*g +
14*c^7*e*h + 15*b*c^6*f*h)/c^6)*x + (168*c^7*e*g + 182*b*c^6*f*g + 168*c^7*
d*h + 182*b*c^6*e*h + 3*b^2*c^5*f*h + 192*a*c^6*f*h)/c^6)*x + (1680*c^7*d*g
 + 1848*b*c^6*e*g + 42*b^2*c^5*f*g + 1960*a*c^6*f*g + 1848*b*c^6*d*h + 42*b
^2*c^5*e*h + 1960*a*c^6*e*h - 27*b^3*c^4*f*h + 132*a*b*c^5*f*h)/c^6)*x + (5
040*b*c^6*d*g + 168*b^2*c^5*e*g + 5376*a*c^6*e*g - 98*b^3*c^4*f*g + 504*a*b
*c^5*f*g + 168*b^2*c^5*d*h + 5376*a*c^6*d*h - 98*b^3*c^4*e*h + 504*a*b*c^5*
e*h + 63*b^4*c^3*f*h - 372*a*b^2*c^4*f*h + 384*a^2*c^5*f*h)/c^6)*x + (1680*
b^2*c^5*d*g + 33600*a*c^6*d*g - 840*b^3*c^4*e*g + 4704*a*b*c^5*e*g + 490*b^
4*c^3*f*g - 3024*a*b^2*c^4*f*g + 3360*a^2*c^5*f*g - 840*b^3*c^4*d*h + 4704*
```

```
a*b*c^5*d*h + 490*b^4*c^3*e*h - 3024*a*b^2*c^4*e*h + 3360*a^2*c^5*e*h - 315
*b^5*c^2*f*h + 2184*a*b^3*c^3*f*h - 3504*a^2*b*c^4*f*h)/c^6)*x - (5040*b^3*
c^4*d*g - 33600*a*b*c^5*d*g - 2520*b^4*c^3*e*g + 16800*a*b^2*c^4*e*g - 2150
4*a^2*c^5*e*g + 1470*b^5*c^2*f*g - 10640*a*b^3*c^3*f*g + 18144*a^2*b*c^4*f*
g - 2520*b^4*c^3*d*h + 16800*a*b^2*c^4*d*h - 21504*a^2*c^5*d*h + 1470*b^5*c
^2*e*h - 10640*a*b^3*c^3*e*h + 18144*a^2*b*c^4*e*h - 945*b^6*c*f*h + 7560*a
*b^4*c^2*f*h - 16464*a^2*b^2*c^3*f*h + 6144*a^3*c^4*f*h)/c^6) - 1/2048*(48*
b^4*c^3*d*g - 384*a*b^2*c^4*d*g + 768*a^2*c^5*d*g - 24*b^5*c^2*e*g + 192*a*
b^3*c^3*e*g - 384*a^2*b*c^4*e*g + 14*b^6*c*f*g - 120*a*b^4*c^2*f*g + 288*a^
2*b^2*c^3*f*g - 128*a^3*c^4*f*g - 24*b^5*c^2*d*h + 192*a*b^3*c^3*d*h - 384*
a^2*b*c^4*d*h + 14*b^6*c*e*h - 120*a*b^4*c^2*e*h + 288*a^2*b^2*c^3*e*h - 12
8*a^3*c^4*e*h - 9*b^7*f*h + 84*a*b^5*c*f*h - 240*a^2*b^3*c^2*f*h + 192*a^3*
b*c^3*f*h)*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(1
1/2)
```

Mupad [F(-1)]

Timed out.

$$\int (g+hx)\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)\,dx=\int (g+h\,x)\,\left(c\,x^2+b\,x+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right)\,dx$$

```
[In] int((g + h*x)*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x)
[Out] int((g + h*x)*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2), x)
```

3.199 $\int (a+bx+cx^2)^{3/2} (d+ex+fx^2)\, dx$

Optimal result

Integrand size = 25, antiderivative size = 236

$$\begin{aligned}\int (a+bx+cx^2)^{3/2} (d+ex+fx^2)\, dx = & -\frac{(b^2-4ac)(24c^2d+7b^2f-4c(3be+af))(b+2cx)\sqrt{a+bx+cx^2}}{512c^4} \\ & + \frac{(24c^2d-12bce+7b^2f-4acf)(b+2cx)(a+bx+cx^2)^{3/2}}{192c^3} \\ & + \frac{(12ce-7bf)(a+bx+cx^2)^{5/2}}{60c^2} + \frac{fx(a+bx+cx^2)^{5/2}}{6c} \\ & + \frac{(b^2-4ac)^2(24c^2d+7b^2f-4c(3be+af))\,\mathrm{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{1024c^{9/2}}\end{aligned}$$

```
[Out] 1/192*(-4*a*c*f+7*b^2*f-12*b*c*e+24*c^2*d)*(2*c*x+b)*(c*x^2+b*x+a)^(3/2)/c^
3+1/60*(-7*b*f+12*c*e)*(c*x^2+b*x+a)^(5/2)/c^2+1/6*f*x*(c*x^2+b*x+a)^(5/2)/
c+1/1024*(-4*a*c+b^2)^2*(24*c^2*d+7*b^2*f-4*c*(a*f+3*b*e))*arctanh(1/2*(2*c
*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(9/2)-1/512*(-4*a*c+b^2)*(24*c^2*d+7*b
^2*f-4*c*(a*f+3*b*e))*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/c^4
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1675, 654, 626, 635, 212}

$$\begin{aligned}\int \left(a+bx+cx^2\right)^{3/2}\left(d+ex\right.&\left.+fx^2\right)dx=\frac{\left(b^2-4ac\right)^2\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-4c(af+3be)+7b^2f+24c^2d\right)}{1024c^{9/2}}\\&-\frac{\left(b^2-4ac\right)(b+2cx)\sqrt{a+bx+cx^2}\left(-4c(af+3be)+7b^2f+24c^2d\right)}{512c^4}\\&+\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}\left(-4acf+7b^2f-12bce+24c^2d\right)}{192c^3}\\&+\frac{\left(a+bx+cx^2\right)^{5/2}(12ce-7bf)}{60c^2}+\frac{fx\left(a+bx+cx^2\right)^{5/2}}{6c}\end{aligned}$$

```
[In] Int[(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

```
[Out] -1/512*((b^2 - 4*a*c)*(24*c^2*d + 7*b^2*f - 4*c*(3*b*e + a*f))*(b + 2*c*x)*
Sqrt[a + b*x + c*x^2])/c^4 + ((24*c^2*d - 12*b*c*e + 7*b^2*f - 4*a*c*f)*(b
+ 2*c*x)*(a + b*x + c*x^2)^(3/2))/(192*c^3) + ((12*c*e - 7*b*f)*(a + b*x +
c*x^2)^(5/2))/(60*c^2) + (f*x*(a + b*x + c*x^2)^(5/2))/(6*c) + ((b^2 - 4*a*
c)^2*(24*c^2*d + 7*b^2*f - 4*c*(3*b*e + a*f))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c
]*Sqrt[a + b*x + c*x^2])])/(1024*c^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{fx(a+bx+cx^2)^{5/2}}{6c}+\frac{\int \left(6cd-af+\frac{1}{2}(12ce-7bf)x\right)\left(a+bx+cx^2\right)^{3/2}\,dx}{6c}\\
&= \frac{(12ce-7bf)\left(a+bx+cx^2\right)^{5/2}}{60c^2}+\frac{fx(a+bx+cx^2)^{5/2}}{6c}\\
&\quad+\frac{\left(2c(6cd-af)-\frac{1}{2}b(12ce-7bf)\right)\int \left(a+bx+cx^2\right)^{3/2}\,dx}{12c^2}\\
&= \frac{(24c^2d-12bce+7b^2f-4acf)\,(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{192c^3}\\
&\quad+\frac{(12ce-7bf)\left(a+bx+cx^2\right)^{5/2}}{60c^2}+\frac{fx(a+bx+cx^2)^{5/2}}{6c}\\
&\quad-\frac{\left((b^2-4ac)\left(24c^2d+7b^2f-4c(3be+af)\right)\right)\int \sqrt{a+bx+cx^2}\,dx}{128c^3}\\
&= -\frac{(b^2-4ac)\left(24c^2d+7b^2f-4c(3be+af)\right)(b+2cx)\sqrt{a+bx+cx^2}}{512c^4}\\
&\quad+\frac{(24c^2d-12bce+7b^2f-4acf)\,(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{192c^3}\\
&\quad+\frac{(12ce-7bf)\left(a+bx+cx^2\right)^{5/2}}{60c^2}+\frac{fx(a+bx+cx^2)^{5/2}}{6c}\\
&\quad+\frac{\left((b^2-4ac)^2\left(24c^2d+7b^2f-4c(3be+af)\right)\right)\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{1024c^4}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{\left(b^2-4ac\right)\left(24c^2d+7b^2f-4c(3be+af)\right)(b+2cx)\sqrt{a+bx+cx^2}}{512c^4} \\
&\quad + \frac{\left(24c^2d-12bce+7b^2f-4acf\right)(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{192c^3} \\
&\quad + \frac{(12ce-7bf)\left(a+bx+cx^2\right)^{5/2}}{60c^2} + \frac{fx\left(a+bx+cx^2\right)^{5/2}}{6c} \\
&\quad + \frac{\left(\left(b^2-4ac\right)^2\left(24c^2d+7b^2f-4c(3be+af)\right)\right)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{512c^4} \\
&= -\frac{\left(b^2-4ac\right)\left(24c^2d+7b^2f-4c(3be+af)\right)(b+2cx)\sqrt{a+bx+cx^2}}{512c^4} \\
&\quad + \frac{\left(24c^2d-12bce+7b^2f-4acf\right)(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{192c^3} \\
&\quad + \frac{(12ce-7bf)\left(a+bx+cx^2\right)^{5/2}}{60c^2} + \frac{fx\left(a+bx+cx^2\right)^{5/2}}{6c} \\
&\quad + \frac{\left(b^2-4ac\right)^2\left(24c^2d+7b^2f-4c(3be+af)\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{1024c^{9/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.24

$$
\int \left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)\,dx = \frac{\sqrt{c}\sqrt{a+x(b+cx)}(-105b^5f+10b^4c(18e+7fx)-8b^3c(45cd-95af+cx(15e+7fx))+48b^2c}{}
$$

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

```
[Out] (Sqrt[c]*Sqrt[a + x*(b + c*x)]*(-105*b^5*f + 10*b^4*c*(18*e + 7*f*x) - 8*b^
3*c*(45*c*d - 95*a*f + c*x*(15*e + 7*f*x)) + 48*b^2*c^2*(-(a*(25*e + 9*f*x)
) + c*x*(5*d + x*(2*e + f*x))) + 16*b*c^2*(-81*a^2*f + 6*a*c*(25*d + x*(7*e
 + 3*f*x)) + 4*c^2*x^2*(45*d + x*(33*e + 26*f*x))) + 32*c^3*(3*a^2*(16*e +
5*f*x) + 4*c^2*x^3*(15*d + 2*x*(6*e + 5*f*x)) + 2*a*c*x*(75*d + x*(48*e + 3
5*f*x)))) + 15*(b^2 - 4*a*c)^2*(24*c^2*d + 7*b^2*f - 4*c*(3*b*e + a*f))*Arc
Tanh[(Sqrt[c]*x)/(-Sqrt[a] + Sqrt[a + x*(b + c*x)])]])/(7680*c^(9/2))
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.69

method	result
risch	$-\frac{\left(-1280c^{5} f x^{5}-1664b c^{4} f x^{4}-1536c^{5} e x^{4}-2240a c^{4} f x^{3}-48b^{2}c^{3} f x^{3}-2112b c^{4} e x^{3}-1920c^{5} d x^{3}-288ab c^{3} f x^{2}-3072a c^{4} e x^{2}+56b\right.}{}$
default	$d\left(\frac{(2cx+b)\left(c x^{2}+bx+a\right)^{\frac{3}{2}}}{8c}+\frac{3\left(4ac-b^{2}\right)\left(\frac{(2cx+b)\sqrt{c x^{2}+bx+a}}{4c}+\frac{\left(4ac-b^{2}\right)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c x^{2}+bx+a}\right)}{8c^{\frac{3}{2}}}\right)}{16c}\right)+f\left(\frac{x\left(c x^{2}+bx+a\right)^{\frac{5}{2}}}{6c}\right.$

```
[In] int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/7680/c^4*(-1280*c^5*f*x^5-1664*b*c^4*f*x^4-1536*c^5*e*x^4-2240*a*c^4*f*x
^3-48*b^2*c^3*f*x^3-2112*b*c^4*e*x^3-1920*c^5*d*x^3-288*a*b*c^3*f*x^2-3072*
a*c^4*e*x^2+56*b^3*c^2*f*x^2-96*b^2*c^3*e*x^2-2880*b*c^4*d*x^2-480*a^2*c^3*
f*x+432*a*b^2*c^2*f*x-672*a*b*c^3*e*x-4800*a*c^4*d*x-70*b^4*c*f*x+120*b^3*c
^2*e*x-240*b^2*c^3*d*x+1296*a^2*b*c^2*f-1536*a^2*c^3*e-760*a*b^3*c*f+1200*a
*b^2*c^2*e-2400*a*b*c^3*d+105*b^5*f-180*b^4*c*e+360*b^3*c^2*d)*(c*x^2+b*x+a
)^(1/2)-1/1024*(64*a^3*c^3*f-144*a^2*b^2*c^2*f+192*a^2*b*c^3*e-384*a^2*c^4*
d+60*a*b^4*c*f-96*a*b^3*c^2*e+192*a*b^2*c^3*d-7*b^6*f+12*b^5*c*e-24*b^4*c^2
*d)/c^(9/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 839, normalized size of antiderivative = 3.56

$$\int (a+bx+cx^2)^{3/2} (d+ex$$

$$+fx^2)\ dx = \Big[-\frac{15\,(24\,(b^4c^2-8\,ab^2c^3+16\,a^2c^4)d-12\,(b^5c-8\,ab^3c^2+16\,a^2bc^3)e+(7\,b^6-60\,ab^4c+144\,a^2b^2}{}$$

$$-\frac{15\,(24\,(b^4c^2-8\,ab^2c^3+16\,a^2c^4)d-12\,(b^5c-8\,ab^3c^2+16\,a^2bc^3)e+(7\,b^6-60\,ab^4c+144\,a^2b^2c^2-64\,a^3c^3)}{}$$

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] [-1/30720*(15*(24*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d - 12*(b^5*c - 8*a*
b^3*c^2 + 16*a^2*b*c^3)*e + (7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c^2 - 64*a^3*
c^3)*f)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2
*c*x + b)*sqrt(c) - 4*a*c) - 4*(1280*c^6*f*x^5 + 128*(12*c^6*e + 13*b*c^5*f
)*x^4 + 16*(120*c^6*d + 132*b*c^5*e + (3*b^2*c^4 + 140*a*c^5)*f)*x^3 + 8*(3
60*b*c^5*d + 12*(b^2*c^4 + 32*a*c^5)*e - (7*b^3*c^3 - 36*a*b*c^4)*f)*x^2 -
120*(3*b^3*c^3 - 20*a*b*c^4)*d + 12*(15*b^4*c^2 - 100*a*b^2*c^3 + 128*a^2*c
^4)*e - (105*b^5*c - 760*a*b^3*c^2 + 1296*a^2*b*c^3)*f + 2*(120*(b^2*c^4 +
20*a*c^5)*d - 12*(5*b^3*c^3 - 28*a*b*c^4)*e + (35*b^4*c^2 - 216*a*b^2*c^3 +
 240*a^2*c^4)*f)*x)*sqrt(c*x^2 + b*x + a))/c^5, -1/15360*(15*(24*(b^4*c^2 -
 8*a*b^2*c^3 + 16*a^2*c^4)*d - 12*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*e +
(7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c^2 - 64*a^3*c^3)*f)*sqrt(-c)*arctan(1/2*
sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(12
80*c^6*f*x^5 + 128*(12*c^6*e + 13*b*c^5*f)*x^4 + 16*(120*c^6*d + 132*b*c^5*
e + (3*b^2*c^4 + 140*a*c^5)*f)*x^3 + 8*(360*b*c^5*d + 12*(b^2*c^4 + 32*a*c^
5)*e - (7*b^3*c^3 - 36*a*b*c^4)*f)*x^2 - 120*(3*b^3*c^3 - 20*a*b*c^4)*d + 1
2*(15*b^4*c^2 - 100*a*b^2*c^3 + 128*a^2*c^4)*e - (105*b^5*c - 760*a*b^3*c^2
 + 1296*a^2*b*c^3)*f + 2*(120*(b^2*c^4 + 20*a*c^5)*d - 12*(5*b^3*c^3 - 28*a
*b*c^4)*e + (35*b^4*c^2 - 216*a*b^2*c^3 + 240*a^2*c^4)*f)*x)*sqrt(c*x^2 + b
*x + a))/c^5]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1360 vs. 2(230) = 460.

Time = 0.56 (sec) , antiderivative size = 1360, normalized size of antiderivative = 5.76

$$\int \left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)\,dx=\text{Too large to display}$$

```
[In] integrate((c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d),x)
```

```
[Out] Piecewise((sqrt(a + b*x + c*x**2)*(c*f*x**5/6 + x**4*(13*b*c*f/12 + c**2*e)
/(5*c) + x**3*(7*a*c*f/6 + b**2*f + 2*b*c*e - 9*b*(13*b*c*f/12 + c**2*e)/(1
0*c) + c**2*d)/(4*c) + x**2*(2*a*b*f + 2*a*c*e - 4*a*(13*b*c*f/12 + c**2*e)
/(5*c) + b**2*e + 2*b*c*d - 7*b*(7*a*c*f/6 + b**2*f + 2*b*c*e - 9*b*(13*b*c
*f/12 + c**2*e)/(10*c) + c**2*d)/(8*c))/(3*c) + x*(a**2*f + 2*a*b*e + 2*a*c
*d - 3*a*(7*a*c*f/6 + b**2*f + 2*b*c*e - 9*b*(13*b*c*f/12 + c**2*e)/(10*c)
+ c**2*d)/(4*c) + b**2*d - 5*b*(2*a*b*f + 2*a*c*e - 4*a*(13*b*c*f/12 + c**2
*e)/(5*c) + b**2*e + 2*b*c*d - 7*b*(7*a*c*f/6 + b**2*f + 2*b*c*e - 9*b*(13*
b*c*f/12 + c**2*e)/(10*c) + c**2*d)/(8*c))/(6*c))/(2*c) + (a**2*e + 2*a*b*d
 - 2*a*(2*a*b*f + 2*a*c*e - 4*a*(13*b*c*f/12 + c**2*e)/(5*c) + b**2*e + 2*b
*c*d - 7*b*(7*a*c*f/6 + b**2*f + 2*b*c*e - 9*b*(13*b*c*f/12 + c**2*e)/(10*c
) + c**2*d)/(8*c))/(3*c) - 3*b*(a**2*f + 2*a*b*e + 2*a*c*d - 3*a*(7*a*c*f/6
 + b**2*f + 2*b*c*e - 9*b*(13*b*c*f/12 + c**2*e)/(10*c) + c**2*d)/(4*c) + b
**2*d - 5*b*(2*a*b*f + 2*a*c*e - 4*a*(13*b*c*f/12 + c**2*e)/(5*c) + b**2*e
+ 2*b*c*d - 7*b*(7*a*c*f/6 + b**2*f + 2*b*c*e - 9*b*(13*b*c*f/12 + c**2*e)/
(10*c) + c**2*d)/(8*c))/(6*c))/(4*c))/c) + (a**2*d - a*(a**2*f + 2*a*b*e +
2*a*c*d - 3*a*(7*a*c*f/6 + b**2*f + 2*b*c*e - 9*b*(13*b*c*f/12 + c**2*e)/(1
0*c) + c**2*d)/(4*c) + b**2*d - 5*b*(2*a*b*f + 2*a*c*e - 4*a*(13*b*c*f/12 +
 c**2*e)/(5*c) + b**2*e + 2*b*c*d - 7*b*(7*a*c*f/6 + b**2*f + 2*b*c*e - 9*b
*(13*b*c*f/12 + c**2*e)/(10*c) + c**2*d)/(8*c))/(6*c))/(2*c) - b*(a**2*e +
2*a*b*d - 2*a*(2*a*b*f + 2*a*c*e - 4*a*(13*b*c*f/12 + c**2*e)/(5*c) + b**2*
e + 2*b*c*d - 7*b*(7*a*c*f/6 + b**2*f + 2*b*c*e - 9*b*(13*b*c*f/12 + c**2*e
)/(10*c) + c**2*d)/(8*c))/(3*c) - 3*b*(a**2*f + 2*a*b*e + 2*a*c*d - 3*a*(7*
a*c*f/6 + b**2*f + 2*b*c*e - 9*b*(13*b*c*f/12 + c**2*e)/(10*c) + c**2*d)/(4
*c) + b**2*d - 5*b*(2*a*b*f + 2*a*c*e - 4*a*(13*b*c*f/12 + c**2*e)/(5*c) +
b**2*e + 2*b*c*d - 7*b*(7*a*c*f/6 + b**2*f + 2*b*c*e - 9*b*(13*b*c*f/12 + c
**2*e)/(10*c) + c**2*d)/(8*c))/(6*c))/(4*c))/(2*c))*Piecewise((log(b + 2*sq
rt(c)*sqrt(a + b*x + c*x**2) + 2*c*x)/sqrt(c), Ne(a - b**2/(4*c), 0)), ((b/
(2*c) + x)*log(b/(2*c) + x)/sqrt(c*(b/(2*c) + x)**2), True)), Ne(c, 0)), (2
*(f*(a + b*x)**(9/2)/(9*b**2) + (a + b*x)**(7/2)*(-2*a*f + b*e)/(7*b**2) +
(a + b*x)**(5/2)*(a**2*f - a*b*e + b**2*d)/(5*b**2))/b, Ne(b, 0)), (a**(3/2
)*(d*x + e*x**2/2 + f*x**3/3), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right)\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.71

$$\int \left(a+bx+cx^2\right)^{3/2} \left(d+ex\right.$$
$$\left.+fx^2\right)\,dx = \frac{1}{7680}\sqrt{cx^2+bx+a}\left(2\left(4\left(2\left(8\left(10\,cfx+\frac{12\,c^6e+13\,bc^5f}{c^5}\right)x+\frac{120\,c^6d+132\,bc^5e+3\,b^2c^4f -}{c^5}\right.\right.\right.\right.\right.$$
$$-\frac{(24\,b^4c^2d-192\,ab^2c^3d+384\,a^2c^4d-12\,b^5ce+96\,ab^3c^2e-192\,a^2bc^3e+7\,b^6f-60\,ab^4cf+144\,a^2b^2c^2f-6}{1024\,c^{\frac{9}{2}}}$$

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] 1/7680*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*c*f*x + (12*c^6*e + 13*b*c^5*f
)/c^5)*x + (120*c^6*d + 132*b*c^5*e + 3*b^2*c^4*f + 140*a*c^5*f)/c^5)*x + (
360*b*c^5*d + 12*b^2*c^4*e + 384*a*c^5*e - 7*b^3*c^3*f + 36*a*b*c^4*f)/c^5)
*x + (120*b^2*c^4*d + 2400*a*c^5*d - 60*b^3*c^3*e + 336*a*b*c^4*e + 35*b^4*
c^2*f - 216*a*b^2*c^3*f + 240*a^2*c^4*f)/c^5)*x - (360*b^3*c^3*d - 2400*a*b
*c^4*d - 180*b^4*c^2*e + 1200*a*b^2*c^3*e - 1536*a^2*c^4*e + 105*b^5*c*f -
760*a*b^3*c^2*f + 1296*a^2*b*c^3*f)/c^5) - 1/1024*(24*b^4*c^2*d - 192*a*b^2
*c^3*d + 384*a^2*c^4*d - 12*b^5*c*e + 96*a*b^3*c^2*e - 192*a^2*b*c^3*e + 7*
b^6*f - 60*a*b^4*c*f + 144*a^2*b^2*c^2*f - 64*a^3*c^3*f)*log(abs(2*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(9/2)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right)\,dx=\int \left(c\,x^2+b\,x+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right)\,dx$$

```
[In] int((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x)
[Out] int((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2), x)
```

3.200 $\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{g+hx}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 660

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{g+hx}\,dx = \frac{(3b^4fh^4+6b^2ch^3(bfg-beh-2afh)+128c^4g^2(fg^2-h(eg-dh))}{}$$

$$-\frac{(8ch(bfg-2cdh)-(8cg-3bh)(2cfg-2ceh+bfh)+6ch(2cfg-2ceh+bfh)x)\,(a+bx+cx^2)^{3/2}}{48c^2h^3}$$

$$+\frac{f(a+bx+cx^2)^{5/2}}{5ch}$$

$$-\frac{\bigl(4ch(2cg-bh)\,(8ch(bg-2ah)(bfg-2cdh)-g(8bcg-3b^2h-4ach)\,(2cfg-2ceh+bfh))-2\bigl(4c^2g^2-}{}$$

$$+\frac{(cg^2-bgh+ah^2)^{3/2}\,(fg^2-h(eg-dh))\operatorname{arctanh}\left(\frac{bg-2ah+(2cg-bh)x}{2\sqrt{cg^2-bgh+ah^2}\sqrt{a+bx+cx^2}}\right)}{h^6}$$

```
[Out] -1/48*(8*c*h*(b*f*g-2*c*d*h)-(-3*b*h+8*c*g)*(b*f*h-2*c*e*h+2*c*f*g)+6*c*h*(
b*f*h-2*c*e*h+2*c*f*g)*x)*(c*x^2+b*x+a)^(3/2)/c^2/h^3+1/5*f*(c*x^2+b*x+a)^(
5/2)/c/h-1/256*(4*c*h*(-b*h+2*c*g)*(8*c*h*(-2*a*h+b*g)*(b*f*g-2*c*d*h)-g*(-
4*a*c*h-3*b^2*h+8*b*c*g)*(b*f*h-2*c*e*h+2*c*f*g))-2*(4*c^2*g^2-1/2*b^2*h^2-
2*c*h*(-a*h+b*g))*(8*c*h*(-b*h+2*c*g)*(b*f*g-2*c*d*h)-(b*f*h-2*c*e*h+2*c*f*
g)*(16*c^2*g^2-3*b^2*h^2-4*c*h*(-3*a*h+2*b*g))))*arctanh(1/2*(2*c*x+b)/c^(1
/2)/(c*x^2+b*x+a)^(1/2))/c^(7/2)/h^6+(a*h^2-b*g*h+c*g^2)^(3/2)*(f*g^2-h*(-d
*h+e*g))*arctanh(1/2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^(1/2)/(
c*x^2+b*x+a)^(1/2))/h^6+1/128*(3*b^4*f*h^4+6*b^2*c*h^3*(-2*a*f*h-b*e*h+b*f*
g)+128*c^4*g^2*(f*g^2-h*(-d*h+e*g))-32*c^3*h*(-4*a*h+5*b*g)*(f*g^2-h*(-d*h+
e*g))-8*b*c^2*h^2*(3*a*h*(-e*h+f*g)-2*b*(d*h^2-e*g*h+f*g^2))+2*c*h*(8*c*h*(
-b*h+2*c*g)*(b*f*g-2*c*d*h)-(b*f*h-2*c*e*h+2*c*f*g)*(16*c^2*g^2-3*b^2*h^2-4
*c*h*(-3*a*h+2*b*g)))*x)*(c*x^2+b*x+a)^(1/2)/c^3/h^5
```

Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 660, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1667, 828, 857, 635, 212, 738}

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{g+hx}\,dx =$$

$$-\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\Big(4ch(2cg-bh)\,(8ch(bg-2ah)(bfg-2cdh)-g(-4ach-3b^2h+8bcg)\,(bfh-2$$

$$+\frac{(ah^2-bgh+cg^2)^{3/2}\,(fg^2-h(eg-dh))\operatorname{arctanh}\left(\frac{-2ah+x(2cg-bh)+bg}{2\sqrt{a+bx+cx^2}\sqrt{ah^2-bgh+cg^2}}\right)}{h^6}$$

$$+\sqrt{a+bx+cx^2}(2chx(8ch(2cg-bh)(bfg-2cdh)-(-4ch(2bg-3ah)-3b^2h^2+16c^2g^2)\,(bfh-2ceh+2$$

$$-\frac{(a+bx+cx^2)^{3/2}\,(8ch(bfg-2cdh)+6chx(bfh-2ceh+2cfg)-(8cg-3bh)(bfh-2ceh+2cfg))}{48c^2h^3}$$

$$+\frac{f(a+bx+cx^2)^{5/2}}{5ch}$$

```
[In] Int[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x),x]
```

```
[Out] ((3*b^4*f*h^4 + 6*b^2*c*h^3*(b*f*g - b*e*h - 2*a*f*h) - 32*c^3*h*(5*b*g - 4
*a*h)*(f*g^2 - h*(e*g - d*h)) + 128*c^4*(f*g^4 - g^2*h*(e*g - d*h)) - 8*b*c
^2*h^2*(3*a*h*(f*g - e*h) - 2*b*(f*g^2 - e*g*h + d*h^2)) + 2*c*h*(8*c*h*(2*
c*g - b*h)*(b*f*g - 2*c*d*h) - (2*c*f*g - 2*c*e*h + b*f*h)*(16*c^2*g^2 - 3*
b^2*h^2 - 4*c*h*(2*b*g - 3*a*h)))*x)*Sqrt[a + b*x + c*x^2])/(128*c^3*h^5) -
 ((8*c*h*(b*f*g - 2*c*d*h) - (8*c*g - 3*b*h)*(2*c*f*g - 2*c*e*h + b*f*h) +
6*c*h*(2*c*f*g - 2*c*e*h + b*f*h)*x)*(a + b*x + c*x^2)^(3/2))/(48*c^2*h^3)
+ (f*(a + b*x + c*x^2)^(5/2))/(5*c*h) - ((4*c*h*(2*c*g - b*h)*(8*c*h*(b*g -
 2*a*h)*(b*f*g - 2*c*d*h) - g*(8*b*c*g - 3*b^2*h - 4*a*c*h)*(2*c*f*g - 2*c*
e*h + b*f*h)) - 2*(4*c^2*g^2 - (b^2*h^2)/2 - 2*c*h*(b*g - a*h))*(8*c*h*(2*c
*g - b*h)*(b*f*g - 2*c*d*h) - (2*c*f*g - 2*c*e*h + b*f*h)*(16*c^2*g^2 - 3*b
^2*h^2 - 4*c*h*(2*b*g - 3*a*h))))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b
*x + c*x^2])])/(256*c^(7/2)*h^6) + ((c*g^2 - b*g*h + a*h^2)^(3/2)*(f*g^2 -
h*(e*g - d*h))*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*
h + a*h^2]*Sqrt[a + b*x + c*x^2])])/h^6
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
```

`[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))`

Rubi steps

$$\text{integral} = \frac{f(a+bx+cx^2)^{5/2}}{5ch} + \frac{\int \frac{\left(-\frac{5}{2}h(bfg-2cdh)-\frac{5}{2}h(2cfg-2ceh+bfh)x\right)(a+bx+cx^2)^{3/2}}{g+hx}\,dx}{5ch^2}$$

$$= -\frac{(8ch(bfg-2cdh)-(8cg-3bh)(2cfg-2ceh+bfh)+6ch(2cfg-2ceh+bfh)x)(a+bx+c}{48c^2h^3}$$

$$+\frac{f(a+bx+cx^2)^{5/2}}{5ch}$$

$$-\frac{\int \frac{(-\frac{5}{4}h(8ch(bg-2ah)(bfg-2cdh)-g(8bcg-3b^2h-4ach)(2cfg-2ceh+bfh))-\frac{5}{4}h(8ch(2cg-bh)(bfg-2cdh)-2(2cfg-2ceh+bfh)}{g+hx}}{40c^2h^4}$$

$$= \frac{(3b^4fh^4+6b^2ch^3(bfg-beh-2afh)-32c^3h(5bg-4ah)(fg^2-h(eg-dh))+128c^4(fg^4-g^2h}{}$$

$$-\frac{(8ch(bfg-2cdh)-(8cg-3bh)(2cfg-2ceh+bfh)+6ch(2cfg-2ceh+bfh)x)(a+bx+c}{48c^2h^3}$$

$$+\frac{f(a+bx+cx^2)^{5/2}}{5ch}$$

$$+\frac{\int \frac{-\frac{5}{8}h(4ch(bg-2ah)(8ch(bg-2ah)(bfg-2cdh)-g(8bcg-3b^2h-4ach)(2cfg-2ceh+bfh))-g(4bcg-b^2h-4ach)(8ch(2cg-bh)(bfg}{}}{}$$

$$= \frac{(3b^4fh^4+6b^2ch^3(bfg-beh-2afh)-32c^3h(5bg-4ah)(fg^2-h(eg-dh))+128c^4(fg^4-g^2h}{}$$

$$-\frac{(8ch(bfg-2cdh)-(8cg-3bh)(2cfg-2ceh+bfh)+6ch(2cfg-2ceh+bfh)x)(a+bx+c}{48c^2h^3}$$

$$+\frac{f(a+bx+cx^2)^{5/2}}{5ch}$$

$$+\frac{\left((cg^2-bgh+ah^2)^2(fg^2-h(eg-dh))\right)\int \frac{1}{(g+hx)\sqrt{a+bx+cx^2}}\,dx}{h^6}$$

$$-\frac{(4ch(2cg-bh)(8ch(bg-2ah)(bfg-2cdh)-g(8bcg-3b^2h-4ach)(2cfg-2ceh+bfh))-}{}$$

$$= \frac{(3b^4fh^4+6b^2ch^3(bfg-beh-2afh)-32c^3h(5bg-4ah)\left(fg^2-h(eg-dh)\right)+128c^4(fg^4-g^2h(e}{}$$

$$- \frac{(8ch(bfg-2cdh)-(8cg-3bh)(2cfg-2ceh+bfh)+6ch(2cfg-2ceh+bfh)x)\left(a+bx+cx\right.}{48c^2h^3}$$

$$+ \frac{f\left(a+bx+cx^2\right)^{5/2}}{5ch}$$

$$- \frac{\left(2\left(cg^2-bgh+ah^2\right)^2\left(fg^2-h(eg-dh)\right)\right)\text{Subst}\left(\int \frac{1}{4cg^2-4bgh+4ah^2-x^2}\,dx,x,\frac{-bg+2ah-(2cg-bh)x}{\sqrt{a+bx+cx^2}}\right)}{h^6}$$

$$- \frac{\left(4ch(2cg-bh)\left(8ch(bg-2ah)(bfg-2cdh)-g\left(8bcg-3b^2h-4ach\right)(2cfg-2ceh+bfh)\right)-2\right.}{}$$

$$= \frac{(3b^4fh^4+6b^2ch^3(bfg-beh-2afh)-32c^3h(5bg-4ah)\left(fg^2-h(eg-dh)\right)+128c^4(fg^4-g^2h(e}{}$$

$$- \frac{(8ch(bfg-2cdh)-(8cg-3bh)(2cfg-2ceh+bfh)+6ch(2cfg-2ceh+bfh)x)\left(a+bx+cx\right.}{48c^2h^3}$$

$$+ \frac{f\left(a+bx+cx^2\right)^{5/2}}{5ch}$$

$$- \frac{\left(4ch(2cg-bh)\left(8ch(bg-2ah)(bfg-2cdh)-g\left(8bcg-3b^2h-4ach\right)(2cfg-2ceh+bfh)\right)-2\right.}{}$$

$$+ \frac{\left(cg^2-bgh+ah^2\right)^{3/2}\left(fg^2-h(eg-dh)\right)\tanh^{-1}\left(\frac{bg-2ah+(2cg-bh)x}{2\sqrt{cg^2-bgh+ah^2}\sqrt{a+bx+cx^2}}\right)}{h^6}$$

Mathematica [A] (verified)

Time = 5.62 (sec) , antiderivative size = 728, normalized size of antiderivative = 1.10

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{g+hx}\,dx = \frac{h\sqrt{a+x(b+cx)}\left(45b^4fh^4-30b^2ch^3(10afh+b(-3fg+3eh+fhx))+12c^2h^2\left(32a^2fh^2+2abh(-\right.\right.}{}$$

```
[In] Integrate[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x),x]

[Out] ((h*Sqrt[a + x*(b + c*x)]*(45*b^4*f*h^4 - 30*b^2*c*h^3*(10*a*f*h + b*(-3*f*
g + 3*e*h + f*h*x)) + 12*c^2*h^2*(32*a^2*f*h^2 + 2*a*b*h*(-25*f*g + 25*e*h
+ 7*f*h*x) + b^2*(5*h*(-4*e*g + 4*d*h + e*h*x) + f*(20*g^2 - 5*g*h*x + 2*h^
2*x^2))) + 32*c^4*(f*(60*g^4 - 30*g^3*h*x + 20*g^2*h^2*x^2 - 15*g*h^3*x^3 +
 12*h^4*x^4) + 5*h*(2*d*h*(6*g^2 - 3*g*h*x + 2*h^2*x^2) + e*(-12*g^3 + 6*g^
2*h*x - 4*g*h^2*x^2 + 3*h^3*x^3))) + 16*c^3*h*(a*h*(5*h*(-32*e*g + 32*d*h +
 15*e*h*x) + f*(160*g^2 - 75*g*h*x + 48*h^2*x^2)) + b*(f*(-150*g^3 + 70*g^2
*h*x - 45*g*h^2*x^2 + 33*h^3*x^3) + 5*h*(2*d*h*(-15*g + 7*h*x) + e*(30*g^2
- 14*g*h*x + 9*h^2*x^2))))))/c^3 + 3840*Sqrt[-(c*g^2) + h*(b*g - a*h)]*(c*g
```

```
^2 + h*(-(b*g) + a*h))*(f*g^2 + h*(-(e*g) + d*h))*ArcTan[(Sqrt[-(c*g^2) + h
*(b*g - a*h)]*x)/(Sqrt[a]*(g + h*x) - g*Sqrt[a + x*(b + c*x)])] - (15*(3*b^
5*f*h^5 - 6*b^3*c*h^4*(-(b*f*g) + b*e*h + 4*a*f*h) - 384*c^4*g*h*(b*g - a*h
)*(f*g^2 + h*(-(e*g) + d*h)) + 256*c^5*(f*g^5 + g^3*h*(-(e*g) + d*h)) + 16*
b*c^2*h^3*(3*a^2*f*h^2 + 3*a*b*h*(-(f*g) + e*h) + b^2*(f*g^2 - e*g*h + d*h^
2)) + 96*c^3*h^2*(a^2*h^2*(f*g - e*h) + b^2*g*(f*g^2 - e*g*h + d*h^2) - 2*a
*b*h*(f*g^2 - e*g*h + d*h^2)))*ArcTanh[(Sqrt[c]*x)/(-Sqrt[a] + Sqrt[a + x*(
b + c*x)])])/c^(7/2))/(1920*h^6)
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 990, normalized size of antiderivative = 1.50

method	result
default	$eh\left(\frac{(2cx+b)\left(c\,x^{2}+bx+a\right)^{\frac{3}{2}}}{8c}+\frac{3\left(4ac-b^{2}\right)\left(\frac{(2cx+b)\sqrt{c\,x^{2}+bx+a}}{4c}+\frac{\left(4ac-b^{2}\right)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx+a}\right)}{8c^{\frac{3}{2}}}\right)}{16c}\right)+fh\Bigg(\frac{\left(c\,x^{2}+bx+a\right)^{\frac{5}{2}}}{5c}-\frac{b\Big(\frac{(2cx+}{}}{}$
risch	Expression too large to display

```
[In] int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g),x,method=_RETURNVERBOSE)
```

```
[Out] 1/h^2*(e*h*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*
c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)
+(c*x^2+b*x+a)^(1/2))))+f*h*(1/5*(c*x^2+b*x+a)^(5/2)/c-1/2*b/c*(1/8*(2*c*x+
b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^
(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))
)-f*g*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b
)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x
^2+b*x+a)^(1/2)))))+(d*h^2-e*g*h+f*g^2)/h^3*(1/3*((x+1/h*g)^2*c+(b*h-2*c*g)
/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(3/2)+1/2*(b*h-2*c*g)/h*(1/4*(2*c*(x+
1/h*g)+(b*h-2*c*g)/h)/c*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h
+c*g^2)/h^2)^(1/2)+1/8*(4*c*(a*h^2-b*g*h+c*g^2)/h^2-(b*h-2*c*g)^2/h^2)/c^(3
/2)*ln((1/2*(b*h-2*c*g)/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g)^2*c+(b*h-2*c*g)/h
*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)))+(a*h^2-b*g*h+c*g^2)/h^2*(((x+1/
h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)+1/2*(b*h-2*
c*g)/h*ln((1/2*(b*h-2*c*g)/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g)^2*c+(b*h-2*c*g
)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/c^(1/2)-(a*h^2-b*g*h+c*g^2)/h
```

```
^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g
)/h*(x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c+(b*h-2*c*g)/
h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}\,(d+ex+fx^2)}{g+hx}\,dx=\text{Timed out}$$

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{(a+bx+cx^2)^{3/2}\,(d+ex+fx^2)}{g+hx}\,dx=\int \frac{(a+bx+cx^2)^{\frac{3}{2}}\,(d+ex+fx^2)}{g+hx}\,dx$$

```
[In] integrate((c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g),x)
```

```
[Out] Integral((a + b*x + c*x**2)**(3/2)*(d + e*x + f*x**2)/(g + h*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx+cx^2)^{3/2}\,(d+ex+fx^2)}{g+hx}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b*h-2*c*g>0)', see `assume?` for mo
re deta
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{g+hx}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{g+hx}\,dx = \int \frac{(c\,x^2+b\,x+a)^{3/2}\,(f\,x^2+e\,x+d)}{g+h\,x}\,dx$$

```
[In] int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x),x)
```

```
[Out] int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x), x)
```

3.201 $\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 754

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^2}\,dx =$$

$$-\frac{(3b^3fh^3+4bch^2(4bfg-2beh-3afh)+64c^3g(5fg^2-h(4eg-3dh))+16c^2h(4ah(2fg-eh)-b(19fg^2 -}{64}$$

$$-\frac{\Big(3bfh^2(bg-ah)+8c^2g(5fg^2-h(4eg-3dh))+ch(8ah(2fg-eh)-b(43fg^2-8h(4eg-3dh)))+6ch^2\Big(}{24ch^3\,(cg^2-bgh+ah^2)}$$

$$-\frac{(fg^2-h(eg-dh))\,(a+bx+cx^2)^{5/2}}{h\,(cg^2-bgh+ah^2)\,(g+hx)}$$

$$+\frac{(3b^4fh^4+8b^2ch^3(2bfg-beh-3afh)+128c^4g^2(5fg^2-h(4eg-3dh))+48c^2h^2(a^2fh^2-2abh(2fg-eh)}{12}$$

$$-\frac{\sqrt{cg^2-bgh+ah^2}(2cg(5fg^2-h(4eg-3dh))+h(2ah(2fg-eh)-b(7fg^2-5egh+3dh^2)))\,\mathrm{arctanh}\Big(\frac{}{2\sqrt{c}}}{2h^6}$$

```
[Out] -1/24*(3*b*f*h^2*(-a*h+b*g)+8*c^2*g*(5*f*g^2-h*(-3*d*h+4*e*g))+c*h*(8*a*h*(
-e*h+2*f*g)-b*(43*f*g^2-8*h*(-3*d*h+4*e*g)))+6*c*h^2*(4*c*e*g+b*f*g-5*c*f*g
^2/h-4*c*d*h-a*f*h)*x)*(c*x^2+b*x+a)^(3/2)/c/h^3/(a*h^2-b*g*h+c*g^2)-(f*g^2
-h*(-d*h+e*g))*(c*x^2+b*x+a)^(5/2)/h/(a*h^2-b*g*h+c*g^2)/(h*x+g)+1/128*(3*b
^4*f*h^4+8*b^2*c*h^3*(-3*a*f*h-b*e*h+2*b*f*g)+128*c^4*g^2*(5*f*g^2-h*(-3*d*
h+4*e*g))+48*c^2*h^2*(a^2*f*h^2-2*a*b*h*(-e*h+2*f*g)+b^2*(d*h^2-2*e*g*h+3*f
*g^2))+192*c^3*h*(a*h*(d*h^2-2*e*g*h+3*f*g^2)-b*g*(2*d*h^2-3*e*g*h+4*f*g^2)
))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(5/2)/h^6-1/2*(2*c*
g*(5*f*g^2-h*(-3*d*h+4*e*g))+h*(2*a*h*(-e*h+2*f*g)-b*(3*d*h^2-5*e*g*h+7*f*g
^2)))*arctanh(1/2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^(1/2)/(c*x
^2+b*x+a)^(1/2))*(a*h^2-b*g*h+c*g^2)^(1/2)/h^6-1/64*(3*b^3*f*h^3+4*b*c*h^2*
```

```
(-3*a*f*h-2*b*e*h+4*b*f*g)+64*c^3*g*(5*f*g^2-h*(-3*d*h+4*e*g))+16*c^2*h*(4*
a*h*(-e*h+2*f*g)-b*(9*d*h^2-14*e*g*h+19*f*g^2))+2*c*h*(3*b^2*f*h^2+4*c*h*(-
3*a*f*h-2*b*e*h+4*b*f*g)-16*c^2*(5*f*g^2-h*(-3*d*h+4*e*g)))*x)*(c*x^2+b*x+a
)^(1/2)/c^2/h^5
```

Rubi [A] (verified)

Time = 1.47 (sec) , antiderivative size = 750, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1664, 828, 857, 635, 212, 738}

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^2}\,dx = \frac{\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(48c^2h^2(a^2fh^2-2abh(2fg-eh)+b^2(dh}{}$$

$$-\frac{\sqrt{ah^2-bgh+cg^2}\operatorname{arctanh}\left(\frac{-2ah+x(2cg-bh)+bg}{2\sqrt{a+bx+cx^2}\sqrt{ah^2-bgh+cg^2}}\right)(2c(5fg^3-gh(4eg-3dh))-h(-2ah(2fg-eh)-b}{2h^6}$$

$$-\frac{\sqrt{a+bx+cx^2}(2chx(4ch(-3afh-2beh+4bfg)+3b^2fh^2-16c^2(5fg^2-h(4eg-3dh)))-16c^2h(-4ah}{}$$

$$-\frac{(a+bx+cx^2)^{3/2}\left(6chx\left(-afh+bfg-4cdh+4ceg-\frac{5cfg^2}{h}\right)-c(-8ah(2fg-eh)-8bh(4eg-3dh)+}{24ch^2(ah^2-bgh+cg^2)}$$

$$-\frac{(a+bx+cx^2)^{5/2}(fg^2-h(eg-dh))}{h(g+hx)(ah^2-bgh+cg^2)}$$

```
[In] Int[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^2,x]
```

```
[Out] -1/64*((3*b^3*f*h^3 + 4*b*c*h^2*(4*b*f*g - 2*b*e*h - 3*a*f*h) + 64*c^3*(5*f
*g^3 - g*h*(4*e*g - 3*d*h)) - 16*c^2*h*(19*b*f*g^2 - b*h*(14*e*g - 9*d*h) -
 4*a*h*(2*f*g - e*h)) + 2*c*h*(3*b^2*f*h^2 + 4*c*h*(4*b*f*g - 2*b*e*h - 3*a
*f*h) - 16*c^2*(5*f*g^2 - h*(4*e*g - 3*d*h)))*x)*Sqrt[a + b*x + c*x^2])/(c^
2*h^5) - ((3*b*f*h*(b*g - a*h) + (8*c^2*(5*f*g^3 - g*h*(4*e*g - 3*d*h)))/h
- c*(43*b*f*g^2 - 8*b*h*(4*e*g - 3*d*h) - 8*a*h*(2*f*g - e*h)) + 6*c*h*(4*c
*e*g + b*f*g - (5*c*f*g^2)/h - 4*c*d*h - a*f*h)*x)*(a + b*x + c*x^2)^(3/2))
/(24*c*h^2*(c*g^2 - b*g*h + a*h^2)) - ((f*g^2 - h*(e*g - d*h))*(a + b*x + c
*x^2)^(5/2))/(h*(c*g^2 - b*g*h + a*h^2)*(g + h*x)) + ((3*b^4*f*h^4 + 8*b^2*
c*h^3*(2*b*f*g - b*e*h - 3*a*f*h) + 128*c^4*(5*f*g^4 - g^2*h*(4*e*g - 3*d*h
)) + 48*c^2*h^2*(a^2*f*h^2 - 2*a*b*h*(2*f*g - e*h) + b^2*(3*f*g^2 - 2*e*g*h
 + d*h^2)) + 192*c^3*h*(a*h*(3*f*g^2 - 2*e*g*h + d*h^2) - b*g*(4*f*g^2 - 3*
e*g*h + 2*d*h^2)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/
(128*c^(5/2)*h^6) - (Sqrt[c*g^2 - b*g*h + a*h^2]*(2*c*(5*f*g^3 - g*h*(4*e*g
 - 3*d*h)) - h*(7*b*f*g^2 - b*h*(5*e*g - 3*d*h) - 2*a*h*(2*f*g - e*h)))*Arc
Tanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[a
+ b*x + c*x^2])])/(2*h^6)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
```

```
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{(fg^2-h(eg-dh))\left(a+bx+cx^2\right)^{5/2}}{h\left(cg^2-bgh+ah^2\right)(g+hx)} - \frac{\int \frac{\left(\frac{1}{2}\left(-2cdg+5beg+2afg-\frac{5bfg^2}{h}-3bdh-2aeh\right)+\left(4ceg+bfg-\frac{5cfg^2}{h}-4cdh-afh\right)x\right)\left(a+bx+cx^2\right)^{3/2}}{g+hx}\,dx}{cg^2-bgh+ah^2}$$

$$= -\frac{\Big(3bfh(bg-ah)+\frac{8c^2\left(5fg^3-gh(4eg-3dh)\right)}{h}-c(43bfg^2-8bh(4eg-3dh)-8ah(2fg-eh))+6ch\Big(}{24ch^2\left(cg^2-bgh+ah^2\right)}$$

$$-\frac{(fg^2-h(eg-dh))\left(a+bx+cx^2\right)^{5/2}}{h\left(cg^2-bgh+ah^2\right)(g+hx)}$$

$$+\frac{\int \frac{\Big(-\frac{\left(cg^2-bgh+ah^2\right)\left(3b^2fgh+4ach(5fg-4eh)-8bc\left(5fg^2-h(4eg-3dh)\right)\right)}{2h}-\frac{\left(cg^2-bgh+ah^2\right)\left(3b^2fh^2+4ch(4bfg-2beh-3afh)-16c^2\left(5fg^2-\right.\right.}{2h}}{g+hx}}{8ch^2\left(cg^2-bgh+ah^2\right)}$$

$$= -\frac{(3b^3fh^3+4bch^2(4bfg-2beh-3afh)+64c^3(5fg^3-gh(4eg-3dh))-16c^2h(19bfg^2-bh(14}{}$$

$$-\frac{\Big(3bfh(bg-ah)+\frac{8c^2\left(5fg^3-gh(4eg-3dh)\right)}{h}-c(43bfg^2-8bh(4eg-3dh)-8ah(2fg-eh))+6ch\Big(}{24ch^2\left(cg^2-bgh+ah^2\right)}$$

$$-\frac{(fg^2-h(eg-dh))\left(a+bx+cx^2\right)^{5/2}}{h\left(cg^2-bgh+ah^2\right)(g+hx)}$$

$$-\frac{\int \frac{-\frac{1}{4}\left(cg^2-bgh+ah^2\right)\Big(4c(bg-2ah)\left(3b^2fgh+4ach(5fg-4eh)-8bc\left(5fg^2-h(4eg-3dh)\right)\right)-\frac{g\left(4bcg-b^2h-4ach\right)\left(3b^2fh^2+4ch(4bfg-2\right.}{h}}{}}{}$$

$$=$$

$$-\frac{(3b^3fh^3+4bch^2(4bfg-2beh-3afh)+64c^3(5fg^3-gh(4eg-3dh))-16c^2h(19bfg^2-bh(14e}{}$$

$$-\frac{\Big(3bfh(bg-ah)+\frac{8c^2(5fg^3-gh(4eg-3dh))}{h}-c(43bfg^2-8bh(4eg-3dh)-8ah(2fg-eh))+6ch\Big(4}{24ch^2\left(cg^2-bgh+ah^2\right)}$$

$$-\frac{\left(fg^2-h(eg-dh)\right)\left(a+bx+cx^2\right)^{5/2}}{h\left(cg^2-bgh+ah^2\right)(g+hx)}$$

$$-\frac{(\left(cg^2-bgh+ah^2\right)(2c(5fg^3-gh(4eg-3dh))-h(7bfg^2-bh(5eg-3dh)-2ah(2fg-eh))))}{2h^6}$$

$$+\frac{\Big(4c(2cg-bh)\left(3b^2fgh+4ach(5fg-4eh)-8bc(5fg^2-h(4eg-3dh))\right)-\frac{(8c^2g^2-b^2h^2-4ch(bg-ah))}{}}{128c^2h^5}$$

$$=$$

$$-\frac{(3b^3fh^3+4bch^2(4bfg-2beh-3afh)+64c^3(5fg^3-gh(4eg-3dh))-16c^2h(19bfg^2-bh(14e}{}$$

$$-\frac{\Big(3bfh(bg-ah)+\frac{8c^2(5fg^3-gh(4eg-3dh))}{h}-c(43bfg^2-8bh(4eg-3dh)-8ah(2fg-eh))+6ch\Big(4}{24ch^2\left(cg^2-bgh+ah^2\right)}$$

$$-\frac{\left(fg^2-h(eg-dh)\right)\left(a+bx+cx^2\right)^{5/2}}{h\left(cg^2-bgh+ah^2\right)(g+hx)}$$

$$+\frac{(\left(cg^2-bgh+ah^2\right)(2c(5fg^3-gh(4eg-3dh))-h(7bfg^2-bh(5eg-3dh)-2ah(2fg-eh))))}{h^6}$$

$$+\frac{\Big(4c(2cg-bh)\left(3b^2fgh+4ach(5fg-4eh)-8bc(5fg^2-h(4eg-3dh))\right)-\frac{(8c^2g^2-b^2h^2-4ch(bg-ah))}{}}{64c^2h^5}$$

$$=$$

$$-\frac{(3b^3fh^3+4bch^2(4bfg-2beh-3afh)+64c^3(5fg^3-gh(4eg-3dh))-16c^2h(19bfg^2-bh(14e}{}$$

$$-\frac{\Big(3bfh(bg-ah)+\frac{8c^2(5fg^3-gh(4eg-3dh))}{h}-c(43bfg^2-8bh(4eg-3dh)-8ah(2fg-eh))+6ch\Big(4}{24ch^2\left(cg^2-bgh+ah^2\right)}$$

$$-\frac{\left(fg^2-h(eg-dh)\right)\left(a+bx+cx^2\right)^{5/2}}{h\left(cg^2-bgh+ah^2\right)(g+hx)}$$

$$+\frac{\Big(4c(2cg-bh)\left(3b^2fgh+4ach(5fg-4eh)-8bc(5fg^2-h(4eg-3dh))\right)-\frac{(8c^2g^2-b^2h^2-4ch(bg-ah))}{}}{128c^{5/2}h^5}$$

$$-\frac{\sqrt{cg^2-bgh+ah^2}(2c(5fg^3-gh(4eg-3dh))-h(7bfg^2-bh(5eg-3dh)-2ah(2fg-eh)))\,\mathrm{ta}}{2h^6}$$

Mathematica [A] (verified)

Time = 4.83 (sec) , antiderivative size = 641, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx+cx^2)^{3/2}\,(d+ex+fx^2)}{(g+hx)^2}\,dx = \frac{h\sqrt{a+x(b+cx)}\left(-9b^3fh^3(g+hx)+6bch^2(g+hx)(10afh+b(-8fg+4eh+fhx))-16c^3\left(f\left(60\right.\right.\right.}{}$$

```
[In] Integrate[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^2,x]
```

```
[Out] ((h*Sqrt[a + x*(b + c*x)]*(-9*b^3*f*h^3*(g + h*x) + 6*b*c*h^2*(g + h*x)*(10
*a*f*h + b*(-8*f*g + 4*e*h + f*h*x)) - 16*c^3*(f*(60*g^4 + 30*g^3*h*x - 10*
g^2*h^2*x^2 + 5*g*h^3*x^3 - 3*h^4*x^4) - 2*h*(3*d*h*(-6*g^2 - 3*g*h*x + h^2
*x^2) + 2*e*(12*g^3 + 6*g^2*h*x - 2*g*h^2*x^2 + h^3*x^3))) + 8*c^2*h*(a*h*(
8*h*(7*e*g - 3*d*h + 4*e*h*x) + f*(-88*g^2 - 49*g*h*x + 15*h^2*x^2)) + b*(f
*(114*g^3 + 62*g^2*h*x - 19*g*h^2*x^2 + 9*h^3*x^3) + 2*h*(3*d*h*(9*g + 5*h*
x) + e*(-42*g^2 - 23*g*h*x + 7*h^2*x^2))))))/(c^2*(g + h*x)) - 192*Sqrt[-(c
*g^2) + h*(b*g - a*h)]*(2*c*(5*f*g^3 + g*h*(-4*e*g + 3*d*h)) + h*(-7*b*f*g^
2 + b*h*(5*e*g - 3*d*h) - 2*a*h*(-2*f*g + e*h)))*ArcTan[(Sqrt[-(c*g^2) + h*
(b*g - a*h)]*x)/(Sqrt[a]*(g + h*x) - g*Sqrt[a + x*(b + c*x)])] + (3*(3*b^4*
f*h^4 - 8*b^2*c*h^3*(-2*b*f*g + b*e*h + 3*a*f*h) + 128*c^4*(5*f*g^4 + g^2*h
*(-4*e*g + 3*d*h)) + 48*c^2*h^2*(a^2*f*h^2 + 2*a*b*h*(-2*f*g + e*h) + b^2*(
3*f*g^2 - 2*e*g*h + d*h^2)) - 192*c^3*h*(-(a*h*(3*f*g^2 - 2*e*g*h + d*h^2))
 + b*g*(4*f*g^2 - 3*e*g*h + 2*d*h^2)))*ArcTanh[(Sqrt[c]*x)/(-Sqrt[a] + Sqrt
[a + x*(b + c*x)])])/c^(5/2))/(192*h^6)
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 1324, normalized size of antiderivative = 1.76

method	result	size
risch	Expression too large to display	1324
default	Expression too large to display	1853

```
[In] int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/192/c^2*(48*c^3*f*h^3*x^3+72*b*c^2*f*h^3*x^2+64*c^3*e*h^3*x^2-128*c^3*f*g
*h^2*x^2+120*a*c^2*f*h^3*x+6*b^2*c*f*h^3*x+112*b*c^2*e*h^3*x-224*b*c^2*f*g*
h^2*x+96*c^3*d*h^3*x-192*c^3*e*g*h^2*x+288*c^3*f*g^2*h*x+60*a*b*c*f*h^3+256
*a*c^2*e*h^3-512*a*c^2*f*g*h^2-9*b^3*f*h^3+24*b^2*c*e*h^3-48*b^2*c*f*g*h^2+
240*b*c^2*d*h^3-480*b*c^2*e*g*h^2+720*b*c^2*f*g^2*h-384*c^3*d*g*h^2+576*c^3
*e*g^2*h-768*c^3*f*g^3)*(c*x^2+b*x+a)^(1/2)/h^5+1/128/h^5/c^2*(-128*c^2/h^2
*(a^2*e*h^5-2*a^2*f*g*h^4+2*a*b*d*h^5-4*a*b*e*g*h^4+6*a*b*f*g^2*h^3-4*a*c*d
*g*h^4+6*a*c*e*g^2*h^3-8*a*c*f*g^3*h^2-2*b^2*d*g*h^4+3*b^2*e*g^2*h^3-4*b^2*
f*g^3*h^2+6*b*c*d*g^2*h^3-8*b*c*e*g^3*h^2+10*b*c*f*g^4*h-4*c^2*d*g^3*h^2+5*
```

```
c^2*e*g^4*h-6*c^2*f*g^5)/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h
+c*g^2)/h^2+(b*h-2*c*g)/h*(x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1
/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g)
)+128*c^2*(a^2*d*h^6-a^2*e*g*h^5+a^2*f*g^2*h^4-2*a*b*d*g*h^5+2*a*b*e*g^2*h^
4-2*a*b*f*g^3*h^3+2*a*c*d*g^2*h^4-2*a*c*e*g^3*h^3+2*a*c*f*g^4*h^2+b^2*d*g^2
*h^4-b^2*e*g^3*h^3+b^2*f*g^4*h^2-2*b*c*d*g^3*h^3+2*b*c*e*g^4*h^2-2*b*c*f*g^
5*h+c^2*d*g^4*h^2-c^2*e*g^5*h+c^2*f*g^6)/h^3*(-1/(a*h^2-b*g*h+c*g^2)*h^2/(x
+1/h*g)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/
2)+1/2*(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln
((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/
h^2)^(1/2)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^
(1/2))/(x+1/h*g)))+(48*a^2*c^2*f*h^4-24*a*b^2*c*f*h^4+96*a*b*c^2*e*h^4-192*
a*b*c^2*f*g*h^3+192*a*c^3*d*h^4-384*a*c^3*e*g*h^3+576*a*c^3*f*g^2*h^2+3*b^4
*f*h^4-8*b^3*c*e*h^4+16*b^3*c*f*g*h^3+48*b^2*c^2*d*h^4-96*b^2*c^2*e*g*h^3+1
44*b^2*c^2*f*g^2*h^2-384*b*c^3*d*g*h^3+576*b*c^3*e*g^2*h^2-768*b*c^3*f*g^3*
h+384*c^4*d*g^2*h^2-512*c^4*e*g^3*h+640*c^4*f*g^4)/h*ln((1/2*b+c*x)/c^(1/2)
+(c*x^2+b*x+a)^(1/2))/c^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^2}\,dx = \text{Timed out}$$

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^2,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^2}\,dx = \int \frac{(a+bx+cx^2)^{\frac{3}{2}}(d+ex+fx^2)}{(g+hx)^2}\,dx$$

```
[In] integrate((c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**2,x)
```

```
[Out] Integral((a + b*x + c*x**2)**(3/2)*(d + e*x + f*x**2)/(g + h*x)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^2,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b*h-2*c*g>0)', see `assume?` for mo
re deta
```

Giac [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^2}\,dx = \text{Timed out}$$

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^2}\,dx = \int \frac{(c\,x^2+b\,x+a)^{3/2}\,(f\,x^2+e\,x+d)}{(g+h\,x)^2}\,dx$$

```
[In] int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^2,x)
```

```
[Out] int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^2, x)
```

3.202 $\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^3}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 824

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^3}\,dx =$$

$$-\frac{(b^2fh^3(bg-ah)-8c^3g^2(10fg^2-3h(2eg-dh))-2c^2h(2ah(19fg^2-9egh+3dh^2)-3bg(22fg^2-12egh}{}$$

$$-\frac{(4cg(6eg-\frac{10fg^2}{h}-3dh)-4ah(7fg-3eh)+b(31fg^2-3h(5eg-dh))+2h(3ceg+2bfg-\frac{5cfg^2}{h}-3cdh}{12h^2(cg^2-bgh+ah^2)(g+hx)}$$

$$-\frac{(fg^2-h(eg-dh))(a+bx+cx^2)^{5/2}}{2h(cg^2-bgh+ah^2)(g+hx)^2}$$

$$-\frac{(b^3fh^3+6bch^2(3bfg-beh-2afh)+16c^3g(10fg^2-3h(2eg-dh))+24c^2h(ah(3fg-eh)-b(6fg^2-3e}{16c^{3/2}h^6}$$

$$+\frac{(8c^2g^2(10fg^2-3h(2eg-dh))+4ch(ah(19fg^2-9egh+3dh^2)-bg(28fg^2-15egh+6dh^2))+h^2(8a^2fh^2}{8h^6\sqrt{cg^2-bgh+ah^2}}$$

```
[Out] -1/12*(4*c*g*(6*e*g-10*f*g^2/h-3*d*h)-4*a*h*(-3*e*h+7*f*g)+b*(31*f*g^2-3*h*
(-d*h+5*e*g))+2*h*(3*c*e*g+2*b*f*g-5*c*f*g^2/h-3*c*d*h-2*a*f*h)*x)*(c*x^2+b
*x+a)^(3/2)/h^2/(a*h^2-b*g*h+c*g^2)/(h*x+g)-1/2*(f*g^2-h*(-d*h+e*g))*(c*x^2
+b*x+a)^(5/2)/h/(a*h^2-b*g*h+c*g^2)/(h*x+g)^2-1/16*(b^3*f*h^3+6*b*c*h^2*(-2
*a*f*h-b*e*h+3*b*f*g)+16*c^3*g*(10*f*g^2-3*h*(-d*h+2*e*g))+24*c^2*h*(a*h*(-
e*h+3*f*g)-b*(d*h^2-3*e*g*h+6*f*g^2)))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2
+b*x+a)^(1/2))/c^(3/2)/h^6+1/8*(8*c^2*g^2*(10*f*g^2-3*h*(-d*h+2*e*g))+4*c*h
*(a*h*(3*d*h^2-9*e*g*h+19*f*g^2)-b*g*(6*d*h^2-15*e*g*h+28*f*g^2))+h^2*(8*a^
2*f*h^2-4*a*b*h*(-3*e*h+10*f*g)+b^2*(35*f*g^2-3*h*(-d*h+5*e*g))))*arctanh(1
/2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

```
)/h^6/(a*h^2-b*g*h+c*g^2)^(1/2)-1/8*(b^2*f*h^3*(-a*h+b*g)-8*c^3*g^2*(10*f*g
^2-3*h*(-d*h+2*e*g))-2*c^2*h*(2*a*h*(3*d*h^2-9*e*g*h+19*f*g^2)-3*b*g*(5*d*h
^2-12*e*g*h+22*f*g^2))-c*h^2*(8*a^2*f*h^2-18*a*b*h*(-e*h+3*f*g)+b^2*(53*f*g
^2-6*h*(-d*h+4*e*g)))+2*c*h*(b*f*h^2*(-a*h+b*g)+2*c^2*g*(10*f*g^2-3*h*(-d*h
+2*e*g))+c*h*(2*a*h*(-3*e*h+7*f*g)-3*b*(d*h^2-3*e*g*h+6*f*g^2)))*x)*(c*x^2+
b*x+a)^(1/2)/c/h^5/(a*h^2-b*g*h+c*g^2)
```

Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 819, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {1664, 826, 828, 857, 635, 212, 738}

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^3}\,dx = -\frac{\left(fg^2-h(eg-dh)\right)\left(cx^2+bx+a\right)^{5/2}}{2h\left(cg^2-bhg+ah^2\right)(g+hx)^2}$$

$$-\frac{\left(31bfg^2+4c\left(-\frac{10fg^2}{h}+6eg-3dh\right)g-3bh(5eg-dh)-4ah(7fg-3eh)+2h\left(-\frac{5cfg^2}{h}+3ceg+2bfg-\right.\right.}{12h^2\left(cg^2-bhg+ah^2\right)(g+hx)}$$

$$-\frac{\left(8g^2\left(-\frac{10fg^2}{h}+6eg-3dh\right)c^3-2\left(2ah\left(19fg^2-9ehg+3dh^2\right)-3bg\left(22fg^2-12ehg+5dh^2\right)\right)c^2-h((53\right.}{}$$

$$-\frac{\left(16\left(10fg^3-3gh(2eg-dh)\right)c^3-24h\left(6bfg^2-bh(3eg-dh)-ah(3fg-eh)\right)c^2+6bh^2(3bfg-beh-2a\right.}{16c^{3/2}h^6}$$

$$+\frac{\left(8\left(10fg^4-3g^2h(2eg-dh)\right)c^2-4h\left(28bfg^3-3bh(5eg-2dh)g-ah\left(19fg^2-9ehg+3dh^2\right)\right)c+h^2((35f\right.}{8h^6\sqrt{cg^2-bhg+a}}$$

```
[In] Int[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^3,x]
```

```
[Out] -1/8*((b^2*f*h^2*(b*g - a*h) + 8*c^3*g^2*(6*e*g - (10*f*g^2)/h - 3*d*h) - 2
*c^2*(2*a*h*(19*f*g^2 - 9*e*g*h + 3*d*h^2) - 3*b*g*(22*f*g^2 - 12*e*g*h + 5
*d*h^2)) - c*h*(8*a^2*f*h^2 - 18*a*b*h*(3*f*g - e*h) + b^2*(53*f*g^2 - 6*h*
(4*e*g - d*h))) + 2*c*(b*f*h^2*(b*g - a*h) + 2*c^2*(10*f*g^3 - 3*g*h*(2*e*g
 - d*h)) + c*h*(2*a*h*(7*f*g - 3*e*h) - 3*b*(6*f*g^2 - 3*e*g*h + d*h^2)))*x
)*Sqrt[a + b*x + c*x^2])/(c*h^4*(c*g^2 - b*g*h + a*h^2)) - ((31*b*f*g^2 + 4
*c*g*(6*e*g - (10*f*g^2)/h - 3*d*h) - 3*b*h*(5*e*g - d*h) - 4*a*h*(7*f*g -
3*e*h) + 2*h*(3*c*e*g + 2*b*f*g - (5*c*f*g^2)/h - 3*c*d*h - 2*a*f*h)*x)*(a
+ b*x + c*x^2)^(3/2))/(12*h^2*(c*g^2 - b*g*h + a*h^2)*(g + h*x)) - ((f*g^2
- h*(e*g - d*h))*(a + b*x + c*x^2)^(5/2))/(2*h*(c*g^2 - b*g*h + a*h^2)*(g +
 h*x)^2) - ((b^3*f*h^3 + 6*b*c*h^2*(3*b*f*g - b*e*h - 2*a*f*h) + 16*c^3*(10
*f*g^3 - 3*g*h*(2*e*g - d*h)) - 24*c^2*h*(6*b*f*g^2 - b*h*(3*e*g - d*h) - a
*h*(3*f*g - e*h)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/
(16*c^(3/2)*h^6) + ((8*c^2*(10*f*g^4 - 3*g^2*h*(2*e*g - d*h)) - 4*c*h*(28*b
*f*g^3 - 3*b*g*h*(5*e*g - 2*d*h) - a*h*(19*f*g^2 - 9*e*g*h + 3*d*h^2)) + h^
2*(8*a^2*f*h^2 - 4*a*b*h*(10*f*g - 3*e*h) + b^2*(35*f*g^2 - 3*h*(5*e*g - d*
```

```
h))))*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2
]*Sqrt[a + b*x + c*x^2])])/(8*h^6*Sqrt[c*g^2 - b*g*h + a*h^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 826

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
```

```
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

integral

$$= -\frac{(fg^2-h(eg-dh))\left(a+bx+cx^2\right)^{5/2}}{2h\left(cg^2-bgh+ah^2\right)(g+hx)^2} - \frac{\int \frac{\left(\frac{1}{2}\left(-4cdg+5beg+4afg-\frac{5bfg^2}{h}-bdh-4aeh\right)+\left(3ceg+2bfg-\frac{5cfg^2}{h}-3cdh-2afh\right)x\right)\left(a+bx+cx^2\right)^{3/2}}{(g+hx)^2}dx}{2\left(cg^2-bgh+ah^2\right)}$$

$$= -\frac{\Big(31bfg^2+4cg\Big(6eg-\frac{10fg^2}{h}-3dh\Big)-3bh(5eg-dh)-4ah(7fg-3eh)+2h\Big(3ceg+2bfg-\frac{5cfg^2}{h}-3}{12h^2\left(cg^2-bgh+ah^2\right)(g+hx)} - \frac{(fg^2-h(eg-dh))\left(a+bx+cx^2\right)^{5/2}}{2h\left(cg^2-bgh+ah^2\right)(g+hx)^2} + \frac{\int \frac{\Big(\frac{1}{2}\Big(2(4bg-2ah)\Big(3ceg+2bfg-\frac{5cfg^2}{h}-3cdh-2afh\Big)+3b\big(5bfg^2-bh(5eg-dh)+4h(cdg-afg+aeh)\big)\Big)-\frac{2\big(bfh^2(bg-ah)+2c^2\big(10fg^3-3gh(2eg}{}}{g+hx}}{4h^2\left(cg^2-bgh+ah^2\right)}$$

$$=$$

$$-\frac{\left(b^2fh^2(bg-ah)+8c^3g^2\left(6eg-\frac{10fg^2}{h}-3dh\right)-2c^2(2ah(19fg^2-9egh+3dh^2)-3bg(22fg^2-12egh+\right.}{}$$

$$-\frac{\left(31bfg^2+4cg\left(6eg-\frac{10fg^2}{h}-3dh\right)-3bh(5eg-dh)-4ah(7fg-3eh)+2h\left(3ceg+2bfg-\frac{5cfg^2}{h}-3c\right.\right.}{12h^2\left(cg^2-bgh+ah^2\right)(g+hx)}$$

$$-\frac{\left(fg^2-h(eg-dh)\right)\left(a+bx+cx^2\right)^{5/2}}{2h\left(cg^2-bgh+ah^2\right)(g+hx)^2}$$

$$-\frac{\int\frac{\frac{\left(cg^2-bgh+ah^2\right)\left(b^3fgh^2-8ach\left(10cfg^2+2afh^2-3ch(2eg-dh)\right)-2b^2ch\left(26fg^2-3h(4eg-dh)\right)+4bc\left(20cfg^3-6cgh(2eg-dh)+ah^2(17fg-6eh)\right)\right)}{h}+\frac{\left(cg^2-bg\right.}{}}{(g+hx)\sqrt{a+bx+cx^2}}}{16ch^4\left(cg^2-bgh+ah^2\right)}$$

$$=$$

$$-\frac{\left(b^2fh^2(bg-ah)+8c^3g^2\left(6eg-\frac{10fg^2}{h}-3dh\right)-2c^2(2ah(19fg^2-9egh+3dh^2)-3bg(22fg^2-12egh+\right.}{}$$

$$-\frac{\left(31bfg^2+4cg\left(6eg-\frac{10fg^2}{h}-3dh\right)-3bh(5eg-dh)-4ah(7fg-3eh)+2h\left(3ceg+2bfg-\frac{5cfg^2}{h}-3c\right.\right.}{12h^2\left(cg^2-bgh+ah^2\right)(g+hx)}$$

$$-\frac{\left(fg^2-h(eg-dh)\right)\left(a+bx+cx^2\right)^{5/2}}{2h\left(cg^2-bgh+ah^2\right)(g+hx)^2}$$

$$-\frac{(b^3fh^3+6bch^2(3bfg-beh-2afh)+16c^3(10fg^3-3gh(2eg-dh))-24c^2h(6bfg^2-bh(3eg-dh)-a}{16ch^6}$$

$$+\frac{(8c^2(10fg^4-3g^2h(2eg-dh))-4ch(28bfg^3-3bgh(5eg-2dh)-ah(19fg^2-9egh+3dh^2))+h^2(8a^2}{8h^6}$$

$$=$$

$$-\frac{\left(b^2fh^2(bg-ah)+8c^3g^2\left(6eg-\frac{10fg^2}{h}-3dh\right)-2c^2(2ah(19fg^2-9egh+3dh^2)-3bg(22fg^2-12egh+\right.}{}$$

$$-\frac{\left(31bfg^2+4cg\left(6eg-\frac{10fg^2}{h}-3dh\right)-3bh(5eg-dh)-4ah(7fg-3eh)+2h\left(3ceg+2bfg-\frac{5cfg^2}{h}-3c\right.\right.}{12h^2\left(cg^2-bgh+ah^2\right)(g+hx)}$$

$$-\frac{\left(fg^2-h(eg-dh)\right)\left(a+bx+cx^2\right)^{5/2}}{2h\left(cg^2-bgh+ah^2\right)(g+hx)^2}$$

$$-\frac{(b^3fh^3+6bch^2(3bfg-beh-2afh)+16c^3(10fg^3-3gh(2eg-dh))-24c^2h(6bfg^2-bh(3eg-dh)-a}{8ch^6}$$

$$-\frac{(8c^2(10fg^4-3g^2h(2eg-dh))-4ch(28bfg^3-3bgh(5eg-2dh)-ah(19fg^2-9egh+3dh^2))+h^2(8a^2}{4}$$

$$=$$

$$-\frac{\left(b^2fh^2(bg-ah)+8c^3g^2\left(6eg-\frac{10fg^2}{h}-3dh\right)-2c^2(2ah(19fg^2-9egh+3dh^2)-3bg(22fg^2-12egh}{}$$

$$-\frac{\left(31bfg^2+4cg\left(6eg-\frac{10fg^2}{h}-3dh\right)-3bh(5eg-dh)-4ah(7fg-3eh)+2h\left(3ceg+2bfg-\frac{5cfg^2}{h}-3\right.\right.}{12h^2\left(cg^2-bgh+ah^2\right)(g+hx)}$$

$$-\frac{\left(fg^2-h(eg-dh)\right)\left(a+bx+cx^2\right)^{5/2}}{2h\left(cg^2-bgh+ah^2\right)(g+hx)^2}$$

$$-\frac{(b^3fh^3+6bch^2(3bfg-beh-2afh)+16c^3(10fg^3-3gh(2eg-dh))-24c^2h(6bfg^2-bh(3eg-dh)-}{16c^{3/2}h^6}$$

$$+\frac{(8c^2(10fg^4-3g^2h(2eg-dh))-4ch(28bfg^3-3bgh(5eg-2dh)-ah(19fg^2-9egh+3dh^2))+h^2(8a}{8h^6\sqrt{cg^2-bgh+}}$$

Mathematica [A] (verified)

Time = 11.63 (sec) , antiderivative size = 817, normalized size of antiderivative = 0.99

$$\int\frac{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^3}\,dx=\frac{4f(a+x(b+cx))^{3/2}-\frac{6\left(fg^2+h(-eg+dh)\right)(a+x(b+cx))^{3/2}}{(g+hx)^2}+\frac{12(2fg-eh)(a+}{g+h}}{}$$

```
[In] Integrate[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^3,x]
```

```
[Out] (4*f*(a + x*(b + c*x))^(3/2) - (6*(f*g^2 + h*(-(e*g) + d*h))*(a + x*(b + c*
x))^(3/2))/(g + h*x)^2 + (12*(2*f*g - e*h)*(a + x*(b + c*x))^(3/2))/(g + h*
x) + (9*(-2*f*g + e*h)*((8*c^2*g^2 + b^2*h^2 + 4*c*h*(-2*b*g + a*h))*ArcTan
h[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] + 2*Sqrt[c]*(h*(-4*c*g + 3
*b*h + 2*c*h*x)*Sqrt[a + x*(b + c*x)] + 2*(2*c*g - b*h)*Sqrt[c*g^2 + h*(-(b
*g) + a*h)]*ArcTanh[(-(b*g) + 2*a*h - 2*c*g*x + b*h*x)/(2*Sqrt[c*g^2 + h*(-
(b*g) + a*h)]*Sqrt[a + x*(b + c*x)])])))/(2*Sqrt[c]*h^3) + (9*(f*g^2 + h*(-
(e*g) + d*h))*(((-2*c*g + b*h)*(a + x*(b + c*x))^(3/2))/(g + h*x) - (Sqrt[a
 + x*(b + c*x)]*(b^2*h^2 + 2*c^2*g*(2*g - h*x) + c*h*(-5*b*g + 2*a*h + b*h*
x)))/h^2 + (4*Sqrt[c]*(2*c*g - b*h)*(c*g^2 + h*(-(b*g) + a*h))*ArcTanh[(b +
 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] + (8*c^2*g^2 + b^2*h^2 + 4*c*h*(
-2*b*g + a*h))*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*ArcTanh[(-(b*g) + 2*a*h - 2*c
*g*x + b*h*x)/(2*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*Sqrt[a + x*(b + c*x)])])/(2
*h^3)))/(-(c*g^2) + h*(b*g - a*h)) - (3*f*((2*c*g - b*h)*(8*c^2*g^2 - b^2*h
^2 + 4*c*h*(-2*b*g + 3*a*h))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b +
 c*x)])] + 2*Sqrt[c]*(h*Sqrt[a + x*(b + c*x)]*(-(b^2*h^2) + 4*c^2*g*(-2*g +
 h*x) - 2*c*h*(-5*b*g + 4*a*h + b*h*x)) + 8*c*(c*g^2 + h*(-(b*g) + a*h))^(3
/2)*ArcTanh[(-(b*g) + 2*a*h - 2*c*g*x + b*h*x)/(2*Sqrt[c*g^2 + h*(-(b*g) +
a*h)]*Sqrt[a + x*(b + c*x)])])))/(4*c^(3/2)*h^3))/(12*h^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1780 vs. 2(792) = 1584.

Time = 1.05 (sec) , antiderivative size = 1781, normalized size of antiderivative = 2.16

method	result	size
risch	Expression too large to display	1781
default	Expression too large to display	3600

```
[In] int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/24/c*(8*c^2*f*h^2*x^2+14*b*c*f*h^2*x+12*c^2*e*h^2*x-36*c^2*f*g*h*x+32*a*c
*f*h^2+3*b^2*f*h^2+30*b*c*e*h^2-90*b*c*f*g*h+24*c^2*d*h^2-72*c^2*e*g*h+144*
c^2*f*g^2)*(c*x^2+b*x+a)^(1/2)/h^5+1/16/c/h^5*((12*a*b*c*f*h^3+24*a*c^2*e*h
^3-72*a*c^2*f*g*h^2-b^3*f*h^3+6*b^2*c*e*h^3-18*b^2*c*f*g*h^2+24*b*c^2*d*h^3
-72*b*c^2*e*g*h^2+144*b*c^2*f*g^2*h-48*c^3*d*g*h^2+96*c^3*e*g^2*h-160*c^3*f
*g^3)/h*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-16*c/h^2*(a^2*f
*h^4+2*a*b*e*h^4-6*a*b*f*g*h^3+2*a*c*d*h^4-6*a*c*e*g*h^3+12*a*c*f*g^2*h^2+b
^2*d*h^4-3*b^2*e*g*h^3+6*b^2*f*g^2*h^2-6*b*c*d*g*h^3+12*b*c*e*g^2*h^2-20*b*
c*f*g^3*h+6*c^2*d*g^2*h^2-10*c^2*e*g^3*h+15*c^2*f*g^4)/((a*h^2-b*g*h+c*g^2)
/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+1/h*g)+2*((a*h^2
-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*
h+c*g^2)/h^2)^(1/2))/(x+1/h*g))+16*c/h^3*(a^2*e*h^5-2*a^2*f*g*h^4+2*a*b*d*h
^5-4*a*b*e*g*h^4+6*a*b*f*g^2*h^3-4*a*c*d*g*h^4+6*a*c*e*g^2*h^3-8*a*c*f*g^3*
h^2-2*b^2*d*g*h^4+3*b^2*e*g^2*h^3-4*b^2*f*g^3*h^2+6*b*c*d*g^2*h^3-8*b*c*e*g
^3*h^2+10*b*c*f*g^4*h-4*c^2*d*g^3*h^2+5*c^2*e*g^4*h-6*c^2*f*g^5)*(-1/(a*h^2
-b*g*h+c*g^2)*h^2/(x+1/h*g)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b
*g*h+c*g^2)/h^2)^(1/2)+1/2*(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)/((a*h^2-b*g*h+
c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+1/h*g)+2*(
(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^
2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g)))+16*c*(a^2*d*h^6-a^2*e*g*h^5+a^2*f*g^
2*h^4-2*a*b*d*g*h^5+2*a*b*e*g^2*h^4-2*a*b*f*g^3*h^3+2*a*c*d*g^2*h^4-2*a*c*e
*g^3*h^3+2*a*c*f*g^4*h^2+b^2*d*g^2*h^4-b^2*e*g^3*h^3+b^2*f*g^4*h^2-2*b*c*d*
g^3*h^3+2*b*c*e*g^4*h^2-2*b*c*f*g^5*h+c^2*d*g^4*h^2-c^2*e*g^5*h+c^2*f*g^6)/
h^4*(-1/2/(a*h^2-b*g*h+c*g^2)*h^2/(x+1/h*g)^2*((x+1/h*g)^2*c+(b*h-2*c*g)/h*
(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)-3/4*(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g
^2)*(-1/(a*h^2-b*g*h+c*g^2)*h^2/(x+1/h*g)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1
/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)+1/2*(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)/
((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h
*(x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(
x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g)))+1/2*c/(a*h^2-b*g*h+c*g
^2)*h^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-
2*c*g)/h*(x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c+(b*h-2*
c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^3}\,dx=\text{Timed out}$$

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^3,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^3}\,dx=\int \frac{\left(a+bx+cx^2\right)^{\frac{3}{2}}\left(d+ex+fx^2\right)}{\left(g+hx\right)^3}\,dx$$

```
[In] integrate((c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**3,x)
```

```
[Out] Integral((a + b*x + c*x**2)**(3/2)*(d + e*x + f*x**2)/(g + h*x)**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^3}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^3,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*h^2-b*g*h>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2628 vs. 2(792) = 1584.

Time = 0.66 (sec) , antiderivative size = 2628, normalized size of antiderivative = 3.19

$$\int \frac{(a+bx+cx^2)^{3/2}\,(d+ex+fx^2)}{(g+hx)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^3,x, algorithm="giac")
```

```
[Out] 1/24*sqrt(c*x^2 + b*x + a)*(2*x*(4*c*f*x/h^3 - (18*c^3*f*g*h^14 - 6*c^3*e*h
^15 - 7*b*c^2*f*h^15)/(c^2*h^18)) + (144*c^3*f*g^2*h^13 - 72*c^3*e*g*h^14 -
 90*b*c^2*f*g*h^14 + 24*c^3*d*h^15 + 30*b*c^2*e*h^15 + 3*b^2*c*f*h^15 + 32*
a*c^2*f*h^15)/(c^2*h^18)) + 1/4*(80*c^2*f*g^4 - 48*c^2*e*g^3*h - 112*b*c*f*
g^3*h + 24*c^2*d*g^2*h^2 + 60*b*c*e*g^2*h^2 + 35*b^2*f*g^2*h^2 + 76*a*c*f*g
^2*h^2 - 24*b*c*d*g*h^3 - 15*b^2*e*g*h^3 - 36*a*c*e*g*h^3 - 40*a*b*f*g*h^3
+ 3*b^2*d*h^4 + 12*a*c*d*h^4 + 12*a*b*e*h^4 + 8*a^2*f*h^4)*arctan(-((sqrt(c
)*x - sqrt(c*x^2 + b*x + a))*h + sqrt(c)*g)/sqrt(-c*g^2 + b*g*h - a*h^2))/(
sqrt(-c*g^2 + b*g*h - a*h^2)*h^6) + 1/16*(160*c^3*f*g^3 - 96*c^3*e*g^2*h -
144*b*c^2*f*g^2*h + 48*c^3*d*g*h^2 + 72*b*c^2*e*g*h^2 + 18*b^2*c*f*g*h^2 +
72*a*c^2*f*g*h^2 - 24*b*c^2*d*h^3 - 6*b^2*c*e*h^3 - 24*a*c^2*e*h^3 + b^3*f*
h^3 - 12*a*b*c*f*h^3)*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c)
 + b))/(c^(3/2)*h^6) + 1/4*(40*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*c^2*f*
g^4*h - 32*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*c^2*e*g^3*h^2 - 48*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^3*b*c*f*g^3*h^2 + 24*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))^3*c^2*d*g^2*h^3 + 36*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b*c*e
*g^2*h^3 + 13*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^2*f*g^2*h^3 + 20*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^3*a*c*f*g^2*h^3 - 24*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^3*b*c*d*g*h^4 - 9*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^2*e
*g*h^4 - 12*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*c*e*g*h^4 - 8*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^3*a*b*f*g*h^4 + 5*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^3*b^2*d*h^5 + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*c*d*h^5 + 4*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b*e*h^5 + 72*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^2*c^(5/2)*f*g^5 - 56*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c^(
5/2)*e*g^4*h - 64*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b*c^(3/2)*f*g^4*h +
 40*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c^(5/2)*d*g^3*h^2 + 44*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))^2*b*c^(3/2)*e*g^3*h^2 + 7*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^2*b^2*sqrt(c)*f*g^3*h^2 - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)^2*a*c^(3/2)*f*g^3*h^2 - 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b*c^(3/2
)*d*g^2*h^3 - 3*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2*sqrt(c)*e*g^2*h^3
 + 12*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*c^(3/2)*e*g^2*h^3 + 24*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^2*a*b*sqrt(c)*f*g^2*h^3 - (sqrt(c)*x - sqrt(c
*x^2 + b*x + a))^2*b^2*sqrt(c)*d*g*h^4 - 20*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))^2*a*c^(3/2)*d*g*h^4 - 20*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b*sqr
t(c)*e*g*h^4 - 16*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*sqrt(c)*f*g*h^4
```

```
 + 16*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b*sqrt(c)*d*h^5 + 8*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^2*a^2*sqrt(c)*e*h^5 + 72*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))*b*c^2*f*g^5 - 56*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b*c^2*e*g
^4*h - 68*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^2*c*f*g^4*h - 104*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))*a*c^2*f*g^4*h + 40*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))*b*c^2*d*g^3*h^2 + 48*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^2*c*e*g^3
*h^2 + 80*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*c^2*e*g^3*h^2 + 11*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))*b^3*f*g^3*h^2 + 124*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))*a*b*c*f*g^3*h^2 - 28*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^2*c*d*g
^2*h^3 - 56*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*c^2*d*g^2*h^3 - 7*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))*b^3*e*g^2*h^3 - 80*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))*a*b*c*e*g^2*h^3 - 19*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^2*f*g
^2*h^3 - 44*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*c*f*g^2*h^3 + 3*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))*b^3*d*g*h^4 + 36*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))*a*b*c*d*g*h^4 + 11*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^2*e*g*h^4
 + 20*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*c*e*g*h^4 + 8*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))*a^2*b*f*g*h^4 - 3*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*
a*b^2*d*h^5 + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*c*d*h^5 - 4*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))*a^2*b*e*h^5 + 18*b^2*c^(3/2)*f*g^5 - 14*b^2*c^
(3/2)*e*g^4*h - 13*b^3*sqrt(c)*f*g^4*h - 52*a*b*c^(3/2)*f*g^4*h + 10*b^2*c^
(3/2)*d*g^3*h^2 + 9*b^3*sqrt(c)*e*g^3*h^2 + 40*a*b*c^(3/2)*e*g^3*h^2 + 45*a
*b^2*sqrt(c)*f*g^3*h^2 + 36*a^2*c^(3/2)*f*g^3*h^2 - 5*b^3*sqrt(c)*d*g^2*h^3
 - 28*a*b*c^(3/2)*d*g^2*h^3 - 29*a*b^2*sqrt(c)*e*g^2*h^3 - 28*a^2*c^(3/2)*e
*g^2*h^3 - 48*a^2*b*sqrt(c)*f*g^2*h^3 + 13*a*b^2*sqrt(c)*d*g*h^4 + 20*a^2*c
^(3/2)*d*g*h^4 + 28*a^2*b*sqrt(c)*e*g*h^4 + 16*a^3*sqrt(c)*f*g*h^4 - 8*a^2*
b*sqrt(c)*d*h^5 - 8*a^3*sqrt(c)*e*h^5)/(((sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)^2*h + 2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c)*g + b*g - a*h)^2*h^6)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}\,(d+ex+fx^2)}{(g+hx)^3}\,dx=\int \frac{{\left(c\,x^2+b\,x+a\right)}^{3/2}\,\left(f\,x^2+e\,x+d\right)}{{\left(g+h\,x\right)}^3}\,dx$$

```
[In] int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^3,x)
[Out] int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^3, x)
```

3.203 $\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^4}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 833

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^4}\,dx =$$

$$-\frac{(8c^2g^2(10fg^2-h(4eg-dh))-2ch(3bg(18fg^2-6egh+dh^2)-2ah(23fg^2-8egh+2dh^2))+h^2(12a^2fh^2}{}$$

$$-\frac{\left(2cg\left(4eg-\frac{10fg^2}{h}-dh\right)-6ah(3fg-eh)+b(17fg^2-h(5eg+dh))+2h\left(2ceg+3bfg-\frac{5cfg^2}{h}-2cdh-\right.\right.}{12h^2\left(cg^2-bgh+ah^2\right)(g+hx)^2}$$

$$-\frac{(fg^2-h(eg-dh))\left(a+bx+cx^2\right)^{5/2}}{3h\left(cg^2-bgh+ah^2\right)(g+hx)^3}$$

$$+\frac{(3b^2fh^2-12ch(4bfg-beh-afh)+8c^2(10fg^2-h(4eg-dh)))\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}h^6}$$

$$-\frac{(16c^3g^3(10fg^2-h(4eg-dh))-bh^3(24a^2fh^2-6abh(10fg-eh)+b^2(35fg^2-5egh-dh^2))+6ch^2(4a^2h}{}$$

```
[Out] -1/12*(2*c*g*(4*e*g-10*f*g^2/h-d*h)-6*a*h*(-e*h+3*f*g)+b*(17*f*g^2-h*(d*h+5
*e*g))+2*h*(2*c*e*g+3*b*f*g-5*c*f*g^2/h-2*c*d*h-3*a*f*h)*x)*(c*x^2+b*x+a)^(
3/2)/h^2/(a*h^2-b*g*h+c*g^2)/(h*x+g)^2-1/3*(f*g^2-h*(-d*h+e*g))*(c*x^2+b*x+
a)^(5/2)/h/(a*h^2-b*g*h+c*g^2)/(h*x+g)^3-1/16*(16*c^3*g^3*(10*f*g^2-h*(-d*h
+4*e*g))-b*h^3*(24*a^2*f*h^2-6*a*b*h*(-e*h+10*f*g)+b^2*(-d*h^2-5*e*g*h+35*f
*g^2))+6*c*h^2*(4*a^2*h^2*(-e*h+4*f*g)+b^2*g*(d*h^2-10*e*g*h+35*f*g^2)-2*a*
b*h*(d*h^2-7*e*g*h+25*f*g^2))-24*c^2*g*h*(b*g*(d*h^2-5*e*g*h+14*f*g^2)-a*h*
(d*h^2-4*e*g*h+11*f*g^2)))*arctanh(1/2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(a*h^2-b*
g*h+c*g^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/h^6/(a*h^2-b*g*h+c*g^2)^(3/2)+1/8*(3*
b^2*f*h^2-12*c*h*(-a*f*h-b*e*h+4*b*f*g)+8*c^2*(10*f*g^2-h*(-d*h+4*e*g)))*ar
```

```
ctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/h^6/c^(1/2)-1/8*(8*c^2*g^2
*(10*f*g^2-h*(-d*h+4*e*g))-2*c*h*(3*b*g*(d*h^2-6*e*g*h+18*f*g^2)-2*a*h*(2*d
*h^2-8*e*g*h+23*f*g^2))+h^2*(12*a^2*f*h^2-6*a*b*h*(-e*h+7*f*g)+b^2*(29*f*g^
2-h*(d*h+5*e*g)))+2*h*(3*b*f*h^2*(-a*h+b*g)+2*c^2*g*(10*f*g^2-h*(-d*h+4*e*g
))+c*h*(6*a*h*(-e*h+3*f*g)-b*(d*h^2-7*e*g*h+22*f*g^2)))*x)*(c*x^2+b*x+a)^(1
/2)/h^5/(a*h^2-b*g*h+c*g^2)/(h*x+g)
```

Rubi [A] (verified)

Time = 1.28 (sec) , antiderivative size = 829, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1664, 826, 857, 635, 212, 738}

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^4}\,dx = -\frac{(fg^2-h(eg-dh))(cx^2+bx+a)^{5/2}}{3h(cg^2-bhg+ah^2)(g+hx)^3}$$

$$-\frac{\left(17bfg^2+2c\left(-\frac{10fg^2}{h}+4eg-dh\right)g-bh(5eg+dh)-6ah(3fg-eh)+2h\left(-\frac{5cfg^2}{h}+2ceg+3bfg-2c\right.\right.}{12h^2(cg^2-bhg+ah^2)(g+hx)^2}$$

$$-\frac{\left(12a^2fh^3-6ab(7fg-eh)h^2+4ac(23fg^2-2h(4eg-dh))h+b^2(29fg^2-h(5eg+dh))h-8c^2g^2\left(-\right.\right.}{}$$

$$+\frac{(8(10fg^2-h(4eg-dh))c^2-12h(4bfg-beh-afh)c+3b^2fh^2)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{cx^2+bx+a}}\right)}{8\sqrt{c}h^6}$$

$$-\frac{(16(10fg^5-g^3h(4eg-dh))c^3-24gh(bg(14fg^2-5ehg+dh^2)-ah(11fg^2-4ehg+dh^2))c^2+6h^2(g(3}{}$$

```
[In] Int[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^4,x]
```

```
[Out] -1/8*((12*a^2*f*h^3 - 8*c^2*g^2*(4*e*g - (10*f*g^2)/h - d*h) - 6*a*b*h^2*(7
*f*g - e*h) + 4*a*c*h*(23*f*g^2 - 2*h*(4*e*g - d*h)) - 6*b*c*g*(18*f*g^2 -
h*(6*e*g - d*h)) + b^2*h*(29*f*g^2 - h*(5*e*g + d*h)) + 2*(3*b*f*h^2*(b*g -
 a*h) + 2*c^2*(10*f*g^3 - g*h*(4*e*g - d*h)) - c*h*(22*b*f*g^2 - b*h*(7*e*g
 - d*h) - 6*a*h*(3*f*g - e*h)))*x)*Sqrt[a + b*x + c*x^2])/(h^4*(c*g^2 - b*g
*h + a*h^2)*(g + h*x)) - ((17*b*f*g^2 + 2*c*g*(4*e*g - (10*f*g^2)/h - d*h)
- b*h*(5*e*g + d*h) - 6*a*h*(3*f*g - e*h) + 2*h*(2*c*e*g + 3*b*f*g - (5*c*f
*g^2)/h - 2*c*d*h - 3*a*f*h)*x)*(a + b*x + c*x^2)^(3/2))/(12*h^2*(c*g^2 - b
*g*h + a*h^2)*(g + h*x)^2) - ((f*g^2 - h*(e*g - d*h))*(a + b*x + c*x^2)^(5/
2))/(3*h*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^3) + ((3*b^2*f*h^2 - 12*c*h*(4*b
*f*g - b*e*h - a*f*h) + 8*c^2*(10*f*g^2 - h*(4*e*g - d*h)))*ArcTanh[(b + 2*
c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*Sqrt[c]*h^6) - ((16*c^3*(10*f*g
^5 - g^3*h*(4*e*g - d*h)) - b*h^3*(24*a^2*f*h^2 - 6*a*b*h*(10*f*g - e*h) +
b^2*(35*f*g^2 - 5*e*g*h - d*h^2)) + 6*c*h^2*(4*a^2*h^2*(4*f*g - e*h) + b^2*
g*(35*f*g^2 - 10*e*g*h + d*h^2) - 2*a*b*h*(25*f*g^2 - 7*e*g*h + d*h^2)) - 2
```

```
4*c^2*g*h*(b*g*(14*f*g^2 - 5*e*g*h + d*h^2) - a*h*(11*f*g^2 - 4*e*g*h + d*h
^2)))*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2
]*Sqrt[a + b*x + c*x^2])])/(16*h^6*(c*g^2 - b*g*h + a*h^2)^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 826

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
```

```
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

integral

$$= -\frac{(fg^2-h(eg-dh))\left(a+bx+cx^2\right)^{5/2}}{3h\left(cg^2-bgh+ah^2\right)(g+hx)^3} - \frac{\int \frac{\left(\frac{1}{2}\left(-6cdg+5beg+6afg-\frac{5bfg^2}{h}+bdh-6aeh\right)+\left(2ceg+3bfg-\frac{5cfg^2}{h}-2cdh-3afh\right)x\right)\left(a+bx+cx^2\right)^{3/2}}{(g+hx)^3}\,dx}{3\left(cg^2-bgh+ah^2\right)}$$

$$= -\frac{\left(17bfg^2+2cg\left(4eg-\frac{10fg^2}{h}-dh\right)-bh(5eg+dh)-6ah(3fg-eh)+2h\left(2ceg+3bfg-\frac{5cfg^2}{h}-2cdh\right.}{12h^2\left(cg^2-bgh+ah^2\right)(g+hx)^2}$$

$$-\frac{(fg^2-h(eg-dh))\left(a+bx+cx^2\right)^{5/2}}{3h\left(cg^2-bgh+ah^2\right)(g+hx)^3}$$

$$+\frac{\int \frac{\left(bh\left(6cdg-5beg-6afg+\frac{5bfg^2}{h}-bdh+6aeh\right)+(4bg-4ah)\left(2ceg+3bfg-\frac{5cfg^2}{h}-2cdh-3afh\right)-\frac{2\left(3bfh^2(bg-ah)+2c^2\left(10fg^3-gh(4eg-dh)\right)-c\right.}{h}\right.}{(g+hx)^2}}{8h^2\left(cg^2-bgh+ah^2\right)}$$

$$= -\frac{\left(12a^2fh^3-8c^2g^2\left(4eg-\frac{10fg^2}{h}-dh\right)-6abh^2(7fg-eh)+4ach(23fg^2-2h(4eg-dh))-6bcg(18fg\right.}{}$$

$$-\frac{\left(17bfg^2+2cg\left(4eg-\frac{10fg^2}{h}-dh\right)-bh(5eg+dh)-6ah(3fg-eh)+2h\left(2ceg+3bfg-\frac{5cfg^2}{h}-2cdh\right.}{12h^2\left(cg^2-bgh+ah^2\right)(g+hx)^2}$$

$$-\frac{(fg^2-h(eg-dh))\left(a+bx+cx^2\right)^{5/2}}{3h\left(cg^2-bgh+ah^2\right)(g+hx)^3}$$

$$-\frac{\int \frac{-\frac{b^3h^2\left(29fg^2-h(5eg+dh)\right)-8ach\left(10cfg^3-cgh(4eg-dh)+3ah^2(3fg-eh)\right)+4b\left(6a^2fh^4+2c^2\left(10fg^4-g^2h(4eg-dh)\right)+3ach^2\left(15fg^2-h(5eg-dh)\right)\right)-6b^2\left(\right.}{h}}{(g+hx)\sqrt{a+bx+cx^2}}}{16h^4\left(cg^2-bgh+ah\right.}$$

$$=$$

$$-\frac{\left(12a^2fh^3-8c^2g^2\left(4eg-\frac{10fg^2}{h}-dh\right)-6abh^2(7fg-eh)+4ach(23fg^2-2h(4eg-dh))-6bcg(18fg^2}{}$$

$$-\frac{\left(17bfg^2+2cg\left(4eg-\frac{10fg^2}{h}-dh\right)-bh(5eg+dh)-6ah(3fg-eh)+2h\left(2ceg+3bfg-\frac{5cfg^2}{h}-2cdh\right.\right.}{12h^2\left(cg^2-bgh+ah^2\right)(g+hx)^2}$$

$$-\frac{\left(fg^2-h(eg-dh)\right)\left(a+bx+cx^2\right)^{5/2}}{3h\left(cg^2-bgh+ah^2\right)(g+hx)^3}$$

$$+\frac{\left(3b^2fh^2-12ch(4bfg-beh-afh)+8c^2(10fg^2-h(4eg-dh))\right)\int\frac{1}{\sqrt{a+bx+cx^2}}\,dx}{8h^6}$$

$$-\frac{(16c^3(10fg^5-g^3h(4eg-dh))-bh^3(24a^2fh^2-6abh(10fg-eh)+b^2(35fg^2-5egh-dh^2))+6ch^2(4a}{}$$

$$=$$

$$-\frac{\left(12a^2fh^3-8c^2g^2\left(4eg-\frac{10fg^2}{h}-dh\right)-6abh^2(7fg-eh)+4ach(23fg^2-2h(4eg-dh))-6bcg(18fg^2}{}$$

$$-\frac{\left(17bfg^2+2cg\left(4eg-\frac{10fg^2}{h}-dh\right)-bh(5eg+dh)-6ah(3fg-eh)+2h\left(2ceg+3bfg-\frac{5cfg^2}{h}-2cdh\right.\right.}{12h^2\left(cg^2-bgh+ah^2\right)(g+hx)^2}$$

$$-\frac{\left(fg^2-h(eg-dh)\right)\left(a+bx+cx^2\right)^{5/2}}{3h\left(cg^2-bgh+ah^2\right)(g+hx)^3}$$

$$+\frac{\left(3b^2fh^2-12ch(4bfg-beh-afh)+8c^2(10fg^2-h(4eg-dh))\right)\operatorname{Subst}\left(\int\frac{1}{4c-x^2}\,dx,x,\frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{4h^6}$$

$$+\frac{(16c^3(10fg^5-g^3h(4eg-dh))-bh^3(24a^2fh^2-6abh(10fg-eh)+b^2(35fg^2-5egh-dh^2))+6ch^2(4a}{}$$

$$=$$

$$-\frac{\left(12a^2fh^3-8c^2g^2\left(4eg-\frac{10fg^2}{h}-dh\right)-6abh^2(7fg-eh)+4ach(23fg^2-2h(4eg-dh))-6bcg(18fg^2}{}$$

$$-\frac{\left(17bfg^2+2cg\left(4eg-\frac{10fg^2}{h}-dh\right)-bh(5eg+dh)-6ah(3fg-eh)+2h\left(2ceg+3bfg-\frac{5cfg^2}{h}-2cdh\right.\right.}{12h^2\left(cg^2-bgh+ah^2\right)(g+hx)^2}$$

$$-\frac{\left(fg^2-h(eg-dh)\right)\left(a+bx+cx^2\right)^{5/2}}{3h\left(cg^2-bgh+ah^2\right)(g+hx)^3}$$

$$+\frac{\left(3b^2fh^2-12ch(4bfg-beh-afh)+8c^2(10fg^2-h(4eg-dh))\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}h^6}$$

$$-\frac{(16c^3(10fg^5-g^3h(4eg-dh))-bh^3(24a^2fh^2-6abh(10fg-eh)+b^2(35fg^2-5egh-dh^2))+6ch^2(4a}{}$$

Mathematica [A] (verified)

Time = 13.34 (sec) , antiderivative size = 983, normalized size of antiderivative = 1.18

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^4}\,dx = \frac{-\frac{4(fg^2+h(-eg+dh))(a+x(b+cx))^{3/2}}{(g+hx)^3}+\frac{6(2fg-eh)(a+x(b+cx))^{3/2}}{(g+hx)^2}-\frac{12f(a+x(b+c}{g+hx}}{}$$

```
[In] Integrate[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^4,x]
```

```
[Out] ((-4*(f*g^2 + h*(-(e*g) + d*h))*(a + x*(b + c*x))^(3/2))/(g + h*x)^3 + (6*(
2*f*g - e*h)*(a + x*(b + c*x))^(3/2))/(g + h*x)^2 - (12*f*(a + x*(b + c*x))
^(3/2))/(g + h*x) + (9*f*(2*h*(-4*c*g + 3*b*h + 2*c*h*x)*Sqrt[a + x*(b + c*
x)] + ((8*c^2*g^2 + b^2*h^2 + 4*c*h*(-2*b*g + a*h))*ArcTanh[(b + 2*c*x)/(2*
Sqrt[c]*Sqrt[a + x*(b + c*x)])])/Sqrt[c] + 4*(2*c*g - b*h)*Sqrt[c*g^2 + h*(
-(b*g) + a*h)]*ArcTanh[(-(b*g) + 2*a*h - 2*c*g*x + b*h*x)/(2*Sqrt[c*g^2 + h
*(-(b*g) + a*h)]*Sqrt[a + x*(b + c*x)])]))/(2*h^3) + (9*(-2*f*g + e*h)*(((-
2*c*g + b*h)*(a + x*(b + c*x))^(3/2))/(g + h*x) - (Sqrt[a + x*(b + c*x)]*(b
^2*h^2 + 2*c^2*g*(2*g - h*x) + c*h*(-5*b*g + 2*a*h + b*h*x)))/h^2 + (4*Sqrt
[c]*(2*c*g - b*h)*(c*g^2 + h*(-(b*g) + a*h))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]
*Sqrt[a + x*(b + c*x)])] + (8*c^2*g^2 + b^2*h^2 + 4*c*h*(-2*b*g + a*h))*Sqr
t[c*g^2 + h*(-(b*g) + a*h)]*ArcTanh[(-(b*g) + 2*a*h - 2*c*g*x + b*h*x)/(2*S
qrt[c*g^2 + h*(-(b*g) + a*h)]*Sqrt[a + x*(b + c*x)])])/(2*h^3)))/(-(c*g^2)
+ h*(b*g - a*h)) + (3*(f*g^2 + h*(-(e*g) + d*h))*((4*(2*c*g - b*h)*(c*g^2 +
 h*(-(b*g) + a*h))*(a + x*(b + c*x))^(3/2))/(g + h*x)^2 + (2*(-4*c^2*g^2 +
b^2*h^2 + 4*c*h*(b*g - 2*a*h))*(a + x*(b + c*x))^(3/2))/(g + h*x) + (-2*c*h
*Sqrt[a + x*(b + c*x)]*(b^3*h^3 + 4*c^3*g^2*(2*g - h*x) + b*c*h^2*(5*b*g -
10*a*h + b*h*x) - 2*c^2*h*(b*g*(7*g - 2*h*x) + 2*a*h*(-3*g + 2*h*x))) + 16*
c^(5/2)*(c*g^2 + h*(-(b*g) + a*h))^2*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a
+ x*(b + c*x)])] + c*(2*c*g - b*h)*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*(8*c^2*g^
2 - b^2*h^2 + 4*c*h*(-2*b*g + 3*a*h))*ArcTanh[(-(b*g) + 2*a*h - 2*c*g*x + b
*h*x)/(2*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*Sqrt[a + x*(b + c*x)])])/(c*h^3)))/
(4*(c*g^2 + h*(-(b*g) + a*h))^2))/(12*h^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2703 vs. 2(801) = 1602.

Time = 1.04 (sec) , antiderivative size = 2704, normalized size of antiderivative = 3.25

method	result	size
risch	Expression too large to display	2704
default	Expression too large to display	6060

```
[In] int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(2*c*f*h*x+5*b*f*h+4*c*e*h-16*c*f*g)*(c*x^2+b*x+a)^(1/2)/h^5+1/8/h^5*((
12*a*c*f*h^2+3*b^2*f*h^2+12*b*c*e*h^2-48*b*c*f*g*h+8*c^2*d*h^2-32*c^2*e*g*h
+80*c^2*f*g^2)/h*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-(16*a*
b*f*h^3+16*a*c*e*h^3-64*a*c*f*g*h^2+8*b^2*e*h^3-32*b^2*f*g*h^2+16*b*c*d*h^3
-64*b*c*e*g*h^2+160*b*c*f*g^2*h-32*c^2*d*g*h^2+80*c^2*e*g^2*h-160*c^2*f*g^3
)/h^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*
c*g)/h*(x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c+(b*h-2*c*
g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g))+(8*a^2*f*h^4+16*a
*b*e*h^4-48*a*b*f*g*h^3+16*a*c*d*h^4-48*a*c*e*g*h^3+96*a*c*f*g^2*h^2+8*b^2*
d*h^4-24*b^2*e*g*h^3+48*b^2*f*g^2*h^2-48*b*c*d*g*h^3+96*b*c*e*g^2*h^2-160*b
*c*f*g^3*h+48*c^2*d*g^2*h^2-80*c^2*e*g^3*h+120*c^2*f*g^4)/h^3*(-1/(a*h^2-b*
g*h+c*g^2)*h^2/(x+1/h*g)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*
h+c*g^2)/h^2)^(1/2)+1/2*(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)/((a*h^2-b*g*h+c*g
^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+1/h*g)+2*((a*
h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b
*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g)))+(8*a^2*e*h^5-16*a^2*f*g*h^4+16*a*b*d*h^
5-32*a*b*e*g*h^4+48*a*b*f*g^2*h^3-32*a*c*d*g*h^4+48*a*c*e*g^2*h^3-64*a*c*f*
g^3*h^2-16*b^2*d*g*h^4+24*b^2*e*g^2*h^3-32*b^2*f*g^3*h^2+48*b*c*d*g^2*h^3-6
4*b*c*e*g^3*h^2+80*b*c*f*g^4*h-32*c^2*d*g^3*h^2+40*c^2*e*g^4*h-48*c^2*f*g^5
)/h^4*(-1/2/(a*h^2-b*g*h+c*g^2)*h^2/(x+1/h*g)^2*((x+1/h*g)^2*c+(b*h-2*c*g)/
h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)-3/4*(b*h-2*c*g)*h/(a*h^2-b*g*h+c
*g^2)*(-1/(a*h^2-b*g*h+c*g^2)*h^2/(x+1/h*g)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x
+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)+1/2*(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2
)/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)
/h*(x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c+(b*h-2*c*g)/h
*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g)))+1/2*c/(a*h^2-b*g*h+c
*g^2)*h^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*
h-2*c*g)/h*(x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c+(b*h-
2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g)))+1/h^5*(8*a^2
*d*h^6-8*a^2*e*g*h^5+8*a^2*f*g^2*h^4-16*a*b*d*g*h^5+16*a*b*e*g^2*h^4-16*a*b
*f*g^3*h^3+16*a*c*d*g^2*h^4-16*a*c*e*g^3*h^3+16*a*c*f*g^4*h^2+8*b^2*d*g^2*h
^4-8*b^2*e*g^3*h^3+8*b^2*f*g^4*h^2-16*b*c*d*g^3*h^3+16*b*c*e*g^4*h^2-16*b*c
*f*g^5*h+8*c^2*d*g^4*h^2-8*c^2*e*g^5*h+8*c^2*f*g^6)*(-1/3/(a*h^2-b*g*h+c*g^
2)*h^2/(x+1/h*g)^3*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^
2)/h^2)^(1/2)-5/6*(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)*(-1/2/(a*h^2-b*g*h+c*g^
2)*h^2/(x+1/h*g)^2*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^
2)/h^2)^(1/2)-3/4*(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)*(-1/(a*h^2-b*g*h+c*g^2)
*h^2/(x+1/h*g)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h
^2)^(1/2)+1/2*(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)/((a*h^2-b*g*h+c*g^2)/h^2)^(
1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+1/h*g)+2*((a*h^2-b*g*h+
c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2
)/h^2)^(1/2))/(x+1/h*g)))+1/2*c/(a*h^2-b*g*h+c*g^2)*h^2/((a*h^2-b*g*h+c*g^2
)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+1/h*g)+2*((a*h^
```

```
2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g
*h+c*g^2)/h^2)^(1/2))/(x+1/h*g)))-2/3*c/(a*h^2-b*g*h+c*g^2)*h^2*(-1/(a*h^2-
b*g*h+c*g^2)*h^2/(x+1/h*g)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*
g*h+c*g^2)/h^2)^(1/2)+1/2*(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)/((a*h^2-b*g*h+c
*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+1/h*g)+2*((
a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2
-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g)))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^4}\,dx = \text{Timed out}$$

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^4,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^4}\,dx = \int \frac{(a+bx+cx^2)^{\frac{3}{2}}(d+ex+fx^2)}{(g+hx)^4}\,dx$$

```
[In] integrate((c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**4,x)
```

```
[Out] Integral((a + b*x + c*x**2)**(3/2)*(d + e*x + f*x**2)/(g + h*x)**4, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^4}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^4,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*h^2-b*g*h>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7155 vs. 2(801) = 1602.

Time = 5.10 (sec) , antiderivative size = 7155, normalized size of antiderivative = 8.59

$$\int \frac{(a+bx+cx^2)^{3/2}\,(d+ex+fx^2)}{(g+hx)^4}\,dx = \text{Too large to display}$$

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^4,x, algorithm="giac")
```

```
[Out] 1/4*sqrt(c*x^2 + b*x + a)*(2*c*f*x/h^4 - (16*c^2*f*g*h^10 - 4*c^2*e*h^11 -
5*b*c*f*h^11)/(c*h^15)) - 1/8*(160*c^3*f*g^5 - 64*c^3*e*g^4*h - 336*b*c^2*f
*g^4*h + 16*c^3*d*g^3*h^2 + 120*b*c^2*e*g^3*h^2 + 210*b^2*c*f*g^3*h^2 + 264
*a*c^2*f*g^3*h^2 - 24*b*c^2*d*g^2*h^3 - 60*b^2*c*e*g^2*h^3 - 96*a*c^2*e*g^2
*h^3 - 35*b^3*f*g^2*h^3 - 300*a*b*c*f*g^2*h^3 + 6*b^2*c*d*g*h^4 + 24*a*c^2*
d*g*h^4 + 5*b^3*e*g*h^4 + 84*a*b*c*e*g*h^4 + 60*a*b^2*f*g*h^4 + 96*a^2*c*f*
g*h^4 + b^3*d*h^5 - 12*a*b*c*d*h^5 - 6*a*b^2*e*h^5 - 24*a^2*c*e*h^5 - 24*a^
2*b*f*h^5)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*h + sqrt(c)*g)/sqrt
(-c*g^2 + b*g*h - a*h^2))/((c*g^2*h^6 - b*g*h^7 + a*h^8)*sqrt(-c*g^2 + b*g*
h - a*h^2)) - 1/24*(480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*c^3*f*g^5*h^2
 - 288*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*c^3*e*g^4*h^3 - 912*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))^5*b*c^2*f*g^4*h^3 + 144*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))^5*c^3*d*g^3*h^4 + 504*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b*c^
2*e*g^3*h^4 + 522*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^2*c*f*g^3*h^4 + 5
52*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*c^2*f*g^3*h^4 - 216*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))^5*b*c^2*d*g^2*h^5 - 252*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^5*b^2*c*e*g^2*h^5 - 288*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*c^
2*e*g^2*h^5 - 87*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^3*f*g^2*h^5 - 540*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b*c*f*g^2*h^5 + 78*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^5*b^2*c*d*g*h^6 + 120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))^5*a*c^2*d*g*h^6 + 33*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^3*e*g*h^6 +
 228*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b*c*e*g*h^6 + 108*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))^5*a*b^2*f*g*h^6 + 96*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^5*a^2*c*f*g*h^6 - 3*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^3*d*h^7 -
 60*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b*c*d*h^7 - 30*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^5*a*b^2*e*h^7 - 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^
5*a^2*c*e*h^7 - 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*b*f*h^7 + 1680
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*c^(7/2)*f*g^6*h - 960*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^4*c^(7/2)*e*g^5*h^2 - 2880*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))^4*b*c^(5/2)*f*g^5*h^2 + 432*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4
*c^(7/2)*d*g^4*h^3 + 1464*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b*c^(5/2)*e
*g^4*h^3 + 1362*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^2*c^(3/2)*f*g^4*h^3
 + 1464*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*c^(5/2)*f*g^4*h^3 - 504*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^4*b*c^(5/2)*d*g^3*h^4 - 540*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^4*b^2*c^(3/2)*e*g^3*h^4 - 672*(sqrt(c)*x - sqrt(c*x^2
```

```
 + b*x + a))^4*a*c^(5/2)*e*g^3*h^4 - 147*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)^4*b^3*sqrt(c)*f*g^3*h^4 - 876*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b*c
^(3/2)*f*g^3*h^4 + 54*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^2*c^(3/2)*d*g
^2*h^5 + 216*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*c^(5/2)*d*g^2*h^5 + 21
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^3*sqrt(c)*e*g^2*h^5 + 180*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^4*a*b*c^(3/2)*e*g^2*h^5 - 36*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^4*a*b^2*sqrt(c)*f*g^2*h^5 - 144*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))^4*a^2*c^(3/2)*f*g^2*h^5 + 33*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
^4*b^3*sqrt(c)*d*g*h^6 + 84*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b*c^(3/
2)*d*g*h^6 + 90*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b^2*sqrt(c)*e*g*h^6
 + 168*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*c^(3/2)*e*g*h^6 + 216*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*b*sqrt(c)*f*g*h^6 - 48*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^4*a*b^2*sqrt(c)*d*h^7 - 96*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))^4*a^2*c^(3/2)*d*h^7 - 96*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^
2*b*sqrt(c)*e*h^7 - 48*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^3*sqrt(c)*f*
h^7 + 1504*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*c^4*f*g^7 - 832*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))^3*c^4*e*g^6*h - 1072*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^3*b*c^3*f*g^6*h + 352*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*c^4*d*g
^5*h^2 + 400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b*c^3*e*g^5*h^2 - 1308*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^2*c^2*f*g^5*h^2 - 656*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^3*a*c^3*f*g^5*h^2 - 16*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^3*b*c^3*d*g^4*h^3 + 840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^2*c^2
*e*g^4*h^3 + 512*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*c^3*e*g^4*h^3 + 10
42*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^3*c*f*g^4*h^3 + 4056*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))^3*a*b*c^2*f*g^4*h^3 - 420*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^3*b^2*c^2*d*g^3*h^4 - 272*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3
*a*c^3*d*g^3*h^4 - 478*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^3*c*e*g^3*h^
4 - 2232*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b*c^2*e*g^3*h^4 - 136*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^3*b^4*f*g^3*h^4 - 2712*(sqrt(c)*x - sqrt(c*
x^2 + b*x + a))^3*a*b^2*c*f*g^3*h^4 - 2208*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))^3*a^2*c^2*f*g^3*h^4 + 106*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^3*c*d
*g^2*h^5 + 840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b*c^2*d*g^2*h^5 + 40
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^4*e*g^2*h^5 + 1092*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))^3*a*b^2*c*e*g^2*h^5 + 1104*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^3*a^2*c^2*e*g^2*h^5 + 328*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*
b^3*f*g^2*h^5 + 1920*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b*c*f*g^2*h^
5 + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^4*d*g*h^6 - 144*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^3*a*b^2*c*d*g*h^6 - 384*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^3*a^2*c^2*d*g*h^6 - 88*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^3*
e*g*h^6 - 576*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b*c*e*g*h^6 - 240*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b^2*f*g*h^6 - 288*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^3*a^3*c*f*g*h^6 - 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
^3*a*b^3*d*h^7 + 48*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b^2*e*h^7 + 4
8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^3*b*f*h^7 + 2256*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^2*b*c^(7/2)*f*g^7 - 1248*(sqrt(c)*x - sqrt(c*x^2 + b*x
```

```
+ a))^2*b*c^(7/2)*e*g^6*h - 3420*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2*
c^(5/2)*f*g^6*h - 2832*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*c^(7/2)*f*g^
6*h + 528*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b*c^(7/2)*d*g^5*h^2 + 1656*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2*c^(5/2)*e*g^5*h^2 + 1536*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^2*a*c^(7/2)*e*g^5*h^2 + 1218*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^2*b^3*c^(3/2)*f*g^5*h^2 + 5976*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))^2*a*b*c^(5/2)*f*g^5*h^2 - 516*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
^2*b^2*c^(5/2)*d*g^4*h^3 - 624*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*c^(7
/2)*d*g^4*h^3 - 414*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^3*c^(3/2)*e*g^4
*h^3 - 2760*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b*c^(5/2)*e*g^4*h^3 - 2
4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^4*sqrt(c)*f*g^4*h^3 - 1944*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^2*a*b^2*c^(3/2)*f*g^4*h^3 - 2208*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))^2*a^2*c^(5/2)*f*g^4*h^3 - 6*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^2*b^3*c^(3/2)*d*g^3*h^4 + 840*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))^2*a*b*c^(5/2)*d*g^3*h^4 - 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^4*
sqrt(c)*e*g^3*h^4 + 420*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b^2*c^(3/2)
*e*g^3*h^4 + 912*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*c^(5/2)*e*g^3*h^
4 - 264*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b^3*sqrt(c)*f*g^3*h^4 + 192
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*b*c^(3/2)*f*g^3*h^4 + 24*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^2*b^4*sqrt(c)*d*g^2*h^5 + 144*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^2*a*b^2*c^(3/2)*d*g^2*h^5 - 288*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^2*a^2*c^(5/2)*d*g^2*h^5 + 168*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))^2*a*b^3*sqrt(c)*e*g^2*h^5 + 432*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a
^2*b*c^(3/2)*e*g^2*h^5 + 720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*b^2*
sqrt(c)*f*g^2*h^5 + 480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*c^(3/2)*f
*g^2*h^5 - 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b^3*sqrt(c)*d*g*h^6 -
 288*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*b*c^(3/2)*d*g*h^6 - 288*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*b^2*sqrt(c)*e*g*h^6 - 384*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))^2*a^3*c^(3/2)*e*g*h^6 - 528*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^2*a^3*b*sqrt(c)*f*g*h^6 + 96*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))^2*a^3*c^(3/2)*d*h^7 + 144*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*b*s
qrt(c)*e*h^7 + 96*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^4*sqrt(c)*f*h^7 +
 1128*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^2*c^3*f*g^7 - 624*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))*b^2*c^3*e*g^6*h - 1776*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))*b^3*c^2*f*g^6*h - 2832*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b*c^3*f
*g^6*h + 264*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^2*c^3*d*g^5*h^2 + 876*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*c^2*e*g^5*h^2 + 1536*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))*a*b*c^3*e*g^5*h^2 + 720*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))*b^4*c*f*g^5*h^2 + 5580*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^2*c^2*f
*g^5*h^2 + 1776*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*c^3*f*g^5*h^2 - 288
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*c^2*d*g^4*h^3 - 624*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))*a*b*c^3*d*g^4*h^3 - 282*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))*b^4*c*e*g^4*h^3 - 2664*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^2*c^2
*e*g^4*h^3 - 960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*c^3*e*g^4*h^3 - 57
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^5*f*g^4*h^3 - 2514*(sqrt(c)*x - sqrt
```

```
(c*x^2 + b*x + a))*a*b^3*c*f*g^4*h^3 - 5688*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))*a^2*b*c^2*f*g^4*h^3 + 36*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^4*c*d*g
^3*h^4 + 852*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^2*c^2*d*g^3*h^4 + 384*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*c^3*d*g^3*h^4 + 15*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))*b^5*e*g^3*h^4 + 894*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
*a*b^3*c*e*g^3*h^4 + 2640*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b*c^2*e*g
^3*h^4 + 198*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^4*f*g^3*h^4 + 3078*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^2*c*f*g^3*h^4 + 1848*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))*a^3*c^2*f*g^3*h^4 + 3*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))*b^5*d*g^2*h^5 - 90*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^3*c*d*g^2*h^
5 - 864*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b*c^2*d*g^2*h^5 - 48*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))*a*b^4*e*g^2*h^5 - 936*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))*a^2*b^2*c*e*g^2*h^5 - 816*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*
a^3*c^2*e*g^2*h^5 - 249*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^3*f*g^2*h
^5 - 1476*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*b*c*f*g^2*h^5 - 6*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))*a*b^4*d*g*h^6 + 90*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))*a^2*b^2*c*d*g*h^6 + 264*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*c^
2*d*g*h^6 + 51*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^3*e*g*h^6 + 300*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*b*c*e*g*h^6 + 132*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))*a^3*b^2*f*g*h^6 + 192*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*
a^4*c*f*g*h^6 + 3*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^3*d*h^7 - 36*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*b*c*d*h^7 - 18*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))*a^3*b^2*e*h^7 + 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^4*c*
e*h^7 - 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^4*b*f*h^7 + 188*b^3*c^(5/2
)*f*g^7 - 104*b^3*c^(5/2)*e*g^6*h - 272*b^4*c^(3/2)*f*g^6*h - 708*a*b^2*c^(
5/2)*f*g^6*h + 44*b^3*c^(5/2)*d*g^5*h^2 + 134*b^4*c^(3/2)*e*g^5*h^2 + 384*a
*b^2*c^(5/2)*e*g^5*h^2 + 87*b^5*sqrt(c)*f*g^5*h^2 + 1214*a*b^3*c^(3/2)*f*g^
5*h^2 + 888*a^2*b*c^(5/2)*f*g^5*h^2 - 44*b^4*c^(3/2)*d*g^4*h^3 - 156*a*b^2*
c^(5/2)*d*g^4*h^3 - 33*b^5*sqrt(c)*e*g^4*h^3 - 578*a*b^3*c^(3/2)*e*g^4*h^3
- 480*a^2*b*c^(5/2)*e*g^4*h^3 - 426*a*b^4*sqrt(c)*f*g^4*h^3 - 2010*a^2*b^2*
c^(3/2)*f*g^4*h^3 - 376*a^3*c^(5/2)*f*g^4*h^3 + 3*b^5*sqrt(c)*d*g^3*h^4 + 1
82*a*b^3*c^(3/2)*d*g^3*h^4 + 192*a^2*b*c^(5/2)*d*g^3*h^4 + 144*a*b^4*sqrt(c
)*e*g^3*h^4 + 936*a^2*b^2*c^(3/2)*e*g^3*h^4 + 208*a^3*c^(5/2)*e*g^3*h^4 + 8
07*a^2*b^3*sqrt(c)*f*g^3*h^4 + 1468*a^3*b*c^(3/2)*f*g^3*h^4 - 6*a*b^4*sqrt(
c)*d*g^2*h^5 - 294*a^2*b^2*c^(3/2)*d*g^2*h^5 - 88*a^3*c^(5/2)*d*g^2*h^5 - 2
37*a^2*b^3*sqrt(c)*e*g^2*h^5 - 676*a^3*b*c^(3/2)*e*g^2*h^5 - 732*a^3*b^2*sq
rt(c)*f*g^2*h^5 - 400*a^4*c^(3/2)*f*g^2*h^5 + 3*a^2*b^3*sqrt(c)*d*g*h^6 + 2
20*a^3*b*c^(3/2)*d*g*h^6 + 174*a^3*b^2*sqrt(c)*e*g*h^6 + 184*a^4*c^(3/2)*e*
g*h^6 + 312*a^4*b*sqrt(c)*f*g*h^6 - 64*a^4*c^(3/2)*d*h^7 - 48*a^4*b*sqrt(c)
*e*h^7 - 48*a^5*sqrt(c)*f*h^7)/((c*g^2*h^6 - b*g*h^7 + a*h^8)*((sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))^2*h + 2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c)
*g + b*g - a*h)^3) - 1/8*(80*c^2*f*g^2 - 32*c^2*e*g*h - 48*b*c*f*g*h + 8*c^
2*d*h^2 + 12*b*c*e*h^2 + 3*b^2*f*h^2 + 12*a*c*f*h^2)*log(abs(-2*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/(sqrt(c)*h^6)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}\,(d+ex+fx^2)}{(g+hx)^4}\,dx=\int \frac{{\left(c\,x^2+b\,x+a\right)}^{3/2}\,\left(f\,x^2+e\,x+d\right)}{{\left(g+h\,x\right)}^4}\,dx$$

```
[In] int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^4,x)
[Out] int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^4, x)
```

3.204 $\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^5}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 1097

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^5}\,dx = \frac{(64c^3g^4(5fg-eh)-16c^2g^2h(bg(41fg-7eh)-8ah(5fg-eh))}{}$$

$$-\frac{(16c^2g^4(5fg-eh)-h^2(16a^2h^2(fg-2eh)-b^2g(35fg^2+5egh+3dh^2)+4abh(7fg^2+7egh+3dh^2))-}{}$$

$$-\frac{(fg^2-h(eg-dh))(a+bx+cx^2)^{5/2}}{4h(cg^2-bgh+ah^2)(g+hx)^4}-\frac{\sqrt{c}(10cfg-2ceh-3bfh)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2h^6}$$

$$+\frac{(128c^4g^5(5fg-eh)-64c^3g^3h(bg(28fg-5eh)-5ah(5fg-eh))+8ch^3(24a^3fh^3-12a^2bh^2(10fg-eh)}{}$$

```
[Out] -1/96*(16*c^2*g^4*(-e*h+5*f*g)-h^2*(16*a^2*h^2*(-2*e*h+f*g)-b^2*g*(3*d*h^2+
5*e*g*h+35*f*g^2)+4*a*b*h*(3*d*h^2+7*e*g*h+7*f*g^2))-4*c*g*h*(b*g*(3*d*h^2-
5*e*g*h+31*f*g^2)-a*h*(9*d*h^2-5*e*g*h+25*f*g^2))+3*h*(8*c^2*g^2*(5*f*g^2-h
*(d*h+e*g))+h^2*(16*a^2*f*h^2-8*a*b*h*(-e*h+6*f*g)+b^2*(-3*d*h^2-5*e*g*h+29
*f*g^2))-4*c*h*(2*b*g*(-d*h^2-2*e*g*h+9*f*g^2)-a*h*(d*h^2-5*e*g*h+17*f*g^2)
))*x)*(c*x^2+b*x+a)^(3/2)/h^3/(a*h^2-b*g*h+c*g^2)^2/(h*x+g)^3-1/4*(f*g^2-h*
(-d*h+e*g))*(c*x^2+b*x+a)^(5/2)/h/(a*h^2-b*g*h+c*g^2)/(h*x+g)^4+1/128*(128*
c^4*g^5*(-e*h+5*f*g)-64*c^3*g^3*h*(b*g*(-5*e*h+28*f*g)-5*a*h*(-e*h+5*f*g))+
8*c*h^3*(24*a^3*f*h^3-12*a^2*b*h^2*(-e*h+10*f*g)-5*b^3*g^2*(-e*h+14*f*g)+3*
a*b^2*h*(-d*h^2-5*e*g*h+55*f*g^2))-48*c^2*h^2*(10*a*b*g^2*h*(-e*h+6*f*g)-5*
b^2*g^3*(-e*h+7*f*g)-a^2*h^2*(d*h^2-5*e*g*h+25*f*g^2))+b^2*h^4*(48*a^2*f*h^
2-8*a*b*h*(e*h+10*f*g)+b^2*(3*d*h^2+5*e*g*h+35*f*g^2)))*arctanh(1/2*(b*g-2*
a*h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/h^6/(a*h
^2-b*g*h+c*g^2)^(5/2)-1/2*(-3*b*f*h-2*c*e*h+10*c*f*g)*arctanh(1/2*(2*c*x+b)
/c^(1/2)/(c*x^2+b*x+a)^(1/2))*c^(1/2)/h^6+1/64*(64*c^3*g^4*(-e*h+5*f*g)-16*
```

```
c^2*g^2*h*(b*g*(-7*e*h+41*f*g)-8*a*h*(-e*h+5*f*g))+4*c*h^2*(2*b^2*g^2*(-5*e
*h+46*f*g)+16*a^2*h^2*(-e*h+5*f*g)-a*b*h*(-3*d*h^2-25*e*g*h+173*f*g^2))-b*h
^3*(48*a^2*f*h^2-8*a*b*h*(e*h+10*f*g)+b^2*(3*d*h^2+5*e*g*h+35*f*g^2))+2*c*h
*(16*c^2*g^3*(-e*h+5*f*g)-4*c*h*(6*b*g^2*(-e*h+6*f*g)-a*h*(35*f*g^2-h*(-3*d
*h+7*e*g)))+h^2*(48*a^2*f*h^2-8*a*b*h*(-e*h+14*f*g)+b^2*(61*f*g^2-h*(3*d*h+
5*e*g))))*x)*(c*x^2+b*x+a)^(1/2)/h^5/(a*h^2-b*g*h+c*g^2)^2/(h*x+g)
```

Rubi [A] (verified)

Time = 1.90 (sec) , antiderivative size = 1096, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {1664, 824, 826, 857, 635, 212, 738}

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^5}\,dx = -\frac{\left(fg^2-h(eg-dh)\right)\left(cx^2+bx+a\right)^{5/2}}{4h\left(cg^2-bhg+ah^2\right)(g+hx)^4}$$

$$-\frac{\left(\frac{16c^2(5fg-eh)g^4}{h}-4c\left(bg(31fg^2-5ehg+3dh^2)-ah(25fg^2-5ehg+9dh^2)\right)g-h(-g(35fg^2+5ehg+3dh\right.}{}$$

$$+\frac{\left(\frac{64c^3(5fg-eh)g^4}{h}-16c^2(bg(41fg-7eh)-8ah(5fg-eh))g^2+4ch(2b^2(46fg-5eh)g^2+16a^2h^2(5fg-eh)-\right.}{}$$

$$-\frac{\sqrt{c}(10cfg-2ceh-3bfh)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{cx^2+bx+a}}\right)}{2h^6}$$

$$+\frac{(128c^4(5fg-eh)g^5-64c^3h(bg(28fg-5eh)-5ah(5fg-eh))g^3+8ch^3(-5g^2(14fg-eh)b^3+3ah(55fg^2}{}$$

```
[In] Int[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^5,x]
```

```
[Out] (((64*c^3*g^4*(5*f*g - e*h))/h - 16*c^2*g^2*(b*g*(41*f*g - 7*e*h) - 8*a*h*(
5*f*g - e*h)) + 4*c*h*(2*b^2*g^2*(46*f*g - 5*e*h) + 16*a^2*h^2*(5*f*g - e*h
) - a*b*h*(173*f*g^2 - 25*e*g*h - 3*d*h^2)) - b*h^2*(48*a^2*f*h^2 - 8*a*b*h
*(10*f*g + e*h) + b^2*(35*f*g^2 + 5*e*g*h + 3*d*h^2)) + 2*c*(16*c^2*g^3*(5*
f*g - e*h) - 4*c*h*(6*b*g^2*(6*f*g - e*h) - a*h*(35*f*g^2 - h*(7*e*g - 3*d*
h))) + h^2*(48*a^2*f*h^2 - 8*a*b*h*(14*f*g - e*h) + b^2*(61*f*g^2 - h*(5*e*
g + 3*d*h))))*x)*Sqrt[a + b*x + c*x^2])/(64*h^4*(c*g^2 - b*g*h + a*h^2)^2*(
g + h*x)) - (((16*c^2*g^4*(5*f*g - e*h))/h - h*(16*a^2*h^2*(f*g - 2*e*h) -
b^2*g*(35*f*g^2 + 5*e*g*h + 3*d*h^2) + 4*a*b*h*(7*f*g^2 + 7*e*g*h + 3*d*h^2
)) - 4*c*g*(b*g*(31*f*g^2 - 5*e*g*h + 3*d*h^2) - a*h*(25*f*g^2 - 5*e*g*h +
9*d*h^2)) + 3*h*((40*c^2*f*g^4)/h + 16*a^2*f*h^3 - 8*c^2*g^2*(e*g + d*h) -
8*a*b*h^2*(6*f*g - e*h) + 4*a*c*h*(17*f*g^2 - h*(5*e*g - d*h)) - 8*b*c*g*(9
*f*g^2 - h*(2*e*g + d*h)) + b^2*h*(29*f*g^2 - h*(5*e*g + 3*d*h))))*x)*(a + b
*x + c*x^2)^(3/2))/(96*h^2*(c*g^2 - b*g*h + a*h^2)^2*(g + h*x)^3) - ((f*g^2
 - h*(e*g - d*h))*(a + b*x + c*x^2)^(5/2))/(4*h*(c*g^2 - b*g*h + a*h^2)*(g
```

```
+ h*x)^4) - (Sqrt[c]*(10*c*f*g - 2*c*e*h - 3*b*f*h)*ArcTanh[(b + 2*c*x)/(2*
Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*h^6) + ((128*c^4*g^5*(5*f*g - e*h) - 64
*c^3*g^3*h*(b*g*(28*f*g - 5*e*h) - 5*a*h*(5*f*g - e*h)) + 8*c*h^3*(24*a^3*f
*h^3 - 12*a^2*b*h^2*(10*f*g - e*h) - 5*b^3*g^2*(14*f*g - e*h) + 3*a*b^2*h*(
55*f*g^2 - 5*e*g*h - d*h^2)) - 48*c^2*h^2*(10*a*b*g^2*h*(6*f*g - e*h) - 5*b
^2*g^3*(7*f*g - e*h) - a^2*h^2*(25*f*g^2 - 5*e*g*h + d*h^2)) + b^2*h^4*(48*
a^2*f*h^2 - 8*a*b*h*(10*f*g + e*h) + b^2*(35*f*g^2 + 5*e*g*h + 3*d*h^2)))*A
rcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[
a + b*x + c*x^2])])/(128*h^6*(c*g^2 - b*g*h + a*h^2)^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 824

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Dist[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)
^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p
 + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*
(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m
 - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &
& LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 826

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
```

```
d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{\left(fg^2-h(eg-dh)\right)\left(a+bx+cx^2\right)^{5/2}}{4h\left(cg^2-bgh+ah^2\right)(g+hx)^4} - \frac{\int \frac{\left(\frac{1}{2}\left(-8cdg+5beg+8afg-\frac{5bfg^2}{h}+3bdh-8aeh\right)+\left(ceg+4bfg-\frac{5cfg^2}{h}-cdh-4afh\right)x\right)\left(a+bx+cx^2\right)^{3/2}}{(g+hx)^4}\,dx}{4\left(cg^2-bgh+ah^2\right)}$$

$$= -\frac{\left(\frac{16c^2g^4(5fg-eh)}{h} - h(16a^2h^2(fg-2eh) - b^2g(35fg^2+5egh+3dh^2) + 4abh(7fg^2+7egh+3dh^2)\right.}{}$$

$$-\frac{\left(fg^2-h(eg-dh)\right)\left(a+bx+cx^2\right)^{5/2}}{4h\left(cg^2-bgh+ah^2\right)(g+hx)^4} + \frac{\int \frac{\left(\frac{1}{4}\left(48a^2bfh^3+\frac{16bc^2g^3(5fg-eh)}{h}+b^3h\left(35fg^2+h(5eg+3dh)\right)+4abch\left(61fg^2-h(17eg+3dh)\right)-16ac\left(5cfg^3-cgh(eg+3dh)+4ah^2(2\right.\right.\right.}{}}{}$$

$$=\frac{\left(\frac{64c^3g^4(5fg-eh)}{h}-16c^2g^2(bg(41fg-7eh)-8ah(5fg-eh))+4ch(2b^2g^2(46fg-5eh)+16a^2h^2(5}{}$$

$$-\frac{\left(\frac{16c^2g^4(5fg-eh)}{h}-h(16a^2h^2(fg-2eh)-b^2g(35fg^2+5egh+3dh^2)+4abh(7fg^2+7egh+3dh^2}{}$$

$$-\frac{(fg^2-h(eg-dh))\left(a+bx+cx^2\right)^{5/2}}{4h\left(cg^2-bgh+ah^2\right)(g+hx)^4}$$

$$-\frac{\int\frac{-\frac{b^4h^3\left(35fg^2+h(5eg+3dh)\right)-32bc\left(cg^2+3ah^2\right)\left(2cg^2(5fg-eh)+ah^2(8fg-eh)\right)-8b^3\left(cg^2h^2(46fg-5eh)+ah^4(10fg+eh)\right)+8b^2\left(6a^2fh^5+2c^2}{4h}}{(g+}}{32h^4\,(c}$$

$$=\frac{\left(\frac{64c^3g^4(5fg-eh)}{h}-16c^2g^2(bg(41fg-7eh)-8ah(5fg-eh))+4ch(2b^2g^2(46fg-5eh)+16a^2h^2(5}{}$$

$$-\frac{\left(\frac{16c^2g^4(5fg-eh)}{h}-h(16a^2h^2(fg-2eh)-b^2g(35fg^2+5egh+3dh^2)+4abh(7fg^2+7egh+3dh^2}{}$$

$$-\frac{(fg^2-h(eg-dh))\left(a+bx+cx^2\right)^{5/2}}{4h\left(cg^2-bgh+ah^2\right)(g+hx)^4}-\frac{(c(10cfg-2ceh-3bfh))\int\frac{1}{\sqrt{a+bx+cx^2}}\,dx}{2h^6}$$

$$+\frac{(128c^4g^5(5fg-eh)-64c^3g^3h(bg(28fg-5eh)-5ah(5fg-eh))+8ch^3(24a^3fh^3-12a^2bh^2(1}{}$$

$$=\frac{\left(\frac{64c^3g^4(5fg-eh)}{h}-16c^2g^2(bg(41fg-7eh)-8ah(5fg-eh))+4ch(2b^2g^2(46fg-5eh)+16a^2h^2(5}{}$$

$$-\frac{\left(\frac{16c^2g^4(5fg-eh)}{h}-h(16a^2h^2(fg-2eh)-b^2g(35fg^2+5egh+3dh^2)+4abh(7fg^2+7egh+3dh^2}{}$$

$$-\frac{(fg^2-h(eg-dh))\left(a+bx+cx^2\right)^{5/2}}{4h\left(cg^2-bgh+ah^2\right)(g+hx)^4}$$

$$-\frac{(c(10cfg-2ceh-3bfh))\text{Subst}\left(\int\frac{1}{4c-x^2}\,dx,x,\frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{h^6}$$

$$-\frac{(128c^4g^5(5fg-eh)-64c^3g^3h(bg(28fg-5eh)-5ah(5fg-eh))+8ch^3(24a^3fh^3-12a^2bh^2(1}{}$$

$$= \frac{\Big(\frac{64c^3g^4(5fg-eh)}{h} - 16c^2g^2(bg(41fg-7eh)-8ah(5fg-eh)) + 4ch(2b^2g^2(46fg-5eh)+16a^2h^2(5f}{}$$

$$- \frac{\Big(\frac{16c^2g^4(5fg-eh)}{h} - h(16a^2h^2(fg-2eh)-b^2g(35fg^2+5egh+3dh^2)+4abh(7fg^2+7egh+3dh^2)}{}$$

$$- \frac{(fg^2-h(eg-dh))\left(a+bx+cx^2\right)^{5/2}}{4h\left(cg^2-bgh+ah^2\right)(g+hx)^4}$$

$$- \frac{\sqrt{c}(10cfg-2ceh-3bfh)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2h^6}$$

$$+ \frac{(128c^4g^5(5fg-eh)-64c^3g^3h(bg(28fg-5eh)-5ah(5fg-eh))+8ch^3(24a^3fh^3-12a^2bh^2(10}{}$$

Mathematica [A] (verified)

Time = 15.59 (sec) , antiderivative size = 1005, normalized size of antiderivative = 0.92

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^5}\,dx = \frac{\frac{128(2fg-eh)(a+x(b+cx))^{3/2}}{(g+hx)^3} - \frac{192f(a+x(b+cx))^{3/2}}{(g+hx)^2} + \frac{48h\big(fg^2+h(-eg+dh)\big)(a+x(b+}{(cg^2+h(-bg-}}{}$$

```
[In] Integrate[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^5,x]
```

```
[Out] ((128*(2*f*g - e*h)*(a + x*(b + c*x))^(3/2))/(g + h*x)^3 - (192*f*(a + x*(b
 + c*x))^(3/2))/(g + h*x)^2 + (48*h*(f*g^2 + h*(-(e*g) + d*h))*(a + x*(b +
c*x))^(3/2)*(-2*a*h + 2*c*g*x + b*(g - h*x)))/((c*g^2 + h*(-(b*g) + a*h))*(
g + h*x)^4) + (288*f*(((-2*c*g + b*h)*(a + x*(b + c*x))^(3/2))/(g + h*x) -
(Sqrt[a + x*(b + c*x)]*(b^2*h^2 + 2*c^2*g*(2*g - h*x) + c*h*(-5*b*g + 2*a*h
 + b*h*x)))/h^2 + (4*Sqrt[c]*(2*c*g - b*h)*(c*g^2 + h*(-(b*g) + a*h))*ArcTa
nh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] + (8*c^2*g^2 + b^2*h^2 +
4*c*h*(-2*b*g + a*h))*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*ArcTanh[(-(b*g) + 2*a*
h - 2*c*g*x + b*h*x)/(2*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*Sqrt[a + x*(b + c*x)
])])/(2*h^3)))/(-(c*g^2) + h*(b*g - a*h)) + (24*(-2*f*g + e*h)*((4*(2*c*g -
 b*h)*(c*g^2 + h*(-(b*g) + a*h))*(a + x*(b + c*x))^(3/2))/(g + h*x)^2 + (2*
(-4*c^2*g^2 + b^2*h^2 + 4*c*h*(b*g - 2*a*h))*(a + x*(b + c*x))^(3/2))/(g +
h*x) + (-2*c*h*Sqrt[a + x*(b + c*x)]*(b^3*h^3 + 4*c^3*g^2*(2*g - h*x) + b*c
*h^2*(5*b*g - 10*a*h + b*h*x) - 2*c^2*h*(b*g*(7*g - 2*h*x) + 2*a*h*(-3*g +
2*h*x))) + 16*c^(5/2)*(c*g^2 + h*(-(b*g) + a*h))^2*ArcTanh[(b + 2*c*x)/(2*S
qrt[c]*Sqrt[a + x*(b + c*x)])] + c*(2*c*g - b*h)*Sqrt[c*g^2 + h*(-(b*g) + a
*h)]*(8*c^2*g^2 - b^2*h^2 + 4*c*h*(-2*b*g + 3*a*h))*ArcTanh[(-(b*g) + 2*a*h
```

```
 - 2*c*g*x + b*h*x)/(2*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*Sqrt[a + x*(b + c*x)]
)])/(c*h^3)))/(c*g^2 + h*(-(b*g) + a*h))^2 - (9*(b^2 - 4*a*c)*h*(f*g^2 + h*
(-(e*g) + d*h))*((2*Sqrt[a + x*(b + c*x)]*(-2*a*h + 2*c*g*x + b*(g - h*x)))
/((c*g^2 + h*(-(b*g) + a*h))*(g + h*x)^2) + ((-b^2 + 4*a*c)*ArcTanh[(-2*a*h
 + 2*c*g*x + b*(g - h*x))/(2*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*Sqrt[a + x*(b +
 c*x)])])/(c*g^2 + h*(-(b*g) + a*h))^(3/2)))/(c*g^2 + h*(-(b*g) + a*h)))/(3
84*h^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 4358 vs. 2(1065) = 2130.

Time = 1.21 (sec) , antiderivative size = 4359, normalized size of antiderivative = 3.97

method	result	size
risch	Expression too large to display	4359
default	Expression too large to display	10038

```
[In] int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^5,x,method=_RETURNVERBOSE)
```

```
[Out] f/h^5*(c*x^2+b*x+a)^(1/2)*c+1/2/h^5*(c^(1/2)*(3*b*f*h+2*c*e*h-10*c*f*g)/h*l
n((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-(4*a*c*f*h^2+2*b^2*f*h^2+4*b*c*e
*h^2-20*b*c*f*g*h+2*c^2*d*h^2-10*c^2*e*g*h+30*c^2*f*g^2)/h^2/((a*h^2-b*g*h+
c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+1/h*g)+2*(
(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^
2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g))+(4*a*b*f*h^3+4*a*c*e*h^3-16*a*c*f*g*h
^2+2*b^2*e*h^3-8*b^2*f*g*h^2+4*b*c*d*h^3-16*b*c*e*g*h^2+40*b*c*f*g^2*h-8*c^
2*d*g*h^2+20*c^2*e*g^2*h-40*c^2*f*g^3)/h^3*(-1/(a*h^2-b*g*h+c*g^2)*h^2/(x+1
/h*g)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)
+1/2*(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((
2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h^
2)^(1/2)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1
/2))/(x+1/h*g)))+(2*a^2*f*h^4+4*a*b*e*h^4-12*a*b*f*g*h^3+4*a*c*d*h^4-12*a*c
*e*g*h^3+24*a*c*f*g^2*h^2+2*b^2*d*h^4-6*b^2*e*g*h^3+12*b^2*f*g^2*h^2-12*b*c
*d*g*h^3+24*b*c*e*g^2*h^2-40*b*c*f*g^3*h+12*c^2*d*g^2*h^2-20*c^2*e*g^3*h+30
*c^2*f*g^4)/h^4*(-1/2/(a*h^2-b*g*h+c*g^2)*h^2/(x+1/h*g)^2*((x+1/h*g)^2*c+(b
*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)-3/4*(b*h-2*c*g)*h/(a*h
^2-b*g*h+c*g^2)*(-1/(a*h^2-b*g*h+c*g^2)*h^2/(x+1/h*g)*((x+1/h*g)^2*c+(b*h-2
*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)+1/2*(b*h-2*c*g)*h/(a*h^2-b
*g*h+c*g^2)/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(
b*h-2*c*g)/h*(x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c+(b*
h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g)))+1/2*c/(a*h
^2-b*g*h+c*g^2)*h^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^
2)/h^2+(b*h-2*c*g)/h*(x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)
^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g)))+(2
```

```
*a^2*e*h^5-4*a^2*f*g*h^4+4*a*b*d*h^5-8*a*b*e*g*h^4+12*a*b*f*g^2*h^3-8*a*c*d
*g*h^4+12*a*c*e*g^2*h^3-16*a*c*f*g^3*h^2-4*b^2*d*g*h^4+6*b^2*e*g^2*h^3-8*b^
2*f*g^3*h^2+12*b*c*d*g^2*h^3-16*b*c*e*g^3*h^2+20*b*c*f*g^4*h-8*c^2*d*g^3*h^
2+10*c^2*e*g^4*h-12*c^2*f*g^5)/h^5*(-1/3/(a*h^2-b*g*h+c*g^2)*h^2/(x+1/h*g)^
3*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)-5/6
*(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)*(-1/2/(a*h^2-b*g*h+c*g^2)*h^2/(x+1/h*g)^
2*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)-3/4
*(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)*(-1/(a*h^2-b*g*h+c*g^2)*h^2/(x+1/h*g)*((
x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)+1/2*(b*
h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2
-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)
*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+
1/h*g)))+1/2*c/(a*h^2-b*g*h+c*g^2)*h^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((
2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h^
2)^(1/2)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1
/2))/(x+1/h*g)))-2/3*c/(a*h^2-b*g*h+c*g^2)*h^2*(-1/(a*h^2-b*g*h+c*g^2)*h^2/
(x+1/h*g)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(
1/2)+1/2*(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*
ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+1/h*g)+2*((a*h^2-b*g*h+c*g^2
)/h^2)^(1/2)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2
)^(1/2))/(x+1/h*g))))+1/h^6*(2*a^2*d*h^6-2*a^2*e*g*h^5+2*a^2*f*g^2*h^4-4*a*
b*d*g*h^5+4*a*b*e*g^2*h^4-4*a*b*f*g^3*h^3+4*a*c*d*g^2*h^4-4*a*c*e*g^3*h^3+4
*a*c*f*g^4*h^2+2*b^2*d*g^2*h^4-2*b^2*e*g^3*h^3+2*b^2*f*g^4*h^2-4*b*c*d*g^3*
h^3+4*b*c*e*g^4*h^2-4*b*c*f*g^5*h+2*c^2*d*g^4*h^2-2*c^2*e*g^5*h+2*c^2*f*g^6
)*(-1/4/(a*h^2-b*g*h+c*g^2)*h^2/(x+1/h*g)^4*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x
+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)-7/8*(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2
)*(-1/3/(a*h^2-b*g*h+c*g^2)*h^2/(x+1/h*g)^3*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x
+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)-5/6*(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2
)*(-1/2/(a*h^2-b*g*h+c*g^2)*h^2/(x+1/h*g)^2*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x
+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)-3/4*(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2
)*(-1/(a*h^2-b*g*h+c*g^2)*h^2/(x+1/h*g)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h
*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)+1/2*(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)/((
a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(
x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+
1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g)))+1/2*c/(a*h^2-b*g*h+c*g^2
)*h^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*
c*g)/h*(x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c+(b*h-2*c*
g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g)))-2/3*c/(a*h^2-b*g
*h+c*g^2)*h^2*(-1/(a*h^2-b*g*h+c*g^2)*h^2/(x+1/h*g)*((x+1/h*g)^2*c+(b*h-2*c
*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)+1/2*(b*h-2*c*g)*h/(a*h^2-b*g
*h+c*g^2)/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*
h-2*c*g)/h*(x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c+(b*h-
2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g))))-3/4*c/(a*h^
2-b*g*h+c*g^2)*h^2*(-1/2/(a*h^2-b*g*h+c*g^2)*h^2/(x+1/h*g)^2*((x+1/h*g)^2*c
+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)-3/4*(b*h-2*c*g)*h/(
```

```
a*h^2-b*g*h+c*g^2)*(-1/(a*h^2-b*g*h+c*g^2)*h^2/(x+1/h*g)*((x+1/h*g)^2*c+(b*
h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)+1/2*(b*h-2*c*g)*h/(a*h^
2-b*g*h+c*g^2)/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^
2+(b*h-2*c*g)/h*(x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c+
(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g)))+1/2*c/(
a*h^2-b*g*h+c*g^2)*h^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c
*g^2)/h^2+(b*h-2*c*g)/h*(x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h
*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g)))
))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^5}\,dx = \text{Timed out}$$

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^5,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^5}\,dx = \int \frac{(a+bx+cx^2)^{\frac{3}{2}}(d+ex+fx^2)}{(g+hx)^5}\,dx$$

```
[In] integrate((c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**5,x)
```

```
[Out] Integral((a + b*x + c*x**2)**(3/2)*(d + e*x + f*x**2)/(g + h*x)**5, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^5}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^5,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*h^2-b*g*h>0)', see `assume?` for
more de
```

Giac [F(-1)]

Timed out.

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^5}\,dx=\text{Timed out}$$

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^5,x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^5}\,dx=\int \frac{{\left(c\,x^2+b\,x+a\right)}^{3/2}\,\left(f\,x^2+e\,x+d\right)}{{\left(g+h\,x\right)}^5}\,dx$$

```
[In] int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^5,x)
[Out] int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^5, x)
```

3.205 $\int \frac{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^6}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 1226

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^6}\,dx =$$

$$-\frac{\left(128c^4fg^7-32c^3fg^5h(11bg-10ah)+8c^2gh^2(38b^2fg^4+2a^2h^2(13fg^2+3dh^2)-abgh(65fg^2+3dh^2))-\right.}{}$$

$$-\frac{(16c^2fg^5-2cgh(13bfg^3-10afg^2h+3bdgh^2-6adh^3)-h^2(4a^2h^2(2fg-3eh)-b^2g(7fg^2+3h(eg+dh}{}$$

$$-\frac{(fg^2-h(eg-dh))\left(a+bx+cx^2\right)^{5/2}}{5h\left(cg^2-bgh+ah^2\right)(g+hx)^5}+\frac{c^{3/2}f\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{h^6}$$

$$-\frac{(256c^5fg^7-896c^4fg^5h(bg-ah)+32c^3gh^2(35b^2fg^4-70abfg^3h+a^2h^2(35fg^2-3dh^2))-16c^2h^3(35b^3f}{}$$

```
[Out] -1/48*(16*c^2*f*g^5-2*c*g*h*(-6*a*d*h^3-10*a*f*g^2*h+3*b*d*g*h^2+13*b*f*g^3
)-h^2*(4*a^2*h^2*(-3*e*h+2*f*g)-b^2*g*(7*f*g^2+3*h*(d*h+e*g))+2*a*b*h*(f*g^
2+3*h*(d*h+2*e*g)))+h*(4*c^2*(-3*d*g^2*h^2+7*f*g^4)+2*c*g*h*(2*a*h*(-3*e*h+
14*f*g)-b*(-6*d*h^2-3*e*g*h+28*f*g^2))+h^2*(16*a^2*f*h^2-2*a*b*h*(-3*e*h+22
*f*g)+b^2*(25*f*g^2-3*h*(d*h+e*g))))*x)*(c*x^2+b*x+a)^(3/2)/h^3/(a*h^2-b*g*
h+c*g^2)^2/(h*x+g)^4-1/5*(f*g^2-h*(-d*h+e*g))*(c*x^2+b*x+a)^(5/2)/h/(a*h^2-
b*g*h+c*g^2)/(h*x+g)^5+c^(3/2)*f*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a
)^(1/2))/h^6-1/256*(256*c^5*f*g^7-896*c^4*f*g^5*h*(-a*h+b*g)+32*c^3*g*h^2*(
35*b^2*f*g^4-70*a*b*f*g^3*h+a^2*h^2*(-3*d*h^2+35*f*g^2))-16*c^2*h^3*(35*b^3
*f*g^4-6*a^3*h^3*(-e*h+6*f*g)+3*a^2*b*h^2*(-d*h^2-e*g*h+35*f*g^2)-3*a*b^2*g
*h*(d*h^2+35*f*g^2))+b^3*h^5*(16*a^2*f*h^2-2*a*b*h*(3*e*h+10*f*g)+b^2*(7*f*
g^2+3*h*(d*h+e*g)))-2*b*c*h^4*(96*a^3*f*h^3-24*a^2*b*h^2*(e*h+8*f*g)-b^3*(-
```

```
3*d*g*h^2+35*f*g^3)+4*a*b^2*h*(35*f*g^2+3*h*(d*h+e*g))))*arctanh(1/2*(b*g-2
*a*h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/h^6/(a*
h^2-b*g*h+c*g^2)^(7/2)-1/128*(128*c^4*f*g^7-32*c^3*f*g^5*h*(-10*a*h+11*b*g)
+8*c^2*g*h^2*(38*b^2*f*g^4+2*a^2*h^2*(3*d*h^2+13*f*g^2)-a*b*g*h*(3*d*h^2+65
*f*g^2))-2*c*h^3*(8*a^3*h^3*(-3*e*h+2*f*g)-2*a*b^2*g^2*h*(3*e*h+34*f*g)+4*a
^2*b*h^2*(3*d*h^2+6*e*g*h+5*f*g^2)+b^3*(-3*d*g^2*h^2+35*f*g^4))-b*h^4*(-2*a
*h+b*g)*(16*a^2*f*h^2-2*a*b*h*(3*e*h+10*f*g)+b^2*(7*f*g^2+3*h*(d*h+e*g)))+h
*(128*c*f*(c*g^2-h*(-a*h+b*g))^3+(-b*h+2*c*g)*(32*c^3*f*g^5-8*c^2*g*h*(3*a*
d*h^3-11*a*f*g^2*h+10*b*f*g^3)+2*c*h^2*(4*a^2*h^2*(-3*e*h+10*f*g)-6*a*b*h*(
-d*h^2-e*g*h+11*f*g^2)+b^2*(3*d*g*h^2+29*f*g^3))-b*h^3*(16*a^2*f*h^2-2*a*b*
h*(3*e*h+10*f*g)+b^2*(7*f*g^2+3*h*(d*h+e*g)))))*x)*(c*x^2+b*x+a)^(1/2)/h^5/
(a*h^2-b*g*h+c*g^2)^3/(h*x+g)^2
```

Rubi [A] (verified)

Time = 2.52 (sec) , antiderivative size = 1223, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1664, 824, 857, 635, 212, 738}

$$\int \frac{(a+bx+cx^2)^{3/2}\,(d+ex+fx^2)}{(g+hx)^6}\,dx = -\frac{(fg^2-h(eg-dh))\,(cx^2+bx+a)^{5/2}}{5h\,(cg^2-bhg+ah^2)\,(g+hx)^5}$$

$$-\frac{\Bigl(16c^2fg^5-2ch(13bfg^3-10afhg^2+3bdh^2g-6adh^3)\,g-h^2(-g(7fg^2+3h(eg+dh))\,b^2+2ah(fg^2+3h}{}$$

$$-\frac{\Bigl(\frac{128c^4fg^7}{h}-32c^3f(11bg-10ah)g^5+8c^2h(38b^2fg^4-abh(65fg^2+3dh^2)\,g+2a^2h^2(13fg^2+3dh^2))\,g-bh^3}{}$$

$$+\frac{c^{3/2}f\operatorname{arctanh}\Bigl(\frac{b+2cx}{2\sqrt{c}\sqrt{cx^2+bx+a}}\Bigr)}{h^6}$$

$$-\frac{(256c^5fg^7-896c^4fh(bg-ah)g^5+32c^3h^2(35b^2fg^4-70abfhg^3+a^2h^2(35fg^2-3dh^2))\,g-16c^2h^3(35b^3fg}{}$$

```
[In] Int[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^6,x]
```

```
[Out] -1/128*(((128*c^4*f*g^7)/h - 32*c^3*f*g^5*(11*b*g - 10*a*h) + 8*c^2*g*h*(38
*b^2*f*g^4 + 2*a^2*h^2*(13*f*g^2 + 3*d*h^2) - a*b*g*h*(65*f*g^2 + 3*d*h^2))
 - b*h^3*(b*g - 2*a*h)*(16*a^2*f*h^2 - 2*a*b*h*(10*f*g + 3*e*h) + b^2*(7*f*
g^2 + 3*h*(e*g + d*h))) - 2*c*h^2*(8*a^3*h^3*(2*f*g - 3*e*h) - 2*a*b^2*g^2*
h*(34*f*g + 3*e*h) + b^3*(35*f*g^4 - 3*d*g^2*h^2) + 4*a^2*b*h^2*(5*f*g^2 +
3*h*(2*e*g + d*h))) + (128*c*f*(c*g^2 - h*(b*g - a*h))^3 + (2*c*g - b*h)*(3
2*c^3*f*g^5 - 8*c^2*g*h*(10*b*f*g^3 - 11*a*f*g^2*h + 3*a*d*h^3) + 2*c*h^2*(
4*a^2*h^2*(10*f*g - 3*e*h) - 6*a*b*h*(11*f*g^2 - e*g*h - d*h^2) + b^2*(29*f
*g^3 + 3*d*g*h^2)) - b*h^3*(16*a^2*f*h^2 - 2*a*b*h*(10*f*g + 3*e*h) + b^2*(
```

```
7*f*g^2 + 3*h*(e*g + d*h)))))*x)*Sqrt[a + b*x + c*x^2])/(h^4*(c*g^2 - b*g*h
 + a*h^2)^3*(g + h*x)^2) - ((16*c^2*f*g^5 - 2*c*g*h*(13*b*f*g^3 - 10*a*f*g^
2*h + 3*b*d*g*h^2 - 6*a*d*h^3) - h^2*(4*a^2*h^2*(2*f*g - 3*e*h) - b^2*g*(7*
f*g^2 + 3*h*(e*g + d*h)) + 2*a*b*h*(f*g^2 + 3*h*(2*e*g + d*h))) + h^2*(16*a
^2*f*h^3 + 4*a*c*g*h*(14*f*g - 3*e*h) + c^2*((28*f*g^4)/h - 12*d*g^2*h) + b
^2*h*(25*f*g^2 - 3*h*(e*g + d*h)) - b*(56*c*f*g^3 - 6*c*g*h*(e*g + 2*d*h) +
 2*a*h^2*(22*f*g - 3*e*h)))*x)*(a + b*x + c*x^2)^(3/2))/(48*h^3*(c*g^2 - b*
g*h + a*h^2)^2*(g + h*x)^4) - ((f*g^2 - h*(e*g - d*h))*(a + b*x + c*x^2)^(5
/2))/(5*h*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^5) + (c^(3/2)*f*ArcTanh[(b + 2*
c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/h^6 - ((256*c^5*f*g^7 - 896*c^4*f*
g^5*h*(b*g - a*h) + 32*c^3*g*h^2*(35*b^2*f*g^4 - 70*a*b*f*g^3*h + a^2*h^2*(
35*f*g^2 - 3*d*h^2)) - 16*c^2*h^3*(35*b^3*f*g^4 - 6*a^3*h^3*(6*f*g - e*h) +
 3*a^2*b*h^2*(35*f*g^2 - e*g*h - d*h^2) - 3*a*b^2*g*h*(35*f*g^2 + d*h^2)) +
 b^3*h^5*(16*a^2*f*h^2 - 2*a*b*h*(10*f*g + 3*e*h) + b^2*(7*f*g^2 + 3*h*(e*g
 + d*h))) - 2*b*c*h^4*(96*a^3*f*h^3 - 24*a^2*b*h^2*(8*f*g + e*h) - b^3*(35*
f*g^3 - 3*d*g*h^2) + 4*a*b^2*h*(35*f*g^2 + 3*h*(e*g + d*h))))*ArcTanh[(b*g
 - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[a + b*x + c*
x^2])])/(256*h^6*(c*g^2 - b*g*h + a*h^2)^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 824

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Dist[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)
^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p
```

```
 + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*
(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m
 - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &
& LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{\left(fg^2-h(eg-dh)\right)\left(a+bx+cx^2\right)^{5/2}}{5h\left(cg^2-bgh+ah^2\right)(g+hx)^5} - \frac{\int \frac{\left(-\frac{5}{2}\left(2cdg-beg-2afg+\frac{bfg^2}{h}-bdh+2aeh\right)+5f\left(bg-\frac{cg^2}{h}-ah\right)x\right)\left(a+bx+cx^2\right)^{3/2}}{(g+hx)^5}\,dx}{5\left(cg^2-bgh+ah^2\right)}$$

$$= -\frac{\left(16c^2fg^5-2cgh(13bfg^3-10afg^2h+3bdgh^2-6adh^3)-h^2(4a^2h^2(2fg-3eh)-b^2g(7fg^2+3\right.}{}$$

$$-\frac{\left(fg^2-h(eg-dh)\right)\left(a+bx+cx^2\right)^{5/2}}{5h\left(cg^2-bgh+ah^2\right)(g+hx)^5} + \frac{\int \frac{\left(\frac{5\left(b^3h^2\left(7fg^2+3h(eg+dh)\right)-24ach\left(ah^2(2fg-eh)+c\left(fg^3-dgh^2\right)\right)-2b^2\left(ah^3(10fg+3eh)+c\left(13fg^3h+3dgh^3\right)\right)+4b\left(4c^2fg^4+4a^2fh^4+ach^2\left(1\right.\right.\right.}{4h}\right.}{(g+hx)^3}}{40h^2\left(cg^2-bgh+ah^2\right)^2}$$

$$=$$

$$-\frac{\left(\frac{128c^4fg^7}{h}-32c^3fg^5(11bg-10ah)+8c^2gh(38b^2fg^4+2a^2h^2(13fg^2+3dh^2)-abgh(65fg^2+3}{}$$

$$-\frac{\left(16c^2fg^5-2cgh(13bfg^3-10afg^2h+3bdgh^2-6adh^3)-h^2(4a^2h^2(2fg-3eh)-b^2g(7fg^2+\right.}{}$$

$$-\frac{(fg^2-h(eg-dh))\left(a+bx+cx^2\right)^{5/2}}{5h\left(cg^2-bgh+ah^2\right)(g+hx)^5}$$

$$-\frac{\int\frac{5\left(b^4\left(70cfg^3h^3-6cdgh^5-20afgh^5-6aeh^6\right)+b^5h^4\left(7fg^2+3h(eg+dh)\right)-16bc\left(8c^3fg^6+44ac^2fg^4h^2+12a^3fh^6+3a^2ch^4\left(19fg^2-egh-dh^2\right)\right)+}{}}{}$$

$$=$$

$$-\frac{\left(\frac{128c^4fg^7}{h}-32c^3fg^5(11bg-10ah)+8c^2gh(38b^2fg^4+2a^2h^2(13fg^2+3dh^2)-abgh(65fg^2+3}{}$$

$$-\frac{\left(16c^2fg^5-2cgh(13bfg^3-10afg^2h+3bdgh^2-6adh^3)-h^2(4a^2h^2(2fg-3eh)-b^2g(7fg^2+\right.}{}$$

$$-\frac{(fg^2-h(eg-dh))\left(a+bx+cx^2\right)^{5/2}}{5h\left(cg^2-bgh+ah^2\right)(g+hx)^5}+\frac{\left(c^2f\right)\int\frac{1}{\sqrt{a+bx+cx^2}}\,dx}{h^6}$$

$$-\frac{(256c^5fg^7-896c^4fg^5h(bg-ah)+32c^3gh^2(35b^2fg^4-70abfg^3h+a^2h^2(35fg^2-3dh^2))-16}{}$$

$$=$$

$$-\frac{\left(\frac{128c^4fg^7}{h}-32c^3fg^5(11bg-10ah)+8c^2gh(38b^2fg^4+2a^2h^2(13fg^2+3dh^2)-abgh(65fg^2+3}{}$$

$$-\frac{\left(16c^2fg^5-2cgh(13bfg^3-10afg^2h+3bdgh^2-6adh^3)-h^2(4a^2h^2(2fg-3eh)-b^2g(7fg^2+\right.}{}$$

$$-\frac{(fg^2-h(eg-dh))\left(a+bx+cx^2\right)^{5/2}}{5h\left(cg^2-bgh+ah^2\right)(g+hx)^5}+\frac{\left(2c^2f\right)\text{Subst}\left(\int\frac{1}{4c-x^2}\,dx,x,\frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{h^6}$$

$$+\frac{(256c^5fg^7-896c^4fg^5h(bg-ah)+32c^3gh^2(35b^2fg^4-70abfg^3h+a^2h^2(35fg^2-3dh^2))-16}{}$$

$$=$$

$$-\frac{\left(\frac{128c^4fg^7}{h}-32c^3fg^5(11bg-10ah)+8c^2gh(38b^2fg^4+2a^2h^2(13fg^2+3dh^2)-abgh(65fg^2+3dh\right.}{}$$

$$-\frac{\left(16c^2fg^5-2cgh(13bfg^3-10afg^2h+3bdgh^2-6adh^3)-h^2(4a^2h^2(2fg-3eh)-b^2g(7fg^2+3\right.}{}$$

$$-\frac{(fg^2-h(eg-dh))\,(a+bx+cx^2)^{5/2}}{5h\,(cg^2-bgh+ah^2)\,(g+hx)^5}+\frac{c^{3/2}f\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{h^6}$$

$$-\frac{(256c^5fg^7-896c^4fg^5h(bg-ah)+32c^3gh^2(35b^2fg^4-70abfg^3h+a^2h^2(35fg^2-3dh^2))-16c^2}{}$$

Mathematica [A] (verified)

Time = 16.34 (sec) , antiderivative size = 1363, normalized size of antiderivative = 1.11

$$\int\frac{(a+bx+cx^2)^{3/2}\,(d+ex+fx^2)}{(g+hx)^6}\,dx=-\frac{f(a+x(b+cx))^{3/2}}{3h^3(g+hx)^3}$$

$$-\frac{(2fg-eh)(bg-2ah+(2cg-bh)x)(a+x(b+cx))^{3/2}}{8h^2\,(cg^2-h(bg-ah))\,(g+hx)^4}$$

$$-\frac{(fg^2-h(eg-dh))\,(a+bx+cx^2)\,(a+x(b+cx))^{3/2}}{5h\,(cg^2-h(bg-ah))\,(g+hx)^5}$$

$$+\frac{(2cg-bh)\,(fg^2-egh+dh^2)\,(a+x(b+cx))^{3/2}\left(\frac{(bg-2ah+(2cg-bh)x)\left(a+bx+cx^2\right)^{3/2}}{8(cg^2-bgh+ah^2)(g+hx)^4}-\frac{3\left(b^2-4ac\right)\left(\frac{(bg-2ah+(2cg-bh)x)\sqrt{}}{4\left(cg^2-bgh+ah^2\right)(g}\right.}{}\right.}{2h^2\,(cg^2-bgh+ah^2)\,(a+bx+cx^2)^{3/2}}$$

$$+\frac{f(a+x(b+cx))^{3/2}\left(-\frac{(-2cg+bh)\left(a+bx+cx^2\right)^{3/2}}{2(cg^2-bgh+ah^2)(g+hx)^2}-\frac{\frac{\left(-cg(2cg-bh)+\frac{1}{2}h\left(2bcg+b^2h-8ach\right)\right)\left(a+bx+cx^2\right)^{3/2}}{\left(-cg^2+bgh-ah^2\right)(g+hx)}+\frac{\left(-c\left(2cg-\frac{bh}{2}\right)\left(4c^2g^2-b^2h^2-4\right.\right.}{}}{}\right.}{}$$

$$+\frac{3(b^2-4ac)\,(2fg-eh)(a+x(b+cx))^{3/2}\left(\frac{2(bg-2ah+(2cg-bh)x)\sqrt{a+bx+cx^2}}{(cg^2-bgh+ah^2)(g+hx)^2}-\frac{(b^2-4ac)\operatorname{arctanh}\left(\frac{bg-2ah+(2cg-bh)x}{2\sqrt{cg^2-h(bg-ah)}\sqrt{a+bx+}}\right.}{(cg^2-h(bg-ah))^{3/2}}\right.}{128h^2\,(cg^2-h(bg-ah))\,(a+bx+cx^2)^{3/2}}$$

```
[In] Integrate[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^6,x]
```

```
[Out] -1/3*(f*(a + x*(b + c*x))^(3/2))/(h^3*(g + h*x)^3) - ((2*f*g - e*h)*(b*g -
2*a*h + (2*c*g - b*h)*x)*(a + x*(b + c*x))^(3/2))/(8*h^2*(c*g^2 - h*(b*g -
a*h))*(g + h*x)^4) - ((f*g^2 - h*(e*g - d*h))*(a + b*x + c*x^2)*(a + x*(b +
 c*x))^(3/2))/(5*h*(c*g^2 - h*(b*g - a*h))*(g + h*x)^5) + ((2*c*g - b*h)*(f
*g^2 - e*g*h + d*h^2)*(a + x*(b + c*x))^(3/2)*(((b*g - 2*a*h + (2*c*g - b*h
)*x)*(a + b*x + c*x^2)^(3/2))/(8*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^4) - (3*
(b^2 - 4*a*c)*(((b*g - 2*a*h + (2*c*g - b*h)*x)*Sqrt[a + b*x + c*x^2])/(4*(
c*g^2 - b*g*h + a*h^2)*(g + h*x)^2) + ((b^2 - 4*a*c)*ArcTanh[(-(b*g) + 2*a*
h - (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[a + b*x + c*x^2])]
)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*(4*c*g^2 - 4*b*g*h + 4*a*h^2))))/(16*(c*g^
2 - b*g*h + a*h^2))))/(2*h^2*(c*g^2 - b*g*h + a*h^2)*(a + b*x + c*x^2)^(3/2
)) + (f*(a + x*(b + c*x))^(3/2)*(-1/2*((-2*c*g + b*h)*(a + b*x + c*x^2)^(3/
2))/((c*g^2 - b*g*h + a*h^2)*(g + h*x)^2) - (((-(c*g*(2*c*g - b*h)) + (h*(2
*b*c*g + b^2*h - 8*a*c*h))/2)*(a + b*x + c*x^2)^(3/2))/((-(c*g^2) + b*g*h -
 a*h^2)*(g + h*x)) + (((-(c*(2*c*g - (b*h)/2)*(4*c^2*g^2 - b^2*h^2 - 4*c*h*
(b*g - 2*a*h))) + (c*h*(-10*b^2*c*g*h + 8*a*c^2*g*h - b^3*h^2 + 4*b*c*(2*c*
g^2 + 3*a*h^2)))/2 + c^2*h*(4*c^2*g^2 - b^2*h^2 - 4*c*h*(b*g - 2*a*h))*x)*S
qrt[a + b*x + c*x^2])/(2*c*h^2) - ((-16*c^(5/2)*(c*g^2 - h*(b*g - a*h))^2*A
rcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])]])/h - (4*Sqrt[c*g^2 -
b*g*h + a*h^2]*(-(c*h*(c*g^2 - b*g*h + a*h^2)*(8*b*c^2*g^2 - 6*b^2*c*g*h -
8*a*c^2*g*h - b^3*h^2 + 12*a*b*c*h^2)) + 16*c^3*g*(c*g^2 - h*(b*g - a*h))^2
)*ArcTanh[(-(b*g) + 2*a*h - (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]
*Sqrt[a + b*x + c*x^2])]])/(h*(4*c*g^2 - 4*b*g*h + 4*a*h^2)))/(4*c*h^2))/(-(
c*g^2) + b*g*h - a*h^2))/(2*(c*g^2 - b*g*h + a*h^2))))/(2*h^3*(a + b*x + c*
x^2)^(3/2)) + (3*(b^2 - 4*a*c)*(2*f*g - e*h)*(a + x*(b + c*x))^(3/2)*((2*(b
*g - 2*a*h + (2*c*g - b*h)*x)*Sqrt[a + b*x + c*x^2])/((c*g^2 - b*g*h + a*h^
2)*(g + h*x)^2) - ((b^2 - 4*a*c)*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2
*Sqrt[c*g^2 - h*(b*g - a*h)]*Sqrt[a + b*x + c*x^2])])/(c*g^2 - h*(b*g - a*h
))^(3/2)))/(128*h^2*(c*g^2 - h*(b*g - a*h))*(a + b*x + c*x^2)^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 13371 vs. 2(1196) = 2392.

Time = 1.39 (sec) , antiderivative size = 13372, normalized size of antiderivative = 10.91

method	result	size
default	Expression too large to display	13372

```
[In] int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^6,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^6}\,dx=\text{Timed out}$$

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^6,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^6}\,dx=\text{Timed out}$$

```
[In] integrate((c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**6,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^6}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^6,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*h^2-b*g*h>0)', see `assume?` for
more de
```

Giac [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}\,(d+ex+fx^2)}{(g+hx)^6}\,dx = \text{Timed out}$$

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^6,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}\,(d+ex+fx^2)}{(g+hx)^6}\,dx = \int \frac{(c\,x^2+b\,x+a)^{3/2}\,(f\,x^2+e\,x+d)}{(g+h\,x)^6}\,dx$$

```
[In] int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^6,x)
```

```
[Out] int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^6, x)
```

3.206 $\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^7}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 657

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^7}\,dx =$$

$$-\frac{(b^2-4ac)\,(24c^2dg^2+24a^2fh^2-12abh(2fg+eh)+b^2(7fg^2+5egh+7dh^2)-4c(3bg(eg+2dh)+a(fg^2}{512\,(cg^2-bgh+ah^2)^4\,(g+hx)^2}$$

$$+\frac{(24c^2dg^2+24a^2fh^2-12abh(2fg+eh)+b^2(7fg^2+5egh+7dh^2)-4c(3bg(eg+2dh)+a(fg^2-7egh+}{192\,(cg^2-bgh+ah^2)^3\,(g+hx)^4}$$

$$-\frac{(fg^2-h(eg-dh))\,(a+bx+cx^2)^{5/2}}{6h\,(cg^2-bgh+ah^2)\,(g+hx)^6}$$

$$+\frac{(2cg(5fg^2+h(eg-7dh))+h(12ah(2fg-eh)-b(17fg^2-5egh-7dh^2)))\,(a+bx+cx^2)^{5/2}}{60h\,(cg^2-bgh+ah^2)^2\,(g+hx)^5}$$

$$+\frac{(b^2-4ac)^2\,(24c^2dg^2+24a^2fh^2-12abh(2fg+eh)+b^2(7fg^2+5egh+7dh^2)-4c(3bg(eg+2dh)+a(fg^2}{1024\,(cg^2-bgh+ah^2)^{9/2}}$$

```
[Out] 1/192*(24*c^2*d*g^2+24*a^2*f*h^2-12*a*b*h*(e*h+2*f*g)+b^2*(7*d*h^2+5*e*g*h+
7*f*g^2)-4*c*(3*b*g*(2*d*h+e*g)+a*(d*h^2-7*e*g*h+f*g^2)))*(b*g-2*a*h+(-b*h+
2*c*g)*x)*(c*x^2+b*x+a)^(3/2)/(a*h^2-b*g*h+c*g^2)^3/(h*x+g)^4-1/6*(f*g^2-h*
(-d*h+e*g))*(c*x^2+b*x+a)^(5/2)/h/(a*h^2-b*g*h+c*g^2)/(h*x+g)^6+1/60*(2*c*g
*(5*f*g^2+h*(-7*d*h+e*g))+h*(12*a*h*(-e*h+2*f*g)-b*(-7*d*h^2-5*e*g*h+17*f*g
^2)))*(c*x^2+b*x+a)^(5/2)/h/(a*h^2-b*g*h+c*g^2)^2/(h*x+g)^5+1/1024*(-4*a*c+
b^2)^2*(24*c^2*d*g^2+24*a^2*f*h^2-12*a*b*h*(e*h+2*f*g)+b^2*(7*d*h^2+5*e*g*h
+7*f*g^2)-4*c*(3*b*g*(2*d*h+e*g)+a*(d*h^2-7*e*g*h+f*g^2)))*arctanh(1/2*(b*g
-2*a*h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/(a*h^
2-b*g*h+c*g^2)^(9/2)-1/512*(-4*a*c+b^2)*(24*c^2*d*g^2+24*a^2*f*h^2-12*a*b*h
```

```
*(e*h+2*f*g)+b^2*(7*d*h^2+5*e*g*h+7*f*g^2)-4*c*(3*b*g*(2*d*h+e*g)+a*(d*h^2-
7*e*g*h+f*g^2)))*(b*g-2*a*h+(-b*h+2*c*g)*x)*(c*x^2+b*x+a)^(1/2)/(a*h^2-b*g*
h+c*g^2)^4/(h*x+g)^2
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 660, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {1664, 820, 734, 738, 212}

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^7}\,dx = \frac{\left(b^2-4ac\right)^2 \text{arctanh}\left(\frac{-2ah+x(2cg-bh)+bg}{2\sqrt{a+bx+cx^2}\sqrt{ah^2-bgh+cg^2}}\right)\left(24a^2fh^2-4c(-ah(\right.}{}$$

$$+\frac{\left(a+bx+cx^2\right)^{3/2}(-2ah+x(2cg-bh)+bg)\left(24a^2fh^2-4c\left(-ah(7eg-dh)+afg^2+3bg(2dh+eg)\right)-1\right.}{192(g+hx)^4\left(ah^2-bgh+cg^2\right)^3}$$

$$-\frac{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}(-2ah+x(2cg-bh)+bg)\left(24a^2fh^2-4c(-ah(7eg-dh)+afg^2+3bg(2dh+\right.}{512(g+hx)^2\left(ah^2-bgh+cg^2\right)^4}$$

$$-\frac{\left(a+bx+cx^2\right)^{5/2}\left(fg^2-h(eg-dh)\right)}{6h(g+hx)^6\left(ah^2-bgh+cg^2\right)}$$

$$+\frac{\left(a+bx+cx^2\right)^{5/2}\left(2c\left(gh(eg-7dh)+5fg^3\right)-h\left(-12ah(2fg-eh)-bh(7dh+5eg)+17bfg^2\right)\right)}{60h(g+hx)^5\left(ah^2-bgh+cg^2\right)^2}$$

```
[In] Int[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^7,x]
```

```
[Out] -1/512*((b^2 - 4*a*c)*(24*c^2*d*g^2 + 24*a^2*f*h^2 - 12*a*b*h*(2*f*g + e*h)
 - 4*c*(a*f*g^2 - a*h*(7*e*g - d*h) + 3*b*g*(e*g + 2*d*h)) + b^2*(7*f*g^2 +
 h*(5*e*g + 7*d*h)))*(b*g - 2*a*h + (2*c*g - b*h)*x)*Sqrt[a + b*x + c*x^2])
/((c*g^2 - b*g*h + a*h^2)^4*(g + h*x)^2) + ((24*c^2*d*g^2 + 24*a^2*f*h^2 -
12*a*b*h*(2*f*g + e*h) - 4*c*(a*f*g^2 - a*h*(7*e*g - d*h) + 3*b*g*(e*g + 2*
d*h)) + b^2*(7*f*g^2 + h*(5*e*g + 7*d*h)))*(b*g - 2*a*h + (2*c*g - b*h)*x)*
(a + b*x + c*x^2)^(3/2))/(192*(c*g^2 - b*g*h + a*h^2)^3*(g + h*x)^4) - ((f*
g^2 - h*(e*g - d*h))*(a + b*x + c*x^2)^(5/2))/(6*h*(c*g^2 - b*g*h + a*h^2)*
(g + h*x)^6) + ((2*c*(5*f*g^3 + g*h*(e*g - 7*d*h)) - h*(17*b*f*g^2 - b*h*(5
*e*g + 7*d*h) - 12*a*h*(2*f*g - e*h)))*(a + b*x + c*x^2)^(5/2))/(60*h*(c*g^
2 - b*g*h + a*h^2)^2*(g + h*x)^5) + ((b^2 - 4*a*c)^2*(24*c^2*d*g^2 + 24*a^2
*f*h^2 - 12*a*b*h*(2*f*g + e*h) - 4*c*(a*f*g^2 - a*h*(7*e*g - d*h) + 3*b*g*
(e*g + 2*d*h)) + b^2*(7*f*g^2 + h*(5*e*g + 7*d*h)))*ArcTanh[(b*g - 2*a*h +
(2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[a + b*x + c*x^2])])/(1
024*(c*g^2 - b*g*h + a*h^2)^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), Int[(d + e*x)^(m + 2)*(a + b*x +
c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
 0] && GtQ[p, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

integral

$$= -\frac{(fg^2-h(eg-dh))\left(a+bx+cx^2\right)^{5/2}}{6h\left(cg^2-bgh+ah^2\right)(g+hx)^6} - \frac{\int \frac{\left(\frac{1}{2}\left(-12cdg+5beg+12afg-\frac{5bfg^2}{h}+7bdh-12aeh\right)-\left(ceg-6bfg+\frac{5cfg^2}{h}-cdh+6afh\right)x\right)\left(a+bx+cx^2\right)^{3/2}}{(g+hx)^6}\,dx}{6\left(cg^2-bgh+ah^2\right)}$$

$$= -\frac{(fg^2-h(eg-dh))\left(a+bx+cx^2\right)^{5/2}}{6h\left(cg^2-bgh+ah^2\right)(g+hx)^6} + \frac{(2c(5fg^3+gh(eg-7dh))-h(17bfg^2-bh(5eg+7dh)-12ah(2fg-eh)))\left(a+bx+cx^2\right)^{5/2}}{60h\left(cg^2-bgh+ah^2\right)^2(g+hx)^5} + \frac{(24c^2dg^2+24a^2fh^2-12abh(2fg+eh)-4c(afg^2-ah(7eg-dh)+3bg(eg+2dh))+b^2(7fg^2+h(5e}{24\left(cg^2-bgh+ah^2\right)^2}$$

$$= \frac{(24c^2dg^2+24a^2fh^2-12abh(2fg+eh)-4c(afg^2-ah(7eg-dh)+3bg(eg+2dh))+b^2(7fg^2+h(5eg}{192\left(cg^2-bgh+ah^2\right)^3(g+hx)^4} - \frac{(fg^2-h(eg-dh))\left(a+bx+cx^2\right)^{5/2}}{6h\left(cg^2-bgh+ah^2\right)(g+hx)^6} + \frac{(2c(5fg^3+gh(eg-7dh))-h(17bfg^2-bh(5eg+7dh)-12ah(2fg-eh)))\left(a+bx+cx^2\right)^{5/2}}{60h\left(cg^2-bgh+ah^2\right)^2(g+hx)^5} - \frac{((b^2-4ac)\left(24c^2dg^2+24a^2fh^2-12abh(2fg+eh)-4c(afg^2-ah(7eg-dh)+3bg(eg+2dh))+b^2(\right.}{128\left(cg^2-bgh+ah^2\right)^3}$$

$$= -\frac{(b^2-4ac)\left(24c^2dg^2+24a^2fh^2-12abh(2fg+eh)-4c(afg^2-ah(7eg-dh)+3bg(eg+2dh))+b^2(7\right.}{512\left(cg^2-bgh+ah^2\right)^4(g+hx)^2} + \frac{(24c^2dg^2+24a^2fh^2-12abh(2fg+eh)-4c(afg^2-ah(7eg-dh)+3bg(eg+2dh))+b^2(7fg^2+h(5e}{192\left(cg^2-bgh+ah^2\right)^3(g+hx)^4} - \frac{(fg^2-h(eg-dh))\left(a+bx+cx^2\right)^{5/2}}{6h\left(cg^2-bgh+ah^2\right)(g+hx)^6} + \frac{(2c(5fg^3+gh(eg-7dh))-h(17bfg^2-bh(5eg+7dh)-12ah(2fg-eh)))\left(a+bx+cx^2\right)^{5/2}}{60h\left(cg^2-bgh+ah^2\right)^2(g+hx)^5} + \frac{\left((b^2-4ac)^2\left(24c^2dg^2+24a^2fh^2-12abh(2fg+eh)-4c(afg^2-ah(7eg-dh)+3bg(eg+2dh))+b^2\right.\right.}{1024\left(cg^2-bgh+ah^2\right)^4}$$

$$=$$

$$-\frac{(b^2-4ac)\,(24c^2dg^2+24a^2fh^2-12abh(2fg+eh)-4c(afg^2-ah(7eg-dh)+3bg(eg+2dh))+b^2(7f}{512\,(cg^2-bgh+ah^2)^4\,(g+hx)^2}$$

$$+\frac{(24c^2dg^2+24a^2fh^2-12abh(2fg+eh)-4c(afg^2-ah(7eg-dh)+3bg(eg+2dh))+b^2(7fg^2+h(5eg}{192\,(cg^2-bgh+ah^2)^3\,(g+hx)^4}$$

$$-\frac{(fg^2-h(eg-dh))\,(a+bx+cx^2)^{5/2}}{6h\,(cg^2-bgh+ah^2)\,(g+hx)^6}$$

$$+\frac{(2c(5fg^3+gh(eg-7dh))-h(17bfg^2-bh(5eg+7dh)-12ah(2fg-eh)))\,(a+bx+cx^2)^{5/2}}{60h\,(cg^2-bgh+ah^2)^2\,(g+hx)^5}$$

$$-\frac{\Big((b^2-4ac)^2\,(24c^2dg^2+24a^2fh^2-12abh(2fg+eh)-4c(afg^2-ah(7eg-dh)+3bg(eg+2dh))+b^2(}{512\,(cg^2-bgh+ah^2)^4}$$

$$=$$

$$-\frac{(b^2-4ac)\,(24c^2dg^2+24a^2fh^2-12abh(2fg+eh)-4c(afg^2-ah(7eg-dh)+3bg(eg+2dh))+b^2(7f}{512\,(cg^2-bgh+ah^2)^4\,(g+hx)^2}$$

$$+\frac{(24c^2dg^2+24a^2fh^2-12abh(2fg+eh)-4c(afg^2-ah(7eg-dh)+3bg(eg+2dh))+b^2(7fg^2+h(5eg}{192\,(cg^2-bgh+ah^2)^3\,(g+hx)^4}$$

$$-\frac{(fg^2-h(eg-dh))\,(a+bx+cx^2)^{5/2}}{6h\,(cg^2-bgh+ah^2)\,(g+hx)^6}$$

$$+\frac{(2c(5fg^3+gh(eg-7dh))-h(17bfg^2-bh(5eg+7dh)-12ah(2fg-eh)))\,(a+bx+cx^2)^{5/2}}{60h\,(cg^2-bgh+ah^2)^2\,(g+hx)^5}$$

$$+\frac{(b^2-4ac)^2\,(24c^2dg^2+24a^2fh^2-12abh(2fg+eh)-4c(afg^2-ah(7eg-dh)+3bg(eg+2dh))+b^2(7}{1024\,(cg^2-bgh+ah^2)^{9/2}}$$

Mathematica [A] (verified)

Time = 16.26 (sec) , antiderivative size = 1222, normalized size of antiderivative = 1.86

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^7}\,dx=\frac{f(bg-2ah+(2cg-bh)x)(a+x(b+cx))^{3/2}}{8h^2\left(cg^2-h(bg-ah)\right)(g+hx)^4}$$

$$-\frac{\left(fg^2-h(eg-dh)\right)\left(a+bx+cx^2\right)(a+x(b+cx))^{3/2}}{6h\left(cg^2-h(bg-ah)\right)(g+hx)^6}$$

$$+\frac{(2fg-eh)\left(a+bx+cx^2\right)(a+x(b+cx))^{3/2}}{5h\left(cg^2-h(bg-ah)\right)(g+hx)^5}$$

$$+\frac{(2cg-bh)(-2fg+eh)(a+x(b+cx))^{3/2}\left(\frac{(bg-2ah+(2cg-bh)x)\left(a+bx+cx^2\right)^{3/2}}{8(cg^2-bgh+ah^2)(g+hx)^4}-\frac{3\left(b^2-4ac\right)\left(\frac{(bg-2ah+(2cg-bh)x)\sqrt{a+bx+c}}{4\left(cg^2-bgh+ah^2\right)(g+hx)^2}\right.}{1}\right.}{2h^2\left(cg^2-bgh+ah^2\right)\left(a+bx+cx^2\right)^{3/2}}$$

$$-\frac{\left(fg^2-egh+dh^2\right)(a+x(b+cx))^{3/2}\left(\frac{\left(cgh-\frac{1}{2}h(-12cg+7bh)\right)\left(a+bx+cx^2\right)^{5/2}}{5(cg^2-bgh+ah^2)(g+hx)^5}-\frac{\left(-2\left(ach^2+\frac{1}{2}cg(-12cg+7bh)\right)+b\left(cgh+\frac{1}{2}h(-\right.\right.}{}\right.}{6h^2\left(cg^2-bgh\right.}$$

$$-\frac{3\left(b^2-4ac\right)f(a+x(b+cx))^{3/2}\left(\frac{2(bg-2ah+(2cg-bh)x)\sqrt{a+bx+cx^2}}{(cg^2-bgh+ah^2)(g+hx)^2}-\frac{\left(b^2-4ac\right)\operatorname{arctanh}\left(\frac{bg-2ah+(2cg-bh)x}{2\sqrt{cg^2-h(bg-ah)}\sqrt{a+bx+cx^2}}\right)}{\left(cg^2-h(bg-ah)\right)^{3/2}}\right)}{128h^2\left(cg^2-h(bg-ah)\right)\left(a+bx+cx^2\right)^{3/2}}$$

```
[In] Integrate[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^7,x]
```

```
[Out] (f*(b*g - 2*a*h + (2*c*g - b*h)*x)*(a + x*(b + c*x))^(3/2))/(8*h^2*(c*g^2 -
 h*(b*g - a*h))*(g + h*x)^4) - ((f*g^2 - h*(e*g - d*h))*(a + b*x + c*x^2)*(
a + x*(b + c*x))^(3/2))/(6*h*(c*g^2 - h*(b*g - a*h))*(g + h*x)^6) + ((2*f*g
 - e*h)*(a + b*x + c*x^2)*(a + x*(b + c*x))^(3/2))/(5*h*(c*g^2 - h*(b*g - a
*h))*(g + h*x)^5) + ((2*c*g - b*h)*(-2*f*g + e*h)*(a + x*(b + c*x))^(3/2)*(
```

```
((b*g - 2*a*h + (2*c*g - b*h)*x)*(a + b*x + c*x^2)^(3/2))/(8*(c*g^2 - b*g*h
 + a*h^2)*(g + h*x)^4) - (3*(b^2 - 4*a*c)*(((b*g - 2*a*h + (2*c*g - b*h)*x)
*Sqrt[a + b*x + c*x^2])/(4*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^2) + ((b^2 - 4
*a*c)*ArcTanh[(-(b*g) + 2*a*h - (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*
h^2]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*(4*c*g^2 - 4*b
*g*h + 4*a*h^2))))/(16*(c*g^2 - b*g*h + a*h^2))))/(2*h^2*(c*g^2 - b*g*h + a
*h^2)*(a + b*x + c*x^2)^(3/2)) - ((f*g^2 - e*g*h + d*h^2)*(a + x*(b + c*x))
^(3/2)*(((c*g*h - (h*(-12*c*g + 7*b*h))/2)*(a + b*x + c*x^2)^(5/2))/(5*(c*g
^2 - b*g*h + a*h^2)*(g + h*x)^5) - ((-2*(a*c*h^2 + (c*g*(-12*c*g + 7*b*h))/
2) + b*(c*g*h + (h*(-12*c*g + 7*b*h))/2))*(((b*g - 2*a*h + (2*c*g - b*h)*x)
*(a + b*x + c*x^2)^(3/2))/(8*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^4) - (3*(b^2
 - 4*a*c)*(((b*g - 2*a*h + (2*c*g - b*h)*x)*Sqrt[a + b*x + c*x^2])/(4*(c*g^
2 - b*g*h + a*h^2)*(g + h*x)^2) + ((b^2 - 4*a*c)*ArcTanh[(-(b*g) + 2*a*h -
(2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[a + b*x + c*x^2])])/(2
*Sqrt[c*g^2 - b*g*h + a*h^2]*(4*c*g^2 - 4*b*g*h + 4*a*h^2))))/(16*(c*g^2 -
b*g*h + a*h^2))))/(2*(c*g^2 - b*g*h + a*h^2))))/(6*h^2*(c*g^2 - b*g*h + a*h
^2)*(a + b*x + c*x^2)^(3/2)) - (3*(b^2 - 4*a*c)*f*(a + x*(b + c*x))^(3/2)*(
(2*(b*g - 2*a*h + (2*c*g - b*h)*x)*Sqrt[a + b*x + c*x^2])/((c*g^2 - b*g*h +
 a*h^2)*(g + h*x)^2) - ((b^2 - 4*a*c)*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*
x)/(2*Sqrt[c*g^2 - h*(b*g - a*h)]*Sqrt[a + b*x + c*x^2])])/(c*g^2 - h*(b*g
- a*h))^(3/2)))/(128*h^2*(c*g^2 - h*(b*g - a*h))*(a + b*x + c*x^2)^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 20683 vs. $2(631) = 1262$.

Time = 1.82 (sec) , antiderivative size = 20684, normalized size of antiderivative = 31.48

method	result	size
default	Expression too large to display	20684

```
[In] int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^7,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}\,(d+ex+fx^2)}{(g+hx)^7}\,dx = \text{Timed out}$$

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^7,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^7}\,dx = \int \frac{(a+bx+cx^2)^{\frac{3}{2}}(d+ex+fx^2)}{(g+hx)^7}\,dx$$

```
[In] integrate((c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**7,x)
```

```
[Out] Integral((a + b*x + c*x**2)**(3/2)*(d + e*x + f*x**2)/(g + h*x)**7, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^7}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^7,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the ’assume’ command before evaluation *may* h
elp (example of legal syntax is ’assume(a*h^2-b*g*h>0)’, see ‘assume?‘ for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48343 vs. $2(631) = 1262$.

Time = 9.70 (sec) , antiderivative size = 48343, normalized size of antiderivative = 73.58

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^7}\,dx = \text{Too large to display}$$

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^7,x, algorithm="giac")
```

```
[Out] 1/512*(24*b^4*c^2*d*g^2 - 192*a*b^2*c^3*d*g^2 + 384*a^2*c^4*d*g^2 - 12*b^5*
c*e*g^2 + 96*a*b^3*c^2*e*g^2 - 192*a^2*b*c^3*e*g^2 + 7*b^6*f*g^2 - 60*a*b^4
*c*f*g^2 + 144*a^2*b^2*c^2*f*g^2 - 64*a^3*c^3*f*g^2 - 24*b^5*c*d*g*h + 192*
a*b^3*c^2*d*g*h - 384*a^2*b*c^3*d*g*h + 5*b^6*e*g*h - 12*a*b^4*c*e*g*h - 14
4*a^2*b^2*c^2*e*g*h + 448*a^3*c^3*e*g*h - 24*a*b^5*f*g*h + 192*a^2*b^3*c*f*
g*h - 384*a^3*b*c^2*f*g*h + 7*b^6*d*h^2 - 60*a*b^4*c*d*h^2 + 144*a^2*b^2*c^
2*d*h^2 - 64*a^3*c^3*d*h^2 - 12*a*b^5*e*h^2 + 96*a^2*b^3*c*e*h^2 - 192*a^3*
b*c^2*e*h^2 + 24*a^2*b^4*f*h^2 - 192*a^3*b^2*c*f*h^2 + 384*a^4*c^2*f*h^2)*a
rctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*h + sqrt(c)*g)/sqrt(-c*g^2 + b*
```

```
g*h - a*h^2))/((c^4*g^8 - 4*b*c^3*g^7*h + 6*b^2*c^2*g^6*h^2 + 4*a*c^3*g^6*h
^2 - 4*b^3*c*g^5*h^3 - 12*a*b*c^2*g^5*h^3 + b^4*g^4*h^4 + 12*a*b^2*c*g^4*h^
4 + 6*a^2*c^2*g^4*h^4 - 4*a*b^3*g^3*h^5 - 12*a^2*b*c*g^3*h^5 + 6*a^2*b^2*g^
2*h^6 + 4*a^3*c*g^2*h^6 - 4*a^3*b*g*h^7 + a^4*h^8)*sqrt(-c*g^2 + b*g*h - a*
h^2)) + 1/7680*(15360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*c^6*f*g^8*h^5
 - 61440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*b*c^5*f*g^7*h^6 + 92160*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^11*b^2*c^4*f*g^6*h^7 + 61440*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^11*a*c^5*f*g^6*h^7 - 61440*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))^11*b^3*c^3*f*g^5*h^8 - 184320*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
^11*a*b*c^4*f*g^5*h^8 + 15360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*b^4*c^
2*f*g^4*h^9 + 184320*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*a*b^2*c^3*f*g^4
*h^9 + 92160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*a^2*c^4*f*g^4*h^9 - 614
40*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*a*b^3*c^2*f*g^3*h^10 - 184320*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^11*a^2*b*c^3*f*g^3*h^10 - 360*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))^11*b^4*c^2*d*g^2*h^11 + 2880*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^11*a*b^2*c^3*d*g^2*h^11 - 5760*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^11*a^2*c^4*d*g^2*h^11 + 180*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*b^
5*c*e*g^2*h^11 - 1440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*a*b^3*c^2*e*g^
2*h^11 + 2880*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*a^2*b*c^3*e*g^2*h^11 -
 105*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*b^6*f*g^2*h^11 + 900*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))^11*a*b^4*c*f*g^2*h^11 + 90000*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))^11*a^2*b^2*c^2*f*g^2*h^11 + 62400*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^11*a^3*c^3*f*g^2*h^11 + 360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
^11*b^5*c*d*g*h^12 - 2880*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*a*b^3*c^2*
d*g*h^12 + 5760*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*a^2*b*c^3*d*g*h^12 -
 75*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*b^6*e*g*h^12 + 180*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^11*a*b^4*c*e*g*h^12 + 2160*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))^11*a^2*b^2*c^2*e*g*h^12 - 6720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))^11*a^3*c^3*e*g*h^12 + 360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*a*b^5*f
*g*h^12 - 2880*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*a^2*b^3*c*f*g*h^12 -
55680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*a^3*b*c^2*f*g*h^12 - 105*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^11*b^6*d*h^13 + 900*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^11*a*b^4*c*d*h^13 - 2160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11
*a^2*b^2*c^2*d*h^13 + 960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*a^3*c^3*d*
h^13 + 180*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*a*b^5*e*h^13 - 1440*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^11*a^2*b^3*c*e*h^13 + 2880*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^11*a^3*b*c^2*e*h^13 - 360*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^11*a^2*b^4*f*h^13 + 2880*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*a^3*b
^2*c*f*h^13 + 9600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*a^4*c^2*f*h^13 +
76800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*c^(13/2)*f*g^9*h^4 + 15360*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^10*c^(13/2)*e*g^8*h^5 - 276480*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))^10*b*c^(11/2)*f*g^8*h^5 - 61440*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^10*b*c^(11/2)*e*g^7*h^6 + 337920*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^10*b^2*c^(9/2)*f*g^7*h^6 + 307200*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))^10*a*c^(11/2)*f*g^7*h^6 + 92160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^
```

10*b^2*c^(9/2)*e*g^6*h^7 + 61440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a*c
^(11/2)*e*g^6*h^7 - 122880*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*b^3*c^(7/
2)*f*g^6*h^7 - 798720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a*b*c^(9/2)*f*
g^6*h^7 - 61440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*b^3*c^(7/2)*e*g^5*h^
8 - 184320*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a*b*c^(9/2)*e*g^5*h^8 - 4
6080*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*b^4*c^(5/2)*f*g^5*h^8 + 552960*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a*b^2*c^(7/2)*f*g^5*h^8 + 460800*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^10*a^2*c^(9/2)*f*g^5*h^8 + 15360*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^10*b^4*c^(5/2)*e*g^4*h^9 + 184320*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^10*a*b^2*c^(7/2)*e*g^4*h^9 + 92160*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^10*a^2*c^(9/2)*e*g^4*h^9 + 30720*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^10*b^5*c^(3/2)*f*g^4*h^9 + 61440*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^10*a*b^3*c^(5/2)*f*g^4*h^9 - 737280*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))^10*a^2*b*c^(7/2)*f*g^4*h^9 - 3960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10
*b^4*c^(5/2)*d*g^3*h^10 + 31680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a*b^
2*c^(7/2)*d*g^3*h^10 - 63360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a^2*c^(
9/2)*d*g^3*h^10 + 1980*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*b^5*c^(3/2)*e
*g^3*h^10 - 77280*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a*b^3*c^(5/2)*e*g^
3*h^10 - 152640*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a^2*b*c^(7/2)*e*g^3*
h^10 - 1155*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*b^6*sqrt(c)*f*g^3*h^10 -
112980*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a*b^4*c^(3/2)*f*g^3*h^10 + 6
8400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a^2*b^2*c^(5/2)*f*g^3*h^10 + 31
7760*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a^3*c^(7/2)*f*g^3*h^10 + 3960*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*b^5*c^(3/2)*d*g^2*h^11 - 31680*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^10*a*b^3*c^(5/2)*d*g^2*h^11 + 63360*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^10*a^2*b*c^(7/2)*d*g^2*h^11 - 825*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^10*b^6*sqrt(c)*e*g^2*h^11 + 1980*(sqrt(c)*x - sqrt(c*
x^2 + b*x + a))^10*a*b^4*c^(3/2)*e*g^2*h^11 + 115920*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^10*a^2*b^2*c^(5/2)*e*g^2*h^11 - 12480*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^10*a^3*c^(7/2)*e*g^2*h^11 + 3960*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^10*a*b^5*sqrt(c)*f*g^2*h^11 + 152640*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))^10*a^2*b^3*c^(3/2)*f*g^2*h^11 - 120960*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))^10*a^3*b*c^(5/2)*f*g^2*h^11 - 1155*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
^10*b^6*sqrt(c)*d*g*h^12 + 9900*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a*b^
4*c^(3/2)*d*g*h^12 - 23760*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a^2*b^2*c
^(5/2)*d*g*h^12 + 10560*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a^3*c^(7/2)*
d*g*h^12 + 1980*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a*b^5*sqrt(c)*e*g*h^
12 - 15840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a^2*b^3*c^(3/2)*e*g*h^12
- 29760*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a^3*b*c^(5/2)*e*g*h^12 - 396
0*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a^2*b^4*sqrt(c)*f*g*h^12 - 91200*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a^3*b^2*c^(3/2)*f*g*h^12 + 13440*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^10*a^4*c^(5/2)*f*g*h^12 + 15360*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))^10*a^4*c^(5/2)*e*h^13 + 30720*(sqrt(c)*x - sqrt(c*
x^2 + b*x + a))^10*a^4*b*c^(3/2)*f*h^13 + 204800*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))^9*c^7*f*g^10*h^3 + 40960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*c^

```
7*e*g^9*h^4 - 680960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*b*c^6*f*g^9*h^4
+ 20480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*c^7*d*g^8*h^5 - 117760*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^9*b*c^6*e*g^8*h^5 + 706560*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^9*b^2*c^5*f*g^8*h^5 + 773120*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^9*a*c^6*f*g^8*h^5 - 81920*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*b*c
^6*d*g^7*h^6 + 61440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*b^2*c^5*e*g^7*h^
6 + 163840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a*c^6*e*g^7*h^6 - 112640*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*b^3*c^4*f*g^7*h^6 - 1720320*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))^9*a*b*c^5*f*g^7*h^6 + 122880*(sqrt(c)*x - sqrt(c*
x^2 + b*x + a))^9*b^2*c^5*d*g^6*h^7 + 81920*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))^9*a*c^6*d*g^6*h^7 + 112640*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*b^3*c
^4*e*g^6*h^7 - 307200*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a*b*c^5*e*g^6*h
^7 - 163840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*b^4*c^3*f*g^6*h^7 + 64512
0*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a*b^2*c^4*f*g^6*h^7 + 1044480*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^9*a^2*c^5*f*g^6*h^7 - 81920*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^9*b^3*c^4*d*g^5*h^8 - 245760*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))^9*a*b*c^5*d*g^5*h^8 - 143360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^
9*b^4*c^3*e*g^5*h^8 - 61440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a*b^2*c^4
*e*g^5*h^8 + 245760*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^2*c^5*e*g^5*h^8
 + 15360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*b^5*c^2*f*g^5*h^8 + 655360*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a*b^3*c^3*f*g^5*h^8 - 1075200*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^9*a^2*b*c^4*f*g^5*h^8 + 2720*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^9*b^4*c^3*d*g^4*h^9 + 387840*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^9*a*b^2*c^4*d*g^4*h^9 - 161280*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^
9*a^2*c^5*d*g^4*h^9 + 54960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*b^5*c^2*e
*g^4*h^9 + 318080*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a*b^3*c^3*e*g^4*h^9
 - 72960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^2*b*c^4*e*g^4*h^9 + 25540*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*b^6*c*f*g^4*h^9 - 186000*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))^9*a*b^4*c^2*f*g^4*h^9 - 905280*(sqrt(c)*x - sqrt(c*
x^2 + b*x + a))^9*a^2*b^2*c^3*f*g^4*h^9 + 590080*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))^9*a^3*c^4*f*g^4*h^9 + 15720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9
*b^5*c^2*d*g^3*h^10 - 207680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a*b^3*c^
3*d*g^3*h^10 + 5760*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^2*b*c^4*d*g^3*h
^10 - 2680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*b^6*c*e*g^3*h^10 - 183600*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a*b^4*c^2*e*g^3*h^10 - 184320*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^9*a^2*b^2*c^3*e*g^3*h^10 - 167680*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))^9*a^3*c^4*e*g^3*h^10 - 595*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^9*b^7*f*g^3*h^10 - 100020*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9
*a*b^5*c*f*g^3*h^10 + 490800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^2*b^3*
c^2*f*g^3*h^10 + 576320*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^3*b*c^3*f*g
^3*h^10 - 3140*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*b^6*c*d*g^2*h^11 + 301
20*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a*b^4*c^2*d*g^2*h^11 + 32640*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^9*a^2*b^2*c^3*d*g^2*h^11 + 161920*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))^9*a^3*c^4*d*g^2*h^11 - 425*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^9*b^7*e*g^2*h^11 + 8880*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9
```

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*a*b^5*c*e*g^2*h^11 + 225840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^2*b^3*
c^2*e*g^2*h^11 + 108160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^3*b*c^3*e*g
^2*h^11 + 2635*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a*b^6*f*g^2*h^11 + 145
140*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^2*b^4*c*f*g^2*h^11 - 519600*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^3*b^2*c^2*f*g^2*h^11 - 269120*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^9*a^4*c^3*f*g^2*h^11 - 595*(sqrt(c)*x - sqrt(c*
x^2 + b*x + a))^9*b^7*d*g*h^12 + 3060*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9
*a*b^5*c*d*g*h^12 + 4080*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^2*b^3*c^2*
d*g*h^12 - 109120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^3*b*c^3*d*g*h^12
+ 1445*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a*b^6*e*g*h^12 - 9180*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^9*a^2*b^4*c*e*g*h^12 - 180240*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^9*a^3*b^2*c^2*e*g*h^12 + 79040*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))^9*a^4*c^3*e*g*h^12 - 4080*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*
a^2*b^5*f*g*h^12 - 90240*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^3*b^3*c*f*
g*h^12 + 257280*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^4*b*c^2*f*g*h^12 +
595*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a*b^6*d*h^13 - 5100*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^9*a^2*b^4*c*d*h^13 + 12240*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))^9*a^3*b^2*c^2*d*h^13 + 15040*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
^9*a^4*c^3*d*h^13 - 1020*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^2*b^5*e*h^
13 + 8160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^3*b^3*c*e*h^13 + 29760*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^4*b*c^2*e*h^13 + 2040*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))^9*a^3*b^4*f*h^13 + 14400*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))^9*a^4*b^2*c*f*h^13 - 13440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^5
*c^2*f*h^13 + 307200*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*c^(15/2)*f*g^11*
h^2 + 61440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*c^(15/2)*e*g^10*h^3 - 829
440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*b*c^(13/2)*f*g^10*h^3 + 30720*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^8*c^(15/2)*d*g^9*h^4 - 138240*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))^8*b*c^(13/2)*e*g^9*h^4 + 368640*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))^8*b^2*c^(11/2)*f*g^9*h^4 + 967680*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^8*a*c^(13/2)*f*g^9*h^4 - 46080*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))^8*b*c^(13/2)*d*g^8*h^5 + 230400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a
*c^(13/2)*e*g^8*h^5 + 691200*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*b^3*c^(9
/2)*f*g^8*h^5 - 1105920*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a*b*c^(11/2)*
f*g^8*h^5 - 122880*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*b^2*c^(11/2)*d*g^7
*h^6 + 122880*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a*c^(13/2)*d*g^7*h^6 +
153600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*b^3*c^(9/2)*e*g^7*h^6 - 245760
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a*b*c^(11/2)*e*g^7*h^6 - 614400*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^8*b^4*c^(7/2)*f*g^7*h^6 - 1935360*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^8*a*b^2*c^(9/2)*f*g^7*h^6 + 798720*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^8*a^2*c^(11/2)*f*g^7*h^6 + 337920*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^8*b^3*c^(9/2)*d*g^6*h^7 - 61440*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))^8*a*b*c^(11/2)*d*g^6*h^7 - 399360*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^8*a*b^2*c^(9/2)*e*g^6*h^7 + 307200*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)^8*a^2*c^(11/2)*e*g^6*h^7 + 2826240*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*
a*b^3*c^(7/2)*f*g^6*h^7 + 1597440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^2
```

```
*b*c^(9/2)*f*g^6*h^7 - 317520*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*b^4*c^(
7/2)*d*g^5*h^8 - 224640*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a*b^2*c^(9/2)
*d*g^5*h^8 - 472320*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^2*c^(11/2)*d*g^
5*h^8 - 117720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*b^5*c^(5/2)*e*g^5*h^8
+ 204480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a*b^3*c^(7/2)*e*g^5*h^8 + 42
0480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^2*b*c^(9/2)*e*g^5*h^8 + 49470*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*b^6*c^(3/2)*f*g^5*h^8 - 219960*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^8*a*b^4*c^(5/2)*f*g^5*h^8 - 4854240*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^8*a^2*b^2*c^(7/2)*f*g^5*h^8 - 228480*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))^8*a^3*c^(9/2)*f*g^5*h^8 + 99480*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^8*b^5*c^(5/2)*d*g^4*h^9 + 617280*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^8*a*b^3*c^(7/2)*d*g^4*h^9 + 455040*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^8*a^2*b*c^(9/2)*d*g^4*h^9 + 62070*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))^8*b^6*c^(3/2)*e*g^4*h^9 + 238920*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a
*b^4*c^(5/2)*e*g^4*h^9 - 344160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^2*b
^2*c^(7/2)*e*g^4*h^9 - 612480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^3*c^(
9/2)*e*g^4*h^9 + 10005*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*b^7*sqrt(c)*f*
g^4*h^9 - 281700*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a*b^5*c^(3/2)*f*g^4*
h^9 + 882480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^2*b^3*c^(5/2)*f*g^4*h^
9 + 3839040*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^3*b*c^(7/2)*f*g^4*h^9 +
 6390*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*b^6*c^(3/2)*d*g^3*h^10 - 333120
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a*b^4*c^(5/2)*d*g^3*h^10 - 836640*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^2*b^2*c^(7/2)*d*g^3*h^10 + 526080*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^3*c^(9/2)*d*g^3*h^10 - 3825*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))^8*b^7*sqrt(c)*e*g^3*h^10 - 225240*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))^8*a*b^5*c^(3/2)*e*g^3*h^10 - 11280*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))^8*a^2*b^3*c^(5/2)*e*g^3*h^10 + 207360*(sqrt(c)*x - sqrt(c*
x^2 + b*x + a))^8*a^3*b*c^(7/2)*e*g^3*h^10 - 37725*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^8*a*b^6*sqrt(c)*f*g^3*h^10 + 564900*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^8*a^2*b^4*c^(3/2)*f*g^3*h^10 - 1203120*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^8*a^3*b^2*c^(5/2)*f*g^3*h^10 - 1442880*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^8*a^4*c^(7/2)*f*g^3*h^10 - 5355*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)^8*b^7*sqrt(c)*d*g^2*h^11 + 27540*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a*
b^5*c^(3/2)*d*g^2*h^11 + 497520*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^2*b
^3*c^(5/2)*d*g^2*h^11 - 60480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^3*b*c
^(7/2)*d*g^2*h^11 + 13005*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a*b^6*sqrt(
c)*e*g^2*h^11 + 286020*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^2*b^4*c^(3/2
)*e*g^2*h^11 - 239760*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^3*b^2*c^(5/2)
*e*g^2*h^11 + 342720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^4*c^(7/2)*e*g^
2*h^11 + 55440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^2*b^5*sqrt(c)*f*g^2*
h^11 - 504960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^3*b^3*c^(3/2)*f*g^2*h
^11 + 610560*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^4*b*c^(5/2)*f*g^2*h^11
 + 5355*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a*b^6*sqrt(c)*d*g*h^12 - 4590
0*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^2*b^4*c^(3/2)*d*g*h^12 - 197040*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^3*b^2*c^(5/2)*d*g*h^12 - 18240*(sqrt
```

```
(c)*x - sqrt(c*x^2 + b*x + a))^8*a^4*c^(7/2)*d*g*h^12 - 9180*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^8*a^2*b^5*sqrt(c)*e*g*h^12 - 172320*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^8*a^3*b^3*c^(3/2)*e*g*h^12 + 22080*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))^8*a^4*b*c^(5/2)*e*g*h^12 - 43080*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^8*a^3*b^4*sqrt(c)*f*g*h^12 + 221760*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))^8*a^4*b^2*c^(3/2)*f*g*h^12 + 32640*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)^8*a^5*c^(5/2)*f*g*h^12 + 76800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^4*
b*c^(5/2)*d*h^13 + 61440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^4*b^2*c^(3
/2)*e*h^13 - 15360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^5*c^(5/2)*e*h^13
 + 15360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^4*b^3*sqrt(c)*f*h^13 - 614
40*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^5*b*c^(3/2)*f*h^13 + 245760*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^7*c^8*f*g^12*h + 49152*(sqrt(c)*x - sqrt(c*
x^2 + b*x + a))^7*c^8*e*g^11*h^2 - 294912*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))^7*b*c^7*f*g^11*h^2 + 24576*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*c^8*d*g
^10*h^3 - 36864*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*b*c^7*e*g^10*h^3 - 93
0816*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*b^2*c^6*f*g^10*h^3 + 405504*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^7*a*c^7*f*g^10*h^3 - 211968*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))^7*b^2*c^6*e*g^9*h^4 + 110592*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))^7*a*c^7*e*g^9*h^4 + 1824768*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^
7*b^3*c^5*f*g^9*h^4 + 1732608*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a*b*c^6
*f*g^9*h^4 - 119808*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*b^2*c^6*d*g^8*h^5
 + 110592*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a*c^7*d*g^8*h^5 + 276480*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))^7*b^3*c^5*e*g^8*h^5 + 368640*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))^7*a*b*c^6*e*g^8*h^5 - 691200*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^7*b^4*c^4*f*g^8*h^5 - 5916672*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))^7*a*b^2*c^5*f*g^8*h^5 - 755712*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a
^2*c^6*f*g^8*h^5 - 12288*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*b^3*c^5*d*g^
7*h^6 + 49152*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a*b*c^6*d*g^7*h^6 + 307
20*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*b^4*c^4*e*g^7*h^6 - 1216512*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^7*a*b^2*c^5*e*g^7*h^6 - 49152*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))^7*a^2*c^6*e*g^7*h^6 - 307200*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))^7*b^5*c^3*f*g^7*h^6 + 3624960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)^7*a*b^3*c^4*f*g^7*h^6 + 6266880*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^2
*b*c^5*f*g^7*h^6 + 336960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*b^4*c^4*d*g
^6*h^7 + 93696*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a*b^2*c^5*d*g^6*h^7 -
605184*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^2*c^6*d*g^6*h^7 - 85536*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^7*b^5*c^3*e*g^6*h^7 + 303360*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^7*a*b^3*c^4*e*g^6*h^7 + 1703424*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))^7*a^2*b*c^5*e*g^6*h^7 + 80616*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^7*b^6*c^2*f*g^6*h^7 + 1354080*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a
*b^4*c^3*f*g^6*h^7 - 7102080*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^2*b^2*
c^4*f*g^6*h^7 - 2029056*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^3*c^5*f*g^6
*h^7 - 419328*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*b^5*c^3*d*g^5*h^8 - 368
640*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a*b^3*c^4*d*g^5*h^8 - 73728*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^7*a^2*b*c^5*d*g^5*h^8 - 18408*(sqrt(c)*x - s
```

```
qrt(c*x^2 + b*x + a))^7*b^6*c^2*e*g^5*h^8 + 211680*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^7*a*b^4*c^3*e*g^5*h^8 - 754560*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))^7*a^2*b^2*c^4*e*g^5*h^8 - 1254912*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7
*a^3*c^5*e*g^5*h^8 + 36648*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*b^7*c*f*g^
5*h^8 - 564192*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a*b^5*c^2*f*g^5*h^8 -
2332800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^2*b^3*c^3*f*g^5*h^8 + 64281
60*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^3*b*c^4*f*g^5*h^8 + 170520*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^7*b^6*c^2*d*g^4*h^9 + 846240*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))^7*a*b^4*c^3*d*g^4*h^9 + 5760*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))^7*a^2*b^2*c^4*d*g^4*h^9 + 1328640*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^7*a^3*c^5*d*g^4*h^9 + 32064*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*b^7
*c*e*g^4*h^9 - 30816*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a*b^5*c^2*e*g^4*
h^9 + 157440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^2*b^3*c^3*e*g^4*h^9 +
23040*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^3*b*c^4*e*g^4*h^9 + 1686*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^7*b^8*f*g^4*h^9 - 194304*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^7*a*b^6*c*f*g^4*h^9 + 1480320*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^7*a^2*b^4*c^2*f*g^4*h^9 + 2188800*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))^7*a^3*b^2*c^3*f*g^4*h^9 - 2557440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7
*a^4*c^4*f*g^4*h^9 - 13896*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*b^7*c*d*g^
3*h^10 - 436416*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a*b^5*c^2*d*g^3*h^10
- 842880*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^2*b^3*c^3*d*g^3*h^10 - 552
960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^3*b*c^4*d*g^3*h^10 - 990*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^7*b^8*e*g^3*h^10 - 129120*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))^7*a*b^6*c*e*g^3*h^10 + 153600*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^7*a^2*b^4*c^2*e*g^3*h^10 - 368640*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))^7*a^3*b^2*c^3*e*g^3*h^10 + 898560*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*
a^4*c^4*e*g^3*h^10 - 4764*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a*b^7*f*g^3
*h^10 + 400176*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^2*b^5*c*f*g^3*h^10 -
 1903680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^3*b^3*c^2*f*g^3*h^10 - 155
5200*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^4*b*c^3*f*g^3*h^10 - 1386*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^7*b^8*d*g^2*h^11 + 21024*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^7*a*b^6*c*d*g^2*h^11 + 638640*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^7*a^2*b^4*c^2*d*g^2*h^11 + 466560*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))^7*a^3*b^2*c^3*d*g^2*h^11 - 172800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7
*a^4*c^4*d*g^2*h^11 + 4356*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a*b^7*e*g^
2*h^11 + 204696*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^2*b^5*c*e*g^2*h^11
- 286080*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^3*b^3*c^2*e*g^2*h^11 + 109
440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^4*b*c^3*e*g^2*h^11 + 2790*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^7*a^2*b^6*f*g^2*h^11 - 399960*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^7*a^3*b^4*c*f*g^2*h^11 + 1360800*(sqrt(c)*x - sqrt(c*
x^2 + b*x + a))^7*a^4*b^2*c^2*f*g^2*h^11 + 777600*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))^7*a^5*c^3*f*g^2*h^11 + 2772*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^
7*a*b^7*d*g*h^12 - 19008*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^2*b^5*c*d*
g*h^12 - 484800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^3*b^3*c^2*d*g*h^12
+ 99840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^4*b*c^3*d*g*h^12 - 5742*(sq
```

```
rt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^2*b^6*e*g*h^12 - 131640*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^7*a^3*b^4*c*e*g*h^12 + 182880*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^7*a^4*b^2*c^2*e*g*h^12 - 174720*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^7*a^5*c^3*e*g*h^12 + 1968*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^3*b
^5*f*g*h^12 + 174720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^4*b^3*c*f*g*h^
12 - 472320*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^5*b*c^2*f*g*h^12 - 1386
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^2*b^6*d*h^13 + 11880*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^7*a^3*b^4*c*d*h^13 + 97440*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))^7*a^4*b^2*c^2*d*h^13 + 24960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
^7*a^5*c^3*d*h^13 + 2376*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^3*b^5*e*h^
13 + 24000*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^4*b^3*c*e*h^13 + 13440*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^5*b*c^2*e*h^13 - 1680*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^7*a^4*b^4*f*h^13 - 17280*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^7*a^5*b^2*c*f*h^13 + 3840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^6
*c^2*f*h^13 + 81920*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*c^(17/2)*f*g^13 +
 16384*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*c^(17/2)*e*g^12*h + 311296*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^6*b*c^(15/2)*f*g^12*h + 8192*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^6*c^(17/2)*d*g^11*h^2 + 69632*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^6*b*c^(15/2)*e*g^11*h^2 - 1416192*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^6*b^2*c^(13/2)*f*g^11*h^2 - 274432*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))^6*a*c^(15/2)*f*g^11*h^2 + 40960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*
b*c^(15/2)*d*g^10*h^3 - 254976*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*b^2*c^
(13/2)*e*g^10*h^3 - 45056*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a*c^(15/2)*
e*g^10*h^3 + 1099776*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*b^3*c^(11/2)*f*g
^10*h^3 + 2973696*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a*b*c^(13/2)*f*g^10
*h^3 - 101376*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*b^2*c^(13/2)*d*g^9*h^4
- 4096*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a*c^(15/2)*d*g^9*h^4 + 92160*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*b^3*c^(11/2)*e*g^9*h^4 + 614400*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^6*a*b*c^(13/2)*e*g^9*h^4 + 691200*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))^6*b^4*c^(9/2)*f*g^9*h^4 - 3938304*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^6*a*b^2*c^(11/2)*f*g^9*h^4 - 1542144*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^6*a^2*c^(13/2)*f*g^9*h^4 - 55296*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^6*b^3*c^(11/2)*d*g^8*h^5 + 405504*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))^6*a*b*c^(13/2)*d*g^8*h^5 + 230400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)^6*b^4*c^(9/2)*e*g^8*h^5 - 681984*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a*
b^2*c^(11/2)*e*g^8*h^5 - 313344*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^2*c
^(13/2)*e*g^8*h^5 - 921600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*b^5*c^(7/2
)*f*g^8*h^5 - 2304000*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a*b^3*c^(9/2)*f
*g^8*h^5 + 4608000*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^2*b*c^(11/2)*f*g
^8*h^5 + 100800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*b^4*c^(9/2)*d*g^7*h^6
 - 450048*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a*b^2*c^(11/2)*d*g^7*h^6 -
549888*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^2*c^(13/2)*d*g^7*h^6 - 84192
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*b^5*c^(7/2)*e*g^7*h^6 - 887040*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^6*a*b^3*c^(9/2)*e*g^7*h^6 + 1356288*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^6*a^2*b*c^(11/2)*e*g^7*h^6 + 23512*(sqrt(c)*x -
```

sqrt(c*x^2 + b*x + a))^6*b^6*c^(5/2)*f*g^7*h^6 + 5108640*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^6*a*b^4*c^(7/2)*f*g^7*h^6 + 2355840*(sqrt(c)*x - sqrt(c*
x^2 + b*x + a))^6*a^2*b^2*c^(9/2)*f*g^7*h^6 - 1710592*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))^6*a^3*c^(11/2)*f*g^7*h^6 + 95424*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^6*b^5*c^(7/2)*d*g^6*h^7 + 698880*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))^6*a*b^3*c^(9/2)*d*g^6*h^7 - 193536*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6
*a^2*b*c^(11/2)*d*g^6*h^7 - 58936*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*b^6
*c^(5/2)*e*g^6*h^7 + 602400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a*b^4*c^(
7/2)*e*g^6*h^7 + 1407360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^2*b^2*c^(9
/2)*e*g^6*h^7 - 1001984*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^3*c^(11/2)*
e*g^6*h^7 + 69776*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*b^7*c^(3/2)*f*g^6*h
^7 - 459264*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a*b^5*c^(5/2)*f*g^6*h^7 -
11447040*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^2*b^3*c^(7/2)*f*g^6*h^7 -
112640*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^3*b*c^(9/2)*f*g^6*h^7 - 242
840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*b^6*c^(5/2)*d*g^5*h^8 - 593760*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a*b^4*c^(7/2)*d*g^5*h^8 - 1011840*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^6*a^2*b^2*c^(9/2)*d*g^5*h^8 + 1564160*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^6*a^3*c^(11/2)*d*g^5*h^8 + 29608*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^6*b^7*c^(3/2)*e*g^5*h^8 + 53088*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))^6*a*b^5*c^(5/2)*e*g^5*h^8 - 1034880*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^6*a^2*b^3*c^(7/2)*e*g^5*h^8 - 2168320*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^6*a^3*b*c^(9/2)*e*g^5*h^8 + 11802*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^6*b^8*sqrt(c)*f*g^5*h^8 - 410168*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)^6*a*b^6*c^(3/2)*f*g^5*h^8 + 1811040*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6
*a^2*b^4*c^(5/2)*f*g^5*h^8 + 13311360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6
*a^3*b^2*c^(7/2)*f*g^5*h^8 - 808960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a
^4*c^(9/2)*f*g^5*h^8 + 115328*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*b^7*c^(
3/2)*d*g^4*h^9 + 793248*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a*b^5*c^(5/2)
*d*g^4*h^9 + 480000*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^2*b^3*c^(7/2)*d
*g^4*h^9 + 739840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^3*b*c^(9/2)*d*g^4
h^9 + 8430(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*b^8*sqrt(c)*e*g^4*h^9 - 1
73800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a*b^6*c^(3/2)*e*g^4*h^9 + 40560
0*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^2*b^4*c^(5/2)*e*g^4*h^9 + 850560*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^3*b^2*c^(7/2)*e*g^4*h^9 + 1715200*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^4*c^(9/2)*e*g^4*h^9 - 64068*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^6*a*b^7*sqrt(c)*f*g^4*h^9 + 967632*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^6*a^2*b^5*c^(3/2)*f*g^4*h^9 - 3060160*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^6*a^3*b^3*c^(5/2)*f*g^4*h^9 - 8643840*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^6*a^4*b*c^(7/2)*f*g^4*h^9 - 9702*(sqrt(c)*x - sqrt(c*
x^2 + b*x + a))^6*b^8*sqrt(c)*d*g^3*h^10 - 403352*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))^6*a*b^6*c^(3/2)*d*g^3*h^10 - 693840*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^6*a^2*b^4*c^(5/2)*d*g^3*h^10 - 756480*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^6*a^3*b^2*c^(7/2)*d*g^3*h^10 - 951040*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^6*a^4*c^(9/2)*d*g^3*h^10 - 30948*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^
6*a*b^7*sqrt(c)*e*g^3*h^10 + 313992*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a

```
^2*b^5*c^(3/2)*e*g^3*h^10 - 997760*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^
3*b^3*c^(5/2)*e*g^3*h^10 + 190080*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^4
*b*c^(7/2)*e*g^3*h^10 + 142410*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^2*b^
6*sqrt(c)*f*g^3*h^10 - 1135400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^3*b^
4*c^(3/2)*f*g^3*h^10 + 2498400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^4*b^
2*c^(5/2)*f*g^3*h^10 + 2663040*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^5*c^
(7/2)*f*g^3*h^10 + 19404*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a*b^7*sqrt(c
)*d*g^2*h^11 + 542784*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^2*b^5*c^(3/2)
*d*g^2*h^11 + 374720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^3*b^3*c^(5/2)*
d*g^2*h^11 + 821760*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^4*b*c^(7/2)*d*g
^2*h^11 + 51966*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^2*b^6*sqrt(c)*e*g^2
*h^11 - 225160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^3*b^4*c^(3/2)*e*g^2*
h^11 + 604320*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^4*b^2*c^(5/2)*e*g^2*h
^11 - 608640*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^5*c^(7/2)*e*g^2*h^11 -
 170544*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^3*b^5*sqrt(c)*f*g^2*h^11 +
690560*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^4*b^3*c^(3/2)*f*g^2*h^11 - 6
64320*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^5*b*c^(5/2)*f*g^2*h^11 - 9702
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^2*b^6*sqrt(c)*d*g*h^12 - 367400*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^3*b^4*c^(3/2)*d*g*h^12 - 300960*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^6*a^4*b^2*c^(5/2)*d*g*h^12 + 51840*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^6*a^5*c^(7/2)*d*g*h^12 - 44808*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^6*a^3*b^5*sqrt(c)*e*g*h^12 + 86080*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))^6*a^4*b^3*c^(3/2)*e*g*h^12 - 28800*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))^6*a^5*b*c^(5/2)*e*g*h^12 + 111120*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^6*a^4*b^4*sqrt(c)*f*g*h^12 - 243840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))^6*a^5*b^2*c^(3/2)*f*g*h^12 - 157440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^
6*a^6*c^(5/2)*f*g*h^12 + 112640*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^4*b
^3*c^(3/2)*d*h^13 + 61440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^5*b*c^(5/
2)*d*h^13 + 15360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^4*b^4*sqrt(c)*e*h
^13 - 30720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^5*b^2*c^(3/2)*e*h^13 +
30720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^6*c^(5/2)*e*h^13 - 30720*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^6*a^5*b^3*sqrt(c)*f*h^13 + 61440*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))^6*a^6*b*c^(3/2)*f*h^13 + 245760*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^5*b*c^8*f*g^13 + 49152*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))^5*b*c^8*e*g^12*h - 294912*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^2*c^7*
f*g^12*h - 245760*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*c^8*f*g^12*h + 24
576*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b*c^8*d*g^11*h^2 - 36864*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^5*b^2*c^7*e*g^11*h^2 - 49152*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^5*a*c^8*e*g^11*h^2 - 930816*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))^5*b^3*c^6*f*g^11*h^2 + 700416*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*
a*b*c^7*f*g^11*h^2 - 24576*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*c^8*d*g^
10*h^3 - 211968*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^3*c^6*e*g^10*h^3 +
147456*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b*c^7*e*g^10*h^3 + 1824768*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^4*c^5*f*g^10*h^3 + 2663424*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^5*a*b^2*c^6*f*g^10*h^3 - 405504*(sqrt(c)*x - sqr
```

```
t(c*x^2 + b*x + a))^5*a^2*c^7*f*g^10*h^3 - 119808*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))^5*b^3*c^6*d*g^9*h^4 + 110592*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
^5*a*b*c^7*d*g^9*h^4 + 276480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^4*c^5
*e*g^9*h^4 + 580608*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b^2*c^6*e*g^9*h
^4 - 110592*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*c^7*e*g^9*h^4 - 69120
0*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^5*c^4*f*g^9*h^4 - 7741440*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^5*a*b^3*c^5*f*g^9*h^4 - 2488320*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^5*a^2*b*c^6*f*g^9*h^4 + 41472*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^5*b^4*c^5*d*g^8*h^5 + 414720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))^5*a*b^2*c^6*d*g^8*h^5 - 110592*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2
*c^7*d*g^8*h^5 + 34560*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^5*c^4*e*g^8*
h^5 - 1492992*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b^3*c^5*e*g^8*h^5 - 3
87072*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*b*c^6*e*g^8*h^5 - 308160*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^6*c^3*f*g^8*h^5 + 4320000*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))^5*a*b^4*c^4*f*g^8*h^5 + 12137472*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^5*a^2*b^2*c^5*f*g^8*h^5 + 755712*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^5*a^3*c^6*f*g^8*h^5 + 87360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)^5*b^5*c^4*d*g^7*h^6 - 600576*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b^3*
c^5*d*g^7*h^6 - 1207296*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*b*c^6*d*g
^7*h^6 - 82656*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^6*c^3*e*g^7*h^6 + 12
2880*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b^4*c^4*e*g^7*h^6 + 3119616*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*b^2*c^5*e*g^7*h^6 - 12288*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^5*a^3*c^6*e*g^7*h^6 + 73416*(sqrt(c)*x - sqrt(c*
x^2 + b*x + a))^5*b^7*c^2*f*g^7*h^6 + 1743840*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))^5*a*b^5*c^3*f*g^7*h^6 - 10796160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
^5*a^2*b^3*c^4*f*g^7*h^6 - 8142336*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^
3*b*c^5*f*g^7*h^6 - 71808*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^6*c^3*d*g
^6*h^7 + 822720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b^4*c^4*d*g^6*h^7 +
 907776*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*b^2*c^5*d*g^6*h^7 + 12195
84*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^3*c^6*d*g^6*h^7 - 1128*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^5*b^7*c^2*e*g^6*h^7 + 312576*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))^5*a*b^5*c^3*e*g^6*h^7 - 597120*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^5*a^2*b^3*c^4*e*g^6*h^7 - 3572736*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))^5*a^3*b*c^5*e*g^6*h^7 + 31248*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^8
*c*f*g^6*h^7 - 592968*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b^6*c^2*f*g^6
*h^7 - 4236000*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*b^4*c^3*f*g^6*h^7
+ 13991040*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^3*b^2*c^4*f*g^6*h^7 + 18
75456*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^4*c^5*f*g^6*h^7 - 47400*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^5*b^7*c^2*d*g^5*h^8 - 362592*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))^5*a*b^5*c^3*d*g^5*h^8 - 2436480*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^5*a^2*b^3*c^4*d*g^5*h^8 + 1033728*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^5*a^3*b*c^5*d*g^5*h^8 + 19944*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^
5*b^8*c*e*g^5*h^8 - 87288*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b^6*c^2*e
*g^5*h^8 - 73440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*b^4*c^3*e*g^5*h^
8 + 132480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^3*b^2*c^4*e*g^5*h^8 + 17
```

```
15712*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^4*c^5*e*g^5*h^8 + 1386*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^5*b^9*f*g^5*h^8 - 181512*(sqrt(c)*x - sqrt(c*
x^2 + b*x + a))^5*a*b^7*c*f*g^5*h^8 + 1906272*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))^5*a^2*b^5*c^2*f*g^5*h^8 + 5788800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)^5*a^3*b^3*c^3*f*g^5*h^8 - 9768960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a
^4*b*c^4*f*g^5*h^8 + 38784*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^8*c*d*g^
4*h^9 + 308424*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b^6*c^2*d*g^4*h^9 +
1435680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*b^4*c^3*d*g^4*h^9 + 11923
20*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^3*b^2*c^4*d*g^4*h^9 - 1374720*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^4*c^5*d*g^4*h^9 + 990*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))^5*b^9*e*g^4*h^9 - 105744*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))^5*a*b^7*c*e*g^4*h^9 + 282816*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a
^2*b^5*c^2*e*g^4*h^9 - 487680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^3*b^3
*c^3*e*g^4*h^9 + 1543680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^4*b*c^4*e*
g^4*h^9 - 8910*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b^8*f*g^4*h^9 + 4386
00*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*b^6*c*f*g^4*h^9 - 3184800*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^5*a^3*b^4*c^2*f*g^4*h^9 - 5212800*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^5*a^4*b^2*c^3*f*g^4*h^9 + 2949120*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^5*a^5*c^4*f*g^4*h^9 - 1686*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))^5*b^9*d*g^3*h^10 - 183000*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a
*b^7*c*d*g^3*h^10 - 545904*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*b^5*c^
2*d*g^3*h^10 - 1482240*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^3*b^3*c^3*d*
g^3*h^10 - 103680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^4*b*c^4*d*g^3*h^1
0 - 2274*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b^8*e*g^3*h^10 + 228336*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*b^6*c*e*g^3*h^10 - 480480*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^5*a^3*b^4*c^2*e*g^3*h^10 + 322560*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^5*a^4*b^2*c^3*e*g^3*h^10 - 1244160*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^5*a^5*c^4*e*g^3*h^10 + 20094*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^5*a^2*b^7*f*g^3*h^10 - 545976*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5
*a^3*b^5*c*f*g^3*h^10 + 3086880*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^4*b
^3*c^2*f*g^3*h^10 + 3335040*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^5*b*c^3
*f*g^3*h^10 + 5058*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b^8*d*g^2*h^11 +
 292248*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*b^6*c*d*g^2*h^11 + 462480
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^3*b^4*c^2*d*g^2*h^11 + 748800*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^5*a^4*b^2*c^3*d*g^2*h^11 + 449280*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^5*a^5*c^4*d*g^2*h^11 + 882*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))^5*a^2*b^7*e*g^2*h^11 - 225936*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^5*a^3*b^5*c*e*g^2*h^11 + 471840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5
*a^4*b^3*c^2*e*g^2*h^11 - 184320*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^5*
b*c^3*e*g^2*h^11 - 20682*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^3*b^6*f*g^
2*h^11 + 328680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^4*b^4*c*f*g^2*h^11
- 1738080*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^5*b^2*c^2*f*g^2*h^11 - 11
11680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^6*c^3*f*g^2*h^11 - 5058*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*b^7*d*g*h^12 - 190632*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^5*a^3*b^5*c*d*g*h^12 - 305760*(sqrt(c)*x - sqrt(c*x^2 +
```

```
 b*x + a))^5*a^4*b^3*c^2*d*g*h^12 - 293760*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))^5*a^5*b*c^3*d*g*h^12 + 1098*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^3*b
^6*e*g*h^12 + 90120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^4*b^4*c*e*g*h^1
2 - 168480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^5*b^2*c^2*e*g*h^12 + 193
920*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^6*c^3*e*g*h^12 + 9792*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^5*a^4*b^5*f*g*h^12 - 53760*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))^5*a^5*b^3*c*f*g*h^12 + 445440*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^5*a^6*b*c^2*f*g*h^12 + 1686*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^3
*b^6*d*h^13 + 42600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^4*b^4*c*d*h^13
+ 128160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^5*b^2*c^2*d*h^13 + 24960*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^6*c^3*d*h^13 - 696*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^5*a^4*b^5*e*h^13 - 6720*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))^5*a^5*b^3*c*e*h^13 - 17280*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^6*b*
c^2*e*h^13 - 1680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^5*b^4*f*h^13 - 17
280*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^6*b^2*c*f*h^13 + 3840*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^5*a^7*c^2*f*h^13 + 307200*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^4*b^2*c^(15/2)*f*g^13 + 61440*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))^4*b^2*c^(15/2)*e*g^12*h - 829440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4
*b^3*c^(13/2)*f*g^12*h - 614400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b*c
^(15/2)*f*g^12*h + 30720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^2*c^(15/2)
*d*g^11*h^2 - 138240*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^3*c^(13/2)*e*g
^11*h^2 - 122880*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b*c^(15/2)*e*g^11*
h^2 + 368640*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^4*c^(11/2)*f*g^11*h^2
+ 2626560*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b^2*c^(13/2)*f*g^11*h^2 +
 307200*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*c^(15/2)*f*g^11*h^2 - 460
80*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^3*c^(13/2)*d*g^10*h^3 - 61440*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b*c^(15/2)*d*g^10*h^3 + 506880*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^4*a*b^2*c^(13/2)*e*g^10*h^3 + 61440*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^4*a^2*c^(15/2)*e*g^10*h^3 + 691200*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^4*b^5*c^(9/2)*f*g^10*h^3 - 1843200*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^4*a*b^3*c^(11/2)*f*g^10*h^3 - 2764800*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^4*a^2*b*c^(13/2)*f*g^10*h^3 - 51840*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))^4*b^4*c^(11/2)*d*g^9*h^4 + 276480*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))^4*a*b^2*c^(13/2)*d*g^9*h^4 + 30720*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^4*a^2*c^(15/2)*d*g^9*h^4 + 164160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
^4*b^5*c^(9/2)*e*g^9*h^4 - 207360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b
^3*c^(11/2)*e*g^9*h^4 - 599040*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*b*
c^(13/2)*e*g^9*h^4 - 614160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^6*c^(7/
2)*f*g^9*h^4 - 3326400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b^4*c^(9/2)*
f*g^9*h^4 + 3375360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*b^2*c^(11/2)*
f*g^9*h^4 + 967680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^3*c^(13/2)*f*g^9
*h^4 + 60480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^5*c^(9/2)*d*g^8*h^5 +
69120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b^3*c^(11/2)*d*g^8*h^5 - 4147
20*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*b*c^(13/2)*d*g^8*h^5 - 38160*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^6*c^(7/2)*e*g^8*h^5 - 907200*(sqrt(c
```

```
)*x - sqrt(c*x^2 + b*x + a))^4*a*b^4*c^(9/2)*e*g^8*h^5 + 794880*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))^4*a^2*b^2*c^(11/2)*e*g^8*h^5 + 230400*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))^4*a^3*c^(13/2)*e*g^8*h^5 - 3240*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))^4*b^7*c^(5/2)*f*g^8*h^5 + 4108320*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^4*a*b^5*c^(7/2)*f*g^8*h^5 + 6134400*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^4*a^2*b^3*c^(9/2)*f*g^8*h^5 - 2695680*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^4*a^3*b*c^(11/2)*f*g^8*h^5 + 15120*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))^4*b^6*c^(7/2)*d*g^7*h^6 - 228480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*
a*b^4*c^(9/2)*d*g^7*h^6 - 864000*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*
b^2*c^(11/2)*d*g^7*h^6 + 245760*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^3*c
^(13/2)*d*g^7*h^6 - 34920*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^7*c^(5/2)
*e*g^7*h^6 + 365760*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b^5*c^(7/2)*e*g
^7*h^6 + 2221440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*b^3*c^(9/2)*e*g^
7*h^6 - 1075200*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^3*b*c^(11/2)*e*g^7*
h^6 + 38220*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^8*c^(3/2)*f*g^7*h^6 - 1
48680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b^6*c^(5/2)*f*g^7*h^6 - 11547
360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*b^4*c^(7/2)*f*g^7*h^6 - 50400
00*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^3*b^2*c^(9/2)*f*g^7*h^6 + 791040
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^4*c^(11/2)*f*g^7*h^6 - 64440*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^4*b^7*c^(5/2)*d*g^6*h^7 + 437280*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))^4*a*b^5*c^(7/2)*d*g^6*h^7 + 670080*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))^4*a^2*b^3*c^(9/2)*d*g^6*h^7 + 2188800*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^4*a^3*b*c^(11/2)*d*g^6*h^7 + 15420*(sqrt(c)*x - sqrt(c*
x^2 + b*x + a))^4*b^8*c^(3/2)*e*g^6*h^7 + 56520*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^4*a*b^6*c^(5/2)*e*g^6*h^7 - 1056480*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^4*a^2*b^4*c^(7/2)*e*g^6*h^7 - 3584640*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))^4*a^3*b^2*c^(9/2)*e*g^6*h^7 + 453120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))^4*a^4*c^(11/2)*e*g^6*h^7 + 6930*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^
9*sqrt(c)*f*g^6*h^7 - 266880*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b^7*c^
(3/2)*f*g^6*h^7 + 849600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*b^5*c^(5
/2)*f*g^6*h^7 + 17756160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^3*b^3*c^(7
/2)*f*g^6*h^7 + 1466880*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^4*b*c^(9/2)
*f*g^6*h^7 + 14460*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^8*c^(3/2)*d*g^5*
h^8 - 81720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b^6*c^(5/2)*d*g^5*h^8 -
 1728000*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*b^4*c^(7/2)*d*g^5*h^8 -
792960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^3*b^2*c^(9/2)*d*g^5*h^8 - 11
98080*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^4*c^(11/2)*d*g^5*h^8 + 4950*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^9*sqrt(c)*e*g^5*h^8 - 96360*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^4*a*b^7*c^(3/2)*e*g^5*h^8 + 327600*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))^4*a^2*b^5*c^(5/2)*e*g^5*h^8 + 1474560*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^4*a^3*b^3*c^(7/2)*e*g^5*h^8 + 3528960*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^4*a^4*b*c^(9/2)*e*g^5*h^8 - 44550*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))^4*a*b^8*sqrt(c)*f*g^5*h^8 + 799620*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^4*a^2*b^6*c^(3/2)*f*g^5*h^8 - 1938960*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^4*a^3*b^4*c^(5/2)*f*g^5*h^8 - 16257600*(sqrt(c)*x - sqrt(c*x^2
```

```
+ b*x + a))^4*a^4*b^2*c^(7/2)*f*g^5*h^8 + 65280*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^4*a^5*c^(9/2)*f*g^5*h^8 + 6930*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
^4*b^9*sqrt(c)*d*g^4*h^9 - 3120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b^7
*c^(3/2)*d*g^4*h^9 + 997200*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*b^5*c
^(5/2)*d*g^4*h^9 + 1948800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^3*b^3*c^
(7/2)*d*g^4*h^9 - 441600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^4*b*c^(9/2
)*d*g^4*h^9 - 26730*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b^8*sqrt(c)*e*g
^4*h^9 + 191940*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*b^6*c^(3/2)*e*g^4
*h^9 - 1174800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^3*b^4*c^(5/2)*e*g^4*
h^9 - 590400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^4*b^2*c^(7/2)*e*g^4*h^
9 - 1424640*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^5*c^(9/2)*e*g^4*h^9 + 1
15830*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*b^7*sqrt(c)*f*g^4*h^9 - 127
0200*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^3*b^5*c^(3/2)*f*g^4*h^9 + 2205
600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^4*b^3*c^(5/2)*f*g^4*h^9 + 86342
40*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^5*b*c^(7/2)*f*g^4*h^9 - 36150*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b^8*sqrt(c)*d*g^3*h^10 - 162060*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*b^6*c^(3/2)*d*g^3*h^10 - 1591200*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^4*a^3*b^4*c^(5/2)*d*g^3*h^10 - 619200*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^4*a^4*b^2*c^(7/2)*d*g^3*h^10 + 744960*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^4*a^5*c^(9/2)*d*g^3*h^10 + 65850*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^4*a^2*b^7*sqrt(c)*e*g^3*h^10 - 128640*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^4*a^3*b^5*c^(3/2)*e*g^3*h^10 + 1293600*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^4*a^4*b^3*c^(5/2)*e*g^3*h^10 - 633600*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))^4*a^5*b*c^(7/2)*e*g^3*h^10 - 164850*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^4*a^3*b^6*sqrt(c)*f*g^3*h^10 + 1146600*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^4*a^4*b^4*c^(3/2)*f*g^3*h^10 - 1029600*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^4*a^5*b^2*c^(5/2)*f*g^3*h^10 - 2079360*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^4*a^6*c^(7/2)*f*g^3*h^10 + 66870*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^4*a^2*b^7*sqrt(c)*d*g^2*h^11 + 337080*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))^4*a^3*b^5*c^(3/2)*d*g^2*h^11 + 928800*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))^4*a^4*b^3*c^(5/2)*d*g^2*h^11 + 5760*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))^4*a^5*b*c^(7/2)*d*g^2*h^11 - 86670*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))^4*a^3*b^6*sqrt(c)*e*g^2*h^11 - 10200*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
^4*a^4*b^4*c^(3/2)*e*g^2*h^11 - 473760*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^
4*a^5*b^2*c^(5/2)*e*g^2*h^11 + 478080*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4
*a^6*c^(7/2)*e*g^2*h^11 + 141120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^4*
b^5*sqrt(c)*f*g^2*h^11 - 637440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^5*b
^3*c^(3/2)*f*g^2*h^11 - 138240*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^6*b*
c^(5/2)*f*g^2*h^11 - 53010*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^3*b^6*sq
rt(c)*d*g*h^12 - 247800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^4*b^4*c^(3/
2)*d*g*h^12 - 280800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^5*b^2*c^(5/2)*
d*g*h^12 - 59520*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^6*c^(7/2)*d*g*h^12
 + 57960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^4*b^5*sqrt(c)*e*g*h^12 + 2
7840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^5*b^3*c^(3/2)*e*g*h^12 + 36480
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^6*b*c^(5/2)*e*g*h^12 - 69840*(sqrt
```

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(c)*x - sqrt(c*x^2 + b*x + a))^4*a^5*b^4*sqrt(c)*f*g*h^12 + 251520*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^4*a^6*b^2*c^(3/2)*f*g*h^12 + 203520*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))^4*a^7*c^(5/2)*f*g*h^12 + 15360*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))^4*a^4*b^5*sqrt(c)*d*h^13 + 61440*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))^4*a^5*b^3*c^(3/2)*d*h^13 + 92160*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))^4*a^6*b*c^(5/2)*d*h^13 - 15360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a
^5*b^4*sqrt(c)*e*h^13 - 30720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^7*c^(
5/2)*e*h^13 + 15360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^6*b^3*sqrt(c)*f
*h^13 - 61440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^7*b*c^(3/2)*f*h^13 +
204800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^3*c^7*f*g^13 + 40960*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^3*b^3*c^7*e*g^12*h - 680960*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^3*b^4*c^6*f*g^12*h - 614400*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))^3*a*b^2*c^7*f*g^12*h + 20480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b
^3*c^7*d*g^11*h^2 - 117760*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^4*c^6*e*
g^11*h^2 - 122880*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^2*c^7*e*g^11*h^
2 + 706560*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^5*c^5*f*g^11*h^2 + 28160
00*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^3*c^6*f*g^11*h^2 + 614400*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b*c^7*f*g^11*h^2 - 43520*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))^3*b^4*c^6*d*g^10*h^3 - 61440*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^3*a*b^2*c^7*d*g^10*h^3 + 80640*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^3*b^5*c^5*e*g^10*h^3 + 532480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a
*b^3*c^6*e*g^10*h^3 + 122880*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b*c^
7*e*g^10*h^3 - 111040*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^6*c^4*f*g^10*
h^3 - 3836160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^4*c^5*f*g^10*h^3 -
4362240*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b^2*c^6*f*g^10*h^3 - 2048
00*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^3*c^7*f*g^10*h^3 - 3840*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^3*b^5*c^5*d*g^9*h^4 + 266240*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^3*a*b^3*c^6*d*g^9*h^4 + 61440*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^3*a^2*b*c^7*d*g^9*h^4 + 36160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^
3*b^6*c^4*e*g^9*h^4 - 579840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^4*c^
5*e*g^9*h^4 - 890880*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b^2*c^6*e*g^
9*h^4 - 40960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^3*c^7*e*g^9*h^4 - 169
760*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^7*c^3*f*g^9*h^4 + 958080*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^5*c^4*f*g^9*h^4 + 8317440*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^3*a^2*b^3*c^5*f*g^9*h^4 + 3000320*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^3*a^3*b*c^6*f*g^9*h^4 + 33920*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^3*b^6*c^4*d*g^8*h^5 - 130560*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3
*a*b^4*c^5*d*g^8*h^5 - 537600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b^2
*c^6*d*g^8*h^5 - 20480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^3*c^7*d*g^8*
h^5 - 25600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^7*c^3*e*g^8*h^5 - 15360
0*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^5*c^4*e*g^8*h^5 + 1628160*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b^3*c^5*e*g^8*h^5 + 655360*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))^3*a^3*b*c^6*e*g^8*h^5 + 21000*(sqrt(c)*x - sqrt(c*
x^2 + b*x + a))^3*b^8*c^2*f*g^8*h^5 + 1230080*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))^3*a*b^6*c^3*f*g^8*h^5 - 3321600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^
```

```
3*a^2*b^4*c^4*f*g^8*h^5 - 9000960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^3
*b^2*c^5*f*g^8*h^5 - 773120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^4*c^6*f
*g^8*h^5 - 18080*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^7*c^3*d*g^7*h^6 +
42240*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^5*c^4*d*g^7*h^6 - 145920*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b^3*c^5*d*g^7*h^6 + 573440*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^3*a^3*b*c^6*d*g^7*h^6 - 4680*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^3*b^8*c^2*e*g^7*h^6 + 158080*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^3*a*b^6*c^3*e*g^7*h^6 + 376320*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^
3*a^2*b^4*c^4*e*g^7*h^6 - 2334720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^3
*b^2*c^5*e*g^7*h^6 - 194560*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^4*c^6*e
*g^7*h^6 + 11480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^9*c*f*g^7*h^6 - 21
2400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^7*c^2*f*g^7*h^6 - 3888960*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b^5*c^3*f*g^7*h^6 + 6173440*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^3*a^3*b^3*c^4*f*g^7*h^6 + 4869120*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))^3*a^4*b*c^5*f*g^7*h^6 - 16320*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))^3*b^8*c^2*d*g^6*h^7 + 129920*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))^3*a*b^6*c^3*d*g^6*h^7 - 52800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^
2*b^4*c^4*d*g^6*h^7 + 1374720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^3*b^2
*c^5*d*g^6*h^7 - 189440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^4*c^6*d*g^6
*h^7 + 7180*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^9*c*e*g^6*h^7 - 18480*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^7*c^2*e*g^6*h^7 - 375840*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^3*a^2*b^5*c^3*e*g^6*h^7 - 1016320*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^3*a^3*b^3*c^4*e*g^6*h^7 + 1681920*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))^3*a^4*b*c^5*e*g^6*h^7 + 595*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^3*b^10*f*g^6*h^7 - 78900*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^8*
c*f*g^6*h^7 + 859320*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b^6*c^2*f*g^
6*h^7 + 7020320*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^3*b^4*c^3*f*g^6*h^7
 - 6579840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^4*b^2*c^4*f*g^6*h^7 - 10
52160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^5*c^5*f*g^6*h^7 + 9440*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^3*b^9*c*d*g^5*h^8 - 14160*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))^3*a*b^7*c^2*d*g^5*h^8 - 454080*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^3*a^2*b^5*c^3*d*g^5*h^8 - 684800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))^3*a^3*b^3*c^4*d*g^5*h^8 - 1827840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*
a^4*b*c^5*d*g^5*h^8 + 425*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^10*e*g^5*
h^8 - 41220*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^8*c*e*g^5*h^8 + 19956
0*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b^6*c^2*e*g^5*h^8 + 541600*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^3*a^3*b^4*c^3*e*g^5*h^8 + 2087040*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^3*a^4*b^2*c^4*e*g^5*h^8 - 499200*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))^3*a^5*c^5*e*g^5*h^8 - 4420*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^3*a*b^9*f*g^5*h^8 + 228600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a
^2*b^7*c*f*g^5*h^8 - 1852320*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^3*b^5*
c^2*f*g^5*h^8 - 8109440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^4*b^3*c^3*f
*g^5*h^8 + 3786240*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^5*b*c^4*f*g^5*h^
8 + 595*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^10*d*g^4*h^9 - 31380*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^8*c*d*g^4*h^9 + 280200*(sqrt(c)*x - sqr
```

```
t(c*x^2 + b*x + a))^3*a^2*b^6*c^2*d*g^4*h^9 + 965600*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^3*a^3*b^4*c^3*d*g^4*h^9 + 873600*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^3*a^4*b^2*c^4*d*g^4*h^9 + 775680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)^3*a^5*c^5*d*g^4*h^9 - 2720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^9*e*
g^4*h^9 + 89760*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b^7*c*e*g^4*h^9 -
 588960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^3*b^5*c^2*e*g^4*h^9 - 62080
0*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^4*b^3*c^3*e*g^4*h^9 - 1989120*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^5*b*c^4*e*g^4*h^9 + 13770*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))^3*a^2*b^8*f*g^4*h^9 - 359840*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^3*a^3*b^6*c*f*g^4*h^9 + 2419200*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))^3*a^4*b^4*c^2*f*g^4*h^9 + 6251520*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)^3*a^5*b^2*c^3*f*g^4*h^9 - 903680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^
6*c^4*f*g^4*h^9 - 2380*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^9*d*g^3*h^
10 + 11880*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b^7*c*d*g^3*h^10 - 645
120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^3*b^5*c^2*d*g^3*h^10 - 944000*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^4*b^3*c^3*d*g^3*h^10 - 61440*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^3*a^5*b*c^4*d*g^3*h^10 + 6630*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^3*a^2*b^8*e*g^3*h^10 - 71200*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))^3*a^3*b^6*c*e*g^3*h^10 + 835200*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))^3*a^4*b^4*c^2*e*g^3*h^10 + 384000*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*
a^5*b^2*c^3*e*g^3*h^10 + 657920*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^6*c
^4*e*g^3*h^10 - 22780*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^3*b^7*f*g^3*h
^10 + 310320*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^4*b^5*c*f*g^3*h^10 - 1
972800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^5*b^3*c^2*f*g^3*h^10 - 30092
80*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^6*b*c^3*f*g^3*h^10 + 3570*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b^8*d*g^2*h^11 + 58240*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^3*a^3*b^6*c*d*g^2*h^11 + 646200*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^3*a^4*b^4*c^2*d*g^2*h^11 + 275520*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^3*a^5*b^2*c^3*d*g^2*h^11 - 149120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))^3*a^6*c^4*d*g^2*h^11 - 7820*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^3*b^
7*e*g^2*h^11 - 20460*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^4*b^5*c*e*g^2*
h^11 - 597600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^5*b^3*c^2*e*g^2*h^11
+ 43840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^6*b*c^3*e*g^2*h^11 + 20995*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^4*b^6*f*g^2*h^11 - 115500*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^3*a^5*b^4*c*f*g^2*h^11 + 914640*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^3*a^6*b^2*c^2*f*g^2*h^11 + 675520*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))^3*a^7*c^3*f*g^2*h^11 - 2380*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))^3*a^3*b^7*d*g*h^12 - 73800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^4*b^
5*c*d*g*h^12 - 309120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^5*b^3*c^2*d*g
*h^12 + 30080*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^6*b*c^3*d*g*h^12 + 45
05*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^4*b^6*e*g*h^12 + 58500*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^3*a^5*b^4*c*e*g*h^12 + 191280*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^3*a^6*b^2*c^2*e*g*h^12 - 105280*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))^3*a^7*c^3*e*g*h^12 - 10200*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3
*a^5*b^5*f*g*h^12 - 10560*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^6*b^3*c*f
```

```
*g*h^12 - 163200*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^7*b*c^2*f*g*h^12 +
 595*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^4*b^6*d*h^13 + 25620*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^3*a^5*b^4*c*d*h^13 + 58320*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))^3*a^6*b^2*c^2*d*h^13 + 15040*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))^3*a^7*c^3*d*h^13 - 1020*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^5*b^5*
e*h^13 - 22560*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^6*b^3*c*e*h^13 - 163
20*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^7*b*c^2*e*h^13 + 2040*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))^3*a^6*b^4*f*h^13 + 14400*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^3*a^7*b^2*c*f*h^13 - 13440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^
3*a^8*c^2*f*h^13 + 76800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^4*c^(13/2)
*f*g^13 + 15360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^4*c^(13/2)*e*g^12*h
 - 276480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^5*c^(11/2)*f*g^12*h - 307
200*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b^3*c^(13/2)*f*g^12*h + 7680*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^4*c^(13/2)*d*g^11*h^2 - 48384*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^2*b^5*c^(11/2)*e*g^11*h^2 - 61440*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))^2*a*b^3*c^(13/2)*e*g^11*h^2 + 343104*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^2*b^6*c^(9/2)*f*g^11*h^2 + 1416960*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^2*a*b^4*c^(11/2)*f*g^11*h^2 + 460800*(sqrt(c)*x - sqrt(c*
x^2 + b*x + a))^2*a^2*b^2*c^(13/2)*f*g^11*h^2 - 18432*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))^2*b^5*c^(11/2)*d*g^10*h^3 - 30720*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))^2*a*b^3*c^(13/2)*d*g^10*h^3 + 43968*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))^2*b^6*c^(9/2)*e*g^10*h^3 + 264960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)^2*a*b^4*c^(11/2)*e*g^10*h^3 + 92160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2
*a^2*b^2*c^(13/2)*e*g^10*h^3 - 143808*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2
*b^7*c^(7/2)*f*g^10*h^3 - 2181888*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b
^5*c^(9/2)*f*g^10*h^3 - 2903040*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*b
^3*c^(11/2)*f*g^10*h^3 - 307200*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*b
*c^(13/2)*f*g^10*h^3 + 3840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^6*c^(9/
2)*d*g^9*h^4 + 126720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b^4*c^(11/2)*
d*g^9*h^4 + 46080*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*b^2*c^(13/2)*d*
g^9*h^4 + 96*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^7*c^(7/2)*e*g^9*h^4 -
336384*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b^5*c^(9/2)*e*g^9*h^4 - 5760
00*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*b^3*c^(11/2)*e*g^9*h^4 - 61440
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*b*c^(13/2)*e*g^9*h^4 - 13896*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^8*c^(5/2)*f*g^9*h^4 + 1150944*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^2*a*b^6*c^(7/2)*f*g^9*h^4 + 5777280*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))^2*a^2*b^4*c^(9/2)*f*g^9*h^4 + 2972160*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))^2*a^3*b^2*c^(11/2)*f*g^9*h^4 + 76800*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^2*a^4*c^(13/2)*f*g^9*h^4 + 11136*(sqrt(c)*x - sqrt(c*
x^2 + b*x + a))^2*b^7*c^(7/2)*d*g^8*h^5 - 88704*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^2*a*b^5*c^(9/2)*d*g^8*h^5 - 322560*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))^2*a^2*b^3*c^(11/2)*d*g^8*h^5 - 30720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))^2*a^3*b*c^(13/2)*d*g^8*h^5 - 8280*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*
b^8*c^(5/2)*e*g^8*h^5 + 43680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b^6*c
^(7/2)*e*g^8*h^5 + 1065600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*b^4*c^
```

```
(9/2)*e*g^8*h^5 + 622080*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*b^2*c^(1
1/2)*e*g^8*h^5 + 15360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^4*c^(13/2)*e
*g^8*h^5 + 6720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^9*c^(3/2)*f*g^8*h^5
 + 83664*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b^7*c^(5/2)*f*g^8*h^5 - 39
63456*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*b^5*c^(7/2)*f*g^8*h^5 - 815
2320*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*b^3*c^(9/2)*f*g^8*h^5 - 1520
640*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^4*b*c^(11/2)*f*g^8*h^5 - 12384*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^8*c^(5/2)*d*g^7*h^6 + 54816*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^2*a*b^6*c^(7/2)*d*g^7*h^6 + 89280*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))^2*a^2*b^4*c^(9/2)*d*g^7*h^6 + 476160*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^2*a^3*b^2*c^(11/2)*d*g^7*h^6 + 15360*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^2*a^4*c^(13/2)*d*g^7*h^6 + 1740*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^2*b^9*c^(3/2)*e*g^7*h^6 + 33264*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^2*a*b^7*c^(5/2)*e*g^7*h^6 - 197856*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)^2*a^2*b^5*c^(7/2)*e*g^7*h^6 - 1835520*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
^2*a^3*b^3*c^(9/2)*e*g^7*h^6 - 360960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2
*a^4*b*c^(11/2)*e*g^7*h^6 + 1785*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^10
*sqrt(c)*f*g^7*h^6 - 58500*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b^8*c^(3
/2)*f*g^7*h^6 - 218520*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*b^6*c^(5/2
)*f*g^7*h^6 + 7686240*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*b^4*c^(7/2)
*f*g^7*h^6 + 6480000*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^4*b^2*c^(9/2)*
f*g^7*h^6 + 314880*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^5*c^(11/2)*f*g^7
*h^6 + 600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^9*c^(3/2)*d*g^6*h^7 + 26
928*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b^7*c^(5/2)*d*g^6*h^7 - 185472*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*b^5*c^(7/2)*d*g^6*h^7 + 357120*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*b^3*c^(9/2)*d*g^6*h^7 - 337920*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^2*a^4*b*c^(11/2)*d*g^6*h^7 + 1275*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^2*b^10*sqrt(c)*e*g^6*h^7 - 16740*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))^2*a*b^8*c^(3/2)*e*g^6*h^7 - 23688*(sqrt(c)*x - sqrt(c*
x^2 + b*x + a))^2*a^2*b^6*c^(5/2)*e*g^6*h^7 + 242400*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^2*a^3*b^4*c^(7/2)*e*g^6*h^7 + 1875840*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^2*a^4*b^2*c^(9/2)*e*g^6*h^7 + 81408*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))^2*a^5*c^(11/2)*e*g^6*h^7 - 13260*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))^2*a*b^9*sqrt(c)*f*g^6*h^7 + 222480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^
2*a^2*b^7*c^(3/2)*f*g^6*h^7 + 346752*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*
a^3*b^5*c^(5/2)*f*g^6*h^7 - 9173760*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a
^4*b^3*c^(7/2)*f*g^6*h^7 - 2761728*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^
5*b*c^(9/2)*f*g^6*h^7 + 1785*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^10*sqr
t(c)*d*g^5*h^8 - 10980*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b^8*c^(3/2)*
d*g^5*h^8 - 14184*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*b^6*c^(5/2)*d*g
^5*h^8 - 26400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*b^4*c^(7/2)*d*g^5*
h^8 - 996480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^4*b^2*c^(9/2)*d*g^5*h^
8 + 105984*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^5*c^(11/2)*d*g^5*h^8 - 8
160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b^9*sqrt(c)*e*g^5*h^8 + 67320*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*b^7*c^(3/2)*e*g^5*h^8 - 62496*(sqr
```

```
t(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*b^5*c^(5/2)*e*g^5*h^8 + 205440*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^2*a^4*b^3*c^(7/2)*e*g^5*h^8 - 1046016*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^2*a^5*b*c^(9/2)*e*g^5*h^8 + 41310*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))^2*a^2*b^8*sqrt(c)*f*g^5*h^8 - 481560*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^2*a^3*b^6*c^(3/2)*f*g^5*h^8 - 433440*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^2*a^4*b^4*c^(5/2)*f*g^5*h^8 + 6754176*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^2*a^5*b^2*c^(7/2)*f*g^5*h^8 + 500736*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))^2*a^6*c^(9/2)*f*g^5*h^8 - 7140*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^2*a*b^9*sqrt(c)*d*g^4*h^9 + 44640*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))^2*a^2*b^7*c^(3/2)*d*g^4*h^9 + 95520*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
^2*a^3*b^5*c^(5/2)*d*g^4*h^9 + 518400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2
*a^4*b^3*c^(7/2)*d*g^4*h^9 + 898560*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a
^5*b*c^(9/2)*d*g^4*h^9 + 19890*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*b^
8*sqrt(c)*e*g^4*h^9 - 143400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*b^6*
c^(3/2)*e*g^4*h^9 + 42720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^4*b^4*c^(
5/2)*e*g^4*h^9 - 749952*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^5*b^2*c^(7/
2)*e*g^4*h^9 + 227328*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^6*c^(9/2)*e*g
^4*h^9 - 68340*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*b^7*sqrt(c)*f*g^4*
h^9 + 642960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^4*b^5*c^(3/2)*f*g^4*h^
9 + 581952*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^5*b^3*c^(5/2)*f*g^4*h^9
- 2834688*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^6*b*c^(7/2)*f*g^4*h^9 + 1
0710*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*b^8*sqrt(c)*d*g^3*h^10 - 104
760*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*b^6*c^(3/2)*d*g^3*h^10 - 2650
80*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^4*b^4*c^(5/2)*d*g^3*h^10 - 55699
2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^5*b^2*c^(7/2)*d*g^3*h^10 - 258432
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^6*c^(9/2)*d*g^3*h^10 - 23460*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*b^7*sqrt(c)*e*g^3*h^10 + 198300*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^2*a^4*b^5*c^(3/2)*e*g^3*h^10 + 123360*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^2*a^5*b^3*c^(5/2)*e*g^3*h^10 + 661440*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^2*a^6*b*c^(7/2)*e*g^3*h^10 + 62985*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))^2*a^4*b^6*sqrt(c)*f*g^3*h^10 - 558660*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^2*a^5*b^4*c^(3/2)*f*g^3*h^10 - 670608*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^2*a^6*b^2*c^(5/2)*f*g^3*h^10 + 524352*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))^2*a^7*c^(7/2)*f*g^3*h^10 - 7140*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^2*a^3*b^7*sqrt(c)*d*g^2*h^11 + 147240*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^2*a^4*b^5*c^(3/2)*d*g^2*h^11 + 252288*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^2*a^5*b^3*c^(5/2)*d*g^2*h^11 + 163968*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))^2*a^6*b*c^(7/2)*d*g^2*h^11 + 13515*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))^2*a^4*b^6*sqrt(c)*e*g^2*h^11 - 193140*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^2*a^5*b^4*c^(3/2)*e*g^2*h^11 - 145008*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))^2*a^6*b^2*c^(5/2)*e*g^2*h^11 - 205248*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))^2*a^7*c^(7/2)*e*g^2*h^11 - 30600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*
a^5*b^5*sqrt(c)*f*g^2*h^11 + 336960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a
^6*b^3*c^(3/2)*f*g^2*h^11 + 432000*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^
7*b*c^(5/2)*f*g^2*h^11 + 1785*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^4*b^6
```

*sqrt(c)*d*g*h^12 - 107460*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^5*b^4*c^
(3/2)*d*g*h^12 - 95376*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^6*b^2*c^(5/2
)*d*g*h^12 + 20544*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^7*c^(7/2)*d*g*h^
12 - 3060*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^5*b^5*sqrt(c)*e*g*h^12 +
116640*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^6*b^3*c^(3/2)*e*g*h^12 + 370
56*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^7*b*c^(5/2)*e*g*h^12 + 6120*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^2*a^6*b^4*sqrt(c)*f*g*h^12 - 141120*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^2*a^7*b^2*c^(3/2)*f*g*h^12 - 107904*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))^2*a^8*c^(5/2)*f*g*h^12 + 30720*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^2*a^6*b^3*c^(3/2)*d*h^13 + 12288*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^2*a^7*b*c^(5/2)*d*h^13 - 30720*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))^2*a^7*b^2*c^(3/2)*e*h^13 + 3072*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a
^8*c^(5/2)*e*h^13 + 30720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^8*b*c^(3/
2)*f*h^13 + 15360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^5*c^6*f*g^13 + 3072
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^5*c^6*e*g^12*h - 56832*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))*b^6*c^5*f*g^12*h - 76800*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))*a*b^4*c^6*f*g^12*h + 1536*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^5*
c^6*d*g^11*h^2 - 9984*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^6*c^5*e*g^11*h^
2 - 15360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^4*c^6*e*g^11*h^2 + 74304*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^7*c^4*f*g^11*h^2 + 347904*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))*a*b^5*c^5*f*g^11*h^2 + 153600*(sqrt(c)*x - sqrt(c*
x^2 + b*x + a))*a^2*b^3*c^6*f*g^11*h^2 - 3840*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))*b^6*c^5*d*g^10*h^3 - 7680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^4*
c^6*d*g^10*h^3 + 9792*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^7*c^4*e*g^10*h^
3 + 64512*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^5*c^5*e*g^10*h^3 + 30720*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^3*c^6*e*g^10*h^3 - 36192*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))*b^8*c^3*f*g^10*h^3 - 545472*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))*a*b^6*c^4*f*g^10*h^3 - 887040*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))*a^2*b^4*c^5*f*g^10*h^3 - 153600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
*a^3*b^2*c^6*f*g^10*h^3 + 1152*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^7*c^4*
d*g^9*h^4 + 29952*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^5*c^5*d*g^9*h^4 +
 15360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^3*c^6*d*g^9*h^4 - 960*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))*b^8*c^3*e*g^9*h^4 - 83520*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))*a*b^6*c^4*e*g^9*h^4 - 172800*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))*a^2*b^4*c^5*e*g^9*h^4 - 30720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*
a^3*b^2*c^6*e*g^9*h^4 + 840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^9*c^2*f*g
^9*h^4 + 320928*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^7*c^3*f*g^9*h^4 + 1
715328*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^5*c^4*f*g^9*h^4 + 1205760*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*b^3*c^5*f*g^9*h^4 + 76800*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))*a^4*b*c^6*f*g^9*h^4 + 2112*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))*b^8*c^3*d*g^8*h^5 - 21888*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)*a*b^6*c^4*d*g^8*h^5 - 92160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^4*c
^5*d*g^8*h^5 - 15360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*b^2*c^6*d*g^8*
h^5 - 1560*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^9*c^2*e*g^8*h^5 + 18048*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^7*c^3*e*g^8*h^5 + 304128*(sqrt(c)*x -

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 sqrt(c*x^2 + b*x + a))*a^2*b^5*c^4*e*g^8*h^5 + 245760*(sqrt(c)*x - sqrt(c*
x^2 + b*x + a))*a^3*b^3*c^5*e*g^8*h^5 + 15360*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))*a^4*b*c^6*e*g^8*h^5 + 1260*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^10*
c*f*g^8*h^5 - 16200*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^8*c^2*f*g^8*h^5
 - 1248960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^6*c^3*f*g^8*h^5 - 2995
200*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*b^4*c^4*f*g^8*h^5 - 921600*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))*a^4*b^2*c^5*f*g^8*h^5 - 15360*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))*a^5*c^6*f*g^8*h^5 - 3120*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))*b^9*c^2*d*g^7*h^6 + 14496*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^
7*c^3*d*g^7*h^6 + 40896*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^5*c^4*d*g
^7*h^6 + 168960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*b^3*c^5*d*g^7*h^6 +
 15360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^4*b*c^6*d*g^7*h^6 + 720*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))*b^10*c*e*g^7*h^6 + 4680*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))*a*b^8*c^2*e*g^7*h^6 - 83616*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))*a^2*b^6*c^3*e*g^7*h^6 - 622080*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*
b^4*c^4*e*g^7*h^6 - 207360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^4*b^2*c^5*
e*g^7*h^6 - 6144*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^5*c^6*e*g^7*h^6 + 10
5*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^11*f*g^7*h^6 - 10260*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))*a*b^9*c*f*g^7*h^6 + 93960*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))*a^2*b^7*c^2*f*g^7*h^6 + 2802144*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)*a^3*b^5*c^3*f*g^7*h^6 + 3139200*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^4*b
^3*c^4*f*g^7*h^6 + 382464*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^5*b*c^5*f*g
^7*h^6 + 900*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^10*c*d*g^6*h^7 + 5040*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^8*c^2*d*g^6*h^7 - 64128*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))*a^2*b^6*c^3*d*g^6*h^7 + 37440*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))*a^3*b^4*c^4*d*g^6*h^7 - 192000*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))*a^4*b^2*c^5*d*g^6*h^7 - 3072*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^5*
c^6*d*g^6*h^7 + 75*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^11*e*g^6*h^7 - 504
0*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^9*c*e*g^6*h^7 + 9720*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))*a^2*b^7*c^2*e*g^6*h^7 + 155328*(sqrt(c)*x - sqrt(c*
x^2 + b*x + a))*a^3*b^5*c^3*e*g^6*h^7 + 812160*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))*a^4*b^3*c^4*e*g^6*h^7 + 104448*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*
a^5*b*c^5*e*g^6*h^7 - 885*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^10*f*g^6*
h^7 + 36540*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^8*c*f*g^6*h^7 - 27228
0*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*b^6*c^2*f*g^6*h^7 - 3991200*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))*a^4*b^4*c^3*f*g^6*h^7 - 2003328*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))*a^5*b^2*c^4*f*g^6*h^7 - 66048*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))*a^6*c^5*f*g^6*h^7 + 105*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*
b^11*d*g^5*h^8 - 4140*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^9*c*d*g^5*h^8
 + 11160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^7*c^2*d*g^5*h^8 + 66720*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*b^5*c^3*d*g^5*h^8 - 236160*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))*a^4*b^3*c^4*d*g^5*h^8 + 115200*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))*a^5*b*c^5*d*g^5*h^8 - 555*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))*a*b^10*e*g^5*h^8 + 14580*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^8
*c*e*g^5*h^8 - 56040*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*b^6*c^2*e*g^5*
```

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h^8 - 112800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^4*b^4*c^3*e*g^5*h^8 - 66
4704*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^5*b^2*c^4*e*g^5*h^8 - 29184*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))*a^6*c^5*e*g^5*h^8 + 3210*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))*a^2*b^9*f*g^5*h^8 - 74520*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))*a^3*b^7*c*f*g^5*h^8 + 456480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^4*b
^5*c^2*f*g^5*h^8 + 3748224*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^5*b^3*c^3*
f*g^5*h^8 + 728064*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^6*b*c^4*f*g^5*h^8
 - 525*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^10*d*g^4*h^9 + 8460*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))*a^2*b^8*c*d*g^4*h^9 - 33960*(sqrt(c)*x - sqrt(c*
x^2 + b*x + a))*a^3*b^6*c^2*d*g^4*h^9 + 66240*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))*a^4*b^4*c^3*d*g^4*h^9 + 337536*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a
^5*b^2*c^4*d*g^4*h^9 - 21504*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^6*c^5*d*
g^4*h^9 + 1650*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^9*e*g^4*h^9 - 2376
0*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*b^7*c*e*g^4*h^9 + 90000*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))*a^4*b^5*c^2*e*g^4*h^9 - 63360*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))*a^5*b^3*c^3*e*g^4*h^9 + 311040*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))*a^6*b*c^4*e*g^4*h^9 - 6450*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^
3*b^8*f*g^4*h^9 + 96660*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^4*b^6*c*f*g^4
*h^9 - 467280*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^5*b^4*c^2*f*g^4*h^9 - 2
285376*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^6*b^2*c^3*f*g^4*h^9 - 112896*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^7*c^4*f*g^4*h^9 + 1050*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))*a^2*b^9*d*g^3*h^10 - 10440*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))*a^3*b^7*c*d*g^3*h^10 + 18120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a
^4*b^5*c^2*d*g^3*h^10 - 195264*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^5*b^3*
c^3*d*g^3*h^10 - 215424*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^6*b*c^4*d*g^3
*h^10 - 2550*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*b^8*e*g^3*h^10 + 24840
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^4*b^6*c*e*g^3*h^10 - 55920*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))*a^5*b^4*c^2*e*g^3*h^10 + 190464*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))*a^6*b^2*c^3*e*g^3*h^10 - 66816*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))*a^7*c^4*e*g^3*h^10 + 7725*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a
^4*b^7*f*g^3*h^10 - 83700*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^5*b^5*c*f*g
^3*h^10 + 283440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^6*b^3*c^2*f*g^3*h^10
 + 816960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^7*b*c^3*f*g^3*h^10 - 1050*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*b^8*d*g^2*h^11 + 8460*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))*a^4*b^6*c*d*g^2*h^11 + 27720*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))*a^5*b^4*c^2*d*g^2*h^11 + 144768*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))*a^6*b^2*c^3*d*g^2*h^11 + 56448*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^7
*c^4*d*g^2*h^11 + 2175*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^4*b^7*e*g^2*h^
11 - 17280*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^5*b^5*c*e*g^2*h^11 - 12240
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^6*b^3*c^2*e*g^2*h^11 - 133248*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))*a^7*b*c^3*e*g^2*h^11 - 5505*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))*a^5*b^6*f*g^2*h^11 + 48420*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))*a^6*b^4*c*f*g^2*h^11 - 77040*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^7
*b^2*c^2*f*g^2*h^11 - 126528*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^8*c^3*f*
g^2*h^11 + 525*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^4*b^7*d*g*h^12 - 4140*
```

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(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^5*b^5*c*d*g*h^12 - 38160*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))*a^6*b^3*c^2*d*g*h^12 - 35904*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))*a^7*b*c^3*d*g*h^12 - 975*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*
a^5*b^6*e*g*h^12 + 7380*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^6*b^4*c*e*g*h
^12 + 33840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^7*b^2*c^2*e*g*h^12 + 3014
4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^8*c^3*e*g*h^12 + 2160*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))*a^6*b^5*f*g*h^12 - 17280*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))*a^7*b^3*c*f*g*h^12 - 11520*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^8
*b*c^2*f*g*h^12 - 105*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^5*b^6*d*h^13 +
900*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^6*b^4*c*d*h^13 + 13200*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))*a^7*b^2*c^2*d*h^13 + 960*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))*a^8*c^3*d*h^13 + 180*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^6*b^
5*e*h^13 - 1440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^7*b^3*c*e*h^13 - 1248
0*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^8*b*c^2*e*h^13 - 360*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))*a^7*b^4*f*h^13 + 2880*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))*a^8*b^2*c*f*h^13 + 9600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^9*c^2*f*
h^13 + 1280*b^6*c^(11/2)*f*g^13 + 256*b^6*c^(11/2)*e*g^12*h - 4736*b^7*c^(9
/2)*f*g^12*h - 7680*a*b^5*c^(11/2)*f*g^12*h + 128*b^6*c^(11/2)*d*g^11*h^2 -
 832*b^7*c^(9/2)*e*g^11*h^2 - 1536*a*b^5*c^(11/2)*e*g^11*h^2 + 6192*b^8*c^(
7/2)*f*g^11*h^2 + 33728*a*b^6*c^(9/2)*f*g^11*h^2 + 19200*a^2*b^4*c^(11/2)*f
*g^11*h^2 - 320*b^7*c^(9/2)*d*g^10*h^3 - 768*a*b^5*c^(11/2)*d*g^10*h^3 + 81
6*b^8*c^(7/2)*e*g^10*h^3 + 6208*a*b^6*c^(9/2)*e*g^10*h^3 + 3840*a^2*b^4*c^(
11/2)*e*g^10*h^3 - 3016*b^9*c^(5/2)*f*g^10*h^3 - 51648*a*b^7*c^(7/2)*f*g^10
*h^3 - 102912*a^2*b^5*c^(9/2)*f*g^10*h^3 - 25600*a^3*b^3*c^(11/2)*f*g^10*h^
3 + 96*b^8*c^(7/2)*d*g^9*h^4 + 2816*a*b^6*c^(9/2)*d*g^9*h^4 + 1920*a^2*b^4*
c^(11/2)*d*g^9*h^4 - 80*b^9*c^(5/2)*e*g^9*h^4 - 7776*a*b^7*c^(7/2)*e*g^9*h^
4 - 19776*a^2*b^5*c^(9/2)*e*g^9*h^4 - 5120*a^3*b^3*c^(11/2)*e*g^9*h^4 + 70*
b^10*c^(3/2)*f*g^9*h^4 + 29760*a*b^8*c^(5/2)*f*g^9*h^4 + 188400*a^2*b^6*c^(
7/2)*f*g^9*h^4 + 174400*a^3*b^4*c^(9/2)*f*g^9*h^4 + 19200*a^4*b^2*c^(11/2)*
f*g^9*h^4 + 176*b^9*c^(5/2)*d*g^8*h^5 - 1920*a*b^7*c^(7/2)*d*g^8*h^5 - 1017
6*a^2*b^5*c^(9/2)*d*g^8*h^5 - 2560*a^3*b^3*c^(11/2)*d*g^8*h^5 - 130*b^10*c^
(3/2)*e*g^8*h^5 + 1584*a*b^8*c^(5/2)*e*g^8*h^5 + 32304*a^2*b^6*c^(7/2)*e*g^
8*h^5 + 34880*a^3*b^4*c^(9/2)*e*g^8*h^5 + 3840*a^4*b^2*c^(11/2)*e*g^8*h^5 +
 105*b^11*sqrt(c)*f*g^8*h^5 - 1420*a*b^9*c^(3/2)*f*g^8*h^5 - 130824*a^2*b^7
*c^(5/2)*f*g^8*h^5 - 392544*a^3*b^5*c^(7/2)*f*g^8*h^5 - 177280*a^4*b^3*c^(9
/2)*f*g^8*h^5 - 7680*a^5*b*c^(11/2)*f*g^8*h^5 - 290*b^10*c^(3/2)*d*g^7*h^6
+ 1272*a*b^8*c^(5/2)*d*g^7*h^6 + 4752*a^2*b^6*c^(7/2)*d*g^7*h^6 + 21760*a^3
*b^4*c^(9/2)*d*g^7*h^6 + 3840*a^4*b^2*c^(11/2)*d*g^7*h^6 + 75*b^11*sqrt(c)*
e*g^7*h^6 + 400*a*b^9*c^(3/2)*e*g^7*h^6 - 8232*a^2*b^7*c^(5/2)*e*g^7*h^6 -
77184*a^3*b^5*c^(7/2)*e*g^7*h^6 - 37760*a^4*b^3*c^(9/2)*e*g^7*h^6 - 3072*a^
5*b*c^(11/2)*e*g^7*h^6 - 885*a*b^10*sqrt(c)*f*g^7*h^6 + 9000*a^2*b^8*c^(3/2
)*f*g^7*h^6 + 337672*a^3*b^6*c^(5/2)*f*g^7*h^6 + 511200*a^4*b^4*c^(7/2)*f*g
^7*h^6 + 108672*a^5*b^2*c^(9/2)*f*g^7*h^6 + 2560*a^6*c^(11/2)*f*g^7*h^6 + 1
05*b^11*sqrt(c)*d*g^6*h^7 + 620*a*b^9*c^(3/2)*d*g^6*h^7 - 7512*a^2*b^7*c^(5
/2)*d*g^6*h^7 + 2592*a^3*b^5*c^(7/2)*d*g^6*h^7 - 35200*a^4*b^3*c^(9/2)*d*g^
```

```
6*h^7 - 1536*a^5*b*c^(11/2)*d*g^6*h^7 - 555*a*b^10*sqrt(c)*e*g^6*h^7 + 1320
*a^2*b^8*c^(3/2)*e*g^6*h^7 + 17912*a^3*b^6*c^(5/2)*e*g^6*h^7 + 119520*a^4*b
^4*c^(7/2)*e*g^6*h^7 + 31104*a^5*b^2*c^(9/2)*e*g^6*h^7 + 512*a^6*c^(11/2)*e
*g^6*h^7 + 3210*a^2*b^9*sqrt(c)*f*g^6*h^7 - 28960*a^3*b^7*c^(3/2)*f*g^6*h^7
 - 566592*a^4*b^5*c^(5/2)*f*g^6*h^7 - 428544*a^5*b^3*c^(7/2)*f*g^6*h^7 - 42
496*a^6*b*c^(9/2)*f*g^6*h^7 - 525*a*b^10*sqrt(c)*d*g^5*h^8 + 1320*a^2*b^8*c
^(3/2)*d*g^5*h^8 + 11704*a^3*b^6*c^(5/2)*d*g^5*h^8 - 22320*a^4*b^4*c^(7/2)*
d*g^5*h^8 + 30720*a^5*b^2*c^(9/2)*d*g^5*h^8 + 256*a^6*c^(11/2)*d*g^5*h^8 +
1650*a^2*b^9*sqrt(c)*e*g^5*h^8 - 8120*a^3*b^7*c^(3/2)*e*g^5*h^8 - 15384*a^4
*b^5*c^(5/2)*e*g^5*h^8 - 130752*a^5*b^3*c^(7/2)*e*g^5*h^8 - 16256*a^6*b*c^(
9/2)*e*g^5*h^8 - 6450*a^3*b^8*sqrt(c)*f*g^5*h^8 + 54670*a^4*b^6*c^(3/2)*f*g
^5*h^8 + 646152*a^5*b^4*c^(5/2)*f*g^5*h^8 + 235296*a^6*b^2*c^(7/2)*f*g^5*h^
8 + 10624*a^7*c^(9/2)*f*g^5*h^8 + 1050*a^2*b^9*sqrt(c)*d*g^4*h^9 - 5680*a^3
*b^7*c^(3/2)*d*g^4*h^9 - 3240*a^4*b^5*c^(5/2)*d*g^4*h^9 + 42048*a^5*b^3*c^(
7/2)*d*g^4*h^9 - 11392*a^6*b*c^(9/2)*d*g^4*h^9 - 2550*a^3*b^8*sqrt(c)*e*g^4
*h^9 + 15830*a^4*b^6*c^(3/2)*e*g^4*h^9 - 3960*a^5*b^4*c^(5/2)*e*g^4*h^9 + 9
6672*a^6*b^2*c^(7/2)*e*g^4*h^9 + 2432*a^7*c^(9/2)*e*g^4*h^9 + 7725*a^4*b^7*
sqrt(c)*f*g^4*h^9 - 63300*a^5*b^5*c^(3/2)*f*g^4*h^9 - 509520*a^6*b^3*c^(5/2
)*f*g^4*h^9 - 85440*a^7*b*c^(7/2)*f*g^4*h^9 - 1050*a^3*b^8*sqrt(c)*d*g^3*h^
10 + 7270*a^4*b^6*c^(3/2)*d*g^3*h^10 - 11232*a^5*b^4*c^(5/2)*d*g^3*h^10 - 4
5600*a^6*b^2*c^(7/2)*d*g^3*h^10 + 1792*a^7*c^(9/2)*d*g^3*h^10 + 2175*a^4*b^
7*sqrt(c)*e*g^3*h^10 - 15240*a^5*b^5*c^(3/2)*e*g^3*h^10 + 20592*a^6*b^3*c^(
5/2)*e*g^3*h^10 - 39168*a^7*b*c^(7/2)*e*g^3*h^10 - 5505*a^5*b^6*sqrt(c)*f*g
^3*h^10 + 44340*a^6*b^4*c^(3/2)*f*g^3*h^10 + 275088*a^7*b^2*c^(5/2)*f*g^3*h
^10 + 17088*a^8*c^(7/2)*f*g^3*h^10 + 525*a^4*b^7*sqrt(c)*d*g^2*h^11 - 4140*
a^5*b^5*c^(3/2)*d*g^2*h^11 + 17136*a^6*b^3*c^(5/2)*d*g^2*h^11 + 25536*a^7*b
*c^(7/2)*d*g^2*h^11 - 975*a^5*b^6*sqrt(c)*e*g^2*h^11 + 7380*a^6*b^4*c^(3/2)
*e*g^2*h^11 - 21456*a^7*b^2*c^(5/2)*e*g^2*h^11 + 5568*a^8*c^(7/2)*e*g^2*h^1
1 + 2160*a^6*b^5*sqrt(c)*f*g^2*h^11 - 17280*a^7*b^3*c^(3/2)*f*g^2*h^11 - 94
464*a^8*b*c^(5/2)*f*g^2*h^11 - 105*a^5*b^6*sqrt(c)*d*g*h^12 + 900*a^6*b^4*c
^(3/2)*d*g*h^12 - 11376*a^7*b^2*c^(5/2)*d*g*h^12 - 5184*a^8*c^(7/2)*d*g*h^1
2 + 180*a^6*b^5*sqrt(c)*e*g*h^12 - 1440*a^7*b^3*c^(3/2)*e*g*h^12 + 12096*a^
8*b*c^(5/2)*e*g*h^12 - 360*a^7*b^4*sqrt(c)*f*g*h^12 + 2880*a^8*b^2*c^(3/2)*
f*g*h^12 + 15744*a^9*c^(5/2)*f*g*h^12 + 3072*a^8*b*c^(5/2)*d*h^13 - 3072*a^
9*c^(5/2)*e*h^13)/((c^4*g^8*h^6 - 4*b*c^3*g^7*h^7 + 6*b^2*c^2*g^6*h^8 + 4*a
*c^3*g^6*h^8 - 4*b^3*c*g^5*h^9 - 12*a*b*c^2*g^5*h^9 + b^4*g^4*h^10 + 12*a*b
^2*c*g^4*h^10 + 6*a^2*c^2*g^4*h^10 - 4*a*b^3*g^3*h^11 - 12*a^2*b*c*g^3*h^11
 + 6*a^2*b^2*g^2*h^12 + 4*a^3*c*g^2*h^12 - 4*a^3*b*g*h^13 + a^4*h^14)*((sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^2*h + 2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
*sqrt(c)*g + b*g - a*h)^6)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^7}\,dx=\int \frac{{\left(c\,x^2+b\,x+a\right)}^{3/2}\,\left(f\,x^2+e\,x+d\right)}{{\left(g+h\,x\right)}^7} \,d x$$

```
[In] int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^7,x)
[Out] int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^7, x)
```

3.207 $\int \frac{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^8}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 1062

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^8}\,dx =$$

$$-\frac{(b^2-4ac)\left(48c^3dg^3-8c^2g(3bg(eg+3dh)+a(fg^2-8egh+3dh^2))-bh(24a^2fh^2-2abh(10fg+7eh)+\right.}{}$$

$$+\frac{\left(48c^3dg^3-8c^2g(3bg(eg+3dh)+a(fg^2-8egh+3dh^2))-bh(24a^2fh^2-2abh(10fg+7eh)+b^2(5fg^2+\right.}{}$$

$$-\frac{(fg^2-h(eg-dh))\left(a+bx+cx^2\right)^{5/2}}{7h\left(cg^2-bgh+ah^2\right)(g+hx)^7}$$

$$+\frac{(2cg(5fg^2+h(2eg-9dh))+h(14ah(2fg-eh)-b(19fg^2-5egh-9dh^2)))\left(a+bx+cx^2\right)^{5/2}}{84h\left(cg^2-bgh+ah^2\right)^2(g+hx)^6}$$

$$+\frac{\left(4c^2g^2(5fg^2+h(2eg-51dh))-7h^2(24a^2fh^2-2abh(10fg+7eh)+b^2(5fg^2+5egh+9dh^2))-2ch(3bg\right.}{840h\left(cg^2-bgh+ah^2\right)^3(g+hx\right.}$$

$$+\frac{(b^2-4ac)^2\left(48c^3dg^3-8c^2g(3bg(eg+3dh)+a(fg^2-8egh+3dh^2))-bh(24a^2fh^2-2abh(10fg+7eh)+\right.}{}$$

```
[Out] 1/384*(48*c^3*d*g^3-8*c^2*g*(3*b*g*(3*d*h+e*g)+a*(3*d*h^2-8*e*g*h+f*g^2))-b
*h*(24*a^2*f*h^2-2*a*b*h*(7*e*h+10*f*g)+b^2*(9*d*h^2+5*e*g*h+5*f*g^2))+2*c*
(4*a^2*h^2*(-e*h+8*f*g)-2*a*b*h*(-3*d*h^2+13*e*g*h+13*f*g^2)+b^2*g*(21*d*h^
2+10*e*g*h+7*f*g^2)))*(b*g-2*a*h+(-b*h+2*c*g)*x)*(c*x^2+b*x+a)^(3/2)/(a*h^2
-b*g*h+c*g^2)^4/(h*x+g)^4-1/7*(f*g^2-h*(-d*h+e*g))*(c*x^2+b*x+a)^(5/2)/h/(a
*h^2-b*g*h+c*g^2)/(h*x+g)^7+1/84*(2*c*g*(5*f*g^2+h*(-9*d*h+2*e*g))+h*(14*a*
h*(-e*h+2*f*g)-b*(-9*d*h^2-5*e*g*h+19*f*g^2)))*(c*x^2+b*x+a)^(5/2)/h/(a*h^2
-b*g*h+c*g^2)^2/(h*x+g)^6+1/840*(4*c^2*g^2*(5*f*g^2+h*(-51*d*h+2*e*g))-7*h^
```

```
2*(24*a^2*f*h^2-2*a*b*h*(7*e*h+10*f*g)+b^2*(9*d*h^2+5*e*g*h+5*f*g^2))-2*c*h
*(3*b*g*(-34*d*h^2-15*e*g*h+8*f*g^2)-2*a*h*(12*d*h^2-61*e*g*h+26*f*g^2)))*(
c*x^2+b*x+a)^(5/2)/h/(a*h^2-b*g*h+c*g^2)^3/(h*x+g)^5+1/2048*(-4*a*c+b^2)^2*
(48*c^3*d*g^3-8*c^2*g*(3*b*g*(3*d*h+e*g)+a*(3*d*h^2-8*e*g*h+f*g^2))-b*h*(24
*a^2*f*h^2-2*a*b*h*(7*e*h+10*f*g)+b^2*(9*d*h^2+5*e*g*h+5*f*g^2))+2*c*(4*a^2
*h^2*(-e*h+8*f*g)-2*a*b*h*(-3*d*h^2+13*e*g*h+13*f*g^2)+b^2*g*(21*d*h^2+10*e
*g*h+7*f*g^2)))*arctanh(1/2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^
(1/2)/(c*x^2+b*x+a)^(1/2))/(a*h^2-b*g*h+c*g^2)^(11/2)-1/1024*(-4*a*c+b^2)*(
48*c^3*d*g^3-8*c^2*g*(3*b*g*(3*d*h+e*g)+a*(3*d*h^2-8*e*g*h+f*g^2))-b*h*(24*
a^2*f*h^2-2*a*b*h*(7*e*h+10*f*g)+b^2*(9*d*h^2+5*e*g*h+5*f*g^2))+2*c*(4*a^2*
h^2*(-e*h+8*f*g)-2*a*b*h*(-3*d*h^2+13*e*g*h+13*f*g^2)+b^2*g*(21*d*h^2+10*e*
g*h+7*f*g^2)))*(b*g-2*a*h+(-b*h+2*c*g)*x)*(c*x^2+b*x+a)^(1/2)/(a*h^2-b*g*h+
c*g^2)^5/(h*x+g)^2
```

Rubi [A] (verified)

Time = 1.77 (sec) , antiderivative size = 1062, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1664, 848, 820, 734, 738, 212}

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^8}\,dx = \frac{(4(5fg^4+h(2eg-51dh)g^2)c^2-2h(3bg(8fg^2-15ehg-34dh^2)-}{}$$

$$+\frac{(2c(5fg^3+h(2eg-9dh)g)-h(19bfg^2-bh(5eg+9dh)-14ah(2fg-eh)))(cx^2+bx+a)^{5/2}}{84h(cg^2-bhg+ah^2)^2(g+hx)^6}$$

$$-\frac{(fg^2-h(eg-dh))(cx^2+bx+a)^{5/2}}{7h(cg^2-bhg+ah^2)(g+hx)^7}$$

$$+\frac{(48c^3dg^3-8c^2(afg^2+3b(eg+3dh)g-ah(8eg-3dh))g-bh((5fg^2+h(5eg+9dh))b^2-2ah(10fg+7e}{}$$

$$-\frac{(b^2-4ac)(48c^3dg^3-8c^2(afg^2+3b(eg+3dh)g-ah(8eg-3dh))g-bh((5fg^2+h(5eg+9dh))b^2-2ah}{}$$

$$+\frac{(b^2-4ac)^2(48c^3dg^3-8c^2(afg^2+3b(eg+3dh)g-ah(8eg-3dh))g-bh((5fg^2+h(5eg+9dh))b^2-2ah}{}$$

```
[In] Int[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^8,x]
```

```
[Out] -1/1024*((b^2 - 4*a*c)*(48*c^3*d*g^3 - 8*c^2*g*(a*f*g^2 - a*h*(8*e*g - 3*d*
h) + 3*b*g*(e*g + 3*d*h)) - b*h*(24*a^2*f*h^2 - 2*a*b*h*(10*f*g + 7*e*h) +
b^2*(5*f*g^2 + h*(5*e*g + 9*d*h))) + 2*c*(4*a^2*h^2*(8*f*g - e*h) - 2*a*b*h
*(13*f*g^2 + h*(13*e*g - 3*d*h)) + b^2*(7*f*g^3 + g*h*(10*e*g + 21*d*h))))*
(b*g - 2*a*h + (2*c*g - b*h)*x)*Sqrt[a + b*x + c*x^2])/((c*g^2 - b*g*h + a*
h^2)^5*(g + h*x)^2) + ((48*c^3*d*g^3 - 8*c^2*g*(a*f*g^2 - a*h*(8*e*g - 3*d*
h) + 3*b*g*(e*g + 3*d*h)) - b*h*(24*a^2*f*h^2 - 2*a*b*h*(10*f*g + 7*e*h) +
```

```
b^2*(5*f*g^2 + h*(5*e*g + 9*d*h))) + 2*c*(4*a^2*h^2*(8*f*g - e*h) - 2*a*b*h
*(13*f*g^2 + h*(13*e*g - 3*d*h)) + b^2*(7*f*g^3 + g*h*(10*e*g + 21*d*h))))*
(b*g - 2*a*h + (2*c*g - b*h)*x)*(a + b*x + c*x^2)^(3/2))/(384*(c*g^2 - b*g*
h + a*h^2)^4*(g + h*x)^4) - ((f*g^2 - h*(e*g - d*h))*(a + b*x + c*x^2)^(5/2
))/(7*h*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^7) + ((2*c*(5*f*g^3 + g*h*(2*e*g
- 9*d*h)) - h*(19*b*f*g^2 - b*h*(5*e*g + 9*d*h) - 14*a*h*(2*f*g - e*h)))*(a
 + b*x + c*x^2)^(5/2))/(84*h*(c*g^2 - b*g*h + a*h^2)^2*(g + h*x)^6) + ((4*c
^2*(5*f*g^4 + g^2*h*(2*e*g - 51*d*h)) - 7*h^2*(24*a^2*f*h^2 - 2*a*b*h*(10*f
*g + 7*e*h) + b^2*(5*f*g^2 + 5*e*g*h + 9*d*h^2)) - 2*c*h*(3*b*g*(8*f*g^2 -
15*e*g*h - 34*d*h^2) - 2*a*h*(26*f*g^2 - 61*e*g*h + 12*d*h^2)))*(a + b*x +
c*x^2)^(5/2))/(840*h*(c*g^2 - b*g*h + a*h^2)^3*(g + h*x)^5) + ((b^2 - 4*a*c
)^2*(48*c^3*d*g^3 - 8*c^2*g*(a*f*g^2 - a*h*(8*e*g - 3*d*h) + 3*b*g*(e*g + 3
*d*h)) - b*h*(24*a^2*f*h^2 - 2*a*b*h*(10*f*g + 7*e*h) + b^2*(5*f*g^2 + h*(5
*e*g + 9*d*h))) + 2*c*(4*a^2*h^2*(8*f*g - e*h) - 2*a*b*h*(13*f*g^2 + h*(13*
e*g - 3*d*h)) + b^2*(7*f*g^3 + g*h*(10*e*g + 21*d*h))))*ArcTanh[(b*g - 2*a*
h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[a + b*x + c*x^2])]
)/(2048*(c*g^2 - b*g*h + a*h^2)^(11/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), Int[(d + e*x)^(m + 2)*(a + b*x +
c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
 0] && GtQ[p, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
```

```
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 848

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

integral

$$= -\frac{(fg^2-h(eg-dh))\left(a+bx+cx^2\right)^{5/2}}{7h\left(cg^2-bgh+ah^2\right)(g+hx)^7} - \frac{\int \frac{\left(\frac{1}{2}\left(-14cdg+5beg+14afg-\frac{5bfg^2}{h}+9bdh-14aeh\right)-\left(2ceg-7bfg+\frac{5cfg^2}{h}-2cdh+7afh\right)x\right)\left(a+bx+cx^2\right)^{3/2}}{(g+hx)^7}\,dx}{7\left(cg^2-bgh+ah^2\right)}$$

$$= -\frac{(fg^2-h(eg-dh))\left(a+bx+cx^2\right)^{5/2}}{7h\left(cg^2-bgh+ah^2\right)(g+hx)^7} + \frac{(2c(5fg^3+gh(2eg-9dh))-h(19bfg^2-bh(5eg+9dh)-14ah(2fg-eh)))\left(a+bx+cx^2\right)^{5/2}}{84h\left(cg^2-bgh+ah^2\right)^2(g+hx)^6} + \frac{\int \frac{\left(\frac{1}{4}\left(168c^2dg^2+168a^2fh^2-2bcg\left(40eg-\frac{5fg^2}{h}+93dh\right)-14abh(10fg+7eh)-24ac\left(2fg^2-h(9eg-2dh)\right)+7b^2\left(5fg^2+h(5eg+9dh)\right)\right)+\frac{c(2c(5fg}{}}{(g+hx)^6}}{42\left(cg^2-bgh+ah^2\right)^2}$$

$$\begin{aligned}
&= -\frac{(fg^2-h(eg-dh))\left(a+bx+cx^2\right)^{5/2}}{7h\left(cg^2-bgh+ah^2\right)(g+hx)^7} \\
&\quad + \frac{(2c(5fg^3+gh(2eg-9dh))-h(19bfg^2-bh(5eg+9dh)-14ah(2fg-eh)))\left(a+bx+cx^2\right)^{5/2}}{84h\left(cg^2-bgh+ah^2\right)^2(g+hx)^6} \\
&\quad + \frac{(4c^2(5fg^4+g^2h(2eg-51dh))-7h^2(24a^2fh^2-2abh(10fg+7eh)+b^2(5fg^2+5egh+9dh^2))-2ch(}{840h\left(cg^2-bgh+ah^2\right)^3(g+} \\
&\quad + \frac{(48c^3dg^3-8c^2g(afg^2-ah(8eg-3dh)+3bg(eg+3dh))-bh(24a^2fh^2-2abh(10fg+7eh)+b^2(5fg}{}
\end{aligned}$$

$$\begin{aligned}
&= \frac{(48c^3dg^3-8c^2g(afg^2-ah(8eg-3dh)+3bg(eg+3dh))-bh(24a^2fh^2-2abh(10fg+7eh)+b^2(5fg^2+}{} \\
&\quad - \frac{(fg^2-h(eg-dh))\left(a+bx+cx^2\right)^{5/2}}{7h\left(cg^2-bgh+ah^2\right)(g+hx)^7} \\
&\quad + \frac{(2c(5fg^3+gh(2eg-9dh))-h(19bfg^2-bh(5eg+9dh)-14ah(2fg-eh)))\left(a+bx+cx^2\right)^{5/2}}{84h\left(cg^2-bgh+ah^2\right)^2(g+hx)^6} \\
&\quad + \frac{(4c^2(5fg^4+g^2h(2eg-51dh))-7h^2(24a^2fh^2-2abh(10fg+7eh)+b^2(5fg^2+5egh+9dh^2))-2ch(}{840h\left(cg^2-bgh+ah^2\right)^3(g+} \\
&\quad - \frac{((b^2-4ac)(48c^3dg^3-8c^2g(afg^2-ah(8eg-3dh)+3bg(eg+3dh))-bh(24a^2fh^2-2abh(10fg+7e}{}
\end{aligned}$$

$$\begin{aligned}
&= \\
&\quad - \frac{(b^2-4ac)(48c^3dg^3-8c^2g(afg^2-ah(8eg-3dh)+3bg(eg+3dh))-bh(24a^2fh^2-2abh(10fg+7eh}{} \\
&\quad + \frac{(48c^3dg^3-8c^2g(afg^2-ah(8eg-3dh)+3bg(eg+3dh))-bh(24a^2fh^2-2abh(10fg+7eh)+b^2(5fg}{} \\
&\quad - \frac{(fg^2-h(eg-dh))\left(a+bx+cx^2\right)^{5/2}}{7h\left(cg^2-bgh+ah^2\right)(g+hx)^7} \\
&\quad + \frac{(2c(5fg^3+gh(2eg-9dh))-h(19bfg^2-bh(5eg+9dh)-14ah(2fg-eh)))\left(a+bx+cx^2\right)^{5/2}}{84h\left(cg^2-bgh+ah^2\right)^2(g+hx)^6} \\
&\quad + \frac{(4c^2(5fg^4+g^2h(2eg-51dh))-7h^2(24a^2fh^2-2abh(10fg+7eh)+b^2(5fg^2+5egh+9dh^2))-2ch(}{840h\left(cg^2-bgh+ah^2\right)^3(g+} \\
&\quad + \frac{\Big(\left(b^2-4ac\right)^2(48c^3dg^3-8c^2g(afg^2-ah(8eg-3dh)+3bg(eg+3dh))-bh(24a^2fh^2-2abh(10fg+7}{}
\end{aligned}$$

$$=$$

$$-\frac{\left(b^2-4ac\right)\left(48c^3dg^3-8c^2g(afg^2-ah(8eg-3dh)+3bg(eg+3dh))-bh(24a^2fh^2-2abh(10fg+7eh)\right.}{}$$

$$+\frac{(48c^3dg^3-8c^2g(afg^2-ah(8eg-3dh)+3bg(eg+3dh))-bh(24a^2fh^2-2abh(10fg+7eh)+b^2(5fg^2}{}$$

$$-\frac{\left(fg^2-h(eg-dh)\right)\left(a+bx+cx^2\right)^{5/2}}{7h\left(cg^2-bgh+ah^2\right)(g+hx)^7}$$

$$+\frac{\left(2c(5fg^3+gh(2eg-9dh))-h(19bfg^2-bh(5eg+9dh)-14ah(2fg-eh))\right)\left(a+bx+cx^2\right)^{5/2}}{84h\left(cg^2-bgh+ah^2\right)^2(g+hx)^6}$$

$$+\frac{(4c^2(5fg^4+g^2h(2eg-51dh))-7h^2(24a^2fh^2-2abh(10fg+7eh)+b^2(5fg^2+5egh+9dh^2))-2ch(3b}{840h\left(cg^2-bgh+ah^2\right)^3(g+h}$$

$$-\frac{\left(\left(b^2-4ac\right)^2\left(48c^3dg^3-8c^2g(afg^2-ah(8eg-3dh)+3bg(eg+3dh))-bh(24a^2fh^2-2abh(10fg+7e\right.\right.}{}$$

$$=$$

$$-\frac{\left(b^2-4ac\right)\left(48c^3dg^3-8c^2g(afg^2-ah(8eg-3dh)+3bg(eg+3dh))-bh(24a^2fh^2-2abh(10fg+7eh)\right.}{}$$

$$+\frac{(48c^3dg^3-8c^2g(afg^2-ah(8eg-3dh)+3bg(eg+3dh))-bh(24a^2fh^2-2abh(10fg+7eh)+b^2(5fg^2}{}$$

$$-\frac{\left(fg^2-h(eg-dh)\right)\left(a+bx+cx^2\right)^{5/2}}{7h\left(cg^2-bgh+ah^2\right)(g+hx)^7}$$

$$+\frac{\left(2c(5fg^3+gh(2eg-9dh))-h(19bfg^2-bh(5eg+9dh)-14ah(2fg-eh))\right)\left(a+bx+cx^2\right)^{5/2}}{84h\left(cg^2-bgh+ah^2\right)^2(g+hx)^6}$$

$$+\frac{(4c^2(5fg^4+g^2h(2eg-51dh))-7h^2(24a^2fh^2-2abh(10fg+7eh)+b^2(5fg^2+5egh+9dh^2))-2ch(3b}{840h\left(cg^2-bgh+ah^2\right)^3(g+h}$$

$$+\frac{\left(b^2-4ac\right)^2\left(48c^3dg^3-8c^2g(afg^2-ah(8eg-3dh)+3bg(eg+3dh))-bh(24a^2fh^2-2abh(10fg+7eh)\right.}{}$$

Mathematica [A] (verified)

Time = 16.32 (sec) , antiderivative size = 1636, normalized size of antiderivative = 1.54

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^8}\,dx =$$

$$-\frac{(fg^2-h(eg-dh))(a+bx+cx^2)(a+x(b+cx))^{3/2}}{7h(cg^2-h(bg-ah))(g+hx)^7}$$

$$+\frac{(2fg-eh)(a+bx+cx^2)(a+x(b+cx))^{3/2}}{6h(cg^2-h(bg-ah))(g+hx)^6}-\frac{f(a+bx+cx^2)(a+x(b+cx))^{3/2}}{5h(cg^2-bgh+ah^2)(g+hx)^5}$$

$$+\frac{f(2cg-bh)(a+x(b+cx))^{3/2}\left(\frac{(bg-2ah+(2cg-bh)x)(a+bx+cx^2)^{3/2}}{8(cg^2-bgh+ah^2)(g+hx)^4}-\frac{3(b^2-4ac)\left(\frac{(bg-2ah+(2cg-bh)x)\sqrt{a+bx+cx^2}}{4(cg^2-bgh+ah^2)(g+hx)^2}+\frac{(b^2-4ac)\mathrm{a}}{2\sqrt{}}\right.}{16(cg^2-bgh+ah}\right.}{2h^2(cg^2-bgh+ah^2)(a+bx+cx^2)^{3/2}}$$

$$-\frac{(-2fg+eh)(a+x(b+cx))^{3/2}\left(\frac{(cgh-\frac{1}{2}h(-12cg+7bh))(a+bx+cx^2)^{5/2}}{5(cg^2-bgh+ah^2)(g+hx)^5}-\frac{(-2(ach^2+\frac{1}{2}cg(-12cg+7bh))+b(cgh+\frac{1}{2}h(-12cg+7}{}\right.}{6h^2(cg^2-bgh+a}$$

$$-\frac{(fg^2-egh+dh^2)(a+x(b+cx))^{3/2}\left(-\frac{(-2cgh+\frac{1}{2}h(-14cg+9bh))(a+bx+cx^2)^{5/2}}{6(cg^2-bgh+ah^2)(g+hx)^6}-\frac{\frac{(-\frac{9}{2}cgh(2cg-bh)-\frac{3}{4}h(56c^2g^2+21b^2h^2-2c}{5(cg^2-bgh+ah^2)(}}{}\right.}{}$$

```
[In] Integrate[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^8,x]
```

```
[Out] -1/7*((f*g^2 - h*(e*g - d*h))*(a + b*x + c*x^2)*(a + x*(b + c*x))^(3/2))/(h
*(c*g^2 - h*(b*g - a*h))*(g + h*x)^7) + ((2*f*g - e*h)*(a + b*x + c*x^2)*(a
 + x*(b + c*x))^(3/2))/(6*h*(c*g^2 - h*(b*g - a*h))*(g + h*x)^6) - (f*(a +
b*x + c*x^2)*(a + x*(b + c*x))^(3/2))/(5*h*(c*g^2 - b*g*h + a*h^2)*(g + h*x
)^5) + (f*(2*c*g - b*h)*(a + x*(b + c*x))^(3/2)*(((b*g - 2*a*h + (2*c*g - b
*h)*x)*(a + b*x + c*x^2)^(3/2))/(8*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^4) - (
3*(b^2 - 4*a*c)*(((b*g - 2*a*h + (2*c*g - b*h)*x)*Sqrt[a + b*x + c*x^2])/(4
*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^2) + ((b^2 - 4*a*c)*ArcTanh[(-(b*g) + 2*
a*h - (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[a + b*x + c*x^2]
)])/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*(4*c*g^2 - 4*b*g*h + 4*a*h^2))))/(16*(c*
g^2 - b*g*h + a*h^2))))/(2*h^2*(c*g^2 - b*g*h + a*h^2)*(a + b*x + c*x^2)^(3
/2)) - ((-2*f*g + e*h)*(a + x*(b + c*x))^(3/2)*(((c*g*h - (h*(-12*c*g + 7*b
*h))/2)*(a + b*x + c*x^2)^(5/2))/(5*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^5) -
((-2*(a*c*h^2 + (c*g*(-12*c*g + 7*b*h))/2) + b*(c*g*h + (h*(-12*c*g + 7*b*h
))/2))*(((b*g - 2*a*h + (2*c*g - b*h)*x)*(a + b*x + c*x^2)^(3/2))/(8*(c*g^2
 - b*g*h + a*h^2)*(g + h*x)^4) - (3*(b^2 - 4*a*c)*(((b*g - 2*a*h + (2*c*g -
 b*h)*x)*Sqrt[a + b*x + c*x^2])/(4*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^2) + (
(b^2 - 4*a*c)*ArcTanh[(-(b*g) + 2*a*h - (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*
g*h + a*h^2]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*(4*c*g
^2 - 4*b*g*h + 4*a*h^2))))/(16*(c*g^2 - b*g*h + a*h^2))))/(2*(c*g^2 - b*g*h
 + a*h^2))))/(6*h^2*(c*g^2 - b*g*h + a*h^2)*(a + b*x + c*x^2)^(3/2)) - ((f*
g^2 - e*g*h + d*h^2)*(a + x*(b + c*x))^(3/2)*(-1/6*((-2*c*g*h + (h*(-14*c*g
 + 9*b*h))/2)*(a + b*x + c*x^2)^(5/2))/((c*g^2 - b*g*h + a*h^2)*(g + h*x)^6
) - ((((-9*c*g*h*(2*c*g - b*h))/2 - (3*h*(56*c^2*g^2 + 21*b^2*h^2 - 2*c*h*(
31*b*g + 8*a*h)))/4)*(a + b*x + c*x^2)^(5/2))/(5*(c*g^2 - b*g*h + a*h^2)*(g
 + h*x)^5) - ((-2*((-9*a*c*h^2*(2*c*g - b*h))/2 + (3*c*g*(56*c^2*g^2 + 21*b
^2*h^2 - 2*c*h*(31*b*g + 8*a*h)))/4) + b*((-9*c*g*h*(2*c*g - b*h))/2 + (3*h
*(56*c^2*g^2 + 21*b^2*h^2 - 2*c*h*(31*b*g + 8*a*h)))/4))*(((b*g - 2*a*h + (
2*c*g - b*h)*x)*(a + b*x + c*x^2)^(3/2))/(8*(c*g^2 - b*g*h + a*h^2)*(g + h*
x)^4) - (3*(b^2 - 4*a*c)*(((b*g - 2*a*h + (2*c*g - b*h)*x)*Sqrt[a + b*x + c
*x^2])/(4*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^2) + ((b^2 - 4*a*c)*ArcTanh[(-(
b*g) + 2*a*h - (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[a + b*x
 + c*x^2])])/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*(4*c*g^2 - 4*b*g*h + 4*a*h^2)))
)/(16*(c*g^2 - b*g*h + a*h^2))))/(2*(c*g^2 - b*g*h + a*h^2)))/(6*(c*g^2 - b
*g*h + a*h^2))))/(7*h^2*(c*g^2 - b*g*h + a*h^2)*(a + b*x + c*x^2)^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 31329 vs. $2(1032) = 2064$.

Time = 2.52 (sec) , antiderivative size = 31330, normalized size of antiderivative = 29.50

method	result	size
default	Expression too large to display	31330

```
[In] int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^8,x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^8}\,dx = \text{Timed out}$$

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^8,x, algorithm="fricas"
)
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^8}\,dx = \text{Timed out}$$

```
[In] integrate((c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**8,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^8}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^8,x, algorithm="maxima"
)
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*h^2-b*g*h>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75375 vs. 2(1032) = 2064.

Time = 31.74 (sec) , antiderivative size = 75375, normalized size of antiderivative = 70.97

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^8}\,dx = \text{Too large to display}$$

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^8,x, algorithm="giac")
```

```
[Out] 1/1024*(48*b^4*c^3*d*g^3 - 384*a*b^2*c^4*d*g^3 + 768*a^2*c^5*d*g^3 - 24*b^5
*c^2*e*g^3 + 192*a*b^3*c^3*e*g^3 - 384*a^2*b*c^4*e*g^3 + 14*b^6*c*f*g^3 - 1
20*a*b^4*c^2*f*g^3 + 288*a^2*b^2*c^3*f*g^3 - 128*a^3*c^4*f*g^3 - 72*b^5*c^2
*d*g^2*h + 576*a*b^3*c^3*d*g^2*h - 1152*a^2*b*c^4*d*g^2*h + 20*b^6*c*e*g^2*
h - 96*a*b^4*c^2*e*g^2*h - 192*a^2*b^2*c^3*e*g^2*h + 1024*a^3*c^4*e*g^2*h -
 5*b^7*f*g^2*h - 12*a*b^5*c*f*g^2*h + 336*a^2*b^3*c^2*f*g^2*h - 832*a^3*b*c
^3*f*g^2*h + 42*b^6*c*d*g*h^2 - 360*a*b^4*c^2*d*g*h^2 + 864*a^2*b^2*c^3*d*g
*h^2 - 384*a^3*c^4*d*g*h^2 - 5*b^7*e*g*h^2 - 12*a*b^5*c*e*g*h^2 + 336*a^2*b
^3*c^2*e*g*h^2 - 832*a^3*b*c^3*e*g*h^2 + 20*a*b^6*f*g*h^2 - 96*a^2*b^4*c*f*
g*h^2 - 192*a^3*b^2*c^2*f*g*h^2 + 1024*a^4*c^3*f*g*h^2 - 9*b^7*d*h^3 + 84*a
*b^5*c*d*h^3 - 240*a^2*b^3*c^2*d*h^3 + 192*a^3*b*c^3*d*h^3 + 14*a*b^6*e*h^3
 - 120*a^2*b^4*c*e*h^3 + 288*a^3*b^2*c^2*e*h^3 - 128*a^4*c^3*e*h^3 - 24*a^2
*b^5*f*h^3 + 192*a^3*b^3*c*f*h^3 - 384*a^4*b*c^2*f*h^3)*arctan(-((sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))*h + sqrt(c)*g)/sqrt(-c*g^2 + b*g*h - a*h^2))/((c^
5*g^10 - 5*b*c^4*g^9*h + 10*b^2*c^3*g^8*h^2 + 5*a*c^4*g^8*h^2 - 10*b^3*c^2*
g^7*h^3 - 20*a*b*c^3*g^7*h^3 + 5*b^4*c*g^6*h^4 + 30*a*b^2*c^2*g^6*h^4 + 10*
a^2*c^3*g^6*h^4 - b^5*g^5*h^5 - 20*a*b^3*c*g^5*h^5 - 30*a^2*b*c^2*g^5*h^5 +
 5*a*b^4*g^4*h^6 + 30*a^2*b^2*c*g^4*h^6 + 10*a^3*c^2*g^4*h^6 - 10*a^2*b^3*g
^3*h^7 - 20*a^3*b*c*g^3*h^7 + 10*a^3*b^2*g^2*h^8 + 5*a^4*c*g^2*h^8 - 5*a^4*
b*g*h^9 + a^5*h^10)*sqrt(-c*g^2 + b*g*h - a*h^2)) - 1/107520*(5040*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^13*b^4*c^3*d*g^3*h^12 - 40320*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^13*a*b^2*c^4*d*g^3*h^12 + 80640*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^13*a^2*c^5*d*g^3*h^12 - 2520*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)^13*b^5*c^2*e*g^3*h^12 + 20160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^13*a*b^
3*c^3*e*g^3*h^12 - 40320*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^13*a^2*b*c^4*e
*g^3*h^12 + 1470*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^13*b^6*c*f*g^3*h^12 -
12600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^13*a*b^4*c^2*f*g^3*h^12 + 30240*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))^13*a^2*b^2*c^3*f*g^3*h^12 - 13440*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^13*a^3*c^4*f*g^3*h^12 - 7560*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^13*b^5*c^2*d*g^2*h^13 + 60480*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))^13*a*b^3*c^3*d*g^2*h^13 - 120960*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))^13*a^2*b*c^4*d*g^2*h^13 + 2100*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^13*
b^6*c*e*g^2*h^13 - 10080*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^13*a*b^4*c^2*e
*g^2*h^13 - 20160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^13*a^2*b^2*c^3*e*g^2*
h^13 + 107520*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^13*a^3*c^4*e*g^2*h^13 - 5
```

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25*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^13*b^7*f*g^2*h^13 - 1260*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))^13*a*b^5*c*f*g^2*h^13 + 35280*(sqrt(c)*x - sqrt(c*
x^2 + b*x + a))^13*a^2*b^3*c^2*f*g^2*h^13 - 87360*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))^13*a^3*b*c^3*f*g^2*h^13 + 4410*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))^13*b^6*c*d*g*h^14 - 37800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^13*a*b^4*c
^2*d*g*h^14 + 90720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^13*a^2*b^2*c^3*d*g*
h^14 - 40320*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^13*a^3*c^4*d*g*h^14 - 525*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^13*b^7*e*g*h^14 - 1260*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^13*a*b^5*c*e*g*h^14 + 35280*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^13*a^2*b^3*c^2*e*g*h^14 - 87360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)^13*a^3*b*c^3*e*g*h^14 + 2100*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^13*a*b^6
*f*g*h^14 - 10080*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^13*a^2*b^4*c*f*g*h^14
 - 20160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^13*a^3*b^2*c^2*f*g*h^14 + 1075
20*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^13*a^4*c^3*f*g*h^14 - 945*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))^13*b^7*d*h^15 + 8820*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^13*a*b^5*c*d*h^15 - 25200*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^13*a^
2*b^3*c^2*d*h^15 + 20160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^13*a^3*b*c^3*d
*h^15 + 1470*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^13*a*b^6*e*h^15 - 12600*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))^13*a^2*b^4*c*e*h^15 + 30240*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^13*a^3*b^2*c^2*e*h^15 - 13440*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^13*a^4*c^3*e*h^15 - 2520*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^
13*a^2*b^5*f*h^15 + 20160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^13*a^3*b^3*c*
f*h^15 - 40320*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^13*a^4*b*c^2*f*h^15 - 21
5040*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^12*c^(15/2)*f*g^10*h^5 + 1075200*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))^12*b*c^(13/2)*f*g^9*h^6 - 2150400*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^12*b^2*c^(11/2)*f*g^8*h^7 - 1075200*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^12*a*c^(13/2)*f*g^8*h^7 + 2150400*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^12*b^3*c^(9/2)*f*g^7*h^8 + 4300800*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^12*a*b*c^(11/2)*f*g^7*h^8 - 1075200*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))^12*b^4*c^(7/2)*f*g^6*h^9 - 6451200*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))^12*a*b^2*c^(9/2)*f*g^6*h^9 - 2150400*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^12*a^2*c^(11/2)*f*g^6*h^9 + 215040*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))^12*b^5*c^(5/2)*f*g^5*h^10 + 4300800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)^12*a*b^3*c^(7/2)*f*g^5*h^10 + 6451200*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
^12*a^2*b*c^(9/2)*f*g^5*h^10 + 65520*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^12
*b^4*c^(7/2)*d*g^4*h^11 - 524160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^12*a*b
^2*c^(9/2)*d*g^4*h^11 + 1048320*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^12*a^2*
c^(11/2)*d*g^4*h^11 - 32760*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^12*b^5*c^(5
/2)*e*g^4*h^11 + 262080*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^12*a*b^3*c^(7/2
)*e*g^4*h^11 - 524160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^12*a^2*b*c^(9/2)*
e*g^4*h^11 + 19110*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^12*b^6*c^(3/2)*f*g^4
*h^11 - 1239000*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^12*a*b^4*c^(5/2)*f*g^4*
h^11 - 6058080*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^12*a^2*b^2*c^(7/2)*f*g^4
*h^11 - 2325120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^12*a^3*c^(9/2)*f*g^4*h^
11 - 98280*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^12*b^5*c^(5/2)*d*g^3*h^12 +
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786240*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^12*a*b^3*c^(7/2)*d*g^3*h^12 - 15
72480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^12*a^2*b*c^(9/2)*d*g^3*h^12 + 273
00*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^12*b^6*c^(3/2)*e*g^3*h^12 - 131040*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))^12*a*b^4*c^(5/2)*e*g^3*h^12 - 262080*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^12*a^2*b^2*c^(7/2)*e*g^3*h^12 + 1397760*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))^12*a^3*c^(9/2)*e*g^3*h^12 - 6825*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^12*b^7*sqrt(c)*f*g^3*h^12 - 16380*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^12*a*b^5*c^(3/2)*f*g^3*h^12 + 2609040*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^12*a^2*b^3*c^(5/2)*f*g^3*h^12 + 3165120*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^12*a^3*b*c^(7/2)*f*g^3*h^12 + 57330*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^12*b^6*c^(3/2)*d*g^2*h^13 - 491400*(sqrt(c)*x - sqrt(c*
x^2 + b*x + a))^12*a*b^4*c^(5/2)*d*g^2*h^13 + 1179360*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))^12*a^2*b^2*c^(7/2)*d*g^2*h^13 - 524160*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))^12*a^3*c^(9/2)*d*g^2*h^13 - 6825*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^12*b^7*sqrt(c)*e*g^2*h^13 - 16380*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))^12*a*b^5*c^(3/2)*e*g^2*h^13 + 458640*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)^12*a^2*b^3*c^(5/2)*e*g^2*h^13 - 1135680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))^12*a^3*b*c^(7/2)*e*g^2*h^13 + 27300*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^
12*a*b^6*sqrt(c)*f*g^2*h^13 - 131040*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^12
*a^2*b^4*c^(3/2)*f*g^2*h^13 - 2412480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^1
2*a^3*b^2*c^(5/2)*f*g^2*h^13 + 322560*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^1
2*a^4*c^(7/2)*f*g^2*h^13 - 12285*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^12*b^7
*sqrt(c)*d*g*h^14 + 114660*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^12*a*b^5*c^(
3/2)*d*g*h^14 - 327600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^12*a^2*b^3*c^(5/
2)*d*g*h^14 + 262080*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^12*a^3*b*c^(7/2)*d
*g*h^14 + 19110*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^12*a*b^6*sqrt(c)*e*g*h^
14 - 163800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^12*a^2*b^4*c^(3/2)*e*g*h^14
 + 393120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^12*a^3*b^2*c^(5/2)*e*g*h^14 -
 174720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^12*a^4*c^(7/2)*e*g*h^14 - 32760
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^12*a^2*b^5*sqrt(c)*f*g*h^14 + 262080*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))^12*a^3*b^3*c^(3/2)*f*g*h^14 + 551040*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^12*a^4*b*c^(5/2)*f*g*h^14 - 215040*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^12*a^5*c^(5/2)*f*h^15 - 716800*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))^11*c^8*f*g^11*h^4 - 286720*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^11*c^8*e*g^10*h^5 + 2938880*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11
*b*c^7*f*g^10*h^5 + 1433600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*b*c^7*e*
g^9*h^6 - 3942400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*b^2*c^6*f*g^9*h^6
- 3584000*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*a*c^7*f*g^9*h^6 - 2867200*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*b^2*c^6*e*g^8*h^7 - 1433600*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^11*a*c^7*e*g^8*h^7 + 716800*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))^11*b^3*c^5*f*g^8*h^7 + 11110400*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))^11*a*b*c^6*f*g^8*h^7 + 2867200*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)^11*b^3*c^5*e*g^7*h^8 + 5734400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*a*b
*c^6*e*g^7*h^8 + 2867200*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*b^4*c^4*f*g
^7*h^8 - 8601600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*a*b^2*c^5*f*g^7*h^8

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 - 7168000*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*a^2*c^6*f*g^7*h^8 - 14336
00*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*b^4*c^4*e*g^6*h^9 - 8601600*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^11*a*b^2*c^5*e*g^6*h^9 - 2867200*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))^11*a^2*c^6*e*g^6*h^9 - 2508800*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))^11*b^5*c^3*f*g^6*h^9 - 5017600*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^11*a*b^3*c^4*f*g^6*h^9 + 15052800*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))^11*a^2*b*c^5*f*g^6*h^9 + 359520*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*
b^4*c^4*d*g^5*h^10 - 2876160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*a*b^2*c
^5*d*g^5*h^10 + 5752320*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*a^2*c^6*d*g^
5*h^10 + 106960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*b^5*c^3*e*g^5*h^10 +
 7172480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*a*b^3*c^4*e*g^5*h^10 + 5725
440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*a^2*b*c^5*e*g^5*h^10 + 749980*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))^11*b^6*c^2*f*g^5*h^10 + 8419600*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^11*a*b^4*c^3*f*g^5*h^10 + 6720*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^11*a^2*b^2*c^4*f*g^5*h^10 - 8126720*(sqrt(c)*x - sqrt(c*
x^2 + b*x + a))^11*a^3*c^5*f*g^5*h^10 - 505680*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^11*b^5*c^3*d*g^4*h^11 + 4045440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
^11*a*b^3*c^4*d*g^4*h^11 - 8090880*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*a
^2*b*c^5*d*g^4*h^11 + 133000*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*b^6*c^2
*e*g^4*h^11 - 2018240*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*a*b^4*c^3*e*g^
4*h^11 - 10308480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*a^2*b^2*c^4*e*g^4*
h^11 + 4802560*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*a^3*c^5*e*g^4*h^11 -
27650*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*b^7*c*f*g^4*h^11 - 3399480*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^11*a*b^5*c^2*f*g^4*h^11 - 9467360*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^11*a^2*b^3*c^3*f*g^4*h^11 + 1563520*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))^11*a^3*b*c^4*f*g^4*h^11 + 264180*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^11*b^6*c^2*d*g^3*h^12 - 2326800*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^11*a*b^4*c^3*d*g^3*h^12 + 5933760*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))^11*a^2*b^2*c^4*d*g^3*h^12 - 3413760*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))^11*a^3*c^5*d*g^3*h^12 - 23450*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*b^
7*c*e*g^3*h^12 - 140280*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*a*b^5*c^2*e*
g^3*h^12 + 5115040*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*a^2*b^3*c^3*e*g^3
*h^12 + 488320*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*a^3*b*c^4*e*g^3*h^12
- 3500*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*b^8*f*g^3*h^12 + 131600*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^11*a*b^6*c*f*g^3*h^12 + 6051360*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))^11*a^2*b^4*c^2*f*g^3*h^12 + 3512320*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))^11*a^3*b^2*c^3*f*g^3*h^12 + 4175360*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^11*a^4*c^4*f*g^3*h^12 - 38010*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^11*b^7*c*d*g^2*h^13 + 427560*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^1
1*a*b^5*c^2*d*g^2*h^13 - 1596000*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*a^2
*b^3*c^3*d*g^2*h^13 + 1975680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*a^3*b*
c^4*d*g^2*h^13 - 3500*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*b^8*e*g^2*h^13
 + 82460*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*a*b^6*c*e*g^2*h^13 - 596400
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*a^2*b^4*c^2*e*g^2*h^13 - 1158080*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))^11*a^3*b^2*c^3*e*g^2*h^13 - 3109120*(sqrt
```

```
(c)*x - sqrt(c*x^2 + b*x + a))^11*a^4*c^4*e*g^2*h^13 + 17500*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^11*a*b^7*f*g^2*h^13 - 238560*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^11*a^2*b^5*c*f*g^2*h^13 - 5382720*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))^11*a^3*b^3*c^2*f*g^2*h^13 - 1218560*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))^11*a^4*b*c^3*f*g^2*h^13 - 6300*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*b
^8*d*g*h^14 + 29400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*a*b^6*c*d*g*h^14
 + 84000*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*a^2*b^4*c^2*d*g*h^14 - 4704
00*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*a^3*b^2*c^3*d*g*h^14 + 268800*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^11*a^4*c^4*d*g*h^14 + 13300*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^11*a*b^7*e*g*h^14 - 75600*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^11*a^2*b^5*c*e*g*h^14 - 33600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^
11*a^3*b^3*c^2*e*g*h^14 + 1926400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*a^
4*b*c^3*e*g*h^14 - 30800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*a^2*b^6*f*g
*h^14 + 201600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*a^3*b^4*c*f*g*h^14 +
3091200*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*a^4*b^2*c^2*f*g*h^14 - 14336
00*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*a^5*c^3*f*g*h^14 + 6300*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^11*a*b^7*d*h^15 - 58800*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^11*a^2*b^5*c*d*h^15 + 168000*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)^11*a^3*b^3*c^2*d*h^15 - 134400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*a^4
*b*c^3*d*h^15 - 9800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*a^2*b^6*e*h^15
+ 84000*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*a^3*b^4*c*e*h^15 - 201600*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))^11*a^4*b^2*c^2*e*h^15 - 197120*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))^11*a^5*c^3*e*h^15 + 16800*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^11*a^3*b^5*f*h^15 - 134400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
^11*a^4*b^3*c*f*h^15 - 376320*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*a^5*b*
c^2*f*h^15 - 1433600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*c^(17/2)*f*g^12
*h^3 - 573440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*c^(17/2)*e*g^11*h^4 +
5232640*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*b*c^(15/2)*f*g^11*h^4 - 4300
80*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*c^(17/2)*d*g^10*h^5 + 1792000*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^10*b*c^(15/2)*e*g^10*h^5 - 5519360*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^10*b^2*c^(13/2)*f*g^10*h^5 - 6737920*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^10*a*c^(15/2)*f*g^10*h^5 + 2150400*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^10*b*c^(15/2)*d*g^9*h^6 - 358400*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))^10*b^2*c^(13/2)*e*g^9*h^6 - 2867200*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^10*a*c^(15/2)*e*g^9*h^6 - 716800*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^10*b^3*c^(11/2)*f*g^9*h^6 + 16844800*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^10*a*b*c^(13/2)*f*g^9*h^6 - 4300800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))^10*b^2*c^(13/2)*d*g^8*h^7 - 2150400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^
10*a*c^(15/2)*d*g^8*h^7 - 5017600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*b^
3*c^(11/2)*e*g^8*h^7 + 6092800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a*b*c
^(13/2)*e*g^8*h^7 + 3584000*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*b^4*c^(9
/2)*f*g^8*h^7 - 4300800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a*b^2*c^(11/
2)*f*g^8*h^7 - 12185600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a^2*c^(13/2)
*f*g^8*h^7 + 4300800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*b^3*c^(11/2)*d*
g^7*h^8 + 8601600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a*b*c^(13/2)*d*g^7
```

```
*h^8 + 7884800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*b^4*c^(9/2)*e*g^7*h^8
 + 4300800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a*b^2*c^(11/2)*e*g^7*h^8
- 5734400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a^2*c^(13/2)*e*g^7*h^8 + 3
58400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*b^5*c^(7/2)*f*g^7*h^8 - 164864
00*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a*b^3*c^(9/2)*f*g^7*h^8 + 1505280
0*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a^2*b*c^(11/2)*f*g^7*h^8 - 1078560
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*b^4*c^(9/2)*d*g^6*h^9 - 21477120*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a*b^2*c^(11/2)*d*g^6*h^9 + 12848640*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a^2*c^(13/2)*d*g^6*h^9 - 5338480*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^10*b^5*c^(7/2)*e*g^6*h^9 - 16499840*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^10*a*b^3*c^(9/2)*e*g^6*h^9 - 2123520*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))^10*a^2*b*c^(11/2)*e*g^6*h^9 - 2052820*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))^10*b^6*c^(5/2)*f*g^6*h^9 + 5205200*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))^10*a*b^4*c^(7/2)*f*g^6*h^9 + 25784640*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^10*a^2*b^2*c^(9/2)*f*g^6*h^9 - 12893440*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))^10*a^3*c^(11/2)*f*g^6*h^9 - 808080*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))^10*b^5*c^(7/2)*d*g^5*h^10 + 18506880*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))^10*a*b^3*c^(9/2)*d*g^5*h^10 - 6908160*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^10*a^2*b*c^(11/2)*d*g^5*h^10 + 1337000*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^10*b^6*c^(5/2)*e*g^5*h^10 + 17971520*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^10*a*b^4*c^(7/2)*e*g^5*h^10 + 7808640*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))^10*a^2*b^2*c^(9/2)*e*g^5*h^10 + 17131520*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^10*a^3*c^(11/2)*e*g^5*h^10 + 856310*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))^10*b^7*c^(3/2)*f*g^5*h^10 + 5904360*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))^10*a*b^5*c^(5/2)*f*g^5*h^10 - 16801120*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))^10*a^2*b^3*c^(7/2)*f*g^5*h^10 - 23148160*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^10*a^3*b*c^(9/2)*f*g^5*h^10 + 383460*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))^10*b^6*c^(5/2)*d*g^4*h^11 - 6123600*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))^10*a*b^4*c^(7/2)*d*g^4*h^11 + 477120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)^10*a^2*b^2*c^(9/2)*d*g^4*h^11 - 18789120*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))^10*a^3*c^(11/2)*d*g^4*h^11 + 42350*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^
10*b^7*c^(3/2)*e*g^4*h^11 - 6198360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*
a*b^5*c^(5/2)*e*g^4*h^11 - 21975520*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*
a^2*b^3*c^(7/2)*e*g^4*h^11 - 7020160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10
*a^3*b*c^(9/2)*e*g^4*h^11 - 38500*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*b^
8*sqrt(c)*f*g^4*h^11 - 4054400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a*b^6
*c^(3/2)*f*g^4*h^11 - 2933280*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a^2*b^
4*c^(5/2)*f*g^4*h^11 + 15760640*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a^3*
b^2*c^(7/2)*f*g^4*h^11 + 20984320*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a^
4*c^(9/2)*f*g^4*h^11 + 122430*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*b^7*c^
(3/2)*d*g^3*h^12 - 341880*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a*b^5*c^(5
/2)*d*g^3*h^12 + 1159200*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a^2*b^3*c^(
7/2)*d*g^3*h^12 + 18802560*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a^3*b*c^(
9/2)*d*g^3*h^12 - 38500*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*b^8*sqrt(c)*
e*g^3*h^12 + 66220*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a*b^6*c^(3/2)*e*g
```

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^3*h^12 + 11398800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a^2*b^4*c^(5/2)*e
*g^3*h^12 + 17272640*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a^3*b^2*c^(7/2)
*e*g^3*h^12 - 13610240*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a^4*c^(9/2)*e
*g^3*h^12 + 192500*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a*b^7*sqrt(c)*f*g
^3*h^12 + 7418880*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a^2*b^5*c^(3/2)*f*
g^3*h^12 - 6229440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a^3*b^3*c^(5/2)*f
*g^3*h^12 - 5393920*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a^4*b*c^(7/2)*f*
g^3*h^12 - 69300*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*b^8*sqrt(c)*d*g^2*h
^13 + 323400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a*b^6*c^(3/2)*d*g^2*h^1
3 + 924000*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a^2*b^4*c^(5/2)*d*g^2*h^1
3 - 9475200*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a^3*b^2*c^(7/2)*d*g^2*h^
13 + 806400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a^4*c^(9/2)*d*g^2*h^13 +
 146300*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a*b^7*sqrt(c)*e*g^2*h^13 - 8
31600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a^2*b^5*c^(3/2)*e*g^2*h^13 - 1
1121600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a^3*b^3*c^(5/2)*e*g^2*h^13 +
 2912000*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a^4*b*c^(7/2)*e*g^2*h^13 -
338800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a^2*b^6*sqrt(c)*f*g^2*h^13 -
6384000*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a^3*b^4*c^(3/2)*f*g^2*h^13 +
 8198400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a^4*b^2*c^(5/2)*f*g^2*h^13
- 7168000*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a^5*c^(7/2)*f*g^2*h^13 + 6
9300*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a*b^7*sqrt(c)*d*g*h^14 - 646800
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a^2*b^5*c^(3/2)*d*g*h^14 + 1848000*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a^3*b^3*c^(5/2)*d*g*h^14 + 672000*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a^4*b*c^(7/2)*d*g*h^14 - 107800*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^10*a^2*b^6*sqrt(c)*e*g*h^14 + 924000*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^10*a^3*b^4*c^(3/2)*e*g*h^14 + 3158400*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^10*a^4*b^2*c^(5/2)*e*g*h^14 + 412160*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))^10*a^5*c^(7/2)*e*g*h^14 + 184800*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^10*a^3*b^5*sqrt(c)*f*g*h^14 + 2822400*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^10*a^4*b^3*c^(3/2)*f*g*h^14 - 1128960*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))^10*a^5*b*c^(5/2)*f*g*h^14 - 430080*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^10*a^5*c^(7/2)*d*h^15 - 1075200*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^10*a^5*b*c^(5/2)*e*h^15 - 860160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^
10*a^5*b^2*c^(3/2)*f*h^15 + 430080*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a
^6*c^(5/2)*f*h^15 - 1720320*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*c^9*f*g^1
3*h^2 - 688128*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*c^9*e*g^12*h^3 + 47022
08*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*b*c^8*f*g^12*h^3 - 516096*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^9*c^9*d*g^11*h^4 + 1519616*(sqrt(c)*x - sqrt(c*
x^2 + b*x + a))^9*b*c^8*e*g^11*h^4 - 157696*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))^9*b^2*c^7*f*g^11*h^4 - 6508544*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a
*c^8*f*g^11*h^4 + 688128*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*b*c^8*d*g^10
*h^5 + 960512*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*b^2*c^7*e*g^10*h^5 - 33
25952*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a*c^8*e*g^10*h^5 - 10135552*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^9*b^3*c^6*f*g^10*h^5 + 5433344*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))^9*a*b*c^7*f*g^10*h^5 + 4300800*(sqrt(c)*x - sqrt(c
```

```
*x^2 + b*x + a))^9*b^2*c^7*d*g^9*h^6 - 2580480*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^9*a*c^8*d*g^9*h^6 - 3512320*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*b
^3*c^6*e*g^9*h^6 + 3584000*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a*b*c^7*e*
g^9*h^6 + 8888320*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*b^4*c^5*f*g^9*h^6 +
 30105600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a*b^2*c^6*f*g^9*h^6 - 67379
20*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^2*c^7*f*g^9*h^6 - 13762560*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^9*b^3*c^6*d*g^8*h^7 + 860160*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))^9*a*b*c^7*d*g^8*h^7 - 1863680*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^9*b^4*c^5*e*g^8*h^7 + 10106880*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))^9*a*b^2*c^6*e*g^8*h^7 - 6307840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a
^2*c^7*e*g^8*h^7 + 716800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*b^5*c^4*f*g
^8*h^7 - 47595520*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a*b^3*c^5*f*g^8*h^7
 - 24299520*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^2*b*c^6*f*g^8*h^7 + 181
93728*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*b^4*c^5*d*g^7*h^8 + 7558656*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^9*a*b^2*c^6*d*g^7*h^8 + 24450048*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^9*a^2*c^7*d*g^7*h^8 + 7790944*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^9*b^5*c^4*e*g^7*h^8 - 2345728*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^9*a*b^3*c^5*e*g^7*h^8 - 15665664*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))^9*a^2*b*c^6*e*g^7*h^8 - 1796984*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*b^
6*c^3*f*g^7*h^8 + 3831520*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a*b^4*c^4*f
*g^7*h^8 + 94969728*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^2*b^2*c^5*f*g^7
*h^8 - 1207808*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^3*c^6*f*g^7*h^8 - 10
063872*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*b^5*c^4*d*g^6*h^9 - 37502976*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a*b^3*c^5*d*g^6*h^9 - 21331968*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^9*a^2*b*c^6*d*g^6*h^9 - 6953408*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^9*b^6*c^3*e*g^6*h^9 - 14416640*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^9*a*b^4*c^4*e*g^6*h^9 + 2128896*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))^9*a^2*b^2*c^5*e*g^6*h^9 + 33746944*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))^9*a^3*c^6*e*g^6*h^9 - 268352*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*b^7*c
^2*f*g^6*h^9 + 9714432*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a*b^5*c^3*f*g^
6*h^9 - 21539840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^2*b^3*c^4*f*g^6*h^
9 - 93986816*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^3*b*c^5*f*g^6*h^9 + 11
20056*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*b^6*c^3*d*g^5*h^10 + 38861760*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a*b^4*c^4*d*g^5*h^10 + 49596288*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^9*a^2*b^2*c^5*d*g^5*h^10 - 46491648*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^9*a^3*c^6*d*g^5*h^10 + 2213764*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^9*b^7*c^2*e*g^5*h^10 + 22977696*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^9*a*b^5*c^3*e*g^5*h^10 - 1211840*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^9*a^2*b^3*c^4*e*g^5*h^10 + 7662592*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)^9*a^3*b*c^5*e*g^5*h^10 + 457156*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*b^8
*c*f*g^5*h^10 - 1568812*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a*b^6*c^2*f*g
^5*h^10 - 13186320*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^2*b^4*c^3*f*g^5*
h^10 + 23047360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^3*b^2*c^4*f*g^5*h^1
0 + 56231168*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^4*c^5*f*g^5*h^10 + 960
960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*b^7*c^2*d*g^4*h^11 - 15219120*(sq
```

```
rt(c)*x - sqrt(c*x^2 + b*x + a))^9*a*b^5*c^3*d*g^4*h^11 - 51676800*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^9*a^2*b^3*c^4*d*g^4*h^11 + 22283520*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))^9*a^3*b*c^5*d*g^4*h^11 - 133070*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^9*b^8*c*e*g^4*h^11 - 9477160*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^9*a*b^6*c^2*e*g^4*h^11 - 23963520*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))^9*a^2*b^4*c^3*e*g^4*h^11 + 9896320*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9
*a^3*b^2*c^4*e*g^4*h^11 - 42595840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^
4*c^5*e*g^4*h^11 - 9905*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*b^9*f*g^4*h^1
1 - 2226350*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a*b^7*c*f*g^4*h^11 + 8711
640*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^2*b^5*c^2*f*g^4*h^11 - 863520*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^3*b^3*c^3*f*g^4*h^11 + 6939520*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^9*a^4*b*c^4*f*g^4*h^11 - 227640*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))^9*b^8*c*d*g^3*h^12 + 1022700*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^9*a*b^6*c^2*d*g^3*h^12 + 22592640*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^9*a^2*b^4*c^3*d*g^3*h^12 + 12519360*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^9*a^3*b^2*c^4*d*g^3*h^12 + 12203520*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))^9*a^4*c^5*d*g^3*h^12 - 9905*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*b^9*e*
g^3*h^12 + 553210*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a*b^7*c*e*g^3*h^12
+ 14901600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^2*b^5*c^2*e*g^3*h^12 + 1
2744480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^3*b^3*c^3*e*g^3*h^12 + 1103
8720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^4*b*c^4*e*g^3*h^12 + 59430*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^9*a*b^8*f*g^3*h^12 + 4386550*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^9*a^2*b^6*c*f*g^3*h^12 - 14366520*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^9*a^3*b^4*c^2*f*g^3*h^12 + 7650720*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^9*a^4*b^2*c^3*f*g^3*h^12 - 26875520*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))^9*a^5*c^4*f*g^3*h^12 - 17829*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
^9*b^9*d*g^2*h^13 + 310842*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a*b^7*c*d*
g^2*h^13 - 2092944*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^2*b^5*c^2*d*g^2*
h^13 - 12536160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^3*b^3*c^3*d*g^2*h^1
3 - 14273280*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^4*b*c^4*d*g^2*h^13 + 4
7544*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a*b^8*e*g^2*h^13 - 634088*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^9*a^2*b^6*c*e*g^2*h^13 - 14439600*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))^9*a^3*b^4*c^2*e*g^2*h^13 - 5349120*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))^9*a^4*b^2*c^3*e*g^2*h^13 + 6334720*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))^9*a^5*c^4*e*g^2*h^13 - 136689*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^9*a^2*b^7*f*g^2*h^13 - 4455276*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^
9*a^3*b^5*c*f*g^2*h^13 + 13172880*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^4
*b^3*c^2*f*g^2*h^13 - 1861440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^5*b*c
^3*f*g^2*h^13 + 35658*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a*b^8*d*g*h^14
- 249606*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^2*b^6*c*d*g*h^14 + 237720*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^3*b^4*c^2*d*g*h^14 + 10412640*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^9*a^4*b^2*c^3*d*g*h^14 - 1276800*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))^9*a^5*c^4*d*g*h^14 - 65373*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^9*a^2*b^7*e*g*h^14 + 451668*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)^9*a^3*b^5*c*e*g*h^14 + 8341200*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^4*
```

b^3*c^2*e*g*h^14 - 3635520*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^5*b*c^3*
e*g*h^14 + 134708*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^3*b^6*f*g*h^14 +
2059680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^4*b^4*c*f*g*h^14 - 6256320*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^5*b^2*c^2*f*g*h^14 + 4121600*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^9*a^6*c^3*f*g*h^14 - 17829*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^9*a^2*b^7*d*h^15 + 166404*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))^9*a^3*b^5*c*d*h^15 - 475440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^4*
b^3*c^2*d*h^15 - 1512000*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^5*b*c^3*d*
h^15 + 27734*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^3*b^6*e*h^15 - 237720*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^4*b^4*c*e*h^15 - 1192800*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))^9*a^5*b^2*c^2*e*h^15 - 138880*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))^9*a^6*c^3*e*h^15 - 47544*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))^9*a^4*b^5*f*h^15 - 221760*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^5*b^3*
c*f*h^15 + 228480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^6*b*c^2*f*h^15 -
1146880*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*c^(19/2)*f*g^14*h - 458752*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))^8*c^(19/2)*e*g^13*h^2 + 745472*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))^8*b*c^(17/2)*f*g^13*h^2 - 344064*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^8*c^(19/2)*d*g^12*h^3 + 57344*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))^8*b*c^(17/2)*e*g^12*h^3 + 8615936*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^8*b^2*c^(15/2)*f*g^12*h^3 - 1949696*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))^8*a*c^(17/2)*f*g^12*h^3 - 258048*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*b
*c^(17/2)*d*g^11*h^4 + 3627008*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*b^2*c^
(15/2)*e*g^11*h^4 - 1261568*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a*c^(17/2
)*e*g^11*h^4 - 15955968*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*b^3*c^(13/2)*
f*g^11*h^4 - 16328704*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a*b*c^(15/2)*f*
g^11*h^4 + 2795520*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*b^2*c^(15/2)*d*g^1
0*h^5 - 2150400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a*c^(17/2)*d*g^10*h^5
 - 4515840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*b^3*c^(13/2)*e*g^10*h^5 -
6952960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a*b*c^(15/2)*e*g^10*h^5 + 451
5840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*b^4*c^(11/2)*f*g^10*h^5 + 572006
40*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a*b^2*c^(13/2)*f*g^10*h^5 + 680960
0*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^2*c^(15/2)*f*g^10*h^5 + 1935360*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*b^3*c^(13/2)*d*g^9*h^6 - 860160*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^8*a*b*c^(15/2)*d*g^9*h^6 - 1720320*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))^8*b^4*c^(11/2)*e*g^9*h^6 + 25374720*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^8*a*b^2*c^(13/2)*e*g^9*h^6 + 573440*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^8*a^2*c^(15/2)*e*g^9*h^6 + 7526400*(sqrt(c)*x - sqrt(c*
x^2 + b*x + a))^8*b^5*c^(9/2)*f*g^9*h^6 - 36986880*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^8*a*b^3*c^(11/2)*f*g^9*h^6 - 64942080*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))^8*a^2*b*c^(13/2)*f*g^9*h^6 - 16695168*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))^8*b^4*c^(11/2)*d*g^8*h^7 - 7719936*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^8*a*b^2*c^(13/2)*d*g^8*h^7 + 23181312*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))^8*a^2*c^(15/2)*d*g^8*h^7 + 2283456*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
^8*b^5*c^(9/2)*e*g^8*h^7 - 7429632*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a*
b^3*c^(11/2)*e*g^8*h^7 - 42556416*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^2

*b*c^(13/2)*e*g^8*h^7 - 2586416*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*b^6*c
^(7/2)*f*g^8*h^7 - 34816320*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a*b^4*c^(
9/2)*f*g^8*h^7 + 98956032*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^2*b^2*c^(
11/2)*f*g^8*h^7 + 18357248*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^3*c^(13/
2)*f*g^8*h^7 + 28144032*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*b^5*c^(9/2)*d
*g^7*h^8 + 15864576*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a*b^3*c^(11/2)*d*
g^7*h^8 + 17299968*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^2*b*c^(13/2)*d*g
^7*h^8 + 2001328*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*b^6*c^(7/2)*e*g^7*h^
8 - 309120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a*b^4*c^(9/2)*e*g^7*h^8 +
12466944*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^2*b^2*c^(11/2)*e*g^7*h^8 +
44097536*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^3*c^(13/2)*e*g^7*h^8 - 12
34268*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*b^7*c^(5/2)*f*g^7*h^8 + 1779556
8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a*b^5*c^(7/2)*f*g^7*h^8 + 67838400*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^2*b^3*c^(9/2)*f*g^7*h^8 - 122961664
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^3*b*c^(11/2)*f*g^7*h^8 - 19619376*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*b^6*c^(7/2)*d*g^6*h^9 - 52150560*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^8*a*b^4*c^(9/2)*d*g^6*h^9 - 15687168*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^8*a^2*b^2*c^(11/2)*d*g^6*h^9 - 83446272*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^8*a^3*c^(13/2)*d*g^6*h^9 - 3935624*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^8*b^7*c^(5/2)*e*g^6*h^9 - 1871856*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^8*a*b^5*c^(7/2)*e*g^6*h^9 - 28358400*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^8*a^2*b^3*c^(9/2)*e*g^6*h^9 + 39243008*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^8*a^3*b*c^(11/2)*e*g^6*h^9 + 610204*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))^8*b^8*c^(3/2)*f*g^6*h^9 + 4332692*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^8*a*b^6*c^(5/2)*f*g^6*h^9 - 42693840*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))^8*a^2*b^4*c^(7/2)*f*g^6*h^9 - 90153280*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))^8*a^3*b^2*c^(9/2)*f*g^6*h^9 + 74244352*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))^8*a^4*c^(11/2)*f*g^6*h^9 + 5388180*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^8*b^7*c^(5/2)*d*g^5*h^10 + 52362240*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))^8*a*b^5*c^(7/2)*d*g^5*h^10 + 69518400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)^8*a^2*b^3*c^(9/2)*d*g^5*h^10 + 41126400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))^8*a^3*b*c^(11/2)*d*g^5*h^10 + 1534330*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)^8*b^8*c^(3/2)*e*g^5*h^10 + 13600160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8
*a*b^6*c^(5/2)*e*g^5*h^10 + 1233120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a
^2*b^4*c^(7/2)*e*g^5*h^10 + 26620160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*
a^3*b^2*c^(9/2)*e*g^5*h^10 - 78525440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8
*a^4*c^(11/2)*e*g^5*h^10 + 125895*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*b^9
*sqrt(c)*f*g^5*h^10 - 4881590*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a*b^7*c
^(3/2)*f*g^5*h^10 + 513240*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^2*b^5*c^
(5/2)*f*g^5*h^10 + 43295840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^3*b^3*c
^(7/2)*f*g^5*h^10 + 108832640*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^4*b*c
^(9/2)*f*g^5*h^10 + 30240*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*b^8*c^(3/2)
*d*g^4*h^11 - 18776100*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a*b^6*c^(5/2)*
d*g^4*h^11 - 86735040*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^2*b^4*c^(7/2)
*d*g^4*h^11 - 39600960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^3*b^2*c^(9/2

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)*d*g^4*h^11 + 38330880*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^4*c^(11/2)*
d*g^4*h^11 - 89145*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*b^9*sqrt(c)*e*g^4*
h^11 - 7294910*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a*b^7*c^(3/2)*e*g^4*h^
11 - 16195200*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^2*b^5*c^(5/2)*e*g^4*h
^11 + 8949920*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^3*b^3*c^(7/2)*e*g^4*h
^11 - 14533120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^4*b*c^(9/2)*e*g^4*h^
11 - 540330*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a*b^8*sqrt(c)*f*g^4*h^11
+ 13802950*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^2*b^6*c^(3/2)*f*g^4*h^11
 - 19135480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^3*b^4*c^(5/2)*f*g^4*h^1
1 - 22266720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^4*b^2*c^(7/2)*f*g^4*h^
11 - 72714880*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^5*c^(9/2)*f*g^4*h^11
- 160461*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*b^9*sqrt(c)*d*g^3*h^12 + 718
578*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a*b^7*c^(3/2)*d*g^3*h^12 + 371243
04*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^2*b^5*c^(5/2)*d*g^3*h^12 + 59676
960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^3*b^3*c^(7/2)*d*g^3*h^12 - 2174
5920*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^4*b*c^(9/2)*d*g^3*h^12 + 42789
6*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a*b^8*sqrt(c)*e*g^3*h^12 + 13296808
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^2*b^6*c^(3/2)*e*g^3*h^12 + 736400*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^3*b^4*c^(5/2)*e*g^3*h^12 + 3333120*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^4*b^2*c^(7/2)*e*g^3*h^12 + 33178880
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^5*c^(9/2)*e*g^3*h^12 + 920199*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^8*a^2*b^7*sqrt(c)*f*g^3*h^12 - 18403084*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^3*b^5*c^(3/2)*f*g^3*h^12 + 32450320*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^4*b^3*c^(5/2)*f*g^3*h^12 - 8688960*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^5*b*c^(7/2)*f*g^3*h^12 + 320922*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^8*a*b^8*sqrt(c)*d*g^2*h^13 - 2246454*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^8*a^2*b^6*c^(3/2)*d*g^2*h^13 - 34417320*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^8*a^3*b^4*c^(5/2)*d*g^2*h^13 - 4129440*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^8*a^4*b^2*c^(7/2)*d*g^2*h^13 - 9340800*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^8*a^5*c^(9/2)*d*g^2*h^13 - 588357*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))^8*a^2*b^7*sqrt(c)*e*g^2*h^13 - 11704588*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))^8*a^3*b^5*c^(3/2)*e*g^2*h^13 + 4824400*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))^8*a^4*b^3*c^(5/2)*e*g^2*h^13 - 16591680*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^8*a^5*b*c^(7/2)*e*g^2*h^13 - 938028*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))^8*a^3*b^6*sqrt(c)*f*g^2*h^13 + 12444320*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))^8*a^4*b^4*c^(3/2)*f*g^2*h^13 - 17599680*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))^8*a^5*b^2*c^(5/2)*f*g^2*h^13 + 18816000*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^8*a^6*c^(7/2)*f*g^2*h^13 - 160461*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^8*a^2*b^7*sqrt(c)*d*g*h^14 + 1497636*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^8*a^3*b^5*c^(3/2)*d*g*h^14 + 13999440*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^8*a^4*b^3*c^(5/2)*d*g*h^14 + 3595200*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))^8*a^5*b*c^(7/2)*d*g*h^14 + 249606*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^8*a^3*b^6*sqrt(c)*e*g*h^14 + 5745320*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))^8*a^4*b^4*c^(3/2)*e*g*h^14 + 1092000*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^8*a^5*b^2*c^(5/2)*e*g*h^14 - 1249920*(sqrt(c)*x - sqrt(c*x^2 + b*x
```

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+ a))^8*a^6*c^(7/2)*e*g*h^14 + 647304*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8
*a^4*b^5*sqrt(c)*f*g*h^14 - 4863040*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a
^5*b^3*c^(3/2)*f*g*h^14 + 981120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^6*
b*c^(5/2)*f*g*h^14 - 3655680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^5*b^2*
c^(5/2)*d*h^15 - 430080*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^6*c^(7/2)*d
*h^15 - 1576960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^5*b^3*c^(3/2)*e*h^1
5 + 215040*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^6*b*c^(5/2)*e*h^15 - 215
040*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^5*b^4*sqrt(c)*f*h^15 + 1290240*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^6*b^2*c^(3/2)*f*h^15 - 645120*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^8*a^7*c^(5/2)*f*h^15 - 327680*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^7*c^10*f*g^15 - 131072*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^7*c^10*e*g^14*h - 1998848*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*b*c^9
*f*g^14*h - 98304*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*c^10*d*g^13*h^2 - 8
68352*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*b*c^9*e*g^13*h^2 + 8507392*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^7*b^2*c^8*f*g^13*h^2 + 1654784*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))^7*a*c^9*f*g^13*h^2 - 737280*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^7*b*c^9*d*g^12*h^3 + 2990080*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))^7*b^2*c^8*e*g^12*h^3 + 524288*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a*c^
9*e*g^12*h^3 - 4761600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*b^3*c^7*f*g^12
*h^3 - 19079168*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a*b*c^8*f*g^12*h^3 +
1683456*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*b^2*c^8*d*g^11*h^4 + 49152*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a*c^9*d*g^11*h^4 - 55296*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^7*b^3*c^7*e*g^11*h^4 - 8216576*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^7*a*b*c^8*e*g^11*h^4 - 11741184*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))^7*b^4*c^6*f*g^11*h^4 + 23531520*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^
7*a*b^2*c^7*f*g^11*h^4 + 10313728*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^2
*c^8*f*g^11*h^4 + 2119680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*b^3*c^7*d*g
^10*h^5 - 8871936*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a*b*c^8*d*g^10*h^5
- 5806080*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*b^4*c^6*e*g^10*h^5 + 881049
6*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a*b^2*c^7*e*g^10*h^5 + 4194304*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^7*a^2*c^8*e*g^10*h^5 + 13547520*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))^7*b^5*c^5*f*g^10*h^5 + 41416704*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^7*a*b^3*c^6*f*g^10*h^5 - 33681408*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^7*a^2*b*c^7*f*g^10*h^5 - 2327808*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))^7*b^4*c^6*d*g^9*h^6 + 12908544*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7
*a*b^2*c^7*d*g^9*h^6 + 15101952*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^2*c
^8*d*g^9*h^6 + 2088576*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*b^5*c^5*e*g^9*
h^6 + 25740288*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a*b^3*c^6*e*g^9*h^6 -
26228736*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^2*b*c^7*e*g^9*h^6 + 430304
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*b^6*c^4*f*g^9*h^6 - 84604800*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^7*a*b^4*c^5*f*g^9*h^6 - 42352128*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^7*a^2*b^2*c^6*f*g^9*h^6 + 13047808*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^7*a^3*c^7*f*g^9*h^6 - 7469952*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))^7*b^5*c^5*d*g^8*h^7 - 37911552*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)^7*a*b^3*c^6*d*g^8*h^7 + 24766464*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^
```

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2*b*c^7*d*g^8*h^7 + 2131136*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*b^6*c^4*e
*g^8*h^7 - 18633216*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a*b^4*c^5*e*g^8*h
^7 - 52061184*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^2*b^2*c^6*e*g^8*h^7 +
 26361856*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^3*c^7*e*g^8*h^7 - 2651056
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*b^7*c^3*f*g^8*h^7 + 8288448*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^7*a*b^5*c^4*f*g^8*h^7 + 218647296*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^7*a^2*b^3*c^5*f*g^8*h^7 - 11648000*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^7*a^3*b*c^6*f*g^8*h^7 + 20156640*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^7*b^6*c^4*d*g^7*h^8 + 43032192*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))^7*a*b^4*c^5*d*g^7*h^8 + 42975744*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7
*a^2*b^2*c^6*d*g^7*h^8 - 77211648*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^3
*c^7*d*g^7*h^8 - 1909360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*b^7*c^3*e*g^
7*h^8 + 5535936*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a*b^5*c^4*e*g^7*h^8 +
 24100608*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^2*b^3*c^5*e*g^7*h^8 + 109
548544*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^3*b*c^6*e*g^7*h^8 + 186920*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*b^8*c^2*f*g^7*h^8 + 12162304*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^7*a*b^6*c^3*f*g^7*h^8 - 40481280*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))^7*a^2*b^4*c^4*f*g^7*h^8 - 310274048*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^7*a^3*b^2*c^5*f*g^7*h^8 + 32872448*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^7*a^4*c^6*f*g^7*h^8 - 16718928*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^7*b^7*c^3*d*g^6*h^9 - 63986496*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*
a*b^5*c^4*d*g^6*h^9 - 39282432*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^2*b^
3*c^5*d*g^6*h^9 - 63544320*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^3*b*c^6*
d*g^6*h^9 - 595552*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*b^8*c^2*e*g^6*h^9
+ 8128288*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a*b^6*c^3*e*g^6*h^9 - 43639
680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^2*b^4*c^4*e*g^6*h^9 - 2390528*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^3*b^2*c^5*e*g^6*h^9 - 94947328*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^7*a^4*c^6*e*g^6*h^9 + 414492*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))^7*b^9*c*f*g^6*h^9 - 2696560*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))^7*a*b^7*c^2*f*g^6*h^9 - 14489664*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))^7*a^2*b^5*c^3*f*g^6*h^9 + 57998080*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^
7*a^3*b^3*c^4*f*g^6*h^9 + 276871168*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a
^4*b*c^5*f*g^6*h^9 + 5471640*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*b^8*c^2*
d*g^5*h^10 + 55872768*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a*b^6*c^3*d*g^5
*h^10 + 66171840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^2*b^4*c^4*d*g^5*h^
10 + 87489024*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^3*b^2*c^5*d*g^5*h^10
+ 90080256*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^4*c^6*d*g^5*h^10 + 54966
0*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*b^9*c*e*g^5*h^10 + 482432*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^7*a*b^7*c^2*e*g^5*h^10 - 1297632*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))^7*a^2*b^5*c^3*e*g^5*h^10 + 72835840*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^7*a^3*b^3*c^4*e*g^5*h^10 - 74299904*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))^7*a^4*b*c^5*e*g^5*h^10 + 15360*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))^7*b^10*f*g^5*h^10 - 2886312*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a*
b^8*c*f*g^5*h^10 + 11171384*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^2*b^6*c
^2*f*g^5*h^10 - 13472480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^3*b^4*c^3*
```

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f*g^5*h^10 - 9878400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^4*b^2*c^4*f*g^
5*h^10 - 120490496*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^5*c^5*f*g^5*h^10
 - 429876*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*b^9*c*d*g^4*h^11 - 19378512
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a*b^7*c^2*d*g^4*h^11 - 79136736*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^7*a^2*b^5*c^3*d*g^4*h^11 - 77790720*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^7*a^3*b^3*c^4*d*g^4*h^11 - 71877120*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))^7*a^4*b*c^5*d*g^4*h^11 - 15360*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^7*b^10*e*g^4*h^11 - 2724384*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))^7*a*b^8*c*e*g^4*h^11 + 1098608*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7
*a^2*b^6*c^2*e*g^4*h^11 - 19040000*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^
3*b^4*c^3*e*g^4*h^11 - 15617280*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^4*b
^2*c^4*e*g^4*h^11 + 77916160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^5*c^5*
e*g^4*h^11 - 46080*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a*b^9*f*g^4*h^11 +
 8193564*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^2*b^7*c*f*g^4*h^11 - 20780
144*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^3*b^5*c^2*f*g^4*h^11 + 40040000
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^4*b^3*c^3*f*g^4*h^11 - 67549440*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^5*b*c^4*f*g^4*h^11 - 27648*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))^7*b^10*d*g^3*h^12 + 988776*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^7*a*b^8*c*d*g^3*h^12 + 33721128*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))^7*a^2*b^6*c^2*d*g^3*h^12 + 69629280*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))^7*a^3*b^4*c^3*d*g^3*h^12 + 57886080*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
^7*a^4*b^2*c^4*d*g^3*h^12 - 23600640*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*
a^5*c^5*d*g^3*h^12 + 89088*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a*b^9*e*g^
3*h^12 + 5864172*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^2*b^7*c*e*g^3*h^12
 - 4613168*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^3*b^5*c^2*e*g^3*h^12 + 1
4638400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^4*b^3*c^3*e*g^3*h^12 - 1131
6480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^5*b*c^4*e*g^3*h^12 + 3072*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^7*a^2*b^8*f*g^3*h^12 - 12396720*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))^7*a^3*b^6*c*f*g^3*h^12 + 23470720*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^7*a^4*b^4*c^2*f*g^3*h^12 - 23520000*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))^7*a^5*b^2*c^3*f*g^3*h^12 + 52899840*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^7*a^6*c^4*f*g^3*h^12 + 82944*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))^7*a*b^9*d*g^2*h^13 - 945972*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^2*b
^7*c*d*g^2*h^13 - 35297136*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^3*b^5*c^
2*d*g^2*h^13 - 32813760*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^4*b^3*c^3*d
*g^2*h^13 - 1962240*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^5*b*c^4*d*g^2*h
^13 - 175104*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^2*b^8*e*g^2*h^13 - 618
3912*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^3*b^6*c*e*g^2*h^13 + 7426720*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^4*b^4*c^2*e*g^2*h^13 - 6384000*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^7*a^5*b^2*c^3*e*g^2*h^13 - 11880960*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^7*a^6*c^4*e*g^2*h^13 + 113664*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^7*a^3*b^7*f*g^2*h^13 + 10115616*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^7*a^4*b^5*c*f*g^2*h^13 - 17373440*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))^7*a^5*b^3*c^2*f*g^2*h^13 + 7795200*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))^7*a^6*b*c^3*f*g^2*h^13 - 82944*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^2
```

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*b^8*d*g*h^14 + 645120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^3*b^6*c*d*g*
h^14 + 18708480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^4*b^4*c^2*d*g*h^14
+ 5591040*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^5*b^2*c^3*d*g*h^14 + 1720
320*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^6*c^4*d*g*h^14 + 144384*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^7*a^3*b^7*e*g*h^14 + 2924544*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^7*a^4*b^5*c*e*g*h^14 - 3368960*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))^7*a^5*b^3*c^2*e*g*h^14 + 6021120*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))^7*a^6*b*c^3*e*g*h^14 - 129024*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^
4*b^6*f*g*h^14 - 3870720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^5*b^4*c*f*
g*h^14 + 5591040*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^6*b^2*c^2*f*g*h^14
 - 6021120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^7*c^3*f*g*h^14 + 27648*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^3*b^7*d*h^15 - 258048*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^7*a^4*b^5*c*d*h^15 - 3225600*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^7*a^5*b^3*c^2*d*h^15 - 2580480*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))^7*a^6*b*c^3*d*h^15 - 43008*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^4*b^
6*e*h^15 - 430080*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^5*b^4*c*e*h^15 -
430080*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^6*b^2*c^2*e*h^15 + 43008*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^5*b^5*f*h^15 + 430080*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^7*a^6*b^3*c*f*h^15 + 430080*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^7*a^7*b*c^2*f*h^15 - 1146880*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6
*b*c^(19/2)*f*g^15 - 458752*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*b*c^(19/2
)*e*g^14*h + 745472*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*b^2*c^(17/2)*f*g^
14*h + 1146880*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a*c^(19/2)*f*g^14*h -
344064*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*b*c^(19/2)*d*g^13*h^2 + 57344*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*b^2*c^(17/2)*e*g^13*h^2 + 458752*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^6*a*c^(19/2)*e*g^13*h^2 + 8615936*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^6*b^3*c^(15/2)*f*g^13*h^2 - 2695168*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))^6*a*b*c^(17/2)*f*g^13*h^2 - 258048*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^6*b^2*c^(17/2)*d*g^12*h^3 + 344064*(sqrt(c)*x - sqrt(c*
x^2 + b*x + a))^6*a*c^(19/2)*d*g^12*h^3 + 3627008*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))^6*b^3*c^(15/2)*e*g^12*h^3 - 1318912*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^6*a*b*c^(17/2)*e*g^12*h^3 - 15955968*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^6*b^4*c^(13/2)*f*g^12*h^3 - 24944640*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))^6*a*b^2*c^(15/2)*f*g^12*h^3 + 1949696*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))^6*a^2*c^(17/2)*f*g^12*h^3 + 2795520*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^
6*b^3*c^(15/2)*d*g^11*h^4 - 1892352*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a
*b*c^(17/2)*d*g^11*h^4 - 4515840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*b^4*
c^(13/2)*e*g^11*h^4 - 10579968*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a*b^2*
c^(15/2)*e*g^11*h^4 + 1261568*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^2*c^(
17/2)*e*g^11*h^4 + 4515840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*b^5*c^(11/
2)*f*g^11*h^4 + 73156608*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a*b^3*c^(13/
2)*f*g^11*h^4 + 23138304*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^2*b*c^(15/
2)*f*g^11*h^4 - 564480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*b^4*c^(13/2)*d
*g^10*h^5 - 11397120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a*b^2*c^(15/2)*d
*g^10*h^5 + 2150400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^2*c^(17/2)*d*g^
```

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10*h^5 - 1975680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*b^5*c^(11/2)*e*g^10*
h^5 + 29352960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a*b^3*c^(13/2)*e*g^10*
h^5 + 6666240*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^2*b*c^(15/2)*e*g^10*h
^5 + 7549920*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*b^6*c^(9/2)*f*g^10*h^5 -
 41489280*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a*b^4*c^(11/2)*f*g^10*h^5 -
 120798720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^2*b^2*c^(13/2)*f*g^10*h^
5 - 6809600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^3*c^(15/2)*f*g^10*h^5 -
 3067008*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*b^5*c^(11/2)*d*g^9*h^6 + 200
20224*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a*b^3*c^(13/2)*d*g^9*h^6 + 4210
4832*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^2*b*c^(15/2)*d*g^9*h^6 + 29688
96*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*b^6*c^(9/2)*e*g^9*h^6 + 1817088*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a*b^4*c^(11/2)*e*g^9*h^6 - 73952256*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^2*b^2*c^(13/2)*e*g^9*h^6 + 1146880*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^3*c^(15/2)*e*g^9*h^6 - 2347856*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^6*b^7*c^(7/2)*f*g^9*h^6 - 45554880*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))^6*a*b^5*c^(9/2)*f*g^9*h^6 + 137286912*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))^6*a^2*b^3*c^(11/2)*f*g^9*h^6 + 78568448*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))^6*a^3*b*c^(13/2)*f*g^9*h^6 + 2198112*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^6*b^6*c^(9/2)*d*g^8*h^7 - 51063936*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^6*a*b^4*c^(11/2)*d*g^8*h^7 - 18525696*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^6*a^2*b^2*c^(13/2)*d*g^8*h^7 - 43825152*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^6*a^3*c^(15/2)*d*g^8*h^7 - 333872*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^6*b^7*c^(7/2)*e*g^8*h^7 - 9326016*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^6*a*b^5*c^(9/2)*e*g^8*h^7 - 1231104*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^6*a^2*b^3*c^(11/2)*e*g^8*h^7 + 109018112*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^6*a^3*b*c^(13/2)*e*g^8*h^7 - 1013768*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^6*b^8*c^(5/2)*f*g^8*h^7 + 19461344*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)^6*a*b^6*c^(7/2)*f*g^8*h^7 + 126564480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
^6*a^2*b^4*c^(9/2)*f*g^8*h^7 - 238368256*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)^6*a^3*b^2*c^(11/2)*f*g^8*h^7 - 13303808*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))^6*a^4*c^(13/2)*f*g^8*h^7 + 5499984*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6
*b^7*c^(7/2)*d*g^7*h^8 + 46511808*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a*b
^5*c^(9/2)*d*g^7*h^8 + 122309376*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^2*
b^3*c^(11/2)*d*g^7*h^8 - 94940160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^3
*b*c^(13/2)*d*g^7*h^8 - 1520624*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*b^8*c
^(5/2)*e*g^7*h^8 + 8135456*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a*b^6*c^(7
/2)*e*g^7*h^8 - 15388800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^2*b^4*c^(9
/2)*e*g^7*h^8 + 59784704*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^3*b^2*c^(1
1/2)*e*g^7*h^8 - 62375936*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^4*c^(13/2
)*e*g^7*h^8 + 551404*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*b^9*c^(3/2)*f*g^
7*h^8 + 2534560*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a*b^7*c^(5/2)*f*g^7*h
^8 - 61154688*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^2*b^5*c^(7/2)*f*g^7*h
^8 - 217853440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^3*b^3*c^(9/2)*f*g^7*
h^8 + 230644736*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^4*b*c^(11/2)*f*g^7*
h^8 - 7011480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*b^8*c^(5/2)*d*g^6*h^9 -
```

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 43348704*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a*b^6*c^(7/2)*d*g^6*h^9 - 1
26651840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^2*b^4*c^(9/2)*d*g^6*h^9 -
39309312*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^3*b^2*c^(11/2)*d*g^6*h^9 +
 94252032*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^4*c^(13/2)*d*g^6*h^9 + 42
2380*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*b^9*c^(3/2)*e*g^6*h^9 + 5884144*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a*b^7*c^(5/2)*e*g^6*h^9 - 19539744*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^2*b^5*c^(7/2)*e*g^6*h^9 + 41431040*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^3*b^3*c^(9/2)*e*g^6*h^9 - 161125888*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^4*b*c^(11/2)*e*g^6*h^9 + 107520*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^6*b^10*sqrt(c)*f*g^6*h^9 - 3953544*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^6*a*b^8*c^(3/2)*f*g^6*h^9 + 7435288*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))^6*a^2*b^6*c^(5/2)*f*g^6*h^9 + 92170400*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))^6*a^3*b^4*c^(7/2)*f*g^6*h^9 + 277683840*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))^6*a^4*b^2*c^(9/2)*f*g^6*h^9 - 92979712*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))^6*a^5*c^(11/2)*f*g^6*h^9 + 2566620*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^6*b^9*c^(3/2)*d*g^5*h^10 + 34171200*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^6*a*b^7*c^(5/2)*d*g^5*h^10 + 86402400*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^6*a^2*b^5*c^(7/2)*d*g^5*h^10 + 119696640*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^6*a^3*b^3*c^(9/2)*d*g^5*h^10 + 32524800*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))^6*a^4*b*c^(11/2)*d*g^5*h^10 + 107520*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^6*b^10*sqrt(c)*e*g^5*h^10 - 3403680*(sqrt(c)*x - sqrt(c*
x^2 + b*x + a))^6*a*b^8*c^(3/2)*e*g^5*h^10 - 5820080*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^6*a^2*b^6*c^(5/2)*e*g^5*h^10 + 30531200*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))^6*a^3*b^4*c^(7/2)*e*g^5*h^10 - 25670400*(sqrt(c)*x - sqrt(c*
x^2 + b*x + a))^6*a^4*b^2*c^(9/2)*e*g^5*h^10 + 103362560*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^6*a^5*c^(11/2)*e*g^5*h^10 - 752640*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^6*a*b^9*sqrt(c)*f*g^5*h^10 + 11935980*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^6*a^2*b^7*c^(3/2)*f*g^5*h^10 - 40625200*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^6*a^3*b^5*c^(5/2)*f*g^5*h^10 - 75535040*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))^6*a^4*b^3*c^(7/2)*f*g^5*h^10 - 246140160*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))^6*a^5*b*c^(9/2)*f*g^5*h^10 - 193536*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^6*b^10*sqrt(c)*d*g^4*h^11 - 11971512*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^6*a*b^8*c^(3/2)*d*g^4*h^11 - 57480696*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))^6*a^2*b^6*c^(5/2)*d*g^4*h^11 - 98733600*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^6*a^3*b^4*c^(7/2)*d*g^4*h^11 - 88287360*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^6*a^4*b^2*c^(9/2)*d*g^4*h^11 - 62522880*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^6*a^5*c^(11/2)*d*g^4*h^11 - 451584*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))^6*a*b^9*sqrt(c)*e*g^4*h^11 + 9023868*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^6*a^2*b^7*c^(3/2)*e*g^4*h^11 - 10687600*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^6*a^3*b^5*c^(5/2)*e*g^4*h^11 - 20081600*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))^6*a^4*b^3*c^(7/2)*e*g^4*h^11 + 45561600*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))^6*a^5*b*c^(9/2)*e*g^4*h^11 + 2171904*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^6*a^2*b^8*sqrt(c)*f*g^4*h^11 - 18374384*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))^6*a^3*b^6*c^(3/2)*f*g^4*h^11 + 74883200*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))^6*a^4*b^4*c^(5/2)*f*g^4*h^11 + 19810560*(sqrt(c)*x - sqrt(c*x^2
```

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+ b*x + a))^6*a^5*b^2*c^(7/2)*f*g^4*h^11 + 97914880*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^6*a^6*c^(9/2)*f*g^4*h^11 + 580608*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^6*a*b^9*sqrt(c)*d*g^3*h^12 + 22178268*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))^6*a^2*b^7*c^(3/2)*d*g^3*h^12 + 51824976*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^6*a^3*b^5*c^(5/2)*d*g^3*h^12 + 91022400*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^6*a^4*b^3*c^(7/2)*d*g^3*h^12 + 42443520*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))^6*a^5*b*c^(9/2)*d*g^3*h^12 + 924672*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))^6*a^2*b^8*sqrt(c)*e*g^3*h^12 - 10673096*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^6*a^3*b^6*c^(3/2)*e*g^3*h^12 + 21677600*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^6*a^4*b^4*c^(5/2)*e*g^3*h^12 - 16329600*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))^6*a^5*b^2*c^(7/2)*e*g^3*h^12 - 39764480*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))^6*a^6*c^(9/2)*e*g^3*h^12 - 3505152*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))^6*a^3*b^7*sqrt(c)*f*g^3*h^12 + 14284704*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^6*a^4*b^5*c^(3/2)*f*g^3*h^12 - 64288000*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^6*a^5*b^3*c^(5/2)*f*g^3*h^12 + 29084160*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))^6*a^6*b*c^(7/2)*f*g^3*h^12 - 580608*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))^6*a^2*b^8*sqrt(c)*d*g^2*h^13 - 21288960*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^6*a^3*b^6*c^(3/2)*d*g^2*h^13 - 36771840*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^6*a^4*b^4*c^(5/2)*d*g^2*h^13 - 41717760*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))^6*a^5*b^2*c^(7/2)*d*g^2*h^13 + 4300800*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))^6*a^6*c^(9/2)*d*g^2*h^13 - 1139712*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^6*a^3*b^7*sqrt(c)*e*g^2*h^13 + 6494208*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^6*a^4*b^5*c^(3/2)*e*g^2*h^13 - 8243200*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^6*a^5*b^3*c^(5/2)*e*g^2*h^13 + 15912960*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))^6*a^6*b*c^(7/2)*e*g^2*h^13 + 3397632*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))^6*a^4*b^6*sqrt(c)*f*g^2*h^13 - 5160960*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))^6*a^5*b^4*c^(3/2)*f*g^2*h^13 + 21504000*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^6*a^6*b^2*c^(5/2)*f*g^2*h^13 - 21504000*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))^6*a^7*c^(7/2)*f*g^2*h^13 + 193536*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)^6*a^3*b^7*sqrt(c)*d*g*h^14 + 11096064*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
^6*a^4*b^5*c^(3/2)*d*g*h^14 + 18708480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^
6*a^5*b^3*c^(5/2)*d*g*h^14 + 1720320*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*
a^6*b*c^(7/2)*d*g*h^14 + 774144*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^4*b
^6*sqrt(c)*e*g*h^14 - 2580480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^5*b^4
*c^(3/2)*e*g*h^14 - 860160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^6*b^2*c^
(5/2)*e*g*h^14 + 1720320*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^7*c^(7/2)*
e*g*h^14 - 1849344*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^5*b^5*sqrt(c)*f*
g*h^14 + 1146880*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^6*b^3*c^(3/2)*f*g*
h^14 - 1290240*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^7*b*c^(5/2)*f*g*h^14
 - 2580480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^5*b^4*c^(3/2)*d*h^15 - 3
440640*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^6*b^2*c^(5/2)*d*h^15 - 86016
0*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^7*c^(7/2)*d*h^15 - 215040*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^6*a^5*b^5*sqrt(c)*e*h^15 + 716800*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))^6*a^6*b^3*c^(3/2)*e*h^15 - 430080*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^6*a^7*b*c^(5/2)*e*h^15 + 430080*(sqrt(c)*x - sqrt(c*x^2
```

```
+ b*x + a))^6*a^6*b^4*sqrt(c)*f*h^15 - 430080*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))^6*a^7*b^2*c^(3/2)*f*h^15 + 860160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)^6*a^8*c^(5/2)*f*h^15 - 1720320*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^2*
c^9*f*g^15 - 688128*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^2*c^9*e*g^14*h
+ 4702208*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^3*c^8*f*g^14*h + 3440640*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b*c^9*f*g^14*h - 516096*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))^5*b^2*c^9*d*g^13*h^2 + 1519616*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))^5*b^3*c^8*e*g^13*h^2 + 1376256*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^5*a*b*c^9*e*g^13*h^2 - 157696*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^
5*b^4*c^7*f*g^13*h^2 - 15912960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b^2
*c^8*f*g^13*h^2 - 1720320*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*c^9*f*g
^13*h^2 + 688128*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^3*c^8*d*g^12*h^3 +
 1032192*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b*c^9*d*g^12*h^3 + 960512*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^4*c^7*e*g^12*h^3 - 6365184*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^5*a*b^2*c^8*e*g^12*h^3 - 688128*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))^5*a^2*c^9*e*g^12*h^3 - 10135552*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^5*b^5*c^6*f*g^12*h^3 + 5748736*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^5*a*b^3*c^7*f*g^12*h^3 + 17719296*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
^5*a^2*b*c^8*f*g^12*h^3 + 1774080*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^4
*c^7*d*g^11*h^4 - 5677056*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b^2*c^8*d
*g^11*h^4 - 516096*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*c^9*d*g^11*h^4
 - 4076800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^5*c^6*e*g^11*h^4 + 37273
6*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b^3*c^7*e*g^11*h^4 + 8171520*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*b*c^8*e*g^11*h^4 + 8859200*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))^5*b^6*c^5*f*g^11*h^4 + 50564864*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^5*a*b^4*c^6*f*g^11*h^4 - 17654784*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^5*a^2*b^2*c^7*f*g^11*h^4 - 6508544*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))^5*a^3*c^8*f*g^11*h^4 - 1881600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)^5*b^5*c^6*d*g^10*h^5 - 4085760*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b^
3*c^7*d*g^10*h^5 + 9289728*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*b*c^8*
d*g^10*h^5 + 815360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^6*c^5*e*g^10*h^
5 + 25088000*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b^4*c^6*e*g^10*h^5 - 1
2300288*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*b^2*c^7*e*g^10*h^5 - 3325
952*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^3*c^8*e*g^10*h^5 + 929600*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^5*b^7*c^4*f*g^10*h^5 - 66796800*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))^5*a*b^5*c^5*f*g^10*h^5 - 93678592*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^5*a^2*b^3*c^6*f*g^10*h^5 + 18694144*(sqrt(c)*x - sqrt(c*
x^2 + b*x + a))^5*a^3*b*c^7*f*g^10*h^5 - 716352*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^5*b^6*c^5*d*g^9*h^6 + 9010176*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^
5*a*b^4*c^6*d*g^9*h^6 + 44448768*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*
b^2*c^7*d*g^9*h^6 - 6021120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^3*c^8*d
*g^9*h^6 + 1485344*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^7*c^4*e*g^9*h^6
- 10418688*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b^5*c^5*e*g^9*h^6 - 7240
0384*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*b^3*c^6*e*g^9*h^6 + 23080960
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^3*b*c^7*e*g^9*h^6 - 1588384*(sqrt(
```

```
c)*x - sqrt(c*x^2 + b*x + a))^5*b^8*c^3*f*g^9*h^6 - 939680*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^5*a*b^6*c^4*f*g^9*h^6 + 213376128*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))^5*a^2*b^4*c^5*f*g^9*h^6 + 66099712*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))^5*a^3*b^2*c^6*f*g^9*h^6 - 6522880*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^5*a^4*c^7*f*g^9*h^6 + 3698688*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b
^7*c^4*d*g^8*h^7 - 32145792*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b^5*c^5
*d*g^8*h^7 - 33008640*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*b^3*c^6*d*g
^8*h^7 - 104380416*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^3*b*c^7*d*g^8*h^
7 - 1255408*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^8*c^3*e*g^8*h^7 + 10165
12*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b^6*c^4*e*g^8*h^7 + 28879872*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*b^4*c^5*e*g^8*h^7 + 148728832*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^5*a^3*b^2*c^6*e*g^8*h^7 - 11038720*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))^5*a^4*c^7*e*g^8*h^7 + 62048*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))^5*b^9*c^2*f*g^8*h^7 + 9606800*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^5*a*b^7*c^3*f*g^8*h^7 - 5651520*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5
*a^2*b^5*c^4*f*g^8*h^7 - 387414272*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^
3*b^3*c^5*f*g^8*h^7 + 480256*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^4*b*c^
6*f*g^8*h^7 - 1320144*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^8*c^3*d*g^7*h
^8 + 24532704*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b^6*c^4*d*g^7*h^8 + 1
20922368*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*b^4*c^5*d*g^7*h^8 + 1018
2144*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^3*b^2*c^6*d*g^7*h^8 + 56168448
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^4*c^7*d*g^7*h^8 - 229936*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^5*b^9*c^2*e*g^7*h^8 + 6378512*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^5*a*b^7*c^3*e*g^7*h^8 - 36575616*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^5*a^2*b^5*c^4*e*g^7*h^8 - 26179328*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^5*a^3*b^3*c^5*e*g^7*h^8 - 171057152*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))^5*a^4*b*c^6*e*g^7*h^8 + 255206*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b
^10*c*f*g^7*h^8 - 1832208*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b^8*c^2*f
*g^7*h^8 - 22544368*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*b^6*c^3*f*g^7
*h^8 + 7322560*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^3*b^4*c^4*f*g^7*h^8
+ 446492928*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^4*b^2*c^5*f*g^7*h^8 - 1
2766208*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^5*c^6*f*g^7*h^8 - 1126440*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^9*c^2*d*g^6*h^9 - 12551952*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^5*a*b^7*c^3*d*g^6*h^9 - 128046912*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^5*a^2*b^5*c^4*d*g^6*h^9 - 108632832*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^5*a^3*b^3*c^5*d*g^6*h^9 + 86166528*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))^5*a^4*b*c^6*d*g^6*h^9 + 243320*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))^5*b^10*c*e*g^6*h^9 + 403872*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*
b^8*c^2*e*g^6*h^9 - 1627360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*b^6*c
^3*e*g^6*h^9 + 95455360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^3*b^4*c^4*e
*g^6*h^9 - 55792128*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^4*b^2*c^5*e*g^6
*h^9 + 73529344*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^5*c^6*e*g^6*h^9 + 9
905*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^11*f*g^6*h^9 - 1845872*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^5*a*b^9*c*f*g^6*h^9 + 10761072*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^5*a^2*b^7*c^2*f*g^6*h^9 + 18340672*(sqrt(c)*x - sqrt(c*
```

```
x^2 + b*x + a))^5*a^3*b^5*c^3*f*g^6*h^9 + 31861760*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^5*a^4*b^3*c^4*f*g^6*h^9 - 307786752*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^5*a^5*b*c^5*f*g^6*h^9 + 641130*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
^5*b^10*c*d*g^5*h^10 + 9335760*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b^8*
c^2*d*g^5*h^10 + 65483376*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*b^6*c^3
*d*g^5*h^10 + 166965120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^3*b^4*c^4*d
*g^5*h^10 + 8563968*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^4*b^2*c^5*d*g^5
*h^10 - 71221248*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^5*c^6*d*g^5*h^10 +
 9905*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^11*e*g^5*h^10 - 1673840*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b^9*c*e*g^5*h^10 - 1025976*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^5*a^2*b^7*c^2*e*g^5*h^10 - 25425344*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^5*a^3*b^5*c^3*e*g^5*h^10 - 75564160*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))^5*a^4*b^3*c^4*e*g^5*h^10 + 125110272*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))^5*a^5*b*c^5*e*g^5*h^10 - 79240*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))^5*a*b^10*f*g^5*h^10 + 5745936*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*
a^2*b^8*c*f*g^5*h^10 - 27611192*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^3*b
^6*c^2*f*g^5*h^10 + 7097440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^4*b^4*c
^3*f*g^5*h^10 - 110006400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^5*b^2*c^4
*f*g^5*h^10 + 93280768*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^6*c^5*f*g^5*
h^10 - 25179*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^11*d*g^4*h^11 - 381292
8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b^9*c*d*g^4*h^11 - 24937584*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*b^7*c^2*d*g^4*h^11 - 107724960*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^5*a^3*b^5*c^3*d*g^4*h^11 - 87494400*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))^5*a^4*b^3*c^4*d*g^4*h^11 - 9515520*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^5*a^5*b*c^5*d*g^4*h^11 - 24346*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^5*a*b^10*e*g^4*h^11 + 4979772*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))^5*a^2*b^8*c*e*g^4*h^11 + 1511440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5
*a^3*b^6*c^2*e*g^4*h^11 + 38147200*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^
4*b^4*c^3*e*g^4*h^11 + 7929600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^5*b^
2*c^4*e*g^4*h^11 - 62397440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^6*c^5*e
*g^4*h^11 + 222446*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*b^9*f*g^4*h^11
 - 9887276*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^3*b^7*c*f*g^4*h^11 + 408
02160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^4*b^5*c^2*f*g^4*h^11 - 141008
00*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^5*b^3*c^3*f*g^4*h^11 + 122940160
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^6*b*c^4*f*g^4*h^11 + 100716*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^5*a*b^10*d*g^3*h^12 + 8360520*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))^5*a^2*b^8*c*d*g^3*h^12 + 33017208*(sqrt(c)*x - sqrt(c*
x^2 + b*x + a))^5*a^3*b^6*c^2*d*g^3*h^12 + 86410800*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^5*a^4*b^4*c^3*d*g^3*h^12 + 40965120*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))^5*a^5*b^2*c^4*d*g^3*h^12 + 21477120*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))^5*a^6*c^5*d*g^3*h^12 - 1666*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^
2*b^9*e*g^3*h^12 - 7567196*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^3*b^7*c*
e*g^3*h^12 + 709800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^4*b^5*c^2*e*g^3
*h^12 - 23206400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^5*b^3*c^3*e*g^3*h^
12 - 5237120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^6*b*c^4*e*g^3*h^12 - 2
```

```
95484*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^3*b^8*f*g^3*h^12 + 9595082*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^4*b^6*c*f*g^3*h^12 - 38812200*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^5*a^5*b^4*c^2*f*g^3*h^12 + 4597600*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))^5*a^6*b^2*c^3*f*g^3*h^12 - 46453120*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^5*a^7*c^4*f*g^3*h^12 - 151074*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^5*a^2*b^9*d*g^2*h^13 - 8615292*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))^5*a^3*b^7*c*d*g^2*h^13 - 26542824*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^
5*a^4*b^5*c^2*d*g^2*h^13 - 42887040*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a
^5*b^3*c^3*d*g^2*h^13 - 19313280*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^6*
b*c^4*d*g^2*h^13 + 52024*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^3*b^8*e*g^
2*h^13 + 5911556*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^4*b^6*c*e*g^2*h^13
 - 1512000*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^5*b^4*c^2*e*g^2*h^13 + 1
2523840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^6*b^2*c^3*e*g^2*h^13 + 1148
6720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^7*c^4*e*g^2*h^13 + 195601*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^5*a^4*b^7*f*g^2*h^13 - 4592196*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))^5*a^5*b^5*c*f*g^2*h^13 + 22402800*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^5*a^6*b^3*c^2*f*g^2*h^13 - 6319040*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))^5*a^7*b*c^3*f*g^2*h^13 + 100716*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^5*a^3*b^8*d*g*h^14 + 4195086*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*
a^4*b^6*c*d*g*h^14 + 13864200*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^5*b^4
*c^2*d*g*h^14 + 14518560*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^6*b^2*c^3*
d*g*h^14 - 1303680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^7*c^4*d*g*h^14 -
 51191*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^4*b^7*e*g*h^14 - 2131332*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^5*b^5*c*e*g*h^14 - 834960*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))^5*a^6*b^3*c^2*e*g*h^14 - 5673920*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^5*a^7*b*c^3*e*g*h^14 - 57764*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^5*a^5*b^6*f*g*h^14 + 507360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a
^6*b^4*c*f*g*h^14 - 5759040*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^7*b^2*c
^2*f*g*h^14 + 4910080*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^8*c^3*f*g*h^1
4 - 25179*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^4*b^7*d*h^15 - 768516*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^5*b^5*c*d*h^15 - 3610320*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^5*a^6*b^3*c^2*d*h^15 - 1928640*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))^5*a^7*b*c^3*d*h^15 + 15274*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))^5*a^5*b^6*e*h^15 + 237720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^6*b^4*
c*e*h^15 + 977760*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^7*b^2*c^2*e*h^15
+ 138880*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^8*c^3*e*h^15 + 4536*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^5*a^6*b^5*f*h^15 + 221760*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))^5*a^7*b^3*c*f*h^15 - 13440*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))^5*a^8*b*c^2*f*h^15 - 1433600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^3
*c^(17/2)*f*g^15 - 573440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^3*c^(17/2
)*e*g^14*h + 5232640*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^4*c^(15/2)*f*g
^14*h + 4300800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b^2*c^(17/2)*f*g^14
*h - 430080*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^3*c^(17/2)*d*g^13*h^2 +
 1792000*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^4*c^(15/2)*e*g^13*h^2 + 17
20320*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b^2*c^(17/2)*e*g^13*h^2 - 551
```

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9360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^5*c^(13/2)*f*g^13*h^2 - 224358
40*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b^3*c^(15/2)*f*g^13*h^2 - 430080
0*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*b*c^(17/2)*f*g^13*h^2 + 967680*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^4*c^(15/2)*d*g^12*h^3 + 1290240*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b^2*c^(17/2)*d*g^12*h^3 - 1078784*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^4*b^5*c^(13/2)*e*g^12*h^3 - 8673280*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^4*a*b^3*c^(15/2)*e*g^12*h^3 - 1720320*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^4*a^2*b*c^(17/2)*e*g^12*h^3 - 834176*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))^4*b^6*c^(11/2)*f*g^12*h^3 + 33241600*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))^4*a*b^4*c^(13/2)*f*g^12*h^3 + 35911680*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))^4*a^2*b^2*c^(15/2)*f*g^12*h^3 + 1433600*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))^4*a^3*c^(17/2)*f*g^12*h^3 + 413952*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))^4*b^5*c^(13/2)*d*g^11*h^4 - 6881280*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^4*a*b^3*c^(15/2)*d*g^11*h^4 - 1290240*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))^4*a^2*b*c^(17/2)*d*g^11*h^4 - 1511552*(sqrt(c)*x - sqrt(c*
x^2 + b*x + a))^4*b^6*c^(11/2)*e*g^11*h^4 + 10286080*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^4*a*b^4*c^(13/2)*e*g^11*h^4 + 15267840*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^4*a^2*b^2*c^(15/2)*e*g^11*h^4 + 573440*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^4*a^3*c^(17/2)*e*g^11*h^4 + 4158112*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))^4*b^7*c^(9/2)*f*g^11*h^4 - 884352*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^4*a*b^5*c^(11/2)*f*g^11*h^4 - 78901760*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^4*a^2*b^3*c^(13/2)*f*g^11*h^4 - 25446400*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^4*a^3*b*c^(15/2)*f*g^11*h^4 - 1430016*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))^4*b^6*c^(11/2)*d*g^10*h^5 + 3198720*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))^4*a*b^4*c^(13/2)*d*g^10*h^5 + 14837760*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))^4*a^2*b^2*c^(15/2)*d*g^10*h^5 + 430080*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))^4*a^3*c^(17/2)*d*g^10*h^5 + 1017856*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
^4*b^7*c^(9/2)*e*g^10*h^5 + 7883904*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a
*b^5*c^(11/2)*e*g^10*h^5 - 34872320*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a
^2*b^3*c^(13/2)*e*g^10*h^5 - 11683840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4
*a^3*b*c^(15/2)*e*g^10*h^5 - 674576*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b
^8*c^(7/2)*f*g^10*h^5 - 32333728*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b^
6*c^(9/2)*f*g^10*h^5 + 23990400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*b
^4*c^(11/2)*f*g^10*h^5 + 92449280*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^3
*b^2*c^(13/2)*f*g^10*h^5 + 6737920*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^
4*c^(15/2)*f*g^10*h^5 + 971040*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^7*c^
(9/2)*d*g^9*h^6 - 2876160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b^5*c^(11
/2)*d*g^9*h^6 + 20912640*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*b^3*c^(1
3/2)*d*g^9*h^6 - 17203200*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^3*b*c^(15
/2)*d*g^9*h^6 + 160720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^8*c^(7/2)*e*
g^9*h^6 - 6294400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b^6*c^(9/2)*e*g^9
*h^6 - 26261760*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*b^4*c^(11/2)*e*g^
9*h^6 + 58634240*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^3*b^2*c^(13/2)*e*g
^9*h^6 + 3727360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^4*c^(15/2)*e*g^9*h
^6 - 503720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^9*c^(5/2)*f*g^9*h^6 + 7
```

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549360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b^7*c^(7/2)*f*g^9*h^6 + 1127
68320*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*b^5*c^(9/2)*f*g^9*h^6 - 827
27680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^3*b^3*c^(11/2)*f*g^9*h^6 - 53
724160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^4*b*c^(13/2)*f*g^9*h^6 + 123
3120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^8*c^(7/2)*d*g^8*h^7 - 9975840*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b^6*c^(9/2)*d*g^8*h^7 - 7566720*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*b^4*c^(11/2)*d*g^8*h^7 - 97574400*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^3*b^2*c^(13/2)*d*g^8*h^7 + 5806080*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^4*c^(15/2)*d*g^8*h^7 - 673400*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^4*b^9*c^(5/2)*e*g^8*h^7 + 3164560*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))^4*a*b^7*c^(7/2)*e*g^8*h^7 + 13702080*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^4*a^2*b^5*c^(9/2)*e*g^8*h^7 + 83767040*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^4*a^3*b^3*c^(11/2)*e*g^8*h^7 - 47272960*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^4*a^4*b*c^(13/2)*e*g^8*h^7 + 200830*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^4*b^10*c^(3/2)*f*g^8*h^7 + 2173920*(sqrt(c)*x - sqrt(c*
x^2 + b*x + a))^4*a*b^8*c^(5/2)*f*g^8*h^7 - 30511040*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^4*a^2*b^6*c^(7/2)*f*g^8*h^7 - 234814720*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))^4*a^3*b^4*c^(9/2)*f*g^8*h^7 + 129615360*(sqrt(c)*x - sqrt(c*
x^2 + b*x + a))^4*a^4*b^2*c^(11/2)*f*g^8*h^7 + 12400640*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))^4*a^5*c^(13/2)*f*g^8*h^7 - 1334256*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^4*b^9*c^(5/2)*d*g^7*h^8 + 7550928*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^4*a*b^7*c^(7/2)*d*g^7*h^8 + 48378624*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^4*a^2*b^5*c^(9/2)*d*g^7*h^8 + 60453120*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^4*a^3*b^3*c^(11/2)*d*g^7*h^8 + 117196800*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^4*a^4*b*c^(13/2)*d*g^7*h^8 + 141400*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))^4*b^10*c^(3/2)*e*g^7*h^8 + 2743776*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
^4*a*b^8*c^(5/2)*e*g^7*h^8 - 22437632*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4
*a^2*b^6*c^(7/2)*e*g^7*h^8 - 23873920*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4
*a^3*b^4*c^(9/2)*e*g^7*h^8 - 167623680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^
4*a^4*b^2*c^(11/2)*e*g^7*h^8 + 15239168*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
^4*a^5*c^(13/2)*e*g^7*h^8 + 49525*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^1
1*sqrt(c)*f*g^7*h^8 - 1702960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b^9*c
^(3/2)*f*g^7*h^8 - 693336*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*b^7*c^(
5/2)*f*g^7*h^8 + 61982816*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^3*b^5*c^(
7/2)*f*g^7*h^8 + 335762560*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^4*b^3*c^
(9/2)*f*g^7*h^8 - 96757248*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^5*b*c^(1
1/2)*f*g^7*h^8 + 195090*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^10*c^(3/2)*
d*g^6*h^9 - 194208*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b^8*c^(5/2)*d*g^
6*h^9 - 56254464*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*b^6*c^(7/2)*d*g^
6*h^9 - 101277120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^3*b^4*c^(9/2)*d*g
^6*h^9 - 39029760*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^4*b^2*c^(11/2)*d*
g^6*h^9 - 46685184*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^5*c^(13/2)*d*g^6
*h^9 + 49525*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^11*sqrt(c)*e*g^6*h^9 -
 1057840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b^9*c^(3/2)*e*g^6*h^9 + 10
24128*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*b^7*c^(5/2)*e*g^6*h^9 + 636
```

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14432*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^3*b^5*c^(7/2)*e*g^6*h^9 + 273
81760*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^4*b^3*c^(9/2)*e*g^6*h^9 + 153
828864*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^5*b*c^(11/2)*e*g^6*h^9 - 396
200*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b^10*sqrt(c)*f*g^6*h^9 + 636552
0*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*b^8*c^(3/2)*f*g^6*h^9 - 1507928
8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^3*b^6*c^(5/2)*f*g^6*h^9 - 7211344
0*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^4*b^4*c^(7/2)*f*g^6*h^9 - 3337770
24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^5*b^2*c^(9/2)*f*g^6*h^9 + 276362
24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^6*c^(11/2)*f*g^6*h^9 + 89145*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^11*sqrt(c)*d*g^5*h^10 + 73920*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^4*a*b^9*c^(3/2)*d*g^5*h^10 + 23256744*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^4*a^2*b^7*c^(5/2)*d*g^5*h^10 + 113116416*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^4*a^3*b^5*c^(7/2)*d*g^5*h^10 + 77831040*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^4*a^4*b^3*c^(9/2)*d*g^5*h^10 - 37997568*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^4*a^5*b*c^(11/2)*d*g^5*h^10 - 336770*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^4*a*b^10*sqrt(c)*e*g^5*h^10 + 2319660*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^4*a^2*b^8*c^(3/2)*e*g^5*h^10 - 23881984*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^4*a^3*b^6*c^(5/2)*e*g^5*h^10 - 83836480*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^4*a^4*b^4*c^(7/2)*e*g^5*h^10 + 10149888*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^4*a^5*b^2*c^(9/2)*e*g^5*h^10 - 50003968*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^4*a^6*c^(11/2)*e*g^5*h^10 + 1327270*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^4*a^2*b^9*sqrt(c)*f*g^5*h^10 - 12664540*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^4*a^3*b^7*c^(3/2)*f*g^5*h^10 + 44371824*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^4*a^4*b^5*c^(5/2)*f*g^5*h^10 + 38424512*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^4*a^5*b^3*c^(7/2)*f*g^5*h^10 + 200297216*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^6*b*c^(9/2)*f*g^5*h^10 - 571620*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b^10*sqrt(c)*d*g^4*h^11 - 4000920*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*b^8*c^(3/2)*d*g^4*h^11 - 58985640*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^4*a^3*b^6*c^(5/2)*d*g^4*h^11 - 99094800*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^4*a^4*b^4*c^(7/2)*d*g^4*h^11 - 6773760*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^4*a^5*b^2*c^(9/2)*d*g^4*h^11 + 28250880*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^6*c^(11/2)*d*g^4*h^11 + 1066870*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*b^9*sqrt(c)*e*g^4*h^11 - 634060*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^4*a^3*b^7*c^(3/2)*e*g^4*h^11 + 48707400*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^4*a^4*b^5*c^(5/2)*e*g^4*h^11 + 44387840*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^4*a^5*b^3*c^(7/2)*e*g^4*h^11 - 45109120*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^4*a^6*b*c^(9/2)*e*g^4*h^11 - 2552620*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^4*a^3*b^8*sqrt(c)*f*g^4*h^11 + 13999090*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^4*a^4*b^6*c^(3/2)*f*g^4*h^11 - 56650440*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^4*a^5*b^4*c^(5/2)*f*g^4*h^11 + 16966880*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^6*b^2*c^(7/2)*f*g^4*h^11 - 52062080*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^7*c^(9/2)*f*g^4*h^11 + 1395030*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*b^9*sqrt(c)*d*g^3*h^12 + 10898580*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^4*a^3*b^7*c^(3/2)*d*g^3*h^12 + 62112120*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^4*a^4*b^5*c^(5/2)*d*g^3*h^12 + 41462400*(sq
```

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rt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^5*b^3*c^(7/2)*d*g^3*h^12 - 2808960*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^6*b*c^(9/2)*d*g^3*h^12 - 1890280*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^4*a^3*b^8*sqrt(c)*e*g^3*h^12 - 3988460*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^4*a^4*b^6*c^(3/2)*e*g^3*h^12 - 41106240*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^4*a^5*b^4*c^(5/2)*e*g^3*h^12 - 1032640*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^4*a^6*b^2*c^(7/2)*e*g^3*h^12 + 23027200*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^7*c^(9/2)*e*g^3*h^12 + 3128405*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^4*a^4*b^7*sqrt(c)*f*g^3*h^12 - 8768340*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^4*a^5*b^5*c^(3/2)*f*g^3*h^12 + 33739440*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^4*a^6*b^3*c^(5/2)*f*g^3*h^12 - 35035840*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^4*a^7*b*c^(7/2)*f*g^3*h^12 - 1646820*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^4*a^3*b^8*sqrt(c)*d*g^2*h^13 - 12355770*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^4*a^4*b^6*c^(3/2)*d*g^2*h^13 - 34156248*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^4*a^5*b^4*c^(5/2)*d*g^2*h^13 - 10928736*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^4*a^6*b^2*c^(7/2)*d*g^2*h^13 - 6346368*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^4*a^7*c^(9/2)*d*g^2*h^13 + 1894445*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^4*a^4*b^7*sqrt(c)*e*g^2*h^13 + 5471340*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^4*a^5*b^5*c^(3/2)*e*g^2*h^13 + 15866928*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^4*a^6*b^3*c^(5/2)*e*g^2*h^13 - 2650816*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^4*a^7*b*c^(7/2)*e*g^2*h^13 - 2439220*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^4*a^5*b^6*sqrt(c)*f*g^2*h^13 + 3612000*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^4*a^6*b^4*c^(3/2)*f*g^2*h^13 - 7119168*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^4*a^7*b^2*c^(5/2)*f*g^2*h^13 + 13411328*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^4*a^8*c^(7/2)*f*g^2*h^13 + 949305*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^4*a^4*b^7*sqrt(c)*d*g*h^14 + 6479340*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))^4*a^5*b^5*c^(3/2)*d*g*h^14 + 12140016*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^4*a^6*b^3*c^(5/2)*d*g*h^14 + 3087168*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^4*a^7*b*c^(7/2)*d*g*h^14 - 998830*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^4*a^5*b^6*sqrt(c)*e*g*h^14 - 2682120*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))^4*a^6*b^4*c^(3/2)*e*g*h^14 - 3798816*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^4*a^7*b^2*c^(5/2)*e*g*h^14 - 1369984*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))^4*a^8*c^(7/2)*e*g*h^14 + 1097880*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
^4*a^6*b^5*sqrt(c)*f*g*h^14 - 1471680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4
*a^7*b^3*c^(3/2)*f*g*h^14 + 233856*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^
8*b*c^(5/2)*f*g*h^14 - 215040*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^5*b^6
*sqrt(c)*d*h^15 - 1290240*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^6*b^4*c^(
3/2)*d*h^15 - 2838528*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^7*b^2*c^(5/2)
*d*h^15 - 172032*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^8*c^(7/2)*d*h^15 +
 215040*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^6*b^5*sqrt(c)*e*h^15 + 4300
80*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^7*b^3*c^(3/2)*e*h^15 + 1118208*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^8*b*c^(5/2)*e*h^15 - 215040*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^4*a^7*b^4*sqrt(c)*f*h^15 + 430080*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^4*a^8*b^2*c^(3/2)*f*h^15 - 473088*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^4*a^9*c^(5/2)*f*h^15 - 716800*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^3*b^4*c^8*f*g^15 - 286720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^
```

```
4*c^8*e*g^14*h + 2938880*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^5*c^7*f*g^
14*h + 2867200*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^3*c^8*f*g^14*h - 2
15040*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^4*c^8*d*g^13*h^2 + 1025024*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^5*c^7*e*g^13*h^2 + 1146880*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))^3*a*b^3*c^8*e*g^13*h^2 - 4152064*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^3*b^6*c^6*f*g^13*h^2 - 15447040*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^3*a*b^4*c^7*f*g^13*h^2 - 4300800*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))^3*a^2*b^2*c^8*f*g^13*h^2 + 580608*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)^3*b^5*c^7*d*g^12*h^3 + 860160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^3
*c^8*d*g^12*h^3 - 1028608*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^6*c^6*e*g
^12*h^3 - 5877760*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^4*c^7*e*g^12*h^
3 - 1720320*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b^2*c^8*e*g^12*h^3 +
1758848*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^7*c^5*f*g^12*h^3 + 28073472
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^5*c^6*f*g^12*h^3 + 32399360*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b^3*c^7*f*g^12*h^3 + 2867200*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^3*a^3*b*c^8*f*g^12*h^3 - 75264*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^3*b^6*c^6*d*g^11*h^4 - 4408320*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^3*a*b^4*c^7*d*g^11*h^4 - 1290240*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^3*a^2*b^2*c^8*d*g^11*h^4 - 184576*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
^3*b^7*c^5*e*g^11*h^4 + 8956416*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^5
*c^6*e*g^11*h^4 + 13260800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b^3*c^
7*e*g^11*h^4 + 1146880*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^3*b*c^8*e*g^
11*h^4 + 790496*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^8*c^4*f*g^11*h^4 -
16752512*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^6*c^5*f*g^11*h^4 - 78650
880*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b^4*c^6*f*g^11*h^4 - 33904640
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^3*b^2*c^7*f*g^11*h^4 - 716800*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^3*a^4*c^8*f*g^11*h^4 - 618240*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^3*b^7*c^5*d*g^10*h^5 + 3763200*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))^3*a*b^5*c^6*d*g^10*h^5 + 11827200*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))^3*a^2*b^3*c^7*d*g^10*h^5 + 860160*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))^3*a^3*b*c^8*d*g^10*h^5 + 459200*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*
b^8*c^4*e*g^10*h^5 - 627200*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^6*c^5
*e*g^10*h^5 - 31610880*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b^4*c^6*e*
g^10*h^5 - 14766080*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^3*b^2*c^7*e*g^1
0*h^5 - 286720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^4*c^8*e*g^10*h^5 - 3
72400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^9*c^3*f*g^10*h^5 - 5608960*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^7*c^4*f*g^10*h^5 + 67361280*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b^5*c^5*f*g^10*h^5 + 116910080*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^3*a^3*b^3*c^6*f*g^10*h^5 + 17704960*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))^3*a^4*b*c^7*f*g^10*h^5 + 913920*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))^3*b^8*c^4*d*g^9*h^6 - 4556160*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^3*a*b^6*c^5*d*g^9*h^6 + 2768640*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
^3*a^2*b^4*c^6*d*g^9*h^6 - 18923520*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a
^3*b^2*c^7*d*g^9*h^6 - 430080*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^4*c^8
*d*g^9*h^6 - 192640*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^9*c^3*e*g^9*h^6
```

- 1395520*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^7*c^4*e*g^9*h^6 + 2593
920*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b^5*c^5*e*g^9*h^6 + 60784640*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^3*b^3*c^6*e*g^9*h^6 + 9103360*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^3*a^4*b*c^7*e*g^9*h^6 - 51940*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^3*b^10*c^2*f*g^9*h^6 + 3133200*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^3*a*b^8*c^3*f*g^9*h^6 + 19473440*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^3*a^2*b^6*c^4*f*g^9*h^6 - 153802880*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))^3*a^3*b^4*c^5*f*g^9*h^6 - 97789440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
^3*a^4*b^2*c^6*f*g^9*h^6 - 3799040*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^
5*c^7*f*g^9*h^6 - 122304*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^9*c^3*d*g^
8*h^7 - 1091328*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^7*c^4*d*g^8*h^7 +
8190336*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b^5*c^5*d*g^8*h^7 - 4569
6000*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^3*b^3*c^6*d*g^8*h^7 + 13547520
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^4*b*c^7*d*g^8*h^7 - 133000*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^3*b^10*c^2*e*g^8*h^7 + 1822464*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))^3*a*b^8*c^3*e*g^8*h^7 - 1425088*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^3*a^2*b^6*c^4*e*g^8*h^7 + 13457920*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))^3*a^3*b^4*c^5*e*g^8*h^7 - 68490240*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^3*a^4*b^2*c^6*e*g^8*h^7 - 2121728*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
^3*a^5*c^7*e*g^8*h^7 + 71750*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^11*c*f
*g^8*h^7 - 41720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^9*c^2*f*g^8*h^7
- 11198544*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b^7*c^3*f*g^8*h^7 - 45
891776*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^3*b^5*c^4*f*g^8*h^7 + 219242
240*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^4*b^3*c^5*f*g^8*h^7 + 44018688*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^5*b*c^6*f*g^8*h^7 - 317100*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^3*b^10*c^2*d*g^7*h^8 + 1983408*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^3*a*b^8*c^3*d*g^7*h^8 + 3599904*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^3*a^2*b^6*c^4*d*g^7*h^8 + 28264320*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^3*a^3*b^4*c^5*d*g^7*h^8 + 95316480*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))^3*a^4*b^2*c^6*d*g^7*h^8 - 4171776*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^
3*a^5*c^7*d*g^7*h^8 + 67550*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^11*c*e*
g^7*h^8 + 460600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^9*c^2*e*g^7*h^8
- 6939408*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b^7*c^3*e*g^7*h^8 + 506
7328*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^3*b^5*c^4*e*g^7*h^8 - 70524160
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^4*b^3*c^5*e*g^7*h^8 + 41051136*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^5*b*c^6*e*g^7*h^8 + 3500*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^3*b^12*f*g^7*h^8 - 595000*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))^3*a*b^10*c*f*g^7*h^8 + 2250360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)^3*a^2*b^8*c^2*f*g^7*h^8 + 21562688*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*
a^3*b^6*c^3*f*g^7*h^8 + 86544640*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^4*
b^4*c^4*f*g^7*h^8 - 192439296*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^5*b^2
*c^5*f*g^7*h^8 - 8587264*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^6*c^6*f*g^
7*h^8 + 117390*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^11*c*d*g^6*h^9 + 417
480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^9*c^2*d*g^6*h^9 - 11087664*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b^7*c^3*d*g^6*h^9 - 31663296*(sqrt(

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c)*x - sqrt(c*x^2 + b*x + a))^3*a^3*b^5*c^4*d*g^6*h^9 - 71581440*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))^3*a^4*b^3*c^5*d*g^6*h^9 - 78769152*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^3*a^5*b*c^6*d*g^6*h^9 + 3500*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^3*b^12*e*g^6*h^9 - 476140*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3
*a*b^10*c*e*g^6*h^9 + 824880*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b^8*
c^2*e*g^6*h^9 + 16853984*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^3*b^6*c^3*
e*g^6*h^9 + 12100480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^4*b^4*c^4*e*g^
6*h^9 + 115594752*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^5*b^2*c^5*e*g^6*h
^9 - 9275392*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^6*c^6*e*g^6*h^9 - 3150
0*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^11*f*g^6*h^9 + 2112740*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^3*a^2*b^9*c*f*g^6*h^9 - 9989840*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))^3*a^3*b^7*c^2*f*g^6*h^9 - 26276544*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))^3*a^4*b^5*c^3*f*g^6*h^9 - 131140352*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^3*a^5*b^3*c^4*f*g^6*h^9 + 94603264*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))^3*a^6*b*c^5*f*g^6*h^9 + 6300*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^
3*b^12*d*g^5*h^10 - 498960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^10*c*d
*g^5*h^10 + 3885000*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b^8*c^2*d*g^5
*h^10 + 32464320*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^3*b^6*c^3*d*g^5*h^
10 + 63218400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^4*b^4*c^4*d*g^5*h^10
+ 47013120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^5*b^2*c^5*d*g^5*h^10 + 2
2095360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^6*c^6*d*g^5*h^10 - 27300*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^11*e*g^5*h^10 + 1304660*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))^3*a^2*b^9*c*e*g^5*h^10 - 7679840*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^3*a^3*b^7*c^2*e*g^5*h^10 - 30399600*(sqrt(c)*x - sqrt(c*
x^2 + b*x + a))^3*a^4*b^5*c^3*e*g^5*h^10 - 33434240*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^3*a^5*b^3*c^4*e*g^5*h^10 - 83229440*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))^3*a^6*b*c^5*e*g^5*h^10 + 121800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))^3*a^2*b^10*f*g^5*h^10 - 4137560*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^
3*b^8*c*f*g^5*h^10 + 22710380*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^4*b^6
*c^2*f*g^5*h^10 + 24684240*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^5*b^4*c^
3*f*g^5*h^10 + 134989120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^6*b^2*c^4*
f*g^5*h^10 - 19989760*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^7*c^5*f*g^5*h
^10 - 31500*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^11*d*g^4*h^11 + 45066
0*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b^9*c*d*g^4*h^11 - 14513520*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^3*b^7*c^2*d*g^4*h^11 - 48768720*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^3*a^4*b^5*c^3*d*g^4*h^11 - 44956800*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^3*a^5*b^3*c^4*d*g^4*h^11 + 1263360*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^3*a^6*b*c^5*d*g^4*h^11 + 84000*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))^3*a^2*b^10*e*g^4*h^11 - 1462160*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^3*a^3*b^8*c*e*g^4*h^11 + 18442760*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))^3*a^4*b^6*c^2*e*g^4*h^11 + 34856640*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^
3*a^5*b^4*c^3*e*g^4*h^11 + 18471040*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a
^6*b^2*c^4*e*g^4*h^11 + 22937600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^7*
c^5*e*g^4*h^11 - 259000*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^3*b^9*f*g^4
*h^11 + 4515910*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^4*b^7*c*f*g^4*h^11
```

```
- 31532760*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^5*b^5*c^2*f*g^4*h^11 - 1
7401440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^6*b^3*c^3*f*g^4*h^11 - 7708
7360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^7*b*c^4*f*g^4*h^11 + 63000*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b^10*d*g^3*h^12 + 839160*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))^3*a^3*b^8*c*d*g^3*h^12 + 22055460*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^3*a^4*b^6*c^2*d*g^3*h^12 + 35771568*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))^3*a^5*b^4*c^3*d*g^3*h^12 + 9917376*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^3*a^6*b^2*c^4*d*g^3*h^12 - 7994112*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^3*a^7*c^5*d*g^3*h^12 - 133000*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^
3*a^3*b^9*e*g^3*h^12 - 39410*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^4*b^7*
c*e*g^3*h^12 - 22245720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^5*b^5*c^2*e
*g^3*h^12 - 19527648*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^6*b^3*c^3*e*g^
3*h^12 + 4176256*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^7*b*c^4*e*g^3*h^12
 + 325500*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^4*b^8*f*g^3*h^12 - 213584
0*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^5*b^6*c*f*g^3*h^12 + 26863200*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^6*b^4*c^2*f*g^3*h^12 + 4101888*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^3*a^7*b^2*c^3*f*g^3*h^12 + 17930752*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))^3*a^8*c^4*f*g^3*h^12 - 63000*(sqrt(c)*x - sqrt(c*
x^2 + b*x + a))^3*a^3*b^9*d*g^2*h^13 - 1992690*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^3*a^4*b^7*c*d*g^2*h^13 - 17538360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))^3*a^5*b^5*c^2*d*g^2*h^13 - 11426016*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^
3*a^6*b^3*c^3*d*g^2*h^13 - 701568*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^7
*b*c^4*d*g^2*h^13 + 115500*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^4*b^8*e*
g^2*h^13 + 1568140*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^5*b^6*c*e*g^2*h^
13 + 14557200*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^6*b^4*c^2*e*g^2*h^13
+ 2431296*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^7*b^2*c^3*e*g^2*h^13 - 40
19456*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^8*c^4*e*g^2*h^13 - 241500*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^5*b^7*f*g^2*h^13 - 517440*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))^3*a^6*b^5*c*f*g^2*h^13 - 12868800*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^3*a^7*b^3*c^2*f*g^2*h^13 + 3945984*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))^3*a^8*b*c^3*f*g^2*h^13 + 31500*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))^3*a^4*b^8*d*g*h^14 + 1455720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a
^5*b^6*c*d*g*h^14 + 7254240*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^6*b^4*c
^2*d*g*h^14 + 1911168*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^7*b^2*c^3*d*g
*h^14 + 763392*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^8*c^4*d*g*h^14 - 525
00*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^5*b^7*e*g*h^14 - 1308720*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^3*a^6*b^5*c*e*g*h^14 - 4858560*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))^3*a^7*b^3*c^2*e*g*h^14 + 897792*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^3*a^8*b*c^3*e*g*h^14 + 98000*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))^3*a^6*b^6*f*g*h^14 + 981120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^7*b
^4*c*f*g*h^14 + 2714880*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^8*b^2*c^2*f
*g*h^14 - 2179072*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^9*c^3*f*g*h^14 -
6300*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^5*b^7*d*h^15 - 371280*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^3*a^6*b^5*c*d*h^15 - 1243200*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^3*a^7*b^3*c^2*d*h^15 - 725760*(sqrt(c)*x - sqrt(c*x^2 + b
```

```
*x + a))^3*a^8*b*c^3*d*h^15 + 9800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^
6*b^6*e*h^15 + 346080*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^7*b^4*c*e*h^1
5 + 631680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^8*b^2*c^2*e*h^15 + 19712
0*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^9*c^3*e*h^15 - 16800*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))^3*a^7*b^5*f*h^15 - 295680*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))^3*a^8*b^3*c*f*h^15 - 53760*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3
*a^9*b*c^2*f*h^15 - 215040*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^5*c^(15/
2)*f*g^15 - 86016*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^5*c^(15/2)*e*g^14
*h + 928256*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^6*c^(13/2)*f*g^14*h + 1
075200*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b^4*c^(15/2)*f*g^14*h - 6451
2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^5*c^(15/2)*d*g^13*h^2 + 326144*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^6*c^(13/2)*e*g^13*h^2 + 430080*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^2*a*b^4*c^(15/2)*e*g^13*h^2 - 1450624*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^2*b^7*c^(11/2)*f*g^13*h^2 - 5795328*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))^2*a*b^5*c^(13/2)*f*g^13*h^2 - 2150400*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))^2*a^2*b^3*c^(15/2)*f*g^13*h^2 + 188160*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))^2*b^6*c^(13/2)*d*g^12*h^3 + 322560*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))^2*a*b^4*c^(15/2)*d*g^12*h^3 - 380800*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^2*b^7*c^(11/2)*e*g^12*h^3 - 2182656*(sqrt(c)*x - sqrt(c*
x^2 + b*x + a))^2*a*b^5*c^(13/2)*e*g^12*h^3 - 860160*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^2*a^2*b^3*c^(15/2)*e*g^12*h^3 + 859040*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^2*b^8*c^(9/2)*f*g^12*h^3 + 11151616*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))^2*a*b^6*c^(11/2)*f*g^12*h^3 + 15052800*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))^2*a^2*b^4*c^(13/2)*f*g^12*h^3 + 2150400*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^2*a^3*b^2*c^(15/2)*f*g^12*h^3 - 64512*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))^2*b^7*c^(11/2)*d*g^11*h^4 - 1580544*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^2*a*b^5*c^(13/2)*d*g^11*h^4 - 645120*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^2*a^2*b^3*c^(15/2)*d*g^11*h^4 + 33152*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))^2*b^8*c^(9/2)*e*g^11*h^4 + 3549952*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
^2*a*b^6*c^(11/2)*e*g^11*h^4 + 6021120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^
2*a^2*b^4*c^(13/2)*e*g^11*h^4 + 860160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^
2*a^3*b^2*c^(15/2)*e*g^11*h^4 + 23408*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2
*b^9*c^(7/2)*f*g^11*h^4 - 8424640*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b
^7*c^(9/2)*f*g^11*h^4 - 36615936*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*
b^5*c^(11/2)*f*g^11*h^4 - 20823040*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^
3*b^3*c^(13/2)*f*g^11*h^4 - 1075200*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a
^4*b*c^(15/2)*f*g^11*h^4 - 168000*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^8
*c^(9/2)*d*g^10*h^5 + 1542912*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b^6*c
^(11/2)*d*g^10*h^5 + 5080320*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*b^4*
c^(13/2)*d*g^10*h^5 + 645120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*b^2*
c^(15/2)*d*g^10*h^5 + 123200*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^9*c^(7
/2)*e*g^10*h^5 - 1036672*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b^7*c^(9/2
)*e*g^10*h^5 - 13980288*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*b^5*c^(11
/2)*e*g^10*h^5 - 8780800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*b^3*c^(1
3/2)*e*g^10*h^5 - 430080*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^4*b*c^(15/
```

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2)*e*g^10*h^5 - 98980*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^10*c^(5/2)*f*
g^10*h^5 + 358512*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b^8*c^(7/2)*f*g^1
0*h^5 + 36090656*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*b^6*c^(9/2)*f*g^
10*h^5 + 66577280*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*b^4*c^(11/2)*f*
g^10*h^5 + 16181760*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^4*b^2*c^(13/2)*
f*g^10*h^5 + 215040*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^5*c^(15/2)*f*g^
10*h^5 + 351456*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^9*c^(7/2)*d*g^9*h^6
 - 1904448*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b^7*c^(9/2)*d*g^9*h^6 -
1217664*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*b^5*c^(11/2)*d*g^9*h^6 -
10106880*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*b^3*c^(13/2)*d*g^9*h^6 -
 645120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^4*b*c^(15/2)*d*g^9*h^6 - 10
4440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^10*c^(5/2)*e*g^9*h^6 - 72576*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b^8*c^(7/2)*e*g^9*h^6 + 4622912*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*b^6*c^(9/2)*e*g^9*h^6 + 31109120*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*b^4*c^(11/2)*e*g^9*h^6 + 7848960*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^2*a^4*b^2*c^(13/2)*e*g^9*h^6 + 172032*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^2*a^5*c^(15/2)*e*g^9*h^6 + 17150*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))^2*b^11*c^(3/2)*f*g^9*h^6 + 665560*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^2*a*b^9*c^(5/2)*f*g^9*h^6 - 3010224*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))^2*a^2*b^7*c^(7/2)*f*g^9*h^6 - 89862976*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))^2*a^3*b^5*c^(9/2)*f*g^9*h^6 - 72692480*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))^2*a^4*b^3*c^(11/2)*f*g^9*h^6 - 6945792*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))^2*a^5*b*c^(13/2)*f*g^9*h^6 - 183372*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^2*b^10*c^(5/2)*d*g^8*h^7 + 137424*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^2*a*b^8*c^(7/2)*d*g^8*h^7 + 5833632*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^2*a^2*b^6*c^(9/2)*d*g^8*h^7 - 11491200*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^2*a^3*b^4*c^(11/2)*d*g^8*h^7 + 11450880*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))^2*a^4*b^2*c^(13/2)*d*g^8*h^7 + 129024*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^2*a^5*c^(15/2)*d*g^8*h^7 + 4550*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2
*b^11*c^(3/2)*e*g^8*h^7 + 624232*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b^
9*c^(5/2)*e*g^8*h^7 - 2626512*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*b^7
*c^(7/2)*e*g^8*h^7 - 4981312*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*b^5*
c^(9/2)*e*g^8*h^7 - 44576000*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^4*b^3*
c^(11/2)*e*g^8*h^7 - 4021248*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^5*b*c^
(13/2)*e*g^8*h^7 + 10500*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^12*sqrt(c)
*f*g^8*h^7 - 200200*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b^10*c^(3/2)*f*
g^8*h^7 - 1679832*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*b^8*c^(5/2)*f*g
^8*h^7 + 8542240*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*b^6*c^(7/2)*f*g^
8*h^7 + 144636800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^4*b^4*c^(9/2)*f*g
^8*h^7 + 48803328*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^5*b^2*c^(11/2)*f*
g^8*h^7 + 1232896*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^6*c^(13/2)*f*g^8*
h^7 - 4410*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^11*c^(3/2)*d*g^7*h^8 + 6
12696*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b^9*c^(5/2)*d*g^7*h^8 - 30888
48*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*b^7*c^(7/2)*d*g^7*h^8 + 245817
6*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*b^5*c^(9/2)*d*g^7*h^8 + 3803520
```

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0*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^4*b^3*c^(11/2)*d*g^7*h^8 - 677376
0*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^5*b*c^(13/2)*d*g^7*h^8 + 10500*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^12*sqrt(c)*e*g^7*h^8 - 81340*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^2*a*b^10*c^(3/2)*e*g^7*h^8 - 1298976*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))^2*a^2*b^8*c^(5/2)*e*g^7*h^8 + 8022112*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))^2*a^3*b^6*c^(7/2)*e*g^7*h^8 - 10917760*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))^2*a^4*b^4*c^(9/2)*e*g^7*h^8 + 38868480*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))^2*a^5*b^2*c^(11/2)*e*g^7*h^8 + 1103872*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))^2*a^6*c^(13/2)*e*g^7*h^8 - 94500*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^2*a*b^11*sqrt(c)*f*g^7*h^8 + 1029140*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))^2*a^2*b^9*c^(3/2)*f*g^7*h^8 + 1421056*(sqrt(c)*x - sqrt(c*
x^2 + b*x + a))^2*a^3*b^7*c^(5/2)*f*g^7*h^8 - 9897216*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))^2*a^4*b^5*c^(7/2)*f*g^7*h^8 - 155330560*(sqrt(c)*x - sqrt(c*
x^2 + b*x + a))^2*a^5*b^3*c^(9/2)*f*g^7*h^8 - 18916352*(sqrt(c)*x - sqrt(c*
x^2 + b*x + a))^2*a^6*b*c^(11/2)*f*g^7*h^8 + 18900*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^2*b^12*sqrt(c)*d*g^6*h^9 - 70560*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^2*a*b^10*c^(3/2)*d*g^6*h^9 - 812616*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))^2*a^2*b^8*c^(5/2)*d*g^6*h^9 + 453600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
^2*a^3*b^6*c^(7/2)*d*g^6*h^9 - 29410080*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
^2*a^4*b^4*c^(9/2)*d*g^6*h^9 - 49142016*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
^2*a^5*b^2*c^(11/2)*d*g^6*h^9 + 1279488*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
^2*a^6*c^(13/2)*d*g^6*h^9 - 81900*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b
^11*sqrt(c)*e*g^6*h^9 + 506660*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*b^
9*c^(3/2)*e*g^6*h^9 + 1086736*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*b^7
*c^(5/2)*e*g^6*h^9 - 5315184*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^4*b^5*
c^(7/2)*e*g^6*h^9 + 39057536*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^5*b^3*
c^(9/2)*e*g^6*h^9 - 18416384*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^6*b*c^
(11/2)*e*g^6*h^9 + 365400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*b^10*sq
rt(c)*f*g^6*h^9 - 3062360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*b^8*c^(
3/2)*f*g^6*h^9 + 1726844*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^4*b^6*c^(5
/2)*f*g^6*h^9 - 3928176*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^5*b^4*c^(7/
2)*f*g^6*h^9 + 107867200*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^6*b^2*c^(9
/2)*f*g^6*h^9 + 3084032*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^7*c^(11/2)*
f*g^6*h^9 - 94500*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b^11*sqrt(c)*d*g^
5*h^10 + 502740*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*b^9*c^(3/2)*d*g^5
*h^10 + 2493792*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*b^7*c^(5/2)*d*g^5
*h^10 + 14939568*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^4*b^5*c^(7/2)*d*g^
5*h^10 + 43344000*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^5*b^3*c^(9/2)*d*g
^5*h^10 + 29304576*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^6*b*c^(11/2)*d*g
^5*h^10 + 252000*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*b^10*sqrt(c)*e*g
^5*h^10 - 1556240*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*b^8*c^(3/2)*e*g
^5*h^10 - 1788472*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^4*b^6*c^(5/2)*e*g
^5*h^10 - 10557120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^5*b^4*c^(7/2)*e*
g^5*h^10 - 48922496*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^6*b^2*c^(9/2)*e
*g^5*h^10 + 3885056*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^7*c^(11/2)*e*g^
```

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5*h^10 - 777000*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*b^9*sqrt(c)*f*g^5
*h^10 + 5764990*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^4*b^7*c^(3/2)*f*g^5
*h^10 - 3872568*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^5*b^5*c^(5/2)*f*g^5
*h^10 + 26243616*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^6*b^3*c^(7/2)*f*g^
5*h^10 - 43121792*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^7*b*c^(9/2)*f*g^5
*h^10 + 189000*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*b^10*sqrt(c)*d*g^4
*h^11 - 1647240*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*b^8*c^(3/2)*d*g^4
*h^11 - 7413420*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^4*b^6*c^(5/2)*d*g^4
*h^11 - 26408592*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^5*b^4*c^(7/2)*d*g^
4*h^11 - 23386944*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^6*b^2*c^(9/2)*d*g
^4*h^11 - 7026432*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^7*c^(11/2)*d*g^4*
h^11 - 399000*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*b^9*sqrt(c)*e*g^4*h
^11 + 3055990*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^4*b^7*c^(3/2)*e*g^4*h
^11 + 5780040*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^5*b^5*c^(5/2)*e*g^4*h
^11 + 20993952*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^6*b^3*c^(7/2)*e*g^4*
h^11 + 28206976*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^7*b*c^(9/2)*e*g^4*h
^11 + 976500*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^4*b^8*sqrt(c)*f*g^4*h^
11 - 7402640*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^5*b^6*c^(3/2)*f*g^4*h^
11 + 339360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^6*b^4*c^(5/2)*f*g^4*h^1
1 - 32589312*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^7*b^2*c^(7/2)*f*g^4*h^
11 + 7286272*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^8*c^(9/2)*f*g^4*h^11 -
 189000*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*b^9*sqrt(c)*d*g^3*h^12 +
3127110*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^4*b^7*c^(3/2)*d*g^3*h^12 +
10408104*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^5*b^5*c^(5/2)*d*g^3*h^12 +
 18144672*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^6*b^3*c^(7/2)*d*g^3*h^12
+ 2061696*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^7*b*c^(9/2)*d*g^3*h^12 +
346500*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^4*b^8*sqrt(c)*e*g^3*h^12 - 4
202660*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^5*b^6*c^(3/2)*e*g^3*h^12 - 8
305584*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^6*b^4*c^(5/2)*e*g^3*h^12 - 1
3465536*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^7*b^2*c^(7/2)*e*g^3*h^12 -
6062336*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^8*c^(9/2)*e*g^3*h^12 - 7245
00*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^5*b^7*sqrt(c)*f*g^3*h^12 + 69585
60*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^6*b^5*c^(3/2)*f*g^3*h^12 + 37511
04*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^7*b^3*c^(5/2)*f*g^3*h^12 + 18385
920*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^8*b*c^(7/2)*f*g^3*h^12 + 94500*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^4*b^8*sqrt(c)*d*g^2*h^13 - 3374280*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^5*b^6*c^(3/2)*d*g^2*h^13 - 7224672*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^6*b^4*c^(5/2)*d*g^2*h^13 - 4416384*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^7*b^2*c^(7/2)*d*g^2*h^13 + 1171968*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^8*c^(9/2)*d*g^2*h^13 - 157500*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^2*a^5*b^7*sqrt(c)*e*g^2*h^13 + 3815280*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^2*a^6*b^5*c^(3/2)*e*g^2*h^13 + 5380032*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^2*a^7*b^3*c^(5/2)*e*g^2*h^13 + 2478336*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^2*a^8*b*c^(7/2)*e*g^2*h^13 + 294000*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^2*a^6*b^6*sqrt(c)*f*g^2*h^13 - 4798080*(sqrt(c)
```

```
*x - sqrt(c*x^2 + b*x + a))^2*a^7*b^4*c^(3/2)*f*g^2*h^13 - 2779392*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^2*a^8*b^2*c^(5/2)*f*g^2*h^13 - 4128768*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^2*a^9*c^(7/2)*f*g^2*h^13 - 18900*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^2*a^5*b^7*sqrt(c)*d*g*h^14 + 1896720*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^2*a^6*b^5*c^(3/2)*d*g*h^14 + 2463552*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^2*a^7*b^3*c^(5/2)*d*g*h^14 - 26880*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^2*a^8*b*c^(7/2)*d*g*h^14 + 29400*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^2*a^6*b^6*sqrt(c)*e*g*h^14 - 1972320*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^2*a^7*b^4*c^(3/2)*e*g*h^14 - 1502592*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))^2*a^8*b^2*c^(5/2)*e*g*h^14 + 419328*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
^2*a^9*c^(7/2)*e*g*h^14 - 50400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^7*b
^5*sqrt(c)*f*g*h^14 + 2123520*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^8*b^3
*c^(3/2)*f*g*h^14 + 440832*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^9*b*c^(5
/2)*f*g*h^14 - 430080*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^7*b^4*c^(3/2)
*d*h^15 - 344064*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^8*b^2*c^(5/2)*d*h^
15 - 86016*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^9*c^(7/2)*d*h^15 + 43008
0*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^8*b^3*c^(3/2)*e*h^15 + 129024*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^9*b*c^(5/2)*e*h^15 - 430080*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))^2*a^9*b^2*c^(3/2)*f*h^15 + 86016*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^2*a^10*c^(5/2)*f*h^15 - 35840*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))*b^6*c^7*f*g^15 - 14336*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^6*c
^7*e*g^14*h + 157696*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^7*c^6*f*g^14*h +
 215040*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^5*c^7*f*g^14*h - 10752*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))*b^6*c^7*d*g^13*h^2 + 55552*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))*b^7*c^6*e*g^13*h^2 + 86016*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))*a*b^5*c^7*e*g^13*h^2 - 254912*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b
^8*c^5*f*g^13*h^2 - 1141504*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^6*c^6*f
*g^13*h^2 - 537600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^4*c^7*f*g^13*h
^2 + 32256*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^7*c^6*d*g^12*h^3 + 64512*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^5*c^7*d*g^12*h^3 - 68096*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))*b^8*c^5*e*g^12*h^3 - 426496*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))*a*b^6*c^6*e*g^12*h^3 - 215040*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))*a^2*b^4*c^7*e*g^12*h^3 + 164416*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^
9*c^4*f*g^12*h^3 + 2208640*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^7*c^5*f*
g^12*h^3 + 3537408*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^5*c^6*f*g^12*h
^3 + 716800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*b^3*c^7*f*g^12*h^3 - 13
440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^8*c^5*d*g^11*h^4 - 301056*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))*a*b^6*c^6*d*g^11*h^4 - 161280*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))*a^2*b^4*c^7*d*g^11*h^4 + 11200*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))*b^9*c^4*e*g^11*h^4 + 695296*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
*a*b^7*c^5*e*g^11*h^4 + 1392384*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^5
*c^6*e*g^11*h^4 + 286720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*b^3*c^7*e*
g^11*h^4 - 9800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^10*c^3*f*g^11*h^4 - 1
752576*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^8*c^4*f*g^11*h^4 - 8351168*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^6*c^5*f*g^11*h^4 - 6083840*(sqrt(c
```

```
)*x - sqrt(c*x^2 + b*x + a))*a^3*b^4*c^6*f*g^11*h^4 - 537600*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))*a^4*b^2*c^7*f*g^11*h^4 - 26880*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))*b^9*c^4*d*g^10*h^5 + 295680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))*a*b^7*c^5*d*g^10*h^5 + 1128960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b
^5*c^6*d*g^10*h^5 + 215040*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*b^3*c^7*
d*g^10*h^5 + 19600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^10*c^3*e*g^10*h^5
- 241920*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^8*c^4*e*g^10*h^5 - 3073280
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^6*c^5*e*g^10*h^5 - 2508800*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))*a^3*b^4*c^6*e*g^10*h^5 - 215040*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))*a^4*b^2*c^7*e*g^10*h^5 - 15680*(sqrt(c)*x - sqrt(c*
x^2 + b*x + a))*b^11*c^2*f*g^10*h^5 + 215600*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))*a*b^9*c^3*f*g^10*h^5 + 8299200*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^
2*b^7*c^4*f*g^10*h^5 + 18000640*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*b^5
*c^5*f*g^10*h^5 + 6272000*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^4*b^3*c^6*f
*g^10*h^5 + 215040*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^5*b*c^7*f*g^10*h^5
 + 66696*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^10*c^3*d*g^9*h^6 - 362208*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^8*c^4*d*g^9*h^6 - 485184*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))*a^2*b^6*c^5*d*g^9*h^6 - 2634240*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))*a^3*b^4*c^6*d*g^9*h^6 - 322560*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))*a^4*b^2*c^7*d*g^9*h^6 - 21980*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*
b^11*c^2*e*g^9*h^6 + 11984*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^9*c^3*e*
g^9*h^6 + 1210272*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^7*c^4*e*g^9*h^6
 + 7840000*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*b^5*c^5*e*g^9*h^6 + 2885
120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^4*b^3*c^6*e*g^9*h^6 + 172032*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))*a^5*b*c^7*e*g^9*h^6 + 5880*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))*b^12*c*f*g^9*h^6 + 93940*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))*a*b^10*c^2*f*g^9*h^6 - 1363824*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^
2*b^8*c^3*f*g^9*h^6 - 23203936*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*b^6*
c^4*f*g^9*h^6 - 24241280*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^4*b^4*c^5*f*
g^9*h^6 - 3988992*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^5*b^2*c^6*f*g^9*h^6
 - 71680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^6*c^7*f*g^9*h^6 - 45360*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))*b^11*c^2*d*g^8*h^7 + 30912*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))*a*b^9*c^3*d*g^8*h^7 + 1489152*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))*a^2*b^7*c^4*d*g^8*h^7 - 1604736*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))*a^3*b^5*c^5*d*g^8*h^7 + 4193280*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^4
*b^3*c^6*d*g^8*h^7 + 129024*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^5*b*c^7*d
*g^8*h^7 + 5250*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^12*c*e*g^8*h^7 + 1262
80*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^10*c^2*e*g^8*h^7 - 707952*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^8*c^3*e*g^8*h^7 - 2102464*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))*a^3*b^6*c^4*e*g^8*h^7 - 13422080*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))*a^4*b^4*c^5*e*g^8*h^7 - 2386944*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))*a^5*b^2*c^6*e*g^8*h^7 - 28672*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
*a^6*c^7*e*g^8*h^7 + 525*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^13*f*g^8*h^7
 - 56070*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^11*c*f*g^8*h^7 - 132440*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^9*c^2*f*g^8*h^7 + 4238192*(sqrt(c)*
```

```
x - sqrt(c*x^2 + b*x + a))*a^3*b^7*c^3*f*g^8*h^7 + 42697536*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))*a^4*b^5*c^4*f*g^8*h^7 + 21462784*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))*a^5*b^3*c^5*f*g^8*h^7 + 1584128*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))*a^6*b*c^6*f*g^8*h^7 + 8820*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^12
*c*d*g^7*h^8 + 142380*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^10*c^2*d*g^7*
h^8 - 934752*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^8*c^3*d*g^7*h^8 - 10
44960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*b^6*c^4*d*g^7*h^8 + 7573440*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^4*b^4*c^5*d*g^7*h^8 - 3612672*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))*a^5*b^2*c^6*d*g^7*h^8 - 21504*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))*a^6*c^7*d*g^7*h^8 + 525*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))*b^13*e*g^7*h^8 - 43470*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^11*c*e*g
^7*h^8 - 179480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^9*c^2*e*g^7*h^8 +
 2410352*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*b^7*c^3*e*g^7*h^8 + 40992*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^4*b^5*c^4*e*g^7*h^8 + 15511552*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))*a^5*b^3*c^5*e*g^7*h^8 + 1229312*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))*a^6*b*c^6*e*g^7*h^8 - 5250*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))*a*b^12*f*g^7*h^8 + 236250*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^
2*b^10*c*f*g^7*h^8 - 416080*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*b^8*c^2
*f*g^7*h^8 - 7306712*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^4*b^6*c^3*f*g^7*
h^8 - 54755232*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^5*b^4*c^4*f*g^7*h^8 -
12557440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^6*b^2*c^5*f*g^7*h^8 - 390656
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^7*c^6*f*g^7*h^8 + 945*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))*b^13*d*g^6*h^9 - 48510*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))*a*b^11*c*d*g^6*h^9 - 35280*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^
9*c^2*d*g^6*h^9 + 1851024*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*b^7*c^3*d
*g^6*h^9 - 4423104*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^4*b^5*c^4*d*g^6*h^
9 - 13520640*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^5*b^3*c^5*d*g^6*h^9 + 13
54752*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^6*b*c^6*d*g^6*h^9 - 4620*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))*a*b^12*e*g^6*h^9 + 149940*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))*a^2*b^10*c*e*g^6*h^9 - 231280*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))*a^3*b^8*c^2*e*g^6*h^9 - 3212384*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
*a^4*b^6*c^3*e*g^6*h^9 + 6410880*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^5*b^
4*c^4*e*g^6*h^9 - 11372032*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^6*b^2*c^5*
e*g^6*h^9 - 186368*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^7*c^6*e*g^6*h^9 +
22995*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^11*f*g^6*h^9 - 576240*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))*a^3*b^9*c*f*g^6*h^9 + 1905680*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))*a^4*b^7*c^2*f*g^6*h^9 + 6869184*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))*a^5*b^5*c^3*f*g^6*h^9 + 48986112*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))*a^6*b^3*c^4*f*g^6*h^9 + 4637696*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)*a^7*b*c^5*f*g^6*h^9 - 5670*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^12*d*g
^5*h^10 + 119070*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^10*c*d*g^5*h^10
- 388080*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*b^8*c^2*d*g^5*h^10 + 22680
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^4*b^6*c^3*d*g^5*h^10 + 11229120*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))*a^5*b^4*c^4*d*g^5*h^10 + 12996480*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))*a^6*b^2*c^5*d*g^5*h^10 - 215040*(sqrt(c)*x - sqr
```

```
t(c*x^2 + b*x + a))*a^7*c^6*d*g^5*h^10 + 16695*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))*a^2*b^11*e*g^5*h^10 - 294000*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^
3*b^9*c*e*g^5*h^10 + 988820*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^4*b^7*c^2
*e*g^5*h^10 + 366240*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^5*b^5*c^3*e*g^5*
h^10 - 12411840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^6*b^3*c^4*e*g^5*h^10
+ 4551680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^7*b*c^5*e*g^5*h^10 - 57120*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*b^10*f*g^5*h^10 + 908460*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))*a^4*b^8*c*f*g^5*h^10 - 3283420*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))*a^5*b^6*c^2*f*g^5*h^10 - 1715280*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))*a^6*b^4*c^3*f*g^5*h^10 - 28983360*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))*a^7*b^2*c^4*f*g^5*h^10 - 904960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*
a^8*c^5*f*g^5*h^10 + 14175*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^11*d*g
^4*h^11 - 176400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*b^9*c*d*g^4*h^11 +
 455280*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^4*b^7*c^2*d*g^4*h^11 - 426283
2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^5*b^5*c^3*d*g^4*h^11 - 11055744*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))*a^6*b^3*c^4*d*g^4*h^11 - 6488832*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))*a^7*b*c^5*d*g^4*h^11 - 32550*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))*a^3*b^10*e*g^4*h^11 + 374850*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))*a^4*b^8*c*e*g^4*h^11 - 1057000*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a
^5*b^6*c^2*e*g^4*h^11 + 4445952*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^6*b^4
*c^3*e*g^4*h^11 + 11362176*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^7*b^2*c^4*
e*g^4*h^11 - 663040*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^8*c^5*e*g^4*h^11
+ 87675*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^4*b^9*f*g^4*h^11 - 983430*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))*a^5*b^7*c*f*g^4*h^11 + 2915640*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))*a^6*b^5*c^2*f*g^4*h^11 - 4113312*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))*a^7*b^3*c^3*f*g^4*h^11 + 10211712*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))*a^8*b*c^4*f*g^4*h^11 - 18900*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))*a^3*b^10*d*g^3*h^12 + 176400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^4*b^
8*c*d*g^3*h^12 + 322140*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^5*b^6*c^2*d*g
^3*h^12 + 5666976*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^6*b^4*c^3*d*g^3*h^1
2 + 5025216*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^7*b^2*c^4*d*g^3*h^12 + 12
47232*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^8*c^5*d*g^3*h^12 + 37275*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))*a^4*b^9*e*g^3*h^12 - 330750*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))*a^5*b^7*c*e*g^3*h^12 + 21840*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))*a^6*b^5*c^2*e*g^3*h^12 - 5754336*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))*a^7*b^3*c^3*e*g^3*h^12 - 5359872*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^8
*b*c^4*e*g^3*h^12 - 85050*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^5*b^8*f*g^3
*h^12 + 751170*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^6*b^6*c*f*g^3*h^12 - 9
71880*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^7*b^4*c^2*f*g^3*h^12 + 5404896*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^8*b^2*c^3*f*g^3*h^12 - 1663872*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))*a^9*c^4*f*g^3*h^12 + 14175*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))*a^4*b^9*d*g^2*h^13 - 119070*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))*a^5*b^7*c*d*g^2*h^13 - 1008000*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a
^6*b^5*c^2*d*g^2*h^13 - 3052896*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^7*b^3
*c^3*d*g^2*h^13 - 698880*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^8*b*c^4*d*g^
```

```
2*h^13 - 25200*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^5*b^8*e*g^2*h^13 + 199
920*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^6*b^6*c*e*g^2*h^13 + 882000*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))*a^7*b^4*c^2*e*g^2*h^13 + 3002496*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))*a^8*b^2*c^3*e*g^2*h^13 + 1080576*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))*a^9*c^4*e*g^2*h^13 + 50925*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))*a^6*b^7*f*g^2*h^13 - 396900*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^7*
b^5*c*f*g^2*h^13 - 559440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^8*b^3*c^2*f
*g^2*h^13 - 2713536*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^9*b*c^3*f*g^2*h^1
3 - 5670*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^5*b^8*d*g*h^14 + 48510*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))*a^6*b^6*c*d*g*h^14 + 746760*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))*a^7*b^4*c^2*d*g*h^14 + 632352*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))*a^8*b^2*c^3*d*g*h^14 - 131712*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)*a^9*c^4*d*g*h^14 + 9345*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^6*b^7*e*g*h
^14 - 74340*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^7*b^5*c*e*g*h^14 - 714000
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^8*b^3*c^2*e*g*h^14 - 595392*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))*a^9*b*c^3*e*g*h^14 - 17220*(sqrt(c)*x - sqrt(c*
x^2 + b*x + a))*a^7*b^6*f*g*h^14 + 131040*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))*a^8*b^4*c*f*g*h^14 + 638400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^9*b^2*
c^2*f*g*h^14 + 494592*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^10*c^3*f*g*h^14
 + 945*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^6*b^7*d*h^15 - 8820*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))*a^7*b^5*c*d*h^15 - 189840*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))*a^8*b^3*c^2*d*h^15 - 20160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
*a^9*b*c^3*d*h^15 - 1470*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^7*b^6*e*h^15
 + 12600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^8*b^4*c*e*h^15 + 184800*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))*a^9*b^2*c^2*e*h^15 + 13440*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))*a^10*c^3*e*h^15 + 2520*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))*a^8*b^5*f*h^15 - 20160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^9*b^3*c*f*
h^15 - 174720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^10*b*c^2*f*h^15 - 2560*
b^7*c^(13/2)*f*g^15 - 1024*b^7*c^(13/2)*e*g^14*h + 11264*b^8*c^(11/2)*f*g^1
4*h + 17920*a*b^6*c^(13/2)*f*g^14*h - 768*b^7*c^(13/2)*d*g^13*h^2 + 3968*b^
8*c^(11/2)*e*g^13*h^2 + 7168*a*b^6*c^(13/2)*e*g^13*h^2 - 18208*b^9*c^(9/2)*
f*g^13*h^2 - 92800*a*b^7*c^(11/2)*f*g^13*h^2 - 53760*a^2*b^5*c^(13/2)*f*g^1
3*h^2 + 2304*b^8*c^(11/2)*d*g^12*h^3 + 5376*a*b^6*c^(13/2)*d*g^12*h^3 - 486
4*b^9*c^(9/2)*e*g^12*h^3 - 34432*a*b^7*c^(11/2)*e*g^12*h^3 - 21504*a^2*b^5*
c^(13/2)*e*g^12*h^3 + 11744*b^10*c^(7/2)*f*g^12*h^3 + 175968*a*b^8*c^(9/2)*
f*g^12*h^3 + 334208*a^2*b^6*c^(11/2)*f*g^12*h^3 + 89600*a^3*b^4*c^(13/2)*f*
g^12*h^3 - 960*b^9*c^(9/2)*d*g^11*h^4 - 23808*a*b^7*c^(11/2)*d*g^11*h^4 - 1
6128*a^2*b^5*c^(13/2)*d*g^11*h^4 + 800*b^10*c^(7/2)*e*g^11*h^4 + 54528*a*b^
8*c^(9/2)*e*g^11*h^4 + 129920*a^2*b^6*c^(11/2)*e*g^11*h^4 + 35840*a^3*b^4*c
^(13/2)*e*g^11*h^4 - 700*b^11*c^(5/2)*f*g^11*h^4 - 136928*a*b^9*c^(7/2)*f*g
^11*h^4 - 754272*a^2*b^7*c^(9/2)*f*g^11*h^4 - 687232*a^3*b^5*c^(11/2)*f*g^1
1*h^4 - 89600*a^4*b^3*c^(13/2)*f*g^11*h^4 - 1920*b^10*c^(7/2)*d*g^10*h^5 +
22080*a*b^8*c^(9/2)*d*g^10*h^5 + 102144*a^2*b^6*c^(11/2)*d*g^10*h^5 + 26880
*a^3*b^4*c^(13/2)*d*g^10*h^5 + 1400*b^11*c^(5/2)*e*g^10*h^5 - 18080*a*b^9*c
^(7/2)*e*g^10*h^5 - 269184*a^2*b^7*c^(9/2)*e*g^10*h^5 - 278656*a^3*b^5*c^(1
```

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1/2)*e*g^10*h^5 - 35840*a^4*b^3*c^(13/2)*e*g^10*h^5 - 1120*b^12*c^(3/2)*f*g
^10*h^5 + 16100*a*b^10*c^(5/2)*f*g^10*h^5 + 717984*a^2*b^8*c^(7/2)*f*g^10*h
^5 + 1882272*a^3*b^6*c^(9/2)*f*g^10*h^5 + 882560*a^4*b^4*c^(11/2)*f*g^10*h^
5 + 53760*a^5*b^2*c^(13/2)*f*g^10*h^5 + 5124*b^11*c^(5/2)*d*g^9*h^6 - 26832
*a*b^9*c^(7/2)*d*g^9*h^6 - 50016*a^2*b^7*c^(9/2)*d*g^9*h^6 - 268800*a^3*b^5
*c^(11/2)*d*g^9*h^6 - 53760*a^4*b^3*c^(13/2)*d*g^9*h^6 - 1750*b^12*c^(3/2)*
e*g^9*h^6 + 896*a*b^10*c^(5/2)*e*g^9*h^6 + 100848*a^2*b^8*c^(7/2)*e*g^9*h^6
 + 779520*a^3*b^6*c^(9/2)*e*g^9*h^6 + 385280*a^4*b^4*c^(11/2)*e*g^9*h^6 + 4
3008*a^5*b^2*c^(13/2)*e*g^9*h^6 + 525*b^13*sqrt(c)*f*g^9*h^6 + 6930*a*b^11*
c^(3/2)*f*g^9*h^6 - 110656*a^2*b^9*c^(5/2)*f*g^9*h^6 - 2251184*a^3*b^7*c^(7
/2)*f*g^9*h^6 - 3017280*a^4*b^5*c^(9/2)*f*g^9*h^6 - 732928*a^5*b^3*c^(11/2)
*f*g^9*h^6 - 35840*a^6*b*c^(13/2)*f*g^9*h^6 - 3780*b^12*c^(3/2)*d*g^8*h^7 -
 756*a*b^10*c^(5/2)*d*g^8*h^7 + 143760*a^2*b^8*c^(7/2)*d*g^8*h^7 - 120288*a
^3*b^6*c^(9/2)*d*g^8*h^7 + 564480*a^4*b^4*c^(11/2)*d*g^8*h^7 + 32256*a^5*b^
2*c^(13/2)*d*g^8*h^7 + 525*b^13*sqrt(c)*e*g^8*h^7 + 11130*a*b^11*c^(3/2)*e*
g^8*h^7 - 62944*a^2*b^9*c^(5/2)*e*g^8*h^7 - 208784*a^3*b^7*c^(7/2)*e*g^8*h^
7 - 1518720*a^4*b^5*c^(9/2)*e*g^8*h^7 - 453376*a^5*b^3*c^(11/2)*e*g^8*h^7 -
 14336*a^6*b*c^(13/2)*e*g^8*h^7 - 5250*a*b^12*sqrt(c)*f*g^8*h^7 - 7350*a^2*
b^10*c^(3/2)*f*g^8*h^7 + 378784*a^3*b^8*c^(5/2)*f*g^8*h^7 + 4713968*a^4*b^6
*c^(7/2)*f*g^8*h^7 + 3264576*a^5*b^4*c^(9/2)*f*g^8*h^7 + 474880*a^6*b^2*c^(
11/2)*f*g^8*h^7 + 5120*a^7*c^(13/2)*f*g^8*h^7 + 945*b^13*sqrt(c)*d*g^7*h^8
 + 14490*a*b^11*c^(3/2)*d*g^7*h^8 - 85176*a^2*b^9*c^(5/2)*d*g^7*h^8 - 183888
*a^3*b^7*c^(7/2)*d*g^7*h^8 + 622944*a^4*b^5*c^(9/2)*d*g^7*h^8 - 634368*a^5*
b^3*c^(11/2)*d*g^7*h^8 - 10752*a^6*b*c^(13/2)*d*g^7*h^8 - 4620*a*b^12*sqrt(
c)*e*g^7*h^8 - 18060*a^2*b^10*c^(3/2)*e*g^7*h^8 + 256816*a^3*b^8*c^(5/2)*e*
g^7*h^8 + 38864*a^4*b^6*c^(7/2)*e*g^7*h^8 + 2220288*a^5*b^4*c^(9/2)*e*g^7*h
^8 + 335104*a^6*b^2*c^(11/2)*e*g^7*h^8 + 2048*a^7*c^(13/2)*e*g^7*h^8 + 2299
5*a^2*b^11*sqrt(c)*f*g^7*h^8 - 58240*a^3*b^9*c^(3/2)*f*g^7*h^8 - 727916*a^4
*b^7*c^(5/2)*f*g^7*h^8 - 6981072*a^5*b^5*c^(7/2)*f*g^7*h^8 - 2583616*a^6*b^
3*c^(9/2)*f*g^7*h^8 - 217856*a^7*b*c^(11/2)*f*g^7*h^8 - 5670*a*b^12*sqrt(c)
*d*g^6*h^9 - 6930*a^2*b^10*c^(3/2)*d*g^6*h^9 + 233184*a^3*b^8*c^(5/2)*d*g^6
*h^9 - 148176*a^4*b^6*c^(7/2)*d*g^6*h^9 - 1401120*a^5*b^4*c^(9/2)*d*g^6*h^9
 + 354816*a^6*b^2*c^(11/2)*d*g^6*h^9 + 1536*a^7*c^(13/2)*d*g^6*h^9 + 16695*
a^2*b^11*sqrt(c)*e*g^6*h^9 - 28000*a^3*b^9*c^(3/2)*e*g^6*h^9 - 439544*a^4*b
^7*c^(5/2)*e*g^6*h^9 + 563472*a^5*b^5*c^(7/2)*e*g^6*h^9 - 2227456*a^6*b^3*c
^(9/2)*e*g^6*h^9 - 101120*a^7*b*c^(11/2)*e*g^6*h^9 - 57120*a^3*b^10*sqrt(c)
*f*g^6*h^9 + 257460*a^4*b^8*c^(3/2)*f*g^6*h^9 + 752164*a^5*b^6*c^(5/2)*f*g^
6*h^9 + 7597296*a^6*b^4*c^(7/2)*f*g^6*h^9 + 1535424*a^7*b^2*c^(9/2)*f*g^6*h
^9 + 27904*a^8*c^(11/2)*f*g^6*h^9 + 14175*a^2*b^11*sqrt(c)*d*g^5*h^10 - 504
00*a^3*b^9*c^(3/2)*d*g^5*h^10 - 219996*a^4*b^7*c^(5/2)*d*g^5*h^10 + 798336*
a^5*b^5*c^(7/2)*d*g^5*h^10 + 1942080*a^6*b^3*c^(9/2)*d*g^5*h^10 - 112128*a^
7*b*c^(11/2)*d*g^5*h^10 - 32550*a^3*b^10*sqrt(c)*e*g^5*h^10 + 143850*a^4*b^
8*c^(3/2)*e*g^5*h^10 + 308896*a^5*b^6*c^(5/2)*e*g^5*h^10 - 1253280*a^6*b^4*
c^(7/2)*e*g^5*h^10 + 1291008*a^7*b^2*c^(9/2)*e*g^5*h^10 + 13312*a^8*c^(11/2
)*e*g^5*h^10 + 87675*a^4*b^9*sqrt(c)*f*g^5*h^10 - 500430*a^5*b^7*c^(3/2)*f*
```

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g^5*h^10 - 234696*a^6*b^5*c^(5/2)*f*g^5*h^10 - 6063648*a^7*b^3*c^(7/2)*f*g^
5*h^10 - 549504*a^8*b*c^(9/2)*f*g^5*h^10 - 18900*a^3*b^10*sqrt(c)*d*g^4*h^1
1 + 113400*a^4*b^8*c^(3/2)*d*g^4*h^11 - 51828*a^5*b^6*c^(5/2)*d*g^4*h^11 -
1272096*a^6*b^4*c^(7/2)*d*g^4*h^11 - 1512768*a^7*b^2*c^(9/2)*d*g^4*h^11 + 1
5360*a^8*c^(11/2)*d*g^4*h^11 + 37275*a^4*b^9*sqrt(c)*e*g^4*h^11 - 225750*a^
5*b^7*c^(3/2)*e*g^4*h^11 + 101808*a^6*b^5*c^(5/2)*e*g^4*h^11 + 1566624*a^7*
b^3*c^(7/2)*e*g^4*h^11 - 372480*a^8*b*c^(9/2)*e*g^4*h^11 - 85050*a^5*b^8*sq
rt(c)*f*g^4*h^11 + 555170*a^6*b^6*c^(3/2)*f*g^4*h^11 - 396648*a^7*b^4*c^(5/
2)*f*g^4*h^11 + 3280608*a^8*b^2*c^(7/2)*f*g^4*h^11 + 64640*a^9*c^(9/2)*f*g^
4*h^11 + 14175*a^4*b^9*sqrt(c)*d*g^3*h^12 - 106470*a^5*b^7*c^(3/2)*d*g^3*h^
12 + 336672*a^6*b^5*c^(5/2)*d*g^3*h^12 + 1055520*a^7*b^3*c^(7/2)*d*g^3*h^12
 + 592896*a^8*b*c^(9/2)*d*g^3*h^12 - 25200*a^5*b^8*sqrt(c)*e*g^3*h^12 + 180
320*a^6*b^6*c^(3/2)*e*g^3*h^12 - 412272*a^7*b^4*c^(5/2)*e*g^3*h^12 - 122227
2*a^8*b^2*c^(7/2)*e*g^3*h^12 + 47360*a^9*c^(9/2)*e*g^3*h^12 + 50925*a^6*b^7
*sqrt(c)*f*g^3*h^12 - 363300*a^7*b^5*c^(3/2)*f*g^3*h^12 + 634032*a^8*b^3*c^
(5/2)*f*g^3*h^12 - 1008576*a^9*b*c^(7/2)*f*g^3*h^12 - 5670*a^5*b^8*sqrt(c)*
d*g^2*h^13 + 48510*a^6*b^6*c^(3/2)*d*g^2*h^13 - 371448*a^7*b^4*c^(5/2)*d*g^
2*h^13 - 405984*a^8*b^2*c^(7/2)*d*g^2*h^13 - 94848*a^9*c^(9/2)*d*g^2*h^13 +
 9345*a^6*b^7*sqrt(c)*e*g^2*h^13 - 74340*a^7*b^5*c^(3/2)*e*g^2*h^13 + 40420
8*a^8*b^3*c^(5/2)*e*g^2*h^13 + 516672*a^9*b*c^(7/2)*e*g^2*h^13 - 17220*a^7*
b^6*sqrt(c)*f*g^2*h^13 + 131040*a^8*b^4*c^(3/2)*f*g^2*h^13 - 479808*a^9*b^2
*c^(5/2)*f*g^2*h^13 + 119808*a^10*c^(7/2)*f*g^2*h^13 + 945*a^6*b^7*sqrt(c)*
d*g*h^14 - 8820*a^7*b^5*c^(3/2)*d*g*h^14 + 197232*a^8*b^3*c^(5/2)*d*g*h^14
+ 28992*a^9*b*c^(7/2)*d*g*h^14 - 1470*a^7*b^6*sqrt(c)*e*g*h^14 + 12600*a^8*
b^4*c^(3/2)*e*g*h^14 - 202272*a^9*b^2*c^(5/2)*e*g*h^14 - 84864*a^10*c^(7/2)
*e*g*h^14 + 2520*a^8*b^5*sqrt(c)*f*g*h^14 - 20160*a^9*b^3*c^(3/2)*f*g*h^14
+ 212352*a^10*b*c^(5/2)*f*g*h^14 - 43008*a^9*b^2*c^(5/2)*d*h^15 + 12288*a^1
0*c^(7/2)*d*h^15 + 43008*a^10*b*c^(5/2)*e*h^15 - 43008*a^11*c^(5/2)*f*h^15)
/((c^5*g^10*h^6 - 5*b*c^4*g^9*h^7 + 10*b^2*c^3*g^8*h^8 + 5*a*c^4*g^8*h^8 -
10*b^3*c^2*g^7*h^9 - 20*a*b*c^3*g^7*h^9 + 5*b^4*c*g^6*h^10 + 30*a*b^2*c^2*g
^6*h^10 + 10*a^2*c^3*g^6*h^10 - b^5*g^5*h^11 - 20*a*b^3*c*g^5*h^11 - 30*a^2
*b*c^2*g^5*h^11 + 5*a*b^4*g^4*h^12 + 30*a^2*b^2*c*g^4*h^12 + 10*a^3*c^2*g^4
*h^12 - 10*a^2*b^3*g^3*h^13 - 20*a^3*b*c*g^3*h^13 + 10*a^3*b^2*g^2*h^14 + 5
*a^4*c*g^2*h^14 - 5*a^4*b*g*h^15 + a^5*h^16)*((sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))^2*h + 2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c)*g + b*g - a*h)^7
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^8}\,dx=\text{Hanged}$$

```
[In] int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^8,x)
[Out] \text{Hanged}
```

3.208 $\int (1+2x)^3\sqrt{2-x+3x^2}(1+3x+4x^2)\,dx$

Optimal result

Integrand size = 32, antiderivative size = 143

$$\begin{aligned}\int (1+2x)^3\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)\,dx &= \frac{5393(1-6x)\sqrt{2-x+3x^2}}{15552}\\ &\quad+\frac{17}{105}(1+2x)^2\left(2-x+3x^2\right)^{3/2}\\ &\quad+\frac{67}{378}(1+2x)^3\left(2-x+3x^2\right)^{3/2}\\ &\quad+\frac{2}{21}(1+2x)^4\left(2-x+3x^2\right)^{3/2}\\ &\quad-\frac{(75295+26982x)\left(2-x+3x^2\right)^{3/2}}{68040}\\ &\quad+\frac{124039\operatorname{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{31104\sqrt{3}}\end{aligned}$$

```
[Out] 17/105*(1+2*x)^2*(3*x^2-x+2)^(3/2)+67/378*(1+2*x)^3*(3*x^2-x+2)^(3/2)+2/21*
(1+2*x)^4*(3*x^2-x+2)^(3/2)-1/68040*(75295+26982*x)*(3*x^2-x+2)^(3/2)+12403
9/93312*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)+5393/15552*(1-6*x)*(3*x^2-x+
2)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1667, 846, 793, 626, 633, 221}

$$\int (1+2x)^3\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)\,dx$$
$$=\frac{124039\text{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{31104\sqrt{3}}+\frac{2}{21}\left(3x^2-x+2\right)^{3/2}(2x+1)^4+\frac{67}{378}\left(3x^2-x+2\right)^{3/2}(2x+1)^3$$
$$+\frac{17}{105}\left(3x^2-x+2\right)^{3/2}(2x+1)^2-\frac{(26982x+75295)\left(3x^2-x+2\right)^{3/2}}{68040}+\frac{5393(1-6x)\sqrt{3x^2-x+2}}{15552}$$

```
[In] Int[(1 + 2*x)^3*Sqrt[2 - x + 3*x^2]*(1 + 3*x + 4*x^2),x]
```

```
[Out] (5393*(1 - 6*x)*Sqrt[2 - x + 3*x^2])/15552 + (17*(1 + 2*x)^2*(2 - x + 3*x^2
)^(3/2))/105 + (67*(1 + 2*x)^3*(2 - x + 3*x^2)^(3/2))/378 + (2*(1 + 2*x)^4*
(2 - x + 3*x^2)^(3/2))/21 - ((75295 + 26982*x)*(2 - x + 3*x^2)^(3/2))/68040
 + (124039*ArcSinh[(1 - 6*x)/Sqrt[23]])/(31104*Sqrt[3])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 846

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2}{21}(1+2x)^4\left(2-x+3x^2\right)^{3/2}+\frac{1}{84}\int(1+2x)^3(-32+268x)\sqrt{2-x+3x^2}\,dx \\
&= \frac{67}{378}(1+2x)^3\left(2-x+3x^2\right)^{3/2} \\
&\quad +\frac{2}{21}(1+2x)^4\left(2-x+3x^2\right)^{3/2}+\frac{\int(1+2x)^2(-3390+3672x)\sqrt{2-x+3x^2}\,dx}{1512} \\
&= \frac{17}{105}(1+2x)^2\left(2-x+3x^2\right)^{3/2}+\frac{67}{378}(1+2x)^3\left(2-x+3x^2\right)^{3/2} \\
&\quad +\frac{2}{21}(1+2x)^4\left(2-x+3x^2\right)^{3/2}+\frac{\int(-74718-53964x)(1+2x)\sqrt{2-x+3x^2}\,dx}{22680} \\
&= \frac{17}{105}(1+2x)^2\left(2-x+3x^2\right)^{3/2} \\
&\quad +\frac{67}{378}(1+2x)^3\left(2-x+3x^2\right)^{3/2}+\frac{2}{21}(1+2x)^4\left(2-x+3x^2\right)^{3/2} \\
&\quad -\frac{(75295+26982x)\left(2-x+3x^2\right)^{3/2}}{68040}-\frac{5393\int\sqrt{2-x+3x^2}\,dx}{1296}
\end{aligned}$$

$$= \frac{5393(1-6x)\sqrt{2-x+3x^2}}{15552} + \frac{17}{105}(1+2x)^2\left(2-x+3x^2\right)^{3/2} + \frac{67}{378}(1+2x)^3\left(2-x+3x^2\right)^{3/2} + \frac{2}{21}(1+2x)^4\left(2-x+3x^2\right)^{3/2} - \frac{(75295+26982x)\left(2-x+3x^2\right)^{3/2}}{68040} - \frac{124039\int\frac{1}{\sqrt{2-x+3x^2}}\,dx}{31104}$$

$$= \frac{5393(1-6x)\sqrt{2-x+3x^2}}{15552} + \frac{17}{105}(1+2x)^2\left(2-x+3x^2\right)^{3/2} + \frac{67}{378}(1+2x)^3\left(2-x+3x^2\right)^{3/2} + \frac{2}{21}(1+2x)^4\left(2-x+3x^2\right)^{3/2} - \frac{(75295+26982x)\left(2-x+3x^2\right)^{3/2}}{68040} - \frac{\left(5393\sqrt{\frac{23}{3}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx, x, -1+6x\right)}{31104}$$

$$= \frac{5393(1-6x)\sqrt{2-x+3x^2}}{15552} + \frac{17}{105}(1+2x)^2\left(2-x+3x^2\right)^{3/2} + \frac{67}{378}(1+2x)^3\left(2-x+3x^2\right)^{3/2} + \frac{2}{21}(1+2x)^4\left(2-x+3x^2\right)^{3/2} - \frac{(75295+26982x)\left(2-x+3x^2\right)^{3/2}}{68040} + \frac{124039\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{31104\sqrt{3}}$$

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.56

$$\int(1+2x)^3\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)\,dx = \frac{6\sqrt{2-x+3x^2}(-543069+1493894x+3280872x^2+5497776x^3+7491456x^4+6462720x^5+2488320x^6)}{3265920}$$

```
[In] Integrate[(1 + 2*x)^3*Sqrt[2 - x + 3*x^2]*(1 + 3*x + 4*x^2),x]
```

```
[Out] (6*Sqrt[2 - x + 3*x^2]*(-543069 + 1493894*x + 3280872*x^2 + 5497776*x^3 + 7
491456*x^4 + 6462720*x^5 + 2488320*x^6) + 4341365*Sqrt[3]*Log[1 - 6*x + 2*S
qrt[6 - 3*x + 9*x^2]])/3265920
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.42

method	result
risch	$\frac{\left(2488320x^{6}+6462720x^{5}+7491456x^{4}+5497776x^{3}+3280872x^{2}+1493894x-543069\right)\sqrt{3x^{2}-x+2}}{544320}-\frac{124039\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\,\left(x-\frac{1}{6}\right.}{23}\right.}{93312}$
trager	$\left(\frac{32}{7}x^{6}+\frac{748}{63}x^{5}+\frac{1858}{135}x^{4}+\frac{38179}{3780}x^{3}+\frac{19529}{3240}x^{2}+\frac{746947}{272160}x-\frac{60341}{60480}\right)\sqrt{3x^{2}-x+2}+\frac{124039\,\operatorname{RootOf}\left(\_Z^{2}-\right.}{}$
default	$-\frac{5393(-1+6x)\sqrt{3x^{2}-x+2}}{15552}-\frac{124039\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\,\left(x-\frac{1}{6}\right)}{23}\right)}{93312}-\frac{45739\left(3x^{2}-x+2\right)^{\frac{3}{2}}}{68040}+\frac{32x^{4}\left(3x^{2}-x+2\right)^{\frac{3}{2}}}{21}+\frac{844x^{3}\left(3x^{2}-\right.}{189}$

```
[In] int((1+2*x)^3*(4*x^2+3*x+1)*(3*x^2-x+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/544320*(2488320*x^6+6462720*x^5+7491456*x^4+5497776*x^3+3280872*x^2+14938
94*x-543069)*(3*x^2-x+2)^(1/2)-124039/93312*3^(1/2)*arcsinh(6/23*23^(1/2)*(
x-1/6))
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.58

$$\int (1+2x)^{3}\sqrt{2-x+3x^{2}}\left(1+3x+4x^{2}\right)\,dx$$
$$=\frac{1}{544320}\left(2488320\,x^{6}+6462720\,x^{5}+7491456\,x^{4}+5497776\,x^{3}+3280872\,x^{2}+1493894\,x-543069\right)\sqrt{3\,x^{2}}$$
$$+\frac{124039}{186624}\,\sqrt{3}\log\left(4\,\sqrt{3}\sqrt{3\,x^{2}-x+2}(6\,x-1)-72\,x^{2}+24\,x-25\right)$$

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)*(3*x^2-x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/544320*(2488320*x^6 + 6462720*x^5 + 7491456*x^4 + 5497776*x^3 + 3280872*x
^2 + 1493894*x - 543069)*sqrt(3*x^2 - x + 2) + 124039/186624*sqrt(3)*log(4*
sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x - 1) - 72*x^2 + 24*x - 25)
```

Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.53

$$\int (1+2x)^3\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)\,dx=\sqrt{3x^2-x+2}\cdot\left(\frac{32x^6}{7}+\frac{748x^5}{63}+\frac{1858x^4}{135}+\frac{38179x^3}{3780}+\frac{19529x^2}{3240}+\frac{746947x}{272160}-\frac{60341}{60480}\right)-\frac{124039\sqrt{3}\operatorname{asinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{93312}$$

```
[In] integrate((1+2*x)**3*(4*x**2+3*x+1)*(3*x**2-x+2)**(1/2),x)
```

```
[Out] sqrt(3*x**2 - x + 2)*(32*x**6/7 + 748*x**5/63 + 1858*x**4/135 + 38179*x**3/
3780 + 19529*x**2/3240 + 746947*x/272160 - 60341/60480) - 124039*sqrt(3)*as
inh(6*sqrt(23)*(x - 1/6)/23)/93312
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.88

$$\begin{aligned}\int (1+2x)^3\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)\,dx &= \frac{32}{21}\left(3x^2-x+2\right)^{\frac{3}{2}}x^4+\frac{844}{189}\left(3x^2-x+2\right)^{\frac{3}{2}}x^3+\frac{1594}{315}\left(3x^2-x+2\right)^{\frac{3}{2}}x^2\\&\quad+\frac{7849}{3780}\left(3x^2-x+2\right)^{\frac{3}{2}}x-\frac{45739}{68040}\left(3x^2-x+2\right)^{\frac{3}{2}}-\frac{5393}{2592}\sqrt{3x^2-x+2}x\\&\quad-\frac{124039}{93312}\sqrt{3}\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}(6x-1)\right)+\frac{5393}{15552}\sqrt{3x^2-x+2}\end{aligned}$$

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)*(3*x^2-x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 32/21*(3*x^2 - x + 2)^(3/2)*x^4 + 844/189*(3*x^2 - x + 2)^(3/2)*x^3 + 1594/
315*(3*x^2 - x + 2)^(3/2)*x^2 + 7849/3780*(3*x^2 - x + 2)^(3/2)*x - 45739/6
8040*(3*x^2 - x + 2)^(3/2) - 5393/2592*sqrt(3*x^2 - x + 2)*x - 124039/93312
*sqrt(3)*arcsinh(1/23*sqrt(23)*(6*x - 1)) + 5393/15552*sqrt(3*x^2 - x + 2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.55

$$\int (1+2x)^3\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)\,dx$$
$$=\frac{1}{544320}\left(2\left(12\left(6\left(8\left(30\left(72\,x+187\right)x+6503\right)x+38179\right)x+136703\right)x+746947\right)x-543069\right)\sqrt{3\,x^2-x+}$$
$$+\frac{124039}{93312}\,\sqrt{3}\log\left(-2\,\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2-x+2}\right)+1\right)$$

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)*(3*x^2-x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/544320*(2*(12*(6*(8*(30*(72*x + 187)*x + 6503)*x + 38179)*x + 136703)*x +
 746947)*x - 543069)*sqrt(3*x^2 - x + 2) + 124039/93312*sqrt(3)*log(-2*sqrt
(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) + 1)
```

Mupad [B] (verification not implemented)

Time = 14.57 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.19

$$\int (1+2x)^3\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)\,dx$$
$$=\frac{1594\,x^2\,{\left(3\,x^2-x+2\right)}^{3/2}}{315}+\frac{844\,x^3\,{\left(3\,x^2-x+2\right)}^{3/2}}{189}$$
$$+\frac{32\,x^4\,{\left(3\,x^2-x+2\right)}^{3/2}}{21}-\frac{137057\,\sqrt{3}\,\ln\left(\sqrt{3\,x^2-x+2}+\frac{\sqrt{3}\,\left(3\,x-\frac{1}{2}\right)}{3}\right)}{136080}$$
$$-\frac{5959\,\left(\frac{x}{2}-\frac{1}{12}\right)\,\sqrt{3\,x^2-x+2}}{1890}-\frac{45739\,\sqrt{3\,x^2-x+2}\,\left(72\,x^2-6\,x+45\right)}{1632960}$$
$$+\frac{7849\,x\,{\left(3\,x^2-x+2\right)}^{3/2}}{3780}-\frac{1051997\,\sqrt{3}\,\ln\left(2\,\sqrt{3\,x^2-x+2}+\frac{\sqrt{3}\,\left(6\,x-1\right)}{3}\right)}{3265920}$$

```
[In] int((2*x + 1)^3*(3*x^2 - x + 2)^(1/2)*(3*x + 4*x^2 + 1),x)
```

```
[Out] (1594*x^2*(3*x^2 - x + 2)^(3/2))/315 + (844*x^3*(3*x^2 - x + 2)^(3/2))/189
+ (32*x^4*(3*x^2 - x + 2)^(3/2))/21 - (137057*3^(1/2)*log((3*x^2 - x + 2)^(
1/2) + (3^(1/2)*(3*x - 1/2))/3))/136080 - (5959*(x/2 - 1/12)*(3*x^2 - x + 2
)^(1/2))/1890 - (45739*(3*x^2 - x + 2)^(1/2)*(72*x^2 - 6*x + 45))/1632960 +
 (7849*x*(3*x^2 - x + 2)^(3/2))/3780 - (1051997*3^(1/2)*log(2*(3*x^2 - x +
2)^(1/2) + (3^(1/2)*(6*x - 1))/3))/3265920
```

3.209 $\int (1+2x)^2\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)\,dx$

Optimal result

Integrand size = 32, antiderivative size = 118

$$\begin{aligned}\int (1+2x)^2\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)\,dx &= \frac{235(1-6x)\sqrt{2-x+3x^2}}{1296}\\ &+\frac{1}{5}(1+2x)^2\left(2-x+3x^2\right)^{3/2}\\ &+\frac{1}{9}(1+2x)^3\left(2-x+3x^2\right)^{3/2}\\ &+\frac{1}{810}(25+306x)\left(2-x+3x^2\right)^{3/2}\\ &+\frac{5405\operatorname{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{2592\sqrt{3}}\end{aligned}$$

```
[Out] 1/5*(1+2*x)^2*(3*x^2-x+2)^(3/2)+1/9*(1+2*x)^3*(3*x^2-x+2)^(3/2)+1/810*(25+3
06*x)*(3*x^2-x+2)^(3/2)+5405/7776*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)+23
5/1296*(1-6*x)*(3*x^2-x+2)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used

$= \{1667, 846, 793, 626, 633, 221\}$

$$\int (1+2x)^2\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)\,dx = \frac{5405\operatorname{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{2592\sqrt{3}} + \frac{1}{9}\left(3x^2-x+2\right)^{3/2}(2x+1)^3 + \frac{1}{5}\left(3x^2-x+2\right)^{3/2}(2x+1)^2 + \frac{1}{810}(306x+25)\left(3x^2-x+2\right)^{3/2} + \frac{235(1-6x)\sqrt{3x^2-x+2}}{1296}$$

```
[In] Int[(1 + 2*x)^2*Sqrt[2 - x + 3*x^2]*(1 + 3*x + 4*x^2),x]
```

```
[Out] (235*(1 - 6*x)*Sqrt[2 - x + 3*x^2])/1296 + ((1 + 2*x)^2*(2 - x + 3*x^2)^(3/
2))/5 + ((1 + 2*x)^3*(2 - x + 3*x^2)^(3/2))/9 + ((25 + 306*x)*(2 - x + 3*x^
2)^(3/2))/810 + (5405*ArcSinh[(1 - 6*x)/Sqrt[23]])/(2592*Sqrt[3])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 846

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
```

```
1)/(c*(m + 2*p + 2))), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{9}(1+2x)^3\left(2-x+3x^2\right)^{3/2}+\frac{1}{72}\int(1+2x)^2(-12+216x)\sqrt{2-x+3x^2}\,dx\\
&= \frac{1}{5}(1+2x)^2\left(2-x+3x^2\right)^{3/2}\\
&\quad +\frac{1}{9}(1+2x)^3\left(2-x+3x^2\right)^{3/2}+\frac{\int(1+2x)(-1584+2448x)\sqrt{2-x+3x^2}\,dx}{1080}\\
&= \frac{1}{5}(1+2x)^2\left(2-x+3x^2\right)^{3/2}+\frac{1}{9}(1+2x)^3\left(2-x+3x^2\right)^{3/2}\\
&\quad +\frac{1}{810}(25+306x)\left(2-x+3x^2\right)^{3/2}-\frac{235}{108}\int\sqrt{2-x+3x^2}\,dx\\
&= \frac{235(1-6x)\sqrt{2-x+3x^2}}{1296}+\frac{1}{5}(1+2x)^2\left(2-x+3x^2\right)^{3/2}\\
&\quad +\frac{1}{9}(1+2x)^3\left(2-x+3x^2\right)^{3/2}+\frac{1}{810}(25+306x)\left(2-x+3x^2\right)^{3/2}-\frac{5405\int\frac{1}{\sqrt{2-x+3x^2}}\,dx}{2592}\\
&= \frac{235(1-6x)\sqrt{2-x+3x^2}}{1296}+\frac{1}{5}(1+2x)^2\left(2-x+3x^2\right)^{3/2}+\frac{1}{9}(1+2x)^3\left(2-x+3x^2\right)^{3/2}\\
&\quad +\frac{1}{810}(25+306x)\left(2-x+3x^2\right)^{3/2}-\frac{\left(235\sqrt{\frac{23}{3}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx,x,-1+6x\right)}{2592}
\end{aligned}
$$

$$= \frac{235(1-6x)\sqrt{2-x+3x^2}}{1296} + \frac{1}{5}(1+2x)^2\left(2-x+3x^2\right)^{3/2}$$
$$+\frac{1}{9}(1+2x)^3\left(2-x+3x^2\right)^{3/2}+\frac{1}{810}(25+306x)\left(2-x+3x^2\right)^{3/2}+\frac{5405\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{2592\sqrt{3}}$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.64

$$\int (1+2x)^2\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)\,dx$$
$$= \frac{6\sqrt{2-x+3x^2}(5607+14638x+22344x^2+33552x^3+35712x^4+17280x^5)+27025\sqrt{3}\log\left(1-6x+2\sqrt{\ }\right.}{38880}$$

```
[In] Integrate[(1 + 2*x)^2*Sqrt[2 - x + 3*x^2]*(1 + 3*x + 4*x^2),x]
```

```
[Out] (6*Sqrt[2 - x + 3*x^2]*(5607 + 14638*x + 22344*x^2 + 33552*x^3 + 35712*x^4
+ 17280*x^5) + 27025*Sqrt[3]*Log[1 - 6*x + 2*Sqrt[6 - 3*x + 9*x^2]])/38880
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.47

method	result
risch	$\frac{(17280x^5+35712x^4+33552x^3+22344x^2+14638x+5607)\sqrt{3x^2-x+2}}{6480}-\frac{5405\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{7776}$
trager	$\left(\frac{8}{3}x^5+\frac{248}{45}x^4+\frac{233}{45}x^3+\frac{931}{270}x^2+\frac{7319}{3240}x+\frac{623}{720}\right)\sqrt{3x^2-x+2}-\frac{5405\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(6\,\mathrm{RootOf}\left(\_Z^2-\right.\right.}{7776}$
default	$-\frac{235(-1+6x)\sqrt{3x^2-x+2}}{1296}-\frac{5405\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{7776}+\frac{277(3x^2-x+2)^{\frac{3}{2}}}{810}+\frac{8x^3(3x^2-x+2)^{\frac{3}{2}}}{9}+\frac{32x^2(3x^2-x+2)^{\frac{3}{2}}}{15}$

```
[In] int((1+2*x)^2*(4*x^2+3*x+1)*(3*x^2-x+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6480*(17280*x^5+35712*x^4+33552*x^3+22344*x^2+14638*x+5607)*(3*x^2-x+2)^(
1/2)-5405/7776*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.66

$$\begin{aligned}\int (1+2x)^2\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)\,dx\\ &= \frac{1}{6480}\left(17280\,x^5+35712\,x^4+33552\,x^3+22344\,x^2+14638\,x+5607\right)\sqrt{3\,x^2-x+2}\\ &\quad+\frac{5405}{15552}\,\sqrt{3}\log\left(4\,\sqrt{3}\sqrt{3\,x^2-x+2}(6\,x-1)-72\,x^2+24\,x-25\right)\end{aligned}$$

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)*(3*x^2-x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6480*(17280*x^5 + 35712*x^4 + 33552*x^3 + 22344*x^2 + 14638*x + 5607)*sqr
t(3*x^2 - x + 2) + 5405/15552*sqrt(3)*log(4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*
x - 1) - 72*x^2 + 24*x - 25)
```

Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.59

$$\int (1+2x)^2\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)\,dx = \sqrt{3x^2-x+2}\cdot\left(\frac{8x^5}{3}+\frac{248x^4}{45}+\frac{233x^3}{45}+\frac{931x^2}{270}+\frac{7319x}{3240}+\frac{623}{720}\right)-\frac{5405\sqrt{3}\operatorname{asinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{7776}$$

```
[In] integrate((1+2*x)**2*(4*x**2+3*x+1)*(3*x**2-x+2)**(1/2),x)
```

```
[Out] sqrt(3*x**2 - x + 2)*(8*x**5/3 + 248*x**4/45 + 233*x**3/45 + 931*x**2/270 +
 7319*x/3240 + 623/720) - 5405*sqrt(3)*asinh(6*sqrt(23)*(x - 1/6)/23)/7776
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.92

$$\int (1+2x)^2\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)\,dx = \frac{8}{9}\left(3x^2-x+2\right)^{\frac{3}{2}}x^3+\frac{32}{15}\left(3x^2-x+2\right)^{\frac{3}{2}}x^2 + \frac{83}{45}\left(3x^2-x+2\right)^{\frac{3}{2}}x + \frac{277}{810}\left(3x^2-x+2\right)^{\frac{3}{2}}-\frac{235}{216}\sqrt{3x^2-x+2}x - \frac{5405}{7776}\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{23}\sqrt{23}(6x-1)\right) + \frac{235}{1296}\sqrt{3x^2-x+2}$$

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)*(3*x^2-x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 8/9*(3*x^2 - x + 2)^(3/2)*x^3 + 32/15*(3*x^2 - x + 2)^(3/2)*x^2 + 83/45*(3*
x^2 - x + 2)^(3/2)*x + 277/810*(3*x^2 - x + 2)^(3/2) - 235/216*sqrt(3*x^2 -
 x + 2)*x - 5405/7776*sqrt(3)*arcsinh(1/23*sqrt(23)*(6*x - 1)) + 235/1296*s
qrt(3*x^2 - x + 2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.62

$$\int (1+2x)^2\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)\,dx = \frac{1}{6480}\left(2\left(12\left(6\left(8\left(15x+31\right)x+233\right)x+931\right)x+7319\right)x+5607\right)\sqrt{3x^2-x+2} + \frac{5405}{7776}\sqrt{3}\log\left(-2\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2-x+2}\right)+1\right)$$

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)*(3*x^2-x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6480*(2*(12*(6*(8*(15*x + 31)*x + 233)*x + 931)*x + 7319)*x + 5607)*sqrt(
3*x^2 - x + 2) + 5405/7776*sqrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 -
 x + 2)) + 1)
```

Mupad [B] (verification not implemented)

Time = 14.20 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.30

$$\int (1+2x)^2\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)\,dx=\frac{32\,x^2\,{\left(3\,x^2-x+2\right)}^{3/2}}{15}+\frac{8\,x^3\,{\left(3\,x^2-x+2\right)}^{3/2}}{9}-\frac{2783\,\sqrt{3}\,\ln\left(\sqrt{3\,x^2-x+2}+\frac{\sqrt{3}\,\left(3\,x-\frac{1}{2}\right)}{3}\right)}{3240}-\frac{121\,\left(\frac{x}{2}-\frac{1}{12}\right)\,\sqrt{3\,x^2-x+2}}{45}+\frac{277\,\sqrt{3\,x^2-x+2}\,\left(72\,x^2-6\,x+45\right)}{19440}+\frac{83\,x\,{\left(3\,x^2-x+2\right)}^{3/2}}{45}+\frac{6371\,\sqrt{3}\,\ln\left(2\,\sqrt{3\,x^2-x+2}+\frac{\sqrt{3}\,\left(6\,x-1\right)}{3}\right)}{38880}$$

```
[In] int((2*x + 1)^2*(3*x^2 - x + 2)^(1/2)*(3*x + 4*x^2 + 1),x)
```

```
[Out] (32*x^2*(3*x^2 - x + 2)^(3/2))/15 + (8*x^3*(3*x^2 - x + 2)^(3/2))/9 - (2783
*3^(1/2)*log((3*x^2 - x + 2)^(1/2) + (3^(1/2)*(3*x - 1/2))/3))/3240 - (121*
(x/2 - 1/12)*(3*x^2 - x + 2)^(1/2))/45 + (277*(3*x^2 - x + 2)^(1/2)*(72*x^2
 - 6*x + 45))/19440 + (83*x*(3*x^2 - x + 2)^(3/2))/45 + (6371*3^(1/2)*log(2
*(3*x^2 - x + 2)^(1/2) + (3^(1/2)*(6*x - 1))/3))/38880
```

3.210 $\int (1+2x)\sqrt{2-x+3x^2}(1+3x+4x^2)\,dx$

Optimal result

Integrand size = 30, antiderivative size = 93

$$\int (1+2x)\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)\,dx = \frac{19(1-6x)\sqrt{2-x+3x^2}}{2592} + \frac{2}{15}(1+2x)^2\left(2-x+3x^2\right)^{3/2} + \frac{(745+738x)\left(2-x+3x^2\right)^{3/2}}{1620} + \frac{437\operatorname{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{5184\sqrt{3}}$$

```
[Out] 2/15*(1+2*x)^2*(3*x^2-x+2)^(3/2)+1/1620*(745+738*x)*(3*x^2-x+2)^(3/2)+437/1
5552*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)+19/2592*(1-6*x)*(3*x^2-x+2)^(1/
2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1667, 793, 626, 633, 221}

$$\int (1+2x)\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)\,dx = \frac{437\operatorname{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{5184\sqrt{3}} + \frac{2}{15}\left(3x^2-x+2\right)^{3/2}(2x+1)^2 + \frac{(738x+745)\left(3x^2-x+2\right)^{3/2}}{1620} + \frac{19(1-6x)\sqrt{3x^2-x+2}}{2592}$$

```
[In] Int[(1 + 2*x)*Sqrt[2 - x + 3*x^2]*(1 + 3*x + 4*x^2),x]
```

```
[Out] (19*(1 - 6*x)*Sqrt[2 - x + 3*x^2])/2592 + (2*(1 + 2*x)^2*(2 - x + 3*x^2)^(3
/2))/15 + ((745 + 738*x)*(2 - x + 3*x^2)^(3/2))/1620 + (437*ArcSinh[(1 - 6*
x)/Sqrt[23]])/(5184*Sqrt[3])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2}{15}(1+2x)^2\left(2-x+3x^2\right)^{3/2}+\frac{1}{60}\int(1+2x)(8+164x)\sqrt{2-x+3x^2}\,dx\\
&= \frac{2}{15}(1+2x)^2\left(2-x+3x^2\right)^{3/2}+\frac{(745+738x)\left(2-x+3x^2\right)^{3/2}}{1620}-\frac{19}{216}\int\sqrt{2-x+3x^2}\,dx\\
&= \frac{19(1-6x)\sqrt{2-x+3x^2}}{2592}+\frac{2}{15}(1+2x)^2\left(2-x+3x^2\right)^{3/2}\\
&\quad+\frac{(745+738x)\left(2-x+3x^2\right)^{3/2}}{1620}-\frac{437\int\frac{1}{\sqrt{2-x+3x^2}}\,dx}{5184}\\
&= \frac{19(1-6x)\sqrt{2-x+3x^2}}{2592}+\frac{2}{15}(1+2x)^2\left(2-x+3x^2\right)^{3/2}\\
&\quad+\frac{(745+738x)\left(2-x+3x^2\right)^{3/2}}{1620}-\frac{\left(19\sqrt{\frac{23}{3}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx,x,-1+6x\right)}{5184}\\
&= \frac{19(1-6x)\sqrt{2-x+3x^2}}{2592}+\frac{2}{15}(1+2x)^2\left(2-x+3x^2\right)^{3/2}\\
&\quad+\frac{(745+738x)\left(2-x+3x^2\right)^{3/2}}{1620}+\frac{437\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{5184\sqrt{3}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.75

$$\int(1+2x)\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)\,dx$$

$$=\frac{6\sqrt{2-x+3x^2}(15471+17374x+24072x^2+31536x^3+20736x^4)+2185\sqrt{3}\log\left(1-6x+2\sqrt{6-3x+9\right.}{77760}$$

```
[In] Integrate[(1 + 2*x)*Sqrt[2 - x + 3*x^2]*(1 + 3*x + 4*x^2),x]
```

```
[Out] (6*Sqrt[2 - x + 3*x^2]*(15471 + 17374*x + 24072*x^2 + 31536*x^3 + 20736*x^4
) + 2185*Sqrt[3]*Log[1 - 6*x + 2*Sqrt[6 - 3*x + 9*x^2]])/77760
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.54

method	result
risch	$\frac{(20736x^4+31536x^3+24072x^2+17374x+15471)\sqrt{3x^2-x+2}}{12960}-\frac{437\sqrt{3}\ \operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{15552}$
trager	$\left(\frac{8}{5}x^4+\frac{73}{30}x^3+\frac{1003}{540}x^2+\frac{8687}{6480}x+\frac{191}{160}\right)\sqrt{3x^2-x+2}-\frac{437\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(6\,\mathrm{RootOf}\left(\_Z^2-3\right)x+6\sqrt{3x^2-}{15552}$
default	$-\frac{19(-1+6x)\sqrt{3x^2-x+2}}{2592}-\frac{437\sqrt{3}\ \operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{15552}+\frac{961\left(3x^2-x+2\right)^{\frac{3}{2}}}{1620}+\frac{8x^2\left(3x^2-x+2\right)^{\frac{3}{2}}}{15}+\frac{89x\left(3x^2-x+2\right)^{\frac{3}{2}}}{90}$

```
[In] int((1+2*x)*(4*x^2+3*x+1)*(3*x^2-x+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12960*(20736*x^4+31536*x^3+24072*x^2+17374*x+15471)*(3*x^2-x+2)^(1/2)-437
/15552*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.78

$$\begin{aligned}\int(1+2x)\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)\,dx\\ &=\frac{1}{12960}\left(20736\,x^4+31536\,x^3+24072\,x^2+17374\,x+15471\right)\sqrt{3\,x^2-x+2}\\ &\quad+\frac{437}{31104}\,\sqrt{3}\log\left(4\,\sqrt{3}\sqrt{3\,x^2-x+2}(6\,x-1)-72\,x^2+24\,x-25\right)\end{aligned}$$

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)*(3*x^2-x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12960*(20736*x^4 + 31536*x^3 + 24072*x^2 + 17374*x + 15471)*sqrt(3*x^2 -
x + 2) + 437/31104*sqrt(3)*log(4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x - 1) - 72
*x^2 + 24*x - 25)
```

Sympy [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.68

$$\begin{aligned}\int(1+2x)\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)\,dx=\sqrt{3x^2-x+2}\\ &\cdot\left(\frac{8x^4}{5}+\frac{73x^3}{30}+\frac{1003x^2}{540}+\frac{8687x}{6480}+\frac{191}{160}\right)\\ &-\frac{437\sqrt{3}\operatorname{asinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{15552}\end{aligned}$$

```
[In] integrate((1+2*x)*(4*x**2+3*x+1)*(3*x**2-x+2)**(1/2),x)
```

```
[Out] sqrt(3*x**2 - x + 2)*(8*x**4/5 + 73*x**3/30 + 1003*x**2/540 + 8687*x/6480 +
 191/160) - 437*sqrt(3)*asinh(6*sqrt(23)*(x - 1/6)/23)/15552
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.99

$$\int (1+2x)\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)\,dx = \frac{8}{15}\left(3\,x^2-x+2\right)^{\frac{3}{2}}x^2+\frac{89}{90}\left(3\,x^2-x+2\right)^{\frac{3}{2}}x+\frac{961}{1620}\left(3\,x^2-x+2\right)^{\frac{3}{2}}-\frac{19}{432}\sqrt{3\,x^2-x+2}x-\frac{437}{15552}\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{23}\sqrt{23}(6\,x-1)\right)+\frac{19}{2592}\sqrt{3\,x^2-x+2}$$

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)*(3*x^2-x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 8/15*(3*x^2 - x + 2)^(3/2)*x^2 + 89/90*(3*x^2 - x + 2)^(3/2)*x + 961/1620*(
3*x^2 - x + 2)^(3/2) - 19/432*sqrt(3*x^2 - x + 2)*x - 437/15552*sqrt(3)*arc
sinh(1/23*sqrt(23)*(6*x - 1)) + 19/2592*sqrt(3*x^2 - x + 2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.73

$$\int (1+2x)\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)\,dx = \frac{1}{12960}\left(2\left(12\left(18\left(48\,x+73\right)x+1003\right)x+8687\right)x+15471\right)\sqrt{3\,x^2-x+2}+\frac{437}{15552}\sqrt{3}\log\left(-2\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2-x+2}\right)+1\right)$$

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)*(3*x^2-x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/12960*(2*(12*(18*(48*x + 73)*x + 1003)*x + 8687)*x + 15471)*sqrt(3*x^2 -
x + 2) + 437/15552*sqrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2))
 + 1)
```

Mupad [B] (verification not implemented)

Time = 13.87 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.46

$$\begin{aligned}\int(1+2x)\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)\,dx &= \frac{8\,x^2\,{\left(3\,x^2-x+2\right)}^{3/2}}{15}-\frac{253\,\sqrt{3}\,\ln\left(\sqrt{3\,x^2-x+2}+\frac{\sqrt{3}\,\left(3\,x-\frac{1}{2}\right)}{3}\right)}{810}\\ &\quad-\frac{44\,\left(\frac{x}{2}-\frac{1}{12}\right)\,\sqrt{3\,x^2-x+2}}{45}+\frac{961\,\sqrt{3\,x^2-x+2}\,\left(72\,x^2-6\,x+45\right)}{38880}\\ &\quad+\frac{89\,x\,{\left(3\,x^2-x+2\right)}^{3/2}}{90}+\frac{22103\,\sqrt{3}\,\ln\left(2\,\sqrt{3\,x^2-x+2}+\frac{\sqrt{3}\,\left(6\,x-1\right)}{3}\right)}{77760}\end{aligned}$$

```
[In] int((2*x + 1)*(3*x^2 - x + 2)^(1/2)*(3*x + 4*x^2 + 1),x)
```

```
[Out] (8*x^2*(3*x^2 - x + 2)^(3/2))/15 - (253*3^(1/2)*log((3*x^2 - x + 2)^(1/2) +
 (3^(1/2)*(3*x - 1/2))/3))/810 - (44*(x/2 - 1/12)*(3*x^2 - x + 2)^(1/2))/45
 + (961*(3*x^2 - x + 2)^(1/2)*(72*x^2 - 6*x + 45))/38880 + (89*x*(3*x^2 - x
 + 2)^(3/2))/90 + (22103*3^(1/2)*log(2*(3*x^2 - x + 2)^(1/2) + (3^(1/2)*(6*
x - 1))/3))/77760
```

3.211 $\int \frac{\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)}{1+2x}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 101

$$\int \frac{\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)}{1+2x}\,dx = \frac{1}{72}(13+30x)\sqrt{2-x+3x^2} + \frac{2}{9}\left(2-x+3x^2\right)^{3/2} - \frac{43\text{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{144\sqrt{3}} - \frac{1}{8}\sqrt{13}\text{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)$$

```
[Out] 2/9*(3*x^2-x+2)^(3/2)-43/432*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)-1/8*arc
tanh(1/26*(9-8*x)*13^(1/2)/(3*x^2-x+2)^(1/2))*13^(1/2)+1/72*(13+30*x)*(3*x^
2-x+2)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {1667, 828, 857, 633, 221, 738, 212}

$$\int \frac{\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)}{1+2x}\,dx = -\frac{43\text{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{144\sqrt{3}} - \frac{1}{8}\sqrt{13}\text{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right) + \frac{2}{9}\left(3x^2-x+2\right)^{3/2} + \frac{1}{72}(30x+13)\sqrt{3x^2-x+2}$$

```
[In] Int[(Sqrt[2 - x + 3*x^2]*(1 + 3*x + 4*x^2))/(1 + 2*x),x]
```

```
[Out] ((13 + 30*x)*Sqrt[2 - x + 3*x^2])/72 + (2*(2 - x + 3*x^2)^(3/2))/9 - (43*Ar
cSinh[(1 - 6*x)/Sqrt[23]])/(144*Sqrt[3]) - (Sqrt[13]*ArcTanh[(9 - 8*x)/(2*S
qrt[13]*Sqrt[2 - x + 3*x^2])])/8
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
```

```
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2}{9}\left(2-x+3x^2\right)^{3/2}+\frac{1}{36}\int \frac{(48+60x)\sqrt{2-x+3x^2}}{1+2x}\,dx \\
&= \frac{1}{72}(13+30x)\sqrt{2-x+3x^2}+\frac{2}{9}\left(2-x+3x^2\right)^{3/2}-\frac{\int \frac{-3324-1032x}{(1+2x)\sqrt{2-x+3x^2}}\,dx}{1728} \\
&= \frac{1}{72}(13+30x)\sqrt{2-x+3x^2}+\frac{2}{9}\left(2-x+3x^2\right)^{3/2} \\
&\quad +\frac{43}{144}\int \frac{1}{\sqrt{2-x+3x^2}}\,dx+\frac{13}{8}\int \frac{1}{(1+2x)\sqrt{2-x+3x^2}}\,dx \\
&= \frac{1}{72}(13+30x)\sqrt{2-x+3x^2}+\frac{2}{9}\left(2-x+3x^2\right)^{3/2} \\
&\quad -\frac{13}{4}\text{Subst}\left(\int \frac{1}{52-x^2}\,dx,x,\frac{9-8x}{\sqrt{2-x+3x^2}}\right) \\
&\quad +\frac{43\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx,x,-1+6x\right)}{144\sqrt{69}} \\
&= \frac{1}{72}(13+30x)\sqrt{2-x+3x^2}+\frac{2}{9}\left(2-x+3x^2\right)^{3/2} \\
&\quad -\frac{43\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{144\sqrt{3}}-\frac{1}{8}\sqrt{13}\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.03

$$\int \frac{\sqrt{2-x+3x^2}(1+3x+4x^2)}{1+2x}\,dx = \frac{1}{432}\left(6\sqrt{2-x+3x^2}(45+14x+48x^2) + 108\sqrt{13}\operatorname{arctanh}\left(\frac{\sqrt{3}+2\sqrt{3}x-2\sqrt{2-x+3x^2}}{\sqrt{13}}\right) - 43\sqrt{3}\log\left(1-6x+2\sqrt{6-3x+9x^2}\right)\right)$$

```
[In] Integrate[(Sqrt[2 - x + 3*x^2]*(1 + 3*x + 4*x^2))/(1 + 2*x),x]
```

```
[Out] (6*Sqrt[2 - x + 3*x^2]*(45 + 14*x + 48*x^2) + 108*Sqrt[13]*ArcTanh[(Sqrt[3]
 + 2*Sqrt[3]*x - 2*Sqrt[2 - x + 3*x^2])/Sqrt[13]] - 43*Sqrt[3]*Log[1 - 6*x
+ 2*Sqrt[6 - 3*x + 9*x^2]])/432
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.69

method	result
risch	$\frac{(48x^2+14x+45)\sqrt{3x^2-x+2}}{72}+\frac{43\sqrt{3}\operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{432}-\frac{\sqrt{13}\operatorname{arctanh}\left(\frac{2\left(\frac{9}{2}-4x\right)\sqrt{13}}{13\sqrt{12\left(x+\frac{1}{2}\right)^2-16x+5}}\right)}{8}$
default	$\frac{5(-1+6x)\sqrt{3x^2-x+2}}{72}+\frac{43\sqrt{3}\operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{432}+\frac{2(3x^2-x+2)^{\frac{3}{2}}}{9}+\frac{\sqrt{12\left(x+\frac{1}{2}\right)^2-16x+5}}{8}-\frac{\sqrt{13}\operatorname{arctanh}\left(\frac{2\left(\frac{9}{2}-4x\right.}{13\sqrt{12\left(x+\frac{1}{2}\right.}}\right.}{8}$
trager	$\left(\frac{2}{3}x^2+\frac{7}{36}x+\frac{5}{8}\right)\sqrt{3x^2-x+2}+\frac{\operatorname{RootOf}\left(\_Z^2-13\right)\ln\left(\frac{8\operatorname{RootOf}\left(\_Z^2-13\right)x+26\sqrt{3x^2-x+2}-9\operatorname{RootOf}\left(\_Z^2-13\right)}{1+2x}\right)}{8}-$

```
[In] int((4*x^2+3*x+1)*(3*x^2-x+2)^(1/2)/(1+2*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/72*(48*x^2+14*x+45)*(3*x^2-x+2)^(1/2)+43/432*3^(1/2)*arcsinh(6/23*23^(1/2
)*(x-1/6))-1/8*13^(1/2)*arctanh(2/13*(9/2-4*x)*13^(1/2)/(12*(x+1/2)^2-16*x+
5)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.14

$$\begin{aligned}\int \frac{\sqrt{2-x+3x^2}(1+3x+4x^2)}{1+2x}\,dx &= \frac{1}{72}\left(48\,x^2+14\,x+45\right)\sqrt{3\,x^2-x+2}\\ &\quad+\frac{43}{864}\,\sqrt{3}\log\left(-4\,\sqrt{3}\sqrt{3\,x^2-x+2}(6\,x-1)-72\,x^2+24\,x-25\right)\\ &\quad+\frac{1}{16}\,\sqrt{13}\log\left(-\frac{4\,\sqrt{13}\sqrt{3\,x^2-x+2}(8\,x-9)+220\,x^2-196\,x+185}{4\,x^2+4\,x+1}\right)\end{aligned}$$

```
[In] integrate((4*x^2+3*x+1)*(3*x^2-x+2)^(1/2)/(1+2*x),x, algorithm="fricas")
```

```
[Out] 1/72*(48*x^2 + 14*x + 45)*sqrt(3*x^2 - x + 2) + 43/864*sqrt(3)*log(-4*sqrt(
3)*sqrt(3*x^2 - x + 2)*(6*x - 1) - 72*x^2 + 24*x - 25) + 1/16*sqrt(13)*log(
-(4*sqrt(13)*sqrt(3*x^2 - x + 2)*(8*x - 9) + 220*x^2 - 196*x + 185)/(4*x^2
+ 4*x + 1))
```

Sympy [F]

$$\int \frac{\sqrt{2-x+3x^2}(1+3x+4x^2)}{1+2x}\,dx = \int \frac{\sqrt{3x^2-x+2}\cdot(4x^2+3x+1)}{2x+1}\,dx$$

```
[In] integrate((4*x**2+3*x+1)*(3*x**2-x+2)**(1/2)/(1+2*x),x)
```

```
[Out] Integral(sqrt(3*x**2 - x + 2)*(4*x**2 + 3*x + 1)/(2*x + 1), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.95

$$\begin{aligned}\int \frac{\sqrt{2-x+3x^2}(1+3x+4x^2)}{1+2x}\,dx &= \frac{2}{9}\left(3\,x^2-x+2\right)^{\frac{3}{2}}+\frac{5}{12}\,\sqrt{3\,x^2-x+2}x\\ &\quad+\frac{43}{432}\,\sqrt{3}\operatorname{arsinh}\left(\frac{6}{23}\,\sqrt{23}x-\frac{1}{23}\,\sqrt{23}\right)\\ &\quad+\frac{1}{8}\,\sqrt{13}\operatorname{arsinh}\left(\frac{8\,\sqrt{23}x}{23\,|2\,x+1|}-\frac{9\,\sqrt{23}}{23\,|2\,x+1|}\right)\\ &\quad+\frac{13}{72}\,\sqrt{3\,x^2-x+2}\end{aligned}$$

```
[In] integrate((4*x^2+3*x+1)*(3*x^2-x+2)^(1/2)/(1+2*x),x, algorithm="maxima")
```

```
[Out] 2/9*(3*x^2 - x + 2)^(3/2) + 5/12*sqrt(3*x^2 - x + 2)*x + 43/432*sqrt(3)*arc
sinh(6/23*sqrt(23)*x - 1/23*sqrt(23)) + 1/8*sqrt(13)*arcsinh(8/23*sqrt(23)*
x/abs(2*x + 1) - 9/23*sqrt(23)/abs(2*x + 1)) + 13/72*sqrt(3*x^2 - x + 2)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.25

$$\int \frac{\sqrt{2-x+3x^2}(1+3x+4x^2)}{1+2x}\,dx$$

$$= \frac{1}{72}\left(2\left(24\,x+7\right)x+45\right)\sqrt{3\,x^2-x+2}-\frac{43}{432}\sqrt{3}\log\left(-6\sqrt{3}x+\sqrt{3}+6\sqrt{3\,x^2-x+2}\right) + \frac{1}{8}\sqrt{13}\log\left(-\frac{\left|-4\sqrt{3}x-2\sqrt{13}-2\sqrt{3}+4\sqrt{3\,x^2-x+2}\right|}{2\left(2\sqrt{3}x-\sqrt{13}+\sqrt{3}-2\sqrt{3\,x^2-x+2}\right)}\right)$$

```
[In] integrate((4*x^2+3*x+1)*(3*x^2-x+2)^(1/2)/(1+2*x),x, algorithm="giac")
```

```
[Out] 1/72*(2*(24*x + 7)*x + 45)*sqrt(3*x^2 - x + 2) - 43/432*sqrt(3)*log(-6*sqrt
(3)*x + sqrt(3) + 6*sqrt(3*x^2 - x + 2)) + 1/8*sqrt(13)*log(-1/2*abs(-4*sqr
t(3)*x - 2*sqrt(13) - 2*sqrt(3) + 4*sqrt(3*x^2 - x + 2))/(2*sqrt(3)*x - sqr
t(13) + sqrt(3) - 2*sqrt(3*x^2 - x + 2)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-x+3x^2}(1+3x+4x^2)}{1+2x}\,dx = \int \frac{\sqrt{3\,x^2-x+2}\,\left(4\,x^2+3\,x+1\right)}{2\,x+1}\,dx$$

```
[In] int(((3*x^2 - x + 2)^(1/2)*(3*x + 4*x^2 + 1))/(2*x + 1),x)
```

```
[Out] int(((3*x^2 - x + 2)^(1/2)*(3*x + 4*x^2 + 1))/(2*x + 1), x)
```

3.212 $\int \frac{\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)}{(1+2x)^2} \, dx$

Optimal result

Integrand size = 32, antiderivative size = 108

$$\int \frac{\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)}{(1+2x)^2} \, dx = -\frac{1}{156}(67-96x)\sqrt{2-x+3x^2}-\frac{\left(2-x+3x^2\right)^{3/2}}{13(1+2x)} - \frac{11\text{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{6\sqrt{3}}+\frac{17\text{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)}{8\sqrt{13}}$$

```
[Out] -1/13*(3*x^2-x+2)^(3/2)/(1+2*x)-11/18*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2
)+17/104*arctanh(1/26*(9-8*x)*13^(1/2)/(3*x^2-x+2)^(1/2))*13^(1/2)-1/156*(6
7-96*x)*(3*x^2-x+2)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {1664, 828, 857, 633, 221, 738, 212}

$$\int \frac{\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)}{(1+2x)^2} \, dx = -\frac{11\text{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{6\sqrt{3}}+\frac{17\text{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{8\sqrt{13}} - \frac{\left(3x^2-x+2\right)^{3/2}}{13(2x+1)}-\frac{1}{156}(67-96x)\sqrt{3x^2-x+2}$$

```
[In] Int[(Sqrt[2 - x + 3*x^2]*(1 + 3*x + 4*x^2))/(1 + 2*x)^2,x]
```

```
[Out] -1/156*((67 - 96*x)*Sqrt[2 - x + 3*x^2]) - (2 - x + 3*x^2)^(3/2)/(13*(1 + 2
*x)) - (11*ArcSinh[(1 - 6*x)/Sqrt[23]])/(6*Sqrt[3]) + (17*ArcTanh[(9 - 8*x)
/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/(8*Sqrt[13])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\left(2-x+3x^2\right)^{3/2}}{13(1+2x)}-\frac{1}{13}\int\frac{\left(-\frac{15}{2}-32x\right)\sqrt{2-x+3x^2}}{1+2x}\,dx\\
&= -\frac{1}{156}(67-96x)\sqrt{2-x+3x^2}-\frac{\left(2-x+3x^2\right)^{3/2}}{13(1+2x)}+\frac{1}{624}\int\frac{-182+2288x}{(1+2x)\sqrt{2-x+3x^2}}\,dx\\
&= -\frac{1}{156}(67-96x)\sqrt{2-x+3x^2}-\frac{\left(2-x+3x^2\right)^{3/2}}{13(1+2x)}\\
&\quad+\frac{11}{6}\int\frac{1}{\sqrt{2-x+3x^2}}\,dx-\frac{17}{8}\int\frac{1}{(1+2x)\sqrt{2-x+3x^2}}\,dx\\
&= -\frac{1}{156}(67-96x)\sqrt{2-x+3x^2}-\frac{\left(2-x+3x^2\right)^{3/2}}{13(1+2x)}\\
&\quad+\frac{17}{4}\text{Subst}\left(\int\frac{1}{52-x^2}\,dx,x,\frac{9-8x}{\sqrt{2-x+3x^2}}\right)\\
&\quad+\frac{11\text{Subst}\left(\int\frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx,x,-1+6x\right)}{6\sqrt{69}}\\
&= -\frac{1}{156}(67-96x)\sqrt{2-x+3x^2}-\frac{\left(2-x+3x^2\right)^{3/2}}{13(1+2x)}\\
&\quad-\frac{11\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{6\sqrt{3}}+\frac{17\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)}{8\sqrt{13}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.02

$$\int \frac{\sqrt{2-x+3x^2}(1+3x+4x^2)}{(1+2x)^2}\,dx = \frac{\sqrt{2-x+3x^2}(-7-2x+12x^2)}{12+24x} - \frac{17\operatorname{arctanh}\left(\frac{\sqrt{3}+2\sqrt{3}x-2\sqrt{2-x+3x^2}}{\sqrt{13}}\right)}{4\sqrt{13}} - \frac{11\log\left(1-6x+2\sqrt{6-3x+9x^2}\right)}{6\sqrt{3}}$$

```
[In] Integrate[(Sqrt[2 - x + 3*x^2]*(1 + 3*x + 4*x^2))/(1 + 2*x)^2,x]
```

```
[Out] (Sqrt[2 - x + 3*x^2]*(-7 - 2*x + 12*x^2))/(12 + 24*x) - (17*ArcTanh[(Sqrt[3
] + 2*Sqrt[3]*x - 2*Sqrt[2 - x + 3*x^2])/Sqrt[13]])/(4*Sqrt[13]) - (11*Log[
1 - 6*x + 2*Sqrt[6 - 3*x + 9*x^2]])/(6*Sqrt[3])
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.81

method	result
risch	$\frac{36x^4-18x^3+5x^2+3x-14}{12(1+2x)\sqrt{3x^2-x+2}}+\frac{11\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\,\left(x-\frac{1}{6}\right)}{23}\right)}{18}+\frac{17\sqrt{13}\,\operatorname{arctanh}\left(\frac{2\left(\frac{9}{2}-4x\right)\sqrt{13}}{13\sqrt{12\left(x+\frac{1}{2}\right)^2-16x+5}}\right)}{104}$
trager	$\frac{(12x^2-2x-7)\sqrt{3x^2-x+2}}{12+24x}+\frac{17\,\mathrm{RootOf}\left(\_Z^2-13\right)\ln\left(-\frac{8\,\mathrm{RootOf}\left(\_Z^2-13\right)x-9\,\mathrm{RootOf}\left(\_Z^2-13\right)-26\sqrt{3x^2-x+2}}{1+2x}\right)}{104}-$ 11 RootO
default	$\frac{(-1+6x)\sqrt{3x^2-x+2}}{12}+\frac{11\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\,\left(x-\frac{1}{6}\right)}{23}\right)}{18}-\frac{\left(3\left(x+\frac{1}{2}\right)^2-4x+\frac{5}{4}\right)^{\frac{3}{2}}}{26\left(x+\frac{1}{2}\right)}-\frac{17\sqrt{12\left(x+\frac{1}{2}\right)^2-16x+5}}{104}+$ $17\sqrt{13}\,\operatorname{arctanh}\Big($

```
[In] int((4*x^2+3*x+1)*(3*x^2-x+2)^(1/2)/(1+2*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*(36*x^4-18*x^3+5*x^2+3*x-14)/(1+2*x)/(3*x^2-x+2)^(1/2)+11/18*3^(1/2)*a
rcsinh(6/23*23^(1/2)*(x-1/6))+17/104*13^(1/2)*arctanh(2/13*(9/2-4*x)*13^(1/
2)/(12*(x+1/2)^2-16*x+5)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.23

$$\int \frac{\sqrt{2-x+3x^2}(1+3x+4x^2)}{(1+2x)^2}\,dx$$

$$= \frac{572\sqrt{3}(2\,x+1)\log\left(-4\sqrt{3}\sqrt{3\,x^2-x+2}(6\,x-1)-72\,x^2+24\,x-25\right)+153\sqrt{13}(2\,x+1)\log\left(\frac{4\sqrt{13}\sqrt{3\,x}}{}\right.}{1872\,(2\,x+1)}$$

```
[In] integrate((4*x^2+3*x+1)*(3*x^2-x+2)^(1/2)/(1+2*x)^2,x, algorithm="fricas")
```

```
[Out] 1/1872*(572*sqrt(3)*(2*x + 1)*log(-4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x - 1)
- 72*x^2 + 24*x - 25) + 153*sqrt(13)*(2*x + 1)*log((4*sqrt(13)*sqrt(3*x^2 -
 x + 2)*(8*x - 9) - 220*x^2 + 196*x - 185)/(4*x^2 + 4*x + 1)) + 156*(12*x^2
 - 2*x - 7)*sqrt(3*x^2 - x + 2))/(2*x + 1)
```

Sympy [F]

$$\int \frac{\sqrt{2-x+3x^2}(1+3x+4x^2)}{(1+2x)^2}\,dx = \int \frac{\sqrt{3x^2-x+2}\cdot(4x^2+3x+1)}{(2x+1)^2}\,dx$$

```
[In] integrate((4*x**2+3*x+1)*(3*x**2-x+2)**(1/2)/(1+2*x)**2,x)
```

```
[Out] Integral(sqrt(3*x**2 - x + 2)*(4*x**2 + 3*x + 1)/(2*x + 1)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.95

$$\begin{aligned}\int \frac{\sqrt{2-x+3x^2}(1+3x+4x^2)}{(1+2x)^2}\,dx &= \frac{1}{2}\sqrt{3\,x^2-x+2}x \\ &\quad + \frac{11}{18}\sqrt{3}\,\mathrm{arsinh}\left(\frac{6}{23}\sqrt{23}x-\frac{1}{23}\sqrt{23}\right) \\ &\quad - \frac{17}{104}\sqrt{13}\,\mathrm{arsinh}\left(\frac{8\sqrt{23}x}{23\,|2\,x+1|}-\frac{9\sqrt{23}}{23\,|2\,x+1|}\right) \\ &\quad - \frac{1}{3}\sqrt{3\,x^2-x+2}-\frac{\sqrt{3\,x^2-x+2}}{4\,(2\,x+1)}\end{aligned}$$

```
[In] integrate((4*x^2+3*x+1)*(3*x^2-x+2)^(1/2)/(1+2*x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(3*x^2 - x + 2)*x + 11/18*sqrt(3)*arcsinh(6/23*sqrt(23)*x - 1/23*sq
rt(23)) - 17/104*sqrt(13)*arcsinh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqrt(
23)/abs(2*x + 1)) - 1/3*sqrt(3*x^2 - x + 2) - 1/4*sqrt(3*x^2 - x + 2)/(2*x
+ 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 380 vs. $2(85) = 170$.

Time = 0.49 (sec) , antiderivative size = 380, normalized size of antiderivative = 3.52

$$\int \frac{\sqrt{2-x+3x^2}(1+3x+4x^2)}{(1+2x)^2}\,dx$$

$$= \frac{17}{104}\sqrt{13}\log\left(\sqrt{13}\left(\sqrt{-\frac{8}{2\,x+1}+\frac{13}{\left(2\,x+1\right)^{2}}+3}+\frac{\sqrt{13}}{2\,x+1}\right)-4\right)\operatorname{sgn}\left(\frac{1}{2\,x+1}\right)$$

$$-\frac{11}{18}\sqrt{3}\log\left(\frac{\left|-2\sqrt{3}+2\sqrt{-\frac{8}{2\,x+1}+\frac{13}{\left(2\,x+1\right)^{2}}+3}+\frac{2\sqrt{13}}{2\,x+1}\right|}{2\left(\sqrt{3}+\sqrt{-\frac{8}{2\,x+1}+\frac{13}{\left(2\,x+1\right)^{2}}+3}+\frac{\sqrt{13}}{2\,x+1}\right)}\right)\operatorname{sgn}\left(\frac{1}{2\,x+1}\right)$$

$$-\frac{1}{8}\sqrt{-\frac{8}{2\,x+1}+\frac{13}{\left(2\,x+1\right)^{2}}+3}\operatorname{sgn}\left(\frac{1}{2\,x+1}\right)$$

$$+\frac{67\left(\sqrt{-\frac{8}{2\,x+1}+\frac{13}{\left(2\,x+1\right)^{2}}+3}+\frac{\sqrt{13}}{2\,x+1}\right)^{3}\operatorname{sgn}\left(\frac{1}{2\,x+1}\right)-57\sqrt{13}\left(\sqrt{-\frac{8}{2\,x+1}+\frac{13}{\left(2\,x+1\right)^{2}}+3}+\frac{\sqrt{13}}{2\,x+1}\right)^{2}\operatorname{sgn}\left(\frac{1}{2\,x+1}\right)+}{12\left(\left(\sqrt{-\frac{8}{2\,x+1}+\frac{13}{\left(2\,x+1\right)^{2}}+3}+\frac{\sqrt{13}}{2\,x+1}\right)^{2}-\right.}$$

```
[In] integrate((4*x^2+3*x+1)*(3*x^2-x+2)^(1/2)/(1+2*x)^2,x, algorithm="giac")
```

```
[Out] 17/104*sqrt(13)*log(sqrt(13)*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqr
t(13)/(2*x + 1)) - 4)*sgn(1/(2*x + 1)) - 11/18*sqrt(3)*log(1/2*abs(-2*sqrt(
3) + 2*sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + 2*sqrt(13)/(2*x + 1))/(sqr
t(3) + sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1)))*sgn(1
/(2*x + 1)) - 1/8*sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3)*sgn(1/(2*x + 1))
+ 1/12*(67*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1))^3
*sgn(1/(2*x + 1)) - 57*sqrt(13)*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) +
sqrt(13)/(2*x + 1))^2*sgn(1/(2*x + 1)) + 129*(sqrt(-8/(2*x + 1) + 13/(2*x +
 1)^2 + 3) + sqrt(13)/(2*x + 1))*sgn(1/(2*x + 1)) + 27*sqrt(13)*sgn(1/(2*x
+ 1)))/((sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1))^2 -
3)^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-x+3x^2}(1+3x+4x^2)}{(1+2x)^2}\,dx=\int \frac{\sqrt{3\,x^2-x+2}\,\left(4\,x^2+3\,x+1\right)}{{\left(2\,x+1\right)}^2}\,dx$$

```
[In] int(((3*x^2 - x + 2)^(1/2)*(3*x + 4*x^2 + 1))/(2*x + 1)^2,x)
[Out] int(((3*x^2 - x + 2)^(1/2)*(3*x + 4*x^2 + 1))/(2*x + 1)^2, x)
```

3.213 $\int \frac{\sqrt{2-x+3x^2}(1+3x+4x^2)}{(1+2x)^3}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 115

$$\int \frac{\sqrt{2-x+3x^2}(1+3x+4x^2)}{(1+2x)^3}\,dx = \frac{11(7+10x)\sqrt{2-x+3x^2}}{104(1+2x)} - \frac{(2-x+3x^2)^{3/2}}{26(1+2x)^2} + \frac{11\text{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{8\sqrt{3}} - \frac{803\text{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)}{208\sqrt{13}}$$

```
[Out] -1/26*(3*x^2-x+2)^(3/2)/(1+2*x)^2+11/24*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1
/2)-803/2704*arctanh(1/26*(9-8*x)*13^(1/2)/(3*x^2-x+2)^(1/2))*13^(1/2)+11/1
04*(7+10*x)*(3*x^2-x+2)^(1/2)/(1+2*x)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {1664, 826, 857, 633, 221, 738, 212}

$$\int \frac{\sqrt{2-x+3x^2}(1+3x+4x^2)}{(1+2x)^3}\,dx = \frac{11\text{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{8\sqrt{3}} - \frac{803\text{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{208\sqrt{13}} - \frac{(3x^2-x+2)^{3/2}}{26(2x+1)^2} + \frac{11(10x+7)\sqrt{3x^2-x+2}}{104(2x+1)}$$

```
[In] Int[(Sqrt[2 - x + 3*x^2]*(1 + 3*x + 4*x^2))/(1 + 2*x)^3,x]
```

```
[Out] (11*(7 + 10*x)*Sqrt[2 - x + 3*x^2])/(104*(1 + 2*x)) - (2 - x + 3*x^2)^(3/2)
/(26*(1 + 2*x)^2) + (11*ArcSinh[(1 - 6*x)/Sqrt[23]])/(8*Sqrt[3]) - (803*Arc
Tanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/(208*Sqrt[13])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 826

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\left(2-x+3x^2\right)^{3/2}}{26(1+2x)^2}-\frac{1}{26}\int \frac{\left(-\frac{33}{2}-55x\right)\sqrt{2-x+3x^2}}{(1+2x)^2}\,dx\\
&= \frac{11(7+10x)\sqrt{2-x+3x^2}}{104(1+2x)}-\frac{\left(2-x+3x^2\right)^{3/2}}{26(1+2x)^2}+\frac{1}{208}\int \frac{517-572x}{(1+2x)\sqrt{2-x+3x^2}}\,dx\\
&= \frac{11(7+10x)\sqrt{2-x+3x^2}}{104(1+2x)}-\frac{\left(2-x+3x^2\right)^{3/2}}{26(1+2x)^2}\\
&\quad -\frac{11}{8}\int \frac{1}{\sqrt{2-x+3x^2}}\,dx+\frac{803}{208}\int \frac{1}{(1+2x)\sqrt{2-x+3x^2}}\,dx\\
&= \frac{11(7+10x)\sqrt{2-x+3x^2}}{104(1+2x)}-\frac{\left(2-x+3x^2\right)^{3/2}}{26(1+2x)^2}\\
&\quad -\frac{803}{104}\text{Subst}\left(\int \frac{1}{52-x^2}\,dx,x,\frac{9-8x}{\sqrt{2-x+3x^2}}\right)\\
&\quad -\frac{11\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx,x,-1+6x\right)}{8\sqrt{69}}\\
&= \frac{11(7+10x)\sqrt{2-x+3x^2}}{104(1+2x)}-\frac{\left(2-x+3x^2\right)^{3/2}}{26(1+2x)^2}\\
&\quad +\frac{11\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{8\sqrt{3}}-\frac{803\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)}{208\sqrt{13}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.97

$$\int \frac{\sqrt{2-x+3x^2}(1+3x+4x^2)}{(1+2x)^3}\,dx = \frac{\frac{39\sqrt{2-x+3x^2}(69+268x+208x^2)}{(1+2x)^2}+2409\sqrt{13}\text{arctanh}\left(\frac{\sqrt{3}+2\sqrt{3}x-2\sqrt{2-x+3x^2}}{\sqrt{13}}\right)+1859\sqrt{3}\log\left(1-6x+2\sqrt{6-3x+9}\right.}{4056}$$

```
[In] Integrate[(Sqrt[2 - x + 3*x^2]*(1 + 3*x + 4*x^2))/(1 + 2*x)^3,x]
```

```
[Out] ((39*Sqrt[2 - x + 3*x^2]*(69 + 268*x + 208*x^2))/(1 + 2*x)^2 + 2409*Sqrt[13
]*ArcTanh[(Sqrt[3] + 2*Sqrt[3]*x - 2*Sqrt[2 - x + 3*x^2])/Sqrt[13]] + 1859*
Sqrt[3]*Log[1 - 6*x + 2*Sqrt[6 - 3*x + 9*x^2]])/4056
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.76

method	result
risch	$\frac{624x^4+596x^3+355x^2+467x+138}{104(1+2x)^2\sqrt{3x^2-x+2}}-\frac{11\sqrt{3}\,\text{arcsinh}\left(\frac{6\sqrt{23}\,\left(x-\frac{1}{6}\right)}{23}\right)}{24}-\frac{803\sqrt{13}\,\text{arctanh}\left(\frac{2\left(\frac{9}{2}-4x\right)\sqrt{13}}{13\sqrt{12\left(x+\frac{1}{2}\right)^2-16x+5}}\right)}{2704}$
trager	$\frac{(208x^2+268x+69)\sqrt{3x^2-x+2}}{104(1+2x)^2}-\frac{11\,\text{RootOf}\left(\_Z^2-3\right)\ln\left(6\,\text{RootOf}\left(\_Z^2-3\right)x+6\sqrt{3x^2-x+2}-\text{RootOf}\left(\_Z^2-3\right)\right)}{24}-\frac{803\,\text{Ro}}{}$
default	$\frac{803\sqrt{12\left(x+\frac{1}{2}\right)^2-16x+5}}{2704}-\frac{11\sqrt{3}\,\text{arcsinh}\left(\frac{6\sqrt{23}\,\left(x-\frac{1}{6}\right)}{23}\right)}{24}-\frac{803\sqrt{13}\,\text{arctanh}\left(\frac{2\left(\frac{9}{2}-4x\right)\sqrt{13}}{13\sqrt{12\left(x+\frac{1}{2}\right)^2-16x+5}}\right)}{2704}+\frac{11\left(3\left(x+\frac{1}{2}\right)^2-4x+\frac{5}{4}\right.}{338\left(x+\frac{1}{2}\right)}$

```
[In] int((4*x^2+3*x+1)*(3*x^2-x+2)^(1/2)/(1+2*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/104*(624*x^4+596*x^3+355*x^2+467*x+138)/(1+2*x)^2/(3*x^2-x+2)^(1/2)-11/24
*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))-803/2704*13^(1/2)*arctanh(2/13*(9/2
-4*x)*13^(1/2)/(12*(x+1/2)^2-16*x+5)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.30

$$\int \frac{\sqrt{2-x+3x^2}(1+3x+4x^2)}{(1+2x)^3}\,dx$$
$$=\frac{3718\,\sqrt{3}(4\,x^{2}+4\,x+1)\log\left(4\,\sqrt{3}\sqrt{3\,x^{2}-x+2}(6\,x-1)-72\,x^{2}+24\,x-25\right)+2409\,\sqrt{13}(4\,x^{2}+4\,x+1)}{16224\,(4\,x^{2}+4\,x+1)}$$

```
[In] integrate((4*x^2+3*x+1)*(3*x^2-x+2)^(1/2)/(1+2*x)^3,x, algorithm="fricas")
```

```
[Out] 1/16224*(3718*sqrt(3)*(4*x^2 + 4*x + 1)*log(4*sqrt(3)*sqrt(3*x^2 - x + 2)*(
6*x - 1) - 72*x^2 + 24*x - 25) + 2409*sqrt(13)*(4*x^2 + 4*x + 1)*log(-(4*sq
rt(13)*sqrt(3*x^2 - x + 2)*(8*x - 9) + 220*x^2 - 196*x + 185)/(4*x^2 + 4*x
+ 1)) + 156*(208*x^2 + 268*x + 69)*sqrt(3*x^2 - x + 2))/(4*x^2 + 4*x + 1)
```

Sympy [F]

$$\int \frac{\sqrt{2-x+3x^2}(1+3x+4x^2)}{(1+2x)^3}\,dx=\int \frac{\sqrt{3x^2-x+2}\cdot(4x^2+3x+1)}{(2x+1)^3}\,dx$$

```
[In] integrate((4*x**2+3*x+1)*(3*x**2-x+2)**(1/2)/(1+2*x)**3,x)
```

```
[Out] Integral(sqrt(3*x**2 - x + 2)*(4*x**2 + 3*x + 1)/(2*x + 1)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.99

$$\begin{aligned}\int \frac{\sqrt{2-x+3x^2}(1+3x+4x^2)}{(1+2x)^3}\,dx=&-\frac{11}{24}\,\sqrt{3}\,\mathrm{arsinh}\left(\frac{6}{23}\,\sqrt{23}x-\frac{1}{23}\,\sqrt{23}\right)\\&+\frac{803}{2704}\,\sqrt{13}\,\mathrm{arsinh}\left(\frac{8\,\sqrt{23}x}{23\,|2\,x+1|}-\frac{9\,\sqrt{23}}{23\,|2\,x+1|}\right)\\&+\frac{55}{104}\,\sqrt{3\,x^{2}-x+2}\\&-\frac{(3\,x^{2}-x+2)^{\frac{3}{2}}}{26\,(4\,x^{2}+4\,x+1)}+\frac{11\,\sqrt{3\,x^{2}-x+2}}{52\,(2\,x+1)}\end{aligned}$$

```
[In] integrate((4*x^2+3*x+1)*(3*x^2-x+2)^(1/2)/(1+2*x)^3,x, algorithm="maxima")
```

```
[Out] -11/24*sqrt(3)*arcsinh(6/23*sqrt(23)*x - 1/23*sqrt(23)) + 803/2704*sqrt(13)
*arcsinh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqrt(23)/abs(2*x + 1)) + 55/10
4*sqrt(3*x^2 - x + 2) - 1/26*(3*x^2 - x + 2)^(3/2)/(4*x^2 + 4*x + 1) + 11/5
2*sqrt(3*x^2 - x + 2)/(2*x + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 251 vs. $2(92) = 184$.

Time = 0.32 (sec) , antiderivative size = 251, normalized size of antiderivative = 2.18

$$\int \frac{\sqrt{2-x+3x^2}(1+3x+4x^2)}{(1+2x)^3}\,dx = \frac{11}{24}\sqrt{3}\log\left(-2\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2-x+2}\right)+1\right) + \frac{803}{2704}\sqrt{13}\log\left(-\frac{\left|-4\sqrt{3}x-2\sqrt{13}-2\sqrt{3}+4\sqrt{3\,x^2-x+2}\right|}{2\left(2\sqrt{3}x-\sqrt{13}+\sqrt{3}-2\sqrt{3\,x^2-x+2}\right)}\right) + \frac{1}{2}\sqrt{3\,x^2-x+2} + \frac{318\left(\sqrt{3}x-\sqrt{3\,x^2-x+2}\right)^3-69\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2-x+2}\right)^2-1241\sqrt{3}x+649\sqrt{3}+1241\sqrt{3\,x^2-}}{104\left(2\left(\sqrt{3}x-\sqrt{3\,x^2-x+2}\right)^2+2\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2-x+2}\right)-5\right)^2}$$

```
[In] integrate((4*x^2+3*x+1)*(3*x^2-x+2)^(1/2)/(1+2*x)^3,x, algorithm="giac")
```

```
[Out] 11/24*sqrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) + 1) + 803/2
704*sqrt(13)*log(-1/2*abs(-4*sqrt(3)*x - 2*sqrt(13) - 2*sqrt(3) + 4*sqrt(3*
x^2 - x + 2))/(2*sqrt(3)*x - sqrt(13) + sqrt(3) - 2*sqrt(3*x^2 - x + 2))) +
 1/2*sqrt(3*x^2 - x + 2) + 1/104*(318*(sqrt(3)*x - sqrt(3*x^2 - x + 2))^3 -
 69*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2))^2 - 1241*sqrt(3)*x + 649*sqrt
(3) + 1241*sqrt(3*x^2 - x + 2))/(2*(sqrt(3)*x - sqrt(3*x^2 - x + 2))^2 + 2*
sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) - 5)^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-x+3x^2}(1+3x+4x^2)}{(1+2x)^3}\,dx = \int \frac{\sqrt{3\,x^2-x+2}\,\left(4\,x^2+3\,x+1\right)}{{\left(2\,x+1\right)}^3}\,dx$$

```
[In] int(((3*x^2 - x + 2)^(1/2)*(3*x + 4*x^2 + 1))/(2*x + 1)^3,x)
```

```
[Out] int(((3*x^2 - x + 2)^(1/2)*(3*x + 4*x^2 + 1))/(2*x + 1)^3, x)
```

3.214 $\int (1+2x)^3 \left(2-x+3x^2\right)^{3/2} \left(1+3x+4x^2\right) dx$

Optimal result

Integrand size = 32, antiderivative size = 158

$$\int (1+2x)^3 \left(2-x+3x^2\right)^{3/2} \left(1+3x+4x^2\right) dx = \frac{1255639(1-6x)\sqrt{2-x+3x^2}}{4478976} + \frac{54593(1-6x)\left(2-x+3x^2\right)^{3/2}}{559872} - \frac{11(283-5850x)\left(2-x+3x^2\right)^{5/2}}{58320} + \frac{913}{486}x^2\left(2-x+3x^2\right)^{5/2} + \frac{77}{81}x^3\left(2-x+3x^2\right)^{5/2} + \frac{2}{27}(1+2x)^4\left(2-x+3x^2\right)^{5/2} + \frac{28879697\operatorname{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{8957952\sqrt{3}}$$

```
[Out] 54593/559872*(1-6*x)*(3*x^2-x+2)^(3/2)-11/58320*(283-5850*x)*(3*x^2-x+2)^(5
/2)+913/486*x^2*(3*x^2-x+2)^(5/2)+77/81*x^3*(3*x^2-x+2)^(5/2)+2/27*(1+2*x)^
4*(3*x^2-x+2)^(5/2)+28879697/26873856*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2
)+1255639/4478976*(1-6*x)*(3*x^2-x+2)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1667, 12, 793, 626, 633, 221}

$$\int (1+2x)^3 \left(2-x+3x^2\right)^{3/2} \left(1+3x+4x^2\right) dx = \frac{28879697\operatorname{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{8957952\sqrt{3}} + \frac{2}{27}\left(3x^2-x+2\right)^{5/2}(2x+1)^4 + \frac{913}{486}x^2\left(3x^2-x+2\right)^{5/2} - \frac{11(283-5850x)\left(3x^2-x+2\right)^{5/2}}{58320} + \frac{54593(1-6x)\left(3x^2-x+2\right)^{3/2}}{559872} + \frac{1255639(1-6x)\sqrt{3x^2-x+2}}{4478976} + \frac{77}{81}x^3\left(3x^2-x+2\right)^{5/2}$$

```
[In] Int[(1 + 2*x)^3*(2 - x + 3*x^2)^(3/2)*(1 + 3*x + 4*x^2),x]
```

```
[Out] (1255639*(1 - 6*x)*Sqrt[2 - x + 3*x^2])/4478976 + (54593*(1 - 6*x)*(2 - x +
 3*x^2)^(3/2))/559872 - (11*(283 - 5850*x)*(2 - x + 3*x^2)^(5/2))/58320 + (
913*x^2*(2 - x + 3*x^2)^(5/2))/486 + (77*x^3*(2 - x + 3*x^2)^(5/2))/81 + (2
*(1 + 2*x)^4*(2 - x + 3*x^2)^(5/2))/27 + (28879697*ArcSinh[(1 - 6*x)/Sqrt[2
3]])/(8957952*Sqrt[3])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
```

```
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2}{27}(1+2x)^4\left(2-x+3x^2\right)^{5/2}+\frac{1}{108}\int 308x(1+2x)^3\left(2-x+3x^2\right)^{3/2}\,dx\\
&= \frac{2}{27}(1+2x)^4\left(2-x+3x^2\right)^{5/2}+\frac{77}{27}\int x(1+2x)^3\left(2-x+3x^2\right)^{3/2}\,dx\\
&= \frac{77}{81}x^3\left(2-x+3x^2\right)^{5/2}\\
&\quad+\frac{2}{27}(1+2x)^4\left(2-x+3x^2\right)^{5/2}+\frac{77}{648}\int x\left(2-x+3x^2\right)^{3/2}\left(24+96x+332x^2\right)\,dx\\
&= \frac{913}{486}x^2\left(2-x+3x^2\right)^{5/2}+\frac{77}{81}x^3\left(2-x+3x^2\right)^{5/2}\\
&\quad+\frac{2}{27}(1+2x)^4\left(2-x+3x^2\right)^{5/2}+\frac{11\int x(-824+3510x)\left(2-x+3x^2\right)^{3/2}\,dx}{1944}\\
&= -\frac{11(283-5850x)\left(2-x+3x^2\right)^{5/2}}{58320}+\frac{913}{486}x^2\left(2-x+3x^2\right)^{5/2}\\
&\quad+\frac{77}{81}x^3\left(2-x+3x^2\right)^{5/2}+\frac{2}{27}(1+2x)^4\left(2-x+3x^2\right)^{5/2}-\frac{54593\int\left(2-x+3x^2\right)^{3/2}\,dx}{23328}\\
&= \frac{54593(1-6x)\left(2-x+3x^2\right)^{3/2}}{559872}-\frac{11(283-5850x)\left(2-x+3x^2\right)^{5/2}}{58320}+\frac{913}{486}x^2\left(2-x+3x^2\right)^{5/2}\\
&\quad+\frac{77}{81}x^3\left(2-x+3x^2\right)^{5/2}+\frac{2}{27}(1+2x)^4\left(2-x+3x^2\right)^{5/2}-\frac{1255639\int\sqrt{2-x+3x^2}\,dx}{373248}\\
&= \frac{1255639(1-6x)\sqrt{2-x+3x^2}}{4478976}+\frac{54593(1-6x)\left(2-x+3x^2\right)^{3/2}}{559872}\\
&\quad-\frac{11(283-5850x)\left(2-x+3x^2\right)^{5/2}}{58320}+\frac{913}{486}x^2\left(2-x+3x^2\right)^{5/2}\\
&\quad+\frac{77}{81}x^3\left(2-x+3x^2\right)^{5/2}+\frac{2}{27}(1+2x)^4\left(2-x+3x^2\right)^{5/2}-\frac{28879697\int\frac{1}{\sqrt{2-x+3x^2}}\,dx}{8957952}\\
&= \frac{1255639(1-6x)\sqrt{2-x+3x^2}}{4478976}+\frac{54593(1-6x)\left(2-x+3x^2\right)^{3/2}}{559872}\\
&\quad-\frac{11(283-5850x)\left(2-x+3x^2\right)^{5/2}}{58320}+\frac{913}{486}x^2\left(2-x+3x^2\right)^{5/2}\\
&\quad+\frac{77}{81}x^3\left(2-x+3x^2\right)^{5/2}+\frac{2}{27}(1+2x)^4\left(2-x+3x^2\right)^{5/2}-\frac{\left(1255639\sqrt{\frac{23}{3}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx,x,-1+6\right.}{8957952}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{1255639(1-6x)\sqrt{2-x+3x^2}}{4478976} + \frac{54593(1-6x)\left(2-x+3x^2\right)^{3/2}}{559872} \\
&\quad - \frac{11(283-5850x)\left(2-x+3x^2\right)^{5/2}}{58320} + \frac{913}{486}x^2\left(2-x+3x^2\right)^{5/2} \\
&\quad + \frac{77}{81}x^3\left(2-x+3x^2\right)^{5/2} + \frac{2}{27}(1+2x)^4\left(2-x+3x^2\right)^{5/2} + \frac{28879697\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{8957952\sqrt{3}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.57

$$
\begin{aligned}
&\int (1+2x)^3\left(2-x+3x^2\right)^{3/2}(1+3x \\
&+4x^2)\ dx = \frac{6\sqrt{2-x+3x^2}(12499587+84014278x+201289704x^2+421626672x^3+649452672x^4+71121}{1}
\end{aligned}
$$

```
[In] Integrate[(1 + 2*x)^3*(2 - x + 3*x^2)^(3/2)*(1 + 3*x + 4*x^2),x]
```

```
[Out] (6*Sqrt[2 - x + 3*x^2]*(12499587 + 84014278*x + 201289704*x^2 + 421626672*x
^3 + 649452672*x^4 + 711210240*x^5 + 635765760*x^6 + 510105600*x^7 + 238878
720*x^8) + 144398485*Sqrt[3]*Log[1 - 6*x + 2*Sqrt[6 - 3*x + 9*x^2]])/134369
280
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.44

method	result
risch	$\frac{(238878720x^8+510105600x^7+635765760x^6+711210240x^5+649452672x^4+421626672x^3+201289704x^2+84014278x+12499587)\sqrt{3}}{22394880}$
trager	$\left(\frac{32}{3}x^8+\frac{205}{9}x^7+\frac{511}{18}x^6+\frac{20579}{648}x^5+\frac{563761}{19440}x^4+\frac{2927963}{155520}x^3+\frac{8387071}{933120}x^2+\frac{42007139}{11197440}x+\frac{1388843}{2488320}\right)\sqrt{3x^2-}$
default	$-\frac{54593(-1+6x)\left(3x^2-x+2\right)^{\frac{3}{2}}}{559872}-\frac{1255639(-1+6x)\sqrt{3x^2-x+2}}{4478976}-\frac{28879697\sqrt{3}\ \operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{26873856}+\frac{1207\left(3x^2-x+2\right)^{\frac{5}{2}}}{58320}$

```
[In] int((1+2*x)^3*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/22394880*(238878720*x^8+510105600*x^7+635765760*x^6+711210240*x^5+6494526
72*x^4+421626672*x^3+201289704*x^2+84014278*x+12499587)*(3*x^2-x+2)^(1/2)-2
8879697/26873856*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.59

$$\int (1+2x)^3 \left(2-x+3x^2\right)^{3/2} (1+3x$$
$$+4x^2\big)\ dx = \frac{1}{22394880}\left(238878720\,x^8+510105600\,x^7+635765760\,x^6+711210240\,x^5+649452672\,x^4+4216\right.$$
$$+\frac{28879697}{53747712}\sqrt{3}\log\left(4\sqrt{3}\sqrt{3\,x^2-x+2}(6\,x-1)-72\,x^2+24\,x-25\right)$$

```
[In] integrate((1+2*x)^3*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x, algorithm="fricas")
```

```
[Out] 1/22394880*(238878720*x^8 + 510105600*x^7 + 635765760*x^6 + 711210240*x^5 +
 649452672*x^4 + 421626672*x^3 + 201289704*x^2 + 84014278*x + 12499587)*sqr
t(3*x^2 - x + 2) + 28879697/53747712*sqrt(3)*log(4*sqrt(3)*sqrt(3*x^2 - x +
 2)*(6*x - 1) - 72*x^2 + 24*x - 25)
```

Sympy [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.57

$$\int (1+2x)^3 \left(2-x+3x^2\right)^{3/2} \left(1+3x+4x^2\right)\,dx = \sqrt{3x^2-x+2}$$
$$\cdot\left(\frac{32x^8}{3}+\frac{205x^7}{9}+\frac{511x^6}{18}+\frac{20579x^5}{648}+\frac{563761x^4}{19440}+\frac{2927963x^3}{155520}\right.$$
$$\left.+\frac{8387071x^2}{933120}+\frac{42007139x}{11197440}+\frac{1388843}{2488320}\right)-\frac{28879697\sqrt{3}\,\mathrm{asinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{26873856}$$

```
[In] integrate((1+2*x)**3*(3*x**2-x+2)**(3/2)*(4*x**2+3*x+1),x)
```

```
[Out] sqrt(3*x**2 - x + 2)*(32*x**8/3 + 205*x**7/9 + 511*x**6/18 + 20579*x**5/648
 + 563761*x**4/19440 + 2927963*x**3/155520 + 8387071*x**2/933120 + 42007139
*x/11197440 + 1388843/2488320) - 28879697*sqrt(3)*asinh(6*sqrt(23)*(x - 1/6
)/23)/26873856
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.98

$$\begin{aligned}\int (1+2x)^3\left(2-x+3x^2\right)^{3/2}\left(1+3x+4x^2\right)\,dx &= \frac{32}{27}\left(3\,x^2-x+2\right)^{\frac{5}{2}}x^4 \\ &+ \frac{269}{81}\left(3\,x^2-x+2\right)^{\frac{5}{2}}x^3+\frac{1777}{486}\left(3\,x^2-x+2\right)^{\frac{5}{2}}x^2+\frac{1099}{648}\left(3\,x^2-x+2\right)^{\frac{5}{2}}x \\ &+ \frac{1207}{58320}\left(3\,x^2-x+2\right)^{\frac{5}{2}}-\frac{54593}{93312}\left(3\,x^2-x+2\right)^{\frac{3}{2}}x \\ &+ \frac{54593}{559872}\left(3\,x^2-x+2\right)^{\frac{3}{2}}-\frac{1255639}{746496}\sqrt{3\,x^2-x+2}x \\ &- \frac{28879697}{26873856}\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{23}\sqrt{23}(6\,x-1)\right)+\frac{1255639}{4478976}\sqrt{3\,x^2-x+2}\end{aligned}$$

```
[In] integrate((1+2*x)^3*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x, algorithm="maxima")
```

```
[Out] 32/27*(3*x^2 - x + 2)^(5/2)*x^4 + 269/81*(3*x^2 - x + 2)^(5/2)*x^3 + 1777/4
86*(3*x^2 - x + 2)^(5/2)*x^2 + 1099/648*(3*x^2 - x + 2)^(5/2)*x + 1207/5832
0*(3*x^2 - x + 2)^(5/2) - 54593/93312*(3*x^2 - x + 2)^(3/2)*x + 54593/55987
2*(3*x^2 - x + 2)^(3/2) - 1255639/746496*sqrt(3*x^2 - x + 2)*x - 28879697/2
6873856*sqrt(3)*arcsinh(1/23*sqrt(23)*(6*x - 1)) + 1255639/4478976*sqrt(3*x
^2 - x + 2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.56

$$\begin{aligned}&\int (1+2x)^3\left(2-x+3x^2\right)^{3/2}(1+3x \\ &+4x^2)\,dx = \frac{1}{22394880}\left(2\left(12\left(6\left(8\left(30\left(36\left(2\left(96\,x+205\right)x+511\right)x+20579\right)x+563761\right)x+2927963\right)x+83\right.\right.\right.\right.\right.\right.\right. \\ &+ \frac{28879697}{26873856}\sqrt{3}\log\left(-2\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2-x+2}\right)+1\right)\end{aligned}$$

```
[In] integrate((1+2*x)^3*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x, algorithm="giac")
```

```
[Out] 1/22394880*(2*(12*(6*(8*(30*(36*(2*(96*x + 205)*x + 511)*x + 20579)*x + 563
761)*x + 2927963)*x + 8387071)*x + 42007139)*x + 12499587)*sqrt(3*x^2 - x +
 2) + 28879697/26873856*sqrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x
+ 2)) + 1)
```

Mupad [F(-1)]

Timed out.

$$\int (1+2x)^3 \left(2-x+3x^2\right)^{3/2} \left(1+3x+4x^2\right)\,dx = \int \left(2\,x+1\right)^3\,\left(3\,x^2-x+2\right)^{3/2}\,\left(4\,x^2+3\,x+1\right) \,dx$$

```
[In] int((2*x + 1)^3*(3*x^2 - x + 2)^(3/2)*(3*x + 4*x^2 + 1),x)
[Out] int((2*x + 1)^3*(3*x^2 - x + 2)^(3/2)*(3*x + 4*x^2 + 1), x)
```

3.215 $\int (1+2x)^2 \left(2-x+3x^2\right)^{3/2} \left(1+3x+4x^2\right) \, dx$

Optimal result

Integrand size = 32, antiderivative size = 141

$$\int (1+2x)^2 \left(2-x+3x^2\right)^{3/2} \left(1+3x+4x^2\right) \, dx = \frac{2093(1-6x)\sqrt{2-x+3x^2}}{27648} + \frac{91(1-6x)\left(2-x+3x^2\right)^{3/2}}{3456} + \frac{8}{63}(1+2x)^2\left(2-x+3x^2\right)^{5/2} + \frac{1}{12}(1+2x)^3\left(2-x+3x^2\right)^{5/2} + \frac{13(29+50x)\left(2-x+3x^2\right)^{5/2}}{2520} + \frac{48139\operatorname{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{55296\sqrt{3}}$$

```
[Out] 91/3456*(1-6*x)*(3*x^2-x+2)^(3/2)+8/63*(1+2*x)^2*(3*x^2-x+2)^(5/2)+1/12*(1+
2*x)^3*(3*x^2-x+2)^(5/2)+13/2520*(29+50*x)*(3*x^2-x+2)^(5/2)+48139/165888*a
rcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)+2093/27648*(1-6*x)*(3*x^2-x+2)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1667, 846, 793, 626, 633, 221}

$$\int (1+2x)^2 \left(2-x+3x^2\right)^{3/2} \left(1+3x+4x^2\right) \, dx = \frac{48139\operatorname{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{55296\sqrt{3}} + \frac{1}{12}\left(3x^2-x+2\right)^{5/2}(2x+1)^3 + \frac{8}{63}\left(3x^2-x+2\right)^{5/2}(2x+1)^2 + \frac{13(50x+29)\left(3x^2-x+2\right)^{5/2}}{2520} + \frac{91(1-6x)\left(3x^2-x+2\right)^{3/2}}{3456} + \frac{2093(1-6x)}{276}$$

```
[In] Int[(1 + 2*x)^2*(2 - x + 3*x^2)^(3/2)*(1 + 3*x + 4*x^2),x]
```

```
[Out] (2093*(1 - 6*x)*Sqrt[2 - x + 3*x^2])/27648 + (91*(1 - 6*x)*(2 - x + 3*x^2)^
(3/2))/3456 + (8*(1 + 2*x)^2*(2 - x + 3*x^2)^(5/2))/63 + ((1 + 2*x)^3*(2 -
x + 3*x^2)^(5/2))/12 + (13*(29 + 50*x)*(2 - x + 3*x^2)^(5/2))/2520 + (48139
*ArcSinh[(1 - 6*x)/Sqrt[23]])/(55296*Sqrt[3])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 846

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
```

```
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{12}(1+2x)^3\left(2-x+3x^2\right)^{5/2}+\frac{1}{96}\int(1+2x)^2(20+256x)\left(2-x+3x^2\right)^{3/2}\,dx\\
&= \frac{8}{63}(1+2x)^2\left(2-x+3x^2\right)^{5/2}\\
&\quad+\frac{1}{12}(1+2x)^3\left(2-x+3x^2\right)^{5/2}+\frac{\int(1+2x)(-988+4680x)\left(2-x+3x^2\right)^{3/2}\,dx}{2016}\\
&= \frac{8}{63}(1+2x)^2\left(2-x+3x^2\right)^{5/2}+\frac{1}{12}(1+2x)^3\left(2-x+3x^2\right)^{5/2}\\
&\quad+\frac{13(29+50x)\left(2-x+3x^2\right)^{5/2}}{2520}-\frac{91}{144}\int\left(2-x+3x^2\right)^{3/2}\,dx\\
&= \frac{91(1-6x)\left(2-x+3x^2\right)^{3/2}}{3456}\\
&\quad+\frac{8}{63}(1+2x)^2\left(2-x+3x^2\right)^{5/2}+\frac{1}{12}(1+2x)^3\left(2-x+3x^2\right)^{5/2}\\
&\quad+\frac{13(29+50x)\left(2-x+3x^2\right)^{5/2}}{2520}-\frac{2093\int\sqrt{2-x+3x^2}\,dx}{2304}\\
&= \frac{2093(1-6x)\sqrt{2-x+3x^2}}{27648}+\frac{91(1-6x)\left(2-x+3x^2\right)^{3/2}}{3456}+\frac{8}{63}(1+2x)^2\left(2-x+3x^2\right)^{5/2}\\
&\quad+\frac{1}{12}(1+2x)^3\left(2-x+3x^2\right)^{5/2}+\frac{13(29+50x)\left(2-x+3x^2\right)^{5/2}}{2520}\\
&\quad-\frac{48139\int\frac{1}{\sqrt{2-x+3x^2}}\,dx}{55296}\\
&= \frac{2093(1-6x)\sqrt{2-x+3x^2}}{27648}+\frac{91(1-6x)\left(2-x+3x^2\right)^{3/2}}{3456}\\
&\quad+\frac{8}{63}(1+2x)^2\left(2-x+3x^2\right)^{5/2}+\frac{1}{12}(1+2x)^3\left(2-x+3x^2\right)^{5/2}\\
&\quad+\frac{13(29+50x)\left(2-x+3x^2\right)^{5/2}}{2520}-\frac{\left(2093\sqrt{\frac{23}{3}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx,x,-1+6x\right)}{55296}
\end{aligned}$$

$$= \frac{2093(1-6x)\sqrt{2-x+3x^2}}{27648} + \frac{91(1-6x)\left(2-x+3x^2\right)^{3/2}}{3456} + \frac{8}{63}(1+2x)^2\left(2-x+3x^2\right)^{5/2} + \frac{1}{12}(1+2x)^3\left(2-x+3x^2\right)^{5/2} + \frac{13(29+50x)\left(2-x+3x^2\right)^{5/2}}{2520} + \frac{48139\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{55296\sqrt{3}}$$

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.60

$$\int (1+2x)^2\left(2-x+3x^2\right)^{3/2}\left(1+3x+4x^2\right)\,dx = \frac{6\sqrt{2-x+3x^2}(1517367+2735918x+5694024x^2+10119792x^3+12173952x^4+10656000x^5+}{5806080}$$

```
[In] Integrate[(1 + 2*x)^2*(2 - x + 3*x^2)^(3/2)*(1 + 3*x + 4*x^2),x]
```

```
[Out] (6*Sqrt[2 - x + 3*x^2]*(1517367 + 2735918*x + 5694024*x^2 + 10119792*x^3 +
12173952*x^4 + 10656000*x^5 + 9262080*x^6 + 5806080*x^7) + 1684865*Sqrt[3]*
Log[1 - 6*x + 2*Sqrt[6 - 3*x + 9*x^2]])/5806080
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.46

method	result
risch	$\frac{\left(5806080x^7+9262080x^6+10656000x^5+12173952x^4+10119792x^3+5694024x^2+2735918x+1517367\right)\sqrt{3x^2-x+2}}{967680} - \frac{48139\sqrt{3}\ \operatorname{arcsinh}}{165}$
trager	$\left(6x^7+\frac{67}{7}x^6+\frac{925}{84}x^5+\frac{4529}{360}x^4+\frac{210829}{20160}x^3+\frac{33893}{5760}x^2+\frac{1367959}{483840}x+\frac{505789}{322560}\right)\sqrt{3x^2-x+2} - \frac{48139\,\mathrm{RootO}}{}$
default	$-\frac{91(-1+6x)\left(3x^2-x+2\right)^{\frac{3}{2}}}{3456} - \frac{2093(-1+6x)\sqrt{3x^2-x+2}}{27648} - \frac{48139\sqrt{3}\ \operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{165888} + \frac{907\left(3x^2-x+2\right)^{\frac{5}{2}}}{2520} + \frac{2x^3\left(3x^2-\right.}{3}$

```
[In] int((1+2*x)^2*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/967680*(5806080*x^7+9262080*x^6+10656000*x^5+12173952*x^4+10119792*x^3+56
94024*x^2+2735918*x+1517367)*(3*x^2-x+2)^(1/2)-48139/165888*3^(1/2)*arcsinh
(6/23*23^(1/2)*(x-1/6))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.62

$$\int (1+2x)^2 \left(2-x+3x^2\right)^{3/2} (1+3x +4x^2)\, dx = \frac{1}{967680}(5806080\,x^7+9262080\,x^6+10656000\,x^5+12173952\,x^4+10119792\,x^3+5694024\,x^2+$$

$$+\frac{48139}{331776}\,\sqrt{3}\log\left(4\,\sqrt{3}\sqrt{3\,x^2-x+2}(6\,x-1)-72\,x^2+24\,x-25\right)$$

```
[In] integrate((1+2*x)^2*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x, algorithm="fricas")
```

```
[Out] 1/967680*(5806080*x^7 + 9262080*x^6 + 10656000*x^5 + 12173952*x^4 + 1011979
2*x^3 + 5694024*x^2 + 2735918*x + 1517367)*sqrt(3*x^2 - x + 2) + 48139/3317
76*sqrt(3)*log(4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x - 1) - 72*x^2 + 24*x - 25
)
```

Sympy [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.58

$$\int (1+2x)^2 \left(2-x+3x^2\right)^{3/2} \left(1+3x+4x^2\right)\, dx = \sqrt{3x^2-x+2} \cdot \left(6x^7+\frac{67x^6}{7}+\frac{925x^5}{84}+\frac{4529x^4}{360}+\frac{210829x^3}{20160}+\frac{33893x^2}{5760}+\frac{1367959x}{483840}+\frac{505789}{322560}\right) - \frac{48139\sqrt{3}\operatorname{asinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{165888}$$

```
[In] integrate((1+2*x)**2*(3*x**2-x+2)**(3/2)*(4*x**2+3*x+1),x)
```

```
[Out] sqrt(3*x**2 - x + 2)*(6*x**7 + 67*x**6/7 + 925*x**5/84 + 4529*x**4/360 + 21
0829*x**3/20160 + 33893*x**2/5760 + 1367959*x/483840 + 505789/322560) - 481
39*sqrt(3)*asinh(6*sqrt(23)*(x - 1/6)/23)/165888
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.98

$$
\begin{aligned}
\int (1+2x)^2 \left(2-x+3x^2\right)^{3/2} \left(1+3x+4x^2\right)\,dx &= \frac{2}{3}\left(3\,x^2-x+2\right)^{\frac{5}{2}}x^3 \\
&+ \frac{95}{63}\left(3\,x^2-x+2\right)^{\frac{5}{2}}x^2 + \frac{319}{252}\left(3\,x^2-x+2\right)^{\frac{5}{2}}x + \frac{907}{2520}\left(3\,x^2-x+2\right)^{\frac{5}{2}} \\
&- \frac{91}{576}\left(3\,x^2-x+2\right)^{\frac{3}{2}}x + \frac{91}{3456}\left(3\,x^2-x+2\right)^{\frac{3}{2}} - \frac{2093}{4608}\sqrt{3\,x^2-x+2}x \\
&- \frac{48139}{165888}\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{23}\sqrt{23}(6\,x-1)\right) + \frac{2093}{27648}\sqrt{3\,x^2-x+2}
\end{aligned}
$$

```
[In] integrate((1+2*x)^2*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x, algorithm="maxima")
```

```
[Out] 2/3*(3*x^2 - x + 2)^(5/2)*x^3 + 95/63*(3*x^2 - x + 2)^(5/2)*x^2 + 319/252*(
3*x^2 - x + 2)^(5/2)*x + 907/2520*(3*x^2 - x + 2)^(5/2) - 91/576*(3*x^2 - x
 + 2)^(3/2)*x + 91/3456*(3*x^2 - x + 2)^(3/2) - 2093/4608*sqrt(3*x^2 - x +
2)*x - 48139/165888*sqrt(3)*arcsinh(1/23*sqrt(23)*(6*x - 1)) + 2093/27648*s
qrt(3*x^2 - x + 2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.59

$$
\begin{aligned}
&\int (1+2x)^2 \left(2-x+3x^2\right)^{3/2} (1+3x \\
&+4x^2)\,dx = \frac{1}{967680}(2\,(12\,(2\,(8\,(30\,(12\,(42\,x+67)x+925)x+31703)x+210829)x+237251)x+1367959)x \\
&+ \frac{48139}{165888}\sqrt{3}\log\left(-2\,\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2-x+2}\right)+1\right)
\end{aligned}
$$

```
[In] integrate((1+2*x)^2*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x, algorithm="giac")
```

```
[Out] 1/967680*(2*(12*(2*(8*(30*(12*(42*x + 67)*x + 925)*x + 31703)*x + 210829)*x
 + 237251)*x + 1367959)*x + 1517367)*sqrt(3*x^2 - x + 2) + 48139/165888*sqr
t(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) + 1)
```

Mupad [F(-1)]

Timed out.

$$\int (1+2x)^2 \left(2-x+3x^2\right)^{3/2} \left(1+3x+4x^2\right)\,dx=\int {\left(2\,x+1\right)}^2\,{\left(3\,x^2-x+2\right)}^{3/2}\,\left(4\,x^2+3\,x+1\right)\,dx$$

```
[In] int((2*x + 1)^2*(3*x^2 - x + 2)^(3/2)*(3*x + 4*x^2 + 1),x)
[Out] int((2*x + 1)^2*(3*x^2 - x + 2)^(3/2)*(3*x + 4*x^2 + 1), x)
```

3.216 $\int (1+2x) \left(2-x+3x^2\right)^{3/2} \left(1+3x+4x^2\right) \, dx$

Optimal result

Integrand size = 30, antiderivative size = 116

$$\begin{aligned}\int (1+2x) \left(2-x+3x^2\right)^{3/2} \left(1+3x+4x^2\right) \, dx = & -\frac{1633(1-6x)\sqrt{2-x+3x^2}}{20736}-\frac{71(1-6x) \left(2-x+3x^2\right)^{3/2}}{2592} \\ & +\frac{2}{21}(1+2x)^2 \left(2-x+3x^2\right)^{5/2}+\frac{1}{378}(109+102x) \left(2-x+3x^2\right)^{5/2}-\frac{37559\text{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{41472\sqrt{3}}\end{aligned}$$

```
[Out] -71/2592*(1-6*x)*(3*x^2-x+2)^(3/2)+2/21*(1+2*x)^2*(3*x^2-x+2)^(5/2)+1/378*(
109+102*x)*(3*x^2-x+2)^(5/2)-37559/124416*arcsinh(1/23*(1-6*x)*23^(1/2))*3^
(1/2)-1633/20736*(1-6*x)*(3*x^2-x+2)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1667, 793, 626, 633, 221}

$$\begin{aligned}\int (1+2x) \left(2-x+3x^2\right)^{3/2} \left(1+3x+4x^2\right) \, dx = & -\frac{37559\text{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{41472\sqrt{3}} \\ & +\frac{2}{21}(2x+1)^2 \left(3x^2-x+2\right)^{5/2}+\frac{1}{378}(102x+109) \left(3x^2-x+2\right)^{5/2} \\ & -\frac{71(1-6x) \left(3x^2-x+2\right)^{3/2}}{2592}-\frac{1633(1-6x)\sqrt{3x^2-x+2}}{20736}\end{aligned}$$

```
[In] Int[(1 + 2*x)*(2 - x + 3*x^2)^(3/2)*(1 + 3*x + 4*x^2),x]
```

```
[Out] (-1633*(1 - 6*x)*Sqrt[2 - x + 3*x^2])/20736 - (71*(1 - 6*x)*(2 - x + 3*x^2)
^(3/2))/2592 + (2*(1 + 2*x)^2*(2 - x + 3*x^2)^(5/2))/21 + ((109 + 102*x)*(2
 - x + 3*x^2)^(5/2))/378 - (37559*ArcSinh[(1 - 6*x)/Sqrt[23]])/(41472*Sqrt[
3])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2}{21}(1+2x)^2\left(2-x+3x^2\right)^{5/2}+\frac{1}{84}\int(1+2x)(40+204x)\left(2-x+3x^2\right)^{3/2}\,dx\\
&= \frac{2}{21}(1+2x)^2\left(2-x+3x^2\right)^{5/2}+\frac{1}{378}(109+102x)\left(2-x+3x^2\right)^{5/2}+\frac{71}{108}\int\left(2-x+3x^2\right)^{3/2}\,dx\\
&= -\frac{71(1-6x)\left(2-x+3x^2\right)^{3/2}}{2592}+\frac{2}{21}(1+2x)^2\left(2-x+3x^2\right)^{5/2}\\
&\quad+\frac{1}{378}(109+102x)\left(2-x+3x^2\right)^{5/2}+\frac{1633\int\sqrt{2-x+3x^2}\,dx}{1728}\\
&= -\frac{1633(1-6x)\sqrt{2-x+3x^2}}{20736}-\frac{71(1-6x)\left(2-x+3x^2\right)^{3/2}}{2592}\\
&\quad+\frac{2}{21}(1+2x)^2\left(2-x+3x^2\right)^{5/2}\\
&\quad+\frac{1}{378}(109+102x)\left(2-x+3x^2\right)^{5/2}+\frac{37559\int\frac{1}{\sqrt{2-x+3x^2}}\,dx}{41472}\\
&= -\frac{1633(1-6x)\sqrt{2-x+3x^2}}{20736}-\frac{71(1-6x)\left(2-x+3x^2\right)^{3/2}}{2592}+\frac{2}{21}(1+2x)^2\left(2-x+3x^2\right)^{5/2}\\
&\quad+\frac{1}{378}(109+102x)\left(2-x+3x^2\right)^{5/2}+\frac{\left(1633\sqrt{\frac{23}{3}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx,x,-1+6x\right)}{41472}\\
&= -\frac{1633(1-6x)\sqrt{2-x+3x^2}}{20736}-\frac{71(1-6x)\left(2-x+3x^2\right)^{3/2}}{2592}\\
&\quad+\frac{2}{21}(1+2x)^2\left(2-x+3x^2\right)^{5/2}\\
&\quad+\frac{1}{378}(109+102x)\left(2-x+3x^2\right)^{5/2}-\frac{37559\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{41472\sqrt{3}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.69

$$
\int(1+2x)\left(2-x+3x^2\right)^{3/2}\left(1+3x+4x^2\right)\,dx=\frac{6\sqrt{2-x+3x^2}(203337+275410x+531384x^2+744336x^3+653184x^4+518400x^5+497664x^6)}{870912}
$$

```
[In] Integrate[(1 + 2*x)*(2 - x + 3*x^2)^(3/2)*(1 + 3*x + 4*x^2),x]
```

```
[Out] (6*Sqrt[2 - x + 3*x^2]*(203337 + 275410*x + 531384*x^2 + 744336*x^3 + 65318
4*x^4 + 518400*x^5 + 497664*x^6) - 262913*Sqrt[3]*Log[1 - 6*x + 2*Sqrt[6 -
3*x + 9*x^2]])/870912
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.52

method	result
risch	$\frac{(497664x^6+518400x^5+653184x^4+744336x^3+531384x^2+275410x+203337)\sqrt{3x^2-x+2}}{145152}+\frac{37559\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\,\left(x-\frac{1}{6}\right)}{23}\right)}{124416}$
trager	$\left(\frac{24}{7}x^6+\frac{25}{7}x^5+\frac{9}{2}x^4+\frac{1723}{336}x^3+\frac{3163}{864}x^2+\frac{137705}{72576}x+\frac{7531}{5376}\right)\sqrt{3x^2-x+2}+\frac{37559\,\operatorname{RootOf}\left(\_Z^2-3\right)\ln\Big(6$
default	$\frac{71(-1+6x)(3x^2-x+2)^{\frac{3}{2}}}{2592}+\frac{1633(-1+6x)\sqrt{3x^2-x+2}}{20736}+\frac{37559\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\,\left(x-\frac{1}{6}\right)}{23}\right)}{124416}+\frac{145(3x^2-x+2)^{\frac{5}{2}}}{378}+\frac{8x^2(3x^2-x}{21}$

```
[In] int((1+2*x)*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/145152*(497664*x^6+518400*x^5+653184*x^4+744336*x^3+531384*x^2+275410*x+2
03337)*(3*x^2-x+2)^(1/2)+37559/124416*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6)
)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.72

$$\int (1+2x)\left(2-x+3x^2\right)^{3/2}\left(1+3x+4x^2\right)\,dx=\frac{1}{145152}\left(497664\,x^6+518400\,x^5+653184\,x^4+744336\,x^3+531384\,x^2+275410\,x+203337\right)\sqrt{3}$$
$$+\frac{37559}{248832}\,\sqrt{3}\log\left(-4\,\sqrt{3}\sqrt{3\,x^2-x+2}(6\,x-1)-72\,x^2+24\,x-25\right)$$

```
[In] integrate((1+2*x)*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x, algorithm="fricas")
```

```
[Out] 1/145152*(497664*x^6 + 518400*x^5 + 653184*x^4 + 744336*x^3 + 531384*x^2 +
275410*x + 203337)*sqrt(3*x^2 - x + 2) + 37559/248832*sqrt(3)*log(-4*sqrt(3
)*sqrt(3*x^2 - x + 2)*(6*x - 1) - 72*x^2 + 24*x - 25)
```

Sympy [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.66

$$\int (1+2x)\left(2-x+3x^2\right)^{3/2}\left(1+3x+4x^2\right)\,dx = \sqrt{3x^2-x+2}\cdot\left(\frac{24x^6}{7}+\frac{25x^5}{7}+\frac{9x^4}{2}+\frac{1723x^3}{336}+\frac{3163x^2}{864}+\frac{137705x}{72576}+\frac{7531}{5376}\right)+\frac{37559\sqrt{3}\operatorname{asinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{124416}$$

```
[In] integrate((1+2*x)*(3*x**2-x+2)**(3/2)*(4*x**2+3*x+1),x)
```

```
[Out] sqrt(3*x**2 - x + 2)*(24*x**6/7 + 25*x**5/7 + 9*x**4/2 + 1723*x**3/336 + 31
63*x**2/864 + 137705*x/72576 + 7531/5376) + 37559*sqrt(3)*asinh(6*sqrt(23)*
(x - 1/6)/23)/124416
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.04

$$\int (1+2x)\left(2-x+3x^2\right)^{3/2}\left(1+3x+4x^2\right)\,dx = \frac{8}{21}\left(3\,x^2-x+2\right)^{\frac{5}{2}}x^2+\frac{41}{63}\left(3\,x^2-x+2\right)^{\frac{5}{2}}x+\frac{145}{378}\left(3\,x^2-x+2\right)^{\frac{5}{2}}+\frac{71}{432}\left(3\,x^2-x+2\right)^{\frac{3}{2}}x-\frac{71}{2592}\left(3\,x^2-x+2\right)^{\frac{3}{2}}+\frac{1633}{3456}\sqrt{3\,x^2-x+2}x+\frac{37559}{124416}\sqrt{3}\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}(6\,x-1)\right)-\frac{1633}{20736}\sqrt{3\,x^2-x+2}$$

```
[In] integrate((1+2*x)*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x, algorithm="maxima")
```

```
[Out] 8/21*(3*x^2 - x + 2)^(5/2)*x^2 + 41/63*(3*x^2 - x + 2)^(5/2)*x + 145/378*(3
*x^2 - x + 2)^(5/2) + 71/432*(3*x^2 - x + 2)^(3/2)*x - 71/2592*(3*x^2 - x +
 2)^(3/2) + 1633/3456*sqrt(3*x^2 - x + 2)*x + 37559/124416*sqrt(3)*arcsinh(
1/23*sqrt(23)*(6*x - 1)) - 1633/20736*sqrt(3*x^2 - x + 2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.67

$$\int (1+2x)\left(2-x+3x^2\right)^{3/2}(1+3x$$
$$+4x^2)\;dx=\frac{1}{145152}(2\,(12\,(18\,(24\,(2\,(24\,x+25)x+63)x+1723)x+22141)x+137705)x+203337)\sqrt{3\,x^2-}$$
$$-\frac{37559}{124416}\sqrt{3}\log\left(-2\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2-x+2}\right)+1\right)$$

```
[In] integrate((1+2*x)*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x, algorithm="giac")
```

```
[Out] 1/145152*(2*(12*(18*(24*(2*(24*x + 25)*x + 63)*x + 1723)*x + 22141)*x + 137
705)*x + 203337)*sqrt(3*x^2 - x + 2) - 37559/124416*sqrt(3)*log(-2*sqrt(3)*
(sqrt(3)*x - sqrt(3*x^2 - x + 2)) + 1)
```

Mupad [F(-1)]

Timed out.

$$\int (1+2x)\,(2-x$$
$$+3x^2)^{3/2}\left(1+3x+4x^2\right)\,dx=\int (2\,x+1)\,\left(3\,x^2-x+2\right)^{3/2}\left(4\,x^2+3\,x+1\right)\,dx$$

```
[In] int((2*x + 1)*(3*x^2 - x + 2)^(3/2)*(3*x + 4*x^2 + 1),x)
```

```
[Out] int((2*x + 1)*(3*x^2 - x + 2)^(3/2)*(3*x + 4*x^2 + 1), x)
```

3.217 $\int \frac{(2-x+3x^2)^{3/2}(1+3x+4x^2)}{1+2x}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 124

$$
\begin{aligned}
\int \frac{(2-x+3x^2)^{3/2}(1+3x+4x^2)}{1+2x}\,dx &= \frac{(869+402x)\sqrt{2-x+3x^2}}{1152} \\
&+ \frac{1}{144}(7+30x)\left(2-x+3x^2\right)^{3/2} \\
&+ \frac{2}{15}\left(2-x+3x^2\right)^{5/2} + \frac{2203\operatorname{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{2304\sqrt{3}} - \frac{13}{32}\sqrt{13}\operatorname{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)
\end{aligned}
$$

```
[Out] 1/144*(7+30*x)*(3*x^2-x+2)^(3/2)+2/15*(3*x^2-x+2)^(5/2)+2203/6912*arcsinh(1
/23*(1-6*x)*23^(1/2))*3^(1/2)-13/32*arctanh(1/26*(9-8*x)*13^(1/2)/(3*x^2-x+
2)^(1/2))*13^(1/2)+1/1152*(869+402*x)*(3*x^2-x+2)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {1667, 828, 857, 633, 221, 738, 212}

$$
\begin{aligned}
\int \frac{(2-x+3x^2)^{3/2}(1+3x+4x^2)}{1+2x}\,dx &= \frac{2203\operatorname{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{2304\sqrt{3}} \\
&- \frac{13}{32}\sqrt{13}\operatorname{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right) + \frac{2}{15}\left(3x^2-x+2\right)^{5/2} \\
&+ \frac{1}{144}(30x+7)\left(3x^2-x+2\right)^{3/2} + \frac{(402x+869)\sqrt{3x^2-x+2}}{1152}
\end{aligned}
$$

```
[In] Int[((2 - x + 3*x^2)^(3/2)*(1 + 3*x + 4*x^2))/(1 + 2*x),x]
```

```
[Out] ((869 + 402*x)*Sqrt[2 - x + 3*x^2])/1152 + ((7 + 30*x)*(2 - x + 3*x^2)^(3/2
))/144 + (2*(2 - x + 3*x^2)^(5/2))/15 + (2203*ArcSinh[(1 - 6*x)/Sqrt[23]])/
(2304*Sqrt[3]) - (13*Sqrt[13]*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*
x^2])])/32
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
```

```
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2}{15}\left(2-x+3x^2\right)^{5/2}+\frac{1}{60}\int \frac{(80+100x)\left(2-x+3x^2\right)^{3/2}}{1+2x}\,dx \\
&= \frac{1}{144}(7+30x)\left(2-x+3x^2\right)^{3/2}+\frac{2}{15}\left(2-x+3x^2\right)^{5/2}-\frac{\int \frac{(-13380-8040x)\sqrt{2-x+3x^2}}{1+2x}\,dx}{5760} \\
&= \frac{(869+402x)\sqrt{2-x+3x^2}}{1152}+\frac{1}{144}(7+30x)\left(2-x+3x^2\right)^{3/2} \\
&\quad +\frac{2}{15}\left(2-x+3x^2\right)^{5/2}+\frac{\int \frac{1195800-528720x}{(1+2x)\sqrt{2-x+3x^2}}\,dx}{276480} \\
&= \frac{(869+402x)\sqrt{2-x+3x^2}}{1152}+\frac{1}{144}(7+30x)\left(2-x+3x^2\right)^{3/2} \\
&\quad +\frac{2}{15}\left(2-x+3x^2\right)^{5/2}-\frac{2203\int \frac{1}{\sqrt{2-x+3x^2}}\,dx}{2304}+\frac{169}{32}\int \frac{1}{(1+2x)\sqrt{2-x+3x^2}}\,dx \\
&= \frac{(869+402x)\sqrt{2-x+3x^2}}{1152}+\frac{1}{144}(7+30x)\left(2-x+3x^2\right)^{3/2}+\frac{2}{15}\left(2-x+3x^2\right)^{5/2} \\
&\quad -\frac{169}{16}\text{Subst}\left(\int \frac{1}{52-x^2}\,dx,x,\frac{9-8x}{\sqrt{2-x+3x^2}}\right)-\frac{2203\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx,x,-1+6x\right)}{2304\sqrt{69}} \\
&= \frac{(869+402x)\sqrt{2-x+3x^2}}{1152}+\frac{1}{144}(7+30x)\left(2-x+3x^2\right)^{3/2} \\
&\quad +\frac{2}{15}\left(2-x+3x^2\right)^{5/2}+\frac{2203\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{2304\sqrt{3}}-\frac{13}{32}\sqrt{13}\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.92

$$\int \frac{(2-x+3x^2)^{3/2}(1+3x+4x^2)}{1+2x}\,dx = \frac{6\sqrt{2-x+3x^2}(7977+1058x+9624x^2-1008x^3+6912x^4)+280}{}$$

```
[In] Integrate[((2 - x + 3*x^2)^(3/2)*(1 + 3*x + 4*x^2))/(1 + 2*x),x]
```

```
[Out] (6*Sqrt[2 - x + 3*x^2]*(7977 + 1058*x + 9624*x^2 - 1008*x^3 + 6912*x^4) + 2
8080*Sqrt[13]*ArcTanh[(Sqrt[3] + 2*Sqrt[3]*x - 2*Sqrt[2 - x + 3*x^2])/Sqrt[
13]] + 11015*Sqrt[3]*Log[1 - 6*x + 2*Sqrt[6 - 3*x + 9*x^2]])/34560
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.65

method	result
risch	$\frac{(6912x^4-1008x^3+9624x^2+1058x+7977)\sqrt{3x^2-x+2}}{5760} - \frac{2203\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{6912} - \frac{13\sqrt{13}\,\operatorname{arctanh}\left(\frac{2\left(\frac{9}{2}-4x\right)\sqrt{13}}{13\sqrt{12\left(x+\frac{1}{2}\right)^2-16x}}\right.}{32}$
trager	$\left(\frac{6}{5}x^4-\frac{7}{40}x^3+\frac{401}{240}x^2+\frac{529}{2880}x+\frac{2659}{1920}\right)\sqrt{3x^2-x+2}+\frac{13\,\mathrm{RootOf}\left(\_Z^2-13\right)\ln\left(\frac{8\,\mathrm{RootOf}\left(\_Z^2-13\right)x+26\sqrt{3x^2}}{1+2}\right.}{32}$
default	$\frac{5(-1+6x)(3x^2-x+2)^{\frac{3}{2}}}{144}+\frac{115(-1+6x)\sqrt{3x^2-x+2}}{1152}-\frac{2203\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{6912}+\frac{2(3x^2-x+2)^{\frac{5}{2}}}{15}+\frac{\left(3\left(x+\frac{1}{2}\right)^2-4x+\right.}{12}$

```
[In] int((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(1+2*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5760*(6912*x^4-1008*x^3+9624*x^2+1058*x+7977)*(3*x^2-x+2)^(1/2)-2203/6912
*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))-13/32*13^(1/2)*arctanh(2/13*(9/2-4*
x)*13^(1/2)/(12*(x+1/2)^2-16*x+5)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.01

$$\int \frac{(2-x+3x^2)^{3/2}\,(1+3x+4x^2)}{1+2x}\,dx = \frac{1}{5760}\left(6912\,x^4-1008\,x^3+9624\,x^2+1058\,x+7977\right)\sqrt{3\,x^2-x+2} + \frac{2203}{13824}\sqrt{3}\log\left(4\sqrt{3}\sqrt{3\,x^2-x+2}(6\,x-1)-72\,x^2+24\,x-25\right) + \frac{13}{64}\sqrt{13}\log\left(-\frac{4\sqrt{13}\sqrt{3\,x^2-x+2}(8\,x-9)+220\,x^2-196\,x+185}{4\,x^2+4\,x+1}\right)$$

```
[In] integrate((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(1+2*x),x, algorithm="fricas")
```

```
[Out] 1/5760*(6912*x^4 - 1008*x^3 + 9624*x^2 + 1058*x + 7977)*sqrt(3*x^2 - x + 2)
 + 2203/13824*sqrt(3)*log(4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x - 1) - 72*x^2
+ 24*x - 25) + 13/64*sqrt(13)*log(-(4*sqrt(13)*sqrt(3*x^2 - x + 2)*(8*x - 9
) + 220*x^2 - 196*x + 185)/(4*x^2 + 4*x + 1))
```

Sympy [F]

$$\int \frac{(2-x+3x^2)^{3/2}\,(1+3x+4x^2)}{1+2x}\,dx = \int \frac{(3x^2-x+2)^{\frac{3}{2}}\cdot(4x^2+3x+1)}{2x+1}\,dx$$

```
[In] integrate((3*x**2-x+2)**(3/2)*(4*x**2+3*x+1)/(1+2*x),x)
```

```
[Out] Integral((3*x**2 - x + 2)**(3/2)*(4*x**2 + 3*x + 1)/(2*x + 1), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.01

$$\int \frac{(2-x+3x^2)^{3/2}\,(1+3x+4x^2)}{1+2x}\,dx = \frac{2}{15}\left(3\,x^2-x+2\right)^{\frac{5}{2}} + \frac{5}{24}\left(3\,x^2-x+2\right)^{\frac{3}{2}}x + \frac{7}{144}\left(3\,x^2-x+2\right)^{\frac{3}{2}} + \frac{67}{192}\sqrt{3\,x^2-x+2}x - \frac{2203}{6912}\sqrt{3}\,\mathrm{arsinh}\left(\frac{6}{23}\sqrt{23}x-\frac{1}{23}\sqrt{23}\right) + \frac{13}{32}\sqrt{13}\,\mathrm{arsinh}\left(\frac{8\sqrt{23}x}{23\,|2\,x+1|}-\frac{9\sqrt{23}}{23\,|2\,x+1|}\right) + \frac{869}{1152}\sqrt{3\,x^2-x+2}$$

```
[In] integrate((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(1+2*x),x, algorithm="maxima")
```

```
[Out] 2/15*(3*x^2 - x + 2)^(5/2) + 5/24*(3*x^2 - x + 2)^(3/2)*x + 7/144*(3*x^2 -
x + 2)^(3/2) + 67/192*sqrt(3*x^2 - x + 2)*x - 2203/6912*sqrt(3)*arcsinh(6/2
3*sqrt(23)*x - 1/23*sqrt(23)) + 13/32*sqrt(13)*arcsinh(8/23*sqrt(23)*x/abs(
2*x + 1) - 9/23*sqrt(23)/abs(2*x + 1)) + 869/1152*sqrt(3*x^2 - x + 2)
```

Giac [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.10

$$\int \frac{(2-x+3x^2)^{3/2}(1+3x+4x^2)}{1+2x}\,dx = \frac{1}{5760}(2(12(6(48x-7)x+401)x+529)x+7977)\sqrt{3x^2-x+2} + \frac{2203}{6912}\sqrt{3}\log\left(-6\sqrt{3}x+\sqrt{3}+6\sqrt{3x^2-x+2}\right) + \frac{13}{32}\sqrt{13}\log\left(-\frac{\left|-4\sqrt{3}x-2\sqrt{13}-2\sqrt{3}+4\sqrt{3x^2-x+2}\right|}{2\left(2\sqrt{3}x-\sqrt{13}+\sqrt{3}-2\sqrt{3x^2-x+2}\right)}\right)$$

```
[In] integrate((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(1+2*x),x, algorithm="giac")
```

```
[Out] 1/5760*(2*(12*(6*(48*x - 7)*x + 401)*x + 529)*x + 7977)*sqrt(3*x^2 - x + 2)
 + 2203/6912*sqrt(3)*log(-6*sqrt(3)*x + sqrt(3) + 6*sqrt(3*x^2 - x + 2)) +
13/32*sqrt(13)*log(-1/2*abs(-4*sqrt(3)*x - 2*sqrt(13) - 2*sqrt(3) + 4*sqrt(
3*x^2 - x + 2))/(2*sqrt(3)*x - sqrt(13) + sqrt(3) - 2*sqrt(3*x^2 - x + 2)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2-x+3x^2)^{3/2}(1+3x+4x^2)}{1+2x}\,dx = \int \frac{(3x^2-x+2)^{3/2}(4x^2+3x+1)}{2x+1}\,dx$$

```
[In] int(((3*x^2 - x + 2)^(3/2)*(3*x + 4*x^2 + 1))/(2*x + 1),x)
```

```
[Out] int(((3*x^2 - x + 2)^(3/2)*(3*x + 4*x^2 + 1))/(2*x + 1), x)
```

3.218 $\int \frac{(2-x+3x^2)^{3/2}(1+3x+4x^2)}{(1+2x)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 131

$$\int \frac{(2-x+3x^2)^{3/2}(1+3x+4x^2)}{(1+2x)^2}\,dx = -\frac{1}{192}(349-294x)\sqrt{2-x+3x^2}-\frac{1}{104}(23-38x)\left(2-x+3x^2\right)^{3/2} -\frac{(2-x+3x^2)^{5/2}}{13(1+2x)}-\frac{2327\operatorname{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{384\sqrt{3}}+\frac{25}{32}\sqrt{13}\operatorname{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)$$

```
[Out] -1/104*(23-38*x)*(3*x^2-x+2)^(3/2)-1/13*(3*x^2-x+2)^(5/2)/(1+2*x)-2327/1152
*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)+25/32*arctanh(1/26*(9-8*x)*13^(1/2)
/(3*x^2-x+2)^(1/2))*13^(1/2)-1/192*(349-294*x)*(3*x^2-x+2)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {1664, 828, 857, 633, 221, 738, 212}

$$\int \frac{(2-x+3x^2)^{3/2}(1+3x+4x^2)}{(1+2x)^2}\,dx = -\frac{2327\operatorname{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{384\sqrt{3}} +\frac{25}{32}\sqrt{13}\operatorname{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)-\frac{(3x^2-x+2)^{5/2}}{13(2x+1)} -\frac{1}{104}(23-38x)\left(3x^2-x+2\right)^{3/2}-\frac{1}{192}(349-294x)\sqrt{3x^2-x+2}$$

```
[In] Int[((2 - x + 3*x^2)^(3/2)*(1 + 3*x + 4*x^2))/(1 + 2*x)^2,x]
```

```
[Out] -1/192*((349 - 294*x)*Sqrt[2 - x + 3*x^2]) - ((23 - 38*x)*(2 - x + 3*x^2)^(
3/2))/104 - (2 - x + 3*x^2)^(5/2)/(13*(1 + 2*x)) - (2327*ArcSinh[(1 - 6*x)/
Sqrt[23]])/(384*Sqrt[3]) + (25*Sqrt[13]*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[
2 - x + 3*x^2])])/32
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
```

```
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\left(2-x+3x^2\right)^{5/2}}{13(1+2x)}-\frac{1}{13}\int\frac{\left(-\frac{13}{2}-38x\right)\left(2-x+3x^2\right)^{3/2}}{1+2x}\,dx\\
&= -\frac{1}{104}(23-38x)\left(2-x+3x^2\right)^{3/2}-\frac{\left(2-x+3x^2\right)^{5/2}}{13(1+2x)}+\frac{\int\frac{(-78+7644x)\sqrt{2-x+3x^2}}{1+2x}\,dx}{1248}\\
&= -\frac{1}{192}(349-294x)\sqrt{2-x+3x^2}-\frac{1}{104}(23-38x)\left(2-x+3x^2\right)^{3/2}\\
&\quad -\frac{\left(2-x+3x^2\right)^{5/2}}{13(1+2x)}-\frac{\int\frac{245388-726024x}{(1+2x)\sqrt{2-x+3x^2}}\,dx}{59904}\\
&= -\frac{1}{192}(349-294x)\sqrt{2-x+3x^2}-\frac{1}{104}(23-38x)\left(2-x+3x^2\right)^{3/2}\\
&\quad -\frac{\left(2-x+3x^2\right)^{5/2}}{13(1+2x)}+\frac{2327}{384}\int\frac{1}{\sqrt{2-x+3x^2}}\,dx-\frac{325}{32}\int\frac{1}{(1+2x)\sqrt{2-x+3x^2}}\,dx\\
&= -\frac{1}{192}(349-294x)\sqrt{2-x+3x^2}-\frac{1}{104}(23-38x)\left(2-x+3x^2\right)^{3/2}-\frac{\left(2-x+3x^2\right)^{5/2}}{13(1+2x)}\\
&\quad +\frac{325}{16}\text{Subst}\left(\int\frac{1}{52-x^2}\,dx,x,\frac{9-8x}{\sqrt{2-x+3x^2}}\right)+\frac{2327\text{Subst}\left(\int\frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx,x,-1+6x\right)}{384\sqrt{69}}\\
&= -\frac{1}{192}(349-294x)\sqrt{2-x+3x^2}-\frac{1}{104}(23-38x)\left(2-x+3x^2\right)^{3/2}\\
&\quad -\frac{\left(2-x+3x^2\right)^{5/2}}{13(1+2x)}-\frac{2327\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{384\sqrt{3}}+\frac{25}{32}\sqrt{13}\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.92

$$\int \frac{(2-x+3x^2)^{3/2}(1+3x+4x^2)}{(1+2x)^2}\,dx = \frac{\frac{6\sqrt{2-x+3x^2}(-493-332x+564x^2-96x^3+288x^4)}{1+2x} - 1800\sqrt{13}\,\text{arctanh}\left(\frac{\sqrt{3}+2\sqrt{3}x-}{\sqrt{}}\right.}{1152}$$

```
[In] Integrate[((2 - x + 3*x^2)^(3/2)*(1 + 3*x + 4*x^2))/(1 + 2*x)^2,x]
```

```
[Out] ((6*Sqrt[2 - x + 3*x^2]*(-493 - 332*x + 564*x^2 - 96*x^3 + 288*x^4))/(1 + 2
*x) - 1800*Sqrt[13]*ArcTanh[(Sqrt[3] + 2*Sqrt[3]*x - 2*Sqrt[2 - x + 3*x^2])
/Sqrt[13]] - 2327*Sqrt[3]*Log[1 - 6*x + 2*Sqrt[6 - 3*x + 9*x^2]])/1152
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.74

method	result
risch	$\frac{864x^6-576x^5+2364x^4-1752x^3-19x^2-171x-986}{192(1+2x)\sqrt{3x^2-x+2}} + \frac{2327\sqrt{3}\,\text{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{1152} + \frac{25\sqrt{13}\,\text{arctanh}\left(\frac{2\left(\frac{9}{2}-4x\right)\sqrt{13}}{13\sqrt{12\left(x+\frac{1}{2}\right)^2-16x+5}}\right)}{32}$
trager	$\frac{(288x^4-96x^3+564x^2-332x-493)\sqrt{3x^2-x+2}}{192+384x} + \frac{25\,\text{RootOf}\left(\_Z^2-13\right)\ln\left(-\frac{8\,\text{RootOf}\left(\_Z^2-13\right)x-9\,\text{RootOf}\left(\_Z^2-13\right)-26\sqrt{3x}}{1+2x}\right.}{32}$
default	$\frac{(-1+6x)(3x^2-x+2)^{\frac{3}{2}}}{24} + \frac{23(-1+6x)\sqrt{3x^2-x+2}}{192} + \frac{2327\sqrt{3}\,\text{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{1152} - \frac{\left(3\left(x+\frac{1}{2}\right)^2-4x+\frac{5}{4}\right)^{\frac{5}{2}}}{26\left(x+\frac{1}{2}\right)} - \frac{25\left(3\left(x+\frac{1}{2}\right)^2\right.}{15}$

```
[In] int((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(1+2*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/192*(864*x^6-576*x^5+2364*x^4-1752*x^3-19*x^2-171*x-986)/(1+2*x)/(3*x^2-x
+2)^(1/2)+2327/1152*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))+25/32*13^(1/2)*a
rctanh(2/13*(9/2-4*x)*13^(1/2)/(12*(x+1/2)^2-16*x+5)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.09

$$\int \frac{(2-x+3x^2)^{3/2}(1+3x+4x^2)}{(1+2x)^2}\,dx = \frac{2327\sqrt{3}(2x+1)\log\left(-4\sqrt{3}\sqrt{3x^2-x+2}(6x-1)-72x^2+24x-\right.}{}$$

```
[In] integrate((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(1+2*x)^2,x, algorithm="fricas")
[Out] 1/2304*(2327*sqrt(3)*(2*x + 1)*log(-4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x - 1)
 - 72*x^2 + 24*x - 25) + 900*sqrt(13)*(2*x + 1)*log((4*sqrt(13)*sqrt(3*x^2
- x + 2)*(8*x - 9) - 220*x^2 + 196*x - 185)/(4*x^2 + 4*x + 1)) + 12*(288*x^
4 - 96*x^3 + 564*x^2 - 332*x - 493)*sqrt(3*x^2 - x + 2))/(2*x + 1)
```

Sympy [F]

$$\int \frac{(2-x+3x^2)^{3/2}(1+3x+4x^2)}{(1+2x)^2}\,dx = \int \frac{(3x^2-x+2)^{\frac{3}{2}}\cdot(4x^2+3x+1)}{(2x+1)^2}\,dx$$

```
[In] integrate((3*x**2-x+2)**(3/2)*(4*x**2+3*x+1)/(1+2*x)**2,x)
[Out] Integral((3*x**2 - x + 2)**(3/2)*(4*x**2 + 3*x + 1)/(2*x + 1)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.01

$$\begin{aligned}\int \frac{(2-x+3x^2)^{3/2}(1+3x+4x^2)}{(1+2x)^2}\,dx &= \frac{1}{4}\left(3x^2-x+2\right)^{\frac{3}{2}}x-\frac{1}{8}\left(3x^2-x+2\right)^{\frac{3}{2}} \\ &+\frac{49}{32}\sqrt{3x^2-x+2}x+\frac{2327}{1152}\sqrt{3}\,\mathrm{arsinh}\left(\frac{6}{23}\sqrt{23}x-\frac{1}{23}\sqrt{23}\right) \\ &-\frac{25}{32}\sqrt{13}\,\mathrm{arsinh}\left(\frac{8\sqrt{23}x}{23\,|2x+1|}-\frac{9\sqrt{23}}{23\,|2x+1|}\right)-\frac{349}{192}\sqrt{3x^2-x+2}-\frac{\left(3x^2-x+2\right)^{\frac{3}{2}}}{4\,(2x+1)}\end{aligned}$$

```
[In] integrate((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(1+2*x)^2,x, algorithm="maxima")
[Out] 1/4*(3*x^2 - x + 2)^(3/2)*x - 1/8*(3*x^2 - x + 2)^(3/2) + 49/32*sqrt(3*x^2
- x + 2)*x + 2327/1152*sqrt(3)*arcsinh(6/23*sqrt(23)*x - 1/23*sqrt(23)) - 2
5/32*sqrt(13)*arcsinh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqrt(23)/abs(2*x
+ 1)) - 349/192*sqrt(3*x^2 - x + 2) - 1/4*(3*x^2 - x + 2)^(3/2)/(2*x + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 570 vs. $2(104) = 208$.

Time = 0.50 (sec) , antiderivative size = 570, normalized size of antiderivative = 4.35

$$\int \frac{(2-x+3x^2)^{3/2}(1+3x+4x^2)}{(1+2x)^2}\,dx = \frac{25}{32}\sqrt{13}\log\left(\sqrt{13}\left(\sqrt{-\frac{8}{2x+1}+\frac{13}{(2x+1)^2}+3}+\frac{\sqrt{13}}{2x+1}\right)-4\right)\operatorname{sgn}\left(\frac{1}{2x+1}\right)$$

$$-\frac{2327}{1152}\sqrt{3}\log\left(\frac{\left|-2\sqrt{3}+2\sqrt{-\frac{8}{2x+1}+\frac{13}{(2x+1)^2}+3}+\frac{2\sqrt{13}}{2x+1}\right|}{2\left(\sqrt{3}+\sqrt{-\frac{8}{2x+1}+\frac{13}{(2x+1)^2}+3}+\frac{\sqrt{13}}{2x+1}\right)}\right)\operatorname{sgn}\left(\frac{1}{2x+1}\right)$$

$$-\frac{13}{32}\sqrt{-\frac{8}{2x+1}+\frac{13}{(2x+1)^2}+3}\operatorname{sgn}\left(\frac{1}{2x+1}\right)$$

$$+\frac{5929\left(\sqrt{-\frac{8}{2x+1}+\frac{13}{(2x+1)^2}+3}+\frac{\sqrt{13}}{2x+1}\right)^7\operatorname{sgn}\left(\frac{1}{2x+1}\right)-7272\sqrt{13}\left(\sqrt{-\frac{8}{2x+1}+\frac{13}{(2x+1)^2}+3}+\frac{\sqrt{13}}{2x+1}\right)^6\operatorname{sgn}\left(\frac{1}{2x+1}\right.}{}$$

```
[In] integrate((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(1+2*x)^2,x, algorithm="giac")
```

```
[Out] 25/32*sqrt(13)*log(sqrt(13)*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt
(13)/(2*x + 1)) - 4)*sgn(1/(2*x + 1)) - 2327/1152*sqrt(3)*log(1/2*abs(-2*sq
rt(3) + 2*sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + 2*sqrt(13)/(2*x + 1))/(
sqrt(3) + sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1)))*sg
n(1/(2*x + 1)) - 13/32*sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3)*sgn(1/(2*x +
 1)) + 1/192*(5929*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x
 + 1))^7*sgn(1/(2*x + 1)) - 7272*sqrt(13)*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)
^2 + 3) + sqrt(13)/(2*x + 1))^6*sgn(1/(2*x + 1)) + 25101*(sqrt(-8/(2*x + 1)
 + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1))^5*sgn(1/(2*x + 1)) - 48*sqrt(1
3)*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1))^4*sgn(1/(
2*x + 1)) + 112359*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x
 + 1))^3*sgn(1/(2*x + 1)) - 69336*sqrt(13)*(sqrt(-8/(2*x + 1) + 13/(2*x + 1
)^2 + 3) + sqrt(13)/(2*x + 1))^2*sgn(1/(2*x + 1)) + 71955*(sqrt(-8/(2*x + 1
) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1))*sgn(1/(2*x + 1)) + 24624*sqrt
(13)*sgn(1/(2*x + 1)))/((sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)
/(2*x + 1))^2 - 3)^4
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2-x+3x^2)^{3/2}(1+3x+4x^2)}{(1+2x)^2}\,dx=\int \frac{{\left(3\,x^2-x+2\right)}^{3/2}\,\left(4\,x^2+3\,x+1\right)}{{\left(2\,x+1\right)}^2}\,dx$$

```
[In] int(((3*x^2 - x + 2)^(3/2)*(3*x + 4*x^2 + 1))/(2*x + 1)^2,x)
[Out] int(((3*x^2 - x + 2)^(3/2)*(3*x + 4*x^2 + 1))/(2*x + 1)^2, x)
```

3.219 $\int \frac{\left(2-x+3x^2\right)^{3/2}\left(1+3x+4x^2\right)}{(1+2x)^3}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 138

$$\int \frac{\left(2-x+3x^2\right)^{3/2}\left(1+3x+4x^2\right)}{(1+2x)^3}\,dx = \frac{1}{624}(1858-771x)\sqrt{2-x+3x^2} + \frac{(151+122x)\left(2-x+3x^2\right)^{3/2}}{312(1+2x)} - \frac{\left(2-x+3x^2\right)^{5/2}}{26(1+2x)^2} + \frac{1519\text{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{192\sqrt{3}} - \frac{1153\text{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)}{64\sqrt{13}}$$

```
[Out] 1/312*(151+122*x)*(3*x^2-x+2)^(3/2)/(1+2*x)-1/26*(3*x^2-x+2)^(5/2)/(1+2*x)^
2+1519/576*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)-1153/832*arctanh(1/26*(9-
8*x)*13^(1/2)/(3*x^2-x+2)^(1/2))*13^(1/2)+1/624*(1858-771*x)*(3*x^2-x+2)^(1
/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1664, 826, 828, 857, 633, 221, 738, 212}

$$\int \frac{\left(2-x+3x^2\right)^{3/2}\left(1+3x+4x^2\right)}{(1+2x)^3}\,dx = \frac{1519\text{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{192\sqrt{3}} - \frac{1153\text{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{64\sqrt{13}} - \frac{\left(3x^2-x+2\right)^{5/2}}{26(2x+1)^2} + \frac{(122x+151)\left(3x^2-x+2\right)^{3/2}}{312(2x+1)} + \frac{1}{624}(1858-771x)\sqrt{3x^2-x+2}$$

```
[In] Int[((2 - x + 3*x^2)^(3/2)*(1 + 3*x + 4*x^2))/(1 + 2*x)^3,x]
```

```
[Out] ((1858 - 771*x)*Sqrt[2 - x + 3*x^2])/624 + ((151 + 122*x)*(2 - x + 3*x^2)^(
3/2))/(312*(1 + 2*x)) - (2 - x + 3*x^2)^(5/2)/(26*(1 + 2*x)^2) + (1519*ArcS
inh[(1 - 6*x)/Sqrt[23]])/(192*Sqrt[3]) - (1153*ArcTanh[(9 - 8*x)/(2*Sqrt[13
]*Sqrt[2 - x + 3*x^2])])/(64*Sqrt[13])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 826

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
```

```
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(2-x+3x^2)^{5/2}}{26(1+2x)^2} - \frac{1}{26}\int \frac{\left(-\frac{31}{2}-61x\right)(2-x+3x^2)^{3/2}}{(1+2x)^2}\,dx \\
&= \frac{(151+122x)(2-x+3x^2)^{3/2}}{312(1+2x)} - \frac{(2-x+3x^2)^{5/2}}{26(1+2x)^2} + \frac{1}{208}\int \frac{(639-1028x)\sqrt{2-x+3x^2}}{1+2x}\,dx \\
&= \frac{1}{624}(1858-771x)\sqrt{2-x+3x^2} + \frac{(151+122x)(2-x+3x^2)^{3/2}}{312(1+2x)} \\
&\quad - \frac{(2-x+3x^2)^{5/2}}{26(1+2x)^2} - \frac{\int \frac{-100880+157976x}{(1+2x)\sqrt{2-x+3x^2}}\,dx}{9984}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{1}{624}(1858-771x)\sqrt{2-x+3x^2}+\frac{(151+122x)\left(2-x+3x^2\right)^{3/2}}{312(1+2x)}-\frac{\left(2-x+3x^2\right)^{5/2}}{26(1+2x)^2} \\
&\quad -\frac{1519}{192}\int \frac{1}{\sqrt{2-x+3x^2}}\,dx+\frac{1153}{64}\int \frac{1}{(1+2x)\sqrt{2-x+3x^2}}\,dx \\
&= \frac{1}{624}(1858-771x)\sqrt{2-x+3x^2}+\frac{(151+122x)\left(2-x+3x^2\right)^{3/2}}{312(1+2x)}-\frac{\left(2-x+3x^2\right)^{5/2}}{26(1+2x)^2} \\
&\quad -\frac{1153}{32}\text{Subst}\left(\int \frac{1}{52-x^2}\,dx,x,\frac{9-8x}{\sqrt{2-x+3x^2}}\right)-\frac{1519\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx,x,-1+6x\right)}{192\sqrt{69}} \\
&= \frac{1}{624}(1858-771x)\sqrt{2-x+3x^2}+\frac{(151+122x)\left(2-x+3x^2\right)^{3/2}}{312(1+2x)} \\
&\quad -\frac{\left(2-x+3x^2\right)^{5/2}}{26(1+2x)^2}+\frac{1519\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{192\sqrt{3}}-\frac{1153\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)}{64\sqrt{13}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.88

$$
\int \frac{\left(2-x+3x^2\right)^{3/2}\left(1+3x+4x^2\right)}{(1+2x)^3}\,dx=\frac{\frac{156\sqrt{2-x+3x^2}\left(182+627x+390x^2-68x^3+96x^4\right)}{(1+2x)^2}+20754\sqrt{13}\text{arctanh}\left(\frac{\sqrt{3}+2\sqrt{3}x-2}{\sqrt{1}}\right.}{7488}
$$

```
[In] Integrate[((2 - x + 3*x^2)^(3/2)*(1 + 3*x + 4*x^2))/(1 + 2*x)^3,x]
```

```
[Out] ((156*Sqrt[2 - x + 3*x^2]*(182 + 627*x + 390*x^2 - 68*x^3 + 96*x^4))/(1 + 2
*x)^2 + 20754*Sqrt[13]*ArcTanh[(Sqrt[3] + 2*Sqrt[3]*x - 2*Sqrt[2 - x + 3*x^
2])/Sqrt[13]] + 19747*Sqrt[3]*Log[1 - 6*x + 2*Sqrt[6 - 3*x + 9*x^2]])/7488
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.70

method	result
risch	$\frac{288x^6-300x^5+1430x^4+1355x^3+699x^2+1072x+364}{48(1+2x)^2\sqrt{3x^2-x+2}}-\frac{1519\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\,\left(x-\frac{1}{6}\right)}{23}\right)}{576}-\frac{1153\sqrt{13}\,\operatorname{arctanh}\left(\frac{2\left(\frac{9}{2}-4x\right)\sqrt{13}}{13\sqrt{12\left(x+\frac{1}{2}\right)^2-16}}\right.}{832}$
trager	$\frac{(96x^4-68x^3+390x^2+627x+182)\sqrt{3x^2-x+2}}{48(1+2x)^2}-\frac{1153\,\mathrm{RootOf}\left(\_Z^2-13\right)\ln\left(-\frac{8\,\mathrm{RootOf}\left(\_Z^2-13\right)x-9\,\mathrm{RootOf}\left(\_Z^2-13\right)-26\sqrt{3}}{1+2x}\right.}{832}$
default	$\frac{1153\left(3\left(x+\frac{1}{2}\right)^2-4x+\frac{5}{4}\right)^{\frac{3}{2}}}{4056}-\frac{257(-1+6x)\sqrt{3\left(x+\frac{1}{2}\right)^2-4x+\frac{5}{4}}}{1248}-\frac{1519\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\,\left(x-\frac{1}{6}\right)}{23}\right)}{576}+\frac{1153\sqrt{12\left(x+\frac{1}{2}\right)^2-16x+5}}{832}$

```
[In] int((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(1+2*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/48*(288*x^6-300*x^5+1430*x^4+1355*x^3+699*x^2+1072*x+364)/(1+2*x)^2/(3*x^
2-x+2)^(1/2)-1519/576*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))-1153/832*13^(1
/2)*arctanh(2/13*(9/2-4*x)*13^(1/2)/(12*(x+1/2)^2-16*x+5)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.15

$$\int \frac{(2-x+3x^2)^{3/2}\,(1+3x+4x^2)}{(1+2x)^3}\,dx=\frac{19747\,\sqrt{3}(4\,x^2+4\,x+1)\log\left(4\,\sqrt{3}\sqrt{3\,x^2-x+2}(6\,x-1)-72\,x^2\right.}{}$$

```
[In] integrate((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(1+2*x)^3,x, algorithm="fricas")
```

```
[Out] 1/14976*(19747*sqrt(3)*(4*x^2 + 4*x + 1)*log(4*sqrt(3)*sqrt(3*x^2 - x + 2)*
(6*x - 1) - 72*x^2 + 24*x - 25) + 10377*sqrt(13)*(4*x^2 + 4*x + 1)*log(-(4*
sqrt(13)*sqrt(3*x^2 - x + 2)*(8*x - 9) + 220*x^2 - 196*x + 185)/(4*x^2 + 4*
x + 1)) + 312*(96*x^4 - 68*x^3 + 390*x^2 + 627*x + 182)*sqrt(3*x^2 - x + 2)
)/(4*x^2 + 4*x + 1)
```

Sympy [F]

$$\int \frac{(2-x+3x^2)^{3/2}\,(1+3x+4x^2)}{(1+2x)^3}\,dx=\int \frac{(3x^2-x+2)^{\frac{3}{2}}\cdot(4x^2+3x+1)}{(2x+1)^3}\,dx$$

```
[In] integrate((3*x**2-x+2)**(3/2)*(4*x**2+3*x+1)/(1+2*x)**3,x)
```

```
[Out] Integral((3*x**2 - x + 2)**(3/2)*(4*x**2 + 3*x + 1)/(2*x + 1)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.04

$$\int \frac{(2-x+3x^2)^{3/2}(1+3x+4x^2)}{(1+2x)^3}\,dx = \frac{61}{312}\left(3\,x^2-x+2\right)^{\frac{3}{2}} - \frac{\left(3\,x^2-x+2\right)^{\frac{5}{2}}}{26\left(4\,x^2+4\,x+1\right)} - \frac{257}{208}\sqrt{3\,x^2-x+2}x - \frac{1519}{576}\sqrt{3}\,\mathrm{arsinh}\left(\frac{6}{23}\sqrt{23}x-\frac{1}{23}\sqrt{23}\right) + \frac{1153}{832}\sqrt{13}\,\mathrm{arsinh}\left(\frac{8\sqrt{23}x}{23\,|2\,x+1|}-\frac{9\sqrt{23}}{23\,|2\,x+1|}\right) + \frac{929}{312}\sqrt{3\,x^2-x+2}+\frac{15\left(3\,x^2-x+2\right)^{\frac{3}{2}}}{52\left(2\,x+1\right)}$$

```
[In] integrate((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(1+2*x)^3,x, algorithm="maxima")
```

```
[Out] 61/312*(3*x^2 - x + 2)^(3/2) - 1/26*(3*x^2 - x + 2)^(5/2)/(4*x^2 + 4*x + 1)
 - 257/208*sqrt(3*x^2 - x + 2)*x - 1519/576*sqrt(3)*arcsinh(6/23*sqrt(23)*x
 - 1/23*sqrt(23)) + 1153/832*sqrt(13)*arcsinh(8/23*sqrt(23)*x/abs(2*x + 1)
- 9/23*sqrt(23)/abs(2*x + 1)) + 929/312*sqrt(3*x^2 - x + 2) + 15/52*(3*x^2
- x + 2)^(3/2)/(2*x + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 261 vs. 2(111) = 222.

Time = 0.31 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.89

$$\int \frac{(2-x+3x^2)^{3/2}(1+3x+4x^2)}{(1+2x)^3}\,dx = \frac{1}{96}\left(2\left(24\,x-41\right)x+265\right)\sqrt{3\,x^2-x+2} + \frac{1519}{576}\sqrt{3}\log\left(-2\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2-x+2}\right)+1\right) + \frac{1153}{832}\sqrt{13}\log\left(-\frac{\left|-4\sqrt{3}x-2\sqrt{13}-2\sqrt{3}+4\sqrt{3\,x^2-x+2}\right|}{2\left(2\sqrt{3}x-\sqrt{13}+\sqrt{3}-2\sqrt{3\,x^2-x+2}\right)}\right) + \frac{446\left(\sqrt{3}x-\sqrt{3\,x^2-x+2}\right)^3-85\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2-x+2}\right)^2-1993\sqrt{3}x+1009\sqrt{3}+1993\sqrt{3\,x^2-x+}}{32\left(2\left(\sqrt{3}x-\sqrt{3\,x^2-x+2}\right)^2+2\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2-x+2}\right)-5\right)^2}$$

```
[In] integrate((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(1+2*x)^3,x, algorithm="giac")
```

```
[Out] 1/96*(2*(24*x - 41)*x + 265)*sqrt(3*x^2 - x + 2) + 1519/576*sqrt(3)*log(-2*
sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) + 1) + 1153/832*sqrt(13)*log(-1/2
*abs(-4*sqrt(3)*x - 2*sqrt(13) - 2*sqrt(3) + 4*sqrt(3*x^2 - x + 2))/(2*sqrt
```

```
(3)*x - sqrt(13) + sqrt(3) - 2*sqrt(3*x^2 - x + 2))) + 1/32*(446*(sqrt(3)*x
 - sqrt(3*x^2 - x + 2))^3 - 85*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2))^2
- 1993*sqrt(3)*x + 1009*sqrt(3) + 1993*sqrt(3*x^2 - x + 2))/(2*(sqrt(3)*x -
 sqrt(3*x^2 - x + 2))^2 + 2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) - 5)^
2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2-x+3x^2)^{3/2}\,(1+3x+4x^2)}{(1+2x)^3}\,dx=\int \frac{{\left(3\,x^2-x+2\right)}^{3/2}\,\left(4\,x^2+3\,x+1\right)}{{\left(2\,x+1\right)}^3}\,dx$$

```
[In] int(((3*x^2 - x + 2)^(3/2)*(3*x + 4*x^2 + 1))/(2*x + 1)^3,x)
```

```
[Out] int(((3*x^2 - x + 2)^(3/2)*(3*x + 4*x^2 + 1))/(2*x + 1)^3, x)
```

3.220 $\int (1+2x)^3 \left(2-x+3x^2\right)^{5/2} \left(1+3x+4x^2\right) \, dx$

Optimal result

Integrand size = 32, antiderivative size = 189

$$\begin{aligned}
\int (1+2x)^3 \left(2-x+3x^2\right)^{5/2} \left(1+3x+4x^2\right) \, dx &= \frac{2692081(1-6x)\sqrt{2-x+3x^2}}{11943936} \\
&\quad + \frac{117047(1-6x)\left(2-x+3x^2\right)^{3/2}}{1492992} + \frac{5089(1-6x)\left(2-x+3x^2\right)^{5/2}}{155520} \\
&\quad - \frac{(26353-21350x)\left(2-x+3x^2\right)^{7/2}}{498960} + \frac{133(1+2x)^2\left(2-x+3x^2\right)^{7/2}}{1485} \\
&\quad + \frac{29}{330}(1+2x)^3\left(2-x+3x^2\right)^{7/2} + \frac{2}{33}(1+2x)^4\left(2-x+3x^2\right)^{7/2} + \frac{61917863\operatorname{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{23887872\sqrt{3}}
\end{aligned}$$

```
[Out] 117047/1492992*(1-6*x)*(3*x^2-x+2)^(3/2)+5089/155520*(1-6*x)*(3*x^2-x+2)^(5
/2)-1/498960*(26353-21350*x)*(3*x^2-x+2)^(7/2)+133/1485*(1+2*x)^2*(3*x^2-x+
2)^(7/2)+29/330*(1+2*x)^3*(3*x^2-x+2)^(7/2)+2/33*(1+2*x)^4*(3*x^2-x+2)^(7/2
)+61917863/71663616*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)+2692081/11943936
*(1-6*x)*(3*x^2-x+2)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used

$$= \{1667, 846, 793, 626, 633, 221\}$$

$$\int (1+2x)^3 \left(2-x+3x^2\right)^{5/2} \left(1+3x+4x^2\right) dx = \frac{61917863\text{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{23887872\sqrt{3}}$$

$$+ \frac{2}{33}\left(3x^2-x+2\right)^{7/2}(2x+1)^4$$

$$+\frac{29}{330}\left(3x^2-x+2\right)^{7/2}(2x+1)^3+\frac{133\left(3x^2-x+2\right)^{7/2}(2x+1)^2}{1485}-\frac{(26353-21350x)\left(3x^2-x+2\right)^{7/2}}{498960}+5089($$

```
[In] Int[(1 + 2*x)^3*(2 - x + 3*x^2)^(5/2)*(1 + 3*x + 4*x^2),x]
```

```
[Out] (2692081*(1 - 6*x)*Sqrt[2 - x + 3*x^2])/11943936 + (117047*(1 - 6*x)*(2 - x
 + 3*x^2)^(3/2))/1492992 + (5089*(1 - 6*x)*(2 - x + 3*x^2)^(5/2))/155520 -
((26353 - 21350*x)*(2 - x + 3*x^2)^(7/2))/498960 + (133*(1 + 2*x)^2*(2 - x
+ 3*x^2)^(7/2))/1485 + (29*(1 + 2*x)^3*(2 - x + 3*x^2)^(7/2))/330 + (2*(1 +
 2*x)^4*(2 - x + 3*x^2)^(7/2))/33 + (61917863*ArcSinh[(1 - 6*x)/Sqrt[23]])/
(23887872*Sqrt[3])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 846

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2}{33}(1+2x)^4\left(2-x+3x^2\right)^{7/2}+\frac{1}{132}\int(1+2x)^3(32+348x)\left(2-x+3x^2\right)^{5/2}\,dx\\
&= \frac{29}{330}(1+2x)^3\left(2-x+3x^2\right)^{7/2}\\
&\quad+\frac{2}{33}(1+2x)^4\left(2-x+3x^2\right)^{7/2}+\frac{\int(1+2x)^2(-1998+9576x)\left(2-x+3x^2\right)^{5/2}\,dx}{3960}\\
&= \frac{133(1+2x)^2\left(2-x+3x^2\right)^{7/2}}{1485}+\frac{29}{330}(1+2x)^3\left(2-x+3x^2\right)^{7/2}\\
&\quad+\frac{2}{33}(1+2x)^4\left(2-x+3x^2\right)^{7/2}+\frac{\int(1+2x)(-97038+54900x)\left(2-x+3x^2\right)^{5/2}\,dx}{106920}\\
&= -\frac{(26353-21350x)\left(2-x+3x^2\right)^{7/2}}{498960}\\
&\quad+\frac{133(1+2x)^2\left(2-x+3x^2\right)^{7/2}}{1485}+\frac{29}{330}(1+2x)^3\left(2-x+3x^2\right)^{7/2}\\
&\quad+\frac{2}{33}(1+2x)^4\left(2-x+3x^2\right)^{7/2}-\frac{5089\int\left(2-x+3x^2\right)^{5/2}\,dx}{4320}
\end{aligned}$$

$$= \frac{5089(1-6x)\left(2-x+3x^2\right)^{5/2}}{155520}$$

$$-\frac{(26353-21350x)\left(2-x+3x^2\right)^{7/2}}{498960}+\frac{133(1+2x)^2\left(2-x+3x^2\right)^{7/2}}{1485}$$

$$+\frac{29}{330}(1+2x)^3\left(2-x+3x^2\right)^{7/2}+\frac{2}{33}(1+2x)^4\left(2-x+3x^2\right)^{7/2}-\frac{117047\int\left(2-x+3x^2\right)^{3/2}\,dx}{62208}$$

$$= \frac{117047(1-6x)\left(2-x+3x^2\right)^{3/2}}{1492992}+\frac{5089(1-6x)\left(2-x+3x^2\right)^{5/2}}{155520}$$

$$-\frac{(26353-21350x)\left(2-x+3x^2\right)^{7/2}}{498960}+\frac{133(1+2x)^2\left(2-x+3x^2\right)^{7/2}}{1485}$$

$$+\frac{29}{330}(1+2x)^3\left(2-x+3x^2\right)^{7/2}+\frac{2}{33}(1+2x)^4\left(2-x+3x^2\right)^{7/2}-\frac{2692081\int\sqrt{2-x+3x^2}\,dx}{995328}$$

$$= \frac{2692081(1-6x)\sqrt{2-x+3x^2}}{11943936}+\frac{117047(1-6x)\left(2-x+3x^2\right)^{3/2}}{1492992}$$

$$+\frac{5089(1-6x)\left(2-x+3x^2\right)^{5/2}}{155520}$$

$$-\frac{(26353-21350x)\left(2-x+3x^2\right)^{7/2}}{498960}+\frac{133(1+2x)^2\left(2-x+3x^2\right)^{7/2}}{1485}$$

$$+\frac{29}{330}(1+2x)^3\left(2-x+3x^2\right)^{7/2}+\frac{2}{33}(1+2x)^4\left(2-x+3x^2\right)^{7/2}-\frac{61917863\int\frac{1}{\sqrt{2-x+3x^2}}\,dx}{23887872}$$

$$= \frac{2692081(1-6x)\sqrt{2-x+3x^2}}{11943936}+\frac{117047(1-6x)\left(2-x+3x^2\right)^{3/2}}{1492992}$$

$$+\frac{5089(1-6x)\left(2-x+3x^2\right)^{5/2}}{155520}$$

$$-\frac{(26353-21350x)\left(2-x+3x^2\right)^{7/2}}{498960}+\frac{133(1+2x)^2\left(2-x+3x^2\right)^{7/2}}{1485}$$

$$+\frac{29}{330}(1+2x)^3\left(2-x+3x^2\right)^{7/2}+\frac{2}{33}(1+2x)^4\left(2-x+3x^2\right)^{7/2}-\frac{\left(2692081\sqrt{\frac{23}{3}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx,\right.}{23887872}$$

$$= \frac{2692081(1-6x)\sqrt{2-x+3x^2}}{11943936}+\frac{117047(1-6x)\left(2-x+3x^2\right)^{3/2}}{1492992}$$

$$+\frac{5089(1-6x)\left(2-x+3x^2\right)^{5/2}}{155520}$$

$$-\frac{(26353-21350x)\left(2-x+3x^2\right)^{7/2}}{498960}+\frac{133(1+2x)^2\left(2-x+3x^2\right)^{7/2}}{1485}$$

$$+\frac{29}{330}(1+2x)^3\left(2-x+3x^2\right)^{7/2}+\frac{2}{33}(1+2x)^4\left(2-x+3x^2\right)^{7/2}+\frac{61917863\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{23887872\sqrt{3}}$$

Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.53

$$\int (1+2x)^3 \left(2-x+3x^2\right)^{5/2} \left(1+3x+4x^2\right) \, dx = \frac{6\sqrt{2-x+3x^2}(9173509857+26646633218x+72088585464x^2+161269204752x^3+2636361342}{}$$

```
[In] Integrate[(1 + 2*x)^3*(2 - x + 3*x^2)^(5/2)*(1 + 3*x + 4*x^2),x]
```

```
[Out] (6*Sqrt[2 - x + 3*x^2]*(9173509857 + 26646633218*x + 72088585464*x^2 + 1612
69204752*x^3 + 263636134272*x^4 + 347247744768*x^5 + 415908006912*x^6 + 419
978151936*x^7 + 308846297088*x^8 + 207681159168*x^9 + 120394874880*x^10) +
23838377255*Sqrt[3]*Log[1 - 6*x + 2*Sqrt[6 - 3*x + 9*x^2]])/27590492160
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.42

method	result
risch	$\frac{(120394874880x^{10}+207681159168x^{9}+308846297088x^{8}+419978151936x^{7}+415908006912x^{6}+347247744768x^{5}+263636134272x^{4}+16}{4598415360}$
trager	$(\frac{288}{11}x^{10}+\frac{2484}{55}x^{9}+\frac{3694}{55}x^{8}+\frac{120557}{1320}x^{7}+\frac{557147}{6160}x^{6}+\frac{50238389}{665280}x^{5}+\frac{32692973}{570240}x^{4}+\frac{1119925033}{31933440}x^{3}+\frac{429098723}{27371520}$
default	$-\frac{5089(-1+6x)(3x^{2}-x+2)^{\frac{5}{2}}}{155520}-\frac{117047(-1+6x)(3x^{2}-x+2)^{\frac{3}{2}}}{1492992}-\frac{2692081(-1+6x)\sqrt{3x^{2}-x+2}}{11943936}-\frac{61917863\sqrt{3}\ \mathrm{arcsinh}\left(\frac{6\sqrt{23}\,(x}{23}\right.}{71663616}$

```
[In] int((1+2*x)^3*(3*x^2-x+2)^(5/2)*(4*x^2+3*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4598415360*(120394874880*x^10+207681159168*x^9+308846297088*x^8+419978151
936*x^7+415908006912*x^6+347247744768*x^5+263636134272*x^4+161269204752*x^3
+72088585464*x^2+26646633218*x+9173509857)*(3*x^2-x+2)^(1/2)-61917863/71663
616*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.54

$$\int (1+2x)^3 \left(2-x+3x^2\right)^{5/2} (1+3x +4x^2)\, dx = \frac{1}{4598415360}\left(120394874880\,x^{10} + 207681159168\,x^{9} + 308846297088\,x^{8} + 419978151936\,x^{7} + 4\right.$$
$$+ \frac{61917863}{143327232}\sqrt{3}\log\left(4\sqrt{3}\sqrt{3\,x^2-x+2}(6\,x-1) - 72\,x^2 + 24\,x - 25\right)$$

```
[In] integrate((1+2*x)^3*(3*x^2-x+2)^(5/2)*(4*x^2+3*x+1),x, algorithm="fricas")
```

```
[Out] 1/4598415360*(120394874880*x^10 + 207681159168*x^9 + 308846297088*x^8 + 419
978151936*x^7 + 415908006912*x^6 + 347247744768*x^5 + 263636134272*x^4 + 16
1269204752*x^3 + 72088585464*x^2 + 26646633218*x + 9173509857)*sqrt(3*x^2 -
 x + 2) + 61917863/143327232*sqrt(3)*log(4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x
 - 1) - 72*x^2 + 24*x - 25)
```

Sympy [A] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.55

$$\int (1+2x)^3 \left(2-x+3x^2\right)^{5/2} \left(1+3x+4x^2\right) dx = \sqrt{3x^2-x+2} \cdot \left(\frac{288x^{10}}{11} + \frac{2484x^9}{55} + \frac{3694x^8}{55} + \frac{120557x^7}{1320} + \frac{557147x^6}{6160} + \frac{50238389x^5}{665280} + \frac{32692973x^4}{570240} + \frac{1119925033x^3}{31933440} + \frac{429098723x^2}{27371520} + \frac{13323316609x}{2299207680} + \frac{1019278873}{510935040}\right) - \frac{61917863\sqrt{3}\operatorname{asinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{71663616}$$

```
[In] integrate((1+2*x)**3*(3*x**2-x+2)**(5/2)*(4*x**2+3*x+1),x)
```

```
[Out] sqrt(3*x**2 - x + 2)*(288*x**10/11 + 2484*x**9/55 + 3694*x**8/55 + 120557*x
**7/1320 + 557147*x**6/6160 + 50238389*x**5/665280 + 32692973*x**4/570240 +
 1119925033*x**3/31933440 + 429098723*x**2/27371520 + 13323316609*x/2299207
680 + 1019278873/510935040) - 61917863*sqrt(3)*asinh(6*sqrt(23)*(x - 1/6)/2
3)/71663616
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.97

$$\begin{aligned}\int (1+2x)^3 \left(2-x+3x^2\right)^{5/2} \left(1+3x+4x^2\right)\,dx &= \frac{32}{33}\left(3\,x^2-x+2\right)^{\frac{7}{2}}x^4 \\ &+ \frac{436}{165}\left(3\,x^2-x+2\right)^{\frac{7}{2}}x^3 + \frac{4258}{1485}\left(3\,x^2-x+2\right)^{\frac{7}{2}}x^2 + \frac{10073}{7128}\left(3\,x^2-x+2\right)^{\frac{7}{2}}x \\ &+ \frac{92423}{498960}\left(3\,x^2-x+2\right)^{\frac{7}{2}} - \frac{5089}{25920}\left(3\,x^2-x+2\right)^{\frac{5}{2}}x + \frac{5089}{155520}\left(3\,x^2-x+2\right)^{\frac{5}{2}} \\ &- \frac{117047}{248832}\left(3\,x^2-x+2\right)^{\frac{3}{2}}x + \frac{117047}{1492992}\left(3\,x^2-x+2\right)^{\frac{3}{2}} - \frac{2692081}{1990656}\sqrt{3\,x^2-x+2}x \\ &- \frac{61917863}{71663616}\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{23}\sqrt{23}(6\,x-1)\right) + \frac{2692081}{11943936}\sqrt{3\,x^2-x+2}\end{aligned}$$

```
[In] integrate((1+2*x)^3*(3*x^2-x+2)^(5/2)*(4*x^2+3*x+1),x, algorithm="maxima")
```

```
[Out] 32/33*(3*x^2 - x + 2)^(7/2)*x^4 + 436/165*(3*x^2 - x + 2)^(7/2)*x^3 + 4258/
1485*(3*x^2 - x + 2)^(7/2)*x^2 + 10073/7128*(3*x^2 - x + 2)^(7/2)*x + 92423
/498960*(3*x^2 - x + 2)^(7/2) - 5089/25920*(3*x^2 - x + 2)^(5/2)*x + 5089/1
55520*(3*x^2 - x + 2)^(5/2) - 117047/248832*(3*x^2 - x + 2)^(3/2)*x + 11704
7/1492992*(3*x^2 - x + 2)^(3/2) - 2692081/1990656*sqrt(3*x^2 - x + 2)*x - 6
1917863/71663616*sqrt(3)*arcsinh(1/23*sqrt(23)*(6*x - 1)) + 2692081/1194393
6*sqrt(3*x^2 - x + 2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.52

$$\begin{aligned}&\int (1+2x)^3 \left(2-x+3x^2\right)^{5/2} \left(1+3x \\ &+4x^2\right)\,dx = \frac{1}{4598415360}\left(2\left(12\left(6\left(8\left(6\left(36\left(14\left(48\left(18\left(40\,x+69\right)x+1847\right)x+120557\right)x+1671441\right)x+5023\right.\right.\right.\right.\right.\right.\right.\right.\right. \\ &+ \frac{61917863}{71663616}\sqrt{3}\log\left(-2\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2-x+2}\right)+1\right)\end{aligned}$$

```
[In] integrate((1+2*x)^3*(3*x^2-x+2)^(5/2)*(4*x^2+3*x+1),x, algorithm="giac")
```

```
[Out] 1/4598415360*(2*(12*(6*(8*(6*(36*(14*(48*(18*(40*x + 69)*x + 1847)*x + 1205
57)*x + 1671441)*x + 50238389)*x + 228850811)*x + 1119925033)*x + 300369106
1)*x + 13323316609)*x + 9173509857)*sqrt(3*x^2 - x + 2) + 61917863/71663616
*sqrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) + 1)
```

Mupad [F(-1)]

Timed out.

$$\int (1+2x)^3 \left(2-x+3x^2\right)^{5/2} \left(1+3x+4x^2\right)\,dx=\int (2\,x+1)^3\,{\left(3\,x^2-x+2\right)}^{5/2}\,\left(4\,x^2+3\,x+1\right)\,dx$$

```
[In] int((2*x + 1)^3*(3*x^2 - x + 2)^(5/2)*(3*x + 4*x^2 + 1),x)
[Out] int((2*x + 1)^3*(3*x^2 - x + 2)^(5/2)*(3*x + 4*x^2 + 1), x)
```

3.221 $\int (1+2x)^2 \left(2-x+3x^2\right)^{5/2} \left(1+3x+4x^2\right) \, dx$

Optimal result

Integrand size = 32, antiderivative size = 164

$$\int (1+2x)^2 \left(2-x+3x^2\right)^{5/2} \left(1+3x+4x^2\right) \, dx = -\frac{154997(1-6x)\sqrt{2-x+3x^2}}{4478976} - \frac{6739(1-6x)\left(2-x+3x^2\right)^{3/2}}{559872} - \frac{293(1-6x)\left(2-x+3x^2\right)^{5/2}}{58320} + \frac{37}{405}(1+2x)^2\left(2-x+3x^2\right)^{7/2} + \frac{1}{15}(1+2x)^3\left(2-x+3x^2\right)^{7/2} + \frac{(2731+3430x)\left(2-x+3x^2\right)^{7/2}}{17010} - \frac{3564931\text{arcsi}}{8957952}$$

```
[Out] -6739/559872*(1-6*x)*(3*x^2-x+2)^(3/2)-293/58320*(1-6*x)*(3*x^2-x+2)^(5/2)+
37/405*(1+2*x)^2*(3*x^2-x+2)^(7/2)+1/15*(1+2*x)^3*(3*x^2-x+2)^(7/2)+1/17010
*(2731+3430*x)*(3*x^2-x+2)^(7/2)-3564931/26873856*arcsinh(1/23*(1-6*x)*23^(
1/2))*3^(1/2)-154997/4478976*(1-6*x)*(3*x^2-x+2)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1667, 846, 793, 626, 633, 221}

$$\int (1+2x)^2 \left(2-x+3x^2\right)^{5/2} \left(1+3x+4x^2\right) \, dx = -\frac{3564931\,\text{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{8957952\sqrt{3}} + \frac{1}{15}(2x+1)^3\left(3x^2-x+2\right)^{7/2} + \frac{37}{405}(2x+1)^2\left(3x^2-x+2\right)^{7/2} + \frac{(3430x+2731)\left(3x^2-x+2\right)^{7/2}}{17010} - \frac{293(1-6x)\left(3x^2-x+2\right)^{5/2}}{58320} - \frac{6739(1-6x)\left(3x^2-x+2\right)^{3/2}}{559872} - \frac{154997(1-6x)\sqrt{3x^2-x+2}}{4478976}$$

```
[In] Int[(1 + 2*x)^2*(2 - x + 3*x^2)^(5/2)*(1 + 3*x + 4*x^2),x]
```

```
[Out] (-154997*(1 - 6*x)*Sqrt[2 - x + 3*x^2])/4478976 - (6739*(1 - 6*x)*(2 - x +
3*x^2)^(3/2))/559872 - (293*(1 - 6*x)*(2 - x + 3*x^2)^(5/2))/58320 + (37*(1
 + 2*x)^2*(2 - x + 3*x^2)^(7/2))/405 + ((1 + 2*x)^3*(2 - x + 3*x^2)^(7/2))/
15 + ((2731 + 3430*x)*(2 - x + 3*x^2)^(7/2))/17010 - (3564931*ArcSinh[(1 -
6*x)/Sqrt[23]])/(8957952*Sqrt[3])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 846

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{15}(1+2x)^3\left(2-x+3x^2\right)^{7/2}+\frac{1}{120}\int(1+2x)^2(52+296x)\left(2-x+3x^2\right)^{5/2}\,dx\\
&= \frac{37}{405}(1+2x)^2\left(2-x+3x^2\right)^{7/2}\\
&\quad+\frac{1}{15}(1+2x)^3\left(2-x+3x^2\right)^{7/2}+\frac{\int(1+2x)(72+7840x)\left(2-x+3x^2\right)^{5/2}\,dx}{3240}\\
&= \frac{37}{405}(1+2x)^2\left(2-x+3x^2\right)^{7/2}+\frac{1}{15}(1+2x)^3\left(2-x+3x^2\right)^{7/2}\\
&\quad+\frac{(2731+3430x)\left(2-x+3x^2\right)^{7/2}}{17010}+\frac{293\int\left(2-x+3x^2\right)^{5/2}\,dx}{1620}\\
&= -\frac{293(1-6x)\left(2-x+3x^2\right)^{5/2}}{58320}\\
&\quad+\frac{37}{405}(1+2x)^2\left(2-x+3x^2\right)^{7/2}+\frac{1}{15}(1+2x)^3\left(2-x+3x^2\right)^{7/2}\\
&\quad+\frac{(2731+3430x)\left(2-x+3x^2\right)^{7/2}}{17010}+\frac{6739\int\left(2-x+3x^2\right)^{3/2}\,dx}{23328}\\
&= -\frac{6739(1-6x)\left(2-x+3x^2\right)^{3/2}}{559872}\\
&\quad-\frac{293(1-6x)\left(2-x+3x^2\right)^{5/2}}{58320}+\frac{37}{405}(1+2x)^2\left(2-x+3x^2\right)^{7/2}\\
&\quad+\frac{1}{15}(1+2x)^3\left(2-x+3x^2\right)^{7/2}+\frac{(2731+3430x)\left(2-x+3x^2\right)^{7/2}}{17010}+\frac{154997\int\sqrt{2-x+3x^2}\,dx}{373248}\\
&= -\frac{154997(1-6x)\sqrt{2-x+3x^2}}{4478976}-\frac{6739(1-6x)\left(2-x+3x^2\right)^{3/2}}{559872}\\
&\quad-\frac{293(1-6x)\left(2-x+3x^2\right)^{5/2}}{58320}+\frac{37}{405}(1+2x)^2\left(2-x+3x^2\right)^{7/2}\\
&\quad+\frac{1}{15}(1+2x)^3\left(2-x+3x^2\right)^{7/2}+\frac{(2731+3430x)\left(2-x+3x^2\right)^{7/2}}{17010}+\frac{3564931\int\frac{1}{\sqrt{2-x+3x^2}}\,dx}{8957952}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{154997(1-6x)\sqrt{2-x+3x^2}}{4478976} - \frac{6739(1-6x)\left(2-x+3x^2\right)^{3/2}}{559872} \\
&\quad - \frac{293(1-6x)\left(2-x+3x^2\right)^{5/2}}{58320} + \frac{37}{405}(1+2x)^2\left(2-x+3x^2\right)^{7/2} \\
&\quad + \frac{1}{15}(1+2x)^3\left(2-x+3x^2\right)^{7/2} + \frac{(2731+3430x)\left(2-x+3x^2\right)^{7/2}}{17010} + \frac{\left(154997\sqrt{\frac{23}{3}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{23}}}}{8957952} \\
&= -\frac{154997(1-6x)\sqrt{2-x+3x^2}}{4478976} - \frac{6739(1-6x)\left(2-x+3x^2\right)^{3/2}}{559872} \\
&\quad - \frac{293(1-6x)\left(2-x+3x^2\right)^{5/2}}{58320} + \frac{37}{405}(1+2x)^2\left(2-x+3x^2\right)^{7/2} \\
&\quad + \frac{1}{15}(1+2x)^3\left(2-x+3x^2\right)^{7/2} + \frac{(2731+3430x)\left(2-x+3x^2\right)^{7/2}}{17010} - \frac{3564931\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{8957952\sqrt{3}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.58

$$
\int (1+2x)^2\left(2-x+3x^2\right)^{5/2}\left(1+3x+4x^2\right)\,dx = \frac{6\sqrt{2-x+3x^2}(387182961+692659234x+1693765752x^2+3096104976x^3+4171579776x^4+}{}
$$

```
[In] Integrate[(1 + 2*x)^2*(2 - x + 3*x^2)^(5/2)*(1 + 3*x + 4*x^2),x]
```

```
[Out] (6*Sqrt[2 - x + 3*x^2]*(387182961 + 692659234*x + 1693765752*x^2 + 30961049
76*x^3 + 4171579776*x^4 + 4996802304*x^5 + 5671627776*x^6 + 4427716608*x^7
+ 2675441664*x^8 + 2257403904*x^9) - 124772585*Sqrt[3]*Log[1 - 6*x + 2*Sqrt
[6 - 3*x + 9*x^2]])/940584960
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.46

method	result
risch	$\frac{(2257403904x^9+2675441664x^8+4427716608x^7+5671627776x^6+4996802304x^5+4171579776x^4+3096104976x^3+1693765752x^2+69}{156764160}$
trager	$\left(\frac{72}{5}x^9+\frac{256}{15}x^8+\frac{1271}{45}x^7+\frac{22793}{630}x^6+\frac{722917}{22680}x^5+\frac{517309}{19440}x^4+\frac{21500729}{1088640}x^3+\frac{10081939}{933120}x^2+\frac{346329617}{78382080}x+\frac{4}{1}\right.$
default	$\frac{293(-1+6x)\left(3x^2-x+2\right)^{\frac{5}{2}}}{58320}+\frac{6739(-1+6x)\left(3x^2-x+2\right)^{\frac{3}{2}}}{559872}+\frac{154997(-1+6x)\sqrt{3x^2-x+2}}{4478976}+\frac{3564931\sqrt{3}\,\text{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{26873856}$

```
[In] int((1+2*x)^2*(3*x^2-x+2)^(5/2)*(4*x^2+3*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/156764160*(2257403904*x^9+2675441664*x^8+4427716608*x^7+5671627776*x^6+49
96802304*x^5+4171579776*x^4+3096104976*x^3+1693765752*x^2+692659234*x+38718
2961)*(3*x^2-x+2)^(1/2)+3564931/26873856*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1
/6))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.60

$$\int (1+2x)^2\left(2-x+3x^2\right)^{5/2}\left(1+3x+4x^2\right)\,dx=\frac{1}{156764160}\left(2257403904\,x^9+2675441664\,x^8+4427716608\,x^7+5671627776\,x^6+4996802304\,x^5\right.$$

$$+\frac{3564931}{53747712}\sqrt{3}\log\left(-4\sqrt{3}\sqrt{3\,x^2-x+2}(6\,x-1)-72\,x^2+24\,x-25\right)$$

```
[In] integrate((1+2*x)^2*(3*x^2-x+2)^(5/2)*(4*x^2+3*x+1),x, algorithm="fricas")
```

```
[Out] 1/156764160*(2257403904*x^9 + 2675441664*x^8 + 4427716608*x^7 + 5671627776*
x^6 + 4996802304*x^5 + 4171579776*x^4 + 3096104976*x^3 + 1693765752*x^2 + 6
92659234*x + 387182961)*sqrt(3*x^2 - x + 2) + 3564931/53747712*sqrt(3)*log(
-4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x - 1) - 72*x^2 + 24*x - 25)
```

Sympy [A] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.59

$$\int (1+2x)^2\left(2-x+3x^2\right)^{5/2}\left(1+3x+4x^2\right)\,dx=\sqrt{3x^2-x+2}\cdot\left(\frac{72x^9}{5}+\frac{256x^8}{15}+\frac{1271x^7}{45}+\frac{22793x^6}{630}+\frac{722917x^5}{22680}+\frac{517309x^4}{19440}+\frac{21500729x^3}{1088640}+\frac{10081939x^2}{933120}+\frac{346329617x}{78382080}+\frac{43020329}{17418240}\right)+\frac{3564931\sqrt{3}\operatorname{asinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{26873856}$$

```
[In] integrate((1+2*x)**2*(3*x**2-x+2)**(5/2)*(4*x**2+3*x+1),x)
```

```
[Out] sqrt(3*x**2 - x + 2)*(72*x**9/5 + 256*x**8/15 + 1271*x**7/45 + 22793*x**6/6
30 + 722917*x**5/22680 + 517309*x**4/19440 + 21500729*x**3/1088640 + 100819
39*x**2/933120 + 346329617*x/78382080 + 43020329/17418240) + 3564931*sqrt(3
)*asinh(6*sqrt(23)*(x - 1/6)/23)/26873856
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.02

$$\begin{aligned}\int (1+2x)^2\left(2-x+3x^2\right)^{5/2}\left(1+3x+4x^2\right)\,dx &= \frac{8}{15}\left(3\,x^2-x+2\right)^{\frac{7}{2}}x^3 \\ &+ \frac{472}{405}\left(3\,x^2-x+2\right)^{\frac{7}{2}}x^2+\frac{235}{243}\left(3\,x^2-x+2\right)^{\frac{7}{2}}x+\frac{5419}{17010}\left(3\,x^2-x+2\right)^{\frac{7}{2}} \\ &+ \frac{293}{9720}\left(3\,x^2-x+2\right)^{\frac{5}{2}}x-\frac{293}{58320}\left(3\,x^2-x+2\right)^{\frac{5}{2}}+\frac{6739}{93312}\left(3\,x^2-x+2\right)^{\frac{3}{2}}x \\ &- \frac{6739}{559872}\left(3\,x^2-x+2\right)^{\frac{3}{2}}+\frac{154997}{746496}\sqrt{3\,x^2-x+2}x \\ &+ \frac{3564931}{26873856}\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{23}\sqrt{23}(6\,x-1)\right)-\frac{154997}{4478976}\sqrt{3\,x^2-x+2}\end{aligned}$$

```
[In] integrate((1+2*x)^2*(3*x^2-x+2)^(5/2)*(4*x^2+3*x+1),x, algorithm="maxima")
```

```
[Out] 8/15*(3*x^2 - x + 2)^(7/2)*x^3 + 472/405*(3*x^2 - x + 2)^(7/2)*x^2 + 235/24
3*(3*x^2 - x + 2)^(7/2)*x + 5419/17010*(3*x^2 - x + 2)^(7/2) + 293/9720*(3*
x^2 - x + 2)^(5/2)*x - 293/58320*(3*x^2 - x + 2)^(5/2) + 6739/93312*(3*x^2
- x + 2)^(3/2)*x - 6739/559872*(3*x^2 - x + 2)^(3/2) + 154997/746496*sqrt(3
*x^2 - x + 2)*x + 3564931/26873856*sqrt(3)*arcsinh(1/23*sqrt(23)*(6*x - 1))
 - 154997/4478976*sqrt(3*x^2 - x + 2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.57

$$\begin{aligned}&\int (1+2x)^2\left(2-x+3x^2\right)^{5/2}(1+3x \\ &+4x^2)\,dx=\frac{1}{156764160}(2\,(12\,(6\,(8\,(6\,(36\,(14\,(24\,(27\,x+32)x+1271)x+22793)x+722917)x+3621163)x \\ &-\frac{3564931}{26873856}\sqrt{3}\log\left(-2\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2-x+2}\right)+1\right)\end{aligned}$$

```
[In] integrate((1+2*x)^2*(3*x^2-x+2)^(5/2)*(4*x^2+3*x+1),x, algorithm="giac")
```

```
[Out] 1/156764160*(2*(12*(6*(8*(6*(36*(14*(24*(27*x + 32)*x + 1271)*x + 22793)*x
+ 722917)*x + 3621163)*x + 21500729)*x + 70573573)*x + 346329617)*x + 38718
2961)*sqrt(3*x^2 - x + 2) - 3564931/26873856*sqrt(3)*log(-2*sqrt(3)*(sqrt(3
)*x - sqrt(3*x^2 - x + 2)) + 1)
```

Mupad [F(-1)]

Timed out.

$$\int (1+2x)^2 \left(2-x+3x^2\right)^{5/2} \left(1+3x+4x^2\right) \, dx = \int \left(2\,x+1\right)^2\,{\left(3\,x^2-x+2\right)}^{5/2}\,\left(4\,x^2+3\,x+1\right) \,d x$$

```
[In] int((2*x + 1)^2*(3*x^2 - x + 2)^(5/2)*(3*x + 4*x^2 + 1),x)
[Out] int((2*x + 1)^2*(3*x^2 - x + 2)^(5/2)*(3*x + 4*x^2 + 1), x)
```

3.222 $\int (1+2x) \left(2-x+3x^2\right)^{5/2} \left(1+3x+4x^2\right) \, dx$

Optimal result

Integrand size = 30, antiderivative size = 139

$$\int (1+2x) \left(2-x+3x^2\right)^{5/2} \left(1+3x+4x^2\right) \, dx = -\frac{1177025(1-6x)\sqrt{2-x+3x^2}}{5971968} - \frac{51175(1-6x) \left(2-x+3x^2\right)^{3/2}}{746496} - \frac{445(1-6x) \left(2-x+3x^2\right)^{5/2}}{15552} + \frac{2}{27}(1+2x)^2 \left(2-x+3x^2\right)^{7/2} + \frac{1}{648}(137+122x) \left(2-x+3x^2\right)^{7/2} - \frac{27071575\text{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{11943936\sqrt{3}}$$

```
[Out] -51175/746496*(1-6*x)*(3*x^2-x+2)^(3/2)-445/15552*(1-6*x)*(3*x^2-x+2)^(5/2)
+2/27*(1+2*x)^2*(3*x^2-x+2)^(7/2)+1/648*(137+122*x)*(3*x^2-x+2)^(7/2)-27071
575/35831808*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)-1177025/5971968*(1-6*x)
*(3*x^2-x+2)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1667, 793, 626, 633, 221}

$$\int (1+2x) \left(2-x+3x^2\right)^{5/2} \left(1+3x+4x^2\right) \, dx = -\frac{27071575\text{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{11943936\sqrt{3}} + \frac{2}{27}(2x+1)^2 \left(3x^2-x+2\right)^{7/2} + \frac{1}{648}(122x+137) \left(3x^2-x+2\right)^{7/2} - \frac{445(1-6x) \left(3x^2-x+2\right)^{5/2}}{15552} - \frac{51175(1-6x) \left(3x^2-x+2\right)^{3/2}}{746496} - \frac{1177025(1-6x)\sqrt{3x^2-x+2}}{5971968}$$

```
[In] Int[(1 + 2*x)*(2 - x + 3*x^2)^(5/2)*(1 + 3*x + 4*x^2),x]
```

```
[Out] (-1177025*(1 - 6*x)*Sqrt[2 - x + 3*x^2])/5971968 - (51175*(1 - 6*x)*(2 - x
+ 3*x^2)^(3/2))/746496 - (445*(1 - 6*x)*(2 - x + 3*x^2)^(5/2))/15552 + (2*(
1 + 2*x)^2*(2 - x + 3*x^2)^(7/2))/27 + ((137 + 122*x)*(2 - x + 3*x^2)^(7/2)
)/648 - (27071575*ArcSinh[(1 - 6*x)/Sqrt[23]])/(11943936*Sqrt[3])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\text{integral} = \frac{2}{27}(1+2x)^2\left(2-x+3x^2\right)^{7/2}+\frac{1}{108}\int(1+2x)(72+244x)\left(2-x+3x^2\right)^{5/2}\,dx$$

$$=\frac{2}{27}(1+2x)^2\left(2-x+3x^2\right)^{7/2}+\frac{1}{648}(137+122x)\left(2-x+3x^2\right)^{7/2}+\frac{445}{432}\int\left(2-x+3x^2\right)^{5/2}\,dx$$

$$=-\frac{445(1-6x)\left(2-x+3x^2\right)^{5/2}}{15552}+\frac{2}{27}(1+2x)^2\left(2-x+3x^2\right)^{7/2}+\frac{1}{648}(137+122x)\left(2-x+3x^2\right)^{7/2}+\frac{51175\int\left(2-x+3x^2\right)^{3/2}\,dx}{31104}$$

$$=-\frac{51175(1-6x)\left(2-x+3x^2\right)^{3/2}}{746496}-\frac{445(1-6x)\left(2-x+3x^2\right)^{5/2}}{15552}+\frac{2}{27}(1+2x)^2\left(2-x+3x^2\right)^{7/2}+\frac{1}{648}(137+122x)\left(2-x+3x^2\right)^{7/2}+\frac{1177025\int\sqrt{2-x+3x^2}\,dx}{497664}$$

$$=-\frac{1177025(1-6x)\sqrt{2-x+3x^2}}{5971968}-\frac{51175(1-6x)\left(2-x+3x^2\right)^{3/2}}{746496}-\frac{445(1-6x)\left(2-x+3x^2\right)^{5/2}}{15552}+\frac{2}{27}(1+2x)^2\left(2-x+3x^2\right)^{7/2}+\frac{1}{648}(137+122x)\left(2-x+3x^2\right)^{7/2}+\frac{27071575\int\frac{1}{\sqrt{2-x+3x^2}}\,dx}{11943936}$$

$$=-\frac{1177025(1-6x)\sqrt{2-x+3x^2}}{5971968}-\frac{51175(1-6x)\left(2-x+3x^2\right)^{3/2}}{746496}-\frac{445(1-6x)\left(2-x+3x^2\right)^{5/2}}{15552}+\frac{2}{27}(1+2x)^2\left(2-x+3x^2\right)^{7/2}+\frac{1}{648}(137+122x)\left(2-x+3x^2\right)^{7/2}+\frac{\left(1177025\sqrt{\frac{23}{3}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx,x,-1+6x\right)}{11943936}$$

$$=-\frac{1177025(1-6x)\sqrt{2-x+3x^2}}{5971968}-\frac{51175(1-6x)\left(2-x+3x^2\right)^{3/2}}{746496}-\frac{445(1-6x)\left(2-x+3x^2\right)^{5/2}}{15552}+\frac{2}{27}(1+2x)^2\left(2-x+3x^2\right)^{7/2}+\frac{1}{648}(137+122x)\left(2-x+3x^2\right)^{7/2}-\frac{27071575\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{11943936\sqrt{3}}$$

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.65

$$\int (1+2x)\left(2-x+3x^2\right)^{5/2}(1+3x+4x^2)\ dx = \frac{6\sqrt{2-x+3x^2}(10960335+19860062x+41031048x^2+58946544x^3+66969216x^4+80034048x}{358318}$$

```
[In] Integrate[(1 + 2*x)*(2 - x + 3*x^2)^(5/2)*(1 + 3*x + 4*x^2),x]
```

```
[Out] (6*Sqrt[2 - x + 3*x^2]*(10960335 + 19860062*x + 41031048*x^2 + 58946544*x^3
 + 66969216*x^4 + 80034048*x^5 + 79377408*x^6 + 30357504*x^7 + 47775744*x^8
) - 27071575*Sqrt[3]*Log[1 - 6*x + 2*Sqrt[6 - 3*x + 9*x^2]])/35831808
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.50

method	result
risch	$\frac{(47775744x^8+30357504x^7+79377408x^6+80034048x^5+66969216x^4+58946544x^3+41031048x^2+19860062x+10960335)\sqrt{3x^2-x+2}}{5971968}$
trager	$\left(8x^8+\frac{61}{12}x^7+\frac{319}{24}x^6+\frac{11579}{864}x^5+\frac{58133}{5184}x^4+\frac{409351}{41472}x^3+\frac{1709627}{248832}x^2+\frac{9930031}{2985984}x+\frac{1217815}{663552}\right)\sqrt{3x^2-x+}$
default	$\frac{445(-1+6x)(3x^2-x+2)^{\frac{5}{2}}}{15552}+\frac{51175(-1+6x)(3x^2-x+2)^{\frac{3}{2}}}{746496}+\frac{1177025(-1+6x)\sqrt{3x^2-x+2}}{5971968}+\frac{27071575\sqrt{3}\ \mathrm{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right.}{35831808}$

```
[In] int((1+2*x)*(3*x^2-x+2)^(5/2)*(4*x^2+3*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5971968*(47775744*x^8+30357504*x^7+79377408*x^6+80034048*x^5+66969216*x^4
+58946544*x^3+41031048*x^2+19860062*x+10960335)*(3*x^2-x+2)^(1/2)+27071575/
35831808*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.67

$$\int (1+2x)\left(2-x+3x^2\right)^{5/2}(1+3x+4x^2)\ dx = \frac{1}{5971968}\left(47775744\,x^8+30357504\,x^7+79377408\,x^6+80034048\,x^5+66969216\,x^4+58946544\,x\right.$$
$$+\frac{27071575}{71663616}\sqrt{3}\log\left(-4\sqrt{3}\sqrt{3\,x^2-x+2}(6\,x-1)-72\,x^2+24\,x-25\right)$$

```
[In] integrate((1+2*x)*(3*x^2-x+2)^(5/2)*(4*x^2+3*x+1),x, algorithm="fricas")
```

```
[Out] 1/5971968*(47775744*x^8 + 30357504*x^7 + 79377408*x^6 + 80034048*x^5 + 6696
9216*x^4 + 58946544*x^3 + 41031048*x^2 + 19860062*x + 10960335)*sqrt(3*x^2
- x + 2) + 27071575/71663616*sqrt(3)*log(-4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*
x - 1) - 72*x^2 + 24*x - 25)
```

Sympy [A] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.63

$$\int (1+2x)\left(2-x+3x^2\right)^{5/2}\left(1+3x+4x^2\right)\,dx=\sqrt{3x^2-x+2}\cdot\left(8x^8+\frac{61x^7}{12}+\frac{319x^6}{24}+\frac{11579x^5}{864}+\frac{58133x^4}{5184}+\frac{409351x^3}{41472}+\frac{1709627x^2}{248832}+\frac{9930031x}{2985984}+\frac{1217815}{663552}\right)+\frac{27071575\sqrt{3}\,\mathrm{asinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{35831808}$$

```
[In] integrate((1+2*x)*(3*x**2-x+2)**(5/2)*(4*x**2+3*x+1),x)
```

```
[Out] sqrt(3*x**2 - x + 2)*(8*x**8 + 61*x**7/12 + 319*x**6/24 + 11579*x**5/864 +
58133*x**4/5184 + 409351*x**3/41472 + 1709627*x**2/248832 + 9930031*x/29859
84 + 1217815/663552) + 27071575*sqrt(3)*asinh(6*sqrt(23)*(x - 1/6)/23)/3583
1808
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.08

$$\int (1+2x)\left(2-x+3x^2\right)^{5/2}\left(1+3x+4x^2\right)\,dx=\frac{8}{27}\left(3x^2-x+2\right)^{\frac{7}{2}}x^2+\frac{157}{324}\left(3x^2-x+2\right)^{\frac{7}{2}}x+\frac{185}{648}\left(3x^2-x+2\right)^{\frac{7}{2}}+\frac{445}{2592}\left(3x^2-x+2\right)^{\frac{5}{2}}x-\frac{445}{15552}\left(3x^2-x+2\right)^{\frac{5}{2}}+\frac{51175}{124416}\left(3x^2-x+2\right)^{\frac{3}{2}}x-\frac{51175}{746496}\left(3x^2-x+2\right)^{\frac{3}{2}}+\frac{1177025}{995328}\sqrt{3x^2-x+2}x+\frac{27071575}{35831808}\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{23}\sqrt{23}(6x-1)\right)-\frac{1177025}{5971968}\sqrt{3x^2-x+2}$$

```
[In] integrate((1+2*x)*(3*x^2-x+2)^(5/2)*(4*x^2+3*x+1),x, algorithm="maxima")
```

```
[Out] 8/27*(3*x^2 - x + 2)^(7/2)*x^2 + 157/324*(3*x^2 - x + 2)^(7/2)*x + 185/648*
(3*x^2 - x + 2)^(7/2) + 445/2592*(3*x^2 - x + 2)^(5/2)*x - 445/15552*(3*x^2
```

```
 - x + 2)^(5/2) + 51175/124416*(3*x^2 - x + 2)^(3/2)*x - 51175/746496*(3*x^
2 - x + 2)^(3/2) + 1177025/995328*sqrt(3*x^2 - x + 2)*x + 27071575/35831808
*sqrt(3)*arcsinh(1/23*sqrt(23)*(6*x - 1)) - 1177025/5971968*sqrt(3*x^2 - x
+ 2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.63

$$\int (1+2x)\left(2-x+3x^2\right)^{5/2}(1+3x+4x^2)\,dx=\frac{1}{5971968}(2(12(6(8(6(36(2(96x+61)x+319)x+11579)x+58133)x+409351)x+1709627)$$
$$-\frac{27071575}{35831808}\sqrt{3}\log\left(-2\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2-x+2}\right)+1\right)$$

```
[In] integrate((1+2*x)*(3*x^2-x+2)^(5/2)*(4*x^2+3*x+1),x, algorithm="giac")
```

```
[Out] 1/5971968*(2*(12*(6*(8*(6*(36*(2*(96*x + 61)*x + 319)*x + 11579)*x + 58133)
*x + 409351)*x + 1709627)*x + 9930031)*x + 10960335)*sqrt(3*x^2 - x + 2) -
27071575/35831808*sqrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2))
+ 1)
```

Mupad [F(-1)]

Timed out.

$$\int (1+2x)\left(2-x+3x^2\right)^{5/2}\left(1+3x+4x^2\right)\,dx=\int \left(2\,x+1\right)\,\left(3\,x^2-x+2\right)^{5/2}\,\left(4\,x^2+3\,x+1\right)\,dx$$

```
[In] int((2*x + 1)*(3*x^2 - x + 2)^(5/2)*(3*x + 4*x^2 + 1),x)
```

```
[Out] int((2*x + 1)*(3*x^2 - x + 2)^(5/2)*(3*x + 4*x^2 + 1), x)
```

3.223 $\int \frac{\left(2-x+3x^2\right)^{5/2}\left(1+3x+4x^2\right)}{1+2x}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 147

$$\int \frac{\left(2-x+3x^2\right)^{5/2}\left(1+3x+4x^2\right)}{1+2x}\,dx = \frac{(221999-17850x)\sqrt{2-x+3x^2}}{82944} + \frac{(2449+2154x)\left(2-x+3x^2\right)^{3/2}}{10368} + \frac{(29+150x)\left(2-x+3x^2\right)^{5/2}}{1080} + \frac{2}{21}\left(2-x+3x^2\right)^{7/2} + \frac{944521\operatorname{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{165888\sqrt{3}} - \frac{169}{128}\sqrt{13}\operatorname{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)$$

```
[Out] 1/10368*(2449+2154*x)*(3*x^2-x+2)^(3/2)+1/1080*(29+150*x)*(3*x^2-x+2)^(5/2)
+2/21*(3*x^2-x+2)^(7/2)+944521/497664*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2
)-169/128*arctanh(1/26*(9-8*x)*13^(1/2)/(3*x^2-x+2)^(1/2))*13^(1/2)+1/82944
*(221999-17850*x)*(3*x^2-x+2)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {1667, 828, 857, 633, 221, 738, 212}

$$\int \frac{(2-x+3x^2)^{5/2}(1+3x+4x^2)}{1+2x}\,dx = \frac{944521\text{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{165888\sqrt{3}} - \frac{169}{128}\sqrt{13}\text{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right) + \frac{2}{21}\left(3x^2-x+2\right)^{7/2} + \frac{(150x+29)\left(3x^2-x+2\right)^{5/2}}{1080} + \frac{(2154x+2449)\left(3x^2-x+2\right)^{3/2}}{10368} + \frac{(221999-17850x)\sqrt{3x^2-x+2}}{82944}$$

```
[In] Int[((2 - x + 3*x^2)^(5/2)*(1 + 3*x + 4*x^2))/(1 + 2*x),x]
```

```
[Out] ((221999 - 17850*x)*Sqrt[2 - x + 3*x^2])/82944 + ((2449 + 2154*x)*(2 - x +
3*x^2)^(3/2))/10368 + ((29 + 150*x)*(2 - x + 3*x^2)^(5/2))/1080 + (2*(2 - x
 + 3*x^2)^(7/2))/21 + (944521*ArcSinh[(1 - 6*x)/Sqrt[23]])/(165888*Sqrt[3])
 - (169*Sqrt[13]*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/128
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\text{integral} = \frac{2}{21}\left(2-x+3x^2\right)^{7/2}+\frac{1}{84}\int \frac{(112+140x)\left(2-x+3x^2\right)^{5/2}}{1+2x}\,dx$$

$$=\frac{(29+150x)\left(2-x+3x^2\right)^{5/2}}{1080}+\frac{2}{21}\left(2-x+3x^2\right)^{7/2}-\frac{\int \frac{(-29708-20104x)\left(2-x+3x^2\right)^{3/2}}{1+2x}\,dx}{12096}$$

$$
\begin{aligned}
&= \frac{(2449+2154x)\left(2-x+3x^2\right)^{3/2}}{10368} + \frac{(29+150x)\left(2-x+3x^2\right)^{5/2}}{1080} \\
&\quad + \frac{2}{21}\left(2-x+3x^2\right)^{7/2} + \frac{\int \frac{(5632872-999600x)\sqrt{2-x+3x^2}}{1+2x}\,dx}{1161216} \\
&= \frac{(221999-17850x)\sqrt{2-x+3x^2}}{82944} + \frac{(2449+2154x)\left(2-x+3x^2\right)^{3/2}}{10368} \\
&\quad + \frac{(29+150x)\left(2-x+3x^2\right)^{5/2}}{1080} + \frac{2}{21}\left(2-x+3x^2\right)^{7/2} - \frac{\int \frac{-639337776+634718112x}{(1+2x)\sqrt{2-x+3x^2}}\,dx}{55738368} \\
&= \frac{(221999-17850x)\sqrt{2-x+3x^2}}{82944} \\
&\quad + \frac{(2449+2154x)\left(2-x+3x^2\right)^{3/2}}{10368} + \frac{(29+150x)\left(2-x+3x^2\right)^{5/2}}{1080} \\
&\quad + \frac{2}{21}\left(2-x+3x^2\right)^{7/2} - \frac{944521\int \frac{1}{\sqrt{2-x+3x^2}}\,dx}{165888} + \frac{2197}{128}\int \frac{1}{(1+2x)\sqrt{2-x+3x^2}}\,dx \\
&= \frac{(221999-17850x)\sqrt{2-x+3x^2}}{82944} + \frac{(2449+2154x)\left(2-x+3x^2\right)^{3/2}}{10368} \\
&\quad + \frac{(29+150x)\left(2-x+3x^2\right)^{5/2}}{1080} + \frac{2}{21}\left(2-x+3x^2\right)^{7/2} \\
&\quad - \frac{2197}{64}\text{Subst}\left(\int \frac{1}{52-x^2}\,dx, x, \frac{9-8x}{\sqrt{2-x+3x^2}}\right) - \frac{944521\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx, x, -1+6x\right)}{165888\sqrt{69}} \\
&= \frac{(221999-17850x)\sqrt{2-x+3x^2}}{82944} + \frac{(2449+2154x)\left(2-x+3x^2\right)^{3/2}}{10368} \\
&\quad + \frac{(29+150x)\left(2-x+3x^2\right)^{5/2}}{1080} + \frac{2}{21}\left(2-x+3x^2\right)^{7/2} \\
&\quad + \frac{944521\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{165888\sqrt{3}} - \frac{169}{128}\sqrt{13}\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.84

$$
\int \frac{\left(2-x+3x^2\right)^{5/2}\left(1+3x+4x^2\right)}{1+2x}\,dx = \frac{6\sqrt{2-x+3x^2}(11665053-2120998x+12466776x^2-3646512x^3+
$$

```
[In] Integrate[((2 - x + 3*x^2)^(5/2)*(1 + 3*x + 4*x^2))/(1 + 2*x),x]
```

```
[Out] (6*Sqrt[2 - x + 3*x^2]*(11665053 - 2120998*x + 12466776*x^2 - 3646512*x^3 +
 15700608*x^4 - 3836160*x^5 + 7464960*x^6) + 45995040*Sqrt[13]*ArcTanh[(Sqr
t[3] + 2*Sqrt[3]*x - 2*Sqrt[2 - x + 3*x^2])/Sqrt[13]] + 33058235*Sqrt[3]*Lo
g[1 - 6*x + 2*Sqrt[6 - 3*x + 9*x^2]])/17418240
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.61

method	result
risch	$\frac{(7464960x^6-3836160x^5+15700608x^4-3646512x^3+12466776x^2-2120998x+11665053)\sqrt{3x^2-x+2}}{2903040} - \frac{944521\sqrt{3}\ \mathrm{arcsinh}\left(\frac{6\sqrt{23}\,(}{23}\right.}{497664}$
trager	$\left(\frac{18}{7}x^6-\frac{37}{28}x^5+\frac{649}{120}x^4-\frac{8441}{6720}x^3+\frac{74207}{17280}x^2-\frac{1060499}{1451520}x+\frac{144013}{35840}\right)\sqrt{3x^2-x+2}+\frac{169\,\mathrm{RootOf}\left(\_Z^2-13\right)}{}$
default	$\frac{5(-1+6x)(3x^2-x+2)^{\frac{5}{2}}}{216}+\frac{575(-1+6x)(3x^2-x+2)^{\frac{3}{2}}}{10368}+\frac{13225(-1+6x)\sqrt{3x^2-x+2}}{82944}-\frac{944521\sqrt{3}\ \mathrm{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{497664}+2$

```
[In] int((3*x^2-x+2)^(5/2)*(4*x^2+3*x+1)/(1+2*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2903040*(7464960*x^6-3836160*x^5+15700608*x^4-3646512*x^3+12466776*x^2-21
20998*x+11665053)*(3*x^2-x+2)^(1/2)-944521/497664*3^(1/2)*arcsinh(6/23*23^(
1/2)*(x-1/6))-169/128*13^(1/2)*arctanh(2/13*(9/2-4*x)*13^(1/2)/(12*(x+1/2)^
2-16*x+5)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.92

$$\int \frac{(2-x+3x^2)^{5/2}\,(1+3x+4x^2)}{1+2x}\,dx = \frac{1}{2903040}\left(7464960\,x^6-3836160\,x^5+15700608\,x^4-3646512\,x^3+\right.$$
$$+\frac{944521}{995328}\sqrt{3}\log\left(4\sqrt{3}\sqrt{3\,x^2-x+2}(6\,x-1)-72\,x^2+24\,x-25\right)$$
$$+\frac{169}{256}\sqrt{13}\log\left(-\frac{4\sqrt{13}\sqrt{3\,x^2-x+2}(8\,x-9)+220\,x^2-196\,x+185}{4\,x^2+4\,x+1}\right)$$

```
[In] integrate((3*x^2-x+2)^(5/2)*(4*x^2+3*x+1)/(1+2*x),x, algorithm="fricas")
```

```
[Out] 1/2903040*(7464960*x^6 - 3836160*x^5 + 15700608*x^4 - 3646512*x^3 + 1246677
6*x^2 - 2120998*x + 11665053)*sqrt(3*x^2 - x + 2) + 944521/995328*sqrt(3)*l
og(4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x - 1) - 72*x^2 + 24*x - 25) + 169/256*
sqrt(13)*log(-(4*sqrt(13)*sqrt(3*x^2 - x + 2)*(8*x - 9) + 220*x^2 - 196*x +
 185)/(4*x^2 + 4*x + 1))
```

Sympy [F]

$$\int \frac{(2-x+3x^2)^{5/2}(1+3x+4x^2)}{1+2x}\,dx = \int \frac{(3x^2-x+2)^{\frac{5}{2}}\cdot(4x^2+3x+1)}{2x+1}\,dx$$

```
[In] integrate((3*x**2-x+2)**(5/2)*(4*x**2+3*x+1)/(1+2*x),x)
[Out] Integral((3*x**2 - x + 2)**(5/2)*(4*x**2 + 3*x + 1)/(2*x + 1), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.05

$$\begin{aligned}\int \frac{(2-x+3x^2)^{5/2}(1+3x+4x^2)}{1+2x}\,dx &= \frac{2}{21}\left(3\,x^2-x+2\right)^{\frac{7}{2}}+\frac{5}{36}\left(3\,x^2-x+2\right)^{\frac{5}{2}}x \\ &+\frac{29}{1080}\left(3\,x^2-x+2\right)^{\frac{5}{2}}+\frac{359}{1728}\left(3\,x^2-x+2\right)^{\frac{3}{2}}x+\frac{2449}{10368}\left(3\,x^2-x+2\right)^{\frac{3}{2}} \\ &-\frac{2975}{13824}\sqrt{3\,x^2-x+2}x-\frac{944521}{497664}\sqrt{3}\,\mathrm{arsinh}\left(\frac{6}{23}\sqrt{23}x-\frac{1}{23}\sqrt{23}\right) \\ &+\frac{169}{128}\sqrt{13}\,\mathrm{arsinh}\left(\frac{8\sqrt{23}x}{23\,|2\,x+1|}-\frac{9\sqrt{23}}{23\,|2\,x+1|}\right)+\frac{221999}{82944}\sqrt{3\,x^2-x+2}\end{aligned}$$

```
[In] integrate((3*x^2-x+2)^(5/2)*(4*x^2+3*x+1)/(1+2*x),x, algorithm="maxima")
[Out] 2/21*(3*x^2 - x + 2)^(7/2) + 5/36*(3*x^2 - x + 2)^(5/2)*x + 29/1080*(3*x^2
 - x + 2)^(5/2) + 359/1728*(3*x^2 - x + 2)^(3/2)*x + 2449/10368*(3*x^2 - x +
 2)^(3/2) - 2975/13824*sqrt(3*x^2 - x + 2)*x - 944521/497664*sqrt(3)*arcsin
h(6/23*sqrt(23)*x - 1/23*sqrt(23)) + 169/128*sqrt(13)*arcsinh(8/23*sqrt(23)
*x/abs(2*x + 1) - 9/23*sqrt(23)/abs(2*x + 1)) + 221999/82944*sqrt(3*x^2 - x
 + 2)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.99

$$\begin{aligned}\int \frac{(2-x+3x^2)^{5/2}(1+3x+4x^2)}{1+2x}\,dx &= \frac{1}{2903040}\left(2\left(12\left(18\left(8\left(30\left(72\,x-37\right)x+4543\right)x-8441\right)x+519449\right)\right. \\ &+\frac{944521}{497664}\sqrt{3}\log\left(-6\sqrt{3}x+\sqrt{3}+6\sqrt{3\,x^2-x+2}\right) \\ &+\frac{169}{128}\sqrt{13}\log\left(-\frac{\left|-4\sqrt{3}x-2\sqrt{13}-2\sqrt{3}+4\sqrt{3\,x^2-x+2}\right|}{2\left(2\sqrt{3}x-\sqrt{13}+\sqrt{3}-2\sqrt{3\,x^2-x+2}\right)}\right)\end{aligned}$$

```
[In] integrate((3*x^2-x+2)^(5/2)*(4*x^2+3*x+1)/(1+2*x),x, algorithm="giac")
```

```
[Out] 1/2903040*(2*(12*(18*(8*(30*(72*x - 37)*x + 4543)*x - 8441)*x + 519449)*x -
 1060499)*x + 11665053)*sqrt(3*x^2 - x + 2) + 944521/497664*sqrt(3)*log(-6*
sqrt(3)*x + sqrt(3) + 6*sqrt(3*x^2 - x + 2)) + 169/128*sqrt(13)*log(-1/2*ab
s(-4*sqrt(3)*x - 2*sqrt(13) - 2*sqrt(3) + 4*sqrt(3*x^2 - x + 2))/(2*sqrt(3)
*x - sqrt(13) + sqrt(3) - 2*sqrt(3*x^2 - x + 2)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2-x+3x^2)^{5/2}\,(1+3x+4x^2)}{1+2x}\,dx=\int \frac{\left(3\,x^2-x+2\right)^{5/2}\,\left(4\,x^2+3\,x+1\right)}{2\,x+1}\,dx$$

```
[In] int(((3*x^2 - x + 2)^(5/2)*(3*x + 4*x^2 + 1))/(2*x + 1),x)
```

```
[Out] int(((3*x^2 - x + 2)^(5/2)*(3*x + 4*x^2 + 1))/(2*x + 1), x)
```

3.224 $\int \frac{(2-x+3x^2)^{5/2}(1+3x+4x^2)}{(1+2x)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 154

$$\begin{aligned}\int \frac{(2-x+3x^2)^{5/2}(1+3x+4x^2)}{(1+2x)^2}\,dx &= -\frac{11(4727-3090x)\sqrt{2-x+3x^2}}{6912} \\ &\quad -\frac{11}{864}(67-78x)\left(2-x+3x^2\right)^{3/2}-\frac{11(37-60x)\left(2-x+3x^2\right)^{5/2}}{2340} \\ &\quad -\frac{(2-x+3x^2)^{7/2}}{13(1+2x)}-\frac{315623\operatorname{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{13824\sqrt{3}}+\frac{429}{128}\sqrt{13}\operatorname{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)\end{aligned}$$

```
[Out] -11/864*(67-78*x)*(3*x^2-x+2)^(3/2)-11/2340*(37-60*x)*(3*x^2-x+2)^(5/2)-1/1
3*(3*x^2-x+2)^(7/2)/(1+2*x)-315623/41472*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(
1/2)+429/128*arctanh(1/26*(9-8*x)*13^(1/2)/(3*x^2-x+2)^(1/2))*13^(1/2)-11/6
912*(4727-3090*x)*(3*x^2-x+2)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {1664, 828, 857, 633, 221, 738, 212}

$$\begin{aligned}\int \frac{(2-x+3x^2)^{5/2}(1+3x+4x^2)}{(1+2x)^2}\,dx = & -\frac{315623\text{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{13824\sqrt{3}} \\ & +\frac{429}{128}\sqrt{13}\text{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right) \\ & -\frac{(3x^2-x+2)^{7/2}}{13(2x+1)}-\frac{11(37-60x)(3x^2-x+2)^{5/2}}{2340} \\ & -\frac{11}{864}(67-78x)\left(3x^2-x+2\right)^{3/2}-\frac{11(4727-3090x)\sqrt{3x^2-x+2}}{6912}\end{aligned}$$

```
[In] Int[((2 - x + 3*x^2)^(5/2)*(1 + 3*x + 4*x^2))/(1 + 2*x)^2,x]
```

```
[Out] (-11*(4727 - 3090*x)*Sqrt[2 - x + 3*x^2])/6912 - (11*(67 - 78*x)*(2 - x + 3
*x^2)^(3/2))/864 - (11*(37 - 60*x)*(2 - x + 3*x^2)^(5/2))/2340 - (2 - x + 3
*x^2)^(7/2)/(13*(1 + 2*x)) - (315623*ArcSinh[(1 - 6*x)/Sqrt[23]])/(13824*Sq
rt[3]) + (429*Sqrt[13]*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])
/128
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{(2-x+3x^2)^{7/2}}{13(1+2x)}-\frac{1}{13}\int\frac{\left(-\frac{11}{2}-44x\right)(2-x+3x^2)^{5/2}}{1+2x}\,dx\\ &= -\frac{11(37-60x)(2-x+3x^2)^{5/2}}{2340}-\frac{(2-x+3x^2)^{7/2}}{13(1+2x)}+\frac{\int\frac{(-286+14872x)(2-x+3x^2)^{3/2}}{1+2x}\,dx}{1872}\end{aligned}$$

$$= -\frac{11}{864}(67-78x)\left(2-x+3x^2\right)^{3/2} - \frac{11(37-60x)\left(2-x+3x^2\right)^{5/2}}{2340} - \frac{\left(2-x+3x^2\right)^{7/2}}{13(1+2x)} - \frac{\int \frac{(641784-3534960x)\sqrt{2-x+3x^2}}{1+2x}\,dx}{179712}$$

$$= -\frac{11(4727-3090x)\sqrt{2-x+3x^2}}{6912} - \frac{11}{864}(67-78x)\left(2-x+3x^2\right)^{3/2} - \frac{11(37-60x)\left(2-x+3x^2\right)^{5/2}}{2340} - \frac{\left(2-x+3x^2\right)^{7/2}}{13(1+2x)} + \frac{\int \frac{-178896432+393897504x}{(1+2x)\sqrt{2-x+3x^2}}\,dx}{8626176}$$

$$= -\frac{11(4727-3090x)\sqrt{2-x+3x^2}}{6912} - \frac{11}{864}(67-78x)\left(2-x+3x^2\right)^{3/2} - \frac{11(37-60x)\left(2-x+3x^2\right)^{5/2}}{2340} - \frac{\left(2-x+3x^2\right)^{7/2}}{13(1+2x)} + \frac{315623\int \frac{1}{\sqrt{2-x+3x^2}}\,dx}{13824} - \frac{5577}{128}\int \frac{1}{(1+2x)\sqrt{2-x+3x^2}}\,dx$$

$$= -\frac{11(4727-3090x)\sqrt{2-x+3x^2}}{6912} - \frac{11}{864}(67-78x)\left(2-x+3x^2\right)^{3/2} - \frac{11(37-60x)\left(2-x+3x^2\right)^{5/2}}{2340} - \frac{\left(2-x+3x^2\right)^{7/2}}{13(1+2x)} + \frac{5577}{64}\text{Subst}\left(\int \frac{1}{52-x^2}\,dx, x, \frac{9-8x}{\sqrt{2-x+3x^2}}\right) + \frac{315623\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx, x, -1+6x\right)}{13824\sqrt{69}}$$

$$= -\frac{11(4727-3090x)\sqrt{2-x+3x^2}}{6912} - \frac{11}{864}(67-78x)\left(2-x+3x^2\right)^{3/2} - \frac{11(37-60x)\left(2-x+3x^2\right)^{5/2}}{2340} - \frac{\left(2-x+3x^2\right)^{7/2}}{13(1+2x)} - \frac{315623\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{13824\sqrt{3}} + \frac{429}{128}\sqrt{13}\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)$$

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.85

$$\int \frac{\left(2-x+3x^2\right)^{5/2}\left(1+3x+4x^2\right)}{(1+2x)^2}\,dx = \frac{\frac{6\sqrt{2-x+3x^2}\left(-364257-322972x+310660x^2-115680x^3+251424x^4-65664x^5+103680x^6\right)}{1+2x}}{}$$

```
[In] Integrate[((2 - x + 3*x^2)^(5/2)*(1 + 3*x + 4*x^2))/(1 + 2*x)^2,x]
```

```
[Out] ((6*Sqrt[2 - x + 3*x^2]*(-364257 - 322972*x + 310660*x^2 - 115680*x^3 + 251
424*x^4 - 65664*x^5 + 103680*x^6))/(1 + 2*x) - 1389960*Sqrt[13]*ArcTanh[(Sq
rt[3] + 2*Sqrt[3]*x - 2*Sqrt[2 - x + 3*x^2])/Sqrt[13]] - 1578115*Sqrt[3]*Lo
g[1 - 6*x + 2*Sqrt[6 - 3*x + 9*x^2]])/207360
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.69

method	result
risch	$\frac{311040x^8-300672x^7+1027296x^6-729792x^5+1550508x^4-1510936x^3-148479x^2-281687x-728514}{34560(1+2x)\sqrt{3x^2-x+2}}+\frac{315623\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\,(x-}{23}\right.}{41472}$
trager	$\frac{(103680x^6-65664x^5+251424x^4-115680x^3+310660x^2-322972x-364257)\sqrt{3x^2-x+2}}{34560+69120x}+\frac{429\,\operatorname{RootOf}\left(\_Z^2-13\right)\ln\left(-\frac{8\,\operatorname{RootOf}\left(\_\right.}{}\right.}{}$
default	$\frac{(-1+6x)(3x^2-x+2)^{\frac{5}{2}}}{36}+\frac{115(-1+6x)(3x^2-x+2)^{\frac{3}{2}}}{1728}+\frac{2645(-1+6x)\sqrt{3x^2-x+2}}{13824}+\frac{315623\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{41472}-\frac{\left(3(x\right.}{}$

```
[In] int((3*x^2-x+2)^(5/2)*(4*x^2+3*x+1)/(1+2*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/34560*(311040*x^8-300672*x^7+1027296*x^6-729792*x^5+1550508*x^4-1510936*x
^3-148479*x^2-281687*x-728514)/(1+2*x)/(3*x^2-x+2)^(1/2)+315623/41472*3^(1/
2)*arcsinh(6/23*23^(1/2)*(x-1/6))+429/128*13^(1/2)*arctanh(2/13*(9/2-4*x)*1
3^(1/2)/(12*(x+1/2)^2-16*x+5)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.99

$$\int \frac{(2-x+3x^2)^{5/2}\,(1+3x+4x^2)}{(1+2x)^2}\,dx=\frac{1578115\,\sqrt{3}(2\,x+1)\log\left(-4\,\sqrt{3}\sqrt{3\,x^2-x+2}(6\,x-1)-72\,x^2+24\right.}{}$$

```
[In] integrate((3*x^2-x+2)^(5/2)*(4*x^2+3*x+1)/(1+2*x)^2,x, algorithm="fricas")
```

```
[Out] 1/414720*(1578115*sqrt(3)*(2*x + 1)*log(-4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x
 - 1) - 72*x^2 + 24*x - 25) + 694980*sqrt(13)*(2*x + 1)*log((4*sqrt(13)*sqr
t(3*x^2 - x + 2)*(8*x - 9) - 220*x^2 + 196*x - 185)/(4*x^2 + 4*x + 1)) + 12
*(103680*x^6 - 65664*x^5 + 251424*x^4 - 115680*x^3 + 310660*x^2 - 322972*x
 - 364257)*sqrt(3*x^2 - x + 2))/(2*x + 1)
```

Sympy [F]

$$\int \frac{(2-x+3x^2)^{5/2}(1+3x+4x^2)}{(1+2x)^2}\,dx = \int \frac{(3x^2-x+2)^{\frac{5}{2}}\cdot(4x^2+3x+1)}{(2x+1)^2}\,dx$$

```
[In] integrate((3*x**2-x+2)**(5/2)*(4*x**2+3*x+1)/(1+2*x)**2,x)
[Out] Integral((3*x**2 - x + 2)**(5/2)*(4*x**2 + 3*x + 1)/(2*x + 1)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.05

$$\begin{aligned}\int \frac{(2-x+3x^2)^{5/2}(1+3x+4x^2)}{(1+2x)^2}\,dx &= \frac{1}{6}\left(3\,x^2-x+2\right)^{\frac{5}{2}}x - \frac{7}{90}\left(3\,x^2-x+2\right)^{\frac{5}{2}} \\ &+ \frac{143}{144}\left(3\,x^2-x+2\right)^{\frac{3}{2}}x - \frac{737}{864}\left(3\,x^2-x+2\right)^{\frac{3}{2}} - \frac{\left(3\,x^2-x+2\right)^{\frac{5}{2}}}{4\,(2\,x+1)} \\ &+ \frac{5665}{1152}\sqrt{3\,x^2-x+2}x + \frac{315623}{41472}\sqrt{3}\,\mathrm{arsinh}\left(\frac{6}{23}\sqrt{23}x - \frac{1}{23}\sqrt{23}\right) \\ &- \frac{429}{128}\sqrt{13}\,\mathrm{arsinh}\left(\frac{8\sqrt{23}x}{23\,|2\,x+1|} - \frac{9\sqrt{23}}{23\,|2\,x+1|}\right) - \frac{51997}{6912}\sqrt{3\,x^2-x+2}\end{aligned}$$

```
[In] integrate((3*x^2-x+2)^(5/2)*(4*x^2+3*x+1)/(1+2*x)^2,x, algorithm="maxima")
[Out] 1/6*(3*x^2 - x + 2)^(5/2)*x - 7/90*(3*x^2 - x + 2)^(5/2) + 143/144*(3*x^2 -
 x + 2)^(3/2)*x - 737/864*(3*x^2 - x + 2)^(3/2) - 1/4*(3*x^2 - x + 2)^(5/2)
/(2*x + 1) + 5665/1152*sqrt(3*x^2 - x + 2)*x + 315623/41472*sqrt(3)*arcsinh
(6/23*sqrt(23)*x - 1/23*sqrt(23)) - 429/128*sqrt(13)*arcsinh(8/23*sqrt(23)*
x/abs(2*x + 1) - 9/23*sqrt(23)/abs(2*x + 1)) - 51997/6912*sqrt(3*x^2 - x +
2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 760 vs. 2(123) = 246.

Time = 0.57 (sec) , antiderivative size = 760, normalized size of antiderivative = 4.94

$$\int \frac{(2-x+3x^2)^{5/2}(1+3x+4x^2)}{(1+2x)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((3*x^2-x+2)^(5/2)*(4*x^2+3*x+1)/(1+2*x)^2,x, algorithm="giac")
```

```
[Out] 429/128*sqrt(13)*log(sqrt(13)*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sq
rt(13)/(2*x + 1)) - 4)*sgn(1/(2*x + 1)) - 315623/41472*sqrt(3)*log(1/2*abs(
-2*sqrt(3) + 2*sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + 2*sqrt(13)/(2*x +
1))/(sqrt(3) + sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1)
))*sgn(1/(2*x + 1)) - 169/128*sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3)*sgn(1
/(2*x + 1)) + 1/34560*(5154065*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + s
qrt(13)/(2*x + 1))^11*sgn(1/(2*x + 1)) - 7837020*sqrt(13)*(sqrt(-8/(2*x + 1
) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1))^10*sgn(1/(2*x + 1)) + 3946881
5*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1))^9*sgn(1/(2
*x + 1)) - 14445540*sqrt(13)*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqr
t(13)/(2*x + 1))^8*sgn(1/(2*x + 1)) + 460893402*(sqrt(-8/(2*x + 1) + 13/(2*
x + 1)^2 + 3) + sqrt(13)/(2*x + 1))^7*sgn(1/(2*x + 1)) - 343084680*sqrt(13)
*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1))^6*sgn(1/(2*
x + 1)) + 944150094*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*
x + 1))^5*sgn(1/(2*x + 1)) - 22871160*sqrt(13)*(sqrt(-8/(2*x + 1) + 13/(2*x
 + 1)^2 + 3) + sqrt(13)/(2*x + 1))^4*sgn(1/(2*x + 1)) + 1397032245*(sqrt(-8
/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1))^3*sgn(1/(2*x + 1)) -
 683367516*sqrt(13)*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*
x + 1))^2*sgn(1/(2*x + 1)) + 392684355*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2
+ 3) + sqrt(13)/(2*x + 1))*sgn(1/(2*x + 1)) + 197538588*sqrt(13)*sgn(1/(2*x
 + 1)))/((sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1))^2 -
 3)^6
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2-x+3x^2)^{5/2}(1+3x+4x^2)}{(1+2x)^2}\,dx = \int \frac{\left(3\,x^2-x+2\right)^{5/2}\,\left(4\,x^2+3\,x+1\right)}{\left(2\,x+1\right)^2}\,dx$$

```
[In] int(((3*x^2 - x + 2)^(5/2)*(3*x + 4*x^2 + 1))/(2*x + 1)^2,x)
```

```
[Out] int(((3*x^2 - x + 2)^(5/2)*(3*x + 4*x^2 + 1))/(2*x + 1)^2, x)
```

3.225 $\int \frac{\left(2-x+3x^2\right)^{5/2}\left(1+3x+4x^2\right)}{(1+2x)^3}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 161

$$\int \frac{\left(2-x+3x^2\right)^{5/2}\left(1+3x+4x^2\right)}{(1+2x)^3}\,dx=\frac{(21317-10470x)\sqrt{2-x+3x^2}}{1536}+\frac{1}{832}(1227-838x)\left(2-x+3x^2\right)^{3/2}+\frac{(257+134x)\left(2-x+3x^2\right)^{5/2}}{520(1+2x)}-\frac{\left(2-x+3x^2\right)^{7/2}}{26(1+2x)^2}+\frac{118423\text{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{3072\sqrt{3}}-\frac{1631}{256}\sqrt{13}\text{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)$$

```
[Out] 1/832*(1227-838*x)*(3*x^2-x+2)^(3/2)+1/520*(257+134*x)*(3*x^2-x+2)^(5/2)/(1
+2*x)-1/26*(3*x^2-x+2)^(7/2)/(1+2*x)^2+118423/9216*arcsinh(1/23*(1-6*x)*23^
(1/2))*3^(1/2)-1631/256*arctanh(1/26*(9-8*x)*13^(1/2)/(3*x^2-x+2)^(1/2))*13
^(1/2)+1/1536*(21317-10470*x)*(3*x^2-x+2)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1664, 826, 828, 857, 633, 221, 738, 212}

$$\int \frac{(2-x+3x^2)^{5/2}(1+3x+4x^2)}{(1+2x)^3}\,dx = \frac{118423\text{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{3072\sqrt{3}} - \frac{1631}{256}\sqrt{13}\text{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right) - \frac{(3x^2-x+2)^{7/2}}{26(2x+1)^2} + \frac{(134x+257)(3x^2-x+2)^{5/2}}{520(2x+1)} + \frac{1}{832}(1227-838x)\left(3x^2-x+2\right)^{3/2} + \frac{(21317-10470x)\sqrt{3x^2-x+2}}{1536}$$

```
[In] Int[((2 - x + 3*x^2)^(5/2)*(1 + 3*x + 4*x^2))/(1 + 2*x)^3,x]
```

```
[Out] ((21317 - 10470*x)*Sqrt[2 - x + 3*x^2])/1536 + ((1227 - 838*x)*(2 - x + 3*x
^2)^(3/2))/832 + ((257 + 134*x)*(2 - x + 3*x^2)^(5/2))/(520*(1 + 2*x)) - (2
 - x + 3*x^2)^(7/2)/(26*(1 + 2*x)^2) + (118423*ArcSinh[(1 - 6*x)/Sqrt[23]])
/(3072*Sqrt[3]) - (1631*Sqrt[13]*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x +
 3*x^2])])/256
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 826

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
```

```
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(2-x+3x^2)^{7/2}}{26(1+2x)^2}-\frac{1}{26}\int\frac{\left(-\frac{29}{2}-67x\right)(2-x+3x^2)^{5/2}}{(1+2x)^2}\,dx\\
&= \frac{(257+134x)(2-x+3x^2)^{5/2}}{520(1+2x)}-\frac{(2-x+3x^2)^{7/2}}{26(1+2x)^2}+\frac{1}{208}\int\frac{(793-1676x)(2-x+3x^2)^{3/2}}{1+2x}\,dx\\
&= \frac{1}{832}(1227-838x)\left(2-x+3x^2\right)^{3/2}+\frac{(257+134x)(2-x+3x^2)^{5/2}}{520(1+2x)}\\
&\quad-\frac{(2-x+3x^2)^{7/2}}{26(1+2x)^2}-\frac{\int\frac{(-236652+544440x)\sqrt{2-x+3x^2}}{1+2x}\,dx}{19968}\\
&= \frac{(21317-10470x)\sqrt{2-x+3x^2}}{1536}+\frac{1}{832}(1227-838x)\left(2-x+3x^2\right)^{3/2}\\
&\quad+\frac{(257+134x)(2-x+3x^2)^{5/2}}{520(1+2x)}-\frac{(2-x+3x^2)^{7/2}}{26(1+2x)^2}+\frac{\int\frac{42436056-73895952x}{(1+2x)\sqrt{2-x+3x^2}}\,dx}{958464}\\
&= \frac{(21317-10470x)\sqrt{2-x+3x^2}}{1536}+\frac{1}{832}(1227-838x)\left(2-x+3x^2\right)^{3/2}\\
&\quad+\frac{(257+134x)(2-x+3x^2)^{5/2}}{520(1+2x)}-\frac{(2-x+3x^2)^{7/2}}{26(1+2x)^2}\\
&\quad-\frac{118423\int\frac{1}{\sqrt{2-x+3x^2}}\,dx}{3072}+\frac{21203}{256}\int\frac{1}{(1+2x)\sqrt{2-x+3x^2}}\,dx\\
&= \frac{(21317-10470x)\sqrt{2-x+3x^2}}{1536}+\frac{1}{832}(1227-838x)\left(2-x+3x^2\right)^{3/2}\\
&\quad+\frac{(257+134x)(2-x+3x^2)^{5/2}}{520(1+2x)}-\frac{(2-x+3x^2)^{7/2}}{26(1+2x)^2}\\
&\quad-\frac{21203}{128}\text{Subst}\left(\int\frac{1}{52-x^2}\,dx,x,\frac{9-8x}{\sqrt{2-x+3x^2}}\right)-\frac{118423\text{Subst}\left(\int\frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx,x,-1+6x\right)}{3072\sqrt{69}}\\
&= \frac{(21317-10470x)\sqrt{2-x+3x^2}}{1536}+\frac{1}{832}(1227-838x)\left(2-x+3x^2\right)^{3/2}\\
&\quad+\frac{(257+134x)(2-x+3x^2)^{5/2}}{520(1+2x)}-\frac{(2-x+3x^2)^{7/2}}{26(1+2x)^2}\\
&\quad+\frac{118423\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{3072\sqrt{3}}-\frac{1631}{256}\sqrt{13}\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.81

$$\int \frac{(2-x+3x^2)^{5/2}(1+3x+4x^2)}{(1+2x)^3}\,dx = \frac{\frac{6\sqrt{2-x+3x^2}(142057+464446x+256564x^2-76200x^3+83616x^4-22464x^5+27648x^6)}{(1+2x)^2}+58}{}$$

```
[In] Integrate[((2 - x + 3*x^2)^(5/2)*(1 + 3*x + 4*x^2))/(1 + 2*x)^3,x]
```

```
[Out] ((6*Sqrt[2 - x + 3*x^2]*(142057 + 464446*x + 256564*x^2 - 76200*x^3 + 83616
*x^4 - 22464*x^5 + 27648*x^6))/(1 + 2*x)^2 + 587160*Sqrt[13]*ArcTanh[(Sqrt[
3] + 2*Sqrt[3]*x - 2*Sqrt[2 - x + 3*x^2])/Sqrt[13]] + 592115*Sqrt[3]*Log[1
- 6*x + 2*Sqrt[6 - 3*x + 9*x^2]])/46080
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.66

method	result
risch	$\frac{82944x^8-95040x^7+328608x^6-357144x^5+1013124x^4+984374x^3+474853x^2+786835x+284114}{7680(1+2x)^2\sqrt{3x^2-x+2}}-\frac{118423\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right.}{23}\right.}{9216}$
trager	$\frac{(27648x^6-22464x^5+83616x^4-76200x^3+256564x^2+464446x+142057)\sqrt{3x^2-x+2}}{7680(1+2x)^2}-\frac{7\operatorname{RootOf}\left(\_Z^2-705757\right)\ln\left(-\frac{8\operatorname{RootOf}\left(\right.}{}\right.}{}$
default	$\frac{1631\left(3\left(x+\frac{1}{2}\right)^2-4x+\frac{5}{4}\right)^{\frac{5}{2}}}{6760}-\frac{419(-1+6x)\left(3\left(x+\frac{1}{2}\right)^2-4x+\frac{5}{4}\right)^{\frac{3}{2}}}{2496}-\frac{1745(-1+6x)\sqrt{3\left(x+\frac{1}{2}\right)^2-4x+\frac{5}{4}}}{1536}-\frac{118423\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{}}{}\right.}{9216}$

```
[In] int((3*x^2-x+2)^(5/2)*(4*x^2+3*x+1)/(1+2*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/7680*(82944*x^8-95040*x^7+328608*x^6-357144*x^5+1013124*x^4+984374*x^3+47
4853*x^2+786835*x+284114)/(1+2*x)^2/(3*x^2-x+2)^(1/2)-118423/9216*3^(1/2)*a
rcsinh(6/23*23^(1/2)*(x-1/6))-1631/256*13^(1/2)*arctanh(2/13*(9/2-4*x)*13^(
1/2)/(12*(x+1/2)^2-16*x+5)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.05

$$\int \frac{(2-x+3x^2)^{5/2}(1+3x+4x^2)}{(1+2x)^3}\,dx = \frac{592115\sqrt{3}(4\,x^2+4\,x+1)\log\left(4\sqrt{3}\sqrt{3\,x^2-x+2}(6\,x-1)-72\,x^2\right.}{}$$

```
[In] integrate((3*x^2-x+2)^(5/2)*(4*x^2+3*x+1)/(1+2*x)^3,x, algorithm="fricas")
```

```
[Out] 1/92160*(592115*sqrt(3)*(4*x^2 + 4*x + 1)*log(4*sqrt(3)*sqrt(3*x^2 - x + 2)
*(6*x - 1) - 72*x^2 + 24*x - 25) + 293580*sqrt(13)*(4*x^2 + 4*x + 1)*log(-(
4*sqrt(13)*sqrt(3*x^2 - x + 2)*(8*x - 9) + 220*x^2 - 196*x + 185)/(4*x^2 +
4*x + 1)) + 12*(27648*x^6 - 22464*x^5 + 83616*x^4 - 76200*x^3 + 256564*x^2
+ 464446*x + 142057)*sqrt(3*x^2 - x + 2))/(4*x^2 + 4*x + 1)
```

Sympy [F]

$$\int \frac{(2-x+3x^2)^{5/2}(1+3x+4x^2)}{(1+2x)^3}\,dx = \int \frac{(3x^2-x+2)^{\frac{5}{2}}\cdot(4x^2+3x+1)}{(2x+1)^3}\,dx$$

```
[In] integrate((3*x**2-x+2)**(5/2)*(4*x**2+3*x+1)/(1+2*x)**3,x)
```

```
[Out] Integral((3*x**2 - x + 2)**(5/2)*(4*x**2 + 3*x + 1)/(2*x + 1)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.07

$$\begin{aligned}\int \frac{(2-x+3x^2)^{5/2}(1+3x+4x^2)}{(1+2x)^3}\,dx &= \frac{67}{520}\left(3\,x^2-x+2\right)^{\frac{5}{2}}-\frac{\left(3\,x^2-x+2\right)^{\frac{7}{2}}}{26\,(4\,x^2+4\,x+1)} \\ &\quad-\frac{419}{416}\left(3\,x^2-x+2\right)^{\frac{3}{2}}x+\frac{1227}{832}\left(3\,x^2-x+2\right)^{\frac{3}{2}}+\frac{19\,(3\,x^2-x+2)^{\frac{5}{2}}}{52\,(2\,x+1)} \\ &\quad-\frac{1745}{256}\sqrt{3\,x^2-x+2}x-\frac{118423}{9216}\sqrt{3}\,\mathrm{arsinh}\left(\frac{6}{23}\sqrt{23}x-\frac{1}{23}\sqrt{23}\right) \\ &\quad+\frac{1631}{256}\sqrt{13}\,\mathrm{arsinh}\left(\frac{8\sqrt{23}x}{23\,|2\,x+1|}-\frac{9\sqrt{23}}{23\,|2\,x+1|}\right)+\frac{21317}{1536}\sqrt{3\,x^2-x+2}\end{aligned}$$

```
[In] integrate((3*x^2-x+2)^(5/2)*(4*x^2+3*x+1)/(1+2*x)^3,x, algorithm="maxima")
```

```
[Out] 67/520*(3*x^2 - x + 2)^(5/2) - 1/26*(3*x^2 - x + 2)^(7/2)/(4*x^2 + 4*x + 1)
 - 419/416*(3*x^2 - x + 2)^(3/2)*x + 1227/832*(3*x^2 - x + 2)^(3/2) + 19/52
*(3*x^2 - x + 2)^(5/2)/(2*x + 1) - 1745/256*sqrt(3*x^2 - x + 2)*x - 118423/
9216*sqrt(3)*arcsinh(6/23*sqrt(23)*x - 1/23*sqrt(23)) + 1631/256*sqrt(13)*a
rcsinh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqrt(23)/abs(2*x + 1)) + 21317/1
536*sqrt(3*x^2 - x + 2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 271 vs. $2(130) = 260$.

Time = 0.31 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.68

$$\int \frac{(2-x+3x^2)^{5/2}\,(1+3x+4x^2)}{(1+2x)^3}\,dx = \frac{1}{7680}\,(6\,(4\,(18\,(16\,x-29)x+1321)x-7937)x+103837)\sqrt{3\,x^2-}$$
$$+\frac{118423}{9216}\,\sqrt{3}\log\left(-2\,\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2-x+2}\right)+1\right)$$
$$+\frac{1631}{256}\,\sqrt{13}\log\left(-\frac{\left|-4\,\sqrt{3}x-2\,\sqrt{13}-2\,\sqrt{3}+4\,\sqrt{3\,x^2-x+2}\right|}{2\,\left(2\,\sqrt{3}x-\sqrt{13}+\sqrt{3}-2\,\sqrt{3\,x^2-x+2}\right)}\right)$$
$$+\frac{13\left(574\,\left(\sqrt{3}x-\sqrt{3\,x^2-x+2}\right)^3-101\,\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2-x+2}\right)^2-2745\,\sqrt{3}x+1369\,\sqrt{3}+2745\,\sqrt{3\,x^2}\right.}{128\left(2\,\left(\sqrt{3}x-\sqrt{3\,x^2-x+2}\right)^2+2\,\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2-x+2}\right)-5\right)^2}$$

```
[In] integrate((3*x^2-x+2)^(5/2)*(4*x^2+3*x+1)/(1+2*x)^3,x, algorithm="giac")
```

```
[Out] 1/7680*(6*(4*(18*(16*x - 29)*x + 1321)*x - 7937)*x + 103837)*sqrt(3*x^2 - x
 + 2) + 118423/9216*sqrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)
) + 1) + 1631/256*sqrt(13)*log(-1/2*abs(-4*sqrt(3)*x - 2*sqrt(13) - 2*sqrt(
3) + 4*sqrt(3*x^2 - x + 2))/(2*sqrt(3)*x - sqrt(13) + sqrt(3) - 2*sqrt(3*x^
2 - x + 2))) + 13/128*(574*(sqrt(3)*x - sqrt(3*x^2 - x + 2))^3 - 101*sqrt(3
)*(sqrt(3)*x - sqrt(3*x^2 - x + 2))^2 - 2745*sqrt(3)*x + 1369*sqrt(3) + 274
5*sqrt(3*x^2 - x + 2))/(2*(sqrt(3)*x - sqrt(3*x^2 - x + 2))^2 + 2*sqrt(3)*(
sqrt(3)*x - sqrt(3*x^2 - x + 2)) - 5)^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2-x+3x^2)^{5/2}\,(1+3x+4x^2)}{(1+2x)^3}\,dx = \int \frac{{\left(3\,x^2-x+2\right)}^{5/2}\,\left(4\,x^2+3\,x+1\right)}{{\left(2\,x+1\right)}^3}\,dx$$

```
[In] int(((3*x^2 - x + 2)^(5/2)*(3*x + 4*x^2 + 1))/(2*x + 1)^3,x)
```

```
[Out] int(((3*x^2 - x + 2)^(5/2)*(3*x + 4*x^2 + 1))/(2*x + 1)^3, x)
```

3.226 $\int \frac{(g+hx)^3(d+ex+fx^2)}{\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 693

$$\begin{aligned}
&\int \frac{(g+hx)^3\,(d+ex+fx^2)}{\sqrt{a+bx+cx^2}}\,dx \\
&= \frac{(63b^2fh^2-2ch(24bfg+35beh+32afh)-c^2(12fg^2-20h(3eg+4dh)))\,(g+hx)^2\sqrt{a+bx+cx^2}}{240c^3h} \\
&\quad -\frac{(9bfh+2c(fg-5eh))(g+hx)^3\sqrt{a+bx+cx^2}}{40c^2h}+\frac{f(g+hx)^4\sqrt{a+bx+cx^2}}{5ch} \\
&\quad +\frac{(945b^4fh^4-64c^4g^2(3fg^2-5h(3eg+16dh))-210b^2ch^3(14afh+5b(3fg+eh))+8c^2h^2(128a^2fh^2+2}{} \\
&\quad +\frac{(256c^5dg^3-63b^5fh^3+70b^3ch^2(3bfg+beh+4afh)-80bc^2h(3a^2fh^2+3abh(3fg+eh)+b^2(3fg^2+3e}{}
\end{aligned}$$

```
[Out] 1/256*(256*c^5*d*g^3-63*b^5*f*h^3+70*b^3*c*h^2*(4*a*f*h+b*e*h+3*b*f*g)-80*b
*c^2*h*(3*a^2*f*h^2+3*a*b*h*(e*h+3*f*g)+b^2*(d*h^2+3*e*g*h+3*f*g^2))-128*c^
4*g*(b*g*(3*d*h+e*g)+a*(f*g^2+3*h*(d*h+e*g)))+96*c^3*(a^2*h^2*(e*h+3*f*g)+b
^2*g*(f*g^2+3*h*(d*h+e*g))+2*a*b*h*(3*f*g^2+h*(d*h+3*e*g))))*arctanh(1/2*(2
*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(11/2)+1/240*(63*b^2*f*h^2-2*c*h*(32
*a*f*h+35*b*e*h+24*b*f*g)-c^2*(12*f*g^2-20*h*(4*d*h+3*e*g)))*(h*x+g)^2*(c*x
^2+b*x+a)^(1/2)/c^3/h-1/40*(9*b*f*h+2*c*(-5*e*h+f*g))*(h*x+g)^3*(c*x^2+b*x+
a)^(1/2)/c^2/h+1/5*f*(h*x+g)^4*(c*x^2+b*x+a)^(1/2)/c/h+1/1920*(945*b^4*f*h^
4-64*c^4*g^2*(3*f*g^2-5*h*(16*d*h+3*e*g))-210*b^2*c*h^3*(14*a*f*h+5*b*(e*h+
3*f*g))+8*c^2*h^2*(128*a^2*f*h^2+275*a*b*h*(e*h+3*f*g)+3*b^2*(129*f*g^2+50*
h*(d*h+3*e*g)))-16*c^3*h*(16*a*h*(13*f*g^2+5*h*(d*h+3*e*g))+b*g*(39*f*g^2+5
*h*(54*d*h+47*e*g)))-2*c*h*(315*b^3*f*h^3-14*b*c*h^2*(46*a*f*h+25*b*e*h+39*
b*f*g)+16*c^3*g*(3*f*g^2-5*h*(10*d*h+3*e*g))+8*c^2*h*(a*h*(45*e*h+71*f*g)+b
*(50*d*h^2+80*e*g*h+21*f*g^2)))*x)*(c*x^2+b*x+a)^(1/2)/c^5/h
```

Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 692, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {1667, 846, 793, 635, 212}

$$\int \frac{(g+hx)^3\left(d+ex+fx^2\right)}{\sqrt{a+bx+cx^2}}\,dx$$

$$= \frac{\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(96c^3\left(a^2h^2(eh+3fg)+2abh\left(h(dh+3eg)+3fg^2\right)+b^2g\left(3h(dh+eg)+fg^2\right)\right)-\right.}{}$$

$$+\frac{\sqrt{a+bx+cx^2}\left(8c^2h^2\left(128a^2fh^2+275abh(eh+3fg)+3b^2\left(50h(dh+3eg)+129fg^2\right)\right)-2chx\left(8c^2h(ah\right.\right.}{}$$

$$+\frac{(g+hx)^2\sqrt{a+bx+cx^2}\left(-2ch(32afh+35beh+24bfg)+63b^2fh^2-\left(c^2\left(12fg^2-20h(4dh+3eg)\right)\right)\right)}{240c^3h}$$

$$-\frac{(g+hx)^3\sqrt{a+bx+cx^2}(9bfh+2c(fg-5eh))}{40c^2h}+\frac{f(g+hx)^4\sqrt{a+bx+cx^2}}{5ch}$$

```
[In] Int[((g + h*x)^3*(d + e*x + f*x^2))/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] ((63*b^2*f*h^2 - 2*c*h*(24*b*f*g + 35*b*e*h + 32*a*f*h) - c^2*(12*f*g^2 - 2
0*h*(3*e*g + 4*d*h)))*(g + h*x)^2*Sqrt[a + b*x + c*x^2])/(240*c^3*h) - ((9*
b*f*h + 2*c*(f*g - 5*e*h))*(g + h*x)^3*Sqrt[a + b*x + c*x^2])/(40*c^2*h) +
(f*(g + h*x)^4*Sqrt[a + b*x + c*x^2])/(5*c*h) + ((945*b^4*f*h^4 - 64*c^4*(3
*f*g^4 - 5*g^2*h*(3*e*g + 16*d*h)) - 210*b^2*c*h^3*(14*a*f*h + 5*b*(3*f*g +
 e*h)) + 8*c^2*h^2*(128*a^2*f*h^2 + 275*a*b*h*(3*f*g + e*h) + 3*b^2*(129*f*
g^2 + 50*h*(3*e*g + d*h))) - 16*c^3*h*(16*a*h*(13*f*g^2 + 5*h*(3*e*g + d*h)
) + b*g*(39*f*g^2 + 5*h*(47*e*g + 54*d*h))) - 2*c*h*(315*b^3*f*h^3 - 14*b*c
*h^2*(39*b*f*g + 25*b*e*h + 46*a*f*h) + 16*c^3*(3*f*g^3 - 5*g*h*(3*e*g + 10
*d*h)) + 8*c^2*h*(21*b*f*g^2 + 10*b*h*(8*e*g + 5*d*h) + a*h*(71*f*g + 45*e*
h)))*x)*Sqrt[a + b*x + c*x^2])/(1920*c^5*h) + ((256*c^5*d*g^3 - 63*b^5*f*h^
3 + 70*b^3*c*h^2*(3*b*f*g + b*e*h + 4*a*f*h) - 128*c^4*g*(a*f*g^2 + 3*a*h*(
e*g + d*h) + b*g*(e*g + 3*d*h)) - 80*b*c^2*h*(3*a^2*f*h^2 + 3*a*b*h*(3*f*g
+ e*h) + b^2*(3*f*g^2 + 3*e*g*h + d*h^2)) + 96*c^3*(a^2*h^2*(3*f*g + e*h) +
 b^2*g*(f*g^2 + 3*h*(e*g + d*h)) + 2*a*b*h*(3*f*g^2 + h*(3*e*g + d*h))))*Ar
cTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(256*c^(11/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
```

```
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 846

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\text{integral} = \frac{f(g+hx)^4\sqrt{a+bx+cx^2}}{5ch} + \frac{\int \frac{(g+hx)^3\left(-\frac{1}{2}h(bfg-10cdh+8afh)-\frac{1}{2}h(2cfg-10ceh+9bfh)x\right)}{\sqrt{a+bx+cx^2}}\,dx}{5ch^2}$$

$$= -\frac{(9bfh+2c(fg-5eh))(g+hx)^3\sqrt{a+bx+cx^2}}{40c^2h} + \frac{f(g+hx)^4\sqrt{a+bx+cx^2}}{5ch}$$

$$+\frac{\int \frac{(g+hx)^2\left(\frac{1}{4}h\left(9b^2fgh+54abfh^2-2bcg(3fg+5eh)+4ch(20cdg-13afg-15aeh)\right)+\frac{1}{4}h\left(63b^2fh^2-2ch(24bfg+35beh+32afh)-c^2(12\right.\right.}{\sqrt{a+bx+cx^2}}}{20c^2h^2}$$

$$= \frac{\left(63b^2fh^2-2ch(24bfg+35beh+32afh)-c^2\left(12fg^2-20h(3eg+4dh)\right)\right)(g+hx)^2\sqrt{a+bx+cx}}{240c^3h}$$

$$-\frac{(9bfh+2c(fg-5eh))(g+hx)^3\sqrt{a+bx+cx^2}}{40c^2h}+\frac{f(g+hx)^4\sqrt{a+bx+cx^2}}{5ch}$$

$$+\frac{\int\frac{(g+hx)\left(-\frac{1}{8}h\left(63b^3fgh^2+4bc\left(6cfg^3+10cgh(3eg+2dh)-5ah^2(29fg+14eh)\right)+2b^2\left(126afh^3-cgh(51fg+35eh)\right)-8ch\left(60c^2dg^2+3\right.\right.\right.}{}}{}$$

$$= \frac{\left(63b^2fh^2-2ch(24bfg+35beh+32afh)-c^2\left(12fg^2-20h(3eg+4dh)\right)\right)(g+hx)^2\sqrt{a+bx+cx}}{240c^3h}$$

$$-\frac{(9bfh+2c(fg-5eh))(g+hx)^3\sqrt{a+bx+cx^2}}{40c^2h}+\frac{f(g+hx)^4\sqrt{a+bx+cx^2}}{5ch}$$

$$+\frac{(945b^4fh^4-64c^4(3fg^4-5g^2h(3eg+16dh))-210b^2ch^3(14afh+5b(3fg+eh))+8c^2h^2(128a}{}$$

$$+\frac{(256c^5dg^3-63b^5fh^3+70b^3ch^2(3bfg+beh+4afh)-128c^4g(afg^2+3ah(eg+dh)+bg(eg+}{}$$

$$= \frac{\left(63b^2fh^2-2ch(24bfg+35beh+32afh)-c^2\left(12fg^2-20h(3eg+4dh)\right)\right)(g+hx)^2\sqrt{a+bx+cx}}{240c^3h}$$

$$-\frac{(9bfh+2c(fg-5eh))(g+hx)^3\sqrt{a+bx+cx^2}}{40c^2h}+\frac{f(g+hx)^4\sqrt{a+bx+cx^2}}{5ch}$$

$$+\frac{(945b^4fh^4-64c^4(3fg^4-5g^2h(3eg+16dh))-210b^2ch^3(14afh+5b(3fg+eh))+8c^2h^2(128a}{}$$

$$+\frac{(256c^5dg^3-63b^5fh^3+70b^3ch^2(3bfg+beh+4afh)-128c^4g(afg^2+3ah(eg+dh)+bg(eg+}{}$$

$$= \frac{\left(63b^2fh^2-2ch(24bfg+35beh+32afh)-c^2\left(12fg^2-20h(3eg+4dh)\right)\right)(g+hx)^2\sqrt{a+bx+cx}}{240c^3h}$$

$$-\frac{(9bfh+2c(fg-5eh))(g+hx)^3\sqrt{a+bx+cx^2}}{40c^2h}+\frac{f(g+hx)^4\sqrt{a+bx+cx^2}}{5ch}$$

$$+\frac{(945b^4fh^4-64c^4(3fg^4-5g^2h(3eg+16dh))-210b^2ch^3(14afh+5b(3fg+eh))+8c^2h^2(128a}{}$$

$$+\frac{(256c^5dg^3-63b^5fh^3+70b^3ch^2(3bfg+beh+4afh)-128c^4g(afg^2+3ah(eg+dh)+bg(eg+}{}$$

Mathematica [A] (verified)

Time = 2.67 (sec) , antiderivative size = 588, normalized size of antiderivative = 0.85

$$\int \frac{(g+hx)^3\,(d+ex+fx^2)}{\sqrt{a+bx+cx^2}}\,dx$$
$$= \frac{2\sqrt{c}\sqrt{a+x(b+cx)}(945b^4fh^3-210b^2ch^2(5beh+14afh+3bf(5g+hx))+32c^4(10dh(18g^2+9ghx+2h}{}$$

```
[In] Integrate[((g + h*x)^3*(d + e*x + f*x^2))/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(945*b^4*f*h^3 - 210*b^2*c*h^2*(5*b*e*h +
14*a*f*h + 3*b*f*(5*g + h*x)) + 32*c^4*(10*d*h*(18*g^2 + 9*g*h*x + 2*h^2*x^
2) + 15*e*(4*g^3 + 6*g^2*h*x + 4*g*h^2*x^2 + h^3*x^3) + 3*f*x*(10*g^3 + 20*
g^2*h*x + 15*g*h^2*x^2 + 4*h^3*x^3)) + 4*c^2*h*(256*a^2*f*h^2 + 2*a*b*h*(82
5*f*g + 275*e*h + 161*f*h*x) + b^2*(25*h*(36*e*g + 12*d*h + 7*e*h*x) + 3*f*
(300*g^2 + 175*g*h*x + 42*h^2*x^2))) - 16*c^3*(a*h*(5*h*(48*e*g + 16*d*h +
9*e*h*x) + f*(240*g^2 + 135*g*h*x + 32*h^2*x^2)) + b*(3*f*(30*g^3 + 50*g^2*
h*x + 35*g*h^2*x^2 + 9*h^3*x^3) + 5*h*(2*d*h*(27*g + 5*h*x) + e*(54*g^2 + 3
0*g*h*x + 7*h^2*x^2))))) + 15*(-256*c^5*d*g^3 + 63*b^5*f*h^3 - 70*b^3*c*h^2
*(3*b*f*g + b*e*h + 4*a*f*h) + 128*c^4*g*(a*f*g^2 + 3*a*h*(e*g + d*h) + b*g
*(e*g + 3*d*h)) + 80*b*c^2*h*(3*a^2*f*h^2 + 3*a*b*h*(3*f*g + e*h) + b^2*(3*
f*g^2 + 3*e*g*h + d*h^2)) - 96*c^3*(a^2*h^2*(3*f*g + e*h) + b^2*g*(f*g^2 +
3*h*(e*g + d*h)) + 2*a*b*h*(3*f*g^2 + h*(3*e*g + d*h))))*Log[b + 2*c*x - 2*
Sqrt[c]*Sqrt[a + x*(b + c*x)]])/(3840*c^(11/2))
```

Maple [A] (verified)

Time = 1.23 (sec) , antiderivative size = 810, normalized size of antiderivative = 1.17

method	result
risch	$\frac{(384f\,h^3c^4x^4-432b\,c^3f\,h^3x^3+480c^4e\,h^3x^3+1440c^4fg\,h^2x^3-512a\,c^3f\,h^3x^2+504b^2c^2f\,h^3x^2-560b\,c^3e\,h^3x^2-1680b\,c^3fg\,h^2x^2+640c^4}{}$
default	Expression too large to display

```
[In] int((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/1920*(384*c^4*f*h^3*x^4-432*b*c^3*f*h^3*x^3+480*c^4*e*h^3*x^3+1440*c^4*f*
g*h^2*x^3-512*a*c^3*f*h^3*x^2+504*b^2*c^2*f*h^3*x^2-560*b*c^3*e*h^3*x^2-168
0*b*c^3*f*g*h^2*x^2+640*c^4*d*h^3*x^2+1920*c^4*e*g*h^2*x^2+1920*c^4*f*g^2*h
*x^2+1288*a*b*c^2*f*h^3*x-720*a*c^3*e*h^3*x-2160*a*c^3*f*g*h^2*x-630*b^3*c*
f*h^3*x+700*b^2*c^2*e*h^3*x+2100*b^2*c^2*f*g*h^2*x-800*b*c^3*d*h^3*x-2400*b
*c^3*e*g*h^2*x-2400*b*c^3*f*g^2*h*x+2880*c^4*d*g*h^2*x+2880*c^4*e*g^2*h*x+9
60*c^4*f*g^3*x+1024*a^2*c^2*f*h^3-2940*a*b^2*c*f*h^3+2200*a*b*c^2*e*h^3+660
```

```
0*a*b*c^2*f*g*h^2-1280*a*c^3*d*h^3-3840*a*c^3*e*g*h^2-3840*a*c^3*f*g^2*h+94
5*b^4*f*h^3-1050*b^3*c*e*h^3-3150*b^3*c*f*g*h^2+1200*b^2*c^2*d*h^3+3600*b^2
*c^2*e*g*h^2+3600*b^2*c^2*f*g^2*h-4320*b*c^3*d*g*h^2-4320*b*c^3*e*g^2*h-144
0*b*c^3*f*g^3+5760*c^4*d*g^2*h+1920*c^4*e*g^3)*(c*x^2+b*x+a)^(1/2)/c^5-1/25
6*(240*a^2*b*c^2*f*h^3-96*a^2*c^3*e*h^3-288*a^2*c^3*f*g*h^2-280*a*b^3*c*f*h
^3+240*a*b^2*c^2*e*h^3+720*a*b^2*c^2*f*g*h^2-192*a*b*c^3*d*h^3-576*a*b*c^3*
e*g*h^2-576*a*b*c^3*f*g^2*h+384*a*c^4*d*g*h^2+384*a*c^4*e*g^2*h+128*a*c^4*f
*g^3+63*b^5*f*h^3-70*b^4*c*e*h^3-210*b^4*c*f*g*h^2+80*b^3*c^2*d*h^3+240*b^3
*c^2*e*g*h^2+240*b^3*c^2*f*g^2*h-288*b^2*c^3*d*g*h^2-288*b^2*c^3*e*g^2*h-96
*b^2*c^3*f*g^3+384*b*c^4*d*g^2*h+128*b*c^4*e*g^3-256*c^5*d*g^3)/c^(11/2)*ln
((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.49 (sec) , antiderivative size = 1435, normalized size of antiderivative = 2.07

$$\int \frac{(g+hx)^3 (d+ex+fx^2)}{\sqrt{a+bx+cx^2}}\, dx = \text{Too large to display}$$

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [-1/7680*(15*(32*(8*c^5*d - 4*b*c^4*e + (3*b^2*c^3 - 4*a*c^4)*f)*g^3 - 48*(
8*b*c^4*d - 2*(3*b^2*c^3 - 4*a*c^4)*e + (5*b^3*c^2 - 12*a*b*c^3)*f)*g^2*h +
 6*(16*(3*b^2*c^3 - 4*a*c^4)*d - 8*(5*b^3*c^2 - 12*a*b*c^3)*e + (35*b^4*c -
 120*a*b^2*c^2 + 48*a^2*c^3)*f)*g*h^2 - (16*(5*b^3*c^2 - 12*a*b*c^3)*d - 2*
(35*b^4*c - 120*a*b^2*c^2 + 48*a^2*c^3)*e + (63*b^5 - 280*a*b^3*c + 240*a^2
*b*c^2)*f)*h^3)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x
 + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(384*c^5*f*h^3*x^4 + 480*(4*c^5*e -
3*b*c^4*f)*g^3 + 240*(24*c^5*d - 18*b*c^4*e + (15*b^2*c^3 - 16*a*c^4)*f)*g^
2*h - 30*(144*b*c^4*d - 8*(15*b^2*c^3 - 16*a*c^4)*e + 5*(21*b^3*c^2 - 44*a*
b*c^3)*f)*g*h^2 + (80*(15*b^2*c^3 - 16*a*c^4)*d - 50*(21*b^3*c^2 - 44*a*b*c
^3)*e + (945*b^4*c - 2940*a*b^2*c^2 + 1024*a^2*c^3)*f)*h^3 + 48*(30*c^5*f*g
*h^2 + (10*c^5*e - 9*b*c^4*f)*h^3)*x^3 + 8*(240*c^5*f*g^2*h + 30*(8*c^5*e -
 7*b*c^4*f)*g*h^2 + (80*c^5*d - 70*b*c^4*e + (63*b^2*c^3 - 64*a*c^4)*f)*h^3
)*x^2 + 2*(480*c^5*f*g^3 + 240*(6*c^5*e - 5*b*c^4*f)*g^2*h + 30*(48*c^5*d -
 40*b*c^4*e + (35*b^2*c^3 - 36*a*c^4)*f)*g*h^2 - (400*b*c^4*d - 10*(35*b^2*
c^3 - 36*a*c^4)*e + 7*(45*b^3*c^2 - 92*a*b*c^3)*f)*h^3)*x)*sqrt(c*x^2 + b*x
 + a))/c^6, -1/3840*(15*(32*(8*c^5*d - 4*b*c^4*e + (3*b^2*c^3 - 4*a*c^4)*f)
*g^3 - 48*(8*b*c^4*d - 2*(3*b^2*c^3 - 4*a*c^4)*e + (5*b^3*c^2 - 12*a*b*c^3)
*f)*g^2*h + 6*(16*(3*b^2*c^3 - 4*a*c^4)*d - 8*(5*b^3*c^2 - 12*a*b*c^3)*e +
(35*b^4*c - 120*a*b^2*c^2 + 48*a^2*c^3)*f)*g*h^2 - (16*(5*b^3*c^2 - 12*a*b*
c^3)*d - 2*(35*b^4*c - 120*a*b^2*c^2 + 48*a^2*c^3)*e + (63*b^5 - 280*a*b^3*
c + 240*a^2*b*c^2)*f)*h^3)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x
```

```
 + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(384*c^5*f*h^3*x^4 + 480*(4*c^5
*e - 3*b*c^4*f)*g^3 + 240*(24*c^5*d - 18*b*c^4*e + (15*b^2*c^3 - 16*a*c^4)*
f)*g^2*h - 30*(144*b*c^4*d - 8*(15*b^2*c^3 - 16*a*c^4)*e + 5*(21*b^3*c^2 -
44*a*b*c^3)*f)*g*h^2 + (80*(15*b^2*c^3 - 16*a*c^4)*d - 50*(21*b^3*c^2 - 44*
a*b*c^3)*e + (945*b^4*c - 2940*a*b^2*c^2 + 1024*a^2*c^3)*f)*h^3 + 48*(30*c^
5*f*g*h^2 + (10*c^5*e - 9*b*c^4*f)*h^3)*x^3 + 8*(240*c^5*f*g^2*h + 30*(8*c^
5*e - 7*b*c^4*f)*g*h^2 + (80*c^5*d - 70*b*c^4*e + (63*b^2*c^3 - 64*a*c^4)*f
)*h^3)*x^2 + 2*(480*c^5*f*g^3 + 240*(6*c^5*e - 5*b*c^4*f)*g^2*h + 30*(48*c^
5*d - 40*b*c^4*e + (35*b^2*c^3 - 36*a*c^4)*f)*g*h^2 - (400*b*c^4*d - 10*(35
*b^2*c^3 - 36*a*c^4)*e + 7*(45*b^3*c^2 - 92*a*b*c^3)*f)*h^3)*x)*sqrt(c*x^2
+ b*x + a))/c^6]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1613 vs. $2(721) = 1442$.

Time = 1.28 (sec) , antiderivative size = 1613, normalized size of antiderivative = 2.33

$$\int \frac{(g+hx)^3\,(d+ex+fx^2)}{\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

```
[In] integrate((h*x+g)**3*(f*x**2+e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Piecewise((sqrt(a + b*x + c*x**2)*(f*h**3*x**4/(5*c) + x**3*(-9*b*f*h**3/(1
0*c) + e*h**3 + 3*f*g*h**2)/(4*c) + x**2*(-4*a*f*h**3/(5*c) - 7*b*(-9*b*f*h
**3/(10*c) + e*h**3 + 3*f*g*h**2)/(8*c) + d*h**3 + 3*e*g*h**2 + 3*f*g**2*h)
/(3*c) + x*(-3*a*(-9*b*f*h**3/(10*c) + e*h**3 + 3*f*g*h**2)/(4*c) - 5*b*(-4
*a*f*h**3/(5*c) - 7*b*(-9*b*f*h**3/(10*c) + e*h**3 + 3*f*g*h**2)/(8*c) + d*
h**3 + 3*e*g*h**2 + 3*f*g**2*h)/(6*c) + 3*d*g*h**2 + 3*e*g**2*h + f*g**3)/(
2*c) + (-2*a*(-4*a*f*h**3/(5*c) - 7*b*(-9*b*f*h**3/(10*c) + e*h**3 + 3*f*g*
h**2)/(8*c) + d*h**3 + 3*e*g*h**2 + 3*f*g**2*h)/(3*c) - 3*b*(-3*a*(-9*b*f*h
**3/(10*c) + e*h**3 + 3*f*g*h**2)/(4*c) - 5*b*(-4*a*f*h**3/(5*c) - 7*b*(-9*
b*f*h**3/(10*c) + e*h**3 + 3*f*g*h**2)/(8*c) + d*h**3 + 3*e*g*h**2 + 3*f*g*
*2*h)/(6*c) + 3*d*g*h**2 + 3*e*g**2*h + f*g**3)/(4*c) + 3*d*g**2*h + e*g**3
)/c) + (-a*(-3*a*(-9*b*f*h**3/(10*c) + e*h**3 + 3*f*g*h**2)/(4*c) - 5*b*(-4
*a*f*h**3/(5*c) - 7*b*(-9*b*f*h**3/(10*c) + e*h**3 + 3*f*g*h**2)/(8*c) + d*
h**3 + 3*e*g*h**2 + 3*f*g**2*h)/(6*c) + 3*d*g*h**2 + 3*e*g**2*h + f*g**3)/(
2*c) - b*(-2*a*(-4*a*f*h**3/(5*c) - 7*b*(-9*b*f*h**3/(10*c) + e*h**3 + 3*f*
g*h**2)/(8*c) + d*h**3 + 3*e*g*h**2 + 3*f*g**2*h)/(3*c) - 3*b*(-3*a*(-9*b*f
*h**3/(10*c) + e*h**3 + 3*f*g*h**2)/(4*c) - 5*b*(-4*a*f*h**3/(5*c) - 7*b*(-
9*b*f*h**3/(10*c) + e*h**3 + 3*f*g*h**2)/(8*c) + d*h**3 + 3*e*g*h**2 + 3*f*
g**2*h)/(6*c) + 3*d*g*h**2 + 3*e*g**2*h + f*g**3)/(4*c) + 3*d*g**2*h + e*g*
*3)/(2*c) + d*g**3)*Piecewise((log(b + 2*sqrt(c)*sqrt(a + b*x + c*x**2) + 2
*c*x)/sqrt(c), Ne(a - b**2/(4*c), 0)), ((b/(2*c) + x)*log(b/(2*c) + x)/sqrt
(c*(b/(2*c) + x)**2), True)), Ne(c, 0)), (2*(f*h**3*(a + b*x)**(11/2)/(11*b
**5) + (a + b*x)**(9/2)*(-5*a*f*h**3 + b*e*h**3 + 3*b*f*g*h**2)/(9*b**5) +
```

```
(a + b*x)**(7/2)*(10*a**2*f*h**3 - 4*a*b*e*h**3 - 12*a*b*f*g*h**2 + b**2*d*
h**3 + 3*b**2*e*g*h**2 + 3*b**2*f*g**2*h)/(7*b**5) + (a + b*x)**(5/2)*(-10*
a**3*f*h**3 + 6*a**2*b*e*h**3 + 18*a**2*b*f*g*h**2 - 3*a*b**2*d*h**3 - 9*a*
b**2*e*g*h**2 - 9*a*b**2*f*g**2*h + 3*b**3*d*g*h**2 + 3*b**3*e*g**2*h + b**
3*f*g**3)/(5*b**5) + (a + b*x)**(3/2)*(5*a**4*f*h**3 - 4*a**3*b*e*h**3 - 12
*a**3*b*f*g*h**2 + 3*a**2*b**2*d*h**3 + 9*a**2*b**2*e*g*h**2 + 9*a**2*b**2*
f*g**2*h - 6*a*b**3*d*g*h**2 - 6*a*b**3*e*g**2*h - 2*a*b**3*f*g**3 + 3*b**4
*d*g**2*h + b**4*e*g**3)/(3*b**5) + sqrt(a + b*x)*(-a**5*f*h**3 + a**4*b*e*
h**3 + 3*a**4*b*f*g*h**2 - a**3*b**2*d*h**3 - 3*a**3*b**2*e*g*h**2 - 3*a**3
*b**2*f*g**2*h + 3*a**2*b**3*d*g*h**2 + 3*a**2*b**3*e*g**2*h + a**2*b**3*f*
g**3 - 3*a*b**4*d*g**2*h - a*b**4*e*g**3 + b**5*d*g**3)/b**5)/b, Ne(b, 0)),
 ((d*g**3*x + f*h**3*x**6/6 + x**5*(e*h**3 + 3*f*g*h**2)/5 + x**4*(d*h**3 +
 3*e*g*h**2 + 3*f*g**2*h)/4 + x**3*(3*d*g*h**2 + 3*e*g**2*h + f*g**3)/3 + x
**2*(3*d*g**2*h + e*g**3)/2)/sqrt(a), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(g+hx)^3\left(d+ex+fx^2\right)}{\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 798, normalized size of antiderivative = 1.15

$$\int \frac{(g+hx)^3\left(d+ex+fx^2\right)}{\sqrt{a+bx+cx^2}}\,dx$$

$$= \frac{1}{1920}\sqrt{cx^2+bx+a}\,(2\,(4\,(6\,(\frac{8\,fh^3x}{c}+\frac{30\,c^4fgh^2+10\,c^4eh^3-9\,bc^3fh^3}{c^5})\,x+ 240\,c^4fg^2h+240\,c^4egh^2$$

$$-\,(256\,c^5dg^3-128\,bc^4eg^3+96\,b^2c^3fg^3-128\,ac^4fg^3-384\,bc^4dg^2h+288\,b^2c^3eg^2h-384\,ac^4eg^2h-240\,b$$

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/1920*sqrt(c*x^2 + b*x + a)*(2*(4*(6*(8*f*h^3*x/c + (30*c^4*f*g*h^2 + 10*c
^4*e*h^3 - 9*b*c^3*f*h^3)/c^5)*x + (240*c^4*f*g^2*h + 240*c^4*e*g*h^2 - 210
*b*c^3*f*g*h^2 + 80*c^4*d*h^3 - 70*b*c^3*e*h^3 + 63*b^2*c^2*f*h^3 - 64*a*c^
3*f*h^3)/c^5)*x + (480*c^4*f*g^3 + 1440*c^4*e*g^2*h - 1200*b*c^3*f*g^2*h +
1440*c^4*d*g*h^2 - 1200*b*c^3*e*g*h^2 + 1050*b^2*c^2*f*g*h^2 - 1080*a*c^3*f
*g*h^2 - 400*b*c^3*d*h^3 + 350*b^2*c^2*e*h^3 - 360*a*c^3*e*h^3 - 315*b^3*c*
f*h^3 + 644*a*b*c^2*f*h^3)/c^5)*x + (1920*c^4*e*g^3 - 1440*b*c^3*f*g^3 + 57
60*c^4*d*g^2*h - 4320*b*c^3*e*g^2*h + 3600*b^2*c^2*f*g^2*h - 3840*a*c^3*f*g
^2*h - 4320*b*c^3*d*g*h^2 + 3600*b^2*c^2*e*g*h^2 - 3840*a*c^3*e*g*h^2 - 315
0*b^3*c*f*g*h^2 + 6600*a*b*c^2*f*g*h^2 + 1200*b^2*c^2*d*h^3 - 1280*a*c^3*d*
h^3 - 1050*b^3*c*e*h^3 + 2200*a*b*c^2*e*h^3 + 945*b^4*f*h^3 - 2940*a*b^2*c*
f*h^3 + 1024*a^2*c^2*f*h^3)/c^5) - 1/256*(256*c^5*d*g^3 - 128*b*c^4*e*g^3 +
 96*b^2*c^3*f*g^3 - 128*a*c^4*f*g^3 - 384*b*c^4*d*g^2*h + 288*b^2*c^3*e*g^2
*h - 384*a*c^4*e*g^2*h - 240*b^3*c^2*f*g^2*h + 576*a*b*c^3*f*g^2*h + 288*b^
2*c^3*d*g*h^2 - 384*a*c^4*d*g*h^2 - 240*b^3*c^2*e*g*h^2 + 576*a*b*c^3*e*g*h
^2 + 210*b^4*c*f*g*h^2 - 720*a*b^2*c^2*f*g*h^2 + 288*a^2*c^3*f*g*h^2 - 80*b
^3*c^2*d*h^3 + 192*a*b*c^3*d*h^3 + 70*b^4*c*e*h^3 - 240*a*b^2*c^2*e*h^3 + 9
6*a^2*c^3*e*h^3 - 63*b^5*f*h^3 + 280*a*b^3*c*f*h^3 - 240*a^2*b*c^2*f*h^3)*l
og(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(11/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(g+hx)^3\,(d+ex+fx^2)}{\sqrt{a+bx+cx^2}}\,dx=\int \frac{(g+h\,x)^3\,(f\,x^2+e\,x+d)}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

```
[In] int(((g + h*x)^3*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(1/2),x)
[Out] int(((g + h*x)^3*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(1/2), x)
```

3.227 $\int \frac{(g+hx)^2(d+ex+fx^2)}{\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 420

$$\int \frac{(g+hx)^2\left(d+ex+fx^2\right)}{\sqrt{a+bx+cx^2}}\,dx$$

$$= -\frac{(2cfg-8ceh+7bfh)(g+hx)^2\sqrt{a+bx+cx^2}}{24c^2h} + \frac{f(g+hx)^3\sqrt{a+bx+cx^2}}{4ch}$$

$$-\frac{(105b^3fh^3+32c^3g(fg^2-4h(eg+3dh))-20bch^2(11afh+6b(2fg+eh))+8c^2h(16ah(2fg+eh)+b}{}$$

$$+\frac{(128c^4dg^2+35b^4fh^2-40b^2ch(2bfg+beh+3afh)-64c^3(bg(eg+2dh)+a(fg^2+2egh+dh^2))+48}{128c^{9/2}}$$

```
[Out] 1/128*(128*c^4*d*g^2+35*b^4*f*h^2-40*b^2*c*h*(3*a*f*h+b*e*h+2*b*f*g)-64*c^3
*(b*g*(2*d*h+e*g)+a*(d*h^2+2*e*g*h+f*g^2))+48*c^2*(a^2*f*h^2+2*a*b*h*(e*h+2
*f*g)+b^2*(d*h^2+2*e*g*h+f*g^2)))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+
a)^(1/2))/c^(9/2)-1/24*(7*b*f*h-8*c*e*h+2*c*f*g)*(h*x+g)^2*(c*x^2+b*x+a)^(1
/2)/c^2/h+1/4*f*(h*x+g)^3*(c*x^2+b*x+a)^(1/2)/c/h-1/192*(105*b^3*f*h^3+32*c
^3*g*(f*g^2-4*h*(3*d*h+e*g))-20*b*c*h^2*(11*a*f*h+6*b*(e*h+2*f*g))+8*c^2*h*
(16*a*h*(e*h+2*f*g)+b*(11*f*g^2+18*h*(d*h+2*e*g)))-2*c*h*(35*b^2*f*h^2-4*c*
h*(9*a*f*h+10*b*e*h+6*b*f*g)-8*c^2*(f*g^2-2*h*(3*d*h+2*e*g)))*x)*(c*x^2+b*x
+a)^(1/2)/c^4/h
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {1667, 846, 793, 635, 212}

$$\begin{aligned}&\int \frac{(g+hx)^2\left(d+ex+fx^2\right)}{\sqrt{a+bx+cx^2}}\,dx\\ =&\ \frac{\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(48c^2\left(a^2fh^2+2abh(eh+2fg)+b^2\left(h(dh+2eg)+fg^2\right)\right)-40b^2ch(3afh+beh+2\right.}{128c^{9/2}}\\ &-\frac{\sqrt{a+bx+cx^2}\left(-2chx\left(-4ch(9afh+10beh+6bfg)+35b^2fh^2-8c^2\left(fg^2-2h(3dh+2eg)\right)\right)+8c^2h(16\right.}{}\\ &-\frac{(g+hx)^2\sqrt{a+bx+cx^2}(7bfh-8ceh+2cfg)}{24c^2h}+\frac{f(g+hx)^3\sqrt{a+bx+cx^2}}{4ch}\end{aligned}$$

```
[In] Int[((g + h*x)^2*(d + e*x + f*x^2))/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] -1/24*((2*c*f*g - 8*c*e*h + 7*b*f*h)*(g + h*x)^2*Sqrt[a + b*x + c*x^2])/(c^
2*h) + (f*(g + h*x)^3*Sqrt[a + b*x + c*x^2])/(4*c*h) - ((105*b^3*f*h^3 + 32
*c^3*(f*g^3 - 4*g*h*(e*g + 3*d*h)) - 20*b*c*h^2*(11*a*f*h + 6*b*(2*f*g + e*
h)) + 8*c^2*h*(11*b*f*g^2 + 18*b*h*(2*e*g + d*h) + 16*a*h*(2*f*g + e*h)) -
2*c*h*(35*b^2*f*h^2 - 4*c*h*(6*b*f*g + 10*b*e*h + 9*a*f*h) - 8*c^2*(f*g^2 -
 2*h*(2*e*g + 3*d*h)))*x)*Sqrt[a + b*x + c*x^2])/(192*c^4*h) + ((128*c^4*d*
g^2 + 35*b^4*f*h^2 - 40*b^2*c*h*(2*b*f*g + b*e*h + 3*a*f*h) - 64*c^3*(a*f*g
^2 + a*h*(2*e*g + d*h) + b*g*(e*g + 2*d*h)) + 48*c^2*(a^2*f*h^2 + 2*a*b*h*(
2*f*g + e*h) + b^2*(f*g^2 + h*(2*e*g + d*h))))*ArcTanh[(b + 2*c*x)/(2*Sqrt[
c]*Sqrt[a + b*x + c*x^2])])/(128*c^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
```

```
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 846

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\text{integral} = \frac{f(g+hx)^3\sqrt{a+bx+cx^2}}{4ch} + \frac{\int \frac{(g+hx)^2\left(-\frac{1}{2}h(bfg-8cdh+6afh)-\frac{1}{2}h(2cfg-8ceh+7bfh)x\right)}{\sqrt{a+bx+cx^2}}\,dx}{4ch^2}$$

$$= -\frac{(2cfg-8ceh+7bfh)(g+hx)^2\sqrt{a+bx+cx^2}}{24c^2h} + \frac{f(g+hx)^3\sqrt{a+bx+cx^2}}{4ch} + \frac{\int \frac{(g+hx)(\frac{1}{4}h(7b^2fgh+28abfh^2-4bcg(fg+2eh)+4ch(12cdg-7afg-8aeh))+\frac{1}{4}h(35b^2fh^2-4ch(6bfg+10beh+9afh)-8c^2(fg^2-2}{\sqrt{a+bx+cx^2}}}{12c^2h^2}$$

$$= -\frac{(2cfg-8ceh+7bfh)(g+hx)^2\sqrt{a+bx+cx^2}}{24c^2h} + \frac{f(g+hx)^3\sqrt{a+bx+cx^2}}{4ch} - \frac{(105b^3fh^3+32c^3(fg^3-4gh(eg+3dh))-20bch^2(11afh+6b(2fg+eh))+8c^2h(11bfg^2+18}{} + \frac{(128c^4dg^2+35b^4fh^2-40b^2ch(2bfg+beh+3afh)-64c^3(afg^2+ah(2eg+dh)+bg(eg+2d}{128c^4}$$

$$= -\frac{(2cfg-8ceh+7bfh)(g+hx)^2\sqrt{a+bx+cx^2}}{24c^2h} + \frac{f(g+hx)^3\sqrt{a+bx+cx^2}}{4ch}$$
$$- \frac{(105b^3fh^3+32c^3(fg^3-4gh(eg+3dh))-20bch^2(11afh+6b(2fg+eh))+8c^2h(11bfg^2+18bl}{}$$
$$+ \frac{(128c^4dg^2+35b^4fh^2-40b^2ch(2bfg+beh+3afh)-64c^3(afg^2+ah(2eg+dh)+bg(eg+2dh)}{64c}$$

$$= -\frac{(2cfg-8ceh+7bfh)(g+hx)^2\sqrt{a+bx+cx^2}}{24c^2h} + \frac{f(g+hx)^3\sqrt{a+bx+cx^2}}{4ch}$$
$$- \frac{(105b^3fh^3+32c^3(fg^3-4gh(eg+3dh))-20bch^2(11afh+6b(2fg+eh))+8c^2h(11bfg^2+18bl}{}$$
$$+ \frac{(128c^4dg^2+35b^4fh^2-40b^2ch(2bfg+beh+3afh)-64c^3(afg^2+ah(2eg+dh)+bg(eg+2dh)}{128c^{9/2}}$$

Mathematica [A] (verified)

Time = 1.26 (sec) , antiderivative size = 341, normalized size of antiderivative = 0.81

$$\int \frac{(g+hx)^2\left(d+ex+fx^2\right)}{\sqrt{a+bx+cx^2}}\,dx$$
$$= \frac{2\sqrt{c}\sqrt{a+x(b+cx)}(-105b^3fh^2+10bch(22afh+b(24fg+12eh+7fhx))+16c^3(6dh(4g+hx)+4e(3g^}{}$$

```
[In] Integrate[((g + h*x)^2*(d + e*x + f*x^2))/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(-105*b^3*f*h^2 + 10*b*c*h*(22*a*f*h + b*(
24*f*g + 12*e*h + 7*f*h*x)) + 16*c^3*(6*d*h*(4*g + h*x) + 4*e*(3*g^2 + 3*g*
h*x + h^2*x^2) + f*x*(6*g^2 + 8*g*h*x + 3*h^2*x^2)) - 8*c^2*(2*b*h*(18*e*g
+ 9*d*h + 5*e*h*x) + a*h*(32*f*g + 16*e*h + 9*f*h*x) + b*f*(18*g^2 + 20*g*h
*x + 7*h^2*x^2))) + 3*(-128*c^4*d*g^2 - 35*b^4*f*h^2 + 40*b^2*c*h*(2*b*f*g
+ b*e*h + 3*a*f*h) + 64*c^3*(a*f*g^2 + a*h*(2*e*g + d*h) + b*g*(e*g + 2*d*h
)) - 48*c^2*(a^2*f*h^2 + 2*a*b*h*(2*f*g + e*h) + b^2*(f*g^2 + h*(2*e*g + d*
h))))*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/(384*c^(9/2))
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 437, normalized size of antiderivative = 1.04

method	result
risch	$\frac{\left(48f h^2 c^3 x^3-56b c^2 f h^2 x^2+64c^3 e h^2 x^2+128c^3 f g h x^2-72a c^2 f h^2 x+70b^2 c f h^2 x-80b c^2 e h^2 x-160b c^2 f g h x+96c^3 d h^2 x+192c^3 e g h x\right.}{}$
default	$\frac{d g^2 \ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c x^2+bx+a}\right)}{\sqrt{c}}+f h^2\left(\frac{x^3\sqrt{c x^2+bx+a}}{4c}-\frac{7b\left(\frac{x^2\sqrt{c x^2+bx+a}}{3c}-\frac{5b\left(\frac{x\sqrt{c x^2+bx+a}}{2c}-\frac{3b\left(\frac{\sqrt{c x^2+bx+a}}{c}-\frac{b\ln\left(\frac{\frac{b}{2}+c}{\sqrt{c}}\right.}{}\right.}{4c}\right.}{}\right.}{}\right.$

```
[In] int((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/192*(48*c^3*f*h^2*x^3-56*b*c^2*f*h^2*x^2+64*c^3*e*h^2*x^2+128*c^3*f*g*h*x
^2-72*a*c^2*f*h^2*x+70*b^2*c*f*h^2*x-80*b*c^2*e*h^2*x-160*b*c^2*f*g*h*x+96*
c^3*d*h^2*x+192*c^3*e*g*h*x+96*c^3*f*g^2*x+220*a*b*c*f*h^2-128*a*c^2*e*h^2-
256*a*c^2*f*g*h-105*b^3*f*h^2+120*b^2*c*e*h^2+240*b^2*c*f*g*h-144*b*c^2*d*h
^2-288*b*c^2*e*g*h-144*b*c^2*f*g^2+384*c^3*d*g*h+192*c^3*e*g^2)*(c*x^2+b*x+
a)^(1/2)/c^4+1/128*(48*a^2*c^2*f*h^2-120*a*b^2*c*f*h^2+96*a*b*c^2*e*h^2+192
*a*b*c^2*f*g*h-64*a*c^3*d*h^2-128*a*c^3*e*g*h-64*a*c^3*f*g^2+35*b^4*f*h^2-4
0*b^3*c*e*h^2-80*b^3*c*f*g*h+48*b^2*c^2*d*h^2+96*b^2*c^2*e*g*h+48*b^2*c^2*f
*g^2-128*b*c^3*d*g*h-64*b*c^3*e*g^2+128*c^4*d*g^2)/c^(9/2)*ln((1/2*b+c*x)/c
^(1/2)+(c*x^2+b*x+a)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 861, normalized size of antiderivative = 2.05

$$\int \frac{(g+hx)^2\left(d+ex+fx^2\right)}{\sqrt{a+bx+cx^2}}\,dx$$

$$= \Bigg[\frac{3\,(16\,(8\,c^4d-4\,bc^3e+(3\,b^2c^2-4\,ac^3)f)g^2-16\,(8\,bc^3d-2\,(3\,b^2c^2-4\,ac^3)e+(5\,b^3c-12\,abc^2)f)gh+(16}{}$$

$$-\frac{3\,(16\,(8\,c^4d-4\,bc^3e+(3\,b^2c^2-4\,ac^3)f)g^2-16\,(8\,bc^3d-2\,(3\,b^2c^2-4\,ac^3)e+(5\,b^3c-12\,abc^2)f)gh+(1}{}$$

```
[In] integrate((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [1/768*(3*(16*(8*c^4*d - 4*b*c^3*e + (3*b^2*c^2 - 4*a*c^3)*f)*g^2 - 16*(8*b
*c^3*d - 2*(3*b^2*c^2 - 4*a*c^3)*e + (5*b^3*c - 12*a*b*c^2)*f)*g*h + (16*(3
*b^2*c^2 - 4*a*c^3)*d - 8*(5*b^3*c - 12*a*b*c^2)*e + (35*b^4 - 120*a*b^2*c
+ 48*a^2*c^2)*f)*h^2)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2
 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(48*c^4*f*h^2*x^3 + 48*(4*c^4*
e - 3*b*c^3*f)*g^2 + 16*(24*c^4*d - 18*b*c^3*e + (15*b^2*c^2 - 16*a*c^3)*f)
*g*h - (144*b*c^3*d - 8*(15*b^2*c^2 - 16*a*c^3)*e + 5*(21*b^3*c - 44*a*b*c^
2)*f)*h^2 + 8*(16*c^4*f*g*h + (8*c^4*e - 7*b*c^3*f)*h^2)*x^2 + 2*(48*c^4*f*
g^2 + 16*(6*c^4*e - 5*b*c^3*f)*g*h + (48*c^4*d - 40*b*c^3*e + (35*b^2*c^2 -
 36*a*c^3)*f)*h^2)*x)*sqrt(c*x^2 + b*x + a))/c^5, -1/384*(3*(16*(8*c^4*d -
4*b*c^3*e + (3*b^2*c^2 - 4*a*c^3)*f)*g^2 - 16*(8*b*c^3*d - 2*(3*b^2*c^2 - 4
*a*c^3)*e + (5*b^3*c - 12*a*b*c^2)*f)*g*h + (16*(3*b^2*c^2 - 4*a*c^3)*d - 8
*(5*b^3*c - 12*a*b*c^2)*e + (35*b^4 - 120*a*b^2*c + 48*a^2*c^2)*f)*h^2)*sqr
t(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*
x + a*c)) - 2*(48*c^4*f*h^2*x^3 + 48*(4*c^4*e - 3*b*c^3*f)*g^2 + 16*(24*c^4
*d - 18*b*c^3*e + (15*b^2*c^2 - 16*a*c^3)*f)*g*h - (144*b*c^3*d - 8*(15*b^2
*c^2 - 16*a*c^3)*e + 5*(21*b^3*c - 44*a*b*c^2)*f)*h^2 + 8*(16*c^4*f*g*h + (
8*c^4*e - 7*b*c^3*f)*h^2)*x^2 + 2*(48*c^4*f*g^2 + 16*(6*c^4*e - 5*b*c^3*f)*
g*h + (48*c^4*d - 40*b*c^3*e + (35*b^2*c^2 - 36*a*c^3)*f)*h^2)*x)*sqrt(c*x^
2 + b*x + a))/c^5]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 910 vs. $2(432) = 864$.

Time = 1.10 (sec) , antiderivative size = 910, normalized size of antiderivative = 2.17

$$\int \frac{(g+hx)^2\,(d+ex+fx^2)}{\sqrt{a+bx+cx^2}}\,dx$$

$$= \begin{cases} \sqrt{a+bx+cx^2}\left(\frac{fh^2x^3}{4c} + \frac{x^2\left(-\frac{7bfh^2}{8c}+eh^2+2fgh\right)}{3c} + \frac{x\left(-\frac{3afh^2}{4c}-\frac{5b\left(-\frac{7bfh^2}{8c}+eh^2+2fgh\right)}{6c}+dh^2+2egh+fg^2\right)}{2c} + \frac{-\frac{2a\left(-\frac{7bfh^2}{8c}+\right.}{3c}}{} \right. \\ \frac{2\left(\frac{fh^2(a+bx)^{\frac{9}{2}}}{9b^4}+\frac{(a+bx)^{\frac{7}{2}}\left(-4afh^2+beh^2+2bfgh\right)}{7b^4}+\frac{(a+bx)^{\frac{5}{2}}\cdot\left(6a^2fh^2-3abeh^2-6abfgh+b^2dh^2+2b^2egh+b^2fg^2\right)}{5b^4}+\frac{(a+bx)^{\frac{3}{2}}\left(-4a^3fh^2+3a^2beh^2+6a^2b\right.}{}\right.}{b} \\ \frac{dg^2x+\frac{fh^2x^5}{5}+\frac{x^4\left(eh^2+2fgh\right)}{4}+\frac{x^3\left(dh^2+2egh+fg^2\right)}{3}+\frac{x^2\cdot\left(2dgh+eg^2\right)}{2}}{\sqrt{a}} \end{cases}$$

```
[In] integrate((h*x+g)**2*(f*x**2+e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Piecewise((sqrt(a + b*x + c*x**2)*(f*h**2*x**3/(4*c) + x**2*(-7*b*f*h**2/(8
*c) + e*h**2 + 2*f*g*h)/(3*c) + x*(-3*a*f*h**2/(4*c) - 5*b*(-7*b*f*h**2/(8*
c) + e*h**2 + 2*f*g*h)/(6*c) + d*h**2 + 2*e*g*h + f*g**2)/(2*c) + (-2*a*(-7
*b*f*h**2/(8*c) + e*h**2 + 2*f*g*h)/(3*c) - 3*b*(-3*a*f*h**2/(4*c) - 5*b*(-
7*b*f*h**2/(8*c) + e*h**2 + 2*f*g*h)/(6*c) + d*h**2 + 2*e*g*h + f*g**2)/(4*
c) + 2*d*g*h + e*g**2)/c) + (-a*(-3*a*f*h**2/(4*c) - 5*b*(-7*b*f*h**2/(8*c)
 + e*h**2 + 2*f*g*h)/(6*c) + d*h**2 + 2*e*g*h + f*g**2)/(2*c) - b*(-2*a*(-7
*b*f*h**2/(8*c) + e*h**2 + 2*f*g*h)/(3*c) - 3*b*(-3*a*f*h**2/(4*c) - 5*b*(-
7*b*f*h**2/(8*c) + e*h**2 + 2*f*g*h)/(6*c) + d*h**2 + 2*e*g*h + f*g**2)/(4*
c) + 2*d*g*h + e*g**2)/(2*c) + d*g**2)*Piecewise((log(b + 2*sqrt(c)*sqrt(a
+ b*x + c*x**2) + 2*c*x)/sqrt(c), Ne(a - b**2/(4*c), 0)), ((b/(2*c) + x)*lo
g(b/(2*c) + x)/sqrt(c*(b/(2*c) + x)**2), True)), Ne(c, 0)), (2*(f*h**2*(a +
 b*x)**(9/2)/(9*b**4) + (a + b*x)**(7/2)*(-4*a*f*h**2 + b*e*h**2 + 2*b*f*g*
h)/(7*b**4) + (a + b*x)**(5/2)*(6*a**2*f*h**2 - 3*a*b*e*h**2 - 6*a*b*f*g*h
+ b**2*d*h**2 + 2*b**2*e*g*h + b**2*f*g**2)/(5*b**4) + (a + b*x)**(3/2)*(-4
*a**3*f*h**2 + 3*a**2*b*e*h**2 + 6*a**2*b*f*g*h - 2*a*b**2*d*h**2 - 4*a*b**
2*e*g*h - 2*a*b**2*f*g**2 + 2*b**3*d*g*h + b**3*e*g**2)/(3*b**4) + sqrt(a +
 b*x)*(a**4*f*h**2 - a**3*b*e*h**2 - 2*a**3*b*f*g*h + a**2*b**2*d*h**2 + 2*
a**2*b**2*e*g*h + a**2*b**2*f*g**2 - 2*a*b**3*d*g*h - a*b**3*e*g**2 + b**4*
d*g**2)/b**4)/b, Ne(b, 0)), ((d*g**2*x + f*h**2*x**5/5 + x**4*(e*h**2 + 2*f
*g*h)/4 + x**3*(d*h**2 + 2*e*g*h + f*g**2)/3 + x**2*(2*d*g*h + e*g**2)/2)/s
qrt(a), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(g+hx)^2 (d+ex+fx^2)}{\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 442, normalized size of antiderivative = 1.05

$$\int \frac{(g+hx)^2 (d+ex+fx^2)}{\sqrt{a+bx+cx^2}}\,dx$$
$$= \frac{1}{192}\sqrt{cx^2+bx+a}\left(2\left(4\left(\frac{6\,fh^2x}{c}+\frac{16\,c^3fgh+8\,c^3eh^2-7\,bc^2fh^2}{c^4}\right)x+\frac{48\,c^3fg^2+96\,c^3egh-80\,bc^2fgh\right.\right.\right.$$
$$-\frac{(128\,c^4dg^2-64\,bc^3eg^2+48\,b^2c^2fg^2-64\,ac^3fg^2-128\,bc^3dgh+96\,b^2c^2egh-128\,ac^3egh-80\,b^3cfgh+}{}$$

```
[In] integrate((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/192*sqrt(c*x^2 + b*x + a)*(2*(4*(6*f*h^2*x/c + (16*c^3*f*g*h + 8*c^3*e*h^
2 - 7*b*c^2*f*h^2)/c^4)*x + (48*c^3*f*g^2 + 96*c^3*e*g*h - 80*b*c^2*f*g*h +
 48*c^3*d*h^2 - 40*b*c^2*e*h^2 + 35*b^2*c*f*h^2 - 36*a*c^2*f*h^2)/c^4)*x +
(192*c^3*e*g^2 - 144*b*c^2*f*g^2 + 384*c^3*d*g*h - 288*b*c^2*e*g*h + 240*b^
2*c*f*g*h - 256*a*c^2*f*g*h - 144*b*c^2*d*h^2 + 120*b^2*c*e*h^2 - 128*a*c^2
*e*h^2 - 105*b^3*f*h^2 + 220*a*b*c*f*h^2)/c^4) - 1/128*(128*c^4*d*g^2 - 64*
b*c^3*e*g^2 + 48*b^2*c^2*f*g^2 - 64*a*c^3*f*g^2 - 128*b*c^3*d*g*h + 96*b^2*
c^2*e*g*h - 128*a*c^3*e*g*h - 80*b^3*c*f*g*h + 192*a*b*c^2*f*g*h + 48*b^2*c
^2*d*h^2 - 64*a*c^3*d*h^2 - 40*b^3*c*e*h^2 + 96*a*b*c^2*e*h^2 + 35*b^4*f*h^
2 - 120*a*b^2*c*f*h^2 + 48*a^2*c^2*f*h^2)*log(abs(2*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))*sqrt(c) + b))/c^(9/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(g+hx)^2\,(d+ex+fx^2)}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{{\left(g+h\,x\right)}^2\,\left(f\,x^2+e\,x+d\right)}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

```
[In] int(((g + h*x)^2*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(1/2),x)
[Out] int(((g + h*x)^2*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(1/2), x)
```

3.228 $\int \frac{(g+hx)(d+ex+fx^2)}{\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 223

$$\int \frac{(g+hx)\left(d+ex+fx^2\right)}{\sqrt{a+bx+cx^2}}\,dx = \frac{f(g+hx)^2\sqrt{a+bx+cx^2}}{3ch}$$
$$+\frac{(15b^2fh^2-8c^2(fg^2-3h(eg+dh))-2ch(8afh+9b(fg+eh))-2ch(2cfg-6ceh+5bfh)x)\sqrt{a+bx}}{24c^3h}$$
$$+\frac{(16c^3dg-5b^3fh-8c^2(beg+afg+bdh+aeh)+6bc(bfg+beh+2afh))\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{7/2}}$$

```
[Out] 1/16*(16*c^3*d*g-5*b^3*f*h-8*c^2*(a*e*h+a*f*g+b*d*h+b*e*g)+6*b*c*(2*a*f*h+b
*e*h+b*f*g))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(7/2)+1/3
*f*(h*x+g)^2*(c*x^2+b*x+a)^(1/2)/c/h+1/24*(15*b^2*f*h^2-8*c^2*(f*g^2-3*h*(d
*h+e*g))-2*c*h*(8*a*f*h+9*b*(e*h+f*g))-2*c*h*(5*b*f*h-6*c*e*h+2*c*f*g)*x)*(
c*x^2+b*x+a)^(1/2)/c^3/h
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1667, 793, 635, 212}

$$\int \frac{(g+hx)\left(d+ex+fx^2\right)}{\sqrt{a+bx+cx^2}}\,dx$$

$$=\frac{\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-8c^2(aeh+afg+bdh+beg)+6bc(2afh+beh+bfg)-5b^3fh+16c^3dg\right)}{16c^{7/2}}$$

$$+\frac{\sqrt{a+bx+cx^2}(-2ch(8afh+9b(eh+fg))+15b^2fh^2-2chx(5bfh-6ceh+2cfg)-8c^2(fg^2-3h(d}{24c^3h}$$

$$+\frac{f(g+hx)^2\sqrt{a+bx+cx^2}}{3ch}$$

```
[In] Int[((g + h*x)*(d + e*x + f*x^2))/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (f*(g + h*x)^2*Sqrt[a + b*x + c*x^2])/(3*c*h) + ((15*b^2*f*h^2 - 8*c^2*(f*g
^2 - 3*h*(e*g + d*h)) - 2*c*h*(8*a*f*h + 9*b*(f*g + e*h)) - 2*c*h*(2*c*f*g
- 6*c*e*h + 5*b*f*h)*x)*Sqrt[a + b*x + c*x^2])/(24*c^3*h) + ((16*c^3*d*g -
5*b^3*f*h - 8*c^2*(b*e*g + a*f*g + b*d*h + a*e*h) + 6*b*c*(b*f*g + b*e*h +
2*a*f*h))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(7/
2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\text{integral} = \frac{f(g+hx)^2\sqrt{a+bx+cx^2}}{3ch} + \frac{\int \frac{(g+hx)\left(-\frac{1}{2}h(bfg-6cdh+4afh)-\frac{1}{2}h(2cfg-6ceh+5bfh)x\right)}{\sqrt{a+bx+cx^2}}\,dx}{3ch^2}$$

$$= \frac{f(g+hx)^2\sqrt{a+bx+cx^2}}{3ch}$$
$$+ \frac{(15b^2fh^2-8c^2(fg^2-3h(eg+dh))-2ch(8afh+9b(fg+eh))-2ch(2cfg-6ceh+5bfh)x)}{24c^3h}$$
$$+ \frac{(16c^3dg-5b^3fh-8c^2(beg+afg+bdh+aeh)+6bc(bfg+beh+2afh))\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{16c^3}$$

$$= \frac{f(g+hx)^2\sqrt{a+bx+cx^2}}{3ch}$$
$$+ \frac{(15b^2fh^2-8c^2(fg^2-3h(eg+dh))-2ch(8afh+9b(fg+eh))-2ch(2cfg-6ceh+5bfh)x)}{24c^3h}$$
$$+ \frac{(16c^3dg-5b^3fh-8c^2(beg+afg+bdh+aeh)+6bc(bfg+beh+2afh))\,\text{Subst}\Bigg(\int \frac{1}{4c-x^2}\,dx, x,}{8c^3}$$

$$= \frac{f(g+hx)^2\sqrt{a+bx+cx^2}}{3ch}$$
$$+ \frac{(15b^2fh^2-8c^2(fg^2-3h(eg+dh))-2ch(8afh+9b(fg+eh))-2ch(2cfg-6ceh+5bfh)x)}{24c^3h}$$
$$+ \frac{(16c^3dg-5b^3fh-8c^2(beg+afg+bdh+aeh)+6bc(bfg+beh+2afh))\tanh^{-1}\Bigg(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}}{16c^{7/2}}$$

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.80

$$\int \frac{(g+hx)\left(d+ex+fx^2\right)}{\sqrt{a+bx+cx^2}}\,dx$$

$$= \frac{2\sqrt{c}\sqrt{a+x(b+cx)}(15b^2fh+4c^2(6eg+6dh+3fgx+3ehx+2fhx^2)-2c(8afh+b(9fg+9eh+5fh}{}$$

```
[In] Integrate[((g + h*x)*(d + e*x + f*x^2))/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(15*b^2*f*h + 4*c^2*(6*e*g + 6*d*h + 3*f*g
*x + 3*e*h*x + 2*f*h*x^2) - 2*c*(8*a*f*h + b*(9*f*g + 9*e*h + 5*f*h*x))) +
3*(-16*c^3*d*g + 5*b^3*f*h + 8*c^2*(b*e*g + a*f*g + b*d*h + a*e*h) - 6*b*c*
(b*f*g + b*e*h + 2*a*f*h))*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]]
)/(48*c^(7/2))
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.86

method	result
risch	$-\frac{\left(-8fh\,c^2x^2+10bcfhx-12c^2ehx-12c^2fgx+16acfh-15b^2fh+18bceh+18bcfg-24c^2dh-24c^2eg\right)\sqrt{c\,x^2+bx+a}}{24c^3}+\frac{(12abcfh-8a}{}$
default	$\frac{dg\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{\sqrt{c}}+fh\left(\frac{x^2\sqrt{c\,x^2+bx+a}}{3c}-\frac{5b\left(\frac{x\sqrt{c\,x^2+bx+a}}{2c}-\frac{3b\left(\frac{\sqrt{c\,x^2+bx+a}}{c}-\frac{b\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{2c^{\frac{3}{2}}}\right)}{4c}-\frac{a\ln}{}\right.}{6c}\right.$

```
[In] int((h*x+g)*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*(-8*c^2*f*h*x^2+10*b*c*f*h*x-12*c^2*e*h*x-12*c^2*f*g*x+16*a*c*f*h-15*
b^2*f*h+18*b*c*e*h+18*b*c*f*g-24*c^2*d*h-24*c^2*e*g)*(c*x^2+b*x+a)^(1/2)/c^
3+1/16*(12*a*b*c*f*h-8*a*c^2*e*h-8*a*c^2*f*g-5*b^3*f*h+6*b^2*c*e*h+6*b^2*c*
f*g-8*b*c^2*d*h-8*b*c^2*e*g+16*c^3*d*g)/c^(7/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x
^2+b*x+a)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 461, normalized size of antiderivative = 2.07

$$\int \frac{(g+hx)(d+ex+fx^2)}{\sqrt{a+bx+cx^2}}\,dx$$

$$= \left[\frac{3\,(2\,(8\,c^3d-4\,bc^2e+(3\,b^2c-4\,ac^2)f)g-(8\,bc^2d-2\,(3\,b^2c-4\,ac^2)e+(5\,b^3-12\,abc)f)h)\sqrt{c}\log\left(-8\,c^2x\right.}{}\right.$$

$$-\frac{3\,(2\,(8\,c^3d-4\,bc^2e+(3\,b^2c-4\,ac^2)f)g-(8\,bc^2d-2\,(3\,b^2c-4\,ac^2)e+(5\,b^3-12\,abc)f)h)\sqrt{-c}\arctan\left(\right.}{}$$

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/96*(3*(2*(8*c^3*d - 4*b*c^2*e + (3*b^2*c - 4*a*c^2)*f)*g - (8*b*c^2*d -
2*(3*b^2*c - 4*a*c^2)*e + (5*b^3 - 12*a*b*c)*f)*h)*sqrt(c)*log(-8*c^2*x^2 -
 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(
8*c^3*f*h*x^2 + 6*(4*c^3*e - 3*b*c^2*f)*g + (24*c^3*d - 18*b*c^2*e + (15*b^
2*c - 16*a*c^2)*f)*h + 2*(6*c^3*f*g + (6*c^3*e - 5*b*c^2*f)*h)*x)*sqrt(c*x^
2 + b*x + a))/c^4, -1/48*(3*(2*(8*c^3*d - 4*b*c^2*e + (3*b^2*c - 4*a*c^2)*f
)*g - (8*b*c^2*d - 2*(3*b^2*c - 4*a*c^2)*e + (5*b^3 - 12*a*b*c)*f)*h)*sqrt(
-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x
+ a*c)) - 2*(8*c^3*f*h*x^2 + 6*(4*c^3*e - 3*b*c^2*f)*g + (24*c^3*d - 18*b*c
^2*e + (15*b^2*c - 16*a*c^2)*f)*h + 2*(6*c^3*f*g + (6*c^3*e - 5*b*c^2*f)*h)
*x)*sqrt(c*x^2 + b*x + a))/c^4]
```

Sympy [A] (verification not implemented)

Time = 0.97 (sec) , antiderivative size = 437, normalized size of antiderivative = 1.96

$$\int \frac{(g+hx)(d+ex+fx^2)}{\sqrt{a+bx+cx^2}}\,dx$$

$$= \begin{cases} \sqrt{a+bx+cx^2}\left(\frac{fhx^2}{3c}+\frac{x\left(-\frac{5bfh}{6c}+eh+fg\right)}{2c}+\frac{-\frac{2afh}{3c}-\frac{3b\left(-\frac{5bfh}{6c}+eh+fg\right)}{4c}+dh+eg}{c}\right)+\left(-\frac{a\left(-\frac{5bfh}{6c}+eh+fg\right)}{2c}-\frac{b\left(-\frac{2afh}{3c}-\right.}{}\right. \\ \frac{2\left(\frac{fh(a+bx)^{\frac{7}{2}}}{7b^3}+\frac{(a+bx)^{\frac{5}{2}}(-3afh+beh+bfg)}{5b^3}+\frac{(a+bx)^{\frac{3}{2}}\cdot\left(3a^2fh-2abeh-2abfg+b^2dh+b^2eg\right)}{3b^3}+\frac{\sqrt{a+bx}\left(-a^3fh+a^2beh+a^2bfg-ab^2dh-ab^2eg+b^3dg\right)}{b^3}\right)}{b} \\ \frac{dgx+\frac{fhx^4}{4}+\frac{x^3(eh+fg)}{3}+\frac{x^2(dh+eg)}{2}}{\sqrt{a}} \end{cases}$$

```
[In] integrate((h*x+g)*(f*x**2+e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Piecewise((sqrt(a + b*x + c*x**2)*(f*h*x**2/(3*c) + x*(-5*b*f*h/(6*c) + e*h
 + f*g)/(2*c) + (-2*a*f*h/(3*c) - 3*b*(-5*b*f*h/(6*c) + e*h + f*g)/(4*c) +
d*h + e*g)/c) + (-a*(-5*b*f*h/(6*c) + e*h + f*g)/(2*c) - b*(-2*a*f*h/(3*c)
- 3*b*(-5*b*f*h/(6*c) + e*h + f*g)/(4*c) + d*h + e*g)/(2*c) + d*g)*Piecewis
e((log(b + 2*sqrt(c)*sqrt(a + b*x + c*x**2) + 2*c*x)/sqrt(c), Ne(a - b**2/(
4*c), 0)), ((b/(2*c) + x)*log(b/(2*c) + x)/sqrt(c*(b/(2*c) + x)**2), True))
, Ne(c, 0)), (2*(f*h*(a + b*x)**(7/2)/(7*b**3) + (a + b*x)**(5/2)*(-3*a*f*h
 + b*e*h + b*f*g)/(5*b**3) + (a + b*x)**(3/2)*(3*a**2*f*h - 2*a*b*e*h - 2*a
*b*f*g + b**2*d*h + b**2*e*g)/(3*b**3) + sqrt(a + b*x)*(-a**3*f*h + a**2*b*
e*h + a**2*b*f*g - a*b**2*d*h - a*b**2*e*g + b**3*d*g)/b**3)/b, Ne(b, 0)),
((d*g*x + f*h*x**4/4 + x**3*(e*h + f*g)/3 + x**2*(d*h + e*g)/2)/sqrt(a), Tr
ue))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(g+hx)\left(d+ex+fx^2\right)}{\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.90

$$\int \frac{(g+hx)\left(d+ex+fx^2\right)}{\sqrt{a+bx+cx^2}}\,dx$$
$$= \frac{1}{24}\sqrt{cx^2+bx+a}\left(2\left(\frac{4fhx}{c}+\frac{6c^2fg+6c^2eh-5bcfh}{c^3}\right)x+\frac{24c^2eg-18bcfg+24c^2dh-18bceh+1}{c^3}\right.$$
$$-\frac{(16c^3dg-8bc^2eg+6b^2cfg-8ac^2fg-8bc^2dh+6b^2ceh-8ac^2eh-5b^3fh+12abcfh)\log\left(\left|2\left(\sqrt{c}\right.\right.\right.}{16c^{\frac{7}{2}}}$$

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*sqrt(c*x^2 + b*x + a)*(2*(4*f*h*x/c + (6*c^2*f*g + 6*c^2*e*h - 5*b*c*f
*h)/c^3)*x + (24*c^2*e*g - 18*b*c*f*g + 24*c^2*d*h - 18*b*c*e*h + 15*b^2*f*
```

```
h - 16*a*c*f*h)/c^3) - 1/16*(16*c^3*d*g - 8*b*c^2*e*g + 6*b^2*c*f*g - 8*a*c
^2*f*g - 8*b*c^2*d*h + 6*b^2*c*e*h - 8*a*c^2*e*h - 5*b^3*f*h + 12*a*b*c*f*h
)*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(7/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(g+hx)\,(d+ex+fx^2)}{\sqrt{a+bx+cx^2}}\,dx=\int \frac{(g+h\,x)\,\left(f\,x^2+e\,x+d\right)}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

```
[In] int(((g + h*x)*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(1/2),x)
```

```
[Out] int(((g + h*x)*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(1/2), x)
```

3.229 $\int \frac{d+ex+fx^2}{\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 116

$$\int \frac{d+ex+fx^2}{\sqrt{a+bx+cx^2}}\,dx=\frac{(4ce-3bf)\sqrt{a+bx+cx^2}}{4c^2}+\frac{fx\sqrt{a+bx+cx^2}}{2c}+\frac{(8c^2d+3b^2f-4c(be+af))\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{5/2}}$$

```
[Out] 1/8*(8*c^2*d+3*b^2*f-4*c*(a*f+b*e))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*
x+a)^(1/2))/c^(5/2)+1/4*(-3*b*f+4*c*e)*(c*x^2+b*x+a)^(1/2)/c^2+1/2*f*x*(c*x
^2+b*x+a)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1675, 654, 635, 212}

$$\int \frac{d+ex+fx^2}{\sqrt{a+bx+cx^2}}\,dx=\frac{\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(-4c(af+be)+3b^2f+8c^2d)}{8c^{5/2}}+\frac{\sqrt{a+bx+cx^2}(4ce-3bf)}{4c^2}+\frac{fx\sqrt{a+bx+cx^2}}{2c}$$

```
[In] Int[(d + e*x + f*x^2)/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] ((4*c*e - 3*b*f)*Sqrt[a + b*x + c*x^2])/(4*c^2) + (f*x*Sqrt[a + b*x + c*x^2
])/(2*c) + ((8*c^2*d + 3*b^2*f - 4*c*(b*e + a*f))*ArcTanh[(b + 2*c*x)/(2*Sq
rt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{fx\sqrt{a+bx+cx^2}}{2c}+\frac{\int \frac{2cd-af+\frac{1}{2}(4ce-3bf)x}{\sqrt{a+bx+cx^2}}\,dx}{2c}\\
&= \frac{(4ce-3bf)\sqrt{a+bx+cx^2}}{4c^2}+\frac{fx\sqrt{a+bx+cx^2}}{2c}\\
&\quad+\frac{\left(2c(2cd-af)-\frac{1}{2}b(4ce-3bf)\right)\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{4c^2}\\
&= \frac{(4ce-3bf)\sqrt{a+bx+cx^2}}{4c^2}+\frac{fx\sqrt{a+bx+cx^2}}{2c}\\
&\quad+\frac{\left(2c(2cd-af)-\frac{1}{2}b(4ce-3bf)\right)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx,x,\frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{2c^2}\\
&= \frac{(4ce-3bf)\sqrt{a+bx+cx^2}}{4c^2}+\frac{fx\sqrt{a+bx+cx^2}}{2c}\\
&\quad+\frac{\left(8c^2d+3b^2f-4c(be+af)\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{5/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.84

$$\int \frac{d+ex+fx^2}{\sqrt{a+bx+cx^2}}\,dx = \frac{\sqrt{c}(4ce-3bf+2cfx)\sqrt{a+x(b+cx)}+\left(8c^2d+3b^2f-4c(be+af)\right)\operatorname{arctanh}\left(\frac{\sqrt{c}x}{-\sqrt{a}+\sqrt{a+x(b+cx)}}\right)}{4c^{5/2}}$$

```
[In] Integrate[(d + e*x + f*x^2)/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (Sqrt[c]*(4*c*e - 3*b*f + 2*c*f*x)*Sqrt[a + x*(b + c*x)] + (8*c^2*d + 3*b^2
*f - 4*c*(b*e + a*f))*ArcTanh[(Sqrt[c]*x)/(-Sqrt[a] + Sqrt[a + x*(b + c*x)]
)])/(4*c^(5/2))
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.74

method	result
risch	$-\frac{(-2cfx+3bf-4ce)\sqrt{c\,x^2+bx+a}}{4c^2}-\frac{(4acf-3b^2f+4bce-8c^2d)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{8c^{\frac{5}{2}}}$
default	$\frac{d\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{\sqrt{c}}+f\left(\frac{x\sqrt{c\,x^2+bx+a}}{2c}-\frac{3b\left(\frac{\sqrt{c\,x^2+bx+a}}{c}-\frac{b\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{2c^{\frac{3}{2}}}\right)}{4c}-\frac{a\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx-}\right.}{2c^{\frac{3}{2}}}\right.$

```
[In] int((f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(-2*c*f*x+3*b*f-4*c*e)*(c*x^2+b*x+a)^(1/2)/c^2-1/8*(4*a*c*f-3*b^2*f+4*
b*c*e-8*c^2*d)/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.96

$$\int \frac{d+ex+fx^2}{\sqrt{a+bx+cx^2}}\,dx$$
$$= \left[-\frac{(8\,c^2d-4\,bce+(3\,b^2-4\,ac)f)\sqrt{c}\log\left(-8\,c^2x^2-8\,bcx-b^2+4\,\sqrt{cx^2+bx+a}(2\,cx+b)\sqrt{c}-4\,ac\right)-4}{16\,c^3}\right.$$
$$-\frac{(8\,c^2d-4\,bce+(3\,b^2-4\,ac)f)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^2+bx+a}(2\,cx+b)\sqrt{-c}}{2\,(c^2x^2+bcx+ac)}\right)-2\,(2\,c^2fx+4\,c^2e-3\,bcf)\sqrt{cx^2+ba}}{8\,c^3}$$

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*((8*c^2*d - 4*b*c*e + (3*b^2 - 4*a*c)*f)*sqrt(c)*log(-8*c^2*x^2 - 8*
b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(2*c
^2*f*x + 4*c^2*e - 3*b*c*f)*sqrt(c*x^2 + b*x + a))/c^3, -1/8*((8*c^2*d - 4*
b*c*e + (3*b^2 - 4*a*c)*f)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x
 + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(2*c^2*f*x + 4*c^2*e - 3*b*c*f)
*sqrt(c*x^2 + b*x + a))/c^3]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. 2(102) = 204.

Time = 0.37 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.95

$$\int \frac{d+ex+fx^2}{\sqrt{a+bx+cx^2}}\,dx$$
$$= \begin{cases} \left(\frac{fx}{2c}+\frac{-\frac{3bf}{4c}+e}{c}\right)\sqrt{a+bx+cx^2}+\left(-\frac{af}{2c}-\frac{b\left(-\frac{3bf}{4c}+e\right)}{2c}+d\right)\left(\begin{cases}\frac{\log\left(b+2\sqrt{c}\sqrt{a+bx+cx^2}+2cx\right)}{\sqrt{c}} & \text{for}\ a-\frac{b^2}{4c}\neq 0\\ \frac{\left(\frac{b}{2c}+x\right)\log\left(\frac{b}{2c}+x\right)}{\sqrt{c\left(\frac{b}{2c}+x\right)^2}} & \text{otherwise}\end{cases}\right) \\ \frac{2d\sqrt{a+bx}+\frac{2e\left(-a\sqrt{a+bx}+\frac{(a+bx)^{\frac{3}{2}}}{3}\right)}{b}+\frac{2f\left(a^2\sqrt{a+bx}-\frac{2a(a+bx)^{\frac{3}{2}}}{3}+\frac{(a+bx)^{\frac{5}{2}}}{5}\right)}{b^2}}{b} \\ \frac{dx+\frac{ex^2}{2}+\frac{fx^3}{3}}{\sqrt{a}} \end{cases}$$

```
[In] integrate((f*x**2+e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Piecewise(((f*x/(2*c) + (-3*b*f/(4*c) + e)/c)*sqrt(a + b*x + c*x**2) + (-a*
f/(2*c) - b*(-3*b*f/(4*c) + e)/(2*c) + d)*Piecewise((log(b + 2*sqrt(c)*sqrt
(a + b*x + c*x**2) + 2*c*x)/sqrt(c), Ne(a - b**2/(4*c), 0)), ((b/(2*c) + x)
```

```
*log(b/(2*c) + x)/sqrt(c*(b/(2*c) + x)**2), True)), Ne(c, 0)), ((2*d*sqrt(a
 + b*x) + 2*e*(-a*sqrt(a + b*x) + (a + b*x)**(3/2)/3)/b + 2*f*(a**2*sqrt(a
+ b*x) - 2*a*(a + b*x)**(3/2)/3 + (a + b*x)**(5/2)/5)/b**2)/b, Ne(b, 0)), (
(d*x + e*x**2/2 + f*x**3/3)/sqrt(a), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex+fx^2}{\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.80

$$\begin{aligned}\int \frac{d+ex+fx^2}{\sqrt{a+bx+cx^2}}\,dx &= \frac{1}{4}\sqrt{cx^2+bx+a}\left(\frac{2\,fx}{c}+\frac{4\,ce-3\,bf}{c^2}\right)\\ &\quad-\frac{\left(8\,c^2d-4\,bce+3\,b^2f-4\,acf\right)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}+b\right|\right)}{8\,c^{\frac{5}{2}}}\end{aligned}$$

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(c*x^2 + b*x + a)*(2*f*x/c + (4*c*e - 3*b*f)/c^2) - 1/8*(8*c^2*d -
4*b*c*e + 3*b^2*f - 4*a*c*f)*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*
sqrt(c) + b))/c^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2}{\sqrt{a+bx+cx^2}}\,dx=\int \frac{f\,x^2+e\,x+d}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

```
[In] int((d + e*x + f*x^2)/(a + b*x + c*x^2)^(1/2),x)
[Out] int((d + e*x + f*x^2)/(a + b*x + c*x^2)^(1/2), x)
```

3.230 $\int \frac{d+ex+fx^2}{(g+hx)\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 179

$$\int \frac{d+ex+fx^2}{(g+hx)\sqrt{a+bx+cx^2}}\,dx=\frac{f\sqrt{a+bx+cx^2}}{ch}-\frac{(2cfg-2ceh+bfh)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}h^2}+\frac{(fg^2-h(eg-dh))\,\text{arctanh}\left(\frac{bg-2ah+(2cg-bh)x}{2\sqrt{cg^2-bgh+ah^2}\sqrt{a+bx+cx^2}}\right)}{h^2\sqrt{cg^2-bgh+ah^2}}$$

```
[Out] -1/2*(b*f*h-2*c*e*h+2*c*f*g)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1
/2))/c^(3/2)/h^2+(f*g^2-h*(-d*h+e*g))*arctanh(1/2*(b*g-2*a*h+(-b*h+2*c*g)*x
)/(a*h^2-b*g*h+c*g^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/h^2/(a*h^2-b*g*h+c*g^2)^(1
/2)+f*(c*x^2+b*x+a)^(1/2)/c/h
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1667, 857, 635, 212, 738}

$$\int \frac{d+ex+fx^2}{(g+hx)\sqrt{a+bx+cx^2}}\,dx=-\frac{\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(bfh-2ceh+2cfg)}{2c^{3/2}h^2}+\frac{(fg^2-h(eg-dh))\,\text{arctanh}\left(\frac{-2ah+x(2cg-bh)+bg}{2\sqrt{a+bx+cx^2}\sqrt{ah^2-bgh+cg^2}}\right)}{h^2\sqrt{ah^2-bgh+cg^2}}+\frac{f\sqrt{a+bx+cx^2}}{ch}$$

```
[In] Int[(d + e*x + f*x^2)/((g + h*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (f*Sqrt[a + b*x + c*x^2])/(c*h) - ((2*c*f*g - 2*c*e*h + b*f*h)*ArcTanh[(b +
 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*c^(3/2)*h^2) + ((f*g^2 - h*(
e*g - d*h))*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h +
 a*h^2]*Sqrt[a + b*x + c*x^2])])/(h^2*Sqrt[c*g^2 - b*g*h + a*h^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{f\sqrt{a+bx+cx^2}}{ch} + \frac{\int \frac{-\frac{1}{2}h(bfg-2cdh)-\frac{1}{2}h(2cfg-2ceh+bfh)x}{(g+hx)\sqrt{a+bx+cx^2}}\,dx}{ch^2} \\
&= \frac{f\sqrt{a+bx+cx^2}}{ch} - \frac{(2cfg-2ceh+bfh)\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{2ch^2} \\
&\quad + \frac{(fg^2-egh+dh^2)\int \frac{1}{(g+hx)\sqrt{a+bx+cx^2}}\,dx}{h^2} \\
&= \frac{f\sqrt{a+bx+cx^2}}{ch} - \frac{(2cfg-2ceh+bfh)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{ch^2} \\
&\quad - \frac{(2(fg^2-egh+dh^2))\,\text{Subst}\left(\int \frac{1}{4cg^2-4bgh+4ah^2-x^2}\,dx, x, \frac{-bg+2ah-(2cg-bh)x}{\sqrt{a+bx+cx^2}}\right)}{h^2} \\
&= \frac{f\sqrt{a+bx+cx^2}}{ch} - \frac{(2cfg-2ceh+bfh)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}h^2} \\
&\quad + \frac{(fg^2-h(eg-dh))\tanh^{-1}\left(\frac{bg-2ah+(2cg-bh)x}{2\sqrt{cg^2-bgh+ah^2}\sqrt{a+bx+cx^2}}\right)}{h^2\sqrt{cg^2-bgh+ah^2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.04

$$
\int \frac{d+ex+fx^2}{(g+hx)\sqrt{a+bx+cx^2}}\,dx
$$

$$
= \frac{\frac{2fh\sqrt{a+x(b+cx)}}{c} + \frac{4\sqrt{-cg^2+bgh-ah^2}\left(fg^2+h(-eg+dh)\right)\arctan\left(\frac{\sqrt{c}(g+hx)-h\sqrt{a+x(b+cx)}}{\sqrt{-cg^2+h(bg-ah)}}\right)}{cg^2+h(-bg+ah)} - \frac{(2cfg-2ceh+bfh)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b}}\right.}{c^{3/2}}}{2h^2}
$$

```
[In] Integrate[(d + e*x + f*x^2)/((g + h*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] ((2*f*h*Sqrt[a + x*(b + c*x)])/c + (4*Sqrt[-(c*g^2) + b*g*h - a*h^2]*(f*g^2
 + h*(-(e*g) + d*h))*ArcTan[(Sqrt[c]*(g + h*x) - h*Sqrt[a + x*(b + c*x)])/S
qrt[-(c*g^2) + h*(b*g - a*h)]])/(c*g^2 + h*(-(b*g) + a*h)) - ((2*c*f*g - 2*
c*e*h + b*f*h)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/c^(3
/2))/(2*h^2)
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.40

method	result
risch	$\frac{f\sqrt{c x^2+bx+a}}{ch}-\frac{\frac{(bfh-2ehc+2cfg)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c x^2+bx+a}\right)}{h\sqrt{c}}+\frac{2(d h^2-egh+f g^2)c\ln\left(\frac{\frac{2a h^2-2bgh+2c g^2}{h^2}+\frac{(bh-2cg)\left(x+\frac{g}{h}\right)}{h}+2\sqrt{\frac{a h^2-bgh-}{h^2}}}{x+}\right.}{h^2\sqrt{\frac{a h^2-bgh+c g^2}{h^2}}}}{2hc}$
default	$\frac{\frac{eh\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c x^2+bx+a}\right)}{\sqrt{c}}+fh\left(\frac{\sqrt{c x^2+bx+a}}{c}-\frac{b\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c x^2+bx+a}\right)}{2c^{\frac{3}{2}}}\right)-\frac{fg\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c x^2+bx+a}\right)}{\sqrt{c}}}{h^2}-\frac{(d h^2-egh+f g^2)\ln\left(\frac{2}{}\right.}{}$

```
[In] int((f*x^2+e*x+d)/(h*x+g)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] f*(c*x^2+b*x+a)^(1/2)/c/h-1/2/h/c*((b*f*h-2*c*e*h+2*c*f*g)/h*ln((1/2*b+c*x)
/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)+2*(d*h^2-e*g*h+f*g^2)*c/h^2/((a*h^2-b
*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+1/h*g
)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+
(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2}{(g+hx)\sqrt{a+bx+cx^2}}\,dx=\text{Timed out}$$

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{d+ex+fx^2}{(g+hx)\sqrt{a+bx+cx^2}}\,dx=\int \frac{d+ex+fx^2}{(g+hx)\sqrt{a+bx+cx^2}}\,dx$$

```
[In] integrate((f*x**2+e*x+d)/(h*x+g)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x + f*x**2)/((g + h*x)*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex+fx^2}{(g+hx)\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((b/h-(2*c*g)/h^2)^2>0)', see `assum
e?` for
```

Giac [F(-2)]

Exception generated.

$$\int \frac{d+ex+fx^2}{(g+hx)\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2}{(g+hx)\sqrt{a+bx+cx^2}}\,dx = \int \frac{f\,x^2+e\,x+d}{(g+h\,x)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

```
[In] int((d + e*x + f*x^2)/((g + h*x)*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] int((d + e*x + f*x^2)/((g + h*x)*(a + b*x + c*x^2)^(1/2)), x)
```

3.231 $\int \frac{d+ex+fx^2}{(g+hx)^2\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 241

$$\begin{aligned}
&\int \frac{d+ex+fx^2}{(g+hx)^2\sqrt{a+bx+cx^2}}\,dx \\
&= -\frac{(fg^2-h(eg-dh))\sqrt{a+bx+cx^2}}{h(cg^2-bgh+ah^2)(g+hx)}+\frac{f\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}h^2} \\
&\quad -\frac{(2c(fg^3-dgh^2)+h(2ah(2fg-eh)-b(3fg^2-egh-dh^2)))\operatorname{arctanh}\left(\frac{bg-2ah+(2cg-bh)x}{2\sqrt{cg^2-bgh+ah^2}\sqrt{a+bx+cx^2}}\right)}{2h^2(cg^2-bgh+ah^2)^{3/2}}
\end{aligned}$$

```
[Out] -1/2*(2*c*(-d*g*h^2+f*g^3)+h*(2*a*h*(-e*h+2*f*g)-b*(-d*h^2-e*g*h+3*f*g^2)))
*arctanh(1/2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^(1/2)/(c*x^2+b*
x+a)^(1/2))/h^2/(a*h^2-b*g*h+c*g^2)^(3/2)+f*arctanh(1/2*(2*c*x+b)/c^(1/2)/(
c*x^2+b*x+a)^(1/2))/h^2/c^(1/2)-(f*g^2-h*(-d*h+e*g))*(c*x^2+b*x+a)^(1/2)/h/
(a*h^2-b*g*h+c*g^2)/(h*x+g)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used

$= \{1664, 857, 635, 212, 738\}$

$$\int \frac{d+ex+fx^2}{(g+hx)^2\sqrt{a+bx+cx^2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{-2ah+x(2cg-bh)+bg}{2\sqrt{a+bx+cx^2}\sqrt{ah^2-bgh+cg^2}}\right)(2c(fg^3-dgh^2)-h(-2ah(2fg-eh)-bh(dh+eg)+3bfg^2))}{2h^2\left(ah^2-bgh+cg^2\right)^{3/2}} + \frac{f\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}h^2} - \frac{\sqrt{a+bx+cx^2}(fg^2-h(eg-dh))}{h(g+hx)\left(ah^2-bgh+cg^2\right)}$$

```
[In] Int[(d + e*x + f*x^2)/((g + h*x)^2*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -(((f*g^2 - h*(e*g - d*h))*Sqrt[a + b*x + c*x^2])/(h*(c*g^2 - b*g*h + a*h^2
)*(g + h*x))) + (f*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/
(Sqrt[c]*h^2) - ((2*c*(f*g^3 - d*g*h^2) - h*(3*b*f*g^2 - b*h*(e*g + d*h) -
2*a*h*(2*f*g - e*h)))*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2
 - b*g*h + a*h^2]*Sqrt[a + b*x + c*x^2])])/(2*h^2*(c*g^2 - b*g*h + a*h^2)^(
3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{(fg^2-h(eg-dh))\sqrt{a+bx+cx^2}}{h\,(cg^2-bgh+ah^2)\,(g+hx)} - \frac{\int \frac{\frac{1}{2}\left(-2cdg+beg+2afg-\frac{bfg^2}{h}+bdh-2aeh\right)+f\left(bg-\frac{cg^2}{h}-ah\right)x}{(g+hx)\sqrt{a+bx+cx^2}}\,dx}{cg^2-bgh+ah^2}$$

$$= -\frac{(fg^2-h(eg-dh))\sqrt{a+bx+cx^2}}{h\,(cg^2-bgh+ah^2)\,(g+hx)} + \frac{f\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{h^2} - \frac{(2c(fg^3-dgh^2)-h(3bfg^2-bh(eg+dh)-2ah(2fg-eh)))\int \frac{1}{(g+hx)\sqrt{a+bx+cx^2}}\,dx}{2h^2\,(cg^2-bgh+ah^2)}$$

$$= -\frac{(fg^2-h(eg-dh))\sqrt{a+bx+cx^2}}{h\,(cg^2-bgh+ah^2)\,(g+hx)} + \frac{(2f)\mathrm{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{h^2} + \frac{(2c(fg^3-dgh^2)-h(3bfg^2-bh(eg+dh)-2ah(2fg-eh)))\,\mathrm{Subst}\left(\int \frac{1}{4cg^2-4bgh+4ah^2-x^2}\,dx, x, -\right.}{h^2\,(cg^2-bgh+ah^2)}$$

$$= -\frac{(fg^2-h(eg-dh))\sqrt{a+bx+cx^2}}{h\,(cg^2-bgh+ah^2)\,(g+hx)} + \frac{f\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}h^2} - \frac{(2c(fg^3-dgh^2)-h(3bfg^2-bh(eg+dh)-2ah(2fg-eh)))\tanh^{-1}\left(\frac{bg-2ah+(2cg-bh)x}{2\sqrt{cg^2-bgh+ah^2}\sqrt{a+bx+cx^2}}\right)}{2h^2\,(cg^2-bgh+ah^2)^{3/2}}$$

Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.98

$$\int \frac{d+ex+fx^2}{(g+hx)^2\sqrt{a+bx+cx^2}}\,dx = -\frac{\frac{h(fg^2+h(-eg+dh))\sqrt{a+x(b+cx)}}{(cg^2+h(-bg+ah))(g+hx)} + \frac{\sqrt{-cg^2+bgh-ah^2}\,(2c(fg^3-dgh^2)+h(-3bfg^2+bh(eg+dh)-2ah(-2fg+eh)))\arctan\left(\frac{\sqrt{c}(g+hx)-h\sqrt{}}{\sqrt{-cg^2+h(}}\right.}{(cg^2+h(-bg+ah))^2}}{h^2}$$

```
[In] Integrate[(d + e*x + f*x^2)/((g + h*x)^2*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -(((h*(f*g^2 + h*(-(e*g) + d*h))*Sqrt[a + x*(b + c*x)])/((c*g^2 + h*(-(b*g)
 + a*h))*(g + h*x)) + (Sqrt[-(c*g^2) + b*g*h - a*h^2]*(2*c*(f*g^3 - d*g*h^2
) + h*(-3*b*f*g^2 + b*h*(e*g + d*h) - 2*a*h*(-2*f*g + e*h)))*ArcTan[(Sqrt[c
]*(g + h*x) - h*Sqrt[a + x*(b + c*x)])/Sqrt[-(c*g^2) + h*(b*g - a*h)]])/(c*
g^2 + h*(-(b*g) + a*h))^2 + (f*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*
x)]])/Sqrt[c])/h^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 484 vs. 2(221) = 442.

Time = 0.84 (sec) , antiderivative size = 485, normalized size of antiderivative = 2.01

method	result
default	$\frac{f\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{h^2\sqrt{c}} - \frac{(eh-2fg)\ln\left(\frac{\frac{2a\,h^2-2bgh+2c\,g^2}{h^2}+\frac{(bh-2cg)\left(x+\frac{g}{h}\right)}{h}+2\sqrt{\frac{a\,h^2-bgh+c\,g^2}{h^2}}\sqrt{\left(x+\frac{g}{h}\right)^2c+\frac{(bh-2cg)\left(x+\frac{g}{h}\right)}{h}+}}{x+\frac{g}{h}}\right.}{h^3\sqrt{\frac{a\,h^2-bgh+c\,g^2}{h^2}}}$

```
[In] int((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] f/h^2*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-1/h^3*(e*h-2*f*g)
/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/
h*(x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*
(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g))+1/h^4*(d*h^2-e*g*h+f*g
^2)*(-1/(a*h^2-b*g*h+c*g^2)*h^2/(x+1/h*g)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1
/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)+1/2*(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)/
((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h
*(x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(
x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2}{(g+hx)^2\sqrt{a+bx+cx^2}}\,dx = \text{Timed out}$$

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{d+ex+fx^2}{(g+hx)^2\sqrt{a+bx+cx^2}}\,dx = \int \frac{d+ex+fx^2}{(g+hx)^2\sqrt{a+bx+cx^2}}\,dx$$

```
[In] integrate((f*x**2+e*x+d)/(h*x+g)**2/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x + f*x**2)/((g + h*x)**2*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex+fx^2}{(g+hx)^2\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((b/h-(2*c*g)/h^2)^2>0)', see `assum
e?` for
```

Giac [F(-2)]

Exception generated.

$$\int \frac{d+ex+fx^2}{(g+hx)^2\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2}{(g+hx)^2\sqrt{a+bx+cx^2}}\,dx = \int \frac{f\,x^2+e\,x+d}{\left(g+h\,x\right)^2\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

```
[In] int((d + e*x + f*x^2)/((g + h*x)^2*(a + b*x + c*x^2)^(1/2)),x)
[Out] int((d + e*x + f*x^2)/((g + h*x)^2*(a + b*x + c*x^2)^(1/2)), x)
```

3.232 $\int \frac{d+ex+fx^2}{(g+hx)^3\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 336

$$\int \frac{d+ex+fx^2}{(g+hx)^3\sqrt{a+bx+cx^2}}\,dx = -\frac{(fg^2-h(eg-dh))\sqrt{a+bx+cx^2}}{2h\,(cg^2-bgh+ah^2)\,(g+hx)^2}$$
$$+\frac{(2cg(fg^2+h(eg-3dh))+h(4ah(2fg-eh)-b(5fg^2-egh-3dh^2)))\sqrt{a+bx+cx^2}}{4h\,(cg^2-bgh+ah^2)^2\,(g+hx)}$$
$$+\frac{(8c^2dg^2+8a^2fh^2-4abh(2fg+eh)+b^2(3fg^2+egh+3dh^2)-4c(bg(eg+2dh)+a(fg^2-3egh+dh^2}{8\,(cg^2-bgh+ah^2)^{5/2}}$$

```
[Out] 1/8*(8*c^2*d*g^2+8*a^2*f*h^2-4*a*b*h*(e*h+2*f*g)+b^2*(3*d*h^2+e*g*h+3*f*g^2
)-4*c*(b*g*(2*d*h+e*g)+a*(d*h^2-3*e*g*h+f*g^2)))*arctanh(1/2*(b*g-2*a*h+(-b
*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/(a*h^2-b*g*h+c*
g^2)^(5/2)-1/2*(f*g^2-h*(-d*h+e*g))*(c*x^2+b*x+a)^(1/2)/h/(a*h^2-b*g*h+c*g^
2)/(h*x+g)^2+1/4*(2*c*g*(f*g^2+h*(-3*d*h+e*g))+h*(4*a*h*(-e*h+2*f*g)-b*(-3*
d*h^2-e*g*h+5*f*g^2)))*(c*x^2+b*x+a)^(1/2)/h/(a*h^2-b*g*h+c*g^2)^2/(h*x+g)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used

$= \{1664, 820, 738, 212\}$

$$\int \frac{d+ex+fx^2}{(g+hx)^3\sqrt{a+bx+cx^2}}\,dx$$

$$= \frac{\operatorname{arctanh}\left(\frac{-2ah+x(2cg-bh)+bg}{2\sqrt{a+bx+cx^2}\sqrt{ah^2-bgh+cg^2}}\right)\left(8a^2fh^2-4c(-ah(3eg-dh)+afg^2+bg(2dh+eg))-4abh(eh+2f\right.}{8\left(ah^2-bgh+cg^2\right)^{5/2}}$$

$$-\frac{\sqrt{a+bx+cx^2}(fg^2-h(eg-dh))}{2h(g+hx)^2\left(ah^2-bgh+cg^2\right)}$$

$$+\frac{\sqrt{a+bx+cx^2}(2c(gh(eg-3dh)+fg^3)-h(-4ah(2fg-eh)-bh(3dh+eg)+5bfg^2))}{4h(g+hx)\left(ah^2-bgh+cg^2\right)^2}$$

```
[In] Int[(d + e*x + f*x^2)/((g + h*x)^3*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -1/2*((f*g^2 - h*(e*g - d*h))*Sqrt[a + b*x + c*x^2])/(h*(c*g^2 - b*g*h + a*
h^2)*(g + h*x)^2) + ((2*c*(f*g^3 + g*h*(e*g - 3*d*h)) - h*(5*b*f*g^2 - b*h*
(e*g + 3*d*h) - 4*a*h*(2*f*g - e*h)))*Sqrt[a + b*x + c*x^2])/(4*h*(c*g^2 -
b*g*h + a*h^2)^2*(g + h*x)) + ((8*c^2*d*g^2 + 8*a^2*f*h^2 - 4*a*b*h*(2*f*g
+ e*h) - 4*c*(a*f*g^2 - a*h*(3*e*g - d*h) + b*g*(e*g + 2*d*h)) + b^2*(3*f*g
^2 + h*(e*g + 3*d*h)))*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^
2 - b*g*h + a*h^2]*Sqrt[a + b*x + c*x^2])])/(8*(c*g^2 - b*g*h + a*h^2)^(5/2
))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{(fg^2-h(eg-dh))\sqrt{a+bx+cx^2}}{2h\left(cg^2-bgh+ah^2\right)(g+hx)^2} - \frac{\int \frac{\frac{1}{2}\left(-4cdg+beg+4afg-\frac{bfg^2}{h}+3bdh-4aeh\right)-\left(ceg-2bfg+\frac{cfg^2}{h}-cdh+2afh\right)x}{(g+hx)^2\sqrt{a+bx+cx^2}}\,dx}{2\left(cg^2-bgh+ah^2\right)}$$

$$= -\frac{(fg^2-h(eg-dh))\sqrt{a+bx+cx^2}}{2h\left(cg^2-bgh+ah^2\right)(g+hx)^2} + \frac{(2c(fg^3+gh(eg-3dh))-h(5bfg^2-bh(eg+3dh)-4ah(2fg-eh)))\sqrt{a+bx+cx^2}}{4h\left(cg^2-bgh+ah^2\right)^2(g+hx)} + \frac{(8c^2dg^2+8a^2fh^2-4abh(2fg+eh)-4c(afg^2-ah(3eg-dh)+bg(eg+2dh))+b^2(3fg^2+h(}{8\left(cg^2-bgh+ah^2\right)^2}$$

$$= -\frac{(fg^2-h(eg-dh))\sqrt{a+bx+cx^2}}{2h\left(cg^2-bgh+ah^2\right)(g+hx)^2} + \frac{(2c(fg^3+gh(eg-3dh))-h(5bfg^2-bh(eg+3dh)-4ah(2fg-eh)))\sqrt{a+bx+cx^2}}{4h\left(cg^2-bgh+ah^2\right)^2(g+hx)} - \frac{(8c^2dg^2+8a^2fh^2-4abh(2fg+eh)-4c(afg^2-ah(3eg-dh)+bg(eg+2dh))+b^2(3fg^2+h(}{4\left(cg^2-bgh+ah^2\right)^2}$$

$$= -\frac{(fg^2-h(eg-dh))\sqrt{a+bx+cx^2}}{2h\left(cg^2-bgh+ah^2\right)(g+hx)^2} + \frac{(2c(fg^3+gh(eg-3dh))-h(5bfg^2-bh(eg+3dh)-4ah(2fg-eh)))\sqrt{a+bx+cx^2}}{4h\left(cg^2-bgh+ah^2\right)^2(g+hx)} + \frac{(8c^2dg^2+8a^2fh^2-4abh(2fg+eh)-4c(afg^2-ah(3eg-dh)+bg(eg+2dh))+b^2(3fg^2+h(}{8\left(cg^2-bgh+ah^2\right)^{5/2}}$$

Mathematica [A] (verified)

Time = 11.21 (sec) , antiderivative size = 471, normalized size of antiderivative = 1.40

$$\int \frac{d+ex+fx^2}{(g+hx)^3\sqrt{a+bx+cx^2}}\,dx =$$

$$-\frac{\frac{4h\left(fg^2+h(-eg+dh)\right)\sqrt{a+x(b+cx)}}{(cg^2+h(-bg+ah))(g+hx)^2}+\frac{8h(-2fg+eh)\sqrt{a+x(b+cx)}}{(cg^2+h(-bg+ah))(g+hx)}+\frac{4(2cg-bh)(2fg-eh)\operatorname{arctanh}\left(\frac{-2ah+2cgx+b(g-hx)}{2\sqrt{cg^2+h(-bg+ah)}\sqrt{a+x(b+cx)}}\right)}{(cg^2+h(-bg+ah))^{3/2}}-}{}$$

```
[In] Integrate[(d + e*x + f*x^2)/((g + h*x)^3*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -1/8*((4*h*(f*g^2 + h*(-(e*g) + d*h))*Sqrt[a + x*(b + c*x)])/((c*g^2 + h*(-
(b*g) + a*h))*(g + h*x)^2) + (8*h*(-2*f*g + e*h)*Sqrt[a + x*(b + c*x)])/((c
*g^2 + h*(-(b*g) + a*h))*(g + h*x)) + (4*(2*c*g - b*h)*(2*f*g - e*h)*ArcTan
h[(-2*a*h + 2*c*g*x + b*(g - h*x))/(2*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*Sqrt[a
 + x*(b + c*x)])])/(c*g^2 + h*(-(b*g) + a*h))^(3/2) - (8*f*ArcTanh[(-2*a*h
+ 2*c*g*x + b*(g - h*x))/(2*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*Sqrt[a + x*(b +
c*x)])])/Sqrt[c*g^2 + h*(-(b*g) + a*h)] + ((f*g^2 + h*(-(e*g) + d*h))*(6*h*
(2*c*g - b*h)*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*Sqrt[a + x*(b + c*x)] - (8*c^2
*g^2 + 3*b^2*h^2 - 4*c*h*(2*b*g + a*h))*(g + h*x)*ArcTanh[(-2*a*h + 2*c*g*x
 + b*(g - h*x))/(2*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*Sqrt[a + x*(b + c*x)])]))
/((c*g^2 + h*(-(b*g) + a*h))^(5/2)*(g + h*x)))/h^2
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1012 vs. 2(318) = 636.

Time = 0.99 (sec) , antiderivative size = 1013, normalized size of antiderivative = 3.01

method	result	size
default	Expression too large to display	1013

```
[In] int((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -f/h^3/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2
*c*g)/h*(x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c+(b*h-2*c
*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g))+(e*h-2*f*g)/h^4*
(-1/(a*h^2-b*g*h+c*g^2)*h^2/(x+1/h*g)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g
)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)+1/2*(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)/((a*
h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+
1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/
h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g)))+(d*h^2-e*g*h+f*g^2)/h^5*(-
1/2/(a*h^2-b*g*h+c*g^2)*h^2/(x+1/h*g)^2*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h
*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)-3/4*(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)*(-
```

```
1/(a*h^2-b*g*h+c*g^2)*h^2/(x+1/h*g)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+
(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)+1/2*(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)/((a*h^
2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+1/
h*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*
g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g)))+1/2*c/(a*h^2-b*g*h+c*g^2)*h^
2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)
/h*(x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c+(b*h-2*c*g)/h
*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 996 vs. $2(318) = 636$.

Time = 34.05 (sec) , antiderivative size = 2034, normalized size of antiderivative = 6.05

$$\int \frac{d+ex+fx^2}{(g+hx)^3\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [1/16*(((8*c^2*d - 4*b*c*e + (3*b^2 - 4*a*c)*f)*g^4 - (8*b*c*d + 8*a*b*f -
(b^2 + 12*a*c)*e)*g^3*h - (4*a*b*e - 8*a^2*f - (3*b^2 - 4*a*c)*d)*g^2*h^2 +
 ((8*c^2*d - 4*b*c*e + (3*b^2 - 4*a*c)*f)*g^2*h^2 - (8*b*c*d + 8*a*b*f - (b
^2 + 12*a*c)*e)*g*h^3 - (4*a*b*e - 8*a^2*f - (3*b^2 - 4*a*c)*d)*h^4)*x^2 +
2*((8*c^2*d - 4*b*c*e + (3*b^2 - 4*a*c)*f)*g^3*h - (8*b*c*d + 8*a*b*f - (b^
2 + 12*a*c)*e)*g^2*h^2 - (4*a*b*e - 8*a^2*f - (3*b^2 - 4*a*c)*d)*g*h^3)*x)*
sqrt(c*g^2 - b*g*h + a*h^2)*log((8*a*b*g*h - 8*a^2*h^2 - (b^2 + 4*a*c)*g^2
- (8*c^2*g^2 - 8*b*c*g*h + (b^2 + 4*a*c)*h^2)*x^2 - 4*sqrt(c*g^2 - b*g*h +
a*h^2)*sqrt(c*x^2 + b*x + a)*(b*g - 2*a*h + (2*c*g - b*h)*x) - 2*(4*b*c*g^2
 + 4*a*b*h^2 - (3*b^2 + 4*a*c)*g*h)*x)/(h^2*x^2 + 2*g*h*x + g^2)) - 4*(2*a^
2*d*h^5 - (4*c^2*e - 3*b*c*f)*g^5 + (8*c^2*d + 5*b*c*e - 3*(b^2 + 2*a*c)*f)
*g^4*h - (13*b*c*d - 9*a*b*f + (b^2 + 2*a*c)*e)*g^3*h^2 - (a*b*e + 6*a^2*f
- 5*(b^2 + 2*a*c)*d)*g^2*h^3 - (7*a*b*d - 2*a^2*e)*g*h^4 - (2*c^2*f*g^5 + (
2*c^2*e - 7*b*c*f)*g^4*h - (6*c^2*d + b*c*e - 5*(b^2 + 2*a*c)*f)*g^3*h^2 +
(9*b*c*d - 13*a*b*f - (b^2 + 2*a*c)*e)*g^2*h^3 + (5*a*b*e + 8*a^2*f - 3*(b^
2 + 2*a*c)*d)*g*h^4 + (3*a*b*d - 4*a^2*e)*h^5)*x)*sqrt(c*x^2 + b*x + a))/(c
^3*g^8 - 3*b*c^2*g^7*h - 3*a^2*b*g^3*h^5 + a^3*g^2*h^6 + 3*(b^2*c + a*c^2)*
g^6*h^2 - (b^3 + 6*a*b*c)*g^5*h^3 + 3*(a*b^2 + a^2*c)*g^4*h^4 + (c^3*g^6*h^
2 - 3*b*c^2*g^5*h^3 - 3*a^2*b*g*h^7 + a^3*h^8 + 3*(b^2*c + a*c^2)*g^4*h^4 -
 (b^3 + 6*a*b*c)*g^3*h^5 + 3*(a*b^2 + a^2*c)*g^2*h^6)*x^2 + 2*(c^3*g^7*h -
3*b*c^2*g^6*h^2 - 3*a^2*b*g^2*h^6 + a^3*g*h^7 + 3*(b^2*c + a*c^2)*g^5*h^3 -
 (b^3 + 6*a*b*c)*g^4*h^4 + 3*(a*b^2 + a^2*c)*g^3*h^5)*x), 1/8*(((8*c^2*d -
4*b*c*e + (3*b^2 - 4*a*c)*f)*g^4 - (8*b*c*d + 8*a*b*f - (b^2 + 12*a*c)*e)*g
^3*h - (4*a*b*e - 8*a^2*f - (3*b^2 - 4*a*c)*d)*g^2*h^2 + ((8*c^2*d - 4*b*c*
```

```
e + (3*b^2 - 4*a*c)*f)*g^2*h^2 - (8*b*c*d + 8*a*b*f - (b^2 + 12*a*c)*e)*g*h
^3 - (4*a*b*e - 8*a^2*f - (3*b^2 - 4*a*c)*d)*h^4)*x^2 + 2*((8*c^2*d - 4*b*c
*e + (3*b^2 - 4*a*c)*f)*g^3*h - (8*b*c*d + 8*a*b*f - (b^2 + 12*a*c)*e)*g^2*
h^2 - (4*a*b*e - 8*a^2*f - (3*b^2 - 4*a*c)*d)*g*h^3)*x)*sqrt(-c*g^2 + b*g*h
 - a*h^2)*arctan(-1/2*sqrt(-c*g^2 + b*g*h - a*h^2)*sqrt(c*x^2 + b*x + a)*(b
*g - 2*a*h + (2*c*g - b*h)*x)/(a*c*g^2 - a*b*g*h + a^2*h^2 + (c^2*g^2 - b*c
*g*h + a*c*h^2)*x^2 + (b*c*g^2 - b^2*g*h + a*b*h^2)*x)) - 2*(2*a^2*d*h^5 -
(4*c^2*e - 3*b*c*f)*g^5 + (8*c^2*d + 5*b*c*e - 3*(b^2 + 2*a*c)*f)*g^4*h - (
13*b*c*d - 9*a*b*f + (b^2 + 2*a*c)*e)*g^3*h^2 - (a*b*e + 6*a^2*f - 5*(b^2 +
 2*a*c)*d)*g^2*h^3 - (7*a*b*d - 2*a^2*e)*g*h^4 - (2*c^2*f*g^5 + (2*c^2*e -
7*b*c*f)*g^4*h - (6*c^2*d + b*c*e - 5*(b^2 + 2*a*c)*f)*g^3*h^2 + (9*b*c*d -
 13*a*b*f - (b^2 + 2*a*c)*e)*g^2*h^3 + (5*a*b*e + 8*a^2*f - 3*(b^2 + 2*a*c)
*d)*g*h^4 + (3*a*b*d - 4*a^2*e)*h^5)*x)*sqrt(c*x^2 + b*x + a))/(c^3*g^8 - 3
*b*c^2*g^7*h - 3*a^2*b*g^3*h^5 + a^3*g^2*h^6 + 3*(b^2*c + a*c^2)*g^6*h^2 -
(b^3 + 6*a*b*c)*g^5*h^3 + 3*(a*b^2 + a^2*c)*g^4*h^4 + (c^3*g^6*h^2 - 3*b*c^
2*g^5*h^3 - 3*a^2*b*g*h^7 + a^3*h^8 + 3*(b^2*c + a*c^2)*g^4*h^4 - (b^3 + 6*
a*b*c)*g^3*h^5 + 3*(a*b^2 + a^2*c)*g^2*h^6)*x^2 + 2*(c^3*g^7*h - 3*b*c^2*g^
6*h^2 - 3*a^2*b*g^2*h^6 + a^3*g*h^7 + 3*(b^2*c + a*c^2)*g^5*h^3 - (b^3 + 6*
a*b*c)*g^4*h^4 + 3*(a*b^2 + a^2*c)*g^3*h^5)*x)]
```

Sympy [F]

$$\int \frac{d+ex+fx^2}{(g+hx)^3\sqrt{a+bx+cx^2}}\,dx=\int \frac{d+ex+fx^2}{\left(g+hx\right)^3\sqrt{a+bx+cx^2}}\,dx$$

```
[In] integrate((f*x**2+e*x+d)/(h*x+g)**3/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x + f*x**2)/((g + h*x)**3*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex+fx^2}{(g+hx)^3\sqrt{a+bx+cx^2}}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*h^2-b*g*h>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2279 vs. 2(318) = 636.

Time = 0.33 (sec) , antiderivative size = 2279, normalized size of antiderivative = 6.78

$$\int \frac{d+ex+fx^2}{(g+hx)^3\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(8*c^2*d*g^2 - 4*b*c*e*g^2 + 3*b^2*f*g^2 - 4*a*c*f*g^2 - 8*b*c*d*g*h +
b^2*e*g*h + 12*a*c*e*g*h - 8*a*b*f*g*h + 3*b^2*d*h^2 - 4*a*c*d*h^2 - 4*a*b*
e*h^2 + 8*a^2*f*h^2)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*h + sqrt(
c)*g)/sqrt(-c*g^2 + b*g*h - a*h^2))/((c^2*g^4 - 2*b*c*g^3*h + b^2*g^2*h^2 +
 2*a*c*g^2*h^2 - 2*a*b*g*h^3 + a^2*h^4)*sqrt(-c*g^2 + b*g*h - a*h^2)) + 1/4
*(8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*c^2*f*g^4*h - 16*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^3*b*c*f*g^3*h^2 - 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
^3*c^2*d*g^2*h^3 + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b*c*e*g^2*h^3 +
5*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^2*f*g^2*h^3 + 20*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^3*a*c*f*g^2*h^3 + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
^3*b*c*d*g*h^4 - (sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^2*e*g*h^4 - 12*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*c*e*g*h^4 - 8*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^3*a*b*f*g*h^4 - 3*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^2*d*h
^5 + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*c*d*h^5 + 4*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^3*a*b*e*h^5 + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c
^(5/2)*f*g^5 + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c^(5/2)*e*g^4*h - 16
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b*c^(3/2)*f*g^4*h - 24*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^2*c^(5/2)*d*g^3*h^2 - 4*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^2*b*c^(3/2)*e*g^3*h^2 - (sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2*s
qrt(c)*f*g^3*h^2 + 28*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*c^(3/2)*f*g^3
*h^2 + 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b*c^(3/2)*d*g^2*h^3 + 5*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2*sqrt(c)*e*g^2*h^3 - 20*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^2*a*c^(3/2)*e*g^2*h^3 + 8*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))^2*a*b*sqrt(c)*f*g^2*h^3 - 9*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2
*b^2*sqrt(c)*d*g*h^4 + 12*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*c^(3/2)*d
*g*h^4 - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b*sqrt(c)*e*g*h^4 - 16*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*sqrt(c)*f*g*h^4 + 8*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^2*a^2*sqrt(c)*e*h^5 + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))*b*c^2*f*g^5 + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b*c^2*e*g^4*h -
20*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^2*c*f*g^4*h - 8*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))*a*c^2*f*g^4*h - 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b*
c^2*d*g^3*h^2 - 16*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*c^2*e*g^3*h^2 + 3*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*f*g^3*h^2 + 60*(sqrt(c)*x - sqrt(c*
x^2 + b*x + a))*a*b*c*f*g^3*h^2 + 20*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^
2*c*d*g^2*h^3 + 40*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*c^2*d*g^2*h^3 + (s
```

```
qrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*e*g^2*h^3 - 16*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))*a*b*c*e*g^2*h^3 - 11*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^
2*f*g^2*h^3 - 44*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*c*f*g^2*h^3 - 5*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*d*g*h^4 - 28*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))*a*b*c*d*g*h^4 + 3*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^2*e*g
*h^4 + 20*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*c*e*g*h^4 + 8*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))*a^2*b*f*g*h^4 + 5*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))*a*b^2*d*h^5 + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*c*d*h^5 - 4*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b*e*h^5 + 2*b^2*c^(3/2)*f*g^5 + 2*b^2*
c^(3/2)*e*g^4*h - 5*b^3*sqrt(c)*f*g^4*h - 4*a*b*c^(3/2)*f*g^4*h - 6*b^2*c^(
3/2)*d*g^3*h^2 + b^3*sqrt(c)*e*g^3*h^2 - 8*a*b*c^(3/2)*e*g^3*h^2 + 21*a*b^2
*sqrt(c)*f*g^3*h^2 + 4*a^2*c^(3/2)*f*g^3*h^2 + 3*b^3*sqrt(c)*d*g^2*h^3 + 20
*a*b*c^(3/2)*d*g^2*h^3 - 5*a*b^2*sqrt(c)*e*g^2*h^3 + 4*a^2*c^(3/2)*e*g^2*h^
3 - 32*a^2*b*sqrt(c)*f*g^2*h^3 - 11*a*b^2*sqrt(c)*d*g*h^4 - 12*a^2*c^(3/2)*
d*g*h^4 + 12*a^2*b*sqrt(c)*e*g*h^4 + 16*a^3*sqrt(c)*f*g*h^4 + 8*a^2*b*sqrt(
c)*d*h^5 - 8*a^3*sqrt(c)*e*h^5)/((c^2*g^4*h^2 - 2*b*c*g^3*h^3 + b^2*g^2*h^4
 + 2*a*c*g^2*h^4 - 2*a*b*g*h^5 + a^2*h^6)*((sqrt(c)*x - sqrt(c*x^2 + b*x +
a))^2*h + 2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c)*g + b*g - a*h)^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2}{(g+hx)^3\sqrt{a+bx+cx^2}}\,dx=\int \frac{f\,x^2+e\,x+d}{{\left(g+h\,x\right)}^3\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

```
[In] int((d + e*x + f*x^2)/((g + h*x)^3*(a + b*x + c*x^2)^(1/2)),x)
[Out] int((d + e*x + f*x^2)/((g + h*x)^3*(a + b*x + c*x^2)^(1/2)), x)
```

3.233 $\int \frac{(g+hx)^3(d+ex+fx^2)}{(a+bx+cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 504

$$\int \frac{(g+hx)^3\,(d+ex+fx^2)}{(a+bx+cx^2)^{3/2}}\,dx = \frac{2\big(c\big(2ae-b\big(d+\frac{af}{c}\big)\big)-(2c^2d-bce+b^2f-2acf)\,x\big)\,(g+hx)^3}{c\,(b^2-4ac)\,\sqrt{a+bx+cx^2}}$$

$$+\frac{(12c^2d-6bce+7b^2f-16acf)\,h(g+hx)^2\sqrt{a+bx+cx^2}}{3c^2\,(b^2-4ac)}$$

$$+\frac{h(192c^4dg^2+105b^4fh^2-10b^2ch(46afh+9b(3fg+eh))-16c^3(3bg(2eg+3dh)+4a(7fg^2+9egh+3dh^2}{}$$

$$-\frac{(35b^3fh^3-30bch^2(3bfg+beh+2afh)-16c^3g(fg^2+3h(eg+dh))+24c^2h(ah(3fg+eh)+b(3fg^2+3e}{16c^{9/2}}$$

```
[Out] -1/16*(35*b^3*f*h^3-30*b*c*h^2*(2*a*f*h+b*e*h+3*b*f*g)-16*c^3*g*(f*g^2+3*h*
(d*h+e*g))+24*c^2*h*(a*h*(e*h+3*f*g)+b*(d*h^2+3*e*g*h+3*f*g^2)))*arctanh(1/
2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(9/2)+2*(c*(2*a*e-b*(d+a*f/c))-(
-2*a*c*f+b^2*f-b*c*e+2*c^2*d)*x)*(h*x+g)^3/c/(-4*a*c+b^2)/(c*x^2+b*x+a)^(1/
2)+1/3*(-16*a*c*f+7*b^2*f-6*b*c*e+12*c^2*d)*h*(h*x+g)^2*(c*x^2+b*x+a)^(1/2)
/c^2/(-4*a*c+b^2)+1/24*h*(192*c^4*d*g^2+105*b^4*f*h^2-10*b^2*c*h*(46*a*f*h+
9*b*(e*h+3*f*g))-16*c^3*(3*b*g*(3*d*h+2*e*g)+4*a*(3*d*h^2+9*e*g*h+7*f*g^2))
+8*c^2*(32*a^2*f*h^2+39*a*b*h*(e*h+3*f*g)+b^2*(20*f*g^2+9*h*(d*h+3*e*g)))+2
*c*h*(48*c^3*d*g-35*b^3*f*h-8*c^2*(9*a*e*h+11*a*f*g+3*b*d*h+3*b*e*g)+2*b*c*
(58*a*f*h+15*b*e*h+17*b*f*g))*x)*(c*x^2+b*x+a)^(1/2)/c^4/(-4*a*c+b^2)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 502, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {1658, 846, 793, 635, 212}

$$\int \frac{(g+hx)^3\left(d+ex+fx^2\right)}{\left(a+bx+cx^2\right)^{3/2}}\,dx = \frac{h\sqrt{a+bx+cx^2}\Big(8c(32a^2fh^2+39abh(eh+3fg)+b^2(9h(dh+3eg)+20}{}$$

$$-\frac{\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(24c^2h(ah(eh+3fg)+bh(dh+3eg)+3bfg^2)-30bch^2(2afh+beh+3bfg)+3}{16c^{9/2}}$$

$$+\frac{2(g+hx)^3\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x(-2acf+b^2f-bce+2c^2d)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

$$+\frac{h(g+hx)^2\sqrt{a+bx+cx^2}(-16acf+7b^2f-6bce+12c^2d)}{3c^2\left(b^2-4ac\right)}$$

```
[In] Int[((g + h*x)^3*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (2*(c*(2*a*e - b*(d + (a*f)/c)) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x)*(g
 + h*x)^3)/(c*(b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]) + ((12*c^2*d - 6*b*c*e +
 7*b^2*f - 16*a*c*f)*h*(g + h*x)^2*Sqrt[a + b*x + c*x^2])/(3*c^2*(b^2 - 4*a
*c)) + (h*(192*c^3*d*g^2 + (105*b^4*f*h^2)/c - 10*b^2*h*(46*a*f*h + 9*b*(3*
f*g + e*h)) - 16*c^2*(3*b*g*(2*e*g + 3*d*h) + 4*a*(7*f*g^2 + 9*e*g*h + 3*d*
h^2)) + 8*c*(32*a^2*f*h^2 + 39*a*b*h*(3*f*g + e*h) + b^2*(20*f*g^2 + 9*h*(3
*e*g + d*h))) + 2*h*(48*c^3*d*g - 35*b^3*f*h - 8*c^2*(3*b*e*g + 11*a*f*g +
3*b*d*h + 9*a*e*h) + 2*b*c*(17*b*f*g + 15*b*e*h + 58*a*f*h))*x)*Sqrt[a + b*
x + c*x^2])/(24*c^3*(b^2 - 4*a*c)) - ((35*b^3*f*h^3 - 30*b*c*h^2*(3*b*f*g +
 b*e*h + 2*a*f*h) - 16*c^3*(f*g^3 + 3*g*h*(e*g + d*h)) + 24*c^2*h*(3*b*f*g^
2 + b*h*(3*e*g + d*h) + a*h*(3*f*g + e*h)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*
Sqrt[a + b*x + c*x^2])])/(16*c^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 846

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1658

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f =
Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[Polynom
ialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x + c
*x^2)^(p + 1)*((f*b - 2*a*g + (2*c*f - b*g)*x)/((p + 1)*(b^2 - 4*a*c))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Q + g*(2*a*e*m + b*d*(2
*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(m + 2*p + 3)*x, x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Integer
Q[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] && Ra
tionalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\text{integral} = \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)(g+hx)^3}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} - \frac{2\int\frac{(g+hx)^2\left(-\frac{b^2fg+6b(cd+af)h-4ac(fg+3eh)}{2c}-\frac{1}{2}\left(12cd-6be-16af+\frac{7b^2f}{c}\right)hx\right)}{\sqrt{a+bx+cx^2}}\,dx}{b^2-4ac}$$

$$=\frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)(g+hx)^3}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

$$+\frac{\left(12c^2d+7b^2f-2c(3be+8af)\right)h(g+hx)^2\sqrt{a+bx+cx^2}}{3c^2\left(b^2-4ac\right)}$$

$$-\frac{2\int\frac{(g+hx)\left(\frac{7b^3fgh-4bch(6cdg+13afg+6aeh)+b^2\left(28afh^2-6cg(fg+eh)\right)-8ac\left(8afh^2-3c\left(fg^2+3egh+2dh^2\right)\right)}{4c}-\frac{h\left(48c^3dg-35b^3fh-8c^2(3b\right.}{}\right.}{\sqrt{a+bx+cx^2}}}{3c\left(b^2-4ac\right)}$$

$$=\frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)(g+hx)^3}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

$$+\frac{\left(12c^2d+7b^2f-2c(3be+8af)\right)h(g+hx)^2\sqrt{a+bx+cx^2}}{3c^2\left(b^2-4ac\right)}$$

$$+\frac{h\left(192c^3dg^2+\frac{105b^4fh^2}{c}-10b^2h(46afh+9b(3fg+eh))-16c^2(3bg(2eg+3dh)+4a(7fg^2+9e\right.}{}$$

$$-\frac{(35b^3fh^3-30bch^2(3bfg+beh+2afh)-16c^3(fg^3+3gh(eg+dh))+24c^2h(3bfg^2+bh(3eg}{16c^4}$$

$$=\frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)(g+hx)^3}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

$$+\frac{\left(12c^2d+7b^2f-2c(3be+8af)\right)h(g+hx)^2\sqrt{a+bx+cx^2}}{3c^2\left(b^2-4ac\right)}$$

$$+\frac{h\left(192c^3dg^2+\frac{105b^4fh^2}{c}-10b^2h(46afh+9b(3fg+eh))-16c^2(3bg(2eg+3dh)+4a(7fg^2+9e\right.}{}$$

$$-\frac{(35b^3fh^3-30bch^2(3bfg+beh+2afh)-16c^3(fg^3+3gh(eg+dh))+24c^2h(3bfg^2+bh(3eg}{8c^4}$$

$$=\frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)(g+hx)^3}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

$$+\frac{\left(12c^2d+7b^2f-2c(3be+8af)\right)h(g+hx)^2\sqrt{a+bx+cx^2}}{3c^2\left(b^2-4ac\right)}$$

$$+\frac{h\left(192c^3dg^2+\frac{105b^4fh^2}{c}-10b^2h(46afh+9b(3fg+eh))-16c^2(3bg(2eg+3dh)+4a(7fg^2+9e\right.}{}$$

$$-\frac{(35b^3fh^3-30bch^2(3bfg+beh+2afh)-16c^3(fg^3+3gh(eg+dh))+24c^2h(3bfg^2+bh(3eg}{16c^{9/2}}$$

Mathematica [A] (verified)

Time = 4.72 (sec) , antiderivative size = 694, normalized size of antiderivative = 1.38

$$\int \frac{(g+hx)^3\left(d+ex+fx^2\right)}{\left(a+bx+cx^2\right)^{3/2}}\,dx =$$

$$-\frac{-105b^5fh^3x-5b^4h^2(21afh+cx(-54fg-18eh+7fhx))+2b^3ch(5ah(27fg+9eh+53fhx)+cx(3h(-}{}$$

$$+\frac{(-35b^3fh^3+30bch^2(3bfg+beh+2afh)+16c^3(fg^3+3gh(eg+dh))-24c^2h(3bfg^2+bh(3eg+dh)+ah}{8c^{9/2}}$$

```
[In] Integrate[((g + h*x)^3*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] -1/24*(-105*b^5*f*h^3*x - 5*b^4*h^2*(21*a*f*h + c*x*(-54*f*g - 18*e*h + 7*f
*h*x)) + 2*b^3*c*h*(5*a*h*(27*f*g + 9*e*h + 53*f*h*x) + c*x*(3*h*(-36*e*g -
 12*d*h + 5*e*h*x) + f*(-108*g^2 + 45*g*h*x + 7*h^2*x^2))) + 16*c^2*(-16*a^
3*f*h^3 + 6*c^3*d*g^3*x + a*c^2*(6*d*h*(-3*g^2 - 3*g*h*x + h^2*x^2) - 3*e*(
2*g^3 + 6*g^2*h*x - 6*g*h^2*x^2 - h^3*x^3) + f*x*(-6*g^3 + 18*g^2*h*x + 9*g
*h^2*x^2 + 2*h^3*x^3)) + a^2*c*h*(f*(36*g^2 + 27*g*h*x - 8*h^2*x^2) + 3*h*(
4*d*h + 3*e*(4*g + h*x)))) + 8*b*c^2*(-6*c^2*g^2*(-(d*g) + e*g*x + 3*d*h*x)
 - a^2*h^2*(117*f*g + 39*e*h + 61*f*h*x) + a*c*(f*(6*g^3 + 90*g^2*h*x - 45*
g*h^2*x^2 - 7*h^3*x^3) + 3*h*(2*d*h*(3*g + 5*h*x) + e*(6*g^2 + 30*g*h*x - 5
*h^2*x^2)))) + 4*b^2*c*(115*a^2*f*h^3 - a*c*h*(3*h*(18*e*g + 6*d*h + 31*e*h
*x) + f*(54*g^2 + 279*g*h*x - 43*h^2*x^2)) - c^2*x*(f*(-12*g^3 + 18*g^2*h*x
 + 9*g*h^2*x^2 + 2*h^3*x^3) + 3*h*(2*d*h*(-6*g + h*x) + e*(-12*g^2 + 6*g*h*
x + h^2*x^2)))))/(c^4*(b^2 - 4*a*c)*Sqrt[a + x*(b + c*x)]) + ((-35*b^3*f*h^
3 + 30*b*c*h^2*(3*b*f*g + b*e*h + 2*a*f*h) + 16*c^3*(f*g^3 + 3*g*h*(e*g + d
*h)) - 24*c^2*h*(3*b*f*g^2 + b*h*(3*e*g + d*h) + a*h*(3*f*g + e*h)))*ArcTan
h[(Sqrt[c]*x)/(-Sqrt[a] + Sqrt[a + x*(b + c*x)])])/(8*c^(9/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 996 vs. 2(484) = 968.

Time = 1.28 (sec) , antiderivative size = 997, normalized size of antiderivative = 1.98

method	result
risch	$-\frac{h\left(-8fh^2c^2x^2+22bcfh^2x-12c^2eh^2x-36c^2fghx+40acfh^2-57b^2fh^2+42bceh^2+126bcfgh-24c^2dh^2-72c^2egh-72c^2fg^2\right)\sqrt{cx^2+}}{24c^4}$
default	Expression too large to display

```
[In] int((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*h*(-8*c^2*f*h^2*x^2+22*b*c*f*h^2*x-12*c^2*e*h^2*x-36*c^2*f*g*h*x+40*a
*c*f*h^2-57*b^2*f*h^2+42*b*c*e*h^2+126*b*c*f*g*h-24*c^2*d*h^2-72*c^2*e*g*h-
72*c^2*f*g^2)*(c*x^2+b*x+a)^(1/2)/c^4+1/16/c^4*(32*c^4*d*g^3*(2*c*x+b)/(4*a
*c-b^2)/(c*x^2+b*x+a)^(1/2)-16*a^2*c^2*e*h^3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b
*x+a)^(1/2)-38*a*b^3*f*h^3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-16*a*b
*c^2*d*h^3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+56*a^2*b*c*f*h^3*(2*c*
x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-48*a^2*c^2*f*g*h^2*(2*c*x+b)/(4*a*c-b^
2)/(c*x^2+b*x+a)^(1/2)+28*a*b^2*c*e*h^3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)
^(1/2)+84*a*b^2*c*f*g*h^2*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-48*a*b*
c^2*e*g*h^2*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-48*a*b*c^2*f*g^2*h*(2
*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+(60*a*b*c^2*f*h^3-24*a*c^3*e*h^3-72
*a*c^3*f*g*h^2-35*b^3*c*f*h^3+30*b^2*c^2*e*h^3+90*b^2*c^2*f*g*h^2-24*b*c^3*
d*h^3-72*b*c^3*e*g*h^2-72*b*c^3*f*g^2*h+48*c^4*d*g*h^2+48*c^4*e*g^2*h+16*c^
4*f*g^3)*(-x/c/(c*x^2+b*x+a)^(1/2)-1/2*b/c*(-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2
*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))+1/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(
c*x^2+b*x+a)^(1/2)))+(16*a^2*c^2*f*h^3+12*a*b^2*c*f*h^3+8*a*b*c^2*e*h^3+24*
a*b*c^2*f*g*h^2-16*a*c^3*d*h^3-48*a*c^3*e*g*h^2-48*a*c^3*f*g^2*h-19*b^4*f*h
^3+14*b^3*c*e*h^3+42*b^3*c*f*g*h^2-8*b^2*c^2*d*h^3-24*b^2*c^2*e*g*h^2-24*b^
2*c^2*f*g^2*h+48*c^4*d*g^2*h+16*c^4*e*g^3)*(-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2
*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1467 vs. 2(482) = 964.

Time = 7.72 (sec) , antiderivative size = 2937, normalized size of antiderivative = 5.83

$$\int \frac{(g+hx)^3\left(d+ex+fx^2\right)}{\left(a+bx+cx^2\right)^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas"
)
```

```
[Out] [1/96*(3*(16*(a*b^2*c^3 - 4*a^2*c^4)*f*g^3 + 24*(2*(a*b^2*c^3 - 4*a^2*c^4)*
e - 3*(a*b^3*c^2 - 4*a^2*b*c^3)*f)*g^2*h + 6*(8*(a*b^2*c^3 - 4*a^2*c^4)*d -
 12*(a*b^3*c^2 - 4*a^2*b*c^3)*e + 3*(5*a*b^4*c - 24*a^2*b^2*c^2 + 16*a^3*c^
3)*f)*g*h^2 - (24*(a*b^3*c^2 - 4*a^2*b*c^3)*d - 6*(5*a*b^4*c - 24*a^2*b^2*c
^2 + 16*a^3*c^3)*e + 5*(7*a*b^5 - 40*a^2*b^3*c + 48*a^3*b*c^2)*f)*h^3 + (16
*(b^2*c^4 - 4*a*c^5)*f*g^3 + 24*(2*(b^2*c^4 - 4*a*c^5)*e - 3*(b^3*c^3 - 4*a
*b*c^4)*f)*g^2*h + 6*(8*(b^2*c^4 - 4*a*c^5)*d - 12*(b^3*c^3 - 4*a*b*c^4)*e
+ 3*(5*b^4*c^2 - 24*a*b^2*c^3 + 16*a^2*c^4)*f)*g*h^2 - (24*(b^3*c^3 - 4*a*b
*c^4)*d - 6*(5*b^4*c^2 - 24*a*b^2*c^3 + 16*a^2*c^4)*e + 5*(7*b^5*c - 40*a*b
^3*c^2 + 48*a^2*b*c^3)*f)*h^3)*x^2 + (16*(b^3*c^3 - 4*a*b*c^4)*f*g^3 + 24*(
2*(b^3*c^3 - 4*a*b*c^4)*e - 3*(b^4*c^2 - 4*a*b^2*c^3)*f)*g^2*h + 6*(8*(b^3*
c^3 - 4*a*b*c^4)*d - 12*(b^4*c^2 - 4*a*b^2*c^3)*e + 3*(5*b^5*c - 24*a*b^3*c
```

```
^2 + 16*a^2*b*c^3)*f)*g*h^2 - (24*(b^4*c^2 - 4*a*b^2*c^3)*d - 6*(5*b^5*c -
24*a*b^3*c^2 + 16*a^2*b*c^3)*e + 5*(7*b^6 - 40*a*b^4*c + 48*a^2*b^2*c^2)*f)
*h^3)*x)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(
2*c*x + b)*sqrt(c) - 4*a*c) + 4*(8*(b^2*c^4 - 4*a*c^5)*f*h^3*x^4 - 48*(b*c^
5*d - 2*a*c^5*e + a*b*c^4*f)*g^3 + 72*(4*a*c^5*d - 2*a*b*c^4*e + (3*a*b^2*c
^3 - 8*a^2*c^4)*f)*g^2*h - 18*(8*a*b*c^4*d - 4*(3*a*b^2*c^3 - 8*a^2*c^4)*e
+ (15*a*b^3*c^2 - 52*a^2*b*c^3)*f)*g*h^2 + (24*(3*a*b^2*c^3 - 8*a^2*c^4)*d
- 6*(15*a*b^3*c^2 - 52*a^2*b*c^3)*e + (105*a*b^4*c - 460*a^2*b^2*c^2 + 256*
a^3*c^3)*f)*h^3 + 2*(18*(b^2*c^4 - 4*a*c^5)*f*g*h^2 + (6*(b^2*c^4 - 4*a*c^5
)*e - 7*(b^3*c^3 - 4*a*b*c^4)*f)*h^3)*x^3 + (72*(b^2*c^4 - 4*a*c^5)*f*g^2*h
 + 18*(4*(b^2*c^4 - 4*a*c^5)*e - 5*(b^3*c^3 - 4*a*b*c^4)*f)*g*h^2 + (24*(b^
2*c^4 - 4*a*c^5)*d - 30*(b^3*c^3 - 4*a*b*c^4)*e + (35*b^4*c^2 - 172*a*b^2*c
^3 + 128*a^2*c^4)*f)*h^3)*x^2 - (48*(2*c^6*d - b*c^5*e + (b^2*c^4 - 2*a*c^5
)*f)*g^3 - 72*(2*b*c^5*d - 2*(b^2*c^4 - 2*a*c^5)*e + (3*b^3*c^3 - 10*a*b*c^
4)*f)*g^2*h + 18*(8*(b^2*c^4 - 2*a*c^5)*d - 4*(3*b^3*c^3 - 10*a*b*c^4)*e +
(15*b^4*c^2 - 62*a*b^2*c^3 + 24*a^2*c^4)*f)*g*h^2 - (24*(3*b^3*c^3 - 10*a*b
*c^4)*d - 6*(15*b^4*c^2 - 62*a*b^2*c^3 + 24*a^2*c^4)*e + (105*b^5*c - 530*a
*b^3*c^2 + 488*a^2*b*c^3)*f)*h^3)*x)*sqrt(c*x^2 + b*x + a))/(a*b^2*c^5 - 4*
a^2*c^6 + (b^2*c^6 - 4*a*c^7)*x^2 + (b^3*c^5 - 4*a*b*c^6)*x), -1/48*(3*(16*
(a*b^2*c^3 - 4*a^2*c^4)*f*g^3 + 24*(2*(a*b^2*c^3 - 4*a^2*c^4)*e - 3*(a*b^3*
c^2 - 4*a^2*b*c^3)*f)*g^2*h + 6*(8*(a*b^2*c^3 - 4*a^2*c^4)*d - 12*(a*b^3*c^
2 - 4*a^2*b*c^3)*e + 3*(5*a*b^4*c - 24*a^2*b^2*c^2 + 16*a^3*c^3)*f)*g*h^2 -
 (24*(a*b^3*c^2 - 4*a^2*b*c^3)*d - 6*(5*a*b^4*c - 24*a^2*b^2*c^2 + 16*a^3*c
^3)*e + 5*(7*a*b^5 - 40*a^2*b^3*c + 48*a^3*b*c^2)*f)*h^3 + (16*(b^2*c^4 - 4
*a*c^5)*f*g^3 + 24*(2*(b^2*c^4 - 4*a*c^5)*e - 3*(b^3*c^3 - 4*a*b*c^4)*f)*g^
2*h + 6*(8*(b^2*c^4 - 4*a*c^5)*d - 12*(b^3*c^3 - 4*a*b*c^4)*e + 3*(5*b^4*c^
2 - 24*a*b^2*c^3 + 16*a^2*c^4)*f)*g*h^2 - (24*(b^3*c^3 - 4*a*b*c^4)*d - 6*(
5*b^4*c^2 - 24*a*b^2*c^3 + 16*a^2*c^4)*e + 5*(7*b^5*c - 40*a*b^3*c^2 + 48*a
^2*b*c^3)*f)*h^3)*x^2 + (16*(b^3*c^3 - 4*a*b*c^4)*f*g^3 + 24*(2*(b^3*c^3 -
4*a*b*c^4)*e - 3*(b^4*c^2 - 4*a*b^2*c^3)*f)*g^2*h + 6*(8*(b^3*c^3 - 4*a*b*c
^4)*d - 12*(b^4*c^2 - 4*a*b^2*c^3)*e + 3*(5*b^5*c - 24*a*b^3*c^2 + 16*a^2*b
*c^3)*f)*g*h^2 - (24*(b^4*c^2 - 4*a*b^2*c^3)*d - 6*(5*b^5*c - 24*a*b^3*c^2
+ 16*a^2*b*c^3)*e + 5*(7*b^6 - 40*a*b^4*c + 48*a^2*b^2*c^2)*f)*h^3)*x)*sqrt
(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x
 + a*c)) - 2*(8*(b^2*c^4 - 4*a*c^5)*f*h^3*x^4 - 48*(b*c^5*d - 2*a*c^5*e + a
*b*c^4*f)*g^3 + 72*(4*a*c^5*d - 2*a*b*c^4*e + (3*a*b^2*c^3 - 8*a^2*c^4)*f)*
g^2*h - 18*(8*a*b*c^4*d - 4*(3*a*b^2*c^3 - 8*a^2*c^4)*e + (15*a*b^3*c^2 - 5
2*a^2*b*c^3)*f)*g*h^2 + (24*(3*a*b^2*c^3 - 8*a^2*c^4)*d - 6*(15*a*b^3*c^2 -
 52*a^2*b*c^3)*e + (105*a*b^4*c - 460*a^2*b^2*c^2 + 256*a^3*c^3)*f)*h^3 + 2
*(18*(b^2*c^4 - 4*a*c^5)*f*g*h^2 + (6*(b^2*c^4 - 4*a*c^5)*e - 7*(b^3*c^3 -
4*a*b*c^4)*f)*h^3)*x^3 + (72*(b^2*c^4 - 4*a*c^5)*f*g^2*h + 18*(4*(b^2*c^4 -
 4*a*c^5)*e - 5*(b^3*c^3 - 4*a*b*c^4)*f)*g*h^2 + (24*(b^2*c^4 - 4*a*c^5)*d
- 30*(b^3*c^3 - 4*a*b*c^4)*e + (35*b^4*c^2 - 172*a*b^2*c^3 + 128*a^2*c^4)*f
)*h^3)*x^2 - (48*(2*c^6*d - b*c^5*e + (b^2*c^4 - 2*a*c^5)*f)*g^3 - 72*(2*b*
c^5*d - 2*(b^2*c^4 - 2*a*c^5)*e + (3*b^3*c^3 - 10*a*b*c^4)*f)*g^2*h + 18*(8
```

```
*(b^2*c^4 - 2*a*c^5)*d - 4*(3*b^3*c^3 - 10*a*b*c^4)*e + (15*b^4*c^2 - 62*a*
b^2*c^3 + 24*a^2*c^4)*f)*g*h^2 - (24*(3*b^3*c^3 - 10*a*b*c^4)*d - 6*(15*b^4
*c^2 - 62*a*b^2*c^3 + 24*a^2*c^4)*e + (105*b^5*c - 530*a*b^3*c^2 + 488*a^2*
b*c^3)*f)*h^3)*x)*sqrt(c*x^2 + b*x + a))/(a*b^2*c^5 - 4*a^2*c^6 + (b^2*c^6
- 4*a*c^7)*x^2 + (b^3*c^5 - 4*a*b*c^6)*x)]
```

Sympy [F]

$$\int \frac{(g+hx)^3\,(d+ex+fx^2)}{(a+bx+cx^2)^{3/2}}\,dx = \int \frac{(g+hx)^3\,(d+ex+fx^2)}{(a+bx+cx^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((h*x+g)**3*(f*x**2+e*x+d)/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral((g + h*x)**3*(d + e*x + f*x**2)/(a + b*x + c*x**2)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(g+hx)^3\,(d+ex+fx^2)}{(a+bx+cx^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1028 vs. 2(482) = 964.

Time = 0.30 (sec) , antiderivative size = 1028, normalized size of antiderivative = 2.04

$$\int \frac{(g+hx)^3\,(d+ex+fx^2)}{(a+bx+cx^2)^{3/2}}\,dx = \frac{((2\,(\frac{4\,(b^2c^3fh^3-4\,ac^4fh^3)x}{b^2c^4-4\,ac^5} + \frac{18\,b^2c^3fgh^2-72\,ac^4fgh^2+6\,b^2c^3eh^3-24\,ac^4eh^3-7\,b^3c^2fh^3+28}{b^2c^4-4\,ac^5}}{} - \frac{(16\,c^3fg^3+48\,c^3eg^2h-72\,bc^2fg^2h+48\,c^3dgh^2-72\,bc^2egh^2+90\,b^2cfgh^2-72\,ac^2fgh^2-24\,bc^2dh^3+3}{16\,c^{\frac{9}{2}}}$$

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/24*(((2*(4*(b^2*c^3*f*h^3 - 4*a*c^4*f*h^3)*x/(b^2*c^4 - 4*a*c^5) + (18*b^
2*c^3*f*g*h^2 - 72*a*c^4*f*g*h^2 + 6*b^2*c^3*e*h^3 - 24*a*c^4*e*h^3 - 7*b^3
*c^2*f*h^3 + 28*a*b*c^3*f*h^3)/(b^2*c^4 - 4*a*c^5))*x + (72*b^2*c^3*f*g^2*h
 - 288*a*c^4*f*g^2*h + 72*b^2*c^3*e*g*h^2 - 288*a*c^4*e*g*h^2 - 90*b^3*c^2*
f*g*h^2 + 360*a*b*c^3*f*g*h^2 + 24*b^2*c^3*d*h^3 - 96*a*c^4*d*h^3 - 30*b^3*
c^2*e*h^3 + 120*a*b*c^3*e*h^3 + 35*b^4*c*f*h^3 - 172*a*b^2*c^2*f*h^3 + 128*
a^2*c^3*f*h^3)/(b^2*c^4 - 4*a*c^5))*x - (96*c^5*d*g^3 - 48*b*c^4*e*g^3 + 48
*b^2*c^3*f*g^3 - 96*a*c^4*f*g^3 - 144*b*c^4*d*g^2*h + 144*b^2*c^3*e*g^2*h -
 288*a*c^4*e*g^2*h - 216*b^3*c^2*f*g^2*h + 720*a*b*c^3*f*g^2*h + 144*b^2*c^
3*d*g*h^2 - 288*a*c^4*d*g*h^2 - 216*b^3*c^2*e*g*h^2 + 720*a*b*c^3*e*g*h^2 +
 270*b^4*c*f*g*h^2 - 1116*a*b^2*c^2*f*g*h^2 + 432*a^2*c^3*f*g*h^2 - 72*b^3*
c^2*d*h^3 + 240*a*b*c^3*d*h^3 + 90*b^4*c*e*h^3 - 372*a*b^2*c^2*e*h^3 + 144*
a^2*c^3*e*h^3 - 105*b^5*f*h^3 + 530*a*b^3*c*f*h^3 - 488*a^2*b*c^2*f*h^3)/(b
^2*c^4 - 4*a*c^5))*x - (48*b*c^4*d*g^3 - 96*a*c^4*e*g^3 + 48*a*b*c^3*f*g^3
- 288*a*c^4*d*g^2*h + 144*a*b*c^3*e*g^2*h - 216*a*b^2*c^2*f*g^2*h + 576*a^2
*c^3*f*g^2*h + 144*a*b*c^3*d*g*h^2 - 216*a*b^2*c^2*e*g*h^2 + 576*a^2*c^3*e*
g*h^2 + 270*a*b^3*c*f*g*h^2 - 936*a^2*b*c^2*f*g*h^2 - 72*a*b^2*c^2*d*h^3 +
192*a^2*c^3*d*h^3 + 90*a*b^3*c*e*h^3 - 312*a^2*b*c^2*e*h^3 - 105*a*b^4*f*h^
3 + 460*a^2*b^2*c*f*h^3 - 256*a^3*c^2*f*h^3)/(b^2*c^4 - 4*a*c^5))/sqrt(c*x^
2 + b*x + a) - 1/16*(16*c^3*f*g^3 + 48*c^3*e*g^2*h - 72*b*c^2*f*g^2*h + 48*
c^3*d*g*h^2 - 72*b*c^2*e*g*h^2 + 90*b^2*c*f*g*h^2 - 72*a*c^2*f*g*h^2 - 24*b
*c^2*d*h^3 + 30*b^2*c*e*h^3 - 24*a*c^2*e*h^3 - 35*b^3*f*h^3 + 60*a*b*c*f*h^
3)*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(9/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(g+hx)^3\left(d+ex+fx^2\right)}{\left(a+bx+cx^2\right)^{3/2}}\,dx=\int \frac{{\left(g+h\,x\right)}^3\,\left(f\,x^2+e\,x+d\right)}{{\left(c\,x^2+b\,x+a\right)}^{3/2}}\,dx$$

```
[In] int(((g + h*x)^3*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(3/2),x)
```

```
[Out] int(((g + h*x)^3*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(3/2), x)
```

3.234 $\int \frac{(g+hx)^2(d+ex+fx^2)}{(a+bx+cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 289

$$\int \frac{(g+hx)^2\,(d+ex+fx^2)}{(a+bx+cx^2)^{3/2}}\,dx = \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-(2c^2d-bce+b^2f-2acf)\,x\right)(g+hx)^2}{c\,(b^2-4ac)\,\sqrt{a+bx+cx^2}}$$

$$+\frac{h(32c^3dg-15b^3fh-8c^2(2beg+8afg+bdh+4aeh)+4bc(6bfg+3beh+13afh)+2c(8c^2d-4bce+5b}{4c^3\,(b^2-4ac)}$$

$$+\frac{(15b^2fh^2-12ch(2bfg+beh+afh)+8c^2(fg^2+h(2eg+dh)))\,\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{7/2}}$$

```
[Out] 1/8*(15*b^2*f*h^2-12*c*h*(a*f*h+b*e*h+2*b*f*g)+8*c^2*(f*g^2+h*(d*h+2*e*g)))
*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(7/2)+2*(c*(2*a*e-b*(
d+a*f/c))-(-2*a*c*f+b^2*f-b*c*e+2*c^2*d)*x)*(h*x+g)^2/c/(-4*a*c+b^2)/(c*x^2
+b*x+a)^(1/2)+1/4*h*(32*c^3*d*g-15*b^3*f*h-8*c^2*(4*a*e*h+8*a*f*g+b*d*h+2*b
*e*g)+4*b*c*(13*a*f*h+3*b*e*h+6*b*f*g)+2*c*(-12*a*c*f+5*b^2*f-4*b*c*e+8*c^2
*d)*h*x)*(c*x^2+b*x+a)^(1/2)/c^3/(-4*a*c+b^2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used

$= \{1658, 793, 635, 212\}$

$$\int \frac{(g+hx)^2\left(d+ex+fx^2\right)}{\left(a+bx+cx^2\right)^{3/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(-12ch(afh+beh+2bfg)+15b^2fh^2+8c^2(h(dh}{8c^{7/2}}$$

$$+\frac{2(g+hx)^2\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x(-2acf+b^2f-bce+2c^2d)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

$$+\frac{h\sqrt{a+bx+cx^2}\Big(2hx(-12acf+5b^2f-4bce+8c^2d)-8c(4aeh+8afg+bdh+2beg)+4b(13afh+3beh}{4c^2\left(b^2-4ac\right)}$$

```
[In] Int[((g + h*x)^2*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (2*(c*(2*a*e - b*(d + (a*f)/c)) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x)*(g
 + h*x)^2)/(c*(b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]) + (h*(32*c^2*d*g - (15*b
^3*f*h)/c - 8*c*(2*b*e*g + 8*a*f*g + b*d*h + 4*a*e*h) + 4*b*(6*b*f*g + 3*b*
e*h + 13*a*f*h) + 2*(8*c^2*d - 4*b*c*e + 5*b^2*f - 12*a*c*f)*h*x)*Sqrt[a +
b*x + c*x^2])/(4*c^2*(b^2 - 4*a*c)) + ((15*b^2*f*h^2 - 12*c*h*(2*b*f*g + b*
e*h + a*f*h) + 8*c^2*(f*g^2 + h*(2*e*g + d*h)))*ArcTanh[(b + 2*c*x)/(2*Sqrt
[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1658

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f =
Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[Polynom
```

```
ialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x + c
*x^2)^(p + 1)*((f*b - 2*a*g + (2*c*f - b*g)*x)/((p + 1)*(b^2 - 4*a*c))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Q + g*(2*a*e*m + b*d*(2
*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(m + 2*p + 3)*x, x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Integer
Q[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] && Ra
tionalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)(g+hx)^2}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} \\
&\quad -\frac{2\int\frac{(g+hx)\left(-\frac{b^2fg+4b(cd+af)h-4ac(fg+2eh)}{2c}-\frac{1}{2}\left(8cd-4be-12af+\frac{5b^2f}{c}\right)hx\right)}{\sqrt{a+bx+cx^2}}\,dx}{b^2-4ac} \\
&= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)(g+hx)^2}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} \\
&\quad +\frac{h\Big(32c^2dg-\frac{15b^3fh}{c}-8c(2beg+8afg+bdh+4aeh)+4b(6bfg+3beh+13afh)+2(8c^2d-4b}{4c^2\left(b^2-4ac\right)} \\
&\quad +\frac{\left(15b^2fh^2-12ch(2bfg+beh+afh)+8c^2\left(fg^2+h(2eg+dh)\right)\right)\int\frac{1}{\sqrt{a+bx+cx^2}}\,dx}{8c^3} \\
&= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)(g+hx)^2}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} \\
&\quad +\frac{h\Big(32c^2dg-\frac{15b^3fh}{c}-8c(2beg+8afg+bdh+4aeh)+4b(6bfg+3beh+13afh)+2(8c^2d-4b}{4c^2\left(b^2-4ac\right)} \\
&\quad +\frac{\left(15b^2fh^2-12ch(2bfg+beh+afh)+8c^2\left(fg^2+h(2eg+dh)\right)\right)\text{Subst}\Big(\int\frac{1}{4c-x^2}\,dx,x,\frac{b+2cx}{\sqrt{a+bx+cx^2}}}{4c^3} \\
&= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)(g+hx)^2}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} \\
&\quad +\frac{h\Big(32c^2dg-\frac{15b^3fh}{c}-8c(2beg+8afg+bdh+4aeh)+4b(6bfg+3beh+13afh)+2(8c^2d-4b}{4c^2\left(b^2-4ac\right)} \\
&\quad +\frac{\left(15b^2fh^2-12ch(2bfg+beh+afh)+8c^2\left(fg^2+h(2eg+dh)\right)\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{7/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 2.45 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.35

$$\int \frac{(g+hx)^2\left(d+ex+fx^2\right)}{\left(a+bx+cx^2\right)^{3/2}}\,dx = -\frac{\sqrt{c}(15b^4fh^2x+b^3h(15afh+cx(-24fg-12eh+5fhx))+4bc(-13a^2fh^2+2c^2g(-egx+d(g-2hx))+ac}{}$$

```
[In] Integrate[((g + h*x)^2*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (-((Sqrt[c]*(15*b^4*f*h^2*x + b^3*h*(15*a*f*h + c*x*(-24*f*g - 12*e*h + 5*f
*h*x)) + 4*b*c*(-13*a^2*f*h^2 + 2*c^2*g*(-(e*g*x) + d*(g - 2*h*x)) + a*c*(2
*h*(2*e*g + d*h + 5*e*h*x) + f*(2*g^2 + 20*g*h*x - 5*h^2*x^2))) - 2*b^2*c*(
a*h*(12*f*g + 6*e*h + 31*f*h*x) + c*x*(2*h*(-4*e*g - 2*d*h + e*h*x) + f*(-4
*g^2 + 4*g*h*x + h^2*x^2))) + 8*c^2*(2*c^2*d*g^2*x + a^2*h*(8*f*g + 4*e*h +
 3*f*h*x) + a*c*(-2*d*h*(2*g + h*x) - 2*e*(g^2 + 2*g*h*x - h^2*x^2) + f*x*(
-2*g^2 + 4*g*h*x + h^2*x^2)))))/((b^2 - 4*a*c)*Sqrt[a + x*(b + c*x)])) + (1
5*b^2*f*h^2 - 12*c*h*(2*b*f*g + b*e*h + a*f*h) + 8*c^2*(f*g^2 + h*(2*e*g +
d*h)))*ArcTanh[(Sqrt[c]*x)/(-Sqrt[a] + Sqrt[a + x*(b + c*x)])])/(4*c^(7/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 547 vs. 2(273) = 546.

Time = 0.96 (sec) , antiderivative size = 548, normalized size of antiderivative = 1.90

method	result
risch	$-\frac{h(-2cfhx+7bfh-4ehc-8cfg)\sqrt{c\,x^2+bx+a}}{4c^3}-\frac{-\frac{16c^3d\,g^2(2cx+b)}{\left(4ac-b^2\right)\sqrt{c\,x^2+bx+a}}+\frac{8a^2cf\,h^2(2cx+b)}{\left(4ac-b^2\right)\sqrt{c\,x^2+bx+a}}-\frac{14a\,b^2f\,h^2(2cx+b)}{\left(4ac-b^2\right)\sqrt{c\,x^2+bx+a}}+\frac{8abce\,h^2}{\left(4ac-b^2\right)\sqrt{}}}{}$
default	$\frac{2d\,g^2(2cx+b)}{(4ac-b^2)\sqrt{c\,x^2+bx+a}}+f\,h^2\left(\frac{x^3}{2c\sqrt{c\,x^2+bx+a}}-\frac{5b\left(\frac{x^2}{c\sqrt{c\,x^2+bx+a}}-\frac{3b\left(-\frac{x}{c\sqrt{c\,x^2+bx+a}}-\frac{b\left(-\frac{1}{c\sqrt{c\,x^2+bx+a}}-\frac{b(2cx+b)}{c\left(4ac-b^2\right)\sqrt{c\,x^2+bx}}\right.}{2c}\right.}{2c}\right.}{}\right.$

```
[In] int((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*h*(-2*c*f*h*x+7*b*f*h-4*c*e*h-8*c*f*g)*(c*x^2+b*x+a)^(1/2)/c^3-1/8/c^3
*(-16*c^3*d*g^2*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+8*a^2*c*f*h^2*(2*
c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-14*a*b^2*f*h^2*(2*c*x+b)/(4*a*c-b^2)
/(c*x^2+b*x+a)^(1/2)+8*a*b*c*e*h^2*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2
)+16*a*b*c*f*g*h*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+(12*a*c^2*f*h^2-
15*b^2*c*f*h^2+12*b*c^2*e*h^2+24*b*c^2*f*g*h-8*c^3*d*h^2-16*c^3*e*g*h-8*c^3
*f*g^2)*(-x/c/(c*x^2+b*x+a)^(1/2)-1/2*b/c*(-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2*
c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))+1/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c
*x^2+b*x+a)^(1/2)))+(-4*a*b*c*f*h^2+8*a*c^2*e*h^2+16*a*c^2*f*g*h-7*b^3*f*h^
2+4*b^2*c*e*h^2+8*b^2*c*f*g*h-16*c^3*d*g*h-8*c^3*e*g^2)*(-1/c/(c*x^2+b*x+a)
^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 883 vs. 2(271) = 542.

Time = 5.98 (sec) , antiderivative size = 1769, normalized size of antiderivative = 6.12

$$\int \frac{(g+hx)^2\left(d+ex+fx^2\right)}{\left(a+bx+cx^2\right)^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas"
)
```

```
[Out] [-1/16*((8*(a*b^2*c^2 - 4*a^2*c^3)*f*g^2 + 8*(2*(a*b^2*c^2 - 4*a^2*c^3)*e -
 3*(a*b^3*c - 4*a^2*b*c^2)*f)*g*h + (8*(a*b^2*c^2 - 4*a^2*c^3)*d - 12*(a*b^
3*c - 4*a^2*b*c^2)*e + 3*(5*a*b^4 - 24*a^2*b^2*c + 16*a^3*c^2)*f)*h^2 + (8*
(b^2*c^3 - 4*a*c^4)*f*g^2 + 8*(2*(b^2*c^3 - 4*a*c^4)*e - 3*(b^3*c^2 - 4*a*b
*c^3)*f)*g*h + (8*(b^2*c^3 - 4*a*c^4)*d - 12*(b^3*c^2 - 4*a*b*c^3)*e + 3*(5
*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*f)*h^2)*x^2 + (8*(b^3*c^2 - 4*a*b*c^3)*
f*g^2 + 8*(2*(b^3*c^2 - 4*a*b*c^3)*e - 3*(b^4*c - 4*a*b^2*c^2)*f)*g*h + (8*
(b^3*c^2 - 4*a*b*c^3)*d - 12*(b^4*c - 4*a*b^2*c^2)*e + 3*(5*b^5 - 24*a*b^3*
c + 16*a^2*b*c^2)*f)*h^2)*x)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqr
t(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(2*(b^2*c^3 - 4*a*c^4)*
f*h^2*x^3 - 8*(b*c^4*d - 2*a*c^4*e + a*b*c^3*f)*g^2 + 8*(4*a*c^4*d - 2*a*b*
c^3*e + (3*a*b^2*c^2 - 8*a^2*c^3)*f)*g*h - (8*a*b*c^3*d - 4*(3*a*b^2*c^2 -
8*a^2*c^3)*e + (15*a*b^3*c - 52*a^2*b*c^2)*f)*h^2 + (8*(b^2*c^3 - 4*a*c^4)*
f*g*h + (4*(b^2*c^3 - 4*a*c^4)*e - 5*(b^3*c^2 - 4*a*b*c^3)*f)*h^2)*x^2 - (8
*(2*c^5*d - b*c^4*e + (b^2*c^3 - 2*a*c^4)*f)*g^2 - 8*(2*b*c^4*d - 2*(b^2*c^
3 - 2*a*c^4)*e + (3*b^3*c^2 - 10*a*b*c^3)*f)*g*h + (8*(b^2*c^3 - 2*a*c^4)*d
 - 4*(3*b^3*c^2 - 10*a*b*c^3)*e + (15*b^4*c - 62*a*b^2*c^2 + 24*a^2*c^3)*f)
*h^2)*x)*sqrt(c*x^2 + b*x + a))/(a*b^2*c^4 - 4*a^2*c^5 + (b^2*c^5 - 4*a*c^6
)*x^2 + (b^3*c^4 - 4*a*b*c^5)*x), -1/8*((8*(a*b^2*c^2 - 4*a^2*c^3)*f*g^2 +
8*(2*(a*b^2*c^2 - 4*a^2*c^3)*e - 3*(a*b^3*c - 4*a^2*b*c^2)*f)*g*h + (8*(a*b
^2*c^2 - 4*a^2*c^3)*d - 12*(a*b^3*c - 4*a^2*b*c^2)*e + 3*(5*a*b^4 - 24*a^2*
```

```
b^2*c + 16*a^3*c^2)*f)*h^2 + (8*(b^2*c^3 - 4*a*c^4)*f*g^2 + 8*(2*(b^2*c^3 -
 4*a*c^4)*e - 3*(b^3*c^2 - 4*a*b*c^3)*f)*g*h + (8*(b^2*c^3 - 4*a*c^4)*d - 1
2*(b^3*c^2 - 4*a*b*c^3)*e + 3*(5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*f)*h^2)
*x^2 + (8*(b^3*c^2 - 4*a*b*c^3)*f*g^2 + 8*(2*(b^3*c^2 - 4*a*b*c^3)*e - 3*(b
^4*c - 4*a*b^2*c^2)*f)*g*h + (8*(b^3*c^2 - 4*a*b*c^3)*d - 12*(b^4*c - 4*a*b
^2*c^2)*e + 3*(5*b^5 - 24*a*b^3*c + 16*a^2*b*c^2)*f)*h^2)*x)*sqrt(-c)*arcta
n(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) -
 2*(2*(b^2*c^3 - 4*a*c^4)*f*h^2*x^3 - 8*(b*c^4*d - 2*a*c^4*e + a*b*c^3*f)*g
^2 + 8*(4*a*c^4*d - 2*a*b*c^3*e + (3*a*b^2*c^2 - 8*a^2*c^3)*f)*g*h - (8*a*b
*c^3*d - 4*(3*a*b^2*c^2 - 8*a^2*c^3)*e + (15*a*b^3*c - 52*a^2*b*c^2)*f)*h^2
 + (8*(b^2*c^3 - 4*a*c^4)*f*g*h + (4*(b^2*c^3 - 4*a*c^4)*e - 5*(b^3*c^2 - 4
*a*b*c^3)*f)*h^2)*x^2 - (8*(2*c^5*d - b*c^4*e + (b^2*c^3 - 2*a*c^4)*f)*g^2
- 8*(2*b*c^4*d - 2*(b^2*c^3 - 2*a*c^4)*e + (3*b^3*c^2 - 10*a*b*c^3)*f)*g*h
+ (8*(b^2*c^3 - 2*a*c^4)*d - 4*(3*b^3*c^2 - 10*a*b*c^3)*e + (15*b^4*c - 62*
a*b^2*c^2 + 24*a^2*c^3)*f)*h^2)*x)*sqrt(c*x^2 + b*x + a))/(a*b^2*c^4 - 4*a^
2*c^5 + (b^2*c^5 - 4*a*c^6)*x^2 + (b^3*c^4 - 4*a*b*c^5)*x)]
```

Sympy [F]

$$\int \frac{(g+hx)^2\,(d+ex+fx^2)}{(a+bx+cx^2)^{3/2}}\,dx=\int \frac{(g+hx)^2\,(d+ex+fx^2)}{(a+bx+cx^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((h*x+g)**2*(f*x**2+e*x+d)/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral((g + h*x)**2*(d + e*x + f*x**2)/(a + b*x + c*x**2)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(g+hx)^2\,(d+ex+fx^2)}{(a+bx+cx^2)^{3/2}}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the ’assume’ command before evaluation *may* h
elp (example of legal syntax is ’assume(4*a*c-b^2>0)’, see ‘assume?‘ for mo
re deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 565 vs. $2(271) = 542$.

Time = 0.29 (sec) , antiderivative size = 565, normalized size of antiderivative = 1.96

$$\int \frac{(g+hx)^2\,(d+ex+fx^2)}{(a+bx+cx^2)^{3/2}}\,dx = \frac{((\frac{2\,(b^2c^2fh^2-4\,ac^3fh^2)x}{b^2c^3-4\,ac^4} + \frac{8\,b^2c^2fgh-32\,ac^3fgh+4\,b^2c^2eh^2-16\,ac^3eh^2-5\,b^3cfh^2+20\,abc^2fh^2}{b^2c^3-4\,ac^4}}{}$$

$$-\frac{(8\,c^2fg^2+16\,c^2egh-24\,bcfgh+8\,c^2dh^2-12\,bceh^2+15\,b^2fh^2-12\,acfh^2)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+}\right.\right.\right.}{8\,c^{\frac{7}{2}}}$$

```
[In] integrate((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*(((2*(b^2*c^2*f*h^2 - 4*a*c^3*f*h^2)*x/(b^2*c^3 - 4*a*c^4) + (8*b^2*c^2
*f*g*h - 32*a*c^3*f*g*h + 4*b^2*c^2*e*h^2 - 16*a*c^3*e*h^2 - 5*b^3*c*f*h^2
+ 20*a*b*c^2*f*h^2)/(b^2*c^3 - 4*a*c^4))*x - (16*c^4*d*g^2 - 8*b*c^3*e*g^2
+ 8*b^2*c^2*f*g^2 - 16*a*c^3*f*g^2 - 16*b*c^3*d*g*h + 16*b^2*c^2*e*g*h - 32
*a*c^3*e*g*h - 24*b^3*c*f*g*h + 80*a*b*c^2*f*g*h + 8*b^2*c^2*d*h^2 - 16*a*c
^3*d*h^2 - 12*b^3*c*e*h^2 + 40*a*b*c^2*e*h^2 + 15*b^4*f*h^2 - 62*a*b^2*c*f*
h^2 + 24*a^2*c^2*f*h^2)/(b^2*c^3 - 4*a*c^4))*x - (8*b*c^3*d*g^2 - 16*a*c^3*
e*g^2 + 8*a*b*c^2*f*g^2 - 32*a*c^3*d*g*h + 16*a*b*c^2*e*g*h - 24*a*b^2*c*f*
g*h + 64*a^2*c^2*f*g*h + 8*a*b*c^2*d*h^2 - 12*a*b^2*c*e*h^2 + 32*a^2*c^2*e*
h^2 + 15*a*b^3*f*h^2 - 52*a^2*b*c*f*h^2)/(b^2*c^3 - 4*a*c^4))/sqrt(c*x^2 +
b*x + a) - 1/8*(8*c^2*f*g^2 + 16*c^2*e*g*h - 24*b*c*f*g*h + 8*c^2*d*h^2 - 1
2*b*c*e*h^2 + 15*b^2*f*h^2 - 12*a*c*f*h^2)*log(abs(2*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))*sqrt(c) + b))/c^(7/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(g+hx)^2\,(d+ex+fx^2)}{(a+bx+cx^2)^{3/2}}\,dx = \int \frac{{(g+h\,x)}^2\,(f\,x^2+e\,x+d)}{{(c\,x^2+b\,x+a)}^{3/2}}\,dx$$

```
[In] int(((g + h*x)^2*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(3/2),x)
```

```
[Out] int(((g + h*x)^2*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(3/2), x)
```

3.235 $\int \frac{(g+hx)(d+ex+fx^2)}{(a+bx+cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 186

$$\int \frac{(g+hx)\left(d+ex+fx^2\right)}{\left(a+bx+cx^2\right)^{3/2}}\,dx = \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)(g+hx)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} + \frac{\left(4c^2d-2bce+3b^2f-8acf\right)h\sqrt{a+bx+cx^2}}{c^2\left(b^2-4ac\right)} - \frac{(3bfh-2c(fg+eh))\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{5/2}}$$

```
[Out] -1/2*(3*b*f*h-2*c*(e*h+f*g))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1
/2))/c^(5/2)+2*(c*(2*a*e-b*(d+a*f/c))-(-2*a*c*f+b^2*f-b*c*e+2*c^2*d)*x)*(h*
x+g)/c/(-4*a*c+b^2)/(c*x^2+b*x+a)^(1/2)+(4*c^2*d+3*b^2*f-2*c*(4*a*f+b*e))*h
*(c*x^2+b*x+a)^(1/2)/c^2/(-4*a*c+b^2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1658, 654, 635, 212}

$$\int \frac{(g+hx)\left(d+ex+fx^2\right)}{\left(a+bx+cx^2\right)^{3/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(3bfh-2c(eh+fg))}{2c^{5/2}} + \frac{2(g+hx)\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} + \frac{h\sqrt{a+bx+cx^2}\left(-8acf+3b^2f-2bce+4c^2d\right)}{c^2\left(b^2-4ac\right)}$$

```
[In] Int[((g + h*x)*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (2*(c*(2*a*e - b*(d + (a*f)/c)) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x)*(g
 + h*x))/(c*(b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]) + ((4*c^2*d - 2*b*c*e + 3*
b^2*f - 8*a*c*f)*h*Sqrt[a + b*x + c*x^2])/(c^2*(b^2 - 4*a*c)) - ((3*b*f*h -
 2*c*(f*g + e*h))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(
2*c^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1658

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f =
Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[Polynom
ialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x + c
*x^2)^(p + 1)*((f*b - 2*a*g + (2*c*f - b*g)*x)/((p + 1)*(b^2 - 4*a*c))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Q + g*(2*a*e*m + b*d*(2
*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(m + 2*p + 3)*x, x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Integer
Q[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] && Ra
tionalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)(g+hx)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} \\
&\quad -\frac{2\int\frac{-\frac{b^2fg+2b(cd+af)h-4ac(fg+eh)}{2c}-\frac{1}{2}\left(4cd-2be-8af+\frac{3b^2f}{c}\right)hx}{\sqrt{a+bx+cx^2}}\,dx}{b^2-4ac} \\
&= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)(g+hx)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} \\
&\quad +\frac{\left(4c^2d+3b^2f-2c(be+4af)\right)h\sqrt{a+bx+cx^2}}{c^2\left(b^2-4ac\right)} \\
&\quad -\frac{\left(3bfh-2c(fg+eh)\right)\int\frac{1}{\sqrt{a+bx+cx^2}}\,dx}{2c^2} \\
&= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)(g+hx)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} \\
&\quad +\frac{\left(4c^2d+3b^2f-2c(be+4af)\right)h\sqrt{a+bx+cx^2}}{c^2\left(b^2-4ac\right)} \\
&\quad -\frac{\left(3bfh-2c(fg+eh)\right)\text{Subst}\left(\int\frac{1}{4c-x^2}\,dx,x,\frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{c^2} \\
&= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)(g+hx)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} \\
&\quad +\frac{\left(4c^2d+3b^2f-2c(be+4af)\right)h\sqrt{a+bx+cx^2}}{c^2\left(b^2-4ac\right)} \\
&\quad -\frac{\left(3bfh-2c(fg+eh)\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{5/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.04

$$
\int\frac{(g+hx)\left(d+ex+fx^2\right)}{\left(a+bx+cx^2\right)^{3/2}}\,dx=\frac{-3b^3fhx+2bc(aeh-cegx+cd(g-hx)+af(g+5hx))+b^2(-3afh+cx}{c^2\left(-b^2+4ac\right.}
$$

$$
+\frac{(-3bfh+2c(fg+eh))\text{arctanh}\left(\frac{\sqrt{c}x}{-\sqrt{a}+\sqrt{a+x(b+cx)}}\right)}{c^{5/2}}
$$

```
[In] Integrate[((g + h*x)*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (-3*b^3*f*h*x + 2*b*c*(a*e*h - c*e*g*x + c*d*(g - h*x) + a*f*(g + 5*h*x)) +
 b^2*(-3*a*f*h + c*x*(2*f*g + 2*e*h - f*h*x)) + 4*c*(2*a^2*f*h + c^2*d*g*x
```

```
- a*c*(d*h + f*x*(g - h*x) + e*(g + h*x))))/(c^2*(-b^2 + 4*a*c)*Sqrt[a + x*
(b + c*x)]) + ((-3*b*f*h + 2*c*(f*g + e*h))*ArcTanh[(Sqrt[c]*x)/(-Sqrt[a] +
 Sqrt[a + x*(b + c*x)])])/c^(5/2)
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.67

method	result
risch	$\frac{fh\sqrt{c\,x^{2}+bx+a}}{c^{2}}-\frac{\frac{2abfh(2cx+b)}{\left(4ac-b^{2}\right)\sqrt{c\,x^{2}+bx+a}}-\frac{4c^{2}dg(2cx+b)}{\left(4ac-b^{2}\right)\sqrt{c\,x^{2}+bx+a}}+\left(3bcfh-2c^{2}eh-2c^{2}fg\right)\left(-\frac{x}{c\sqrt{c\,x^{2}+bx+a}}-\frac{b\left(-\frac{1}{c\sqrt{c\,x^{2}+bx+a}}-\frac{}{c\left(\right.}\right.}{2}\right.}{}$
default	$\frac{2dg(2cx+b)}{(4ac-b^{2})\sqrt{c\,x^{2}+bx+a}}+fh\left(\frac{x^{2}}{c\sqrt{c\,x^{2}+bx+a}}-\frac{3b\left(-\frac{x}{c\sqrt{c\,x^{2}+bx+a}}-\frac{b\left(-\frac{1}{c\sqrt{c\,x^{2}+bx+a}}-\frac{b(2cx+b)}{c\left(4ac-b^{2}\right)\sqrt{c\,x^{2}+bx+a}}\right)}{2c}+\frac{\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{}\right.}{c}\right.}{2c}\right.$

```
[In] int((h*x+g)*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] f*h/c^2*(c*x^2+b*x+a)^(1/2)-1/2/c^2*(2*a*b*f*h*(2*c*x+b)/(4*a*c-b^2)/(c*x^2
+b*x+a)^(1/2)-4*c^2*d*g*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+(3*b*c*f*
h-2*c^2*e*h-2*c^2*f*g)*(-x/c/(c*x^2+b*x+a)^(1/2)-1/2*b/c*(-1/c/(c*x^2+b*x+a
)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))+1/c^(3/2)*ln((1/2*b+
c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))+(2*a*c*f*h+b^2*f*h-2*c^2*d*h-2*c^2*e*g)*
(-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 451 vs. $2(171) = 342$.

Time = 3.87 (sec) , antiderivative size = 905, normalized size of antiderivative = 4.87

$$\int \frac{(g+hx)\,(d+ex+fx^{2})}{(a+bx+cx^{2})^{3/2}}\,dx = \left[-\frac{(2\,(ab^{2}c-4\,a^{2}c^{2})fg+(2\,(b^{2}c^{2}-4\,ac^{3})fg+(2\,(b^{2}c^{2}-4\,ac^{3})e-3\,(b^{3}c}{}\right.$$

$$-\frac{(2\,(ab^{2}c-4\,a^{2}c^{2})fg+(2\,(b^{2}c^{2}-4\,ac^{3})fg+(2\,(b^{2}c^{2}-4\,ac^{3})e-3\,(b^{3}c-4\,abc^{2})f)h)x^{2}+(2\,(ab^{2}c-4\,a^{2}c^{2}}{}$$

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((2*(a*b^2*c - 4*a^2*c^2)*f*g + (2*(b^2*c^2 - 4*a*c^3)*f*g + (2*(b^2*
c^2 - 4*a*c^3)*e - 3*(b^3*c - 4*a*b*c^2)*f)*h)*x^2 + (2*(a*b^2*c - 4*a^2*c^
```

```
2)*e - 3*(a*b^3 - 4*a^2*b*c)*f)*h + (2*(b^3*c - 4*a*b*c^2)*f*g + (2*(b^3*c
- 4*a*b*c^2)*e - 3*(b^4 - 4*a*b^2*c)*f)*h)*x)*sqrt(c)*log(-8*c^2*x^2 - 8*b*
c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*((b^2*
c^2 - 4*a*c^3)*f*h*x^2 - 2*(b*c^3*d - 2*a*c^3*e + a*b*c^2*f)*g + (4*a*c^3*d
 - 2*a*b*c^2*e + (3*a*b^2*c - 8*a^2*c^2)*f)*h - (2*(2*c^4*d - b*c^3*e + (b^
2*c^2 - 2*a*c^3)*f)*g - (2*b*c^3*d - 2*(b^2*c^2 - 2*a*c^3)*e + (3*b^3*c - 1
0*a*b*c^2)*f)*h)*x)*sqrt(c*x^2 + b*x + a))/(a*b^2*c^3 - 4*a^2*c^4 + (b^2*c^
4 - 4*a*c^5)*x^2 + (b^3*c^3 - 4*a*b*c^4)*x), -1/2*((2*(a*b^2*c - 4*a^2*c^2)
*f*g + (2*(b^2*c^2 - 4*a*c^3)*f*g + (2*(b^2*c^2 - 4*a*c^3)*e - 3*(b^3*c - 4
*a*b*c^2)*f)*h)*x^2 + (2*(a*b^2*c - 4*a^2*c^2)*e - 3*(a*b^3 - 4*a^2*b*c)*f)
*h + (2*(b^3*c - 4*a*b*c^2)*f*g + (2*(b^3*c - 4*a*b*c^2)*e - 3*(b^4 - 4*a*b
^2*c)*f)*h)*x)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-
c)/(c^2*x^2 + b*c*x + a*c)) - 2*((b^2*c^2 - 4*a*c^3)*f*h*x^2 - 2*(b*c^3*d -
 2*a*c^3*e + a*b*c^2*f)*g + (4*a*c^3*d - 2*a*b*c^2*e + (3*a*b^2*c - 8*a^2*c
^2)*f)*h - (2*(2*c^4*d - b*c^3*e + (b^2*c^2 - 2*a*c^3)*f)*g - (2*b*c^3*d -
2*(b^2*c^2 - 2*a*c^3)*e + (3*b^3*c - 10*a*b*c^2)*f)*h)*x)*sqrt(c*x^2 + b*x
+ a))/(a*b^2*c^3 - 4*a^2*c^4 + (b^2*c^4 - 4*a*c^5)*x^2 + (b^3*c^3 - 4*a*b*c
^4)*x)]
```

Sympy [F]

$$\int \frac{(g+hx)\left(d+ex+fx^2\right)}{\left(a+bx+cx^2\right)^{3/2}}\,dx=\int \frac{(g+hx)\left(d+ex+fx^2\right)}{\left(a+bx+cx^2\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((h*x+g)*(f*x**2+e*x+d)/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral((g + h*x)*(d + e*x + f*x**2)/(a + b*x + c*x**2)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(g+hx)\left(d+ex+fx^2\right)}{\left(a+bx+cx^2\right)^{3/2}}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.41

$$\int \frac{(g+hx)\left(d+ex+fx^2\right)}{\left(a+bx+cx^2\right)^{3/2}}\,dx = \frac{\left(\frac{(b^2cfh-4\,ac^2fh)x}{b^2c^2-4\,ac^3} - \frac{4\,c^3dg-2\,bc^2eg+2\,b^2cfg-4\,ac^2fg-2\,bc^2dh+2\,b^2ceh-4\,ac^2eh-3\,b^3fh+10\,ab}{b^2c^2-4\,ac^3}}{\sqrt{cx^2+bx+a}}$$

$$-\frac{(2\,cfg+2\,ceh-3\,bfh)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}+b\right|\right)}{2\,c^{\frac{5}{2}}}$$

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] (((b^2*c*f*h - 4*a*c^2*f*h)*x/(b^2*c^2 - 4*a*c^3) - (4*c^3*d*g - 2*b*c^2*e*
g + 2*b^2*c*f*g - 4*a*c^2*f*g - 2*b*c^2*d*h + 2*b^2*c*e*h - 4*a*c^2*e*h - 3
*b^3*f*h + 10*a*b*c*f*h)/(b^2*c^2 - 4*a*c^3))*x - (2*b*c^2*d*g - 4*a*c^2*e*
g + 2*a*b*c*f*g - 4*a*c^2*d*h + 2*a*b*c*e*h - 3*a*b^2*f*h + 8*a^2*c*f*h)/(b
^2*c^2 - 4*a*c^3))/sqrt(c*x^2 + b*x + a) - 1/2*(2*c*f*g + 2*c*e*h - 3*b*f*h
)*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(g+hx)\left(d+ex+fx^2\right)}{\left(a+bx+cx^2\right)^{3/2}}\,dx = \int \frac{(g+h\,x)\,\left(f\,x^2+e\,x+d\right)}{{\left(c\,x^2+b\,x+a\right)}^{3/2}} \,d x$$

```
[In] int(((g + h*x)*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(3/2),x)
```

```
[Out] int(((g + h*x)*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(3/2), x)
```

3.236 $\int \frac{d+ex+fx^2}{\left(a+bx+cx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 111

$$\int \frac{d+ex+fx^2}{\left(a+bx+cx^2\right)^{3/2}}\,dx = \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} + \frac{f\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}}$$

```
[Out] f*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(3/2)+2*(c*(2*a*e-b*
(d+a*f/c))-(-2*a*c*f+b^2*f-b*c*e+2*c^2*d)*x)/c/(-4*a*c+b^2)/(c*x^2+b*x+a)^(
1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1674, 12, 635, 212}

$$\int \frac{d+ex+fx^2}{\left(a+bx+cx^2\right)^{3/2}}\,dx = \frac{f\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}} + \frac{2\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

```
[In] Int[(d + e*x + f*x^2)/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (2*(c*(2*a*e - b*(d + (a*f)/c)) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x))/(
c*(b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]) + (f*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*
Sqrt[a + b*x + c*x^2])])/c^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}-\frac{2\int -\frac{\left(b^2-4ac\right)f}{2c\sqrt{a+bx+cx^2}}\,dx}{b^2-4ac}\\
&= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}+\frac{f\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{c}\\
&= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}+\frac{(2f)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx,x,\frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{c}\\
&= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}+\frac{f\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.95

$$\int \frac{d+ex+fx^2}{\left(a+bx+cx^2\right)^{3/2}}\,dx=\frac{2(abf+2c^2dx+b^2fx+bc(d-ex)-2ac(e+fx))}{c\left(-b^2+4ac\right)\sqrt{a+x(b+cx)}}+\frac{2f\operatorname{arctanh}\left(\frac{\sqrt{c}x}{-\sqrt{a}+\sqrt{a+x(b+cx)}}\right)}{c^{3/2}}$$

```
[In] Integrate[(d + e*x + f*x^2)/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (2*(a*b*f + 2*c^2*d*x + b^2*f*x + b*c*(d - e*x) - 2*a*c*(e + f*x)))/(c*(-b^
2 + 4*a*c)*Sqrt[a + x*(b + c*x)]) + (2*f*ArcTanh[(Sqrt[c]*x)/(-Sqrt[a] + Sq
rt[a + x*(b + c*x)])])/c^(3/2)
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.81

method	result
default	$\frac{2d(2cx+b)}{(4ac-b^2)\sqrt{c\,x^2+bx+a}}+f\left(-\frac{x}{c\sqrt{c\,x^2+bx+a}}-\frac{b\left(-\frac{1}{c\sqrt{c\,x^2+bx+a}}-\frac{b(2cx+b)}{c\left(4ac-b^2\right)\sqrt{c\,x^2+bx+a}}\right)}{2c}+\frac{\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{c^{\frac{3}{2}}}\right)+$

```
[In] int((f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*d*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+f*(-x/c/(c*x^2+b*x+a)^(1/2)-1
/2*b/c*(-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1
/2))+1/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))+e*(-1/c/(c*x^2+
b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 213 vs. 2(99) = 198.

Time = 0.47 (sec) , antiderivative size = 429, normalized size of antiderivative = 3.86

$$\int \frac{d+ex+fx^2}{\left(a+bx+cx^2\right)^{3/2}}\,dx=\left[\frac{((b^2c-4\,ac^2)fx^2+(b^3-4\,abc)fx+(ab^2-4\,a^2c)f)\sqrt{c}\log\left(-8\,c^2x^2-8\,bcx-b^2\right.}{2\,(ab^2c^2-}\right.$$

$$-\frac{((b^2c-4\,ac^2)fx^2+(b^3-4\,abc)fx+(ab^2-4\,a^2c)f)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^2+bx+a}(2\,cx+b)\sqrt{-c}}{2\,(c^2x^2+bcx+ac)}\right)+2\,(bc^2d-2\,ac^2e-}{ab^2c^2-4\,a^2c^3+(b^2c^3-4\,ac^4)x^2+(b^3c^2-4\,abc^3)x}$$

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(((b^2*c - 4*a*c^2)*f*x^2 + (b^3 - 4*a*b*c)*f*x + (a*b^2 - 4*a^2*c)*f)
*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x +
b)*sqrt(c) - 4*a*c) - 4*(b*c^2*d - 2*a*c^2*e + a*b*c*f + (2*c^3*d - b*c^2*e
 + (b^2*c - 2*a*c^2)*f)*x)*sqrt(c*x^2 + b*x + a))/(a*b^2*c^2 - 4*a^2*c^3 +
(b^2*c^3 - 4*a*c^4)*x^2 + (b^3*c^2 - 4*a*b*c^3)*x), -(((b^2*c - 4*a*c^2)*f*
x^2 + (b^3 - 4*a*b*c)*f*x + (a*b^2 - 4*a^2*c)*f)*sqrt(-c)*arctan(1/2*sqrt(c
*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(b*c^2*d
- 2*a*c^2*e + a*b*c*f + (2*c^3*d - b*c^2*e + (b^2*c - 2*a*c^2)*f)*x)*sqrt(c
*x^2 + b*x + a))/(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^2 + (b^3*c^
2 - 4*a*b*c^3)*x)]
```

Sympy [F]

$$\int \frac{d+ex+fx^2}{(a+bx+cx^2)^{3/2}}\,dx = \int \frac{d+ex+fx^2}{(a+bx+cx^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((f*x**2+e*x+d)/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral((d + e*x + f*x**2)/(a + b*x + c*x**2)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex+fx^2}{(a+bx+cx^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.06

$$\int \frac{d+ex+fx^2}{(a+bx+cx^2)^{3/2}}\,dx = -\frac{2\left(\frac{(2\,c^2d-bce+b^2f-2\,acf)x}{b^2c-4\,ac^2}+\frac{bcd-2\,ace+abf}{b^2c-4\,ac^2}\right)}{\sqrt{cx^2+bx+a}} - \frac{f\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}+b\right|\right)}{c^{\frac{3}{2}}}$$

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -2*((2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x/(b^2*c - 4*a*c^2) + (b*c*d - 2*a*
c*e + a*b*f)/(b^2*c - 4*a*c^2))/sqrt(c*x^2 + b*x + a) - f*log(abs(2*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(3/2)
```

Mupad [B] (verification not implemented)

Time = 13.60 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.29

$$\int \frac{d+ex+fx^2}{(a+bx+cx^2)^{3/2}}\,dx = \frac{f\,\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x+a}\right)}{c^{3/2}} - \frac{e\,(4\,a+2\,b\,x)}{(4\,a\,c-b^2)\,\sqrt{c\,x^2+b\,x+a}} + \frac{d\,\left(\frac{b}{2}+c\,x\right)}{\left(a\,c-\frac{b^2}{4}\right)\,\sqrt{c\,x^2+b\,x+a}} + \frac{f\,\left(\frac{a\,b}{2}-x\,\left(a\,c-\frac{b^2}{2}\right)\right)}{c\,\left(a\,c-\frac{b^2}{4}\right)\,\sqrt{c\,x^2+b\,x+a}}$$

```
[In] int((d + e*x + f*x^2)/(a + b*x + c*x^2)^(3/2),x)
```

```
[Out] (f*log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2)))/c^(3/2) - (e*(4*a +
2*b*x))/((4*a*c - b^2)*(a + b*x + c*x^2)^(1/2)) + (d*(b/2 + c*x))/((a*c - b
^2/4)*(a + b*x + c*x^2)^(1/2)) + (f*((a*b)/2 - x*(a*c - b^2/2)))/(c*(a*c -
b^2/4)*(a + b*x + c*x^2)^(1/2))
```

3.237 $\int \frac{d+ex+fx^2}{(g+hx)\left(a+bx+cx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 225

$$\int \frac{d+ex+fx^2}{(g+hx)\left(a+bx+cx^2\right)^{3/2}}\,dx=\frac{2(b^2dh-b(cdg+afg+aeh)+2a(ceg-cdh+afh)-(2c^2dg+bf(bg}{(b^2-4ac)\left(cg^2-bgh+ah^2\right)\sqrt{a+bx}}$$
$$+\frac{(fg^2-h(eg-dh))\operatorname{arctanh}\left(\frac{bg-2ah+(2cg-bh)x}{2\sqrt{cg^2-bgh+ah^2}\sqrt{a+bx+cx^2}}\right)}{\left(cg^2-bgh+ah^2\right)^{3/2}}$$

```
[Out] (f*g^2-h*(-d*h+e*g))*arctanh(1/2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*
g^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/(a*h^2-b*g*h+c*g^2)^(3/2)+2*(b^2*d*h-b*(a*e
*h+a*f*g+c*d*g)+2*a*(a*f*h-c*d*h+c*e*g)-(2*c^2*d*g+b*f*(-a*h+b*g)-c*(-2*a*e
*h+2*a*f*g+b*d*h+b*e*g))*x)/(-4*a*c+b^2)/(a*h^2-b*g*h+c*g^2)/(c*x^2+b*x+a)^
(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1660, 12, 738, 212}

$$\int \frac{d+ex+fx^2}{(g+hx)\left(a+bx+cx^2\right)^{3/2}}\,dx=\frac{(fg^2-h(eg-dh))\operatorname{arctanh}\left(\frac{-2ah+x(2cg-bh)+bg}{2\sqrt{a+bx+cx^2}\sqrt{ah^2-bgh+cg^2}}\right)}{\left(ah^2-bgh+cg^2\right)^{3/2}}$$
$$+\frac{2(-x(-c(-2aeh+2afg+bdh+beg)+bf(bg-ah)+2c^2dg)-b(aeh+afg+cdg)+2a(afh-cdh+c}{(b^2-4ac)\sqrt{a+bx+cx^2}\left(ah^2-bgh+cg^2\right)}$$

```
[In] Int[(d + e*x + f*x^2)/((g + h*x)*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (2*(b^2*d*h - b*(c*d*g + a*f*g + a*e*h) + 2*a*(c*e*g - c*d*h + a*f*h) - (2*
c^2*d*g + b*f*(b*g - a*h) - c*(b*e*g + 2*a*f*g + b*d*h - 2*a*e*h))*x))/((b^
2 - 4*a*c)*(c*g^2 - b*g*h + a*h^2)*Sqrt[a + b*x + c*x^2]) + ((f*g^2 - h*(e*
g - d*h))*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a
*h^2]*Sqrt[a + b*x + c*x^2])])/(c*g^2 - b*g*h + a*h^2)^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1660

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^(p + 1)/((p
+ 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

integral

$$= \frac{2(b^2dh - b(cdg+afg+aeh) + 2a(ceg-cdh+afh) - (2c^2dg + bf(bg-ah) - c(beg+2afg+bdh-2ae}{(b^2-4ac)\left(cg^2-bgh+ah^2\right)\sqrt{a+bx+cx^2}}$$

$$- \frac{2\int -\frac{(b^2-4ac)\left(fg^2-h(eg-dh)\right)}{2(cg^2-bgh+ah^2)(g+hx)\sqrt{a+bx+cx^2}}\,dx}{b^2-4ac}$$

$$= \frac{2(b^2dh - b(cdg + afg + aeh) + 2a(ceg - cdh + afh) - (2c^2dg + bf(bg - ah) - c(beg + 2afg +}{(b^2 - 4ac)(cg^2 - bgh + ah^2)\sqrt{a + bx + cx^2}} + \frac{(fg^2 - h(eg - dh))\int \frac{1}{(g+hx)\sqrt{a+bx+cx^2}}\,dx}{cg^2 - bgh + ah^2}$$

$$= \frac{2(b^2dh - b(cdg + afg + aeh) + 2a(ceg - cdh + afh) - (2c^2dg + bf(bg - ah) - c(beg + 2afg +}{(b^2 - 4ac)(cg^2 - bgh + ah^2)\sqrt{a + bx + cx^2}} - \frac{(2(fg^2 - h(eg - dh)))\,\text{Subst}\left(\int \frac{1}{4cg^2-4bgh+4ah^2-x^2}\,dx, x, \frac{-bg+2ah-(2cg-bh)x}{\sqrt{a+bx+cx^2}}\right)}{cg^2 - bgh + ah^2}$$

$$= \frac{2(b^2dh - b(cdg + afg + aeh) + 2a(ceg - cdh + afh) - (2c^2dg + bf(bg - ah) - c(beg + 2afg +}{(b^2 - 4ac)(cg^2 - bgh + ah^2)\sqrt{a + bx + cx^2}} + \frac{(fg^2 - h(eg - dh))\tanh^{-1}\left(\frac{bg-2ah+(2cg-bh)x}{2\sqrt{cg^2-bgh+ah^2}\sqrt{a+bx+cx^2}}\right)}{(cg^2 - bgh + ah^2)^{3/2}}$$

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.04

$$\int \frac{d + ex + fx^2}{(g + hx)(a + bx + cx^2)^{3/2}}\,dx = 2\left(\frac{-2a^2fh + 2c^2dgx + b^2(-dh + fgx) + 2ac(-eg + dh - fgx + ehx) -}{(b^2 - 4ac)(-cg^2 + h(bg - ah))\sqrt{a}} + \frac{\sqrt{-cg^2 + bgh - ah^2}(fg^2 + h(-eg + dh))\arctan\left(\frac{\sqrt{c}(g+hx)-h\sqrt{a+x(b+cx)}}{\sqrt{-cg^2+h(bg-ah)}}\right)}{(cg^2 + h(-bg + ah))^2}\right)$$

```
[In] Integrate[(d + e*x + f*x^2)/((g + h*x)*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] 2*((-2*a^2*f*h + 2*c^2*d*g*x + b^2*(-(d*h) + f*g*x) + 2*a*c*(-(e*g) + d*h -
 f*g*x + e*h*x) + b*c*(-(e*g*x) + d*(g - h*x)) + a*b*(e*h + f*(g - h*x)))/(
(b^2 - 4*a*c)*(-(c*g^2) + h*(b*g - a*h))*Sqrt[a + x*(b + c*x)]) + (Sqrt[-(c
*g^2) + b*g*h - a*h^2]*(f*g^2 + h*(-(e*g) + d*h))*ArcTan[(Sqrt[c]*(g + h*x)
 - h*Sqrt[a + x*(b + c*x)])/Sqrt[-(c*g^2) + h*(b*g - a*h)]])/(c*g^2 + h*(-(
b*g) + a*h))^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 546 vs. $2(215) = 430$.

Time = 0.76 (sec) , antiderivative size = 547, normalized size of antiderivative = 2.43

method	result
default	$\frac{\frac{2eh(2cx+b)}{\left(4ac-b^{2}\right)\sqrt{c\,x^{2}+bx+a}}+fh\left(-\frac{1}{c\sqrt{c\,x^{2}+bx+a}}-\frac{b(2cx+b)}{c\left(4ac-b^{2}\right)\sqrt{c\,x^{2}+bx+a}}\right)-\frac{2fg(2cx+b)}{\left(4ac-b^{2}\right)\sqrt{c\,x^{2}+bx+a}}}{h^{2}}+\frac{\left(d\,h^{2}-egh+f\,g^{2}\right)\left(\frac{}{\left(a\,h^{2}-bgh+c\,g^{2}\right)}\right.}{}$

```
[In] int((f*x^2+e*x+d)/(h*x+g)/(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/h^2*(2*e*h*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+f*h*(-1/c/(c*x^2+b*x
+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))-2*f*g*(2*c*x+b)/(4
*a*c-b^2)/(c*x^2+b*x+a)^(1/2))+(d*h^2-e*g*h+f*g^2)/h^3*(1/(a*h^2-b*g*h+c*g^
2)*h^2/((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2
)-(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)*(2*c*(x+1/h*g)+(b*h-2*c*g)/h)/(4*c*(a*h
^2-b*g*h+c*g^2)/h^2-(b*h-2*c*g)^2/h^2)/((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*
g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)-1/(a*h^2-b*g*h+c*g^2)*h^2/((a*h^2-b*g*h+c
*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+1/h*g)+2*((
a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2
-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 931 vs. $2(215) = 430$.

Time = 9.79 (sec) , antiderivative size = 1905, normalized size of antiderivative = 8.47

$$\int \frac{d+ex+fx^2}{(g+hx)\left(a+bx+cx^2\right)^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(((a*b^2 - 4*a^2*c)*f*g^2 - (a*b^2 - 4*a^2*c)*e*g*h + (a*b^2 - 4*a^2*c
)*d*h^2 + ((b^2*c - 4*a*c^2)*f*g^2 - (b^2*c - 4*a*c^2)*e*g*h + (b^2*c - 4*a
*c^2)*d*h^2)*x^2 + ((b^3 - 4*a*b*c)*f*g^2 - (b^3 - 4*a*b*c)*e*g*h + (b^3 -
4*a*b*c)*d*h^2)*x)*sqrt(c*g^2 - b*g*h + a*h^2)*log((8*a*b*g*h - 8*a^2*h^2 -
 (b^2 + 4*a*c)*g^2 - (8*c^2*g^2 - 8*b*c*g*h + (b^2 + 4*a*c)*h^2)*x^2 - 4*sq
rt(c*g^2 - b*g*h + a*h^2)*sqrt(c*x^2 + b*x + a)*(b*g - 2*a*h + (2*c*g - b*h
)*x) - 2*(4*b*c*g^2 + 4*a*b*h^2 - (3*b^2 + 4*a*c)*g*h)*x)/(h^2*x^2 + 2*g*h*
x + g^2)) - 4*((b*c^2*d - 2*a*c^2*e + a*b*c*f)*g^3 + (3*a*b*c*e - 2*(b^2*c
- a*c^2)*d - (a*b^2 + 2*a^2*c)*f)*g^2*h + (3*a^2*b*f + (b^3 - a*b*c)*d - (a
```

```
*b^2 + 2*a^2*c)*e)*g*h^2 + (a^2*b*e - 2*a^3*f - (a*b^2 - 2*a^2*c)*d)*h^3 +
((2*c^3*d - b*c^2*e + (b^2*c - 2*a*c^2)*f)*g^3 - (3*b*c^2*d - (b^2*c + 2*a*
c^2)*e + (b^3 - a*b*c)*f)*g^2*h - (3*a*b*c*e - (b^2*c + 2*a*c^2)*d - 2*(a*b
^2 - a^2*c)*f)*g*h^2 - (a*b*c*d - 2*a^2*c*e + a^2*b*f)*h^3)*x)*sqrt(c*x^2 +
 b*x + a))/((a*b^2*c^2 - 4*a^2*c^3)*g^4 - 2*(a*b^3*c - 4*a^2*b*c^2)*g^3*h +
 (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*g^2*h^2 - 2*(a^2*b^3 - 4*a^3*b*c)*g*h^3
+ (a^3*b^2 - 4*a^4*c)*h^4 + ((b^2*c^3 - 4*a*c^4)*g^4 - 2*(b^3*c^2 - 4*a*b*c
^3)*g^3*h + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*g^2*h^2 - 2*(a*b^3*c - 4*a^2*
b*c^2)*g*h^3 + (a^2*b^2*c - 4*a^3*c^2)*h^4)*x^2 + ((b^3*c^2 - 4*a*b*c^3)*g^
4 - 2*(b^4*c - 4*a*b^2*c^2)*g^3*h + (b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*g^2*h^2
 - 2*(a*b^4 - 4*a^2*b^2*c)*g*h^3 + (a^2*b^3 - 4*a^3*b*c)*h^4)*x), (((a*b^2
- 4*a^2*c)*f*g^2 - (a*b^2 - 4*a^2*c)*e*g*h + (a*b^2 - 4*a^2*c)*d*h^2 + ((b^
2*c - 4*a*c^2)*f*g^2 - (b^2*c - 4*a*c^2)*e*g*h + (b^2*c - 4*a*c^2)*d*h^2)*x
^2 + ((b^3 - 4*a*b*c)*f*g^2 - (b^3 - 4*a*b*c)*e*g*h + (b^3 - 4*a*b*c)*d*h^2
)*x)*sqrt(-c*g^2 + b*g*h - a*h^2)*arctan(-1/2*sqrt(-c*g^2 + b*g*h - a*h^2)*
sqrt(c*x^2 + b*x + a)*(b*g - 2*a*h + (2*c*g - b*h)*x)/(a*c*g^2 - a*b*g*h +
a^2*h^2 + (c^2*g^2 - b*c*g*h + a*c*h^2)*x^2 + (b*c*g^2 - b^2*g*h + a*b*h^2)
*x)) - 2*((b*c^2*d - 2*a*c^2*e + a*b*c*f)*g^3 + (3*a*b*c*e - 2*(b^2*c - a*c
^2)*d - (a*b^2 + 2*a^2*c)*f)*g^2*h + (3*a^2*b*f + (b^3 - a*b*c)*d - (a*b^2
+ 2*a^2*c)*e)*g*h^2 + (a^2*b*e - 2*a^3*f - (a*b^2 - 2*a^2*c)*d)*h^3 + ((2*c
^3*d - b*c^2*e + (b^2*c - 2*a*c^2)*f)*g^3 - (3*b*c^2*d - (b^2*c + 2*a*c^2)*
e + (b^3 - a*b*c)*f)*g^2*h - (3*a*b*c*e - (b^2*c + 2*a*c^2)*d - 2*(a*b^2 -
a^2*c)*f)*g*h^2 - (a*b*c*d - 2*a^2*c*e + a^2*b*f)*h^3)*x)*sqrt(c*x^2 + b*x
+ a))/((a*b^2*c^2 - 4*a^2*c^3)*g^4 - 2*(a*b^3*c - 4*a^2*b*c^2)*g^3*h + (a*b
^4 - 2*a^2*b^2*c - 8*a^3*c^2)*g^2*h^2 - 2*(a^2*b^3 - 4*a^3*b*c)*g*h^3 + (a^
3*b^2 - 4*a^4*c)*h^4 + ((b^2*c^3 - 4*a*c^4)*g^4 - 2*(b^3*c^2 - 4*a*b*c^3)*g
^3*h + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*g^2*h^2 - 2*(a*b^3*c - 4*a^2*b*c^2
)*g*h^3 + (a^2*b^2*c - 4*a^3*c^2)*h^4)*x^2 + ((b^3*c^2 - 4*a*b*c^3)*g^4 - 2
*(b^4*c - 4*a*b^2*c^2)*g^3*h + (b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*g^2*h^2 - 2*
(a*b^4 - 4*a^2*b^2*c)*g*h^3 + (a^2*b^3 - 4*a^3*b*c)*h^4)*x)]
```

Sympy [F]

$$\int \frac{d+ex+fx^2}{(g+hx)\left(a+bx+cx^2\right)^{3/2}}\,dx=\int \frac{d+ex+fx^2}{(g+hx)\left(a+bx+cx^2\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((f*x**2+e*x+d)/(h*x+g)/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral((d + e*x + f*x**2)/((g + h*x)*(a + b*x + c*x**2)**(3/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex+fx^2}{(g+hx)\left(a+bx+cx^2\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((b/h-(2*c*g)/h^2)^2>0)', see `assum
e?` for
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 708 vs. 2(215) = 430.

Time = 0.31 (sec) , antiderivative size = 708, normalized size of antiderivative = 3.15

$$\int \frac{d+ex+fx^2}{(g+hx)\left(a+bx+cx^2\right)^{3/2}}\,dx =$$

$$-\frac{2\left(\frac{(2\,c^3dg^3-bc^2eg^3+b^2cfg^3-2\,ac^2fg^3-3\,bc^2dg^2h+b^2ceg^2h+2\,ac^2eg^2h-b^3fg^2h+abcfg^2h+b^2cdgh^2+2\,ac^2dgh^2-3\,abcegh^2+2\,ab^2fgh^2-2\,a^2cfg}{b^2c^2g^4-4\,ac^3g^4-2\,b^3cg^3h+8\,abc^2g^3h+b^4g^2h^2-2\,ab^2cg^2h^2-8\,a^2c^2g^2h^2-2\,ab^3gh^3+8\,a^2bcgh^3+a^2b^2h^4-4\,a^3ch}\right.}{}$$

$$+\frac{2\left(fg^2-egh+dh^2\right)\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)h+\sqrt{c}g}{\sqrt{-cg^2+bgh-ah^2}}\right)}{\left(cg^2-bgh+ah^2\right)\sqrt{-cg^2+bgh-ah^2}}$$

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -2*((2*c^3*d*g^3 - b*c^2*e*g^3 + b^2*c*f*g^3 - 2*a*c^2*f*g^3 - 3*b*c^2*d*g^
2*h + b^2*c*e*g^2*h + 2*a*c^2*e*g^2*h - b^3*f*g^2*h + a*b*c*f*g^2*h + b^2*c
*d*g*h^2 + 2*a*c^2*d*g*h^2 - 3*a*b*c*e*g*h^2 + 2*a*b^2*f*g*h^2 - 2*a^2*c*f*
g*h^2 - a*b*c*d*h^3 + 2*a^2*c*e*h^3 - a^2*b*f*h^3)*x/(b^2*c^2*g^4 - 4*a*c^3
*g^4 - 2*b^3*c*g^3*h + 8*a*b*c^2*g^3*h + b^4*g^2*h^2 - 2*a*b^2*c*g^2*h^2 -
8*a^2*c^2*g^2*h^2 - 2*a*b^3*g*h^3 + 8*a^2*b*c*g*h^3 + a^2*b^2*h^4 - 4*a^3*c
*h^4) + (b*c^2*d*g^3 - 2*a*c^2*e*g^3 + a*b*c*f*g^3 - 2*b^2*c*d*g^2*h + 2*a*
c^2*d*g^2*h + 3*a*b*c*e*g^2*h - a*b^2*f*g^2*h - 2*a^2*c*f*g^2*h + b^3*d*g*h
^2 - a*b*c*d*g*h^2 - a*b^2*e*g*h^2 - 2*a^2*c*e*g*h^2 + 3*a^2*b*f*g*h^2 - a*
b^2*d*h^3 + 2*a^2*c*d*h^3 + a^2*b*e*h^3 - 2*a^3*f*h^3)/(b^2*c^2*g^4 - 4*a*c
^3*g^4 - 2*b^3*c*g^3*h + 8*a*b*c^2*g^3*h + b^4*g^2*h^2 - 2*a*b^2*c*g^2*h^2
- 8*a^2*c^2*g^2*h^2 - 2*a*b^3*g*h^3 + 8*a^2*b*c*g*h^3 + a^2*b^2*h^4 - 4*a^3
*c*h^4))/sqrt(c*x^2 + b*x + a) + 2*(f*g^2 - e*g*h + d*h^2)*arctan(-((sqrt(c
)*x - sqrt(c*x^2 + b*x + a))*h + sqrt(c)*g)/sqrt(-c*g^2 + b*g*h - a*h^2))/(
(c*g^2 - b*g*h + a*h^2)*sqrt(-c*g^2 + b*g*h - a*h^2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2}{(g+hx)\,(a+bx+cx^2)^{3/2}}\,dx = \int \frac{f\,x^2+e\,x+d}{(g+h\,x)\,\left(c\,x^2+b\,x+a\right)^{3/2}}\,dx$$

```
[In] int((d + e*x + f*x^2)/((g + h*x)*(a + b*x + c*x^2)^(3/2)),x)
[Out] int((d + e*x + f*x^2)/((g + h*x)*(a + b*x + c*x^2)^(3/2)), x)
```

3.238 $\int \frac{d+ex+fx^2}{(g+hx)^2\left(a+bx+cx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 421

$$\int \frac{d+ex+fx^2}{(g+hx)^2\left(a+bx+cx^2\right)^{3/2}}\,dx =$$

$$-\frac{2(b^3dh^2-b^2h(2cdg+aeh)-2ac(cg(eg-2dh)+ah(2fg-eh))+b(c^2dg^2+a^2fh^2+ac(fg^2+2egh-3d}{(b^2-4ac)\left(cg^2-bgh\right.}$$

$$-\frac{h(fg^2-h(eg-dh))\sqrt{a+bx+cx^2}}{\left(cg^2-bgh+ah^2\right)^2(g+hx)}$$

$$+\frac{(2cg(fg^2-h(2eg-3dh))-h(2ah(2fg-eh)-b(fg^2+egh-3dh^2)))\operatorname{arctanh}\left(\frac{bg-2ah+(2cg-bh)x}{2\sqrt{cg^2-bgh+ah^2}\sqrt{a+bx+cx^2}}\right)}{2\left(cg^2-bgh+ah^2\right)^{5/2}}$$

```
[Out] 1/2*(2*c*g*(f*g^2-h*(-3*d*h+2*e*g))-h*(2*a*h*(-e*h+2*f*g)-b*(-3*d*h^2+e*g*h
+f*g^2)))*arctanh(1/2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^(1/2)/
(c*x^2+b*x+a)^(1/2))/(a*h^2-b*g*h+c*g^2)^(5/2)-2*(b^3*d*h^2-b^2*h*(a*e*h+2*
c*d*g)-2*a*c*(c*g*(-2*d*h+e*g)+a*h*(-e*h+2*f*g))+b*(c^2*d*g^2+a^2*f*h^2+a*c
*(-3*d*h^2+2*e*g*h+f*g^2))+c*(2*c^2*d*g^2+2*a^2*f*h^2-a*b*h*(e*h+2*f*g)+b^2
*(d*h^2+f*g^2)-c*(b*g*(2*d*h+e*g)+2*a*(d*h^2-2*e*g*h+f*g^2)))*x)/(-4*a*c+b^
2)/(a*h^2-b*g*h+c*g^2)^2/(c*x^2+b*x+a)^(1/2)-h*(f*g^2-h*(-d*h+e*g))*(c*x^2+
b*x+a)^(1/2)/(a*h^2-b*g*h+c*g^2)^2/(h*x+g)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 418, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1660, 820, 738, 212}

$$\int \frac{d+ex+fx^2}{(g+hx)^2\left(a+bx+cx^2\right)^{3/2}}\,dx =$$

$$-\frac{2(cx(2a^2fh^2-c(2a(dh^2-2egh+fg^2)+bg(2dh+eg))-abh(eh+2fg)+b^2(dh^2+fg^2)+2c^2dg^2)+b(}{(b^2-4ac)\sqrt{a+bx+}}$$

$$+\frac{\operatorname{arctanh}\left(\frac{-2ah+x(2cg-bh)+bg}{2\sqrt{a+bx+cx^2}\sqrt{ah^2-bgh+cg^2}}\right)(h(-2ah(2fg-eh)+bh(eg-3dh)+bfg^2)+2c(fg^3-gh(2eg-3dh))}{2\left(ah^2-bgh+cg^2\right)^{5/2}}$$

$$-\frac{h\sqrt{a+bx+cx^2}(fg^2-h(eg-dh))}{(g+hx)\left(ah^2-bgh+cg^2\right)^2}$$

```
[In] Int[(d + e*x + f*x^2)/((g + h*x)^2*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (-2*(b^3*d*h^2 - b^2*h*(2*c*d*g + a*e*h) - 2*a*c*(c*g*(e*g - 2*d*h) + a*h*(
2*f*g - e*h)) + b*(c^2*d*g^2 + a^2*f*h^2 + a*c*(f*g^2 + 2*e*g*h - 3*d*h^2))
 + c*(2*c^2*d*g^2 + 2*a^2*f*h^2 - a*b*h*(2*f*g + e*h) + b^2*(f*g^2 + d*h^2)
 - c*(b*g*(e*g + 2*d*h) + 2*a*(f*g^2 - 2*e*g*h + d*h^2)))*x))/((b^2 - 4*a*c
)*(c*g^2 - b*g*h + a*h^2)^2*Sqrt[a + b*x + c*x^2]) - (h*(f*g^2 - h*(e*g - d
*h))*Sqrt[a + b*x + c*x^2])/((c*g^2 - b*g*h + a*h^2)^2*(g + h*x)) + ((2*c*(
f*g^3 - g*h*(2*e*g - 3*d*h)) + h*(b*f*g^2 + b*h*(e*g - 3*d*h) - 2*a*h*(2*f*
g - e*h)))*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h +
a*h^2]*Sqrt[a + b*x + c*x^2])])/(2*(c*g^2 - b*g*h + a*h^2)^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
```

```
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 1660

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^(p + 1)/((p
+ 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

integral =

$$-\frac{2(b^3dh^2-b^2h(2cdg+aeh)-2ac(cg(eg-2dh)+ah(2fg-eh))+b(c^2dg^2+a^2fh^2+ac(fg^2+2egh-}{(b^2-4ac)\left(cg^2-bg\right.}$$

$$-\frac{2\int\frac{-\frac{\left(b^2-4ac\right)\left(c\left(fg^4-g^2h(2eg-3dh)\right)-h^2\left(afg^2-adh^2-bg(eg-2dh)\right)\right)}{2\left(cg^2-bgh+ah^2\right)^2}+\frac{\left(b^2-4ac\right)h^2\left(cg(eg-2dh)+ah(2fg-eh)-b\left(fg^2-dh^2\right)\right)x}{2\left(cg^2-bgh+ah^2\right)^2}}{(g+hx)^2\sqrt{a+bx+cx^2}}\,dx}{b^2-4ac}$$

$$=-\frac{2(b^3dh^2-b^2h(2cdg+aeh)-2ac(cg(eg-2dh)+ah(2fg-eh))+b(c^2dg^2+a^2fh^2+ac(fg^2+}{(b^2-4ac)\left(\right.}$$

$$-\frac{h(fg^2-h(eg-dh))\sqrt{a+bx+cx^2}}{\left(cg^2-bgh+ah^2\right)^2(g+hx)}$$

$$+\frac{(2c(fg^3-gh(2eg-3dh))+h(bfg^2+bh(eg-3dh)-2ah(2fg-eh)))\int\frac{1}{(g+hx)\sqrt{a+bx+cx^2}}\,dx}{2\left(cg^2-bgh+ah^2\right)^2}$$

$$=-\frac{2(b^3dh^2-b^2h(2cdg+aeh)-2ac(cg(eg-2dh)+ah(2fg-eh))+b(c^2dg^2+a^2fh^2+ac(fg^2+}{(b^2-4ac)\left(\right.}$$

$$-\frac{h(fg^2-h(eg-dh))\sqrt{a+bx+cx^2}}{\left(cg^2-bgh+ah^2\right)^2(g+hx)}$$

$$-\frac{(2c(fg^3-gh(2eg-3dh))+h(bfg^2+bh(eg-3dh)-2ah(2fg-eh)))\,\mathrm{Subst}\left(\int\frac{1}{4cg^2-4bgh+4ah^2-x}\right.}{\left(cg^2-bgh+ah^2\right)^2}$$

$$= -\frac{2(b^3dh^2 - b^2h(2cdg + aeh) - 2ac(cg(eg - 2dh) + ah(2fg - eh)) + b(c^2dg^2 + a^2fh^2 + ac(fg^2 -}{(b^2 - 4ac)}$$

$$- \frac{h(fg^2 - h(eg - dh))\sqrt{a + bx + cx^2}}{(cg^2 - bgh + ah^2)^2 (g + hx)}$$

$$+ \frac{(2c(fg^3 - gh(2eg - 3dh)) + h(bfg^2 + bh(eg - 3dh) - 2ah(2fg - eh))) \tanh^{-1}\left(\frac{bg-2ah+(2cg}{2\sqrt{cg^2-bgh+ah^2}\sqrt{}}\right.}{2\,(cg^2 - bgh + ah^2)^{5/2}}$$

Mathematica [A] (verified)

Time = 10.97 (sec) , antiderivative size = 518, normalized size of antiderivative = 1.23

$$\int \frac{d + ex + fx^2}{(g + hx)^2 (a + bx + cx^2)^{3/2}}\, dx = -\frac{2f(b + 2cx)}{(b^2 - 4ac)\, h^2 \sqrt{a + x(b + cx)}}$$
$$+ \frac{2(fg^2 + h(-eg + dh))\,(-b^2h + 2c(ah + cgx) + bc(g - hx))}{(b^2 - 4ac)\, h^2\, (-cg^2 + h(bg - ah))\,(g + hx)\sqrt{a + x(b + cx)}}$$
$$- \frac{2(-2fg + eh)\,(b^2h - 2c(ah + cgx) + bc(-g + hx))}{(b^2 - 4ac)\, h^2\, (-cg^2 + h(bg - ah))\sqrt{a + x(b + cx)}}$$
$$- \frac{(fg^2 + h(-eg + dh))\left(\frac{2(4c^2g^2+3b^2h^2-4ch(bg+2ah))\sqrt{a+x(b+cx)}}{(cg^2+h(-bg+ah))(g+hx)} - \frac{3(b^2-4ac)h(-2cg+bh)\text{arctanh}\left(\frac{-bg+2ah-2cgx+bhx}{2\sqrt{cg^2+h(-bg+ah)}\sqrt{a+x(b}}\right.}{(cg^2+h(-bg+ah))^{3/2}}\right.}{2\,(b^2 - 4ac)\, h\, (cg^2 + h(-bg + ah))}$$
$$- \frac{(2fg - eh)\text{arctanh}\left(\frac{-2ah+2cgx+b(g-hx)}{2\sqrt{cg^2+h(-bg+ah)}\sqrt{a+x(b+cx)}}\right)}{(cg^2 + h(-bg + ah))^{3/2}}$$

```
[In] Integrate[(d + e*x + f*x^2)/((g + h*x)^2*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (-2*f*(b + 2*c*x))/((b^2 - 4*a*c)*h^2*Sqrt[a + x*(b + c*x)]) + (2*(f*g^2 +
h*(-(e*g) + d*h))*(-(b^2*h) + 2*c*(a*h + c*g*x) + b*c*(g - h*x)))/((b^2 - 4
*a*c)*h^2*(-(c*g^2) + h*(b*g - a*h))*(g + h*x)*Sqrt[a + x*(b + c*x)]) - (2*
(-2*f*g + e*h)*(b^2*h - 2*c*(a*h + c*g*x) + b*c*(-g + h*x)))/((b^2 - 4*a*c)
*h^2*(-(c*g^2) + h*(b*g - a*h))*Sqrt[a + x*(b + c*x)]) - ((f*g^2 + h*(-(e*g
) + d*h))*((2*(4*c^2*g^2 + 3*b^2*h^2 - 4*c*h*(b*g + 2*a*h))*Sqrt[a + x*(b +
 c*x)])/((c*g^2 + h*(-(b*g) + a*h))*(g + h*x)) - (3*(b^2 - 4*a*c)*h*(-2*c*g
 + b*h)*ArcTanh[(-(b*g) + 2*a*h - 2*c*g*x + b*h*x)/(2*Sqrt[c*g^2 + h*(-(b*g
) + a*h)]*Sqrt[a + x*(b + c*x)])])/(c*g^2 + h*(-(b*g) + a*h))^(3/2)))/(2*(b
^2 - 4*a*c)*h*(c*g^2 + h*(-(b*g) + a*h))) - ((2*f*g - e*h)*ArcTanh[(-2*a*h
+ 2*c*g*x + b*(g - h*x))/(2*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*Sqrt[a + x*(b +
c*x)])])/(c*g^2 + h*(-(b*g) + a*h))^(3/2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1114 vs. 2(407) = 814.

Time = 0.76 (sec) , antiderivative size = 1115, normalized size of antiderivative = 2.65

method	result	size
default	Expression too large to display	1115

```
[In] int((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*f/h^2*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+1/h^3*(e*h-2*f*g)*(1/(a*h
^2-b*g*h+c*g^2)*h^2/((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g
^2)/h^2)^(1/2)-(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)*(2*c*(x+1/h*g)+(b*h-2*c*g)
/h)/(4*c*(a*h^2-b*g*h+c*g^2)/h^2-(b*h-2*c*g)^2/h^2)/((x+1/h*g)^2*c+(b*h-2*c
*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)-1/(a*h^2-b*g*h+c*g^2)*h^2/((
a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(
x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+
1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g)))+1/h^4*(d*h^2-e*g*h+f*g^2
)*(-1/(a*h^2-b*g*h+c*g^2)*h^2/(x+1/h*g)/((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h
*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)-3/2*(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)*(1
/(a*h^2-b*g*h+c*g^2)*h^2/((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*
h+c*g^2)/h^2)^(1/2)-(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)*(2*c*(x+1/h*g)+(b*h-2
*c*g)/h)/(4*c*(a*h^2-b*g*h+c*g^2)/h^2-(b*h-2*c*g)^2/h^2)/((x+1/h*g)^2*c+(b*
h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)-1/(a*h^2-b*g*h+c*g^2)*h
^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g
)/h*(x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c+(b*h-2*c*g)/
h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g)))-4*c/(a*h^2-b*g*h+c*
g^2)*h^2*(2*c*(x+1/h*g)+(b*h-2*c*g)/h)/(4*c*(a*h^2-b*g*h+c*g^2)/h^2-(b*h-2*
c*g)^2/h^2)/((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)
^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2528 vs. 2(407) = 814.

Time = 35.52 (sec) , antiderivative size = 5098, normalized size of antiderivative = 12.11

$$\int \frac{d+ex+fx^2}{(g+hx)^2\left(a+bx+cx^2\right)^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas"
)
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2}{(g+hx)^2\left(a+bx+cx^2\right)^{3/2}}\,dx=\text{Timed out}$$

```
[In] integrate((f*x**2+e*x+d)/(h*x+g)**2/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex+fx^2}{(g+hx)^2\left(a+bx+cx^2\right)^{3/2}}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((b/h-(2*c*g)/h^2)^2>0)', see `assum
e?` for
```

Giac [F]

$$\int \frac{d+ex+fx^2}{(g+hx)^2\left(a+bx+cx^2\right)^{3/2}}\,dx=\int \frac{fx^2+ex+d}{\left(cx^2+bx+a\right)^{\frac{3}{2}}(hx+g)^2}\,dx$$

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2}{(g+hx)^2\left(a+bx+cx^2\right)^{3/2}}\,dx=\int \frac{f\,x^2+e\,x+d}{{\left(g+h\,x\right)}^2\,{\left(c\,x^2+b\,x+a\right)}^{3/2}}\,dx$$

```
[In] int((d + e*x + f*x^2)/((g + h*x)^2*(a + b*x + c*x^2)^(3/2)),x)
```

```
[Out] int((d + e*x + f*x^2)/((g + h*x)^2*(a + b*x + c*x^2)^(3/2)), x)
```

3.239 $\int \frac{d+ex+fx^2}{(g+hx)^3\left(a+bx+cx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 713

$$\int \frac{d+ex+fx^2}{(g+hx)^3\left(a+bx+cx^2\right)^{3/2}}\,dx = \frac{2(b^4dh^3-b^3h^2(3cdg+aeh)+b^2h(3c^2dg^2+a^2fh^2+ach(3eg-4dh))-}{}$$

$$-\frac{h(fg^2-h(eg-dh))\sqrt{a+bx+cx^2}}{2\left(cg^2-bgh+ah^2\right)^2(g+hx)^2}$$

$$-\frac{h(2cg(3fg^2-h(5eg-7dh))-h(4ah(2fg-eh)-b(fg^2+3egh-7dh^2)))\sqrt{a+bx+cx^2}}{4\left(cg^2-bgh+ah^2\right)^3(g+hx)}$$

$$+\frac{(8c^2g^2(fg^2-3egh+6dh^2)+h^2(8a^2fh^2+4abh(2fg-3eh)-b^2(fg^2+3h(eg-5dh)))-4ch(ah(11fg^2-}{8\left(cg^2-bgh+ah^2\right)^{7/2}}$$

```
[Out] 1/8*(8*c^2*g^2*(6*d*h^2-3*e*g*h+f*g^2)+h^2*(8*a^2*f*h^2+4*a*b*h*(-3*e*h+2*f
*g)-b^2*(f*g^2+3*h*(-5*d*h+e*g)))-4*c*h*(a*h*(3*d*h^2-9*e*g*h+11*f*g^2)-b*g
*(2*f*g^2+3*h*(-4*d*h+e*g))))*arctanh(1/2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(a*h^2
-b*g*h+c*g^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/(a*h^2-b*g*h+c*g^2)^(7/2)+2*(b^4*d
*h^3-b^3*h^2*(a*e*h+3*c*d*g)+b^2*h*(3*c^2*d*g^2+a^2*f*h^2+a*c*h*(-4*d*h+3*e
*g))-b*c*(c^2*d*g^3+3*a^2*h^2*(-e*h+f*g)+a*c*g*(-9*d*h^2+3*e*g*h+f*g^2))-2*
a*c*(a^2*f*h^3-c^2*g^2*(-3*d*h+e*g)-a*c*h*(d*h^2-3*e*g*h+3*f*g^2))-c*(2*c^3
*d*g^3-b*(a^2*f-a*b*e+b^2*d)*h^3-c^2*g*(b*g*(3*d*h+e*g)+2*a*(3*d*h^2-3*e*g*
h+f*g^2))+c*(2*a^2*h^2*(-e*h+3*f*g)-3*a*b*h*(-d*h^2+e*g*h+f*g^2)+b^2*(3*d*g
*h^2+f*g^3)))*x)/(-4*a*c+b^2)/(a*h^2-b*g*h+c*g^2)^3/(c*x^2+b*x+a)^(1/2)-1/2
*h*(f*g^2-h*(-d*h+e*g))*(c*x^2+b*x+a)^(1/2)/(a*h^2-b*g*h+c*g^2)^2/(h*x+g)^2
-1/4*h*(2*c*g*(3*f*g^2-h*(-7*d*h+5*e*g))-h*(4*a*h*(-e*h+2*f*g)-b*(-7*d*h^2+
3*e*g*h+f*g^2)))*(c*x^2+b*x+a)^(1/2)/(a*h^2-b*g*h+c*g^2)^3/(h*x+g)
```

Rubi [A] (verified)

Time = 2.00 (sec) , antiderivative size = 707, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {1660, 1664, 820, 738, 212}

$$\int \frac{d+ex+fx^2}{(g+hx)^3\left(a+bx+cx^2\right)^{3/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{-2ah+x(2cg-bh)+bg}{2\sqrt{a+bx+cx^2}\sqrt{ah^2-bgh+cg^2}}\right)\left(h^2\left(8a^2fh^2+4abh(2fg-3eh)-(b^2\right.\right.}{}$$

$$+\frac{2\left(b^2h\left(a^2fh^2+ach(3eg-4dh)+3c^2dg^2\right)-cx\left(c\left(2a^2h^2(3fg-eh)-3abh\left(h(eg-dh)+fg^2\right)+b^2\left(3dgh^2\right.\right.\right.\right.}{}$$

$$-\frac{h\sqrt{a+bx+cx^2}\left(fg^2-h(eg-dh)\right)}{2(g+hx)^2\left(ah^2-bgh+cg^2\right)^2}$$

$$-\frac{h\sqrt{a+bx+cx^2}\left(-4ah^2(2fg-eh)+bh\left(h(3eg-7dh)+fg^2\right)-2cgh(5eg-7dh)+6cfg^3\right)}{4(g+hx)\left(ah^2-bgh+cg^2\right)^3}$$

```
[In] Int[(d + e*x + f*x^2)/((g + h*x)^3*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (2*(b^4*d*h^3 - b^3*h^2*(3*c*d*g + a*e*h) + b^2*h*(3*c^2*d*g^2 + a^2*f*h^2
+ a*c*h*(3*e*g - 4*d*h)) - b*c*(c^2*d*g^3 + 3*a^2*h^2*(f*g - e*h) + a*c*g*(
f*g^2 + 3*e*g*h - 9*d*h^2)) - 2*a*c*(a^2*f*h^3 - c^2*g^2*(e*g - 3*d*h) - a*
c*h*(3*f*g^2 - 3*e*g*h + d*h^2)) - c*(2*c^3*d*g^3 - b*(b^2*d - a*b*e + a^2*
f)*h^3 - c^2*g*(2*a*f*g^2 - 6*a*h*(e*g - d*h) + b*g*(e*g + 3*d*h)) + c*(2*a
^2*h^2*(3*f*g - e*h) + b^2*(f*g^3 + 3*d*g*h^2) - 3*a*b*h*(f*g^2 + h*(e*g -
d*h))))*x))/((b^2 - 4*a*c)*(c*g^2 - b*g*h + a*h^2)^3*Sqrt[a + b*x + c*x^2])
 - (h*(f*g^2 - h*(e*g - d*h))*Sqrt[a + b*x + c*x^2])/(2*(c*g^2 - b*g*h + a*
h^2)^2*(g + h*x)^2) - (h*(6*c*f*g^3 - 2*c*g*h*(5*e*g - 7*d*h) - 4*a*h^2*(2*
f*g - e*h) + b*h*(f*g^2 + h*(3*e*g - 7*d*h)))*Sqrt[a + b*x + c*x^2])/(4*(c*
g^2 - b*g*h + a*h^2)^3*(g + h*x)) + ((8*c^2*g^2*(f*g^2 - 3*e*g*h + 6*d*h^2)
 + 4*c*h*(2*b*f*g^3 + 3*b*g*h*(e*g - 4*d*h) - a*h*(11*f*g^2 - 9*e*g*h + 3*d
*h^2)) + h^2*(8*a^2*f*h^2 + 4*a*b*h*(2*f*g - 3*e*h) - b^2*(f*g^2 + 3*h*(e*g
 - 5*d*h))))*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h
+ a*h^2]*Sqrt[a + b*x + c*x^2])])/(8*(c*g^2 - b*g*h + a*h^2)^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 1660

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^(p + 1)/((p
+ 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{2(b^4dh^3-b^3h^2(3cdg+aeh)+b^2h(3c^2dg^2+a^2fh^2+ach(3eg-4dh))-bc(c^2dg^3+3a^2h^2(fg-eh)+acg(}{}$$

$$-\frac{2\int \frac{-\frac{\left(b^2-4ac\right)\left(c^2g^4\left(fg^2-3egh+6dh^2\right)-cg^2h^2\left(3afg^2-bg(3eg-8dh)-ah(eg+3dh)\right)+h^3\left(a^2dh^3-b^2g^2(eg-3dh)+abg\left(fg^2-3dh^2\right)\right)\right)}{2\left(cg^2-bgh+ah^2\right)^3}+\frac{\left(b^2-4ac\right)h^2\left(c^2g^3\right.}{}}{}}{}$$

$$= \frac{2(b^4dh^3 - b^3h^2(3cdg + aeh) + b^2h(3c^2dg^2 + a^2fh^2 + ach(3eg - 4dh)) - bc(c^2dg^3 + 3a^2h^2(fg - e}{}$$

$$- \frac{h(fg^2 - h(eg - dh))\sqrt{a + bx + cx^2}}{2(cg^2 - bgh + ah^2)^2 (g + hx)^2}$$

$$+ \frac{\int \frac{\frac{(b^2-4ac)(4c^2g^3(fg^2-3egh+6dh^2)-h^2(4a^2h^2(fg-eh)+b^2g(fg^2+3egh-11dh^2)-abh(9fg^2-5egh-7dh^2))+cgh(bg(fg^2+11egh-31dh^2)}{4(cg^2-bgh+ah^2)^2}}{(g+hx)^2\sqrt{}}}{(b^2 - 4ac)(cg}$$

$$= \frac{2(b^4dh^3 - b^3h^2(3cdg + aeh) + b^2h(3c^2dg^2 + a^2fh^2 + ach(3eg - 4dh)) - bc(c^2dg^3 + 3a^2h^2(fg - e}{}$$

$$- \frac{h(fg^2 - h(eg - dh))\sqrt{a + bx + cx^2}}{2(cg^2 - bgh + ah^2)^2 (g + hx)^2}$$

$$- \frac{h(6cfg^3 - 2cgh(5eg - 7dh) - 4ah^2(2fg - eh) + bh(fg^2 + h(3eg - 7dh)))\sqrt{a + bx + cx^2}}{4(cg^2 - bgh + ah^2)^3 (g + hx)}$$

$$+ \frac{(8c^2g^2(fg^2 - 3egh + 6dh^2) + 4ch(2bfg^3 + 3bgh(eg - 4dh) - ah(11fg^2 - 9egh + 3dh^2)) + h^2(}{8(cg^2 - bgh + ah^2)}$$

$$= \frac{2(b^4dh^3 - b^3h^2(3cdg + aeh) + b^2h(3c^2dg^2 + a^2fh^2 + ach(3eg - 4dh)) - bc(c^2dg^3 + 3a^2h^2(fg - e}{}$$

$$- \frac{h(fg^2 - h(eg - dh))\sqrt{a + bx + cx^2}}{2(cg^2 - bgh + ah^2)^2 (g + hx)^2}$$

$$- \frac{h(6cfg^3 - 2cgh(5eg - 7dh) - 4ah^2(2fg - eh) + bh(fg^2 + h(3eg - 7dh)))\sqrt{a + bx + cx^2}}{4(cg^2 - bgh + ah^2)^3 (g + hx)}$$

$$- \frac{(8c^2g^2(fg^2 - 3egh + 6dh^2) + 4ch(2bfg^3 + 3bgh(eg - 4dh) - ah(11fg^2 - 9egh + 3dh^2)) + h^2(}{4(cg}$$

$$= \frac{2(b^4dh^3 - b^3h^2(3cdg + aeh) + b^2h(3c^2dg^2 + a^2fh^2 + ach(3eg - 4dh)) - bc(c^2dg^3 + 3a^2h^2(fg - e}{}$$

$$- \frac{h(fg^2 - h(eg - dh))\sqrt{a + bx + cx^2}}{2(cg^2 - bgh + ah^2)^2 (g + hx)^2}$$

$$- \frac{h(6cfg^3 - 2cgh(5eg - 7dh) - 4ah^2(2fg - eh) + bh(fg^2 + h(3eg - 7dh)))\sqrt{a + bx + cx^2}}{4(cg^2 - bgh + ah^2)^3 (g + hx)}$$

$$+ \frac{(8c^2g^2(fg^2 - 3egh + 6dh^2) + 4ch(2bfg^3 + 3bgh(eg - 4dh) - ah(11fg^2 - 9egh + 3dh^2)) + h^2(}{8(cg^2 - bgh +}$$

Mathematica [A] (verified)

Time = 11.87 (sec) , antiderivative size = 847, normalized size of antiderivative = 1.19

$$\int \frac{d+ex+fx^2}{(g+hx)^3\left(a+bx+cx^2\right)^{3/2}}\,dx=\frac{2(fg^2+h(-eg+dh))\left(-b^2h+2c(ah+cgx)+bc(g-hx)\right)}{\left(b^2-4ac\right)h^2\left(-cg^2+h(bg-ah)\right)(g+hx)^2\sqrt{a+x(b+cx)}}$$
$$-\frac{2f\left(b^2h-2c(ah+cgx)+bc(-g+hx)\right)}{\left(b^2-4ac\right)h^2\left(-cg^2+h(bg-ah)\right)\sqrt{a+x(b+cx)}}$$
$$-\frac{2(-2fg+eh)\left(b^2h-2c(ah+cgx)+bc(-g+hx)\right)}{\left(b^2-4ac\right)h^2\left(-cg^2+h(bg-ah)\right)(g+hx)\sqrt{a+x(b+cx)}}$$
$$-\frac{(-2fg+eh)\left(\frac{2\left(4c^2g^2+3b^2h^2-4ch(bg+2ah)\right)\sqrt{a+x(b+cx)}}{(cg^2+h(-bg+ah))(g+hx)}-\frac{3\left(b^2-4ac\right)h(-2cg+bh)\operatorname{arctanh}\left(\frac{-bg+2ah-2cgx+bhx}{2\sqrt{cg^2+h(-bg+ah)}\sqrt{a+x(b+cx)}}\right)}{\left(cg^2+h(-bg+ah)\right)^{3/2}}\right)}{2\left(b^2-4ac\right)h\left(cg^2+h(-bg+ah)\right)}$$
$$-\frac{(fg^2+h(-eg+dh))\left(\frac{4\left(8c^2g^2+5b^2h^2-4ch(2bg+3ah)\right)\sqrt{a+x(b+cx)}}{(g+hx)^2}+\frac{2(2cg-bh)\left(8c^2g^2+15b^2h^2-4ch(2bg+13ah)\right)\sqrt{a+x(b+cx)}}{(cg^2+h(-bg+ah))(g+hx)}+\right.}{8\left(b^2-4ac\right)h\left(cg^2+h(-bg+ah)\right)^2}$$
$$+\frac{f\operatorname{arctanh}\left(\frac{-2ah+2cgx+b(g-hx)}{2\sqrt{cg^2+h(-bg+ah)}\sqrt{a+x(b+cx)}}\right)}{\left(cg^2+h(-bg+ah)\right)^{3/2}}$$

```
[In] Integrate[(d + e*x + f*x^2)/((g + h*x)^3*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (2*(f*g^2 + h*(-(e*g) + d*h))*(-(b^2*h) + 2*c*(a*h + c*g*x) + b*c*(g - h*x)
))/((b^2 - 4*a*c)*h^2*(-(c*g^2) + h*(b*g - a*h))*(g + h*x)^2*Sqrt[a + x*(b
+ c*x)]) - (2*f*(b^2*h - 2*c*(a*h + c*g*x) + b*c*(-g + h*x)))/((b^2 - 4*a*c
)*h^2*(-(c*g^2) + h*(b*g - a*h))*Sqrt[a + x*(b + c*x)]) - (2*(-2*f*g + e*h)
*(b^2*h - 2*c*(a*h + c*g*x) + b*c*(-g + h*x)))/((b^2 - 4*a*c)*h^2*(-(c*g^2)
 + h*(b*g - a*h))*(g + h*x)*Sqrt[a + x*(b + c*x)]) - ((-2*f*g + e*h)*((2*(4
*c^2*g^2 + 3*b^2*h^2 - 4*c*h*(b*g + 2*a*h))*Sqrt[a + x*(b + c*x)])/((c*g^2
+ h*(-(b*g) + a*h))*(g + h*x)) - (3*(b^2 - 4*a*c)*h*(-2*c*g + b*h)*ArcTanh[
(-(b*g) + 2*a*h - 2*c*g*x + b*h*x)/(2*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*Sqrt[a
 + x*(b + c*x)])])/(c*g^2 + h*(-(b*g) + a*h))^(3/2)))/(2*(b^2 - 4*a*c)*h*(c
*g^2 + h*(-(b*g) + a*h))) - ((f*g^2 + h*(-(e*g) + d*h))*((4*(8*c^2*g^2 + 5*
b^2*h^2 - 4*c*h*(2*b*g + 3*a*h))*Sqrt[a + x*(b + c*x)])/(g + h*x)^2 + (2*(2
*c*g - b*h)*(8*c^2*g^2 + 15*b^2*h^2 - 4*c*h*(2*b*g + 13*a*h))*Sqrt[a + x*(b
 + c*x)])/((c*g^2 + h*(-(b*g) + a*h))*(g + h*x)) + (3*(b^2 - 4*a*c)*h*(16*c
^2*g^2 + 5*b^2*h^2 - 4*c*h*(4*b*g + a*h))*ArcTanh[(-(b*g) + 2*a*h - 2*c*g*x
 + b*h*x)/(2*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*Sqrt[a + x*(b + c*x)])])/(c*g^2
 + h*(-(b*g) + a*h))^(3/2)))/(8*(b^2 - 4*a*c)*h*(c*g^2 + h*(-(b*g) + a*h))^
2) + (f*ArcTanh[(-2*a*h + 2*c*g*x + b*(g - h*x))/(2*Sqrt[c*g^2 + h*(-(b*g)
+ a*h)]*Sqrt[a + x*(b + c*x)])])/(c*g^2 + h*(-(b*g) + a*h))^(3/2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2267 vs. 2(693) = 1386.

Time = 0.90 (sec) , antiderivative size = 2268, normalized size of antiderivative = 3.18

method	result	size
default	Expression too large to display	2268

```
[In] int((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] f/h^3*(1/(a*h^2-b*g*h+c*g^2)*h^2/((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*
h^2-b*g*h+c*g^2)/h^2)^(1/2)-(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)*(2*c*(x+1/h*g
)+(b*h-2*c*g)/h)/(4*c*(a*h^2-b*g*h+c*g^2)/h^2-(b*h-2*c*g)^2/h^2)/((x+1/h*g)
^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)-1/(a*h^2-b*g*h+
c*g^2)*h^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b
*h-2*c*g)/h*(x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c+(b*h
-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g)))+(e*h-2*f*g)
/h^4*(-1/(a*h^2-b*g*h+c*g^2)*h^2/(x+1/h*g)/((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+
1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)-3/2*(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)
*(1/(a*h^2-b*g*h+c*g^2)*h^2/((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b
*g*h+c*g^2)/h^2)^(1/2)-(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)*(2*c*(x+1/h*g)+(b*
h-2*c*g)/h)/(4*c*(a*h^2-b*g*h+c*g^2)/h^2-(b*h-2*c*g)^2/h^2)/((x+1/h*g)^2*c+
(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)-1/(a*h^2-b*g*h+c*g^2
)*h^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*
c*g)/h*(x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c+(b*h-2*c*
g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g)))-4*c/(a*h^2-b*g*h
+c*g^2)*h^2*(2*c*(x+1/h*g)+(b*h-2*c*g)/h)/(4*c*(a*h^2-b*g*h+c*g^2)/h^2-(b*h
-2*c*g)^2/h^2)/((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h
^2)^(1/2))+(d*h^2-e*g*h+f*g^2)/h^5*(-1/2/(a*h^2-b*g*h+c*g^2)*h^2/(x+1/h*g)^
2/((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)-5/4
*(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)*(-1/(a*h^2-b*g*h+c*g^2)*h^2/(x+1/h*g)/((
x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)-3/2*(b*
h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)*(1/(a*h^2-b*g*h+c*g^2)*h^2/((x+1/h*g)^2*c+(b
*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)-(b*h-2*c*g)*h/(a*h^2-b
*g*h+c*g^2)*(2*c*(x+1/h*g)+(b*h-2*c*g)/h)/(4*c*(a*h^2-b*g*h+c*g^2)/h^2-(b*h
-2*c*g)^2/h^2)/((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h
^2)^(1/2)-1/(a*h^2-b*g*h+c*g^2)*h^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(
a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^
(1/2)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)
)/(x+1/h*g)))-4*c/(a*h^2-b*g*h+c*g^2)*h^2*(2*c*(x+1/h*g)+(b*h-2*c*g)/h)/(4*
c*(a*h^2-b*g*h+c*g^2)/h^2-(b*h-2*c*g)^2/h^2)/((x+1/h*g)^2*c+(b*h-2*c*g)/h*(
x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))-3/2*c/(a*h^2-b*g*h+c*g^2)*h^2*(1/(
a*h^2-b*g*h+c*g^2)*h^2/((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+
c*g^2)/h^2)^(1/2)-(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)*(2*c*(x+1/h*g)+(b*h-2*c
*g)/h)/(4*c*(a*h^2-b*g*h+c*g^2)/h^2-(b*h-2*c*g)^2/h^2)/((x+1/h*g)^2*c+(b*h-
```

```
2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)-1/(a*h^2-b*g*h+c*g^2)*h^2
/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/
h*(x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*
(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5149 vs. $2(693) = 1386$.

Time = 162.17 (sec) , antiderivative size = 10340, normalized size of antiderivative = 14.50

$$\int \frac{d+ex+fx^2}{(g+hx)^3\left(a+bx+cx^2\right)^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas"
)
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2}{(g+hx)^3\left(a+bx+cx^2\right)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((f*x**2+e*x+d)/(h*x+g)**3/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex+fx^2}{(g+hx)^3\left(a+bx+cx^2\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*h^2-b*g*h>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5562 vs. $2(693) = 1386$.

Time = 0.41 (sec) , antiderivative size = 5562, normalized size of antiderivative = 7.80

$$\int \frac{d+ex+fx^2}{(g+hx)^3\left(a+bx+cx^2\right)^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -2*((2*c^7*d*g^9 - b*c^6*e*g^9 + b^2*c^5*f*g^9 - 2*a*c^6*f*g^9 - 9*b*c^6*d*
g^8*h + 3*b^2*c^5*e*g^8*h + 6*a*c^6*e*g^8*h - 3*b^3*c^4*f*g^8*h + 3*a*b*c^5
*f*g^8*h + 18*b^2*c^5*d*g^7*h^2 - 3*b^3*c^4*e*g^7*h^2 - 24*a*b*c^5*e*g^7*h^
2 + 3*b^4*c^3*f*g^7*h^2 + 6*a*b^2*c^4*f*g^7*h^2 - 21*b^3*c^4*d*g^6*h^3 + b^
4*c^3*e*g^6*h^3 + 34*a*b^2*c^4*e*g^6*h^3 + 16*a^2*c^5*e*g^6*h^3 - b^5*c^2*f
*g^6*h^3 - 13*a*b^3*c^3*f*g^6*h^3 - 16*a^2*b*c^4*f*g^6*h^3 + 15*b^4*c^3*d*g
^5*h^4 + 6*a*b^2*c^4*d*g^5*h^4 - 12*a^2*c^5*d*g^5*h^4 - 21*a*b^3*c^3*e*g^5*
h^4 - 42*a^2*b*c^4*e*g^5*h^4 + 6*a*b^4*c^2*f*g^5*h^4 + 36*a^2*b^2*c^3*f*g^5
*h^4 + 12*a^3*c^4*f*g^5*h^4 - 6*b^5*c^2*d*g^4*h^5 - 15*a*b^3*c^3*d*g^4*h^5
+ 30*a^2*b*c^4*d*g^4*h^5 + 6*a*b^4*c^2*e*g^4*h^5 + 36*a^2*b^2*c^3*e*g^4*h^5
 + 12*a^3*c^4*e*g^4*h^5 - 21*a^2*b^3*c^2*f*g^4*h^5 - 42*a^3*b*c^3*f*g^4*h^5
 + b^6*c*d*g^3*h^6 + 12*a*b^4*c^2*d*g^3*h^6 - 18*a^2*b^2*c^3*d*g^3*h^6 - 16
*a^3*c^4*d*g^3*h^6 - a*b^5*c*e*g^3*h^6 - 13*a^2*b^3*c^2*e*g^3*h^6 - 16*a^3*
b*c^3*e*g^3*h^6 + a^2*b^4*c*f*g^3*h^6 + 34*a^3*b^2*c^2*f*g^3*h^6 + 16*a^4*c
^3*f*g^3*h^6 - 3*a*b^5*c*d*g^2*h^7 - 3*a^2*b^3*c^2*d*g^2*h^7 + 24*a^3*b*c^3
*d*g^2*h^7 + 3*a^2*b^4*c*e*g^2*h^7 + 6*a^3*b^2*c^2*e*g^2*h^7 - 3*a^3*b^3*c*
f*g^2*h^7 - 24*a^4*b*c^2*f*g^2*h^7 + 3*a^2*b^4*c*d*g*h^8 - 6*a^3*b^2*c^2*d*
g*h^8 - 6*a^4*c^3*d*g*h^8 - 3*a^3*b^3*c*e*g*h^8 + 3*a^4*b*c^2*e*g*h^8 + 3*a
^4*b^2*c*f*g*h^8 + 6*a^5*c^2*f*g*h^8 - a^3*b^3*c*d*h^9 + 3*a^4*b*c^2*d*h^9
+ a^4*b^2*c*e*h^9 - 2*a^5*c^2*e*h^9 - a^5*b*c*f*h^9)*x/(b^2*c^6*g^12 - 4*a*
c^7*g^12 - 6*b^3*c^5*g^11*h + 24*a*b*c^6*g^11*h + 15*b^4*c^4*g^10*h^2 - 54*
a*b^2*c^5*g^10*h^2 - 24*a^2*c^6*g^10*h^2 - 20*b^5*c^3*g^9*h^3 + 50*a*b^3*c^
4*g^9*h^3 + 120*a^2*b*c^5*g^9*h^3 + 15*b^6*c^2*g^8*h^4 - 225*a^2*b^2*c^4*g^
8*h^4 - 60*a^3*c^5*g^8*h^4 - 6*b^7*c*g^7*h^5 - 36*a*b^5*c^2*g^7*h^5 + 180*a
^2*b^3*c^3*g^7*h^5 + 240*a^3*b*c^4*g^7*h^5 + b^8*g^6*h^6 + 26*a*b^6*c*g^6*h
^6 - 30*a^2*b^4*c^2*g^6*h^6 - 340*a^3*b^2*c^3*g^6*h^6 - 80*a^4*c^4*g^6*h^6
- 6*a*b^7*g^5*h^7 - 36*a^2*b^5*c*g^5*h^7 + 180*a^3*b^3*c^2*g^5*h^7 + 240*a^
4*b*c^3*g^5*h^7 + 15*a^2*b^6*g^4*h^8 - 225*a^4*b^2*c^2*g^4*h^8 - 60*a^5*c^3
*g^4*h^8 - 20*a^3*b^5*g^3*h^9 + 50*a^4*b^3*c*g^3*h^9 + 120*a^5*b*c^2*g^3*h^
9 + 15*a^4*b^4*g^2*h^10 - 54*a^5*b^2*c*g^2*h^10 - 24*a^6*c^2*g^2*h^10 - 6*a
^5*b^3*g*h^11 + 24*a^6*b*c*g*h^11 + a^6*b^2*h^12 - 4*a^7*c*h^12) + (b*c^6*d
*g^9 - 2*a*c^6*e*g^9 + a*b*c^5*f*g^9 - 6*b^2*c^5*d*g^8*h + 6*a*c^6*d*g^8*h
+ 9*a*b*c^5*e*g^8*h - 3*a*b^2*c^4*f*g^8*h - 6*a^2*c^5*f*g^8*h + 15*b^3*c^4*
d*g^7*h^2 - 24*a*b*c^5*d*g^7*h^2 - 18*a*b^2*c^4*e*g^7*h^2 + 3*a*b^3*c^3*f*g
^7*h^2 + 24*a^2*b*c^4*f*g^7*h^2 - 20*b^4*c^3*d*g^6*h^3 + 34*a*b^2*c^4*d*g^6
```

```
*h^3 + 16*a^2*c^5*d*g^6*h^3 + 21*a*b^3*c^3*e*g^6*h^3 - a*b^4*c^2*f*g^6*h^3
- 34*a^2*b^2*c^3*f*g^6*h^3 - 16*a^3*c^4*f*g^6*h^3 + 15*b^5*c^2*d*g^5*h^4 -
15*a*b^3*c^3*d*g^5*h^4 - 54*a^2*b*c^4*d*g^5*h^4 - 15*a*b^4*c^2*e*g^5*h^4 -
6*a^2*b^2*c^3*e*g^5*h^4 + 12*a^3*c^4*e*g^5*h^4 + 21*a^2*b^3*c^2*f*g^5*h^4 +
 42*a^3*b*c^3*f*g^5*h^4 - 6*b^6*c*d*g^4*h^5 - 9*a*b^4*c^2*d*g^4*h^5 + 66*a^
2*b^2*c^3*d*g^4*h^5 + 12*a^3*c^4*d*g^4*h^5 + 6*a*b^5*c*e*g^4*h^5 + 15*a^2*b
^3*c^2*e*g^4*h^5 - 30*a^3*b*c^3*e*g^4*h^5 - 6*a^2*b^4*c*f*g^4*h^5 - 36*a^3*
b^2*c^2*f*g^4*h^5 - 12*a^4*c^3*f*g^4*h^5 + b^7*d*g^3*h^6 + 11*a*b^5*c*d*g^3
*h^6 - 31*a^2*b^3*c^2*d*g^3*h^6 - 32*a^3*b*c^3*d*g^3*h^6 - a*b^6*e*g^3*h^6
- 12*a^2*b^4*c*e*g^3*h^6 + 18*a^3*b^2*c^2*e*g^3*h^6 + 16*a^4*c^3*e*g^3*h^6
+ a^2*b^5*f*g^3*h^6 + 13*a^3*b^3*c*f*g^3*h^6 + 16*a^4*b*c^2*f*g^3*h^6 - 3*a
*b^6*d*g^2*h^7 + 30*a^3*b^2*c^2*d*g^2*h^7 + 3*a^2*b^5*e*g^2*h^7 + 3*a^3*b^3
*c*e*g^2*h^7 - 24*a^4*b*c^2*e*g^2*h^7 - 3*a^3*b^4*f*g^2*h^7 - 6*a^4*b^2*c*f
*g^2*h^7 + 3*a^2*b^5*d*g*h^8 - 9*a^3*b^3*c*d*g*h^8 - 3*a^4*b*c^2*d*g*h^8 -
3*a^3*b^4*e*g*h^8 + 6*a^4*b^2*c*e*g*h^8 + 6*a^5*c^2*e*g*h^8 + 3*a^4*b^3*f*g
*h^8 - 3*a^5*b*c*f*g*h^8 - a^3*b^4*d*h^9 + 4*a^4*b^2*c*d*h^9 - 2*a^5*c^2*d*
h^9 + a^4*b^3*e*h^9 - 3*a^5*b*c*e*h^9 - a^5*b^2*f*h^9 + 2*a^6*c*f*h^9)/(b^2
*c^6*g^12 - 4*a*c^7*g^12 - 6*b^3*c^5*g^11*h + 24*a*b*c^6*g^11*h + 15*b^4*c^
4*g^10*h^2 - 54*a*b^2*c^5*g^10*h^2 - 24*a^2*c^6*g^10*h^2 - 20*b^5*c^3*g^9*h
^3 + 50*a*b^3*c^4*g^9*h^3 + 120*a^2*b*c^5*g^9*h^3 + 15*b^6*c^2*g^8*h^4 - 22
5*a^2*b^2*c^4*g^8*h^4 - 60*a^3*c^5*g^8*h^4 - 6*b^7*c*g^7*h^5 - 36*a*b^5*c^2
*g^7*h^5 + 180*a^2*b^3*c^3*g^7*h^5 + 240*a^3*b*c^4*g^7*h^5 + b^8*g^6*h^6 +
26*a*b^6*c*g^6*h^6 - 30*a^2*b^4*c^2*g^6*h^6 - 340*a^3*b^2*c^3*g^6*h^6 - 80*
a^4*c^4*g^6*h^6 - 6*a*b^7*g^5*h^7 - 36*a^2*b^5*c*g^5*h^7 + 180*a^3*b^3*c^2*
g^5*h^7 + 240*a^4*b*c^3*g^5*h^7 + 15*a^2*b^6*g^4*h^8 - 225*a^4*b^2*c^2*g^4*
h^8 - 60*a^5*c^3*g^4*h^8 - 20*a^3*b^5*g^3*h^9 + 50*a^4*b^3*c*g^3*h^9 + 120*
a^5*b*c^2*g^3*h^9 + 15*a^4*b^4*g^2*h^10 - 54*a^5*b^2*c*g^2*h^10 - 24*a^6*c^
2*g^2*h^10 - 6*a^5*b^3*g*h^11 + 24*a^6*b*c*g*h^11 + a^6*b^2*h^12 - 4*a^7*c*
h^12))/sqrt(c*x^2 + b*x + a) + 1/4*(8*c^2*f*g^4 - 24*c^2*e*g^3*h + 8*b*c*f*
g^3*h + 48*c^2*d*g^2*h^2 + 12*b*c*e*g^2*h^2 - b^2*f*g^2*h^2 - 44*a*c*f*g^2*
h^2 - 48*b*c*d*g*h^3 - 3*b^2*e*g*h^3 + 36*a*c*e*g*h^3 + 8*a*b*f*g*h^3 + 15*
b^2*d*h^4 - 12*a*c*d*h^4 - 12*a*b*e*h^4 + 8*a^2*f*h^4)*arctan(-((sqrt(c)*x
- sqrt(c*x^2 + b*x + a))*h + sqrt(c)*g)/sqrt(-c*g^2 + b*g*h - a*h^2))/((c^3
*g^6 - 3*b*c^2*g^5*h + 3*b^2*c*g^4*h^2 + 3*a*c^2*g^4*h^2 - b^3*g^3*h^3 - 6*
a*b*c*g^3*h^3 + 3*a*b^2*g^2*h^4 + 3*a^2*c*g^2*h^4 - 3*a^2*b*g*h^5 + a^3*h^6
)*sqrt(-c*g^2 + b*g*h - a*h^2)) - 1/4*(8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)^3*c^2*f*g^4*h - 16*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*c^2*e*g^3*h^2 +
24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*c^2*d*g^2*h^3 + 12*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))^3*b*c*e*g^2*h^3 - (sqrt(c)*x - sqrt(c*x^2 + b*x + a))^
3*b^2*f*g^2*h^3 - 20*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*c*f*g^2*h^3 -
24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b*c*d*g*h^4 - 3*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^3*b^2*e*g*h^4 + 12*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*
a*c*e*g*h^4 + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b*f*g*h^4 + 7*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^3*b^2*d*h^5 - 4*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^3*a*c*d*h^5 - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b*e*h^5 + 2
```

```
4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c^(5/2)*f*g^5 - 40*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^2*c^(5/2)*e*g^4*h - 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))^2*b*c^(3/2)*f*g^4*h + 56*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c^(5/2)*d
*g^3*h^2 + 28*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b*c^(3/2)*e*g^3*h^2 + 5
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2*sqrt(c)*f*g^3*h^2 - 44*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^2*a*c^(3/2)*f*g^3*h^2 - 48*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))^2*b*c^(3/2)*d*g^2*h^3 - 9*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)^2*b^2*sqrt(c)*e*g^2*h^3 + 36*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*c^(3
/2)*e*g^2*h^3 + 13*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2*sqrt(c)*d*g*h^
4 - 28*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*c^(3/2)*d*g*h^4 - 4*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^2*a*b*sqrt(c)*e*g*h^4 + 16*(sqrt(c)*x - sqrt(c*
x^2 + b*x + a))^2*a^2*sqrt(c)*f*g*h^4 + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))^2*a*b*sqrt(c)*d*h^5 - 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*sqrt(c
)*e*h^5 + 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b*c^2*f*g^5 - 40*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))*b*c^2*e*g^4*h - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))*b^2*c*f*g^4*h - 40*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*c^2*f*g^4*h
+ 56*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b*c^2*d*g^3*h^2 + 24*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))*b^2*c*e*g^3*h^2 + 64*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))*a*c^2*e*g^3*h^2 + (sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*f*g^3*h^2 -
28*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b*c*f*g^3*h^2 - 44*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))*b^2*c*d*g^2*h^3 - 88*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))*a*c^2*d*g^2*h^3 - 5*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*e*g^2*h^3 -
16*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b*c*e*g^2*h^3 + 7*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))*a*b^2*f*g^2*h^3 + 44*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)*a^2*c*f*g^2*h^3 + 9*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*d*g*h^4 + 60*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b*c*d*g*h^4 + (sqrt(c)*x - sqrt(c*x^2
 + b*x + a))*a*b^2*e*g*h^4 - 20*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*c*e
*g*h^4 - 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b*f*g*h^4 - 9*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))*a*b^2*d*h^5 - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))*a^2*c*d*h^5 + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b*e*h^5 + 6*b^2*
c^(3/2)*f*g^5 - 10*b^2*c^(3/2)*e*g^4*h + b^3*sqrt(c)*f*g^4*h - 20*a*b*c^(3/
2)*f*g^4*h + 14*b^2*c^(3/2)*d*g^3*h^2 + 3*b^3*sqrt(c)*e*g^3*h^2 + 32*a*b*c^
(3/2)*e*g^3*h^2 - 9*a*b^2*sqrt(c)*f*g^3*h^2 + 12*a^2*c^(3/2)*f*g^3*h^2 - 7*
b^3*sqrt(c)*d*g^2*h^3 - 44*a*b*c^(3/2)*d*g^2*h^3 - 7*a*b^2*sqrt(c)*e*g^2*h^
3 - 20*a^2*c^(3/2)*e*g^2*h^3 + 24*a^2*b*sqrt(c)*f*g^2*h^3 + 23*a*b^2*sqrt(c
)*d*g*h^4 + 28*a^2*c^(3/2)*d*g*h^4 - 4*a^2*b*sqrt(c)*e*g*h^4 - 16*a^3*sqrt(
c)*f*g*h^4 - 16*a^2*b*sqrt(c)*d*h^5 + 8*a^3*sqrt(c)*e*h^5)/((c^3*g^6 - 3*b*
c^2*g^5*h + 3*b^2*c*g^4*h^2 + 3*a*c^2*g^4*h^2 - b^3*g^3*h^3 - 6*a*b*c*g^3*h
^3 + 3*a*b^2*g^2*h^4 + 3*a^2*c*g^2*h^4 - 3*a^2*b*g*h^5 + a^3*h^6)*((sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^2*h + 2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqr
t(c)*g + b*g - a*h)^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2}{(g+hx)^3\left(a+bx+cx^2\right)^{3/2}}\,dx=\int \frac{f\,x^2+e\,x+d}{{\left(g+h\,x\right)}^3\,{\left(c\,x^2+b\,x+a\right)}^{3/2}} \,d x$$

```
[In] int((d + e*x + f*x^2)/((g + h*x)^3*(a + b*x + c*x^2)^(3/2)),x)
[Out] int((d + e*x + f*x^2)/((g + h*x)^3*(a + b*x + c*x^2)^(3/2)), x)
```

3.240 $\int \frac{(1+2x)^3(1+3x+4x^2)}{\sqrt{2-x+3x^2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 120

$$\int \frac{(1+2x)^3\,(1+3x+4x^2)}{\sqrt{2-x+3x^2}}\,dx = \frac{44}{135}(1+2x)^2\sqrt{2-x+3x^2}+\frac{19}{60}(1+2x)^3\sqrt{2-x+3x^2} + \frac{2}{15}(1+2x)^4\sqrt{2-x+3x^2} - \frac{(24897+6298x)\sqrt{2-x+3x^2}}{3240}+\frac{9211\operatorname{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{1296\sqrt{3}}$$

```
[Out] 9211/3888*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)+44/135*(1+2*x)^2*(3*x^2-x+
2)^(1/2)+19/60*(1+2*x)^3*(3*x^2-x+2)^(1/2)+2/15*(1+2*x)^4*(3*x^2-x+2)^(1/2)
-1/3240*(24897+6298*x)*(3*x^2-x+2)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1667, 846, 793, 633, 221}

$$\int \frac{(1+2x)^3\,(1+3x+4x^2)}{\sqrt{2-x+3x^2}}\,dx = \frac{9211\operatorname{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{1296\sqrt{3}}+\frac{2}{15}\sqrt{3x^2-x+2}(2x+1)^4 + \frac{19}{60}\sqrt{3x^2-x+2}(2x+1)^3+\frac{44}{135}\sqrt{3x^2-x+2}(2x+1)^2 - \frac{(6298x+24897)\sqrt{3x^2-x+2}}{3240}$$

```
[In] Int[((1 + 2*x)^3*(1 + 3*x + 4*x^2))/Sqrt[2 - x + 3*x^2],x]
```

```
[Out] (44*(1 + 2*x)^2*Sqrt[2 - x + 3*x^2])/135 + (19*(1 + 2*x)^3*Sqrt[2 - x + 3*x
^2])/60 + (2*(1 + 2*x)^4*Sqrt[2 - x + 3*x^2])/15 - ((24897 + 6298*x)*Sqrt[2
 - x + 3*x^2])/3240 + (9211*ArcSinh[(1 - 6*x)/Sqrt[23]])/(1296*Sqrt[3])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 846

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
```

```
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2}{15}(1+2x)^4\sqrt{2-x+3x^2}+\frac{1}{60}\int \frac{(1+2x)^3(-64+228x)}{\sqrt{2-x+3x^2}}\,dx \\
&= \frac{19}{60}(1+2x)^3\sqrt{2-x+3x^2}+\frac{2}{15}(1+2x)^4\sqrt{2-x+3x^2} \\
&\quad +\frac{1}{720}\int \frac{(1+2x)^2(-3390+2112x)}{\sqrt{2-x+3x^2}}\,dx \\
&= \frac{44}{135}(1+2x)^2\sqrt{2-x+3x^2}+\frac{19}{60}(1+2x)^3\sqrt{2-x+3x^2} \\
&\quad +\frac{2}{15}(1+2x)^4\sqrt{2-x+3x^2}+\frac{\int \frac{(-46350-37788x)(1+2x)}{\sqrt{2-x+3x^2}}\,dx}{6480} \\
&= \frac{44}{135}(1+2x)^2\sqrt{2-x+3x^2}+\frac{19}{60}(1+2x)^3\sqrt{2-x+3x^2}+\frac{2}{15}(1+2x)^4\sqrt{2-x+3x^2} \\
&\quad -\frac{(24897+6298x)\sqrt{2-x+3x^2}}{3240}-\frac{9211\int \frac{1}{\sqrt{2-x+3x^2}}\,dx}{1296} \\
&= \frac{44}{135}(1+2x)^2\sqrt{2-x+3x^2}+\frac{19}{60}(1+2x)^3\sqrt{2-x+3x^2}+\frac{2}{15}(1+2x)^4\sqrt{2-x+3x^2} \\
&\quad -\frac{(24897+6298x)\sqrt{2-x+3x^2}}{3240}-\frac{9211\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx,x,-1+6x\right)}{1296\sqrt{69}} \\
&= \frac{44}{135}(1+2x)^2\sqrt{2-x+3x^2}+\frac{19}{60}(1+2x)^3\sqrt{2-x+3x^2}+\frac{2}{15}(1+2x)^4\sqrt{2-x+3x^2} \\
&\quad -\frac{(24897+6298x)\sqrt{2-x+3x^2}}{3240}+\frac{9211\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{1296\sqrt{3}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.58

$$\int \frac{(1+2x)^3\left(1+3x+4x^2\right)}{\sqrt{2-x+3x^2}}\,dx = \frac{6\sqrt{2-x+3x^2}(-22383+7538x+26904x^2+22032x^3+6912x^4)+46055\sqrt{3}\log\left(1-6x+2\sqrt{6-3x+}\right.}{19440}$$

```
[In] Integrate[((1 + 2*x)^3*(1 + 3*x + 4*x^2))/Sqrt[2 - x + 3*x^2],x]
```

```
[Out] (6*Sqrt[2 - x + 3*x^2]*(-22383 + 7538*x + 26904*x^2 + 22032*x^3 + 6912*x^4)
 + 46055*Sqrt[3]*Log[1 - 6*x + 2*Sqrt[6 - 3*x + 9*x^2]])/19440
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.42

method	result
risch	$\frac{(6912x^4+22032x^3+26904x^2+7538x-22383)\sqrt{3x^2-x+2}}{3240} - \frac{9211\sqrt{3}\ \operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{3888}$
trager	$\left(\frac{32}{15}x^4+\frac{34}{5}x^3+\frac{1121}{135}x^2+\frac{3769}{1620}x-\frac{829}{120}\right)\sqrt{3x^2-x+2} - \frac{9211\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(6\,\mathrm{RootOf}\left(\_Z^2-3\right)x+6\sqrt{3x^2}}{3888}$
default	$-\frac{9211\sqrt{3}\ \operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{3888} - \frac{829\sqrt{3x^2-x+2}}{120} + \frac{32x^4\sqrt{3x^2-x+2}}{15} + \frac{34x^3\sqrt{3x^2-x+2}}{5} + \frac{1121x^2\sqrt{3x^2-x+2}}{135} + \frac{3769x\sqrt{}}{16}$

```
[In] int((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2-x+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3240*(6912*x^4+22032*x^3+26904*x^2+7538*x-22383)*(3*x^2-x+2)^(1/2)-9211/3
888*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.61

$$\int \frac{(1+2x)^3\left(1+3x+4x^2\right)}{\sqrt{2-x+3x^2}}\,dx = \frac{1}{3240}\left(6912\,x^4+22032\,x^3+26904\,x^2+7538\,x-22383\right)\sqrt{3\,x^2-x+2} + \frac{9211}{7776}\sqrt{3}\log\left(4\sqrt{3}\sqrt{3\,x^2-x+2}(6\,x-1)-72\,x^2+24\,x-25\right)$$

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2-x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3240*(6912*x^4 + 22032*x^3 + 26904*x^2 + 7538*x - 22383)*sqrt(3*x^2 - x +
 2) + 9211/7776*sqrt(3)*log(4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x - 1) - 72*x^
2 + 24*x - 25)
```

Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.52

$$\int \frac{(1+2x)^3\left(1+3x+4x^2\right)}{\sqrt{2-x+3x^2}}\,dx = \sqrt{3x^2-x+2}\cdot\left(\frac{32x^4}{15}+\frac{34x^3}{5}+\frac{1121x^2}{135}+\frac{3769x}{1620}-\frac{829}{120}\right) - \frac{9211\sqrt{3}\operatorname{asinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{3888}$$

```
[In] integrate((1+2*x)**3*(4*x**2+3*x+1)/(3*x**2-x+2)**(1/2),x)
```

```
[Out] sqrt(3*x**2 - x + 2)*(32*x**4/15 + 34*x**3/5 + 1121*x**2/135 + 3769*x/1620
- 829/120) - 9211*sqrt(3)*asinh(6*sqrt(23)*(x - 1/6)/23)/3888
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.81

$$\begin{aligned}\int \frac{(1+2x)^3(1+3x+4x^2)}{\sqrt{2-x+3x^2}}\,dx =& \frac{32}{15}\sqrt{3\,x^2-x+2}x^4+\frac{34}{5}\sqrt{3\,x^2-x+2}x^3\\&+\frac{1121}{135}\sqrt{3\,x^2-x+2}x^2+\frac{3769}{1620}\sqrt{3\,x^2-x+2}x\\&-\frac{9211}{3888}\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{23}\sqrt{23}(6\,x-1)\right)-\frac{829}{120}\sqrt{3\,x^2-x+2}\end{aligned}$$

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2-x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 32/15*sqrt(3*x^2 - x + 2)*x^4 + 34/5*sqrt(3*x^2 - x + 2)*x^3 + 1121/135*sqr
t(3*x^2 - x + 2)*x^2 + 3769/1620*sqrt(3*x^2 - x + 2)*x - 9211/3888*sqrt(3)*
arcsinh(1/23*sqrt(23)*(6*x - 1)) - 829/120*sqrt(3*x^2 - x + 2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.57

$$\begin{aligned}&\int \frac{(1+2x)^3(1+3x+4x^2)}{\sqrt{2-x+3x^2}}\,dx\\&=\frac{1}{3240}(2\,(12\,(18\,(16\,x+51)x+1121)x+3769)x-22383)\sqrt{3\,x^2-x+2}\\&\quad+\frac{9211}{3888}\sqrt{3}\log\left(-2\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2-x+2}\right)+1\right)\end{aligned}$$

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2-x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3240*(2*(12*(18*(16*x + 51)*x + 1121)*x + 3769)*x - 22383)*sqrt(3*x^2 - x
 + 2) + 9211/3888*sqrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2))
+ 1)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+2x)^3\,(1+3x+4x^2)}{\sqrt{2-x+3x^2}}\,dx = \int \frac{{\left(2\,x+1\right)}^3\,\left(4\,x^2+3\,x+1\right)}{\sqrt{3\,x^2-x+2}}\,dx$$

```
[In] int(((2*x + 1)^3*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(1/2),x)
[Out] int(((2*x + 1)^3*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(1/2), x)
```

3.241 $\int \frac{(1+2x)^2(1+3x+4x^2)}{\sqrt{2-x+3x^2}}\,dx$

Optimal result

Integrand size $= 32$, antiderivative size $= 95$

$$\int \frac{(1+2x)^2(1+3x+4x^2)}{\sqrt{2-x+3x^2}}\,dx = -\frac{143}{324}(3-2x)\sqrt{2-x+3x^2}+\frac{11}{27}(1+2x)^2\sqrt{2-x+3x^2} + \frac{1}{6}(1+2x)^3\sqrt{2-x+3x^2}+\frac{4147\operatorname{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{648\sqrt{3}}$$

```
[Out] 4147/1944*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)-143/324*(3-2*x)*(3*x^2-x+2
)^(1/2)+11/27*(1+2*x)^2*(3*x^2-x+2)^(1/2)+1/6*(1+2*x)^3*(3*x^2-x+2)^(1/2)
```

Rubi [A] (verified)

Time $= 0.06$ (sec), antiderivative size $= 95$, normalized size of antiderivative $= 1.00$, number of steps used $= 5$, number of rules used $= 5$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used $= \{1667, 846, 793, 633, 221\}$

$$\int \frac{(1+2x)^2(1+3x+4x^2)}{\sqrt{2-x+3x^2}}\,dx = \frac{4147\operatorname{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{648\sqrt{3}}+\frac{1}{6}\sqrt{3x^2-x+2}(2x+1)^3 + \frac{11}{27}\sqrt{3x^2-x+2}(2x+1)^2-\frac{143}{324}(3-2x)\sqrt{3x^2-x+2}$$

```
[In] Int[((1 + 2*x)^2*(1 + 3*x + 4*x^2))/Sqrt[2 - x + 3*x^2],x]
```

```
[Out] (-143*(3 - 2*x)*Sqrt[2 - x + 3*x^2])/324 + (11*(1 + 2*x)^2*Sqrt[2 - x + 3*x
^2])/27 + ((1 + 2*x)^3*Sqrt[2 - x + 3*x^2])/6 + (4147*ArcSinh[(1 - 6*x)/Sqr
t[23]])/(648*Sqrt[3])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 846

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\text{integral} = \frac{1}{6}(1+2x)^3\sqrt{2-x+3x^2} + \frac{1}{48}\int \frac{(1+2x)^2(-44+176x)}{\sqrt{2-x+3x^2}}\,dx$$

$$
\begin{aligned}
&= \frac{11}{27}(1+2x)^2\sqrt{2-x+3x^2}+\frac{1}{6}(1+2x)^3\sqrt{2-x+3x^2}+\frac{1}{432}\int \frac{(1+2x)(-1716+1144x)}{\sqrt{2-x+3x^2}}\,dx \\
&= -\frac{143}{324}(3-2x)\sqrt{2-x+3x^2}+\frac{11}{27}(1+2x)^2\sqrt{2-x+3x^2} \\
&\quad +\frac{1}{6}(1+2x)^3\sqrt{2-x+3x^2}-\frac{4147}{648}\int \frac{1}{\sqrt{2-x+3x^2}}\,dx \\
&= -\frac{143}{324}(3-2x)\sqrt{2-x+3x^2}+\frac{11}{27}(1+2x)^2\sqrt{2-x+3x^2} \\
&\quad +\frac{1}{6}(1+2x)^3\sqrt{2-x+3x^2}-\frac{4147\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx,x,-1+6x\right)}{648\sqrt{69}} \\
&= -\frac{143}{324}(3-2x)\sqrt{2-x+3x^2}+\frac{11}{27}(1+2x)^2\sqrt{2-x+3x^2} \\
&\quad +\frac{1}{6}(1+2x)^3\sqrt{2-x+3x^2}+\frac{4147\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{648\sqrt{3}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.68

$$
\int \frac{(1+2x)^2\left(1+3x+4x^2\right)}{\sqrt{2-x+3x^2}}\,dx = \frac{6\sqrt{2-x+3x^2}\left(-243+1138x+1176x^2+432x^3\right)+4147\sqrt{3}\log\left(1-6x+2\sqrt{6-3x+9x^2}\right)}{1944}
$$

```
[In] Integrate[((1 + 2*x)^2*(1 + 3*x + 4*x^2))/Sqrt[2 - x + 3*x^2],x]
[Out] (6*Sqrt[2 - x + 3*x^2]*(-243 + 1138*x + 1176*x^2 + 432*x^3) + 4147*Sqrt[3]*
Log[1 - 6*x + 2*Sqrt[6 - 3*x + 9*x^2]])/1944
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.47

method	result
risch	$\frac{(432x^3+1176x^2+1138x-243)\sqrt{3x^2-x+2}}{324}-\frac{4147\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{1944}$
trager	$\left(\frac{4}{3}x^3+\frac{98}{27}x^2+\frac{569}{162}x-\frac{3}{4}\right)\sqrt{3x^2-x+2}-\frac{4147\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(6\,\mathrm{RootOf}\left(\_Z^2-3\right)x+6\sqrt{3x^2-x+2}-\mathrm{RootOf}\right.}{1944}$
default	$-\frac{4147\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{1944}-\frac{3\sqrt{3x^2-x+2}}{4}+\frac{4x^3\sqrt{3x^2-x+2}}{3}+\frac{98x^2\sqrt{3x^2-x+2}}{27}+\frac{569x\sqrt{3x^2-x+2}}{162}$

```
[In] int((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/324*(432*x^3+1176*x^2+1138*x-243)*(3*x^2-x+2)^(1/2)-4147/1944*3^(1/2)*arc
sinh(6/23*23^(1/2)*(x-1/6))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.72

$$\int \frac{(1+2x)^2\left(1+3x+4x^2\right)}{\sqrt{2-x+3x^2}}\,dx = \frac{1}{324}\left(432\,x^3+1176\,x^2+1138\,x-243\right)\sqrt{3\,x^2-x+2} + \frac{4147}{3888}\,\sqrt{3}\log\left(4\,\sqrt{3}\sqrt{3\,x^2-x+2}(6\,x-1)-72\,x^2 + 24\,x-25\right)$$

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/324*(432*x^3 + 1176*x^2 + 1138*x - 243)*sqrt(3*x^2 - x + 2) + 4147/3888*s
qrt(3)*log(4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x - 1) - 72*x^2 + 24*x - 25)
```

Sympy [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.59

$$\int \frac{(1+2x)^2\left(1+3x+4x^2\right)}{\sqrt{2-x+3x^2}}\,dx = \sqrt{3x^2-x+2}\cdot\left(\frac{4x^3}{3}+\frac{98x^2}{27}+\frac{569x}{162}-\frac{3}{4}\right) - \frac{4147\sqrt{3}\operatorname{asinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{1944}$$

```
[In] integrate((1+2*x)**2*(4*x**2+3*x+1)/(3*x**2-x+2)**(1/2),x)
```

```
[Out] sqrt(3*x**2 - x + 2)*(4*x**3/3 + 98*x**2/27 + 569*x/162 - 3/4) - 4147*sqrt(
3)*asinh(6*sqrt(23)*(x - 1/6)/23)/1944
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.84

$$\int \frac{(1+2x)^2\,(1+3x+4x^2)}{\sqrt{2-x+3x^2}}\,dx = \frac{4}{3}\,\sqrt{3\,x^2-x+2}x^3+\frac{98}{27}\,\sqrt{3\,x^2-x+2}x^2 + \frac{569}{162}\,\sqrt{3\,x^2-x+2}x - \frac{4147}{1944}\,\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{23}\,\sqrt{23}(6\,x-1)\right)-\frac{3}{4}\,\sqrt{3\,x^2-x+2}$$

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 4/3*sqrt(3*x^2 - x + 2)*x^3 + 98/27*sqrt(3*x^2 - x + 2)*x^2 + 569/162*sqrt(
3*x^2 - x + 2)*x - 4147/1944*sqrt(3)*arcsinh(1/23*sqrt(23)*(6*x - 1)) - 3/4
*sqrt(3*x^2 - x + 2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.66

$$\int \frac{(1+2x)^2\,(1+3x+4x^2)}{\sqrt{2-x+3x^2}}\,dx = \frac{1}{324}\,(2\,(12\,(18\,x+49)x+569)x-243)\sqrt{3\,x^2-x+2} + \frac{4147}{1944}\,\sqrt{3}\log\left(-2\,\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2-x+2}\right)+1\right)$$

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/324*(2*(12*(18*x + 49)*x + 569)*x - 243)*sqrt(3*x^2 - x + 2) + 4147/1944*
sqrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) + 1)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+2x)^2\,(1+3x+4x^2)}{\sqrt{2-x+3x^2}}\,dx = \int \frac{{(2\,x+1)}^2\,\left(4\,x^2+3\,x+1\right)}{\sqrt{3\,x^2-x+2}}\,dx$$

```
[In] int(((2*x + 1)^2*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(1/2),x)
```

```
[Out] int(((2*x + 1)^2*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(1/2), x)
```

3.242 $\int \frac{(1+2x)(1+3x+4x^2)}{\sqrt{2-x+3x^2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 70

$$\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\sqrt{2-x+3x^2}}\,dx=\frac{2}{9}(1+2x)^2\sqrt{2-x+3x^2}+\frac{1}{54}(69+62x)\sqrt{2-x+3x^2}+\frac{251\text{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{108\sqrt{3}}$$

```
[Out] 251/324*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)+2/9*(1+2*x)^2*(3*x^2-x+2)^(1
/2)+1/54*(69+62*x)*(3*x^2-x+2)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1667, 793, 633, 221}

$$\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\sqrt{2-x+3x^2}}\,dx=\frac{251\text{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{108\sqrt{3}}+\frac{2}{9}\sqrt{3x^2-x+2}(2x+1)^2+\frac{1}{54}(62x+69)\sqrt{3x^2-x+2}$$

```
[In] Int[((1 + 2*x)*(1 + 3*x + 4*x^2))/Sqrt[2 - x + 3*x^2],x]
```

```
[Out] (2*(1 + 2*x)^2*Sqrt[2 - x + 3*x^2])/9 + ((69 + 62*x)*Sqrt[2 - x + 3*x^2])/5
4 + (251*ArcSinh[(1 - 6*x)/Sqrt[23]])/(108*Sqrt[3])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2}{9}(1+2x)^2\sqrt{2-x+3x^2}+\frac{1}{36}\int \frac{(1+2x)(-24+124x)}{\sqrt{2-x+3x^2}}\,dx \\
&= \frac{2}{9}(1+2x)^2\sqrt{2-x+3x^2}+\frac{1}{54}(69+62x)\sqrt{2-x+3x^2}-\frac{251}{108}\int \frac{1}{\sqrt{2-x+3x^2}}\,dx \\
&= \frac{2}{9}(1+2x)^2\sqrt{2-x+3x^2}+\frac{1}{54}(69+62x)\sqrt{2-x+3x^2} \\
&\quad -\frac{251\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx,x,-1+6x\right)}{108\sqrt{69}} \\
&= \frac{2}{9}(1+2x)^2\sqrt{2-x+3x^2}+\frac{1}{54}(69+62x)\sqrt{2-x+3x^2}+\frac{251\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{108\sqrt{3}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.86

$$\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\sqrt{2-x+3x^2}}\,dx = \frac{1}{324}\left(6\sqrt{2-x+3x^2}\left(81+110x+48x^2\right)\right.$$
$$\left.+251\sqrt{3}\log\left(1-6x+2\sqrt{6-3x+9x^2}\right)\right)$$

```
[In] Integrate[((1 + 2*x)*(1 + 3*x + 4*x^2))/Sqrt[2 - x + 3*x^2],x]
```

```
[Out] (6*Sqrt[2 - x + 3*x^2]*(81 + 110*x + 48*x^2) + 251*Sqrt[3]*Log[1 - 6*x + 2*
Sqrt[6 - 3*x + 9*x^2]])/324
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.57

method	result
risch	$\frac{\left(48x^2+110x+81\right)\sqrt{3x^2-x+2}}{54}-\frac{251\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{324}$
default	$-\frac{251\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{324}+\frac{3\sqrt{3x^2-x+2}}{2}+\frac{8x^2\sqrt{3x^2-x+2}}{9}+\frac{55x\sqrt{3x^2-x+2}}{27}$
trager	$\left(\frac{8}{9}x^2+\frac{55}{27}x+\frac{3}{2}\right)\sqrt{3x^2-x+2}+\frac{251\operatorname{RootOf}\left(\_Z^2-3\right)\ln(-6\operatorname{RootOf}\left(\_Z^2-3\right)x+\operatorname{RootOf}\left(\_Z^2-3\right)+6\sqrt{3x^2-x+2}}{324}$

```
[In] int((1+2*x)*(4*x^2+3*x+1)/(3*x^2-x+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/54*(48*x^2+110*x+81)*(3*x^2-x+2)^(1/2)-251/324*3^(1/2)*arcsinh(6/23*23^(1
/2)*(x-1/6))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.90

$$\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\sqrt{2-x+3x^2}}\,dx = \frac{1}{54}\left(48\,x^2+110\,x+81\right)\sqrt{3\,x^2-x+2}$$
$$+\frac{251}{648}\sqrt{3}\log\left(4\sqrt{3}\sqrt{3\,x^2-x+2}(6\,x-1)-72\,x^2+24\,x\right.$$
$$\left.-25\right)$$

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2-x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/54*(48*x^2 + 110*x + 81)*sqrt(3*x^2 - x + 2) + 251/648*sqrt(3)*log(4*sqrt
(3)*sqrt(3*x^2 - x + 2)*(6*x - 1) - 72*x^2 + 24*x - 25)
```

Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.70

$$\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\sqrt{2-x+3x^2}}\,dx = \left(\frac{8x^2}{9}+\frac{55x}{27}+\frac{3}{2}\right)\sqrt{3x^2-x+2}-\frac{251\sqrt{3}\operatorname{asinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{324}$$

```
[In] integrate((1+2*x)*(4*x**2+3*x+1)/(3*x**2-x+2)**(1/2),x)
```

```
[Out] (8*x**2/9 + 55*x/27 + 3/2)*sqrt(3*x**2 - x + 2) - 251*sqrt(3)*asinh(6*sqrt(
23)*(x - 1/6)/23)/324
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.90

$$\begin{aligned}\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\sqrt{2-x+3x^2}}\,dx &= \frac{8}{9}\sqrt{3x^2-x+2}x^2+\frac{55}{27}\sqrt{3x^2-x+2}x \\ &\quad -\frac{251}{324}\sqrt{3}\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}(6x-1)\right)+\frac{3}{2}\sqrt{3x^2-x+2}\end{aligned}$$

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2-x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 8/9*sqrt(3*x^2 - x + 2)*x^2 + 55/27*sqrt(3*x^2 - x + 2)*x - 251/324*sqrt(3)
*arcsinh(1/23*sqrt(23)*(6*x - 1)) + 3/2*sqrt(3*x^2 - x + 2)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.83

$$\begin{aligned}\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\sqrt{2-x+3x^2}}\,dx &= \frac{1}{54}\left(2\left(24x+55\right)x+81\right)\sqrt{3x^2-x+2} \\ &\quad +\frac{251}{324}\sqrt{3}\log\left(-2\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2-x+2}\right)+1\right)\end{aligned}$$

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2-x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/54*(2*(24*x + 55)*x + 81)*sqrt(3*x^2 - x + 2) + 251/324*sqrt(3)*log(-2*sq
rt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) + 1)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+2x)\,(1+3x+4x^2)}{\sqrt{2-x+3x^2}}\,dx=\int \frac{(2\,x+1)\,\left(4\,x^2+3\,x+1\right)}{\sqrt{3\,x^2-x+2}}\,dx$$

```
[In] int(((2*x + 1)*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(1/2),x)
[Out] int(((2*x + 1)*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(1/2), x)
```

3.243 $\int \frac{1+3x+4x^2}{(1+2x)\sqrt{2-x+3x^2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 78

$$\int \frac{1+3x+4x^2}{(1+2x)\sqrt{2-x+3x^2}}\,dx = \frac{2}{3}\sqrt{2-x+3x^2}-\frac{5\operatorname{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{6\sqrt{3}}-\frac{\operatorname{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)}{2\sqrt{13}}$$

```
[Out] -5/18*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)-1/26*arctanh(1/26*(9-8*x)*13^(
1/2)/(3*x^2-x+2)^(1/2))*13^(1/2)+2/3*(3*x^2-x+2)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1667, 857, 633, 221, 738, 212}

$$\int \frac{1+3x+4x^2}{(1+2x)\sqrt{2-x+3x^2}}\,dx = -\frac{5\operatorname{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{6\sqrt{3}}-\frac{\operatorname{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{2\sqrt{13}}+\frac{2}{3}\sqrt{3x^2-x+2}$$

```
[In] Int[(1 + 3*x + 4*x^2)/((1 + 2*x)*Sqrt[2 - x + 3*x^2]),x]
```

```
[Out] (2*Sqrt[2 - x + 3*x^2])/3 - (5*ArcSinh[(1 - 6*x)/Sqrt[23]])/(6*Sqrt[3]) - A
rcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])]/(2*Sqrt[13])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2}{3}\sqrt{2-x+3x^2}+\frac{1}{12}\int\frac{16+20x}{(1+2x)\sqrt{2-x+3x^2}}\,dx\\
&= \frac{2}{3}\sqrt{2-x+3x^2}+\frac{1}{2}\int\frac{1}{(1+2x)\sqrt{2-x+3x^2}}\,dx+\frac{5}{6}\int\frac{1}{\sqrt{2-x+3x^2}}\,dx
\end{aligned}$$

$$
\begin{aligned}
&= \frac{2}{3}\sqrt{2-x+3x^2} + \frac{5\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx, x, -1+6x\right)}{6\sqrt{69}} \\
&\quad - \text{Subst}\left(\int \frac{1}{52-x^2}\,dx, x, \frac{9-8x}{\sqrt{2-x+3x^2}}\right) \\
&= \frac{2}{3}\sqrt{2-x+3x^2} - \frac{5\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{6\sqrt{3}} - \frac{\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)}{2\sqrt{13}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.19

$$
\int \frac{1+3x+4x^2}{(1+2x)\sqrt{2-x+3x^2}}\,dx = \frac{2}{3}\sqrt{2-x+3x^2} + \frac{\text{arctanh}\left(\frac{\sqrt{3}+2\sqrt{3}x-2\sqrt{2-x+3x^2}}{\sqrt{13}}\right)}{\sqrt{13}} - \frac{5\log\left(1-6x+2\sqrt{6-3x+9x^2}\right)}{6\sqrt{3}}
$$

```
[In] Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)*Sqrt[2 - x + 3*x^2]),x]
```

```
[Out] (2*Sqrt[2 - x + 3*x^2])/3 + ArcTanh[(Sqrt[3] + 2*Sqrt[3]*x - 2*Sqrt[2 - x +
 3*x^2])/Sqrt[13]]/Sqrt[13] - (5*Log[1 - 6*x + 2*Sqrt[6 - 3*x + 9*x^2]])/(6
*Sqrt[3])
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.77

method	result
default	$\frac{5\sqrt{3}\,\text{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{18}+\frac{2\sqrt{3x^2-x+2}}{3}-\frac{\sqrt{13}\,\text{arctanh}\left(\frac{2\left(\frac{9}{2}-4x\right)\sqrt{13}}{13\sqrt{12\left(x+\frac{1}{2}\right)^2-16x+5}}\right)}{26}$
risch	$\frac{5\sqrt{3}\,\text{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{18}+\frac{2\sqrt{3x^2-x+2}}{3}-\frac{\sqrt{13}\,\text{arctanh}\left(\frac{2\left(\frac{9}{2}-4x\right)\sqrt{13}}{13\sqrt{12\left(x+\frac{1}{2}\right)^2-16x+5}}\right)}{26}$
trager	$\frac{2\sqrt{3x^2-x+2}}{3}+\frac{\text{RootOf}\left(\_Z^2-13\right)\ln\left(\frac{8\,\text{RootOf}\left(\_Z^2-13\right)x+26\sqrt{3x^2-x+2}-9\,\text{RootOf}\left(\_Z^2-13\right)}{1+2x}\right)}{26}+5\,\text{RootOf}\left(\_Z^2-3\right)\ln($

```
[In] int((4*x^2+3*x+1)/(1+2*x)/(3*x^2-x+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 5/18*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))+2/3*(3*x^2-x+2)^(1/2)-1/26*13^(
1/2)*arctanh(2/13*(9/2-4*x)*13^(1/2)/(12*(x+1/2)^2-16*x+5)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time $=0.27$ (sec) , antiderivative size $=105$, normalized size of antiderivative $=1.35$

$$\begin{aligned}\int \frac{1+3x+4x^2}{(1+2x)\sqrt{2-x+3x^2}}\,dx &= \frac{5}{36}\,\sqrt{3}\log\left(-4\,\sqrt{3}\sqrt{3\,x^2-x+2}(6\,x-1)-72\,x^2+24\,x-25\right)\\ &\quad+\frac{1}{52}\,\sqrt{13}\log\left(-\frac{4\,\sqrt{13}\sqrt{3\,x^2-x+2}(8\,x-9)+220\,x^2-196\,x+185}{4\,x^2+4\,x+1}\right)\\ &\quad+\frac{2}{3}\,\sqrt{3\,x^2-x+2}\end{aligned}$$

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2-x+2)^(1/2),x, algorithm="fricas")
[Out] 5/36*sqrt(3)*log(-4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x - 1) - 72*x^2 + 24*x -
 25) + 1/52*sqrt(13)*log(-(4*sqrt(13)*sqrt(3*x^2 - x + 2)*(8*x - 9) + 220*x
^2 - 196*x + 185)/(4*x^2 + 4*x + 1)) + 2/3*sqrt(3*x^2 - x + 2)
```

Sympy [F]

$$\int \frac{1+3x+4x^2}{(1+2x)\sqrt{2-x+3x^2}}\,dx = \int \frac{4x^2+3x+1}{(2x+1)\sqrt{3x^2-x+2}}\,dx$$

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)/(3*x**2-x+2)**(1/2),x)
[Out] Integral((4*x**2 + 3*x + 1)/((2*x + 1)*sqrt(3*x**2 - x + 2)), x)
```

Maxima [A] (verification not implemented)

none

Time $=0.29$ (sec) , antiderivative size $=67$, normalized size of antiderivative $=0.86$

$$\begin{aligned}\int \frac{1+3x+4x^2}{(1+2x)\sqrt{2-x+3x^2}}\,dx &= \frac{5}{18}\,\sqrt{3}\,\mathrm{arsinh}\left(\frac{6}{23}\,\sqrt{23}x-\frac{1}{23}\,\sqrt{23}\right)\\ &\quad+\frac{1}{26}\,\sqrt{13}\,\mathrm{arsinh}\left(\frac{8\,\sqrt{23}x}{23\,|2\,x+1|}-\frac{9\,\sqrt{23}}{23\,|2\,x+1|}\right)\\ &\quad+\frac{2}{3}\,\sqrt{3\,x^2-x+2}\end{aligned}$$

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2-x+2)^(1/2),x, algorithm="maxima")
[Out] 5/18*sqrt(3)*arcsinh(6/23*sqrt(23)*x - 1/23*sqrt(23)) + 1/26*sqrt(13)*arcsi
nh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqrt(23)/abs(2*x + 1)) + 2/3*sqrt(3*
x^2 - x + 2)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.49

$$\begin{aligned}&\int \frac{1+3x+4x^2}{(1+2x)\sqrt{2-x+3x^2}}\,dx\\&=-\frac{5}{18}\sqrt{3}\log\left(-6\sqrt{3}x+\sqrt{3}+6\sqrt{3x^2-x+2}\right)\\&\quad+\frac{1}{26}\sqrt{13}\log\left(-\frac{\left|-4\sqrt{3}x-2\sqrt{13}-2\sqrt{3}+4\sqrt{3x^2-x+2}\right|}{2\left(2\sqrt{3}x-\sqrt{13}+\sqrt{3}-2\sqrt{3x^2-x+2}\right)}\right)+\frac{2}{3}\sqrt{3x^2-x+2}\end{aligned}$$

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2-x+2)^(1/2),x, algorithm="giac")
```

```
[Out] -5/18*sqrt(3)*log(-6*sqrt(3)*x + sqrt(3) + 6*sqrt(3*x^2 - x + 2)) + 1/26*sq
rt(13)*log(-1/2*abs(-4*sqrt(3)*x - 2*sqrt(13) - 2*sqrt(3) + 4*sqrt(3*x^2 -
x + 2))/(2*sqrt(3)*x - sqrt(13) + sqrt(3) - 2*sqrt(3*x^2 - x + 2))) + 2/3*s
qrt(3*x^2 - x + 2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+3x+4x^2}{(1+2x)\sqrt{2-x+3x^2}}\,dx=\int \frac{4x^2+3x+1}{(2x+1)\sqrt{3x^2-x+2}}\,dx$$

```
[In] int((3*x + 4*x^2 + 1)/((2*x + 1)*(3*x^2 - x + 2)^(1/2)),x)
```

```
[Out] int((3*x + 4*x^2 + 1)/((2*x + 1)*(3*x^2 - x + 2)^(1/2)), x)
```

3.244 $\int \frac{1+3x+4x^2}{(1+2x)^2\sqrt{2-x+3x^2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 83

$$\int \frac{1+3x+4x^2}{(1+2x)^2\sqrt{2-x+3x^2}}\,dx=-\frac{\sqrt{2-x+3x^2}}{13(1+2x)}-\frac{\operatorname{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{\sqrt{3}}+\frac{9\operatorname{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)}{26\sqrt{13}}$$

```
[Out] -1/3*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)+9/338*arctanh(1/26*(9-8*x)*13^(
1/2)/(3*x^2-x+2)^(1/2))*13^(1/2)-1/13*(3*x^2-x+2)^(1/2)/(1+2*x)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1664, 857, 633, 221, 738, 212}

$$\int \frac{1+3x+4x^2}{(1+2x)^2\sqrt{2-x+3x^2}}\,dx=-\frac{\operatorname{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{\sqrt{3}}+\frac{9\operatorname{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{26\sqrt{13}}-\frac{\sqrt{3x^2-x+2}}{13(2x+1)}$$

```
[In] Int[(1 + 3*x + 4*x^2)/((1 + 2*x)^2*Sqrt[2 - x + 3*x^2]),x]
```

```
[Out] -1/13*Sqrt[2 - x + 3*x^2]/(1 + 2*x) - ArcSinh[(1 - 6*x)/Sqrt[23]]/Sqrt[3] +
 (9*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/(26*Sqrt[13])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\sqrt{2-x+3x^2}}{13(1+2x)}-\frac{1}{13}\int\frac{-\frac{17}{2}-26x}{(1+2x)\sqrt{2-x+3x^2}}\,dx\\ &= -\frac{\sqrt{2-x+3x^2}}{13(1+2x)}-\frac{9}{26}\int\frac{1}{(1+2x)\sqrt{2-x+3x^2}}\,dx+\int\frac{1}{\sqrt{2-x+3x^2}}\,dx\end{aligned}$$

$$= -\frac{\sqrt{2-x+3x^2}}{13(1+2x)} + \frac{9}{13}\text{Subst}\left(\int \frac{1}{52-x^2}\,dx, x, \frac{9-8x}{\sqrt{2-x+3x^2}}\right) + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx, x, -1+6x\right)}{\sqrt{69}}$$

$$= -\frac{\sqrt{2-x+3x^2}}{13(1+2x)} - \frac{\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{\sqrt{3}} + \frac{9\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)}{26\sqrt{13}}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.19

$$\int \frac{1+3x+4x^2}{(1+2x)^2\sqrt{2-x+3x^2}}\,dx = -\frac{\sqrt{2-x+3x^2}}{13+26x} - \frac{9\text{arctanh}\left(\frac{\sqrt{3}+2\sqrt{3}x-2\sqrt{2-x+3x^2}}{\sqrt{13}}\right)}{13\sqrt{13}} - \frac{\log\left(1-6x+2\sqrt{6-3x+9x^2}\right)}{\sqrt{3}}$$

```
[In] Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)^2*Sqrt[2 - x + 3*x^2]),x]
```

```
[Out] -(Sqrt[2 - x + 3*x^2]/(13 + 26*x)) - (9*ArcTanh[(Sqrt[3] + 2*Sqrt[3]*x - 2*
Sqrt[2 - x + 3*x^2])/Sqrt[13]])/(13*Sqrt[13]) - Log[1 - 6*x + 2*Sqrt[6 - 3*
x + 9*x^2]]/Sqrt[3]
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.81

method	result
default	$\frac{\sqrt{3}\,\text{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{3} - \frac{\sqrt{3\left(x+\frac{1}{2}\right)^2-4x+\frac{5}{4}}}{26\left(x+\frac{1}{2}\right)} + \frac{9\sqrt{13}\,\text{arctanh}\left(\frac{2\left(\frac{9}{2}-4x\right)\sqrt{13}}{13\sqrt{12\left(x+\frac{1}{2}\right)^2-16x+5}}\right)}{338}$
risch	$-\frac{\sqrt{3x^2-x+2}}{13(1+2x)} + \frac{\sqrt{3}\,\text{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{3} + \frac{9\sqrt{13}\,\text{arctanh}\left(\frac{2\left(\frac{9}{2}-4x\right)\sqrt{13}}{13\sqrt{12\left(x+\frac{1}{2}\right)^2-16x+5}}\right)}{338}$
trager	$-\frac{\sqrt{3x^2-x+2}}{13(1+2x)} + \frac{9\,\text{RootOf}\left(\_Z^2-13\right)\ln\left(-\frac{8\,\text{RootOf}\left(\_Z^2-13\right)x-9\,\text{RootOf}\left(\_Z^2-13\right)-26\sqrt{3x^2-x+2}}{1+2x}\right)}{338} + \text{RootOf}\left(\_Z^2-3\right)\ln$

```
[In] int((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2-x+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))-1/26/(x+1/2)*(3*(x+1/2)^2-4*x+5/
4)^(1/2)+9/338*13^(1/2)*arctanh(2/13*(9/2-4*x)*13^(1/2)/(12*(x+1/2)^2-16*x+
5)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.48

$$\int \frac{1+3x+4x^2}{(1+2x)^2\sqrt{2-x+3x^2}}\,dx$$
$$= \frac{338\sqrt{3}(2x+1)\log\left(-4\sqrt{3}\sqrt{3x^2-x+2}(6x-1)-72x^2+24x-25\right)+27\sqrt{13}(2x+1)\log\left(\frac{4\sqrt{13}\sqrt{3x^2}}{}\right.}{2028(2x+1)}$$

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2-x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2028*(338*sqrt(3)*(2*x + 1)*log(-4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x - 1)
- 72*x^2 + 24*x - 25) + 27*sqrt(13)*(2*x + 1)*log((4*sqrt(13)*sqrt(3*x^2 -
x + 2)*(8*x - 9) - 220*x^2 + 196*x - 185)/(4*x^2 + 4*x + 1)) - 156*sqrt(3*x
^2 - x + 2))/(2*x + 1)
```

Sympy [F]

$$\int \frac{1+3x+4x^2}{(1+2x)^2\sqrt{2-x+3x^2}}\,dx = \int \frac{4x^2+3x+1}{(2x+1)^2\sqrt{3x^2-x+2}}\,dx$$

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)**2/(3*x**2-x+2)**(1/2),x)
```

```
[Out] Integral((4*x**2 + 3*x + 1)/((2*x + 1)**2*sqrt(3*x**2 - x + 2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.89

$$\begin{aligned}\int \frac{1+3x+4x^2}{(1+2x)^2\sqrt{2-x+3x^2}}\,dx &= \frac{1}{3}\sqrt{3}\,\mathrm{arsinh}\left(\frac{6}{23}\sqrt{23}x-\frac{1}{23}\sqrt{23}\right)\\ &\quad-\frac{9}{338}\sqrt{13}\,\mathrm{arsinh}\left(\frac{8\sqrt{23}x}{23\,|2x+1|}-\frac{9\sqrt{23}}{23\,|2x+1|}\right)\\ &\quad-\frac{\sqrt{3x^2-x+2}}{13\,(2x+1)}\end{aligned}$$

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2-x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arcsinh(6/23*sqrt(23)*x - 1/23*sqrt(23)) - 9/338*sqrt(13)*arcsi
nh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqrt(23)/abs(2*x + 1)) - 1/13*sqrt(3
*x^2 - x + 2)/(2*x + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 191 vs. $2(66) = 132$.

Time = 0.42 (sec) , antiderivative size = 191, normalized size of antiderivative = 2.30

$$\int \frac{1+3x+4x^2}{(1+2x)^2\sqrt{2-x+3x^2}}\,dx = \frac{9\sqrt{13}\log\left(\sqrt{13}\left(\sqrt{-\frac{8}{2x+1}+\frac{13}{(2x+1)^2}+3}+\frac{\sqrt{13}}{2x+1}\right)-4\right)}{338\,\mathrm{sgn}\left(\frac{1}{2x+1}\right)} - \frac{\sqrt{3}\log\left(\frac{\left|-2\sqrt{3}+2\sqrt{-\frac{8}{2x+1}+\frac{13}{(2x+1)^2}+3}+\frac{2\sqrt{13}}{2x+1}\right|}{2\left(\sqrt{3}+\sqrt{-\frac{8}{2x+1}+\frac{13}{(2x+1)^2}+3}+\frac{\sqrt{13}}{2x+1}\right)}\right)}{3\,\mathrm{sgn}\left(\frac{1}{2x+1}\right)} - \frac{\sqrt{-\frac{8}{2x+1}+\frac{13}{(2x+1)^2}+3}}{26\,\mathrm{sgn}\left(\frac{1}{2x+1}\right)}$$

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2-x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 9/338*sqrt(13)*log(sqrt(13)*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt
(13)/(2*x + 1)) - 4)/sgn(1/(2*x + 1)) - 1/3*sqrt(3)*log(1/2*abs(-2*sqrt(3)
+ 2*sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + 2*sqrt(13)/(2*x + 1))/(sqrt(3
) + sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1)))/sgn(1/(2
*x + 1)) - 1/26*sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3)/sgn(1/(2*x + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+3x+4x^2}{(1+2x)^2\sqrt{2-x+3x^2}}\,dx = \int \frac{4\,x^2+3\,x+1}{{\left(2\,x+1\right)}^2\,\sqrt{3\,x^2-x+2}}\,dx$$

```
[In] int((3*x + 4*x^2 + 1)/((2*x + 1)^2*(3*x^2 - x + 2)^(1/2)),x)
```

```
[Out] int((3*x + 4*x^2 + 1)/((2*x + 1)^2*(3*x^2 - x + 2)^(1/2)), x)
```

3.245 $\int \frac{1+3x+4x^2}{(1+2x)^3\sqrt{2-x+3x^2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 89

$$\int \frac{1+3x+4x^2}{(1+2x)^3\sqrt{2-x+3x^2}}\,dx=-\frac{\sqrt{2-x+3x^2}}{26(1+2x)^2}+\frac{7\sqrt{2-x+3x^2}}{169(1+2x)}-\frac{581\operatorname{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)}{676\sqrt{13}}$$

```
[Out] -581/8788*arctanh(1/26*(9-8*x)*13^(1/2)/(3*x^2-x+2)^(1/2))*13^(1/2)-1/26*(3
*x^2-x+2)^(1/2)/(1+2*x)^2+7/169*(3*x^2-x+2)^(1/2)/(1+2*x)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1664, 820, 738, 212}

$$\int \frac{1+3x+4x^2}{(1+2x)^3\sqrt{2-x+3x^2}}\,dx=-\frac{581\operatorname{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{676\sqrt{13}}+\frac{7\sqrt{3x^2-x+2}}{169(2x+1)}-\frac{\sqrt{3x^2-x+2}}{26(2x+1)^2}$$

```
[In] Int[(1 + 3*x + 4*x^2)/((1 + 2*x)^3*Sqrt[2 - x + 3*x^2]),x]
```

```
[Out] -1/26*Sqrt[2 - x + 3*x^2]/(1 + 2*x)^2 + (7*Sqrt[2 - x + 3*x^2])/(169*(1 + 2
*x)) - (581*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/(676*Sqrt[
13])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\sqrt{2-x+3x^2}}{26(1+2x)^2}-\frac{1}{26}\int\frac{-\frac{35}{2}-49x}{(1+2x)^2\sqrt{2-x+3x^2}}\,dx\\
&= -\frac{\sqrt{2-x+3x^2}}{26(1+2x)^2}+\frac{7\sqrt{2-x+3x^2}}{169(1+2x)}+\frac{581}{676}\int\frac{1}{(1+2x)\sqrt{2-x+3x^2}}\,dx\\
&= -\frac{\sqrt{2-x+3x^2}}{26(1+2x)^2}+\frac{7\sqrt{2-x+3x^2}}{169(1+2x)}-\frac{581}{338}\text{Subst}\left(\int\frac{1}{52-x^2}\,dx,x,\frac{9-8x}{\sqrt{2-x+3x^2}}\right)\\
&= -\frac{\sqrt{2-x+3x^2}}{26(1+2x)^2}+\frac{7\sqrt{2-x+3x^2}}{169(1+2x)}-\frac{581\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)}{676\sqrt{13}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.87

$$\int \frac{1+3x+4x^2}{(1+2x)^3\sqrt{2-x+3x^2}}\,dx = \frac{\frac{13(1+28x)\sqrt{2-x+3x^2}}{(1+2x)^2}+581\sqrt{13}\operatorname{arctanh}\left(\frac{\sqrt{3}+2\sqrt{3}x-2\sqrt{2-x+3x^2}}{\sqrt{13}}\right)}{4394}$$

```
[In] Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)^3*Sqrt[2 - x + 3*x^2]),x]
```

```
[Out] ((13*(1 + 28*x)*Sqrt[2 - x + 3*x^2])/(1 + 2*x)^2 + 581*Sqrt[13]*ArcTanh[(Sq
rt[3] + 2*Sqrt[3]*x - 2*Sqrt[2 - x + 3*x^2])/Sqrt[13]])/4394
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.76

method	result	size
risch	$\frac{84x^3-25x^2+55x+2}{338(1+2x)^2\sqrt{3x^2-x+2}}-\frac{581\sqrt{13}\,\operatorname{arctanh}\left(\frac{2\left(\frac{9}{2}-4x\right)\sqrt{13}}{13\sqrt{12\left(x+\frac{1}{2}\right)^2-16x+5}}\right)}{8788}$	68
default	$-\frac{581\sqrt{13}\,\operatorname{arctanh}\left(\frac{2\left(\frac{9}{2}-4x\right)\sqrt{13}}{13\sqrt{12\left(x+\frac{1}{2}\right)^2-16x+5}}\right)}{8788}+\frac{7\sqrt{3\left(x+\frac{1}{2}\right)^2-4x+\frac{5}{4}}}{338\left(x+\frac{1}{2}\right)}-\frac{\sqrt{3\left(x+\frac{1}{2}\right)^2-4x+\frac{5}{4}}}{104\left(x+\frac{1}{2}\right)^2}$	74
trager	$\frac{(28x+1)\sqrt{3x^2-x+2}}{338(1+2x)^2}-\frac{581\,\operatorname{RootOf}\left(\_Z^2-13\right)\ln\left(-\frac{8\,\operatorname{RootOf}\left(\_Z^2-13\right)x-9\,\operatorname{RootOf}\left(\_Z^2-13\right)-26\sqrt{3x^2-x+2}}{1+2x}\right)}{8788}$	78

```
[In] int((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2-x+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/338*(84*x^3-25*x^2+55*x+2)/(1+2*x)^2/(3*x^2-x+2)^(1/2)-581/8788*13^(1/2)*
arctanh(2/13*(9/2-4*x)*13^(1/2)/(12*(x+1/2)^2-16*x+5)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.08

$$\int \frac{1+3x+4x^2}{(1+2x)^3\sqrt{2-x+3x^2}}\,dx = \frac{581\,\sqrt{13}(4\,x^2+4\,x+1)\log\left(-\frac{4\,\sqrt{13}\sqrt{3\,x^2-x+2}(8\,x-9)+220\,x^2-196\,x+185}{4\,x^2+4\,x+1}\right)+52\,\sqrt{3\,x^2-x+2}(28\,x+1)}{17576\,(4\,x^2+4\,x+1)}$$

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2-x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/17576*(581*sqrt(13)*(4*x^2 + 4*x + 1)*log(-(4*sqrt(13)*sqrt(3*x^2 - x + 2
)*(8*x - 9) + 220*x^2 - 196*x + 185)/(4*x^2 + 4*x + 1)) + 52*sqrt(3*x^2 - x
 + 2)*(28*x + 1))/(4*x^2 + 4*x + 1)
```

Sympy [F]

$$\int \frac{1+3x+4x^2}{(1+2x)^3\sqrt{2-x+3x^2}}\,dx = \int \frac{4x^2+3x+1}{(2x+1)^3\sqrt{3x^2-x+2}}\,dx$$

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)**3/(3*x**2-x+2)**(1/2),x)
```

```
[Out] Integral((4*x**2 + 3*x + 1)/((2*x + 1)**3*sqrt(3*x**2 - x + 2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.92

$$\int \frac{1+3x+4x^2}{(1+2x)^3\sqrt{2-x+3x^2}}\,dx = \frac{581}{8788}\,\sqrt{13}\,\mathrm{arsinh}\left(\frac{8\,\sqrt{23}x}{23\,|2\,x+1|}-\frac{9\,\sqrt{23}}{23\,|2\,x+1|}\right) - \frac{\sqrt{3\,x^2-x+2}}{26\,(4\,x^2+4\,x+1)} + \frac{7\,\sqrt{3\,x^2-x+2}}{169\,(2\,x+1)}$$

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2-x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 581/8788*sqrt(13)*arcsinh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqrt(23)/abs(
2*x + 1)) - 1/26*sqrt(3*x^2 - x + 2)/(4*x^2 + 4*x + 1) + 7/169*sqrt(3*x^2 -
 x + 2)/(2*x + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 204 vs. 2(71) = 142.

Time = 0.32 (sec) , antiderivative size = 204, normalized size of antiderivative = 2.29

$$\int \frac{1+3x+4x^2}{(1+2x)^3\sqrt{2-x+3x^2}}\,dx = \frac{581}{8788}\,\sqrt{13}\log\left(-\frac{\left|-4\,\sqrt{3}x-2\,\sqrt{13}-2\,\sqrt{3}+4\,\sqrt{3\,x^2-x+2}\right|}{2\,\left(2\,\sqrt{3}x-\sqrt{13}+\sqrt{3}-2\,\sqrt{3\,x^2-x+2}\right)}\right) + \frac{190\,\left(\sqrt{3}x-\sqrt{3\,x^2-x+2}\right)^3-53\,\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2-x+2}\right)^2-489\,\sqrt{3}x+289\,\sqrt{3}+489\,\sqrt{3\,x^2-x+}}{338\,\left(2\,\left(\sqrt{3}x-\sqrt{3\,x^2-x+2}\right)^2+2\,\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2-x+2}\right)-5\right)^2}$$

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2-x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 581/8788*sqrt(13)*log(-1/2*abs(-4*sqrt(3)*x - 2*sqrt(13) - 2*sqrt(3) + 4*sq
rt(3*x^2 - x + 2))/(2*sqrt(3)*x - sqrt(13) + sqrt(3) - 2*sqrt(3*x^2 - x + 2
))) + 1/338*(190*(sqrt(3)*x - sqrt(3*x^2 - x + 2))^3 - 53*sqrt(3)*(sqrt(3)*
x - sqrt(3*x^2 - x + 2))^2 - 489*sqrt(3)*x + 289*sqrt(3) + 489*sqrt(3*x^2 -
 x + 2))/(2*(sqrt(3)*x - sqrt(3*x^2 - x + 2))^2 + 2*sqrt(3)*(sqrt(3)*x - sq
rt(3*x^2 - x + 2)) - 5)^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+3x+4x^2}{(1+2x)^3\sqrt{2-x+3x^2}}\,dx=\int \frac{4\,x^2+3\,x+1}{{\left(2\,x+1\right)}^3\,\sqrt{3\,x^2-x+2}}\,dx$$

```
[In] int((3*x + 4*x^2 + 1)/((2*x + 1)^3*(3*x^2 - x + 2)^(1/2)),x)
```

```
[Out] int((3*x + 4*x^2 + 1)/((2*x + 1)^3*(3*x^2 - x + 2)^(1/2)), x)
```

3.246 $\int \frac{(1+2x)^3(1+3x+4x^2)}{(2-x+3x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 103

$$\int \frac{(1+2x)^3\,(1+3x+4x^2)}{(2-x+3x^2)^{3/2}}\,dx = \frac{2(12839-3871x)}{1863\sqrt{2-x+3x^2}}+\frac{746}{81}\sqrt{2-x+3x^2}+\frac{412}{81}x\sqrt{2-x+3x^2}+\frac{32}{27}x^2\sqrt{2-x+3x^2}+\frac{353\text{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{81\sqrt{3}}$$

```
[Out] 353/243*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)+2/1863*(12839-3871*x)/(3*x^2
-x+2)^(1/2)+746/81*(3*x^2-x+2)^(1/2)+412/81*x*(3*x^2-x+2)^(1/2)+32/27*x^2*(
3*x^2-x+2)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1674, 1675, 654, 633, 221}

$$\int \frac{(1+2x)^3\,(1+3x+4x^2)}{(2-x+3x^2)^{3/2}}\,dx = \frac{353\text{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{81\sqrt{3}}+\frac{32}{27}\sqrt{3x^2-x+2}x^2+\frac{412}{81}\sqrt{3x^2-x+2}x+\frac{746}{81}\sqrt{3x^2-x+2}+\frac{2(12839-3871x)}{1863\sqrt{3x^2-x+2}}$$

```
[In] Int[((1 + 2*x)^3*(1 + 3*x + 4*x^2))/(2 - x + 3*x^2)^(3/2),x]
```

```
[Out] (2*(12839 - 3871*x))/(1863*Sqrt[2 - x + 3*x^2]) + (746*Sqrt[2 - x + 3*x^2])
/81 + (412*x*Sqrt[2 - x + 3*x^2])/81 + (32*x^2*Sqrt[2 - x + 3*x^2])/27 + (3
53*ArcSinh[(1 - 6*x)/Sqrt[23]])/(81*Sqrt[3])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2(12839-3871x)}{1863\sqrt{2-x+3x^2}}+\frac{2}{23}\int\frac{\frac{1127}{81}+\frac{7682x}{27}+\frac{2852x^2}{9}+\frac{368x^3}{3}}{\sqrt{2-x+3x^2}}\,dx\\ &= \frac{2(12839-3871x)}{1863\sqrt{2-x+3x^2}}+\frac{32}{27}x^2\sqrt{2-x+3x^2}+\frac{2}{207}\int\frac{\frac{1127}{9}+2070x+\frac{9476x^2}{3}}{\sqrt{2-x+3x^2}}\,dx\end{aligned}$$

$$=\frac{2(12839-3871x)}{1863\sqrt{2-x+3x^2}}+\frac{412}{81}x\sqrt{2-x+3x^2}+\frac{32}{27}x^2\sqrt{2-x+3x^2}+\frac{1}{621}\int\frac{-5566+17158x}{\sqrt{2-x+3x^2}}\,dx$$

$$=\frac{2(12839-3871x)}{1863\sqrt{2-x+3x^2}}+\frac{746}{81}\sqrt{2-x+3x^2}+\frac{412}{81}x\sqrt{2-x+3x^2}$$
$$+\frac{32}{27}x^2\sqrt{2-x+3x^2}-\frac{353}{81}\int\frac{1}{\sqrt{2-x+3x^2}}\,dx$$

$$=\frac{2(12839-3871x)}{1863\sqrt{2-x+3x^2}}+\frac{746}{81}\sqrt{2-x+3x^2}+\frac{412}{81}x\sqrt{2-x+3x^2}$$
$$+\frac{32}{27}x^2\sqrt{2-x+3x^2}-\frac{353\text{Subst}\left(\int\frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx,x,-1+6x\right)}{81\sqrt{69}}$$

$$=\frac{2(12839-3871x)}{1863\sqrt{2-x+3x^2}}+\frac{746}{81}\sqrt{2-x+3x^2}+\frac{412}{81}x\sqrt{2-x+3x^2}$$
$$+\frac{32}{27}x^2\sqrt{2-x+3x^2}+\frac{353\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{81\sqrt{3}}$$

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.68

$$\int\frac{(1+2x)^3\,(1+3x+4x^2)}{(2-x+3x^2)^{3/2}}\,dx=\frac{\frac{6(29997-2974x+23207x^2+13110x^3+3312x^4)}{\sqrt{2-x+3x^2}}+8119\sqrt{3}\log\bigl(1-6x+2\sqrt{6-3x+9x^2}}{5589}$$

```
[In] Integrate[((1 + 2*x)^3*(1 + 3*x + 4*x^2))/(2 - x + 3*x^2)^(3/2),x]
```

```
[Out] ((6*(29997 - 2974*x + 23207*x^2 + 13110*x^3 + 3312*x^4))/Sqrt[2 - x + 3*x^2
] + 8119*Sqrt[3]*Log[1 - 6*x + 2*Sqrt[6 - 3*x + 9*x^2]])/5589
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.49

method	result
risch	$\frac{\frac{32}{9}x^4+\frac{380}{27}x^3+\frac{2018}{81}x^2-\frac{5948}{1863}x+\frac{2222}{69}}{\sqrt{3x^2-x+2}}-\frac{353\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{243}$
trager	$\frac{\frac{32}{9}x^4+\frac{380}{27}x^3+\frac{2018}{81}x^2-\frac{5948}{1863}x+\frac{2222}{69}}{\sqrt{3x^2-x+2}}-\frac{353\,\text{RootOf}\left(\_Z^2-3\right)\ln\left(6\,\text{RootOf}\left(\_Z^2-3\right)x+6\sqrt{3x^2-x+2}-\text{RootOf}\left(\_Z^2-3\right)\right)}{243}$
default	$-\frac{521(-1+6x)}{414\sqrt{3x^2-x+2}}+\frac{557}{18\sqrt{3x^2-x+2}}+\frac{32x^4}{9\sqrt{3x^2-x+2}}+\frac{380x^3}{27\sqrt{3x^2-x+2}}+\frac{2018x^2}{81\sqrt{3x^2-x+2}}+\frac{353x}{81\sqrt{3x^2-x+2}}-\frac{353\sqrt{3}\,\operatorname{arcsinh}\Bigl(6}{243}$

```
[In] int((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2-x+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/1863*(3312*x^4+13110*x^3+23207*x^2-2974*x+29997)/(3*x^2-x+2)^(1/2)-353/24
3*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.94

$$\int \frac{(1+2x)^3\,(1+3x+4x^2)}{(2-x+3x^2)^{3/2}}\,dx = \frac{8119\,\sqrt{3}(3\,x^2-x+2)\log\,(4\,\sqrt{3}\sqrt{3\,x^2-x+2}(6\,x-1)-72\,x^2+24\,x-2}{11178\,(3\,x}$$

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2-x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/11178*(8119*sqrt(3)*(3*x^2 - x + 2)*log(4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*
x - 1) - 72*x^2 + 24*x - 25) + 12*(3312*x^4 + 13110*x^3 + 23207*x^2 - 2974*
x + 29997)*sqrt(3*x^2 - x + 2))/(3*x^2 - x + 2)
```

Sympy [F]

$$\int \frac{(1+2x)^3\,(1+3x+4x^2)}{(2-x+3x^2)^{3/2}}\,dx = \int \frac{(2x+1)^3\cdot(4x^2+3x+1)}{(3x^2-x+2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((1+2*x)**3*(4*x**2+3*x+1)/(3*x**2-x+2)**(3/2),x)
```

```
[Out] Integral((2*x + 1)**3*(4*x**2 + 3*x + 1)/(3*x**2 - x + 2)**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.94

$$\begin{aligned}\int \frac{(1+2x)^3\,(1+3x+4x^2)}{(2-x+3x^2)^{3/2}}\,dx &= \frac{32\,x^4}{9\,\sqrt{3\,x^2-x+2}} + \frac{380\,x^3}{27\,\sqrt{3\,x^2-x+2}} \\ &+ \frac{2018\,x^2}{81\,\sqrt{3\,x^2-x+2}} - \frac{353}{243}\,\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{23}\,\sqrt{23}(6\,x-1)\right) - \frac{5948\,x}{1863\,\sqrt{3\,x^2-x+2}} \\ &+ \frac{2222}{69\,\sqrt{3\,x^2-x+2}}\end{aligned}$$

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2-x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 32/9*x^4/sqrt(3*x^2 - x + 2) + 380/27*x^3/sqrt(3*x^2 - x + 2) + 2018/81*x^2
/sqrt(3*x^2 - x + 2) - 353/243*sqrt(3)*arcsinh(1/23*sqrt(23)*(6*x - 1)) - 5
948/1863*x/sqrt(3*x^2 - x + 2) + 2222/69/sqrt(3*x^2 - x + 2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.65

$$\int \frac{(1+2x)^3\,(1+3x+4x^2)}{(2-x+3x^2)^{3/2}}\,dx = \frac{353}{243}\,\sqrt{3}\log\left(-2\,\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2-x+2}\right)+1\right) + \frac{2\,((23\,(6\,(24\,x+95)x+1009)x-2974)x+29997)}{1863\,\sqrt{3\,x^2-x+2}}$$

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2-x+2)^(3/2),x, algorithm="giac")
[Out] 353/243*sqrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) + 1) + 2/1
863*((23*(6*(24*x + 95)*x + 1009)*x - 2974)*x + 29997)/sqrt(3*x^2 - x + 2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+2x)^3\,(1+3x+4x^2)}{(2-x+3x^2)^{3/2}}\,dx = \int \frac{(2\,x+1)^3\,(4\,x^2+3\,x+1)}{(3\,x^2-x+2)^{3/2}}\,dx$$

```
[In] int(((2*x + 1)^3*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(3/2),x)
[Out] int(((2*x + 1)^3*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(3/2), x)
```

3.247 $\int \frac{(1+2x)^2(1+3x+4x^2)}{(2-x+3x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 82

$$\int \frac{(1+2x)^2(1+3x+4x^2)}{(2-x+3x^2)^{3/2}}\,dx=\frac{2(1249-2273x)}{621\sqrt{2-x+3x^2}}+\frac{112}{27}\sqrt{2-x+3x^2}+\frac{8}{9}x\sqrt{2-x+3x^2}-\frac{64\operatorname{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{9\sqrt{3}}$$

```
[Out] -64/27*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)+2/621*(1249-2273*x)/(3*x^2-x+
2)^(1/2)+112/27*(3*x^2-x+2)^(1/2)+8/9*x*(3*x^2-x+2)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1674, 1675, 654, 633, 221}

$$\int \frac{(1+2x)^2(1+3x+4x^2)}{(2-x+3x^2)^{3/2}}\,dx=-\frac{64\operatorname{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{9\sqrt{3}}+\frac{2(1249-2273x)}{621\sqrt{3x^2-x+2}}+\frac{8}{9}x\sqrt{3x^2-x+2}+\frac{112}{27}\sqrt{3x^2-x+2}$$

```
[In] Int[((1 + 2*x)^2*(1 + 3*x + 4*x^2))/(2 - x + 3*x^2)^(3/2),x]
```

```
[Out] (2*(1249 - 2273*x))/(621*Sqrt[2 - x + 3*x^2]) + (112*Sqrt[2 - x + 3*x^2])/2
7 + (8*x*Sqrt[2 - x + 3*x^2])/9 - (64*ArcSinh[(1 - 6*x)/Sqrt[23]])/(9*Sqrt[
3])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2(1249-2273x)}{621\sqrt{2-x+3x^2}}+\frac{2}{23}\int\frac{\frac{2116}{27}+\frac{1150x}{9}+\frac{184x^2}{3}}{\sqrt{2-x+3x^2}}\,dx\\ &= \frac{2(1249-2273x)}{621\sqrt{2-x+3x^2}}+\frac{8}{9}x\sqrt{2-x+3x^2}+\frac{1}{69}\int\frac{\frac{3128}{9}+\frac{2576x}{3}}{\sqrt{2-x+3x^2}}\,dx\end{aligned}$$

$$= \frac{2(1249-2273x)}{621\sqrt{2-x+3x^2}}+\frac{112}{27}\sqrt{2-x+3x^2}+\frac{8}{9}x\sqrt{2-x+3x^2}+\frac{64}{9}\int\frac{1}{\sqrt{2-x+3x^2}}\,dx$$

$$= \frac{2(1249-2273x)}{621\sqrt{2-x+3x^2}}+\frac{112}{27}\sqrt{2-x+3x^2}+\frac{8}{9}x\sqrt{2-x+3x^2} + \frac{64\text{Subst}\left(\int\frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx,x,-1+6x\right)}{9\sqrt{69}}$$

$$= \frac{2(1249-2273x)}{621\sqrt{2-x+3x^2}}+\frac{112}{27}\sqrt{2-x+3x^2}+\frac{8}{9}x\sqrt{2-x+3x^2}-\frac{64\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{9\sqrt{3}}$$

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.79

$$\int\frac{(1+2x)^2\left(1+3x+4x^2\right)}{\left(2-x+3x^2\right)^{3/2}}\,dx = \frac{2(1275-1003x+1196x^2+276x^3)}{207\sqrt{2-x+3x^2}} - \frac{64\log\left(1-6x+2\sqrt{6-3x+9x^2}\right)}{9\sqrt{3}}$$

```
[In] Integrate[((1 + 2*x)^2*(1 + 3*x + 4*x^2))/(2 - x + 3*x^2)^(3/2),x]
```

```
[Out] (2*(1275 - 1003*x + 1196*x^2 + 276*x^3))/(207*Sqrt[2 - x + 3*x^2]) - (64*Lo
g[1 - 6*x + 2*Sqrt[6 - 3*x + 9*x^2]])/(9*Sqrt[3])
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.55

method	result	size
risch	$\frac{\frac{8}{3}x^3+\frac{104}{9}x^2-\frac{2006}{207}x+\frac{850}{69}}{\sqrt{3x^2-x+2}}+\frac{64\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{27}$	45
trager	$\frac{\frac{8}{3}x^3+\frac{104}{9}x^2-\frac{2006}{207}x+\frac{850}{69}}{\sqrt{3x^2-x+2}}-\frac{64\,\text{RootOf}\left(\_Z^2-3\right)\ln\left(-6\,\text{RootOf}\left(\_Z^2-3\right)x+\text{RootOf}\left(\_Z^2-3\right)+6\sqrt{3x^2-x+2}\right)}{27}$	70
default	$-\frac{89(-1+6x)}{207\sqrt{3x^2-x+2}}+\frac{107}{9\sqrt{3x^2-x+2}}+\frac{8x^3}{3\sqrt{3x^2-x+2}}+\frac{104x^2}{9\sqrt{3x^2-x+2}}-\frac{64x}{9\sqrt{3x^2-x+2}}+\frac{64\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{27}$	98

```
[In] int((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/207*(276*x^3+1196*x^2-1003*x+1275)/(3*x^2-x+2)^(1/2)+64/27*3^(1/2)*arcsin
h(6/23*23^(1/2)*(x-1/6))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.12

$$\int \frac{(1+2x)^2\left(1+3x+4x^2\right)}{\left(2-x+3x^2\right)^{3/2}}\,dx = \frac{2\left(368\sqrt{3}(3\,x^2-x+2)\log\left(-4\sqrt{3}\sqrt{3\,x^2-x+2}(6\,x-1)-72\,x^2+24\,x-\right.\right.}{621\left(3\,x^2-x+\right.}$$

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/621*(368*sqrt(3)*(3*x^2 - x + 2)*log(-4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x
- 1) - 72*x^2 + 24*x - 25) + 3*(276*x^3 + 1196*x^2 - 1003*x + 1275)*sqrt(3*
x^2 - x + 2))/(3*x^2 - x + 2)
```

Sympy [F]

$$\int \frac{(1+2x)^2\left(1+3x+4x^2\right)}{\left(2-x+3x^2\right)^{3/2}}\,dx = \int \frac{\left(2x+1\right)^2\cdot\left(4x^2+3x+1\right)}{\left(3x^2-x+2\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((1+2*x)**2*(4*x**2+3*x+1)/(3*x**2-x+2)**(3/2),x)
```

```
[Out] Integral((2*x + 1)**2*(4*x**2 + 3*x + 1)/(3*x**2 - x + 2)**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.98

$$\begin{aligned}\int \frac{(1+2x)^2\left(1+3x+4x^2\right)}{\left(2-x+3x^2\right)^{3/2}}\,dx &= \frac{8\,x^3}{3\sqrt{3\,x^2-x+2}}+\frac{104\,x^2}{9\sqrt{3\,x^2-x+2}} \\ &\quad +\frac{64}{27}\sqrt{3}\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}(6\,x-1)\right)-\frac{2006\,x}{207\sqrt{3\,x^2-x+2}}+\frac{850}{69\sqrt{3\,x^2-x+2}}\end{aligned}$$

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 8/3*x^3/sqrt(3*x^2 - x + 2) + 104/9*x^2/sqrt(3*x^2 - x + 2) + 64/27*sqrt(3)
*arcsinh(1/23*sqrt(23)*(6*x - 1)) - 2006/207*x/sqrt(3*x^2 - x + 2) + 850/69
/sqrt(3*x^2 - x + 2)
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.76

$$\int \frac{(1+2x)^2(1+3x+4x^2)}{(2-x+3x^2)^{3/2}}\,dx = -\frac{64}{27}\sqrt{3}\log\left(-2\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2-x+2}\right)+1\right) + \frac{2\,((92\,(3\,x+13)x-1003)x+1275)}{207\,\sqrt{3\,x^2-x+2}}$$

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(3/2),x, algorithm="giac")
[Out] -64/27*sqrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) + 1) + 2/20
7*((92*(3*x + 13)*x - 1003)*x + 1275)/sqrt(3*x^2 - x + 2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+2x)^2(1+3x+4x^2)}{(2-x+3x^2)^{3/2}}\,dx = \int \frac{(2\,x+1)^2\,(4\,x^2+3\,x+1)}{(3\,x^2-x+2)^{3/2}}\,dx$$

```
[In] int(((2*x + 1)^2*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(3/2),x)
[Out] int(((2*x + 1)^2*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(3/2), x)
```

3.248 $\int \frac{(1+2x)(1+3x+4x^2)}{(2-x+3x^2)^{3/2}} \, dx$

Optimal result

Integrand size = 30, antiderivative size = 63

$$\int \frac{(1+2x)(1+3x+4x^2)}{(2-x+3x^2)^{3/2}} \, dx = -\frac{2(73+367x)}{207\sqrt{2-x+3x^2}} + \frac{8}{9}\sqrt{2-x+3x^2} - \frac{14\operatorname{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{3\sqrt{3}}$$

```
[Out] -14/9*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)-2/207*(73+367*x)/(3*x^2-x+2)^(
1/2)+8/9*(3*x^2-x+2)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1674, 654, 633, 221}

$$\int \frac{(1+2x)(1+3x+4x^2)}{(2-x+3x^2)^{3/2}} \, dx = -\frac{14\operatorname{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{3\sqrt{3}} - \frac{2(367x+73)}{207\sqrt{3x^2-x+2}} + \frac{8}{9}\sqrt{3x^2-x+2}$$

```
[In] Int[((1 + 2*x)*(1 + 3*x + 4*x^2))/(2 - x + 3*x^2)^(3/2),x]
```

```
[Out] (-2*(73 + 367*x))/(207*Sqrt[2 - x + 3*x^2]) + (8*Sqrt[2 - x + 3*x^2])/9 - (
14*ArcSinh[(1 - 6*x)/Sqrt[23]])/(3*Sqrt[3])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2(73+367x)}{207\sqrt{2-x+3x^2}}+\frac{2}{23}\int\frac{\frac{437}{9}+\frac{92x}{3}}{\sqrt{2-x+3x^2}}\,dx\\
&= -\frac{2(73+367x)}{207\sqrt{2-x+3x^2}}+\frac{8}{9}\sqrt{2-x+3x^2}+\frac{14}{3}\int\frac{1}{\sqrt{2-x+3x^2}}\,dx\\
&= -\frac{2(73+367x)}{207\sqrt{2-x+3x^2}}+\frac{8}{9}\sqrt{2-x+3x^2}+\frac{14\text{Subst}\left(\int\frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx,x,-1+6x\right)}{3\sqrt{69}}\\
&= -\frac{2(73+367x)}{207\sqrt{2-x+3x^2}}+\frac{8}{9}\sqrt{2-x+3x^2}-\frac{14\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{3\sqrt{3}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95

$$\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\left(2-x+3x^2\right)^{3/2}}\,dx=\frac{2\left(37-153x+92x^2\right)}{69\sqrt{2-x+3x^2}}-\frac{14\log\left(1-6x+2\sqrt{6-3x+9x^2}\right)}{3\sqrt{3}}$$

```
[In] Integrate[((1 + 2*x)*(1 + 3*x + 4*x^2))/(2 - x + 3*x^2)^(3/2),x]
[Out] (2*(37 - 153*x + 92*x^2))/(69*Sqrt[2 - x + 3*x^2]) - (14*Log[1 - 6*x + 2*Sq
rt[6 - 3*x + 9*x^2]])/(3*Sqrt[3])
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.63

method	result	size
risch	$\frac{\frac{8}{3}x^2-\frac{102}{23}x+\frac{74}{69}}{\sqrt{3x^2-x+2}}+\frac{14\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{9}$	40
trager	$\frac{\frac{8}{3}x^2-\frac{102}{23}x+\frac{74}{69}}{\sqrt{3x^2-x+2}}+\frac{14\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(6\,\mathrm{RootOf}\left(\_Z^2-3\right)x+6\sqrt{3x^2-x+2}-\mathrm{RootOf}\left(\_Z^2-3\right)\right)}{9}$	67
default	$\frac{-\frac{8}{207}+\frac{16x}{69}}{\sqrt{3x^2-x+2}}+\frac{10}{9\sqrt{3x^2-x+2}}+\frac{8x^2}{3\sqrt{3x^2-x+2}}-\frac{14x}{3\sqrt{3x^2-x+2}}+\frac{14\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{9}$	81

```
[In] int((1+2*x)*(4*x^2+3*x+1)/(3*x^2-x+2)^(3/2),x,method=_RETURNVERBOSE)
[Out] 2/69*(92*x^2-153*x+37)/(3*x^2-x+2)^(1/2)+14/9*3^(1/2)*arcsinh(6/23*23^(1/2)
*(x-1/6))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.38

$$\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\left(2-x+3x^2\right)^{3/2}}\,dx=\frac{161\sqrt{3}\left(3\,x^2-x+2\right)\log\left(-4\sqrt{3}\sqrt{3\,x^2-x+2}(6\,x-1)-72\,x^2+24\,x-25\right.}{207\left(3\,x^2-x+2\right)}$$

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2-x+2)^(3/2),x, algorithm="fricas")
[Out] 1/207*(161*sqrt(3)*(3*x^2 - x + 2)*log(-4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x
- 1) - 72*x^2 + 24*x - 25) + 6*(92*x^2 - 153*x + 37)*sqrt(3*x^2 - x + 2))/(
3*x^2 - x + 2)
```

Sympy [F]

$$\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\left(2-x+3x^2\right)^{3/2}}\,dx=\int \frac{(2x+1)\left(4x^2+3x+1\right)}{\left(3x^2-x+2\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((1+2*x)*(4*x**2+3*x+1)/(3*x**2-x+2)**(3/2),x)
```

```
[Out] Integral((2*x + 1)*(4*x**2 + 3*x + 1)/(3*x**2 - x + 2)**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\left(2-x+3x^2\right)^{3/2}}\,dx=\frac{8\,x^2}{3\,\sqrt{3\,x^2-x+2}}+\frac{14}{9}\,\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{23}\,\sqrt{23}(6\,x-1)\right)-\frac{102\,x}{23\,\sqrt{3\,x^2-x+2}}+\frac{74}{69\,\sqrt{3\,x^2-x+2}}$$

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2-x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 8/3*x^2/sqrt(3*x^2 - x + 2) + 14/9*sqrt(3)*arcsinh(1/23*sqrt(23)*(6*x - 1))
 - 102/23*x/sqrt(3*x^2 - x + 2) + 74/69/sqrt(3*x^2 - x + 2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

$$\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\left(2-x+3x^2\right)^{3/2}}\,dx=-\frac{14}{9}\,\sqrt{3}\log\left(-2\,\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2-x+2}\right)+1\right)+\frac{2\,((92\,x-153)x+37)}{69\,\sqrt{3\,x^2-x+2}}$$

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2-x+2)^(3/2),x, algorithm="giac")
```

```
[Out] -14/9*sqrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) + 1) + 2/69*
((92*x - 153)*x + 37)/sqrt(3*x^2 - x + 2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\left(2-x+3x^2\right)^{3/2}}\,dx=\int \frac{(2\,x+1)\,\left(4\,x^2+3\,x+1\right)}{{\left(3\,x^2-x+2\right)}^{3/2}} \,d x$$

```
[In] int(((2*x + 1)*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(3/2),x)
[Out] int(((2*x + 1)*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(3/2), x)
```

3.249 $\int \frac{1+3x+4x^2}{(1+2x)\left(2-x+3x^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 62

$$\int \frac{1+3x+4x^2}{(1+2x)\left(2-x+3x^2\right)^{3/2}}\,dx=-\frac{2(101-77x)}{299\sqrt{2-x+3x^2}}-\frac{2\operatorname{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)}{13\sqrt{13}}$$

```
[Out] -2/169*arctanh(1/26*(9-8*x)*13^(1/2)/(3*x^2-x+2)^(1/2))*13^(1/2)-2/299*(101
-77*x)/(3*x^2-x+2)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1660, 12, 738, 212}

$$\int \frac{1+3x+4x^2}{(1+2x)\left(2-x+3x^2\right)^{3/2}}\,dx=-\frac{2\operatorname{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{13\sqrt{13}}-\frac{2(101-77x)}{299\sqrt{3x^2-x+2}}$$

```
[In] Int[(1 + 3*x + 4*x^2)/((1 + 2*x)*(2 - x + 3*x^2)^(3/2)),x]
```

```
[Out] (-2*(101 - 77*x))/(299*Sqrt[2 - x + 3*x^2]) - (2*ArcTanh[(9 - 8*x)/(2*Sqrt[
13]*Sqrt[2 - x + 3*x^2])])/(13*Sqrt[13])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1660

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^(p + 1)/((p
+ 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2(101-77x)}{299\sqrt{2-x+3x^2}}+\frac{2}{23}\int\frac{23}{13(1+2x)\sqrt{2-x+3x^2}}\,dx\\
&= -\frac{2(101-77x)}{299\sqrt{2-x+3x^2}}+\frac{2}{13}\int\frac{1}{(1+2x)\sqrt{2-x+3x^2}}\,dx\\
&= -\frac{2(101-77x)}{299\sqrt{2-x+3x^2}}-\frac{4}{13}\text{Subst}\left(\int\frac{1}{52-x^2}\,dx,x,\frac{9-8x}{\sqrt{2-x+3x^2}}\right)\\
&= -\frac{2(101-77x)}{299\sqrt{2-x+3x^2}}-\frac{2\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)}{13\sqrt{13}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.13

$$\int \frac{1+3x+4x^2}{(1+2x)\left(2-x+3x^2\right)^{3/2}}\,dx=\frac{2(-101+77x)}{299\sqrt{2-x+3x^2}}+\frac{4\text{arctanh}\left(\frac{\sqrt{3}+2\sqrt{3}x-2\sqrt{2-x+3x^2}}{\sqrt{13}}\right)}{13\sqrt{13}}$$

```
[In] Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)*(2 - x + 3*x^2)^(3/2)),x]
```

```
[Out] (2*(-101 + 77*x))/(299*Sqrt[2 - x + 3*x^2]) + (4*ArcTanh[(Sqrt[3] + 2*Sqrt[
3]*x - 2*Sqrt[2 - x + 3*x^2])/Sqrt[13]])/(13*Sqrt[13])
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.82

method	result	siz
risch	$\frac{-\frac{202}{299}+\frac{154x}{299}}{\sqrt{3x^2-x+2}}-\frac{2\sqrt{13}\,\text{arctanh}\left(\frac{2\left(\frac{9}{2}-4x\right)\sqrt{13}}{13\sqrt{12\left(x+\frac{1}{2}\right)^2-16x+5}}\right)}{169}$	51
trager	$\frac{-\frac{202}{299}+\frac{154x}{299}}{\sqrt{3x^2-x+2}}-\frac{2\,\text{RootOf}\left(\_Z^2-13\right)\ln\left(-\frac{8\,\text{RootOf}\left(\_Z^2-13\right)x-9\,\text{RootOf}\left(\_Z^2-13\right)-26\sqrt{3x^2-x+2}}{1+2x}\right)}{169}$	71
default	$\frac{-\frac{5}{69}+\frac{10x}{23}}{\sqrt{3x^2-x+2}}-\frac{2}{3\sqrt{3x^2-x+2}}+\frac{1}{13\sqrt{3\left(x+\frac{1}{2}\right)^2-4x+\frac{5}{4}}}+\frac{-\frac{4}{299}+\frac{24x}{299}}{\sqrt{3\left(x+\frac{1}{2}\right)^2-4x+\frac{5}{4}}}-\frac{2\sqrt{13}\,\text{arctanh}\left(\frac{2\left(\frac{9}{2}-4x\right)\sqrt{13}}{13\sqrt{12\left(x+\frac{1}{2}\right)^2-16x+5}}\right)}{169}$	10

```
[In] int((4*x^2+3*x+1)/(1+2*x)/(3*x^2-x+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/299*(-101+77*x)/(3*x^2-x+2)^(1/2)-2/169*13^(1/2)*arctanh(2/13*(9/2-4*x)*1
3^(1/2)/(12*(x+1/2)^2-16*x+5)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.55

$$\int \frac{1+3x+4x^2}{(1+2x)\left(2-x+3x^2\right)^{3/2}}\,dx=\frac{23\sqrt{13}(3x^2-x+2)\log\left(-\frac{4\sqrt{13}\sqrt{3x^2-x+2}(8x-9)+220x^2-196x+185}{4x^2+4x+1}\right)+26\sqrt{3}}{3887\left(3x^2-x+2\right)}$$

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2-x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3887*(23*sqrt(13)*(3*x^2 - x + 2)*log(-(4*sqrt(13)*sqrt(3*x^2 - x + 2)*(8
*x - 9) + 220*x^2 - 196*x + 185)/(4*x^2 + 4*x + 1)) + 26*sqrt(3*x^2 - x + 2
)*(77*x - 101))/(3*x^2 - x + 2)
```

Sympy [F]

$$\int \frac{1+3x+4x^2}{(1+2x)\left(2-x+3x^2\right)^{3/2}}\,dx=\int \frac{4x^2+3x+1}{(2x+1)\left(3x^2-x+2\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)/(3*x**2-x+2)**(3/2),x)
```

```
[Out] Integral((4*x**2 + 3*x + 1)/((2*x + 1)*(3*x**2 - x + 2)**(3/2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.03

$$\int \frac{1+3x+4x^2}{(1+2x)\left(2-x+3x^2\right)^{3/2}}\,dx=\frac{2}{169}\sqrt{13}\,\operatorname{arsinh}\left(\frac{8\sqrt{23}x}{23\,|2\,x+1|}-\frac{9\sqrt{23}}{23\,|2\,x+1|}\right)+\frac{154\,x}{299\sqrt{3\,x^2-x+2}}-\frac{202}{299\sqrt{3\,x^2-x+2}}$$

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2-x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/169*sqrt(13)*arcsinh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqrt(23)/abs(2*x
 + 1)) + 154/299*x/sqrt(3*x^2 - x + 2) - 202/299/sqrt(3*x^2 - x + 2)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.47

$$\int \frac{1+3x+4x^2}{(1+2x)\left(2-x+3x^2\right)^{3/2}}\,dx=\frac{2}{169}\sqrt{13}\log\left(-\frac{\left|-4\sqrt{3}x-2\sqrt{13}-2\sqrt{3}+4\sqrt{3\,x^2-x+2}\right|}{2\left(2\sqrt{3}x-\sqrt{13}+\sqrt{3}-2\sqrt{3\,x^2-x+2}\right)}\right)+\frac{2\,(77\,x-101)}{299\sqrt{3\,x^2-x+2}}$$

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2-x+2)^(3/2),x, algorithm="giac")
```

```
[Out] 2/169*sqrt(13)*log(-1/2*abs(-4*sqrt(3)*x - 2*sqrt(13) - 2*sqrt(3) + 4*sqrt(
3*x^2 - x + 2))/(2*sqrt(3)*x - sqrt(13) + sqrt(3) - 2*sqrt(3*x^2 - x + 2)))
 + 2/299*(77*x - 101)/sqrt(3*x^2 - x + 2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+3x+4x^2}{(1+2x)\left(2-x+3x^2\right)^{3/2}}\,dx=\int \frac{4\,x^2+3\,x+1}{\left(2\,x+1\right)\,{\left(3\,x^2-x+2\right)}^{3/2}} \,d x$$

```
[In] int((3*x + 4*x^2 + 1)/((2*x + 1)*(3*x^2 - x + 2)^(3/2)),x)
[Out] int((3*x + 4*x^2 + 1)/((2*x + 1)*(3*x^2 - x + 2)^(3/2)), x)
```

3.250 $\int \frac{1+3x+4x^2}{(1+2x)^2\left(2-x+3x^2\right)^{3/2}} \, dx$

Optimal result

Integrand size = 32, antiderivative size = 87

$$\int \frac{1+3x+4x^2}{(1+2x)^2\left(2-x+3x^2\right)^{3/2}} \, dx=-\frac{2(197-837x)}{3887\sqrt{2-x+3x^2}}-\frac{4\sqrt{2-x+3x^2}}{169(1+2x)}+\frac{2\text{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)}{169\sqrt{13}}$$

```
[Out] 2/2197*arctanh(1/26*(9-8*x)*13^(1/2)/(3*x^2-x+2)^(1/2))*13^(1/2)-2/3887*(19
7-837*x)/(3*x^2-x+2)^(1/2)-4/169*(3*x^2-x+2)^(1/2)/(1+2*x)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1660, 820, 738, 212}

$$\int \frac{1+3x+4x^2}{(1+2x)^2\left(2-x+3x^2\right)^{3/2}} \, dx=\frac{2\text{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{169\sqrt{13}}-\frac{2(197-837x)}{3887\sqrt{3x^2-x+2}}-\frac{4\sqrt{3x^2-x+2}}{169(2x+1)}$$

```
[In] Int[(1 + 3*x + 4*x^2)/((1 + 2*x)^2*(2 - x + 3*x^2)^(3/2)),x]
```

```
[Out] (-2*(197 - 837*x))/(3887*Sqrt[2 - x + 3*x^2]) - (4*Sqrt[2 - x + 3*x^2])/(16
9*(1 + 2*x)) + (2*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/(169
*Sqrt[13])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 1660

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^(p + 1)/((p
+ 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2(197-837x)}{3887\sqrt{2-x+3x^2}}+\frac{2}{23}\int\frac{\frac{184}{169}-\frac{230x}{169}}{(1+2x)^2\sqrt{2-x+3x^2}}\,dx\\
&= -\frac{2(197-837x)}{3887\sqrt{2-x+3x^2}}-\frac{4\sqrt{2-x+3x^2}}{169(1+2x)}-\frac{2}{169}\int\frac{1}{(1+2x)\sqrt{2-x+3x^2}}\,dx\\
&= -\frac{2(197-837x)}{3887\sqrt{2-x+3x^2}}-\frac{4\sqrt{2-x+3x^2}}{169(1+2x)}+\frac{4}{169}\text{Subst}\left(\int\frac{1}{52-x^2}\,dx,x,\frac{9-8x}{\sqrt{2-x+3x^2}}\right)
\end{aligned}$$

$$= -\frac{2(197-837x)}{3887\sqrt{2-x+3x^2}} - \frac{4\sqrt{2-x+3x^2}}{169(1+2x)} + \frac{2\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)}{169\sqrt{13}}$$

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.03

$$\int \frac{1+3x+4x^2}{(1+2x)^2\left(2-x+3x^2\right)^{3/2}}\,dx = \frac{2\sqrt{2-x+3x^2}(-289+489x+1536x^2)}{3887\left(2+3x+x^2+6x^3\right)} - \frac{4\text{arctanh}\left(\frac{\sqrt{3}+2\sqrt{3}x-2\sqrt{2-x+3x^2}}{\sqrt{13}}\right)}{169\sqrt{13}}$$

```
[In] Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)^2*(2 - x + 3*x^2)^(3/2)),x]
```

```
[Out] (2*Sqrt[2 - x + 3*x^2]*(-289 + 489*x + 1536*x^2))/(3887*(2 + 3*x + x^2 + 6*
x^3)) - (4*ArcTanh[(Sqrt[3] + 2*Sqrt[3]*x - 2*Sqrt[2 - x + 3*x^2])/Sqrt[13]
])/(169*Sqrt[13])
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.72

method	result
risch	$\frac{\frac{3072}{3887}x^2+\frac{978}{3887}x-\frac{578}{3887}}{(1+2x)\sqrt{3x^2-x+2}}+\frac{2\sqrt{13}\,\operatorname{arctanh}\left(\frac{2\left(\frac{9}{2}-4x\right)\sqrt{13}}{13\sqrt{12\left(x+\frac{1}{2}\right)^2-16x+5}}\right)}{2197}$
trager	$\frac{2\left(1536x^2+489x-289\right)\sqrt{3x^2-x+2}}{3887(6x^3+x^2+3x+2)}+\frac{2\operatorname{RootOf}\left(\_Z^2-13\right)\ln\left(-\frac{8\operatorname{RootOf}\left(\_Z^2-13\right)x-9\operatorname{RootOf}\left(\_Z^2-13\right)-26\sqrt{3x^2-x+2}}{1+2x}\right)}{2197}$
default	$\frac{-\frac{2}{23}+\frac{12x}{23}}{\sqrt{3x^2-x+2}}-\frac{1}{26\left(x+\frac{1}{2}\right)\sqrt{3\left(x+\frac{1}{2}\right)^2-4x+\frac{5}{4}}}-\frac{1}{169\sqrt{3\left(x+\frac{1}{2}\right)^2-4x+\frac{5}{4}}}-\frac{82(-1+6x)}{3887\sqrt{3\left(x+\frac{1}{2}\right)^2-4x+\frac{5}{4}}}+\frac{2\sqrt{13}\,\operatorname{arctanh}\left(\frac{2\left(\frac{9}{2}-4\right.}{13\sqrt{12\left(x+\right.}}\right.}{2197}$

```
[In] int((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2-x+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3887*(1536*x^2+489*x-289)/(1+2*x)/(3*x^2-x+2)^(1/2)+2/2197*13^(1/2)*arcta
nh(2/13*(9/2-4*x)*13^(1/2)/(12*(x+1/2)^2-16*x+5)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.22

$$\int \frac{1+3x+4x^2}{(1+2x)^2\,(2-x+3x^2)^{3/2}}\,dx = \frac{23\,\sqrt{13}(6\,x^3+x^2+3\,x+2)\log\left(\frac{4\,\sqrt{13}\sqrt{3\,x^2-x+2}(8\,x-9)-220\,x^2+196\,x-185}{4\,x^2+4\,x+1}\right)+}{50531\,(6\,x^3+x^2+3\,x+2)}$$

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2-x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/50531*(23*sqrt(13)*(6*x^3 + x^2 + 3*x + 2)*log((4*sqrt(13)*sqrt(3*x^2 - x
 + 2)*(8*x - 9) - 220*x^2 + 196*x - 185)/(4*x^2 + 4*x + 1)) + 26*(1536*x^2
+ 489*x - 289)*sqrt(3*x^2 - x + 2))/(6*x^3 + x^2 + 3*x + 2)
```

Sympy [F]

$$\int \frac{1+3x+4x^2}{(1+2x)^2\,(2-x+3x^2)^{3/2}}\,dx = \int \frac{4x^2+3x+1}{\left(2x+1\right)^2\left(3x^2-x+2\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)**2/(3*x**2-x+2)**(3/2),x)
```

```
[Out] Integral((4*x**2 + 3*x + 1)/((2*x + 1)**2*(3*x**2 - x + 2)**(3/2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.10

$$\int \frac{1+3x+4x^2}{(1+2x)^2\,(2-x+3x^2)^{3/2}}\,dx = -\frac{2}{2197}\,\sqrt{13}\,\mathrm{arsinh}\left(\frac{8\,\sqrt{23}x}{23\,|2\,x+1|}-\frac{9\,\sqrt{23}}{23\,|2\,x+1|}\right) + \frac{1536\,x}{3887\,\sqrt{3\,x^2-x+2}} - \frac{279}{3887\,\sqrt{3\,x^2-x+2}} - \frac{1}{13\,\left(2\,\sqrt{3\,x^2-x+2}x+\sqrt{3\,x^2-x+2}\right)}$$

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2-x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] -2/2197*sqrt(13)*arcsinh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqrt(23)/abs(2
*x + 1)) + 1536/3887*x/sqrt(3*x^2 - x + 2) - 279/3887/sqrt(3*x^2 - x + 2) -
 1/13/(2*sqrt(3*x^2 - x + 2)*x + sqrt(3*x^2 - x + 2))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. 2(69) = 138.

Time = 0.30 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.93

$$\int \frac{1+3x+4x^2}{(1+2x)^2\left(2-x+3x^2\right)^{3/2}}\,dx = -\frac{2}{50531}\sqrt{13}\left(256\sqrt{13}\sqrt{3}+23\log\left(\sqrt{13}\sqrt{3}-4\right)\right)\operatorname{sgn}\left(\frac{1}{2x+1}\right) - \frac{2\left(\frac{\frac{1047}{\operatorname{sgn}\left(\frac{1}{2x+1}\right)}+\frac{299}{(2x+1)\operatorname{sgn}\left(\frac{1}{2x+1}\right)}}{2x+1}-\frac{768}{\operatorname{sgn}\left(\frac{1}{2x+1}\right)}\right)}{3887\sqrt{-\frac{8}{2x+1}+\frac{13}{(2x+1)^2}+3}} + \frac{2\sqrt{13}\log\left(\sqrt{13}\left(\sqrt{-\frac{8}{2x+1}+\frac{13}{(2x+1)^2}+3}+\frac{\sqrt{13}}{2x+1}\right)-4\right)}{2197\operatorname{sgn}\left(\frac{1}{2x+1}\right)}$$

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2-x+2)^(3/2),x, algorithm="giac")
```

```
[Out] -2/50531*sqrt(13)*(256*sqrt(13)*sqrt(3) + 23*log(sqrt(13)*sqrt(3) - 4))*sgn
(1/(2*x + 1)) - 2/3887*((1047/sgn(1/(2*x + 1)) + 299/((2*x + 1)*sgn(1/(2*x
 + 1))))/(2*x + 1) - 768/sgn(1/(2*x + 1)))/sqrt(-8/(2*x + 1) + 13/(2*x + 1)^
2 + 3) + 2/2197*sqrt(13)*log(sqrt(13)*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 +
 3) + sqrt(13)/(2*x + 1)) - 4)/sgn(1/(2*x + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+3x+4x^2}{(1+2x)^2\left(2-x+3x^2\right)^{3/2}}\,dx = \int \frac{4x^2+3x+1}{(2x+1)^2\left(3x^2-x+2\right)^{3/2}}\,dx$$

```
[In] int((3*x + 4*x^2 + 1)/((2*x + 1)^2*(3*x^2 - x + 2)^(3/2)),x)
```

```
[Out] int((3*x + 4*x^2 + 1)/((2*x + 1)^2*(3*x^2 - x + 2)^(3/2)), x)
```

3.251 $\int \frac{1+3x+4x^2}{(1+2x)^3\left(2-x+3x^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 112

$$\int \frac{1+3x+4x^2}{(1+2x)^3\left(2-x+3x^2\right)^{3/2}}\,dx=\frac{2(2363+3693x)}{50531\sqrt{2-x+3x^2}}-\frac{2\sqrt{2-x+3x^2}}{169(1+2x)^2}-\frac{4\sqrt{2-x+3x^2}}{2197(1+2x)}-\frac{487\text{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)}{2197\sqrt{13}}$$

```
[Out] -487/28561*arctanh(1/26*(9-8*x)*13^(1/2)/(3*x^2-x+2)^(1/2))*13^(1/2)+2/5053
1*(2363+3693*x)/(3*x^2-x+2)^(1/2)-2/169*(3*x^2-x+2)^(1/2)/(1+2*x)^2-4/2197*
(3*x^2-x+2)^(1/2)/(1+2*x)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1660, 1664, 820, 738, 212}

$$\int \frac{1+3x+4x^2}{(1+2x)^3\left(2-x+3x^2\right)^{3/2}}\,dx=-\frac{487\text{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{2197\sqrt{13}}+\frac{2(3693x+2363)}{50531\sqrt{3x^2-x+2}}-\frac{4\sqrt{3x^2-x+2}}{2197(2x+1)}-\frac{2\sqrt{3x^2-x+2}}{169(2x+1)^2}$$

```
[In] Int[(1 + 3*x + 4*x^2)/((1 + 2*x)^3*(2 - x + 3*x^2)^(3/2)),x]
```

```
[Out] (2*(2363 + 3693*x))/(50531*Sqrt[2 - x + 3*x^2]) - (2*Sqrt[2 - x + 3*x^2])/(
169*(1 + 2*x)^2) - (4*Sqrt[2 - x + 3*x^2])/(2197*(1 + 2*x)) - (487*ArcTanh[
(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/(2197*Sqrt[13])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 1660

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^(p + 1)/((p
+ 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2(2363+3693x)}{50531\sqrt{2-x+3x^2}}+\frac{2}{23}\int\frac{\frac{8349}{2197}+\frac{20838x}{2197}+\frac{23828x^2}{2197}}{(1+2x)^3\sqrt{2-x+3x^2}}\,dx\\
&= \frac{2(2363+3693x)}{50531\sqrt{2-x+3x^2}}-\frac{2\sqrt{2-x+3x^2}}{169(1+2x)^2}-\frac{1}{299}\int\frac{-\frac{11615}{169}-\frac{22034x}{169}}{(1+2x)^2\sqrt{2-x+3x^2}}\,dx\\
&= \frac{2(2363+3693x)}{50531\sqrt{2-x+3x^2}}-\frac{2\sqrt{2-x+3x^2}}{169(1+2x)^2}-\frac{4\sqrt{2-x+3x^2}}{2197(1+2x)}+\frac{487\int\frac{1}{(1+2x)\sqrt{2-x+3x^2}}\,dx}{2197}\\
&= \frac{2(2363+3693x)}{50531\sqrt{2-x+3x^2}}-\frac{2\sqrt{2-x+3x^2}}{169(1+2x)^2}\\
&\quad-\frac{4\sqrt{2-x+3x^2}}{2197(1+2x)}-\frac{974\text{Subst}\left(\int\frac{1}{52-x^2}\,dx,x,\frac{9-8x}{\sqrt{2-x+3x^2}}\right)}{2197}\\
&= \frac{2(2363+3693x)}{50531\sqrt{2-x+3x^2}}-\frac{2\sqrt{2-x+3x^2}}{169(1+2x)^2}-\frac{4\sqrt{2-x+3x^2}}{2197(1+2x)}-\frac{487\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)}{2197\sqrt{13}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.78

$$
\begin{aligned}
\int\frac{1+3x+4x^2}{(1+2x)^3\,(2-x+3x^2)^{3/2}}\,dx &= \frac{2(1673+13306x+23281x^2+14496x^3)}{50531(1+2x)^2\sqrt{2-x+3x^2}}\\
&\quad+\frac{974\text{arctanh}\left(\frac{\sqrt{3}+2\sqrt{3}x-2\sqrt{2-x+3x^2}}{\sqrt{13}}\right)}{2197\sqrt{13}}
\end{aligned}
$$

```
[In] Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)^3*(2 - x + 3*x^2)^(3/2)),x]
```

```
[Out] (2*(1673 + 13306*x + 23281*x^2 + 14496*x^3))/(50531*(1 + 2*x)^2*Sqrt[2 - x
+ 3*x^2]) + (974*ArcTanh[(Sqrt[3] + 2*Sqrt[3]*x - 2*Sqrt[2 - x + 3*x^2])/Sq
rt[13]])/(2197*Sqrt[13])
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.61

method	result
risch	$\frac{\frac{28992}{50531}x^3+\frac{46562}{50531}x^2+\frac{26612}{50531}x+\frac{3346}{50531}}{(1+2x)^2\sqrt{3x^2-x+2}}-\frac{487\sqrt{13}\,\operatorname{arctanh}\left(\frac{2\left(\frac{9}{2}-4x\right)\sqrt{13}}{13\sqrt{12\left(x+\frac{1}{2}\right)^2-16x+5}}\right)}{28561}$
trager	$\frac{\frac{28992}{50531}x^3+\frac{46562}{50531}x^2+\frac{26612}{50531}x+\frac{3346}{50531}}{(1+2x)^2\sqrt{3x^2-x+2}}+\frac{487\operatorname{RootOf}\left(\_Z^2-13\right)\ln\left(\frac{8\operatorname{RootOf}\left(\_Z^2-13\right)x+26\sqrt{3x^2-x+2}-9\operatorname{RootOf}\left(\_Z^2-13\right)}{1+2x}\right)}{28561}$
default	$\frac{487}{4394\sqrt{3\left(x+\frac{1}{2}\right)^2-4x+\frac{5}{4}}}+\frac{-\frac{1208}{50531}+\frac{7248x}{50531}}{\sqrt{3\left(x+\frac{1}{2}\right)^2-4x+\frac{5}{4}}}-\frac{487\sqrt{13}\,\operatorname{arctanh}\left(\frac{2\left(\frac{9}{2}-4x\right)\sqrt{13}}{13\sqrt{12\left(x+\frac{1}{2}\right)^2-16x+5}}\right)}{28561}+\frac{3}{338\left(x+\frac{1}{2}\right)\sqrt{3\left(x+\frac{1}{2}\right)^2-4x+\frac{5}{4}}}-$

```
[In] int((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2-x+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/50531*(14496*x^3+23281*x^2+13306*x+1673)/(1+2*x)^2/(3*x^2-x+2)^(1/2)-487/
28561*13^(1/2)*arctanh(2/13*(9/2-4*x)*13^(1/2)/(12*(x+1/2)^2-16*x+5)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.12

$$\int \frac{1+3x+4x^2}{(1+2x)^3\left(2-x+3x^2\right)^{3/2}}\,dx=\frac{11201\sqrt{13}(12\,x^4+8\,x^3+7\,x^2+7\,x+2)\log\left(-\frac{4\sqrt{13}\sqrt{3\,x^2-x+2}(8\,x-9)+220\,x^2}{4\,x^2+4\,x+1}\right.}{1313806\,(12\,x^4+8\,x^3}$$

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2-x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/1313806*(11201*sqrt(13)*(12*x^4 + 8*x^3 + 7*x^2 + 7*x + 2)*log(-(4*sqrt(1
3)*sqrt(3*x^2 - x + 2)*(8*x - 9) + 220*x^2 - 196*x + 185)/(4*x^2 + 4*x + 1)
) + 52*(14496*x^3 + 23281*x^2 + 13306*x + 1673)*sqrt(3*x^2 - x + 2))/(12*x^
4 + 8*x^3 + 7*x^2 + 7*x + 2)
```

Sympy [F]

$$\int \frac{1+3x+4x^2}{(1+2x)^3\left(2-x+3x^2\right)^{3/2}}\,dx=\int \frac{4x^2+3x+1}{\left(2x+1\right)^3\left(3x^2-x+2\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)**3/(3*x**2-x+2)**(3/2),x)
```

```
[Out] Integral((4*x**2 + 3*x + 1)/((2*x + 1)**3*(3*x**2 - x + 2)**(3/2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.29

$$\int \frac{1+3x+4x^2}{(1+2x)^3\left(2-x+3x^2\right)^{3/2}}\,dx = \frac{487}{28561}\sqrt{13}\,\operatorname{arsinh}\left(\frac{8\sqrt{23}x}{23\,|2\,x+1|} - \frac{9\sqrt{23}}{23\,|2\,x+1|}\right) + \frac{7248\,x}{50531\sqrt{3\,x^2-x+2}} + \frac{8785}{101062\sqrt{3\,x^2-x+2}} - \frac{1}{26\left(4\sqrt{3\,x^2-x+2}x^2 + 4\sqrt{3\,x^2-x+2}x + \sqrt{3\,x^2-x+2}\right)} + \frac{3}{169\left(2\sqrt{3\,x^2-x+2}x + \sqrt{3\,x^2-x+2}\right)}$$

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2-x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 487/28561*sqrt(13)*arcsinh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqrt(23)/abs
(2*x + 1)) + 7248/50531*x/sqrt(3*x^2 - x + 2) + 8785/101062/sqrt(3*x^2 - x
+ 2) - 1/26/(4*sqrt(3*x^2 - x + 2)*x^2 + 4*sqrt(3*x^2 - x + 2)*x + sqrt(3*x
^2 - x + 2)) + 3/169/(2*sqrt(3*x^2 - x + 2)*x + sqrt(3*x^2 - x + 2))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. 2(90) = 180.

Time = 0.32 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.99

$$\int \frac{1+3x+4x^2}{(1+2x)^3\left(2-x+3x^2\right)^{3/2}}\,dx = \frac{487}{28561}\sqrt{13}\log\left(-\frac{\left|-4\sqrt{3}x - 2\sqrt{13} - 2\sqrt{3} + 4\sqrt{3\,x^2-x+2}\right|}{2\left(2\sqrt{3}x - \sqrt{13} + \sqrt{3} - 2\sqrt{3\,x^2-x+2}\right)}\right) + \frac{2\,(3693\,x+2363)}{50531\sqrt{3\,x^2-x+2}} + \frac{2\bigg(62\left(\sqrt{3}x-\sqrt{3\,x^2-x+2}\right)^3 - 37\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2-x+2}\right)^2 + 263\sqrt{3}x - 71\sqrt{3} - 263\sqrt{3\,x^2-x+2}}{2197\left(2\left(\sqrt{3}x-\sqrt{3\,x^2-x+2}\right)^2 + 2\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2-x+2}\right) - 5\right)^2}$$

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2-x+2)^(3/2),x, algorithm="giac")
```

```
[Out] 487/28561*sqrt(13)*log(-1/2*abs(-4*sqrt(3)*x - 2*sqrt(13) - 2*sqrt(3) + 4*s
qrt(3*x^2 - x + 2))/(2*sqrt(3)*x - sqrt(13) + sqrt(3) - 2*sqrt(3*x^2 - x +
2))) + 2/50531*(3693*x + 2363)/sqrt(3*x^2 - x + 2) + 2/2197*(62*(sqrt(3)*x
- sqrt(3*x^2 - x + 2))^3 - 37*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2))^2 +
 263*sqrt(3)*x - 71*sqrt(3) - 263*sqrt(3*x^2 - x + 2))/(2*(sqrt(3)*x - sqrt
(3*x^2 - x + 2))^2 + 2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) - 5)^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+3x+4x^2}{(1+2x)^3\left(2-x+3x^2\right)^{3/2}}\,dx=\int \frac{4\,x^2+3\,x+1}{{\left(2\,x+1\right)}^3\,{\left(3\,x^2-x+2\right)}^{3/2}} \,d x$$

```
[In] int((3*x + 4*x^2 + 1)/((2*x + 1)^3*(3*x^2 - x + 2)^(3/2)),x)
[Out] int((3*x + 4*x^2 + 1)/((2*x + 1)^3*(3*x^2 - x + 2)^(3/2)), x)
```

3.252 $\int \frac{(1+2x)^3(1+3x+4x^2)}{(2-x+3x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 86

$$\int \frac{(1+2x)^3(1+3x+4x^2)}{(2-x+3x^2)^{5/2}}\,dx = \frac{2(12839-3871x)}{5589(2-x+3x^2)^{3/2}} - \frac{28(35809+42240x)}{128547\sqrt{2-x+3x^2}} + \frac{32}{27}\sqrt{2-x+3x^2} - \frac{296\operatorname{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{27\sqrt{3}}$$

```
[Out] 2/5589*(12839-3871*x)/(3*x^2-x+2)^(3/2)-296/81*arcsinh(1/23*(1-6*x)*23^(1/2
))*3^(1/2)-28/128547*(35809+42240*x)/(3*x^2-x+2)^(1/2)+32/27*(3*x^2-x+2)^(1
/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1674, 654, 633, 221}

$$\int \frac{(1+2x)^3(1+3x+4x^2)}{(2-x+3x^2)^{5/2}}\,dx = -\frac{296\operatorname{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{27\sqrt{3}} + \frac{2(12839-3871x)}{5589(3x^2-x+2)^{3/2}} + \frac{32}{27}\sqrt{3x^2-x+2} - \frac{28(42240x+35809)}{128547\sqrt{3x^2-x+2}}$$

```
[In] Int[((1 + 2*x)^3*(1 + 3*x + 4*x^2))/(2 - x + 3*x^2)^(5/2),x]
```

```
[Out] (2*(12839 - 3871*x))/(5589*(2 - x + 3*x^2)^(3/2)) - (28*(35809 + 42240*x))/
(128547*Sqrt[2 - x + 3*x^2]) + (32*Sqrt[2 - x + 3*x^2])/27 - (296*ArcSinh[(
1 - 6*x)/Sqrt[23]])/(27*Sqrt[3])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2(12839-3871x)}{5589\left(2-x+3x^2\right)^{3/2}}+\frac{2}{69}\int \frac{-\frac{4361}{81}+\frac{7682x}{9}+\frac{2852x^2}{3}+368x^3}{\left(2-x+3x^2\right)^{3/2}}\,dx \\
&= \frac{2(12839-3871x)}{5589\left(2-x+3x^2\right)^{3/2}}-\frac{28(35809+42240x)}{128547\sqrt{2-x+3x^2}}+\frac{4\int \frac{\frac{37030}{9}+\frac{4232x}{3}}{\sqrt{2-x+3x^2}}\,dx}{1587} \\
&= \frac{2(12839-3871x)}{5589\left(2-x+3x^2\right)^{3/2}}-\frac{28(35809+42240x)}{128547\sqrt{2-x+3x^2}}+\frac{32}{27}\sqrt{2-x+3x^2}+\frac{296}{27}\int \frac{1}{\sqrt{2-x+3x^2}}\,dx \\
&= \frac{2(12839-3871x)}{5589\left(2-x+3x^2\right)^{3/2}}-\frac{28(35809+42240x)}{128547\sqrt{2-x+3x^2}} \\
&\quad +\frac{32}{27}\sqrt{2-x+3x^2}+\frac{296\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx,x,-1+6x\right)}{27\sqrt{69}}
\end{aligned}$$

$$=\frac{2(12839-3871x)}{5589\left(2-x+3x^2\right)^{3/2}}-\frac{28(35809+42240x)}{128547\sqrt{2-x+3x^2}}+\frac{32}{27}\sqrt{2-x+3x^2}-\frac{296\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{27\sqrt{3}}$$

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.81

$$\int\frac{(1+2x)^3\left(1+3x+4x^2\right)}{\left(2-x+3x^2\right)^{5/2}}\,dx=\frac{2(-44739-119459x+8630x^2-247904x^3+76176x^4)}{14283\left(2-x+3x^2\right)^{3/2}}-\frac{296\log\left(1-6x+2\sqrt{6-3x+9x^2}\right)}{27\sqrt{3}}$$

```
[In] Integrate[((1 + 2*x)^3*(1 + 3*x + 4*x^2))/(2 - x + 3*x^2)^(5/2),x]
```

```
[Out] (2*(-44739 - 119459*x + 8630*x^2 - 247904*x^3 + 76176*x^4))/(14283*(2 - x +
 3*x^2)^(3/2)) - (296*Log[1 - 6*x + 2*Sqrt[6 - 3*x + 9*x^2]])/(27*Sqrt[3])
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.58

method	result
risch	$\frac{\frac{32}{3}x^4-\frac{495808}{14283}x^3+\frac{17260}{14283}x^2-\frac{238918}{14283}x-\frac{3314}{529}}{(3x^2-x+2)^{\frac{3}{2}}}+\frac{296\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{81}$
trager	$\frac{\frac{32}{3}x^4-\frac{495808}{14283}x^3+\frac{17260}{14283}x^2-\frac{238918}{14283}x-\frac{3314}{529}}{(3x^2-x+2)^{\frac{3}{2}}}-\frac{296\operatorname{RootOf}\left(\_Z^2-3\right)\ln\left(-6\operatorname{RootOf}\left(\_Z^2-3\right)x+\operatorname{RootOf}\left(\_Z^2-3\right)+6\sqrt{3x^2-x-}\right.}{81}$
default	$\frac{-\frac{13763}{33534}+\frac{13763x}{5589}}{(3x^2-x+2)^{\frac{3}{2}}}+\frac{-\frac{65264}{128547}+\frac{130528x}{42849}}{\sqrt{3x^2-x+2}}-\frac{1727}{1458(3x^2-x+2)^{\frac{3}{2}}}+\frac{32x^4}{3(3x^2-x+2)^{\frac{3}{2}}}-\frac{296x^3}{27(3x^2-x+2)^{\frac{3}{2}}}+\frac{8x^2}{27(3x^2-x+2)^{\frac{3}{2}}}-\frac{}{81(3}$

```
[In] int((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2-x+2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/14283*(76176*x^4-247904*x^3+8630*x^2-119459*x-44739)/(3*x^2-x+2)^(3/2)+29
6/81*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.36

$$\int \frac{(1+2x)^3\left(1+3x+4x^2\right)}{\left(2-x+3x^2\right)^{5/2}}\,dx = \frac{2\left(39146\sqrt{3}(9\,x^4-6\,x^3+13\,x^2-4\,x+4)\log\left(-4\sqrt{3}\sqrt{3\,x^2-x+2}(6\,x-\right.\right.}{42849\,(}$$

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2-x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/42849*(39146*sqrt(3)*(9*x^4 - 6*x^3 + 13*x^2 - 4*x + 4)*log(-4*sqrt(3)*sq
rt(3*x^2 - x + 2)*(6*x - 1) - 72*x^2 + 24*x - 25) + 3*(76176*x^4 - 247904*x
^3 + 8630*x^2 - 119459*x - 44739)*sqrt(3*x^2 - x + 2))/(9*x^4 - 6*x^3 + 13*
x^2 - 4*x + 4)
```

Sympy [F]

$$\int \frac{(1+2x)^3\left(1+3x+4x^2\right)}{\left(2-x+3x^2\right)^{5/2}}\,dx = \int \frac{(2x+1)^3\cdot\left(4x^2+3x+1\right)}{\left(3x^2-x+2\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((1+2*x)**3*(4*x**2+3*x+1)/(3*x**2-x+2)**(5/2),x)
```

```
[Out] Integral((2*x + 1)**3*(4*x**2 + 3*x + 1)/(3*x**2 - x + 2)**(5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. 2(69) = 138.

Time = 0.29 (sec) , antiderivative size = 202, normalized size of antiderivative = 2.35

$$\begin{aligned}\int \frac{(1+2x)^3\left(1+3x+4x^2\right)}{\left(2-x+3x^2\right)^{5/2}}\,dx &= \frac{32\,x^4}{3\,(3\,x^2-x+2)^{\frac{3}{2}}}\\ &+\frac{296}{42849}\,x\left(\frac{426\,x}{\sqrt{3\,x^2-x+2}}-\frac{4761\,x^2}{(3\,x^2-x+2)^{\frac{3}{2}}}-\frac{71}{\sqrt{3\,x^2-x+2}}+\frac{805\,x}{(3\,x^2-x+2)^{\frac{3}{2}}}-\frac{2162}{(3\,x^2-x+2)^{\frac{3}{2}}}\right)\\ &+\frac{296}{81}\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{23}\sqrt{23}(6\,x-1)\right)-\frac{42032}{42849}\sqrt{3\,x^2-x+2}-\frac{47072\,x}{42849\sqrt{3\,x^2-x+2}}\\ &+\frac{52\,x^2}{9\,(3\,x^2-x+2)^{\frac{3}{2}}}-\frac{23104}{14283\sqrt{3\,x^2-x+2}}-\frac{7742\,x}{1863\,(3\,x^2-x+2)^{\frac{3}{2}}}+\frac{1666}{1863\,(3\,x^2-x+2)^{\frac{3}{2}}}\end{aligned}$$

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2-x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 32/3*x^4/(3*x^2 - x + 2)^(3/2) + 296/42849*x*(426*x/sqrt(3*x^2 - x + 2) - 4
761*x^2/(3*x^2 - x + 2)^(3/2) - 71/sqrt(3*x^2 - x + 2) + 805*x/(3*x^2 - x +
```

```
 2)^(3/2) - 2162/(3*x^2 - x + 2)^(3/2)) + 296/81*sqrt(3)*arcsinh(1/23*sqrt(
23)*(6*x - 1)) - 42032/42849*sqrt(3*x^2 - x + 2) - 47072/42849*x/sqrt(3*x^2
 - x + 2) + 52/9*x^2/(3*x^2 - x + 2)^(3/2) - 23104/14283/sqrt(3*x^2 - x + 2
) - 7742/1863*x/(3*x^2 - x + 2)^(3/2) + 1666/1863/(3*x^2 - x + 2)^(3/2)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.78

$$\int \frac{(1+2x)^3(1+3x+4x^2)}{(2-x+3x^2)^{5/2}}\,dx = -\frac{296}{81}\sqrt{3}\log\left(-2\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2-x+2}\right)+1\right) + \frac{2\left((2\left(8\left(4761x-15494\right)x+4315\right)x-119459)x-44739\right)}{14283\left(3x^2-x+2\right)^{\frac{3}{2}}}$$

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2-x+2)^(5/2),x, algorithm="giac")
[Out] -296/81*sqrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) + 1) + 2/1
4283*((2*(8*(4761*x - 15494)*x + 4315)*x - 119459)*x - 44739)/(3*x^2 - x +
2)^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+2x)^3(1+3x+4x^2)}{(2-x+3x^2)^{5/2}}\,dx = \int \frac{(2x+1)^3\left(4x^2+3x+1\right)}{\left(3x^2-x+2\right)^{5/2}}\,dx$$

```
[In] int(((2*x + 1)^3*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(5/2),x)
[Out] int(((2*x + 1)^3*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(5/2), x)
```

3.253 $\int \frac{(1+2x)^2(1+3x+4x^2)}{(2-x+3x^2)^{5/2}} \, dx$

Optimal result

Integrand size = 32, antiderivative size = 68

$$\int \frac{(1+2x)^2\left(1+3x+4x^2\right)}{\left(2-x+3x^2\right)^{5/2}} \, dx = \frac{2(1249-2273x)}{1863\left(2-x+3x^2\right)^{3/2}} - \frac{8(23257-1473x)}{42849\sqrt{2-x+3x^2}} - \frac{16\operatorname{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{9\sqrt{3}}$$

```
[Out] 2/1863*(1249-2273*x)/(3*x^2-x+2)^(3/2)-16/27*arcsinh(1/23*(1-6*x)*23^(1/2))
*3^(1/2)-8/42849*(23257-1473*x)/(3*x^2-x+2)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1674, 12, 633, 221}

$$\int \frac{(1+2x)^2\left(1+3x+4x^2\right)}{\left(2-x+3x^2\right)^{5/2}} \, dx = -\frac{16\operatorname{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{9\sqrt{3}} + \frac{2(1249-2273x)}{1863\left(3x^2-x+2\right)^{3/2}} - \frac{8(23257-1473x)}{42849\sqrt{3x^2-x+2}}$$

```
[In] Int[((1 + 2*x)^2*(1 + 3*x + 4*x^2))/(2 - x + 3*x^2)^(5/2),x]
```

```
[Out] (2*(1249 - 2273*x))/(1863*(2 - x + 3*x^2)^(3/2)) - (8*(23257 - 1473*x))/(42
849*Sqrt[2 - x + 3*x^2]) - (16*ArcSinh[(1 - 6*x)/Sqrt[23]])/(9*Sqrt[3])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2(1249-2273x)}{1863\left(2-x+3x^2\right)^{3/2}}+\frac{2}{69}\int \frac{\frac{1802}{27}+\frac{1150x}{3}+184x^2}{\left(2-x+3x^2\right)^{3/2}}\,dx\\
&= \frac{2(1249-2273x)}{1863\left(2-x+3x^2\right)^{3/2}}-\frac{8(23257-1473x)}{42849\sqrt{2-x+3x^2}}+\frac{4\int \frac{2116}{3\sqrt{2-x+3x^2}}\,dx}{1587}\\
&= \frac{2(1249-2273x)}{1863\left(2-x+3x^2\right)^{3/2}}-\frac{8(23257-1473x)}{42849\sqrt{2-x+3x^2}}+\frac{16}{9}\int \frac{1}{\sqrt{2-x+3x^2}}\,dx\\
&= \frac{2(1249-2273x)}{1863\left(2-x+3x^2\right)^{3/2}}-\frac{8(23257-1473x)}{42849\sqrt{2-x+3x^2}}+\frac{16\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx,x,-1+6x\right)}{9\sqrt{69}}\\
&= \frac{2(1249-2273x)}{1863\left(2-x+3x^2\right)^{3/2}}-\frac{8(23257-1473x)}{42849\sqrt{2-x+3x^2}}-\frac{16\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{9\sqrt{3}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.96

$$\int \frac{(1+2x)^2\left(1+3x+4x^2\right)}{\left(2-x+3x^2\right)^{5/2}}\,dx=\frac{2(-17481+5837x-31664x^2+1964x^3)}{4761\left(2-x+3x^2\right)^{3/2}}-\frac{16\log\left(1-6x+2\sqrt{6-3x+9x^2}\right)}{9\sqrt{3}}$$

```
[In] Integrate[((1 + 2*x)^2*(1 + 3*x + 4*x^2))/(2 - x + 3*x^2)^(5/2),x]
[Out] (2*(-17481 + 5837*x - 31664*x^2 + 1964*x^3))/(4761*(2 - x + 3*x^2)^(3/2)) -
 (16*Log[1 - 6*x + 2*Sqrt[6 - 3*x + 9*x^2]])/(9*Sqrt[3])
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.66

method	result
risch	$\frac{\frac{3928}{4761}x^3-\frac{63328}{4761}x^2+\frac{11674}{4761}x-\frac{11654}{1587}}{(3x^2-x+2)^{\frac{3}{2}}}+\frac{16\sqrt{3}\ \operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{27}$
trager	$\frac{\frac{3928}{4761}x^3-\frac{63328}{4761}x^2+\frac{11674}{4761}x-\frac{11654}{1587}}{(3x^2-x+2)^{\frac{3}{2}}}+\frac{16\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(6\,\mathrm{RootOf}\left(\_Z^2-3\right)x+6\sqrt{3x^2-x+2}-\mathrm{RootOf}\left(\_Z^2-3\right)\right)}{27}$
default	$\frac{-\frac{4585}{11178}+\frac{4585x}{1863}}{(3x^2-x+2)^{\frac{3}{2}}}+\frac{-\frac{18892}{42849}+\frac{37784x}{14283}}{\sqrt{3x^2-x+2}}-\frac{2653}{486(3x^2-x+2)^{\frac{3}{2}}}-\frac{16x^3}{9(3x^2-x+2)^{\frac{3}{2}}}-\frac{92x^2}{9(3x^2-x+2)^{\frac{3}{2}}}-\frac{67x}{27(3x^2-x+2)^{\frac{3}{2}}}-\frac{16x}{9\sqrt{3x^2-x+}}$

```
[In] int((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(5/2),x,method=_RETURNVERBOSE)
[Out] 2/4761*(1964*x^3-31664*x^2+5837*x-17481)/(3*x^2-x+2)^(3/2)+16/27*3^(1/2)*ar
csinh(6/23*23^(1/2)*(x-1/6))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(55) = 110.

Time = 0.25 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.65

$$\int \frac{(1+2x)^2\left(1+3x+4x^2\right)}{\left(2-x+3x^2\right)^{5/2}}\,dx=\frac{2\left(2116\sqrt{3}(9x^4-6x^3+13x^2-4x+4)\log\left(-4\sqrt{3}\sqrt{3x^2-x+2}(6x-1\right.\right.}{14283\,(9x^4-6}$$

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/14283*(2116*sqrt(3)*(9*x^4 - 6*x^3 + 13*x^2 - 4*x + 4)*log(-4*sqrt(3)*sqr
t(3*x^2 - x + 2)*(6*x - 1) - 72*x^2 + 24*x - 25) + 3*(1964*x^3 - 31664*x^2
+ 5837*x - 17481)*sqrt(3*x^2 - x + 2))/(9*x^4 - 6*x^3 + 13*x^2 - 4*x + 4)
```

Sympy [F]

$$\int \frac{(1+2x)^2\,(1+3x+4x^2)}{(2-x+3x^2)^{5/2}}\,dx = \int \frac{(2x+1)^2\cdot(4x^2+3x+1)}{(3x^2-x+2)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((1+2*x)**2*(4*x**2+3*x+1)/(3*x**2-x+2)**(5/2),x)
```

```
[Out] Integral((2*x + 1)**2*(4*x**2 + 3*x + 1)/(3*x**2 - x + 2)**(5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 185 vs. 2(55) = 110.

Time = 0.29 (sec) , antiderivative size = 185, normalized size of antiderivative = 2.72

$$\int \frac{(1+2x)^2\,(1+3x+4x^2)}{(2-x+3x^2)^{5/2}}\,dx = \frac{16}{14283}\,x\Bigg(\frac{426\,x}{\sqrt{3\,x^2-x+2}} - \frac{4761\,x^2}{(3\,x^2-x+2)^{\frac{3}{2}}} - \frac{71}{\sqrt{3\,x^2-x+2}} + \frac{805}{(3\,x^2-x}$$
$$+\frac{16}{27}\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{23}\sqrt{23}(6\,x-1)\right) - \frac{2272}{14283}\sqrt{3\,x^2-x+2} + \frac{28184\,x}{14283\sqrt{3\,x^2-x+2}}$$
$$-\frac{28\,x^2}{3\,(3\,x^2-x+2)^{\frac{3}{2}}} - \frac{2956}{4761\sqrt{3\,x^2-x+2}} - \frac{106\,x}{621\,(3\,x^2-x+2)^{\frac{3}{2}}} - \frac{3394}{621\,(3\,x^2-x+2)^{\frac{3}{2}}}$$

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 16/14283*x*(426*x/sqrt(3*x^2 - x + 2) - 4761*x^2/(3*x^2 - x + 2)^(3/2) - 71
/sqrt(3*x^2 - x + 2) + 805*x/(3*x^2 - x + 2)^(3/2) - 2162/(3*x^2 - x + 2)^(
3/2)) + 16/27*sqrt(3)*arcsinh(1/23*sqrt(23)*(6*x - 1)) - 2272/14283*sqrt(3*
x^2 - x + 2) + 28184/14283*x/sqrt(3*x^2 - x + 2) - 28/3*x^2/(3*x^2 - x + 2)
^(3/2) - 2956/4761/sqrt(3*x^2 - x + 2) - 106/621*x/(3*x^2 - x + 2)^(3/2) -
3394/621/(3*x^2 - x + 2)^(3/2)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.91

$$\int \frac{(1+2x)^2 (1+3x+4x^2)}{(2-x+3x^2)^{5/2}}\,dx = -\frac{16}{27}\sqrt{3}\log\left(-2\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2-x+2}\right)+1\right) + \frac{2\left((4\,(491\,x-7916)x+5837)x-17481\right)}{4761\,(3\,x^2-x+2)^{\frac{3}{2}}}$$

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(5/2),x, algorithm="giac")
[Out] -16/27*sqrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) + 1) + 2/47
61*((4*(491*x - 7916)*x + 5837)*x - 17481)/(3*x^2 - x + 2)^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+2x)^2 (1+3x+4x^2)}{(2-x+3x^2)^{5/2}}\,dx = \int \frac{(2\,x+1)^2\,(4\,x^2+3\,x+1)}{(3\,x^2-x+2)^{5/2}}\,dx$$

```
[In] int(((2*x + 1)^2*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(5/2),x)
[Out] int(((2*x + 1)^2*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(5/2), x)
```

3.254 $\int \frac{(1+2x)(1+3x+4x^2)}{(2-x+3x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 47

$$\int \frac{(1+2x)(1+3x+4x^2)}{(2-x+3x^2)^{5/2}}\,dx = -\frac{2(73+367x)}{621(2-x+3x^2)^{3/2}} - \frac{4(3889-4290x)}{14283\sqrt{2-x+3x^2}}$$

```
[Out] -2/621*(73+367*x)/(3*x^2-x+2)^(3/2)-4/14283*(3889-4290*x)/(3*x^2-x+2)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1674, 650}

$$\int \frac{(1+2x)(1+3x+4x^2)}{(2-x+3x^2)^{5/2}}\,dx = -\frac{4(3889-4290x)}{14283\sqrt{3x^2-x+2}} - \frac{2(367x+73)}{621(3x^2-x+2)^{3/2}}$$

```
[In] Int[((1 + 2*x)*(1 + 3*x + 4*x^2))/(2 - x + 3*x^2)^(5/2),x]
```

```
[Out] (-2*(73 + 367*x))/(621*(2 - x + 3*x^2)^(3/2)) - (4*(3889 - 4290*x))/(14283*
Sqrt[2 - x + 3*x^2])
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-2*((b*d - 2*a*e + (2*c*d - b*e)*x)/((b^2 - 4*a*c)*Sqrt[a + b*x
+ c*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2(73+367x)}{621\left(2-x+3x^2\right)^{3/2}}+\frac{2}{69}\int\frac{\frac{577}{9}+92x}{\left(2-x+3x^2\right)^{3/2}}\,dx\\ &= -\frac{2(73+367x)}{621\left(2-x+3x^2\right)^{3/2}}-\frac{4(3889-4290x)}{14283\sqrt{2-x+3x^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.70

$$\int\frac{(1+2x)\left(1+3x+4x^2\right)}{\left(2-x+3x^2\right)^{5/2}}\,dx=\frac{2(-1915+1833x-3546x^2+2860x^3)}{1587\left(2-x+3x^2\right)^{3/2}}$$

```
[In] Integrate[((1 + 2*x)*(1 + 3*x + 4*x^2))/(2 - x + 3*x^2)^(5/2),x]
[Out] (2*(-1915 + 1833*x - 3546*x^2 + 2860*x^3))/(1587*(2 - x + 3*x^2)^(3/2))
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.64

method	result	size
gosper	$\frac{\frac{5720}{1587}x^3-\frac{2364}{529}x^2+\frac{1222}{529}x-\frac{3830}{1587}}{(3x^2-x+2)^{\frac{3}{2}}}$	30
trager	$\frac{\frac{5720}{1587}x^3-\frac{2364}{529}x^2+\frac{1222}{529}x-\frac{3830}{1587}}{(3x^2-x+2)^{\frac{3}{2}}}$	30
risch	$\frac{\frac{5720}{1587}x^3-\frac{2364}{529}x^2+\frac{1222}{529}x-\frac{3830}{1587}}{(3x^2-x+2)^{\frac{3}{2}}}$	30
default	$\frac{-\frac{715}{3726}+\frac{715x}{621}}{(3x^2-x+2)^{\frac{3}{2}}}+\frac{-\frac{2860}{14283}+\frac{5720x}{4761}}{\sqrt{3x^2-x+2}}-\frac{295}{162(3x^2-x+2)^{\frac{3}{2}}}-\frac{8x^2}{3(3x^2-x+2)^{\frac{3}{2}}}-\frac{13x}{9(3x^2-x+2)^{\frac{3}{2}}}$	86

```
[In] int((1+2*x)*(4*x^2+3*x+1)/(3*x^2-x+2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/1587/(3*x^2-x+2)^(3/2)*(2860*x^3-3546*x^2+1833*x-1915)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.09

$$\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\left(2-x+3x^2\right)^{5/2}}\,dx=\frac{2\,\left(2860\,x^3-3546\,x^2+1833\,x-1915\right)\sqrt{3\,x^2-x+2}}{1587\,\left(9\,x^4-6\,x^3+13\,x^2-4\,x+4\right)}$$

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2-x+2)^(5/2),x, algorithm="fricas")
[Out] 2/1587*(2860*x^3 - 3546*x^2 + 1833*x - 1915)*sqrt(3*x^2 - x + 2)/(9*x^4 - 6
*x^3 + 13*x^2 - 4*x + 4)
```

Sympy [F]

$$\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\left(2-x+3x^2\right)^{5/2}}\,dx=\int \frac{(2x+1)\left(4x^2+3x+1\right)}{\left(3x^2-x+2\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((1+2*x)*(4*x**2+3*x+1)/(3*x**2-x+2)**(5/2),x)
[Out] Integral((2*x + 1)*(4*x**2 + 3*x + 1)/(3*x**2 - x + 2)**(5/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.62

$$\begin{aligned}\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\left(2-x+3x^2\right)^{5/2}}\,dx&=\frac{5720\,x}{4761\,\sqrt{3\,x^2-x+2}}-\frac{8\,x^2}{3\,\left(3\,x^2-x+2\right)^{\frac{3}{2}}}\\&\quad-\frac{2860}{14283\,\sqrt{3\,x^2-x+2}}-\frac{182\,x}{621\,\left(3\,x^2-x+2\right)^{\frac{3}{2}}}-\frac{1250}{621\,\left(3\,x^2-x+2\right)^{\frac{3}{2}}}\end{aligned}$$

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2-x+2)^(5/2),x, algorithm="maxima")
[Out] 5720/4761*x/sqrt(3*x^2 - x + 2) - 8/3*x^2/(3*x^2 - x + 2)^(3/2) - 2860/1428
3/sqrt(3*x^2 - x + 2) - 182/621*x/(3*x^2 - x + 2)^(3/2) - 1250/621/(3*x^2 -
 x + 2)^(3/2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.60

$$\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\left(2-x+3x^2\right)^{5/2}}\,dx=\frac{2\,\left(\left(2\,\left(1430\,x-1773\right)x+1833\right)x-1915\right)}{1587\,\left(3\,x^2-x+2\right)^{\frac{3}{2}}}$$

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2-x+2)^(5/2),x, algorithm="giac")
[Out] 2/1587*((2*(1430*x - 1773)*x + 1833)*x - 1915)/(3*x^2 - x + 2)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 13.34 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.04

$$\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\left(2-x+3x^2\right)^{5/2}}\,dx=-\frac{442\,x-5720\,x\,\left(3\,x^2-x+2\right)+15556\,x^2+11490}{\sqrt{3\,x^2-x+2}\,\left(14283\,x^2-4761\,x+9522\right)}$$

```
[In] int(((2*x + 1)*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(5/2),x)
[Out] -(442*x - 5720*x*(3*x^2 - x + 2) + 15556*x^2 + 11490)/((3*x^2 - x + 2)^(1/2
)*(14283*x^2 - 4761*x + 9522))
```

3.255 $\int \frac{1+3x+4x^2}{(1+2x)\left(2-x+3x^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 85

$$\int \frac{1+3x+4x^2}{(1+2x)\left(2-x+3x^2\right)^{5/2}}\,dx = -\frac{2(101-77x)}{897\left(2-x+3x^2\right)^{3/2}} - \frac{4(691-13668x)}{268203\sqrt{2-x+3x^2}} - \frac{8\operatorname{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)}{169\sqrt{13}}$$

```
[Out] -2/897*(101-77*x)/(3*x^2-x+2)^(3/2)-8/2197*arctanh(1/26*(9-8*x)*13^(1/2)/(3
*x^2-x+2)^(1/2))*13^(1/2)-4/268203*(691-13668*x)/(3*x^2-x+2)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {1660, 836, 12, 738, 212}

$$\int \frac{1+3x+4x^2}{(1+2x)\left(2-x+3x^2\right)^{5/2}}\,dx = -\frac{8\operatorname{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{169\sqrt{13}} - \frac{4(691-13668x)}{268203\sqrt{3x^2-x+2}} - \frac{2(101-77x)}{897\left(3x^2-x+2\right)^{3/2}}$$

```
[In] Int[(1 + 3*x + 4*x^2)/((1 + 2*x)*(2 - x + 3*x^2)^(5/2)),x]
```

```
[Out] (-2*(101 - 77*x))/(897*(2 - x + 3*x^2)^(3/2)) - (4*(691 - 13668*x))/(268203
*Sqrt[2 - x + 3*x^2]) - (8*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2
])])/(169*Sqrt[13])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 836

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1660

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^(p + 1)/((p
+ 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2(101-77x)}{897\left(2-x+3x^2\right)^{3/2}}+\frac{2}{69}\int \frac{\frac{223}{13}+\frac{308x}{13}}{(1+2x)\left(2-x+3x^2\right)^{3/2}}\,dx\\
&= -\frac{2(101-77x)}{897\left(2-x+3x^2\right)^{3/2}}-\frac{4(691-13668x)}{268203\sqrt{2-x+3x^2}}+\frac{4\int \frac{3174}{13(1+2x)\sqrt{2-x+3x^2}}\,dx}{20631}\\
&= -\frac{2(101-77x)}{897\left(2-x+3x^2\right)^{3/2}}-\frac{4(691-13668x)}{268203\sqrt{2-x+3x^2}}+\frac{8}{169}\int \frac{1}{(1+2x)\sqrt{2-x+3x^2}}\,dx\\
&= -\frac{2(101-77x)}{897\left(2-x+3x^2\right)^{3/2}}-\frac{4(691-13668x)}{268203\sqrt{2-x+3x^2}}-\frac{16}{169}\text{Subst}\left(\int \frac{1}{52-x^2}\,dx,x,\frac{9-8x}{\sqrt{2-x+3x^2}}\right)\\
&= -\frac{2(101-77x)}{897\left(2-x+3x^2\right)^{3/2}}-\frac{4(691-13668x)}{268203\sqrt{2-x+3x^2}}-\frac{8\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)}{169\sqrt{13}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.94

$$
\int \frac{1+3x+4x^2}{(1+2x)\left(2-x+3x^2\right)^{5/2}}\,dx = \frac{2\left(-32963+79077x-31482x^2+82008x^3\right)}{268203\left(2-x+3x^2\right)^{3/2}} + \frac{16\operatorname{arctanh}\left(\frac{\sqrt{3}+2\sqrt{3}x-2\sqrt{2-x+3x^2}}{\sqrt{13}}\right)}{169\sqrt{13}}
$$

```
[In] Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)*(2 - x + 3*x^2)^(5/2)),x]
```

```
[Out] (2*(-32963 + 79077*x - 31482*x^2 + 82008*x^3))/(268203*(2 - x + 3*x^2)^(3/2
)) + (16*ArcTanh[(Sqrt[3] + 2*Sqrt[3]*x - 2*Sqrt[2 - x + 3*x^2])/Sqrt[13]])
/(169*Sqrt[13])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.66 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.94

method	result
trager	$\frac{\frac{54672}{89401}x^3-\frac{20988}{89401}x^2+\frac{52718}{89401}x-\frac{65926}{268203}}{(3x^2-x+2)^{\frac{3}{2}}}+\frac{8\,\mathrm{RootOf}\left(\_Z^2-13\right)\ln\left(\frac{8\,\mathrm{RootOf}\left(\_Z^2-13\right)x+26\sqrt{3x^2-x+2}-9\,\mathrm{RootOf}\left(\_Z^2-13\right)}{1+2x}\right)}{2197}$
default	$\frac{-\frac{5}{207}+\frac{10x}{69}}{(3x^2-x+2)^{\frac{3}{2}}}+\frac{-\frac{40}{1587}+\frac{80x}{529}}{\sqrt{3x^2-x+2}}-\frac{2}{9(3x^2-x+2)^{\frac{3}{2}}}+\frac{1}{39\left(3\left(x+\frac{1}{2}\right)^2-4x+\frac{5}{4}\right)^{\frac{3}{2}}}+\frac{-\frac{4}{897}+\frac{8x}{299}}{\left(3\left(x+\frac{1}{2}\right)^2-4x+\frac{5}{4}\right)^{\frac{3}{2}}}+\frac{-\frac{784}{89401}+\frac{4704x}{89401}}{\sqrt{3\left(x+\frac{1}{2}\right)^2-4x+\frac{5}{4}}}+$

```
[In] int((4*x^2+3*x+1)/(1+2*x)/(3*x^2-x+2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/268203*(82008*x^3-31482*x^2+79077*x-32963)/(3*x^2-x+2)^(3/2)+8/2197*RootO
f(_Z^2-13)*ln((8*RootOf(_Z^2-13)*x+26*(3*x^2-x+2)^(1/2)-9*RootOf(_Z^2-13))/
(1+2*x))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.48

$$\int \frac{1+3x+4x^2}{(1+2x)\left(2-x+3x^2\right)^{5/2}}\,dx=\frac{2\Bigg(3174\sqrt{13}(9x^4-6x^3+13x^2-4x+4)\log\Bigg(-\frac{4\sqrt{13}\sqrt{3x^2-x+2}(8x-9)+220x}{4x^2+4x+1}}{3486639\,(9x^4-6x}$$

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2-x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3486639*(3174*sqrt(13)*(9*x^4 - 6*x^3 + 13*x^2 - 4*x + 4)*log(-(4*sqrt(13
)*sqrt(3*x^2 - x + 2)*(8*x - 9) + 220*x^2 - 196*x + 185)/(4*x^2 + 4*x + 1))
 + 13*(82008*x^3 - 31482*x^2 + 79077*x - 32963)*sqrt(3*x^2 - x + 2))/(9*x^4
 - 6*x^3 + 13*x^2 - 4*x + 4)
```

Sympy [F]

$$\int \frac{1+3x+4x^2}{(1+2x)\left(2-x+3x^2\right)^{5/2}}\,dx=\int \frac{4x^2+3x+1}{(2x+1)\left(3x^2-x+2\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)/(3*x**2-x+2)**(5/2),x)
```

```
[Out] Integral((4*x**2 + 3*x + 1)/((2*x + 1)*(3*x**2 - x + 2)**(5/2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.09

$$\int \frac{1+3x+4x^2}{(1+2x)\left(2-x+3x^2\right)^{5/2}}\,dx = \frac{8}{2197}\,\sqrt{13}\,\operatorname{arsinh}\left(\frac{8\,\sqrt{23}x}{23\,|2\,x+1|}-\frac{9\,\sqrt{23}}{23\,|2\,x+1|}\right) + \frac{18224\,x}{89401\,\sqrt{3\,x^2-x+2}}-\frac{2764}{268203\,\sqrt{3\,x^2-x+2}} + \frac{154\,x}{897\,\left(3\,x^2-x+2\right)^{\frac{3}{2}}}-\frac{202}{897\,\left(3\,x^2-x+2\right)^{\frac{3}{2}}}$$

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2-x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 8/2197*sqrt(13)*arcsinh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqrt(23)/abs(2*
x + 1)) + 18224/89401*x/sqrt(3*x^2 - x + 2) - 2764/268203/sqrt(3*x^2 - x +
2) + 154/897*x/(3*x^2 - x + 2)^(3/2) - 202/897/(3*x^2 - x + 2)^(3/2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.19

$$\int \frac{1+3x+4x^2}{(1+2x)\left(2-x+3x^2\right)^{5/2}}\,dx = \frac{8}{2197}\,\sqrt{13}\log\left(-\frac{\left|-4\,\sqrt{3}x-2\,\sqrt{13}-2\,\sqrt{3}+4\,\sqrt{3\,x^2-x+2}\right|}{2\,\left(2\,\sqrt{3}x-\sqrt{13}+\sqrt{3}-2\,\sqrt{3\,x^2-x+2}\right)}\right) + \frac{2\,(3\,(6\,(4556\,x-1749)x+26359)x-32963)}{268203\,\left(3\,x^2-x+2\right)^{\frac{3}{2}}}$$

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2-x+2)^(5/2),x, algorithm="giac")
```

```
[Out] 8/2197*sqrt(13)*log(-1/2*abs(-4*sqrt(3)*x - 2*sqrt(13) - 2*sqrt(3) + 4*sqrt
(3*x^2 - x + 2))/(2*sqrt(3)*x - sqrt(13) + sqrt(3) - 2*sqrt(3*x^2 - x + 2))
) + 2/268203*(3*(6*(4556*x - 1749)*x + 26359)*x - 32963)/(3*x^2 - x + 2)^(3
/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+3x+4x^2}{(1+2x)\,(2-x+3x^2)^{5/2}}\,dx=\int \frac{4\,x^2+3\,x+1}{(2\,x+1)\,\left(3\,x^2-x+2\right)^{5/2}}\,dx$$

```
[In] int((3*x + 4*x^2 + 1)/((2*x + 1)*(3*x^2 - x + 2)^(5/2)),x)
[Out] int((3*x + 4*x^2 + 1)/((2*x + 1)*(3*x^2 - x + 2)^(5/2)), x)
```

3.256 $\int \frac{1+3x+4x^2}{(1+2x)^2(2-x+3x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 110

$$\int \frac{1+3x+4x^2}{(1+2x)^2\left(2-x+3x^2\right)^{5/2}}\,dx = -\frac{2(197-837x)}{11661\left(2-x+3x^2\right)^{3/2}} - \frac{24(841-6633x)}{1162213\sqrt{2-x+3x^2}} - \frac{16\sqrt{2-x+3x^2}}{2197(1+2x)} - \frac{56\operatorname{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)}{2197\sqrt{13}}$$

```
[Out] -2/11661*(197-837*x)/(3*x^2-x+2)^(3/2)-56/28561*arctanh(1/26*(9-8*x)*13^(1/
2)/(3*x^2-x+2)^(1/2))*13^(1/2)-24/1162213*(841-6633*x)/(3*x^2-x+2)^(1/2)-16
/2197*(3*x^2-x+2)^(1/2)/(1+2*x)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1660, 820, 738, 212}

$$\int \frac{1+3x+4x^2}{(1+2x)^2\left(2-x+3x^2\right)^{5/2}}\,dx = -\frac{56\operatorname{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{2197\sqrt{13}} - \frac{24(841-6633x)}{1162213\sqrt{3x^2-x+2}} - \frac{16\sqrt{3x^2-x+2}}{2197(2x+1)} - \frac{2(197-837x)}{11661\left(3x^2-x+2\right)^{3/2}}$$

```
[In] Int[(1 + 3*x + 4*x^2)/((1 + 2*x)^2*(2 - x + 3*x^2)^(5/2)),x]
```

```
[Out] (-2*(197 - 837*x))/(11661*(2 - x + 3*x^2)^(3/2)) - (24*(841 - 6633*x))/(116
2213*Sqrt[2 - x + 3*x^2]) - (16*Sqrt[2 - x + 3*x^2])/(2197*(1 + 2*x)) - (56
*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/(2197*Sqrt[13])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 1660

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^(p + 1)/((p
+ 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2(197-837x)}{11661\left(2-x+3x^2\right)^{3/2}}+\frac{2}{69}\int \frac{\frac{2226}{169}+\frac{462x}{13}+\frac{6696x^2}{169}}{(1+2x)^2\left(2-x+3x^2\right)^{3/2}}\,dx\\ &= -\frac{2(197-837x)}{11661\left(2-x+3x^2\right)^{3/2}}-\frac{24(841-6633x)}{1162213\sqrt{2-x+3x^2}}+\frac{4\int \frac{\frac{50784}{2197}+\frac{19044x}{2197}}{(1+2x)^2\sqrt{2-x+3x^2}}\,dx}{1587}\end{aligned}$$

$$= -\frac{2(197-837x)}{11661\left(2-x+3x^2\right)^{3/2}}-\frac{24(841-6633x)}{1162213\sqrt{2-x+3x^2}} -\frac{16\sqrt{2-x+3x^2}}{2197(1+2x)}+\frac{56\int\frac{1}{(1+2x)\sqrt{2-x+3x^2}}\,dx}{2197}$$

$$= -\frac{2(197-837x)}{11661\left(2-x+3x^2\right)^{3/2}}-\frac{24(841-6633x)}{1162213\sqrt{2-x+3x^2}} -\frac{16\sqrt{2-x+3x^2}}{2197(1+2x)}-\frac{112\text{Subst}\left(\int\frac{1}{52-x^2}\,dx,x,\frac{9-8x}{\sqrt{2-x+3x^2}}\right)}{2197}$$

$$= -\frac{2(197-837x)}{11661\left(2-x+3x^2\right)^{3/2}}-\frac{24(841-6633x)}{1162213\sqrt{2-x+3x^2}} -\frac{16\sqrt{2-x+3x^2}}{2197(1+2x)}-\frac{56\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)}{2197\sqrt{13}}$$

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.84

$$\int\frac{1+3x+4x^2}{(1+2x)^2\left(2-x+3x^2\right)^{5/2}}\,dx=\frac{2(-170239+569989x+1021566x^2+133308x^3+1318464x^4)}{3486639(1+2x)\left(2-x+3x^2\right)^{3/2}} +\frac{112\text{arctanh}\left(\frac{\sqrt{3}+2\sqrt{3}x-2\sqrt{2-x+3x^2}}{\sqrt{13}}\right)}{2197\sqrt{13}}$$

```
[In] Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)^2*(2 - x + 3*x^2)^(5/2)),x]
```

```
[Out] (2*(-170239 + 569989*x + 1021566*x^2 + 133308*x^3 + 1318464*x^4))/(3486639*
(1 + 2*x)*(2 - x + 3*x^2)^(3/2)) + (112*ArcTanh[(Sqrt[3] + 2*Sqrt[3]*x - 2*
Sqrt[2 - x + 3*x^2])/Sqrt[13]])/(2197*Sqrt[13])
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.66

method	result
risch	$\frac{\frac{878976}{1162213}x^4+\frac{168}{2197}x^3+\frac{52388}{89401}x^2+\frac{1139978}{3486639}x-\frac{340478}{3486639}}{(1+2x)(3x^2-x+2)^{\frac{3}{2}}}-\frac{56\sqrt{13}\,\operatorname{arctanh}\left(\frac{2\left(\frac{9}{2}-4x\right)\sqrt{13}}{13\sqrt{12\left(x+\frac{1}{2}\right)^2-16x+5}}\right)}{28561}$
trager	$\frac{\frac{878976}{1162213}x^4+\frac{168}{2197}x^3+\frac{52388}{89401}x^2+\frac{1139978}{3486639}x-\frac{340478}{3486639}}{(1+2x)(3x^2-x+2)^{\frac{3}{2}}}+\frac{56\,\mathrm{RootOf}\left(\_Z^2-13\right)\ln\left(\frac{8\,\mathrm{RootOf}\left(\_Z^2-13\right)x+26\sqrt{3x^2-x+2}-9\,\mathrm{RootOf}\left(\_Z\right.}{1+2x}\right.}{28561}$
default	$\frac{-\frac{2}{69}+\frac{4x}{23}}{(3x^2-x+2)^{\frac{3}{2}}}+\frac{-\frac{16}{529}+\frac{96x}{529}}{\sqrt{3x^2-x+2}}-\frac{1}{26\left(x+\frac{1}{2}\right)\left(3\left(x+\frac{1}{2}\right)^2-4x+\frac{5}{4}\right)^{\frac{3}{2}}}+\frac{7}{507\left(3\left(x+\frac{1}{2}\right)^2-4x+\frac{5}{4}\right)^{\frac{3}{2}}}-\frac{128(-1+6x)}{11661\left(3\left(x+\frac{1}{2}\right)^2-4x+\frac{5}{4}\right)^{\frac{3}{2}}}-$

```
[In] int((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2-x+2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3486639*(1318464*x^4+133308*x^3+1021566*x^2+569989*x-170239)/(3*x^2-x+2)^
(3/2)/(1+2*x)-56/28561*13^(1/2)*arctanh(2/13*(9/2-4*x)*13^(1/2)/(12*(x+1/2)
^2-16*x+5)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.28

$$\int \frac{1+3x+4x^2}{(1+2x)^2\left(2-x+3x^2\right)^{5/2}}\,dx=\frac{2\left(22218\sqrt{13}(18x^5-3x^4+20x^3+5x^2+4x+4)\log\left(-\frac{4\sqrt{13}\sqrt{3x^2-x+2}(8}{4}\right.\right.}{45326307}$$

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2-x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/45326307*(22218*sqrt(13)*(18*x^5 - 3*x^4 + 20*x^3 + 5*x^2 + 4*x + 4)*log(
-(4*sqrt(13)*sqrt(3*x^2 - x + 2)*(8*x - 9) + 220*x^2 - 196*x + 185)/(4*x^2
+ 4*x + 1)) + 13*(1318464*x^4 + 133308*x^3 + 1021566*x^2 + 569989*x - 17023
9)*sqrt(3*x^2 - x + 2))/(18*x^5 - 3*x^4 + 20*x^3 + 5*x^2 + 4*x + 4)
```

Sympy [F]

$$\int \frac{1+3x+4x^2}{(1+2x)^2\left(2-x+3x^2\right)^{5/2}}\,dx=\int \frac{4x^2+3x+1}{\left(2x+1\right)^2\left(3x^2-x+2\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)**2/(3*x**2-x+2)**(5/2),x)
```

```
[Out] Integral((4*x**2 + 3*x + 1)/((2*x + 1)**2*(3*x**2 - x + 2)**(5/2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.14

$$\int \frac{1+3x+4x^2}{(1+2x)^2\left(2-x+3x^2\right)^{5/2}}\,dx = \frac{56}{28561}\,\sqrt{13}\,\mathrm{arsinh}\left(\frac{8\,\sqrt{23}x}{23\,|2\,x+1|}-\frac{9\,\sqrt{23}}{23\,|2\,x+1|}\right) + \frac{146496\,x}{1162213\,\sqrt{3\,x^2-x+2}} - \frac{9604}{1162213\,\sqrt{3\,x^2-x+2}} + \frac{420\,x}{3887\,\left(3\,x^2-x+2\right)^{\frac{3}{2}}} - \frac{1}{13\,\left(2\,\left(3\,x^2-x+2\right)^{\frac{3}{2}}x+\left(3\,x^2-x+2\right)^{\frac{3}{2}}\right)} - \frac{49}{11661\,\left(3\,x^2-x+2\right)^{\frac{3}{2}}}$$

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2-x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 56/28561*sqrt(13)*arcsinh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqrt(23)/abs(
2*x + 1)) + 146496/1162213*x/sqrt(3*x^2 - x + 2) - 9604/1162213/sqrt(3*x^2
- x + 2) + 420/3887*x/(3*x^2 - x + 2)^(3/2) - 1/13/(2*(3*x^2 - x + 2)^(3/2)
*x + (3*x^2 - x + 2)^(3/2)) - 49/11661/(3*x^2 - x + 2)^(3/2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. 2(88) = 176.

Time = 0.32 (sec) , antiderivative size = 233, normalized size of antiderivative = 2.12

$$\int \frac{1+3x+4x^2}{(1+2x)^2\left(2-x+3x^2\right)^{5/2}}\,dx = -\frac{56}{15108769}\,\sqrt{13}\left(872\,\sqrt{13}\sqrt{3}-529\,\log\left(\sqrt{13}\sqrt{3}-4\right)\right)\mathrm{sgn}\left(\frac{1}{2\,x+1}\right) - \frac{56\,\sqrt{13}\log\left(\sqrt{13}\left(\sqrt{-\frac{8}{2\,x+1}+\frac{13}{(2\,x+1)^2}+3}+\frac{\sqrt{13}}{2\,x+1}\right)-4\right)}{28561\,\mathrm{sgn}\left(\frac{1}{2\,x+1}\right)} + \frac{8\left(\frac{\frac{\frac{13\left(\frac{77756}{\mathrm{sgn}\left(\frac{1}{2\,x+1}\right)}+\frac{20631}{(2\,x+1)\mathrm{sgn}\left(\frac{1}{2\,x+1}\right)}\right)}{2\,x+1}-\frac{1399650}{\mathrm{sgn}\left(\frac{1}{2\,x+1}\right)}}{2\,x+1}+\frac{625905}{\mathrm{sgn}\left(\frac{1}{2\,x+1}\right)}}{2\,x+1}-\frac{164808}{\mathrm{sgn}\left(\frac{1}{2\,x+1}\right)}\right)}{3486639\left(\frac{8}{2\,x+1}-\frac{13}{(2\,x+1)^2}-3\right)\sqrt{-\frac{8}{2\,x+1}+\frac{13}{(2\,x+1)^2}+3}}$$

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2-x+2)^(5/2),x, algorithm="giac")
```

```
[Out] -56/15108769*sqrt(13)*(872*sqrt(13)*sqrt(3) - 529*log(sqrt(13)*sqrt(3) - 4)
)*sgn(1/(2*x + 1)) - 56/28561*sqrt(13)*log(sqrt(13)*(sqrt(-8/(2*x + 1) + 13
/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1)) - 4)/sgn(1/(2*x + 1)) + 8/3486639*(
((13*(77756/sgn(1/(2*x + 1)) + 20631/((2*x + 1)*sgn(1/(2*x + 1))))/(2*x + 1
) - 1399650/sgn(1/(2*x + 1)))/(2*x + 1) + 625905/sgn(1/(2*x + 1)))/(2*x + 1
) - 164808/sgn(1/(2*x + 1)))/((8/(2*x + 1) - 13/(2*x + 1)^2 - 3)*sqrt(-8/(2
*x + 1) + 13/(2*x + 1)^2 + 3))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+3x+4x^2}{(1+2x)^2\left(2-x+3x^2\right)^{5/2}}\,dx=\int \frac{4\,x^2+3\,x+1}{{\left(2\,x+1\right)}^2\,{\left(3\,x^2-x+2\right)}^{5/2}}\,dx$$

```
[In] int((3*x + 4*x^2 + 1)/((2*x + 1)^2*(3*x^2 - x + 2)^(5/2)),x)
```

```
[Out] int((3*x + 4*x^2 + 1)/((2*x + 1)^2*(3*x^2 - x + 2)^(5/2)), x)
```

3.257 $\int \frac{1+3x+4x^2}{(1+2x)^3\left(2-x+3x^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 135

$$\int \frac{1+3x+4x^2}{(1+2x)^3\left(2-x+3x^2\right)^{5/2}}\,dx=\frac{2(2363+3693x)}{151593\left(2-x+3x^2\right)^{3/2}}+\frac{12(25771+103526x)}{15108769\sqrt{2-x+3x^2}}-\frac{8\sqrt{2-x+3x^2}}{2197(1+2x)^2}-\frac{144\sqrt{2-x+3x^2}}{28561(1+2x)}-\frac{2084\operatorname{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)}{28561\sqrt{13}}$$

```
[Out] 2/151593*(2363+3693*x)/(3*x^2-x+2)^(3/2)-2084/371293*arctanh(1/26*(9-8*x)*1
3^(1/2)/(3*x^2-x+2)^(1/2))*13^(1/2)+12/15108769*(25771+103526*x)/(3*x^2-x+2
)^(1/2)-8/2197*(3*x^2-x+2)^(1/2)/(1+2*x)^2-144/28561*(3*x^2-x+2)^(1/2)/(1+2
*x)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1660, 1664, 820, 738, 212}

$$\int \frac{1+3x+4x^2}{(1+2x)^3\left(2-x+3x^2\right)^{5/2}}\,dx=-\frac{2084\operatorname{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{28561\sqrt{13}}+\frac{2(3693x+2363)}{151593\left(3x^2-x+2\right)^{3/2}}-\frac{144\sqrt{3x^2-x+2}}{28561(2x+1)}-\frac{8\sqrt{3x^2-x+2}}{2197(2x+1)^2}+\frac{12(103526x+25771)}{15108769\sqrt{3x^2-x+2}}$$

```
[In] Int[(1 + 3*x + 4*x^2)/((1 + 2*x)^3*(2 - x + 3*x^2)^(5/2)),x]
```

```
[Out] (2*(2363 + 3693*x))/(151593*(2 - x + 3*x^2)^(3/2)) + (12*(25771 + 103526*x)
)/(15108769*Sqrt[2 - x + 3*x^2]) - (8*Sqrt[2 - x + 3*x^2])/(2197*(1 + 2*x)^
2) - (144*Sqrt[2 - x + 3*x^2])/(28561*(1 + 2*x)) - (2084*ArcTanh[(9 - 8*x)/
(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/(28561*Sqrt[13])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 1660

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^(p + 1)/((p
+ 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
```

```
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2(2363+3693x)}{151593\left(2-x+3x^2\right)^{3/2}}+\frac{2}{69}\int \frac{\frac{32433}{2197}+\frac{106830x}{2197}+\frac{160116x^2}{2197}+\frac{59088x^3}{2197}}{(1+2x)^3\left(2-x+3x^2\right)^{3/2}}\,dx \\
&= \frac{2(2363+3693x)}{151593\left(2-x+3x^2\right)^{3/2}}+\frac{12(25771+103526x)}{15108769\sqrt{2-x+3x^2}}+\frac{4\int \frac{\frac{1434648}{28561}+\frac{3345396x}{28561}+\frac{3097824x^2}{28561}}{(1+2x)^3\sqrt{2-x+3x^2}}\,dx}{1587} \\
&= \frac{2(2363+3693x)}{151593\left(2-x+3x^2\right)^{3/2}}+\frac{12(25771+103526x)}{15108769\sqrt{2-x+3x^2}}-\frac{8\sqrt{2-x+3x^2}}{2197(1+2x)^2}-\frac{2\int \frac{-\frac{2167842}{2197}-\frac{2850252x}{2197}}{(1+2x)^2\sqrt{2-x+3x^2}}\,dx}{20631} \\
&= \frac{2(2363+3693x)}{151593\left(2-x+3x^2\right)^{3/2}}+\frac{12(25771+103526x)}{15108769\sqrt{2-x+3x^2}} \\
&\quad -\frac{8\sqrt{2-x+3x^2}}{2197(1+2x)^2}-\frac{144\sqrt{2-x+3x^2}}{28561(1+2x)}+\frac{2084\int \frac{1}{(1+2x)\sqrt{2-x+3x^2}}\,dx}{28561} \\
&= \frac{2(2363+3693x)}{151593\left(2-x+3x^2\right)^{3/2}}+\frac{12(25771+103526x)}{15108769\sqrt{2-x+3x^2}}-\frac{8\sqrt{2-x+3x^2}}{2197(1+2x)^2} \\
&\quad -\frac{144\sqrt{2-x+3x^2}}{28561(1+2x)}-\frac{4168\,\text{Subst}\left(\int \frac{1}{52-x^2}\,dx,x,\frac{9-8x}{\sqrt{2-x+3x^2}}\right)}{28561} \\
&= \frac{2(2363+3693x)}{151593\left(2-x+3x^2\right)^{3/2}}+\frac{12(25771+103526x)}{15108769\sqrt{2-x+3x^2}}-\frac{8\sqrt{2-x+3x^2}}{2197(1+2x)^2} \\
&\quad -\frac{144\sqrt{2-x+3x^2}}{28561(1+2x)}-\frac{2084\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)}{28561\sqrt{13}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.78

$$\int \frac{1+3x+4x^2}{(1+2x)^3\left(2-x+3x^2\right)^{5/2}}\,dx=\frac{2\sqrt{2-x+3x^2}(847141+10777477x+21890266x^2+19381992x^3+200}{45326307\left(2+3x+x^2+6x^3\right)^2}$$

$$+\frac{4168\,\text{arctanh}\left(\frac{\sqrt{3}+2\sqrt{3}x-2\sqrt{2-x+3x^2}}{\sqrt{13}}\right)}{28561\sqrt{13}}$$

```
[In] Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)^3*(2 - x + 3*x^2)^(5/2)),x]
```

```
[Out] (2*Sqrt[2 - x + 3*x^2]*(847141 + 10777477*x + 21890266*x^2 + 19381992*x^3 +
 20074356*x^4 + 20304864*x^5))/(45326307*(2 + 3*x + x^2 + 6*x^3)^2) + (4168
*ArcTanh[(Sqrt[3] + 2*Sqrt[3]*x - 2*Sqrt[2 - x + 3*x^2])/Sqrt[13]])/(28561*
Sqrt[13])
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.58

method	result
risch	$\frac{\frac{13536576}{15108769}x^5+\frac{13382904}{15108769}x^4+\frac{12921328}{15108769}x^3+\frac{43780532}{45326307}x^2+\frac{21554954}{45326307}x+\frac{1694282}{45326307}}{(1+2x)^2(3x^2-x+2)^{\frac{3}{2}}}-\frac{2084\sqrt{13}\,\operatorname{arctanh}\left(\frac{2\left(\frac{9}{2}-4x\right)\sqrt{13}}{13\sqrt{12\left(x+\frac{1}{2}\right)^2-16x+5}}\right)}{371293}$
trager	$\frac{2(20304864x^5+20074356x^4+19381992x^3+21890266x^2+10777477x+847141)\sqrt{3x^2-x+2}}{45326307(6x^3+x^2+3x+2)^2}+\frac{2084\,\mathrm{RootOf}\left(\_Z^2-13\right)\ln\left(\frac{8\,\mathrm{RootOf}\left(\right.}{}\right.}{}$
default	$\frac{521}{13182\left(3\left(x+\frac{1}{2}\right)^2-4x+\frac{5}{4}\right)^{\frac{3}{2}}}+\frac{-\frac{886}{151593}+\frac{1772x}{50531}}{\left(3\left(x+\frac{1}{2}\right)^2-4x+\frac{5}{4}\right)^{\frac{3}{2}}}+\frac{-\frac{188008}{15108769}+\frac{1128048x}{15108769}}{\sqrt{3\left(x+\frac{1}{2}\right)^2-4x+\frac{5}{4}}}+\frac{1042}{28561\sqrt{3\left(x+\frac{1}{2}\right)^2-4x+\frac{5}{4}}}-\frac{2084\sqrt{13}\,\operatorname{arctanh}\left(\right.}{3}$

```
[In] int((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2-x+2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/45326307*(20304864*x^5+20074356*x^4+19381992*x^3+21890266*x^2+10777477*x+
847141)/(3*x^2-x+2)^(3/2)/(1+2*x)^2-2084/371293*13^(1/2)*arctanh(2/13*(9/2-
4*x)*13^(1/2)/(12*(x+1/2)^2-16*x+5)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.16

$$\int \frac{1+3x+4x^2}{(1+2x)^3\left(2-x+3x^2\right)^{5/2}}\,dx=\frac{2\left(826827\sqrt{13}(36x^6+12x^5+37x^4+30x^3+13x^2+12x+4)\log\left(-\frac{4}{}\right.\right.}{}$$

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2-x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/589241991*(826827*sqrt(13)*(36*x^6 + 12*x^5 + 37*x^4 + 30*x^3 + 13*x^2 +
12*x + 4)*log(-(4*sqrt(13)*sqrt(3*x^2 - x + 2)*(8*x - 9) + 220*x^2 - 196*x
+ 185)/(4*x^2 + 4*x + 1)) + 13*(20304864*x^5 + 20074356*x^4 + 19381992*x^3
+ 21890266*x^2 + 10777477*x + 847141)*sqrt(3*x^2 - x + 2))/(36*x^6 + 12*x^5
 + 37*x^4 + 30*x^3 + 13*x^2 + 12*x + 4)
```

Sympy [F]

$$\int \frac{1+3x+4x^2}{(1+2x)^3 (2-x+3x^2)^{5/2}}\,dx = \int \frac{4x^2+3x+1}{(2x+1)^3 (3x^2-x+2)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)**3/(3*x**2-x+2)**(5/2),x)
```

```
[Out] Integral((4*x**2 + 3*x + 1)/((2*x + 1)**3*(3*x**2 - x + 2)**(5/2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.29

$$\int \frac{1+3x+4x^2}{(1+2x)^3 (2-x+3x^2)^{5/2}}\,dx = \frac{2084}{371293}\sqrt{13}\,\mathrm{arsinh}\left(\frac{8\sqrt{23}x}{23\,|2\,x+1|}-\frac{9\sqrt{23}}{23\,|2\,x+1|}\right) + \frac{1128048\,x}{15108769\sqrt{3\,x^2-x+2}} + \frac{363210}{15108769\sqrt{3\,x^2-x+2}} + \frac{1772\,x}{50531\,(3\,x^2-x+2)^{\frac{3}{2}}} - \frac{1}{26\left(4\,(3\,x^2-x+2)^{\frac{3}{2}}x^2+4\,(3\,x^2-x+2)^{\frac{3}{2}}x+(3\,x^2-x+2)^{\frac{3}{2}}\right)} - \frac{1}{169\left(2\,(3\,x^2-x+2)^{\frac{3}{2}}x+(3\,x^2-x+2)^{\frac{3}{2}}\right)} + \frac{10211}{303186\,(3\,x^2-x+2)^{\frac{3}{2}}}$$

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2-x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 2084/371293*sqrt(13)*arcsinh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqrt(23)/a
bs(2*x + 1)) + 1128048/15108769*x/sqrt(3*x^2 - x + 2) + 363210/15108769/sqr
t(3*x^2 - x + 2) + 1772/50531*x/(3*x^2 - x + 2)^(3/2) - 1/26/(4*(3*x^2 - x
+ 2)^(3/2)*x^2 + 4*(3*x^2 - x + 2)^(3/2)*x + (3*x^2 - x + 2)^(3/2)) - 1/169
/(2*(3*x^2 - x + 2)^(3/2)*x + (3*x^2 - x + 2)^(3/2)) + 10211/303186/(3*x^2
- x + 2)^(3/2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(109) = 218$.

Time = 0.31 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.73

$$\int \frac{1+3x+4x^2}{(1+2x)^3\left(2-x+3x^2\right)^{5/2}}\,dx = \frac{2084}{371293}\sqrt{13}\log\left(-\frac{\left|-4\sqrt{3}x-2\sqrt{13}-2\sqrt{3}+4\sqrt{3\,x^2-x+2}\right|}{2\left(2\sqrt{3}x-\sqrt{13}+\sqrt{3}-2\sqrt{3\,x^2-x+2}\right)}\right) + \frac{2\left(3\left(6\left(310578\,x-26213\right)x+1455755\right)x+1634293\right)}{45326307\left(3\,x^2-x+2\right)^{\frac{3}{2}}} - \frac{8\left(66\left(\sqrt{3}x-\sqrt{3\,x^2-x+2}\right)^3+21\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2-x+2}\right)^2-1015\sqrt{3}x+431\sqrt{3}+1015\sqrt{3\,x^2-x-}\right.}{28561\left(2\left(\sqrt{3}x-\sqrt{3\,x^2-x+2}\right)^2+2\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2-x+2}\right)-5\right)^2}$$

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2-x+2)^(5/2),x, algorithm="giac")
```

```
[Out] 2084/371293*sqrt(13)*log(-1/2*abs(-4*sqrt(3)*x - 2*sqrt(13) - 2*sqrt(3) + 4
*sqrt(3*x^2 - x + 2))/(2*sqrt(3)*x - sqrt(13) + sqrt(3) - 2*sqrt(3*x^2 - x
+ 2))) + 2/45326307*(3*(6*(310578*x - 26213)*x + 1455755)*x + 1634293)/(3*x
^2 - x + 2)^(3/2) - 8/28561*(66*(sqrt(3)*x - sqrt(3*x^2 - x + 2))^3 + 21*sq
rt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2))^2 - 1015*sqrt(3)*x + 431*sqrt(3) +
1015*sqrt(3*x^2 - x + 2))/(2*(sqrt(3)*x - sqrt(3*x^2 - x + 2))^2 + 2*sqrt(3
)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) - 5)^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+3x+4x^2}{(1+2x)^3\left(2-x+3x^2\right)^{5/2}}\,dx = \int \frac{4\,x^2+3\,x+1}{\left(2\,x+1\right)^3\left(3\,x^2-x+2\right)^{5/2}}\,dx$$

```
[In] int((3*x + 4*x^2 + 1)/((2*x + 1)^3*(3*x^2 - x + 2)^(5/2)),x)
```

```
[Out] int((3*x + 4*x^2 + 1)/((2*x + 1)^3*(3*x^2 - x + 2)^(5/2)), x)
```

3.258 $\int \frac{d+ex+fx^2}{(g+hx)\left(-cg^2+bgh+bh^2x+ch^2x^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 47, antiderivative size = 208

$$\begin{aligned}\int \frac{d+ex+fx^2}{(g+hx)\left(-cg^2+bgh+bh^2x+ch^2x^2\right)^{3/2}}\,dx &= \\ &-\frac{f}{ch^3\sqrt{-g(cg-bh)+bh^2x+ch^2x^2}} \\ &+\frac{\left(6bceh^2-3b^2fh^2+4c^2(fg^2-h(eg+2dh))\right)(b+2cx)}{3ch^2(2cg-bh)^3\sqrt{-g(cg-bh)+bh^2x+ch^2x^2}} \\ &+\frac{2(fg^2-egh+dh^2)}{3h^3(2cg-bh)(g+hx)\sqrt{-g(cg-bh)+bh^2x+ch^2x^2}}\end{aligned}$$

```
[Out] -f/c/h^3/(-g*(-b*h+c*g)+b*h^2*x+c*h^2*x^2)^(1/2)+1/3*(6*b*c*e*h^2-3*b^2*f*h
^2+4*c^2*(f*g^2-h*(2*d*h+e*g)))*(2*c*x+b)/c/h^2/(-b*h+2*c*g)^3/(-g*(-b*h+c*
g)+b*h^2*x+c*h^2*x^2)^(1/2)+2/3*(d*h^2-e*g*h+f*g^2)/h^3/(-b*h+2*c*g)/(h*x+g
)/(-g*(-b*h+c*g)+b*h^2*x+c*h^2*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.064, Rules used = {1652, 806, 627}

$$\begin{aligned}\int \frac{d+ex+fx^2}{(g+hx)\left(-cg^2+bgh+bh^2x+ch^2x^2\right)^{3/2}}\,dx &= \frac{(b+2cx)\left(-3b^2fh^2+6bceh^2+4c^2(fg^2-h(2dh+eg))\right)}{3ch^2(2cg-bh)^3\sqrt{-g(cg-bh)+bh^2x+ch^2x^2}} \\ &+\frac{2(dh^2-egh+fg^2)}{3h^3(g+hx)(2cg-bh)\sqrt{-g(cg-bh)+bh^2x+ch^2x^2}}-\frac{f}{ch^3\sqrt{-g(cg-bh)+bh^2x+ch^2x^2}}\end{aligned}$$

```
[In] Int[(d + e*x + f*x^2)/((g + h*x)*(-(c*g^2) + b*g*h + b*h^2*x + c*h^2*x^2)^(
3/2)),x]
```

```
[Out] -(f/(c*h^3*Sqrt[-(g*(c*g - b*h)) + b*h^2*x + c*h^2*x^2])) + ((6*b*c*e*h^2 -
 3*b^2*f*h^2 + 4*c^2*(f*g^2 - h*(e*g + 2*d*h)))*(b + 2*c*x))/(3*c*h^2*(2*c*
g - b*h)^3*Sqrt[-(g*(c*g - b*h)) + b*h^2*x + c*h^2*x^2]) + (2*(f*g^2 - e*g*
h + d*h^2))/(3*h^3*(2*c*g - b*h)*(g + h*x)*Sqrt[-(g*(c*g - b*h)) + b*h^2*x
+ c*h^2*x^2])
```

Rule 627

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 1652

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q + e*f*(m + p + q)*(d + e*x)^(q - 2)*(b*d - 2*a*e + (2*c*d - b
*e)*x), x], x], x] /; NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m,
p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2,
 0]
```

Rubi steps

$$\text{integral} = -\frac{f}{ch^3\sqrt{-g(cg-bh)+bh^2x+ch^2x^2}} - \frac{\int \frac{\frac{1}{2}h^3(bfg-2cdh)+\frac{1}{2}h^3(2cfg-2ceh+bfh)x}{(g+hx)(-cg^2+bgh+bh^2x+ch^2x^2)^{3/2}}\,dx}{ch^4}$$

$$
\begin{aligned}
&= -\frac{f}{ch^3\sqrt{-g(cg-bh)+bh^2x+ch^2x^2}} \\
&\quad+\frac{2(fg^2-egh+dh^2)}{3h^3(2cg-bh)(g+hx)\sqrt{-g(cg-bh)+bh^2x+ch^2x^2}} \\
&\quad-\frac{(6bceh^2-3b^2fh^2+4c^2(fg^2-h(eg+2dh)))\int\frac{1}{(-cg^2+bgh+bh^2x+ch^2x^2)^{3/2}}dx}{6ch^2(2cg-bh)}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{f}{ch^3\sqrt{-g(cg-bh)+bh^2x+ch^2x^2}} \\
&\quad+\frac{(6bceh^2-3b^2fh^2+4c^2(fg^2-h(eg+2dh)))\,(b+2cx)}{3ch^2(2cg-bh)^3\sqrt{-g(cg-bh)+bh^2x+ch^2x^2}} \\
&\quad+\frac{2(fg^2-egh+dh^2)}{3h^3(2cg-bh)(g+hx)\sqrt{-g(cg-bh)+bh^2x+ch^2x^2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.05

$$
\int\frac{d+ex+fx^2}{(g+hx)\,(-cg^2+bgh+bh^2x+ch^2x^2)^{3/2}}\,dx = \frac{2b^2h^2(-h(2eg+dh+3ehx)+f(8g^2+12ghx+3h^2x^2))}{}
$$

```
[In] Integrate[(d + e*x + f*x^2)/((g + h*x)*(-(c*g^2) + b*g*h + b*h^2*x + c*h^2*
x^2)^(3/2)),x]
```

```
[Out] (2*b^2*h^2*(-(h*(2*e*g + d*h + 3*e*h*x)) + f*(8*g^2 + 12*g*h*x + 3*h^2*x^2)
) + 8*c^2*(f*g^2*(2*g^2 + 2*g*h*x - h^2*x^2) + h*(e*g*(g^2 + g*h*x + h^2*x^
2) + d*h*(-g^2 + 2*g*h*x + 2*h^2*x^2))) - 4*b*c*h*(2*f*g^2*(4*g + 5*h*x) +
h*(-2*d*h*(2*g + h*x) + e*(g^2 + 2*g*h*x + 3*h^2*x^2))))/(3*h^3*(-2*c*g + b
*h)^3*(g + h*x)*Sqrt[(g + h*x)*(-(c*g) + b*h + c*h*x)])
```

Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.56

method	result
gosper	$-\frac{2(chx+bh-cg)(-3b^2fh^4x^2+6bceh^4x^2-8c^2dh^4x^2-4c^2egh^3x^2+4c^2fg^2h^2x^2+3b^2eh^4x-12b^2fgh^3x-4bcdh^4x+4bcegh^3x+20b}{3(b^3h^3-6b^2cgh^2+12bc^2g^2}$
trager	$-\frac{2(-3b^2fh^4x^2+6bceh^4x^2-8c^2dh^4x^2-4c^2egh^3x^2+4c^2fg^2h^2x^2+3b^2eh^4x-12b^2fgh^3x-4bcdh^4x+4bcegh^3x+20bcfg^2h^2x-8c^2}{3h^3(b^2h^2-4bcg}$
default	$\frac{\frac{2eh(2ch^2x+bh^2)}{(4ch^2(bgh-cg^2)-b^2h^4)\sqrt{cx^2h^2+bxh^2+bgh-cg^2}}+fh\left(-\frac{1}{ch^2\sqrt{cx^2h^2+bxh^2+bgh-cg^2}}-\frac{b(2ch^2x+bh^2)}{c(4ch^2(bgh-cg^2)-b^2h^4)\sqrt{cx^2h^2+bxh^2+bgh-}}\right.}{h^2}$

```
[In] int((f*x^2+e*x+d)/(h*x+g)/(c*h^2*x^2+b*h^2*x+b*g*h-c*g^2)^(3/2),x,method=_R
ETURNVERBOSE)
```

```
[Out] -2/3*(c*h*x+b*h-c*g)*(-3*b^2*f*h^4*x^2+6*b*c*e*h^4*x^2-8*c^2*d*h^4*x^2-4*c^
2*e*g*h^3*x^2+4*c^2*f*g^2*h^2*x^2+3*b^2*e*h^4*x-12*b^2*f*g*h^3*x-4*b*c*d*h^
4*x+4*b*c*e*g*h^3*x+20*b*c*f*g^2*h^2*x-8*c^2*d*g*h^3*x-4*c^2*e*g^2*h^2*x-8*
c^2*f*g^3*h*x+b^2*d*h^4+2*b^2*e*g*h^3-8*b^2*f*g^2*h^2-8*b*c*d*g*h^3+2*b*c*e
*g^2*h^2+16*b*c*f*g^3*h+4*c^2*d*g^2*h^2-4*c^2*e*g^3*h-8*c^2*f*g^4)/(b^3*h^3
-6*b^2*c*g*h^2+12*b*c^2*g^2*h-8*c^3*g^3)/h^3/(c*h^2*x^2+b*h^2*x+b*g*h-c*g^2
)^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 465 vs. 2(198) = 396.

Time = 13.84 (sec) , antiderivative size = 465, normalized size of antiderivative = 2.24

$$\int \frac{d+ex+fx^2}{(g+hx)\left(-cg^2+bgh+bh^2x+ch^2x^2\right)^{3/2}}\,dx = \frac{2\left(8\,c^2fg^4-b^2dh^4+4\left(c^2e-4\,bcf\right)g^3h-2\left(2\,c^2d+bce-\right.\right.}{3\left(8\,c^4g^6h^3-20\,bc^3g^5h^4+18\,b^2c^2g^4h^5-7\,b^3c\right.}$$

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)/(c*h^2*x^2+b*h^2*x+b*g*h-c*g^2)^(3/2),x, al
gorithm="fricas")
```

```
[Out] 2/3*(8*c^2*f*g^4 - b^2*d*h^4 + 4*(c^2*e - 4*b*c*f)*g^3*h - 2*(2*c^2*d + b*c
*e - 4*b^2*f)*g^2*h^2 + 2*(4*b*c*d - b^2*e)*g*h^3 - (4*c^2*f*g^2*h^2 - 4*c^
2*e*g*h^3 - (8*c^2*d - 6*b*c*e + 3*b^2*f)*h^4)*x^2 + (8*c^2*f*g^3*h + 4*(c^
2*e - 5*b*c*f)*g^2*h^2 + 4*(2*c^2*d - b*c*e + 3*b^2*f)*g*h^3 + (4*b*c*d - 3
*b^2*e)*h^4)*x)*sqrt(c*h^2*x^2 + b*h^2*x - c*g^2 + b*g*h)/(8*c^4*g^6*h^3 -
20*b*c^3*g^5*h^4 + 18*b^2*c^2*g^4*h^5 - 7*b^3*c*g^3*h^6 + b^4*g^2*h^7 - (8*
c^4*g^3*h^6 - 12*b*c^3*g^2*h^7 + 6*b^2*c^2*g*h^8 - b^3*c*h^9)*x^3 - (8*c^4*
g^4*h^5 - 4*b*c^3*g^3*h^6 - 6*b^2*c^2*g^2*h^7 + 5*b^3*c*g*h^8 - b^4*h^9)*x^
2 + (8*c^4*g^5*h^4 - 28*b*c^3*g^4*h^5 + 30*b^2*c^2*g^3*h^6 - 13*b^3*c*g^2*h
^7 + 2*b^4*g*h^8)*x)
```

Sympy [F]

$$\int \frac{d+ex+fx^2}{(g+hx)\left(-cg^2+bgh+bh^2x+ch^2x^2\right)^{3/2}}\,dx = \int \frac{d+ex+fx^2}{\left((g+hx)\left(bh-cg+chx\right)\right)^{\frac{3}{2}}(g+hx)}\,dx$$

```
[In] integrate((f*x**2+e*x+d)/(h*x+g)/(c*h**2*x**2+b*h**2*x+b*g*h-c*g**2)**(3/2)
,x)
```

```
[Out] Integral((d + e*x + f*x**2)/(((g + h*x)*(b*h - c*g + c*h*x))**(3/2)*(g + h*
x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex+fx^2}{(g+hx)\left(-cg^2+bgh+bh^2x+ch^2x^2\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)/(c*h^2*x^2+b*h^2*x+b*g*h-c*g^2)^(3/2),x, al
gorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b*h-2*c*g>0)', see `assume?` for mo
re deta
```

Giac [F]

$$\int \frac{d+ex+fx^2}{(g+hx)\left(-cg^2+bgh+bh^2x+ch^2x^2\right)^{3/2}}\,dx = \int \frac{fx^2+ex+d}{\left(ch^2x^2+bh^2x-cg^2+bgh\right)^{\frac{3}{2}}(hx+g)}\,dx$$

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)/(c*h^2*x^2+b*h^2*x+b*g*h-c*g^2)^(3/2),x, al
gorithm="giac")
```

```
[Out] integrate((f*x^2 + e*x + d)/((c*h^2*x^2 + b*h^2*x - c*g^2 + b*g*h)^(3/2)*(h
*x + g)), x)
```

Mupad [B] (verification not implemented)

Time = 14.73 (sec) , antiderivative size = 1089, normalized size of antiderivative = 5.24

$$\int \frac{d+ex+fx^2}{(g+hx)\left(-cg^2+bgh+bh^2x+ch^2x^2\right)^{3/2}}\,dx = 16\,c^2\,f\,g^4\,\sqrt{-c\,g^2+b\,g\,h+c\,h^2\,x^2+b\,h^2\,x}-2\,b^2\,d\,h^4\,\sqrt{-}$$

```
[In] int((d + e*x + f*x^2)/((g + h*x)*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^(3/2
)),x)
```

```
[Out] (16*c^2*f*g^4*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^(1/2) - 2*b^2*d*h^4*(b*
h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^(1/2) - 8*c^2*d*g^2*h^2*(b*h^2*x - c*g^2
 + c*h^2*x^2 + b*g*h)^(1/2) + 16*b^2*f*g^2*h^2*(b*h^2*x - c*g^2 + c*h^2*x^2
 + b*g*h)^(1/2) + 16*c^2*d*h^4*x^2*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^(1
/2) + 6*b^2*f*h^4*x^2*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^(1/2) - 4*b^2*e
*g*h^3*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^(1/2) + 8*c^2*e*g^3*h*(b*h^2*x
 - c*g^2 + c*h^2*x^2 + b*g*h)^(1/2) - 6*b^2*e*h^4*x*(b*h^2*x - c*g^2 + c*h^
```

```
2*x^2 + b*g*h)^(1/2) + 8*b*c*d*h^4*x*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^
(1/2) - 8*c^2*f*g^2*h^2*x^2*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^(1/2) - 4
*b*c*e*g^2*h^2*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^(1/2) - 12*b*c*e*h^4*x
^2*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^(1/2) + 16*c^2*d*g*h^3*x*(b*h^2*x
- c*g^2 + c*h^2*x^2 + b*g*h)^(1/2) + 24*b^2*f*g*h^3*x*(b*h^2*x - c*g^2 + c*
h^2*x^2 + b*g*h)^(1/2) + 16*c^2*f*g^3*h*x*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*
g*h)^(1/2) + 8*c^2*e*g^2*h^2*x*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^(1/2)
+ 8*c^2*e*g*h^3*x^2*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^(1/2) + 16*b*c*d*
g*h^3*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^(1/2) - 32*b*c*f*g^3*h*(b*h^2*x
 - c*g^2 + c*h^2*x^2 + b*g*h)^(1/2) - 8*b*c*e*g*h^3*x*(b*h^2*x - c*g^2 + c*
h^2*x^2 + b*g*h)^(1/2) - 40*b*c*f*g^2*h^2*x*(b*h^2*x - c*g^2 + c*h^2*x^2 +
b*g*h)^(1/2))/(3*b^4*g^2*h^7 + 24*c^4*g^6*h^3 + 3*b^4*h^9*x^2 - 60*b*c^3*g^
5*h^4 - 21*b^3*c*g^3*h^6 + 3*b^3*c*h^9*x^3 + 24*c^4*g^5*h^4*x + 54*b^2*c^2*
g^4*h^5 - 24*c^4*g^4*h^5*x^2 - 24*c^4*g^3*h^6*x^3 + 6*b^4*g*h^8*x + 18*b^2*
c^2*g^2*h^7*x^2 - 84*b*c^3*g^4*h^5*x - 39*b^3*c*g^2*h^7*x - 15*b^3*c*g*h^8*
x^2 + 90*b^2*c^2*g^3*h^6*x + 12*b*c^3*g^3*h^6*x^2 + 36*b*c^3*g^2*h^7*x^3 -
18*b^2*c^2*g*h^8*x^3)
```

3.259 $\int \sqrt{d+ex}\sqrt{a+bx+cx^2}(A+Bx+Cx^2)\,dx$

Optimal result

Integrand size = 34, antiderivative size = 906

$$\int \sqrt{d+ex}\sqrt{a+bx+cx^2}\left(A+Bx+Cx^2\right)\,dx$$

$$= \frac{2\sqrt{d+ex}(8b^3Ce^3-3bce^2(bCd+4bBe-aCe)+c^3d(8Cd^2-3e(4Bd-7Ae))+3c^2e(ae(Cd-5Be)-b}{}$$

$$-\frac{2(2cCd-3Bce+2bCe)\sqrt{d+ex}(a+bx+cx^2)^{3/2}}{21c^2e}+\frac{2C(d+ex)^{3/2}\left(a+bx+cx^2\right)^{3/2}}{9ce}$$

$$+\frac{\sqrt{2}\sqrt{b^2-4ac}\left(2\left(4c^2d^2-b^2e^2-\frac{3}{2}ce(bd-2ae)\right)\left(8b^2Ce^2-ce(bCd+12bBe+7aCe)-c^2(2Cd^2-3e(\right.\right.}{}$$

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}(cd^2-bde+ae^2)\left(8b^3Ce^3-3c^2e^2(bBd+2aCd-7Abe-10aBe)+3bce^2(bCd-4bBe\right.}{}$$

```
[Out] 2/9*C*(e*x+d)^(3/2)*(c*x^2+b*x+a)^(3/2)/c/e-2/21*(-3*B*c*e+2*C*b*e+2*C*c*d)
*(c*x^2+b*x+a)^(3/2)*(e*x+d)^(1/2)/c^2/e+2/315*(8*b^3*C*e^3-3*b*c*e^2*(4*B*
b*e-C*a*e+C*b*d)+c^3*d*(8*C*d^2-3*e*(-7*A*e+4*B*d))+3*c^2*e*(a*e*(-5*B*e+C*
d)-b*(-7*A*e^2-2*B*d*e+C*d^2))+3*c*e*(8*b^2*C*e^2-c*e*(12*B*b*e+7*C*a*e+C*b
*d)-c^2*(2*C*d^2-3*e*(7*A*e+B*d)))*x)*(e*x+d)^(1/2)*(c*x^2+b*x+a)^(1/2)/c^3
/e^3+1/315*(2*(4*c^2*d^2-b^2*e^2-3/2*c*e*(-2*a*e+b*d))*(8*b^2*C*e^2-c*e*(12
*B*b*e+7*C*a*e+C*b*d)-c^2*(2*C*d^2-3*e*(7*A*e+B*d)))-5*c*e*(-b*e+2*c*d)*(6*
b^2*C*d*e+c*e*(21*A*c*d-3*B*a*e-5*C*a*d)+b*(2*a*C*e^2-c*d*(9*B*e+C*d))))*El
lipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2)
,(-2*e*(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*
```

```
(-4*a*c+b^2)^(1/2)*(e*x+d)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/c^4/
e^4/(c*x^2+b*x+a)^(1/2)/(c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)-
2/315*(a*e^2-b*d*e+c*d^2)*(8*b^3*C*e^3-3*c^2*e^2*(-7*A*b*e-10*B*a*e+B*b*d+2
*C*a*d)+3*b*c*e^2*(-4*B*b*e-9*C*a*e+C*b*d)-2*c^3*d*(8*C*d^2-3*e*(-7*A*e+4*B
*d)))*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)
*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*
2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(e*x+d)
/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c^4/e^4/(e*x+d)^(1/2)/(c*x^2+b*x+a
)^(1/2)
```

Rubi [A] (verified)

Time = 1.59 (sec) , antiderivative size = 905, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.206$, Rules used = {1667, 846, 828, 857, 732, 435, 430}

$$\int \sqrt{d+ex}\sqrt{a+bx+cx^2}\left(A+Bx+Cx^2\right)\,dx$$

$$= \frac{2C(d+ex)^{3/2}\left(cx^2+bx+a\right)^{3/2}}{9ce} - \frac{2(2cCd-3Bce+2bCe)\sqrt{d+ex}(cx^2+bx+a)^{3/2}}{21c^2e}$$

$$+ \frac{2\sqrt{d+ex}((8Cd^3-3de(4Bd-7Ae))\,c^3-3e(bCd^2-be(2Bd+7Ae)-ae(Cd-5Be))\,c^2-3be^2(bCd+}{}$$

$$- \frac{\sqrt{2}\sqrt{b^2-4ac}\big(5ce(2cd-be)\,(6Cdeb^2+2aCe^2b-cd(Cd+9Be)b+ce(21Acd-5aCd-3aBe))-2\big(4c}{}$$

$$- \frac{2\sqrt{2}\sqrt{b^2-4ac}(cd^2-bed+ae^2)\,(-2(8Cd^3-3de(4Bd-7Ae))\,c^3-3e^2(bBd+2aCd-7Abe-10aBe)}{}$$

```
[In] Int[Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2),x]
```

```
[Out] (2*Sqrt[d + e*x]*(8*b^3*C*e^3 - 3*b*c*e^2*(b*C*d + 4*b*B*e - a*C*e) + c^3*(
8*C*d^3 - 3*d*e*(4*B*d - 7*A*e)) - 3*c^2*e*(b*C*d^2 - b*e*(2*B*d + 7*A*e) -
 a*e*(C*d - 5*B*e)) + 3*c*e*(8*b^2*C*e^2 - c*e*(b*C*d + 12*b*B*e + 7*a*C*e)
 - c^2*(2*C*d^2 - 3*e*(B*d + 7*A*e)))*x)*Sqrt[a + b*x + c*x^2])/(315*c^3*e^
3) - (2*(2*c*C*d - 3*B*c*e + 2*b*C*e)*Sqrt[d + e*x]*(a + b*x + c*x^2)^(3/2)
)/(21*c^2*e) + (2*C*(d + e*x)^(3/2)*(a + b*x + c*x^2)^(3/2))/(9*c*e) - (Sqr
t[2]*Sqrt[b^2 - 4*a*c]*(5*c*e*(2*c*d - b*e)*(6*b^2*C*d*e + 2*a*b*C*e^2 - b*
c*d*(C*d + 9*B*e) + c*e*(21*A*c*d - 5*a*C*d - 3*a*B*e)) - 2*(4*c^2*d^2 - b^
2*e^2 - (3*c*e*(b*d - 2*a*e))/2)*(8*b^2*C*e^2 - c*e*(b*C*d + 12*b*B*e + 7*a
*C*e) - c^2*(2*C*d^2 - 3*e*(B*d + 7*A*e))))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*
```

```
x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] +
2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b +
Sqrt[b^2 - 4*a*c])*e)])/(315*c^4*e^4*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[
b^2 - 4*a*c])*e)]*Sqrt[a + b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c*
d^2 - b*d*e + a*e^2)*(8*b^3*C*e^3 - 3*c^2*e^2*(b*B*d + 2*a*C*d - 7*A*b*e -
10*a*B*e) + 3*b*c*e^2*(b*C*d - 4*b*B*e - 9*a*C*e) - 2*c^3*(8*C*d^3 - 3*d*e*
(4*B*d - 7*A*e)))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*S
qrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt
[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e
)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(315*c^4*e^4*Sqrt[d + e*x]*Sqrt[a +
 b*x + c*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
```

```
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 846

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2C(d+ex)^{3/2}\left(a+bx+cx^2\right)^{3/2}}{9ce}\\ &\quad+\frac{2\int\sqrt{d+ex}\left(-\frac{3}{2}e(bCd-3Ace+aCe)-\frac{3}{2}e(2cCd-3Bce+2bCe)x\right)\sqrt{a+bx+cx^2}\,dx}{9ce^2}\end{aligned}$$

$$= -\frac{2(2cCd-3Bce+2bCe)\sqrt{d+ex}(a+bx+cx^2)^{3/2}}{21c^2e}+\frac{2C(d+ex)^{3/2}(a+bx+cx^2)^{3/2}}{9ce}$$
$$+\frac{4\int\frac{(\frac{3}{4}e(6b^2Cde+2abCe^2-bcd(Cd+9Be)+ce(21Acd-5aCd-3aBe))+\frac{3}{4}e(8b^2Ce^2-ce(bCd+12bBe+7aCe)-c^2(2Cd^2-3e(Bd+7}{\sqrt{d+ex}}}{63c^2e^2}$$

$$= \frac{2\sqrt{d+ex}(8b^3Ce^3-3bce^2(bCd+4bBe-aCe)+c^3(8Cd^3-3de(4Bd-7Ae))-3c^2e(bCd^2-be(}{}$$
$$-\frac{2(2cCd-3Bce+2bCe)\sqrt{d+ex}(a+bx+cx^2)^{3/2}}{21c^2e}$$
$$+\frac{2C(d+ex)^{3/2}(a+bx+cx^2)^{3/2}}{9ce}$$
$$-\frac{8\int\frac{\frac{3}{8}e(5ce(bd-2ae)(6b^2Cde+2abCe^2-bcd(Cd+9Be)+ce(21Acd-5aCd-3aBe))-2(\frac{1}{2}bd(4cd-be)-ae(cd+\frac{be}{2}))(8b^2Ce^2-ce(b}{}}{}$$

$$= \frac{2\sqrt{d+ex}(8b^3Ce^3-3bce^2(bCd+4bBe-aCe)+c^3(8Cd^3-3de(4Bd-7Ae))-3c^2e(bCd^2-be(}{}$$
$$-\frac{2(2cCd-3Bce+2bCe)\sqrt{d+ex}(a+bx+cx^2)^{3/2}}{21c^2e}$$
$$+\frac{2C(d+ex)^{3/2}(a+bx+cx^2)^{3/2}}{9ce}$$
$$-\frac{((cd^2-bde+ae^2)(8b^3Ce^3-3c^2e^2(bBd+2aCd-7Abe-10aBe)+3bce^2(bCd-4bBe-9aC}{315c^3e^4}$$
$$-\frac{(5ce(2cd-be)(6b^2Cde+2abCe^2-bcd(Cd+9Be)+ce(21Acd-5aCd-3aBe))-2(4c^2d^2-}{}$$

$$= \frac{2\sqrt{d+ex}(8b^3Ce^3-3bce^2(bCd+4bBe-aCe)+c^3(8Cd^3-3de(4Bd-7Ae))-3c^2e(bCd^2-be(}{}$$
$$-\frac{2(2cCd-3Bce+2bCe)\sqrt{d+ex}(a+bx+cx^2)^{3/2}}{21c^2e}$$
$$+\frac{2C(d+ex)^{3/2}(a+bx+cx^2)^{3/2}}{9ce}$$
$$-\frac{(\sqrt{2}\sqrt{b^2-4ac}(5ce(2cd-be)(6b^2Cde+2abCe^2-bcd(Cd+9Be)+ce(21Acd-5aCd-3aB}{}$$
$$-\frac{(2\sqrt{2}\sqrt{b^2-4ac}(cd^2-bde+ae^2)(8b^3Ce^3-3c^2e^2(bBd+2aCd-7Abe-10aBe)+3bce^2(bC}{}$$

$$= \frac{2\sqrt{d+ex}(8b^3Ce^3-3bce^2(bCd+4bBe-aCe)+c^3(8Cd^3-3de(4Bd-7Ae))-3c^2e(bCd^2-be(2}{}$$

$$- \frac{2(2cCd-3Bce+2bCe)\sqrt{d+ex}\left(a+bx+cx^2\right)^{3/2}}{21c^2e}$$

$$+ \frac{2C(d+ex)^{3/2}\left(a+bx+cx^2\right)^{3/2}}{9ce}$$

$$- \frac{\sqrt{2}\sqrt{b^2-4ac}\left(5ce(2cd-be)\left(6b^2Cde+2abCe^2-bcd(Cd+9Be)+ce(21Acd-5aCd-3aBe)\right)}{}$$

$$- \frac{2\sqrt{2}\sqrt{b^2-4ac}\left(cd^2-bde+ae^2\right)\left(8b^3Ce^3-3c^2e^2(bBd+2aCd-7Abe-10aBe)+3bce^2(bCd-\right.}{}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 35.80 (sec) , antiderivative size = 15669, normalized size of antiderivative = 17.29

$$\int \sqrt{d+ex}\sqrt{a+bx+cx^2}\left(A+Bx+Cx^2\right)\,dx = \text{Result too large to show}$$

```
[In] Integrate[Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2),x]
```

```
[Out] Result too large to show
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1735 vs. 2(834) = 1668.

Time = 3.46 (sec) , antiderivative size = 1736, normalized size of antiderivative = 1.92

method	result	size
elliptic	Expression too large to display	1736
risch	Expression too large to display	7113
default	Expression too large to display	19955

```
[In] int((e*x+d)^(1/2)*(C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE
)
```

```
[Out] ((e*x+d)*(c*x^2+b*x+a))^(1/2)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)*(2/9*C*x^3*
(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)+2/7*(B*c*e+C*b*e+C*c*d-2/9*
(4*b*e+4*c*d)*C)/c/e*x^2*(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)+2/
5*(A*c*e+B*b*e+B*c*d+C*a*e+C*b*d-2/9*C*(7/2*a*e+7/2*b*d)-2/7*(B*c*e+C*b*e+C
*c*d-2/9*(4*b*e+4*c*d)*C)/c/e*(3*b*e+3*c*d))/c/e*x*(c*e*x^3+b*e*x^2+c*d*x^2
+a*e*x+b*d*x+a*d)^(1/2)+2/3*(A*b*e+A*c*d+B*a*e+B*b*d+1/3*C*a*d-2/7*(B*c*e+C
*b*e+C*c*d-2/9*(4*b*e+4*c*d)*C)/c/e*(5/2*a*e+5/2*b*d)-2/5*(A*c*e+B*b*e+B*c*
d+C*a*e+C*b*d-2/9*C*(7/2*a*e+7/2*b*d)-2/7*(B*c*e+C*b*e+C*c*d-2/9*(4*b*e+4*c
*d)*C)/c/e*(3*b*e+3*c*d))/c/e*(2*b*e+2*c*d))/c/e*(c*e*x^3+b*e*x^2+c*d*x^2+a
*e*x+b*d*x+a*d)^(1/2)+2*(d*A*a-2/5*(A*c*e+B*b*e+B*c*d+C*a*e+C*b*d-2/9*C*(7/
2*a*e+7/2*b*d)-2/7*(B*c*e+C*b*e+C*c*d-2/9*(4*b*e+4*c*d)*C)/c/e*(3*b*e+3*c*d
))/c/e*a*d-2/3*(A*b*e+A*c*d+B*a*e+B*b*d+1/3*C*a*d-2/7*(B*c*e+C*b*e+C*c*d-2/
9*(4*b*e+4*c*d)*C)/c/e*(5/2*a*e+5/2*b*d)-2/5*(A*c*e+B*b*e+B*c*d+C*a*e+C*b*d
-2/9*C*(7/2*a*e+7/2*b*d)-2/7*(B*c*e+C*b*e+C*c*d-2/9*(4*b*e+4*c*d)*C)/c/e*(3
*b*e+3*c*d))/c/e*(2*b*e+2*c*d))/c/e*(1/2*a*e+1/2*b*d))*(d/e-1/2*(b+(-4*a*c+
b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c
*(-b+(-4*a*c+b^2)^(1/2)))/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1
/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*
e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)*EllipticF(((x+d/e)/(d/e-1/2*(b
+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e-1
/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+2*(a*A*e+A*b*d+B*a*d-4/7*(B*c*e+C*b*e
+C*c*d-2/9*(4*b*e+4*c*d)*C)/c/e*a*d-2/5*(A*c*e+B*b*e+B*c*d+C*a*e+C*b*d-2/9*
C*(7/2*a*e+7/2*b*d)-2/7*(B*c*e+C*b*e+C*c*d-2/9*(4*b*e+4*c*d)*C)/c/e*(3*b*e+
3*c*d))/c/e*(3/2*a*e+3/2*b*d)-2/3*(A*b*e+A*c*d+B*a*e+B*b*d+1/3*C*a*d-2/7*(B
*c*e+C*b*e+C*c*d-2/9*(4*b*e+4*c*d)*C)/c/e*(5/2*a*e+5/2*b*d)-2/5*(A*c*e+B*b*
e+B*c*d+C*a*e+C*b*d-2/9*C*(7/2*a*e+7/2*b*d)-2/7*(B*c*e+C*b*e+C*c*d-2/9*(4*b
*e+4*c*d)*C)/c/e*(3*b*e+3*c*d))/c/e*(2*b*e+2*c*d))/c/e*(b*e+c*d))*(d/e-1/2*
(b+(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2
)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^
(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c)
)^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)*((-d/e-1/2/c*(-b+(-
4*a*c+b^2)^(1/2)))*EllipticE(((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(
1/2),((-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2
))))^(1/2))+1/2/c*(-b+(-4*a*c+b^2)^(1/2))*EllipticF(((x+d/e)/(d/e-1/2*(b+(-
4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e-1/2/
c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 1023, normalized size of antiderivative = 1.13

$$\int \sqrt{d+ex}\sqrt{a+bx+cx^2}\left(A+Bx+Cx^2\right)\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)^(1/2)*(C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2),x, algorithm="fri
cas")
```

```
[Out] 2/945*((16*C*c^5*d^5 - 8*(2*C*b*c^4 + 3*B*c^5)*d^4*e - (5*C*b^2*c^3 - 42*A*
c^5 - 3*(10*C*a + 9*B*b)*c^4)*d^3*e^2 - (5*C*b^3*c^2 + 3*(22*B*a + 21*A*b)*
c^4 - 3*(7*C*a*b + 4*B*b^2)*c^3)*d^2*e^3 - (16*C*b^4*c - 378*A*a*c^4 + 3*(2
2*C*a^2 + 41*B*a*b + 21*A*b^2)*c^3 - 3*(28*C*a*b^2 + 9*B*b^3)*c^2)*d*e^4 +
(16*C*b^5 - 9*(10*B*a^2 + 21*A*a*b)*c^3 + 3*(41*C*a^2*b + 41*B*a*b^2 + 14*A
*b^3)*c^2 - 24*(4*C*a*b^3 + B*b^4)*c)*e^5)*sqrt(c*e)*weierstrassPInverse(4/
3*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b
*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(c^3*e^3),
1/3*(3*c*e*x + c*d + b*e)/(c*e)) + 3*(16*C*c^5*d^4*e - 8*(C*b*c^4 + 3*B*c^5
)*d^3*e^2 - 3*(2*C*b^2*c^3 - 14*A*c^5 - (6*C*a + 5*B*b)*c^4)*d^2*e^3 - (8*C
*b^3*c^2 + 6*(8*B*a + 7*A*b)*c^4 - 15*(2*C*a*b + B*b^2)*c^3)*d*e^4 + (16*C*
b^4*c - 126*A*a*c^4 + 3*(14*C*a^2 + 29*B*a*b + 14*A*b^2)*c^3 - 24*(3*C*a*b^
2 + B*b^3)*c^2)*e^5)*sqrt(c*e)*weierstrassZeta(4/3*(c^2*d^2 - b*c*d*e + (b^
2 - 3*a*c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*
a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(c^3*e^3), weierstrassPInverse(4/3*(c
^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2
*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(c^3*e^3), 1/3*
(3*c*e*x + c*d + b*e)/(c*e))) + 3*(35*C*c^5*e^5*x^3 + 8*C*c^5*d^3*e^2 - 3*(
C*b*c^4 + 4*B*c^5)*d^2*e^3 - (3*C*b^2*c^3 - 21*A*c^5 - 2*(4*C*a + 3*B*b)*c^
4)*d*e^4 + (8*C*b^3*c^2 + 3*(10*B*a + 7*A*b)*c^4 - 3*(9*C*a*b + 4*B*b^2)*c^
3)*e^5 + 5*(C*c^5*d*e^4 + (C*b*c^4 + 9*B*c^5)*e^5)*x^2 - (6*C*c^5*d^2*e^3 -
 (2*C*b*c^4 + 9*B*c^5)*d*e^4 + (6*C*b^2*c^3 - 63*A*c^5 - (14*C*a + 9*B*b)*c
^4)*e^5)*x)*sqrt(c*x^2 + b*x + a)*sqrt(e*x + d))/(c^5*e^5)
```

Sympy [F]

$$\int \sqrt{d+ex}\sqrt{a+bx+cx^2}\left(A+Bx+Cx^2\right)\,dx = \int \sqrt{d+ex}\left(A+Bx+Cx^2\right)\sqrt{a+bx+cx^2}\,dx$$

```
[In] integrate((e*x+d)**(1/2)*(C*x**2+B*x+A)*(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral(sqrt(d + e*x)*(A + B*x + C*x**2)*sqrt(a + b*x + c*x**2), x)
```

Maxima [F]

$$\int \sqrt{d+ex}\sqrt{a+bx+cx^2}\left(A+Bx+Cx^2\right)\,dx$$
$$=\int \left(Cx^2+Bx+A\right)\sqrt{cx^2+bx+a}\sqrt{ex+d}\,dx$$

```
[In] integrate((e*x+d)^(1/2)*(C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)*sqrt(e*x + d), x)
```

Giac [F]

$$\int \sqrt{d+ex}\sqrt{a+bx+cx^2}\left(A+Bx+Cx^2\right)\,dx$$
$$=\int \left(Cx^2+Bx+A\right)\sqrt{cx^2+bx+a}\sqrt{ex+d}\,dx$$

```
[In] integrate((e*x+d)^(1/2)*(C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)*sqrt(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d+ex}\sqrt{a+bx+cx^2}\left(A+Bx+Cx^2\right)\,dx$$
$$=\int \sqrt{d+e\,x}\,\left(C\,x^2+B\,x+A\right)\,\sqrt{c\,x^2+b\,x+a}\,dx$$

```
[In] int((d + e*x)^(1/2)*(A + B*x + C*x^2)*(a + b*x + c*x^2)^(1/2),x)
```

```
[Out] int((d + e*x)^(1/2)*(A + B*x + C*x^2)*(a + b*x + c*x^2)^(1/2), x)
```

3.260 $\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{\sqrt{d+ex}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 668

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{\sqrt{d+ex}}\,dx =$$

$$-\frac{2\sqrt{d+ex}(5ce(3bCd-7Ace+aCe)-(4cd-be)(6cCd-7Bce+4bCe)+3ce(6cCd-7Bce+4bCe)x}{105c^2e^3}$$

$$+\frac{2C\sqrt{d+ex}(a+bx+cx^2)^{3/2}}{7ce}$$

$$+\frac{\sqrt{2}\sqrt{b^2-4ac}(5ce(2cd-be)(3bCd-7Ace+aCe)-(6cCd-7Bce+4bCe)\left(8c^2d^2-2b^2e^2-3ce(bd-\right.}{105c^3e^4\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{a+}}$$

$$+\frac{2\sqrt{2}\sqrt{b^2-4ac}(cd^2-bde+ae^2)\left(4b^2Ce^2+ce(8bCd-7bBe-10aCe)+c^2(48Cd^2-14e(4Bd-5Ae))\right)}{105c^3e^4\sqrt{d+ex}\sqrt{a+}}$$

```
[Out] 2/7*C*(c*x^2+b*x+a)^(3/2)*(e*x+d)^(1/2)/c/e-2/105*(5*c*e*(-7*A*c*e+C*a*e+3*
C*b*d)-(-b*e+4*c*d)*(-7*B*c*e+4*C*b*e+6*C*c*d)+3*c*e*(-7*B*c*e+4*C*b*e+6*C*
c*d)*x)*(e*x+d)^(1/2)*(c*x^2+b*x+a)^(1/2)/c^2/e^3+1/105*(5*c*e*(-b*e+2*c*d)
*(-7*A*c*e+C*a*e+3*C*b*d)-(-7*B*c*e+4*C*b*e+6*C*c*d)*(8*c^2*d^2-2*b^2*e^2-3
*c*e*(-2*a*e+b*d)))*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2
)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1
/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(e*x+d)^(1/2)*(-c*(c*x^2+b*x+a)/(-
4*a*c+b^2))^(1/2)/c^3/e^4/(c*x^2+b*x+a)^(1/2)/(c*(e*x+d)/(2*c*d-e*(b+(-4*a*
c+b^2)^(1/2))))^(1/2)+2/105*(a*e^2-b*d*e+c*d^2)*(4*b^2*C*e^2+c*e*(-7*B*b*e-
10*C*a*e+8*C*b*d)+c^2*(48*C*d^2-14*e*(-5*A*e+4*B*d)))*EllipticF(1/2*((b+2*c
```

```
*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)
^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*
(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(
1/2))))^(1/2)/c^3/e^4/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 668, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1667, 828, 857, 732, 435, 430}

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{\sqrt{d+ex}}\,dx$$

$$= \frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(ae^2-bde+cd^2)\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}(ce(-10aCe-7bBe+8bCd)+c^2(48C}{105c^3e^4\sqrt{d+ex}\sqrt{a+}}$$

$$+\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(5ce(2cd-be)(aCe-7Ace+3bCd)-(-3ce(bd-2ae)-2b^2e^2+}{105c^3e^4\sqrt{a+bx+cx^2}\sqrt{\frac{c}{2cd-e(}}}$$

$$-\frac{2\sqrt{d+ex}\sqrt{a+bx+cx^2}(5ce(aCe-7Ace+3bCd)+3cex(4bCe-7Bce+6cCd)-(4cd-be)(4bCe-}{105c^2e^3}$$

$$+\frac{2C\sqrt{d+ex}(a+bx+cx^2)^{3/2}}{7ce}$$

```
[In] Int[(Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2))/Sqrt[d + e*x],x]
```

```
[Out] (-2*Sqrt[d + e*x]*(5*c*e*(3*b*C*d - 7*A*c*e + a*C*e) - (4*c*d - b*e)*(6*c*C
*d - 7*B*c*e + 4*b*C*e) + 3*c*e*(6*c*C*d - 7*B*c*e + 4*b*C*e)*x)*Sqrt[a + b
*x + c*x^2])/(105*c^2*e^3) + (2*C*Sqrt[d + e*x]*(a + b*x + c*x^2)^(3/2))/(7
*c*e) + (Sqrt[2]*Sqrt[b^2 - 4*a*c]*(5*c*e*(2*c*d - b*e)*(3*b*C*d - 7*A*c*e
+ a*C*e) - (6*c*C*d - 7*B*c*e + 4*b*C*e)*(8*c^2*d^2 - 2*b^2*e^2 - 3*c*e*(b*
d - 2*a*e)))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*Ell
ipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[
2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(105*c^
3*e^4*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a + b*x
+ c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)*(4*b^2*C*e
^2 + c*e*(8*b*C*d - 7*b*B*e - 10*a*C*e) + c^2*(48*C*d^2 - 14*e*(4*B*d - 5*A
*e)))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((c*(a
+ b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c
] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (
```

```
b + Sqrt[b^2 - 4*a*c])*e)])/(105*c^3*e^4*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2
])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
```

```
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\text{integral} = \frac{2C\sqrt{d+ex}\left(a+bx+cx^2\right)^{3/2}}{7ce} + \frac{2\int \frac{\left(-\frac{1}{2}e(3bCd-7Ace+aCe)-\frac{1}{2}e(6cCd-7Bce+4bCe)x\right)\sqrt{a+bx+cx^2}}{\sqrt{d+ex}}\,dx}{7ce^2}$$

$$= -\frac{2\sqrt{d+ex}(5ce(3bCd-7Ace+aCe)-(4cd-be)(6cCd-7Bce+4bCe)+3ce(6cCd-7Bce}{105c^2e^3} + \frac{2C\sqrt{d+ex}\left(a+bx+cx^2\right)^{3/2}}{7ce} - \frac{4\int \frac{-\frac{1}{4}e\left(5ce(bd-2ae)(3bCd-7Ace+aCe)-(6cCd-7Bce+4bCe)\left(4bcd^2-b^2de-2acde-abe^2\right)\right)-\frac{1}{4}e\left(5ce(2cd-be)(3bCd-7Ace+aC\right.}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}}{105c^2e^4}$$

$$= -\frac{2\sqrt{d+ex}(5ce(3bCd-7Ace+aCe)-(4cd-be)(6cCd-7Bce+4bCe)+3ce(6cCd-7Bce}{105c^2e^3} + \frac{2C\sqrt{d+ex}\left(a+bx+cx^2\right)^{3/2}}{7ce} + \frac{(5ce(2cd-be)(3bCd-7Ace+aCe)-(6cCd-7Bce+4bCe)\left(8c^2d^2-2b^2e^2-3ce(bd-2ae)\right)}{105c^2e^4} + \frac{\left(\left(cd^2-bde+ae^2\right)\left(4b^2Ce^2+ce(8bCd-7bBe-10aCe)+c^2(48Cd^2-14e(4Bd-5Ae))\right)\right)\int}{105c^2e^4}$$

$$=$$

$$-\frac{2\sqrt{d+ex}(5ce(3bCd-7Ace+aCe)-(4cd-be)(6cCd-7Bce+4bCe)+3ce(6cCd-7Bce+}{105c^2e^3}$$

$$+\frac{2C\sqrt{d+ex}(a+bx+cx^2)^{3/2}}{7ce}$$

$$+\frac{\Big(\sqrt{2}\sqrt{b^2-4ac}(5ce(2cd-be)(3bCd-7Ace+aCe)-(6cCd-7Bce+4bCe)\left(8c^2d^2-2b^2e^2-\right.}{105c^3e^4\sqrt{\frac{c(d+ex)}{2cd-be-\sqrt{b^2-4}}}}$$

$$+\frac{\Big(2\sqrt{2}\sqrt{b^2-4ac}(cd^2-bde+ae^2)\left(4b^2Ce^2+ce(8bCd-7bBe-10aCe)+c^2(48Cd^2-14e(4Bd\right.}{105c^3e^4\sqrt{d+}}$$

$$=$$

$$-\frac{2\sqrt{d+ex}(5ce(3bCd-7Ace+aCe)-(4cd-be)(6cCd-7Bce+4bCe)+3ce(6cCd-7Bce+}{105c^2e^3}$$

$$+\frac{2C\sqrt{d+ex}(a+bx+cx^2)^{3/2}}{7ce}$$

$$+\frac{\sqrt{2}\sqrt{b^2-4ac}(5ce(2cd-be)(3bCd-7Ace+aCe)-(6cCd-7Bce+4bCe)\left(8c^2d^2-2b^2e^2-3c\right.}{105c^3e^4\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}}$$

$$+\frac{2\sqrt{2}\sqrt{b^2-4ac}(cd^2-bde+ae^2)\left(4b^2Ce^2+ce(8bCd-7bBe-10aCe)+c^2(48Cd^2-14e(4Bd-\right.}{105c^3e^4\sqrt{d+ex}\sqrt{}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 35.38 (sec) , antiderivative size = 9965, normalized size of antiderivative = 14.92

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{\sqrt{d+ex}}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2))/Sqrt[d + e*x],x]
[Out] Result too large to show
```

Maple [A] (verified)

Time = 2.45 (sec) , antiderivative size = 1202, normalized size of antiderivative = 1.80

method	result	size
elliptic	Expression too large to display	1202
risch	Expression too large to display	4505
default	Expression too large to display	12761

```
[In] int((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(1/2),x,method=_RETURNVERBOSE
)
```

```
[Out] ((e*x+d)*(c*x^2+b*x+a))^(1/2)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)*(2/7*C/e*x^
2*(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)+2/5*(B*c+b*C-2/7*C/e*(3*b
*e+3*c*d))/c/e*x*(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)+2/3*(A*c+B
*b+C*a-2/7*C/e*(5/2*a*e+5/2*b*d)-2/5*(B*c+b*C-2/7*C/e*(3*b*e+3*c*d))/c/e*(2
*b*e+2*c*d))/c/e*(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)+2*(a*A-2/5
*(B*c+b*C-2/7*C/e*(3*b*e+3*c*d))/c/e*a*d-2/3*(A*c+B*b+C*a-2/7*C/e*(5/2*a*e+
5/2*b*d)-2/5*(B*c+b*C-2/7*C/e*(3*b*e+3*c*d))/c/e*(2*b*e+2*c*d))/c/e*(1/2*a*
e+1/2*b*d))*(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(b+(-4*a*c
+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d/e-1/2/c*(-b+(
-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e+1/2*(b+(
-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2
)*EllipticF(((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2*(
b+(-4*a*c+b^2)^(1/2))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+2*(A*
b+B*a-4/7*a*d/e*C-2/5*(B*c+b*C-2/7*C/e*(3*b*e+3*c*d))/c/e*(3/2*a*e+3/2*b*d)
-2/3*(A*c+B*b+C*a-2/7*C/e*(5/2*a*e+5/2*b*d)-2/5*(B*c+b*C-2/7*C/e*(3*b*e+3*c
*d))/c/e*(2*b*e+2*c*d))/c/e*(b*e+c*d))*(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*(
(x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2
)^(1/2)))/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^
2)^(1/2))/c)/(-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*e*x^3+b*e*x^2+c*
d*x^2+a*e*x+b*d*x+a*d)^(1/2)*((-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*Elliptic
E(((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2*(b+(-4*a*c+
b^2)^(1/2))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+1/2/c*(-b+(-4*a
*c+b^2)^(1/2))*EllipticF(((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)
,((-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))
^(1/2))))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 764, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{\sqrt{d+ex}}\,dx$$

$$= \frac{2\Big((48\,Cc^4d^4 - 8\,(5\,Cbc^3 + 7\,Bc^4)d^3e - (10\,Cb^2c^2 - 70\,Ac^4 - (62\,Ca + 49\,Bb)c^3)d^2e^2 - (5\,Cb^3c + 14\,(6\,Bc}{}$$

```
[In] integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(1/2),x, algorithm="fri
cas")
```

```
[Out] 2/315*((48*C*c^4*d^4 - 8*(5*C*b*c^3 + 7*B*c^4)*d^3*e - (10*C*b^2*c^2 - 70*A
*c^4 - (62*C*a + 49*B*b)*c^3)*d^2*e^2 - (5*C*b^3*c + 14*(6*B*a + 5*A*b)*c^3
 - 2*(11*C*a*b + 7*B*b^2)*c^2)*d*e^3 - (8*C*b^4 - 210*A*a*c^3 + (30*C*a^2 +
 63*B*a*b + 35*A*b^2)*c^2 - (41*C*a*b^2 + 14*B*b^3)*c)*e^4)*sqrt(c*e)*weier
strassPInverse(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/27
*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)
*e^3)/(c^3*e^3), 1/3*(3*c*e*x + c*d + b*e)/(c*e)) + 3*(48*C*c^4*d^3*e - 8*(
2*C*b*c^3 + 7*B*c^4)*d^2*e^2 - (9*C*b^2*c^2 - 70*A*c^4 - (26*C*a + 21*B*b)*
c^3)*d*e^3 - (8*C*b^3*c + 7*(6*B*a + 5*A*b)*c^3 - (29*C*a*b + 14*B*b^2)*c^2
)*e^4)*sqrt(c*e)*weierstrassZeta(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2
)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 +
 (2*b^3 - 9*a*b*c)*e^3)/(c^3*e^3), weierstrassPInverse(4/3*(c^2*d^2 - b*c*d
*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^
2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(c^3*e^3), 1/3*(3*c*e*x + c*d
 + b*e)/(c*e))) + 3*(15*C*c^4*e^4*x^2 + 24*C*c^4*d^2*e^2 - (5*C*b*c^3 + 28*
B*c^4)*d*e^3 - (4*C*b^2*c^2 - 35*A*c^4 - (10*C*a + 7*B*b)*c^3)*e^4 - 3*(6*C
*c^4*d*e^3 - (C*b*c^3 + 7*B*c^4)*e^4)*x)*sqrt(c*x^2 + b*x + a)*sqrt(e*x + d
))/(c^4*e^5)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{\sqrt{d+ex}}\,dx = \int \frac{(A+Bx+Cx^2)\sqrt{a+bx+cx^2}}{\sqrt{d+ex}}\,dx$$

```
[In] integrate((C*x**2+B*x+A)*(c*x**2+b*x+a)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] Integral((A + B*x + C*x**2)*sqrt(a + b*x + c*x**2)/sqrt(d + e*x), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{\sqrt{d+ex}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{cx^2+bx+a}}{\sqrt{ex+d}}\,dx$$

```
[In] integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)/sqrt(e*x + d), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{\sqrt{d+ex}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{cx^2+bx+a}}{\sqrt{ex+d}}\,dx$$

```
[In] integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)/sqrt(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{\sqrt{d+ex}}\,dx = \int \frac{(C\,x^2+B\,x+A)\,\sqrt{c\,x^2+b\,x+a}}{\sqrt{d+e\,x}}\,dx$$

```
[In] int(((A + B*x + C*x^2)*(a + b*x + c*x^2)^(1/2))/(d + e*x)^(1/2),x)
```

```
[Out] int(((A + B*x + C*x^2)*(a + b*x + c*x^2)^(1/2))/(d + e*x)^(1/2), x)
```

3.261 $\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{3/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 749

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{3/2}}\,dx =$$

$$-\frac{2\sqrt{d+ex}\Big(bCe^2(bd-ae)+c^2d(24Cd^2-5e(4Bd-3Ae))+ce(ae(9Cd-5Be)-5b(5Cd^2-4Bde+3Ae^2}{15ce^3\left(cd^2-bde+ae^2\right)}$$

$$-\frac{2(Cd^2-e(Bd-Ae))\left(a+bx+cx^2\right)^{3/2}}{e\left(cd^2-bde+ae^2\right)\sqrt{d+ex}}$$

$$-\frac{\sqrt{2}\sqrt{b^2-4ac}(2b^2Ce^2+ce(8bCd-5bBe-6aCe)-c^2(48Cd^2-10e(4Bd-3Ae)))\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2}{b^2-4ac}}}{15c^2e^4\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{a+bx+cx^2}}$$

$$+\frac{2\sqrt{2}\sqrt{b^2-4ac}(bCe^2(bd-ae)-2c^2d(24Cd^2-5e(4Bd-3Ae))-ce(2ae(9Cd-5Be)-b(32Cd^2-5e(5B}{15c^2e^4\sqrt{d+ex}\sqrt{}}$$

```
[Out] -2*(C*d^2-e*(-A*e+B*d))*(c*x^2+b*x+a)^(3/2)/e/(a*e^2-b*d*e+c*d^2)/(e*x+d)^(
1/2)-2/15*(b*C*e^2*(-a*e+b*d)+c^2*d*(24*C*d^2-5*e*(-3*A*e+4*B*d))+c*e*(a*e*
(-5*B*e+9*C*d)-5*b*(3*A*e^2-4*B*d*e+5*C*d^2))+3*c*e^2*(5*B*c*d+C*b*d-6*c*C*
d^2/e-5*A*c*e-C*a*e)*x)*(e*x+d)^(1/2)*(c*x^2+b*x+a)^(1/2)/c/e^3/(a*e^2-b*d*
e+c*d^2)-1/15*(2*b^2*C*e^2+c*e*(-5*B*b*e-6*C*a*e+8*C*b*d)-c^2*(48*C*d^2-10*
e*(-3*A*e+4*B*d)))*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)
^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/
2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(e*x+d)^(1/2)*(-c*(c*x^2+b*x+a)/(-4
```

```
*a*c+b^2))^(1/2)/c^2/e^4/(c*x^2+b*x+a)^(1/2)/(c*(e*x+d)/(2*c*d-e*(b+(-4*a*c
+b^2)^(1/2))))^(1/2)+2/15*(b*C*e^2*(-a*e+b*d)-2*c^2*d*(24*C*d^2-5*e*(-3*A*e
+4*B*d))-c*e*(2*a*e*(-5*B*e+9*C*d)-b*(32*C*d^2-5*e*(-3*A*e+5*B*d))))*Ellipt
icF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2
*e*(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*
a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(e*x+d)/(2*c*d-e*(b
+(-4*a*c+b^2)^(1/2))))^(1/2)/c^2/e^4/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 746, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1664, 828, 857, 732, 435, 430}

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{3/2}}\,dx = \frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}(ce(-2ae(9Cd-5}{}$$

$$-\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(ce(-6aCe-5bBe+8bCd)-(c^2(48Cd^2-10e(4Bd-3Ae)))+2b}{15c^2e^4\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}$$

$$-\frac{2\sqrt{d+ex}\sqrt{a+bx+cx^2}\left(3ce^2x\left(-aCe-5Ace+bCd+5Bcd-\frac{6cCd^2}{e}\right)+ce(ae(9Cd-5Be)-5b(3Ae^2}{15ce^3\left(ae^2-bde+cd^2\right)}$$

$$-\frac{2\left(a+bx+cx^2\right)^{3/2}\left(Cd^2-e(Bd-Ae)\right)}{e\sqrt{d+ex}\left(ae^2-bde+cd^2\right)}$$

```
[In] Int[(Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2))/(d + e*x)^(3/2),x]
```

```
[Out] (-2*Sqrt[d + e*x]*(b*C*e^2*(b*d - a*e) + c^2*(24*C*d^3 - 5*d*e*(4*B*d - 3*A
*e)) + c*e*(a*e*(9*C*d - 5*B*e) - 5*b*(5*C*d^2 - 4*B*d*e + 3*A*e^2)) + 3*c*
e^2*(5*B*c*d + b*C*d - (6*c*C*d^2)/e - 5*A*c*e - a*C*e)*x)*Sqrt[a + b*x + c
*x^2])/(15*c*e^3*(c*d^2 - b*d*e + a*e^2)) - (2*(C*d^2 - e*(B*d - A*e))*(a +
 b*x + c*x^2)^(3/2))/(e*(c*d^2 - b*d*e + a*e^2)*Sqrt[d + e*x]) - (Sqrt[2]*S
qrt[b^2 - 4*a*c]*(2*b^2*C*e^2 + c*e*(8*b*C*d - 5*b*B*e - 6*a*C*e) - c^2*(48
*C*d^2 - 10*e*(4*B*d - 3*A*e)))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/
(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[
b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4
*a*c])*e)])/(15*c^2*e^4*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])
*e)]*Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(b*C*e^2*(b*d -
a*e) - 2*c^2*d*(24*C*d^2 - 5*e*(4*B*d - 3*A*e)) + c*e*(32*b*C*d^2 - 5*b*e*(
5*B*d - 3*A*e) - 2*a*e*(9*C*d - 5*B*e)))*Sqrt[(c*(d + e*x))/(2*c*d - (b + S
```

```
qrt[b^2 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*Elliptic
F[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]],
(-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(15*c^2*e^4*
Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
```

```
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{2(Cd^2-e(Bd-Ae))\left(a+bx+cx^2\right)^{3/2}}{e\left(cd^2-bde+ae^2\right)\sqrt{d+ex}} - \frac{2\int \frac{\left(-\frac{3bCd^2-be(3Bd-2Ae)+e(Acd-aCd+aBe)}{2e}+\frac{1}{2}\left(5Bcd+bCd-\frac{6cCd^2}{e}-5Ace-aCe\right)x\right)\sqrt{a+bx+cx^2}}{\sqrt{d+ex}}\,dx}{cd^2-bde+ae^2}$$

$$= -\frac{2\sqrt{d+ex}\Big(bCe^2(bd-ae)+c^2(24Cd^3-5de(4Bd-3Ae))+ce(ae(9Cd-5Be)-5b(5Cd^2-4}{15ce^3\left(cd^2-bde+a\right.}$$

$$-\frac{2(Cd^2-e(Bd-Ae))\left(a+bx+cx^2\right)^{3/2}}{e\left(cd^2-bde+ae^2\right)\sqrt{d+ex}}$$

$$+\frac{4\int \frac{-\frac{\left(cd^2-bde+ae^2\right)\left(b^2Cde+abCe^2+2ace(6Cd-5Be)-bc\left(24Cd^2-5e(4Bd-3Ae)\right)\right)}{4e}-\frac{\left(cd^2-bde+ae^2\right)\left(2b^2Ce^2+ce(8bCd-5bBe-6aCe)-c^2(\right.}{4e}}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}}{15ce^2\left(cd^2-bde+ae^2\right)}$$

$$= -\frac{2\sqrt{d+ex}\Big(bCe^2(bd-ae)+c^2(24Cd^3-5de(4Bd-3Ae))+ce(ae(9Cd-5Be)-5b(5Cd^2-4}{15ce^3\left(cd^2-bde+a\right.}$$

$$-\frac{2(Cd^2-e(Bd-Ae))\left(a+bx+cx^2\right)^{3/2}}{e\left(cd^2-bde+ae^2\right)\sqrt{d+ex}}$$

$$-\frac{(2b^2Ce^2+ce(8bCd-5bBe-6aCe)-c^2(48Cd^2-10e(4Bd-3Ae)))\int\frac{\sqrt{d+ex}}{\sqrt{a+bx+cx^2}}\,dx}{15ce^4}$$

$$+\frac{(bCe^2(bd-ae)-2c^2d(24Cd^2-5e(4Bd-3Ae))+ce(32bCd^2-5be(5Bd-3Ae)-2ae(9Cd-}{15ce^4}$$

$$=$$

$$-\frac{2\sqrt{d+ex}\Big(bCe^2(bd-ae)+c^2(24Cd^3-5de(4Bd-3Ae))+ce(ae(9Cd-5Be)-5b(5Cd^2-4B}{15ce^3\left(cd^2-bde+ae\right.}$$

$$-\frac{2(Cd^2-e(Bd-Ae))\left(a+bx+cx^2\right)^{3/2}}{e\left(cd^2-bde+ae^2\right)\sqrt{d+ex}}$$

$$-\frac{\Bigg(\sqrt{2}\sqrt{b^2-4ac}(2b^2Ce^2+ce(8bCd-5bBe-6aCe)-c^2(48Cd^2-10e(4Bd-3Ae)))\sqrt{d+ex}\sqrt{}}{15c^2e^4\sqrt{\frac{c(d+ex)}{2cd-be-\sqrt{b^2-4ac}e}}\sqrt{a+bx+}}$$

$$+\frac{\Bigg(2\sqrt{2}\sqrt{b^2-4ac}(bCe^2(bd-ae)-2c^2d(24Cd^2-5e(4Bd-3Ae))+ce(32bCd^2-5be(5Bd-3A}{15c^2e^4}$$

$$=$$

$$-\frac{2\sqrt{d+ex}\Big(bCe^2(bd-ae)+c^2(24Cd^3-5de(4Bd-3Ae))+ce(ae(9Cd-5Be)-5b(5Cd^2-4B}{15ce^3\left(cd^2-bde+ae\right.}$$

$$-\frac{2(Cd^2-e(Bd-Ae))\left(a+bx+cx^2\right)^{3/2}}{e\left(cd^2-bde+ae^2\right)\sqrt{d+ex}}$$

$$-\frac{\sqrt{2}\sqrt{b^2-4ac}(2b^2Ce^2+ce(8bCd-5bBe-6aCe)-c^2(48Cd^2-10e(4Bd-3Ae)))\sqrt{d+ex}\sqrt{-}}{15c^2e^4\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{a+bx+cx}}$$

$$+\frac{2\sqrt{2}\sqrt{b^2-4ac}(bCe^2(bd-ae)-2c^2d(24Cd^2-5e(4Bd-3Ae))+ce(32bCd^2-5be(5Bd-3Ae)}{15c^2e^4\sqrt{d}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 35.14 (sec) , antiderivative size = 1276, normalized size of antiderivative = 1.70

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{3/2}}\,dx=\sqrt{d+ex}\sqrt{a+x(b+cx)}\Bigg(\frac{2(-9cCd+5Bce+bCe)}{15ce^3}+\frac{2Cx}{5e^2}-\frac{2(Cd^2-Bde+Ae^2)}{e^3(d+ex)}\Bigg)$$

$$+\,\frac{(d+ex)^{3/2}\sqrt{a+x(b+cx)}\Bigg(-4\sqrt{\frac{cd^2+e(-bd+ae)}{-2cd+be+\sqrt{(b^2-4ac)e^2}}}(2b^2Ce^2+ce(8bCd-5bBe-6aCe)+c^2(-48Cd^2+}{}$$

```
[In] Integrate[(Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2))/(d + e*x)^(3/2),x]
```

```
[Out] Sqrt[d + e*x]*Sqrt[a + x*(b + c*x)]*((2*(-9*c*C*d + 5*B*c*e + b*C*e))/(15*c
*e^3) + (2*C*x)/(5*e^2) - (2*(C*d^2 - B*d*e + A*e^2))/(e^3*(d + e*x))) + ((
d + e*x)^(3/2)*Sqrt[a + x*(b + c*x)]*(-4*Sqrt[(c*d^2 + e*(-(b*d) + a*e))/(-
2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*(2*b^2*C*e^2 + c*e*(8*b*C*d - 5*b*B
*e - 6*a*C*e) + c^2*(-48*C*d^2 + 10*e*(4*B*d - 3*A*e)))*(c*(-1 + d/(d + e*x
))^2 + (e*(b - (b*d)/(d + e*x) + (a*e)/(d + e*x)))/(d + e*x)) + (I*Sqrt[2]*
(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(2*b^2*C*e^2 + c*e*(8*b*C*d - 5*b*B
*e - 6*a*C*e) + c^2*(-48*C*d^2 + 10*e*(4*B*d - 3*A*e)))*Sqrt[(Sqrt[(b^2 - 4
*a*c)*e^2] - (2*a*e^2)/(d + e*x) - 2*c*d*(-1 + d/(d + e*x)) + b*e*(-1 + (2*
d)/(d + e*x)))/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*Sqrt[(Sqrt[(b^2 - 4
*a*c)*e^2] + (2*a*e^2)/(d + e*x) + 2*c*d*(-1 + d/(d + e*x)) + b*(e - (2*d*e
)/(d + e*x)))/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*EllipticE[I*ArcSinh
[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e
^2])])/Sqrt[d + e*x]], -((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*c*d -
b*e + Sqrt[(b^2 - 4*a*c)*e^2]))])/Sqrt[d + e*x] - (I*Sqrt[2]*(-2*b^3*C*e^3
+ b^2*e^2*(-6*c*C*d + 5*B*c*e + 2*C*Sqrt[(b^2 - 4*a*c)*e^2]) + b*(8*a*c*C*e
^3 + c*e*Sqrt[(b^2 - 4*a*c)*e^2]*(8*C*d - 5*B*e)) - 2*c*(a*e^2*(-12*c*C*d +
 10*B*c*e + 3*C*Sqrt[(b^2 - 4*a*c)*e^2]) + c*Sqrt[(b^2 - 4*a*c)*e^2]*(24*C*
d^2 + 5*e*(-4*B*d + 3*A*e))))*Sqrt[(Sqrt[(b^2 - 4*a*c)*e^2] - (2*a*e^2)/(d
+ e*x) - 2*c*d*(-1 + d/(d + e*x)) + b*e*(-1 + (2*d)/(d + e*x)))/(2*c*d - b*
e + Sqrt[(b^2 - 4*a*c)*e^2])]*Sqrt[(Sqrt[(b^2 - 4*a*c)*e^2] + (2*a*e^2)/(d
+ e*x) + 2*c*d*(-1 + d/(d + e*x)) + b*(e - (2*d*e)/(d + e*x)))/(-2*c*d + b*
```

```
e + Sqrt[(b^2 - 4*a*c)*e^2])]*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*
d*e + a*e^2)/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((
-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e
^2]))])/Sqrt[d + e*x]))/(30*c^2*e^5*Sqrt[(c*d^2 + e*(-(b*d) + a*e))/(-2*c*d
 + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*Sqrt[a + b*x + c*x^2]*Sqrt[((d + e*x)^2*
(c*(-1 + d/(d + e*x))^2 + (e*(b - (b*d)/(d + e*x) + (a*e)/(d + e*x)))/(d +
e*x)))/e^2])
```

Maple [A] (verified)

Time = 4.02 (sec) , antiderivative size = 1215, normalized size of antiderivative = 1.62

method	result	size
elliptic	Expression too large to display	1215
risch	Expression too large to display	1864
default	Expression too large to display	8221

```
[In] int((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(3/2),x,method=_RETURNVERBOSE
)
```

```
[Out] ((e*x+d)*(c*x^2+b*x+a))^(1/2)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)*(-2*(c*e*x^
2+b*e*x+a*e)*(A*e^2-B*d*e+C*d^2)/e^4/((x+d/e)*(c*e*x^2+b*e*x+a*e))^(1/2)+2/
5*C/e^2*x*(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)+2/3*(1/e^2*(B*c*e
+C*b*e-C*c*d)-2/5/e^2*(2*b*e+2*c*d)*C)/c/e*(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b
*d*x+a*d)^(1/2)+2*((A*b*e^3-A*c*d*e^2+B*a*e^3-B*b*d*e^2+B*c*d^2*e-C*a*d*e^2
+C*b*d^2*e-C*c*d^3)/e^4-(A*e^2-B*d*e+C*d^2)/e^4*(b*e-c*d)+b/e^3*(A*e^2-B*d*
e+C*d^2)-2/5*a*d/e^2*C-2/3*(1/e^2*(B*c*e+C*b*e-C*c*d)-2/5/e^2*(2*b*e+2*c*d)
*C)/c/e*(1/2*a*e+1/2*b*d))*(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e
-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-
d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)
/(-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+
b*d*x+a*d)^(1/2)*EllipticF(((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/
2),((-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))
))^(1/2))+2*(1/e^3*(A*c*e^2+B*b*e^2-B*c*d*e+C*a*e^2-C*b*d*e+C*c*d^2)+(A*e^2
-B*d*e+C*d^2)/e^3*c-2/5*C/e^2*(3/2*a*e+3/2*b*d)-2/3*(1/e^2*(B*c*e+C*b*e-C*c
*d)-2/5/e^2*(2*b*e+2*c*d)*C)/c/e*(b*e+c*d))*(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))
/c)*((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*
c+b^2)^(1/2)))/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a
*c+b^2)^(1/2))/c)/(-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*e*x^3+b*e*x
^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)*((-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*Ell
ipticE(((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2*(b+(-4
*a*c+b^2)^(1/2))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+1/2/c*(-b+
(-4*a*c+b^2)^(1/2))*EllipticF(((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^
(1/2),((-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/
2))))^(1/2))))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 814, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{3/2}}\,dx =$$

$$-\frac{2\Big((48\,Cc^3d^4 - 8\,(4\,Cbc^2 + 5\,Bc^3)d^3e - (7\,Cb^2c - 30\,Ac^3 - (42\,Ca + 25\,Bb)c^2)d^2e^2 - (2\,Cb^3 + 15\,(2\,Ba +}{}$$

```
[In] integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(3/2),x, algorithm="fri
cas")
```

```
[Out] -2/45*((48*C*c^3*d^4 - 8*(4*C*b*c^2 + 5*B*c^3)*d^3*e - (7*C*b^2*c - 30*A*c^
3 - (42*C*a + 25*B*b)*c^2)*d^2*e^2 - (2*C*b^3 + 15*(2*B*a + A*b)*c^2 - (9*C
*a*b + 5*B*b^2)*c)*d*e^3 + (48*C*c^3*d^3*e - 8*(4*C*b*c^2 + 5*B*c^3)*d^2*e^
2 - (7*C*b^2*c - 30*A*c^3 - (42*C*a + 25*B*b)*c^2)*d*e^3 - (2*C*b^3 + 15*(2
*B*a + A*b)*c^2 - (9*C*a*b + 5*B*b^2)*c)*e^4)*x)*sqrt(c*e)*weierstrassPInve
rse(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3
 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(c^3*
e^3), 1/3*(3*c*e*x + c*d + b*e)/(c*e)) + 3*(48*C*c^3*d^3*e - 8*(C*b*c^2 + 5
*B*c^3)*d^2*e^2 - (2*C*b^2*c - 30*A*c^3 - (6*C*a + 5*B*b)*c^2)*d*e^3 + (48*
C*c^3*d^2*e^2 - 8*(C*b*c^2 + 5*B*c^3)*d*e^3 - (2*C*b^2*c - 30*A*c^3 - (6*C*
a + 5*B*b)*c^2)*e^4)*x)*sqrt(c*e)*weierstrassZeta(4/3*(c^2*d^2 - b*c*d*e +
(b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c -
 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(c^3*e^3), weierstrassPInverse(4/3
*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*
c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(c^3*e^3), 1
/3*(3*c*e*x + c*d + b*e)/(c*e))) - 3*(3*C*c^3*e^4*x^2 - 24*C*c^3*d^2*e^2 -
15*A*c^3*e^4 + (C*b*c^2 + 20*B*c^3)*d*e^3 - (6*C*c^3*d*e^3 - (C*b*c^2 + 5*B
*c^3)*e^4)*x)*sqrt(c*x^2 + b*x + a)*sqrt(e*x + d))/(c^3*e^6*x + c^3*d*e^5)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{3/2}}\,dx = \int \frac{(A+Bx+Cx^2)\sqrt{a+bx+cx^2}}{(d+ex)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((C*x**2+B*x+A)*(c*x**2+b*x+a)**(1/2)/(e*x+d)**(3/2),x)
```

```
[Out] Integral((A + B*x + C*x**2)*sqrt(a + b*x + c*x**2)/(d + e*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{3/2}}\,dx=\int \frac{(Cx^2+Bx+A)\sqrt{cx^2+bx+a}}{(ex+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(3/2),x, algorithm="max
ima")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)/(e*x + d)^(3/2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{3/2}}\,dx=\int \frac{(Cx^2+Bx+A)\sqrt{cx^2+bx+a}}{(ex+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(3/2),x, algorithm="gia
c")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)/(e*x + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{3/2}}\,dx=\int \frac{\left(C\,x^2+B\,x+A\right)\,\sqrt{c\,x^2+b\,x+a}}{{\left(d+e\,x\right)}^{3/2}}\,dx$$

```
[In] int(((A + B*x + C*x^2)*(a + b*x + c*x^2)^(1/2))/(d + e*x)^(3/2),x)
```

```
[Out] int(((A + B*x + C*x^2)*(a + b*x + c*x^2)^(1/2))/(d + e*x)^(3/2), x)
```

3.262 $\int \frac{\sqrt{a+bx+cx^2}\left(A+Bx+Cx^2\right)}{(d+ex)^{5/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 712

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{5/2}}\,dx =$$

$$-\frac{2\left(e(bd-ae)(7Cd-3Be)-cd(8Cd^2-e(4Bd-Ae))+e^2\left(Bcd+bCd-\frac{2cCd^2}{e}-Ace-aCe\right)x\right)\sqrt{a+}}{3e^3\left(cd^2-bde+ae^2\right)\sqrt{d+ex}}$$

$$-\frac{2(Cd^2-e(Bd-Ae))\left(a+bx+cx^2\right)^{3/2}}{3e\left(cd^2-bde+ae^2\right)(d+ex)^{3/2}}$$

$$+\frac{\sqrt{2}\sqrt{b^2-4ac}\left(2\left(4cd-\frac{be}{2}\right)\left(Bcd+bCd-\frac{2cCd^2}{e}-Ace-aCe\right)+6c(bd(Cd-Be)+e(Acd-aCd+aBe}{3ce^3\left(cd^2-bde+ae^2\right)\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{}}$$

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}(e(8bCd-3bBe-2aCe)-2c(8Cd^2-e(4Bd-Ae)))\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\mathrm{E}}{3ce^4\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

```
[Out] -2/3*(C*d^2-e*(-A*e+B*d))*(c*x^2+b*x+a)^(3/2)/e/(a*e^2-b*d*e+c*d^2)/(e*x+d)
^(3/2)-2/3*(e*(-a*e+b*d)*(-3*B*e+7*C*d)-c*d*(8*C*d^2-e*(-A*e+4*B*d))+e^2*(B
*c*d+C*b*d-2*c*C*d^2/e-A*c*e-C*a*e)*x)*(c*x^2+b*x+a)^(1/2)/e^3/(a*e^2-b*d*e
+c*d^2)/(e*x+d)^(1/2)+1/3*(2*(4*c*d-1/2*b*e)*(B*c*d+C*b*d-2*c*C*d^2/e-A*c*e
-C*a*e)+6*c*(b*d*(-B*e+C*d)+e*(A*c*d+B*a*e-C*a*d)))*EllipticE(1/2*((b+2*c*x
+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(
1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(e
*x+d)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/c/e^3/(a*e^2-b*d*e+c*d^2)
```

```
/(c*x^2+b*x+a)^(1/2)/(c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)-2/3
*(e*(-3*B*b*e-2*C*a*e+8*C*b*d)-2*c*(8*C*d^2-e*(-A*e+4*B*d)))*EllipticF(1/2*
((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a
*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)
^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(e*x+d)/(2*c*d-e*(b+(-4*a*c
+b^2)^(1/2))))^(1/2)/c/e^4/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 711, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1664, 826, 857, 732, 435, 430}

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{5/2}}\,dx =$$

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}(e(-2aCe-3bBe+8bCd)-2c(8Cd^2-e(4Bd-Ae)))\,\mathrm{El}}{3ce^4\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

$$+\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\left(2\left(4cd-\frac{be}{2}\right)\left(-aCe-Ace+bCd+Bcd-\frac{2cCd^2}{e}\right)+6c(e(aBe-aC}{3ce^3\sqrt{a+bx+cx^2}\,(ae^2-bde+cd^2)\sqrt{\frac{}{2cd-}}}$$

$$-\frac{2(a+bx+cx^2)^{3/2}\,(Cd^2-e(Bd-Ae))}{3e(d+ex)^{3/2}\,(ae^2-bde+cd^2)}$$

$$+\frac{2\sqrt{a+bx+cx^2}\left(-ex\left(-aCe-Ace+bCd+Bcd-\frac{2cCd^2}{e}\right)-(bd-ae)(7Cd-3Be)-cd(4Bd-Ae)+\frac{8c}{}\right.}{3e^2\sqrt{d+ex}\,(ae^2-bde+cd^2)}$$

```
[In] Int[(Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2))/(d + e*x)^(5/2),x]
```

```
[Out] (2*((8*c*C*d^3)/e - c*d*(4*B*d - A*e) - (b*d - a*e)*(7*C*d - 3*B*e) - e*(B*
c*d + b*C*d - (2*c*C*d^2)/e - A*c*e - a*C*e)*x)*Sqrt[a + b*x + c*x^2])/(3*e
^2*(c*d^2 - b*d*e + a*e^2)*Sqrt[d + e*x]) - (2*(C*d^2 - e*(B*d - A*e))*(a +
 b*x + c*x^2)^(3/2))/(3*e*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^(3/2)) + (Sqrt[
2]*Sqrt[b^2 - 4*a*c]*(2*(4*c*d - (b*e)/2)*(B*c*d + b*C*d - (2*c*C*d^2)/e -
A*c*e - a*C*e) + 6*c*(b*d*(C*d - B*e) + e*(A*c*d - a*C*d + a*B*e)))*Sqrt[d
+ e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(
b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 -
4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(3*c*e^3*(c*d^2 - b*d*e + a
*e^2)*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a + b*x
+ c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(e*(8*b*C*d - 3*b*B*e - 2*a*C*e) -
 2*c*(8*C*d^2 - e*(4*B*d - A*e)))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2
```

```
 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSi
n[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqr
t[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(3*c*e^4*Sqrt[d + e
*x]*Sqrt[a + b*x + c*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 826

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
```

```
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{2(Cd^2-e(Bd-Ae))\left(a+bx+cx^2\right)^{3/2}}{3e\left(cd^2-bde+ae^2\right)(d+ex)^{3/2}} - \frac{2\int \frac{\left(-\frac{3(bd(Cd-Be)+e(Acd-aCd+aBe))}{2e}+\frac{3}{2}\left(Bcd+bCd-\frac{2cCd^2}{e}-Ace-aCe\right)x\right)\sqrt{a+bx+cx^2}}{(d+ex)^{3/2}}\,dx}{3\left(cd^2-bde+ae^2\right)}$$

$$= \frac{2\left(\frac{8cCd^3}{e}-cd(4Bd-Ae)-(bd-ae)(7Cd-3Be)-e\left(Bcd+bCd-\frac{2cCd^2}{e}-Ace-aCe\right)x\right)\sqrt{a-}}{3e^2\left(cd^2-bde+ae^2\right)\sqrt{d+ex}} - \frac{2(Cd^2-e(Bd-Ae))\left(a+bx+cx^2\right)^{3/2}}{3e\left(cd^2-bde+ae^2\right)(d+ex)^{3/2}} + \frac{4\int \frac{\frac{3}{4}\left(2(2bd-ae)\left(Bcd+bCd-\frac{2cCd^2}{e}-Ace-aCe\right)+3b(bd(Cd-Be)+e(Acd-aCd+aBe))\right)+\frac{3}{4}\left(2\left(4cd-\frac{be}{2}\right)\left(Bcd+bCd-\frac{2cCd^2}{e}-Ac\right.\right.}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}}{9e^2\left(cd^2-bde+ae^2\right)}$$

$$= \frac{2\left(\frac{8cCd^3}{e}-cd(4Bd-Ae)-(bd-ae)(7Cd-3Be)-e\left(Bcd+bCd-\frac{2cCd^2}{e}-Ace-aCe\right)x\right)\sqrt{a-}}{3e^2\left(cd^2-bde+ae^2\right)\sqrt{d+ex}} - \frac{2(Cd^2-e(Bd-Ae))\left(a+bx+cx^2\right)^{3/2}}{3e\left(cd^2-bde+ae^2\right)(d+ex)^{3/2}} - \frac{(e(8bCd-3bBe-2aCe)-2c(8Cd^2-e(4Bd-Ae)))\int \frac{1}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx}{3e^4} - \frac{(bCe^2(bd-ae)+2c^2(8Cd^3-de(4Bd-Ae))-ce(16bCd^2-be(7Bd-Ae)-2ae(7Cd-3Be))}{3e^4\left(cd^2-bde+ae^2\right)}$$

$$= \frac{2\left(\frac{8cCd^3}{e}-cd(4Bd-Ae)-(bd-ae)(7Cd-3Be)-e\left(Bcd+bCd-\frac{2cCd^2}{e}-Ace-aCe\right)x\right)\sqrt{a}}{3e^2\left(cd^2-bde+ae^2\right)\sqrt{d+ex}}$$

$$-\frac{2(Cd^2-e(Bd-Ae))\left(a+bx+cx^2\right)^{3/2}}{3e\left(cd^2-bde+ae^2\right)(d+ex)^{3/2}}$$

$$-\frac{\left(\sqrt{2}\sqrt{b^2-4ac}(bCe^2(bd-ae)+2c^2(8Cd^3-de(4Bd-Ae))-ce(16bCd^2-be(7Bd-Ae)-2\right.}{3ce^4\left(cd^2-bde+ae^2\right)\sqrt{\frac{}{2cd}}}$$

$$-\frac{\left(2\sqrt{2}\sqrt{b^2-4ac}(e(8bCd-3bBe-2aCe)-2c(8Cd^2-e(4Bd-Ae)))\sqrt{\frac{c(d+ex)}{2cd-be-\sqrt{b^2-4ac}e}}\sqrt{-\frac{c(}{}}\right.}{3ce^4\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

$$= \frac{2\left(\frac{8cCd^3}{e}-cd(4Bd-Ae)-(bd-ae)(7Cd-3Be)-e\left(Bcd+bCd-\frac{2cCd^2}{e}-Ace-aCe\right)x\right)\sqrt{a}}{3e^2\left(cd^2-bde+ae^2\right)\sqrt{d+ex}}$$

$$-\frac{2(Cd^2-e(Bd-Ae))\left(a+bx+cx^2\right)^{3/2}}{3e\left(cd^2-bde+ae^2\right)(d+ex)^{3/2}}$$

$$-\frac{\sqrt{2}\sqrt{b^2-4ac}(bCe^2(bd-ae)+2c^2(8Cd^3-de(4Bd-Ae))-ce(16bCd^2-be(7Bd-Ae)-2a}{3ce^4\left(cd^2-bde+ae^2\right)\sqrt{\frac{c}{2cd-\left(b\right.}}}$$

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}(e(8bCd-3bBe-2aCe)-2c(8Cd^2-e(4Bd-Ae)))\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c(a}{}}}{3ce^4\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 35.08 (sec) , antiderivative size = 8456, normalized size of antiderivative = 11.88

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{5/2}}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2))/(d + e*x)^(5/2),x]
```

```
[Out] Result too large to show
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1339 vs. 2(646) = 1292.

Time = 4.86 (sec) , antiderivative size = 1340, normalized size of antiderivative = 1.88

method	result	size
elliptic	Expression too large to display	1340
risch	Expression too large to display	2764
default	Expression too large to display	21038

```
[In] int((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(5/2),x,method=_RETURNVERBOSE
)
```

```
[Out] ((e*x+d)*(c*x^2+b*x+a))^(1/2)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)*(-2/3*(A*e^
2-B*d*e+C*d^2)/e^5*(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)/(x+d/e)^
2-2/3*(c*e*x^2+b*e*x+a*e)/e^4/(a*e^2-b*d*e+c*d^2)*(A*b*e^3-2*A*c*d*e^2+3*B*
a*e^3-4*B*b*d*e^2+5*B*c*d^2*e-6*C*a*d*e^2+7*C*b*d^2*e-8*C*c*d^3)/((x+d/e)*(
c*e*x^2+b*e*x+a*e))^(1/2)+2/3*C/e^3*(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*
d)^(1/2)+2*((A*c*e^2+B*b*e^2-2*B*c*d*e+C*a*e^2-2*C*b*d*e+3*C*c*d^2)/e^4-1/3
*(A*e^2-B*d*e+C*d^2)/e^4*c-1/3/e^4*(b*e-c*d)*(A*b*e^3-2*A*c*d*e^2+3*B*a*e^3
-4*B*b*d*e^2+5*B*c*d^2*e-6*C*a*d*e^2+7*C*b*d^2*e-8*C*c*d^3)/(a*e^2-b*d*e+c*
d^2)+1/3*b/e^3/(a*e^2-b*d*e+c*d^2)*(A*b*e^3-2*A*c*d*e^2+3*B*a*e^3-4*B*b*d*e
^2+5*B*c*d^2*e-6*C*a*d*e^2+7*C*b*d^2*e-8*C*c*d^3)-2/3*C/e^3*(1/2*a*e+1/2*b*
d))*(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1
/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d/e-1/2/c*(-b+(-4*a*c+b
^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e+1/2*(b+(-4*a*c+b
^2)^(1/2))/c))^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)*Ellipt
icF(((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2*(b+(-4*a*
c+b^2)^(1/2))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+2*(1/e^3*(B*c
*e+C*b*e-2*C*c*d)+1/3*c/e^3*(A*b*e^3-2*A*c*d*e^2+3*B*a*e^3-4*B*b*d*e^2+5*B*
c*d^2*e-6*C*a*d*e^2+7*C*b*d^2*e-8*C*c*d^3)/(a*e^2-b*d*e+c*d^2)-2/3*C/e^3*(b
*e+c*d))*(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^
2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d/e-1/2/c*(-b+(-4*
a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e+1/2*(b+(-4*
a*c+b^2)^(1/2))/c))^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)*(
(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*EllipticE(((x+d/e)/(d/e-1/2*(b+(-4*a*c
+b^2)^(1/2))/c))^(1/2),((-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e-1/2/c*(-b
+(-4*a*c+b^2)^(1/2))))^(1/2))+1/2/c*(-b+(-4*a*c+b^2)^(1/2))*EllipticF(((x+d
/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2*(b+(-4*a*c+b^2)^(1
/2))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.15 (sec) , antiderivative size = 1385, normalized size of antiderivative = 1.95

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(5/2),x, algorithm="fri
cas")
```

```
[Out] 2/9*((16*C*c^3*d^6 - 8*(3*C*b*c^2 + B*c^3)*d^5*e + (6*C*b^2*c + 2*A*c^3 + (
26*C*a + 11*B*b)*c^2)*d^4*e^2 + (C*b^3 - 2*(6*B*a + A*b)*c^2 - 2*(7*C*a*b +
 B*b^2)*c)*d^3*e^3 - (C*a*b^2 - 6*A*a*c^2 - (6*C*a^2 + 3*B*a*b - A*b^2)*c)*
d^2*e^4 + (16*C*c^3*d^4*e^2 - 8*(3*C*b*c^2 + B*c^3)*d^3*e^3 + (6*C*b^2*c +
2*A*c^3 + (26*C*a + 11*B*b)*c^2)*d^2*e^4 + (C*b^3 - 2*(6*B*a + A*b)*c^2 - 2
*(7*C*a*b + B*b^2)*c)*d*e^5 - (C*a*b^2 - 6*A*a*c^2 - (6*C*a^2 + 3*B*a*b - A
*b^2)*c)*e^6)*x^2 + 2*(16*C*c^3*d^5*e - 8*(3*C*b*c^2 + B*c^3)*d^4*e^2 + (6*
C*b^2*c + 2*A*c^3 + (26*C*a + 11*B*b)*c^2)*d^3*e^3 + (C*b^3 - 2*(6*B*a + A*
b)*c^2 - 2*(7*C*a*b + B*b^2)*c)*d^2*e^4 - (C*a*b^2 - 6*A*a*c^2 - (6*C*a^2 +
 3*B*a*b - A*b^2)*c)*d*e^5)*x)*sqrt(c*e)*weierstrassPInverse(4/3*(c^2*d^2 -
 b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e -
 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(c^3*e^3), 1/3*(3*c*e*x
 + c*d + b*e)/(c*e)) + 3*(16*C*c^3*d^5*e - 8*(2*C*b*c^2 + B*c^3)*d^4*e^2 +
(C*b^2*c + 2*A*c^3 + 7*(2*C*a + B*b)*c^2)*d^3*e^3 - (C*a*b*c + (6*B*a + A*b
)*c^2)*d^2*e^4 + (16*C*c^3*d^3*e^3 - 8*(2*C*b*c^2 + B*c^3)*d^2*e^4 + (C*b^2
*c + 2*A*c^3 + 7*(2*C*a + B*b)*c^2)*d*e^5 - (C*a*b*c + (6*B*a + A*b)*c^2)*e
^6)*x^2 + 2*(16*C*c^3*d^4*e^2 - 8*(2*C*b*c^2 + B*c^3)*d^3*e^3 + (C*b^2*c +
2*A*c^3 + 7*(2*C*a + B*b)*c^2)*d^2*e^4 - (C*a*b*c + (6*B*a + A*b)*c^2)*d*e^
5)*x)*sqrt(c*e)*weierstrassZeta(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)
/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 +
(2*b^3 - 9*a*b*c)*e^3)/(c^3*e^3), weierstrassPInverse(4/3*(c^2*d^2 - b*c*d*
e + (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2
*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(c^3*e^3), 1/3*(3*c*e*x + c*d
+ b*e)/(c*e))) + 3*(8*C*c^3*d^4*e^2 - 2*B*a*c^2*d*e^5 - A*a*c^2*e^6 - (7*C*
b*c^2 + 4*B*c^3)*d^3*e^3 + (A*c^3 + 3*(2*C*a + B*b)*c^2)*d^2*e^4 + (C*c^3*d
^2*e^4 - C*b*c^2*d*e^5 + C*a*c^2*e^6)*x^2 + (10*C*c^3*d^3*e^3 - (3*B*a + A*
b)*c^2*e^6 - (9*C*b*c^2 + 5*B*c^3)*d^2*e^4 + 2*(A*c^3 + 2*(2*C*a + B*b)*c^2
)*d*e^5)*x)*sqrt(c*x^2 + b*x + a)*sqrt(e*x + d))/(c^3*d^4*e^5 - b*c^2*d^3*e
^6 + a*c^2*d^2*e^7 + (c^3*d^2*e^7 - b*c^2*d*e^8 + a*c^2*e^9)*x^2 + 2*(c^3*d
^3*e^6 - b*c^2*d^2*e^7 + a*c^2*d*e^8)*x)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{5/2}}\,dx = \int \frac{(A+Bx+Cx^2)\sqrt{a+bx+cx^2}}{(d+ex)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((C*x**2+B*x+A)*(c*x**2+b*x+a)**(1/2)/(e*x+d)**(5/2),x)
[Out] Integral((A + B*x + C*x**2)*sqrt(a + b*x + c*x**2)/(d + e*x)**(5/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{5/2}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{cx^2+bx+a}}{(ex+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(5/2),x, algorithm="max
ima")
[Out] integrate((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)/(e*x + d)^(5/2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{5/2}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{cx^2+bx+a}}{(ex+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(5/2),x, algorithm="gia
c")
[Out] integrate((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)/(e*x + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{5/2}}\,dx = \int \frac{(C\,x^2+B\,x+A)\,\sqrt{c\,x^2+b\,x+a}}{(d+e\,x)^{5/2}}\,dx$$

```
[In] int(((A + B*x + C*x^2)*(a + b*x + c*x^2)^(1/2))/(d + e*x)^(5/2),x)
[Out] int(((A + B*x + C*x^2)*(a + b*x + c*x^2)^(1/2))/(d + e*x)^(5/2), x)
```

3.263 $\int \frac{\sqrt{a+bx+cx^2}\left(A+Bx+Cx^2\right)}{(d+ex)^{7/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 992

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{7/2}}\,dx =$$

$$-\frac{2(c^2d^3(24Cd^2-e(4Bd+Ae))+e^2(15b^2Cd^3+5a^2e^2(Cd+Be)-abe(22Cd^2+3Bde+2Ae^2))-cde(bd}{}$$

$$-\frac{2(Cd^2-e(Bd-Ae))\left(a+bx+cx^2\right)^{3/2}}{5e\left(cd^2-bde+ae^2\right)(d+ex)^{5/2}}$$

$$+\frac{\sqrt{2}\sqrt{b^2-4ac}(2c^2d^2(24Cd^2-e(4Bd+Ae))+e^2(30a^2Ce^2-5abe(14Cd-Be)+b^2(38Cd^2-3Bde-2A}{15e^4\left(cd^2\right.}$$

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}(15bCe^2(bd-ae)+2c^2d(24Cd^2-e(4Bd+Ae))+ce(10ae(5Cd-Be)-b(64Cd^2-9Bd}{15ce^4\left(cd^2-bde+ae^2\right)\surd}$$

```
[Out] -2/5*(C*d^2-e*(-A*e+B*d))*(c*x^2+b*x+a)^(3/2)/e/(a*e^2-b*d*e+c*d^2)/(e*x+d)
^(5/2)-2/15*(c^2*d^3*(24*C*d^2-e*(A*e+4*B*d))+e^2*(15*b^2*C*d^3+5*a^2*e^2*(
B*e+C*d)-a*b*e*(2*A*e^2+3*B*d*e+22*C*d^2))-c*d*e*(b*d*(A*e^2-6*B*d*e+41*C*d
^2)-a*e*(7*A*e^2-7*B*d*e+37*C*d^2))+e*(5*c^2*d^2*(6*C*d^2-e*(A*e+B*d))+e^2*
(15*a^2*C*e^2-5*a*b*e*(-B*e+8*C*d)+b^2*(-2*A*e^2-3*B*d*e+23*C*d^2))-c*e*(5*
b*d*(-A*e^2-2*B*d*e+11*C*d^2)-a*e*(3*A*e^2-13*B*d*e+53*C*d^2)))*x)*(c*x^2+b
*x+a)^(1/2)/e^3/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)^(3/2)+1/15*(2*c^2*d^2*(24*C*d
^2-e*(A*e+4*B*d))+e^2*(30*a^2*C*e^2-5*a*b*e*(-B*e+14*C*d)+b^2*(-2*A*e^2-3*B
*d*e+38*C*d^2))-c*e*(b*d*(-2*A*e^2-13*B*d*e+88*C*d^2)-2*a*e*(3*A*e^2-8*B*d*
```

```
e+43*C*d^2)))*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2
))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))
^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(e*x+d)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+
b^2))^(1/2)/e^4/(a*e^2-b*d*e+c*d^2)^2/(c*x^2+b*x+a)^(1/2)/(c*(e*x+d)/(2*c*d
-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)-2/15*(15*b*C*e^2*(-a*e+b*d)+2*c^2*d*(24*C
*d^2-e*(A*e+4*B*d))+c*e*(10*a*e*(-B*e+5*C*d)-b*(-A*e^2-9*B*d*e+64*C*d^2)))*
EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/
2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2
)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(e*x+d)/(2*c*
d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c/e^4/(a*e^2-b*d*e+c*d^2)/(e*x+d)^(1/2)/
(c*x^2+b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 989, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1664, 824, 857, 732, 435, 430}

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{7/2}}\,dx = -\frac{2(Cd^2-e(Bd-Ae))\left(cx^2+bx+a\right)^{3/2}}{5e\left(cd^2-bed+ae^2\right)(d+ex)^{5/2}}$$

$$-\frac{2\Big((24Cd^5-d^3e(4Bd+Ae))\,c^2-de(bd(41Cd^2-6Bed+Ae^2)-ae(37Cd^2-7Bed+7Ae^2))\,c+e^2(15b^2C}{}$$

$$+\frac{\sqrt{2}\sqrt{b^2-4ac}((48Cd^4-2d^2e(4Bd+Ae))\,c^2-e(bd(88Cd^2-13Bed-2Ae^2)-2ae(43Cd^2-8Bed+3Ae^2}{15e^4\left(cd^2-\right.}$$

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}((48Cd^3-2de(4Bd+Ae))\,c^2-e(64bCd^2-be(9Bd+Ae)-10ae(5Cd-Be))\,c+15bCe^2(}{15ce^4\left(cd^2-bed+ae^2\right)\sqrt{}}$$

```
[In] Int[(Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2))/(d + e*x)^(7/2),x]
```

```
[Out] (-2*(c^2*(24*C*d^5 - d^3*e*(4*B*d + A*e)) + e^2*(15*b^2*C*d^3 + 5*a^2*e^2*(
C*d + B*e) - a*b*e*(22*C*d^2 + 3*B*d*e + 2*A*e^2)) - c*d*e*(b*d*(41*C*d^2 -
 6*B*d*e + A*e^2) - a*e*(37*C*d^2 - 7*B*d*e + 7*A*e^2)) + e^2*((30*c^2*C*d^
4)/e + 15*a^2*C*e^3 - 5*c^2*d^2*(B*d + A*e) - 5*a*b*e^2*(8*C*d - B*e) + a*c
*e*(53*C*d^2 - e*(13*B*d - 3*A*e)) - 5*b*c*d*(11*C*d^2 - e*(2*B*d + A*e)) +
 b^2*e*(23*C*d^2 - e*(3*B*d + 2*A*e)))*x)*Sqrt[a + b*x + c*x^2])/(15*e^3*(c
*d^2 - b*d*e + a*e^2)^2*(d + e*x)^(3/2)) - (2*(C*d^2 - e*(B*d - A*e))*(a +
b*x + c*x^2)^(3/2))/(5*e*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^(5/2)) + (Sqrt[2
]*Sqrt[b^2 - 4*a*c]*(c^2*(48*C*d^4 - 2*d^2*e*(4*B*d + A*e)) + e^2*(30*a^2*C
```

```
*e^2 - 5*a*b*e*(14*C*d - B*e) + b^2*(38*C*d^2 - 3*B*d*e - 2*A*e^2)) - c*e*(
b*d*(88*C*d^2 - 13*B*d*e - 2*A*e^2) - 2*a*e*(43*C*d^2 - 8*B*d*e + 3*A*e^2))
)*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcS
in[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sq
rt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(15*e^4*(c*d^2 - b
*d*e + a*e^2)^2*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqr
t[a + b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(15*b*C*e^2*(b*d - a*e)
+ c^2*(48*C*d^3 - 2*d*e*(4*B*d + A*e)) - c*e*(64*b*C*d^2 - b*e*(9*B*d + A*e
) - 10*a*e*(5*C*d - B*e)))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*
c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[
(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 -
 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(15*c*e^4*(c*d^2 - b*d*e +
 a*e^2)*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 824

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Dist[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)
^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p
```

```
 + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*
(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m
 - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &
& LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{2(Cd^2-e(Bd-Ae))\left(a+bx+cx^2\right)^{3/2}}{5e\left(cd^2-bde+ae^2\right)(d+ex)^{5/2}} - \frac{2\int \frac{\left(-\frac{3bCd^2-be(3Bd+2Ae)+5e(Acd-aCd+aBe)}{2e}+\frac{1}{2}\left(Bcd+5bCd-\frac{6cCd^2}{e}-Ace-5aCe\right)x\right)\sqrt{a+bx+cx^2}}{(d+ex)^{5/2}}\,dx}{5\left(cd^2-bde+ae^2\right)}$$

$$= -\frac{2\Big(c^2(24Cd^5-d^3e(4Bd+Ae))+e^2(15b^2Cd^3+5a^2e^2(Cd+Be)-abe(22Cd^2+3Bde+2Ae^2))}{}$$

$$- \frac{2(Cd^2-e(Bd-Ae))\left(a+bx+cx^2\right)^{3/2}}{5e\left(cd^2-bde+ae^2\right)(d+ex)^{5/2}} + \frac{4\int \frac{\frac{15b^3Cd^2e^2-b^2de\left(41cCd^2+30aCe^2-ce(6Bd-Ae)\right)-2ace\left(6cCd^3-cde(Bd+4Ae)+5ae^2(2Cd-Be)\right)+b\left(15a^2Ce^4+c^2\left(24Cd^4-d^2e(4Bd+Ae)\right)\right.}{4e}}{}}{}$$

$$=$$

$$-\frac{2\left(c^2(24Cd^5-d^3e(4Bd+Ae))+e^2(15b^2Cd^3+5a^2e^2(Cd+Be)-abe(22Cd^2+3Bde+2Ae^2)\right.}{}$$

$$-\frac{2(Cd^2-e(Bd-Ae))\left(a+bx+cx^2\right)^{3/2}}{5e\left(cd^2-bde+ae^2\right)(d+ex)^{5/2}}$$

$$-\frac{(15bCe^2(bd-ae)+c^2(48Cd^3-2de(4Bd+Ae))-ce(64bCd^2-be(9Bd+Ae)-10ae(5Cd-}{15e^4\left(cd^2-bde+ae^2\right)}$$

$$+\frac{(c(c^2(48Cd^4-2d^2e(4Bd+Ae))+e^2(30a^2Ce^2-5abe(14Cd-Be)+b^2(38Cd^2-3Bde-2Ae}{15e^4\left(cd^2-bde\right.}$$

$$=$$

$$-\frac{2\left(c^2(24Cd^5-d^3e(4Bd+Ae))+e^2(15b^2Cd^3+5a^2e^2(Cd+Be)-abe(22Cd^2+3Bde+2Ae^2)\right.}{}$$

$$-\frac{2(Cd^2-e(Bd-Ae))\left(a+bx+cx^2\right)^{3/2}}{5e\left(cd^2-bde+ae^2\right)(d+ex)^{5/2}}$$

$$+\frac{\left(\sqrt{2}\sqrt{b^2-4ac}(c^2(48Cd^4-2d^2e(4Bd+Ae))+e^2(30a^2Ce^2-5abe(14Cd-Be)+b^2(38Cd^2-\right.}{}$$

$$-\frac{\left(2\sqrt{2}\sqrt{b^2-4ac}(15bCe^2(bd-ae)+c^2(48Cd^3-2de(4Bd+Ae))-ce(64bCd^2-be(9Bd+Ae\right.}{15ce^4\left(cd^2-bc\right.}$$

$$=$$

$$-\frac{2\left(c^2(24Cd^5-d^3e(4Bd+Ae))+e^2(15b^2Cd^3+5a^2e^2(Cd+Be)-abe(22Cd^2+3Bde+2Ae^2)\right.}{}$$

$$-\frac{2(Cd^2-e(Bd-Ae))\left(a+bx+cx^2\right)^{3/2}}{5e\left(cd^2-bde+ae^2\right)(d+ex)^{5/2}}$$

$$+\frac{\sqrt{2}\sqrt{b^2-4ac}(c^2(48Cd^4-2d^2e(4Bd+Ae))+e^2(30a^2Ce^2-5abe(14Cd-Be)+b^2(38Cd^2-3}{}$$

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}(15bCe^2(bd-ae)+c^2(48Cd^3-2de(4Bd+Ae))-ce(64bCd^2-be(9Bd+Ae)}{15ce^4\left(cd^2-bde+\right.}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 35.86 (sec) , antiderivative size = 12997, normalized size of antiderivative = 13.10

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{7/2}}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2))/(d + e*x)^(7/2),x]
```

```
[Out] Result too large to show
```

Maple [A] (verified)

Time = 3.22 (sec) , antiderivative size = 1766, normalized size of antiderivative = 1.78

method	result	size
elliptic	Expression too large to display	1766
default	Expression too large to display	48427

```
[In] int((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(7/2),x,method=_RETURNVERBOSE
)
```

```
[Out] ((e*x+d)*(c*x^2+b*x+a))^(1/2)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)*(-2/5*(A*e^
2-B*d*e+C*d^2)/e^6*(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)/(x+d/e)^
3-2/15*(A*b*e^3-2*A*c*d*e^2+5*B*a*e^3-6*B*b*d*e^2+7*B*c*d^2*e-10*C*a*d*e^2+
11*C*b*d^2*e-12*C*c*d^3)/e^5/(a*e^2-b*d*e+c*d^2)*(c*e*x^3+b*e*x^2+c*d*x^2+a
*e*x+b*d*x+a*d)^(1/2)/(x+d/e)^2-2/15*(c*e*x^2+b*e*x+a*e)/e^4/(a*e^2-b*d*e+c
*d^2)^2*(6*A*a*c*e^4-2*A*b^2*e^4+2*A*b*c*d*e^3-2*A*c^2*d^2*e^2+5*B*a*b*e^4-
16*B*a*c*d*e^3-3*B*b^2*d*e^3+13*B*b*c*d^2*e^2-8*B*c^2*d^3*e+15*C*a^2*e^4-40
*C*a*b*d*e^3+56*C*a*c*d^2*e^2+23*C*b^2*d^2*e^2-58*C*b*c*d^3*e+33*C*c^2*d^4)
/((x+d/e)*(c*e*x^2+b*e*x+a*e))^(1/2)+2*((B*c*e+C*b*e-3*C*c*d)/e^4-1/15*c*(A
*b*e^3-2*A*c*d*e^2+5*B*a*e^3-6*B*b*d*e^2+7*B*c*d^2*e-10*C*a*d*e^2+11*C*b*d^
2*e-12*C*c*d^3)/e^4/(a*e^2-b*d*e+c*d^2)-1/15/e^4*(b*e-c*d)*(6*A*a*c*e^4-2*A
*b^2*e^4+2*A*b*c*d*e^3-2*A*c^2*d^2*e^2+5*B*a*b*e^4-16*B*a*c*d*e^3-3*B*b^2*d
*e^3+13*B*b*c*d^2*e^2-8*B*c^2*d^3*e+15*C*a^2*e^4-40*C*a*b*d*e^3+56*C*a*c*d^
2*e^2+23*C*b^2*d^2*e^2-58*C*b*c*d^3*e+33*C*c^2*d^4)/(a*e^2-b*d*e+c*d^2)^2+1
/15*b/e^3/(a*e^2-b*d*e+c*d^2)^2*(6*A*a*c*e^4-2*A*b^2*e^4+2*A*b*c*d*e^3-2*A*
c^2*d^2*e^2+5*B*a*b*e^4-16*B*a*c*d*e^3-3*B*b^2*d*e^3+13*B*b*c*d^2*e^2-8*B*c
^2*d^3*e+15*C*a^2*e^4-40*C*a*b*d*e^3+56*C*a*c*d^2*e^2+23*C*b^2*d^2*e^2-58*C
*b*c*d^3*e+33*C*c^2*d^4))*(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-
1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d
/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/
(-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b
*d*x+a*d)^(1/2)*EllipticF(((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2
```

```
),((-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))
)^(1/2))+2*(c*C/e^3+1/15*c/e^3*(6*A*a*c*e^4-2*A*b^2*e^4+2*A*b*c*d*e^3-2*A*c
^2*d^2*e^2+5*B*a*b*e^4-16*B*a*c*d*e^3-3*B*b^2*d*e^3+13*B*b*c*d^2*e^2-8*B*c^
2*d^3*e+15*C*a^2*e^4-40*C*a*b*d*e^3+56*C*a*c*d^2*e^2+23*C*b^2*d^2*e^2-58*C*
b*c*d^3*e+33*C*c^2*d^4)/(a*e^2-b*d*e+c*d^2)^2)*(d/e-1/2*(b+(-4*a*c+b^2)^(1/
2))/c)*((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4
*a*c+b^2)^(1/2)))/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-
4*a*c+b^2)^(1/2))/c)/(-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*e*x^3+b*
e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)*((-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*
EllipticE(((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2*(b+
(-4*a*c+b^2)^(1/2))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+1/2/c*(
-b+(-4*a*c+b^2)^(1/2))*EllipticF(((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c
))^(1/2),((-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^
(1/2))))^(1/2))))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.31 (sec) , antiderivative size = 2430, normalized size of antiderivative = 2.45

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{7/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(7/2),x, algorithm="fri
cas")
```

```
[Out] -2/45*((48*C*c^3*d^8 - 8*(14*C*b*c^2 + B*c^3)*d^7*e + (73*C*b^2*c - 2*A*c^3
 + (122*C*a + 17*B*b)*c^2)*d^6*e^2 - (7*C*b^3 + (22*B*a - 3*A*b)*c^2 + (161
*C*a*b + 8*B*b^2)*c)*d^5*e^3 + (20*C*a*b^2 - 3*B*b^3 - 18*A*a*c^2 + (90*C*a
^2 + 31*B*a*b + 3*A*b^2)*c)*d^4*e^4 - (15*C*a^2*b - 5*B*a*b^2 + 2*A*b^3 + 3
*(10*B*a^2 - 3*A*a*b)*c)*d^3*e^5 + (48*C*c^3*d^5*e^3 - 8*(14*C*b*c^2 + B*c^
3)*d^4*e^4 + (73*C*b^2*c - 2*A*c^3 + (122*C*a + 17*B*b)*c^2)*d^3*e^5 - (7*C
*b^3 + (22*B*a - 3*A*b)*c^2 + (161*C*a*b + 8*B*b^2)*c)*d^2*e^6 + (20*C*a*b^
2 - 3*B*b^3 - 18*A*a*c^2 + (90*C*a^2 + 31*B*a*b + 3*A*b^2)*c)*d*e^7 - (15*C
*a^2*b - 5*B*a*b^2 + 2*A*b^3 + 3*(10*B*a^2 - 3*A*a*b)*c)*e^8)*x^3 + 3*(48*C
*c^3*d^6*e^2 - 8*(14*C*b*c^2 + B*c^3)*d^5*e^3 + (73*C*b^2*c - 2*A*c^3 + (12
2*C*a + 17*B*b)*c^2)*d^4*e^4 - (7*C*b^3 + (22*B*a - 3*A*b)*c^2 + (161*C*a*b
 + 8*B*b^2)*c)*d^3*e^5 + (20*C*a*b^2 - 3*B*b^3 - 18*A*a*c^2 + (90*C*a^2 + 3
1*B*a*b + 3*A*b^2)*c)*d^2*e^6 - (15*C*a^2*b - 5*B*a*b^2 + 2*A*b^3 + 3*(10*B
*a^2 - 3*A*a*b)*c)*d*e^7)*x^2 + 3*(48*C*c^3*d^7*e - 8*(14*C*b*c^2 + B*c^3)*
d^6*e^2 + (73*C*b^2*c - 2*A*c^3 + (122*C*a + 17*B*b)*c^2)*d^5*e^3 - (7*C*b^
3 + (22*B*a - 3*A*b)*c^2 + (161*C*a*b + 8*B*b^2)*c)*d^4*e^4 + (20*C*a*b^2 -
 3*B*b^3 - 18*A*a*c^2 + (90*C*a^2 + 31*B*a*b + 3*A*b^2)*c)*d^3*e^5 - (15*C*
a^2*b - 5*B*a*b^2 + 2*A*b^3 + 3*(10*B*a^2 - 3*A*a*b)*c)*d^2*e^6)*x)*sqrt(c*
```

```
e)*weierstrassPInverse(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2
), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 -
9*a*b*c)*e^3)/(c^3*e^3), 1/3*(3*c*e*x + c*d + b*e)/(c*e)) + 3*(48*C*c^3*d^7
*e - 8*(11*C*b*c^2 + B*c^3)*d^6*e^2 + (38*C*b^2*c - 2*A*c^3 + (86*C*a + 13*
B*b)*c^2)*d^5*e^3 - (2*(8*B*a - A*b)*c^2 + (70*C*a*b + 3*B*b^2)*c)*d^4*e^4
+ (6*A*a*c^2 + (30*C*a^2 + 5*B*a*b - 2*A*b^2)*c)*d^3*e^5 + (48*C*c^3*d^4*e^
4 - 8*(11*C*b*c^2 + B*c^3)*d^3*e^5 + (38*C*b^2*c - 2*A*c^3 + (86*C*a + 13*B
*b)*c^2)*d^2*e^6 - (2*(8*B*a - A*b)*c^2 + (70*C*a*b + 3*B*b^2)*c)*d*e^7 + (
6*A*a*c^2 + (30*C*a^2 + 5*B*a*b - 2*A*b^2)*c)*e^8)*x^3 + 3*(48*C*c^3*d^5*e^
3 - 8*(11*C*b*c^2 + B*c^3)*d^4*e^4 + (38*C*b^2*c - 2*A*c^3 + (86*C*a + 13*B
*b)*c^2)*d^3*e^5 - (2*(8*B*a - A*b)*c^2 + (70*C*a*b + 3*B*b^2)*c)*d^2*e^6 +
 (6*A*a*c^2 + (30*C*a^2 + 5*B*a*b - 2*A*b^2)*c)*d*e^7)*x^2 + 3*(48*C*c^3*d^
6*e^2 - 8*(11*C*b*c^2 + B*c^3)*d^5*e^3 + (38*C*b^2*c - 2*A*c^3 + (86*C*a +
13*B*b)*c^2)*d^4*e^4 - (2*(8*B*a - A*b)*c^2 + (70*C*a*b + 3*B*b^2)*c)*d^3*e
^5 + (6*A*a*c^2 + (30*C*a^2 + 5*B*a*b - 2*A*b^2)*c)*d^2*e^6)*x)*sqrt(c*e)*w
eierstrassZeta(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/27
*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)
*e^3)/(c^3*e^3), weierstrassPInverse(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)
*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e
^2 + (2*b^3 - 9*a*b*c)*e^3)/(c^3*e^3), 1/3*(3*c*e*x + c*d + b*e)/(c*e))) +
3*(24*C*c^3*d^6*e^2 + 3*A*a^2*c*e^8 + (2*B*a^2 - 5*A*a*b)*c*d*e^7 - (41*C*b
*c^2 + 4*B*c^3)*d^5*e^3 + (15*C*b^2*c - A*c^3 + 2*(20*C*a + 3*B*b)*c^2)*d^4
*e^4 - (25*C*a*b*c + (10*B*a + A*b)*c^2)*d^3*e^5 + 2*(4*C*a^2*c + 5*A*a*c^2
)*d^2*e^6 + (33*C*c^3*d^4*e^4 - 2*(29*C*b*c^2 + 4*B*c^3)*d^3*e^5 + (23*C*b^
2*c - 2*A*c^3 + (56*C*a + 13*B*b)*c^2)*d^2*e^6 - (2*(8*B*a - A*b)*c^2 + (40
*C*a*b + 3*B*b^2)*c)*d*e^7 + (6*A*a*c^2 + (15*C*a^2 + 5*B*a*b - 2*A*b^2)*c)
*e^8)*x^2 + (54*C*c^3*d^5*e^3 + (5*B*a^2 + A*a*b)*c*e^8 - 3*(31*C*b*c^2 + 3
*B*c^3)*d^4*e^4 + (35*C*b^2*c - 6*A*c^3 + (90*C*a + 13*B*b)*c^2)*d^3*e^5 -
(59*C*a*b*c + (20*B*a - 7*A*b)*c^2)*d^2*e^6 + (10*A*a*c^2 + (20*C*a^2 - B*a
*b - 5*A*b^2)*c)*d*e^7)*x)*sqrt(c*x^2 + b*x + a)*sqrt(e*x + d))/(c^3*d^7*e^
5 - 2*b*c^2*d^6*e^6 - 2*a*b*c*d^4*e^8 + a^2*c*d^3*e^9 + (b^2*c + 2*a*c^2)*d
^5*e^7 + (c^3*d^4*e^8 - 2*b*c^2*d^3*e^9 - 2*a*b*c*d*e^11 + a^2*c*e^12 + (b^
2*c + 2*a*c^2)*d^2*e^10)*x^3 + 3*(c^3*d^5*e^7 - 2*b*c^2*d^4*e^8 - 2*a*b*c*d
^2*e^10 + a^2*c*d*e^11 + (b^2*c + 2*a*c^2)*d^3*e^9)*x^2 + 3*(c^3*d^6*e^6 -
2*b*c^2*d^5*e^7 - 2*a*b*c*d^3*e^9 + a^2*c*d^2*e^10 + (b^2*c + 2*a*c^2)*d^4*
e^8)*x)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{7/2}}\,dx = \int \frac{(A+Bx+Cx^2)\sqrt{a+bx+cx^2}}{(d+ex)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((C*x**2+B*x+A)*(c*x**2+b*x+a)**(1/2)/(e*x+d)**(7/2),x)
```

```
[Out] Integral((A + B*x + C*x**2)*sqrt(a + b*x + c*x**2)/(d + e*x)**(7/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{7/2}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{cx^2+bx+a}}{(ex+d)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(7/2),x, algorithm="max
ima")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)/(e*x + d)^(7/2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{7/2}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{cx^2+bx+a}}{(ex+d)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(7/2),x, algorithm="gia
c")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)/(e*x + d)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{7/2}}\,dx = \int \frac{(C\,x^2+B\,x+A)\,\sqrt{c\,x^2+b\,x+a}}{{(d+e\,x)}^{7/2}}\,dx$$

```
[In] int(((A + B*x + C*x^2)*(a + b*x + c*x^2)^(1/2))/(d + e*x)^(7/2),x)
```

```
[Out] int(((A + B*x + C*x^2)*(a + b*x + c*x^2)^(1/2))/(d + e*x)^(7/2), x)
```

3.264 $\int \frac{\sqrt{a+bx+cx^2}\left(A+Bx+Cx^2\right)}{(d+ex)^{9/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 1363

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{9/2}}\,dx = \frac{2(2c^3d^3(24Cd^2+e(4Bd+3Ae))-be^3(35a^2Ce^2-14abe(3Cd+Be)}{}$$

$$-\frac{2(c^2d^3(24Cd^2+e(4Bd+3Ae))-e^2(7a^2e^2(Cd-3Be)-b^2d(15Cd^2+6Bde+8Ae^2)+abe(12Cd^2+23B}{}$$

$$-\frac{2(Cd^2-e(Bd-Ae))\left(a+bx+cx^2\right)^{3/2}}{7e\left(cd^2-bde+ae^2\right)(d+ex)^{7/2}}$$

$$-\frac{\sqrt{2}\sqrt{b^2-4ac}(2c^3d^3(24Cd^2+e(4Bd+3Ae))-be^3(35a^2Ce^2-14abe(3Cd+Be)+b^2(15Cd^2+6Bde+8A}{}$$

$$+\frac{2\sqrt{2}\sqrt{b^2-4ac}(2c^2d^2(24Cd^2+e(4Bd+3Ae))+ce(2ae(51Cd^2+e(12Bd-5Ae))-bd(104Cd^2+3e(5Bd}{}$$

```
[Out] -2/7*(C*d^2-e*(-A*e+B*d))*(c*x^2+b*x+a)^(3/2)/e/(a*e^2-b*d*e+c*d^2)/(e*x+d)
^(7/2)-2/105*(c^2*d^3*(24*C*d^2+e*(3*A*e+4*B*d))-e^2*(7*a^2*e^2*(-3*B*e+C*d
)-b^2*d*(8*A*e^2+6*B*d*e+15*C*d^2)+a*b*e*(12*A*e^2+23*B*d*e+12*C*d^2))-c*d*
e*(b*d*(15*A*e^2+6*B*d*e+43*C*d^2)-a*e*(19*A*e^2+9*B*d*e+33*C*d^2))+e*(7*c^
2*d^2*(6*C*d^2+e*(-3*A*e+B*d))+e^2*(35*a^2*C*e^2-7*a*b*e*(-B*e+12*C*d)+b^2*
(-4*A*e^2-3*B*d*e+45*C*d^2))+c*e*(a*e*(-5*A*e^2-9*B*d*e+93*C*d^2)-b*(-21*A*
d*e^2+91*C*d^3)))*x)*(c*x^2+b*x+a)^(1/2)/e^3/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)^
(5/2)+2/105*(2*c^3*d^3*(24*C*d^2+e*(3*A*e+4*B*d))-b*e^3*(35*a^2*C*e^2-14*a*
b*e*(B*e+3*C*d)+b^2*(8*A*e^2+6*B*d*e+15*C*d^2))+c^2*d*e*(2*a*e*(69*C*d^2+e*
```

```
(-29*A*e+15*B*d))-b*d*(128*C*d^2+e*(9*A*e+19*B*d)))+c*e^2*(14*a^2*e^2*(-3*B
*e+11*C*d)-a*b*e*(237*C*d^2+e*(-29*A*e+B*d))+b^2*d*(103*C*d^2+e*(19*A*e+9*B
*d))))*(c*x^2+b*x+a)^(1/2)/e^3/(a*e^2-b*d*e+c*d^2)^3/(e*x+d)^(1/2)-1/105*(2
*c^3*d^3*(24*C*d^2+e*(3*A*e+4*B*d))-b*e^3*(35*a^2*C*e^2-14*a*b*e*(B*e+3*C*d
)+b^2*(8*A*e^2+6*B*d*e+15*C*d^2))+c^2*d*e*(2*a*e*(69*C*d^2+e*(-29*A*e+15*B*
d))-b*d*(128*C*d^2+e*(9*A*e+19*B*d)))+c*e^2*(14*a^2*e^2*(-3*B*e+11*C*d)-a*b
*e*(237*C*d^2+e*(-29*A*e+B*d))+b^2*d*(103*C*d^2+e*(19*A*e+9*B*d))))*Ellipti
cE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*
e*(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a
*c+b^2)^(1/2)*(e*x+d)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/e^4/(a*e^
2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)^(1/2)/(c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1
/2))))^(1/2)+2/105*(2*c^2*d^2*(24*C*d^2+e*(3*A*e+4*B*d))+c*e*(2*a*e*(51*C*d
^2+e*(-5*A*e+12*B*d))-b*d*(104*C*d^2+3*e*(2*A*e+5*B*d)))+e^2*(70*a^2*C*e^2-
7*a*b*e*(B*e+18*C*d)+b^2*(60*C*d^2+e*(4*A*e+3*B*d))))*EllipticF(1/2*((b+2*c
*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)
^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*
(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(
1/2))))^(1/2)/e^4/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 2.43 (sec) , antiderivative size = 1363, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.206$, Rules used = {1664, 824, 848, 857, 732, 435, 430}

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{9/2}}\,dx = -\frac{2(Cd^2-e(Bd-Ae))\left(cx^2+bx+a\right)^{3/2}}{7e\left(cd^2-bed+ae^2\right)(d+ex)^{7/2}}$$

$$-\frac{2((24Cd^5+e(4Bd+3Ae)d^3)\,c^2-de(bd(43Cd^2+6Bed+15Ae^2)-ae(33Cd^2+9Bed+19Ae^2))\,c-e^2(}{}$$

$$+\frac{2((48Cd^5+2e(4Bd+3Ae)d^3)\,c^3+de(2ae(69Cd^2+e(15Bd-29Ae))-b(128Cd^3+e(19Bd+9Ae)d))\,c}{}$$

$$-\frac{\sqrt{2}\sqrt{b^2-4ac}(2(24Cd^5+e(4Bd+3Ae)d^3)\,c^3+de(2ae(69Cd^2+e(15Bd-29Ae))-b(128Cd^3+e(19Bc}{}$$

$$+\frac{2\sqrt{2}\sqrt{b^2-4ac}((48Cd^4+2e(4Bd+3Ae)d^2)\,c^2+e(2ae(51Cd^2+e(12Bd-5Ae))-b(104Cd^3+3e(5Bd-}{}$$

```
[In] Int[(Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2))/(d + e*x)^(9/2),x]
```

```
[Out] (2*(c^3*(48*C*d^5 + 2*d^3*e*(4*B*d + 3*A*e)) - b*e^3*(35*a^2*C*e^2 - 14*a*b
*e*(3*C*d + B*e) + b^2*(15*C*d^2 + 6*B*d*e + 8*A*e^2)) + c^2*d*e*(2*a*e*(69
*C*d^2 + e*(15*B*d - 29*A*e)) - b*(128*C*d^3 + d*e*(19*B*d + 9*A*e))) + c*e
^2*(14*a^2*e^2*(11*C*d - 3*B*e) - a*b*e*(237*C*d^2 + e*(B*d - 29*A*e)) + b^
2*(103*C*d^3 + d*e*(9*B*d + 19*A*e))))*Sqrt[a + b*x + c*x^2])/(105*e^3*(c*d
^2 - b*d*e + a*e^2)^3*Sqrt[d + e*x]) - (2*(c^2*(24*C*d^5 + d^3*e*(4*B*d + 3
*A*e)) - e^2*(7*a^2*e^2*(C*d - 3*B*e) - b^2*d*(15*C*d^2 + 6*B*d*e + 8*A*e^2
) + a*b*e*(12*C*d^2 + 23*B*d*e + 12*A*e^2)) - c*d*e*(b*d*(43*C*d^2 + 6*B*d*
e + 15*A*e^2) - a*e*(33*C*d^2 + 9*B*d*e + 19*A*e^2)) + e*(7*c^2*(6*C*d^4 +
d^2*e*(B*d - 3*A*e)) + e^2*(35*a^2*C*e^2 - 7*a*b*e*(12*C*d - B*e) + b^2*(45
*C*d^2 - 3*B*d*e - 4*A*e^2)) + c*e*(a*e*(93*C*d^2 - 9*B*d*e - 5*A*e^2) - b*
(91*C*d^3 - 21*A*d*e^2)))*x)*Sqrt[a + b*x + c*x^2])/(105*e^3*(c*d^2 - b*d*e
 + a*e^2)^2*(d + e*x)^(5/2)) - (2*(C*d^2 - e*(B*d - A*e))*(a + b*x + c*x^2)
^(3/2))/(7*e*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^(7/2)) - (Sqrt[2]*Sqrt[b^2 -
 4*a*c]*(2*c^3*(24*C*d^5 + d^3*e*(4*B*d + 3*A*e)) - b*e^3*(35*a^2*C*e^2 - 1
4*a*b*e*(3*C*d + B*e) + b^2*(15*C*d^2 + 6*B*d*e + 8*A*e^2)) + c^2*d*e*(2*a*
e*(69*C*d^2 + e*(15*B*d - 29*A*e)) - b*(128*C*d^3 + d*e*(19*B*d + 9*A*e)))
+ c*e^2*(14*a^2*e^2*(11*C*d - 3*B*e) - a*b*e*(237*C*d^2 + e*(B*d - 29*A*e))
 + b^2*(103*C*d^3 + d*e*(9*B*d + 19*A*e))))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*
x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] +
2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b +
Sqrt[b^2 - 4*a*c])*e)])/(105*e^4*(c*d^2 - b*d*e + a*e^2)^3*Sqrt[(c*(d + e*x
))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2]
*Sqrt[b^2 - 4*a*c]*(c^2*(48*C*d^4 + 2*d^2*e*(4*B*d + 3*A*e)) + c*e*(2*a*e*(
51*C*d^2 + e*(12*B*d - 5*A*e)) - b*(104*C*d^3 + 3*d*e*(5*B*d + 2*A*e))) + e
^2*(70*a^2*C*e^2 - 7*a*b*e*(18*C*d + B*e) + b^2*(60*C*d^2 + e*(3*B*d + 4*A*
e))))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((c*(a
+ b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c
] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (
b + Sqrt[b^2 - 4*a*c])*e)])/(105*e^4*(c*d^2 - b*d*e + a*e^2)^2*Sqrt[d + e*x
]*Sqrt[a + b*x + c*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 824

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Dist[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)
^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p
 + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*
(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m
 - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &
& LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 848

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
```

```
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{2(Cd^2-e(Bd-Ae))\left(a+bx+cx^2\right)^{3/2}}{7e\left(cd^2-bde+ae^2\right)(d+ex)^{7/2}} - \frac{2\int \frac{\left(-\frac{3bCd^2-be(3Bd+4Ae)+7e(Acd-aCd+aBe)}{2e}-\frac{1}{2}\left(Bcd-7bCd+\frac{6cCd^2}{e}-Ace+7aCe\right)x\right)\sqrt{a+bx+cx^2}}{(d+ex)^{7/2}}dx}{7\left(cd^2-bde+ae^2\right)}$$

$$= -\frac{2(c^2(24Cd^5+d^3e(4Bd+3Ae))-e^2(7a^2e^2(Cd-3Be)-b^2d(15Cd^2+6Bde+8Ae^2)+abe(12C}{}$$

$$- \frac{2(Cd^2-e(Bd-Ae))\left(a+bx+cx^2\right)^{3/2}}{7e\left(cd^2-bde+ae^2\right)(d+ex)^{7/2}} + \frac{4\int \frac{\frac{b^3e^2\left(15Cd^2+6Bde+8Ae^2\right)-6ace\left(7ae^2(2Cd-Be)+cd\left(6Cd^2+Bde-8Ae^2\right)\right)+b\left(35a^2Ce^4+ace^2\left(111Cd^2-6Bde-29Ae^2\right)+c^2d^2(24Cd^2+4Bd}{4e}}{}}{}$$

$$= \frac{2(c^3(48Cd^5+2d^3e(4Bd+3Ae))-be^3(35a^2Ce^2-14abe(3Cd+Be)+b^2(15Cd^2+6Bde+8Ae^2))}{}$$

$$- \frac{2(c^2(24Cd^5+d^3e(4Bd+3Ae))-e^2(7a^2e^2(Cd-3Be)-b^2d(15Cd^2+6Bde+8Ae^2)+abe(12C}{}$$

$$- \frac{2(Cd^2-e(Bd-Ae))\left(a+bx+cx^2\right)^{3/2}}{7e\left(cd^2-bde+ae^2\right)(d+ex)^{7/2}} - \frac{8\int \frac{\frac{c\left(b^3de^2\left(45Cd^2-e(3Bd+4Ae)\right)-b^2\left(cd^2e\left(61Cd^2+9Bde-9Ae^2\right)+4ae^3\left(36Cd^2-Bde+Ae^2\right)\right)+b\left(7a^2e^4(23Cd+Be)+c^2d^3(24Cd^2+4Bde+3Ae}{8e}}{}}{}$$

$$= \frac{2(c^3(48Cd^5 + 2d^3e(4Bd + 3Ae)) - be^3(35a^2Ce^2 - 14abe(3Cd + Be) + b^2(15Cd^2 + 6Bde + 8Ae^2}{}$$

$$- \frac{2(c^2(24Cd^5 + d^3e(4Bd + 3Ae)) - e^2(7a^2e^2(Cd - 3Be) - b^2d(15Cd^2 + 6Bde + 8Ae^2) + abe(12}{}$$

$$- \frac{2(Cd^2 - e(Bd - Ae))\left(a + bx + cx^2\right)^{3/2}}{7e\left(cd^2 - bde + ae^2\right)(d + ex)^{7/2}}$$

$$- \frac{(c(c^3(48Cd^5 + 2d^3e(4Bd + 3Ae)) - be^3(35a^2Ce^2 - 14abe(3Cd + Be) + b^2(15Cd^2 + 6Bde + 8}{}$$

$$- \frac{\left(8\left(-\frac{cd\left(c^3\left(48Cd^5+2d^3e(4Bd+3Ae)\right)-be^3\left(35a^2Ce^2-14abe(3Cd+Be)+b^2\left(15Cd^2+6Bde+8Ae^2\right)\right)+c^2de\left(2ae\left(69Cd^2+15Bde-\right.\right.\right.\right.\right.}{}\right.\right.}{}$$

= Too large to display

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 36.64 (sec) , antiderivative size = 19853, normalized size of antiderivative = 14.57

$$\int \frac{\sqrt{a + bx + cx^2}(A + Bx + Cx^2)}{(d + ex)^{9/2}}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2))/(d + e*x)^(9/2),x]
```

```
[Out] Result too large to show
```

Maple [A] (verified)

Time = 4.21 (sec) , antiderivative size = 2484, normalized size of antiderivative = 1.82

method	result	size
elliptic	Expression too large to display	2484
default	Expression too large to display	88790

```
[In] int((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(9/2),x,method=_RETURNVERBOSE
)
```

```
[Out] ((e*x+d)*(c*x^2+b*x+a))^(1/2)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)*(-2/7*(A*e^
2-B*d*e+C*d^2)/e^7*(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)/(x+d/e)^
4-2/35*(A*b*e^3-2*A*c*d*e^2+7*B*a*e^3-8*B*b*d*e^2+9*B*c*d^2*e-14*C*a*d*e^2+
15*C*b*d^2*e-16*C*c*d^3)/(a*e^2-b*d*e+c*d^2)/e^6*(c*e*x^3+b*e*x^2+c*d*x^2+a
*e*x+b*d*x+a*d)^(1/2)/(x+d/e)^3-2/105*(10*A*a*c*e^4-4*A*b^2*e^4+6*A*b*c*d*e
```

```
^3-6*A*c^2*d^2*e^2+7*B*a*b*e^4-24*B*a*c*d*e^3-3*B*b^2*d*e^3+15*B*b*c*d^2*e^
2-8*B*c^2*d^3*e+35*C*a^2*e^4-84*C*a*b*d*e^3+108*C*a*c*d^2*e^2+45*C*b^2*d^2*
e^2-106*C*b*c*d^3*e+57*C*c^2*d^4)/e^5/(a*e^2-b*d*e+c*d^2)^2*(c*e*x^3+b*e*x^
2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)/(x+d/e)^2+2/105*(c*e*x^2+b*e*x+a*e)/e^4/(a
*e^2-b*d*e+c*d^2)^3*(29*A*a*b*c*e^5-58*A*a*c^2*d*e^4-8*A*b^3*e^5+19*A*b^2*c
*d*e^4-9*A*b*c^2*d^2*e^3+6*A*c^3*d^3*e^2-42*B*a^2*c*e^5+14*B*a*b^2*e^5-B*a*
b*c*d*e^4+30*B*a*c^2*d^2*e^3-6*B*b^3*d*e^4+9*B*b^2*c*d^2*e^3-19*B*b*c^2*d^3
*e^2+8*B*c^3*d^4*e-35*C*a^2*b*e^5+154*C*a^2*c*d*e^4+42*C*a*b^2*d*e^4-237*C*
a*b*c*d^2*e^3+138*C*a*c^2*d^3*e^2-15*C*b^3*d^2*e^3+103*C*b^2*c*d^3*e^2-128*
C*b*c^2*d^4*e+48*C*c^3*d^5)/((x+d/e)*(c*e*x^2+b*e*x+a*e))^(1/2)+2*(c*C/e^4-
1/105*c*(10*A*a*c*e^4-4*A*b^2*e^4+6*A*b*c*d*e^3-6*A*c^2*d^2*e^2+7*B*a*b*e^4
-24*B*a*c*d*e^3-3*B*b^2*d*e^3+15*B*b*c*d^2*e^2-8*B*c^2*d^3*e+35*C*a^2*e^4-8
4*C*a*b*d*e^3+108*C*a*c*d^2*e^2+45*C*b^2*d^2*e^2-106*C*b*c*d^3*e+57*C*c^2*d
^4)/e^4/(a*e^2-b*d*e+c*d^2)^2+1/105/e^4*(b*e-c*d)*(29*A*a*b*c*e^5-58*A*a*c^
2*d*e^4-8*A*b^3*e^5+19*A*b^2*c*d*e^4-9*A*b*c^2*d^2*e^3+6*A*c^3*d^3*e^2-42*B
*a^2*c*e^5+14*B*a*b^2*e^5-B*a*b*c*d*e^4+30*B*a*c^2*d^2*e^3-6*B*b^3*d*e^4+9*
B*b^2*c*d^2*e^3-19*B*b*c^2*d^3*e^2+8*B*c^3*d^4*e-35*C*a^2*b*e^5+154*C*a^2*c
*d*e^4+42*C*a*b^2*d*e^4-237*C*a*b*c*d^2*e^3+138*C*a*c^2*d^3*e^2-15*C*b^3*d^
2*e^3+103*C*b^2*c*d^3*e^2-128*C*b*c^2*d^4*e+48*C*c^3*d^5)/(a*e^2-b*d*e+c*d^
2)^3-1/105*b/e^3/(a*e^2-b*d*e+c*d^2)^3*(29*A*a*b*c*e^5-58*A*a*c^2*d*e^4-8*A
*b^3*e^5+19*A*b^2*c*d*e^4-9*A*b*c^2*d^2*e^3+6*A*c^3*d^3*e^2-42*B*a^2*c*e^5+
14*B*a*b^2*e^5-B*a*b*c*d*e^4+30*B*a*c^2*d^2*e^3-6*B*b^3*d*e^4+9*B*b^2*c*d^2
*e^3-19*B*b*c^2*d^3*e^2+8*B*c^3*d^4*e-35*C*a^2*b*e^5+154*C*a^2*c*d*e^4+42*C
*a*b^2*d*e^4-237*C*a*b*c*d^2*e^3+138*C*a*c^2*d^3*e^2-15*C*b^3*d^2*e^3+103*C
*b^2*c*d^3*e^2-128*C*b*c^2*d^4*e+48*C*c^3*d^5))*(d/e-1/2*(b+(-4*a*c+b^2)^(1
/2))/c)*((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-
4*a*c+b^2)^(1/2)))/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(
-4*a*c+b^2)^(1/2))/c)/(-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*e*x^3+b
*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)*EllipticF(((x+d/e)/(d/e-1/2*(b+(-4*a*
c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e-1/2/c*(-
b+(-4*a*c+b^2)^(1/2))))^(1/2))-2/105*c/e^3*(29*A*a*b*c*e^5-58*A*a*c^2*d*e^4
-8*A*b^3*e^5+19*A*b^2*c*d*e^4-9*A*b*c^2*d^2*e^3+6*A*c^3*d^3*e^2-42*B*a^2*c*
e^5+14*B*a*b^2*e^5-B*a*b*c*d*e^4+30*B*a*c^2*d^2*e^3-6*B*b^3*d*e^4+9*B*b^2*c
*d^2*e^3-19*B*b*c^2*d^3*e^2+8*B*c^3*d^4*e-35*C*a^2*b*e^5+154*C*a^2*c*d*e^4+
42*C*a*b^2*d*e^4-237*C*a*b*c*d^2*e^3+138*C*a*c^2*d^3*e^2-15*C*b^3*d^2*e^3+1
03*C*b^2*c*d^3*e^2-128*C*b*c^2*d^4*e+48*C*c^3*d^5)/(a*e^2-b*d*e+c*d^2)^3*(d
/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c
))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1
/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e+1/2*(b+(-4*a*c+b^2)^(1
/2))/c))^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)*((-d/e-1/2/c
*(-b+(-4*a*c+b^2)^(1/2)))*EllipticE(((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2)
)/c))^(1/2),((-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^
2)^(1/2))))^(1/2))+1/2/c*(-b+(-4*a*c+b^2)^(1/2))*EllipticF(((x+d/e)/(d/e-1/
2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d
/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.78 (sec) , antiderivative size = 4543, normalized size of antiderivative = 3.33

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{9/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(9/2),x, algorithm="fri
cas")

[Out] 2/315*((48*C*c^4*d^10 - 8*(19*C*b*c^3 - B*c^4)*d^9*e + (158*C*b^2*c^2 + 6*A
*c^4 + (174*C*a - 23*B*b)*c^3)*d^8*e^2 - (47*C*b^3*c - 12*(3*B*a - A*b)*c^3
 + (384*C*a*b - 17*B*b^2)*c^2)*d^7*e^3 - (15*C*b^4 - 104*A*a*c^3 - (208*C*a
^2 - 106*B*a*b - 17*A*b^2)*c^2 - 3*(79*C*a*b^2 + 4*B*b^3)*c)*d^6*e^4 + (42*
C*a*b^3 - 6*B*b^4 + 52*(3*B*a^2 - 2*A*a*b)*c^2 - (364*C*a^2*b - B*a*b^2 - 2
3*A*b^3)*c)*d^5*e^5 - (35*C*a^2*b^2 - 14*B*a*b^3 + 8*A*b^4 + 30*A*a^2*c^2 -
 (210*C*a^3 - 63*B*a^2*b + 41*A*a*b^2)*c)*d^4*e^6 + (48*C*c^4*d^6*e^4 - 8*(
19*C*b*c^3 - B*c^4)*d^5*e^5 + (158*C*b^2*c^2 + 6*A*c^4 + (174*C*a - 23*B*b)
*c^3)*d^4*e^6 - (47*C*b^3*c - 12*(3*B*a - A*b)*c^3 + (384*C*a*b - 17*B*b^2)
*c^2)*d^3*e^7 - (15*C*b^4 - 104*A*a*c^3 - (208*C*a^2 - 106*B*a*b - 17*A*b^2
)*c^2 - 3*(79*C*a*b^2 + 4*B*b^3)*c)*d^2*e^8 + (42*C*a*b^3 - 6*B*b^4 + 52*(3
*B*a^2 - 2*A*a*b)*c^2 - (364*C*a^2*b - B*a*b^2 - 23*A*b^3)*c)*d*e^9 - (35*C
*a^2*b^2 - 14*B*a*b^3 + 8*A*b^4 + 30*A*a^2*c^2 - (210*C*a^3 - 63*B*a^2*b +
41*A*a*b^2)*c)*e^10)*x^4 + 4*(48*C*c^4*d^7*e^3 - 8*(19*C*b*c^3 - B*c^4)*d^6
*e^4 + (158*C*b^2*c^2 + 6*A*c^4 + (174*C*a - 23*B*b)*c^3)*d^5*e^5 - (47*C*b
^3*c - 12*(3*B*a - A*b)*c^3 + (384*C*a*b - 17*B*b^2)*c^2)*d^4*e^6 - (15*C*b
^4 - 104*A*a*c^3 - (208*C*a^2 - 106*B*a*b - 17*A*b^2)*c^2 - 3*(79*C*a*b^2 +
 4*B*b^3)*c)*d^3*e^7 + (42*C*a*b^3 - 6*B*b^4 + 52*(3*B*a^2 - 2*A*a*b)*c^2 -
 (364*C*a^2*b - B*a*b^2 - 23*A*b^3)*c)*d^2*e^8 - (35*C*a^2*b^2 - 14*B*a*b^3
 + 8*A*b^4 + 30*A*a^2*c^2 - (210*C*a^3 - 63*B*a^2*b + 41*A*a*b^2)*c)*d*e^9)
*x^3 + 6*(48*C*c^4*d^8*e^2 - 8*(19*C*b*c^3 - B*c^4)*d^7*e^3 + (158*C*b^2*c^
2 + 6*A*c^4 + (174*C*a - 23*B*b)*c^3)*d^6*e^4 - (47*C*b^3*c - 12*(3*B*a - A
*b)*c^3 + (384*C*a*b - 17*B*b^2)*c^2)*d^5*e^5 - (15*C*b^4 - 104*A*a*c^3 - (
208*C*a^2 - 106*B*a*b - 17*A*b^2)*c^2 - 3*(79*C*a*b^2 + 4*B*b^3)*c)*d^4*e^6
 + (42*C*a*b^3 - 6*B*b^4 + 52*(3*B*a^2 - 2*A*a*b)*c^2 - (364*C*a^2*b - B*a*
b^2 - 23*A*b^3)*c)*d^3*e^7 - (35*C*a^2*b^2 - 14*B*a*b^3 + 8*A*b^4 + 30*A*a^
2*c^2 - (210*C*a^3 - 63*B*a^2*b + 41*A*a*b^2)*c)*d^2*e^8)*x^2 + 4*(48*C*c^4
*d^9*e - 8*(19*C*b*c^3 - B*c^4)*d^8*e^2 + (158*C*b^2*c^2 + 6*A*c^4 + (174*C
*a - 23*B*b)*c^3)*d^7*e^3 - (47*C*b^3*c - 12*(3*B*a - A*b)*c^3 + (384*C*a*b
 - 17*B*b^2)*c^2)*d^6*e^4 - (15*C*b^4 - 104*A*a*c^3 - (208*C*a^2 - 106*B*a*
b - 17*A*b^2)*c^2 - 3*(79*C*a*b^2 + 4*B*b^3)*c)*d^5*e^5 + (42*C*a*b^3 - 6*B
*b^4 + 52*(3*B*a^2 - 2*A*a*b)*c^2 - (364*C*a^2*b - B*a*b^2 - 23*A*b^3)*c)*d
^4*e^6 - (35*C*a^2*b^2 - 14*B*a*b^3 + 8*A*b^4 + 30*A*a^2*c^2 - (210*C*a^3 -
 63*B*a^2*b + 41*A*a*b^2)*c)*d^3*e^7)*x)*sqrt(c*e)*weierstrassPInverse(4/3*
```

```
(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c
^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(c^3*e^3), 1/
3*(3*c*e*x + c*d + b*e)/(c*e)) + 3*(48*C*c^4*d^9*e - 8*(16*C*b*c^3 - B*c^4)
*d^8*e^2 + (103*C*b^2*c^2 + 6*A*c^4 + (138*C*a - 19*B*b)*c^3)*d^7*e^3 - 3*(
5*C*b^3*c - (10*B*a - 3*A*b)*c^3 + (79*C*a*b - 3*B*b^2)*c^2)*d^6*e^4 - (58*
A*a*c^3 - (154*C*a^2 - B*a*b + 19*A*b^2)*c^2 - 6*(7*C*a*b^2 - B*b^3)*c)*d^5
*e^5 - ((42*B*a^2 - 29*A*a*b)*c^2 + (35*C*a^2*b - 14*B*a*b^2 + 8*A*b^3)*c)*
d^4*e^6 + (48*C*c^4*d^5*e^5 - 8*(16*C*b*c^3 - B*c^4)*d^4*e^6 + (103*C*b^2*c
^2 + 6*A*c^4 + (138*C*a - 19*B*b)*c^3)*d^3*e^7 - 3*(5*C*b^3*c - (10*B*a - 3
*A*b)*c^3 + (79*C*a*b - 3*B*b^2)*c^2)*d^2*e^8 - (58*A*a*c^3 - (154*C*a^2 -
B*a*b + 19*A*b^2)*c^2 - 6*(7*C*a*b^2 - B*b^3)*c)*d*e^9 - ((42*B*a^2 - 29*A*
a*b)*c^2 + (35*C*a^2*b - 14*B*a*b^2 + 8*A*b^3)*c)*e^10)*x^4 + 4*(48*C*c^4*d
^6*e^4 - 8*(16*C*b*c^3 - B*c^4)*d^5*e^5 + (103*C*b^2*c^2 + 6*A*c^4 + (138*C
*a - 19*B*b)*c^3)*d^4*e^6 - 3*(5*C*b^3*c - (10*B*a - 3*A*b)*c^3 + (79*C*a*b
 - 3*B*b^2)*c^2)*d^3*e^7 - (58*A*a*c^3 - (154*C*a^2 - B*a*b + 19*A*b^2)*c^2
 - 6*(7*C*a*b^2 - B*b^3)*c)*d^2*e^8 - ((42*B*a^2 - 29*A*a*b)*c^2 + (35*C*a^
2*b - 14*B*a*b^2 + 8*A*b^3)*c)*d*e^9)*x^3 + 6*(48*C*c^4*d^7*e^3 - 8*(16*C*b
*c^3 - B*c^4)*d^6*e^4 + (103*C*b^2*c^2 + 6*A*c^4 + (138*C*a - 19*B*b)*c^3)*
d^5*e^5 - 3*(5*C*b^3*c - (10*B*a - 3*A*b)*c^3 + (79*C*a*b - 3*B*b^2)*c^2)*d
^4*e^6 - (58*A*a*c^3 - (154*C*a^2 - B*a*b + 19*A*b^2)*c^2 - 6*(7*C*a*b^2 -
B*b^3)*c)*d^3*e^7 - ((42*B*a^2 - 29*A*a*b)*c^2 + (35*C*a^2*b - 14*B*a*b^2 +
 8*A*b^3)*c)*d^2*e^8)*x^2 + 4*(48*C*c^4*d^8*e^2 - 8*(16*C*b*c^3 - B*c^4)*d^
7*e^3 + (103*C*b^2*c^2 + 6*A*c^4 + (138*C*a - 19*B*b)*c^3)*d^6*e^4 - 3*(5*C
*b^3*c - (10*B*a - 3*A*b)*c^3 + (79*C*a*b - 3*B*b^2)*c^2)*d^5*e^5 - (58*A*a
*c^3 - (154*C*a^2 - B*a*b + 19*A*b^2)*c^2 - 6*(7*C*a*b^2 - B*b^3)*c)*d^4*e^
6 - ((42*B*a^2 - 29*A*a*b)*c^2 + (35*C*a^2*b - 14*B*a*b^2 + 8*A*b^3)*c)*d^3
*e^7)*x)*sqrt(c*e)*weierstrassZeta(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e
^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2
 + (2*b^3 - 9*a*b*c)*e^3)/(c^3*e^3), weierstrassPInverse(4/3*(c^2*d^2 - b*c
*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(
b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(c^3*e^3), 1/3*(3*c*e*x + c
*d + b*e)/(c*e))) + 3*(24*C*c^4*d^8*e^2 - 15*A*a^3*c*e^10 - 21*(2*B*a^2 - 5
*A*a*b)*c^2*d^3*e^7 - 6*(B*a^3 - 7*A*a^2*b)*c*d*e^9 - (61*C*b*c^3 - 4*B*c^4
)*d^7*e^3 + 3*(15*C*b^2*c^2 + A*c^4 + (22*C*a - 3*B*b)*c^3)*d^6*e^4 + ((32*
B*a + 9*A*b)*c^3 - (119*C*a*b + 3*B*b^2)*c^2)*d^5*e^5 - (95*A*a*c^3 - (98*C
*a^2 + 7*B*a*b - 4*A*b^2)*c^2)*d^4*e^6 - (49*A*a^2*c^2 + (8*C*a^3 - 14*B*a^
2*b + 35*A*a*b^2)*c)*d^2*e^8 + (48*C*c^4*d^5*e^5 - 8*(16*C*b*c^3 - B*c^4)*d
^4*e^6 + (103*C*b^2*c^2 + 6*A*c^4 + (138*C*a - 19*B*b)*c^3)*d^3*e^7 - 3*(5*
C*b^3*c - (10*B*a - 3*A*b)*c^3 + (79*C*a*b - 3*B*b^2)*c^2)*d^2*e^8 - (58*A*
a*c^3 - (154*C*a^2 - B*a*b + 19*A*b^2)*c^2 - 6*(7*C*a*b^2 - B*b^3)*c)*d*e^9
 - ((42*B*a^2 - 29*A*a*b)*c^2 + (35*C*a^2*b - 14*B*a*b^2 + 8*A*b^3)*c)*e^10
)*x^3 + (87*C*c^4*d^6*e^4 - (221*C*b*c^3 - 32*B*c^4)*d^5*e^5 + (158*C*b^2*c
^2 + 24*A*c^4 + (249*C*a - 80*B*b)*c^3)*d^4*e^6 + ((122*B*a - 39*A*b)*c^3 -
 (413*C*a*b - 45*B*b^2)*c^2)*d^3*e^7 - (178*A*a*c^3 - (319*C*a^2 - 49*B*a*b
 + 67*A*b^2)*c^2 + 3*(C*a*b^2 + 7*B*b^3)*c)*d^2*e^8 - ((102*B*a^2 - 91*A*a*
```

```
b)*c^2 - 2*(7*C*a^2*b + 26*B*a*b^2 - 14*A*b^3)*c)*d*e^9 - (10*A*a^2*c^2 + (
35*C*a^3 + 7*B*a^2*b - 4*A*a*b^2)*c)*e^10)*x^2 + (78*C*c^4*d^7*e^3 - 3*(7*B
*a^3 + A*a^2*b)*c*e^10 - (199*C*b*c^3 - 13*B*c^4)*d^6*e^4 + (145*C*b^2*c^2
+ 36*A*c^4 + (222*C*a - 25*B*b)*c^3)*d^5*e^5 + ((79*B*a - 66*A*b)*c^3 - (38
5*C*a*b + 12*B*b^2)*c^2)*d^4*e^6 - (170*A*a*c^3 - (308*C*a^2 + 49*B*a*b + 8
9*A*b^2)*c^2)*d^3*e^7 - (7*(21*B*a^2 - 11*A*a*b)*c^2 - (4*C*a^2*b - 7*B*a*b
^2 - 35*A*b^3)*c)*d^2*e^8 - 2*(7*A*a^2*c^2 + (14*C*a^3 - 26*B*a^2*b - 7*A*a
*b^2)*c)*d*e^9)*x)*sqrt(c*x^2 + b*x + a)*sqrt(e*x + d))/(c^4*d^10*e^5 - 3*b
*c^3*d^9*e^6 - 3*a^2*b*c*d^5*e^10 + a^3*c*d^4*e^11 + 3*(b^2*c^2 + a*c^3)*d^
8*e^7 - (b^3*c + 6*a*b*c^2)*d^7*e^8 + 3*(a*b^2*c + a^2*c^2)*d^6*e^9 + (c^4*
d^6*e^9 - 3*b*c^3*d^5*e^10 - 3*a^2*b*c*d*e^14 + a^3*c*e^15 + 3*(b^2*c^2 + a
*c^3)*d^4*e^11 - (b^3*c + 6*a*b*c^2)*d^3*e^12 + 3*(a*b^2*c + a^2*c^2)*d^2*e
^13)*x^4 + 4*(c^4*d^7*e^8 - 3*b*c^3*d^6*e^9 - 3*a^2*b*c*d^2*e^13 + a^3*c*d*
e^14 + 3*(b^2*c^2 + a*c^3)*d^5*e^10 - (b^3*c + 6*a*b*c^2)*d^4*e^11 + 3*(a*b
^2*c + a^2*c^2)*d^3*e^12)*x^3 + 6*(c^4*d^8*e^7 - 3*b*c^3*d^7*e^8 - 3*a^2*b*
c*d^3*e^12 + a^3*c*d^2*e^13 + 3*(b^2*c^2 + a*c^3)*d^6*e^9 - (b^3*c + 6*a*b*
c^2)*d^5*e^10 + 3*(a*b^2*c + a^2*c^2)*d^4*e^11)*x^2 + 4*(c^4*d^9*e^6 - 3*b*
c^3*d^8*e^7 - 3*a^2*b*c*d^4*e^11 + a^3*c*d^3*e^12 + 3*(b^2*c^2 + a*c^3)*d^7
*e^8 - (b^3*c + 6*a*b*c^2)*d^6*e^9 + 3*(a*b^2*c + a^2*c^2)*d^5*e^10)*x)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{9/2}}\,dx=\int \frac{(A+Bx+Cx^2)\sqrt{a+bx+cx^2}}{(d+ex)^{\frac{9}{2}}}\,dx$$

```
[In] integrate((C*x**2+B*x+A)*(c*x**2+b*x+a)**(1/2)/(e*x+d)**(9/2),x)
```

```
[Out] Integral((A + B*x + C*x**2)*sqrt(a + b*x + c*x**2)/(d + e*x)**(9/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{9/2}}\,dx=\int \frac{(Cx^2+Bx+A)\sqrt{cx^2+bx+a}}{(ex+d)^{\frac{9}{2}}}\,dx$$

```
[In] integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(9/2),x, algorithm="max
ima")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)/(e*x + d)^(9/2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{9/2}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{cx^2+bx+a}}{(ex+d)^{\frac{9}{2}}}\,dx$$

```
[In] integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(9/2),x, algorithm="gia
c")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)/(e*x + d)^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{9/2}}\,dx = \int \frac{(C\,x^2+B\,x+A)\,\sqrt{c\,x^2+b\,x+a}}{(d+e\,x)^{9/2}}\,dx$$

```
[In] int(((A + B*x + C*x^2)*(a + b*x + c*x^2)^(1/2))/(d + e*x)^(9/2),x)
```

```
[Out] int(((A + B*x + C*x^2)*(a + b*x + c*x^2)^(1/2))/(d + e*x)^(9/2), x)
```

3.265 $\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{11/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 1904

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{11/2}}\,dx = \frac{2(2c^3d^3(8Cd^2+e(4Bd+5Ae))+3c^2de(2ae(9Cd^2+7Bde-9Ae^2}{}$$

$$+\frac{2(2c^4d^4(8Cd^2+e(4Bd+5Ae))+2b^2e^4(21a^2Ce^2-6abe(3Cd+2Be)+b^2(5Cd^2+4Bde+8Ae^2))-6c^2e}{}$$

$$-\frac{2(c^2d^3(8Cd^2+e(4Bd+5Ae))-e^2(3a^2e^2(3Cd-5Be)-abe(2Cd^2-17Bde-10Ae^2)-b^2d(5Cd^2+4B}{}$$

$$-\frac{2(Cd^2-e(Bd-Ae))\left(a+bx+cx^2\right)^{3/2}}{9e\left(cd^2-bde+ae^2\right)(d+ex)^{9/2}}$$

$$-\frac{\sqrt{2}\sqrt{b^2-4ac}(2c^4d^4(8Cd^2+e(4Bd+5Ae))+2b^2e^4(21a^2Ce^2-6abe(3Cd+2Be)+b^2(5Cd^2+4Bde+8}{}$$

$$+\frac{2\sqrt{2}\sqrt{b^2-4ac}(2c^3d^3(8Cd^2+e(4Bd+5Ae))+3c^2de(2ae(9Cd^2+7Bde-9Ae^2)-bd(16Cd^2+7Bde+5}{}$$

```
[Out] -2/9*(C*d^2-e*(-A*e+B*d))*(c*x^2+b*x+a)^(3/2)/e/(a*e^2-b*d*e+c*d^2)/(e*x+d)
^(9/2)+2/315*(2*c^3*d^3*(8*C*d^2+e*(5*A*e+4*B*d))+3*c^2*d*e*(2*a*e*(-9*A*e^
2+7*B*d*e+9*C*d^2)-b*d*(5*A*e^2+7*B*d*e+16*C*d^2))+3*c*e^2*(2*a^2*e^2*(-5*B
*e+17*C*d)-a*b*e*(-9*A*e^2+5*B*d*e+41*C*d^2)+b^2*d*(7*A*e^2+3*B*d*e+15*C*d^
2))-b*e^3*(21*a^2*C*e^2-6*a*b*e*(2*B*e+3*C*d)+b^2*(8*A*e^2+4*B*d*e+5*C*d^2)
))*(c*x^2+b*x+a)^(1/2)/e^3/(a*e^2-b*d*e+c*d^2)^3/(e*x+d)^(3/2)-2/105*(c^2*d
```

```
^3*(8*C*d^2+e*(5*A*e+4*B*d))-e^2*(3*a^2*e^2*(-5*B*e+3*C*d)-a*b*e*(-10*A*e^2
-17*B*d*e+2*C*d^2)-b^2*d*(8*A*e^2+4*B*d*e+5*C*d^2))-c*d*e*(3*b*d*(5*A*e^2+2
*B*d*e+5*C*d^2)-a*e*(13*A*e^2+11*B*d*e+7*C*d^2))+e*(3*c^2*d^2*(6*C*d^2+e*(-
5*A*e+3*B*d))+c*e*(a*e*(-7*A*e^2+B*d*e+47*C*d^2)-3*b*d*(-5*A*e^2+2*B*d*e+15
*C*d^2))+e^2*(21*a^2*C*e^2-3*a*b*e*(-B*e+16*C*d)+b^2*(25*C*d^2-e*(2*A*e+B*d
))))*x)*(c*x^2+b*x+a)^(1/2)/e^3/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)^(7/2)+2/315*(
2*c^4*d^4*(8*C*d^2+e*(5*A*e+4*B*d))+2*b^2*e^4*(21*a^2*C*e^2-6*a*b*e*(2*B*e+
3*C*d)+b^2*(8*A*e^2+4*B*d*e+5*C*d^2))-6*c^2*e^2*(a*b*d*e*(-34*A*e^2-5*B*d*e
+30*C*d^2)-a^2*e^2*(7*A*e^2-36*B*d*e+30*C*d^2)-b^2*d^2*(11*A*e^2+3*B*d*e+11
*C*d^2))-c*e^3*(126*a^3*C*e^3-3*a^2*b*e^2*(29*B*e+12*C*d)-6*a*b^2*e*(-12*A*
e^2+7*B*d*e+5*C*d^2)+b^3*d*(56*A*e^2+25*B*d*e+20*C*d^2))+c^3*d^2*e*(6*a*e*(
-34*A*e^2+8*B*d*e+11*C*d^2)-b*d*(56*C*d^2+5*e*(4*A*e+5*B*d))))*(c*x^2+b*x+a
)^(1/2)/e^3/(a*e^2-b*d*e+c*d^2)^4/(e*x+d)^(1/2)-1/315*(2*c^4*d^4*(8*C*d^2+e
*(5*A*e+4*B*d))+2*b^2*e^4*(21*a^2*C*e^2-6*a*b*e*(2*B*e+3*C*d)+b^2*(8*A*e^2+
4*B*d*e+5*C*d^2))-6*c^2*e^2*(a*b*d*e*(-34*A*e^2-5*B*d*e+30*C*d^2)-a^2*e^2*(
7*A*e^2-36*B*d*e+30*C*d^2)-b^2*d^2*(11*A*e^2+3*B*d*e+11*C*d^2))-c*e^3*(126*
a^3*C*e^3-3*a^2*b*e^2*(29*B*e+12*C*d)-6*a*b^2*e*(-12*A*e^2+7*B*d*e+5*C*d^2)
+b^3*d*(56*A*e^2+25*B*d*e+20*C*d^2))+c^3*d^2*e*(6*a*e*(-34*A*e^2+8*B*d*e+11
*C*d^2)-b*d*(56*C*d^2+5*e*(4*A*e+5*B*d))))*EllipticE(1/2*((b+2*c*x+(-4*a*c+
b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*c
*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(e*x+d)^(1/
2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/e^4/(a*e^2-b*d*e+c*d^2)^4/(c*x^2+b
*x+a)^(1/2)/(c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)+2/315*(2*c^3
*d^3*(8*C*d^2+e*(5*A*e+4*B*d))+3*c^2*d*e*(2*a*e*(-9*A*e^2+7*B*d*e+9*C*d^2)-
b*d*(5*A*e^2+7*B*d*e+16*C*d^2))+3*c*e^2*(2*a^2*e^2*(-5*B*e+17*C*d)-a*b*e*(-
9*A*e^2+5*B*d*e+41*C*d^2)+b^2*d*(7*A*e^2+3*B*d*e+15*C*d^2))-b*e^3*(21*a^2*C
*e^2-6*a*b*e*(2*B*e+3*C*d)+b^2*(8*A*e^2+4*B*d*e+5*C*d^2)))*EllipticF(1/2*((
b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c
+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(
1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b
^2)^(1/2))))^(1/2)/e^4/(a*e^2-b*d*e+c*d^2)^3/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1
/2)
```

Rubi [A] (verified)

Time = 3.79 (sec) , antiderivative size = 1904, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.206$, Rules used

$$= \{1664, 824, 848, 857, 732, 435, 430\}$$

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{11/2}}\,dx = -\frac{2(Cd^2-e(Bd-Ae))\,(cx^2+bx+a)^{3/2}}{9e\,(cd^2-bed+ae^2)\,(d+ex)^{9/2}}$$

$$-\frac{2\Big((8Cd^5+e(4Bd+5Ae)d^3)\,c^2-de(3bd(5Cd^2+2Bed+5Ae^2)-ae(7Cd^2+11Bed+13Ae^2))\,c-e^2(-}{}$$

$$+\frac{2(2(8Cd^6+e(4Bd+5Ae)d^4)\,c^4-d^2e(56bCd^3+5be(5Bd+4Ae)d-6ae(11Cd^2+8Bed-34Ae^2))\,c^3-}{}$$

$$+\frac{2(2(8Cd^5+e(4Bd+5Ae)d^3)\,c^3+3de(2ae(9Cd^2+7Bed-9Ae^2)-bd(16Cd^2+7Bed+5Ae^2))\,c^2+3e^2(}{}$$

$$-\frac{\sqrt{2}\sqrt{b^2-4ac}(2(8Cd^6+e(4Bd+5Ae)d^4)\,c^4-d^2e(56bCd^3+5be(5Bd+4Ae)d-6ae(11Cd^2+8Bed-}{}$$

$$+\frac{2\sqrt{2}\sqrt{b^2-4ac}(2(8Cd^5+e(4Bd+5Ae)d^3)\,c^3+3de(2ae(9Cd^2+7Bed-9Ae^2)-bd(16Cd^2+7Bed+5A}{}$$

```
[In] Int[(Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2))/(d + e*x)^(11/2),x]
```

```
[Out] (2*(2*c^3*(8*C*d^5 + d^3*e*(4*B*d + 5*A*e)) + 3*c^2*d*e*(2*a*e*(9*C*d^2 + 7
*B*d*e - 9*A*e^2) - b*d*(16*C*d^2 + 7*B*d*e + 5*A*e^2)) + 3*c*e^2*(2*a^2*e^
2*(17*C*d - 5*B*e) - a*b*e*(41*C*d^2 + 5*B*d*e - 9*A*e^2) + b^2*d*(15*C*d^2
 + 3*B*d*e + 7*A*e^2)) - b*e^3*(21*a^2*C*e^2 - 6*a*b*e*(3*C*d + 2*B*e) + b^
2*(5*C*d^2 + 4*B*d*e + 8*A*e^2)))*Sqrt[a + b*x + c*x^2])/(315*e^3*(c*d^2 -
b*d*e + a*e^2)^3*(d + e*x)^(3/2)) + (2*(2*c^4*(8*C*d^6 + d^4*e*(4*B*d + 5*A
*e)) - c^3*d^2*e*(56*b*C*d^3 + 5*b*d*e*(5*B*d + 4*A*e) - 6*a*e*(11*C*d^2 +
8*B*d*e - 34*A*e^2)) + 2*b^2*e^4*(21*a^2*C*e^2 - 6*a*b*e*(3*C*d + 2*B*e) +
b^2*(5*C*d^2 + 4*B*d*e + 8*A*e^2)) - 6*c^2*e^2*(a*b*d*e*(30*C*d^2 - 5*B*d*e
 - 34*A*e^2) - a^2*e^2*(30*C*d^2 - 36*B*d*e + 7*A*e^2) - b^2*d^2*(11*C*d^2
+ 3*B*d*e + 11*A*e^2)) - c*e^3*(126*a^3*C*e^3 - 3*a^2*b*e^2*(12*C*d + 29*B*
e) - 6*a*b^2*e*(5*C*d^2 + 7*B*d*e - 12*A*e^2) + b^3*d*(20*C*d^2 + 25*B*d*e
+ 56*A*e^2)))*Sqrt[a + b*x + c*x^2])/(315*e^3*(c*d^2 - b*d*e + a*e^2)^4*Sqr
t[d + e*x]) - (2*(c^2*(8*C*d^5 + d^3*e*(4*B*d + 5*A*e)) - e^2*(3*a^2*e^2*(3
*C*d - 5*B*e) - a*b*e*(2*C*d^2 - 17*B*d*e - 10*A*e^2) - b^2*d*(5*C*d^2 + 4*
B*d*e + 8*A*e^2)) - c*d*e*(3*b*d*(5*C*d^2 + 2*B*d*e + 5*A*e^2) - a*e*(7*C*d
^2 + 11*B*d*e + 13*A*e^2)) + e^2*((3*c^2*(6*C*d^4 + d^2*e*(3*B*d - 5*A*e)))
/e + c*(a*e*(47*C*d^2 + e*(B*d - 7*A*e)) - 3*b*(15*C*d^3 + d*e*(2*B*d - 5*A
*e))) + e*(21*a^2*C*e^2 - 3*a*b*e*(16*C*d - B*e) + b^2*(25*C*d^2 - e*(B*d +
 2*A*e))))*x)*Sqrt[a + b*x + c*x^2])/(105*e^3*(c*d^2 - b*d*e + a*e^2)^2*(d
+ e*x)^(7/2)) - (2*(C*d^2 - e*(B*d - A*e))*(a + b*x + c*x^2)^(3/2))/(9*e*(c
```

```
*d^2 - b*d*e + a*e^2)*(d + e*x)^(9/2)) - (Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*c^4*
(8*C*d^6 + d^4*e*(4*B*d + 5*A*e)) - c^3*d^2*e*(56*b*C*d^3 + 5*b*d*e*(5*B*d
+ 4*A*e) - 6*a*e*(11*C*d^2 + 8*B*d*e - 34*A*e^2)) + 2*b^2*e^4*(21*a^2*C*e^2
 - 6*a*b*e*(3*C*d + 2*B*e) + b^2*(5*C*d^2 + 4*B*d*e + 8*A*e^2)) - 6*c^2*e^2
*(a*b*d*e*(30*C*d^2 - 5*B*d*e - 34*A*e^2) - a^2*e^2*(30*C*d^2 - 36*B*d*e +
7*A*e^2) - b^2*d^2*(11*C*d^2 + 3*B*d*e + 11*A*e^2)) - c*e^3*(126*a^3*C*e^3
- 3*a^2*b*e^2*(12*C*d + 29*B*e) - 6*a*b^2*e*(5*C*d^2 + 7*B*d*e - 12*A*e^2)
+ b^3*d*(20*C*d^2 + 25*B*d*e + 56*A*e^2)))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x
 + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2
*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + S
qrt[b^2 - 4*a*c])*e)])/(315*e^4*(c*d^2 - b*d*e + a*e^2)^4*Sqrt[(c*(d + e*x)
)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2]*
Sqrt[b^2 - 4*a*c]*(2*c^3*(8*C*d^5 + d^3*e*(4*B*d + 5*A*e)) + 3*c^2*d*e*(2*a
*e*(9*C*d^2 + 7*B*d*e - 9*A*e^2) - b*d*(16*C*d^2 + 7*B*d*e + 5*A*e^2)) + 3*
c*e^2*(2*a^2*e^2*(17*C*d - 5*B*e) - a*b*e*(41*C*d^2 + 5*B*d*e - 9*A*e^2) +
b^2*d*(15*C*d^2 + 3*B*d*e + 7*A*e^2)) - b*e^3*(21*a^2*C*e^2 - 6*a*b*e*(3*C*
d + 2*B*e) + b^2*(5*C*d^2 + 4*B*d*e + 8*A*e^2)))*Sqrt[(c*(d + e*x))/(2*c*d
- (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*
EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sq
rt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(315
*e^4*(c*d^2 - b*d*e + a*e^2)^3*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 824

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Dist[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)
^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p
 + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*
(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m
 - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &
& LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 848

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{2(Cd^2-e(Bd-Ae))\left(a+bx+cx^2\right)^{3/2}}{9e\left(cd^2-bde+ae^2\right)(d+ex)^{9/2}} - \frac{2\int \frac{\left(-\frac{3\left(bCd^2-be(Bd+2Ae)+3e(Acd-aCd+aBe)\right)}{2e}-\frac{3}{2}\left(Bcd-3bCd+\frac{2cCd^2}{e}-Ace+3aCe\right)x\right)\sqrt{a+bx+cx^2}}{(d+ex)^{9/2}}dx}{9\left(cd^2-bde+ae^2\right)}$$

$$= -\frac{2\left(c^2(8Cd^5+d^3e(4Bd+5Ae))-e^2(3a^2e^2(3Cd-5Be)-abe(2Cd^2-17Bde-10Ae^2)-b^2d(5\right.}{}$$

$$-\frac{2(Cd^2-e(Bd-Ae))\left(a+bx+cx^2\right)^{3/2}}{9e\left(cd^2-bde+ae^2\right)(d+ex)^{9/2}}$$

$$+\frac{4\int \frac{\frac{3\left(b^3e^2\left(5Cd^2+4e(Bd+2Ae)\right)-10ace\left(3ae^2(2Cd-Be)+cd\left(2Cd^2+Bde-4Ae^2\right)\right)-3b^2\left(2ae^3(3Cd+2Be)+cde\left(5Cd^2+2Bde+5Ae^2\right)\right)+b\left(21a^2Ce\right.\right.}{4e}}{}}{}$$

$$= \frac{2(2c^3(8Cd^5+d^3e(4Bd+5Ae))+3c^2de(2ae(9Cd^2+7Bde-9Ae^2)-bd(16Cd^2+7Bde+5Ae^2))}{}$$

$$-\frac{2\left(c^2(8Cd^5+d^3e(4Bd+5Ae))-e^2(3a^2e^2(3Cd-5Be)-abe(2Cd^2-17Bde-10Ae^2)-b^2d(5\right.}{}$$

$$-\frac{2(Cd^2-e(Bd-Ae))\left(a+bx+cx^2\right)^{3/2}}{9e\left(cd^2-bde+ae^2\right)(d+ex)^{9/2}}$$

$$-\frac{8\int \frac{3\left(2b^4e^3\left(5Cd^2+4e(Bd+2Ae)\right)+bc\left(3a^2e^4(19Cd+29Be)-3acde^2\left(19Cd^2-15Bde-59Ae^2\right)-c^2d^3\left(8Cd^2+4Bde+5Ae^2\right)\right)-6ace\left(21a^2Ce^4-c^2d^2\right.\right.}{}}{}$$

$$= \text{Too large to display}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 38.31 (sec) , antiderivative size = 29140, normalized size of antiderivative = 15.30

$$\int \frac{\sqrt{a+bx+cx^2}\left(A+Bx+Cx^2\right)}{(d+ex)^{11/2}}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2))/(d + e*x)^(11/2),x]
[Out] Result too large to show
```

Maple [A] (verified)

Time = 4.81 (sec) , antiderivative size = 3498, normalized size of antiderivative = 1.84

method	result	size
elliptic	Expression too large to display	3498
default	Expression too large to display	153623

```
[In] int((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(11/2),x,method=_RETURNVERBOS
E)
```

```
[Out] ((e*x+d)*(c*x^2+b*x+a))^(1/2)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)*(-2/9*(A*e^
2-B*d*e+C*d^2)/e^8*(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)/(x+d/e)^
5-2/63*(A*b*e^3-2*A*c*d*e^2+9*B*a*e^3-10*B*b*d*e^2+11*B*c*d^2*e-18*C*a*d*e^
2+19*C*b*d^2*e-20*C*c*d^3)/(a*e^2-b*d*e+c*d^2)/e^7*(c*e*x^3+b*e*x^2+c*d*x^2
+a*e*x+b*d*x+a*d)^(1/2)/(x+d/e)^4-2/315*(14*A*a*c*e^4-6*A*b^2*e^4+10*A*b*c*
d*e^3-10*A*c^2*d^2*e^2+9*B*a*b*e^4-32*B*a*c*d*e^3-3*B*b^2*d*e^3+17*B*b*c*d^
2*e^2-8*B*c^2*d^3*e+63*C*a^2*e^4-144*C*a*b*d*e^3+176*C*a*c*d^2*e^2+75*C*b^2
*d^2*e^2-170*C*b*c*d^3*e+89*C*c^2*d^4)/e^6/(a*e^2-b*d*e+c*d^2)^2*(c*e*x^3+b
*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)/(x+d/e)^3+2/315*(27*A*a*b*c*e^5-54*A*
a*c^2*d*e^4-8*A*b^3*e^5+21*A*b^2*c*d*e^4-15*A*b*c^2*d^2*e^3+10*A*c^3*d^3*e^
2-30*B*a^2*c*e^5+12*B*a*b^2*e^5-15*B*a*b*c*d*e^4+42*B*a*c^2*d^2*e^3-4*B*b^3
*d*e^4+9*B*b^2*c*d^2*e^3-21*B*b*c^2*d^3*e^2+8*B*c^3*d^4*e-21*C*a^2*b*e^5+10
2*C*a^2*c*d*e^4+18*C*a*b^2*d*e^4-123*C*a*b*c*d^2*e^3+54*C*a*c^2*d^3*e^2-5*C
*b^3*d^2*e^3+45*C*b^2*c*d^3*e^2-48*C*b*c^2*d^4*e+16*C*c^3*d^5)/e^5/(a*e^2-b
*d*e+c*d^2)^3*(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)/(x+d/e)^2+2/3
15*(c*e*x^2+b*e*x+a*e)/e^4/(a*e^2-b*d*e+c*d^2)^4*(42*A*a^2*c^2*e^6-72*A*a*b
^2*c*e^6+204*A*a*b*c^2*d*e^5-204*A*a*c^3*d^2*e^4+16*A*b^4*e^6-56*A*b^3*c*d*
e^5+66*A*b^2*c^2*d^2*e^4-20*A*b*c^3*d^3*e^3+10*A*c^4*d^4*e^2+87*B*a^2*b*c*e
^6-216*B*a^2*c^2*d*e^5-24*B*a*b^3*e^6+42*B*a*b^2*c*d*e^5+30*B*a*b*c^2*d^2*e
^4+48*B*a*c^3*d^3*e^3+8*B*b^4*d*e^5-25*B*b^3*c*d^2*e^4+18*B*b^2*c^2*d^3*e^3
-25*B*b*c^3*d^4*e^2+8*B*c^4*d^5*e-126*C*a^3*c*e^6+42*C*a^2*b^2*e^6+36*C*a^2
*b*c*d*e^5+180*C*a^2*c^2*d^2*e^4-36*C*a*b^3*d*e^5+30*C*a*b^2*c*d^2*e^4-180*
C*a*b*c^2*d^3*e^3+66*C*a*c^3*d^4*e^2+10*C*b^4*d^2*e^4-20*C*b^3*c*d^3*e^3+66
*C*b^2*c^2*d^4*e^2-56*C*b*c^3*d^5*e+16*C*c^4*d^6)/((x+d/e)*(c*e*x^2+b*e*x+a
*e))^(1/2)+2*(1/315*c*(27*A*a*b*c*e^5-54*A*a*c^2*d*e^4-8*A*b^3*e^5+21*A*b^2
*c*d*e^4-15*A*b*c^2*d^2*e^3+10*A*c^3*d^3*e^2-30*B*a^2*c*e^5+12*B*a*b^2*e^5-
15*B*a*b*c*d*e^4+42*B*a*c^2*d^2*e^3-4*B*b^3*d*e^4+9*B*b^2*c*d^2*e^3-21*B*b*
c^2*d^3*e^2+8*B*c^3*d^4*e-21*C*a^2*b*e^5+102*C*a^2*c*d*e^4+18*C*a*b^2*d*e^4
-123*C*a*b*c*d^2*e^3+54*C*a*c^2*d^3*e^2-5*C*b^3*d^2*e^3+45*C*b^2*c*d^3*e^2-
48*C*b*c^2*d^4*e+16*C*c^3*d^5)/e^4/(a*e^2-b*d*e+c*d^2)^3+1/315/e^4*(b*e-c*d
)*(42*A*a^2*c^2*e^6-72*A*a*b^2*c*e^6+204*A*a*b*c^2*d*e^5-204*A*a*c^3*d^2*e^
4+16*A*b^4*e^6-56*A*b^3*c*d*e^5+66*A*b^2*c^2*d^2*e^4-20*A*b*c^3*d^3*e^3+10*
A*c^4*d^4*e^2+87*B*a^2*b*c*e^6-216*B*a^2*c^2*d*e^5-24*B*a*b^3*e^6+42*B*a*b^
2*c*d*e^5+30*B*a*b*c^2*d^2*e^4+48*B*a*c^3*d^3*e^3+8*B*b^4*d*e^5-25*B*b^3*c*
```

```
d^2*e^4+18*B*b^2*c^2*d^3*e^3-25*B*b*c^3*d^4*e^2+8*B*c^4*d^5*e-126*C*a^3*c*e
^6+42*C*a^2*b^2*e^6+36*C*a^2*b*c*d*e^5+180*C*a^2*c^2*d^2*e^4-36*C*a*b^3*d*e
^5+30*C*a*b^2*c*d^2*e^4-180*C*a*b*c^2*d^3*e^3+66*C*a*c^3*d^4*e^2+10*C*b^4*d
^2*e^4-20*C*b^3*c*d^3*e^3+66*C*b^2*c^2*d^4*e^2-56*C*b*c^3*d^5*e+16*C*c^4*d^
6)/(a*e^2-b*d*e+c*d^2)^4-1/315*b/e^3/(a*e^2-b*d*e+c*d^2)^4*(42*A*a^2*c^2*e^
6-72*A*a*b^2*c*e^6+204*A*a*b*c^2*d*e^5-204*A*a*c^3*d^2*e^4+16*A*b^4*e^6-56*
A*b^3*c*d*e^5+66*A*b^2*c^2*d^2*e^4-20*A*b*c^3*d^3*e^3+10*A*c^4*d^4*e^2+87*B
*a^2*b*c*e^6-216*B*a^2*c^2*d*e^5-24*B*a*b^3*e^6+42*B*a*b^2*c*d*e^5+30*B*a*b
*c^2*d^2*e^4+48*B*a*c^3*d^3*e^3+8*B*b^4*d*e^5-25*B*b^3*c*d^2*e^4+18*B*b^2*c
^2*d^3*e^3-25*B*b*c^3*d^4*e^2+8*B*c^4*d^5*e-126*C*a^3*c*e^6+42*C*a^2*b^2*e^
6+36*C*a^2*b*c*d*e^5+180*C*a^2*c^2*d^2*e^4-36*C*a*b^3*d*e^5+30*C*a*b^2*c*d^
2*e^4-180*C*a*b*c^2*d^3*e^3+66*C*a*c^3*d^4*e^2+10*C*b^4*d^2*e^4-20*C*b^3*c*
d^3*e^3+66*C*b^2*c^2*d^4*e^2-56*C*b*c^3*d^5*e+16*C*c^4*d^6))*(d/e-1/2*(b+(-
4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x
-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)
*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/
2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)*EllipticF(((x+d/e)/(d/e-
1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(
-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))-2/315*c/e^3*(42*A*a^2*c^2*e^6-7
2*A*a*b^2*c*e^6+204*A*a*b*c^2*d*e^5-204*A*a*c^3*d^2*e^4+16*A*b^4*e^6-56*A*b
^3*c*d*e^5+66*A*b^2*c^2*d^2*e^4-20*A*b*c^3*d^3*e^3+10*A*c^4*d^4*e^2+87*B*a^
2*b*c*e^6-216*B*a^2*c^2*d*e^5-24*B*a*b^3*e^6+42*B*a*b^2*c*d*e^5+30*B*a*b*c^
2*d^2*e^4+48*B*a*c^3*d^3*e^3+8*B*b^4*d*e^5-25*B*b^3*c*d^2*e^4+18*B*b^2*c^2*
d^3*e^3-25*B*b*c^3*d^4*e^2+8*B*c^4*d^5*e-126*C*a^3*c*e^6+42*C*a^2*b^2*e^6+3
6*C*a^2*b*c*d*e^5+180*C*a^2*c^2*d^2*e^4-36*C*a*b^3*d*e^5+30*C*a*b^2*c*d^2*e
^4-180*C*a*b*c^2*d^3*e^3+66*C*a*c^3*d^4*e^2+10*C*b^4*d^2*e^4-20*C*b^3*c*d^3
*e^3+66*C*b^2*c^2*d^4*e^2-56*C*b*c^3*d^5*e+16*C*c^4*d^6)/(a*e^2-b*d*e+c*d^2
)^4*(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1
/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d/e-1/2/c*(-b+(-4*a*c+b
^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e+1/2*(b+(-4*a*c+b
^2)^(1/2))/c))^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)*((-d/e
-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*EllipticE(((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)
^(1/2))/c))^(1/2),((-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e-1/2/c*(-b+(-4*
a*c+b^2)^(1/2))))^(1/2))+1/2/c*(-b+(-4*a*c+b^2)^(1/2))*EllipticF(((x+d/e)/(
d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/
c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.28 (sec) , antiderivative size = 7780, normalized size of antiderivative = 4.09

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{11/2}}\,dx=\text{Too large to display}$$

```
[In] integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(11/2),x, algorithm="fr
icas")
```

```
[Out] Too large to include
```

Sympy [F]

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{11/2}}\,dx=\int \frac{(A+Bx+Cx^2)\sqrt{a+bx+cx^2}}{(d+ex)^{\frac{11}{2}}}\,dx$$

```
[In] integrate((C*x**2+B*x+A)*(c*x**2+b*x+a)**(1/2)/(e*x+d)**(11/2),x)
```

```
[Out] Integral((A + B*x + C*x**2)*sqrt(a + b*x + c*x**2)/(d + e*x)**(11/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{11/2}}\,dx=\int \frac{(Cx^2+Bx+A)\sqrt{cx^2+bx+a}}{(ex+d)^{\frac{11}{2}}}\,dx$$

```
[In] integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(11/2),x, algorithm="ma
xima")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)/(e*x + d)^(11/2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{11/2}}\,dx=\int \frac{(Cx^2+Bx+A)\sqrt{cx^2+bx+a}}{(ex+d)^{\frac{11}{2}}}\,dx$$

```
[In] integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(11/2),x, algorithm="gi
ac")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)/(e*x + d)^(11/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{11/2}}\,dx=\int \frac{\left(C\,x^2+B\,x+A\right)\,\sqrt{c\,x^2+b\,x+a}}{{\left(d+e\,x\right)}^{11/2}}\,dx$$

```
[In] int(((A + B*x + C*x^2)*(a + b*x + c*x^2)^(1/2))/(d + e*x)^(11/2),x)
[Out] int(((A + B*x + C*x^2)*(a + b*x + c*x^2)^(1/2))/(d + e*x)^(11/2), x)
```

3.266 $\int \frac{(d+ex)^{3/2}(A+Bx+Cx^2)}{\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 724

$$\int \frac{(d+ex)^{3/2}\left(A+Bx+Cx^2\right)}{\sqrt{a+bx+cx^2}}\,dx = \frac{2(24b^2Ce^2-ce(15bCd+28bBe+25aCe)-c^2(6Cd^2-7e(3Bd+5A}{105c^3e}$$

$$-\frac{2(2cCd-7Bce+6bCe)(d+ex)^{3/2}\sqrt{a+bx+cx^2}}{35c^2e}+\frac{2C(d+ex)^{5/2}\sqrt{a+bx+cx^2}}{7ce}$$

$$-\frac{\sqrt{2}\sqrt{b^2-4ac}(48b^3Ce^3-8bce^2(9bCd+7bBe+13aCe)+c^3d(6Cd^2-7e(3Bd+20Ae))+c^2e(ae(82Cd-}{105c^4e^2\sqrt{\frac{c(}{2cd-\left(b\right.}}}$$

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}(cd^2-bde+ae^2)\left(24b^2Ce^2-ce(15bCd+28bBe+25aCe)-c^2(6Cd^2-7e(3Bd+5Ae))\right)}{105c^4e^2\sqrt{d+ex}\sqrt{a+}}$$

```
[Out] -2/35*(-7*B*c*e+6*C*b*e+2*C*c*d)*(e*x+d)^(3/2)*(c*x^2+b*x+a)^(1/2)/c^2/e+2/
7*C*(e*x+d)^(5/2)*(c*x^2+b*x+a)^(1/2)/c/e+2/105*(24*b^2*C*e^2-c*e*(28*B*b*e
+25*C*a*e+15*C*b*d)-c^2*(6*C*d^2-7*e*(5*A*e+3*B*d)))*(e*x+d)^(1/2)*(c*x^2+b
*x+a)^(1/2)/c^3/e-1/105*(48*b^3*C*e^3-8*b*c*e^2*(7*B*b*e+13*C*a*e+9*C*b*d)+
c^3*d*(6*C*d^2-7*e*(20*A*e+3*B*d))+c^2*e*(a*e*(63*B*e+82*C*d)+b*(70*A*e^2+9
1*B*d*e+12*C*d^2)))*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2
)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1
/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(e*x+d)^(1/2)*(-c*(c*x^2+b*x+a)/(-
4*a*c+b^2))^(1/2)/c^4/e^2/(c*x^2+b*x+a)^(1/2)/(c*(e*x+d)/(2*c*d-e*(b+(-4*a*
c+b^2)^(1/2))))^(1/2)-2/105*(a*e^2-b*d*e+c*d^2)*(24*b^2*C*e^2-c*e*(28*B*b*e
+25*C*a*e+15*C*b*d)-c^2*(6*C*d^2-7*e*(5*A*e+3*B*d)))*EllipticF(1/2*((b+2*c*
x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^
```

```
(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(
-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1
/2))))^(1/2)/c^4/e^2/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 724, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1667, 846, 857, 732, 435, 430}

$$\int \frac{(d+ex)^{3/2}\,(A+Bx+Cx^2)}{\sqrt{a+bx+cx^2}}\,dx =$$

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(ae^2-bde+cd^2)\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}(-ce(25aCe+28bBe+15bCd)-(c^2(6C}{105c^4e^2\sqrt{d+ex}\sqrt{a+}}$$

$$-\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(c^2e(ae(63Be+82Cd)+b(70Ae^2+91Bde+12Cd^2))-8bce^2(13aCe}{105c^4e^2\sqrt{a+bx+}}$$

$$+\frac{2\sqrt{d+ex}\sqrt{a+bx+cx^2}(-ce(25aCe+28bBe+15bCd)-(c^2(6Cd^2-7e(5Ae+3Bd)))+24b^2Ce^2)}{105c^3e}$$

$$-\frac{2(d+ex)^{3/2}\sqrt{a+bx+cx^2}(6bCe-7Bce+2cCd)}{35c^2e}+\frac{2C(d+ex)^{5/2}\sqrt{a+bx+cx^2}}{7ce}$$

```
[In] Int[((d + e*x)^(3/2)*(A + B*x + C*x^2))/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (2*(24*b^2*C*e^2 - c*e*(15*b*C*d + 28*b*B*e + 25*a*C*e) - c^2*(6*C*d^2 - 7*
e*(3*B*d + 5*A*e)))*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])/(105*c^3*e) - (2*(
2*c*C*d - 7*B*c*e + 6*b*C*e)*(d + e*x)^(3/2)*Sqrt[a + b*x + c*x^2])/(35*c^2
*e) + (2*C*(d + e*x)^(5/2)*Sqrt[a + b*x + c*x^2])/(7*c*e) - (Sqrt[2]*Sqrt[b
^2 - 4*a*c]*(48*b^3*C*e^3 - 8*b*c*e^2*(9*b*C*d + 7*b*B*e + 13*a*C*e) + c^3*
(6*C*d^3 - 7*d*e*(3*B*d + 20*A*e)) + c^2*e*(a*e*(82*C*d + 63*B*e) + b*(12*C
*d^2 + 91*B*d*e + 70*A*e^2)))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b
^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^
2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a
*c])*e)])/(105*c^4*e^2*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*
e)]*Sqrt[a + b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e +
a*e^2)*(24*b^2*C*e^2 - c*e*(15*b*C*d + 28*b*B*e + 25*a*C*e) - c^2*(6*C*d^2
- 7*e*(3*B*d + 5*A*e)))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])
*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b
+ Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*
```

```
a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(105*c^4*e^2*Sqrt[d + e*x]*Sq
rt[a + b*x + c*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 846

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

integral

$$= \frac{2C(d+ex)^{5/2}\sqrt{a+bx+cx^2}}{7ce} + \frac{2\int \frac{(d+ex)^{3/2}\left(-\frac{1}{2}e(bCd-7Ace+5aCe)-\frac{1}{2}e(2cCd-7Bce+6bCe)x\right)}{\sqrt{a+bx+cx^2}}\,dx}{7ce^2}$$

$$= -\frac{2(2cCd-7Bce+6bCe)(d+ex)^{3/2}\sqrt{a+bx+cx^2}}{35c^2e} + \frac{2C(d+ex)^{5/2}\sqrt{a+bx+cx^2}}{7ce}$$
$$+ \frac{4\int \frac{\sqrt{d+ex}\left(\frac{1}{4}e\left(6b^2Cde+18abCe^2-bcd(3Cd+7Be)+ce(35Acd-19aCd-21aBe)\right)+\frac{1}{4}e\left(24b^2Ce^2-ce(15bCd+28bBe+25aCe)-c^2\left(6Cd^2-7e(3B\right.\right.\right.}{\sqrt{a+bx+cx^2}}}{35c^2e^2}$$

$$= \frac{2\left(24b^2Ce^2-ce(15bCd+28bBe+25aCe)-c^2\left(6Cd^2-7e(3Bd+5Ae)\right)\right)\sqrt{d+ex}\sqrt{a+bx+cx^2}}{105c^3e}$$
$$- \frac{2(2cCd-7Bce+6bCe)(d+ex)^{3/2}\sqrt{a+bx+cx^2}}{35c^2e} + \frac{2C(d+ex)^{5/2}\sqrt{a+bx+cx^2}}{7ce}$$
$$+ \frac{8\int \frac{-\frac{1}{8}e\left(24b^3Cde^2+bc\left(3cCd^3+7cde(6Bd+5Ae)-2ae^2(47Cd+14Be)\right)+b^2\left(24aCe^3-cde(33Cd+28Be)\right)-ce\left(35Ac\left(3cd^2-ae^2\right)-a\left(51cCd^2+84\right.\right.\right.}{\sqrt{d+e}}}{1}$$

$$= \frac{2\left(24b^2Ce^2-ce(15bCd+28bBe+25aCe)-c^2\left(6Cd^2-7e(3Bd+5Ae)\right)\right)\sqrt{d+ex}\sqrt{a+bx+cx^2}}{105c^3e}$$
$$- \frac{2(2cCd-7Bce+6bCe)(d+ex)^{3/2}\sqrt{a+bx+cx^2}}{35c^2e} + \frac{2C(d+ex)^{5/2}\sqrt{a+bx+cx^2}}{7ce}$$
$$- \frac{\left(\left(cd^2-bde+ae^2\right)\left(24b^2Ce^2-ce(15bCd+28bBe+25aCe)-c^2\left(6Cd^2-7e(3Bd+5Ae)\right)\right)\right)\int \frac{1}{\sqrt{d+ex}\sqrt{a+}}}{105c^3e^2}$$
$$- \frac{\left(48b^3Ce^3-8bce^2(9bCd+7bBe+13aCe)+c^3\left(6Cd^3-7de(3Bd+20Ae)\right)+c^2e\left(ae(82Cd+63Be)+b(\right.\right.}{105c^3e^2}$$

$$= \frac{2(24b^2Ce^2 - ce(15bCd + 28bBe + 25aCe) - c^2(6Cd^2 - 7e(3Bd + 5Ae)))\sqrt{d+ex}\sqrt{a+bx+cx^2}}{105c^3e}$$

$$- \frac{2(2cCd - 7Bce + 6bCe)(d+ex)^{3/2}\sqrt{a+bx+cx^2}}{35c^2e} + \frac{2C(d+ex)^{5/2}\sqrt{a+bx+cx^2}}{7ce}$$

$$- \frac{\Big(\sqrt{2}\sqrt{b^2-4ac}(48b^3Ce^3 - 8bce^2(9bCd + 7bBe + 13aCe) + c^3(6Cd^3 - 7de(3Bd + 20Ae)) + c^2e(ae(82}{105c^4e^2\sqrt{\frac{}{2}}}$$

$$- \frac{\Big(2\sqrt{2}\sqrt{b^2-4ac}(cd^2 - bde + ae^2)\left(24b^2Ce^2 - ce(15bCd + 28bBe + 25aCe) - c^2(6Cd^2 - 7e(3Bd + 5A\right.}{105c^4e^2\sqrt{d+ex}\sqrt{a}}$$

$$= \frac{2(24b^2Ce^2 - ce(15bCd + 28bBe + 25aCe) - c^2(6Cd^2 - 7e(3Bd + 5Ae)))\sqrt{d+ex}\sqrt{a+bx+cx^2}}{105c^3e}$$

$$- \frac{2(2cCd - 7Bce + 6bCe)(d+ex)^{3/2}\sqrt{a+bx+cx^2}}{35c^2e} + \frac{2C(d+ex)^{5/2}\sqrt{a+bx+cx^2}}{7ce}$$

$$- \frac{\sqrt{2}\sqrt{b^2-4ac}(48b^3Ce^3 - 8bce^2(9bCd + 7bBe + 13aCe) + c^3(6Cd^3 - 7de(3Bd + 20Ae)) + c^2e(ae(82C}{105c^4e^2\sqrt{\frac{}{2cd-}}}$$

$$- \frac{2\sqrt{2}\sqrt{b^2-4ac}(cd^2 - bde + ae^2)\left(24b^2Ce^2 - ce(15bCd + 28bBe + 25aCe) - c^2(6Cd^2 - 7e(3Bd + 5Ae\right.}{105c^4e^2\sqrt{d+ex}\sqrt{a+bx}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 33.48 (sec) , antiderivative size = 1314, normalized size of antiderivative = 1.81

$$\int \frac{(d+ex)^{3/2}\left(A+Bx+Cx^2\right)}{\sqrt{a+bx+cx^2}}\,dx = \frac{\sqrt{d+ex}\left(a+bx+cx^2\right)\left(\frac{2\left(3c^2Cd^2+42Bc^2de-33bcCde-28bBce^2+35Ac^2e^2+24b^2Ce^2-\right.}{105c^3e}\right.}{\sqrt{a+x(b+cx)}}$$

$$+\frac{2(d+ex)^{3/2}\sqrt{a+bx+cx^2}\left(-\left(\left(48b^3Ce^3-8bce^2(9bCd+7bBe+13aCe)+c^3\left(6Cd^3-7de(3Bd+20Ae)\right)\right.\right.\right.}{}$$

```
[In] Integrate[((d + e*x)^(3/2)*(A + B*x + C*x^2))/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (Sqrt[d + e*x]*(a + b*x + c*x^2)*((2*(3*c^2*C*d^2 + 42*B*c^2*d*e - 33*b*c*C
*d*e - 28*b*B*c*e^2 + 35*A*c^2*e^2 + 24*b^2*C*e^2 - 25*a*c*C*e^2))/(105*c^3
*e) + (2*(8*c*C*d + 7*B*c*e - 6*b*C*e)*x)/(35*c^2) + (2*C*e*x^2)/(7*c)))/Sq
rt[a + x*(b + c*x)] + (2*(d + e*x)^(3/2)*Sqrt[a + b*x + c*x^2]*(-((48*b^3*C
*e^3 - 8*b*c*e^2*(9*b*C*d + 7*b*B*e + 13*a*C*e) + c^3*(6*C*d^3 - 7*d*e*(3*B
*d + 20*A*e)) + c^2*e*(a*e*(82*C*d + 63*B*e) + b*(12*C*d^2 + 91*B*d*e + 70*
A*e^2)))*(c*(-1 + d/(d + e*x))^2 + (e*(b - (b*d)/(d + e*x) + (a*e)/(d + e*x
)))/(d + e*x))) + ((I/2)*Sqrt[1 - (2*(c*d^2 + e*(-(b*d) + a*e)))/((2*c*d -
b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*Sqrt[1 + (2*(c*d^2 + e*(-(b*d) +
 a*e)))/((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*((2*c*d - b*e
 + Sqrt[(b^2 - 4*a*c)*e^2])*(48*b^3*C*e^3 - 8*b*c*e^2*(9*b*C*d + 7*b*B*e +
13*a*C*e) + c^3*(6*C*d^3 - 7*d*e*(3*B*d + 20*A*e)) + c^2*e*(a*e*(82*C*d + 6
3*B*e) + b*(12*C*d^2 + 91*B*d*e + 70*A*e^2)))*EllipticE[I*ArcSinh[(Sqrt[2]*
Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqr
t[d + e*x]], -((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*c*d - b*e + Sqrt
[(b^2 - 4*a*c)*e^2]))] - (-48*b^4*C*e^4 + 8*b^3*e^3*(15*c*C*d + 7*B*c*e + 6
*C*Sqrt[(b^2 - 4*a*c)*e^2]) - b^2*c*e^2*(78*c*C*d^2 - 152*a*C*e^2 + 7*c*e*(
21*B*d + 10*A*e) + 8*Sqrt[(b^2 - 4*a*c)*e^2]*(9*C*d + 7*B*e)) + c^2*(-50*a^
2*C*e^4 - 3*c*d^2*Sqrt[(b^2 - 4*a*c)*e^2]*(-2*C*d + 7*B*e) - 70*A*c*e^2*(3*
c*d^2 - a*e^2 + 2*d*Sqrt[(b^2 - 4*a*c)*e^2]) + a*e^2*(6*c*d*(17*C*d + 28*B*
e) + Sqrt[(b^2 - 4*a*c)*e^2]*(82*C*d + 63*B*e))) + b*(7*B*c^2*e^2*(15*c*d^2
 - 17*a*e^2 + 13*d*Sqrt[(b^2 - 4*a*c)*e^2]) + 2*c*e*(6*c*C*d^2*Sqrt[(b^2 -
4*a*c)*e^2] + 35*A*c*e^2*(3*c*d + Sqrt[(b^2 - 4*a*c)*e^2]) - a*C*e^2*(135*c
*d + 52*Sqrt[(b^2 - 4*a*c)*e^2]))))*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^
```

```
2 - b*d*e + a*e^2)/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]
], -((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*c*d - b*e + Sqrt[(b^2 - 4*
a*c)*e^2]))]))/(Sqrt[2]*Sqrt[(c*d^2 + e*(-(b*d) + a*e))/(-2*c*d + b*e + Sqr
t[(b^2 - 4*a*c)*e^2])]*Sqrt[d + e*x])))/(105*c^4*e^3*Sqrt[a + x*(b + c*x)]*
Sqrt[((d + e*x)^2*(c*(-1 + d/(d + e*x))^2 + (e*(b - (b*d)/(d + e*x) + (a*e)
/(d + e*x)))/(d + e*x)))/e^2])
```

Maple [A] (verified)

Time = 3.45 (sec) , antiderivative size = 1261, normalized size of antiderivative = 1.74

method	result	size
elliptic	Expression too large to display	1261
risch	Expression too large to display	4796
default	Expression too large to display	14084

```
[In] int((e*x+d)^(3/2)*(C*x^2+B*x+A)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE
)
```

```
[Out] ((e*x+d)*(c*x^2+b*x+a))^(1/2)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)*(2/7*C/c*e*
x^2*(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)+2/5*(B*e^2+2*d*e*C-2/7*
C/c*e*(3*b*e+3*c*d))/c/e*x*(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)+
2/3*(A*e^2+2*B*d*e+C*d^2-2/7*C/c*e*(5/2*a*e+5/2*b*d)-2/5*(B*e^2+2*d*e*C-2/7
*C/c*e*(3*b*e+3*c*d))/c/e*(2*b*e+2*c*d))/c/e*(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x
+b*d*x+a*d)^(1/2)+2*(A*d^2-2/5*(B*e^2+2*d*e*C-2/7*C/c*e*(3*b*e+3*c*d))/c/e*
a*d-2/3*(A*e^2+2*B*d*e+C*d^2-2/7*C/c*e*(5/2*a*e+5/2*b*d)-2/5*(B*e^2+2*d*e*C
-2/7*C/c*e*(3*b*e+3*c*d))/c/e*(2*b*e+2*c*d))/c/e*(1/2*a*e+1/2*b*d))*(d/e-1/
2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1
/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))
)^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/
c))^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)*EllipticF(((x+d/e
)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2*(b+(-4*a*c+b^2)^(1/2
))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+2*(2*A*d*e+B*d^2-4/7*a/c
*d*e*C-2/5*(B*e^2+2*d*e*C-2/7*C/c*e*(3*b*e+3*c*d))/c/e*(3/2*a*e+3/2*b*d)-2/
3*(A*e^2+2*B*d*e+C*d^2-2/7*C/c*e*(5/2*a*e+5/2*b*d)-2/5*(B*e^2+2*d*e*C-2/7*C
/c*e*(3*b*e+3*c*d))/c/e*(2*b*e+2*c*d))/c/e*(b*e+c*d))*(d/e-1/2*(b+(-4*a*c+b
^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*
(-b+(-4*a*c+b^2)^(1/2)))/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/
2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*e
*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)*((-d/e-1/2/c*(-b+(-4*a*c+b^2)^(
1/2)))*EllipticE(((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+
1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+
1/2/c*(-b+(-4*a*c+b^2)^(1/2))*EllipticF(((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(
1/2))/c))^(1/2),((-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e-1/2/c*(-b+(-4*a*
c+b^2)^(1/2))))^(1/2))))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 761, normalized size of antiderivative = 1.05

$$\int \frac{(d+ex)^{3/2}\,(A+Bx+Cx^2)}{\sqrt{a+bx+cx^2}}\,dx = \frac{2\Big((6\,Cc^4d^4+3\,(3\,Cbc^3-7\,Bc^4)d^3e+(39\,Cb^2c^2+175\,Ac^4-(71\,Ca+}{}$$

```
[In] integrate((e*x+d)^(3/2)*(C*x^2+B*x+A)/(c*x^2+b*x+a)^(1/2),x, algorithm="fri
cas")
```

```
[Out] 2/315*((6*C*c^4*d^4 + 3*(3*C*b*c^3 - 7*B*c^4)*d^3*e + (39*C*b^2*c^2 + 175*A
*c^4 - (71*C*a + 56*B*b)*c^3)*d^2*e^2 - (96*C*b^3*c + 7*(27*B*a + 25*A*b)*c
^3 - (260*C*a*b + 119*B*b^2)*c^2)*d*e^3 + (48*C*b^4 - 105*A*a*c^3 + (75*C*a
^2 + 147*B*a*b + 70*A*b^2)*c^2 - 8*(22*C*a*b^2 + 7*B*b^3)*c)*e^4)*sqrt(c*e)
*weierstrassPInverse(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2),
 -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*
a*b*c)*e^3)/(c^3*e^3), 1/3*(3*c*e*x + c*d + b*e)/(c*e)) + 3*(6*C*c^4*d^3*e
+ 3*(4*C*b*c^3 - 7*B*c^4)*d^2*e^2 - (72*C*b^2*c^2 + 140*A*c^4 - (82*C*a + 9
1*B*b)*c^3)*d*e^3 + (48*C*b^3*c + 7*(9*B*a + 10*A*b)*c^3 - 8*(13*C*a*b + 7*
B*b^2)*c^2)*e^4)*sqrt(c*e)*weierstrassZeta(4/3*(c^2*d^2 - b*c*d*e + (b^2 -
3*a*c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^
2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(c^3*e^3), weierstrassPInverse(4/3*(c^2*d
^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2
*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(c^3*e^3), 1/3*(3*c
*e*x + c*d + b*e)/(c*e))) + 3*(15*C*c^4*e^4*x^2 + 3*C*c^4*d^2*e^2 - 3*(11*C
*b*c^3 - 14*B*c^4)*d*e^3 + (24*C*b^2*c^2 + 35*A*c^4 - (25*C*a + 28*B*b)*c^3
)*e^4 + 3*(8*C*c^4*d*e^3 - (6*C*b*c^3 - 7*B*c^4)*e^4)*x)*sqrt(c*x^2 + b*x +
 a)*sqrt(e*x + d))/(c^5*e^3)
```

Sympy [F]

$$\int \frac{(d+ex)^{3/2}\,(A+Bx+Cx^2)}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{(d+ex)^{\frac{3}{2}}\,(A+Bx+Cx^2)}{\sqrt{a+bx+cx^2}}\,dx$$

```
[In] integrate((e*x+d)**(3/2)*(C*x**2+B*x+A)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x)**(3/2)*(A + B*x + C*x**2)/sqrt(a + b*x + c*x**2), x)
```

Maxima [F]

$$\int \frac{(d+ex)^{3/2}\left(A+Bx+Cx^2\right)}{\sqrt{a+bx+cx^2}}\,dx=\int \frac{(Cx^2+Bx+A)(ex+d)^{\frac{3}{2}}}{\sqrt{cx^2+bx+a}}\,dx$$

```
[In] integrate((e*x+d)^(3/2)*(C*x^2+B*x+A)/(c*x^2+b*x+a)^(1/2),x, algorithm="max
ima")
[Out] integrate((C*x^2 + B*x + A)*(e*x + d)^(3/2)/sqrt(c*x^2 + b*x + a), x)
```

Giac [F]

$$\int \frac{(d+ex)^{3/2}\left(A+Bx+Cx^2\right)}{\sqrt{a+bx+cx^2}}\,dx=\int \frac{(Cx^2+Bx+A)(ex+d)^{\frac{3}{2}}}{\sqrt{cx^2+bx+a}}\,dx$$

```
[In] integrate((e*x+d)^(3/2)*(C*x^2+B*x+A)/(c*x^2+b*x+a)^(1/2),x, algorithm="gia
c")
[Out] integrate((C*x^2 + B*x + A)*(e*x + d)^(3/2)/sqrt(c*x^2 + b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}\left(A+Bx+Cx^2\right)}{\sqrt{a+bx+cx^2}}\,dx=\int \frac{{\left(d+e\,x\right)}^{3/2}\,\left(C\,x^2+B\,x+A\right)}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

```
[In] int(((d + e*x)^(3/2)*(A + B*x + C*x^2))/(a + b*x + c*x^2)^(1/2),x)
[Out] int(((d + e*x)^(3/2)*(A + B*x + C*x^2))/(a + b*x + c*x^2)^(1/2), x)
```

3.267 $\int \frac{\sqrt{d+ex}\left(A+Bx+Cx^2\right)}{\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 557

$$\begin{aligned}
&\int \frac{\sqrt{d+ex}\left(A+Bx+Cx^2\right)}{\sqrt{a+bx+cx^2}}\,dx \\
&= -\frac{2(2cCd-5Bce+4bCe)\sqrt{d+ex}\sqrt{a+bx+cx^2}}{15c^2e}+\frac{2C(d+ex)^{3/2}\sqrt{a+bx+cx^2}}{5ce} \\
&\quad+\frac{\sqrt{2}\sqrt{b^2-4ac}\left(8b^2Ce^2-ce(3bCd+10bBe+9aCe)-c^2\left(2Cd^2-5e(Bd+3Ae)\right)\right)\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}}{15c^3e^2\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{a+bx+cx^2}} \\
&\quad+\frac{2\sqrt{2}\sqrt{b^2-4ac}(2cCd-5Bce+4bCe)\left(cd^2-bde+ae^2\right)\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\text{EllipticF}\Bigg(\text{arc}}{15c^3e^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}
\end{aligned}$$

```
[Out] 2/5*C*(e*x+d)^(3/2)*(c*x^2+b*x+a)^(1/2)/c/e-2/15*(-5*B*c*e+4*C*b*e+2*C*c*d)
*(e*x+d)^(1/2)*(c*x^2+b*x+a)^(1/2)/c^2/e+1/15*(8*b^2*C*e^2-c*e*(10*B*b*e+9*
C*a*e+3*C*b*d)-c^2*(2*C*d^2-5*e*(3*A*e+B*d)))*EllipticE(1/2*((b+2*c*x+(-4*a
*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(
2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(e*x+d)^
(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/c^3/e^2/(c*x^2+b*x+a)^(1/2)/(c*
(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)+2/15*(-5*B*c*e+4*C*b*e+2*C*
c*d)*(a*e^2-b*d*e+c*d^2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*
c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^
2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2
))^(1/2)*(c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c^3/e^2/(e*x+d)
^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 557, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1667, 846, 857, 732, 435, 430}

$$\int \frac{\sqrt{d+ex}(A+Bx+Cx^2)}{\sqrt{a+bx+cx^2}}\,dx$$

$$= \frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(-ce(9aCe+10bBe+3bCd)-(c^2(2Cd^2-5e(3Ae+Bd)))+8b^2C}{15c^3e^2\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}$$

$$+ \frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(ae^2-bde+cd^2)\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}(4bCe-5Bce+2cCd)\,\text{EllipticF}\Bigg(\text{ar}}{15c^3e^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

$$- \frac{2\sqrt{d+ex}\sqrt{a+bx+cx^2}(4bCe-5Bce+2cCd)}{15c^2e} + \frac{2C(d+ex)^{3/2}\sqrt{a+bx+cx^2}}{5ce}$$

```
[In] Int[(Sqrt[d + e*x]*(A + B*x + C*x^2))/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (-2*(2*c*C*d - 5*B*c*e + 4*b*C*e)*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])/(15*
c^2*e) + (2*C*(d + e*x)^(3/2)*Sqrt[a + b*x + c*x^2])/(5*c*e) + (Sqrt[2]*Sqr
t[b^2 - 4*a*c]*(8*b^2*C*e^2 - c*e*(3*b*C*d + 10*b*B*e + 9*a*C*e) - c^2*(2*C
*d^2 - 5*e*(B*d + 3*A*e)))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2
- 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 -
 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c]
)*e)])/(15*c^3*e^2*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*
Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*c*C*d - 5*B*c*e +
4*b*C*e)*(c*d^2 - b*d*e + a*e^2)*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2
- 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin
[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt
[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(15*c^3*e^2*Sqrt[d +
 e*x]*Sqrt[a + b*x + c*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 846

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
```

```
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\text{integral} = \frac{2C(d+ex)^{3/2}\sqrt{a+bx+cx^2}}{5ce} + \frac{2\int \frac{\sqrt{d+ex}\left(-\frac{1}{2}e(bCd-5Ace+3aCe)-\frac{1}{2}e(2cCd-5Bce+4bCe)x\right)}{\sqrt{a+bx+cx^2}}\,dx}{5ce^2}$$

$$= -\frac{2(2cCd-5Bce+4bCe)\sqrt{d+ex}\sqrt{a+bx+cx^2}}{15c^2e} + \frac{2C(d+ex)^{3/2}\sqrt{a+bx+cx^2}}{5ce} + \frac{4\int \frac{\frac{1}{4}e\left(4b^2Cde+4abCe^2-bcd(Cd+5Be)+ce(15Acd-7aCd-5aBe)\right)+\frac{1}{4}e\left(8b^2Ce^2-ce(3bCd+10bBe+9aCe)-c^2\left(2Cd^2-5e(Bd+3\right.\right.}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}}{15c^2e^2}$$

$$= -\frac{2(2cCd-5Bce+4bCe)\sqrt{d+ex}\sqrt{a+bx+cx^2}}{15c^2e} + \frac{2C(d+ex)^{3/2}\sqrt{a+bx+cx^2}}{5ce} + \frac{\left((2cCd-5Bce+4bCe)\left(cd^2-bde+ae^2\right)\right)\int \frac{1}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx}{15c^2e^2} + \frac{\left(8b^2Ce^2-ce(3bCd+10bBe+9aCe)-c^2(2Cd^2-5e(Bd+3Ae))\right)\int \frac{\sqrt{d+ex}}{\sqrt{a+bx+cx^2}}\,dx}{15c^2e^2}$$

$$= -\frac{2(2cCd-5Bce+4bCe)\sqrt{d+ex}\sqrt{a+bx+cx^2}}{15c^2e} + \frac{2C(d+ex)^{3/2}\sqrt{a+bx+cx^2}}{5ce} + \frac{\left(\sqrt{2}\sqrt{b^2-4ac}\left(8b^2Ce^2-ce(3bCd+10bBe+9aCe)-c^2(2Cd^2-5e(Bd+3Ae))\right)\sqrt{d+ex}\sqrt{-}\right.}{15c^3e^2\sqrt{\frac{c(d+ex)}{2cd-be-\sqrt{b^2-4ac}e}}\sqrt{a+bx+}} + \frac{\left(2\sqrt{2}\sqrt{b^2-4ac}(2cCd-5Bce+4bCe)\left(cd^2-bde+ae^2\right)\sqrt{\frac{c(d+ex)}{2cd-be-\sqrt{b^2-4ac}e}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right)\text{Su}}{15c^3e^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

$$= -\frac{2(2cCd-5Bce+4bCe)\sqrt{d+ex}\sqrt{a+bx+cx^2}}{15c^2e} + \frac{2C(d+ex)^{3/2}\sqrt{a+bx+cx^2}}{5ce} + \frac{\sqrt{2}\sqrt{b^2-4ac}\left(8b^2Ce^2-ce(3bCd+10bBe+9aCe)-c^2(2Cd^2-5e(Bd+3Ae))\right)\sqrt{d+ex}\sqrt{-}}{15c^3e^2\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{a+bx+c}} + \frac{2\sqrt{2}\sqrt{b^2-4ac}(2cCd-5Bce+4bCe)\left(cd^2-bde+ae^2\right)\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\text{si}\right.}{15c^3e^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 31.14 (sec) , antiderivative size = 862, normalized size of antiderivative = 1.55

$$\int \frac{\sqrt{d+ex}(A+Bx+Cx^2)}{\sqrt{a+bx+cx^2}}\,dx$$

$$= \frac{\sqrt{d+ex}\left(\frac{2(a+x(b+cx))(5Bce-4bCe+cC(d+3ex))}{c^2e} - \frac{2(d+ex)\left(\frac{e^2\left(-8b^2Ce^2+ce(3bCd+10bBe+9aCe)+c^2\left(2Cd^2-5e(Bd+3Ae)\right)\right)(a+x(b+cx))}{(d+ex)^2}+ \frac{i}{}\right.}{}\right.}{}$$

```
[In] Integrate[(Sqrt[d + e*x]*(A + B*x + C*x^2))/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (Sqrt[d + e*x]*((2*(a + x*(b + c*x))*(5*B*c*e - 4*b*C*e + c*C*(d + 3*e*x)))
/(c^2*e) - (2*(d + e*x)*((e^2*(-8*b^2*C*e^2 + c*e*(3*b*C*d + 10*b*B*e + 9*a
*C*e) + c^2*(2*C*d^2 - 5*e*(B*d + 3*A*e)))*(a + x*(b + c*x)))/(d + e*x)^2 +
 ((I/2)*Sqrt[1 - (2*(c*d^2 + e*(-(b*d) + a*e)))/((2*c*d - b*e + Sqrt[(b^2 -
 4*a*c)*e^2])*(d + e*x))]*Sqrt[1 + (2*(c*d^2 + e*(-(b*d) + a*e)))/((-2*c*d
+ b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*((2*c*d - b*e + Sqrt[(b^2 - 4*
a*c)*e^2])*(8*b^2*C*e^2 - c*e*(3*b*C*d + 10*b*B*e + 9*a*C*e) + c^2*(-2*C*d^
2 + 5*e*(B*d + 3*A*e)))*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e +
a*e^2)/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c*d
 + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))]
 + (8*b^3*C*e^3 - b^2*e^2*(11*c*C*d + 10*B*c*e + 8*C*Sqrt[(b^2 - 4*a*c)*e^2
]) + c*(c*d*Sqrt[(b^2 - 4*a*c)*e^2]*(2*C*d - 5*B*e) - 15*A*c*e^2*(2*c*d + S
qrt[(b^2 - 4*a*c)*e^2]) + a*e^2*(14*c*C*d + 10*B*c*e + 9*C*Sqrt[(b^2 - 4*a*
c)*e^2])) + b*c*e*(15*A*c*e^2 - 17*a*C*e^2 + 3*C*d*Sqrt[(b^2 - 4*a*c)*e^2]
+ 5*B*(3*c*d*e + 2*e*Sqrt[(b^2 - 4*a*c)*e^2])))*EllipticF[I*ArcSinh[(Sqrt[2
]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/S
qrt[d + e*x]], -((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*c*d - b*e + Sq
rt[(b^2 - 4*a*c)*e^2]))]))/(Sqrt[2]*Sqrt[(c*d^2 + e*(-(b*d) + a*e))/(-2*c*d
 + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*Sqrt[d + e*x])))/(c^3*e^3)))/(15*Sqrt[a
+ x*(b + c*x)])
```

Maple [A] (verified)

Time = 2.56 (sec) , antiderivative size = 955, normalized size of antiderivative = 1.71

method	result
elliptic	$\sqrt{(ex+d)(c\,x^2+bx+a)}\left(\frac{2Cx\sqrt{ce\,x^3+be\,x^2+cd\,x^2+aex+bdx+ad}}{5c}+\frac{2\left(Be+Cd-\frac{2(2be+2cd)C}{5c}\right)\sqrt{ce\,x^3+be\,x^2+cd\,x^2+aex+bdx+ad}}{3ce}+\right.$ 2 $\left(dA-\right.$
risch	Expression too large to display
default	Expression too large to display

```
[In] int((e*x+d)^(1/2)*(C*x^2+B*x+A)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE
)
```

```
[Out] ((e*x+d)*(c*x^2+b*x+a))^(1/2)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)*(2/5*C/c*x*
(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)+2/3*(B*e+C*d-2/5/c*(2*b*e+2
*c*d)*C)/c/e*(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)+2*(d*A-2/5*a/c
*d*C-2/3*(B*e+C*d-2/5/c*(2*b*e+2*c*d)*C)/c/e*(1/2*a*e+1/2*b*d))*(d/e-1/2*(b
+(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*
((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1
/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^
(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)*EllipticF(((x+d/e)/(d
/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c
)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+2*(A*e+B*d-2/5*C/c*(3/2*a*e+
3/2*b*d)-2/3*(B*e+C*d-2/5/c*(2*b*e+2*c*d)*C)/c/e*(b*e+c*d))*(d/e-1/2*(b+(-4
*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-
1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*
((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2
)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)*((-d/e-1/2/c*(-b+(-4*a*c+
b^2)^(1/2)))*EllipticE(((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),(
(-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(
1/2))+1/2/c*(-b+(-4*a*c+b^2)^(1/2))*EllipticF(((x+d/e)/(d/e-1/2*(b+(-4*a*c+
b^2)^(1/2))/c))^(1/2),((-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e-1/2/c*(-b+
(-4*a*c+b^2)^(1/2))))^(1/2))))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 571, normalized size of antiderivative = 1.03

$$\int \frac{\sqrt{d+ex}(A+Bx+Cx^2)}{\sqrt{a+bx+cx^2}}\,dx$$

$$= \frac{2\left((2\,Cc^3d^3+(2\,Cbc^2-5\,Bc^3)d^2e+(7\,Cb^2c+30\,Ac^3-2\,(6\,Ca+5\,Bb)c^2)de^2-(8\,Cb^3+15\,(Ba+Ab)c^2\right.}{}$$

```
[In] integrate((e*x+d)^(1/2)*(C*x^2+B*x+A)/(c*x^2+b*x+a)^(1/2),x, algorithm="fri
cas")
```

```
[Out] 2/45*((2*C*c^3*d^3 + (2*C*b*c^2 - 5*B*c^3)*d^2*e + (7*C*b^2*c + 30*A*c^3 -
2*(6*C*a + 5*B*b)*c^2)*d*e^2 - (8*C*b^3 + 15*(B*a + A*b)*c^2 - (21*C*a*b +
10*B*b^2)*c)*e^3)*sqrt(c*e)*weierstrassPInverse(4/3*(c^2*d^2 - b*c*d*e + (b
^2 - 3*a*c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6
*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(c^3*e^3), 1/3*(3*c*e*x + c*d + b*e)
/(c*e)) + 3*(2*C*c^3*d^2*e + (3*C*b*c^2 - 5*B*c^3)*d*e^2 - (8*C*b^2*c + 15*
A*c^3 - (9*C*a + 10*B*b)*c^2)*e^3)*sqrt(c*e)*weierstrassZeta(4/3*(c^2*d^2 -
 b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e -
 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(c^3*e^3), weierstrassP
Inverse(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/27*(2*c^3
*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(
c^3*e^3), 1/3*(3*c*e*x + c*d + b*e)/(c*e))) + 3*(3*C*c^3*e^3*x + C*c^3*d*e^
2 - (4*C*b*c^2 - 5*B*c^3)*e^3)*sqrt(c*x^2 + b*x + a)*sqrt(e*x + d))/(c^4*e^
3)
```

Sympy [F]

$$\int \frac{\sqrt{d+ex}(A+Bx+Cx^2)}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{\sqrt{d+ex}(A+Bx+Cx^2)}{\sqrt{a+bx+cx^2}}\,dx$$

```
[In] integrate((e*x+d)**(1/2)*(C*x**2+B*x+A)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral(sqrt(d + e*x)*(A + B*x + C*x**2)/sqrt(a + b*x + c*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{d+ex}(A+Bx+Cx^2)}{\sqrt{a+bx+cx^2}}\,dx=\int \frac{(Cx^2+Bx+A)\sqrt{ex+d}}{\sqrt{cx^2+bx+a}}\,dx$$

```
[In] integrate((e*x+d)^(1/2)*(C*x^2+B*x+A)/(c*x^2+b*x+a)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(e*x + d)/sqrt(c*x^2 + b*x + a), x)
```

Giac [F]

$$\int \frac{\sqrt{d+ex}(A+Bx+Cx^2)}{\sqrt{a+bx+cx^2}}\,dx=\int \frac{(Cx^2+Bx+A)\sqrt{ex+d}}{\sqrt{cx^2+bx+a}}\,dx$$

```
[In] integrate((e*x+d)^(1/2)*(C*x^2+B*x+A)/(c*x^2+b*x+a)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(e*x + d)/sqrt(c*x^2 + b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}(A+Bx+Cx^2)}{\sqrt{a+bx+cx^2}}\,dx=\int \frac{\sqrt{d+e\,x}\,\left(C\,x^2+B\,x+A\right)}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

```
[In] int(((d + e*x)^(1/2)*(A + B*x + C*x^2))/(a + b*x + c*x^2)^(1/2),x)
```

```
[Out] int(((d + e*x)^(1/2)*(A + B*x + C*x^2))/(a + b*x + c*x^2)^(1/2), x)
```

3.268 $\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 471

$$\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \frac{2C\sqrt{d+ex}\sqrt{a+bx+cx^2}}{3ce}$$

$$-\frac{\sqrt{2}\sqrt{b^2-4ac}(2cCd-3Bce+2bCe)\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,E\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4}\right.}\right.}{3c^2e^2\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{a+bx+cx^2}}$$

$$+\frac{2\sqrt{2}\sqrt{b^2-4ac}(Ce(bd-ae)+c(2Cd^2-3e(Bd-Ae)))\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\mathrm{EllipticF}\left(\mathrm{arc}\right.}{3c^2e^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

```
[Out] 2/3*C*(e*x+d)^(1/2)*(c*x^2+b*x+a)^(1/2)/c/e-1/3*(-3*B*c*e+2*C*b*e+2*C*c*d)*
EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/
2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2
)*(-4*a*c+b^2)^(1/2)*(e*x+d)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/c^
2/e^2/(c*x^2+b*x+a)^(1/2)/(c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2
)+2/3*(C*e*(-a*e+b*d)+c*(2*C*d^2-3*e*(-A*e+B*d)))*EllipticF(1/2*((b+2*c*x+(
-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/
2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*
(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)
)))^(1/2)/c^2/e^2/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 470, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {1667, 857, 732, 435, 430}

$$\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx$$

$$= \frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}(Ce(bd-ae)-3ce(Bd-Ae)+2cCd^2)\,\text{EllipticF}\left(\text{arcsi}\right.}{3c^2e^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

$$- \frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(2bCe-3Bce+2cCd)E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4a}}{2cd-\left(b+\sqrt{b^2-}\right.}\right.}{3c^2e^2\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}$$

$$+ \frac{2C\sqrt{d+ex}\sqrt{a+bx+cx^2}}{3ce}$$

```
[In] Int[(A + B*x + C*x^2)/(Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (2*C*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])/(3*c*e) - (Sqrt[2]*Sqrt[b^2 - 4*a
*c]*(2*c*C*d - 3*B*c*e + 2*b*C*e)*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqr
t[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 -
 4*a*c])*e)])/(3*c^2*e^2*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c]
)*e)]*Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*c*C*d^2 + C*
e*(b*d - a*e) - 3*c*e*(B*d - A*e))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^
2 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcS
in[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sq
rt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(3*c^2*e^2*Sqrt[d
+ e*x]*Sqrt[a + b*x + c*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
```

```
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2C\sqrt{d+ex}\sqrt{a+bx+cx^2}}{3ce}+\frac{2\int \frac{-\frac{1}{2}e(bCd-3Ace+aCe)-\frac{1}{2}e(2cCd-3Bce+2bCe)x}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx}{3ce^2}\\
&= \frac{2C\sqrt{d+ex}\sqrt{a+bx+cx^2}}{3ce}-\frac{(2cCd-3Bce+2bCe)\int \frac{\sqrt{d+ex}}{\sqrt{a+bx+cx^2}}\,dx}{3ce^2}\\
&\quad+\frac{(2cCd^2+Ce(bd-ae)-3ce(Bd-Ae))\int \frac{1}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx}{3ce^2}
\end{aligned}
$$

$$= \frac{2C\sqrt{d+ex}\sqrt{a+bx+cx^2}}{3ce} - \frac{\left(\sqrt{2}\sqrt{b^2-4ac}(2cCd-3Bce+2bCe)\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right)\mathrm{Subst}\left(\int \frac{\sqrt{1+\frac{2\sqrt{b^2-4ac}ex^2}{2cd-be-\sqrt{b^2-4ac}e}}}{\sqrt{1-x^2}}\,dx\right.}{3c^2e^2\sqrt{\frac{c(d+ex)}{2cd-be-\sqrt{b^2-4ac}e}}\sqrt{a+bx+cx^2}} + \frac{\left(2\sqrt{2}\sqrt{b^2-4ac}(2cCd^2+Ce(bd-ae)-3ce(Bd-Ae))\sqrt{\frac{c(d+ex)}{2cd-be-\sqrt{b^2-4ac}e}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right)\mathrm{Sub}}{3c^2e^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

$$= \frac{2C\sqrt{d+ex}\sqrt{a+bx+cx^2}}{3ce} - \frac{\sqrt{2}\sqrt{b^2-4ac}(2cCd-3Bce+2bCe)\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{}{2cd-}\right.}{3c^2e^2\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{a+bx+cx^2}} + \frac{2\sqrt{2}\sqrt{b^2-4ac}(2cCd^2+Ce(bd-ae)-3ce(Bd-Ae))\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin\right.}{3c^2e^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 30.18 (sec) , antiderivative size = 980, normalized size of antiderivative = 2.08

$$\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx$$

$$= \frac{\sqrt{d+ex}\left(4cCe^2(a+x(b+cx)) + \frac{(d+ex)\left(-\frac{4e^2(2cCd-3Bce+2bCe)\sqrt{\frac{cd^2+e(-bd+ae)}{-2cd+be+\sqrt{\left(b^2-4ac\right)e^2}}}(a+x(b+cx))}{(d+ex)^2}+\frac{i\sqrt{2}(2cCd-3Bce+2bCe)\left(2c\right.}{}\right.}{}\right.}{}$$

```
[In] Integrate[(A + B*x + C*x^2)/(Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (Sqrt[d + e*x]*(4*c*C*e^2*(a + x*(b + c*x)) + ((d + e*x)*((-4*e^2*(2*c*C*d
 - 3*B*c*e + 2*b*C*e)*Sqrt[(c*d^2 + e*(-(b*d) + a*e))/(-2*c*d + b*e + Sqrt[(
b^2 - 4*a*c)*e^2])]*(a + x*(b + c*x)))/(d + e*x)^2 + (I*Sqrt[2]*(2*c*C*d -
3*B*c*e + 2*b*C*e)*(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*Sqrt[(-2*a*e^2 +
 d*Sqrt[(b^2 - 4*a*c)*e^2] + 2*c*d*e*x + e*Sqrt[(b^2 - 4*a*c)*e^2]*x + b*e*
(d - e*x))/((2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*Sqrt[(2*a*e
^2 + d*Sqrt[(b^2 - 4*a*c)*e^2] - 2*c*d*e*x + e*Sqrt[(b^2 - 4*a*c)*e^2]*x +
b*e*(-d + e*x))/((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*Ellip
ticE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*c*d + b*e + Sqrt[(
b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e
^2])/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))])/Sqrt[d + e*x] + (I*Sqrt[2]*
(2*b^2*C*e^2 - b*e*(3*B*c*e + 2*C*Sqrt[(b^2 - 4*a*c)*e^2]) + c*(6*A*c*e^2 -
 2*a*C*e^2 + Sqrt[(b^2 - 4*a*c)*e^2]*(-2*C*d + 3*B*e)))*Sqrt[(-2*a*e^2 + d*
Sqrt[(b^2 - 4*a*c)*e^2] + 2*c*d*e*x + e*Sqrt[(b^2 - 4*a*c)*e^2]*x + b*e*(d
- e*x))/((2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*Sqrt[(2*a*e^2
+ d*Sqrt[(b^2 - 4*a*c)*e^2] - 2*c*d*e*x + e*Sqrt[(b^2 - 4*a*c)*e^2]*x + b*e
*(-d + e*x))/((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*Elliptic
F[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*c*d + b*e + Sqrt[(b^2
 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2]
)/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))])/Sqrt[d + e*x]))/Sqrt[(c*d^2 +
e*(-(b*d) + a*e))/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]))/(6*c^2*e^3*Sq
rt[a + x*(b + c*x)])
```

Maple [A] (verified)

Time = 3.82 (sec) , antiderivative size = 823, normalized size of antiderivative = 1.75

method	result
elliptic	$\frac{\sqrt{(ex+d)(c\,x^2+bx+a)}\left(\frac{2C\sqrt{ce\,x^3+be\,x^2+cd\,x^2+aex+bdx+ad}}{3ce}+\frac{2\left(A-\frac{2C\left(\frac{ae}{2}+\frac{bd}{2}\right)}{3ce}\right)\left(\frac{d}{e}-\frac{b+\sqrt{-4ac+b^2}}{2c}\right)\sqrt{\frac{x+\frac{d}{e}}{\frac{d}{e}-\frac{b+\sqrt{-4ac+b^2}}{2c}}}\sqrt{\frac{x-\frac{-b+}{}}{-\frac{d}{e}-\frac{-b}{}}}}{\sqrt{ce\,x^3+be}}\right.}{}$
risch	Expression too large to display
default	Expression too large to display

```
[In] int((C*x^2+B*x+A)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE
)
```

```
[Out] ((e*x+d)*(c*x^2+b*x+a))^(1/2)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)*(2/3*C/c/e*
(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)+2*(A-2/3*C/c/e*(1/2*a*e+1/2
*b*d))*(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)
^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d/e-1/2/c*(-b+(-4*a*
c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e+1/2*(b+(-4*a*
c+b^2)^(1/2))/c))^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)*Ell
ipticF(((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2*(b+(-4
*a*c+b^2)^(1/2))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+2*(B-2/3*C
/c/e*(b*e+c*d))*(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(b+(-4
*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d/e-1/2/c*(
-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e+1/2*
(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^
(1/2)*((-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*EllipticE(((x+d/e)/(d/e-1/2*(b+
(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e-1/
2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+1/2/c*(-b+(-4*a*c+b^2)^(1/2))*Elliptic
F(((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2*(b+(-4*a*c+
b^2)^(1/2))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 451, normalized size of antiderivative = 0.96

$$\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx$$
$$= \frac{2\left(3\sqrt{cx^2+bx+a}\sqrt{ex+d}Cc^2e^2+(2\,Cc^2d^2+(Cbc-3\,Bc^2)de+(2\,Cb^2+9\,Ac^2-3\,(Ca+Bb)c)e^2)\sqrt{ce}\right.}{}$$

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fri
cas")
```

```
[Out] 2/9*(3*sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)*C*c^2*e^2 + (2*C*c^2*d^2 + (C*b*
c - 3*B*c^2)*d*e + (2*C*b^2 + 9*A*c^2 - 3*(C*a + B*b)*c)*e^2)*sqrt(c*e)*wei
erstrassPInverse(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/
27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*
c)*e^3)/(c^3*e^3), 1/3*(3*c*e*x + c*d + b*e)/(c*e)) + 3*(2*C*c^2*d*e + (2*C
*b*c - 3*B*c^2)*e^2)*sqrt(c*e)*weierstrassZeta(4/3*(c^2*d^2 - b*c*d*e + (b^
2 - 3*a*c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*
a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(c^3*e^3), weierstrassPInverse(4/3*(c
^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2
*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(c^3*e^3), 1/3*
(3*c*e*x + c*d + b*e)/(c*e))))/(c^3*e^3)
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \int \frac{A+Bx+Cx^2}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx$$

```
[In] integrate((C*x**2+B*x+A)/(e*x+d)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((A + B*x + C*x**2)/(sqrt(d + e*x)*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \int \frac{Cx^2+Bx+A}{\sqrt{cx^2+bx+a}\sqrt{ex+d}}\,dx$$

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate((C*x^2 + B*x + A)/(sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx=\int \frac{Cx^2+Bx+A}{\sqrt{cx^2+bx+a}\sqrt{ex+d}}\,dx$$

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate((C*x^2 + B*x + A)/(sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx=\int \frac{C\,x^2+B\,x+A}{\sqrt{d+e\,x}\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

```
[In] int((A + B*x + C*x^2)/((d + e*x)^(1/2)*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] int((A + B*x + C*x^2)/((d + e*x)^(1/2)*(a + b*x + c*x^2)^(1/2)), x)
```

3.269 $\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 508

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}\sqrt{a+bx+cx^2}}\,dx = -\frac{2(Cd^2-e(Bd-Ae))\sqrt{a+bx+cx^2}}{e\left(cd^2-bde+ae^2\right)\sqrt{d+ex}}$$

$$-\frac{\sqrt{2}\sqrt{b^2-4ac}(Ce(bd-ae)-c(2Cd^2-e(Bd-Ae)))\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\right.}{ce^2\left(cd^2-bde+ae^2\right)\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{a+bx+cx^2}}$$

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}(2Cd-Be)\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\right.}{ce^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

```
[Out] -2*(C*d^2-e*(-A*e+B*d))*(c*x^2+b*x+a)^(1/2)/e/(a*e^2-b*d*e+c*d^2)/(e*x+d)^(
1/2)-(C*e*(-a*e+b*d)-c*(2*C*d^2-e*(-A*e+B*d)))*EllipticE(1/2*((b+2*c*x+(-4*
a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/
(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(e*x+d)
^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/c/e^2/(a*e^2-b*d*e+c*d^2)/(c*x
^2+b*x+a)^(1/2)/(c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)-2*(-B*e+
2*C*d)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2
)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))
*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(e*x+d
)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c/e^2/(e*x+d)^(1/2)/(c*x^2+b*x+a)
^(1/2)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 506, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {1664, 857, 732, 435, 430}

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}\sqrt{a+bx+cx^2}}\,dx = \frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(-Ce(bd-ae)-ce(Bd-Ae)+2c}{ce^2\sqrt{a+bx+cx^2}\,(ae^2-bde+cd^2)}$$

$$-\frac{2\sqrt{a+bx+cx^2}(Cd^2-e(Bd-Ae))}{e\sqrt{d+ex}\,(ae^2-bde+cd^2)}$$

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(2Cd-Be)\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{}}{2cd-(}\right.}{ce^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

```
[In] Int[(A + B*x + C*x^2)/((d + e*x)^(3/2)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (-2*(C*d^2 - e*(B*d - A*e))*Sqrt[a + b*x + c*x^2])/(e*(c*d^2 - b*d*e + a*e^
2)*Sqrt[d + e*x]) + (Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*c*C*d^2 - C*e*(b*d - a*e)
 - c*e*(B*d - A*e))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c
))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]
]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/
(c*e^2*(c*d^2 - b*d*e + a*e^2)*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 -
4*a*c])*e)]*Sqrt[a + b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*C*d -
B*e)*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((c*(a +
 b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c]
 + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b
 + Sqrt[b^2 - 4*a*c])*e)])/(c*e^2*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2(Cd^2-e(Bd-Ae))\sqrt{a+bx+cx^2}}{e\left(cd^2-bde+ae^2\right)\sqrt{d+ex}} \\
&\quad -\frac{2\int\frac{-\frac{bd(Cd-Be)+e(Acd-aCd+aBe)}{2e}+\frac{1}{2}\left(Bcd+bCd-\frac{2cCd^2}{e}-Ace-aCe\right)x}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx}{cd^2-bde+ae^2} \\
&= -\frac{2(Cd^2-e(Bd-Ae))\sqrt{a+bx+cx^2}}{e\left(cd^2-bde+ae^2\right)\sqrt{d+ex}}-\frac{(2Cd-Be)\int\frac{1}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx}{e^2} \\
&\quad -\frac{\left(Bcd+bCd-\frac{2cCd^2}{e}-Ace-aCe\right)\int\frac{\sqrt{d+ex}}{\sqrt{a+bx+cx^2}}\,dx}{e\left(cd^2-bde+ae^2\right)}
\end{aligned}$$

$$= -\frac{2(Cd^2-e(Bd-Ae))\sqrt{a+bx+cx^2}}{e\left(cd^2-bde+ae^2\right)\sqrt{d+ex}} - \frac{\left(\sqrt{2}\sqrt{b^2-4ac}\left(Bcd+bCd-\frac{2cCd^2}{e}-Ace-aCe\right)\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right)\text{Subst}\Bigg(\int \frac{\sqrt{1+\frac{2\sqrt{}}{2cd-}}}{\sqrt{}}}{ce\left(cd^2-bde+ae^2\right)\sqrt{\frac{c(d+ex)}{2cd-be-\sqrt{b^2-4ac}e}}\sqrt{a+bx+cx^2}} - \frac{\left(2\sqrt{2}\sqrt{b^2-4ac}(2Cd-Be)\sqrt{\frac{c(d+ex)}{2cd-be-\sqrt{b^2-4ac}e}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right)\text{Subst}\Bigg(\int \frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2\sqrt{b^2-4ac}ex^2}{2cd-be-\sqrt{b^2-4ac}e}}}}{ce^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

$$= -\frac{2(Cd^2-e(Bd-Ae))\sqrt{a+bx+cx^2}}{e\left(cd^2-bde+ae^2\right)\sqrt{d+ex}} - \frac{\sqrt{2}\sqrt{b^2-4ac}\left(Bcd+bCd-\frac{2cCd^2}{e}-Ace-aCe\right)\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\Bigg(\sin^{-1}\Bigg(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}}{ce\left(cd^2-bde+ae^2\right)\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{a+bx+cx^2}} - \frac{2\sqrt{2}\sqrt{b^2-4ac}(2Cd-Be)\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2}{2cd-}\right.}{ce^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 26.43 (sec) , antiderivative size = 772, normalized size of antiderivative = 1.52

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}\sqrt{a+bx+cx^2}}\,dx = \frac{2\Bigg(-e^2(Cd^2+e(-Bd+Ae))\left(a+x(b+cx)\right)+\frac{e^2\left(2cCd^2+Ce(-bd+ae)+ce(-B\right.}{c}}{}$$

```
[In] Integrate[(A + B*x + C*x^2)/((d + e*x)^(3/2)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (2*(-(e^2*(C*d^2 + e*(-(B*d) + A*e))*(a + x*(b + c*x))) + (e^2*(2*c*C*d^2 +
 C*e*(-(b*d) + a*e) + c*e*(-(B*d) + A*e))*(a + x*(b + c*x)))/c - ((I/2)*(d
+ e*x)^(3/2)*Sqrt[1 - (2*(c*d^2 + e*(-(b*d) + a*e)))/((2*c*d - b*e + Sqrt[(
```

```
b^2 - 4*a*c)*e^2])*(d + e*x))]*Sqrt[1 + (2*(c*d^2 + e*(-(b*d) + a*e)))/((-2
*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*((2*c*d - b*e + Sqrt[(b^2
 - 4*a*c)*e^2])*(2*c*C*d^2 + C*e*(-(b*d) + a*e) + c*e*(-(B*d) + A*e))*Ellip
ticE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*c*d + b*e + Sqrt[(
b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e
^2])/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))] + (-(b^2*C*d*e^2) + 2*a*c*C*
d*e^2 - 2*a*B*c*e^3 - 2*c*C*d^2*Sqrt[(b^2 - 4*a*c)*e^2] + B*c*d*e*Sqrt[(b^2
 - 4*a*c)*e^2] - a*C*e^2*Sqrt[(b^2 - 4*a*c)*e^2] - A*c*e^2*(2*c*d + Sqrt[(b
^2 - 4*a*c)*e^2]) + b*(B*c*d*e^2 + A*c*e^3 + a*C*e^3 + C*d*e*Sqrt[(b^2 - 4*
a*c)*e^2]))*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*c
*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c*d + b*e + Sqr
t[(b^2 - 4*a*c)*e^2])/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])))]))/(Sqrt[2]*
c*Sqrt[(c*d^2 + e*(-(b*d) + a*e))/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]
)))/(e^3*(c*d^2 + e*(-(b*d) + a*e))*Sqrt[d + e*x]*Sqrt[a + x*(b + c*x)])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 964 vs. 2(456) = 912.

Time = 4.17 (sec) , antiderivative size = 965, normalized size of antiderivative = 1.90

method	result
elliptic	$\sqrt{(ex+d)(c\,x^2+bx+a)}\left(-\frac{2\left(ce\,x^2+bex+ae\right)\left(A\,e^2-Bde+C\,d^2\right)}{e^2\left(e^2a-bde+c\,d^2\right)\sqrt{\left(x+\frac{d}{e}\right)\left(ce\,x^2+bex+ae\right)}}+\frac{2\left(\frac{Be-Cd}{e^2}-\frac{(be-cd)\left(A\,e^2-Bde+C\,d^2\right)}{e^2\left(e^2a-bde+c\,d^2\right)}+\frac{b\left(A\,e^2-Bde+C\,d^2\right)}{e\left(e^2a-bde+c\,d^2\right)}\right)\left(\frac{d}{e}-\right.}{}\right.$
default	Expression too large to display

```
[In] int((C*x^2+B*x+A)/(e*x+d)^(3/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE
)
```

```
[Out] ((e*x+d)*(c*x^2+b*x+a))^(1/2)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)*(-2*(c*e*x^
2+b*e*x+a*e)/e^2/(a*e^2-b*d*e+c*d^2)*(A*e^2-B*d*e+C*d^2)/((x+d/e)*(c*e*x^2+
b*e*x+a*e))^(1/2)+2*((B*e-C*d)/e^2-(b*e-c*d)/e^2*(A*e^2-B*d*e+C*d^2)/(a*e^2
-b*d*e+c*d^2)+b/e/(a*e^2-b*d*e+c*d^2)*(A*e^2-B*d*e+C*d^2))*(d/e-1/2*(b+(-4*
a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1
/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*(
(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)
/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)*EllipticF(((x+d/e)/(d/e-1/
2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d
```

```
/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+2*(C/e+c/e*(A*e^2-B*d*e+C*d^2)/(a
*e^2-b*d*e+c*d^2))*(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(b+
(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d/e-1/2/
c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e+1
/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*
d)^(1/2)*((-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*EllipticE(((x+d/e)/(d/e-1/2*
(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e
-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+1/2/c*(-b+(-4*a*c+b^2)^(1/2))*Ellip
ticF(((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2*(b+(-4*a
*c+b^2)^(1/2))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 708, normalized size of antiderivative = 1.39

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}\sqrt{a+bx+cx^2}}\,dx =$$

$$-\frac{2\left(\left(2\,Cc^2d^4-(2\,Cbc+Bc^2)d^3e-(Cb^2+2\,Ac^2-2\,(2\,Ca+Bb)c)d^2e^2+(Cab-(3\,Ba-Ab)c)de^3+(2\right.\right.}{}$$

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^(3/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fri
cas")
```

```
[Out] -2/3*((2*C*c^2*d^4 - (2*C*b*c + B*c^2)*d^3*e - (C*b^2 + 2*A*c^2 - 2*(2*C*a
+ B*b)*c)*d^2*e^2 + (C*a*b - (3*B*a - A*b)*c)*d*e^3 + (2*C*c^2*d^3*e - (2*C
*b*c + B*c^2)*d^2*e^2 - (C*b^2 + 2*A*c^2 - 2*(2*C*a + B*b)*c)*d*e^3 + (C*a*
b - (3*B*a - A*b)*c)*e^4)*x)*sqrt(c*e)*weierstrassPInverse(4/3*(c^2*d^2 - b
*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3
*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(c^3*e^3), 1/3*(3*c*e*x +
 c*d + b*e)/(c*e)) + 3*(2*C*c^2*d^3*e - (C*b*c + B*c^2)*d^2*e^2 + (C*a*c +
A*c^2)*d*e^3 + (2*C*c^2*d^2*e^2 - (C*b*c + B*c^2)*d*e^3 + (C*a*c + A*c^2)*e
^4)*x)*sqrt(c*e)*weierstrassZeta(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2
)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 +
 (2*b^3 - 9*a*b*c)*e^3)/(c^3*e^3), weierstrassPInverse(4/3*(c^2*d^2 - b*c*d
*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^
2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(c^3*e^3), 1/3*(3*c*e*x + c*d
 + b*e)/(c*e))) + 3*(C*c^2*d^2*e^2 - B*c^2*d*e^3 + A*c^2*e^4)*sqrt(c*x^2 +
b*x + a)*sqrt(e*x + d))/(c^3*d^3*e^3 - b*c^2*d^2*e^4 + a*c^2*d*e^5 + (c^3*d
^2*e^4 - b*c^2*d*e^5 + a*c^2*e^6)*x)
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}\sqrt{a+bx+cx^2}}\,dx=\int \frac{A+Bx+Cx^2}{(d+ex)^{\frac{3}{2}}\sqrt{a+bx+cx^2}}\,dx$$

```
[In] integrate((C*x**2+B*x+A)/(e*x+d)**(3/2)/(c*x**2+b*x+a)**(1/2),x)
[Out] Integral((A + B*x + C*x**2)/((d + e*x)**(3/2)*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}\sqrt{a+bx+cx^2}}\,dx=\int \frac{Cx^2+Bx+A}{\sqrt{cx^2+bx+a}(ex+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^(3/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="max
ima")
[Out] integrate((C*x^2 + B*x + A)/(sqrt(c*x^2 + b*x + a)*(e*x + d)^(3/2)), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}\sqrt{a+bx+cx^2}}\,dx=\int \frac{Cx^2+Bx+A}{\sqrt{cx^2+bx+a}(ex+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^(3/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="gia
c")
[Out] integrate((C*x^2 + B*x + A)/(sqrt(c*x^2 + b*x + a)*(e*x + d)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}\sqrt{a+bx+cx^2}}\,dx=\int \frac{C\,x^2+B\,x+A}{{\left(d+e\,x\right)}^{3/2}\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

```
[In] int((A + B*x + C*x^2)/((d + e*x)^(3/2)*(a + b*x + c*x^2)^(1/2)),x)
[Out] int((A + B*x + C*x^2)/((d + e*x)^(3/2)*(a + b*x + c*x^2)^(1/2)), x)
```

3.270 $\int \frac{A+Bx+Cx^2}{(d+ex)^{5/2}\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 684

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{5/2}\sqrt{a+bx+cx^2}}\,dx=-\frac{2(Cd^2-e(Bd-Ae))\sqrt{a+bx+cx^2}}{3e\left(cd^2-bde+ae^2\right)(d+ex)^{3/2}}$$

$$+\frac{2(cd(2Cd^2+e(Bd-4Ae))+e(3ae(2Cd-Be)-b(4Cd^2-Bde-2Ae^2)))\sqrt{a+bx+cx^2}}{3e\left(cd^2-bde+ae^2\right)^2\sqrt{d+ex}}$$

$$-\frac{\sqrt{2}\sqrt{b^2-4ac}(cd(2Cd^2+e(Bd-4Ae))+e(3ae(2Cd-Be)-b(4Cd^2-Bde-2Ae^2)))\sqrt{d+ex}\sqrt{-\frac{c(a+}{b^2}}}{3e^2\left(cd^2-bde+ae^2\right)^2\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{a+bx+}}$$

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}(3Ce(bd-ae)-c(2Cd^2+e(Bd-Ae)))\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\operatorname{EllipticF}\left(\text{arcs}\right.}{3ce^2\left(cd^2-bde+ae^2\right)\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

```
[Out] -2/3*(C*d^2-e*(-A*e+B*d))*(c*x^2+b*x+a)^(1/2)/e/(a*e^2-b*d*e+c*d^2)/(e*x+d)
^(3/2)+2/3*(c*d*(2*C*d^2+e*(-4*A*e+B*d))+e*(3*a*e*(-B*e+2*C*d)-b*(-2*A*e^2-
B*d*e+4*C*d^2)))*(c*x^2+b*x+a)^(1/2)/e/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)^(1/2)-
1/3*(c*d*(2*C*d^2+e*(-4*A*e+B*d))+e*(3*a*e*(-B*e+2*C*d)-b*(-2*A*e^2-B*d*e+4
*C*d^2)))*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(
1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/
2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(e*x+d)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2)
)^(1/2)/e^2/(a*e^2-b*d*e+c*d^2)^2/(c*x^2+b*x+a)^(1/2)/(c*(e*x+d)/(2*c*d-e*(
b+(-4*a*c+b^2)^(1/2))))^(1/2)-2/3*(3*C*e*(-a*e+b*d)-c*(2*C*d^2+e*(-A*e+B*d)
))*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^
(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(
```

```
1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(e*x+d)/(2
*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c/e^2/(a*e^2-b*d*e+c*d^2)/(e*x+d)^(1/
2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 680, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1664, 848, 857, 732, 435, 430}

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{5/2}\sqrt{a+bx+cx^2}}\,dx = \frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}(-3Ce(bd-ae)+ce(Bd}{3ce^2\sqrt{d+ex}\sqrt{a+bx}}$$

$$-\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(3ae^2(2Cd-Be)-be(4Cd^2-e(2Ae+Bd))+cde(Bd-4Ae)+2cC}{3e^2\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)^2\sqrt{\frac{c(d+ex)}{2cd-e(\sqrt{b^2-4ac}+b}}}$$

$$-\frac{2\sqrt{a+bx+cx^2}(Cd^2-e(Bd-Ae))}{3e(d+ex)^{3/2}\left(ae^2-bde+cd^2\right)}$$

$$+\frac{2\sqrt{a+bx+cx^2}(3ae^2(2Cd-Be)-be(4Cd^2-e(2Ae+Bd))+cde(Bd-4Ae)+2cCd^3)}{3e\sqrt{d+ex}\left(ae^2-bde+cd^2\right)^2}$$

```
[In] Int[(A + B*x + C*x^2)/((d + e*x)^(5/2)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (-2*(C*d^2 - e*(B*d - A*e))*Sqrt[a + b*x + c*x^2])/(3*e*(c*d^2 - b*d*e + a*
e^2)*(d + e*x)^(3/2)) + (2*(2*c*C*d^3 + c*d*e*(B*d - 4*A*e) + 3*a*e^2*(2*C*
d - B*e) - b*e*(4*C*d^2 - e*(B*d + 2*A*e)))*Sqrt[a + b*x + c*x^2])/(3*e*(c*
d^2 - b*d*e + a*e^2)^2*Sqrt[d + e*x]) - (Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*c*C*d
^3 + c*d*e*(B*d - 4*A*e) + 3*a*e^2*(2*C*d - B*e) - b*e*(4*C*d^2 - e*(B*d +
2*A*e)))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*Ellipti
cE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]],
 (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(3*e^2*(c*d
^2 - b*d*e + a*e^2)^2*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e
)]*Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*c*C*d^2 - 3*C*e
*(b*d - a*e) + c*e*(B*d - A*e))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 -
 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[
Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[
b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(3*c*e^2*(c*d^2 - b*d
*e + a*e^2)*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 848

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
```

```
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{2(Cd^2-e(Bd-Ae))\sqrt{a+bx+cx^2}}{3e\left(cd^2-bde+ae^2\right)(d+ex)^{3/2}} - \frac{2\int \frac{-\frac{bCd^2-be(Bd+2Ae)+3e(Acd-aCd+aBe)}{2e}-\frac{1}{2}\left(Bcd-3bCd+\frac{2cCd^2}{e}-Ace+3aCe\right)x}{(d+ex)^{3/2}\sqrt{a+bx+cx^2}}\,dx}{3\left(cd^2-bde+ae^2\right)}$$

$$= -\frac{2(Cd^2-e(Bd-Ae))\sqrt{a+bx+cx^2}}{3e\left(cd^2-bde+ae^2\right)(d+ex)^{3/2}} + \frac{2(2cCd^3+cde(Bd-4Ae)+3ae^2(2Cd-Be)-be(4Cd^2-e(Bd+2Ae)))\sqrt{a+bx+cx^2}}{3e\left(cd^2-bde+ae^2\right)^2\sqrt{d+ex}} + \frac{4\int \frac{\frac{3b^2Cd^2e-bd\left(cCd^2+6aCe^2+ce(2Bd+Ae)\right)+e\left(Ac\left(3cd^2-ae^2\right)+a\left(3aCe^2-cd(Cd-4Be)\right)\right)}{4e}-\frac{c\bigl(2cCd^3+cde(Bd-4Ae)+3ae^2(2Cd-Be)-be\bigl(4C}{4e}}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}}{3\left(cd^2-bde+ae^2\right)^2}$$

$$= -\frac{2(Cd^2-e(Bd-Ae))\sqrt{a+bx+cx^2}}{3e\left(cd^2-bde+ae^2\right)(d+ex)^{3/2}} + \frac{2(2cCd^3+cde(Bd-4Ae)+3ae^2(2Cd-Be)-be(4Cd^2-e(Bd+2Ae)))\sqrt{a+bx+cx^2}}{3e\left(cd^2-bde+ae^2\right)^2\sqrt{d+ex}} + \frac{(2cCd^2-3Ce(bd-ae)+ce(Bd-Ae))\int \frac{1}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx}{3e^2\left(cd^2-bde+ae^2\right)} - \frac{(c(2cCd^3+cde(Bd-4Ae)+3ae^2(2Cd-Be)-be(4Cd^2-e(Bd+2Ae))))\int \frac{\sqrt{d+ex}}{\sqrt{a+bx+cx^2}}\,dx}{3e^2\left(cd^2-bde+ae^2\right)^2}$$

$$= -\frac{2(Cd^2-e(Bd-Ae))\sqrt{a+bx+cx^2}}{3e\left(cd^2-bde+ae^2\right)(d+ex)^{3/2}}$$

$$+\frac{2(2cCd^3+cde(Bd-4Ae)+3ae^2(2Cd-Be)-be(4Cd^2-e(Bd+2Ae)))\sqrt{a+bx+cx^2}}{3e\left(cd^2-bde+ae^2\right)^2\sqrt{d+ex}}$$

$$-\frac{\left(\sqrt{2}\sqrt{b^2-4ac}(2cCd^3+cde(Bd-4Ae)+3ae^2(2Cd-Be)-be(4Cd^2-e(Bd+2Ae)))\sqrt{d+}\right.}{3e^2\left(cd^2-bde+ae^2\right)^2\sqrt{\frac{c(d+ex)}{2cd-be-\sqrt{b^2-4ace}}}}$$

$$+\frac{\left(2\sqrt{2}\sqrt{b^2-4ac}(2cCd^2-3Ce(bd-ae)+ce(Bd-Ae))\sqrt{\frac{c(d+ex)}{2cd-be-\sqrt{b^2-4ace}}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right)\text{Sub}}{3ce^2\left(cd^2-bde+ae^2\right)\sqrt{d+ex}\sqrt{a+bx+cx}}$$

$$= -\frac{2(Cd^2-e(Bd-Ae))\sqrt{a+bx+cx^2}}{3e\left(cd^2-bde+ae^2\right)(d+ex)^{3/2}}$$

$$+\frac{2(2cCd^3+cde(Bd-4Ae)+3ae^2(2Cd-Be)-be(4Cd^2-e(Bd+2Ae)))\sqrt{a+bx+cx^2}}{3e\left(cd^2-bde+ae^2\right)^2\sqrt{d+ex}}$$

$$-\frac{\sqrt{2}\sqrt{b^2-4ac}(2cCd^3+cde(Bd-4Ae)+3ae^2(2Cd-Be)-be(4Cd^2-e(Bd+2Ae)))\sqrt{d+e}}{3e^2\left(cd^2-bde+ae^2\right)^2\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{}}$$

$$+\frac{2\sqrt{2}\sqrt{b^2-4ac}(2cCd^2-3Ce(bd-ae)+ce(Bd-Ae))\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin\right.}{3ce^2\left(cd^2-bde+ae^2\right)\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 31.72 (sec) , antiderivative size = 1194, normalized size of antiderivative = 1.75

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{5/2}\sqrt{a+bx+cx^2}}\,dx = \frac{\sqrt{d+ex}(a+bx+cx^2)\left(-\frac{2(Cd^2-Bde+Ae^2)}{3e(cd^2-bde+ae^2)(d+ex)^2}-\frac{2(-2cCd^3-Bcd^2e+4bCd^2e-bB}{3e(cd^2-bd}\right.}{\sqrt{a+x(b+cx)}}$$

$$+\frac{2(d+ex)^{3/2}\sqrt{a+bx+cx^2}\left(-\left((2cCd^3+cde(Bd-4Ae)-3ae^2(-2Cd+Be)+be(-4Cd^2+e(Bd+2Ae\right.\right.}{}$$

```
[In] Integrate[(A + B*x + C*x^2)/((d + e*x)^(5/2)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (Sqrt[d + e*x]*(a + b*x + c*x^2)*((-2*(C*d^2 - B*d*e + A*e^2))/(3*e*(c*d^2
- b*d*e + a*e^2)*(d + e*x)^2) - (2*(-2*c*C*d^3 - B*c*d^2*e + 4*b*C*d^2*e -
b*B*d*e^2 + 4*A*c*d*e^2 - 6*a*C*d*e^2 - 2*A*b*e^3 + 3*a*B*e^3))/(3*e*(c*d^2
 - b*d*e + a*e^2)^2*(d + e*x))))/Sqrt[a + x*(b + c*x)] + (2*(d + e*x)^(3/2)
*Sqrt[a + b*x + c*x^2]*(-((2*c*C*d^3 + c*d*e*(B*d - 4*A*e) - 3*a*e^2*(-2*C*
d + B*e) + b*e*(-4*C*d^2 + e*(B*d + 2*A*e)))*(c*(-1 + d/(d + e*x))^2 + (e*(
b - (b*d)/(d + e*x) + (a*e)/(d + e*x)))/(d + e*x))) + ((I/2)*Sqrt[1 - (2*(c
*d^2 + e*(-(b*d) + a*e)))/((2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x
))]*Sqrt[1 + (2*(c*d^2 + e*(-(b*d) + a*e)))/((-2*c*d + b*e + Sqrt[(b^2 - 4*
a*c)*e^2])*(d + e*x))]*((2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(c*d*(2*C*d
^2 + e*(B*d - 4*A*e)) + e*(-4*b*C*d^2 + b*e*(B*d + 2*A*e) - 3*a*e*(-2*C*d +
 B*e)))*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*c*d +
 b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c*d + b*e + Sqrt[(b
^2 - 4*a*c)*e^2])/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))] - (2*a*c*C*d^2*
e^2 - 8*a*B*c*d*e^3 - 6*a^2*C*e^4 + 2*c*C*d^3*Sqrt[(b^2 - 4*a*c)*e^2] + B*c
*d^2*e*Sqrt[(b^2 - 4*a*c)*e^2] + 6*a*C*d*e^2*Sqrt[(b^2 - 4*a*c)*e^2] - 3*a*
B*e^3*Sqrt[(b^2 - 4*a*c)*e^2] + 2*A*c*e^2*(-3*c*d^2 + a*e^2 - 2*d*Sqrt[(b^2
 - 4*a*c)*e^2]) - b^2*e^2*(2*C*d^2 + e*(B*d + 2*A*e)) + b*e*(2*A*e^2*(3*c*d
 + Sqrt[(b^2 - 4*a*c)*e^2]) + 2*C*d*(3*a*e^2 - 2*d*Sqrt[(b^2 - 4*a*c)*e^2])
 + B*e*(3*c*d^2 + 3*a*e^2 + d*Sqrt[(b^2 - 4*a*c)*e^2])))*EllipticF[I*ArcSin
h[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*
e^2])])/Sqrt[d + e*x]], -((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*c*d -
 b*e + Sqrt[(b^2 - 4*a*c)*e^2]))]))/(Sqrt[2]*Sqrt[(c*d^2 + e*(-(b*d) + a*e)
)/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*Sqrt[d + e*x])))/(3*e^3*(c*d^2
```

```
- b*d*e + a*e^2)^2*Sqrt[a + x*(b + c*x)]*Sqrt[((d + e*x)^2*(c*(-1 + d/(d +
e*x))^2 + (e*(b - (b*d)/(d + e*x) + (a*e)/(d + e*x)))/(d + e*x)))/e^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1247 vs. 2(620) = 1240.

Time = 3.61 (sec) , antiderivative size = 1248, normalized size of antiderivative = 1.82

method	result	size
elliptic	Expression too large to display	1248
default	Expression too large to display	20481

```
[In] int((C*x^2+B*x+A)/(e*x+d)^(5/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE
)
```

```
[Out] ((e*x+d)*(c*x^2+b*x+a))^(1/2)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)*(-2/3/e^3/(
a*e^2-b*d*e+c*d^2)*(A*e^2-B*d*e+C*d^2)*(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x
+a*d)^(1/2)/(x+d/e)^2+2/3*(c*e*x^2+b*e*x+a*e)/e^2/(a*e^2-b*d*e+c*d^2)^2*(2*
A*b*e^3-4*A*c*d*e^2-3*B*a*e^3+B*b*d*e^2+B*c*d^2*e+6*C*a*d*e^2-4*C*b*d^2*e+2
*C*c*d^3)/((x+d/e)*(c*e*x^2+b*e*x+a*e))^(1/2)+2*(C/e^2-1/3*c/e^2*(A*e^2-B*d
*e+C*d^2)/(a*e^2-b*d*e+c*d^2)+1/3*(b*e-c*d)/e^2*(2*A*b*e^3-4*A*c*d*e^2-3*B*
a*e^3+B*b*d*e^2+B*c*d^2*e+6*C*a*d*e^2-4*C*b*d^2*e+2*C*c*d^3)/(a*e^2-b*d*e+c
*d^2)^2-1/3*b/e/(a*e^2-b*d*e+c*d^2)^2*(2*A*b*e^3-4*A*c*d*e^2-3*B*a*e^3+B*b*
d*e^2+B*c*d^2*e+6*C*a*d*e^2-4*C*b*d^2*e+2*C*c*d^3))*(d/e-1/2*(b+(-4*a*c+b^2
)^(1/2))/c)*((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-
b+(-4*a*c+b^2)^(1/2)))/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*
(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*e*x
^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)*EllipticF(((x+d/e)/(d/e-1/2*(b+(-
4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e-1/2/
c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))-2/3*c/e*(2*A*b*e^3-4*A*c*d*e^2-3*B*a*e^3
+B*b*d*e^2+B*c*d^2*e+6*C*a*d*e^2-4*C*b*d^2*e+2*C*c*d^3)/(a*e^2-b*d*e+c*d^2)
^2*(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/
2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d/e-1/2/c*(-b+(-4*a*c+b^
2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e+1/2*(b+(-4*a*c+b^
2)^(1/2))/c))^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)*((-d/e-
1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*EllipticE(((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^
(1/2))/c))^(1/2),((-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e-1/2/c*(-b+(-4*a
*c+b^2)^(1/2))))^(1/2))+1/2/c*(-b+(-4*a*c+b^2)^(1/2))*EllipticF(((x+d/e)/(d
/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c
)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.15 (sec) , antiderivative size = 1305, normalized size of antiderivative = 1.91

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{5/2}\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^(5/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fri
cas")
```

```
[Out] 2/9*((2*C*c^2*d^6 - (5*C*b*c - B*c^2)*d^5*e + (5*C*b^2 + 5*A*c^2 + (3*C*a -
 4*B*b)*c)*d^4*e^2 - (12*C*a*b - B*b^2 - (9*B*a - 5*A*b)*c)*d^3*e^3 + (9*C*
a^2 - 3*B*a*b + 2*A*b^2 - 3*A*a*c)*d^2*e^4 + (2*C*c^2*d^4*e^2 - (5*C*b*c -
B*c^2)*d^3*e^3 + (5*C*b^2 + 5*A*c^2 + (3*C*a - 4*B*b)*c)*d^2*e^4 - (12*C*a*
b - B*b^2 - (9*B*a - 5*A*b)*c)*d*e^5 + (9*C*a^2 - 3*B*a*b + 2*A*b^2 - 3*A*a
*c)*e^6)*x^2 + 2*(2*C*c^2*d^5*e - (5*C*b*c - B*c^2)*d^4*e^2 + (5*C*b^2 + 5*
A*c^2 + (3*C*a - 4*B*b)*c)*d^3*e^3 - (12*C*a*b - B*b^2 - (9*B*a - 5*A*b)*c)
*d^2*e^4 + (9*C*a^2 - 3*B*a*b + 2*A*b^2 - 3*A*a*c)*d*e^5)*x)*sqrt(c*e)*weie
rstrassPInverse(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/2
7*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c
)*e^3)/(c^3*e^3), 1/3*(3*c*e*x + c*d + b*e)/(c*e)) + 3*(2*C*c^2*d^5*e - (3*
B*a - 2*A*b)*c*d^2*e^4 - (4*C*b*c - B*c^2)*d^4*e^2 - (4*A*c^2 - (6*C*a + B*
b)*c)*d^3*e^3 + (2*C*c^2*d^3*e^3 - (3*B*a - 2*A*b)*c*e^6 - (4*C*b*c - B*c^2
)*d^2*e^4 - (4*A*c^2 - (6*C*a + B*b)*c)*d*e^5)*x^2 + 2*(2*C*c^2*d^4*e^2 - (
3*B*a - 2*A*b)*c*d*e^5 - (4*C*b*c - B*c^2)*d^3*e^3 - (4*A*c^2 - (6*C*a + B*
b)*c)*d^2*e^4)*x)*sqrt(c*e)*weierstrassZeta(4/3*(c^2*d^2 - b*c*d*e + (b^2 -
 3*a*c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c
^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(c^3*e^3), weierstrassPInverse(4/3*(c^2*
d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^
2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(c^3*e^3), 1/3*(3*
c*e*x + c*d + b*e)/(c*e))) + 3*(C*c^2*d^4*e^2 - A*a*c*e^6 - (2*B*a - 3*A*b)
*c*d*e^5 - (3*C*b*c - 2*B*c^2)*d^3*e^3 + 5*(C*a*c - A*c^2)*d^2*e^4 + (2*C*c
^2*d^3*e^3 - (3*B*a - 2*A*b)*c*e^6 - (4*C*b*c - B*c^2)*d^2*e^4 - (4*A*c^2 -
 (6*C*a + B*b)*c)*d*e^5)*x)*sqrt(c*x^2 + b*x + a)*sqrt(e*x + d))/(c^3*d^6*e
^3 - 2*b*c^2*d^5*e^4 - 2*a*b*c*d^3*e^6 + a^2*c*d^2*e^7 + (b^2*c + 2*a*c^2)*
d^4*e^5 + (c^3*d^4*e^5 - 2*b*c^2*d^3*e^6 - 2*a*b*c*d*e^8 + a^2*c*e^9 + (b^2
*c + 2*a*c^2)*d^2*e^7)*x^2 + 2*(c^3*d^5*e^4 - 2*b*c^2*d^4*e^5 - 2*a*b*c*d^2
*e^7 + a^2*c*d*e^8 + (b^2*c + 2*a*c^2)*d^3*e^6)*x)
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{5/2}\sqrt{a+bx+cx^2}}\,dx = \int \frac{A+Bx+Cx^2}{(d+ex)^{\frac{5}{2}}\sqrt{a+bx+cx^2}}\,dx$$

```
[In] integrate((C*x**2+B*x+A)/(e*x+d)**(5/2)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((A + B*x + C*x**2)/((d + e*x)**(5/2)*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{5/2}\sqrt{a+bx+cx^2}}\,dx = \int \frac{Cx^2+Bx+A}{\sqrt{cx^2+bx+a}(ex+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^(5/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate((C*x^2 + B*x + A)/(sqrt(c*x^2 + b*x + a)*(e*x + d)^(5/2)), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{5/2}\sqrt{a+bx+cx^2}}\,dx = \int \frac{Cx^2+Bx+A}{\sqrt{cx^2+bx+a}(ex+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^(5/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate((C*x^2 + B*x + A)/(sqrt(c*x^2 + b*x + a)*(e*x + d)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{5/2}\sqrt{a+bx+cx^2}}\,dx = \int \frac{C\,x^2+B\,x+A}{(d+e\,x)^{5/2}\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

```
[In] int((A + B*x + C*x^2)/((d + e*x)^(5/2)*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] int((A + B*x + C*x^2)/((d + e*x)^(5/2)*(a + b*x + c*x^2)^(1/2)), x)
```

3.271 $\int \frac{A+Bx+Cx^2}{(d+ex)^{7/2}\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 944

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{7/2}\sqrt{a+bx+cx^2}}\,dx = -\frac{2(Cd^2-e(Bd-Ae))\sqrt{a+bx+cx^2}}{5e\,(cd^2-bde+ae^2)\,(d+ex)^{5/2}}$$

$$+\frac{2(cd(2Cd^2+e(3Bd-8Ae))+e(5ae(2Cd-Be)-b(6Cd^2-Bde-4Ae^2)))\sqrt{a+bx+cx^2}}{15e\,(cd^2-bde+ae^2)^2\,(d+ex)^{3/2}}$$

$$+\frac{2(c^2d^2(2Cd^2+e(3Bd-23Ae))-e^2(15a^2Ce^2-10abe(Cd+Be)+b^2(3Cd^2+2Bde+8Ae^2))-ce(bd(7C}{15e\,(cd^2-bde+ae^2)^3\,\sqrt{d+ex}}$$

$$-\frac{\sqrt{2}\sqrt{b^2-4ac}(c^2d^2(2Cd^2+e(3Bd-23Ae))-e^2(15a^2Ce^2-10abe(Cd+Be)+b^2(3Cd^2+2Bde+8Ae^2))}{15e^2\,(cd^2-}$$

$$+\frac{2\sqrt{2}\sqrt{b^2-4ac}(cd(2Cd^2+e(3Bd-8Ae))+e(5ae(2Cd-Be)-b(6Cd^2-Bde-4Ae^2)))\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right.}}}{15e^2\,(cd^2-bde+ae^2)^2\,\sqrt{d+ex}\sqrt{a+b}}$$

```
[Out] -2/5*(C*d^2-e*(-A*e+B*d))*(c*x^2+b*x+a)^(1/2)/e/(a*e^2-b*d*e+c*d^2)/(e*x+d)
^(5/2)+2/15*(c*d*(2*C*d^2+e*(-8*A*e+3*B*d))+e*(5*a*e*(-B*e+2*C*d)-b*(-4*A*e
^2-B*d*e+6*C*d^2)))*(c*x^2+b*x+a)^(1/2)/e/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)^(3/
2)+2/15*(c^2*d^2*(2*C*d^2+e*(-23*A*e+3*B*d))-e^2*(15*a^2*C*e^2-10*a*b*e*(B*
e+C*d)+b^2*(8*A*e^2+2*B*d*e+3*C*d^2))-c*e*(b*d*(-23*A*e^2-7*B*d*e+7*C*d^2)-
a*e*(9*A*e^2-29*B*d*e+19*C*d^2)))*(c*x^2+b*x+a)^(1/2)/e/(a*e^2-b*d*e+c*d^2)
^3/(e*x+d)^(1/2)-1/15*(c^2*d^2*(2*C*d^2+e*(-23*A*e+3*B*d))-e^2*(15*a^2*C*e^
2-10*a*b*e*(B*e+C*d)+b^2*(8*A*e^2+2*B*d*e+3*C*d^2))-c*e*(b*d*(-23*A*e^2-7*B
*d*e+7*C*d^2)-a*e*(9*A*e^2-29*B*d*e+19*C*d^2)))*EllipticE(1/2*((b+2*c*x+(-4
```

```
*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)
/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(e*x+d
)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/e^2/(a*e^2-b*d*e+c*d^2)^3/(c*
x^2+b*x+a)^(1/2)/(c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)+2/15*(c
*d*(2*C*d^2+e*(-8*A*e+3*B*d))+e*(5*a*e*(-B*e+2*C*d)-b*(-4*A*e^2-B*d*e+6*C*d
^2)))*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)
*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*
2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(e*x+d)
/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/e^2/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)^
(1/2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 1.32 (sec) , antiderivative size = 942, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1664, 848, 857, 732, 435, 430}

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{7/2}\sqrt{a+bx+cx^2}}\,dx = -\frac{2\sqrt{cx^2+bx+a}(Cd^2-e(Bd-Ae))}{5e\left(cd^2-bed+ae^2\right)(d+ex)^{5/2}}$$

$$-\frac{\sqrt{2}\sqrt{b^2-4ac}\left(\left(2Cd^4+e(3Bd-23Ae)d^2\right)c^2-e(bd(7Cd^2-7Bed-23Ae^2)-ae(19Cd^2-29Bed+9Ae}{15e^2\left(cd^2 -\right.}$$

$$+\frac{2\sqrt{2}\sqrt{b^2-4ac}(2cCd^3+ce(3Bd-8Ae)d+5ae^2(2Cd-Be)-be(6Cd^2-e(Bd+4Ae)))\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4a\right.}}}}{15e^2\left(cd^2-bed+ae^2\right)^2\sqrt{d+ex}\sqrt{cx^2 -}}$$

$$+\frac{2\left(\left(2Cd^4+e(3Bd-23Ae)d^2\right)c^2-e(bd(7Cd^2-7Bed-23Ae^2)-ae(19Cd^2-29Bed+9Ae^2))c-e^2((3}{15e\left(cd^2-bed+ae^2\right)^3\sqrt{d+ex}}$$

$$+\frac{2(2cCd^3+ce(3Bd-8Ae)d+5ae^2(2Cd-Be)-be(6Cd^2-e(Bd+4Ae)))\sqrt{cx^2+bx+a}}{15e\left(cd^2-bed+ae^2\right)^2(d+ex)^{3/2}}$$

```
[In] Int[(A + B*x + C*x^2)/((d + e*x)^(7/2)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (-2*(C*d^2 - e*(B*d - A*e))*Sqrt[a + b*x + c*x^2])/(5*e*(c*d^2 - b*d*e + a*
e^2)*(d + e*x)^(5/2)) + (2*(2*c*C*d^3 + c*d*e*(3*B*d - 8*A*e) + 5*a*e^2*(2*
C*d - B*e) - b*e*(6*C*d^2 - e*(B*d + 4*A*e)))*Sqrt[a + b*x + c*x^2])/(15*e*
(c*d^2 - b*d*e + a*e^2)^2*(d + e*x)^(3/2)) + (2*(c^2*(2*C*d^4 + d^2*e*(3*B*
d - 23*A*e)) - e^2*(15*a^2*C*e^2 - 10*a*b*e*(C*d + B*e) + b^2*(3*C*d^2 + 2*
B*d*e + 8*A*e^2)) - c*e*(b*d*(7*C*d^2 - 7*B*d*e - 23*A*e^2) - a*e*(19*C*d^2
 - 29*B*d*e + 9*A*e^2)))*Sqrt[a + b*x + c*x^2])/(15*e*(c*d^2 - b*d*e + a*e^
2)^3*Sqrt[d + e*x]) - (Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c^2*(2*C*d^4 + d^2*e*(3*B
```

```
*d - 23*A*e)) - e^2*(15*a^2*C*e^2 - 10*a*b*e*(C*d + B*e) + b^2*(3*C*d^2 + 2
*B*d*e + 8*A*e^2)) - c*e*(b*d*(7*C*d^2 - 7*B*d*e - 23*A*e^2) - a*e*(19*C*d^
2 - 29*B*d*e + 9*A*e^2)))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 -
 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 -
4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])
*e)])/(15*e^2*(c*d^2 - b*d*e + a*e^2)^3*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sq
rt[b^2 - 4*a*c])*e)]*Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*
(2*c*C*d^3 + c*d*e*(3*B*d - 8*A*e) + 5*a*e^2*(2*C*d - B*e) - b*e*(6*C*d^2 -
 e*(B*d + 4*A*e)))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*
Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqr
t[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*
e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(15*e^2*(c*d^2 - b*d*e + a*e^2)^2*
Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 848

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
```

```
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2(Cd^2-e(Bd-Ae))\sqrt{a+bx+cx^2}}{5e\left(cd^2-bde+ae^2\right)(d+ex)^{5/2}} \\
&\quad -\frac{2\int\frac{-\frac{bCd^2-be(Bd+4Ae)+5e(Acd-aCd+aBe)}{2e}-\frac{1}{2}\left(3Bcd-5bCd+\frac{2cCd^2}{e}-3Ace+5aCe\right)x}{(d+ex)^{5/2}\sqrt{a+bx+cx^2}}\,dx}{5\left(cd^2-bde+ae^2\right)} \\
&= -\frac{2(Cd^2-e(Bd-Ae))\sqrt{a+bx+cx^2}}{5e\left(cd^2-bde+ae^2\right)(d+ex)^{5/2}} \\
&\quad +\frac{2(2cCd^3+cde(3Bd-8Ae)+5ae^2(2Cd-Be)-be(6Cd^2-e(Bd+4Ae)))\sqrt{a+bx+cx^2}}{15e\left(cd^2-bde+ae^2\right)^2(d+ex)^{3/2}} \\
&\quad +\frac{4\int\frac{\frac{b^2e\left(3Cd^2+2e(Bd+4Ae)\right)+b\left(cCd^3-cde(6Bd+19Ae)-10ae^2(Cd+Be)\right)+3e\left(Ac\left(5cd^2-3ae^2\right)+a\left(5aCe^2-cd(3Cd-8Be)\right)\right)}{4e}+\frac{c\big(2cCd^3+cde\big(}{}}{(d+ex)^{3/2}\sqrt{a+bx+cx^2}}}{15\left(cd^2-bde+ae^2\right)^2}
\end{aligned}$$

$$= -\frac{2(Cd^2 - e(Bd - Ae))\sqrt{a + bx + cx^2}}{5e\left(cd^2 - bde + ae^2\right)(d + ex)^{5/2}}$$
$$+ \frac{2(2cCd^3 + cde(3Bd - 8Ae) + 5ae^2(2Cd - Be) - be(6Cd^2 - e(Bd + 4Ae)))\sqrt{a + bx + cx^2}}{15e\left(cd^2 - bde + ae^2\right)^2 (d + ex)^{3/2}}$$
$$+ \frac{2(c^2(2Cd^4 + d^2e(3Bd - 23Ae)) - e^2(15a^2Ce^2 - 10abe(Cd + Be) + b^2(3Cd^2 + 2Bde + 8Ae^2)) -}{15e\left(cd^2 - bde + ae^2\right)^3 \sqrt{}}$$
$$- \frac{8\int -\frac{c\left(b^2de\left(9Cd^2+e(Bd+4Ae)\right)-b\left(cCd^4+26aCd^2e^2+4ae^3(Bd-Ae)+cd^2e(9Bd+11Ae)\right)+e\left(Acd\left(15cd^2-17ae^2\right)-a\left(cd^2(7Cd-27Be)-5ae^2(5C\right.\right.\right.}{8e}}{15\,(}$$

$$= -\frac{2(Cd^2 - e(Bd - Ae))\sqrt{a + bx + cx^2}}{5e\left(cd^2 - bde + ae^2\right)(d + ex)^{5/2}}$$
$$+ \frac{2(2cCd^3 + cde(3Bd - 8Ae) + 5ae^2(2Cd - Be) - be(6Cd^2 - e(Bd + 4Ae)))\sqrt{a + bx + cx^2}}{15e\left(cd^2 - bde + ae^2\right)^2 (d + ex)^{3/2}}$$
$$+ \frac{2(c^2(2Cd^4 + d^2e(3Bd - 23Ae)) - e^2(15a^2Ce^2 - 10abe(Cd + Be) + b^2(3Cd^2 + 2Bde + 8Ae^2)) -}{15e\left(cd^2 - bde + ae^2\right)^3 \sqrt{}}$$
$$+ \frac{(c(2cCd^3 + cde(3Bd - 8Ae) + 5ae^2(2Cd - Be) - be(6Cd^2 - e(Bd + 4Ae))))\int \frac{1}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\, d}{15e^2\left(cd^2 - bde + ae^2\right)^2}$$
$$- \frac{(c(c^2(2Cd^4 + d^2e(3Bd - 23Ae)) - e^2(15a^2Ce^2 - 10abe(Cd + Be) + b^2(3Cd^2 + 2Bde + 8Ae^2))}{15e^2\left(cd^2 - bde + a\right.}$$

$$= -\frac{2(Cd^2 - e(Bd - Ae))\sqrt{a + bx + cx^2}}{5e\left(cd^2 - bde + ae^2\right)(d + ex)^{5/2}}$$
$$+ \frac{2(2cCd^3 + cde(3Bd - 8Ae) + 5ae^2(2Cd - Be) - be(6Cd^2 - e(Bd + 4Ae)))\sqrt{a + bx + cx^2}}{15e\left(cd^2 - bde + ae^2\right)^2 (d + ex)^{3/2}}$$
$$+ \frac{2(c^2(2Cd^4 + d^2e(3Bd - 23Ae)) - e^2(15a^2Ce^2 - 10abe(Cd + Be) + b^2(3Cd^2 + 2Bde + 8Ae^2)) -}{15e\left(cd^2 - bde + ae^2\right)^3 \sqrt{}}$$
$$- \frac{\left(\sqrt{2}\sqrt{b^2 - 4ac}(c^2(2Cd^4 + d^2e(3Bd - 23Ae)) - e^2(15a^2Ce^2 - 10abe(Cd + Be) + b^2(3Cd^2 + 2B\right.}{}$$
$$+ \frac{\left(2\sqrt{2}\sqrt{b^2 - 4ac}(2cCd^3 + cde(3Bd - 8Ae) + 5ae^2(2Cd - Be) - be(6Cd^2 - e(Bd + 4Ae)))\sqrt{\frac{}{2c}}\right.}{15e^2\left(cd^2 - bde + ae^2\right)^2 \sqrt{d +}}$$

$$= -\frac{2(Cd^2 - e(Bd - Ae))\sqrt{a + bx + cx^2}}{5e\left(cd^2 - bde + ae^2\right)(d + ex)^{5/2}}$$

$$+\frac{2(2cCd^3 + cde(3Bd - 8Ae) + 5ae^2(2Cd - Be) - be(6Cd^2 - e(Bd + 4Ae)))\sqrt{a + bx + cx^2}}{15e\left(cd^2 - bde + ae^2\right)^2(d + ex)^{3/2}}$$

$$+\frac{2(c^2(2Cd^4 + d^2e(3Bd - 23Ae)) - e^2(15a^2Ce^2 - 10abe(Cd + Be) + b^2(3Cd^2 + 2Bde + 8Ae^2))}{15e\left(cd^2 - bde + ae^2\right)^3}$$

$$-\frac{\sqrt{2}\sqrt{b^2 - 4ac}(c^2(2Cd^4 + d^2e(3Bd - 23Ae)) - e^2(15a^2Ce^2 - 10abe(Cd + Be) + b^2(3Cd^2 + 2B}{1}$$

$$+\frac{2\sqrt{2}\sqrt{b^2 - 4ac}(2cCd^3 + cde(3Bd - 8Ae) + 5ae^2(2Cd - Be) - be(6Cd^2 - e(Bd + 4Ae)))\sqrt{\frac{}{2cd}}}{15e^2\left(cd^2 - bde + ae^2\right)^2\sqrt{d + ex}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 34.76 (sec) , antiderivative size = 1746, normalized size of antiderivative = 1.85

$$\int \frac{A + Bx + Cx^2}{(d + ex)^{7/2}\sqrt{a + bx + cx^2}}\,dx = \frac{\sqrt{d + ex}(a + bx + cx^2)\left(-\frac{2(Cd^2 - Bde + Ae^2)}{5e(cd^2 - bde + ae^2)(d + ex)^3} - \frac{2(-2cCd^3 - 3Bcd^2e + 6bCd^2e - b}{15e(cd^2 -}\right.}{}$$

$$+\frac{2(d + ex)^{3/2}\sqrt{a + bx + cx^2}\left(-\left((c^2(2Cd^4 + d^2e(3Bd - 23Ae)) - e^2(15a^2Ce^2 - 10abe(Cd + Be) + b^2(3C\right.\right.}{}$$

```
[In] Integrate[(A + B*x + C*x^2)/((d + e*x)^(7/2)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (Sqrt[d + e*x]*(a + b*x + c*x^2)*((-2*(C*d^2 - B*d*e + A*e^2))/(5*e*(c*d^2
 - b*d*e + a*e^2)*(d + e*x)^3) - (2*(-2*c*C*d^3 - 3*B*c*d^2*e + 6*b*C*d^2*e
 - b*B*d*e^2 + 8*A*c*d*e^2 - 10*a*C*d*e^2 - 4*A*b*e^3 + 5*a*B*e^3))/(15*e*(c
*d^2 - b*d*e + a*e^2)^2*(d + e*x)^2) - (2*(-2*c^2*C*d^4 - 3*B*c^2*d^3*e + 7
*b*c*C*d^3*e - 7*b*B*c*d^2*e^2 + 23*A*c^2*d^2*e^2 + 3*b^2*C*d^2*e^2 - 19*a*
c*C*d^2*e^2 + 2*b^2*B*d*e^3 - 23*A*b*c*d*e^3 + 29*a*B*c*d*e^3 - 10*a*b*C*d*
```

```
e^3 + 8*A*b^2*e^4 - 10*a*b*B*e^4 - 9*a*A*c*e^4 + 15*a^2*C*e^4))/(15*e*(c*d^
2 - b*d*e + a*e^2)^3*(d + e*x))))/Sqrt[a + x*(b + c*x)] + (2*(d + e*x)^(3/2
)*Sqrt[a + b*x + c*x^2]*(-((c^2*(2*C*d^4 + d^2*e*(3*B*d - 23*A*e)) - e^2*(1
5*a^2*C*e^2 - 10*a*b*e*(C*d + B*e) + b^2*(3*C*d^2 + 2*B*d*e + 8*A*e^2)) + c
*e*(a*e*(19*C*d^2 - 29*B*d*e + 9*A*e^2) + b*d*(-7*C*d^2 + 7*B*d*e + 23*A*e^
2)))*(c*(-1 + d/(d + e*x))^2 + (e*(b - (b*d)/(d + e*x) + (a*e)/(d + e*x)))/
(d + e*x))) - ((I/2)*Sqrt[1 - (2*(c*d^2 + e*(-(b*d) + a*e)))/((2*c*d - b*e
+ Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*Sqrt[1 + (2*(c*d^2 + e*(-(b*d) + a*e
)))/((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*((2*c*d - b*e + S
qrt[(b^2 - 4*a*c)*e^2])*(c^2*(-2*C*d^4 + d^2*e*(-3*B*d + 23*A*e)) + e^2*(15
*a^2*C*e^2 - 10*a*b*e*(C*d + B*e) + b^2*(3*C*d^2 + 2*B*d*e + 8*A*e^2)) - c*
e*(a*e*(19*C*d^2 - 29*B*d*e + 9*A*e^2) + b*d*(-7*C*d^2 + 7*B*d*e + 23*A*e^2
)))*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*c*d + b*e
 + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c*d + b*e + Sqrt[(b^2 -
 4*a*c)*e^2])/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))] + (-30*A*c^3*d^3*e^
2 + 14*a*c^2*C*d^3*e^2 - 54*a*B*c^2*d^2*e^3 + 34*a*A*c^2*d*e^4 - 50*a^2*c*C
*d*e^4 + 10*a^2*B*c*e^5 + 2*c^2*C*d^4*Sqrt[(b^2 - 4*a*c)*e^2] + 3*B*c^2*d^3
*e*Sqrt[(b^2 - 4*a*c)*e^2] - 23*A*c^2*d^2*e^2*Sqrt[(b^2 - 4*a*c)*e^2] + 19*
a*c*C*d^2*e^2*Sqrt[(b^2 - 4*a*c)*e^2] - 29*a*B*c*d*e^3*Sqrt[(b^2 - 4*a*c)*e
^2] + 9*a*A*c*e^4*Sqrt[(b^2 - 4*a*c)*e^2] - 15*a^2*C*e^4*Sqrt[(b^2 - 4*a*c)
*e^2] + b^3*e^3*(3*C*d^2 + 2*e*(B*d + 4*A*e)) - b^2*e^2*(11*c*C*d^3 + c*d*e
*(9*B*d + 31*A*e) + 10*a*e^2*(C*d + B*e) + Sqrt[(b^2 - 4*a*c)*e^2]*(3*C*d^2
 + 2*e*(B*d + 4*A*e))) + b*(A*c*e^3*(45*c*d^2 - 17*a*e^2 + 23*d*Sqrt[(b^2 -
 4*a*c)*e^2]) + C*e*(15*a^2*e^4 - 7*c*d^3*Sqrt[(b^2 - 4*a*c)*e^2] + a*d*e^2
*(33*c*d + 10*Sqrt[(b^2 - 4*a*c)*e^2])) + B*e^2*(15*c^2*d^3 + 10*a*e^2*Sqrt
[(b^2 - 4*a*c)*e^2] + c*d*(37*a*e^2 + 7*d*Sqrt[(b^2 - 4*a*c)*e^2]))))*Ellip
ticF[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*c*d + b*e + Sqrt[(
b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e
^2])/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))]))/(Sqrt[2]*Sqrt[(c*d^2 + e*(
-(b*d) + a*e))/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*Sqrt[d + e*x]))/(
15*e^3*(c*d^2 - b*d*e + a*e^2)^3*Sqrt[a + x*(b + c*x)]*Sqrt[((d + e*x)^2*(c
*(-1 + d/(d + e*x))^2 + (e*(b - (b*d)/(d + e*x) + (a*e)/(d + e*x)))/(d + e*
x)))/e^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1756 vs. $2(874) = 1748$.

Time = 4.39 (sec) , antiderivative size = 1757, normalized size of antiderivative = 1.86

method	result	size
elliptic	Expression too large to display	1757
default	Expression too large to display	46695

```
[In] int((C*x^2+B*x+A)/(e*x+d)^(7/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE
```

```
)

[Out] ((e*x+d)*(c*x^2+b*x+a))^(1/2)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)*(-2/5/e^4/(
a*e^2-b*d*e+c*d^2)*(A*e^2-B*d*e+C*d^2)*(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x
+a*d)^(1/2)/(x+d/e)^3+2/15*(4*A*b*e^3-8*A*c*d*e^2-5*B*a*e^3+B*b*d*e^2+3*B*c
*d^2*e+10*C*a*d*e^2-6*C*b*d^2*e+2*C*c*d^3)/e^3/(a*e^2-b*d*e+c*d^2)^2*(c*e*x
^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)/(x+d/e)^2+2/15*(c*e*x^2+b*e*x+a*e
)/e^2/(a*e^2-b*d*e+c*d^2)^3*(9*A*a*c*e^4-8*A*b^2*e^4+23*A*b*c*d*e^3-23*A*c^
2*d^2*e^2+10*B*a*b*e^4-29*B*a*c*d*e^3-2*B*b^2*d*e^3+7*B*b*c*d^2*e^2+3*B*c^2
*d^3*e-15*C*a^2*e^4+10*C*a*b*d*e^3+19*C*a*c*d^2*e^2-3*C*b^2*d^2*e^2-7*C*b*c
*d^3*e+2*C*c^2*d^4)/((x+d/e)*(c*e*x^2+b*e*x+a*e))^(1/2)+2*(1/15*c*(4*A*b*e^
3-8*A*c*d*e^2-5*B*a*e^3+B*b*d*e^2+3*B*c*d^2*e+10*C*a*d*e^2-6*C*b*d^2*e+2*C*
c*d^3)/e^2/(a*e^2-b*d*e+c*d^2)^2+1/15*(b*e-c*d)/e^2*(9*A*a*c*e^4-8*A*b^2*e^
4+23*A*b*c*d*e^3-23*A*c^2*d^2*e^2+10*B*a*b*e^4-29*B*a*c*d*e^3-2*B*b^2*d*e^3
+7*B*b*c*d^2*e^2+3*B*c^2*d^3*e-15*C*a^2*e^4+10*C*a*b*d*e^3+19*C*a*c*d^2*e^2
-3*C*b^2*d^2*e^2-7*C*b*c*d^3*e+2*C*c^2*d^4)/(a*e^2-b*d*e+c*d^2)^3-1/15*b/e/
(a*e^2-b*d*e+c*d^2)^3*(9*A*a*c*e^4-8*A*b^2*e^4+23*A*b*c*d*e^3-23*A*c^2*d^2*
e^2+10*B*a*b*e^4-29*B*a*c*d*e^3-2*B*b^2*d*e^3+7*B*b*c*d^2*e^2+3*B*c^2*d^3*e
-15*C*a^2*e^4+10*C*a*b*d*e^3+19*C*a*c*d^2*e^2-3*C*b^2*d^2*e^2-7*C*b*c*d^3*e
+2*C*c^2*d^4))*(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(b+(-4*
a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d/e-1/2/c*(-
b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e+1/2*(
b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(
1/2)*EllipticF(((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/
2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))-2/
15*c/e*(9*A*a*c*e^4-8*A*b^2*e^4+23*A*b*c*d*e^3-23*A*c^2*d^2*e^2+10*B*a*b*e^
4-29*B*a*c*d*e^3-2*B*b^2*d*e^3+7*B*b*c*d^2*e^2+3*B*c^2*d^3*e-15*C*a^2*e^4+1
0*C*a*b*d*e^3+19*C*a*c*d^2*e^2-3*C*b^2*d^2*e^2-7*C*b*c*d^3*e+2*C*c^2*d^4)/(
a*e^2-b*d*e+c*d^2)^3*(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(
b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d/e-1/
2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e
+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+
a*d)^(1/2)*((-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*EllipticE(((x+d/e)/(d/e-1/
2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d
/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+1/2/c*(-b+(-4*a*c+b^2)^(1/2))*Ell
ipticF(((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2*(b+(-4
*a*c+b^2)^(1/2))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.40 (sec) , antiderivative size = 2656, normalized size of antiderivative = 2.81

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{7/2}\sqrt{a+bx+cx^2}}\,dx=\text{Too large to display}$$

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^(7/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fri
cas")
```

```
[Out] 2/45*((2*C*c^3*d^8 - (8*C*b*c^2 - 3*B*c^3)*d^7*e + (17*C*b^2*c + 22*A*c^3 -
 (2*C*a + 17*B*b)*c^2)*d^6*e^2 - (3*C*b^3 - (52*B*a - 33*A*b)*c^2 + (49*C*a
*b - 8*B*b^2)*c)*d^5*e^3 + (10*C*a*b^2 - 2*B*b^3 - 42*A*a*c^2 + (60*C*a^2 -
 31*B*a*b + 27*A*b^2)*c)*d^4*e^4 - (15*C*a^2*b - 10*B*a*b^2 + 8*A*b^3 + 3*(
5*B*a^2 - 7*A*a*b)*c)*d^3*e^5 + (2*C*c^3*d^5*e^3 - (8*C*b*c^2 - 3*B*c^3)*d^
4*e^4 + (17*C*b^2*c + 22*A*c^3 - (2*C*a + 17*B*b)*c^2)*d^3*e^5 - (3*C*b^3 -
 (52*B*a - 33*A*b)*c^2 + (49*C*a*b - 8*B*b^2)*c)*d^2*e^6 + (10*C*a*b^2 - 2*
B*b^3 - 42*A*a*c^2 + (60*C*a^2 - 31*B*a*b + 27*A*b^2)*c)*d*e^7 - (15*C*a^2*
b - 10*B*a*b^2 + 8*A*b^3 + 3*(5*B*a^2 - 7*A*a*b)*c)*e^8)*x^3 + 3*(2*C*c^3*d
^6*e^2 - (8*C*b*c^2 - 3*B*c^3)*d^5*e^3 + (17*C*b^2*c + 22*A*c^3 - (2*C*a +
17*B*b)*c^2)*d^4*e^4 - (3*C*b^3 - (52*B*a - 33*A*b)*c^2 + (49*C*a*b - 8*B*b
^2)*c)*d^3*e^5 + (10*C*a*b^2 - 2*B*b^3 - 42*A*a*c^2 + (60*C*a^2 - 31*B*a*b
+ 27*A*b^2)*c)*d^2*e^6 - (15*C*a^2*b - 10*B*a*b^2 + 8*A*b^3 + 3*(5*B*a^2 -
7*A*a*b)*c)*d*e^7)*x^2 + 3*(2*C*c^3*d^7*e - (8*C*b*c^2 - 3*B*c^3)*d^6*e^2 +
 (17*C*b^2*c + 22*A*c^3 - (2*C*a + 17*B*b)*c^2)*d^5*e^3 - (3*C*b^3 - (52*B*
a - 33*A*b)*c^2 + (49*C*a*b - 8*B*b^2)*c)*d^4*e^4 + (10*C*a*b^2 - 2*B*b^3 -
 42*A*a*c^2 + (60*C*a^2 - 31*B*a*b + 27*A*b^2)*c)*d^3*e^5 - (15*C*a^2*b - 1
0*B*a*b^2 + 8*A*b^3 + 3*(5*B*a^2 - 7*A*a*b)*c)*d^2*e^6)*x)*sqrt(c*e)*weiers
trassPInverse(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/27*
(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*
e^3)/(c^3*e^3), 1/3*(3*c*e*x + c*d + b*e)/(c*e)) + 3*(2*C*c^3*d^7*e - (7*C*
b*c^2 - 3*B*c^3)*d^6*e^2 - (3*C*b^2*c + 23*A*c^3 - (19*C*a + 7*B*b)*c^2)*d^
5*e^3 - ((29*B*a - 23*A*b)*c^2 - 2*(5*C*a*b - B*b^2)*c)*d^4*e^4 + (9*A*a*c^
2 - (15*C*a^2 - 10*B*a*b + 8*A*b^2)*c)*d^3*e^5 + (2*C*c^3*d^4*e^4 - (7*C*b*
c^2 - 3*B*c^3)*d^3*e^5 - (3*C*b^2*c + 23*A*c^3 - (19*C*a + 7*B*b)*c^2)*d^2*
e^6 - ((29*B*a - 23*A*b)*c^2 - 2*(5*C*a*b - B*b^2)*c)*d*e^7 + (9*A*a*c^2 -
(15*C*a^2 - 10*B*a*b + 8*A*b^2)*c)*e^8)*x^3 + 3*(2*C*c^3*d^5*e^3 - (7*C*b*c
^2 - 3*B*c^3)*d^4*e^4 - (3*C*b^2*c + 23*A*c^3 - (19*C*a + 7*B*b)*c^2)*d^3*e
^5 - ((29*B*a - 23*A*b)*c^2 - 2*(5*C*a*b - B*b^2)*c)*d^2*e^6 + (9*A*a*c^2 -
 (15*C*a^2 - 10*B*a*b + 8*A*b^2)*c)*d*e^7)*x^2 + 3*(2*C*c^3*d^6*e^2 - (7*C*
b*c^2 - 3*B*c^3)*d^5*e^3 - (3*C*b^2*c + 23*A*c^3 - (19*C*a + 7*B*b)*c^2)*d^
4*e^4 - ((29*B*a - 23*A*b)*c^2 - 2*(5*C*a*b - B*b^2)*c)*d^3*e^5 + (9*A*a*c^
2 - (15*C*a^2 - 10*B*a*b + 8*A*b^2)*c)*d^2*e^6)*x)*sqrt(c*e)*weierstrassZet
a(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 -
```

```
 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(c^3*e^
3), weierstrassPInverse(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^
2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 -
 9*a*b*c)*e^3)/(c^3*e^3), 1/3*(3*c*e*x + c*d + b*e)/(c*e))) + 3*(C*c^3*d^6*
e^2 - 3*A*a^2*c*e^8 - (25*B*a - 41*A*b)*c^2*d^3*e^5 - 2*(B*a^2 - 5*A*a*b)*c
*d*e^7 - 9*(C*b*c^2 - B*c^3)*d^5*e^3 - (34*A*c^3 - (25*C*a - B*b)*c^2)*d^4*
e^4 - (5*A*a*c^2 + (8*C*a^2 - 10*B*a*b + 15*A*b^2)*c)*d^2*e^6 + (2*C*c^3*d^
4*e^4 - (7*C*b*c^2 - 3*B*c^3)*d^3*e^5 - (3*C*b^2*c + 23*A*c^3 - (19*C*a + 7
*B*b)*c^2)*d^2*e^6 - ((29*B*a - 23*A*b)*c^2 - 2*(5*C*a*b - B*b^2)*c)*d*e^7
+ (9*A*a*c^2 - (15*C*a^2 - 10*B*a*b + 8*A*b^2)*c)*e^8)*x^2 + (6*C*c^3*d^5*e
^3 - (5*B*a^2 - 4*A*a*b)*c*e^8 - (22*C*b*c^2 - 9*B*c^3)*d^4*e^4 - 2*(27*A*c
^3 - (25*C*a + 6*B*b)*c^2)*d^3*e^5 - (2*(30*B*a - 29*A*b)*c^2 - (4*C*a*b -
5*B*b^2)*c)*d^2*e^6 + 2*(5*A*a*c^2 - (10*C*a^2 - 13*B*a*b + 10*A*b^2)*c)*d*
e^7)*x)*sqrt(c*x^2 + b*x + a)*sqrt(e*x + d))/(c^4*d^9*e^3 - 3*b*c^3*d^8*e^4
 - 3*a^2*b*c*d^4*e^8 + a^3*c*d^3*e^9 + 3*(b^2*c^2 + a*c^3)*d^7*e^5 - (b^3*c
 + 6*a*b*c^2)*d^6*e^6 + 3*(a*b^2*c + a^2*c^2)*d^5*e^7 + (c^4*d^6*e^6 - 3*b*
c^3*d^5*e^7 - 3*a^2*b*c*d*e^11 + a^3*c*e^12 + 3*(b^2*c^2 + a*c^3)*d^4*e^8 -
 (b^3*c + 6*a*b*c^2)*d^3*e^9 + 3*(a*b^2*c + a^2*c^2)*d^2*e^10)*x^3 + 3*(c^4
*d^7*e^5 - 3*b*c^3*d^6*e^6 - 3*a^2*b*c*d^2*e^10 + a^3*c*d*e^11 + 3*(b^2*c^2
 + a*c^3)*d^5*e^7 - (b^3*c + 6*a*b*c^2)*d^4*e^8 + 3*(a*b^2*c + a^2*c^2)*d^3
*e^9)*x^2 + 3*(c^4*d^8*e^4 - 3*b*c^3*d^7*e^5 - 3*a^2*b*c*d^3*e^9 + a^3*c*d^
2*e^10 + 3*(b^2*c^2 + a*c^3)*d^6*e^6 - (b^3*c + 6*a*b*c^2)*d^5*e^7 + 3*(a*b
^2*c + a^2*c^2)*d^4*e^8)*x)
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{7/2}\sqrt{a+bx+cx^2}}\,dx = \int \frac{A+Bx+Cx^2}{(d+ex)^{\frac{7}{2}}\sqrt{a+bx+cx^2}}\,dx$$

```
[In] integrate((C*x**2+B*x+A)/(e*x+d)**(7/2)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((A + B*x + C*x**2)/((d + e*x)**(7/2)*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{7/2}\sqrt{a+bx+cx^2}}\,dx = \int \frac{Cx^2+Bx+A}{\sqrt{cx^2+bx+a}(ex+d)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^(7/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate((C*x^2 + B*x + A)/(sqrt(c*x^2 + b*x + a)*(e*x + d)^(7/2)), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{7/2}\sqrt{a+bx+cx^2}}\,dx = \int \frac{Cx^2+Bx+A}{\sqrt{cx^2+bx+a}(ex+d)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^(7/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate((C*x^2 + B*x + A)/(sqrt(c*x^2 + b*x + a)*(e*x + d)^(7/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{7/2}\sqrt{a+bx+cx^2}}\,dx = \int \frac{C\,x^2+B\,x+A}{(d+e\,x)^{7/2}\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

```
[In] int((A + B*x + C*x^2)/((d + e*x)^(7/2)*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] int((A + B*x + C*x^2)/((d + e*x)^(7/2)*(a + b*x + c*x^2)^(1/2)), x)
```

3.272 $\int (g+hx)^m \left(a+bx+cx^2\right)^p \left(d+ex+fx^2\right) dx$

Optimal result

Integrand size = 30, antiderivative size = 510

$$\int (g+hx)^m \left(a+bx+cx^2\right)^p \left(d+ex+fx^2\right)\, dx = \frac{f(g+hx)^{1+m}\left(a+bx+cx^2\right)^{1+p}}{ch(3+m+2p)}$$

$$+\frac{(fh(bg-ah)(1+m)+c(2fg^2(1+p)-h(eg-dh)(3+m+2p)))\,(g+hx)^{1+m}\left(a+bx+cx^2\right)^p \Bigg(1-}{ch^3(1}$$

$$-\frac{(bfh(2+m+p)+c(2fg(1+p)-eh(3+m+2p)))(g+hx)^{2+m}\left(a+bx+cx^2\right)^p \Bigg(1-\frac{2c(g+hx)}{2cg-\left(b-\sqrt{b^2-4ac}\right)}}{ch^3(2+m)(}$$

```
[Out] f*(h*x+g)^(1+m)*(c*x^2+b*x+a)^(p+1)/c/h/(3+m+2*p)+(f*h*(-a*h+b*g)*(1+m)+c*(
2*f*g^2*(p+1)-h*(-d*h+e*g)*(3+m+2*p)))*(h*x+g)^(1+m)*(c*x^2+b*x+a)^p*Appell
F1(1+m,-p,-p,2+m,2*c*(h*x+g)/(2*c*g-h*(b-(-4*a*c+b^2)^(1/2))),2*c*(h*x+g)/(
2*c*g-h*(b+(-4*a*c+b^2)^(1/2))))/c/h^3/(1+m)/(3+m+2*p)/((1-2*c*(h*x+g)/(2*c
*g-h*(b-(-4*a*c+b^2)^(1/2))))^p)/((1-2*c*(h*x+g)/(2*c*g-h*(b+(-4*a*c+b^2)^(
1/2))))^p)-(b*f*h*(2+m+p)+c*(2*f*g*(p+1)-e*h*(3+m+2*p)))*(h*x+g)^(2+m)*(c*x
^2+b*x+a)^p*AppellF1(2+m,-p,-p,3+m,2*c*(h*x+g)/(2*c*g-h*(b-(-4*a*c+b^2)^(1/
2))),2*c*(h*x+g)/(2*c*g-h*(b+(-4*a*c+b^2)^(1/2))))/c/h^3/(2+m)/(3+m+2*p)/((
1-2*c*(h*x+g)/(2*c*g-h*(b-(-4*a*c+b^2)^(1/2))))^p)/((1-2*c*(h*x+g)/(2*c*g-h
*(b+(-4*a*c+b^2)^(1/2))))^p)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 508, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1667, 857, 773, 138}

$$\int (g+hx)^m \left(a+bx+cx^2\right)^p \left(d+ex+fx^2\right)\,dx$$

$$= \frac{(g+hx)^{m+1}\left(a+bx+cx^2\right)^p \left(1-\frac{2c(g+hx)}{2cg-h\left(b-\sqrt{b^2-4ac}\right)}\right)^{-p} \left(1-\frac{2c(g+hx)}{2cg-h\left(\sqrt{b^2-4ac}+b\right)}\right)^{-p} \text{AppellF1}\left(m+1,-p,-\right.}{ch^3(m+}$$

$$- \frac{(g+hx)^{m+2}\left(a+bx+cx^2\right)^p \left(1-\frac{2c(g+hx)}{2cg-h\left(b-\sqrt{b^2-4ac}\right)}\right)^{-p} \left(1-\frac{2c(g+hx)}{2cg-h\left(\sqrt{b^2-4ac}+b\right)}\right)^{-p} \left(bfh(m+p+2)-ce\right.}{ch^3(m+2)(m}$$

$$+ \frac{f(g+hx)^{m+1}\left(a+bx+cx^2\right)^{p+1}}{ch(m+2p+3)}$$

```
[In] Int[(g + h*x)^m*(a + b*x + c*x^2)^p*(d + e*x + f*x^2),x]
```

```
[Out] (f*(g + h*x)^(1 + m)*(a + b*x + c*x^2)^(1 + p))/(c*h*(3 + m + 2*p)) + ((f*h
*(b*g - a*h)*(1 + m) + 2*c*f*g^2*(1 + p) - c*h*(e*g - d*h)*(3 + m + 2*p))*(
g + h*x)^(1 + m)*(a + b*x + c*x^2)^p*AppellF1[1 + m, -p, -p, 2 + m, (2*c*(g
 + h*x))/(2*c*g - (b - Sqrt[b^2 - 4*a*c])*h), (2*c*(g + h*x))/(2*c*g - (b +
 Sqrt[b^2 - 4*a*c])*h)])/(c*h^3*(1 + m)*(3 + m + 2*p)*(1 - (2*c*(g + h*x))/
(2*c*g - (b - Sqrt[b^2 - 4*a*c])*h))^p*(1 - (2*c*(g + h*x))/(2*c*g - (b + S
qrt[b^2 - 4*a*c])*h))^p) - ((2*c*f*g*(1 + p) + b*f*h*(2 + m + p) - c*e*h*(3
 + m + 2*p))*(g + h*x)^(2 + m)*(a + b*x + c*x^2)^p*AppellF1[2 + m, -p, -p,
3 + m, (2*c*(g + h*x))/(2*c*g - (b - Sqrt[b^2 - 4*a*c])*h), (2*c*(g + h*x))
/(2*c*g - (b + Sqrt[b^2 - 4*a*c])*h)])/(c*h^3*(2 + m)*(3 + m + 2*p)*(1 - (2
*c*(g + h*x))/(2*c*g - (b - Sqrt[b^2 - 4*a*c])*h))^p*(1 - (2*c*(g + h*x))/(
2*c*g - (b + Sqrt[b^2 - 4*a*c])*h))^p)
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 773

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - e*((b - q)/(2*c))))^p*(1 - (d + e*x)/(d - e*((b + q)/(2*c))))
^p), Subst[Int[x^m*Simp[1 - x/(d - e*((b - q)/(2*c))), x]^p*Simp[1 - x/(d -
 e*((b + q)/(2*c))), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m,
```

```
 p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*
d - b*e, 0] &&  !IntegerQ[p]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{f(g+hx)^{1+m}\left(a+bx+cx^2\right)^{1+p}}{ch(3+m+2p)} \\
&\quad + \frac{\int (g+hx)^m(-h(afh(1+m)+bfg(1+p)-cdh(3+m+2p))-h(2cfg(1+p)+bfh(2+m+p)-c}{ch^2(3+m+2p)} \\
&= \frac{f(g+hx)^{1+m}\left(a+bx+cx^2\right)^{1+p}}{ch(3+m+2p)} \\
&\quad - \frac{(2cfg(1+p)+bfh(2+m+p)-ceh(3+m+2p))\int (g+hx)^{1+m}\left(a+bx+cx^2\right)^p\,dx}{ch^2(3+m+2p)} \\
&\quad + \frac{(fh(bg-ah)(1+m)+2cfg^2(1+p)-ch(eg-dh)(3+m+2p))\int (g+hx)^m\left(a+bx+cx^2\right)^p}{ch^2(3+m+2p)}
\end{aligned}$$

$$= \frac{f(g+hx)^{1+m}\left(a+bx+cx^2\right)^{1+p}}{ch(3+m+2p)}$$

$$- \frac{\left((2cfg(1+p)+bfh(2+m+p)-ceh(3+m+2p))\left(a+bx+cx^2\right)^p\left(1-\frac{g+hx}{g-\frac{\left(b-\sqrt{b^2-4ac}\right)h}{2c}}\right)^{-p}\left(\right.\right.}{ch^3(}$$

$$+ \frac{\left((fh(bg-ah)(1+m)+2cfg^2(1+p)-ch(eg-dh)(3+m+2p))\left(a+bx+cx^2\right)^p\left(1-\frac{g}{g-\frac{\left(b-\right.}{}}\right.\right.}{}$$

$$= \frac{f(g+hx)^{1+m}\left(a+bx+cx^2\right)^{1+p}}{ch(3+m+2p)}$$

$$+ \frac{(fh(bg-ah)(1+m)+2cfg^2(1+p)-ch(eg-dh)(3+m+2p))(g+hx)^{1+m}\left(a+bx+cx^2\right)^p\left(\right.}{ch^3}$$

$$- \frac{(2cfg(1+p)+bfh(2+m+p)-ceh(3+m+2p))(g+hx)^{2+m}\left(a+bx+cx^2\right)^p\left(1-\frac{2c(g+}{2cg-\left(b-\sqrt{b}\right.}\right.}{ch^3(2+m}$$

Mathematica [F]

$$\int (g+hx)^m\left(a+bx+cx^2\right)^p\left(d+ex+fx^2\right)\,dx$$
$$= \int (g+hx)^m\left(a+bx+cx^2\right)^p\left(d+ex+fx^2\right)\,dx$$

```
[In] Integrate[(g + h*x)^m*(a + b*x + c*x^2)^p*(d + e*x + f*x^2),x]
[Out] Integrate[(g + h*x)^m*(a + b*x + c*x^2)^p*(d + e*x + f*x^2), x]
```

Maple [F]

$$\int (hx+g)^m\left(c\,x^2+bx+a\right)^p\left(f\,x^2+ex+d\right)dx$$

```
[In] int((h*x+g)^m*(c*x^2+b*x+a)^p*(f*x^2+e*x+d),x)
[Out] int((h*x+g)^m*(c*x^2+b*x+a)^p*(f*x^2+e*x+d),x)
```

Fricas [F]

$$\begin{aligned}&\int (g+hx)^m \left(a+bx+cx^2\right)^p \left(d+ex+fx^2\right) \, dx\\&=\int \left(fx^2+ex+d\right) \left(cx^2+bx+a\right)^p (hx+g)^m \, dx\end{aligned}$$

```
[In] integrate((h*x+g)^m*(c*x^2+b*x+a)^p*(f*x^2+e*x+d),x, algorithm="fricas")
[Out] integral((f*x^2 + e*x + d)*(c*x^2 + b*x + a)^p*(h*x + g)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (g+hx)^m \left(a+bx+cx^2\right)^p \left(d+ex+fx^2\right) \, dx = \text{Timed out}$$

```
[In] integrate((h*x+g)**m*(c*x**2+b*x+a)**p*(f*x**2+e*x+d),x)
[Out] Timed out
```

Maxima [F]

$$\begin{aligned}&\int (g+hx)^m \left(a+bx+cx^2\right)^p \left(d+ex+fx^2\right) \, dx\\&=\int \left(fx^2+ex+d\right) \left(cx^2+bx+a\right)^p (hx+g)^m \, dx\end{aligned}$$

```
[In] integrate((h*x+g)^m*(c*x^2+b*x+a)^p*(f*x^2+e*x+d),x, algorithm="maxima")
[Out] integrate((f*x^2 + e*x + d)*(c*x^2 + b*x + a)^p*(h*x + g)^m, x)
```

Giac [F]

$$\begin{aligned}&\int (g+hx)^m \left(a+bx+cx^2\right)^p \left(d+ex+fx^2\right) \, dx\\&=\int \left(fx^2+ex+d\right) \left(cx^2+bx+a\right)^p (hx+g)^m \, dx\end{aligned}$$

```
[In] integrate((h*x+g)^m*(c*x^2+b*x+a)^p*(f*x^2+e*x+d),x, algorithm="giac")
[Out] integrate((f*x^2 + e*x + d)*(c*x^2 + b*x + a)^p*(h*x + g)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (g+hx)^m\left(a+bx+cx^2\right)^p\left(d+ex+fx^2\right)\,dx$$
$$=\int (g+h\,x)^m\,\left(c\,x^2+b\,x+a\right)^p\,\left(f\,x^2+e\,x+d\right)\,dx$$

```
[In] int((g + h*x)^m*(a + b*x + c*x^2)^p*(d + e*x + f*x^2),x)
[Out] int((g + h*x)^m*(a + b*x + c*x^2)^p*(d + e*x + f*x^2), x)
```

3.273 $\int (g+hx)^m\sqrt{a+bx+cx^2}(d+ex+fx^2)\,dx$

Optimal result

Integrand size = 32, antiderivative size = 496

$$\int (g+hx)^m\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx=\frac{f(g+hx)^{1+m}\left(a+bx+cx^2\right)^{3/2}}{ch(4+m)}$$

$$+\frac{\left(fh(bg-ah)(1+m)+c\left(3fg^2-h(eg-dh)(4+m)\right)\right)(g+hx)^{1+m}\sqrt{a+bx+cx^2}\,\mathrm{AppellF1}\left(1+m,-\right.}{ch^3(1+m)(4+m)\sqrt{1-\frac{2c(g+hx)}{2cg-\left(b-\sqrt{b^2-4ac}\right)h}}\sqrt{1-\frac{2c(g+h}{2cg-\left(b+\sqrt{b^2\right.}}}$$

$$-\frac{(bfh(5+2m)+c(6fg-2eh(4+m)))(g+hx)^{2+m}\sqrt{a+bx+cx^2}\,\mathrm{AppellF1}\left(2+m,-\frac{1}{2},-\frac{1}{2},3+m,\frac{}{2c}\right.}{2ch^3(2+m)(4+m)\sqrt{1-\frac{2c(g+hx)}{2cg-\left(b-\sqrt{b^2-4ac}\right)h}}\sqrt{1-\frac{2c(g+hx)}{2cg-\left(b+\sqrt{b^2-4ac}\right)h}}}$$

```
[Out] f*(h*x+g)^(1+m)*(c*x^2+b*x+a)^(3/2)/c/h/(4+m)+(f*h*(-a*h+b*g)*(1+m)+c*(3*f*
g^2-h*(-d*h+e*g)*(4+m)))*(h*x+g)^(1+m)*AppellF1(1+m,-1/2,-1/2,2+m,2*c*(h*x+
g)/(2*c*g-h*(b-(-4*a*c+b^2)^(1/2))),2*c*(h*x+g)/(2*c*g-h*(b+(-4*a*c+b^2)^(1
/2))))*(c*x^2+b*x+a)^(1/2)/c/h^3/(1+m)/(4+m)/(1-2*c*(h*x+g)/(2*c*g-h*(b-(-4
*a*c+b^2)^(1/2))))^(1/2)/(1-2*c*(h*x+g)/(2*c*g-h*(b+(-4*a*c+b^2)^(1/2))))^(
1/2)-1/2*(b*f*h*(5+2*m)+c*(6*f*g-2*e*h*(4+m)))*(h*x+g)^(2+m)*AppellF1(2+m,-
1/2,-1/2,3+m,2*c*(h*x+g)/(2*c*g-h*(b-(-4*a*c+b^2)^(1/2))),2*c*(h*x+g)/(2*c*
g-h*(b+(-4*a*c+b^2)^(1/2))))*(c*x^2+b*x+a)^(1/2)/c/h^3/(2+m)/(4+m)/(1-2*c*(
h*x+g)/(2*c*g-h*(b-(-4*a*c+b^2)^(1/2))))^(1/2)/(1-2*c*(h*x+g)/(2*c*g-h*(b+(
-4*a*c+b^2)^(1/2))))^(1/2)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 494, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1667, 857, 773, 138}

$$\int (g+hx)^m\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx$$

$$=\frac{\sqrt{a+bx+cx^2}(g+hx)^{m+1}\,\text{AppellF1}\left(m+1,-\frac{1}{2},-\frac{1}{2},m+2,\frac{2c(g+hx)}{2cg-\left(b-\sqrt{b^2-4ac}\right)h},\frac{2c(g+hx)}{2cg-\left(b+\sqrt{b^2-4ac}\right)h}\right)(fh(m+}{ch^3(m+1)(m+4)\sqrt{1-\frac{2c(g+hx)}{2cg-h\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(g+hx)}{2cg-h\left(\sqrt{b^2-4ac}+\right.}}}$$

$$-\frac{\sqrt{a+bx+cx^2}(g+hx)^{m+2}(bfh(2m+5)-2ceh(m+4)+6cfg)\,\text{AppellF1}\left(m+2,-\frac{1}{2},-\frac{1}{2},m+3,\frac{}{2cg-}\right.}{2ch^3(m+2)(m+4)\sqrt{1-\frac{2c(g+hx)}{2cg-h\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(g+hx)}{2cg-h\left(\sqrt{b^2-4ac}+b\right)}}}$$

$$+\frac{f\left(a+bx+cx^2\right)^{3/2}(g+hx)^{m+1}}{ch(m+4)}$$

```
[In] Int[(g + h*x)^m*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] (f*(g + h*x)^(1 + m)*(a + b*x + c*x^2)^(3/2))/(c*h*(4 + m)) + ((3*c*f*g^2 +
 f*h*(b*g - a*h)*(1 + m) - c*h*(e*g - d*h)*(4 + m))*(g + h*x)^(1 + m)*Sqrt[
a + b*x + c*x^2]*AppellF1[1 + m, -1/2, -1/2, 2 + m, (2*c*(g + h*x))/(2*c*g
- (b - Sqrt[b^2 - 4*a*c])*h), (2*c*(g + h*x))/(2*c*g - (b + Sqrt[b^2 - 4*a*
c])*h)])/(c*h^3*(1 + m)*(4 + m)*Sqrt[1 - (2*c*(g + h*x))/(2*c*g - (b - Sqrt
[b^2 - 4*a*c])*h)]*Sqrt[1 - (2*c*(g + h*x))/(2*c*g - (b + Sqrt[b^2 - 4*a*c]
)*h)]) - ((6*c*f*g - 2*c*e*h*(4 + m) + b*f*h*(5 + 2*m))*(g + h*x)^(2 + m)*S
qrt[a + b*x + c*x^2]*AppellF1[2 + m, -1/2, -1/2, 3 + m, (2*c*(g + h*x))/(2*
c*g - (b - Sqrt[b^2 - 4*a*c])*h), (2*c*(g + h*x))/(2*c*g - (b + Sqrt[b^2 -
4*a*c])*h)])/(2*c*h^3*(2 + m)*(4 + m)*Sqrt[1 - (2*c*(g + h*x))/(2*c*g - (b
- Sqrt[b^2 - 4*a*c])*h)]*Sqrt[1 - (2*c*(g + h*x))/(2*c*g - (b + Sqrt[b^2 -
4*a*c])*h)])
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 773

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(a + b*x + c*x^2)^p/(e*(1 - (
```

```
d + e*x)/(d - e*((b - q)/(2*c))))^p*(1 - (d + e*x)/(d - e*((b + q)/(2*c))))
^p), Subst[Int[x^m*Simp[1 - x/(d - e*((b - q)/(2*c))), x]^p*Simp[1 - x/(d -
 e*((b + q)/(2*c))), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*
d - b*e, 0] &&  !IntegerQ[p]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{f(g+hx)^{1+m}\left(a+bx+cx^2\right)^{3/2}}{ch(4+m)} \\ &\quad+\frac{\int (g+hx)^m\left(-\frac{1}{2}h(3bfg+2afh(1+m)-2cdh(4+m))-\frac{1}{2}h(6cfg-2ceh(4+m)+bfh(5+2m))x\right)}{ch^2(4+m)} \\ &= \frac{f(g+hx)^{1+m}\left(a+bx+cx^2\right)^{3/2}}{ch(4+m)} \\ &\quad+\frac{\left(3cfg^2+fh(bg-ah)(1+m)-ch(eg-dh)(4+m)\right)\int (g+hx)^m\sqrt{a+bx+cx^2}\,dx}{ch^2(4+m)} \\ &\quad-\frac{(6cfg-2ceh(4+m)+bfh(5+2m))\int (g+hx)^{1+m}\sqrt{a+bx+cx^2}\,dx}{2ch^2(4+m)}\end{aligned}$$

$$= \frac{f(g+hx)^{1+m}\left(a+bx+cx^2\right)^{3/2}}{ch(4+m)}$$

$$+ \frac{\left(\left(3cfg^2+fh(bg-ah)(1+m)-ch(eg-dh)(4+m)\right)\sqrt{a+bx+cx^2}\right)\text{Subst}\left(\int x^m\sqrt{1-\frac{}{2cg-(b}}\right.}{ch^3(4+m)\sqrt{1-\frac{g+hx}{g-\frac{\left(b-\sqrt{b^2-4ac}\right)h}{2c}}}\sqrt{1-\frac{g+hx}{g-\frac{(b+\sqrt{b^2-4ac}}{2c}}}}$$

$$- \frac{\left(\left(6cfg-2ceh(4+m)+bfh(5+2m)\right)\sqrt{a+bx+cx^2}\right)\text{Subst}\left(\int x^{1+m}\sqrt{1-\frac{2cx}{2cg-\left(b-\sqrt{b^2-4ac}\right)h}}\sqrt{1}\right.}{2ch^3(4+m)\sqrt{1-\frac{g+hx}{g-\frac{\left(b-\sqrt{b^2-4ac}\right)h}{2c}}}\sqrt{1-\frac{g+hx}{g-\frac{\left(b+\sqrt{b^2-4ac}\right)h}{2c}}}}$$

$$= \frac{f(g+hx)^{1+m}\left(a+bx+cx^2\right)^{3/2}}{ch(4+m)}$$

$$+ \frac{\left(3cfg^2+fh(bg-ah)(1+m)-ch(eg-dh)(4+m)\right)(g+hx)^{1+m}\sqrt{a+bx+cx^2}\,F_1\left(1+m;-\frac{1}{2}\right.}{ch^3(1+m)(4+m)\sqrt{1-\frac{2c(g+hx)}{2cg-\left(b-\sqrt{b^2-4ac}\right)h}}\sqrt{1-\frac{2c(g-}{2cg-(b+\sqrt{}}}}$$

$$- \frac{\left(6cfg-2ceh(4+m)+bfh(5+2m)\right)(g+hx)^{2+m}\sqrt{a+bx+cx^2}\,F_1\left(2+m;-\frac{1}{2},-\frac{1}{2};3+m;\frac{}{2cg-}\right.}{2ch^3(2+m)(4+m)\sqrt{1-\frac{2c(g+hx)}{2cg-\left(b-\sqrt{b^2-4ac}\right)h}}\sqrt{1-\frac{2c(g+hx)}{2cg-\left(b+\sqrt{b^2-4ac}\right)h}}}$$

Mathematica [F]

$$\int (g+hx)^m\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx = \int (g+hx)^m\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx$$

```
[In] Integrate[(g + h*x)^m*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2),x]
[Out] Integrate[(g + h*x)^m*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2), x]
```

Maple [F]

$$\int (hx+g)^m\left(f\,x^2+ex+d\right)\sqrt{c\,x^2+bx+a}dx$$

```
[In] int((h*x+g)^m*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x)
[Out] int((h*x+g)^m*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x)
```

Fricas [F]

$$\int (g+hx)^m\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx=\int \sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)(hx+g)^m\,dx$$

```
[In] integrate((h*x+g)^m*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(c*x^2 + b*x + a)*(f*x^2 + e*x + d)*(h*x + g)^m, x)
```

Sympy [F]

$$\int (g+hx)^m\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx=\int (g+hx)^m\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx$$

```
[In] integrate((h*x+g)**m*(f*x**2+e*x+d)*(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((g + h*x)**m*sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2), x)
```

Maxima [F]

$$\int (g+hx)^m\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx=\int \sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)(hx+g)^m\,dx$$

```
[In] integrate((h*x+g)^m*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)*(f*x^2 + e*x + d)*(h*x + g)^m, x)
```

Giac [F]

$$\int (g+hx)^m\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx=\int \sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)(hx+g)^m\,dx$$

```
[In] integrate((h*x+g)^m*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)*(f*x^2 + e*x + d)*(h*x + g)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (g+hx)^m\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx$$
$$=\int (g+h\,x)^m\,\sqrt{c\,x^2+b\,x+a}\,\left(f\,x^2+e\,x+d\right)\,dx$$

```
[In] int((g + h*x)^m*(a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2),x)
[Out] int((g + h*x)^m*(a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2), x)
```

3.274 $\int (g+hx)^{-3-2p}\left(a+bx+cx^2\right)^p\left(d+ex+fx^2\right)\,dx$

Optimal result

Integrand size = 34, antiderivative size = 590

$$
\begin{aligned}
&\int (g+hx)^{-3-2p}\left(a+bx+cx^2\right)^p\left(d+ex+fx^2\right)\,dx \\
&= -\frac{\left(fg^2-h(eg-dh)\right)(g+hx)^{-2(1+p)}\left(a+bx+cx^2\right)^{1+p}}{2h\left(cg^2-bgh+ah^2\right)(1+p)} \\
&\quad -\frac{f(g+hx)^{-2p}\left(a+bx+cx^2\right)^p\left(1-\frac{2c(g+hx)}{2cg-\left(b-\sqrt{b^2-4ac}\right)h}\right)^{-p}\left(1-\frac{2c(g+hx)}{2cg-\left(b+\sqrt{b^2-4ac}\right)h}\right)^{-p}\operatorname{AppellF1}\left(-2p,-p\right.}{2h^3p} \\
&\quad -\frac{\left(2c\left(fg^3-dgh^2\right)+h\left(2ah(2fg-eh)-b\left(3fg^2-egh-dh^2\right)\right)\right)\left(b-\sqrt{b^2-4ac}+2cx\right)\left(\frac{\left(2cg-\left(b-\sqrt{b^2-4ac}\right.\right.}{\left(2cg-\left(b+\sqrt{b^2-4ac}\right.\right.}\right.}{2h^2\left(2cg-\left(b-\right.\right.}
\end{aligned}
$$

```
[Out] -1/2*(f*g^2-h*(-d*h+e*g))*(c*x^2+b*x+a)^(p+1)/h/(a*h^2-b*g*h+c*g^2)/(p+1)/(
(h*x+g)^(2+2*p))-1/2*(2*c*(-d*g*h^2+f*g^3)+h*(2*a*h*(-e*h+2*f*g)-b*(-d*h^2-
e*g*h+3*f*g^2)))*(h*x+g)^(-1-2*p)*(c*x^2+b*x+a)^p*hypergeom([-p, -1-2*p],[-
2*p],-4*c*(h*x+g)*(-4*a*c+b^2)^(1/2)/(b+2*c*x-(-4*a*c+b^2)^(1/2))/(2*c*g-h*
(b+(-4*a*c+b^2)^(1/2))))*(b+2*c*x-(-4*a*c+b^2)^(1/2))/h^2/(a*h^2-b*g*h+c*g^
2)/(1+2*p)/(2*c*g-h*(b-(-4*a*c+b^2)^(1/2)))/(((2*c*g-h*(b-(-4*a*c+b^2)^(1/2
)))*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(b+2*c*x-(-4*a*c+b^2)^(1/2))/(2*c*g-h*(b+(
-4*a*c+b^2)^(1/2))))^p)-1/2*f*(c*x^2+b*x+a)^p*AppellF1(-2*p,-p,-p,1-2*p,2*c
*(h*x+g)/(2*c*g-h*(b-(-4*a*c+b^2)^(1/2))),2*c*(h*x+g)/(2*c*g-h*(b+(-4*a*c+b
^2)^(1/2))))/h^3/p/((h*x+g)^(2*p))/((1-2*c*(h*x+g)/(2*c*g-h*(b-(-4*a*c+b^2)
^(1/2))))^p)/((1-2*c*(h*x+g)/(2*c*g-h*(b+(-4*a*c+b^2)^(1/2))))^p)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 588, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {1669, 773, 138, 820, 740}

$$\int (g+hx)^{-3-2p}\left(a+bx+cx^2\right)^p\left(d+ex+fx^2\right)\,dx =$$

$$-\frac{f(g+hx)^{-2p}\left(a+bx+cx^2\right)^p\left(1-\frac{2c(g+hx)}{2cg-h\left(b-\sqrt{b^2-4ac}\right)}\right)^{-p}\left(1-\frac{2c(g+hx)}{2cg-h\left(\sqrt{b^2-4ac}+b\right)}\right)^{-p}\text{AppellF1}\Big(-2p,-p,}{2h^3p}$$

$$-\frac{\left(-\sqrt{b^2-4ac}+b+2cx\right)(g+hx)^{-2p-1}\left(a+bx+cx^2\right)^p\left(\frac{\left(\sqrt{b^2-4ac}+b+2cx\right)\left(2cg-h\left(b-\sqrt{b^2-4ac}\right)\right)}{\left(-\sqrt{b^2-4ac}+b+2cx\right)\left(2cg-h\left(\sqrt{b^2-4ac}+b\right)\right)}\right)^{-p}(2c(fg^3}{2h^2(2p+1)\,(}$$

$$-\frac{(g+hx)^{-2(p+1)}\left(a+bx+cx^2\right)^{p+1}\left(fg^2-h(eg-dh)\right)}{2h(p+1)\left(ah^2-bgh+cg^2\right)}$$

```
[In] Int[(g + h*x)^(-3 - 2*p)*(a + b*x + c*x^2)^p*(d + e*x + f*x^2),x]
```

```
[Out] -1/2*((f*g^2 - h*(e*g - d*h))*(a + b*x + c*x^2)^(1 + p))/(h*(c*g^2 - b*g*h
+ a*h^2)*(1 + p)*(g + h*x)^(2*(1 + p))) - (f*(a + b*x + c*x^2)^p*AppellF1[-
2*p, -p, -p, 1 - 2*p, (2*c*(g + h*x))/(2*c*g - (b - Sqrt[b^2 - 4*a*c])*h),
(2*c*(g + h*x))/(2*c*g - (b + Sqrt[b^2 - 4*a*c])*h)])/(2*h^3*p*(g + h*x)^(2
*p)*(1 - (2*c*(g + h*x))/(2*c*g - (b - Sqrt[b^2 - 4*a*c])*h))^p*(1 - (2*c*(
g + h*x))/(2*c*g - (b + Sqrt[b^2 - 4*a*c])*h))^p) - ((2*c*(f*g^3 - d*g*h^2)
 - h*(3*b*f*g^2 - b*h*(e*g + d*h) - 2*a*h*(2*f*g - e*h)))*(b - Sqrt[b^2 - 4
*a*c] + 2*c*x)*(g + h*x)^(-1 - 2*p)*(a + b*x + c*x^2)^p*Hypergeometric2F1[-
1 - 2*p, -p, -2*p, (-4*c*Sqrt[b^2 - 4*a*c]*(g + h*x))/((2*c*g - (b + Sqrt[b
^2 - 4*a*c])*h)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))])/(2*h^2*(2*c*g - (b - Sqr
t[b^2 - 4*a*c])*h)*(c*g^2 - b*g*h + a*h^2)*(1 + 2*p)*(((2*c*g - (b - Sqrt[b
^2 - 4*a*c])*h)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/((2*c*g - (b + Sqrt[b^2 -
4*a*c])*h)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x)))^p)
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(b - Rt[b^2 - 4*a*c, 2] + 2*c*x))*(d + e*x)^(m + 1)*((a +
b*x + c*x^2)^p/((m + 1)*(2*c*d - b*e + e*Rt[b^2 - 4*a*c, 2])*((2*c*d - b*e
```

```
+ e*Rt[b^2 - 4*a*c, 2])*((b + Rt[b^2 - 4*a*c, 2] + 2*c*x)/((2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])*(b - Rt[b^2 - 4*a*c, 2] + 2*c*x))))^p))*Hypergeometric
2F1[m + 1, -p, m + 2, -4*c*Rt[b^2 - 4*a*c, 2]*((d + e*x)/((2*c*d - b*e - e*
Rt[b^2 - 4*a*c, 2])*(b - Rt[b^2 - 4*a*c, 2] + 2*c*x)))], x] /; FreeQ[{a, b,
 c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[2*c*d - b*e, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 773

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - e*((b - q)/(2*c))))^p*(1 - (d + e*x)/(d - e*((b + q)/(2*c))))
^p), Subst[Int[x^m*Simp[1 - x/(d - e*((b - q)/(2*c))), x]^p*Simp[1 - x/(d -
 e*((b + q)/(2*c))), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*
d - b*e, 0] &&  !IntegerQ[p]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 1669

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> With[{q = Expon[Pq, x]}, Dist[Coeff[Pq, x, q]/e^q, Int[(d
 + e*x)^(m + q)*(a + b*x + c*x^2)^p, x], x] + Dist[1/e^q, Int[(d + e*x)^m*(
a + b*x + c*x^2)^p*ExpandToSum[e^q*Pq - Coeff[Pq, x, q]*(d + e*x)^q, x], x]
, x]] /; FreeQ[{a, b, c, d, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ[m, 0] && RationalQ[a, b, c
, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\int (g+hx)^{-3-2p}\left(-fg^2+dh^2-h(2fg-eh)x\right)\left(a+bx+cx^2\right)^p\,dx}{h^2}\\&\quad+\frac{f\int (g+hx)^{-1-2p}\left(a+bx+cx^2\right)^p\,dx}{h^2}\end{aligned}$$

$$= -\frac{\left(fg^2 - h(eg - dh)\right)(g + hx)^{-2(1+p)}\left(a + bx + cx^2\right)^{1+p}}{2h\left(cg^2 - bgh + ah^2\right)(1 + p)}$$
$$- \frac{\left(2c(fg^3 - dgh^2) - h(3bfg^2 - bh(eg + dh) - 2ah(2fg - eh))\right)\int (g + hx)^{-2-2p}\left(a + bx + cx^2\right)^p dx}{2h^2\left(cg^2 - bgh + ah^2\right)}$$
$$+ \frac{\left(f\left(a + bx + cx^2\right)^p\left(1 - \frac{g+hx}{g - \frac{\left(b-\sqrt{b^2-4ac}\right)h}{2c}}\right)^{-p}\left(1 - \frac{g+hx}{g - \frac{\left(b+\sqrt{b^2-4ac}\right)h}{2c}}\right)^{-p}\right)\mathrm{Subst}\left(\int x^{-1-2p}\left(1 - \frac{}{2cg-}\right.\right.}{h^3}$$

$$= -\frac{\left(fg^2 - h(eg - dh)\right)(g + hx)^{-2(1+p)}\left(a + bx + cx^2\right)^{1+p}}{2h\left(cg^2 - bgh + ah^2\right)(1 + p)}$$
$$- \frac{f(g + hx)^{-2p}\left(a + bx + cx^2\right)^p\left(1 - \frac{2c(g+hx)}{2cg - \left(b-\sqrt{b^2-4ac}\right)h}\right)^{-p}\left(1 - \frac{2c(g+hx)}{2cg - \left(b+\sqrt{b^2-4ac}\right)h}\right)^{-p} F_1\left(-2p; -p,\right.}{2h^3p}$$
$$- \frac{\left(2c(fg^3 - dgh^2) - h(3bfg^2 - bh(eg + dh) - 2ah(2fg - eh))\right)\left(b - \sqrt{b^2 - 4ac} + 2cx\right)\left(\frac{\left(2cg - \left(b - \sqrt{}\right.\right.}{\left(2cg - \left(b + \sqrt{}\right.\right.}\right.}{2h^2\left(2cg - \left(b - \sqrt{}\right.\right.}$$

Mathematica [F]

$$\int (g + hx)^{-3-2p}\left(a + bx + cx^2\right)^p\left(d + ex + fx^2\right) dx$$
$$= \int (g + hx)^{-3-2p}\left(a + bx + cx^2\right)^p\left(d + ex + fx^2\right) dx$$

```
[In] Integrate[(g + h*x)^(-3 - 2*p)*(a + b*x + c*x^2)^p*(d + e*x + f*x^2),x]
[Out] Integrate[(g + h*x)^(-3 - 2*p)*(a + b*x + c*x^2)^p*(d + e*x + f*x^2), x]
```

Maple [F]

$$\int (hx + g)^{-3-2p}\left(c\,x^2 + bx + a\right)^p\left(f\,x^2 + ex + d\right) dx$$

```
[In] int((h*x+g)^(-3-2*p)*(c*x^2+b*x+a)^p*(f*x^2+e*x+d),x)
[Out] int((h*x+g)^(-3-2*p)*(c*x^2+b*x+a)^p*(f*x^2+e*x+d),x)
```

Fricas [F]

$$\int (g+hx)^{-3-2p}\left(a+bx+cx^2\right)^p\left(d+ex+fx^2\right)\,dx$$
$$=\int \left(fx^2+ex+d\right)\left(cx^2+bx+a\right)^p(hx+g)^{-2\,p-3}\,dx$$

```
[In] integrate((h*x+g)^(-3-2*p)*(c*x^2+b*x+a)^p*(f*x^2+e*x+d),x, algorithm="fric
as")
[Out] integral((f*x^2 + e*x + d)*(c*x^2 + b*x + a)^p*(h*x + g)^(-2*p - 3), x)
```

Sympy [F(-1)]

Timed out.

$$\int (g+hx)^{-3-2p}\left(a+bx+cx^2\right)^p\left(d+ex+fx^2\right)\,dx=\text{Timed out}$$

```
[In] integrate((h*x+g)**(-3-2*p)*(c*x**2+b*x+a)**p*(f*x**2+e*x+d),x)
[Out] Timed out
```

Maxima [F]

$$\int (g+hx)^{-3-2p}\left(a+bx+cx^2\right)^p\left(d+ex+fx^2\right)\,dx$$
$$=\int \left(fx^2+ex+d\right)\left(cx^2+bx+a\right)^p(hx+g)^{-2\,p-3}\,dx$$

```
[In] integrate((h*x+g)^(-3-2*p)*(c*x^2+b*x+a)^p*(f*x^2+e*x+d),x, algorithm="maxi
ma")
[Out] integrate((f*x^2 + e*x + d)*(c*x^2 + b*x + a)^p*(h*x + g)^(-2*p - 3), x)
```

Giac [F]

$$\int (g+hx)^{-3-2p}\left(a+bx+cx^2\right)^p\left(d+ex+fx^2\right)\,dx$$
$$=\int\left(fx^2+ex+d\right)\left(cx^2+bx+a\right)^p(hx+g)^{-2\,p-3}\,dx$$

```
[In] integrate((h*x+g)^(-3-2*p)*(c*x^2+b*x+a)^p*(f*x^2+e*x+d),x, algorithm="giac
")
```

```
[Out] integrate((f*x^2 + e*x + d)*(c*x^2 + b*x + a)^p*(h*x + g)^(-2*p - 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int (g+hx)^{-3-2p}\left(a+bx+cx^2\right)^p\left(d+ex+fx^2\right)\,dx=\int\frac{{\left(c\,x^2+b\,x+a\right)}^p\,\left(f\,x^2+e\,x+d\right)}{{\left(g+h\,x\right)}^{2\,p+3}}\,dx$$

```
[In] int(((a + b*x + c*x^2)^p*(d + e*x + f*x^2))/(g + h*x)^(2*p + 3),x)
```

```
[Out] int(((a + b*x + c*x^2)^p*(d + e*x + f*x^2))/(g + h*x)^(2*p + 3), x)
```

3.275 $\int (d+fx^2)^p (2cdf+2bf^2(3+2p)x+2cf^2(3+2p)x$

Optimal result

Integrand size = 42, antiderivative size = 41

$$\int (d+fx^2)^p \left(2cdf+2bf^2(3+2p)x+2cf^2(3+2p)x^2\right) dx = \frac{bf(3+2p)\left(d+fx^2\right)^{1+p}}{1+p}+2cfx\left(d+fx^2\right)^{1+p}$$

```
[Out] b*f*(3+2*p)*(f*x^2+d)^(p+1)/(p+1)+2*c*f*x*(f*x^2+d)^(p+1)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1829, 12, 267}

$$\int (d+fx^2)^p \left(2cdf+2bf^2(3+2p)x+2cf^2(3+2p)x^2\right) dx = \frac{bf(2p+3)\left(d+fx^2\right)^{p+1}}{p+1}+2cfx\left(d+fx^2\right)^{p+1}$$

```
[In] Int[(d + f*x^2)^p*(2*c*d*f + 2*b*f^2*(3 + 2*p)*x + 2*c*f^2*(3 + 2*p)*x^2),x
]
```

```
[Out] (b*f*(3 + 2*p)*(d + f*x^2)^(1 + p))/(1 + p) + 2*c*f*x*(d + f*x^2)^(1 + p)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1829

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= 2cfx\left(d+fx^2\right)^{1+p}+\frac{\int 2bf^3(3+2p)^2x(d+fx^2)^p\,dx}{f(3+2p)}\\ &= 2cfx\left(d+fx^2\right)^{1+p}+\left(2bf^2(3+2p)\right)\int x\left(d+fx^2\right)^p\,dx\\ &= \frac{bf(3+2p)\left(d+fx^2\right)^{1+p}}{1+p}+2cfx\left(d+fx^2\right)^{1+p}\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.28 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.90

$$\int \left(d+fx^2\right)^p\left(2cdf+2bf^2(3+2p)x+2cf^2(3+2p)x^2\right)\,dx$$

$$=\frac{f(d+fx^2)^p\left(1+\frac{fx^2}{d}\right)^{-p}\left(6cd(1+p)x\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{fx^2}{d}\right)+(3+2p)\left(3b(d+fx^2)\left(1+\right.\right.\right.}{3(1+p)}$$

```
[In] Integrate[(d + f*x^2)^p*(2*c*d*f + 2*b*f^2*(3 + 2*p)*x + 2*c*f^2*(3 + 2*p)*
x^2),x]
```

```
[Out] (f*(d + f*x^2)^p*(6*c*d*(1 + p)*x*Hypergeometric2F1[1/2, -p, 3/2, -((f*x^2)
/d)] + (3 + 2*p)*(3*b*(d + f*x^2)*(1 + (f*x^2)/d)^p + 2*c*f*(1 + p)*x^3*Hyp
ergeometric2F1[3/2, -p, 5/2, -((f*x^2)/d)])))/(3*(1 + p)*(1 + (f*x^2)/d)^p)
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

method	result
gosper	$\frac{f\left(f\,x^{2}+d\right)^{1+p}\left(2pcx+2bp+2cx+3b\right)}{1+p}$
risch	$\frac{f\left(2cfp\,x^{3}+2bfp\,x^{2}+2cf\,x^{3}+3bf\,x^{2}+2cdpx+2bdp+2cdx+3bd\right)\left(f\,x^{2}+d\right)^{p}}{1+p}$
norman	$\frac{bdf(3+2p)\mathrm{e}^{p\ln\left(f\,x^{2}+d\right)}}{1+p}+\frac{b\,f^{2}(3+2p)x^{2}\mathrm{e}^{p\ln\left(f\,x^{2}+d\right)}}{1+p}+2c\,f^{2}x^{3}\mathrm{e}^{p\ln\left(f\,x^{2}+d\right)}+2cdfx\,\mathrm{e}^{p\ln\left(f\,x^{2}+d\right)}$
parallelrisch	$\frac{2x^{3}\left(f\,x^{2}+d\right)^{p}c\,f^{3}p+2x^{3}\left(f\,x^{2}+d\right)^{p}c\,f^{3}+2x^{2}\left(f\,x^{2}+d\right)^{p}b\,f^{3}p+3x^{2}\left(f\,x^{2}+d\right)^{p}b\,f^{3}+2x\left(f\,x^{2}+d\right)^{p}cd\,f^{2}p+2x\left(f\,x^{2}+d\right)^{p}cd\,f^{2}+2}{f(1+p)}$

```
[In] int((f*x^2+d)^p*(2*c*d*f+2*b*f^2*(3+2*p)*x+2*c*f^2*(3+2*p)*x^2),x,method=_R
ETURNVERBOSE)
```

```
[Out] f/(1+p)*(f*x^2+d)^(1+p)*(2*c*p*x+2*b*p+2*c*x+3*b)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.83

$$\int \left(d+fx^2\right)^p \left(2cdf+2bf^2(3+2p)x+2cf^2(3+2p)x^2\right)\,dx = \frac{(2\,bdfp+2\,(cf^2p+cf^2)x^3+3\,bdf+(2\,bf^2p+3\,bf^2)x^2+2\,(cdfp+cdf)x)(fx^2+d)^p}{p+1}$$

```
[In] integrate((f*x^2+d)^p*(2*c*d*f+2*b*f^2*(3+2*p)*x+2*c*f^2*(3+2*p)*x^2),x, al
gorithm="fricas")
```

```
[Out] (2*b*d*f*p + 2*(c*f^2*p + c*f^2)*x^3 + 3*b*d*f + (2*b*f^2*p + 3*b*f^2)*x^2
+ 2*(c*d*f*p + c*d*f)*x)*(f*x^2 + d)^p/(p + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 211 vs. 2(37) = 74.

Time = 3.23 (sec) , antiderivative size = 211, normalized size of antiderivative = 5.15

$$\int \left(d+fx^2\right)^p \left(2cdf+2bf^2(3+2p)x+2cf^2(3+2p)x^2\right)\,dx = \begin{cases} \frac{2bdfp(d+fx^2)^p}{p+1}+\frac{3bdf(d+fx^2)^p}{p+1}+\frac{2bf^2px^2(d+fx^2)^p}{p+1}+\frac{3bf^2x^2(d+fx^2)^p}{p+1}+\frac{2cdfpx(d+fx^2)^p}{p+1}+\frac{2cdfx(d+fx^2)^p}{p+1}+\frac{2cf^2px^3(d+f}{p+1} \\ bf\log\left(x-\sqrt{-\frac{d}{f}}\right)+bf\log\left(x+\sqrt{-\frac{d}{f}}\right)+2cfx \end{cases}$$

```
[In] integrate((f*x**2+d)**p*(2*c*d*f+2*b*f**2*(3+2*p)*x+2*c*f**2*(3+2*p)*x**2),
x)
```

```
[Out] Piecewise((2*b*d*f*p*(d + f*x**2)**p/(p + 1) + 3*b*d*f*(d + f*x**2)**p/(p +
 1) + 2*b*f**2*p*x**2*(d + f*x**2)**p/(p + 1) + 3*b*f**2*x**2*(d + f*x**2)*
*p/(p + 1) + 2*c*d*f*p*x*(d + f*x**2)**p/(p + 1) + 2*c*d*f*x*(d + f*x**2)**
p/(p + 1) + 2*c*f**2*p*x**3*(d + f*x**2)**p/(p + 1) + 2*c*f**2*x**3*(d + f*
x**2)**p/(p + 1), Ne(p, -1)), (b*f*log(x - sqrt(-d/f)) + b*f*log(x + sqrt(-
d/f)) + 2*c*f*x, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.44

$$\int \left(d+fx^2\right)^p \left(2cdf+2bf^2(3+2p)x+2cf^2(3+2p)x^2\right)\,dx = \frac{(2\,cf^2(p+1)x^3+bf^2(2\,p+3)x^2+2\,cdf(p+1)x+bdf(2\,p+3))(fx^2+d)^p}{p+1}$$

```
[In] integrate((f*x^2+d)^p*(2*c*d*f+2*b*f^2*(3+2*p)*x+2*c*f^2*(3+2*p)*x^2),x, al
gorithm="maxima")
```

```
[Out] (2*c*f^2*(p + 1)*x^3 + b*f^2*(2*p + 3)*x^2 + 2*c*d*f*(p + 1)*x + b*d*f*(2*p
 + 3))*(f*x^2 + d)^p/(p + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(41) = 82$.

Time = 0.29 (sec) , antiderivative size = 141, normalized size of antiderivative = 3.44

$$\int \left(d+fx^2\right)^p \left(2cdf+2bf^2(3+2p)x+2cf^2(3+2p)x^2\right)\,dx = \frac{2\,(fx^2+d)^p cf^2px^3+2\,(fx^2+d)^p bf^2px^2+2\,(fx^2+d)^p cf^2x^3+2\,(fx^2+d)^p cdfpx+3\,(fx^2+d)^p bf^2x^2+}{p+1}$$

```
[In] integrate((f*x^2+d)^p*(2*c*d*f+2*b*f^2*(3+2*p)*x+2*c*f^2*(3+2*p)*x^2),x, al
gorithm="giac")
```

```
[Out] (2*(f*x^2 + d)^p*c*f^2*p*x^3 + 2*(f*x^2 + d)^p*b*f^2*p*x^2 + 2*(f*x^2 + d)^
p*c*f^2*x^3 + 2*(f*x^2 + d)^p*c*d*f*p*x + 3*(f*x^2 + d)^p*b*f^2*x^2 + 2*(f*
x^2 + d)^p*b*d*f*p + 2*(f*x^2 + d)^p*c*d*f*x + 3*(f*x^2 + d)^p*b*d*f)/(p +
1)
```

Mupad [B] (verification not implemented)

Time = 13.56 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.41

$$\int \left(d+fx^2\right)^p \left(2cdf+2bf^2(3+2p)x+2cf^2(3+2p)x^2\right)\,dx$$
$$=\left(f\,x^2+d\right)^p\,\left(2\,c\,f^2\,x^3+2\,c\,d\,f\,x+\frac{b\,f^2\,x^2\,\left(2\,p+3\right)}{p+1}+\frac{b\,d\,f\,\left(2\,p+3\right)}{p+1}\right)$$

```
[In] int((d + f*x^2)^p*(2*c*d*f + 2*b*f^2*x*(2*p + 3) + 2*c*f^2*x^2*(2*p + 3)),x
)
```

```
[Out] (d + f*x^2)^p*(2*c*f^2*x^3 + 2*c*d*f*x + (b*f^2*x^2*(2*p + 3))/(p + 1) + (b
*d*f*(2*p + 3))/(p + 1))
```

3.276 $\int (d + ex + fx^2)^p (-2ce^2 + 2cdf - ce^2p + 2cf^2(3 +$

Optimal result

Integrand size = 46, antiderivative size = 46

$$\int (d + ex + fx^2)^p \left(-2ce^2 + 2cdf - ce^2p + 2cf^2(3 + 2p)x^2\right) dx = -\frac{ce(2 + p)\left(d + ex + fx^2\right)^{1+p}}{1 + p} + 2cfx\left(d + ex + fx^2\right)^{1+p}$$

```
[Out] -c*e*(2+p)*(f*x^2+e*x+d)^(p+1)/(p+1)+2*c*f*x*(f*x^2+e*x+d)^(p+1)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1675, 643}

$$\int (d + ex + fx^2)^p \left(-2ce^2 + 2cdf - ce^2p + 2cf^2(3 + 2p)x^2\right) dx = 2cfx\left(d + ex + fx^2\right)^{p+1} - \frac{ce(p + 2)\left(d + ex + fx^2\right)^{p+1}}{p + 1}$$

```
[In] Int[(d + e*x + f*x^2)^p*(-2*c*e^2 + 2*c*d*f - c*e^2*p + 2*c*f^2*(3 + 2*p)*x
^2),x]
```

```
[Out] -((c*e*(2 + p)*(d + e*x + f*x^2)^(1 + p))/(1 + p)) + 2*c*f*x*(d + e*x + f*x
^2)^(1 + p)
```

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 2cfx\left(d+ex+fx^2\right)^{1+p} \\
&\quad + \frac{\int \left(-ce^2f(2+p)(3+2p)-2cef^2(2+p)(3+2p)x\right)\left(d+ex+fx^2\right)^p\,dx}{f(3+2p)} \\
&= -\frac{ce(2+p)\left(d+ex+fx^2\right)^{1+p}}{1+p}+2cfx\left(d+ex+fx^2\right)^{1+p}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74

$$
\int \left(d+ex+fx^2\right)^p\left(-2ce^2+2cdf-ce^2p+2cf^2(3+2p)x^2\right)\,dx = \frac{c(-e(2+p)+2f(1+p)x)(d+x(e+fx))^{1+p}}{1+p}
$$

```
[In] Integrate[(d + e*x + f*x^2)^p*(-2*c*e^2 + 2*c*d*f - c*e^2*p + 2*c*f^2*(3 +
2*p)*x^2),x]
[Out] (c*(-(e*(2 + p)) + 2*f*(1 + p)*x)*(d + x*(e + f*x))^(1 + p))/(1 + p)
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

method	result
gosper	$-\frac{c\left(f x^2+ex+d\right)^{1+p}\left(-2fpx+ep-2fx+2e\right)}{1+p}$
risch	$-\frac{c\left(-2p f^2x^3-efp x^2-2f^2x^3-2dfpx+e^2px+dep-2dfx+2e^2x+2de\right)\left(f x^2+ex+d\right)^p}{1+p}$
norman	$\frac{c\left(2dfp-e^2p+2df-2e^2\right)x\,\mathrm{e}^{p\ln\left(f x^2+ex+d\right)}}{1+p}+\frac{cefp\,x^2\mathrm{e}^{p\ln\left(f x^2+ex+d\right)}}{1+p}+2c f^2x^3\mathrm{e}^{p\ln\left(f x^2+ex+d\right)}-\frac{cde(2+p)\mathrm{e}^{p\ln(f x}}{1+p}$
parallelrisch	$\frac{2x^3\left(f x^2+ex+d\right)^p cd f^2p+2x^3\left(f x^2+ex+d\right)^p cd f^2+x^2\left(f x^2+ex+d\right)^p cdefp+2x\left(f x^2+ex+d\right)^p c d^2fp-x\left(f x^2+ex+d\right)^p cd e^2p}{(1+p)d}$

```
[In] int((f*x^2+e*x+d)^p*(-2*c*e^2+2*c*d*f-c*e^2*p+2*c*f^2*(3+2*p)*x^2),x,method
=_RETURNVERBOSE)
```

```
[Out] -c/(1+p)*(f*x^2+e*x+d)^(1+p)*(-2*f*p*x+e*p-2*f*x+2*e)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.80

$$\int \left(d+ex+fx^2\right)^p \left(-2ce^2+2cdf-ce^2p+2cf^2(3+2p)x^2\right)\,dx \\ =\frac{(cefpx^2-cdep+2\,(cf^2p+cf^2)x^3-2\,cde-(2\,ce^2-2\,cdf+(ce^2-2\,cdf)p)x)(fx^2+ex+d)^p}{p+1}$$

```
[In] integrate((f*x^2+e*x+d)^p*(-2*c*e^2+2*c*d*f-c*e^2*p+2*c*f^2*(3+2*p)*x^2),x,
 algorithm="fricas")
```

```
[Out] (c*e*f*p*x^2 - c*d*e*p + 2*(c*f^2*p + c*f^2)*x^3 - 2*c*d*e - (2*c*e^2 - 2*c
*d*f + (c*e^2 - 2*c*d*f)*p)*x)*(f*x^2 + e*x + d)^p/(p + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 280 vs. 2(42) = 84.

Time = 53.17 (sec) , antiderivative size = 280, normalized size of antiderivative = 6.09

$$\int \left(d+ex+fx^2\right)^p \left(-2ce^2+2cdf-ce^2p+2cf^2(3+2p)x^2\right)\,dx \\ =\begin{cases} -\frac{cdep(d+ex+fx^2)^p}{p+1}-\frac{2cde(d+ex+fx^2)^p}{p+1}+\frac{2cdfpx(d+ex+fx^2)^p}{p+1}+\frac{2cdfx(d+ex+fx^2)^p}{p+1}-\frac{ce^2px(d+ex+fx^2)^p}{p+1}-\frac{2ce^2x(d+ex+fx^2}{p+1} \\ -ce\log\left(\frac{e}{2f}+x-\frac{\sqrt{-4df+e^2}}{2f}\right)-ce\log\left(\frac{e}{2f}+x+\frac{\sqrt{-4df+e^2}}{2f}\right)+2cfx \end{cases}$$

```
[In] integrate((f*x**2+e*x+d)**p*(-2*c*e**2+2*c*d*f-c*e**2*p+2*c*f**2*(3+2*p)*x*
*2),x)
```

```
[Out] Piecewise((-c*d*e*p*(d + e*x + f*x**2)**p/(p + 1) - 2*c*d*e*(d + e*x + f*x*
*2)**p/(p + 1) + 2*c*d*f*p*x*(d + e*x + f*x**2)**p/(p + 1) + 2*c*d*f*x*(d +
 e*x + f*x**2)**p/(p + 1) - c*e**2*p*x*(d + e*x + f*x**2)**p/(p + 1) - 2*c*
e**2*x*(d + e*x + f*x**2)**p/(p + 1) + c*e*f*p*x**2*(d + e*x + f*x**2)**p/(
p + 1) + 2*c*f**2*p*x**3*(d + e*x + f*x**2)**p/(p + 1) + 2*c*f**2*x**3*(d +
 e*x + f*x**2)**p/(p + 1), Ne(p, -1)), (-c*e*log(e/(2*f) + x - sqrt(-4*d*f
+ e**2)/(2*f)) - c*e*log(e/(2*f) + x + sqrt(-4*d*f + e**2)/(2*f)) + 2*c*f*x
, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.43

$$\int \left(d+ex+fx^2\right)^p \left(-2ce^2+2cdf-ce^2p+2cf^2(3+2p)x^2\right)\,dx = \frac{(2\,cf^2(p+1)x^3+cefpx^2-cde(p+2)-(e^2(p+2)-2\,df(p+1))cx)(fx^2+ex+d)^p}{p+1}$$

```
[In] integrate((f*x^2+e*x+d)^p*(-2*c*e^2+2*c*d*f-c*e^2*p+2*c*f^2*(3+2*p)*x^2),x,
 algorithm="maxima")
```

```
[Out] (2*c*f^2*(p + 1)*x^3 + c*e*f*p*x^2 - c*d*e*(p + 2) - (e^2*(p + 2) - 2*d*f*(
p + 1))*c*x)*(f*x^2 + e*x + d)^p/(p + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 181 vs. 2(46) = 92.

Time = 0.31 (sec) , antiderivative size = 181, normalized size of antiderivative = 3.93

$$\int \left(d+ex+fx^2\right)^p \left(-2ce^2+2cdf-ce^2p+2cf^2(3+2p)x^2\right)\,dx = \frac{2\,(fx^2+ex+d)^p cf^2px^3+(fx^2+ex+d)^p cefpx^2+2\,(fx^2+ex+d)^p cf^2x^3-(fx^2+ex+d)^p ce^2px+2}{}$$

```
[In] integrate((f*x^2+e*x+d)^p*(-2*c*e^2+2*c*d*f-c*e^2*p+2*c*f^2*(3+2*p)*x^2),x,
 algorithm="giac")
```

```
[Out] (2*(f*x^2 + e*x + d)^p*c*f^2*p*x^3 + (f*x^2 + e*x + d)^p*c*e*f*p*x^2 + 2*(f
*x^2 + e*x + d)^p*c*f^2*x^3 - (f*x^2 + e*x + d)^p*c*e^2*p*x + 2*(f*x^2 + e*
x + d)^p*c*d*f*p*x - (f*x^2 + e*x + d)^p*c*d*e*p - 2*(f*x^2 + e*x + d)^p*c*
e^2*x + 2*(f*x^2 + e*x + d)^p*c*d*f*x - 2*(f*x^2 + e*x + d)^p*c*d*e)/(p + 1
)
```

Mupad [B] (verification not implemented)

Time = 13.49 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.70

$$\int \left(d+ex+fx^2\right)^p \left(-2ce^2+2cdf-ce^2p+2cf^2(3+2p)x^2\right)\,dx = \left(f\,x^2+e\,x+d\right)^p\,\left(2\,c\,f^2\,x^3+\frac{c\,x\,\left(2\,d\,f-e^2\,p-2\,e^2+2\,d\,f\,p\right)}{p+1}-\frac{c\,d\,e\,\left(p+2\right)}{p+1}+\frac{c\,e\,f\,p\,x^2}{p+1}\right)$$

```
[In] int(-(d + e*x + f*x^2)^p*(2*c*e^2 - 2*c*d*f + c*e^2*p - 2*c*f^2*x^2*(2*p +
3)),x)
```

```
[Out] (d + e*x + f*x^2)^p*(2*c*f^2*x^3 + (c*x*(2*d*f - e^2*p - 2*e^2 + 2*d*f*p))/
(p + 1) - (c*d*e*(p + 2))/(p + 1) + (c*e*f*p*x^2)/(p + 1))
```

3.277 $\int (d+ex+fx^2)^p \left(-2ce^2+2cdf+3bef-ce^2p+2\right.$

Optimal result

Integrand size = 69, antiderivative size = 57

$$\int \left(d+ex+fx^2\right)^p \left(-2ce^2+2cdf+3bef-ce^2p+2befp+2bf^2(3+2p)x+2cf^2(3+2p)x^2\right)\,dx = -\frac{(ce(2+p)-bf(3+2p))\left(d+ex+fx^2\right)^{1+p}}{1+p}+2cfx\left(d+ex+fx^2\right)^{1+p}$$

```
[Out] -(c*e*(2+p)-b*f*(3+2*p))*(f*x^2+e*x+d)^(p+1)/(p+1)+2*c*f*x*(f*x^2+e*x+d)^(p
+1)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {1675, 643}

$$\int \left(d+ex+fx^2\right)^p \left(-2ce^2+2cdf+3bef-ce^2p+2befp+2bf^2(3+2p)x+2cf^2(3+2p)x^2\right)\,dx = 2cfx\left(d+ex+fx^2\right)^{p+1}-\frac{(ce(p+2)-bf(2p+3))\left(d+ex+fx^2\right)^{p+1}}{p+1}$$

```
[In] Int[(d + e*x + f*x^2)^p*(-2*c*e^2 + 2*c*d*f + 3*b*e*f - c*e^2*p + 2*b*e*f*p
 + 2*b*f^2*(3 + 2*p)*x + 2*c*f^2*(3 + 2*p)*x^2),x]
```

```
[Out] -(((c*e*(2 + p) - b*f*(3 + 2*p))*(d + e*x + f*x^2)^(1 + p))/(1 + p)) + 2*c*
f*x*(d + e*x + f*x^2)^(1 + p)
```

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
```

```
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= 2cfx\left(d+ex+fx^2\right)^{1+p} + \frac{\int \left(-ef(3+2p)(ce(2+p)-bf(3+2p))-2f^2(3+2p)(ce(2+p)-bf(3+2p))x\right)\left(d+ex+fx^2\right)^p\,dx}{f(3+2p)}\\ &= -\frac{(ce(2+p)-bf(3+2p))\left(d+ex+fx^2\right)^{1+p}}{1+p}+2cfx\left(d+ex+fx^2\right)^{1+p}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.75

$$\int \left(d+ex+fx^2\right)^p\left(-2ce^2+2cdf+3bef-ce^2p+2befp+2bf^2(3+2p)x+2cf^2(3+2p)x^2\right)\,dx=\frac{(-ce(2+p)+bf(3+2p)+2cf(1+p)x)(d+x(e+fx))^{1+p}}{1+p}$$

```
[In] Integrate[(d + e*x + f*x^2)^p*(-2*c*e^2 + 2*c*d*f + 3*b*e*f - c*e^2*p + 2*b
*e*f*p + 2*b*f^2*(3 + 2*p)*x + 2*c*f^2*(3 + 2*p)*x^2),x]
```

```
[Out] ((-(c*e*(2 + p)) + b*f*(3 + 2*p) + 2*c*f*(1 + p)*x)*(d + x*(e + f*x))^(1 +
p))/(1 + p)
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.89

method	result
gosper	$\frac{(f\,x^2+ex+d)^{1+p}(2cfxp+2bfp-cep+2cfx+3bf-2ce)}{1+p}$
risch	$\frac{(2pc\,f^2x^3+2b\,f^2p\,x^2+cefp\,x^2+2c\,f^2x^3+2befpx+3b\,f^2x^2+2cdfpx-c\,e^2px+2bdfp+3befx-cdep+2cdfx-2c\,e^2x+3bdf-2cde)}{1+p}$
norman	$\frac{d(2bfp-cep+3bf-2ce)e^{p\ln\left(f\,x^2+ex+d\right)}}{1+p}+\frac{(2befp+2cdfp-c\,e^2p+3bef+2cdf-2c\,e^2)x\,e^{p\ln\left(f\,x^2+ex+d\right)}}{1+p}+$ f(2bfp+cep+3b
parallelrisch	$2x^3(f\,x^2+ex+d)^p cd\,f^2p+2x^3(f\,x^2+ex+d)^p cd\,f^2+2x^2(f\,x^2+ex+d)^p bd\,f^2p+x^2(f\,x^2+ex+d)^p cdefp+3x^2(f\,x^2+ex+d)^p bd$

```
[In] int((f*x^2+e*x+d)^p*(-2*c*e^2+2*c*d*f+3*b*e*f-c*e^2*p+2*b*e*f*p+2*b*f^2*(3+
2*p)*x+2*c*f^2*(3+2*p)*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(1+p)*(f*x^2+e*x+d)^(1+p)*(2*c*f*p*x+2*b*f*p-c*e*p+2*c*f*x+3*b*f-2*c*e)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 123 vs. 2(56) = 112.

Time = 0.26 (sec) , antiderivative size = 123, normalized size of antiderivative = 2.16

$$\int \left(d+ex+fx^2\right)^p\left(-2ce^2+2cdf+3bef-ce^2p+2befp+2bf^2(3+2p)x+2cf^2(3+2p)x^2\right)\,dx$$
$$=\frac{(2\,(cf^2p+cf^2)x^3-2\,cde+3\,bdf+(3\,bf^2+(cef+2\,bf^2)p)x^2-(cde-2\,bdf)p-(2\,ce^2-(2\,cd+3\,be)f}{p+1}$$

```
[In] integrate((f*x^2+e*x+d)^p*(-2*c*e^2+2*c*d*f+3*b*e*f-c*e^2*p+2*b*e*f*p+2*b*f
^2*(3+2*p)*x+2*c*f^2*(3+2*p)*x^2),x, algorithm="fricas")
```

```
[Out] (2*(c*f^2*p + c*f^2)*x^3 - 2*c*d*e + 3*b*d*f + (3*b*f^2 + (c*e*f + 2*b*f^2)
*p)*x^2 - (c*d*e - 2*b*d*f)*p - (2*c*e^2 - (2*c*d + 3*b*e)*f + (c*e^2 - 2*(
c*d + b*e)*f)*p)*x)*(f*x^2 + e*x + d)^p/(p + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 483 vs. 2(51) = 102.

Time = 58.82 (sec) , antiderivative size = 483, normalized size of antiderivative = 8.47

$$\int \left(d+ex+fx^2\right)^p\left(-2ce^2+2cdf+3bef-ce^2p+2befp+2bf^2(3+2p)x+2cf^2(3+2p)x^2\right)\,dx$$
$$=\begin{cases}\frac{2bdfp(d+ex+fx^2)^p}{p+1}+\frac{3bdf(d+ex+fx^2)^p}{p+1}+\frac{2befpx(d+ex+fx^2)^p}{p+1}+\frac{3befx(d+ex+fx^2)^p}{p+1}+\frac{2bf^2px^2(d+ex+fx^2)^p}{p+1}+\frac{3bf^2x^2(d+ex}{p+1} \\ bf\log\left(\frac{e}{2f}+x-\frac{\sqrt{-4df+e^2}}{2f}\right)+bf\log\left(\frac{e}{2f}+x+\frac{\sqrt{-4df+e^2}}{2f}\right)-ce\log\left(\frac{e}{2f}+x-\frac{\sqrt{-4df+e^2}}{2f}\right)-ce\log\left(\frac{e}{2f}+\right.\end{cases}$$

```
[In] integrate((f*x**2+e*x+d)**p*(-2*c*e**2+2*c*d*f+3*b*e*f-c*e**2*p+2*b*e*f*p+2
*b*f**2*(3+2*p)*x+2*c*f**2*(3+2*p)*x**2),x)
```

```
[Out] Piecewise((2*b*d*f*p*(d + e*x + f*x**2)**p/(p + 1) + 3*b*d*f*(d + e*x + f*x
**2)**p/(p + 1) + 2*b*e*f*p*x*(d + e*x + f*x**2)**p/(p + 1) + 3*b*e*f*x*(d
+ e*x + f*x**2)**p/(p + 1) + 2*b*f**2*p*x**2*(d + e*x + f*x**2)**p/(p + 1)
+ 3*b*f**2*x**2*(d + e*x + f*x**2)**p/(p + 1) - c*d*e*p*(d + e*x + f*x**2)*
*p/(p + 1) - 2*c*d*e*(d + e*x + f*x**2)**p/(p + 1) + 2*c*d*f*p*x*(d + e*x +
 f*x**2)**p/(p + 1) + 2*c*d*f*x*(d + e*x + f*x**2)**p/(p + 1) - c*e**2*p*x*
(d + e*x + f*x**2)**p/(p + 1) - 2*c*e**2*x*(d + e*x + f*x**2)**p/(p + 1) +
c*e*f*p*x**2*(d + e*x + f*x**2)**p/(p + 1) + 2*c*f**2*p*x**3*(d + e*x + f*x
**2)**p/(p + 1) + 2*c*f**2*x**3*(d + e*x + f*x**2)**p/(p + 1), Ne(p, -1)),
(b*f*log(e/(2*f) + x - sqrt(-4*d*f + e**2)/(2*f)) + b*f*log(e/(2*f) + x + s
qrt(-4*d*f + e**2)/(2*f)) - c*e*log(e/(2*f) + x - sqrt(-4*d*f + e**2)/(2*f)
) - c*e*log(e/(2*f) + x + sqrt(-4*d*f + e**2)/(2*f)) + 2*c*f*x, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.72

$$\int \left(d+ex+fx^2\right)^p \left(-2ce^2+2cdf+3bef-ce^2p+2befp+2bf^2(3+2p)x+2cf^2(3+2p)x^2\right)\,dx$$
$$=\frac{(2\,cf^2(p+1)x^3+bdf(2\,p+3)-cde(p+2)+(bf^2(2\,p+3)+cefp)x^2+(bef(2\,p+3)-(e^2(p+2)-2\,df}{p+1}$$

```
[In] integrate((f*x^2+e*x+d)^p*(-2*c*e^2+2*c*d*f+3*b*e*f-c*e^2*p+2*b*e*f*p+2*b*f
^2*(3+2*p)*x+2*c*f^2*(3+2*p)*x^2),x, algorithm="maxima")
```

```
[Out] (2*c*f^2*(p + 1)*x^3 + b*d*f*(2*p + 3) - c*d*e*(p + 2) + (b*f^2*(2*p + 3) +
 c*e*f*p)*x^2 + (b*e*f*(2*p + 3) - (e^2*(p + 2) - 2*d*f*(p + 1))*c)*x)*(f*x
^2 + e*x + d)^p/(p + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 296 vs. 2(56) = 112.

Time = 0.30 (sec) , antiderivative size = 296, normalized size of antiderivative = 5.19

$$\int \left(d+ex+fx^2\right)^p \left(-2ce^2+2cdf+3bef-ce^2p+2befp+2bf^2(3+2p)x+2cf^2(3+2p)x^2\right)\,dx$$
$$=\frac{2\,(fx^2+ex+d)^p cf^2px^3+(fx^2+ex+d)^p cefpx^2+2\,(fx^2+ex+d)^p bf^2px^2+2\,(fx^2+ex+d)^p cf^2x^3-}{}$$

```
[In] integrate((f*x^2+e*x+d)^p*(-2*c*e^2+2*c*d*f+3*b*e*f-c*e^2*p+2*b*e*f*p+2*b*f
^2*(3+2*p)*x+2*c*f^2*(3+2*p)*x^2),x, algorithm="giac")
```

```
[Out] (2*(f*x^2 + e*x + d)^p*c*f^2*p*x^3 + (f*x^2 + e*x + d)^p*c*e*f*p*x^2 + 2*(f
*x^2 + e*x + d)^p*b*f^2*p*x^2 + 2*(f*x^2 + e*x + d)^p*c*f^2*x^3 - (f*x^2 +
```

```
e*x + d)^p*c*e^2*p*x + 2*(f*x^2 + e*x + d)^p*c*d*f*p*x + 2*(f*x^2 + e*x + d
)^p*b*e*f*p*x + 3*(f*x^2 + e*x + d)^p*b*f^2*x^2 - (f*x^2 + e*x + d)^p*c*d*e
*p + 2*(f*x^2 + e*x + d)^p*b*d*f*p - 2*(f*x^2 + e*x + d)^p*c*e^2*x + 2*(f*x
^2 + e*x + d)^p*c*d*f*x + 3*(f*x^2 + e*x + d)^p*b*e*f*x - 2*(f*x^2 + e*x +
d)^p*c*d*e + 3*(f*x^2 + e*x + d)^p*b*d*f)/(p + 1)
```

Mupad [B] (verification not implemented)

Time = 13.57 (sec) , antiderivative size = 120, normalized size of antiderivative = 2.11

$$\int (d+ex+fx^2)^p \left(-2ce^2+2cdf+3bef-ce^2p+2befp+2bf^2(3+2p)x+2cf^2(3+2p)x^2\right)\,dx = \left(f\,x^2+e\,x+d\right)^p\left(\frac{x^2\,(3\,b\,f^2+2\,b\,f^2\,p+c\,e\,f\,p)}{p+1}+2\,c\,f^2\,x^3+\frac{d\,(3\,b\,f-2\,c\,e+2\,b\,f\,p-c\,e\,p)}{p+1}+\frac{x\,(3\,b\,e\,f-2\,c\,e^2+2\,c\,d\,f-c\,e^2\,p+2\,b\,e\,f\,p+2\,c\,d\,f\,p)}{p+1}\right)$$

```
[In] int((d + e*x + f*x^2)^p*(3*b*e*f - 2*c*e^2 + 2*c*d*f - c*e^2*p + 2*b*f^2*x*
(2*p + 3) + 2*c*f^2*x^2*(2*p + 3) + 2*b*e*f*p),x)
```

```
[Out] (d + e*x + f*x^2)^p*((x^2*(3*b*f^2 + 2*b*f^2*p + c*e*f*p))/(p + 1) + 2*c*f^
2*x^3 + (d*(3*b*f - 2*c*e + 2*b*f*p - c*e*p))/(p + 1) + (x*(3*b*e*f - 2*c*e
^2 + 2*c*d*f - c*e^2*p + 2*b*e*f*p + 2*c*d*f*p))/(p + 1))
```

3.278 $\int (d+ex)^3 \left(a+bx+cx^2\right)^5 \left(d(6bd+5ae)+\left(12cd^2+1\right.\right.$

Optimal result

Integrand size = 75, antiderivative size = 20

$$\int (d+ex)^3 \left(a+bx+cx^2\right)^5 \left(d(6bd+5ae)+\left(12cd^2+17bde+5ae^2\right) x + e(29cd+11be)x^2+17ce^2x^3\right) \, dx = (d+ex)^5 \left(a+bx+cx^2\right)^6$$

```
[Out] (e*x+d)^5*(c*x^2+b*x+a)^6
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.027, Rules used = {1638, 1604}

$$\int (d+ex)^3 \left(a+bx+cx^2\right)^5 \left(d(6bd+5ae)+\left(12cd^2+17bde+5ae^2\right) x + e(29cd+11be)x^2+17ce^2x^3\right) \, dx = (d+ex)^5 \left(a+bx+cx^2\right)^6$$

```
[In] Int[(d + e*x)^3*(a + b*x + c*x^2)^5*(d*(6*b*d + 5*a*e) + (12*c*d^2 + 17*b*d
*e + 5*a*e^2)*x + e*(29*c*d + 11*b*e)*x^2 + 17*c*e^2*x^3),x]
```

```
[Out] (d + e*x)^5*(a + b*x + c*x^2)^6
```

Rule 1604

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[Coeff[Pp, x, p]*x^(p - q - r + 1)*Qq
^(m + 1)*(Rr^(n + 1)/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]))
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
```

```
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rule 1638

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[(d + e*x)^(m + 1)*PolynomialQuotient[Pq, d + e*x, x]*
(a + b*x + c*x^2)^p, x] /; FreeQ[{a, b, c, d, e, m, p}, x] && PolyQ[Pq, x]
&& EqQ[PolynomialRemainder[Pq, d + e*x, x], 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int (d+ex)^4 \left(a+bx+cx^2\right)^5 \left(6bd+5ae+(12cd+11be)x+17cex^2\right)\,dx\\ &= (d+ex)^5 \left(a+bx+cx^2\right)^6\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 167 vs. 2(20) = 40.

Time = 0.30 (sec) , antiderivative size = 167, normalized size of antiderivative = 8.35

$$\begin{aligned}&\int (d+ex)^3 \left(a+bx+cx^2\right)^5 \left(d(6bd+5ae)+\left(12cd^2+17bde+5ae^2\right)x+e(29cd+11be)x^2\right.\\ &\left.+17ce^2x^3\right)\,dx = x\big(6a^5(b+cx)(d+ex)^5+15a^4x(b+cx)^2(d+ex)^5+20a^3x^2(b+cx)^3(d+ex)^5\\ &+15a^2x^3(b+cx)^4(d+ex)^5+6ax^4(b+cx)^5(d+ex)^5+x^5(b+cx)^6(d+ex)^5\\ &+a^6e\big(5d^4+10d^3ex+10d^2e^2x^2+5de^3x^3+e^4x^4\big)\big)\end{aligned}$$

```
[In] Integrate[(d + e*x)^3*(a + b*x + c*x^2)^5*(d*(6*b*d + 5*a*e) + (12*c*d^2 +
17*b*d*e + 5*a*e^2)*x + e*(29*c*d + 11*b*e)*x^2 + 17*c*e^2*x^3),x]
```

```
[Out] x*(6*a^5*(b + c*x)*(d + e*x)^5 + 15*a^4*x*(b + c*x)^2*(d + e*x)^5 + 20*a^3*
x^2*(b + c*x)^3*(d + e*x)^5 + 15*a^2*x^3*(b + c*x)^4*(d + e*x)^5 + 6*a*x^4*
(b + c*x)^5*(d + e*x)^5 + x^5*(b + c*x)^6*(d + e*x)^5 + a^6*e*(5*d^4 + 10*d
^3*e*x + 10*d^2*e^2*x^2 + 5*d*e^3*x^3 + e^4*x^4))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2051 vs. 2(20) = 40.

Time = 0.91 (sec) , antiderivative size = 2052, normalized size of antiderivative = 102.60

method	result	size
norman	Expression too large to display	2052
gosper	Expression too large to display	2460
risch	Expression too large to display	2468
parallelrisch	Expression too large to display	2468
default	Expression too large to display	8419

```
[In] int((e*x+d)^3*(c*x^2+b*x+a)^5*(d*(5*a*e+6*b*d)+(5*a*e^2+17*b*d*e+12*c*d^2)*
x+e*(11*b*e+29*c*d)*x^2+17*c*e^2*x^3),x,method=_RETURNVERBOSE)
```

```
[Out] (5*a^6*d^4*e+6*a^5*b*d^5)*x+(10*a^6*d^3*e^2+30*a^5*b*d^4*e+6*a^5*c*d^5+15*a
^4*b^2*d^5)*x^2+(10*a^6*d^2*e^3+60*a^5*b*d^3*e^2+30*a^5*c*d^4*e+75*a^4*b^2*
d^4*e+30*a^4*b*c*d^5+20*a^3*b^3*d^5)*x^3+(5*a^6*d*e^4+60*a^5*b*d^2*e^3+60*a
^5*c*d^3*e^2+150*a^4*b^2*d^3*e^2+150*a^4*b*c*d^4*e+15*a^4*c^2*d^5+100*a^3*b
^3*d^4*e+60*a^3*b^2*c*d^5+15*a^2*b^4*d^5)*x^4+(a^6*e^5+30*a^5*b*d*e^4+60*a^
5*c*d^2*e^3+150*a^4*b^2*d^2*e^3+300*a^4*b*c*d^3*e^2+75*a^4*c^2*d^4*e+200*a^
3*b^3*d^3*e^2+300*a^3*b^2*c*d^4*e+60*a^3*b*c^2*d^5+75*a^2*b^4*d^4*e+60*a^2*
b^3*c*d^5+6*a*b^5*d^5)*x^5+(6*a^5*b*e^5+30*a^5*c*d*e^4+75*a^4*b^2*d*e^4+300
*a^4*b*c*d^2*e^3+150*a^4*c^2*d^3*e^2+200*a^3*b^3*d^2*e^3+600*a^3*b^2*c*d^3*
e^2+300*a^3*b*c^2*d^4*e+20*a^3*c^3*d^5+150*a^2*b^4*d^3*e^2+300*a^2*b^3*c*d^
4*e+90*a^2*b^2*c^2*d^5+30*a*b^5*d^4*e+30*a*b^4*c*d^5+b^6*d^5)*x^6+(6*a^5*c*
e^5+15*a^4*b^2*e^5+150*a^4*b*c*d*e^4+150*a^4*c^2*d^2*e^3+100*a^3*b^3*d*e^4+
600*a^3*b^2*c*d^2*e^3+600*a^3*b*c^2*d^3*e^2+100*a^3*c^3*d^4*e+150*a^2*b^4*d
^2*e^3+600*a^2*b^3*c*d^3*e^2+450*a^2*b^2*c^2*d^4*e+60*a^2*b*c^3*d^5+60*a*b^
5*d^3*e^2+150*a*b^4*c*d^4*e+60*a*b^3*c^2*d^5+5*b^6*d^4*e+6*b^5*c*d^5)*x^7+(
30*a^4*b*c*e^5+75*a^4*c^2*d*e^4+20*a^3*b^3*e^5+300*a^3*b^2*c*d*e^4+600*a^3*
b*c^2*d^2*e^3+200*a^3*c^3*d^3*e^2+75*a^2*b^4*d*e^4+600*a^2*b^3*c*d^2*e^3+90
0*a^2*b^2*c^2*d^3*e^2+300*a^2*b*c^3*d^4*e+15*a^2*c^4*d^5+60*a*b^5*d^2*e^3+3
00*a*b^4*c*d^3*e^2+300*a*b^3*c^2*d^4*e+60*a*b^2*c^3*d^5+10*b^6*d^3*e^2+30*b
^5*c*d^4*e+15*b^4*c^2*d^5)*x^8+(15*a^4*c^2*e^5+60*a^3*b^2*c*e^5+300*a^3*b*c
^2*d*e^4+200*a^3*c^3*d^2*e^3+15*a^2*b^4*e^5+300*a^2*b^3*c*d*e^4+900*a^2*b^2
*c^2*d^2*e^3+600*a^2*b*c^3*d^3*e^2+75*a^2*c^4*d^4*e+30*a*b^5*d*e^4+300*a*b^
4*c*d^2*e^3+600*a*b^3*c^2*d^3*e^2+300*a*b^2*c^3*d^4*e+30*a*b*c^4*d^5+10*b^6
*d^2*e^3+60*b^5*c*d^3*e^2+75*b^4*c^2*d^4*e+20*b^3*c^3*d^5)*x^9+(60*a^3*b*c^
2*e^5+100*a^3*c^3*d*e^4+60*a^2*b^3*c*e^5+450*a^2*b^2*c^2*d*e^4+600*a^2*b*c^
3*d^2*e^3+150*a^2*c^4*d^3*e^2+6*a*b^5*e^5+150*a*b^4*c*d*e^4+600*a*b^3*c^2*d
^2*e^3+600*a*b^2*c^3*d^3*e^2+150*a*b*c^4*d^4*e+6*a*c^5*d^5+5*b^6*d*e^4+60*b
^5*c*d^2*e^3+150*b^4*c^2*d^3*e^2+100*b^3*c^3*d^4*e+15*b^2*c^4*d^5)*x^10+(20
*a^3*c^3*e^5+90*a^2*b^2*c^2*e^5+300*a^2*b*c^3*d*e^4+150*a^2*c^4*d^2*e^3+30*
a*b^4*c*e^5+300*a*b^3*c^2*d*e^4+600*a*b^2*c^3*d^2*e^3+300*a*b*c^4*d^3*e^2+3
0*a*c^5*d^4*e+b^6*e^5+30*b^5*c*d*e^4+150*b^4*c^2*d^2*e^3+200*b^3*c^3*d^3*e^
2+75*b^2*c^4*d^4*e+6*b*c^5*d^5)*x^11+(60*a^2*b*c^3*e^5+75*a^2*c^4*d*e^4+60*
a*b^3*c^2*e^5+300*a*b^2*c^3*d*e^4+300*a*b*c^4*d^2*e^3+60*a*c^5*d^3*e^2+6*b^
5*c*e^5+75*b^4*c^2*d*e^4+200*b^3*c^3*d^2*e^3+150*b^2*c^4*d^3*e^2+30*b*c^5*d
```

```
^4*e+c^6*d^5)*x^12+(15*a^2*c^4*e^5+60*a*b^2*c^3*e^5+150*a*b*c^4*d*e^4+60*a*
c^5*d^2*e^3+15*b^4*c^2*e^5+100*b^3*c^3*d*e^4+150*b^2*c^4*d^2*e^3+60*b*c^5*d
^3*e^2+5*c^6*d^4*e)*x^13+(30*a*b*c^4*e^5+30*a*c^5*d*e^4+20*b^3*c^3*e^5+75*b
^2*c^4*d*e^4+60*b*c^5*d^2*e^3+10*c^6*d^3*e^2)*x^14+(6*a*c^5*e^5+15*b^2*c^4*
e^5+30*b*c^5*d*e^4+10*c^6*d^2*e^3)*x^15+(6*b*c^5*e^5+5*c^6*d*e^4)*x^16+c^6*
e^5*x^17
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1779 vs. $2(20) = 40$.

Time = 0.27 (sec) , antiderivative size = 1779, normalized size of antiderivative = 88.95

$$\int (d+ex)^3 \left(a+bx+cx^2\right)^5 \left(d(6bd+5ae)+\left(12cd^2+17bde+5ae^2\right)x \right.$$
$$\left. +e(29cd+11be)x^2+17ce^2x^3\right)\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)^3*(c*x^2+b*x+a)^5*(d*(5*a*e+6*b*d)+(5*a*e^2+17*b*d*e+12*c
*d^2)*x+e*(11*b*e+29*c*d)*x^2+17*c*e^2*x^3),x, algorithm="fricas")
```

```
[Out] c^6*e^5*x^17 + (5*c^6*d*e^4 + 6*b*c^5*e^5)*x^16 + (10*c^6*d^2*e^3 + 30*b*c^
5*d*e^4 + 3*(5*b^2*c^4 + 2*a*c^5)*e^5)*x^15 + 5*(2*c^6*d^3*e^2 + 12*b*c^5*d
^2*e^3 + 3*(5*b^2*c^4 + 2*a*c^5)*d*e^4 + 2*(2*b^3*c^3 + 3*a*b*c^4)*e^5)*x^1
4 + 5*(c^6*d^4*e + 12*b*c^5*d^3*e^2 + 6*(5*b^2*c^4 + 2*a*c^5)*d^2*e^3 + 10*
(2*b^3*c^3 + 3*a*b*c^4)*d*e^4 + 3*(b^4*c^2 + 4*a*b^2*c^3 + a^2*c^4)*e^5)*x^
13 + (c^6*d^5 + 30*b*c^5*d^4*e + 30*(5*b^2*c^4 + 2*a*c^5)*d^3*e^2 + 100*(2*
b^3*c^3 + 3*a*b*c^4)*d^2*e^3 + 75*(b^4*c^2 + 4*a*b^2*c^3 + a^2*c^4)*d*e^4 +
 6*(b^5*c + 10*a*b^3*c^2 + 10*a^2*b*c^3)*e^5)*x^12 + (6*b*c^5*d^5 + 15*(5*b
^2*c^4 + 2*a*c^5)*d^4*e + 100*(2*b^3*c^3 + 3*a*b*c^4)*d^3*e^2 + 150*(b^4*c^
2 + 4*a*b^2*c^3 + a^2*c^4)*d^2*e^3 + 30*(b^5*c + 10*a*b^3*c^2 + 10*a^2*b*c^
3)*d*e^4 + (b^6 + 30*a*b^4*c + 90*a^2*b^2*c^2 + 20*a^3*c^3)*e^5)*x^11 + (3*
(5*b^2*c^4 + 2*a*c^5)*d^5 + 50*(2*b^3*c^3 + 3*a*b*c^4)*d^4*e + 150*(b^4*c^2
 + 4*a*b^2*c^3 + a^2*c^4)*d^3*e^2 + 60*(b^5*c + 10*a*b^3*c^2 + 10*a^2*b*c^3
)*d^2*e^3 + 5*(b^6 + 30*a*b^4*c + 90*a^2*b^2*c^2 + 20*a^3*c^3)*d*e^4 + 6*(a
*b^5 + 10*a^2*b^3*c + 10*a^3*b*c^2)*e^5)*x^10 + 5*(2*(2*b^3*c^3 + 3*a*b*c^4
)*d^5 + 15*(b^4*c^2 + 4*a*b^2*c^3 + a^2*c^4)*d^4*e + 12*(b^5*c + 10*a*b^3*c
^2 + 10*a^2*b*c^3)*d^3*e^2 + 2*(b^6 + 30*a*b^4*c + 90*a^2*b^2*c^2 + 20*a^3*
c^3)*d^2*e^3 + 6*(a*b^5 + 10*a^2*b^3*c + 10*a^3*b*c^2)*d*e^4 + 3*(a^2*b^4 +
 4*a^3*b^2*c + a^4*c^2)*e^5)*x^9 + 5*(3*(b^4*c^2 + 4*a*b^2*c^3 + a^2*c^4)*d
^5 + 6*(b^5*c + 10*a*b^3*c^2 + 10*a^2*b*c^3)*d^4*e + 2*(b^6 + 30*a*b^4*c +
90*a^2*b^2*c^2 + 20*a^3*c^3)*d^3*e^2 + 12*(a*b^5 + 10*a^2*b^3*c + 10*a^3*b*
c^2)*d^2*e^3 + 15*(a^2*b^4 + 4*a^3*b^2*c + a^4*c^2)*d*e^4 + 2*(2*a^3*b^3 +
3*a^4*b*c)*e^5)*x^8 + (6*(b^5*c + 10*a*b^3*c^2 + 10*a^2*b*c^3)*d^5 + 5*(b^6
 + 30*a*b^4*c + 90*a^2*b^2*c^2 + 20*a^3*c^3)*d^4*e + 60*(a*b^5 + 10*a^2*b^3
*c + 10*a^3*b*c^2)*d^3*e^2 + 150*(a^2*b^4 + 4*a^3*b^2*c + a^4*c^2)*d^2*e^3
```

```
+ 50*(2*a^3*b^3 + 3*a^4*b*c)*d*e^4 + 3*(5*a^4*b^2 + 2*a^5*c)*e^5)*x^7 + (6*
a^5*b*e^5 + (b^6 + 30*a*b^4*c + 90*a^2*b^2*c^2 + 20*a^3*c^3)*d^5 + 30*(a*b^
5 + 10*a^2*b^3*c + 10*a^3*b*c^2)*d^4*e + 150*(a^2*b^4 + 4*a^3*b^2*c + a^4*c
^2)*d^3*e^2 + 100*(2*a^3*b^3 + 3*a^4*b*c)*d^2*e^3 + 15*(5*a^4*b^2 + 2*a^5*c
)*d*e^4)*x^6 + (30*a^5*b*d*e^4 + a^6*e^5 + 6*(a*b^5 + 10*a^2*b^3*c + 10*a^3
*b*c^2)*d^5 + 75*(a^2*b^4 + 4*a^3*b^2*c + a^4*c^2)*d^4*e + 100*(2*a^3*b^3 +
 3*a^4*b*c)*d^3*e^2 + 30*(5*a^4*b^2 + 2*a^5*c)*d^2*e^3)*x^5 + 5*(12*a^5*b*d
^2*e^3 + a^6*d*e^4 + 3*(a^2*b^4 + 4*a^3*b^2*c + a^4*c^2)*d^5 + 10*(2*a^3*b^
3 + 3*a^4*b*c)*d^4*e + 6*(5*a^4*b^2 + 2*a^5*c)*d^3*e^2)*x^4 + 5*(12*a^5*b*d
^3*e^2 + 2*a^6*d^2*e^3 + 2*(2*a^3*b^3 + 3*a^4*b*c)*d^5 + 3*(5*a^4*b^2 + 2*a
^5*c)*d^4*e)*x^3 + (30*a^5*b*d^4*e + 10*a^6*d^3*e^2 + 3*(5*a^4*b^2 + 2*a^5*
c)*d^5)*x^2 + (6*a^5*b*d^5 + 5*a^6*d^4*e)*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2281 vs. $2(17) = 34$.

Time = 0.16 (sec) , antiderivative size = 2281, normalized size of antiderivative = 114.05

$$\int (d+ex)^3 \left(a+bx+cx^2\right)^5 \left(d(6bd+5ae)+\left(12cd^2+17bde+5ae^2\right)x + e(29cd+11be)x^2+17ce^2x^3\right)\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)**3*(c*x**2+b*x+a)**5*(d*(5*a*e+6*b*d)+(5*a*e**2+17*b*d*e+
12*c*d**2)*x+e*(11*b*e+29*c*d)*x**2+17*c*e**2*x**3),x)
```

```
[Out] c**6*e**5*x**17 + x**16*(6*b*c**5*e**5 + 5*c**6*d*e**4) + x**15*(6*a*c**5*e
**5 + 15*b**2*c**4*e**5 + 30*b*c**5*d*e**4 + 10*c**6*d**2*e**3) + x**14*(30
*a*b*c**4*e**5 + 30*a*c**5*d*e**4 + 20*b**3*c**3*e**5 + 75*b**2*c**4*d*e**4
 + 60*b*c**5*d**2*e**3 + 10*c**6*d**3*e**2) + x**13*(15*a**2*c**4*e**5 + 60
*a*b**2*c**3*e**5 + 150*a*b*c**4*d*e**4 + 60*a*c**5*d**2*e**3 + 15*b**4*c**
2*e**5 + 100*b**3*c**3*d*e**4 + 150*b**2*c**4*d**2*e**3 + 60*b*c**5*d**3*e*
*2 + 5*c**6*d**4*e) + x**12*(60*a**2*b*c**3*e**5 + 75*a**2*c**4*d*e**4 + 60
*a*b**3*c**2*e**5 + 300*a*b**2*c**3*d*e**4 + 300*a*b*c**4*d**2*e**3 + 60*a*
c**5*d**3*e**2 + 6*b**5*c*e**5 + 75*b**4*c**2*d*e**4 + 200*b**3*c**3*d**2*e
**3 + 150*b**2*c**4*d**3*e**2 + 30*b*c**5*d**4*e + c**6*d**5) + x**11*(20*a
**3*c**3*e**5 + 90*a**2*b**2*c**2*e**5 + 300*a**2*b*c**3*d*e**4 + 150*a**2*
c**4*d**2*e**3 + 30*a*b**4*c*e**5 + 300*a*b**3*c**2*d*e**4 + 600*a*b**2*c**
3*d**2*e**3 + 300*a*b*c**4*d**3*e**2 + 30*a*c**5*d**4*e + b**6*e**5 + 30*b*
*5*c*d*e**4 + 150*b**4*c**2*d**2*e**3 + 200*b**3*c**3*d**3*e**2 + 75*b**2*c
**4*d**4*e + 6*b*c**5*d**5) + x**10*(60*a**3*b*c**2*e**5 + 100*a**3*c**3*d*
e**4 + 60*a**2*b**3*c*e**5 + 450*a**2*b**2*c**2*d*e**4 + 600*a**2*b*c**3*d*
*2*e**3 + 150*a**2*c**4*d**3*e**2 + 6*a*b**5*e**5 + 150*a*b**4*c*d*e**4 + 6
00*a*b**3*c**2*d**2*e**3 + 600*a*b**2*c**3*d**3*e**2 + 150*a*b*c**4*d**4*e
+ 6*a*c**5*d**5 + 5*b**6*d*e**4 + 60*b**5*c*d**2*e**3 + 150*b**4*c**2*d**3*
```

```
e**2 + 100*b**3*c**3*d**4*e + 15*b**2*c**4*d**5) + x**9*(15*a**4*c**2*e**5
+ 60*a**3*b**2*c*e**5 + 300*a**3*b*c**2*d*e**4 + 200*a**3*c**3*d**2*e**3 +
15*a**2*b**4*e**5 + 300*a**2*b**3*c*d*e**4 + 900*a**2*b**2*c**2*d**2*e**3 +
 600*a**2*b*c**3*d**3*e**2 + 75*a**2*c**4*d**4*e + 30*a*b**5*d*e**4 + 300*a
*b**4*c*d**2*e**3 + 600*a*b**3*c**2*d**3*e**2 + 300*a*b**2*c**3*d**4*e + 30
*a*b*c**4*d**5 + 10*b**6*d**2*e**3 + 60*b**5*c*d**3*e**2 + 75*b**4*c**2*d**
4*e + 20*b**3*c**3*d**5) + x**8*(30*a**4*b*c*e**5 + 75*a**4*c**2*d*e**4 + 2
0*a**3*b**3*e**5 + 300*a**3*b**2*c*d*e**4 + 600*a**3*b*c**2*d**2*e**3 + 200
*a**3*c**3*d**3*e**2 + 75*a**2*b**4*d*e**4 + 600*a**2*b**3*c*d**2*e**3 + 90
0*a**2*b**2*c**2*d**3*e**2 + 300*a**2*b*c**3*d**4*e + 15*a**2*c**4*d**5 + 6
0*a*b**5*d**2*e**3 + 300*a*b**4*c*d**3*e**2 + 300*a*b**3*c**2*d**4*e + 60*a
*b**2*c**3*d**5 + 10*b**6*d**3*e**2 + 30*b**5*c*d**4*e + 15*b**4*c**2*d**5)
 + x**7*(6*a**5*c*e**5 + 15*a**4*b**2*e**5 + 150*a**4*b*c*d*e**4 + 150*a**4
*c**2*d**2*e**3 + 100*a**3*b**3*d*e**4 + 600*a**3*b**2*c*d**2*e**3 + 600*a*
*3*b*c**2*d**3*e**2 + 100*a**3*c**3*d**4*e + 150*a**2*b**4*d**2*e**3 + 600*
a**2*b**3*c*d**3*e**2 + 450*a**2*b**2*c**2*d**4*e + 60*a**2*b*c**3*d**5 + 6
0*a*b**5*d**3*e**2 + 150*a*b**4*c*d**4*e + 60*a*b**3*c**2*d**5 + 5*b**6*d**
4*e + 6*b**5*c*d**5) + x**6*(6*a**5*b*e**5 + 30*a**5*c*d*e**4 + 75*a**4*b**
2*d*e**4 + 300*a**4*b*c*d**2*e**3 + 150*a**4*c**2*d**3*e**2 + 200*a**3*b**3
*d**2*e**3 + 600*a**3*b**2*c*d**3*e**2 + 300*a**3*b*c**2*d**4*e + 20*a**3*c
**3*d**5 + 150*a**2*b**4*d**3*e**2 + 300*a**2*b**3*c*d**4*e + 90*a**2*b**2*
c**2*d**5 + 30*a*b**5*d**4*e + 30*a*b**4*c*d**5 + b**6*d**5) + x**5*(a**6*e
**5 + 30*a**5*b*d*e**4 + 60*a**5*c*d**2*e**3 + 150*a**4*b**2*d**2*e**3 + 30
0*a**4*b*c*d**3*e**2 + 75*a**4*c**2*d**4*e + 200*a**3*b**3*d**3*e**2 + 300*
a**3*b**2*c*d**4*e + 60*a**3*b*c**2*d**5 + 75*a**2*b**4*d**4*e + 60*a**2*b*
*3*c*d**5 + 6*a*b**5*d**5) + x**4*(5*a**6*d*e**4 + 60*a**5*b*d**2*e**3 + 60
*a**5*c*d**3*e**2 + 150*a**4*b**2*d**3*e**2 + 150*a**4*b*c*d**4*e + 15*a**4
*c**2*d**5 + 100*a**3*b**3*d**4*e + 60*a**3*b**2*c*d**5 + 15*a**2*b**4*d**5
) + x**3*(10*a**6*d**2*e**3 + 60*a**5*b*d**3*e**2 + 30*a**5*c*d**4*e + 75*a
**4*b**2*d**4*e + 30*a**4*b*c*d**5 + 20*a**3*b**3*d**5) + x**2*(10*a**6*d**
3*e**2 + 30*a**5*b*d**4*e + 6*a**5*c*d**5 + 15*a**4*b**2*d**5) + x*(5*a**6*
d**4*e + 6*a**5*b*d**5)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1779 vs. $2(20) = 40$.

Time = 0.21 (sec) , antiderivative size = 1779, normalized size of antiderivative = 88.95

$$\int (d+ex)^3 \left(a+bx+cx^2\right)^5 \left(d(6bd+5ae)+\left(12cd^2+17bde+5ae^2\right)x + e(29cd+11be)x^2+17ce^2x^3\right) \, dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)^3*(c*x^2+b*x+a)^5*(d*(5*a*e+6*b*d)+(5*a*e^2+17*b*d*e+12*c
*d^2)*x+e*(11*b*e+29*c*d)*x^2+17*c*e^2*x^3),x, algorithm="maxima")
```

```
[Out] c^6*e^5*x^17 + (5*c^6*d*e^4 + 6*b*c^5*e^5)*x^16 + (10*c^6*d^2*e^3 + 30*b*c^
5*d*e^4 + 3*(5*b^2*c^4 + 2*a*c^5)*e^5)*x^15 + 5*(2*c^6*d^3*e^2 + 12*b*c^5*d
^2*e^3 + 3*(5*b^2*c^4 + 2*a*c^5)*d*e^4 + 2*(2*b^3*c^3 + 3*a*b*c^4)*e^5)*x^1
4 + 5*(c^6*d^4*e + 12*b*c^5*d^3*e^2 + 6*(5*b^2*c^4 + 2*a*c^5)*d^2*e^3 + 10*
(2*b^3*c^3 + 3*a*b*c^4)*d*e^4 + 3*(b^4*c^2 + 4*a*b^2*c^3 + a^2*c^4)*e^5)*x^
13 + (c^6*d^5 + 30*b*c^5*d^4*e + 30*(5*b^2*c^4 + 2*a*c^5)*d^3*e^2 + 100*(2*
b^3*c^3 + 3*a*b*c^4)*d^2*e^3 + 75*(b^4*c^2 + 4*a*b^2*c^3 + a^2*c^4)*d*e^4 +
 6*(b^5*c + 10*a*b^3*c^2 + 10*a^2*b*c^3)*e^5)*x^12 + (6*b*c^5*d^5 + 15*(5*b
^2*c^4 + 2*a*c^5)*d^4*e + 100*(2*b^3*c^3 + 3*a*b*c^4)*d^3*e^2 + 150*(b^4*c^
2 + 4*a*b^2*c^3 + a^2*c^4)*d^2*e^3 + 30*(b^5*c + 10*a*b^3*c^2 + 10*a^2*b*c^
3)*d*e^4 + (b^6 + 30*a*b^4*c + 90*a^2*b^2*c^2 + 20*a^3*c^3)*e^5)*x^11 + (3*
(5*b^2*c^4 + 2*a*c^5)*d^5 + 50*(2*b^3*c^3 + 3*a*b*c^4)*d^4*e + 150*(b^4*c^2
 + 4*a*b^2*c^3 + a^2*c^4)*d^3*e^2 + 60*(b^5*c + 10*a*b^3*c^2 + 10*a^2*b*c^3
)*d^2*e^3 + 5*(b^6 + 30*a*b^4*c + 90*a^2*b^2*c^2 + 20*a^3*c^3)*d*e^4 + 6*(a
*b^5 + 10*a^2*b^3*c + 10*a^3*b*c^2)*e^5)*x^10 + 5*(2*(2*b^3*c^3 + 3*a*b*c^4
)*d^5 + 15*(b^4*c^2 + 4*a*b^2*c^3 + a^2*c^4)*d^4*e + 12*(b^5*c + 10*a*b^3*c
^2 + 10*a^2*b*c^3)*d^3*e^2 + 2*(b^6 + 30*a*b^4*c + 90*a^2*b^2*c^2 + 20*a^3*
c^3)*d^2*e^3 + 6*(a*b^5 + 10*a^2*b^3*c + 10*a^3*b*c^2)*d*e^4 + 3*(a^2*b^4 +
 4*a^3*b^2*c + a^4*c^2)*e^5)*x^9 + 5*(3*(b^4*c^2 + 4*a*b^2*c^3 + a^2*c^4)*d
^5 + 6*(b^5*c + 10*a*b^3*c^2 + 10*a^2*b*c^3)*d^4*e + 2*(b^6 + 30*a*b^4*c +
90*a^2*b^2*c^2 + 20*a^3*c^3)*d^3*e^2 + 12*(a*b^5 + 10*a^2*b^3*c + 10*a^3*b*
c^2)*d^2*e^3 + 15*(a^2*b^4 + 4*a^3*b^2*c + a^4*c^2)*d*e^4 + 2*(2*a^3*b^3 +
3*a^4*b*c)*e^5)*x^8 + (6*(b^5*c + 10*a*b^3*c^2 + 10*a^2*b*c^3)*d^5 + 5*(b^6
 + 30*a*b^4*c + 90*a^2*b^2*c^2 + 20*a^3*c^3)*d^4*e + 60*(a*b^5 + 10*a^2*b^3
*c + 10*a^3*b*c^2)*d^3*e^2 + 150*(a^2*b^4 + 4*a^3*b^2*c + a^4*c^2)*d^2*e^3
+ 50*(2*a^3*b^3 + 3*a^4*b*c)*d*e^4 + 3*(5*a^4*b^2 + 2*a^5*c)*e^5)*x^7 + (6*
a^5*b*e^5 + (b^6 + 30*a*b^4*c + 90*a^2*b^2*c^2 + 20*a^3*c^3)*d^5 + 30*(a*b^
5 + 10*a^2*b^3*c + 10*a^3*b*c^2)*d^4*e + 150*(a^2*b^4 + 4*a^3*b^2*c + a^4*c
^2)*d^3*e^2 + 100*(2*a^3*b^3 + 3*a^4*b*c)*d^2*e^3 + 15*(5*a^4*b^2 + 2*a^5*c
)*d*e^4)*x^6 + (30*a^5*b*d*e^4 + a^6*e^5 + 6*(a*b^5 + 10*a^2*b^3*c + 10*a^3
*b*c^2)*d^5 + 75*(a^2*b^4 + 4*a^3*b^2*c + a^4*c^2)*d^4*e + 100*(2*a^3*b^3 +
 3*a^4*b*c)*d^3*e^2 + 30*(5*a^4*b^2 + 2*a^5*c)*d^2*e^3)*x^5 + 5*(12*a^5*b*d
^2*e^3 + a^6*d*e^4 + 3*(a^2*b^4 + 4*a^3*b^2*c + a^4*c^2)*d^5 + 10*(2*a^3*b^
3 + 3*a^4*b*c)*d^4*e + 6*(5*a^4*b^2 + 2*a^5*c)*d^3*e^2)*x^4 + 5*(12*a^5*b*d
^3*e^2 + 2*a^6*d^2*e^3 + 2*(2*a^3*b^3 + 3*a^4*b*c)*d^5 + 3*(5*a^4*b^2 + 2*a
^5*c)*d^4*e)*x^3 + (30*a^5*b*d^4*e + 10*a^6*d^3*e^2 + 3*(5*a^4*b^2 + 2*a^5*
c)*d^5)*x^2 + (6*a^5*b*d^5 + 5*a^6*d^4*e)*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2467 vs. $2(20) = 40$.

Time = 0.28 (sec) , antiderivative size = 2467, normalized size of antiderivative = 123.35

$$\int (d+ex)^3 \left(a+bx+cx^2\right)^5 \left(d(6bd+5ae)+\left(12cd^2+17bde+5ae^2\right)x + e(29cd+11be)x^2+17ce^2x^3\right)\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)^3*(c*x^2+b*x+a)^5*(d*(5*a*e+6*b*d)+(5*a*e^2+17*b*d*e+12*c
*d^2)*x+e*(11*b*e+29*c*d)*x^2+17*c*e^2*x^3),x, algorithm="giac")

[Out] c^6*e^5*x^17 + 5*c^6*d*e^4*x^16 + 6*b*c^5*e^5*x^16 + 10*c^6*d^2*e^3*x^15 +
30*b*c^5*d*e^4*x^15 + 15*b^2*c^4*e^5*x^15 + 6*a*c^5*e^5*x^15 + 10*c^6*d^3*e
^2*x^14 + 60*b*c^5*d^2*e^3*x^14 + 75*b^2*c^4*d*e^4*x^14 + 30*a*c^5*d*e^4*x^
14 + 20*b^3*c^3*e^5*x^14 + 30*a*b*c^4*e^5*x^14 + 5*c^6*d^4*e*x^13 + 60*b*c^
5*d^3*e^2*x^13 + 150*b^2*c^4*d^2*e^3*x^13 + 60*a*c^5*d^2*e^3*x^13 + 100*b^3
*c^3*d*e^4*x^13 + 150*a*b*c^4*d*e^4*x^13 + 15*b^4*c^2*e^5*x^13 + 60*a*b^2*c
^3*e^5*x^13 + 15*a^2*c^4*e^5*x^13 + c^6*d^5*x^12 + 30*b*c^5*d^4*e*x^12 + 15
0*b^2*c^4*d^3*e^2*x^12 + 60*a*c^5*d^3*e^2*x^12 + 200*b^3*c^3*d^2*e^3*x^12 +
 300*a*b*c^4*d^2*e^3*x^12 + 75*b^4*c^2*d*e^4*x^12 + 300*a*b^2*c^3*d*e^4*x^1
2 + 75*a^2*c^4*d*e^4*x^12 + 6*b^5*c*e^5*x^12 + 60*a*b^3*c^2*e^5*x^12 + 60*a
^2*b*c^3*e^5*x^12 + 6*b*c^5*d^5*x^11 + 75*b^2*c^4*d^4*e*x^11 + 30*a*c^5*d^4
*e*x^11 + 200*b^3*c^3*d^3*e^2*x^11 + 300*a*b*c^4*d^3*e^2*x^11 + 150*b^4*c^2
*d^2*e^3*x^11 + 600*a*b^2*c^3*d^2*e^3*x^11 + 150*a^2*c^4*d^2*e^3*x^11 + 30*
b^5*c*d*e^4*x^11 + 300*a*b^3*c^2*d*e^4*x^11 + 300*a^2*b*c^3*d*e^4*x^11 + b^
6*e^5*x^11 + 30*a*b^4*c*e^5*x^11 + 90*a^2*b^2*c^2*e^5*x^11 + 20*a^3*c^3*e^5
*x^11 + 15*b^2*c^4*d^5*x^10 + 6*a*c^5*d^5*x^10 + 100*b^3*c^3*d^4*e*x^10 + 1
50*a*b*c^4*d^4*e*x^10 + 150*b^4*c^2*d^3*e^2*x^10 + 600*a*b^2*c^3*d^3*e^2*x^
10 + 150*a^2*c^4*d^3*e^2*x^10 + 60*b^5*c*d^2*e^3*x^10 + 600*a*b^3*c^2*d^2*e
^3*x^10 + 600*a^2*b*c^3*d^2*e^3*x^10 + 5*b^6*d*e^4*x^10 + 150*a*b^4*c*d*e^4
*x^10 + 450*a^2*b^2*c^2*d*e^4*x^10 + 100*a^3*c^3*d*e^4*x^10 + 6*a*b^5*e^5*x
^10 + 60*a^2*b^3*c*e^5*x^10 + 60*a^3*b*c^2*e^5*x^10 + 20*b^3*c^3*d^5*x^9 +
30*a*b*c^4*d^5*x^9 + 75*b^4*c^2*d^4*e*x^9 + 300*a*b^2*c^3*d^4*e*x^9 + 75*a^
2*c^4*d^4*e*x^9 + 60*b^5*c*d^3*e^2*x^9 + 600*a*b^3*c^2*d^3*e^2*x^9 + 600*a^
2*b*c^3*d^3*e^2*x^9 + 10*b^6*d^2*e^3*x^9 + 300*a*b^4*c*d^2*e^3*x^9 + 900*a^
2*b^2*c^2*d^2*e^3*x^9 + 200*a^3*c^3*d^2*e^3*x^9 + 30*a*b^5*d*e^4*x^9 + 300*
a^2*b^3*c*d*e^4*x^9 + 300*a^3*b*c^2*d*e^4*x^9 + 15*a^2*b^4*e^5*x^9 + 60*a^3
*b^2*c*e^5*x^9 + 15*a^4*c^2*e^5*x^9 + 15*b^4*c^2*d^5*x^8 + 60*a*b^2*c^3*d^5
*x^8 + 15*a^2*c^4*d^5*x^8 + 30*b^5*c*d^4*e*x^8 + 300*a*b^3*c^2*d^4*e*x^8 +
300*a^2*b*c^3*d^4*e*x^8 + 10*b^6*d^3*e^2*x^8 + 300*a*b^4*c*d^3*e^2*x^8 + 90
0*a^2*b^2*c^2*d^3*e^2*x^8 + 200*a^3*c^3*d^3*e^2*x^8 + 60*a*b^5*d^2*e^3*x^8
+ 600*a^2*b^3*c*d^2*e^3*x^8 + 600*a^3*b*c^2*d^2*e^3*x^8 + 75*a^2*b^4*d*e^4*
x^8 + 300*a^3*b^2*c*d*e^4*x^8 + 75*a^4*c^2*d*e^4*x^8 + 20*a^3*b^3*e^5*x^8 +
 30*a^4*b*c*e^5*x^8 + 6*b^5*c*d^5*x^7 + 60*a*b^3*c^2*d^5*x^7 + 60*a^2*b*c^3
```

```
*d^5*x^7 + 5*b^6*d^4*e*x^7 + 150*a*b^4*c*d^4*e*x^7 + 450*a^2*b^2*c^2*d^4*e*
x^7 + 100*a^3*c^3*d^4*e*x^7 + 60*a*b^5*d^3*e^2*x^7 + 600*a^2*b^3*c*d^3*e^2*
x^7 + 600*a^3*b*c^2*d^3*e^2*x^7 + 150*a^2*b^4*d^2*e^3*x^7 + 600*a^3*b^2*c*d
^2*e^3*x^7 + 150*a^4*c^2*d^2*e^3*x^7 + 100*a^3*b^3*d*e^4*x^7 + 150*a^4*b*c*
d*e^4*x^7 + 15*a^4*b^2*e^5*x^7 + 6*a^5*c*e^5*x^7 + b^6*d^5*x^6 + 30*a*b^4*c
*d^5*x^6 + 90*a^2*b^2*c^2*d^5*x^6 + 20*a^3*c^3*d^5*x^6 + 30*a*b^5*d^4*e*x^6
 + 300*a^2*b^3*c*d^4*e*x^6 + 300*a^3*b*c^2*d^4*e*x^6 + 150*a^2*b^4*d^3*e^2*
x^6 + 600*a^3*b^2*c*d^3*e^2*x^6 + 150*a^4*c^2*d^3*e^2*x^6 + 200*a^3*b^3*d^2
*e^3*x^6 + 300*a^4*b*c*d^2*e^3*x^6 + 75*a^4*b^2*d*e^4*x^6 + 30*a^5*c*d*e^4*
x^6 + 6*a^5*b*e^5*x^6 + 6*a*b^5*d^5*x^5 + 60*a^2*b^3*c*d^5*x^5 + 60*a^3*b*c
^2*d^5*x^5 + 75*a^2*b^4*d^4*e*x^5 + 300*a^3*b^2*c*d^4*e*x^5 + 75*a^4*c^2*d^
4*e*x^5 + 200*a^3*b^3*d^3*e^2*x^5 + 300*a^4*b*c*d^3*e^2*x^5 + 150*a^4*b^2*d
^2*e^3*x^5 + 60*a^5*c*d^2*e^3*x^5 + 30*a^5*b*d*e^4*x^5 + a^6*e^5*x^5 + 15*a
^2*b^4*d^5*x^4 + 60*a^3*b^2*c*d^5*x^4 + 15*a^4*c^2*d^5*x^4 + 100*a^3*b^3*d^
4*e*x^4 + 150*a^4*b*c*d^4*e*x^4 + 150*a^4*b^2*d^3*e^2*x^4 + 60*a^5*c*d^3*e^
2*x^4 + 60*a^5*b*d^2*e^3*x^4 + 5*a^6*d*e^4*x^4 + 20*a^3*b^3*d^5*x^3 + 30*a^
4*b*c*d^5*x^3 + 75*a^4*b^2*d^4*e*x^3 + 30*a^5*c*d^4*e*x^3 + 60*a^5*b*d^3*e^
2*x^3 + 10*a^6*d^2*e^3*x^3 + 15*a^4*b^2*d^5*x^2 + 6*a^5*c*d^5*x^2 + 30*a^5*
b*d^4*e*x^2 + 10*a^6*d^3*e^2*x^2 + 6*a^5*b*d^5*x + 5*a^6*d^4*e*x
```

Mupad [B] (verification not implemented)

Time = 14.16 (sec) , antiderivative size = 2026, normalized size of antiderivative = 101.30

$$\int (d+ex)^3\left(a+bx+cx^2\right)^5\left(d(6bd+5ae)+\left(12cd^2+17bde+5ae^2\right)x\right.$$
$$\left.+e(29cd+11be)x^2+17ce^2x^3\right)\,dx=\text{Too large to display}$$

```
[In] int((d + e*x)^3*(a + b*x + c*x^2)^5*(d*(5*a*e + 6*b*d) + x*(5*a*e^2 + 12*c*
d^2 + 17*b*d*e) + e*x^2*(11*b*e + 29*c*d) + 17*c*e^2*x^3),x)
```

```
[Out] x^6*(b^6*d^5 + 6*a^5*b*e^5 + 20*a^3*c^3*d^5 + 75*a^4*b^2*d*e^4 + 90*a^2*b^2
*c^2*d^5 + 150*a^2*b^4*d^3*e^2 + 200*a^3*b^3*d^2*e^3 + 150*a^4*c^2*d^3*e^2
+ 30*a*b^4*c*d^5 + 30*a*b^5*d^4*e + 30*a^5*c*d*e^4 + 300*a^2*b^3*c*d^4*e +
300*a^3*b*c^2*d^4*e + 300*a^4*b*c*d^2*e^3 + 600*a^3*b^2*c*d^3*e^2) + x^11*(
b^6*e^5 + 6*b*c^5*d^5 + 20*a^3*c^3*e^5 + 75*b^2*c^4*d^4*e + 90*a^2*b^2*c^2*
e^5 + 150*a^2*c^4*d^2*e^3 + 200*b^3*c^3*d^3*e^2 + 150*b^4*c^2*d^2*e^3 + 30*
a*b^4*c*e^5 + 30*a*c^5*d^4*e + 30*b^5*c*d*e^4 + 300*a*b*c^4*d^3*e^2 + 300*a
*b^3*c^2*d*e^4 + 300*a^2*b*c^3*d*e^4 + 600*a*b^2*c^3*d^2*e^3) + x^5*(a^6*e^
5 + 6*a*b^5*d^5 + 60*a^2*b^3*c*d^5 + 60*a^3*b*c^2*d^5 + 75*a^2*b^4*d^4*e +
75*a^4*c^2*d^4*e + 60*a^5*c*d^2*e^3 + 200*a^3*b^3*d^3*e^2 + 150*a^4*b^2*d^2
*e^3 + 30*a^5*b*d*e^4 + 300*a^3*b^2*c*d^4*e + 300*a^4*b*c*d^3*e^2) + x^3*(2
0*a^3*b^3*d^5 + 10*a^6*d^2*e^3 + 75*a^4*b^2*d^4*e + 60*a^5*b*d^3*e^2 + 30*a
^4*b*c*d^5 + 30*a^5*c*d^4*e) + x^12*(c^6*d^5 + 6*b^5*c*e^5 + 60*a*b^3*c^2*e
^5 + 60*a^2*b*c^3*e^5 + 60*a*c^5*d^3*e^2 + 75*a^2*c^4*d*e^4 + 75*b^4*c^2*d*
```

```
e^4 + 150*b^2*c^4*d^3*e^2 + 200*b^3*c^3*d^2*e^3 + 30*b*c^5*d^4*e + 300*a*b*
c^4*d^2*e^3 + 300*a*b^2*c^3*d*e^4) + x^7*(6*a^5*c*e^5 + 6*b^5*c*d^5 + 5*b^6
*d^4*e + 15*a^4*b^2*e^5 + 60*a*b^3*c^2*d^5 + 60*a^2*b*c^3*d^5 + 60*a*b^5*d^
3*e^2 + 100*a^3*b^3*d*e^4 + 100*a^3*c^3*d^4*e + 150*a^2*b^4*d^2*e^3 + 150*a
^4*c^2*d^2*e^3 + 150*a*b^4*c*d^4*e + 150*a^4*b*c*d*e^4 + 450*a^2*b^2*c^2*d^
4*e + 600*a^2*b^3*c*d^3*e^2 + 600*a^3*b*c^2*d^3*e^2 + 600*a^3*b^2*c*d^2*e^3
) + x^10*(6*a*b^5*e^5 + 6*a*c^5*d^5 + 5*b^6*d*e^4 + 15*b^2*c^4*d^5 + 60*a^2
*b^3*c*e^5 + 60*a^3*b*c^2*e^5 + 100*a^3*c^3*d*e^4 + 100*b^3*c^3*d^4*e + 60*
b^5*c*d^2*e^3 + 150*a^2*c^4*d^3*e^2 + 150*b^4*c^2*d^3*e^2 + 150*a*b*c^4*d^4
*e + 150*a*b^4*c*d*e^4 + 600*a*b^2*c^3*d^3*e^2 + 600*a*b^3*c^2*d^2*e^3 + 60
0*a^2*b*c^3*d^2*e^3 + 450*a^2*b^2*c^2*d*e^4) + x^8*(15*a^2*c^4*d^5 + 20*a^3
*b^3*e^5 + 15*b^4*c^2*d^5 + 10*b^6*d^3*e^2 + 60*a*b^2*c^3*d^5 + 60*a*b^5*d^
2*e^3 + 75*a^2*b^4*d*e^4 + 75*a^4*c^2*d*e^4 + 200*a^3*c^3*d^3*e^2 + 30*a^4*
b*c*e^5 + 30*b^5*c*d^4*e + 900*a^2*b^2*c^2*d^3*e^2 + 300*a*b^3*c^2*d^4*e +
300*a*b^4*c*d^3*e^2 + 300*a^2*b*c^3*d^4*e + 300*a^3*b^2*c*d*e^4 + 600*a^2*b
^3*c*d^2*e^3 + 600*a^3*b*c^2*d^2*e^3) + x^9*(15*a^2*b^4*e^5 + 15*a^4*c^2*e^
5 + 20*b^3*c^3*d^5 + 10*b^6*d^2*e^3 + 60*a^3*b^2*c*e^5 + 75*a^2*c^4*d^4*e +
 75*b^4*c^2*d^4*e + 60*b^5*c*d^3*e^2 + 200*a^3*c^3*d^2*e^3 + 30*a*b*c^4*d^5
 + 30*a*b^5*d*e^4 + 900*a^2*b^2*c^2*d^2*e^3 + 300*a*b^2*c^3*d^4*e + 300*a*b
^4*c*d^2*e^3 + 300*a^2*b^3*c*d*e^4 + 300*a^3*b*c^2*d*e^4 + 600*a*b^3*c^2*d^
3*e^2 + 600*a^2*b*c^3*d^3*e^2) + x^4*(5*a^6*d*e^4 + 15*a^2*b^4*d^5 + 15*a^4
*c^2*d^5 + 60*a^3*b^2*c*d^5 + 100*a^3*b^3*d^4*e + 60*a^5*b*d^2*e^3 + 60*a^5
*c*d^3*e^2 + 150*a^4*b^2*d^3*e^2 + 150*a^4*b*c*d^4*e) + x^13*(5*c^6*d^4*e +
 15*a^2*c^4*e^5 + 15*b^4*c^2*e^5 + 60*a*b^2*c^3*e^5 + 60*a*c^5*d^2*e^3 + 60
*b*c^5*d^3*e^2 + 100*b^3*c^3*d*e^4 + 150*b^2*c^4*d^2*e^3 + 150*a*b*c^4*d*e^
4) + c^6*e^5*x^17 + a^5*d^4*x*(5*a*e + 6*b*d) + 5*c^3*e^2*x^14*(4*b^3*e^3 +
 2*c^3*d^3 + 6*a*b*c*e^3 + 6*a*c^2*d*e^2 + 12*b*c^2*d^2*e + 15*b^2*c*d*e^2)
 + c^5*e^4*x^16*(6*b*e + 5*c*d) + a^4*d^3*x^2*(10*a^2*e^2 + 15*b^2*d^2 + 6*
a*c*d^2 + 30*a*b*d*e) + c^4*e^3*x^15*(15*b^2*e^2 + 10*c^2*d^2 + 6*a*c*e^2 +
 30*b*c*d*e)
```

3.279 $\int \frac{x^2+x^3}{-2+x+x^2} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 26

$$\int \frac{x^2+x^3}{-2+x+x^2} \, dx = \frac{x^2}{2} + \frac{2}{3} \log(1-x) + \frac{4}{3} \log(2+x)$$

```
[Out] 1/2*x^2+2/3*ln(1-x)+4/3*ln(2+x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1607, 814, 646, 31}

$$\int \frac{x^2+x^3}{-2+x+x^2} \, dx = \frac{x^2}{2} + \frac{2}{3} \log(1-x) + \frac{4}{3} \log(x+2)$$

```
[In] Int[(x^2 + x^3)/(-2 + x + x^2),x]
```

```
[Out] x^2/2 + (2*Log[1 - x])/3 + (4*Log[2 + x])/3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
```

```
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{x^2(1+x)}{-2+x+x^2}\,dx\\ &= \int \left(x+\frac{2x}{-2+x+x^2}\right)dx\\ &= \frac{x^2}{2}+2\int \frac{x}{-2+x+x^2}\,dx\\ &= \frac{x^2}{2}+\frac{2}{3}\int \frac{1}{-1+x}\,dx+\frac{4}{3}\int \frac{1}{2+x}\,dx\\ &= \frac{x^2}{2}+\frac{2}{3}\log(1-x)+\frac{4}{3}\log(2+x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{x^2+x^3}{-2+x+x^2}\,dx=\frac{x^2}{2}+\frac{2}{3}\log(1-x)+\frac{4}{3}\log(2+x)$$

```
[In] Integrate[(x^2 + x^3)/(-2 + x + x^2),x]
[Out] x^2/2 + (2*Log[1 - x])/3 + (4*Log[2 + x])/3
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.73

method	result	size
default	$\frac{x^{2}}{2}+\frac{2 \ln \left(-1+x\right)}{3}+\frac{4 \ln \left(2+x\right)}{3}$	19
norman	$\frac{x^{2}}{2}+\frac{2 \ln \left(-1+x\right)}{3}+\frac{4 \ln \left(2+x\right)}{3}$	19
risch	$\frac{x^{2}}{2}+\frac{2 \ln \left(-1+x\right)}{3}+\frac{4 \ln \left(2+x\right)}{3}$	19
parallelrisch	$\frac{x^{2}}{2}+\frac{2 \ln \left(-1+x\right)}{3}+\frac{4 \ln \left(2+x\right)}{3}$	19

```
[In] int((x^3+x^2)/(x^2+x-2),x,method=_RETURNVERBOSE)
[Out] 1/2*x^2+2/3*ln(-1+x)+4/3*ln(2+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.69

$$\int \frac{x^2+x^3}{-2+x+x^2}\,dx=\frac{1}{2}\,x^{2}+\frac{4}{3}\,\log\left(x+2\right)+\frac{2}{3}\,\log\left(x-1\right)$$

```
[In] integrate((x^3+x^2)/(x^2+x-2),x, algorithm="fricas")
[Out] 1/2*x^2 + 4/3*log(x + 2) + 2/3*log(x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{x^2+x^3}{-2+x+x^2}\,dx=\frac{x^{2}}{2}+\frac{2\log\left(x-1\right)}{3}+\frac{4\log\left(x+2\right)}{3}$$

```
[In] integrate((x**3+x**2)/(x**2+x-2),x)
[Out] x**2/2 + 2*log(x - 1)/3 + 4*log(x + 2)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.69

$$\int \frac{x^2+x^3}{-2+x+x^2}\,dx = \frac{1}{2}\,x^2 + \frac{4}{3}\,\log\left(x+2\right) + \frac{2}{3}\,\log\left(x-1\right)$$

```
[In] integrate((x^3+x^2)/(x^2+x-2),x, algorithm="maxima")
[Out] 1/2*x^2 + 4/3*log(x + 2) + 2/3*log(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{x^2+x^3}{-2+x+x^2}\,dx = \frac{1}{2}\,x^2 + \frac{4}{3}\,\log\left(|x+2|\right) + \frac{2}{3}\,\log\left(|x-1|\right)$$

```
[In] integrate((x^3+x^2)/(x^2+x-2),x, algorithm="giac")
[Out] 1/2*x^2 + 4/3*log(abs(x + 2)) + 2/3*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.69

$$\int \frac{x^2+x^3}{-2+x+x^2}\,dx = \frac{2\,\ln\left(x-1\right)}{3} + \frac{4\,\ln\left(x+2\right)}{3} + \frac{x^2}{2}$$

```
[In] int((x^2 + x^3)/(x + x^2 - 2),x)
[Out] (2*log(x - 1))/3 + (4*log(x + 2))/3 + x^2/2
```

3.280 $\int \frac{x^2\left(d+ex+fx^2+gx^3\right)}{\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 346

$$\int \frac{x^2(d+ex+fx^2+gx^3)}{\sqrt{a+bx+cx^2}}\,dx = \frac{(80c^2e-70bcf+63b^2g-64acg)\,x^2\sqrt{a+bx+cx^2}}{240c^3}$$
$$+\frac{(10cf-9bg)x^3\sqrt{a+bx+cx^2}}{40c^2}+\frac{gx^4\sqrt{a+bx+cx^2}}{5c}$$
$$-\frac{(1050b^3cf+40bc^2(36cd-55af)-945b^4g-60b^2c(20ce-49ag)+256ac^2(5ce-4ag)-2c(480c^3d-40}{1920c^5}$$
$$+\frac{(70b^4cf+48b^2c^2(2cd-5af)-32ac^3(4cd-3af)-63b^5g-40b^3c(2ce-7ag)+48abc^2(4ce-5ag))\,\text{arct}}{256c^{11/2}}$$

```
[Out] 1/256*(70*b^4*c*f+48*b^2*c^2*(-5*a*f+2*c*d)-32*a*c^3*(-3*a*f+4*c*d)-63*b^5*
g-40*b^3*c*(-7*a*g+2*c*e)+48*a*b*c^2*(-5*a*g+4*c*e))*arctanh(1/2*(2*c*x+b)/
c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(11/2)+1/240*(-64*a*c*g+63*b^2*g-70*b*c*f+80
*c^2*e)*x^2*(c*x^2+b*x+a)^(1/2)/c^3+1/40*(-9*b*g+10*c*f)*x^3*(c*x^2+b*x+a)^
(1/2)/c^2+1/5*g*x^4*(c*x^2+b*x+a)^(1/2)/c-1/1920*(1050*b^3*c*f+40*b*c^2*(-5
5*a*f+36*c*d)-945*b^4*g-60*b^2*c*(-49*a*g+20*c*e)+256*a*c^2*(-4*a*g+5*c*e)-
2*c*(480*c^3*d-40*c^2*(9*a*f+10*b*e)-315*b^3*g+14*b*c*(46*a*g+25*b*f))*x)*(
c*x^2+b*x+a)^(1/2)/c^5
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {1667, 846, 793, 635, 212}

$$\begin{aligned}
&\int \frac{x^2(d+ex+fx^2+gx^3)}{\sqrt{a+bx+cx^2}}\,dx \\
&= \frac{\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(-40b^3c(2ce-7ag)+48b^2c^2(2cd-5af)+48abc^2(4ce-5ag)-32ac^3(4cd-3a}{256c^{11/2}} \\
&\quad+\frac{x^2\sqrt{a+bx+cx^2}(-64acg+63b^2g-70bcf+80c^2e)}{240c^3} \\
&\quad-\frac{\sqrt{a+bx+cx^2}(-2cx(-40c^2(9af+10be)+14bc(46ag+25bf)-315b^3g+480c^3d)-60b^2c(20ce-49a}{1920c^5} \\
&\quad+\frac{x^3\sqrt{a+bx+cx^2}(10cf-9bg)}{40c^2}+\frac{gx^4\sqrt{a+bx+cx^2}}{5c}
\end{aligned}$$

```
[In] Int[(x^2*(d + e*x + f*x^2 + g*x^3))/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] ((80*c^2*e - 70*b*c*f + 63*b^2*g - 64*a*c*g)*x^2*Sqrt[a + b*x + c*x^2])/(24
0*c^3) + ((10*c*f - 9*b*g)*x^3*Sqrt[a + b*x + c*x^2])/(40*c^2) + (g*x^4*Sqr
t[a + b*x + c*x^2])/(5*c) - ((1050*b^3*c*f + 40*b*c^2*(36*c*d - 55*a*f) - 9
45*b^4*g - 60*b^2*c*(20*c*e - 49*a*g) + 256*a*c^2*(5*c*e - 4*a*g) - 2*c*(48
0*c^3*d - 40*c^2*(10*b*e + 9*a*f) - 315*b^3*g + 14*b*c*(25*b*f + 46*a*g))*x
)*Sqrt[a + b*x + c*x^2])/(1920*c^5) + ((70*b^4*c*f + 48*b^2*c^2*(2*c*d - 5*
a*f) - 32*a*c^3*(4*c*d - 3*a*f) - 63*b^5*g - 40*b^3*c*(2*c*e - 7*a*g) + 48*
a*b*c^2*(4*c*e - 5*a*g))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^
2])])/(256*c^(11/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
```

```
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 846

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{gx^4\sqrt{a+bx+cx^2}}{5c}+\frac{\int \frac{x^2\left(5cd+(5ce-4ag)x+\frac{1}{2}(10cf-9bg)x^2\right)}{\sqrt{a+bx+cx^2}}\,dx}{5c}\\
&= \frac{(10cf-9bg)x^3\sqrt{a+bx+cx^2}}{40c^2}+\frac{gx^4\sqrt{a+bx+cx^2}}{5c}\\
&\quad+\frac{\int \frac{x^2\left(\frac{1}{2}\left(40c^2d-30acf+27abg\right)+\frac{1}{4}\left(80c^2e-70bcf+63b^2g-64acg\right)x\right)}{\sqrt{a+bx+cx^2}}\,dx}{20c^2}\\
&= \frac{\left(80c^2e-70bcf+63b^2g-64acg\right)x^2\sqrt{a+bx+cx^2}}{240c^3}\\
&\quad+\frac{(10cf-9bg)x^3\sqrt{a+bx+cx^2}}{40c^2}+\frac{gx^4\sqrt{a+bx+cx^2}}{5c}\\
&\quad+\frac{\int \frac{x\left(-\frac{1}{2}a\left(80c^2e-70bcf+63b^2g-64acg\right)+\frac{1}{8}\left(480c^3d-40c^2(10be+9af)-315b^3g+14bc(25bf+46ag)\right)x\right)}{\sqrt{a+bx+cx^2}}\,dx}{60c^3}
\end{aligned}
$$

$$= \frac{(80c^2e - 70bcf + 63b^2g - 64acg)\,x^2\sqrt{a+bx+cx^2}}{240c^3} + \frac{(10cf - 9bg)x^3\sqrt{a+bx+cx^2}}{40c^2} + \frac{gx^4\sqrt{a+bx+cx^2}}{5c} - \frac{(1050b^3cf + 40bc^2(36cd - 55af) - 945b^4g - 60b^2c(20ce - 49ag) + 256ac^2(5ce - 4ag) - 2c(48}{1920c^5} + \frac{(70b^4cf + 48b^2c^2(2cd - 5af) - 32ac^3(4cd - 3af) - 63b^5g - 40b^3c(2ce - 7ag) + 48abc^2(4ce -}{256c^5}$$

$$= \frac{(80c^2e - 70bcf + 63b^2g - 64acg)\,x^2\sqrt{a+bx+cx^2}}{240c^3} + \frac{(10cf - 9bg)x^3\sqrt{a+bx+cx^2}}{40c^2} + \frac{gx^4\sqrt{a+bx+cx^2}}{5c} - \frac{(1050b^3cf + 40bc^2(36cd - 55af) - 945b^4g - 60b^2c(20ce - 49ag) + 256ac^2(5ce - 4ag) - 2c(48}{1920c^5} + \frac{(70b^4cf + 48b^2c^2(2cd - 5af) - 32ac^3(4cd - 3af) - 63b^5g - 40b^3c(2ce - 7ag) + 48abc^2(4ce -}{128c^5}$$

$$= \frac{(80c^2e - 70bcf + 63b^2g - 64acg)\,x^2\sqrt{a+bx+cx^2}}{240c^3} + \frac{(10cf - 9bg)x^3\sqrt{a+bx+cx^2}}{40c^2} + \frac{gx^4\sqrt{a+bx+cx^2}}{5c} - \frac{(1050b^3cf + 40bc^2(36cd - 55af) - 945b^4g - 60b^2c(20ce - 49ag) + 256ac^2(5ce - 4ag) - 2c(48}{1920c^5} + \frac{(70b^4cf + 48b^2c^2(2cd - 5af) - 32ac^3(4cd - 3af) - 63b^5g - 40b^3c(2ce - 7ag) + 48abc^2(4ce -}{256c^{11/2}}$$

Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 282, normalized size of antiderivative = 0.82

$$\int \frac{x^2(d + ex + fx^2 + gx^3)}{\sqrt{a + bx + cx^2}}\,dx = \frac{2\sqrt{c}\sqrt{a + x(b + cx)}(945b^4g - 210b^3c(5f + 3gx) + 4b^2c(300ce - 735ag + 7cx(25f + 18gx)) - 8bc^2(-a(2}{}$$

```
[In] Integrate[(x^2*(d + e*x + f*x^2 + g*x^3))/Sqrt[a + b*x + c*x^2],x]
[Out] (2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(945*b^4*g - 210*b^3*c*(5*f + 3*g*x) + 4*b
^2*c*(300*c*e - 735*a*g + 7*c*x*(25*f + 18*g*x)) - 8*b*c^2*(-(a*(275*f + 16
1*g*x)) + 2*c*(90*d + x*(50*e + 35*f*x + 27*g*x^2))) + 16*c^2*(64*a^2*g - a
*c*(80*e + x*(45*f + 32*g*x)) + 2*c^2*x*(30*d + x*(20*e + 3*x*(5*f + 4*g*x)
)))) + 15*(-70*b^4*c*f - 48*b^2*c^2*(2*c*d - 5*a*f) + 32*a*c^3*(4*c*d - 3*a
*f) + 63*b^5*g + 40*b^3*c*(2*c*e - 7*a*g) + 48*a*b*c^2*(-4*c*e + 5*a*g))*Lo
g[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/(3840*c^(11/2))
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 314, normalized size of antiderivative = 0.91

method	result
risch	$\frac{(384g\,c^4x^4-432b\,c^3g\,x^3+480c^4f\,x^3-512a\,c^3g\,x^2+504b^2c^2g\,x^2-560b\,c^3f\,x^2+640c^4e\,x^2+1288ab\,c^2gx-720a\,c^3fx-630gc\,b^3x+700f\,c^2}{1920}$
default	Expression too large to display

```
[In] int(x^2*(g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/1920*(384*c^4*g*x^4-432*b*c^3*g*x^3+480*c^4*f*x^3-512*a*c^3*g*x^2+504*b^2
*c^2*g*x^2-560*b*c^3*f*x^2+640*c^4*e*x^2+1288*a*b*c^2*g*x-720*a*c^3*f*x-630
*b^3*c*g*x+700*b^2*c^2*f*x-800*b*c^3*e*x+960*c^4*d*x+1024*a^2*c^2*g-2940*a*
b^2*c*g+2200*a*b*c^2*f-1280*a*c^3*e+945*b^4*g-1050*b^3*c*f+1200*b^2*c^2*e-1
440*b*c^3*d)*(c*x^2+b*x+a)^(1/2)/c^5-1/256*(240*a^2*b*c^2*g-96*a^2*c^3*f-28
0*a*b^3*c*g+240*a*b^2*c^2*f-192*a*b*c^3*e+128*a*c^4*d+63*b^5*g-70*b^4*c*f+8
0*b^3*c^2*e-96*b^2*c^3*d)/c^(11/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/
2))
```

Fricas [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 701, normalized size of antiderivative = 2.03

$$\int \frac{x^2(d+ex+fx^2+gx^3)}{\sqrt{a+bx+cx^2}}\,dx$$

$$= \Bigg[-\frac{15\,(32\,(3\,b^2c^3-4\,ac^4)d-16\,(5\,b^3c^2-12\,abc^3)e+2\,(35\,b^4c-120\,ab^2c^2+48\,a^2c^3)f-(63\,b^5-280\,ab^3c}{}$$

$$-\frac{15\,(32\,(3\,b^2c^3-4\,ac^4)d-16\,(5\,b^3c^2-12\,abc^3)e+2\,(35\,b^4c-120\,ab^2c^2+48\,a^2c^3)f-(63\,b^5-280\,ab^3c+}{}$$

```
[In] integrate(x^2*(g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [-1/7680*(15*(32*(3*b^2*c^3 - 4*a*c^4)*d - 16*(5*b^3*c^2 - 12*a*b*c^3)*e +
2*(35*b^4*c - 120*a*b^2*c^2 + 48*a^2*c^3)*f - (63*b^5 - 280*a*b^3*c + 240*a
^2*b*c^2)*g)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x +
a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(384*c^5*g*x^4 - 1440*b*c^4*d + 48*(10*
c^5*f - 9*b*c^4*g)*x^3 + 8*(80*c^5*e - 70*b*c^4*f + (63*b^2*c^3 - 64*a*c^4)
*g)*x^2 + 80*(15*b^2*c^3 - 16*a*c^4)*e - 50*(21*b^3*c^2 - 44*a*b*c^3)*f + (
945*b^4*c - 2940*a*b^2*c^2 + 1024*a^2*c^3)*g + 2*(480*c^5*d - 400*b*c^4*e +
```

```
 10*(35*b^2*c^3 - 36*a*c^4)*f - 7*(45*b^3*c^2 - 92*a*b*c^3)*g)*x)*sqrt(c*x^
2 + b*x + a))/c^6, -1/3840*(15*(32*(3*b^2*c^3 - 4*a*c^4)*d - 16*(5*b^3*c^2
- 12*a*b*c^3)*e + 2*(35*b^4*c - 120*a*b^2*c^2 + 48*a^2*c^3)*f - (63*b^5 - 2
80*a*b^3*c + 240*a^2*b*c^2)*g)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2
*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(384*c^5*g*x^4 - 1440*b*c^4
*d + 48*(10*c^5*f - 9*b*c^4*g)*x^3 + 8*(80*c^5*e - 70*b*c^4*f + (63*b^2*c^3
 - 64*a*c^4)*g)*x^2 + 80*(15*b^2*c^3 - 16*a*c^4)*e - 50*(21*b^3*c^2 - 44*a*
b*c^3)*f + (945*b^4*c - 2940*a*b^2*c^2 + 1024*a^2*c^3)*g + 2*(480*c^5*d - 4
00*b*c^4*e + 10*(35*b^2*c^3 - 36*a*c^4)*f - 7*(45*b^3*c^2 - 92*a*b*c^3)*g)*
x)*sqrt(c*x^2 + b*x + a))/c^6]
```

Sympy [A] (verification not implemented)

Time = 0.87 (sec) , antiderivative size = 700, normalized size of antiderivative = 2.02

$$\int \frac{x^2(d+ex+fx^2+gx^3)}{\sqrt{a+bx+cx^2}}\,dx$$

$$= \begin{cases} \Bigg(-\frac{a\left(-\frac{3a\left(-\frac{9bg}{10c}+f\right)}{4c}-\frac{5b\left(-\frac{4ag}{5c}-\frac{7b\left(-\frac{9bg}{10c}+f\right)}{8c}+e\right)}{6c}+d\right)}{2c} - \frac{b\Big(-\frac{2a\left(-\frac{4ag}{5c}-\frac{7b\left(-\frac{9bg}{10c}+f\right)}{8c}+e\right)}{3c}-\frac{3b\Big(-\frac{3a\left(-\frac{9bg}{10c}+f\right)}{4c}-\frac{5b\Big(-\frac{4ag}{5c}-\frac{7b\big(-\frac{9bg}{10c}+f}{8c}}{6c}}{4c}}{2c} \\ \frac{2\left(\frac{g(a+bx)^{\frac{11}{2}}}{11b^3}+\frac{(a+bx)^{\frac{9}{2}}(-5ag+bf)}{9b^3}+\frac{(a+bx)^{\frac{7}{2}}\cdot\left(10a^2g-4abf+b^2e\right)}{7b^3}+\frac{(a+bx)^{\frac{5}{2}}\left(-10a^3g+6a^2bf-3ab^2e+b^3d\right)}{5b^3}+\frac{(a+bx)^{\frac{3}{2}}\cdot\left(5a^4g-4a^3bf+3a^2b^2e-2ab^3d\right)}{3b^3}}{b^3} \\ \frac{\frac{dx^3}{3}+\frac{ex^4}{4}+\frac{fx^5}{5}+\frac{gx^6}{6}}{\sqrt{a}} \end{cases}$$

```
[In] integrate(x**2*(g*x**3+f*x**2+e*x+d)/(c*x**2+b*x+a)**(1/2),x)

[Out] Piecewise(((-a*(-3*a*(-9*b*g/(10*c) + f)/(4*c) - 5*b*(-4*a*g/(5*c) - 7*b*(-
9*b*g/(10*c) + f)/(8*c) + e)/(6*c) + d)/(2*c) - b*(-2*a*(-4*a*g/(5*c) - 7*b
*(-9*b*g/(10*c) + f)/(8*c) + e)/(3*c) - 3*b*(-3*a*(-9*b*g/(10*c) + f)/(4*c)
 - 5*b*(-4*a*g/(5*c) - 7*b*(-9*b*g/(10*c) + f)/(8*c) + e)/(6*c) + d)/(4*c))
/(2*c))*Piecewise((log(b + 2*sqrt(c)*sqrt(a + b*x + c*x**2) + 2*c*x)/sqrt(c
), Ne(a - b**2/(4*c), 0)), ((b/(2*c) + x)*log(b/(2*c) + x)/sqrt(c*(b/(2*c)
+ x)**2), True)) + sqrt(a + b*x + c*x**2)*(g*x**4/(5*c) + x**3*(-9*b*g/(10*
```

```
c) + f)/(4*c) + x**2*(-4*a*g/(5*c) - 7*b*(-9*b*g/(10*c) + f)/(8*c) + e)/(3*
c) + x*(-3*a*(-9*b*g/(10*c) + f)/(4*c) - 5*b*(-4*a*g/(5*c) - 7*b*(-9*b*g/(1
0*c) + f)/(8*c) + e)/(6*c) + d)/(2*c) + (-2*a*(-4*a*g/(5*c) - 7*b*(-9*b*g/(
10*c) + f)/(8*c) + e)/(3*c) - 3*b*(-3*a*(-9*b*g/(10*c) + f)/(4*c) - 5*b*(-4
*a*g/(5*c) - 7*b*(-9*b*g/(10*c) + f)/(8*c) + e)/(6*c) + d)/(4*c))/c), Ne(c,
 0)), (2*(g*(a + b*x)**(11/2)/(11*b**3) + (a + b*x)**(9/2)*(-5*a*g + b*f)/(
9*b**3) + (a + b*x)**(7/2)*(10*a**2*g - 4*a*b*f + b**2*e)/(7*b**3) + (a + b
*x)**(5/2)*(-10*a**3*g + 6*a**2*b*f - 3*a*b**2*e + b**3*d)/(5*b**3) + (a +
b*x)**(3/2)*(5*a**4*g - 4*a**3*b*f + 3*a**2*b**2*e - 2*a*b**3*d)/(3*b**3) +
 sqrt(a + b*x)*(-a**5*g + a**4*b*f - a**3*b**2*e + a**2*b**3*d)/b**3)/b**3,
 Ne(b, 0)), ((d*x**3/3 + e*x**4/4 + f*x**5/5 + g*x**6/6)/sqrt(a), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(d+ex+fx^2+gx^3)}{\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2*(g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 321, normalized size of antiderivative = 0.93

$$\begin{aligned}&\int \frac{x^2(d+ex+fx^2+gx^3)}{\sqrt{a+bx+cx^2}}\,dx\\&=\frac{1}{1920}\sqrt{cx^2+bx+a}\left(2\left(4\left(6\left(\frac{8\,gx}{c}+\frac{10\,c^4f-9\,bc^3g}{c^5}\right)x+\frac{80\,c^4e-70\,bc^3f+63\,b^2c^2g-64\,ac^3g}{c^5}\right)x+48\right.\right.\\&\quad-\frac{(96\,b^2c^3d-128\,ac^4d-80\,b^3c^2e+192\,abc^3e+70\,b^4cf-240\,ab^2c^2f+96\,a^2c^3f-63\,b^5g+280\,ab^3cg-24}{256\,c^{\frac{11}{2}}}\end{aligned}$$

```
[In] integrate(x^2*(g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/1920*sqrt(c*x^2 + b*x + a)*(2*(4*(6*(8*g*x/c + (10*c^4*f - 9*b*c^3*g)/c^5
)*x + (80*c^4*e - 70*b*c^3*f + 63*b^2*c^2*g - 64*a*c^3*g)/c^5)*x + (480*c^4
*d - 400*b*c^3*e + 350*b^2*c^2*f - 360*a*c^3*f - 315*b^3*c*g + 644*a*b*c^2*
```

```
g)/c^5)*x - (1440*b*c^3*d - 1200*b^2*c^2*e + 1280*a*c^3*e + 1050*b^3*c*f -
2200*a*b*c^2*f - 945*b^4*g + 2940*a*b^2*c*g - 1024*a^2*c^2*g)/c^5) - 1/256*
(96*b^2*c^3*d - 128*a*c^4*d - 80*b^3*c^2*e + 192*a*b*c^3*e + 70*b^4*c*f - 2
40*a*b^2*c^2*f + 96*a^2*c^3*f - 63*b^5*g + 280*a*b^3*c*g - 240*a^2*b*c^2*g)
*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(11/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(d+ex+fx^2+gx^3)}{\sqrt{a+bx+cx^2}}\,dx=\int \frac{x^2\,(g\,x^3+f\,x^2+e\,x+d)}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

```
[In] int((x^2*(d + e*x + f*x^2 + g*x^3))/(a + b*x + c*x^2)^(1/2),x)
[Out] int((x^2*(d + e*x + f*x^2 + g*x^3))/(a + b*x + c*x^2)^(1/2), x)
```

3.281 $\int \frac{x\left(d+ex+fx^2+gx^3\right)}{\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 245

$$\begin{aligned}\int \frac{x(d+ex+fx^2+gx^3)}{\sqrt{a+bx+cx^2}}\,dx &= \frac{(8cf-7bg)x^2\sqrt{a+bx+cx^2}}{24c^2}+\frac{gx^3\sqrt{a+bx+cx^2}}{4c}\\ &+\frac{(192c^3d-16c^2(9be+8af)-105b^3g+20bc(6bf+11ag)+2c(48c^2e-40bcf+35b^2g-36acg)\,x)\sqrt{a+}}{192c^4}\\ &-\frac{(40b^3cf+32bc^2(2cd-3af)-35b^4g-24b^2c(2ce-5ag)+16ac^2(4ce-3ag))\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{128c^{9/2}}\end{aligned}$$

```
[Out] -1/128*(40*b^3*c*f+32*b*c^2*(-3*a*f+2*c*d)-35*b^4*g-24*b^2*c*(-5*a*g+2*c*e)
+16*a*c^2*(-3*a*g+4*c*e))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2)
)/c^(9/2)+1/24*(-7*b*g+8*c*f)*x^2*(c*x^2+b*x+a)^(1/2)/c^2+1/4*g*x^3*(c*x^2+
b*x+a)^(1/2)/c+1/192*(192*c^3*d-16*c^2*(8*a*f+9*b*e)-105*b^3*g+20*b*c*(11*a
*g+6*b*f)+2*c*(-36*a*c*g+35*b^2*g-40*b*c*f+48*c^2*e)*x)*(c*x^2+b*x+a)^(1/2)
/c^4
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used

$= \{1667, 793, 635, 212\}$

$$\int \frac{x(d+ex+fx^2+gx^3)}{\sqrt{a+bx+cx^2}}\,dx =$$

$$-\frac{\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(-24b^2c(2ce-5ag)+32bc^2(2cd-3af)+16ac^2(4ce-3ag)-35b^4g+40b^3cf}{128c^{9/2}}$$

$$+\frac{\sqrt{a+bx+cx^2}(2cx(-36acg+35b^2g-40bcf+48c^2e)-16c^2(8af+9be)+20bc(11ag+6bf)-105b^3}{192c^4}$$

$$+\frac{x^2\sqrt{a+bx+cx^2}(8cf-7bg)}{24c^2}+\frac{gx^3\sqrt{a+bx+cx^2}}{4c}$$

```
[In] Int[(x*(d + e*x + f*x^2 + g*x^3))/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] ((8*c*f - 7*b*g)*x^2*Sqrt[a + b*x + c*x^2])/(24*c^2) + (g*x^3*Sqrt[a + b*x
+ c*x^2])/(4*c) + ((192*c^3*d - 16*c^2*(9*b*e + 8*a*f) - 105*b^3*g + 20*b*c
*(6*b*f + 11*a*g) + 2*c*(48*c^2*e - 40*b*c*f + 35*b^2*g - 36*a*c*g)*x)*Sqrt
[a + b*x + c*x^2])/(192*c^4) - ((40*b^3*c*f + 32*b*c^2*(2*c*d - 3*a*f) - 35
*b^4*g - 24*b^2*c*(2*c*e - 5*a*g) + 16*a*c^2*(4*c*e - 3*a*g))*ArcTanh[(b +
2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(128*c^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
```

```
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\text{integral} = \frac{gx^3\sqrt{a+bx+cx^2}}{4c} + \frac{\int \frac{x\left(4cd+(4ce-3ag)x+\frac{1}{2}(8cf-7bg)x^2\right)}{\sqrt{a+bx+cx^2}}\,dx}{4c}$$

$$= \frac{(8cf-7bg)x^2\sqrt{a+bx+cx^2}}{24c^2} + \frac{gx^3\sqrt{a+bx+cx^2}}{4c} + \frac{\int \frac{x\left(12c^2d-8acf+7abg+\frac{1}{4}\left(48c^2e-40bcf+35b^2g-36acg\right)x\right)}{\sqrt{a+bx+cx^2}}\,dx}{12c^2}$$

$$= \frac{(8cf-7bg)x^2\sqrt{a+bx+cx^2}}{24c^2} + \frac{gx^3\sqrt{a+bx+cx^2}}{4c} + \frac{(192c^3d-16c^2(9be+8af)-105b^3g+20bc(6bf+11ag)+2c(48c^2e-40bcf+35b^2g-36acg)x}{192c^4} - \frac{(40b^3cf+32bc^2(2cd-3af)-35b^4g-24b^2c(2ce-5ag)+16ac^2(4ce-3ag))\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{128c^4}$$

$$= \frac{(8cf-7bg)x^2\sqrt{a+bx+cx^2}}{24c^2} + \frac{gx^3\sqrt{a+bx+cx^2}}{4c} + \frac{(192c^3d-16c^2(9be+8af)-105b^3g+20bc(6bf+11ag)+2c(48c^2e-40bcf+35b^2g-36acg)x}{192c^4} - \frac{(40b^3cf+32bc^2(2cd-3af)-35b^4g-24b^2c(2ce-5ag)+16ac^2(4ce-3ag))\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx,\right.}{64c^4}$$

$$= \frac{(8cf-7bg)x^2\sqrt{a+bx+cx^2}}{24c^2} + \frac{gx^3\sqrt{a+bx+cx^2}}{4c} + \frac{(192c^3d-16c^2(9be+8af)-105b^3g+20bc(6bf+11ag)+2c(48c^2e-40bcf+35b^2g-36acg)x}{192c^4} - \frac{(40b^3cf+32bc^2(2cd-3af)-35b^4g-24b^2c(2ce-5ag)+16ac^2(4ce-3ag))\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+}}\right.}{128c^{9/2}}$$

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.81

$$\int \frac{x(d+ex+fx^2+gx^3)}{\sqrt{a+bx+cx^2}}\,dx$$
$$= \frac{2\sqrt{c}\sqrt{a+x(b+cx)}(-105b^3g+10bc(12bf+22ag+7bgx)-8c^2(18be+16af+10bfx+9agx+7bgx^2)}{}$$

```
[In] Integrate[(x*(d + e*x + f*x^2 + g*x^3))/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(-105*b^3*g + 10*b*c*(12*b*f + 22*a*g + 7*
b*g*x) - 8*c^2*(18*b*e + 16*a*f + 10*b*f*x + 9*a*g*x + 7*b*g*x^2) + 16*c^3*
(12*d + x*(6*e + 4*f*x + 3*g*x^2))) + 3*(40*b^3*c*f + 32*b*c^2*(2*c*d - 3*a
*f) - 35*b^4*g - 24*b^2*c*(2*c*e - 5*a*g) + 16*a*c^2*(4*c*e - 3*a*g))*Log[b
 + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/(384*c^(9/2))
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.86

method	result
risch	$\frac{\left(48g\,c^3x^3-56b\,c^2g\,x^2+64c^3f\,x^2-72a\,c^2gx+70b^2cgx-80b\,c^2fx+96c^3ex+220abcg-128a\,c^2f-105b^3g+120b^2cf-144b\,c^2e+192c^3d\right)\sqrt{c}}{192c^4}$
default	$g\left(\frac{x^3\sqrt{c\,x^2+bx+a}}{4c}-\frac{7b\left(\frac{x^2\sqrt{c\,x^2+bx+a}}{3c}-\frac{5b\left(\frac{x\sqrt{c\,x^2+bx+a}}{2c}-\frac{3b\left(\frac{\sqrt{c\,x^2+bx+a}}{c}-\frac{b\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{2c^{\frac{3}{2}}}\right)}{4c}-\frac{a\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx}\right.}{2c^{\frac{3}{2}}}\right.}{6c}\right.}{8c}\right.$

```
[In] int(x*(g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/192*(48*c^3*g*x^3-56*b*c^2*g*x^2+64*c^3*f*x^2-72*a*c^2*g*x+70*b^2*c*g*x-8
0*b*c^2*f*x+96*c^3*e*x+220*a*b*c*g-128*a*c^2*f-105*b^3*g+120*b^2*c*f-144*b*
c^2*e+192*c^3*d)*(c*x^2+b*x+a)^(1/2)/c^4+1/128*(48*a^2*c^2*g-120*a*b^2*c*g+
96*a*b*c^2*f-64*a*c^3*e+35*b^4*g-40*b^3*c*f+48*b^2*c^2*e-64*b*c^3*d)/c^(9/2
)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 499, normalized size of antiderivative = 2.04

$$\int \frac{x(d+ex+fx^2+gx^3)}{\sqrt{a+bx+cx^2}}\,dx$$

$$=\left[-\frac{3\left(64\,bc^3d-16\left(3\,b^2c^2-4\,ac^3\right)e+8\left(5\,b^3c-12\,abc^2\right)f-\left(35\,b^4-120\,ab^2c+48\,a^2c^2\right)g\right)\sqrt{c}\log\left(-8\,c^2x^2\right.}{}\right.$$

```
[In] integrate(x*(g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/768*(3*(64*b*c^3*d - 16*(3*b^2*c^2 - 4*a*c^3)*e + 8*(5*b^3*c - 12*a*b*c
^2)*f - (35*b^4 - 120*a*b^2*c + 48*a^2*c^2)*g)*sqrt(c)*log(-8*c^2*x^2 - 8*b
*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(48*c
^4*g*x^3 + 192*c^4*d - 144*b*c^3*e + 8*(8*c^4*f - 7*b*c^3*g)*x^2 + 8*(15*b^
2*c^2 - 16*a*c^3)*f - 5*(21*b^3*c - 44*a*b*c^2)*g + 2*(48*c^4*e - 40*b*c^3*
f + (35*b^2*c^2 - 36*a*c^3)*g)*x)*sqrt(c*x^2 + b*x + a))/c^5, 1/384*(3*(64*
b*c^3*d - 16*(3*b^2*c^2 - 4*a*c^3)*e + 8*(5*b^3*c - 12*a*b*c^2)*f - (35*b^4
 - 120*a*b^2*c + 48*a^2*c^2)*g)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(
2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(48*c^4*g*x^3 + 192*c^4*d
- 144*b*c^3*e + 8*(8*c^4*f - 7*b*c^3*g)*x^2 + 8*(15*b^2*c^2 - 16*a*c^3)*f -
 5*(21*b^3*c - 44*a*b*c^2)*g + 2*(48*c^4*e - 40*b*c^3*f + (35*b^2*c^2 - 36*
a*c^3)*g)*x)*sqrt(c*x^2 + b*x + a))/c^5]
```

Sympy [A] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 479, normalized size of antiderivative = 1.96

$$\int \frac{x(d+ex+fx^2+gx^3)}{\sqrt{a+bx+cx^2}}\,dx = \begin{cases} \left(-\frac{a\left(-\frac{3ag}{4c}-\frac{5b\left(-\frac{7bg}{8c}+f\right)}{6c}+e\right)}{2c}-\frac{b\left(-\frac{2a\left(-\frac{7bg}{8c}+f\right)}{3c}-\frac{3b\left(-\frac{3ag}{4c}-\frac{5b\left(-\frac{7bg}{8c}+f\right)}{6c}+e\right)}{4c}+d\right)}{2c}\right)\left(\begin{cases} \frac{\log\left(b+2\sqrt{c}\sqrt{a+bx+cx^2}+2cx\right)}{\sqrt{c}} & \text{for} \\ \frac{\left(\frac{b}{2c}+x\right)\log\left(\frac{b}{2c}+x\right)}{\sqrt{c\left(\frac{b}{2c}+x\right)^2}} & \text{oth} \end{cases}\right. \\ \frac{2\left(\frac{g(a+bx)^{\frac{9}{2}}}{9b^3}+\frac{(a+bx)^{\frac{7}{2}}(-4ag+bf)}{7b^3}+\frac{(a+bx)^{\frac{5}{2}}\cdot\left(6a^2g-3abf+b^2e\right)}{5b^3}+\frac{(a+bx)^{\frac{3}{2}}\left(-4a^3g+3a^2bf-2ab^2e+b^3d\right)}{3b^3}+\frac{\sqrt{a+bx}\left(a^4g-a^3bf+a^2b^2e-ab^3d\right)}{b^3}\right)}{b^2} \\ \frac{\frac{dx^2}{2}+\frac{ex^3}{3}+\frac{fx^4}{4}+\frac{gx^5}{5}}{\sqrt{a}} \end{cases}$$

```
[In] integrate(x*(g*x**3+f*x**2+e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Piecewise(((-a*(-3*a*g/(4*c) - 5*b*(-7*b*g/(8*c) + f)/(6*c) + e)/(2*c) - b*
(-2*a*(-7*b*g/(8*c) + f)/(3*c) - 3*b*(-3*a*g/(4*c) - 5*b*(-7*b*g/(8*c) + f)
/(6*c) + e)/(4*c) + d)/(2*c))*Piecewise((log(b + 2*sqrt(c)*sqrt(a + b*x + c
*x**2) + 2*c*x)/sqrt(c), Ne(a - b**2/(4*c), 0)), ((b/(2*c) + x)*log(b/(2*c)
 + x)/sqrt(c*(b/(2*c) + x)**2), True)) + sqrt(a + b*x + c*x**2)*(g*x**3/(4*
c) + x**2*(-7*b*g/(8*c) + f)/(3*c) + x*(-3*a*g/(4*c) - 5*b*(-7*b*g/(8*c) +
f)/(6*c) + e)/(2*c) + (-2*a*(-7*b*g/(8*c) + f)/(3*c) - 3*b*(-3*a*g/(4*c) -
5*b*(-7*b*g/(8*c) + f)/(6*c) + e)/(4*c) + d)/c), Ne(c, 0)), (2*(g*(a + b*x)
**(9/2)/(9*b**3) + (a + b*x)**(7/2)*(-4*a*g + b*f)/(7*b**3) + (a + b*x)**(5
/2)*(6*a**2*g - 3*a*b*f + b**2*e)/(5*b**3) + (a + b*x)**(3/2)*(-4*a**3*g +
```

```
3*a**2*b*f - 2*a*b**2*e + b**3*d)/(3*b**3) + sqrt(a + b*x)*(a**4*g - a**3*b
*f + a**2*b**2*e - a*b**3*d)/b**3)/b**2, Ne(b, 0)), ((d*x**2/2 + e*x**3/3 +
 f*x**4/4 + g*x**5/5)/sqrt(a), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x(d+ex+fx^2+gx^3)}{\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x*(g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.90

$$\int \frac{x(d+ex+fx^2+gx^3)}{\sqrt{a+bx+cx^2}}\,dx$$

$$= \frac{1}{192}\sqrt{cx^2+bx+a}\Bigg(2\Bigg(4\left(\frac{6\,gx}{c}+\frac{8\,c^3f-7\,bc^2g}{c^4}\right)x+\frac{48\,c^3e-40\,bc^2f+35\,b^2cg-36\,ac^2g}{c^4}\Bigg)x+\frac{192\,c^3d-}{}$$

$$+\frac{(64\,bc^3d-48\,b^2c^2e+64\,ac^3e+40\,b^3cf-96\,abc^2f-35\,b^4g+120\,ab^2cg-48\,a^2c^2g)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2}\right.\right.\right.}{128\,c^{\frac{9}{2}}}$$

```
[In] integrate(x*(g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/192*sqrt(c*x^2 + b*x + a)*(2*(4*(6*g*x/c + (8*c^3*f - 7*b*c^2*g)/c^4)*x +
 (48*c^3*e - 40*b*c^2*f + 35*b^2*c*g - 36*a*c^2*g)/c^4)*x + (192*c^3*d - 14
4*b*c^2*e + 120*b^2*c*f - 128*a*c^2*f - 105*b^3*g + 220*a*b*c*g)/c^4) + 1/1
28*(64*b*c^3*d - 48*b^2*c^2*e + 64*a*c^3*e + 40*b^3*c*f - 96*a*b*c^2*f - 35
*b^4*g + 120*a*b^2*c*g - 48*a^2*c^2*g)*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))*sqrt(c) + b))/c^(9/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(d+ex+fx^2+gx^3)}{\sqrt{a+bx+cx^2}}\,dx=\int \frac{x\,\left(g\,x^3+f\,x^2+e\,x+d\right)}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

```
[In] int((x*(d + e*x + f*x^2 + g*x^3))/(a + b*x + c*x^2)^(1/2),x)
[Out] int((x*(d + e*x + f*x^2 + g*x^3))/(a + b*x + c*x^2)^(1/2), x)
```

3.282 $\int \frac{d+ex+fx^2+gx^3}{\sqrt{a+bx+cx^2}} dx$

Optimal result

Integrand size = 30, antiderivative size = 177

$$\begin{aligned}\int \frac{d+ex+fx^2+gx^3}{\sqrt{a+bx+cx^2}}\,dx &= \frac{(24c^2e-18bcf+15b^2g-16acg)\sqrt{a+bx+cx^2}}{24c^3} \\ &+ \frac{(6cf-5bg)x\sqrt{a+bx+cx^2}}{12c^2}+\frac{gx^2\sqrt{a+bx+cx^2}}{3c} \\ &+ \frac{(16c^3d-8c^2(be+af)-5b^3g+6bc(bf+2ag))\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{7/2}}\end{aligned}$$

```
[Out] 1/16*(16*c^3*d-8*c^2*(a*f+b*e)-5*b^3*g+6*b*c*(2*a*g+b*f))*arctanh(1/2*(2*c*
x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(7/2)+1/24*(-16*a*c*g+15*b^2*g-18*b*c*f
+24*c^2*e)*(c*x^2+b*x+a)^(1/2)/c^3+1/12*(-5*b*g+6*c*f)*x*(c*x^2+b*x+a)^(1/2
)/c^2+1/3*g*x^2*(c*x^2+b*x+a)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used

$= \{1675, 654, 635, 212\}$

$$\int \frac{d+ex+fx^2+gx^3}{\sqrt{a+bx+cx^2}}\,dx$$
$$= \frac{\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-8c^2(af+be)+6bc(2ag+bf)-5b^3g+16c^3d\right)}{16c^{7/2}}$$
$$+ \frac{\sqrt{a+bx+cx^2}(-16acg+15b^2g-18bcf+24c^2e)}{24c^3}$$
$$+ \frac{x\sqrt{a+bx+cx^2}(6cf-5bg)}{12c^2} + \frac{gx^2\sqrt{a+bx+cx^2}}{3c}$$

```
[In] Int[(d + e*x + f*x^2 + g*x^3)/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] ((24*c^2*e - 18*b*c*f + 15*b^2*g - 16*a*c*g)*Sqrt[a + b*x + c*x^2])/(24*c^3
) + ((6*c*f - 5*b*g)*x*Sqrt[a + b*x + c*x^2])/(12*c^2) + (g*x^2*Sqrt[a + b*
x + c*x^2])/(3*c) + ((16*c^3*d - 8*c^2*(b*e + a*f) - 5*b^3*g + 6*b*c*(b*f +
 2*a*g))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(7/2
))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{gx^2\sqrt{a+bx+cx^2}}{3c} + \frac{\int \frac{3cd+(3ce-2ag)x+\frac{1}{2}(6cf-5bg)x^2}{\sqrt{a+bx+cx^2}}\,dx}{3c} \\
&= \frac{(6cf-5bg)x\sqrt{a+bx+cx^2}}{12c^2} + \frac{gx^2\sqrt{a+bx+cx^2}}{3c} \\
&\quad + \frac{\int \frac{\frac{1}{2}(12c^2d-6acf+5abg)+\frac{1}{4}(24c^2e-18bcf+15b^2g-16acg)x}{\sqrt{a+bx+cx^2}}\,dx}{6c^2} \\
&= \frac{(24c^2e-18bcf+15b^2g-16acg)\sqrt{a+bx+cx^2}}{24c^3} + \frac{(6cf-5bg)x\sqrt{a+bx+cx^2}}{12c^2} \\
&\quad + \frac{gx^2\sqrt{a+bx+cx^2}}{3c} + \frac{(16c^3d-8c^2(be+af)-5b^3g+6bc(bf+2ag))\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{16c^3} \\
&= \frac{(24c^2e-18bcf+15b^2g-16acg)\sqrt{a+bx+cx^2}}{24c^3} \\
&\quad + \frac{(6cf-5bg)x\sqrt{a+bx+cx^2}}{12c^2} + \frac{gx^2\sqrt{a+bx+cx^2}}{3c} \\
&\quad + \frac{(16c^3d-8c^2(be+af)-5b^3g+6bc(bf+2ag))\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{8c^3} \\
&= \frac{(24c^2e-18bcf+15b^2g-16acg)\sqrt{a+bx+cx^2}}{24c^3} \\
&\quad + \frac{(6cf-5bg)x\sqrt{a+bx+cx^2}}{12c^2} + \frac{gx^2\sqrt{a+bx+cx^2}}{3c} \\
&\quad + \frac{(16c^3d-8c^2(be+af)-5b^3g+6bc(bf+2ag))\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{7/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.79

$$\int \frac{d+ex+fx^2+gx^3}{\sqrt{a+bx+cx^2}}\,dx$$

$$= \frac{2\sqrt{c}\sqrt{a+x(b+cx)}(15b^2g-2c(9bf+8ag+5bgx)+4c^2(6e+x(3f+2gx)))+3(-16c^3d+8c^2(be+af)}{48c^{7/2}}$$

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3)/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(15*b^2*g - 2*c*(9*b*f + 8*a*g + 5*b*g*x)
+ 4*c^2*(6*e + x*(3*f + 2*g*x))) + 3*(-16*c^3*d + 8*c^2*(b*e + a*f) + 5*b^3
*g - 6*b*c*(b*f + 2*a*g))*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])
/(48*c^(7/2))
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.76

method	result
risch	$-\frac{(-8gc^2x^2+10bcgx-12c^2fx+16acg-15b^2g+18bcf-24c^2e)\sqrt{cx^2+bx+a}}{24c^3}+\frac{(12abcg-8ac^2f-5b^3g+6b^2cf-8bc^2e+16c^3d)\ln\Big(\frac{b}{2}}{16c^{\frac{7}{2}}}$
default	$\frac{d\ln\Big(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{cx^2+bx+a}\Big)}{\sqrt{c}}+g\Bigg(\frac{x^2\sqrt{cx^2+bx+a}}{3c}-\frac{5b\Big(\frac{x\sqrt{cx^2+bx+a}}{2c}-\frac{3b\Big(\frac{\sqrt{cx^2+bx+a}}{c}-\frac{b\ln\big(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{cx^2+bx+a}\big)}{2c^{\frac{3}{2}}}\Big)}{4c}-\frac{a\ln\big(\frac{b}{2}}{}}{6c}$

```
[In] int((g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*(-8*c^2*g*x^2+10*b*c*g*x-12*c^2*f*x+16*a*c*g-15*b^2*g+18*b*c*f-24*c^2
*e)*(c*x^2+b*x+a)^(1/2)/c^3+1/16*(12*a*b*c*g-8*a*c^2*f-5*b^3*g+6*b^2*c*f-8*
b*c^2*e+16*c^3*d)/c^(7/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.93

$$\int \frac{d+ex+fx^2+gx^3}{\sqrt{a+bx+cx^2}}\,dx$$

$$= \Bigg[\frac{3\,(16\,c^3d-8\,bc^2e+2\,(3\,b^2c-4\,ac^2)f-(5\,b^3-12\,abc)g)\sqrt{c}\log\big(-8\,c^2x^2-8\,bcx-b^2-4\,\sqrt{cx^2+bx+}}{}$$

$$-\frac{3\,(16\,c^3d-8\,bc^2e+2\,(3\,b^2c-4\,ac^2)f-(5\,b^3-12\,abc)g)\sqrt{-c}\arctan\Big(\frac{\sqrt{cx^2+bx+a}(2\,cx+b)\sqrt{-c}}{2\,(c^2x^2+bcx+ac)}\Big)-2\,(8\,c^3gx}{48\,c^4}$$

```
[In] integrate((g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/96*(3*(16*c^3*d - 8*b*c^2*e + 2*(3*b^2*c - 4*a*c^2)*f - (5*b^3 - 12*a*b*
c)*g)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c
*x + b)*sqrt(c) - 4*a*c) + 4*(8*c^3*g*x^2 + 24*c^3*e - 18*b*c^2*f + (15*b^2
*c - 16*a*c^2)*g + 2*(6*c^3*f - 5*b*c^2*g)*x)*sqrt(c*x^2 + b*x + a))/c^4, -
```

```
1/48*(3*(16*c^3*d - 8*b*c^2*e + 2*(3*b^2*c - 4*a*c^2)*f - (5*b^3 - 12*a*b*c
)*g)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^
2 + b*c*x + a*c)) - 2*(8*c^3*g*x^2 + 24*c^3*e - 18*b*c^2*f + (15*b^2*c - 16
*a*c^2)*g + 2*(6*c^3*f - 5*b*c^2*g)*x)*sqrt(c*x^2 + b*x + a))/c^4]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 347 vs. 2(172) = 344.

Time = 0.48 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.96

$$\int \frac{d+ex+fx^2+gx^3}{\sqrt{a+bx+cx^2}}\,dx = \begin{cases} \sqrt{a+bx+cx^2}\left(\frac{gx^2}{3c}+\frac{x\left(-\frac{5bg}{6c}+f\right)}{2c}+\frac{-\frac{2ag}{3c}-\frac{3b\left(-\frac{5bg}{6c}+f\right)}{4c}+e}{c}\right)+\left(-\frac{a\left(-\frac{5bg}{6c}+f\right)}{2c}-\frac{b\left(-\frac{2ag}{3c}-\frac{3b\left(-\frac{5bg}{6c}+f\right)}{4c}+e\right)}{2c}+d\right)\Bigg(\\ \frac{2d\sqrt{a+bx}+\frac{2e\left(-a\sqrt{a+bx}+\frac{(a+bx)^{\frac{3}{2}}}{3}\right)}{b}+\frac{2f\left(a^2\sqrt{a+bx}-\frac{2a(a+bx)^{\frac{3}{2}}}{3}+\frac{(a+bx)^{\frac{5}{2}}}{5}\right)}{b^2}+\frac{2g\left(-a^3\sqrt{a+bx}+a^2(a+bx)^{\frac{3}{2}}-\frac{3a(a+bx)^{\frac{5}{2}}}{5}+\frac{(a+bx)^{\frac{7}{2}}}{7}\right)}{b^3}}{b} \\ \frac{dx+\frac{ex^2}{2}+\frac{fx^3}{3}+\frac{gx^4}{4}}{\sqrt{a}} \end{cases}$$

```
[In] integrate((g*x**3+f*x**2+e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Piecewise((sqrt(a + b*x + c*x**2)*(g*x**2/(3*c) + x*(-5*b*g/(6*c) + f)/(2*c
) + (-2*a*g/(3*c) - 3*b*(-5*b*g/(6*c) + f)/(4*c) + e)/c) + (-a*(-5*b*g/(6*c
) + f)/(2*c) - b*(-2*a*g/(3*c) - 3*b*(-5*b*g/(6*c) + f)/(4*c) + e)/(2*c) +
d)*Piecewise((log(b + 2*sqrt(c)*sqrt(a + b*x + c*x**2) + 2*c*x)/sqrt(c), Ne
(a - b**2/(4*c), 0)), ((b/(2*c) + x)*log(b/(2*c) + x)/sqrt(c*(b/(2*c) + x)*
*2), True)), Ne(c, 0)), ((2*d*sqrt(a + b*x) + 2*e*(-a*sqrt(a + b*x) + (a +
b*x)**(3/2)/3)/b + 2*f*(a**2*sqrt(a + b*x) - 2*a*(a + b*x)**(3/2)/3 + (a +
b*x)**(5/2)/5)/b**2 + 2*g*(-a**3*sqrt(a + b*x) + a**2*(a + b*x)**(3/2) - 3*
a*(a + b*x)**(5/2)/5 + (a + b*x)**(7/2)/7)/b**3)/b, Ne(b, 0)), ((d*x + e*x*
*2/2 + f*x**3/3 + g*x**4/4)/sqrt(a), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex+fx^2+gx^3}{\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.81

$$\begin{aligned}&\int \frac{d+ex+fx^2+gx^3}{\sqrt{a+bx+cx^2}}\,dx\\&=\frac{1}{24}\sqrt{cx^2+bx+a}\left(2\left(\frac{4\,gx}{c}+\frac{6\,c^2f-5\,bcg}{c^3}\right)x+\frac{24\,c^2e-18\,bcf+15\,b^2g-16\,acg}{c^3}\right)\\&\quad-\frac{(16\,c^3d-8\,bc^2e+6\,b^2cf-8\,ac^2f-5\,b^3g+12\,abcg)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}+b\right|\right)}{16\,c^{\frac{7}{2}}}\end{aligned}$$

```
[In] integrate((g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*sqrt(c*x^2 + b*x + a)*(2*(4*g*x/c + (6*c^2*f - 5*b*c*g)/c^3)*x + (24*c
^2*e - 18*b*c*f + 15*b^2*g - 16*a*c*g)/c^3) - 1/16*(16*c^3*d - 8*b*c^2*e +
6*b^2*c*f - 8*a*c^2*f - 5*b^3*g + 12*a*b*c*g)*log(abs(2*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))*sqrt(c) + b))/c^(7/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2+gx^3}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{g\,x^3+f\,x^2+e\,x+d}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

```
[In] int((d + e*x + f*x^2 + g*x^3)/(a + b*x + c*x^2)^(1/2),x)
```

```
[Out] int((d + e*x + f*x^2 + g*x^3)/(a + b*x + c*x^2)^(1/2), x)
```

3.283 $\int \frac{d+ex+fx^2+gx^3}{x\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 155

$$\int \frac{d+ex+fx^2+gx^3}{x\sqrt{a+bx+cx^2}}\,dx = \frac{(4cf-3bg)\sqrt{a+bx+cx^2}}{4c^2}+\frac{gx\sqrt{a+bx+cx^2}}{2c} - \frac{d\,\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}} + \frac{(8c^2e+3b^2g-4c(bf+ag))\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{5/2}}$$

```
[Out] 1/8*(8*c^2*e+3*b^2*g-4*c*(a*g+b*f))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*
x+a)^(1/2))/c^(5/2)-d*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/a^
(1/2)+1/4*(-3*b*g+4*c*f)*(c*x^2+b*x+a)^(1/2)/c^2+1/2*g*x*(c*x^2+b*x+a)^(1/2
)/c
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {1667, 857, 635, 212, 738}

$$\int \frac{d+ex+fx^2+gx^3}{x\sqrt{a+bx+cx^2}}\,dx = \frac{\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(-4c(ag+bf)+3b^2g+8c^2e)}{8c^{5/2}} - \frac{d\,\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}} + \frac{\sqrt{a+bx+cx^2}(4cf-3bg)}{4c^2}+\frac{gx\sqrt{a+bx+cx^2}}{2c}$$

```
[In] Int[(d + e*x + f*x^2 + g*x^3)/(x*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] ((4*c*f - 3*b*g)*Sqrt[a + b*x + c*x^2])/(4*c^2) + (g*x*Sqrt[a + b*x + c*x^2
])/(2*c) - (d*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/Sqrt[
a] + ((8*c^2*e + 3*b^2*g - 4*c*(b*f + a*g))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*
Sqrt[a + b*x + c*x^2])])/(8*c^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{gx\sqrt{a+bx+cx^2}}{2c}+\frac{\int \frac{2cd+(2ce-ag)x+\frac{1}{2}(4cf-3bg)x^2}{x\sqrt{a+bx+cx^2}}\,dx}{2c}\\
&= \frac{(4cf-3bg)\sqrt{a+bx+cx^2}}{4c^2}+\frac{gx\sqrt{a+bx+cx^2}}{2c}+\frac{\int \frac{2c^2d+\frac{1}{4}(8c^2e+3b^2g-4c(bf+ag))x}{x\sqrt{a+bx+cx^2}}\,dx}{2c^2}\\
&= \frac{(4cf-3bg)\sqrt{a+bx+cx^2}}{4c^2}+\frac{gx\sqrt{a+bx+cx^2}}{2c}\\
&\quad +d\int\frac{1}{x\sqrt{a+bx+cx^2}}\,dx+\frac{(8c^2e+3b^2g-4c(bf+ag))\int\frac{1}{\sqrt{a+bx+cx^2}}\,dx}{8c^2}\\
&= \frac{(4cf-3bg)\sqrt{a+bx+cx^2}}{4c^2}+\frac{gx\sqrt{a+bx+cx^2}}{2c}\\
&\quad -(2d)\mathrm{Subst}\left(\int\frac{1}{4a-x^2}\,dx,x,\frac{2a+bx}{\sqrt{a+bx+cx^2}}\right)\\
&\quad +\frac{(8c^2e+3b^2g-4c(bf+ag))\,\mathrm{Subst}\left(\int\frac{1}{4c-x^2}\,dx,x,\frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{4c^2}\\
&= \frac{(4cf-3bg)\sqrt{a+bx+cx^2}}{4c^2}+\frac{gx\sqrt{a+bx+cx^2}}{2c}-\frac{d\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}}\\
&\quad +\frac{(8c^2e+3b^2g-4c(bf+ag))\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{5/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.87

$$
\begin{aligned}
&\int\frac{d+ex+fx^2+gx^3}{x\sqrt{a+bx+cx^2}}\,dx\\
&=\frac{1}{8}\left(\frac{2(4cf-3bg+2cgx)\sqrt{a+x(b+cx)}}{c^2}+\frac{16d\operatorname{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)}{\sqrt{a}}\right.\\
&\quad\left.+\frac{(-8c^2e-3b^2g+4c(bf+ag))\log\left(c^2\left(b+2cx-2\sqrt{c}\sqrt{a+x(b+cx)}\right)\right)}{c^{5/2}}\right)
\end{aligned}
$$

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3)/(x*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] ((2*(4*c*f - 3*b*g + 2*c*g*x)*Sqrt[a + x*(b + c*x)])/c^2 + (16*d*ArcTanh[(S
qrt[c]*x - Sqrt[a + x*(b + c*x)])/Sqrt[a]])/Sqrt[a] + ((-8*c^2*e - 3*b^2*g
+ 4*c*(b*f + a*g))*Log[c^2*(b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/
c^(5/2))/8
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.44

method	result
default	$\frac{e\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx+a}\right)}{\sqrt{c}}+g\left(\frac{x\sqrt{c\,x^{2}+bx+a}}{2c}-\frac{3b\left(\frac{\sqrt{c\,x^{2}+bx+a}}{c}-\frac{b\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx+a}\right)}{2c^{\frac{3}{2}}}\right)}{4c}-\frac{a\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx+\right.}}{2c^{\frac{3}{2}}}\right.$

```
[In] int((g*x^3+f*x^2+e*x+d)/x/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] e*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)+g*(1/2*x/c*(c*x^2+b*x
+a)^(1/2)-3/4*b/c*(1/c*(c*x^2+b*x+a)^(1/2)-1/2*b/c^(3/2)*ln((1/2*b+c*x)/c^(
1/2)+(c*x^2+b*x+a)^(1/2)))-1/2*a/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+
a)^(1/2)))+f*(1/c*(c*x^2+b*x+a)^(1/2)-1/2*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+
(c*x^2+b*x+a)^(1/2)))-d/a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/
x)
```

Fricas [A] (verification not implemented)

none

Time = 2.08 (sec) , antiderivative size = 733, normalized size of antiderivative = 4.73

$$\int \frac{d+ex+fx^2+gx^3}{x\sqrt{a+bx+cx^2}}\,dx = \left[\frac{8\sqrt{a}c^3d\log\left(-\frac{8\,abx+(b^2+4\,ac)x^2-4\,\sqrt{cx^2+bx+a}(bx+2\,a)\sqrt{a}+8\,a^2}{x^2}\right)-(8\,ac^2e-4\,abcf+(3\,ab^2-4\,a^2c)g)\sqrt{c}\log\left(-\right.}{16}\right.$$

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(8*sqrt(a)*c^3*d*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 + b
*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) - (8*a*c^2*e - 4*a*b*c*f + (3*a*b
^2 - 4*a^2*c)*g)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*
x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(2*a*c^2*g*x + 4*a*c^2*f - 3*a*b*c*
g)*sqrt(c*x^2 + b*x + a))/(a*c^3), 1/8*(4*sqrt(a)*c^3*d*log(-(8*a*b*x + (b^
2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2)
- (8*a*c^2*e - 4*a*b*c*f + (3*a*b^2 - 4*a^2*c)*g)*sqrt(-c)*arctan(1/2*sqrt(
c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(2*a*c^2
*g*x + 4*a*c^2*f - 3*a*b*c*g)*sqrt(c*x^2 + b*x + a))/(a*c^3), 1/16*(16*sqrt
(-a)*c^3*d*arctan(1/2*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2 +
 a*b*x + a^2)) - (8*a*c^2*e - 4*a*b*c*f + (3*a*b^2 - 4*a^2*c)*g)*sqrt(c)*lo
```

```
g(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c)
- 4*a*c) + 4*(2*a*c^2*g*x + 4*a*c^2*f - 3*a*b*c*g)*sqrt(c*x^2 + b*x + a))/(
a*c^3), 1/8*(8*sqrt(-a)*c^3*d*arctan(1/2*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*
sqrt(-a)/(a*c*x^2 + a*b*x + a^2)) - (8*a*c^2*e - 4*a*b*c*f + (3*a*b^2 - 4*a
^2*c)*g)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^
2*x^2 + b*c*x + a*c)) + 2*(2*a*c^2*g*x + 4*a*c^2*f - 3*a*b*c*g)*sqrt(c*x^2
+ b*x + a))/(a*c^3)]
```

Sympy [F]

$$\int \frac{d + ex + fx^2 + gx^3}{x\sqrt{a + bx + cx^2}}\,dx = \int \frac{d + ex + fx^2 + gx^3}{x\sqrt{a + bx + cx^2}}\,dx$$

```
[In] integrate((g*x**3+f*x**2+e*x+d)/x/(c*x**2+b*x+a)**(1/2),x)
[Out] Integral((d + e*x + f*x**2 + g*x**3)/(x*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d + ex + fx^2 + gx^3}{x\sqrt{a + bx + cx^2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [F(-2)]

Exception generated.

$$\int \frac{d + ex + fx^2 + gx^3}{x\sqrt{a + bx + cx^2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2+gx^3}{x\sqrt{a+bx+cx^2}}\,dx=\int \frac{g\,x^3+f\,x^2+e\,x+d}{x\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

```
[In] int((d + e*x + f*x^2 + g*x^3)/(x*(a + b*x + c*x^2)^(1/2)),x)
[Out] int((d + e*x + f*x^2 + g*x^3)/(x*(a + b*x + c*x^2)^(1/2)), x)
```

3.284 $\int \frac{d+ex+fx^2+gx^3}{x^2\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 139

$$\int \frac{d+ex+fx^2+gx^3}{x^2\sqrt{a+bx+cx^2}}\,dx = \frac{g\sqrt{a+bx+cx^2}}{c} - \frac{d\sqrt{a+bx+cx^2}}{ax} + \frac{(bd-2ae)\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{3/2}} + \frac{(2cf-bg)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}}$$

```
[Out] 1/2*(-2*a*e+b*d)*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/a^(3/2)
+1/2*(-b*g+2*c*f)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(3/2
)+g*(c*x^2+b*x+a)^(1/2)/c-d*(c*x^2+b*x+a)^(1/2)/a/x
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1664, 1667, 857, 635, 212, 738}

$$\int \frac{d+ex+fx^2+gx^3}{x^2\sqrt{a+bx+cx^2}}\,dx = \frac{(bd-2ae)\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{3/2}} + \frac{(2cf-bg)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}} - \frac{d\sqrt{a+bx+cx^2}}{ax} + \frac{g\sqrt{a+bx+cx^2}}{c}$$

```
[In] Int[(d + e*x + f*x^2 + g*x^3)/(x^2*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (g*Sqrt[a + b*x + c*x^2])/c - (d*Sqrt[a + b*x + c*x^2])/(a*x) + ((b*d - 2*a
*e)*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(2*a^(3/2)) + (
(2*c*f - b*g)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*c^
(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d\sqrt{a+bx+cx^2}}{ax} - \frac{\int \frac{\frac{1}{2}(bd-2ae)-afx-agx^2}{x\sqrt{a+bx+cx^2}}\,dx}{a} \\
&= \frac{g\sqrt{a+bx+cx^2}}{c} - \frac{d\sqrt{a+bx+cx^2}}{ax} - \frac{\int \frac{\frac{1}{2}c(bd-2ae)-\frac{1}{2}a(2cf-bg)x}{x\sqrt{a+bx+cx^2}}\,dx}{ac} \\
&= \frac{g\sqrt{a+bx+cx^2}}{c} - \frac{d\sqrt{a+bx+cx^2}}{ax} - \frac{(bd-2ae)\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx}{2a} + \frac{(2cf-bg)\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{2c} \\
&= \frac{g\sqrt{a+bx+cx^2}}{c} - \frac{d\sqrt{a+bx+cx^2}}{ax} + \frac{(bd-2ae)\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx}{\sqrt{a+bx+cx^2}}\right)}{a} \\
&\quad + \frac{(2cf-bg)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{c} \\
&= \frac{g\sqrt{a+bx+cx^2}}{c} - \frac{d\sqrt{a+bx+cx^2}}{ax} + \frac{(bd-2ae)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{3/2}} \\
&\quad + \frac{(2cf-bg)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.05

$$\begin{aligned}
\int \frac{d+ex+fx^2+gx^3}{x^2\sqrt{a+bx+cx^2}}\,dx &= \frac{(-cd+agx)\sqrt{a+x(b+cx)}}{acx} \\
&\quad + \frac{(2cf-bg)\text{arctanh}\left(\frac{\sqrt{c}x}{-\sqrt{a}+\sqrt{a+x(b+cx)}}\right)}{c^{3/2}} + \frac{(bd-2ae)\log(x)}{2a^{3/2}} \\
&\quad + \frac{(-bd+2ae)\log\left(a\left(2a+bx-2\sqrt{a}\sqrt{a+x(b+cx)}\right)\right)}{2a^{3/2}}
\end{aligned}$$

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3)/(x^2*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] ((-(c*d) + a*g*x)*Sqrt[a + x*(b + c*x)])/(a*c*x) + ((2*c*f - b*g)*ArcTanh[(
Sqrt[c]*x)/(-Sqrt[a] + Sqrt[a + x*(b + c*x)])])/c^(3/2) + ((b*d - 2*a*e)*Lo
g[x])/(2*a^(3/2)) + ((-(b*d) + 2*a*e)*Log[a*(2*a + b*x - 2*Sqrt[a]*Sqrt[a +
 x*(b + c*x)])])/(2*a^(3/2))
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.12

method	result
risch	$-\frac{d\sqrt{c\,x^{2}+bx+a}}{ax}+\frac{\frac{2fa\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx+a}\right)}{\sqrt{c}}+2ag\left(\frac{\sqrt{c\,x^{2}+bx+a}}{c}-\frac{b\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx+a}\right)}{2c^{\frac{3}{2}}}\right)-\frac{(2ae-bd)\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^{2}+}}{x}\right.}{\sqrt{a}}}{2a}$
default	$\frac{f\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx+a}\right)}{\sqrt{c}}+g\left(\frac{\sqrt{c\,x^{2}+bx+a}}{c}-\frac{b\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx+a}\right)}{2c^{\frac{3}{2}}}\right)-\frac{e\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^{2}+bx+a}}{x}\right)}{\sqrt{a}}+d\left(-\right.$

```
[In] int((g*x^3+f*x^2+e*x+d)/x^2/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -d*(c*x^2+b*x+a)^(1/2)/a/x+1/2/a*(2*f*a*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a
)^(1/2))/c^(1/2)+2*a*g*(1/c*(c*x^2+b*x+a)^(1/2)-1/2*b/c^(3/2)*ln((1/2*b+c*x
)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))-(2*a*e-b*d)/a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(
c*x^2+b*x+a)^(1/2))/x))
```

Fricas [A] (verification not implemented)

none

Time = 1.34 (sec) , antiderivative size = 703, normalized size of antiderivative = 5.06

$$\int \frac{d+ex+fx^2+gx^3}{x^2\sqrt{a+bx+cx^2}}\,dx$$

$$= \left[-\frac{(2\,a^2cf-a^2bg)\sqrt{c}x\log\left(-8\,c^2x^2-8\,bcx-b^2+4\,\sqrt{cx^2+bx+a}(2\,cx+b)\sqrt{c}-4\,ac\right)+(bc^2d-2\,ac^2e}{4\,a^2c^2x}\right.$$

$$-\frac{2\,(2\,a^2cf-a^2bg)\sqrt{-c}x\arctan\left(\frac{\sqrt{cx^2+bx+a}(2\,cx+b)\sqrt{-c}}{2\,(c^2x^2+bcx+ac)}\right)+(bc^2d-2\,ac^2e)\sqrt{a}x\log\left(-\frac{8\,abx+(b^2+4\,ac)x^2-4\,\sqrt{cx}}{x}\right.}{4\,a^2c^2x}$$

$$-\frac{2\,(bc^2d-2\,ac^2e)\sqrt{-a}x\arctan\left(\frac{\sqrt{cx^2+bx+a}(bx+2\,a)\sqrt{-a}}{2\,(acx^2+abx+a^2)}\right)+(2\,a^2cf-a^2bg)\sqrt{c}x\log\left(-8\,c^2x^2-8\,bcx-b^2\right.}{4\,a^2c^2x}$$

$$-\frac{(bc^2d-2\,ac^2e)\sqrt{-a}x\arctan\left(\frac{\sqrt{cx^2+bx+a}(bx+2\,a)\sqrt{-a}}{2\,(acx^2+abx+a^2)}\right)+(2\,a^2cf-a^2bg)\sqrt{-c}x\arctan\left(\frac{\sqrt{cx^2+bx+a}(2\,cx+b)\sqrt{}}{2\,(c^2x^2+bcx+ac)}\right.}{2\,a^2c^2x}$$

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x^2/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [-1/4*((2*a^2*c*f - a^2*b*g)*sqrt(c)*x*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*s
qrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + (b*c^2*d - 2*a*c^2*e)*s
qrt(a)*x*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 + b*x + a)*(b*x +
 2*a)*sqrt(a) + 8*a^2)/x^2) - 4*(a^2*c*g*x - a*c^2*d)*sqrt(c*x^2 + b*x + a)
)/(a^2*c^2*x), -1/4*(2*(2*a^2*c*f - a^2*b*g)*sqrt(-c)*x*arctan(1/2*sqrt(c*x
^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + (b*c^2*d - 2*
a*c^2*e)*sqrt(a)*x*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 + b*x +
 a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) - 4*(a^2*c*g*x - a*c^2*d)*sqrt(c*x^2
+ b*x + a))/(a^2*c^2*x), -1/4*(2*(b*c^2*d - 2*a*c^2*e)*sqrt(-a)*x*arctan(1/
2*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2)) + (2*
a^2*c*f - a^2*b*g)*sqrt(c)*x*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2
+ b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(a^2*c*g*x - a*c^2*d)*sqrt(c*x^
2 + b*x + a))/(a^2*c^2*x), -1/2*((b*c^2*d - 2*a*c^2*e)*sqrt(-a)*x*arctan(1/
2*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2)) + (2*
a^2*c*f - a^2*b*g)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*
sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(a^2*c*g*x - a*c^2*d)*sqrt(c*x^2 + b*
x + a))/(a^2*c^2*x)]
```

Sympy [F]

$$\int \frac{d+ex+fx^2+gx^3}{x^2\sqrt{a+bx+cx^2}}\,dx=\int \frac{d+ex+fx^2+gx^3}{x^2\sqrt{a+bx+cx^2}}\,dx$$

```
[In] integrate((g*x**3+f*x**2+e*x+d)/x**2/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x + f*x**2 + g*x**3)/(x**2*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex+fx^2+gx^3}{x^2\sqrt{a+bx+cx^2}}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x^2/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.22

$$\int \frac{d+ex+fx^2+gx^3}{x^2\sqrt{a+bx+cx^2}}\,dx = \frac{\sqrt{cx^2+bx+a}\,g}{c} - \frac{(bd-2\,ae)\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a} - \frac{(2\,cf-bg)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}+b\right|\right)}{2\,c^{\frac{3}{2}}} + \frac{\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)bd+2\,a\sqrt{c}d}{\left(\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^2-a\right)a}$$

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x^2/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(c*x^2 + b*x + a)*g/c - (b*d - 2*a*e)*arctan(-(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))/sqrt(-a))/(sqrt(-a)*a) - 1/2*(2*c*f - b*g)*log(abs(2*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(3/2) + ((sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))*b*d + 2*a*sqrt(c)*d)/(((sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2 - a
)*a)
```

Mupad [B] (verification not implemented)

Time = 13.67 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.19

$$\int \frac{d+ex+fx^2+gx^3}{x^2\sqrt{a+bx+cx^2}}\,dx = \frac{g\,\sqrt{c\,x^2+b\,x+a}}{c} - \frac{e\,\ln\left(\frac{b}{2}+\frac{a}{x}+\frac{\sqrt{a}\,\sqrt{c\,x^2+b\,x+a}}{x}\right)}{\sqrt{a}} + \frac{f\,\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x+a}\right)}{\sqrt{c}} - \frac{b\,g\,\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x+a}\right)}{2\,c^{3/2}} - \frac{d\,\sqrt{c\,x^2+b\,x+a}}{a\,x} + \frac{b\,d\,\mathrm{atanh}\left(\frac{a+\frac{b\,x}{2}}{\sqrt{a}\,\sqrt{c\,x^2+b\,x+a}}\right)}{2\,a^{3/2}}$$

```
[In] int((d + e*x + f*x^2 + g*x^3)/(x^2*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] (g*(a + b*x + c*x^2)^(1/2))/c - (e*log(b/2 + a/x + (a^(1/2)*(a + b*x + c*x^
2)^(1/2))/x))/a^(1/2) + (f*log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2
)))/c^(1/2) - (b*g*log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2)))/(2*c
^(3/2)) - (d*(a + b*x + c*x^2)^(1/2))/(a*x) + (b*d*atanh((a + (b*x)/2)/(a^(
1/2)*(a + b*x + c*x^2)^(1/2))))/(2*a^(3/2))
```

3.285 $\int \frac{d+ex+fx^2+gx^3}{x^3\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 159

$$\int \frac{d+ex+fx^2+gx^3}{x^3\sqrt{a+bx+cx^2}}\,dx = -\frac{d\sqrt{a+bx+cx^2}}{2ax^2}+\frac{(3bd-4ae)\sqrt{a+bx+cx^2}}{4a^2x} - \frac{(3b^2d-4acd-4abe+8a^2f)\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{5/2}} + \frac{g\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}}$$

```
[Out] -1/8*(8*a^2*f-4*a*b*e-4*a*c*d+3*b^2*d)*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2
+b*x+a)^(1/2))/a^(5/2)+g*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))
/c^(1/2)-1/2*d*(c*x^2+b*x+a)^(1/2)/a/x^2+1/4*(-4*a*e+3*b*d)*(c*x^2+b*x+a)^(
1/2)/a^2/x
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {1664, 857, 635, 212, 738}

$$\int \frac{d+ex+fx^2+gx^3}{x^3\sqrt{a+bx+cx^2}}\,dx = \frac{\sqrt{a+bx+cx^2}(3bd-4ae)}{4a^2x} - \frac{\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)(8a^2f-4abe-4acd+3b^2d)}{8a^{5/2}} + \frac{g\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}} - \frac{d\sqrt{a+bx+cx^2}}{2ax^2}$$

```
[In] Int[(d + e*x + f*x^2 + g*x^3)/(x^3*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -1/2*(d*Sqrt[a + b*x + c*x^2])/(a*x^2) + ((3*b*d - 4*a*e)*Sqrt[a + b*x + c*
x^2])/(4*a^2*x) - ((3*b^2*d - 4*a*c*d - 4*a*b*e + 8*a^2*f)*ArcTanh[(2*a + b
*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(8*a^(5/2)) + (g*ArcTanh[(b + 2*c*x
)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/Sqrt[c]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{d\sqrt{a+bx+cx^2}}{2ax^2} - \frac{\int \frac{\frac{1}{2}(3bd-4ae)+(cd-2af)x-2agx^2}{x^2\sqrt{a+bx+cx^2}}\,dx}{2a} \\
&= -\frac{d\sqrt{a+bx+cx^2}}{2ax^2} + \frac{(3bd-4ae)\sqrt{a+bx+cx^2}}{4a^2x} + \frac{\int \frac{\frac{1}{4}\left(3b^2d-4abe-4a(cd-2af)\right)+2a^2gx}{x\sqrt{a+bx+cx^2}}\,dx}{2a^2} \\
&= -\frac{d\sqrt{a+bx+cx^2}}{2ax^2} + \frac{(3bd-4ae)\sqrt{a+bx+cx^2}}{4a^2x} \\
&\quad + \frac{(3b^2d-4acd-4abe+8a^2f)\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx}{8a^2} + g\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx \\
&= -\frac{d\sqrt{a+bx+cx^2}}{2ax^2} + \frac{(3bd-4ae)\sqrt{a+bx+cx^2}}{4a^2x} \\
&\quad - \frac{(3b^2d-4acd-4abe+8a^2f)\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx}{\sqrt{a+bx+cx^2}}\right)}{4a^2} \\
&\quad + (2g)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right) \\
&= -\frac{d\sqrt{a+bx+cx^2}}{2ax^2} + \frac{(3bd-4ae)\sqrt{a+bx+cx^2}}{4a^2x} \\
&\quad - \frac{(3b^2d-4acd-4abe+8a^2f)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{5/2}} + \frac{g\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.13 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.07

$$
\int \frac{d+ex+fx^2+gx^3}{x^3\sqrt{a+bx+cx^2}}\,dx
$$

$$
= \frac{\frac{\sqrt{a}\sqrt{a+x(b+cx)}(3bdx-2a(d+2ex))}{x^2} + (3b^2d+8a^2f)\,\text{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right) + 4a(cd+be)\text{arctanh}\left(\frac{-\sqrt{c}x+\sqrt{a+x(b+}}{\sqrt{a}}\right.}{4a^{5/2}}
$$

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3)/(x^3*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] ((Sqrt[a]*Sqrt[a + x*(b + c*x)]*(3*b*d*x - 2*a*(d + 2*e*x)))/x^2 + (3*b^2*d
 + 8*a^2*f)*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)])/Sqrt[a]] + 4*a*(c*d
 + b*e)*ArcTanh[(-(Sqrt[c]*x) + Sqrt[a + x*(b + c*x)])/Sqrt[a]] - (4*a^(5/2
)*g*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/Sqrt[c])/(4*a^(5/2))
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.84

method	result
risch	$-\frac{\sqrt{c\,x^{2}+bx+a}\,(4aex-3bdx+2ad)}{4a^{2}x^{2}}+\frac{\frac{8a^{2}g\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx+a}\right)}{\sqrt{c}}-\frac{\left(8a^{2}f-4abe-4acd+3b^{2}d\right)\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^{2}+bx+a}}{x}\right)}{\sqrt{a}}}{8a^{2}}$
default	$\frac{g\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx+a}\right)}{\sqrt{c}}-\frac{f\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^{2}+bx+a}}{x}\right)}{\sqrt{a}}+d\left(-\frac{\sqrt{c\,x^{2}+bx+a}}{2a\,x^{2}}-\frac{3b\left(-\frac{\sqrt{c\,x^{2}+bx+a}}{ax}+\frac{b\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{}}{x}\right.}{2a^{\frac{3}{2}}}\right.}{4a}\right.$

```
[In] int((g*x^3+f*x^2+e*x+d)/x^3/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(c*x^2+b*x+a)^(1/2)*(4*a*e*x-3*b*d*x+2*a*d)/a^2/x^2+1/8/a^2*(8*a^2*g*l
n((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-(8*a^2*f-4*a*b*e-4*a*c*d
+3*b^2*d)/a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x))
```

Fricas [A] (verification not implemented)

none

Time = 1.74 (sec) , antiderivative size = 783, normalized size of antiderivative = 4.92

$$\int \frac{d+ex+fx^2+gx^3}{x^3\sqrt{a+bx+cx^2}}\,dx$$

$$=\left[\frac{8\,a^{3}\sqrt{c}gx^{2}\log\left(-8\,c^{2}x^{2}-8\,bcx-b^{2}-4\,\sqrt{cx^{2}+bx+a}(2\,cx+b)\sqrt{c}-4\,ac\right)-(4\,abce-8\,a^{2}cf-(3\,b^{2}c}{}\right.$$

$$-\frac{16\,a^{3}\sqrt{-c}gx^{2}\arctan\left(\frac{\sqrt{cx^{2}+bx+a}(2\,cx+b)\sqrt{-c}}{2\,(c^{2}x^{2}+bcx+ac)}\right)+(4\,abce-8\,a^{2}cf-(3\,b^{2}c-4\,ac^{2})d)\sqrt{a}x^{2}\log\left(-\frac{8\,abx+(b^{2}+}{}\right.}{16\,a^{3}cx^{2}}$$

$$-\frac{8\,a^{3}\sqrt{-c}gx^{2}\arctan\left(\frac{\sqrt{cx^{2}+bx+a}(2\,cx+b)\sqrt{-c}}{2\,(c^{2}x^{2}+bcx+ac)}\right)+(4\,abce-8\,a^{2}cf-(3\,b^{2}c-4\,ac^{2})d)\sqrt{-a}x^{2}\arctan\left(\frac{\sqrt{cx^{2}+b}}{2\,(a}\right.}{8\,a^{3}cx^{2}}$$

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x^3/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [1/16*(8*a^3*sqrt(c)*g*x^2*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 +
b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - (4*a*b*c*e - 8*a^2*c*f - (3*b^2*c -
 4*a*c^2)*d)*sqrt(a)*x^2*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 +
 b*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) - 4*(2*a^2*c*d - (3*a*b*c*d - 4
*a^2*c*e)*x)*sqrt(c*x^2 + b*x + a))/(a^3*c*x^2), -1/16*(16*a^3*sqrt(-c)*g*x
```

```
^2*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x +
 a*c)) + (4*a*b*c*e - 8*a^2*c*f - (3*b^2*c - 4*a*c^2)*d)*sqrt(a)*x^2*log(-(
8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(a) +
 8*a^2)/x^2) + 4*(2*a^2*c*d - (3*a*b*c*d - 4*a^2*c*e)*x)*sqrt(c*x^2 + b*x +
 a))/(a^3*c*x^2), 1/8*(4*a^3*sqrt(c)*g*x^2*log(-8*c^2*x^2 - 8*b*c*x - b^2 -
 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - (4*a*b*c*e - 8*a^2*
c*f - (3*b^2*c - 4*a*c^2)*d)*sqrt(-a)*x^2*arctan(1/2*sqrt(c*x^2 + b*x + a)*
(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2)) - 2*(2*a^2*c*d - (3*a*b*c*d -
 4*a^2*c*e)*x)*sqrt(c*x^2 + b*x + a))/(a^3*c*x^2), -1/8*(8*a^3*sqrt(-c)*g*x
^2*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x +
 a*c)) + (4*a*b*c*e - 8*a^2*c*f - (3*b^2*c - 4*a*c^2)*d)*sqrt(-a)*x^2*arcta
n(1/2*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2)) +
 2*(2*a^2*c*d - (3*a*b*c*d - 4*a^2*c*e)*x)*sqrt(c*x^2 + b*x + a))/(a^3*c*x^
2)]
```

Sympy [F]

$$\int \frac{d+ex+fx^2+gx^3}{x^3\sqrt{a+bx+cx^2}}\,dx=\int \frac{d+ex+fx^2+gx^3}{x^3\sqrt{a+bx+cx^2}}\,dx$$

```
[In] integrate((g*x**3+f*x**2+e*x+d)/x**3/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x + f*x**2 + g*x**3)/(x**3*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex+fx^2+gx^3}{x^3\sqrt{a+bx+cx^2}}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x^3/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 346 vs. 2(133) = 266.

Time = 0.35 (sec) , antiderivative size = 346, normalized size of antiderivative = 2.18

$$\int \frac{d+ex+fx^2+gx^3}{x^3\sqrt{a+bx+cx^2}}\,dx = -\frac{g\log\left(\left|-2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}-b\right|\right)}{\sqrt{c}} + \frac{\left(3\,b^2d-4\,acd-4\,abe+8\,a^2f\right)\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+bx+a}}{\sqrt{-a}}\right)}{4\,\sqrt{-a}a^2} - \frac{3\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^3b^2d-4\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^3acd-4\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^3abe-8\,\bigl(}{}$$

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x^3/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] -g*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/sqrt(c) + 1
/4*(3*b^2*d - 4*a*c*d - 4*a*b*e + 8*a^2*f)*arctan(-(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))/sqrt(-a))/(sqrt(-a)*a^2) - 1/4*(3*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))^3*b^2*d - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*c*d - 4*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^3*a*b*e - 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
^2*a^2*sqrt(c)*e - 5*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^2*d - 4*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))*a^2*c*d + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))*a^2*b*e - 8*a^2*b*sqrt(c)*d + 8*a^3*sqrt(c)*e)/(((sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^2 - a)^2*a^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2+gx^3}{x^3\sqrt{a+bx+cx^2}}\,dx = \int \frac{g\,x^3+f\,x^2+e\,x+d}{x^3\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

```
[In] int((d + e*x + f*x^2 + g*x^3)/(x^3*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] int((d + e*x + f*x^2 + g*x^3)/(x^3*(a + b*x + c*x^2)^(1/2)), x)
```

3.286 $\int \frac{d+ex+fx^2+gx^3}{x^4\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 186

$$\begin{aligned}\int \frac{d+ex+fx^2+gx^3}{x^4\sqrt{a+bx+cx^2}}\,dx &= -\frac{d\sqrt{a+bx+cx^2}}{3ax^3}+\frac{(5bd-6ae)\sqrt{a+bx+cx^2}}{12a^2x^2}\\ &\quad-\frac{(15b^2d-16acd-18abe+24a^2f)\sqrt{a+bx+cx^2}}{24a^3x}\\ &\quad+\frac{(5b^3d-6ab^2e-4ab(3cd-2af)+8a^2(ce-2ag))\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{16a^{7/2}}\end{aligned}$$

```
[Out] 1/16*(5*b^3*d-6*a*b^2*e-4*a*b*(-2*a*f+3*c*d)+8*a^2*(-2*a*g+c*e))*arctanh(1/
2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/a^(7/2)-1/3*d*(c*x^2+b*x+a)^(1/2)/
a/x^3+1/12*(-6*a*e+5*b*d)*(c*x^2+b*x+a)^(1/2)/a^2/x^2-1/24*(24*a^2*f-18*a*b
*e-16*a*c*d+15*b^2*d)*(c*x^2+b*x+a)^(1/2)/a^3/x
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used

$= \{1664, 820, 738, 212\}$

$$\begin{aligned}\int \frac{d+ex+fx^2+gx^3}{x^4\sqrt{a+bx+cx^2}}\,dx \\ = \frac{\sqrt{a+bx+cx^2}(5bd-6ae)}{12a^2x^2} \\ + \frac{\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)(8a^2(ce-2ag)-6ab^2e-4ab(3cd-2af)+5b^3d)}{16a^{7/2}} \\ - \frac{\sqrt{a+bx+cx^2}(24a^2f-18abe-16acd+15b^2d)}{24a^3x} - \frac{d\sqrt{a+bx+cx^2}}{3ax^3}\end{aligned}$$

```
[In] Int[(d + e*x + f*x^2 + g*x^3)/(x^4*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -1/3*(d*Sqrt[a + b*x + c*x^2])/(a*x^3) + ((5*b*d - 6*a*e)*Sqrt[a + b*x + c*
x^2])/(12*a^2*x^2) - ((15*b^2*d - 16*a*c*d - 18*a*b*e + 24*a^2*f)*Sqrt[a +
b*x + c*x^2])/(24*a^3*x) + ((5*b^3*d - 6*a*b^2*e - 4*a*b*(3*c*d - 2*a*f) +
8*a^2*(c*e - 2*a*g))*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])]
)/(16*a^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
```

```
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d\sqrt{a+bx+cx^2}}{3ax^3} - \frac{\int \frac{\frac{1}{2}(5bd-6ae)+(2cd-3af)x-3agx^2}{x^3\sqrt{a+bx+cx^2}}\,dx}{3a} \\
&= -\frac{d\sqrt{a+bx+cx^2}}{3ax^3} + \frac{(5bd-6ae)\sqrt{a+bx+cx^2}}{12a^2x^2} \\
&\quad + \frac{\int \frac{\frac{1}{4}\left(15b^2d-16acd-18abe+24a^2f\right)+\frac{1}{2}\left(5bcd-6ace+12a^2g\right)x}{x^2\sqrt{a+bx+cx^2}}\,dx}{6a^2} \\
&= -\frac{d\sqrt{a+bx+cx^2}}{3ax^3} + \frac{(5bd-6ae)\sqrt{a+bx+cx^2}}{12a^2x^2} \\
&\quad - \frac{\left(15b^2d-16acd-18abe+24a^2f\right)\sqrt{a+bx+cx^2}}{24a^3x} \\
&\quad - \frac{\left(5b^3d-6ab^2e-4ab(3cd-2af)+8a^2(ce-2ag)\right)\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx}{16a^3} \\
&= -\frac{d\sqrt{a+bx+cx^2}}{3ax^3} + \frac{(5bd-6ae)\sqrt{a+bx+cx^2}}{12a^2x^2} \\
&\quad - \frac{\left(15b^2d-16acd-18abe+24a^2f\right)\sqrt{a+bx+cx^2}}{24a^3x} \\
&\quad + \frac{\left(5b^3d-6ab^2e-4ab(3cd-2af)+8a^2(ce-2ag)\right)\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx}{\sqrt{a+bx+cx^2}}\right)}{8a^3} \\
&= -\frac{d\sqrt{a+bx+cx^2}}{3ax^3} + \frac{(5bd-6ae)\sqrt{a+bx+cx^2}}{12a^2x^2} \\
&\quad - \frac{\left(15b^2d-16acd-18abe+24a^2f\right)\sqrt{a+bx+cx^2}}{24a^3x} \\
&\quad + \frac{\left(5b^3d-6ab^2e-4ab(3cd-2af)+8a^2(ce-2ag)\right)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{16a^{7/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.96

$$\int \frac{d+ex+fx^2+gx^3}{x^4\sqrt{a+bx+cx^2}}\,dx$$
$$=\frac{\frac{\sqrt{a}\sqrt{a+x(b+cx)}(-15b^2dx^2+2ax(5bd+8cdx+9bex)-4a^2(2d+3x(e+2fx)))}{x^3}+3(-5b^3d+16a^3g)\operatorname{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)+}{24a^{7/2}}$$

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3)/(x^4*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] ((Sqrt[a]*Sqrt[a + x*(b + c*x)]*(-15*b^2*d*x^2 + 2*a*x*(5*b*d + 8*c*d*x + 9
*b*e*x) - 4*a^2*(2*d + 3*x*(e + 2*f*x))))/x^3 + 3*(-5*b^3*d + 16*a^3*g)*Arc
Tanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)])/Sqrt[a]] + 6*a*(-6*b*c*d - 3*b^2*e
 + 4*a*c*e + 4*a*b*f)*ArcTanh[(-(Sqrt[c]*x) + Sqrt[a + x*(b + c*x)])/Sqrt[a
]])/(24*a^(7/2))
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.81

method	result
risch	$-\frac{\sqrt{c\,x^2+bx+a}\,(24a^2f\,x^2-18abe\,x^2-16acd\,x^2+15b^2d\,x^2+12a^2ex-10abdx+8a^2d)}{24a^3x^3}-\frac{(16a^3g-8a^2bf-8a^2ce+6a\,b^2e+12abcd-5b^3}{16a^{\frac{7}{2}}}$
default	$-\frac{g\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)}{\sqrt{a}}+e\left(-\frac{\sqrt{c\,x^2+bx+a}}{2a\,x^2}-\frac{3b\left(-\frac{\sqrt{c\,x^2+bx+a}}{ax}+\frac{b\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)}{2a^{\frac{3}{2}}}\right)}{4a}+\frac{c\ln\left(\frac{2a+bx}{}\right.}{}\right.$

```
[In] int((g*x^3+f*x^2+e*x+d)/x^4/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*(c*x^2+b*x+a)^(1/2)*(24*a^2*f*x^2-18*a*b*e*x^2-16*a*c*d*x^2+15*b^2*d*
x^2+12*a^2*e*x-10*a*b*d*x+8*a^2*d)/a^3/x^3-1/16*(16*a^3*g-8*a^2*b*f-8*a^2*c
*e+6*a*b^2*e+12*a*b*c*d-5*b^3*d)/a^(7/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a
)^(1/2))/x)
```

Fricas [A] (verification not implemented)

none

Time = 1.89 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.96

$$\int \frac{d+ex+fx^2+gx^3}{x^4\sqrt{a+bx+cx^2}}\,dx$$

$$= \Bigg[-\frac{3\,(8\,a^2bf-16\,a^3g+(5\,b^3-12\,abc)d-2\,(3\,ab^2-4\,a^2c)e)\sqrt{a}x^3\log\Bigg(-\frac{8\,abx+(b^2+4\,ac)x^2-4\,\sqrt{cx^2+bx+a}(bx+2\,a)\sqrt{}}{x^2}}{96\,a^4}$$

$$-\frac{3\,(8\,a^2bf-16\,a^3g+(5\,b^3-12\,abc)d-2\,(3\,ab^2-4\,a^2c)e)\sqrt{-a}x^3\arctan\left(\frac{\sqrt{cx^2+bx+a}(bx+2\,a)\sqrt{-a}}{2\,(acx^2+abx+a^2)}\right)+2\,(8\,a^3}{48\,a^4x^3}$$

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x^4/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [-1/96*(3*(8*a^2*b*f - 16*a^3*g + (5*b^3 - 12*a*b*c)*d - 2*(3*a*b^2 - 4*a^2
*c)*e)*sqrt(a)*x^3*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 + b*x +
 a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) + 4*(8*a^3*d - (18*a^2*b*e - 24*a^3*f
 - (15*a*b^2 - 16*a^2*c)*d)*x^2 - 2*(5*a^2*b*d - 6*a^3*e)*x)*sqrt(c*x^2 + b
*x + a))/(a^4*x^3), -1/48*(3*(8*a^2*b*f - 16*a^3*g + (5*b^3 - 12*a*b*c)*d -
 2*(3*a*b^2 - 4*a^2*c)*e)*sqrt(-a)*x^3*arctan(1/2*sqrt(c*x^2 + b*x + a)*(b*
x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2)) + 2*(8*a^3*d - (18*a^2*b*e - 24*
a^3*f - (15*a*b^2 - 16*a^2*c)*d)*x^2 - 2*(5*a^2*b*d - 6*a^3*e)*x)*sqrt(c*x^
2 + b*x + a))/(a^4*x^3)]
```

Sympy [F]

$$\int \frac{d+ex+fx^2+gx^3}{x^4\sqrt{a+bx+cx^2}}\,dx = \int \frac{d+ex+fx^2+gx^3}{x^4\sqrt{a+bx+cx^2}}\,dx$$

```
[In] integrate((g*x**3+f*x**2+e*x+d)/x**4/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x + f*x**2 + g*x**3)/(x**4*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex+fx^2+gx^3}{x^4\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x^4/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 680 vs. $2(164) = 328$.

Time = 0.31 (sec) , antiderivative size = 680, normalized size of antiderivative = 3.66

$$\int \frac{d+ex+fx^2+gx^3}{x^4\sqrt{a+bx+cx^2}}\,dx$$
$$= -\frac{\left(5\,b^3d-12\,abcd-6\,ab^2e+8\,a^2ce+8\,a^2bf-16\,a^3g\right)\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+bx+a}}{\sqrt{-a}}\right)}{8\,\sqrt{-a}a^3}$$
$$+ 15\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^5 b^3d-36\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^5 abcd-18\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^5 ab^2e$$

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x^4/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*(5*b^3*d - 12*a*b*c*d - 6*a*b^2*e + 8*a^2*c*e + 8*a^2*b*f - 16*a^3*g)*
arctan(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))/sqrt(-a))/(sqrt(-a)*a^3) + 1/24
*(15*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^3*d - 36*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))^5*a*b*c*d - 18*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b^2*e
 + 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*c*e + 24*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^5*a^2*b*f + 48*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^3*
sqrt(c)*f - 40*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^3*d + 96*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^3*a^2*b*c*d + 48*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))^3*a^2*b^2*e - 48*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^3*b*f + 96*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*c^(3/2)*d + 48*(sqrt(c)*x - sqrt(c*
x^2 + b*x + a))^2*a^3*b*sqrt(c)*e - 96*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^
2*a^4*sqrt(c)*f + 33*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^3*d + 36*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*b*c*d - 30*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))*a^3*b^2*e - 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^4*c*e + 24*(s
```

```
qrt(c)*x - sqrt(c*x^2 + b*x + a))*a^4*b*f + 48*a^3*b^2*sqrt(c)*d - 32*a^4*c
^(3/2)*d - 48*a^4*b*sqrt(c)*e + 48*a^5*sqrt(c)*f)/(((sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^2 - a)^3*a^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2+gx^3}{x^4\sqrt{a+bx+cx^2}}\,dx=\int \frac{g\,x^3+f\,x^2+e\,x+d}{x^4\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

```
[In] int((d + e*x + f*x^2 + g*x^3)/(x^4*(a + b*x + c*x^2)^(1/2)),x)
[Out] int((d + e*x + f*x^2 + g*x^3)/(x^4*(a + b*x + c*x^2)^(1/2)), x)
```

3.287 $\int \frac{d+ex+fx^2+gx^3}{x^5\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 270

$$\begin{aligned}\int \frac{d+ex+fx^2+gx^3}{x^5\sqrt{a+bx+cx^2}}\,dx &= -\frac{d\sqrt{a+bx+cx^2}}{4ax^4}+\frac{(7bd-8ae)\sqrt{a+bx+cx^2}}{24a^2x^3} \\ &- \frac{(35b^2d-36acd-40abe+48a^2f)\sqrt{a+bx+cx^2}}{96a^3x^2} \\ &+ \frac{(105b^3d-120ab^2e-4ab(55cd-36af)+64a^2(2ce-3ag))\sqrt{a+bx+cx^2}}{192a^4x} \\ &- \frac{(35b^4d-40ab^3e+16a^2c(3cd-4af)-24ab^2(5cd-2af)+32a^2b(3ce-2ag))\,\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{128a^{9/2}}\end{aligned}$$

```
[Out] -1/128*(35*b^4*d-40*a*b^3*e+16*a^2*c*(-4*a*f+3*c*d)-24*a*b^2*(-2*a*f+5*c*d)
+32*a^2*b*(-2*a*g+3*c*e))*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2)
)/a^(9/2)-1/4*d*(c*x^2+b*x+a)^(1/2)/a/x^4+1/24*(-8*a*e+7*b*d)*(c*x^2+b*x+a)
^(1/2)/a^2/x^3-1/96*(48*a^2*f-40*a*b*e-36*a*c*d+35*b^2*d)*(c*x^2+b*x+a)^(1/
2)/a^3/x^2+1/192*(105*b^3*d-120*a*b^2*e-4*a*b*(-36*a*f+55*c*d)+64*a^2*(-3*a
*g+2*c*e))*(c*x^2+b*x+a)^(1/2)/a^4/x
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used

$= \{1664, 848, 820, 738, 212\}$

$$\int \frac{d+ex+fx^2+gx^3}{x^5\sqrt{a+bx+cx^2}}\,dx = \frac{\sqrt{a+bx+cx^2}(7bd-8ae)}{24a^2x^3} - \frac{\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)(32a^2b(3ce-2ag)+16a^2c(3cd-4af)-40ab^3e-24ab^2(5cd-2af)+35b^4d)}{128a^{9/2}} + \frac{\sqrt{a+bx+cx^2}(64a^2(2ce-3ag)-120ab^2e-4ab(55cd-36af)+105b^3d)}{192a^4x} - \frac{\sqrt{a+bx+cx^2}(48a^2f-40abe-36acd+35b^2d)}{96a^3x^2} - \frac{d\sqrt{a+bx+cx^2}}{4ax^4}$$

```
[In] Int[(d + e*x + f*x^2 + g*x^3)/(x^5*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -1/4*(d*Sqrt[a + b*x + c*x^2])/(a*x^4) + ((7*b*d - 8*a*e)*Sqrt[a + b*x + c*
x^2])/(24*a^2*x^3) - ((35*b^2*d - 36*a*c*d - 40*a*b*e + 48*a^2*f)*Sqrt[a +
b*x + c*x^2])/(96*a^3*x^2) + ((105*b^3*d - 120*a*b^2*e - 4*a*b*(55*c*d - 36
*a*f) + 64*a^2*(2*c*e - 3*a*g))*Sqrt[a + b*x + c*x^2])/(192*a^4*x) - ((35*b
^4*d - 40*a*b^3*e + 16*a^2*c*(3*c*d - 4*a*f) - 24*a*b^2*(5*c*d - 2*a*f) + 3
2*a^2*b*(3*c*e - 2*a*g))*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^
2])])/(128*a^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 848

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d\sqrt{a+bx+cx^2}}{4ax^4}-\frac{\int\frac{\frac{1}{2}(7bd-8ae)+(3cd-4af)x-4agx^2}{x^4\sqrt{a+bx+cx^2}}\,dx}{4a}\\
&= -\frac{d\sqrt{a+bx+cx^2}}{4ax^4}+\frac{(7bd-8ae)\sqrt{a+bx+cx^2}}{24a^2x^3}\\
&\quad+\frac{\int\frac{\frac{1}{4}\left(35b^2d-40abe-12a(3cd-4af)\right)+\left(7bcd-8ace+12a^2g\right)x}{x^3\sqrt{a+bx+cx^2}}\,dx}{12a^2}\\
&= -\frac{d\sqrt{a+bx+cx^2}}{4ax^4}+\frac{(7bd-8ae)\sqrt{a+bx+cx^2}}{24a^2x^3}\\
&\quad-\frac{\left(35b^2d-36acd-40abe+48a^2f\right)\sqrt{a+bx+cx^2}}{96a^3x^2}\\
&\quad-\frac{\int\frac{\frac{1}{8}\left(105b^3d-220abcd-120ab^2e+128a^2ce+144a^2bf-192a^3g\right)+\frac{1}{4}c\left(35b^2d-40abe-12a(3cd-4af)\right)x}{x^2\sqrt{a+bx+cx^2}}\,dx}{24a^3}\\
&= -\frac{d\sqrt{a+bx+cx^2}}{4ax^4}+\frac{(7bd-8ae)\sqrt{a+bx+cx^2}}{24a^2x^3}\\
&\quad-\frac{\left(35b^2d-36acd-40abe+48a^2f\right)\sqrt{a+bx+cx^2}}{96a^3x^2}\\
&\quad+\frac{\left(105b^3d-120ab^2e-4ab(55cd-36af)+64a^2(2ce-3ag)\right)\sqrt{a+bx+cx^2}}{192a^4x}\\
&\quad+\frac{\left(35b^4d-40ab^3e+16a^2c(3cd-4af)-24ab^2(5cd-2af)+32a^2b(3ce-2ag)\right)\int\frac{1}{x\sqrt{a+bx+cx^2}}\,dx}{128a^4}
\end{aligned}$$

$$= -\frac{d\sqrt{a+bx+cx^2}}{4ax^4} + \frac{(7bd-8ae)\sqrt{a+bx+cx^2}}{24a^2x^3} - \frac{\left(35b^2d-36acd-40abe+48a^2f\right)\sqrt{a+bx+cx^2}}{96a^3x^2} + \frac{\left(105b^3d-120ab^2e-4ab(55cd-36af)+64a^2(2ce-3ag)\right)\sqrt{a+bx+cx^2}}{192a^4x} - \frac{\left(35b^4d-40ab^3e+16a^2c(3cd-4af)-24ab^2(5cd-2af)+32a^2b(3ce-2ag)\right)\text{Subst}\left(\int\frac{1}{4a-x^2}\,dx\right.}{64a^4}$$

$$= -\frac{d\sqrt{a+bx+cx^2}}{4ax^4} + \frac{(7bd-8ae)\sqrt{a+bx+cx^2}}{24a^2x^3} - \frac{\left(35b^2d-36acd-40abe+48a^2f\right)\sqrt{a+bx+cx^2}}{96a^3x^2} + \frac{\left(105b^3d-120ab^2e-4ab(55cd-36af)+64a^2(2ce-3ag)\right)\sqrt{a+bx+cx^2}}{192a^4x} - \frac{\left(35b^4d-40ab^3e+16a^2c(3cd-4af)-24ab^2(5cd-2af)+32a^2b(3ce-2ag)\right)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx}}\right.}{128a^{9/2}}$$

Mathematica [A] (verified)

Time = 1.57 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.88

$$\int\frac{d+ex+fx^2+gx^3}{x^5\sqrt{a+bx+cx^2}}\,dx = \frac{\frac{\sqrt{a}\sqrt{a+x(b+cx)}\left(105b^3dx^3-10abx^2(7bd+22cdx+12bex)+8a^2x(7bd+cx(9d+16ex)+2bx(5e+9fx))-16a^3\left(3d+4ex+6x^2(f+2gx)\right)\right)}{x^4}+105b^4da}{}$$

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3)/(x^5*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] ((Sqrt[a]*Sqrt[a + x*(b + c*x)]*(105*b^3*d*x^3 - 10*a*b*x^2*(7*b*d + 22*c*d
*x + 12*b*e*x) + 8*a^2*x*(7*b*d + c*x*(9*d + 16*e*x) + 2*b*x*(5*e + 9*f*x))
 - 16*a^3*(3*d + 4*e*x + 6*x^2*(f + 2*g*x))))/x^4 + 105*b^4*d*ArcTanh[(Sqrt
[c]*x - Sqrt[a + x*(b + c*x)])/Sqrt[a]] + 24*a*(5*b^3*e + 3*b^2*(5*c*d - 2*
a*f) + 2*a*c*(-3*c*d + 4*a*f) + 4*a*b*(-3*c*e + 2*a*g))*ArcTanh[(-(Sqrt[c]*
x) + Sqrt[a + x*(b + c*x)])/Sqrt[a]])/(192*a^(9/2))
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.87

method	result
risch	$-\frac{\sqrt{c\,x^2+bx+a}\,(192a^3 g\,x^3-144a^2 bf\,x^3-128x^3 a^2 ce+120x^3 a\,b^2 e+220x^3 abcd-105x^3 b^3 d+96a^3 f\,x^2-80x^2 a^2 be-72x^2 a^2 cd+70x^2 b^2 a}{192a^4 x^4}$
default	$d\left(-\frac{\sqrt{c\,x^2+bx+a}}{4a\,x^4}-\frac{7b\left(-\frac{\sqrt{c\,x^2+bx+a}}{3a\,x^3}-\frac{5b\left(-\frac{\sqrt{c\,x^2+bx+a}}{2a\,x^2}-\frac{3b\left(-\frac{\sqrt{c\,x^2+bx+a}}{ax}+\frac{b\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)}{2a^{\frac{3}{2}}}\right)}{4a}+\frac{c\ln\left(\frac{2a+bx+2}{}\right.}{}\right.}{6a}\right.}{8a}\right.$

```
[In] int((g*x^3+f*x^2+e*x+d)/x^5/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/192*(c*x^2+b*x+a)^(1/2)*(192*a^3*g*x^3-144*a^2*b*f*x^3-128*a^2*c*e*x^3+1
20*a*b^2*e*x^3+220*a*b*c*d*x^3-105*b^3*d*x^3+96*a^3*f*x^2-80*a^2*b*e*x^2-72
*a^2*c*d*x^2+70*a*b^2*d*x^2+64*a^3*e*x-56*a^2*b*d*x+48*a^3*d)/a^4/x^4+1/128
*(64*a^3*b*g+64*a^3*c*f-48*a^2*b^2*f-96*a^2*b*c*e-48*a^2*c^2*d+40*a*b^3*e+1
20*a*b^2*c*d-35*b^4*d)/a^(9/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x
)
```

Fricas [A] (verification not implemented)

none

Time = 4.41 (sec) , antiderivative size = 525, normalized size of antiderivative = 1.94

$$\int \frac{d+ex+fx^2+gx^3}{x^5\sqrt{a+bx+cx^2}}\,dx$$

$$= \left[\frac{3\,(64\,a^3bg-(35\,b^4-120\,ab^2c+48\,a^2c^2)d+8\,(5\,ab^3-12\,a^2bc)e-16\,(3\,a^2b^2-4\,a^3c)f)\sqrt{a}x^4\log\left(-\frac{8\,abx+}{}\right.}{}\right.$$

$$-\frac{3\,(64\,a^3bg-(35\,b^4-120\,ab^2c+48\,a^2c^2)d+8\,(5\,ab^3-12\,a^2bc)e-16\,(3\,a^2b^2-4\,a^3c)f)\sqrt{-a}x^4\arctan\left(\right.}{}$$

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x^5/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [1/768*(3*(64*a^3*b*g - (35*b^4 - 120*a*b^2*c + 48*a^2*c^2)*d + 8*(5*a*b^3
- 12*a^2*b*c)*e - 16*(3*a^2*b^2 - 4*a^3*c)*f)*sqrt(a)*x^4*log(-(8*a*b*x + (
b^2 + 4*a*c)*x^2 + 4*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2
) - 4*(48*a^4*d - (144*a^3*b*f - 192*a^4*g + 5*(21*a*b^3 - 44*a^2*b*c)*d -
8*(15*a^2*b^2 - 16*a^3*c)*e)*x^3 - 2*(40*a^3*b*e - 48*a^4*f - (35*a^2*b^2 -
 36*a^3*c)*d)*x^2 - 8*(7*a^3*b*d - 8*a^4*e)*x)*sqrt(c*x^2 + b*x + a))/(a^5*
x^4), -1/384*(3*(64*a^3*b*g - (35*b^4 - 120*a*b^2*c + 48*a^2*c^2)*d + 8*(5*
a*b^3 - 12*a^2*b*c)*e - 16*(3*a^2*b^2 - 4*a^3*c)*f)*sqrt(-a)*x^4*arctan(1/2
*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2)) + 2*(4
8*a^4*d - (144*a^3*b*f - 192*a^4*g + 5*(21*a*b^3 - 44*a^2*b*c)*d - 8*(15*a^
2*b^2 - 16*a^3*c)*e)*x^3 - 2*(40*a^3*b*e - 48*a^4*f - (35*a^2*b^2 - 36*a^3*
c)*d)*x^2 - 8*(7*a^3*b*d - 8*a^4*e)*x)*sqrt(c*x^2 + b*x + a))/(a^5*x^4)]
```

Sympy [F]

$$\int \frac{d+ex+fx^2+gx^3}{x^5\sqrt{a+bx+cx^2}}\,dx = \int \frac{d+ex+fx^2+gx^3}{x^5\sqrt{a+bx+cx^2}}\,dx$$

```
[In] integrate((g*x**3+f*x**2+e*x+d)/x**5/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x + f*x**2 + g*x**3)/(x**5*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex+fx^2+gx^3}{x^5\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x^5/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1433 vs. 2(244) = 488.

Time = 0.33 (sec) , antiderivative size = 1433, normalized size of antiderivative = 5.31

$$\int \frac{d+ex+fx^2+gx^3}{x^5\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x^5/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/64*(35*b^4*d - 120*a*b^2*c*d + 48*a^2*c^2*d - 40*a*b^3*e + 96*a^2*b*c*e +
 48*a^2*b^2*f - 64*a^3*c*f - 64*a^3*b*g)*arctan(-(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))/sqrt(-a))/(sqrt(-a)*a^4) - 1/192*(105*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^7*b^4*d - 360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a*b^2*c*d + 14
4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^2*c^2*d - 120*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))^7*a*b^3*e + 288*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^2*
b*c*e + 144*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^2*b^2*f - 192*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^7*a^3*c*f - 192*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))^7*a^3*b*g - 384*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^4*sqrt(c)*g - 3
85*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b^4*d + 1320*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))^5*a^2*b^2*c*d - 528*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*
a^3*c^2*d + 440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*b^3*e - 1056*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^5*a^3*b*c*e - 528*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))^5*a^3*b^2*f + 192*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^4*c*f
+ 576*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^4*b*g - 768*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^4*a^4*c^(3/2)*e - 384*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)^4*a^4*b*sqrt(c)*f + 1152*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^5*sqrt(c
)*g + 511*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b^4*d - 1752*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))^3*a^3*b^2*c*d - 528*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))^3*a^4*c^2*d - 584*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^3*b^3*e +
```

```
480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^4*b*c*e + 624*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^3*a^4*b^2*f + 192*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*
a^5*c*f - 576*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^5*b*g - 2048*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^2*a^4*b*c^(3/2)*d - 384*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^2*a^4*b^2*sqrt(c)*e + 1024*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
^2*a^5*c^(3/2)*e + 768*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^5*b*sqrt(c)*
f - 1152*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^6*sqrt(c)*g - 279*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))*a^3*b^4*d - 360*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))*a^4*b^2*c*d + 144*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^5*c^2*d + 264*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^4*b^3*e + 288*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))*a^5*b*c*e - 240*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^5*b^2*f
- 192*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^6*c*f + 192*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))*a^6*b*g - 384*a^4*b^3*sqrt(c)*d + 512*a^5*b*c^(3/2)*d + 38
4*a^5*b^2*sqrt(c)*e - 256*a^6*c^(3/2)*e - 384*a^6*b*sqrt(c)*f + 384*a^7*sqr
t(c)*g)/(((sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2 - a)^4*a^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2+gx^3}{x^5\sqrt{a+bx+cx^2}}\,dx=\int \frac{g\,x^3+f\,x^2+e\,x+d}{x^5\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

```
[In] int((d + e*x + f*x^2 + g*x^3)/(x^5*(a + b*x + c*x^2)^(1/2)),x)
[Out] int((d + e*x + f*x^2 + g*x^3)/(x^5*(a + b*x + c*x^2)^(1/2)), x)
```

3.288 $\int \frac{d+ex+fx^2+gx^3}{x^6\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 371

$$\begin{aligned}\int \frac{d+ex+fx^2+gx^3}{x^6\sqrt{a+bx+cx^2}}\,dx &= -\frac{d\sqrt{a+bx+cx^2}}{5ax^5}+\frac{(9bd-10ae)\sqrt{a+bx+cx^2}}{40a^2x^4}\\ &-\frac{(63b^2d-64acd-70abe+80a^2f)\sqrt{a+bx+cx^2}}{240a^3x^3}\\ &+\frac{(315b^3d-350ab^2e-4ab(161cd-100af)+120a^2(3ce-4ag))\sqrt{a+bx+cx^2}}{960a^4x^2}\\ &-\frac{(945b^4d-1050ab^3e-60ab^2(49cd-20af)+256a^2c(4cd-5af)+40a^2b(55ce-36ag))\sqrt{a+bx+cx^2}}{1920a^5x}\\ &+\frac{(63b^5d-70ab^4e+48a^2bc(5cd-4af)-40ab^3(7cd-2af)-32a^3c(3ce-4ag)+48a^2b^2(5ce-2ag))\,\mathrm{ar}}{256a^{11/2}}\end{aligned}$$

```
[Out] 1/256*(63*b^5*d-70*a*b^4*e+48*a^2*b*c*(-4*a*f+5*c*d)-40*a*b^3*(-2*a*f+7*c*d
)-32*a^3*c*(-4*a*g+3*c*e)+48*a^2*b^2*(-2*a*g+5*c*e))*arctanh(1/2*(b*x+2*a)/
a^(1/2)/(c*x^2+b*x+a)^(1/2))/a^(11/2)-1/5*d*(c*x^2+b*x+a)^(1/2)/a/x^5+1/40*
(-10*a*e+9*b*d)*(c*x^2+b*x+a)^(1/2)/a^2/x^4-1/240*(80*a^2*f-70*a*b*e-64*a*c
*d+63*b^2*d)*(c*x^2+b*x+a)^(1/2)/a^3/x^3+1/960*(315*b^3*d-350*a*b^2*e-4*a*b
*(-100*a*f+161*c*d)+120*a^2*(-4*a*g+3*c*e))*(c*x^2+b*x+a)^(1/2)/a^4/x^2-1/1
920*(945*b^4*d-1050*a*b^3*e-60*a*b^2*(-20*a*f+49*c*d)+256*a^2*c*(-5*a*f+4*c
*d)+40*a^2*b*(-36*a*g+55*c*e))*(c*x^2+b*x+a)^(1/2)/a^5/x
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {1664, 848, 820, 738, 212}

$$\begin{aligned}
\int \frac{d+ex+fx^2+gx^3}{x^6\sqrt{a+bx+cx^2}}\,dx = {} & \frac{\sqrt{a+bx+cx^2}(9bd-10ae)}{40a^2x^4} \\
& - \frac{\sqrt{a+bx+cx^2}(40a^2b(55ce-36ag)+256a^2c(4cd-5af)-1050ab^3e-60ab^2(49cd-20af)+945b^4d)}{1920a^5x} \\
& + \frac{\sqrt{a+bx+cx^2}(120a^2(3ce-4ag)-350ab^2e-4ab(161cd-100af)+315b^3d)}{960a^4x^2} \\
& - \frac{\sqrt{a+bx+cx^2}(80a^2f-70abe-64acd+63b^2d)}{240a^3x^3} \\
& + \frac{\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)(-32a^3c(3ce-4ag)+48a^2b^2(5ce-2ag)+48a^2bc(5cd-4af)-70ab^4e-40ab}{256a^{11/2}} \\
& - \frac{d\sqrt{a+bx+cx^2}}{5ax^5}
\end{aligned}$$

```
[In] Int[(d + e*x + f*x^2 + g*x^3)/(x^6*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -1/5*(d*Sqrt[a + b*x + c*x^2])/(a*x^5) + ((9*b*d - 10*a*e)*Sqrt[a + b*x + c
*x^2])/(40*a^2*x^4) - ((63*b^2*d - 64*a*c*d - 70*a*b*e + 80*a^2*f)*Sqrt[a +
 b*x + c*x^2])/(240*a^3*x^3) + ((315*b^3*d - 350*a*b^2*e - 4*a*b*(161*c*d -
 100*a*f) + 120*a^2*(3*c*e - 4*a*g))*Sqrt[a + b*x + c*x^2])/(960*a^4*x^2) -
 ((945*b^4*d - 1050*a*b^3*e - 60*a*b^2*(49*c*d - 20*a*f) + 256*a^2*c*(4*c*d
 - 5*a*f) + 40*a^2*b*(55*c*e - 36*a*g))*Sqrt[a + b*x + c*x^2])/(1920*a^5*x)
 + ((63*b^5*d - 70*a*b^4*e + 48*a^2*b*c*(5*c*d - 4*a*f) - 40*a*b^3*(7*c*d -
 2*a*f) - 32*a^3*c*(3*c*e - 4*a*g) + 48*a^2*b^2*(5*c*e - 2*a*g))*ArcTanh[(2
*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(256*a^(11/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 848

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{d\sqrt{a+bx+cx^2}}{5ax^5}-\frac{\int \frac{\frac{1}{2}(9bd-10ae)+(4cd-5af)x-5agx^2}{x^5\sqrt{a+bx+cx^2}}\,dx}{5a}\\
&= -\frac{d\sqrt{a+bx+cx^2}}{5ax^5}+\frac{(9bd-10ae)\sqrt{a+bx+cx^2}}{40a^2x^4}\\
&\quad+\frac{\int \frac{\frac{1}{4}\left(63b^2d-64acd-70abe+80a^2f\right)+\frac{1}{2}\left(27bcd-30ace+40a^2g\right)x}{x^4\sqrt{a+bx+cx^2}}\,dx}{20a^2}\\
&= -\frac{d\sqrt{a+bx+cx^2}}{5ax^5}+\frac{(9bd-10ae)\sqrt{a+bx+cx^2}}{40a^2x^4}\\
&\quad-\frac{\left(63b^2d-64acd-70abe+80a^2f\right)\sqrt{a+bx+cx^2}}{240a^3x^3}\\
&\quad-\frac{\int \frac{\frac{1}{8}\left(315b^3d-644abcd-350ab^2e+360a^2ce+400a^2bf-480a^3g\right)+\frac{1}{2}c\left(63b^2d-70abe-16a(4cd-5af)\right)x}{x^3\sqrt{a+bx+cx^2}}\,dx}{60a^3}
\end{aligned}
$$

$$= -\frac{d\sqrt{a+bx+cx^2}}{5ax^5} + \frac{(9bd-10ae)\sqrt{a+bx+cx^2}}{40a^2x^4} - \frac{(63b^2d-64acd-70abe+80a^2f)\sqrt{a+bx+cx^2}}{240a^3x^3} + \frac{(315b^3d-350ab^2e-4ab(161cd-100af)+120a^2(3ce-4ag))\sqrt{a+bx+cx^2}}{960a^4x^2} + \frac{\int \frac{\frac{1}{16}(945b^4d-1050ab^3e-60ab^2(49cd-20af)+256a^2c(4cd-5af)+40a^2b(55ce-36ag))+\frac{1}{8}c(315b^3d-350ab^2e-4ab(161cd-100af)+1}{x^2\sqrt{a+bx+cx^2}}}{120a^4}$$

$$= -\frac{d\sqrt{a+bx+cx^2}}{5ax^5} + \frac{(9bd-10ae)\sqrt{a+bx+cx^2}}{40a^2x^4} - \frac{(63b^2d-64acd-70abe+80a^2f)\sqrt{a+bx+cx^2}}{240a^3x^3} + \frac{(315b^3d-350ab^2e-4ab(161cd-100af)+120a^2(3ce-4ag))\sqrt{a+bx+cx^2}}{960a^4x^2} - \frac{(945b^4d-1050ab^3e-60ab^2(49cd-20af)+256a^2c(4cd-5af)+40a^2b(55ce-36ag))\sqrt{a+bx}}{1920a^5x} - \frac{(63b^5d-70ab^4e+48a^2bc(5cd-4af)-40ab^3(7cd-2af)-32a^3c(3ce-4ag)+48a^2b^2(5ce-2}{256a^5}$$

$$= -\frac{d\sqrt{a+bx+cx^2}}{5ax^5} + \frac{(9bd-10ae)\sqrt{a+bx+cx^2}}{40a^2x^4} - \frac{(63b^2d-64acd-70abe+80a^2f)\sqrt{a+bx+cx^2}}{240a^3x^3} + \frac{(315b^3d-350ab^2e-4ab(161cd-100af)+120a^2(3ce-4ag))\sqrt{a+bx+cx^2}}{960a^4x^2} - \frac{(945b^4d-1050ab^3e-60ab^2(49cd-20af)+256a^2c(4cd-5af)+40a^2b(55ce-36ag))\sqrt{a+bx}}{1920a^5x} + \frac{(63b^5d-70ab^4e+48a^2bc(5cd-4af)-40ab^3(7cd-2af)-32a^3c(3ce-4ag)+48a^2b^2(5ce-2}{128a^5}$$

$$= -\frac{d\sqrt{a+bx+cx^2}}{5ax^5} + \frac{(9bd-10ae)\sqrt{a+bx+cx^2}}{40a^2x^4} - \frac{(63b^2d-64acd-70abe+80a^2f)\sqrt{a+bx+cx^2}}{240a^3x^3} + \frac{(315b^3d-350ab^2e-4ab(161cd-100af)+120a^2(3ce-4ag))\sqrt{a+bx+cx^2}}{960a^4x^2} - \frac{(945b^4d-1050ab^3e-60ab^2(49cd-20af)+256a^2c(4cd-5af)+40a^2b(55ce-36ag))\sqrt{a+bx}}{1920a^5x} + \frac{(63b^5d-70ab^4e+48a^2bc(5cd-4af)-40ab^3(7cd-2af)-32a^3c(3ce-4ag)+48a^2b^2(5ce-2}{256a^{11/2}}$$

Mathematica [A] (verified)

Time = 2.14 (sec) , antiderivative size = 328, normalized size of antiderivative = 0.88

$$\int \frac{d+ex+fx^2+gx^3}{x^6\sqrt{a+bx+cx^2}}\,dx$$
$$=\frac{\frac{\sqrt{a}\sqrt{a+x(b+cx)}(-945b^4dx^4+210ab^2x^3(3bd+14cdx+5bex)-32a^4(12d+5x(3e+4fx+6gx^2))-4a^2x^2(256c^2dx^2+2bcx(161d+275ex)+b^2(126d+2}{x^5}}{}$$

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3)/(x^6*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] ((Sqrt[a]*Sqrt[a + x*(b + c*x)]*(-945*b^4*d*x^4 + 210*a*b^2*x^3*(3*b*d + 14
*c*d*x + 5*b*e*x) - 32*a^4*(12*d + 5*x*(3*e + 4*f*x + 6*g*x^2)) - 4*a^2*x^2
*(256*c^2*d*x^2 + 2*b*c*x*(161*d + 275*e*x) + b^2*(126*d + 25*x*(7*e + 12*f
*x))) + 16*a^3*x*(c*x*(32*d + 5*x*(9*e + 16*f*x)) + b*(27*d + 5*x*(7*e + 2*
x*(5*f + 9*g*x))))))/x^5 - 15*(63*b^5*d + 128*a^4*c*g)*ArcTanh[(Sqrt[c]*x -
 Sqrt[a + x*(b + c*x)])/Sqrt[a]] - 30*a*(35*b^4*e + 48*a^2*c^2*e + 20*b^3*(
7*c*d - 2*a*f) + 24*a*b*c*(-5*c*d + 4*a*f) + 24*a*b^2*(-5*c*e + 2*a*g))*Arc
Tanh[(-(Sqrt[c]*x) + Sqrt[a + x*(b + c*x)])/Sqrt[a]])/(1920*a^(11/2))
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 350, normalized size of antiderivative = 0.94

method	result
risch	$-\frac{\sqrt{c\,x^2+bx+a}\,(-1440a^3bg\,x^4-1280a^3cf\,x^4+1200a^2b^2f\,x^4+2200x^4a^2bce+1024x^4a^2c^2d-1050x^4a\,b^3e-2940x^4a\,b^2cd+945x^4b^4d+}{}$
default	Expression too large to display

```
[In] int((g*x^3+f*x^2+e*x+d)/x^6/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/1920*(c*x^2+b*x+a)^(1/2)*(-1440*a^3*b*g*x^4-1280*a^3*c*f*x^4+1200*a^2*b^
2*f*x^4+2200*a^2*b*c*e*x^4+1024*a^2*c^2*d*x^4-1050*a*b^3*e*x^4-2940*a*b^2*c
*d*x^4+945*b^4*d*x^4+960*a^4*g*x^3-800*a^3*b*f*x^3-720*a^3*c*e*x^3+700*a^2*
b^2*e*x^3+1288*a^2*b*c*d*x^3-630*a*b^3*d*x^3+640*a^4*f*x^2-560*a^3*b*e*x^2-
512*a^3*c*d*x^2+504*a^2*b^2*d*x^2+480*a^4*e*x-432*a^3*b*d*x+384*a^4*d)/a^5/
x^5+1/256*(128*a^4*c*g-96*a^3*b^2*g-192*a^3*b*c*f-96*a^3*c^2*e+80*a^2*b^3*f
+240*a^2*b^2*c*e+240*a^2*b*c^2*d-70*a*b^4*e-280*a*b^3*c*d+63*b^5*d)/a^(11/2
)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)
```

Fricas [A] (verification not implemented)

none

Time = 11.94 (sec) , antiderivative size = 727, normalized size of antiderivative = 1.96

$$\int \frac{d+ex+fx^2+gx^3}{x^6\sqrt{a+bx+cx^2}}\,dx$$

$$= \left[\frac{15\left((63\,b^5-280\,ab^3c+240\,a^2bc^2)d-2\,(35\,ab^4-120\,a^2b^2c+48\,a^3c^2)e+16\,(5\,a^2b^3-12\,a^3bc)f-32\,(3\,a^3\right.}{} \right.$$

$$-\frac{15\left((63\,b^5-280\,ab^3c+240\,a^2bc^2)d-2\,(35\,ab^4-120\,a^2b^2c+48\,a^3c^2)e+16\,(5\,a^2b^3-12\,a^3bc)f-32\,(3\,a\right.}{}$$

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x^6/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [1/7680*(15*((63*b^5 - 280*a*b^3*c + 240*a^2*b*c^2)*d - 2*(35*a*b^4 - 120*a
^2*b^2*c + 48*a^3*c^2)*e + 16*(5*a^2*b^3 - 12*a^3*b*c)*f - 32*(3*a^3*b^2 -
4*a^4*c)*g)*sqrt(a)*x^5*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 + 4*sqrt(c*x^2 +
b*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) - 4*(384*a^5*d - (1440*a^4*b*g -
 (945*a*b^4 - 2940*a^2*b^2*c + 1024*a^3*c^2)*d + 50*(21*a^2*b^3 - 44*a^3*b*
c)*e - 80*(15*a^3*b^2 - 16*a^4*c)*f)*x^4 - 2*(400*a^4*b*f - 480*a^5*g + 7*(
45*a^2*b^3 - 92*a^3*b*c)*d - 10*(35*a^3*b^2 - 36*a^4*c)*e)*x^3 - 8*(70*a^4*
b*e - 80*a^5*f - (63*a^3*b^2 - 64*a^4*c)*d)*x^2 - 48*(9*a^4*b*d - 10*a^5*e)
*x)*sqrt(c*x^2 + b*x + a))/(a^6*x^5), -1/3840*(15*((63*b^5 - 280*a*b^3*c +
240*a^2*b*c^2)*d - 2*(35*a*b^4 - 120*a^2*b^2*c + 48*a^3*c^2)*e + 16*(5*a^2*
b^3 - 12*a^3*b*c)*f - 32*(3*a^3*b^2 - 4*a^4*c)*g)*sqrt(-a)*x^5*arctan(1/2*s
qrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2)) + 2*(384
*a^5*d - (1440*a^4*b*g - (945*a*b^4 - 2940*a^2*b^2*c + 1024*a^3*c^2)*d + 50
*(21*a^2*b^3 - 44*a^3*b*c)*e - 80*(15*a^3*b^2 - 16*a^4*c)*f)*x^4 - 2*(400*a
^4*b*f - 480*a^5*g + 7*(45*a^2*b^3 - 92*a^3*b*c)*d - 10*(35*a^3*b^2 - 36*a^
4*c)*e)*x^3 - 8*(70*a^4*b*e - 80*a^5*f - (63*a^3*b^2 - 64*a^4*c)*d)*x^2 - 4
8*(9*a^4*b*d - 10*a^5*e)*x)*sqrt(c*x^2 + b*x + a))/(a^6*x^5)]
```

Sympy [F]

$$\int \frac{d+ex+fx^2+gx^3}{x^6\sqrt{a+bx+cx^2}}\,dx=\int \frac{d+ex+fx^2+gx^3}{x^6\sqrt{a+bx+cx^2}}\,dx$$

```
[In] integrate((g*x**3+f*x**2+e*x+d)/x**6/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x + f*x**2 + g*x**3)/(x**6*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex+fx^2+gx^3}{x^6\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x^6/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the ’assume’ command before evaluation *may* h
elp (example of legal syntax is ’assume(4*a*c-b^2>0)’, see ‘assume?‘ for mo
re deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2155 vs. $2(341) = 682$.

Time = 0.31 (sec) , antiderivative size = 2155, normalized size of antiderivative = 5.81

$$\int \frac{d+ex+fx^2+gx^3}{x^6\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x^6/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/128*(63*b^5*d - 280*a*b^3*c*d + 240*a^2*b*c^2*d - 70*a*b^4*e + 240*a^2*b
^2*c*e - 96*a^3*c^2*e + 80*a^2*b^3*f - 192*a^3*b*c*f - 96*a^3*b^2*g + 128*a
^4*c*g)*arctan(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))/sqrt(-a))/(sqrt(-a)*a^5
) + 1/1920*(945*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*b^5*d - 4200*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^9*a*b^3*c*d + 3600*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^9*a^2*b*c^2*d - 1050*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a*b^4*e
+ 3600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^2*b^2*c*e - 1440*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))^9*a^3*c^2*e + 1200*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))^9*a^2*b^3*f - 2880*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^3*b*c*f - 1
440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^3*b^2*g + 1920*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^9*a^4*c*g - 4410*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*
a*b^5*d + 19600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^2*b^3*c*d - 16800*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^3*b*c^2*d + 4900*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))^7*a^2*b^4*e - 16800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*
a^3*b^2*c*e + 6720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^4*c^2*e - 5600*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^3*b^3*f + 13440*(sqrt(c)*x - sqrt(c*
x^2 + b*x + a))^7*a^4*b*c*f + 6720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^
4*b^2*g - 3840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^5*c*g + 7680*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^6*a^5*c^(3/2)*f + 3840*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^6*a^5*b*sqrt(c)*g + 8064*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5
```

```
*a^2*b^5*d - 35840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^3*b^3*c*d + 3072
0*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^4*b*c^2*d - 8960*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^5*a^3*b^4*e + 30720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
^5*a^4*b^2*c*e + 10240*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^4*b^3*f - 15
360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^5*b*c*f - 11520*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))^5*a^5*b^2*g + 20480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)^4*a^5*c^(5/2)*d + 20480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^5*b*c^(3/
2)*e + 3840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^5*b^2*sqrt(c)*f - 17920
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^6*c^(3/2)*f - 11520*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^4*a^6*b*sqrt(c)*g - 7110*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^3*a^3*b^5*d + 31600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^4*b^3*c
*d + 16800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^5*b*c^2*d + 7900*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^3*a^4*b^4*e - 13920*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))^3*a^5*b^2*c*e - 6720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^6*c^
2*e - 8480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^5*b^3*f + 1920*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^3*a^6*b*c*f + 8640*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))^3*a^6*b^2*g + 3840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^7*c*g + 3
8400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^5*b^2*c^(3/2)*d - 10240*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^2*a^6*c^(5/2)*d + 3840*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^2*a^5*b^3*sqrt(c)*e - 25600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))^2*a^6*b*c^(3/2)*e - 7680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^6*b^2*s
qrt(c)*f + 12800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^7*c^(3/2)*f + 1152
0*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^7*b*sqrt(c)*g + 2895*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))*a^4*b^5*d + 4200*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)*a^5*b^3*c*d - 3600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^6*b*c^2*d - 2790
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^5*b^4*e - 3600*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))*a^6*b^2*c*e + 1440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^7*c
^2*e + 2640*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^6*b^3*f + 2880*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))*a^7*b*c*f - 2400*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))*a^7*b^2*g - 1920*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^8*c*g + 3840*a^5
*b^4*sqrt(c)*d - 7680*a^6*b^2*c^(3/2)*d + 2048*a^7*c^(5/2)*d - 3840*a^6*b^3
*sqrt(c)*e + 5120*a^7*b*c^(3/2)*e + 3840*a^7*b^2*sqrt(c)*f - 2560*a^8*c^(3/
2)*f - 3840*a^8*b*sqrt(c)*g)/(((sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2 - a)^5
*a^5)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2+gx^3}{x^6\sqrt{a+bx+cx^2}}\,dx=\int \frac{g\,x^3+f\,x^2+e\,x+d}{x^6\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

```
[In] int((d + e*x + f*x^2 + g*x^3)/(x^6*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] int((d + e*x + f*x^2 + g*x^3)/(x^6*(a + b*x + c*x^2)^(1/2)), x)
```

3.289 $\int (d+ex)^3 \left(3+2x+5x^2\right) \left(2+x+3x^2-5x^3+4x^4\right)$

Optimal result

Integrand size = 36, antiderivative size = 258

$$
\begin{aligned}
\int (d+ex)^3 \left(3+2x+5x^2\right) \left(2+x+3x^2-5x^3+4x^4\right)\,dx
&= \frac{(5d^2-2de+3e^2)\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)(d+ex)^4}{4e^7} \\
&\quad - \frac{\left(120d^5+85d^4e+68d^3e^2+12d^2e^3+42de^4-7e^5\right)(d+ex)^5}{5e^7} \\
&\quad + \frac{\left(300d^4+170d^3e+102d^2e^2+12de^3+21e^4\right)(d+ex)^6}{6e^7} \\
&\quad - \frac{2\left(200d^3+85d^2e+34de^2+2e^3\right)(d+ex)^7}{7e^7} \\
&\quad + \frac{\left(300d^2+85de+17e^2\right)(d+ex)^8}{8e^7} - \frac{(120d+17e)(d+ex)^9}{9e^7} + \frac{2(d+ex)^{10}}{e^7}
\end{aligned}
$$

```
[Out] 1/4*(5*d^2-2*d*e+3*e^2)*(4*d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)*(e*x+d)^4/e^7
-1/5*(120*d^5+85*d^4*e+68*d^3*e^2+12*d^2*e^3+42*d*e^4-7*e^5)*(e*x+d)^5/e^7+
1/6*(300*d^4+170*d^3*e+102*d^2*e^2+12*d*e^3+21*e^4)*(e*x+d)^6/e^7-2/7*(200*
d^3+85*d^2*e+34*d*e^2+2*e^3)*(e*x+d)^7/e^7+1/8*(300*d^2+85*d*e+17*e^2)*(e*x
+d)^8/e^7-1/9*(120*d+17*e)*(e*x+d)^9/e^7+2*(e*x+d)^10/e^7
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {1642}

$$\begin{aligned}
&\int (d+ex)^3\left(3+2x+5x^2\right)\left(2+x+3x^2-5x^3+4x^4\right)\,dx\\
&=\frac{\left(300d^2+85de+17e^2\right)(d+ex)^8}{8e^7}-\frac{2\left(200d^3+85d^2e+34de^2+2e^3\right)(d+ex)^7}{7e^7}\\
&\quad+\frac{\left(300d^4+170d^3e+102d^2e^2+12de^3+21e^4\right)(d+ex)^6}{6e^7}\\
&\quad+\frac{\left(5d^2-2de+3e^2\right)\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)(d+ex)^4}{4e^7}\\
&\quad-\frac{\left(120d^5+85d^4e+68d^3e^2+12d^2e^3+42de^4-7e^5\right)(d+ex)^5}{5e^7}\\
&\quad+\frac{2(d+ex)^{10}}{e^7}-\frac{(120d+17e)(d+ex)^9}{9e^7}
\end{aligned}$$

```
[In] Int[(d + e*x)^3*(3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

```
[Out] ((5*d^2 - 2*d*e + 3*e^2)*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*(d +
 e*x)^4)/(4*e^7) - ((120*d^5 + 85*d^4*e + 68*d^3*e^2 + 12*d^2*e^3 + 42*d*e^
4 - 7*e^5)*(d + e*x)^5)/(5*e^7) + ((300*d^4 + 170*d^3*e + 102*d^2*e^2 + 12*
d*e^3 + 21*e^4)*(d + e*x)^6)/(6*e^7) - (2*(200*d^3 + 85*d^2*e + 34*d*e^2 +
2*e^3)*(d + e*x)^7)/(7*e^7) + ((300*d^2 + 85*d*e + 17*e^2)*(d + e*x)^8)/(8*
e^7) - ((120*d + 17*e)*(d + e*x)^9)/(9*e^7) + (2*(d + e*x)^10)/e^7
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral}=\int\Bigg(&\frac{\left(20d^6+17d^5e+17d^4e^2+4d^3e^3+21d^2e^4-7de^5+6e^6\right)(d+ex)^3}{e^6}\\
&+\frac{\left(-120d^5-85d^4e-68d^3e^2-12d^2e^3-42de^4+7e^5\right)(d+ex)^4}{e^6}\\
&+\frac{\left(300d^4+170d^3e+102d^2e^2+12de^3+21e^4\right)(d+ex)^5}{e^6}\\
&-\frac{2\left(200d^3+85d^2e+34de^2+2e^3\right)(d+ex)^6}{e^6}+\frac{\left(300d^2+85de+17e^2\right)(d+ex)^7}{e^6}\\
&+\frac{(-120d-17e)(d+ex)^8}{e^6}+\frac{20(d+ex)^9}{e^6}\Bigg)\,dx
\end{aligned}$$

$$
\begin{aligned}
&= \frac{(5d^2-2de+3e^2)\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)(d+ex)^4}{4e^7} \\
&- \frac{(120d^5+85d^4e+68d^3e^2+12d^2e^3+42de^4-7e^5)(d+ex)^5}{5e^7} \\
&+ \frac{(300d^4+170d^3e+102d^2e^2+12de^3+21e^4)(d+ex)^6}{6e^7} \\
&- \frac{2(200d^3+85d^2e+34de^2+2e^3)(d+ex)^7}{7e^7} \\
&+ \frac{(300d^2+85de+17e^2)(d+ex)^8}{8e^7} - \frac{(120d+17e)(d+ex)^9}{9e^7} + \frac{2(d+ex)^{10}}{e^7}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.82

$$
\begin{aligned}
&\int (d+ex)^3\left(3+2x+5x^2\right)\left(2+x+3x^2-5x^3+4x^4\right)\,dx \\
&= 6d^3x+\frac{1}{2}d^2(7d+18e)x^2+d\left(7d^2+7de+6e^2\right)x^3 \\
&+\frac{1}{4}\left(-4d^3+63d^2e+21de^2+6e^3\right)x^4+\frac{1}{5}\left(17d^3-12d^2e+63de^2+7e^3\right)x^5 \\
&+\frac{1}{6}\left(-17d^3+51d^2e-12de^2+21e^3\right)x^6+\frac{1}{7}\left(20d^3-51d^2e+51de^2-4e^3\right)x^7 \\
&+\frac{1}{8}e\left(60d^2-51de+17e^2\right)x^8+\frac{1}{9}(60d-17e)e^2x^9+2e^3x^{10}
\end{aligned}
$$

```
[In] Integrate[(d + e*x)^3*(3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

```
[Out] 6*d^3*x + (d^2*(7*d + 18*e)*x^2)/2 + d*(7*d^2 + 7*d*e + 6*e^2)*x^3 + ((-4*d
^3 + 63*d^2*e + 21*d*e^2 + 6*e^3)*x^4)/4 + ((17*d^3 - 12*d^2*e + 63*d*e^2 +
 7*e^3)*x^5)/5 + ((-17*d^3 + 51*d^2*e - 12*d*e^2 + 21*e^3)*x^6)/6 + ((20*d^
3 - 51*d^2*e + 51*d*e^2 - 4*e^3)*x^7)/7 + (e*(60*d^2 - 51*d*e + 17*e^2)*x^8
)/8 + ((60*d - 17*e)*e^2*x^9)/9 + 2*e^3*x^10
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.78

method	result
norman	$2e^{3}x^{10}+\left(\frac{20}{3}d\,e^{2}-\frac{17}{9}e^{3}\right)x^{9}+\left(\frac{15}{2}d^{2}e-\frac{51}{8}d\,e^{2}+\frac{17}{8}e^{3}\right)x^{8}+\left(\frac{20}{7}d^{3}-\frac{51}{7}d^{2}e+\frac{51}{7}d\,e^{2}-\frac{4}{7}e^{3}\right)x^{7}$
default	$2e^{3}x^{10}+\frac{(60d\,e^{2}-17e^{3})x^{9}}{9}+\frac{(60d^{2}e-51d\,e^{2}+17e^{3})x^{8}}{8}+\frac{(20d^{3}-51d^{2}e+51d\,e^{2}-4e^{3})x^{7}}{7}+\frac{(-17d^{3}+51d^{2}e-12d\,e^{2}+21e}{6}$
gosper	$7d^{2}e\,x^{3}+\frac{17}{2}x^{6}d^{2}e-2x^{6}d\,e^{2}-\frac{12}{5}x^{5}d^{2}e+\frac{63}{5}x^{5}d\,e^{2}+\frac{63}{4}x^{4}d^{2}e+\frac{21}{4}x^{4}d\,e^{2}-\frac{51}{7}x^{7}d^{2}e+\frac{51}{7}x^{7}d\,e$
risch	$7d^{2}e\,x^{3}+\frac{17}{2}x^{6}d^{2}e-2x^{6}d\,e^{2}-\frac{12}{5}x^{5}d^{2}e+\frac{63}{5}x^{5}d\,e^{2}+\frac{63}{4}x^{4}d^{2}e+\frac{21}{4}x^{4}d\,e^{2}-\frac{51}{7}x^{7}d^{2}e+\frac{51}{7}x^{7}d\,e$
parallelrisch	$7d^{2}e\,x^{3}+\frac{17}{2}x^{6}d^{2}e-2x^{6}d\,e^{2}-\frac{12}{5}x^{5}d^{2}e+\frac{63}{5}x^{5}d\,e^{2}+\frac{63}{4}x^{4}d^{2}e+\frac{21}{4}x^{4}d\,e^{2}-\frac{51}{7}x^{7}d^{2}e+\frac{51}{7}x^{7}d\,e$

```
[In] int((e*x+d)^3*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x,method=_RETURNVERBOSE
)
```

```
[Out] 2*e^3*x^10+(20/3*d*e^2-17/9*e^3)*x^9+(15/2*d^2*e-51/8*d*e^2+17/8*e^3)*x^8+(
20/7*d^3-51/7*d^2*e+51/7*d*e^2-4/7*e^3)*x^7+(-17/6*d^3+17/2*d^2*e-2*d*e^2+7
/2*e^3)*x^6+(17/5*d^3-12/5*d^2*e+63/5*d*e^2+7/5*e^3)*x^5+(-d^3+63/4*d^2*e+2
1/4*d*e^2+3/2*e^3)*x^4+(7*d^3+7*d^2*e+6*d*e^2)*x^3+(7/2*d^3+9*d^2*e)*x^2+6*
x*d^3
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.80

$$
\begin{aligned}
&\int (d+ex)^3\left(3+2x+5x^2\right)\left(2+x+3x^2-5x^3+4x^4\right)\,dx\\
&=2\,e^3x^{10}+\frac{1}{9}\left(60\,de^2-17\,e^3\right)x^9+\frac{1}{8}\left(60\,d^2e-51\,de^2+17\,e^3\right)x^8\\
&\quad+\frac{1}{7}\left(20\,d^3-51\,d^2e+51\,de^2-4\,e^3\right)x^7-\frac{1}{6}\left(17\,d^3-51\,d^2e+12\,de^2-21\,e^3\right)x^6\\
&\quad+\frac{1}{5}\left(17\,d^3-12\,d^2e+63\,de^2+7\,e^3\right)x^5-\frac{1}{4}\left(4\,d^3-63\,d^2e-21\,de^2-6\,e^3\right)x^4\\
&\quad+6\,d^3x+\left(7\,d^3+7\,d^2e+6\,de^2\right)x^3+\frac{1}{2}\left(7\,d^3+18\,d^2e\right)x^2
\end{aligned}
$$

```
[In] integrate((e*x+d)^3*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="fri
cas")
```

```
[Out] 2*e^3*x^10 + 1/9*(60*d*e^2 - 17*e^3)*x^9 + 1/8*(60*d^2*e - 51*d*e^2 + 17*e^
3)*x^8 + 1/7*(20*d^3 - 51*d^2*e + 51*d*e^2 - 4*e^3)*x^7 - 1/6*(17*d^3 - 51*
d^2*e + 12*d*e^2 - 21*e^3)*x^6 + 1/5*(17*d^3 - 12*d^2*e + 63*d*e^2 + 7*e^3)
*x^5 - 1/4*(4*d^3 - 63*d^2*e - 21*d*e^2 - 6*e^3)*x^4 + 6*d^3*x + (7*d^3 + 7
*d^2*e + 6*d*e^2)*x^3 + 1/2*(7*d^3 + 18*d^2*e)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.89

$$
\begin{aligned}
&\int (d+ex)^3\left(3+2x+5x^2\right)\left(2+x+3x^2-5x^3+4x^4\right)\,dx\\
&=6d^3x+2e^3x^{10}+x^9\cdot\left(\frac{20de^2}{3}-\frac{17e^3}{9}\right)+x^8\cdot\left(\frac{15d^2e}{2}-\frac{51de^2}{8}+\frac{17e^3}{8}\right)+x^7\\
&\cdot\left(\frac{20d^3}{7}-\frac{51d^2e}{7}+\frac{51de^2}{7}-\frac{4e^3}{7}\right)+x^6\left(-\frac{17d^3}{6}+\frac{17d^2e}{2}-2de^2+\frac{7e^3}{2}\right)\\
&+x^5\cdot\left(\frac{17d^3}{5}-\frac{12d^2e}{5}+\frac{63de^2}{5}+\frac{7e^3}{5}\right)+x^4\left(-d^3+\frac{63d^2e}{4}+\frac{21de^2}{4}+\frac{3e^3}{2}\right)\\
&+x^3\cdot\left(7d^3+7d^2e+6de^2\right)+x^2\cdot\left(\frac{7d^3}{2}+9d^2e\right)
\end{aligned}
$$

```
[In] integrate((e*x+d)**3*(5*x**2+2*x+3)*(4*x**4-5*x**3+3*x**2+x+2),x)
```

```
[Out] 6*d**3*x + 2*e**3*x**10 + x**9*(20*d*e**2/3 - 17*e**3/9) + x**8*(15*d**2*e/
2 - 51*d*e**2/8 + 17*e**3/8) + x**7*(20*d**3/7 - 51*d**2*e/7 + 51*d*e**2/7
- 4*e**3/7) + x**6*(-17*d**3/6 + 17*d**2*e/2 - 2*d*e**2 + 7*e**3/2) + x**5*
(17*d**3/5 - 12*d**2*e/5 + 63*d*e**2/5 + 7*e**3/5) + x**4*(-d**3 + 63*d**2*
e/4 + 21*d*e**2/4 + 3*e**3/2) + x**3*(7*d**3 + 7*d**2*e + 6*d*e**2) + x**2*
(7*d**3/2 + 9*d**2*e)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.80

$$
\begin{aligned}
&\int (d+ex)^3\left(3+2x+5x^2\right)\left(2+x+3x^2-5x^3+4x^4\right)\,dx\\
&=2\,e^3x^{10}+\frac{1}{9}\left(60\,de^2-17\,e^3\right)x^9+\frac{1}{8}\left(60\,d^2e-51\,de^2+17\,e^3\right)x^8\\
&+\frac{1}{7}\left(20\,d^3-51\,d^2e+51\,de^2-4\,e^3\right)x^7-\frac{1}{6}\left(17\,d^3-51\,d^2e+12\,de^2-21\,e^3\right)x^6\\
&+\frac{1}{5}\left(17\,d^3-12\,d^2e+63\,de^2+7\,e^3\right)x^5-\frac{1}{4}\left(4\,d^3-63\,d^2e-21\,de^2-6\,e^3\right)x^4\\
&+6\,d^3x+\left(7\,d^3+7\,d^2e+6\,de^2\right)x^3+\frac{1}{2}\left(7\,d^3+18\,d^2e\right)x^2
\end{aligned}
$$

```
[In] integrate((e*x+d)^3*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="max
ima")
```

```
[Out] 2*e^3*x^10 + 1/9*(60*d*e^2 - 17*e^3)*x^9 + 1/8*(60*d^2*e - 51*d*e^2 + 17*e^
3)*x^8 + 1/7*(20*d^3 - 51*d^2*e + 51*d*e^2 - 4*e^3)*x^7 - 1/6*(17*d^3 - 51*
```

```
d^2*e + 12*d*e^2 - 21*e^3)*x^6 + 1/5*(17*d^3 - 12*d^2*e + 63*d*e^2 + 7*e^3)
*x^5 - 1/4*(4*d^3 - 63*d^2*e - 21*d*e^2 - 6*e^3)*x^4 + 6*d^3*x + (7*d^3 + 7
*d^2*e + 6*d*e^2)*x^3 + 1/2*(7*d^3 + 18*d^2*e)*x^2
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.92

$$\begin{aligned}\int (d+ex)^3\left(3+2x+5x^2\right)\left(2+x+3x^2-5x^3+4x^4\right)\,dx&\\ =2\,e^3x^{10}+\frac{20}{3}\,de^2x^9-\frac{17}{9}\,e^3x^9+\frac{15}{2}\,d^2ex^8-\frac{51}{8}\,de^2x^8+\frac{17}{8}\,e^3x^8+\frac{20}{7}\,d^3x^7&\\ -\frac{51}{7}\,d^2ex^7+\frac{51}{7}\,de^2x^7-\frac{4}{7}\,e^3x^7-\frac{17}{6}\,d^3x^6+\frac{17}{2}\,d^2ex^6-2\,de^2x^6+\frac{7}{2}\,e^3x^6&\\ +\frac{17}{5}\,d^3x^5-\frac{12}{5}\,d^2ex^5+\frac{63}{5}\,de^2x^5+\frac{7}{5}\,e^3x^5-d^3x^4+\frac{63}{4}\,d^2ex^4+\frac{21}{4}\,de^2x^4&\\ +\frac{3}{2}\,e^3x^4+7\,d^3x^3+7\,d^2ex^3+6\,de^2x^3+\frac{7}{2}\,d^3x^2+9\,d^2ex^2+6\,d^3x&\end{aligned}$$

```
[In] integrate((e*x+d)^3*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="gia
c")
```

```
[Out] 2*e^3*x^10 + 20/3*d*e^2*x^9 - 17/9*e^3*x^9 + 15/2*d^2*e*x^8 - 51/8*d*e^2*x^
8 + 17/8*e^3*x^8 + 20/7*d^3*x^7 - 51/7*d^2*e*x^7 + 51/7*d*e^2*x^7 - 4/7*e^3
*x^7 - 17/6*d^3*x^6 + 17/2*d^2*e*x^6 - 2*d*e^2*x^6 + 7/2*e^3*x^6 + 17/5*d^3
*x^5 - 12/5*d^2*e*x^5 + 63/5*d*e^2*x^5 + 7/5*e^3*x^5 - d^3*x^4 + 63/4*d^2*e
*x^4 + 21/4*d*e^2*x^4 + 3/2*e^3*x^4 + 7*d^3*x^3 + 7*d^2*e*x^3 + 6*d*e^2*x^3
 + 7/2*d^3*x^2 + 9*d^2*e*x^2 + 6*d^3*x
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.76

$$\begin{aligned}\int (d+ex)^3\left(3+2x+5x^2\right)\left(2+x+3x^2-5x^3+4x^4\right)\,dx&\\ =6\,d^3\,x+x^8\left(\frac{15\,d^2\,e}{2}-\frac{51\,d\,e^2}{8}+\frac{17\,e^3}{8}\right)-x^6\left(\frac{17\,d^3}{6}-\frac{17\,d^2\,e}{2}+2\,d\,e^2-\frac{7\,e^3}{2}\right)&\\ +x^4\left(-d^3+\frac{63\,d^2\,e}{4}+\frac{21\,d\,e^2}{4}+\frac{3\,e^3}{2}\right)+x^5\left(\frac{17\,d^3}{5}-\frac{12\,d^2\,e}{5}+\frac{63\,d\,e^2}{5}+\frac{7\,e^3}{5}\right)&\\ +x^7\left(\frac{20\,d^3}{7}-\frac{51\,d^2\,e}{7}+\frac{51\,d\,e^2}{7}-\frac{4\,e^3}{7}\right)+2\,e^3\,x^{10}&\\ +d\,x^3\left(7\,d^2+7\,d\,e+6\,e^2\right)+\frac{d^2\,x^2\,(7\,d+18\,e)}{2}+\frac{e^2\,x^9\,(60\,d-17\,e)}{9}&\end{aligned}$$

```
[In] int((d + e*x)^3*(2*x + 5*x^2 + 3)*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2),x)
```

```
[Out] 6*d^3*x + x^8*((15*d^2*e)/2 - (51*d*e^2)/8 + (17*e^3)/8) - x^6*(2*d*e^2 - (
17*d^2*e)/2 + (17*d^3)/6 - (7*e^3)/2) + x^4*((21*d*e^2)/4 + (63*d^2*e)/4 -
d^3 + (3*e^3)/2) + x^5*((63*d*e^2)/5 - (12*d^2*e)/5 + (17*d^3)/5 + (7*e^3)/
5) + x^7*((51*d*e^2)/7 - (51*d^2*e)/7 + (20*d^3)/7 - (4*e^3)/7) + 2*e^3*x^1
0 + d*x^3*(7*d*e + 7*d^2 + 6*e^2) + (d^2*x^2*(7*d + 18*e))/2 + (e^2*x^9*(60
*d - 17*e))/9
```

3.290 $\int (d+ex)^2 \left(3+2x+5x^2\right) \left(2+x+3x^2-5x^3+4x^4\right) d$

Optimal result

Integrand size = 36, antiderivative size = 157

$$\begin{aligned}\int (d+ex)^2 \left(3+2x+5x^2\right) \left(2+x+3x^2-5x^3+4x^4\right) dx &= 6d^2x+\frac{1}{2}d(7d+12e)x^2+\frac{1}{3}\left(21d^2+14de+6e^2\right)x^3-\frac{1}{4}\left(4d^2-42de-7e^2\right)x^4\\ &\quad+\frac{1}{5}\left(17d^2-8de+21e^2\right)x^5-\frac{1}{6}\left(17d^2-34de+4e^2\right)x^6\\ &\quad+\frac{1}{7}\left(20d^2-34de+17e^2\right)x^7+\frac{1}{8}(40d-17e)ex^8+\frac{20e^2x^9}{9}\end{aligned}$$

```
[Out] 6*d^2*x+1/2*d*(7*d+12*e)*x^2+1/3*(21*d^2+14*d*e+6*e^2)*x^3-1/4*(4*d^2-42*d*
e-7*e^2)*x^4+1/5*(17*d^2-8*d*e+21*e^2)*x^5-1/6*(17*d^2-34*d*e+4*e^2)*x^6+1/
7*(20*d^2-34*d*e+17*e^2)*x^7+1/8*(40*d-17*e)*e*x^8+20/9*e^2*x^9
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {1642}

$$\begin{aligned}\int (d+ex)^2 \left(3+2x+5x^2\right) \left(2+x+3x^2-5x^3+4x^4\right) dx &= \frac{1}{7}x^7\left(20d^2-34de+17e^2\right)-\frac{1}{6}x^6\left(17d^2-34de+4e^2\right)\\ &\quad+\frac{1}{5}x^5\left(17d^2-8de+21e^2\right)-\frac{1}{4}x^4\left(4d^2-42de-7e^2\right)+\frac{1}{3}x^3\left(21d^2+14de+6e^2\right)\\ &\quad+6d^2x+\frac{1}{8}ex^8(40d-17e)+\frac{1}{2}dx^2(7d+12e)+\frac{20e^2x^9}{9}\end{aligned}$$

```
[In] Int[(d + e*x)^2*(3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

```
[Out] 6*d^2*x + (d*(7*d + 12*e)*x^2)/2 + ((21*d^2 + 14*d*e + 6*e^2)*x^3)/3 - ((4*
d^2 - 42*d*e - 7*e^2)*x^4)/4 + ((17*d^2 - 8*d*e + 21*e^2)*x^5)/5 - ((17*d^2
 - 34*d*e + 4*e^2)*x^6)/6 + ((20*d^2 - 34*d*e + 17*e^2)*x^7)/7 + ((40*d - 1
7*e)*e*x^8)/8 + (20*e^2*x^9)/9
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(6d^2+d(7d+12e)x+\left(21d^2+14de+6e^2\right)x^2-\left(4d^2-42de-7e^2\right)x^3\right.\\ &\qquad +\left(17d^2-8de+21e^2\right)x^4-\left(17d^2-34de+4e^2\right)x^5+\left(20d^2-34de+17e^2\right)x^6\\ &\qquad \left.+(40d-17e)ex^7+20e^2x^8\right)\,dx\\ &=6d^2x+\frac{1}{2}d(7d+12e)x^2+\frac{1}{3}\left(21d^2+14de+6e^2\right)x^3-\frac{1}{4}\left(4d^2-42de-7e^2\right)x^4\\ &\quad+\frac{1}{5}\left(17d^2-8de+21e^2\right)x^5-\frac{1}{6}\left(17d^2-34de+4e^2\right)x^6\\ &\quad+\frac{1}{7}\left(20d^2-34de+17e^2\right)x^7+\frac{1}{8}(40d-17e)ex^8+\frac{20e^2x^9}{9}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.87

$$\begin{aligned}&\int (d+ex)^2\left(3+2x+5x^2\right)\left(2+x+3x^2-5x^3+4x^4\right)\,dx\\ &=\frac{e^2x^3(5040+4410x+10584x^2-1680x^3+6120x^4-5355x^5+5600x^6)}{2520}\\ &\quad+d^2\left(6x+\frac{7x^2}{2}+7x^3-x^4+\frac{17x^5}{5}-\frac{17x^6}{6}+\frac{20x^7}{7}\right)\\ &\quad+de\left(6x^2+\frac{14x^3}{3}+\frac{21x^4}{2}-\frac{8x^5}{5}+\frac{17x^6}{3}-\frac{34x^7}{7}+5x^8\right)\end{aligned}$$

```
[In] Integrate[(d + e*x)^2*(3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

```
[Out] (e^2*x^3*(5040 + 4410*x + 10584*x^2 - 1680*x^3 + 6120*x^4 - 5355*x^5 + 5600
*x^6))/2520 + d^2*(6*x + (7*x^2)/2 + 7*x^3 - x^4 + (17*x^5)/5 - (17*x^6)/6
+ (20*x^7)/7) + d*e*(6*x^2 + (14*x^3)/3 + (21*x^4)/2 - (8*x^5)/5 + (17*x^6)
/3 - (34*x^7)/7 + 5*x^8)
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.89

method	result
norman	$\frac{20e^2x^9}{9}+\left(5de-\frac{17}{8}e^2\right)x^8+\left(\frac{20}{7}d^2-\frac{34}{7}de+\frac{17}{7}e^2\right)x^7+\left(-\frac{17}{6}d^2+\frac{17}{3}de-\frac{2}{3}e^2\right)x^6+\left(\frac{17}{5}d^2-\frac{8}{5}d\right.$
default	$\frac{20e^2x^9}{9}+\frac{(40de-17e^2)x^8}{8}+\frac{(20d^2-34de+17e^2)x^7}{7}+\frac{(-17d^2+34de-4e^2)x^6}{6}+\frac{(17d^2-8de+21e^2)x^5}{5}+\frac{(-4d^2+42de+}{4}$
gosper	$\frac{20}{9}e^2x^9+5x^8de-\frac{17}{8}x^8e^2+\frac{20}{7}x^7d^2-\frac{34}{7}x^7de+\frac{17}{7}x^7e^2-\frac{17}{6}x^6d^2+\frac{17}{3}x^6de-\frac{2}{3}x^6e^2+\frac{17}{5}x^5d^2-$
risch	$\frac{20}{9}e^2x^9+5x^8de-\frac{17}{8}x^8e^2+\frac{20}{7}x^7d^2-\frac{34}{7}x^7de+\frac{17}{7}x^7e^2-\frac{17}{6}x^6d^2+\frac{17}{3}x^6de-\frac{2}{3}x^6e^2+\frac{17}{5}x^5d^2-$
parallelrisch	$\frac{20}{9}e^2x^9+5x^8de-\frac{17}{8}x^8e^2+\frac{20}{7}x^7d^2-\frac{34}{7}x^7de+\frac{17}{7}x^7e^2-\frac{17}{6}x^6d^2+\frac{17}{3}x^6de-\frac{2}{3}x^6e^2+\frac{17}{5}x^5d^2-$

```
[In] int((e*x+d)^2*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x,method=_RETURNVERBOSE
)
```

```
[Out] 20/9*e^2*x^9+(5*d*e-17/8*e^2)*x^8+(20/7*d^2-34/7*d*e+17/7*e^2)*x^7+(-17/6*d
^2+17/3*d*e-2/3*e^2)*x^6+(17/5*d^2-8/5*d*e+21/5*e^2)*x^5+(-d^2+21/2*d*e+7/4
*e^2)*x^4+(7*d^2+14/3*d*e+2*e^2)*x^3+(7/2*d^2+6*d*e)*x^2+6*x*d^2
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.92

$$\begin{aligned}\int (d+ex)^2\left(3+2x+5x^2\right)\left(2+x+3x^2-5x^3+4x^4\right)\,dx &= \frac{20}{9}\,e^2x^9+\frac{1}{8}\left(40\,de-17\,e^2\right)x^8+\frac{1}{7}\left(20\,d^2-34\,de+17\,e^2\right)x^7\\&\quad-\frac{1}{6}\left(17\,d^2-34\,de+4\,e^2\right)x^6+\frac{1}{5}\left(17\,d^2-8\,de+21\,e^2\right)x^5-\frac{1}{4}\left(4\,d^2-42\,de-7\,e^2\right)x^4\\&\quad+\frac{1}{3}\left(21\,d^2+14\,de+6\,e^2\right)x^3+6\,d^2x+\frac{1}{2}\left(7\,d^2+12\,de\right)x^2\end{aligned}$$

```
[In] integrate((e*x+d)^2*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="fri
cas")
```

```
[Out] 20/9*e^2*x^9 + 1/8*(40*d*e - 17*e^2)*x^8 + 1/7*(20*d^2 - 34*d*e + 17*e^2)*x
^7 - 1/6*(17*d^2 - 34*d*e + 4*e^2)*x^6 + 1/5*(17*d^2 - 8*d*e + 21*e^2)*x^5
- 1/4*(4*d^2 - 42*d*e - 7*e^2)*x^4 + 1/3*(21*d^2 + 14*d*e + 6*e^2)*x^3 + 6*
d^2*x + 1/2*(7*d^2 + 12*d*e)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.01

$$\begin{aligned}
&\int (d+ex)^2 \left(3+2x+5x^2\right) \left(2+x+3x^2-5x^3+4x^4\right) \, dx \\
&= 6d^2x + \frac{20e^2x^9}{9} + x^8 \cdot \left(5de - \frac{17e^2}{8}\right) + x^7 \cdot \left(\frac{20d^2}{7} - \frac{34de}{7} + \frac{17e^2}{7}\right) \\
&\quad + x^6\left(-\frac{17d^2}{6} + \frac{17de}{3} - \frac{2e^2}{3}\right) + x^5 \cdot \left(\frac{17d^2}{5} - \frac{8de}{5} + \frac{21e^2}{5}\right) \\
&\quad + x^4\left(-d^2 + \frac{21de}{2} + \frac{7e^2}{4}\right) + x^3 \cdot \left(7d^2 + \frac{14de}{3} + 2e^2\right) + x^2 \cdot \left(\frac{7d^2}{2} + 6de\right)
\end{aligned}$$

```
[In] integrate((e*x+d)**2*(5*x**2+2*x+3)*(4*x**4-5*x**3+3*x**2+x+2),x)
```

```
[Out] 6*d**2*x + 20*e**2*x**9/9 + x**8*(5*d*e - 17*e**2/8) + x**7*(20*d**2/7 - 34
*d*e/7 + 17*e**2/7) + x**6*(-17*d**2/6 + 17*d*e/3 - 2*e**2/3) + x**5*(17*d*
*2/5 - 8*d*e/5 + 21*e**2/5) + x**4*(-d**2 + 21*d*e/2 + 7*e**2/4) + x**3*(7*
d**2 + 14*d*e/3 + 2*e**2) + x**2*(7*d**2/2 + 6*d*e)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.92

$$\begin{aligned}
&\int (d+ex)^2 \left(3+2x+5x^2\right) \left(2+x+3x^2-5x^3+4x^4\right) \, dx \\
&= \frac{20}{9}\,e^2x^9 + \frac{1}{8}\left(40\,de - 17\,e^2\right)x^8 + \frac{1}{7}\left(20\,d^2 - 34\,de + 17\,e^2\right)x^7 \\
&\quad - \frac{1}{6}\left(17\,d^2 - 34\,de + 4\,e^2\right)x^6 + \frac{1}{5}\left(17\,d^2 - 8\,de + 21\,e^2\right)x^5 - \frac{1}{4}\left(4\,d^2 - 42\,de - 7\,e^2\right)x^4 \\
&\quad + \frac{1}{3}\left(21\,d^2 + 14\,de + 6\,e^2\right)x^3 + 6\,d^2x + \frac{1}{2}\left(7\,d^2 + 12\,de\right)x^2
\end{aligned}$$

```
[In] integrate((e*x+d)^2*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="max
ima")
```

```
[Out] 20/9*e^2*x^9 + 1/8*(40*d*e - 17*e^2)*x^8 + 1/7*(20*d^2 - 34*d*e + 17*e^2)*x
^7 - 1/6*(17*d^2 - 34*d*e + 4*e^2)*x^6 + 1/5*(17*d^2 - 8*d*e + 21*e^2)*x^5
- 1/4*(4*d^2 - 42*d*e - 7*e^2)*x^4 + 1/3*(21*d^2 + 14*d*e + 6*e^2)*x^3 + 6*
d^2*x + 1/2*(7*d^2 + 12*d*e)*x^2
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.02

$$\begin{aligned}\int(d+ex)^2\left(3+2x+5x^2\right)\left(2+x+3x^2-5x^3+4x^4\right)\,dx&\\=\frac{20}{9}e^2x^9+5\,dex^8-\frac{17}{8}e^2x^8+\frac{20}{7}d^2x^7-\frac{34}{7}dex^7+\frac{17}{7}e^2x^7-\frac{17}{6}d^2x^6&\\+\frac{17}{3}dex^6-\frac{2}{3}e^2x^6+\frac{17}{5}d^2x^5-\frac{8}{5}dex^5+\frac{21}{5}e^2x^5-d^2x^4+\frac{21}{2}dex^4&\\+\frac{7}{4}e^2x^4+7\,d^2x^3+\frac{14}{3}dex^3+2\,e^2x^3+\frac{7}{2}d^2x^2+6\,dex^2+6\,d^2x&\end{aligned}$$

```
[In] integrate((e*x+d)^2*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="gia
c")
```

```
[Out] 20/9*e^2*x^9 + 5*d*e*x^8 - 17/8*e^2*x^8 + 20/7*d^2*x^7 - 34/7*d*e*x^7 + 17/
7*e^2*x^7 - 17/6*d^2*x^6 + 17/3*d*e*x^6 - 2/3*e^2*x^6 + 17/5*d^2*x^5 - 8/5*
d*e*x^5 + 21/5*e^2*x^5 - d^2*x^4 + 21/2*d*e*x^4 + 7/4*e^2*x^4 + 7*d^2*x^3 +
 14/3*d*e*x^3 + 2*e^2*x^3 + 7/2*d^2*x^2 + 6*d*e*x^2 + 6*d^2*x
```

Mupad [B] (verification not implemented)

Time = 13.31 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.87

$$\begin{aligned}\int(d+ex)^2\left(3+2x+5x^2\right)\left(2+x+3x^2-5x^3+4x^4\right)\,dx&\\=x^3\left(7\,d^2+\frac{14\,d\,e}{3}+2\,e^2\right)+x^4\left(-d^2+\frac{21\,d\,e}{2}+\frac{7\,e^2}{4}\right)-x^6\left(\frac{17\,d^2}{6}-\frac{17\,d\,e}{3}+\frac{2\,e^2}{3}\right)&\\+x^5\left(\frac{17\,d^2}{5}-\frac{8\,d\,e}{5}+\frac{21\,e^2}{5}\right)+x^7\left(\frac{20\,d^2}{7}-\frac{34\,d\,e}{7}+\frac{17\,e^2}{7}\right)&\\+6\,d^2\,x+\frac{20\,e^2\,x^9}{9}+\frac{d\,x^2\,(7\,d+12\,e)}{2}+\frac{e\,x^8\,(40\,d-17\,e)}{8}&\end{aligned}$$

```
[In] int((d + e*x)^2*(2*x + 5*x^2 + 3)*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2),x)
```

```
[Out] x^3*((14*d*e)/3 + 7*d^2 + 2*e^2) + x^4*((21*d*e)/2 - d^2 + (7*e^2)/4) - x^6
*((17*d^2)/6 - (17*d*e)/3 + (2*e^2)/3) + x^5*((17*d^2)/5 - (8*d*e)/5 + (21*
e^2)/5) + x^7*((20*d^2)/7 - (34*d*e)/7 + (17*e^2)/7) + 6*d^2*x + (20*e^2*x^
9)/9 + (d*x^2*(7*d + 12*e))/2 + (e*x^8*(40*d - 17*e))/8
```

3.291 $\int (d+ex) \left(3+2x+5x^2\right) \left(2+x+3x^2-5x^3+4x^4\right)$

Optimal result

Integrand size = 34, antiderivative size = 93

$$\begin{aligned}
&\int (d+ex) \left(3+2x+5x^2\right) \left(2+x+3x^2-5x^3+4x^4\right) \, dx \\
&= 6dx+\frac{1}{2}(7d+6e)x^2+\frac{7}{3}(3d+e)x^3-\frac{1}{4}(4d-21e)x^4 \\
&\quad +\frac{1}{5}(17d-4e)x^5-\frac{17}{6}(d-e)x^6+\frac{1}{7}(20d-17e)x^7+\frac{5ex^8}{2}
\end{aligned}$$

```
[Out] 6*d*x+1/2*(7*d+6*e)*x^2+7/3*(3*d+e)*x^3-1/4*(4*d-21*e)*x^4+1/5*(17*d-4*e)*x
^5-17/6*(d-e)*x^6+1/7*(20*d-17*e)*x^7+5/2*e*x^8
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {1642}

$$\begin{aligned}
&\int (d+ex) \left(3+2x+5x^2\right) \left(2+x+3x^2-5x^3+4x^4\right) \, dx \\
&= \frac{1}{7}x^7(20d-17e)-\frac{17}{6}x^6(d-e)+\frac{1}{5}x^5(17d-4e) \\
&\quad -\frac{1}{4}x^4(4d-21e)+\frac{7}{3}x^3(3d+e)+\frac{1}{2}x^2(7d+6e)+6dx+\frac{5ex^8}{2}
\end{aligned}$$

```
[In] Int[(d + e*x)*(3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

```
[Out] 6*d*x + ((7*d + 6*e)*x^2)/2 + (7*(3*d + e)*x^3)/3 - ((4*d - 21*e)*x^4)/4 +
((17*d - 4*e)*x^5)/5 - (17*(d - e)*x^6)/6 + ((20*d - 17*e)*x^7)/7 + (5*e*x^
8)/2
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(6d+(7d+6e)x+7(3d+e)x^2-(4d-21e)x^3+(17d-4e)x^4-17(d-e)x^5\right. \\ &\qquad \left.+(20d-17e)x^6+20ex^7\right)\,dx \\ &= 6dx+\frac{1}{2}(7d+6e)x^2+\frac{7}{3}(3d+e)x^3-\frac{1}{4}(4d-21e)x^4 \\ &\quad +\frac{1}{5}(17d-4e)x^5-\frac{17}{6}(d-e)x^6+\frac{1}{7}(20d-17e)x^7+\frac{5ex^8}{2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00

$$\begin{aligned}&\int (d+ex)\left(3+2x+5x^2\right)\left(2+x+3x^2-5x^3+4x^4\right)\,dx \\ &= 6dx+\frac{1}{2}(7d+6e)x^2+\frac{7}{3}(3d+e)x^3+\frac{1}{4}(-4d+21e)x^4 \\ &\quad +\frac{1}{5}(17d-4e)x^5-\frac{17}{6}(d-e)x^6+\frac{1}{7}(20d-17e)x^7+\frac{5ex^8}{2}\end{aligned}$$

```
[In] Integrate[(d + e*x)*(3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

```
[Out] 6*d*x + ((7*d + 6*e)*x^2)/2 + (7*(3*d + e)*x^3)/3 + ((-4*d + 21*e)*x^4)/4 +
 ((17*d - 4*e)*x^5)/5 - (17*(d - e)*x^6)/6 + ((20*d - 17*e)*x^7)/7 + (5*e*x
^8)/2
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.84

method	result
norman	$\frac{5e\,x^8}{2}+\left(\frac{20d}{7}-\frac{17e}{7}\right)x^7+\left(-\frac{17d}{6}+\frac{17e}{6}\right)x^6+\left(\frac{17d}{5}-\frac{4e}{5}\right)x^5+\left(-d+\frac{21e}{4}\right)x^4+\left(7d+\frac{7e}{3}\right)x^3+($
gosper	$\frac{5}{2}e\,x^8+\frac{20}{7}d\,x^7-\frac{17}{7}e\,x^7-\frac{17}{6}d\,x^6+\frac{17}{6}e\,x^6+\frac{17}{5}d\,x^5-\frac{4}{5}e\,x^5-d\,x^4+\frac{21}{4}e\,x^4+7d\,x^3+\frac{7}{3}e\,x^3+$
default	$\frac{5e\,x^8}{2}+\frac{(20d-17e)x^7}{7}+\frac{(-17d+17e)x^6}{6}+\frac{(17d-4e)x^5}{5}+\frac{(-4d+21e)x^4}{4}+\frac{(21d+7e)x^3}{3}+\frac{(7d+6e)x^2}{2}+6dx$
risch	$\frac{5}{2}e\,x^8+\frac{20}{7}d\,x^7-\frac{17}{7}e\,x^7-\frac{17}{6}d\,x^6+\frac{17}{6}e\,x^6+\frac{17}{5}d\,x^5-\frac{4}{5}e\,x^5-d\,x^4+\frac{21}{4}e\,x^4+7d\,x^3+\frac{7}{3}e\,x^3+$
parallelrisch	$\frac{5}{2}e\,x^8+\frac{20}{7}d\,x^7-\frac{17}{7}e\,x^7-\frac{17}{6}d\,x^6+\frac{17}{6}e\,x^6+\frac{17}{5}d\,x^5-\frac{4}{5}e\,x^5-d\,x^4+\frac{21}{4}e\,x^4+7d\,x^3+\frac{7}{3}e\,x^3+$

```
[In] int((e*x+d)*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x,method=_RETURNVERBOSE)
```

```
[Out] 5/2*e*x^8+(20/7*d-17/7*e)*x^7+(-17/6*d+17/6*e)*x^6+(17/5*d-4/5*e)*x^5+(-d+2
1/4*e)*x^4+(7*d+7/3*e)*x^3+(7/2*d+3*e)*x^2+6*d*x
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.85

$$\begin{aligned}\int (d+ex)\left(3+2x+5x^2\right)\left(2+x+3x^2-5x^3+4x^4\right)\,dx &= \frac{5}{2}\,ex^8+\frac{1}{7}\,(20\,d-17\,e)x^7-\frac{17}{6}\,(d-e)x^6+\frac{1}{5}\,(17\,d-4\,e)x^5\\ &\quad -\frac{1}{4}\,(4\,d-21\,e)x^4+\frac{7}{3}\,(3\,d+e)x^3+\frac{1}{2}\,(7\,d+6\,e)x^2+6\,dx\end{aligned}$$

```
[In] integrate((e*x+d)*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="frica
s")
```

```
[Out] 5/2*e*x^8 + 1/7*(20*d - 17*e)*x^7 - 17/6*(d - e)*x^6 + 1/5*(17*d - 4*e)*x^5
 - 1/4*(4*d - 21*e)*x^4 + 7/3*(3*d + e)*x^3 + 1/2*(7*d + 6*e)*x^2 + 6*d*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.94

$$\begin{aligned}\int (d+ex)\left(3+2x+5x^2\right)\left(2+x+3x^2-5x^3+4x^4\right)\,dx &= 6dx+\frac{5ex^8}{2}+x^7\cdot\left(\frac{20d}{7}-\frac{17e}{7}\right)+x^6\left(-\frac{17d}{6}+\frac{17e}{6}\right)+x^5\\ &\quad\cdot\left(\frac{17d}{5}-\frac{4e}{5}\right)+x^4\left(-d+\frac{21e}{4}\right)+x^3\cdot\left(7d+\frac{7e}{3}\right)+x^2\cdot\left(\frac{7d}{2}+3e\right)\end{aligned}$$

```
[In] integrate((e*x+d)*(5*x**2+2*x+3)*(4*x**4-5*x**3+3*x**2+x+2),x)
```

```
[Out] 6*d*x + 5*e*x**8/2 + x**7*(20*d/7 - 17*e/7) + x**6*(-17*d/6 + 17*e/6) + x**
5*(17*d/5 - 4*e/5) + x**4*(-d + 21*e/4) + x**3*(7*d + 7*e/3) + x**2*(7*d/2
+ 3*e)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.85

$$\begin{aligned}\int (d+ex)\left(3+2x+5x^2\right)\left(2+x+3x^2-5x^3+4x^4\right)\,dx\\=\frac{5}{2}\,ex^8+\frac{1}{7}\,(20\,d-17\,e)x^7-\frac{17}{6}\,(d-e)x^6+\frac{1}{5}\,(17\,d-4\,e)x^5\\-\frac{1}{4}\,(4\,d-21\,e)x^4+\frac{7}{3}\,(3\,d+e)x^3+\frac{1}{2}\,(7\,d+6\,e)x^2+6\,dx\end{aligned}$$

```
[In] integrate((e*x+d)*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="maxim
a")
```

```
[Out] 5/2*e*x^8 + 1/7*(20*d - 17*e)*x^7 - 17/6*(d - e)*x^6 + 1/5*(17*d - 4*e)*x^5
 - 1/4*(4*d - 21*e)*x^4 + 7/3*(3*d + e)*x^3 + 1/2*(7*d + 6*e)*x^2 + 6*d*x
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.89

$$\begin{aligned}\int (d+ex)\left(3+2x+5x^2\right)\left(2+x+3x^2-5x^3+4x^4\right)\,dx\\=\frac{5}{2}\,ex^8+\frac{20}{7}\,dx^7-\frac{17}{7}\,ex^7-\frac{17}{6}\,dx^6+\frac{17}{6}\,ex^6+\frac{17}{5}\,dx^5-\frac{4}{5}\,ex^5\\-dx^4+\frac{21}{4}\,ex^4+7\,dx^3+\frac{7}{3}\,ex^3+\frac{7}{2}\,dx^2+3\,ex^2+6\,dx\end{aligned}$$

```
[In] integrate((e*x+d)*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="giac"
)
```

```
[Out] 5/2*e*x^8 + 20/7*d*x^7 - 17/7*e*x^7 - 17/6*d*x^6 + 17/6*e*x^6 + 17/5*d*x^5
- 4/5*e*x^5 - d*x^4 + 21/4*e*x^4 + 7*d*x^3 + 7/3*e*x^3 + 7/2*d*x^2 + 3*e*x^
2 + 6*d*x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.83

$$\int (d+ex)\left(3+2x+5x^2\right)\left(2+x+3x^2-5x^3+4x^4\right)\,dx$$
$$=\frac{5\,e\,x^8}{2}+\left(\frac{20\,d}{7}-\frac{17\,e}{7}\right)\,x^7+\left(\frac{17\,e}{6}-\frac{17\,d}{6}\right)\,x^6+\left(\frac{17\,d}{5}-\frac{4\,e}{5}\right)\,x^5$$
$$+\left(\frac{21\,e}{4}-d\right)\,x^4+\left(7\,d+\frac{7\,e}{3}\right)\,x^3+\left(\frac{7\,d}{2}+3\,e\right)\,x^2+6\,d\,x$$

```
[In] int((d + e*x)*(2*x + 5*x^2 + 3)*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2),x)
```

```
[Out] x^2*((7*d)/2 + 3*e) + x^3*(7*d + (7*e)/3) + x^5*((17*d)/5 - (4*e)/5) - x^6*
((17*d)/6 - (17*e)/6) + x^7*((20*d)/7 - (17*e)/7) + 6*d*x + (5*e*x^8)/2 - x
^4*(d - (21*e)/4)
```

3.292 $\int (3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)\,dx$

Optimal result

Integrand size = 29, antiderivative size = 42

$$\int (3+2x+5x^2)\left(2+x+3x^2-5x^3+4x^4\right)\,dx=6x+\frac{7x^2}{2}+7x^3-x^4+\frac{17x^5}{5}-\frac{17x^6}{6}+\frac{20x^7}{7}$$

```
[Out] 6*x+7/2*x^2+7*x^3-x^4+17/5*x^5-17/6*x^6+20/7*x^7
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {1671}

$$\int (3+2x+5x^2)\left(2+x+3x^2-5x^3+4x^4\right)\,dx=\frac{20x^7}{7}-\frac{17x^6}{6}+\frac{17x^5}{5}-x^4+7x^3+\frac{7x^2}{2}+6x$$

```
[In] Int[(3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

```
[Out] 6*x + (7*x^2)/2 + 7*x^3 - x^4 + (17*x^5)/5 - (17*x^6)/6 + (20*x^7)/7
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\int \left(6+7x+21x^2-4x^3+17x^4-17x^5+20x^6\right)\,dx\\&=6x+\frac{7x^2}{2}+7x^3-x^4+\frac{17x^5}{5}-\frac{17x^6}{6}+\frac{20x^7}{7}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int (3+2x+5x^2) (2+x+3x^2-5x^3+4x^4) \, dx = 6x+\frac{7x^2}{2}+7x^3-x^4+\frac{17x^5}{5}-\frac{17x^6}{6}+\frac{20x^7}{7}$$

```
[In] Integrate[(3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
[Out] 6*x + (7*x^2)/2 + 7*x^3 - x^4 + (17*x^5)/5 - (17*x^6)/6 + (20*x^7)/7
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.83

method	result	size
gosper	$6x+\frac{7}{2}x^2+7x^3-x^4+\frac{17}{5}x^5-\frac{17}{6}x^6+\frac{20}{7}x^7$	35
default	$6x+\frac{7}{2}x^2+7x^3-x^4+\frac{17}{5}x^5-\frac{17}{6}x^6+\frac{20}{7}x^7$	35
norman	$6x+\frac{7}{2}x^2+7x^3-x^4+\frac{17}{5}x^5-\frac{17}{6}x^6+\frac{20}{7}x^7$	35
risch	$6x+\frac{7}{2}x^2+7x^3-x^4+\frac{17}{5}x^5-\frac{17}{6}x^6+\frac{20}{7}x^7$	35
parallelrisch	$6x+\frac{7}{2}x^2+7x^3-x^4+\frac{17}{5}x^5-\frac{17}{6}x^6+\frac{20}{7}x^7$	35

```
[In] int((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x,method=_RETURNVERBOSE)
[Out] 6*x+7/2*x^2+7*x^3-x^4+17/5*x^5-17/6*x^6+20/7*x^7
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.81

$$\int (3+2x+5x^2) (2+x+3x^2-5x^3+4x^4) \, dx = \frac{20}{7}x^7-\frac{17}{6}x^6+\frac{17}{5}x^5-x^4+7x^3+\frac{7}{2}x^2+6x$$

```
[In] integrate((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="fricas")
[Out] 20/7*x^7 - 17/6*x^6 + 17/5*x^5 - x^4 + 7*x^3 + 7/2*x^2 + 6*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

$$\int \left(3+2x+5x^2\right)\left(2+x+3x^2-5x^3+4x^4\right)\,dx=\frac{20x^7}{7}-\frac{17x^6}{6}+\frac{17x^5}{5}-x^4+7x^3+\frac{7x^2}{2}+6x$$

```
[In] integrate((5*x**2+2*x+3)*(4*x**4-5*x**3+3*x**2+x+2),x)
[Out] 20*x**7/7 - 17*x**6/6 + 17*x**5/5 - x**4 + 7*x**3 + 7*x**2/2 + 6*x
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.81

$$\begin{aligned}&\int \left(3+2x+5x^2\right)\left(2+x+3x^2-5x^3+4x^4\right)\,dx\\&=\frac{20}{7}\,x^7-\frac{17}{6}\,x^6+\frac{17}{5}\,x^5-x^4+7\,x^3+\frac{7}{2}\,x^2+6\,x\end{aligned}$$

```
[In] integrate((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="maxima")
[Out] 20/7*x^7 - 17/6*x^6 + 17/5*x^5 - x^4 + 7*x^3 + 7/2*x^2 + 6*x
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.81

$$\begin{aligned}&\int \left(3+2x+5x^2\right)\left(2+x+3x^2-5x^3+4x^4\right)\,dx\\&=\frac{20}{7}\,x^7-\frac{17}{6}\,x^6+\frac{17}{5}\,x^5-x^4+7\,x^3+\frac{7}{2}\,x^2+6\,x\end{aligned}$$

```
[In] integrate((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="giac")
[Out] 20/7*x^7 - 17/6*x^6 + 17/5*x^5 - x^4 + 7*x^3 + 7/2*x^2 + 6*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.81

$$\int \left(3+2x+5x^2\right)\left(2+x+3x^2-5x^3+4x^4\right)\,dx$$
$$=\frac{20\,x^7}{7}-\frac{17\,x^6}{6}+\frac{17\,x^5}{5}-x^4+7\,x^3+\frac{7\,x^2}{2}+6\,x$$

```
[In] int((2*x + 5*x^2 + 3)*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2),x)
[Out] 6*x + (7*x^2)/2 + 7*x^3 - x^4 + (17*x^5)/5 - (17*x^6)/6 + (20*x^7)/7
```

3.293 $\int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{d+ex}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 228

$$\begin{aligned}\int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{d+ex}\,dx &= -\frac{(20d^5+17d^4e+17d^3e^2+4d^2e^3+21de^4-7e^5)\,x}{e^6} \\ &+\frac{(20d^4+17d^3e+17d^2e^2+4de^3+21e^4)\,x^2}{2e^5}-\frac{(20d^3+17d^2e+17de^2+4e^3)\,x^3}{3e^4} \\ &+\frac{(20d^2+17de+17e^2)\,x^4}{4e^3}-\frac{(20d+17e)x^5}{5e^2}+\frac{10x^6}{3e} \\ &+\frac{(5d^2-2de+3e^2)(4d^4+5d^3e+3d^2e^2-de^3+2e^4)\log(d+ex)}{e^7}\end{aligned}$$

```
[Out] -(20*d^5+17*d^4*e+17*d^3*e^2+4*d^2*e^3+21*d*e^4-7*e^5)*x/e^6+1/2*(20*d^4+17
*d^3*e+17*d^2*e^2+4*d*e^3+21*e^4)*x^2/e^5-1/3*(20*d^3+17*d^2*e+17*d*e^2+4*e
^3)*x^3/e^4+1/4*(20*d^2+17*d*e+17*e^2)*x^4/e^3-1/5*(20*d+17*e)*x^5/e^2+10/3
*x^6/e+(5*d^2-2*d*e+3*e^2)*(4*d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)*ln(e*x+d)/
e^7
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used

$= \{1642\}$

$$\int \frac{(3+2x+5x^2)\,(2+x+3x^2-5x^3+4x^4)}{d+ex}\,dx$$
$$= \frac{x^4(20d^2+17de+17e^2)}{4e^3} - \frac{x^3(20d^3+17d^2e+17de^2+4e^3)}{3e^4} + \frac{(5d^2-2de+3e^2)\,(4d^4+5d^3e+3d^2e^2-de^3+2e^4)\log(d+ex)}{e^7} + \frac{x^2(20d^4+17d^3e+17d^2e^2+4de^3+21e^4)}{2e^5} - \frac{x(20d^5+17d^4e+17d^3e^2+4d^2e^3+21de^4-7e^5)}{e^6} - \frac{x^5(20d+17e)}{5e^2} + \frac{10x^6}{3e}$$

```
[In] Int[((3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x),x]
```

```
[Out] -(((20*d^5 + 17*d^4*e + 17*d^3*e^2 + 4*d^2*e^3 + 21*d*e^4 - 7*e^5)*x)/e^6)
+ ((20*d^4 + 17*d^3*e + 17*d^2*e^2 + 4*d*e^3 + 21*e^4)*x^2)/(2*e^5) - ((20*
d^3 + 17*d^2*e + 17*d*e^2 + 4*e^3)*x^3)/(3*e^4) + ((20*d^2 + 17*d*e + 17*e^
2)*x^4)/(4*e^3) - ((20*d + 17*e)*x^5)/(5*e^2) + (10*x^6)/(3*e) + ((5*d^2 -
2*d*e + 3*e^2)*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*Log[d + e*x])/
e^7
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\text{integral} = \int \left(\frac{-20d^5-17d^4e-17d^3e^2-4d^2e^3-21de^4+7e^5}{e^6} + \frac{(20d^4+17d^3e+17d^2e^2+4de^3+21e^4)\,x}{e^5} - \frac{(20d^3+17d^2e+17de^2+4e^3)\,x^2}{e^4} + \frac{(20d^2+17de+17e^2)\,x^3}{e^3} - \frac{(20d+17e)x^4}{e^2} + \frac{20x^5}{e} + \frac{20d^6+17d^5e+17d^4e^2+4d^3e^3+21d^2e^4-7de^5+6e^6}{e^6(d+ex)}\right) dx$$
$$= -\frac{(20d^5+17d^4e+17d^3e^2+4d^2e^3+21de^4-7e^5)\,x}{e^6} + \frac{(20d^4+17d^3e+17d^2e^2+4de^3+21e^4)\,x^2}{2e^5} - \frac{(20d^3+17d^2e+17de^2+4e^3)\,x^3}{3e^4} + \frac{(20d^2+17de+17e^2)\,x^4}{4e^3} - \frac{(20d+17e)x^5}{5e^2} + \frac{10x^6}{3e} + \frac{(5d^2-2de+3e^2)\,(4d^4+5d^3e+3d^2e^2-de^3+2e^4)\log(d+ex)}{e^7}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.79

$$\int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{d+ex}\,dx$$
$$= \frac{ex(-1200d^5+60d^4e(-17+10x)-10d^3e^2(102-51x+40x^2)+10d^2e^3(-24+51x-34x^2+30x^3)-5de}{}$$

```
[In] Integrate[((3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x),x]
```

```
[Out] (e*x*(-1200*d^5 + 60*d^4*e*(-17 + 10*x) - 10*d^3*e^2*(102 - 51*x + 40*x^2)
+ 10*d^2*e^3*(-24 + 51*x - 34*x^2 + 30*x^3) - 5*d*e^4*(252 - 24*x + 68*x^2
- 51*x^3 + 48*x^4) + e^5*(420 + 630*x - 80*x^2 + 255*x^3 - 204*x^4 + 200*x^
5)) + 60*(20*d^6 + 17*d^5*e + 17*d^4*e^2 + 4*d^3*e^3 + 21*d^2*e^4 - 7*d*e^5
 + 6*e^6)*Log[d + e*x])/(60*e^7)
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.96

method	result
norman	$\frac{10x^6}{3e}-\frac{(20d+17e)x^5}{5e^2}+\frac{(20d^2+17de+17e^2)x^4}{4e^3}-\frac{(20d^3+17d^2e+17d\,e^2+4e^3)x^3}{3e^4}+\frac{(20d^4+17d^3e+17d^2e^2+4d\,e^3+21e^4)x}{2e^5}$
default	$-\frac{-\frac{10}{3}e^5x^6+4d\,e^4x^5+\frac{17}{5}e^5x^5-5d^2e^3x^4-\frac{17}{4}d\,e^4x^4-\frac{17}{4}e^5x^4+\frac{20}{3}d^3e^2x^3+\frac{17}{3}d^2e^3x^3+\frac{17}{3}d\,e^4x^3+\frac{4}{3}e^5x^3-10d^4e\,x^2-\frac{17}{2}d^3e^2x^2-}{e^6}$
parallelrisch	$-204e^6x^5+200e^6x^6-400d^3x^3e^3-1200d^5ex+600d^4e^2x^2+300d^2e^4x^4-240d\,e^5x^5+255x^4e^6-80x^3e^6+630x^2e^6+420x\,e^6+1200\ln$
risch	$-\frac{17x^5}{5e}+\frac{17\ln(ex+d)d^4}{e^5}+\frac{4\ln(ex+d)d^3}{e^4}+\frac{21\ln(ex+d)d^2}{e^3}-\frac{7\ln(ex+d)d}{e^2}+\frac{20\ln(ex+d)d^6}{e^7}+\frac{17\ln(ex+d)d^5}{e^6}+\frac{17d^3}{2e}$

```
[In] int((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 10/3*x^6/e-1/5*(20*d+17*e)*x^5/e^2+1/4*(20*d^2+17*d*e+17*e^2)*x^4/e^3-1/3*(
20*d^3+17*d^2*e+17*d*e^2+4*e^3)*x^3/e^4+1/2*(20*d^4+17*d^3*e+17*d^2*e^2+4*d
*e^3+21*e^4)*x^2/e^5-(20*d^5+17*d^4*e+17*d^3*e^2+4*d^2*e^3+21*d*e^4-7*e^5)*
x/e^6+(20*d^6+17*d^5*e+17*d^4*e^2+4*d^3*e^3+21*d^2*e^4-7*d*e^5+6*e^6)/e^7*l
n(e*x+d)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.01

$$\int \frac{(3+2x+5x^2)\,(2+x+3x^2-5x^3+4x^4)}{d+ex}\,dx$$
$$=\frac{200\,e^6x^6-12\,(20\,de^5+17\,e^6)x^5+15\,(20\,d^2e^4+17\,de^5+17\,e^6)x^4-20\,(20\,d^3e^3+17\,d^2e^4+17\,de^5+4\,e^6)}{}$$

```
[In] integrate((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d),x, algorithm="frica
s")
```

```
[Out] 1/60*(200*e^6*x^6 - 12*(20*d*e^5 + 17*e^6)*x^5 + 15*(20*d^2*e^4 + 17*d*e^5
+ 17*e^6)*x^4 - 20*(20*d^3*e^3 + 17*d^2*e^4 + 17*d*e^5 + 4*e^6)*x^3 + 30*(2
0*d^4*e^2 + 17*d^3*e^3 + 17*d^2*e^4 + 4*d*e^5 + 21*e^6)*x^2 - 60*(20*d^5*e
+ 17*d^4*e^2 + 17*d^3*e^3 + 4*d^2*e^4 + 21*d*e^5 - 7*e^6)*x + 60*(20*d^6 +
17*d^5*e + 17*d^4*e^2 + 4*d^3*e^3 + 21*d^2*e^4 - 7*d*e^5 + 6*e^6)*log(e*x +
 d))/e^7
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.03

$$\int \frac{(3+2x+5x^2)\,(2+x+3x^2-5x^3+4x^4)}{d+ex}\,dx$$
$$=x^5\left(-\frac{4d}{e^2}-\frac{17}{5e}\right)+x^4\cdot\left(\frac{5d^2}{e^3}+\frac{17d}{4e^2}+\frac{17}{4e}\right)+x^3\left(-\frac{20d^3}{3e^4}-\frac{17d^2}{3e^3}-\frac{17d}{3e^2}-\frac{4}{3e}\right)+x^2$$
$$\cdot\left(\frac{10d^4}{e^5}+\frac{17d^3}{2e^4}+\frac{17d^2}{2e^3}+\frac{2d}{e^2}+\frac{21}{2e}\right)+x\left(-\frac{20d^5}{e^6}-\frac{17d^4}{e^5}-\frac{17d^3}{e^4}-\frac{4d^2}{e^3}-\frac{21d}{e^2}+\frac{7}{e}\right)$$
$$+\frac{10x^6}{3e}+\frac{(5d^2-2de+3e^2)\,(4d^4+5d^3e+3d^2e^2-de^3+2e^4)\log{(d+ex)}}{e^7}$$

```
[In] integrate((5*x**2+2*x+3)*(4*x**4-5*x**3+3*x**2+x+2)/(e*x+d),x)
```

```
[Out] x**5*(-4*d/e**2 - 17/(5*e)) + x**4*(5*d**2/e**3 + 17*d/(4*e**2) + 17/(4*e))
 + x**3*(-20*d**3/(3*e**4) - 17*d**2/(3*e**3) - 17*d/(3*e**2) - 4/(3*e)) +
x**2*(10*d**4/e**5 + 17*d**3/(2*e**4) + 17*d**2/(2*e**3) + 2*d/e**2 + 21/(2
*e)) + x*(-20*d**5/e**6 - 17*d**4/e**5 - 17*d**3/e**4 - 4*d**2/e**3 - 21*d/
e**2 + 7/e) + 10*x**6/(3*e) + (5*d**2 - 2*d*e + 3*e**2)*(4*d**4 + 5*d**3*e
+ 3*d**2*e**2 - d*e**3 + 2*e**4)*log(d + e*x)/e**7
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.00

$$\int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{d+ex}\,dx$$
$$= \frac{200\,e^5x^6 - 12\,(20\,de^4 + 17\,e^5)x^5 + 15\,(20\,d^2e^3 + 17\,de^4 + 17\,e^5)x^4 - 20\,(20\,d^3e^2 + 17\,d^2e^3 + 17\,de^4 + 4\,e^5)x}{6}$$
$$+ \frac{(20\,d^6 + 17\,d^5e + 17\,d^4e^2 + 4\,d^3e^3 + 21\,d^2e^4 - 7\,de^5 + 6\,e^6)\log(ex+d)}{e^7}$$

```
[In] integrate((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d),x, algorithm="maxim
a")
```

```
[Out] 1/60*(200*e^5*x^6 - 12*(20*d*e^4 + 17*e^5)*x^5 + 15*(20*d^2*e^3 + 17*d*e^4
+ 17*e^5)*x^4 - 20*(20*d^3*e^2 + 17*d^2*e^3 + 17*d*e^4 + 4*e^5)*x^3 + 30*(2
0*d^4*e + 17*d^3*e^2 + 17*d^2*e^3 + 4*d*e^4 + 21*e^5)*x^2 - 60*(20*d^5 + 17
*d^4*e + 17*d^3*e^2 + 4*d^2*e^3 + 21*d*e^4 - 7*e^5)*x)/e^6 + (20*d^6 + 17*d
^5*e + 17*d^4*e^2 + 4*d^3*e^3 + 21*d^2*e^4 - 7*d*e^5 + 6*e^6)*log(e*x + d)/
e^7
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.09

$$\int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{d+ex}\,dx$$
$$= 200\,e^5x^6 - 240\,de^4x^5 - 204\,e^5x^5 + 300\,d^2e^3x^4 + 255\,de^4x^4 + 255\,e^5x^4 - 400\,d^3e^2x^3 - 340\,d^2e^3x^3 - 340\,de^4$$
$$+ \frac{(20\,d^6 + 17\,d^5e + 17\,d^4e^2 + 4\,d^3e^3 + 21\,d^2e^4 - 7\,de^5 + 6\,e^6)\log(|ex+d|)}{e^7}$$

```
[In] integrate((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d),x, algorithm="giac"
)
```

```
[Out] 1/60*(200*e^5*x^6 - 240*d*e^4*x^5 - 204*e^5*x^5 + 300*d^2*e^3*x^4 + 255*d*e
^4*x^4 + 255*e^5*x^4 - 400*d^3*e^2*x^3 - 340*d^2*e^3*x^3 - 340*d*e^4*x^3 -
80*e^5*x^3 + 600*d^4*e*x^2 + 510*d^3*e^2*x^2 + 510*d^2*e^3*x^2 + 120*d*e^4*
x^2 + 630*e^5*x^2 - 1200*d^5*x - 1020*d^4*e*x - 1020*d^3*e^2*x - 240*d^2*e^
3*x - 1260*d*e^4*x + 420*e^5*x)/e^6 + (20*d^6 + 17*d^5*e + 17*d^4*e^2 + 4*d
^3*e^3 + 21*d^2*e^4 - 7*d*e^5 + 6*e^6)*log(abs(e*x + d))/e^7
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.14

$$\int \frac{(3+2x+5x^2)\left(2+x+3x^2-5x^3+4x^4\right)}{d+ex}\,dx$$

$$= x\left(\frac{7}{e}-\frac{d\left(\frac{21}{e}+\frac{d\left(\frac{4}{e}+\frac{d\left(\frac{17}{e}+\frac{d\left(\frac{20\,d}{e^2}+\frac{17}{e}\right)}{e}\right)}{e}\right)}{e}\right)}{e}\right)-x^5\left(\frac{4\,d}{e^2}+\frac{17}{5\,e}\right)+x^4\left(\frac{17}{4\,e}+\frac{d\left(\frac{20\,d}{e^2}+\frac{17}{e}\right)}{4\,e}\right)$$

$$-x^3\left(\frac{4}{3\,e}+\frac{d\left(\frac{17}{e}+\frac{d\left(\frac{20\,d}{e^2}+\frac{17}{e}\right)}{e}\right)}{3\,e}\right)+x^2\left(\frac{21}{2\,e}+\frac{d\left(\frac{4}{e}+\frac{d\left(\frac{17}{e}+\frac{d\left(\frac{20\,d}{e^2}+\frac{17}{e}\right)}{e}\right)}{e}\right)}{2\,e}\right)$$

$$+\frac{10\,x^6}{3\,e}+\frac{\ln\left(d+e\,x\right)\,\left(20\,d^6+17\,d^5\,e+17\,d^4\,e^2+4\,d^3\,e^3+21\,d^2\,e^4-7\,d\,e^5+6\,e^6\right)}{e^7}$$

```
[In] int(((2*x + 5*x^2 + 3)*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(d + e*x),x)
```

```
[Out] x*(7/e - (d*(21/e + (d*(4/e + (d*(17/e + (d*((20*d)/e^2 + 17/e))/e))/e))/e))/e
)/e) - x^5*((4*d)/e^2 + 17/(5*e)) + x^4*(17/(4*e) + (d*((20*d)/e^2 + 17/e))
/(4*e)) - x^3*(4/(3*e) + (d*(17/e + (d*((20*d)/e^2 + 17/e))/e))/(3*e)) + x^
2*(21/(2*e) + (d*(4/e + (d*(17/e + (d*((20*d)/e^2 + 17/e))/e))/e))/(2*e)) +
 (10*x^6)/(3*e) + (log(d + e*x)*(17*d^5*e - 7*d*e^5 + 20*d^6 + 6*e^6 + 21*d
^2*e^4 + 4*d^3*e^3 + 17*d^4*e^2))/e^7
```

3.294 $\int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{(d+ex)^2} \, dx$

Optimal result

Integrand size = 36, antiderivative size = 228

$$
\begin{aligned}
&\int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{(d+ex)^2} \, dx \\
&= \frac{(100d^4+68d^3e+51d^2e^2+8de^3+21e^4)\,x}{e^6} - \frac{(80d^3+51d^2e+34de^2+4e^3)\,x^2}{2e^5} \\
&\quad + \frac{(60d^2+34de+17e^2)\,x^3}{3e^4} - \frac{(40d+17e)x^4}{4e^3} + \frac{4x^5}{e^2} \\
&\quad - \frac{(5d^2-2de+3e^2)(4d^4+5d^3e+3d^2e^2-de^3+2e^4)}{e^7(d+ex)} \\
&\quad - \frac{(120d^5+85d^4e+68d^3e^2+12d^2e^3+42de^4-7e^5)\log(d+ex)}{e^7}
\end{aligned}
$$

```
[Out] (100*d^4+68*d^3*e+51*d^2*e^2+8*d*e^3+21*e^4)*x/e^6-1/2*(80*d^3+51*d^2*e+34*
d*e^2+4*e^3)*x^2/e^5+1/3*(60*d^2+34*d*e+17*e^2)*x^3/e^4-1/4*(40*d+17*e)*x^4
/e^3+4*x^5/e^2-(5*d^2-2*d*e+3*e^2)*(4*d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)/e^
7/(e*x+d)-(120*d^5+85*d^4*e+68*d^3*e^2+12*d^2*e^3+42*d*e^4-7*e^5)*ln(e*x+d)
/e^7
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.028$, Rules used

$= \{1642\}$

$$\begin{aligned}
&\int \frac{(3+2x+5x^2)\left(2+x+3x^2-5x^3+4x^4\right)}{(d+ex)^2}\,dx \\
&\quad= \frac{x^3(60d^2+34de+17e^2)}{3e^4} - \frac{x^2(80d^3+51d^2e+34de^2+4e^3)}{2e^5} \\
&\qquad - \frac{(5d^2-2de+3e^2)\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)}{e^7(d+ex)} \\
&\qquad + \frac{x(100d^4+68d^3e+51d^2e^2+8de^3+21e^4)}{e^6} \\
&\qquad - \frac{(120d^5+85d^4e+68d^3e^2+12d^2e^3+42de^4-7e^5)\log(d+ex)}{e^7} - \frac{x^4(40d+17e)}{4e^3} + \frac{4x^5}{e^2}
\end{aligned}$$

```
[In] Int[((3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x)^2,x]
```

```
[Out] ((100*d^4 + 68*d^3*e + 51*d^2*e^2 + 8*d*e^3 + 21*e^4)*x)/e^6 - ((80*d^3 + 5
1*d^2*e + 34*d*e^2 + 4*e^3)*x^2)/(2*e^5) + ((60*d^2 + 34*d*e + 17*e^2)*x^3)
/(3*e^4) - ((40*d + 17*e)*x^4)/(4*e^3) + (4*x^5)/e^2 - ((5*d^2 - 2*d*e + 3*
e^2)*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4))/(e^7*(d + e*x)) - ((120
*d^5 + 85*d^4*e + 68*d^3*e^2 + 12*d^2*e^3 + 42*d*e^4 - 7*e^5)*Log[d + e*x])
/e^7
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{100d^4+68d^3e+51d^2e^2+8de^3+21e^4}{e^6} - \frac{(80d^3+51d^2e+34de^2+4e^3)\,x}{e^5} \right. \\
&\qquad + \frac{(60d^2+34de+17e^2)\,x^2}{e^4} - \frac{(40d+17e)x^3}{e^3} + \frac{20x^4}{e^2} \\
&\qquad + \frac{20d^6+17d^5e+17d^4e^2+4d^3e^3+21d^2e^4-7de^5+6e^6}{e^6(d+ex)^2} \\
&\qquad \left. + \frac{-120d^5-85d^4e-68d^3e^2-12d^2e^3-42de^4+7e^5}{e^6(d+ex)}\right) dx \\
&= \frac{(100d^4+68d^3e+51d^2e^2+8de^3+21e^4)\,x}{e^6} - \frac{(80d^3+51d^2e+34de^2+4e^3)\,x^2}{2e^5} \\
&\quad + \frac{(60d^2+34de+17e^2)\,x^3}{3e^4} - \frac{(40d+17e)x^4}{4e^3} + \frac{4x^5}{e^2} \\
&\quad - \frac{(5d^2-2de+3e^2)\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)}{e^7(d+ex)} \\
&\quad - \frac{(120d^5+85d^4e+68d^3e^2+12d^2e^3+42de^4-7e^5)\log(d+ex)}{e^7}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.98

$$\int \frac{(3+2x+5x^2)\,(2+x+3x^2-5x^3+4x^4)}{(d+ex)^2}\,dx$$

$$=\frac{12e(100d^4+68d^3e+51d^2e^2+8de^3+21e^4)\,x-6e^2(80d^3+51d^2e+34de^2+4e^3)\,x^2+4e^3(60d^2+34de+}{}$$

```
[In] Integrate[((3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x)^2,x
]
```

```
[Out] (12*e*(100*d^4 + 68*d^3*e + 51*d^2*e^2 + 8*d*e^3 + 21*e^4)*x - 6*e^2*(80*d^
3 + 51*d^2*e + 34*d*e^2 + 4*e^3)*x^2 + 4*e^3*(60*d^2 + 34*d*e + 17*e^2)*x^3
 - 3*e^4*(40*d + 17*e)*x^4 + 48*e^5*x^5 - (12*(20*d^6 + 17*d^5*e + 17*d^4*e
^2 + 4*d^3*e^3 + 21*d^2*e^4 - 7*d*e^5 + 6*e^6))/(d + e*x) - 12*(120*d^5 + 8
5*d^4*e + 68*d^3*e^2 + 12*d^2*e^3 + 42*d*e^4 - 7*e^5)*Log[d + e*x])/(12*e^7
)
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.02

method	result
norman	$\frac{\frac{\left(120d^6+85d^5e+68d^4e^2+12d^3e^3+42d^2e^4-7d\,e^5+6e^6\right)x}{e^6d}+\frac{4x^6}{e}-\frac{(24d+17e)x^5}{4e^2}+\frac{\left(120d^2+85de+68e^2\right)x^4}{12e^3}-\frac{\left(120d^3+85d^2e+68d\,e^2+12e^3\right)x^3}{6e^4}+}{ex+d}$
default	$\frac{4e^4x^5-10d\,e^3x^4-\frac{17}{4}e^4x^4+20d^2e^2x^3+\frac{34}{3}d\,e^3x^3+\frac{17}{3}e^4x^3-40d^3e\,x^2-\frac{51}{2}d^2e^2x^2-17d\,e^3x^2-2e^4x^2+100d^4x+68d^3ex+51d^2e^2x+}{e^6}$
risch	$\frac{7\ln(ex+d)}{e^2}-\frac{17x^4}{4e^2}-\frac{120\ln(ex+d)d^5}{e^7}-\frac{85\ln(ex+d)d^4}{e^6}-\frac{68\ln(ex+d)d^3}{e^5}-\frac{12\ln(ex+d)d^2}{e^4}-\frac{42\ln(ex+d)d}{e^3}-\frac{10d\,x^4}{e^3}$
parallelrisch	$-\frac{51e^6x^5+72e^6-48e^6x^6+1020d^5e+504\ln(ex+d)xd\,e^5+1440\ln(ex+d)x\,d^5e+1020\ln(ex+d)x\,d^4e^2+816\ln(ex+d)x\,d^3e^3+144\ln}{}$

```
[In] int((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2,x,method=_RETURNVERBOSE
)
```

```
[Out] ((120*d^6+85*d^5*e+68*d^4*e^2+12*d^3*e^3+42*d^2*e^4-7*d*e^5+6*e^6)/e^6/d*x+
4*x^6/e-1/4*(24*d+17*e)/e^2*x^5+1/12*(120*d^2+85*d*e+68*e^2)/e^3*x^4-1/6*(1
20*d^3+85*d^2*e+68*d*e^2+12*e^3)/e^4*x^3+1/2*(120*d^4+85*d^3*e+68*d^2*e^2+1
2*d*e^3+42*e^4)/e^5*x^2)/(e*x+d)-(120*d^5+85*d^4*e+68*d^3*e^2+12*d^2*e^3+42
*d*e^4-7*e^5)*ln(e*x+d)/e^7
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.40

$$\int \frac{(3+2x+5x^2)\,(2+x+3x^2-5x^3+4x^4)}{(d+ex)^2}\,dx$$
$$= \frac{48\,e^6x^6 - 240\,d^6 - 204\,d^5e - 204\,d^4e^2 - 48\,d^3e^3 - 252\,d^2e^4 + 84\,de^5 - 72\,e^6 - 3\,(24\,de^5 + 17\,e^6)x^5 + (120}{}$$

```
[In] integrate((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2,x, algorithm="fri
cas")
```

```
[Out] 1/12*(48*e^6*x^6 - 240*d^6 - 204*d^5*e - 204*d^4*e^2 - 48*d^3*e^3 - 252*d^2
*e^4 + 84*d*e^5 - 72*e^6 - 3*(24*d*e^5 + 17*e^6)*x^5 + (120*d^2*e^4 + 85*d*
e^5 + 68*e^6)*x^4 - 2*(120*d^3*e^3 + 85*d^2*e^4 + 68*d*e^5 + 12*e^6)*x^3 +
6*(120*d^4*e^2 + 85*d^3*e^3 + 68*d^2*e^4 + 12*d*e^5 + 42*e^6)*x^2 + 12*(100
*d^5*e + 68*d^4*e^2 + 51*d^3*e^3 + 8*d^2*e^4 + 21*d*e^5)*x - 12*(120*d^6 +
85*d^5*e + 68*d^4*e^2 + 12*d^3*e^3 + 42*d^2*e^4 - 7*d*e^5 + (120*d^5*e + 85
*d^4*e^2 + 68*d^3*e^3 + 12*d^2*e^4 + 42*d*e^5 - 7*e^6)*x)*log(e*x + d))/(e^
8*x + d*e^7)
```

Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.04

$$\begin{aligned}\int \frac{(3+2x+5x^2)\,(2+x+3x^2-5x^3+4x^4)}{(d+ex)^2}\,dx &= x^4\left(-\frac{10d}{e^3}-\frac{17}{4e^2}\right)+x^3\cdot\left(\frac{20d^2}{e^4}+\frac{34d}{3e^3}+\frac{17}{3e^2}\right)\\ &\quad+x^2\left(-\frac{40d^3}{e^5}-\frac{51d^2}{2e^4}-\frac{17d}{e^3}-\frac{2}{e^2}\right)+x\left(\frac{100d^4}{e^6}+\frac{68d^3}{e^5}+\frac{51d^2}{e^4}+\frac{8d}{e^3}+\frac{21}{e^2}\right)\\ &\quad+\frac{-20d^6-17d^5e-17d^4e^2-4d^3e^3-21d^2e^4+7de^5-6e^6}{de^7+e^8x}+\frac{4x^5}{e^2}\\ &\quad-\frac{(120d^5+85d^4e+68d^3e^2+12d^2e^3+42de^4-7e^5)\log{(d+ex)}}{e^7}\end{aligned}$$

```
[In] integrate((5*x**2+2*x+3)*(4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)**2,x)
```

```
[Out] x**4*(-10*d/e**3 - 17/(4*e**2)) + x**3*(20*d**2/e**4 + 34*d/(3*e**3) + 17/(
3*e**2)) + x**2*(-40*d**3/e**5 - 51*d**2/(2*e**4) - 17*d/e**3 - 2/e**2) + x
*(100*d**4/e**6 + 68*d**3/e**5 + 51*d**2/e**4 + 8*d/e**3 + 21/e**2) + (-20*
d**6 - 17*d**5*e - 17*d**4*e**2 - 4*d**3*e**3 - 21*d**2*e**4 + 7*d*e**5 - 6
*e**6)/(d*e**7 + e**8*x) + 4*x**5/e**2 - (120*d**5 + 85*d**4*e + 68*d**3*e*
*2 + 12*d**2*e**3 + 42*d*e**4 - 7*e**5)*log(d + e*x)/e**7
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.03

$$\int \frac{(3+2x+5x^2)\,(2+x+3x^2-5x^3+4x^4)}{(d+ex)^2}\,dx$$
$$= -\frac{20\,d^6+17\,d^5e+17\,d^4e^2+4\,d^3e^3+21\,d^2e^4-7\,de^5+6\,e^6}{e^8x+de^7}$$
$$+\frac{48\,e^4x^5-3\,(40\,de^3+17\,e^4)x^4+4\,(60\,d^2e^2+34\,de^3+17\,e^4)x^3-6\,(80\,d^3e+51\,d^2e^2+34\,de^3+4\,e^4)x^2+}{12\,e^6}$$
$$-\frac{(120\,d^5+85\,d^4e+68\,d^3e^2+12\,d^2e^3+42\,de^4-7\,e^5)\log{(ex+d)}}{e^7}$$

```
[In] integrate((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2,x, algorithm="max
ima")
```

```
[Out] -(20*d^6 + 17*d^5*e + 17*d^4*e^2 + 4*d^3*e^3 + 21*d^2*e^4 - 7*d*e^5 + 6*e^6
)/(e^8*x + d*e^7) + 1/12*(48*e^4*x^5 - 3*(40*d*e^3 + 17*e^4)*x^4 + 4*(60*d^
2*e^2 + 34*d*e^3 + 17*e^4)*x^3 - 6*(80*d^3*e + 51*d^2*e^2 + 34*d*e^3 + 4*e^
4)*x^2 + 12*(100*d^4 + 68*d^3*e + 51*d^2*e^2 + 8*d*e^3 + 21*e^4)*x)/e^6 - (
120*d^5 + 85*d^4*e + 68*d^3*e^2 + 12*d^2*e^3 + 42*d*e^4 - 7*e^5)*log(e*x +
d)/e^7
```

Giac [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.43

$$\int \frac{(3+2x+5x^2)\,(2+x+3x^2-5x^3+4x^4)}{(d+ex)^2}\,dx =$$
$$-\frac{(ex+d)^5\left(\frac{3\,(120\,de+17\,e^2)}{(ex+d)e}-\frac{4\,(300\,d^2e^2+85\,de^3+17\,e^4)}{(ex+d)^2e^2}+\frac{12\,(200\,d^3e^3+85\,d^2e^4+34\,de^5+2\,e^6)}{(ex+d)^3e^3}-\frac{12\,(300\,d^4e^4+170\,d^3e^5+102\,d^2e^6-}{(ex+d)^4e^4}\right.}{12\,e^7}$$
$$+\frac{(120\,d^5+85\,d^4e+68\,d^3e^2+12\,d^2e^3+42\,de^4-7\,e^5)\log\left(\frac{|ex+d|}{(ex+d)^2|e|}\right)}{e^7}$$
$$-\frac{\frac{20\,d^6e^5}{ex+d}+\frac{17\,d^5e^6}{ex+d}+\frac{17\,d^4e^7}{ex+d}+\frac{4\,d^3e^8}{ex+d}+\frac{21\,d^2e^9}{ex+d}-\frac{7\,de^{10}}{ex+d}+\frac{6\,e^{11}}{ex+d}}{e^{12}}$$

```
[In] integrate((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2,x, algorithm="gia
c")
```

```
[Out] -1/12*(e*x + d)^5*(3*(120*d*e + 17*e^2)/((e*x + d)*e) - 4*(300*d^2*e^2 + 85
*d*e^3 + 17*e^4)/((e*x + d)^2*e^2) + 12*(200*d^3*e^3 + 85*d^2*e^4 + 34*d*e^
```

```
5 + 2*e^6)/((e*x + d)^3*e^3) - 12*(300*d^4*e^4 + 170*d^3*e^5 + 102*d^2*e^6
+ 12*d*e^7 + 21*e^8)/((e*x + d)^4*e^4) - 48)/e^7 + (120*d^5 + 85*d^4*e + 68
*d^3*e^2 + 12*d^2*e^3 + 42*d*e^4 - 7*e^5)*log(abs(e*x + d)/((e*x + d)^2*abs
(e)))/e^7 - (20*d^6*e^5/(e*x + d) + 17*d^5*e^6/(e*x + d) + 17*d^4*e^7/(e*x
+ d) + 4*d^3*e^8/(e*x + d) + 21*d^2*e^9/(e*x + d) - 7*d*e^10/(e*x + d) + 6*
e^11/(e*x + d))/e^12
```

Mupad [B] (verification not implemented)

Time = 13.31 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.59

$$
\begin{aligned}
&\int \frac{(3+2x+5x^2)\,(2+x+3x^2-5x^3+4x^4)}{(d+ex)^2}\,dx \\
&= x^3\left(\frac{17}{3\,e^2}-\frac{20\,d^2}{3\,e^4}+\frac{2\,d\,\left(\frac{40\,d}{e^3}+\frac{17}{e^2}\right)}{3\,e}\right) \\
&\quad -x^2\left(\frac{2}{e^2}+\frac{d\left(\frac{17}{e^2}-\frac{20\,d^2}{e^4}+\frac{2\,d\,\left(\frac{40\,d}{e^3}+\frac{17}{e^2}\right)}{e}\right)}{e}-\frac{d^2\,\left(\frac{40\,d}{e^3}+\frac{17}{e^2}\right)}{2\,e^2}\right)-x^4\left(\frac{10\,d}{e^3}+\frac{17}{4\,e^2}\right)+x\left(\frac{21}{e^2}\right. \\
&\quad \left.+\frac{2\,d\left(\frac{4}{e^2}+\frac{2\,d\left(\frac{17}{e^2}-\frac{20\,d^2}{e^4}+\frac{2\,d\,\left(\frac{40\,d}{e^3}+\frac{17}{e^2}\right)}{e}\right)}{e}-\frac{d^2\,\left(\frac{40\,d}{e^3}+\frac{17}{e^2}\right)}{e^2}\right)}{e}-\frac{d^2\left(\frac{17}{e^2}-\frac{20\,d^2}{e^4}+\frac{2\,d\,\left(\frac{40\,d}{e^3}+\frac{17}{e^2}\right)}{e}\right)}{e^2}\right) \\
&\quad +\frac{4\,x^5}{e^2}-\frac{\ln\left(d+e\,x\right)\,\left(120\,d^5+85\,d^4\,e+68\,d^3\,e^2+12\,d^2\,e^3+42\,d\,e^4-7\,e^5\right)}{e^7} \\
&\quad -\frac{20\,d^6+17\,d^5\,e+17\,d^4\,e^2+4\,d^3\,e^3+21\,d^2\,e^4-7\,d\,e^5+6\,e^6}{e\,\left(x\,e^7+d\,e^6\right)}
\end{aligned}
$$

```
[In] int(((2*x + 5*x^2 + 3)*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(d + e*x)^2,x)
```

```
[Out] x^3*(17/(3*e^2) - (20*d^2)/(3*e^4) + (2*d*((40*d)/e^3 + 17/e^2))/(3*e)) - x
^2*(2/e^2 + (d*(17/e^2 - (20*d^2)/e^4 + (2*d*((40*d)/e^3 + 17/e^2))/e))/e -
 (d^2*((40*d)/e^3 + 17/e^2))/(2*e^2)) - x^4*((10*d)/e^3 + 17/(4*e^2)) + x*(
21/e^2 + (2*d*(4/e^2 + (2*d*(17/e^2 - (20*d^2)/e^4 + (2*d*((40*d)/e^3 + 17/
e^2))/e))/e - (d^2*((40*d)/e^3 + 17/e^2))/e^2))/e - (d^2*(17/e^2 - (20*d^2)
```

```
/e^4 + (2*d*((40*d)/e^3 + 17/e^2))/e))/e^2) + (4*x^5)/e^2 - (log(d + e*x)*(
42*d*e^4 + 85*d^4*e + 120*d^5 - 7*e^5 + 12*d^2*e^3 + 68*d^3*e^2))/e^7 - (17
*d^5*e - 7*d*e^5 + 20*d^6 + 6*e^6 + 21*d^2*e^4 + 4*d^3*e^3 + 17*d^4*e^2)/(e
*(d*e^6 + e^7*x))
```

3.295 $\int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{(d+ex)^3}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 231

$$\begin{aligned}
\int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{(d+ex)^3}\,dx
&= -\frac{(200d^3+102d^2e+51de^2+4e^3)\,x}{e^6}+\frac{(120d^2+51de+17e^2)\,x^2}{2e^5} \\
&\quad -\frac{(60d+17e)x^3}{3e^4}+\frac{5x^4}{e^3}-\frac{(5d^2-2de+3e^2)(4d^4+5d^3e+3d^2e^2-de^3+2e^4)}{2e^7(d+ex)^2} \\
&\quad +\frac{120d^5+85d^4e+68d^3e^2+12d^2e^3+42de^4-7e^5}{e^7(d+ex)} \\
&\quad +\frac{(300d^4+170d^3e+102d^2e^2+12de^3+21e^4)\log(d+ex)}{e^7}
\end{aligned}$$

```
[Out] -(200*d^3+102*d^2*e+51*d*e^2+4*e^3)*x/e^6+1/2*(120*d^2+51*d*e+17*e^2)*x^2/e
^5-1/3*(60*d+17*e)*x^3/e^4+5*x^4/e^3-1/2*(5*d^2-2*d*e+3*e^2)*(4*d^4+5*d^3*e
+3*d^2*e^2-d*e^3+2*e^4)/e^7/(e*x+d)^2+(120*d^5+85*d^4*e+68*d^3*e^2+12*d^2*e
^3+42*d*e^4-7*e^5)/e^7/(e*x+d)+(300*d^4+170*d^3*e+102*d^2*e^2+12*d*e^3+21*e
^4)*ln(e*x+d)/e^7
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {1642}

$$\begin{aligned}\int \frac{(3+2x+5x^2)\,(2+x+3x^2-5x^3+4x^4)}{(d+ex)^3}\,dx &= \frac{x^2(120d^2+51de+17e^2)}{2e^5}-\frac{x(200d^3+102d^2e+51de^2+4e^3)}{e^6}\\ &\quad-\frac{(5d^2-2de+3e^2)\,(4d^4+5d^3e+3d^2e^2-de^3+2e^4)}{2e^7(d+ex)^2}\\ &\quad+\frac{(300d^4+170d^3e+102d^2e^2+12de^3+21e^4)\log(d+ex)}{e^7}\\ &\quad+\frac{120d^5+85d^4e+68d^3e^2+12d^2e^3+42de^4-7e^5}{e^7(d+ex)}-\frac{x^3(60d+17e)}{3e^4}+\frac{5x^4}{e^3}\end{aligned}$$

```
[In] Int[((3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x)^3,x]
```

```
[Out] -(((200*d^3 + 102*d^2*e + 51*d*e^2 + 4*e^3)*x)/e^6) + ((120*d^2 + 51*d*e +
17*e^2)*x^2)/(2*e^5) - ((60*d + 17*e)*x^3)/(3*e^4) + (5*x^4)/e^3 - ((5*d^2
- 2*d*e + 3*e^2)*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4))/(2*e^7*(d +
 e*x)^2) + (120*d^5 + 85*d^4*e + 68*d^3*e^2 + 12*d^2*e^3 + 42*d*e^4 - 7*e^5
)/(e^7*(d + e*x)) + ((300*d^4 + 170*d^3*e + 102*d^2*e^2 + 12*d*e^3 + 21*e^4
)*Log[d + e*x])/e^7
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{-200d^3-102d^2e-51de^2-4e^3}{e^6}+\frac{(120d^2+51de+17e^2)\,x}{e^5}-\frac{(60d+17e)x^2}{e^4}\right.\\ &\quad+\frac{20x^3}{e^3}+\frac{20d^6+17d^5e+17d^4e^2+4d^3e^3+21d^2e^4-7de^5+6e^6}{e^6(d+ex)^3}\\ &\quad+\frac{-120d^5-85d^4e-68d^3e^2-12d^2e^3-42de^4+7e^5}{e^6(d+ex)^2}\\ &\quad\left.+\frac{300d^4+170d^3e+102d^2e^2+12de^3+21e^4}{e^6(d+ex)}\right)dx\end{aligned}$$

$$= -\frac{(200d^3+102d^2e+51de^2+4e^3)\,x}{e^6}+\frac{(120d^2+51de+17e^2)\,x^2}{2e^5} - \frac{(60d+17e)x^3}{3e^4}+\frac{5x^4}{e^3}-\frac{(5d^2-2de+3e^2)\,(4d^4+5d^3e+3d^2e^2-de^3+2e^4)}{2e^7(d+ex)^2} + \frac{120d^5+85d^4e+68d^3e^2+12d^2e^3+42de^4-7e^5}{e^7(d+ex)} + \frac{(300d^4+170d^3e+102d^2e^2+12de^3+21e^4)\log(d+ex)}{e^7}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.88

$$\int \frac{(3+2x+5x^2)\,(2+x+3x^2-5x^3+4x^4)}{(d+ex)^3}\,dx = \frac{660d^6+d^5e(459-480x)-51d^4e^2(-7+2x+40x^2)-3d^3e^3(-20-34x+357x^2+200x^3)+d^2e^4(189+}{}$$

```
[In] Integrate[((3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x)^3,x
]
```

```
[Out] (660*d^6 + d^5*e*(459 - 480*x) - 51*d^4*e^2*(-7 + 2*x + 40*x^2) - 3*d^3*e^3
*(-20 - 34*x + 357*x^2 + 200*x^3) + d^2*e^4*(189 + 48*x - 561*x^2 - 340*x^3
 + 150*x^4) - d*e^5*(21 - 252*x + 48*x^2 + 204*x^3 - 85*x^4 + 60*x^5) + e^6
*(-18 - 42*x - 24*x^3 + 51*x^4 - 34*x^5 + 30*x^6) + 6*(300*d^4 + 170*d^3*e
+ 102*d^2*e^2 + 12*d*e^3 + 21*e^4)*(d + e*x)^2*Log[d + e*x])/(6*e^7*(d + e*
x)^2)
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.97

method	result
norman	$\frac{\frac{\left(600d^5+340d^4e+204d^3e^2+24d^2e^3+42d\,e^4-7e^5\right)x}{e^6}+\frac{5x^6}{e}+\frac{900d^6+510d^5e+306d^4e^2+36d^3e^3+63d^2e^4-7d\,e^5-6e^6}{2e^7}-\frac{(30d+17e)x^5}{3e^2}+ (150d^2}{(ex+d)^2}$
default	$-\frac{-5e^3x^4+20d\,e^2x^3+\frac{17}{3}e^3x^3-60d^2e\,x^2-\frac{51}{2}d\,e^2x^2-\frac{17}{2}e^3x^2+200x\,d^3+102d^2ex+51d\,e^2x+4e^3x}{e^6}-\frac{-120d^5-85d^4e-68d^3e^2}{e^7(ex}$
risch	$\frac{5x^4}{e^3}-\frac{20d\,x^3}{e^4}-\frac{17x^3}{3e^3}+\frac{60d^2x^2}{e^5}+\frac{51d\,x^2}{2e^4}+\frac{17x^2}{2e^3}-\frac{200x\,d^3}{e^6}-\frac{102d^2x}{e^5}-\frac{51dx}{e^4}-\frac{4x}{e^3}+\frac{(120d^5+85d^4e+68d^3e^2}{}$
parallelrisch	$\frac{-34e^6x^5-18e^6+30e^6x^6+1530d^5e+1800\ln(ex+d)x^2d^4e^2+612\ln(ex+d)x^2d^2e^4+72\ln(ex+d)x^2d\,e^5+1020\ln(ex+d)x^2d^3e^3+}{}$

```
[In] int((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3,x,method=_RETURNVERBOSE
)
```

```
[Out] ((600*d^5+340*d^4*e+204*d^3*e^2+24*d^2*e^3+42*d*e^4-7*e^5)/e^6*x+5*x^6/e+1/
2*(900*d^6+510*d^5*e+306*d^4*e^2+36*d^3*e^3+63*d^2*e^4-7*d*e^5-6*e^6)/e^7-1
/3*(30*d+17*e)/e^2*x^5+1/6*(150*d^2+85*d*e+51*e^2)/e^3*x^4-2/3*(150*d^3+85*
d^2*e+51*d*e^2+6*e^3)/e^4*x^3)/(e*x+d)^2+(300*d^4+170*d^3*e+102*d^2*e^2+12*
d*e^3+21*e^4)*ln(e*x+d)/e^7
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.56

$$\int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{(d+ex)^3}\,dx$$
$$=\frac{30\,e^6x^6+660\,d^6+459\,d^5e+357\,d^4e^2+60\,d^3e^3+189\,d^2e^4-21\,de^5-18\,e^6-2\,(30\,de^5+17\,e^6)x^5+(150\,d}{}$$

```
[In] integrate((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3,x, algorithm="fri
cas")
```

```
[Out] 1/6*(30*e^6*x^6 + 660*d^6 + 459*d^5*e + 357*d^4*e^2 + 60*d^3*e^3 + 189*d^2*
e^4 - 21*d*e^5 - 18*e^6 - 2*(30*d*e^5 + 17*e^6)*x^5 + (150*d^2*e^4 + 85*d*e
^5 + 51*e^6)*x^4 - 4*(150*d^3*e^3 + 85*d^2*e^4 + 51*d*e^5 + 6*e^6)*x^3 - 3*
(680*d^4*e^2 + 357*d^3*e^3 + 187*d^2*e^4 + 16*d*e^5)*x^2 - 6*(80*d^5*e + 17
*d^4*e^2 - 17*d^3*e^3 - 8*d^2*e^4 - 42*d*e^5 + 7*e^6)*x + 6*(300*d^6 + 170*
d^5*e + 102*d^4*e^2 + 12*d^3*e^3 + 21*d^2*e^4 + (300*d^4*e^2 + 170*d^3*e^3
+ 102*d^2*e^4 + 12*d*e^5 + 21*e^6)*x^2 + 2*(300*d^5*e + 170*d^4*e^2 + 102*d
^3*e^3 + 12*d^2*e^4 + 21*d*e^5)*x)*log(e*x + d))/(e^9*x^2 + 2*d*e^8*x + d^2
*e^7)
```

Sympy [A] (verification not implemented)

Time = 0.85 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.07

$$\int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{(d+ex)^3}\,dx$$
$$=x^3\left(-\frac{20d}{e^4}-\frac{17}{3e^3}\right)+x^2\cdot\left(\frac{60d^2}{e^5}+\frac{51d}{2e^4}+\frac{17}{2e^3}\right)+x\left(-\frac{200d^3}{e^6}-\frac{102d^2}{e^5}-\frac{51d}{e^4}-\frac{4}{e^3}\right)$$
$$+\frac{220d^6+153d^5e+119d^4e^2+20d^3e^3+63d^2e^4-7de^5-6e^6+x(240d^5e+170d^4e^2+136d^3e^3+24d^2e^4+}{2d^2e^7+4de^8x+2e^9x^2}$$
$$+\frac{5x^4}{e^3}+\frac{(300d^4+170d^3e+102d^2e^2+12de^3+21e^4)\log{(d+ex)}}{e^7}$$

```
[In] integrate((5*x**2+2*x+3)*(4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)**3,x)
```

```
[Out] x**3*(-20*d/e**4 - 17/(3*e**3)) + x**2*(60*d**2/e**5 + 51*d/(2*e**4) + 17/(
2*e**3)) + x*(-200*d**3/e**6 - 102*d**2/e**5 - 51*d/e**4 - 4/e**3) + (220*d
**6 + 153*d**5*e + 119*d**4*e**2 + 20*d**3*e**3 + 63*d**2*e**4 - 7*d*e**5 -
 6*e**6 + x*(240*d**5*e + 170*d**4*e**2 + 136*d**3*e**3 + 24*d**2*e**4 + 84
*d*e**5 - 14*e**6))/(2*d**2*e**7 + 4*d*e**8*x + 2*e**9*x**2) + 5*x**4/e**3
+ (300*d**4 + 170*d**3*e + 102*d**2*e**2 + 12*d*e**3 + 21*e**4)*log(d + e*x
)/e**7
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.04

$$\int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{(d+ex)^3}\,dx$$
$$= \frac{220\,d^6+153\,d^5e+119\,d^4e^2+20\,d^3e^3+63\,d^2e^4-7\,de^5-6\,e^6+2\,(120\,d^5e+85\,d^4e^2+68\,d^3e^3+12\,d^2e^4 -}{2\,(e^9x^2+2\,de^8x+d^2e^7)}$$
$$+\frac{30\,e^3x^4-2\,(60\,de^2+17\,e^3)x^3+3\,(120\,d^2e+51\,de^2+17\,e^3)x^2-6\,(200\,d^3+102\,d^2e+51\,de^2+4\,e^3)x}{6\,e^6}$$
$$+\frac{(300\,d^4+170\,d^3e+102\,d^2e^2+12\,de^3+21\,e^4)\log{(ex+d)}}{e^7}$$

```
[In] integrate((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3,x, algorithm="max
ima")
```

```
[Out] 1/2*(220*d^6 + 153*d^5*e + 119*d^4*e^2 + 20*d^3*e^3 + 63*d^2*e^4 - 7*d*e^5
- 6*e^6 + 2*(120*d^5*e + 85*d^4*e^2 + 68*d^3*e^3 + 12*d^2*e^4 + 42*d*e^5 -
7*e^6)*x)/(e^9*x^2 + 2*d*e^8*x + d^2*e^7) + 1/6*(30*e^3*x^4 - 2*(60*d*e^2 +
 17*e^3)*x^3 + 3*(120*d^2*e + 51*d*e^2 + 17*e^3)*x^2 - 6*(200*d^3 + 102*d^2
*e + 51*d*e^2 + 4*e^3)*x)/e^6 + (300*d^4 + 170*d^3*e + 102*d^2*e^2 + 12*d*e
^3 + 21*e^4)*log(e*x + d)/e^7
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.03

$$\int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{(d+ex)^3}\,dx$$
$$= \frac{(300\,d^4+170\,d^3e+102\,d^2e^2+12\,de^3+21\,e^4)\log{(|ex+d|)}}{e^7}$$
$$+\frac{220\,d^6+153\,d^5e+119\,d^4e^2+20\,d^3e^3+63\,d^2e^4-7\,de^5-6\,e^6+2\,(120\,d^5e+85\,d^4e^2+68\,d^3e^3+12\,d^2e}{2\,(ex+d)^2e^7}$$
$$+\frac{30\,e^9x^4-120\,de^8x^3-34\,e^9x^3+360\,d^2e^7x^2+153\,de^8x^2+51\,e^9x^2-1200\,d^3e^6x-612\,d^2e^7x-306\,de^8x}{6\,e^{12}}$$

```
[In] integrate((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3,x, algorithm="gia
c")
```

```
[Out] (300*d^4 + 170*d^3*e + 102*d^2*e^2 + 12*d*e^3 + 21*e^4)*log(abs(e*x + d))/e
^7 + 1/2*(220*d^6 + 153*d^5*e + 119*d^4*e^2 + 20*d^3*e^3 + 63*d^2*e^4 - 7*d
*e^5 - 6*e^6 + 2*(120*d^5*e + 85*d^4*e^2 + 68*d^3*e^3 + 12*d^2*e^4 + 42*d*e
^5 - 7*e^6)*x)/((e*x + d)^2*e^7) + 1/6*(30*e^9*x^4 - 120*d*e^8*x^3 - 34*e^9
*x^3 + 360*d^2*e^7*x^2 + 153*d*e^8*x^2 + 51*e^9*x^2 - 1200*d^3*e^6*x - 612*
d^2*e^7*x - 306*d*e^8*x - 24*e^9*x)/e^12
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.29

$$\begin{aligned}
&\int \frac{(3+2x+5x^2)\,(2+x+3x^2-5x^3+4x^4)}{(d+ex)^3}\,dx \\
&= x^2\left(\frac{17}{2\,e^3}-\frac{30\,d^2}{e^5}+\frac{3\,d\,\left(\frac{60\,d}{e^4}+\frac{17}{e^3}\right)}{2\,e}\right)-x^3\left(\frac{20\,d}{e^4}+\frac{17}{3\,e^3}\right) \\
&\quad+\frac{x\,(120\,d^5+85\,d^4\,e+68\,d^3\,e^2+12\,d^2\,e^3+42\,d\,e^4-7\,e^5)+\frac{220\,d^6+153\,d^5\,e+119\,d^4\,e^2+20\,d^3\,e^3+63\,d^2\,e^4-7\,d\,e^5-6\,e^6}{2\,e}}{d^2\,e^6+2\,d\,e^7\,x+e^8\,x^2} \\
&\quad-x\left(\frac{4}{e^3}+\frac{20\,d^3}{e^6}+\frac{3\,d\,\left(\frac{17}{e^3}-\frac{60\,d^2}{e^5}+\frac{3\,d\,\left(\frac{60\,d}{e^4}+\frac{17}{e^3}\right)}{e}\right)}{e}-\frac{3\,d^2\,\left(\frac{60\,d}{e^4}+\frac{17}{e^3}\right)}{e^2}\right) \\
&\quad+\frac{5\,x^4}{e^3}+\frac{\ln(d+e\,x)\,\left(300\,d^4+170\,d^3\,e+102\,d^2\,e^2+12\,d\,e^3+21\,e^4\right)}{e^7}
\end{aligned}$$

```
[In] int(((2*x + 5*x^2 + 3)*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(d + e*x)^3,x)
```

```
[Out] x^2*(17/(2*e^3) - (30*d^2)/e^5 + (3*d*((60*d)/e^4 + 17/e^3))/(2*e)) - x^3*(
(20*d)/e^4 + 17/(3*e^3)) + (x*(42*d*e^4 + 85*d^4*e + 120*d^5 - 7*e^5 + 12*d
^2*e^3 + 68*d^3*e^2) + (153*d^5*e - 7*d*e^5 + 220*d^6 - 6*e^6 + 63*d^2*e^4
+ 20*d^3*e^3 + 119*d^4*e^2)/(2*e))/(d^2*e^6 + e^8*x^2 + 2*d*e^7*x) - x*(4/e
^3 + (20*d^3)/e^6 + (3*d*(17/e^3 - (60*d^2)/e^5 + (3*d*((60*d)/e^4 + 17/e^3
))/e))/e - (3*d^2*((60*d)/e^4 + 17/e^3))/e^2) + (5*x^4)/e^3 + (log(d + e*x)
*(12*d*e^3 + 170*d^3*e + 300*d^4 + 21*e^4 + 102*d^2*e^2))/e^7
```

3.296 $\int (d+ex)^3 \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right)$

Optimal result

Integrand size = 38, antiderivative size = 391

$$
\begin{aligned}
&\int (d+ex)^3 \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right)\,dx \\
&= \frac{\left(5d^2-2de+3e^2\right)^2 \left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)(d+ex)^4}{4e^9} \\
&\quad -\frac{\left(5d^2-2de+3e^2\right)\left(160d^5+127d^4e+88d^3e^2-4d^2e^3+64de^4-11e^5\right)(d+ex)^5}{5e^9} \\
&\quad +\frac{\left(2800d^6+945d^5e+1665d^4e^2+370d^3e^3+888d^2e^4-195de^5+107e^6\right)(d+ex)^6}{6e^9} \\
&\quad -\frac{\left(5600d^5+1575d^4e+2220d^3e^2+370d^2e^3+592de^4-65e^5\right)(d+ex)^7}{7e^9} \\
&\quad +\frac{\left(7000d^4+1575d^3e+1665d^2e^2+185de^3+148e^4\right)(d+ex)^8}{8e^9} \\
&\quad -\frac{\left(5600d^3+945d^2e+666de^2+37e^3\right)(d+ex)^9}{9e^9} \\
&\quad +\frac{\left(2800d^2+315de+111e^2\right)(d+ex)^{10}}{10e^9}-\frac{5(160d+9e)(d+ex)^{11}}{11e^9}+\frac{25(d+ex)^{12}}{3e^9}
\end{aligned}
$$

```
[Out] 1/4*(5*d^2-2*d*e+3*e^2)^2*(4*d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)*(e*x+d)^4/e
^9-1/5*(5*d^2-2*d*e+3*e^2)*(160*d^5+127*d^4*e+88*d^3*e^2-4*d^2*e^3+64*d*e^4
-11*e^5)*(e*x+d)^5/e^9+1/6*(2800*d^6+945*d^5*e+1665*d^4*e^2+370*d^3*e^3+888
*d^2*e^4-195*d*e^5+107*e^6)*(e*x+d)^6/e^9-1/7*(5600*d^5+1575*d^4*e+2220*d^3
*e^2+370*d^2*e^3+592*d*e^4-65*e^5)*(e*x+d)^7/e^9+1/8*(7000*d^4+1575*d^3*e+1
665*d^2*e^2+185*d*e^3+148*e^4)*(e*x+d)^8/e^9-1/9*(5600*d^3+945*d^2*e+666*d*
e^2+37*e^3)*(e*x+d)^9/e^9+1/10*(2800*d^2+315*d*e+111*e^2)*(e*x+d)^10/e^9-5/
11*(160*d+9*e)*(e*x+d)^11/e^9+25/3*(e*x+d)^12/e^9
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {1642}

$$
\begin{aligned}
&\int (d+ex)^3\left(3+2x+5x^2\right)^2\left(2+x+3x^2-5x^3+4x^4\right)\,dx\\
&=\frac{\left(2800d^2+315de+111e^2\right)(d+ex)^{10}}{10e^9}-\frac{\left(5600d^3+945d^2e+666de^2+37e^3\right)(d+ex)^9}{9e^9}\\
&\quad+\frac{\left(7000d^4+1575d^3e+1665d^2e^2+185de^3+148e^4\right)(d+ex)^8}{8e^9}\\
&\quad+\frac{\left(5d^2-2de+3e^2\right)^2\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)(d+ex)^4}{4e^9}\\
&\quad-\frac{\left(5600d^5+1575d^4e+2220d^3e^2+370d^2e^3+592de^4-65e^5\right)(d+ex)^7}{7e^9}\\
&\quad-\frac{\left(5d^2-2de+3e^2\right)\left(160d^5+127d^4e+88d^3e^2-4d^2e^3+64de^4-11e^5\right)(d+ex)^5}{5e^9}\\
&\quad+\frac{\left(2800d^6+945d^5e+1665d^4e^2+370d^3e^3+888d^2e^4-195de^5+107e^6\right)(d+ex)^6}{6e^9}\\
&\quad+\frac{25(d+ex)^{12}}{3e^9}-\frac{5(160d+9e)(d+ex)^{11}}{11e^9}
\end{aligned}
$$

```
[In] Int[(d + e*x)^3*(3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

```
[Out] ((5*d^2 - 2*d*e + 3*e^2)^2*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*(d
 + e*x)^4)/(4*e^9) - ((5*d^2 - 2*d*e + 3*e^2)*(160*d^5 + 127*d^4*e + 88*d^3
*e^2 - 4*d^2*e^3 + 64*d*e^4 - 11*e^5)*(d + e*x)^5)/(5*e^9) + ((2800*d^6 + 9
45*d^5*e + 1665*d^4*e^2 + 370*d^3*e^3 + 888*d^2*e^4 - 195*d*e^5 + 107*e^6)*
(d + e*x)^6)/(6*e^9) - ((5600*d^5 + 1575*d^4*e + 2220*d^3*e^2 + 370*d^2*e^3
 + 592*d*e^4 - 65*e^5)*(d + e*x)^7)/(7*e^9) + ((7000*d^4 + 1575*d^3*e + 166
5*d^2*e^2 + 185*d*e^3 + 148*e^4)*(d + e*x)^8)/(8*e^9) - ((5600*d^3 + 945*d^
2*e + 666*d*e^2 + 37*e^3)*(d + e*x)^9)/(9*e^9) + ((2800*d^2 + 315*d*e + 111
*e^2)*(d + e*x)^10)/(10*e^9) - (5*(160*d + 9*e)*(d + e*x)^11)/(11*e^9) + (2
5*(d + e*x)^12)/(3*e^9)
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{(5d^2-2de+3e^2)^2(4d^4+5d^3e+3d^2e^2-de^3+2e^4)(d+ex)^3}{e^8}\right.\\
&\quad+\frac{(-800d^7-315d^6e-666d^5e^2-185d^4e^3-592d^3e^4+195d^2e^5-214de^6+33e^7)(d+ex)^4}{e^8}\\
&\quad+\frac{(2800d^6+945d^5e+1665d^4e^2+370d^3e^3+888d^2e^4-195de^5+107e^6)(d+ex)^5}{e^8}\\
&\quad+\frac{(-5600d^5-1575d^4e-2220d^3e^2-370d^2e^3-592de^4+65e^5)(d+ex)^6}{e^8}\\
&\quad+\frac{(7000d^4+1575d^3e+1665d^2e^2+185de^3+148e^4)(d+ex)^7}{e^8}\\
&\quad+\frac{(-5600d^3-945d^2e-666de^2-37e^3)(d+ex)^8}{e^8}+\frac{(2800d^2+315de+111e^2)(d+ex)^9}{e^8}\\
&\quad\left.-\frac{5(160d+9e)(d+ex)^{10}}{e^8}+\frac{100(d+ex)^{11}}{e^8}\right)dx\\
&= \frac{(5d^2-2de+3e^2)^2(4d^4+5d^3e+3d^2e^2-de^3+2e^4)(d+ex)^4}{4e^9}\\
&\quad-\frac{(5d^2-2de+3e^2)(160d^5+127d^4e+88d^3e^2-4d^2e^3+64de^4-11e^5)(d+ex)^5}{5e^9}\\
&\quad+\frac{(2800d^6+945d^5e+1665d^4e^2+370d^3e^3+888d^2e^4-195de^5+107e^6)(d+ex)^6}{6e^9}\\
&\quad-\frac{(5600d^5+1575d^4e+2220d^3e^2+370d^2e^3+592de^4-65e^5)(d+ex)^7}{7e^9}\\
&\quad+\frac{(7000d^4+1575d^3e+1665d^2e^2+185de^3+148e^4)(d+ex)^8}{8e^9}\\
&\quad-\frac{(5600d^3+945d^2e+666de^2+37e^3)(d+ex)^9}{9e^9}\\
&\quad+\frac{(2800d^2+315de+111e^2)(d+ex)^{10}}{10e^9}-\frac{5(160d+9e)(d+ex)^{11}}{11e^9}+\frac{25(d+ex)^{12}}{3e^9}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.71

$$
\begin{aligned}
&\int (d+ex)^3 \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right)\,dx \\
&= 18d^3x+\frac{3}{2}d^2(11d+18e)x^2+\frac{1}{3}d\left(107d^2+99de+54e^2\right)x^3 \\
&\quad+\frac{1}{4}\left(65d^3+321d^2e+99de^2+18e^3\right)x^4+\frac{1}{5}\left(148d^3+195d^2e+321de^2+33e^3\right)x^5 \\
&\quad+\frac{1}{6}\left(-37d^3+444d^2e+195de^2+107e^3\right)x^6+\frac{1}{7}\left(111d^3-111d^2e+444de^2+65e^3\right)x^7 \\
&\quad+\frac{1}{8}\left(-45d^3+333d^2e-111de^2+148e^3\right)x^8+\frac{1}{9}\left(100d^3-135d^2e+333de^2-37e^3\right)x^9 \\
&\quad+\frac{3}{10}e\left(100d^2-45de+37e^2\right)x^{10}+\frac{15}{11}(20d-3e)e^2x^{11}+\frac{25e^3x^{12}}{3}
\end{aligned}
$$

```
[In] Integrate[(d + e*x)^3*(3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x
]
```

```
[Out] 18*d^3*x + (3*d^2*(11*d + 18*e)*x^2)/2 + (d*(107*d^2 + 99*d*e + 54*e^2)*x^3
)/3 + ((65*d^3 + 321*d^2*e + 99*d*e^2 + 18*e^3)*x^4)/4 + ((148*d^3 + 195*d^
2*e + 321*d*e^2 + 33*e^3)*x^5)/5 + ((-37*d^3 + 444*d^2*e + 195*d*e^2 + 107*
e^3)*x^6)/6 + ((111*d^3 - 111*d^2*e + 444*d*e^2 + 65*e^3)*x^7)/7 + ((-45*d^
3 + 333*d^2*e - 111*d*e^2 + 148*e^3)*x^8)/8 + ((100*d^3 - 135*d^2*e + 333*d
*e^2 - 37*e^3)*x^9)/9 + (3*e*(100*d^2 - 45*d*e + 37*e^2)*x^10)/10 + (15*(20
*d - 3*e)*e^2*x^11)/11 + (25*e^3*x^12)/3
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.65

method	result
norman	$\frac{25e^3x^{12}}{3}+\left(\frac{300}{11}d\,e^2-\frac{45}{11}e^3\right)x^{11}+\left(30d^2e-\frac{27}{2}d\,e^2+\frac{111}{10}e^3\right)x^{10}+\left(\frac{100}{9}d^3-15d^2e+37d\,e^2-\frac{37}{9}e$
default	$\frac{25e^3x^{12}}{3}+\frac{\left(300d\,e^2-45e^3\right)x^{11}}{11}+\frac{\left(300d^2e-135d\,e^2+111e^3\right)x^{10}}{10}+\frac{\left(100d^3-135d^2e+333d\,e^2-37e^3\right)x^9}{9}+$
gosper	$33d^2e\,x^3+74x^6d^2e+\frac{65}{2}x^6d\,e^2+39x^5d^2e+\frac{321}{5}x^5d\,e^2+\frac{321}{4}x^4d^2e+\frac{99}{4}x^4d\,e^2-\frac{111}{7}x^7d^2e+\frac{444}{7}$
risch	$33d^2e\,x^3+74x^6d^2e+\frac{65}{2}x^6d\,e^2+39x^5d^2e+\frac{321}{5}x^5d\,e^2+\frac{321}{4}x^4d^2e+\frac{99}{4}x^4d\,e^2-\frac{111}{7}x^7d^2e+\frac{444}{7}$
parallelrisch	$33d^2e\,x^3+74x^6d^2e+\frac{65}{2}x^6d\,e^2+39x^5d^2e+\frac{321}{5}x^5d\,e^2+\frac{321}{4}x^4d^2e+\frac{99}{4}x^4d\,e^2-\frac{111}{7}x^7d^2e+\frac{444}{7}$

```
[In] int((e*x+d)^3*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x,method=_RETURNVERBO
SE)
```

```
[Out] 25/3*e^3*x^12+(300/11*d*e^2-45/11*e^3)*x^11+(30*d^2*e-27/2*d*e^2+111/10*e^3
)*x^10+(100/9*d^3-15*d^2*e+37*d*e^2-37/9*e^3)*x^9+(-45/8*d^3+333/8*d^2*e-11
```

```
1/8*d*e^2+37/2*e^3)*x^8+(111/7*d^3-111/7*d^2*e+444/7*d*e^2+65/7*e^3)*x^7+(-
37/6*d^3+74*d^2*e+65/2*d*e^2+107/6*e^3)*x^6+(148/5*d^3+39*d^2*e+321/5*d*e^2
+33/5*e^3)*x^5+(65/4*d^3+321/4*d^2*e+99/4*d*e^2+9/2*e^3)*x^4+(107/3*d^3+33*
d^2*e+18*d*e^2)*x^3+(33/2*d^3+27*d^2*e)*x^2+18*x*d^3
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.67

$$\begin{aligned}\int (d+ex)^3 \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right) \, dx &= \frac{25}{3}\, e^3 x^{12} + \frac{15}{11}\, \left(20\, de^2 - 3\, e^3\right) x^{11} + \frac{3}{10}\, \left(100\, d^2 e - 45\, de^2 + 37\, e^3\right) x^{10} \\ &\quad + \frac{1}{9}\, \left(100\, d^3 - 135\, d^2 e + 333\, de^2 - 37\, e^3\right) x^9 - \frac{1}{8}\, \left(45\, d^3 - 333\, d^2 e + 111\, de^2 - 148\, e^3\right) x^8 \\ &\quad + \frac{1}{7}\, \left(111\, d^3 - 111\, d^2 e + 444\, de^2 + 65\, e^3\right) x^7 - \frac{1}{6}\, \left(37\, d^3 - 444\, d^2 e - 195\, de^2 - 107\, e^3\right) x^6 \\ &\quad + \frac{1}{5}\, \left(148\, d^3 + 195\, d^2 e + 321\, de^2 + 33\, e^3\right) x^5 + \frac{1}{4}\, \left(65\, d^3 + 321\, d^2 e + 99\, de^2 + 18\, e^3\right) x^4 \\ &\quad + 18\, d^3 x + \frac{1}{3}\, \left(107\, d^3 + 99\, d^2 e + 54\, de^2\right) x^3 + \frac{3}{2}\, \left(11\, d^3 + 18\, d^2 e\right) x^2\end{aligned}$$

```
[In] integrate((e*x+d)^3*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="f
ricas")
```

```
[Out] 25/3*e^3*x^12 + 15/11*(20*d*e^2 - 3*e^3)*x^11 + 3/10*(100*d^2*e - 45*d*e^2
+ 37*e^3)*x^10 + 1/9*(100*d^3 - 135*d^2*e + 333*d*e^2 - 37*e^3)*x^9 - 1/8*(
45*d^3 - 333*d^2*e + 111*d*e^2 - 148*e^3)*x^8 + 1/7*(111*d^3 - 111*d^2*e +
444*d*e^2 + 65*e^3)*x^7 - 1/6*(37*d^3 - 444*d^2*e - 195*d*e^2 - 107*e^3)*x^
6 + 1/5*(148*d^3 + 195*d^2*e + 321*d*e^2 + 33*e^3)*x^5 + 1/4*(65*d^3 + 321*
d^2*e + 99*d*e^2 + 18*e^3)*x^4 + 18*d^3*x + 1/3*(107*d^3 + 99*d^2*e + 54*d*
e^2)*x^3 + 3/2*(11*d^3 + 18*d^2*e)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 298, normalized size of antiderivative = 0.76

$$\begin{aligned}&\int (d+ex)^3 \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right)\,dx\\&=18d^3x+\frac{25e^3x^{12}}{3}+x^{11}\cdot\left(\frac{300de^2}{11}-\frac{45e^3}{11}\right)+x^{10}\cdot\left(30d^2e-\frac{27de^2}{2}+\frac{111e^3}{10}\right)+x^9\\&\quad\cdot\left(\frac{100d^3}{9}-15d^2e+37de^2-\frac{37e^3}{9}\right)+x^8\left(-\frac{45d^3}{8}+\frac{333d^2e}{8}-\frac{111de^2}{8}+\frac{37e^3}{2}\right)+x^7\\&\quad\cdot\left(\frac{111d^3}{7}-\frac{111d^2e}{7}+\frac{444de^2}{7}+\frac{65e^3}{7}\right)+x^6\left(-\frac{37d^3}{6}+74d^2e+\frac{65de^2}{2}+\frac{107e^3}{6}\right)\\&\quad+x^5\cdot\left(\frac{148d^3}{5}+39d^2e+\frac{321de^2}{5}+\frac{33e^3}{5}\right)+x^4\cdot\left(\frac{65d^3}{4}+\frac{321d^2e}{4}+\frac{99de^2}{4}+\frac{9e^3}{2}\right)\\&\quad+x^3\cdot\left(\frac{107d^3}{3}+33d^2e+18de^2\right)+x^2\cdot\left(\frac{33d^3}{2}+27d^2e\right)\end{aligned}$$

```
[In] integrate((e*x+d)**3*(5*x**2+2*x+3)**2*(4*x**4-5*x**3+3*x**2+x+2),x)
```

```
[Out] 18*d**3*x + 25*e**3*x**12/3 + x**11*(300*d*e**2/11 - 45*e**3/11) + x**10*(3
0*d**2*e - 27*d*e**2/2 + 111*e**3/10) + x**9*(100*d**3/9 - 15*d**2*e + 37*d
*e**2 - 37*e**3/9) + x**8*(-45*d**3/8 + 333*d**2*e/8 - 111*d*e**2/8 + 37*e*
*3/2) + x**7*(111*d**3/7 - 111*d**2*e/7 + 444*d*e**2/7 + 65*e**3/7) + x**6*
(-37*d**3/6 + 74*d**2*e + 65*d*e**2/2 + 107*e**3/6) + x**5*(148*d**3/5 + 39
*d**2*e + 321*d*e**2/5 + 33*e**3/5) + x**4*(65*d**3/4 + 321*d**2*e/4 + 99*d
*e**2/4 + 9*e**3/2) + x**3*(107*d**3/3 + 33*d**2*e + 18*d*e**2) + x**2*(33*
d**3/2 + 27*d**2*e)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.67

$$\begin{aligned}&\int (d+ex)^3 \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right)\,dx\\&=\frac{25}{3}\,e^3x^{12}+\frac{15}{11}\left(20\,de^2-3\,e^3\right)x^{11}+\frac{3}{10}\left(100\,d^2e-45\,de^2+37\,e^3\right)x^{10}\\&\quad+\frac{1}{9}\left(100\,d^3-135\,d^2e+333\,de^2-37\,e^3\right)x^9-\frac{1}{8}\left(45\,d^3-333\,d^2e+111\,de^2-148\,e^3\right)x^8\\&\quad+\frac{1}{7}\left(111\,d^3-111\,d^2e+444\,de^2+65\,e^3\right)x^7-\frac{1}{6}\left(37\,d^3-444\,d^2e-195\,de^2-107\,e^3\right)x^6\\&\quad+\frac{1}{5}\left(148\,d^3+195\,d^2e+321\,de^2+33\,e^3\right)x^5+\frac{1}{4}\left(65\,d^3+321\,d^2e+99\,de^2+18\,e^3\right)x^4\\&\quad+18\,d^3x+\frac{1}{3}\left(107\,d^3+99\,d^2e+54\,de^2\right)x^3+\frac{3}{2}\left(11\,d^3+18\,d^2e\right)x^2\end{aligned}$$

```
[In] integrate((e*x+d)^3*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="m
axima")
```

```
[Out] 25/3*e^3*x^12 + 15/11*(20*d*e^2 - 3*e^3)*x^11 + 3/10*(100*d^2*e - 45*d*e^2
+ 37*e^3)*x^10 + 1/9*(100*d^3 - 135*d^2*e + 333*d*e^2 - 37*e^3)*x^9 - 1/8*(
45*d^3 - 333*d^2*e + 111*d*e^2 - 148*e^3)*x^8 + 1/7*(111*d^3 - 111*d^2*e +
444*d*e^2 + 65*e^3)*x^7 - 1/6*(37*d^3 - 444*d^2*e - 195*d*e^2 - 107*e^3)*x^
6 + 1/5*(148*d^3 + 195*d^2*e + 321*d*e^2 + 33*e^3)*x^5 + 1/4*(65*d^3 + 321*
d^2*e + 99*d*e^2 + 18*e^3)*x^4 + 18*d^3*x + 1/3*(107*d^3 + 99*d^2*e + 54*d*
e^2)*x^3 + 3/2*(11*d^3 + 18*d^2*e)*x^2
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 305, normalized size of antiderivative = 0.78

$$\begin{aligned}&\int (d+ex)^3 \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right)\,dx\\&=\frac{25}{3}e^3x^{12}+\frac{300}{11}de^2x^{11}-\frac{45}{11}e^3x^{11}+30\,d^2ex^{10}-\frac{27}{2}de^2x^{10}+\frac{111}{10}e^3x^{10}+\frac{100}{9}d^3x^9\\&\quad-15\,d^2ex^9+37\,de^2x^9-\frac{37}{9}e^3x^9-\frac{45}{8}d^3x^8+\frac{333}{8}d^2ex^8-\frac{111}{8}de^2x^8+\frac{37}{2}e^3x^8\\&\quad+\frac{111}{7}d^3x^7-\frac{111}{7}d^2ex^7+\frac{444}{7}de^2x^7+\frac{65}{7}e^3x^7-\frac{37}{6}d^3x^6+74\,d^2ex^6+\frac{65}{2}de^2x^6\\&\quad+\frac{107}{6}e^3x^6+\frac{148}{5}d^3x^5+39\,d^2ex^5+\frac{321}{5}de^2x^5+\frac{33}{5}e^3x^5+\frac{65}{4}d^3x^4+\frac{321}{4}d^2ex^4\\&\quad+\frac{99}{4}de^2x^4+\frac{9}{2}e^3x^4+\frac{107}{3}d^3x^3+33\,d^2ex^3+18\,de^2x^3+\frac{33}{2}d^3x^2+27\,d^2ex^2+18\,d^3x\end{aligned}$$

```
[In] integrate((e*x+d)^3*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="g
iac")
```

```
[Out] 25/3*e^3*x^12 + 300/11*d*e^2*x^11 - 45/11*e^3*x^11 + 30*d^2*e*x^10 - 27/2*d
*e^2*x^10 + 111/10*e^3*x^10 + 100/9*d^3*x^9 - 15*d^2*e*x^9 + 37*d*e^2*x^9 -
 37/9*e^3*x^9 - 45/8*d^3*x^8 + 333/8*d^2*e*x^8 - 111/8*d*e^2*x^8 + 37/2*e^3
*x^8 + 111/7*d^3*x^7 - 111/7*d^2*e*x^7 + 444/7*d*e^2*x^7 + 65/7*e^3*x^7 - 3
7/6*d^3*x^6 + 74*d^2*e*x^6 + 65/2*d*e^2*x^6 + 107/6*e^3*x^6 + 148/5*d^3*x^5
 + 39*d^2*e*x^5 + 321/5*d*e^2*x^5 + 33/5*e^3*x^5 + 65/4*d^3*x^4 + 321/4*d^2
*e*x^4 + 99/4*d*e^2*x^4 + 9/2*e^3*x^4 + 107/3*d^3*x^3 + 33*d^2*e*x^3 + 18*d
*e^2*x^3 + 33/2*d^3*x^2 + 27*d^2*e*x^2 + 18*d^3*x
```

Mupad [B] (verification not implemented)

Time = 13.41 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.64

$$\int (d+ex)^3 \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right) \, dx$$
$$= 18\,d^3\,x + x^3\,\left(\frac{107\,d^3}{3} + 33\,d^2\,e + 18\,d\,e^2\right) + x^9\,\left(\frac{100\,d^3}{9} - 15\,d^2\,e + 37\,d\,e^2 - \frac{37\,e^3}{9}\right)$$
$$+ x^6\,\left(-\frac{37\,d^3}{6} + 74\,d^2\,e + \frac{65\,d\,e^2}{2} + \frac{107\,e^3}{6}\right) + x^4\,\left(\frac{65\,d^3}{4} + \frac{321\,d^2\,e}{4} + \frac{99\,d\,e^2}{4} + \frac{9\,e^3}{2}\right)$$
$$- x^8\,\left(\frac{45\,d^3}{8} - \frac{333\,d^2\,e}{8} + \frac{111\,d\,e^2}{8} - \frac{37\,e^3}{2}\right) + x^5\,\left(\frac{148\,d^3}{5} + 39\,d^2\,e + \frac{321\,d\,e^2}{5} + \frac{33\,e^3}{5}\right)$$
$$+ x^7\,\left(\frac{111\,d^3}{7} - \frac{111\,d^2\,e}{7} + \frac{444\,d\,e^2}{7} + \frac{65\,e^3}{7}\right) + \frac{25\,e^3\,x^{12}}{3}$$
$$+ \frac{3\,e\,x^{10}\,\left(100\,d^2 - 45\,d\,e + 37\,e^2\right)}{10} + \frac{3\,d^2\,x^2\,\left(11\,d + 18\,e\right)}{2} + \frac{15\,e^2\,x^{11}\,\left(20\,d - 3\,e\right)}{11}$$

```
[In] int((d + e*x)^3*(2*x + 5*x^2 + 3)^2*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2),x)
```

```
[Out] 18*d^3*x + x^3*(18*d*e^2 + 33*d^2*e + (107*d^3)/3) + x^9*(37*d*e^2 - 15*d^2
*e + (100*d^3)/9 - (37*e^3)/9) + x^6*((65*d*e^2)/2 + 74*d^2*e - (37*d^3)/6
+ (107*e^3)/6) + x^4*((99*d*e^2)/4 + (321*d^2*e)/4 + (65*d^3)/4 + (9*e^3)/2
) - x^8*((111*d*e^2)/8 - (333*d^2*e)/8 + (45*d^3)/8 - (37*e^3)/2) + x^5*((3
21*d*e^2)/5 + 39*d^2*e + (148*d^3)/5 + (33*e^3)/5) + x^7*((444*d*e^2)/7 - (
111*d^2*e)/7 + (111*d^3)/7 + (65*e^3)/7) + (25*e^3*x^12)/3 + (3*e*x^10*(100
*d^2 - 45*d*e + 37*e^2))/10 + (3*d^2*x^2*(11*d + 18*e))/2 + (15*e^2*x^11*(2
0*d - 3*e))/11
```

3.297 $\int (d+ex)^2 \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right)$

Optimal result

Integrand size = 38, antiderivative size = 201

$$\begin{aligned}\int (d+ex)^2 \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right)\,dx &= 18d^2x+\frac{3}{2}d(11d+12e)x^2+\frac{1}{3}\left(107d^2+66de+18e^2\right)x^3+\frac{1}{4}\left(65d^2+214de+33e^2\right)x^4\\ &\quad+\frac{1}{5}\left(148d^2+130de+107e^2\right)x^5-\frac{1}{6}\left(37d^2-296de-65e^2\right)x^6+\frac{37}{7}\left(3d^2-2de+4e^2\right)x^7\\ &\quad-\frac{1}{8}\left(45d^2-222de+37e^2\right)x^8+\frac{1}{9}\left(100d^2-90de+111e^2\right)x^9+\frac{1}{2}(40d-9e)ex^{10}+\frac{100e^2x^{11}}{11}\end{aligned}$$

```
[Out] 18*d^2*x+3/2*d*(11*d+12*e)*x^2+1/3*(107*d^2+66*d*e+18*e^2)*x^3+1/4*(65*d^2+
214*d*e+33*e^2)*x^4+1/5*(148*d^2+130*d*e+107*e^2)*x^5-1/6*(37*d^2-296*d*e-6
5*e^2)*x^6+37/7*(3*d^2-2*d*e+4*e^2)*x^7-1/8*(45*d^2-222*d*e+37*e^2)*x^8+1/9
*(100*d^2-90*d*e+111*e^2)*x^9+1/2*(40*d-9*e)*e*x^10+100/11*e^2*x^11
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {1642}

$$\begin{aligned}\int (d+ex)^2 \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right)\,dx &= \frac{1}{9}x^9\left(100d^2-90de+111e^2\right)-\frac{1}{8}x^8\left(45d^2-222de+37e^2\right)+\frac{37}{7}x^7\left(3d^2-2de+4e^2\right)\\ &\quad-\frac{1}{6}x^6\left(37d^2-296de-65e^2\right)+\frac{1}{5}x^5\left(148d^2+130de+107e^2\right)+\frac{1}{4}x^4\left(65d^2+214de+33e^2\right)\\ &\quad+\frac{1}{3}x^3\left(107d^2+66de+18e^2\right)+18d^2x+\frac{1}{2}ex^{10}(40d-9e)+\frac{3}{2}dx^2(11d+12e)+\frac{100e^2x^{11}}{11}\end{aligned}$$

```
[In] Int[(d + e*x)^2*(3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

```
[Out] 18*d^2*x + (3*d*(11*d + 12*e)*x^2)/2 + ((107*d^2 + 66*d*e + 18*e^2)*x^3)/3
+ ((65*d^2 + 214*d*e + 33*e^2)*x^4)/4 + ((148*d^2 + 130*d*e + 107*e^2)*x^5)
/5 - ((37*d^2 - 296*d*e - 65*e^2)*x^6)/6 + (37*(3*d^2 - 2*d*e + 4*e^2)*x^7)
/7 - ((45*d^2 - 222*d*e + 37*e^2)*x^8)/8 + ((100*d^2 - 90*d*e + 111*e^2)*x^
9)/9 + ((40*d - 9*e)*e*x^10)/2 + (100*e^2*x^11)/11
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(18d^2+3d(11d+12e)x+\left(107d^2+66de+18e^2\right)x^2+\left(65d^2+214de+33e^2\right)x^3\right.\\ &\qquad +\left(148d^2+130de+107e^2\right)x^4-\left(37d^2-296de-65e^2\right)x^5\\ &\qquad +37\left(3d^2-2de+4e^2\right)x^6-\left(45d^2-222de+37e^2\right)x^7\\ &\qquad \left.+\left(100d^2-90de+111e^2\right)x^8+5(40d-9e)ex^9+100e^2x^{10}\right)\,dx\\ &= 18d^2x+\frac{3}{2}d(11d+12e)x^2+\frac{1}{3}\left(107d^2+66de+18e^2\right)x^3+\frac{1}{4}\left(65d^2+214de+33e^2\right)x^4\\ &\quad +\frac{1}{5}\left(148d^2+130de+107e^2\right)x^5-\frac{1}{6}\left(37d^2-296de-65e^2\right)x^6+\frac{37}{7}\left(3d^2-2de+4e^2\right)x^7\\ &\quad -\frac{1}{8}\left(45d^2-222de+37e^2\right)x^8+\frac{1}{9}\left(100d^2-90de+111e^2\right)x^9+\frac{1}{2}(40d-9e)ex^{10}+\frac{100e^2x^{11}}{11}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.00

$$\begin{aligned}&\int (d+ex)^2\left(3+2x+5x^2\right)^2\left(2+x+3x^2-5x^3+4x^4\right)\,dx\\ &= 18d^2x+\frac{3}{2}d(11d+12e)x^2+\frac{1}{3}\left(107d^2+66de+18e^2\right)x^3+\frac{1}{4}\left(65d^2+214de+33e^2\right)x^4\\ &\quad +\frac{1}{5}\left(148d^2+130de+107e^2\right)x^5+\frac{1}{6}\left(-37d^2+296de+65e^2\right)x^6+\frac{37}{7}\left(3d^2-2de+4e^2\right)x^7\\ &\quad +\frac{1}{8}\left(-45d^2+222de-37e^2\right)x^8+\frac{1}{9}\left(100d^2-90de+111e^2\right)x^9+\frac{1}{2}(40d-9e)ex^{10}+\frac{100e^2x^{11}}{11}\end{aligned}$$

```
[In] Integrate[(d + e*x)^2*(3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x
]
```

```
[Out] 18*d^2*x + (3*d*(11*d + 12*e)*x^2)/2 + ((107*d^2 + 66*d*e + 18*e^2)*x^3)/3
+ ((65*d^2 + 214*d*e + 33*e^2)*x^4)/4 + ((148*d^2 + 130*d*e + 107*e^2)*x^5)
/5 + ((-37*d^2 + 296*d*e + 65*e^2)*x^6)/6 + (37*(3*d^2 - 2*d*e + 4*e^2)*x^7
)/7 + ((-45*d^2 + 222*d*e - 37*e^2)*x^8)/8 + ((100*d^2 - 90*d*e + 111*e^2)*
x^9)/9 + ((40*d - 9*e)*e*x^10)/2 + (100*e^2*x^11)/11
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.88

method	result
norman	$\frac{100e^2x^{11}}{11} + \left(20de - \frac{9}{2}e^2\right)x^{10} + \left(\frac{100}{9}d^2 - 10de + \frac{37}{3}e^2\right)x^9 + \left(-\frac{45}{8}d^2 + \frac{111}{4}de - \frac{37}{8}e^2\right)x^8 + \big(\frac{111}{7}$
default	$\frac{100e^2x^{11}}{11} + \frac{(200de-45e^2)x^{10}}{10} + \frac{(100d^2-90de+111e^2)x^9}{9} + \frac{(-45d^2+222de-37e^2)x^8}{8} + \frac{(111d^2-74de+148e^2)x^7}{7} +$
gosper	$\frac{111}{4}x^8de - \frac{74}{7}x^7de + \frac{148}{3}x^6de + \frac{107}{2}x^4de + 22x^3de + 26x^5de - 10x^9de + 20x^{10}de - \frac{45}{8}x^8d^2 +$
risch	$\frac{111}{4}x^8de - \frac{74}{7}x^7de + \frac{148}{3}x^6de + \frac{107}{2}x^4de + 22x^3de + 26x^5de - 10x^9de + 20x^{10}de - \frac{45}{8}x^8d^2 +$
parallelrisch	$\frac{111}{4}x^8de - \frac{74}{7}x^7de + \frac{148}{3}x^6de + \frac{107}{2}x^4de + 22x^3de + 26x^5de - 10x^9de + 20x^{10}de - \frac{45}{8}x^8d^2 +$

```
[In] int((e*x+d)^2*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x,method=_RETURNVERBO
SE)
```

```
[Out] 100/11*e^2*x^11+(20*d*e-9/2*e^2)*x^10+(100/9*d^2-10*d*e+37/3*e^2)*x^9+(-45/
8*d^2+111/4*d*e-37/8*e^2)*x^8+(111/7*d^2-74/7*d*e+148/7*e^2)*x^7+(-37/6*d^2
+148/3*d*e+65/6*e^2)*x^6+(148/5*d^2+26*d*e+107/5*e^2)*x^5+(65/4*d^2+107/2*d
*e+33/4*e^2)*x^4+(107/3*d^2+22*d*e+6*e^2)*x^3+(33/2*d^2+18*d*e)*x^2+18*x*d^
2
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.92

$$\begin{aligned}
\int (d+ex)^2 \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right)\,dx
&= \frac{100}{11}\,e^2x^{11} + \frac{1}{2}\left(40\,de - 9\,e^2\right)x^{10} + \frac{1}{9}\left(100\,d^2 - 90\,de + 111\,e^2\right)x^9 \\
&\quad - \frac{1}{8}\left(45\,d^2 - 222\,de + 37\,e^2\right)x^8 + \frac{37}{7}\left(3\,d^2 - 2\,de + 4\,e^2\right)x^7 - \frac{1}{6}\left(37\,d^2 - 296\,de - 65\,e^2\right)x^6 \\
&\quad + \frac{1}{5}\left(148\,d^2 + 130\,de + 107\,e^2\right)x^5 + \frac{1}{4}\left(65\,d^2 + 214\,de + 33\,e^2\right)x^4 \\
&\quad + \frac{1}{3}\left(107\,d^2 + 66\,de + 18\,e^2\right)x^3 + 18\,d^2x + \frac{3}{2}\left(11\,d^2 + 12\,de\right)x^2
\end{aligned}$$

```
[In] integrate((e*x+d)^2*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="f
ricas")
```

```
[Out] 100/11*e^2*x^11 + 1/2*(40*d*e - 9*e^2)*x^10 + 1/9*(100*d^2 - 90*d*e + 111*e
^2)*x^9 - 1/8*(45*d^2 - 222*d*e + 37*e^2)*x^8 + 37/7*(3*d^2 - 2*d*e + 4*e^2
)*x^7 - 1/6*(37*d^2 - 296*d*e - 65*e^2)*x^6 + 1/5*(148*d^2 + 130*d*e + 107*
e^2)*x^5 + 1/4*(65*d^2 + 214*d*e + 33*e^2)*x^4 + 1/3*(107*d^2 + 66*d*e + 18
*e^2)*x^3 + 18*d^2*x + 3/2*(11*d^2 + 12*d*e)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.02

$$\begin{aligned}
&\int (d+ex)^2\left(3+2x+5x^2\right)^2\left(2+x+3x^2-5x^3+4x^4\right)\,dx \\
&= 18d^2x+\frac{100e^2x^{11}}{11}+x^{10}\cdot\left(20de-\frac{9e^2}{2}\right)+x^9\cdot\left(\frac{100d^2}{9}-10de+\frac{37e^2}{3}\right) \\
&\quad +x^8\left(-\frac{45d^2}{8}+\frac{111de}{4}-\frac{37e^2}{8}\right)+x^7\cdot\left(\frac{111d^2}{7}-\frac{74de}{7}+\frac{148e^2}{7}\right) \\
&\quad +x^6\left(-\frac{37d^2}{6}+\frac{148de}{3}+\frac{65e^2}{6}\right)+x^5\cdot\left(\frac{148d^2}{5}+26de+\frac{107e^2}{5}\right)+x^4 \\
&\quad \cdot\left(\frac{65d^2}{4}+\frac{107de}{2}+\frac{33e^2}{4}\right)+x^3\cdot\left(\frac{107d^2}{3}+22de+6e^2\right)+x^2\cdot\left(\frac{33d^2}{2}+18de\right)
\end{aligned}$$

```
[In] integrate((e*x+d)**2*(5*x**2+2*x+3)**2*(4*x**4-5*x**3+3*x**2+x+2),x)
```

```
[Out] 18*d**2*x + 100*e**2*x**11/11 + x**10*(20*d*e - 9*e**2/2) + x**9*(100*d**2/
9 - 10*d*e + 37*e**2/3) + x**8*(-45*d**2/8 + 111*d*e/4 - 37*e**2/8) + x**7*
(111*d**2/7 - 74*d*e/7 + 148*e**2/7) + x**6*(-37*d**2/6 + 148*d*e/3 + 65*e*
*2/6) + x**5*(148*d**2/5 + 26*d*e + 107*e**2/5) + x**4*(65*d**2/4 + 107*d*e
/2 + 33*e**2/4) + x**3*(107*d**2/3 + 22*d*e + 6*e**2) + x**2*(33*d**2/2 + 1
8*d*e)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.92

$$\begin{aligned}
&\int (d+ex)^2\left(3+2x+5x^2\right)^2\left(2+x+3x^2-5x^3+4x^4\right)\,dx \\
&= \frac{100}{11}\,e^2x^{11}+\frac{1}{2}\left(40\,de-9\,e^2\right)x^{10}+\frac{1}{9}\left(100\,d^2-90\,de+111\,e^2\right)x^9 \\
&\quad -\frac{1}{8}\left(45\,d^2-222\,de+37\,e^2\right)x^8+\frac{37}{7}\left(3\,d^2-2\,de+4\,e^2\right)x^7-\frac{1}{6}\left(37\,d^2-296\,de-65\,e^2\right)x^6 \\
&\quad +\frac{1}{5}\left(148\,d^2+130\,de+107\,e^2\right)x^5+\frac{1}{4}\left(65\,d^2+214\,de+33\,e^2\right)x^4 \\
&\quad +\frac{1}{3}\left(107\,d^2+66\,de+18\,e^2\right)x^3+18\,d^2x+\frac{3}{2}\left(11\,d^2+12\,de\right)x^2
\end{aligned}$$

```
[In] integrate((e*x+d)^2*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="m
axima")
```

```
[Out] 100/11*e^2*x^11 + 1/2*(40*d*e - 9*e^2)*x^10 + 1/9*(100*d^2 - 90*d*e + 111*e
^2)*x^9 - 1/8*(45*d^2 - 222*d*e + 37*e^2)*x^8 + 37/7*(3*d^2 - 2*d*e + 4*e^2
```

```
)*x^7 - 1/6*(37*d^2 - 296*d*e - 65*e^2)*x^6 + 1/5*(148*d^2 + 130*d*e + 107*
e^2)*x^5 + 1/4*(65*d^2 + 214*d*e + 33*e^2)*x^4 + 1/3*(107*d^2 + 66*d*e + 18
*e^2)*x^3 + 18*d^2*x + 3/2*(11*d^2 + 12*d*e)*x^2
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.02

$$\begin{aligned}&\int (d+ex)^2\left(3+2x+5x^2\right)^2\left(2+x+3x^2-5x^3+4x^4\right)\,dx\\&=\frac{100}{11}\,e^2x^{11}+20\,dex^{10}-\frac{9}{2}\,e^2x^{10}+\frac{100}{9}\,d^2x^9-10\,dex^9+\frac{37}{3}\,e^2x^9-\frac{45}{8}\,d^2x^8\\&\quad+\frac{111}{4}\,dex^8-\frac{37}{8}\,e^2x^8+\frac{111}{7}\,d^2x^7-\frac{74}{7}\,dex^7+\frac{148}{7}\,e^2x^7-\frac{37}{6}\,d^2x^6\\&\quad+\frac{148}{3}\,dex^6+\frac{65}{6}\,e^2x^6+\frac{148}{5}\,d^2x^5+26\,dex^5+\frac{107}{5}\,e^2x^5+\frac{65}{4}\,d^2x^4+\frac{107}{2}\,dex^4\\&\quad+\frac{33}{4}\,e^2x^4+\frac{107}{3}\,d^2x^3+22\,dex^3+6\,e^2x^3+\frac{33}{2}\,d^2x^2+18\,dex^2+18\,d^2x\end{aligned}$$

```
[In] integrate((e*x+d)^2*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="g
iac")
```

```
[Out] 100/11*e^2*x^11 + 20*d*e*x^10 - 9/2*e^2*x^10 + 100/9*d^2*x^9 - 10*d*e*x^9 +
 37/3*e^2*x^9 - 45/8*d^2*x^8 + 111/4*d*e*x^8 - 37/8*e^2*x^8 + 111/7*d^2*x^7
 - 74/7*d*e*x^7 + 148/7*e^2*x^7 - 37/6*d^2*x^6 + 148/3*d*e*x^6 + 65/6*e^2*x
^6 + 148/5*d^2*x^5 + 26*d*e*x^5 + 107/5*e^2*x^5 + 65/4*d^2*x^4 + 107/2*d*e*
x^4 + 33/4*e^2*x^4 + 107/3*d^2*x^3 + 22*d*e*x^3 + 6*e^2*x^3 + 33/2*d^2*x^2
+ 18*d*e*x^2 + 18*d^2*x
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.87

$$\begin{aligned}&\int (d+ex)^2\left(3+2x+5x^2\right)^2\left(2+x+3x^2-5x^3+4x^4\right)\,dx\\&=x^3\left(\frac{107\,d^2}{3}+22\,d\,e+6\,e^2\right)+x^9\left(\frac{100\,d^2}{9}-10\,d\,e+\frac{37\,e^2}{3}\right)+x^4\left(\frac{65\,d^2}{4}+\frac{107\,d\,e}{2}+\frac{33\,e^2}{4}\right)\\&\quad-x^8\left(\frac{45\,d^2}{8}-\frac{111\,d\,e}{4}+\frac{37\,e^2}{8}\right)+x^6\left(-\frac{37\,d^2}{6}+\frac{148\,d\,e}{3}+\frac{65\,e^2}{6}\right)\\&\quad+x^5\left(\frac{148\,d^2}{5}+26\,d\,e+\frac{107\,e^2}{5}\right)+x^7\left(\frac{111\,d^2}{7}-\frac{74\,d\,e}{7}+\frac{148\,e^2}{7}\right)\\&\quad+18\,d^2\,x+\frac{100\,e^2\,x^{11}}{11}+\frac{3\,d\,x^2\,(11\,d+12\,e)}{2}+\frac{e\,x^{10}\,(40\,d-9\,e)}{2}\end{aligned}$$

```
[In] int((d + e*x)^2*(2*x + 5*x^2 + 3)^2*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2),x)
```

```
[Out] x^3*(22*d*e + (107*d^2)/3 + 6*e^2) + x^9*((100*d^2)/9 - 10*d*e + (37*e^2)/3
) + x^4*((107*d*e)/2 + (65*d^2)/4 + (33*e^2)/4) - x^8*((45*d^2)/8 - (111*d*
e)/4 + (37*e^2)/8) + x^6*((148*d*e)/3 - (37*d^2)/6 + (65*e^2)/6) + x^5*(26*
d*e + (148*d^2)/5 + (107*e^2)/5) + x^7*((111*d^2)/7 - (74*d*e)/7 + (148*e^2
)/7) + 18*d^2*x + (100*e^2*x^11)/11 + (3*d*x^2*(11*d + 12*e))/2 + (e*x^10*(
40*d - 9*e))/2
```

3.298 $\int (d+ex) \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right)$

Optimal result

Integrand size = 36, antiderivative size = 121

$$\begin{aligned}
&\int (d+ex) \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right) \, dx \\
&= 18dx + \frac{3}{2}(11d+6e)x^2 + \frac{1}{3}(107d+33e)x^3 + \frac{1}{4}(65d+107e)x^4 + \frac{1}{5}(148d+65e)x^5 \\
&\quad - \frac{37}{6}(d-4e)x^6 + \frac{37}{7}(3d-e)x^7 - \frac{3}{8}(15d-37e)x^8 + \frac{5}{9}(20d-9e)x^9 + 10ex^{10}
\end{aligned}$$

```
[Out] 18*d*x+3/2*(11*d+6*e)*x^2+1/3*(107*d+33*e)*x^3+1/4*(65*d+107*e)*x^4+1/5*(14
8*d+65*e)*x^5-37/6*(d-4*e)*x^6+37/7*(3*d-e)*x^7-3/8*(15*d-37*e)*x^8+5/9*(20
*d-9*e)*x^9+10*e*x^10
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {1642}

$$\begin{aligned}
&\int (d+ex) \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right) \, dx \\
&= \frac{5}{9}x^9(20d-9e) - \frac{3}{8}x^8(15d-37e) + \frac{37}{7}x^7(3d-e) - \frac{37}{6}x^6(d-4e) + \frac{1}{5}x^5(148d+65e) \\
&\quad + \frac{1}{4}x^4(65d+107e) + \frac{1}{3}x^3(107d+33e) + \frac{3}{2}x^2(11d+6e) + 18dx + 10ex^{10}
\end{aligned}$$

```
[In] Int[(d + e*x)*(3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

```
[Out] 18*d*x + (3*(11*d + 6*e)*x^2)/2 + ((107*d + 33*e)*x^3)/3 + ((65*d + 107*e)*
x^4)/4 + ((148*d + 65*e)*x^5)/5 - (37*(d - 4*e)*x^6)/6 + (37*(3*d - e)*x^7)
/7 - (3*(15*d - 37*e)*x^8)/8 + (5*(20*d - 9*e)*x^9)/9 + 10*e*x^10
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(18d+3(11d+6e)x+(107d+33e)x^2+(65d+107e)x^3+(148d+65e)x^4\right.\\&\quad\left.-37(d-4e)x^5+37(3d-e)x^6-3(15d-37e)x^7+5(20d-9e)x^8+100ex^9\right)\,dx\\&=18dx+\frac{3}{2}(11d+6e)x^2+\frac{1}{3}(107d+33e)x^3+\frac{1}{4}(65d+107e)x^4+\frac{1}{5}(148d+65e)x^5\\&\quad-\frac{37}{6}(d-4e)x^6+\frac{37}{7}(3d-e)x^7-\frac{3}{8}(15d-37e)x^8+\frac{5}{9}(20d-9e)x^9+10ex^{10}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00

$$\begin{aligned}&\int (d+ex)\left(3+2x+5x^2\right)^2\left(2+x+3x^2-5x^3+4x^4\right)\,dx\\&=18dx+\frac{3}{2}(11d+6e)x^2+\frac{1}{3}(107d+33e)x^3+\frac{1}{4}(65d+107e)x^4+\frac{1}{5}(148d+65e)x^5\\&\quad-\frac{37}{6}(d-4e)x^6+\frac{37}{7}(3d-e)x^7-\frac{3}{8}(15d-37e)x^8+\frac{5}{9}(20d-9e)x^9+10ex^{10}\end{aligned}$$

```
[In] Integrate[(d + e*x)*(3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

```
[Out] 18*d*x + (3*(11*d + 6*e)*x^2)/2 + ((107*d + 33*e)*x^3)/3 + ((65*d + 107*e)*
x^4)/4 + ((148*d + 65*e)*x^5)/5 - (37*(d - 4*e)*x^6)/6 + (37*(3*d - e)*x^7)
/7 - (3*(15*d - 37*e)*x^8)/8 + (5*(20*d - 9*e)*x^9)/9 + 10*e*x^10
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.83

method	result
norman	$10e\,x^{10}+\left(\frac{100d}{9}-5e\right)x^{9}+\left(-\frac{45d}{8}+\frac{111e}{8}\right)x^{8}+\left(\frac{111d}{7}-\frac{37e}{7}\right)x^{7}+\left(-\frac{37d}{6}+\frac{74e}{3}\right)x^{6}+\left(\frac{148d}{5}+$
gosper	$10e\,x^{10}+\frac{100}{9}d\,x^{9}-5e\,x^{9}-\frac{45}{8}d\,x^{8}+\frac{111}{8}e\,x^{8}+\frac{111}{7}d\,x^{7}-\frac{37}{7}e\,x^{7}-\frac{37}{6}d\,x^{6}+\frac{74}{3}e\,x^{6}+\frac{148}{5}d\,x^{5}$
default	$10e\,x^{10}+\frac{(100d-45e)x^{9}}{9}+\frac{(-45d+111e)x^{8}}{8}+\frac{(111d-37e)x^{7}}{7}+\frac{(-37d+148e)x^{6}}{6}+\frac{(148d+65e)x^{5}}{5}+$
risch	$10e\,x^{10}+\frac{100}{9}d\,x^{9}-5e\,x^{9}-\frac{45}{8}d\,x^{8}+\frac{111}{8}e\,x^{8}+\frac{111}{7}d\,x^{7}-\frac{37}{7}e\,x^{7}-\frac{37}{6}d\,x^{6}+\frac{74}{3}e\,x^{6}+\frac{148}{5}d\,x^{5}$
parallelrisch	$10e\,x^{10}+\frac{100}{9}d\,x^{9}-5e\,x^{9}-\frac{45}{8}d\,x^{8}+\frac{111}{8}e\,x^{8}+\frac{111}{7}d\,x^{7}-\frac{37}{7}e\,x^{7}-\frac{37}{6}d\,x^{6}+\frac{74}{3}e\,x^{6}+\frac{148}{5}d\,x^{5}$

```
[In] int((e*x+d)*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x,method=_RETURNVERBOSE
)
```

```
[Out] 10*e*x^10+(100/9*d-5*e)*x^9+(-45/8*d+111/8*e)*x^8+(111/7*d-37/7*e)*x^7+(-37
/6*d+74/3*e)*x^6+(148/5*d+13*e)*x^5+(65/4*d+107/4*e)*x^4+(107/3*d+11*e)*x^3
+(33/2*d+9*e)*x^2+18*d*x
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.87

$$\begin{aligned}\int (d+ex)\left(3+2x+5x^2\right)^2\left(2+x+3x^2-5x^3+4x^4\right)\,dx\\ &= 10\,ex^{10}+\frac{5}{9}\,(20\,d-9\,e)x^9-\frac{3}{8}\,(15\,d-37\,e)x^8+\frac{37}{7}\,(3\,d-e)x^7-\frac{37}{6}\,(d-4\,e)x^6\\ &\quad+\frac{1}{5}\,(148\,d+65\,e)x^5+\frac{1}{4}\,(65\,d+107\,e)x^4+\frac{1}{3}\,(107\,d+33\,e)x^3+\frac{3}{2}\,(11\,d+6\,e)x^2+18\,dx\end{aligned}$$

```
[In] integrate((e*x+d)*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="fri
cas")
```

```
[Out] 10*e*x^10 + 5/9*(20*d - 9*e)*x^9 - 3/8*(15*d - 37*e)*x^8 + 37/7*(3*d - e)*x
^7 - 37/6*(d - 4*e)*x^6 + 1/5*(148*d + 65*e)*x^5 + 1/4*(65*d + 107*e)*x^4 +
 1/3*(107*d + 33*e)*x^3 + 3/2*(11*d + 6*e)*x^2 + 18*d*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.93

$$\begin{aligned}&\int (d+ex)\left(3+2x+5x^2\right)^2\left(2+x+3x^2-5x^3+4x^4\right)\,dx\\&=18dx+10ex^{10}+x^9\cdot\left(\frac{100d}{9}-5e\right)+x^8\left(-\frac{45d}{8}+\frac{111e}{8}\right)+x^7\\&\cdot\left(\frac{111d}{7}-\frac{37e}{7}\right)+x^6\left(-\frac{37d}{6}+\frac{74e}{3}\right)+x^5\cdot\left(\frac{148d}{5}+13e\right)\\&+x^4\cdot\left(\frac{65d}{4}+\frac{107e}{4}\right)+x^3\cdot\left(\frac{107d}{3}+11e\right)+x^2\cdot\left(\frac{33d}{2}+9e\right)\end{aligned}$$

```
[In] integrate((e*x+d)*(5*x**2+2*x+3)**2*(4*x**4-5*x**3+3*x**2+x+2),x)
```

```
[Out] 18*d*x + 10*e*x**10 + x**9*(100*d/9 - 5*e) + x**8*(-45*d/8 + 111*e/8) + x**
7*(111*d/7 - 37*e/7) + x**6*(-37*d/6 + 74*e/3) + x**5*(148*d/5 + 13*e) + x*
*4*(65*d/4 + 107*e/4) + x**3*(107*d/3 + 11*e) + x**2*(33*d/2 + 9*e)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.87

$$\begin{aligned}&\int (d+ex)\left(3+2x+5x^2\right)^2\left(2+x+3x^2-5x^3+4x^4\right)\,dx\\&=10\,ex^{10}+\frac{5}{9}\,(20\,d-9\,e)x^9-\frac{3}{8}\,(15\,d-37\,e)x^8+\frac{37}{7}\,(3\,d-e)x^7-\frac{37}{6}\,(d-4\,e)x^6\\&+\frac{1}{5}\,(148\,d+65\,e)x^5+\frac{1}{4}\,(65\,d+107\,e)x^4+\frac{1}{3}\,(107\,d+33\,e)x^3+\frac{3}{2}\,(11\,d+6\,e)x^2+18\,dx\end{aligned}$$

```
[In] integrate((e*x+d)*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="max
ima")
```

```
[Out] 10*e*x^10 + 5/9*(20*d - 9*e)*x^9 - 3/8*(15*d - 37*e)*x^8 + 37/7*(3*d - e)*x
^7 - 37/6*(d - 4*e)*x^6 + 1/5*(148*d + 65*e)*x^5 + 1/4*(65*d + 107*e)*x^4 +
 1/3*(107*d + 33*e)*x^3 + 3/2*(11*d + 6*e)*x^2 + 18*d*x
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.88

$$\begin{aligned}&\int (d+ex)\left(3+2x+5x^2\right)^2\left(2+x+3x^2-5x^3+4x^4\right)\,dx\\&=10\,ex^{10}+\frac{100}{9}\,dx^9-5\,ex^9-\frac{45}{8}\,dx^8+\frac{111}{8}\,ex^8+\frac{111}{7}\,dx^7-\frac{37}{7}\,ex^7-\frac{37}{6}\,dx^6+\frac{74}{3}\,ex^6\\&\quad+\frac{148}{5}\,dx^5+13\,ex^5+\frac{65}{4}\,dx^4+\frac{107}{4}\,ex^4+\frac{107}{3}\,dx^3+11\,ex^3+\frac{33}{2}\,dx^2+9\,ex^2+18\,dx\end{aligned}$$

```
[In] integrate((e*x+d)*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="gia
c")
```

```
[Out] 10*e*x^10 + 100/9*d*x^9 - 5*e*x^9 - 45/8*d*x^8 + 111/8*e*x^8 + 111/7*d*x^7
- 37/7*e*x^7 - 37/6*d*x^6 + 74/3*e*x^6 + 148/5*d*x^5 + 13*e*x^5 + 65/4*d*x^
4 + 107/4*e*x^4 + 107/3*d*x^3 + 11*e*x^3 + 33/2*d*x^2 + 9*e*x^2 + 18*d*x
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.83

$$\begin{aligned}&\int (d+ex)\left(3+2x+5x^2\right)^2\left(2+x+3x^2-5x^3+4x^4\right)\,dx\\&=10\,e\,x^{10}+\left(\frac{100\,d}{9}-5\,e\right)\,x^9+\left(\frac{111\,e}{8}-\frac{45\,d}{8}\right)\,x^8+\left(\frac{111\,d}{7}-\frac{37\,e}{7}\right)\,x^7+\left(\frac{74\,e}{3}-\frac{37\,d}{6}\right)\,x^6\\&\quad+\left(\frac{148\,d}{5}+13\,e\right)\,x^5+\left(\frac{65\,d}{4}+\frac{107\,e}{4}\right)\,x^4+\left(\frac{107\,d}{3}+11\,e\right)\,x^3+\left(\frac{33\,d}{2}+9\,e\right)\,x^2+18\,d\,x\end{aligned}$$

```
[In] int((d + e*x)*(2*x + 5*x^2 + 3)^2*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2),x)
```

```
[Out] x^2*((33*d)/2 + 9*e) + x^9*((100*d)/9 - 5*e) + x^3*((107*d)/3 + 11*e) - x^6
*((37*d)/6 - (74*e)/3) + x^7*((111*d)/7 - (37*e)/7) + x^5*((148*d)/5 + 13*e
) - x^8*((45*d)/8 - (111*e)/8) + x^4*((65*d)/4 + (107*e)/4) + 18*d*x + 10*e
*x^10
```

3.299 $\int \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right) dx$

Optimal result

Integrand size = 31, antiderivative size = 60

$$\int \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right) dx = 18x+\frac{33x^2}{2}+\frac{107x^3}{3}+\frac{65x^4}{4}+\frac{148x^5}{5}-\frac{37x^6}{6}+\frac{111x^7}{7}-\frac{45x^8}{8}+\frac{100x^9}{9}$$

```
[Out] 18*x+33/2*x^2+107/3*x^3+65/4*x^4+148/5*x^5-37/6*x^6+111/7*x^7-45/8*x^8+100/
9*x^9
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.032, Rules used = {1671}

$$\int \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right) dx = \frac{100x^9}{9}-\frac{45x^8}{8}+\frac{111x^7}{7}-\frac{37x^6}{6}+\frac{148x^5}{5}+\frac{65x^4}{4}+\frac{107x^3}{3}+\frac{33x^2}{2}+18x$$

```
[In] Int[(3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

```
[Out] 18*x + (33*x^2)/2 + (107*x^3)/3 + (65*x^4)/4 + (148*x^5)/5 - (37*x^6)/6 + (
111*x^7)/7 - (45*x^8)/8 + (100*x^9)/9
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(18+33x+107x^2+65x^3+148x^4-37x^5+111x^6-45x^7+100x^8\right)\,dx\\ &= 18x+\frac{33x^2}{2}+\frac{107x^3}{3}+\frac{65x^4}{4}+\frac{148x^5}{5}-\frac{37x^6}{6}+\frac{111x^7}{7}-\frac{45x^8}{8}+\frac{100x^9}{9}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\begin{aligned}&\int \left(3+2x+5x^2\right)^2\left(2+x+3x^2-5x^3+4x^4\right)\,dx\\ &=18x+\frac{33x^2}{2}+\frac{107x^3}{3}+\frac{65x^4}{4}+\frac{148x^5}{5}-\frac{37x^6}{6}+\frac{111x^7}{7}-\frac{45x^8}{8}+\frac{100x^9}{9}\end{aligned}$$

```
[In] Integrate[(3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

```
[Out] 18*x + (33*x^2)/2 + (107*x^3)/3 + (65*x^4)/4 + (148*x^5)/5 - (37*x^6)/6 + (
111*x^7)/7 - (45*x^8)/8 + (100*x^9)/9
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.75

method	result	size
gosper	$18x+\frac{33}{2}x^2+\frac{107}{3}x^3+\frac{65}{4}x^4+\frac{148}{5}x^5-\frac{37}{6}x^6+\frac{111}{7}x^7-\frac{45}{8}x^8+\frac{100}{9}x^9$	45
default	$18x+\frac{33}{2}x^2+\frac{107}{3}x^3+\frac{65}{4}x^4+\frac{148}{5}x^5-\frac{37}{6}x^6+\frac{111}{7}x^7-\frac{45}{8}x^8+\frac{100}{9}x^9$	45
norman	$18x+\frac{33}{2}x^2+\frac{107}{3}x^3+\frac{65}{4}x^4+\frac{148}{5}x^5-\frac{37}{6}x^6+\frac{111}{7}x^7-\frac{45}{8}x^8+\frac{100}{9}x^9$	45
risch	$18x+\frac{33}{2}x^2+\frac{107}{3}x^3+\frac{65}{4}x^4+\frac{148}{5}x^5-\frac{37}{6}x^6+\frac{111}{7}x^7-\frac{45}{8}x^8+\frac{100}{9}x^9$	45
parallelrisch	$18x+\frac{33}{2}x^2+\frac{107}{3}x^3+\frac{65}{4}x^4+\frac{148}{5}x^5-\frac{37}{6}x^6+\frac{111}{7}x^7-\frac{45}{8}x^8+\frac{100}{9}x^9$	45

```
[In] int((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x,method=_RETURNVERBOSE)
```

```
[Out] 18*x+33/2*x^2+107/3*x^3+65/4*x^4+148/5*x^5-37/6*x^6+111/7*x^7-45/8*x^8+100/
9*x^9
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.73

$$\int \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right)\,dx = \frac{100}{9}\,x^9-\frac{45}{8}\,x^8+\frac{111}{7}\,x^7-\frac{37}{6}\,x^6+\frac{148}{5}\,x^5+\frac{65}{4}\,x^4+\frac{107}{3}\,x^3+\frac{33}{2}\,x^2+18\,x$$

```
[In] integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="fricas")
```

```
[Out] 100/9*x^9 - 45/8*x^8 + 111/7*x^7 - 37/6*x^6 + 148/5*x^5 + 65/4*x^4 + 107/3*
x^3 + 33/2*x^2 + 18*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.93

$$\int \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right)\,dx = \frac{100x^9}{9}-\frac{45x^8}{8}+\frac{111x^7}{7}-\frac{37x^6}{6}+\frac{148x^5}{5}+\frac{65x^4}{4}+\frac{107x^3}{3}+\frac{33x^2}{2}+18x$$

```
[In] integrate((5*x**2+2*x+3)**2*(4*x**4-5*x**3+3*x**2+x+2),x)
```

```
[Out] 100*x**9/9 - 45*x**8/8 + 111*x**7/7 - 37*x**6/6 + 148*x**5/5 + 65*x**4/4 +
107*x**3/3 + 33*x**2/2 + 18*x
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.73

$$\int \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right)\,dx = \frac{100}{9}\,x^9-\frac{45}{8}\,x^8+\frac{111}{7}\,x^7-\frac{37}{6}\,x^6+\frac{148}{5}\,x^5+\frac{65}{4}\,x^4+\frac{107}{3}\,x^3+\frac{33}{2}\,x^2+18\,x$$

```
[In] integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="maxima")
```

```
[Out] 100/9*x^9 - 45/8*x^8 + 111/7*x^7 - 37/6*x^6 + 148/5*x^5 + 65/4*x^4 + 107/3*
x^3 + 33/2*x^2 + 18*x
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.73

$$\int \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right)\,dx = \frac{100}{9}\,x^9-\frac{45}{8}\,x^8+\frac{111}{7}\,x^7-\frac{37}{6}\,x^6+\frac{148}{5}\,x^5+\frac{65}{4}\,x^4+\frac{107}{3}\,x^3+\frac{33}{2}\,x^2+18\,x$$

```
[In] integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="giac")
```

```
[Out] 100/9*x^9 - 45/8*x^8 + 111/7*x^7 - 37/6*x^6 + 148/5*x^5 + 65/4*x^4 + 107/3*
x^3 + 33/2*x^2 + 18*x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.73

$$\int \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right)\,dx = \frac{100\,x^9}{9}-\frac{45\,x^8}{8}+\frac{111\,x^7}{7}-\frac{37\,x^6}{6}+\frac{148\,x^5}{5}+\frac{65\,x^4}{4}+\frac{107\,x^3}{3}+\frac{33\,x^2}{2}+18\,x$$

```
[In] int((2*x + 5*x^2 + 3)^2*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2),x)
```

```
[Out] 18*x + (33*x^2)/2 + (107*x^3)/3 + (65*x^4)/4 + (148*x^5)/5 - (37*x^6)/6 + (
111*x^7)/7 - (45*x^8)/8 + (100*x^9)/9
```

3.300 $\int \frac{(3+2x+5x^2)^2 (2+x+3x^2-5x^3+4x^4)}{d+ex}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 352

$$
\begin{aligned}
\int \frac{(3+2x+5x^2)^2 (2+x+3x^2-5x^3+4x^4)}{d+ex}\,dx
&= -\frac{(100d^7+45d^6e+111d^5e^2+37d^4e^3+148d^3e^4-65d^2e^5+107de^6-33e^7)\,x}{e^8} \\
&+ \frac{(100d^6+45d^5e+111d^4e^2+37d^3e^3+148d^2e^4-65de^5+107e^6)\,x^2}{2e^7} \\
&- \frac{(100d^5+45d^4e+111d^3e^2+37d^2e^3+148de^4-65e^5)\,x^3}{3e^6} \\
&+ \frac{(100d^4+45d^3e+111d^2e^2+37de^3+148e^4)\,x^4}{4e^5} - \frac{(100d^3+45d^2e+111de^2+37e^3)\,x^5}{5e^4} \\
&+ \frac{(100d^2+45de+111e^2)\,x^6}{6e^3} - \frac{5(20d+9e)x^7}{7e^2} + \frac{25x^8}{2e} \\
&+ \frac{(5d^2-2de+3e^2)^2 (4d^4+5d^3e+3d^2e^2-de^3+2e^4)\log(d+ex)}{e^9}
\end{aligned}
$$

```
[Out] -(100*d^7+45*d^6*e+111*d^5*e^2+37*d^4*e^3+148*d^3*e^4-65*d^2*e^5+107*d*e^6-
33*e^7)*x/e^8+1/2*(100*d^6+45*d^5*e+111*d^4*e^2+37*d^3*e^3+148*d^2*e^4-65*d
*e^5+107*e^6)*x^2/e^7-1/3*(100*d^5+45*d^4*e+111*d^3*e^2+37*d^2*e^3+148*d*e^
4-65*e^5)*x^3/e^6+1/4*(100*d^4+45*d^3*e+111*d^2*e^2+37*d*e^3+148*e^4)*x^4/e
^5-1/5*(100*d^3+45*d^2*e+111*d*e^2+37*e^3)*x^5/e^4+1/6*(100*d^2+45*d*e+111*
e^2)*x^6/e^3-5/7*(20*d+9*e)*x^7/e^2+25/2*x^8/e+(5*d^2-2*d*e+3*e^2)^2*(4*d^4
+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)*ln(e*x+d)/e^9
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$, Rules used = {1642}

$$\begin{aligned}
&\int \frac{(3+2x+5x^2)^2 (2+x+3x^2-5x^3+4x^4)}{d+ex}\,dx \\
&= \frac{x^6(100d^2+45de+111e^2)}{6e^3} - \frac{x^5(100d^3+45d^2e+111de^2+37e^3)}{5e^4} \\
&\quad + \frac{(5d^2-2de+3e^2)^2 (4d^4+5d^3e+3d^2e^2-de^3+2e^4)\log(d+ex)}{e^9} \\
&\quad + \frac{x^4(100d^4+45d^3e+111d^2e^2+37de^3+148e^4)}{4e^5} \\
&\quad - \frac{x^3(100d^5+45d^4e+111d^3e^2+37d^2e^3+148de^4-65e^5)}{3e^6} \\
&\quad + \frac{x^2(100d^6+45d^5e+111d^4e^2+37d^3e^3+148d^2e^4-65de^5+107e^6)}{2e^7} \\
&\quad - \frac{x(100d^7+45d^6e+111d^5e^2+37d^4e^3+148d^3e^4-65d^2e^5+107de^6-33e^7)}{e^8} \\
&\quad - \frac{5x^7(20d+9e)}{7e^2} + \frac{25x^8}{2e}
\end{aligned}$$

```
[In] Int[((3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x),x]
```

```
[Out] -(((100*d^7 + 45*d^6*e + 111*d^5*e^2 + 37*d^4*e^3 + 148*d^3*e^4 - 65*d^2*e^
5 + 107*d*e^6 - 33*e^7)*x)/e^8) + ((100*d^6 + 45*d^5*e + 111*d^4*e^2 + 37*d
^3*e^3 + 148*d^2*e^4 - 65*d*e^5 + 107*e^6)*x^2)/(2*e^7) - ((100*d^5 + 45*d^
4*e + 111*d^3*e^2 + 37*d^2*e^3 + 148*d*e^4 - 65*e^5)*x^3)/(3*e^6) + ((100*d
^4 + 45*d^3*e + 111*d^2*e^2 + 37*d*e^3 + 148*e^4)*x^4)/(4*e^5) - ((100*d^3
+ 45*d^2*e + 111*d*e^2 + 37*e^3)*x^5)/(5*e^4) + ((100*d^2 + 45*d*e + 111*e^
2)*x^6)/(6*e^3) - (5*(20*d + 9*e)*x^7)/(7*e^2) + (25*x^8)/(2*e) + ((5*d^2 -
 2*d*e + 3*e^2)^2*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*Log[d + e*x
])/e^9
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{-100d^7-45d^6e-111d^5e^2-37d^4e^3-148d^3e^4+65d^2e^5-107de^6+33e^7}{e^8}\right. \\ &\quad +\frac{(100d^6+45d^5e+111d^4e^2+37d^3e^3+148d^2e^4-65de^5+107e^6)\,x}{e^7} \\ &\quad +\frac{(-100d^5-45d^4e-111d^3e^2-37d^2e^3-148de^4+65e^5)\,x^2}{e^6} \\ &\quad +\frac{(100d^4+45d^3e+111d^2e^2+37de^3+148e^4)\,x^3}{e^5} \\ &\quad -\frac{(100d^3+45d^2e+111de^2+37e^3)\,x^4}{e^4}+\frac{(100d^2+45de+111e^2)\,x^5}{e^3} \\ &\quad -\frac{5(20d+9e)x^6}{e^2}+\frac{100x^7}{e} \\ &\quad \left.+\frac{(5d^2-2de+3e^2)^2\,(4d^4+5d^3e+3d^2e^2-de^3+2e^4)}{e^8(d+ex)}\right)dx \\ &= -\frac{(100d^7+45d^6e+111d^5e^2+37d^4e^3+148d^3e^4-65d^2e^5+107de^6-33e^7)\,x}{e^8} \\ &\quad +\frac{(100d^6+45d^5e+111d^4e^2+37d^3e^3+148d^2e^4-65de^5+107e^6)\,x^2}{2e^7} \\ &\quad -\frac{(100d^5+45d^4e+111d^3e^2+37d^2e^3+148de^4-65e^5)\,x^3}{3e^6} \\ &\quad +\frac{(100d^4+45d^3e+111d^2e^2+37de^3+148e^4)\,x^4}{4e^5} \\ &\quad -\frac{(100d^3+45d^2e+111de^2+37e^3)\,x^5}{5e^4} \\ &\quad +\frac{(100d^2+45de+111e^2)\,x^6}{6e^3}-\frac{5(20d+9e)x^7}{7e^2}+\frac{25x^8}{2e} \\ &\quad +\frac{(5d^2-2de+3e^2)^2\,(4d^4+5d^3e+3d^2e^2-de^3+2e^4)\log(d+ex)}{e^9}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.74

$$\begin{aligned}&\int \frac{(3+2x+5x^2)^2\,(2+x+3x^2-5x^3+4x^4)}{d+ex}\,dx \\ &= \frac{x(-42000d^7+2100d^6e(-9+10x)-70d^5e^2(666-135x+200x^2)+210d^4e^3(-74+111x-30x^2+50x^3)}{} \\ &\quad +\frac{(5d^2-2de+3e^2)^2\,(4d^4+5d^3e+3d^2e^2-de^3+2e^4)\log(d+ex)}{e^9}\end{aligned}$$

```
[In] Integrate[((3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x),x
]
```

```
[Out] (x*(-42000*d^7 + 2100*d^6*e*(-9 + 10*x) - 70*d^5*e^2*(666 - 135*x + 200*x^2
) + 210*d^4*e^3*(-74 + 111*x - 30*x^2 + 50*x^3) - 105*d^3*e^4*(592 - 74*x +
 148*x^2 - 45*x^3 + 80*x^4) + 35*d^2*e^5*(780 + 888*x - 148*x^2 + 333*x^3 -
 108*x^4 + 200*x^5) - d*e^6*(44940 + 13650*x + 20720*x^2 - 3885*x^3 + 9324*
x^4 - 3150*x^5 + 6000*x^6) + 2*e^7*(6930 + 11235*x + 4550*x^2 + 7770*x^3 -
1554*x^4 + 3885*x^5 - 1350*x^6 + 2625*x^7)))/(420*e^8) + ((5*d^2 - 2*d*e +
3*e^2)^2*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*Log[d + e*x])/e^9
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.01

method	result
norman	$\frac{25x^8}{2e}-\frac{5(20d+9e)x^7}{7e^2}+\frac{(100d^2+45de+111e^2)x^6}{6e^3}-\frac{(100d^3+45d^2e+111d\,e^2+37e^3)x^5}{5e^4}+\frac{(100d^4+45d^3e+111d^2e^2+37d}{4e^5}$
default	$-\frac{-\frac{37}{2}e^7x^6+\frac{45}{7}e^7x^7+37d^3x^3e^4+45d^6ex-\frac{45}{2}d^5e^2x^2+15d^4e^3x^3-\frac{45}{4}d^3e^4x^4+9d^2e^5x^5-\frac{15}{2}d\,e^6x^6-\frac{25}{2}e^7x^8+107d\,e^6x-65d^2e^5}{}$
parallelrisch	$\frac{42000\ln(ex+d)d^8+7560\ln(ex+d)e^8+7770x^6e^8-3108x^5e^8+15540x^4e^8+9100x^3e^8+22470x^2e^8+13860x\,e^8+11655x^4d^2e^6+21}{}$
risch	$\frac{18\ln(ex+d)}{e}-\frac{37d^3x^3}{e^4}-\frac{45d^6x}{e^7}+\frac{45d^5x^2}{2e^6}-\frac{15d^4x^3}{e^5}+\frac{45d^3x^4}{4e^4}-\frac{9d^2x^5}{e^3}+\frac{15d\,x^6}{2e^2}-\frac{107dx}{e^2}+\frac{65d^2x}{e^3}-\frac{148d^3x}{e^4}$

```
[In] int((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d),x,method=_RETURNVERBOSE
)
```

```
[Out] 25/2*x^8/e-5/7*(20*d+9*e)*x^7/e^2+1/6*(100*d^2+45*d*e+111*e^2)*x^6/e^3-1/5*
(100*d^3+45*d^2*e+111*d*e^2+37*e^3)*x^5/e^4+1/4*(100*d^4+45*d^3*e+111*d^2*e
^2+37*d*e^3+148*e^4)*x^4/e^5-1/3*(100*d^5+45*d^4*e+111*d^3*e^2+37*d^2*e^3+1
48*d*e^4-65*e^5)*x^3/e^6+1/2*(100*d^6+45*d^5*e+111*d^4*e^2+37*d^3*e^3+148*d
^2*e^4-65*d*e^5+107*e^6)*x^2/e^7-(100*d^7+45*d^6*e+111*d^5*e^2+37*d^4*e^3+1
48*d^3*e^4-65*d^2*e^5+107*d*e^6-33*e^7)*x/e^8+(100*d^8+45*d^7*e+111*d^6*e^2
+37*d^5*e^3+148*d^4*e^4-65*d^3*e^5+107*d^2*e^6-33*d*e^7+18*e^8)/e^9*ln(e*x+
d)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.05

$$\int \frac{(3+2x+5x^2)^2\,(2+x+3x^2-5x^3+4x^4)}{d+ex}\,dx$$
$$=\frac{5250\,e^8x^8-300\,(20\,de^7+9\,e^8)x^7+70\,(100\,d^2e^6+45\,de^7+111\,e^8)x^6-84\,(100\,d^3e^5+45\,d^2e^6+111\,de^7}{}$$

```
[In] integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d),x, algorithm="fri
cas")
```

```
[Out] 1/420*(5250*e^8*x^8 - 300*(20*d*e^7 + 9*e^8)*x^7 + 70*(100*d^2*e^6 + 45*d*e
^7 + 111*e^8)*x^6 - 84*(100*d^3*e^5 + 45*d^2*e^6 + 111*d*e^7 + 37*e^8)*x^5
+ 105*(100*d^4*e^4 + 45*d^3*e^5 + 111*d^2*e^6 + 37*d*e^7 + 148*e^8)*x^4 - 1
40*(100*d^5*e^3 + 45*d^4*e^4 + 111*d^3*e^5 + 37*d^2*e^6 + 148*d*e^7 - 65*e^
8)*x^3 + 210*(100*d^6*e^2 + 45*d^5*e^3 + 111*d^4*e^4 + 37*d^3*e^5 + 148*d^2
*e^6 - 65*d*e^7 + 107*e^8)*x^2 - 420*(100*d^7*e + 45*d^6*e^2 + 111*d^5*e^3
+ 37*d^4*e^4 + 148*d^3*e^5 - 65*d^2*e^6 + 107*d*e^7 - 33*e^8)*x + 420*(100*
d^8 + 45*d^7*e + 111*d^6*e^2 + 37*d^5*e^3 + 148*d^4*e^4 - 65*d^3*e^5 + 107*
d^2*e^6 - 33*d*e^7 + 18*e^8)*log(e*x + d))/e^9
```

Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.06

$$
\begin{aligned}
&\int \frac{(3+2x+5x^2)^2\,(2+x+3x^2-5x^3+4x^4)}{d+ex}\,dx \\
&= x^7\left(-\frac{100d}{7e^2}-\frac{45}{7e}\right)+x^6\cdot\left(\frac{50d^2}{3e^3}+\frac{15d}{2e^2}+\frac{37}{2e}\right)+x^5\left(-\frac{20d^3}{e^4}-\frac{9d^2}{e^3}-\frac{111d}{5e^2}-\frac{37}{5e}\right)+x^4 \\
&\cdot\left(\frac{25d^4}{e^5}+\frac{45d^3}{4e^4}+\frac{111d^2}{4e^3}+\frac{37d}{4e^2}+\frac{37}{e}\right)+x^3\left(-\frac{100d^5}{3e^6}-\frac{15d^4}{e^5}-\frac{37d^3}{e^4}-\frac{37d^2}{3e^3}-\frac{148d}{3e^2}+\frac{65}{3e}\right) \\
&+x^2\cdot\left(\frac{50d^6}{e^7}+\frac{45d^5}{2e^6}+\frac{111d^4}{2e^5}+\frac{37d^3}{2e^4}+\frac{74d^2}{e^3}-\frac{65d}{2e^2}+\frac{107}{2e}\right) \\
&+x\left(-\frac{100d^7}{e^8}-\frac{45d^6}{e^7}-\frac{111d^5}{e^6}-\frac{37d^4}{e^5}-\frac{148d^3}{e^4}+\frac{65d^2}{e^3}-\frac{107d}{e^2}+\frac{33}{e}\right) \\
&+\frac{25x^8}{2e}+\frac{(5d^2-2de+3e^2)^2\cdot(4d^4+5d^3e+3d^2e^2-de^3+2e^4)\log(d+ex)}{e^9}
\end{aligned}
$$

```
[In] integrate((5*x**2+2*x+3)**2*(4*x**4-5*x**3+3*x**2+x+2)/(e*x+d),x)
```

```
[Out] x**7*(-100*d/(7*e**2) - 45/(7*e)) + x**6*(50*d**2/(3*e**3) + 15*d/(2*e**2)
+ 37/(2*e)) + x**5*(-20*d**3/e**4 - 9*d**2/e**3 - 111*d/(5*e**2) - 37/(5*e)
) + x**4*(25*d**4/e**5 + 45*d**3/(4*e**4) + 111*d**2/(4*e**3) + 37*d/(4*e**
2) + 37/e) + x**3*(-100*d**5/(3*e**6) - 15*d**4/e**5 - 37*d**3/e**4 - 37*d*
*2/(3*e**3) - 148*d/(3*e**2) + 65/(3*e)) + x**2*(50*d**6/e**7 + 45*d**5/(2*
e**6) + 111*d**4/(2*e**5) + 37*d**3/(2*e**4) + 74*d**2/e**3 - 65*d/(2*e**2)
 + 107/(2*e)) + x*(-100*d**7/e**8 - 45*d**6/e**7 - 111*d**5/e**6 - 37*d**4/
e**5 - 148*d**3/e**4 + 65*d**2/e**3 - 107*d/e**2 + 33/e) + 25*x**8/(2*e) +
(5*d**2 - 2*d*e + 3*e**2)**2*(4*d**4 + 5*d**3*e + 3*d**2*e**2 - d*e**3 + 2*
e**4)*log(d + e*x)/e**9
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.04

$$\int \frac{(3+2x+5x^2)^2\,(2+x+3x^2-5x^3+4x^4)}{d+ex}\,dx$$

$$= \frac{5250\,e^7x^8 - 300\,(20\,de^6+9\,e^7)x^7 + 70\,(100\,d^2e^5+45\,de^6+111\,e^7)x^6 - 84\,(100\,d^3e^4+45\,d^2e^5+111\,de^6 -}{}$$

$$+ \frac{(100\,d^8+45\,d^7e+111\,d^6e^2+37\,d^5e^3+148\,d^4e^4-65\,d^3e^5+107\,d^2e^6-33\,de^7+18\,e^8)\log{(ex+d)}}{e^9}$$

```
[In] integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d),x, algorithm="max
ima")
```

```
[Out] 1/420*(5250*e^7*x^8 - 300*(20*d*e^6 + 9*e^7)*x^7 + 70*(100*d^2*e^5 + 45*d*e
^6 + 111*e^7)*x^6 - 84*(100*d^3*e^4 + 45*d^2*e^5 + 111*d*e^6 + 37*e^7)*x^5
+ 105*(100*d^4*e^3 + 45*d^3*e^4 + 111*d^2*e^5 + 37*d*e^6 + 148*e^7)*x^4 - 1
40*(100*d^5*e^2 + 45*d^4*e^3 + 111*d^3*e^4 + 37*d^2*e^5 + 148*d*e^6 - 65*e^
7)*x^3 + 210*(100*d^6*e + 45*d^5*e^2 + 111*d^4*e^3 + 37*d^3*e^4 + 148*d^2*e
^5 - 65*d*e^6 + 107*e^7)*x^2 - 420*(100*d^7 + 45*d^6*e + 111*d^5*e^2 + 37*d
^4*e^3 + 148*d^3*e^4 - 65*d^2*e^5 + 107*d*e^6 - 33*e^7)*x)/e^8 + (100*d^8 +
 45*d^7*e + 111*d^6*e^2 + 37*d^5*e^3 + 148*d^4*e^4 - 65*d^3*e^5 + 107*d^2*e
^6 - 33*d*e^7 + 18*e^8)*log(e*x + d)/e^9
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.18

$$\int \frac{(3+2x+5x^2)^2\,(2+x+3x^2-5x^3+4x^4)}{d+ex}\,dx$$

$$= \frac{5250\,e^7x^8 - 6000\,de^6x^7 - 2700\,e^7x^7 + 7000\,d^2e^5x^6 + 3150\,de^6x^6 + 7770\,e^7x^6 - 8400\,d^3e^4x^5 - 3780\,d^2e^5x^5}{}$$

$$+ \frac{(100\,d^8+45\,d^7e+111\,d^6e^2+37\,d^5e^3+148\,d^4e^4-65\,d^3e^5+107\,d^2e^6-33\,de^7+18\,e^8)\log{(|ex+d|)}}{e^9}$$

```
[In] integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d),x, algorithm="gia
c")
```

```
[Out] 1/420*(5250*e^7*x^8 - 6000*d*e^6*x^7 - 2700*e^7*x^7 + 7000*d^2*e^5*x^6 + 31
50*d*e^6*x^6 + 7770*e^7*x^6 - 8400*d^3*e^4*x^5 - 3780*d^2*e^5*x^5 - 9324*d*
e^6*x^5 - 3108*e^7*x^5 + 10500*d^4*e^3*x^4 + 4725*d^3*e^4*x^4 + 11655*d^2*e
^5*x^4 + 3885*d*e^6*x^4 + 15540*e^7*x^4 - 14000*d^5*e^2*x^3 - 6300*d^4*e^3*
```

```
x^3 - 15540*d^3*e^4*x^3 - 5180*d^2*e^5*x^3 - 20720*d*e^6*x^3 + 9100*e^7*x^3
 + 21000*d^6*e*x^2 + 9450*d^5*e^2*x^2 + 23310*d^4*e^3*x^2 + 7770*d^3*e^4*x^
2 + 31080*d^2*e^5*x^2 - 13650*d*e^6*x^2 + 22470*e^7*x^2 - 42000*d^7*x - 189
00*d^6*e*x - 46620*d^5*e^2*x - 15540*d^4*e^3*x - 62160*d^3*e^4*x + 27300*d^
2*e^5*x - 44940*d*e^6*x + 13860*e^7*x)/e^8 + (100*d^8 + 45*d^7*e + 111*d^6*
e^2 + 37*d^5*e^3 + 148*d^4*e^4 - 65*d^3*e^5 + 107*d^2*e^6 - 33*d*e^7 + 18*e
^8)*log(abs(e*x + d))/e^9
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 434, normalized size of antiderivative = 1.23

$$\int \frac{(3+2x+5x^2)^2 (2+x+3x^2-5x^3+4x^4)}{d+ex}\,dx$$

$$= x\left(\frac{33}{e}-\frac{d\left(\frac{107}{e}-\frac{d\left(\frac{65}{e}-\frac{d\left(\frac{148}{e}+\frac{d\left(\frac{37}{e}+\frac{d\left(\frac{111}{e}+\frac{d\left(\frac{100\,d}{e^2}+\frac{45}{e}\right)}{e}\right)}{e}\right)}{e}\right)}{e}\right)}{e}\right)}{e}\right.$$

```
[In] int(((2*x + 5*x^2 + 3)^2*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(d + e*x),x)
```

```
[Out] x*(33/e - (d*(107/e - (d*(65/e - (d*(148/e + (d*(37/e + (d*(111/e + (d*((10
0*d)/e^2 + 45/e))/e))/e))/e))/e))/e))/e) - x^7*((100*d)/(7*e^2) + 45/(7*e))
 + x^6*(37/(2*e) + (d*((100*d)/e^2 + 45/e))/(6*e)) - x^5*(37/(5*e) + (d*(11
1/e + (d*((100*d)/e^2 + 45/e))/e))/(5*e)) + x^4*(37/e + (d*(37/e + (d*(111/
e + (d*((100*d)/e^2 + 45/e))/e))/e))/(4*e)) + x^3*(65/(3*e) - (d*(148/e + (
d*(37/e + (d*(111/e + (d*((100*d)/e^2 + 45/e))/e))/e))/e))/(3*e)) + x^2*(10
7/(2*e) - (d*(65/e - (d*(148/e + (d*(37/e + (d*(111/e + (d*((100*d)/e^2 + 4
5/e))/e))/e))/e))/e))/(2*e)) + (25*x^8)/(2*e) + (log(d + e*x)*(45*d^7*e - 3
3*d*e^7 + 100*d^8 + 18*e^8 + 107*d^2*e^6 - 65*d^3*e^5 + 148*d^4*e^4 + 37*d^
5*e^3 + 111*d^6*e^2))/e^9
```

3.301 $\int \frac{(3+2x+5x^2)^2(2+x+3x^2-5x^3+4x^4)}{(d+ex)^2}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 353

$$
\begin{aligned}
&\int \frac{(3+2x+5x^2)^2(2+x+3x^2-5x^3+4x^4)}{(d+ex)^2}\,dx \\
&= \frac{(700d^6+270d^5e+555d^4e^2+148d^3e^3+444d^2e^4-130de^5+107e^6)\,x}{e^8} \\
&\quad - \frac{(600d^5+225d^4e+444d^3e^2+111d^2e^3+296de^4-65e^5)\,x^2}{2e^7} \\
&\quad + \frac{(500d^4+180d^3e+333d^2e^2+74de^3+148e^4)\,x^3}{3e^6} \\
&\quad - \frac{(400d^3+135d^2e+222de^2+37e^3)\,x^4}{4e^5} + \frac{3(100d^2+30de+37e^2)\,x^5}{5e^4} \\
&\quad - \frac{5(40d+9e)x^6}{6e^3} + \frac{100x^7}{7e^2} - \frac{(5d^2-2de+3e^2)^2\,(4d^4+5d^3e+3d^2e^2-de^3+2e^4)}{e^9(d+ex)} \\
&\quad - \frac{(5d^2-2de+3e^2)\,(160d^5+127d^4e+88d^3e^2-4d^2e^3+64de^4-11e^5)\log(d+ex)}{e^9}
\end{aligned}
$$

```
[Out] (700*d^6+270*d^5*e+555*d^4*e^2+148*d^3*e^3+444*d^2*e^4-130*d*e^5+107*e^6)*x
/e^8-1/2*(600*d^5+225*d^4*e+444*d^3*e^2+111*d^2*e^3+296*d*e^4-65*e^5)*x^2/e
^7+1/3*(500*d^4+180*d^3*e+333*d^2*e^2+74*d*e^3+148*e^4)*x^3/e^6-1/4*(400*d^
3+135*d^2*e+222*d*e^2+37*e^3)*x^4/e^5+3/5*(100*d^2+30*d*e+37*e^2)*x^5/e^4-5
/6*(40*d+9*e)*x^6/e^3+100/7*x^7/e^2-(5*d^2-2*d*e+3*e^2)^2*(4*d^4+5*d^3*e+3*
d^2*e^2-d*e^3+2*e^4)/e^9/(e*x+d)-(5*d^2-2*d*e+3*e^2)*(160*d^5+127*d^4*e+88*
d^3*e^2-4*d^2*e^3+64*d*e^4-11*e^5)*ln(e*x+d)/e^9
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$, Rules used = {1642}

$$\begin{aligned}
&\int \frac{(3+2x+5x^2)^2\,(2+x+3x^2-5x^3+4x^4)}{(d+ex)^2}\,dx \\
&= \frac{3x^5(100d^2+30de+37e^2)}{5e^4} - \frac{x^4(400d^3+135d^2e+222de^2+37e^3)}{4e^5} \\
&\quad - \frac{(5d^2-2de+3e^2)^2\,(4d^4+5d^3e+3d^2e^2-de^3+2e^4)}{e^9(d+ex)} \\
&\quad + \frac{x^3(500d^4+180d^3e+333d^2e^2+74de^3+148e^4)}{3e^6} \\
&\quad - \frac{(5d^2-2de+3e^2)\,(160d^5+127d^4e+88d^3e^2-4d^2e^3+64de^4-11e^5)\log(d+ex)}{e^9} \\
&\quad - \frac{x^2(600d^5+225d^4e+444d^3e^2+111d^2e^3+296de^4-65e^5)}{2e^7} \\
&\quad + \frac{x(700d^6+270d^5e+555d^4e^2+148d^3e^3+444d^2e^4-130de^5+107e^6)}{e^8} \\
&\quad - \frac{5x^6(40d+9e)}{6e^3} + \frac{100x^7}{7e^2}
\end{aligned}$$

```
[In] Int[((3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x)^2,x]
```

```
[Out] ((700*d^6 + 270*d^5*e + 555*d^4*e^2 + 148*d^3*e^3 + 444*d^2*e^4 - 130*d*e^5
 + 107*e^6)*x)/e^8 - ((600*d^5 + 225*d^4*e + 444*d^3*e^2 + 111*d^2*e^3 + 29
6*d*e^4 - 65*e^5)*x^2)/(2*e^7) + ((500*d^4 + 180*d^3*e + 333*d^2*e^2 + 74*d
*e^3 + 148*e^4)*x^3)/(3*e^6) - ((400*d^3 + 135*d^2*e + 222*d*e^2 + 37*e^3)*
x^4)/(4*e^5) + (3*(100*d^2 + 30*d*e + 37*e^2)*x^5)/(5*e^4) - (5*(40*d + 9*e
)*x^6)/(6*e^3) + (100*x^7)/(7*e^2) - ((5*d^2 - 2*d*e + 3*e^2)^2*(4*d^4 + 5*
d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4))/(e^9*(d + e*x)) - ((5*d^2 - 2*d*e + 3*e
^2)*(160*d^5 + 127*d^4*e + 88*d^3*e^2 - 4*d^2*e^3 + 64*d*e^4 - 11*e^5)*Log[
d + e*x])/e^9
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
&\text{integral}\\
&=\int\left(\frac{700d^6+270d^5e+555d^4e^2+148d^3e^3+444d^2e^4-130de^5+107e^6}{e^8}\right.\\
&\quad+\frac{\left(-600d^5-225d^4e-444d^3e^2-111d^2e^3-296de^4+65e^5\right)x}{e^7}\\
&\quad+\frac{\left(500d^4+180d^3e+333d^2e^2+74de^3+148e^4\right)x^2}{e^6}\\
&\quad-\frac{\left(400d^3+135d^2e+222de^2+37e^3\right)x^3}{e^5}+\frac{3\left(100d^2+30de+37e^2\right)x^4}{e^4}-\frac{5(40d+9e)x^5}{e^3}\\
&\quad+\frac{100x^6}{e^2}+\frac{\left(5d^2-2de+3e^2\right)^2\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)}{e^8(d+ex)^2}\\
&\quad\left.+\frac{-800d^7-315d^6e-666d^5e^2-185d^4e^3-592d^3e^4+195d^2e^5-214de^6+33e^7}{e^8(d+ex)}\right)dx\\
&=\frac{\left(700d^6+270d^5e+555d^4e^2+148d^3e^3+444d^2e^4-130de^5+107e^6\right)x}{e^8}\\
&\quad-\frac{\left(600d^5+225d^4e+444d^3e^2+111d^2e^3+296de^4-65e^5\right)x^2}{2e^7}\\
&\quad+\frac{\left(500d^4+180d^3e+333d^2e^2+74de^3+148e^4\right)x^3}{3e^6}\\
&\quad-\frac{\left(400d^3+135d^2e+222de^2+37e^3\right)x^4}{4e^5}+\frac{3\left(100d^2+30de+37e^2\right)x^5}{5e^4}\\
&\quad-\frac{5(40d+9e)x^6}{6e^3}+\frac{100x^7}{7e^2}-\frac{\left(5d^2-2de+3e^2\right)^2\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)}{e^9(d+ex)}\\
&\quad-\frac{\left(5d^2-2de+3e^2\right)\left(160d^5+127d^4e+88d^3e^2-4d^2e^3+64de^4-11e^5\right)\log(d+ex)}{e^9}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 342, normalized size of antiderivative = 0.97

$$\int\frac{\left(3+2x+5x^2\right)^2\left(2+x+3x^2-5x^3+4x^4\right)}{(d+ex)^2}\,dx$$

$$=\frac{420e\left(700d^6+270d^5e+555d^4e^2+148d^3e^3+444d^2e^4-130de^5+107e^6\right)x-210e^2(600d^5+225d^4e+444}{}$$

```
[In] Integrate[((3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x)^2
,x]
```

```
[Out] (420*e*(700*d^6 + 270*d^5*e + 555*d^4*e^2 + 148*d^3*e^3 + 444*d^2*e^4 - 130
*d*e^5 + 107*e^6)*x - 210*e^2*(600*d^5 + 225*d^4*e + 444*d^3*e^2 + 111*d^2*
```

```
e^3 + 296*d*e^4 - 65*e^5)*x^2 + 140*e^3*(500*d^4 + 180*d^3*e + 333*d^2*e^2
+ 74*d*e^3 + 148*e^4)*x^3 - 105*e^4*(400*d^3 + 135*d^2*e + 222*d*e^2 + 37*e
^3)*x^4 + 252*e^5*(100*d^2 + 30*d*e + 37*e^2)*x^5 - 350*e^6*(40*d + 9*e)*x^
6 + 6000*e^7*x^7 - (420*(5*d^2 - 2*d*e + 3*e^2)^2*(4*d^4 + 5*d^3*e + 3*d^2*
e^2 - d*e^3 + 2*e^4))/(d + e*x) - 420*(800*d^7 + 315*d^6*e + 666*d^5*e^2 +
185*d^4*e^3 + 592*d^3*e^4 - 195*d^2*e^5 + 214*d*e^6 - 33*e^7)*Log[d + e*x])
/(420*e^9)
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.04

method	result
norman	$\frac{\frac{\left(800d^8+315d^7e+666d^6e^2+185d^5e^3+592d^4e^4-195d^3e^5+214d^2e^6-33d\,e^7+18e^8\right)x}{e^8d}+\frac{100x^8}{7e}-\frac{5(160d+63e)x^7}{42e^2}+\frac{\left(800d^2+315de+666e^2\right)x^6}{30e^3}-}{}$
default	$\frac{\frac{111}{5}e^6x^5-\frac{15}{2}e^6x^6+60d^3x^3e^3+270d^5ex-\frac{225}{2}d^4e^2x^2-\frac{135}{4}d^2e^4x^4+18d\,e^5x^5+\frac{100}{7}e^6x^7-\frac{37}{4}x^4e^6+\frac{148}{3}x^3e^6+\frac{65}{2}x^2e^6+107x\,e^6+70}{}$
risch	$-\frac{111d^6}{e^7(ex+d)}-\frac{37d^5}{e^6(ex+d)}-\frac{148d^4}{e^5(ex+d)}+\frac{65d^3}{e^4(ex+d)}-\frac{107d^2}{e^3(ex+d)}+\frac{33d}{e^2(ex+d)}-\frac{15x^6}{2e^2}+\frac{60d^3x^3}{e^5}+\frac{270d^5x}{e^7}-\frac{225d^4}{2e^6}$
parallelrisch	$-\frac{336000\ln(ex+d)d^8-9324x^6e^8+3885x^5e^8-20720x^4e^8-13650x^3e^8-44940x^2e^8-13860d\,e^7-81900d^3e^5+89880d^2e^6+77700d^5}{}$

```
[In] int((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2,x,method=_RETURNVERBO
SE)
```

```
[Out] ((800*d^8+315*d^7*e+666*d^6*e^2+185*d^5*e^3+592*d^4*e^4-195*d^3*e^5+214*d^2
*e^6-33*d*e^7+18*e^8)/e^8/d*x+100/7*x^8/e-5/42*(160*d+63*e)/e^2*x^7+1/30*(8
00*d^2+315*d*e+666*e^2)/e^3*x^6-1/20*(800*d^3+315*d^2*e+666*d*e^2+185*e^3)/
e^4*x^5+1/12*(800*d^4+315*d^3*e+666*d^2*e^2+185*d*e^3+592*e^4)/e^5*x^4-1/6*
(800*d^5+315*d^4*e+666*d^3*e^2+185*d^2*e^3+592*d*e^4-195*e^5)/e^6*x^3+1/2*(
800*d^6+315*d^5*e+666*d^4*e^2+185*d^3*e^3+592*d^2*e^4-195*d*e^5+214*e^6)/e^
7*x^2)/(e*x+d)-(800*d^7+315*d^6*e+666*d^5*e^2+185*d^4*e^3+592*d^3*e^4-195*d
^2*e^5+214*d*e^6-33*e^7)/e^9*ln(e*x+d)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 490, normalized size of antiderivative = 1.39

$$\int \frac{(3+2x+5x^2)^2\,(2+x+3x^2-5x^3+4x^4)}{(d+ex)^2}\,dx$$
$$=\frac{6000\,e^8x^8-42000\,d^8-18900\,d^7e-46620\,d^6e^2-15540\,d^5e^3-62160\,d^4e^4+27300\,d^3e^5-44940\,d^2e^6+13}{}$$

```
[In] integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2,x, algorithm="f
ricas")
```

```
[Out] 1/420*(6000*e^8*x^8 - 42000*d^8 - 18900*d^7*e - 46620*d^6*e^2 - 15540*d^5*e
^3 - 62160*d^4*e^4 + 27300*d^3*e^5 - 44940*d^2*e^6 + 13860*d*e^7 - 7560*e^8
 - 50*(160*d*e^7 + 63*e^8)*x^7 + 14*(800*d^2*e^6 + 315*d*e^7 + 666*e^8)*x^6
 - 21*(800*d^3*e^5 + 315*d^2*e^6 + 666*d*e^7 + 185*e^8)*x^5 + 35*(800*d^4*e
^4 + 315*d^3*e^5 + 666*d^2*e^6 + 185*d*e^7 + 592*e^8)*x^4 - 70*(800*d^5*e^3
 + 315*d^4*e^4 + 666*d^3*e^5 + 185*d^2*e^6 + 592*d*e^7 - 195*e^8)*x^3 + 210
*(800*d^6*e^2 + 315*d^5*e^3 + 666*d^4*e^4 + 185*d^3*e^5 + 592*d^2*e^6 - 195
*d*e^7 + 214*e^8)*x^2 + 420*(700*d^7*e + 270*d^6*e^2 + 555*d^5*e^3 + 148*d^
4*e^4 + 444*d^3*e^5 - 130*d^2*e^6 + 107*d*e^7)*x - 420*(800*d^8 + 315*d^7*e
 + 666*d^6*e^2 + 185*d^5*e^3 + 592*d^4*e^4 - 195*d^3*e^5 + 214*d^2*e^6 - 33
*d*e^7 + (800*d^7*e + 315*d^6*e^2 + 666*d^5*e^3 + 185*d^4*e^4 + 592*d^3*e^5
 - 195*d^2*e^6 + 214*d*e^7 - 33*e^8)*x)*log(e*x + d))/(e^10*x + d*e^9)
```

Sympy [A] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 393, normalized size of antiderivative = 1.11

$$
\begin{aligned}
&\int \frac{(3+2x+5x^2)^2\,(2+x+3x^2-5x^3+4x^4)}{(d+ex)^2}\,dx \\
&= x^6\left(-\frac{100d}{3e^3}-\frac{15}{2e^2}\right)+x^5\cdot\left(\frac{60d^2}{e^4}+\frac{18d}{e^3}+\frac{111}{5e^2}\right) \\
&\quad+x^4\left(-\frac{100d^3}{e^5}-\frac{135d^2}{4e^4}-\frac{111d}{2e^3}-\frac{37}{4e^2}\right)+x^3\cdot\left(\frac{500d^4}{3e^6}+\frac{60d^3}{e^5}+\frac{111d^2}{e^4}+\frac{74d}{3e^3}+\frac{148}{3e^2}\right) \\
&\quad+x^2\left(-\frac{300d^5}{e^7}-\frac{225d^4}{2e^6}-\frac{222d^3}{e^5}-\frac{111d^2}{2e^4}-\frac{148d}{e^3}+\frac{65}{2e^2}\right) \\
&\quad+x\left(\frac{700d^6}{e^8}+\frac{270d^5}{e^7}+\frac{555d^4}{e^6}+\frac{148d^3}{e^5}+\frac{444d^2}{e^4}-\frac{130d}{e^3}+\frac{107}{e^2}\right) \\
&\quad+\frac{-100d^8-45d^7e-111d^6e^2-37d^5e^3-148d^4e^4+65d^3e^5-107d^2e^6+33de^7-18e^8}{de^9+e^{10}x} \\
&\quad+\frac{100x^7}{7e^2} \\
&\quad-\frac{(5d^2-2de+3e^2)\,(160d^5+127d^4e+88d^3e^2-4d^2e^3+64de^4-11e^5)\log(d+ex)}{e^9}
\end{aligned}
$$

```
[In] integrate((5*x**2+2*x+3)**2*(4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)**2,x)
```

```
[Out] x**6*(-100*d/(3*e**3) - 15/(2*e**2)) + x**5*(60*d**2/e**4 + 18*d/e**3 + 111
/(5*e**2)) + x**4*(-100*d**3/e**5 - 135*d**2/(4*e**4) - 111*d/(2*e**3) - 37
/(4*e**2)) + x**3*(500*d**4/(3*e**6) + 60*d**3/e**5 + 111*d**2/e**4 + 74*d/
(3*e**3) + 148/(3*e**2)) + x**2*(-300*d**5/e**7 - 225*d**4/(2*e**6) - 222*d
**3/e**5 - 111*d**2/(2*e**4) - 148*d/e**3 + 65/(2*e**2)) + x*(700*d**6/e**8
 + 270*d**5/e**7 + 555*d**4/e**6 + 148*d**3/e**5 + 444*d**2/e**4 - 130*d/e*
*3 + 107/e**2) + (-100*d**8 - 45*d**7*e - 111*d**6*e**2 - 37*d**5*e**3 - 14
8*d**4*e**4 + 65*d**3*e**5 - 107*d**2*e**6 + 33*d*e**7 - 18*e**8)/(d*e**9 +
```

```
 e**10*x) + 100*x**7/(7*e**2) - (5*d**2 - 2*d*e + 3*e**2)*(160*d**5 + 127*d
**4*e + 88*d**3*e**2 - 4*d**2*e**3 + 64*d*e**4 - 11*e**5)*log(d + e*x)/e**9
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.05

$$\int \frac{(3+2x+5x^2)^2\,(2+x+3x^2-5x^3+4x^4)}{(d+ex)^2}\,dx$$
$$= -\frac{100\,d^8+45\,d^7e+111\,d^6e^2+37\,d^5e^3+148\,d^4e^4-65\,d^3e^5+107\,d^2e^6-33\,de^7+18\,e^8}{e^{10}x+de^9}$$
$$+\frac{6000\,e^6x^7-350\,(40\,de^5+9\,e^6)x^6+252\,(100\,d^2e^4+30\,de^5+37\,e^6)x^5-105\,(400\,d^3e^3+135\,d^2e^4+222\,d}{}$$
$$-\frac{(800\,d^7+315\,d^6e+666\,d^5e^2+185\,d^4e^3+592\,d^3e^4-195\,d^2e^5+214\,de^6-33\,e^7)\log{(ex+d)}}{e^9}$$

```
[In] integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2,x, algorithm="m
axima")
```

```
[Out] -(100*d^8 + 45*d^7*e + 111*d^6*e^2 + 37*d^5*e^3 + 148*d^4*e^4 - 65*d^3*e^5
+ 107*d^2*e^6 - 33*d*e^7 + 18*e^8)/(e^10*x + d*e^9) + 1/420*(6000*e^6*x^7 -
 350*(40*d*e^5 + 9*e^6)*x^6 + 252*(100*d^2*e^4 + 30*d*e^5 + 37*e^6)*x^5 - 1
05*(400*d^3*e^3 + 135*d^2*e^4 + 222*d*e^5 + 37*e^6)*x^4 + 140*(500*d^4*e^2
+ 180*d^3*e^3 + 333*d^2*e^4 + 74*d*e^5 + 148*e^6)*x^3 - 210*(600*d^5*e + 22
5*d^4*e^2 + 444*d^3*e^3 + 111*d^2*e^4 + 296*d*e^5 - 65*e^6)*x^2 + 420*(700*
d^6 + 270*d^5*e + 555*d^4*e^2 + 148*d^3*e^3 + 444*d^2*e^4 - 130*d*e^5 + 107
*e^6)*x)/e^8 - (800*d^7 + 315*d^6*e + 666*d^5*e^2 + 185*d^4*e^3 + 592*d^3*e
^4 - 195*d^2*e^5 + 214*d*e^6 - 33*e^7)*log(e*x + d)/e^9
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 491, normalized size of antiderivative = 1.39

$$\int \frac{(3+2x+5x^2)^2\,(2+x+3x^2-5x^3+4x^4)}{(d+ex)^2}\,dx =$$
$$-\frac{(ex+d)^7\left(\frac{350\,(160\,de+9\,e^2)}{(ex+d)e}-\frac{84\,(2800\,d^2e^2+315\,de^3+111\,e^4)}{(ex+d)^2e^2}+\frac{105\,(5600\,d^3e^3+945\,d^2e^4+666\,de^5+37\,e^6)}{(ex+d)^3e^3}-\frac{140\,(7000\,d^4e^4+1575}{}\right.}{}$$
$$+\frac{(800\,d^7+315\,d^6e+666\,d^5e^2+185\,d^4e^3+592\,d^3e^4-195\,d^2e^5+214\,de^6-33\,e^7)\log\left(\frac{|ex+d|}{(ex+d)^2|e|}\right)}{e^9}$$
$$-\frac{\frac{100\,d^8e^7}{ex+d}+\frac{45\,d^7e^8}{ex+d}+\frac{111\,d^6e^9}{ex+d}+\frac{37\,d^5e^{10}}{ex+d}+\frac{148\,d^4e^{11}}{ex+d}-\frac{65\,d^3e^{12}}{ex+d}+\frac{107\,d^2e^{13}}{ex+d}-\frac{33\,de^{14}}{ex+d}+\frac{18\,e^{15}}{ex+d}}{e^{16}}$$

```
[In] integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2,x, algorithm="g
iac")
```

```
[Out] -1/420*(e*x + d)^7*(350*(160*d*e + 9*e^2)/((e*x + d)*e) - 84*(2800*d^2*e^2
+ 315*d*e^3 + 111*e^4)/((e*x + d)^2*e^2) + 105*(5600*d^3*e^3 + 945*d^2*e^4
+ 666*d*e^5 + 37*e^6)/((e*x + d)^3*e^3) - 140*(7000*d^4*e^4 + 1575*d^3*e^5
+ 1665*d^2*e^6 + 185*d*e^7 + 148*e^8)/((e*x + d)^4*e^4) + 210*(5600*d^5*e^5
 + 1575*d^4*e^6 + 2220*d^3*e^7 + 370*d^2*e^8 + 592*d*e^9 - 65*e^10)/((e*x +
 d)^5*e^5) - 420*(2800*d^6*e^6 + 945*d^5*e^7 + 1665*d^4*e^8 + 370*d^3*e^9 +
 888*d^2*e^10 - 195*d*e^11 + 107*e^12)/((e*x + d)^6*e^6) - 6000)/e^9 + (800
*d^7 + 315*d^6*e + 666*d^5*e^2 + 185*d^4*e^3 + 592*d^3*e^4 - 195*d^2*e^5 +
214*d*e^6 - 33*e^7)*log(abs(e*x + d)/((e*x + d)^2*abs(e)))/e^9 - (100*d^8*e
^7/(e*x + d) + 45*d^7*e^8/(e*x + d) + 111*d^6*e^9/(e*x + d) + 37*d^5*e^10/(
e*x + d) + 148*d^4*e^11/(e*x + d) - 65*d^3*e^12/(e*x + d) + 107*d^2*e^13/(e
*x + d) - 33*d*e^14/(e*x + d) + 18*e^15/(e*x + d))/e^16
```

Mupad [B] (verification not implemented)

Time = 13.41 (sec) , antiderivative size = 939, normalized size of antiderivative = 2.66

$$\int \frac{(3+2x+5x^2)^2\,(2+x+3x^2-5x^3+4x^4)}{(d+ex)^2}\,dx = x^2\left(\frac{65}{2\,e^2} - \frac{d\left(\frac{148}{e^2}+\frac{2\,d\left(\frac{37}{e^2}+\frac{2\,d\left(\frac{111}{e^2}-\frac{100\,d^2}{e^4}+\frac{2\,d\left(\frac{200\,d}{e^3}+\frac{45}{e^2}\right)}{e}\right)}{e}-\frac{d^2\left(\frac{200\,d}{e^3}+\frac{45}{e^2}\right)}{e^2}\right)}{e}-\frac{d^2\left(\frac{111}{e^2}-\frac{100\,d^2}{e^4}+\frac{2\,d\left(\frac{200\,d}{e^3}+\frac{45}{e^2}\right)}{e}\right)}{e^2}\right)}{e} + \frac{d^2\left(\frac{37}{e^2}+\frac{2\,d\left(\frac{111}{e^2}-\frac{100\,d^2}{e^4}+\frac{2\,d\left(\frac{200\,d}{e^3}+\frac{45}{e^2}\right)}{e}\right)}{e}-\frac{d^2\left(\frac{200\,d}{e^3}+\frac{45}{e^2}\right)}{e^2}\right)}{2\,e^2}\right)$$

$$+\,x^3\left(148 \quad 2\,d\left(\frac{37}{e^2}+\frac{2\,d\left(\frac{111}{e^2}-\frac{100\,d^2}{e^4}+\frac{2\,d\left(\frac{200\,d}{e^3}+\frac{45}{e^2}\right)}{e}\right)}{e}-\frac{d^2\left(\frac{200\,d}{e^3}+\frac{45}{e^2}\right)}{e^2}\right)\right.$$

```
[In] int(((2*x + 5*x^2 + 3)^2*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(d + e*x)^2,x)
```

```
[Out] x^2*(65/(2*e^2) - (d*(148/e^2 + (2*d*(37/e^2 + (2*d*(111/e^2 - (100*d^2)/e^
4 + (2*d*((200*d)/e^3 + 45/e^2))/e))/e - (d^2*((200*d)/e^3 + 45/e^2))/e^2))
/e - (d^2*(111/e^2 - (100*d^2)/e^4 + (2*d*((200*d)/e^3 + 45/e^2))/e))/e^2))
/e + (d^2*(37/e^2 + (2*d*(111/e^2 - (100*d^2)/e^4 + (2*d*((200*d)/e^3 + 45/
e^2))/e))/e - (d^2*((200*d)/e^3 + 45/e^2))/e^2))/(2*e^2)) + x^3*(148/(3*e^2
) + (2*d*(37/e^2 + (2*d*(111/e^2 - (100*d^2)/e^4 + (2*d*((200*d)/e^3 + 45/e
^2))/e))/e - (d^2*((200*d)/e^3 + 45/e^2))/e^2))/(3*e) - (d^2*(111/e^2 - (10
0*d^2)/e^4 + (2*d*((200*d)/e^3 + 45/e^2))/e))/(3*e^2)) - x^4*(37/(4*e^2) +
(d*(111/e^2 - (100*d^2)/e^4 + (2*d*((200*d)/e^3 + 45/e^2))/e))/(2*e) - (d^2
*((200*d)/e^3 + 45/e^2))/(4*e^2)) + x^5*(111/(5*e^2) - (20*d^2)/e^4 + (2*d*
((200*d)/e^3 + 45/e^2))/(5*e)) - x^6*((100*d)/(3*e^3) + 15/(2*e^2)) - x*((2
*d*(65/e^2 - (2*d*(148/e^2 + (2*d*(37/e^2 + (2*d*(111/e^2 - (100*d^2)/e^4 +
 (2*d*((200*d)/e^3 + 45/e^2))/e))/e - (d^2*((200*d)/e^3 + 45/e^2))/e^2))/e
- (d^2*(111/e^2 - (100*d^2)/e^4 + (2*d*((200*d)/e^3 + 45/e^2))/e))/e^2))/e
+ (d^2*(37/e^2 + (2*d*(111/e^2 - (100*d^2)/e^4 + (2*d*((200*d)/e^3 + 45/e^2
))/e))/e - (d^2*((200*d)/e^3 + 45/e^2))/e^2))/e^2))/e - 107/e^2 + (d^2*(148
/e^2 + (2*d*(37/e^2 + (2*d*(111/e^2 - (100*d^2)/e^4 + (2*d*((200*d)/e^3 + 4
5/e^2))/e))/e - (d^2*((200*d)/e^3 + 45/e^2))/e^2))/e - (d^2*(111/e^2 - (100
*d^2)/e^4 + (2*d*((200*d)/e^3 + 45/e^2))/e))/e^2))/e^2) + (100*x^7)/(7*e^2)
 - (45*d^7*e - 33*d*e^7 + 100*d^8 + 18*e^8 + 107*d^2*e^6 - 65*d^3*e^5 + 148
*d^4*e^4 + 37*d^5*e^3 + 111*d^6*e^2)/(e*(d*e^8 + e^9*x)) - (log(d + e*x)*(2
14*d*e^6 + 315*d^6*e + 800*d^7 - 33*e^7 - 195*d^2*e^5 + 592*d^3*e^4 + 185*d
^4*e^3 + 666*d^5*e^2))/e^9
```

3.302 $\int \frac{(3+2x+5x^2)^2(2+x+3x^2-5x^3+4x^4)}{(d+ex)^3}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 354

$$
\begin{aligned}
&\int \frac{(3+2x+5x^2)^2\,(2+x+3x^2-5x^3+4x^4)}{(d+ex)^3}\,dx \\
&= -\frac{(2100d^5+675d^4e+1110d^3e^2+222d^2e^3+444de^4-65e^5)\,x}{e^8} \\
&\quad+\frac{(1500d^4+450d^3e+666d^2e^2+111de^3+148e^4)\,x^2}{2e^7} \\
&\quad-\frac{(1000d^3+270d^2e+333de^2+37e^3)\,x^3}{3e^6}+\frac{3(200d^2+45de+37e^2)\,x^4}{4e^5} \\
&\quad-\frac{3(20d+3e)x^5}{e^4}+\frac{50x^6}{3e^3}-\frac{(5d^2-2de+3e^2)^2\,(4d^4+5d^3e+3d^2e^2-de^3+2e^4)}{2e^9(d+ex)^2} \\
&\quad+\frac{(5d^2-2de+3e^2)\,(160d^5+127d^4e+88d^3e^2-4d^2e^3+64de^4-11e^5)}{e^9(d+ex)} \\
&\quad+\frac{(2800d^6+945d^5e+1665d^4e^2+370d^3e^3+888d^2e^4-195de^5+107e^6)\log(d+ex)}{e^9}
\end{aligned}
$$

```
[Out] -(2100*d^5+675*d^4*e+1110*d^3*e^2+222*d^2*e^3+444*d*e^4-65*e^5)*x/e^8+1/2*(
1500*d^4+450*d^3*e+666*d^2*e^2+111*d*e^3+148*e^4)*x^2/e^7-1/3*(1000*d^3+270
*d^2*e+333*d*e^2+37*e^3)*x^3/e^6+3/4*(200*d^2+45*d*e+37*e^2)*x^4/e^5-3*(20*
d+3*e)*x^5/e^4+50/3*x^6/e^3-1/2*(5*d^2-2*d*e+3*e^2)^2*(4*d^4+5*d^3*e+3*d^2*
e^2-d*e^3+2*e^4)/e^9/(e*x+d)^2+(5*d^2-2*d*e+3*e^2)*(160*d^5+127*d^4*e+88*d^
3*e^2-4*d^2*e^3+64*d*e^4-11*e^5)/e^9/(e*x+d)+(2800*d^6+945*d^5*e+1665*d^4*e
^2+370*d^3*e^3+888*d^2*e^4-195*d*e^5+107*e^6)*ln(e*x+d)/e^9
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {1642}

$$\begin{aligned}
&\int \frac{(3+2x+5x^2)^2(2+x+3x^2-5x^3+4x^4)}{(d+ex)^3}\,dx\\
&\quad=\frac{3x^4(200d^2+45de+37e^2)}{4e^5}-\frac{x^3(1000d^3+270d^2e+333de^2+37e^3)}{3e^6}\\
&\quad-\frac{(5d^2-2de+3e^2)^2(4d^4+5d^3e+3d^2e^2-de^3+2e^4)}{2e^9(d+ex)^2}\\
&\quad+\frac{x^2(1500d^4+450d^3e+666d^2e^2+111de^3+148e^4)}{2e^7}\\
&\quad+\frac{(5d^2-2de+3e^2)(160d^5+127d^4e+88d^3e^2-4d^2e^3+64de^4-11e^5)}{e^9(d+ex)}\\
&\quad-\frac{x(2100d^5+675d^4e+1110d^3e^2+222d^2e^3+444de^4-65e^5)}{e^8}\\
&\quad+\frac{(2800d^6+945d^5e+1665d^4e^2+370d^3e^3+888d^2e^4-195de^5+107e^6)\log(d+ex)}{e^9}\\
&\quad-\frac{3x^5(20d+3e)}{e^4}+\frac{50x^6}{3e^3}
\end{aligned}$$

```
[In] Int[((3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x)^3,x]
```

```
[Out] -(((2100*d^5 + 675*d^4*e + 1110*d^3*e^2 + 222*d^2*e^3 + 444*d*e^4 - 65*e^5)
*x)/e^8) + ((1500*d^4 + 450*d^3*e + 666*d^2*e^2 + 111*d*e^3 + 148*e^4)*x^2)
/(2*e^7) - ((1000*d^3 + 270*d^2*e + 333*d*e^2 + 37*e^3)*x^3)/(3*e^6) + (3*(
200*d^2 + 45*d*e + 37*e^2)*x^4)/(4*e^5) - (3*(20*d + 3*e)*x^5)/e^4 + (50*x^
6)/(3*e^3) - ((5*d^2 - 2*d*e + 3*e^2)^2*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^
3 + 2*e^4))/(2*e^9*(d + e*x)^2) + ((5*d^2 - 2*d*e + 3*e^2)*(160*d^5 + 127*d
^4*e + 88*d^3*e^2 - 4*d^2*e^3 + 64*d*e^4 - 11*e^5))/(e^9*(d + e*x)) + ((280
0*d^6 + 945*d^5*e + 1665*d^4*e^2 + 370*d^3*e^3 + 888*d^2*e^4 - 195*d*e^5 +
107*e^6)*Log[d + e*x])/e^9
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{-2100d^5-675d^4e-1110d^3e^2-222d^2e^3-444de^4+65e^5}{e^8} + \frac{(1500d^4+450d^3e+666d^2e^2+111de^3+148e^4)\,x}{e^7} - \frac{(1000d^3+270d^2e+333de^2+37e^3)\,x^2}{e^6} + \frac{3(200d^2+45de+37e^2)\,x^3}{e^5} - \frac{15(20d+3e)x^4}{e^4} + \frac{100x^5}{e^3} + \frac{(5d^2-2de+3e^2)^2\,(4d^4+5d^3e+3d^2e^2-de^3+2e^4)}{e^8(d+ex)^3} + \frac{-800d^7-315d^6e-666d^5e^2-185d^4e^3-592d^3e^4+195d^2e^5-214de^6+33e^7}{e^8(d+ex)^2} + \frac{2800d^6+945d^5e+1665d^4e^2+370d^3e^3+888d^2e^4-195de^5+107e^6}{e^8(d+ex)} \right) dx \\ &= -\frac{(2100d^5+675d^4e+1110d^3e^2+222d^2e^3+444de^4-65e^5)\,x}{e^8} + \frac{(1500d^4+450d^3e+666d^2e^2+111de^3+148e^4)\,x^2}{2e^7} - \frac{(1000d^3+270d^2e+333de^2+37e^3)\,x^3}{3e^6} + \frac{3(200d^2+45de+37e^2)\,x^4}{4e^5} - \frac{3(20d+3e)x^5}{e^4} + \frac{50x^6}{3e^3} - \frac{(5d^2-2de+3e^2)^2\,(4d^4+5d^3e+3d^2e^2-de^3+2e^4)}{2e^9(d+ex)^2} + \frac{(5d^2-2de+3e^2)\,(160d^5+127d^4e+88d^3e^2-4d^2e^3+64de^4-11e^5)}{e^9(d+ex)} + \frac{(2800d^6+945d^5e+1665d^4e^2+370d^3e^3+888d^2e^4-195de^5+107e^6)\log(d+ex)}{e^9}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 311, normalized size of antiderivative = 0.88

$$\int \frac{(3+2x+5x^2)^2\,(2+x+3x^2-5x^3+4x^4)}{(d+ex)^3}\,dx$$

$$= 9000d^8-390d^7e(-9+40x)-18d^6e^2(-407+240x+2300x^2)-2d^5e^3(-999+2664x+6750x^2+5600x^3)$$

```
[In] Integrate[((3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x)^3
,x]

[Out] (9000*d^8 - 390*d^7*e*(-9 + 40*x) - 18*d^6*e^2*(-407 + 240*x + 2300*x^2) -
2*d^5*e^3*(-999 + 2664*x + 6750*x^2 + 5600*x^3) + 4*d^4*e^4*(1554 - 111*x -
 5661*x^2 - 945*x^3 + 700*x^4) - d^3*e^5*(1950 - 1776*x + 4662*x^2 + 6660*x
```

```
^3 - 945*x^4 + 1120*x^5) + d^2*e^6*(1926 - 1560*x - 9768*x^2 - 1480*x^3 + 1
665*x^4 - 378*x^5 + 560*x^6) + d*e^7*(-198 + 2568*x + 1560*x^2 - 3552*x^3 +
 370*x^4 - 666*x^5 + 189*x^6 - 320*x^7) + e^8*(-108 - 396*x + 780*x^3 + 888
*x^4 - 148*x^5 + 333*x^6 - 108*x^7 + 200*x^8) + 12*(2800*d^6 + 945*d^5*e +
1665*d^4*e^2 + 370*d^3*e^3 + 888*d^2*e^4 - 195*d*e^5 + 107*e^6)*(d + e*x)^2
*Log[d + e*x])/(12*e^9*(d + e*x)^2)
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.01

method	result
norman	$\frac{\frac{\left(5600 d^{7}+1890 d^{6} e+3330 d^{5} e^{2}+740 d^{4} e^{3}+1776 d^{3} e^{4}-390 d^{2} e^{5}+214 d\,e^{6}-33 e^{7}\right) x}{e^{8}}+\frac{50 x^{8}}{3 e}+\frac{8400 d^{8}+2835 d^{7} e+4995 d^{6} e^{2}+1110 d^{5} e^{3}+2664 d^{4} e^{4}}{2 e^{9}}}{}$
default	$-\frac{-\frac{50}{3} e^{5} x^{6}+60 d\,e^{4} x^{5}+9 e^{5} x^{5}-150 d^{2} e^{3} x^{4}-\frac{135}{4} d\,e^{4} x^{4}-\frac{111}{4} e^{5} x^{4}+\frac{1000}{3} d^{3} e^{2} x^{3}+90 d^{2} e^{3} x^{3}+111 d\,e^{4} x^{3}+\frac{37}{3} e^{5} x^{3}-750 d^{4} e\,x^{2}}{e^{8}}$
risch	$\frac{2800 \ln \left(e x+d\right) d^{6}}{e^{9}}+\frac{945 \ln \left(e x+d\right) d^{5}}{e^{8}}-\frac{37 x^{3}}{3 e^{3}}+\frac{\left(800 d^{7}+315 d^{6} e+666 d^{5} e^{2}+185 d^{4} e^{3}+592 d^{3} e^{4}-195 d^{2} e^{5}+214 d\,e^{6}-33 e^{7}\right) x}{e^{8}}$
parallelrisch	$\frac{33600 \ln \left(e x+d\right) d^{8}+333 x^{6} e^{8}-148 x^{5} e^{8}+888 x^{4} e^{8}+780 x^{3} e^{8}-396 x\,e^{8}-198 d\,e^{7}-3510 d^{3} e^{5}+1926 d^{2} e^{6}+6660 d^{5} e^{3}+15984 d^{4} e^{4}}{}$

```
[In] int((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3,x,method=_RETURNVERBO
SE)
```

```
[Out] ((5600*d^7+1890*d^6*e+3330*d^5*e^2+740*d^4*e^3+1776*d^3*e^4-390*d^2*e^5+214
*d*e^6-33*e^7)/e^8*x+50/3*x^8/e+1/2*(8400*d^8+2835*d^7*e+4995*d^6*e^2+1110*
d^5*e^3+2664*d^4*e^4-585*d^3*e^5+321*d^2*e^6-33*d*e^7-18*e^8)/e^9-1/3*(80*d
+27*e)/e^2*x^7+1/12*(560*d^2+189*d*e+333*e^2)/e^3*x^6-1/6*(560*d^3+189*d^2*
e+333*d*e^2+74*e^3)/e^4*x^5+1/12*(2800*d^4+945*d^3*e+1665*d^2*e^2+370*d*e^3
+888*e^4)/e^5*x^4-1/3*(2800*d^5+945*d^4*e+1665*d^3*e^2+370*d^2*e^3+888*d*e^
4-195*e^5)/e^6*x^3)/(e*x+d)^2+(2800*d^6+945*d^5*e+1665*d^4*e^2+370*d^3*e^3+
888*d^2*e^4-195*d*e^5+107*e^6)*ln(e*x+d)/e^9
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 545, normalized size of antiderivative = 1.54

$$\int \frac{\left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right)}{(d+ex)^3}\,dx$$
$$=\frac{200\,e^8x^8+9000\,d^8+3510\,d^7e+7326\,d^6e^2+1998\,d^5e^3+6216\,d^4e^4-1950\,d^3e^5+1926\,d^2e^6-198\,de^7-}{}$$

```
[In] integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3,x, algorithm="f
ricas")
```

```
[Out] 1/12*(200*e^8*x^8 + 9000*d^8 + 3510*d^7*e + 7326*d^6*e^2 + 1998*d^5*e^3 + 6
216*d^4*e^4 - 1950*d^3*e^5 + 1926*d^2*e^6 - 198*d*e^7 - 108*e^8 - 4*(80*d*e
^7 + 27*e^8)*x^7 + (560*d^2*e^6 + 189*d*e^7 + 333*e^8)*x^6 - 2*(560*d^3*e^5
 + 189*d^2*e^6 + 333*d*e^7 + 74*e^8)*x^5 + (2800*d^4*e^4 + 945*d^3*e^5 + 16
65*d^2*e^6 + 370*d*e^7 + 888*e^8)*x^4 - 4*(2800*d^5*e^3 + 945*d^4*e^4 + 166
5*d^3*e^5 + 370*d^2*e^6 + 888*d*e^7 - 195*e^8)*x^3 - 6*(6900*d^6*e^2 + 2250
*d^5*e^3 + 3774*d^4*e^4 + 777*d^3*e^5 + 1628*d^2*e^6 - 260*d*e^7)*x^2 - 12*
(1300*d^7*e + 360*d^6*e^2 + 444*d^5*e^3 + 37*d^4*e^4 - 148*d^3*e^5 + 130*d^
2*e^6 - 214*d*e^7 + 33*e^8)*x + 12*(2800*d^8 + 945*d^7*e + 1665*d^6*e^2 + 3
70*d^5*e^3 + 888*d^4*e^4 - 195*d^3*e^5 + 107*d^2*e^6 + (2800*d^6*e^2 + 945*
d^5*e^3 + 1665*d^4*e^4 + 370*d^3*e^5 + 888*d^2*e^6 - 195*d*e^7 + 107*e^8)*x
^2 + 2*(2800*d^7*e + 945*d^6*e^2 + 1665*d^5*e^3 + 370*d^4*e^4 + 888*d^3*e^5
 - 195*d^2*e^6 + 107*d*e^7)*x)*log(e*x + d))/(e^11*x^2 + 2*d*e^10*x + d^2*e
^9)
```

Sympy [A] (verification not implemented)

Time = 1.36 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.11

$$
\begin{aligned}
&\int \frac{(3+2x+5x^2)^2\,(2+x+3x^2-5x^3+4x^4)}{(d+ex)^3}\,dx \\
&= x^5\left(-\frac{60d}{e^4}-\frac{9}{e^3}\right)+x^4\cdot\left(\frac{150d^2}{e^5}+\frac{135d}{4e^4}+\frac{111}{4e^3}\right) \\
&\quad + x^3\left(-\frac{1000d^3}{3e^6}-\frac{90d^2}{e^5}-\frac{111d}{e^4}-\frac{37}{3e^3}\right)+x^2\cdot\left(\frac{750d^4}{e^7}+\frac{225d^3}{e^6}+\frac{333d^2}{e^5}+\frac{111d}{2e^4}+\frac{74}{e^3}\right) \\
&\quad + x\left(-\frac{2100d^5}{e^8}-\frac{675d^4}{e^7}-\frac{1110d^3}{e^6}-\frac{222d^2}{e^5}-\frac{444d}{e^4}+\frac{65}{e^3}\right) \\
&\quad + \frac{1500d^8+585d^7e+1221d^6e^2+333d^5e^3+1036d^4e^4-325d^3e^5+321d^2e^6-33de^7-18e^8+x(1600d^7e+}{2d^2e^9+4de^{10}x+2e^{11}x^2} \\
&\quad + \frac{50x^6}{3e^3} \\
&\quad + \frac{(2800d^6+945d^5e+1665d^4e^2+370d^3e^3+888d^2e^4-195de^5+107e^6)\log(d+ex)}{e^9}
\end{aligned}
$$

```
[In] integrate((5*x**2+2*x+3)**2*(4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)**3,x)
```

```
[Out] x**5*(-60*d/e**4 - 9/e**3) + x**4*(150*d**2/e**5 + 135*d/(4*e**4) + 111/(4*
e**3)) + x**3*(-1000*d**3/(3*e**6) - 90*d**2/e**5 - 111*d/e**4 - 37/(3*e**3
)) + x**2*(750*d**4/e**7 + 225*d**3/e**6 + 333*d**2/e**5 + 111*d/(2*e**4) +
 74/e**3) + x*(-2100*d**5/e**8 - 675*d**4/e**7 - 1110*d**3/e**6 - 222*d**2/
e**5 - 444*d/e**4 + 65/e**3) + (1500*d**8 + 585*d**7*e + 1221*d**6*e**2 + 3
33*d**5*e**3 + 1036*d**4*e**4 - 325*d**3*e**5 + 321*d**2*e**6 - 33*d*e**7 -
 18*e**8 + x*(1600*d**7*e + 630*d**6*e**2 + 1332*d**5*e**3 + 370*d**4*e**4
+ 1184*d**3*e**5 - 390*d**2*e**6 + 428*d*e**7 - 66*e**8))/(2*d**2*e**9 + 4*
```

```
d*e**10*x + 2*e**11*x**2) + 50*x**6/(3*e**3) + (2800*d**6 + 945*d**5*e + 16
65*d**4*e**2 + 370*d**3*e**3 + 888*d**2*e**4 - 195*d*e**5 + 107*e**6)*log(d
 + e*x)/e**9
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.07

$$\int \frac{(3+2x+5x^2)^2\,(2+x+3x^2-5x^3+4x^4)}{(d+ex)^3}\,dx$$
$$= \frac{1500\,d^8+585\,d^7e+1221\,d^6e^2+333\,d^5e^3+1036\,d^4e^4-325\,d^3e^5+321\,d^2e^6-33\,de^7-18\,e^8+2\,(800\,d^7e}{2\,(e^{11}x^2+2\,de^{10}x+d^2e^9)}$$
$$+\frac{200\,e^5x^6-36\,(20\,de^4+3\,e^5)x^5+9\,(200\,d^2e^3+45\,de^4+37\,e^5)x^4-4\,(1000\,d^3e^2+270\,d^2e^3+333\,de^4+}{}$$
$$+\frac{(2800\,d^6+945\,d^5e+1665\,d^4e^2+370\,d^3e^3+888\,d^2e^4-195\,de^5+107\,e^6)\log(ex+d)}{e^9}$$

```
[In] integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3,x, algorithm="m
axima")
```

```
[Out] 1/2*(1500*d^8 + 585*d^7*e + 1221*d^6*e^2 + 333*d^5*e^3 + 1036*d^4*e^4 - 325
*d^3*e^5 + 321*d^2*e^6 - 33*d*e^7 - 18*e^8 + 2*(800*d^7*e + 315*d^6*e^2 + 6
66*d^5*e^3 + 185*d^4*e^4 + 592*d^3*e^5 - 195*d^2*e^6 + 214*d*e^7 - 33*e^8)*
x)/(e^11*x^2 + 2*d*e^10*x + d^2*e^9) + 1/12*(200*e^5*x^6 - 36*(20*d*e^4 + 3
*e^5)*x^5 + 9*(200*d^2*e^3 + 45*d*e^4 + 37*e^5)*x^4 - 4*(1000*d^3*e^2 + 270
*d^2*e^3 + 333*d*e^4 + 37*e^5)*x^3 + 6*(1500*d^4*e + 450*d^3*e^2 + 666*d^2*
e^3 + 111*d*e^4 + 148*e^5)*x^2 - 12*(2100*d^5 + 675*d^4*e + 1110*d^3*e^2 +
222*d^2*e^3 + 444*d*e^4 - 65*e^5)*x)/e^8 + (2800*d^6 + 945*d^5*e + 1665*d^4
*e^2 + 370*d^3*e^3 + 888*d^2*e^4 - 195*d*e^5 + 107*e^6)*log(e*x + d)/e^9
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.11

$$\int \frac{(3+2x+5x^2)^2\,(2+x+3x^2-5x^3+4x^4)}{(d+ex)^3}\,dx$$
$$= \frac{(2800\,d^6+945\,d^5e+1665\,d^4e^2+370\,d^3e^3+888\,d^2e^4-195\,de^5+107\,e^6)\log(|ex+d|)}{e^9}$$
$$+\frac{1500\,d^8+585\,d^7e+1221\,d^6e^2+333\,d^5e^3+1036\,d^4e^4-325\,d^3e^5+321\,d^2e^6-33\,de^7-18\,e^8+2\,(800\,d}{2\,(ex+d)^2e^9}$$
$$+\frac{200\,e^{15}x^6-720\,de^{14}x^5-108\,e^{15}x^5+1800\,d^2e^{13}x^4+405\,de^{14}x^4+333\,e^{15}x^4-4000\,d^3e^{12}x^3-1080\,d^2e^1}{}$$

```
[In] integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3,x, algorithm="g
iac")
```

```
[Out] (2800*d^6 + 945*d^5*e + 1665*d^4*e^2 + 370*d^3*e^3 + 888*d^2*e^4 - 195*d*e^
5 + 107*e^6)*log(abs(e*x + d))/e^9 + 1/2*(1500*d^8 + 585*d^7*e + 1221*d^6*e
^2 + 333*d^5*e^3 + 1036*d^4*e^4 - 325*d^3*e^5 + 321*d^2*e^6 - 33*d*e^7 - 18
*e^8 + 2*(800*d^7*e + 315*d^6*e^2 + 666*d^5*e^3 + 185*d^4*e^4 + 592*d^3*e^5
 - 195*d^2*e^6 + 214*d*e^7 - 33*e^8)*x)/((e*x + d)^2*e^9) + 1/12*(200*e^15*
x^6 - 720*d*e^14*x^5 - 108*e^15*x^5 + 1800*d^2*e^13*x^4 + 405*d*e^14*x^4 +
333*e^15*x^4 - 4000*d^3*e^12*x^3 - 1080*d^2*e^13*x^3 - 1332*d*e^14*x^3 - 14
8*e^15*x^3 + 9000*d^4*e^11*x^2 + 2700*d^3*e^12*x^2 + 3996*d^2*e^13*x^2 + 66
6*d*e^14*x^2 + 888*e^15*x^2 - 25200*d^5*e^10*x - 8100*d^4*e^11*x - 13320*d^
3*e^12*x - 2664*d^2*e^13*x - 5328*d*e^14*x + 780*e^15*x)/e^18
```

Mupad [B] (verification not implemented)

Time = 13.33 (sec) , antiderivative size = 771, normalized size of antiderivative = 2.18

$$\int \frac{(3+2x+5x^2)^2\,(2+x+3x^2-5x^3+4x^4)}{(d+ex)^3}\,dx = x^4\left(\frac{111}{4\,e^3}-\frac{75\,d^2}{e^5}+\frac{3\,d\left(\frac{300\,d}{e^4}+\frac{45}{e^3}\right)}{4\,e}\right)$$

$$-\,x^3\left(\frac{37}{3\,e^3}+\frac{100\,d^3}{3\,e^6}+\frac{d\left(\frac{111}{e^3}-\frac{300\,d^2}{e^5}+\frac{3\,d\left(\frac{300\,d}{e^4}+\frac{45}{e^3}\right)}{e}\right)}{e}-\frac{d^2\left(\frac{300\,d}{e^4}+\frac{45}{e^3}\right)}{e^2}\right)$$

$$-\,x^5\left(\frac{60\,d}{e^4}+\frac{9}{e^3}\right)+x\left(\frac{65}{e^3}\right.$$

$$-\,\frac{3\,d\left(\frac{148}{e^3}+\frac{3\,d\left(\frac{37}{e^3}+\frac{100\,d^3}{e^6}+\frac{3\,d\left(\frac{111}{e^3}-\frac{300\,d^2}{e^5}+\frac{3\,d\left(\frac{300\,d}{e^4}+\frac{45}{e^3}\right)}{e}\right)}{e}-\frac{3\,d^2\left(\frac{300\,d}{e^4}+\frac{45}{e^3}\right)}{e^2}\right)}{e}-\frac{3\,d^2\left(\frac{111}{e^3}-\frac{300\,d^2}{e^5}+\frac{3\,d\left(\frac{300\,d}{e^4}+\frac{45}{e^3}\right)}{e}\right)}{e^2}+\frac{d^3\left(\right.}{}\right.}{e}$$

$$+\,\frac{3\,d^2\left(\frac{37}{e^3}+\frac{100\,d^3}{e^6}+\frac{3\,d\left(\frac{111}{e^3}-\frac{300\,d^2}{e^5}+\frac{3\,d\left(\frac{300\,d}{e^4}+\frac{45}{e^3}\right)}{e}\right)}{e}-\frac{3\,d^2\left(\frac{300\,d}{e^4}+\frac{45}{e^3}\right)}{e^2}\right)}{e^2}$$

$$\left.-\,d^3\left(\frac{111}{e^3}-\frac{300\,d^2}{e^5}+\frac{3\,d\left(\frac{300\,d}{e^4}+\frac{45}{e^3}\right)}{e}\right)\right)$$

```
[In] int(((2*x + 5*x^2 + 3)^2*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(d + e*x)^3,x)
```

```
[Out] x^4*(111/(4*e^3) - (75*d^2)/e^5 + (3*d*((300*d)/e^4 + 45/e^3))/(4*e)) - x^3
*(37/(3*e^3) + (100*d^3)/(3*e^6) + (d*(111/e^3 - (300*d^2)/e^5 + (3*d*((300
*d)/e^4 + 45/e^3))/e))/e - (d^2*((300*d)/e^4 + 45/e^3))/e^2) - x^5*((60*d)/
e^4 + 9/e^3) + x*(65/e^3 - (3*d*(148/e^3 + (3*d*(37/e^3 + (100*d^3)/e^6 + (
3*d*(111/e^3 - (300*d^2)/e^5 + (3*d*((300*d)/e^4 + 45/e^3))/e))/e - (3*d^2*
((300*d)/e^4 + 45/e^3))/e^2))/e - (3*d^2*(111/e^3 - (300*d^2)/e^5 + (3*d*((
300*d)/e^4 + 45/e^3))/e))/e^2 + (d^3*((300*d)/e^4 + 45/e^3))/e^3))/e + (3*d
^2*(37/e^3 + (100*d^3)/e^6 + (3*d*(111/e^3 - (300*d^2)/e^5 + (3*d*((300*d)/
e^4 + 45/e^3))/e))/e - (3*d^2*((300*d)/e^4 + 45/e^3))/e^2))/e^2 - (d^3*(111
/e^3 - (300*d^2)/e^5 + (3*d*((300*d)/e^4 + 45/e^3))/e))/e^3) + (x*(214*d*e^
6 + 315*d^6*e + 800*d^7 - 33*e^7 - 195*d^2*e^5 + 592*d^3*e^4 + 185*d^4*e^3
+ 666*d^5*e^2) + (585*d^7*e - 33*d*e^7 + 1500*d^8 - 18*e^8 + 321*d^2*e^6 -
325*d^3*e^5 + 1036*d^4*e^4 + 333*d^5*e^3 + 1221*d^6*e^2)/(2*e))/(d^2*e^8 +
e^10*x^2 + 2*d*e^9*x) + (50*x^6)/(3*e^3) + x^2*(74/e^3 + (3*d*(37/e^3 + (10
0*d^3)/e^6 + (3*d*(111/e^3 - (300*d^2)/e^5 + (3*d*((300*d)/e^4 + 45/e^3))/e
))/e - (3*d^2*((300*d)/e^4 + 45/e^3))/e^2))/(2*e) - (3*d^2*(111/e^3 - (300*
d^2)/e^5 + (3*d*((300*d)/e^4 + 45/e^3))/e))/(2*e^2) + (d^3*((300*d)/e^4 + 4
5/e^3))/(2*e^3)) + (log(d + e*x)*(945*d^5*e - 195*d*e^5 + 2800*d^6 + 107*e^
6 + 888*d^2*e^4 + 370*d^3*e^3 + 1665*d^4*e^2))/e^9
```

3.303 $\int \frac{(3+2x+5x^2)^2(2+x+3x^2-5x^3+4x^4)}{(d+ex)^4}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 360

$$
\begin{aligned}
&\int \frac{(3+2x+5x^2)^2(2+x+3x^2-5x^3+4x^4)}{(d+ex)^4}\,dx \\
&= \frac{2(1750d^4+450d^3e+555d^2e^2+74de^3+74e^4)\,x}{e^8} \\
&\quad - \frac{(2000d^3+450d^2e+444de^2+37e^3)\,x^2}{2e^7} + \frac{(1000d^2+180de+111e^2)\,x^3}{3e^6} \\
&\quad - \frac{5(80d+9e)x^4}{4e^5} + \frac{20x^5}{e^4} - \frac{(5d^2-2de+3e^2)^2\,(4d^4+5d^3e+3d^2e^2-de^3+2e^4)}{3e^9(d+ex)^3} \\
&\quad + \frac{(5d^2-2de+3e^2)\,(160d^5+127d^4e+88d^3e^2-4d^2e^3+64de^4-11e^5)}{2e^9(d+ex)^2} \\
&\quad - \frac{2800d^6+945d^5e+1665d^4e^2+370d^3e^3+888d^2e^4-195de^5+107e^6}{e^9(d+ex)} \\
&\quad - \frac{(5600d^5+1575d^4e+2220d^3e^2+370d^2e^3+592de^4-65e^5)\log(d+ex)}{e^9}
\end{aligned}
$$

```
[Out] 2*(1750*d^4+450*d^3*e+555*d^2*e^2+74*d*e^3+74*e^4)*x/e^8-1/2*(2000*d^3+450*
d^2*e+444*d*e^2+37*e^3)*x^2/e^7+1/3*(1000*d^2+180*d*e+111*e^2)*x^3/e^6-5/4*
(80*d+9*e)*x^4/e^5+20*x^5/e^4-1/3*(5*d^2-2*d*e+3*e^2)^2*(4*d^4+5*d^3*e+3*d^
2*e^2-d*e^3+2*e^4)/e^9/(e*x+d)^3+1/2*(5*d^2-2*d*e+3*e^2)*(160*d^5+127*d^4*e
+88*d^3*e^2-4*d^2*e^3+64*d*e^4-11*e^5)/e^9/(e*x+d)^2+(-2800*d^6-945*d^5*e-1
665*d^4*e^2-370*d^3*e^3-888*d^2*e^4+195*d*e^5-107*e^6)/e^9/(e*x+d)-(5600*d^
5+1575*d^4*e+2220*d^3*e^2+370*d^2*e^3+592*d*e^4-65*e^5)*ln(e*x+d)/e^9
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {1642}

$$\begin{aligned}&\int \frac{(3+2x+5x^2)^2\,(2+x+3x^2-5x^3+4x^4)}{(d+ex)^4}\,dx\\&=\frac{x^3(1000d^2+180de+111e^2)}{3e^6}-\frac{x^2(2000d^3+450d^2e+444de^2+37e^3)}{2e^7}\\&-\frac{(5d^2-2de+3e^2)^2\,(4d^4+5d^3e+3d^2e^2-de^3+2e^4)}{3e^9(d+ex)^3}\\&+\frac{2x(1750d^4+450d^3e+555d^2e^2+74de^3+74e^4)}{e^8}\\&+\frac{(5d^2-2de+3e^2)\,(160d^5+127d^4e+88d^3e^2-4d^2e^3+64de^4-11e^5)}{2e^9(d+ex)^2}\\&-\frac{(5600d^5+1575d^4e+2220d^3e^2+370d^2e^3+592de^4-65e^5)\log(d+ex)}{e^9}\\&-\frac{2800d^6+945d^5e+1665d^4e^2+370d^3e^3+888d^2e^4-195de^5+107e^6}{e^9(d+ex)}\\&-\frac{5x^4(80d+9e)}{4e^5}+\frac{20x^5}{e^4}\end{aligned}$$

```
[In] Int[((3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x)^4,x]
```

```
[Out] (2*(1750*d^4 + 450*d^3*e + 555*d^2*e^2 + 74*d*e^3 + 74*e^4)*x)/e^8 - ((2000
*d^3 + 450*d^2*e + 444*d*e^2 + 37*e^3)*x^2)/(2*e^7) + ((1000*d^2 + 180*d*e
+ 111*e^2)*x^3)/(3*e^6) - (5*(80*d + 9*e)*x^4)/(4*e^5) + (20*x^5)/e^4 - ((5
*d^2 - 2*d*e + 3*e^2)^2*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4))/(3*e
^9*(d + e*x)^3) + ((5*d^2 - 2*d*e + 3*e^2)*(160*d^5 + 127*d^4*e + 88*d^3*e^
2 - 4*d^2*e^3 + 64*d*e^4 - 11*e^5))/(2*e^9*(d + e*x)^2) - (2800*d^6 + 945*d
^5*e + 1665*d^4*e^2 + 370*d^3*e^3 + 888*d^2*e^4 - 195*d*e^5 + 107*e^6)/(e^9
*(d + e*x)) - ((5600*d^5 + 1575*d^4*e + 2220*d^3*e^2 + 370*d^2*e^3 + 592*d*
e^4 - 65*e^5)*Log[d + e*x])/e^9
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{2(1750d^4+450d^3e+555d^2e^2+74de^3+74e^4)}{e^8} - \frac{(2000d^3+450d^2e+444de^2+37e^3)\,x}{e^7} + \frac{(1000d^2+180de+111e^2)\,x^2}{e^6} - \frac{5(80d+9e)x^3}{e^5} + \frac{100x^4}{e^4} + \frac{(5d^2-2de+3e^2)^2\,(4d^4+5d^3e+3d^2e^2-de^3+2e^4)}{e^8(d+ex)^4} + \frac{-800d^7-315d^6e-666d^5e^2-185d^4e^3-592d^3e^4+195d^2e^5-214de^6+33e^7}{e^8(d+ex)^3} + \frac{2800d^6+945d^5e+1665d^4e^2+370d^3e^3+888d^2e^4-195de^5+107e^6}{e^8(d+ex)^2} + \frac{-5600d^5-1575d^4e-2220d^3e^2-370d^2e^3-592de^4+65e^5}{e^8(d+ex)}\right) dx \\
&= \frac{2(1750d^4+450d^3e+555d^2e^2+74de^3+74e^4)\,x}{e^8} - \frac{(2000d^3+450d^2e+444de^2+37e^3)\,x^2}{2e^7} + \frac{(1000d^2+180de+111e^2)\,x^3}{3e^6} - \frac{5(80d+9e)x^4}{4e^5} + \frac{20x^5}{e^4} - \frac{(5d^2-2de+3e^2)^2\,(4d^4+5d^3e+3d^2e^2-de^3+2e^4)}{3e^9(d+ex)^3} + \frac{(5d^2-2de+3e^2)\,(160d^5+127d^4e+88d^3e^2-4d^2e^3+64de^4-11e^5)}{2e^9(d+ex)^2} - \frac{2800d^6+945d^5e+1665d^4e^2+370d^3e^3+888d^2e^4-195de^5+107e^6}{e^9(d+ex)} - \frac{(5600d^5+1575d^4e+2220d^3e^2+370d^2e^3+592de^4-65e^5)\log(d+ex)}{e^9}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 344, normalized size of antiderivative = 0.96

$$
\int \frac{(3+2x+5x^2)^2\,(2+x+3x^2-5x^3+4x^4)}{(d+ex)^4}\,dx
$$

$$
= \frac{24e(1750d^4+450d^3e+555d^2e^2+74de^3+74e^4)\,x-6e^2(2000d^3+450d^2e+444de^2+37e^3)\,x^2+4e^3(100}{}
$$

```
[In] Integrate[((3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x)^4
,x]
```

```
[Out] (24*e*(1750*d^4 + 450*d^3*e + 555*d^2*e^2 + 74*d*e^3 + 74*e^4)*x - 6*e^2*(2
000*d^3 + 450*d^2*e + 444*d*e^2 + 37*e^3)*x^2 + 4*e^3*(1000*d^2 + 180*d*e +
```

```
111*e^2)*x^3 - 15*e^4*(80*d + 9*e)*x^4 + 240*e^5*x^5 - (4*(5*d^2 - 2*d*e +
 3*e^2)^2*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4))/(d + e*x)^3 + (6*(
800*d^7 + 315*d^6*e + 666*d^5*e^2 + 185*d^4*e^3 + 592*d^3*e^4 - 195*d^2*e^5
 + 214*d*e^6 - 33*e^7))/(d + e*x)^2 - (12*(2800*d^6 + 945*d^5*e + 1665*d^4*
e^2 + 370*d^3*e^3 + 888*d^2*e^4 - 195*d*e^5 + 107*e^6))/(d + e*x) - 12*(560
0*d^5 + 1575*d^4*e + 2220*d^3*e^2 + 370*d^2*e^3 + 592*d*e^4 - 65*e^5)*Log[d
 + e*x])/(12*e^9)
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.00

method	result
norman	$\frac{\frac{20x^8}{e}-\frac{61600d^8+17325d^7e+24420d^6e^2+4070d^5e^3+6512d^4e^4-715d^3e^5+214d^2e^6+33d\,e^7+36e^8}{6e^9}-\frac{5(32d+9e)x^7}{4e^2}+\frac{\left(1120d^2+315de+444e^2\right)x^6}{12e^3}}{}$
default	$\frac{20e^4x^5-100d\,e^3x^4-\frac{45}{4}e^4x^4+\frac{1000}{3}d^2e^2x^3+60d\,e^3x^3+37e^4x^3-1000d^3e\,x^2-225d^2e^2x^2-222d\,e^3x^2-\frac{37}{2}e^4x^2+3500d^4x+900d^3ex}{e^8}$
risch	$\frac{20x^5}{e^4}-\frac{100d\,x^4}{e^5}-\frac{45x^4}{4e^4}+\frac{1000d^2x^3}{3e^6}+\frac{60d\,x^3}{e^5}+\frac{37x^3}{e^4}-\frac{1000d^3x^2}{e^7}-\frac{225d^2x^2}{e^6}-\frac{222d\,x^2}{e^5}-\frac{37x^2}{2e^4}+\frac{3500d^4x}{e^8}+$
parallelrisch	$-\frac{67200\ln(ex+d)d^8-444x^6e^8+222x^5e^8-1776x^4e^8+1284x^2e^8+198x\,e^8+66d\,e^7-1430d^3e^5+428d^2e^6+8140d^5e^3+13024d^4e^4+}{}$

```
[In] int((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^4,x,method=_RETURNVERBO
SE)
```

```
[Out] (20*x^8/e-1/6*(61600*d^8+17325*d^7*e+24420*d^6*e^2+4070*d^5*e^3+6512*d^4*e^
4-715*d^3*e^5+214*d^2*e^6+33*d*e^7+36*e^8)/e^9-5/4*(32*d+9*e)/e^2*x^7+1/12*
(1120*d^2+315*d*e+444*e^2)/e^3*x^6-1/4*(1120*d^3+315*d^2*e+444*d*e^2+74*e^3
)/e^4*x^5+1/4*(5600*d^4+1575*d^3*e+2220*d^2*e^2+370*d*e^3+592*e^4)/e^5*x^4-
(16800*d^6+4725*d^5*e+6660*d^4*e^2+1110*d^3*e^3+1776*d^2*e^4-195*d*e^5+107*
e^6)/e^7*x^2-1/2*(50400*d^7+14175*d^6*e+19980*d^5*e^2+3330*d^4*e^3+5328*d^3
*e^4-585*d^2*e^5+214*d*e^6+33*e^7)/e^8*x)/(e*x+d)^3-(5600*d^5+1575*d^4*e+22
20*d^3*e^2+370*d^2*e^3+592*d*e^4-65*e^5)*ln(e*x+d)/e^9
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 587, normalized size of antiderivative = 1.63

$$\int \frac{(3+2x+5x^2)^2\left(2+x+3x^2-5x^3+4x^4\right)}{(d+ex)^4}\,dx$$
$$=\frac{240\,e^8x^8-29200\,d^8-9630\,d^7e-16428\,d^6e^2-3478\,d^5e^3-7696\,d^4e^4+1430\,d^3e^5-428\,d^2e^6-66\,de^7-72}{}$$

```
[In] integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^4,x, algorithm="f
ricas")
```

```
[Out] 1/12*(240*e^8*x^8 - 29200*d^8 - 9630*d^7*e - 16428*d^6*e^2 - 3478*d^5*e^3 -
 7696*d^4*e^4 + 1430*d^3*e^5 - 428*d^2*e^6 - 66*d*e^7 - 72*e^8 - 15*(32*d*e
^7 + 9*e^8)*x^7 + (1120*d^2*e^6 + 315*d*e^7 + 444*e^8)*x^6 - 3*(1120*d^3*e^
5 + 315*d^2*e^6 + 444*d*e^7 + 74*e^8)*x^5 + 3*(5600*d^4*e^4 + 1575*d^3*e^5
+ 2220*d^2*e^6 + 370*d*e^7 + 592*e^8)*x^4 + 2*(47000*d^5*e^3 + 12510*d^4*e^
4 + 16206*d^3*e^5 + 2331*d^2*e^6 + 2664*d*e^7)*x^3 + 6*(13400*d^6*e^2 + 306
0*d^5*e^3 + 2886*d^4*e^4 + 111*d^3*e^5 - 888*d^2*e^6 + 390*d*e^7 - 214*e^8)
*x^2 - 6*(3400*d^7*e + 1665*d^6*e^2 + 3774*d^5*e^3 + 999*d^4*e^4 + 2664*d^3
*e^5 - 585*d^2*e^6 + 214*d*e^7 + 33*e^8)*x - 12*(5600*d^8 + 1575*d^7*e + 22
20*d^6*e^2 + 370*d^5*e^3 + 592*d^4*e^4 - 65*d^3*e^5 + (5600*d^5*e^3 + 1575*
d^4*e^4 + 2220*d^3*e^5 + 370*d^2*e^6 + 592*d*e^7 - 65*e^8)*x^3 + 3*(5600*d^
6*e^2 + 1575*d^5*e^3 + 2220*d^4*e^4 + 370*d^3*e^5 + 592*d^2*e^6 - 65*d*e^7)
*x^2 + 3*(5600*d^7*e + 1575*d^6*e^2 + 2220*d^5*e^3 + 370*d^4*e^4 + 592*d^3*
e^5 - 65*d^2*e^6)*x)*log(e*x + d))/(e^12*x^3 + 3*d*e^11*x^2 + 3*d^2*e^10*x
+ d^3*e^9)
```

Sympy [A] (verification not implemented)

Time = 2.54 (sec) , antiderivative size = 401, normalized size of antiderivative = 1.11

$$
\begin{aligned}
&\int \frac{(3+2x+5x^2)^2\,(2+x+3x^2-5x^3+4x^4)}{(d+ex)^4}\,dx \\
&= x^4\left(-\frac{100d}{e^5}-\frac{45}{4e^4}\right)+x^3\cdot\left(\frac{1000d^2}{3e^6}+\frac{60d}{e^5}+\frac{37}{e^4}\right) \\
&\quad + x^2\left(-\frac{1000d^3}{e^7}-\frac{225d^2}{e^6}-\frac{222d}{e^5}-\frac{37}{2e^4}\right)+x\left(\frac{3500d^4}{e^8}+\frac{900d^3}{e^7}+\frac{1110d^2}{e^6}+\frac{148d}{e^5}+\frac{148}{e^4}\right) \\
&\quad + \frac{-14600d^8-4815d^7e-8214d^6e^2-1739d^5e^3-3848d^4e^4+715d^3e^5-214d^2e^6-33de^7-36e^8+x^2(-1}{} \\
&\quad + \frac{20x^5}{e^4}-\frac{(5600d^5+1575d^4e+2220d^3e^2+370d^2e^3+592de^4-65e^5)\log(d+ex)}{e^9}
\end{aligned}
$$

```
[In] integrate((5*x**2+2*x+3)**2*(4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)**4,x)
```

```
[Out] x**4*(-100*d/e**5 - 45/(4*e**4)) + x**3*(1000*d**2/(3*e**6) + 60*d/e**5 + 3
7/e**4) + x**2*(-1000*d**3/e**7 - 225*d**2/e**6 - 222*d/e**5 - 37/(2*e**4))
 + x*(3500*d**4/e**8 + 900*d**3/e**7 + 1110*d**2/e**6 + 148*d/e**5 + 148/e*
*4) + (-14600*d**8 - 4815*d**7*e - 8214*d**6*e**2 - 1739*d**5*e**3 - 3848*d
**4*e**4 + 715*d**3*e**5 - 214*d**2*e**6 - 33*d*e**7 - 36*e**8 + x**2*(-168
00*d**6*e**2 - 5670*d**5*e**3 - 9990*d**4*e**4 - 2220*d**3*e**5 - 5328*d**2
*e**6 + 1170*d*e**7 - 642*e**8) + x*(-31200*d**7*e - 10395*d**6*e**2 - 1798
2*d**5*e**3 - 3885*d**4*e**4 - 8880*d**3*e**5 + 1755*d**2*e**6 - 642*d*e**7
 - 99*e**8))/(6*d**3*e**9 + 18*d**2*e**10*x + 18*d*e**11*x**2 + 6*e**12*x**
```

```
3) + 20*x**5/e**4 - (5600*d**5 + 1575*d**4*e + 2220*d**3*e**2 + 370*d**2*e*
*3 + 592*d*e**4 - 65*e**5)*log(d + e*x)/e**9
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.08

$$\int \frac{(3+2x+5x^2)^2\,(2+x+3x^2-5x^3+4x^4)}{(d+ex)^4}\,dx =$$

$$-\frac{14600\,d^8+4815\,d^7e+8214\,d^6e^2+1739\,d^5e^3+3848\,d^4e^4-715\,d^3e^5+214\,d^2e^6+33\,de^7+36\,e^8+6\,(280}{}$$

$$+\frac{240\,e^4x^5-15\,(80\,de^3+9\,e^4)x^4+4\,(1000\,d^2e^2+180\,de^3+111\,e^4)x^3-6\,(2000\,d^3e+450\,d^2e^2+444\,de^3}{12\,e^8}$$

$$-\frac{(5600\,d^5+1575\,d^4e+2220\,d^3e^2+370\,d^2e^3+592\,de^4-65\,e^5)\log(ex+d)}{e^9}$$

```
[In] integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^4,x, algorithm="m
axima")
```

```
[Out] -1/6*(14600*d^8 + 4815*d^7*e + 8214*d^6*e^2 + 1739*d^5*e^3 + 3848*d^4*e^4 -
 715*d^3*e^5 + 214*d^2*e^6 + 33*d*e^7 + 36*e^8 + 6*(2800*d^6*e^2 + 945*d^5*
e^3 + 1665*d^4*e^4 + 370*d^3*e^5 + 888*d^2*e^6 - 195*d*e^7 + 107*e^8)*x^2 +
 3*(10400*d^7*e + 3465*d^6*e^2 + 5994*d^5*e^3 + 1295*d^4*e^4 + 2960*d^3*e^5
 - 585*d^2*e^6 + 214*d*e^7 + 33*e^8)*x)/(e^12*x^3 + 3*d*e^11*x^2 + 3*d^2*e^
10*x + d^3*e^9) + 1/12*(240*e^4*x^5 - 15*(80*d*e^3 + 9*e^4)*x^4 + 4*(1000*d
^2*e^2 + 180*d*e^3 + 111*e^4)*x^3 - 6*(2000*d^3*e + 450*d^2*e^2 + 444*d*e^3
 + 37*e^4)*x^2 + 24*(1750*d^4 + 450*d^3*e + 555*d^2*e^2 + 74*d*e^3 + 74*e^4
)*x)/e^8 - (5600*d^5 + 1575*d^4*e + 2220*d^3*e^2 + 370*d^2*e^3 + 592*d*e^4
- 65*e^5)*log(e*x + d)/e^9
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.06

$$\int \frac{(3+2x+5x^2)^2\,(2+x+3x^2-5x^3+4x^4)}{(d+ex)^4}\,dx$$

$$=-\frac{(5600\,d^5+1575\,d^4e+2220\,d^3e^2+370\,d^2e^3+592\,de^4-65\,e^5)\log(|ex+d|)}{e^9}$$

$$-\frac{14600\,d^8+4815\,d^7e+8214\,d^6e^2+1739\,d^5e^3+3848\,d^4e^4-715\,d^3e^5+214\,d^2e^6+33\,de^7+36\,e^8+6\,(280}{}$$

$$+\frac{240\,e^{16}x^5-1200\,de^{15}x^4-135\,e^{16}x^4+4000\,d^2e^{14}x^3+720\,de^{15}x^3+444\,e^{16}x^3-12000\,d^3e^{13}x^2-2700\,d^2e}{12}$$

```
[In] integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^4,x, algorithm="g
iac")
```

```
[Out] -(5600*d^5 + 1575*d^4*e + 2220*d^3*e^2 + 370*d^2*e^3 + 592*d*e^4 - 65*e^5)*
log(abs(e*x + d))/e^9 - 1/6*(14600*d^8 + 4815*d^7*e + 8214*d^6*e^2 + 1739*d
^5*e^3 + 3848*d^4*e^4 - 715*d^3*e^5 + 214*d^2*e^6 + 33*d*e^7 + 36*e^8 + 6*(
2800*d^6*e^2 + 945*d^5*e^3 + 1665*d^4*e^4 + 370*d^3*e^5 + 888*d^2*e^6 - 195
*d*e^7 + 107*e^8)*x^2 + 3*(10400*d^7*e + 3465*d^6*e^2 + 5994*d^5*e^3 + 1295
*d^4*e^4 + 2960*d^3*e^5 - 585*d^2*e^6 + 214*d*e^7 + 33*e^8)*x)/((e*x + d)^3
*e^9) + 1/12*(240*e^16*x^5 - 1200*d*e^15*x^4 - 135*e^16*x^4 + 4000*d^2*e^14
*x^3 + 720*d*e^15*x^3 + 444*e^16*x^3 - 12000*d^3*e^13*x^2 - 2700*d^2*e^14*x
^2 - 2664*d*e^15*x^2 - 222*e^16*x^2 + 42000*d^4*e^12*x + 10800*d^3*e^13*x +
 13320*d^2*e^14*x + 1776*d*e^15*x + 1776*e^16*x)/e^20
```

Mupad [B] (verification not implemented)

Time = 13.26 (sec) , antiderivative size = 560, normalized size of antiderivative = 1.56

$$
\begin{aligned}
&\int \frac{(3+2x+5x^2)^2\,(2+x+3x^2-5x^3+4x^4)}{(d+ex)^4}\,dx = x^3\left(\frac{37}{e^4}-\frac{200\,d^2}{e^6}+\frac{4\,d\left(\frac{400\,d}{e^5}+\frac{45}{e^4}\right)}{3\,e}\right) \\
&\quad - x^2\left(\frac{37}{2\,e^4}+\frac{200\,d^3}{e^7}+\frac{2\,d\left(\frac{111}{e^4}-\frac{600\,d^2}{e^6}+\frac{4\,d\left(\frac{400\,d}{e^5}+\frac{45}{e^4}\right)}{e}\right)}{e}-\frac{3\,d^2\left(\frac{400\,d}{e^5}+\frac{45}{e^4}\right)}{e^2}\right) \\
&\quad - \frac{x\left(5200\,d^7+\frac{3465\,d^6\,e}{2}+2997\,d^5\,e^2+\frac{1295\,d^4\,e^3}{2}+1480\,d^3\,e^4-\frac{585\,d^2\,e^5}{2}+107\,d\,e^6+\frac{33\,e^7}{2}\right)+\frac{14600\,d^8+4815\,d^7\,e+}{}}{d^3} \\
&\quad - x^4\left(\frac{100\,d}{e^5}+\frac{45}{4\,e^4}\right) \\
&\quad + x\left(\frac{148}{e^4}-\frac{100\,d^4}{e^8}+\frac{4\,d\left(\frac{37}{e^4}+\frac{400\,d^3}{e^7}+\frac{4\,d\left(\frac{111}{e^4}-\frac{600\,d^2}{e^6}+\frac{4\,d\left(\frac{400\,d}{e^5}+\frac{45}{e^4}\right)}{e}\right)}{e}-\frac{6\,d^2\left(\frac{400\,d}{e^5}+\frac{45}{e^4}\right)}{e^2}\right)}{e}\right. \\
&\qquad \left. -\frac{6\,d^2\left(\frac{111}{e^4}-\frac{600\,d^2}{e^6}+\frac{4\,d\left(\frac{400\,d}{e^5}+\frac{45}{e^4}\right)}{e}\right)}{e^2}+\frac{4\,d^3\left(\frac{400\,d}{e^5}+\frac{45}{e^4}\right)}{e^3}\right) \\
&\quad + \frac{20\,x^5}{e^4}-\frac{\ln(d+e\,x)\,\left(5600\,d^5+1575\,d^4\,e+2220\,d^3\,e^2+370\,d^2\,e^3+592\,d\,e^4-65\,e^5\right)}{e^9}
\end{aligned}
$$

```
[In] int(((2*x + 5*x^2 + 3)^2*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(d + e*x)^4,x)
```

```
[Out] x^3*(37/e^4 - (200*d^2)/e^6 + (4*d*((400*d)/e^5 + 45/e^4))/(3*e)) - x^2*(37
/(2*e^4) + (200*d^3)/e^7 + (2*d*(111/e^4 - (600*d^2)/e^6 + (4*d*((400*d)/e^
5 + 45/e^4))/e))/e - (3*d^2*((400*d)/e^5 + 45/e^4))/e^2) - (x*(107*d*e^6 +
(3465*d^6*e)/2 + 5200*d^7 + (33*e^7)/2 - (585*d^2*e^5)/2 + 1480*d^3*e^4 + (
1295*d^4*e^3)/2 + 2997*d^5*e^2) + (33*d*e^7 + 4815*d^7*e + 14600*d^8 + 36*e
^8 + 214*d^2*e^6 - 715*d^3*e^5 + 3848*d^4*e^4 + 1739*d^5*e^3 + 8214*d^6*e^2
)/(6*e) + x^2*(2800*d^6*e - 195*d*e^6 + 107*e^7 + 888*d^2*e^5 + 370*d^3*e^4
```

```
 + 1665*d^4*e^3 + 945*d^5*e^2))/(d^3*e^8 + e^11*x^3 + 3*d^2*e^9*x + 3*d*e^1
0*x^2) - x^4*((100*d)/e^5 + 45/(4*e^4)) + x*(148/e^4 - (100*d^4)/e^8 + (4*d
*(37/e^4 + (400*d^3)/e^7 + (4*d*(111/e^4 - (600*d^2)/e^6 + (4*d*((400*d)/e^
5 + 45/e^4))/e))/e - (6*d^2*((400*d)/e^5 + 45/e^4))/e^2))/e - (6*d^2*(111/e
^4 - (600*d^2)/e^6 + (4*d*((400*d)/e^5 + 45/e^4))/e))/e^2 + (4*d^3*((400*d)
/e^5 + 45/e^4))/e^3) + (20*x^5)/e^4 - (log(d + e*x)*(592*d*e^4 + 1575*d^4*e
 + 5600*d^5 - 65*e^5 + 370*d^2*e^3 + 2220*d^3*e^2))/e^9
```

3.304 $\int \frac{(d+ex)^3(2+x+3x^2-5x^3+4x^4)}{3+2x+5x^2}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 221

$$\begin{aligned}
&\int \frac{(d+ex)^3\left(2+x+3x^2-5x^3+4x^4\right)}{3+2x+5x^2}\,dx \\
&= \frac{\left(10125d^3+34350d^2e-13215de^2-5108e^3\right)x}{15625} \\
&\quad - \frac{\left(4125d^3-6075d^2e-6870de^2+881e^3\right)x^2}{6250} + \frac{\left(500d^3-2475d^2e+1215de^2+458e^3\right)x^3}{1875} \\
&\quad + \frac{3}{500}e\left(100d^2-165de+27e^2\right)x^4 + \frac{3}{125}(20d-11e)e^2x^5 + \frac{2e^3x^6}{15} \\
&\quad - \frac{\left(52875d^3+449175d^2e-274845de^2-53189e^3\right)\arctan\left(\frac{1+5x}{\sqrt{14}}\right)}{78125\sqrt{14}} \\
&\quad + \frac{\left(57250d^3-66075d^2e-76620de^2+23431e^3\right)\log\left(3+2x+5x^2\right)}{156250}
\end{aligned}$$

```
[Out] 1/15625*(10125*d^3+34350*d^2*e-13215*d*e^2-5108*e^3)*x-1/6250*(4125*d^3-607
5*d^2*e-6870*d*e^2+881*e^3)*x^2+1/1875*(500*d^3-2475*d^2*e+1215*d*e^2+458*e
^3)*x^3+3/500*e*(100*d^2-165*d*e+27*e^2)*x^4+3/125*(20*d-11*e)*e^2*x^5+2/15
*e^3*x^6+1/156250*(57250*d^3-66075*d^2*e-76620*d*e^2+23431*e^3)*ln(5*x^2+2*
x+3)-1/1093750*(52875*d^3+449175*d^2*e-274845*d*e^2-53189*e^3)*arctan(1/14*
(1+5*x)*14^(1/2))*14^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {1642, 648, 632, 210, 642}

$$\begin{aligned}
&\int \frac{(d+ex)^3\left(2+x+3x^2-5x^3+4x^4\right)}{3+2x+5x^2}\,dx \\
&=-\frac{\arctan\left(\frac{5x+1}{\sqrt{14}}\right)\left(52875d^3+449175d^2e-274845de^2-53189e^3\right)}{78125\sqrt{14}} \\
&\quad+\frac{3}{500}ex^4\left(100d^2-165de+27e^2\right)+\frac{x^3\left(500d^3-2475d^2e+1215de^2+458e^3\right)}{1875} \\
&\quad-\frac{x^2\left(4125d^3-6075d^2e-6870de^2+881e^3\right)}{6250} \\
&\quad+\frac{\left(57250d^3-66075d^2e-76620de^2+23431e^3\right)\log\left(5x^2+2x+3\right)}{156250} \\
&\quad+\frac{x\left(10125d^3+34350d^2e-13215de^2-5108e^3\right)}{15625}+\frac{3}{125}e^2x^5(20d-11e)+\frac{2e^3x^6}{15}
\end{aligned}$$

```
[In] Int[((d + e*x)^3*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2),x]
```

```
[Out] ((10125*d^3 + 34350*d^2*e - 13215*d*e^2 - 5108*e^3)*x)/15625 - ((4125*d^3 -
 6075*d^2*e - 6870*d*e^2 + 881*e^3)*x^2)/6250 + ((500*d^3 - 2475*d^2*e + 12
15*d*e^2 + 458*e^3)*x^3)/1875 + (3*e*(100*d^2 - 165*d*e + 27*e^2)*x^4)/500
+ (3*(20*d - 11*e)*e^2*x^5)/125 + (2*e^3*x^6)/15 - ((52875*d^3 + 449175*d^2
*e - 274845*d*e^2 - 53189*e^3)*ArcTan[(1 + 5*x)/Sqrt[14]])/(78125*Sqrt[14])
 + ((57250*d^3 - 66075*d^2*e - 76620*d*e^2 + 23431*e^3)*Log[3 + 2*x + 5*x^2
])/156250
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{10125d^3+34350d^2e-13215de^2-5108e^3}{15625} - \frac{(4125d^3-6075d^2e-6870de^2+881e^3)\,x}{3125} + \frac{1}{625}\left(500d^3-2475d^2e+1215de^2+458e^3\right)x^2 \right.\\ &\quad + \frac{3}{125}e\left(100d^2-165de+27e^2\right)x^3 + \frac{3}{25}(20d-11e)e^2x^4 + \frac{4e^3x^5}{5}\\ &\quad \left. + \frac{875d^3-103050d^2e+39645de^2+15324e^3+(57250d^3-66075d^2e-76620de^2+23431e^3)\,x}{15625\,(3+2x+5x^2)}\right) dx\\ &= \frac{(10125d^3+34350d^2e-13215de^2-5108e^3)\,x}{15625}\\ &\quad - \frac{(4125d^3-6075d^2e-6870de^2+881e^3)\,x^2}{6250}\\ &\quad + \frac{(500d^3-2475d^2e+1215de^2+458e^3)\,x^3}{1875}\\ &\quad + \frac{3}{500}e\left(100d^2-165de+27e^2\right)x^4 + \frac{3}{125}(20d-11e)e^2x^5\\ &\quad + \frac{2e^3x^6}{15} + \frac{\int \frac{875d^3-103050d^2e+39645de^2+15324e^3+(57250d^3-66075d^2e-76620de^2+23431e^3)x}{3+2x+5x^2}\,dx}{15625}\\ &= \frac{(10125d^3+34350d^2e-13215de^2-5108e^3)\,x}{15625}\\ &\quad - \frac{(4125d^3-6075d^2e-6870de^2+881e^3)\,x^2}{6250}\\ &\quad + \frac{(500d^3-2475d^2e+1215de^2+458e^3)\,x^3}{1875}\\ &\quad + \frac{3}{500}e\left(100d^2-165de+27e^2\right)x^4 + \frac{3}{125}(20d-11e)e^2x^5\\ &\quad + \frac{2e^3x^6}{15} + \frac{(57250d^3-66075d^2e-76620de^2+23431e^3)\int \frac{2+10x}{3+2x+5x^2}\,dx}{156250}\\ &\quad + \frac{(-52875d^3-449175d^2e+274845de^2+53189e^3)\int \frac{1}{3+2x+5x^2}\,dx}{78125}\end{aligned}$$

$$
\begin{aligned}
&= \frac{(10125d^3+34350d^2e-13215de^2-5108e^3)\,x}{15625} \\
&\quad - \frac{(4125d^3-6075d^2e-6870de^2+881e^3)\,x^2}{6250} \\
&\quad + \frac{(500d^3-2475d^2e+1215de^2+458e^3)\,x^3}{1875} \\
&\quad + \frac{3}{500}e\big(100d^2-165de+27e^2\big)\,x^4 + \frac{3}{125}(20d-11e)e^2x^5 \\
&\quad + \frac{2e^3x^6}{15} + \frac{(57250d^3-66075d^2e-76620de^2+23431e^3)\log\left(3+2x+5x^2\right)}{156250} \\
&\quad + \frac{(2(52875d^3+449175d^2e-274845de^2-53189e^3))\,\mathrm{Subst}\left(\int \frac{1}{-56-x^2}\,dx, x, 2+10x\right)}{78125} \\
&= \frac{(10125d^3+34350d^2e-13215de^2-5108e^3)\,x}{15625} \\
&\quad - \frac{(4125d^3-6075d^2e-6870de^2+881e^3)\,x^2}{6250} \\
&\quad + \frac{(500d^3-2475d^2e+1215de^2+458e^3)\,x^3}{1875} \\
&\quad + \frac{3}{500}e\big(100d^2-165de+27e^2\big)\,x^4 + \frac{3}{125}(20d-11e)e^2x^5 \\
&\quad + \frac{2e^3x^6}{15} - \frac{(52875d^3+449175d^2e-274845de^2-53189e^3)\tan^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{78125\sqrt{14}} \\
&\quad + \frac{(57250d^3-66075d^2e-76620de^2+23431e^3)\log\left(3+2x+5x^2\right)}{156250}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.81

$$
\int \frac{(d+ex)^3\left(2+x+3x^2-5x^3+4x^4\right)}{3+2x+5x^2}\,dx
$$

$$
= 35x(250d^3(486-495x+200x^2)+450d^2e(916+405x-550x^2+250x^3)+45de^2(-3524+4580x+2700
$$

```
[In] Integrate[((d + e*x)^3*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2),x
]

[Out] (35*x*(250*d^3*(486 - 495*x + 200*x^2) + 450*d^2*e*(916 + 405*x - 550*x^2 +
 250*x^3) + 45*d*e^2*(-3524 + 4580*x + 2700*x^2 - 4125*x^3 + 2000*x^4) + e^
3*(-61296 - 26430*x + 45800*x^2 + 30375*x^3 - 49500*x^4 + 25000*x^5)) - 6*S
qrt[14]*(52875*d^3 + 449175*d^2*e - 274845*d*e^2 - 53189*e^3)*ArcTan[(1 + 5
*x)/Sqrt[14]] + 42*(57250*d^3 - 66075*d^2*e - 76620*d*e^2 + 23431*e^3)*Log[
3 + 2*x + 5*x^2])/6562500
```

Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.00

method	result
default	$\frac{2e^{3}x^{6}}{15}+\frac{12x^{5}d\,e^{2}}{25}-\frac{33x^{5}e^{3}}{125}+\frac{3x^{4}d^{2}e}{5}-\frac{99x^{4}d\,e^{2}}{100}+\frac{81e^{3}x^{4}}{500}+\frac{4d^{3}x^{3}}{15}-\frac{33d^{2}e\,x^{3}}{25}+\frac{81d\,e^{2}x^{3}}{125}+\frac{458e^{3}x^{3}}{1875}-\frac{33x^{2}d^{3}}{50}+$
risch	$-\frac{33d^{2}e\,x^{3}}{25}+\frac{12x^{5}d\,e^{2}}{25}+\frac{3x^{4}d^{2}e}{5}-\frac{99x^{4}d\,e^{2}}{100}+\frac{458e^{3}x^{3}}{1875}-\frac{17967\sqrt{14}\,d^{2}e\arctan\left(\frac{5\sqrt{14}\,x}{14}+\frac{\sqrt{14}}{14}\right)}{43750}+\frac{54969\sqrt{14}\,d\,e^{2}\arctan(}{218750}$

```
[In] int((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x,method=_RETURNVERBOSE
)
```

```
[Out] 2/15*e^3*x^6+12/25*x^5*d*e^2-33/125*x^5*e^3+3/5*x^4*d^2*e-99/100*x^4*d*e^2+
81/500*e^3*x^4+4/15*d^3*x^3-33/25*d^2*e*x^3+81/125*d*e^2*x^3+458/1875*e^3*x
^3-33/50*x^2*d^3+243/250*d^2*e*x^2+687/625*d*e^2*x^2-881/6250*e^3*x^2+81/12
5*x*d^3+1374/625*d^2*e*x-2643/3125*d*e^2*x-5108/15625*e^3*x+1/156250*(57250
*d^3-66075*d^2*e-76620*d*e^2+23431*e^3)*ln(5*x^2+2*x+3)+1/218750*(-10575*d^
3-89835*d^2*e+54969*d*e^2+53189/5*e^3)*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/
2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.93

$$\begin{aligned}\int \frac{(d+ex)^{3}\left(2+x+3x^{2}-5x^{3}+4x^{4}\right)}{3+2x+5x^{2}}\,dx &= \frac{2}{15}\,e^{3}x^{6}+\frac{3}{125}\left(20\,de^{2}-11\,e^{3}\right)x^{5}\\ &\quad+\frac{3}{500}\left(100\,d^{2}e-165\,de^{2}+27\,e^{3}\right)x^{4}+\frac{1}{1875}\left(500\,d^{3}-2475\,d^{2}e+1215\,de^{2}+458\,e^{3}\right)x^{3}\\ &\quad-\frac{1}{6250}\left(4125\,d^{3}-6075\,d^{2}e-6870\,de^{2}+881\,e^{3}\right)x^{2}\\ &\quad-\frac{1}{1093750}\sqrt{14}\left(52875\,d^{3}+449175\,d^{2}e-274845\,de^{2}-53189\,e^{3}\right)\arctan\left(\frac{1}{14}\sqrt{14}(5\,x+1)\right)\\ &\quad+\frac{1}{15625}\left(10125\,d^{3}+34350\,d^{2}e-13215\,de^{2}-5108\,e^{3}\right)x\\ &\quad+\frac{1}{156250}\left(57250\,d^{3}-66075\,d^{2}e-76620\,de^{2}+23431\,e^{3}\right)\log\left(5\,x^{2}+2\,x+3\right)\end{aligned}$$

```
[In] integrate((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="fri
cas")
```

```
[Out] 2/15*e^3*x^6 + 3/125*(20*d*e^2 - 11*e^3)*x^5 + 3/500*(100*d^2*e - 165*d*e^2
 + 27*e^3)*x^4 + 1/1875*(500*d^3 - 2475*d^2*e + 1215*d*e^2 + 458*e^3)*x^3 -
 1/6250*(4125*d^3 - 6075*d^2*e - 6870*d*e^2 + 881*e^3)*x^2 - 1/1093750*sqrt
(14)*(52875*d^3 + 449175*d^2*e - 274845*d*e^2 - 53189*e^3)*arctan(1/14*sqrt
```

```
(14)*(5*x + 1)) + 1/15625*(10125*d^3 + 34350*d^2*e - 13215*d*e^2 - 5108*e^3
)*x + 1/156250*(57250*d^3 - 66075*d^2*e - 76620*d*e^2 + 23431*e^3)*log(5*x^
2 + 2*x + 3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.75 (sec) , antiderivative size = 450, normalized size of antiderivative = 2.04

$$
\begin{aligned}
&\int \frac{(d+ex)^3\,(2+x+3x^2-5x^3+4x^4)}{3+2x+5x^2}\,dx \\
&= \frac{2e^3x^6}{15} + x^5\cdot\left(\frac{12de^2}{25} - \frac{33e^3}{125}\right) + x^4\cdot\left(\frac{3d^2e}{5} - \frac{99de^2}{100} + \frac{81e^3}{500}\right) \\
&\quad + x^3\cdot\left(\frac{4d^3}{15} - \frac{33d^2e}{25} + \frac{81de^2}{125} + \frac{458e^3}{1875}\right) + x^2\left(-\frac{33d^3}{50} + \frac{243d^2e}{250} + \frac{687de^2}{625} - \frac{881e^3}{6250}\right) \\
&\quad + x\left(\frac{81d^3}{125} + \frac{1374d^2e}{625} - \frac{2643de^2}{3125} - \frac{5108e^3}{15625}\right) + \left(\frac{229d^3}{625} - \frac{2643d^2e}{6250} - \frac{7662de^2}{15625} + \frac{23431e^3}{156250}\right. \\
&\quad \left. - \frac{\sqrt{14}i(52875d^3+449175d^2e-274845de^2-53189e^3)}{2187500}\right)\log\left(x + \frac{10575d^3+89835d^2e-54969de^2-}{52875d^3+449175d}\right. \\
&\quad + \left(\frac{229d^3}{625} - \frac{2643d^2e}{6250} - \frac{7662de^2}{15625} + \frac{23431e^3}{156250}\right. \\
&\quad \left. + \frac{\sqrt{14}i(52875d^3+449175d^2e-274845de^2-53189e^3)}{2187500}\right)\log\left(x + \frac{10575d^3+89835d^2e-54969de^2-}{52875d^3+449175d}\right.
\end{aligned}
$$

```
[In] integrate((e*x+d)**3*(4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3),x)
```

```
[Out] 2*e**3*x**6/15 + x**5*(12*d*e**2/25 - 33*e**3/125) + x**4*(3*d**2*e/5 - 99*
d*e**2/100 + 81*e**3/500) + x**3*(4*d**3/15 - 33*d**2*e/25 + 81*d*e**2/125
+ 458*e**3/1875) + x**2*(-33*d**3/50 + 243*d**2*e/250 + 687*d*e**2/625 - 88
1*e**3/6250) + x*(81*d**3/125 + 1374*d**2*e/625 - 2643*d*e**2/3125 - 5108*e
**3/15625) + (229*d**3/625 - 2643*d**2*e/6250 - 7662*d*e**2/15625 + 23431*e
**3/156250 - sqrt(14)*I*(52875*d**3 + 449175*d**2*e - 274845*d*e**2 - 53189
*e**3)/2187500)*log(x + (10575*d**3 + 89835*d**2*e - 54969*d*e**2 - 53189*e
**3/5 + sqrt(14)*I*(52875*d**3 + 449175*d**2*e - 274845*d*e**2 - 53189*e**3
)/5)/(52875*d**3 + 449175*d**2*e - 274845*d*e**2 - 53189*e**3)) + (229*d**3
/625 - 2643*d**2*e/6250 - 7662*d*e**2/15625 + 23431*e**3/156250 + sqrt(14)*
I*(52875*d**3 + 449175*d**2*e - 274845*d*e**2 - 53189*e**3)/2187500)*log(x
+ (10575*d**3 + 89835*d**2*e - 54969*d*e**2 - 53189*e**3/5 - sqrt(14)*I*(52
875*d**3 + 449175*d**2*e - 274845*d*e**2 - 53189*e**3)/5)/(52875*d**3 + 449
175*d**2*e - 274845*d*e**2 - 53189*e**3))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.93

$$\begin{aligned}\int \frac{(d+ex)^3\,(2+x+3x^2-5x^3+4x^4)}{3+2x+5x^2}\,dx &= \frac{2}{15}\,e^3x^6+\frac{3}{125}\,\left(20\,de^2-11\,e^3\right)x^5\\ &\quad+\frac{3}{500}\,\left(100\,d^2e-165\,de^2+27\,e^3\right)x^4+\frac{1}{1875}\,\left(500\,d^3-2475\,d^2e+1215\,de^2+458\,e^3\right)x^3\\ &\quad-\frac{1}{6250}\,\left(4125\,d^3-6075\,d^2e-6870\,de^2+881\,e^3\right)x^2\\ &\quad-\frac{1}{1093750}\,\sqrt{14}\left(52875\,d^3+449175\,d^2e-274845\,de^2-53189\,e^3\right)\arctan\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right)\\ &\quad+\frac{1}{15625}\,\left(10125\,d^3+34350\,d^2e-13215\,de^2-5108\,e^3\right)x\\ &\quad+\frac{1}{156250}\,\left(57250\,d^3-66075\,d^2e-76620\,de^2+23431\,e^3\right)\log\left(5\,x^2+2\,x+3\right)\end{aligned}$$

```
[In] integrate((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="max
ima")
```

```
[Out] 2/15*e^3*x^6 + 3/125*(20*d*e^2 - 11*e^3)*x^5 + 3/500*(100*d^2*e - 165*d*e^2
 + 27*e^3)*x^4 + 1/1875*(500*d^3 - 2475*d^2*e + 1215*d*e^2 + 458*e^3)*x^3 -
 1/6250*(4125*d^3 - 6075*d^2*e - 6870*d*e^2 + 881*e^3)*x^2 - 1/1093750*sqrt
(14)*(52875*d^3 + 449175*d^2*e - 274845*d*e^2 - 53189*e^3)*arctan(1/14*sqrt
(14)*(5*x + 1)) + 1/15625*(10125*d^3 + 34350*d^2*e - 13215*d*e^2 - 5108*e^3
)*x + 1/156250*(57250*d^3 - 66075*d^2*e - 76620*d*e^2 + 23431*e^3)*log(5*x^
2 + 2*x + 3)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.00

$$\begin{aligned}\int \frac{(d+ex)^3\,(2+x+3x^2-5x^3+4x^4)}{3+2x+5x^2}\,dx &= \frac{2}{15}\,e^3x^6+\frac{12}{25}\,de^2x^5-\frac{33}{125}\,e^3x^5+\frac{3}{5}\,d^2ex^4\\ &\quad-\frac{99}{100}\,de^2x^4+\frac{81}{500}\,e^3x^4+\frac{4}{15}\,d^3x^3-\frac{33}{25}\,d^2ex^3+\frac{81}{125}\,de^2x^3+\frac{458}{1875}\,e^3x^3-\frac{33}{50}\,d^3x^2\\ &\quad+\frac{243}{250}\,d^2ex^2+\frac{687}{625}\,de^2x^2-\frac{881}{6250}\,e^3x^2+\frac{81}{125}\,d^3x+\frac{1374}{625}\,d^2ex-\frac{2643}{3125}\,de^2x-\frac{5108}{15625}\,e^3x\\ &\quad-\frac{1}{1093750}\,\sqrt{14}\left(52875\,d^3+449175\,d^2e-274845\,de^2-53189\,e^3\right)\arctan\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right)\\ &\quad+\frac{1}{156250}\,\left(57250\,d^3-66075\,d^2e-76620\,de^2+23431\,e^3\right)\log\left(5\,x^2+2\,x+3\right)\end{aligned}$$

```
[In] integrate((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="gia
c")
```

```
[Out] 2/15*e^3*x^6 + 12/25*d*e^2*x^5 - 33/125*e^3*x^5 + 3/5*d^2*e*x^4 - 99/100*d*
e^2*x^4 + 81/500*e^3*x^4 + 4/15*d^3*x^3 - 33/25*d^2*e*x^3 + 81/125*d*e^2*x^
3 + 458/1875*e^3*x^3 - 33/50*d^3*x^2 + 243/250*d^2*e*x^2 + 687/625*d*e^2*x^
2 - 881/6250*e^3*x^2 + 81/125*d^3*x + 1374/625*d^2*e*x - 2643/3125*d*e^2*x
- 5108/15625*e^3*x - 1/1093750*sqrt(14)*(52875*d^3 + 449175*d^2*e - 274845*
d*e^2 - 53189*e^3)*arctan(1/14*sqrt(14)*(5*x + 1)) + 1/156250*(57250*d^3 -
66075*d^2*e - 76620*d*e^2 + 23431*e^3)*log(5*x^2 + 2*x + 3)
```

Mupad [B] (verification not implemented)

Time = 13.40 (sec) , antiderivative size = 397, normalized size of antiderivative = 1.80

$$
\begin{aligned}
&\int \frac{(d+ex)^3 \left(2+x+3x^2-5x^3+4x^4\right)}{3+2x+5x^2}\,dx \\
&= x^2 \left(\frac{26\,e^2\,(12\,d-5\,e)}{625}-\frac{33\,e\,(4\,d^2-5\,d\,e+e^2)}{250}-\frac{3\,d\,e^2}{50}+\frac{3\,d^2\,e}{2}-\frac{33\,d^3}{50}+\frac{622\,e^3}{3125}\right) \\
&\quad - x^3 \left(\frac{11\,e^2\,(12\,d-5\,e)}{375}+\frac{2\,e\,(4\,d^2-5\,d\,e+e^2)}{25}-\frac{3\,d\,e^2}{5}+d^2\,e-\frac{4\,d^3}{15}-\frac{111\,e^3}{625}\right) \\
&\quad + x^5 \left(\frac{e^2\,(12\,d-5\,e)}{25}-\frac{8\,e^3}{125}\right) \\
&\quad - \ln\left(5\,x^2+2\,x+3\right) \left(-\frac{229\,d^3}{625}+\frac{2643\,d^2\,e}{6250}+\frac{7662\,d\,e^2}{15625}-\frac{23431\,e^3}{156250}\right) \\
&\quad - x^4 \left(\frac{e^2\,(12\,d-5\,e)}{50}-\frac{3\,e\,(4\,d^2-5\,d\,e+e^2)}{20}+\frac{11\,e^3}{125}\right)+\frac{2\,e^3\,x^6}{15}+x \left(\frac{61\,e^2\,(12\,d-5\,e)}{3125}\right. \\
&\qquad \left. +\frac{3\,d\,(d^2+d\,e+2\,e^2)}{5}+\frac{156\,e\,(4\,d^2-5\,d\,e+e^2)}{625}-\frac{129\,d\,e^2}{125}+\frac{3\,d^2\,e}{5}+\frac{6\,d^3}{125}-\frac{7483\,e^3}{15625}\right) \\
&\quad + \frac{\sqrt{14}\,\mathrm{atan}\left(\frac{\frac{\sqrt{14}\,\left(-52875\,d^3-449175\,d^2\,e+274845\,d\,e^2+53189\,e^3\right)}{1093750}+\frac{\sqrt{14}\,x\,\left(-52875\,d^3-449175\,d^2\,e+274845\,d\,e^2+53189\,e^3\right)}{218750}}{-\frac{423\,d^3}{625}-\frac{17967\,d^2\,e}{3125}+\frac{54969\,d\,e^2}{15625}+\frac{53189\,e^3}{78125}}\right)\,\left(-52875\,d^3-4\right.}{1093750}
\end{aligned}
$$

```
[In] int(((d + e*x)^3*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(2*x + 5*x^2 + 3),x)
```

```
[Out] x^2*((26*e^2*(12*d - 5*e))/625 - (33*e*(4*d^2 - 5*d*e + e^2))/250 - (3*d*e^
2)/50 + (3*d^2*e)/2 - (33*d^3)/50 + (622*e^3)/3125) - x^3*((11*e^2*(12*d -
5*e))/375 + (2*e*(4*d^2 - 5*d*e + e^2))/25 - (3*d*e^2)/5 + d^2*e - (4*d^3)/
15 - (111*e^3)/625) + x^5*((e^2*(12*d - 5*e))/25 - (8*e^3)/125) - log(2*x +
 5*x^2 + 3)*((7662*d*e^2)/15625 + (2643*d^2*e)/6250 - (229*d^3)/625 - (2343
1*e^3)/156250) - x^4*((e^2*(12*d - 5*e))/50 - (3*e*(4*d^2 - 5*d*e + e^2))/2
0 + (11*e^3)/125) + (2*e^3*x^6)/15 + x*((61*e^2*(12*d - 5*e))/3125 + (3*d*(
d*e + d^2 + 2*e^2))/5 + (156*e*(4*d^2 - 5*d*e + e^2))/625 - (129*d*e^2)/125
 + (3*d^2*e)/5 + (6*d^3)/125 - (7483*e^3)/15625) + (14^(1/2)*atan(((14^(1/2
```

```
)*(274845*d*e^2 - 449175*d^2*e - 52875*d^3 + 53189*e^3))/1093750 + (14^(1/2
)*x*(274845*d*e^2 - 449175*d^2*e - 52875*d^3 + 53189*e^3))/218750)/((54969*
d*e^2)/15625 - (17967*d^2*e)/3125 - (423*d^3)/625 + (53189*e^3)/78125))*(27
4845*d*e^2 - 449175*d^2*e - 52875*d^3 + 53189*e^3))/1093750
```

3.305 $\int \frac{(d+ex)^2(2+x+3x^2-5x^3+4x^4)}{3+2x+5x^2} dx$

Optimal result

Integrand size = 38, antiderivative size = 156

$$\begin{aligned}\int \frac{(d+ex)^2\,(2+x+3x^2-5x^3+4x^4)}{3+2x+5x^2}\,dx &= \frac{(2025d^2+4580de-881e^2)\,x}{3125}-\frac{(825d^2-810de-458e^2)\,x^2}{1250}\\ &\quad+\frac{1}{375}\left(100d^2-330de+81e^2\right)x^3+\frac{1}{100}(40d-33e)ex^4\\ &\quad+\frac{4e^2x^5}{25}-\frac{(10575d^2+59890de-18323e^2)\arctan\left(\frac{1+5x}{\sqrt{14}}\right)}{15625\sqrt{14}}\\ &\quad+\frac{(5725d^2-4405de-2554e^2)\log\left(3+2x+5x^2\right)}{15625}\end{aligned}$$

```
[Out] 1/3125*(2025*d^2+4580*d*e-881*e^2)*x-1/1250*(825*d^2-810*d*e-458*e^2)*x^2+1
/375*(100*d^2-330*d*e+81*e^2)*x^3+1/100*(40*d-33*e)*e*x^4+4/25*e^2*x^5+1/15
625*(5725*d^2-4405*d*e-2554*e^2)*ln(5*x^2+2*x+3)-1/218750*(10575*d^2+59890*
d*e-18323*e^2)*arctan(1/14*(1+5*x)*14^(1/2))*14^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used

$= \{1642,\ 648,\ 632,\ 210,\ 642\}$

$$\int \frac{(d+ex)^2\,(2+x+3x^2-5x^3+4x^4)}{3+2x+5x^2}\,dx$$
$$= -\frac{\arctan\left(\frac{5x+1}{\sqrt{14}}\right)(10575d^2+59890de-18323e^2)}{15625\sqrt{14}} + \frac{1}{375}x^3\left(100d^2-330de+81e^2\right)$$
$$- \frac{x^2(825d^2-810de-458e^2)}{1250} + \frac{(5725d^2-4405de-2554e^2)\log\left(5x^2+2x+3\right)}{15625}$$
$$+ \frac{x(2025d^2+4580de-881e^2)}{3125} + \frac{1}{100}ex^4(40d-33e) + \frac{4e^2x^5}{25}$$

```
[In] Int[((d + e*x)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2),x]
```

```
[Out] ((2025*d^2 + 4580*d*e - 881*e^2)*x)/3125 - ((825*d^2 - 810*d*e - 458*e^2)*x
^2)/1250 + ((100*d^2 - 330*d*e + 81*e^2)*x^3)/375 + ((40*d - 33*e)*e*x^4)/1
00 + (4*e^2*x^5)/25 - ((10575*d^2 + 59890*d*e - 18323*e^2)*ArcTan[(1 + 5*x)
/Sqrt[14]])/(15625*Sqrt[14]) + ((5725*d^2 - 4405*d*e - 2554*e^2)*Log[3 + 2*
x + 5*x^2])/15625
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{2025d^2+4580de-881e^2}{3125}-\frac{1}{625}\left(825d^2-810de-458e^2\right)x+\frac{1}{125}\left(100d^2-330de+81e^2\right)x^2+\frac{1}{25}(40d-33e)ex^3+\frac{4e^2x^4}{5}+\frac{175d^2-13740de+2643e^2+2\left(5725d^2-4405de-2554e^2\right)x}{3125\left(3+2x+5x^2\right)}\right)dx\\
&= \frac{\left(2025d^2+4580de-881e^2\right)x}{3125}-\frac{\left(825d^2-810de-458e^2\right)x^2}{1250}+\frac{1}{375}\left(100d^2-330de+81e^2\right)x^3+\frac{1}{100}(40d-33e)ex^4+\frac{4e^2x^5}{25}+\frac{\int \frac{175d^2-13740de+2643e^2+2\left(5725d^2-4405de-2554e^2\right)x}{3+2x+5x^2}\,dx}{3125}\\
&= \frac{\left(2025d^2+4580de-881e^2\right)x}{3125}-\frac{\left(825d^2-810de-458e^2\right)x^2}{1250}+\frac{1}{375}\left(100d^2-330de+81e^2\right)x^3+\frac{1}{100}(40d-33e)ex^4+\frac{4e^2x^5}{25}+\frac{\left(5725d^2-4405de-2554e^2\right)\int \frac{2+10x}{3+2x+5x^2}\,dx}{15625}+\frac{\left(-10575d^2-59890de+18323e^2\right)\int \frac{1}{3+2x+5x^2}\,dx}{15625}\\
&= \frac{\left(2025d^2+4580de-881e^2\right)x}{3125}-\frac{\left(825d^2-810de-458e^2\right)x^2}{1250}+\frac{1}{375}\left(100d^2-330de+81e^2\right)x^3+\frac{1}{100}(40d-33e)ex^4+\frac{4e^2x^5}{25}+\frac{\left(5725d^2-4405de-2554e^2\right)\log\left(3+2x+5x^2\right)}{15625}+\frac{\left(2\left(10575d^2+59890de-18323e^2\right)\right)\text{Subst}\left(\int \frac{1}{-56-x^2}\,dx,x,2+10x\right)}{15625}\\
&= \frac{\left(2025d^2+4580de-881e^2\right)x}{3125}-\frac{\left(825d^2-810de-458e^2\right)x^2}{1250}+\frac{1}{375}\left(100d^2-330de+81e^2\right)x^3+\frac{1}{100}(40d-33e)ex^4+\frac{4e^2x^5}{25}-\frac{\left(10575d^2+59890de-18323e^2\right)\tan^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{15625\sqrt{14}}+\frac{\left(5725d^2-4405de-2554e^2\right)\log\left(3+2x+5x^2\right)}{15625}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.83

$$\int \frac{(d+ex)^2\left(2+x+3x^2-5x^3+4x^4\right)}{3+2x+5x^2}\,dx$$
$$=\frac{35x(50d^2(486-495x+200x^2)+60de(916+405x-550x^2+250x^3)+3e^2(-3524+4580x+2700x^2-4}{}$$

```
[In] Integrate[((d + e*x)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2),x
]
```

```
[Out] (35*x*(50*d^2*(486 - 495*x + 200*x^2) + 60*d*e*(916 + 405*x - 550*x^2 + 250
*x^3) + 3*e^2*(-3524 + 4580*x + 2700*x^2 - 4125*x^3 + 2000*x^4)) - 6*Sqrt[1
4]*(10575*d^2 + 59890*d*e - 18323*e^2)*ArcTan[(1 + 5*x)/Sqrt[14]] + 84*(572
5*d^2 - 4405*d*e - 2554*e^2)*Log[3 + 2*x + 5*x^2])/1312500
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.94

method	result
default	$\frac{4e^2x^5}{25}+\frac{2x^4de}{5}-\frac{33x^4e^2}{100}+\frac{4x^3d^2}{15}-\frac{22x^3de}{25}+\frac{27x^3e^2}{125}-\frac{33d^2x^2}{50}+\frac{81de\,x^2}{125}+\frac{229e^2x^2}{625}+\frac{81x\,d^2}{125}+\frac{916dex}{625}-\frac{881e^2x}{3125}$
risch	$\frac{2x^4de}{5}-\frac{22x^3de}{25}-\frac{5989\sqrt{14}\,de\arctan\left(\frac{5\sqrt{14}\,x}{14}+\frac{\sqrt{14}}{14}\right)}{21875}+\frac{229e^2x^2}{625}-\frac{881e^2x}{3125}-\frac{33d^2x^2}{50}-\frac{33x^4e^2}{100}+\frac{4x^3d^2}{15}-\frac{881de\ln(350}{3}$

```
[In] int((e*x+d)^2*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x,method=_RETURNVERBOSE
)
```

```
[Out] 4/25*e^2*x^5+2/5*x^4*d*e-33/100*x^4*e^2+4/15*x^3*d^2-22/25*x^3*d*e+27/125*x
^3*e^2-33/50*d^2*x^2+81/125*d*e*x^2+229/625*e^2*x^2+81/125*x*d^2+916/625*d*
e*x-881/3125*e^2*x+1/31250*(11450*d^2-8810*d*e-5108*e^2)*ln(5*x^2+2*x+3)+1/
43750*(-2115*d^2-11978*d*e+18323/5*e^2)*14^(1/2)*arctan(1/28*(10*x+2)*14^(1
/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.90

$$\begin{aligned}&\int \frac{(d+ex)^2\left(2+x+3x^2-5x^3+4x^4\right)}{3+2x+5x^2}\,dx\\&=\frac{4}{25}\,e^2x^5+\frac{1}{100}\,\left(40\,de-33\,e^2\right)x^4+\frac{1}{375}\,\left(100\,d^2-330\,de+81\,e^2\right)x^3\\&\quad-\frac{1}{1250}\,\left(825\,d^2-810\,de-458\,e^2\right)x^2\\&\quad-\frac{1}{218750}\,\sqrt{14}\left(10575\,d^2+59890\,de-18323\,e^2\right)\arctan\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right)\\&\quad+\frac{1}{3125}\,\left(2025\,d^2+4580\,de-881\,e^2\right)x\\&\quad+\frac{1}{15625}\,\left(5725\,d^2-4405\,de-2554\,e^2\right)\log\left(5\,x^2+2\,x+3\right)\end{aligned}$$

```
[In] integrate((e*x+d)^2*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="fri
cas")
```

```
[Out] 4/25*e^2*x^5 + 1/100*(40*d*e - 33*e^2)*x^4 + 1/375*(100*d^2 - 330*d*e + 81*
e^2)*x^3 - 1/1250*(825*d^2 - 810*d*e - 458*e^2)*x^2 - 1/218750*sqrt(14)*(10
575*d^2 + 59890*d*e - 18323*e^2)*arctan(1/14*sqrt(14)*(5*x + 1)) + 1/3125*(
2025*d^2 + 4580*d*e - 881*e^2)*x + 1/15625*(5725*d^2 - 4405*d*e - 2554*e^2)
*log(5*x^2 + 2*x + 3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.94

$$\begin{aligned}&\int \frac{(d+ex)^2\left(2+x+3x^2-5x^3+4x^4\right)}{3+2x+5x^2}\,dx\\&=\frac{4e^2x^5}{25}+x^4\cdot\left(\frac{2de}{5}-\frac{33e^2}{100}\right)+x^3\cdot\left(\frac{4d^2}{15}-\frac{22de}{25}+\frac{27e^2}{125}\right)+x^2\left(-\frac{33d^2}{50}+\frac{81de}{125}+\frac{229e^2}{625}\right)\\&\quad+x\left(\frac{81d^2}{125}+\frac{916de}{625}-\frac{881e^2}{3125}\right)+\left(\frac{229d^2}{625}-\frac{881de}{3125}-\frac{2554e^2}{15625}\right.\\&\quad\left.-\frac{\sqrt{14}i\left(10575d^2+59890de-18323e^2\right)}{437500}\right)\log\left(x+\frac{2115d^2+11978de-\frac{18323e^2}{5}+\frac{\sqrt{14}i(10575d^2+59890de-1832}{5}}{10575d^2+59890de-18323e^2}\right.\\&\quad+\left(\frac{229d^2}{625}-\frac{881de}{3125}-\frac{2554e^2}{15625}\right.\\&\quad\left.+\frac{\sqrt{14}i\left(10575d^2+59890de-18323e^2\right)}{437500}\right)\log\left(x+\frac{2115d^2+11978de-\frac{18323e^2}{5}-\frac{\sqrt{14}i(10575d^2+59890de-1832}{5}}{10575d^2+59890de-18323e^2}\right.\end{aligned}$$

```
[In] integrate((e*x+d)**2*(4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3),x)
```

```
[Out] 4*e**2*x**5/25 + x**4*(2*d*e/5 - 33*e**2/100) + x**3*(4*d**2/15 - 22*d*e/25
 + 27*e**2/125) + x**2*(-33*d**2/50 + 81*d*e/125 + 229*e**2/625) + x*(81*d*
*2/125 + 916*d*e/625 - 881*e**2/3125) + (229*d**2/625 - 881*d*e/3125 - 2554
*e**2/15625 - sqrt(14)*I*(10575*d**2 + 59890*d*e - 18323*e**2)/437500)*log(
x + (2115*d**2 + 11978*d*e - 18323*e**2/5 + sqrt(14)*I*(10575*d**2 + 59890*
d*e - 18323*e**2)/5)/(10575*d**2 + 59890*d*e - 18323*e**2)) + (229*d**2/625
 - 881*d*e/3125 - 2554*e**2/15625 + sqrt(14)*I*(10575*d**2 + 59890*d*e - 18
323*e**2)/437500)*log(x + (2115*d**2 + 11978*d*e - 18323*e**2/5 - sqrt(14)*
I*(10575*d**2 + 59890*d*e - 18323*e**2)/5)/(10575*d**2 + 59890*d*e - 18323*
e**2))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.90

$$\begin{aligned}
\int \frac{(d+ex)^2\left(2+x+3x^2-5x^3+4x^4\right)}{3+2x+5x^2}\,dx
&= \frac{4}{25}\,e^2x^5 + \frac{1}{100}\,\left(40\,de-33\,e^2\right)x^4 + \frac{1}{375}\,\left(100\,d^2-330\,de+81\,e^2\right)x^3 \\
&\quad - \frac{1}{1250}\,\left(825\,d^2-810\,de-458\,e^2\right)x^2 \\
&\quad - \frac{1}{218750}\,\sqrt{14}\left(10575\,d^2+59890\,de-18323\,e^2\right)\arctan\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right) \\
&\quad + \frac{1}{3125}\,\left(2025\,d^2+4580\,de-881\,e^2\right)x \\
&\quad + \frac{1}{15625}\,\left(5725\,d^2-4405\,de-2554\,e^2\right)\log\left(5\,x^2+2\,x+3\right)
\end{aligned}$$

```
[In] integrate((e*x+d)^2*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="max
ima")
```

```
[Out] 4/25*e^2*x^5 + 1/100*(40*d*e - 33*e^2)*x^4 + 1/375*(100*d^2 - 330*d*e + 81*
e^2)*x^3 - 1/1250*(825*d^2 - 810*d*e - 458*e^2)*x^2 - 1/218750*sqrt(14)*(10
575*d^2 + 59890*d*e - 18323*e^2)*arctan(1/14*sqrt(14)*(5*x + 1)) + 1/3125*(
2025*d^2 + 4580*d*e - 881*e^2)*x + 1/15625*(5725*d^2 - 4405*d*e - 2554*e^2)
*log(5*x^2 + 2*x + 3)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.94

$$\begin{aligned}\int \frac{(d+ex)^2 (2+x+3x^2-5x^3+4x^4)}{3+2x+5x^2}\,dx &= \frac{4}{25}\,e^2x^5+\frac{2}{5}\,dex^4-\frac{33}{100}\,e^2x^4+\frac{4}{15}\,d^2x^3-\frac{22}{25}\,dex^3+\frac{27}{125}\,e^2x^3\\ &\quad-\frac{33}{50}\,d^2x^2+\frac{81}{125}\,dex^2+\frac{229}{625}\,e^2x^2+\frac{81}{125}\,d^2x+\frac{916}{625}\,dex-\frac{881}{3125}\,e^2x\\ &\quad-\frac{1}{218750}\,\sqrt{14}\big(10575\,d^2+59890\,de-18323\,e^2\big)\arctan\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right)\\ &\quad+\frac{1}{15625}\,\big(5725\,d^2-4405\,de-2554\,e^2\big)\log\left(5\,x^2+2\,x+3\right)\end{aligned}$$

```
[In] integrate((e*x+d)^2*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="gia
c")
```

```
[Out] 4/25*e^2*x^5 + 2/5*d*e*x^4 - 33/100*e^2*x^4 + 4/15*d^2*x^3 - 22/25*d*e*x^3
+ 27/125*e^2*x^3 - 33/50*d^2*x^2 + 81/125*d*e*x^2 + 229/625*e^2*x^2 + 81/12
5*d^2*x + 916/625*d*e*x - 881/3125*e^2*x - 1/218750*sqrt(14)*(10575*d^2 + 5
9890*d*e - 18323*e^2)*arctan(1/14*sqrt(14)*(5*x + 1)) + 1/15625*(5725*d^2 -
 4405*d*e - 2554*e^2)*log(5*x^2 + 2*x + 3)
```

Mupad [B] (verification not implemented)

Time = 13.74 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.43

$$\begin{aligned}\int \frac{(d+ex)^2 (2+x+3x^2-5x^3+4x^4)}{3+2x+5x^2}\,dx &= x\left(\frac{4\,d\,e}{5}+\frac{52\,e\,(8\,d-5\,e)}{625}+\frac{81\,d^2}{125}+\frac{419\,e^2}{3125}\right)\\ &\quad-\ln\left(5\,x^2+2\,x+3\right)\left(-\frac{229\,d^2}{625}+\frac{881\,d\,e}{3125}+\frac{2554\,e^2}{15625}\right)\\ &\quad+x^4\left(\frac{e\,(8\,d-5\,e)}{20}-\frac{2\,e^2}{25}\right)-x^3\left(\frac{2\,d\,e}{3}+\frac{2\,e\,(8\,d-5\,e)}{75}-\frac{4\,d^2}{15}-\frac{31\,e^2}{375}\right)\\ &\quad+x^2\left(d\,e-\frac{11\,e\,(8\,d-5\,e)}{250}-\frac{33\,d^2}{50}+\frac{183\,e^2}{1250}\right)+\frac{4\,e^2\,x^5}{25}\\ &\quad-\frac{\sqrt{14}\,\mathrm{atan}\left(\frac{\frac{\sqrt{14}\,\left(10575\,d^2+59890\,d\,e-18323\,e^2\right)}{218750}+\frac{\sqrt{14}\,x\,\left(10575\,d^2+59890\,d\,e-18323\,e^2\right)}{43750}}{\frac{423\,d^2}{625}+\frac{11978\,d\,e}{3125}-\frac{18323\,e^2}{15625}}\right)\,\left(10575\,d^2+59890\,d\,e-18323\,e^2\right)}{218750}\end{aligned}$$

```
[In] int(((d + e*x)^2*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(2*x + 5*x^2 + 3),x)
```

```
[Out] x*((4*d*e)/5 + (52*e*(8*d - 5*e))/625 + (81*d^2)/125 + (419*e^2)/3125) - lo
g(2*x + 5*x^2 + 3)*((881*d*e)/3125 - (229*d^2)/625 + (2554*e^2)/15625) + x^
4*((e*(8*d - 5*e))/20 - (2*e^2)/25) - x^3*((2*d*e)/3 + (2*e*(8*d - 5*e))/75
 - (4*d^2)/15 - (31*e^2)/375) + x^2*(d*e - (11*e*(8*d - 5*e))/250 - (33*d^2
)/50 + (183*e^2)/1250) + (4*e^2*x^5)/25 - (14^(1/2)*atan(((14^(1/2)*(59890*
d*e + 10575*d^2 - 18323*e^2))/218750 + (14^(1/2)*x*(59890*d*e + 10575*d^2 -
 18323*e^2))/43750)/((11978*d*e)/3125 + (423*d^2)/625 - (18323*e^2)/15625))
*(59890*d*e + 10575*d^2 - 18323*e^2))/218750
```

3.306 $\int \frac{(d+ex)(2+x+3x^2-5x^3+4x^4)}{3+2x+5x^2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 99

$$\begin{aligned}\int \frac{(d+ex)\left(2+x+3x^2-5x^3+4x^4\right)}{3+2x+5x^2}\,dx &= \frac{1}{625}(405d+458e)x-\frac{3}{250}(55d-27e)x^2\\&\quad+\frac{1}{75}(20d-33e)x^3+\frac{ex^4}{5}\\&\quad-\frac{(2115d+5989e)\arctan\left(\frac{1+5x}{\sqrt{14}}\right)}{3125\sqrt{14}}\\&\quad+\frac{(2290d-881e)\log\left(3+2x+5x^2\right)}{6250}\end{aligned}$$

```
[Out] 1/625*(405*d+458*e)*x-3/250*(55*d-27*e)*x^2+1/75*(20*d-33*e)*x^3+1/5*e*x^4+
1/6250*(2290*d-881*e)*ln(5*x^2+2*x+3)-1/43750*(2115*d+5989*e)*arctan(1/14*(
1+5*x)*14^(1/2))*14^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$, Rules used = {1642, 648, 632, 210, 642}

$$\begin{aligned}\int \frac{(d+ex)\left(2+x+3x^2-5x^3+4x^4\right)}{3+2x+5x^2}\,dx &= -\frac{\arctan\left(\frac{5x+1}{\sqrt{14}}\right)(2115d+5989e)}{3125\sqrt{14}}\\&\quad+\frac{1}{75}x^3(20d-33e)-\frac{3}{250}x^2(55d-27e)\\&\quad+\frac{(2290d-881e)\log\left(5x^2+2x+3\right)}{6250}\\&\quad+\frac{1}{625}x(405d+458e)+\frac{ex^4}{5}\end{aligned}$$

```
[In] Int[((d + e*x)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2),x]
```

```
[Out] ((405*d + 458*e)*x)/625 - (3*(55*d - 27*e)*x^2)/250 + ((20*d - 33*e)*x^3)/7
5 + (e*x^4)/5 - ((2115*d + 5989*e)*ArcTan[(1 + 5*x)/Sqrt[14]])/(3125*Sqrt[1
4]) + ((2290*d - 881*e)*Log[3 + 2*x + 5*x^2])/6250
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{625}(405d+458e)-\frac{3}{125}(55d-27e)x+\frac{1}{25}(20d-33e)x^2+\frac{4ex^3}{5}\right.\\
&\qquad\left.+\frac{35d-1374e+(2290d-881e)x}{625\left(3+2x+5x^2\right)}\right)dx\\
&=\frac{1}{625}(405d+458e)x-\frac{3}{250}(55d-27e)x^2+\frac{1}{75}(20d-33e)x^3\\
&\quad+\frac{ex^4}{5}+\frac{1}{625}\int\frac{35d-1374e+(2290d-881e)x}{3+2x+5x^2}\,dx
\end{aligned}$$

$$
\begin{aligned}
&= \frac{1}{625}(405d+458e)x - \frac{3}{250}(55d-27e)x^2 + \frac{1}{75}(20d-33e)x^3 + \frac{ex^4}{5} \\
&\quad + \frac{(-2115d-5989e)\int \frac{1}{3+2x+5x^2}\,dx}{3125} + \frac{(2290d-881e)\int \frac{2+10x}{3+2x+5x^2}\,dx}{6250} \\
&= \frac{1}{625}(405d+458e)x - \frac{3}{250}(55d-27e)x^2 + \frac{1}{75}(20d-33e)x^3 \\
&\quad + \frac{ex^4}{5} + \frac{(2290d-881e)\log\left(3+2x+5x^2\right)}{6250} \\
&\quad + \frac{(2(2115d+5989e))\text{Subst}\left(\int \frac{1}{-56-x^2}\,dx, x, 2+10x\right)}{3125} \\
&= \frac{1}{625}(405d+458e)x - \frac{3}{250}(55d-27e)x^2 + \frac{1}{75}(20d-33e)x^3 + \frac{ex^4}{5} \\
&\quad - \frac{(2115d+5989e)\tan^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{3125\sqrt{14}} + \frac{(2290d-881e)\log\left(3+2x+5x^2\right)}{6250}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.87

$$
\begin{aligned}
&\int \frac{(d+ex)\left(2+x+3x^2-5x^3+4x^4\right)}{3+2x+5x^2}\,dx \\
&= \frac{35x(5d(486-495x+200x^2)+3e(916+405x-550x^2+250x^3)) - 3\sqrt{14}(2115d+5989e)\arctan\left(\frac{1+5x}{\sqrt{14}}\right)}{131250}
\end{aligned}
$$

```
[In] Integrate[((d + e*x)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2),x]
```

```
[Out] (35*x*(5*d*(486 - 495*x + 200*x^2) + 3*e*(916 + 405*x - 550*x^2 + 250*x^3))
 - 3*Sqrt[14]*(2115*d + 5989*e)*ArcTan[(1 + 5*x)/Sqrt[14]] + 21*(2290*d - 8
81*e)*Log[3 + 2*x + 5*x^2])/131250
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.84

method	result
default	$\frac{e\,x^4}{5}+\frac{4d\,x^3}{15}-\frac{11e\,x^3}{25}-\frac{33d\,x^2}{50}+\frac{81e\,x^2}{250}+\frac{81dx}{125}+\frac{458ex}{625}+\frac{(2290d-881e)\ln\left(5x^2+2x+3\right)}{6250}+\frac{\left(-423d-\frac{5989e}{5}\right)\sqrt{14}\ \text{arct}}{8750}$
risch	$\frac{e\,x^4}{5}+\frac{4d\,x^3}{15}-\frac{11e\,x^3}{25}-\frac{33d\,x^2}{50}+\frac{81e\,x^2}{250}+\frac{81dx}{125}+\frac{458ex}{625}+\frac{229d\ln\left(350x^2+140x+210\right)}{625}-\frac{881e\ln\left(350x^2+140x+210\right)}{6250}$

```
[In] int((e*x+d)*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*e*x^4+4/15*d*x^3-11/25*e*x^3-33/50*d*x^2+81/250*e*x^2+81/125*d*x+458/62
5*e*x+1/6250*(2290*d-881*e)*ln(5*x^2+2*x+3)+1/8750*(-423*d-5989/5*e)*14^(1/
2)*arctan(1/28*(10*x+2)*14^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.85

$$\begin{aligned}\int \frac{(d+ex)\left(2+x+3x^2-5x^3+4x^4\right)}{3+2x+5x^2}\,dx &= \frac{1}{5}\,ex^4+\frac{1}{75}\,(20\,d-33\,e)x^3-\frac{3}{250}\,(55\,d-27\,e)x^2\\ &\quad -\frac{1}{43750}\,\sqrt{14}(2115\,d+5989\,e)\arctan\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right)\\ &\quad +\frac{1}{625}\,(405\,d+458\,e)x+\frac{1}{6250}\,(2290\,d-881\,e)\log\left(5\,x^2+2\,x+3\right)\end{aligned}$$

```
[In] integrate((e*x+d)*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="frica
s")
```

```
[Out] 1/5*e*x^4 + 1/75*(20*d - 33*e)*x^3 - 3/250*(55*d - 27*e)*x^2 - 1/43750*sqrt
(14)*(2115*d + 5989*e)*arctan(1/14*sqrt(14)*(5*x + 1)) + 1/625*(405*d + 458
*e)*x + 1/6250*(2290*d - 881*e)*log(5*x^2 + 2*x + 3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.65

$$\begin{aligned}\int \frac{(d+ex)\left(2+x+3x^2-5x^3+4x^4\right)}{3+2x+5x^2}\,dx &= \frac{ex^4}{5}+x^3\cdot\left(\frac{4d}{15}-\frac{11e}{25}\right)+x^2\left(-\frac{33d}{50}+\frac{81e}{250}\right)+x\left(\frac{81d}{125}+\frac{458e}{625}\right)\\ &\quad +\left(\frac{229d}{625}-\frac{881e}{6250}-\frac{\sqrt{14}i(2115d+5989e)}{87500}\right)\log\left(x+\frac{423d+\frac{5989e}{5}+\frac{\sqrt{14}i(2115d+5989e)}{5}}{2115d+5989e}\right)\\ &\quad +\left(\frac{229d}{625}-\frac{881e}{6250}+\frac{\sqrt{14}i(2115d+5989e)}{87500}\right)\log\left(x+\frac{423d+\frac{5989e}{5}-\frac{\sqrt{14}i(2115d+5989e)}{5}}{2115d+5989e}\right)\end{aligned}$$

```
[In] integrate((e*x+d)*(4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3),x)
```

```
[Out] e*x**4/5 + x**3*(4*d/15 - 11*e/25) + x**2*(-33*d/50 + 81*e/250) + x*(81*d/1
25 + 458*e/625) + (229*d/625 - 881*e/6250 - sqrt(14)*I*(2115*d + 5989*e)/87
```

```
500)*log(x + (423*d + 5989*e/5 + sqrt(14)*I*(2115*d + 5989*e)/5)/(2115*d +
5989*e)) + (229*d/625 - 881*e/6250 + sqrt(14)*I*(2115*d + 5989*e)/87500)*lo
g(x + (423*d + 5989*e/5 - sqrt(14)*I*(2115*d + 5989*e)/5)/(2115*d + 5989*e)
)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.85

$$\begin{aligned}
&\int \frac{(d+ex)\left(2+x+3x^2-5x^3+4x^4\right)}{3+2x+5x^2}\,dx \\
&= \frac{1}{5}\,ex^4 + \frac{1}{75}\,(20\,d-33\,e)x^3 - \frac{3}{250}\,(55\,d-27\,e)x^2 \\
&\quad - \frac{1}{43750}\,\sqrt{14}(2115\,d+5989\,e)\arctan\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right) \\
&\quad + \frac{1}{625}\,(405\,d+458\,e)x + \frac{1}{6250}\,(2290\,d-881\,e)\log\left(5\,x^2+2\,x+3\right)
\end{aligned}$$

```
[In] integrate((e*x+d)*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="maxim
a")
```

```
[Out] 1/5*e*x^4 + 1/75*(20*d - 33*e)*x^3 - 3/250*(55*d - 27*e)*x^2 - 1/43750*sqrt
(14)*(2115*d + 5989*e)*arctan(1/14*sqrt(14)*(5*x + 1)) + 1/625*(405*d + 458
*e)*x + 1/6250*(2290*d - 881*e)*log(5*x^2 + 2*x + 3)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.83

$$\begin{aligned}
&\int \frac{(d+ex)\left(2+x+3x^2-5x^3+4x^4\right)}{3+2x+5x^2}\,dx \\
&= \frac{1}{5}\,ex^4 + \frac{4}{15}\,dx^3 - \frac{11}{25}\,ex^3 - \frac{33}{50}\,dx^2 + \frac{81}{250}\,ex^2 \\
&\quad - \frac{1}{43750}\,\sqrt{14}(2115\,d+5989\,e)\arctan\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right) \\
&\quad + \frac{81}{125}\,dx + \frac{458}{625}\,ex + \frac{1}{6250}\,(2290\,d-881\,e)\log\left(5\,x^2+2\,x+3\right)
\end{aligned}$$

```
[In] integrate((e*x+d)*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="giac"
)
```

```
[Out] 1/5*e*x^4 + 4/15*d*x^3 - 11/25*e*x^3 - 33/50*d*x^2 + 81/250*e*x^2 - 1/43750
*sqrt(14)*(2115*d + 5989*e)*arctan(1/14*sqrt(14)*(5*x + 1)) + 81/125*d*x +
458/625*e*x + 1/6250*(2290*d - 881*e)*log(5*x^2 + 2*x + 3)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.08

$$
\begin{aligned}
&\int \frac{(d+ex)\left(2+x+3x^2-5x^3+4x^4\right)}{3+2x+5x^2}\,dx \\
&= x^3\left(\frac{4\,d}{15}-\frac{11\,e}{25}\right)-x^2\left(\frac{33\,d}{50}-\frac{81\,e}{250}\right)+\ln\left(5\,x^2+2\,x+3\right)\,\left(\frac{229\,d}{625}-\frac{881\,e}{6250}\right)+\frac{e\,x^4}{5} \\
&\quad +x\left(\frac{81\,d}{125}+\frac{458\,e}{625}\right)-\frac{\sqrt{14}\,\mathrm{atan}\left(\frac{\frac{\sqrt{14}\,\left(2115\,d+5989\,e\right)}{43750}+\frac{\sqrt{14}\,x\,\left(2115\,d+5989\,e\right)}{8750}}{\frac{423\,d}{625}+\frac{5989\,e}{3125}}\right)\,\left(2115\,d+5989\,e\right)}{43750}
\end{aligned}
$$

```
[In] int(((d + e*x)*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(2*x + 5*x^2 + 3),x)
[Out] x^3*((4*d)/15 - (11*e)/25) - x^2*((33*d)/50 - (81*e)/250) + log(2*x + 5*x^2
 + 3)*((229*d)/625 - (881*e)/6250) + (e*x^4)/5 + x*((81*d)/125 + (458*e)/62
5) - (14^(1/2)*atan(((14^(1/2)*(2115*d + 5989*e))/43750 + (14^(1/2)*x*(2115
*d + 5989*e))/8750)/((423*d)/625 + (5989*e)/3125))*(2115*d + 5989*e))/43750
```

3.307 $\int \frac{2+x+3x^2-5x^3+4x^4}{3+2x+5x^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 56

$$\int \frac{2+x+3x^2-5x^3+4x^4}{3+2x+5x^2}\,dx=\frac{81x}{125}-\frac{33x^2}{50}+\frac{4x^3}{15}-\frac{423\arctan\left(\frac{1+5x}{\sqrt{14}}\right)}{625\sqrt{14}}+\frac{229}{625}\log\left(3+2x+5x^2\right)$$

```
[Out] 81/125*x-33/50*x^2+4/15*x^3+229/625*ln(5*x^2+2*x+3)-423/8750*arctan(1/14*(1
+5*x)*14^(1/2))*14^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {1671, 648, 632, 210, 642}

$$\int \frac{2+x+3x^2-5x^3+4x^4}{3+2x+5x^2}\,dx=-\frac{423\arctan\left(\frac{5x+1}{\sqrt{14}}\right)}{625\sqrt{14}}+\frac{4x^3}{15}-\frac{33x^2}{50}+\frac{229}{625}\log\left(5x^2+2x+3\right)+\frac{81x}{125}$$

```
[In] Int[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/(3 + 2*x + 5*x^2),x]
```

```
[Out] (81*x)/125 - (33*x^2)/50 + (4*x^3)/15 - (423*ArcTan[(1 + 5*x)/Sqrt[14]])/(6
25*Sqrt[14]) + (229*Log[3 + 2*x + 5*x^2])/625
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{81}{125}-\frac{33x}{25}+\frac{4x^2}{5}+\frac{7+458x}{125\,(3+2x+5x^2)}\right)dx\\
&= \frac{81x}{125}-\frac{33x^2}{50}+\frac{4x^3}{15}+\frac{1}{125}\int \frac{7+458x}{3+2x+5x^2}\,dx\\
&= \frac{81x}{125}-\frac{33x^2}{50}+\frac{4x^3}{15}+\frac{229}{625}\int \frac{2+10x}{3+2x+5x^2}\,dx-\frac{423}{625}\int \frac{1}{3+2x+5x^2}\,dx\\
&= \frac{81x}{125}-\frac{33x^2}{50}+\frac{4x^3}{15}+\frac{229}{625}\log\left(3+2x+5x^2\right)+\frac{846}{625}\text{Subst}\left(\int \frac{1}{-56-x^2}\,dx,x,2+10x\right)\\
&= \frac{81x}{125}-\frac{33x^2}{50}+\frac{4x^3}{15}-\frac{423\tan^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{625\sqrt{14}}+\frac{229}{625}\log\left(3+2x+5x^2\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.89

$$\int \frac{2+x+3x^2-5x^3+4x^4}{3+2x+5x^2}\,dx = \frac{35x(486-495x+200x^2)-1269\sqrt{14}\arctan\left(\frac{1+5x}{\sqrt{14}}\right)+9618\log\left(3+2x+5x^2\right)}{26250}$$

```
[In] Integrate[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/(3 + 2*x + 5*x^2),x]
```

```
[Out] (35*x*(486 - 495*x + 200*x^2) - 1269*Sqrt[14]*ArcTan[(1 + 5*x)/Sqrt[14]] +
9618*Log[3 + 2*x + 5*x^2])/26250
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{4x^3}{15}-\frac{33x^2}{50}+\frac{81x}{125}+\frac{229\ln\left(5x^2+2x+3\right)}{625}-\frac{423\sqrt{14}\,\arctan\left(\frac{(10x+2)\sqrt{14}}{28}\right)}{8750}$	44
risch	$\frac{4x^3}{15}-\frac{33x^2}{50}+\frac{81x}{125}+\frac{229\ln\left(25x^2+10x+15\right)}{625}-\frac{423\arctan\left(\frac{(1+5x)\sqrt{14}}{14}\right)\sqrt{14}}{8750}$	44

```
[In] int((4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x,method=_RETURNVERBOSE)
```

```
[Out] 4/15*x^3-33/50*x^2+81/125*x+229/625*ln(5*x^2+2*x+3)-423/8750*14^(1/2)*arcta
n(1/28*(10*x+2)*14^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.77

$$\int \frac{2+x+3x^2-5x^3+4x^4}{3+2x+5x^2}\,dx = \frac{4}{15}\,x^3-\frac{33}{50}\,x^2-\frac{423}{8750}\,\sqrt{14}\arctan\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right) + \frac{81}{125}\,x+\frac{229}{625}\,\log\left(5\,x^2+2\,x+3\right)$$

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="fricas")
```

```
[Out] 4/15*x^3 - 33/50*x^2 - 423/8750*sqrt(14)*arctan(1/14*sqrt(14)*(5*x + 1)) +
81/125*x + 229/625*log(5*x^2 + 2*x + 3)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.09

$$\int \frac{2+x+3x^2-5x^3+4x^4}{3+2x+5x^2}\,dx = \frac{4x^3}{15}-\frac{33x^2}{50}+\frac{81x}{125}+\frac{229\log\left(x^2+\frac{2x}{5}+\frac{3}{5}\right)}{625} - \frac{423\sqrt{14}\operatorname{atan}\left(\frac{5\sqrt{14}x}{14}+\frac{\sqrt{14}}{14}\right)}{8750}$$

```
[In] integrate((4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3),x)
```

```
[Out] 4*x**3/15 - 33*x**2/50 + 81*x/125 + 229*log(x**2 + 2*x/5 + 3/5)/625 - 423*s
qrt(14)*atan(5*sqrt(14)*x/14 + sqrt(14)/14)/8750
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.77

$$\int \frac{2+x+3x^2-5x^3+4x^4}{3+2x+5x^2}\,dx = \frac{4}{15}\,x^3-\frac{33}{50}\,x^2-\frac{423}{8750}\,\sqrt{14}\arctan\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right) + \frac{81}{125}\,x+\frac{229}{625}\,\log\left(5\,x^2+2\,x+3\right)$$

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="maxima")
```

```
[Out] 4/15*x^3 - 33/50*x^2 - 423/8750*sqrt(14)*arctan(1/14*sqrt(14)*(5*x + 1)) +
81/125*x + 229/625*log(5*x^2 + 2*x + 3)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.77

$$\int \frac{2+x+3x^2-5x^3+4x^4}{3+2x+5x^2}\,dx = \frac{4}{15}\,x^3-\frac{33}{50}\,x^2-\frac{423}{8750}\,\sqrt{14}\arctan\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right) + \frac{81}{125}\,x+\frac{229}{625}\,\log\left(5\,x^2+2\,x+3\right)$$

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="giac")
```

```
[Out] 4/15*x^3 - 33/50*x^2 - 423/8750*sqrt(14)*arctan(1/14*sqrt(14)*(5*x + 1)) +
81/125*x + 229/625*log(5*x^2 + 2*x + 3)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.80

$$\int \frac{2+x+3x^2-5x^3+4x^4}{3+2x+5x^2}\,dx = \frac{81\,x}{125}+\frac{229\,\ln\left(5\,x^2+2\,x+3\right)}{625}-\frac{423\,\sqrt{14}\,\mathrm{atan}\left(\frac{5\,\sqrt{14}\,x}{14}+\frac{\sqrt{14}}{14}\right)}{8750}-\frac{33\,x^2}{50}+\frac{4\,x^3}{15}$$

```
[In] int((x + 3*x^2 - 5*x^3 + 4*x^4 + 2)/(2*x + 5*x^2 + 3),x)
```

```
[Out] (81*x)/125 + (229*log(2*x + 5*x^2 + 3))/625 - (423*14^(1/2)*atan((5*14^(1/2
)*x)/14 + 14^(1/2)/14))/8750 - (33*x^2)/50 + (4*x^3)/15
```

3.308 $\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)(3+2x+5x^2)}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 168

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)(3+2x+5x^2)}\,dx = -\frac{(20d+33e)x}{25e^2}+\frac{2x^2}{5e}-\frac{(423d-1367e)\arctan\left(\frac{1+5x}{\sqrt{14}}\right)}{125\sqrt{14}\,(5d^2-2de+3e^2)}+\frac{(4d^4+5d^3e+3d^2e^2-de^3+2e^4)\log(d+ex)}{e^3\,(5d^2-2de+3e^2)}+\frac{(458d-7e)\log(3+2x+5x^2)}{250\,(5d^2-2de+3e^2)}$$

```
[Out] -1/25*(20*d+33*e)*x/e^2+2/5*x^2/e+(4*d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)*ln(
e*x+d)/e^3/(5*d^2-2*d*e+3*e^2)+1/250*(458*d-7*e)*ln(5*x^2+2*x+3)/(5*d^2-2*d
*e+3*e^2)-1/1750*(423*d-1367*e)*arctan(1/14*(1+5*x)*14^(1/2))/(5*d^2-2*d*e+
3*e^2)*14^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {1642, 648, 632, 210, 642}

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)(3+2x+5x^2)}\,dx = -\frac{\arctan\left(\frac{5x+1}{\sqrt{14}}\right)(423d-1367e)}{125\sqrt{14}\,(5d^2-2de+3e^2)}+\frac{(458d-7e)\log(5x^2+2x+3)}{250\,(5d^2-2de+3e^2)}+\frac{(4d^4+5d^3e+3d^2e^2-de^3+2e^4)\log(d+ex)}{e^3\,(5d^2-2de+3e^2)}-\frac{x(20d+33e)}{25e^2}+\frac{2x^2}{5e}$$

```
[In] Int[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)*(3 + 2*x + 5*x^2)),x]
```

```
[Out] -1/25*((20*d + 33*e)*x)/e^2 + (2*x^2)/(5*e) - ((423*d - 1367*e)*ArcTan[(1 +
 5*x)/Sqrt[14]])/(125*Sqrt[14]*(5*d^2 - 2*d*e + 3*e^2)) + ((4*d^4 + 5*d^3*e
 + 3*d^2*e^2 - d*e^3 + 2*e^4)*Log[d + e*x])/(e^3*(5*d^2 - 2*d*e + 3*e^2)) +
 ((458*d - 7*e)*Log[3 + 2*x + 5*x^2])/(250*(5*d^2 - 2*d*e + 3*e^2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{-20d-33e}{25e^2}+\frac{4x}{5e}+\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{e^2\left(5d^2-2de+3e^2\right)(d+ex)}\right.\\ &\qquad\left.+\frac{7d+272e+(458d-7e)x}{25\left(5d^2-2de+3e^2\right)\left(3+2x+5x^2\right)}\right)dx\\ &= -\frac{(20d+33e)x}{25e^2}+\frac{2x^2}{5e}+\frac{\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)\log(d+ex)}{e^3\left(5d^2-2de+3e^2\right)}\\ &\quad+\frac{\int \frac{7d+272e+(458d-7e)x}{3+2x+5x^2}\,dx}{25\left(5d^2-2de+3e^2\right)}\end{aligned}$$

$$
\begin{aligned}
&= -\frac{(20d+33e)x}{25e^2} + \frac{2x^2}{5e} + \frac{(4d^4+5d^3e+3d^2e^2-de^3+2e^4)\log(d+ex)}{e^3(5d^2-2de+3e^2)} \\
&\quad - \frac{(423d-1367e)\int \frac{1}{3+2x+5x^2}\,dx}{125(5d^2-2de+3e^2)} + \frac{(458d-7e)\int \frac{2+10x}{3+2x+5x^2}\,dx}{250(5d^2-2de+3e^2)} \\
&= -\frac{(20d+33e)x}{25e^2} + \frac{2x^2}{5e} + \frac{(4d^4+5d^3e+3d^2e^2-de^3+2e^4)\log(d+ex)}{e^3(5d^2-2de+3e^2)} \\
&\quad + \frac{(458d-7e)\log(3+2x+5x^2)}{250(5d^2-2de+3e^2)} \\
&\quad + \frac{(2(423d-1367e))\text{Subst}\left(\int \frac{1}{-56-x^2}\,dx, x, 2+10x\right)}{125(5d^2-2de+3e^2)} \\
&= -\frac{(20d+33e)x}{25e^2} + \frac{2x^2}{5e} - \frac{(423d-1367e)\tan^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{125\sqrt{14}(5d^2-2de+3e^2)} \\
&\quad + \frac{(4d^4+5d^3e+3d^2e^2-de^3+2e^4)\log(d+ex)}{e^3(5d^2-2de+3e^2)} + \frac{(458d-7e)\log(3+2x+5x^2)}{250(5d^2-2de+3e^2)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.87

$$
\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)(3+2x+5x^2)}\,dx
= \frac{70e(5d^2-2de+3e^2)\,x(-20d+e(-33+10x)) - \sqrt{14}(423d-1367e)e^3\arctan\left(\frac{1+5x}{\sqrt{14}}\right) + 1750(4d^4+5d^3e}{1750e^3(5d^2-2de+3e^2)}
$$

```
[In] Integrate[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)*(3 + 2*x + 5*x^2)),x]
```

```
[Out] (70*e*(5*d^2 - 2*d*e + 3*e^2)*x*(-20*d + e*(-33 + 10*x)) - Sqrt[14]*(423*d
- 1367*e)*e^3*ArcTan[(1 + 5*x)/Sqrt[14]] + 1750*(4*d^4 + 5*d^3*e + 3*d^2*e^
2 - d*e^3 + 2*e^4)*Log[d + e*x] + 7*(458*d - 7*e)*e^3*Log[3 + 2*x + 5*x^2])
/(1750*e^3*(5*d^2 - 2*d*e + 3*e^2))
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.85

method	result
default	$-\frac{-10e\,x^2+20dx+33ex}{25e^2}+\frac{\frac{(458d-7e)\ln\left(5x^2+2x+3\right)}{10}+\frac{\left(-\frac{423d}{5}+\frac{1367e}{5}\right)\sqrt{14}\,\arctan\left(\frac{(10x+2)\sqrt{14}}{28}\right)}{14}}{125d^2-50de+75e^2}+\frac{(4d^4+5d^3e+3d^2e^2-d\,e^3+2e^4)\ln(}{e^3(5d^2-2de+3e^2)}$
risch	Expression too large to display

```
[In] int((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/25/e^2*(-10*e*x^2+20*d*x+33*e*x)+1/(125*d^2-50*d*e+75*e^2)*(1/10*(458*d-
7*e)*ln(5*x^2+2*x+3)+1/14*(-423/5*d+1367/5*e)*14^(1/2)*arctan(1/28*(10*x+2)
*14^(1/2)))+(4*d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)*ln(e*x+d)/e^3/(5*d^2-2*d*
e+3*e^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.02

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)\,(3+2x+5x^2)}\,dx = \frac{700\,(5\,d^2e^2-2\,de^3+3\,e^4)x^2-\sqrt{14}(423\,de^3-1367\,e^4)\arctan\left(\frac{1}{14}\sqrt{14}(5\,x+1)\right)-70\,(100\,d^3e+125\,d^2e^2}{1750\,(5\,d^2}$$

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3),x, algorithm="frica
s")
```

```
[Out] 1/1750*(700*(5*d^2*e^2 - 2*d*e^3 + 3*e^4)*x^2 - sqrt(14)*(423*d*e^3 - 1367*
e^4)*arctan(1/14*sqrt(14)*(5*x + 1)) - 70*(100*d^3*e + 125*d^2*e^2 - 6*d*e^
3 + 99*e^4)*x + 1750*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*log(e*x
+ d) + 7*(458*d*e^3 - 7*e^4)*log(5*x^2 + 2*x + 3))/(5*d^2*e^3 - 2*d*e^4 + 3
*e^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)\,(3+2x+5x^2)}\,dx = \text{Timed out}$$

```
[In] integrate((4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)/(5*x**2+2*x+3),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.95

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)\left(3+2x+5x^2\right)}\,dx = -\frac{\sqrt{14}(423\,d-1367\,e)\arctan\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right)}{1750\,(5\,d^2-2\,de+3\,e^2)} + \frac{(4\,d^4+5\,d^3e+3\,d^2e^2-de^3+2\,e^4)\log\left(ex+d\right)}{5\,d^2e^3-2\,de^4+3\,e^5} + \frac{(458\,d-7\,e)\log\left(5\,x^2+2\,x+3\right)}{250\,(5\,d^2-2\,de+3\,e^2)} + \frac{10\,ex^2-(20\,d+33\,e)x}{25\,e^2}$$

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3),x, algorithm="maxim
a")
```

```
[Out] -1/1750*sqrt(14)*(423*d - 1367*e)*arctan(1/14*sqrt(14)*(5*x + 1))/(5*d^2 -
2*d*e + 3*e^2) + (4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*log(e*x + d)
/(5*d^2*e^3 - 2*d*e^4 + 3*e^5) + 1/250*(458*d - 7*e)*log(5*x^2 + 2*x + 3)/(
5*d^2 - 2*d*e + 3*e^2) + 1/25*(10*e*x^2 - (20*d + 33*e)*x)/e^2
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.95

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)\left(3+2x+5x^2\right)}\,dx = -\frac{\sqrt{14}(423\,d-1367\,e)\arctan\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right)}{1750\,(5\,d^2-2\,de+3\,e^2)} + \frac{(458\,d-7\,e)\log\left(5\,x^2+2\,x+3\right)}{250\,(5\,d^2-2\,de+3\,e^2)} + \frac{(4\,d^4+5\,d^3e+3\,d^2e^2-de^3+2\,e^4)\log\left(|ex+d|\right)}{5\,d^2e^3-2\,de^4+3\,e^5} + \frac{10\,ex^2-20\,dx-33\,ex}{25\,e^2}$$

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3),x, algorithm="giac"
)
```

```
[Out] -1/1750*sqrt(14)*(423*d - 1367*e)*arctan(1/14*sqrt(14)*(5*x + 1))/(5*d^2 -
2*d*e + 3*e^2) + 1/250*(458*d - 7*e)*log(5*x^2 + 2*x + 3)/(5*d^2 - 2*d*e +
3*e^2) + (4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*log(abs(e*x + d))/(5
*d^2*e^3 - 2*d*e^4 + 3*e^5) + 1/25*(10*e*x^2 - 20*d*x - 33*e*x)/e^2
```

Mupad [B] (verification not implemented)

Time = 15.97 (sec) , antiderivative size = 713, normalized size of antiderivative = 4.24

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)\left(3+2x+5x^2\right)}\,dx$$

$$=\frac{2\,x^2}{5\,e}-\ln\left(d+e\,x\right)\,\left(\frac{\frac{458\,d}{125}-\frac{7\,e}{125}}{5\,d^2-2\,d\,e+3\,e^2}-\frac{100\,d^2+165\,d\,e+81\,e^2}{125\,e^3}\right)-x\,\left(\frac{4\,\left(5\,d+2\,e\right)}{25\,e^2}+\frac{1}{e}\right)$$

$$-\ln\Bigg(\frac{-28\,d^3+1053\,d^2\,e+1791\,d\,e^2+916\,e^3}{25\,e^2}-\frac{x\,\left(1832\,d^3+2318\,d^2\,e+321\,d\,e^2-2249\,e^3\right)}{25\,e^2}+\frac{\left(d\,\left(\frac{423\,\sqrt{14}}{3500}-\frac{229}{125}\mathrm{i}\right)-e\,\left(\frac{1367\,\sqrt{14}}{3500}-\frac{7}{250}\mathrm{i}\right)\right)\Bigg(\frac{-10}{}}{}$$

$$+\ln\Bigg(\frac{-28\,d^3+1053\,d^2\,e+1791\,d\,e^2+916\,e^3}{25\,e^2}-\frac{x\,\left(1832\,d^3+2318\,d^2\,e+321\,d\,e^2-2249\,e^3\right)}{25\,e^2}-\frac{\left(d\,\left(\frac{423\,\sqrt{14}}{3500}+\frac{229}{125}\mathrm{i}\right)-e\,\left(\frac{1367\,\sqrt{14}}{3500}+\frac{7}{250}\mathrm{i}\right)\right)\Bigg(\frac{-10}{}}{}$$

```
[In] int((x + 3*x^2 - 5*x^3 + 4*x^4 + 2)/((d + e*x)*(2*x + 5*x^2 + 3)),x)
```

```
[Out] (2*x^2)/(5*e) - log(d + e*x)*(((458*d)/125 - (7*e)/125)/(5*d^2 - 2*d*e + 3*
e^2) - (165*d*e + 100*d^2 + 81*e^2)/(125*e^3)) - x*((4*(5*d + 2*e))/(25*e^2
) + 1/e) - (log((1791*d*e^2 + 1053*d^2*e - 28*d^3 + 916*e^3)/(25*e^2) - (x*
(321*d*e^2 + 2318*d^2*e + 1832*d^3 - 2249*e^3))/(25*e^2) + ((d*((423*14^(1/
2))/3500 - 229i/125) - e*((1367*14^(1/2))/3500 - 7i/250))*((4751*d*e^3 + 43
50*d^3*e - 1000*d^4 + 874*e^4 + 8490*d^2*e^2)/(25*e^2) + (x*(8200*d*e^3 - 6
250*d^3*e - 5000*d^4 + 2917*e^4 + 1850*d^2*e^2))/(25*e^2) - (((750*e^5 - 14
500*d*e^4 + 1250*d^2*e^3)/(25*e^2) - (x*(2500*d*e^4 + 10250*e^5 - 6250*d^2*
e^3))/(25*e^2))*(d*((423*14^(1/2))/3500 - 229i/125) - e*((1367*14^(1/2))/35
00 - 7i/250)))/(d^2*5i - d*e*2i + e^2*3i)))/(d^2*5i - d*e*2i + e^2*3i))*(d*
((423*14^(1/2))/3500 - 229i/125) - e*((1367*14^(1/2))/3500 - 7i/250)))/(d^2
*5i - d*e*2i + e^2*3i) + (log((1791*d*e^2 + 1053*d^2*e - 28*d^3 + 916*e^3)/
(25*e^2) - (x*(321*d*e^2 + 2318*d^2*e + 1832*d^3 - 2249*e^3))/(25*e^2) - ((
d*((423*14^(1/2))/3500 + 229i/125) - e*((1367*14^(1/2))/3500 + 7i/250))*((4
751*d*e^3 + 4350*d^3*e - 1000*d^4 + 874*e^4 + 8490*d^2*e^2)/(25*e^2) + (x*(
8200*d*e^3 - 6250*d^3*e - 5000*d^4 + 2917*e^4 + 1850*d^2*e^2))/(25*e^2) + (
((750*e^5 - 14500*d*e^4 + 1250*d^2*e^3)/(25*e^2) - (x*(2500*d*e^4 + 10250*e
^5 - 6250*d^2*e^3))/(25*e^2))*(d*((423*14^(1/2))/3500 + 229i/125) - e*((136
7*14^(1/2))/3500 + 7i/250)))/(d^2*5i - d*e*2i + e^2*3i)))/(d^2*5i - d*e*2i
```

```
+ e^2*3i))*(d*((423*14^(1/2))/3500 + 229i/125) - e*((1367*14^(1/2))/3500 +
7i/250)))/(d^2*5i - d*e*2i + e^2*3i)
```

3.309 $\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2(3+2x+5x^2)}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 233

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2(3+2x+5x^2)}\,dx = \frac{4x}{5e^2}-\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{e^3(5d^2-2de+3e^2)(d+ex)} - \frac{(423d^2-2734de+293e^2)\arctan\left(\frac{1+5x}{\sqrt{14}}\right)}{25\sqrt{14}(5d^2-2de+3e^2)^2} - \frac{(40d^5+d^4e+28d^3e^2+44d^2e^3-2de^4+e^5)\log(d+ex)}{e^3(5d^2-2de+3e^2)^2} + \frac{(229d^2-7de-136e^2)\log(3+2x+5x^2)}{25(5d^2-2de+3e^2)^2}$$

```
[Out] 4/5*x/e^2+(-4*d^4-5*d^3*e-3*d^2*e^2+d*e^3-2*e^4)/e^3/(5*d^2-2*d*e+3*e^2)/(e
*x+d)-(40*d^5+d^4*e+28*d^3*e^2+44*d^2*e^3-2*d*e^4+e^5)*ln(e*x+d)/e^3/(5*d^2
-2*d*e+3*e^2)^2+1/25*(229*d^2-7*d*e-136*e^2)*ln(5*x^2+2*x+3)/(5*d^2-2*d*e+3
*e^2)^2-1/350*(423*d^2-2734*d*e+293*e^2)*arctan(1/14*(1+5*x)*14^(1/2))/(5*d
^2-2*d*e+3*e^2)^2*14^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used

$= \{1642, 648, 632, 210, 642\}$

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2(3+2x+5x^2)}\,dx = -\frac{\arctan\left(\frac{5x+1}{\sqrt{14}}\right)(423d^2-2734de+293e^2)}{25\sqrt{14}\,(5d^2-2de+3e^2)^2} + \frac{(229d^2-7de-136e^2)\log(5x^2+2x+3)}{25\,(5d^2-2de+3e^2)^2} - \frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{e^3(5d^2-2de+3e^2)(d+ex)} - \frac{(40d^5+d^4e+28d^3e^2+44d^2e^3-2de^4+e^5)\log(d+ex)}{e^3(5d^2-2de+3e^2)^2} + \frac{4x}{5e^2}$$

```
[In] Int[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)^2*(3 + 2*x + 5*x^2)),x]
```

```
[Out] (4*x)/(5*e^2) - (4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)/(e^3*(5*d^2 -
 2*d*e + 3*e^2)*(d + e*x)) - ((423*d^2 - 2734*d*e + 293*e^2)*ArcTan[(1 + 5*
x)/Sqrt[14]])/(25*Sqrt[14]*(5*d^2 - 2*d*e + 3*e^2)^2) - ((40*d^5 + d^4*e +
28*d^3*e^2 + 44*d^2*e^3 - 2*d*e^4 + e^5)*Log[d + e*x])/(e^3*(5*d^2 - 2*d*e
+ 3*e^2)^2) + ((229*d^2 - 7*d*e - 136*e^2)*Log[3 + 2*x + 5*x^2])/(25*(5*d^2
 - 2*d*e + 3*e^2)^2)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{4}{5e^2}+\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{e^2\left(5d^2-2de+3e^2\right)(d+ex)^2}\right.\\
&\qquad+\frac{-40d^5-d^4e-28d^3e^2-44d^2e^3+2de^4-e^5}{e^2\left(5d^2-2de+3e^2\right)^2(d+ex)}\\
&\qquad\left.+\frac{7d^2+544de-113e^2+2(229d^2-7de-136e^2)\,x}{5\left(5d^2-2de+3e^2\right)^2\left(3+2x+5x^2\right)}\right)dx\\
&= \frac{4x}{5e^2}-\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{e^3\left(5d^2-2de+3e^2\right)(d+ex)}\\
&\quad-\frac{\left(40d^5+d^4e+28d^3e^2+44d^2e^3-2de^4+e^5\right)\log(d+ex)}{e^3\left(5d^2-2de+3e^2\right)^2}\\
&\quad+\frac{\int \frac{7d^2+544de-113e^2+2(229d^2-7de-136e^2)x}{3+2x+5x^2}\,dx}{5\left(5d^2-2de+3e^2\right)^2}\\
&= \frac{4x}{5e^2}-\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{e^3\left(5d^2-2de+3e^2\right)(d+ex)}\\
&\quad-\frac{\left(40d^5+d^4e+28d^3e^2+44d^2e^3-2de^4+e^5\right)\log(d+ex)}{e^3\left(5d^2-2de+3e^2\right)^2}\\
&\quad+\frac{\left(229d^2-7de-136e^2\right)\int \frac{2+10x}{3+2x+5x^2}\,dx}{25\left(5d^2-2de+3e^2\right)^2}-\frac{\left(423d^2-2734de+293e^2\right)\int \frac{1}{3+2x+5x^2}\,dx}{25\left(5d^2-2de+3e^2\right)^2}\\
&= \frac{4x}{5e^2}-\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{e^3\left(5d^2-2de+3e^2\right)(d+ex)}\\
&\quad-\frac{\left(40d^5+d^4e+28d^3e^2+44d^2e^3-2de^4+e^5\right)\log(d+ex)}{e^3\left(5d^2-2de+3e^2\right)^2}\\
&\quad+\frac{\left(229d^2-7de-136e^2\right)\log\left(3+2x+5x^2\right)}{25\left(5d^2-2de+3e^2\right)^2}\\
&\quad+\frac{\left(2(423d^2-2734de+293e^2)\right)\operatorname{Subst}\left(\int \frac{1}{-56-x^2}\,dx,x,2+10x\right)}{25\left(5d^2-2de+3e^2\right)^2}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{4x}{5e^2} - \frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{e^3(5d^2-2de+3e^2)(d+ex)} - \frac{(423d^2-2734de+293e^2)\tan^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{25\sqrt{14}\,(5d^2-2de+3e^2)^2} \\
&\quad - \frac{(40d^5+d^4e+28d^3e^2+44d^2e^3-2de^4+e^5)\log(d+ex)}{e^3(5d^2-2de+3e^2)^2} \\
&\quad + \frac{(229d^2-7de-136e^2)\log(3+2x+5x^2)}{25(5d^2-2de+3e^2)^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.00

$$
\begin{aligned}
&\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2(3+2x+5x^2)}\,dx \\
&= \frac{4x}{5e^2} + \frac{-4d^4-5d^3e-3d^2e^2+de^3-2e^4}{e^3(5d^2-2de+3e^2)(d+ex)} + \frac{(-423d^2+2734de-293e^2)\arctan\left(\frac{1+5x}{\sqrt{14}}\right)}{25\sqrt{14}\,(5d^2-2de+3e^2)^2} \\
&\quad + \frac{(-40d^5-d^4e-28d^3e^2-44d^2e^3+2de^4-e^5)\log(d+ex)}{e^3(5d^2-2de+3e^2)^2} \\
&\quad + \frac{(229d^2-7de-136e^2)\log(3+2x+5x^2)}{25(5d^2-2de+3e^2)^2}
\end{aligned}
$$

```
[In] Integrate[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)^2*(3 + 2*x + 5*x^2)),x
]

[Out] (4*x)/(5*e^2) + (-4*d^4 - 5*d^3*e - 3*d^2*e^2 + d*e^3 - 2*e^4)/(e^3*(5*d^2
- 2*d*e + 3*e^2)*(d + e*x)) + ((-423*d^2 + 2734*d*e - 293*e^2)*ArcTan[(1 +
5*x)/Sqrt[14]])/(25*Sqrt[14]*(5*d^2 - 2*d*e + 3*e^2)^2) + ((-40*d^5 - d^4*e
 - 28*d^3*e^2 - 44*d^2*e^3 + 2*d*e^4 - e^5)*Log[d + e*x])/(e^3*(5*d^2 - 2*d
*e + 3*e^2)^2) + ((229*d^2 - 7*d*e - 136*e^2)*Log[3 + 2*x + 5*x^2])/(25*(5*
d^2 - 2*d*e + 3*e^2)^2)
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.91

method	result
default	$\frac{4x}{5e^2}+\frac{\frac{\left(458d^2-14de-272e^2\right)\ln\left(5x^2+2x+3\right)}{10}+\frac{\left(-\frac{423}{5}d^2+\frac{2734}{5}de-\frac{293}{5}e^2\right)\sqrt{14}\,\arctan\left(\frac{(10x+2)\sqrt{14}}{28}\right)}{14}}{5(5d^2-2de+3e^2)^2}-\frac{4d^4+5d^3e+3d^2e^2-d\,e^3+2e^4}{e^3(5d^2-2de+3e^2)(ex+d)}+$
risch	Expression too large to display

```
[In] int((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2/(5*x^2+2*x+3),x,method=_RETURNVERBOSE
)
```

```
[Out] 4/5*x/e^2+1/5/(5*d^2-2*d*e+3*e^2)^2*(1/10*(458*d^2-14*d*e-272*e^2)*ln(5*x^2
+2*x+3)+1/14*(-423/5*d^2+2734/5*d*e-293/5*e^2)*14^(1/2)*arctan(1/28*(10*x+2
)*14^(1/2)))-1/e^3*(4*d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)/(5*d^2-2*d*e+3*e^2
)/(e*x+d)+(-40*d^5-d^4*e-28*d^3*e^2-44*d^2*e^3+2*d*e^4-e^5)/e^3/(5*d^2-2*d*
e+3*e^2)^2*ln(e*x+d)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.79

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2\left(3+2x+5x^2\right)}\,dx = -\frac{7000\,d^6+5950\,d^5e+5950\,d^4e^2+1400\,d^3e^3+7350\,d^2e^4-2450\,de^5+2100\,e^6-280\left(25\,d^4e^2-20\,d^3e^3\right.}{}$$

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2/(5*x^2+2*x+3),x, algorithm="fri
cas")
```

```
[Out] -1/350*(7000*d^6 + 5950*d^5*e + 5950*d^4*e^2 + 1400*d^3*e^3 + 7350*d^2*e^4
- 2450*d*e^5 + 2100*e^6 - 280*(25*d^4*e^2 - 20*d^3*e^3 + 34*d^2*e^4 - 12*d*
e^5 + 9*e^6)*x^2 + sqrt(14)*(423*d^3*e^3 - 2734*d^2*e^4 + 293*d*e^5 + (423*
d^2*e^4 - 2734*d*e^5 + 293*e^6)*x)*arctan(1/14*sqrt(14)*(5*x + 1)) - 280*(2
5*d^5*e - 20*d^4*e^2 + 34*d^3*e^3 - 12*d^2*e^4 + 9*d*e^5)*x + 350*(40*d^6 +
 d^5*e + 28*d^4*e^2 + 44*d^3*e^3 - 2*d^2*e^4 + d*e^5 + (40*d^5*e + d^4*e^2
+ 28*d^3*e^3 + 44*d^2*e^4 - 2*d*e^5 + e^6)*x)*log(e*x + d) - 14*(229*d^3*e^
3 - 7*d^2*e^4 - 136*d*e^5 + (229*d^2*e^4 - 7*d*e^5 - 136*e^6)*x)*log(5*x^2
+ 2*x + 3))/(25*d^5*e^3 - 20*d^4*e^4 + 34*d^3*e^5 - 12*d^2*e^6 + 9*d*e^7 +
(25*d^4*e^4 - 20*d^3*e^5 + 34*d^2*e^6 - 12*d*e^7 + 9*e^8)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2\left(3+2x+5x^2\right)}\,dx = \text{Timed out}$$

```
[In] integrate((4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)**2/(5*x**2+2*x+3),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.26

$$\begin{aligned}\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2\,(3+2x+5x^2)}\,dx &= -\frac{\sqrt{14}(423\,d^2-2734\,de+293\,e^2)\arctan\left(\frac{1}{14}\sqrt{14}(5\,x+1)\right)}{350\,(25\,d^4-20\,d^3e+34\,d^2e^2-12\,de^3+9\,e^4)}\\&\quad-\frac{(40\,d^5+d^4e+28\,d^3e^2+44\,d^2e^3-2\,de^4+e^5)\log\left(ex+d\right)}{25\,d^4e^3-20\,d^3e^4+34\,d^2e^5-12\,de^6+9\,e^7}\\&\quad+\frac{(229\,d^2-7\,de-136\,e^2)\log\left(5\,x^2+2\,x+3\right)}{25\,(25\,d^4-20\,d^3e+34\,d^2e^2-12\,de^3+9\,e^4)}\\&\quad-\frac{4\,d^4+5\,d^3e+3\,d^2e^2-de^3+2\,e^4}{5\,d^3e^3-2\,d^2e^4+3\,de^5+(5\,d^2e^4-2\,de^5+3\,e^6)x}+\frac{4\,x}{5\,e^2}\end{aligned}$$

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2/(5*x^2+2*x+3),x, algorithm="max
ima")
```

```
[Out] -1/350*sqrt(14)*(423*d^2 - 2734*d*e + 293*e^2)*arctan(1/14*sqrt(14)*(5*x +
1))/(25*d^4 - 20*d^3*e + 34*d^2*e^2 - 12*d*e^3 + 9*e^4) - (40*d^5 + d^4*e +
 28*d^3*e^2 + 44*d^2*e^3 - 2*d*e^4 + e^5)*log(e*x + d)/(25*d^4*e^3 - 20*d^3
*e^4 + 34*d^2*e^5 - 12*d*e^6 + 9*e^7) + 1/25*(229*d^2 - 7*d*e - 136*e^2)*lo
g(5*x^2 + 2*x + 3)/(25*d^4 - 20*d^3*e + 34*d^2*e^2 - 12*d*e^3 + 9*e^4) - (4
*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)/(5*d^3*e^3 - 2*d^2*e^4 + 3*d*e^
5 + (5*d^2*e^4 - 2*d*e^5 + 3*e^6)*x) + 4/5*x/e^2
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.53

$$\begin{aligned}\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2\,(3+2x+5x^2)}\,dx &= \frac{(229\,d^2-7\,de-136\,e^2)\log\left(-\frac{10\,d}{ex+d}+\frac{5\,d^2}{(ex+d)^2}+\frac{2\,e}{ex+d}-\frac{2\,de}{(ex+d)^2}+\frac{3\,e^2}{(ex+d)^2}+5\right)}{25\,(25\,d^4-20\,d^3e+34\,d^2e^2-12\,de^3+9\,e^4)}\\&\quad-\frac{\frac{4\,d^4e^3}{ex+d}+\frac{5\,d^3e^4}{ex+d}+\frac{3\,d^2e^5}{ex+d}-\frac{de^6}{ex+d}+\frac{2\,e^7}{ex+d}}{5\,d^2e^6-2\,de^7+3\,e^8}\\&\quad-\frac{\sqrt{14}(423\,d^2e^2-2734\,de^3+293\,e^4)\arctan\left(\frac{\sqrt{14}\left(5\,d-\frac{5\,d^2}{ex+d}+\frac{2\,de}{ex+d}-e-\frac{3\,e^2}{ex+d}\right)}{14\,e}\right)}{350\,(25\,d^4-20\,d^3e+34\,d^2e^2-12\,de^3+9\,e^4)e^2}\\&\quad+\frac{(40\,d+33\,e)\log\left(\frac{|ex+d|}{(ex+d)^2|e|}\right)}{25\,e^3}+\frac{4\,(ex+d)}{5\,e^3}\end{aligned}$$

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2/(5*x^2+2*x+3),x, algorithm="gia
c")
```

```
[Out] 1/25*(229*d^2 - 7*d*e - 136*e^2)*log(-10*d/(e*x + d) + 5*d^2/(e*x + d)^2 +
2*e/(e*x + d) - 2*d*e/(e*x + d)^2 + 3*e^2/(e*x + d)^2 + 5)/(25*d^4 - 20*d^3
*e + 34*d^2*e^2 - 12*d*e^3 + 9*e^4) - (4*d^4*e^3/(e*x + d) + 5*d^3*e^4/(e*x
 + d) + 3*d^2*e^5/(e*x + d) - d*e^6/(e*x + d) + 2*e^7/(e*x + d))/(5*d^2*e^6
 - 2*d*e^7 + 3*e^8) - 1/350*sqrt(14)*(423*d^2*e^2 - 2734*d*e^3 + 293*e^4)*a
rctan(1/14*sqrt(14)*(5*d - 5*d^2/(e*x + d) + 2*d*e/(e*x + d) - e - 3*e^2/(e
*x + d))/e)/((25*d^4 - 20*d^3*e + 34*d^2*e^2 - 12*d*e^3 + 9*e^4)*e^2) + 1/2
5*(40*d + 33*e)*log(abs(e*x + d)/((e*x + d)^2*abs(e)))/e^3 + 4/5*(e*x + d)/
e^3
```

Mupad [B] (verification not implemented)

Time = 14.22 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.34

$$\begin{aligned}\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2\,(3+2x+5x^2)}\,dx &= \frac{4\,x}{5\,e^2}\\ &\quad-\frac{\ln\left(x+\frac{1}{5}-\frac{\sqrt{14}\,1\mathrm{i}}{5}\right)\,\left(\left(\frac{423\,\sqrt{14}}{700}-\frac{229}{25}\mathrm{i}\right)\,d^2+\left(-\frac{1367\,\sqrt{14}}{350}+\frac{7}{25}\mathrm{i}\right)\,d\,e+\left(\frac{293\,\sqrt{14}}{700}+\frac{136}{25}\mathrm{i}\right)\,e^2\right)}{d^4\,25\mathrm{i}-d^3\,e\,20\mathrm{i}+d^2\,e^2\,34\mathrm{i}-d\,e^3\,12\mathrm{i}+e^4\,9\mathrm{i}}\\ &\quad+\frac{\ln\left(x+\frac{1}{5}+\frac{\sqrt{14}\,1\mathrm{i}}{5}\right)\,\left(\left(\frac{423\,\sqrt{14}}{700}+\frac{229}{25}\mathrm{i}\right)\,d^2+\left(-\frac{1367\,\sqrt{14}}{350}-\frac{7}{25}\mathrm{i}\right)\,d\,e+\left(\frac{293\,\sqrt{14}}{700}-\frac{136}{25}\mathrm{i}\right)\,e^2\right)}{d^4\,25\mathrm{i}-d^3\,e\,20\mathrm{i}+d^2\,e^2\,34\mathrm{i}-d\,e^3\,12\mathrm{i}+e^4\,9\mathrm{i}}\\ &\quad-\frac{5\,\left(4\,d^4+5\,d^3\,e+3\,d^2\,e^2-d\,e^3+2\,e^4\right)}{e\,\left(5\,x\,e^3+5\,d\,e^2\right)\,\left(5\,d^2-2\,d\,e+3\,e^2\right)}\\ &\quad-\frac{\ln\left(d+e\,x\right)\,\left(40\,d^5+d^4\,e+28\,d^3\,e^2+44\,d^2\,e^3-2\,d\,e^4+e^5\right)}{e^3\,{\left(5\,d^2-2\,d\,e+3\,e^2\right)}^2}\end{aligned}$$

```
[In] int((x + 3*x^2 - 5*x^3 + 4*x^4 + 2)/((d + e*x)^2*(2*x + 5*x^2 + 3)),x)
```

```
[Out] (4*x)/(5*e^2) - (log(x - (14^(1/2)*1i)/5 + 1/5)*(d^2*((423*14^(1/2))/700 -
229i/25) + e^2*((293*14^(1/2))/700 + 136i/25) - d*e*((1367*14^(1/2))/350 -
7i/25)))/(d^4*25i - d^3*e*20i - d*e^3*12i + e^4*9i + d^2*e^2*34i) + (log(x
+ (14^(1/2)*1i)/5 + 1/5)*(d^2*((423*14^(1/2))/700 + 229i/25) + e^2*((293*14
^(1/2))/700 - 136i/25) - d*e*((1367*14^(1/2))/350 + 7i/25)))/(d^4*25i - d^3
*e*20i - d*e^3*12i + e^4*9i + d^2*e^2*34i) - (5*(5*d^3*e - d*e^3 + 4*d^4 +
2*e^4 + 3*d^2*e^2))/(e*(5*d*e^2 + 5*e^3*x)*(5*d^2 - 2*d*e + 3*e^2)) - (log(
d + e*x)*(d^4*e - 2*d*e^4 + 40*d^5 + e^5 + 44*d^2*e^3 + 28*d^3*e^2))/(e^3*(
5*d^2 - 2*d*e + 3*e^2)^2)
```

3.310 $\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^3(3+2x+5x^2)}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 317

$$\begin{aligned}
&\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^3(3+2x+5x^2)}\,dx \\
&= -\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{2e^3(5d^2-2de+3e^2)(d+ex)^2}+\frac{40d^5+d^4e+28d^3e^2+44d^2e^3-2de^4+e^5}{e^3(5d^2-2de+3e^2)^2(d+ex)} \\
&\quad -\frac{(423d^3-4101d^2e+879de^2+703e^3)\arctan\left(\frac{1+5x}{\sqrt{14}}\right)}{5\sqrt{14}\,(5d^2-2de+3e^2)^3} \\
&\quad +\frac{(100d^6-120d^5e+228d^4e^2-242d^3e^3+141d^2e^4+120de^5-e^6)\log(d+ex)}{e^3(5d^2-2de+3e^2)^3} \\
&\quad +\frac{(458d^3-21d^2e-816de^2+113e^3)\log(3+2x+5x^2)}{10(5d^2-2de+3e^2)^3}
\end{aligned}$$

```
[Out] 1/2*(-4*d^4-5*d^3*e-3*d^2*e^2+d*e^3-2*e^4)/e^3/(5*d^2-2*d*e+3*e^2)/(e*x+d)^
2+(40*d^5+d^4*e+28*d^3*e^2+44*d^2*e^3-2*d*e^4+e^5)/e^3/(5*d^2-2*d*e+3*e^2)^
2/(e*x+d)+(100*d^6-120*d^5*e+228*d^4*e^2-242*d^3*e^3+141*d^2*e^4+120*d*e^5-
e^6)*ln(e*x+d)/e^3/(5*d^2-2*d*e+3*e^2)^3+1/10*(458*d^3-21*d^2*e-816*d*e^2+1
13*e^3)*ln(5*x^2+2*x+3)/(5*d^2-2*d*e+3*e^2)^3-1/70*(423*d^3-4101*d^2*e+879*
d*e^2+703*e^3)*arctan(1/14*(1+5*x)*14^(1/2))/(5*d^2-2*d*e+3*e^2)^3*14^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {1642, 648, 632, 210, 642}

$$
\begin{aligned}
&\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^3\,(3+2x+5x^2)}\,dx \\
&= -\frac{\arctan\left(\frac{5x+1}{\sqrt{14}}\right)\left(423d^3-4101d^2e+879de^2+703e^3\right)}{5\sqrt{14}\,\left(5d^2-2de+3e^2\right)^3} \\
&\quad+\frac{\left(458d^3-21d^2e-816de^2+113e^3\right)\log\left(5x^2+2x+3\right)}{10\left(5d^2-2de+3e^2\right)^3} \\
&\quad-\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{2e^3\left(5d^2-2de+3e^2\right)(d+ex)^2}+\frac{40d^5+d^4e+28d^3e^2+44d^2e^3-2de^4+e^5}{e^3\left(5d^2-2de+3e^2\right)^2(d+ex)} \\
&\quad+\frac{\left(100d^6-120d^5e+228d^4e^2-242d^3e^3+141d^2e^4+120de^5-e^6\right)\log(d+ex)}{e^3\left(5d^2-2de+3e^2\right)^3}
\end{aligned}
$$

```
[In] Int[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)^3*(3 + 2*x + 5*x^2)),x]
```

```
[Out] -1/2*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)/(e^3*(5*d^2 - 2*d*e + 3*
e^2)*(d + e*x)^2) + (40*d^5 + d^4*e + 28*d^3*e^2 + 44*d^2*e^3 - 2*d*e^4 + e
^5)/(e^3*(5*d^2 - 2*d*e + 3*e^2)^2*(d + e*x)) - ((423*d^3 - 4101*d^2*e + 87
9*d*e^2 + 703*e^3)*ArcTan[(1 + 5*x)/Sqrt[14]])/(5*Sqrt[14]*(5*d^2 - 2*d*e +
 3*e^2)^3) + ((100*d^6 - 120*d^5*e + 228*d^4*e^2 - 242*d^3*e^3 + 141*d^2*e^
4 + 120*d*e^5 - e^6)*Log[d + e*x])/(e^3*(5*d^2 - 2*d*e + 3*e^2)^3) + ((458*
d^3 - 21*d^2*e - 816*d*e^2 + 113*e^3)*Log[3 + 2*x + 5*x^2])/(10*(5*d^2 - 2*
d*e + 3*e^2)^3)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{e^2\left(5d^2-2de+3e^2\right)(d+ex)^3}+\frac{-40d^5-d^4e-28d^3e^2-44d^2e^3+2de^4-e^5}{e^2\left(5d^2-2de+3e^2\right)^2(d+ex)^2}\right.\\
&\quad+\frac{100d^6-120d^5e+228d^4e^2-242d^3e^3+141d^2e^4+120de^5-e^6}{e^2\left(5d^2-2de+3e^2\right)^3(d+ex)}\\
&\quad\left.+\frac{7d^3+816d^2e-339de^2-118e^3+\left(458d^3-21d^2e-816de^2+113e^3\right)x}{\left(5d^2-2de+3e^2\right)^3\left(3+2x+5x^2\right)}\right)dx\\
&= -\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{2e^3\left(5d^2-2de+3e^2\right)(d+ex)^2}+\frac{40d^5+d^4e+28d^3e^2+44d^2e^3-2de^4+e^5}{e^3\left(5d^2-2de+3e^2\right)^2(d+ex)}\\
&\quad+\frac{\left(100d^6-120d^5e+228d^4e^2-242d^3e^3+141d^2e^4+120de^5-e^6\right)\log(d+ex)}{e^3\left(5d^2-2de+3e^2\right)^3}\\
&\quad+\frac{\int\frac{7d^3+816d^2e-339de^2-118e^3+\left(458d^3-21d^2e-816de^2+113e^3\right)x}{3+2x+5x^2}\,dx}{\left(5d^2-2de+3e^2\right)^3}\\
&= -\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{2e^3\left(5d^2-2de+3e^2\right)(d+ex)^2}+\frac{40d^5+d^4e+28d^3e^2+44d^2e^3-2de^4+e^5}{e^3\left(5d^2-2de+3e^2\right)^2(d+ex)}\\
&\quad+\frac{\left(100d^6-120d^5e+228d^4e^2-242d^3e^3+141d^2e^4+120de^5-e^6\right)\log(d+ex)}{e^3\left(5d^2-2de+3e^2\right)^3}\\
&\quad+\frac{\left(458d^3-21d^2e-816de^2+113e^3\right)\int\frac{2+10x}{3+2x+5x^2}\,dx}{10\left(5d^2-2de+3e^2\right)^3}\\
&\quad-\frac{\left(423d^3-4101d^2e+879de^2+703e^3\right)\int\frac{1}{3+2x+5x^2}\,dx}{5\left(5d^2-2de+3e^2\right)^3}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{2e^3\left(5d^2-2de+3e^2\right)(d+ex)^2}+\frac{40d^5+d^4e+28d^3e^2+44d^2e^3-2de^4+e^5}{e^3\left(5d^2-2de+3e^2\right)^2(d+ex)}\\
&\quad+\frac{\left(100d^6-120d^5e+228d^4e^2-242d^3e^3+141d^2e^4+120de^5-e^6\right)\log(d+ex)}{e^3\left(5d^2-2de+3e^2\right)^3}\\
&\quad+\frac{\left(458d^3-21d^2e-816de^2+113e^3\right)\log\left(3+2x+5x^2\right)}{10\left(5d^2-2de+3e^2\right)^3}\\
&\quad+\frac{\left(2\left(423d^3-4101d^2e+879de^2+703e^3\right)\right)\text{Subst}\left(\int\frac{1}{-56-x^2}\,dx,x,2+10x\right)}{5\left(5d^2-2de+3e^2\right)^3}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{2e^3\left(5d^2-2de+3e^2\right)(d+ex)^2}+\frac{40d^5+d^4e+28d^3e^2+44d^2e^3-2de^4+e^5}{e^3\left(5d^2-2de+3e^2\right)^2(d+ex)}\\
&\quad-\frac{\left(423d^3-4101d^2e+879de^2+703e^3\right)\tan^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{5\sqrt{14}\left(5d^2-2de+3e^2\right)^3}\\
&\quad+\frac{\left(100d^6-120d^5e+228d^4e^2-242d^3e^3+141d^2e^4+120de^5-e^6\right)\log(d+ex)}{e^3\left(5d^2-2de+3e^2\right)^3}\\
&\quad+\frac{\left(458d^3-21d^2e-816de^2+113e^3\right)\log\left(3+2x+5x^2\right)}{10\left(5d^2-2de+3e^2\right)^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 278, normalized size of antiderivative = 0.88

$$
\int\frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^3\left(3+2x+5x^2\right)}\,dx=
$$

$$
-\frac{\frac{35\left(5d^2-2de+3e^2\right)^2\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)}{e^3(d+ex)^2}-\frac{70\left(5d^2-2de+3e^2\right)\left(40d^5+d^4e+28d^3e^2+44d^2e^3-2de^4+e^5\right)}{e^3(d+ex)}+\sqrt{14}(423d^3-41}{}
$$

```
[In] Integrate[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)^3*(3 + 2*x + 5*x^2)),x
]
```

```
[Out] -1/70*((35*(5*d^2 - 2*d*e + 3*e^2)^2*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 +
 2*e^4))/(e^3*(d + e*x)^2) - (70*(5*d^2 - 2*d*e + 3*e^2)*(40*d^5 + d^4*e +
28*d^3*e^2 + 44*d^2*e^3 - 2*d*e^4 + e^5))/(e^3*(d + e*x)) + Sqrt[14]*(423*d
^3 - 4101*d^2*e + 879*d*e^2 + 703*e^3)*ArcTan[(1 + 5*x)/Sqrt[14]] + (70*(-1
00*d^6 + 120*d^5*e - 228*d^4*e^2 + 242*d^3*e^3 - 141*d^2*e^4 - 120*d*e^5 +
e^6)*Log[d + e*x])/e^3 - 7*(458*d^3 - 21*d^2*e - 816*d*e^2 + 113*e^3)*Log[3
 + 2*x + 5*x^2])/(5*d^2 - 2*d*e + 3*e^2)^3
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 298, normalized size of antiderivative = 0.94

method	result
default	$\frac{\frac{\left(458d^3-21d^2e-816d\,e^2+113e^3\right)\ln\left(5x^2+2x+3\right)}{10}+\frac{\left(-\frac{423}{5}d^3+\frac{4101}{5}d^2e-\frac{879}{5}d\,e^2-\frac{703}{5}e^3\right)\sqrt{14}\ \arctan\left(\frac{(10x+2)\sqrt{14}}{28}\right)}{14}}{\left(5d^2-2de+3e^2\right)^3}-\frac{4d^4+5d^3e+3d^2e^2-d\,e}{2e^3\left(5d^2-2de+3e^2\right)(e}$
risch	Expression too large to display

```
[In] int((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3/(5*x^2+2*x+3),x,method=_RETURNVERBOSE
)
```

```
[Out] 1/(5*d^2-2*d*e+3*e^2)^3*(1/10*(458*d^3-21*d^2*e-816*d*e^2+113*e^3)*ln(5*x^2
+2*x+3)+1/14*(-423/5*d^3+4101/5*d^2*e-879/5*d*e^2-703/5*e^3)*14^(1/2)*arcta
n(1/28*(10*x+2)*14^(1/2)))-1/2*(4*d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)/e^3/(5
*d^2-2*d*e+3*e^2)/(e*x+d)^2-(-40*d^5-d^4*e-28*d^3*e^2-44*d^2*e^3+2*d*e^4-e^
5)/e^3/(5*d^2-2*d*e+3*e^2)^2/(e*x+d)+(100*d^6-120*d^5*e+228*d^4*e^2-242*d^3
*e^3+141*d^2*e^4+120*d*e^5-e^6)*ln(e*x+d)/e^3/(5*d^2-2*d*e+3*e^2)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 698 vs. 2(308) = 616.

Time = 0.40 (sec) , antiderivative size = 698, normalized size of antiderivative = 2.20

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^3\,(3+2x+5x^2)}\,dx$$
$$=\frac{10500\,d^8-6825\,d^7e+14175\,d^6e^2+10395\,d^5e^3-6160\,d^4e^4+12145\,d^3e^5-4305\,d^2e^6+1365\,de^7-630\,e^8-}{}$$

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3/(5*x^2+2*x+3),x, algorithm="fri
cas")
```

```
[Out] 1/70*(10500*d^8 - 6825*d^7*e + 14175*d^6*e^2 + 10395*d^5*e^3 - 6160*d^4*e^4
 + 12145*d^3*e^5 - 4305*d^2*e^6 + 1365*d*e^7 - 630*e^8 - sqrt(14)*(423*d^5*
e^3 - 4101*d^4*e^4 + 879*d^3*e^5 + 703*d^2*e^6 + (423*d^3*e^5 - 4101*d^2*e^
6 + 879*d*e^7 + 703*e^8)*x^2 + 2*(423*d^4*e^4 - 4101*d^3*e^5 + 879*d^2*e^6
+ 703*d*e^7)*x)*arctan(1/14*sqrt(14)*(5*x + 1)) + 70*(200*d^7*e - 75*d^6*e^
2 + 258*d^5*e^3 + 167*d^4*e^4 - 14*d^3*e^5 + 141*d^2*e^6 - 8*d*e^7 + 3*e^8)
*x + 70*(100*d^8 - 120*d^7*e + 228*d^6*e^2 - 242*d^5*e^3 + 141*d^4*e^4 + 12
0*d^3*e^5 - d^2*e^6 + (100*d^6*e^2 - 120*d^5*e^3 + 228*d^4*e^4 - 242*d^3*e^
5 + 141*d^2*e^6 + 120*d*e^7 - e^8)*x^2 + 2*(100*d^7*e - 120*d^6*e^2 + 228*d
^5*e^3 - 242*d^4*e^4 + 141*d^3*e^5 + 120*d^2*e^6 - d*e^7)*x)*log(e*x + d) +
 7*(458*d^5*e^3 - 21*d^4*e^4 - 816*d^3*e^5 + 113*d^2*e^6 + (458*d^3*e^5 - 2
1*d^2*e^6 - 816*d*e^7 + 113*e^8)*x^2 + 2*(458*d^4*e^4 - 21*d^3*e^5 - 816*d^
```

```
2*e^6 + 113*d*e^7)*x)*log(5*x^2 + 2*x + 3))/(125*d^8*e^3 - 150*d^7*e^4 + 28
5*d^6*e^5 - 188*d^5*e^6 + 171*d^4*e^7 - 54*d^3*e^8 + 27*d^2*e^9 + (125*d^6*
e^5 - 150*d^5*e^6 + 285*d^4*e^7 - 188*d^3*e^8 + 171*d^2*e^9 - 54*d*e^10 + 2
7*e^11)*x^2 + 2*(125*d^7*e^4 - 150*d^6*e^5 + 285*d^5*e^6 - 188*d^4*e^7 + 17
1*d^3*e^8 - 54*d^2*e^9 + 27*d*e^10)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^3\,(3+2x+5x^2)}\,dx = \text{Timed out}$$

```
[In] integrate((4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)**3/(5*x**2+2*x+3),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 498, normalized size of antiderivative = 1.57

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^3\,(3+2x+5x^2)}\,dx$$
$$= -\frac{\sqrt{14}(423\,d^3-4101\,d^2e+879\,de^2+703\,e^3)\arctan\left(\frac{1}{14}\sqrt{14}(5\,x+1)\right)}{70\,(125\,d^6-150\,d^5e+285\,d^4e^2-188\,d^3e^3+171\,d^2e^4-54\,de^5+27\,e^6)}$$
$$+\frac{(100\,d^6-120\,d^5e+228\,d^4e^2-242\,d^3e^3+141\,d^2e^4+120\,de^5-e^6)\log\left(ex+d\right)}{125\,d^6e^3-150\,d^5e^4+285\,d^4e^5-188\,d^3e^6+171\,d^2e^7-54\,de^8+27\,e^9}$$
$$+\frac{(458\,d^3-21\,d^2e-816\,de^2+113\,e^3)\log\left(5\,x^2+2\,x+3\right)}{10\,(125\,d^6-150\,d^5e+285\,d^4e^2-188\,d^3e^3+171\,d^2e^4-54\,de^5+27\,e^6)}$$
$$+\frac{60\,d^6-15\,d^5e+39\,d^4e^2+84\,d^3e^3-25\,d^2e^4+9\,de^5-6\,e^6+2\,(40\,d^5e+d^4e^2+28}{2\,(25\,d^6e^3-20\,d^5e^4+34\,d^4e^5-12\,d^3e^6+9\,d^2e^7+(25\,d^4e^5-20\,d^3e^6+34\,d^2e^7-12\,de^8+9\,e^9)x^2+2}$$

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3/(5*x^2+2*x+3),x, algorithm="max
ima")
```

```
[Out] -1/70*sqrt(14)*(423*d^3 - 4101*d^2*e + 879*d*e^2 + 703*e^3)*arctan(1/14*sqr
t(14)*(5*x + 1))/(125*d^6 - 150*d^5*e + 285*d^4*e^2 - 188*d^3*e^3 + 171*d^2
*e^4 - 54*d*e^5 + 27*e^6) + (100*d^6 - 120*d^5*e + 228*d^4*e^2 - 242*d^3*e^
3 + 141*d^2*e^4 + 120*d*e^5 - e^6)*log(e*x + d)/(125*d^6*e^3 - 150*d^5*e^4
+ 285*d^4*e^5 - 188*d^3*e^6 + 171*d^2*e^7 - 54*d*e^8 + 27*e^9) + 1/10*(458*
d^3 - 21*d^2*e - 816*d*e^2 + 113*e^3)*log(5*x^2 + 2*x + 3)/(125*d^6 - 150*d
^5*e + 285*d^4*e^2 - 188*d^3*e^3 + 171*d^2*e^4 - 54*d*e^5 + 27*e^6) + 1/2*(
60*d^6 - 15*d^5*e + 39*d^4*e^2 + 84*d^3*e^3 - 25*d^2*e^4 + 9*d*e^5 - 6*e^6
+ 2*(40*d^5*e + d^4*e^2 + 28*d^3*e^3 + 44*d^2*e^4 - 2*d*e^5 + e^6)*x)/(25*d
```

```
^6*e^3 - 20*d^5*e^4 + 34*d^4*e^5 - 12*d^3*e^6 + 9*d^2*e^7 + (25*d^4*e^5 - 2
0*d^3*e^6 + 34*d^2*e^7 - 12*d*e^8 + 9*e^9)*x^2 + 2*(25*d^5*e^4 - 20*d^4*e^5
 + 34*d^3*e^6 - 12*d^2*e^7 + 9*d*e^8)*x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.38

$$
\begin{aligned}
&\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^3\left(3+2x+5x^2\right)}\,dx \\
&= -\frac{\sqrt{14}(423\,d^{3}-4101\,d^{2}e+879\,de^{2}+703\,e^{3})\arctan\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right)}{70\,(125\,d^{6}-150\,d^{5}e+285\,d^{4}e^{2}-188\,d^{3}e^{3}+171\,d^{2}e^{4}-54\,de^{5}+27\,e^{6})} \\
&\quad+\frac{(458\,d^{3}-21\,d^{2}e-816\,de^{2}+113\,e^{3})\log\left(5\,x^{2}+2\,x+3\right)}{10\,(125\,d^{6}-150\,d^{5}e+285\,d^{4}e^{2}-188\,d^{3}e^{3}+171\,d^{2}e^{4}-54\,de^{5}+27\,e^{6})} \\
&\quad+\frac{(100\,d^{6}-120\,d^{5}e+228\,d^{4}e^{2}-242\,d^{3}e^{3}+141\,d^{2}e^{4}+120\,de^{5}-e^{6})\log\left(|ex+d|\right)}{125\,d^{6}e^{3}-150\,d^{5}e^{4}+285\,d^{4}e^{5}-188\,d^{3}e^{6}+171\,d^{2}e^{7}-54\,de^{8}+27\,e^{9}} \\
&\quad+\frac{2\,(200\,d^{7}-75\,d^{6}e+258\,d^{5}e^{2}+167\,d^{4}e^{3}-14\,d^{3}e^{4}+141\,d^{2}e^{5}-8\,de^{6}+3\,e^{7})x+\frac{300\,d^{8}-195\,d^{7}e+405\,d^{6}e^{2}+297}{}}{2\,(5\,d^{2}-2\,de+3\,e^{2})^{3}(ex+d)^{2}e^{2}}
\end{aligned}
$$

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3/(5*x^2+2*x+3),x, algorithm="gia
c")
```

```
[Out] -1/70*sqrt(14)*(423*d^3 - 4101*d^2*e + 879*d*e^2 + 703*e^3)*arctan(1/14*sqr
t(14)*(5*x + 1))/(125*d^6 - 150*d^5*e + 285*d^4*e^2 - 188*d^3*e^3 + 171*d^2
*e^4 - 54*d*e^5 + 27*e^6) + 1/10*(458*d^3 - 21*d^2*e - 816*d*e^2 + 113*e^3)
*log(5*x^2 + 2*x + 3)/(125*d^6 - 150*d^5*e + 285*d^4*e^2 - 188*d^3*e^3 + 17
1*d^2*e^4 - 54*d*e^5 + 27*e^6) + (100*d^6 - 120*d^5*e + 228*d^4*e^2 - 242*d
^3*e^3 + 141*d^2*e^4 + 120*d*e^5 - e^6)*log(abs(e*x + d))/(125*d^6*e^3 - 15
0*d^5*e^4 + 285*d^4*e^5 - 188*d^3*e^6 + 171*d^2*e^7 - 54*d*e^8 + 27*e^9) +
1/2*(2*(200*d^7 - 75*d^6*e + 258*d^5*e^2 + 167*d^4*e^3 - 14*d^3*e^4 + 141*d
^2*e^5 - 8*d*e^6 + 3*e^7)*x + (300*d^8 - 195*d^7*e + 405*d^6*e^2 + 297*d^5*
e^3 - 176*d^4*e^4 + 347*d^3*e^5 - 123*d^2*e^6 + 39*d*e^7 - 18*e^8)/e)/((5*d
^2 - 2*d*e + 3*e^2)^3*(e*x + d)^2*e^2)
```

Mupad [B] (verification not implemented)

Time = 13.77 (sec) , antiderivative size = 493, normalized size of antiderivative = 1.56

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^3\,(3+2x+5x^2)}\,dx$$

$$= \frac{\frac{60\,d^6-15\,d^5\,e+39\,d^4\,e^2+84\,d^3\,e^3-25\,d^2\,e^4+9\,d\,e^5-6\,e^6}{2\,e^3\,(25\,d^4-20\,d^3\,e+34\,d^2\,e^2-12\,d\,e^3+9\,e^4)}+\frac{x\,(40\,d^5+d^4\,e+28\,d^3\,e^2+44\,d^2\,e^3-2\,d\,e^4+e^5)}{e^2\,(25\,d^4-20\,d^3\,e+34\,d^2\,e^2-12\,d\,e^3+9\,e^4)}}{d^2+2\,d\,e\,x+e^2\,x^2}$$
$$-\frac{\ln\left(x+\frac{1}{5}-\frac{\sqrt{14}\,1\mathrm{i}}{5}\right)\,\Bigl(\Bigl(\frac{423\,\sqrt{14}}{140}-\frac{229}{5}\mathrm{i}\Bigr)\,d^3+\Bigl(-\frac{4101\,\sqrt{14}}{140}+\frac{21}{10}\mathrm{i}\Bigr)\,d^2\,e+\Bigl(\frac{879\,\sqrt{14}}{140}+\frac{408}{5}\mathrm{i}\Bigr)\,d\,e^2+\Bigl(\frac{703\,\sqrt{14}}{140}-}{d^6\,125\mathrm{i}-d^5\,e\,150\mathrm{i}+d^4\,e^2\,285\mathrm{i}-d^3\,e^3\,188\mathrm{i}+d^2\,e^4\,171\mathrm{i}-d\,e^5\,54\mathrm{i}+e^6\,27\mathrm{i}}$$
$$+\frac{\ln\left(x+\frac{1}{5}+\frac{\sqrt{14}\,1\mathrm{i}}{5}\right)\,\Bigl(\Bigl(\frac{423\,\sqrt{14}}{140}+\frac{229}{5}\mathrm{i}\Bigr)\,d^3+\Bigl(-\frac{4101\,\sqrt{14}}{140}-\frac{21}{10}\mathrm{i}\Bigr)\,d^2\,e+\Bigl(\frac{879\,\sqrt{14}}{140}-\frac{408}{5}\mathrm{i}\Bigr)\,d\,e^2+\Bigl(\frac{703\,\sqrt{14}}{140}+}{d^6\,125\mathrm{i}-d^5\,e\,150\mathrm{i}+d^4\,e^2\,285\mathrm{i}-d^3\,e^3\,188\mathrm{i}+d^2\,e^4\,171\mathrm{i}-d\,e^5\,54\mathrm{i}+e^6\,27\mathrm{i}}$$
$$+\frac{\ln\left(d+e\,x\right)\,\left(100\,d^6-120\,d^5\,e+228\,d^4\,e^2-242\,d^3\,e^3+141\,d^2\,e^4+120\,d\,e^5-e^6\right)}{e^3\,{\left(5\,d^2-2\,d\,e+3\,e^2\right)}^3}$$

```
[In] int((x + 3*x^2 - 5*x^3 + 4*x^4 + 2)/((d + e*x)^3*(2*x + 5*x^2 + 3)),x)
```

```
[Out] ((9*d*e^5 - 15*d^5*e + 60*d^6 - 6*e^6 - 25*d^2*e^4 + 84*d^3*e^3 + 39*d^4*e^
2)/(2*e^3*(25*d^4 - 20*d^3*e - 12*d*e^3 + 9*e^4 + 34*d^2*e^2)) + (x*(d^4*e
- 2*d*e^4 + 40*d^5 + e^5 + 44*d^2*e^3 + 28*d^3*e^2))/(e^2*(25*d^4 - 20*d^3*
e - 12*d*e^3 + 9*e^4 + 34*d^2*e^2)))/(d^2 + e^2*x^2 + 2*d*e*x) - (log(x - (
14^(1/2)*1i)/5 + 1/5)*(d^3*((423*14^(1/2))/140 - 229i/5) + e^3*((703*14^(1/
2))/140 - 113i/10) + d*e^2*((879*14^(1/2))/140 + 408i/5) - d^2*e*((4101*14^
(1/2))/140 - 21i/10)))/(d^6*125i - d^5*e*150i - d*e^5*54i + e^6*27i + d^2*e
^4*171i - d^3*e^3*188i + d^4*e^2*285i) + (log(x + (14^(1/2)*1i)/5 + 1/5)*(d
^3*((423*14^(1/2))/140 + 229i/5) + e^3*((703*14^(1/2))/140 + 113i/10) + d*e
^2*((879*14^(1/2))/140 - 408i/5) - d^2*e*((4101*14^(1/2))/140 + 21i/10)))/(
d^6*125i - d^5*e*150i - d*e^5*54i + e^6*27i + d^2*e^4*171i - d^3*e^3*188i +
 d^4*e^2*285i) + (log(d + e*x)*(120*d*e^5 - 120*d^5*e + 100*d^6 - e^6 + 141
*d^2*e^4 - 242*d^3*e^3 + 228*d^4*e^2))/(e^3*(5*d^2 - 2*d*e + 3*e^2)^3)
```

3.311 $\int \frac{(d+ex)^3\left(2+x+3x^2-5x^3+4x^4\right)}{\left(3+2x+5x^2\right)^2}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 189

$$\begin{aligned}\int \frac{(d+ex)^3\left(2+x+3x^2-5x^3+4x^4\right)}{\left(3+2x+5x^2\right)^2}\,dx &= \frac{\left(2800d^3-17220d^2e+9921de^2+6053e^3\right)x}{17500}+\frac{e\left(840d^2-1722de+373e^2\right)x^2}{3500}\\ &\quad+\frac{1}{375}(60d-41e)e^2x^3+\frac{e^3x^4}{25}-\frac{(1367+423x)(d+ex)^3}{3500\left(3+2x+5x^2\right)}\\ &\quad+\frac{\left(32825d^3+317565d^2e-221643de^2-67499e^3\right)\arctan\left(\frac{1+5x}{\sqrt{14}}\right)}{87500\sqrt{14}}\\ &\quad-\frac{\left(1025d^3-1545d^2e-2601de^2+832e^3\right)\log\left(3+2x+5x^2\right)}{6250}\end{aligned}$$

```
[Out] 1/17500*(2800*d^3-17220*d^2*e+9921*d*e^2+6053*e^3)*x+1/3500*e*(840*d^2-1722
*d*e+373*e^2)*x^2+1/375*(60*d-41*e)*e^2*x^3+1/25*e^3*x^4-1/3500*(1367+423*x
)*(e*x+d)^3/(5*x^2+2*x+3)-1/6250*(1025*d^3-1545*d^2*e-2601*d*e^2+832*e^3)*l
n(5*x^2+2*x+3)+1/1225000*(32825*d^3+317565*d^2*e-221643*d*e^2-67499*e^3)*ar
ctan(1/14*(1+5*x)*14^(1/2))*14^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1658, 1642, 648, 632, 210, 642}

$$\begin{aligned}
&\int \frac{(d+ex)^3(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2}\,dx \\
&= \frac{\arctan\left(\frac{5x+1}{\sqrt{14}}\right)(32825d^3+317565d^2e-221643de^2-67499e^3)}{87500\sqrt{14}} \\
&\quad+\frac{ex^2(840d^2-1722de+373e^2)}{3500} \\
&\quad-\frac{(1025d^3-1545d^2e-2601de^2+832e^3)\log(5x^2+2x+3)}{6250} \\
&\quad+\frac{x(2800d^3-17220d^2e+9921de^2+6053e^3)}{17500} \\
&\quad+\frac{1}{375}e^2x^3(60d-41e)-\frac{(423x+1367)(d+ex)^3}{3500(5x^2+2x+3)}+\frac{e^3x^4}{25}
\end{aligned}$$

```
[In] Int[((d + e*x)^3*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^2,x]
```

```
[Out] ((2800*d^3 - 17220*d^2*e + 9921*d*e^2 + 6053*e^3)*x)/17500 + (e*(840*d^2 -
1722*d*e + 373*e^2)*x^2)/3500 + ((60*d - 41*e)*e^2*x^3)/375 + (e^3*x^4)/25
- ((1367 + 423*x)*(d + e*x)^3)/(3500*(3 + 2*x + 5*x^2)) + ((32825*d^3 + 317
565*d^2*e - 221643*d*e^2 - 67499*e^3)*ArcTan[(1 + 5*x)/Sqrt[14]])/(87500*Sq
rt[14]) - ((1025*d^3 - 1545*d^2*e - 2601*d*e^2 + 832*e^3)*Log[3 + 2*x + 5*x
^2])/6250
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1658

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f =
Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[Polynom
ialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x + c
*x^2)^(p + 1)*((f*b - 2*a*g + (2*c*f - b*g)*x)/((p + 1)*(b^2 - 4*a*c))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Q + g*(2*a*e*m + b*d*(2
*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(m + 2*p + 3)*x, x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Integer
Q[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] && Ra
tionalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(1367+423x)(d+ex)^3}{3500\,(3+2x+5x^2)} \\
&\quad +\frac{1}{56}\int\frac{(d+ex)^2\left(\frac{6}{125}(615d+1367e)-\frac{12}{125}(770d-519e)x+\frac{56}{25}(20d-33e)x^2+\frac{224ex^3}{5}\right)}{3+2x+5x^2}\,dx \\
&= -\frac{(1367+423x)(d+ex)^3}{3500\,(3+2x+5x^2)}+\frac{1}{56}\int\left(\frac{2}{625}\left(2800d^3-17220d^2e+9921de^2+6053e^3\right)\right. \\
&\qquad +\frac{4}{125}e\left(840d^2-1722de+373e^2\right)x+\frac{56}{125}(60d-41e)e^2x^2+\frac{224e^3x^3}{25} \\
&\qquad \left.+\frac{2(3(275d^3+24055d^2e-9921de^2-6053e^3)-28(1025d^3-1545d^2e-2601de^2+832e^3)\,x)}{625\,(3+2x+5x^2)}\right)dx
\end{aligned}
$$

$$= \frac{(2800d^3-17220d^2e+9921de^2+6053e^3)\,x}{17500}+\frac{e(840d^2-1722de+373e^2)\,x^2}{3500} + \frac{1}{375}(60d-41e)e^2x^3+\frac{e^3x^4}{25}-\frac{(1367+423x)(d+ex)^3}{3500\,(3+2x+5x^2)} + \frac{\int \frac{3(275d^3+24055d^2e-9921de^2-6053e^3)-28(1025d^3-1545d^2e-2601de^2+832e^3)x}{3+2x+5x^2}\,dx}{17500}$$

$$= \frac{(2800d^3-17220d^2e+9921de^2+6053e^3)\,x}{17500}+\frac{e(840d^2-1722de+373e^2)\,x^2}{3500} + \frac{1}{375}(60d-41e)e^2x^3+\frac{e^3x^4}{25}-\frac{(1367+423x)(d+ex)^3}{3500\,(3+2x+5x^2)} + \frac{(32825d^3+317565d^2e-221643de^2-67499e^3)\int\frac{1}{3+2x+5x^2}\,dx}{87500} + \frac{(-1025d^3+1545d^2e+2601de^2-832e^3)\int\frac{2+10x}{3+2x+5x^2}\,dx}{6250}$$

$$= \frac{(2800d^3-17220d^2e+9921de^2+6053e^3)\,x}{17500}+\frac{e(840d^2-1722de+373e^2)\,x^2}{3500} + \frac{1}{375}(60d-41e)e^2x^3+\frac{e^3x^4}{25}-\frac{(1367+423x)(d+ex)^3}{3500\,(3+2x+5x^2)} - \frac{(1025d^3-1545d^2e-2601de^2+832e^3)\log(3+2x+5x^2)}{6250} + \frac{(-32825d^3-317565d^2e+221643de^2+67499e^3)\,\mathrm{Subst}\left(\int\frac{1}{-56-x^2}\,dx,x,2+10x\right)}{43750}$$

$$= \frac{(2800d^3-17220d^2e+9921de^2+6053e^3)\,x}{17500}+\frac{e(840d^2-1722de+373e^2)\,x^2}{3500} + \frac{1}{375}(60d-41e)e^2x^3+\frac{e^3x^4}{25}-\frac{(1367+423x)(d+ex)^3}{3500\,(3+2x+5x^2)} + \frac{(32825d^3+317565d^2e-221643de^2-67499e^3)\tan^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{87500\sqrt{14}} - \frac{(1025d^3-1545d^2e-2601de^2+832e^3)\log(3+2x+5x^2)}{6250}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.11

$$\int \frac{(d+ex)^3\,(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2}\,dx$$

$$= \frac{5880(500d^3-3075d^2e+1545de^2+867e^3)\,x+14700e(300d^2-615de+103e^2)\,x^2+49000(60d-41e)e^2}{}$$

```
[In] Integrate[((d + e*x)^3*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^2
,x]
```

```
[Out] (5880*(500*d^3 - 3075*d^2*e + 1545*d*e^2 + 867*e^3)*x + 14700*e*(300*d^2 -
615*d*e + 103*e^2)*x^2 + 49000*(60*d - 41*e)*e^2*x^3 + 735000*e^3*x^4 - (42
*(e^3*(54969 - 53189*x) + 125*d^3*(1367 + 423*x) + 75*d^2*e*(-1269 + 5989*x
) - 15*d*e^2*(17967 + 18323*x)))/(3 + 2*x + 5*x^2) + 15*Sqrt[14]*(32825*d^3
 + 317565*d^2*e - 221643*d*e^2 - 67499*e^3)*ArcTan[(1 + 5*x)/Sqrt[14]] + 29
40*(-1025*d^3 + 1545*d^2*e + 2601*d*e^2 - 832*e^3)*Log[3 + 2*x + 5*x^2])/18
375000
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.13

method	result
default	$\frac{e^3x^4}{25}+\frac{4d\,e^2x^3}{25}-\frac{41e^3x^3}{375}+\frac{6d^2e\,x^2}{25}-\frac{123d\,e^2x^2}{250}+\frac{103e^3x^2}{1250}+\frac{4x\,d^3}{25}-\frac{123d^2ex}{125}+\frac{309d\,e^2x}{625}+\frac{867e^3x}{3125}-$
risch	$\frac{309d^2e\ln\left(350x^2+140x+210\right)}{1250}+\frac{2601d\,e^2\ln\left(350x^2+140x+210\right)}{6250}+\frac{1313\sqrt{14}\,d^3\arctan\left(\frac{5\sqrt{14}\,x}{14}+\frac{\sqrt{14}}{14}\right)}{49000}-\frac{41e^3x^3}{375}+$

```
[In] int((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x,method=_RETURNVERBO
SE)
```

```
[Out] 1/25*e^3*x^4+4/25*d*e^2*x^3-41/375*e^3*x^3+6/25*d^2*e*x^2-123/250*d*e^2*x^2
+103/1250*e^3*x^2+4/25*x*d^3-123/125*d^2*e*x+309/625*d*e^2*x+867/3125*e^3*x
-1/3125*((2115/28*d^3+17967/28*d^2*e-54969/140*d*e^2-53189/700*e^3)*x+6835/
28*d^3-3807/28*d^2*e-53901/140*d*e^2+54969/700*e^3)/(x^2+2/5*x+3/5)-1/17500
0*(28700*d^3-43260*d^2*e-72828*d*e^2+23296*e^3)*ln(5*x^2+2*x+3)-1/245000*(-
6565*d^3-63513*d^2*e+221643/5*d*e^2+67499/5*e^3)*14^(1/2)*arctan(1/28*(10*x
+2)*14^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 350 vs. 2(172) = 344.

Time = 0.26 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.85

$$\int \frac{(d+ex)^3\,(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2}\,dx$$

$$= 3675000\,e^3x^6+1225000\,(12\,de^2-7\,e^3)x^5+122500\,(180\,d^2e-321\,de^2+47\,e^3)x^4+147000\,(100\,d^3-555\,d$$

```
[In] integrate((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="f
ricas")
```

```
[Out] 1/18375000*(3675000*e^3*x^6 + 1225000*(12*d*e^2 - 7*e^3)*x^5 + 122500*(180*
d^2*e - 321*d*e^2 + 47*e^3)*x^4 + 147000*(100*d^3 - 555*d^2*e + 246*d*e^2 +
 153*e^3)*x^3 - 7176750*d^3 + 3997350*d^2*e + 11319210*d*e^2 - 2308698*e^3
+ 2940*(2000*d^3 - 7800*d^2*e - 3045*d*e^2 + 5013*e^3)*x^2 + 15*sqrt(14)*(9
8475*d^3 + 952695*d^2*e - 664929*d*e^2 - 202497*e^3 + 5*(32825*d^3 + 317565
*d^2*e - 221643*d*e^2 - 67499*e^3)*x^2 + 2*(32825*d^3 + 317565*d^2*e - 2216
43*d*e^2 - 67499*e^3)*x)*arctan(1/14*sqrt(14)*(5*x + 1)) + 42*(157125*d^3 -
 1740675*d^2*e + 923745*d*e^2 + 417329*e^3)*x - 2940*(3075*d^3 - 4635*d^2*e
 - 7803*d*e^2 + 2496*e^3 + 5*(1025*d^3 - 1545*d^2*e - 2601*d*e^2 + 832*e^3)
*x^2 + 2*(1025*d^3 - 1545*d^2*e - 2601*d*e^2 + 832*e^3)*x)*log(5*x^2 + 2*x
+ 3))/(5*x^2 + 2*x + 3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.35 (sec) , antiderivative size = 444, normalized size of antiderivative = 2.35

$$
\begin{aligned}
&\int \frac{(d+ex)^3\left(2+x+3x^2-5x^3+4x^4\right)}{\left(3+2x+5x^2\right)^2}\,dx \\
&= \frac{e^3x^4}{25} + x^3\cdot\left(\frac{4de^2}{25}-\frac{41e^3}{375}\right) + x^2\cdot\left(\frac{6d^2e}{25}-\frac{123de^2}{250}+\frac{103e^3}{1250}\right) \\
&\quad + x\left(\frac{4d^3}{25}-\frac{123d^2e}{125}+\frac{309de^2}{625}+\frac{867e^3}{3125}\right) + \Bigg(-\frac{41d^3}{250}+\frac{309d^2e}{1250}+\frac{2601de^2}{6250}-\frac{416e^3}{3125} \\
&\quad - \frac{\sqrt{14}i(32825d^3+317565d^2e-221643de^2-67499e^3)}{2450000}\Bigg)\log\Bigg(x+\frac{6565d^3+63513d^2e-\frac{221643de^2}{5}-\frac{67499}{5}}{32825d^3+317565d^2} \\
&\quad + \Bigg(-\frac{41d^3}{250}+\frac{309d^2e}{1250}+\frac{2601de^2}{6250}-\frac{416e^3}{3125} \\
&\quad + \frac{\sqrt{14}i(32825d^3+317565d^2e-221643de^2-67499e^3)}{2450000}\Bigg)\log\Bigg(x+\frac{6565d^3+63513d^2e-\frac{221643de^2}{5}-\frac{67499}{5}}{32825d^3+317565d^2} \\
&\quad + \frac{-170875d^3+95175d^2e+269505de^2-54969e^3+x(-52875d^3-449175d^2e+274845de^2+53189e^3)}{2187500x^2+875000x+1312500}
\end{aligned}
$$

```
[In] integrate((e*x+d)**3*(4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3)**2,x)
```

```
[Out] e**3*x**4/25 + x**3*(4*d*e**2/25 - 41*e**3/375) + x**2*(6*d**2*e/25 - 123*d
*e**2/250 + 103*e**3/1250) + x*(4*d**3/25 - 123*d**2*e/125 + 309*d*e**2/625
 + 867*e**3/3125) + (-41*d**3/250 + 309*d**2*e/1250 + 2601*d*e**2/6250 - 41
6*e**3/3125 - sqrt(14)*I*(32825*d**3 + 317565*d**2*e - 221643*d*e**2 - 6749
9*e**3)/2450000)*log(x + (6565*d**3 + 63513*d**2*e - 221643*d*e**2/5 - 6749
9*e**3/5 - sqrt(14)*I*(32825*d**3 + 317565*d**2*e - 221643*d*e**2 - 67499*e
**3)/5)/(32825*d**3 + 317565*d**2*e - 221643*d*e**2 - 67499*e**3)) + (-41*d
```

```
**3/250 + 309*d**2*e/1250 + 2601*d*e**2/6250 - 416*e**3/3125 + sqrt(14)*I*(
32825*d**3 + 317565*d**2*e - 221643*d*e**2 - 67499*e**3)/2450000)*log(x + (
6565*d**3 + 63513*d**2*e - 221643*d*e**2/5 - 67499*e**3/5 + sqrt(14)*I*(328
25*d**3 + 317565*d**2*e - 221643*d*e**2 - 67499*e**3)/5)/(32825*d**3 + 3175
65*d**2*e - 221643*d*e**2 - 67499*e**3)) + (-170875*d**3 + 95175*d**2*e + 2
69505*d*e**2 - 54969*e**3 + x*(-52875*d**3 - 449175*d**2*e + 274845*d*e**2
+ 53189*e**3))/(2187500*x**2 + 875000*x + 1312500)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.12

$$
\begin{aligned}
&\int \frac{(d+ex)^3\,(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2}\,dx \\
&= \frac{1}{25}\,e^3x^4 + \frac{1}{375}\,\left(60\,de^2 - 41\,e^3\right)x^3 + \frac{1}{1250}\,\left(300\,d^2e - 615\,de^2 + 103\,e^3\right)x^2 \\
&\quad + \frac{1}{1225000}\,\sqrt{14}\left(32825\,d^3 + 317565\,d^2e - 221643\,de^2 - 67499\,e^3\right)\arctan\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right) \\
&\quad + \frac{1}{3125}\,\left(500\,d^3 - 3075\,d^2e + 1545\,de^2 + 867\,e^3\right)x \\
&\quad - \frac{1}{6250}\,\left(1025\,d^3 - 1545\,d^2e - 2601\,de^2 + 832\,e^3\right)\log\left(5\,x^2 + 2\,x + 3\right) \\
&\quad - \frac{170875\,d^3 - 95175\,d^2e - 269505\,de^2 + 54969\,e^3 + \left(52875\,d^3 + 449175\,d^2e - 274845\,de^2 - 53189\,e^3\right)x}{437500\,\left(5\,x^2 + 2\,x + 3\right)}
\end{aligned}
$$

```
[In] integrate((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="m
axima")
```

```
[Out] 1/25*e^3*x^4 + 1/375*(60*d*e^2 - 41*e^3)*x^3 + 1/1250*(300*d^2*e - 615*d*e^
2 + 103*e^3)*x^2 + 1/1225000*sqrt(14)*(32825*d^3 + 317565*d^2*e - 221643*d*
e^2 - 67499*e^3)*arctan(1/14*sqrt(14)*(5*x + 1)) + 1/3125*(500*d^3 - 3075*d
^2*e + 1545*d*e^2 + 867*e^3)*x - 1/6250*(1025*d^3 - 1545*d^2*e - 2601*d*e^2
 + 832*e^3)*log(5*x^2 + 2*x + 3) - 1/437500*(170875*d^3 - 95175*d^2*e - 269
505*d*e^2 + 54969*e^3 + (52875*d^3 + 449175*d^2*e - 274845*d*e^2 - 53189*e^
3)*x)/(5*x^2 + 2*x + 3)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.14

$$\int \frac{(d+ex)^3 (2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2}\,dx = \frac{1}{25}\,e^3x^4+\frac{4}{25}\,de^2x^3-\frac{41}{375}\,e^3x^3 + \frac{6}{25}\,d^2ex^2-\frac{123}{250}\,de^2x^2+\frac{103}{1250}\,e^3x^2+\frac{4}{25}\,d^3x-\frac{123}{125}\,d^2ex+\frac{309}{625}\,de^2x+\frac{867}{3125}\,e^3x + \frac{1}{1225000}\,\sqrt{14}\left(32825\,d^3+317565\,d^2e-221643\,de^2-67499\,e^3\right)\arctan\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right) - \frac{1}{6250}\,\left(1025\,d^3-1545\,d^2e-2601\,de^2+832\,e^3\right)\log\left(5\,x^2+2\,x+3\right) - \frac{170875\,d^3-95175\,d^2e-269505\,de^2+54969\,e^3+(52875\,d^3+449175\,d^2e-274845\,de^2-53189\,e^3)x}{437500\,(5\,x^2+2\,x+3)}$$

```
[In] integrate((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="g
iac")
```

```
[Out] 1/25*e^3*x^4 + 4/25*d*e^2*x^3 - 41/375*e^3*x^3 + 6/25*d^2*e*x^2 - 123/250*d
*e^2*x^2 + 103/1250*e^3*x^2 + 4/25*d^3*x - 123/125*d^2*e*x + 309/625*d*e^2*
x + 867/3125*e^3*x + 1/1225000*sqrt(14)*(32825*d^3 + 317565*d^2*e - 221643*
d*e^2 - 67499*e^3)*arctan(1/14*sqrt(14)*(5*x + 1)) - 1/6250*(1025*d^3 - 154
5*d^2*e - 2601*d*e^2 + 832*e^3)*log(5*x^2 + 2*x + 3) - 1/437500*(170875*d^3
 - 95175*d^2*e - 269505*d*e^2 + 54969*e^3 + (52875*d^3 + 449175*d^2*e - 274
845*d*e^2 - 53189*e^3)*x)/(5*x^2 + 2*x + 3)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.76

$$\int \frac{(d+ex)^3\,(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2}\,dx$$
$$= \frac{\frac{53901\,d\,e^2}{28}+\frac{19035\,d^2\,e}{28}+x\left(-\frac{10575\,d^3}{28}-\frac{89835\,d^2\,e}{28}+\frac{54969\,d\,e^2}{28}+\frac{53189\,e^3}{140}\right)-\frac{34175\,d^3}{28}-\frac{54969\,e^3}{140}}{15625\,x^2+6250\,x+9375}$$
$$+x^3\left(\frac{e^2\,(12\,d-5\,e)}{75}-\frac{16\,e^3}{375}\right)$$
$$-x\left(\frac{18\,e^2\,(12\,d-5\,e)}{625}+\frac{12\,e\,(4\,d^2-5\,d\,e+e^2)}{125}-\frac{9\,d\,e^2}{25}+\frac{3\,d^2\,e}{5}-\frac{4\,d^3}{25}-\frac{717\,e^3}{3125}\right)$$
$$+\ln\left(5\,x^2+2\,x+3\right)\left(-\frac{41\,d^3}{250}+\frac{309\,d^2\,e}{1250}+\frac{2601\,d\,e^2}{6250}-\frac{416\,e^3}{3125}\right)$$
$$-x^2\left(\frac{2\,e^2\,(12\,d-5\,e)}{125}-\frac{3\,e\,(4\,d^2-5\,d\,e+e^2)}{50}+\frac{36\,e^3}{625}\right)+\frac{e^3\,x^4}{25}$$
$$-\frac{\sqrt{14}\,\mathrm{atan}\left(\frac{\frac{\sqrt{14}\,\left(-32825\,d^3-317565\,d^2\,e+221643\,d\,e^2+67499\,e^3\right)}{1225000}+\frac{\sqrt{14}\,x\,\left(-32825\,d^3-317565\,d^2\,e+221643\,d\,e^2+67499\,e^3\right)}{245000}}{-\frac{1313\,d^3}{3500}-\frac{63513\,d^2\,e}{17500}+\frac{221643\,d\,e^2}{87500}+\frac{67499\,e^3}{87500}}\right)\,(-32825\,d^3-31}{1225000}$$

```
[In] int(((d + e*x)^3*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(2*x + 5*x^2 + 3)^2,x)
```

```
[Out] ((53901*d*e^2)/28 + (19035*d^2*e)/28 + x*((54969*d*e^2)/28 - (89835*d^2*e)/
28 - (10575*d^3)/28 + (53189*e^3)/140) - (34175*d^3)/28 - (54969*e^3)/140)/
(6250*x + 15625*x^2 + 9375) + x^3*((e^2*(12*d - 5*e))/75 - (16*e^3)/375) -
x*((18*e^2*(12*d - 5*e))/625 + (12*e*(4*d^2 - 5*d*e + e^2))/125 - (9*d*e^2)
/25 + (3*d^2*e)/5 - (4*d^3)/25 - (717*e^3)/3125) + log(2*x + 5*x^2 + 3)*((2
601*d*e^2)/6250 + (309*d^2*e)/1250 - (41*d^3)/250 - (416*e^3)/3125) - x^2*(
(2*e^2*(12*d - 5*e))/125 - (3*e*(4*d^2 - 5*d*e + e^2))/50 + (36*e^3)/625) +
 (e^3*x^4)/25 - (14^(1/2)*atan(((14^(1/2)*(221643*d*e^2 - 317565*d^2*e - 32
825*d^3 + 67499*e^3))/1225000 + (14^(1/2)*x*(221643*d*e^2 - 317565*d^2*e -
32825*d^3 + 67499*e^3))/245000)/((221643*d*e^2)/87500 - (63513*d^2*e)/17500
 - (1313*d^3)/3500 + (67499*e^3)/87500))*(221643*d*e^2 - 317565*d^2*e - 328
25*d^3 + 67499*e^3))/1225000
```

3.312 $\int \frac{(d+ex)^2\left(2+x+3x^2-5x^3+4x^4\right)}{\left(3+2x+5x^2\right)^2}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 140

$$\begin{aligned}
&\int \frac{(d+ex)^2\left(2+x+3x^2-5x^3+4x^4\right)}{\left(3+2x+5x^2\right)^2}\,dx \\
&=\frac{\left(2800d^2-11480de+3307e^2\right)x}{17500}+\frac{1}{250}(40d-41e)ex^2+\frac{4e^2x^3}{75} \\
&\quad-\frac{(1367+423x)(d+ex)^2}{3500\left(3+2x+5x^2\right)}+\frac{\left(32825d^2+211710de-73881e^2\right)\arctan\left(\frac{1+5x}{\sqrt{14}}\right)}{87500\sqrt{14}} \\
&\quad-\frac{\left(1025d^2-1030de-867e^2\right)\log\left(3+2x+5x^2\right)}{6250}
\end{aligned}$$

```
[Out] 1/17500*(2800*d^2-11480*d*e+3307*e^2)*x+1/250*(40*d-41*e)*e*x^2+4/75*e^2*x^
3-1/3500*(1367+423*x)*(e*x+d)^2/(5*x^2+2*x+3)-1/6250*(1025*d^2-1030*d*e-867
*e^2)*ln(5*x^2+2*x+3)+1/1225000*(32825*d^2+211710*d*e-73881*e^2)*arctan(1/1
4*(1+5*x)*14^(1/2))*14^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used

$= \{1658, 1642, 648, 632, 210, 642\}$

$$\begin{aligned}&\int \frac{(d+ex)^2\left(2+x+3x^2-5x^3+4x^4\right)}{\left(3+2x+5x^2\right)^2}\,dx\\&=\frac{\arctan\left(\frac{5x+1}{\sqrt{14}}\right)\left(32825d^2+211710de-73881e^2\right)}{87500\sqrt{14}}\\&\quad-\frac{\left(1025d^2-1030de-867e^2\right)\log\left(5x^2+2x+3\right)}{6250}+\frac{x\left(2800d^2-11480de+3307e^2\right)}{17500}\\&\quad+\frac{1}{250}ex^2(40d-41e)-\frac{(423x+1367)(d+ex)^2}{3500\left(5x^2+2x+3\right)}+\frac{4e^2x^3}{75}\end{aligned}$$

```
[In] Int[((d + e*x)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^2,x]
```

```
[Out] ((2800*d^2 - 11480*d*e + 3307*e^2)*x)/17500 + ((40*d - 41*e)*e*x^2)/250 + (
4*e^2*x^3)/75 - ((1367 + 423*x)*(d + e*x)^2)/(3500*(3 + 2*x + 5*x^2)) + ((3
2825*d^2 + 211710*d*e - 73881*e^2)*ArcTan[(1 + 5*x)/Sqrt[14]])/(87500*Sqrt[
14]) - ((1025*d^2 - 1030*d*e - 867*e^2)*Log[3 + 2*x + 5*x^2])/6250
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
```

```
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1658

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f =
Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[Polynom
ialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x + c
*x^2)^(p + 1)*((f*b - 2*a*g + (2*c*f - b*g)*x)/((p + 1)*(b^2 - 4*a*c))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Q + g*(2*a*e*m + b*d*(2
*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(m + 2*p + 3)*x, x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Integer
Q[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] && Ra
tionalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(1367+423x)(d+ex)^2}{3500\,(3+2x+5x^2)} \\
&\quad +\frac{1}{56}\int \frac{(d+ex)\left(\frac{2}{125}(1845d+2734e)-\frac{6}{125}(1540d-897e)x+\frac{56}{25}(20d-33e)x^2+\frac{224ex^3}{5}\right)}{3+2x+5x^2}\,dx \\
&= -\frac{(1367+423x)(d+ex)^2}{3500\,(3+2x+5x^2)}+\frac{1}{56}\int\left(\frac{2}{625}(2800d^2-11480de+3307e^2)+\frac{56}{125}(40d-41e)ex\right. \\
&\quad \left.+\frac{224e^2x^2}{25}+\frac{2(825d^2+48110de-9921e^2-28(1025d^2-1030de-867e^2)\,x)}{625\,(3+2x+5x^2)}\right)dx \\
&= \frac{(2800d^2-11480de+3307e^2)\,x}{17500}+\frac{1}{250}(40d-41e)ex^2+\frac{4e^2x^3}{75} \\
&\quad -\frac{(1367+423x)(d+ex)^2}{3500\,(3+2x+5x^2)}+\frac{\int\frac{825d^2+48110de-9921e^2-28(1025d^2-1030de-867e^2)x}{3+2x+5x^2}\,dx}{17500} \\
&= \frac{(2800d^2-11480de+3307e^2)\,x}{17500}+\frac{1}{250}(40d-41e)ex^2+\frac{4e^2x^3}{75} \\
&\quad -\frac{(1367+423x)(d+ex)^2}{3500\,(3+2x+5x^2)}+\frac{(32825d^2+211710de-73881e^2)\int\frac{1}{3+2x+5x^2}\,dx}{87500} \\
&\quad +\frac{(-1025d^2+1030de+867e^2)\int\frac{2+10x}{3+2x+5x^2}\,dx}{6250}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{(2800d^2 - 11480de + 3307e^2)\,x}{17500} + \frac{1}{250}(40d - 41e)ex^2 + \frac{4e^2x^3}{75} \\
&\quad - \frac{(1367 + 423x)(d + ex)^2}{3500\,(3 + 2x + 5x^2)} - \frac{(1025d^2 - 1030de - 867e^2)\log(3 + 2x + 5x^2)}{6250} \\
&\quad + \frac{(-32825d^2 - 211710de + 73881e^2)\,\mathrm{Subst}\left(\int \frac{1}{-56-x^2}\,dx, x, 2 + 10x\right)}{43750} \\
&= \frac{(2800d^2 - 11480de + 3307e^2)\,x}{17500} + \frac{1}{250}(40d - 41e)ex^2 + \frac{4e^2x^3}{75} \\
&\quad - \frac{(1367 + 423x)(d + ex)^2}{3500\,(3 + 2x + 5x^2)} + \frac{(32825d^2 + 211710de - 73881e^2)\tan^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{87500\sqrt{14}} \\
&\quad - \frac{(1025d^2 - 1030de - 867e^2)\log(3 + 2x + 5x^2)}{6250}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.07

$$
\int \frac{(d + ex)^2\,(2 + x + 3x^2 - 5x^3 + 4x^4)}{(3 + 2x + 5x^2)^2}\,dx
$$

$$
= 5880(100d^2 - 410de + 103e^2)\,x + 14700(40d - 41e)ex^2 + 196000e^2x^3 - \frac{42(25d^2(1367+423x)+10de(-1269+5989x)-}{3+2x+5x^2}
$$

```
[In] Integrate[((d + e*x)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^2
,x]
```

```
[Out] (5880*(100*d^2 - 410*d*e + 103*e^2)*x + 14700*(40*d - 41*e)*e*x^2 + 196000*
e^2*x^3 - (42*(25*d^2*(1367 + 423*x) + 10*d*e*(-1269 + 5989*x) - e^2*(17967
 + 18323*x)))/(3 + 2*x + 5*x^2) + 3*Sqrt[14]*(32825*d^2 + 211710*d*e - 7388
1*e^2)*ArcTan[(1 + 5*x)/Sqrt[14]] + 588*(-1025*d^2 + 1030*d*e + 867*e^2)*Lo
g[3 + 2*x + 5*x^2])/3675000
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.04

method	result
default	$\frac{4x^3e^2}{75} + \frac{4de\,x^2}{25} - \frac{41e^2x^2}{250} + \frac{4x\,d^2}{25} - \frac{82dex}{125} + \frac{103e^2x}{625} - \frac{\left(\frac{423}{28}d^2 + \frac{5989}{70}de - \frac{18323}{700}e^2\right)x + \frac{1367d^2}{28} - \frac{1269de}{70} - \frac{17967e^2}{700}}{625\left(x^2 + \frac{2}{5}x + \frac{3}{5}\right)} -$ (28700d
risch	$\frac{103de\ln(350x^2+140x+210)}{625} + \frac{21171\sqrt{14}\,de\arctan\left(\frac{5\sqrt{14}\,x}{14} + \frac{\sqrt{14}}{14}\right)}{122500} - \frac{41e^2x^2}{250} + \frac{103e^2x}{625} - \frac{82dex}{125} +$

```
[In] int((e*x+d)^2*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x,method=_RETURNVERBO
SE)
```

```
[Out] 4/75*x^3*e^2+4/25*d*e*x^2-41/250*e^2*x^2+4/25*x*d^2-82/125*d*e*x+103/625*e^
2*x-1/625*((423/28*d^2+5989/70*d*e-18323/700*e^2)*x+1367/28*d^2-1269/70*d*e
-17967/700*e^2)/(x^2+2/5*x+3/5)-1/175000*(28700*d^2-28840*d*e-24276*e^2)*ln
(5*x^2+2*x+3)-1/245000*(-6565*d^2-42342*d*e+73881/5*e^2)*14^(1/2)*arctan(1/
28*(10*x+2)*14^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.75

$$\int \frac{(d+ex)^2\,(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2}\,dx$$
$$= \frac{980000\,e^2x^5+24500\,(120\,de-107\,e^2)x^4+58800\,(50\,d^2-185\,de+41\,e^2)x^3+2940\,(400\,d^2-1040\,de-2}{}$$

```
[In] integrate((e*x+d)^2*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="f
ricas")
```

```
[Out] 1/3675000*(980000*e^2*x^5 + 24500*(120*d*e - 107*e^2)*x^4 + 58800*(50*d^2 -
 185*d*e + 41*e^2)*x^3 + 2940*(400*d^2 - 1040*d*e - 203*e^2)*x^2 + 3*sqrt(1
4)*(5*(32825*d^2 + 211710*d*e - 73881*e^2)*x^2 + 98475*d^2 + 635130*d*e - 2
21643*e^2 + 2*(32825*d^2 + 211710*d*e - 73881*e^2)*x)*arctan(1/14*sqrt(14)*
(5*x + 1)) - 1435350*d^2 + 532980*d*e + 754614*e^2 + 42*(31425*d^2 - 232090
*d*e + 61583*e^2)*x - 588*(5*(1025*d^2 - 1030*d*e - 867*e^2)*x^2 + 3075*d^2
 - 3090*d*e - 2601*e^2 + 2*(1025*d^2 - 1030*d*e - 867*e^2)*x)*log(5*x^2 + 2
*x + 3))/(5*x^2 + 2*x + 3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.92 (sec) , antiderivative size = 298, normalized size of antiderivative = 2.13

$$\int \frac{(d+ex)^2\,(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2}\,dx$$

$$= \frac{4e^2x^3}{75} + x^2\cdot\left(\frac{4de}{25}-\frac{41e^2}{250}\right) + x\left(\frac{4d^2}{25}-\frac{82de}{125}+\frac{103e^2}{625}\right) + \left(-\frac{41d^2}{250}+\frac{103de}{625}+\frac{867e^2}{6250}\right.$$

$$\left.-\frac{\sqrt{14}i(32825d^2+211710de-73881e^2)}{2450000}\right)\log\left(x+\frac{6565d^2+42342de-\frac{73881e^2}{5}-\frac{\sqrt{14}i(32825d^2+211710de-738}{5}}{32825d^2+211710de-73881e^2}\right.$$

$$+\left(-\frac{41d^2}{250}+\frac{103de}{625}+\frac{867e^2}{6250}\right.$$

$$\left.+\frac{\sqrt{14}i(32825d^2+211710de-73881e^2)}{2450000}\right)\log\left(x+\frac{6565d^2+42342de-\frac{73881e^2}{5}+\frac{\sqrt{14}i(32825d^2+211710de-738}{5}}{32825d^2+211710de-73881e^2}\right.$$

$$+\frac{-34175d^2+12690de+17967e^2+x(-10575d^2-59890de+18323e^2)}{437500x^2+175000x+262500}$$

```
[In] integrate((e*x+d)**2*(4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3)**2,x)
```

```
[Out] 4*e**2*x**3/75 + x**2*(4*d*e/25 - 41*e**2/250) + x*(4*d**2/25 - 82*d*e/125
+ 103*e**2/625) + (-41*d**2/250 + 103*d*e/625 + 867*e**2/6250 - sqrt(14)*I*
(32825*d**2 + 211710*d*e - 73881*e**2)/2450000)*log(x + (6565*d**2 + 42342*
d*e - 73881*e**2/5 - sqrt(14)*I*(32825*d**2 + 211710*d*e - 73881*e**2)/5)/(
32825*d**2 + 211710*d*e - 73881*e**2)) + (-41*d**2/250 + 103*d*e/625 + 867*
e**2/6250 + sqrt(14)*I*(32825*d**2 + 211710*d*e - 73881*e**2)/2450000)*log(
x + (6565*d**2 + 42342*d*e - 73881*e**2/5 + sqrt(14)*I*(32825*d**2 + 211710
*d*e - 73881*e**2)/5)/(32825*d**2 + 211710*d*e - 73881*e**2)) + (-34175*d**
2 + 12690*d*e + 17967*e**2 + x*(-10575*d**2 - 59890*d*e + 18323*e**2))/(437
500*x**2 + 175000*x + 262500)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.05

$$\begin{aligned}\int \frac{(d+ex)^2\,(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2}\,dx\\ =\frac{4}{75}\,e^2x^3+\frac{1}{250}\,\left(40\,de-41\,e^2\right)x^2\\ +\frac{1}{1225000}\,\sqrt{14}\left(32825\,d^2+211710\,de-73881\,e^2\right)\arctan\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right)\\ +\frac{1}{625}\,\left(100\,d^2-410\,de+103\,e^2\right)x\\ -\frac{1}{6250}\,\left(1025\,d^2-1030\,de-867\,e^2\right)\log\left(5\,x^2+2\,x+3\right)\\ -\frac{34175\,d^2-12690\,de-17967\,e^2+(10575\,d^2+59890\,de-18323\,e^2)x}{87500\,(5\,x^2+2\,x+3)}\end{aligned}$$

```
[In] integrate((e*x+d)^2*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="m
axima")
```

```
[Out] 4/75*e^2*x^3 + 1/250*(40*d*e - 41*e^2)*x^2 + 1/1225000*sqrt(14)*(32825*d^2
+ 211710*d*e - 73881*e^2)*arctan(1/14*sqrt(14)*(5*x + 1)) + 1/625*(100*d^2
- 410*d*e + 103*e^2)*x - 1/6250*(1025*d^2 - 1030*d*e - 867*e^2)*log(5*x^2 +
 2*x + 3) - 1/87500*(34175*d^2 - 12690*d*e - 17967*e^2 + (10575*d^2 + 59890
*d*e - 18323*e^2)*x)/(5*x^2 + 2*x + 3)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.04

$$\begin{aligned}\int \frac{(d+ex)^2\,(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2}\,dx\\ =\frac{4}{75}\,e^2x^3+\frac{4}{25}\,dex^2-\frac{41}{250}\,e^2x^2+\frac{4}{25}\,d^2x-\frac{82}{125}\,dex+\frac{103}{625}\,e^2x\\ +\frac{1}{1225000}\,\sqrt{14}\left(32825\,d^2+211710\,de-73881\,e^2\right)\arctan\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right)\\ -\frac{1}{6250}\,\left(1025\,d^2-1030\,de-867\,e^2\right)\log\left(5\,x^2+2\,x+3\right)\\ -\frac{34175\,d^2-12690\,de-17967\,e^2+(10575\,d^2+59890\,de-18323\,e^2)x}{87500\,(5\,x^2+2\,x+3)}\end{aligned}$$

```
[In] integrate((e*x+d)^2*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="g
iac")
```

```
[Out] 4/75*e^2*x^3 + 4/25*d*e*x^2 - 41/250*e^2*x^2 + 4/25*d^2*x - 82/125*d*e*x +
103/625*e^2*x + 1/1225000*sqrt(14)*(32825*d^2 + 211710*d*e - 73881*e^2)*arc
```

```
tan(1/14*sqrt(14)*(5*x + 1)) - 1/6250*(1025*d^2 - 1030*d*e - 867*e^2)*log(5
*x^2 + 2*x + 3) - 1/87500*(34175*d^2 - 12690*d*e - 17967*e^2 + (10575*d^2 +
 59890*d*e - 18323*e^2)*x)/(5*x^2 + 2*x + 3)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.51

$$\int \frac{(d+ex)^2\,(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2}\,dx = \ln\left(5\,x^2+2\,x+3\right)\,\left(-\frac{41\,d^2}{250}+\frac{103\,d\,e}{625}+\frac{867\,e^2}{6250}\right) - x\left(\frac{2\,d\,e}{5}+\frac{4\,e\,(8\,d-5\,e)}{125}-\frac{4\,d^2}{25}-\frac{3\,e^2}{625}\right)+x^2\left(\frac{e\,(8\,d-5\,e)}{50}-\frac{8\,e^2}{125}\right) + \frac{\frac{1269\,d\,e}{14}-x\left(\frac{2115\,d^2}{28}+\frac{5989\,d\,e}{14}-\frac{18323\,e^2}{140}\right)-\frac{6835\,d^2}{28}+\frac{17967\,e^2}{140}}{3125\,x^2+1250\,x+1875}+\frac{4\,e^2\,x^3}{75} + \frac{\sqrt{14}\,\mathrm{atan}\left(\frac{\frac{\sqrt{14}\,\left(32825\,d^2+211710\,d\,e-73881\,e^2\right)}{1225000}+\frac{\sqrt{14}\,x\,\left(32825\,d^2+211710\,d\,e-73881\,e^2\right)}{245000}}{\frac{1313\,d^2}{3500}+\frac{21171\,d\,e}{8750}-\frac{73881\,e^2}{87500}}\right)\,\left(32825\,d^2+211710\,d\,e-73881\,e^2\right)}{1225000}$$

```
[In] int(((d + e*x)^2*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(2*x + 5*x^2 + 3)^2,x)
```

```
[Out] log(2*x + 5*x^2 + 3)*((103*d*e)/625 - (41*d^2)/250 + (867*e^2)/6250) - x*((
2*d*e)/5 + (4*e*(8*d - 5*e))/125 - (4*d^2)/25 - (3*e^2)/625) + x^2*((e*(8*d
 - 5*e))/50 - (8*e^2)/125) + ((1269*d*e)/14 - x*((5989*d*e)/14 + (2115*d^2)
/28 - (18323*e^2)/140) - (6835*d^2)/28 + (17967*e^2)/140)/(1250*x + 3125*x^
2 + 1875) + (4*e^2*x^3)/75 + (14^(1/2)*atan(((14^(1/2)*(211710*d*e + 32825*
d^2 - 73881*e^2))/1225000 + (14^(1/2)*x*(211710*d*e + 32825*d^2 - 73881*e^2
))/245000)/((21171*d*e)/8750 + (1313*d^2)/3500 - (73881*e^2)/87500))*(21171
0*d*e + 32825*d^2 - 73881*e^2))/1225000
```

3.313 $\int \frac{(d+ex)\left(2+x+3x^2-5x^3+4x^4\right)}{\left(3+2x+5x^2\right)^2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 97

$$\begin{aligned}\int \frac{(d+ex)\left(2+x+3x^2-5x^3+4x^4\right)}{\left(3+2x+5x^2\right)^2}\,dx &= \frac{1}{125}(20d-41e)x+\frac{2ex^2}{25}\\ &\quad-\frac{(1367+423x)(d+ex)}{3500\left(3+2x+5x^2\right)}\\ &\quad+\frac{(6565d+21171e)\arctan\left(\frac{1+5x}{\sqrt{14}}\right)}{17500\sqrt{14}}\\ &\quad-\frac{(205d-103e)\log\left(3+2x+5x^2\right)}{1250}\end{aligned}$$

```
[Out] 1/125*(20*d-41*e)*x+2/25*e*x^2-1/3500*(1367+423*x)*(e*x+d)/(5*x^2+2*x+3)-1/
1250*(205*d-103*e)*ln(5*x^2+2*x+3)+1/245000*(6565*d+21171*e)*arctan(1/14*(1
+5*x)*14^(1/2))*14^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1658, 1671, 648, 632, 210, 642}

$$\int \frac{(d+ex)\left(2+x+3x^2-5x^3+4x^4\right)}{\left(3+2x+5x^2\right)^2}\,dx=\frac{\arctan\left(\frac{5x+1}{\sqrt{14}}\right)(6565d+21171e)}{17500\sqrt{14}}-\frac{(423x+1367)(d+ex)}{3500\left(5x^2+2x+3\right)}-\frac{(205d-103e)\log\left(5x^2+2x+3\right)}{1250}+\frac{1}{125}x(20d-41e)+\frac{2ex^2}{25}$$

```
[In] Int[((d + e*x)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^2,x]
```

```
[Out] ((20*d - 41*e)*x)/125 + (2*e*x^2)/25 - ((1367 + 423*x)*(d + e*x))/(3500*(3
+ 2*x + 5*x^2)) + ((6565*d + 21171*e)*ArcTan[(1 + 5*x)/Sqrt[14]])/(17500*Sq
rt[14]) - ((205*d - 103*e)*Log[3 + 2*x + 5*x^2])/1250
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1658

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f =
```

```
Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[Polynom
ialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x + c
*x^2)^(p + 1)*((f*b - 2*a*g + (2*c*f - b*g)*x)/((p + 1)*(b^2 - 4*a*c)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Q + g*(2*a*e*m + b*d*(2
*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(m + 2*p + 3)*x, x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Integer
Q[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] && Ra
tionalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(1367+423x)(d+ex)}{3500\,(3+2x+5x^2)} \\
&\quad+\frac{1}{56}\int\frac{\frac{2}{125}(1845d+1367e)-\frac{168}{125}(55d-27e)x+\frac{56}{25}(20d-33e)x^2+\frac{224ex^3}{5}}{3+2x+5x^2}\,dx \\
&= -\frac{(1367+423x)(d+ex)}{3500\,(3+2x+5x^2)} \\
&\quad+\frac{1}{56}\int\left(\frac{56}{125}(20d-41e)+\frac{224ex}{25}+\frac{2(165d+4811e-28(205d-103e)x)}{125\,(3+2x+5x^2)}\right)dx \\
&= \frac{1}{125}(20d-41e)x+\frac{2ex^2}{25}-\frac{(1367+423x)(d+ex)}{3500\,(3+2x+5x^2)}+\frac{\int\frac{165d+4811e-28(205d-103e)x}{3+2x+5x^2}\,dx}{3500} \\
&= \frac{1}{125}(20d-41e)x+\frac{2ex^2}{25}-\frac{(1367+423x)(d+ex)}{3500\,(3+2x+5x^2)} \\
&\quad+\frac{(-205d+103e)\int\frac{2+10x}{3+2x+5x^2}\,dx}{1250}+\frac{(6565d+21171e)\int\frac{1}{3+2x+5x^2}\,dx}{17500} \\
&= \frac{1}{125}(20d-41e)x+\frac{2ex^2}{25}-\frac{(1367+423x)(d+ex)}{3500\,(3+2x+5x^2)} \\
&\quad-\frac{(205d-103e)\log\left(3+2x+5x^2\right)}{1250} \\
&\quad+\frac{(-6565d-21171e)\operatorname{Subst}\left(\int\frac{1}{-56-x^2}\,dx,x,2+10x\right)}{8750}
\end{aligned}
$$

$$= \frac{1}{125}(20d - 41e)x + \frac{2ex^2}{25} - \frac{(1367 + 423x)(d + ex)}{3500\,(3 + 2x + 5x^2)} + \frac{(6565d + 21171e)\tan^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{17500\sqrt{14}} - \frac{(205d - 103e)\log\left(3 + 2x + 5x^2\right)}{1250}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.99

$$\int \frac{(d + ex)\left(2 + x + 3x^2 - 5x^3 + 4x^4\right)}{\left(3 + 2x + 5x^2\right)^2}\,dx$$

$$= \frac{1960(20d - 41e)x + 19600ex^2 - \frac{14(5d(1367+423x)+e(-1269+5989x))}{3+2x+5x^2} + \sqrt{14}(6565d + 21171e)\arctan\left(\frac{1+5x}{\sqrt{14}}\right) + 19}{245000}$$

```
[In] Integrate[((d + e*x)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^2,x
]
```

```
[Out] (1960*(20*d - 41*e)*x + 19600*e*x^2 - (14*(5*d*(1367 + 423*x) + e*(-1269 +
5989*x)))/(3 + 2*x + 5*x^2) + Sqrt[14]*(6565*d + 21171*e)*ArcTan[(1 + 5*x)/
Sqrt[14]] + 196*(-205*d + 103*e)*Log[3 + 2*x + 5*x^2])/245000
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.90

method	result
default	$\frac{2e\,x^2}{25} + \frac{4dx}{25} - \frac{41ex}{125} - \frac{\left(\frac{423d}{140}+\frac{5989e}{700}\right)x+\frac{1367d}{140}-\frac{1269e}{700}}{125\left(x^2+\frac{2}{5}x+\frac{3}{5}\right)} - \frac{(5740d-2884e)\ln\left(5x^2+2x+3\right)}{35000} - \frac{\left(-1313d-\frac{21171e}{5}\right)\sqrt{14}\,\arctan(}{49000}$
risch	$\frac{2e\,x^2}{25} + \frac{4dx}{25} - \frac{41ex}{125} + \frac{\frac{\left(-\frac{423d}{140}-\frac{5989e}{700}\right)x}{125}-\frac{1367d}{17500}+\frac{1269e}{87500}}{x^2+\frac{2}{5}x+\frac{3}{5}} - \frac{41d\ln\left(350x^2+140x+210\right)}{250} + \frac{103e\ln\left(350x^2+140x+210\right)}{1250} + 1313$

```
[In] int((e*x+d)*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x,method=_RETURNVERBOSE
)
```

```
[Out] 2/25*e*x^2+4/25*d*x-41/125*e*x-1/125*((423/140*d+5989/700*e)*x+1367/140*d-1
269/700*e)/(x^2+2/5*x+3/5)-1/35000*(5740*d-2884*e)*ln(5*x^2+2*x+3)-1/49000*
(-1313*d-21171/5*e)*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.52

$$\int \frac{(d+ex)\left(2+x+3x^2-5x^3+4x^4\right)}{\left(3+2x+5x^2\right)^2}\,dx$$
$$= \frac{98000\,ex^4+9800\,(20\,d-37\,e)x^3+7840\,(10\,d-13\,e)x^2+\sqrt{14}(5\,(6565\,d+21171\,e)x^2+2\,(6565\,d+2117}{}$$

```
[In] integrate((e*x+d)*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="fri
cas")
```

```
[Out] 1/245000*(98000*e*x^4 + 9800*(20*d - 37*e)*x^3 + 7840*(10*d - 13*e)*x^2 + s
qrt(14)*(5*(6565*d + 21171*e)*x^2 + 2*(6565*d + 21171*e)*x + 19695*d + 6351
3*e)*arctan(1/14*sqrt(14)*(5*x + 1)) + 14*(6285*d - 23209*e)*x - 196*(5*(20
5*d - 103*e)*x^2 + 2*(205*d - 103*e)*x + 615*d - 309*e)*log(5*x^2 + 2*x + 3
) - 95690*d + 17766*e)/(5*x^2 + 2*x + 3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.62 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.70

$$\int \frac{(d+ex)\left(2+x+3x^2-5x^3+4x^4\right)}{\left(3+2x+5x^2\right)^2}\,dx$$
$$= \frac{2ex^2}{25}+x\left(\frac{4d}{25}-\frac{41e}{125}\right)+\frac{-6835d+1269e+x(-2115d-5989e)}{87500x^2+35000x+52500}+\left(-\frac{41d}{250}+\frac{103e}{1250}-\frac{\sqrt{14}i(6565d+21171e)}{490000}\right)\log\left(x+\frac{1313d+\frac{21171e}{5}-\frac{\sqrt{14}i(6565d+21171e)}{5}}{6565d+21171e}\right)+\left(-\frac{41d}{250}+\frac{103e}{1250}+\frac{\sqrt{14}i(6565d+21171e)}{490000}\right)\log\left(x+\frac{1313d+\frac{21171e}{5}+\frac{\sqrt{14}i(6565d+21171e)}{5}}{6565d+21171e}\right)$$

```
[In] integrate((e*x+d)*(4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3)**2,x)
```

```
[Out] 2*e*x**2/25 + x*(4*d/25 - 41*e/125) + (-6835*d + 1269*e + x*(-2115*d - 5989
*e))/(87500*x**2 + 35000*x + 52500) + (-41*d/250 + 103*e/1250 - sqrt(14)*I*
(6565*d + 21171*e)/490000)*log(x + (1313*d + 21171*e/5 - sqrt(14)*I*(6565*d
 + 21171*e)/5)/(6565*d + 21171*e)) + (-41*d/250 + 103*e/1250 + sqrt(14)*I*(
6565*d + 21171*e)/490000)*log(x + (1313*d + 21171*e/5 + sqrt(14)*I*(6565*d
+ 21171*e)/5)/(6565*d + 21171*e))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.93

$$\begin{aligned}&\int \frac{(d+ex)\left(2+x+3x^2-5x^3+4x^4\right)}{\left(3+2x+5x^2\right)^2}\,dx\\&=\frac{2}{25}\,ex^2+\frac{1}{245000}\,\sqrt{14}(6565\,d+21171\,e)\arctan\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right)+\frac{1}{125}\,(20\,d-41\,e)x\\&\quad-\frac{1}{1250}\,(205\,d-103\,e)\log\left(5\,x^2+2\,x+3\right)-\frac{(2115\,d+5989\,e)x+6835\,d-1269\,e}{17500\,(5\,x^2+2\,x+3)}\end{aligned}$$

```
[In] integrate((e*x+d)*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="max
ima")
```

```
[Out] 2/25*e*x^2 + 1/245000*sqrt(14)*(6565*d + 21171*e)*arctan(1/14*sqrt(14)*(5*x
 + 1)) + 1/125*(20*d - 41*e)*x - 1/1250*(205*d - 103*e)*log(5*x^2 + 2*x + 3
) - 1/17500*((2115*d + 5989*e)*x + 6835*d - 1269*e)/(5*x^2 + 2*x + 3)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.91

$$\begin{aligned}&\int \frac{(d+ex)\left(2+x+3x^2-5x^3+4x^4\right)}{\left(3+2x+5x^2\right)^2}\,dx\\&=\frac{2}{25}\,ex^2+\frac{1}{245000}\,\sqrt{14}(6565\,d+21171\,e)\arctan\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right)+\frac{4}{25}\,dx-\frac{41}{125}\,ex\\&\quad-\frac{1}{1250}\,(205\,d-103\,e)\log\left(5\,x^2+2\,x+3\right)-\frac{(2115\,d+5989\,e)x+6835\,d-1269\,e}{17500\,(5\,x^2+2\,x+3)}\end{aligned}$$

```
[In] integrate((e*x+d)*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="gia
c")
```

```
[Out] 2/25*e*x^2 + 1/245000*sqrt(14)*(6565*d + 21171*e)*arctan(1/14*sqrt(14)*(5*x
 + 1)) + 4/25*d*x - 41/125*e*x - 1/1250*(205*d - 103*e)*log(5*x^2 + 2*x + 3
) - 1/17500*((2115*d + 5989*e)*x + 6835*d - 1269*e)/(5*x^2 + 2*x + 3)
```

Mupad [B] (verification not implemented)

Time = 13.44 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.19

$$\begin{aligned}\int \frac{(d+ex)\left(2+x+3x^2-5x^3+4x^4\right)}{\left(3+2x+5x^2\right)^2}\,dx &= \frac{2\,e\,x^2}{25}-\ln\left(5\,x^2+2\,x+3\right)\left(\frac{41\,d}{250}-\frac{103\,e}{1250}\right)\\ &\quad +x\left(\frac{4\,d}{25}-\frac{41\,e}{125}\right)-\frac{\frac{1367\,d}{28}-\frac{1269\,e}{140}+x\left(\frac{423\,d}{28}+\frac{5989\,e}{140}\right)}{625\,x^2+250\,x+375}\\ &\quad +\frac{\sqrt{14}\,\mathrm{atan}\left(\frac{\frac{\sqrt{14}\,(6565\,d+21171\,e)}{245000}+\frac{\sqrt{14}\,x\,(6565\,d+21171\,e)}{49000}}{\frac{1313\,d}{3500}+\frac{21171\,e}{17500}}\right)(6565\,d+21171\,e)}{245000}\end{aligned}$$

```
[In] int(((d + e*x)*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(2*x + 5*x^2 + 3)^2,x)
```

```
[Out] (2*e*x^2)/25 - log(2*x + 5*x^2 + 3)*((41*d)/250 - (103*e)/1250) + x*((4*d)/
25 - (41*e)/125) - ((1367*d)/28 - (1269*e)/140 + x*((423*d)/28 + (5989*e)/1
40))/(250*x + 625*x^2 + 375) + (14^(1/2)*atan(((14^(1/2)*(6565*d + 21171*e)
)/245000 + (14^(1/2)*x*(6565*d + 21171*e))/49000)/((1313*d)/3500 + (21171*e
)/17500))*(6565*d + 21171*e))/245000
```

3.314 $\int \frac{2+x+3x^2-5x^3+4x^4}{\left(3+2x+5x^2\right)^2} \, dx$

Optimal result

Integrand size = 31, antiderivative size = 63

$$\int \frac{2+x+3x^2-5x^3+4x^4}{\left(3+2x+5x^2\right)^2} \, dx = \frac{4x}{25}-\frac{1367+423x}{3500\left(3+2x+5x^2\right)} +\frac{1313\arctan\left(\frac{1+5x}{\sqrt{14}}\right)}{3500\sqrt{14}}-\frac{41}{250}\log\left(3+2x+5x^2\right)$$

```
[Out] 4/25*x+1/3500*(-1367-423*x)/(5*x^2+2*x+3)-41/250*ln(5*x^2+2*x+3)+1313/49000
*arctan(1/14*(1+5*x)*14^(1/2))*14^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {1674, 1671, 648, 632, 210, 642}

$$\int \frac{2+x+3x^2-5x^3+4x^4}{\left(3+2x+5x^2\right)^2} \, dx = \frac{1313\arctan\left(\frac{5x+1}{\sqrt{14}}\right)}{3500\sqrt{14}}-\frac{423x+1367}{3500\left(5x^2+2x+3\right)} -\frac{41}{250}\log\left(5x^2+2x+3\right)+\frac{4x}{25}$$

```
[In] Int[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/(3 + 2*x + 5*x^2)^2,x]
```

```
[Out] (4*x)/25 - (1367 + 423*x)/(3500*(3 + 2*x + 5*x^2)) + (1313*ArcTan[(1 + 5*x)
/Sqrt[14]])/(3500*Sqrt[14]) - (41*Log[3 + 2*x + 5*x^2])/250
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\text{integral} = -\frac{1367+423x}{3500\left(3+2x+5x^2\right)}+\frac{1}{56}\int\frac{\frac{738}{25}-\frac{1848x}{25}+\frac{224x^2}{5}}{3+2x+5x^2}\,dx$$

$$
\begin{aligned}
&=-\frac{1367+423x}{3500\left(3+2x+5x^2\right)}+\frac{1}{56}\int\left(\frac{224}{25}+\frac{2(33-1148x)}{25\left(3+2x+5x^2\right)}\right)dx\\
&=\frac{4x}{25}-\frac{1367+423x}{3500\left(3+2x+5x^2\right)}+\frac{1}{700}\int\frac{33-1148x}{3+2x+5x^2}\,dx\\
&=\frac{4x}{25}-\frac{1367+423x}{3500\left(3+2x+5x^2\right)}-\frac{41}{250}\int\frac{2+10x}{3+2x+5x^2}\,dx+\frac{1313\int\frac{1}{3+2x+5x^2}\,dx}{3500}\\
&=\frac{4x}{25}-\frac{1367+423x}{3500\left(3+2x+5x^2\right)}-\frac{41}{250}\log\left(3+2x+5x^2\right)-\frac{1313\text{Subst}\left(\int\frac{1}{-56-x^2}\,dx,x,2+10x\right)}{1750}\\
&=\frac{4x}{25}-\frac{1367+423x}{3500\left(3+2x+5x^2\right)}+\frac{1313\tan^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{3500\sqrt{14}}-\frac{41}{250}\log\left(3+2x+5x^2\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.94

$$
\int\frac{2+x+3x^2-5x^3+4x^4}{\left(3+2x+5x^2\right)^2}\,dx
=\frac{7840x-\frac{14(1367+423x)}{3+2x+5x^2}+1313\sqrt{14}\arctan\left(\frac{1+5x}{\sqrt{14}}\right)-8036\log\left(3+2x+5x^2\right)}{49000}
$$

```
[In] Integrate[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/(3 + 2*x + 5*x^2)^2,x]
```

```
[Out] (7840*x - (14*(1367 + 423*x))/(3 + 2*x + 5*x^2) + 1313*Sqrt[14]*ArcTan[(1 +
 5*x)/Sqrt[14]] - 8036*Log[3 + 2*x + 5*x^2])/49000
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.79

method	result	size
risch	$\frac{4x}{25}+\frac{-\frac{423x}{17500}-\frac{1367}{17500}}{x^2+\frac{2}{5}x+\frac{3}{5}}-\frac{41\ln\left(25x^2+10x+15\right)}{250}+\frac{1313\arctan\left(\frac{(1+5x)\sqrt{14}}{14}\right)\sqrt{14}}{49000}$	50
default	$\frac{4x}{25}-\frac{\frac{423x}{700}+\frac{1367}{700}}{25\left(x^2+\frac{2}{5}x+\frac{3}{5}\right)}-\frac{41\ln\left(5x^2+2x+3\right)}{250}+\frac{1313\sqrt{14}\,\arctan\left(\frac{(10x+2)\sqrt{14}}{28}\right)}{49000}$	51

```
[In] int((4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 4/25*x+(-423/17500*x-1367/17500)/(x^2+2/5*x+3/5)-41/250*ln(25*x^2+10*x+15)+
1313/49000*arctan(1/14*(1+5*x)*14^(1/2))*14^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.24

$$\int \frac{2+x+3x^2-5x^3+4x^4}{\left(3+2x+5x^2\right)^2}\,dx$$
$$= \frac{39200\,x^3+1313\,\sqrt{14}(5\,x^2+2\,x+3)\arctan\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right)+15680\,x^2-8036\,(5\,x^2+2\,x+3)\log\left(5\,x\right.}{49000\,(5\,x^2+2\,x+3)}$$

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="fricas")
```

```
[Out] 1/49000*(39200*x^3 + 1313*sqrt(14)*(5*x^2 + 2*x + 3)*arctan(1/14*sqrt(14)*(
5*x + 1)) + 15680*x^2 - 8036*(5*x^2 + 2*x + 3)*log(5*x^2 + 2*x + 3) + 17598
*x - 19138)/(5*x^2 + 2*x + 3)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.03

$$\int \frac{2+x+3x^2-5x^3+4x^4}{\left(3+2x+5x^2\right)^2}\,dx = \frac{4x}{25}+\frac{-423x-1367}{17500x^2+7000x+10500} - \frac{41\log\left(x^2+\frac{2x}{5}+\frac{3}{5}\right)}{250}+\frac{1313\sqrt{14}\,\mathrm{atan}\left(\frac{5\sqrt{14}x}{14}+\frac{\sqrt{14}}{14}\right)}{49000}$$

```
[In] integrate((4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3)**2,x)
```

```
[Out] 4*x/25 + (-423*x - 1367)/(17500*x**2 + 7000*x + 10500) - 41*log(x**2 + 2*x/
5 + 3/5)/250 + 1313*sqrt(14)*atan(5*sqrt(14)*x/14 + sqrt(14)/14)/49000
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

$$\int \frac{2+x+3x^2-5x^3+4x^4}{\left(3+2x+5x^2\right)^2}\,dx = \frac{1313}{49000}\,\sqrt{14}\arctan\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right)+\frac{4}{25}\,x - \frac{423\,x+1367}{3500\,(5\,x^2+2\,x+3)}-\frac{41}{250}\,\log\left(5\,x^2+2\,x+3\right)$$

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="maxima")
```

```
[Out] 1313/49000*sqrt(14)*arctan(1/14*sqrt(14)*(5*x + 1)) + 4/25*x - 1/3500*(423*
x + 1367)/(5*x^2 + 2*x + 3) - 41/250*log(5*x^2 + 2*x + 3)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(3+2x+5x^2)^2}\,dx = \frac{1313}{49000}\,\sqrt{14}\arctan\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right)+\frac{4}{25}\,x - \frac{423\,x+1367}{3500\,(5\,x^2+2\,x+3)}-\frac{41}{250}\,\log\left(5\,x^2+2\,x+3\right)$$

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="giac")
```

```
[Out] 1313/49000*sqrt(14)*arctan(1/14*sqrt(14)*(5*x + 1)) + 4/25*x - 1/3500*(423*
x + 1367)/(5*x^2 + 2*x + 3) - 41/250*log(5*x^2 + 2*x + 3)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(3+2x+5x^2)^2}\,dx = \frac{4\,x}{25}-\frac{41\,\ln\left(5\,x^2+2\,x+3\right)}{250}-\frac{\frac{423\,x}{17500}+\frac{1367}{17500}}{x^2+\frac{2\,x}{5}+\frac{3}{5}} + \frac{1313\,\sqrt{14}\,\mathrm{atan}\left(\frac{5\,\sqrt{14}\,x}{14}+\frac{\sqrt{14}}{14}\right)}{49000}$$

```
[In] int((x + 3*x^2 - 5*x^3 + 4*x^4 + 2)/(2*x + 5*x^2 + 3)^2,x)
```

```
[Out] (4*x)/25 - (41*log(2*x + 5*x^2 + 3))/250 - ((423*x)/17500 + 1367/17500)/((2
*x)/5 + x^2 + 3/5) + (1313*14^(1/2)*atan((5*14^(1/2)*x)/14 + 14^(1/2)/14))/
49000
```

3.315 $\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)(3+2x+5x^2)^2}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 224

$$\begin{aligned}\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)(3+2x+5x^2)^2}\,dx = &-\frac{1367d-293e+(423d-1367e)x}{700(5d^2-2de+3e^2)(3+2x+5x^2)}\\ &+\frac{(6565d^3-26423d^2e+11089de^2-6623e^3)\arctan\left(\frac{1+5x}{\sqrt{14}}\right)}{700\sqrt{14}\,(5d^2-2de+3e^2)^2}\\ &+\frac{(4d^4+5d^3e+3d^2e^2-de^3+2e^4)\log(d+ex)}{e\,(5d^2-2de+3e^2)^2}\\ &-\frac{(205d^3-61d^2e+23de^2+14e^3)\log(3+2x+5x^2)}{50\,(5d^2-2de+3e^2)^2}\end{aligned}$$

```
[Out] 1/700*(-1367*d+293*e-(423*d-1367*e)*x)/(5*d^2-2*d*e+3*e^2)/(5*x^2+2*x+3)+(4
*d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)*ln(e*x+d)/e/(5*d^2-2*d*e+3*e^2)^2-1/50*
(205*d^3-61*d^2*e+23*d*e^2+14*e^3)*ln(5*x^2+2*x+3)/(5*d^2-2*d*e+3*e^2)^2+1/
9800*(6565*d^3-26423*d^2*e+11089*d*e^2-6623*e^3)*arctan(1/14*(1+5*x)*14^(1/
2))/(5*d^2-2*d*e+3*e^2)^2*14^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used

$= \{1660, 1642, 648, 632, 210, 642\}$

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)\left(3+2x+5x^2\right)^2}\,dx = \frac{\arctan\left(\frac{5x+1}{\sqrt{14}}\right)\left(6565d^3-26423d^2e+11089de^2-6623e^3\right)}{700\sqrt{14}\left(5d^2-2de+3e^2\right)^2} - \frac{x(423d-1367e)+1367d-293e}{700\left(5x^2+2x+3\right)\left(5d^2-2de+3e^2\right)} - \frac{\left(205d^3-61d^2e+23de^2+14e^3\right)\log\left(5x^2+2x+3\right)}{50\left(5d^2-2de+3e^2\right)^2} + \frac{\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)\log(d+ex)}{e\left(5d^2-2de+3e^2\right)^2}$$

```
[In] Int[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)*(3 + 2*x + 5*x^2)^2),x]
```

```
[Out] -1/700*(1367*d - 293*e + (423*d - 1367*e)*x)/((5*d^2 - 2*d*e + 3*e^2)*(3 +
2*x + 5*x^2)) + ((6565*d^3 - 26423*d^2*e + 11089*d*e^2 - 6623*e^3)*ArcTan[(
1 + 5*x)/Sqrt[14]])/(700*Sqrt[14]*(5*d^2 - 2*d*e + 3*e^2)^2) + ((4*d^4 + 5*
d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*Log[d + e*x])/(e*(5*d^2 - 2*d*e + 3*e^2)
^2) - ((205*d^3 - 61*d^2*e + 23*d*e^2 + 14*e^3)*Log[3 + 2*x + 5*x^2])/(50*(
5*d^2 - 2*d*e + 3*e^2)^2)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1660

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^(p + 1)/((p
+ 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1367d-293e+(423d-1367e)x}{700\left(5d^2-2de+3e^2\right)\left(3+2x+5x^2\right)} \\
&\quad+\frac{1}{56}\int\frac{\frac{2\left(369d^2-421de+280e^2\right)}{5(5d^2-2de+3e^2)}-\frac{2\left(924d^2-285de+281e^2\right)x}{5(5d^2-2de+3e^2)}+\frac{224x^2}{5}}{(d+ex)\left(3+2x+5x^2\right)}\,dx \\
&= -\frac{1367d-293e+(423d-1367e)x}{700\left(5d^2-2de+3e^2\right)\left(3+2x+5x^2\right)}+\frac{1}{56}\int\left(\frac{56(4d^4+5d^3e+3d^2e^2-de^3+2e^4)}{\left(5d^2-2de+3e^2\right)^2(d+ex)}\right. \\
&\quad\left.+\frac{2(165d^3-4943d^2e+2089de^2-1403e^3-28(205d^3-61d^2e+23de^2+14e^3)\,x)}{5\left(5d^2-2de+3e^2\right)^2\left(3+2x+5x^2\right)}\right)dx \\
&= -\frac{1367d-293e+(423d-1367e)x}{700\left(5d^2-2de+3e^2\right)\left(3+2x+5x^2\right)} \\
&\quad+\frac{\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)\log(d+ex)}{e\left(5d^2-2de+3e^2\right)^2} \\
&\quad+\frac{\int\frac{165d^3-4943d^2e+2089de^2-1403e^3-28\left(205d^3-61d^2e+23de^2+14e^3\right)x}{3+2x+5x^2}\,dx}{140\left(5d^2-2de+3e^2\right)^2}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{1367d-293e+(423d-1367e)x}{700\left(5d^2-2de+3e^2\right)\left(3+2x+5x^2\right)} \\
&\quad+\frac{\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)\log(d+ex)}{e\left(5d^2-2de+3e^2\right)^2} \\
&\quad+\frac{\left(6565d^3-26423d^2e+11089de^2-6623e^3\right)\int\frac{1}{3+2x+5x^2}\,dx}{700\left(5d^2-2de+3e^2\right)^2} \\
&\quad-\frac{\left(205d^3-61d^2e+23de^2+14e^3\right)\int\frac{2+10x}{3+2x+5x^2}\,dx}{50\left(5d^2-2de+3e^2\right)^2}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{1367d-293e+(423d-1367e)x}{700\left(5d^2-2de+3e^2\right)\left(3+2x+5x^2\right)} \\
&\quad+\frac{\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)\log(d+ex)}{e\left(5d^2-2de+3e^2\right)^2} \\
&\quad-\frac{\left(205d^3-61d^2e+23de^2+14e^3\right)\log\left(3+2x+5x^2\right)}{50\left(5d^2-2de+3e^2\right)^2} \\
&\quad-\frac{\left(6565d^3-26423d^2e+11089de^2-6623e^3\right)\operatorname{Subst}\left(\int\frac{1}{-56-x^2}\,dx,x,2+10x\right)}{350\left(5d^2-2de+3e^2\right)^2}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{1367d-293e+(423d-1367e)x}{700\left(5d^2-2de+3e^2\right)\left(3+2x+5x^2\right)} \\
&\quad+\frac{\left(6565d^3-26423d^2e+11089de^2-6623e^3\right)\tan^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{700\sqrt{14}\left(5d^2-2de+3e^2\right)^2} \\
&\quad+\frac{\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)\log(d+ex)}{e\left(5d^2-2de+3e^2\right)^2} \\
&\quad-\frac{\left(205d^3-61d^2e+23de^2+14e^3\right)\log\left(3+2x+5x^2\right)}{50\left(5d^2-2de+3e^2\right)^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.83

$$
\int\frac{2+x+3x^2-5x^3+4x^4}{(d+ex)\left(3+2x+5x^2\right)^2}\,dx
$$

$$
=\frac{\frac{14\left(5d^2-2de+3e^2\right)\left(-d(1367+423x)+e(293+1367x)\right)}{3+2x+5x^2}+\sqrt{14}\left(6565d^3-26423d^2e+11089de^2-6623e^3\right)\arctan\left(\frac{1+5x}{\sqrt{14}}\right)+}{9800\left(5d^2-2de+3\right.}
$$

```
[In] Integrate[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)*(3 + 2*x + 5*x^2)^2),x
]
```

```
[Out] ((14*(5*d^2 - 2*d*e + 3*e^2)*(-(d*(1367 + 423*x)) + e*(293 + 1367*x)))/(3 +
 2*x + 5*x^2) + Sqrt[14]*(6565*d^3 - 26423*d^2*e + 11089*d*e^2 - 6623*e^3)*
```

```
ArcTan[(1 + 5*x)/Sqrt[14]] + (9800*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2
*e^4)*Log[d + e*x])/e - 196*(205*d^3 - 61*d^2*e + 23*d*e^2 + 14*e^3)*Log[3
+ 2*x + 5*x^2])/(9800*(5*d^2 - 2*d*e + 3*e^2)^2)
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.96

method	result
default	$-\frac{\frac{\left(\frac{423}{700}d^3-\frac{7681}{3500}d^2e+\frac{4003}{3500}d\,e^2-\frac{4101}{3500}e^3\right)x+\frac{1367d^3}{700}-\frac{4199d^2e}{3500}+\frac{4687d\,e^2}{3500}-\frac{879e^3}{3500}}{x^2+\frac{2}{5}x+\frac{3}{5}}+\frac{\left(5740d^3-1708d^2e+644d\,e^2+392e^3\right)\ln\left(5x^2+2x+3\right)}{1400}+\left(-13}{\left(5d^2-2de+3e^2\right)^2}$
risch	Expression too large to display

```
[In] int((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3)^2,x,method=_RETURNVERBOSE
)
```

```
[Out] -1/(5*d^2-2*d*e+3*e^2)^2*(((423/700*d^3-7681/3500*d^2*e+4003/3500*d*e^2-410
1/3500*e^3)*x+1367/700*d^3-4199/3500*d^2*e+4687/3500*d*e^2-879/3500*e^3)/(x
^2+2/5*x+3/5)+1/1400*(5740*d^3-1708*d^2*e+644*d*e^2+392*e^3)*ln(5*x^2+2*x+3
)+1/1960*(-1313*d^3+26423/5*d^2*e-11089/5*d*e^2+6623/5*e^3)*14^(1/2)*arctan
(1/28*(10*x+2)*14^(1/2)))+(4*d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)*ln(e*x+d)/e
/(5*d^2-2*d*e+3*e^2)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 479 vs. 2(215) = 430.

Time = 0.32 (sec) , antiderivative size = 479, normalized size of antiderivative = 2.14

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)\left(3+2x+5x^2\right)^2}\,dx = -\frac{95690\,d^3e-58786\,d^2e^2+65618\,de^3-12306\,e^4-\sqrt{14}(19695\,d^3e-79269\,d^2e^2+33267\,de^3-19869\,e^4}{}$$

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3)^2,x, algorithm="fri
cas")
```

```
[Out] -1/9800*(95690*d^3*e - 58786*d^2*e^2 + 65618*d*e^3 - 12306*e^4 - sqrt(14)*(
19695*d^3*e - 79269*d^2*e^2 + 33267*d*e^3 - 19869*e^4 + 5*(6565*d^3*e - 264
23*d^2*e^2 + 11089*d*e^3 - 6623*e^4)*x^2 + 2*(6565*d^3*e - 26423*d^2*e^2 +
11089*d*e^3 - 6623*e^4)*x)*arctan(1/14*sqrt(14)*(5*x + 1)) + 14*(2115*d^3*e
 - 7681*d^2*e^2 + 4003*d*e^3 - 4101*e^4)*x - 9800*(12*d^4 + 15*d^3*e + 9*d^
2*e^2 - 3*d*e^3 + 6*e^4 + 5*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*x
^2 + 2*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*x)*log(e*x + d) + 196*
```

```
(615*d^3*e - 183*d^2*e^2 + 69*d*e^3 + 42*e^4 + 5*(205*d^3*e - 61*d^2*e^2 +
23*d*e^3 + 14*e^4)*x^2 + 2*(205*d^3*e - 61*d^2*e^2 + 23*d*e^3 + 14*e^4)*x)*
log(5*x^2 + 2*x + 3))/(75*d^4*e - 60*d^3*e^2 + 102*d^2*e^3 - 36*d*e^4 + 27*
e^5 + 5*(25*d^4*e - 20*d^3*e^2 + 34*d^2*e^3 - 12*d*e^4 + 9*e^5)*x^2 + 2*(25
*d^4*e - 20*d^3*e^2 + 34*d^2*e^3 - 12*d*e^4 + 9*e^5)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)\left(3+2x+5x^2\right)^2}\,dx=\text{Timed out}$$

```
[In] integrate((4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)/(5*x**2+2*x+3)**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.29

$$\begin{aligned}\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)\left(3+2x+5x^2\right)^2}\,dx &= \frac{\sqrt{14}(6565\,d^3-26423\,d^2e+11089\,de^2-6623\,e^3)\arctan\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right)}{9800\,(25\,d^4-20\,d^3e+34\,d^2e^2-12\,de^3+9\,e^4)}\\ &\quad+\frac{(4\,d^4+5\,d^3e+3\,d^2e^2-de^3+2\,e^4)\log\left(ex+d\right)}{25\,d^4e-20\,d^3e^2+34\,d^2e^3-12\,de^4+9\,e^5}\\ &\quad-\frac{(205\,d^3-61\,d^2e+23\,de^2+14\,e^3)\log\left(5\,x^2+2\,x+3\right)}{50\,(25\,d^4-20\,d^3e+34\,d^2e^2-12\,de^3+9\,e^4)}\\ &\quad-\frac{(423\,d-1367\,e)x+1367\,d-293\,e}{700\,(5\,(5\,d^2-2\,de+3\,e^2)x^2+15\,d^2-6\,de+9\,e^2+2\,(5\,d^2-2\,de+3\,e^2)x)}\end{aligned}$$

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3)^2,x, algorithm="max
ima")
```

```
[Out] 1/9800*sqrt(14)*(6565*d^3 - 26423*d^2*e + 11089*d*e^2 - 6623*e^3)*arctan(1/
14*sqrt(14)*(5*x + 1))/(25*d^4 - 20*d^3*e + 34*d^2*e^2 - 12*d*e^3 + 9*e^4)
+ (4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*log(e*x + d)/(25*d^4*e - 20
*d^3*e^2 + 34*d^2*e^3 - 12*d*e^4 + 9*e^5) - 1/50*(205*d^3 - 61*d^2*e + 23*d
*e^2 + 14*e^3)*log(5*x^2 + 2*x + 3)/(25*d^4 - 20*d^3*e + 34*d^2*e^2 - 12*d*
e^3 + 9*e^4) - 1/700*((423*d - 1367*e)*x + 1367*d - 293*e)/(5*(5*d^2 - 2*d*
e + 3*e^2)*x^2 + 15*d^2 - 6*d*e + 9*e^2 + 2*(5*d^2 - 2*d*e + 3*e^2)*x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.32

$$
\begin{aligned}
\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)\left(3+2x+5x^2\right)^2}\,dx
&= \frac{\sqrt{14}(6565\,d^3-26423\,d^2e+11089\,de^2-6623\,e^3)\arctan\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right)}{9800\,(25\,d^4-20\,d^3e+34\,d^2e^2-12\,de^3+9\,e^4)} \\
&\quad - \frac{(205\,d^3-61\,d^2e+23\,de^2+14\,e^3)\log\left(5\,x^2+2\,x+3\right)}{50\,(25\,d^4-20\,d^3e+34\,d^2e^2-12\,de^3+9\,e^4)} \\
&\quad + \frac{(4\,d^4+5\,d^3e+3\,d^2e^2-de^3+2\,e^4)\log\left(|ex+d|\right)}{25\,d^4e-20\,d^3e^2+34\,d^2e^3-12\,de^4+9\,e^5} \\
&\quad - \frac{6835\,d^3-4199\,d^2e+4687\,de^2-879\,e^3+(2115\,d^3-7681\,d^2e+4003\,de^2-4101\,e^3)x}{700\,(5\,d^2-2\,de+3\,e^2)^2(5\,x^2+2\,x+3)}
\end{aligned}
$$

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3)^2,x, algorithm="gia
c")
```

```
[Out] 1/9800*sqrt(14)*(6565*d^3 - 26423*d^2*e + 11089*d*e^2 - 6623*e^3)*arctan(1/
14*sqrt(14)*(5*x + 1))/(25*d^4 - 20*d^3*e + 34*d^2*e^2 - 12*d*e^3 + 9*e^4)
- 1/50*(205*d^3 - 61*d^2*e + 23*d*e^2 + 14*e^3)*log(5*x^2 + 2*x + 3)/(25*d^
4 - 20*d^3*e + 34*d^2*e^2 - 12*d*e^3 + 9*e^4) + (4*d^4 + 5*d^3*e + 3*d^2*e^
2 - d*e^3 + 2*e^4)*log(abs(e*x + d))/(25*d^4*e - 20*d^3*e^2 + 34*d^2*e^3 -
12*d*e^4 + 9*e^5) - 1/700*(6835*d^3 - 4199*d^2*e + 4687*d*e^2 - 879*e^3 + (
2115*d^3 - 7681*d^2*e + 4003*d*e^2 - 4101*e^3)*x)/((5*d^2 - 2*d*e + 3*e^2)^
2*(5*x^2 + 2*x + 3))
```

Mupad [B] (verification not implemented)

Time = 14.22 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.47

$$
\begin{aligned}
\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)\left(3+2x+5x^2\right)^2}\,dx &= \frac{\ln\left(d+e\,x\right)\,\left(4\,d^4+5\,d^3\,e+3\,d^2\,e^2-d\,e^3+2\,e^4\right)}{e\,{\left(5\,d^2-2\,d\,e+3\,e^2\right)}^2} \\
&\quad + \frac{\ln\left(x+\frac{1}{5}-\frac{\sqrt{14}\,1{}\mathrm{i}}{5}\right)\,\Bigg(\left(\frac{1313\,\sqrt{14}}{3920}-\frac{41}{10}{}\mathrm{i}\right)\,d^3+\left(-\frac{26423\,\sqrt{14}}{19600}+\frac{61}{50}{}\mathrm{i}\right)\,d^2\,e+\left(\frac{11089\,\sqrt{14}}{19600}-\frac{23}{50}{}\mathrm{i}\right)\,d\,e^2+\Bigg(-\frac{6623\,\sqrt{1}}{19600}}{d^4\,25{}\mathrm{i}-d^3\,e\,20{}\mathrm{i}+d^2\,e^2\,34{}\mathrm{i}-d\,e^3\,12{}\mathrm{i}+e^4\,9{}\mathrm{i}} \\
&\quad - \frac{\ln\left(x+\frac{1}{5}+\frac{\sqrt{14}\,1{}\mathrm{i}}{5}\right)\,\Bigg(\left(\frac{1313\,\sqrt{14}}{3920}+\frac{41}{10}{}\mathrm{i}\right)\,d^3+\left(-\frac{26423\,\sqrt{14}}{19600}-\frac{61}{50}{}\mathrm{i}\right)\,d^2\,e+\left(\frac{11089\,\sqrt{14}}{19600}+\frac{23}{50}{}\mathrm{i}\right)\,d\,e^2+\Bigg(-\frac{6623\,\sqrt{1}}{19600}}{d^4\,25{}\mathrm{i}-d^3\,e\,20{}\mathrm{i}+d^2\,e^2\,34{}\mathrm{i}-d\,e^3\,12{}\mathrm{i}+e^4\,9{}\mathrm{i}} \\
&\quad - \frac{\frac{1367\,d-293\,e}{700\,\left(5\,d^2-2\,d\,e+3\,e^2\right)}+\frac{x\,\left(423\,d-1367\,e\right)}{700\,\left(5\,d^2-2\,d\,e+3\,e^2\right)}}{5\,x^2+2\,x+3}
\end{aligned}
$$

```
[In] int((x + 3*x^2 - 5*x^3 + 4*x^4 + 2)/((d + e*x)*(2*x + 5*x^2 + 3)^2),x)
```

```
[Out] (log(x - (14^(1/2)*1i)/5 + 1/5)*(d^3*((1313*14^(1/2))/3920 - 41i/10) - e^3*
((6623*14^(1/2))/19600 + 7i/25) + d*e^2*((11089*14^(1/2))/19600 - 23i/50) -
 d^2*e*((26423*14^(1/2))/19600 - 61i/50)))/(d^4*25i - d^3*e*20i - d*e^3*12i
 + e^4*9i + d^2*e^2*34i) - ((1367*d - 293*e)/(700*(5*d^2 - 2*d*e + 3*e^2))
+ (x*(423*d - 1367*e))/(700*(5*d^2 - 2*d*e + 3*e^2)))/(2*x + 5*x^2 + 3) - (
log(x + (14^(1/2)*1i)/5 + 1/5)*(d^3*((1313*14^(1/2))/3920 + 41i/10) - e^3*(
(6623*14^(1/2))/19600 - 7i/25) + d*e^2*((11089*14^(1/2))/19600 + 23i/50) -
d^2*e*((26423*14^(1/2))/19600 + 61i/50)))/(d^4*25i - d^3*e*20i - d*e^3*12i
+ e^4*9i + d^2*e^2*34i) + (log(d + e*x)*(5*d^3*e - d*e^3 + 4*d^4 + 2*e^4 +
3*d^2*e^2))/(e*(5*d^2 - 2*d*e + 3*e^2)^2)
```

3.316 $\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2(3+2x+5x^2)^2}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 313

$$
\begin{aligned}
&\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2\,(3+2x+5x^2)^2}\,dx \\
&= -\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{e\,(5d^2-2de+3e^2)^2\,(d+ex)} \\
&\quad -\frac{1367d^2-586de-703e^2+(423d^2-2734de+293e^2)\,x}{140\,(5d^2-2de+3e^2)^2\,(3+2x+5x^2)} \\
&\quad +\frac{(1313d^4-10044d^3e+4290d^2e^2+156de^3-271e^4)\arctan\left(\frac{1+5x}{\sqrt{14}}\right)}{28\sqrt{14}\,(5d^2-2de+3e^2)^3} \\
&\quad +\frac{(41d^4-8d^3e-60d^2e^2+24de^3-5e^4)\log(d+ex)}{(5d^2-2de+3e^2)^3} \\
&\quad -\frac{(41d^4-8d^3e-60d^2e^2+24de^3-5e^4)\log(3+2x+5x^2)}{2\,(5d^2-2de+3e^2)^3}
\end{aligned}
$$

```
[Out] (-4*d^4-5*d^3*e-3*d^2*e^2+d*e^3-2*e^4)/e/(5*d^2-2*d*e+3*e^2)^2/(e*x+d)+1/14
0*(-1367*d^2+586*d*e+703*e^2-(423*d^2-2734*d*e+293*e^2)*x)/(5*d^2-2*d*e+3*e
^2)^2/(5*x^2+2*x+3)+(41*d^4-8*d^3*e-60*d^2*e^2+24*d*e^3-5*e^4)*ln(e*x+d)/(5
*d^2-2*d*e+3*e^2)^3-1/2*(41*d^4-8*d^3*e-60*d^2*e^2+24*d*e^3-5*e^4)*ln(5*x^2
+2*x+3)/(5*d^2-2*d*e+3*e^2)^3+1/392*(1313*d^4-10044*d^3*e+4290*d^2*e^2+156*
d*e^3-271*e^4)*arctan(1/14*(1+5*x)*14^(1/2))/(5*d^2-2*d*e+3*e^2)^3*14^(1/2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1660, 1642, 648, 632, 210, 642}

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2(3+2x+5x^2)^2}\,dx$$
$$= \frac{\arctan\left(\frac{5x+1}{\sqrt{14}}\right)(1313d^4-10044d^3e+4290d^2e^2+156de^3-271e^4)}{28\sqrt{14}\,(5d^2-2de+3e^2)^3}$$
$$- \frac{x(423d^2-2734de+293e^2)+1367d^2-586de-703e^2}{140\,(5x^2+2x+3)\,(5d^2-2de+3e^2)^2}$$
$$- \frac{(41d^4-8d^3e-60d^2e^2+24de^3-5e^4)\log(5x^2+2x+3)}{2\,(5d^2-2de+3e^2)^3}$$
$$- \frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{e\,(5d^2-2de+3e^2)^2\,(d+ex)} + \frac{(41d^4-8d^3e-60d^2e^2+24de^3-5e^4)\log(d+ex)}{(5d^2-2de+3e^2)^3}$$

```
[In] Int[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)^2*(3 + 2*x + 5*x^2)^2),x]
```

```
[Out] -((4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)/(e*(5*d^2 - 2*d*e + 3*e^2)^
2*(d + e*x))) - (1367*d^2 - 586*d*e - 703*e^2 + (423*d^2 - 2734*d*e + 293*e
^2)*x)/(140*(5*d^2 - 2*d*e + 3*e^2)^2*(3 + 2*x + 5*x^2)) + ((1313*d^4 - 100
44*d^3*e + 4290*d^2*e^2 + 156*d*e^3 - 271*e^4)*ArcTan[(1 + 5*x)/Sqrt[14]])/
(28*Sqrt[14]*(5*d^2 - 2*d*e + 3*e^2)^3) + ((41*d^4 - 8*d^3*e - 60*d^2*e^2 +
 24*d*e^3 - 5*e^4)*Log[d + e*x])/(5*d^2 - 2*d*e + 3*e^2)^3 - ((41*d^4 - 8*d
^3*e - 60*d^2*e^2 + 24*d*e^3 - 5*e^4)*Log[3 + 2*x + 5*x^2])/(2*(5*d^2 - 2*d
*e + 3*e^2)^3)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1660

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^(p + 1)/((p
+ 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{1367d^2-586de-703e^2+(423d^2-2734de+293e^2)\,x}{140\,(5d^2-2de+3e^2)^2\,(3+2x+5x^2)}\\
&\quad+\frac{1}{56}\int\frac{\frac{2(369d^4-842d^3e+787d^2e^2-224de^3+168e^4)}{(5d^2-2de+3e^2)^2}-\frac{4(462d^4-285d^3e+338d^2e^2-171de^3+14e^4)x}{(5d^2-2de+3e^2)^2}+\frac{2(560d^4-448d^3e+677d^2e^2+278de}{(5d^2-2de+3e^2)^2}}{(d+ex)^2\,(3+2x+5x^2)}\\
&= -\frac{1367d^2-586de-703e^2+(423d^2-2734de+293e^2)\,x}{140\,(5d^2-2de+3e^2)^2\,(3+2x+5x^2)}\\
&\quad+\frac{1}{56}\int\Bigg(\frac{56(4d^4+5d^3e+3d^2e^2-de^3+2e^4)}{(5d^2-2de+3e^2)^2\,(d+ex)^2}\\
&\qquad-\frac{56e(-41d^4+8d^3e+60d^2e^2-24de^3+5e^4)}{(5d^2-2de+3e^2)^3\,(d+ex)}\\
&\qquad+\frac{2(165d^4-9820d^3e+5970d^2e^2-516de^3-131e^4-140(41d^4-8d^3e-60d^2e^2+24de^3-5e^4)\,x}{(5d^2-2de+3e^2)^3\,(3+2x+5x^2)}\end{aligned}$$

$$
\begin{aligned}
&= -\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{e\left(5d^2-2de+3e^2\right)^2(d+ex)} \\
&\quad - \frac{1367d^2-586de-703e^2+\left(423d^2-2734de+293e^2\right)x}{140\left(5d^2-2de+3e^2\right)^2\left(3+2x+5x^2\right)} \\
&\quad + \frac{\left(41d^4-8d^3e-60d^2e^2+24de^3-5e^4\right)\log(d+ex)}{\left(5d^2-2de+3e^2\right)^3} \\
&\quad + \frac{\int \frac{165d^4-9820d^3e+5970d^2e^2-516de^3-131e^4-140\left(41d^4-8d^3e-60d^2e^2+24de^3-5e^4\right)x}{3+2x+5x^2}\,dx}{28\left(5d^2-2de+3e^2\right)^3}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{e\left(5d^2-2de+3e^2\right)^2(d+ex)} \\
&\quad - \frac{1367d^2-586de-703e^2+\left(423d^2-2734de+293e^2\right)x}{140\left(5d^2-2de+3e^2\right)^2\left(3+2x+5x^2\right)} \\
&\quad + \frac{\left(41d^4-8d^3e-60d^2e^2+24de^3-5e^4\right)\log(d+ex)}{\left(5d^2-2de+3e^2\right)^3} \\
&\quad + \frac{\left(1313d^4-10044d^3e+4290d^2e^2+156de^3-271e^4\right)\int \frac{1}{3+2x+5x^2}\,dx}{28\left(5d^2-2de+3e^2\right)^3} \\
&\quad - \frac{\left(41d^4-8d^3e-60d^2e^2+24de^3-5e^4\right)\int \frac{2+10x}{3+2x+5x^2}\,dx}{2\left(5d^2-2de+3e^2\right)^3}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{e\left(5d^2-2de+3e^2\right)^2(d+ex)} \\
&\quad - \frac{1367d^2-586de-703e^2+\left(423d^2-2734de+293e^2\right)x}{140\left(5d^2-2de+3e^2\right)^2\left(3+2x+5x^2\right)} \\
&\quad + \frac{\left(41d^4-8d^3e-60d^2e^2+24de^3-5e^4\right)\log(d+ex)}{\left(5d^2-2de+3e^2\right)^3} \\
&\quad - \frac{\left(41d^4-8d^3e-60d^2e^2+24de^3-5e^4\right)\log\left(3+2x+5x^2\right)}{2\left(5d^2-2de+3e^2\right)^3} \\
&\quad - \frac{\left(1313d^4-10044d^3e+4290d^2e^2+156de^3-271e^4\right)\operatorname{Subst}\left(\int \frac{1}{-56-x^2}\,dx, x, 2+10x\right)}{14\left(5d^2-2de+3e^2\right)^3}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{e\left(5d^2-2de+3e^2\right)^2(d+ex)} \\
&\quad - \frac{1367d^2-586de-703e^2+\left(423d^2-2734de+293e^2\right)x}{140\left(5d^2-2de+3e^2\right)^2\left(3+2x+5x^2\right)} \\
&\quad + \frac{\left(1313d^4-10044d^3e+4290d^2e^2+156de^3-271e^4\right)\tan^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{28\sqrt{14}\left(5d^2-2de+3e^2\right)^3} \\
&\quad + \frac{\left(41d^4-8d^3e-60d^2e^2+24de^3-5e^4\right)\log(d+ex)}{\left(5d^2-2de+3e^2\right)^3} \\
&\quad - \frac{\left(41d^4-8d^3e-60d^2e^2+24de^3-5e^4\right)\log\left(3+2x+5x^2\right)}{2\left(5d^2-2de+3e^2\right)^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.86

$$
\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2\left(3+2x+5x^2\right)^2}\,dx
$$

$$
= \frac{-\frac{1960\left(5d^2-2de+3e^2\right)\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)}{e(d+ex)} - \frac{14\left(5d^2-2de+3e^2\right)\left(e^2(-703+293x)+d^2(1367+423x)-2de(293+1367x)\right)}{3+2x+5x^2} + 5\sqrt{14}(}{}
$$

```
[In] Integrate[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)^2*(3 + 2*x + 5*x^2)^2)
,x]
```

```
[Out] ((-1960*(5*d^2 - 2*d*e + 3*e^2)*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^
4))/(e*(d + e*x)) - (14*(5*d^2 - 2*d*e + 3*e^2)*(e^2*(-703 + 293*x) + d^2*(
1367 + 423*x) - 2*d*e*(293 + 1367*x)))/(3 + 2*x + 5*x^2) + 5*Sqrt[14]*(1313
*d^4 - 10044*d^3*e + 4290*d^2*e^2 + 156*d*e^3 - 271*e^4)*ArcTan[(1 + 5*x)/S
qrt[14]] + 1960*(41*d^4 - 8*d^3*e - 60*d^2*e^2 + 24*d*e^3 - 5*e^4)*Log[d +
e*x] + 980*(-41*d^4 + 8*d^3*e + 60*d^2*e^2 - 24*d*e^3 + 5*e^4)*Log[3 + 2*x
+ 5*x^2])/(1960*(5*d^2 - 2*d*e + 3*e^2)^3)
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.97

method	result
default	$-\frac{\left(\frac{423}{140}d^4-\frac{3629}{175}d^3e+\frac{4101}{350}d^2e^2-\frac{2197}{175}de^3+\frac{879}{700}e^4\right)x+\frac{1367d^4}{140}-\frac{1416d^3e}{175}+\frac{879d^2e^2}{350}-\frac{88de^3}{175}-\frac{2109e^4}{700}}{x^2+\frac{2}{5}x+\frac{3}{5}} + \frac{(5740d^4-1120d^3e-8400d^2e^2+3360de^3}{280}$ $\left(5d^2-2de+3e^2\right)^3$
risch	Expression too large to display

```
[In] int((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2/(5*x^2+2*x+3)^2,x,method=_RETURNVERBO
SE)
```

```
[Out] -1/(5*d^2-2*d*e+3*e^2)^3*(((423/140*d^4-3629/175*d^3*e+4101/350*d^2*e^2-219
7/175*d*e^3+879/700*e^4)*x+1367/140*d^4-1416/175*d^3*e+879/350*d^2*e^2-88/1
75*d*e^3-2109/700*e^4)/(x^2+2/5*x+3/5)+1/280*(5740*d^4-1120*d^3*e-8400*d^2*
e^2+3360*d*e^3-700*e^4)*ln(5*x^2+2*x+3)+1/392*(-1313*d^4+10044*d^3*e-4290*d
^2*e^2-156*d*e^3+271*e^4)*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2)))-(4*d^4+5
*d^3*e+3*d^2*e^2-d*e^3+2*e^4)/(5*d^2-2*d*e+3*e^2)^2/e/(e*x+d)+(41*d^4-8*d^3
*e-60*d^2*e^2+24*d*e^3-5*e^4)*ln(e*x+d)/(5*d^2-2*d*e+3*e^2)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 910 vs. $2(304) = 608$.

Time = 0.35 (sec) , antiderivative size = 910, normalized size of antiderivative = 2.91

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2\left(3+2x+5x^2\right)^2}\,dx = -\frac{117600\,d^6+195650\,d^5e+20664\,d^4e^2+48132\,d^3e^3+118552\,d^2e^4-70686\,de^5+35280\,e^6+14\,(14000\,d^6}{}$$

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2/(5*x^2+2*x+3)^2,x, algorithm="f
ricas")
```

```
[Out] -1/1960*(117600*d^6 + 195650*d^5*e + 20664*d^4*e^2 + 48132*d^3*e^3 + 118552
*d^2*e^4 - 70686*d*e^5 + 35280*e^6 + 14*(14000*d^6 + 11900*d^5*e + 14015*d^
4*e^2 - 11716*d^3*e^3 + 22902*d^2*e^4 - 13688*d*e^5 + 5079*e^6)*x^2 - 5*sqr
t(14)*(3939*d^5*e - 30132*d^4*e^2 + 12870*d^3*e^3 + 468*d^2*e^4 - 813*d*e^5
 + 5*(1313*d^4*e^2 - 10044*d^3*e^3 + 4290*d^2*e^4 + 156*d*e^5 - 271*e^6)*x^
3 + (6565*d^5*e - 47594*d^4*e^2 + 1362*d^3*e^3 + 9360*d^2*e^4 - 1043*d*e^5
- 542*e^6)*x^2 + (2626*d^5*e - 16149*d^4*e^2 - 21552*d^3*e^3 + 13182*d^2*e^
4 - 74*d*e^5 - 813*e^6)*x)*arctan(1/14*sqrt(14)*(5*x + 1)) + 14*(5600*d^6 +
 6875*d^5*e - 2921*d^4*e^2 + 3658*d^3*e^3 - 1150*d^2*e^4 - 1433*d*e^5 - 429
*e^6)*x - 1960*(123*d^5*e - 24*d^4*e^2 - 180*d^3*e^3 + 72*d^2*e^4 - 15*d*e^
5 + 5*(41*d^4*e^2 - 8*d^3*e^3 - 60*d^2*e^4 + 24*d*e^5 - 5*e^6)*x^3 + (205*d
^5*e + 42*d^4*e^2 - 316*d^3*e^3 + 23*d*e^5 - 10*e^6)*x^2 + (82*d^5*e + 107*
d^4*e^2 - 144*d^3*e^3 - 132*d^2*e^4 + 62*d*e^5 - 15*e^6)*x)*log(e*x + d) +
980*(123*d^5*e - 24*d^4*e^2 - 180*d^3*e^3 + 72*d^2*e^4 - 15*d*e^5 + 5*(41*d
^4*e^2 - 8*d^3*e^3 - 60*d^2*e^4 + 24*d*e^5 - 5*e^6)*x^3 + (205*d^5*e + 42*d
^4*e^2 - 316*d^3*e^3 + 23*d*e^5 - 10*e^6)*x^2 + (82*d^5*e + 107*d^4*e^2 - 1
44*d^3*e^3 - 132*d^2*e^4 + 62*d*e^5 - 15*e^6)*x)*log(5*x^2 + 2*x + 3))/(375
*d^7*e - 450*d^6*e^2 + 855*d^5*e^3 - 564*d^4*e^4 + 513*d^3*e^5 - 162*d^2*e^
6 + 81*d*e^7 + 5*(125*d^6*e^2 - 150*d^5*e^3 + 285*d^4*e^4 - 188*d^3*e^5 + 1
71*d^2*e^6 - 54*d*e^7 + 27*e^8)*x^3 + (625*d^7*e - 500*d^6*e^2 + 1125*d^5*e
^3 - 370*d^4*e^4 + 479*d^3*e^5 + 72*d^2*e^6 + 27*d*e^7 + 54*e^8)*x^2 + (250
```

```
*d^7*e + 75*d^6*e^2 + 120*d^5*e^3 + 479*d^4*e^4 - 222*d^3*e^5 + 405*d^2*e^6
 - 108*d*e^7 + 81*e^8)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2\left(3+2x+5x^2\right)^2}\,dx = \text{Timed out}$$

```
[In] integrate((4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)**2/(5*x**2+2*x+3)**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 548, normalized size of antiderivative = 1.75

$$\begin{aligned}&\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2\left(3+2x+5x^2\right)^2}\,dx\\&= \frac{\sqrt{14}(1313\,d^4-10044\,d^3e+4290\,d^2e^2+156\,de^3-271\,e^4)\arctan\left(\frac{1}{14}\sqrt{14}(5\,x+1)\right)}{392\,(125\,d^6-150\,d^5e+285\,d^4e^2-188\,d^3e^3+171\,d^2e^4-54\,de^5+27\,e^6)}\\&\quad+\frac{(41\,d^4-8\,d^3e-60\,d^2e^2+24\,de^3-5\,e^4)\log\left(ex+d\right)}{125\,d^6-150\,d^5e+285\,d^4e^2-188\,d^3e^3+171\,d^2e^4-54\,de^5+27\,e^6}\\&\quad-\frac{(41\,d^4-8\,d^3e-60\,d^2e^2+24\,de^3-5\,e^4)\log\left(5\,x^2+2\,x+3\right)}{2\,(125\,d^6-150\,d^5e+285\,d^4e^2-188\,d^3e^3+171\,d^2e^4-54\,de^5+27\,e^6)}\\&\quad-\frac{1680\,d^4+3467\,d^3e+674\,d^2e^2-1123\,de^3+840\,e^4+(2800\,d^4+3500\,d^3e+}{140\,(75\,d^5e-60\,d^4e^2+102\,d^3e^3-36\,d^2e^4+27\,de^5+5\,(25\,d^4e^2-20\,d^3e^3+34\,d^2e^4-12\,de^5+9\,e^6)x^3}\end{aligned}$$

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2/(5*x^2+2*x+3)^2,x, algorithm="m
axima")
```

```
[Out] 1/392*sqrt(14)*(1313*d^4 - 10044*d^3*e + 4290*d^2*e^2 + 156*d*e^3 - 271*e^4
)*arctan(1/14*sqrt(14)*(5*x + 1))/(125*d^6 - 150*d^5*e + 285*d^4*e^2 - 188*
d^3*e^3 + 171*d^2*e^4 - 54*d*e^5 + 27*e^6) + (41*d^4 - 8*d^3*e - 60*d^2*e^2
 + 24*d*e^3 - 5*e^4)*log(e*x + d)/(125*d^6 - 150*d^5*e + 285*d^4*e^2 - 188*
d^3*e^3 + 171*d^2*e^4 - 54*d*e^5 + 27*e^6) - 1/2*(41*d^4 - 8*d^3*e - 60*d^2
*e^2 + 24*d*e^3 - 5*e^4)*log(5*x^2 + 2*x + 3)/(125*d^6 - 150*d^5*e + 285*d^
4*e^2 - 188*d^3*e^3 + 171*d^2*e^4 - 54*d*e^5 + 27*e^6) - 1/140*(1680*d^4 +
3467*d^3*e + 674*d^2*e^2 - 1123*d*e^3 + 840*e^4 + (2800*d^4 + 3500*d^3*e +
2523*d^2*e^2 - 3434*d*e^3 + 1693*e^4)*x^2 + (1120*d^4 + 1823*d^3*e - 527*d^
2*e^2 - 573*d*e^3 - 143*e^4)*x)/(75*d^5*e - 60*d^4*e^2 + 102*d^3*e^3 - 36*d
^2*e^4 + 27*d*e^5 + 5*(25*d^4*e^2 - 20*d^3*e^3 + 34*d^2*e^4 - 12*d*e^5 + 9*
```

```
e^6)*x^3 + (125*d^5*e - 50*d^4*e^2 + 130*d^3*e^3 + 8*d^2*e^4 + 21*d*e^5 + 1
8*e^6)*x^2 + (50*d^5*e + 35*d^4*e^2 + 8*d^3*e^3 + 78*d^2*e^4 - 18*d*e^5 + 2
7*e^6)*x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 584, normalized size of antiderivative = 1.87

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2\,(3+2x+5x^2)^2}\,dx = -\frac{(41\,d^4-8\,d^3e-60\,d^2e^2+24\,de^3-5\,e^4)\log\left(-\frac{10\,d}{ex+d}+\frac{5\,d^2}{(ex+d)^2}+\frac{2\,e}{ex+d}-\frac{2\,de}{(ex+d)^2}+\frac{3\,e^2}{(ex+d)^2}+5\right)}{2\,(125\,d^6-150\,d^5e+285\,d^4e^2-188\,d^3e^3+171\,d^2e^4-54\,de^5+27\,e^6)} - \frac{\frac{4\,d^4e^3}{ex+d}+\frac{5\,d^3e^4}{ex+d}+\frac{3\,d^2e^5}{ex+d}-\frac{de^6}{ex+d}+\frac{2\,e^7}{ex+d}}{25\,d^4e^4-20\,d^3e^5+34\,d^2e^6-12\,de^7+9\,e^8} + \frac{\sqrt{14}(1313\,d^4e^2-10044\,d^3e^3+4290\,d^2e^4+156\,de^5-271\,e^6)\arctan\left(\frac{\sqrt{14}\left(5\,d-\frac{5\,d^2}{ex+d}+\frac{2\,de}{ex+d}-e-\frac{3\,e^2}{ex+d}\right)}{14\,e}\right)}{392\,(125\,d^6-150\,d^5e+285\,d^4e^2-188\,d^3e^3+171\,d^2e^4-54\,de^5+27\,e^6)e^2} + \frac{\frac{423\,d^3e-4101\,d^2e^2+879\,de^3+703\,e^4}{5\,d^2-2\,de+3\,e^2}-\frac{423\,d^4e^2-5468\,d^3e^3+1758\,d^2e^4+2812\,de^5-457\,e^6}{(5\,d^2-2\,de+3\,e^2)(ex+d)e}}{28\,(5\,d^2-2\,de+3\,e^2)^2\left(\frac{10\,d}{ex+d}-\frac{5\,d^2}{(ex+d)^2}-\frac{2\,e}{ex+d}+\frac{2\,de}{(ex+d)^2}-\frac{3\,e^2}{(ex+d)^2}-5\right)}$$

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2/(5*x^2+2*x+3)^2,x, algorithm="g
iac")
```

```
[Out] -1/2*(41*d^4 - 8*d^3*e - 60*d^2*e^2 + 24*d*e^3 - 5*e^4)*log(-10*d/(e*x + d)
 + 5*d^2/(e*x + d)^2 + 2*e/(e*x + d) - 2*d*e/(e*x + d)^2 + 3*e^2/(e*x + d)^
2 + 5)/(125*d^6 - 150*d^5*e + 285*d^4*e^2 - 188*d^3*e^3 + 171*d^2*e^4 - 54*
d*e^5 + 27*e^6) - (4*d^4*e^3/(e*x + d) + 5*d^3*e^4/(e*x + d) + 3*d^2*e^5/(e
*x + d) - d*e^6/(e*x + d) + 2*e^7/(e*x + d))/(25*d^4*e^4 - 20*d^3*e^5 + 34*
d^2*e^6 - 12*d*e^7 + 9*e^8) + 1/392*sqrt(14)*(1313*d^4*e^2 - 10044*d^3*e^3
+ 4290*d^2*e^4 + 156*d*e^5 - 271*e^6)*arctan(1/14*sqrt(14)*(5*d - 5*d^2/(e*
x + d) + 2*d*e/(e*x + d) - e - 3*e^2/(e*x + d))/e)/((125*d^6 - 150*d^5*e +
285*d^4*e^2 - 188*d^3*e^3 + 171*d^2*e^4 - 54*d*e^5 + 27*e^6)*e^2) + 1/28*((
423*d^3*e - 4101*d^2*e^2 + 879*d*e^3 + 703*e^4)/(5*d^2 - 2*d*e + 3*e^2) - (
423*d^4*e^2 - 5468*d^3*e^3 + 1758*d^2*e^4 + 2812*d*e^5 - 457*e^6)/((5*d^2 -
 2*d*e + 3*e^2)*(e*x + d)*e))/((5*d^2 - 2*d*e + 3*e^2)^2*(10*d/(e*x + d) -
5*d^2/(e*x + d)^2 - 2*e/(e*x + d) + 2*d*e/(e*x + d)^2 - 3*e^2/(e*x + d)^2 -
 5))
```

Mupad [B] (verification not implemented)

Time = 14.28 (sec) , antiderivative size = 601, normalized size of antiderivative = 1.92

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2\left(3+2x+5x^2\right)^2}\,dx = \ln(d+e\,x)\left(\frac{41}{25\,\left(5\,d^2-2\,d\,e+3\,e^2\right)} - \frac{4\,e^3\,(423\,d-1367\,e)}{125\,\left(5\,d^2-2\,d\,e+3\,e^2\right)^3} + \frac{2\,e\,(310\,d-1323\,e)}{125\,\left(5\,d^2-2\,d\,e+3\,e^2\right)^2}\right)$$

$$- \frac{\frac{1680\,d^4+3467\,d^3\,e+674\,d^2\,e^2-1123\,d\,e^3+840\,e^4}{140\,e\,\left(25\,d^4-20\,d^3\,e+34\,d^2\,e^2-12\,d\,e^3+9\,e^4\right)} - \frac{x\,\left(-1120\,d^4-1823\,d^3\,e+527\,d^2\,e^2+573\,d\,e^3+143\,e^4\right)}{140\,e\,\left(25\,d^4-20\,d^3\,e+34\,d^2\,e^2-12\,d\,e^3+9\,e^4\right)} + \frac{x^2\,\left(2800\,d^4+3500\,d^3\,e+2523\,d^2}{140\,e\,\left(25\,d^4-20\,d^3\,e+34\,d^2}}{5\,e\,x^3+(5\,d+2\,e)\,x^2+(2\,d+3\,e)\,x+3\,d}$$

$$+ \frac{\ln\left(x+\frac{1}{5}-\frac{\sqrt{14}\,1\mathrm{i}}{5}\right)\,\left(\left(\frac{1313\,\sqrt{14}}{784}-\frac{41}{2}\mathrm{i}\right)\,d^4+\left(-\frac{2511\,\sqrt{14}}{196}+4\mathrm{i}\right)\,d^3\,e+\left(\frac{2145\,\sqrt{14}}{392}+30\mathrm{i}\right)\,d^2\,e^2+\left(\frac{39\,\sqrt{14}}{196}-1\right.\right.}{d^6\,125\mathrm{i}-d^5\,e\,150\mathrm{i}+d^4\,e^2\,285\mathrm{i}-d^3\,e^3\,188\mathrm{i}+d^2\,e^4\,171\mathrm{i}-d\,e^5\,54\mathrm{i}+e^6\,27}$$

$$- \frac{\ln\left(x+\frac{1}{5}+\frac{\sqrt{14}\,1\mathrm{i}}{5}\right)\,\left(\left(\frac{1313\,\sqrt{14}}{784}+\frac{41}{2}\mathrm{i}\right)\,d^4+\left(-\frac{2511\,\sqrt{14}}{196}-4\mathrm{i}\right)\,d^3\,e+\left(\frac{2145\,\sqrt{14}}{392}-30\mathrm{i}\right)\,d^2\,e^2+\left(\frac{39\,\sqrt{14}}{196}+1\right.\right.}{d^6\,125\mathrm{i}-d^5\,e\,150\mathrm{i}+d^4\,e^2\,285\mathrm{i}-d^3\,e^3\,188\mathrm{i}+d^2\,e^4\,171\mathrm{i}-d\,e^5\,54\mathrm{i}+e^6\,27}$$

```
[In] int((x + 3*x^2 - 5*x^3 + 4*x^4 + 2)/((d + e*x)^2*(2*x + 5*x^2 + 3)^2),x)
```

```
[Out] log(d + e*x)*(41/(25*(5*d^2 - 2*d*e + 3*e^2)) - (4*e^3*(423*d - 1367*e))/(1
25*(5*d^2 - 2*d*e + 3*e^2)^3) + (2*e*(310*d - 1323*e))/(125*(5*d^2 - 2*d*e
+ 3*e^2)^2)) - ((3467*d^3*e - 1123*d*e^3 + 1680*d^4 + 840*e^4 + 674*d^2*e^2
)/(140*e*(25*d^4 - 20*d^3*e - 12*d*e^3 + 9*e^4 + 34*d^2*e^2)) - (x*(573*d*e
^3 - 1823*d^3*e - 1120*d^4 + 143*e^4 + 527*d^2*e^2))/(140*e*(25*d^4 - 20*d^
3*e - 12*d*e^3 + 9*e^4 + 34*d^2*e^2)) + (x^2*(3500*d^3*e - 3434*d*e^3 + 280
0*d^4 + 1693*e^4 + 2523*d^2*e^2))/(140*e*(25*d^4 - 20*d^3*e - 12*d*e^3 + 9*
e^4 + 34*d^2*e^2)))/(3*d + x^2*(5*d + 2*e) + 5*e*x^3 + x*(2*d + 3*e)) + (lo
g(x - (14^(1/2)*1i)/5 + 1/5)*(d^4*((1313*14^(1/2))/784 - 41i/2) - e^4*((271
*14^(1/2))/784 - 5i/2) + d^2*e^2*((2145*14^(1/2))/392 + 30i) + d*e^3*((39*1
4^(1/2))/196 - 12i) - d^3*e*((2511*14^(1/2))/196 - 4i)))/(d^6*125i - d^5*e*
150i - d*e^5*54i + e^6*27i + d^2*e^4*171i - d^3*e^3*188i + d^4*e^2*285i) -
(log(x + (14^(1/2)*1i)/5 + 1/5)*(d^4*((1313*14^(1/2))/784 + 41i/2) - e^4*((
271*14^(1/2))/784 + 5i/2) + d^2*e^2*((2145*14^(1/2))/392 - 30i) + d*e^3*((3
9*14^(1/2))/196 + 12i) - d^3*e*((2511*14^(1/2))/196 + 4i)))/(d^6*125i - d^5
*e*150i - d*e^5*54i + e^6*27i + d^2*e^4*171i - d^3*e^3*188i + d^4*e^2*285i)
```

3.317 $\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^3\left(3+2x+5x^2\right)^2}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 412

$$\begin{aligned}
&\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^3\left(3+2x+5x^2\right)^2}\,dx \\
&= -\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{2e\left(5d^2-2de+3e^2\right)^2(d+ex)^2}-\frac{41d^4-8d^3e-60d^2e^2+24de^3-5e^4}{\left(5d^2-2de+3e^2\right)^3(d+ex)} \\
&\quad -\frac{1367d^3-879d^2e-2109de^2+457e^3+\left(423d^3-4101d^2e+879de^2+703e^3\right)x}{28\left(5d^2-2de+3e^2\right)^3\left(3+2x+5x^2\right)} \\
&\quad +\frac{\left(6565d^5-74017d^4e+35022d^3e^2+42858d^2e^3-17247de^4+579e^5\right)\arctan\left(\frac{1+5x}{\sqrt{14}}\right)}{28\sqrt{14}\left(5d^2-2de+3e^2\right)^4} \\
&\quad +\frac{\left(205d^5-19d^4e-846d^3e^2+396d^2e^3+57de^4-21e^5\right)\log(d+ex)}{\left(5d^2-2de+3e^2\right)^4} \\
&\quad -\frac{\left(205d^5-19d^4e-846d^3e^2+396d^2e^3+57de^4-21e^5\right)\log\left(3+2x+5x^2\right)}{2\left(5d^2-2de+3e^2\right)^4}
\end{aligned}$$

```
[Out] 1/2*(-4*d^4-5*d^3*e-3*d^2*e^2+d*e^3-2*e^4)/e/(5*d^2-2*d*e+3*e^2)^2/(e*x+d)^
2+(-41*d^4+8*d^3*e+60*d^2*e^2-24*d*e^3+5*e^4)/(5*d^2-2*d*e+3*e^2)^3/(e*x+d)
+1/28*(-1367*d^3+879*d^2*e+2109*d*e^2-457*e^3-(423*d^3-4101*d^2*e+879*d*e^2
+703*e^3)*x)/(5*d^2-2*d*e+3*e^2)^3/(5*x^2+2*x+3)+(205*d^5-19*d^4*e-846*d^3*
e^2+396*d^2*e^3+57*d*e^4-21*e^5)*ln(e*x+d)/(5*d^2-2*d*e+3*e^2)^4-1/2*(205*d
^5-19*d^4*e-846*d^3*e^2+396*d^2*e^3+57*d*e^4-21*e^5)*ln(5*x^2+2*x+3)/(5*d^2
-2*d*e+3*e^2)^4+1/392*(6565*d^5-74017*d^4*e+35022*d^3*e^2+42858*d^2*e^3-172
47*d*e^4+579*e^5)*arctan(1/14*(1+5*x)*14^(1/2))/(5*d^2-2*d*e+3*e^2)^4*14^(1
/2)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1660, 1642, 648, 632, 210, 642}

$$\begin{aligned}
&\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^3(3+2x+5x^2)^2}\,dx \\
&= \frac{\arctan\left(\frac{5x+1}{\sqrt{14}}\right)(6565d^5-74017d^4e+35022d^3e^2+42858d^2e^3-17247de^4+579e^5)}{28\sqrt{14}\left(5d^2-2de+3e^2\right)^4} \\
&\quad - \frac{1367d^3-879d^2e+x(423d^3-4101d^2e+879de^2+703e^3)-2109de^2+457e^3}{28\left(5x^2+2x+3\right)\left(5d^2-2de+3e^2\right)^3} \\
&\quad - \frac{41d^4-8d^3e-60d^2e^2+24de^3-5e^4}{\left(5d^2-2de+3e^2\right)^3(d+ex)} - \frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{2e\left(5d^2-2de+3e^2\right)^2(d+ex)^2} \\
&\quad - \frac{(205d^5-19d^4e-846d^3e^2+396d^2e^3+57de^4-21e^5)\log\left(5x^2+2x+3\right)}{2\left(5d^2-2de+3e^2\right)^4} \\
&\quad + \frac{(205d^5-19d^4e-846d^3e^2+396d^2e^3+57de^4-21e^5)\log(d+ex)}{\left(5d^2-2de+3e^2\right)^4}
\end{aligned}$$

```
[In] Int[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)^3*(3 + 2*x + 5*x^2)^2),x]
```

```
[Out] -1/2*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)/(e*(5*d^2 - 2*d*e + 3*e^
2)^2*(d + e*x)^2) - (41*d^4 - 8*d^3*e - 60*d^2*e^2 + 24*d*e^3 - 5*e^4)/((5*
d^2 - 2*d*e + 3*e^2)^3*(d + e*x)) - (1367*d^3 - 879*d^2*e - 2109*d*e^2 + 45
7*e^3 + (423*d^3 - 4101*d^2*e + 879*d*e^2 + 703*e^3)*x)/(28*(5*d^2 - 2*d*e
+ 3*e^2)^3*(3 + 2*x + 5*x^2)) + ((6565*d^5 - 74017*d^4*e + 35022*d^3*e^2 +
42858*d^2*e^3 - 17247*d*e^4 + 579*e^5)*ArcTan[(1 + 5*x)/Sqrt[14]])/(28*Sqrt
[14]*(5*d^2 - 2*d*e + 3*e^2)^4) + ((205*d^5 - 19*d^4*e - 846*d^3*e^2 + 396*
d^2*e^3 + 57*d*e^4 - 21*e^5)*Log[d + e*x])/(5*d^2 - 2*d*e + 3*e^2)^4 - ((20
5*d^5 - 19*d^4*e - 846*d^3*e^2 + 396*d^2*e^3 + 57*d*e^4 - 21*e^5)*Log[3 + 2
*x + 5*x^2])/(2*(5*d^2 - 2*d*e + 3*e^2)^4)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1660

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^(p + 1)/((p
+ 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\text{integral} = -\frac{1367d^3-879d^2e-2109de^2+457e^3+(423d^3-4101d^2e+879de^2+703e^3)\,x}{28\,(5d^2-2de+3e^2)^3\,(3+2x+5x^2)}$$

$$+\frac{1}{56}\int\frac{\frac{6(615d^6-2105d^5e+2535d^4e^2-1037d^3e^3+1064d^2e^4-336de^5+168e^6)}{(5d^2-2de+3e^2)^3}-\frac{2(4620d^6-4275d^5e+5925d^4e^2-5651d^3e^3-663d^2e^4-168de^5-}{(5d^2-2de+3e^2)^3}}{(d+ex)^3\,(3+}$$

$$= -\frac{1367d^3-879d^2e-2109de^2+457e^3+\left(423d^3-4101d^2e+879de^2+703e^3\right)x}{28\left(5d^2-2de+3e^2\right)^3\left(3+2x+5x^2\right)}$$

$$+\frac{1}{56}\int\Bigg(\frac{56(4d^4+5d^3e+3d^2e^2-de^3+2e^4)}{\left(5d^2-2de+3e^2\right)^2(d+ex)^3}$$

$$-\frac{56e(-41d^4+8d^3e+60d^2e^2-24de^3+5e^4)}{\left(5d^2-2de+3e^2\right)^3(d+ex)^2}$$

$$-\frac{56e(-205d^5+19d^4e+846d^3e^2-396d^2e^3-57de^4+21e^5)}{\left(5d^2-2de+3e^2\right)^4(d+ex)}$$

$$+\frac{2(3(275d^5-24495d^4e+19570d^3e^2+10590d^2e^3-6281de^4+389e^5)-140(205d^5-19d^4e-8}{\left(5d^2-2de+3e^2\right)^4\left(3+2x+5x^2\right)}$$

$$= -\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{2e\left(5d^2-2de+3e^2\right)^2(d+ex)^2}-\frac{41d^4-8d^3e-60d^2e^2+24de^3-5e^4}{\left(5d^2-2de+3e^2\right)^3(d+ex)}$$

$$-\frac{1367d^3-879d^2e-2109de^2+457e^3+\left(423d^3-4101d^2e+879de^2+703e^3\right)x}{28\left(5d^2-2de+3e^2\right)^3\left(3+2x+5x^2\right)}$$

$$+\frac{\left(205d^5-19d^4e-846d^3e^2+396d^2e^3+57de^4-21e^5\right)\log(d+ex)}{\left(5d^2-2de+3e^2\right)^4}$$

$$+\frac{\int\frac{3\left(275d^5-24495d^4e+19570d^3e^2+10590d^2e^3-6281de^4+389e^5\right)-140\left(205d^5-19d^4e-846d^3e^2+396d^2e^3+57de^4-21e^5\right)x}{3+2x+5x^2}\,dx}{28\left(5d^2-2de+3e^2\right)^4}$$

$$= -\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{2e\left(5d^2-2de+3e^2\right)^2(d+ex)^2}-\frac{41d^4-8d^3e-60d^2e^2+24de^3-5e^4}{\left(5d^2-2de+3e^2\right)^3(d+ex)}$$

$$-\frac{1367d^3-879d^2e-2109de^2+457e^3+\left(423d^3-4101d^2e+879de^2+703e^3\right)x}{28\left(5d^2-2de+3e^2\right)^3\left(3+2x+5x^2\right)}$$

$$+\frac{\left(205d^5-19d^4e-846d^3e^2+396d^2e^3+57de^4-21e^5\right)\log(d+ex)}{\left(5d^2-2de+3e^2\right)^4}$$

$$-\frac{\left(205d^5-19d^4e-846d^3e^2+396d^2e^3+57de^4-21e^5\right)\int\frac{2+10x}{3+2x+5x^2}\,dx}{2\left(5d^2-2de+3e^2\right)^4}$$

$$+\frac{\left(6565d^5-74017d^4e+35022d^3e^2+42858d^2e^3-17247de^4+579e^5\right)\int\frac{1}{3+2x+5x^2}\,dx}{28\left(5d^2-2de+3e^2\right)^4}$$

$$= -\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{2e\left(5d^2-2de+3e^2\right)^2(d+ex)^2}-\frac{41d^4-8d^3e-60d^2e^2+24de^3-5e^4}{\left(5d^2-2de+3e^2\right)^3(d+ex)}$$

$$-\frac{1367d^3-879d^2e-2109de^2+457e^3+\left(423d^3-4101d^2e+879de^2+703e^3\right)x}{28\left(5d^2-2de+3e^2\right)^3\left(3+2x+5x^2\right)}$$

$$+\frac{\left(205d^5-19d^4e-846d^3e^2+396d^2e^3+57de^4-21e^5\right)\log(d+ex)}{\left(5d^2-2de+3e^2\right)^4}$$

$$-\frac{\left(205d^5-19d^4e-846d^3e^2+396d^2e^3+57de^4-21e^5\right)\log\left(3+2x+5x^2\right)}{2\left(5d^2-2de+3e^2\right)^4}$$

$$-\frac{\left(6565d^5-74017d^4e+35022d^3e^2+42858d^2e^3-17247de^4+579e^5\right)\text{Subst}\big(\int\frac{1}{-56-x^2}\,dx,x,2+1}{14\left(5d^2-2de+3e^2\right)^4}$$

$$
\begin{aligned}
&= -\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{2e\left(5d^2-2de+3e^2\right)^2(d+ex)^2} - \frac{41d^4-8d^3e-60d^2e^2+24de^3-5e^4}{\left(5d^2-2de+3e^2\right)^3(d+ex)} \\
&\quad - \frac{1367d^3-879d^2e-2109de^2+457e^3+\left(423d^3-4101d^2e+879de^2+703e^3\right)x}{28\left(5d^2-2de+3e^2\right)^3\left(3+2x+5x^2\right)} \\
&\quad + \frac{\left(6565d^5-74017d^4e+35022d^3e^2+42858d^2e^3-17247de^4+579e^5\right)\tan^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{28\sqrt{14}\left(5d^2-2de+3e^2\right)^4} \\
&\quad + \frac{\left(205d^5-19d^4e-846d^3e^2+396d^2e^3+57de^4-21e^5\right)\log(d+ex)}{\left(5d^2-2de+3e^2\right)^4} \\
&\quad - \frac{\left(205d^5-19d^4e-846d^3e^2+396d^2e^3+57de^4-21e^5\right)\log\left(3+2x+5x^2\right)}{2\left(5d^2-2de+3e^2\right)^4}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 363, normalized size of antiderivative = 0.88

$$
\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^3\left(3+2x+5x^2\right)^2}\,dx
$$

$$
= \frac{-\frac{196(5d^2-2de+3e^2)^2(4d^4+5d^3e+3d^2e^2-de^3+2e^4)}{e(d+ex)^2} + \frac{392(5d^2-2de+3e^2)(-41d^4+8d^3e+60d^2e^2-24de^3+5e^4)}{d+ex} - 14(5d^2-2de+3e^2)(3de^2(-}{}
$$

```
[In] Integrate[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)^3*(3 + 2*x + 5*x^2)^2)
,x]
```

```
[Out] ((-196*(5*d^2 - 2*d*e + 3*e^2)^2*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e
^4))/(e*(d + e*x)^2) + (392*(5*d^2 - 2*d*e + 3*e^2)*(-41*d^4 + 8*d^3*e + 60
*d^2*e^2 - 24*d*e^3 + 5*e^4))/(d + e*x) - (14*(5*d^2 - 2*d*e + 3*e^2)*(3*d*
e^2*(-703 + 293*x) + d^3*(1367 + 423*x) + e^3*(457 + 703*x) - 3*d^2*e*(293
+ 1367*x)))/(3 + 2*x + 5*x^2) + Sqrt[14]*(6565*d^5 - 74017*d^4*e + 35022*d^
3*e^2 + 42858*d^2*e^3 - 17247*d*e^4 + 579*e^5)*ArcTan[(1 + 5*x)/Sqrt[14]] +
 392*(205*d^5 - 19*d^4*e - 846*d^3*e^2 + 396*d^2*e^3 + 57*d*e^4 - 21*e^5)*L
og[d + e*x] + 196*(-205*d^5 + 19*d^4*e + 846*d^3*e^2 - 396*d^2*e^3 - 57*d*e
^4 + 21*e^5)*Log[3 + 2*x + 5*x^2])/(392*(5*d^2 - 2*d*e + 3*e^2)^4)
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 400, normalized size of antiderivative = 0.97

method	result
default	$-\frac{\left(\frac{423}{28}d^5-\frac{21351}{140}d^4e+\frac{6933}{70}d^3e^2-\frac{5273}{70}d^2e^3+\frac{1231}{140}d\,e^4+\frac{2109}{140}e^5\right)x+\frac{1367d^5}{28}-\frac{7129d^4e}{140}-\frac{2343d^3e^2}{70}+\frac{1933d^2e^3}{70}-\frac{7241d\,e^4}{140}+\frac{1371e^5}{140}}{x^2+\frac{2}{5}x+\frac{3}{5}}+\left(28700d^5-\right.$
risch	Expression too large to display

```
[In] int((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3/(5*x^2+2*x+3)^2,x,method=_RETURNVERBO
SE)
```

```
[Out] -1/(5*d^2-2*d*e+3*e^2)^4*(((423/28*d^5-21351/140*d^4*e+6933/70*d^3*e^2-5273
/70*d^2*e^3+1231/140*d*e^4+2109/140*e^5)*x+1367/28*d^5-7129/140*d^4*e-2343/
70*d^3*e^2+1933/70*d^2*e^3-7241/140*d*e^4+1371/140*e^5)/(x^2+2/5*x+3/5)+1/2
80*(28700*d^5-2660*d^4*e-118440*d^3*e^2+55440*d^2*e^3+7980*d*e^4-2940*e^5)*
ln(5*x^2+2*x+3)+1/392*(-6565*d^5+74017*d^4*e-35022*d^3*e^2-42858*d^2*e^3+17
247*d*e^4-579*e^5)*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2)))-1/2*(4*d^4+5*d^
3*e+3*d^2*e^2-d*e^3+2*e^4)/(5*d^2-2*d*e+3*e^2)^2/e/(e*x+d)^2-(41*d^4-8*d^3*
e-60*d^2*e^2+24*d*e^3-5*e^4)/(5*d^2-2*d*e+3*e^2)^3/(e*x+d)+(205*d^5-19*d^4*
e-846*d^3*e^2+396*d^2*e^3+57*d*e^4-21*e^5)*ln(e*x+d)/(5*d^2-2*d*e+3*e^2)^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1499 vs. $2(401) = 802$.

Time = 0.45 (sec) , antiderivative size = 1499, normalized size of antiderivative = 3.64

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^3\,(3+2x+5x^2)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3/(5*x^2+2*x+3)^2,x, algorithm="f
ricas")
```

```
[Out] -1/392*(58800*d^8 + 363230*d^7*e - 178010*d^6*e^2 - 233184*d^5*e^3 + 395164
*d^4*e^4 - 437122*d^3*e^5 + 178542*d^2*e^6 - 37044*d*e^7 + 10584*e^8 + 14*(
28700*d^6*e^2 - 14965*d^5*e^3 - 43891*d^4*e^4 + 44106*d^3*e^5 - 45966*d^2*e
^6 + 12711*d*e^7 + 9*e^8)*x^3 + 14*(7000*d^8 + 31850*d^7*e + 6400*d^6*e^2 -
 62649*d^5*e^3 + 52187*d^4*e^4 - 53652*d^3*e^5 + 11130*d^2*e^6 - 2841*d*e^7
 + 1791*e^8)*x^2 - sqrt(14)*(19695*d^7*e - 222051*d^6*e^2 + 105066*d^5*e^3
+ 128574*d^4*e^4 - 51741*d^3*e^5 + 1737*d^2*e^6 + 5*(6565*d^5*e^3 - 74017*d
^4*e^4 + 35022*d^3*e^5 + 42858*d^2*e^6 - 17247*d*e^7 + 579*e^8)*x^4 + 2*(32
825*d^6*e^2 - 363520*d^5*e^3 + 101093*d^4*e^4 + 249312*d^3*e^5 - 43377*d^2*
e^6 - 14352*d*e^7 + 579*e^8)*x^3 + (32825*d^7*e - 343825*d^6*e^2 - 101263*d
^5*e^3 + 132327*d^4*e^4 + 190263*d^3*e^5 + 62481*d^2*e^6 - 49425*d*e^7 + 17
37*e^8)*x^2 + 2*(6565*d^7*e - 54322*d^6*e^2 - 187029*d^5*e^3 + 147924*d^4*e
^4 + 111327*d^3*e^5 - 51162*d^2*e^6 + 1737*d*e^7)*x)*arctan(1/14*sqrt(14)*(
5*x + 1)) + 14*(2800*d^8 + 14855*d^7*e + 5815*d^6*e^2 - 18620*d^5*e^3 - 172
```

```
02*d^4*e^4 + 11119*d^3*e^5 - 26037*d^2*e^6 + 7866*d*e^7 - 756*e^8)*x - 392*
(615*d^7*e - 57*d^6*e^2 - 2538*d^5*e^3 + 1188*d^4*e^4 + 171*d^3*e^5 - 63*d^
2*e^6 + 5*(205*d^5*e^3 - 19*d^4*e^4 - 846*d^3*e^5 + 396*d^2*e^6 + 57*d*e^7
- 21*e^8)*x^4 + 2*(1025*d^6*e^2 + 110*d^5*e^3 - 4249*d^4*e^4 + 1134*d^3*e^5
 + 681*d^2*e^6 - 48*d*e^7 - 21*e^8)*x^3 + (1025*d^7*e + 725*d^6*e^2 - 3691*
d^5*e^3 - 1461*d^4*e^4 - 669*d^3*e^5 + 1311*d^2*e^6 + 87*d*e^7 - 63*e^8)*x^
2 + 2*(205*d^7*e + 596*d^6*e^2 - 903*d^5*e^3 - 2142*d^4*e^4 + 1245*d^3*e^5
+ 150*d^2*e^6 - 63*d*e^7)*x)*log(e*x + d) + 196*(615*d^7*e - 57*d^6*e^2 - 2
538*d^5*e^3 + 1188*d^4*e^4 + 171*d^3*e^5 - 63*d^2*e^6 + 5*(205*d^5*e^3 - 19
*d^4*e^4 - 846*d^3*e^5 + 396*d^2*e^6 + 57*d*e^7 - 21*e^8)*x^4 + 2*(1025*d^6
*e^2 + 110*d^5*e^3 - 4249*d^4*e^4 + 1134*d^3*e^5 + 681*d^2*e^6 - 48*d*e^7 -
 21*e^8)*x^3 + (1025*d^7*e + 725*d^6*e^2 - 3691*d^5*e^3 - 1461*d^4*e^4 - 66
9*d^3*e^5 + 1311*d^2*e^6 + 87*d*e^7 - 63*e^8)*x^2 + 2*(205*d^7*e + 596*d^6*
e^2 - 903*d^5*e^3 - 2142*d^4*e^4 + 1245*d^3*e^5 + 150*d^2*e^6 - 63*d*e^7)*x
)*log(5*x^2 + 2*x + 3))/(1875*d^10*e - 3000*d^9*e^2 + 6300*d^8*e^3 - 5880*d
^7*e^4 + 6258*d^6*e^5 - 3528*d^5*e^6 + 2268*d^4*e^7 - 648*d^3*e^8 + 243*d^2
*e^9 + 5*(625*d^8*e^3 - 1000*d^7*e^4 + 2100*d^6*e^5 - 1960*d^5*e^6 + 2086*d
^4*e^7 - 1176*d^3*e^8 + 756*d^2*e^9 - 216*d*e^10 + 81*e^11)*x^4 + 2*(3125*d
^9*e^2 - 4375*d^8*e^3 + 9500*d^7*e^4 - 7700*d^6*e^5 + 8470*d^5*e^6 - 3794*d
^4*e^7 + 2604*d^3*e^8 - 324*d^2*e^9 + 189*d*e^10 + 81*e^11)*x^3 + (3125*d^1
0*e - 2500*d^9*e^2 + 8375*d^8*e^3 - 4400*d^7*e^4 + 8890*d^6*e^5 - 3416*d^5*
e^6 + 5334*d^4*e^7 - 1584*d^3*e^8 + 1809*d^2*e^9 - 324*d*e^10 + 243*e^11)*x
^2 + 2*(625*d^10*e + 875*d^9*e^2 - 900*d^8*e^3 + 4340*d^7*e^4 - 3794*d^6*e^
5 + 5082*d^5*e^6 - 2772*d^4*e^7 + 2052*d^3*e^8 - 567*d^2*e^9 + 243*d*e^10)*
x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^3\left(3+2x+5x^2\right)^2}\,dx=\text{Timed out}$$

```
[In] integrate((4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)**3/(5*x**2+2*x+3)**2,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 851 vs. 2(401) = 802.

Time = 0.30 (sec) , antiderivative size = 851, normalized size of antiderivative = 2.07

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^3\left(3+2x+5x^2\right)^2}\,dx$$
$$=\frac{\sqrt{14}(6565\,d^5-74017\,d^4e+35022\,d^3e^2+42858\,d^2e^3-17247\,de^4+579\,e^5)\arctan\left(\frac{1}{14}\sqrt{14}(5\,x+1)\right)}{392\,(625\,d^8-1000\,d^7e+2100\,d^6e^2-1960\,d^5e^3+2086\,d^4e^4-1176\,d^3e^5+756\,d^2e^6-216\,de^7+81\,e^8)}$$
$$+\frac{(205\,d^5-19\,d^4e-846\,d^3e^2+396\,d^2e^3+57\,de^4-21\,e^5)\log(ex+d)}{625\,d^8-1000\,d^7e+2100\,d^6e^2-1960\,d^5e^3+2086\,d^4e^4-1176\,d^3e^5+756\,d^2e^6-216\,de^7+81\,e^8}$$
$$-\frac{(205\,d^5-19\,d^4e-846\,d^3e^2+396\,d^2e^3+57\,de^4-21\,e^5)\log(5\,x^2+2\,x+3)}{2\,(625\,d^8-1000\,d^7e+2100\,d^6e^2-1960\,d^5e^3+2086\,d^4e^4-1176\,d^3e^5+756\,d^2e^6-216\,de^7+81\,e^8)}$$
$$-\frac{840\,d^6+5525\,d^5e-837\,d^4e^2-69}{28\,(375\,d^8e-450\,d^7e^2+855\,d^6e^3-564\,d^5e^4+513\,d^4e^5-162\,d^3e^6+81\,d^2e^7+5\,(125\,d^6e^3-150\,d^5e^4+}$$

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3/(5*x^2+2*x+3)^2,x, algorithm="m
axima")
```

```
[Out] 1/392*sqrt(14)*(6565*d^5 - 74017*d^4*e + 35022*d^3*e^2 + 42858*d^2*e^3 - 17
247*d*e^4 + 579*e^5)*arctan(1/14*sqrt(14)*(5*x + 1))/(625*d^8 - 1000*d^7*e
+ 2100*d^6*e^2 - 1960*d^5*e^3 + 2086*d^4*e^4 - 1176*d^3*e^5 + 756*d^2*e^6 -
 216*d*e^7 + 81*e^8) + (205*d^5 - 19*d^4*e - 846*d^3*e^2 + 396*d^2*e^3 + 57
*d*e^4 - 21*e^5)*log(e*x + d)/(625*d^8 - 1000*d^7*e + 2100*d^6*e^2 - 1960*d
^5*e^3 + 2086*d^4*e^4 - 1176*d^3*e^5 + 756*d^2*e^6 - 216*d*e^7 + 81*e^8) -
1/2*(205*d^5 - 19*d^4*e - 846*d^3*e^2 + 396*d^2*e^3 + 57*d*e^4 - 21*e^5)*lo
g(5*x^2 + 2*x + 3)/(625*d^8 - 1000*d^7*e + 2100*d^6*e^2 - 1960*d^5*e^3 + 20
86*d^4*e^4 - 1176*d^3*e^5 + 756*d^2*e^6 - 216*d*e^7 + 81*e^8) - 1/28*(840*d
^6 + 5525*d^5*e - 837*d^4*e^2 - 6981*d^3*e^3 + 3355*d^2*e^4 - 714*d*e^5 + 2
52*e^6 + (5740*d^4*e^2 - 697*d^3*e^3 - 12501*d^2*e^4 + 4239*d*e^5 + 3*e^6)*
x^3 + (1400*d^6 + 6930*d^5*e + 3212*d^4*e^2 - 15403*d^3*e^3 + 2349*d^2*e^4
- 549*d*e^5 + 597*e^6)*x^2 + (560*d^6 + 3195*d^5*e + 2105*d^4*e^2 - 4799*d^
3*e^3 - 6623*d^2*e^4 + 2454*d*e^5 - 252*e^6)*x)/(375*d^8*e - 450*d^7*e^2 +
855*d^6*e^3 - 564*d^5*e^4 + 513*d^4*e^5 - 162*d^3*e^6 + 81*d^2*e^7 + 5*(125
*d^6*e^3 - 150*d^5*e^4 + 285*d^4*e^5 - 188*d^3*e^6 + 171*d^2*e^7 - 54*d*e^8
 + 27*e^9)*x^4 + 2*(625*d^7*e^2 - 625*d^6*e^3 + 1275*d^5*e^4 - 655*d^4*e^5
+ 667*d^3*e^6 - 99*d^2*e^7 + 81*d*e^8 + 27*e^9)*x^3 + (625*d^8*e - 250*d^7*
e^2 + 1200*d^6*e^3 - 250*d^5*e^4 + 958*d^4*e^5 - 150*d^3*e^6 + 432*d^2*e^7
- 54*d*e^8 + 81*e^9)*x^2 + 2*(125*d^8*e + 225*d^7*e^2 - 165*d^6*e^3 + 667*d
^5*e^4 - 393*d^4*e^5 + 459*d^3*e^6 - 135*d^2*e^7 + 81*d*e^8)*x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 648, normalized size of antiderivative = 1.57

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^3\left(3+2x+5x^2\right)^2}\,dx$$
$$= \frac{\sqrt{14}(6565\,d^5-74017\,d^4e+35022\,d^3e^2+42858\,d^2e^3-17247\,de^4+579\,e^5)\arctan\left(\frac{1}{14}\sqrt{14}(5\,x+1)\right)}{392\,(625\,d^8-1000\,d^7e+2100\,d^6e^2-1960\,d^5e^3+2086\,d^4e^4-1176\,d^3e^5+756\,d^2e^6-216\,de^7+81\,e^8)}$$
$$-\frac{(205\,d^5-19\,d^4e-846\,d^3e^2+396\,d^2e^3+57\,de^4-21\,e^5)\log\left(5\,x^2+2\,x+3\right)}{2\,(625\,d^8-1000\,d^7e+2100\,d^6e^2-1960\,d^5e^3+2086\,d^4e^4-1176\,d^3e^5+756\,d^2e^6-216\,de^7+81\,e^8)}$$
$$+\frac{(205\,d^5e-19\,d^4e^2-846\,d^3e^3+396\,d^2e^4+57\,de^5-21\,e^6)\log\left(|ex+d|\right)}{625\,d^8e-1000\,d^7e^2+2100\,d^6e^3-1960\,d^5e^4+2086\,d^4e^5-1176\,d^3e^6+756\,d^2e^7-216\,de^8+81\,e^9}$$
$$-\frac{4200\,d^8+25945\,d^7e-12715\,d^6e^2-16656\,d^5e^3+28226\,d^4e^4-31223\,d^3e^5+12753\,d^2e^6-2646\,de^7+75}{}$$

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3/(5*x^2+2*x+3)^2,x, algorithm="g
iac")
```

```
[Out] 1/392*sqrt(14)*(6565*d^5 - 74017*d^4*e + 35022*d^3*e^2 + 42858*d^2*e^3 - 17
247*d*e^4 + 579*e^5)*arctan(1/14*sqrt(14)*(5*x + 1))/(625*d^8 - 1000*d^7*e
+ 2100*d^6*e^2 - 1960*d^5*e^3 + 2086*d^4*e^4 - 1176*d^3*e^5 + 756*d^2*e^6 -
 216*d*e^7 + 81*e^8) - 1/2*(205*d^5 - 19*d^4*e - 846*d^3*e^2 + 396*d^2*e^3
+ 57*d*e^4 - 21*e^5)*log(5*x^2 + 2*x + 3)/(625*d^8 - 1000*d^7*e + 2100*d^6*
e^2 - 1960*d^5*e^3 + 2086*d^4*e^4 - 1176*d^3*e^5 + 756*d^2*e^6 - 216*d*e^7
+ 81*e^8) + (205*d^5*e - 19*d^4*e^2 - 846*d^3*e^3 + 396*d^2*e^4 + 57*d*e^5
- 21*e^6)*log(abs(e*x + d))/(625*d^8*e - 1000*d^7*e^2 + 2100*d^6*e^3 - 1960
*d^5*e^4 + 2086*d^4*e^5 - 1176*d^3*e^6 + 756*d^2*e^7 - 216*d*e^8 + 81*e^9)
- 1/28*(4200*d^8 + 25945*d^7*e - 12715*d^6*e^2 - 16656*d^5*e^3 + 28226*d^4*
e^4 - 31223*d^3*e^5 + 12753*d^2*e^6 - 2646*d*e^7 + 756*e^8 + (28700*d^6*e^2
 - 14965*d^5*e^3 - 43891*d^4*e^4 + 44106*d^3*e^5 - 45966*d^2*e^6 + 12711*d*
e^7 + 9*e^8)*x^3 + (7000*d^8 + 31850*d^7*e + 6400*d^6*e^2 - 62649*d^5*e^3 +
 52187*d^4*e^4 - 53652*d^3*e^5 + 11130*d^2*e^6 - 2841*d*e^7 + 1791*e^8)*x^2
 + (2800*d^8 + 14855*d^7*e + 5815*d^6*e^2 - 18620*d^5*e^3 - 17202*d^4*e^4 +
 11119*d^3*e^5 - 26037*d^2*e^6 + 7866*d*e^7 - 756*e^8)*x)/((5*d^2 - 2*d*e +
 3*e^2)^4*(e*x + d)^2*(5*x^2 + 2*x + 3)*e)
```

Mupad [B] (verification not implemented)

Time = 14.36 (sec) , antiderivative size = 887, normalized size of antiderivative = 2.15

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^3\left(3+2x+5x^2\right)^2}\,dx$$

$$= \ln\left(d+e\,x\right)\,\left(\frac{\frac{41\,d}{5}+\frac{29\,e}{5}}{{\left(5\,d^2-2\,d\,e+3\,e^2\right)}^2}+\frac{168\,e^4\,\left(458\,d-7\,e\right)}{125\,{\left(5\,d^2-2\,d\,e+3\,e^2\right)}^4}-\frac{2\,e^2\,\left(12610\,d+1329\,e\right)}{125\,{\left(5\,d^2-2\,d\,e+3\,e^2\right)}^3}\right)$$
$$-\frac{\frac{840\,d^6+5525\,d^5\,e-837\,d^4\,e^2-6981\,d^3\,e^3+3355\,d^2\,e^4-714\,d\,e^5+252\,e^6}{28\,e\,\left(125\,d^6-150\,d^5\,e+285\,d^4\,e^2-188\,d^3\,e^3+171\,d^2\,e^4-54\,d\,e^5+27\,e^6\right)}+\frac{x^3\,\left(5740\,d^4\,e-697\,d^3\,e^2-12501\,d^2\,e^3+4239\,d\,e^4+3\,e^5\right)}{28\,\left(125\,d^6-150\,d^5\,e+285\,d^4\,e^2-188\,d^3\,e^3+171\,d^2\,e^4-54\,d\,e^5+27\,e^6}}{x^2\,\left(5\,d^2+4\,d\,e+3\,e^2\right)+x\,\left(2}$$
$$+\frac{\ln\left(x+\frac{1}{5}-\frac{\sqrt{14}\,1\mathrm{i}}{5}\right)\,\left(\left(\frac{6565\,\sqrt{14}}{784}-\frac{205}{2}\mathrm{i}\right)\,d^5+\left(-\frac{74017\,\sqrt{14}}{784}+\frac{19}{2}\mathrm{i}\right)\,d^4\,e+\left(\frac{17511\,\sqrt{14}}{392}+423\mathrm{i}\right)\,d^3\,e^2+\left(\frac{21429}{39}\right.\right.}{d^8\,625\mathrm{i}-d^7\,e\,1000\mathrm{i}+d^6\,e^2\,2100\mathrm{i}-d^5\,e^3\,1960\mathrm{i}+d^4\,e^4\,2086\mathrm{i}-d^3\,e^5\,117}$$
$$-\frac{\ln\left(x+\frac{1}{5}+\frac{\sqrt{14}\,1\mathrm{i}}{5}\right)\,\left(\left(\frac{6565\,\sqrt{14}}{784}+\frac{205}{2}\mathrm{i}\right)\,d^5+\left(-\frac{74017\,\sqrt{14}}{784}-\frac{19}{2}\mathrm{i}\right)\,d^4\,e+\left(\frac{17511\,\sqrt{14}}{392}-423\mathrm{i}\right)\,d^3\,e^2+\left(\frac{21429}{39}\right.\right.}{d^8\,625\mathrm{i}-d^7\,e\,1000\mathrm{i}+d^6\,e^2\,2100\mathrm{i}-d^5\,e^3\,1960\mathrm{i}+d^4\,e^4\,2086\mathrm{i}-d^3\,e^5\,117}$$

```
[In] int((x + 3*x^2 - 5*x^3 + 4*x^4 + 2)/((d + e*x)^3*(2*x + 5*x^2 + 3)^2),x)
```

```
[Out] log(d + e*x)*(((41*d)/5 + (29*e)/5)/(5*d^2 - 2*d*e + 3*e^2)^2 + (168*e^4*(4
58*d - 7*e))/(125*(5*d^2 - 2*d*e + 3*e^2)^4) - (2*e^2*(12610*d + 1329*e))/(
125*(5*d^2 - 2*d*e + 3*e^2)^3)) - ((5525*d^5*e - 714*d*e^5 + 840*d^6 + 252*
e^6 + 3355*d^2*e^4 - 6981*d^3*e^3 - 837*d^4*e^2)/(28*e*(125*d^6 - 150*d^5*e
 - 54*d*e^5 + 27*e^6 + 171*d^2*e^4 - 188*d^3*e^3 + 285*d^4*e^2)) + (x^3*(42
39*d*e^4 + 5740*d^4*e + 3*e^5 - 12501*d^2*e^3 - 697*d^3*e^2))/(28*(125*d^6
- 150*d^5*e - 54*d*e^5 + 27*e^6 + 171*d^2*e^4 - 188*d^3*e^3 + 285*d^4*e^2))
 + (x^2*(6930*d^5*e - 549*d*e^5 + 1400*d^6 + 597*e^6 + 2349*d^2*e^4 - 15403
*d^3*e^3 + 3212*d^4*e^2))/(28*e*(125*d^6 - 150*d^5*e - 54*d*e^5 + 27*e^6 +
171*d^2*e^4 - 188*d^3*e^3 + 285*d^4*e^2)) + (x*(2454*d*e^5 + 3195*d^5*e + 5
60*d^6 - 252*e^6 - 6623*d^2*e^4 - 4799*d^3*e^3 + 2105*d^4*e^2))/(28*e*(125*
d^6 - 150*d^5*e - 54*d*e^5 + 27*e^6 + 171*d^2*e^4 - 188*d^3*e^3 + 285*d^4*e
^2)))/(x^2*(4*d*e + 5*d^2 + 3*e^2) + x*(6*d*e + 2*d^2) + 3*d^2 + x^3*(10*d*
e + 2*e^2) + 5*e^2*x^4) + (log(x - (14^(1/2)*1i)/5 + 1/5)*(d^5*((6565*14^(1
/2))/784 - 205i/2) + e^5*((579*14^(1/2))/784 + 21i/2) + d^3*e^2*((17511*14^
(1/2))/392 + 423i) + d^2*e^3*((21429*14^(1/2))/392 - 198i) - d*e^4*((17247*
14^(1/2))/784 + 57i/2) - d^4*e*((74017*14^(1/2))/784 - 19i/2)))/(d^8*625i -
 d^7*e*1000i - d*e^7*216i + e^8*81i + d^2*e^6*756i - d^3*e^5*1176i + d^4*e^
4*2086i - d^5*e^3*1960i + d^6*e^2*2100i) - (log(x + (14^(1/2)*1i)/5 + 1/5)*
(d^5*((6565*14^(1/2))/784 + 205i/2) + e^5*((579*14^(1/2))/784 - 21i/2) + d^
3*e^2*((17511*14^(1/2))/392 - 423i) + d^2*e^3*((21429*14^(1/2))/392 + 198i)
 - d*e^4*((17247*14^(1/2))/784 - 57i/2) - d^4*e*((74017*14^(1/2))/784 + 19i
/2)))/(d^8*625i - d^7*e*1000i - d*e^7*216i + e^8*81i + d^2*e^6*756i - d^3*e
^5*1176i + d^4*e^4*2086i - d^5*e^3*1960i + d^6*e^2*2100i)
```

3.318 $\int \frac{(d+ex)^3(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^3}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 171

$$
\begin{aligned}
&\int \frac{(d+ex)^3(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^3}\,dx \\
&= \frac{(83065d-126009e)e^2x}{980000} + \frac{2e^3x^2}{125} - \frac{(1367+423x)(d+ex)^3}{7000\,(3+2x+5x^2)^2} \\
&\quad + \frac{(d+ex)^2(3(11449d-2105e)+(11015d+49177e)x)}{196000\,(3+2x+5x^2)} \\
&\quad + \frac{3(353125d^3-855175d^2e+74085de^2+556349e^3)\arctan\left(\frac{1+5x}{\sqrt{14}}\right)}{4900000\sqrt{14}} \\
&\quad + \frac{3e(100d^2-245de+47e^2)\log(3+2x+5x^2)}{6250}
\end{aligned}
$$

```
[Out] 1/980000*(83065*d-126009*e)*e^2*x+2/125*e^3*x^2-1/7000*(1367+423*x)*(e*x+d)
^3/(5*x^2+2*x+3)^2+1/196000*(e*x+d)^2*(34347*d-6315*e+(11015*d+49177*e)*x)/
(5*x^2+2*x+3)+3/6250*e*(100*d^2-245*d*e+47*e^2)*ln(5*x^2+2*x+3)+3/68600000*
(353125*d^3-855175*d^2*e+74085*d*e^2+556349*e^3)*arctan(1/14*(1+5*x)*14^(1/
2))*14^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1658, 1642, 648, 632, 210, 642}

$$
\begin{aligned}
&\int \frac{(d+ex)^3 (2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^3} \, dx \\
&= \frac{3\arctan\left(\frac{5x+1}{\sqrt{14}}\right)(353125d^3-855175d^2e+74085de^2+556349e^3)}{4900000\sqrt{14}} \\
&\quad + \frac{3e(100d^2-245de+47e^2)\log(5x^2+2x+3)}{6250} + \frac{e^2x(83065d-126009e)}{980000} \\
&\quad + \frac{(d+ex)^2(x(11015d+49177e)+3(11449d-2105e))}{196000(5x^2+2x+3)} \\
&\quad - \frac{(423x+1367)(d+ex)^3}{7000(5x^2+2x+3)^2} + \frac{2e^3x^2}{125}
\end{aligned}
$$

```
[In] Int[((d + e*x)^3*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^3,x]
```

```
[Out] ((83065*d - 126009*e)*e^2*x)/980000 + (2*e^3*x^2)/125 - ((1367 + 423*x)*(d
+ e*x)^3)/(7000*(3 + 2*x + 5*x^2)^2) + ((d + e*x)^2*(3*(11449*d - 2105*e) +
 (11015*d + 49177*e)*x))/(196000*(3 + 2*x + 5*x^2)) + (3*(353125*d^3 - 8551
75*d^2*e + 74085*d*e^2 + 556349*e^3)*ArcTan[(1 + 5*x)/Sqrt[14]])/(4900000*S
qrt[14]) + (3*e*(100*d^2 - 245*d*e + 47*e^2)*Log[3 + 2*x + 5*x^2])/6250
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1658

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f =
Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[Polynom
ialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x + c
*x^2)^(p + 1)*((f*b - 2*a*g + (2*c*f - b*g)*x)/((p + 1)*(b^2 - 4*a*c))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Q + g*(2*a*e*m + b*d*(2
*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(m + 2*p + 3)*x, x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Integer
Q[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] && Ra
tionalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\text{integral} = -\frac{(1367+423x)(d+ex)^3}{7000\left(3+2x+5x^2\right)^2} + \frac{1}{112}\int \frac{(d+ex)^2\left(\frac{6}{125}(1089d+1367e)-\frac{336}{125}(55d-27e)x+\frac{112}{25}(20d-33e)x^2+\frac{448ex^3}{5}\right)}{\left(3+2x+5x^2\right)^2}\,dx$$

$$= -\frac{(1367+423x)(d+ex)^3}{7000\left(3+2x+5x^2\right)^2} + \frac{(d+ex)^2(3(11449d-2105e)+(11015d+49177e)x)}{196000\left(3+2x+5x^2\right)} + \frac{\int \frac{(d+ex)\left(\frac{12}{25}\left(2825d^2-5587de+842e^2\right)+\frac{4}{25}(10341d-22693e)ex+\frac{25088e^2x^2}{25}\right)}{3+2x+5x^2}\,dx}{6272}$$

$$= -\frac{(1367+423x)(d+ex)^3}{7000\left(3+2x+5x^2\right)^2} + \frac{(d+ex)^2(3(11449d-2105e)+(11015d+49177e)x)}{196000\left(3+2x+5x^2\right)} + \frac{\int \left(\frac{4}{625}(83065d-126009e)e^2+\frac{25088e^3x}{125}+\frac{12\left(70625d^3-139675d^2e-62015de^2+126009e^3+1568e\left(100d^2-245de+47e^2\right.\right.}{625\left(3+2x+5x^2\right)}\right.}{6272}$$

$$
\begin{aligned}
&= \frac{(83065d-126009e)e^2x}{980000}+\frac{2e^3x^2}{125}-\frac{(1367+423x)(d+ex)^3}{7000\left(3+2x+5x^2\right)^2} \\
&\quad+\frac{(d+ex)^2(3(11449d-2105e)+(11015d+49177e)x)}{196000\left(3+2x+5x^2\right)} \\
&\quad+\frac{3\int \frac{70625d^3-139675d^2e-62015de^2+126009e^3+1568e\left(100d^2-245de+47e^2\right)x}{3+2x+5x^2}\,dx}{980000} \\
&= \frac{(83065d-126009e)e^2x}{980000}+\frac{2e^3x^2}{125}-\frac{(1367+423x)(d+ex)^3}{7000\left(3+2x+5x^2\right)^2} \\
&\quad+\frac{(d+ex)^2(3(11449d-2105e)+(11015d+49177e)x)}{196000\left(3+2x+5x^2\right)} \\
&\quad+\frac{\left(3e\left(100d^2-245de+47e^2\right)\right)\int \frac{2+10x}{3+2x+5x^2}\,dx}{6250} \\
&\quad+\frac{\left(3\left(353125d^3-855175d^2e+74085de^2+556349e^3\right)\right)\int \frac{1}{3+2x+5x^2}\,dx}{4900000} \\
&= \frac{(83065d-126009e)e^2x}{980000}+\frac{2e^3x^2}{125}-\frac{(1367+423x)(d+ex)^3}{7000\left(3+2x+5x^2\right)^2} \\
&\quad+\frac{(d+ex)^2(3(11449d-2105e)+(11015d+49177e)x)}{196000\left(3+2x+5x^2\right)} \\
&\quad+\frac{3e\left(100d^2-245de+47e^2\right)\log\left(3+2x+5x^2\right)}{6250} \\
&\quad-\frac{\left(3\left(353125d^3-855175d^2e+74085de^2+556349e^3\right)\right)\text{Subst}\left(\int \frac{1}{-56-x^2}\,dx,x,2+10x\right)}{2450000} \\
&= \frac{(83065d-126009e)e^2x}{980000}+\frac{2e^3x^2}{125}-\frac{(1367+423x)(d+ex)^3}{7000\left(3+2x+5x^2\right)^2} \\
&\quad+\frac{(d+ex)^2(3(11449d-2105e)+(11015d+49177e)x)}{196000\left(3+2x+5x^2\right)} \\
&\quad+\frac{3\left(353125d^3-855175d^2e+74085de^2+556349e^3\right)\tan^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{4900000\sqrt{14}} \\
&\quad+\frac{3e\left(100d^2-245de+47e^2\right)\log\left(3+2x+5x^2\right)}{6250}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.22

$$\int \frac{(d+ex)^3 (2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^3} \, dx$$
$$= \frac{548800(60d-49e)e^2x+5488000e^3x^2-\frac{392(e^3(54969-53189x)+125d^3(1367+423x)+75d^2e(-1269+5989x)-15de^2(17967+18323}{(3+2x+5x^2)^2}}{}$$

```
[In] Integrate[((d + e*x)^3*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^3
,x]
```

```
[Out] (548800*(60*d - 49*e)*e^2*x + 5488000*e^3*x^2 - (392*(e^3*(54969 - 53189*x)
 + 125*d^3*(1367 + 423*x) + 75*d^2*e*(-1269 + 5989*x) - 15*d*e^2*(17967 + 1
8323*x)))/(3 + 2*x + 5*x^2)^2 + (14*(e^3*(2639639 - 3109005*x) + 125*d^3*(3
4347 + 11015*x) + 75*d^2*e*(-44399 + 181765*x) - 15*d*e^2*(809167 + 647195*
x)))/(3 + 2*x + 5*x^2) + 15*Sqrt[14]*(353125*d^3 - 855175*d^2*e + 74085*d*e
^2 + 556349*e^3)*ArcTan[(1 + 5*x)/Sqrt[14]] + 164640*e*(100*d^2 - 245*d*e +
 47*e^2)*Log[3 + 2*x + 5*x^2])/343000000
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.22

method	result
default	$\frac{2e^3x^2}{125}+\frac{12d\,e^2x}{125}-\frac{49e^3x}{625}+\frac{(\frac{11015}{1568}d^3+\frac{109059}{1568}d^2e-\frac{388317}{7840}d\,e^2-\frac{621801}{39200}e^3)x^3+(\frac{38753}{1568}d^3+\frac{84921}{7840}d^2e-\frac{640827}{7840}d\,e^2+\frac{1396037}{196000}e^3)x^2+(}{25(5x^2+2x+3}$
risch	$\frac{44451\sqrt{14}\,d\,e^2\arctan\left(\frac{5\sqrt{14}\,x}{14}+\frac{\sqrt{14}}{14}\right)}{13720000}+\frac{1669047\sqrt{14}\,e^3\arctan\left(\frac{5\sqrt{14}\,x}{14}+\frac{\sqrt{14}}{14}\right)}{68600000}+\frac{6d^2e\ln(350x^2+140x+210)}{125}-\frac{147d\,e^2\ln(350x^2}{1250}$

```
[In] int((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x,method=_RETURNVERBO
SE)
```

```
[Out] 2/125*e^3*x^2+12/125*d*e^2*x-49/625*e^3*x+1/25*((11015/1568*d^3+109059/1568
*d^2*e-388317/7840*d*e^2-621801/39200*e^3)*x^3+(38753/1568*d^3+84921/7840*d
^2*e-640827/7840*d*e^2+1396037/196000*e^3)*x^2+(17979/1568*d^3+173283/7840*
d^2*e-73125/1568*d*e^2-511689/196000*e^3)*x+12953/1568*d^3-58599/7840*d^2*e
-230931/7840*d*e^2+1275957/196000*e^3)/(5*x^2+2*x+3)^2+3/9800000*(156800*d^
2*e-384160*d*e^2+73696*e^3)*ln(5*x^2+2*x+3)+3/13720000*(70625*d^3-171035*d^
2*e+14817*d*e^2+556349/5*e^3)*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 441 vs. $2(153) = 306$.

Time = 0.25 (sec) , antiderivative size = 441, normalized size of antiderivative = 2.58

$$\int \frac{(d+ex)^3\,(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^3}\,dx$$
$$= \frac{27440000\,e^3x^6 + 2744000\,(60\,de^2 - 41\,e^3)x^5 + 8780800\,(15\,de^2 - 8\,e^3)x^4 + 70\,(275375\,d^3 + 2726475\,d^2e +}{}$$

```
[In] integrate((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x, algorithm="f
ricas")
```

```
[Out] 1/68600000*(27440000*e^3*x^6 + 2744000*(60*d*e^2 - 41*e^3)*x^5 + 8780800*(1
5*d*e^2 - 8*e^3)*x^4 + 70*(275375*d^3 + 2726475*d^2*e + 1257135*d*e^2 - 304
5929*e^3)*x^3 + 22667750*d^3 - 20509650*d^2*e - 80825850*d*e^2 + 17863398*e
^3 + 14*(4844125*d^3 + 2123025*d^2*e - 10375875*d*e^2 - 2508283*e^3)*x^2 +
3*sqrt(14)*(25*(353125*d^3 - 855175*d^2*e + 74085*d*e^2 + 556349*e^3)*x^4 +
 20*(353125*d^3 - 855175*d^2*e + 74085*d*e^2 + 556349*e^3)*x^3 + 3178125*d^
3 - 7696575*d^2*e + 666765*d*e^2 + 5007141*e^3 + 34*(353125*d^3 - 855175*d^
2*e + 74085*d*e^2 + 556349*e^3)*x^2 + 12*(353125*d^3 - 855175*d^2*e + 74085
*d*e^2 + 556349*e^3)*x)*arctan(1/14*sqrt(14)*(5*x + 1)) + 42*(749125*d^3 +
1444025*d^2*e - 1635675*d*e^2 - 1323043*e^3)*x + 32928*(25*(100*d^2*e - 245
*d*e^2 + 47*e^3)*x^4 + 20*(100*d^2*e - 245*d*e^2 + 47*e^3)*x^3 + 900*d^2*e
- 2205*d*e^2 + 423*e^3 + 34*(100*d^2*e - 245*d*e^2 + 47*e^3)*x^2 + 12*(100*
d^2*e - 245*d*e^2 + 47*e^3)*x)*log(5*x^2 + 2*x + 3))/(25*x^4 + 20*x^3 + 34*
x^2 + 12*x + 9)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.95 (sec) , antiderivative size = 469, normalized size of antiderivative = 2.74

$$\int \frac{(d+ex)^3\,(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^3}\,dx$$

$$= \frac{2e^3x^2}{125} + x\left(\frac{12de^2}{125} - \frac{49e^3}{625}\right) + \left(\frac{3e(100d^2-245de+47e^2)}{6250}\right.$$

$$\left. - \frac{3\sqrt{14}i(353125d^3-855175d^2e+74085de^2+556349e^3)}{137200000}\right)\log\left(x + \frac{211875d^3-1830225d^2e+3271395d}{1059}\right.$$

$$+ \left(\frac{3e(100d^2-245de+47e^2)}{6250}\right.$$

$$\left. + \frac{3\sqrt{14}i(353125d^3-855175d^2e+74085de^2+556349e^3)}{137200000}\right)\log\left(x + \frac{211875d^3-1830225d^2e+3271395d}{1059}\right.$$

$$+ \frac{1619125d^3-1464975d^2e-5773275de^2+1275957e^3+x^3\cdot(1376875d^3+13632375d^2e-9707925de^2-}{122500000x^4+98000}$$

```
[In] integrate((e*x+d)**3*(4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3)**3,x)
```

```
[Out] 2*e**3*x**2/125 + x*(12*d*e**2/125 - 49*e**3/625) + (3*e*(100*d**2 - 245*d*
e + 47*e**2)/6250 - 3*sqrt(14)*I*(353125*d**3 - 855175*d**2*e + 74085*d*e**
2 + 556349*e**3)/137200000)*log(x + (211875*d**3 - 1830225*d**2*e + 3271395
*d*e**2 - 285237*e**3 + 65856*e*(100*d**2 - 245*d*e + 47*e**2)/5 - 3*sqrt(1
4)*I*(353125*d**3 - 855175*d**2*e + 74085*d*e**2 + 556349*e**3)/5)/(1059375
*d**3 - 2565525*d**2*e + 222255*d*e**2 + 1669047*e**3)) + (3*e*(100*d**2 -
245*d*e + 47*e**2)/6250 + 3*sqrt(14)*I*(353125*d**3 - 855175*d**2*e + 74085
*d*e**2 + 556349*e**3)/137200000)*log(x + (211875*d**3 - 1830225*d**2*e + 3
271395*d*e**2 - 285237*e**3 + 65856*e*(100*d**2 - 245*d*e + 47*e**2)/5 + 3*
sqrt(14)*I*(353125*d**3 - 855175*d**2*e + 74085*d*e**2 + 556349*e**3)/5)/(1
059375*d**3 - 2565525*d**2*e + 222255*d*e**2 + 1669047*e**3)) + (1619125*d*
*3 - 1464975*d**2*e - 5773275*d*e**2 + 1275957*e**3 + x**3*(1376875*d**3 +
13632375*d**2*e - 9707925*d*e**2 - 3109005*e**3) + x**2*(4844125*d**3 + 212
3025*d**2*e - 16020675*d*e**2 + 1396037*e**3) + x*(2247375*d**3 + 4332075*d
**2*e - 9140625*d*e**2 - 511689*e**3))/(122500000*x**4 + 98000000*x**3 + 16
6600000*x**2 + 58800000*x + 44100000)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.30

$$\int \frac{(d+ex)^3\,(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^3}\,dx = \frac{2}{125}\,e^3x^2$$
$$+\frac{3}{68600000}\,\sqrt{14}\big(353125\,d^3-855175\,d^2e+74085\,de^2+556349\,e^3\big)\arctan\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right)$$
$$+\frac{1}{625}\,\big(60\,de^2-49\,e^3\big)x+\frac{3}{6250}\,\big(100\,d^2e-245\,de^2+47\,e^3\big)\log\big(5\,x^2+2\,x+3\big)$$
$$+\frac{5\,(275375\,d^3+2726475\,d^2e-1941585\,de^2-621801\,e^3)x^3+1619125\,d^3-1464975\,d^2e-5773275\,de^2+}{49000}$$

```
[In] integrate((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x, algorithm="m
axima")
```

```
[Out] 2/125*e^3*x^2 + 3/68600000*sqrt(14)*(353125*d^3 - 855175*d^2*e + 74085*d*e^
2 + 556349*e^3)*arctan(1/14*sqrt(14)*(5*x + 1)) + 1/625*(60*d*e^2 - 49*e^3)
*x + 3/6250*(100*d^2*e - 245*d*e^2 + 47*e^3)*log(5*x^2 + 2*x + 3) + 1/49000
00*(5*(275375*d^3 + 2726475*d^2*e - 1941585*d*e^2 - 621801*e^3)*x^3 + 16191
25*d^3 - 1464975*d^2*e - 5773275*d*e^2 + 1275957*e^3 + (4844125*d^3 + 21230
25*d^2*e - 16020675*d*e^2 + 1396037*e^3)*x^2 + 3*(749125*d^3 + 1444025*d^2*
e - 3046875*d*e^2 - 170563*e^3)*x)/(25*x^4 + 20*x^3 + 34*x^2 + 12*x + 9)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.23

$$\int \frac{(d+ex)^3\,(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^3}\,dx = \frac{2}{125}\,e^3x^2+\frac{12}{125}\,de^2x-\frac{49}{625}\,e^3x$$
$$+\frac{3}{68600000}\,\sqrt{14}\big(353125\,d^3-855175\,d^2e+74085\,de^2+556349\,e^3\big)\arctan\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right)$$
$$+\frac{3}{6250}\,\big(100\,d^2e-245\,de^2+47\,e^3\big)\log\big(5\,x^2+2\,x+3\big)$$
$$+\frac{5\,(275375\,d^3+2726475\,d^2e-1941585\,de^2-621801\,e^3)x^3+1619125\,d^3-1464975\,d^2e-5773275\,de^2+}{}$$

```
[In] integrate((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x, algorithm="g
iac")
```

```
[Out] 2/125*e^3*x^2 + 12/125*d*e^2*x - 49/625*e^3*x + 3/68600000*sqrt(14)*(353125
*d^3 - 855175*d^2*e + 74085*d*e^2 + 556349*e^3)*arctan(1/14*sqrt(14)*(5*x +
```

```
 1)) + 3/6250*(100*d^2*e - 245*d*e^2 + 47*e^3)*log(5*x^2 + 2*x + 3) + 1/490
0000*(5*(275375*d^3 + 2726475*d^2*e - 1941585*d*e^2 - 621801*e^3)*x^3 + 161
9125*d^3 - 1464975*d^2*e - 5773275*d*e^2 + 1275957*e^3 + (4844125*d^3 + 212
3025*d^2*e - 16020675*d*e^2 + 1396037*e^3)*x^2 + 3*(749125*d^3 + 1444025*d^
2*e - 3046875*d*e^2 - 170563*e^3)*x)/(5*x^2 + 2*x + 3)^2
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.75

$$\int \frac{(d+ex)^3\left(2+x+3x^2-5x^3+4x^4\right)}{\left(3+2x+5x^2\right)^3}\,dx = x\left(\frac{e^2\,\left(12\,d-5\,e\right)}{125}-\frac{24\,e^3}{625}\right)$$
$$-\frac{\frac{1154655\,d\,e^2}{1568}+\frac{292995\,d^2\,e}{1568}+x\left(-\frac{449475\,d^3}{1568}-\frac{866415\,d^2\,e}{1568}+\frac{1828125\,d\,e^2}{1568}+\frac{511689\,e^3}{7840}\right)-\frac{323825\,d^3}{1568}-\frac{1275957\,e^3}{7840}+x^3\left(-\right.}{15625\,x^4+12500\,x^3+21250}$$
$$+\ln\left(5\,x^2+2\,x+3\right)\,\left(\frac{6\,d^2\,e}{125}-\frac{147\,d\,e^2}{1250}+\frac{141\,e^3}{6250}\right)+\frac{2\,e^3\,x^2}{125}$$
$$+\frac{3\,\sqrt{14}\,\mathrm{atan}\left(\frac{\frac{3\,\sqrt{14}\,\left(353125\,d^3-855175\,d^2\,e+74085\,d\,e^2+556349\,e^3\right)}{68600000}+\frac{3\,\sqrt{14}\,x\,\left(353125\,d^3-855175\,d^2\,e+74085\,d\,e^2+556349\,e^3\right)}{13720000}}{\frac{339\,d^3}{1568}-\frac{102621\,d^2\,e}{196000}+\frac{44451\,d\,e^2}{980000}+\frac{1669047\,e^3}{4900000}}\right)\,\left(353125\,d^3-\right.}{68600000}$$

```
[In] int(((d + e*x)^3*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(2*x + 5*x^2 + 3)^3,x)
```

```
[Out] x*((e^2*(12*d - 5*e))/125 - (24*e^3)/625) - ((1154655*d*e^2)/1568 + (292995
*d^2*e)/1568 + x*((1828125*d*e^2)/1568 - (866415*d^2*e)/1568 - (449475*d^3)
/1568 + (511689*e^3)/7840) - (323825*d^3)/1568 - (1275957*e^3)/7840 + x^3*(
(1941585*d*e^2)/1568 - (2726475*d^2*e)/1568 - (275375*d^3)/1568 + (621801*e
^3)/1568) - x^2*((424605*d^2*e)/1568 - (3204135*d*e^2)/1568 + (968825*d^3)/
1568 + (1396037*e^3)/7840))/(7500*x + 21250*x^2 + 12500*x^3 + 15625*x^4 + 5
625) + log(2*x + 5*x^2 + 3)*((6*d^2*e)/125 - (147*d*e^2)/1250 + (141*e^3)/6
250) + (2*e^3*x^2)/125 + (3*14^(1/2)*atan(((3*14^(1/2)*(74085*d*e^2 - 85517
5*d^2*e + 353125*d^3 + 556349*e^3))/68600000 + (3*14^(1/2)*x*(74085*d*e^2 -
 855175*d^2*e + 353125*d^3 + 556349*e^3))/13720000)/((44451*d*e^2)/980000 -
 (102621*d^2*e)/196000 + (339*d^3)/1568 + (1669047*e^3)/4900000))*(74085*d*
e^2 - 855175*d^2*e + 353125*d^3 + 556349*e^3))/68600000
```

3.319 $\int \frac{(d+ex)^2\left(2+x+3x^2-5x^3+4x^4\right)}{\left(3+2x+5x^2\right)^3}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 134

$$\begin{aligned}\int \frac{(d+ex)^2\left(2+x+3x^2-5x^3+4x^4\right)}{\left(3+2x+5x^2\right)^3}\,dx &= \frac{4e^2x}{125}-\frac{(1367+423x)(d+ex)^2}{7000\left(3+2x+5x^2\right)^2}+\frac{(d+ex)(34347d-6413e+5(2203d+8553e)x)}{196000\left(3+2x+5x^2\right)} \\ &\quad+\frac{\left(211875d^2-342070de+14817e^2\right)\arctan\left(\frac{1+5x}{\sqrt{14}}\right)}{980000\sqrt{14}}+\frac{(40d-49e)e\log\left(3+2x+5x^2\right)}{1250}\end{aligned}$$

```
[Out] 4/125*x*e^2-1/7000*(1367+423*x)*(e*x+d)^2/(5*x^2+2*x+3)^2+1/196000*(e*x+d)*
(34347*d-6413*e+5*(2203*d+8553*e)*x)/(5*x^2+2*x+3)+1/1250*(40*d-49*e)*e*ln(
5*x^2+2*x+3)+1/13720000*(211875*d^2-342070*d*e+14817*e^2)*arctan(1/14*(1+5*
x)*14^(1/2))*14^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1658, 1671, 648, 632, 210, 642}

$$\begin{aligned}&\int \frac{(d+ex)^2\left(2+x+3x^2-5x^3+4x^4\right)}{\left(3+2x+5x^2\right)^3}\,dx\\&=\frac{\arctan\left(\frac{5x+1}{\sqrt{14}}\right)\left(211875d^2-342070de+14817e^2\right)}{980000\sqrt{14}}\\&\quad+\frac{(d+ex)(5x(2203d+8553e)+34347d-6413e)}{196000\left(5x^2+2x+3\right)}\\&\quad-\frac{(423x+1367)(d+ex)^2}{7000\left(5x^2+2x+3\right)^2}+\frac{e(40d-49e)\log\left(5x^2+2x+3\right)}{1250}+\frac{4e^2x}{125}\end{aligned}$$

```
[In] Int[((d + e*x)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^3,x]
```

```
[Out] (4*e^2*x)/125 - ((1367 + 423*x)*(d + e*x)^2)/(7000*(3 + 2*x + 5*x^2)^2) + (
(d + e*x)*(34347*d - 6413*e + 5*(2203*d + 8553*e)*x))/(196000*(3 + 2*x + 5*
x^2)) + ((211875*d^2 - 342070*d*e + 14817*e^2)*ArcTan[(1 + 5*x)/Sqrt[14]])/
(980000*Sqrt[14]) + ((40*d - 49*e)*e*Log[3 + 2*x + 5*x^2])/1250
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1658

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f =
Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[Polynom
ialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x + c
*x^2)^(p + 1)*((f*b - 2*a*g + (2*c*f - b*g)*x)/((p + 1)*(b^2 - 4*a*c))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Q + g*(2*a*e*m + b*d*(2
*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(m + 2*p + 3)*x, x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Integer
Q[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] && Ra
tionalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(1367+423x)(d+ex)^2}{7000\left(3+2x+5x^2\right)^2} \\
&\quad +\frac{1}{112}\int \frac{(d+ex)\left(\frac{2}{125}(3267d+2734e)-\frac{6}{125}(3080d-1371e)x+\frac{112}{25}(20d-33e)x^2+\frac{448ex^3}{5}\right)}{\left(3+2x+5x^2\right)^2}\,dx \\
&= -\frac{(1367+423x)(d+ex)^2}{7000\left(3+2x+5x^2\right)^2}+\frac{(d+ex)(34347d-6413e+5(2203d+8553e)x)}{196000\left(3+2x+5x^2\right)} \\
&\quad +\frac{\int \frac{\frac{4}{125}\left(42375d^2-55870de+6413e^2\right)+\frac{6272}{125}(40d-41e)ex+\frac{25088e^2x^2}{25}}{3+2x+5x^2}\,dx}{6272} \\
&= -\frac{(1367+423x)(d+ex)^2}{7000\left(3+2x+5x^2\right)^2}+\frac{(d+ex)(34347d-6413e+5(2203d+8553e)x)}{196000\left(3+2x+5x^2\right)} \\
&\quad +\frac{\int \left(\frac{25088e^2}{125}+\frac{4\left(42375d^2-55870de-12403e^2+1568(40d-49e)ex\right)}{125\left(3+2x+5x^2\right)}\right)dx}{6272} \\
&= \frac{4e^2x}{125}-\frac{(1367+423x)(d+ex)^2}{7000\left(3+2x+5x^2\right)^2}+\frac{(d+ex)(34347d-6413e+5(2203d+8553e)x)}{196000\left(3+2x+5x^2\right)} \\
&\quad +\frac{\int \frac{42375d^2-55870de-12403e^2+1568(40d-49e)ex}{3+2x+5x^2}\,dx}{196000}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{4e^2x}{125} - \frac{(1367+423x)(d+ex)^2}{7000\,(3+2x+5x^2)^2} + \frac{(d+ex)(34347d-6413e+5(2203d+8553e)x)}{196000\,(3+2x+5x^2)} \\
&\quad + \frac{((40d-49e)e)\int \frac{2+10x}{3+2x+5x^2}\,dx}{1250} + \frac{(211875d^2-342070de+14817e^2)\int \frac{1}{3+2x+5x^2}\,dx}{980000} \\
&= \frac{4e^2x}{125} - \frac{(1367+423x)(d+ex)^2}{7000\,(3+2x+5x^2)^2} + \frac{(d+ex)(34347d-6413e+5(2203d+8553e)x)}{196000\,(3+2x+5x^2)} \\
&\quad + \frac{(40d-49e)e\log(3+2x+5x^2)}{1250} \\
&\quad + \frac{(-211875d^2+342070de-14817e^2)\,\mathrm{Subst}\left(\int \frac{1}{-56-x^2}\,dx, x, 2+10x\right)}{490000} \\
&= \frac{4e^2x}{125} - \frac{(1367+423x)(d+ex)^2}{7000\,(3+2x+5x^2)^2} + \frac{(d+ex)(34347d-6413e+5(2203d+8553e)x)}{196000\,(3+2x+5x^2)} \\
&\quad + \frac{(211875d^2-342070de+14817e^2)\tan^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{980000\sqrt{14}} + \frac{(40d-49e)e\log(3+2x+5x^2)}{1250}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.09

$$
\int \frac{(d+ex)^2\,(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^3}\,dx
= \frac{5\sqrt{14}(211875d^2-342070de+14817e^2)\arctan\left(\frac{1+5x}{\sqrt{14}}\right)+70\Bigl(\frac{5(5d^2(12953+17979x+38753x^2+11015x^3)+2de(-19533+57}{}}{686}
$$

```
[In] Integrate[((d + e*x)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^3
,x]
```

```
[Out] (5*Sqrt[14]*(211875*d^2 - 342070*d*e + 14817*e^2)*ArcTan[(1 + 5*x)/Sqrt[14]
] + 70*((5*(5*d^2*(12953 + 17979*x + 38753*x^2 + 11015*x^3) + 2*d*e*(-19533
 + 57761*x + 28307*x^2 + 181765*x^3) + e^2*(-76977 - 65427*x - 138345*x^2 +
 83809*x^3 + 125440*x^4 + 156800*x^5)))/(3 + 2*x + 5*x^2)^2 + 784*(40*d - 4
9*e)*e*Log[3 + 2*x + 5*x^2]))/68600000
```

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.09

method	result
default	$\frac{4e^2x}{125}+\frac{\left(\frac{2203}{1568}d^2+\frac{36353}{3920}de-\frac{129439}{39200}e^2\right)x^3+\left(\frac{38753}{7840}d^2+\frac{28307}{19600}de-\frac{213609}{39200}e^2\right)x^2+\left(\frac{17979}{7840}d^2+\frac{57761}{19600}de-\frac{4875}{1568}e^2\right)x+\frac{12953d^2}{7840}-\frac{19533de}{19600}-}{5\left(5x^2+2x+3\right)^2}$
risch	$\frac{4e^2x}{125}+\frac{\frac{\left(\frac{2203}{1568}d^2+\frac{36353}{3920}de-\frac{129439}{39200}e^2\right)x^3}{5}+\frac{\left(\frac{38753}{7840}d^2+\frac{28307}{19600}de-\frac{213609}{39200}e^2\right)x^2}{5}+\frac{\left(\frac{17979}{7840}d^2+\frac{57761}{19600}de-\frac{4875}{1568}e^2\right)x}{5}+\frac{12953d^2}{39200}-\frac{19533de}{98000}-}{\left(5x^2+2x+3\right)^2}$

```
[In] int((e*x+d)^2*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x,method=_RETURNVERBO
SE)
```

```
[Out] 4/125*e^2*x+1/5*((2203/1568*d^2+36353/3920*d*e-129439/39200*e^2)*x^3+(38753
/7840*d^2+28307/19600*d*e-213609/39200*e^2)*x^2+(17979/7840*d^2+57761/19600
*d*e-4875/1568*e^2)*x+12953/7840*d^2-19533/19600*d*e-76977/39200*e^2)/(5*x^
2+2*x+3)^2+1/1960000*(62720*d*e-76832*e^2)*ln(5*x^2+2*x+3)+1/2744000*(42375
*d^2-68414*d*e+14817/5*e^2)*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 302 vs. 2(121) = 242.

Time = 0.26 (sec) , antiderivative size = 302, normalized size of antiderivative = 2.25

$$\int \frac{(d+ex)^2\left(2+x+3x^2-5x^3+4x^4\right)}{\left(3+2x+5x^2\right)^3}\,dx$$
$$=\frac{10976000\,e^2x^5+8780800\,e^2x^4+70\,(55075\,d^2+363530\,de+83809\,e^2)x^3+70\,(193765\,d^2+56614\,de-}{}$$

```
[In] integrate((e*x+d)^2*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x, algorithm="f
ricas")
```

```
[Out] 1/13720000*(10976000*e^2*x^5 + 8780800*e^2*x^4 + 70*(55075*d^2 + 363530*d*e
 + 83809*e^2)*x^3 + 70*(193765*d^2 + 56614*d*e - 138345*e^2)*x^2 + sqrt(14)
*(25*(211875*d^2 - 342070*d*e + 14817*e^2)*x^4 + 20*(211875*d^2 - 342070*d*
e + 14817*e^2)*x^3 + 34*(211875*d^2 - 342070*d*e + 14817*e^2)*x^2 + 1906875
*d^2 - 3078630*d*e + 133353*e^2 + 12*(211875*d^2 - 342070*d*e + 14817*e^2)*
x)*arctan(1/14*sqrt(14)*(5*x + 1)) + 4533550*d^2 - 2734620*d*e - 5388390*e^
2 + 70*(89895*d^2 + 115522*d*e - 65427*e^2)*x + 10976*(25*(40*d*e - 49*e^2)
*x^4 + 20*(40*d*e - 49*e^2)*x^3 + 34*(40*d*e - 49*e^2)*x^2 + 360*d*e - 441*
e^2 + 12*(40*d*e - 49*e^2)*x)*log(5*x^2 + 2*x + 3))/(25*x^4 + 20*x^3 + 34*x
^2 + 12*x + 9)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.88 (sec) , antiderivative size = 304, normalized size of antiderivative = 2.27

$$\int \frac{(d+ex)^2\left(2+x+3x^2-5x^3+4x^4\right)}{\left(3+2x+5x^2\right)^3}\,dx = \frac{4e^2x}{125} + \left(\frac{e(40d-49e)}{1250} - \frac{\sqrt{14}i(211875d^2-342070de+14817e^2)}{27440000}\right)\log\left(x+\frac{42375d^2-244030de+218093e^2+\frac{21952e(40d-49e)}{5}-}{211875d^2-342070de+148}\right.$$

$$+\left(\frac{e(40d-49e)}{1250} + \frac{\sqrt{14}i(211875d^2-342070de+14817e^2)}{27440000}\right)\log\left(x+\frac{42375d^2-244030de+218093e^2+\frac{21952e(40d-49e)}{5}+}{211875d^2-342070de+148}\right.$$

$$+\frac{64765d^2-39066de-76977e^2+x^3\cdot(55075d^2+363530de-129439e^2)+x^2\cdot(193765d^2+56614de-2}{4900000x^4+3920000x^3+6664000x^2+2352000x+17640}$$

```
[In] integrate((e*x+d)**2*(4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3)**3,x)
```

```
[Out] 4*e**2*x/125 + (e*(40*d - 49*e)/1250 - sqrt(14)*I*(211875*d**2 - 342070*d*e
 + 14817*e**2)/27440000)*log(x + (42375*d**2 - 244030*d*e + 218093*e**2 + 2
1952*e*(40*d - 49*e)/5 - sqrt(14)*I*(211875*d**2 - 342070*d*e + 14817*e**2)
/5)/(211875*d**2 - 342070*d*e + 14817*e**2)) + (e*(40*d - 49*e)/1250 + sqrt
(14)*I*(211875*d**2 - 342070*d*e + 14817*e**2)/27440000)*log(x + (42375*d**
2 - 244030*d*e + 218093*e**2 + 21952*e*(40*d - 49*e)/5 + sqrt(14)*I*(211875
*d**2 - 342070*d*e + 14817*e**2)/5)/(211875*d**2 - 342070*d*e + 14817*e**2)
) + (64765*d**2 - 39066*d*e - 76977*e**2 + x**3*(55075*d**2 + 363530*d*e -
129439*e**2) + x**2*(193765*d**2 + 56614*d*e - 213609*e**2) + x*(89895*d**2
 + 115522*d*e - 121875*e**2))/(4900000*x**4 + 3920000*x**3 + 6664000*x**2 +
 2352000*x + 1764000)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.16

$$\int \frac{(d+ex)^2\left(2+x+3x^2-5x^3+4x^4\right)}{\left(3+2x+5x^2\right)^3}\,dx$$

$$= \frac{4}{125}\,e^2x + \frac{1}{13720000}\,\sqrt{14}\left(211875\,d^2-342070\,de+14817\,e^2\right)\arctan\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right) + \frac{1}{1250}\left(40\,de-49\,e^2\right)\log\left(5\,x^2+2\,x+3\right) + \frac{(55075\,d^2+363530\,de-129439\,e^2)x^3+(193765\,d^2+56614\,de-213609\,e^2)x^2+64765\,d^2-39066\,de-}{196000\left(25\,x^4+20\,x^3+34\,x^2+12\,x+9\right)}$$

```
[In] integrate((e*x+d)^2*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x, algorithm="m
axima")
```

```
[Out] 4/125*e^2*x + 1/13720000*sqrt(14)*(211875*d^2 - 342070*d*e + 14817*e^2)*arc
tan(1/14*sqrt(14)*(5*x + 1)) + 1/1250*(40*d*e - 49*e^2)*log(5*x^2 + 2*x + 3
) + 1/196000*((55075*d^2 + 363530*d*e - 129439*e^2)*x^3 + (193765*d^2 + 566
14*d*e - 213609*e^2)*x^2 + 64765*d^2 - 39066*d*e - 76977*e^2 + (89895*d^2 +
 115522*d*e - 121875*e^2)*x)/(25*x^4 + 20*x^3 + 34*x^2 + 12*x + 9)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.08

$$\int \frac{(d+ex)^2\,(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^3}\,dx$$

$$= \frac{4}{125}\,e^2x + \frac{1}{13720000}\,\sqrt{14}\big(211875\,d^2 - 342070\,de + 14817\,e^2\big)\arctan\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right) + \frac{1}{1250}\,\big(40\,de - 49\,e^2\big)\log\big(5\,x^2+2\,x+3\big) + \frac{(55075\,d^2 + 363530\,de - 129439\,e^2)x^3 + (193765\,d^2 + 56614\,de - 213609\,e^2)x^2 + 64765\,d^2 - 39066\,de -}{196000\,(5\,x^2+2\,x+3)^2}$$

```
[In] integrate((e*x+d)^2*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x, algorithm="g
iac")
```

```
[Out] 4/125*e^2*x + 1/13720000*sqrt(14)*(211875*d^2 - 342070*d*e + 14817*e^2)*arc
tan(1/14*sqrt(14)*(5*x + 1)) + 1/1250*(40*d*e - 49*e^2)*log(5*x^2 + 2*x + 3
) + 1/196000*((55075*d^2 + 363530*d*e - 129439*e^2)*x^3 + (193765*d^2 + 566
14*d*e - 213609*e^2)*x^2 + 64765*d^2 - 39066*d*e - 76977*e^2 + (89895*d^2 +
 115522*d*e - 121875*e^2)*x)/(5*x^2 + 2*x + 3)^2
```

Mupad [B] (verification not implemented)

Time = 13.25 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.51

$$\int \frac{(d+ex)^2\,(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^3}\,dx$$

$$= \frac{x^3\left(\frac{55075\,d^2}{1568}+\frac{181765\,d\,e}{784}-\frac{129439\,e^2}{1568}\right)+x^2\left(\frac{193765\,d^2}{1568}+\frac{28307\,d\,e}{784}-\frac{213609\,e^2}{1568}\right)-\frac{19533\,d\,e}{784}+x\,(\frac{89895\,d^2}{1568}+\frac{57761\,d\,e}{784}-}{3125\,x^4+2500\,x^3+4250\,x^2+1500\,x+1125} + \frac{4\,e^2\,x}{125} + \ln\left(5\,x^2+2\,x+3\right)\,\left(\frac{4\,d\,e}{125}-\frac{49\,e^2}{1250}\right) + \frac{\sqrt{14}\,\mathrm{atan}\left(\frac{\frac{\sqrt{14}\,\left(211875\,d^2-342070\,d\,e+14817\,e^2\right)}{13720000}+\frac{\sqrt{14}\,x\,\left(211875\,d^2-342070\,d\,e+14817\,e^2\right)}{2744000}}{\frac{339\,d^2}{1568}-\frac{34207\,d\,e}{98000}+\frac{14817\,e^2}{980000}}\right)\,(211875\,d^2-342070\,d\,e+14817\,e^2}{13720000}$$

```
[In] int(((d + e*x)^2*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(2*x + 5*x^2 + 3)^3,x)
```

```
[Out] (x^3*((181765*d*e)/784 + (55075*d^2)/1568 - (129439*e^2)/1568) + x^2*((2830
7*d*e)/784 + (193765*d^2)/1568 - (213609*e^2)/1568) - (19533*d*e)/784 + x*(
(57761*d*e)/784 + (89895*d^2)/1568 - (121875*e^2)/1568) + (64765*d^2)/1568
- (76977*e^2)/1568)/(1500*x + 4250*x^2 + 2500*x^3 + 3125*x^4 + 1125) + (4*e
^2*x)/125 + log(2*x + 5*x^2 + 3)*((4*d*e)/125 - (49*e^2)/1250) + (14^(1/2)*
atan(((14^(1/2)*(211875*d^2 - 342070*d*e + 14817*e^2))/13720000 + (14^(1/2)
*x*(211875*d^2 - 342070*d*e + 14817*e^2))/2744000)/((339*d^2)/1568 - (34207
*d*e)/98000 + (14817*e^2)/980000))*(211875*d^2 - 342070*d*e + 14817*e^2))/1
3720000
```

3.320 $\int \frac{(d+ex)\left(2+x+3x^2-5x^3+4x^4\right)}{\left(3+2x+5x^2\right)^3}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 103

$$\begin{aligned}\int \frac{(d+ex)\left(2+x+3x^2-5x^3+4x^4\right)}{\left(3+2x+5x^2\right)^3}\,dx = &-\frac{(1367+423x)(d+ex)}{7000\left(3+2x+5x^2\right)^2}\\&+\frac{34347d-6511e+(11015d+36353e)x}{196000\left(3+2x+5x^2\right)}\\&+\frac{(42375d-34207e)\arctan\left(\frac{1+5x}{\sqrt{14}}\right)}{196000\sqrt{14}}\\&+\frac{2}{125}e\log\left(3+2x+5x^2\right)\end{aligned}$$

```
[Out] -1/7000*(1367+423*x)*(e*x+d)/(5*x^2+2*x+3)^2+1/196000*(34347*d-6511*e+(1101
5*d+36353*e)*x)/(5*x^2+2*x+3)+2/125*e*ln(5*x^2+2*x+3)+1/2744000*(42375*d-34
207*e)*arctan(1/14*(1+5*x)*14^(1/2))*14^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used

$= \{1658, 1674, 648, 632, 210, 642\}$

$$\int \frac{(d+ex)\left(2+x+3x^2-5x^3+4x^4\right)}{\left(3+2x+5x^2\right)^3}\,dx = \frac{\arctan\left(\frac{5x+1}{\sqrt{14}}\right)(42375d-34207e)}{196000\sqrt{14}} - \frac{(423x+1367)(d+ex)}{7000\left(5x^2+2x+3\right)^2} + \frac{x(11015d+36353e)+34347d-6511e}{196000\left(5x^2+2x+3\right)} + \frac{2}{125}e\log\left(5x^2+2x+3\right)$$

```
[In] Int[((d + e*x)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^3,x]
```

```
[Out] -1/7000*((1367 + 423*x)*(d + e*x))/(3 + 2*x + 5*x^2)^2 + (34347*d - 6511*e
+ (11015*d + 36353*e)*x)/(196000*(3 + 2*x + 5*x^2)) + ((42375*d - 34207*e)*
ArcTan[(1 + 5*x)/Sqrt[14]])/(196000*Sqrt[14]) + (2*e*Log[3 + 2*x + 5*x^2])/
125
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1658

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f =
```

```
Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[Polynom
ialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x + c
*x^2)^(p + 1)*((f*b - 2*a*g + (2*c*f - b*g)*x)/((p + 1)*(b^2 - 4*a*c))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Q + g*(2*a*e*m + b*d*(2
*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(m + 2*p + 3)*x, x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Integer
Q[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] && Ra
tionalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(1367+423x)(d+ex)}{7000\left(3+2x+5x^2\right)^2} \\
&\quad +\frac{1}{112}\int \frac{\frac{2}{125}(3267d+1367e)-\frac{12}{25}(308d-123e)x+\frac{112}{25}(20d-33e)x^2+\frac{448ex^3}{5}}{\left(3+2x+5x^2\right)^2}\,dx \\
&= -\frac{(1367+423x)(d+ex)}{7000\left(3+2x+5x^2\right)^2}+\frac{34347d-6511e+(11015d+36353e)x}{196000\left(3+2x+5x^2\right)} \\
&\quad +\frac{\int \frac{\frac{4}{25}(8475d-5587e)+\frac{25088ex}{25}}{3+2x+5x^2}\,dx}{6272} \\
&= -\frac{(1367+423x)(d+ex)}{7000\left(3+2x+5x^2\right)^2}+\frac{34347d-6511e+(11015d+36353e)x}{196000\left(3+2x+5x^2\right)} \\
&\quad +\frac{(42375d-34207e)\int \frac{1}{3+2x+5x^2}\,dx}{196000}+\frac{1}{125}(2e)\int \frac{2+10x}{3+2x+5x^2}\,dx \\
&= -\frac{(1367+423x)(d+ex)}{7000\left(3+2x+5x^2\right)^2}+\frac{34347d-6511e+(11015d+36353e)x}{196000\left(3+2x+5x^2\right)} \\
&\quad +\frac{2}{125}e\log\left(3+2x+5x^2\right)+\frac{(-42375d+34207e)\text{Subst}\left(\int \frac{1}{-56-x^2}\,dx,x,2+10x\right)}{98000}
\end{aligned}$$

$$= -\frac{(1367+423x)(d+ex)}{7000\left(3+2x+5x^2\right)^2}+\frac{34347d-6511e+(11015d+36353e)x}{196000\left(3+2x+5x^2\right)} + \frac{(42375d-34207e)\tan^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{196000\sqrt{14}}+\frac{2}{125}e\log\left(3+2x+5x^2\right)$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.04

$$\int \frac{(d+ex)\left(2+x+3x^2-5x^3+4x^4\right)}{\left(3+2x+5x^2\right)^3}\,dx = \frac{-6835d+1269e-2115dx-5989ex}{35000\left(3+2x+5x^2\right)^2} + \frac{171735d-44399e+55075dx+181765ex}{980000\left(3+2x+5x^2\right)} + \frac{(42375d-34207e)\arctan\left(\frac{1+5x}{\sqrt{14}}\right)}{196000\sqrt{14}} + \frac{2}{125}e\log\left(3+2x+5x^2\right)$$

```
[In] Integrate[((d + e*x)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^3,x
]
```

```
[Out] (-6835*d + 1269*e - 2115*d*x - 5989*e*x)/(35000*(3 + 2*x + 5*x^2)^2) + (171
735*d - 44399*e + 55075*d*x + 181765*e*x)/(980000*(3 + 2*x + 5*x^2)) + ((42
375*d - 34207*e)*ArcTan[(1 + 5*x)/Sqrt[14]])/(196000*Sqrt[14]) + (2*e*Log[3
 + 2*x + 5*x^2])/125
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.88

method	result
default	$\frac{25\left(\frac{36353e}{980000}+\frac{2203d}{196000}\right)x^3+25\left(\frac{28307e}{4900000}+\frac{38753d}{980000}\right)x^2+25\left(\frac{57761e}{4900000}+\frac{17979d}{980000}\right)x+\frac{12953d}{39200}-\frac{19533e}{196000}}{\left(5x^2+2x+3\right)^2}+\frac{2e\ln\left(5x^2+2x+3\right)}{125}+\left(8475d-\frac{34207e}{5}\right.$
risch	$\frac{25\left(\frac{36353e}{980000}+\frac{2203d}{196000}\right)x^3+25\left(\frac{28307e}{4900000}+\frac{38753d}{980000}\right)x^2+25\left(\frac{57761e}{4900000}+\frac{17979d}{980000}\right)x+\frac{12953d}{39200}-\frac{19533e}{196000}}{\left(5x^2+2x+3\right)^2}+\frac{2e\ln\left(350x^2+140x+210\right)}{125}+339\sqrt{14}$

```
[In] int((e*x+d)*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x,method=_RETURNVERBOSE
)
```

```
[Out] 25*((36353/980000*e+2203/196000*d)*x^3+(28307/4900000*e+38753/980000*d)*x^2
+(57761/4900000*e+17979/980000*d)*x+12953/980000*d-19533/4900000*e)/(5*x^2+
2*x+3)^2+2/125*e*ln(5*x^2+2*x+3)+1/548800*(8475*d-34207/5*e)*14^(1/2)*arcta
n(1/28*(10*x+2)*14^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.67

$$\int \frac{(d+ex)\left(2+x+3x^2-5x^3+4x^4\right)}{\left(3+2x+5x^2\right)^3}\,dx$$
$$= \frac{70\,(11015\,d+36353\,e)x^3+14\,(193765\,d+28307\,e)x^2+\sqrt{14}(25\,(42375\,d-34207\,e)x^4+20\,(42375\,d-3}{}$$

```
[In] integrate((e*x+d)*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x, algorithm="fri
cas")
```

```
[Out] 1/2744000*(70*(11015*d + 36353*e)*x^3 + 14*(193765*d + 28307*e)*x^2 + sqrt(
14)*(25*(42375*d - 34207*e)*x^4 + 20*(42375*d - 34207*e)*x^3 + 34*(42375*d
- 34207*e)*x^2 + 12*(42375*d - 34207*e)*x + 381375*d - 307863*e)*arctan(1/1
4*sqrt(14)*(5*x + 1)) + 14*(89895*d + 57761*e)*x + 43904*(25*e*x^4 + 20*e*x
^3 + 34*e*x^2 + 12*e*x + 9*e)*log(5*x^2 + 2*x + 3) + 906710*d - 273462*e)/(
25*x^4 + 20*x^3 + 34*x^2 + 12*x + 9)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.88 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.58

$$\int \frac{(d+ex)\left(2+x+3x^2-5x^3+4x^4\right)}{\left(3+2x+5x^2\right)^3}\,dx$$
$$= \left(\frac{2e}{125}-\frac{\sqrt{14}i(42375d-34207e)}{5488000}\right)\log\left(x+\frac{8475d-\frac{34207e}{5}-\frac{\sqrt{14}i(42375d-34207e)}{5}}{42375d-34207e}\right)$$
$$+\left(\frac{2e}{125}+\frac{\sqrt{14}i(42375d-34207e)}{5488000}\right)\log\left(x+\frac{8475d-\frac{34207e}{5}+\frac{\sqrt{14}i(42375d-34207e)}{5}}{42375d-34207e}\right)$$
$$+\frac{64765d-19533e+x^3\cdot(55075d+181765e)+x^2\cdot(193765d+28307e)+x(89895d+57761e)}{4900000x^4+3920000x^3+6664000x^2+2352000x+1764000}$$

```
[In] integrate((e*x+d)*(4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3)**3,x)
```

```
[Out] (2*e/125 - sqrt(14)*I*(42375*d - 34207*e)/5488000)*log(x + (8475*d - 34207*
e/5 - sqrt(14)*I*(42375*d - 34207*e)/5)/(42375*d - 34207*e)) + (2*e/125 + s
qrt(14)*I*(42375*d - 34207*e)/5488000)*log(x + (8475*d - 34207*e/5 + sqrt(1
4)*I*(42375*d - 34207*e)/5)/(42375*d - 34207*e)) + (64765*d - 19533*e + x**
3*(55075*d + 181765*e) + x**2*(193765*d + 28307*e) + x*(89895*d + 57761*e))
/(4900000*x**4 + 3920000*x**3 + 6664000*x**2 + 2352000*x + 1764000)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.98

$$\int \frac{(d+ex)\left(2+x+3x^2-5x^3+4x^4\right)}{(3+2x+5x^2)^3}\,dx$$
$$=\frac{1}{2744000}\sqrt{14}(42375\,d-34207\,e)\arctan\left(\frac{1}{14}\sqrt{14}(5\,x+1)\right)+\frac{2}{125}\,e\log\left(5\,x^2+2\,x+3\right)$$
$$+\frac{5\,(11015\,d+36353\,e)x^3+(193765\,d+28307\,e)x^2+(89895\,d+57761\,e)x+64765\,d-19533\,e}{196000\,(25\,x^4+20\,x^3+34\,x^2+12\,x+9)}$$

```
[In] integrate((e*x+d)*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x, algorithm="max
ima")
```

```
[Out] 1/2744000*sqrt(14)*(42375*d - 34207*e)*arctan(1/14*sqrt(14)*(5*x + 1)) + 2/
125*e*log(5*x^2 + 2*x + 3) + 1/196000*(5*(11015*d + 36353*e)*x^3 + (193765*
d + 28307*e)*x^2 + (89895*d + 57761*e)*x + 64765*d - 19533*e)/(25*x^4 + 20*
x^3 + 34*x^2 + 12*x + 9)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.88

$$\int \frac{(d+ex)\left(2+x+3x^2-5x^3+4x^4\right)}{(3+2x+5x^2)^3}\,dx$$
$$=\frac{1}{2744000}\sqrt{14}(42375\,d-34207\,e)\arctan\left(\frac{1}{14}\sqrt{14}(5\,x+1)\right)+\frac{2}{125}\,e\log\left(5\,x^2+2\,x+3\right)$$
$$+\frac{5\,(11015\,d+36353\,e)x^3+(193765\,d+28307\,e)x^2+(89895\,d+57761\,e)x+64765\,d-19533\,e}{196000\,(5\,x^2+2\,x+3)^2}$$

```
[In] integrate((e*x+d)*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x, algorithm="gia
c")
```

```
[Out] 1/2744000*sqrt(14)*(42375*d - 34207*e)*arctan(1/14*sqrt(14)*(5*x + 1)) + 2/
125*e*log(5*x^2 + 2*x + 3) + 1/196000*(5*(11015*d + 36353*e)*x^3 + (193765*
d + 28307*e)*x^2 + (89895*d + 57761*e)*x + 64765*d - 19533*e)/(5*x^2 + 2*x
+ 3)^2
```

Mupad [B] (verification not implemented)

Time = 13.55 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.21

$$\int \frac{(d+ex)\left(2+x+3x^2-5x^3+4x^4\right)}{\left(3+2x+5x^2\right)^3}\,dx$$
$$=\frac{\left(\frac{2203\,d}{7840}+\frac{36353\,e}{39200}\right)\,x^3+\left(\frac{38753\,d}{39200}+\frac{28307\,e}{196000}\right)\,x^2+\left(\frac{17979\,d}{39200}+\frac{57761\,e}{196000}\right)\,x+\frac{12953\,d}{39200}-\frac{19533\,e}{196000}}{25\,x^4+20\,x^3+34\,x^2+12\,x+9}$$
$$+\frac{2\,e\,\ln\left(5\,x^2+2\,x+3\right)}{125}$$
$$+\frac{\sqrt{14}\,\mathrm{atan}\left(\frac{\frac{\sqrt{14}\,\left(42375\,d-34207\,e\right)}{2744000}+\frac{\sqrt{14}\,x\,\left(42375\,d-34207\,e\right)}{548800}}{\frac{339\,d}{1568}-\frac{34207\,e}{196000}}\right)\,\left(42375\,d-34207\,e\right)}{2744000}$$

```
[In] int(((d + e*x)*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(2*x + 5*x^2 + 3)^3,x)
```

```
[Out] ((12953*d)/39200 - (19533*e)/196000 + x^3*((2203*d)/7840 + (36353*e)/39200)
 + x^2*((38753*d)/39200 + (28307*e)/196000) + x*((17979*d)/39200 + (57761*e
)/196000))/(12*x + 34*x^2 + 20*x^3 + 25*x^4 + 9) + (2*e*log(2*x + 5*x^2 + 3
))/125 + (14^(1/2)*atan(((14^(1/2)*(42375*d - 34207*e))/2744000 + (14^(1/2)
*x*(42375*d - 34207*e))/548800)/((339*d)/1568 - (34207*e)/196000))*(42375*d
 - 34207*e))/2744000
```

3.321 $\int \frac{2+x+3x^2-5x^3+4x^4}{(3+2x+5x^2)^3}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 64

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(3+2x+5x^2)^3}\,dx = -\frac{1367+423x}{7000\,(3+2x+5x^2)^2} + \frac{34347+11015x}{196000\,(3+2x+5x^2)} + \frac{339\arctan\left(\frac{1+5x}{\sqrt{14}}\right)}{1568\sqrt{14}}$$

```
[Out] 1/7000*(-1367-423*x)/(5*x^2+2*x+3)^2+1/196000*(34347+11015*x)/(5*x^2+2*x+3)
+339/21952*arctan(1/14*(1+5*x)*14^(1/2))*14^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {1674, 12, 632, 210}

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(3+2x+5x^2)^3}\,dx = \frac{339\arctan\left(\frac{5x+1}{\sqrt{14}}\right)}{1568\sqrt{14}} - \frac{423x+1367}{7000\,(5x^2+2x+3)^2} + \frac{11015x+34347}{196000\,(5x^2+2x+3)}$$

```
[In] Int[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/(3 + 2*x + 5*x^2)^3,x]
```

```
[Out] -1/7000*(1367 + 423*x)/(3 + 2*x + 5*x^2)^2 + (34347 + 11015*x)/(196000*(3 +
 2*x + 5*x^2)) + (339*ArcTan[(1 + 5*x)/Sqrt[14]])/(1568*Sqrt[14])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1367+423x}{7000\left(3+2x+5x^2\right)^2}+\frac{1}{112}\int\frac{\frac{6534}{125}-\frac{3696x}{25}+\frac{448x^2}{5}}{\left(3+2x+5x^2\right)^2}\,dx\\
&= -\frac{1367+423x}{7000\left(3+2x+5x^2\right)^2}+\frac{34347+11015x}{196000\left(3+2x+5x^2\right)}+\frac{\int\frac{1356}{3+2x+5x^2}\,dx}{6272}\\
&= -\frac{1367+423x}{7000\left(3+2x+5x^2\right)^2}+\frac{34347+11015x}{196000\left(3+2x+5x^2\right)}+\frac{339\int\frac{1}{3+2x+5x^2}\,dx}{1568}\\
&= -\frac{1367+423x}{7000\left(3+2x+5x^2\right)^2}+\frac{34347+11015x}{196000\left(3+2x+5x^2\right)}-\frac{339}{784}\text{Subst}\left(\int\frac{1}{-56-x^2}\,dx,x,2+10x\right)\\
&= -\frac{1367+423x}{7000\left(3+2x+5x^2\right)^2}+\frac{34347+11015x}{196000\left(3+2x+5x^2\right)}+\frac{339\tan^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{1568\sqrt{14}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.83

$$\int \frac{2+x+3x^2-5x^3+4x^4}{\left(3+2x+5x^2\right)^3}\,dx=\frac{\frac{14\left(12953+17979x+38753x^2+11015x^3\right)}{\left(3+2x+5x^2\right)^2}+8475\sqrt{14}\arctan\left(\frac{1+5x}{\sqrt{14}}\right)}{548800}$$

```
[In] Integrate[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/(3 + 2*x + 5*x^2)^3,x]
```

```
[Out] ((14*(12953 + 17979*x + 38753*x^2 + 11015*x^3))/(3 + 2*x + 5*x^2)^2 + 8475*
Sqrt[14]*ArcTan[(1 + 5*x)/Sqrt[14]])/548800
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.73

method	result	size
default	$\frac{\frac{2203}{7840}x^3+\frac{38753}{39200}x^2+\frac{17979}{39200}x+\frac{12953}{39200}}{\left(5x^2+2x+3\right)^2}+\frac{339\sqrt{14}\,\arctan\left(\frac{(10x+2)\sqrt{14}}{28}\right)}{21952}$	47
risch	$\frac{\frac{2203}{7840}x^3+\frac{38753}{39200}x^2+\frac{17979}{39200}x+\frac{12953}{39200}}{\left(5x^2+2x+3\right)^2}+\frac{339\arctan\left(\frac{(1+5x)\sqrt{14}}{14}\right)\sqrt{14}}{21952}$	47

```
[In] int((4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 25*(2203/196000*x^3+38753/980000*x^2+17979/980000*x+12953/980000)/(5*x^2+2*
x+3)^2+339/21952*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.17

$$\int \frac{2+x+3x^2-5x^3+4x^4}{\left(3+2x+5x^2\right)^3}\,dx$$
$$=\frac{154210\,x^3+8475\,\sqrt{14}\left(25\,x^4+20\,x^3+34\,x^2+12\,x+9\right)\arctan\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right)+542542\,x^2+251706\,x}{548800\left(25\,x^4+20\,x^3+34\,x^2+12\,x+9\right)}$$

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x, algorithm="fricas")
```

```
[Out] 1/548800*(154210*x^3 + 8475*sqrt(14)*(25*x^4 + 20*x^3 + 34*x^2 + 12*x + 9)*
arctan(1/14*sqrt(14)*(5*x + 1)) + 542542*x^2 + 251706*x + 181342)/(25*x^4 +
 20*x^3 + 34*x^2 + 12*x + 9)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.95

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(3+2x+5x^2)^3}\,dx = \frac{11015x^3+38753x^2+17979x+12953}{980000x^4+784000x^3+1332800x^2+470400x+352800} + \frac{339\sqrt{14}\,\mathrm{atan}\left(\frac{5\sqrt{14}x}{14}+\frac{\sqrt{14}}{14}\right)}{21952}$$

```
[In] integrate((4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3)**3,x)
```

```
[Out] (11015*x**3 + 38753*x**2 + 17979*x + 12953)/(980000*x**4 + 784000*x**3 + 13
32800*x**2 + 470400*x + 352800) + 339*sqrt(14)*atan(5*sqrt(14)*x/14 + sqrt(
14)/14)/21952
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.88

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(3+2x+5x^2)^3}\,dx = \frac{339}{21952}\sqrt{14}\arctan\left(\frac{1}{14}\sqrt{14}(5x+1)\right) + \frac{11015x^3+38753x^2+17979x+12953}{39200(25x^4+20x^3+34x^2+12x+9)}$$

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x, algorithm="maxima")
```

```
[Out] 339/21952*sqrt(14)*arctan(1/14*sqrt(14)*(5*x + 1)) + 1/39200*(11015*x^3 + 3
8753*x^2 + 17979*x + 12953)/(25*x^4 + 20*x^3 + 34*x^2 + 12*x + 9)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.72

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(3+2x+5x^2)^3}\,dx = \frac{339}{21952}\sqrt{14}\arctan\left(\frac{1}{14}\sqrt{14}(5x+1)\right) + \frac{11015x^3+38753x^2+17979x+12953}{39200(5x^2+2x+3)^2}$$

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x, algorithm="giac")
```

```
[Out] 339/21952*sqrt(14)*arctan(1/14*sqrt(14)*(5*x + 1)) + 1/39200*(11015*x^3 + 3
8753*x^2 + 17979*x + 12953)/(5*x^2 + 2*x + 3)^2
```

Mupad [B] (verification not implemented)

Time = 13.34 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.86

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(3+2x+5x^2)^3}\,dx = \frac{339\sqrt{14}\,\mathrm{atan}\left(\frac{5\sqrt{14}\,x}{14}+\frac{\sqrt{14}}{14}\right)}{21952} + \frac{\frac{2203\,x^3}{196000}+\frac{38753\,x^2}{980000}+\frac{17979\,x}{980000}+\frac{12953}{980000}}{x^4+\frac{4\,x^3}{5}+\frac{34\,x^2}{25}+\frac{12\,x}{25}+\frac{9}{25}}$$

```
[In] int((x + 3*x^2 - 5*x^3 + 4*x^4 + 2)/(2*x + 5*x^2 + 3)^3,x)
```

```
[Out] (339*14^(1/2)*atan((5*14^(1/2)*x)/14 + 14^(1/2)/14))/21952 + ((17979*x)/980
000 + (38753*x^2)/980000 + (2203*x^3)/196000 + 12953/980000)/((12*x)/25 + (
34*x^2)/25 + (4*x^3)/5 + x^4 + 9/25)
```

3.322 $\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)\left(3+2x+5x^2\right)^3}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 329

$$
\begin{aligned}
\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)\left(3+2x+5x^2\right)^3}\,dx &= -\frac{1367d-293e+(423d-1367e)x}{1400\left(5d^2-2de+3e^2\right)\left(3+2x+5x^2\right)^2} \\
&+\frac{171735d^3-92989d^2e+36207de^2+1831e^3+25\left(2203d^3-9033d^2e+3635de^2-1829e^3\right)x}{39200\left(5d^2-2de+3e^2\right)^2\left(3+2x+5x^2\right)} \\
&+\frac{\left(42375d^5-16643d^4e+58530d^3e^2-56058d^2e^3+31811de^4-8623e^5\right)\arctan\left(\frac{1+5x}{\sqrt{14}}\right)}{1568\sqrt{14}\left(5d^2-2de+3e^2\right)^3} \\
&+\frac{e\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)\log(d+ex)}{\left(5d^2-2de+3e^2\right)^3} \\
&-\frac{e\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)\log\left(3+2x+5x^2\right)}{2\left(5d^2-2de+3e^2\right)^3}
\end{aligned}
$$

```
[Out] 1/1400*(-1367*d+293*e-(423*d-1367*e)*x)/(5*d^2-2*d*e+3*e^2)/(5*x^2+2*x+3)^2
+1/39200*(171735*d^3-92989*d^2*e+36207*d*e^2+1831*e^3+25*(2203*d^3-9033*d^2
*e+3635*d*e^2-1829*e^3)*x)/(5*d^2-2*d*e+3*e^2)^2/(5*x^2+2*x+3)+e*(4*d^4+5*d
^3*e+3*d^2*e^2-d*e^3+2*e^4)*ln(e*x+d)/(5*d^2-2*d*e+3*e^2)^3-1/2*e*(4*d^4+5*
d^3*e+3*d^2*e^2-d*e^3+2*e^4)*ln(5*x^2+2*x+3)/(5*d^2-2*d*e+3*e^2)^3+1/21952*
(42375*d^5-16643*d^4*e+58530*d^3*e^2-56058*d^2*e^3+31811*d*e^4-8623*e^5)*ar
ctan(1/14*(1+5*x)*14^(1/2))/(5*d^2-2*d*e+3*e^2)^3*14^(1/2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1660, 814, 648, 632, 210, 642}

$$\begin{aligned}
&\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)\left(3+2x+5x^2\right)^3}\,dx \\
&= \frac{\arctan\left(\frac{5x+1}{\sqrt{14}}\right)\left(42375d^5-16643d^4e+58530d^3e^2-56058d^2e^3+31811de^4-8623e^5\right)}{1568\sqrt{14}\left(5d^2-2de+3e^2\right)^3} \\
&\quad-\frac{x(423d-1367e)+1367d-293e}{1400\left(5x^2+2x+3\right)^2\left(5d^2-2de+3e^2\right)} \\
&\quad+\frac{171735d^3-92989d^2e+25x\left(2203d^3-9033d^2e+3635de^2-1829e^3\right)+36207de^2+1831e^3}{39200\left(5x^2+2x+3\right)\left(5d^2-2de+3e^2\right)^2} \\
&\quad-\frac{e\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)\log\left(5x^2+2x+3\right)}{2\left(5d^2-2de+3e^2\right)^3} \\
&\quad+\frac{e\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)\log(d+ex)}{\left(5d^2-2de+3e^2\right)^3}
\end{aligned}$$

```
[In] Int[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)*(3 + 2*x + 5*x^2)^3),x]
```

```
[Out] -1/1400*(1367*d - 293*e + (423*d - 1367*e)*x)/((5*d^2 - 2*d*e + 3*e^2)*(3 +
 2*x + 5*x^2)^2) + (171735*d^3 - 92989*d^2*e + 36207*d*e^2 + 1831*e^3 + 25*
(2203*d^3 - 9033*d^2*e + 3635*d*e^2 - 1829*e^3)*x)/(39200*(5*d^2 - 2*d*e +
3*e^2)^2*(3 + 2*x + 5*x^2)) + ((42375*d^5 - 16643*d^4*e + 58530*d^3*e^2 - 5
6058*d^2*e^3 + 31811*d*e^4 - 8623*e^5)*ArcTan[(1 + 5*x)/Sqrt[14]])/(1568*Sq
rt[14]*(5*d^2 - 2*d*e + 3*e^2)^3) + (e*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3
 + 2*e^4)*Log[d + e*x])/(5*d^2 - 2*d*e + 3*e^2)^3 - (e*(4*d^4 + 5*d^3*e + 3
*d^2*e^2 - d*e^3 + 2*e^4)*Log[3 + 2*x + 5*x^2])/(2*(5*d^2 - 2*d*e + 3*e^2)^
3)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1660

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^(p + 1)/((p
+ 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{1367d-293e+(423d-1367e)x}{1400\left(5d^2-2de+3e^2\right)\left(3+2x+5x^2\right)^2}\\&\quad+\frac{1}{112}\int\frac{\frac{2\left(3267d^2-2843de+2800e^2\right)}{25(5d^2-2de+3e^2)}-\frac{6\left(3080d^2-809de+481e^2\right)x}{25(5d^2-2de+3e^2)}+\frac{448x^2}{5}}{(d+ex)\left(3+2x+5x^2\right)^2}\,dx\\&=-\frac{1367d-293e+(423d-1367e)x}{1400\left(5d^2-2de+3e^2\right)\left(3+2x+5x^2\right)^2}\\&\quad+\frac{171735d^3-92989d^2e+36207de^2+1831e^3+25(2203d^3-9033d^2e+3635de^2-1829e^3)x}{39200\left(5d^2-2de+3e^2\right)^2\left(3+2x+5x^2\right)}\\&\quad+\frac{\int\frac{\frac{4\left(8475d^4-1193d^3e+8339d^2e^2-3397de^3+3136e^4\right)}{\left(5d^2-2de+3e^2\right)^2}+\frac{4e\left(2203d^3-9033d^2e+3635de^2-1829e^3\right)x}{\left(5d^2-2de+3e^2\right)^2}}{(d+ex)(3+2x+5x^2)}\,dx}{6272}\end{aligned}$$

$$= -\frac{1367d - 293e + (423d - 1367e)x}{1400\left(5d^2 - 2de + 3e^2\right)\left(3 + 2x + 5x^2\right)^2} + \frac{171735d^3 - 92989d^2e + 36207de^2 + 1831e^3 + 25(2203d^3 - 9033d^2e + 3635de^2 - 1829e^3)\,x}{39200\left(5d^2 - 2de + 3e^2\right)^2\left(3 + 2x + 5x^2\right)} + \frac{\int \left(\frac{6272e^2\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)}{\left(5d^2-2de+3e^2\right)^3(d+ex)} + \frac{4\left(42375d^5-22915d^4e+50690d^3e^2-60762d^2e^3+33379de^4-11759e^5-7840e\left(4d^4+5d^3e\right.\right.}{\left(5d^2-2de+3e^2\right)^3\left(3+2x+5x^2\right)} \right.}{6272}$$

$$= -\frac{1367d - 293e + (423d - 1367e)x}{1400\left(5d^2 - 2de + 3e^2\right)\left(3 + 2x + 5x^2\right)^2} + \frac{171735d^3 - 92989d^2e + 36207de^2 + 1831e^3 + 25(2203d^3 - 9033d^2e + 3635de^2 - 1829e^3)\,x}{39200\left(5d^2 - 2de + 3e^2\right)^2\left(3 + 2x + 5x^2\right)} + \frac{e(4d^4 + 5d^3e + 3d^2e^2 - de^3 + 2e^4)\log(d + ex)}{\left(5d^2 - 2de + 3e^2\right)^3} + \frac{\int \frac{42375d^5-22915d^4e+50690d^3e^2-60762d^2e^3+33379de^4-11759e^5-7840e\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)x}{3+2x+5x^2}\,dx}{1568\left(5d^2 - 2de + 3e^2\right)^3}$$

$$= -\frac{1367d - 293e + (423d - 1367e)x}{1400\left(5d^2 - 2de + 3e^2\right)\left(3 + 2x + 5x^2\right)^2} + \frac{171735d^3 - 92989d^2e + 36207de^2 + 1831e^3 + 25(2203d^3 - 9033d^2e + 3635de^2 - 1829e^3)\,x}{39200\left(5d^2 - 2de + 3e^2\right)^2\left(3 + 2x + 5x^2\right)} + \frac{e(4d^4 + 5d^3e + 3d^2e^2 - de^3 + 2e^4)\log(d + ex)}{\left(5d^2 - 2de + 3e^2\right)^3} - \frac{\left(e(4d^4 + 5d^3e + 3d^2e^2 - de^3 + 2e^4)\right)\int \frac{2+10x}{3+2x+5x^2}\,dx}{2\left(5d^2 - 2de + 3e^2\right)^3} + \frac{(42375d^5 - 16643d^4e + 58530d^3e^2 - 56058d^2e^3 + 31811de^4 - 8623e^5)\int \frac{1}{3+2x+5x^2}\,dx}{1568\left(5d^2 - 2de + 3e^2\right)^3}$$

$$= -\frac{1367d - 293e + (423d - 1367e)x}{1400\left(5d^2 - 2de + 3e^2\right)\left(3 + 2x + 5x^2\right)^2} + \frac{171735d^3 - 92989d^2e + 36207de^2 + 1831e^3 + 25(2203d^3 - 9033d^2e + 3635de^2 - 1829e^3)\,x}{39200\left(5d^2 - 2de + 3e^2\right)^2\left(3 + 2x + 5x^2\right)} + \frac{e(4d^4 + 5d^3e + 3d^2e^2 - de^3 + 2e^4)\log(d + ex)}{\left(5d^2 - 2de + 3e^2\right)^3} - \frac{e(4d^4 + 5d^3e + 3d^2e^2 - de^3 + 2e^4)\log\left(3 + 2x + 5x^2\right)}{2\left(5d^2 - 2de + 3e^2\right)^3} - \frac{(42375d^5 - 16643d^4e + 58530d^3e^2 - 56058d^2e^3 + 31811de^4 - 8623e^5)\,\mathrm{Subst}\left(\int \frac{1}{-56-x^2}\,dx, x, 2 + \right.}{784\left(5d^2 - 2de + 3e^2\right)^3}$$

$$= -\frac{1367d - 293e + (423d - 1367e)x}{1400\left(5d^2 - 2de + 3e^2\right)\left(3 + 2x + 5x^2\right)^2}$$
$$+ \frac{171735d^3 - 92989d^2e + 36207de^2 + 1831e^3 + 25(2203d^3 - 9033d^2e + 3635de^2 - 1829e^3)\,x}{39200\left(5d^2 - 2de + 3e^2\right)^2\left(3 + 2x + 5x^2\right)}$$
$$+ \frac{(42375d^5 - 16643d^4e + 58530d^3e^2 - 56058d^2e^3 + 31811de^4 - 8623e^5)\tan^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{1568\sqrt{14}\left(5d^2 - 2de + 3e^2\right)^3}$$
$$+ \frac{e(4d^4 + 5d^3e + 3d^2e^2 - de^3 + 2e^4)\log(d + ex)}{\left(5d^2 - 2de + 3e^2\right)^3}$$
$$- \frac{e(4d^4 + 5d^3e + 3d^2e^2 - de^3 + 2e^4)\log\left(3 + 2x + 5x^2\right)}{2\left(5d^2 - 2de + 3e^2\right)^3}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 282, normalized size of antiderivative = 0.86

$$\int \frac{2 + x + 3x^2 - 5x^3 + 4x^4}{(d + ex)\left(3 + 2x + 5x^2\right)^3}\,dx$$
$$= \frac{\frac{392\left(5d^2-2de+3e^2\right)^2\left(-d(1367+423x)+e(293+1367x)\right)}{\left(3+2x+5x^2\right)^2} + \frac{14\left(5d^2-2de+3e^2\right)\left(e^3(1831-45725x)+5d^3(34347+11015x)+de^2(36207+90875x)-d}{3+2x+5x^2}}{}$$

```
[In] Integrate[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)*(3 + 2*x + 5*x^2)^3),x
]
```

```
[Out] ((392*(5*d^2 - 2*d*e + 3*e^2)^2*(-(d*(1367 + 423*x)) + e*(293 + 1367*x)))/(
3 + 2*x + 5*x^2)^2 + (14*(5*d^2 - 2*d*e + 3*e^2)*(e^3*(1831 - 45725*x) + 5*
d^3*(34347 + 11015*x) + d*e^2*(36207 + 90875*x) - d^2*e*(92989 + 225825*x))
)/(3 + 2*x + 5*x^2) + 25*Sqrt[14]*(42375*d^5 - 16643*d^4*e + 58530*d^3*e^2
- 56058*d^2*e^3 + 31811*d*e^4 - 8623*e^5)*ArcTan[(1 + 5*x)/Sqrt[14]] + 5488
00*e*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*Log[d + e*x] - 274400*e*
(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*Log[3 + 2*x + 5*x^2])/(548800
*(5*d^2 - 2*d*e + 3*e^2)^3)
```

Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.09

method	result
default	$\frac{25\left(\frac{2203}{1568}d^5-\frac{49571}{7840}d^4e+\frac{4285}{784}d^3e^2-\frac{21757}{3920}d^2e^3+\frac{14563}{7840}de^4-\frac{5487}{7840}e^5\right)x^3+25\left(\frac{38753}{7840}d^5-\frac{10433}{1568}d^4e+\frac{655359}{98000}d^3e^2-\frac{388683}{98000}d^2e^3+\frac{250589}{196000}de^4-\frac{493}{196}\right.}{}$
risch	Expression too large to display

```
[In] int((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3)^3,x,method=_RETURNVERBOSE
)
```

```
[Out] 1/(5*d^2-2*d*e+3*e^2)^3*(25*((2203/1568*d^5-49571/7840*d^4*e+4285/784*d^3*e
^2-21757/3920*d^2*e^3+14563/7840*d*e^4-5487/7840*e^5)*x^3+(38753/7840*d^5-1
0433/1568*d^4*e+655359/98000*d^3*e^2-388683/98000*d^2*e^3+250589/196000*d*e
^4-49377/196000*e^5)*x^2+(17979/7840*d^5-33127/7840*d^4*e+380997/98000*d^3*
e^2-250449/98000*d^2*e^3+147247/196000*d*e^4-11211/196000*e^5)*x+12953/7840
*d^5-11637/7840*d^4*e+118119/98000*d^3*e^2-28843/98000*d^2*e^3-25611/196000
*d*e^4+18063/196000*e^5)/(5*x^2+2*x+3)^2+1/15680*(-31360*d^4*e-39200*d^3*e^
2-23520*d^2*e^3+7840*d*e^4-15680*e^5)*ln(5*x^2+2*x+3)+1/21952*(42375*d^5-16
643*d^4*e+58530*d^3*e^2-56058*d^2*e^3+31811*d*e^4-8623*e^5)*14^(1/2)*arctan
(1/28*(10*x+2)*14^(1/2)))+e*(4*d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)*ln(e*x+d)
/(5*d^2-2*d*e+3*e^2)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1052 vs. 2(318) = 636.

Time = 0.39 (sec) , antiderivative size = 1052, normalized size of antiderivative = 3.20

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)\left(3+2x+5x^2\right)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3)^3,x, algorithm="fri
cas")
```

```
[Out] 1/109760*(4533550*d^5 - 4072950*d^4*e + 3307332*d^3*e^2 - 807604*d^2*e^3 -
358554*d*e^4 + 252882*e^5 + 350*(11015*d^5 - 49571*d^4*e + 42850*d^3*e^2 -
43514*d^2*e^3 + 14563*d*e^4 - 5487*e^5)*x^3 + 14*(968825*d^5 - 1304125*d^4*
e + 1310718*d^3*e^2 - 777366*d^2*e^3 + 250589*d*e^4 - 49377*e^5)*x^2 + 5*sq
rt(14)*(381375*d^5 - 149787*d^4*e + 526770*d^3*e^2 - 504522*d^2*e^3 + 28629
9*d*e^4 - 77607*e^5 + 25*(42375*d^5 - 16643*d^4*e + 58530*d^3*e^2 - 56058*d
^2*e^3 + 31811*d*e^4 - 8623*e^5)*x^4 + 20*(42375*d^5 - 16643*d^4*e + 58530*
d^3*e^2 - 56058*d^2*e^3 + 31811*d*e^4 - 8623*e^5)*x^3 + 34*(42375*d^5 - 166
43*d^4*e + 58530*d^3*e^2 - 56058*d^2*e^3 + 31811*d*e^4 - 8623*e^5)*x^2 + 12
*(42375*d^5 - 16643*d^4*e + 58530*d^3*e^2 - 56058*d^2*e^3 + 31811*d*e^4 - 8
623*e^5)*x)*arctan(1/14*sqrt(14)*(5*x + 1)) + 14*(449475*d^5 - 828175*d^4*e
 + 761994*d^3*e^2 - 500898*d^2*e^3 + 147247*d*e^4 - 11211*e^5)*x + 109760*(
36*d^4*e + 45*d^3*e^2 + 27*d^2*e^3 - 9*d*e^4 + 18*e^5 + 25*(4*d^4*e + 5*d^3
*e^2 + 3*d^2*e^3 - d*e^4 + 2*e^5)*x^4 + 20*(4*d^4*e + 5*d^3*e^2 + 3*d^2*e^3
 - d*e^4 + 2*e^5)*x^3 + 34*(4*d^4*e + 5*d^3*e^2 + 3*d^2*e^3 - d*e^4 + 2*e^5
)*x^2 + 12*(4*d^4*e + 5*d^3*e^2 + 3*d^2*e^3 - d*e^4 + 2*e^5)*x)*log(e*x + d
) - 54880*(36*d^4*e + 45*d^3*e^2 + 27*d^2*e^3 - 9*d*e^4 + 18*e^5 + 25*(4*d^
4*e + 5*d^3*e^2 + 3*d^2*e^3 - d*e^4 + 2*e^5)*x^4 + 20*(4*d^4*e + 5*d^3*e^2
+ 3*d^2*e^3 - d*e^4 + 2*e^5)*x^3 + 34*(4*d^4*e + 5*d^3*e^2 + 3*d^2*e^3 - d*
```

```
e^4 + 2*e^5)*x^2 + 12*(4*d^4*e + 5*d^3*e^2 + 3*d^2*e^3 - d*e^4 + 2*e^5)*x)*
log(5*x^2 + 2*x + 3))/(1125*d^6 - 1350*d^5*e + 2565*d^4*e^2 - 1692*d^3*e^3
+ 1539*d^2*e^4 - 486*d*e^5 + 243*e^6 + 25*(125*d^6 - 150*d^5*e + 285*d^4*e^
2 - 188*d^3*e^3 + 171*d^2*e^4 - 54*d*e^5 + 27*e^6)*x^4 + 20*(125*d^6 - 150*
d^5*e + 285*d^4*e^2 - 188*d^3*e^3 + 171*d^2*e^4 - 54*d*e^5 + 27*e^6)*x^3 +
34*(125*d^6 - 150*d^5*e + 285*d^4*e^2 - 188*d^3*e^3 + 171*d^2*e^4 - 54*d*e^
5 + 27*e^6)*x^2 + 12*(125*d^6 - 150*d^5*e + 285*d^4*e^2 - 188*d^3*e^3 + 171
*d^2*e^4 - 54*d*e^5 + 27*e^6)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)\left(3+2x+5x^2\right)^3}\,dx = \text{Timed out}$$

```
[In] integrate((4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)/(5*x**2+2*x+3)**3,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 571, normalized size of antiderivative = 1.74

$$\begin{aligned}\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)\left(3+2x+5x^2\right)^3}\,dx &= \frac{\sqrt{14}(42375\,d^5-16643\,d^4e+58530\,d^3e^2-56058\,d^2e^3+31811\,de^4-8623\,e^5)\arctan\left(\frac{1}{14}\sqrt{14}(5\,x+1)\right)}{21952\,(125\,d^6-150\,d^5e+285\,d^4e^2-188\,d^3e^3+171\,d^2e^4-54\,de^5+27\,e^6)} \\ &+ \frac{(4\,d^4e+5\,d^3e^2+3\,d^2e^3-de^4+2\,e^5)\log(ex+d)}{125\,d^6-150\,d^5e+285\,d^4e^2-188\,d^3e^3+171\,d^2e^4-54\,de^5+27\,e^6} \\ &- \frac{(4\,d^4e+5\,d^3e^2+3\,d^2e^3-de^4+2\,e^5)\log(5\,x^2+2\,x+3)}{2\,(125\,d^6-150\,d^5e+285\,d^4e^2-188\,d^3e^3+171\,d^2e^4-54\,de^5+27\,e^6)} \\ &+ \frac{25\,(2203\,d^3-9033\,d^2e+3635\,de^2-1829\,e^3)x^3+64765\,d^3-32279\,d^2e-452}{7840\,(25\,(25\,d^4-20\,d^3e+34\,d^2e^2-12\,de^3+9\,e^4)x^4+225\,d^4-180\,d^3e+306\,d^2e^2-108\,de^3+81\,e^4+}\end{aligned}$$

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3)^3,x, algorithm="max
ima")
```

```
[Out] 1/21952*sqrt(14)*(42375*d^5 - 16643*d^4*e + 58530*d^3*e^2 - 56058*d^2*e^3 +
 31811*d*e^4 - 8623*e^5)*arctan(1/14*sqrt(14)*(5*x + 1))/(125*d^6 - 150*d^5
*e + 285*d^4*e^2 - 188*d^3*e^3 + 171*d^2*e^4 - 54*d*e^5 + 27*e^6) + (4*d^4*
e + 5*d^3*e^2 + 3*d^2*e^3 - d*e^4 + 2*e^5)*log(e*x + d)/(125*d^6 - 150*d^5*
e + 285*d^4*e^2 - 188*d^3*e^3 + 171*d^2*e^4 - 54*d*e^5 + 27*e^6) - 1/2*(4*d
```

```
^4*e + 5*d^3*e^2 + 3*d^2*e^3 - d*e^4 + 2*e^5)*log(5*x^2 + 2*x + 3)/(125*d^6
 - 150*d^5*e + 285*d^4*e^2 - 188*d^3*e^3 + 171*d^2*e^4 - 54*d*e^5 + 27*e^6)
 + 1/7840*(25*(2203*d^3 - 9033*d^2*e + 3635*d*e^2 - 1829*e^3)*x^3 + 64765*d
^3 - 32279*d^2*e - 4523*d*e^2 + 6021*e^3 + (193765*d^3 - 183319*d^2*e + 725
57*d*e^2 - 16459*e^3)*x^2 + (89895*d^3 - 129677*d^2*e + 46591*d*e^2 - 3737*
e^3)*x)/(25*(25*d^4 - 20*d^3*e + 34*d^2*e^2 - 12*d*e^3 + 9*e^4)*x^4 + 225*d
^4 - 180*d^3*e + 306*d^2*e^2 - 108*d*e^3 + 81*e^4 + 20*(25*d^4 - 20*d^3*e +
 34*d^2*e^2 - 12*d*e^3 + 9*e^4)*x^3 + 34*(25*d^4 - 20*d^3*e + 34*d^2*e^2 -
12*d*e^3 + 9*e^4)*x^2 + 12*(25*d^4 - 20*d^3*e + 34*d^2*e^2 - 12*d*e^3 + 9*e
^4)*x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 495, normalized size of antiderivative = 1.50

$$\begin{aligned}
&\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)\left(3+2x+5x^2\right)^3}\,dx \\
&= \frac{\sqrt{14}(42375\,d^5-16643\,d^4e+58530\,d^3e^2-56058\,d^2e^3+31811\,de^4-8623\,e^5)\arctan\left(\frac{1}{14}\sqrt{14}(5\,x+1)\right)}{21952\,(125\,d^6-150\,d^5e+285\,d^4e^2-188\,d^3e^3+171\,d^2e^4-54\,de^5+27\,e^6)} \\
&\quad-\frac{(4\,d^4e+5\,d^3e^2+3\,d^2e^3-de^4+2\,e^5)\log\left(5\,x^2+2\,x+3\right)}{2\,(125\,d^6-150\,d^5e+285\,d^4e^2-188\,d^3e^3+171\,d^2e^4-54\,de^5+27\,e^6)} \\
&\quad+\frac{(4\,d^4e^2+5\,d^3e^3+3\,d^2e^4-de^5+2\,e^6)\log\left(|ex+d|\right)}{125\,d^6e-150\,d^5e^2+285\,d^4e^3-188\,d^3e^4+171\,d^2e^5-54\,de^6+27\,e^7} \\
&\quad+\frac{323825\,d^5-290925\,d^4e+236238\,d^3e^2-57686\,d^2e^3-25611\,de^4+18063\,e^5+25\,(11015\,d^5-49571\,d^4e-}{}
\end{aligned}$$

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3)^3,x, algorithm="gia
c")
```

```
[Out] 1/21952*sqrt(14)*(42375*d^5 - 16643*d^4*e + 58530*d^3*e^2 - 56058*d^2*e^3 +
 31811*d*e^4 - 8623*e^5)*arctan(1/14*sqrt(14)*(5*x + 1))/(125*d^6 - 150*d^5
*e + 285*d^4*e^2 - 188*d^3*e^3 + 171*d^2*e^4 - 54*d*e^5 + 27*e^6) - 1/2*(4*
d^4*e + 5*d^3*e^2 + 3*d^2*e^3 - d*e^4 + 2*e^5)*log(5*x^2 + 2*x + 3)/(125*d^
6 - 150*d^5*e + 285*d^4*e^2 - 188*d^3*e^3 + 171*d^2*e^4 - 54*d*e^5 + 27*e^6
) + (4*d^4*e^2 + 5*d^3*e^3 + 3*d^2*e^4 - d*e^5 + 2*e^6)*log(abs(e*x + d))/(
125*d^6*e - 150*d^5*e^2 + 285*d^4*e^3 - 188*d^3*e^4 + 171*d^2*e^5 - 54*d*e^
6 + 27*e^7) + 1/7840*(323825*d^5 - 290925*d^4*e + 236238*d^3*e^2 - 57686*d^
2*e^3 - 25611*d*e^4 + 18063*e^5 + 25*(11015*d^5 - 49571*d^4*e + 42850*d^3*e
^2 - 43514*d^2*e^3 + 14563*d*e^4 - 5487*e^5)*x^3 + (968825*d^5 - 1304125*d^
4*e + 1310718*d^3*e^2 - 777366*d^2*e^3 + 250589*d*e^4 - 49377*e^5)*x^2 + (4
49475*d^5 - 828175*d^4*e + 761994*d^3*e^2 - 500898*d^2*e^3 + 147247*d*e^4 -
 11211*e^5)*x)/((5*d^2 - 2*d*e + 3*e^2)^3*(5*x^2 + 2*x + 3)^2)
```

Mupad [B] (verification not implemented)

Time = 13.84 (sec) , antiderivative size = 641, normalized size of antiderivative = 1.95

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)\left(3+2x+5x^2\right)^3}\,dx$$

$$= \frac{\frac{x\,(89895\,d^3-129677\,d^2\,e+46591\,d\,e^2-3737\,e^3)}{7840\,(25\,d^4-20\,d^3\,e+34\,d^2\,e^2-12\,d\,e^3+9\,e^4)} - \frac{-64765\,d^3+32279\,d^2\,e+4523\,d\,e^2-6021\,e^3}{7840\,(25\,d^4-20\,d^3\,e+34\,d^2\,e^2-12\,d\,e^3+9\,e^4)} + \frac{5\,x^3\,(2203\,d^3-9033\,d^2\,e+3635\,d\,e^2-1829\,e^3)}{1568\,(25\,d^4-20\,d^3\,e+34\,d^2\,e^2-12\,d\,e^3+9\,e^4)}}{25\,x^4+20\,x^3+34\,x^2+12\,x+9}$$

$$+\ln(d+e\,x)\left(\frac{4\,e}{25\,\left(5\,d^2-2\,d\,e+3\,e^2\right)}+\frac{e^2\,(205\,d+21\,e)}{125\,\left(5\,d^2-2\,d\,e+3\,e^2\right)^2} - \frac{e^4\,(458\,d-7\,e)}{125\,\left(5\,d^2-2\,d\,e+3\,e^2\right)^3}\right)$$

$$-\frac{\ln\left(x+\frac{1}{5}-\frac{\sqrt{14}\,1\mathrm{i}}{5}\right)\left(-\frac{42375\,\sqrt{14}\,d^5}{43904}+\left(\frac{16643\,\sqrt{14}}{43904}+2\mathrm{i}\right)d^4\,e+\left(-\frac{29265\,\sqrt{14}}{21952}+\frac{5}{2}\mathrm{i}\right)d^3\,e^2+\left(\frac{28029\,\sqrt{14}}{21952}+\frac{3}{2}\mathrm{i}\right)\right.}{d^6\,125\mathrm{i}-d^5\,e\,150\mathrm{i}+d^4\,e^2\,285\mathrm{i}-d^3\,e^3\,188\mathrm{i}+d^2\,e^4\,171\mathrm{i}-d\,e^5}$$

$$+\frac{\ln\left(x+\frac{1}{5}+\frac{\sqrt{14}\,1\mathrm{i}}{5}\right)\left(-\frac{42375\,\sqrt{14}\,d^5}{43904}+\left(\frac{16643\,\sqrt{14}}{43904}-2\mathrm{i}\right)d^4\,e+\left(-\frac{29265\,\sqrt{14}}{21952}-\frac{5}{2}\mathrm{i}\right)d^3\,e^2+\left(\frac{28029\,\sqrt{14}}{21952}-\frac{3}{2}\mathrm{i}\right)\right.}{d^6\,125\mathrm{i}-d^5\,e\,150\mathrm{i}+d^4\,e^2\,285\mathrm{i}-d^3\,e^3\,188\mathrm{i}+d^2\,e^4\,171\mathrm{i}-d\,e^5}$$

```
[In] int((x + 3*x^2 - 5*x^3 + 4*x^4 + 2)/((d + e*x)*(2*x + 5*x^2 + 3)^3),x)
```

```
[Out] ((x*(46591*d*e^2 - 129677*d^2*e + 89895*d^3 - 3737*e^3))/(7840*(25*d^4 - 20
*d^3*e - 12*d*e^3 + 9*e^4 + 34*d^2*e^2)) - (4523*d*e^2 + 32279*d^2*e - 6476
5*d^3 - 6021*e^3)/(7840*(25*d^4 - 20*d^3*e - 12*d*e^3 + 9*e^4 + 34*d^2*e^2)
) + (5*x^3*(3635*d*e^2 - 9033*d^2*e + 2203*d^3 - 1829*e^3))/(1568*(25*d^4 -
 20*d^3*e - 12*d*e^3 + 9*e^4 + 34*d^2*e^2)) + (x^2*(72557*d*e^2 - 183319*d^
2*e + 193765*d^3 - 16459*e^3))/(7840*(25*d^4 - 20*d^3*e - 12*d*e^3 + 9*e^4
+ 34*d^2*e^2)))/(12*x + 34*x^2 + 20*x^3 + 25*x^4 + 9) + log(d + e*x)*((4*e)
/(25*(5*d^2 - 2*d*e + 3*e^2)) + (e^2*(205*d + 21*e))/(125*(5*d^2 - 2*d*e +
3*e^2)^2) - (e^4*(458*d - 7*e))/(125*(5*d^2 - 2*d*e + 3*e^2)^3)) - (log(x -
 (14^(1/2)*1i)/5 + 1/5)*(e^5*((8623*14^(1/2))/43904 + 1i) - (42375*14^(1/2)
*d^5)/43904 + d^2*e^3*((28029*14^(1/2))/21952 + 3i/2) - d^3*e^2*((29265*14^
(1/2))/21952 - 5i/2) + d^4*e*((16643*14^(1/2))/43904 + 2i) - d*e^4*((31811*
14^(1/2))/43904 + 1i/2)))/(d^6*125i - d^5*e*150i - d*e^5*54i + e^6*27i + d^
2*e^4*171i - d^3*e^3*188i + d^4*e^2*285i) + (log(x + (14^(1/2)*1i)/5 + 1/5)
*(e^5*((8623*14^(1/2))/43904 - 1i) - (42375*14^(1/2)*d^5)/43904 + d^2*e^3*(
(28029*14^(1/2))/21952 - 3i/2) - d^3*e^2*((29265*14^(1/2))/21952 + 5i/2) +
d^4*e*((16643*14^(1/2))/43904 - 2i) - d*e^4*((31811*14^(1/2))/43904 - 1i/2)
))/(d^6*125i - d^5*e*150i - d*e^5*54i + e^6*27i + d^2*e^4*171i - d^3*e^3*18
8i + d^4*e^2*285i)
```

3.323 $\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2\left(3+2x+5x^2\right)^3}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 443

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2\left(3+2x+5x^2\right)^3}\,dx = -\frac{e(4d^4+5d^3e+3d^2e^2-de^3+2e^4)}{(5d^2-2de+3e^2)^3\,(d+ex)}$$
$$-\frac{1367d^2-586de-703e^2+(423d^2-2734de+293e^2)\,x}{280\,(5d^2-2de+3e^2)^2\,(3+2x+5x^2)^2}$$
$$+\frac{171735d^4-117284d^3e-200502d^2e^2+104428de^3-23189e^4+5(11015d^4-85924d^3e+34698d^2e^2+10}{7840\,(5d^2-2de+3e^2)^3\,(3+2x+5x^2)}$$
$$+\frac{(211875d^6+3070d^5e+209039d^4e^2-921444d^3e^3+380621d^2e^4-49586de^5-43695e^6)\arctan\left(\frac{1+5x}{\sqrt{14}}\right)}{1568\sqrt{14}\,(5d^2-2de+3e^2)^4}$$
$$+\frac{e(40d^5+83d^4e+12d^3e^2-76d^2e^3+46de^4-9e^5)\log(d+ex)}{(5d^2-2de+3e^2)^4}$$
$$-\frac{e(40d^5+83d^4e+12d^3e^2-76d^2e^3+46de^4-9e^5)\log\left(3+2x+5x^2\right)}{2\,(5d^2-2de+3e^2)^4}$$

```
[Out] -e*(4*d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)/(5*d^2-2*d*e+3*e^2)^3/(e*x+d)+1/28
0*(-1367*d^2+586*d*e+703*e^2-(423*d^2-2734*d*e+293*e^2)*x)/(5*d^2-2*d*e+3*e
^2)^2/(5*x^2+2*x+3)^2+1/7840*(171735*d^4-117284*d^3*e-200502*d^2*e^2+104428
*d*e^3-23189*e^4+5*(11015*d^4-85924*d^3*e+34698*d^2*e^2+10348*d*e^3-3589*e^
4)*x)/(5*d^2-2*d*e+3*e^2)^3/(5*x^2+2*x+3)+e*(40*d^5+83*d^4*e+12*d^3*e^2-76*
d^2*e^3+46*d*e^4-9*e^5)*ln(e*x+d)/(5*d^2-2*d*e+3*e^2)^4-1/2*e*(40*d^5+83*d^
4*e+12*d^3*e^2-76*d^2*e^3+46*d*e^4-9*e^5)*ln(5*x^2+2*x+3)/(5*d^2-2*d*e+3*e^
2)^4+1/21952*(211875*d^6+3070*d^5*e+209039*d^4*e^2-921444*d^3*e^3+380621*d^
2*e^4-49586*d*e^5-43695*e^6)*arctan(1/14*(1+5*x)*14^(1/2))/(5*d^2-2*d*e+3*e
^2)^4*14^(1/2)
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 443, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1660, 1642, 648, 632, 210, 642}

$$\begin{aligned}
&\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2(3+2x+5x^2)^3}\,dx \\
&= \frac{\arctan\left(\frac{5x+1}{\sqrt{14}}\right)(211875d^6+3070d^5e+209039d^4e^2-921444d^3e^3+380621d^2e^4-49586de^5-43695e^6)}{1568\sqrt{14}\,(5d^2-2de+3e^2)^4} \\
&\quad - \frac{x(423d^2-2734de+293e^2)+1367d^2-586de-703e^2}{280\,(5x^2+2x+3)^2\,(5d^2-2de+3e^2)^2} \\
&\quad + \frac{171735d^4-117284d^3e-200502d^2e^2+5x(11015d^4-85924d^3e+34698d^2e^2+10348de^3-3589e^4)+1}{7840\,(5x^2+2x+3)\,(5d^2-2de+3e^2)^3} \\
&\quad - \frac{e(4d^4+5d^3e+3d^2e^2-de^3+2e^4)}{(5d^2-2de+3e^2)^3\,(d+ex)} \\
&\quad - \frac{e(40d^5+83d^4e+12d^3e^2-76d^2e^3+46de^4-9e^5)\log\left(5x^2+2x+3\right)}{2\,(5d^2-2de+3e^2)^4} \\
&\quad + \frac{e(40d^5+83d^4e+12d^3e^2-76d^2e^3+46de^4-9e^5)\log(d+ex)}{(5d^2-2de+3e^2)^4}
\end{aligned}$$

```
[In] Int[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)^2*(3 + 2*x + 5*x^2)^3),x]
```

```
[Out] -((e*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4))/((5*d^2 - 2*d*e + 3*e^2
)^3*(d + e*x))) - (1367*d^2 - 586*d*e - 703*e^2 + (423*d^2 - 2734*d*e + 293
*e^2)*x)/(280*(5*d^2 - 2*d*e + 3*e^2)^2*(3 + 2*x + 5*x^2)^2) + (171735*d^4
- 117284*d^3*e - 200502*d^2*e^2 + 104428*d*e^3 - 23189*e^4 + 5*(11015*d^4 -
 85924*d^3*e + 34698*d^2*e^2 + 10348*d*e^3 - 3589*e^4)*x)/(7840*(5*d^2 - 2*
d*e + 3*e^2)^3*(3 + 2*x + 5*x^2)) + ((211875*d^6 + 3070*d^5*e + 209039*d^4*
e^2 - 921444*d^3*e^3 + 380621*d^2*e^4 - 49586*d*e^5 - 43695*e^6)*ArcTan[(1
+ 5*x)/Sqrt[14]])/(1568*Sqrt[14]*(5*d^2 - 2*d*e + 3*e^2)^4) + (e*(40*d^5 +
83*d^4*e + 12*d^3*e^2 - 76*d^2*e^3 + 46*d*e^4 - 9*e^5)*Log[d + e*x])/(5*d^2
 - 2*d*e + 3*e^2)^4 - (e*(40*d^5 + 83*d^4*e + 12*d^3*e^2 - 76*d^2*e^3 + 46*
d*e^4 - 9*e^5)*Log[3 + 2*x + 5*x^2])/(2*(5*d^2 - 2*d*e + 3*e^2)^4)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1660

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^(p + 1)/((p
+ 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\text{integral} = -\frac{1367d^2-586de-703e^2+\left(423d^2-2734de+293e^2\right)x}{280\left(5d^2-2de+3e^2\right)^2\left(3+2x+5x^2\right)^2} + \frac{1}{112}\int \frac{\frac{2\left(3267d^4-5686d^3e+7577d^2e^2-2240de^3+1680e^4\right)}{5\left(5d^2-2de+3e^2\right)^2} - \frac{4\left(4620d^4-2427d^3e+646d^2e^2-1417de^3+140e^4\right)x}{5\left(5d^2-2de+3e^2\right)^2} + \frac{2\left(5600d^4-4480d^3e+634\right.}{5\left(5d^2-\right.}}{(d+ex)^2\left(3+2x+5x^2\right)^2}$$

$$= -\frac{1367d^2 - 586de - 703e^2 + (423d^2 - 2734de + 293e^2)\,x}{280\,(5d^2 - 2de + 3e^2)^2\,(3 + 2x + 5x^2)^2} + \frac{171735d^4 - 117284d^3e - 200502d^2e^2 + 104428de^3 - 23189e^4 + 5(11015d^4 - 85924d^3e + 34698}{7840\,(5d^2 - 2de + 3e^2)^3\,(3 + 2x + 5x^2)} + \frac{\int \frac{\frac{4\left(42375d^6+5020d^5e+48810d^4e^2-77460d^3e^3+66971d^2e^4-18816de^5+9408e^6\right)}{\left(5d^2-2de+3e^2\right)^3}+\frac{8e\left(11015d^5-53780d^4e+28426d^3e^2-36692d^2e^3+15227de^4-3\right.}{\left(5d^2-2de+3e^2\right)^3}}{(d+ex)^2(3+2x+5x^2)}}{6272}$$

$$= -\frac{1367d^2 - 586de - 703e^2 + (423d^2 - 2734de + 293e^2)\,x}{280\,(5d^2 - 2de + 3e^2)^2\,(3 + 2x + 5x^2)^2} + \frac{171735d^4 - 117284d^3e - 200502d^2e^2 + 104428de^3 - 23189e^4 + 5(11015d^4 - 85924d^3e + 34698}{7840\,(5d^2 - 2de + 3e^2)^3\,(3 + 2x + 5x^2)} + \frac{\int \left(\frac{6272e^2\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)}{(5d^2-2de+3e^2)^3(d+ex)^2} - \frac{6272e^2\left(-40d^5-83d^4e-12d^3e^2+76d^2e^3-46de^4+9e^5\right)}{(5d^2-2de+3e^2)^4(d+ex)} + \frac{4(211875d^6-59650d^5e+7}{}\right.}{6272}$$

$$= -\frac{e(4d^4 + 5d^3e + 3d^2e^2 - de^3 + 2e^4)}{(5d^2 - 2de + 3e^2)^3\,(d + ex)} - \frac{1367d^2 - 586de - 703e^2 + (423d^2 - 2734de + 293e^2)\,x}{280\,(5d^2 - 2de + 3e^2)^2\,(3 + 2x + 5x^2)^2} + \frac{171735d^4 - 117284d^3e - 200502d^2e^2 + 104428de^3 - 23189e^4 + 5(11015d^4 - 85924d^3e + 34698}{7840\,(5d^2 - 2de + 3e^2)^3\,(3 + 2x + 5x^2)} + \frac{e(40d^5 + 83d^4e + 12d^3e^2 - 76d^2e^3 + 46de^4 - 9e^5)\log(d + ex)}{(5d^2 - 2de + 3e^2)^4} + \frac{\int \frac{211875d^6-59650d^5e+78895d^4e^2-940260d^3e^3+499789d^2e^4-121714de^5-29583e^6-7840e\left(40d^5+83d^4e+12d^3e^2-76d^2e^3+46de\right.}{3+2x+5x^2}}{1568\,(5d^2 - 2de + 3e^2)^4}$$

$$= -\frac{e(4d^4 + 5d^3e + 3d^2e^2 - de^3 + 2e^4)}{(5d^2 - 2de + 3e^2)^3\,(d + ex)} - \frac{1367d^2 - 586de - 703e^2 + (423d^2 - 2734de + 293e^2)\,x}{280\,(5d^2 - 2de + 3e^2)^2\,(3 + 2x + 5x^2)^2} + \frac{171735d^4 - 117284d^3e - 200502d^2e^2 + 104428de^3 - 23189e^4 + 5(11015d^4 - 85924d^3e + 34698}{7840\,(5d^2 - 2de + 3e^2)^3\,(3 + 2x + 5x^2)} + \frac{e(40d^5 + 83d^4e + 12d^3e^2 - 76d^2e^3 + 46de^4 - 9e^5)\log(d + ex)}{(5d^2 - 2de + 3e^2)^4} - \frac{(e(40d^5 + 83d^4e + 12d^3e^2 - 76d^2e^3 + 46de^4 - 9e^5))\int \frac{2+10x}{3+2x+5x^2}\,dx}{2\,(5d^2 - 2de + 3e^2)^4} + \frac{(211875d^6 + 3070d^5e + 209039d^4e^2 - 921444d^3e^3 + 380621d^2e^4 - 49586de^5 - 43695e^6)\int \frac{1}{3+2x}}{1568\,(5d^2 - 2de + 3e^2)^4}$$

$$
\begin{aligned}
&= -\frac{e(4d^4+5d^3e+3d^2e^2-de^3+2e^4)}{(5d^2-2de+3e^2)^3(d+ex)} \\
&- \frac{1367d^2-586de-703e^2+(423d^2-2734de+293e^2)x}{280(5d^2-2de+3e^2)^2(3+2x+5x^2)^2} \\
&+ \frac{171735d^4-117284d^3e-200502d^2e^2+104428de^3-23189e^4+5(11015d^4-85924d^3e+34698d}{7840(5d^2-2de+3e^2)^3(3+2x+5x^2)} \\
&+ \frac{e(40d^5+83d^4e+12d^3e^2-76d^2e^3+46de^4-9e^5)\log(d+ex)}{(5d^2-2de+3e^2)^4} \\
&- \frac{e(40d^5+83d^4e+12d^3e^2-76d^2e^3+46de^4-9e^5)\log(3+2x+5x^2)}{2(5d^2-2de+3e^2)^4} \\
&- \frac{(211875d^6+3070d^5e+209039d^4e^2-921444d^3e^3+380621d^2e^4-49586de^5-43695e^6)\,\mathrm{Subst}(\int}{784(5d^2-2de+3e^2)^4}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{e(4d^4+5d^3e+3d^2e^2-de^3+2e^4)}{(5d^2-2de+3e^2)^3(d+ex)} \\
&- \frac{1367d^2-586de-703e^2+(423d^2-2734de+293e^2)x}{280(5d^2-2de+3e^2)^2(3+2x+5x^2)^2} \\
&+ \frac{171735d^4-117284d^3e-200502d^2e^2+104428de^3-23189e^4+5(11015d^4-85924d^3e+34698d}{7840(5d^2-2de+3e^2)^3(3+2x+5x^2)} \\
&+ \frac{(211875d^6+3070d^5e+209039d^4e^2-921444d^3e^3+380621d^2e^4-49586de^5-43695e^6)\tan^{-1}(}{1568\sqrt{14}(5d^2-2de+3e^2)^4} \\
&+ \frac{e(40d^5+83d^4e+12d^3e^2-76d^2e^3+46de^4-9e^5)\log(d+ex)}{(5d^2-2de+3e^2)^4} \\
&- \frac{e(40d^5+83d^4e+12d^3e^2-76d^2e^3+46de^4-9e^5)\log(3+2x+5x^2)}{2(5d^2-2de+3e^2)^4}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 389, normalized size of antiderivative = 0.88

$$
\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2(3+2x+5x^2)^3}\,dx
$$

$$
= \frac{-\frac{109760e(5d^2-2de+3e^2)(4d^4+5d^3e+3d^2e^2-de^3+2e^4)}{d+ex} - \frac{392(5d^2-2de+3e^2)^2(e^2(-703+293x)+d^2(1367+423x)-2de(293+1367x))}{(3+2x+5x^2)^2} + 14(5}{}
$$

```
[In] Integrate[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)^2*(3 + 2*x + 5*x^2)^3)
,x]
```

```
[Out] ((-109760*e*(5*d^2 - 2*d*e + 3*e^2)*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 +
2*e^4))/(d + e*x) - (392*(5*d^2 - 2*d*e + 3*e^2)^2*(e^2*(-703 + 293*x) + d^
```

```
2*(1367 + 423*x) - 2*d*e*(293 + 1367*x)))/(3 + 2*x + 5*x^2)^2 + (14*(5*d^2
- 2*d*e + 3*e^2)*(5*d^4*(34347 + 11015*x) + 4*d*e^3*(26107 + 12935*x) - e^4
*(23189 + 17945*x) + 6*d^2*e^2*(-33417 + 28915*x) - 4*d^3*e*(29321 + 107405
*x)))/(3 + 2*x + 5*x^2) + 5*Sqrt[14]*(211875*d^6 + 3070*d^5*e + 209039*d^4*
e^2 - 921444*d^3*e^3 + 380621*d^2*e^4 - 49586*d*e^5 - 43695*e^6)*ArcTan[(1
+ 5*x)/Sqrt[14]] + 109760*e*(40*d^5 + 83*d^4*e + 12*d^3*e^2 - 76*d^2*e^3 +
46*d*e^4 - 9*e^5)*Log[d + e*x] - 54880*e*(40*d^5 + 83*d^4*e + 12*d^3*e^2 -
76*d^2*e^3 + 46*d*e^4 - 9*e^5)*Log[3 + 2*x + 5*x^2])/(109760*(5*d^2 - 2*d*e
 + 3*e^2)^4)
```

Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 473, normalized size of antiderivative = 1.07

method	result
default	$\frac{25\left(\frac{11015}{1568}d^6-\frac{45165}{784}d^5e+\frac{378383}{7840}d^4e^2-\frac{68857}{1960}d^3e^3+\frac{65453}{7840}d^2e^4+\frac{19111}{3920}de^5-\frac{10767}{7840}e^6\right)x^3+25\left(\frac{38753}{1568}d^6-\frac{183319}{3920}d^5e+\frac{504029}{39200}d^4e^2+\frac{5109}{9800}d^3e^3-\right.}{}$
risch	Expression too large to display

```
[In] int((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2/(5*x^2+2*x+3)^3,x,method=_RETURNVERBO
SE)
```

```
[Out] 1/(5*d^2-2*d*e+3*e^2)^4*(25*((11015/1568*d^6-45165/784*d^5*e+378383/7840*d^
4*e^2-68857/1960*d^3*e^3+65453/7840*d^2*e^4+19111/3920*d*e^5-10767/7840*e^6
)*x^3+(38753/1568*d^6-183319/3920*d^5*e+504029/39200*d^4*e^2+5109/9800*d^3*
e^3-795401/39200*d^2*e^4+218053/19600*d*e^5-91101/39200*e^6)*x^2+(17979/156
8*d^6-129677/3920*d^5*e+606287/39200*d^4*e^2-3993/9800*d^3*e^3-86999/7840*d
^2*e^4+208007/19600*d*e^5-14979/7840*e^6)*x+12953/1568*d^6-32279/3920*d^5*e
-379131/39200*d^4*e^2+116869/9800*d^3*e^3-530209/39200*d^2*e^4+19809/3920*d
*e^5-6309/39200*e^6)/(5*x^2+2*x+3)^2+1/15680*(-313600*d^5*e-650720*d^4*e^2-
94080*d^3*e^3+595840*d^2*e^4-360640*d*e^5+70560*e^6)*ln(5*x^2+2*x+3)+1/2195
2*(211875*d^6+3070*d^5*e+209039*d^4*e^2-921444*d^3*e^3+380621*d^2*e^4-49586
*d*e^5-43695*e^6)*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2)))+e*(40*d^5+83*d^4
*e+12*d^3*e^2-76*d^2*e^3+46*d*e^4-9*e^5)*ln(e*x+d)/(5*d^2-2*d*e+3*e^2)^4-e*
(4*d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)/(5*d^2-2*d*e+3*e^2)^3/(e*x+d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1734 vs. 2(432) = 864.

Time = 0.52 (sec) , antiderivative size = 1734, normalized size of antiderivative = 3.91

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2\left(3+2x+5x^2\right)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2/(5*x^2+2*x+3)^3,x, algorithm="f
ricas")
```

```
[Out] 1/21952*(4533550*d^7 - 8470420*d^6*e - 8666490*d^5*e^2 + 3186008*d^4*e^3 -
8213198*d^3*e^4 - 1375668*d^2*e^5 + 1294650*d*e^6 - 1185408*e^7 - 70*(10172
5*d^6*e + 584930*d^5*e^2 - 245103*d^4*e^3 + 306788*d^3*e^4 + 99187*d^2*e^5
- 93102*d*e^6 + 57807*e^7)*x^4 + 14*(275375*d^7 - 1916625*d^6*e - 474395*d^
5*e^2 - 1406231*d^4*e^3 + 222261*d^3*e^4 - 1262851*d^2*e^5 + 601791*d*e^6 -
 279261*e^7)*x^3 + 14*(968825*d^7 - 2449955*d^6*e - 1699045*d^5*e^2 - 27958
1*d^4*e^3 - 1024621*d^3*e^4 - 1118441*d^2*e^5 + 698097*d*e^6 - 394767*e^7)*
x^2 + sqrt(14)*(1906875*d^7 + 27630*d^6*e + 1881351*d^5*e^2 - 8292996*d^4*e
^3 + 3425589*d^3*e^4 - 446274*d^2*e^5 - 393255*d*e^6 + 25*(211875*d^6*e + 3
070*d^5*e^2 + 209039*d^4*e^3 - 921444*d^3*e^4 + 380621*d^2*e^5 - 49586*d*e^
6 - 43695*e^7)*x^5 + 5*(1059375*d^7 + 862850*d^6*e + 1057475*d^5*e^2 - 3771
064*d^4*e^3 - 1782671*d^3*e^4 + 1274554*d^2*e^5 - 416819*d*e^6 - 174780*e^7
)*x^4 + 2*(2118750*d^7 + 3632575*d^6*e + 2142580*d^5*e^2 - 5660777*d^4*e^3
- 11858338*d^3*e^4 + 5974697*d^2*e^5 - 1279912*d*e^6 - 742815*e^7)*x^3 + 2*
(3601875*d^7 + 1323440*d^6*e + 3572083*d^5*e^2 - 14410314*d^4*e^3 + 941893*
d^3*e^4 + 1440764*d^2*e^5 - 1040331*d*e^6 - 262170*e^7)*x^2 + 3*(847500*d^7
 + 647905*d^6*e + 845366*d^5*e^2 - 3058659*d^4*e^3 - 1241848*d^3*e^4 + 9435
19*d^2*e^5 - 323538*d*e^6 - 131085*e^7)*x)*arctan(1/14*sqrt(14)*(5*x + 1))
+ 42*(149825*d^7 - 449755*d^6*e - 12125*d^5*e^2 - 238325*d^4*e^3 - 14261*d^
3*e^4 - 169777*d^2*e^5 + 84969*d*e^6 - 39735*e^7)*x + 21952*(360*d^6*e + 74
7*d^5*e^2 + 108*d^4*e^3 - 684*d^3*e^4 + 414*d^2*e^5 - 81*d*e^6 + 25*(40*d^5
*e^2 + 83*d^4*e^3 + 12*d^3*e^4 - 76*d^2*e^5 + 46*d*e^6 - 9*e^7)*x^5 + 5*(20
0*d^6*e + 575*d^5*e^2 + 392*d^4*e^3 - 332*d^3*e^4 - 74*d^2*e^5 + 139*d*e^6
- 36*e^7)*x^4 + 2*(400*d^6*e + 1510*d^5*e^2 + 1531*d^4*e^3 - 556*d^3*e^4 -
832*d^2*e^5 + 692*d*e^6 - 153*e^7)*x^3 + 2*(680*d^6*e + 1651*d^5*e^2 + 702*
d^4*e^3 - 1220*d^3*e^4 + 326*d^2*e^5 + 123*d*e^6 - 54*e^7)*x^2 + 3*(160*d^6
*e + 452*d^5*e^2 + 297*d^4*e^3 - 268*d^3*e^4 - 44*d^2*e^5 + 102*d*e^6 - 27*
e^7)*x)*log(e*x + d) - 10976*(360*d^6*e + 747*d^5*e^2 + 108*d^4*e^3 - 684*d
^3*e^4 + 414*d^2*e^5 - 81*d*e^6 + 25*(40*d^5*e^2 + 83*d^4*e^3 + 12*d^3*e^4
- 76*d^2*e^5 + 46*d*e^6 - 9*e^7)*x^5 + 5*(200*d^6*e + 575*d^5*e^2 + 392*d^4
*e^3 - 332*d^3*e^4 - 74*d^2*e^5 + 139*d*e^6 - 36*e^7)*x^4 + 2*(400*d^6*e +
1510*d^5*e^2 + 1531*d^4*e^3 - 556*d^3*e^4 - 832*d^2*e^5 + 692*d*e^6 - 153*e
^7)*x^3 + 2*(680*d^6*e + 1651*d^5*e^2 + 702*d^4*e^3 - 1220*d^3*e^4 + 326*d^
2*e^5 + 123*d*e^6 - 54*e^7)*x^2 + 3*(160*d^6*e + 452*d^5*e^2 + 297*d^4*e^3
```

```
- 268*d^3*e^4 - 44*d^2*e^5 + 102*d*e^6 - 27*e^7)*x)*log(5*x^2 + 2*x + 3))/(
5625*d^9 - 9000*d^8*e + 18900*d^7*e^2 - 17640*d^6*e^3 + 18774*d^5*e^4 - 105
84*d^4*e^5 + 6804*d^3*e^6 - 1944*d^2*e^7 + 729*d*e^8 + 25*(625*d^8*e - 1000
*d^7*e^2 + 2100*d^6*e^3 - 1960*d^5*e^4 + 2086*d^4*e^5 - 1176*d^3*e^6 + 756*
d^2*e^7 - 216*d*e^8 + 81*e^9)*x^5 + 5*(3125*d^9 - 2500*d^8*e + 6500*d^7*e^2
 - 1400*d^6*e^3 + 2590*d^5*e^4 + 2464*d^4*e^5 - 924*d^3*e^6 + 1944*d^2*e^7
- 459*d*e^8 + 324*e^9)*x^4 + 2*(6250*d^9 + 625*d^8*e + 4000*d^7*e^2 + 16100
*d^6*e^3 - 12460*d^5*e^4 + 23702*d^4*e^5 - 12432*d^3*e^6 + 10692*d^2*e^7 -
2862*d*e^8 + 1377*e^9)*x^3 + 2*(10625*d^9 - 13250*d^8*e + 29700*d^7*e^2 - 2
0720*d^6*e^3 + 23702*d^5*e^4 - 7476*d^4*e^5 + 5796*d^3*e^6 + 864*d^2*e^7 +
81*d*e^8 + 486*e^9)*x^2 + 3*(2500*d^9 - 2125*d^8*e + 5400*d^7*e^2 - 1540*d^
6*e^3 + 2464*d^5*e^4 + 1554*d^4*e^5 - 504*d^3*e^6 + 1404*d^2*e^7 - 324*d*e^
8 + 243*e^9)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2\left(3+2x+5x^2\right)^3}\,dx=\text{Timed out}$$

```
[In] integrate((4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)**2/(5*x**2+2*x+3)**3,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 916 vs. $2(432) = 864$.

Time = 0.31 (sec) , antiderivative size = 916, normalized size of antiderivative = 2.07

$$\begin{aligned}&\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2\left(3+2x+5x^2\right)^3}\,dx\\&=\frac{\sqrt{14}(211875\,d^6+3070\,d^5e+209039\,d^4e^2-921444\,d^3e^3+380621\,d^2e^4-49586\,de^5-43695\,e^6)\arctan\left(\right.}{21952\,(625\,d^8-1000\,d^7e+2100\,d^6e^2-1960\,d^5e^3+2086\,d^4e^4-1176\,d^3e^5+756\,d^2e^6-216\,de^7+}\\&\quad+\frac{(40\,d^5e+83\,d^4e^2+12\,d^3e^3-76\,d^2e^4+46\,de^5-9\,e^6)\log\left(ex+d\right)}{625\,d^8-1000\,d^7e+2100\,d^6e^2-1960\,d^5e^3+2086\,d^4e^4-1176\,d^3e^5+756\,d^2e^6-216\,de^7+81\,e^8}\\&\quad-\frac{(40\,d^5e+83\,d^4e^2+12\,d^3e^3-76\,d^2e^4+46\,de^5-9\,e^6)\log\left(5\,x^2+2\,x+3\right)}{2\,(625\,d^8-1000\,d^7e+2100\,d^6e^2-1960\,d^5e^3+2086\,d^4e^4-1176\,d^3e^5+756\,d^2e^6-216\,de^7+81\,e^8)}\\&\quad+\frac{64765\,d^5-95100\,d^4e-200706\,d^3e^2+}{1568\,(1125\,d^7-1350\,d^6e+2565\,d^5e^2-1692\,d^4e^3+1539\,d^3e^4-486\,d^2e^5+243\,de^6+25\,(125\,d^6e-}\end{aligned}$$

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2/(5*x^2+2*x+3)^3,x, algorithm="m
axima")
```

```
[Out] 1/21952*sqrt(14)*(211875*d^6 + 3070*d^5*e + 209039*d^4*e^2 - 921444*d^3*e^3
 + 380621*d^2*e^4 - 49586*d*e^5 - 43695*e^6)*arctan(1/14*sqrt(14)*(5*x + 1)
)/(625*d^8 - 1000*d^7*e + 2100*d^6*e^2 - 1960*d^5*e^3 + 2086*d^4*e^4 - 1176
*d^3*e^5 + 756*d^2*e^6 - 216*d*e^7 + 81*e^8) + (40*d^5*e + 83*d^4*e^2 + 12*
d^3*e^3 - 76*d^2*e^4 + 46*d*e^5 - 9*e^6)*log(e*x + d)/(625*d^8 - 1000*d^7*e
 + 2100*d^6*e^2 - 1960*d^5*e^3 + 2086*d^4*e^4 - 1176*d^3*e^5 + 756*d^2*e^6
- 216*d*e^7 + 81*e^8) - 1/2*(40*d^5*e + 83*d^4*e^2 + 12*d^3*e^3 - 76*d^2*e^
4 + 46*d*e^5 - 9*e^6)*log(5*x^2 + 2*x + 3)/(625*d^8 - 1000*d^7*e + 2100*d^6
*e^2 - 1960*d^5*e^3 + 2086*d^4*e^4 - 1176*d^3*e^5 + 756*d^2*e^6 - 216*d*e^7
 + 81*e^8) + 1/1568*(64765*d^5 - 95100*d^4*e - 200706*d^3*e^2 + 22292*d^2*e
^3 + 12009*d*e^4 - 28224*e^5 - 5*(20345*d^4*e + 125124*d^3*e^2 - 11178*d^2*
e^3 - 18188*d*e^4 + 19269*e^5)*x^4 + (55075*d^5 - 361295*d^4*e - 272442*d^3
*e^2 - 173446*d^2*e^3 + 138539*d*e^4 - 93087*e^5)*x^3 + (193765*d^5 - 41248
5*d^4*e - 621062*d^3*e^2 - 56850*d^2*e^3 + 144973*d*e^4 - 131589*e^5)*x^2 +
 3*(29965*d^5 - 77965*d^4*e - 51590*d^3*e^2 - 21522*d^2*e^3 + 19493*d*e^4 -
 13245*e^5)*x)/(1125*d^7 - 1350*d^6*e + 2565*d^5*e^2 - 1692*d^4*e^3 + 1539*
d^3*e^4 - 486*d^2*e^5 + 243*d*e^6 + 25*(125*d^6*e - 150*d^5*e^2 + 285*d^4*e
^3 - 188*d^3*e^4 + 171*d^2*e^5 - 54*d*e^6 + 27*e^7)*x^5 + 5*(625*d^7 - 250*
d^6*e + 825*d^5*e^2 + 200*d^4*e^3 + 103*d^3*e^4 + 414*d^2*e^5 - 81*d*e^6 +
108*e^7)*x^4 + 2*(1250*d^7 + 625*d^6*e + 300*d^5*e^2 + 2965*d^4*e^3 - 1486*
d^3*e^4 + 2367*d^2*e^5 - 648*d*e^6 + 459*e^7)*x^3 + 2*(2125*d^7 - 1800*d^6*
e + 3945*d^5*e^2 - 1486*d^4*e^3 + 1779*d^3*e^4 + 108*d^2*e^5 + 135*d*e^6 +
162*e^7)*x^2 + 3*(500*d^7 - 225*d^6*e + 690*d^5*e^2 + 103*d^4*e^3 + 120*d^3
*e^4 + 297*d^2*e^5 - 54*d*e^6 + 81*e^7)*x)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 806, normalized size of antiderivative = 1.82

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2\left(3+2x+5x^2\right)^3}\,dx =$$

$$-\frac{\left(40\,d^5e+83\,d^4e^2+12\,d^3e^3-76\,d^2e^4+46\,de^5-9\,e^6\right)\log\left(-\frac{10\,d}{ex+d}+\frac{5\,d^2}{(ex+d)^2}+\frac{2\,e}{ex+d}-\frac{2\,de}{(ex+d)^2}+\frac{3\,e^2}{(ex+d)^2}+\right.}{2\left(625\,d^8-1000\,d^7e+2100\,d^6e^2-1960\,d^5e^3+2086\,d^4e^4-1176\,d^3e^5+756\,d^2e^6-216\,de^7+81\,e^8\right)}$$

$$-\frac{\frac{4\,d^4e^7}{ex+d}+\frac{5\,d^3e^8}{ex+d}+\frac{3\,d^2e^9}{ex+d}-\frac{de^{10}}{ex+d}+\frac{2\,e^{11}}{ex+d}}{125\,d^6e^6-150\,d^5e^7+285\,d^4e^8-188\,d^3e^9+171\,d^2e^{10}-54\,de^{11}+27\,e^{12}}$$

$$+\frac{\sqrt{14}\left(211875\,d^6e^2+3070\,d^5e^3+209039\,d^4e^4-921444\,d^3e^5+380621\,d^2e^6-49586\,de^7-43695\,e^8\right)\mathrm{arcta}}{21952\left(625\,d^8-1000\,d^7e+2100\,d^6e^2-1960\,d^5e^3+2086\,d^4e^4-1176\,d^3e^5+756\,d^2e^6-\right.}$$

$$+\frac{275375\,d^5e-3006775\,d^4e^2+1394650\,d^3e^3+1835350\,d^2e^4-734925\,de^5+17525\,e^6-\frac{5\left(165225\,d^6e^2-1997830\right.}{}}{}$$

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2/(5*x^2+2*x+3)^3,x, algorithm="g
iac")
```

```
[Out] -1/2*(40*d^5*e + 83*d^4*e^2 + 12*d^3*e^3 - 76*d^2*e^4 + 46*d*e^5 - 9*e^6)*l
og(-10*d/(e*x + d) + 5*d^2/(e*x + d)^2 + 2*e/(e*x + d) - 2*d*e/(e*x + d)^2
+ 3*e^2/(e*x + d)^2 + 5)/(625*d^8 - 1000*d^7*e + 2100*d^6*e^2 - 1960*d^5*e^
3 + 2086*d^4*e^4 - 1176*d^3*e^5 + 756*d^2*e^6 - 216*d*e^7 + 81*e^8) - (4*d^
4*e^7/(e*x + d) + 5*d^3*e^8/(e*x + d) + 3*d^2*e^9/(e*x + d) - d*e^10/(e*x +
 d) + 2*e^11/(e*x + d))/(125*d^6*e^6 - 150*d^5*e^7 + 285*d^4*e^8 - 188*d^3*
e^9 + 171*d^2*e^10 - 54*d*e^11 + 27*e^12) + 1/21952*sqrt(14)*(211875*d^6*e^
2 + 3070*d^5*e^3 + 209039*d^4*e^4 - 921444*d^3*e^5 + 380621*d^2*e^6 - 49586
*d*e^7 - 43695*e^8)*arctan(1/14*sqrt(14)*(5*d - 5*d^2/(e*x + d) + 2*d*e/(e*
x + d) - e - 3*e^2/(e*x + d))/e)/((625*d^8 - 1000*d^7*e + 2100*d^6*e^2 - 19
60*d^5*e^3 + 2086*d^4*e^4 - 1176*d^3*e^5 + 756*d^2*e^6 - 216*d*e^7 + 81*e^8
)*e^2) + 1/1568*(275375*d^5*e - 3006775*d^4*e^2 + 1394650*d^3*e^3 + 1835350
*d^2*e^4 - 734925*d*e^5 + 17525*e^6 - 5*(165225*d^6*e^2 - 1997830*d^5*e^3 +
 1218421*d^4*e^4 + 1520564*d^3*e^5 - 947049*d^2*e^6 + 93386*d*e^7 + 7963*e^
8)/((e*x + d)*e) + (826125*d^7*e^3 - 10957975*d^6*e^4 + 8449735*d^5*e^5 + 8
211175*d^4*e^6 - 7879025*d^3*e^7 + 2996315*d^2*e^8 - 443947*d*e^9 - 67267*e
^10)/((e*x + d)^2*e^2) - (275375*d^8*e^4 - 3975600*d^7*e^5 + 3752280*d^6*e^
6 + 2119880*d^5*e^7 - 3655050*d^4*e^8 + 4008480*d^3*e^9 - 1453312*d^2*e^10
- 197784*d*e^11 + 66483*e^12)/((e*x + d)^3*e^3))/((5*d^2 - 2*d*e + 3*e^2)^4
*(10*d/(e*x + d) - 5*d^2/(e*x + d)^2 - 2*e/(e*x + d) + 2*d*e/(e*x + d)^2 -
3*e^2/(e*x + d)^2 - 5)^2)
```

Mupad [B] (verification not implemented)

Time = 14.10 (sec) , antiderivative size = 965, normalized size of antiderivative = 2.18

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2\left(3+2x+5x^2\right)^3}\,dx=\text{Too large to display}$$

```
[In] int((x + 3*x^2 - 5*x^3 + 4*x^4 + 2)/((d + e*x)^2*(2*x + 5*x^2 + 3)^3),x)
```

```
[Out] log(d + e*x)*((2*e^3*(620*d - 2417*e))/(125*(5*d^2 - 2*d*e + 3*e^2)^3) - (6
*e^5*(423*d - 1367*e))/(125*(5*d^2 - 2*d*e + 3*e^2)^4) + (e*(8*d + 23*e))/(
5*(5*d^2 - 2*d*e + 3*e^2)^2)) - ((3*x*(77965*d^4*e - 19493*d*e^4 - 29965*d^
5 + 13245*e^5 + 21522*d^2*e^3 + 51590*d^3*e^2))/(1568*(125*d^6 - 150*d^5*e
- 54*d*e^5 + 27*e^6 + 171*d^2*e^4 - 188*d^3*e^3 + 285*d^4*e^2)) - (12009*d*
e^4 - 95100*d^4*e + 64765*d^5 - 28224*e^5 + 22292*d^2*e^3 - 200706*d^3*e^2)
/(1568*(125*d^6 - 150*d^5*e - 54*d*e^5 + 27*e^6 + 171*d^2*e^4 - 188*d^3*e^3
 + 285*d^4*e^2)) + (5*x^4*(20345*d^4*e - 18188*d*e^4 + 19269*e^5 - 11178*d^
2*e^3 + 125124*d^3*e^2))/(1568*(125*d^6 - 150*d^5*e - 54*d*e^5 + 27*e^6 + 1
71*d^2*e^4 - 188*d^3*e^3 + 285*d^4*e^2)) + (x^3*(361295*d^4*e - 138539*d*e^
4 - 55075*d^5 + 93087*e^5 + 173446*d^2*e^3 + 272442*d^3*e^2))/(1568*(125*d^
```

```
6 - 150*d^5*e - 54*d*e^5 + 27*e^6 + 171*d^2*e^4 - 188*d^3*e^3 + 285*d^4*e^2
)) + (x^2*(412485*d^4*e - 144973*d*e^4 - 193765*d^5 + 131589*e^5 + 56850*d^
2*e^3 + 621062*d^3*e^2))/(1568*(125*d^6 - 150*d^5*e - 54*d*e^5 + 27*e^6 + 1
71*d^2*e^4 - 188*d^3*e^3 + 285*d^4*e^2)))/(9*d + x^2*(34*d + 12*e) + x^4*(2
5*d + 20*e) + x^3*(20*d + 34*e) + 25*e*x^5 + x*(12*d + 9*e)) + (log(x - (14
^(1/2)*1i)/5 + 1/5)*((211875*14^(1/2)*d^6)/43904 - e^6*((43695*14^(1/2))/43
904 - 9i/2) - d^3*e^3*((230361*14^(1/2))/10976 + 6i) + d^4*e^2*((209039*14^
(1/2))/43904 - 83i/2) + d^2*e^4*((380621*14^(1/2))/43904 + 38i) + d^5*e*((1
535*14^(1/2))/21952 - 20i) - d*e^5*((24793*14^(1/2))/21952 + 23i)))/(d^8*62
5i - d^7*e*1000i - d*e^7*216i + e^8*81i + d^2*e^6*756i - d^3*e^5*1176i + d^
4*e^4*2086i - d^5*e^3*1960i + d^6*e^2*2100i) - (log(x + (14^(1/2)*1i)/5 + 1
/5)*((211875*14^(1/2)*d^6)/43904 - e^6*((43695*14^(1/2))/43904 + 9i/2) - d^
3*e^3*((230361*14^(1/2))/10976 - 6i) + d^4*e^2*((209039*14^(1/2))/43904 + 8
3i/2) + d^2*e^4*((380621*14^(1/2))/43904 - 38i) + d^5*e*((1535*14^(1/2))/21
952 + 20i) - d*e^5*((24793*14^(1/2))/21952 - 23i)))/(d^8*625i - d^7*e*1000i
 - d*e^7*216i + e^8*81i + d^2*e^6*756i - d^3*e^5*1176i + d^4*e^4*2086i - d^
5*e^3*1960i + d^6*e^2*2100i)
```

3.324 $\int (5+2x)\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)\,dx$

Optimal result

Integrand size = 38, antiderivative size = 143

$$\begin{aligned}
&\int (5+2x)\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)\,dx \\
&= -\frac{51435(1-4x)\sqrt{3-x+2x^2}}{32768}+\frac{11433(5+2x)^2\left(3-x+2x^2\right)^{3/2}}{4480} \\
&\quad -\frac{823(5+2x)^3\left(3-x+2x^2\right)^{3/2}}{1344}+\frac{5}{112}(5+2x)^4\left(3-x+2x^2\right)^{3/2} \\
&\quad -\frac{(1005757+295276x)\left(3-x+2x^2\right)^{3/2}}{71680}-\frac{1183005\text{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{65536\sqrt{2}}
\end{aligned}$$

```
[Out] 11433/4480*(5+2*x)^2*(2*x^2-x+3)^(3/2)-823/1344*(5+2*x)^3*(2*x^2-x+3)^(3/2)
+5/112*(5+2*x)^4*(2*x^2-x+3)^(3/2)-1/71680*(1005757+295276*x)*(2*x^2-x+3)^(
3/2)-1183005/131072*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-51435/32768*(1-4
*x)*(2*x^2-x+3)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used

$= \{1667, 793, 626, 633, 221\}$

$$\int (5+2x)\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)\,dx$$

$$= -\frac{1183005\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{65536\sqrt{2}} + \frac{5}{112}\left(2x^2-x+3\right)^{3/2}(2x+5)^4 - \frac{823(2x^2-x+3)^{3/2}(2x+5)^3}{1344} + \frac{11433(2x^2-x+3)^{3/2}(2x+5)^2}{4480} - \frac{(295276x+1005757)\left(2x^2-x+3\right)^{3/2}}{71680} - \frac{51435(1-4x)\sqrt{2x^2-x+3}}{32768}$$

```
[In] Int[(5 + 2*x)*Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4),x]
```

```
[Out] (-51435*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/32768 + (11433*(5 + 2*x)^2*(3 - x +
2*x^2)^(3/2))/4480 - (823*(5 + 2*x)^3*(3 - x + 2*x^2)^(3/2))/1344 + (5*(5 +
 2*x)^4*(3 - x + 2*x^2)^(3/2))/112 - ((1005757 + 295276*x)*(3 - x + 2*x^2)^
(3/2))/71680 - (1183005*ArcSinh[(1 - 4*x)/Sqrt[23]])/(65536*Sqrt[2])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{5}{112}(5+2x)^4\left(3-x+2x^2\right)^{3/2} \\
&\quad+\frac{1}{224}\int(5+2x)\sqrt{3-x+2x^2}\left(-3677-7826x-10788x^2-6584x^3\right)\,dx \\
&= -\frac{823(5+2x)^3\left(3-x+2x^2\right)^{3/2}}{1344}+\frac{5}{112}(5+2x)^4\left(3-x+2x^2\right)^{3/2} \\
&\quad+\frac{\int(5+2x)\sqrt{3-x+2x^2}\left(338328+907872x+1097568x^2\right)\,dx}{21504} \\
&= \frac{11433(5+2x)^2\left(3-x+2x^2\right)^{3/2}}{4480}-\frac{823(5+2x)^3\left(3-x+2x^2\right)^{3/2}}{1344} \\
&\quad+\frac{5}{112}(5+2x)^4\left(3-x+2x^2\right)^{3/2} \\
&\quad+\frac{\int(3655008-14173248x)(5+2x)\sqrt{3-x+2x^2}\,dx}{860160} \\
&= \frac{11433(5+2x)^2\left(3-x+2x^2\right)^{3/2}}{4480}-\frac{823(5+2x)^3\left(3-x+2x^2\right)^{3/2}}{1344} \\
&\quad+\frac{5}{112}(5+2x)^4\left(3-x+2x^2\right)^{3/2} \\
&\quad-\frac{(1005757+295276x)\left(3-x+2x^2\right)^{3/2}}{71680}+\frac{51435\int\sqrt{3-x+2x^2}\,dx}{4096} \\
&= -\frac{51435(1-4x)\sqrt{3-x+2x^2}}{32768}+\frac{11433(5+2x)^2\left(3-x+2x^2\right)^{3/2}}{4480} \\
&\quad-\frac{823(5+2x)^3\left(3-x+2x^2\right)^{3/2}}{1344}+\frac{5}{112}(5+2x)^4\left(3-x+2x^2\right)^{3/2} \\
&\quad-\frac{(1005757+295276x)\left(3-x+2x^2\right)^{3/2}}{71680}+\frac{1183005\int\frac{1}{\sqrt{3-x+2x^2}}\,dx}{65536}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{51435(1-4x)\sqrt{3-x+2x^2}}{32768}+\frac{11433(5+2x)^2\left(3-x+2x^2\right)^{3/2}}{4480} \\
&\quad -\frac{823(5+2x)^3\left(3-x+2x^2\right)^{3/2}}{1344} \\
&\quad +\frac{5}{112}(5+2x)^4\left(3-x+2x^2\right)^{3/2}-\frac{(1005757+295276x)\left(3-x+2x^2\right)^{3/2}}{71680} \\
&\quad +\frac{\left(51435\sqrt{\frac{23}{2}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx,x,-1+4x\right)}{65536} \\
&= -\frac{51435(1-4x)\sqrt{3-x+2x^2}}{32768}+\frac{11433(5+2x)^2\left(3-x+2x^2\right)^{3/2}}{4480} \\
&\quad -\frac{823(5+2x)^3\left(3-x+2x^2\right)^{3/2}}{1344}+\frac{5}{112}(5+2x)^4\left(3-x+2x^2\right)^{3/2} \\
&\quad -\frac{(1005757+295276x)\left(3-x+2x^2\right)^{3/2}}{71680}-\frac{1183005\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{65536\sqrt{2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.56

$$
\begin{aligned}
&\int(5+2x)\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)\,dx \\
&= \frac{4\sqrt{3-x+2x^2}(6231117+14742332x+11357024x^2+20304768x^3+1390592x^4+12984320x^5+4915200}{13762560}
\end{aligned}
$$

```
[In] Integrate[(5 + 2*x)*Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4),x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(6231117 + 14742332*x + 11357024*x^2 + 20304768*x^3
+ 1390592*x^4 + 12984320*x^5 + 4915200*x^6) - 124215525*Sqrt[2]*Log[1 - 4*x
 + 2*Sqrt[6 - 2*x + 4*x^2]])/13762560
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.42

method	result
risch	$\frac{(4915200x^6+12984320x^5+1390592x^4+20304768x^3+11357024x^2+14742332x+6231117)\sqrt{2x^2-x+3}}{3440640} + \frac{1183005\sqrt{2}\ \mathrm{arcsinh}\left(\frac{4\sqrt{23}}{}\right.}{131072}$
trager	$\left(\frac{10}{7}x^6 + \frac{317}{84}x^5 + \frac{97}{240}x^4 + \frac{52877}{8960}x^3 + \frac{50701}{15360}x^2 + \frac{3685583}{860160}x + \frac{2077039}{1146880}\right)\sqrt{2x^2-x+3} - \frac{1183005\,\mathrm{RootOf}\left(\_\right.}{}$
default	$\frac{51435(-1+4x)\sqrt{2x^2-x+3}}{32768} + \frac{1183005\sqrt{2}\ \mathrm{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{131072} + \frac{283x^2(2x^2-x+3)^{\frac{3}{2}}}{1120} - \frac{5179x(2x^2-x+3)^{\frac{3}{2}}}{17920} + \frac{242329(2x}{2150}$

```
[In] int((5+2*x)*(5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2),x,method=_RETURNVERBOSE
)
```

```
[Out] 1/3440640*(4915200*x^6+12984320*x^5+1390592*x^4+20304768*x^3+11357024*x^2+1
4742332*x+6231117)*(2*x^2-x+3)^(1/2)+1183005/131072*2^(1/2)*arcsinh(4/23*23
^(1/2)*(x-1/4))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.58

$$\int (5+2x)\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)\,dx$$
$$= \frac{1}{3440640}\left(4915200\,x^6 + 12984320\,x^5 + 1390592\,x^4 + 20304768\,x^3 + 11357024\,x^2 + 14742332\,x + 6231117\right.$$
$$+ \frac{1183005}{262144}\sqrt{2}\log\left(-4\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1) - 32\,x^2 + 16\,x - 25\right)$$

```
[In] integrate((5+2*x)*(5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2),x, algorithm="fri
cas")
```

```
[Out] 1/3440640*(4915200*x^6 + 12984320*x^5 + 1390592*x^4 + 20304768*x^3 + 113570
24*x^2 + 14742332*x + 6231117)*sqrt(2*x^2 - x + 3) + 1183005/262144*sqrt(2)
*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25)
```

Sympy [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.53

$$\int (5+2x)\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)\,dx$$
$$= \sqrt{2x^2-x+3}\cdot\left(\frac{10x^6}{7} + \frac{317x^5}{84} + \frac{97x^4}{240} + \frac{52877x^3}{8960} + \frac{50701x^2}{15360} + \frac{3685583x}{860160} + \frac{2077039}{1146880}\right)$$
$$+ \frac{1183005\sqrt{2}\,\mathrm{asinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{131072}$$

```
[In] integrate((5+2*x)*(5*x**4-x**3+3*x**2+x+2)*(2*x**2-x+3)**(1/2),x)
```

```
[Out] sqrt(2*x**2 - x + 3)*(10*x**6/7 + 317*x**5/84 + 97*x**4/240 + 52877*x**3/89
60 + 50701*x**2/15360 + 3685583*x/860160 + 2077039/1146880) + 1183005*sqrt(
2)*asinh(4*sqrt(23)*(x - 1/4)/23)/131072
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.88

$$\begin{aligned}\int (5+2x)\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)\,dx &= \frac{5}{7}\left(2\,x^{2}-x+3\right)^{\frac{3}{2}}x^{4}+\frac{377}{168}\left(2\,x^{2}-x+3\right)^{\frac{3}{2}}x^{3}+\frac{283}{1120}\left(2\,x^{2}-x+3\right)^{\frac{3}{2}}x^{2}\\ &\quad -\frac{5179}{17920}\left(2\,x^{2}-x+3\right)^{\frac{3}{2}}x+\frac{242329}{215040}\left(2\,x^{2}-x+3\right)^{\frac{3}{2}}+\frac{51435}{8192}\sqrt{2\,x^{2}-x+3}x\\ &\quad +\frac{1183005}{131072}\sqrt{2}\,\mathrm{arsinh}\left(\frac{1}{23}\sqrt{23}(4\,x-1)\right)-\frac{51435}{32768}\sqrt{2\,x^{2}-x+3}\end{aligned}$$

```
[In] integrate((5+2*x)*(5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2),x, algorithm="max
ima")
```

```
[Out] 5/7*(2*x^2 - x + 3)^(3/2)*x^4 + 377/168*(2*x^2 - x + 3)^(3/2)*x^3 + 283/112
0*(2*x^2 - x + 3)^(3/2)*x^2 - 5179/17920*(2*x^2 - x + 3)^(3/2)*x + 242329/2
15040*(2*x^2 - x + 3)^(3/2) + 51435/8192*sqrt(2*x^2 - x + 3)*x + 1183005/13
1072*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 51435/32768*sqrt(2*x^2 - x
+ 3)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.55

$$\begin{aligned}&\int (5+2x)\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)\,dx\\ &=\frac{1}{3440640}\left(4\left(8\left(4\left(16\left(20\left(120\,x+317\right)x+679\right)x+158631\right)x+354907\right)x+3685583\right)x+6231117\right)\sqrt{2\,x^{2}-}\\ &\quad -\frac{1183005}{131072}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^{2}-x+3}\right)+1\right)\end{aligned}$$

```
[In] integrate((5+2*x)*(5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2),x, algorithm="gia
c")
```

```
[Out] 1/3440640*(4*(8*(4*(16*(20*(120*x + 317)*x + 679)*x + 158631)*x + 354907)*x
 + 3685583)*x + 6231117)*sqrt(2*x^2 - x + 3) - 1183005/131072*sqrt(2)*log(-
2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)
```

Mupad [B] (verification not implemented)

Time = 1.75 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.19

$$\int (5+2x)\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)\,dx$$
$$=\frac{283\,x^2\,{\left(2\,x^2-x+3\right)}^{3/2}}{1120}+\frac{377\,x^3\,{\left(2\,x^2-x+3\right)}^{3/2}}{168}+\frac{5\,x^4\,{\left(2\,x^2-x+3\right)}^{3/2}}{7}+\frac{4478951\,\sqrt{2}\,\ln\left(\sqrt{2\,x^2-x+3}+\frac{\sqrt{2}\,\left(2\,x-\frac{1}{2}\right)}{2}\right)}{573440}+\frac{194737\,\left(\frac{x}{2}-\frac{1}{8}\right)\,\sqrt{2\,x^2-x+3}}{17920}+\frac{242329\,\sqrt{2\,x^2-x+3}\,\left(32\,x^2-4\,x+45\right)}{3440640}-\frac{5179\,x\,{\left(2\,x^2-x+3\right)}^{3/2}}{17920}+\frac{5573567\,\sqrt{2}\,\ln\left(2\,\sqrt{2\,x^2-x+3}+\frac{\sqrt{2}\,\left(4\,x-1\right)}{2}\right)}{4587520}$$

```
[In] int((2*x + 5)*(2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2),x)
```

```
[Out] (283*x^2*(2*x^2 - x + 3)^(3/2))/1120 + (377*x^3*(2*x^2 - x + 3)^(3/2))/168
+ (5*x^4*(2*x^2 - x + 3)^(3/2))/7 + (4478951*2^(1/2)*log((2*x^2 - x + 3)^(1
/2) + (2^(1/2)*(2*x - 1/2))/2))/573440 + (194737*(x/2 - 1/8)*(2*x^2 - x + 3
)^(1/2))/17920 + (242329*(2*x^2 - x + 3)^(1/2)*(32*x^2 - 4*x + 45))/3440640
 - (5179*x*(2*x^2 - x + 3)^(3/2))/17920 + (5573567*2^(1/2)*log(2*(2*x^2 - x
 + 3)^(1/2) + (2^(1/2)*(4*x - 1))/2))/4587520
```

3.325 $\int \sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 124

$$\begin{aligned}\int \sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)\,dx &= -\frac{4609(1-4x)\sqrt{3-x+2x^2}}{16384}+\frac{287\left(3-x+2x^2\right)^{3/2}}{5120}-\frac{71x\left(3-x+2x^2\right)^{3/2}}{1280}\\ &\quad+\frac{7}{80}x^2\left(3-x+2x^2\right)^{3/2}+\frac{5}{12}x^3\left(3-x+2x^2\right)^{3/2}-\frac{106007\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{32768\sqrt{2}}\end{aligned}$$

```
[Out] 287/5120*(2*x^2-x+3)^(3/2)-71/1280*x*(2*x^2-x+3)^(3/2)+7/80*x^2*(2*x^2-x+3)
^(3/2)+5/12*x^3*(2*x^2-x+3)^(3/2)-106007/65536*arcsinh(1/23*(1-4*x)*23^(1/2
))*2^(1/2)-4609/16384*(1-4*x)*(2*x^2-x+3)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {1675, 654, 626, 633, 221}

$$\begin{aligned}\int \sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)\,dx &= -\frac{106007\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{32768\sqrt{2}}+\frac{7}{80}\left(2x^2-x+3\right)^{3/2}x^2-\frac{71\left(2x^2-x+3\right)^{3/2}x}{1280}\\ &\quad+\frac{287\left(2x^2-x+3\right)^{3/2}}{5120}-\frac{4609(1-4x)\sqrt{2x^2-x+3}}{16384}+\frac{5}{12}\left(2x^2-x+3\right)^{3/2}x^3\end{aligned}$$

```
[In] Int[Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4),x]
```

```
[Out] (-4609*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/16384 + (287*(3 - x + 2*x^2)^(3/2))/5
120 - (71*x*(3 - x + 2*x^2)^(3/2))/1280 + (7*x^2*(3 - x + 2*x^2)^(3/2))/80
+ (5*x^3*(3 - x + 2*x^2)^(3/2))/12 - (106007*ArcSinh[(1 - 4*x)/Sqrt[23]])/(
32768*Sqrt[2])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{5}{12}x^3\left(3-x+2x^2\right)^{3/2}+\frac{1}{12}\int\sqrt{3-x+2x^2}\left(24+12x-9x^2+\frac{21x^3}{2}\right)\,dx\\
&= \frac{7}{80}x^2\left(3-x+2x^2\right)^{3/2}+\frac{5}{12}x^3\left(3-x+2x^2\right)^{3/2}+\frac{1}{120}\int\left(240+57x-\frac{213x^2}{4}\right)\sqrt{3-x+2x^2}\,dx
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{71x(3-x+2x^2)^{3/2}}{1280} + \frac{7}{80}x^2\left(3-x+2x^2\right)^{3/2} \\
&\quad + \frac{5}{12}x^3\left(3-x+2x^2\right)^{3/2} + \frac{1}{960}\int \left(\frac{8319}{4} + \frac{2583x}{8}\right)\sqrt{3-x+2x^2}\,dx
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{287(3-x+2x^2)^{3/2}}{5120} - \frac{71x(3-x+2x^2)^{3/2}}{1280} \\
&\quad + \frac{7}{80}x^2\left(3-x+2x^2\right)^{3/2} + \frac{5}{12}x^3\left(3-x+2x^2\right)^{3/2} + \frac{4609\int \sqrt{3-x+2x^2}\,dx}{2048}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{4609(1-4x)\sqrt{3-x+2x^2}}{16384} + \frac{287(3-x+2x^2)^{3/2}}{5120} - \frac{71x(3-x+2x^2)^{3/2}}{1280} \\
&\quad + \frac{7}{80}x^2\left(3-x+2x^2\right)^{3/2} + \frac{5}{12}x^3\left(3-x+2x^2\right)^{3/2} + \frac{106007\int \frac{1}{\sqrt{3-x+2x^2}}\,dx}{32768}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{4609(1-4x)\sqrt{3-x+2x^2}}{16384} + \frac{287(3-x+2x^2)^{3/2}}{5120} \\
&\quad - \frac{71x(3-x+2x^2)^{3/2}}{1280} + \frac{7}{80}x^2\left(3-x+2x^2\right)^{3/2} \\
&\quad + \frac{5}{12}x^3\left(3-x+2x^2\right)^{3/2} + \frac{\left(4609\sqrt{\frac{23}{2}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx, x, -1+4x\right)}{32768}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{4609(1-4x)\sqrt{3-x+2x^2}}{16384} + \frac{287(3-x+2x^2)^{3/2}}{5120} - \frac{71x(3-x+2x^2)^{3/2}}{1280} \\
&\quad + \frac{7}{80}x^2\left(3-x+2x^2\right)^{3/2} + \frac{5}{12}x^3\left(3-x+2x^2\right)^{3/2} - \frac{106007\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{32768\sqrt{2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.60

$$
\int \sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)\,dx
$$

$$
= \frac{4\sqrt{3-x+2x^2}(-27807+221868x+105696x^2+258432x^3-59392x^4+204800x^5) - 1590105\sqrt{2}\log (1 -}{983040}
$$

```
[In] Integrate[Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4),x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(-27807 + 221868*x + 105696*x^2 + 258432*x^3 - 59392
*x^4 + 204800*x^5) - 1590105*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]]
)/983040
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.44

method	result
risch	$\frac{\left(204800x^{5}-59392x^{4}+258432x^{3}+105696x^{2}+221868x-27807\right)\sqrt{2x^{2}-x+3}}{245760}+\frac{106007\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\,\left(x-\frac{1}{4}\right)}{23}\right)}{65536}$
trager	$\left(\frac{5}{6}x^{5}-\frac{29}{120}x^{4}+\frac{673}{640}x^{3}+\frac{1101}{2560}x^{2}+\frac{18489}{20480}x-\frac{9269}{81920}\right)\sqrt{2x^{2}-x+3}+\frac{106007\,\operatorname{RootOf}\left(\_Z^{2}-2\right)\ln\left(4\,\operatorname{RootOf}\left(\_\right.\right.}{}$
default	$\frac{287\left(2x^{2}-x+3\right)^{\frac{3}{2}}}{5120}+\frac{4609(-1+4x)\sqrt{2x^{2}-x+3}}{16384}+\frac{106007\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\,\left(x-\frac{1}{4}\right)}{23}\right)}{65536}+\frac{5x^{3}\left(2x^{2}-x+3\right)^{\frac{3}{2}}}{12}+\frac{7x^{2}\left(2x^{2}-x+3\right)^{\frac{3}{2}}}{80}$

```
[In] int((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/245760*(204800*x^5-59392*x^4+258432*x^3+105696*x^2+221868*x-27807)*(2*x^2
-x+3)^(1/2)+106007/65536*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.63

$$\int \sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)\,dx$$
$$=\frac{1}{245760}\left(204800\,x^{5}-59392\,x^{4}+258432\,x^{3}+105696\,x^{2}+221868\,x-27807\right)\sqrt{2\,x^{2}-x+3}$$
$$+\frac{106007}{131072}\,\sqrt{2}\log\left(-4\,\sqrt{2}\sqrt{2\,x^{2}-x+3}(4\,x-1)-32\,x^{2}+16\,x-25\right)$$

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/245760*(204800*x^5 - 59392*x^4 + 258432*x^3 + 105696*x^2 + 221868*x - 278
07)*sqrt(2*x^2 - x + 3) + 106007/131072*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 -
 x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25)
```

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.56

$$\int \sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)\,dx=\sqrt{2x^{2}-x+3}\cdot\left(\frac{5x^{5}}{6}-\frac{29x^{4}}{120}+\frac{673x^{3}}{640}\right.$$
$$\left.+\frac{1101x^{2}}{2560}+\frac{18489x}{20480}-\frac{9269}{81920}\right)$$
$$+\frac{106007\sqrt{2}\operatorname{asinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{65536}$$

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)*(2*x**2-x+3)**(1/2),x)
```

```
[Out] sqrt(2*x**2 - x + 3)*(5*x**5/6 - 29*x**4/120 + 673*x**3/640 + 1101*x**2/256
0 + 18489*x/20480 - 9269/81920) + 106007*sqrt(2)*asinh(4*sqrt(23)*(x - 1/4)
/23)/65536
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.88

$$\begin{aligned}\int \sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)\,dx &= \frac{5}{12}\left(2\,x^2-x+3\right)^{\frac{3}{2}}x^3+\frac{7}{80}\left(2\,x^2-x+3\right)^{\frac{3}{2}}x^2-\frac{71}{1280}\left(2\,x^2-x+3\right)^{\frac{3}{2}}x\\&\quad+\frac{287}{5120}\left(2\,x^2-x+3\right)^{\frac{3}{2}}+\frac{4609}{4096}\sqrt{2\,x^2-x+3}x\\&\quad+\frac{106007}{65536}\sqrt{2}\,\mathrm{arsinh}\left(\frac{1}{23}\sqrt{23}(4\,x-1)\right)-\frac{4609}{16384}\sqrt{2\,x^2-x+3}\end{aligned}$$

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2),x, algorithm="maxima")
```

```
[Out] 5/12*(2*x^2 - x + 3)^(3/2)*x^3 + 7/80*(2*x^2 - x + 3)^(3/2)*x^2 - 71/1280*(
2*x^2 - x + 3)^(3/2)*x + 287/5120*(2*x^2 - x + 3)^(3/2) + 4609/4096*sqrt(2*
x^2 - x + 3)*x + 106007/65536*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 46
09/16384*sqrt(2*x^2 - x + 3)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.59

$$\begin{aligned}\int \sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)\,dx &= \frac{1}{245760}\,(4\,(8\,(4\,(16\,(100\,x-29)x+2019)x+3303)x+55467)x-27807)\sqrt{2\,x^2-x+3}\\&\quad-\frac{106007}{65536}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)+1\right)\end{aligned}$$

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/245760*(4*(8*(4*(16*(100*x - 29)*x + 2019)*x + 3303)*x + 55467)*x - 27807
)*sqrt(2*x^2 - x + 3) - 106007/65536*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sq
rt(2*x^2 - x + 3)) + 1)
```

Mupad [B] (verification not implemented)

Time = 13.96 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.23

$$\int \sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)\,dx$$
$$=\frac{7\,x^2\,{\left(2\,x^2-x+3\right)}^{3/2}}{80}+\frac{5\,x^3\,{\left(2\,x^2-x+3\right)}^{3/2}}{12}$$
$$+\frac{63779\,\sqrt{2}\,\ln\left(\sqrt{2\,x^2-x+3}+\frac{\sqrt{2}\,\left(2\,x-\frac{1}{2}\right)}{2}\right)}{40960}+\frac{2773\,\left(\frac{x}{2}-\frac{1}{8}\right)\,\sqrt{2\,x^2-x+3}}{1280}$$
$$+\frac{287\,\sqrt{2\,x^2-x+3}\,\left(32\,x^2-4\,x+45\right)}{81920}-\frac{71\,x\,{\left(2\,x^2-x+3\right)}^{3/2}}{1280}$$
$$+\frac{19803\,\sqrt{2}\,\ln\left(2\,\sqrt{2\,x^2-x+3}+\frac{\sqrt{2}\,\left(4\,x-1\right)}{2}\right)}{327680}$$

```
[In] int((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2),x)
```

```
[Out] (7*x^2*(2*x^2 - x + 3)^(3/2))/80 + (5*x^3*(2*x^2 - x + 3)^(3/2))/12 + (6377
9*2^(1/2)*log((2*x^2 - x + 3)^(1/2) + (2^(1/2)*(2*x - 1/2))/2))/40960 + (27
73*(x/2 - 1/8)*(2*x^2 - x + 3)^(1/2))/1280 + (287*(2*x^2 - x + 3)^(1/2)*(32
*x^2 - 4*x + 45))/81920 - (71*x*(2*x^2 - x + 3)^(3/2))/1280 + (19803*2^(1/2
)*log(2*(2*x^2 - x + 3)^(1/2) + (2^(1/2)*(4*x - 1))/2))/327680
```

3.326 $\int \frac{\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)}{5+2x}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 149

$$\int \frac{\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)}{5+2x}\,dx = \frac{(489587-80844x)\sqrt{3-x+2x^2}}{4096} + \frac{4535}{768}\left(3-x+2x^2\right)^{3/2} - \frac{127}{128}(5+2x)\left(3-x+2x^2\right)^{3/2} + \frac{1}{16}(5+2x)^2\left(3-x+2x^2\right)^{3/2} + \frac{5627989\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{8192\sqrt{2}} - \frac{11001\operatorname{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{16\sqrt{2}}$$

```
[Out] 4535/768*(2*x^2-x+3)^(3/2)-127/128*(5+2*x)*(2*x^2-x+3)^(3/2)+1/16*(5+2*x)^2
*(2*x^2-x+3)^(3/2)+5627989/16384*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-110
01/32*arctanh(1/24*(17-22*x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)+1/4096*(489
587-80844*x)*(2*x^2-x+3)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {1667, 828, 857, 633, 221, 738, 212}

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{5+2x}\,dx = \frac{5627989\text{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{8192\sqrt{2}} - \frac{11001\text{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{16\sqrt{2}} + \frac{1}{16}\left(2x^2-x+3\right)^{3/2}(2x+5)^2 - \frac{127}{128}\left(2x^2-x+3\right)^{3/2}(2x+5) + \frac{4535}{768}\left(2x^2-x+3\right)^{3/2} + \frac{(489587-80844x)\sqrt{2x^2-x+3}}{4096}$$

```
[In] Int[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x),x]
```

```
[Out] ((489587 - 80844*x)*Sqrt[3 - x + 2*x^2])/4096 + (4535*(3 - x + 2*x^2)^(3/2)
)/768 - (127*(5 + 2*x)*(3 - x + 2*x^2)^(3/2))/128 + ((5 + 2*x)^2*(3 - x + 2
*x^2)^(3/2))/16 + (5627989*ArcSinh[(1 - 4*x)/Sqrt[23]])/(8192*Sqrt[2]) - (1
1001*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(16*Sqrt[2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{16}(5+2x)^2\left(3-x+2x^2\right)^{3/2}\\
&\quad+\frac{1}{160}\int\frac{\sqrt{3-x+2x^2}(-805-6490x-9300x^2-5080x^3)}{5+2x}\,dx
\end{aligned}
$$

$$= -\frac{127}{128}(5+2x)\left(3-x+2x^2\right)^{3/2} + \frac{1}{16}(5+2x)^2\left(3-x+2x^2\right)^{3/2} + \frac{\int \frac{\sqrt{3-x+2x^2}\left(-127720+824160x+725600x^2\right)}{5+2x}\,dx}{10240}$$

$$= \frac{4535}{768}\left(3-x+2x^2\right)^{3/2} - \frac{127}{128}(5+2x)\left(3-x+2x^2\right)^{3/2} + \frac{1}{16}(5+2x)^2\left(3-x+2x^2\right)^{3/2} + \frac{\int \frac{(7818720-19402560x)\sqrt{3-x+2x^2}}{5+2x}\,dx}{245760}$$

$$= \frac{(489587-80844x)\sqrt{3-x+2x^2}}{4096} + \frac{4535}{768}\left(3-x+2x^2\right)^{3/2} - \frac{127}{128}(5+2x)\left(3-x+2x^2\right)^{3/2} + \frac{1}{16}(5+2x)^2\left(3-x+2x^2\right)^{3/2} - \frac{\int \frac{-5428921920+10805738880x}{(5+2x)\sqrt{3-x+2x^2}}\,dx}{7864320}$$

$$= \frac{(489587-80844x)\sqrt{3-x+2x^2}}{4096} + \frac{4535}{768}\left(3-x+2x^2\right)^{3/2} - \frac{127}{128}(5+2x)\left(3-x+2x^2\right)^{3/2} + \frac{1}{16}(5+2x)^2\left(3-x+2x^2\right)^{3/2} - \frac{5627989\int \frac{1}{\sqrt{3-x+2x^2}}\,dx}{8192} + \frac{33003}{8}\int \frac{1}{(5+2x)\sqrt{3-x+2x^2}}\,dx$$

$$= \frac{(489587-80844x)\sqrt{3-x+2x^2}}{4096} + \frac{4535}{768}\left(3-x+2x^2\right)^{3/2} - \frac{127}{128}(5+2x)\left(3-x+2x^2\right)^{3/2} + \frac{1}{16}(5+2x)^2\left(3-x+2x^2\right)^{3/2} - \frac{33003}{4}\text{Subst}\left(\int \frac{1}{288-x^2}\,dx, x, \frac{17-22x}{\sqrt{3-x+2x^2}}\right) - \frac{5627989\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx, x, -1+4x\right)}{8192\sqrt{46}}$$

$$= \frac{(489587-80844x)\sqrt{3-x+2x^2}}{4096} + \frac{4535}{768}\left(3-x+2x^2\right)^{3/2} - \frac{127}{128}(5+2x)\left(3-x+2x^2\right)^{3/2} + \frac{1}{16}(5+2x)^2\left(3-x+2x^2\right)^{3/2} + \frac{5627989\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{8192\sqrt{2}} - \frac{11001\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{16\sqrt{2}}$$

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.69

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{5+2x}\,dx$$
$$= \frac{4\sqrt{3-x+2x^2}(1561161-300404x+79840x^2-21120x^3+6144x^4)+33795072\sqrt{2}\text{arctanh}\big(\tfrac{1}{6}(5+2x-}{49152}$$

```
[In] Integrate[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x),x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(1561161 - 300404*x + 79840*x^2 - 21120*x^3 + 6144*x
^4) + 33795072*Sqrt[2]*ArcTanh[(5 + 2*x - Sqrt[6 - 2*x + 4*x^2])/6] + 16883
967*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/49152
```

Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.85

$$-\frac{20211(4x-1)\sqrt{2x^2-x+3}}{4096}-\frac{5627989\sqrt{2}\ \text{arcsinh}\left(\frac{4\sqrt{23}\,(x-\frac{1}{4})}{23}\right)}{16384}+\frac{1925(2x^2-x+3)^{\frac{3}{2}}}{768}+\frac{x^2(2x^2-x+3)^{\frac{3}{2}}}{4}-$$

```
[In] int((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x),x)
```

```
[Out] -20211/4096*(4*x-1)*(2*x^2-x+3)^(1/2)-5627989/16384*2^(1/2)*arcsinh(4/23*23
^(1/2)*(x-1/4))+1925/768*(2*x^2-x+3)^(3/2)+1/4*x^2*(2*x^2-x+3)^(3/2)-47/64*
x*(2*x^2-x+3)^(3/2)+3667/32*(2*(x+5/2)^2-11*x-19/2)^(1/2)-11001/32*2^(1/2)*
arctanh(1/12*(17/2-11*x)*2^(1/2)/(2*(x+5/2)^2-11*x-19/2)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.84

$$\begin{aligned}
&\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{5+2x}\,dx \\
&= \frac{1}{12288}\left(6144\,x^4-21120\,x^3+79840\,x^2-300404\,x+1561161\right)\sqrt{2\,x^2-x+3} \\
&\quad+\frac{5627989}{32768}\sqrt{2}\log\left(4\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1)-32\,x^2+16\,x-25\right) \\
&\quad+\frac{11001}{64}\sqrt{2}\log\left(-\frac{24\sqrt{2}\sqrt{2\,x^2-x+3}(22\,x-17)+1060\,x^2-1036\,x+1153}{4\,x^2+20\,x+25}\right)
\end{aligned}$$

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x),x, algorithm="fri
cas")
```

```
[Out] 1/12288*(6144*x^4 - 21120*x^3 + 79840*x^2 - 300404*x + 1561161)*sqrt(2*x^2
- x + 3) + 5627989/32768*sqrt(2)*log(4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1
) - 32*x^2 + 16*x - 25) + 11001/64*sqrt(2)*log(-(24*sqrt(2)*sqrt(2*x^2 - x
+ 3)*(22*x - 17) + 1060*x^2 - 1036*x + 1153)/(4*x^2 + 20*x + 25))
```

Sympy [F]

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{5+2x}\,dx$$
$$=\int \frac{\sqrt{2x^2-x+3}\cdot(5x^4-x^3+3x^2+x+2)}{2x+5}\,dx$$

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)*(2*x**2-x+3)**(1/2)/(5+2*x),x)
```

```
[Out] Integral(sqrt(2*x**2 - x + 3)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5), x
)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.86

$$\begin{aligned}\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{5+2x}\,dx &= \frac{1}{4}\left(2\,x^2-x+3\right)^{\frac{3}{2}}x^2-\frac{47}{64}\left(2\,x^2-x+3\right)^{\frac{3}{2}}x\\&\quad+\frac{1925}{768}\left(2\,x^2-x+3\right)^{\frac{3}{2}}\\&\quad-\frac{20211}{1024}\sqrt{2\,x^2-x+3}x\\&\quad-\frac{5627989}{16384}\sqrt{2}\,\mathrm{arsinh}\left(\frac{4}{23}\sqrt{23}x-\frac{1}{23}\sqrt{23}\right)\\&\quad+\frac{11001}{32}\sqrt{2}\,\mathrm{arsinh}\left(\frac{22\sqrt{23}x}{23\,|2\,x+5|}\right.\\&\quad\left.-\frac{17\sqrt{23}}{23\,|2\,x+5|}\right)+\frac{489587}{4096}\sqrt{2\,x^2-x+3}\end{aligned}$$

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x),x, algorithm="max
ima")
```

```
[Out] 1/4*(2*x^2 - x + 3)^(3/2)*x^2 - 47/64*(2*x^2 - x + 3)^(3/2)*x + 1925/768*(2
*x^2 - x + 3)^(3/2) - 20211/1024*sqrt(2*x^2 - x + 3)*x - 5627989/16384*sqrt
```

```
(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) + 11001/32*sqrt(2)*arcsinh(22/
23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)/abs(2*x + 5)) + 489587/4096*sqr
t(2*x^2 - x + 3)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.87

$$\begin{aligned}\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{5+2x}\,dx &= \frac{1}{12288}\left(4\left(8\left(12\left(16\,x-55\right)x+2495\right)x-75101\right)x+1561161\right)\sqrt{2\,x^2-x+3} \\ &\quad+\frac{5627989}{16384}\sqrt{2}\log\left(-4\sqrt{2}x+\sqrt{2}+4\sqrt{2\,x^2-x+3}\right) \\ &\quad-\frac{11001}{32}\sqrt{2}\log\left(\left|-2\sqrt{2}x+\sqrt{2}+2\sqrt{2\,x^2-x+3}\right|\right) \\ &\quad+\frac{11001}{32}\sqrt{2}\log\left(\left|-2\sqrt{2}x-11\sqrt{2}+2\sqrt{2\,x^2-x+3}\right|\right)\end{aligned}$$

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x),x, algorithm="gia
c")
```

```
[Out] 1/12288*(4*(8*(12*(16*x - 55)*x + 2495)*x - 75101)*x + 1561161)*sqrt(2*x^2
- x + 3) + 5627989/16384*sqrt(2)*log(-4*sqrt(2)*x + sqrt(2) + 4*sqrt(2*x^2
- x + 3)) - 11001/32*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2
- x + 3))) + 11001/32*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2) + 2*sqrt(2*
x^2 - x + 3)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{5+2x}\,dx = \int \frac{\sqrt{2\,x^2-x+3}\,\left(5\,x^4-x^3+3\,x^2+x+2\right)}{2\,x+5}\,dx$$

```
[In] int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5),x)
```

```
[Out] int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5), x)
```

3.327 $\int \frac{\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^2} \, dx$

Optimal result

Integrand size = 40, antiderivative size = 149

$$
\begin{aligned}
\int \frac{\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^2} \, dx = & -\frac{(1996953-333380x)\sqrt{3-x+2x^2}}{18432} \\
& -\frac{541}{384}\left(3-x+2x^2\right)^{3/2} \\
& -\frac{3667\left(3-x+2x^2\right)^{3/2}}{576(5+2x)} \\
& +\frac{5}{64}(5+2x)\left(3-x+2x^2\right)^{3/2} \\
& -\frac{2551847\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{4096\sqrt{2}} \\
& +\frac{239201\operatorname{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{384\sqrt{2}}
\end{aligned}
$$

```
[Out] -541/384*(2*x^2-x+3)^(3/2)-3667/576*(2*x^2-x+3)^(3/2)/(5+2*x)+5/64*(5+2*x)*
(2*x^2-x+3)^(3/2)-2551847/8192*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)+23920
1/768*arctanh(1/24*(17-22*x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)-1/18432*(19
96953-333380*x)*(2*x^2-x+3)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1664, 1667, 828, 857, 633, 221, 738, 212}

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^2}\,dx = -\frac{2551847\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{4096\sqrt{2}} + \frac{239201\operatorname{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{384\sqrt{2}} + \frac{5}{64}(2x+5)\left(2x^2-x+3\right)^{3/2} - \frac{3667(2x^2-x+3)^{3/2}}{576(2x+5)} - \frac{541}{384}\left(2x^2-x+3\right)^{3/2} - \frac{(1996953-333380x)\sqrt{2x^2-x+3}}{18432}$$

```
[In] Int[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^2,x]
```

```
[Out] -1/18432*((1996953 - 333380*x)*Sqrt[3 - x + 2*x^2]) - (541*(3 - x + 2*x^2)^
(3/2))/384 - (3667*(3 - x + 2*x^2)^(3/2))/(576*(5 + 2*x)) + (5*(5 + 2*x)*(3
 - x + 2*x^2)^(3/2))/64 - (2551847*ArcSinh[(1 - 4*x)/Sqrt[23]])/(4096*Sqrt[
2]) + (239201*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(384*S
qrt[2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
```

```
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

<u>Rubi steps</u>

$$\begin{aligned}
\text{integral} &= -\frac{3667(3-x+2x^2)^{3/2}}{576(5+2x)}-\frac{1}{72}\int\frac{\sqrt{3-x+2x^2}\left(\frac{19341}{16}-\frac{6313x}{2}+486x^2-180x^3\right)}{5+2x}\,dx\\
&= -\frac{3667(3-x+2x^2)^{3/2}}{576(5+2x)}+\frac{5}{64}(5+2x)\left(3-x+2x^2\right)^{3/2}-\frac{\int\frac{\sqrt{3-x+2x^2}\left(74664-158096x+77904x^2\right)}{5+2x}\,dx}{4608}\\
&= -\frac{541}{384}\left(3-x+2x^2\right)^{3/2}-\frac{3667(3-x+2x^2)^{3/2}}{576(5+2x)}\\
&\quad+\frac{5}{64}(5+2x)\left(3-x+2x^2\right)^{3/2}-\frac{\int\frac{(2960496-8001120x)\sqrt{3-x+2x^2}}{5+2x}\,dx}{110592}\\
&= -\frac{(1996953-333380x)\sqrt{3-x+2x^2}}{18432}-\frac{541}{384}\left(3-x+2x^2\right)^{3/2}\\
&\quad-\frac{3667(3-x+2x^2)^{3/2}}{576(5+2x)}+\frac{5}{64}(5+2x)\left(3-x+2x^2\right)^{3/2}+\frac{\int\frac{-2202879456+4409591616x}{(5+2x)\sqrt{3-x+2x^2}}\,dx}{3538944}\\
&= -\frac{(1996953-333380x)\sqrt{3-x+2x^2}}{18432}-\frac{541}{384}\left(3-x+2x^2\right)^{3/2}\\
&\quad-\frac{3667(3-x+2x^2)^{3/2}}{576(5+2x)}+\frac{5}{64}(5+2x)\left(3-x+2x^2\right)^{3/2}\\
&\quad+\frac{2551847\int\frac{1}{\sqrt{3-x+2x^2}}\,dx}{4096}-\frac{239201}{64}\int\frac{1}{(5+2x)\sqrt{3-x+2x^2}}\,dx\\
&= -\frac{(1996953-333380x)\sqrt{3-x+2x^2}}{18432}-\frac{541}{384}\left(3-x+2x^2\right)^{3/2}\\
&\quad-\frac{3667(3-x+2x^2)^{3/2}}{576(5+2x)}+\frac{5}{64}(5+2x)\left(3-x+2x^2\right)^{3/2}\\
&\quad+\frac{239201}{32}\text{Subst}\left(\int\frac{1}{288-x^2}\,dx,x,\frac{17-22x}{\sqrt{3-x+2x^2}}\right)+\frac{2551847\text{Subst}\left(\int\frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx,x,-1+4x\right)}{4096\sqrt{46}}\\
&= -\frac{(1996953-333380x)\sqrt{3-x+2x^2}}{18432}-\frac{541}{384}\left(3-x+2x^2\right)^{3/2}-\frac{3667(3-x+2x^2)^{3/2}}{576(5+2x)}\\
&\quad+\frac{5}{64}(5+2x)\left(3-x+2x^2\right)^{3/2}-\frac{2551847\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{4096\sqrt{2}}+\frac{239201\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{384\sqrt{2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^2}\,dx$$
$$= \frac{\frac{4\sqrt{3-x+2x^2}(-3539439-728410x+94936x^2-17344x^3+3840x^4)}{5+2x} - 15308864\sqrt{2}\operatorname{arctanh}\bigl(\frac{1}{6}(5+2x-\sqrt{6-2x+4x^2})\bigr) - 7}{24576}$$

```
[In] Integrate[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^2,x
]
```

```
[Out] ((4*Sqrt[3 - x + 2*x^2]*(-3539439 - 728410*x + 94936*x^2 - 17344*x^3 + 3840
*x^4))/(5 + 2*x) - 15308864*Sqrt[2]*ArcTanh[(5 + 2*x - Sqrt[6 - 2*x + 4*x^2
])/6] - 7655541*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/24576
```

Maple [F(-1)]

Timed out.

hanged

```
[In] int((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^2,x)
```

```
[Out] int((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^2,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^2}\,dx$$
$$= \frac{7655541\sqrt{2}(2x+5)\log\bigl(-4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-32x^2+16x-25\bigr)+7654432\sqrt{2}(2x+5)\log\bigl(}{49}$$

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^2,x, algorithm="f
ricas")
```

```
[Out] 1/49152*(7655541*sqrt(2)*(2*x + 5)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x
- 1) - 32*x^2 + 16*x - 25) + 7654432*sqrt(2)*(2*x + 5)*log((24*sqrt(2)*sqrt
(2*x^2 - x + 3)*(22*x - 17) - 1060*x^2 + 1036*x - 1153)/(4*x^2 + 20*x + 25)
) + 8*(3840*x^4 - 17344*x^3 + 94936*x^2 - 728410*x - 3539439)*sqrt(2*x^2 -
x + 3))/(2*x + 5)
```

Sympy [F]

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^2}\,dx = \int \frac{\sqrt{2x^2-x+3}\cdot(5x^4-x^3+3x^2+x+2)}{(2x+5)^2}\,dx$$

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)*(2*x**2-x+3)**(1/2)/(5+2*x)**2,x)
```

```
[Out] Integral(sqrt(2*x**2 - x + 3)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)**2
, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^2}\,dx = \frac{5}{32}\left(2x^2-x+3\right)^{\frac{3}{2}}x - \frac{391}{384}\left(2x^2-x+3\right)^{\frac{3}{2}} + \frac{6001}{512}\sqrt{2x^2-x+3}x + \frac{2551847}{8192}\sqrt{2}\,\mathrm{arsinh}\left(\frac{4}{23}\sqrt{23}x - \frac{1}{23}\sqrt{23}\right) - \frac{239201}{768}\sqrt{2}\,\mathrm{arsinh}\left(\frac{22\sqrt{23}x}{23\,|2x+5|} - \frac{17\sqrt{23}}{23\,|2x+5|}\right) - \frac{182769}{2048}\sqrt{2x^2-x+3} - \frac{3667\sqrt{2x^2-x+3}}{32\,(2x+5)}$$

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^2,x, algorithm="m
axima")
```

```
[Out] 5/32*(2*x^2 - x + 3)^(3/2)*x - 391/384*(2*x^2 - x + 3)^(3/2) + 6001/512*sqr
t(2*x^2 - x + 3)*x + 2551847/8192*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sq
rt(23)) - 239201/768*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*
sqrt(23)/abs(2*x + 5)) - 182769/2048*sqrt(2*x^2 - x + 3) - 3667/32*sqrt(2*x
^2 - x + 3)/(2*x + 5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 531 vs. $2(118) = 236$.

Time = 0.33 (sec) , antiderivative size = 531, normalized size of antiderivative = 3.56

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^2}\,dx$$
$$= \frac{1}{24576}\sqrt{2}\left(7654432\log\left(12\sqrt{-\frac{11}{2x+5}+\frac{36}{(2x+5)^2}+1}+\frac{72}{2x+5}-11\right)\operatorname{sgn}\left(\frac{1}{2x+5}\right)+7655541\log\right.$$

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^2,x, algorithm="g
iac")
```

```
[Out] 1/24576*sqrt(2)*(7654432*log(12*sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) +
72/(2*x + 5) - 11)*sgn(1/(2*x + 5)) + 7655541*log(abs(sqrt(-11/(2*x + 5) +
36/(2*x + 5)^2 + 1) + 6/(2*x + 5) + 1))*sgn(1/(2*x + 5)) - 7655541*log(abs(
sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5) - 1))*sgn(1/(2*x + 5
)) - 1408128*sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1)*sgn(1/(2*x + 5)) + 2*
(16367883*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^7*sgn(1/
(2*x + 5)) - 34896384*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x +
5))^6*sgn(1/(2*x + 5)) - 93395*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) +
6/(2*x + 5))^5*sgn(1/(2*x + 5)) + 25574400*(sqrt(-11/(2*x + 5) + 36/(2*x +
5)^2 + 1) + 6/(2*x + 5))^4*sgn(1/(2*x + 5)) + 19752365*(sqrt(-11/(2*x + 5)
+ 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^3*sgn(1/(2*x + 5)) - 31921920*(sqrt(-1
1/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^2*sgn(1/(2*x + 5)) - 24458
13*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))*sgn(1/(2*x + 5)
) + 7663104*sgn(1/(2*x + 5)))/((sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) +
6/(2*x + 5))^2 - 1)^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^2}\,dx$$
$$= \int \frac{\sqrt{2\,x^2-x+3}\,\left(5\,x^4-x^3+3\,x^2+x+2\right)}{{\left(2\,x+5\right)}^2}\,dx$$

```
[In] int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^2,x)
```

```
[Out] int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^2, x)
```

3.328 $\int \frac{\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^3}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 151

$$\begin{aligned}\int \frac{\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^3}\,dx &= \frac{5(661065-110099x)\sqrt{3-x+2x^2}}{82944} \\ &\quad+\frac{5}{48}\left(3-x+2x^2\right)^{3/2}-\frac{3667\left(3-x+2x^2\right)^{3/2}}{1152(5+2x)^2} \\ &\quad+\frac{357391\left(3-x+2x^2\right)^{3/2}}{82944(5+2x)} \\ &\quad+\frac{117315\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{512\sqrt{2}} \\ &\quad-\frac{12670805\operatorname{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{55296\sqrt{2}}\end{aligned}$$

```
[Out] 5/48*(2*x^2-x+3)^(3/2)-3667/1152*(2*x^2-x+3)^(3/2)/(5+2*x)^2+357391/82944*(
2*x^2-x+3)^(3/2)/(5+2*x)+117315/1024*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)
-12670805/110592*arctanh(1/24*(17-22*x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)+
5/82944*(661065-110099*x)*(2*x^2-x+3)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1664, 1667, 828, 857, 633, 221, 738, 212}

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^3}\,dx = \frac{117315\text{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{512\sqrt{2}} - \frac{12670805\text{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{55296\sqrt{2}} + \frac{357391(2x^2-x+3)^{3/2}}{82944(2x+5)} - \frac{3667(2x^2-x+3)^{3/2}}{1152(2x+5)^2} + \frac{5}{48}\left(2x^2-x+3\right)^{3/2} + \frac{5(661065-110099x)\sqrt{2x^2-x+3}}{82944}$$

```
[In] Int[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^3,x]
```

```
[Out] (5*(661065 - 110099*x)*Sqrt[3 - x + 2*x^2])/82944 + (5*(3 - x + 2*x^2)^(3/2
))/48 - (3667*(3 - x + 2*x^2)^(3/2))/(1152*(5 + 2*x)^2) + (357391*(3 - x +
2*x^2)^(3/2))/(82944*(5 + 2*x)) + (117315*ArcSinh[(1 - 4*x)/Sqrt[23]])/(512
*Sqrt[2]) - (12670805*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])]
)/(55296*Sqrt[2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
```

```
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{3667(3-x+2x^2)^{3/2}}{1152(5+2x)^2} - \frac{1}{144}\int \frac{\sqrt{3-x+2x^2}\left(\frac{27681}{16}-\frac{14251x}{4}+972x^2-360x^3\right)}{(5+2x)^2}\,dx \\
&= -\frac{3667(3-x+2x^2)^{3/2}}{1152(5+2x)^2} + \frac{357391(3-x+2x^2)^{3/2}}{82944(5+2x)} + \frac{\int \frac{\sqrt{3-x+2x^2}\left(\frac{1531305}{16}-\frac{492175x}{2}+12960x^2\right)}{5+2x}\,dx}{10368} \\
&= \frac{5}{48}\left(3-x+2x^2\right)^{3/2} - \frac{3667(3-x+2x^2)^{3/2}}{1152(5+2x)^2} \\
&\quad + \frac{357391(3-x+2x^2)^{3/2}}{82944(5+2x)} + \frac{\int \frac{\left(\frac{4982715}{2}-6605940x\right)\sqrt{3-x+2x^2}}{5+2x}\,dx}{248832} \\
&= \frac{5(661065-110099x)\sqrt{3-x+2x^2}}{82944} + \frac{5}{48}\left(3-x+2x^2\right)^{3/2} \\
&\quad - \frac{3667(3-x+2x^2)^{3/2}}{1152(5+2x)^2} + \frac{357391(3-x+2x^2)^{3/2}}{82944(5+2x)} - \frac{\int \frac{-1825161120+3648965760x}{(5+2x)\sqrt{3-x+2x^2}}\,dx}{7962624} \\
&= \frac{5(661065-110099x)\sqrt{3-x+2x^2}}{82944} + \frac{5}{48}\left(3-x+2x^2\right)^{3/2} - \frac{3667(3-x+2x^2)^{3/2}}{1152(5+2x)^2} \\
&\quad + \frac{357391(3-x+2x^2)^{3/2}}{82944(5+2x)} - \frac{117315}{512}\int \frac{1}{\sqrt{3-x+2x^2}}\,dx + \frac{12670805\int \frac{1}{(5+2x)\sqrt{3-x+2x^2}}\,dx}{9216} \\
&= \frac{5(661065-110099x)\sqrt{3-x+2x^2}}{82944} + \frac{5}{48}\left(3-x+2x^2\right)^{3/2} - \frac{3667(3-x+2x^2)^{3/2}}{1152(5+2x)^2} \\
&\quad + \frac{357391(3-x+2x^2)^{3/2}}{82944(5+2x)} - \frac{12670805\,\text{Subst}\left(\int \frac{1}{288-x^2}\,dx, x, \frac{17-22x}{\sqrt{3-x+2x^2}}\right)}{4608} \\
&\quad - \frac{117315\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx, x, -1+4x\right)}{512\sqrt{46}} \\
&= \frac{5(661065-110099x)\sqrt{3-x+2x^2}}{82944} + \frac{5}{48}\left(3-x+2x^2\right)^{3/2} - \frac{3667(3-x+2x^2)^{3/2}}{1152(5+2x)^2} \\
&\quad + \frac{357391(3-x+2x^2)^{3/2}}{82944(5+2x)} + \frac{117315\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{512\sqrt{2}} - \frac{12670805\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{55296\sqrt{2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.73

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^3}\,dx = \frac{\frac{12\sqrt{3-x+2x^2}(4880551+2959330x+272520x^2-25632x^3+3840x^4)}{(5+2x)^2}+12670805\sqrt{2}\operatorname{arctanh}\left(\frac{1}{6}\left(5+2x-\sqrt{6-2x+4x^2}\right)\right)+6}{55296}$$

```
[In] Integrate[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^3,x
]
```

```
[Out] ((12*Sqrt[3 - x + 2*x^2]*(4880551 + 2959330*x + 272520*x^2 - 25632*x^3 + 38
40*x^4))/(5 + 2*x)^2 + 12670805*Sqrt[2]*ArcTanh[(5 + 2*x - Sqrt[6 - 2*x + 4
*x^2])/6] + 6335010*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/55296
```

Maple [F(-1)]

Timed out.

hanged

```
[In] int((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^3,x)
```

```
[Out] int((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^3,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.05

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^3}\,dx = \frac{12670020\,\sqrt{2}(4\,x^2+20\,x+25)\log\left(4\,\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1)-32\,x^2+16\,x-25\right)+12670805\,\sqrt{2}(4\,x^2}{}$$

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^3,x, algorithm="f
ricas")
```

```
[Out] 1/221184*(12670020*sqrt(2)*(4*x^2 + 20*x + 25)*log(4*sqrt(2)*sqrt(2*x^2 - x
 + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 12670805*sqrt(2)*(4*x^2 + 20*x + 25
)*log(-(24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) + 1060*x^2 - 1036*x + 11
53)/(4*x^2 + 20*x + 25)) + 48*(3840*x^4 - 25632*x^3 + 272520*x^2 + 2959330*
x + 4880551)*sqrt(2*x^2 - x + 3))/(4*x^2 + 20*x + 25)
```

Sympy [F]

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^3}\,dx = \int \frac{\sqrt{2x^2-x+3}\cdot(5x^4-x^3+3x^2+x+2)}{(2x+5)^3}\,dx$$

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)*(2*x**2-x+3)**(1/2)/(5+2*x)**3,x)
```

```
[Out] Integral(sqrt(2*x**2 - x + 3)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)**3
, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^3}\,dx = \frac{5}{48}\left(2x^2-x+3\right)^{\frac{3}{2}} - \frac{149}{64}\sqrt{2x^2-x+3}x - \frac{117315}{1024}\sqrt{2}\operatorname{arsinh}\left(\frac{4}{23}\sqrt{23}x - \frac{1}{23}\sqrt{23}\right) + \frac{12670805}{110592}\sqrt{2}\operatorname{arsinh}\left(\frac{22\sqrt{23}x}{23\,|2x+5|} - \frac{17\sqrt{23}}{23\,|2x+5|}\right) + \frac{3877}{144}\sqrt{2x^2-x+3} - \frac{3667\left(2x^2-x+3\right)^{\frac{3}{2}}}{1152\left(4x^2+20x+25\right)} + \frac{357391\sqrt{2x^2-x+3}}{4608\,(2x+5)}$$

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^3,x, algorithm="m
axima")
```

```
[Out] 5/48*(2*x^2 - x + 3)^(3/2) - 149/64*sqrt(2*x^2 - x + 3)*x - 117315/1024*sqr
t(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) + 12670805/110592*sqrt(2)*arc
sinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)/abs(2*x + 5)) + 3877/14
4*sqrt(2*x^2 - x + 3) - 3667/1152*(2*x^2 - x + 3)^(3/2)/(4*x^2 + 20*x + 25)
 + 357391/4608*sqrt(2*x^2 - x + 3)/(2*x + 5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 258 vs. $2(120) = 240$.

Time = 0.30 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.71

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^3}\,dx = \frac{1}{768}\,(4\,(40\,x-467)x+19695)\sqrt{2\,x^2-x+3}$$

$$+\frac{117315}{1024}\,\sqrt{2}\log\left(-2\,\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)+1\right)$$

$$-\frac{12670805}{110592}\,\sqrt{2}\log\left(\left|-2\,\sqrt{2}x+\sqrt{2}+2\,\sqrt{2\,x^2-x+3}\right|\right)$$

$$+\frac{12670805}{110592}\,\sqrt{2}\log\left(\left|-2\,\sqrt{2}x-11\,\sqrt{2}+2\,\sqrt{2\,x^2-x+3}\right|\right)$$

$$+\frac{\sqrt{2}(10693526\,\sqrt{2}(\sqrt{2}x-\sqrt{2\,x^2-x+3})^3+79895946\,(\sqrt{2}x-\sqrt{2\,x^2-x+3})^2-124044603\,\sqrt{2}(\sqrt{2}x}{9216\,\left(2\,(\sqrt{2}x-\sqrt{2\,x^2-x+3})^2+10\,\sqrt{2}(\sqrt{2}x-\sqrt{2\,x^2-x+3})-11\right)^2}$$

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^3,x, algorithm="g
iac")
```

```
[Out] 1/768*(4*(40*x - 467)*x + 19695)*sqrt(2*x^2 - x + 3) + 117315/1024*sqrt(2)*
log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) - 12670805/110592*sqr
t(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 - x + 3))) + 12670805/11
0592*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2) + 2*sqrt(2*x^2 - x + 3))) +
1/9216*sqrt(2)*(10693526*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^3 + 7989
5946*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 - 124044603*sqrt(2)*(sqrt(2)*x - s
qrt(2*x^2 - x + 3)) + 80334011)/(2*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 10
*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) - 11)^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^3}\,dx$$
$$=\int \frac{\sqrt{2\,x^2-x+3}\,(5\,x^4-x^3+3\,x^2+x+2)}{{(2\,x+5)}^3}\,dx$$

```
[In] int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^3,x)
```

```
[Out] int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^3, x)
```

3.329 $\int \frac{\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^4}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 158

$$
\begin{aligned}
\int \frac{\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^4}\,dx
&= -\frac{(44378877-7400779x)\sqrt{3-x+2x^2}}{5971968}-\frac{3667\left(3-x+2x^2\right)^{3/2}}{1728(5+2x)^3}\\
&\quad+\frac{158527\left(3-x+2x^2\right)^{3/2}}{82944(5+2x)^2}-\frac{6467659\left(3-x+2x^2\right)^{3/2}}{5971968(5+2x)}\\
&\quad-\frac{10939\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{256\sqrt{2}}+\frac{170114729\operatorname{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{3981312\sqrt{2}}
\end{aligned}
$$

```
[Out] -3667/1728*(2*x^2-x+3)^(3/2)/(5+2*x)^3+158527/82944*(2*x^2-x+3)^(3/2)/(5+2*
x)^2-6467659/5971968*(2*x^2-x+3)^(3/2)/(5+2*x)-10939/512*arcsinh(1/23*(1-4*
x)*23^(1/2))*2^(1/2)+170114729/7962624*arctanh(1/24*(17-22*x)*2^(1/2)/(2*x^
2-x+3)^(1/2))*2^(1/2)-1/5971968*(44378877-7400779*x)*(2*x^2-x+3)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used

$= \{1664, 828, 857, 633, 221, 738, 212\}$

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^4}\,dx$$
$$= -\frac{10939\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{256\sqrt{2}} + \frac{170114729\operatorname{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{3981312\sqrt{2}}$$
$$- \frac{6467659(2x^2-x+3)^{3/2}}{5971968(2x+5)} + \frac{158527(2x^2-x+3)^{3/2}}{82944(2x+5)^2}$$
$$- \frac{3667(2x^2-x+3)^{3/2}}{1728(2x+5)^3} - \frac{(44378877-7400779x)\sqrt{2x^2-x+3}}{5971968}$$

```
[In] Int[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^4,x]
```

```
[Out] -1/5971968*((44378877 - 7400779*x)*Sqrt[3 - x + 2*x^2]) - (3667*(3 - x + 2*
x^2)^(3/2))/(1728*(5 + 2*x)^3) + (158527*(3 - x + 2*x^2)^(3/2))/(82944*(5 +
 2*x)^2) - (6467659*(3 - x + 2*x^2)^(3/2))/(5971968*(5 + 2*x)) - (10939*Arc
Sinh[(1 - 4*x)/Sqrt[23]])/(256*Sqrt[2]) + (170114729*ArcTanh[(17 - 22*x)/(1
2*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(3981312*Sqrt[2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

integral

$$= -\frac{3667(3-x+2x^2)^{3/2}}{1728(5+2x)^3} - \frac{1}{216}\int \frac{\sqrt{3-x+2x^2}\left(\frac{36021}{16}-3969x+1458x^2-540x^3\right)}{(5+2x)^3}\,dx$$

$$= -\frac{3667(3-x+2x^2)^{3/2}}{1728(5+2x)^3} + \frac{158527(3-x+2x^2)^{3/2}}{82944(5+2x)^2} + \frac{\int \frac{\sqrt{3-x+2x^2}\left(\frac{2672127}{16}-\frac{1284285x}{4}+38880x^2\right)}{(5+2x)^2}\,dx}{31104}$$

$$= -\frac{3667(3-x+2x^2)^{3/2}}{1728(5+2x)^3} + \frac{158527(3-x+2x^2)^{3/2}}{82944(5+2x)^2} - \frac{6467659(3-x+2x^2)^{3/2}}{5971968(5+2x)} - \frac{\int \frac{\left(\frac{66297447}{16}-\frac{22202337x}{2}\right)\sqrt{3-x+2x^2}}{5+2x}\,dx}{2239488}$$

$$= -\frac{(44378877-7400779x)\sqrt{3-x+2x^2}}{5971968}-\frac{3667\left(3-x+2x^2\right)^{3/2}}{1728(5+2x)^3}$$
$$+\frac{158527\left(3-x+2x^2\right)^{3/2}}{82944(5+2x)^2}-\frac{6467659\left(3-x+2x^2\right)^{3/2}}{5971968(5+2x)}+\frac{\int\frac{-3061291212+6124439808x}{(5+2x)\sqrt{3-x+2x^2}}\,dx}{71663616}$$

$$= -\frac{(44378877-7400779x)\sqrt{3-x+2x^2}}{5971968}-\frac{3667\left(3-x+2x^2\right)^{3/2}}{1728(5+2x)^3}+\frac{158527\left(3-x+2x^2\right)^{3/2}}{82944(5+2x)^2}$$
$$-\frac{6467659\left(3-x+2x^2\right)^{3/2}}{5971968(5+2x)}+\frac{10939}{256}\int\frac{1}{\sqrt{3-x+2x^2}}\,dx-\frac{170114729\int\frac{1}{(5+2x)\sqrt{3-x+2x^2}}\,dx}{663552}$$

$$= -\frac{(44378877-7400779x)\sqrt{3-x+2x^2}}{5971968}-\frac{3667\left(3-x+2x^2\right)^{3/2}}{1728(5+2x)^3}$$
$$+\frac{158527\left(3-x+2x^2\right)^{3/2}}{82944(5+2x)^2}-\frac{6467659\left(3-x+2x^2\right)^{3/2}}{5971968(5+2x)}$$
$$+\frac{170114729\text{Subst}\left(\int\frac{1}{288-x^2}\,dx,x,\frac{17-22x}{\sqrt{3-x+2x^2}}\right)}{331776}+\frac{10939\text{Subst}\left(\int\frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx,x,-1+4x\right)}{256\sqrt{46}}$$

$$= -\frac{(44378877-7400779x)\sqrt{3-x+2x^2}}{5971968}-\frac{3667\left(3-x+2x^2\right)^{3/2}}{1728(5+2x)^3}+\frac{158527\left(3-x+2x^2\right)^{3/2}}{82944(5+2x)^2}$$
$$-\frac{6467659\left(3-x+2x^2\right)^{3/2}}{5971968(5+2x)}-\frac{10939\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{256\sqrt{2}}+\frac{170114729\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{3981312\sqrt{2}}$$

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.70

$$\int\frac{\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^4}\,dx$$
$$=\frac{\frac{12\sqrt{3-x+2x^2}\left(-327735797-329667508x-97682900x^2-5453568x^3+414720x^4\right)}{(5+2x)^3}-170114729\sqrt{2}\text{arctanh}\left(\frac{1}{6}\left(5+2x-\sqrt{6-2x+}\right.\right.}{3981312}$$

```
[In] Integrate[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^4,x
]
```

```
[Out] ((12*Sqrt[3 - x + 2*x^2]*(-327735797 - 329667508*x - 97682900*x^2 - 5453568
*x^3 + 414720*x^4))/(5 + 2*x)^3 - 170114729*Sqrt[2]*ArcTanh[(5 + 2*x - Sqrt
[6 - 2*x + 4*x^2])/6] - 85061664*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x
^2]])/3981312
```

Maple [F(-1)]

Timed out.

hanged

```
[In] int((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^4,x)
```

```
[Out] int((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^4,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^4}\,dx$$

$$= \frac{170123328\sqrt{2}(8x^3+60x^2+150x+125)\log\left(-4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-32x^2+16x-25\right)+170}{}$$

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^4,x, algorithm="f
ricas")
```

```
[Out] 1/15925248*(170123328*sqrt(2)*(8*x^3 + 60*x^2 + 150*x + 125)*log(-4*sqrt(2)
*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 170114729*sqrt(2)*(8
*x^3 + 60*x^2 + 150*x + 125)*log((24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17
) - 1060*x^2 + 1036*x - 1153)/(4*x^2 + 20*x + 25)) + 48*(414720*x^4 - 54535
68*x^3 - 97682900*x^2 - 329667508*x - 327735797)*sqrt(2*x^2 - x + 3))/(8*x^
3 + 60*x^2 + 150*x + 125)
```

Sympy [F]

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^4}\,dx = \int \frac{\sqrt{2x^2-x+3}\cdot(5x^4-x^3+3x^2+x+2)}{(2x+5)^4}\,dx$$

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)*(2*x**2-x+3)**(1/2)/(5+2*x)**4,x)
```

```
[Out] Integral(sqrt(2*x**2 - x + 3)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)**4
, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.01

$$\begin{aligned}&\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^4}\,dx\\&=\frac{5}{32}\sqrt{2\,x^2-x+3}x+\frac{10939}{512}\sqrt{2}\operatorname{arsinh}\left(\frac{4}{23}\sqrt{23}x-\frac{1}{23}\sqrt{23}\right)\\&\quad-\frac{170114729}{7962624}\sqrt{2}\operatorname{arsinh}\left(\frac{22\sqrt{23}x}{23\,|2\,x+5|}-\frac{17\sqrt{23}}{23\,|2\,x+5|}\right)-\frac{693775}{165888}\sqrt{2\,x^2-x+3}\\&\quad-\frac{3667\,(2\,x^2-x+3)^{\frac{3}{2}}}{1728\,(8\,x^3+60\,x^2+150\,x+125)}+\frac{158527\,(2\,x^2-x+3)^{\frac{3}{2}}}{82944\,(4\,x^2+20\,x+25)}-\frac{6467659\sqrt{2\,x^2-x+3}}{331776\,(2\,x+5)}\end{aligned}$$

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^4,x, algorithm="m
axima")
```

```
[Out] 5/32*sqrt(2*x^2 - x + 3)*x + 10939/512*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/
23*sqrt(23)) - 170114729/7962624*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x +
 5) - 17/23*sqrt(23)/abs(2*x + 5)) - 693775/165888*sqrt(2*x^2 - x + 3) - 36
67/1728*(2*x^2 - x + 3)^(3/2)/(8*x^3 + 60*x^2 + 150*x + 125) + 158527/82944
*(2*x^2 - x + 3)^(3/2)/(4*x^2 + 20*x + 25) - 6467659/331776*sqrt(2*x^2 - x
+ 3)/(2*x + 5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 304 vs. 2(127) = 254.

Time = 0.30 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.92

$$\begin{aligned}&\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^4}\,dx\\&=\frac{1}{128}\sqrt{2\,x^2-x+3}(20\,x-413)-\frac{10939}{512}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)+1\right)\\&\quad+\frac{170114729}{7962624}\sqrt{2}\log\left(\left|-2\sqrt{2}x+\sqrt{2}+2\sqrt{2\,x^2-x+3}\right|\right)\\&\quad-\frac{170114729}{7962624}\sqrt{2}\log\left(\left|-2\sqrt{2}x-11\sqrt{2}+2\sqrt{2\,x^2-x+3}\right|\right)\\&\quad-\frac{\sqrt{2}\Big(575810908\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)^5+9206213116\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)^4+9688786604\sqrt{2}(\sqrt{}}{663552\Big(2\left(\sqrt{2}x-\sqrt{2\,x^2-}\right.}\end{aligned}$$

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^4,x, algorithm="g
iac")
```

```
[Out] 1/128*sqrt(2*x^2 - x + 3)*(20*x - 413) - 10939/512*sqrt(2)*log(-2*sqrt(2)*(
sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) + 170114729/7962624*sqrt(2)*log(abs(-
2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 - x + 3))) - 170114729/7962624*sqrt(2)
*log(abs(-2*sqrt(2)*x - 11*sqrt(2) + 2*sqrt(2*x^2 - x + 3))) - 1/663552*sqr
t(2)*(575810908*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^5 + 9206213116*(s
qrt(2)*x - sqrt(2*x^2 - x + 3))^4 + 9688786604*sqrt(2)*(sqrt(2)*x - sqrt(2*
x^2 - x + 3))^3 - 73157325092*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 4948195
2947*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) - 20269228621)/(2*(sqrt(2)*x
 - sqrt(2*x^2 - x + 3))^2 + 10*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) -
11)^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^4}\,dx = \int \frac{\sqrt{2\,x^2-x+3}\,\left(5\,x^4-x^3+3\,x^2+x+2\right)}{{\left(2\,x+5\right)}^4} \,d x$$

```
[In] int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^4,x)
[Out] int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^4, x)
```

3.330 $\int \frac{\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^5} \, dx$

Optimal result

Integrand size = 40, antiderivative size = 165

$$\begin{aligned}\int \frac{\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^5} \, dx &= \frac{7(52836655+9616196x)\sqrt{3-x+2x^2}}{95551488(5+2x)}-\frac{3667\left(3-x+2x^2\right)^{3/2}}{2304(5+2x)^4}+\frac{593771\left(3-x+2x^2\right)^{3/2}}{497664(5+2x)^3} \\ &\quad -\frac{9363383\left(3-x+2x^2\right)^{3/2}}{23887872(5+2x)^2}+\frac{259\text{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{64\sqrt{2}}-\frac{4640586097\text{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{1146617856\sqrt{2}}\end{aligned}$$

```
[Out] -3667/2304*(2*x^2-x+3)^(3/2)/(5+2*x)^4+593771/497664*(2*x^2-x+3)^(3/2)/(5+2
*x)^3-9363383/23887872*(2*x^2-x+3)^(3/2)/(5+2*x)^2+259/128*arcsinh(1/23*(1-
4*x)*23^(1/2))*2^(1/2)-4640586097/2293235712*arctanh(1/24*(17-22*x)*2^(1/2)
/(2*x^2-x+3)^(1/2))*2^(1/2)+7/95551488*(52836655+9616196*x)*(2*x^2-x+3)^(1/
2)/(5+2*x)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used

$= \{1664, 826, 857, 633, 221, 738, 212\}$

$$\begin{aligned}
&\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^5}\,dx \\
&= \frac{259\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{64\sqrt{2}} - \frac{4640586097\operatorname{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{1146617856\sqrt{2}} \\
&\quad - \frac{9363383(2x^2-x+3)^{3/2}}{23887872(2x+5)^2} + \frac{593771(2x^2-x+3)^{3/2}}{497664(2x+5)^3} \\
&\quad - \frac{3667(2x^2-x+3)^{3/2}}{2304(2x+5)^4} + \frac{7(9616196x+52836655)\sqrt{2x^2-x+3}}{95551488(2x+5)}
\end{aligned}$$

```
[In] Int[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^5,x]
```

```
[Out] (7*(52836655 + 9616196*x)*Sqrt[3 - x + 2*x^2])/(95551488*(5 + 2*x)) - (3667
*(3 - x + 2*x^2)^(3/2))/(2304*(5 + 2*x)^4) + (593771*(3 - x + 2*x^2)^(3/2))
/(497664*(5 + 2*x)^3) - (9363383*(3 - x + 2*x^2)^(3/2))/(23887872*(5 + 2*x)
^2) + (259*ArcSinh[(1 - 4*x)/Sqrt[23]])/(64*Sqrt[2]) - (4640586097*ArcTanh[
(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(1146617856*Sqrt[2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 826

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

integral

$$= -\frac{3667(3-x+2x^2)^{3/2}}{2304(5+2x)^4} - \frac{1}{288}\int \frac{\sqrt{3-x+2x^2}\left(\frac{44361}{16} - \frac{17501x}{4} + 1944x^2 - 720x^3\right)}{(5+2x)^4}\,dx$$

$$= -\frac{3667(3-x+2x^2)^{3/2}}{2304(5+2x)^4} + \frac{593771(3-x+2x^2)^{3/2}}{497664(5+2x)^3} + \frac{\int \frac{\sqrt{3-x+2x^2}\left(\frac{4140069}{16} - 404352x + 77760x^2\right)}{(5+2x)^3}\,dx}{62208}$$

$$= -\frac{3667(3-x+2x^2)^{3/2}}{2304(5+2x)^4} + \frac{593771(3-x+2x^2)^{3/2}}{497664(5+2x)^3} - \frac{9363383(3-x+2x^2)^{3/2}}{23887872(5+2x)^2} - \frac{\int \frac{\left(\frac{99869175}{16} - \frac{50485029x}{4}\right)\sqrt{3-x+2x^2}}{(5+2x)^2}\,dx}{8957952}$$

$$= \frac{7(52836655+9616196x)\sqrt{3-x+2x^2}}{95551488(5+2x)} - \frac{3667(3-x+2x^2)^{3/2}}{2304(5+2x)^4} + \frac{593771(3-x+2x^2)^{3/2}}{497664(5+2x)^3} - \frac{9363383(3-x+2x^2)^{3/2}}{23887872(5+2x)^2} + \frac{\int \frac{\frac{2321210451}{8}-580027392x}{(5+2x)\sqrt{3-x+2x^2}}\,dx}{71663616}$$

$$= \frac{7(52836655+9616196x)\sqrt{3-x+2x^2}}{95551488(5+2x)} - \frac{3667(3-x+2x^2)^{3/2}}{2304(5+2x)^4} + \frac{593771(3-x+2x^2)^{3/2}}{497664(5+2x)^3} - \frac{9363383(3-x+2x^2)^{3/2}}{23887872(5+2x)^2} - \frac{259}{64}\int \frac{1}{\sqrt{3-x+2x^2}}\,dx + \frac{4640586097\int \frac{1}{(5+2x)\sqrt{3-x+2x^2}}\,dx}{191102976}$$

$$= \frac{7(52836655+9616196x)\sqrt{3-x+2x^2}}{95551488(5+2x)} - \frac{3667(3-x+2x^2)^{3/2}}{2304(5+2x)^4} + \frac{593771(3-x+2x^2)^{3/2}}{497664(5+2x)^3} - \frac{9363383(3-x+2x^2)^{3/2}}{23887872(5+2x)^2} - \frac{4640586097\text{Subst}\left(\int \frac{1}{288-x^2}\,dx, x, \frac{17-22x}{\sqrt{3-x+2x^2}}\right)}{95551488} - \frac{259\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx, x, -1+4x\right)}{64\sqrt{46}}$$

$$= \frac{7(52836655+9616196x)\sqrt{3-x+2x^2}}{95551488(5+2x)} - \frac{3667(3-x+2x^2)^{3/2}}{2304(5+2x)^4} + \frac{593771(3-x+2x^2)^{3/2}}{497664(5+2x)^3} - \frac{9363383(3-x+2x^2)^{3/2}}{23887872(5+2x)^2} + \frac{259\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{64\sqrt{2}} - \frac{4640586097\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{1146617856\sqrt{2}}$$

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^5}\,dx = \frac{\frac{12\sqrt{3-x+2x^2}(44676885233+62847867486x+31323229164x^2+6105343976x^3+238878720x^4)}{(5+2x)^4} + 4640586097\sqrt{2}\text{arctanh}\big(\frac{1}{6}\big(5+2x}{1146617856}$$

```
[In] Integrate[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^5,x
]
```

```
[Out] ((12*Sqrt[3 - x + 2*x^2]*(44676885233 + 62847867486*x + 31323229164*x^2 + 6
105343976*x^3 + 238878720*x^4))/(5 + 2*x)^4 + 4640586097*Sqrt[2]*ArcTanh[(5
 + 2*x - Sqrt[6 - 2*x + 4*x^2])/6] + 2320109568*Sqrt[2]*Log[1 - 4*x + 2*Sqr
t[6 - 2*x + 4*x^2]])/1146617856
```

Maple [F(-1)]

Timed out.

hanged

```
[In] int((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^5,x)
```

```
[Out] int((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^5,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.15

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^5}\,dx$$
$$= \frac{4640219136\,\sqrt{2}(16\,x^4+160\,x^3+600\,x^2+1000\,x+625)\log\,(4\,\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1)-32\,x^2+16\,x-}{}$$

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^5,x, algorithm="f
ricas")
```

```
[Out] 1/4586471424*(4640219136*sqrt(2)*(16*x^4 + 160*x^3 + 600*x^2 + 1000*x + 625
)*log(4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 46405
86097*sqrt(2)*(16*x^4 + 160*x^3 + 600*x^2 + 1000*x + 625)*log(-(24*sqrt(2)*
sqrt(2*x^2 - x + 3)*(22*x - 17) + 1060*x^2 - 1036*x + 1153)/(4*x^2 + 20*x +
 25)) + 48*(238878720*x^4 + 6105343976*x^3 + 31323229164*x^2 + 62847867486*
x + 44676885233)*sqrt(2*x^2 - x + 3))/(16*x^4 + 160*x^3 + 600*x^2 + 1000*x
+ 625)
```

Sympy [F]

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^5}\,dx$$
$$= \int \frac{\sqrt{2x^2-x+3}\cdot(5x^4-x^3+3x^2+x+2)}{(2x+5)^5}\,dx$$

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)*(2*x**2-x+3)**(1/2)/(5+2*x)**5,x)
```

```
[Out] Integral(sqrt(2*x**2 - x + 3)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)**5
, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.10

$$\begin{aligned}\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^5}\,dx &= -\frac{259}{128}\sqrt{2}\operatorname{arsinh}\left(\frac{4}{23}\sqrt{23}x-\frac{1}{23}\sqrt{23}\right)\\ &\quad+\frac{4640586097}{2293235712}\sqrt{2}\operatorname{arsinh}\left(\frac{22\sqrt{23}x}{23\,|2\,x+5|}-\frac{17\sqrt{23}}{23\,|2\,x+5|}\right)+\frac{16828343}{47775744}\sqrt{2\,x^2-x+3}\\ &\quad-\frac{3667\,(2\,x^2-x+3)^{\frac{3}{2}}}{2304\,(16\,x^4+160\,x^3+600\,x^2+1000\,x+625)}+\frac{593771\,(2\,x^2-x+3)^{\frac{3}{2}}}{497664\,(8\,x^3+60\,x^2+150\,x+125)}\\ &\quad-\frac{9363383\,(2\,x^2-x+3)^{\frac{3}{2}}}{23887872\,(4\,x^2+20\,x+25)}+\frac{201573155\sqrt{2\,x^2-x+3}}{95551488\,(2\,x+5)}\end{aligned}$$

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^5,x, algorithm="m
axima")
```

```
[Out] -259/128*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) + 4640586097/2293
235712*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)/abs(2
*x + 5)) + 16828343/47775744*sqrt(2*x^2 - x + 3) - 3667/2304*(2*x^2 - x + 3
)^(3/2)/(16*x^4 + 160*x^3 + 600*x^2 + 1000*x + 625) + 593771/497664*(2*x^2
- x + 3)^(3/2)/(8*x^3 + 60*x^2 + 150*x + 125) - 9363383/23887872*(2*x^2 - x
 + 3)^(3/2)/(4*x^2 + 20*x + 25) + 201573155/95551488*sqrt(2*x^2 - x + 3)/(2
*x + 5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 327 vs. 2(134) = 268.

Time = 0.32 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.98

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^5}\,dx =$$

$$-\frac{1}{2293235712}\sqrt{2}\left(4640586097\log\left(12\sqrt{-\frac{11}{2\,x+5}+\frac{36}{(2\,x+5)^2}+1}+\frac{72}{2\,x+5}-11\right)\operatorname{sgn}\left(\frac{1}{2\,x+5}\right)+\right.$$

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^5,x, algorithm="g
iac")
```

```
[Out] -1/2293235712*sqrt(2)*(4640586097*log(12*sqrt(-11/(2*x + 5) + 36/(2*x + 5)^
2 + 1) + 72/(2*x + 5) - 11)*sgn(1/(2*x + 5)) + 4640219136*log(abs(sqrt(-11/
(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5) + 1))*sgn(1/(2*x + 5)) - 4640
219136*log(abs(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5) - 1))
*sgn(1/(2*x + 5)) + 12*(24*(144*(792072*sgn(1/(2*x + 5))/(2*x + 5) - 835793
*sgn(1/(2*x + 5)))/(2*x + 5) + 57384361*sgn(1/(2*x + 5)))/(2*x + 5) - 46456
9597*sgn(1/(2*x + 5)))*sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 179159040
*(11*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))*sgn(1/(2*x +
5)) - 12*sgn(1/(2*x + 5)))/((sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(
2*x + 5))^2 - 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^5}\,dx = \int \frac{\sqrt{2\,x^2-x+3}\,\left(5\,x^4-x^3+3\,x^2+x+2\right)}{{\left(2\,x+5\right)}^5}\,dx$$

```
[In] int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^5,x)
```

```
[Out] int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^5, x)
```

3.331 $\int \frac{\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^6}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 165

$$
\begin{aligned}
\int \frac{\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^6}\,dx
&= -\frac{(4583087983+3174439702x)\sqrt{3-x+2x^2}}{6879707136(5+2x)^2}-\frac{3667\left(3-x+2x^2\right)^{3/2}}{2880(5+2x)^5} \\
&\quad+\frac{711961\left(3-x+2x^2\right)^{3/2}}{829440(5+2x)^4}-\frac{38732321\left(3-x+2x^2\right)^{3/2}}{179159040(5+2x)^3} \\
&\quad-\frac{5\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{32\sqrt{2}}+\frac{12895597463\operatorname{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{82556485632\sqrt{2}}
\end{aligned}
$$

```
[Out] -3667/2880*(2*x^2-x+3)^(3/2)/(5+2*x)^5+711961/829440*(2*x^2-x+3)^(3/2)/(5+2
*x)^4-38732321/179159040*(2*x^2-x+3)^(3/2)/(5+2*x)^3-5/64*arcsinh(1/23*(1-4
*x)*23^(1/2))*2^(1/2)+12895597463/165112971264*arctanh(1/24*(17-22*x)*2^(1/
2)/(2*x^2-x+3)^(1/2))*2^(1/2)-1/6879707136*(4583087983+3174439702*x)*(2*x^2
-x+3)^(1/2)/(5+2*x)^2
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used

$= \{1664, 824, 857, 633, 221, 738, 212\}$

$$
\begin{aligned}
&\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^6}\,dx \\
&= -\frac{5\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{32\sqrt{2}} + \frac{12895597463\operatorname{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{82556485632\sqrt{2}} \\
&\quad - \frac{38732321(2x^2-x+3)^{3/2}}{179159040(2x+5)^3} + \frac{711961(2x^2-x+3)^{3/2}}{829440(2x+5)^4} \\
&\quad - \frac{3667(2x^2-x+3)^{3/2}}{2880(2x+5)^5} - \frac{(3174439702x+4583087983)\sqrt{2x^2-x+3}}{6879707136(2x+5)^2}
\end{aligned}
$$

```
[In] Int[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^6,x]
```

```
[Out] -1/6879707136*((4583087983 + 3174439702*x)*Sqrt[3 - x + 2*x^2])/(5 + 2*x)^2
 - (3667*(3 - x + 2*x^2)^(3/2))/(2880*(5 + 2*x)^5) + (711961*(3 - x + 2*x^2
)^(3/2))/(829440*(5 + 2*x)^4) - (38732321*(3 - x + 2*x^2)^(3/2))/(179159040
*(5 + 2*x)^3) - (5*ArcSinh[(1 - 4*x)/Sqrt[23]])/(32*Sqrt[2]) + (12895597463
*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(82556485632*Sqrt[2
])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 824

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Dist[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)
^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p
 + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*
(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m
 - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &
& LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

integral

$$= -\frac{3667\left(3-x+2x^2\right)^{3/2}}{2880(5+2x)^5} - \frac{1}{360}\int \frac{\sqrt{3-x+2x^2}\left(\frac{52701}{16}-\frac{9563x}{2}+2430x^2-900x^3\right)}{(5+2x)^5}\,dx$$

$$= -\frac{3667\left(3-x+2x^2\right)^{3/2}}{2880(5+2x)^5} + \frac{711961\left(3-x+2x^2\right)^{3/2}}{829440(5+2x)^4} + \frac{\int \frac{\sqrt{3-x+2x^2}\left(\frac{5935131}{16}-\frac{1983719x}{4}+129600x^2\right)}{(5+2x)^4}\,dx}{103680}$$

$$= -\frac{3667\left(3-x+2x^2\right)^{3/2}}{2880(5+2x)^5} + \frac{711961\left(3-x+2x^2\right)^{3/2}}{829440(5+2x)^4} - \frac{38732321\left(3-x+2x^2\right)^{3/2}}{179159040(5+2x)^3} - \frac{\int \frac{\left(\frac{138672015}{16}-13996800x\right)\sqrt{3-x+2x^2}}{(5+2x)^3}\,dx}{22394880}$$

$$= -\frac{(4583087983+3174439702x)\sqrt{3-x+2x^2}}{6879707136(5+2x)^2} - \frac{3667(3-x+2x^2)^{3/2}}{2880(5+2x)^5} + \frac{711961(3-x+2x^2)^{3/2}}{829440(5+2x)^4} - \frac{38732321(3-x+2x^2)^{3/2}}{179159040(5+2x)^3} + \frac{\int \frac{-\frac{32190825945}{8}+8062156800x}{(5+2x)\sqrt{3-x+2x^2}}\,dx}{25798901760}$$

$$= -\frac{(4583087983+3174439702x)\sqrt{3-x+2x^2}}{6879707136(5+2x)^2} - \frac{3667(3-x+2x^2)^{3/2}}{2880(5+2x)^5} + \frac{711961(3-x+2x^2)^{3/2}}{829440(5+2x)^4} - \frac{38732321(3-x+2x^2)^{3/2}}{179159040(5+2x)^3} + \frac{5}{32}\int \frac{1}{\sqrt{3-x+2x^2}}\,dx - \frac{12895597463\int \frac{1}{(5+2x)\sqrt{3-x+2x^2}}\,dx}{13759414272}$$

$$= -\frac{(4583087983+3174439702x)\sqrt{3-x+2x^2}}{6879707136(5+2x)^2} - \frac{3667(3-x+2x^2)^{3/2}}{2880(5+2x)^5} + \frac{711961(3-x+2x^2)^{3/2}}{829440(5+2x)^4} - \frac{38732321(3-x+2x^2)^{3/2}}{179159040(5+2x)^3} + \frac{12895597463\text{Subst}\left(\int \frac{1}{288-x^2}\,dx, x, \frac{17-22x}{\sqrt{3-x+2x^2}}\right)}{6879707136} + \frac{5\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx, x, -1+4x\right)}{32\sqrt{46}}$$

$$= -\frac{(4583087983+3174439702x)\sqrt{3-x+2x^2}}{6879707136(5+2x)^2} - \frac{3667(3-x+2x^2)^{3/2}}{2880(5+2x)^5} + \frac{711961(3-x+2x^2)^{3/2}}{829440(5+2x)^4} - \frac{38732321(3-x+2x^2)^{3/2}}{179159040(5+2x)^3} - \frac{5\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{32\sqrt{2}} + \frac{12895597463\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{82556485632\sqrt{2}}$$

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^6}\,dx = \frac{-\frac{12\sqrt{3-x+2x^2}(3110673952831+5608297138216x+3919478861832x^2+1285267446304x^3+186470433136x^4)}{(5+2x)^5} - 64477987315\sqrt{2}\text{arctan}}{412782428160}$$

```
[In] Integrate[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^6,x
]
```

```
[Out] ((-12*Sqrt[3 - x + 2*x^2]*(3110673952831 + 5608297138216*x + 3919478861832*
x^2 + 1285267446304*x^3 + 186470433136*x^4))/(5 + 2*x)^5 - 64477987315*Sqrt
[2]*ArcTanh[(5 + 2*x - Sqrt[6 - 2*x + 4*x^2])/6] - 32248627200*Sqrt[2]*Log[
1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/412782428160
```

Maple [F(-1)]

Timed out.

hanged

```
[In] int((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^6,x)
```

```
[Out] int((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^6,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.23

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^6}\,dx$$
$$= \frac{64497254400 \sqrt{2}(32 x^{5} + 400 x^{4} + 2000 x^{3} + 5000 x^{2} + 6250 x + 3125) \log\left(-4 \sqrt{2}\sqrt{2 x^{2} - x + 3}(4 x - 1)\right.}{}$$

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^6,x, algorithm="f
ricas")
```

```
[Out] 1/1651129712640*(64497254400*sqrt(2)*(32*x^5 + 400*x^4 + 2000*x^3 + 5000*x^
2 + 6250*x + 3125)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 +
16*x - 25) + 64477987315*sqrt(2)*(32*x^5 + 400*x^4 + 2000*x^3 + 5000*x^2 +
6250*x + 3125)*log((24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) - 1060*x^2 +
 1036*x - 1153)/(4*x^2 + 20*x + 25)) - 48*(186470433136*x^4 + 1285267446304
*x^3 + 3919478861832*x^2 + 5608297138216*x + 3110673952831)*sqrt(2*x^2 - x
+ 3))/(32*x^5 + 400*x^4 + 2000*x^3 + 5000*x^2 + 6250*x + 3125)
```

Sympy [F]

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^6}\,dx$$
$$= \int \frac{\sqrt{2x^2-x+3}\cdot(5x^4-x^3+3x^2+x+2)}{(2x+5)^6}\,dx$$

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)*(2*x**2-x+3)**(1/2)/(5+2*x)**6,x)
```

```
[Out] Integral(sqrt(2*x**2 - x + 3)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)**6
, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.35

$$
\begin{aligned}
&\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^6}\,dx \\
&= \frac{5}{64}\sqrt{2}\,\mathrm{arsinh}\left(\frac{4}{23}\sqrt{23}x-\frac{1}{23}\sqrt{23}\right) \\
&\quad -\frac{12895597463}{165112971264}\sqrt{2}\,\mathrm{arsinh}\left(\frac{22\sqrt{23}x}{23\,|2\,x+5|}-\frac{17\sqrt{23}}{23\,|2\,x+5|}\right)-\frac{46569601}{3439853568}\sqrt{2\,x^2-x+3} \\
&\quad -\frac{3667\,(2\,x^2-x+3)^{\frac{3}{2}}}{2880\,(32\,x^5+400\,x^4+2000\,x^3+5000\,x^2+6250\,x+3125)} \\
&\quad +\frac{711961\,(2\,x^2-x+3)^{\frac{3}{2}}}{829440\,(16\,x^4+160\,x^3+600\,x^2+1000\,x+625)} \\
&\quad -\frac{38732321\,(2\,x^2-x+3)^{\frac{3}{2}}}{179159040\,(8\,x^3+60\,x^2+150\,x+125)} \\
&\quad +\frac{46569601\,(2\,x^2-x+3)^{\frac{3}{2}}}{1719926784\,(4\,x^2+20\,x+25)}-\frac{562688629\sqrt{2\,x^2-x+3}}{6879707136\,(2\,x+5)}
\end{aligned}
$$

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^6,x, algorithm="m
axima")
```

```
[Out] 5/64*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) - 12895597463/1651129
71264*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)/abs(2*
x + 5)) - 46569601/3439853568*sqrt(2*x^2 - x + 3) - 3667/2880*(2*x^2 - x +
3)^(3/2)/(32*x^5 + 400*x^4 + 2000*x^3 + 5000*x^2 + 6250*x + 3125) + 711961/
829440*(2*x^2 - x + 3)^(3/2)/(16*x^4 + 160*x^3 + 600*x^2 + 1000*x + 625) -
38732321/179159040*(2*x^2 - x + 3)^(3/2)/(8*x^3 + 60*x^2 + 150*x + 125) + 4
6569601/1719926784*(2*x^2 - x + 3)^(3/2)/(4*x^2 + 20*x + 25) - 562688629/68
79707136*sqrt(2*x^2 - x + 3)/(2*x + 5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 387 vs. 2(134) = 268.

Time = 0.30 (sec) , antiderivative size = 387, normalized size of antiderivative = 2.35

$$\begin{aligned}&\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^6}\,dx\\
&=-\frac{5}{64}\,\sqrt{2}\log\left(-2\,\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)+1\right)\\
&\quad+\frac{12895597463}{165112971264}\,\sqrt{2}\log\left(\left|-2\,\sqrt{2}x+\sqrt{2}+2\,\sqrt{2\,x^2-x+3}\right|\right)\\
&\quad-\frac{12895597463}{165112971264}\,\sqrt{2}\log\left(\left|-2\,\sqrt{2}x-11\,\sqrt{2}+2\,\sqrt{2\,x^2-x+3}\right|\right)\\
&\quad-\frac{\sqrt{2}\Big(4368922304720\,\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)^9+124570969998480\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)^8+6378}{}\end{aligned}$$

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^6,x, algorithm="g
iac")
```

```
[Out] -5/64*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) + 12895
597463/165112971264*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 -
 x + 3))) - 12895597463/165112971264*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt
(2) + 2*sqrt(2*x^2 - x + 3))) - 1/68797071360*sqrt(2)*(4368922304720*sqrt(2
)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^9 + 124570969998480*(sqrt(2)*x - sqrt(2
*x^2 - x + 3))^8 + 637804348664160*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)
)^7 + 1828845222532320*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^6 - 37631893001870
16*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^5 - 10794416351958120*(sqrt(2)
*x - sqrt(2*x^2 - x + 3))^4 + 25049834283305880*sqrt(2)*(sqrt(2)*x - sqrt(2
*x^2 - x + 3))^3 - 34708488692384520*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 +
10654664764755165*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) - 2507056315485
767)/(2*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 10*sqrt(2)*(sqrt(2)*x - sqrt(
2*x^2 - x + 3)) - 11)^5
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^6}\,dx=\int \frac{\sqrt{2\,x^2-x+3}\,\left(5\,x^4-x^3+3\,x^2+x+2\right)}{{\left(2\,x+5\right)}^6}\,dx$$

```
[In] int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^6,x)
```

```
[Out] int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^6, x)
```

3.332 $\int \frac{\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^7}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 169

$$\begin{aligned}\int \frac{\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^7}\,dx &= -\frac{1172725(17-22x)\sqrt{3-x+2x^2}}{330225942528(5+2x)^2}-\frac{3667\left(3-x+2x^2\right)^{3/2}}{3456(5+2x)^6}\\ &\quad+\frac{92239\left(3-x+2x^2\right)^{3/2}}{138240(5+2x)^5}-\frac{5703277\left(3-x+2x^2\right)^{3/2}}{39813120(5+2x)^4}\\ &\quad+\frac{87677717\left(3-x+2x^2\right)^{3/2}}{8599633920(5+2x)^3}-\frac{26972675\operatorname{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{3962711310336\sqrt{2}}\end{aligned}$$

```
[Out] -3667/3456*(2*x^2-x+3)^(3/2)/(5+2*x)^6+92239/138240*(2*x^2-x+3)^(3/2)/(5+2*
x)^5-5703277/39813120*(2*x^2-x+3)^(3/2)/(5+2*x)^4+87677717/8599633920*(2*x^
2-x+3)^(3/2)/(5+2*x)^3-26972675/7925422620672*arctanh(1/24*(17-22*x)*2^(1/2
)/(2*x^2-x+3)^(1/2))*2^(1/2)-1172725/330225942528*(17-22*x)*(2*x^2-x+3)^(1/
2)/(5+2*x)^2
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used

$= \{1664, 820, 734, 738, 212\}$

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^7}\,dx$$
$$= -\frac{26972675\text{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{3962711310336\sqrt{2}} + \frac{87677717(2x^2-x+3)^{3/2}}{8599633920(2x+5)^3}$$
$$- \frac{5703277(2x^2-x+3)^{3/2}}{39813120(2x+5)^4} + \frac{92239(2x^2-x+3)^{3/2}}{138240(2x+5)^5}$$
$$- \frac{3667(2x^2-x+3)^{3/2}}{3456(2x+5)^6} - \frac{1172725(17-22x)\sqrt{2x^2-x+3}}{330225942528(2x+5)^2}$$

```
[In] Int[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^7,x]
```

```
[Out] (-1172725*(17 - 22*x)*Sqrt[3 - x + 2*x^2])/(330225942528*(5 + 2*x)^2) - (36
67*(3 - x + 2*x^2)^(3/2))/(3456*(5 + 2*x)^6) + (92239*(3 - x + 2*x^2)^(3/2)
)/(138240*(5 + 2*x)^5) - (5703277*(3 - x + 2*x^2)^(3/2))/(39813120*(5 + 2*x
)^4) + (87677717*(3 - x + 2*x^2)^(3/2))/(8599633920*(5 + 2*x)^3) - (2697267
5*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(3962711310336*Sqr
t[2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), Int[(d + e*x)^(m + 2)*(a + b*x +
c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
 0] && GtQ[p, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

integral

$$= -\frac{3667(3-x+2x^2)^{3/2}}{3456(5+2x)^6} - \frac{1}{432}\int \frac{\sqrt{3-x+2x^2}\left(\frac{61041}{16} - \frac{20751x}{4} + 2916x^2 - 1080x^3\right)}{(5+2x)^6}\,dx$$

$$= -\frac{3667(3-x+2x^2)^{3/2}}{3456(5+2x)^6} + \frac{92239(3-x+2x^2)^{3/2}}{138240(5+2x)^5} + \frac{\int \frac{\sqrt{3-x+2x^2}\left(\frac{8057313}{16} - \frac{1191609x}{2} + 194400x^2\right)}{(5+2x)^5}\,dx}{155520}$$

$$= -\frac{3667(3-x+2x^2)^{3/2}}{3456(5+2x)^6} + \frac{92239(3-x+2x^2)^{3/2}}{138240(5+2x)^5} - \frac{5703277(3-x+2x^2)^{3/2}}{39813120(5+2x)^4} - \frac{\int \frac{\left(\frac{182650383}{16} - \frac{60644907x}{4}\right)\sqrt{3-x+2x^2}}{(5+2x)^4}\,dx}{44789760}$$

$$= -\frac{3667(3-x+2x^2)^{3/2}}{3456(5+2x)^6} + \frac{92239(3-x+2x^2)^{3/2}}{138240(5+2x)^5} - \frac{5703277(3-x+2x^2)^{3/2}}{39813120(5+2x)^4} + \frac{87677717(3-x+2x^2)^{3/2}}{8599633920(5+2x)^3} + \frac{1172725\int \frac{\sqrt{3-x+2x^2}}{(5+2x)^3}\,dx}{1146617856}$$

$$= -\frac{1172725(17-22x)\sqrt{3-x+2x^2}}{330225942528(5+2x)^2} - \frac{3667(3-x+2x^2)^{3/2}}{3456(5+2x)^6} + \frac{92239(3-x+2x^2)^{3/2}}{138240(5+2x)^5} - \frac{5703277(3-x+2x^2)^{3/2}}{39813120(5+2x)^4} + \frac{87677717(3-x+2x^2)^{3/2}}{8599633920(5+2x)^3} + \frac{26972675\int \frac{1}{(5+2x)\sqrt{3-x+2x^2}}\,dx}{660451885056}$$

$$= -\frac{1172725(17-22x)\sqrt{3-x+2x^2}}{330225942528(5+2x)^2} - \frac{3667\left(3-x+2x^2\right)^{3/2}}{3456(5+2x)^6}$$
$$+ \frac{92239\left(3-x+2x^2\right)^{3/2}}{138240(5+2x)^5} - \frac{5703277\left(3-x+2x^2\right)^{3/2}}{39813120(5+2x)^4}$$
$$+ \frac{87677717\left(3-x+2x^2\right)^{3/2}}{8599633920(5+2x)^3} - \frac{26972675\text{Subst}\left(\int \frac{1}{288-x^2}\,dx, x, \frac{17-22x}{\sqrt{3-x+2x^2}}\right)}{330225942528}$$

$$= -\frac{1172725(17-22x)\sqrt{3-x+2x^2}}{330225942528(5+2x)^2} - \frac{3667\left(3-x+2x^2\right)^{3/2}}{3456(5+2x)^6}$$
$$+ \frac{92239\left(3-x+2x^2\right)^{3/2}}{138240(5+2x)^5} - \frac{5703277\left(3-x+2x^2\right)^{3/2}}{39813120(5+2x)^4}$$
$$+ \frac{87677717\left(3-x+2x^2\right)^{3/2}}{8599633920(5+2x)^3} - \frac{26972675\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{3962711310336\sqrt{2}}$$

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.51

$$\int \frac{\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^7}\,dx$$
$$= \frac{\frac{12\sqrt{3-x+2x^2}\left(-219337079305+27245373694x+158340720344x^2+397498825328x^3+12256250416x^4+271409942624x^5\right)}{(5+2x)^6} + 134863375\sqrt{2}}{19813556551680}$$

```
[In] Integrate[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^7,x
]
```

```
[Out] ((12*Sqrt[3 - x + 2*x^2]*(-219337079305 + 27245373694*x + 158340720344*x^2
+ 397498825328*x^3 + 12256250416*x^4 + 271409942624*x^5))/(5 + 2*x)^6 + 134
863375*Sqrt[2]*ArcTanh[(5 + 2*x - Sqrt[6 - 2*x + 4*x^2])/6])/19813556551680
```

Maple [A] (verified)

Time = 1.15 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.52

method	result
risch	$\frac{542819885248x^7-246897441792x^6+1596971228112x^5-44048633392x^4+1088646503028x^3+9102628728x^2+301073200387x-658011}{1651129712640(5+2x)^6\sqrt{2x^2-x+3}}$
trager	$\frac{(271409942624x^5+12256250416x^4+397498825328x^3+158340720344x^2+27245373694x-219337079305)\sqrt{2x^2-x+3}}{1651129712640(5+2x)^6}+\frac{26972675\,\mathrm{Ro}}{}$
default	$-\frac{5703277\left(2\left(x+\frac{5}{2}\right)^2-11x-\frac{19}{2}\right)^{\frac{3}{2}}}{637009920\left(x+\frac{5}{2}\right)^4}+\frac{87677717\left(2\left(x+\frac{5}{2}\right)^2-11x-\frac{19}{2}\right)^{\frac{3}{2}}}{68797071360\left(x+\frac{5}{2}\right)^3}-\frac{1172725\left(2\left(x+\frac{5}{2}\right)^2-11x-\frac{19}{2}\right)^{\frac{3}{2}}}{330225942528\left(x+\frac{5}{2}\right)^2}-\frac{12899975\left(2\left(x+\frac{5}{2}\right)\right.}{118881339310}$

```
[In] int((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^7,x,method=_RETURNVERBO
SE)
```

```
[Out] 1/1651129712640*(542819885248*x^7-246897441792*x^6+1596971228112*x^5-440486
33392*x^4+1088646503028*x^3+9102628728*x^2+301073200387*x-658011237915)/(5+
2*x)^6/(2*x^2-x+3)^(1/2)-26972675/7925422620672*2^(1/2)*arctanh(1/12*(17/2-
11*x)*2^(1/2)/(2*(x+5/2)^2-11*x-19/2)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^7}\,dx$$
$$=\frac{134863375\sqrt{2}(64\,x^6+960\,x^5+6000\,x^4+20000\,x^3+37500\,x^2+37500\,x+15625)\log\left(-\frac{24\sqrt{2}\sqrt{2\,x^2-x+3}(22}{4\,x}\right.}{79254226206720\,(6}$$

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^7,x, algorithm="f
ricas")
```

```
[Out] 1/79254226206720*(134863375*sqrt(2)*(64*x^6 + 960*x^5 + 6000*x^4 + 20000*x^
3 + 37500*x^2 + 37500*x + 15625)*log(-(24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x
 - 17) + 1060*x^2 - 1036*x + 1153)/(4*x^2 + 20*x + 25)) + 48*(271409942624*
x^5 + 12256250416*x^4 + 397498825328*x^3 + 158340720344*x^2 + 27245373694*x
 - 219337079305)*sqrt(2*x^2 - x + 3))/(64*x^6 + 960*x^5 + 6000*x^4 + 20000*
x^3 + 37500*x^2 + 37500*x + 15625)
```

Sympy [F]

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^7}\,dx = \int \frac{\sqrt{2x^2-x+3}\cdot(5x^4-x^3+3x^2+x+2)}{(2x+5)^7}\,dx$$

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)*(2*x**2-x+3)**(1/2)/(5+2*x)**7,x)
```

```
[Out] Integral(sqrt(2*x**2 - x + 3)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)**7
, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.48

$$\begin{aligned}&\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^7}\,dx\\&= \frac{26972675}{7925422620672}\sqrt{2}\,\mathrm{arsinh}\left(\frac{22\sqrt{23}x}{23\,|2\,x+5|}-\frac{17\sqrt{23}}{23\,|2\,x+5|}\right)+\frac{1172725}{165112971264}\sqrt{2\,x^2-x+3}\\&\quad-\frac{3667\,(2\,x^2-x+3)^{\frac{3}{2}}}{3456\,(64\,x^6+960\,x^5+6000\,x^4+20000\,x^3+37500\,x^2+37500\,x+15625)}\\&\quad+\frac{92239\,(2\,x^2-x+3)^{\frac{3}{2}}}{138240\,(32\,x^5+400\,x^4+2000\,x^3+5000\,x^2+6250\,x+3125)}\\&\quad-\frac{5703277\,(2\,x^2-x+3)^{\frac{3}{2}}}{39813120\,(16\,x^4+160\,x^3+600\,x^2+1000\,x+625)}\\&\quad+\frac{87677717\,(2\,x^2-x+3)^{\frac{3}{2}}}{8599633920\,(8\,x^3+60\,x^2+150\,x+125)}\\&\quad-\frac{1172725\,(2\,x^2-x+3)^{\frac{3}{2}}}{82556485632\,(4\,x^2+20\,x+25)}-\frac{12899975\sqrt{2\,x^2-x+3}}{330225942528\,(2\,x+5)}\end{aligned}$$

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^7,x, algorithm="m
axima")
```

```
[Out] 26972675/7925422620672*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/2
3*sqrt(23)/abs(2*x + 5)) + 1172725/165112971264*sqrt(2*x^2 - x + 3) - 3667/
3456*(2*x^2 - x + 3)^(3/2)/(64*x^6 + 960*x^5 + 6000*x^4 + 20000*x^3 + 37500
*x^2 + 37500*x + 15625) + 92239/138240*(2*x^2 - x + 3)^(3/2)/(32*x^5 + 400*
x^4 + 2000*x^3 + 5000*x^2 + 6250*x + 3125) - 5703277/39813120*(2*x^2 - x +
```

```
3)^(3/2)/(16*x^4 + 160*x^3 + 600*x^2 + 1000*x + 625) + 87677717/8599633920*
(2*x^2 - x + 3)^(3/2)/(8*x^3 + 60*x^2 + 150*x + 125) - 1172725/82556485632*
(2*x^2 - x + 3)^(3/2)/(4*x^2 + 20*x + 25) - 12899975/330225942528*sqrt(2*x^
2 - x + 3)/(2*x + 5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 405 vs. 2(139) = 278.

Time = 0.29 (sec) , antiderivative size = 405, normalized size of antiderivative = 2.40

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^7}\,dx$$
$$= -\frac{26972675}{7925422620672}\sqrt{2}\log\left(\left|-2\sqrt{2}x+\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right)$$
$$+\frac{26972675}{7925422620672}\sqrt{2}\log\left(\left|-2\sqrt{2}x-11\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right)$$
$$+\frac{\sqrt{2}\Big(16506981498400\sqrt{2}\big(\sqrt{2}x-\sqrt{2x^2-x+3}\big)^{11}+389429252643040\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^{10}+226}{}$$

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^7,x, algorithm="g
iac")
```

```
[Out] -26972675/7925422620672*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x
^2 - x + 3))) + 26972675/7925422620672*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sq
rt(2) + 2*sqrt(2*x^2 - x + 3))) + 1/3302259425280*sqrt(2)*(16506981498400*s
qrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^11 + 389429252643040*(sqrt(2)*x -
sqrt(2*x^2 - x + 3))^10 + 2263923918689840*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2
- x + 3))^9 + 11663651054548560*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^8 + 90221
2326134736*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^7 - 84192729519861840*
(sqrt(2)*x - sqrt(2*x^2 - x + 3))^6 - 4317200555009448*sqrt(2)*(sqrt(2)*x -
 sqrt(2*x^2 - x + 3))^5 + 351543414066518760*(sqrt(2)*x - sqrt(2*x^2 - x +
3))^4 - 376787166452923830*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^3 + 35
6306707647610982*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 - 82348353128195465*sq
rt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 15499394004553969)/(2*(sqrt(2)*x
- sqrt(2*x^2 - x + 3))^2 + 10*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) - 1
1)^6
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^7}\,dx = \int \frac{\sqrt{2\,x^2-x+3}\,\left(5\,x^4-x^3+3\,x^2+x+2\right)}{{\left(2\,x+5\right)}^7} \,dx$$

```
[In] int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^7,x)
[Out] int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^7, x)
```

3.333 $\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^8}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 194

$$\begin{aligned}
&\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^8}\,dx \\
&= -\frac{12568315(17-22x)\sqrt{3-x+2x^2}}{23776267862016(5+2x)^2} - \frac{3667(3-x+2x^2)^{3/2}}{4032(5+2x)^7} \\
&\quad + \frac{948341(3-x+2x^2)^{3/2}}{1741824(5+2x)^6} - \frac{1464037(3-x+2x^2)^{3/2}}{13934592(5+2x)^5} + \frac{19414831(3-x+2x^2)^{3/2}}{4013162496(5+2x)^4} \\
&\quad + \frac{246159769(3-x+2x^2)^{3/2}}{866843099136(5+2x)^3} - \frac{289071245\,\text{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{285315214344192\sqrt{2}}
\end{aligned}$$

```
[Out] -3667/4032*(2*x^2-x+3)^(3/2)/(5+2*x)^7+948341/1741824*(2*x^2-x+3)^(3/2)/(5+
2*x)^6-1464037/13934592*(2*x^2-x+3)^(3/2)/(5+2*x)^5+19414831/4013162496*(2*
x^2-x+3)^(3/2)/(5+2*x)^4+246159769/866843099136*(2*x^2-x+3)^(3/2)/(5+2*x)^3
-289071245/570630428688384*arctanh(1/24*(17-22*x)*2^(1/2)/(2*x^2-x+3)^(1/2)
)*2^(1/2)-12568315/23776267862016*(17-22*x)*(2*x^2-x+3)^(1/2)/(5+2*x)^2
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used

$= \{1664, 848, 820, 734, 738, 212\}$

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^8}\,dx$$
$$= -\frac{289071245\,\mathrm{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{285315214344192\sqrt{2}} + \frac{246159769(2x^2-x+3)^{3/2}}{866843099136(2x+5)^3}$$
$$+ \frac{19414831(2x^2-x+3)^{3/2}}{4013162496(2x+5)^4} - \frac{1464037(2x^2-x+3)^{3/2}}{13934592(2x+5)^5} + \frac{948341(2x^2-x+3)^{3/2}}{1741824(2x+5)^6}$$
$$- \frac{3667(2x^2-x+3)^{3/2}}{4032(2x+5)^7} - \frac{12568315(17-22x)\sqrt{2x^2-x+3}}{23776267862016(2x+5)^2}$$

```
[In] Int[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^8,x]
```

```
[Out] (-12568315*(17 - 22*x)*Sqrt[3 - x + 2*x^2])/(23776267862016*(5 + 2*x)^2) -
(3667*(3 - x + 2*x^2)^(3/2))/(4032*(5 + 2*x)^7) + (948341*(3 - x + 2*x^2)^(
3/2))/(1741824*(5 + 2*x)^6) - (1464037*(3 - x + 2*x^2)^(3/2))/(13934592*(5
+ 2*x)^5) + (19414831*(3 - x + 2*x^2)^(3/2))/(4013162496*(5 + 2*x)^4) + (24
6159769*(3 - x + 2*x^2)^(3/2))/(866843099136*(5 + 2*x)^3) - (289071245*ArcT
anh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(285315214344192*Sqrt[2]
)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), Int[(d + e*x)^(m + 2)*(a + b*x +
c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
 0] && GtQ[p, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 848

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{3667\left(3-x+2x^2\right)^{3/2}}{4032(5+2x)^7} \\
&\quad -\frac{1}{504}\int\frac{\sqrt{3-x+2x^2}\left(\frac{69381}{16}-5594x+3402x^2-1260x^3\right)}{(5+2x)^7}\,dx \\
&= -\frac{3667\left(3-x+2x^2\right)^{3/2}}{4032(5+2x)^7}+\frac{948341\left(3-x+2x^2\right)^{3/2}}{1741824(5+2x)^6}+\frac{\int\frac{\sqrt{3-x+2x^2}\left(\frac{10506615}{16}-\frac{2815905x}{4}+272160x^2\right)}{(5+2x)^6}\,dx}{217728} \\
&= -\frac{3667\left(3-x+2x^2\right)^{3/2}}{4032(5+2x)^7}+\frac{948341\left(3-x+2x^2\right)^{3/2}}{1741824(5+2x)^6} \\
&\quad -\frac{1464037\left(3-x+2x^2\right)^{3/2}}{13934592(5+2x)^5}-\frac{\int\frac{\left(\frac{231748695}{16}-\frac{32095935x}{2}\right)\sqrt{3-x+2x^2}}{(5+2x)^5}\,dx}{78382080}
\end{aligned}$$

$$\begin{aligned}&= -\frac{3667(3-x+2x^2)^{3/2}}{4032(5+2x)^7} + \frac{948341(3-x+2x^2)^{3/2}}{1741824(5+2x)^6} - \frac{1464037(3-x+2x^2)^{3/2}}{13934592(5+2x)^5} \\ &\quad + \frac{19414831(3-x+2x^2)^{3/2}}{4013162496(5+2x)^4} + \frac{\int \frac{\left(-\frac{2340515655}{16}+\frac{873667395x}{4}\right)\sqrt{3-x+2x^2}}{(5+2x)^4}\,dx}{22574039040}\end{aligned}$$

$$\begin{aligned}&= -\frac{3667(3-x+2x^2)^{3/2}}{4032(5+2x)^7} + \frac{948341(3-x+2x^2)^{3/2}}{1741824(5+2x)^6} - \frac{1464037(3-x+2x^2)^{3/2}}{13934592(5+2x)^5} \\ &\quad + \frac{19414831(3-x+2x^2)^{3/2}}{4013162496(5+2x)^4} + \frac{246159769(3-x+2x^2)^{3/2}}{866843099136(5+2x)^3} + \frac{12568315\int \frac{\sqrt{3-x+2x^2}}{(5+2x)^3}\,dx}{82556485632}\end{aligned}$$

$$\begin{aligned}&= -\frac{12568315(17-22x)\sqrt{3-x+2x^2}}{23776267862016(5+2x)^2} - \frac{3667(3-x+2x^2)^{3/2}}{4032(5+2x)^7} \\ &\quad + \frac{948341(3-x+2x^2)^{3/2}}{1741824(5+2x)^6} - \frac{1464037(3-x+2x^2)^{3/2}}{13934592(5+2x)^5} + \frac{19414831(3-x+2x^2)^{3/2}}{4013162496(5+2x)^4} \\ &\quad + \frac{246159769(3-x+2x^2)^{3/2}}{866843099136(5+2x)^3} + \frac{289071245\int \frac{1}{(5+2x)\sqrt{3-x+2x^2}}\,dx}{47552535724032}\end{aligned}$$

$$\begin{aligned}&= -\frac{12568315(17-22x)\sqrt{3-x+2x^2}}{23776267862016(5+2x)^2} - \frac{3667(3-x+2x^2)^{3/2}}{4032(5+2x)^7} \\ &\quad + \frac{948341(3-x+2x^2)^{3/2}}{1741824(5+2x)^6} - \frac{1464037(3-x+2x^2)^{3/2}}{13934592(5+2x)^5} + \frac{19414831(3-x+2x^2)^{3/2}}{4013162496(5+2x)^4} \\ &\quad + \frac{246159769(3-x+2x^2)^{3/2}}{866843099136(5+2x)^3} - \frac{289071245\text{Subst}\left(\int \frac{1}{288-x^2}\,dx, x, \frac{17-22x}{\sqrt{3-x+2x^2}}\right)}{23776267862016}\end{aligned}$$

$$\begin{aligned}&= -\frac{12568315(17-22x)\sqrt{3-x+2x^2}}{23776267862016(5+2x)^2} - \frac{3667(3-x+2x^2)^{3/2}}{4032(5+2x)^7} \\ &\quad + \frac{948341(3-x+2x^2)^{3/2}}{1741824(5+2x)^6} - \frac{1464037(3-x+2x^2)^{3/2}}{13934592(5+2x)^5} + \frac{19414831(3-x+2x^2)^{3/2}}{4013162496(5+2x)^4} \\ &\quad + \frac{246159769(3-x+2x^2)^{3/2}}{866843099136(5+2x)^3} - \frac{289071245\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{285315214344192\sqrt{2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.47

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^8}\,dx$$
$$= \frac{\frac{12\sqrt{3-x+2x^2}(-20465234808721+590492177460x+14716683780036x^2+41058010262368x^3+4982916071952x^4+27976951397184x^5+1574342277}{(5+2x)^7}}{1997206500409344}$$

```
[In] Integrate[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^8,x
]
```

```
[Out] ((12*Sqrt[3 - x + 2*x^2]*(-20465234808721 + 590492177460*x + 14716683780036
*x^2 + 41058010262368*x^3 + 4982916071952*x^4 + 27976951397184*x^5 + 157434
2277056*x^6))/(5 + 2*x)^7 + 2023498715*Sqrt[2]*ArcTanh[(5 + 2*x - Sqrt[6 -
2*x + 4*x^2])/6])/1997206500409344
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.48

method	result
risch	$\frac{3148684554112x^8+54379560517312x^7-13288092422112x^6+161063958644336x^5+3324105513560x^4+109638331361988x^3+26290895}{166433875034112(5+2x)^7\sqrt{2x^2-x+3}}$
trager	$\frac{(1574342277056x^6+27976951397184x^5+4982916071952x^4+41058010262368x^3+14716683780036x^2+590492177460x-204652348087}{166433875034112(5+2x)^7}$
default	$\frac{19414831\left(2\left(x+\frac{5}{2}\right)^2-11x-\frac{19}{2}\right)^{\frac{3}{2}}}{64210599936\left(x+\frac{5}{2}\right)^4}+\frac{246159769\left(2\left(x+\frac{5}{2}\right)^2-11x-\frac{19}{2}\right)^{\frac{3}{2}}}{6934744793088\left(x+\frac{5}{2}\right)^3}-\frac{12568315\left(2\left(x+\frac{5}{2}\right)^2-11x-\frac{19}{2}\right)^{\frac{3}{2}}}{23776267862016\left(x+\frac{5}{2}\right)^2}-\frac{138251465\left(2(x+}{855945643032}$

```
[In] int((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^8,x,method=_RETURNVERBO
SE)
```

```
[Out] 1/166433875034112*(3148684554112*x^8+54379560517312*x^7-13288092422112*x^6+
161063958644336*x^5+3324105513560*x^4+109638331361988*x^3+2629089545206*x^2
+22236711341101*x-61395704426163)/(5+2*x)^7/(2*x^2-x+3)^(1/2)-289071245/570
630428688384*2^(1/2)*arctanh(1/12*(17/2-11*x)*2^(1/2)/(2*(x+5/2)^2-11*x-19/
2)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^8}\,dx$$
$$= \frac{2023498715\,\sqrt{2}(128\,x^7+2240\,x^6+16800\,x^5+70000\,x^4+175000\,x^3+262500\,x^2+218750\,x+78125)\,\mathrm{lo}}{7}$$

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^8,x, algorithm="f
ricas")
```

```
[Out] 1/7988826001637376*(2023498715*sqrt(2)*(128*x^7 + 2240*x^6 + 16800*x^5 + 70
000*x^4 + 175000*x^3 + 262500*x^2 + 218750*x + 78125)*log(-(24*sqrt(2)*sqrt
(2*x^2 - x + 3)*(22*x - 17) + 1060*x^2 - 1036*x + 1153)/(4*x^2 + 20*x + 25)
) + 48*(1574342277056*x^6 + 27976951397184*x^5 + 4982916071952*x^4 + 410580
10262368*x^3 + 14716683780036*x^2 + 590492177460*x - 20465234808721)*sqrt(2
*x^2 - x + 3))/(128*x^7 + 2240*x^6 + 16800*x^5 + 70000*x^4 + 175000*x^3 + 2
62500*x^2 + 218750*x + 78125)
```

Sympy [F]

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^8}\,dx$$
$$= \int \frac{\sqrt{2x^2-x+3}\cdot(5x^4-x^3+3x^2+x+2)}{(2x+5)^8}\,dx$$

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)*(2*x**2-x+3)**(1/2)/(5+2*x)**8,x)
```

```
[Out] Integral(sqrt(2*x**2 - x + 3)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)**8
, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.55

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^8}\,dx$$
$$= \frac{289071245}{570630428688384}\sqrt{2}\operatorname{arsinh}\left(\frac{22\sqrt{23}x}{23\,|2\,x+5|}-\frac{17\sqrt{23}}{23\,|2\,x+5|}\right)$$
$$+\frac{12568315}{11888133931008}\sqrt{2\,x^2-x+3}$$
$$-\frac{3667\,(2\,x^2-x+3)^{\frac{3}{2}}}{4032\,(128\,x^7+2240\,x^6+16800\,x^5+70000\,x^4+175000\,x^3+262500\,x^2+218750\,x+78125)}$$
$$+\frac{948341\,(2\,x^2-x+3)^{\frac{3}{2}}}{1741824\,(64\,x^6+960\,x^5+6000\,x^4+20000\,x^3+37500\,x^2+37500\,x+15625)}$$
$$-\frac{1464037\,(2\,x^2-x+3)^{\frac{3}{2}}}{13934592\,(32\,x^5+400\,x^4+2000\,x^3+5000\,x^2+6250\,x+3125)}$$
$$+\frac{19414831\,(2\,x^2-x+3)^{\frac{3}{2}}}{4013162496\,(16\,x^4+160\,x^3+600\,x^2+1000\,x+625)}$$
$$+\frac{246159769\,(2\,x^2-x+3)^{\frac{3}{2}}}{866843099136\,(8\,x^3+60\,x^2+150\,x+125)}$$
$$-\frac{12568315\,(2\,x^2-x+3)^{\frac{3}{2}}}{5944066965504\,(4\,x^2+20\,x+25)}-\frac{138251465\sqrt{2\,x^2-x+3}}{23776267862016\,(2\,x+5)}$$

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^8,x, algorithm="m
axima")
```

```
[Out] 289071245/570630428688384*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 1
7/23*sqrt(23)/abs(2*x + 5)) + 12568315/11888133931008*sqrt(2*x^2 - x + 3) -
 3667/4032*(2*x^2 - x + 3)^(3/2)/(128*x^7 + 2240*x^6 + 16800*x^5 + 70000*x^
4 + 175000*x^3 + 262500*x^2 + 218750*x + 78125) + 948341/1741824*(2*x^2 - x
 + 3)^(3/2)/(64*x^6 + 960*x^5 + 6000*x^4 + 20000*x^3 + 37500*x^2 + 37500*x
+ 15625) - 1464037/13934592*(2*x^2 - x + 3)^(3/2)/(32*x^5 + 400*x^4 + 2000*
x^3 + 5000*x^2 + 6250*x + 3125) + 19414831/4013162496*(2*x^2 - x + 3)^(3/2)
/(16*x^4 + 160*x^3 + 600*x^2 + 1000*x + 625) + 246159769/866843099136*(2*x^
2 - x + 3)^(3/2)/(8*x^3 + 60*x^2 + 150*x + 125) - 12568315/5944066965504*(2
*x^2 - x + 3)^(3/2)/(4*x^2 + 20*x + 25) - 138251465/23776267862016*sqrt(2*x
^2 - x + 3)/(2*x + 5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 456 vs. $2(160) = 320$.

Time = 0.31 (sec) , antiderivative size = 456, normalized size of antiderivative = 2.35

$$
\begin{aligned}
&\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^8}\,dx \\
&= -\frac{289071245}{570630428688384}\sqrt{2}\log\left(\left|-2\sqrt{2}x+\sqrt{2}+2\sqrt{2\,x^2-x+3}\right|\right) \\
&\quad + \frac{289071245}{570630428688384}\sqrt{2}\log\left(\left|-2\sqrt{2}x-11\sqrt{2}+2\sqrt{2\,x^2-x+3}\right|\right) \\
&\quad - \frac{\sqrt{2}\Big(129503917760\sqrt{2}\big(\sqrt{2}x-\sqrt{2\,x^2-x+3}\big)^{13} - 3320259746027840\,\big(\sqrt{2}x-\sqrt{2\,x^2-x+3}\big)^{12} - 239}{}
\end{aligned}
$$

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^8,x, algorithm="g
iac")
```

```
[Out] -289071245/570630428688384*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(
2*x^2 - x + 3))) + 289071245/570630428688384*sqrt(2)*log(abs(-2*sqrt(2)*x -
 11*sqrt(2) + 2*sqrt(2*x^2 - x + 3))) - 1/332867750068224*sqrt(2)*(12950391
7760*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^13 - 3320259746027840*(sqrt(
2)*x - sqrt(2*x^2 - x + 3))^12 - 23966708071916736*sqrt(2)*(sqrt(2)*x - sqr
t(2*x^2 - x + 3))^11 - 186055342532355520*(sqrt(2)*x - sqrt(2*x^2 - x + 3))
^10 - 274256644494948976*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^9 + 7961
35370176031760*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^8 + 2531523139171005408*sq
rt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^7 - 4610393811900786336*(sqrt(2)*x
- sqrt(2*x^2 - x + 3))^6 - 7997126854300052364*sqrt(2)*(sqrt(2)*x - sqrt(2*
x^2 - x + 3))^5 + 30842713619423538868*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^4
- 21873571601855032556*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^3 + 162047
0696060466810 0*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 - 3196254593191113265*sq
rt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 536799032216117911)/(2*(sqrt(2)*x
 - sqrt(2*x^2 - x + 3))^2 + 10*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) -
11)^7
```

Mupad [F(-1)]

Timed out.

$$
\begin{aligned}
&\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^8}\,dx \\
&= \int \frac{\sqrt{2\,x^2-x+3}\,(5\,x^4-x^3+3\,x^2+x+2)}{{(2\,x+5)}^8}\,dx
\end{aligned}
$$

```
[In] int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^8,x)
```

```
[Out] int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^8, x)
```

3.334 $\int (5+2x) \left(3-x+2x^2\right)^{3/2} \left(2+x+3x^2-x^3+5x^4\right) d$

Optimal result

Integrand size = 38, antiderivative size = 166

$$\begin{aligned}
&\int (5+2x) \left(3-x+2x^2\right)^{3/2} \left(2+x+3x^2-x^3+5x^4\right) dx = \\
&-\frac{6398163(1-4x)\sqrt{3-x+2x^2}}{2097152}-\frac{92727(1-4x) \left(3-x+2x^2\right)^{3/2}}{131072} \\
&+\frac{69415(5+2x)^2 \left(3-x+2x^2\right)^{5/2}}{32256}-\frac{1121(5+2x)^3 \left(3-x+2x^2\right)^{5/2}}{2304} \\
&+\frac{5}{144}(5+2x)^4 \left(3-x+2x^2\right)^{5/2}-\frac{3(661397+215900x) \left(3-x+2x^2\right)^{5/2}}{143360}-\frac{147157749 \operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{4194304\sqrt{2}}
\end{aligned}$$

```
[Out] -92727/131072*(1-4*x)*(2*x^2-x+3)^(3/2)+69415/32256*(5+2*x)^2*(2*x^2-x+3)^(
5/2)-1121/2304*(5+2*x)^3*(2*x^2-x+3)^(5/2)+5/144*(5+2*x)^4*(2*x^2-x+3)^(5/2
)-3/143360*(661397+215900*x)*(2*x^2-x+3)^(5/2)-147157749/8388608*arcsinh(1/
23*(1-4*x)*23^(1/2))*2^(1/2)-6398163/2097152*(1-4*x)*(2*x^2-x+3)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used

$= \{1667, 793, 626, 633, 221\}$

$$\int (5+2x)\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)\,dx = -\frac{147157749\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{4194304\sqrt{2}} + \frac{5}{144}\left(2x^2-x+3\right)^{5/2}(2x+5)^4 - \frac{1121(2x^2-x+3)^{5/2}(2x+5)^3}{2304} + \frac{69415(2x^2-x+3)^{5/2}(2x+5)^2}{32256} - \frac{3(215900x+661397)\left(2x^2-x+3\right)^{5/2}}{143360} - \frac{92727(1-4x)\left(2x^2-x+3\right)^{3/2}}{131072} - \frac{6398163(1-4x)\sqrt{2x^2-x+3}}{2097152}$$

```
[In] Int[(5 + 2*x)*(3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4),x]
```

```
[Out] (-6398163*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/2097152 - (92727*(1 - 4*x)*(3 - x
+ 2*x^2)^(3/2))/131072 + (69415*(5 + 2*x)^2*(3 - x + 2*x^2)^(5/2))/32256 -
(1121*(5 + 2*x)^3*(3 - x + 2*x^2)^(5/2))/2304 + (5*(5 + 2*x)^4*(3 - x + 2*x
^2)^(5/2))/144 - (3*(661397 + 215900*x)*(3 - x + 2*x^2)^(5/2))/143360 - (14
7157749*ArcSinh[(1 - 4*x)/Sqrt[23]])/(4194304*Sqrt[2])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{5}{144}(5+2x)^4\left(3-x+2x^2\right)^{5/2} \\
&\quad + \frac{1}{288}\int (5+2x)\left(3-x+2x^2\right)^{3/2}\left(-2299-11262x-15996x^2-8968x^3\right)\,dx \\
&= -\frac{1121(5+2x)^3\left(3-x+2x^2\right)^{5/2}}{2304} + \frac{5}{144}(5+2x)^4\left(3-x+2x^2\right)^{5/2} \\
&\quad + \frac{\int (5+2x)\left(3-x+2x^2\right)^{3/2}\left(198968+2253280x+2221280x^2\right)\,dx}{36864} \\
&= \frac{69415(5+2x)^2\left(3-x+2x^2\right)^{5/2}}{32256} - \frac{1121(5+2x)^3\left(3-x+2x^2\right)^{5/2}}{2304} \\
&\quad + \frac{5}{144}(5+2x)^4\left(3-x+2x^2\right)^{5/2} \\
&\quad + \frac{\int (13363488-55961280x)(5+2x)\left(3-x+2x^2\right)^{3/2}\,dx}{2064384} \\
&= \frac{69415(5+2x)^2\left(3-x+2x^2\right)^{5/2}}{32256} - \frac{1121(5+2x)^3\left(3-x+2x^2\right)^{5/2}}{2304} \\
&\quad + \frac{5}{144}(5+2x)^4\left(3-x+2x^2\right)^{5/2} \\
&\quad - \frac{3(661397+215900x)\left(3-x+2x^2\right)^{5/2}}{143360} + \frac{92727\int \left(3-x+2x^2\right)^{3/2}\,dx}{8192} \\
&= -\frac{92727(1-4x)\left(3-x+2x^2\right)^{3/2}}{131072} \\
&\quad + \frac{69415(5+2x)^2\left(3-x+2x^2\right)^{5/2}}{32256} - \frac{1121(5+2x)^3\left(3-x+2x^2\right)^{5/2}}{2304} \\
&\quad + \frac{5}{144}(5+2x)^4\left(3-x+2x^2\right)^{5/2} - \frac{3(661397+215900x)\left(3-x+2x^2\right)^{5/2}}{143360} + \frac{6398163\int \sqrt{3-x+2x^2}}{262144}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{6398163(1-4x)\sqrt{3-x+2x^2}}{2097152} - \frac{92727(1-4x)\left(3-x+2x^2\right)^{3/2}}{131072} \\
&\quad + \frac{69415(5+2x)^2\left(3-x+2x^2\right)^{5/2}}{32256} - \frac{1121(5+2x)^3\left(3-x+2x^2\right)^{5/2}}{2304} \\
&\quad + \frac{5}{144}(5+2x)^4\left(3-x+2x^2\right)^{5/2} - \frac{3(661397+215900x)\left(3-x+2x^2\right)^{5/2}}{143360} + \frac{147157749\int\frac{1}{\sqrt{3-x+2x^2}}\,dx}{4194304} \\
&= -\frac{6398163(1-4x)\sqrt{3-x+2x^2}}{2097152} - \frac{92727(1-4x)\left(3-x+2x^2\right)^{3/2}}{131072} \\
&\quad + \frac{69415(5+2x)^2\left(3-x+2x^2\right)^{5/2}}{32256} - \frac{1121(5+2x)^3\left(3-x+2x^2\right)^{5/2}}{2304} \\
&\quad + \frac{5}{144}(5+2x)^4\left(3-x+2x^2\right)^{5/2} - \frac{3(661397+215900x)\left(3-x+2x^2\right)^{5/2}}{143360} + \frac{\left(6398163\sqrt{\frac{23}{2}}\right)\text{Subst}\Bigg(}{419} \\
&= -\frac{6398163(1-4x)\sqrt{3-x+2x^2}}{2097152} - \frac{92727(1-4x)\left(3-x+2x^2\right)^{3/2}}{131072} \\
&\quad + \frac{69415(5+2x)^2\left(3-x+2x^2\right)^{5/2}}{32256} - \frac{1121(5+2x)^3\left(3-x+2x^2\right)^{5/2}}{2304} \\
&\quad + \frac{5}{144}(5+2x)^4\left(3-x+2x^2\right)^{5/2} - \frac{3(661397+215900x)\left(3-x+2x^2\right)^{5/2}}{143360} - \frac{147157749\sinh^{-1}\Big(\frac{1-4x}{\sqrt{23}}}{4194304\sqrt{2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.54

$$
\int (5+2x)\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)\,dx = \frac{4\sqrt{3-x+2x^2}(1592737263+12357760788x+4870637856x^2+12669290112x^3+379086848x}{}
$$

```
[In] Integrate[(5 + 2*x)*(3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4),x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(1592737263 + 12357760788*x + 4870637856*x^2 + 12669
290112*x^3 + 379086848*x^4 + 12117893120*x^5 + 1033175040*x^6 + 2926837760*
x^7 + 1468006400*x^8) - 46354690935*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x +
4*x^2]])/2642411520
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.42

method	result
risch	$\frac{(1468006400x^8+2926837760x^7+1033175040x^6+12117893120x^5+379086848x^4+12669290112x^3+4870637856x^2+12357760788x+159}{660602880}$
trager	$\left(\frac{20}{9}x^8+\frac{319}{72}x^7+\frac{1051}{672}x^6+\frac{295847}{16128}x^5+\frac{26443}{46080}x^4+\frac{32992943}{1720320}x^3+\frac{2415991}{327680}x^2+\frac{343271133}{18350080}x+\frac{176970807}{73400320}\right)\sqrt{2x}$
default	$\frac{92727(4x-1)\left(2x^2-x+3\right)^{\frac{3}{2}}}{131072}+\frac{6398163\sqrt{2x^2-x+3}\,(4x-1)}{2097152}+\frac{147157749\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{8388608}+\frac{2005x^2\left(2x^2-x+3\right)^{\frac{5}{2}}}{8064}+$

```
[In] int((5+2*x)*(2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2),x,method=_RETURNVERBOSE
)
```

```
[Out] 1/660602880*(1468006400*x^8+2926837760*x^7+1033175040*x^6+12117893120*x^5+3
79086848*x^4+12669290112*x^3+4870637856*x^2+12357760788*x+1592737263)*(2*x^
2-x+3)^(1/2)+147157749/8388608*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.56

$$\int (5+2x)\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)dx=\frac{1}{660602880}\left(1468006400\,x^8+2926837760\,x^7+1033175040\,x^6+12117893120\,x^5+379086848\,x^4\right.$$
$$\left.+\frac{147157749}{16777216}\sqrt{2}\log\left(-4\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1)-32\,x^2+16\,x-25\right)\right.$$

```
[In] integrate((5+2*x)*(2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2),x, algorithm="fri
cas")
```

```
[Out] 1/660602880*(1468006400*x^8 + 2926837760*x^7 + 1033175040*x^6 + 12117893120
*x^5 + 379086848*x^4 + 12669290112*x^3 + 4870637856*x^2 + 12357760788*x + 1
592737263)*sqrt(2*x^2 - x + 3) + 147157749/16777216*sqrt(2)*log(-4*sqrt(2)*
sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25)
```

Sympy [A] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.54

$$\int (5+2x) \left(3-x+2x^2\right)^{3/2} \left(2+x+3x^2-x^3+5x^4\right) \, dx = \sqrt{2x^2-x+3} \cdot \left(\frac{20x^8}{9}+\frac{319x^7}{72}+\frac{1051x^6}{672}+\frac{295847x^5}{16128}+\frac{26443x^4}{46080}+\frac{32992943x^3}{1720320} + \frac{2415991x^2}{327680}+\frac{343271133x}{18350080}+\frac{176970807}{73400320}\right)+\frac{147157749\sqrt{2}\operatorname{asinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{8388608}$$

```
[In] integrate((5+2*x)*(2*x**2-x+3)**(3/2)*(5*x**4-x**3+3*x**2+x+2),x)
```

```
[Out] sqrt(2*x**2 - x + 3)*(20*x**8/9 + 319*x**7/72 + 1051*x**6/672 + 295847*x**5
/16128 + 26443*x**4/46080 + 32992943*x**3/1720320 + 2415991*x**2/327680 + 3
43271133*x/18350080 + 176970807/73400320) + 147157749*sqrt(2)*asinh(4*sqrt(
23)*(x - 1/4)/23)/8388608
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.93

$$\int (5+2x) \left(3-x+2x^2\right)^{3/2} \left(2+x+3x^2-x^3+5x^4\right) \, dx = \frac{5}{9} \left(2x^2-x+3\right)^{\frac{5}{2}} x^4 + \frac{479}{288} \left(2x^2-x+3\right)^{\frac{5}{2}} x^3 + \frac{2005}{8064} \left(2x^2-x+3\right)^{\frac{5}{2}} x^2 + \frac{5645}{21504} \left(2x^2-x+3\right)^{\frac{5}{2}} x + \frac{120809}{143360} \left(2x^2-x+3\right)^{\frac{5}{2}} + \frac{92727}{32768} \left(2x^2-x+3\right)^{\frac{3}{2}} x - \frac{92727}{131072} \left(2x^2-x+3\right)^{\frac{3}{2}} + \frac{6398163}{524288} \sqrt{2x^2-x+3}x + \frac{147157749}{8388608} \sqrt{2}\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}(4x-1)\right) - \frac{6398163}{2097152} \sqrt{2x^2-x+3}$$

```
[In] integrate((5+2*x)*(2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2),x, algorithm="max
ima")
```

```
[Out] 5/9*(2*x^2 - x + 3)^(5/2)*x^4 + 479/288*(2*x^2 - x + 3)^(5/2)*x^3 + 2005/80
64*(2*x^2 - x + 3)^(5/2)*x^2 + 5645/21504*(2*x^2 - x + 3)^(5/2)*x + 120809/
143360*(2*x^2 - x + 3)^(5/2) + 92727/32768*(2*x^2 - x + 3)^(3/2)*x - 92727/
131072*(2*x^2 - x + 3)^(3/2) + 6398163/524288*sqrt(2*x^2 - x + 3)*x + 14715
7749/8388608*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 6398163/2097152*sqr
t(2*x^2 - x + 3)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.53

$$\int (5+2x)\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)\,dx = \frac{1}{660602880}\left(4\left(8\left(4\left(16\left(20\left(8\left(28\left(160\,x+319\right)x+3153\right)x+295847\right)x+185101\right)x+98978829\right)x\right.\right.\right. - \frac{147157749}{8388608}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)+1\right)$$

```
[In] integrate((5+2*x)*(2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2),x, algorithm="gia
c")
```

```
[Out] 1/660602880*(4*(8*(4*(16*(20*(8*(28*(160*x + 319)*x + 3153)*x + 295847)*x +
 185101)*x + 98978829)*x + 152207433)*x + 3089440197)*x + 1592737263)*sqrt(
2*x^2 - x + 3) - 147157749/8388608*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt
(2*x^2 - x + 3)) + 1)
```

Mupad [F(-1)]

Timed out.

$$\int (5+2x)\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)\,dx = \int (2\,x+5)\left(2\,x^2-x+3\right)^{3/2}\left(5\,x^4-x^3+3\,x^2+x+2\right)\,dx$$

```
[In] int((2*x + 5)*(2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2),x)
```

```
[Out] int((2*x + 5)*(2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2), x)
```

3.335 $\int \left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 147

$$\begin{aligned}\int \left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)\,dx &= -\frac{593193(1-4x)\sqrt{3-x+2x^2}}{1048576}\\ &\quad-\frac{8597(1-4x)\left(3-x+2x^2\right)^{3/2}}{65536}+\frac{1167(3-x+2x^2)^{5/2}}{14336}+\frac{125x(3-x+2x^2)^{5/2}}{3584}\\ &\quad+\frac{23}{448}x^2\left(3-x+2x^2\right)^{5/2}+\frac{5}{16}x^3\left(3-x+2x^2\right)^{5/2}-\frac{13643439\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{2097152\sqrt{2}}\end{aligned}$$

```
[Out] -8597/65536*(1-4*x)*(2*x^2-x+3)^(3/2)+1167/14336*(2*x^2-x+3)^(5/2)+125/3584
*x*(2*x^2-x+3)^(5/2)+23/448*x^2*(2*x^2-x+3)^(5/2)+5/16*x^3*(2*x^2-x+3)^(5/2
)-13643439/4194304*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-593193/1048576*(1
-4*x)*(2*x^2-x+3)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {1675, 654, 626, 633, 221}

$$\begin{aligned}&\int \left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)\,dx =\\ &-\frac{13643439\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{2097152\sqrt{2}}+\frac{23}{448}\left(2x^2-x+3\right)^{5/2}x^2+\frac{125(2x^2-x+3)^{5/2}x}{3584}\\ &+\frac{1167(2x^2-x+3)^{5/2}}{14336}-\frac{8597(1-4x)\left(2x^2-x+3\right)^{3/2}}{65536}\\ &-\frac{593193(1-4x)\sqrt{2x^2-x+3}}{1048576}+\frac{5}{16}\left(2x^2-x+3\right)^{5/2}x^3\end{aligned}$$

```
[In] Int[(3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4),x]
```

```
[Out] (-593193*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/1048576 - (8597*(1 - 4*x)*(3 - x +
2*x^2)^(3/2))/65536 + (1167*(3 - x + 2*x^2)^(5/2))/14336 + (125*x*(3 - x +
2*x^2)^(5/2))/3584 + (23*x^2*(3 - x + 2*x^2)^(5/2))/448 + (5*x^3*(3 - x + 2
*x^2)^(5/2))/16 - (13643439*ArcSinh[(1 - 4*x)/Sqrt[23]])/(2097152*Sqrt[2])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{5}{16}x^3\left(3-x+2x^2\right)^{5/2}+\frac{1}{16}\int\left(3-x+2x^2\right)^{3/2}\left(32+16x+3x^2+\frac{23x^3}{2}\right)\,dx$$

$$
\begin{aligned}
&= \frac{23}{448}x^2\left(3-x+2x^2\right)^{5/2} \\
&\quad + \frac{5}{16}x^3\left(3-x+2x^2\right)^{5/2} + \frac{1}{224}\int \left(3-x+2x^2\right)^{3/2}\left(448+155x+\frac{375x^2}{4}\right)\,dx
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{125x\left(3-x+2x^2\right)^{5/2}}{3584} + \frac{23}{448}x^2\left(3-x+2x^2\right)^{5/2} \\
&\quad + \frac{5}{16}x^3\left(3-x+2x^2\right)^{5/2} + \frac{\int \left(\frac{20379}{4}+\frac{17505x}{8}\right)\left(3-x+2x^2\right)^{3/2}\,dx}{2688}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{1167\left(3-x+2x^2\right)^{5/2}}{14336} + \frac{125x\left(3-x+2x^2\right)^{5/2}}{3584} \\
&\quad + \frac{23}{448}x^2\left(3-x+2x^2\right)^{5/2} + \frac{5}{16}x^3\left(3-x+2x^2\right)^{5/2} + \frac{8597\int \left(3-x+2x^2\right)^{3/2}\,dx}{4096}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{8597(1-4x)\left(3-x+2x^2\right)^{3/2}}{65536} + \frac{1167\left(3-x+2x^2\right)^{5/2}}{14336} + \frac{125x\left(3-x+2x^2\right)^{5/2}}{3584} \\
&\quad + \frac{23}{448}x^2\left(3-x+2x^2\right)^{5/2} + \frac{5}{16}x^3\left(3-x+2x^2\right)^{5/2} + \frac{593193\int \sqrt{3-x+2x^2}\,dx}{131072}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{593193(1-4x)\sqrt{3-x+2x^2}}{1048576} - \frac{8597(1-4x)\left(3-x+2x^2\right)^{3/2}}{65536} \\
&\quad + \frac{1167\left(3-x+2x^2\right)^{5/2}}{14336} + \frac{125x\left(3-x+2x^2\right)^{5/2}}{3584} \\
&\quad + \frac{23}{448}x^2\left(3-x+2x^2\right)^{5/2} + \frac{5}{16}x^3\left(3-x+2x^2\right)^{5/2} + \frac{13643439\int \frac{1}{\sqrt{3-x+2x^2}}\,dx}{2097152}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{593193(1-4x)\sqrt{3-x+2x^2}}{1048576} - \frac{8597(1-4x)\left(3-x+2x^2\right)^{3/2}}{65536} \\
&\quad + \frac{1167\left(3-x+2x^2\right)^{5/2}}{14336} + \frac{125x\left(3-x+2x^2\right)^{5/2}}{3584} \\
&\quad + \frac{23}{448}x^2\left(3-x+2x^2\right)^{5/2} + \frac{5}{16}x^3\left(3-x+2x^2\right)^{5/2} + \frac{\left(593193\sqrt{\frac{23}{2}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx, x, -1+4x\right)}{2097152}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{593193(1-4x)\sqrt{3-x+2x^2}}{1048576} - \frac{8597(1-4x)\left(3-x+2x^2\right)^{3/2}}{65536} \\
&\quad + \frac{1167\left(3-x+2x^2\right)^{5/2}}{14336} + \frac{125x\left(3-x+2x^2\right)^{5/2}}{3584} \\
&\quad + \frac{23}{448}x^2\left(3-x+2x^2\right)^{5/2} + \frac{5}{16}x^3\left(3-x+2x^2\right)^{5/2} - \frac{13643439\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{2097152\sqrt{2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.58

$$\int \left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)\,dx=\frac{4\sqrt{3-x+2x^2}(-1663407+27845612x+3845856x^2+27023744x^3-7497728x^4+29335552x^5}{29360128}$$

```
[In] Integrate[(3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4),x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(-1663407 + 27845612*x + 3845856*x^2 + 27023744*x^3
- 7497728*x^4 + 29335552*x^5 - 7667712*x^6 + 9175040*x^7) - 95504073*Sqrt[2
]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/29360128
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.44

method	result
risch	$\frac{(9175040x^7-7667712x^6+29335552x^5-7497728x^4+27023744x^3+3845856x^2+27845612x-1663407)\sqrt{2x^2-x+3}}{7340032}+\frac{13643439\sqrt{2}\ \mathrm{arcs}}{41}$
trager	$\left(\frac{5}{4}x^7-\frac{117}{112}x^6+\frac{3581}{896}x^5-\frac{523}{512}x^4+\frac{211123}{57344}x^3+\frac{17169}{32768}x^2+\frac{6961403}{1835008}x-\frac{1663407}{7340032}\right)\sqrt{2x^2-x+3}-\frac{13643439}{}$
default	$\frac{1167(2x^2-x+3)^{\frac{5}{2}}}{14336}+\frac{8597(4x-1)(2x^2-x+3)^{\frac{3}{2}}}{65536}+\frac{593193\sqrt{2x^2-x+3}\,(4x-1)}{1048576}+\frac{13643439\sqrt{2}\ \mathrm{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{4194304}+\frac{5x^3(2x}{}$

```
[In] int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/7340032*(9175040*x^7-7667712*x^6+29335552*x^5-7497728*x^4+27023744*x^3+38
45856*x^2+27845612*x-1663407)*(2*x^2-x+3)^(1/2)+13643439/4194304*2^(1/2)*ar
csinh(4/23*23^(1/2)*(x-1/4))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.60

$$\int \left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)\,dx=\frac{1}{7340032}\left(9175040\,x^7-7667712\,x^6+29335552\,x^5-7497728\,x^4+27023744\,x^3+3845856\,x^2+2\right.$$
$$+\frac{13643439}{8388608}\sqrt{2}\log\left(-4\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1)-32\,x^2+16\,x-25\right)$$

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2),x, algorithm="fricas")
```

```
[Out] 1/7340032*(9175040*x^7 - 7667712*x^6 + 29335552*x^5 - 7497728*x^4 + 2702374
4*x^3 + 3845856*x^2 + 27845612*x - 1663407)*sqrt(2*x^2 - x + 3) + 13643439/
8388608*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*
x - 25)
```

Sympy [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.56

$$\int \left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)\,dx=\sqrt{2x^2-x+3}\cdot\left(\frac{5x^7}{4}-\frac{117x^6}{112}+\frac{3581x^5}{896}-\frac{523x^4}{512}+\frac{211123x^3}{57344}+\frac{17169x^2}{32768}+\frac{6961403x}{1835008}-\frac{1663407}{7340032}\right)+\frac{13643439\sqrt{2}\operatorname{asinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{4194304}$$

```
[In] integrate((2*x**2-x+3)**(3/2)*(5*x**4-x**3+3*x**2+x+2),x)
```

```
[Out] sqrt(2*x**2 - x + 3)*(5*x**7/4 - 117*x**6/112 + 3581*x**5/896 - 523*x**4/51
2 + 211123*x**3/57344 + 17169*x**2/32768 + 6961403*x/1835008 - 1663407/7340
032) + 13643439*sqrt(2)*asinh(4*sqrt(23)*(x - 1/4)/23)/4194304
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.94

$$\int \left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)\,dx=\frac{5}{16}\left(2x^2-x+3\right)^{\frac{5}{2}}x^3+\frac{23}{448}\left(2x^2-x+3\right)^{\frac{5}{2}}x^2+\frac{125}{3584}\left(2x^2-x+3\right)^{\frac{5}{2}}x+\frac{1167}{14336}\left(2x^2-x+3\right)^{\frac{5}{2}}+\frac{8597}{16384}\left(2x^2-x+3\right)^{\frac{3}{2}}x-\frac{8597}{65536}\left(2x^2-x+3\right)^{\frac{3}{2}}+\frac{593193}{262144}\sqrt{2x^2-x+3}\,x+\frac{13643439}{4194304}\sqrt{2}\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}(4x-1)\right)-\frac{593193}{1048576}\sqrt{2x^2-x+3}$$

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2),x, algorithm="maxima")
```

```
[Out] 5/16*(2*x^2 - x + 3)^(5/2)*x^3 + 23/448*(2*x^2 - x + 3)^(5/2)*x^2 + 125/358
4*(2*x^2 - x + 3)^(5/2)*x + 1167/14336*(2*x^2 - x + 3)^(5/2) + 8597/16384*(
2*x^2 - x + 3)^(3/2)*x - 8597/65536*(2*x^2 - x + 3)^(3/2) + 593193/262144*s
qrt(2*x^2 - x + 3)*x + 13643439/4194304*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x
- 1)) - 593193/1048576*sqrt(2*x^2 - x + 3)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.56

$$\int \left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3\right.$$
$$\left.+5x^4\right)\,dx=\frac{1}{7340032}\left(4\left(8\left(4\left(16\left(4\left(8\left(140\,x-117\right)x+3581\right)x-3661\right)x+211123\right)x+120183\right)x+6961403\right)$$
$$-\frac{13643439}{4194304}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)+1\right)$$

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2),x, algorithm="giac")
```

```
[Out] 1/7340032*(4*(8*(4*(16*(4*(8*(140*x - 117)*x + 3581)*x - 3661)*x + 211123)*
x + 120183)*x + 6961403)*x - 1663407)*sqrt(2*x^2 - x + 3) - 13643439/419430
4*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)
```

Mupad [F(-1)]

Timed out.

$$\int (3-x$$
$$+2x^2)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)\,dx=\int \left(2\,x^2-x+3\right)^{3/2}\left(5\,x^4-x^3+3\,x^2+x+2\right)\,dx$$

```
[In] int((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2),x)
```

```
[Out] int((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2), x)
```

3.336 $\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{5+2x}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 172

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{5+2x}\,dx = \frac{(141051019-23482924x)\sqrt{3-x+2x^2}}{65536} + \frac{(500141-123060x)(3-x+2x^2)^{3/2}}{12288} + \frac{3505}{896}(3-x+2x^2)^{5/2} - \frac{311}{448}(5+2x)(3-x+2x^2)^{5/2} + \frac{5}{112}(5+2x)^2(3-x+2x^2)^{5/2} + \frac{1622009981\,\mathrm{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{131072\sqrt{2}} - \frac{99009\,\mathrm{arctanh}\left(\frac{}{12}\right.}{8\sqrt{2}}$$

```
[Out] 1/12288*(500141-123060*x)*(2*x^2-x+3)^(3/2)+3505/896*(2*x^2-x+3)^(5/2)-311/
448*(5+2*x)*(2*x^2-x+3)^(5/2)+5/112*(5+2*x)^2*(2*x^2-x+3)^(5/2)+1622009981/
262144*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-99009/16*arctanh(1/24*(17-22*
x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)+1/65536*(141051019-23482924*x)*(2*x^2
-x+3)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {1667, 828, 857, 633, 221, 738, 212}

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{5+2x}\,dx = \frac{1622009981\,\mathrm{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{131072\sqrt{2}} - \frac{99009\,\mathrm{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{8\sqrt{2}} + \frac{5}{112}(2x+5)^2(2x^2-x+3)^{5/2} - \frac{311}{448}(2x+5)(2x^2-x+3)^{5/2} + \frac{3505}{896}(2x^2-x+3)^{5/2} + \frac{(500141-123060x)(2x^2-x+3)^{3/2}}{12288} + \frac{(141051019-23482924x)\sqrt{2x^2-x+3}}{65536}$$

```
[In] Int[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x),x]
```

```
[Out] ((141051019 - 23482924*x)*Sqrt[3 - x + 2*x^2])/65536 + ((500141 - 123060*x)
*(3 - x + 2*x^2)^(3/2))/12288 + (3505*(3 - x + 2*x^2)^(5/2))/896 - (311*(5
+ 2*x)*(3 - x + 2*x^2)^(5/2))/448 + (5*(5 + 2*x)^2*(3 - x + 2*x^2)^(5/2))/1
12 + (1622009981*ArcSinh[(1 - 4*x)/Sqrt[23]])/(131072*Sqrt[2]) - (99009*Arc
Tanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(8*Sqrt[2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{5}{112}(5+2x)^2\left(3-x+2x^2\right)^{5/2} \\
&\quad +\frac{1}{224}\int \frac{\left(3-x+2x^2\right)^{3/2}\left(573-9926x-14508x^2-7464x^3\right)}{5+2x}\,dx \\
&= -\frac{311}{448}(5+2x)\left(3-x+2x^2\right)^{5/2} \\
&\quad +\frac{5}{112}(5+2x)^2\left(3-x+2x^2\right)^{5/2}+\frac{\int \frac{\left(3-x+2x^2\right)^{3/2}\left(-430152+2062560x+1682400x^2\right)}{5+2x}\,dx}{21504} \\
&= \frac{3505}{896}\left(3-x+2x^2\right)^{5/2}-\frac{311}{448}(5+2x)\left(3-x+2x^2\right)^{5/2} \\
&\qquad +\frac{5}{112}(5+2x)^2\left(3-x+2x^2\right)^{5/2}+\frac{\int \frac{(24853920-68913600x)\left(3-x+2x^2\right)^{3/2}}{5+2x}\,dx}{860160} \\
&= \frac{(500141-123060x)\left(3-x+2x^2\right)^{3/2}}{12288}+\frac{3505}{896}\Big(3-x \\
&\qquad +2x^2\Big)^{5/2}-\frac{311}{448}(5+2x)\left(3-x+2x^2\right)^{5/2} \\
&\quad +\frac{5}{112}(5+2x)^2\left(3-x+2x^2\right)^{5/2}-\frac{\int \frac{(-29846322240+78902624640x)\sqrt{3-x+2x^2}}{5+2x}\,dx}{55050240}
\end{aligned}$$

$$= \frac{(141051019-23482924x)\sqrt{3-x+2x^2}}{65536}+\frac{(500141-123060x)\left(3-x+2x^2\right)^{3/2}}{12288} + \frac{3505}{896}\left(3-x+2x^2\right)^{5/2}-\frac{311}{448}(5+2x)\left(3-x+2x^2\right)^{5/2} + \frac{5}{112}(5+2x)^2\left(3-x+2x^2\right)^{5/2}+\frac{\int \frac{21812190368640-43599628289280x}{(5+2x)\sqrt{3-x+2x^2}}\,dx}{1761607680}$$

$$= \frac{(141051019-23482924x)\sqrt{3-x+2x^2}}{65536}+\frac{(500141-123060x)\left(3-x+2x^2\right)^{3/2}}{12288} + \frac{3505}{896}\left(3-x+2x^2\right)^{5/2}-\frac{311}{448}(5+2x)\left(3-x+2x^2\right)^{5/2} + \frac{5}{112}(5+2x)^2\left(3-x+2x^2\right)^{5/2}-\frac{1622009981\int \frac{1}{\sqrt{3-x+2x^2}}\,dx}{131072}+\frac{297027}{4}\int \frac{1}{(5+2x)\sqrt{3-x+2x^2}}\,dx$$

$$= \frac{(141051019-23482924x)\sqrt{3-x+2x^2}}{65536}+\frac{(500141-123060x)\left(3-x+2x^2\right)^{3/2}}{12288} + \frac{3505}{896}\left(3-x+2x^2\right)^{5/2}-\frac{311}{448}(5+2x)\left(3-x+2x^2\right)^{5/2} + \frac{5}{112}(5+2x)^2\left(3-x+2x^2\right)^{5/2}-\frac{297027}{2}\text{Subst}\left(\int \frac{1}{288-x^2}\,dx,x,\frac{17-22x}{\sqrt{3-x+2x^2}}\right)-\frac{1622009981\text{Su}}{}$$

$$= \frac{(141051019-23482924x)\sqrt{3-x+2x^2}}{65536}+\frac{(500141-123060x)\left(3-x+2x^2\right)^{3/2}}{12288} + \frac{3505}{896}\left(3-x+2x^2\right)^{5/2}-\frac{311}{448}(5+2x)\left(3-x+2x^2\right)^{5/2} + \frac{5}{112}(5+2x)^2\left(3-x+2x^2\right)^{5/2}+\frac{1622009981\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{131072\sqrt{2}}-\frac{99009\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{8\sqrt{2}}$$

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.66

$$\int \frac{\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)}{5+2x}\,dx = \frac{4\sqrt{3-x+2x^2}(3149403255-609499532x+159973408x}{}$$

```
[In] Integrate[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x),x
]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(3149403255 - 609499532*x + 159973408*x^2 - 46476672
*x^3 + 14493696*x^4 - 3710976*x^5 + 983040*x^6) + 68130865152*Sqrt[2]*ArcTa
nh[(5 + 2*x - Sqrt[6 - 2*x + 4*x^2])/6] + 34062209601*Sqrt[2]*Log[1 - 4*x +
 2*Sqrt[6 - 2*x + 4*x^2]])/5505024
```

Maple [F(-1)]

Timed out.

hanged

```
[In] int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x),x)
```

```
[Out] int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.78

$$\int \frac{(3-x+2x^2)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)}{5+2x}\,dx = \frac{1}{1376256}\big(983040\,x^6 - 3710976\,x^5 + 14493696\,x^4 - 4647$$
$$+ \frac{1622009981}{524288}\,\sqrt{2}\log\left(4\,\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1) - 32\,x^2 + 16\,x - 25\right)$$
$$+ \frac{99009}{32}\,\sqrt{2}\log\left(-\frac{24\,\sqrt{2}\sqrt{2\,x^2-x+3}(22\,x-17) + 1060\,x^2 - 1036\,x + 1153}{4\,x^2 + 20\,x + 25}\right)$$

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x),x, algorithm="fri
cas")
```

```
[Out] 1/1376256*(983040*x^6 - 3710976*x^5 + 14493696*x^4 - 46476672*x^3 + 1599734
08*x^2 - 609499532*x + 3149403255)*sqrt(2*x^2 - x + 3) + 1622009981/524288*
sqrt(2)*log(4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) +
 99009/32*sqrt(2)*log(-(24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) + 1060*x
^2 - 1036*x + 1153)/(4*x^2 + 20*x + 25))
```

Sympy [F]

$$\int \frac{(3-x+2x^2)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)}{5+2x}\,dx = \int \frac{(2x^2-x+3)^{\frac{3}{2}}\cdot(5x^4-x^3+3x^2+x+2)}{2x+5}\,dx$$

```
[In] integrate((2*x**2-x+3)**(3/2)*(5*x**4-x**3+3*x**2+x+2)/(5+2*x),x)
```

```
[Out] Integral((2*x**2 - x + 3)**(3/2)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)
, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.91

$$\int \frac{(3-x+2x^2)^{3/2}\,(2+x+3x^2-x^3+5x^4)}{5+2x}\,dx = \frac{5}{28}\left(2\,x^2-x+3\right)^{\frac{5}{2}}x^2 - \frac{111}{224}\left(2\,x^2-x+3\right)^{\frac{5}{2}}x + \frac{1395}{896}\left(2\,x^2-x+3\right)^{\frac{5}{2}} - \frac{10255}{1024}\left(2\,x^2-x+3\right)^{\frac{3}{2}}x + \frac{500141}{12288}\left(2\,x^2-x+3\right)^{\frac{3}{2}} - \frac{5870731}{16384}\sqrt{2\,x^2-x+3}x - \frac{1622009981}{262144}\sqrt{2}\,\mathrm{arsinh}\left(\frac{4}{23}\sqrt{23}x-\frac{1}{23}\sqrt{23}\right) + \frac{99009}{16}\sqrt{2}\,\mathrm{arsinh}\left(\frac{22\sqrt{23}x}{23\,|2\,x+5|}-\frac{17\sqrt{23}}{23\,|2\,x+5|}\right) + \frac{141051019}{65536}\sqrt{2\,x^2-x+3}$$

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x),x, algorithm="max
ima")
```

```
[Out] 5/28*(2*x^2 - x + 3)^(5/2)*x^2 - 111/224*(2*x^2 - x + 3)^(5/2)*x + 1395/896
*(2*x^2 - x + 3)^(5/2) - 10255/1024*(2*x^2 - x + 3)^(3/2)*x + 500141/12288*
(2*x^2 - x + 3)^(3/2) - 5870731/16384*sqrt(2*x^2 - x + 3)*x - 1622009981/26
2144*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) + 99009/16*sqrt(2)*ar
csinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)/abs(2*x + 5)) + 141051
019/65536*sqrt(2*x^2 - x + 3)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.81

$$\int \frac{(3-x+2x^2)^{3/2}\,(2+x+3x^2-x^3+5x^4)}{5+2x}\,dx = \frac{1}{1376256}\left(4\left(8\left(12\left(16\left(4\left(40\,x-151\right)x+2359\right)x-121033\right)\right.\right.\right.\right. + \frac{1622009981}{262144}\sqrt{2}\log\left(-4\sqrt{2}x+\sqrt{2}+4\sqrt{2\,x^2-x+3}\right) - \frac{99009}{16}\sqrt{2}\log\left(\left|-2\sqrt{2}x+\sqrt{2}+2\sqrt{2\,x^2-x+3}\right|\right) + \frac{99009}{16}\sqrt{2}\log\left(\left|-2\sqrt{2}x-11\sqrt{2}+2\sqrt{2\,x^2-x+3}\right|\right)$$

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x),x, algorithm="gia
c")
```

```
[Out] 1/1376256*(4*(8*(12*(16*(4*(40*x - 151)*x + 2359)*x - 121033)*x + 4999169)*
x - 152374883)*x + 3149403255)*sqrt(2*x^2 - x + 3) + 1622009981/262144*sqrt
```

```
(2)*log(-4*sqrt(2)*x + sqrt(2) + 4*sqrt(2*x^2 - x + 3)) - 99009/16*sqrt(2)*
log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 - x + 3))) + 99009/16*sqrt(2)
*log(abs(-2*sqrt(2)*x - 11*sqrt(2) + 2*sqrt(2*x^2 - x + 3)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(3-x+2x^2)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)}{5+2x}\,dx=\int \frac{{\left(2\,x^2-x+3\right)}^{3/2}\,\left(5\,x^4-x^3+3\,x^2+x+2\right)}{2\,x+5}\,dx$$

```
[In] int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5),x)
[Out] int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5), x)
```

3.337 $\int \frac{\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^2}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 172

$$\int \frac{\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^2}\,dx = -\frac{(85448933-14243732x)\sqrt{3-x+2x^2}}{32768} - \frac{(909513-226052x)\left(3-x+2x^2\right)^{3/2}}{18432} - \frac{839}{960}\left(3-x+2x^2\right)^{5/2} - \frac{3667\left(3-x+2x^2\right)^{5/2}}{576(5+2x)} + \frac{5}{96}(5+2x)\left(3-x+2x^2\right)^{5/2} - \frac{982669459\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{65536\sqrt{2}} + \frac{959625\operatorname{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{64\sqrt{2}}$$

```
[Out] -1/18432*(909513-226052*x)*(2*x^2-x+3)^(3/2)-839/960*(2*x^2-x+3)^(5/2)-3667
/576*(2*x^2-x+3)^(5/2)/(5+2*x)+5/96*(5+2*x)*(2*x^2-x+3)^(5/2)-982669459/131
072*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)+959625/128*arctanh(1/24*(17-22*x
)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)-1/32768*(85448933-14243732*x)*(2*x^2-x
+3)^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used

$= \{1664, 1667, 828, 857, 633, 221, 738, 212\}$

$$\int \frac{\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^2}\,dx = -\frac{982669459\text{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{65536\sqrt{2}} + \frac{959625\text{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{64\sqrt{2}} + \frac{5}{96}(2x+5)\left(2x^2-x+3\right)^{5/2} - \frac{3667\left(2x^2-x+3\right)^{5/2}}{576(2x+5)} - \frac{839}{960}\left(2x^2-x+3\right)^{5/2} - \frac{(909513-226052x)\left(2x^2-x+3\right)^{3/2}}{18432} - \frac{(85448933-14243732x)\sqrt{2x^2-x+3}}{32768}$$

```
[In] Int[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^2,x]
```

```
[Out] -1/32768*((85448933 - 14243732*x)*Sqrt[3 - x + 2*x^2]) - ((909513 - 226052*
x)*(3 - x + 2*x^2)^(3/2))/18432 - (839*(3 - x + 2*x^2)^(5/2))/960 - (3667*(
3 - x + 2*x^2)^(5/2))/(576*(5 + 2*x)) + (5*(5 + 2*x)*(3 - x + 2*x^2)^(5/2))
/96 - (982669459*ArcSinh[(1 - 4*x)/Sqrt[23]])/(65536*Sqrt[2]) + (959625*Arc
Tanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(64*Sqrt[2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
```

```
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

integral

$$= -\frac{3667(3-x+2x^2)^{5/2}}{576(5+2x)} - \frac{1}{72}\int \frac{(3-x+2x^2)^{3/2}\left(\frac{26675}{16}-4990x+486x^2-180x^3\right)}{5+2x}\,dx$$

$$= -\frac{3667(3-x+2x^2)^{5/2}}{576(5+2x)}$$
$$+\frac{5}{96}(5+2x)\left(3-x+2x^2\right)^{5/2} - \frac{\int \frac{(3-x+2x^2)^{3/2}(148350-406320x+120816x^2)}{5+2x}\,dx}{6912}$$

$$= -\frac{839}{960}\left(3-x+2x^2\right)^{5/2} - \frac{3667(3-x+2x^2)^{5/2}}{576(5+2x)}$$
$$+\frac{5}{96}(5+2x)\left(3-x+2x^2\right)^{5/2} - \frac{\int \frac{(8954400-27126240x)(3-x+2x^2)^{3/2}}{5+2x}\,dx}{276480}$$

$$= -\frac{(909513-226052x)\left(3-x+2x^2\right)^{3/2}}{18432} - \frac{839}{960}\left(3-x+2x^2\right)^{5/2} - \frac{3667(3-x+2x^2)^{5/2}}{576(5+2x)}$$
$$+\frac{5}{96}(5+2x)\left(3-x+2x^2\right)^{5/2} + \frac{\int \frac{(-11522887200+30766461120x)\sqrt{3-x+2x^2}}{5+2x}\,dx}{17694720}$$

$$= -\frac{(85448933-14243732x)\sqrt{3-x+2x^2}}{32768} - \frac{(909513-226052x)\left(3-x+2x^2\right)^{3/2}}{18432}$$
$$-\frac{839}{960}\left(3-x+2x^2\right)^{5/2} - \frac{3667(3-x+2x^2)^{5/2}}{576(5+2x)}$$
$$+\frac{5}{96}(5+2x)\left(3-x+2x^2\right)^{5/2} - \frac{\int \frac{8489566411200-16980528251520x}{(5+2x)\sqrt{3-x+2x^2}}\,dx}{566231040}$$

$$= -\frac{(85448933-14243732x)\sqrt{3-x+2x^2}}{32768} - \frac{(909513-226052x)\left(3-x+2x^2\right)^{3/2}}{18432}$$
$$-\frac{839}{960}\left(3-x+2x^2\right)^{5/2} - \frac{3667(3-x+2x^2)^{5/2}}{576(5+2x)}$$
$$+\frac{5}{96}(5+2x)\left(3-x+2x^2\right)^{5/2} + \frac{982669459\int \frac{1}{\sqrt{3-x+2x^2}}\,dx}{65536} - \frac{2878875}{32}\int \frac{1}{(5+2x)\sqrt{3-x+2x^2}}\,dx$$

$$= -\frac{(85448933-14243732x)\sqrt{3-x+2x^2}}{32768} - \frac{(909513-226052x)\left(3-x+2x^2\right)^{3/2}}{18432}$$
$$-\frac{839}{960}\left(3-x+2x^2\right)^{5/2} - \frac{3667(3-x+2x^2)^{5/2}}{576(5+2x)}$$
$$+\frac{5}{96}(5+2x)\left(3-x+2x^2\right)^{5/2} + \frac{2878875}{16}\text{Subst}\left(\int \frac{1}{288-x^2}\,dx, x, \frac{17-22x}{\sqrt{3-x+2x^2}}\right) + \frac{982669459\text{Subst}\Big(\int -}{6553}$$

$$
\begin{aligned}
&= -\frac{(85448933 - 14243732x)\sqrt{3 - x + 2x^2}}{32768} - \frac{(909513 - 226052x)\left(3 - x + 2x^2\right)^{3/2}}{18432} \\
&\quad - \frac{839}{960}\left(3 - x + 2x^2\right)^{5/2} - \frac{3667\left(3 - x + 2x^2\right)^{5/2}}{576(5 + 2x)} \\
&\quad + \frac{5}{96}(5+2x)\left(3-x+2x^2\right)^{5/2} - \frac{982669459\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{65536\sqrt{2}} + \frac{959625\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{64\sqrt{2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.70

$$
\int \frac{\left(3 - x + 2x^2\right)^{3/2}\left(2 + x + 3x^2 - x^3 + 5x^4\right)}{(5 + 2x)^2}\, dx = \frac{\frac{4\sqrt{3-x+2x^2}(-6814208295-1404323114x+182033816x^2-35369408x^3+82839}{5+2x}}{}
$$

```
[In] Integrate[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^2
,x]
```

```
[Out] ((4*Sqrt[3 - x + 2*x^2]*(-6814208295 - 1404323114*x + 182033816*x^2 - 35369
408*x^3 + 8283904*x^4 - 1798144*x^5 + 409600*x^6))/(5 + 2*x) - 29479680000*
Sqrt[2]*ArcTanh[(5 + 2*x - Sqrt[6 - 2*x + 4*x^2])/6] - 14740041885*Sqrt[2]*
Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/1966080
```

Maple [F(-1)]

Timed out.

hanged

```
[In] int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2,x)
```

```
[Out] int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.89

$$
\int \frac{\left(3 - x + 2x^2\right)^{3/2}\left(2 + x + 3x^2 - x^3 + 5x^4\right)}{(5 + 2x)^2}\, dx = \frac{14740041885\sqrt{2}(2x + 5)\log\left(-4\sqrt{2}\sqrt{2x^2 - x + 3}(4x - \right.}{}
$$

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2,x, algorithm="f
ricas")
```

```
[Out] 1/3932160*(14740041885*sqrt(2)*(2*x + 5)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)
*(4*x - 1) - 32*x^2 + 16*x - 25) + 14739840000*sqrt(2)*(2*x + 5)*log((24*sq
rt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) - 1060*x^2 + 1036*x - 1153)/(4*x^2 +
20*x + 25)) + 8*(409600*x^6 - 1798144*x^5 + 8283904*x^4 - 35369408*x^3 + 18
2033816*x^2 - 1404323114*x - 6814208295)*sqrt(2*x^2 - x + 3))/(2*x + 5)
```

Sympy [F]

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^2}\,dx = \int \frac{(2x^2-x+3)^{\frac{3}{2}}\cdot(5x^4-x^3+3x^2+x+2)}{(2x+5)^2}\,dx$$

```
[In] integrate((2*x**2-x+3)**(3/2)*(5*x**4-x**3+3*x**2+x+2)/(5+2*x)**2,x)
```

```
[Out] Integral((2*x**2 - x + 3)**(3/2)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)
**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.94

$$\begin{aligned}\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^2}\,dx &= \frac{5}{48}\left(2x^2-x+3\right)^{\frac{5}{2}}x \\ &- \frac{589}{960}\left(2x^2-x+3\right)^{\frac{5}{2}} + \frac{9059}{1536}\left(2x^2-x+3\right)^{\frac{3}{2}}x - \frac{185827}{6144}\left(2x^2-x+3\right)^{\frac{3}{2}} \\ &+ \frac{3560933}{8192}\sqrt{2x^2-x+3}\,x + \frac{982669459}{131072}\sqrt{2}\,\operatorname{arsinh}\left(\frac{4}{23}\sqrt{23}x - \frac{1}{23}\sqrt{23}\right) \\ &- \frac{959625}{128}\sqrt{2}\,\operatorname{arsinh}\left(\frac{22\sqrt{23}x}{23\,|2x+5|} - \frac{17\sqrt{23}}{23\,|2x+5|}\right) \\ &- \frac{85448933}{32768}\sqrt{2x^2-x+3} - \frac{3667\left(2x^2-x+3\right)^{\frac{3}{2}}}{32\,(2x+5)}\end{aligned}$$

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2,x, algorithm="m
axima")
```

```
[Out] 5/48*(2*x^2 - x + 3)^(5/2)*x - 589/960*(2*x^2 - x + 3)^(5/2) + 9059/1536*(2
*x^2 - x + 3)^(3/2)*x - 185827/6144*(2*x^2 - x + 3)^(3/2) + 3560933/8192*sq
rt(2*x^2 - x + 3)*x + 982669459/131072*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/
23*sqrt(23)) - 959625/128*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 1
7/23*sqrt(23)/abs(2*x + 5)) - 85448933/32768*sqrt(2*x^2 - x + 3) - 3667/32*
(2*x^2 - x + 3)^(3/2)/(2*x + 5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 707 vs. $2(137) = 274$.

Time = 0.36 (sec) , antiderivative size = 707, normalized size of antiderivative = 4.11

$$\int \frac{(3-x+2x^2)^{3/2}\,(2+x+3x^2-x^3+5x^4)}{(5+2x)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2,x, algorithm="g
iac")
```

```
[Out] 1/1966080*sqrt(2)*(14739840000*log(12*sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 +
 1) + 72/(2*x + 5) - 11)*sgn(1/(2*x + 5)) + 14740041885*log(abs(sqrt(-11/(2
*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5) + 1))*sgn(1/(2*x + 5)) - 147400
41885*log(abs(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5) - 1))*
sgn(1/(2*x + 5)) - 2027704320*sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1)*sgn(
1/(2*x + 5)) + 2*(45496763235*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6
/(2*x + 5))^11*sgn(1/(2*x + 5)) - 126553743360*(sqrt(-11/(2*x + 5) + 36/(2*
x + 5)^2 + 1) + 6/(2*x + 5))^10*sgn(1/(2*x + 5)) + 44062768335*(sqrt(-11/(2
*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^9*sgn(1/(2*x + 5)) + 331789824
00*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^8*sgn(1/(2*x +
5)) + 294206421582*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))
^7*sgn(1/(2*x + 5)) - 463672074240*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1
) + 6/(2*x + 5))^6*sgn(1/(2*x + 5)) + 35099942478*(sqrt(-11/(2*x + 5) + 36/
(2*x + 5)^2 + 1) + 6/(2*x + 5))^5*sgn(1/(2*x + 5)) + 171324610560*(sqrt(-11
/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^4*sgn(1/(2*x + 5)) + 600592
81615*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^3*sgn(1/(2*x
 + 5)) - 105051009024*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x +
5))^2*sgn(1/(2*x + 5)) - 5210329245*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 +
1) + 6/(2*x + 5))*sgn(1/(2*x + 5)) + 17058392064*sgn(1/(2*x + 5)))/((sqrt(-
11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^2 - 1)^6)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(3-x+2x^2)^{3/2}\,(2+x+3x^2-x^3+5x^4)}{(5+2x)^2}\,dx = \int \frac{{\left(2\,x^2-x+3\right)}^{3/2}\,\left(5\,x^4-x^3+3\,x^2+x+2\right)}{{\left(2\,x+5\right)}^2}\,dx$$

```
[In] int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^2,x)
```

```
[Out] int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^2, x)
```

3.338 $\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^3}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 174

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^3}\,dx = \frac{(33741483-5623292x)\sqrt{3-x+2x^2}}{24576} + \frac{(2154633-534617x)(3-x+2x^2)^{3/2}}{82944} + \frac{1}{16}(3-x+2x^2)^{5/2} - \frac{3667(3-x+2x^2)^{5/2}}{1152(5+2x)^2} + \frac{438065(3-x+2x^2)^{5/2}}{82944(5+2x)} + \frac{129342063\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{16384\sqrt{2}} - \frac{8083915\operatorname{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{1024\sqrt{2}}$$

```
[Out] 1/82944*(2154633-534617*x)*(2*x^2-x+3)^(3/2)+1/16*(2*x^2-x+3)^(5/2)-3667/11
52*(2*x^2-x+3)^(5/2)/(5+2*x)^2+438065/82944*(2*x^2-x+3)^(5/2)/(5+2*x)+12934
2063/32768*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-8083915/2048*arctanh(1/24
*(17-22*x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)+1/24576*(33741483-5623292*x)*
(2*x^2-x+3)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used

$= \{1664, 1667, 828, 857, 633, 221, 738, 212\}$

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^3}\,dx = \frac{129342063\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{16384\sqrt{2}} - \frac{8083915\operatorname{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{1024\sqrt{2}} + \frac{438065(2x^2-x+3)^{5/2}}{82944(2x+5)} - \frac{3667(2x^2-x+3)^{5/2}}{1152(2x+5)^2} + \frac{1}{16}(2x^2-x+3)^{5/2} + \frac{(2154633-534617x)(2x^2-x+3)^{3/2}}{82944} + \frac{(33741483-5623292x)\sqrt{2x^2-x+3}}{24576}$$

```
[In] Int[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^3,x]
```

```
[Out] ((33741483 - 5623292*x)*Sqrt[3 - x + 2*x^2])/24576 + ((2154633 - 534617*x)*
(3 - x + 2*x^2)^(3/2))/82944 + (3 - x + 2*x^2)^(5/2)/16 - (3667*(3 - x + 2*
x^2)^(5/2))/(1152*(5 + 2*x)^2) + (438065*(3 - x + 2*x^2)^(5/2))/(82944*(5 +
 2*x)) + (129342063*ArcSinh[(1 - 4*x)/Sqrt[23]])/(16384*Sqrt[2]) - (8083915
*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(1024*Sqrt[2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
```

```
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{3667(3-x+2x^2)^{5/2}}{1152(5+2x)^2} \\
&\quad -\frac{1}{144}\int \frac{(3-x+2x^2)^{3/2}\left(\frac{35015}{16}-\frac{21585x}{4}+972x^2-360x^3\right)}{(5+2x)^2}\,dx \\
&= -\frac{3667(3-x+2x^2)^{5/2}}{1152(5+2x)^2}+\frac{438065(3-x+2x^2)^{5/2}}{82944(5+2x)}+\frac{\int \frac{(3-x+2x^2)^{3/2}\left(\frac{2737465}{16}-505457x+12960x^2\right)}{5+2x}\,dx}{10368} \\
&= \frac{1}{16}\left(3-x+2x^2\right)^{5/2}-\frac{3667(3-x+2x^2)^{5/2}}{1152(5+2x)^2} \\
&\quad +\frac{438065(3-x+2x^2)^{5/2}}{82944(5+2x)}+\frac{\int \frac{\left(\frac{14335325}{2}-21384680x\right)(3-x+2x^2)^{3/2}}{5+2x}\,dx}{414720} \\
&= \frac{(2154633-534617x)\left(3-x+2x^2\right)^{3/2}}{82944}+\frac{1}{16}\left(3-x+2x^2\right)^{5/2}-\frac{3667(3-x+2x^2)^{5/2}}{1152(5+2x)^2} \\
&\quad +\frac{438065(3-x+2x^2)^{5/2}}{82944(5+2x)}-\frac{\int \frac{(-9113472000+24292621440x)\sqrt{3-x+2x^2}}{5+2x}\,dx}{26542080} \\
&= \frac{(33741483-5623292x)\sqrt{3-x+2x^2}}{24576}+\frac{(2154633-534617x)\left(3-x+2x^2\right)^{3/2}}{82944} \\
&\quad +\frac{1}{16}\left(3-x+2x^2\right)^{5/2}-\frac{3667(3-x+2x^2)^{5/2}}{1152(5+2x)^2} \\
&\quad +\frac{438065(3-x+2x^2)^{5/2}}{82944(5+2x)}+\frac{\int \frac{6705272016000-13410185091840x}{(5+2x)\sqrt{3-x+2x^2}}\,dx}{849346560} \\
&= \frac{(33741483-5623292x)\sqrt{3-x+2x^2}}{24576}+\frac{(2154633-534617x)\left(3-x+2x^2\right)^{3/2}}{82944} \\
&\quad +\frac{1}{16}\left(3-x+2x^2\right)^{5/2}-\frac{3667(3-x+2x^2)^{5/2}}{1152(5+2x)^2}+\frac{438065(3-x+2x^2)^{5/2}}{82944(5+2x)} \\
&\quad -\frac{129342063\int \frac{1}{\sqrt{3-x+2x^2}}\,dx}{16384}+\frac{24251745}{512}\int \frac{1}{(5+2x)\sqrt{3-x+2x^2}}\,dx \\
&= \frac{(33741483-5623292x)\sqrt{3-x+2x^2}}{24576}+\frac{(2154633-534617x)\left(3-x+2x^2\right)^{3/2}}{82944} \\
&\quad +\frac{1}{16}\left(3-x+2x^2\right)^{5/2}-\frac{3667(3-x+2x^2)^{5/2}}{1152(5+2x)^2}+\frac{438065(3-x+2x^2)^{5/2}}{82944(5+2x)} \\
&\quad -\frac{24251745}{256}\text{Subst}\left(\int \frac{1}{288-x^2}\,dx,x,\frac{17-22x}{\sqrt{3-x+2x^2}}\right)-\frac{129342063\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx,x,-1+4x\right)}{16384\sqrt{46}}
\end{aligned}
$$

$$= \frac{(33741483-5623292x)\sqrt{3-x+2x^2}}{24576}+\frac{(2154633-534617x)\left(3-x+2x^2\right)^{3/2}}{82944}$$
$$+\frac{1}{16}\left(3-x+2x^2\right)^{5/2}-\frac{3667\left(3-x+2x^2\right)^{5/2}}{1152(5+2x)^2}+\frac{438065\left(3-x+2x^2\right)^{5/2}}{82944(5+2x)}$$
$$+\frac{129342063\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{16384\sqrt{2}}-\frac{8083915\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{1024\sqrt{2}}$$

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.69

$$\int\frac{\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^3}\,dx=\frac{\frac{4\sqrt{3-x+2x^2}\bigl(298966737+181223072x+16667188x^2-1620944x^3+253312x^4-}{(5+2x)^2}}{}$$

```
[In] Integrate[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^3
,x]
```

```
[Out] ((4*Sqrt[3 - x + 2*x^2]*(298966737 + 181223072*x + 16667188*x^2 - 1620944*x
^3 + 253312*x^4 - 43520*x^5 + 8192*x^6))/(5 + 2*x)^2 + 258685280*Sqrt[2]*Ar
cTanh[(5 + 2*x - Sqrt[6 - 2*x + 4*x^2])/6] + 129342063*Sqrt[2]*Log[1 - 4*x
+ 2*Sqrt[6 - 2*x + 4*x^2]])/32768
```

Maple [F(-1)]

Timed out.

hanged

```
[In] int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3,x)
```

```
[Out] int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.97

$$\int\frac{\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^3}\,dx=\frac{129342063\sqrt{2}(4x^2+20x+25)\log\bigl(4\sqrt{2}\sqrt{2x^2-x+3}{}$$

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3,x, algorithm="f
ricas")
```

```
[Out] 1/65536*(129342063*sqrt(2)*(4*x^2 + 20*x + 25)*log(4*sqrt(2)*sqrt(2*x^2 - x
 + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 129342640*sqrt(2)*(4*x^2 + 20*x + 2
5)*log(-(24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) + 1060*x^2 - 1036*x + 1
153)/(4*x^2 + 20*x + 25)) + 8*(8192*x^6 - 43520*x^5 + 253312*x^4 - 1620944*
x^3 + 16667188*x^2 + 181223072*x + 298966737)*sqrt(2*x^2 - x + 3))/(4*x^2 +
 20*x + 25)
```

Sympy [F]

$$\int \frac{(3-x+2x^2)^{3/2}\,(2+x+3x^2-x^3+5x^4)}{(5+2x)^3}\,dx = \int \frac{(2x^2-x+3)^{\frac{3}{2}}\cdot(5x^4-x^3+3x^2+x+2)}{(2x+5)^3}\,dx$$

```
[In] integrate((2*x**2-x+3)**(3/2)*(5*x**4-x**3+3*x**2+x+2)/(5+2*x)**3,x)
```

```
[Out] Integral((2*x**2 - x + 3)**(3/2)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)
**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.99

$$\begin{aligned}\int \frac{(3-x+2x^2)^{3/2}\,(2+x+3x^2-x^3+5x^4)}{(5+2x)^3}\,dx &= \frac{1}{16}\left(2\,x^2-x+3\right)^{\frac{5}{2}} \\ &- \frac{149}{128}\left(2\,x^2-x+3\right)^{\frac{3}{2}}x + \frac{46691}{4608}\left(2\,x^2-x+3\right)^{\frac{3}{2}} - \frac{3667\,(2\,x^2-x+3)^{\frac{5}{2}}}{1152\,(4\,x^2+20\,x+25)} \\ &- \frac{1405823}{6144}\sqrt{2\,x^2-x+3}x - \frac{129342063}{32768}\sqrt{2}\,\mathrm{arsinh}\left(\frac{4}{23}\sqrt{23}x - \frac{1}{23}\sqrt{23}\right) \\ &+ \frac{8083915}{2048}\sqrt{2}\,\mathrm{arsinh}\left(\frac{22\sqrt{23}x}{23\,|2\,x+5|} - \frac{17\sqrt{23}}{23\,|2\,x+5|}\right) \\ &+ \frac{11247161}{8192}\sqrt{2\,x^2-x+3} + \frac{438065\,(2\,x^2-x+3)^{\frac{3}{2}}}{4608\,(2\,x+5)}\end{aligned}$$

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3,x, algorithm="m
axima")
```

```
[Out] 1/16*(2*x^2 - x + 3)^(5/2) - 149/128*(2*x^2 - x + 3)^(3/2)*x + 46691/4608*(
2*x^2 - x + 3)^(3/2) - 3667/1152*(2*x^2 - x + 3)^(5/2)/(4*x^2 + 20*x + 25)
- 1405823/6144*sqrt(2*x^2 - x + 3)*x - 129342063/32768*sqrt(2)*arcsinh(4/23
*sqrt(23)*x - 1/23*sqrt(23)) + 8083915/2048*sqrt(2)*arcsinh(22/23*sqrt(23)*
x/abs(2*x + 5) - 17/23*sqrt(23)/abs(2*x + 5)) + 11247161/8192*sqrt(2*x^2 -
x + 3) + 438065/4608*(2*x^2 - x + 3)^(3/2)/(2*x + 5)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.54

$$\int \frac{(3-x+2x^2)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^3}\,dx = \frac{1}{8192}\left(4\left(8\left(4\left(16x-165\right)x+4879\right)x-263469\right)x+8460\right.$$

$$+\frac{129342063}{32768}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right)$$

$$-\frac{8083915}{2048}\sqrt{2}\log\left(\left|-2\sqrt{2}x+\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right)$$

$$+\frac{8083915}{2048}\sqrt{2}\log\left(\left|-2\sqrt{2}x-11\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right)$$

$$+\frac{\sqrt{2}\left(14243182\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^3+109906674\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^2-170996871\sqrt{2}\left(\sqrt{2}x-\right.\right.}{512\left(2\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^2+10\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)-11\right)^2}$$

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3,x, algorithm="g
iac")
```

```
[Out] 1/8192*(4*(8*(4*(16*x - 165)*x + 4879)*x - 263469)*x + 8460377)*sqrt(2*x^2
 - x + 3) + 129342063/32768*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 -
 x + 3)) + 1) - 8083915/2048*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqr
t(2*x^2 - x + 3))) + 8083915/2048*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2)
 + 2*sqrt(2*x^2 - x + 3))) + 1/512*sqrt(2)*(14243182*sqrt(2)*(sqrt(2)*x - s
qrt(2*x^2 - x + 3))^3 + 109906674*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 - 170
996871*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 110506087)/(2*(sqrt(2)*x
 - sqrt(2*x^2 - x + 3))^2 + 10*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) -
11)^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(3-x+2x^2)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^3}\,dx = \int \frac{\left(2x^2-x+3\right)^{3/2}\left(5x^4-x^3+3x^2+x+2\right)}{\left(2x+5\right)^3}\,dx$$

```
[In] int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^3,x)
```

```
[Out] int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^3, x)
```

3.339 $\int \frac{\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^4}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 181

$$\begin{aligned}\int \frac{\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^4}\,dx =& -\frac{(135068604-22512089x)\sqrt{3-x+2x^2}}{331776} \\ &-\frac{(138006843-34265045x)\left(3-x+2x^2\right)^{3/2}}{17915904}-\frac{3667\left(3-x+2x^2\right)^{5/2}}{1728(5+2x)^3} \\ &+\frac{556255\left(3-x+2x^2\right)^{5/2}}{248832(5+2x)^2}-\frac{32865365\left(3-x+2x^2\right)^{5/2}}{17915904(5+2x)} \\ &-\frac{19176431\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{8192\sqrt{2}}+\frac{517762327\operatorname{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{221184\sqrt{2}}\end{aligned}$$

```
[Out] -1/17915904*(138006843-34265045*x)*(2*x^2-x+3)^(3/2)-3667/1728*(2*x^2-x+3)^
(5/2)/(5+2*x)^3+556255/248832*(2*x^2-x+3)^(5/2)/(5+2*x)^2-32865365/17915904
*(2*x^2-x+3)^(5/2)/(5+2*x)-19176431/16384*arcsinh(1/23*(1-4*x)*23^(1/2))*2^
(1/2)+517762327/442368*arctanh(1/24*(17-22*x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^
(1/2)-1/331776*(135068604-22512089*x)*(2*x^2-x+3)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$, Rules used = {1664, 828, 857, 633, 221, 738, 212}

$$\begin{aligned}
&\int \frac{(3-x+2x^2)^{3/2}\,(2+x+3x^2-x^3+5x^4)}{(5+2x)^4}\,dx = \\
&-\frac{19176431\text{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{8192\sqrt{2}}+\frac{517762327\text{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{221184\sqrt{2}} \\
&-\frac{32865365(2x^2-x+3)^{5/2}}{17915904(2x+5)}+\frac{556255(2x^2-x+3)^{5/2}}{248832(2x+5)^2} \\
&-\frac{3667(2x^2-x+3)^{5/2}}{1728(2x+5)^3}-\frac{(138006843-34265045x)\,(2x^2-x+3)^{3/2}}{17915904} \\
&-\frac{(135068604-22512089x)\sqrt{2x^2-x+3}}{331776}
\end{aligned}$$

```
[In] Int[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^4,x]
```

```
[Out] -1/331776*((135068604 - 22512089*x)*Sqrt[3 - x + 2*x^2]) - ((138006843 - 34
265045*x)*(3 - x + 2*x^2)^(3/2))/17915904 - (3667*(3 - x + 2*x^2)^(5/2))/(1
728*(5 + 2*x)^3) + (556255*(3 - x + 2*x^2)^(5/2))/(248832*(5 + 2*x)^2) - (3
2865365*(3 - x + 2*x^2)^(5/2))/(17915904*(5 + 2*x)) - (19176431*ArcSinh[(1
- 4*x)/Sqrt[23]])/(8192*Sqrt[2]) + (517762327*ArcTanh[(17 - 22*x)/(12*Sqrt[
2]*Sqrt[3 - x + 2*x^2])])/(221184*Sqrt[2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{3667(3-x+2x^2)^{5/2}}{1728(5+2x)^3}\\ &\quad -\frac{1}{216}\int\frac{(3-x+2x^2)^{3/2}\left(\frac{43355}{16}-\frac{11605x}{2}+1458x^2-540x^3\right)}{(5+2x)^3}\,dx\end{aligned}$$

$$= -\frac{3667\left(3-x+2x^2\right)^{5/2}}{1728(5+2x)^3} + \frac{556255\left(3-x+2x^2\right)^{5/2}}{248832(5+2x)^2} + \frac{\int \frac{\left(3-x+2x^2\right)^{3/2}\left(\frac{4202675}{16}-\frac{2477469x}{4}+38880x^2\right)}{(5+2x)^2}\,dx}{31104}$$

$$= -\frac{3667\left(3-x+2x^2\right)^{5/2}}{1728(5+2x)^3} + \frac{556255\left(3-x+2x^2\right)^{5/2}}{248832(5+2x)^2} - \frac{32865365\left(3-x+2x^2\right)^{5/2}}{17915904(5+2x)} - \frac{\int \frac{\left(\frac{182685181}{16}-34265045x\right)\left(3-x+2x^2\right)^{3/2}}{5+2x}\,dx}{2239488}$$

$$= -\frac{(138006843-34265045x)\left(3-x+2x^2\right)^{3/2}}{17915904} - \frac{3667\left(3-x+2x^2\right)^{5/2}}{1728(5+2x)^3} + \frac{556255\left(3-x+2x^2\right)^{5/2}}{248832(5+2x)^2} - \frac{32865365\left(3-x+2x^2\right)^{5/2}}{17915904(5+2x)} + \frac{\int \frac{(-14584438152+38900889792x)\sqrt{3-x+2x^2}}{5+2x}\,dx}{143327232}$$

$$= -\frac{(135068604-22512089x)\sqrt{3-x+2x^2}}{331776} - \frac{(138006843-34265045x)\left(3-x+2x^2\right)^{3/2}}{17915904} - \frac{3667\left(3-x+2x^2\right)^{5/2}}{1728(5+2x)^3} + \frac{556255\left(3-x+2x^2\right)^{5/2}}{248832(5+2x)^2} - \frac{32865365\left(3-x+2x^2\right)^{5/2}}{17915904(5+2x)} - \frac{\int \frac{10736183791872-21472693553664x}{(5+2x)\sqrt{3-x+2x^2}}\,dx}{4586471424}$$

$$= -\frac{(135068604-22512089x)\sqrt{3-x+2x^2}}{331776} - \frac{(138006843-34265045x)\left(3-x+2x^2\right)^{3/2}}{17915904} - \frac{3667\left(3-x+2x^2\right)^{5/2}}{1728(5+2x)^3} + \frac{556255\left(3-x+2x^2\right)^{5/2}}{248832(5+2x)^2} - \frac{32865365\left(3-x+2x^2\right)^{5/2}}{17915904(5+2x)} + \frac{19176431\int \frac{1}{\sqrt{3-x+2x^2}}\,dx}{8192} - \frac{517762327\int \frac{1}{(5+2x)\sqrt{3-x+2x^2}}\,dx}{36864}$$

$$= -\frac{(135068604-22512089x)\sqrt{3-x+2x^2}}{331776} - \frac{(138006843-34265045x)\left(3-x+2x^2\right)^{3/2}}{17915904} - \frac{3667\left(3-x+2x^2\right)^{5/2}}{1728(5+2x)^3} + \frac{556255\left(3-x+2x^2\right)^{5/2}}{248832(5+2x)^2} - \frac{32865365\left(3-x+2x^2\right)^{5/2}}{17915904(5+2x)} + \frac{517762327\text{Subst}\left(\int \frac{1}{288-x^2}\,dx, x, \frac{17-22x}{\sqrt{3-x+2x^2}}\right)}{18432} + \frac{19176431\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx, x, -1+4x\right)}{8192\sqrt{46}}$$

$$
\begin{aligned}
&= -\frac{(135068604-22512089x)\sqrt{3-x+2x^2}}{331776} \\
&\quad -\frac{(138006843-34265045x)\left(3-x+2x^2\right)^{3/2}}{17915904}-\frac{3667\left(3-x+2x^2\right)^{5/2}}{1728(5+2x)^3} \\
&\quad +\frac{556255\left(3-x+2x^2\right)^{5/2}}{248832(5+2x)^2}-\frac{32865365\left(3-x+2x^2\right)^{5/2}}{17915904(5+2x)} \\
&\quad -\frac{19176431\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{8192\sqrt{2}}+\frac{517762327\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{221184\sqrt{2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.66

$$
\int \frac{\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^4}\,dx=\frac{\frac{12\sqrt{3-x+2x^2}\left(-1994650739-2006873194x-594798908x^2-33595416x^3+2626\right.}{(5+2x)^3}}{}
$$

```
[In] Integrate[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^4
,x]
```

```
[Out] ((12*Sqrt[3 - x + 2*x^2]*(-1994650739 - 2006873194*x - 594798908*x^2 - 3359
5416*x^3 + 2626848*x^4 - 315648*x^5 + 46080*x^6))/(5 + 2*x)^3 - 1035524654*
Sqrt[2]*ArcTanh[(5 + 2*x - Sqrt[6 - 2*x + 4*x^2])/6] - 517763637*Sqrt[2]*Lo
g[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/442368
```

Maple [F(-1)]

Timed out.

hanged

```
[In] int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4,x)
```

```
[Out] int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.01

$$
\int \frac{\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^4}\,dx=\frac{517763637\sqrt{2}(8x^3+60x^2+150x+125)\log\left(-4\sqrt{2}\sqrt{2}\right.}{}
$$

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4,x, algorithm="f
ricas")
```

```
[Out] 1/884736*(517763637*sqrt(2)*(8*x^3 + 60*x^2 + 150*x + 125)*log(-4*sqrt(2)*s
qrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 517762327*sqrt(2)*(8*x
^3 + 60*x^2 + 150*x + 125)*log((24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17)
- 1060*x^2 + 1036*x - 1153)/(4*x^2 + 20*x + 25)) + 24*(46080*x^6 - 315648*x
^5 + 2626848*x^4 - 33595416*x^3 - 594798908*x^2 - 2006873194*x - 1994650739
)*sqrt(2*x^2 - x + 3))/(8*x^3 + 60*x^2 + 150*x + 125)
```

Sympy [F]

$$\int \frac{(3-x+2x^2)^{3/2}\,(2+x+3x^2-x^3+5x^4)}{(5+2x)^4}\,dx = \int \frac{(2x^2-x+3)^{\frac{3}{2}}\cdot(5x^4-x^3+3x^2+x+2)}{(2x+5)^4}\,dx$$

```
[In] integrate((2*x**2-x+3)**(3/2)*(5*x**4-x**3+3*x**2+x+2)/(5+2*x)**4,x)
```

```
[Out] Integral((2*x**2 - x + 3)**(3/2)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)
**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.04

$$\begin{aligned}\int \frac{(3-x+2x^2)^{3/2}\,(2+x+3x^2-x^3+5x^4)}{(5+2x)^4}\,dx &= \frac{5}{64}\left(2\,x^2-x+3\right)^{\frac{3}{2}}x \\ &- \frac{1094743}{497664}\left(2\,x^2-x+3\right)^{\frac{3}{2}} - \frac{3667\left(2\,x^2-x+3\right)^{\frac{5}{2}}}{1728\,(8\,x^3+60\,x^2+150\,x+125)} \\ &+ \frac{556255\left(2\,x^2-x+3\right)^{\frac{5}{2}}}{248832\,(4\,x^2+20\,x+25)} + \frac{22512089}{331776}\sqrt{2\,x^2-x+3}x \\ &+ \frac{19176431}{16384}\sqrt{2}\,\mathrm{arsinh}\left(\frac{4}{23}\sqrt{23}x-\frac{1}{23}\sqrt{23}\right) \\ &- \frac{517762327}{442368}\sqrt{2}\,\mathrm{arsinh}\left(\frac{22\sqrt{23}x}{23\,|2\,x+5|}-\frac{17\sqrt{23}}{23\,|2\,x+5|}\right) \\ &- \frac{11255717}{27648}\sqrt{2\,x^2-x+3} - \frac{32865365\left(2\,x^2-x+3\right)^{\frac{3}{2}}}{995328\,(2\,x+5)}\end{aligned}$$

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4,x, algorithm="m
axima")
```

```
[Out] 5/64*(2*x^2 - x + 3)^(3/2)*x - 1094743/497664*(2*x^2 - x + 3)^(3/2) - 3667/
1728*(2*x^2 - x + 3)^(5/2)/(8*x^3 + 60*x^2 + 150*x + 125) + 556255/248832*(
```

```
2*x^2 - x + 3)^(5/2)/(4*x^2 + 20*x + 25) + 22512089/331776*sqrt(2*x^2 - x +
 3)*x + 19176431/16384*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) - 5
17762327/442368*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(
23)/abs(2*x + 5)) - 11255717/27648*sqrt(2*x^2 - x + 3) - 32865365/995328*(2
*x^2 - x + 3)^(3/2)/(2*x + 5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 314 vs. $2(146) = 292$.

Time = 0.29 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.73

$$\int \frac{(3-x+2x^2)^{3/2}\,(2+x+3x^2-x^3+5x^4)}{(5+2x)^4}\,dx = \frac{1}{4096}\left(4\left(8\left(20\,x-287\right)x+23341\right)x-1004633\right)\sqrt{2\,x^2-x}$$

$$-\frac{19176431}{16384}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)+1\right)$$

$$+\frac{517762327}{442368}\sqrt{2}\log\left(\left|-2\sqrt{2}x+\sqrt{2}+2\sqrt{2\,x^2-x+3}\right|\right)$$

$$-\frac{517762327}{442368}\sqrt{2}\log\left(\left|-2\sqrt{2}x-11\sqrt{2}+2\sqrt{2\,x^2-x+3}\right|\right)$$

$$-\frac{\sqrt{2}\Big(1092794276\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)^5+18284336132\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)^4+20314214356\sqrt{2}\Big(}{36864\Big(2\left(\sqrt{2}x-\sqrt{2\,x^2-}\right.}$$

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4,x, algorithm="g
iac")
```

```
[Out] 1/4096*(4*(8*(20*x - 287)*x + 23341)*x - 1004633)*sqrt(2*x^2 - x + 3) - 191
76431/16384*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) +
 517762327/442368*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 - x
 + 3))) - 517762327/442368*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2) + 2*sq
rt(2*x^2 - x + 3))) - 1/36864*sqrt(2)*(1092794276*sqrt(2)*(sqrt(2)*x - sqrt
(2*x^2 - x + 3))^5 + 18284336132*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^4 + 2031
4214356*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^3 - 151449344092*(sqrt(2)
*x - sqrt(2*x^2 - x + 3))^2 + 102529692109*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2
- x + 3)) - 41882448755)/(2*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 10*sqrt(2
)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) - 11)^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(3-x+2x^2)^{3/2}\,(2+x+3x^2-x^3+5x^4)}{(5+2x)^4}\,dx=\int \frac{{\left(2\,x^2-x+3\right)}^{3/2}\,\left(5\,x^4-x^3+3\,x^2+x+2\right)}{{\left(2\,x+5\right)}^4}\,dx$$

```
[In] int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^4,x)
[Out] int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^4, x)
```

3.340 $\int \frac{\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^5}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 188

$$\begin{aligned}
&\int \frac{\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^5}\,dx=\frac{(2339916063-389975609x)\sqrt{3-x+2x^2}}{31850496}\\
&+\frac{(762984903+67865260x)\left(3-x+2x^2\right)^{3/2}}{95551488(5+2x)}-\frac{3667\left(3-x+2x^2\right)^{5/2}}{2304(5+2x)^4}\\
&+\frac{224815\left(3-x+2x^2\right)^{5/2}}{165888(5+2x)^3}-\frac{14477995\left(3-x+2x^2\right)^{5/2}}{23887872(5+2x)^2}\\
&+\frac{432565\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{1024\sqrt{2}}-\frac{8969688643\operatorname{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{21233664\sqrt{2}}
\end{aligned}$$

```
[Out] 1/95551488*(762984903+67865260*x)*(2*x^2-x+3)^(3/2)/(5+2*x)-3667/2304*(2*x^
2-x+3)^(5/2)/(5+2*x)^4+224815/165888*(2*x^2-x+3)^(5/2)/(5+2*x)^3-14477995/2
3887872*(2*x^2-x+3)^(5/2)/(5+2*x)^2+432565/2048*arcsinh(1/23*(1-4*x)*23^(1/
2))*2^(1/2)-8969688643/42467328*arctanh(1/24*(17-22*x)*2^(1/2)/(2*x^2-x+3)^
(1/2))*2^(1/2)+1/31850496*(2339916063-389975609*x)*(2*x^2-x+3)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used

$= \{1664, 826, 828, 857, 633, 221, 738, 212\}$

$$\int \frac{(3-x+2x^2)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^5}\,dx = \frac{432565\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{1024\sqrt{2}} - \frac{8969688643\operatorname{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{21233664\sqrt{2}} - \frac{14477995(2x^2-x+3)^{5/2}}{23887872(2x+5)^2} + \frac{224815(2x^2-x+3)^{5/2}}{165888(2x+5)^3} - \frac{3667(2x^2-x+3)^{5/2}}{2304(2x+5)^4} + \frac{(67865260x+762984903)\left(2x^2-x+3\right)^{3/2}}{95551488(2x+5)} + \frac{(2339916063-389975609x)\sqrt{2x^2-x+3}}{31850496}$$

```
[In] Int[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^5,x]
```

```
[Out] ((2339916063 - 389975609*x)*Sqrt[3 - x + 2*x^2])/31850496 + ((762984903 + 6
7865260*x)*(3 - x + 2*x^2)^(3/2))/(95551488*(5 + 2*x)) - (3667*(3 - x + 2*x
^2)^(5/2))/(2304*(5 + 2*x)^4) + (224815*(3 - x + 2*x^2)^(5/2))/(165888*(5 +
 2*x)^3) - (14477995*(3 - x + 2*x^2)^(5/2))/(23887872*(5 + 2*x)^2) + (43256
5*ArcSinh[(1 - 4*x)/Sqrt[23]])/(1024*Sqrt[2]) - (8969688643*ArcTanh[(17 - 2
2*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(21233664*Sqrt[2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 826

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
```

```
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} = & -\frac{3667(3-x+2x^2)^{5/2}}{2304(5+2x)^4} \\
& -\frac{1}{288}\int \frac{(3-x+2x^2)^{3/2}\left(\frac{51695}{16}-\frac{24835x}{4}+1944x^2-720x^3\right)}{(5+2x)^4}\,dx \\
= & -\frac{3667(3-x+2x^2)^{5/2}}{2304(5+2x)^4}+\frac{224815(3-x+2x^2)^{5/2}}{165888(5+2x)^3}+\frac{\int \frac{(3-x+2x^2)^{3/2}\left(\frac{5995005}{16}-\frac{1483149x}{2}+77760x^2\right)}{(5+2x)^3}\,dx}{62208} \\
= & -\frac{3667(3-x+2x^2)^{5/2}}{2304(5+2x)^4}+\frac{224815(3-x+2x^2)^{5/2}}{165888(5+2x)^3} \\
& -\frac{14477995(3-x+2x^2)^{5/2}}{23887872(5+2x)^2}-\frac{\int \frac{\left(\frac{252996909}{16}-\frac{152696835x}{4}\right)(3-x+2x^2)^{3/2}}{(5+2x)^2}\,dx}{8957952} \\
= & \frac{(762984903+67865260x)(3-x+2x^2)^{3/2}}{95551488(5+2x)} \\
& -\frac{3667(3-x+2x^2)^{5/2}}{2304(5+2x)^4}+\frac{224815(3-x+2x^2)^{5/2}}{165888(5+2x)^3} \\
& -\frac{14477995(3-x+2x^2)^{5/2}}{23887872(5+2x)^2}+\frac{\int \frac{\left(\frac{10531588167}{8}-3509780481x\right)\sqrt{3-x+2x^2}}{5+2x}\,dx}{71663616} \\
= & \frac{(2339916063-389975609x)\sqrt{3-x+2x^2}}{31850496} \\
& +\frac{(762984903+67865260x)(3-x+2x^2)^{3/2}}{95551488(5+2x)} \\
& -\frac{3667(3-x+2x^2)^{5/2}}{2304(5+2x)^4}+\frac{224815(3-x+2x^2)^{5/2}}{165888(5+2x)^3} \\
& -\frac{14477995(3-x+2x^2)^{5/2}}{23887872(5+2x)^2}-\frac{\int \frac{-968737607064+1937448253440x}{(5+2x)\sqrt{3-x+2x^2}}\,dx}{2293235712} \\
= & \frac{(2339916063-389975609x)\sqrt{3-x+2x^2}}{31850496} \\
& +\frac{(762984903+67865260x)(3-x+2x^2)^{3/2}}{95551488(5+2x)}-\frac{3667(3-x+2x^2)^{5/2}}{2304(5+2x)^4} \\
& +\frac{224815(3-x+2x^2)^{5/2}}{165888(5+2x)^3}-\frac{14477995(3-x+2x^2)^{5/2}}{23887872(5+2x)^2} \\
& -\frac{432565\int \frac{1}{\sqrt{3-x+2x^2}}\,dx}{1024}+\frac{8969688643\int \frac{1}{(5+2x)\sqrt{3-x+2x^2}}\,dx}{3538944}
\end{aligned}$$

$$= \frac{(2339916063-389975609x)\sqrt{3-x+2x^2}}{31850496}+\frac{(762984903+67865260x)\left(3-x+2x^2\right)^{3/2}}{95551488(5+2x)} - \frac{3667\left(3-x+2x^2\right)^{5/2}}{2304(5+2x)^4}+\frac{224815\left(3-x+2x^2\right)^{5/2}}{165888(5+2x)^3}-\frac{14477995\left(3-x+2x^2\right)^{5/2}}{23887872(5+2x)^2} - \frac{8969688643\text{Subst}\left(\int\frac{1}{288-x^2}\,dx,x,\frac{17-22x}{\sqrt{3-x+2x^2}}\right)}{1769472}-\frac{432565\text{Subst}\left(\int\frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx,x,-1+4x\right)}{1024\sqrt{46}}$$

$$= \frac{(2339916063-389975609x)\sqrt{3-x+2x^2}}{31850496} + \frac{(762984903+67865260x)\left(3-x+2x^2\right)^{3/2}}{95551488(5+2x)}-\frac{3667\left(3-x+2x^2\right)^{5/2}}{2304(5+2x)^4} + \frac{224815\left(3-x+2x^2\right)^{5/2}}{165888(5+2x)^3}-\frac{14477995\left(3-x+2x^2\right)^{5/2}}{23887872(5+2x)^2} + \frac{432565\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{1024\sqrt{2}}-\frac{8969688643\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{21233664\sqrt{2}}$$

Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.64

$$\int\frac{\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^5}\,dx = \frac{\frac{12\sqrt{3-x+2x^2}\left(86386856771+121473790266x+60528581892x^2+11761910072x^3\right.}{(5+2x)^4}}{}$$

```
[In] Integrate[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^5
,x]
```

```
[Out] ((12*Sqrt[3 - x + 2*x^2]*(86386856771 + 121473790266*x + 60528581892*x^2 +
11761910072*x^3 + 468043776*x^4 - 29270016*x^5 + 2949120*x^6))/(5 + 2*x)^4
+ 8969688643*Sqrt[2]*ArcTanh[(5 + 2*x - Sqrt[6 - 2*x + 4*x^2])/6] + 4484833
920*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/21233664
```

Maple [F(-1)]

Timed out.

hanged

```
[In] int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^5,x)
```

```
[Out] int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^5,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.06

$$\int \frac{(3-x+2x^2)^{3/2}\,(2+x+3x^2-x^3+5x^4)}{(5+2x)^5}\,dx = \frac{8969667840\,\sqrt{2}(16\,x^4+160\,x^3+600\,x^2+1000\,x+625)}{}$$

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^5,x, algorithm="f
ricas")
```

```
[Out] 1/84934656*(8969667840*sqrt(2)*(16*x^4 + 160*x^3 + 600*x^2 + 1000*x + 625)*
log(4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 8969688
643*sqrt(2)*(16*x^4 + 160*x^3 + 600*x^2 + 1000*x + 625)*log(-(24*sqrt(2)*sq
rt(2*x^2 - x + 3)*(22*x - 17) + 1060*x^2 - 1036*x + 1153)/(4*x^2 + 20*x + 2
5)) + 48*(2949120*x^6 - 29270016*x^5 + 468043776*x^4 + 11761910072*x^3 + 60
528581892*x^2 + 121473790266*x + 86386856771)*sqrt(2*x^2 - x + 3))/(16*x^4
+ 160*x^3 + 600*x^2 + 1000*x + 625)
```

Sympy [F]

$$\int \frac{(3-x+2x^2)^{3/2}\,(2+x+3x^2-x^3+5x^4)}{(5+2x)^5}\,dx = \int \frac{(2x^2-x+3)^{\frac{3}{2}} \cdot (5x^4-x^3+3x^2+x+2)}{(2x+5)^5}\,dx$$

```
[In] integrate((2*x**2-x+3)**(3/2)*(5*x**4-x**3+3*x**2+x+2)/(5+2*x)**5,x)
```

```
[Out] Integral((2*x**2 - x + 3)**(3/2)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)
**5, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.12

$$\int \frac{(3-x+2x^2)^{3/2}\,(2+x+3x^2-x^3+5x^4)}{(5+2x)^5}\,dx = \frac{16966315}{47775744}\left(2\,x^2-x+3\right)^{\frac{3}{2}}$$
$$-\frac{3667\left(2\,x^2-x+3\right)^{\frac{5}{2}}}{2304\left(16\,x^4+160\,x^3+600\,x^2+1000\,x+625\right)}$$
$$+\frac{224815\left(2\,x^2-x+3\right)^{\frac{5}{2}}}{165888\left(8\,x^3+60\,x^2+150\,x+125\right)}-\frac{14477995\left(2\,x^2-x+3\right)^{\frac{5}{2}}}{23887872\left(4\,x^2+20\,x+25\right)}$$
$$-\frac{389975609}{31850496}\sqrt{2\,x^2-x+3}x-\frac{432565}{2048}\sqrt{2}\,\mathrm{arsinh}\left(\frac{4}{23}\sqrt{23}x-\frac{1}{23}\sqrt{23}\right)$$
$$+\frac{8969688643}{42467328}\sqrt{2}\,\mathrm{arsinh}\left(\frac{22\sqrt{23}x}{23\,|2\,x+5|}-\frac{17\sqrt{23}}{23\,|2\,x+5|}\right)$$
$$+\frac{779972021}{10616832}\sqrt{2\,x^2-x+3}+\frac{593321753\left(2\,x^2-x+3\right)^{\frac{3}{2}}}{95551488\left(2\,x+5\right)}$$

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^5,x, algorithm="m
axima")
```

```
[Out] 16966315/47775744*(2*x^2 - x + 3)^(3/2) - 3667/2304*(2*x^2 - x + 3)^(5/2)/(
16*x^4 + 160*x^3 + 600*x^2 + 1000*x + 625) + 224815/165888*(2*x^2 - x + 3)^
(5/2)/(8*x^3 + 60*x^2 + 150*x + 125) - 14477995/23887872*(2*x^2 - x + 3)^(5
/2)/(4*x^2 + 20*x + 25) - 389975609/31850496*sqrt(2*x^2 - x + 3)*x - 432565
/2048*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) + 8969688643/4246732
8*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)/abs(2*x +
5)) + 779972021/10616832*sqrt(2*x^2 - x + 3) + 593321753/95551488*(2*x^2 -
x + 3)^(3/2)/(2*x + 5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 503 vs. 2(153) = 306.

Time = 0.36 (sec) , antiderivative size = 503, normalized size of antiderivative = 2.68

$$\int \frac{(3-x+2x^2)^{3/2}\,(2+x+3x^2-x^3+5x^4)}{(5+2x)^5}\,dx =$$
$$-\frac{1}{42467328}\sqrt{2}\left(8969688643\,\log\left(12\sqrt{-\frac{11}{2\,x+5}+\frac{36}{\left(2\,x+5\right)^2}+1}+\frac{72}{2\,x+5}-11\right)\mathrm{sgn}\left(\frac{1}{2\,x+5}\right)+89696\right.$$

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^5,x, algorithm="g
iac")
```

```
[Out] -1/42467328*sqrt(2)*(8969688643*log(12*sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2
+ 1) + 72/(2*x + 5) - 11)*sgn(1/(2*x + 5)) + 8969667840*log(abs(sqrt(-11/(2
*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5) + 1))*sgn(1/(2*x + 5)) - 896966
7840*log(abs(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5) - 1))*s
gn(1/(2*x + 5)) + 12*(24*(1296*(29336*sgn(1/(2*x + 5))/(2*x + 5) - 42907*sg
n(1/(2*x + 5)))/(2*x + 5) + 39923563*sgn(1/(2*x + 5)))/(2*x + 5) - 54131203
9*sgn(1/(2*x + 5)))*sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 13824*(80624
1*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^5*sgn(1/(2*x + 5
)) - 1152288*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^4*sgn
(1/(2*x + 5)) - 957352*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x +
 5))^3*sgn(1/(2*x + 5)) + 1529280*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1)
 + 6/(2*x + 5))^2*sgn(1/(2*x + 5)) + 394431*(sqrt(-11/(2*x + 5) + 36/(2*x +
 5)^2 + 1) + 6/(2*x + 5))*sgn(1/(2*x + 5)) - 620352*sgn(1/(2*x + 5)))/((sqr
t(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^2 - 1)^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(3-x+2x^2)^{3/2}\,(2+x+3x^2-x^3+5x^4)}{(5+2x)^5}\,dx = \int \frac{(2\,x^2-x+3)^{3/2}\,(5\,x^4-x^3+3\,x^2+x+2)}{(2\,x+5)^5}\,dx$$

```
[In] int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^5,x)
```

```
[Out] int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^5, x)
```

3.341 $\int \frac{\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^6}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 195

$$
\begin{aligned}
\int \frac{\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^6}\,dx = \\
&-\frac{(5658774871+1028823716x)\sqrt{3-x+2x^2}}{127401984(5+2x)} \\
&+\frac{(246012435+44773976x)\left(3-x+2x^2\right)^{3/2}}{95551488(5+2x)^2}-\frac{3667\left(3-x+2x^2\right)^{5/2}}{2880(5+2x)^5} \\
&+\frac{158527\left(3-x+2x^2\right)^{5/2}}{165888(5+2x)^4}-\frac{3730507\left(3-x+2x^2\right)^{5/2}}{11943936(5+2x)^3} \\
&-\frac{23775\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{512\sqrt{2}}+\frac{70991525167\operatorname{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{1528823808\sqrt{2}}
\end{aligned}
$$

```
[Out] 1/95551488*(246012435+44773976*x)*(2*x^2-x+3)^(3/2)/(5+2*x)^2-3667/2880*(2*
x^2-x+3)^(5/2)/(5+2*x)^5+158527/165888*(2*x^2-x+3)^(5/2)/(5+2*x)^4-3730507/
11943936*(2*x^2-x+3)^(5/2)/(5+2*x)^3-23775/1024*arcsinh(1/23*(1-4*x)*23^(1/
2))*2^(1/2)+70991525167/3057647616*arctanh(1/24*(17-22*x)*2^(1/2)/(2*x^2-x+
3)^(1/2))*2^(1/2)-1/127401984*(5658774871+1028823716*x)*(2*x^2-x+3)^(1/2)/(
5+2*x)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {1664, 826, 857, 633, 221, 738, 212}

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^6}\,dx = -\frac{23775\text{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{512\sqrt{2}}+\frac{70991525167\text{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{1528823808\sqrt{2}} -\frac{3730507(2x^2-x+3)^{5/2}}{11943936(2x+5)^3}+\frac{158527(2x^2-x+3)^{5/2}}{165888(2x+5)^4} -\frac{3667(2x^2-x+3)^{5/2}}{2880(2x+5)^5}+\frac{(44773976x+246012435)(2x^2-x+3)^{3/2}}{95551488(2x+5)^2} -\frac{(1028823716x+5658774871)\sqrt{2x^2-x+3}}{127401984(2x+5)}$$

```
[In] Int[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^6,x]
```

```
[Out] -1/127401984*((5658774871 + 1028823716*x)*Sqrt[3 - x + 2*x^2])/(5 + 2*x) +
((246012435 + 44773976*x)*(3 - x + 2*x^2)^(3/2))/(95551488*(5 + 2*x)^2) - (
3667*(3 - x + 2*x^2)^(5/2))/(2880*(5 + 2*x)^5) + (158527*(3 - x + 2*x^2)^(5
/2))/(165888*(5 + 2*x)^4) - (3730507*(3 - x + 2*x^2)^(5/2))/(11943936*(5 +
2*x)^3) - (23775*ArcSinh[(1 - 4*x)/Sqrt[23]])/(512*Sqrt[2]) + (70991525167*
ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(1528823808*Sqrt[2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 826

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} = {} & -\frac{3667(3-x+2x^2)^{5/2}}{2880(5+2x)^5} \\
& -\frac{1}{360}\int \frac{(3-x+2x^2)^{3/2}\left(\frac{60035}{16}-6615x+2430x^2-900x^3\right)}{(5+2x)^5}\,dx
\end{aligned}
$$

$$= -\frac{3667(3-x+2x^2)^{5/2}}{2880(5+2x)^5} + \frac{158527(3-x+2x^2)^{5/2}}{165888(5+2x)^4} + \frac{\int \frac{(3-x+2x^2)^{3/2}\left(\frac{8114455}{16}-\frac{3488315x}{4}+129600x^2\right)}{(5+2x)^4}\,dx}{103680}$$

$$= -\frac{3667(3-x+2x^2)^{5/2}}{2880(5+2x)^5} + \frac{158527(3-x+2x^2)^{5/2}}{165888(5+2x)^4} - \frac{3730507(3-x+2x^2)^{5/2}}{11943936(5+2x)^3} - \frac{\int \frac{\left(\frac{332138325}{16}-\frac{83951205x}{2}\right)(3-x+2x^2)^{3/2}}{(5+2x)^3}\,dx}{22394880}$$

$$= \frac{(246012435+44773976x)\left(3-x+2x^2\right)^{3/2}}{95551488(5+2x)^2} - \frac{3667(3-x+2x^2)^{5/2}}{2880(5+2x)^5} + \frac{158527(3-x+2x^2)^{5/2}}{165888(5+2x)^4} - \frac{3730507(3-x+2x^2)^{5/2}}{11943936(5+2x)^3} + \frac{\int \frac{\left(\frac{7719844365}{4}-3858088935x\right)\sqrt{3-x+2x^2}}{(5+2x)^2}\,dx}{238878720}$$

$$= -\frac{(5658774871+1028823716x)\sqrt{3-x+2x^2}}{127401984(5+2x)} + \frac{(246012435+44773976x)\left(3-x+2x^2\right)^{3/2}}{95551488(5+2x)^2} - \frac{3667(3-x+2x^2)^{5/2}}{2880(5+2x)^5} + \frac{158527(3-x+2x^2)^{5/2}}{165888(5+2x)^4} - \frac{3730507(3-x+2x^2)^{5/2}}{11943936(5+2x)^3} - \frac{\int \frac{\frac{177475757505}{2}-177479424000x}{(5+2x)\sqrt{3-x+2x^2}}\,dx}{1911029760}$$

$$= -\frac{(5658774871+1028823716x)\sqrt{3-x+2x^2}}{127401984(5+2x)} + \frac{(246012435+44773976x)\left(3-x+2x^2\right)^{3/2}}{95551488(5+2x)^2} - \frac{3667(3-x+2x^2)^{5/2}}{2880(5+2x)^5} + \frac{158527(3-x+2x^2)^{5/2}}{165888(5+2x)^4} - \frac{3730507(3-x+2x^2)^{5/2}}{11943936(5+2x)^3} + \frac{23775}{512}\int \frac{1}{\sqrt{3-x+2x^2}}\,dx - \frac{70991525167\int \frac{1}{(5+2x)\sqrt{3-x+2x^2}}\,dx}{254803968}$$

$$= -\frac{(5658774871+1028823716x)\sqrt{3-x+2x^2}}{127401984(5+2x)} + \frac{(246012435+44773976x)\left(3-x+2x^2\right)^{3/2}}{95551488(5+2x)^2} - \frac{3667(3-x+2x^2)^{5/2}}{2880(5+2x)^5} + \frac{158527(3-x+2x^2)^{5/2}}{165888(5+2x)^4} - \frac{3730507(3-x+2x^2)^{5/2}}{11943936(5+2x)^3} + \frac{70991525167\text{Subst}\left(\int \frac{1}{288-x^2}\,dx, x, \frac{17-22x}{\sqrt{3-x+2x^2}}\right)}{127401984} + \frac{23775\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx, x, -1+4x\right)}{512\sqrt{46}}$$

$$
\begin{aligned}
&= -\frac{(5658774871 + 1028823716x)\sqrt{3-x+2x^2}}{127401984(5+2x)} \\
&\quad + \frac{(246012435 + 44773976x)\left(3-x+2x^2\right)^{3/2}}{95551488(5+2x)^2} - \frac{3667\left(3-x+2x^2\right)^{5/2}}{2880(5+2x)^5} \\
&\quad + \frac{158527\left(3-x+2x^2\right)^{5/2}}{165888(5+2x)^4} - \frac{3730507\left(3-x+2x^2\right)^{5/2}}{11943936(5+2x)^3} \\
&\quad - \frac{23775\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{512\sqrt{2}} + \frac{70991525167\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{1528823808\sqrt{2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.62

$$\int \frac{\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^6}\,dx = \frac{\frac{12\sqrt{3-x+2x^2}(-17093312738327-30872393829992x-21590439797064x^2-711}{(5+2x)^5}}{}$$

```
[In] Integrate[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^6
,x]
```

```
[Out] ((12*Sqrt[3 - x + 2*x^2]*(-17093312738327 - 30872393829992*x - 215904397970
64*x^2 - 7117092892448*x^3 - 1023534029552*x^4 - 30496849920*x^5 + 15925248
00*x^6))/(5 + 2*x)^5 - 354957625835*Sqrt[2]*ArcTanh[(5 + 2*x - Sqrt[6 - 2*x
 + 4*x^2])/6] - 177479424000*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]]
)/7644119040
```

Maple [F(-1)]

Timed out.

hanged

```
[In] int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^6,x)
```

```
[Out] int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^6,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.09

$$\int \frac{\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^6}\,dx = \frac{354958848000\sqrt{2}(32x^5+400x^4+2000x^3+5000x^2+6}{}$$

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^6,x, algorithm="f
ricas")
```

```
[Out] 1/30576476160*(354958848000*sqrt(2)*(32*x^5 + 400*x^4 + 2000*x^3 + 5000*x^2
 + 6250*x + 3125)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 1
6*x - 25) + 354957625835*sqrt(2)*(32*x^5 + 400*x^4 + 2000*x^3 + 5000*x^2 +
6250*x + 3125)*log((24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) - 1060*x^2 +
 1036*x - 1153)/(4*x^2 + 20*x + 25)) + 48*(1592524800*x^6 - 30496849920*x^5
 - 1023534029552*x^4 - 7117092892448*x^3 - 21590439797064*x^2 - 30872393829
992*x - 17093312738327)*sqrt(2*x^2 - x + 3))/(32*x^5 + 400*x^4 + 2000*x^3 +
 5000*x^2 + 6250*x + 3125)
```

Sympy [F]

$$\int \frac{(3-x+2x^2)^{3/2}\,(2+x+3x^2-x^3+5x^4)}{(5+2x)^6}\,dx = \int \frac{(2x^2-x+3)^{\frac{3}{2}}\cdot(5x^4-x^3+3x^2+x+2)}{(2x+5)^6}\,dx$$

```
[In] integrate((2*x**2-x+3)**(3/2)*(5*x**4-x**3+3*x**2+x+2)/(5+2*x)**6,x)
```

```
[Out] Integral((2*x**2 - x + 3)**(3/2)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)
**6, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.29

$$\begin{aligned}\int \frac{(3-x+2x^2)^{3/2}\,(2+x+3x^2-x^3+5x^4)}{(5+2x)^6}\,dx = & -\frac{134077495}{3439853568}\,(2\,x^2-x+3)^{\frac{3}{2}} \\ & -\frac{3667\,(2\,x^2-x+3)^{\frac{5}{2}}}{2880\,(32\,x^5+400\,x^4+2000\,x^3+5000\,x^2+6250\,x+3125)} \\ & +\frac{158527\,(2\,x^2-x+3)^{\frac{5}{2}}}{165888\,(16\,x^4+160\,x^3+600\,x^2+1000\,x+625)} \\ & -\frac{3730507\,(2\,x^2-x+3)^{\frac{5}{2}}}{11943936\,(8\,x^3+60\,x^2+150\,x+125)}+\frac{134077495\,(2\,x^2-x+3)^{\frac{5}{2}}}{1719926784\,(4\,x^2+20\,x+25)} \\ & +\frac{3086715581}{2293235712}\,\sqrt{2\,x^2-x+3}x+\frac{23775}{1024}\,\sqrt{2}\,\mathrm{arsinh}\left(\frac{4}{23}\,\sqrt{23}x-\frac{1}{23}\,\sqrt{23}\right) \\ & -\frac{70991525167}{3057647616}\,\sqrt{2}\,\mathrm{arsinh}\left(\frac{22\,\sqrt{23}x}{23\,|2\,x+5|}-\frac{17\,\sqrt{23}}{23\,|2\,x+5|}\right) \\ & -\frac{6173186729}{764411904}\,\sqrt{2\,x^2-x+3}-\frac{4698578717\,(2\,x^2-x+3)^{\frac{3}{2}}}{6879707136\,(2\,x+5)}\end{aligned}$$

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^6,x, algorithm="m
axima")
```

```
[Out] -134077495/3439853568*(2*x^2 - x + 3)^(3/2) - 3667/2880*(2*x^2 - x + 3)^(5/
2)/(32*x^5 + 400*x^4 + 2000*x^3 + 5000*x^2 + 6250*x + 3125) + 158527/165888
*(2*x^2 - x + 3)^(5/2)/(16*x^4 + 160*x^3 + 600*x^2 + 1000*x + 625) - 373050
7/11943936*(2*x^2 - x + 3)^(5/2)/(8*x^3 + 60*x^2 + 150*x + 125) + 134077495
/1719926784*(2*x^2 - x + 3)^(5/2)/(4*x^2 + 20*x + 25) + 3086715581/22932357
12*sqrt(2*x^2 - x + 3)*x + 23775/1024*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/2
3*sqrt(23)) - 70991525167/3057647616*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2
*x + 5) - 17/23*sqrt(23)/abs(2*x + 5)) - 6173186729/764411904*sqrt(2*x^2 -
x + 3) - 4698578717/6879707136*(2*x^2 - x + 3)^(3/2)/(2*x + 5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 406 vs. 2(160) = 320.

Time = 0.31 (sec) , antiderivative size = 406, normalized size of antiderivative = 2.08

$$
\begin{aligned}
\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^6}\,dx &= \frac{1}{256}\sqrt{2\,x^2-x+3}(20\,x-633) \\
&- \frac{23775}{1024}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)+1\right) \\
&+ \frac{70991525167}{3057647616}\sqrt{2}\log\left(\left|-2\sqrt{2}x+\sqrt{2}+2\sqrt{2\,x^2-x+3}\right|\right) \\
&- \frac{70991525167}{3057647616}\sqrt{2}\log\left(\left|-2\sqrt{2}x-11\sqrt{2}+2\sqrt{2\,x^2-x+3}\right|\right) \\
&- \frac{\sqrt{2}\left(8281387393360\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)^9+275661428628240\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)^8+15603827\right.}{}
\end{aligned}
$$

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^6,x, algorithm="g
iac")
```

```
[Out] 1/256*sqrt(2*x^2 - x + 3)*(20*x - 633) - 23775/1024*sqrt(2)*log(-2*sqrt(2)*
(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) + 70991525167/3057647616*sqrt(2)*log
(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 - x + 3))) - 70991525167/3057647
616*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2) + 2*sqrt(2*x^2 - x + 3))) - 1
/1274019840*sqrt(2)*(8281387393360*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)
)^9 + 275661428628240*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^8 + 156038270334576
0*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^7 + 4938646760855520*(sqrt(2)*x
 - sqrt(2*x^2 - x + 3))^6 - 9673562837036232*sqrt(2)*(sqrt(2)*x - sqrt(2*x^
2 - x + 3))^5 - 30647310393849000*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^4 + 700
60241036847960*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^3 - 97730658088823
880*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 30180638363071845*sqrt(2)*(sqrt(2
)*x - sqrt(2*x^2 - x + 3)) - 7096913381268319)/(2*(sqrt(2)*x - sqrt(2*x^2 -
 x + 3))^2 + 10*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) - 11)^5
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(3-x+2x^2)^{3/2}\,(2+x+3x^2-x^3+5x^4)}{(5+2x)^6}\,dx=\int \frac{{\left(2\,x^2-x+3\right)}^{3/2}\,\left(5\,x^4-x^3+3\,x^2+x+2\right)}{{\left(2\,x+5\right)}^6}\,dx$$

```
[In] int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^6,x)
[Out] int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^6, x)
```

3.342 $\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^7}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 195

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^7}\,dx = \frac{(151764102421+27596573612x)\sqrt{3-x+2x^2}}{55037657088(5+2x)} - \frac{(9802984711+6793718806x)(3-x+2x^2)^{3/2}}{13759414272(5+2x)^3} - \frac{3667(3-x+2x^2)^{5/2}}{3456(5+2x)^6} + \frac{182165(3-x+2x^2)^{5/2}}{248832(5+2x)^5} - \frac{14087245(3-x+2x^2)^{5/2}}{71663616(5+2x)^4} + \frac{369\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{128\sqrt{2}} - \frac{1903976002333\operatorname{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{660451885056\sqrt{2}}$$

```
[Out] -1/13759414272*(9802984711+6793718806*x)*(2*x^2-x+3)^(3/2)/(5+2*x)^3-3667/3
456*(2*x^2-x+3)^(5/2)/(5+2*x)^6+182165/248832*(2*x^2-x+3)^(5/2)/(5+2*x)^5-1
4087245/71663616*(2*x^2-x+3)^(5/2)/(5+2*x)^4+369/256*arcsinh(1/23*(1-4*x)*2
3^(1/2))*2^(1/2)-1903976002333/1320903770112*arctanh(1/24*(17-22*x)*2^(1/2)
/(2*x^2-x+3)^(1/2))*2^(1/2)+1/55037657088*(151764102421+27596573612*x)*(2*x
^2-x+3)^(1/2)/(5+2*x)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1664, 824, 826, 857, 633, 221, 738, 212}

$$\begin{aligned}\int \frac{(3-x+2x^2)^{3/2}\,(2+x+3x^2-x^3+5x^4)}{(5+2x)^7}\,dx &= \frac{369\text{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{128\sqrt{2}}\\ &\quad-\frac{1903976002333\text{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{660451885056\sqrt{2}}-\frac{14087245(2x^2-x+3)^{5/2}}{71663616(2x+5)^4}\\ &\quad+\frac{182165(2x^2-x+3)^{5/2}}{248832(2x+5)^5}-\frac{3667(2x^2-x+3)^{5/2}}{3456(2x+5)^6}\\ &\quad-\frac{(6793718806x+9802984711)\,(2x^2-x+3)^{3/2}}{13759414272(2x+5)^3}\\ &\quad+\frac{(27596573612x+151764102421)\sqrt{2x^2-x+3}}{55037657088(2x+5)}\end{aligned}$$

```
[In] Int[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^7,x]
```

```
[Out] ((151764102421 + 27596573612*x)*Sqrt[3 - x + 2*x^2])/(55037657088*(5 + 2*x)
) - ((9802984711 + 6793718806*x)*(3 - x + 2*x^2)^(3/2))/(13759414272*(5 + 2
*x)^3) - (3667*(3 - x + 2*x^2)^(5/2))/(3456*(5 + 2*x)^6) + (182165*(3 - x +
 2*x^2)^(5/2))/(248832*(5 + 2*x)^5) - (14087245*(3 - x + 2*x^2)^(5/2))/(716
63616*(5 + 2*x)^4) + (369*ArcSinh[(1 - 4*x)/Sqrt[23]])/(128*Sqrt[2]) - (190
3976002333*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(66045188
5056*Sqrt[2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 824

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Dist[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)
^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p
 + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*
(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m
 - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &
& LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 826

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
```

```
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{3667(3-x+2x^2)^{5/2}}{3456(5+2x)^6} - \frac{1}{432}\int \frac{(3-x+2x^2)^{3/2}\left(\frac{68375}{16}-\frac{28085x}{4}+2916x^2-1080x^3\right)}{(5+2x)^6}\,dx \\
&= -\frac{3667(3-x+2x^2)^{5/2}}{3456(5+2x)^6} + \frac{182165(3-x+2x^2)^{5/2}}{248832(5+2x)^5} + \frac{\int \frac{(3-x+2x^2)^{3/2}\left(\frac{10561025}{16}-1010880x+194400x^2\right)}{(5+2x)^5}\,dx}{155520} \\
&= -\frac{3667(3-x+2x^2)^{5/2}}{3456(5+2x)^6} + \frac{182165(3-x+2x^2)^{5/2}}{248832(5+2x)^5} \\
&\quad - \frac{14087245(3-x+2x^2)^{5/2}}{71663616(5+2x)^4} - \frac{\int \frac{\left(\frac{420053845}{16}-\frac{182410625x}{4}\right)(3-x+2x^2)^{3/2}}{(5+2x)^4}\,dx}{44789760} \\
&= -\frac{(9802984711+6793718806x)(3-x+2x^2)^{3/2}}{13759414272(5+2x)^3} \\
&\quad - \frac{3667(3-x+2x^2)^{5/2}}{3456(5+2x)^6} + \frac{182165(3-x+2x^2)^{5/2}}{248832(5+2x)^5} \\
&\quad - \frac{14087245(3-x+2x^2)^{5/2}}{71663616(5+2x)^4} + \frac{\int \frac{\left(-\frac{206718515415}{8}+\frac{103487151045x}{2}\right)\sqrt{3-x+2x^2}}{(5+2x)^2}\,dx}{51597803520} \\
&= \frac{(151764102421+27596573612x)\sqrt{3-x+2x^2}}{55037657088(5+2x)} \\
&\quad - \frac{(9802984711+6793718806x)(3-x+2x^2)^{3/2}}{13759414272(5+2x)^3} \\
&\quad - \frac{3667(3-x+2x^2)^{5/2}}{3456(5+2x)^6} + \frac{182165(3-x+2x^2)^{5/2}}{248832(5+2x)^5} \\
&\quad - \frac{14087245(3-x+2x^2)^{5/2}}{71663616(5+2x)^4} - \frac{\int \frac{-\frac{4760153161395}{4}+2379948687360x}{(5+2x)\sqrt{3-x+2x^2}}\,dx}{412782428160}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{(151764102421+27596573612x)\sqrt{3-x+2x^2}}{55037657088(5+2x)} \\
&\quad - \frac{(9802984711+6793718806x)\left(3-x+2x^2\right)^{3/2}}{13759414272(5+2x)^3} - \frac{3667\left(3-x+2x^2\right)^{5/2}}{3456(5+2x)^6} \\
&\quad + \frac{182165\left(3-x+2x^2\right)^{5/2}}{248832(5+2x)^5} - \frac{14087245\left(3-x+2x^2\right)^{5/2}}{71663616(5+2x)^4} \\
&\quad - \frac{369}{128}\int \frac{1}{\sqrt{3-x+2x^2}}\,dx + \frac{1903976002333\int \frac{1}{(5+2x)\sqrt{3-x+2x^2}}\,dx}{110075314176} \\
&= \frac{(151764102421+27596573612x)\sqrt{3-x+2x^2}}{55037657088(5+2x)} \\
&\quad - \frac{(9802984711+6793718806x)\left(3-x+2x^2\right)^{3/2}}{13759414272(5+2x)^3} - \frac{3667\left(3-x+2x^2\right)^{5/2}}{3456(5+2x)^6} \\
&\quad + \frac{182165\left(3-x+2x^2\right)^{5/2}}{248832(5+2x)^5} - \frac{14087245\left(3-x+2x^2\right)^{5/2}}{71663616(5+2x)^4} \\
&\quad - \frac{1903976002333\text{Subst}\left(\int \frac{1}{288-x^2}\,dx, x, \frac{17-22x}{\sqrt{3-x+2x^2}}\right)}{55037657088} \\
&\quad - \frac{369\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx, x, -1+4x\right)}{128\sqrt{46}} \\
&= \frac{(151764102421+27596573612x)\sqrt{3-x+2x^2}}{55037657088(5+2x)} \\
&\quad - \frac{(9802984711+6793718806x)\left(3-x+2x^2\right)^{3/2}}{13759414272(5+2x)^3} - \frac{3667\left(3-x+2x^2\right)^{5/2}}{3456(5+2x)^6} \\
&\quad + \frac{182165\left(3-x+2x^2\right)^{5/2}}{248832(5+2x)^5} - \frac{14087245\left(3-x+2x^2\right)^{5/2}}{71663616(5+2x)^4} \\
&\quad + \frac{369\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{128\sqrt{2}} - \frac{1903976002333\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{660451885056\sqrt{2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.62

$$
\int \frac{\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^7}\,dx = \frac{\frac{12\sqrt{3-x+2x^2}\left(458411625354581+1011372787716826x+910256842473992x^2+4\right.}{(5+}}{}
$$

```
[In] Integrate[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^7
,x]
```

```
[Out] ((12*Sqrt[3 - x + 2*x^2]*(458411625354581 + 1011372787716826*x + 9102568424
73992*x^2 + 422554114856528*x^3 + 103803827945872*x^4 + 11854023276320*x^5
+ 275188285440*x^6))/(5 + 2*x)^6 + 1903976002333*Sqrt[2]*ArcTanh[(5 + 2*x -
 Sqrt[6 - 2*x + 4*x^2])/6] + 951979474944*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 -
2*x + 4*x^2]])/660451885056
```

Maple [F(-1)]

Timed out.

hanged

```
[In] int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^7,x)
```

```
[Out] int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^7,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.17

$$\int \frac{(3-x+2x^2)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^7}\,dx = \frac{1903958949888\,\sqrt{2}(64\,x^6+960\,x^5+6000\,x^4+20000\,x^3}{}$$

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^7,x, algorithm="f
ricas")
```

```
[Out] 1/2641807540224*(1903958949888*sqrt(2)*(64*x^6 + 960*x^5 + 6000*x^4 + 20000
*x^3 + 37500*x^2 + 37500*x + 15625)*log(4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x
- 1) - 32*x^2 + 16*x - 25) + 1903976002333*sqrt(2)*(64*x^6 + 960*x^5 + 6000
*x^4 + 20000*x^3 + 37500*x^2 + 37500*x + 15625)*log(-(24*sqrt(2)*sqrt(2*x^2
 - x + 3)*(22*x - 17) + 1060*x^2 - 1036*x + 1153)/(4*x^2 + 20*x + 25)) + 48
*(275188285440*x^6 + 11854023276320*x^5 + 103803827945872*x^4 + 42255411485
6528*x^3 + 910256842473992*x^2 + 1011372787716826*x + 458411625354581)*sqrt
(2*x^2 - x + 3))/(64*x^6 + 960*x^5 + 6000*x^4 + 20000*x^3 + 37500*x^2 + 375
00*x + 15625)
```

Sympy [F]

$$\int \frac{\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^7}\,dx=\int \frac{\left(2x^2-x+3\right)^{\frac{3}{2}}\cdot\left(5x^4-x^3+3x^2+x+2\right)}{\left(2x+5\right)^7}\,dx$$

```
[In] integrate((2*x**2-x+3)**(3/2)*(5*x**4-x**3+3*x**2+x+2)/(5+2*x)**7,x)
```

```
[Out] Integral((2*x**2 - x + 3)**(3/2)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)
**7, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.52

$$\begin{aligned}\int \frac{\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^7}\,dx &= \frac{3607708597}{1486016741376}\left(2\,x^2-x+3\right)^{\frac{3}{2}}\\
&-\frac{3667\left(2\,x^2-x+3\right)^{\frac{5}{2}}}{3456\left(64\,x^6+960\,x^5+6000\,x^4+20000\,x^3+37500\,x^2+37500\,x+15625\right)}\\
&+\frac{182165\left(2\,x^2-x+3\right)^{\frac{5}{2}}}{248832\left(32\,x^5+400\,x^4+2000\,x^3+5000\,x^2+6250\,x+3125\right)}\\
&-\frac{14087245\left(2\,x^2-x+3\right)^{\frac{5}{2}}}{71663616\left(16\,x^4+160\,x^3+600\,x^2+1000\,x+625\right)}\\
&+\frac{149610673\left(2\,x^2-x+3\right)^{\frac{5}{2}}}{5159780352\left(8\,x^3+60\,x^2+150\,x+125\right)}-\frac{3607708597\left(2\,x^2-x+3\right)^{\frac{5}{2}}}{743008370688\left(4\,x^2+20\,x+25\right)}\\
&-\frac{82772668391}{990677827584}\sqrt{2\,x^2-x+3}x-\frac{369}{256}\sqrt{2}\,\mathrm{arsinh}\left(\frac{4}{23}\sqrt{23}x-\frac{1}{23}\sqrt{23}\right)\\
&+\frac{1903976002333}{1320903770112}\sqrt{2}\,\mathrm{arsinh}\left(\frac{22\sqrt{23}x}{23\,|2\,x+5|}-\frac{17\sqrt{23}}{23\,|2\,x+5|}\right)\\
&+\frac{165562389227}{330225942528}\sqrt{2\,x^2-x+3}+\frac{125860542215\left(2\,x^2-x+3\right)^{\frac{3}{2}}}{2972033482752\left(2\,x+5\right)}\end{aligned}$$

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^7,x, algorithm="m
axima")
```

```
[Out] 3607708597/1486016741376*(2*x^2 - x + 3)^(3/2) - 3667/3456*(2*x^2 - x + 3)^
(5/2)/(64*x^6 + 960*x^5 + 6000*x^4 + 20000*x^3 + 37500*x^2 + 37500*x + 1562
5) + 182165/248832*(2*x^2 - x + 3)^(5/2)/(32*x^5 + 400*x^4 + 2000*x^3 + 500
0*x^2 + 6250*x + 3125) - 14087245/71663616*(2*x^2 - x + 3)^(5/2)/(16*x^4 +
160*x^3 + 600*x^2 + 1000*x + 625) + 149610673/5159780352*(2*x^2 - x + 3)^(5
```

```
/2)/(8*x^3 + 60*x^2 + 150*x + 125) - 3607708597/743008370688*(2*x^2 - x + 3
)^(5/2)/(4*x^2 + 20*x + 25) - 82772668391/990677827584*sqrt(2*x^2 - x + 3)*
x - 369/256*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) + 190397600233
3/1320903770112*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(
23)/abs(2*x + 5)) + 165562389227/330225942528*sqrt(2*x^2 - x + 3) + 1258605
42215/2972033482752*(2*x^2 - x + 3)^(3/2)/(2*x + 5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 452 vs. $2(160) = 320$.

Time = 0.30 (sec) , antiderivative size = 452, normalized size of antiderivative = 2.32

$$\begin{aligned}\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^7}\,dx &= \frac{369}{256}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)\right.\\ &\left.+1\right)-\frac{1903976002333}{1320903770112}\sqrt{2}\log\left(\left|-2\sqrt{2}x+\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right)\\ &+\frac{1903976002333}{1320903770112}\sqrt{2}\log\left(\left|-2\sqrt{2}x-11\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right)+\frac{5}{64}\sqrt{2x^2-x+3}\\ &+\frac{\sqrt{2}\left(159278433934432\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^{11}+6347903280912544\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^{10}+485\right.}{}\end{aligned}$$

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^7,x, algorithm="g
iac")
```

```
[Out] 369/256*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) - 190
3976002333/1320903770112*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*
x^2 - x + 3))) + 1903976002333/1320903770112*sqrt(2)*log(abs(-2*sqrt(2)*x -
 11*sqrt(2) + 2*sqrt(2*x^2 - x + 3))) + 5/64*sqrt(2*x^2 - x + 3) + 1/110075
314176*sqrt(2)*(159278433934432*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^1
1 + 6347903280912544*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^10 + 485445268408334
24*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^9 + 305716670132783088*(sqrt(2
)*x - sqrt(2*x^2 - x + 3))^8 + 883138211359110 24*sqrt(2)*(sqrt(2)*x - sqrt(
2*x^2 - x + 3))^7 - 2423668581998843376*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^6
 - 3972111316970320 56*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^5 + 1170889
723253229957 6*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^4 - 12803484860728491138*sq
rt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^3 + 12593033197867577234*(sqrt(2)*x
 - sqrt(2*x^2 - x + 3))^2 - 3042533760672408875*sqrt(2)*(sqrt(2)*x - sqrt(2
*x^2 - x + 3)) + 5895262632497801 95)/(2*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2
 + 10*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) - 11)^6
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(3-x+2x^2)^{3/2}\,(2+x+3x^2-x^3+5x^4)}{(5+2x)^7}\,dx=\int \frac{\left(2\,x^2-x+3\right)^{3/2}\,\left(5\,x^4-x^3+3\,x^2+x+2\right)}{{\left(2\,x+5\right)}^7}\,dx$$

```
[In] int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^7,x)
[Out] int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^7, x)
```

3.343 $\int \frac{\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^8}\,dx$

Optimal result

Integrand size $=40$, antiderivative size $=195$

$$\int \frac{\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^8}\,dx = -\frac{(146583836191+101679102454x)\sqrt{3-x+2x^2}}{440301256704(5+2x)^2} - \frac{(463558457+411822458x)\left(3-x+2x^2\right)^{3/2}}{2293235712(5+2x)^4} - \frac{3667(3-x+2x^2)^{5/2}}{4032(5+2x)^7} + \frac{114335(3-x+2x^2)^{5/2}}{193536(5+2x)^6} - \frac{1930441(3-x+2x^2)^{5/2}}{13934592(5+2x)^5} - \frac{5\,\mathrm{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{64\sqrt{2}} + \frac{412760561351\,\mathrm{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{5283615080448\sqrt{2}}$$

```
[Out] -1/2293235712*(463558457+411822458*x)*(2*x^2-x+3)^(3/2)/(5+2*x)^4-3667/4032
*(2*x^2-x+3)^(5/2)/(5+2*x)^7+114335/193536*(2*x^2-x+3)^(5/2)/(5+2*x)^6-1930
441/13934592*(2*x^2-x+3)^(5/2)/(5+2*x)^5-5/128*arcsinh(1/23*(1-4*x)*23^(1/2
))*2^(1/2)+412760561351/10567230160896*arctanh(1/24*(17-22*x)*2^(1/2)/(2*x^
2-x+3)^(1/2))*2^(1/2)-1/440301256704*(146583836191+101679102454*x)*(2*x^2-x
+3)^(1/2)/(5+2*x)^2
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {1664, 824, 857, 633, 221, 738, 212}

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^8}\,dx = -\frac{5\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{64\sqrt{2}}+\frac{412760561351\operatorname{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{5283615080448\sqrt{2}} -\frac{1930441(2x^2-x+3)^{5/2}}{13934592(2x+5)^5}+\frac{114335(2x^2-x+3)^{5/2}}{193536(2x+5)^6} -\frac{3667(2x^2-x+3)^{5/2}}{4032(2x+5)^7}-\frac{(411822458x+463558457)(2x^2-x+3)^{3/2}}{2293235712(2x+5)^4} -\frac{(101679102454x+146583836191)\sqrt{2x^2-x+3}}{440301256704(2x+5)^2}$$

```
[In] Int[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^8,x]
```

```
[Out] -1/440301256704*((146583836191 + 101679102454*x)*Sqrt[3 - x + 2*x^2])/(5 +
2*x)^2 - ((463558457 + 411822458*x)*(3 - x + 2*x^2)^(3/2))/(2293235712*(5 +
 2*x)^4) - (3667*(3 - x + 2*x^2)^(5/2))/(4032*(5 + 2*x)^7) + (114335*(3 - x
 + 2*x^2)^(5/2))/(193536*(5 + 2*x)^6) - (1930441*(3 - x + 2*x^2)^(5/2))/(13
934592*(5 + 2*x)^5) - (5*ArcSinh[(1 - 4*x)/Sqrt[23]])/(64*Sqrt[2]) + (41276
0561351*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(52836150804
48*Sqrt[2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 824

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Dist[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)
^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p
 + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*
(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m
 - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &
& LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{3667(3-x+2x^2)^{5/2}}{4032(5+2x)^7}\\ &\quad -\frac{1}{504}\int\frac{(3-x+2x^2)^{3/2}\left(\frac{76715}{16}-\frac{14855x}{2}+3402x^2-1260x^3\right)}{(5+2x)^7}\,dx\end{aligned}$$

$$=-\frac{3667\left(3-x+2x^2\right)^{5/2}}{4032(5+2x)^7}+\frac{114335\left(3-x+2x^2\right)^{5/2}}{193536(5+2x)^6}+\frac{\int \frac{\left(3-x+2x^2\right)^{3/2}\left(\frac{13334715}{16}-\frac{4631913x}{4}+272160x^2\right)}{(5+2x)^6}\,dx}{217728}$$

$$=-\frac{3667\left(3-x+2x^2\right)^{5/2}}{4032(5+2x)^7}+\frac{114335\left(3-x+2x^2\right)^{5/2}}{193536(5+2x)^6}$$
$$-\frac{1930441\left(3-x+2x^2\right)^{5/2}}{13934592(5+2x)^5}-\frac{\int \frac{\left(\frac{516687885}{16}-48988800x\right)\left(3-x+2x^2\right)^{3/2}}{(5+2x)^5}\,dx}{78382080}$$

$$=-\frac{(463558457+411822458x)\left(3-x+2x^2\right)^{3/2}}{2293235712(5+2x)^4}$$
$$-\frac{3667\left(3-x+2x^2\right)^{5/2}}{4032(5+2x)^7}+\frac{114335\left(3-x+2x^2\right)^{5/2}}{193536(5+2x)^6}$$
$$-\frac{1930441\left(3-x+2x^2\right)^{5/2}}{13934592(5+2x)^5}+\frac{\int \frac{\left(-\frac{283730747265}{8}+56435097600x\right)\sqrt{3-x+2x^2}}{(5+2x)^3}\,dx}{180592312320}$$

$$=-\frac{(146583836191+101679102454x)\sqrt{3-x+2x^2}}{440301256704(5+2x)^2}$$
$$-\frac{(463558457+411822458x)\left(3-x+2x^2\right)^{3/2}}{2293235712(5+2x)^4}$$
$$-\frac{3667\left(3-x+2x^2\right)^{5/2}}{4032(5+2x)^7}+\frac{114335\left(3-x+2x^2\right)^{5/2}}{193536(5+2x)^6}$$
$$-\frac{1930441\left(3-x+2x^2\right)^{5/2}}{13934592(5+2x)^5}-\frac{\int \frac{\frac{64992568300695}{4}-32506616217600x}{(5+2x)\sqrt{3-x+2x^2}}\,dx}{208042343792640}$$

$$=-\frac{(146583836191+101679102454x)\sqrt{3-x+2x^2}}{440301256704(5+2x)^2}$$
$$-\frac{(463558457+411822458x)\left(3-x+2x^2\right)^{3/2}}{2293235712(5+2x)^4}-\frac{3667\left(3-x+2x^2\right)^{5/2}}{4032(5+2x)^7}$$
$$+\frac{114335\left(3-x+2x^2\right)^{5/2}}{193536(5+2x)^6}-\frac{1930441\left(3-x+2x^2\right)^{5/2}}{13934592(5+2x)^5}$$
$$+\frac{5}{64}\int \frac{1}{\sqrt{3-x+2x^2}}\,dx-\frac{412760561351\int \frac{1}{(5+2x)\sqrt{3-x+2x^2}}\,dx}{880602513408}$$

$$= -\frac{(146583836191+101679102454x)\sqrt{3-x+2x^2}}{440301256704(5+2x)^2} - \frac{(463558457+411822458x)\left(3-x+2x^2\right)^{3/2}}{2293235712(5+2x)^4} - \frac{3667\left(3-x+2x^2\right)^{5/2}}{4032(5+2x)^7} + \frac{114335\left(3-x+2x^2\right)^{5/2}}{193536(5+2x)^6} - \frac{1930441\left(3-x+2x^2\right)^{5/2}}{13934592(5+2x)^5} + \frac{412760561351\text{Subst}\left(\int \frac{1}{288-x^2}\,dx, x, \frac{17-22x}{\sqrt{3-x+2x^2}}\right)}{440301256704} + \frac{5\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx, x, -1+4x\right)}{64\sqrt{46}}$$

$$= -\frac{(146583836191+101679102454x)\sqrt{3-x+2x^2}}{440301256704(5+2x)^2} - \frac{(463558457+411822458x)\left(3-x+2x^2\right)^{3/2}}{2293235712(5+2x)^4} - \frac{3667\left(3-x+2x^2\right)^{5/2}}{4032(5+2x)^7} + \frac{114335\left(3-x+2x^2\right)^{5/2}}{193536(5+2x)^6} - \frac{1930441\left(3-x+2x^2\right)^{5/2}}{13934592(5+2x)^5} - \frac{5\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{64\sqrt{2}} + \frac{412760561351\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{5283615080448\sqrt{2}}$$

Mathematica [A] (verified)

Time = 1.19 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.62

$$\int \frac{\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^8}\,dx = -\frac{12\sqrt{3-x+2x^2}(3479517268702637+9065154700300572x+99760653674981}{}$$

```
[In] Integrate[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^8
,x]
```

```
[Out] ((-12*Sqrt[3 - x + 2*x^2]*(3479517268702637 + 9065154700300572*x + 99760653
67498188*x^2 + 5966329646300704*x^3 + 2069947287085104*x^4 + 40225582273171
2*x^5 + 38463671680832*x^6))/(5 + 2*x)^7 - 2889323929457*Sqrt[2]*ArcTanh[(5
 + 2*x - Sqrt[6 - 2*x + 4*x^2])/6] - 1444738498560*Sqrt[2]*Log[1 - 4*x + 2*
Sqrt[6 - 2*x + 4*x^2]])/36985305563136
```

Maple [F(-1)]

Timed out.

hanged

```
[In] int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^8,x)
```

```
[Out] int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^8,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.25

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^8}\,dx = \frac{2889476997120\sqrt{2}(128\,x^7+2240\,x^6+16800\,x^5+70000}{}$$

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^8,x, algorithm="f
ricas")
```

```
[Out] 1/147941222252544*(2889476997120*sqrt(2)*(128*x^7 + 2240*x^6 + 16800*x^5 +
70000*x^4 + 175000*x^3 + 262500*x^2 + 218750*x + 78125)*log(-4*sqrt(2)*sqrt
(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 2889323929457*sqrt(2)*(12
8*x^7 + 2240*x^6 + 16800*x^5 + 70000*x^4 + 175000*x^3 + 262500*x^2 + 218750
*x + 78125)*log((24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) - 1060*x^2 + 10
36*x - 1153)/(4*x^2 + 20*x + 25)) - 48*(38463671680832*x^6 + 40225582273171
2*x^5 + 2069947287085104*x^4 + 5966329646300704*x^3 + 9976065367498188*x^2
+ 9065154700300572*x + 3479517268702637)*sqrt(2*x^2 - x + 3))/(128*x^7 + 22
40*x^6 + 16800*x^5 + 70000*x^4 + 175000*x^3 + 262500*x^2 + 218750*x + 78125
)
```

Sympy [F]

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^8}\,dx = \int \frac{(2x^2-x+3)^{\frac{3}{2}}\cdot(5x^4-x^3+3x^2+x+2)}{(2x+5)^8}\,dx$$

```
[In] integrate((2*x**2-x+3)**(3/2)*(5*x**4-x**3+3*x**2+x+2)/(5+2*x)**8,x)
```

```
[Out] Integral((2*x**2 - x + 3)**(3/2)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)
**8, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 348 vs. 2(160) = 320.

Time = 0.34 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.78

$$\begin{aligned}
\int \frac{(3-x+2x^2)^{3/2}\,(2+x+3x^2-x^3+5x^4)}{(5+2x)^8}\,dx &= -\frac{769352975}{11888133931008}\left(2\,x^2-x+3\right)^{\frac{3}{2}} \\
&- \frac{3667\left(2\,x^2-x+3\right)^{\frac{5}{2}}}{4032\left(128\,x^7+2240\,x^6+16800\,x^5+70000\,x^4+175000\,x^3+262500\,x^2+218750\,x+78125\right)} \\
&+ \frac{114335\left(2\,x^2-x+3\right)^{\frac{5}{2}}}{193536\left(64\,x^6+960\,x^5+6000\,x^4+20000\,x^3+37500\,x^2+37500\,x+15625\right)} \\
&- \frac{1930441\left(2\,x^2-x+3\right)^{\frac{5}{2}}}{13934592\left(32\,x^5+400\,x^4+2000\,x^3+5000\,x^2+6250\,x+3125\right)} \\
&+ \frac{7861079\left(2\,x^2-x+3\right)^{\frac{5}{2}}}{573308928\left(16\,x^4+160\,x^3+600\,x^2+1000\,x+625\right)} \\
&- \frac{32967491\left(2\,x^2-x+3\right)^{\frac{5}{2}}}{41278242816\left(8\,x^3+60\,x^2+150\,x+125\right)} + \frac{769352975\left(2\,x^2-x+3\right)^{\frac{5}{2}}}{5944066965504\left(4\,x^2+20\,x+25\right)} \\
&+ \frac{17957520133}{7925422620672}\sqrt{2\,x^2-x+3}x + \frac{5}{128}\sqrt{2}\operatorname{arsinh}\left(\frac{4}{23}\sqrt{23}x-\frac{1}{23}\sqrt{23}\right) \\
&- \frac{412760561351}{10567230160896}\sqrt{2}\operatorname{arsinh}\left(\frac{22\sqrt{23}x}{23\left|2\,x+5\right|}-\frac{17\sqrt{23}}{23\left|2\,x+5\right|}\right) \\
&- \frac{35893173457}{2641807540224}\sqrt{2\,x^2-x+3} - \frac{27452157541\left(2\,x^2-x+3\right)^{\frac{3}{2}}}{23776267862016\left(2\,x+5\right)}
\end{aligned}$$

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^8,x, algorithm="m
axima")
```

```
[Out] -769352975/11888133931008*(2*x^2 - x + 3)^(3/2) - 3667/4032*(2*x^2 - x + 3)
^(5/2)/(128*x^7 + 2240*x^6 + 16800*x^5 + 70000*x^4 + 175000*x^3 + 262500*x^
2 + 218750*x + 78125) + 114335/193536*(2*x^2 - x + 3)^(5/2)/(64*x^6 + 960*x
^5 + 6000*x^4 + 20000*x^3 + 37500*x^2 + 37500*x + 15625) - 1930441/13934592
*(2*x^2 - x + 3)^(5/2)/(32*x^5 + 400*x^4 + 2000*x^3 + 5000*x^2 + 6250*x + 3
125) + 7861079/573308928*(2*x^2 - x + 3)^(5/2)/(16*x^4 + 160*x^3 + 600*x^2
+ 1000*x + 625) - 32967491/41278242816*(2*x^2 - x + 3)^(5/2)/(8*x^3 + 60*x^
2 + 150*x + 125) + 769352975/5944066965504*(2*x^2 - x + 3)^(5/2)/(4*x^2 + 2
0*x + 25) + 17957520133/7925422620672*sqrt(2*x^2 - x + 3)*x + 5/128*sqrt(2)
*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) - 412760561351/10567230160896*sqr
t(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)/abs(2*x + 5)) -
 35893173457/2641807540224*sqrt(2*x^2 - x + 3) - 27452157541/23776267862016
*(2*x^2 - x + 3)^(3/2)/(2*x + 5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 489 vs. $2(160) = 320$.

Time = 0.31 (sec) , antiderivative size = 489, normalized size of antiderivative = 2.51

$$\int \frac{(3-x+2x^2)^{3/2}\,(2+x+3x^2-x^3+5x^4)}{(5+2x)^8}\,dx =$$

$$-\frac{5}{128}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right)$$

$$+\frac{412760561351}{10567230160896}\sqrt{2}\log\left(\left|-2\sqrt{2}x+\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right)$$

$$-\frac{412760561351}{10567230160896}\sqrt{2}\log\left(\left|-2\sqrt{2}x-11\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right)$$

$$-\frac{\sqrt{2}\Big(1121897398412224\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^{13}+48260296303776704\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^{12}+}{}$$

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^8,x, algorithm="g
iac")
```

```
[Out] -5/128*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) + 4127
60561351/10567230160896*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x
^2 - x + 3))) - 412760561351/10567230160896*sqrt(2)*log(abs(-2*sqrt(2)*x -
11*sqrt(2) + 2*sqrt(2*x^2 - x + 3))) - 1/6164217593856*sqrt(2)*(11218973984
12224*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^13 + 48260296303776704*(sqr
t(2)*x - sqrt(2*x^2 - x + 3))^12 + 444673458321712704*sqrt(2)*(sqrt(2)*x -
sqrt(2*x^2 - x + 3))^11 + 3996455936659982656*(sqrt(2)*x - sqrt(2*x^2 - x +
 3))^10 + 6725227967167489360*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^9 -
 171326610284839480080*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^8 - 63713012094737246112*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^7 + 106515880136064432096*(
sqrt(2)*x - sqrt(2*x^2 - x + 3))^6 + 226947197958946260516*sqrt(2)*(sqrt(2)
*x - sqrt(2*x^2 - x + 3))^5 - 856601202771483308188*(sqrt(2)*x - sqrt(2*x^2
 - x + 3))^4 + 617998258357377713732*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x +
3))^3 - 467121785339763351756*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 9229208
0735560562227*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) - 1516171609382750
1349)/(2*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 10*sqrt(2)*(sqrt(2)*x - sqrt(
2*x^2 - x + 3)) - 11)^7
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^8}\,dx=\int \frac{{\left(2\,x^2-x+3\right)}^{3/2}\,\left(5\,x^4-x^3+3\,x^2+x+2\right)}{{\left(2\,x+5\right)}^8}\,dx$$

```
[In] int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^8,x)
[Out] int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^8, x)
```

3.344 $\int \frac{(5+2x)(2+x+3x^2-x^3+5x^4)}{\sqrt{3-x+2x^2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 120

$$\begin{aligned}\int \frac{(5+2x)\left(2+x+3x^2-x^3+5x^4\right)}{\sqrt{3-x+2x^2}}\,dx &= \frac{761}{256}(5+2x)^2\sqrt{3-x+2x^2}\\ &\quad-\frac{105}{128}(5+2x)^3\sqrt{3-x+2x^2}\\ &\quad+\frac{1}{16}(5+2x)^4\sqrt{3-x+2x^2}\\ &\quad-\frac{(19227+4676x)\sqrt{3-x+2x^2}}{2048}\\ &\quad-\frac{85429\text{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{4096\sqrt{2}}\end{aligned}$$

```
[Out] -85429/8192*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)+761/256*(5+2*x)^2*(2*x^2
-x+3)^(1/2)-105/128*(5+2*x)^3*(2*x^2-x+3)^(1/2)+1/16*(5+2*x)^4*(2*x^2-x+3)^
(1/2)-1/2048*(19227+4676*x)*(2*x^2-x+3)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used

$= \{1667, 793, 633, 221\}$

$$\int \frac{(5+2x)\left(2+x+3x^2-x^3+5x^4\right)}{\sqrt{3-x+2x^2}}\,dx = -\frac{85429\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{4096\sqrt{2}} + \frac{1}{16}\sqrt{2x^2-x+3}(2x+5)^4 - \frac{105}{128}\sqrt{2x^2-x+3}(2x+5)^3 + \frac{761}{256}\sqrt{2x^2-x+3}(2x+5)^2 - \frac{(4676x+19227)\sqrt{2x^2-x+3}}{2048}$$

```
[In] Int[((5 + 2*x)*(2 + x + 3*x^2 - x^3 + 5*x^4))/Sqrt[3 - x + 2*x^2],x]
```

```
[Out] (761*(5 + 2*x)^2*Sqrt[3 - x + 2*x^2])/256 - (105*(5 + 2*x)^3*Sqrt[3 - x + 2
*x^2])/128 + ((5 + 2*x)^4*Sqrt[3 - x + 2*x^2])/16 - ((19227 + 4676*x)*Sqrt[
3 - x + 2*x^2])/2048 - (85429*ArcSinh[(1 - 4*x)/Sqrt[23]])/(4096*Sqrt[2])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
```

```
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

integral

$$= \frac{1}{16}(5+2x)^4\sqrt{3-x+2x^2} + \frac{1}{160}\int \frac{(5+2x)\left(-5055-4390x-5580x^2-4200x^3\right)}{\sqrt{3-x+2x^2}}\,dx$$

$$= -\frac{105}{128}(5+2x)^3\sqrt{3-x+2x^2} + \frac{1}{16}(5+2x)^4\sqrt{3-x+2x^2} + \frac{\int \frac{(5+2x)\left(327480+105440x+365280x^2\right)}{\sqrt{3-x+2x^2}}\,dx}{10240}$$

$$= \frac{761}{256}(5+2x)^2\sqrt{3-x+2x^2} - \frac{105}{128}(5+2x)^3\sqrt{3-x+2x^2} + \frac{1}{16}(5+2x)^4\sqrt{3-x+2x^2} + \frac{\int \frac{(919200-1122240x)(5+2x)}{\sqrt{3-x+2x^2}}\,dx}{245760}$$

$$= \frac{761}{256}(5+2x)^2\sqrt{3-x+2x^2} - \frac{105}{128}(5+2x)^3\sqrt{3-x+2x^2} + \frac{1}{16}(5+2x)^4\sqrt{3-x+2x^2} - \frac{(19227+4676x)\sqrt{3-x+2x^2}}{2048} + \frac{85429\int \frac{1}{\sqrt{3-x+2x^2}}\,dx}{4096}$$

$$= \frac{761}{256}(5+2x)^2\sqrt{3-x+2x^2} - \frac{105}{128}(5+2x)^3\sqrt{3-x+2x^2} + \frac{1}{16}(5+2x)^4\sqrt{3-x+2x^2} - \frac{(19227+4676x)\sqrt{3-x+2x^2}}{2048} + \frac{85429\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx, x, -1+4x\right)}{4096\sqrt{46}}$$

$$= \frac{761}{256}(5+2x)^2\sqrt{3-x+2x^2} - \frac{105}{128}(5+2x)^3\sqrt{3-x+2x^2} + \frac{1}{16}(5+2x)^4\sqrt{3-x+2x^2} - \frac{(19227+4676x)\sqrt{3-x+2x^2}}{2048} - \frac{85429\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{4096\sqrt{2}}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.58

$$\int \frac{(5+2x)\left(2+x+3x^2-x^3+5x^4\right)}{\sqrt{3-x+2x^2}}\,dx = \frac{4\sqrt{3-x+2x^2}\left(2973-6916x+352x^2+7040x^3+2048x^4\right)-85429\sqrt{2}\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{8192}$$

```
[In] Integrate[((5 + 2*x)*(2 + x + 3*x^2 - x^3 + 5*x^4))/Sqrt[3 - x + 2*x^2],x]
[Out] (4*Sqrt[3 - x + 2*x^2]*(2973 - 6916*x + 352*x^2 + 7040*x^3 + 2048*x^4) - 85
429*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/8192
```

Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.42

method	result
risch	$\frac{(2048x^4+7040x^3+352x^2-6916x+2973)\sqrt{2x^2-x+3}}{2048}+\frac{85429\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{8192}$
trager	$\left(x^4+\frac{55}{16}x^3+\frac{11}{64}x^2-\frac{1729}{512}x+\frac{2973}{2048}\right)\sqrt{2x^2-x+3}-\frac{85429\,\mathrm{RootOf}\left(\_Z^2-2\right)\ln\left(-4\,\mathrm{RootOf}\left(\_Z^2-2\right)x+\mathrm{RootO}\right.}{8192}$
default	$\frac{85429\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{8192}+\frac{11x^2\sqrt{2x^2-x+3}}{64}-\frac{1729x\sqrt{2x^2-x+3}}{512}+\frac{2973\sqrt{2x^2-x+3}}{2048}+x^4\sqrt{2x^2-x+3}+$

```
[In] int((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(1/2),x,method=_RETURNVERBOSE
)
```

```
[Out] 1/2048*(2048*x^4+7040*x^3+352*x^2-6916*x+2973)*(2*x^2-x+3)^(1/2)+85429/8192
*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.61

$$\begin{aligned}\int \frac{(5+2x)\left(2+x+3x^2-x^3+5x^4\right)}{\sqrt{3-x+2x^2}}\,dx\\ &= \frac{1}{2048}\left(2048\,x^4+7040\,x^3+352\,x^2-6916\,x+2973\right)\sqrt{2\,x^2-x+3}\\ &\quad+\frac{85429}{16384}\sqrt{2}\log\left(-4\,\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1)-32\,x^2+16\,x-25\right)\end{aligned}$$

```
[In] integrate((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(1/2),x, algorithm="fri
cas")
```

```
[Out] 1/2048*(2048*x^4 + 7040*x^3 + 352*x^2 - 6916*x + 2973)*sqrt(2*x^2 - x + 3)
+ 85429/16384*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2
 + 16*x - 25)
```

Sympy [A] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.50

$$\int \frac{(5+2x)\left(2+x+3x^2-x^3+5x^4\right)}{\sqrt{3-x+2x^2}}\,dx = \sqrt{2x^2-x+3}\left(x^4+\frac{55x^3}{16}+\frac{11x^2}{64}-\frac{1729x}{512}+\frac{2973}{2048}\right)+\frac{85429\sqrt{2}\operatorname{asinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{8192}$$

```
[In] integrate((5+2*x)*(5*x**4-x**3+3*x**2+x+2)/(2*x**2-x+3)**(1/2),x)
```

```
[Out] sqrt(2*x**2 - x + 3)*(x**4 + 55*x**3/16 + 11*x**2/64 - 1729*x/512 + 2973/20
48) + 85429*sqrt(2)*asinh(4*sqrt(23)*(x - 1/4)/23)/8192
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.80

$$\int \frac{(5+2x)\left(2+x+3x^2-x^3+5x^4\right)}{\sqrt{3-x+2x^2}}\,dx = \sqrt{2\,x^2-x+3}x^4+\frac{55}{16}\sqrt{2\,x^2-x+3}x^3+\frac{11}{64}\sqrt{2\,x^2-x+3}x^2-\frac{1729}{512}\sqrt{2\,x^2-x+3}x+\frac{85429}{8192}\sqrt{2}\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}(4\,x-1)\right)+\frac{2973}{2048}\sqrt{2\,x^2-x+3}$$

```
[In] integrate((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(1/2),x, algorithm="max
ima")
```

```
[Out] sqrt(2*x^2 - x + 3)*x^4 + 55/16*sqrt(2*x^2 - x + 3)*x^3 + 11/64*sqrt(2*x^2
- x + 3)*x^2 - 1729/512*sqrt(2*x^2 - x + 3)*x + 85429/8192*sqrt(2)*arcsinh(
1/23*sqrt(23)*(4*x - 1)) + 2973/2048*sqrt(2*x^2 - x + 3)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.57

$$\int \frac{(5+2x)\left(2+x+3x^2-x^3+5x^4\right)}{\sqrt{3-x+2x^2}}\,dx = \frac{1}{2048}\left(4\left(8\left(4\left(16\,x+55\right)x+11\right)x-1729\right)x+2973\right)\sqrt{2\,x^2-x+3}-\frac{85429}{8192}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)+1\right)$$

```
[In] integrate((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(1/2),x, algorithm="gia
c")
```

```
[Out] 1/2048*(4*(8*(4*(16*x + 55)*x + 11)*x - 1729)*x + 2973)*sqrt(2*x^2 - x + 3)
 - 85429/8192*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(5+2x)\left(2+x+3x^2-x^3+5x^4\right)}{\sqrt{3-x+2x^2}}\,dx=\int \frac{\left(2\,x+5\right)\,\left(5\,x^4-x^3+3\,x^2+x+2\right)}{\sqrt{2\,x^2-x+3}}\,dx$$

```
[In] int(((2*x + 5)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x^2 - x + 3)^(1/2),x)
```

```
[Out] int(((2*x + 5)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x^2 - x + 3)^(1/2), x)
```

3.345 $\int \frac{2+x+3x^2-x^3+5x^4}{\sqrt{3-x+2x^2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 101

$$\begin{aligned}\int \frac{2+x+3x^2-x^3+5x^4}{\sqrt{3-x+2x^2}}\,dx =& -\frac{505\sqrt{3-x+2x^2}}{1024}-\frac{409}{768}x\sqrt{3-x+2x^2}\\&+\frac{19}{96}x^2\sqrt{3-x+2x^2}+\frac{5}{8}x^3\sqrt{3-x+2x^2}\\&-\frac{6863\text{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{2048\sqrt{2}}\end{aligned}$$

```
[Out] -6863/4096*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-505/1024*(2*x^2-x+3)^(1/2
)-409/768*x*(2*x^2-x+3)^(1/2)+19/96*x^2*(2*x^2-x+3)^(1/2)+5/8*x^3*(2*x^2-x+
3)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {1675, 654, 633, 221}

$$\begin{aligned}\int \frac{2+x+3x^2-x^3+5x^4}{\sqrt{3-x+2x^2}}\,dx =& -\frac{6863\text{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{2048\sqrt{2}}\\&+\frac{19}{96}\sqrt{2x^2-x+3}x^2-\frac{409}{768}\sqrt{2x^2-x+3}x\\&-\frac{505\sqrt{2x^2-x+3}}{1024}+\frac{5}{8}\sqrt{2x^2-x+3}x^3\end{aligned}$$

```
[In] Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/Sqrt[3 - x + 2*x^2],x]
```

```
[Out] (-505*Sqrt[3 - x + 2*x^2])/1024 - (409*x*Sqrt[3 - x + 2*x^2])/768 + (19*x^2
*Sqrt[3 - x + 2*x^2])/96 + (5*x^3*Sqrt[3 - x + 2*x^2])/8 - (6863*ArcSinh[(1
 - 4*x)/Sqrt[23]])/(2048*Sqrt[2])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{5}{8}x^3\sqrt{3-x+2x^2}+\frac{1}{8}\int \frac{16+8x-21x^2+\frac{19x^3}{2}}{\sqrt{3-x+2x^2}}\,dx \\
&= \frac{19}{96}x^2\sqrt{3-x+2x^2}+\frac{5}{8}x^3\sqrt{3-x+2x^2}+\frac{1}{48}\int \frac{96-9x-\frac{409x^2}{4}}{\sqrt{3-x+2x^2}}\,dx \\
&= -\frac{409}{768}x\sqrt{3-x+2x^2}+\frac{19}{96}x^2\sqrt{3-x+2x^2}+\frac{5}{8}x^3\sqrt{3-x+2x^2}+\frac{1}{192}\int \frac{\frac{2763}{4}-\frac{1515x}{8}}{\sqrt{3-x+2x^2}}\,dx \\
&= -\frac{505\sqrt{3-x+2x^2}}{1024}-\frac{409}{768}x\sqrt{3-x+2x^2}+\frac{19}{96}x^2\sqrt{3-x+2x^2} \\
&\quad +\frac{5}{8}x^3\sqrt{3-x+2x^2}+\frac{6863\int \frac{1}{\sqrt{3-x+2x^2}}\,dx}{2048}
\end{aligned}
$$

$$= -\frac{505\sqrt{3-x+2x^2}}{1024} - \frac{409}{768}x\sqrt{3-x+2x^2} + \frac{19}{96}x^2\sqrt{3-x+2x^2} + \frac{5}{8}x^3\sqrt{3-x+2x^2} + \frac{6863\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx, x, -1+4x\right)}{2048\sqrt{46}}$$

$$= -\frac{505\sqrt{3-x+2x^2}}{1024} - \frac{409}{768}x\sqrt{3-x+2x^2} + \frac{19}{96}x^2\sqrt{3-x+2x^2} + \frac{5}{8}x^3\sqrt{3-x+2x^2} - \frac{6863\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{2048\sqrt{2}}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.64

$$\int \frac{2+x+3x^2-x^3+5x^4}{\sqrt{3-x+2x^2}}\,dx = \frac{4\sqrt{3-x+2x^2}(-1515-1636x+608x^2+1920x^3) - 20589\sqrt{2}\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{12288}$$

```
[In] Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/Sqrt[3 - x + 2*x^2],x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(-1515 - 1636*x + 608*x^2 + 1920*x^3) - 20589*Sqrt[2
]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/12288
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.45

method	result
risch	$\frac{(1920x^3+608x^2-1636x-1515)\sqrt{2x^2-x+3}}{3072} + \frac{6863\sqrt{2}\,\text{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{4096}$
trager	$\left(\frac{5}{8}x^3 + \frac{19}{96}x^2 - \frac{409}{768}x - \frac{505}{1024}\right)\sqrt{2x^2-x+3} + \frac{6863\,\text{RootOf}\left(\_Z^2-2\right)\ln\left(4\,\text{RootOf}\left(\_Z^2-2\right)x+4\sqrt{2x^2-x+3}-\text{Root}\right.}{4096}$
default	$-\frac{505\sqrt{2x^2-x+3}}{1024} + \frac{6863\sqrt{2}\,\text{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{4096} + \frac{5x^3\sqrt{2x^2-x+3}}{8} + \frac{19x^2\sqrt{2x^2-x+3}}{96} - \frac{409x\sqrt{2x^2-x+3}}{768}$

```
[In] int((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3072*(1920*x^3+608*x^2-1636*x-1515)*(2*x^2-x+3)^(1/2)+6863/4096*2^(1/2)*a
rcsinh(4/23*23^(1/2)*(x-1/4))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.67

$$\int \frac{2+x+3x^2-x^3+5x^4}{\sqrt{3-x+2x^2}}\,dx = \frac{1}{3072}\left(1920\,x^3+608\,x^2-1636\,x-1515\right)\sqrt{2\,x^2-x+3} + \frac{6863}{8192}\,\sqrt{2}\log\Big(-4\,\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1)-32\,x^2 + 16\,x-25\Big)$$

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(1/2),x, algorithm="fricas")
[Out] 1/3072*(1920*x^3 + 608*x^2 - 1636*x - 1515)*sqrt(2*x^2 - x + 3) + 6863/8192
*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25)
```

Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.55

$$\int \frac{2+x+3x^2-x^3+5x^4}{\sqrt{3-x+2x^2}}\,dx = \sqrt{2x^2-x+3}\cdot\left(\frac{5x^3}{8}+\frac{19x^2}{96}-\frac{409x}{768}-\frac{505}{1024}\right) + \frac{6863\sqrt{2}\operatorname{asinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{4096}$$

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)/(2*x**2-x+3)**(1/2),x)
[Out] sqrt(2*x**2 - x + 3)*(5*x**3/8 + 19*x**2/96 - 409*x/768 - 505/1024) + 6863*
sqrt(2)*asinh(4*sqrt(23)*(x - 1/4)/23)/4096
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.79

$$\int \frac{2+x+3x^2-x^3+5x^4}{\sqrt{3-x+2x^2}}\,dx = \frac{5}{8}\,\sqrt{2\,x^2-x+3}x^3+\frac{19}{96}\,\sqrt{2\,x^2-x+3}x^2 - \frac{409}{768}\,\sqrt{2\,x^2-x+3}x + \frac{6863}{4096}\,\sqrt{2}\operatorname{arsinh}\left(\frac{1}{23}\,\sqrt{23}(4\,x-1)\right)-\frac{505}{1024}\,\sqrt{2\,x^2-x+3}$$

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(1/2),x, algorithm="maxima")
[Out] 5/8*sqrt(2*x^2 - x + 3)*x^3 + 19/96*sqrt(2*x^2 - x + 3)*x^2 - 409/768*sqrt(
2*x^2 - x + 3)*x + 6863/4096*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 505
/1024*sqrt(2*x^2 - x + 3)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.62

$$\int \frac{2+x+3x^2-x^3+5x^4}{\sqrt{3-x+2x^2}}\,dx = \frac{1}{3072}\left(4\left(8\left(60\,x+19\right)x-409\right)x-1515\right)\sqrt{2\,x^2-x+3} - \frac{6863}{4096}\,\sqrt{2}\log\left(-2\,\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)+1\right)$$

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3072*(4*(8*(60*x + 19)*x - 409)*x - 1515)*sqrt(2*x^2 - x + 3) - 6863/4096
*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2+x+3x^2-x^3+5x^4}{\sqrt{3-x+2x^2}}\,dx = \int \frac{5\,x^4-x^3+3\,x^2+x+2}{\sqrt{2\,x^2-x+3}}\,dx$$

```
[In] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/(2*x^2 - x + 3)^(1/2),x)
```

```
[Out] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/(2*x^2 - x + 3)^(1/2), x)
```

3.346 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)\sqrt{3-x+2x^2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 126

$$\begin{aligned}\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)\sqrt{3-x+2x^2}}\,dx &= \frac{1669}{128}\sqrt{3-x+2x^2}-\frac{337}{192}(5+2x)\sqrt{3-x+2x^2}\\ &\quad+\frac{5}{48}(5+2x)^2\sqrt{3-x+2x^2}+\frac{9657\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{256\sqrt{2}}\\ &\quad-\frac{3667\operatorname{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{96\sqrt{2}}\end{aligned}$$

```
[Out] 9657/512*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-3667/192*arctanh(1/24*(17-2
2*x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)+1669/128*(2*x^2-x+3)^(1/2)-337/192*
(5+2*x)*(2*x^2-x+3)^(1/2)+5/48*(5+2*x)^2*(2*x^2-x+3)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1667, 857, 633, 221, 738, 212}

$$\begin{aligned}\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)\sqrt{3-x+2x^2}}\,dx &= \frac{9657\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{256\sqrt{2}}-\frac{3667\operatorname{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{96\sqrt{2}}\\ &\quad+\frac{5}{48}\sqrt{2x^2-x+3}(2x+5)^2\\ &\quad-\frac{337}{192}\sqrt{2x^2-x+3}(2x+5)+\frac{1669}{128}\sqrt{2x^2-x+3}\end{aligned}$$

```
[In] Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)*Sqrt[3 - x + 2*x^2]),x]
```

```
[Out] (1669*Sqrt[3 - x + 2*x^2])/128 - (337*(5 + 2*x)*Sqrt[3 - x + 2*x^2])/192 +
(5*(5 + 2*x)^2*Sqrt[3 - x + 2*x^2])/48 + (9657*ArcSinh[(1 - 4*x)/Sqrt[23]])
/(256*Sqrt[2]) - (3667*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])
])/(96*Sqrt[2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
```

```
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{5}{48}(5+2x)^2\sqrt{3-x+2x^2}+\frac{1}{96}\int\frac{-2183-3054x-4092x^2-2696x^3}{(5+2x)\sqrt{3-x+2x^2}}\,dx\\
&= -\frac{337}{192}(5+2x)\sqrt{3-x+2x^2}+\frac{5}{48}(5+2x)^2\sqrt{3-x+2x^2}+\frac{\int\frac{24504+128736x+160224x^2}{(5+2x)\sqrt{3-x+2x^2}}\,dx}{3072}\\
&= \frac{1669}{128}\sqrt{3-x+2x^2}-\frac{337}{192}(5+2x)\sqrt{3-x+2x^2}\\
&\quad+\frac{5}{48}(5+2x)^2\sqrt{3-x+2x^2}+\frac{\int\frac{997152-1854144x}{(5+2x)\sqrt{3-x+2x^2}}\,dx}{24576}\\
&= \frac{1669}{128}\sqrt{3-x+2x^2}-\frac{337}{192}(5+2x)\sqrt{3-x+2x^2}+\frac{5}{48}(5+2x)^2\sqrt{3-x+2x^2}\\
&\quad-\frac{9657}{256}\int\frac{1}{\sqrt{3-x+2x^2}}\,dx+\frac{3667}{16}\int\frac{1}{(5+2x)\sqrt{3-x+2x^2}}\,dx\\
&= \frac{1669}{128}\sqrt{3-x+2x^2}-\frac{337}{192}(5+2x)\sqrt{3-x+2x^2}+\frac{5}{48}(5+2x)^2\sqrt{3-x+2x^2}\\
&\quad-\frac{3667}{8}\text{Subst}\left(\int\frac{1}{288-x^2}\,dx,x,\frac{17-22x}{\sqrt{3-x+2x^2}}\right)-\frac{9657\text{Subst}\left(\int\frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx,x,-1+4x\right)}{256\sqrt{46}}\\
&= \frac{1669}{128}\sqrt{3-x+2x^2}-\frac{337}{192}(5+2x)\sqrt{3-x+2x^2}+\frac{5}{48}(5+2x)^2\sqrt{3-x+2x^2}\\
&\quad+\frac{9657\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{256\sqrt{2}}-\frac{3667\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{96\sqrt{2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.74

$$\int\frac{2+x+3x^2-x^3+5x^4}{(5+2x)\sqrt{3-x+2x^2}}\,dx$$

$$=\frac{4\sqrt{3-x+2x^2}(2637-548x+160x^2)+58672\sqrt{2}\text{arctanh}\left(\frac{1}{6}\left(5+2x-\sqrt{6-2x+4x^2}\right)\right)+28971\sqrt{2}\log (}{1536}$$

```
[In] Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)*Sqrt[3 - x + 2*x^2]),x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(2637 - 548*x + 160*x^2) + 58672*Sqrt[2]*ArcTanh[(5
+ 2*x - Sqrt[6 - 2*x + 4*x^2])/6] + 28971*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 -
2*x + 4*x^2]])/1536
```

Maple [F(-1)]

Timed out.

hanged

```
[In] int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(1/2),x)
```

```
[Out] int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.91

$$\begin{aligned}&\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)\sqrt{3-x+2x^2}}\,dx\\&=\frac{1}{384}\left(160\,x^2-548\,x+2637\right)\sqrt{2\,x^2-x+3}\\&\quad+\frac{9657}{1024}\,\sqrt{2}\log\left(4\,\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1)-32\,x^2+16\,x-25\right)\\&\quad+\frac{3667}{384}\,\sqrt{2}\log\left(-\frac{24\,\sqrt{2}\sqrt{2\,x^2-x+3}(22\,x-17)+1060\,x^2-1036\,x+1153}{4\,x^2+20\,x+25}\right)\end{aligned}$$

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(1/2),x, algorithm="fri
cas")
```

```
[Out] 1/384*(160*x^2 - 548*x + 2637)*sqrt(2*x^2 - x + 3) + 9657/1024*sqrt(2)*log(
4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 3667/384*sq
rt(2)*log(-(24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) + 1060*x^2 - 1036*x
+ 1153)/(4*x^2 + 20*x + 25))
```

Sympy [F]

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)\sqrt{3-x+2x^2}}\,dx=\int \frac{5x^4-x^3+3x^2+x+2}{(2x+5)\sqrt{2x^2-x+3}}\,dx$$

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)/(2*x**2-x+3)**(1/2),x)
```

```
[Out] Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)*sqrt(2*x**2 - x + 3)),
 x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.79

$$\begin{aligned}\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)\sqrt{3-x+2x^2}}\,dx &= \frac{5}{12}\,\sqrt{2\,x^2-x+3}x^2-\frac{137}{96}\,\sqrt{2\,x^2-x+3}x\\&\quad-\frac{9657}{512}\,\sqrt{2}\,\mathrm{arsinh}\left(\frac{4}{23}\,\sqrt{23}x-\frac{1}{23}\,\sqrt{23}\right)\\&\quad+\frac{3667}{192}\,\sqrt{2}\,\mathrm{arsinh}\left(\frac{22\,\sqrt{23}x}{23\,|2\,x+5|}-\frac{17\,\sqrt{23}}{23\,|2\,x+5|}\right)\\&\quad+\frac{879}{128}\,\sqrt{2\,x^2-x+3}\end{aligned}$$

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(1/2),x, algorithm="max
ima")
```

```
[Out] 5/12*sqrt(2*x^2 - x + 3)*x^2 - 137/96*sqrt(2*x^2 - x + 3)*x - 9657/512*sqrt
(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) + 3667/192*sqrt(2)*arcsinh(22/
23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)/abs(2*x + 5)) + 879/128*sqrt(2*
x^2 - x + 3)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.94

$$\begin{aligned}\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)\sqrt{3-x+2x^2}}\,dx &= \frac{1}{384}\,(4\,(40\,x-137)x+2637)\sqrt{2\,x^2-x+3}\\&\quad+\frac{9657}{512}\,\sqrt{2}\log\left(-4\,\sqrt{2}x+\sqrt{2}+4\,\sqrt{2\,x^2-x+3}\right)\\&\quad-\frac{3667}{192}\,\sqrt{2}\log\left(\left|-2\,\sqrt{2}x+\sqrt{2}+2\,\sqrt{2\,x^2-x+3}\right|\right)\\&\quad+\frac{3667}{192}\,\sqrt{2}\log\left(\left|-2\,\sqrt{2}x-11\,\sqrt{2}+2\,\sqrt{2\,x^2-x+3}\right|\right)\end{aligned}$$

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(1/2),x, algorithm="gia
c")
```

```
[Out] 1/384*(4*(40*x - 137)*x + 2637)*sqrt(2*x^2 - x + 3) + 9657/512*sqrt(2)*log(
-4*sqrt(2)*x + sqrt(2) + 4*sqrt(2*x^2 - x + 3)) - 3667/192*sqrt(2)*log(abs(
-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 - x + 3))) + 3667/192*sqrt(2)*log(abs
(-2*sqrt(2)*x - 11*sqrt(2) + 2*sqrt(2*x^2 - x + 3)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)\sqrt{3-x+2x^2}}\,dx = \int \frac{5\,x^4-x^3+3\,x^2+x+2}{(2\,x+5)\,\sqrt{2\,x^2-x+3}}\,dx$$

```
[In] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)*(2*x^2 - x + 3)^(1/2)),x)
[Out] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)*(2*x^2 - x + 3)^(1/2)), x)
```

3.347 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2\sqrt{3-x+2x^2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 126

$$\begin{aligned}\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2\sqrt{3-x+2x^2}}\,dx &= -\frac{243}{64}\sqrt{3-x+2x^2}-\frac{3667\sqrt{3-x+2x^2}}{576(5+2x)}\\ &\quad+\frac{5}{32}(5+2x)\sqrt{3-x+2x^2}-\frac{2943\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{128\sqrt{2}}\\ &\quad+\frac{158527\operatorname{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{6912\sqrt{2}}\end{aligned}$$

```
[Out] -2943/256*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)+158527/13824*arctanh(1/24*
(17-22*x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)-243/64*(2*x^2-x+3)^(1/2)-3667/
576*(2*x^2-x+3)^(1/2)/(5+2*x)+5/32*(5+2*x)*(2*x^2-x+3)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {1664, 1667, 857, 633, 221, 738, 212}

$$\begin{aligned}\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2\sqrt{3-x+2x^2}}\,dx &= -\frac{2943\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{128\sqrt{2}}+\frac{158527\operatorname{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{6912\sqrt{2}}\\ &\quad+\frac{5}{32}\sqrt{2x^2-x+3}(2x+5)\\ &\quad-\frac{243}{64}\sqrt{2x^2-x+3}-\frac{3667\sqrt{2x^2-x+3}}{576(2x+5)}\end{aligned}$$

```
[In] Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^2*Sqrt[3 - x + 2*x^2]),x]
```

```
[Out] (-243*Sqrt[3 - x + 2*x^2])/64 - (3667*Sqrt[3 - x + 2*x^2])/(576*(5 + 2*x))
+ (5*(5 + 2*x)*Sqrt[3 - x + 2*x^2])/32 - (2943*ArcSinh[(1 - 4*x)/Sqrt[23]])
/(128*Sqrt[2]) + (158527*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2
])])/(6912*Sqrt[2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
```

```
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{3667\sqrt{3-x+2x^2}}{576(5+2x)} - \frac{1}{72}\int \frac{\frac{12007}{16}-1323x+486x^2-180x^3}{(5+2x)\sqrt{3-x+2x^2}}\,dx \\
&= -\frac{3667\sqrt{3-x+2x^2}}{576(5+2x)} + \frac{5}{32}(5+2x)\sqrt{3-x+2x^2} - \frac{\int \frac{30314-27216x+34992x^2}{(5+2x)\sqrt{3-x+2x^2}}\,dx}{2304} \\
&= -\frac{243}{64}\sqrt{3-x+2x^2} - \frac{3667\sqrt{3-x+2x^2}}{576(5+2x)} + \frac{5}{32}(5+2x)\sqrt{3-x+2x^2} - \frac{\int \frac{417472-847584x}{(5+2x)\sqrt{3-x+2x^2}}\,dx}{18432} \\
&= -\frac{243}{64}\sqrt{3-x+2x^2} - \frac{3667\sqrt{3-x+2x^2}}{576(5+2x)} + \frac{5}{32}(5+2x)\sqrt{3-x+2x^2} \\
&\quad + \frac{2943}{128}\int \frac{1}{\sqrt{3-x+2x^2}}\,dx - \frac{158527\int \frac{1}{(5+2x)\sqrt{3-x+2x^2}}\,dx}{1152} \\
&= -\frac{243}{64}\sqrt{3-x+2x^2} - \frac{3667\sqrt{3-x+2x^2}}{576(5+2x)} + \frac{5}{32}(5+2x)\sqrt{3-x+2x^2} \\
&\quad + \frac{158527}{576}\text{Subst}\left(\int \frac{1}{288-x^2}\,dx, x, \frac{17-22x}{\sqrt{3-x+2x^2}}\right) \\
&\quad + \frac{2943\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx, x, -1+4x\right)}{128\sqrt{46}} \\
&= -\frac{243}{64}\sqrt{3-x+2x^2} - \frac{3667\sqrt{3-x+2x^2}}{576(5+2x)} + \frac{5}{32}(5+2x)\sqrt{3-x+2x^2} \\
&\quad - \frac{2943\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{128\sqrt{2}} + \frac{158527\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{6912\sqrt{2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.79

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2\sqrt{3-x+2x^2}}\,dx$$
$$= \frac{\frac{24\sqrt{3-x+2x^2}(-6176-1287x+180x^2)}{5+2x} - 158527\sqrt{2}\operatorname{arctanh}\left(\frac{1}{6}\left(5+2x-\sqrt{6-2x+4x^2}\right)\right) - 79461\sqrt{2}\log(1-4x+}{6912}$$

```
[In] Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^2*Sqrt[3 - x + 2*x^2]),x
]
```

```
[Out] ((24*Sqrt[3 - x + 2*x^2]*(-6176 - 1287*x + 180*x^2))/(5 + 2*x) - 158527*Sqr
t[2]*ArcTanh[(5 + 2*x - Sqrt[6 - 2*x + 4*x^2])/6] - 79461*Sqrt[2]*Log[1 - 4
*x + 2*Sqrt[6 - 2*x + 4*x^2]])/6912
```

Maple [F(-1)]

Timed out.

hanged

```
[In] int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(1/2),x)
```

```
[Out] int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.06

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2\sqrt{3-x+2x^2}}\,dx$$
$$= \frac{158922\sqrt{2}(2x+5)\log\left(-4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-32x^2+16x-25\right)+158527\sqrt{2}(2x+5)\log(\frac{24}{}}{27648\,(2x+5)}$$

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(1/2),x, algorithm="f
ricas")
```

```
[Out] 1/27648*(158922*sqrt(2)*(2*x + 5)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x -
 1) - 32*x^2 + 16*x - 25) + 158527*sqrt(2)*(2*x + 5)*log((24*sqrt(2)*sqrt(2
*x^2 - x + 3)*(22*x - 17) - 1060*x^2 + 1036*x - 1153)/(4*x^2 + 20*x + 25))
+ 96*(180*x^2 - 1287*x - 6176)*sqrt(2*x^2 - x + 3))/(2*x + 5)
```

Sympy [F]

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2\sqrt{3-x+2x^2}}\,dx = \int \frac{5x^4-x^3+3x^2+x+2}{(2x+5)^2\sqrt{2x^2-x+3}}\,dx$$

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)**2/(2*x**2-x+3)**(1/2),x)
```

```
[Out] Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)**2*sqrt(2*x**2 - x + 3
)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.82

$$\begin{aligned}\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2\sqrt{3-x+2x^2}}\,dx = {} & \frac{5}{16}\sqrt{2\,x^2-x+3}x + \frac{2943}{256}\sqrt{2}\operatorname{arsinh}\left(\frac{4}{23}\sqrt{23}x - \frac{1}{23}\sqrt{23}\right) \\ & - \frac{158527}{13824}\sqrt{2}\operatorname{arsinh}\left(\frac{22\sqrt{23}x}{23\,|2\,x+5|} - \frac{17\sqrt{23}}{23\,|2\,x+5|}\right) \\ & - \frac{193}{64}\sqrt{2\,x^2-x+3} - \frac{3667\sqrt{2\,x^2-x+3}}{576\,(2\,x+5)}\end{aligned}$$

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(1/2),x, algorithm="m
axima")
```

```
[Out] 5/16*sqrt(2*x^2 - x + 3)*x + 2943/256*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/2
3*sqrt(23)) - 158527/13824*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) -
17/23*sqrt(23)/abs(2*x + 5)) - 193/64*sqrt(2*x^2 - x + 3) - 3667/576*sqrt(2
*x^2 - x + 3)/(2*x + 5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 339 vs. 2(99) = 198.

Time = 0.34 (sec) , antiderivative size = 339, normalized size of antiderivative = 2.69

$$\begin{aligned}&\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2\sqrt{3-x+2x^2}}\,dx \\ &= \frac{1}{13824}\sqrt{2}\left(\frac{158527\,\log\left(12\sqrt{-\frac{11}{2\,x+5}+\frac{36}{(2\,x+5)^2}+1}+\frac{72}{2\,x+5}-11\right)}{\operatorname{sgn}\left(\frac{1}{2\,x+5}\right)} + \frac{158922\,\log\left(\left|\sqrt{-\frac{11}{2\,x+5}+\frac{36}{(2\,x+5)^2}+1\right.\right.}{\operatorname{sgn}\left(\frac{1}{2\,x+5}\right)}\right.\end{aligned}$$

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(1/2),x, algorithm="g
iac")
```

```
[Out] 1/13824*sqrt(2)*(158527*log(12*sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 7
2/(2*x + 5) - 11)/sgn(1/(2*x + 5)) + 158922*log(abs(sqrt(-11/(2*x + 5) + 36
/(2*x + 5)^2 + 1) + 6/(2*x + 5) + 1))/sgn(1/(2*x + 5)) - 158922*log(abs(sqr
t(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5) - 1))/sgn(1/(2*x + 5))
- 44004*sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1)/sgn(1/(2*x + 5)) + 108*(33
93*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^3 - 4896*(sqrt(
-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^2 - 743*sqrt(-11/(2*x +
5) + 36/(2*x + 5)^2 + 1) - 4458/(2*x + 5) + 2256)/(((sqrt(-11/(2*x + 5) + 3
6/(2*x + 5)^2 + 1) + 6/(2*x + 5))^2 - 1)^2*sgn(1/(2*x + 5))))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2\sqrt{3-x+2x^2}}\,dx=\int \frac{5\,x^4-x^3+3\,x^2+x+2}{{\left(2\,x+5\right)}^2\,\sqrt{2\,x^2-x+3}}\,dx$$

```
[In] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^2*(2*x^2 - x + 3)^(1/2)),x)
```

```
[Out] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^2*(2*x^2 - x + 3)^(1/2)), x)
```

3.348 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3\sqrt{3-x+2x^2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 128

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3\sqrt{3-x+2x^2}}\,dx=\frac{5}{16}\sqrt{3-x+2x^2}-\frac{3667\sqrt{3-x+2x^2}}{1152(5+2x)^2}+\frac{92239\sqrt{3-x+2x^2}}{27648(5+2x)}+\frac{149\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{32\sqrt{2}}-\frac{1546507\operatorname{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{331776\sqrt{2}}$$

```
[Out] 149/64*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-1546507/663552*arctanh(1/24*(
17-22*x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)+5/16*(2*x^2-x+3)^(1/2)-3667/115
2*(2*x^2-x+3)^(1/2)/(5+2*x)^2+92239/27648*(2*x^2-x+3)^(1/2)/(5+2*x)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {1664, 1667, 857, 633, 221, 738, 212}

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3\sqrt{3-x+2x^2}}\,dx=\frac{149\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{32\sqrt{2}}-\frac{1546507\operatorname{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{331776\sqrt{2}}+\frac{92239\sqrt{2x^2-x+3}}{27648(2x+5)}-\frac{3667\sqrt{2x^2-x+3}}{1152(2x+5)^2}+\frac{5}{16}\sqrt{2x^2-x+3}$$

```
[In] Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^3*Sqrt[3 - x + 2*x^2]),x]
```

```
[Out] (5*Sqrt[3 - x + 2*x^2])/16 - (3667*Sqrt[3 - x + 2*x^2])/(1152*(5 + 2*x)^2)
+ (92239*Sqrt[3 - x + 2*x^2])/(27648*(5 + 2*x)) + (149*ArcSinh[(1 - 4*x)/Sq
rt[23]])/(32*Sqrt[2]) - (1546507*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x
 + 2*x^2])])/(331776*Sqrt[2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{3667\sqrt{3-x+2x^2}}{1152(5+2x)^2}-\frac{1}{144}\int\frac{\frac{20347}{16}-\frac{6917x}{4}+972x^2-360x^3}{(5+2x)^2\sqrt{3-x+2x^2}}\,dx\\
&= -\frac{3667\sqrt{3-x+2x^2}}{1152(5+2x)^2}+\frac{92239\sqrt{3-x+2x^2}}{27648(5+2x)}+\frac{\int\frac{\frac{647841}{16}-67392x+12960x^2}{(5+2x)\sqrt{3-x+2x^2}}\,dx}{10368}\\
&= \frac{5}{16}\sqrt{3-x+2x^2}-\frac{3667\sqrt{3-x+2x^2}}{1152(5+2x)^2}+\frac{92239\sqrt{3-x+2x^2}}{27648(5+2x)}+\frac{\int\frac{\frac{777441}{2}-772416x}{(5+2x)\sqrt{3-x+2x^2}}\,dx}{82944}\\
&= \frac{5}{16}\sqrt{3-x+2x^2}-\frac{3667\sqrt{3-x+2x^2}}{1152(5+2x)^2}+\frac{92239\sqrt{3-x+2x^2}}{27648(5+2x)}\\
&\quad -\frac{149}{32}\int\frac{1}{\sqrt{3-x+2x^2}}\,dx+\frac{1546507\int\frac{1}{(5+2x)\sqrt{3-x+2x^2}}\,dx}{55296}\\
&= \frac{5}{16}\sqrt{3-x+2x^2}-\frac{3667\sqrt{3-x+2x^2}}{1152(5+2x)^2}+\frac{92239\sqrt{3-x+2x^2}}{27648(5+2x)}\\
&\quad -\frac{1546507\text{Subst}\left(\int\frac{1}{288-x^2}\,dx,x,\frac{17-22x}{\sqrt{3-x+2x^2}}\right)}{27648}-\frac{149\text{Subst}\left(\int\frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx,x,-1+4x\right)}{32\sqrt{46}}\\
&= \frac{5}{16}\sqrt{3-x+2x^2}-\frac{3667\sqrt{3-x+2x^2}}{1152(5+2x)^2}+\frac{92239\sqrt{3-x+2x^2}}{27648(5+2x)}\\
&\quad +\frac{149\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{32\sqrt{2}}-\frac{1546507\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{331776\sqrt{2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.78

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3\sqrt{3-x+2x^2}}\,dx = \frac{\frac{12\sqrt{3-x+2x^2}(589187+357278x+34560x^2)}{(5+2x)^2}+1546507\sqrt{2}\text{arctanh}\left(\frac{1}{6}\left(5+2x-\sqrt{6-2x+4x^2}\right)\right)+772416\sqrt{2}\log (1-}{331776}$$

```
[In] Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^3*Sqrt[3 - x + 2*x^2]),x
]
```

```
[Out] ((12*Sqrt[3 - x + 2*x^2]*(589187 + 357278*x + 34560*x^2))/(5 + 2*x)^2 + 154
6507*Sqrt[2]*ArcTanh[(5 + 2*x - Sqrt[6 - 2*x + 4*x^2])/6] + 772416*Sqrt[2]*
Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/331776
```

Maple [F(-1)]

Timed out.

hanged

```
[In] int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(1/2),x)
```

```
[Out] int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.16

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3\sqrt{3-x+2x^2}}\,dx = \frac{1544832\sqrt{2}(4x^2+20x+25)\log\left(4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-32x^2+16x-25\right)+1546507\sqrt{2}(4x^2+}{1327104\,(4}$$

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(1/2),x, algorithm="f
ricas")
```

```
[Out] 1/1327104*(1544832*sqrt(2)*(4*x^2 + 20*x + 25)*log(4*sqrt(2)*sqrt(2*x^2 - x
 + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 1546507*sqrt(2)*(4*x^2 + 20*x + 25)
*log(-(24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) + 1060*x^2 - 1036*x + 115
3)/(4*x^2 + 20*x + 25)) + 48*(34560*x^2 + 357278*x + 589187)*sqrt(2*x^2 - x
 + 3))/(4*x^2 + 20*x + 25)
```

Sympy [F]

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3\sqrt{3-x+2x^2}}\,dx=\int \frac{5x^4-x^3+3x^2+x+2}{(2x+5)^3\sqrt{2x^2-x+3}}\,dx$$

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)**3/(2*x**2-x+3)**(1/2),x)
```

```
[Out] Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)**3*sqrt(2*x**2 - x + 3
)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.89

$$\begin{aligned}\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3\sqrt{3-x+2x^2}}\,dx=&-\frac{149}{64}\sqrt{2}\,\mathrm{arsinh}\left(\frac{4}{23}\sqrt{23}x-\frac{1}{23}\sqrt{23}\right)\\&+\frac{1546507}{663552}\sqrt{2}\,\mathrm{arsinh}\left(\frac{22\sqrt{23}x}{23\,|2\,x+5|}-\frac{17\sqrt{23}}{23\,|2\,x+5|}\right)\\&+\frac{5}{16}\sqrt{2\,x^2-x+3}-\frac{3667\sqrt{2\,x^2-x+3}}{1152\,(4\,x^2+20\,x+25)}\\&+\frac{92239\sqrt{2\,x^2-x+3}}{27648\,(2\,x+5)}\end{aligned}$$

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(1/2),x, algorithm="m
axima")
```

```
[Out] -149/64*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) + 1546507/663552*s
qrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)/abs(2*x + 5))
 + 5/16*sqrt(2*x^2 - x + 3) - 3667/1152*sqrt(2*x^2 - x + 3)/(4*x^2 + 20*x +
 25) + 92239/27648*sqrt(2*x^2 - x + 3)/(2*x + 5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(101) = 202$.

Time = 0.30 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.94

$$\begin{aligned}\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3\sqrt{3-x+2x^2}}\,dx &= \frac{149}{64}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right)\\ &\quad-\frac{1546507}{663552}\sqrt{2}\log\left(\left|-2\sqrt{2}x+\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right)\\ &\quad+\frac{1546507}{663552}\sqrt{2}\log\left(\left|-2\sqrt{2}x-11\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right)+\frac{5}{16}\sqrt{2x^2-x+3}\\ &\quad+\frac{\sqrt{2}\Big(2381290\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^3+16628406\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^2-25697445\sqrt{2}\big(\sqrt{2}x-}{55296\left(2\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^2+10\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)-11\right)^2}\end{aligned}$$

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(1/2),x, algorithm="g
iac")
```

```
[Out] 149/64*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) - 1546
507/663552*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 - x + 3)))
 + 1546507/663552*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2) + 2*sqrt(2*x^2
- x + 3))) + 5/16*sqrt(2*x^2 - x + 3) + 1/55296*sqrt(2)*(2381290*sqrt(2)*(s
qrt(2)*x - sqrt(2*x^2 - x + 3))^3 + 16628406*(sqrt(2)*x - sqrt(2*x^2 - x +
3))^2 - 25697445*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 16720645)/(2*(
sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 10*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x
 + 3)) - 11)^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3\sqrt{3-x+2x^2}}\,dx=\int \frac{5\,x^4-x^3+3\,x^2+x+2}{\left(2\,x+5\right)^3\,\sqrt{2\,x^2-x+3}}\,dx$$

```
[In] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^3*(2*x^2 - x + 3)^(1/2)),x)
```

```
[Out] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^3*(2*x^2 - x + 3)^(1/2)), x)
```

3.349 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4\sqrt{3-x+2x^2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 135

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4\sqrt{3-x+2x^2}}\,dx = -\frac{3667\sqrt{3-x+2x^2}}{1728(5+2x)^3}+\frac{394907\sqrt{3-x+2x^2}}{248832(5+2x)^2} - \frac{3163415\sqrt{3-x+2x^2}}{5971968(5+2x)}-\frac{5\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{16\sqrt{2}} + \frac{22389491\operatorname{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{71663616\sqrt{2}}$$

```
[Out] -5/32*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)+22389491/143327232*arctanh(1/2
4*(17-22*x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)-3667/1728*(2*x^2-x+3)^(1/2)/
(5+2*x)^3+394907/248832*(2*x^2-x+3)^(1/2)/(5+2*x)^2-3163415/5971968*(2*x^2-
x+3)^(1/2)/(5+2*x)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1664, 857, 633, 221, 738, 212}

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4\sqrt{3-x+2x^2}}\,dx = -\frac{5\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{16\sqrt{2}}+\frac{22389491\operatorname{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{71663616\sqrt{2}} - \frac{3163415\sqrt{2x^2-x+3}}{5971968(2x+5)} + \frac{394907\sqrt{2x^2-x+3}}{248832(2x+5)^2}-\frac{3667\sqrt{2x^2-x+3}}{1728(2x+5)^3}$$

```
[In] Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^4*Sqrt[3 - x + 2*x^2]),x]
```

```
[Out] (-3667*Sqrt[3 - x + 2*x^2])/(1728*(5 + 2*x)^3) + (394907*Sqrt[3 - x + 2*x^2
])/(248832*(5 + 2*x)^2) - (3163415*Sqrt[3 - x + 2*x^2])/(5971968*(5 + 2*x))
 - (5*ArcSinh[(1 - 4*x)/Sqrt[23]])/(16*Sqrt[2]) + (22389491*ArcTanh[(17 - 2
2*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(71663616*Sqrt[2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
```

```
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{3667\sqrt{3-x+2x^2}}{1728(5+2x)^3} - \frac{1}{216}\int \frac{\frac{28687}{16} - \frac{4271x}{2} + 1458x^2 - 540x^3}{(5+2x)^3\sqrt{3-x+2x^2}}\,dx \\
&= -\frac{3667\sqrt{3-x+2x^2}}{1728(5+2x)^3} + \frac{394907\sqrt{3-x+2x^2}}{248832(5+2x)^2} + \frac{\int \frac{\frac{1464275}{16} - \frac{413797x}{4} + 38880x^2}{(5+2x)^2\sqrt{3-x+2x^2}}\,dx}{31104} \\
&= -\frac{3667\sqrt{3-x+2x^2}}{1728(5+2x)^3} + \frac{394907\sqrt{3-x+2x^2}}{248832(5+2x)^2} - \frac{3163415\sqrt{3-x+2x^2}}{5971968(5+2x)} - \frac{\int \frac{\frac{11181273}{16} - 1399680x}{(5+2x)\sqrt{3-x+2x^2}}\,dx}{2239488} \\
&= -\frac{3667\sqrt{3-x+2x^2}}{1728(5+2x)^3} + \frac{394907\sqrt{3-x+2x^2}}{248832(5+2x)^2} - \frac{3163415\sqrt{3-x+2x^2}}{5971968(5+2x)} \\
&\quad + \frac{5}{16}\int \frac{1}{\sqrt{3-x+2x^2}}\,dx - \frac{22389491\int \frac{1}{(5+2x)\sqrt{3-x+2x^2}}\,dx}{11943936} \\
&= -\frac{3667\sqrt{3-x+2x^2}}{1728(5+2x)^3} + \frac{394907\sqrt{3-x+2x^2}}{248832(5+2x)^2} - \frac{3163415\sqrt{3-x+2x^2}}{5971968(5+2x)} \\
&\quad + \frac{22389491\text{Subst}\left(\int \frac{1}{288-x^2}\,dx, x, \frac{17-22x}{\sqrt{3-x+2x^2}}\right)}{5971968} + \frac{5\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx, x, -1+4x\right)}{16\sqrt{46}} \\
&= -\frac{3667\sqrt{3-x+2x^2}}{1728(5+2x)^3} + \frac{394907\sqrt{3-x+2x^2}}{248832(5+2x)^2} - \frac{3163415\sqrt{3-x+2x^2}}{5971968(5+2x)} \\
&\quad - \frac{5\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{16\sqrt{2}} + \frac{22389491\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{71663616\sqrt{2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.74

$$
\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4\sqrt{3-x+2x^2}}\,dx
= \frac{-\frac{12\sqrt{3-x+2x^2}(44369687+44312764x+12653660x^2)}{(5+2x)^3} - 22389491\sqrt{2}\text{arctanh}\left(\frac{1}{6}\left(5+2x-\sqrt{6-2x+4x^2}\right)\right) - 1119744}{71663616}
$$

```
[In] Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^4*Sqrt[3 - x + 2*x^2]),x
]
```

```
[Out] ((-12*Sqrt[3 - x + 2*x^2]*(44369687 + 44312764*x + 12653660*x^2))/(5 + 2*x)
^3 - 22389491*Sqrt[2]*ArcTanh[(5 + 2*x - Sqrt[6 - 2*x + 4*x^2])/6] - 111974
40*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/71663616
```

Maple [F(-1)]

Timed out.

hanged

```
[In] int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(1/2),x)
```

```
[Out] int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.21

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4\sqrt{3-x+2x^2}}\,dx$$
$$= \frac{22394880\,\sqrt{2}(8\,x^3+60\,x^2+150\,x+125)\log\left(-4\,\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1)-32\,x^2+16\,x-25\right)+22389}{28}$$

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(1/2),x, algorithm="f
ricas")
```

```
[Out] 1/286654464*(22394880*sqrt(2)*(8*x^3 + 60*x^2 + 150*x + 125)*log(-4*sqrt(2)
*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 22389491*sqrt(2)*(8*
x^3 + 60*x^2 + 150*x + 125)*log((24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17)
 - 1060*x^2 + 1036*x - 1153)/(4*x^2 + 20*x + 25)) - 48*(12653660*x^2 + 4431
2764*x + 44369687)*sqrt(2*x^2 - x + 3))/(8*x^3 + 60*x^2 + 150*x + 125)
```

Sympy [F]

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4\sqrt{3-x+2x^2}}\,dx = \int \frac{5x^4-x^3+3x^2+x+2}{(2x+5)^4\,\sqrt{2x^2-x+3}}\,dx$$

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)**4/(2*x**2-x+3)**(1/2),x)
```

```
[Out] Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)**4*sqrt(2*x**2 - x + 3
)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.97

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4\sqrt{3-x+2x^2}}\,dx = \frac{5}{32}\,\sqrt{2}\,\mathrm{arsinh}\left(\frac{4}{23}\,\sqrt{23}x-\frac{1}{23}\,\sqrt{23}\right) - \frac{22389491}{143327232}\,\sqrt{2}\,\mathrm{arsinh}\left(\frac{22\,\sqrt{23}x}{23\,|2\,x+5|}-\frac{17\,\sqrt{23}}{23\,|2\,x+5|}\right) - \frac{3667\,\sqrt{2\,x^2-x+3}}{1728\,(8\,x^3+60\,x^2+150\,x+125)} + \frac{394907\,\sqrt{2\,x^2-x+3}}{248832\,(4\,x^2+20\,x+25)} - \frac{3163415\,\sqrt{2\,x^2-x+3}}{5971968\,(2\,x+5)}$$

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(1/2),x, algorithm="m
axima")
```

```
[Out] 5/32*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) - 22389491/143327232*
sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)/abs(2*x + 5)
) - 3667/1728*sqrt(2*x^2 - x + 3)/(8*x^3 + 60*x^2 + 150*x + 125) + 394907/2
48832*sqrt(2*x^2 - x + 3)/(4*x^2 + 20*x + 25) - 3163415/5971968*sqrt(2*x^2
- x + 3)/(2*x + 5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 285 vs. 2(108) = 216.

Time = 0.28 (sec) , antiderivative size = 285, normalized size of antiderivative = 2.11

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4\sqrt{3-x+2x^2}}\,dx = -\frac{5}{32}\,\sqrt{2}\log\left(-2\,\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)+1\right) + \frac{22389491}{143327232}\,\sqrt{2}\log\left(\left|-2\,\sqrt{2}x+\sqrt{2}+2\,\sqrt{2\,x^2-x+3}\right|\right) - \frac{22389491}{143327232}\,\sqrt{2}\log\left(\left|-2\,\sqrt{2}x-11\,\sqrt{2}+2\,\sqrt{2\,x^2-x+3}\right|\right) - \frac{\sqrt{2}\Big(215012404\,\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)^5+3010410772\,\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)^4+2740802468\,\sqrt{2}(}{11943936\,\Big(2\,\left(\sqrt{2}x-\sqrt{2\,x^2}\right.}$$

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(1/2),x, algorithm="g
iac")
```

```
[Out] -5/32*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) + 22389
491/143327232*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 - x + 3
```

```
))) - 22389491/143327232*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2) + 2*sqrt
(2*x^2 - x + 3))) - 1/11943936*sqrt(2)*(215012404*sqrt(2)*(sqrt(2)*x - sqrt
(2*x^2 - x + 3))^5 + 3010410772*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^4 + 27408
02468*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^3 - 21459328844*(sqrt(2)*x
- sqrt(2*x^2 - x + 3))^2 + 14434519361*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x
+ 3)) - 5957650879)/(2*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 10*sqrt(2)*(sq
rt(2)*x - sqrt(2*x^2 - x + 3)) - 11)^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4\sqrt{3-x+2x^2}}\,dx=\int \frac{5\,x^4-x^3+3\,x^2+x+2}{{\left(2\,x+5\right)}^4\,\sqrt{2\,x^2-x+3}}\,dx$$

```
[In] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^4*(2*x^2 - x + 3)^(1/2)),x)
[Out] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^4*(2*x^2 - x + 3)^(1/2)), x)
```

3.350 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^5\sqrt{3-x+2x^2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 139

$$\begin{aligned}\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^5\sqrt{3-x+2x^2}}\,dx &= -\frac{3667\sqrt{3-x+2x^2}}{2304(5+2x)^4}+\frac{513097\sqrt{3-x+2x^2}}{497664(5+2x)^3}\\ &\quad-\frac{16295969\sqrt{3-x+2x^2}}{71663616(5+2x)^2}+\frac{26800085\sqrt{3-x+2x^2}}{1719926784(5+2x)}\\ &\quad+\frac{2053207\operatorname{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{20639121408\sqrt{2}}\end{aligned}$$

```
[Out] 2053207/41278242816*arctanh(1/24*(17-22*x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/
2)-3667/2304*(2*x^2-x+3)^(1/2)/(5+2*x)^4+513097/497664*(2*x^2-x+3)^(1/2)/(5
+2*x)^3-16295969/71663616*(2*x^2-x+3)^(1/2)/(5+2*x)^2+26800085/1719926784*(
2*x^2-x+3)^(1/2)/(5+2*x)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1664, 820, 738, 212}

$$\begin{aligned}\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^5\sqrt{3-x+2x^2}}\,dx &= \frac{2053207\operatorname{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{20639121408\sqrt{2}}\\ &\quad+\frac{26800085\sqrt{2x^2-x+3}}{1719926784(2x+5)}-\frac{16295969\sqrt{2x^2-x+3}}{71663616(2x+5)^2}\\ &\quad+\frac{513097\sqrt{2x^2-x+3}}{497664(2x+5)^3}-\frac{3667\sqrt{2x^2-x+3}}{2304(2x+5)^4}\end{aligned}$$

```
[In] Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^5*Sqrt[3 - x + 2*x^2]),x]
```

```
[Out] (-3667*Sqrt[3 - x + 2*x^2])/(2304*(5 + 2*x)^4) + (513097*Sqrt[3 - x + 2*x^2
])/(497664*(5 + 2*x)^3) - (16295969*Sqrt[3 - x + 2*x^2])/(71663616*(5 + 2*x
)^2) + (26800085*Sqrt[3 - x + 2*x^2])/(1719926784*(5 + 2*x)) + (2053207*Arc
Tanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(20639121408*Sqrt[2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{3667\sqrt{3-x+2x^2}}{2304(5+2x)^4}-\frac{1}{288}\int\frac{\frac{37027}{16}-\frac{10167x}{4}+1944x^2-720x^3}{(5+2x)^4\sqrt{3-x+2x^2}}\,dx\\ &= -\frac{3667\sqrt{3-x+2x^2}}{2304(5+2x)^4}+\frac{513097\sqrt{3-x+2x^2}}{497664(5+2x)^3}+\frac{\int\frac{\frac{2607829}{16}-\frac{295607x}{2}+77760x^2}{(5+2x)^3\sqrt{3-x+2x^2}}\,dx}{62208}\end{aligned}$$

$$
\begin{aligned}
&= -\frac{3667\sqrt{3-x+2x^2}}{2304(5+2x)^4} + \frac{513097\sqrt{3-x+2x^2}}{497664(5+2x)^3} \\
&\quad - \frac{16295969\sqrt{3-x+2x^2}}{71663616(5+2x)^2} - \frac{\int \frac{\frac{19411145}{16}-\frac{6098911x}{4}}{(5+2x)^2\sqrt{3-x+2x^2}}\,dx}{8957952} \\
&= -\frac{3667\sqrt{3-x+2x^2}}{2304(5+2x)^4} + \frac{513097\sqrt{3-x+2x^2}}{497664(5+2x)^3} - \frac{16295969\sqrt{3-x+2x^2}}{71663616(5+2x)^2} \\
&\quad + \frac{26800085\sqrt{3-x+2x^2}}{1719926784(5+2x)} - \frac{2053207\int \frac{1}{(5+2x)\sqrt{3-x+2x^2}}\,dx}{3439853568} \\
&= -\frac{3667\sqrt{3-x+2x^2}}{2304(5+2x)^4} + \frac{513097\sqrt{3-x+2x^2}}{497664(5+2x)^3} - \frac{16295969\sqrt{3-x+2x^2}}{71663616(5+2x)^2} \\
&\quad + \frac{26800085\sqrt{3-x+2x^2}}{1719926784(5+2x)} + \frac{2053207\text{Subst}\left(\int \frac{1}{288-x^2}\,dx, x, \frac{17-22x}{\sqrt{3-x+2x^2}}\right)}{1719926784} \\
&= -\frac{3667\sqrt{3-x+2x^2}}{2304(5+2x)^4} + \frac{513097\sqrt{3-x+2x^2}}{497664(5+2x)^3} - \frac{16295969\sqrt{3-x+2x^2}}{71663616(5+2x)^2} \\
&\quad + \frac{26800085\sqrt{3-x+2x^2}}{1719926784(5+2x)} + \frac{2053207\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{20639121408\sqrt{2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.55

$$
\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^5\sqrt{3-x+2x^2}}\,dx = \frac{\frac{12\sqrt{3-x+2x^2}(-298655447-255525906x+43592076x^2+214400680x^3)}{(5+2x)^4} - 2053207\sqrt{2}\text{arctanh}\left(\frac{1}{6}\left(5+2x-\sqrt{6-2x+4x^2}\right)\right)}{20639121408}
$$

```
[In] Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^5*Sqrt[3 - x + 2*x^2]),x
]
```

```
[Out] ((12*Sqrt[3 - x + 2*x^2]*(-298655447 - 255525906*x + 43592076*x^2 + 2144006
80*x^3))/(5 + 2*x)^4 - 2053207*Sqrt[2]*ArcTanh[(5 + 2*x - Sqrt[6 - 2*x + 4*
x^2])/6])/20639121408
```

Maple [F(-1)]

Timed out.

hanged

```
[In] int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^5/(2*x^2-x+3)^(1/2),x)
```

```
[Out] int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^5/(2*x^2-x+3)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.90

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^5\sqrt{3-x+2x^2}}\,dx$$
$$= \frac{2053207\,\sqrt{2}(16\,x^4+160\,x^3+600\,x^2+1000\,x+625)\log\left(\frac{24\,\sqrt{2}\sqrt{2\,x^2-x+3}(22\,x-17)-1060\,x^2+1036\,x-1153}{4\,x^2+20\,x+25}\right)+48\,(21}{82556485632\,(16\,x^4+160\,x^3+600\,x^2+1000}$$

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^5/(2*x^2-x+3)^(1/2),x, algorithm="f
ricas")
```

```
[Out] 1/82556485632*(2053207*sqrt(2)*(16*x^4 + 160*x^3 + 600*x^2 + 1000*x + 625)*
log((24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) - 1060*x^2 + 1036*x - 1153)
/(4*x^2 + 20*x + 25)) + 48*(214400680*x^3 + 43592076*x^2 - 255525906*x - 29
8655447)*sqrt(2*x^2 - x + 3))/(16*x^4 + 160*x^3 + 600*x^2 + 1000*x + 625)
```

Sympy [F]

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^5\sqrt{3-x+2x^2}}\,dx = \int \frac{5x^4-x^3+3x^2+x+2}{(2x+5)^5\sqrt{2x^2-x+3}}\,dx$$

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)**5/(2*x**2-x+3)**(1/2),x)
```

```
[Out] Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)**5*sqrt(2*x**2 - x + 3
)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.07

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^5\sqrt{3-x+2x^2}}\,dx = -\frac{2053207}{41278242816}\sqrt{2}\,\mathrm{arsinh}\left(\frac{22\sqrt{23}x}{23\,|2\,x+5|}-\frac{17\sqrt{23}}{23\,|2\,x+5|}\right) - \frac{3667\sqrt{2\,x^2-x+3}}{2304\,(16\,x^4+160\,x^3+600\,x^2+1000\,x+625)} + \frac{513097\sqrt{2\,x^2-x+3}}{497664\,(8\,x^3+60\,x^2+150\,x+125)} - \frac{16295969\sqrt{2\,x^2-x+3}}{71663616\,(4\,x^2+20\,x+25)} + \frac{26800085\sqrt{2\,x^2-x+3}}{1719926784\,(2\,x+5)}$$

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^5/(2*x^2-x+3)^(1/2),x, algorithm="m
axima")
```

```
[Out] -2053207/41278242816*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*
sqrt(23)/abs(2*x + 5)) - 3667/2304*sqrt(2*x^2 - x + 3)/(16*x^4 + 160*x^3 +
600*x^2 + 1000*x + 625) + 513097/497664*sqrt(2*x^2 - x + 3)/(8*x^3 + 60*x^2
 + 150*x + 125) - 16295969/71663616*sqrt(2*x^2 - x + 3)/(4*x^2 + 20*x + 25)
 + 26800085/1719926784*sqrt(2*x^2 - x + 3)/(2*x + 5)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.18

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^5\sqrt{3-x+2x^2}}\,dx = \frac{1}{41278242816}\sqrt{2}\left(12\left(\frac{24\left(\frac{144\left(\frac{513097}{\mathrm{sgn}\left(\frac{1}{2\,x+5}\right)}-\frac{792072}{(2\,x+5)\mathrm{sgn}\left(\frac{1}{2\,x+5}\right)}\right)}{2\,x+5}-\frac{16295969}{\mathrm{sgn}\left(\frac{1}{2\,x+5}\right)}\right)}{2\,x+5}+\frac{26800085}{\mathrm{sgn}\left(\frac{1}{2\,x+5}\right)}\right)\sqrt{-\frac{11}{2\,x+5}+}$$

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^5/(2*x^2-x+3)^(1/2),x, algorithm="g
iac")
```

```
[Out] 1/41278242816*sqrt(2)*(12*(24*(144*(513097/sgn(1/(2*x + 5)) - 792072/((2*x
+ 5)*sgn(1/(2*x + 5))))/(2*x + 5) - 16295969/sgn(1/(2*x + 5)))/(2*x + 5) +
```

```
26800085/sgn(1/(2*x + 5)))*sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 20532
07*log(12*sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 72/(2*x + 5) - 11)/sgn
(1/(2*x + 5)) - 321601020*sgn(1/(2*x + 5)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^5\sqrt{3-x+2x^2}}\,dx = \int \frac{5\,x^4-x^3+3\,x^2+x+2}{{\left(2\,x+5\right)}^5\,\sqrt{2\,x^2-x+3}}\,dx$$

```
[In] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^5*(2*x^2 - x + 3)^(1/2)),x)
[Out] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^5*(2*x^2 - x + 3)^(1/2)), x)
```

3.351 $\int \frac{(5+2x)^2\left(2+x+3x^2-x^3+5x^4\right)}{\left(3-x+2x^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 124

$$\int \frac{(5+2x)^2\left(2+x+3x^2-x^3+5x^4\right)}{\left(3-x+2x^2\right)^{3/2}}\,dx = -\frac{4(346-533x)}{23\sqrt{3-x+2x^2}}-\frac{13153}{512}\sqrt{3-x+2x^2} +\frac{2645}{128}x\sqrt{3-x+2x^2}+\frac{153}{16}x^2\sqrt{3-x+2x^2}+\frac{5}{4}x^3\sqrt{3-x+2x^2}+\frac{144217\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{1024\sqrt{2}}$$

```
[Out] 144217/2048*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-4/23*(346-533*x)/(2*x^2-
x+3)^(1/2)-13153/512*(2*x^2-x+3)^(1/2)+2645/128*x*(2*x^2-x+3)^(1/2)+153/16*
x^2*(2*x^2-x+3)^(1/2)+5/4*x^3*(2*x^2-x+3)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1674, 1675, 654, 633, 221}

$$\int \frac{(5+2x)^2\left(2+x+3x^2-x^3+5x^4\right)}{\left(3-x+2x^2\right)^{3/2}}\,dx = \frac{144217\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{1024\sqrt{2}}+\frac{153}{16}\sqrt{2x^2-x+3}x^2 +\frac{2645}{128}\sqrt{2x^2-x+3}x-\frac{13153}{512}\sqrt{2x^2-x+3}-\frac{4(346-533x)}{23\sqrt{2x^2-x+3}}+\frac{5}{4}\sqrt{2x^2-x+3}x^3$$

```
[In] Int[((5 + 2*x)^2*(2 + x + 3*x^2 - x^3 + 5*x^4))/(3 - x + 2*x^2)^(3/2),x]
```

```
[Out] (-4*(346 - 533*x))/(23*Sqrt[3 - x + 2*x^2]) - (13153*Sqrt[3 - x + 2*x^2])/5
12 + (2645*x*Sqrt[3 - x + 2*x^2])/128 + (153*x^2*Sqrt[3 - x + 2*x^2])/16 +
```

```
(5*x^3*Sqrt[3 - x + 2*x^2])/4 + (144217*ArcSinh[(1 - 4*x)/Sqrt[23]])/(1024*
Sqrt[2])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\text{integral} = -\frac{4(346-533x)}{23\sqrt{3-x+2x^2}}+\frac{2}{23}\int\frac{-759-\frac{575x}{2}+805x^2+\frac{1219x^3}{2}+115x^4}{\sqrt{3-x+2x^2}}\,dx$$

$$
\begin{aligned}
&= -\frac{4(346-533x)}{23\sqrt{3-x+2x^2}}+\frac{5}{4}x^3\sqrt{3-x+2x^2}+\frac{1}{92}\int \frac{-6072-2300x+5405x^2+\frac{10557x^3}{2}}{\sqrt{3-x+2x^2}}\,dx \\
&= -\frac{4(346-533x)}{23\sqrt{3-x+2x^2}}+\frac{153}{16}x^2\sqrt{3-x+2x^2}+\frac{5}{4}x^3\sqrt{3-x+2x^2} \\
&\quad +\frac{1}{552}\int \frac{-36432-45471x+\frac{182505x^2}{4}}{\sqrt{3-x+2x^2}}\,dx \\
&= -\frac{4(346-533x)}{23\sqrt{3-x+2x^2}}+\frac{2645}{128}x\sqrt{3-x+2x^2}+\frac{153}{16}x^2\sqrt{3-x+2x^2} \\
&\quad +\frac{5}{4}x^3\sqrt{3-x+2x^2}+\frac{\int \frac{-\frac{1130427}{4}-\frac{907557x}{8}}{\sqrt{3-x+2x^2}}\,dx}{2208} \\
&= -\frac{4(346-533x)}{23\sqrt{3-x+2x^2}}-\frac{13153}{512}\sqrt{3-x+2x^2}+\frac{2645}{128}x\sqrt{3-x+2x^2} \\
&\quad +\frac{153}{16}x^2\sqrt{3-x+2x^2}+\frac{5}{4}x^3\sqrt{3-x+2x^2}-\frac{144217\int \frac{1}{\sqrt{3-x+2x^2}}\,dx}{1024} \\
&= -\frac{4(346-533x)}{23\sqrt{3-x+2x^2}}-\frac{13153}{512}\sqrt{3-x+2x^2} \\
&\quad +\frac{2645}{128}x\sqrt{3-x+2x^2}+\frac{153}{16}x^2\sqrt{3-x+2x^2} \\
&\quad +\frac{5}{4}x^3\sqrt{3-x+2x^2}-\frac{144217\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx,x,-1+4x\right)}{1024\sqrt{46}} \\
&= -\frac{4(346-533x)}{23\sqrt{3-x+2x^2}}-\frac{13153}{512}\sqrt{3-x+2x^2}+\frac{2645}{128}x\sqrt{3-x+2x^2} \\
&\quad +\frac{153}{16}x^2\sqrt{3-x+2x^2}+\frac{5}{4}x^3\sqrt{3-x+2x^2}+\frac{144217\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{1024\sqrt{2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.60

$$
\int \frac{(5+2x)^2\left(2+x+3x^2-x^3+5x^4\right)}{\left(3-x+2x^2\right)^{3/2}}\,dx=\frac{\frac{4\left(-1616165+2124123x-510554x^2+418232x^3+210496x^4+29440x^5\right)}{\sqrt{3-x+2x^2}}+3316991\sqrt{}}{47104}
$$

```
[In] Integrate[((5 + 2*x)^2*(2 + x + 3*x^2 - x^3 + 5*x^4))/(3 - x + 2*x^2)^(3/2)
,x]
```

```
[Out] ((4*(-1616165 + 2124123*x - 510554*x^2 + 418232*x^3 + 210496*x^4 + 29440*x^
5))/Sqrt[3 - x + 2*x^2] + 3316991*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*
x^2]])/47104
```

Maple [F(-1)]

Timed out.

hanged

```
[In] int((5+2*x)^2*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x)
```

```
[Out] int((5+2*x)^2*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.82

$$\int \frac{(5+2x)^2\left(2+x+3x^2-x^3+5x^4\right)}{\left(3-x+2x^2\right)^{3/2}}\,dx = \frac{3316991\,\sqrt{2}(2\,x^2-x+3)\log\left(4\,\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1)-32\right.}{}$$

```
[In] integrate((5+2*x)^2*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x, algorithm="f
ricas")
```

```
[Out] 1/94208*(3316991*sqrt(2)*(2*x^2 - x + 3)*log(4*sqrt(2)*sqrt(2*x^2 - x + 3)*
(4*x - 1) - 32*x^2 + 16*x - 25) + 8*(29440*x^5 + 210496*x^4 + 418232*x^3 -
510554*x^2 + 2124123*x - 1616165)*sqrt(2*x^2 - x + 3))/(2*x^2 - x + 3)
```

Sympy [F]

$$\int \frac{(5+2x)^2\left(2+x+3x^2-x^3+5x^4\right)}{\left(3-x+2x^2\right)^{3/2}}\,dx = \int \frac{\left(2x+5\right)^2\cdot\left(5x^4-x^3+3x^2+x+2\right)}{\left(2x^2-x+3\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((5+2*x)**2*(5*x**4-x**3+3*x**2+x+2)/(2*x**2-x+3)**(3/2),x)
```

```
[Out] Integral((2*x + 5)**2*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x**2 - x + 3)**(3
/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.92

$$\begin{aligned}\int \frac{(5+2x)^2\left(2+x+3x^2-x^3+5x^4\right)}{\left(3-x+2x^2\right)^{3/2}}\,dx &= \frac{5\,x^5}{2\,\sqrt{2\,x^2-x+3}}+\frac{143\,x^4}{8\,\sqrt{2\,x^2-x+3}}\\ &+\frac{2273\,x^3}{64\,\sqrt{2\,x^2-x+3}}-\frac{11099\,x^2}{256\,\sqrt{2\,x^2-x+3}}-\frac{144217}{2048}\,\sqrt{2}\,\mathrm{arsinh}\left(\frac{1}{23}\,\sqrt{23}(4\,x-1)\right)\\ &+\frac{2124123\,x}{11776\,\sqrt{2\,x^2-x+3}}-\frac{1616165}{11776\,\sqrt{2\,x^2-x+3}}\end{aligned}$$

```
[In] integrate((5+2*x)^2*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x, algorithm="m
axima")
```

```
[Out] 5/2*x^5/sqrt(2*x^2 - x + 3) + 143/8*x^4/sqrt(2*x^2 - x + 3) + 2273/64*x^3/s
qrt(2*x^2 - x + 3) - 11099/256*x^2/sqrt(2*x^2 - x + 3) - 144217/2048*sqrt(2
)*arcsinh(1/23*sqrt(23)*(4*x - 1)) + 2124123/11776*x/sqrt(2*x^2 - x + 3) -
1616165/11776/sqrt(2*x^2 - x + 3)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.58

$$\int \frac{(5+2x)^2\left(2+x+3x^2-x^3+5x^4\right)}{(3-x+2x^2)^{3/2}}\,dx=\frac{144217}{2048}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)+1\right)+\frac{(46\,(4\,(8\,(20\,x+143)x+2273)x-11099)x+2124123)x-1616165}{11776\,\sqrt{2\,x^2-x+3}}$$

```
[In] integrate((5+2*x)^2*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x, algorithm="g
iac")
```

```
[Out] 144217/2048*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) +
 1/11776*((46*(4*(8*(20*x + 143)*x + 2273)*x - 11099)*x + 2124123)*x - 1616
165)/sqrt(2*x^2 - x + 3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(5+2x)^2\left(2+x+3x^2-x^3+5x^4\right)}{(3-x+2x^2)^{3/2}}\,dx=\int \frac{{\left(2\,x+5\right)}^2\,\left(5\,x^4-x^3+3\,x^2+x+2\right)}{{\left(2\,x^2-x+3\right)}^{3/2}} \,d x$$

```
[In] int(((2*x + 5)^2*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x^2 - x + 3)^(3/2),x)
```

```
[Out] int(((2*x + 5)^2*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x^2 - x + 3)^(3/2), x)
```

3.352 $\int \frac{(5+2x)(2+x+3x^2-x^3+5x^4)}{(3-x+2x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 103

$$\int \frac{(5+2x)\left(2+x+3x^2-x^3+5x^4\right)}{\left(3-x+2x^2\right)^{3/2}}\,dx = \frac{-53+373x}{23\sqrt{3-x+2x^2}}+\frac{33}{64}\sqrt{3-x+2x^2} + \frac{193}{48}x\sqrt{3-x+2x^2}+\frac{5}{6}x^2\sqrt{3-x+2x^2}+\frac{3111\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{128\sqrt{2}}$$

```
[Out] 3111/256*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)+1/23*(-53+373*x)/(2*x^2-x+3
)^(1/2)+33/64*(2*x^2-x+3)^(1/2)+193/48*x*(2*x^2-x+3)^(1/2)+5/6*x^2*(2*x^2-x
+3)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {1674, 1675, 654, 633, 221}

$$\int \frac{(5+2x)\left(2+x+3x^2-x^3+5x^4\right)}{\left(3-x+2x^2\right)^{3/2}}\,dx = \frac{3111\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{128\sqrt{2}} + \frac{5}{6}\sqrt{2x^2-x+3}x^2+\frac{193}{48}\sqrt{2x^2-x+3}x+\frac{33}{64}\sqrt{2x^2-x+3}-\frac{53-373x}{23\sqrt{2x^2-x+3}}$$

```
[In] Int[((5 + 2*x)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(3 - x + 2*x^2)^(3/2),x]
```

```
[Out] -1/23*(53 - 373*x)/Sqrt[3 - x + 2*x^2] + (33*Sqrt[3 - x + 2*x^2])/64 + (193
*x*Sqrt[3 - x + 2*x^2])/48 + (5*x^2*Sqrt[3 - x + 2*x^2])/6 + (3111*ArcSinh[
(1 - 4*x)/Sqrt[23]])/(128*Sqrt[2])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{53-373x}{23\sqrt{3-x+2x^2}}+\frac{2}{23}\int\frac{-\frac{575}{4}+161x^2+\frac{115x^3}{2}}{\sqrt{3-x+2x^2}}\,dx\\ &= -\frac{53-373x}{23\sqrt{3-x+2x^2}}+\frac{5}{6}x^2\sqrt{3-x+2x^2}+\frac{1}{69}\int\frac{-\frac{1725}{2}-345x+\frac{4439x^2}{4}}{\sqrt{3-x+2x^2}}\,dx\end{aligned}$$

$$
\begin{aligned}
&= -\frac{53-373x}{23\sqrt{3-x+2x^2}} + \frac{193}{48}x\sqrt{3-x+2x^2} + \frac{5}{6}x^2\sqrt{3-x+2x^2} + \frac{1}{276}\int \frac{-\frac{27117}{4}+\frac{2277x}{8}}{\sqrt{3-x+2x^2}}\,dx \\
&= -\frac{53-373x}{23\sqrt{3-x+2x^2}} + \frac{33}{64}\sqrt{3-x+2x^2} + \frac{193}{48}x\sqrt{3-x+2x^2} \\
&\quad + \frac{5}{6}x^2\sqrt{3-x+2x^2} - \frac{3111}{128}\int \frac{1}{\sqrt{3-x+2x^2}}\,dx \\
&= -\frac{53-373x}{23\sqrt{3-x+2x^2}} + \frac{33}{64}\sqrt{3-x+2x^2} + \frac{193}{48}x\sqrt{3-x+2x^2} \\
&\quad + \frac{5}{6}x^2\sqrt{3-x+2x^2} - \frac{3111\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx, x, -1+4x\right)}{128\sqrt{46}} \\
&= -\frac{53-373x}{23\sqrt{3-x+2x^2}} + \frac{33}{64}\sqrt{3-x+2x^2} + \frac{193}{48}x\sqrt{3-x+2x^2} \\
&\quad + \frac{5}{6}x^2\sqrt{3-x+2x^2} + \frac{3111\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{128\sqrt{2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.68

$$
\int \frac{(5+2x)\left(2+x+3x^2-x^3+5x^4\right)}{\left(3-x+2x^2\right)^{3/2}}\,dx = \frac{-3345+122607x-2162x^2+31832x^3+7360x^4}{4416\sqrt{3-x+2x^2}} + \frac{3111\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{128\sqrt{2}}
$$

```
[In] Integrate[((5 + 2*x)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(3 - x + 2*x^2)^(3/2),x
]
```

```
[Out] (-3345 + 122607*x - 2162*x^2 + 31832*x^3 + 7360*x^4)/(4416*Sqrt[3 - x + 2*x
^2]) + (3111*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/(128*Sqrt[2])
```

Maple [A] (verified)

Time = 1.21 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.49

method	result
risch	$\frac{7360x^4+31832x^3-2162x^2+122607x-3345}{4416\sqrt{2x^2-x+3}}-\frac{3111\sqrt{2}\,\mathrm{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{256}$
trager	$\frac{7360x^4+31832x^3-2162x^2+122607x-3345}{4416\sqrt{2x^2-x+3}}+\frac{3111\,\mathrm{RootOf}\left(\_Z^2-2\right)\ln\left(-4\,\mathrm{RootOf}\left(\_Z^2-2\right)x+\mathrm{RootOf}\left(\_Z^2-2\right)+4\sqrt{2x^2-}\right.}{256}$
default	$\frac{\frac{10185x}{2944}-\frac{10185}{11776}}{\sqrt{2x^2-x+3}}-\frac{47x^2}{96\sqrt{2x^2-x+3}}+\frac{3111x}{128\sqrt{2x^2-x+3}}+\frac{55}{512\sqrt{2x^2-x+3}}-\frac{3111\sqrt{2}\,\mathrm{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{256}+\frac{5x^4}{3\sqrt{2x^2-x+3}}+$

```
[In] int((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x,method=_RETURNVERBOSE
)
```

```
[Out] 1/4416*(7360*x^4+31832*x^3-2162*x^2+122607*x-3345)/(2*x^2-x+3)^(1/2)-3111/2
56*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.94

$$\int \frac{(5+2x)\left(2+x+3x^2-x^3+5x^4\right)}{\left(3-x+2x^2\right)^{3/2}}\,dx=\frac{214659\sqrt{2}(2x^2-x+3)\log\left(4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-32x\right.}{3}$$

```
[In] integrate((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x, algorithm="fri
cas")
```

```
[Out] 1/35328*(214659*sqrt(2)*(2*x^2 - x + 3)*log(4*sqrt(2)*sqrt(2*x^2 - x + 3)*(
4*x - 1) - 32*x^2 + 16*x - 25) + 8*(7360*x^4 + 31832*x^3 - 2162*x^2 + 12260
7*x - 3345)*sqrt(2*x^2 - x + 3))/(2*x^2 - x + 3)
```

Sympy [F]

$$\int \frac{(5+2x)\left(2+x+3x^2-x^3+5x^4\right)}{\left(3-x+2x^2\right)^{3/2}}\,dx=\int \frac{(2x+5)\left(5x^4-x^3+3x^2+x+2\right)}{\left(2x^2-x+3\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((5+2*x)*(5*x**4-x**3+3*x**2+x+2)/(2*x**2-x+3)**(3/2),x)
```

```
[Out] Integral((2*x + 5)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x**2 - x + 3)**(3/2)
, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.94

$$\int \frac{(5+2x)\left(2+x+3x^2-x^3+5x^4\right)}{\left(3-x+2x^2\right)^{3/2}}\,dx = \frac{5\,x^4}{3\,\sqrt{2\,x^2-x+3}} + \frac{173\,x^3}{24\,\sqrt{2\,x^2-x+3}} - \frac{47\,x^2}{96\,\sqrt{2\,x^2-x+3}} - \frac{3111}{256}\,\sqrt{2}\,\mathrm{arsinh}\left(\frac{1}{23}\,\sqrt{23}(4\,x-1)\right) + \frac{40869\,x}{1472\,\sqrt{2\,x^2-x+3}} - \frac{1115}{1472\,\sqrt{2\,x^2-x+3}}$$

```
[In] integrate((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x, algorithm="max
ima")
```

```
[Out] 5/3*x^4/sqrt(2*x^2 - x + 3) + 173/24*x^3/sqrt(2*x^2 - x + 3) - 47/96*x^2/sq
rt(2*x^2 - x + 3) - 3111/256*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) + 408
69/1472*x/sqrt(2*x^2 - x + 3) - 1115/1472/sqrt(2*x^2 - x + 3)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.65

$$\int \frac{(5+2x)\left(2+x+3x^2-x^3+5x^4\right)}{\left(3-x+2x^2\right)^{3/2}}\,dx = \frac{3111}{256}\,\sqrt{2}\log\left(-2\,\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)+1\right) + \frac{(46\,(4\,(40\,x+173)x-47)x+122607)x-3345}{4416\,\sqrt{2\,x^2-x+3}}$$

```
[In] integrate((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x, algorithm="gia
c")
```

```
[Out] 3111/256*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) + 1/
4416*((46*(4*(40*x + 173)*x - 47)*x + 122607)*x - 3345)/sqrt(2*x^2 - x + 3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(5+2x)\left(2+x+3x^2-x^3+5x^4\right)}{\left(3-x+2x^2\right)^{3/2}}\,dx=\int \frac{\left(2\,x+5\right)\,\left(5\,x^4-x^3+3\,x^2+x+2\right)}{{\left(2\,x^2-x+3\right)}^{3/2}} \,d x$$

```
[In] int(((2*x + 5)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x^2 - x + 3)^(3/2),x)
[Out] int(((2*x + 5)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x^2 - x + 3)^(3/2), x)
```

3.353 $\int \frac{2+x+3x^2-x^3+5x^4}{\left(3-x+2x^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 82

$$\int \frac{2+x+3x^2-x^3+5x^4}{\left(3-x+2x^2\right)^{3/2}}\,dx=\frac{89+219x}{92\sqrt{3-x+2x^2}}+\frac{27}{32}\sqrt{3-x+2x^2}+\frac{5}{8}x\sqrt{3-x+2x^2}+\frac{213\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{64\sqrt{2}}$$

```
[Out] 213/128*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)+1/92*(89+219*x)/(2*x^2-x+3)^
(1/2)+27/32*(2*x^2-x+3)^(1/2)+5/8*x*(2*x^2-x+3)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {1674, 1675, 654, 633, 221}

$$\int \frac{2+x+3x^2-x^3+5x^4}{\left(3-x+2x^2\right)^{3/2}}\,dx=\frac{213\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{64\sqrt{2}}+\frac{5}{8}\sqrt{2x^2-x+3}x+\frac{27}{32}\sqrt{2x^2-x+3}+\frac{219x+89}{92\sqrt{2x^2-x+3}}$$

```
[In] Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/(3 - x + 2*x^2)^(3/2),x]
```

```
[Out] (89 + 219*x)/(92*Sqrt[3 - x + 2*x^2]) + (27*Sqrt[3 - x + 2*x^2])/32 + (5*x*
Sqrt[3 - x + 2*x^2])/8 + (213*ArcSinh[(1 - 4*x)/Sqrt[23]])/(64*Sqrt[2])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{89+219x}{92\sqrt{3-x+2x^2}}+\frac{2}{23}\int\frac{-\frac{345}{16}+\frac{69x}{8}+\frac{115x^2}{4}}{\sqrt{3-x+2x^2}}\,dx\\
&= \frac{89+219x}{92\sqrt{3-x+2x^2}}+\frac{5}{8}x\sqrt{3-x+2x^2}+\frac{1}{46}\int\frac{-\frac{345}{2}+\frac{621x}{8}}{\sqrt{3-x+2x^2}}\,dx\\
&= \frac{89+219x}{92\sqrt{3-x+2x^2}}+\frac{27}{32}\sqrt{3-x+2x^2}+\frac{5}{8}x\sqrt{3-x+2x^2}-\frac{213}{64}\int\frac{1}{\sqrt{3-x+2x^2}}\,dx
\end{aligned}$$

$$= \frac{89+219x}{92\sqrt{3-x+2x^2}} + \frac{27}{32}\sqrt{3-x+2x^2} + \frac{5}{8}x\sqrt{3-x+2x^2} - \frac{213\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx, x, -1+4x\right)}{64\sqrt{46}}$$

$$= \frac{89+219x}{92\sqrt{3-x+2x^2}} + \frac{27}{32}\sqrt{3-x+2x^2} + \frac{5}{8}x\sqrt{3-x+2x^2} + \frac{213\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{64\sqrt{2}}$$

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.79

$$\int \frac{2+x+3x^2-x^3+5x^4}{(3-x+2x^2)^{3/2}}\,dx = \frac{2575+2511x+782x^2+920x^3}{736\sqrt{3-x+2x^2}} + \frac{213\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{64\sqrt{2}}$$

```
[In] Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/(3 - x + 2*x^2)^(3/2),x]
```

```
[Out] (2575 + 2511*x + 782*x^2 + 920*x^3)/(736*Sqrt[3 - x + 2*x^2]) + (213*Log[1
- 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/(64*Sqrt[2])
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.55

method	result	size
risch	$\frac{920x^3+782x^2+2511x+2575}{736\sqrt{2x^2-x+3}} - \frac{213\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{128}$	45
trager	$\frac{920x^3+782x^2+2511x+2575}{736\sqrt{2x^2-x+3}} - \frac{213\,\operatorname{RootOf}\left(\_Z^2-2\right)\ln\left(4\operatorname{RootOf}\left(\_Z^2-2\right)x+4\sqrt{2x^2-x+3}-\operatorname{RootOf}\left(\_Z^2-2\right)\right)}{128}$	72
default	$\frac{901}{256\sqrt{2x^2-x+3}} + \frac{\frac{123x}{1472}-\frac{123}{5888}}{\sqrt{2x^2-x+3}} + \frac{5x^3}{4\sqrt{2x^2-x+3}} + \frac{17x^2}{16\sqrt{2x^2-x+3}} + \frac{213x}{64\sqrt{2x^2-x+3}} - \frac{213\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{128}$	98

```
[In] int((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/736*(920*x^3+782*x^2+2511*x+2575)/(2*x^2-x+3)^(1/2)-213/128*2^(1/2)*arcsi
nh(4/23*23^(1/2)*(x-1/4))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.12

$$\int \frac{2+x+3x^2-x^3+5x^4}{(3-x+2x^2)^{3/2}}\,dx = \frac{4899\,\sqrt{2}(2\,x^2-x+3)\log\left(4\,\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1)-32\,x^2+16\,x-25}{5888\,(2\,x^2-x+3)}$$

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/5888*(4899*sqrt(2)*(2*x^2 - x + 3)*log(4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x
 - 1) - 32*x^2 + 16*x - 25) + 8*(920*x^3 + 782*x^2 + 2511*x + 2575)*sqrt(2*
x^2 - x + 3))/(2*x^2 - x + 3)
```

Sympy [F]

$$\int \frac{2+x+3x^2-x^3+5x^4}{(3-x+2x^2)^{3/2}}\,dx = \int \frac{5x^4-x^3+3x^2+x+2}{(2x^2-x+3)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)/(2*x**2-x+3)**(3/2),x)
```

```
[Out] Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x**2 - x + 3)**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.98

$$\begin{aligned}\int \frac{2+x+3x^2-x^3+5x^4}{(3-x+2x^2)^{3/2}}\,dx &= \frac{5\,x^3}{4\,\sqrt{2\,x^2-x+3}} + \frac{17\,x^2}{16\,\sqrt{2\,x^2-x+3}} \\ &\quad - \frac{213}{128}\,\sqrt{2}\,\mathrm{arsinh}\left(\frac{1}{23}\,\sqrt{23}(4\,x-1)\right) + \frac{2511\,x}{736\,\sqrt{2\,x^2-x+3}} + \frac{2575}{736\,\sqrt{2\,x^2-x+3}}\end{aligned}$$

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x, algorithm="maxima")
```

```
[Out] 5/4*x^3/sqrt(2*x^2 - x + 3) + 17/16*x^2/sqrt(2*x^2 - x + 3) - 213/128*sqrt(
2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) + 2511/736*x/sqrt(2*x^2 - x + 3) + 2575
/736/sqrt(2*x^2 - x + 3)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.76

$$\int \frac{2+x+3x^2-x^3+5x^4}{(3-x+2x^2)^{3/2}}\,dx = \frac{213}{128}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right) + \frac{(46(20x+17)x+2511)x+2575}{736\sqrt{2x^2-x+3}}$$

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x, algorithm="giac")
```

```
[Out] 213/128*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) + 1/7
36*((46*(20*x + 17)*x + 2511)*x + 2575)/sqrt(2*x^2 - x + 3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2+x+3x^2-x^3+5x^4}{(3-x+2x^2)^{3/2}}\,dx = \int \frac{5x^4-x^3+3x^2+x+2}{(2x^2-x+3)^{3/2}}\,dx$$

```
[In] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/(2*x^2 - x + 3)^(3/2),x)
```

```
[Out] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/(2*x^2 - x + 3)^(3/2), x)
```

3.354 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)\left(3-x+2x^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 101

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)\left(3-x+2x^2\right)^{3/2}}\,dx=\frac{1191+917x}{3312\sqrt{3-x+2x^2}}+\frac{5}{8}\sqrt{3-x+2x^2}+\frac{39\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{16\sqrt{2}}-\frac{3667\operatorname{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{1728\sqrt{2}}$$

```
[Out] 39/32*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-3667/3456*arctanh(1/24*(17-22*
x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)+1/3312*(1191+917*x)/(2*x^2-x+3)^(1/2)
+5/8*(2*x^2-x+3)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {1660, 1667, 857, 633, 221, 738, 212}

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)\left(3-x+2x^2\right)^{3/2}}\,dx=\frac{39\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{16\sqrt{2}}-\frac{3667\operatorname{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{1728\sqrt{2}}+\frac{917x+1191}{3312\sqrt{2x^2-x+3}}+\frac{5}{8}\sqrt{2x^2-x+3}$$

```
[In] Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)*(3 - x + 2*x^2)^(3/2)),x]
```

```
[Out] (1191 + 917*x)/(3312*Sqrt[3 - x + 2*x^2]) + (5*Sqrt[3 - x + 2*x^2])/8 + (39
*ArcSinh[(1 - 4*x)/Sqrt[23]])/(16*Sqrt[2]) - (3667*ArcTanh[(17 - 22*x)/(12*
Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(1728*Sqrt[2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1660

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^(p + 1)/((p
+ 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1191+917x}{3312\sqrt{3-x+2x^2}}+\frac{2}{23}\int \frac{-\frac{6739}{576}+\frac{69x}{8}+\frac{115x^2}{4}}{(5+2x)\sqrt{3-x+2x^2}}\,dx\\
&= \frac{1191+917x}{3312\sqrt{3-x+2x^2}}+\frac{5}{8}\sqrt{3-x+2x^2}+\frac{1}{92}\int \frac{\frac{3611}{72}-\frac{897x}{2}}{(5+2x)\sqrt{3-x+2x^2}}\,dx\\
&= \frac{1191+917x}{3312\sqrt{3-x+2x^2}}+\frac{5}{8}\sqrt{3-x+2x^2}\\
&\quad -\frac{39}{16}\int \frac{1}{\sqrt{3-x+2x^2}}\,dx+\frac{3667}{288}\int \frac{1}{(5+2x)\sqrt{3-x+2x^2}}\,dx\\
&= \frac{1191+917x}{3312\sqrt{3-x+2x^2}}+\frac{5}{8}\sqrt{3-x+2x^2}\\
&\quad -\frac{3667}{144}\text{Subst}\left(\int \frac{1}{288-x^2}\,dx,x,\frac{17-22x}{\sqrt{3-x+2x^2}}\right)\\
&\quad -\frac{39\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx,x,-1+4x\right)}{16\sqrt{46}}\\
&= \frac{1191+917x}{3312\sqrt{3-x+2x^2}}+\frac{5}{8}\sqrt{3-x+2x^2}+\frac{39\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{16\sqrt{2}}-\frac{3667\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{1728\sqrt{2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.92

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)\left(3-x+2x^2\right)^{3/2}}\,dx=\frac{\frac{12\left(7401-1153x+4140x^2\right)}{\sqrt{3-x+2x^2}}+84341\sqrt{2}\text{arctanh}\left(\frac{1}{6}\left(5+2x-\sqrt{6-2x+4x^2}\right)\right)+}{39744}$$

```
[In] Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)*(3 - x + 2*x^2)^(3/2)),x
]
```

```
[Out] ((12*(7401 - 1153*x + 4140*x^2))/Sqrt[3 - x + 2*x^2] + 84341*Sqrt[2]*ArcTan
h[(5 + 2*x - Sqrt[6 - 2*x + 4*x^2])/6] + 48438*Sqrt[2]*Log[1 - 4*x + 2*Sqrt
[6 - 2*x + 4*x^2]])/39744
```

Maple [F(-1)]

Timed out.

hanged

```
[In] int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(3/2),x)
```

```
[Out] int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(3/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.48

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)\left(3-x+2x^2\right)^{3/2}}\,dx = \frac{96876\sqrt{2}(2x^2-x+3)\log\left(4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-32x^2+16x-\right.}{}$$

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(3/2),x, algorithm="fri
cas")
```

```
[Out] 1/158976*(96876*sqrt(2)*(2*x^2 - x + 3)*log(4*sqrt(2)*sqrt(2*x^2 - x + 3)*(
4*x - 1) - 32*x^2 + 16*x - 25) + 84341*sqrt(2)*(2*x^2 - x + 3)*log(-(24*sqr
t(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) + 1060*x^2 - 1036*x + 1153)/(4*x^2 + 2
0*x + 25)) + 48*(4140*x^2 - 1153*x + 7401)*sqrt(2*x^2 - x + 3))/(2*x^2 - x
+ 3)
```

Sympy [F]

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)\left(3-x+2x^2\right)^{3/2}}\,dx = \int \frac{5x^4-x^3+3x^2+x+2}{(2x+5)\left(2x^2-x+3\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)/(2*x**2-x+3)**(3/2),x)
```

```
[Out] Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)*(2*x**2 - x + 3)**(3/2
)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.98

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)\left(3-x+2x^2\right)^{3/2}}\,dx = \frac{5\,x^2}{4\,\sqrt{2\,x^2-x+3}} - \frac{39}{32}\,\sqrt{2}\,\mathrm{arsinh}\left(\frac{4}{23}\,\sqrt{23}x - \frac{1}{23}\,\sqrt{23}\right) + \frac{3667}{3456}\,\sqrt{2}\,\mathrm{arsinh}\left(\frac{22\,\sqrt{23}x}{23\,|2\,x+5|} - \frac{17\,\sqrt{23}}{23\,|2\,x+5|}\right) - \frac{1153\,x}{3312\,\sqrt{2\,x^2-x+3}} + \frac{2467}{1104\,\sqrt{2\,x^2-x+3}}$$

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(3/2),x, algorithm="max
ima")
```

```
[Out] 5/4*x^2/sqrt(2*x^2 - x + 3) - 39/32*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/23*
sqrt(23)) + 3667/3456*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23
*sqrt(23)/abs(2*x + 5)) - 1153/3312*x/sqrt(2*x^2 - x + 3) + 2467/1104/sqrt(
2*x^2 - x + 3)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.17

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)\left(3-x+2x^2\right)^{3/2}}\,dx = \frac{39}{32}\,\sqrt{2}\log\left(-4\,\sqrt{2}x + \sqrt{2} + 4\,\sqrt{2\,x^2-x+3}\right) - \frac{3667}{3456}\,\sqrt{2}\log\left(\left|-2\,\sqrt{2}x + \sqrt{2} + 2\,\sqrt{2\,x^2-x+3}\right|\right) + \frac{3667}{3456}\,\sqrt{2}\log\left(\left|-2\,\sqrt{2}x - 11\,\sqrt{2} + 2\,\sqrt{2\,x^2-x+3}\right|\right) + \frac{(4140\,x-1153)x+7401}{3312\,\sqrt{2\,x^2-x+3}}$$

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(3/2),x, algorithm="gia
c")
```

```
[Out] 39/32*sqrt(2)*log(-4*sqrt(2)*x + sqrt(2) + 4*sqrt(2*x^2 - x + 3)) - 3667/34
56*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 - x + 3))) + 3667/
3456*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2) + 2*sqrt(2*x^2 - x + 3))) +
1/3312*((4140*x - 1153)*x + 7401)/sqrt(2*x^2 - x + 3)
```

Mupad [**F(-1)**]

Timed out.

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)\left(3-x+2x^2\right)^{3/2}}\,dx=\int \frac{5\,x^4-x^3+3\,x^2+x+2}{(2\,x+5)\,\left(2\,x^2-x+3\right)^{3/2}}\,dx$$

```
[In] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)*(2*x^2 - x + 3)^(3/2)),x)
[Out] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)*(2*x^2 - x + 3)^(3/2)), x)
```

3.355 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2(3-x+2x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 108

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2(3-x+2x^2)^{3/2}}\,dx = \frac{9897+2203x}{119232\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-x+2x^2}}{10368(5+2x)} - \frac{5\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{8\sqrt{2}} + \frac{25951\operatorname{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{41472\sqrt{2}}$$

```
[Out] -5/16*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)+25951/82944*arctanh(1/24*(17-2
2*x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)+1/119232*(9897+2203*x)/(2*x^2-x+3)^
(1/2)-3667/10368*(2*x^2-x+3)^(1/2)/(5+2*x)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$, Rules used = {1660, 1664, 857, 633, 221, 738, 212}

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2(3-x+2x^2)^{3/2}}\,dx = -\frac{5\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{8\sqrt{2}} + \frac{25951\operatorname{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{41472\sqrt{2}} + \frac{2203x+9897}{119232\sqrt{2x^2-x+3}} - \frac{3667\sqrt{2x^2-x+3}}{10368(2x+5)}$$

```
[In] Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^2*(3 - x + 2*x^2)^(3/2)),x]
```

```
[Out] (9897 + 2203*x)/(119232*Sqrt[3 - x + 2*x^2]) - (3667*Sqrt[3 - x + 2*x^2])/(
10368*(5 + 2*x)) - (5*ArcSinh[(1 - 4*x)/Sqrt[23]])/(8*Sqrt[2]) + (25951*Arc
Tanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(41472*Sqrt[2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1660

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^(p + 1)/((p
+ 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{9897+2203x}{119232\sqrt{3-x+2x^2}}+\frac{2}{23}\int\frac{-\frac{33649}{20736}+\frac{131215x}{10368}+\frac{115x^2}{4}}{(5+2x)^2\sqrt{3-x+2x^2}}\,dx\\
&= \frac{9897+2203x}{119232\sqrt{3-x+2x^2}}-\frac{3667\sqrt{3-x+2x^2}}{10368(5+2x)}-\frac{1}{828}\int\frac{\frac{100073}{192}-1035x}{(5+2x)\sqrt{3-x+2x^2}}\,dx\\
&= \frac{9897+2203x}{119232\sqrt{3-x+2x^2}}-\frac{3667\sqrt{3-x+2x^2}}{10368(5+2x)}\\
&\quad+\frac{5}{8}\int\frac{1}{\sqrt{3-x+2x^2}}\,dx-\frac{25951\int\frac{1}{(5+2x)\sqrt{3-x+2x^2}}\,dx}{6912}\\
&= \frac{9897+2203x}{119232\sqrt{3-x+2x^2}}-\frac{3667\sqrt{3-x+2x^2}}{10368(5+2x)}\\
&\quad+\frac{25951\text{Subst}\left(\int\frac{1}{288-x^2}\,dx,x,\frac{17-22x}{\sqrt{3-x+2x^2}}\right)}{3456}+\frac{5\text{Subst}\left(\int\frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx,x,-1+4x\right)}{8\sqrt{46}}\\
&= \frac{9897+2203x}{119232\sqrt{3-x+2x^2}}-\frac{3667\sqrt{3-x+2x^2}}{10368(5+2x)}\\
&\quad-\frac{5\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{8\sqrt{2}}+\frac{25951\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{41472\sqrt{2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.02

$$\int\frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2\left(3-x+2x^2\right)^{3/2}}\,dx=-\frac{\sqrt{3-x+2x^2}\left(51351-48653x+53290x^2\right)}{79488\left(15+x+8x^2+4x^3\right)}-\frac{25951\text{arctanh}\left(\frac{1}{6}\left(5+2x-\sqrt{6-2x+4x^2}\right)\right)}{20736\sqrt{2}}-\frac{5\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{8\sqrt{2}}$$

```
[In] Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^2*(3 - x + 2*x^2)^(3/2))
,x]
```

```
[Out] -1/79488*(Sqrt[3 - x + 2*x^2]*(51351 - 48653*x + 53290*x^2))/(15 + x + 8*x^
2 + 4*x^3) - (25951*ArcTanh[(5 + 2*x - Sqrt[6 - 2*x + 4*x^2])/6])/(20736*Sq
rt[2]) - (5*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/(8*Sqrt[2])
```

Maple [F(-1)]

Timed out.

hanged

```
[In] int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(3/2),x)
```

```
[Out] int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(3/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.45

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2\left(3-x+2x^2\right)^{3/2}}\,dx = \frac{596160\sqrt{2}(4\,x^3+8\,x^2+x+15)\log\left(-4\,\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1)-32\right.}{}$$

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(3/2),x, algorithm="f
ricas")
```

```
[Out] 1/3815424*(596160*sqrt(2)*(4*x^3 + 8*x^2 + x + 15)*log(-4*sqrt(2)*sqrt(2*x^
2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 596873*sqrt(2)*(4*x^3 + 8*x^2
+ x + 15)*log((24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) - 1060*x^2 + 1036
*x - 1153)/(4*x^2 + 20*x + 25)) - 48*(53290*x^2 - 48653*x + 51351)*sqrt(2*x
^2 - x + 3))/(4*x^3 + 8*x^2 + x + 15)
```

Sympy [F]

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2\left(3-x+2x^2\right)^{3/2}}\,dx = \int \frac{5x^4-x^3+3x^2+x+2}{\left(2x+5\right)^2\left(2x^2-x+3\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)**2/(2*x**2-x+3)**(3/2),x)
```

```
[Out] Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)**2*(2*x**2 - x + 3)**(
3/2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.07

$$\begin{aligned}\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2\left(3-x+2x^2\right)^{3/2}}\,dx &= \frac{5}{16}\,\sqrt{2}\,\mathrm{arsinh}\left(\frac{4}{23}\,\sqrt{23}x-\frac{1}{23}\,\sqrt{23}\right)\\ &\quad -\frac{25951}{82944}\,\sqrt{2}\,\mathrm{arsinh}\left(\frac{22\,\sqrt{23}x}{23\,|2\,x+5|}-\frac{17\,\sqrt{23}}{23\,|2\,x+5|}\right)-\frac{26645\,x}{79488\,\sqrt{2\,x^2-x+3}}\\ &\quad +\frac{30313}{26496\,\sqrt{2\,x^2-x+3}}-\frac{3667}{576\,\left(2\,\sqrt{2\,x^2-x+3}x+5\,\sqrt{2\,x^2-x+3}\right)}\end{aligned}$$

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(3/2),x, algorithm="m
axima")
```

```
[Out] 5/16*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) - 25951/82944*sqrt(2)
*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)/abs(2*x + 5)) - 266
45/79488*x/sqrt(2*x^2 - x + 3) + 30313/26496/sqrt(2*x^2 - x + 3) - 3667/576
/(2*sqrt(2*x^2 - x + 3)*x + 5*sqrt(2*x^2 - x + 3))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 225 vs. 2(85) = 170.

Time = 0.35 (sec) , antiderivative size = 225, normalized size of antiderivative = 2.08

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2\left(3-x+2x^2\right)^{3/2}}\,dx = \frac{1}{1907712}\,\sqrt{2}\left(\frac{12\left(\frac{\frac{315103}{\mathrm{sgn}\left(\frac{1}{2\,x+5}\right)}-\frac{1012092}{(2\,x+5)\mathrm{sgn}\left(\frac{1}{2\,x+5}\right)}}{2\,x+5}-\frac{26645}{\mathrm{sgn}\left(\frac{1}{2\,x+5}\right)}\right)}{\sqrt{-\frac{11}{2\,x+5}+\frac{36}{(2\,x+5)^2}+1}}+\frac{596873\,\log(}{}\right.$$

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(3/2),x, algorithm="g
iac")
```

```
[Out] 1/1907712*sqrt(2)*(12*((315103/sgn(1/(2*x + 5)) - 1012092/((2*x + 5)*sgn(1/
(2*x + 5))))/(2*x + 5) - 26645/sgn(1/(2*x + 5)))/sqrt(-11/(2*x + 5) + 36/(2
*x + 5)^2 + 1) + 596873*log(12*sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 7
2/(2*x + 5) - 11)/sgn(1/(2*x + 5)) + 596160*log(abs(sqrt(-11/(2*x + 5) + 36
/(2*x + 5)^2 + 1) + 6/(2*x + 5) + 1))/sgn(1/(2*x + 5)) - 596160*log(abs(sqr
t(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5) - 1))/sgn(1/(2*x + 5)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2\left(3-x+2x^2\right)^{3/2}}\,dx=\int \frac{5\,x^4-x^3+3\,x^2+x+2}{{\left(2\,x+5\right)}^2\,{\left(2\,x^2-x+3\right)}^{3/2}}\,dx$$

```
[In] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^2*(2*x^2 - x + 3)^(3/2)),x)
[Out] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^2*(2*x^2 - x + 3)^(3/2)), x)
```

3.356 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3\left(3-x+2x^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 112

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3\left(3-x+2x^2\right)^{3/2}}\,dx=\frac{65991-8779x}{4292352\sqrt{3-x+2x^2}}-\frac{3667\sqrt{3-x+2x^2}}{20736(5+2x)^2}+\frac{115369\sqrt{3-x+2x^2}}{1492992(5+2x)}-\frac{52631\text{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{5971968\sqrt{2}}$$

```
[Out] -52631/11943936*arctanh(1/24*(17-22*x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)+1
/4292352*(65991-8779*x)/(2*x^2-x+3)^(1/2)-3667/20736*(2*x^2-x+3)^(1/2)/(5+2
*x)^2+115369/1492992*(2*x^2-x+3)^(1/2)/(5+2*x)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1660, 1664, 820, 738, 212}

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3\left(3-x+2x^2\right)^{3/2}}\,dx=-\frac{52631\text{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{5971968\sqrt{2}}+\frac{65991-8779x}{4292352\sqrt{2x^2-x+3}}+\frac{115369\sqrt{2x^2-x+3}}{1492992(2x+5)}-\frac{3667\sqrt{2x^2-x+3}}{20736(2x+5)^2}$$

```
[In] Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^3*(3 - x + 2*x^2)^(3/2)),x]
```

```
[Out] (65991 - 8779*x)/(4292352*Sqrt[3 - x + 2*x^2]) - (3667*Sqrt[3 - x + 2*x^2])
/(20736*(5 + 2*x)^2) + (115369*Sqrt[3 - x + 2*x^2])/(1492992*(5 + 2*x)) - (
```

```
52631*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(5971968*Sqrt[
2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 1660

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^(p + 1)/((p
+ 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
```

```
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

<u>Rubi steps</u>

$$\begin{aligned}
\text{integral} &= \frac{65991-8779x}{4292352\sqrt{3-x+2x^2}}+\frac{2}{23}\int\frac{\frac{5168261}{746496}+\frac{3637795x}{186624}+\frac{5620625x^2}{186624}}{(5+2x)^3\sqrt{3-x+2x^2}}\,dx\\
&= \frac{65991-8779x}{4292352\sqrt{3-x+2x^2}}-\frac{3667\sqrt{3-x+2x^2}}{20736(5+2x)^2}-\frac{\int\frac{\frac{842237}{1296}-\frac{4102487x}{2592}}{(5+2x)^2\sqrt{3-x+2x^2}}\,dx}{1656}\\
&= \frac{65991-8779x}{4292352\sqrt{3-x+2x^2}}-\frac{3667\sqrt{3-x+2x^2}}{20736(5+2x)^2}\\
&\quad+\frac{115369\sqrt{3-x+2x^2}}{1492992(5+2x)}+\frac{52631\int\frac{1}{(5+2x)\sqrt{3-x+2x^2}}\,dx}{995328}\\
&= \frac{65991-8779x}{4292352\sqrt{3-x+2x^2}}-\frac{3667\sqrt{3-x+2x^2}}{20736(5+2x)^2}\\
&\quad+\frac{115369\sqrt{3-x+2x^2}}{1492992(5+2x)}-\frac{52631\text{Subst}\left(\int\frac{1}{288-x^2}\,dx,x,\frac{17-22x}{\sqrt{3-x+2x^2}}\right)}{497664}\\
&= \frac{65991-8779x}{4292352\sqrt{3-x+2x^2}}-\frac{3667\sqrt{3-x+2x^2}}{20736(5+2x)^2}\\
&\quad+\frac{115369\sqrt{3-x+2x^2}}{1492992(5+2x)}-\frac{52631\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{5971968\sqrt{2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.68

$$\int\frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3\left(3-x+2x^2\right)^{3/2}}\,dx=\frac{\frac{12(11594283+5842933x+3263288x^2+3444340x^3)}{(5+2x)^2\sqrt{3-x+2x^2}}+1210513\sqrt{2}\text{arctanh}(\frac{1}{6}(5+2x-}{137355264}$$

```
[In] Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^3*(3 - x + 2*x^2)^(3/2))
,x]
```

```
[Out] ((12*(11594283 + 5842933*x + 3263288*x^2 + 3444340*x^3))/((5 + 2*x)^2*Sqrt[
3 - x + 2*x^2]) + 1210513*Sqrt[2]*ArcTanh[(5 + 2*x - Sqrt[6 - 2*x + 4*x^2])
/6])/137355264
```

Maple [F(-1)]

Timed out.

hanged

```
[In] int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(3/2),x)
```

```
[Out] int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(3/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.12

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3\left(3-x+2x^2\right)^{3/2}}\,dx = \frac{1210513\sqrt{2}(8\,x^4+36\,x^3+42\,x^2+35\,x+75)\log\left(-\frac{24\,\sqrt{2}\sqrt{2\,x^2-x+3}(22\,x-17)}{4\,x^2+20}\right.}{549421056\,(8}$$

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(3/2),x, algorithm="f
ricas")
```

```
[Out] 1/549421056*(1210513*sqrt(2)*(8*x^4 + 36*x^3 + 42*x^2 + 35*x + 75)*log(-(24
*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) + 1060*x^2 - 1036*x + 1153)/(4*x^2
 + 20*x + 25)) + 48*(3444340*x^3 + 3263288*x^2 + 5842933*x + 11594283)*sqrt
(2*x^2 - x + 3))/(8*x^4 + 36*x^3 + 42*x^2 + 35*x + 75)
```

Sympy [F]

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3\left(3-x+2x^2\right)^{3/2}}\,dx = \int \frac{5x^4-x^3+3x^2+x+2}{\left(2x+5\right)^3\left(2x^2-x+3\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)**3/(2*x**2-x+3)**(3/2),x)
```

```
[Out] Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)**3*(2*x**2 - x + 3)**(
3/2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.33

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3\,(3-x+2x^2)^{3/2}}\,dx = \frac{52631}{11943936}\,\sqrt{2}\operatorname{arsinh}\left(\frac{22\,\sqrt{23}x}{23\,|2\,x+5|}-\frac{17\,\sqrt{23}}{23\,|2\,x+5|}\right) + \frac{861085\,x}{11446272\,\sqrt{2\,x^2-x+3}} - \frac{1163201}{3815424\,\sqrt{2\,x^2-x+3}} - \frac{3667}{1152\,\left(4\,\sqrt{2\,x^2-x+3}x^2+20\,\sqrt{2\,x^2-x+3}x+25\,\sqrt{2\,x^2-x+3}\right)} + \frac{196043}{82944\,\left(2\,\sqrt{2\,x^2-x+3}x+5\,\sqrt{2\,x^2-x+3}\right)}$$

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(3/2),x, algorithm="m
axima")
```

```
[Out] 52631/11943936*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(2
3)/abs(2*x + 5)) + 861085/11446272*x/sqrt(2*x^2 - x + 3) - 1163201/3815424/
sqrt(2*x^2 - x + 3) - 3667/1152/(4*sqrt(2*x^2 - x + 3)*x^2 + 20*sqrt(2*x^2
- x + 3)*x + 25*sqrt(2*x^2 - x + 3)) + 196043/82944/(2*sqrt(2*x^2 - x + 3)*
x + 5*sqrt(2*x^2 - x + 3))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 220 vs. 2(90) = 180.

Time = 0.30 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.96

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3\,(3-x+2x^2)^{3/2}}\,dx = -\frac{52631}{11943936}\,\sqrt{2}\log\left(\left|-2\,\sqrt{2}x+\sqrt{2}+2\,\sqrt{2\,x^2-x+3}\right|\right) + \frac{52631}{11943936}\,\sqrt{2}\log\left(\left|-2\,\sqrt{2}x-11\,\sqrt{2}+2\,\sqrt{2\,x^2-x+3}\right|\right) - \frac{8779\,x-65991}{4292352\,\sqrt{2\,x^2-x+3}} + \frac{\sqrt{2}\left(3594214\,\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)^3+19874490\,\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)^2-30140067\,\sqrt{2}\left(\sqrt{2}x-\right.\right.}{2985984\,\left(2\,\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)^2+10\,\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)-11\right)^2}$$

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(3/2),x, algorithm="g
iac")
```

```
[Out] -52631/11943936*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 - x +
 3))) + 52631/11943936*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2) + 2*sqrt(2
*x^2 - x + 3))) - 1/4292352*(8779*x - 65991)/sqrt(2*x^2 - x + 3) + 1/298598
```

```
4*sqrt(2)*(3594214*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^3 + 19874490*(
sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 - 30140067*sqrt(2)*(sqrt(2)*x - sqrt(2*x
^2 - x + 3)) + 19989859)/(2*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 10*sqrt(2
)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) - 11)^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3\left(3-x+2x^2\right)^{3/2}}\,dx=\int \frac{5\,x^4-x^3+3\,x^2+x+2}{{\left(2\,x+5\right)}^3\,{\left(2\,x^2-x+3\right)}^{3/2}} \,d x$$

```
[In] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^3*(2*x^2 - x + 3)^(3/2)),x)
[Out] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^3*(2*x^2 - x + 3)^(3/2)), x)
```

3.357 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4\left(3-x+2x^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 137

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4\left(3-x+2x^2\right)^{3/2}}\,dx=\frac{369609-175877x}{154524672\sqrt{3-x+2x^2}}-\frac{3667\sqrt{3-x+2x^2}}{31104(5+2x)^3}+\frac{152885\sqrt{3-x+2x^2}}{4478976(5+2x)^2}+\frac{430799\sqrt{3-x+2x^2}}{107495424(5+2x)}-\frac{3505819\text{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{1289945088\sqrt{2}}$$

```
[Out] -3505819/2579890176*arctanh(1/24*(17-22*x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/
2)+1/154524672*(369609-175877*x)/(2*x^2-x+3)^(1/2)-3667/31104*(2*x^2-x+3)^(
1/2)/(5+2*x)^3+152885/4478976*(2*x^2-x+3)^(1/2)/(5+2*x)^2+430799/107495424*
(2*x^2-x+3)^(1/2)/(5+2*x)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1660, 1664, 820, 738, 212}

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4\left(3-x+2x^2\right)^{3/2}}\,dx=-\frac{3505819\text{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{1289945088\sqrt{2}}+\frac{369609-175877x}{154524672\sqrt{2x^2-x+3}}+\frac{430799\sqrt{2x^2-x+3}}{107495424(2x+5)}+\frac{152885\sqrt{2x^2-x+3}}{4478976(2x+5)^2}-\frac{3667\sqrt{2x^2-x+3}}{31104(2x+5)^3}$$

```
[In] Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^4*(3 - x + 2*x^2)^(3/2)),x]
```

```
[Out] (369609 - 175877*x)/(154524672*Sqrt[3 - x + 2*x^2]) - (3667*Sqrt[3 - x + 2*
x^2])/(31104*(5 + 2*x)^3) + (152885*Sqrt[3 - x + 2*x^2])/(4478976*(5 + 2*x)
^2) + (430799*Sqrt[3 - x + 2*x^2])/(107495424*(5 + 2*x)) - (3505819*ArcTanh
[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(1289945088*Sqrt[2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 1660

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^(p + 1)/((p
+ 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
```

```
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{369609-175877x}{154524672\sqrt{3-x+2x^2}}+\frac{2}{23}\int\frac{\frac{348877271}{26873856}+\frac{119871055x}{4478976}+\frac{73960295x^2}{2239488}+\frac{1302559x^3}{3359232}}{(5+2x)^4\sqrt{3-x+2x^2}}\,dx\\
&= \frac{369609-175877x}{154524672\sqrt{3-x+2x^2}}-\frac{3667\sqrt{3-x+2x^2}}{31104(5+2x)^3}-\frac{\int\frac{\frac{79609325}{124416}-\frac{71248733x}{31104}-\frac{1302559x^2}{31104}}{(5+2x)^3\sqrt{3-x+2x^2}}\,dx}{2484}\\
&= \frac{369609-175877x}{154524672\sqrt{3-x+2x^2}}-\frac{3667\sqrt{3-x+2x^2}}{31104(5+2x)^3}+\frac{152885\sqrt{3-x+2x^2}}{4478976(5+2x)^2}+\frac{\int\frac{\frac{29340847}{1728}+\frac{1481453x}{54}}{(5+2x)^2\sqrt{3-x+2x^2}}\,dx}{357696}\\
&= \frac{369609-175877x}{154524672\sqrt{3-x+2x^2}}-\frac{3667\sqrt{3-x+2x^2}}{31104(5+2x)^3}+\frac{152885\sqrt{3-x+2x^2}}{4478976(5+2x)^2}\\
&\quad+\frac{430799\sqrt{3-x+2x^2}}{107495424(5+2x)}+\frac{3505819\int\frac{1}{(5+2x)\sqrt{3-x+2x^2}}\,dx}{214990848}\\
&= \frac{369609-175877x}{154524672\sqrt{3-x+2x^2}}-\frac{3667\sqrt{3-x+2x^2}}{31104(5+2x)^3}+\frac{152885\sqrt{3-x+2x^2}}{4478976(5+2x)^2}\\
&\quad+\frac{430799\sqrt{3-x+2x^2}}{107495424(5+2x)}-\frac{3505819\text{Subst}\left(\int\frac{1}{288-x^2}\,dx,x,\frac{17-22x}{\sqrt{3-x+2x^2}}\right)}{107495424}\\
&= \frac{369609-175877x}{154524672\sqrt{3-x+2x^2}}-\frac{3667\sqrt{3-x+2x^2}}{31104(5+2x)^3}+\frac{152885\sqrt{3-x+2x^2}}{4478976(5+2x)^2}\\
&\quad+\frac{430799\sqrt{3-x+2x^2}}{107495424(5+2x)}-\frac{3505819\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{1289945088\sqrt{2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.59

$$\int\frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4\left(3-x+2x^2\right)^{3/2}}\,dx=\frac{\frac{12\left(1873786587+1257975811x+441046842x^2+572739684x^3+56754760x^4\right)}{(5+2x)^3\sqrt{3-x+2x^2}}+80633837\sqrt{2}\mathrm{ar}}{29668737024}$$

```
[In] Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^4*(3 - x + 2*x^2)^(3/2))
,x]
```

```
[Out] ((12*(1873786587 + 1257975811*x + 441046842*x^2 + 572739684*x^3 + 56754760*
x^4))/((5 + 2*x)^3*Sqrt[3 - x + 2*x^2]) + 80633837*Sqrt[2]*ArcTanh[(5 + 2*x
 - Sqrt[6 - 2*x + 4*x^2])/6])/29668737024
```

Maple [F(-1)]

Timed out.

hanged

```
[In] int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(3/2),x)
```

```
[Out] int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(3/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.03

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4\,(3-x+2x^2)^{3/2}}\,dx = \frac{80633837\,\sqrt{2}(16\,x^5+112\,x^4+264\,x^3+280\,x^2+325\,x+375)\log\left(-\frac{24\sqrt{2}}{}\right.}{11}$$

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(3/2),x, algorithm="f
ricas")
```

```
[Out] 1/118674948096*(80633837*sqrt(2)*(16*x^5 + 112*x^4 + 264*x^3 + 280*x^2 + 32
5*x + 375)*log(-(24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) + 1060*x^2 - 10
36*x + 1153)/(4*x^2 + 20*x + 25)) + 48*(56754760*x^4 + 572739684*x^3 + 4410
46842*x^2 + 1257975811*x + 1873786587)*sqrt(2*x^2 - x + 3))/(16*x^5 + 112*x
^4 + 264*x^3 + 280*x^2 + 325*x + 375)
```

Sympy [F]

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4\,(3-x+2x^2)^{3/2}}\,dx = \int \frac{5x^4-x^3+3x^2+x+2}{(2x+5)^4\,(2x^2-x+3)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)**4/(2*x**2-x+3)**(3/2),x)
```

```
[Out] Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)**4*(2*x**2 - x + 3)**(
3/2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.58

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4\left(3-x+2x^2\right)^{3/2}}\,dx = \frac{3505819}{2579890176}\sqrt{2}\,\operatorname{arsinh}\left(\frac{22\sqrt{23}x}{23\,|2\,x+5|}-\frac{17\sqrt{23}}{23\,|2\,x+5|}\right)$$
$$+\frac{7094345\,x}{2472394752\sqrt{2\,x^2-x+3}}+\frac{6128291}{824131584\sqrt{2\,x^2-x+3}}$$
$$-\frac{3667}{1728\left(8\sqrt{2\,x^2-x+3}x^3+60\sqrt{2\,x^2-x+3}x^2+150\sqrt{2\,x^2-x+3}x+125\sqrt{2\,x^2-x+3}\right)}$$
$$+\frac{314233}{248832\left(4\sqrt{2\,x^2-x+3}x^2+20\sqrt{2\,x^2-x+3}x+25\sqrt{2\,x^2-x+3}\right)}$$
$$-\frac{3127169}{17915904\left(2\sqrt{2\,x^2-x+3}x+5\sqrt{2\,x^2-x+3}\right)}$$

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(3/2),x, algorithm="m
axima")
```

```
[Out] 3505819/2579890176*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sq
rt(23)/abs(2*x + 5)) + 7094345/2472394752*x/sqrt(2*x^2 - x + 3) + 6128291/8
24131584/sqrt(2*x^2 - x + 3) - 3667/1728/(8*sqrt(2*x^2 - x + 3)*x^3 + 60*sq
rt(2*x^2 - x + 3)*x^2 + 150*sqrt(2*x^2 - x + 3)*x + 125*sqrt(2*x^2 - x + 3)
) + 314233/248832/(4*sqrt(2*x^2 - x + 3)*x^2 + 20*sqrt(2*x^2 - x + 3)*x + 2
5*sqrt(2*x^2 - x + 3)) - 3127169/17915904/(2*sqrt(2*x^2 - x + 3)*x + 5*sqrt
(2*x^2 - x + 3))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 271 vs. 2(111) = 222.

Time = 0.28 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.98

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4\left(3-x+2x^2\right)^{3/2}}\,dx =$$
$$-\frac{3505819}{2579890176}\sqrt{2}\log\left(\left|-2\sqrt{2}x+\sqrt{2}+2\sqrt{2\,x^2-x+3}\right|\right)$$
$$+\frac{3505819}{2579890176}\sqrt{2}\log\left(\left|-2\sqrt{2}x-11\sqrt{2}+2\sqrt{2\,x^2-x+3}\right|\right)-\frac{175877\,x-369609}{154524672\sqrt{2\,x^2-x+3}}$$
$$-\frac{\sqrt{2}\Bigl(10398764\sqrt{2}\bigl(\sqrt{2}x-\sqrt{2\,x^2-x+3}\bigr)^5-303070900\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)^4-529738052\sqrt{2}\bigl(\sqrt{2}x}{214990848\Bigl(2\left(\sqrt{2}x-\sqrt{2\,x^2-x}\right.}$$

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(3/2),x, algorithm="g
iac")
```

```
[Out] -3505819/2579890176*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 -
 x + 3))) + 3505819/2579890176*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2) +
2*sqrt(2*x^2 - x + 3))) - 1/154524672*(175877*x - 369609)/sqrt(2*x^2 - x +
3) - 1/214990848*sqrt(2)*(10398764*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)
)^5 - 303070900*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^4 - 529738052*sqrt(2)*(sq
rt(2)*x - sqrt(2*x^2 - x + 3))^3 + 3644644652*(sqrt(2)*x - sqrt(2*x^2 - x +
 3))^2 - 2612608649*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1052284471)
/(2*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 10*sqrt(2)*(sqrt(2)*x - sqrt(2*x^
2 - x + 3)) - 11)^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4\left(3-x+2x^2\right)^{3/2}}\,dx=\int \frac{5\,x^4-x^3+3\,x^2+x+2}{{\left(2\,x+5\right)}^4\,{\left(2\,x^2-x+3\right)}^{3/2}}\,dx$$

```
[In] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^4*(2*x^2 - x + 3)^(3/2)),x)
```

```
[Out] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^4*(2*x^2 - x + 3)^(3/2)), x)
```

3.358 $\int \frac{(5+2x)^2(2+x+3x^2-x^3+5x^4)}{(3-x+2x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 105

$$\int \frac{(5+2x)^2(2+x+3x^2-x^3+5x^4)}{(3-x+2x^2)^{5/2}}\,dx = -\frac{4(346-533x)}{69(3-x+2x^2)^{3/2}} + \frac{4(18982-20383x)}{1587\sqrt{3-x+2x^2}} + \frac{247}{16}\sqrt{3-x+2x^2} + \frac{5}{4}x\sqrt{3-x+2x^2} - \frac{1471\text{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{32\sqrt{2}}$$

```
[Out] -4/69*(346-533*x)/(2*x^2-x+3)^(3/2)-1471/64*arcsinh(1/23*(1-4*x)*23^(1/2))*
2^(1/2)+4/1587*(18982-20383*x)/(2*x^2-x+3)^(1/2)+247/16*(2*x^2-x+3)^(1/2)+5
/4*x*(2*x^2-x+3)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1674, 1675, 654, 633, 221}

$$\int \frac{(5+2x)^2(2+x+3x^2-x^3+5x^4)}{(3-x+2x^2)^{5/2}}\,dx = -\frac{1471\text{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{32\sqrt{2}} + \frac{4(18982-20383x)}{1587\sqrt{2x^2-x+3}} + \frac{5}{4}x\sqrt{2x^2-x+3} + \frac{247}{16}\sqrt{2x^2-x+3} - \frac{4(346-533x)}{69(2x^2-x+3)^{3/2}}$$

```
[In] Int[((5 + 2*x)^2*(2 + x + 3*x^2 - x^3 + 5*x^4))/(3 - x + 2*x^2)^(5/2),x]
```

```
[Out] (-4*(346 - 533*x))/(69*(3 - x + 2*x^2)^(3/2)) + (4*(18982 - 20383*x))/(1587
*Sqrt[3 - x + 2*x^2]) + (247*Sqrt[3 - x + 2*x^2])/16 + (5*x*Sqrt[3 - x + 2*
x^2])/4 - (1471*ArcSinh[(1 - 4*x)/Sqrt[23]])/(32*Sqrt[2])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\text{integral} = -\frac{4(346-533x)}{69\left(3-x+2x^2\right)^{3/2}}+\frac{2}{69}\int\frac{-145-\frac{1725x}{2}+2415x^2+\frac{3657x^3}{2}+345x^4}{\left(3-x+2x^2\right)^{3/2}}\,dx$$

$$= -\frac{4(346-533x)}{69\left(3-x+2x^2\right)^{3/2}}+\frac{4(18982-20383x)}{1587\sqrt{3-x+2x^2}}+\frac{4\int\frac{\frac{33327}{2}+\frac{46023x}{4}+\frac{7935x^2}{4}}{\sqrt{3-x+2x^2}}\,dx}{1587}$$

$$
\begin{aligned}
&= -\frac{4(346-533x)}{69\left(3-x+2x^2\right)^{3/2}}+\frac{4(18982-20383x)}{1587\sqrt{3-x+2x^2}}+\frac{5}{4}x\sqrt{3-x+2x^2}+\frac{\int \frac{\frac{242811}{4}+\frac{391989x}{8}}{\sqrt{3-x+2x^2}}\,dx}{1587}\\
&= -\frac{4(346-533x)}{69\left(3-x+2x^2\right)^{3/2}}+\frac{4(18982-20383x)}{1587\sqrt{3-x+2x^2}}+\frac{247}{16}\sqrt{3-x+2x^2}\\
&\quad+\frac{5}{4}x\sqrt{3-x+2x^2}+\frac{1471}{32}\int\frac{1}{\sqrt{3-x+2x^2}}\,dx\\
&= -\frac{4(346-533x)}{69\left(3-x+2x^2\right)^{3/2}}+\frac{4(18982-20383x)}{1587\sqrt{3-x+2x^2}}+\frac{247}{16}\sqrt{3-x+2x^2}\\
&\quad+\frac{5}{4}x\sqrt{3-x+2x^2}+\frac{1471\text{Subst}\left(\int\frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx,x,-1+4x\right)}{32\sqrt{46}}\\
&= -\frac{4(346-533x)}{69\left(3-x+2x^2\right)^{3/2}}+\frac{4(18982-20383x)}{1587\sqrt{3-x+2x^2}}\\
&\quad+\frac{247}{16}\sqrt{3-x+2x^2}+\frac{5}{4}x\sqrt{3-x+2x^2}-\frac{1471\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{32\sqrt{2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.71

$$
\int\frac{(5+2x)^2\left(2+x+3x^2-x^3+5x^4\right)}{\left(3-x+2x^2\right)^{5/2}}\,dx=\frac{6663133-6410082x+8639625x^2-3764360x^3+1440996x^4+}{25392\left(3-x+2x^2\right)^{3/2}}
$$

$$
-\frac{1471\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{32\sqrt{2}}
$$

```
[In] Integrate[((5 + 2*x)^2*(2 + x + 3*x^2 - x^3 + 5*x^4))/(3 - x + 2*x^2)^(5/2)
,x]
```

```
[Out] (6663133 - 6410082*x + 8639625*x^2 - 3764360*x^3 + 1440996*x^4 + 126960*x^5
)/(25392*(3 - x + 2*x^2)^(3/2)) - (1471*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^
2]])/(32*Sqrt[2])
```

Maple [F(-1)]

Timed out.

hanged

```
[In] int((5+2*x)^2*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x)
```

```
[Out] int((5+2*x)^2*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.16

$$\int \frac{(5+2x)^2\left(2+x+3x^2-x^3+5x^4\right)}{\left(3-x+2x^2\right)^{5/2}}\,dx = 2334477\sqrt{2}(4x^4-4x^3+13x^2-6x+9)\log\left(-4\sqrt{2}\sqrt{2x^2-x\right.$$

```
[In] integrate((5+2*x)^2*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x, algorithm="f
ricas")
```

```
[Out] 1/203136*(2334477*sqrt(2)*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*log(-4*sqrt(2)
*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 8*(126960*x^5 + 1440
996*x^4 - 3764360*x^3 + 8639625*x^2 - 6410082*x + 6663133)*sqrt(2*x^2 - x +
 3))/(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)
```

Sympy [F]

$$\int \frac{(5+2x)^2\left(2+x+3x^2-x^3+5x^4\right)}{\left(3-x+2x^2\right)^{5/2}}\,dx = \int \frac{\left(2x+5\right)^{2}\cdot\left(5x^{4}-x^{3}+3x^{2}+x+2\right)}{\left(2x^{2}-x+3\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((5+2*x)**2*(5*x**4-x**3+3*x**2+x+2)/(2*x**2-x+3)**(5/2),x)
```

```
[Out] Integral((2*x + 5)**2*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x**2 - x + 3)**(5
/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(84) = 168$.

Time = 0.29 (sec) , antiderivative size = 219, normalized size of antiderivative = 2.09

$$\int \frac{(5+2x)^2\,(2+x+3x^2-x^3+5x^4)}{(3-x+2x^2)^{5/2}}\,dx = \frac{5\,x^5}{(2\,x^2-x+3)^{\frac{3}{2}}} + \frac{227\,x^4}{4\,(2\,x^2-x+3)^{\frac{3}{2}}} + \frac{1471}{50784}\,x\left(\frac{284\,x}{\sqrt{2\,x^2-x+3}} - \frac{3174\,x^2}{(2\,x^2-x+3)^{\frac{3}{2}}} - \frac{71}{\sqrt{2\,x^2-x+3}} + \frac{805\,x}{(2\,x^2-x+3)^{\frac{3}{2}}} - \frac{3243}{(2\,x^2-x+3)^{\frac{3}{2}}}\right) + \frac{1471}{64}\,\sqrt{2}\,\mathrm{arsinh}\left(\frac{1}{23}\,\sqrt{23}(4\,x-1)\right) - \frac{104441}{25392}\,\sqrt{2\,x^2-x+3} - \frac{383581\,x}{12696\,\sqrt{2\,x^2-x+3}} + \frac{321\,x^2}{(2\,x^2-x+3)^{\frac{3}{2}}} - \frac{15965}{4232\,\sqrt{2\,x^2-x+3}} - \frac{4147\,x}{46\,(2\,x^2-x+3)^{\frac{3}{2}}} + \frac{42883}{138\,(2\,x^2-x+3)^{\frac{3}{2}}}$$

```
[In] integrate((5+2*x)^2*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x, algorithm="m
axima")
```

```
[Out] 5*x^5/(2*x^2 - x + 3)^(3/2) + 227/4*x^4/(2*x^2 - x + 3)^(3/2) + 1471/50784*
x*(284*x/sqrt(2*x^2 - x + 3) - 3174*x^2/(2*x^2 - x + 3)^(3/2) - 71/sqrt(2*x
^2 - x + 3) + 805*x/(2*x^2 - x + 3)^(3/2) - 3243/(2*x^2 - x + 3)^(3/2)) + 1
471/64*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 104441/25392*sqrt(2*x^2 -
 x + 3) - 383581/12696*x/sqrt(2*x^2 - x + 3) + 321*x^2/(2*x^2 - x + 3)^(3/2
) - 15965/4232/sqrt(2*x^2 - x + 3) - 4147/46*x/(2*x^2 - x + 3)^(3/2) + 4288
3/138/(2*x^2 - x + 3)^(3/2)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.68

$$\int \frac{(5+2x)^2\,(2+x+3x^2-x^3+5x^4)}{(3-x+2x^2)^{5/2}}\,dx = -\frac{1471}{64}\,\sqrt{2}\log\left(-2\,\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)+1\right) + \frac{((4\,(1587\,(20\,x+227)x-941090)x+8639625)x-6410082)x+6663133}{25392\,(2\,x^2-x+3)^{\frac{3}{2}}}$$

```
[In] integrate((5+2*x)^2*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x, algorithm="g
iac")
```

```
[Out] -1471/64*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) + 1/
25392*(((4*(1587*(20*x + 227)*x - 941090)*x + 8639625)*x - 6410082)*x + 666
3133)/(2*x^2 - x + 3)^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(5+2x)^2\,(2+x+3x^2-x^3+5x^4)}{(3-x+2x^2)^{5/2}}\,dx = \int \frac{{\left(2\,x+5\right)}^2\,\left(5\,x^4-x^3+3\,x^2+x+2\right)}{{\left(2\,x^2-x+3\right)}^{5/2}}\,dx$$

```
[In] int(((2*x + 5)^2*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x^2 - x + 3)^(5/2),x)
[Out] int(((2*x + 5)^2*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x^2 - x + 3)^(5/2), x)
```

3.359 $\int \frac{(5+2x)\left(2+x+3x^2-x^3+5x^4\right)}{\left(3-x+2x^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 86

$$\int \frac{(5+2x)\left(2+x+3x^2-x^3+5x^4\right)}{\left(3-x+2x^2\right)^{5/2}}\,dx = \frac{-53+373x}{69\left(3-x+2x^2\right)^{3/2}} + \frac{6055-28981x}{3174\sqrt{3-x+2x^2}} + \frac{5}{4}\sqrt{3-x+2x^2} - \frac{71\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{8\sqrt{2}}$$

```
[Out] 1/69*(-53+373*x)/(2*x^2-x+3)^(3/2)-71/16*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(
1/2)+1/3174*(6055-28981*x)/(2*x^2-x+3)^(1/2)+5/4*(2*x^2-x+3)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1674, 654, 633, 221}

$$\int \frac{(5+2x)\left(2+x+3x^2-x^3+5x^4\right)}{\left(3-x+2x^2\right)^{5/2}}\,dx = -\frac{71\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{8\sqrt{2}} + \frac{6055-28981x}{3174\sqrt{2x^2-x+3}} + \frac{5}{4}\sqrt{2x^2-x+3} - \frac{53-373x}{69\left(2x^2-x+3\right)^{3/2}}$$

```
[In] Int[((5 + 2*x)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(3 - x + 2*x^2)^(5/2),x]
```

```
[Out] -1/69*(53 - 373*x)/(3 - x + 2*x^2)^(3/2) + (6055 - 28981*x)/(3174*Sqrt[3 -
x + 2*x^2]) + (5*Sqrt[3 - x + 2*x^2])/4 - (71*ArcSinh[(1 - 4*x)/Sqrt[23]])/
(8*Sqrt[2])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{53-373x}{69\left(3-x+2x^2\right)^{3/2}}+\frac{2}{69}\int\frac{-\frac{233}{4}+483x^2+\frac{345x^3}{2}}{\left(3-x+2x^2\right)^{3/2}}\,dx\\
&= -\frac{53-373x}{69\left(3-x+2x^2\right)^{3/2}}+\frac{6055-28981x}{3174\sqrt{3-x+2x^2}}+\frac{4\int\frac{\frac{52371}{16}+\frac{7935x}{8}}{\sqrt{3-x+2x^2}}\,dx}{1587}\\
&= -\frac{53-373x}{69\left(3-x+2x^2\right)^{3/2}}+\frac{6055-28981x}{3174\sqrt{3-x+2x^2}}+\frac{5}{4}\sqrt{3-x+2x^2}+\frac{71}{8}\int\frac{1}{\sqrt{3-x+2x^2}}\,dx\\
&= -\frac{53-373x}{69\left(3-x+2x^2\right)^{3/2}}+\frac{6055-28981x}{3174\sqrt{3-x+2x^2}}\\
&\quad+\frac{5}{4}\sqrt{3-x+2x^2}+\frac{71\text{Subst}\left(\int\frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx,x,-1+4x\right)}{8\sqrt{46}}
\end{aligned}$$

$$= -\frac{53-373x}{69\left(3-x+2x^2\right)^{3/2}} + \frac{6055-28981x}{3174\sqrt{3-x+2x^2}} + \frac{5}{4}\sqrt{3-x+2x^2} - \frac{71\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{8\sqrt{2}}$$

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.81

$$\int \frac{(5+2x)\left(2+x+3x^2-x^3+5x^4\right)}{\left(3-x+2x^2\right)^{5/2}}\,dx = \frac{102869-199290x+185337x^2-147664x^3+31740x^4}{6348\left(3-x+2x^2\right)^{3/2}} - \frac{71\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{8\sqrt{2}}$$

```
[In] Integrate[((5 + 2*x)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(3 - x + 2*x^2)^(5/2),x
]
```

```
[Out] (102869 - 199290*x + 185337*x^2 - 147664*x^3 + 31740*x^4)/(6348*(3 - x + 2*
x^2)^(3/2)) - (71*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/(8*Sqrt[2])
```

Maple [F(-1)]

Timed out.

hanged

```
[In] int((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x)
```

```
[Out] int((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.36

$$\int \frac{(5+2x)\left(2+x+3x^2-x^3+5x^4\right)}{\left(3-x+2x^2\right)^{5/2}}\,dx = \frac{112677\sqrt{2}(4x^4-4x^3+13x^2-6x+9)\log\left(-4\sqrt{2}\sqrt{2x^2-x-}\right.}{}$$

```
[In] integrate((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x, algorithm="fri
cas")
```

```
[Out] 1/50784*(112677*sqrt(2)*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*log(-4*sqrt(2)*s
qrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 8*(31740*x^4 - 147664*
x^3 + 185337*x^2 - 199290*x + 102869)*sqrt(2*x^2 - x + 3))/(4*x^4 - 4*x^3 +
 13*x^2 - 6*x + 9)
```

Sympy [F]

$$\int \frac{(5+2x)\left(2+x+3x^2-x^3+5x^4\right)}{\left(3-x+2x^2\right)^{5/2}}\,dx=\int \frac{(2x+5)\left(5x^4-x^3+3x^2+x+2\right)}{\left(2x^2-x+3\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((5+2*x)*(5*x**4-x**3+3*x**2+x+2)/(2*x**2-x+3)**(5/2),x)
```

```
[Out] Integral((2*x + 5)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x**2 - x + 3)**(5/2)
, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(69) = 138$.

Time = 0.29 (sec) , antiderivative size = 202, normalized size of antiderivative = 2.35

$$\int \frac{(5+2x)\left(2+x+3x^2-x^3+5x^4\right)}{\left(3-x+2x^2\right)^{5/2}}\,dx=\frac{5\,x^4}{\left(2\,x^2-x+3\right)^{\frac{3}{2}}}+\frac{71}{12696}\,x\left(\frac{284\,x}{\sqrt{2\,x^2-x+3}}-\frac{3174\,x^2}{\left(2\,x^2-x+3\right)^{\frac{3}{2}}}-\frac{71}{\sqrt{2\,x^2-x+3}}+\frac{805\,x}{\left(2\,x^2-x+3\right)^{\frac{3}{2}}}-\frac{3243}{\left(2\,x^2-x+3\right)^{\frac{3}{2}}}\right)+\frac{71}{16}\,\sqrt{2}\,\mathrm{arsinh}\left(\frac{1}{23}\,\sqrt{23}(4\,x-1)\right)-\frac{5041}{6348}\,\sqrt{2\,x^2-x+3}-\frac{10007\,x}{3174\,\sqrt{2\,x^2-x+3}}+\frac{59\,x^2}{2\left(2\,x^2-x+3\right)^{\frac{3}{2}}}-\frac{2959}{2116\,\sqrt{2\,x^2-x+3}}-\frac{807\,x}{92\left(2\,x^2-x+3\right)^{\frac{3}{2}}}+\frac{7603}{276\left(2\,x^2-x+3\right)^{\frac{3}{2}}}$$

```
[In] integrate((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x, algorithm="max
ima")
```

```
[Out] 5*x^4/(2*x^2 - x + 3)^(3/2) + 71/12696*x*(284*x/sqrt(2*x^2 - x + 3) - 3174*
x^2/(2*x^2 - x + 3)^(3/2) - 71/sqrt(2*x^2 - x + 3) + 805*x/(2*x^2 - x + 3)^
(3/2) - 3243/(2*x^2 - x + 3)^(3/2)) + 71/16*sqrt(2)*arcsinh(1/23*sqrt(23)*(
4*x - 1)) - 5041/6348*sqrt(2*x^2 - x + 3) - 10007/3174*x/sqrt(2*x^2 - x + 3
) + 59/2*x^2/(2*x^2 - x + 3)^(3/2) - 2959/2116/sqrt(2*x^2 - x + 3) - 807/92
*x/(2*x^2 - x + 3)^(3/2) + 7603/276/(2*x^2 - x + 3)^(3/2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.77

$$\int \frac{(5+2x)\left(2+x+3x^2-x^3+5x^4\right)}{\left(3-x+2x^2\right)^{5/2}}\,dx = -\frac{71}{16}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right) + \frac{((4(7935x-36916)x+185337)x-199290)x+102869}{6348\left(2x^2-x+3\right)^{\frac{3}{2}}}$$

```
[In] integrate((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x, algorithm="gia
c")
```

```
[Out] -71/16*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) + 1/63
48*(((4*(7935*x - 36916)*x + 185337)*x - 199290)*x + 102869)/(2*x^2 - x + 3
)^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(5+2x)\left(2+x+3x^2-x^3+5x^4\right)}{\left(3-x+2x^2\right)^{5/2}}\,dx = \int \frac{(2x+5)\left(5x^4-x^3+3x^2+x+2\right)}{\left(2x^2-x+3\right)^{5/2}}\,dx$$

```
[In] int(((2*x + 5)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x^2 - x + 3)^(5/2),x)
```

```
[Out] int(((2*x + 5)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x^2 - x + 3)^(5/2), x)
```

3.360 $\int \frac{2+x+3x^2-x^3+5x^4}{\left(3-x+2x^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 68

$$\int \frac{2+x+3x^2-x^3+5x^4}{\left(3-x+2x^2\right)^{5/2}}\,dx=\frac{89+219x}{276\left(3-x+2x^2\right)^{3/2}}-\frac{1465+2604x}{2116\sqrt{3-x+2x^2}}-\frac{5\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{4\sqrt{2}}$$

```
[Out] 1/276*(89+219*x)/(2*x^2-x+3)^(3/2)-5/8*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/
2)+1/2116*(-1465-2604*x)/(2*x^2-x+3)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {1674, 12, 633, 221}

$$\int \frac{2+x+3x^2-x^3+5x^4}{\left(3-x+2x^2\right)^{5/2}}\,dx=-\frac{5\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{4\sqrt{2}}+\frac{219x+89}{276\left(2x^2-x+3\right)^{3/2}}-\frac{2604x+1465}{2116\sqrt{2x^2-x+3}}$$

```
[In] Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/(3 - x + 2*x^2)^(5/2),x]
```

```
[Out] (89 + 219*x)/(276*(3 - x + 2*x^2)^(3/2)) - (1465 + 2604*x)/(2116*Sqrt[3 - x
 + 2*x^2]) - (5*ArcSinh[(1 - 4*x)/Sqrt[23]])/(4*Sqrt[2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{89+219x}{276\left(3-x+2x^2\right)^{3/2}}+\frac{2}{69}\int \frac{-\frac{159}{16}+\frac{207x}{8}+\frac{345x^2}{4}}{\left(3-x+2x^2\right)^{3/2}}\,dx \\
&= \frac{89+219x}{276\left(3-x+2x^2\right)^{3/2}}-\frac{1465+2604x}{2116\sqrt{3-x+2x^2}}+\frac{4\int \frac{7935}{16\sqrt{3-x+2x^2}}\,dx}{1587} \\
&= \frac{89+219x}{276\left(3-x+2x^2\right)^{3/2}}-\frac{1465+2604x}{2116\sqrt{3-x+2x^2}}+\frac{5}{4}\int \frac{1}{\sqrt{3-x+2x^2}}\,dx \\
&= \frac{89+219x}{276\left(3-x+2x^2\right)^{3/2}}-\frac{1465+2604x}{2116\sqrt{3-x+2x^2}}+\frac{5\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx,x,-1+4x\right)}{4\sqrt{46}} \\
&= \frac{89+219x}{276\left(3-x+2x^2\right)^{3/2}}-\frac{1465+2604x}{2116\sqrt{3-x+2x^2}}-\frac{5\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{4\sqrt{2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.96

$$\int \frac{2+x+3x^2-x^3+5x^4}{(3-x+2x^2)^{5/2}}\,dx = \frac{-5569-7002x-489x^2-7812x^3}{3174\,(3-x+2x^2)^{3/2}} - \frac{5\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{4\sqrt{2}}$$

```
[In] Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/(3 - x + 2*x^2)^(5/2),x]
```

```
[Out] (-5569 - 7002*x - 489*x^2 - 7812*x^3)/(3174*(3 - x + 2*x^2)^(3/2)) - (5*Log
[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/(4*Sqrt[2])
```

Maple [F(-1)]

Timed out.

hanged

```
[In] int((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x)
```

```
[Out] int((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(55) = 110.

Time = 0.27 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.65

$$\int \frac{2+x+3x^2-x^3+5x^4}{(3-x+2x^2)^{5/2}}\,dx = \frac{7935\sqrt{2}(4x^4-4x^3+13x^2-6x+9)\log\left(-4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-\right.}{25392\,(4x^4-4x^3+}$$

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/25392*(7935*sqrt(2)*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*log(-4*sqrt(2)*sqr
t(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) - 8*(7812*x^3 + 489*x^2 +
7002*x + 5569)*sqrt(2*x^2 - x + 3))/(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)
```

Sympy [F]

$$\int \frac{2+x+3x^2-x^3+5x^4}{(3-x+2x^2)^{5/2}}\,dx = \int \frac{5x^4-x^3+3x^2+x+2}{(2x^2-x+3)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)/(2*x**2-x+3)**(5/2),x)
[Out] Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x**2 - x + 3)**(5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 185 vs. $2(55) = 110$.

Time = 0.29 (sec) , antiderivative size = 185, normalized size of antiderivative = 2.72

$$\int \frac{2+x+3x^2-x^3+5x^4}{(3-x+2x^2)^{5/2}}\,dx = \frac{5}{6348}\,x\left(\frac{284\,x}{\sqrt{2\,x^2-x+3}} - \frac{3174\,x^2}{(2\,x^2-x+3)^{\frac{3}{2}}} - \frac{71}{\sqrt{2\,x^2-x+3}} + \frac{805\,x}{(2\,x^2-x+\right.$$

$$+\frac{5}{8}\sqrt{2}\,\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}(4\,x-1)\right) - \frac{355}{3174}\sqrt{2\,x^2-x+3} - \frac{58\,x}{1587\sqrt{2\,x^2-x+3}}$$

$$+\frac{x^2}{2\,(2\,x^2-x+3)^{\frac{3}{2}}} - \frac{1897}{6348\sqrt{2\,x^2-x+3}} - \frac{95\,x}{276\,(2\,x^2-x+3)^{\frac{3}{2}}} + \frac{41}{276\,(2\,x^2-x+3)^{\frac{3}{2}}}$$

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x, algorithm="maxima")
[Out] 5/6348*x*(284*x/sqrt(2*x^2 - x + 3) - 3174*x^2/(2*x^2 - x + 3)^(3/2) - 71/s
qrt(2*x^2 - x + 3) + 805*x/(2*x^2 - x + 3)^(3/2) - 3243/(2*x^2 - x + 3)^(3/
2)) + 5/8*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 355/3174*sqrt(2*x^2 -
x + 3) - 58/1587*x/sqrt(2*x^2 - x + 3) + 1/2*x^2/(2*x^2 - x + 3)^(3/2) - 18
97/6348/sqrt(2*x^2 - x + 3) - 95/276*x/(2*x^2 - x + 3)^(3/2) + 41/276/(2*x^
2 - x + 3)^(3/2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.91

$$\int \frac{2+x+3x^2-x^3+5x^4}{(3-x+2x^2)^{5/2}}\,dx =$$
$$-\frac{5}{8}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)+1\right) - \frac{3\left((2604\,x+163)x+2334\right)x+5569}{3174\,(2\,x^2-x+3)^{\frac{3}{2}}}$$

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x, algorithm="giac")
[Out] -5/8*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) - 1/3174
*(3*((2604*x + 163)*x + 2334)*x + 5569)/(2*x^2 - x + 3)^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2+x+3x^2-x^3+5x^4}{\left(3-x+2x^2\right)^{5/2}}\,dx=\int \frac{5\,x^4-x^3+3\,x^2+x+2}{\left(2\,x^2-x+3\right)^{5/2}}\,dx$$

```
[In] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/(2*x^2 - x + 3)^(5/2),x)
[Out] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/(2*x^2 - x + 3)^(5/2), x)
```

3.361 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)(3-x+2x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 85

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)\left(3-x+2x^2\right)^{5/2}}\,dx=\frac{1191+917x}{9936\left(3-x+2x^2\right)^{3/2}}-\frac{335337+146729x}{1371168\sqrt{3-x+2x^2}}-\frac{3667\operatorname{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{31104\sqrt{2}}$$

```
[Out] 1/9936*(1191+917*x)/(2*x^2-x+3)^(3/2)-3667/62208*arctanh(1/24*(17-22*x)*2^(
1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)+1/1371168*(-335337-146729*x)/(2*x^2-x+3)^(1
/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1660, 12, 738, 212}

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)\left(3-x+2x^2\right)^{5/2}}\,dx=-\frac{3667\operatorname{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{31104\sqrt{2}}+\frac{917x+1191}{9936\left(2x^2-x+3\right)^{3/2}}-\frac{146729x+335337}{1371168\sqrt{2x^2-x+3}}$$

```
[In] Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)*(3 - x + 2*x^2)^(5/2)),x]
```

```
[Out] (1191 + 917*x)/(9936*(3 - x + 2*x^2)^(3/2)) - (335337 + 146729*x)/(1371168*
Sqrt[3 - x + 2*x^2]) - (3667*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2
*x^2])])/(31104*Sqrt[2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1660

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^(p + 1)/((p
+ 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1191+917x}{9936\left(3-x+2x^2\right)^{3/2}}+\frac{2}{69}\int \frac{-\frac{1877}{576}+\frac{695x}{18}+\frac{345x^2}{4}}{(5+2x)\left(3-x+2x^2\right)^{3/2}}\,dx\\
&= \frac{1191+917x}{9936\left(3-x+2x^2\right)^{3/2}}-\frac{335337+146729x}{1371168\sqrt{3-x+2x^2}}+\frac{4\int \frac{1939843}{6912(5+2x)\sqrt{3-x+2x^2}}\,dx}{1587}\\
&= \frac{1191+917x}{9936\left(3-x+2x^2\right)^{3/2}}-\frac{335337+146729x}{1371168\sqrt{3-x+2x^2}}+\frac{3667\int \frac{1}{(5+2x)\sqrt{3-x+2x^2}}\,dx}{5184}\\
&= \frac{1191+917x}{9936\left(3-x+2x^2\right)^{3/2}}-\frac{335337+146729x}{1371168\sqrt{3-x+2x^2}}-\frac{3667\text{Subst}\left(\int \frac{1}{288-x^2}\,dx,x,\frac{17-22x}{\sqrt{3-x+2x^2}}\right)}{2592}
\end{aligned}
$$

$$= \frac{1191 + 917x}{9936\,(3 - x + 2x^2)^{3/2}} - \frac{335337 + 146729x}{1371168\sqrt{3 - x + 2x^2}} - \frac{3667\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{31104\sqrt{2}}$$

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.81

$$\int \frac{2 + x + 3x^2 - x^3 + 5x^4}{(5 + 2x)\,(3 - x + 2x^2)^{5/2}}\,dx = \frac{-841653 + 21696x - 523945x^2 - 293458x^3}{1371168\,(3 - x + 2x^2)^{3/2}} + \frac{3667\text{arctanh}\left(\frac{1}{6}\left(5 + 2x - \sqrt{6 - 2x + 4x^2}\right)\right)}{15552\sqrt{2}}$$

```
[In] Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)*(3 - x + 2*x^2)^(5/2)),x
]
```

```
[Out] (-841653 + 21696*x - 523945*x^2 - 293458*x^3)/(1371168*(3 - x + 2*x^2)^(3/2
)) + (3667*ArcTanh[(5 + 2*x - Sqrt[6 - 2*x + 4*x^2])/6])/(15552*Sqrt[2])
```

Maple [F(-1)]

Timed out.

hanged

```
[In] int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(5/2),x)
```

```
[Out] int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(5/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.48

$$\int \frac{2 + x + 3x^2 - x^3 + 5x^4}{(5 + 2x)\,(3 - x + 2x^2)^{5/2}}\,dx = \frac{1939843\,\sqrt{2}(4\,x^4 - 4\,x^3 + 13\,x^2 - 6\,x + 9)\log\left(-\frac{24\,\sqrt{2}\sqrt{2\,x^2-x+3}(22\,x-17)+10}{4\,x^2+20\,x+2}\right.}{65816064\,(4\,x^4 -}$$

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(5/2),x, algorithm="fri
cas")
```

```
[Out] 1/65816064*(1939843*sqrt(2)*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*log(-(24*sqr
t(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) + 1060*x^2 - 1036*x + 1153)/(4*x^2 + 2
0*x + 25)) - 48*(293458*x^3 + 523945*x^2 - 21696*x + 841653)*sqrt(2*x^2 - x
 + 3))/(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)
```

Sympy [F]

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)\left(3-x+2x^2\right)^{5/2}}\,dx=\int \frac{5x^4-x^3+3x^2+x+2}{(2x+5)\left(2x^2-x+3\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)/(2*x**2-x+3)**(5/2),x)
```

```
[Out] Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)*(2*x**2 - x + 3)**(5/2
)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.29

$$\begin{aligned}\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)\left(3-x+2x^2\right)^{5/2}}\,dx&=\frac{3667}{62208}\sqrt{2}\,\mathrm{arsinh}\left(\frac{22\sqrt{23}x}{23\,|2\,x+5|}-\frac{17\sqrt{23}}{23\,|2\,x+5|}\right)\\&-\frac{146729\,x}{1371168\sqrt{2\,x^2-x+3}}-\frac{5\,x^2}{4\left(2\,x^2-x+3\right)^{\frac{3}{2}}}+\frac{173881}{457056\sqrt{2\,x^2-x+3}}\\&+\frac{7127\,x}{9936\left(2\,x^2-x+3\right)^{\frac{3}{2}}}-\frac{5813}{3312\left(2\,x^2-x+3\right)^{\frac{3}{2}}}\end{aligned}$$

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(5/2),x, algorithm="max
ima")
```

```
[Out] 3667/62208*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)/a
bs(2*x + 5)) - 146729/1371168*x/sqrt(2*x^2 - x + 3) - 5/4*x^2/(2*x^2 - x +
3)^(3/2) + 173881/457056/sqrt(2*x^2 - x + 3) + 7127/9936*x/(2*x^2 - x + 3)^
(3/2) - 5813/3312/(2*x^2 - x + 3)^(3/2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.08

$$\begin{aligned}&\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)\left(3-x+2x^2\right)^{5/2}}\,dx=\\&-\frac{3667}{62208}\sqrt{2}\log\left(\left|-2\sqrt{2}x+\sqrt{2}+2\sqrt{2\,x^2-x+3}\right|\right)\\&+\frac{3667}{62208}\sqrt{2}\log\left(\left|-2\sqrt{2}x-11\sqrt{2}+2\sqrt{2\,x^2-x+3}\right|\right)\\&-\frac{((293458\,x+523945)x-21696)x+841653}{1371168\left(2\,x^2-x+3\right)^{\frac{3}{2}}}\end{aligned}$$

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(5/2),x, algorithm="gia
c")
```

```
[Out] -3667/62208*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 - x + 3))
) + 3667/62208*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2) + 2*sqrt(2*x^2 - x
 + 3))) - 1/1371168*(((293458*x + 523945)*x - 21696)*x + 841653)/(2*x^2 - x
 + 3)^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)\,(3-x+2x^2)^{5/2}}\,dx=\int \frac{5\,x^4-x^3+3\,x^2+x+2}{(2\,x+5)\,\left(2\,x^2-x+3\right)^{5/2}}\,dx$$

```
[In] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)*(2*x^2 - x + 3)^(5/2)),x)
```

```
[Out] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)*(2*x^2 - x + 3)^(5/2)), x)
```

3.362 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2\left(3-x+2x^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 110

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2\left(3-x+2x^2\right)^{5/2}}\,dx=\frac{9897+2203x}{357696\left(3-x+2x^2\right)^{3/2}}-\frac{1255878-62021x}{24681024\sqrt{3-x+2x^2}}-\frac{3667\sqrt{3-x+2x^2}}{186624(5+2x)}-\frac{2821\operatorname{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{2239488\sqrt{2}}$$

```
[Out] 1/357696*(9897+2203*x)/(2*x^2-x+3)^(3/2)-2821/4478976*arctanh(1/24*(17-22*x
)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)+1/24681024*(-1255878+62021*x)/(2*x^2-x
+3)^(1/2)-3667/186624*(2*x^2-x+3)^(1/2)/(5+2*x)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1660, 820, 738, 212}

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2\left(3-x+2x^2\right)^{5/2}}\,dx=-\frac{2821\operatorname{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{2239488\sqrt{2}}-\frac{1255878-62021x}{24681024\sqrt{2x^2-x+3}}-\frac{3667\sqrt{2x^2-x+3}}{186624(2x+5)}+\frac{2203x+9897}{357696\left(2x^2-x+3\right)^{3/2}}$$

```
[In] Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^2*(3 - x + 2*x^2)^(5/2)),x]
```

```
[Out] (9897 + 2203*x)/(357696*(3 - x + 2*x^2)^(3/2)) - (1255878 - 62021*x)/(24681
024*Sqrt[3 - x + 2*x^2]) - (3667*Sqrt[3 - x + 2*x^2])/(186624*(5 + 2*x)) -
```

```
(2821*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(2239488*Sqrt[
2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 1660

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^(p + 1)/((p
+ 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{9897+2203x}{357696\left(3-x+2x^2\right)^{3/2}}+\frac{2}{69}\int \frac{\frac{119353}{20736}+\frac{481765x}{10368}+\frac{113983x^2}{1296}}{(5+2x)^2\left(3-x+2x^2\right)^{3/2}}\,dx\\
&= \frac{9897+2203x}{357696\left(3-x+2x^2\right)^{3/2}}-\frac{1255878-62021x}{24681024\sqrt{3-x+2x^2}}+\frac{4\int \frac{\frac{10109719}{124416}-\frac{4961491x}{62208}}{(5+2x)^2\sqrt{3-x+2x^2}}\,dx}{1587}
\end{aligned}
$$

$$= \frac{9897+2203x}{357696\left(3-x+2x^2\right)^{3/2}} - \frac{1255878-62021x}{24681024\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-x+2x^2}}{186624(5+2x)} + \frac{2821\int \frac{1}{(5+2x)\sqrt{3-x+2x^2}}\,dx}{373248}$$

$$= \frac{9897+2203x}{357696\left(3-x+2x^2\right)^{3/2}} - \frac{1255878-62021x}{24681024\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-x+2x^2}}{186624(5+2x)} - \frac{2821\text{Subst}\left(\int \frac{1}{288-x^2}\,dx, x, \frac{17-22x}{\sqrt{3-x+2x^2}}\right)}{186624}$$

$$= \frac{9897+2203x}{357696\left(3-x+2x^2\right)^{3/2}} - \frac{1255878-62021x}{24681024\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-x+2x^2}}{186624(5+2x)} - \frac{2821\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{2239488\sqrt{2}}$$

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.74

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2\left(3-x+2x^2\right)^{5/2}}\,dx = \frac{-\frac{12(79153407-18840090x+63941915x^2+10350004x^3+6767036x^4)}{(5+2x)(3-x+2x^2)^{3/2}} + 1492309\sqrt{2}\text{arctanh}(}{1184689152}$$

```
[In] Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^2*(3 - x + 2*x^2)^(5/2))
,x]
```

```
[Out] ((-12*(79153407 - 18840090*x + 63941915*x^2 + 10350004*x^3 + 6767036*x^4))/
((5 + 2*x)*(3 - x + 2*x^2)^(3/2)) + 1492309*Sqrt[2]*ArcTanh[(5 + 2*x - Sqrt
[6 - 2*x + 4*x^2])/6])/1184689152
```

Maple [F(-1)]

Timed out.

hanged

```
[In] int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(5/2),x)
```

```
[Out] int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(5/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.28

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2\left(3-x+2x^2\right)^{5/2}}\,dx = \frac{1492309\,\sqrt{2}(8\,x^5+12\,x^4+6\,x^3+53\,x^2-12\,x+45)\log\left(-\frac{24\,\sqrt{2}\sqrt{2\,x^2-x+}}{}\right.}{4738}$$

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(5/2),x, algorithm="f
ricas")
```

```
[Out] 1/4738756608*(1492309*sqrt(2)*(8*x^5 + 12*x^4 + 6*x^3 + 53*x^2 - 12*x + 45)
*log(-(24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) + 1060*x^2 - 1036*x + 115
3)/(4*x^2 + 20*x + 25)) - 48*(6767036*x^4 + 10350004*x^3 + 63941915*x^2 - 1
8840090*x + 79153407)*sqrt(2*x^2 - x + 3))/(8*x^5 + 12*x^4 + 6*x^3 + 53*x^2
 - 12*x + 45)
```

Sympy [F]

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2\left(3-x+2x^2\right)^{5/2}}\,dx = \int \frac{5x^4-x^3+3x^2+x+2}{(2x+5)^2\,(2x^2-x+3)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)**2/(2*x**2-x+3)**(5/2),x)
```

```
[Out] Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)**2*(2*x**2 - x + 3)**(
5/2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.15

$$\begin{aligned}\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2\left(3-x+2x^2\right)^{5/2}}\,dx = {} & \frac{2821}{4478976}\,\sqrt{2}\,\mathrm{arsinh}\left(\frac{22\,\sqrt{23}x}{23\,|2\,x+5|}-\frac{17\,\sqrt{23}}{23\,|2\,x+5|}\right) \\ & -\frac{1691759\,x}{98724096\,\sqrt{2\,x^2-x+3}}+\frac{265339}{32908032\,\sqrt{2\,x^2-x+3}}-\frac{248617\,x}{715392\,(2\,x^2-x+3)^{\frac{3}{2}}} \\ & -\frac{3667}{576\left(2\,(2\,x^2-x+3)^{\frac{3}{2}}x+5\,(2\,x^2-x+3)^{\frac{3}{2}}\right)}+\frac{259621}{238464\,(2\,x^2-x+3)^{\frac{3}{2}}}\end{aligned}$$

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(5/2),x, algorithm="m
axima")
```

```
[Out] 2821/4478976*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)
/abs(2*x + 5)) - 1691759/98724096*x/sqrt(2*x^2 - x + 3) + 265339/32908032/s
qrt(2*x^2 - x + 3) - 248617/715392*x/(2*x^2 - x + 3)^(3/2) - 3667/576/(2*(2
*x^2 - x + 3)^(3/2)*x + 5*(2*x^2 - x + 3)^(3/2)) + 259621/238464/(2*x^2 - x
 + 3)^(3/2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 206 vs. $2(88) = 176$.

Time = 0.35 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.87

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2\left(3-x+2x^2\right)^{5/2}}\,dx =$$

$$-\frac{1}{2369378304}\sqrt{2}\Biggl(\frac{1492309\log\left(12\sqrt{-\frac{11}{2\,x+5}+\frac{36}{(2\,x+5)^2}+1}+\frac{72}{2\,x+5}-11\right)}{\operatorname{sgn}\left(\frac{1}{2\,x+5}\right)}+\frac{12\Biggl(\frac{\frac{\frac{48\bigl(\frac{23642785}{\operatorname{sgn}\left(\frac{1}{2\,x+5}\right)}-\frac{52375761}{(2\,x+5)\operatorname{sgn}\left(\frac{1}{2\,x+5}\right)}}{2\,x+5}}{2\,x+5}}{2\,x+5}}{\Bigl(\frac{11}{2\,x+5}-\frac{36}{(2\,x+5)^2}-1}$$

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(5/2),x, algorithm="g
iac")
```

```
[Out] -1/2369378304*sqrt(2)*(1492309*log(12*sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 +
 1) + 72/(2*x + 5) - 11)/sgn(1/(2*x + 5)) + 12*(((48*(23642785/sgn(1/(2*x +
 5)) - 52375761/((2*x + 5)*sgn(1/(2*x + 5))))/(2*x + 5) - 240080735/sgn(1/(
2*x + 5)))/(2*x + 5) + 28660178/sgn(1/(2*x + 5)))/(2*x + 5) - 1691759/sgn(1
/(2*x + 5)))/((11/(2*x + 5) - 36/(2*x + 5)^2 - 1)*sqrt(-11/(2*x + 5) + 36/(
2*x + 5)^2 + 1)) - 20301108*sgn(1/(2*x + 5)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2\,(3-x+2x^2)^{5/2}}\,dx=\int \frac{5\,x^4-x^3+3\,x^2+x+2}{{\left(2\,x+5\right)}^2\,{\left(2\,x^2-x+3\right)}^{5/2}}\,dx$$

```
[In] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^2*(2*x^2 - x + 3)^(5/2)),x)
[Out] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^2*(2*x^2 - x + 3)^(5/2)), x)
```

3.363 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3\left(3-x+2x^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 135

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3\left(3-x+2x^2\right)^{5/2}}\,dx = \frac{65991-8779x}{12877056\left(3-x+2x^2\right)^{3/2}} - \frac{4679797-2148263x}{592344576\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-x+2x^2}}{373248(5+2x)^2} - \frac{45979\sqrt{3-x+2x^2}}{26873856(5+2x)} + \frac{774079\operatorname{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{322486272\sqrt{2}}$$

```
[Out] 1/12877056*(65991-8779*x)/(2*x^2-x+3)^(3/2)+774079/644972544*arctanh(1/24*(
17-22*x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)+1/592344576*(-4679797+2148263*x
)/(2*x^2-x+3)^(1/2)-3667/373248*(2*x^2-x+3)^(1/2)/(5+2*x)^2-45979/26873856*
(2*x^2-x+3)^(1/2)/(5+2*x)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1660, 1664, 820, 738, 212}

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3\left(3-x+2x^2\right)^{5/2}}\,dx = \frac{774079\operatorname{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{322486272\sqrt{2}} - \frac{4679797-2148263x}{592344576\sqrt{2x^2-x+3}} - \frac{45979\sqrt{2x^2-x+3}}{26873856(2x+5)} - \frac{3667\sqrt{2x^2-x+3}}{373248(2x+5)^2} + \frac{65991-8779x}{12877056\left(2x^2-x+3\right)^{3/2}}$$

```
[In] Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^3*(3 - x + 2*x^2)^(5/2)),x]
```

```
[Out] (65991 - 8779*x)/(12877056*(3 - x + 2*x^2)^(3/2)) - (4679797 - 2148263*x)/(
592344576*Sqrt[3 - x + 2*x^2]) - (3667*Sqrt[3 - x + 2*x^2])/(373248*(5 + 2*
x)^2) - (45979*Sqrt[3 - x + 2*x^2])/(26873856*(5 + 2*x)) + (774079*ArcTanh[
(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(322486272*Sqrt[2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 1660

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^(p + 1)/((p
+ 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
```

```
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{65991-8779x}{12877056\left(3-x+2x^2\right)^{3/2}}+\frac{2}{69}\int\frac{\frac{11115283}{746496}+\frac{3198845x}{62208}+\frac{605005x^2}{6912}-\frac{8779x^3}{23328}}{(5+2x)^3\left(3-x+2x^2\right)^{3/2}}\,dx\\
&= \frac{65991-8779x}{12877056\left(3-x+2x^2\right)^{3/2}}-\frac{4679797-2148263x}{592344576\sqrt{3-x+2x^2}}+\frac{4\int\frac{-\frac{171639869}{2985984}-\frac{142392517x}{746496}-\frac{16570925x^2}{746496}}{(5+2x)^3\sqrt{3-x+2x^2}}\,dx}{1587}\\
&= \frac{65991-8779x}{12877056\left(3-x+2x^2\right)^{3/2}}-\frac{4679797-2148263x}{592344576\sqrt{3-x+2x^2}}\\
&\quad-\frac{3667\sqrt{3-x+2x^2}}{373248(5+2x)^2}-\frac{\int\frac{\frac{34040621}{10368}+\frac{28209983x}{10368}}{(5+2x)^2\sqrt{3-x+2x^2}}\,dx}{57132}\\
&= \frac{65991-8779x}{12877056\left(3-x+2x^2\right)^{3/2}}-\frac{4679797-2148263x}{592344576\sqrt{3-x+2x^2}}\\
&\quad-\frac{3667\sqrt{3-x+2x^2}}{373248(5+2x)^2}-\frac{45979\sqrt{3-x+2x^2}}{26873856(5+2x)}-\frac{774079\int\frac{1}{(5+2x)\sqrt{3-x+2x^2}}\,dx}{53747712}\\
&= \frac{65991-8779x}{12877056\left(3-x+2x^2\right)^{3/2}}-\frac{4679797-2148263x}{592344576\sqrt{3-x+2x^2}}-\frac{3667\sqrt{3-x+2x^2}}{373248(5+2x)^2}\\
&\quad-\frac{45979\sqrt{3-x+2x^2}}{26873856(5+2x)}+\frac{774079\text{Subst}\left(\int\frac{1}{288-x^2}\,dx,x,\frac{17-22x}{\sqrt{3-x+2x^2}}\right)}{26873856}\\
&= \frac{65991-8779x}{12877056\left(3-x+2x^2\right)^{3/2}}-\frac{4679797-2148263x}{592344576\sqrt{3-x+2x^2}}-\frac{3667\sqrt{3-x+2x^2}}{373248(5+2x)^2}\\
&\quad-\frac{45979\sqrt{3-x+2x^2}}{26873856(5+2x)}+\frac{774079\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{322486272\sqrt{2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.70

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3\left(3-x+2x^2\right)^{5/2}}\,dx = \frac{\frac{12\sqrt{3-x+2x^2}\,(-8953831359+2280511668x-5919924791x^2-1503926130x^3+107028732x^4+217883}{(15+x+8x^2+4x^3)^2}}{17059523788}$$

```
[In] Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^3*(3 - x + 2*x^2)^(5/2))
,x]
```

```
[Out] ((12*Sqrt[3 - x + 2*x^2]*(-8953831359 + 2280511668*x - 5919924791*x^2 - 150
3926130*x^3 + 107028732*x^4 + 217883368*x^5))/(15 + x + 8*x^2 + 4*x^3)^2 -
409487791*Sqrt[2]*ArcTanh[(5 + 2*x - Sqrt[6 - 2*x + 4*x^2])/6])/17059523788
8
```

Maple [F(-1)]

Timed out.

hanged

```
[In] int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(5/2),x)
```

```
[Out] int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(5/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.15

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3\left(3-x+2x^2\right)^{5/2}}\,dx = \frac{409487791\,\sqrt{2}(16\,x^6+64\,x^5+72\,x^4+136\,x^3+241\,x^2+30\,x+225)\log}{}$$

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(5/2),x, algorithm="f
ricas")
```

```
[Out] 1/682380951552*(409487791*sqrt(2)*(16*x^6 + 64*x^5 + 72*x^4 + 136*x^3 + 241
*x^2 + 30*x + 225)*log((24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) - 1060*x
^2 + 1036*x - 1153)/(4*x^2 + 20*x + 25)) + 48*(217883368*x^5 + 107028732*x^
4 - 1503926130*x^3 - 5919924791*x^2 + 2280511668*x - 8953831359)*sqrt(2*x^2
 - x + 3))/(16*x^6 + 64*x^5 + 72*x^4 + 136*x^3 + 241*x^2 + 30*x + 225)
```

Sympy [F]

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3\left(3-x+2x^2\right)^{5/2}}\,dx=\int \frac{5x^4-x^3+3x^2+x+2}{(2x+5)^3\left(2x^2-x+3\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)**3/(2*x**2-x+3)**(5/2),x)
```

```
[Out] Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)**3*(2*x**2 - x + 3)**(
5/2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.32

$$\begin{aligned}
&\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3\left(3-x+2x^2\right)^{5/2}}\,dx=\\
&-\frac{774079}{644972544}\sqrt{2}\operatorname{arsinh}\left(\frac{22\sqrt{23}x}{23\,|2\,x+5|}-\frac{17\sqrt{23}}{23\,|2\,x+5|}\right)+\frac{27235421\,x}{14216269824\sqrt{2\,x^2-x+3}}\\
&-\frac{36393601}{4738756608\sqrt{2\,x^2-x+3}}+\frac{2323723\,x}{103016448\,(2\,x^2-x+3)^{\frac{3}{2}}}\\
&-\frac{3667}{1152\left(4\,(2\,x^2-x+3)^{\frac{3}{2}}x^2+20\,(2\,x^2-x+3)^{\frac{3}{2}}x+25\,(2\,x^2-x+3)^{\frac{3}{2}}\right)}\\
&+\frac{115369}{82944\left(2\,(2\,x^2-x+3)^{\frac{3}{2}}x+5\,(2\,x^2-x+3)^{\frac{3}{2}}\right)}-\frac{5254255}{34338816\,(2\,x^2-x+3)^{\frac{3}{2}}}
\end{aligned}$$

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(5/2),x, algorithm="m
axima")
```

```
[Out] -774079/644972544*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqr
t(23)/abs(2*x + 5)) + 27235421/14216269824*x/sqrt(2*x^2 - x + 3) - 36393601
/4738756608/sqrt(2*x^2 - x + 3) + 2323723/103016448*x/(2*x^2 - x + 3)^(3/2)
 - 3667/1152/(4*(2*x^2 - x + 3)^(3/2)*x^2 + 20*(2*x^2 - x + 3)^(3/2)*x + 25
*(2*x^2 - x + 3)^(3/2)) + 115369/82944/(2*(2*x^2 - x + 3)^(3/2)*x + 5*(2*x^
2 - x + 3)^(3/2)) - 5254255/34338816/(2*x^2 - x + 3)^(3/2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 228 vs. 2(109) = 218.

Time = 0.29 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.69

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3\,(3-x+2x^2)^{5/2}}\,dx = \frac{774079}{644972544}\,\sqrt{2}\log\left(\left|-2\,\sqrt{2}x+\sqrt{2}+2\,\sqrt{2\,x^2-x+3}\right|\right)$$

$$-\frac{774079}{644972544}\,\sqrt{2}\log\left(\left|-2\,\sqrt{2}x-11\,\sqrt{2}+2\,\sqrt{2\,x^2-x+3}\right|\right)$$

$$+\frac{\sqrt{2}\Big(44558\,\sqrt{2}\big(\sqrt{2}x-\sqrt{2\,x^2-x+3}\big)^3-10136238\,\big(\sqrt{2}x-\sqrt{2\,x^2-x+3}\big)^2+16812201\,\sqrt{2}\big(\sqrt{2}x-\sqrt{2\,x}}{53747712\,\Big(2\,\big(\sqrt{2}x-\sqrt{2\,x^2-x+3}\big)^2+10\,\sqrt{2}\big(\sqrt{2}x-\sqrt{2\,x^2-x+3}\big)-11\Big)^2}$$

$$+\frac{((4296526\,x-11507857)x+10720752)x-11003805}{592344576\,(2\,x^2-x+3)^{\frac{3}{2}}}$$

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(5/2),x, algorithm="g
iac")
```

```
[Out] 774079/644972544*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 - x
+ 3))) - 774079/644972544*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2) + 2*sqr
t(2*x^2 - x + 3))) + 1/53747712*sqrt(2)*(44558*sqrt(2)*(sqrt(2)*x - sqrt(2*
x^2 - x + 3))^3 - 10136238*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 16812201*s
qrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) - 10182217)/(2*(sqrt(2)*x - sqrt(2
*x^2 - x + 3))^2 + 10*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) - 11)^2 + 1
/592344576*(((4296526*x - 11507857)*x + 10720752)*x - 11003805)/(2*x^2 - x
+ 3)^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3\,(3-x+2x^2)^{5/2}}\,dx = \int \frac{5\,x^4-x^3+3\,x^2+x+2}{{\left(2\,x+5\right)}^3\,{\left(2\,x^2-x+3\right)}^{5/2}}\,dx$$

```
[In] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^3*(2*x^2 - x + 3)^(5/2)),x)
```

```
[Out] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^3*(2*x^2 - x + 3)^(5/2)), x)
```

3.364 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4\left(3-x+2x^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 160

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4\left(3-x+2x^2\right)^{5/2}}\,dx = \frac{369609-175877x}{463574016\left(3-x+2x^2\right)^{3/2}} - \frac{27754539-31190998x}{31986607104\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-x+2x^2}}{559872(5+2x)^3} - \frac{89137\sqrt{3-x+2x^2}}{80621568(5+2x)^2} + \frac{475357\sqrt{3-x+2x^2}}{1934917632(5+2x)} + \frac{4778789\,\text{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{7739670528\sqrt{2}}$$

```
[Out] 1/463574016*(369609-175877*x)/(2*x^2-x+3)^(3/2)+4778789/15479341056*arctanh
(1/24*(17-22*x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)+1/31986607104*(-27754539
+31190998*x)/(2*x^2-x+3)^(1/2)-3667/559872*(2*x^2-x+3)^(1/2)/(5+2*x)^3-8913
7/80621568*(2*x^2-x+3)^(1/2)/(5+2*x)^2+475357/1934917632*(2*x^2-x+3)^(1/2)/
(5+2*x)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1660, 1664, 820, 738, 212}

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4\left(3-x+2x^2\right)^{5/2}}\,dx = \frac{4778789\,\text{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{7739670528\sqrt{2}} - \frac{27754539-31190998x}{31986607104\sqrt{2x^2-x+3}} + \frac{475357\sqrt{2x^2-x+3}}{1934917632(2x+5)} - \frac{89137\sqrt{2x^2-x+3}}{80621568(2x+5)^2} - \frac{3667\sqrt{2x^2-x+3}}{559872(2x+5)^3} + \frac{369609-175877x}{463574016\left(2x^2-x+3\right)^{3/2}}$$

```
[In] Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^4*(3 - x + 2*x^2)^(5/2)),x]
```

```
[Out] (369609 - 175877*x)/(463574016*(3 - x + 2*x^2)^(3/2)) - (27754539 - 3119099
8*x)/(31986607104*Sqrt[3 - x + 2*x^2]) - (3667*Sqrt[3 - x + 2*x^2])/(559872
*(5 + 2*x)^3) - (89137*Sqrt[3 - x + 2*x^2])/(80621568*(5 + 2*x)^2) + (47535
7*Sqrt[3 - x + 2*x^2])/(1934917632*(5 + 2*x)) + (4778789*ArcTanh[(17 - 22*x
)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(7739670528*Sqrt[2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 1660

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^(p + 1)/((p
+ 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
```

```
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{369609-175877x}{463574016\,(3-x+2x^2)^{3/2}} \\
&\quad + \frac{2}{69}\int \frac{\frac{606939313}{26873856}+\frac{727085495x}{13436928}+\frac{186705485x^2}{2239488}-\frac{10162483x^3}{3359232}-\frac{175877x^4}{419904}}{(5+2x)^4\,(3-x+2x^2)^{3/2}}\,dx \\
&= \frac{369609-175877x}{463574016\,(3-x+2x^2)^{3/2}}-\frac{27754539-31190998x}{31986607104\sqrt{3-x+2x^2}} \\
&\quad + \frac{4\int \frac{-\frac{4811736919}{40310784}-\frac{3560904781x}{13436928}-\frac{87176555x^2}{1679616}-\frac{39913579x^3}{10077696}}{(5+2x)^4\sqrt{3-x+2x^2}}\,dx}{1587} \\
&= \frac{369609-175877x}{463574016\,(3-x+2x^2)^{3/2}}-\frac{27754539-31190998x}{31986607104\sqrt{3-x+2x^2}} \\
&\quad -\frac{3667\sqrt{3-x+2x^2}}{559872(5+2x)^3}-\frac{\int \frac{\frac{86989289}{11664}+\frac{1265556853x}{186624}+\frac{39913579x^2}{93312}}{(5+2x)^3\sqrt{3-x+2x^2}}\,dx}{85698} \\
&= \frac{369609-175877x}{463574016\,(3-x+2x^2)^{3/2}}-\frac{27754539-31190998x}{31986607104\sqrt{3-x+2x^2}} \\
&\quad -\frac{3667\sqrt{3-x+2x^2}}{559872(5+2x)^3}-\frac{89137\sqrt{3-x+2x^2}}{80621568(5+2x)^2}+\frac{\int \frac{-\frac{5274322027}{20736}-\frac{301114735x}{5184}}{(5+2x)^2\sqrt{3-x+2x^2}}\,dx}{12340512} \\
&= \frac{369609-175877x}{463574016\,(3-x+2x^2)^{3/2}}-\frac{27754539-31190998x}{31986607104\sqrt{3-x+2x^2}}-\frac{3667\sqrt{3-x+2x^2}}{559872(5+2x)^3} \\
&\quad -\frac{89137\sqrt{3-x+2x^2}}{80621568(5+2x)^2}+\frac{475357\sqrt{3-x+2x^2}}{1934917632(5+2x)}-\frac{4778789\int \frac{1}{(5+2x)\sqrt{3-x+2x^2}}\,dx}{1289945088} \\
&= \frac{369609-175877x}{463574016\,(3-x+2x^2)^{3/2}}-\frac{27754539-31190998x}{31986607104\sqrt{3-x+2x^2}}-\frac{3667\sqrt{3-x+2x^2}}{559872(5+2x)^3} \\
&\quad -\frac{89137\sqrt{3-x+2x^2}}{80621568(5+2x)^2}+\frac{475357\sqrt{3-x+2x^2}}{1934917632(5+2x)}+\frac{4778789\text{Subst}\left(\int \frac{1}{288-x^2}\,dx,x,\frac{17-22x}{\sqrt{3-x+2x^2}}\right)}{644972544}
\end{aligned}$$

$$= \frac{369609 - 175877x}{463574016\,(3 - x + 2x^2)^{3/2}} - \frac{27754539 - 31190998x}{31986607104\sqrt{3 - x + 2x^2}} - \frac{3667\sqrt{3 - x + 2x^2}}{559872(5 + 2x)^3} - \frac{89137\sqrt{3 - x + 2x^2}}{80621568(5 + 2x)^2} + \frac{475357\sqrt{3 - x + 2x^2}}{1934917632(5 + 2x)} + \frac{4778789\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{7739670528\sqrt{2}}$$

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.57

$$\int \frac{2 + x + 3x^2 - x^3 + 5x^4}{(5 + 2x)^4\,(3 - x + 2x^2)^{5/2}}\,dx = \frac{\frac{12(-95241881529+73621973154x-6702882569x^2+27484986184x^3+46210466520x^4+34872810880}{(5+2x)^3(3-x+2x^2)^{3/2}}}{4094285}$$

```
[In] Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^4*(3 - x + 2*x^2)^(5/2))
,x]
[Out] ((12*(-95241881529 + 73621973154*x - 6702882569*x^2 + 27484986184*x^3 + 462
10466520*x^4 + 34872810880*x^5 + 6664404208*x^6))/((5 + 2*x)^3*(3 - x + 2*x
^2)^(3/2)) - 2527979381*Sqrt[2]*ArcTanh[(5 + 2*x - Sqrt[6 - 2*x + 4*x^2])/6
])/4094285709312
```

Maple [F(-1)]

Timed out.

hanged

```
[In] int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(5/2),x)
[Out] int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(5/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.06

$$\int \frac{2 + x + 3x^2 - x^3 + 5x^4}{(5 + 2x)^4\,(3 - x + 2x^2)^{5/2}}\,dx = \frac{2527979381\,\sqrt{2}(32\,x^7 + 208\,x^6 + 464\,x^5 + 632\,x^4 + 1162\,x^3 + 1265\,x^2 +}{}$$

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(5/2),x, algorithm="f
ricas")
[Out] 1/16377142837248*(2527979381*sqrt(2)*(32*x^7 + 208*x^6 + 464*x^5 + 632*x^4
+ 1162*x^3 + 1265*x^2 + 600*x + 1125)*log((24*sqrt(2)*sqrt(2*x^2 - x + 3)*(
22*x - 17) - 1060*x^2 + 1036*x - 1153)/(4*x^2 + 20*x + 25)) + 48*(666440420
8*x^6 + 34872810880*x^5 + 46210466520*x^4 + 27484986184*x^3 - 6702882569*x^
2 + 73621973154*x - 95241881529)*sqrt(2*x^2 - x + 3))/(32*x^7 + 208*x^6 + 4
64*x^5 + 632*x^4 + 1162*x^3 + 1265*x^2 + 600*x + 1125)
```

Sympy [F]

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4\left(3-x+2x^2\right)^{5/2}}\,dx=\int \frac{5x^4-x^3+3x^2+x+2}{(2x+5)^4\left(2x^2-x+3\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)**4/(2*x**2-x+3)**(5/2),x)
```

```
[Out] Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)**4*(2*x**2 - x + 3)**(
5/2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.54

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4\left(3-x+2x^2\right)^{5/2}}\,dx=-\frac{4778789}{15479341056}\,\sqrt{2}\,\operatorname{arsinh}\left(\frac{22\,\sqrt{23}x}{23\,|2\,x+5|}-\frac{17\,\sqrt{23}}{23\,|2\,x+5|}\right)$$
$$+\frac{416525263\,x}{341190475776\,\sqrt{2\,x^2-x+3}}-\frac{245375387}{113730158592\,\sqrt{2\,x^2-x+3}}+\frac{16932905\,x}{2472394752\,(2\,x^2-x+3)^{\frac{3}{2}}}$$
$$-\frac{3667}{1728\left(8\,(2\,x^2-x+3)^{\frac{3}{2}}x^3+60\,(2\,x^2-x+3)^{\frac{3}{2}}x^2+150\,(2\,x^2-x+3)^{\frac{3}{2}}x+125\,(2\,x^2-x+3)^{\frac{3}{2}}\right)}$$
$$+\frac{25951}{27648\left(4\,(2\,x^2-x+3)^{\frac{3}{2}}x^2+20\,(2\,x^2-x+3)^{\frac{3}{2}}x+25\,(2\,x^2-x+3)^{\frac{3}{2}}\right)}$$
$$-\frac{34861}{1990656\left(2\,(2\,x^2-x+3)^{\frac{3}{2}}x+5\,(2\,x^2-x+3)^{\frac{3}{2}}\right)}-\frac{10570421}{824131584\,(2\,x^2-x+3)^{\frac{3}{2}}}$$

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(5/2),x, algorithm="m
axima")
```

```
[Out] -4778789/15479341056*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*
sqrt(23)/abs(2*x + 5)) + 416525263/341190475776*x/sqrt(2*x^2 - x + 3) - 245
375387/113730158592/sqrt(2*x^2 - x + 3) + 16932905/2472394752*x/(2*x^2 - x
+ 3)^(3/2) - 3667/1728/(8*(2*x^2 - x + 3)^(3/2)*x^3 + 60*(2*x^2 - x + 3)^(3
/2)*x^2 + 150*(2*x^2 - x + 3)^(3/2)*x + 125*(2*x^2 - x + 3)^(3/2)) + 25951/
27648/(4*(2*x^2 - x + 3)^(3/2)*x^2 + 20*(2*x^2 - x + 3)^(3/2)*x + 25*(2*x^2
 - x + 3)^(3/2)) - 34861/1990656/(2*(2*x^2 - x + 3)^(3/2)*x + 5*(2*x^2 - x
+ 3)^(3/2)) - 10570421/824131584/(2*x^2 - x + 3)^(3/2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 279 vs. $2(130) = 260$.

Time = 0.31 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.74

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4 (3-x+2x^2)^{5/2}}\,dx = \frac{4778789}{15479341056}\sqrt{2}\log\left(\left|-2\sqrt{2}x+\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right)$$
$$-\frac{4778789}{15479341056}\sqrt{2}\log\left(\left|-2\sqrt{2}x-11\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right)$$
$$+\frac{((15595499x-21675019)x+27298005)x-14440149}{7996651776\,(2x^2-x+3)^{\frac{3}{2}}}$$
$$+\frac{\sqrt{2}\Big(38030012\sqrt{2}\big(\sqrt{2}x-\sqrt{2x^2-x+3}\big)^5+734231900\,\big(\sqrt{2}x-\sqrt{2x^2-x+3}\big)^4+122834956\sqrt{2}\big(\sqrt{2}x -}{3869835264\,\Big(2\,\big(\sqrt{2}x-\sqrt{2x^2-x+}}$$

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(5/2),x, algorithm="g
iac")
```

```
[Out] 4778789/15479341056*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 -
 x + 3))) - 4778789/15479341056*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2) +
 2*sqrt(2*x^2 - x + 3))) + 1/7996651776*(((15595499*x - 21675019)*x + 27298
005)*x - 14440149)/(2*x^2 - x + 3)^(3/2) + 1/3869835264*sqrt(2)*(38030012*s
qrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^5 + 734231900*(sqrt(2)*x - sqrt(2*
x^2 - x + 3))^4 + 122834956*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^3 - 2
154595396*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 1659431083*sqrt(2)*(sqrt(2)
*x - sqrt(2*x^2 - x + 3)) - 760577429)/(2*(sqrt(2)*x - sqrt(2*x^2 - x + 3))
^2 + 10*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) - 11)^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4 (3-x+2x^2)^{5/2}}\,dx = \int \frac{5x^4-x^3+3x^2+x+2}{(2x+5)^4\,(2x^2-x+3)^{5/2}}\,dx$$

```
[In] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^4*(2*x^2 - x + 3)^(5/2)),x)
```

```
[Out] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^4*(2*x^2 - x + 3)^(5/2)), x)
```

3.365 $\int \frac{f+gx+hx^2+ix^3+jx^4}{(a+bx+cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 354

$$\int \frac{f+gx+hx^2+ix^3+jx^4}{(a+bx+cx^2)^{5/2}}\,dx = \frac{2(ab^2ci+2ac^2(cg-ai)-ab^3j-bc(c^2f+ach-3a^2j)-(2c^4f-c^3(bg+}{3c^3\,(b^2-4ac)\,(a+bx+c}$$

$$-\frac{2(b^4ci+24a^2c^3i+2b^2c^2(2cg-3ai)-b^5j-b^3c(ch-10aj)-4bc^2(2c^2f+ach+8a^2j)-c(16c^4f-c^3(8bg}{3c^3\,(b^2-4ac)^2\,\sqrt{a+bx+cx^2}}$$

$$+\frac{j\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{5/2}}$$

```
[Out] 2/3*(a*b^2*c*i+2*a*c^2*(-a*i+c*g)-a*b^3*j-b*c*(-3*a^2*j+a*c*h+c^2*f)-(2*c^4
*f-c^3*(2*a*h+b*g)+b^4*j-b^2*c*(4*a*j+b*i)+c^2*(2*a^2*j+3*a*b*i+b^2*h))*x)/
c^3/(-4*a*c+b^2)/(c*x^2+b*x+a)^(3/2)+j*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2
+b*x+a)^(1/2))/c^(5/2)-2/3*(b^4*c*i+24*a^2*c^3*i+2*b^2*c^2*(-3*a*i+2*c*g)-b
^5*j-b^3*c*(-10*a*j+c*h)-4*b*c^2*(8*a^2*j+a*c*h+2*c^2*f)-c*(16*c^4*f-c^3*(-
8*a*h+8*b*g)-4*b^4*j+b^2*c*(28*a*j+b*i)+2*c^2*(-16*a^2*j-6*a*b*i+b^2*h))*x)
/c^3/(-4*a*c+b^2)^2/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used

$= \{1674, 12, 635, 212\}$

$$\int \frac{f+gx+hx^2+ix^3+jx^4}{(a+bx+cx^2)^{5/2}}\,dx = \frac{2(-x(c^2(2a^2j+3abi+b^2h)-b^2c(4aj+bi)-c^3(2ah+bg)+b^4j+2c^4f}{3c^3\,(b^2-4ac)\,(a+bx}$$
$$-\frac{2(-cx(2c^2(-16a^2j-6abi+b^2h)+b^2c(28aj+bi)-c^3(8bg-8ah)-4b^4j+16c^4f)-4bc^2(8a^2j+ach+}{3c^3\,(b^2-4ac)^2\sqrt{a+bx+cx^2}}$$
$$+\frac{j\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{5/2}}$$

```
[In] Int[(f + g*x + h*x^2 + i*x^3 + j*x^4)/(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (2*(a*b^2*c*i + 2*a*c^2*(c*g - a*i) - a*b^3*j - b*c*(c^2*f + a*c*h - 3*a^2*
j) - (2*c^4*f - c^3*(b*g + 2*a*h) + b^4*j - b^2*c*(b*i + 4*a*j) + c^2*(b^2*
h + 3*a*b*i + 2*a^2*j))*x))/(3*c^3*(b^2 - 4*a*c)*(a + b*x + c*x^2)^(3/2)) -
 (2*(b^4*c*i + 24*a^2*c^3*i + 2*b^2*c^2*(2*c*g - 3*a*i) - b^5*j - b^3*c*(c*
h - 10*a*j) - 4*b*c^2*(2*c^2*f + a*c*h + 8*a^2*j) - c*(16*c^4*f - c^3*(8*b*
g - 8*a*h) - 4*b^4*j + b^2*c*(b*i + 28*a*j) + 2*c^2*(b^2*h - 6*a*b*i - 16*a
^2*j))*x))/(3*c^3*(b^2 - 4*a*c)^2*Sqrt[a + b*x + c*x^2]) + (j*ArcTanh[(b +
2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/c^(5/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
```

```
- 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

integral

$$=\frac{2(ab^2ci+2ac^2(cg-ai)-ab^3j-bc(c^2f+ach-3a^2j)-(2c^4f-c^3(bg+2ah)+b^4j-b^2c(bi+4aj)+c^2(}{3c^3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}-\frac{2\int\frac{\frac{8c^4f-c^3(4bg-4ah)+b^4j-b^2c(bi+aj)+c^2\left(b^2h-4a^2j\right)}{2c^3}-\frac{3\left(b^2-4ac\right)(ci-bj)x}{2c^2}+\frac{3}{2}\left(4a-\frac{b^2}{c}\right)jx^2}{\left(a+bx+cx^2\right)^{3/2}}\,dx}{3\left(b^2-4ac\right)}$$

$$=\frac{2(ab^2ci+2ac^2(cg-ai)-ab^3j-bc(c^2f+ach-3a^2j)-(2c^4f-c^3(bg+2ah)+b^4j-b^2c(bi+4a}{3c^3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}-\frac{2(b^4ci+24a^2c^3i+2b^2c^2(2cg-3ai)-b^5j-b^3c(ch-10aj)-4bc^2(2c^2f+ach+8a^2j)-c(16c^4}{3c^3\left(b^2-4ac\right)^2\sqrt{a+bx+}}+\frac{4\int\frac{3\left(b^2-4ac\right)^2j}{4c^2\sqrt{a+bx+cx^2}}\,dx}{3\left(b^2-4ac\right)^2}$$

$$=\frac{2(ab^2ci+2ac^2(cg-ai)-ab^3j-bc(c^2f+ach-3a^2j)-(2c^4f-c^3(bg+2ah)+b^4j-b^2c(bi+4a}{3c^3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}-\frac{2(b^4ci+24a^2c^3i+2b^2c^2(2cg-3ai)-b^5j-b^3c(ch-10aj)-4bc^2(2c^2f+ach+8a^2j)-c(16c^4}{3c^3\left(b^2-4ac\right)^2\sqrt{a+bx+}}+\frac{j\int\frac{1}{\sqrt{a+bx+cx^2}}\,dx}{c^2}$$

$$=\frac{2(ab^2ci+2ac^2(cg-ai)-ab^3j-bc(c^2f+ach-3a^2j)-(2c^4f-c^3(bg+2ah)+b^4j-b^2c(bi+4a}{3c^3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}-\frac{2(b^4ci+24a^2c^3i+2b^2c^2(2cg-3ai)-b^5j-b^3c(ch-10aj)-4bc^2(2c^2f+ach+8a^2j)-c(16c^4}{3c^3\left(b^2-4ac\right)^2\sqrt{a+bx+}}+\frac{(2j)\text{Subst}\left(\int\frac{1}{4c-x^2}\,dx,x,\frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{c^2}$$

$$=\frac{2(ab^2ci+2ac^2(cg-ai)-ab^3j-bc(c^2f+ach-3a^2j)-(2c^4f-c^3(bg+2ah)+b^4j-b^2c(bi+4a}{3c^3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}-\frac{2(b^4ci+24a^2c^3i+2b^2c^2(2cg-3ai)-b^5j-b^3c(ch-10aj)-4bc^2(2c^2f+ach+8a^2j)-c(16c^4}{3c^3\left(b^2-4ac\right)^2\sqrt{a+bx+}}+\frac{j\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{5/2}}$$

Mathematica [A] (verified)

Time = 1.97 (sec) , antiderivative size = 315, normalized size of antiderivative = 0.89

$$\int \frac{f+gx+hx^2+ix^3+jx^4}{(a+bx+cx^2)^{5/2}}\,dx = \frac{2(-3b^5jx^2-2b^4jx(3a+2cx^2)+b^3(-3a^2j+18acjx^2+c^2(-f-3gx+}{}$$

$$+\frac{2j\operatorname{arctanh}\left(\frac{\sqrt{c}x}{-\sqrt{a}+\sqrt{a+x(b+cx)}}\right)}{c^{5/2}}$$

```
[In] Integrate[(f + g*x + h*x^2 + i*x^3 + j*x^4)/(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (2*(-3*b^5*j*x^2 - 2*b^4*j*x*(3*a + 2*c*x^2) + b^3*(-3*a^2*j + 18*a*c*j*x^2
 + c^2*(-f - 3*g*x + 3*h*x^2 + i*x^3)) + 2*b^2*c*(21*a^2*j*x + c^2*x*(3*f -
 6*g*x + h*x^2) - a*c*(g - 6*h*x + 3*i*x^2 - 14*j*x^3)) - 8*c^2*(-2*c^3*f*x
^3 + a^3*(2*i + 3*j*x) - a*c^2*x*(3*f + h*x^2) + a^2*c*(g + 3*i*x^2 + 4*j*x
^3)) + 4*b*c*(5*a^3*j - 2*c^3*x^2*(-3*f + g*x) + 2*a^2*c*(h - 3*i*x) + 3*a*
c^2*(f - x*(g - h*x + i*x^2)))))/(3*c^2*(b^2 - 4*a*c)^2*(a + x*(b + c*x))^(
3/2)) + (2*j*ArcTanh[(Sqrt[c]*x)/(-Sqrt[a] + Sqrt[a + x*(b + c*x)])])/c^(5/
2)
```

Maple [F(-1)]

Timed out.

hanged

```
[In] int((j*x^4+i*x^3+h*x^2+g*x+f)/(c*x^2+b*x+a)^(5/2),x)
```

```
[Out] int((j*x^4+i*x^3+h*x^2+g*x+f)/(c*x^2+b*x+a)^(5/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 685 vs. $2(337) = 674$.

Time = 22.78 (sec) , antiderivative size = 1373, normalized size of antiderivative = 3.88

$$\int \frac{f+gx+hx^2+ix^3+jx^4}{(a+bx+cx^2)^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((j*x^4+i*x^3+h*x^2+g*x+f)/(c*x^2+b*x+a)^(5/2),x, algorithm="frica
s")
```

```
[Out] [1/6*(3*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*j*x^4 + 2*(b^5*c - 8*a*b^3*c^
2 + 16*a^2*b*c^3)*j*x^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*j*x^2 + 2*(a*b^5 -
 8*a^2*b^3*c + 16*a^3*b*c^2)*j*x + (a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2)*j)*
sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b
```

```
)*sqrt(c) - 4*a*c) + 4*(8*a^2*b*c^3*h - 16*a^3*c^3*i + (16*c^6*f - 8*b*c^5*
g + 2*(b^2*c^4 + 4*a*c^5)*h + (b^3*c^3 - 12*a*b*c^4)*i - 4*(b^4*c^2 - 7*a*b
^2*c^3 + 8*a^2*c^4)*j)*x^3 + 3*(8*b*c^5*f - 4*b^2*c^4*g + (b^3*c^3 + 4*a*b*
c^4)*h - 2*(a*b^2*c^3 + 4*a^2*c^4)*i - (b^5*c - 6*a*b^3*c^2)*j)*x^2 - (b^3*
c^3 - 12*a*b*c^4)*f - 2*(a*b^2*c^3 + 4*a^2*c^4)*g - (3*a^2*b^3*c - 20*a^3*b
*c^2)*j + 3*(4*a*b^2*c^3*h - 8*a^2*b*c^3*i + 2*(b^2*c^4 + 4*a*c^5)*f - (b^3
*c^3 + 4*a*b*c^4)*g - 2*(a*b^4*c - 7*a^2*b^2*c^2 + 4*a^3*c^3)*j)*x)*sqrt(c*
x^2 + b*x + a))/(a^2*b^4*c^3 - 8*a^3*b^2*c^4 + 16*a^4*c^5 + (b^4*c^5 - 8*a*
b^2*c^6 + 16*a^2*c^7)*x^4 + 2*(b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)*x^3 +
(b^6*c^3 - 6*a*b^4*c^4 + 32*a^3*c^6)*x^2 + 2*(a*b^5*c^3 - 8*a^2*b^3*c^4 + 1
6*a^3*b*c^5)*x), -1/3*(3*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*j*x^4 + 2*(b
^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*j*x^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*j
*x^2 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*j*x + (a^2*b^4 - 8*a^3*b^2*c
+ 16*a^4*c^2)*j)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt
(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(8*a^2*b*c^3*h - 16*a^3*c^3*i + (16*c^6*f
 - 8*b*c^5*g + 2*(b^2*c^4 + 4*a*c^5)*h + (b^3*c^3 - 12*a*b*c^4)*i - 4*(b^4*
c^2 - 7*a*b^2*c^3 + 8*a^2*c^4)*j)*x^3 + 3*(8*b*c^5*f - 4*b^2*c^4*g + (b^3*c
^3 + 4*a*b*c^4)*h - 2*(a*b^2*c^3 + 4*a^2*c^4)*i - (b^5*c - 6*a*b^3*c^2)*j)*
x^2 - (b^3*c^3 - 12*a*b*c^4)*f - 2*(a*b^2*c^3 + 4*a^2*c^4)*g - (3*a^2*b^3*c
 - 20*a^3*b*c^2)*j + 3*(4*a*b^2*c^3*h - 8*a^2*b*c^3*i + 2*(b^2*c^4 + 4*a*c^
5)*f - (b^3*c^3 + 4*a*b*c^4)*g - 2*(a*b^4*c - 7*a^2*b^2*c^2 + 4*a^3*c^3)*j)
*x)*sqrt(c*x^2 + b*x + a))/(a^2*b^4*c^3 - 8*a^3*b^2*c^4 + 16*a^4*c^5 + (b^4
*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*x^4 + 2*(b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*
c^6)*x^3 + (b^6*c^3 - 6*a*b^4*c^4 + 32*a^3*c^6)*x^2 + 2*(a*b^5*c^3 - 8*a^2*
b^3*c^4 + 16*a^3*b*c^5)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{f + gx + hx^2 + ix^3 + jx^4}{(a + bx + cx^2)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((j*x**4+i*x**3+h*x**2+g*x+f)/(c*x**2+b*x+a)**(5/2),x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{f+gx+hx^2+ix^3+jx^4}{(a+bx+cx^2)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((j*x^4+i*x^3+h*x^2+g*x+f)/(c*x^2+b*x+a)^(5/2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 463, normalized size of antiderivative = 1.31

$$\int \frac{f+gx+hx^2+ix^3+jx^4}{(a+bx+cx^2)^{5/2}}\,dx = \frac{2\Bigg(\Bigg(\Bigg(\frac{(16\,c^5f-8\,bc^4g+2\,b^2c^3h+8\,ac^4h+b^3c^2i-12\,abc^3i-4\,b^4cj+28\,ab^2c^2j-32\,a^2c^3j)x}{b^4c^2-8\,ab^2c^3+16\,a^2c^4}+\frac{3\,(8\,bc}{}}{}$$

$$-\frac{j\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}+b\right|\right)}{c^{\frac{5}{2}}}$$

```
[In] integrate((j*x^4+i*x^3+h*x^2+g*x+f)/(c*x^2+b*x+a)^(5/2),x, algorithm="giac"
)
```

```
[Out] 2/3*((((16*c^5*f - 8*b*c^4*g + 2*b^2*c^3*h + 8*a*c^4*h + b^3*c^2*i - 12*a*b
*c^3*i - 4*b^4*c*j + 28*a*b^2*c^2*j - 32*a^2*c^3*j)*x/(b^4*c^2 - 8*a*b^2*c^
3 + 16*a^2*c^4) + 3*(8*b*c^4*f - 4*b^2*c^3*g + b^3*c^2*h + 4*a*b*c^3*h - 2*
a*b^2*c^2*i - 8*a^2*c^3*i - b^5*j + 6*a*b^3*c*j)/(b^4*c^2 - 8*a*b^2*c^3 + 1
6*a^2*c^4))*x + 3*(2*b^2*c^3*f + 8*a*c^4*f - b^3*c^2*g - 4*a*b*c^3*g + 4*a*
b^2*c^2*h - 8*a^2*b*c^2*i - 2*a*b^4*j + 14*a^2*b^2*c*j - 8*a^3*c^2*j)/(b^4*
c^2 - 8*a*b^2*c^3 + 16*a^2*c^4))*x - (b^3*c^2*f - 12*a*b*c^3*f + 2*a*b^2*c^
2*g + 8*a^2*c^3*g - 8*a^2*b*c^2*h + 16*a^3*c^2*i + 3*a^2*b^3*j - 20*a^3*b*c
*j)/(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4))/(c*x^2 + b*x + a)^(3/2) - j*log(a
bs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{f+gx+hx^2+ix^3+jx^4}{(a+bx+cx^2)^{5/2}}\,dx = \int \frac{j\,x^4+i\,x^3+h\,x^2+g\,x+f}{(c\,x^2+b\,x+a)^{5/2}}\,dx$$

```
[In] int((f + g*x + h*x^2 + i*x^3 + j*x^4)/(a + b*x + c*x^2)^(5/2),x)
[Out] int((f + g*x + h*x^2 + i*x^3 + j*x^4)/(a + b*x + c*x^2)^(5/2), x)
```

3.366 $\int \frac{f+gx+hx^2+ix^3+jx^4}{(a+bx-cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 353

$$\int \frac{f+gx+hx^2+ix^3+jx^4}{(a+bx-cx^2)^{5/2}}\,dx=\frac{2(ab^2ci+2ac^2(cg+ai)+ab^3j-bc(c^2f-ach-3a^2j)+(2c^4f+c^3(bg}{3c^3\,(b^2+4ac)\,(a+bx-}$$
$$-\frac{2(b^4ci+24a^2c^3i+2b^2c^2(2cg+3ai)+b^5j+b^3c(ch+10aj)+4bc^2(2c^2f-ach+8a^2j)-c(16c^4f+8c^3(b}{3c^3\,(b^2+4ac)^2\sqrt{a+bx-cx^2}}$$
$$-\frac{j\arctan\left(\frac{b-2cx}{2\sqrt{c}\sqrt{a+bx-cx^2}}\right)}{c^{5/2}}$$

```
[Out] 2/3*(a*b^2*c*i+2*a*c^2*(a*i+c*g)+a*b^3*j-b*c*(-3*a^2*j-a*c*h+c^2*f)+(2*c^4*
f+c^3*(2*a*h+b*g)+b^4*j+b^2*c*(4*a*j+b*i)+c^2*(2*a^2*j+3*a*b*i+b^2*h))*x)/c
^3/(4*a*c+b^2)/(-c*x^2+b*x+a)^(3/2)-j*arctan(1/2*(-2*c*x+b)/c^(1/2)/(-c*x^2
+b*x+a)^(1/2))/c^(5/2)-2/3*(b^4*c*i+24*a^2*c^3*i+2*b^2*c^2*(3*a*i+2*c*g)+b^
5*j+b^3*c*(10*a*j+c*h)+4*b*c^2*(8*a^2*j-a*c*h+2*c^2*f)-c*(16*c^4*f+8*c^3*(-
a*h+b*g)-4*b^4*j-b^2*c*(28*a*j+b*i)+2*c^2*(-16*a^2*j-6*a*b*i+b^2*h))*x)/c^3
/(4*a*c+b^2)^2/(-c*x^2+b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used

$= \{1674, 12, 635, 210\}$

$$\int \frac{f+gx+hx^2+ix^3+jx^4}{(a+bx-cx^2)^{5/2}}\,dx = \frac{2(x(c^2(2a^2j+3abi+b^2h)+b^2c(4aj+bi)+c^3(2ah+bg)+b^4j+2c^4f)-}{3c^3\,(4ac+b^2)\,(a+bx-}$$

$$-\frac{2(-cx(2c^2(-16a^2j-6abi+b^2h)-b^2c(28aj+bi)+8c^3(bg-ah)-4b^4j+16c^4f)+4bc^2(8a^2j-ach+2c}{3c^3\,(4ac+b^2)^2\sqrt{a+bx-cx^2}}$$

$$-\frac{j\arctan\left(\frac{b-2cx}{2\sqrt{c}\sqrt{a+bx-cx^2}}\right)}{c^{5/2}}$$

```
[In] Int[(f + g*x + h*x^2 + i*x^3 + j*x^4)/(a + b*x - c*x^2)^(5/2),x]
```

```
[Out] (2*(a*b^2*c*i + 2*a*c^2*(c*g + a*i) + a*b^3*j - b*c*(c^2*f - a*c*h - 3*a^2*
j) + (2*c^4*f + c^3*(b*g + 2*a*h) + b^4*j + b^2*c*(b*i + 4*a*j) + c^2*(b^2*
h + 3*a*b*i + 2*a^2*j))*x))/(3*c^3*(b^2 + 4*a*c)*(a + b*x - c*x^2)^(3/2)) -
 (2*(b^4*c*i + 24*a^2*c^3*i + 2*b^2*c^2*(2*c*g + 3*a*i) + b^5*j + b^3*c*(c*
h + 10*a*j) + 4*b*c^2*(2*c^2*f - a*c*h + 8*a^2*j) - c*(16*c^4*f + 8*c^3*(b*
g - a*h) - 4*b^4*j - b^2*c*(b*i + 28*a*j) + 2*c^2*(b^2*h - 6*a*b*i - 16*a^2
*j))*x))/(3*c^3*(b^2 + 4*a*c)^2*Sqrt[a + b*x - c*x^2]) - (j*ArcTan[(b - 2*c
*x)/(2*Sqrt[c]*Sqrt[a + b*x - c*x^2])])/c^(5/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
```

```
- 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

integral

$$= \frac{2(ab^2ci + 2ac^2(cg+ai) + ab^3j - bc(c^2f - ach - 3a^2j) + (2c^4f + c^3(bg+2ah) + b^4j + b^2c(bi+4aj) + c}{3c^3\left(b^2+4ac\right)\left(a+bx-cx^2\right)^{3/2}}$$

$$- \frac{2\int \frac{-\frac{8c^4f+4c^3(bg-ah)+b^4j+b^2c(bi+aj)+c^2\left(b^2h-4a^2j\right)}{2c^3}+\frac{3\left(b^2+4ac\right)(ci+bj)x}{2c^2}+\frac{3\left(b^2+4ac\right)jx^2}{2c}}{(a+bx-cx^2)^{3/2}}\,dx}{3\left(b^2+4ac\right)}$$

$$= \frac{2(ab^2ci + 2ac^2(cg+ai) + ab^3j - bc(c^2f - ach - 3a^2j) + (2c^4f + c^3(bg+2ah) + b^4j + b^2c(bi + 4}{3c^3\left(b^2+4ac\right)\left(a+bx-cx^2\right)^{3/2}}$$

$$- \frac{2(b^4ci + 24a^2c^3i + 2b^2c^2(2cg+3ai) + b^5j + b^3c(ch+10aj) + 4bc^2(2c^2f - ach + 8a^2j) - c(16c}{3c^3\left(b^2+4ac\right)^2\sqrt{a+bx-}}$$

$$+ \frac{4\int \frac{3\left(b^2+4ac\right)^2j}{4c^2\sqrt{a+bx-cx^2}}\,dx}{3\left(b^2+4ac\right)^2}$$

$$= \frac{2(ab^2ci + 2ac^2(cg+ai) + ab^3j - bc(c^2f - ach - 3a^2j) + (2c^4f + c^3(bg+2ah) + b^4j + b^2c(bi + 4}{3c^3\left(b^2+4ac\right)\left(a+bx-cx^2\right)^{3/2}}$$

$$- \frac{2(b^4ci + 24a^2c^3i + 2b^2c^2(2cg+3ai) + b^5j + b^3c(ch+10aj) + 4bc^2(2c^2f - ach + 8a^2j) - c(16c}{3c^3\left(b^2+4ac\right)^2\sqrt{a+bx-}}$$

$$+ \frac{j\int \frac{1}{\sqrt{a+bx-cx^2}}\,dx}{c^2}$$

$$= \frac{2(ab^2ci + 2ac^2(cg+ai) + ab^3j - bc(c^2f - ach - 3a^2j) + (2c^4f + c^3(bg+2ah) + b^4j + b^2c(bi + 4}{3c^3\left(b^2+4ac\right)\left(a+bx-cx^2\right)^{3/2}}$$

$$- \frac{2(b^4ci + 24a^2c^3i + 2b^2c^2(2cg+3ai) + b^5j + b^3c(ch+10aj) + 4bc^2(2c^2f - ach + 8a^2j) - c(16c}{3c^3\left(b^2+4ac\right)^2\sqrt{a+bx-}}$$

$$+ \frac{(2j)\text{Subst}\left(\int \frac{1}{-4c-x^2}\,dx, x, \frac{b-2cx}{\sqrt{a+bx-cx^2}}\right)}{c^2}$$

$$= \frac{2(ab^2ci + 2ac^2(cg+ai) + ab^3j - bc(c^2f - ach - 3a^2j) + (2c^4f + c^3(bg+2ah) + b^4j + b^2c(bi + 4}{3c^3\left(b^2+4ac\right)\left(a+bx-cx^2\right)^{3/2}}$$

$$- \frac{2(b^4ci + 24a^2c^3i + 2b^2c^2(2cg+3ai) + b^5j + b^3c(ch+10aj) + 4bc^2(2c^2f - ach + 8a^2j) - c(16c}{3c^3\left(b^2+4ac\right)^2\sqrt{a+bx-}}$$

$$- \frac{j\tan^{-1}\left(\frac{b-2cx}{2\sqrt{c}\sqrt{a+bx-cx^2}}\right)}{c^{5/2}}$$

Mathematica [A] (verified)

Time = 1.97 (sec) , antiderivative size = 316, normalized size of antiderivative = 0.90

$$\int \frac{f+gx+hx^2+ix^3+jx^4}{(a+bx-cx^2)^{5/2}}\,dx =$$

$$-\frac{2(3b^5jx^2+b^4(6ajx-4cjx^3)+b^3(3a^2j+18acjx^2+c^2(f+3gx-x^2(3h+ix)))+8c^2(2c^3fx^3+a^3(2i+3}{}$$

$$+\frac{2j\arctan\left(\frac{\sqrt{c}x}{-\sqrt{a}+\sqrt{a+x(b-cx)}}\right)}{c^{5/2}}$$

```
[In] Integrate[(f + g*x + h*x^2 + i*x^3 + j*x^4)/(a + b*x - c*x^2)^(5/2),x]
```

```
[Out] (-2*(3*b^5*j*x^2 + b^4*(6*a*j*x - 4*c*j*x^3) + b^3*(3*a^2*j + 18*a*c*j*x^2
+ c^2*(f + 3*g*x - x^2*(3*h + i*x))) + 8*c^2*(2*c^3*f*x^3 + a^3*(2*i + 3*j*
x) - a*c^2*x*(3*f + h*x^2) - a^2*c*(g + x^2*(3*i + 4*j*x))) + 4*b*c*(5*a^3*
j + 2*c^3*x^2*(-3*f + g*x) - 2*a^2*c*(h - 3*i*x) + 3*a*c^2*(f - x*(g - h*x
+ i*x^2))) + 2*b^2*c*(21*a^2*j*x + c^2*x*(3*f + x*(-6*g + h*x)) + a*c*(g +
x*(-6*h + 3*i*x - 14*j*x^2))))))/(3*c^2*(b^2 + 4*a*c)^2*(a + x*(b - c*x))^(3
/2)) + (2*j*ArcTan[(Sqrt[c]*x)/(-Sqrt[a] + Sqrt[a + x*(b - c*x)])])/c^(5/2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1146 vs. 2(337) = 674.

Time = 2.46 (sec) , antiderivative size = 1147, normalized size of antiderivative = 3.25

method	result	size
default	Expression too large to display	1147

```
[In] int((j*x^4+i*x^3+h*x^2+g*x+f)/(-c*x^2+b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] f*(2/3*(-2*c*x+b)/(-4*a*c-b^2)/(-c*x^2+b*x+a)^(3/2)-16/3*c/(-4*a*c-b^2)^2*(
-2*c*x+b)/(-c*x^2+b*x+a)^(1/2))+j*(1/3*x^3/c/(-c*x^2+b*x+a)^(3/2)+1/2*b/c*(
x^2/c/(-c*x^2+b*x+a)^(3/2)-1/2*b/c*(1/2*x/c/(-c*x^2+b*x+a)^(3/2)+1/4*b/c*(1
/3/c/(-c*x^2+b*x+a)^(3/2)+1/2*b/c*(2/3*(-2*c*x+b)/(-4*a*c-b^2)/(-c*x^2+b*x+
a)^(3/2)-16/3*c/(-4*a*c-b^2)^2*(-2*c*x+b)/(-c*x^2+b*x+a)^(1/2)))-1/2*a/c*(2
/3*(-2*c*x+b)/(-4*a*c-b^2)/(-c*x^2+b*x+a)^(3/2)-16/3*c/(-4*a*c-b^2)^2*(-2*c
*x+b)/(-c*x^2+b*x+a)^(1/2)))-2*a/c*(1/3/c/(-c*x^2+b*x+a)^(3/2)+1/2*b/c*(2/3
*(-2*c*x+b)/(-4*a*c-b^2)/(-c*x^2+b*x+a)^(3/2)-16/3*c/(-4*a*c-b^2)^2*(-2*c*x
+b)/(-c*x^2+b*x+a)^(1/2))))-1/c*(x/c/(-c*x^2+b*x+a)^(1/2)+1/2*b/c*(1/c/(-c*
x^2+b*x+a)^(1/2)+b/c*(-2*c*x+b)/(-4*a*c-b^2)/(-c*x^2+b*x+a)^(1/2))-1/c^(3/2
)*arctan(c^(1/2)*(x-1/2*b/c)/(-c*x^2+b*x+a)^(1/2))))+i*(x^2/c/(-c*x^2+b*x+a
)^(3/2)-1/2*b/c*(1/2*x/c/(-c*x^2+b*x+a)^(3/2)+1/4*b/c*(1/3/c/(-c*x^2+b*x+a)
^(3/2)+1/2*b/c*(2/3*(-2*c*x+b)/(-4*a*c-b^2)/(-c*x^2+b*x+a)^(3/2)-16/3*c/(-4
```

```
*a*c-b^2)^2*(-2*c*x+b)/(-c*x^2+b*x+a)^(1/2)))-1/2*a/c*(2/3*(-2*c*x+b)/(-4*a
*c-b^2)/(-c*x^2+b*x+a)^(3/2)-16/3*c/(-4*a*c-b^2)^2*(-2*c*x+b)/(-c*x^2+b*x+a
)^(1/2)))-2*a/c*(1/3/c/(-c*x^2+b*x+a)^(3/2)+1/2*b/c*(2/3*(-2*c*x+b)/(-4*a*c
-b^2)/(-c*x^2+b*x+a)^(3/2)-16/3*c/(-4*a*c-b^2)^2*(-2*c*x+b)/(-c*x^2+b*x+a)^
(1/2))))+h*(1/2*x/c/(-c*x^2+b*x+a)^(3/2)+1/4*b/c*(1/3/c/(-c*x^2+b*x+a)^(3/2
)+1/2*b/c*(2/3*(-2*c*x+b)/(-4*a*c-b^2)/(-c*x^2+b*x+a)^(3/2)-16/3*c/(-4*a*c-
b^2)^2*(-2*c*x+b)/(-c*x^2+b*x+a)^(1/2)))-1/2*a/c*(2/3*(-2*c*x+b)/(-4*a*c-b^
2)/(-c*x^2+b*x+a)^(3/2)-16/3*c/(-4*a*c-b^2)^2*(-2*c*x+b)/(-c*x^2+b*x+a)^(1/
2)))+g*(1/3/c/(-c*x^2+b*x+a)^(3/2)+1/2*b/c*(2/3*(-2*c*x+b)/(-4*a*c-b^2)/(-c
*x^2+b*x+a)^(3/2)-16/3*c/(-4*a*c-b^2)^2*(-2*c*x+b)/(-c*x^2+b*x+a)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 689 vs. $2(339) = 678$.

Time = 23.13 (sec) , antiderivative size = 1385, normalized size of antiderivative = 3.92

$$\int \frac{f + gx + hx^2 + ix^3 + jx^4}{(a + bx - cx^2)^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((j*x^4+i*x^3+h*x^2+g*x+f)/(-c*x^2+b*x+a)^(5/2),x, algorithm="fric
as")
```

```
[Out] [-1/6*(3*((b^4*c^2 + 8*a*b^2*c^3 + 16*a^2*c^4)*j*x^4 - 2*(b^5*c + 8*a*b^3*c
^2 + 16*a^2*b*c^3)*j*x^3 + (b^6 + 6*a*b^4*c - 32*a^3*c^3)*j*x^2 + 2*(a*b^5
+ 8*a^2*b^3*c + 16*a^3*b*c^2)*j*x + (a^2*b^4 + 8*a^3*b^2*c + 16*a^4*c^2)*j)
*sqrt(-c)*log(8*c^2*x^2 - 8*b*c*x + b^2 - 4*sqrt(-c*x^2 + b*x + a)*(2*c*x -
 b)*sqrt(-c) - 4*a*c) - 4*(8*a^2*b*c^3*h - 16*a^3*c^3*i - (16*c^6*f + 8*b*c
^5*g + 2*(b^2*c^4 - 4*a*c^5)*h - (b^3*c^3 + 12*a*b*c^4)*i - 4*(b^4*c^2 + 7*
a*b^2*c^3 + 8*a^2*c^4)*j)*x^3 + 3*(8*b*c^5*f + 4*b^2*c^4*g + (b^3*c^3 - 4*a
*b*c^4)*h - 2*(a*b^2*c^3 - 4*a^2*c^4)*i - (b^5*c + 6*a*b^3*c^2)*j)*x^2 - (b
^3*c^3 + 12*a*b*c^4)*f - 2*(a*b^2*c^3 - 4*a^2*c^4)*g - (3*a^2*b^3*c + 20*a^
3*b*c^2)*j + 3*(4*a*b^2*c^3*h - 8*a^2*b*c^3*i - 2*(b^2*c^4 - 4*a*c^5)*f - (
b^3*c^3 - 4*a*b*c^4)*g - 2*(a*b^4*c + 7*a^2*b^2*c^2 + 4*a^3*c^3)*j)*x)*sqrt
(-c*x^2 + b*x + a))/(a^2*b^4*c^3 + 8*a^3*b^2*c^4 + 16*a^4*c^5 + (b^4*c^5 +
8*a*b^2*c^6 + 16*a^2*c^7)*x^4 - 2*(b^5*c^4 + 8*a*b^3*c^5 + 16*a^2*b*c^6)*x^
3 + (b^6*c^3 + 6*a*b^4*c^4 - 32*a^3*c^6)*x^2 + 2*(a*b^5*c^3 + 8*a^2*b^3*c^4
 + 16*a^3*b*c^5)*x), -1/3*(3*((b^4*c^2 + 8*a*b^2*c^3 + 16*a^2*c^4)*j*x^4 -
2*(b^5*c + 8*a*b^3*c^2 + 16*a^2*b*c^3)*j*x^3 + (b^6 + 6*a*b^4*c - 32*a^3*c^
3)*j*x^2 + 2*(a*b^5 + 8*a^2*b^3*c + 16*a^3*b*c^2)*j*x + (a^2*b^4 + 8*a^3*b^
2*c + 16*a^4*c^2)*j)*sqrt(c)*arctan(1/2*sqrt(-c*x^2 + b*x + a)*(2*c*x - b)*
sqrt(c)/(c^2*x^2 - b*c*x - a*c)) - 2*(8*a^2*b*c^3*h - 16*a^3*c^3*i - (16*c^
6*f + 8*b*c^5*g + 2*(b^2*c^4 - 4*a*c^5)*h - (b^3*c^3 + 12*a*b*c^4)*i - 4*(b
^4*c^2 + 7*a*b^2*c^3 + 8*a^2*c^4)*j)*x^3 + 3*(8*b*c^5*f + 4*b^2*c^4*g + (b^
3*c^3 - 4*a*b*c^4)*h - 2*(a*b^2*c^3 - 4*a^2*c^4)*i - (b^5*c + 6*a*b^3*c^2)*
```

```
j)*x^2 - (b^3*c^3 + 12*a*b*c^4)*f - 2*(a*b^2*c^3 - 4*a^2*c^4)*g - (3*a^2*b^
3*c + 20*a^3*b*c^2)*j + 3*(4*a*b^2*c^3*h - 8*a^2*b*c^3*i - 2*(b^2*c^4 - 4*a
*c^5)*f - (b^3*c^3 - 4*a*b*c^4)*g - 2*(a*b^4*c + 7*a^2*b^2*c^2 + 4*a^3*c^3)
*j)*x)*sqrt(-c*x^2 + b*x + a))/(a^2*b^4*c^3 + 8*a^3*b^2*c^4 + 16*a^4*c^5 +
(b^4*c^5 + 8*a*b^2*c^6 + 16*a^2*c^7)*x^4 - 2*(b^5*c^4 + 8*a*b^3*c^5 + 16*a^
2*b*c^6)*x^3 + (b^6*c^3 + 6*a*b^4*c^4 - 32*a^3*c^6)*x^2 + 2*(a*b^5*c^3 + 8*
a^2*b^3*c^4 + 16*a^3*b*c^5)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{f+gx+hx^2+ix^3+jx^4}{(a+bx-cx^2)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((j*x**4+i*x**3+h*x**2+g*x+f)/(-c*x**2+b*x+a)**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{f+gx+hx^2+ix^3+jx^4}{(a+bx-cx^2)^{5/2}}\,dx = \int \frac{jx^4+ix^3+hx^2+gx+f}{(-cx^2+bx+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((j*x^4+i*x^3+h*x^2+g*x+f)/(-c*x^2+b*x+a)^(5/2),x, algorithm="maxi
ma")
```

```
[Out] -1/3*i*(32*a*b*x/(sqrt(-c*x^2 + b*x + a)*(b^2 + 4*a*c)^2) - 16*a*b^2/(sqrt(
-c*x^2 + b*x + a)*(b^2 + 4*a*c)^2*c) + b^3*x/((-c*x^2 + b*x + a)^(3/2)*(b^2
 + 4*a*c)*c^2) + 2*(b^2 - 4*a*c)*b*x/(sqrt(-c*x^2 + b*x + a)*(b^2 + 4*a*c)^
2*c) + 6*a*b*x/((-c*x^2 + b*x + a)^(3/2)*(b^2 + 4*a*c)*c) - 3*x^2/((-c*x^2
+ b*x + a)^(3/2)*c) - (b^2 - 4*a*c)*b^2/(sqrt(-c*x^2 + b*x + a)*(b^2 + 4*a*
c)^2*c^2) - a*b^2/((-c*x^2 + b*x + a)^(3/2)*(b^2 + 4*a*c)*c^2) + 2*a/((-c*x
^2 + b*x + a)^(3/2)*c^2)) + 1/3*g*(16*b*c*x/(sqrt(-c*x^2 + b*x + a)*(b^2 +
4*a*c)^2) - 8*b^2/(sqrt(-c*x^2 + b*x + a)*(b^2 + 4*a*c)^2) + 2*b*x/((-c*x^2
 + b*x + a)^(3/2)*(b^2 + 4*a*c)) - b^2/((-c*x^2 + b*x + a)^(3/2)*(b^2 + 4*a
*c)*c) + 1/((-c*x^2 + b*x + a)^(3/2)*c)) + 2/3*f*(16*c^2*x/(sqrt(-c*x^2 + b
*x + a)*(b^2 + 4*a*c)^2) - 8*b*c/(sqrt(-c*x^2 + b*x + a)*(b^2 + 4*a*c)^2) +
 2*c*x/((-c*x^2 + b*x + a)^(3/2)*(b^2 + 4*a*c)) - b/((-c*x^2 + b*x + a)^(3/
2)*(b^2 + 4*a*c))) + 2/3*h*(2*(b^2 - 4*a*c)*x/(sqrt(-c*x^2 + b*x + a)*(b^2
+ 4*a*c)^2) + 2*a*x/((-c*x^2 + b*x + a)^(3/2)*(b^2 + 4*a*c)) + b^2*x/((-c*x
^2 + b*x + a)^(3/2)*(b^2 + 4*a*c)*c) - (b^2 - 4*a*c)*b/(sqrt(-c*x^2 + b*x +
 a)*(b^2 + 4*a*c)^2*c) + a*b/((-c*x^2 + b*x + a)^(3/2)*(b^2 + 4*a*c)*c)) +
j*integrate(x^4/((c^2*x^4 - 2*b*c*x^3 + 2*a*b*x + (b^2 - 2*a*c)*x^2 + a^2)*
sqrt(-c*x^2 + b*x + a)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 488, normalized size of antiderivative = 1.38

$$\int \frac{f+gx+hx^2+ix^3+jx^4}{(a+bx-cx^2)^{5/2}}\,dx =$$

$$-\frac{2\sqrt{-cx^2+bx+a}\Bigg(\Bigg(\Bigg(\frac{(16\,c^5f+8\,bc^4g+2\,b^2c^3h-8\,ac^4h-b^3c^2i-12\,abc^3i-4\,b^4cj-28\,ab^2c^2j-32\,a^2c^3j)x}{b^4c^2+8\,ab^2c^3+16\,a^2c^4}-\frac{3\,(8\,bc^4f+4\,b^2c^3g+b^3c^2h-4}{b^4c^2+}}{}$$

$$-\frac{j\log\left(\left|2\left(\sqrt{-c}x-\sqrt{-cx^2+bx+a}\right)\sqrt{-c}+b\right|\right)}{\sqrt{-c}c^2}$$

```
[In] integrate((j*x^4+i*x^3+h*x^2+g*x+f)/(-c*x^2+b*x+a)^(5/2),x, algorithm="giac
")
```

```
[Out] -2/3*sqrt(-c*x^2 + b*x + a)*((((16*c^5*f + 8*b*c^4*g + 2*b^2*c^3*h - 8*a*c^
4*h - b^3*c^2*i - 12*a*b*c^3*i - 4*b^4*c*j - 28*a*b^2*c^2*j - 32*a^2*c^3*j)
*x/(b^4*c^2 + 8*a*b^2*c^3 + 16*a^2*c^4) - 3*(8*b*c^4*f + 4*b^2*c^3*g + b^3*
c^2*h - 4*a*b*c^3*h - 2*a*b^2*c^2*i + 8*a^2*c^3*i - b^5*j - 6*a*b^3*c*j)/(b
^4*c^2 + 8*a*b^2*c^3 + 16*a^2*c^4))*x + 3*(2*b^2*c^3*f - 8*a*c^4*f + b^3*c^
2*g - 4*a*b*c^3*g - 4*a*b^2*c^2*h + 8*a^2*b*c^2*i + 2*a*b^4*j + 14*a^2*b^2*
c*j + 8*a^3*c^2*j)/(b^4*c^2 + 8*a*b^2*c^3 + 16*a^2*c^4))*x + (b^3*c^2*f + 1
2*a*b*c^3*f + 2*a*b^2*c^2*g - 8*a^2*c^3*g - 8*a^2*b*c^2*h + 16*a^3*c^2*i +
3*a^2*b^3*j + 20*a^3*b*c*j)/(b^4*c^2 + 8*a*b^2*c^3 + 16*a^2*c^4))/(c*x^2 -
b*x - a)^2 - j*log(abs(2*(sqrt(-c)*x - sqrt(-c*x^2 + b*x + a))*sqrt(-c) + b
))/(sqrt(-c)*c^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{f+gx+hx^2+ix^3+jx^4}{(a+bx-cx^2)^{5/2}}\,dx = \int \frac{j\,x^4+i\,x^3+h\,x^2+g\,x+f}{{\left(-c\,x^2+b\,x+a\right)}^{5/2}} \,d x$$

```
[In] int((f + g*x + h*x^2 + i*x^3 + j*x^4)/(a + b*x - c*x^2)^(5/2),x)
```

```
[Out] int((f + g*x + h*x^2 + i*x^3 + j*x^4)/(a + b*x - c*x^2)^(5/2), x)
```

3.367 $\int (d+ex)^m \left(3+2x+5x^2\right)^3 \left(2+x+3x^2-5x^3+4x^4\right)$

Optimal result

Integrand size = 38, antiderivative size = 588

$$\begin{aligned}
&\int (d+ex)^m \left(3+2x+5x^2\right)^3 \left(2+x+3x^2-5x^3+4x^4\right)\,dx \\
&= \frac{\left(5d^2-2de+3e^2\right)^3 \left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)(d+ex)^{1+m}}{e^{11}(1+m)} \\
&\quad - \frac{\left(5d^2-2de+3e^2\right)^2 \left(200d^5+169d^4e+108d^3e^2-20d^2e^3+86de^4-15e^5\right)(d+ex)^{2+m}}{e^{11}(2+m)} \\
&\quad + \frac{3\left(5d^2-2de+3e^2\right)\left(1500d^6+660d^5e+792d^4e^2+58d^3e^3+547d^2e^4-156de^5+53e^6\right)(d+ex)^{3+m}}{e^{11}(3+m)} \\
&\quad - \frac{2\left(30000d^7+1050d^6e+21420d^5e^2+1715d^4e^3+9990d^3e^4-2550d^2e^5+2218de^6-287e^7\right)(d+ex)^{4+m}}{e^{11}(4+m)} \\
&\quad + \frac{\left(105000d^6+3150d^5e+53550d^4e^2+3430d^3e^3+14985d^2e^4-2550de^5+1109e^6\right)(d+ex)^{5+m}}{e^{11}(5+m)} \\
&\quad - \frac{6\left(21000d^5+525d^4e+7140d^3e^2+343d^2e^3+999de^4-85e^5\right)(d+ex)^{6+m}}{e^{11}(6+m)} \\
&\quad + \frac{\left(105000d^4+2100d^3e+21420d^2e^2+686de^3+999e^4\right)(d+ex)^{7+m}}{e^{11}(7+m)} \\
&\quad - \frac{2\left(30000d^3+450d^2e+3060de^2+49e^3\right)(d+ex)^{8+m}}{e^{11}(8+m)} \\
&\quad + \frac{45\left(500d^2+5de+17e^2\right)(d+ex)^{9+m}}{e^{11}(9+m)} - \frac{25(200d+e)(d+ex)^{10+m}}{e^{11}(10+m)} + \frac{500(d+ex)^{11+m}}{e^{11}(11+m)}
\end{aligned}$$

```
[Out] (5*d^2-2*d*e+3*e^2)^3*(4*d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)*(e*x+d)^(1+m)/e
^11/(1+m)-(5*d^2-2*d*e+3*e^2)^2*(200*d^5+169*d^4*e+108*d^3*e^2-20*d^2*e^3+8
6*d*e^4-15*e^5)*(e*x+d)^(2+m)/e^11/(2+m)+3*(5*d^2-2*d*e+3*e^2)*(1500*d^6+66
0*d^5*e+792*d^4*e^2+58*d^3*e^3+547*d^2*e^4-156*d*e^5+53*e^6)*(e*x+d)^(3+m)/
```

```
e^11/(3+m)-2*(30000*d^7+1050*d^6*e+21420*d^5*e^2+1715*d^4*e^3+9990*d^3*e^4-
2550*d^2*e^5+2218*d*e^6-287*e^7)*(e*x+d)^(4+m)/e^11/(4+m)+(105000*d^6+3150*
d^5*e+53550*d^4*e^2+3430*d^3*e^3+14985*d^2*e^4-2550*d*e^5+1109*e^6)*(e*x+d)
^(5+m)/e^11/(5+m)-6*(21000*d^5+525*d^4*e+7140*d^3*e^2+343*d^2*e^3+999*d*e^4
-85*e^5)*(e*x+d)^(6+m)/e^11/(6+m)+(105000*d^4+2100*d^3*e+21420*d^2*e^2+686*
d*e^3+999*e^4)*(e*x+d)^(7+m)/e^11/(7+m)-2*(30000*d^3+450*d^2*e+3060*d*e^2+4
9*e^3)*(e*x+d)^(8+m)/e^11/(8+m)+45*(500*d^2+5*d*e+17*e^2)*(e*x+d)^(9+m)/e^1
1/(9+m)-25*(200*d+e)*(e*x+d)^(10+m)/e^11/(10+m)+500*(e*x+d)^(11+m)/e^11/(11
+m)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 588, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$, Rules used = {1642}

$$
\begin{aligned}
&\int (d+ex)^m \left(3+2x+5x^2\right)^3 \left(2+x+3x^2-5x^3+4x^4\right)\,dx \\
&= \frac{45(500d^2+5de+17e^2)\,(d+ex)^{m+9}}{e^{11}(m+9)} \\
&\quad - \frac{2(30000d^3+450d^2e+3060de^2+49e^3)\,(d+ex)^{m+8}}{e^{11}(m+8)} \\
&\quad + \frac{(5d^2-2de+3e^2)^3\,(4d^4+5d^3e+3d^2e^2-de^3+2e^4)\,(d+ex)^{m+1}}{e^{11}(m+1)} \\
&\quad + \frac{(105000d^4+2100d^3e+21420d^2e^2+686de^3+999e^4)\,(d+ex)^{m+7}}{e^{11}(m+7)} \\
&\quad - \frac{(5d^2-2de+3e^2)^2\,(200d^5+169d^4e+108d^3e^2-20d^2e^3+86de^4-15e^5)\,(d+ex)^{m+2}}{e^{11}(m+2)} \\
&\quad - \frac{6(21000d^5+525d^4e+7140d^3e^2+343d^2e^3+999de^4-85e^5)\,(d+ex)^{m+6}}{e^{11}(m+6)} \\
&\quad + \frac{3(5d^2-2de+3e^2)\,(1500d^6+660d^5e+792d^4e^2+58d^3e^3+547d^2e^4-156de^5+53e^6)\,(d+ex)^{m+3}}{e^{11}(m+3)} \\
&\quad + \frac{(105000d^6+3150d^5e+53550d^4e^2+3430d^3e^3+14985d^2e^4-2550de^5+1109e^6)\,(d+ex)^{m+5}}{e^{11}(m+5)} \\
&\quad - \frac{2(30000d^7+1050d^6e+21420d^5e^2+1715d^4e^3+9990d^3e^4-2550d^2e^5+2218de^6-287e^7)\,(d+ex)^{m+4}}{e^{11}(m+4)} \\
&\quad - \frac{25(200d+e)(d+ex)^{m+10}}{e^{11}(m+10)} + \frac{500(d+ex)^{m+11}}{e^{11}(m+11)}
\end{aligned}
$$

```
[In] Int[(d + e*x)^m*(3 + 2*x + 5*x^2)^3*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

```
[Out] ((5*d^2 - 2*d*e + 3*e^2)^3*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*(d
 + e*x)^(1 + m))/(e^11*(1 + m)) - ((5*d^2 - 2*d*e + 3*e^2)^2*(200*d^5 + 169
```

```
*d^4*e + 108*d^3*e^2 - 20*d^2*e^3 + 86*d*e^4 - 15*e^5)*(d + e*x)^(2 + m))/(
e^11*(2 + m)) + (3*(5*d^2 - 2*d*e + 3*e^2)*(1500*d^6 + 660*d^5*e + 792*d^4*
e^2 + 58*d^3*e^3 + 547*d^2*e^4 - 156*d*e^5 + 53*e^6)*(d + e*x)^(3 + m))/(e^
11*(3 + m)) - (2*(30000*d^7 + 1050*d^6*e + 21420*d^5*e^2 + 1715*d^4*e^3 + 9
990*d^3*e^4 - 2550*d^2*e^5 + 2218*d*e^6 - 287*e^7)*(d + e*x)^(4 + m))/(e^11
*(4 + m)) + ((105000*d^6 + 3150*d^5*e + 53550*d^4*e^2 + 3430*d^3*e^3 + 1498
5*d^2*e^4 - 2550*d*e^5 + 1109*e^6)*(d + e*x)^(5 + m))/(e^11*(5 + m)) - (6*(
21000*d^5 + 525*d^4*e + 7140*d^3*e^2 + 343*d^2*e^3 + 999*d*e^4 - 85*e^5)*(d
 + e*x)^(6 + m))/(e^11*(6 + m)) + ((105000*d^4 + 2100*d^3*e + 21420*d^2*e^2
 + 686*d*e^3 + 999*e^4)*(d + e*x)^(7 + m))/(e^11*(7 + m)) - (2*(30000*d^3 +
 450*d^2*e + 3060*d*e^2 + 49*e^3)*(d + e*x)^(8 + m))/(e^11*(8 + m)) + (45*(
500*d^2 + 5*d*e + 17*e^2)*(d + e*x)^(9 + m))/(e^11*(9 + m)) - (25*(200*d +
e)*(d + e*x)^(10 + m))/(e^11*(10 + m)) + (500*(d + e*x)^(11 + m))/(e^11*(11
 + m))
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} = \int \Bigg(& \frac{(5d^2-2de+3e^2)^3\,(4d^4+5d^3e+3d^2e^2-de^3+2e^4)\,(d+ex)^m}{e^{10}} \\
& + \frac{(5d^2-2de+3e^2)^2\,(-200d^5-169d^4e-108d^3e^2+20d^2e^3-86de^4+15e^5)\,(d+ex)^{1+m}}{e^{10}} \\
& + \frac{3(7500d^8+300d^7e+7140d^6e^2+686d^5e^3+4995d^4e^4-1700d^3e^5+2218d^2e^6-574de^7+159e^8)\,(d+ex}{e^{10}} \\
& + \frac{2(-30000d^7-1050d^6e-21420d^5e^2-1715d^4e^3-9990d^3e^4+2550d^2e^5-2218de^6+287e^7)\,(d+ex)^{3+m}}{e^{10}} \\
& + \frac{(105000d^6+3150d^5e+53550d^4e^2+3430d^3e^3+14985d^2e^4-2550de^5+1109e^6)\,(d+ex)^{4+m}}{e^{10}} \\
& + \frac{6(-21000d^5-525d^4e-7140d^3e^2-343d^2e^3-999de^4+85e^5)\,(d+ex)^{5+m}}{e^{10}} \\
& + \frac{(105000d^4+2100d^3e+21420d^2e^2+686de^3+999e^4)\,(d+ex)^{6+m}}{e^{10}} \\
& - \frac{2(30000d^3+450d^2e+3060de^2+49e^3)\,(d+ex)^{7+m}}{e^{10}} \\
& + \frac{45(500d^2+5de+17e^2)\,(d+ex)^{8+m}}{e^{10}} - \frac{25(200d+e)(d+ex)^{9+m}}{e^{10}} + \frac{500(d+ex)^{10+m}}{e^{10}} \Bigg)\,dx
\end{aligned}$$

$$
\begin{aligned}
&= \frac{\left(5d^2-2de+3e^2\right)^3\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)(d+ex)^{1+m}}{e^{11}(1+m)} \\
&\quad - \frac{\left(5d^2-2de+3e^2\right)^2\left(200d^5+169d^4e+108d^3e^2-20d^2e^3+86de^4-15e^5\right)(d+ex)^{2+m}}{e^{11}(2+m)} \\
&\quad + \frac{3(5d^2-2de+3e^2)\left(1500d^6+660d^5e+792d^4e^2+58d^3e^3+547d^2e^4-156de^5+53e^6\right)(d+ex)}{e^{11}(3+m)} \\
&\quad - \frac{2(30000d^7+1050d^6e+21420d^5e^2+1715d^4e^3+9990d^3e^4-2550d^2e^5+2218de^6-287e^7)(d+}{e^{11}(4+m)} \\
&\quad + \frac{\left(105000d^6+3150d^5e+53550d^4e^2+3430d^3e^3+14985d^2e^4-2550de^5+1109e^6\right)(d+ex)^{5+m}}{e^{11}(5+m)} \\
&\quad - \frac{6(21000d^5+525d^4e+7140d^3e^2+343d^2e^3+999de^4-85e^5)(d+ex)^{6+m}}{e^{11}(6+m)} \\
&\quad + \frac{\left(105000d^4+2100d^3e+21420d^2e^2+686de^3+999e^4\right)(d+ex)^{7+m}}{e^{11}(7+m)} \\
&\quad - \frac{2(30000d^3+450d^2e+3060de^2+49e^3)(d+ex)^{8+m}}{e^{11}(8+m)} \\
&\quad + \frac{45(500d^2+5de+17e^2)(d+ex)^{9+m}}{e^{11}(9+m)} \\
&\quad - \frac{25(200d+e)(d+ex)^{10+m}}{e^{11}(10+m)} + \frac{500(d+ex)^{11+m}}{e^{11}(11+m)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 537, normalized size of antiderivative = 0.91

$$
\int (d+ex)^m\left(3+2x+5x^2\right)^3\left(2+x+3x^2-5x^3+4x^4\right)\,dx
$$

$$
= \frac{(d+ex)^{1+m}\left(\frac{\left(5d^2-2de+3e^2\right)^3\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)}{1+m} - \frac{\left(5d^2-2de+3e^2\right)^2\left(200d^5+169d^4e+108d^3e^2-20d^2e^3+86de^4-15e^5\right)(d+e}{2+m}\right.}{}
$$

```
[In] Integrate[(d + e*x)^m*(3 + 2*x + 5*x^2)^3*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x
]

[Out] ((d + e*x)^(1 + m)*(((5*d^2 - 2*d*e + 3*e^2)^3*(4*d^4 + 5*d^3*e + 3*d^2*e^2
 - d*e^3 + 2*e^4))/(1 + m) - ((5*d^2 - 2*d*e + 3*e^2)^2*(200*d^5 + 169*d^4*
e + 108*d^3*e^2 - 20*d^2*e^3 + 86*d*e^4 - 15*e^5)*(d + e*x))/(2 + m) + (3*(
5*d^2 - 2*d*e + 3*e^2)*(1500*d^6 + 660*d^5*e + 792*d^4*e^2 + 58*d^3*e^3 + 5
47*d^2*e^4 - 156*d*e^5 + 53*e^6)*(d + e*x)^2)/(3 + m) - (2*(30000*d^7 + 105
0*d^6*e + 21420*d^5*e^2 + 1715*d^4*e^3 + 9990*d^3*e^4 - 2550*d^2*e^5 + 2218
*d*e^6 - 287*e^7)*(d + e*x)^3)/(4 + m) + ((105000*d^6 + 3150*d^5*e + 53550*
d^4*e^2 + 3430*d^3*e^3 + 14985*d^2*e^4 - 2550*d*e^5 + 1109*e^6)*(d + e*x)^4
)/(5 + m) - (6*(21000*d^5 + 525*d^4*e + 7140*d^3*e^2 + 343*d^2*e^3 + 999*d*
```

```
e^4 - 85*e^5)*(d + e*x)^5)/(6 + m) + ((105000*d^4 + 2100*d^3*e + 21420*d^2*
e^2 + 686*d*e^3 + 999*e^4)*(d + e*x)^6)/(7 + m) - (2*(30000*d^3 + 450*d^2*e
 + 3060*d*e^2 + 49*e^3)*(d + e*x)^7)/(8 + m) + (45*(500*d^2 + 5*d*e + 17*e^
2)*(d + e*x)^8)/(9 + m) - (25*(200*d + e)*(d + e*x)^9)/(10 + m) + (500*(d +
 e*x)^10)/(11 + m)))/e^11
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 5923 vs. 2(588) = 1176.

Time = 0.45 (sec) , antiderivative size = 5924, normalized size of antiderivative = 10.07

method	result	size
gosper	Expression too large to display	5924
risch	Expression too large to display	6934
parallelrisch	Expression too large to display	11277

```
[In] int((e*x+d)^m*(5*x^2+2*x+3)^3*(4*x^4-5*x^3+3*x^2+x+2),x,method=_RETURNVERBO
SE)
```

```
[Out] result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4795 vs. 2(588) = 1176.

Time = 0.35 (sec) , antiderivative size = 4795, normalized size of antiderivative = 8.15

$$\int (d+ex)^m \left(3+2x+5x^2\right)^3 \left(2+x+3x^2-5x^3+4x^4\right)\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)^m*(5*x^2+2*x+3)^3*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="f
ricas")
```

```
[Out] (54*d*e^10*m^10 + 500*(e^11*m^10 + 55*e^11*m^9 + 1320*e^11*m^8 + 18150*e^11
*m^7 + 157773*e^11*m^6 + 902055*e^11*m^5 + 3416930*e^11*m^4 + 8409500*e^11*
m^3 + 12753576*e^11*m^2 + 10628640*e^11*m + 3628800*e^11)*x^11 + 1814400000
*d^11 + 99792000*d^10*e + 3392928000*d^9*e^2 + 488980800*d^8*e^3 + 56966976
00*d^7*e^4 - 3392928000*d^6*e^5 + 8853546240*d^5*e^6 - 5728060800*d^4*e^7 +
 6346771200*d^3*e^8 - 2694384000*d^2*e^9 + 2155507200*d*e^10 - 25*(3991680*
e^11 - (20*d*e^10 - e^11)*m^10 - 4*(225*d*e^10 - 14*e^11)*m^9 - 15*(1160*d*
e^10 - 91*e^11)*m^8 - 60*(3150*d*e^10 - 317*e^11)*m^7 - 21*(60260*d*e^10 -
7963*e^11)*m^6 - 84*(64125*d*e^10 - 11492*e^11)*m^5 - 5*(2894720*d*e^10 - 7
37251*e^11)*m^4 - 20*(1172700*d*e^10 - 456659*e^11)*m^3 - 36*(570320*d*e^10
 - 386841*e^11)*m^2 - 144*(50400*d*e^10 - 80939*e^11)*m)*x^10 - 135*(d^2*e^
9 - 26*d*e^10)*m^9 + 5*(678585600*e^11 - (5*d*e^10 - 153*e^11)*m^10 - (1000
```

```
*d^2*e^9 + 235*d*e^10 - 8721*e^11)*m^9 - 6*(6000*d^2*e^9 + 785*d*e^10 - 360
06*e^11)*m^8 - 6*(91000*d^2*e^9 + 8785*d*e^10 - 509031*e^11)*m^7 - 105*(432
00*d^2*e^9 + 3445*d*e^10 - 259029*e^11)*m^6 - 21*(1069000*d^2*e^9 + 74815*d
*e^10 - 7560189*e^11)*m^5 - 2*(33642000*d^2*e^9 + 2145620*d*e^10 - 30603656
7*e^11)*m^4 - 4*(29531000*d^2*e^9 + 1761185*d*e^10 - 382172121*e^11)*m^3 -
72*(1522000*d^2*e^9 + 86510*d*e^10 - 32587351*e^11)*m^2 - 1440*(28000*d^2*e
^9 + 1540*d*e^10 - 1370727*e^11)*m)*x^9 + 9*(106*d^3*e^8 - 945*d^2*e^9 + 11
160*d*e^10)*m^8 - (488980800*e^11 - (765*d*e^10 - 98*e^11)*m^10 - (225*d^2*
e^9 + 37485*d*e^10 - 5684*e^11)*m^9 - 3*(15000*d^3*e^8 + 2925*d^2*e^9 + 260
100*d*e^10 - 47726*e^11)*m^8 - 42*(30000*d^3*e^8 + 3375*d^2*e^9 + 214965*d*
e^10 - 48958*e^11)*m^7 - 63*(230000*d^3*e^8 + 19650*d^2*e^9 + 1012095*d*e^1
0 - 294882*e^11)*m^6 - 63*(1400000*d^3*e^8 + 101175*d^2*e^9 + 4503555*d*e^1
0 - 1743812*e^11)*m^5 - (304605000*d^3*e^8 + 19707975*d^2*e^9 + 790573950*d
*e^10 - 428393182*e^11)*m^4 - 4*(147735000*d^3*e^8 + 8860500*d^2*e^9 + 3297
12705*d*e^10 - 270109021*e^11)*m^3 - 36*(16335000*d^3*e^8 + 929925*d^2*e^9
+ 32795550*d*e^10 - 46438966*e^11)*m^2 - 5040*(45000*d^3*e^8 + 2475*d^2*e^9
 + 84150*d*e^10 - 280861*e^11)*m)*x^8 - 6*(574*d^4*e^7 - 9540*d^3*e^8 + 390
15*d^2*e^9 - 277290*d*e^10)*m^7 + (5696697600*e^11 - (98*d*e^10 - 999*e^11)
*m^10 - 3*(2040*d^2*e^9 + 1666*d*e^10 - 19647*e^11)*m^9 - 24*(75*d^3*e^8 +
10710*d^2*e^9 + 4508*d*e^10 - 62937*e^11)*m^8 - 6*(60000*d^4*e^7 + 9600*d^3
*e^8 + 740520*d^2*e^9 + 216482*d*e^10 - 3677319*e^11)*m^7 - 3*(2520000*d^4*
e^7 + 243600*d^3*e^8 + 13708800*d^2*e^9 + 3161774*d*e^10 - 67539393*e^11)*m
^6 - 21*(3000000*d^4*e^7 + 228000*d^3*e^8 + 10581480*d^2*e^9 + 2069662*d*e^
10 - 57933009*e^11)*m^5 - 2*(132300000*d^4*e^7 + 8738100*d^3*e^8 + 35715708
0*d^2*e^9 + 62076434*d*e^10 - 2405021571*e^11)*m^4 - 36*(16240000*d^4*e^7 +
 981400*d^3*e^8 + 36788680*d^2*e^9 + 5871278*d*e^10 - 341095341*e^11)*m^3 -
 72*(8820000*d^4*e^7 + 503100*d^3*e^8 + 17778600*d^2*e^9 + 2670010*d*e^10 -
 266622111*e^11)*m^2 - 12960*(20000*d^4*e^7 + 1100*d^3*e^8 + 37400*d^2*e^9
+ 5390*d*e^10 - 1264623*e^11)*m)*x^7 + 6*(4436*d^5*e^6 - 32144*d^4*e^7 + 24
7086*d^3*e^8 - 615195*d^2*e^9 + 2939517*d*e^10)*m^6 + (3392928000*e^11 + 3*
(333*d*e^10 + 170*e^11)*m^10 + (686*d^2*e^9 + 52947*d*e^10 + 30600*e^11)*m^
9 + 6*(7140*d^3*e^8 + 5145*d^2*e^9 + 198801*d*e^10 + 133025*e^11)*m^8 + 6*(
2100*d^4*e^7 + 257040*d^3*e^8 + 95354*d^2*e^9 + 2484513*d*e^10 + 1978800*e^
11)*m^7 + 3*(840000*d^5*e^6 + 109200*d^4*e^7 + 7282800*d^3*e^8 + 1886500*d^
2*e^9 + 37725237*d*e^10 + 37016310*e^11)*m^6 + 3*(12600000*d^5*e^6 + 105000
0*d^4*e^7 + 52264800*d^3*e^8 + 10813418*d^2*e^9 + 179179641*d*e^10 + 226287
000*e^11)*m^5 + 42*(5100000*d^5*e^6 + 348000*d^4*e^7 + 14635980*d^3*e^8 + 2
609495*d^2*e^9 + 37733562*d*e^10 + 64999925*e^11)*m^4 + 4*(141750000*d^5*e^
6 + 8659350*d^4*e^7 + 327983040*d^3*e^8 + 52869334*d^2*e^9 + 692643663*d*e^
10 + 1769460300*e^11)*m^3 + 120*(5754000*d^5*e^6 + 329070*d^4*e^7 + 1165962
0*d^3*e^8 + 1755817*d^2*e^9 + 21444534*d*e^10 + 93454763*e^11)*m^2 + 7200*(
42000*d^5*e^6 + 2310*d^4*e^7 + 78540*d^3*e^8 + 11319*d^2*e^9 + 131868*d*e^1
0 + 1344547*e^11)*m)*x^6 - 3*(20400*d^6*e^5 - 452472*d^5*e^6 + 1526840*d^4*
e^7 - 7212240*d^3*e^8 + 12236805*d^2*e^9 - 41597010*d*e^10)*m^5 + (88535462
40*e^11 + (510*d*e^10 + 1109*e^11)*m^10 - (5994*d^2*e^9 - 28050*d*e^10 - 67
```

```
649*e^11)*m^9 - 12*(343*d^3*e^8 + 23976*d^2*e^9 - 54825*d*e^10 - 149715*e^1
1)*m^8 - 6*(42840*d^4*e^7 + 27440*d^3*e^8 + 953046*d^2*e^9 - 1430550*d*e^10
 - 4541355*e^11)*m^7 - 3*(25200*d^5*e^6 + 2656080*d^4*e^7 + 869848*d^3*e^8
+ 20283696*d^2*e^9 - 22710810*d*e^10 - 86713819*e^11)*m^6 - 3*(5040000*d^6*
e^5 + 529200*d^5*e^6 + 30416400*d^4*e^7 + 6969760*d^3*e^8 + 124932942*d^2*e
^9 - 112732950*d*e^10 - 541448179*e^11)*m^5 - 2*(75600000*d^6*e^5 + 5481000
*d^5*e^6 + 242260200*d^4*e^7 + 45047562*d^3*e^8 + 675619704*d^2*e^9 - 51950
1300*d*e^10 - 3335910815*e^11)*m^4 - 4*(132300000*d^6*e^5 + 8221500*d^5*e^6
 + 316416240*d^4*e^7 + 51779280*d^3*e^8 + 688165146*d^2*e^9 - 470707050*d*e
^10 - 4412539105*e^11)*m^3 - 72*(10500000*d^6*e^5 + 602700*d^5*e^6 + 214342
80*d^4*e^7 + 3239978*d^3*e^8 + 39724236*d^2*e^9 - 25005980*d*e^10 - 3955614
47*e^11)*m^2 - 288*(1260000*d^6*e^5 + 69300*d^5*e^6 + 2356200*d^4*e^7 + 339
570*d^3*e^8 + 3956040*d^2*e^9 - 2356200*d*e^10 - 86687203*e^11)*m)*x^5 + 3*
(239760*d^7*e^4 - 918000*d^6*e^5 + 9537400*d^5*e^6 - 19929280*d^4*e^7 + 648
36702*d^3*e^8 - 79518915*d^2*e^9 + 198514620*d*e^10)*m^4 + (5728060800*e^11
 + (1109*d*e^10 + 574*e^11)*m^10 - (2550*d^2*e^9 - 63213*d*e^10 - 35588*e^1
1)*m^9 + 6*(4995*d^3*e^8 - 21675*d^2*e^9 + 257288*d*e^10 + 160433*e^11)*m^8
 + 6*(3430*d^4*e^7 + 219780*d^3*e^8 - 461550*d^2*e^9 + 3512203*d*e^10 + 248
3698*e^11)*m^7 + 15*(85680*d^5*e^6 + 49392*d^4*e^7 + 1554444*d^3*e^8 - 2122
620*d^2*e^9 + 11723239*d*e^10 + 9703470*e^11)*m^6 + 3*(126000*d^6*e^5 + 115
66800*d^5*e^6 + 3361400*d^4*e^7 + 70329600*d^3*e^8 - 71101650*d^2*e^9 + 306
983399*d*e^10 + 310583364*e^11)*m^5 + 2*(37800000*d^7*e^4 + 3213000*d^6*e^5
 + 158722200*d^5*e^6 + 32104800*d^4*e^7 + 515019465*d^3*e^8 - 418887225*d^2
*e^9 + 1494010421*d*e^10 + 1964946361*e^11)*m^4 + 4*(113400000*d^7*e^4 + 72
76500*d^6*e^5 + 288206100*d^5*e^6 + 48409305*d^4*e^7 + 659010330*d^3*e^8 -
460978800*d^2*e^9 + 1424518263*d*e^10 + 2670494533*e^11)*m^3 + 72*(11550000
*d^7*e^4 + 666750*d^6*e^5 + 23847600*d^5*e^6 + 3625510*d^4*e^7 + 44710245*d
^3*e^8 - 28312225*d^2*e^9 + 79001833*d*e^10 + 245697543*e^11)*m^2 + 720*(63
0000*d^7*e^4 + 34650*d^6*e^5 + 1178100*d^5*e^6 + 169785*d^4*e^7 + 1978020*d
^3*e^8 - 1178100*d^2*e^9 + 3074148*d*e^10 + 22036147*e^11)*m)*x^4 + 12*(411
60*d^8*e^3 + 2277720*d^7*e^4 - 4105500*d^6*e^5 + 26582730*d^5*e^6 - 3858686
3*d^4*e^7 + 91855890*d^3*e^8 - 84312180*d^2*e^9 + 157352130*d*e^10)*m^3 + (
6346771200*e^11 + (574*d*e^10 + 477*e^11)*m^10 - (4436*d^2*e^9 - 33866*d*e^
10 - 30051*e^11)*m^9 + 24*(425*d^3*e^8 - 9981*d^2*e^9 + 35875*d*e^10 + 3450
3*e^11)*m^8 - 6*(19980*d^4*e^7 - 81600*d^3*e^8 + 909380*d^2*e^9 - 2053198*d
*e^10 - 2183229*e^11)*m^7 - 3*(27440*d^5*e^6 + 1638360*d^4*e^7 - 3202800*d^
3*e^8 + 22641344*d^2*e^9 - 36198162*d*e^10 - 43730883*e^11)*m^6 - 3*(171360
0*d^6*e^5 + 905520*d^5*e^6 + 26173800*d^4*e^7 - 32844000*d^3*e^8 + 16654074
8*d^2*e^9 - 201988878*d*e^10 - 288179073*e^11)*m^5 - 2*(756000*d^7*e^4 + 61
689600*d^6*e^5 + 16093560*d^5*e^6 + 304195500*d^4*e^7 - 278811900*d^3*e^8 +
 1092467028*d^2*e^9 - 1055996410*d*e^10 - 1884673269*e^11)*m^4 - 4*(7560000
0*d^8*e^3 + 5292000*d^7*e^4 + 224910000*d^6*e^5 + 40069260*d^5*e^6 + 573745
680*d^4*e^7 - 419556600*d^3*e^8 + 1349320300*d^2*e^9 - 1086499918*d*e^10 -
2657980899*e^11)*m^3 - 24*(37800000*d^8*e^3 + 2205000*d^7*e^4 + 79682400*d^
6*e^5 + 12238240*d^5*e^6 + 152467380*d^4*e^7 - 97540900*d^3*e^8 + 275018692
```

```
*d^2*e^9 - 193842670*d*e^10 - 763013811*e^11)*m^2 - 4320*(140000*d^8*e^3 +
7700*d^7*e^4 + 261800*d^6*e^5 + 37730*d^5*e^6 + 439560*d^4*e^7 - 261800*d^3
*e^8 + 683144*d^2*e^9 - 441980*d*e^10 - 3946963*e^11)*m)*x^3 + 12*(2570400*
d^9*e^2 + 1234800*d^8*e^3 + 32307660*d^7*e^4 - 36490500*d^6*e^5 + 165294232
*d^5*e^6 - 177258088*d^4*e^7 + 320238402*d^3*e^8 - 224755965*d^2*e^9 + 3163
09212*d*e^10)*m^2 + 3*(898128000*e^11 + 3*(53*d*e^10 + 15*e^11)*m^10 - (574
*d^2*e^9 - 9699*d*e^10 - 2880*e^11)*m^9 + (4436*d^3*e^8 - 32718*d^2*e^9 + 2
56626*d*e^10 + 80865*e^11)*m^8 - 2*(5100*d^4*e^7 - 115336*d^3*e^8 + 397782*
d^2*e^9 - 1926603*d*e^10 - 654210*e^11)*m^7 + (119880*d^5*e^6 - 469200*d^4*
e^7 + 4994936*d^3*e^8 - 10728060*d^2*e^9 + 36024471*d*e^10 + 13467195*e^11)
*m^6 + (82320*d^6*e^5 + 4675320*d^5*e^6 - 8670000*d^4*e^7 + 57934160*d^3*e^
8 - 87138366*d^2*e^9 + 216130131*d*e^10 + 91755720*e^11)*m^5 + (5140800*d^7
*e^4 + 2551920*d^6*e^5 + 69170760*d^5*e^6 - 81192000*d^4*e^7 + 383753924*d^
3*e^8 - 431689902*d^2*e^9 + 824188584*d*e^10 + 416767635*e^11)*m^4 + 4*(378
000*d^8*e^3 + 28274400*d^7*e^4 + 6770820*d^6*e^5 + 117512370*d^5*e^6 - 9880
9950*d^4*e^7 + 354356552*d^3*e^8 - 312153254*d^2*e^9 + 473899341*d*e^10 + 3
09068145*e^11)*m^3 + 12*(25200000*d^9*e^2 + 1512000*d^8*e^3 + 56120400*d^7*
e^4 + 8842540*d^6*e^5 + 112906980*d^5*e^6 - 73978900*d^4*e^7 + 213535732*d^
3*e^8 - 154064470*d^2*e^9 + 192742980*d*e^10 + 188672355*e^11)*m^2 + 2160*(
140000*d^9*e^2 + 7700*d^8*e^3 + 261800*d^7*e^4 + 37730*d^6*e^5 + 439560*d^5
*e^6 - 261800*d^4*e^7 + 683144*d^3*e^8 - 441980*d^2*e^9 + 489720*d*e^10 + 1
047765*e^11)*m)*x^2 + 144*(63000*d^10*e + 4498200*d^9*e^2 + 1025570*d^8*e^3
 + 16893090*d^7*e^4 - 13427450*d^6*e^5 + 45284906*d^5*e^6 - 37254035*d^4*e^
7 + 52296690*d^3*e^8 - 28438425*d^2*e^9 + 30235140*d*e^10)*m + 3*(718502400
*e^11 + 9*(5*d*e^10 + 2*e^11)*m^10 - 3*(106*d^2*e^9 - 945*d*e^10 - 390*e^11
)*m^9 + 2*(574*d^3*e^8 - 9540*d^2*e^9 + 39015*d*e^10 + 16740*e^11)*m^8 - 2*
(4436*d^4*e^7 - 32144*d^3*e^8 + 247086*d^2*e^9 - 615195*d*e^10 - 277290*e^1
1)*m^7 + (20400*d^5*e^6 - 452472*d^4*e^7 + 1526840*d^3*e^8 - 7212240*d^2*e^
9 + 12236805*d*e^10 + 5879034*e^11)*m^6 - (239760*d^6*e^5 - 918000*d^5*e^6
+ 9537400*d^4*e^7 - 19929280*d^3*e^8 + 64836702*d^2*e^9 - 79518915*d*e^10 -
 41597010*e^11)*m^5 - 4*(41160*d^7*e^4 + 2277720*d^6*e^5 - 4105500*d^5*e^6
+ 26582730*d^4*e^7 - 38586863*d^3*e^8 + 91855890*d^2*e^9 - 84312180*d*e^10
- 49628655*e^11)*m^4 - 4*(2570400*d^8*e^3 + 1234800*d^7*e^4 + 32307660*d^6*
e^5 - 36490500*d^5*e^6 + 165294232*d^4*e^7 - 177258088*d^3*e^8 + 320238402*
d^2*e^9 - 224755965*d*e^10 - 157352130*e^11)*m^3 - 48*(63000*d^9*e^2 + 4498
200*d^8*e^3 + 1025570*d^7*e^4 + 16893090*d^6*e^5 - 13427450*d^5*e^6 + 45284
906*d^4*e^7 - 37254035*d^3*e^8 + 52296690*d^2*e^9 - 28438425*d*e^10 - 26359
101*e^11)*m^2 - 8640*(70000*d^10*e + 3850*d^9*e^2 + 130900*d^8*e^3 + 18865*
d^7*e^4 + 219780*d^6*e^5 - 130900*d^5*e^6 + 341572*d^4*e^7 - 220990*d^3*e^8
 + 244860*d^2*e^9 - 103950*d*e^10 - 167973*e^11)*m)*x)*(e*x + d)^m/(e^11*m^
11 + 66*e^11*m^10 + 1925*e^11*m^9 + 32670*e^11*m^8 + 357423*e^11*m^7 + 2637
558*e^11*m^6 + 13339535*e^11*m^5 + 45995730*e^11*m^4 + 105258076*e^11*m^3 +
 150917976*e^11*m^2 + 120543840*e^11*m + 39916800*e^11)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136733 vs. 2(564) = 1128.

Time = 38.08 (sec) , antiderivative size = 136733, normalized size of antiderivative = 232.54

$$\int (d+ex)^m \left(3+2x+5x^2\right)^3 \left(2+x+3x^2-5x^3+4x^4\right)\, dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)**m*(5*x**2+2*x+3)**3*(4*x**4-5*x**3+3*x**2+x+2),x)
```

```
[Out] Piecewise((d**m*(500*x**11/11 - 5*x**10/2 + 85*x**9 - 49*x**8/4 + 999*x**7/
7 + 85*x**6 + 1109*x**5/5 + 287*x**4/2 + 159*x**3 + 135*x**2/2 + 54*x), Eq(
e, 0)), (1260000*d**10*log(d/e + x)/(2520*d**10*e**11 + 25200*d**9*e**12*x
+ 113400*d**8*e**13*x**2 + 302400*d**7*e**14*x**3 + 529200*d**6*e**15*x**4
+ 635040*d**5*e**16*x**5 + 529200*d**4*e**17*x**6 + 302400*d**3*e**18*x**7
+ 113400*d**2*e**19*x**8 + 25200*d*e**20*x**9 + 2520*e**21*x**10) + 3690500
*d**10/(2520*d**10*e**11 + 25200*d**9*e**12*x + 113400*d**8*e**13*x**2 + 30
2400*d**7*e**14*x**3 + 529200*d**6*e**15*x**4 + 635040*d**5*e**16*x**5 + 52
9200*d**4*e**17*x**6 + 302400*d**3*e**18*x**7 + 113400*d**2*e**19*x**8 + 25
200*d*e**20*x**9 + 2520*e**21*x**10) + 12600000*d**9*e*x*log(d/e + x)/(2520
*d**10*e**11 + 25200*d**9*e**12*x + 113400*d**8*e**13*x**2 + 302400*d**7*e*
*14*x**3 + 529200*d**6*e**15*x**4 + 635040*d**5*e**16*x**5 + 529200*d**4*e*
*17*x**6 + 302400*d**3*e**18*x**7 + 113400*d**2*e**19*x**8 + 25200*d*e**20*
x**9 + 2520*e**21*x**10) + 35645000*d**9*e*x/(2520*d**10*e**11 + 25200*d**9
*e**12*x + 113400*d**8*e**13*x**2 + 302400*d**7*e**14*x**3 + 529200*d**6*e*
*15*x**4 + 635040*d**5*e**16*x**5 + 529200*d**4*e**17*x**6 + 302400*d**3*e*
*18*x**7 + 113400*d**2*e**19*x**8 + 25200*d*e**20*x**9 + 2520*e**21*x**10)
+ 6300*d**9*e/(2520*d**10*e**11 + 25200*d**9*e**12*x + 113400*d**8*e**13*x*
*2 + 302400*d**7*e**14*x**3 + 529200*d**6*e**15*x**4 + 635040*d**5*e**16*x*
*5 + 529200*d**4*e**17*x**6 + 302400*d**3*e**18*x**7 + 113400*d**2*e**19*x*
*8 + 25200*d*e**20*x**9 + 2520*e**21*x**10) + 56700000*d**8*e**2*x**2*log(d
/e + x)/(2520*d**10*e**11 + 25200*d**9*e**12*x + 113400*d**8*e**13*x**2 + 3
02400*d**7*e**14*x**3 + 529200*d**6*e**15*x**4 + 635040*d**5*e**16*x**5 + 5
29200*d**4*e**17*x**6 + 302400*d**3*e**18*x**7 + 113400*d**2*e**19*x**8 + 2
5200*d*e**20*x**9 + 2520*e**21*x**10) + 154102500*d**8*e**2*x**2/(2520*d**1
0*e**11 + 25200*d**9*e**12*x + 113400*d**8*e**13*x**2 + 302400*d**7*e**14*x
**3 + 529200*d**6*e**15*x**4 + 635040*d**5*e**16*x**5 + 529200*d**4*e**17*x
**6 + 302400*d**3*e**18*x**7 + 113400*d**2*e**19*x**8 + 25200*d*e**20*x**9
+ 2520*e**21*x**10) + 63000*d**8*e**2*x/(2520*d**10*e**11 + 25200*d**9*e**1
2*x + 113400*d**8*e**13*x**2 + 302400*d**7*e**14*x**3 + 529200*d**6*e**15*x
**4 + 635040*d**5*e**16*x**5 + 529200*d**4*e**17*x**6 + 302400*d**3*e**18*x
**7 + 113400*d**2*e**19*x**8 + 25200*d*e**20*x**9 + 2520*e**21*x**10) - 214
20*d**8*e**2/(2520*d**10*e**11 + 25200*d**9*e**12*x + 113400*d**8*e**13*x**
2 + 302400*d**7*e**14*x**3 + 529200*d**6*e**15*x**4 + 635040*d**5*e**16*x**
5 + 529200*d**4*e**17*x**6 + 302400*d**3*e**18*x**7 + 113400*d**2*e**19*x**
```

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8 + 25200*d*e**20*x**9 + 2520*e**21*x**10) + 151200000*d**7*e**3*x**3*log(d
/e + x)/(2520*d**10*e**11 + 25200*d**9*e**12*x + 113400*d**8*e**13*x**2 + 3
02400*d**7*e**14*x**3 + 529200*d**6*e**15*x**4 + 635040*d**5*e**16*x**5 + 5
29200*d**4*e**17*x**6 + 302400*d**3*e**18*x**7 + 113400*d**2*e**19*x**8 + 2
5200*d*e**20*x**9 + 2520*e**21*x**10) + 392040000*d**7*e**3*x**3/(2520*d**1
0*e**11 + 25200*d**9*e**12*x + 113400*d**8*e**13*x**2 + 302400*d**7*e**14*x
**3 + 529200*d**6*e**15*x**4 + 635040*d**5*e**16*x**5 + 529200*d**4*e**17*x
**6 + 302400*d**3*e**18*x**7 + 113400*d**2*e**19*x**8 + 25200*d*e**20*x**9
 + 2520*e**21*x**10) + 283500*d**7*e**3*x**2/(2520*d**10*e**11 + 25200*d**9*
e**12*x + 113400*d**8*e**13*x**2 + 302400*d**7*e**14*x**3 + 529200*d**6*e**
15*x**4 + 635040*d**5*e**16*x**5 + 529200*d**4*e**17*x**6 + 302400*d**3*e**
18*x**7 + 113400*d**2*e**19*x**8 + 25200*d*e**20*x**9 + 2520*e**21*x**10) -
 214200*d**7*e**3*x/(2520*d**10*e**11 + 25200*d**9*e**12*x + 113400*d**8*e*
*13*x**2 + 302400*d**7*e**14*x**3 + 529200*d**6*e**15*x**4 + 635040*d**5*e*
*16*x**5 + 529200*d**4*e**17*x**6 + 302400*d**3*e**18*x**7 + 113400*d**2*e*
*19*x**8 + 25200*d*e**20*x**9 + 2520*e**21*x**10) + 686*d**7*e**3/(2520*d**
10*e**11 + 25200*d**9*e**12*x + 113400*d**8*e**13*x**2 + 302400*d**7*e**14*
x**3 + 529200*d**6*e**15*x**4 + 635040*d**5*e**16*x**5 + 529200*d**4*e**17*
x**6 + 302400*d**3*e**18*x**7 + 113400*d**2*e**19*x**8 + 25200*d*e**20*x**9
 + 2520*e**21*x**10) + 264600000*d**6*e**4*x**4*log(d/e + x)/(2520*d**10*e*
*11 + 25200*d**9*e**12*x + 113400*d**8*e**13*x**2 + 302400*d**7*e**14*x**3
+ 529200*d**6*e**15*x**4 + 635040*d**5*e**16*x**5 + 529200*d**4*e**17*x**6
+ 302400*d**3*e**18*x**7 + 113400*d**2*e**19*x**8 + 25200*d*e**20*x**9 + 25
20*e**21*x**10) + 648270000*d**6*e**4*x**4/(2520*d**10*e**11 + 25200*d**9*e
**12*x + 113400*d**8*e**13*x**2 + 302400*d**7*e**14*x**3 + 529200*d**6*e**1
5*x**4 + 635040*d**5*e**16*x**5 + 529200*d**4*e**17*x**6 + 302400*d**3*e**1
8*x**7 + 113400*d**2*e**19*x**8 + 25200*d*e**20*x**9 + 2520*e**21*x**10) +
756000*d**6*e**4*x**3/(2520*d**10*e**11 + 25200*d**9*e**12*x + 113400*d**8*
e**13*x**2 + 302400*d**7*e**14*x**3 + 529200*d**6*e**15*x**4 + 635040*d**5*
e**16*x**5 + 529200*d**4*e**17*x**6 + 302400*d**3*e**18*x**7 + 113400*d**2*
e**19*x**8 + 25200*d*e**20*x**9 + 2520*e**21*x**10) - 963900*d**6*e**4*x**2
/(2520*d**10*e**11 + 25200*d**9*e**12*x + 113400*d**8*e**13*x**2 + 302400*d
**7*e**14*x**3 + 529200*d**6*e**15*x**4 + 635040*d**5*e**16*x**5 + 529200*d
**4*e**17*x**6 + 302400*d**3*e**18*x**7 + 113400*d**2*e**19*x**8 + 25200*d*
e**20*x**9 + 2520*e**21*x**10) + 6860*d**6*e**4*x/(2520*d**10*e**11 + 25200
*d**9*e**12*x + 113400*d**8*e**13*x**2 + 302400*d**7*e**14*x**3 + 529200*d*
*6*e**15*x**4 + 635040*d**5*e**16*x**5 + 529200*d**4*e**17*x**6 + 302400*d*
*3*e**18*x**7 + 113400*d**2*e**19*x**8 + 25200*d*e**20*x**9 + 2520*e**21*x*
*10) - 2997*d**6*e**4/(2520*d**10*e**11 + 25200*d**9*e**12*x + 113400*d**8*
e**13*x**2 + 302400*d**7*e**14*x**3 + 529200*d**6*e**15*x**4 + 635040*d**5*
e**16*x**5 + 529200*d**4*e**17*x**6 + 302400*d**3*e**18*x**7 + 113400*d**2*
e**19*x**8 + 25200*d*e**20*x**9 + 2520*e**21*x**10) + 317520000*d**5*e**5*x
**5*log(d/e + x)/(2520*d**10*e**11 + 25200*d**9*e**12*x + 113400*d**8*e**13
*x**2 + 302400*d**7*e**14*x**3 + 529200*d**6*e**15*x**4 + 635040*d**5*e**16
*x**5 + 529200*d**4*e**17*x**6 + 302400*d**3*e**18*x**7 + 113400*d**2*e**19
```

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*x**8 + 25200*d*e**20*x**9 + 2520*e**21*x**10) + 725004000*d**5*e**5*x**5/(
2520*d**10*e**11 + 25200*d**9*e**12*x + 113400*d**8*e**13*x**2 + 302400*d**
7*e**14*x**3 + 529200*d**6*e**15*x**4 + 635040*d**5*e**16*x**5 + 529200*d**
4*e**17*x**6 + 302400*d**3*e**18*x**7 + 113400*d**2*e**19*x**8 + 25200*d*e*
*20*x**9 + 2520*e**21*x**10) + 1323000*d**5*e**5*x**4/(2520*d**10*e**11 + 2
5200*d**9*e**12*x + 113400*d**8*e**13*x**2 + 302400*d**7*e**14*x**3 + 52920
0*d**6*e**15*x**4 + 635040*d**5*e**16*x**5 + 529200*d**4*e**17*x**6 + 30240
0*d**3*e**18*x**7 + 113400*d**2*e**19*x**8 + 25200*d*e**20*x**9 + 2520*e**2
1*x**10) - 2570400*d**5*e**5*x**3/(2520*d**10*e**11 + 25200*d**9*e**12*x +
113400*d**8*e**13*x**2 + 302400*d**7*e**14*x**3 + 529200*d**6*e**15*x**4 +
635040*d**5*e**16*x**5 + 529200*d**4*e**17*x**6 + 302400*d**3*e**18*x**7 +
113400*d**2*e**19*x**8 + 25200*d*e**20*x**9 + 2520*e**21*x**10) + 30870*d**
5*e**5*x**2/(2520*d**10*e**11 + 25200*d**9*e**12*x + 113400*d**8*e**13*x**2
 + 302400*d**7*e**14*x**3 + 529200*d**6*e**15*x**4 + 635040*d**5*e**16*x**5
 + 529200*d**4*e**17*x**6 + 302400*d**3*e**18*x**7 + 113400*d**2*e**19*x**8
 + 25200*d*e**20*x**9 + 2520*e**21*x**10) - 29970*d**5*e**5*x/(2520*d**10*e
**11 + 25200*d**9*e**12*x + 113400*d**8*e**13*x**2 + 302400*d**7*e**14*x**3
 + 529200*d**6*e**15*x**4 + 635040*d**5*e**16*x**5 + 529200*d**4*e**17*x**6
 + 302400*d**3*e**18*x**7 + 113400*d**2*e**19*x**8 + 25200*d*e**20*x**9 + 2
520*e**21*x**10) - 1020*d**5*e**5/(2520*d**10*e**11 + 25200*d**9*e**12*x +
113400*d**8*e**13*x**2 + 302400*d**7*e**14*x**3 + 529200*d**6*e**15*x**4 +
635040*d**5*e**16*x**5 + 529200*d**4*e**17*x**6 + 302400*d**3*e**18*x**7 +
113400*d**2*e**19*x**8 + 25200*d*e**20*x**9 + 2520*e**21*x**10) + 264600000
*d**4*e**6*x**6*log(d/e + x)/(2520*d**10*e**11 + 25200*d**9*e**12*x + 11340
0*d**8*e**13*x**2 + 302400*d**7*e**14*x**3 + 529200*d**6*e**15*x**4 + 63504
0*d**5*e**16*x**5 + 529200*d**4*e**17*x**6 + 302400*d**3*e**18*x**7 + 11340
0*d**2*e**19*x**8 + 25200*d*e**20*x**9 + 2520*e**21*x**10) + 551250000*d**4
*e**6*x**6/(2520*d**10*e**11 + 25200*d**9*e**12*x + 113400*d**8*e**13*x**2
+ 302400*d**7*e**14*x**3 + 529200*d**6*e**15*x**4 + 635040*d**5*e**16*x**5
+ 529200*d**4*e**17*x**6 + 302400*d**3*e**18*x**7 + 113400*d**2*e**19*x**8
+ 25200*d*e**20*x**9 + 2520*e**21*x**10) + 1587600*d**4*e**6*x**5/(2520*d**
10*e**11 + 25200*d**9*e**12*x + 113400*d**8*e**13*x**2 + 302400*d**7*e**14*
x**3 + 529200*d**6*e**15*x**4 + 635040*d**5*e**16*x**5 + 529200*d**4*e**17*
x**6 + 302400*d**3*e**18*x**7 + 113400*d**2*e**19*x**8 + 25200*d*e**20*x**9
 + 2520*e**21*x**10) - 4498200*d**4*e**6*x**4/(2520*d**10*e**11 + 25200*d**
9*e**12*x + 113400*d**8*e**13*x**2 + 302400*d**7*e**14*x**3 + 529200*d**6*e
**15*x**4 + 635040*d**5*e**16*x**5 + 529200*d**4*e**17*x**6 + 302400*d**3*e
**18*x**7 + 113400*d**2*e**19*x**8 + 25200*d*e**20*x**9 + 2520*e**21*x**10)
 + 82320*d**4*e**6*x**3/(2520*d**10*e**11 + 25200*d**9*e**12*x + 113400*d**
8*e**13*x**2 + 302400*d**7*e**14*x**3 + 529200*d**6*e**15*x**4 + 635040*d**
5*e**16*x**5 + 529200*d**4*e**17*x**6 + 302400*d**3*e**18*x**7 + 113400*d**
2*e**19*x**8 + 25200*d*e**20*x**9 + 2520*e**21*x**10) - 134865*d**4*e**6*x*
*2/(2520*d**10*e**11 + 25200*d**9*e**12*x + 113400*d**8*e**13*x**2 + 302400
*d**7*e**14*x**3 + 529200*d**6*e**15*x**4 + 635040*d**5*e**16*x**5 + 529200
*d**4*e**17*x**6 + 302400*d**3*e**18*x**7 + 113400*d**2*e**19*x**8 + 25200*
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d*e**20*x**9 + 2520*e**21*x**10) - 10200*d**4*e**6*x/(2520*d**10*e**11 + 25
200*d**9*e**12*x + 113400*d**8*e**13*x**2 + 302400*d**7*e**14*x**3 + 529200
*d**6*e**15*x**4 + 635040*d**5*e**16*x**5 + 529200*d**4*e**17*x**6 + 302400
*d**3*e**18*x**7 + 113400*d**2*e**19*x**8 + 25200*d*e**20*x**9 + 2520*e**21
*x**10) - 2218*d**4*e**6/(2520*d**10*e**11 + 25200*d**9*e**12*x + 113400*d*
*8*e**13*x**2 + 302400*d**7*e**14*x**3 + 529200*d**6*e**15*x**4 + 635040*d*
*5*e**16*x**5 + 529200*d**4*e**17*x**6 + 302400*d**3*e**18*x**7 + 113400*d*
*2*e**19*x**8 + 25200*d*e**20*x**9 + 2520*e**21*x**10) + 151200000*d**3*e**
7*x**7*log(d/e + x)/(2520*d**10*e**11 + 25200*d**9*e**12*x + 113400*d**8*e*
*13*x**2 + 302400*d**7*e**14*x**3 + 529200*d**6*e**15*x**4 + 635040*d**5*e*
*16*x**5 + 529200*d**4*e**17*x**6 + 302400*d**3*e**18*x**7 + 113400*d**2*e*
*19*x**8 + 25200*d*e**20*x**9 + 2520*e**21*x**10) + 277200000*d**3*e**7*x**
7/(2520*d**10*e**11 + 25200*d**9*e**12*x + 113400*d**8*e**13*x**2 + 302400*
d**7*e**14*x**3 + 529200*d**6*e**15*x**4 + 635040*d**5*e**16*x**5 + 529200*
d**4*e**17*x**6 + 302400*d**3*e**18*x**7 + 113400*d**2*e**19*x**8 + 25200*d
*e**20*x**9 + 2520*e**21*x**10) + 1323000*d**3*e**7*x**6/(2520*d**10*e**11
+ 25200*d**9*e**12*x + 113400*d**8*e**13*x**2 + 302400*d**7*e**14*x**3 + 52
9200*d**6*e**15*x**4 + 635040*d**5*e**16*x**5 + 529200*d**4*e**17*x**6 + 30
2400*d**3*e**18*x**7 + 113400*d**2*e**19*x**8 + 25200*d*e**20*x**9 + 2520*e
**21*x**10) - 5397840*d**3*e**7*x**5/(2520*d**10*e**11 + 25200*d**9*e**12*x
 + 113400*d**8*e**13*x**2 + 302400*d**7*e**14*x**3 + 529200*d**6*e**15*x**4
 + 635040*d**5*e**16*x**5 + 529200*d**4*e**17*x**6 + 302400*d**3*e**18*x**7
 + 113400*d**2*e**19*x**8 + 25200*d*e**20*x**9 + 2520*e**21*x**10) + 144060
*d**3*e**7*x**4/(2520*d**10*e**11 + 25200*d**9*e**12*x + 113400*d**8*e**13*
x**2 + 302400*d**7*e**14*x**3 + 529200*d**6*e**15*x**4 + 635040*d**5*e**16*
x**5 + 529200*d**4*e**17*x**6 + 302400*d**3*e**18*x**7 + 113400*d**2*e**19*
x**8 + 25200*d*e**20*x**9 + 2520*e**21*x**10) - 359640*d**3*e**7*x**3/(2520
*d**10*e**11 + 25200*d**9*e**12*x + 113400*d**8*e**13*x**2 + 302400*d**7*e*
*14*x**3 + 529200*d**6*e**15*x**4 + 635040*d**5*e**16*x**5 + 529200*d**4*e*
*17*x**6 + 302400*d**3*e**18*x**7 + 113400*d**2*e**19*x**8 + 25200*d*e**20*
x**9 + 2520*e**21*x**10) - 45900*d**3*e**7*x**2/(2520*d**10*e**11 + 25200*d
**9*e**12*x + 113400*d**8*e**13*x**2 + 302400*d**7*e**14*x**3 + 529200*d**6
*e**15*x**4 + 635040*d**5*e**16*x**5 + 529200*d**4*e**17*x**6 + 302400*d**3
*e**18*x**7 + 113400*d**2*e**19*x**8 + 25200*d*e**20*x**9 + 2520*e**21*x**1
0) - 22180*d**3*e**7*x/(2520*d**10*e**11 + 25200*d**9*e**12*x + 113400*d**8
*e**13*x**2 + 302400*d**7*e**14*x**3 + 529200*d**6*e**15*x**4 + 635040*d**5
*e**16*x**5 + 529200*d**4*e**17*x**6 + 302400*d**3*e**18*x**7 + 113400*d**2
*e**19*x**8 + 25200*d*e**20*x**9 + 2520*e**21*x**10) - 1722*d**3*e**7/(2520
*d**10*e**11 + 25200*d**9*e**12*x + 113400*d**8*e**13*x**2 + 302400*d**7*e*
*14*x**3 + 529200*d**6*e**15*x**4 + 635040*d**5*e**16*x**5 + 529200*d**4*e*
*17*x**6 + 302400*d**3*e**18*x**7 + 113400*d**2*e**19*x**8 + 25200*d*e**20*
x**9 + 2520*e**21*x**10) + 56700000*d**2*e**8*x**8*log(d/e + x)/(2520*d**10
*e**11 + 25200*d**9*e**12*x + 113400*d**8*e**13*x**2 + 302400*d**7*e**14*x*
*3 + 529200*d**6*e**15*x**4 + 635040*d**5*e**16*x**5 + 529200*d**4*e**17*x*
*6 + 302400*d**3*e**18*x**7 + 113400*d**2*e**19*x**8 + 25200*d*e**20*x**9 +
```

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 2520*e**21*x**10) + 85050000*d**2*e**8*x**8/(2520*d**10*e**11 + 25200*d**9
*e**12*x + 113400*d**8*e**13*x**2 + 302400*d**7*e**14*x**3 + 529200*d**6*e*
*15*x**4 + 635040*d**5*e**16*x**5 + 529200*d**4*e**17*x**6 + 302400*d**3*e*
*18*x**7 + 113400*d**2*e**19*x**8 + 25200*d*e**20*x**9 + 2520*e**21*x**10)
+ 756000*d**2*e**8*x**7/(2520*d**10*e**11 + 25200*d**9*e**12*x + 113400*d**
8*e**13*x**2 + 302400*d**7*e**14*x**3 + 529200*d**6*e**15*x**4 + 635040*d**
5*e**16*x**5 + 529200*d**4*e**17*x**6 + 302400*d**3*e**18*x**7 + 113400*d**
2*e**19*x**8 + 25200*d*e**20*x**9 + 2520*e**21*x**10) - 4498200*d**2*e**8*x
**6/(2520*d**10*e**11 + 25200*d**9*e**12*x + 113400*d**8*e**13*x**2 + 30240
0*d**7*e**14*x**3 + 529200*d**6*e**15*x**4 + 635040*d**5*e**16*x**5 + 52920
0*d**4*e**17*x**6 + 302400*d**3*e**18*x**7 + 113400*d**2*e**19*x**8 + 25200
*d*e**20*x**9 + 2520*e**21*x**10) + 172872*d**2*e**8*x**5/(2520*d**10*e**11
 + 25200*d**9*e**12*x + 113400*d**8*e**13*x**2 + 302400*d**7*e**14*x**3 + 5
29200*d**6*e**15*x**4 + 635040*d**5*e**16*x**5 + 529200*d**4*e**17*x**6 + 3
02400*d**3*e**18*x**7 + 113400*d**2*e**19*x**8 + 25200*d*e**20*x**9 + 2520*
e**21*x**10) - 629370*d**2*e**8*x**4/(2520*d**10*e**11 + 25200*d**9*e**12*x
 + 113400*d**8*e**13*x**2 + 302400*d**7*e**14*x**3 + 529200*d**6*e**15*x**4
 + 635040*d**5*e**16*x**5 + 529200*d**4*e**17*x**6 + 302400*d**3*e**18*x**7
 + 113400*d**2*e**19*x**8 + 25200*d*e**20*x**9 + 2520*e**21*x**10) - 122400
*d**2*e**8*x**3/(2520*d**10*e**11 + 25200*d**9*e**12*x + 113400*d**8*e**13*
x**2 + 302400*d**7*e**14*x**3 + 529200*d**6*e**15*x**4 + 635040*d**5*e**16*
x**5 + 529200*d**4*e**17*x**6 + 302400*d**3*e**18*x**7 + 113400*d**2*e**19*
x**8 + 25200*d*e**20*x**9 + 2520*e**21*x**10) - 99810*d**2*e**8*x**2/(2520*
d**10*e**11 + 25200*d**9*e**12*x + 113400*d**8*e**13*x**2 + 302400*d**7*e**
14*x**3 + 529200*d**6*e**15*x**4 + 635040*d**5*e**16*x**5 + 529200*d**4*e**
17*x**6 + 302400*d**3*e**18*x**7 + 113400*d**2*e**19*x**8 + 25200*d*e**20*x
**9 + 2520*e**21*x**10) - 17220*d**2*e**8*x/(2520*d**10*e**11 + 25200*d**9*
e**12*x + 113400*d**8*e**13*x**2 + 302400*d**7*e**14*x**3 + 529200*d**6*e**
15*x**4 + 635040*d**5*e**16*x**5 + 529200*d**4*e**17*x**6 + 302400*d**3*e**
18*x**7 + 113400*d**2*e**19*x**8 + 25200*d*e**20*x**9 + 2520*e**21*x**10) -
 3339*d**2*e**8/(2520*d**10*e**11 + 25200*d**9*e**12*x + 113400*d**8*e**13*
x**2 + 302400*d**7*e**14*x**3 + 529200*d**6*e**15*x**4 + 635040*d**5*e**16*
x**5 + 529200*d**4*e**17*x**6 + 302400*d**3*e**18*x**7 + 113400*d**2*e**19*
x**8 + 25200*d*e**20*x**9 + 2520*e**21*x**10) + 12600000*d*e**9*x**9*log(d/
e + x)/(2520*d**10*e**11 + 25200*d**9*e**12*x + 113400*d**8*e**13*x**2 + 30
2400*d**7*e**14*x**3 + 529200*d**6*e**15*x**4 + 635040*d**5*e**16*x**5 + 52
9200*d**4*e**17*x**6 + 302400*d**3*e**18*x**7 + 113400*d**2*e**19*x**8 + 25
200*d*e**20*x**9 + 2520*e**21*x**10) + 12600000*d*e**9*x**9/(2520*d**10*e**
11 + 25200*d**9*e**12*x + 113400*d**8*e**13*x**2 + 302400*d**7*e**14*x**3 +
 529200*d**6*e**15*x**4 + 635040*d**5*e**16*x**5 + 529200*d**4*e**17*x**6 +
 302400*d**3*e**18*x**7 + 113400*d**2*e**19*x**8 + 25200*d*e**20*x**9 + 252
0*e**21*x**10) + 283500*d*e**9*x**8/(2520*d**10*e**11 + 25200*d**9*e**12*x
+ 113400*d**8*e**13*x**2 + 302400*d**7*e**14*x**3 + 529200*d**6*e**15*x**4
+ 635040*d**5*e**16*x**5 + 529200*d**4*e**17*x**6 + 302400*d**3*e**18*x**7
+ 113400*d**2*e**19*x**8 + 25200*d*e**20*x**9 + 2520*e**21*x**10) - 2570400
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*d*e**9*x**7/(2520*d**10*e**11 + 25200*d**9*e**12*x + 113400*d**8*e**13*x**
2 + 302400*d**7*e**14*x**3 + 529200*d**6*e**15*x**4 + 635040*d**5*e**16*x**
5 + 529200*d**4*e**17*x**6 + 302400*d**3*e**18*x**7 + 113400*d**2*e**19*x**
8 + 25200*d*e**20*x**9 + 2520*e**21*x**10) + 144060*d*e**9*x**6/(2520*d**10
*e**11 + 25200*d**9*e**12*x + 113400*d**8*e**13*x**2 + 302400*d**7*e**14*x*
*3 + 529200*d**6*e**15*x**4 + 635040*d**5*e**16*x**5 + 529200*d**4*e**17*x*
*6 + 302400*d**3*e**18*x**7 + 113400*d**2*e**19*x**8 + 25200*d*e**20*x**9 +
 2520*e**21*x**10) - 755244*d*e**9*x**5/(2520*d**10*e**11 + 25200*d**9*e**1
2*x + 113400*d**8*e**13*x**2 + 302400*d**7*e**14*x**3 + 529200*d**6*e**15*x
**4 + 635040*d**5*e**16*x**5 + 529200*d**4*e**17*x**6 + 302400*d**3*e**18*x
**7 + 113400*d**2*e**19*x**8 + 25200*d*e**20*x**9 + 2520*e**21*x**10) - 214
200*d*e**9*x**4/(2520*d**10*e**11 + 25200*d**9*e**12*x + 113400*d**8*e**13*
x**2 + 302400*d**7*e**14*x**3 + 529200*d**6*e**15*x**4 + 635040*d**5*e**16*
x**5 + 529200*d**4*e**17*x**6 + 302400*d**3*e**18*x**7 + 113400*d**2*e**19*
x**8 + 25200*d*e**20*x**9 + 2520*e**21*x**10) - 266160*d*e**9*x**3/(2520*d*
*10*e**11 + 25200*d**9*e**12*x + 113400*d**8*e**13*x**2 + 302400*d**7*e**14
*x**3 + 529200*d**6*e**15*x**4 + 635040*d**5*e**16*x**5 + 529200*d**4*e**17
*x**6 + 302400*d**3*e**18*x**7 + 113400*d**2*e**19*x**8 + 25200*d*e**20*x**
9 + 2520*e**21*x**10) - 77490*d*e**9*x**2/(2520*d**10*e**11 + 25200*d**9*e*
*12*x + 113400*d**8*e**13*x**2 + 302400*d**7*e**14*x**3 + 529200*d**6*e**15
*x**4 + 635040*d**5*e**16*x**5 + 529200*d**4*e**17*x**6 + 302400*d**3*e**18
*x**7 + 113400*d**2*e**19*x**8 + 25200*d*e**20*x**9 + 2520*e**21*x**10) - 3
3390*d*e**9*x/(2520*d**10*e**11 + 25200*d**9*e**12*x + 113400*d**8*e**13*x*
*2 + 302400*d**7*e**14*x**3 + 529200*d**6*e**15*x**4 + 635040*d**5*e**16*x*
*5 + 529200*d**4*e**17*x**6 + 302400*d**3*e**18*x**7 + 113400*d**2*e**19*x*
*8 + 25200*d*e**20*x**9 + 2520*e**21*x**10) - 3780*d*e**9/(2520*d**10*e**11
 + 25200*d**9*e**12*x + 113400*d**8*e**13*x**2 + 302400*d**7*e**14*x**3 + 5
29200*d**6*e**15*x**4 + 635040*d**5*e**16*x**5 + 529200*d**4*e**17*x**6 + 3
02400*d**3*e**18*x**7 + 113400*d**2*e**19*x**8 + 25200*d*e**20*x**9 + 2520*
e**21*x**10) + 1260000*e**10*x**10*log(d/e + x)/(2520*d**10*e**11 + 25200*d
**9*e**12*x + 113400*d**8*e**13*x**2 + 302400*d**7*e**14*x**3 + 529200*d**6
*e**15*x**4 + 635040*d**5*e**16*x**5 + 529200*d**4*e**17*x**6 + 302400*d**3
*e**18*x**7 + 113400*d**2*e**19*x**8 + 25200*d*e**20*x**9 + 2520*e**21*x**1
0) + 63000*e**10*x**9/(2520*d**10*e**11 + 25200*d**9*e**12*x + 113400*d**8*
e**13*x**2 + 302400*d**7*e**14*x**3 + 529200*d**6*e**15*x**4 + 635040*d**5*
e**16*x**5 + 529200*d**4*e**17*x**6 + 302400*d**3*e**18*x**7 + 113400*d**2*
e**19*x**8 + 25200*d*e**20*x**9 + 2520*e**21*x**10) - 963900*e**10*x**8/(25
20*d**10*e**11 + 25200*d**9*e**12*x + 113400*d**8*e**13*x**2 + 302400*d**7*
e**14*x**3 + 529200*d**6*e**15*x**4 + 635040*d**5*e**16*x**5 + 529200*d**4*
e**17*x**6 + 302400*d**3*e**18*x**7 + 113400*d**2*e**19*x**8 + 25200*d*e**2
0*x**9 + 2520*e**21*x**10) + 82320*e**10*x**7/(2520*d**10*e**11 + 25200*d**
9*e**12*x + 113400*d**8*e**13*x**2 + 302400*d**7*e**14*x**3 + 529200*d**6*e
**15*x**4 + 635040*d**5*e**16*x**5 + 529200*d**4*e**17*x**6 + 302400*d**3*e
**18*x**7 + 113400*d**2*e**19*x**8 + 25200*d*e**20*x**9 + 2520*e**21*x**10)
 - 629370*e**10*x**6/(2520*d**10*e**11 + 25200*d**9*e**12*x + 113400*d**8*e
```

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**13*x**2 + 302400*d**7*e**14*x**3 + 529200*d**6*e**15*x**4 + 635040*d**5*e
**16*x**5 + 529200*d**4*e**17*x**6 + 302400*d**3*e**18*x**7 + 113400*d**2*e
**19*x**8 + 25200*d*e**20*x**9 + 2520*e**21*x**10) - 257040*e**10*x**5/(252
0*d**10*e**11 + 25200*d**9*e**12*x + 113400*d**8*e**13*x**2 + 302400*d**7*e
**14*x**3 + 529200*d**6*e**15*x**4 + 635040*d**5*e**16*x**5 + 529200*d**4*e
**17*x**6 + 302400*d**3*e**18*x**7 + 113400*d**2*e**19*x**8 + 25200*d*e**20
*x**9 + 2520*e**21*x**10) - 465780*e**10*x**4/(2520*d**10*e**11 + 25200*d**
9*e**12*x + 113400*d**8*e**13*x**2 + 302400*d**7*e**14*x**3 + 529200*d**6*e
**15*x**4 + 635040*d**5*e**16*x**5 + 529200*d**4*e**17*x**6 + 302400*d**3*e
**18*x**7 + 113400*d**2*e**19*x**8 + 25200*d*e**20*x**9 + 2520*e**21*x**10)
 - 206640*e**10*x**3/(2520*d**10*e**11 + 25200*d**9*e**12*x + 113400*d**8*e
**13*x**2 + 302400*d**7*e**14*x**3 + 529200*d**6*e**15*x**4 + 635040*d**5*e
**16*x**5 + 529200*d**4*e**17*x**6 + 302400*d**3*e**18*x**7 + 113400*d**2*e
**19*x**8 + 25200*d*e**20*x**9 + 2520*e**21*x**10) - 150255*e**10*x**2/(252
0*d**10*e**11 + 25200*d**9*e**12*x + 113400*d**8*e**13*x**2 + 302400*d**7*e
**14*x**3 + 529200*d**6*e**15*x**4 + 635040*d**5*e**16*x**5 + 529200*d**4*e
**17*x**6 + 302400*d**3*e**18*x**7 + 113400*d**2*e**19*x**8 + 25200*d*e**20
*x**9 + 2520*e**21*x**10) - 37800*e**10*x/(2520*d**10*e**11 + 25200*d**9*e*
*12*x + 113400*d**8*e**13*x**2 + 302400*d**7*e**14*x**3 + 529200*d**6*e**15
*x**4 + 635040*d**5*e**16*x**5 + 529200*d**4*e**17*x**6 + 302400*d**3*e**18
*x**7 + 113400*d**2*e**19*x**8 + 25200*d*e**20*x**9 + 2520*e**21*x**10) - 1
3608*e**10/(2520*d**10*e**11 + 25200*d**9*e**12*x + 113400*d**8*e**13*x**2
+ 302400*d**7*e**14*x**3 + 529200*d**6*e**15*x**4 + 635040*d**5*e**16*x**5
+ 529200*d**4*e**17*x**6 + 302400*d**3*e**18*x**7 + 113400*d**2*e**19*x**8
+ 25200*d*e**20*x**9 + 2520*e**21*x**10), Eq(m, -11)), (-12600000*d**10*log
(d/e + x)/(2520*d**9*e**11 + 22680*d**8*e**12*x + 90720*d**7*e**13*x**2 + 2
11680*d**6*e**14*x**3 + 317520*d**5*e**15*x**4 + 317520*d**4*e**16*x**5 + 2
11680*d**3*e**17*x**6 + 90720*d**2*e**18*x**7 + 22680*d*e**19*x**8 + 2520*e
**20*x**9) - 35645000*d**10/(2520*d**9*e**11 + 22680*d**8*e**12*x + 90720*d
**7*e**13*x**2 + 211680*d**6*e**14*x**3 + 317520*d**5*e**15*x**4 + 317520*d
**4*e**16*x**5 + 211680*d**3*e**17*x**6 + 90720*d**2*e**18*x**7 + 22680*d*e
**19*x**8 + 2520*e**20*x**9) - 113400000*d**9*e*x*log(d/e + x)/(2520*d**9*e
**11 + 22680*d**8*e**12*x + 90720*d**7*e**13*x**2 + 211680*d**6*e**14*x**3
+ 317520*d**5*e**15*x**4 + 317520*d**4*e**16*x**5 + 211680*d**3*e**17*x**6
+ 90720*d**2*e**18*x**7 + 22680*d*e**19*x**8 + 2520*e**20*x**9) - 308205000
*d**9*e*x/(2520*d**9*e**11 + 22680*d**8*e**12*x + 90720*d**7*e**13*x**2 + 2
11680*d**6*e**14*x**3 + 317520*d**5*e**15*x**4 + 317520*d**4*e**16*x**5 + 2
11680*d**3*e**17*x**6 + 90720*d**2*e**18*x**7 + 22680*d*e**19*x**8 + 2520*e
**20*x**9) - 63000*d**9*e*log(d/e + x)/(2520*d**9*e**11 + 22680*d**8*e**12*
x + 90720*d**7*e**13*x**2 + 211680*d**6*e**14*x**3 + 317520*d**5*e**15*x**4
 + 317520*d**4*e**16*x**5 + 211680*d**3*e**17*x**6 + 90720*d**2*e**18*x**7
+ 22680*d*e**19*x**8 + 2520*e**20*x**9) - 178225*d**9*e/(2520*d**9*e**11 +
22680*d**8*e**12*x + 90720*d**7*e**13*x**2 + 211680*d**6*e**14*x**3 + 31752
0*d**5*e**15*x**4 + 317520*d**4*e**16*x**5 + 211680*d**3*e**17*x**6 + 90720
*d**2*e**18*x**7 + 22680*d*e**19*x**8 + 2520*e**20*x**9) - 453600000*d**8*e
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**2*x**2*log(d/e + x)/(2520*d**9*e**11 + 22680*d**8*e**12*x + 90720*d**7*e*
*13*x**2 + 211680*d**6*e**14*x**3 + 317520*d**5*e**15*x**4 + 317520*d**4*e*
*16*x**5 + 211680*d**3*e**17*x**6 + 90720*d**2*e**18*x**7 + 22680*d*e**19*x
**8 + 2520*e**20*x**9) - 1176120000*d**8*e**2*x**2/(2520*d**9*e**11 + 22680
*d**8*e**12*x + 90720*d**7*e**13*x**2 + 211680*d**6*e**14*x**3 + 317520*d**
5*e**15*x**4 + 317520*d**4*e**16*x**5 + 211680*d**3*e**17*x**6 + 90720*d**2
*e**18*x**7 + 22680*d*e**19*x**8 + 2520*e**20*x**9) - 567000*d**8*e**2*x*lo
g(d/e + x)/(2520*d**9*e**11 + 22680*d**8*e**12*x + 90720*d**7*e**13*x**2 +
211680*d**6*e**14*x**3 + 317520*d**5*e**15*x**4 + 317520*d**4*e**16*x**5 +
211680*d**3*e**17*x**6 + 90720*d**2*e**18*x**7 + 22680*d*e**19*x**8 + 2520*
e**20*x**9) - 1541025*d**8*e**2*x/(2520*d**9*e**11 + 22680*d**8*e**12*x + 9
0720*d**7*e**13*x**2 + 211680*d**6*e**14*x**3 + 317520*d**5*e**15*x**4 + 31
7520*d**4*e**16*x**5 + 211680*d**3*e**17*x**6 + 90720*d**2*e**18*x**7 + 226
80*d*e**19*x**8 + 2520*e**20*x**9) - 214200*d**8*e**2/(2520*d**9*e**11 + 22
680*d**8*e**12*x + 90720*d**7*e**13*x**2 + 211680*d**6*e**14*x**3 + 317520*
d**5*e**15*x**4 + 317520*d**4*e**16*x**5 + 211680*d**3*e**17*x**6 + 90720*d
**2*e**18*x**7 + 22680*d*e**19*x**8 + 2520*e**20*x**9) - 1058400000*d**7*e*
*3*x**3*log(d/e + x)/(2520*d**9*e**11 + 22680*d**8*e**12*x + 90720*d**7*e**
13*x**2 + 211680*d**6*e**14*x**3 + 317520*d**5*e**15*x**4 + 317520*d**4*e**
16*x**5 + 211680*d**3*e**17*x**6 + 90720*d**2*e**18*x**7 + 22680*d*e**19*x*
*8 + 2520*e**20*x**9) - 2593080000*d**7*e**3*x**3/(2520*d**9*e**11 + 22680*
d**8*e**12*x + 90720*d**7*e**13*x**2 + 211680*d**6*e**14*x**3 + 317520*d**5
*e**15*x**4 + 317520*d**4*e**16*x**5 + 211680*d**3*e**17*x**6 + 90720*d**2*
e**18*x**7 + 22680*d*e**19*x**8 + 2520*e**20*x**9) - 2268000*d**7*e**3*x**2
*log(d/e + x)/(2520*d**9*e**11 + 22680*d**8*e**12*x + 90720*d**7*e**13*x**2
 + 211680*d**6*e**14*x**3 + 317520*d**5*e**15*x**4 + 317520*d**4*e**16*x**5
 + 211680*d**3*e**17*x**6 + 90720*d**2*e**18*x**7 + 22680*d*e**19*x**8 + 25
20*e**20*x**9) - 5880600*d**7*e**3*x**2/(2520*d**9*e**11 + 22680*d**8*e**12
*x + 90720*d**7*e**13*x**2 + 211680*d**6*e**14*x**3 + 317520*d**5*e**15*x**
4 + 317520*d**4*e**16*x**5 + 211680*d**3*e**17*x**6 + 90720*d**2*e**18*x**7
 + 22680*d*e**19*x**8 + 2520*e**20*x**9) - 1927800*d**7*e**3*x/(2520*d**9*e
**11 + 22680*d**8*e**12*x + 90720*d**7*e**13*x**2 + 211680*d**6*e**14*x**3
+ 317520*d**5*e**15*x**4 + 317520*d**4*e**16*x**5 + 211680*d**3*e**17*x**6
+ 90720*d**2*e**18*x**7 + 22680*d*e**19*x**8 + 2520*e**20*x**9) + 3430*d**7
*e**3/(2520*d**9*e**11 + 22680*d**8*e**12*x + 90720*d**7*e**13*x**2 + 21168
0*d**6*e**14*x**3 + 317520*d**5*e**15*x**4 + 317520*d**4*e**16*x**5 + 21168
0*d**3*e**17*x**6 + 90720*d**2*e**18*x**7 + 22680*d*e**19*x**8 + 2520*e**20
*x**9) - 1587600000*d**6*e**4*x**4*log(d/e + x)/(2520*d**9*e**11 + 22680*d*
*8*e**12*x + 90720*d**7*e**13*x**2 + 211680*d**6*e**14*x**3 + 317520*d**5*e
**15*x**4 + 317520*d**4*e**16*x**5 + 211680*d**3*e**17*x**6 + 90720*d**2*e*
*18*x**7 + 22680*d*e**19*x**8 + 2520*e**20*x**9) - 3625020000*d**6*e**4*x**
4/(2520*d**9*e**11 + 22680*d**8*e**12*x + 90720*d**7*e**13*x**2 + 211680*d*
*6*e**14*x**3 + 317520*d**5*e**15*x**4 + 317520*d**4*e**16*x**5 + 211680*d*
*3*e**17*x**6 + 90720*d**2*e**18*x**7 + 22680*d*e**19*x**8 + 2520*e**20*x**
9) - 5292000*d**6*e**4*x**3*log(d/e + x)/(2520*d**9*e**11 + 22680*d**8*e**1
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2*x + 90720*d**7*e**13*x**2 + 211680*d**6*e**14*x**3 + 317520*d**5*e**15*x*
*4 + 317520*d**4*e**16*x**5 + 211680*d**3*e**17*x**6 + 90720*d**2*e**18*x**
7 + 22680*d*e**19*x**8 + 2520*e**20*x**9) - 12965400*d**6*e**4*x**3/(2520*d
**9*e**11 + 22680*d**8*e**12*x + 90720*d**7*e**13*x**2 + 211680*d**6*e**14*
x**3 + 317520*d**5*e**15*x**4 + 317520*d**4*e**16*x**5 + 211680*d**3*e**17*
x**6 + 90720*d**2*e**18*x**7 + 22680*d*e**19*x**8 + 2520*e**20*x**9) - 7711
200*d**6*e**4*x**2/(2520*d**9*e**11 + 22680*d**8*e**12*x + 90720*d**7*e**13
*x**2 + 211680*d**6*e**14*x**3 + 317520*d**5*e**15*x**4 + 317520*d**4*e**16
*x**5 + 211680*d**3*e**17*x**6 + 90720*d**2*e**18*x**7 + 22680*d*e**19*x**8
 + 2520*e**20*x**9) + 30870*d**6*e**4*x/(2520*d**9*e**11 + 22680*d**8*e**12
*x + 90720*d**7*e**13*x**2 + 211680*d**6*e**14*x**3 + 317520*d**5*e**15*x**
4 + 317520*d**4*e**16*x**5 + 211680*d**3*e**17*x**6 + 90720*d**2*e**18*x**7
 + 22680*d*e**19*x**8 + 2520*e**20*x**9) - 9990*d**6*e**4/(2520*d**9*e**11
+ 22680*d**8*e**12*x + 90720*d**7*e**13*x**2 + 211680*d**6*e**14*x**3 + 317
520*d**5*e**15*x**4 + 317520*d**4*e**16*x**5 + 211680*d**3*e**17*x**6 + 907
20*d**2*e**18*x**7 + 22680*d*e**19*x**8 + 2520*e**20*x**9) - 1587600000*d**
5*e**5*x**5*log(d/e + x)/(2520*d**9*e**11 + 22680*d**8*e**12*x + 90720*d**7
*e**13*x**2 + 211680*d**6*e**14*x**3 + 317520*d**5*e**15*x**4 + 317520*d**4
*e**16*x**5 + 211680*d**3*e**17*x**6 + 90720*d**2*e**18*x**7 + 22680*d*e**1
9*x**8 + 2520*e**20*x**9) - 3307500000*d**5*e**5*x**5/(2520*d**9*e**11 + 22
680*d**8*e**12*x + 90720*d**7*e**13*x**2 + 211680*d**6*e**14*x**3 + 317520*
d**5*e**15*x**4 + 317520*d**4*e**16*x**5 + 211680*d**3*e**17*x**6 + 90720*d
**2*e**18*x**7 + 22680*d*e**19*x**8 + 2520*e**20*x**9) - 7938000*d**5*e**5*
x**4*log(d/e + x)/(2520*d**9*e**11 + 22680*d**8*e**12*x + 90720*d**7*e**13*
x**2 + 211680*d**6*e**14*x**3 + 317520*d**5*e**15*x**4 + 317520*d**4*e**16*
x**5 + 211680*d**3*e**17*x**6 + 90720*d**2*e**18*x**7 + 22680*d*e**19*x**8
+ 2520*e**20*x**9) - 18125100*d**5*e**5*x**4/(2520*d**9*e**11 + 22680*d**8*
e**12*x + 90720*d**7*e**13*x**2 + 211680*d**6*e**14*x**3 + 317520*d**5*e**1
5*x**4 + 317520*d**4*e**16*x**5 + 211680*d**3*e**17*x**6 + 90720*d**2*e**18
*x**7 + 22680*d*e**19*x**8 + 2520*e**20*x**9) - 17992800*d**5*e**5*x**3/(25
20*d**9*e**11 + 22680*d**8*e**12*x + 90720*d**7*e**13*x**2 + 211680*d**6*e*
*14*x**3 + 317520*d**5*e**15*x**4 + 317520*d**4*e**16*x**5 + 211680*d**3*e*
*17*x**6 + 90720*d**2*e**18*x**7 + 22680*d*e**19*x**8 + 2520*e**20*x**9) +
123480*d**5*e**5*x**2/(2520*d**9*e**11 + 22680*d**8*e**12*x + 90720*d**7*e*
*13*x**2 + 211680*d**6*e**14*x**3 + 317520*d**5*e**15*x**4 + 317520*d**4*e*
*16*x**5 + 211680*d**3*e**17*x**6 + 90720*d**2*e**18*x**7 + 22680*d*e**19*x
**8 + 2520*e**20*x**9) - 89910*d**5*e**5*x/(2520*d**9*e**11 + 22680*d**8*e*
*12*x + 90720*d**7*e**13*x**2 + 211680*d**6*e**14*x**3 + 317520*d**5*e**15*
x**4 + 317520*d**4*e**16*x**5 + 211680*d**3*e**17*x**6 + 90720*d**2*e**18*x
**7 + 22680*d*e**19*x**8 + 2520*e**20*x**9) - 2550*d**5*e**5/(2520*d**9*e**
11 + 22680*d**8*e**12*x + 90720*d**7*e**13*x**2 + 211680*d**6*e**14*x**3 +
317520*d**5*e**15*x**4 + 317520*d**4*e**16*x**5 + 211680*d**3*e**17*x**6 +
90720*d**2*e**18*x**7 + 22680*d*e**19*x**8 + 2520*e**20*x**9) - 1058400000*
d**4*e**6*x**6*log(d/e + x)/(2520*d**9*e**11 + 22680*d**8*e**12*x + 90720*d
**7*e**13*x**2 + 211680*d**6*e**14*x**3 + 317520*d**5*e**15*x**4 + 317520*d
```

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**4*e**16*x**5 + 211680*d**3*e**17*x**6 + 90720*d**2*e**18*x**7 + 22680*d*e
**19*x**8 + 2520*e**20*x**9) - 1940400000*d**4*e**6*x**6/(2520*d**9*e**11 +
 22680*d**8*e**12*x + 90720*d**7*e**13*x**2 + 211680*d**6*e**14*x**3 + 3175
20*d**5*e**15*x**4 + 317520*d**4*e**16*x**5 + 211680*d**3*e**17*x**6 + 9072
0*d**2*e**18*x**7 + 22680*d*e**19*x**8 + 2520*e**20*x**9) - 7938000*d**4*e*
*6*x**5*log(d/e + x)/(2520*d**9*e**11 + 22680*d**8*e**12*x + 90720*d**7*e**
13*x**2 + 211680*d**6*e**14*x**3 + 317520*d**5*e**15*x**4 + 317520*d**4*e**
16*x**5 + 211680*d**3*e**17*x**6 + 90720*d**2*e**18*x**7 + 22680*d*e**19*x*
*8 + 2520*e**20*x**9) - 16537500*d**4*e**6*x**5/(2520*d**9*e**11 + 22680*d*
*8*e**12*x + 90720*d**7*e**13*x**2 + 211680*d**6*e**14*x**3 + 317520*d**5*e
**15*x**4 + 317520*d**4*e**16*x**5 + 211680*d**3*e**17*x**6 + 90720*d**2*e*
*18*x**7 + 22680*d*e**19*x**8 + 2520*e**20*x**9) - 26989200*d**4*e**6*x**4/
(2520*d**9*e**11 + 22680*d**8*e**12*x + 90720*d**7*e**13*x**2 + 211680*d**6
*e**14*x**3 + 317520*d**5*e**15*x**4 + 317520*d**4*e**16*x**5 + 211680*d**3
*e**17*x**6 + 90720*d**2*e**18*x**7 + 22680*d*e**19*x**8 + 2520*e**20*x**9)
 + 288120*d**4*e**6*x**3/(2520*d**9*e**11 + 22680*d**8*e**12*x + 90720*d**7
*e**13*x**2 + 211680*d**6*e**14*x**3 + 317520*d**5*e**15*x**4 + 317520*d**4
*e**16*x**5 + 211680*d**3*e**17*x**6 + 90720*d**2*e**18*x**7 + 22680*d*e**1
9*x**8 + 2520*e**20*x**9) - 359640*d**4*e**6*x**2/(2520*d**9*e**11 + 22680*
d**8*e**12*x + 90720*d**7*e**13*x**2 + 211680*d**6*e**14*x**3 + 317520*d**5
*e**15*x**4 + 317520*d**4*e**16*x**5 + 211680*d**3*e**17*x**6 + 90720*d**2*
e**18*x**7 + 22680*d*e**19*x**8 + 2520*e**20*x**9) - 22950*d**4*e**6*x/(252
0*d**9*e**11 + 22680*d**8*e**12*x + 90720*d**7*e**13*x**2 + 211680*d**6*e**
14*x**3 + 317520*d**5*e**15*x**4 + 317520*d**4*e**16*x**5 + 211680*d**3*e**
17*x**6 + 90720*d**2*e**18*x**7 + 22680*d*e**19*x**8 + 2520*e**20*x**9) - 4
436*d**4*e**6/(2520*d**9*e**11 + 22680*d**8*e**12*x + 90720*d**7*e**13*x**2
 + 211680*d**6*e**14*x**3 + 317520*d**5*e**15*x**4 + 317520*d**4*e**16*x**5
 + 211680*d**3*e**17*x**6 + 90720*d**2*e**18*x**7 + 22680*d*e**19*x**8 + 25
20*e**20*x**9) - 453600000*d**3*e**7*x**7*log(d/e + x)/(2520*d**9*e**11 + 2
2680*d**8*e**12*x + 90720*d**7*e**13*x**2 + 211680*d**6*e**14*x**3 + 317520
*d**5*e**15*x**4 + 317520*d**4*e**16*x**5 + 211680*d**3*e**17*x**6 + 90720*
d**2*e**18*x**7 + 22680*d*e**19*x**8 + 2520*e**20*x**9) - 680400000*d**3*e*
*7*x**7/(2520*d**9*e**11 + 22680*d**8*e**12*x + 90720*d**7*e**13*x**2 + 211
680*d**6*e**14*x**3 + 317520*d**5*e**15*x**4 + 317520*d**4*e**16*x**5 + 211
680*d**3*e**17*x**6 + 90720*d**2*e**18*x**7 + 22680*d*e**19*x**8 + 2520*e**
20*x**9) - 5292000*d**3*e**7*x**6*log(d/e + x)/(2520*d**9*e**11 + 22680*d**
8*e**12*x + 90720*d**7*e**13*x**2 + 211680*d**6*e**14*x**3 + 317520*d**5*e*
*15*x**4 + 317520*d**4*e**16*x**5 + 211680*d**3*e**17*x**6 + 90720*d**2*e**
18*x**7 + 22680*d*e**19*x**8 + 2520*e**20*x**9) - 9702000*d**3*e**7*x**6/(2
520*d**9*e**11 + 22680*d**8*e**12*x + 90720*d**7*e**13*x**2 + 211680*d**6*e
**14*x**3 + 317520*d**5*e**15*x**4 + 317520*d**4*e**16*x**5 + 211680*d**3*e
**17*x**6 + 90720*d**2*e**18*x**7 + 22680*d*e**19*x**8 + 2520*e**20*x**9) -
 26989200*d**3*e**7*x**5/(2520*d**9*e**11 + 22680*d**8*e**12*x + 90720*d**7
*e**13*x**2 + 211680*d**6*e**14*x**3 + 317520*d**5*e**15*x**4 + 317520*d**4
*e**16*x**5 + 211680*d**3*e**17*x**6 + 90720*d**2*e**18*x**7 + 22680*d*e**1
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9*x**8 + 2520*e**20*x**9) + 432180*d**3*e**7*x**4/(2520*d**9*e**11 + 22680*
d**8*e**12*x + 90720*d**7*e**13*x**2 + 211680*d**6*e**14*x**3 + 317520*d**5
*e**15*x**4 + 317520*d**4*e**16*x**5 + 211680*d**3*e**17*x**6 + 90720*d**2*
e**18*x**7 + 22680*d*e**19*x**8 + 2520*e**20*x**9) - 839160*d**3*e**7*x**3/
(2520*d**9*e**11 + 22680*d**8*e**12*x + 90720*d**7*e**13*x**2 + 211680*d**6
*e**14*x**3 + 317520*d**5*e**15*x**4 + 317520*d**4*e**16*x**5 + 211680*d**3
*e**17*x**6 + 90720*d**2*e**18*x**7 + 22680*d*e**19*x**8 + 2520*e**20*x**9)
 - 91800*d**3*e**7*x**2/(2520*d**9*e**11 + 22680*d**8*e**12*x + 90720*d**7*
e**13*x**2 + 211680*d**6*e**14*x**3 + 317520*d**5*e**15*x**4 + 317520*d**4*
e**16*x**5 + 211680*d**3*e**17*x**6 + 90720*d**2*e**18*x**7 + 22680*d*e**19
*x**8 + 2520*e**20*x**9) - 39924*d**3*e**7*x/(2520*d**9*e**11 + 22680*d**8*
e**12*x + 90720*d**7*e**13*x**2 + 211680*d**6*e**14*x**3 + 317520*d**5*e**1
5*x**4 + 317520*d**4*e**16*x**5 + 211680*d**3*e**17*x**6 + 90720*d**2*e**18
*x**7 + 22680*d*e**19*x**8 + 2520*e**20*x**9) - 2870*d**3*e**7/(2520*d**9*e
**11 + 22680*d**8*e**12*x + 90720*d**7*e**13*x**2 + 211680*d**6*e**14*x**3
+ 317520*d**5*e**15*x**4 + 317520*d**4*e**16*x**5 + 211680*d**3*e**17*x**6
+ 90720*d**2*e**18*x**7 + 22680*d*e**19*x**8 + 2520*e**20*x**9) - 113400000
*d**2*e**8*x**8*log(d/e + x)/(2520*d**9*e**11 + 22680*d**8*e**12*x + 90720*
d**7*e**13*x**2 + 211680*d**6*e**14*x**3 + 317520*d**5*e**15*x**4 + 317520*
d**4*e**16*x**5 + 211680*d**3*e**17*x**6 + 90720*d**2*e**18*x**7 + 22680*d*
e**19*x**8 + 2520*e**20*x**9) - 113400000*d**2*e**8*x**8/(2520*d**9*e**11 +
 22680*d**8*e**12*x + 90720*d**7*e**13*x**2 + 211680*d**6*e**14*x**3 + 3175
20*d**5*e**15*x**4 + 317520*d**4*e**16*x**5 + 211680*d**3*e**17*x**6 + 9072
0*d**2*e**18*x**7 + 22680*d*e**19*x**8 + 2520*e**20*x**9) - 2268000*d**2*e*
*8*x**7*log(d/e + x)/(2520*d**9*e**11 + 22680*d**8*e**12*x + 90720*d**7*e**
13*x**2 + 211680*d**6*e**14*x**3 + 317520*d**5*e**15*x**4 + 317520*d**4*e**
16*x**5 + 211680*d**3*e**17*x**6 + 90720*d**2*e**18*x**7 + 22680*d*e**19*x*
*8 + 2520*e**20*x**9) - 3402000*d**2*e**8*x**7/(2520*d**9*e**11 + 22680*d**
8*e**12*x + 90720*d**7*e**13*x**2 + 211680*d**6*e**14*x**3 + 317520*d**5*e*
*15*x**4 + 317520*d**4*e**16*x**5 + 211680*d**3*e**17*x**6 + 90720*d**2*e**
18*x**7 + 22680*d*e**19*x**8 + 2520*e**20*x**9) - 17992800*d**2*e**8*x**6/(
2520*d**9*e**11 + 22680*d**8*e**12*x + 90720*d**7*e**13*x**2 + 211680*d**6*
e**14*x**3 + 317520*d**5*e**15*x**4 + 317520*d**4*e**16*x**5 + 211680*d**3*
e**17*x**6 + 90720*d**2*e**18*x**7 + 22680*d*e**19*x**8 + 2520*e**20*x**9)
+ 432180*d**2*e**8*x**5/(2520*d**9*e**11 + 22680*d**8*e**12*x + 90720*d**7*
e**13*x**2 + 211680*d**6*e**14*x**3 + 317520*d**5*e**15*x**4 + 317520*d**4*
e**16*x**5 + 211680*d**3*e**17*x**6 + 90720*d**2*e**18*x**7 + 22680*d*e**19
*x**8 + 2520*e**20*x**9) - 1258740*d**2*e**8*x**4/(2520*d**9*e**11 + 22680*
d**8*e**12*x + 90720*d**7*e**13*x**2 + 211680*d**6*e**14*x**3 + 317520*d**5
*e**15*x**4 + 317520*d**4*e**16*x**5 + 211680*d**3*e**17*x**6 + 90720*d**2*
e**18*x**7 + 22680*d*e**19*x**8 + 2520*e**20*x**9) - 214200*d**2*e**8*x**3/
(2520*d**9*e**11 + 22680*d**8*e**12*x + 90720*d**7*e**13*x**2 + 211680*d**6
*e**14*x**3 + 317520*d**5*e**15*x**4 + 317520*d**4*e**16*x**5 + 211680*d**3
*e**17*x**6 + 90720*d**2*e**18*x**7 + 22680*d*e**19*x**8 + 2520*e**20*x**9)
 - 159696*d**2*e**8*x**2/(2520*d**9*e**11 + 22680*d**8*e**12*x + 90720*d**7
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*e**13*x**2 + 211680*d**6*e**14*x**3 + 317520*d**5*e**15*x**4 + 317520*d**4
*e**16*x**5 + 211680*d**3*e**17*x**6 + 90720*d**2*e**18*x**7 + 22680*d*e**1
9*x**8 + 2520*e**20*x**9) - 25830*d**2*e**8*x/(2520*d**9*e**11 + 22680*d**8
*e**12*x + 90720*d**7*e**13*x**2 + 211680*d**6*e**14*x**3 + 317520*d**5*e**
15*x**4 + 317520*d**4*e**16*x**5 + 211680*d**3*e**17*x**6 + 90720*d**2*e**1
8*x**7 + 22680*d*e**19*x**8 + 2520*e**20*x**9) - 4770*d**2*e**8/(2520*d**9*
e**11 + 22680*d**8*e**12*x + 90720*d**7*e**13*x**2 + 211680*d**6*e**14*x**3
 + 317520*d**5*e**15*x**4 + 317520*d**4*e**16*x**5 + 211680*d**3*e**17*x**6
 + 90720*d**2*e**18*x**7 + 22680*d*e**19*x**8 + 2520*e**20*x**9) - 12600000
*d*e**9*x**9*log(d/e + x)/(2520*d**9*e**11 + 22680*d**8*e**12*x + 90720*d**
7*e**13*x**2 + 211680*d**6*e**14*x**3 + 317520*d**5*e**15*x**4 + 317520*d**
4*e**16*x**5 + 211680*d**3*e**17*x**6 + 90720*d**2*e**18*x**7 + 22680*d*e**
19*x**8 + 2520*e**20*x**9) - 567000*d*e**9*x**8*log(d/e + x)/(2520*d**9*e**
11 + 22680*d**8*e**12*x + 90720*d**7*e**13*x**2 + 211680*d**6*e**14*x**3 +
317520*d**5*e**15*x**4 + 317520*d**4*e**16*x**5 + 211680*d**3*e**17*x**6 +
90720*d**2*e**18*x**7 + 22680*d*e**19*x**8 + 2520*e**20*x**9) - 567000*d*e*
*9*x**8/(2520*d**9*e**11 + 22680*d**8*e**12*x + 90720*d**7*e**13*x**2 + 211
680*d**6*e**14*x**3 + 317520*d**5*e**15*x**4 + 317520*d**4*e**16*x**5 + 211
680*d**3*e**17*x**6 + 90720*d**2*e**18*x**7 + 22680*d*e**19*x**8 + 2520*e**
20*x**9) - 7711200*d*e**9*x**7/(2520*d**9*e**11 + 22680*d**8*e**12*x + 9072
0*d**7*e**13*x**2 + 211680*d**6*e**14*x**3 + 317520*d**5*e**15*x**4 + 31752
0*d**4*e**16*x**5 + 211680*d**3*e**17*x**6 + 90720*d**2*e**18*x**7 + 22680*
d*e**19*x**8 + 2520*e**20*x**9) + 288120*d*e**9*x**6/(2520*d**9*e**11 + 226
80*d**8*e**12*x + 90720*d**7*e**13*x**2 + 211680*d**6*e**14*x**3 + 317520*d
**5*e**15*x**4 + 317520*d**4*e**16*x**5 + 211680*d**3*e**17*x**6 + 90720*d*
*2*e**18*x**7 + 22680*d*e**19*x**8 + 2520*e**20*x**9) - 1258740*d*e**9*x**5
/(2520*d**9*e**11 + 22680*d**8*e**12*x + 90720*d**7*e**13*x**2 + 211680*d**
6*e**14*x**3 + 317520*d**5*e**15*x**4 + 317520*d**4*e**16*x**5 + 211680*d**
3*e**17*x**6 + 90720*d**2*e**18*x**7 + 22680*d*e**19*x**8 + 2520*e**20*x**9
) - 321300*d*e**9*x**4/(2520*d**9*e**11 + 22680*d**8*e**12*x + 90720*d**7*e
**13*x**2 + 211680*d**6*e**14*x**3 + 317520*d**5*e**15*x**4 + 317520*d**4*e
**16*x**5 + 211680*d**3*e**17*x**6 + 90720*d**2*e**18*x**7 + 22680*d*e**19*
x**8 + 2520*e**20*x**9) - 372624*d*e**9*x**3/(2520*d**9*e**11 + 22680*d**8*
e**12*x + 90720*d**7*e**13*x**2 + 211680*d**6*e**14*x**3 + 317520*d**5*e**1
5*x**4 + 317520*d**4*e**16*x**5 + 211680*d**3*e**17*x**6 + 90720*d**2*e**18
*x**7 + 22680*d*e**19*x**8 + 2520*e**20*x**9) - 103320*d*e**9*x**2/(2520*d*
*9*e**11 + 22680*d**8*e**12*x + 90720*d**7*e**13*x**2 + 211680*d**6*e**14*x
**3 + 317520*d**5*e**15*x**4 + 317520*d**4*e**16*x**5 + 211680*d**3*e**17*x
**6 + 90720*d**2*e**18*x**7 + 22680*d*e**19*x**8 + 2520*e**20*x**9) - 42930
*d*e**9*x/(2520*d**9*e**11 + 22680*d**8*e**12*x + 90720*d**7*e**13*x**2 + 2
11680*d**6*e**14*x**3 + 317520*d**5*e**15*x**4 + 317520*d**4*e**16*x**5 + 2
11680*d**3*e**17*x**6 + 90720*d**2*e**18*x**7 + 22680*d*e**19*x**8 + 2520*e
**20*x**9) - 4725*d*e**9/(2520*d**9*e**11 + 22680*d**8*e**12*x + 90720*d**7
*e**13*x**2 + 211680*d**6*e**14*x**3 + 317520*d**5*e**15*x**4 + 317520*d**4
*e**16*x**5 + 211680*d**3*e**17*x**6 + 90720*d**2*e**18*x**7 + 22680*d*e**1
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9*x**8 + 2520*e**20*x**9) + 1260000*e**10*x**10/(2520*d**9*e**11 + 22680*d*
*8*e**12*x + 90720*d**7*e**13*x**2 + 211680*d**6*e**14*x**3 + 317520*d**5*e
**15*x**4 + 317520*d**4*e**16*x**5 + 211680*d**3*e**17*x**6 + 90720*d**2*e*
*18*x**7 + 22680*d*e**19*x**8 + 2520*e**20*x**9) - 63000*e**10*x**9*log(d/e
 + x)/(2520*d**9*e**11 + 22680*d**8*e**12*x + 90720*d**7*e**13*x**2 + 21168
0*d**6*e**14*x**3 + 317520*d**5*e**15*x**4 + 317520*d**4*e**16*x**5 + 21168
0*d**3*e**17*x**6 + 90720*d**2*e**18*x**7 + 22680*d*e**19*x**8 + 2520*e**20
*x**9) - 1927800*e**10*x**8/(2520*d**9*e**11 + 22680*d**8*e**12*x + 90720*d
**7*e**13*x**2 + 211680*d**6*e**14*x**3 + 317520*d**5*e**15*x**4 + 317520*d
**4*e**16*x**5 + 211680*d**3*e**17*x**6 + 90720*d**2*e**18*x**7 + 22680*d*e
**19*x**8 + 2520*e**20*x**9) + 123480*e**10*x**7/(2520*d**9*e**11 + 22680*d
**8*e**12*x + 90720*d**7*e**13*x**2 + 211680*d**6*e**14*x**3 + 317520*d**5*
e**15*x**4 + 317520*d**4*e**16*x**5 + 211680*d**3*e**17*x**6 + 90720*d**2*e
**18*x**7 + 22680*d*e**19*x**8 + 2520*e**20*x**9) - 839160*e**10*x**6/(2520
*d**9*e**11 + 22680*d**8*e**12*x + 90720*d**7*e**13*x**2 + 211680*d**6*e**1
4*x**3 + 317520*d**5*e**15*x**4 + 317520*d**4*e**16*x**5 + 211680*d**3*e**1
7*x**6 + 90720*d**2*e**18*x**7 + 22680*d*e**19*x**8 + 2520*e**20*x**9) - 32
1300*e**10*x**5/(2520*d**9*e**11 + 22680*d**8*e**12*x + 90720*d**7*e**13*x*
*2 + 211680*d**6*e**14*x**3 + 317520*d**5*e**15*x**4 + 317520*d**4*e**16*x*
*5 + 211680*d**3*e**17*x**6 + 90720*d**2*e**18*x**7 + 22680*d*e**19*x**8 +
2520*e**20*x**9) - 558936*e**10*x**4/(2520*d**9*e**11 + 22680*d**8*e**12*x
+ 90720*d**7*e**13*x**2 + 211680*d**6*e**14*x**3 + 317520*d**5*e**15*x**4 +
 317520*d**4*e**16*x**5 + 211680*d**3*e**17*x**6 + 90720*d**2*e**18*x**7 +
22680*d*e**19*x**8 + 2520*e**20*x**9) - 241080*e**10*x**3/(2520*d**9*e**11
+ 22680*d**8*e**12*x + 90720*d**7*e**13*x**2 + 211680*d**6*e**14*x**3 + 317
520*d**5*e**15*x**4 + 317520*d**4*e**16*x**5 + 211680*d**3*e**17*x**6 + 907
20*d**2*e**18*x**7 + 22680*d*e**19*x**8 + 2520*e**20*x**9) - 171720*e**10*x
**2/(2520*d**9*e**11 + 22680*d**8*e**12*x + 90720*d**7*e**13*x**2 + 211680*
d**6*e**14*x**3 + 317520*d**5*e**15*x**4 + 317520*d**4*e**16*x**5 + 211680*
d**3*e**17*x**6 + 90720*d**2*e**18*x**7 + 22680*d*e**19*x**8 + 2520*e**20*x
**9) - 42525*e**10*x/(2520*d**9*e**11 + 22680*d**8*e**12*x + 90720*d**7*e**
13*x**2 + 211680*d**6*e**14*x**3 + 317520*d**5*e**15*x**4 + 317520*d**4*e**
16*x**5 + 211680*d**3*e**17*x**6 + 90720*d**2*e**18*x**7 + 22680*d*e**19*x*
*8 + 2520*e**20*x**9) - 15120*e**10/(2520*d**9*e**11 + 22680*d**8*e**12*x +
 90720*d**7*e**13*x**2 + 211680*d**6*e**14*x**3 + 317520*d**5*e**15*x**4 +
317520*d**4*e**16*x**5 + 211680*d**3*e**17*x**6 + 90720*d**2*e**18*x**7 + 2
2680*d*e**19*x**8 + 2520*e**20*x**9), Eq(m, -10)), (6300000*d**10*log(d/e +
 x)/(280*d**8*e**11 + 2240*d**7*e**12*x + 7840*d**6*e**13*x**2 + 15680*d**5
*e**14*x**3 + 19600*d**4*e**15*x**4 + 15680*d**3*e**16*x**5 + 7840*d**2*e**
17*x**6 + 2240*d*e**18*x**7 + 280*e**19*x**8) + 17122500*d**10/(280*d**8*e*
*11 + 2240*d**7*e**12*x + 7840*d**6*e**13*x**2 + 15680*d**5*e**14*x**3 + 19
600*d**4*e**15*x**4 + 15680*d**3*e**16*x**5 + 7840*d**2*e**17*x**6 + 2240*d
*e**18*x**7 + 280*e**19*x**8) + 50400000*d**9*e*x*log(d/e + x)/(280*d**8*e*
*11 + 2240*d**7*e**12*x + 7840*d**6*e**13*x**2 + 15680*d**5*e**14*x**3 + 19
600*d**4*e**15*x**4 + 15680*d**3*e**16*x**5 + 7840*d**2*e**17*x**6 + 2240*d
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*e**18*x**7 + 280*e**19*x**8) + 130680000*d**9*e*x/(280*d**8*e**11 + 2240*d
**7*e**12*x + 7840*d**6*e**13*x**2 + 15680*d**5*e**14*x**3 + 19600*d**4*e**
15*x**4 + 15680*d**3*e**16*x**5 + 7840*d**2*e**17*x**6 + 2240*d*e**18*x**7
+ 280*e**19*x**8) + 63000*d**9*e*log(d/e + x)/(280*d**8*e**11 + 2240*d**7*e
**12*x + 7840*d**6*e**13*x**2 + 15680*d**5*e**14*x**3 + 19600*d**4*e**15*x*
*4 + 15680*d**3*e**16*x**5 + 7840*d**2*e**17*x**6 + 2240*d*e**18*x**7 + 280
*e**19*x**8) + 171225*d**9*e/(280*d**8*e**11 + 2240*d**7*e**12*x + 7840*d**
6*e**13*x**2 + 15680*d**5*e**14*x**3 + 19600*d**4*e**15*x**4 + 15680*d**3*e
**16*x**5 + 7840*d**2*e**17*x**6 + 2240*d*e**18*x**7 + 280*e**19*x**8) + 17
6400000*d**8*e**2*x**2*log(d/e + x)/(280*d**8*e**11 + 2240*d**7*e**12*x + 7
840*d**6*e**13*x**2 + 15680*d**5*e**14*x**3 + 19600*d**4*e**15*x**4 + 15680
*d**3*e**16*x**5 + 7840*d**2*e**17*x**6 + 2240*d*e**18*x**7 + 280*e**19*x**
8) + 432180000*d**8*e**2*x**2/(280*d**8*e**11 + 2240*d**7*e**12*x + 7840*d*
*6*e**13*x**2 + 15680*d**5*e**14*x**3 + 19600*d**4*e**15*x**4 + 15680*d**3*
e**16*x**5 + 7840*d**2*e**17*x**6 + 2240*d*e**18*x**7 + 280*e**19*x**8) + 5
04000*d**8*e**2*x*log(d/e + x)/(280*d**8*e**11 + 2240*d**7*e**12*x + 7840*d
**6*e**13*x**2 + 15680*d**5*e**14*x**3 + 19600*d**4*e**15*x**4 + 15680*d**3
*e**16*x**5 + 7840*d**2*e**17*x**6 + 2240*d*e**18*x**7 + 280*e**19*x**8) +
1306800*d**8*e**2*x/(280*d**8*e**11 + 2240*d**7*e**12*x + 7840*d**6*e**13*x
**2 + 15680*d**5*e**14*x**3 + 19600*d**4*e**15*x**4 + 15680*d**3*e**16*x**5
 + 7840*d**2*e**17*x**6 + 2240*d*e**18*x**7 + 280*e**19*x**8) + 214200*d**8
*e**2*log(d/e + x)/(280*d**8*e**11 + 2240*d**7*e**12*x + 7840*d**6*e**13*x*
*2 + 15680*d**5*e**14*x**3 + 19600*d**4*e**15*x**4 + 15680*d**3*e**16*x**5
+ 7840*d**2*e**17*x**6 + 2240*d*e**18*x**7 + 280*e**19*x**8) + 582165*d**8*
e**2/(280*d**8*e**11 + 2240*d**7*e**12*x + 7840*d**6*e**13*x**2 + 15680*d**
5*e**14*x**3 + 19600*d**4*e**15*x**4 + 15680*d**3*e**16*x**5 + 7840*d**2*e*
*17*x**6 + 2240*d*e**18*x**7 + 280*e**19*x**8) + 352800000*d**7*e**3*x**3*l
og(d/e + x)/(280*d**8*e**11 + 2240*d**7*e**12*x + 7840*d**6*e**13*x**2 + 15
680*d**5*e**14*x**3 + 19600*d**4*e**15*x**4 + 15680*d**3*e**16*x**5 + 7840*
d**2*e**17*x**6 + 2240*d*e**18*x**7 + 280*e**19*x**8) + 805560000*d**7*e**3
*x**3/(280*d**8*e**11 + 2240*d**7*e**12*x + 7840*d**6*e**13*x**2 + 15680*d*
*5*e**14*x**3 + 19600*d**4*e**15*x**4 + 15680*d**3*e**16*x**5 + 7840*d**2*e
**17*x**6 + 2240*d*e**18*x**7 + 280*e**19*x**8) + 1764000*d**7*e**3*x**2*lo
g(d/e + x)/(280*d**8*e**11 + 2240*d**7*e**12*x + 7840*d**6*e**13*x**2 + 156
80*d**5*e**14*x**3 + 19600*d**4*e**15*x**4 + 15680*d**3*e**16*x**5 + 7840*d
**2*e**17*x**6 + 2240*d*e**18*x**7 + 280*e**19*x**8) + 4321800*d**7*e**3*x*
*2/(280*d**8*e**11 + 2240*d**7*e**12*x + 7840*d**6*e**13*x**2 + 15680*d**5*
e**14*x**3 + 19600*d**4*e**15*x**4 + 15680*d**3*e**16*x**5 + 7840*d**2*e**1
7*x**6 + 2240*d*e**18*x**7 + 280*e**19*x**8) + 1713600*d**7*e**3*x*log(d/e
+ x)/(280*d**8*e**11 + 2240*d**7*e**12*x + 7840*d**6*e**13*x**2 + 15680*d**
5*e**14*x**3 + 19600*d**4*e**15*x**4 + 15680*d**3*e**16*x**5 + 7840*d**2*e*
*17*x**6 + 2240*d*e**18*x**7 + 280*e**19*x**8) + 4443120*d**7*e**3*x/(280*d
**8*e**11 + 2240*d**7*e**12*x + 7840*d**6*e**13*x**2 + 15680*d**5*e**14*x**
3 + 19600*d**4*e**15*x**4 + 15680*d**3*e**16*x**5 + 7840*d**2*e**17*x**6 +
2240*d*e**18*x**7 + 280*e**19*x**8) + 3430*d**7*e**3/(280*d**8*e**11 + 2240
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*d**7*e**12*x + 7840*d**6*e**13*x**2 + 15680*d**5*e**14*x**3 + 19600*d**4*e
**15*x**4 + 15680*d**3*e**16*x**5 + 7840*d**2*e**17*x**6 + 2240*d*e**18*x**
7 + 280*e**19*x**8) + 441000000*d**6*e**4*x**4*log(d/e + x)/(280*d**8*e**11
 + 2240*d**7*e**12*x + 7840*d**6*e**13*x**2 + 15680*d**5*e**14*x**3 + 19600
*d**4*e**15*x**4 + 15680*d**3*e**16*x**5 + 7840*d**2*e**17*x**6 + 2240*d*e*
*18*x**7 + 280*e**19*x**8) + 918750000*d**6*e**4*x**4/(280*d**8*e**11 + 224
0*d**7*e**12*x + 7840*d**6*e**13*x**2 + 15680*d**5*e**14*x**3 + 19600*d**4*
e**15*x**4 + 15680*d**3*e**16*x**5 + 7840*d**2*e**17*x**6 + 2240*d*e**18*x*
*7 + 280*e**19*x**8) + 3528000*d**6*e**4*x**3*log(d/e + x)/(280*d**8*e**11
+ 2240*d**7*e**12*x + 7840*d**6*e**13*x**2 + 15680*d**5*e**14*x**3 + 19600*
d**4*e**15*x**4 + 15680*d**3*e**16*x**5 + 7840*d**2*e**17*x**6 + 2240*d*e**
18*x**7 + 280*e**19*x**8) + 8055600*d**6*e**4*x**3/(280*d**8*e**11 + 2240*d
**7*e**12*x + 7840*d**6*e**13*x**2 + 15680*d**5*e**14*x**3 + 19600*d**4*e**
15*x**4 + 15680*d**3*e**16*x**5 + 7840*d**2*e**17*x**6 + 2240*d*e**18*x**7
+ 280*e**19*x**8) + 5997600*d**6*e**4*x**2*log(d/e + x)/(280*d**8*e**11 + 2
240*d**7*e**12*x + 7840*d**6*e**13*x**2 + 15680*d**5*e**14*x**3 + 19600*d**
4*e**15*x**4 + 15680*d**3*e**16*x**5 + 7840*d**2*e**17*x**6 + 2240*d*e**18*
x**7 + 280*e**19*x**8) + 14694120*d**6*e**4*x**2/(280*d**8*e**11 + 2240*d**
7*e**12*x + 7840*d**6*e**13*x**2 + 15680*d**5*e**14*x**3 + 19600*d**4*e**15
*x**4 + 15680*d**3*e**16*x**5 + 7840*d**2*e**17*x**6 + 2240*d*e**18*x**7 +
280*e**19*x**8) + 27440*d**6*e**4*x/(280*d**8*e**11 + 2240*d**7*e**12*x + 7
840*d**6*e**13*x**2 + 15680*d**5*e**14*x**3 + 19600*d**4*e**15*x**4 + 15680
*d**3*e**16*x**5 + 7840*d**2*e**17*x**6 + 2240*d*e**18*x**7 + 280*e**19*x**
8) - 4995*d**6*e**4/(280*d**8*e**11 + 2240*d**7*e**12*x + 7840*d**6*e**13*x
**2 + 15680*d**5*e**14*x**3 + 19600*d**4*e**15*x**4 + 15680*d**3*e**16*x**5
 + 7840*d**2*e**17*x**6 + 2240*d*e**18*x**7 + 280*e**19*x**8) + 352800000*d
**5*e**5*x**5*log(d/e + x)/(280*d**8*e**11 + 2240*d**7*e**12*x + 7840*d**6*
e**13*x**2 + 15680*d**5*e**14*x**3 + 19600*d**4*e**15*x**4 + 15680*d**3*e**
16*x**5 + 7840*d**2*e**17*x**6 + 2240*d*e**18*x**7 + 280*e**19*x**8) + 6468
00000*d**5*e**5*x**5/(280*d**8*e**11 + 2240*d**7*e**12*x + 7840*d**6*e**13*
x**2 + 15680*d**5*e**14*x**3 + 19600*d**4*e**15*x**4 + 15680*d**3*e**16*x**
5 + 7840*d**2*e**17*x**6 + 2240*d*e**18*x**7 + 280*e**19*x**8) + 4410000*d*
*5*e**5*x**4*log(d/e + x)/(280*d**8*e**11 + 2240*d**7*e**12*x + 7840*d**6*e
**13*x**2 + 15680*d**5*e**14*x**3 + 19600*d**4*e**15*x**4 + 15680*d**3*e**1
6*x**5 + 7840*d**2*e**17*x**6 + 2240*d*e**18*x**7 + 280*e**19*x**8) + 91875
00*d**5*e**5*x**4/(280*d**8*e**11 + 2240*d**7*e**12*x + 7840*d**6*e**13*x**
2 + 15680*d**5*e**14*x**3 + 19600*d**4*e**15*x**4 + 15680*d**3*e**16*x**5 +
 7840*d**2*e**17*x**6 + 2240*d*e**18*x**7 + 280*e**19*x**8) + 11995200*d**5
*e**5*x**3*log(d/e + x)/(280*d**8*e**11 + 2240*d**7*e**12*x + 7840*d**6*e**
13*x**2 + 15680*d**5*e**14*x**3 + 19600*d**4*e**15*x**4 + 15680*d**3*e**16*
x**5 + 7840*d**2*e**17*x**6 + 2240*d*e**18*x**7 + 280*e**19*x**8) + 2738904
0*d**5*e**5*x**3/(280*d**8*e**11 + 2240*d**7*e**12*x + 7840*d**6*e**13*x**2
 + 15680*d**5*e**14*x**3 + 19600*d**4*e**15*x**4 + 15680*d**3*e**16*x**5 +
7840*d**2*e**17*x**6 + 2240*d*e**18*x**7 + 280*e**19*x**8) + 96040*d**5*e**
5*x**2/(280*d**8*e**11 + 2240*d**7*e**12*x + 7840*d**6*e**13*x**2 + 15680*d
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**5*e**14*x**3 + 19600*d**4*e**15*x**4 + 15680*d**3*e**16*x**5 + 7840*d**2*
e**17*x**6 + 2240*d*e**18*x**7 + 280*e**19*x**8) - 39960*d**5*e**5*x/(280*d
**8*e**11 + 2240*d**7*e**12*x + 7840*d**6*e**13*x**2 + 15680*d**5*e**14*x**
3 + 19600*d**4*e**15*x**4 + 15680*d**3*e**16*x**5 + 7840*d**2*e**17*x**6 +
2240*d*e**18*x**7 + 280*e**19*x**8) - 850*d**5*e**5/(280*d**8*e**11 + 2240*
d**7*e**12*x + 7840*d**6*e**13*x**2 + 15680*d**5*e**14*x**3 + 19600*d**4*e*
*15*x**4 + 15680*d**3*e**16*x**5 + 7840*d**2*e**17*x**6 + 2240*d*e**18*x**7
 + 280*e**19*x**8) + 176400000*d**4*e**6*x**6*log(d/e + x)/(280*d**8*e**11
+ 2240*d**7*e**12*x + 7840*d**6*e**13*x**2 + 15680*d**5*e**14*x**3 + 19600*
d**4*e**15*x**4 + 15680*d**3*e**16*x**5 + 7840*d**2*e**17*x**6 + 2240*d*e**
18*x**7 + 280*e**19*x**8) + 264600000*d**4*e**6*x**6/(280*d**8*e**11 + 2240
*d**7*e**12*x + 7840*d**6*e**13*x**2 + 15680*d**5*e**14*x**3 + 19600*d**4*e
**15*x**4 + 15680*d**3*e**16*x**5 + 7840*d**2*e**17*x**6 + 2240*d*e**18*x**
7 + 280*e**19*x**8) + 3528000*d**4*e**6*x**5*log(d/e + x)/(280*d**8*e**11 +
 2240*d**7*e**12*x + 7840*d**6*e**13*x**2 + 15680*d**5*e**14*x**3 + 19600*d
**4*e**15*x**4 + 15680*d**3*e**16*x**5 + 7840*d**2*e**17*x**6 + 2240*d*e**1
8*x**7 + 280*e**19*x**8) + 6468000*d**4*e**6*x**5/(280*d**8*e**11 + 2240*d*
*7*e**12*x + 7840*d**6*e**13*x**2 + 15680*d**5*e**14*x**3 + 19600*d**4*e**1
5*x**4 + 15680*d**3*e**16*x**5 + 7840*d**2*e**17*x**6 + 2240*d*e**18*x**7 +
 280*e**19*x**8) + 14994000*d**4*e**6*x**4*log(d/e + x)/(280*d**8*e**11 + 2
240*d**7*e**12*x + 7840*d**6*e**13*x**2 + 15680*d**5*e**14*x**3 + 19600*d**
4*e**15*x**4 + 15680*d**3*e**16*x**5 + 7840*d**2*e**17*x**6 + 2240*d*e**18*
x**7 + 280*e**19*x**8) + 31237500*d**4*e**6*x**4/(280*d**8*e**11 + 2240*d**
7*e**12*x + 7840*d**6*e**13*x**2 + 15680*d**5*e**14*x**3 + 19600*d**4*e**15
*x**4 + 15680*d**3*e**16*x**5 + 7840*d**2*e**17*x**6 + 2240*d*e**18*x**7 +
280*e**19*x**8) + 192080*d**4*e**6*x**3/(280*d**8*e**11 + 2240*d**7*e**12*x
 + 7840*d**6*e**13*x**2 + 15680*d**5*e**14*x**3 + 19600*d**4*e**15*x**4 + 1
5680*d**3*e**16*x**5 + 7840*d**2*e**17*x**6 + 2240*d*e**18*x**7 + 280*e**19
*x**8) - 139860*d**4*e**6*x**2/(280*d**8*e**11 + 2240*d**7*e**12*x + 7840*d
**6*e**13*x**2 + 15680*d**5*e**14*x**3 + 19600*d**4*e**15*x**4 + 15680*d**3
*e**16*x**5 + 7840*d**2*e**17*x**6 + 2240*d*e**18*x**7 + 280*e**19*x**8) -
6800*d**4*e**6*x/(280*d**8*e**11 + 2240*d**7*e**12*x + 7840*d**6*e**13*x**2
 + 15680*d**5*e**14*x**3 + 19600*d**4*e**15*x**4 + 15680*d**3*e**16*x**5 +
7840*d**2*e**17*x**6 + 2240*d*e**18*x**7 + 280*e**19*x**8) - 1109*d**4*e**6
/(280*d**8*e**11 + 2240*d**7*e**12*x + 7840*d**6*e**13*x**2 + 15680*d**5*e*
*14*x**3 + 19600*d**4*e**15*x**4 + 15680*d**3*e**16*x**5 + 7840*d**2*e**17*
x**6 + 2240*d*e**18*x**7 + 280*e**19*x**8) + 50400000*d**3*e**7*x**7*log(d/
e + x)/(280*d**8*e**11 + 2240*d**7*e**12*x + 7840*d**6*e**13*x**2 + 15680*d
**5*e**14*x**3 + 19600*d**4*e**15*x**4 + 15680*d**3*e**16*x**5 + 7840*d**2*
e**17*x**6 + 2240*d*e**18*x**7 + 280*e**19*x**8) + 50400000*d**3*e**7*x**7/
(280*d**8*e**11 + 2240*d**7*e**12*x + 7840*d**6*e**13*x**2 + 15680*d**5*e**
14*x**3 + 19600*d**4*e**15*x**4 + 15680*d**3*e**16*x**5 + 7840*d**2*e**17*x
**6 + 2240*d*e**18*x**7 + 280*e**19*x**8) + 1764000*d**3*e**7*x**6*log(d/e
+ x)/(280*d**8*e**11 + 2240*d**7*e**12*x + 7840*d**6*e**13*x**2 + 15680*d**
5*e**14*x**3 + 19600*d**4*e**15*x**4 + 15680*d**3*e**16*x**5 + 7840*d**2*e*
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*17*x**6 + 2240*d*e**18*x**7 + 280*e**19*x**8) + 2646000*d**3*e**7*x**6/(28
0*d**8*e**11 + 2240*d**7*e**12*x + 7840*d**6*e**13*x**2 + 15680*d**5*e**14*
x**3 + 19600*d**4*e**15*x**4 + 15680*d**3*e**16*x**5 + 7840*d**2*e**17*x**6
 + 2240*d*e**18*x**7 + 280*e**19*x**8) + 11995200*d**3*e**7*x**5*log(d/e +
x)/(280*d**8*e**11 + 2240*d**7*e**12*x + 7840*d**6*e**13*x**2 + 15680*d**5*
e**14*x**3 + 19600*d**4*e**15*x**4 + 15680*d**3*e**16*x**5 + 7840*d**2*e**1
7*x**6 + 2240*d*e**18*x**7 + 280*e**19*x**8) + 21991200*d**3*e**7*x**5/(280
*d**8*e**11 + 2240*d**7*e**12*x + 7840*d**6*e**13*x**2 + 15680*d**5*e**14*x
**3 + 19600*d**4*e**15*x**4 + 15680*d**3*e**16*x**5 + 7840*d**2*e**17*x**6
+ 2240*d*e**18*x**7 + 280*e**19*x**8) + 240100*d**3*e**7*x**4/(280*d**8*e**
11 + 2240*d**7*e**12*x + 7840*d**6*e**13*x**2 + 15680*d**5*e**14*x**3 + 196
00*d**4*e**15*x**4 + 15680*d**3*e**16*x**5 + 7840*d**2*e**17*x**6 + 2240*d*
e**18*x**7 + 280*e**19*x**8) - 279720*d**3*e**7*x**3/(280*d**8*e**11 + 2240
*d**7*e**12*x + 7840*d**6*e**13*x**2 + 15680*d**5*e**14*x**3 + 19600*d**4*e
**15*x**4 + 15680*d**3*e**16*x**5 + 7840*d**2*e**17*x**6 + 2240*d*e**18*x**
7 + 280*e**19*x**8) - 23800*d**3*e**7*x**2/(280*d**8*e**11 + 2240*d**7*e**1
2*x + 7840*d**6*e**13*x**2 + 15680*d**5*e**14*x**3 + 19600*d**4*e**15*x**4
+ 15680*d**3*e**16*x**5 + 7840*d**2*e**17*x**6 + 2240*d*e**18*x**7 + 280*e*
*19*x**8) - 8872*d**3*e**7*x/(280*d**8*e**11 + 2240*d**7*e**12*x + 7840*d**
6*e**13*x**2 + 15680*d**5*e**14*x**3 + 19600*d**4*e**15*x**4 + 15680*d**3*e
**16*x**5 + 7840*d**2*e**17*x**6 + 2240*d*e**18*x**7 + 280*e**19*x**8) - 57
4*d**3*e**7/(280*d**8*e**11 + 2240*d**7*e**12*x + 7840*d**6*e**13*x**2 + 15
680*d**5*e**14*x**3 + 19600*d**4*e**15*x**4 + 15680*d**3*e**16*x**5 + 7840*
d**2*e**17*x**6 + 2240*d*e**18*x**7 + 280*e**19*x**8) + 6300000*d**2*e**8*x
**8*log(d/e + x)/(280*d**8*e**11 + 2240*d**7*e**12*x + 7840*d**6*e**13*x**2
 + 15680*d**5*e**14*x**3 + 19600*d**4*e**15*x**4 + 15680*d**3*e**16*x**5 +
7840*d**2*e**17*x**6 + 2240*d*e**18*x**7 + 280*e**19*x**8) + 504000*d**2*e*
*8*x**7*log(d/e + x)/(280*d**8*e**11 + 2240*d**7*e**12*x + 7840*d**6*e**13*
x**2 + 15680*d**5*e**14*x**3 + 19600*d**4*e**15*x**4 + 15680*d**3*e**16*x**
5 + 7840*d**2*e**17*x**6 + 2240*d*e**18*x**7 + 280*e**19*x**8) + 504000*d**
2*e**8*x**7/(280*d**8*e**11 + 2240*d**7*e**12*x + 7840*d**6*e**13*x**2 + 15
680*d**5*e**14*x**3 + 19600*d**4*e**15*x**4 + 15680*d**3*e**16*x**5 + 7840*
d**2*e**17*x**6 + 2240*d*e**18*x**7 + 280*e**19*x**8) + 5997600*d**2*e**8*x
**6*log(d/e + x)/(280*d**8*e**11 + 2240*d**7*e**12*x + 7840*d**6*e**13*x**2
 + 15680*d**5*e**14*x**3 + 19600*d**4*e**15*x**4 + 15680*d**3*e**16*x**5 +
7840*d**2*e**17*x**6 + 2240*d*e**18*x**7 + 280*e**19*x**8) + 8996400*d**2*e
**8*x**6/(280*d**8*e**11 + 2240*d**7*e**12*x + 7840*d**6*e**13*x**2 + 15680
*d**5*e**14*x**3 + 19600*d**4*e**15*x**4 + 15680*d**3*e**16*x**5 + 7840*d**
2*e**17*x**6 + 2240*d*e**18*x**7 + 280*e**19*x**8) + 192080*d**2*e**8*x**5/
(280*d**8*e**11 + 2240*d**7*e**12*x + 7840*d**6*e**13*x**2 + 15680*d**5*e**
14*x**3 + 19600*d**4*e**15*x**4 + 15680*d**3*e**16*x**5 + 7840*d**2*e**17*x
**6 + 2240*d*e**18*x**7 + 280*e**19*x**8) - 349650*d**2*e**8*x**4/(280*d**8
*e**11 + 2240*d**7*e**12*x + 7840*d**6*e**13*x**2 + 15680*d**5*e**14*x**3 +
 19600*d**4*e**15*x**4 + 15680*d**3*e**16*x**5 + 7840*d**2*e**17*x**6 + 224
0*d*e**18*x**7 + 280*e**19*x**8) - 47600*d**2*e**8*x**3/(280*d**8*e**11 + 2
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240*d**7*e**12*x + 7840*d**6*e**13*x**2 + 15680*d**5*e**14*x**3 + 19600*d**
4*e**15*x**4 + 15680*d**3*e**16*x**5 + 7840*d**2*e**17*x**6 + 2240*d*e**18*
x**7 + 280*e**19*x**8) - 31052*d**2*e**8*x**2/(280*d**8*e**11 + 2240*d**7*e
**12*x + 7840*d**6*e**13*x**2 + 15680*d**5*e**14*x**3 + 19600*d**4*e**15*x*
*4 + 15680*d**3*e**16*x**5 + 7840*d**2*e**17*x**6 + 2240*d*e**18*x**7 + 280
*e**19*x**8) - 4592*d**2*e**8*x/(280*d**8*e**11 + 2240*d**7*e**12*x + 7840*
d**6*e**13*x**2 + 15680*d**5*e**14*x**3 + 19600*d**4*e**15*x**4 + 15680*d**
3*e**16*x**5 + 7840*d**2*e**17*x**6 + 2240*d*e**18*x**7 + 280*e**19*x**8) -
 795*d**2*e**8/(280*d**8*e**11 + 2240*d**7*e**12*x + 7840*d**6*e**13*x**2 +
 15680*d**5*e**14*x**3 + 19600*d**4*e**15*x**4 + 15680*d**3*e**16*x**5 + 78
40*d**2*e**17*x**6 + 2240*d*e**18*x**7 + 280*e**19*x**8) - 700000*d*e**9*x*
*9/(280*d**8*e**11 + 2240*d**7*e**12*x + 7840*d**6*e**13*x**2 + 15680*d**5*
e**14*x**3 + 19600*d**4*e**15*x**4 + 15680*d**3*e**16*x**5 + 7840*d**2*e**1
7*x**6 + 2240*d*e**18*x**7 + 280*e**19*x**8) + 63000*d*e**9*x**8*log(d/e +
x)/(280*d**8*e**11 + 2240*d**7*e**12*x + 7840*d**6*e**13*x**2 + 15680*d**5*
e**14*x**3 + 19600*d**4*e**15*x**4 + 15680*d**3*e**16*x**5 + 7840*d**2*e**1
7*x**6 + 2240*d*e**18*x**7 + 280*e**19*x**8) + 1713600*d*e**9*x**7*log(d/e
+ x)/(280*d**8*e**11 + 2240*d**7*e**12*x + 7840*d**6*e**13*x**2 + 15680*d**
5*e**14*x**3 + 19600*d**4*e**15*x**4 + 15680*d**3*e**16*x**5 + 7840*d**2*e*
*17*x**6 + 2240*d*e**18*x**7 + 280*e**19*x**8) + 1713600*d*e**9*x**7/(280*d
**8*e**11 + 2240*d**7*e**12*x + 7840*d**6*e**13*x**2 + 15680*d**5*e**14*x**
3 + 19600*d**4*e**15*x**4 + 15680*d**3*e**16*x**5 + 7840*d**2*e**17*x**6 +
2240*d*e**18*x**7 + 280*e**19*x**8) + 96040*d*e**9*x**6/(280*d**8*e**11 + 2
240*d**7*e**12*x + 7840*d**6*e**13*x**2 + 15680*d**5*e**14*x**3 + 19600*d**
4*e**15*x**4 + 15680*d**3*e**16*x**5 + 7840*d**2*e**17*x**6 + 2240*d*e**18*
x**7 + 280*e**19*x**8) - 279720*d*e**9*x**5/(280*d**8*e**11 + 2240*d**7*e**
12*x + 7840*d**6*e**13*x**2 + 15680*d**5*e**14*x**3 + 19600*d**4*e**15*x**4
 + 15680*d**3*e**16*x**5 + 7840*d**2*e**17*x**6 + 2240*d*e**18*x**7 + 280*e
**19*x**8) - 59500*d*e**9*x**4/(280*d**8*e**11 + 2240*d**7*e**12*x + 7840*d
**6*e**13*x**2 + 15680*d**5*e**14*x**3 + 19600*d**4*e**15*x**4 + 15680*d**3
*e**16*x**5 + 7840*d**2*e**17*x**6 + 2240*d*e**18*x**7 + 280*e**19*x**8) -
62104*d*e**9*x**3/(280*d**8*e**11 + 2240*d**7*e**12*x + 7840*d**6*e**13*x**
2 + 15680*d**5*e**14*x**3 + 19600*d**4*e**15*x**4 + 15680*d**3*e**16*x**5 +
 7840*d**2*e**17*x**6 + 2240*d*e**18*x**7 + 280*e**19*x**8) - 16072*d*e**9*
x**2/(280*d**8*e**11 + 2240*d**7*e**12*x + 7840*d**6*e**13*x**2 + 15680*d**
5*e**14*x**3 + 19600*d**4*e**15*x**4 + 15680*d**3*e**16*x**5 + 7840*d**2*e*
*17*x**6 + 2240*d*e**18*x**7 + 280*e**19*x**8) - 6360*d*e**9*x/(280*d**8*e*
*11 + 2240*d**7*e**12*x + 7840*d**6*e**13*x**2 + 15680*d**5*e**14*x**3 + 19
600*d**4*e**15*x**4 + 15680*d**3*e**16*x**5 + 7840*d**2*e**17*x**6 + 2240*d
*e**18*x**7 + 280*e**19*x**8) - 675*d*e**9/(280*d**8*e**11 + 2240*d**7*e**1
2*x + 7840*d**6*e**13*x**2 + 15680*d**5*e**14*x**3 + 19600*d**4*e**15*x**4
+ 15680*d**3*e**16*x**5 + 7840*d**2*e**17*x**6 + 2240*d*e**18*x**7 + 280*e*
*19*x**8) + 70000*e**10*x**10/(280*d**8*e**11 + 2240*d**7*e**12*x + 7840*d*
*6*e**13*x**2 + 15680*d**5*e**14*x**3 + 19600*d**4*e**15*x**4 + 15680*d**3*
e**16*x**5 + 7840*d**2*e**17*x**6 + 2240*d*e**18*x**7 + 280*e**19*x**8) - 7
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000*e**10*x**9/(280*d**8*e**11 + 2240*d**7*e**12*x + 7840*d**6*e**13*x**2 +
 15680*d**5*e**14*x**3 + 19600*d**4*e**15*x**4 + 15680*d**3*e**16*x**5 + 78
40*d**2*e**17*x**6 + 2240*d*e**18*x**7 + 280*e**19*x**8) + 214200*e**10*x**
8*log(d/e + x)/(280*d**8*e**11 + 2240*d**7*e**12*x + 7840*d**6*e**13*x**2 +
 15680*d**5*e**14*x**3 + 19600*d**4*e**15*x**4 + 15680*d**3*e**16*x**5 + 78
40*d**2*e**17*x**6 + 2240*d*e**18*x**7 + 280*e**19*x**8) + 27440*e**10*x**7
/(280*d**8*e**11 + 2240*d**7*e**12*x + 7840*d**6*e**13*x**2 + 15680*d**5*e*
*14*x**3 + 19600*d**4*e**15*x**4 + 15680*d**3*e**16*x**5 + 7840*d**2*e**17*
x**6 + 2240*d*e**18*x**7 + 280*e**19*x**8) - 139860*e**10*x**6/(280*d**8*e*
*11 + 2240*d**7*e**12*x + 7840*d**6*e**13*x**2 + 15680*d**5*e**14*x**3 + 19
600*d**4*e**15*x**4 + 15680*d**3*e**16*x**5 + 7840*d**2*e**17*x**6 + 2240*d
*e**18*x**7 + 280*e**19*x**8) - 47600*e**10*x**5/(280*d**8*e**11 + 2240*d**
7*e**12*x + 7840*d**6*e**13*x**2 + 15680*d**5*e**14*x**3 + 19600*d**4*e**15
*x**4 + 15680*d**3*e**16*x**5 + 7840*d**2*e**17*x**6 + 2240*d*e**18*x**7 +
280*e**19*x**8) - 77630*e**10*x**4/(280*d**8*e**11 + 2240*d**7*e**12*x + 78
40*d**6*e**13*x**2 + 15680*d**5*e**14*x**3 + 19600*d**4*e**15*x**4 + 15680*
d**3*e**16*x**5 + 7840*d**2*e**17*x**6 + 2240*d*e**18*x**7 + 280*e**19*x**8
) - 32144*e**10*x**3/(280*d**8*e**11 + 2240*d**7*e**12*x + 7840*d**6*e**13*
x**2 + 15680*d**5*e**14*x**3 + 19600*d**4*e**15*x**4 + 15680*d**3*e**16*x**
5 + 7840*d**2*e**17*x**6 + 2240*d*e**18*x**7 + 280*e**19*x**8) - 22260*e**1
0*x**2/(280*d**8*e**11 + 2240*d**7*e**12*x + 7840*d**6*e**13*x**2 + 15680*d
**5*e**14*x**3 + 19600*d**4*e**15*x**4 + 15680*d**3*e**16*x**5 + 7840*d**2*
e**17*x**6 + 2240*d*e**18*x**7 + 280*e**19*x**8) - 5400*e**10*x/(280*d**8*e
**11 + 2240*d**7*e**12*x + 7840*d**6*e**13*x**2 + 15680*d**5*e**14*x**3 + 1
9600*d**4*e**15*x**4 + 15680*d**3*e**16*x**5 + 7840*d**2*e**17*x**6 + 2240*
d*e**18*x**7 + 280*e**19*x**8) - 1890*e**10/(280*d**8*e**11 + 2240*d**7*e**
12*x + 7840*d**6*e**13*x**2 + 15680*d**5*e**14*x**3 + 19600*d**4*e**15*x**4
 + 15680*d**3*e**16*x**5 + 7840*d**2*e**17*x**6 + 2240*d*e**18*x**7 + 280*e
**19*x**8), Eq(m, -9)), (-12600000*d**10*log(d/e + x)/(210*d**7*e**11 + 147
0*d**6*e**12*x + 4410*d**5*e**13*x**2 + 7350*d**4*e**14*x**3 + 7350*d**3*e*
*15*x**4 + 4410*d**2*e**16*x**5 + 1470*d*e**17*x**6 + 210*e**18*x**7) - 326
70000*d**10/(210*d**7*e**11 + 1470*d**6*e**12*x + 4410*d**5*e**13*x**2 + 73
50*d**4*e**14*x**3 + 7350*d**3*e**15*x**4 + 4410*d**2*e**16*x**5 + 1470*d*e
**17*x**6 + 210*e**18*x**7) - 88200000*d**9*e*x*log(d/e + x)/(210*d**7*e**1
1 + 1470*d**6*e**12*x + 4410*d**5*e**13*x**2 + 7350*d**4*e**14*x**3 + 7350*
d**3*e**15*x**4 + 4410*d**2*e**16*x**5 + 1470*d*e**17*x**6 + 210*e**18*x**7
) - 216090000*d**9*e*x/(210*d**7*e**11 + 1470*d**6*e**12*x + 4410*d**5*e**1
3*x**2 + 7350*d**4*e**14*x**3 + 7350*d**3*e**15*x**4 + 4410*d**2*e**16*x**5
 + 1470*d*e**17*x**6 + 210*e**18*x**7) - 189000*d**9*e*log(d/e + x)/(210*d*
*7*e**11 + 1470*d**6*e**12*x + 4410*d**5*e**13*x**2 + 7350*d**4*e**14*x**3
+ 7350*d**3*e**15*x**4 + 4410*d**2*e**16*x**5 + 1470*d*e**17*x**6 + 210*e**
18*x**7) - 490050*d**9*e/(210*d**7*e**11 + 1470*d**6*e**12*x + 4410*d**5*e*
*13*x**2 + 7350*d**4*e**14*x**3 + 7350*d**3*e**15*x**4 + 4410*d**2*e**16*x*
*5 + 1470*d*e**17*x**6 + 210*e**18*x**7) - 264600000*d**8*e**2*x**2*log(d/e
 + x)/(210*d**7*e**11 + 1470*d**6*e**12*x + 4410*d**5*e**13*x**2 + 7350*d**
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4*e**14*x**3 + 7350*d**3*e**15*x**4 + 4410*d**2*e**16*x**5 + 1470*d*e**17*x
**6 + 210*e**18*x**7) - 604170000*d**8*e**2*x**2/(210*d**7*e**11 + 1470*d**
6*e**12*x + 4410*d**5*e**13*x**2 + 7350*d**4*e**14*x**3 + 7350*d**3*e**15*x
**4 + 4410*d**2*e**16*x**5 + 1470*d*e**17*x**6 + 210*e**18*x**7) - 1323000*
d**8*e**2*x*log(d/e + x)/(210*d**7*e**11 + 1470*d**6*e**12*x + 4410*d**5*e*
*13*x**2 + 7350*d**4*e**14*x**3 + 7350*d**3*e**15*x**4 + 4410*d**2*e**16*x*
*5 + 1470*d*e**17*x**6 + 210*e**18*x**7) - 3241350*d**8*e**2*x/(210*d**7*e*
*11 + 1470*d**6*e**12*x + 4410*d**5*e**13*x**2 + 7350*d**4*e**14*x**3 + 735
0*d**3*e**15*x**4 + 4410*d**2*e**16*x**5 + 1470*d*e**17*x**6 + 210*e**18*x*
*7) - 1285200*d**8*e**2*log(d/e + x)/(210*d**7*e**11 + 1470*d**6*e**12*x +
4410*d**5*e**13*x**2 + 7350*d**4*e**14*x**3 + 7350*d**3*e**15*x**4 + 4410*d
**2*e**16*x**5 + 1470*d*e**17*x**6 + 210*e**18*x**7) - 3332340*d**8*e**2/(2
10*d**7*e**11 + 1470*d**6*e**12*x + 4410*d**5*e**13*x**2 + 7350*d**4*e**14*
x**3 + 7350*d**3*e**15*x**4 + 4410*d**2*e**16*x**5 + 1470*d*e**17*x**6 + 21
0*e**18*x**7) - 441000000*d**7*e**3*x**3*log(d/e + x)/(210*d**7*e**11 + 147
0*d**6*e**12*x + 4410*d**5*e**13*x**2 + 7350*d**4*e**14*x**3 + 7350*d**3*e*
*15*x**4 + 4410*d**2*e**16*x**5 + 1470*d*e**17*x**6 + 210*e**18*x**7) - 918
750000*d**7*e**3*x**3/(210*d**7*e**11 + 1470*d**6*e**12*x + 4410*d**5*e**13
*x**2 + 7350*d**4*e**14*x**3 + 7350*d**3*e**15*x**4 + 4410*d**2*e**16*x**5
+ 1470*d*e**17*x**6 + 210*e**18*x**7) - 3969000*d**7*e**3*x**2*log(d/e + x)
/(210*d**7*e**11 + 1470*d**6*e**12*x + 4410*d**5*e**13*x**2 + 7350*d**4*e**
14*x**3 + 7350*d**3*e**15*x**4 + 4410*d**2*e**16*x**5 + 1470*d*e**17*x**6 +
 210*e**18*x**7) - 9062550*d**7*e**3*x**2/(210*d**7*e**11 + 1470*d**6*e**12
*x + 4410*d**5*e**13*x**2 + 7350*d**4*e**14*x**3 + 7350*d**3*e**15*x**4 + 4
410*d**2*e**16*x**5 + 1470*d*e**17*x**6 + 210*e**18*x**7) - 8996400*d**7*e*
*3*x*log(d/e + x)/(210*d**7*e**11 + 1470*d**6*e**12*x + 4410*d**5*e**13*x**
2 + 7350*d**4*e**14*x**3 + 7350*d**3*e**15*x**4 + 4410*d**2*e**16*x**5 + 14
70*d*e**17*x**6 + 210*e**18*x**7) - 22041180*d**7*e**3*x/(210*d**7*e**11 +
1470*d**6*e**12*x + 4410*d**5*e**13*x**2 + 7350*d**4*e**14*x**3 + 7350*d**3
*e**15*x**4 + 4410*d**2*e**16*x**5 + 1470*d*e**17*x**6 + 210*e**18*x**7) -
20580*d**7*e**3*log(d/e + x)/(210*d**7*e**11 + 1470*d**6*e**12*x + 4410*d**
5*e**13*x**2 + 7350*d**4*e**14*x**3 + 7350*d**3*e**15*x**4 + 4410*d**2*e**1
6*x**5 + 1470*d*e**17*x**6 + 210*e**18*x**7) - 53361*d**7*e**3/(210*d**7*e*
*11 + 1470*d**6*e**12*x + 4410*d**5*e**13*x**2 + 7350*d**4*e**14*x**3 + 735
0*d**3*e**15*x**4 + 4410*d**2*e**16*x**5 + 1470*d*e**17*x**6 + 210*e**18*x*
*7) - 441000000*d**6*e**4*x**4*log(d/e + x)/(210*d**7*e**11 + 1470*d**6*e**
12*x + 4410*d**5*e**13*x**2 + 7350*d**4*e**14*x**3 + 7350*d**3*e**15*x**4 +
 4410*d**2*e**16*x**5 + 1470*d*e**17*x**6 + 210*e**18*x**7) - 808500000*d**
6*e**4*x**4/(210*d**7*e**11 + 1470*d**6*e**12*x + 4410*d**5*e**13*x**2 + 73
50*d**4*e**14*x**3 + 7350*d**3*e**15*x**4 + 4410*d**2*e**16*x**5 + 1470*d*e
**17*x**6 + 210*e**18*x**7) - 6615000*d**6*e**4*x**3*log(d/e + x)/(210*d**7
*e**11 + 1470*d**6*e**12*x + 4410*d**5*e**13*x**2 + 7350*d**4*e**14*x**3 +
7350*d**3*e**15*x**4 + 4410*d**2*e**16*x**5 + 1470*d*e**17*x**6 + 210*e**18
*x**7) - 13781250*d**6*e**4*x**3/(210*d**7*e**11 + 1470*d**6*e**12*x + 4410
*d**5*e**13*x**2 + 7350*d**4*e**14*x**3 + 7350*d**3*e**15*x**4 + 4410*d**2*
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e**16*x**5 + 1470*d*e**17*x**6 + 210*e**18*x**7) - 26989200*d**6*e**4*x**2*
log(d/e + x)/(210*d**7*e**11 + 1470*d**6*e**12*x + 4410*d**5*e**13*x**2 + 7
350*d**4*e**14*x**3 + 7350*d**3*e**15*x**4 + 4410*d**2*e**16*x**5 + 1470*d*
e**17*x**6 + 210*e**18*x**7) - 61625340*d**6*e**4*x**2/(210*d**7*e**11 + 14
70*d**6*e**12*x + 4410*d**5*e**13*x**2 + 7350*d**4*e**14*x**3 + 7350*d**3*e
**15*x**4 + 4410*d**2*e**16*x**5 + 1470*d*e**17*x**6 + 210*e**18*x**7) - 14
4060*d**6*e**4*x*log(d/e + x)/(210*d**7*e**11 + 1470*d**6*e**12*x + 4410*d*
*5*e**13*x**2 + 7350*d**4*e**14*x**3 + 7350*d**3*e**15*x**4 + 4410*d**2*e**
16*x**5 + 1470*d*e**17*x**6 + 210*e**18*x**7) - 352947*d**6*e**4*x/(210*d**
7*e**11 + 1470*d**6*e**12*x + 4410*d**5*e**13*x**2 + 7350*d**4*e**14*x**3 +
 7350*d**3*e**15*x**4 + 4410*d**2*e**16*x**5 + 1470*d*e**17*x**6 + 210*e**1
8*x**7) - 29970*d**6*e**4/(210*d**7*e**11 + 1470*d**6*e**12*x + 4410*d**5*e
**13*x**2 + 7350*d**4*e**14*x**3 + 7350*d**3*e**15*x**4 + 4410*d**2*e**16*x
**5 + 1470*d*e**17*x**6 + 210*e**18*x**7) - 264600000*d**5*e**5*x**5*log(d/
e + x)/(210*d**7*e**11 + 1470*d**6*e**12*x + 4410*d**5*e**13*x**2 + 7350*d*
*4*e**14*x**3 + 7350*d**3*e**15*x**4 + 4410*d**2*e**16*x**5 + 1470*d*e**17*
x**6 + 210*e**18*x**7) - 396900000*d**5*e**5*x**5/(210*d**7*e**11 + 1470*d*
*6*e**12*x + 4410*d**5*e**13*x**2 + 7350*d**4*e**14*x**3 + 7350*d**3*e**15*
x**4 + 4410*d**2*e**16*x**5 + 1470*d*e**17*x**6 + 210*e**18*x**7) - 6615000
*d**5*e**5*x**4*log(d/e + x)/(210*d**7*e**11 + 1470*d**6*e**12*x + 4410*d**
5*e**13*x**2 + 7350*d**4*e**14*x**3 + 7350*d**3*e**15*x**4 + 4410*d**2*e**1
6*x**5 + 1470*d*e**17*x**6 + 210*e**18*x**7) - 12127500*d**5*e**5*x**4/(210
*d**7*e**11 + 1470*d**6*e**12*x + 4410*d**5*e**13*x**2 + 7350*d**4*e**14*x*
*3 + 7350*d**3*e**15*x**4 + 4410*d**2*e**16*x**5 + 1470*d*e**17*x**6 + 210*
e**18*x**7) - 44982000*d**5*e**5*x**3*log(d/e + x)/(210*d**7*e**11 + 1470*d
**6*e**12*x + 4410*d**5*e**13*x**2 + 7350*d**4*e**14*x**3 + 7350*d**3*e**15
*x**4 + 4410*d**2*e**16*x**5 + 1470*d*e**17*x**6 + 210*e**18*x**7) - 937125
00*d**5*e**5*x**3/(210*d**7*e**11 + 1470*d**6*e**12*x + 4410*d**5*e**13*x**
2 + 7350*d**4*e**14*x**3 + 7350*d**3*e**15*x**4 + 4410*d**2*e**16*x**5 + 14
70*d*e**17*x**6 + 210*e**18*x**7) - 432180*d**5*e**5*x**2*log(d/e + x)/(210
*d**7*e**11 + 1470*d**6*e**12*x + 4410*d**5*e**13*x**2 + 7350*d**4*e**14*x*
*3 + 7350*d**3*e**15*x**4 + 4410*d**2*e**16*x**5 + 1470*d*e**17*x**6 + 210*
e**18*x**7) - 986811*d**5*e**5*x**2/(210*d**7*e**11 + 1470*d**6*e**12*x + 4
410*d**5*e**13*x**2 + 7350*d**4*e**14*x**3 + 7350*d**3*e**15*x**4 + 4410*d*
*2*e**16*x**5 + 1470*d*e**17*x**6 + 210*e**18*x**7) - 209790*d**5*e**5*x/(2
10*d**7*e**11 + 1470*d**6*e**12*x + 4410*d**5*e**13*x**2 + 7350*d**4*e**14*
x**3 + 7350*d**3*e**15*x**4 + 4410*d**2*e**16*x**5 + 1470*d*e**17*x**6 + 21
0*e**18*x**7) - 2550*d**5*e**5/(210*d**7*e**11 + 1470*d**6*e**12*x + 4410*d
**5*e**13*x**2 + 7350*d**4*e**14*x**3 + 7350*d**3*e**15*x**4 + 4410*d**2*e*
*16*x**5 + 1470*d*e**17*x**6 + 210*e**18*x**7) - 88200000*d**4*e**6*x**6*lo
g(d/e + x)/(210*d**7*e**11 + 1470*d**6*e**12*x + 4410*d**5*e**13*x**2 + 735
0*d**4*e**14*x**3 + 7350*d**3*e**15*x**4 + 4410*d**2*e**16*x**5 + 1470*d*e*
*17*x**6 + 210*e**18*x**7) - 88200000*d**4*e**6*x**6/(210*d**7*e**11 + 1470
*d**6*e**12*x + 4410*d**5*e**13*x**2 + 7350*d**4*e**14*x**3 + 7350*d**3*e**
15*x**4 + 4410*d**2*e**16*x**5 + 1470*d*e**17*x**6 + 210*e**18*x**7) - 3969
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000*d**4*e**6*x**5*log(d/e + x)/(210*d**7*e**11 + 1470*d**6*e**12*x + 4410*
d**5*e**13*x**2 + 7350*d**4*e**14*x**3 + 7350*d**3*e**15*x**4 + 4410*d**2*e
**16*x**5 + 1470*d*e**17*x**6 + 210*e**18*x**7) - 5953500*d**4*e**6*x**5/(2
10*d**7*e**11 + 1470*d**6*e**12*x + 4410*d**5*e**13*x**2 + 7350*d**4*e**14*
x**3 + 7350*d**3*e**15*x**4 + 4410*d**2*e**16*x**5 + 1470*d*e**17*x**6 + 21
0*e**18*x**7) - 44982000*d**4*e**6*x**4*log(d/e + x)/(210*d**7*e**11 + 1470
*d**6*e**12*x + 4410*d**5*e**13*x**2 + 7350*d**4*e**14*x**3 + 7350*d**3*e**
15*x**4 + 4410*d**2*e**16*x**5 + 1470*d*e**17*x**6 + 210*e**18*x**7) - 8246
7000*d**4*e**6*x**4/(210*d**7*e**11 + 1470*d**6*e**12*x + 4410*d**5*e**13*x
**2 + 7350*d**4*e**14*x**3 + 7350*d**3*e**15*x**4 + 4410*d**2*e**16*x**5 +
1470*d*e**17*x**6 + 210*e**18*x**7) - 720300*d**4*e**6*x**3*log(d/e + x)/(2
10*d**7*e**11 + 1470*d**6*e**12*x + 4410*d**5*e**13*x**2 + 7350*d**4*e**14*
x**3 + 7350*d**3*e**15*x**4 + 4410*d**2*e**16*x**5 + 1470*d*e**17*x**6 + 21
0*e**18*x**7) - 1500625*d**4*e**6*x**3/(210*d**7*e**11 + 1470*d**6*e**12*x
+ 4410*d**5*e**13*x**2 + 7350*d**4*e**14*x**3 + 7350*d**3*e**15*x**4 + 4410
*d**2*e**16*x**5 + 1470*d*e**17*x**6 + 210*e**18*x**7) - 629370*d**4*e**6*x
**2/(210*d**7*e**11 + 1470*d**6*e**12*x + 4410*d**5*e**13*x**2 + 7350*d**4*
e**14*x**3 + 7350*d**3*e**15*x**4 + 4410*d**2*e**16*x**5 + 1470*d*e**17*x**
6 + 210*e**18*x**7) - 17850*d**4*e**6*x/(210*d**7*e**11 + 1470*d**6*e**12*x
 + 4410*d**5*e**13*x**2 + 7350*d**4*e**14*x**3 + 7350*d**3*e**15*x**4 + 441
0*d**2*e**16*x**5 + 1470*d*e**17*x**6 + 210*e**18*x**7) - 2218*d**4*e**6/(2
10*d**7*e**11 + 1470*d**6*e**12*x + 4410*d**5*e**13*x**2 + 7350*d**4*e**14*
x**3 + 7350*d**3*e**15*x**4 + 4410*d**2*e**16*x**5 + 1470*d*e**17*x**6 + 21
0*e**18*x**7) - 12600000*d**3*e**7*x**7*log(d/e + x)/(210*d**7*e**11 + 1470
*d**6*e**12*x + 4410*d**5*e**13*x**2 + 7350*d**4*e**14*x**3 + 7350*d**3*e**
15*x**4 + 4410*d**2*e**16*x**5 + 1470*d*e**17*x**6 + 210*e**18*x**7) - 1323
000*d**3*e**7*x**6*log(d/e + x)/(210*d**7*e**11 + 1470*d**6*e**12*x + 4410*
d**5*e**13*x**2 + 7350*d**4*e**14*x**3 + 7350*d**3*e**15*x**4 + 4410*d**2*e
**16*x**5 + 1470*d*e**17*x**6 + 210*e**18*x**7) - 1323000*d**3*e**7*x**6/(2
10*d**7*e**11 + 1470*d**6*e**12*x + 4410*d**5*e**13*x**2 + 7350*d**4*e**14*
x**3 + 7350*d**3*e**15*x**4 + 4410*d**2*e**16*x**5 + 1470*d*e**17*x**6 + 21
0*e**18*x**7) - 26989200*d**3*e**7*x**5*log(d/e + x)/(210*d**7*e**11 + 1470
*d**6*e**12*x + 4410*d**5*e**13*x**2 + 7350*d**4*e**14*x**3 + 7350*d**3*e**
15*x**4 + 4410*d**2*e**16*x**5 + 1470*d*e**17*x**6 + 210*e**18*x**7) - 4048
3800*d**3*e**7*x**5/(210*d**7*e**11 + 1470*d**6*e**12*x + 4410*d**5*e**13*x
**2 + 7350*d**4*e**14*x**3 + 7350*d**3*e**15*x**4 + 4410*d**2*e**16*x**5 +
1470*d*e**17*x**6 + 210*e**18*x**7) - 720300*d**3*e**7*x**4*log(d/e + x)/(2
10*d**7*e**11 + 1470*d**6*e**12*x + 4410*d**5*e**13*x**2 + 7350*d**4*e**14*
x**3 + 7350*d**3*e**15*x**4 + 4410*d**2*e**16*x**5 + 1470*d*e**17*x**6 + 21
0*e**18*x**7) - 1320550*d**3*e**7*x**4/(210*d**7*e**11 + 1470*d**6*e**12*x
+ 4410*d**5*e**13*x**2 + 7350*d**4*e**14*x**3 + 7350*d**3*e**15*x**4 + 4410
*d**2*e**16*x**5 + 1470*d*e**17*x**6 + 210*e**18*x**7) - 1048950*d**3*e**7*
x**3/(210*d**7*e**11 + 1470*d**6*e**12*x + 4410*d**5*e**13*x**2 + 7350*d**4
*e**14*x**3 + 7350*d**3*e**15*x**4 + 4410*d**2*e**16*x**5 + 1470*d*e**17*x*
*6 + 210*e**18*x**7) - 53550*d**3*e**7*x**2/(210*d**7*e**11 + 1470*d**6*e**
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12*x + 4410*d**5*e**13*x**2 + 7350*d**4*e**14*x**3 + 7350*d**3*e**15*x**4 +
 4410*d**2*e**16*x**5 + 1470*d*e**17*x**6 + 210*e**18*x**7) - 15526*d**3*e*
*7*x/(210*d**7*e**11 + 1470*d**6*e**12*x + 4410*d**5*e**13*x**2 + 7350*d**4
*e**14*x**3 + 7350*d**3*e**15*x**4 + 4410*d**2*e**16*x**5 + 1470*d*e**17*x*
*6 + 210*e**18*x**7) - 861*d**3*e**7/(210*d**7*e**11 + 1470*d**6*e**12*x +
4410*d**5*e**13*x**2 + 7350*d**4*e**14*x**3 + 7350*d**3*e**15*x**4 + 4410*d
**2*e**16*x**5 + 1470*d*e**17*x**6 + 210*e**18*x**7) + 1575000*d**2*e**8*x*
*8/(210*d**7*e**11 + 1470*d**6*e**12*x + 4410*d**5*e**13*x**2 + 7350*d**4*e
**14*x**3 + 7350*d**3*e**15*x**4 + 4410*d**2*e**16*x**5 + 1470*d*e**17*x**6
 + 210*e**18*x**7) - 189000*d**2*e**8*x**7*log(d/e + x)/(210*d**7*e**11 + 1
470*d**6*e**12*x + 4410*d**5*e**13*x**2 + 7350*d**4*e**14*x**3 + 7350*d**3*
e**15*x**4 + 4410*d**2*e**16*x**5 + 1470*d*e**17*x**6 + 210*e**18*x**7) - 8
996400*d**2*e**8*x**6*log(d/e + x)/(210*d**7*e**11 + 1470*d**6*e**12*x + 44
10*d**5*e**13*x**2 + 7350*d**4*e**14*x**3 + 7350*d**3*e**15*x**4 + 4410*d**
2*e**16*x**5 + 1470*d*e**17*x**6 + 210*e**18*x**7) - 8996400*d**2*e**8*x**6
/(210*d**7*e**11 + 1470*d**6*e**12*x + 4410*d**5*e**13*x**2 + 7350*d**4*e**
14*x**3 + 7350*d**3*e**15*x**4 + 4410*d**2*e**16*x**5 + 1470*d*e**17*x**6 +
 210*e**18*x**7) - 432180*d**2*e**8*x**5*log(d/e + x)/(210*d**7*e**11 + 147
0*d**6*e**12*x + 4410*d**5*e**13*x**2 + 7350*d**4*e**14*x**3 + 7350*d**3*e*
*15*x**4 + 4410*d**2*e**16*x**5 + 1470*d*e**17*x**6 + 210*e**18*x**7) - 648
270*d**2*e**8*x**5/(210*d**7*e**11 + 1470*d**6*e**12*x + 4410*d**5*e**13*x*
*2 + 7350*d**4*e**14*x**3 + 7350*d**3*e**15*x**4 + 4410*d**2*e**16*x**5 + 1
470*d*e**17*x**6 + 210*e**18*x**7) - 1048950*d**2*e**8*x**4/(210*d**7*e**11
 + 1470*d**6*e**12*x + 4410*d**5*e**13*x**2 + 7350*d**4*e**14*x**3 + 7350*d
**3*e**15*x**4 + 4410*d**2*e**16*x**5 + 1470*d*e**17*x**6 + 210*e**18*x**7)
 - 89250*d**2*e**8*x**3/(210*d**7*e**11 + 1470*d**6*e**12*x + 4410*d**5*e**
13*x**2 + 7350*d**4*e**14*x**3 + 7350*d**3*e**15*x**4 + 4410*d**2*e**16*x**
5 + 1470*d*e**17*x**6 + 210*e**18*x**7) - 46578*d**2*e**8*x**2/(210*d**7*e*
*11 + 1470*d**6*e**12*x + 4410*d**5*e**13*x**2 + 7350*d**4*e**14*x**3 + 735
0*d**3*e**15*x**4 + 4410*d**2*e**16*x**5 + 1470*d*e**17*x**6 + 210*e**18*x*
*7) - 6027*d**2*e**8*x/(210*d**7*e**11 + 1470*d**6*e**12*x + 4410*d**5*e**1
3*x**2 + 7350*d**4*e**14*x**3 + 7350*d**3*e**15*x**4 + 4410*d**2*e**16*x**5
 + 1470*d*e**17*x**6 + 210*e**18*x**7) - 954*d**2*e**8/(210*d**7*e**11 + 14
70*d**6*e**12*x + 4410*d**5*e**13*x**2 + 7350*d**4*e**14*x**3 + 7350*d**3*e
**15*x**4 + 4410*d**2*e**16*x**5 + 1470*d*e**17*x**6 + 210*e**18*x**7) - 17
5000*d*e**9*x**9/(210*d**7*e**11 + 1470*d**6*e**12*x + 4410*d**5*e**13*x**2
 + 7350*d**4*e**14*x**3 + 7350*d**3*e**15*x**4 + 4410*d**2*e**16*x**5 + 147
0*d*e**17*x**6 + 210*e**18*x**7) + 23625*d*e**9*x**8/(210*d**7*e**11 + 1470
*d**6*e**12*x + 4410*d**5*e**13*x**2 + 7350*d**4*e**14*x**3 + 7350*d**3*e**
15*x**4 + 4410*d**2*e**16*x**5 + 1470*d*e**17*x**6 + 210*e**18*x**7) - 1285
200*d*e**9*x**7*log(d/e + x)/(210*d**7*e**11 + 1470*d**6*e**12*x + 4410*d**
5*e**13*x**2 + 7350*d**4*e**14*x**3 + 7350*d**3*e**15*x**4 + 4410*d**2*e**1
6*x**5 + 1470*d*e**17*x**6 + 210*e**18*x**7) - 144060*d*e**9*x**6*log(d/e +
 x)/(210*d**7*e**11 + 1470*d**6*e**12*x + 4410*d**5*e**13*x**2 + 7350*d**4*
e**14*x**3 + 7350*d**3*e**15*x**4 + 4410*d**2*e**16*x**5 + 1470*d*e**17*x**
```

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6 + 210*e**18*x**7) - 144060*d*e**9*x**6/(210*d**7*e**11 + 1470*d**6*e**12*
x + 4410*d**5*e**13*x**2 + 7350*d**4*e**14*x**3 + 7350*d**3*e**15*x**4 + 44
10*d**2*e**16*x**5 + 1470*d*e**17*x**6 + 210*e**18*x**7) - 629370*d*e**9*x*
*5/(210*d**7*e**11 + 1470*d**6*e**12*x + 4410*d**5*e**13*x**2 + 7350*d**4*e
**14*x**3 + 7350*d**3*e**15*x**4 + 4410*d**2*e**16*x**5 + 1470*d*e**17*x**6
 + 210*e**18*x**7) - 89250*d*e**9*x**4/(210*d**7*e**11 + 1470*d**6*e**12*x
+ 4410*d**5*e**13*x**2 + 7350*d**4*e**14*x**3 + 7350*d**3*e**15*x**4 + 4410
*d**2*e**16*x**5 + 1470*d*e**17*x**6 + 210*e**18*x**7) - 77630*d*e**9*x**3/
(210*d**7*e**11 + 1470*d**6*e**12*x + 4410*d**5*e**13*x**2 + 7350*d**4*e**1
4*x**3 + 7350*d**3*e**15*x**4 + 4410*d**2*e**16*x**5 + 1470*d*e**17*x**6 +
210*e**18*x**7) - 18081*d*e**9*x**2/(210*d**7*e**11 + 1470*d**6*e**12*x + 4
410*d**5*e**13*x**2 + 7350*d**4*e**14*x**3 + 7350*d**3*e**15*x**4 + 4410*d*
*2*e**16*x**5 + 1470*d*e**17*x**6 + 210*e**18*x**7) - 6678*d*e**9*x/(210*d*
*7*e**11 + 1470*d**6*e**12*x + 4410*d**5*e**13*x**2 + 7350*d**4*e**14*x**3
+ 7350*d**3*e**15*x**4 + 4410*d**2*e**16*x**5 + 1470*d*e**17*x**6 + 210*e**
18*x**7) - 675*d*e**9/(210*d**7*e**11 + 1470*d**6*e**12*x + 4410*d**5*e**13
*x**2 + 7350*d**4*e**14*x**3 + 7350*d**3*e**15*x**4 + 4410*d**2*e**16*x**5
+ 1470*d*e**17*x**6 + 210*e**18*x**7) + 35000*e**10*x**10/(210*d**7*e**11 +
 1470*d**6*e**12*x + 4410*d**5*e**13*x**2 + 7350*d**4*e**14*x**3 + 7350*d**
3*e**15*x**4 + 4410*d**2*e**16*x**5 + 1470*d*e**17*x**6 + 210*e**18*x**7) -
 2625*e**10*x**9/(210*d**7*e**11 + 1470*d**6*e**12*x + 4410*d**5*e**13*x**2
 + 7350*d**4*e**14*x**3 + 7350*d**3*e**15*x**4 + 4410*d**2*e**16*x**5 + 147
0*d*e**17*x**6 + 210*e**18*x**7) + 160650*e**10*x**8/(210*d**7*e**11 + 1470
*d**6*e**12*x + 4410*d**5*e**13*x**2 + 7350*d**4*e**14*x**3 + 7350*d**3*e**
15*x**4 + 4410*d**2*e**16*x**5 + 1470*d*e**17*x**6 + 210*e**18*x**7) - 2058
0*e**10*x**7*log(d/e + x)/(210*d**7*e**11 + 1470*d**6*e**12*x + 4410*d**5*e
**13*x**2 + 7350*d**4*e**14*x**3 + 7350*d**3*e**15*x**4 + 4410*d**2*e**16*x
**5 + 1470*d*e**17*x**6 + 210*e**18*x**7) - 209790*e**10*x**6/(210*d**7*e**
11 + 1470*d**6*e**12*x + 4410*d**5*e**13*x**2 + 7350*d**4*e**14*x**3 + 7350
*d**3*e**15*x**4 + 4410*d**2*e**16*x**5 + 1470*d*e**17*x**6 + 210*e**18*x**
7) - 53550*e**10*x**5/(210*d**7*e**11 + 1470*d**6*e**12*x + 4410*d**5*e**13
*x**2 + 7350*d**4*e**14*x**3 + 7350*d**3*e**15*x**4 + 4410*d**2*e**16*x**5
+ 1470*d*e**17*x**6 + 210*e**18*x**7) - 77630*e**10*x**4/(210*d**7*e**11 +
1470*d**6*e**12*x + 4410*d**5*e**13*x**2 + 7350*d**4*e**14*x**3 + 7350*d**3
*e**15*x**4 + 4410*d**2*e**16*x**5 + 1470*d*e**17*x**6 + 210*e**18*x**7) -
30135*e**10*x**3/(210*d**7*e**11 + 1470*d**6*e**12*x + 4410*d**5*e**13*x**2
 + 7350*d**4*e**14*x**3 + 7350*d**3*e**15*x**4 + 4410*d**2*e**16*x**5 + 147
0*d*e**17*x**6 + 210*e**18*x**7) - 20034*e**10*x**2/(210*d**7*e**11 + 1470*
d**6*e**12*x + 4410*d**5*e**13*x**2 + 7350*d**4*e**14*x**3 + 7350*d**3*e**1
5*x**4 + 4410*d**2*e**16*x**5 + 1470*d*e**17*x**6 + 210*e**18*x**7) - 4725*
e**10*x/(210*d**7*e**11 + 1470*d**6*e**12*x + 4410*d**5*e**13*x**2 + 7350*d
**4*e**14*x**3 + 7350*d**3*e**15*x**4 + 4410*d**2*e**16*x**5 + 1470*d*e**17
*x**6 + 210*e**18*x**7) - 1620*e**10/(210*d**7*e**11 + 1470*d**6*e**12*x +
4410*d**5*e**13*x**2 + 7350*d**4*e**14*x**3 + 7350*d**3*e**15*x**4 + 4410*d
**2*e**16*x**5 + 1470*d*e**17*x**6 + 210*e**18*x**7), Eq(m, -8)), (6300000*
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d**10*log(d/e + x)/(60*d**6*e**11 + 360*d**5*e**12*x + 900*d**4*e**13*x**2
+ 1200*d**3*e**14*x**3 + 900*d**2*e**15*x**4 + 360*d*e**16*x**5 + 60*e**17*
x**6) + 15435000*d**10/(60*d**6*e**11 + 360*d**5*e**12*x + 900*d**4*e**13*x
**2 + 1200*d**3*e**14*x**3 + 900*d**2*e**15*x**4 + 360*d*e**16*x**5 + 60*e*
*17*x**6) + 37800000*d**9*e*x*log(d/e + x)/(60*d**6*e**11 + 360*d**5*e**12*
x + 900*d**4*e**13*x**2 + 1200*d**3*e**14*x**3 + 900*d**2*e**15*x**4 + 360*
d*e**16*x**5 + 60*e**17*x**6) + 86310000*d**9*e*x/(60*d**6*e**11 + 360*d**5
*e**12*x + 900*d**4*e**13*x**2 + 1200*d**3*e**14*x**3 + 900*d**2*e**15*x**4
 + 360*d*e**16*x**5 + 60*e**17*x**6) + 126000*d**9*e*log(d/e + x)/(60*d**6*
e**11 + 360*d**5*e**12*x + 900*d**4*e**13*x**2 + 1200*d**3*e**14*x**3 + 900
*d**2*e**15*x**4 + 360*d*e**16*x**5 + 60*e**17*x**6) + 308700*d**9*e/(60*d*
*6*e**11 + 360*d**5*e**12*x + 900*d**4*e**13*x**2 + 1200*d**3*e**14*x**3 +
900*d**2*e**15*x**4 + 360*d*e**16*x**5 + 60*e**17*x**6) + 94500000*d**8*e**
2*x**2*log(d/e + x)/(60*d**6*e**11 + 360*d**5*e**12*x + 900*d**4*e**13*x**2
 + 1200*d**3*e**14*x**3 + 900*d**2*e**15*x**4 + 360*d*e**16*x**5 + 60*e**17
*x**6) + 196875000*d**8*e**2*x**2/(60*d**6*e**11 + 360*d**5*e**12*x + 900*d
**4*e**13*x**2 + 1200*d**3*e**14*x**3 + 900*d**2*e**15*x**4 + 360*d*e**16*x
**5 + 60*e**17*x**6) + 756000*d**8*e**2*x*log(d/e + x)/(60*d**6*e**11 + 360
*d**5*e**12*x + 900*d**4*e**13*x**2 + 1200*d**3*e**14*x**3 + 900*d**2*e**15
*x**4 + 360*d*e**16*x**5 + 60*e**17*x**6) + 1726200*d**8*e**2*x/(60*d**6*e*
*11 + 360*d**5*e**12*x + 900*d**4*e**13*x**2 + 1200*d**3*e**14*x**3 + 900*d
**2*e**15*x**4 + 360*d*e**16*x**5 + 60*e**17*x**6) + 1285200*d**8*e**2*log(
d/e + x)/(60*d**6*e**11 + 360*d**5*e**12*x + 900*d**4*e**13*x**2 + 1200*d**
3*e**14*x**3 + 900*d**2*e**15*x**4 + 360*d*e**16*x**5 + 60*e**17*x**6) + 31
48740*d**8*e**2/(60*d**6*e**11 + 360*d**5*e**12*x + 900*d**4*e**13*x**2 + 1
200*d**3*e**14*x**3 + 900*d**2*e**15*x**4 + 360*d*e**16*x**5 + 60*e**17*x**
6) + 126000000*d**7*e**3*x**3*log(d/e + x)/(60*d**6*e**11 + 360*d**5*e**12*
x + 900*d**4*e**13*x**2 + 1200*d**3*e**14*x**3 + 900*d**2*e**15*x**4 + 360*
d*e**16*x**5 + 60*e**17*x**6) + 231000000*d**7*e**3*x**3/(60*d**6*e**11 + 3
60*d**5*e**12*x + 900*d**4*e**13*x**2 + 1200*d**3*e**14*x**3 + 900*d**2*e**
15*x**4 + 360*d*e**16*x**5 + 60*e**17*x**6) + 1890000*d**7*e**3*x**2*log(d/
e + x)/(60*d**6*e**11 + 360*d**5*e**12*x + 900*d**4*e**13*x**2 + 1200*d**3*
e**14*x**3 + 900*d**2*e**15*x**4 + 360*d*e**16*x**5 + 60*e**17*x**6) + 3937
500*d**7*e**3*x**2/(60*d**6*e**11 + 360*d**5*e**12*x + 900*d**4*e**13*x**2
+ 1200*d**3*e**14*x**3 + 900*d**2*e**15*x**4 + 360*d*e**16*x**5 + 60*e**17*
x**6) + 7711200*d**7*e**3*x*log(d/e + x)/(60*d**6*e**11 + 360*d**5*e**12*x
+ 900*d**4*e**13*x**2 + 1200*d**3*e**14*x**3 + 900*d**2*e**15*x**4 + 360*d*
e**16*x**5 + 60*e**17*x**6) + 17607240*d**7*e**3*x/(60*d**6*e**11 + 360*d**
5*e**12*x + 900*d**4*e**13*x**2 + 1200*d**3*e**14*x**3 + 900*d**2*e**15*x**
4 + 360*d*e**16*x**5 + 60*e**17*x**6) + 41160*d**7*e**3*log(d/e + x)/(60*d*
*6*e**11 + 360*d**5*e**12*x + 900*d**4*e**13*x**2 + 1200*d**3*e**14*x**3 +
900*d**2*e**15*x**4 + 360*d*e**16*x**5 + 60*e**17*x**6) + 100842*d**7*e**3/
(60*d**6*e**11 + 360*d**5*e**12*x + 900*d**4*e**13*x**2 + 1200*d**3*e**14*x
**3 + 900*d**2*e**15*x**4 + 360*d*e**16*x**5 + 60*e**17*x**6) + 94500000*d*
*6*e**4*x**4*log(d/e + x)/(60*d**6*e**11 + 360*d**5*e**12*x + 900*d**4*e**1
```

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3*x**2 + 1200*d**3*e**14*x**3 + 900*d**2*e**15*x**4 + 360*d*e**16*x**5 + 60
*e**17*x**6) + 141750000*d**6*e**4*x**4/(60*d**6*e**11 + 360*d**5*e**12*x +
 900*d**4*e**13*x**2 + 1200*d**3*e**14*x**3 + 900*d**2*e**15*x**4 + 360*d*e
**16*x**5 + 60*e**17*x**6) + 2520000*d**6*e**4*x**3*log(d/e + x)/(60*d**6*e
**11 + 360*d**5*e**12*x + 900*d**4*e**13*x**2 + 1200*d**3*e**14*x**3 + 900*
d**2*e**15*x**4 + 360*d*e**16*x**5 + 60*e**17*x**6) + 4620000*d**6*e**4*x**
3/(60*d**6*e**11 + 360*d**5*e**12*x + 900*d**4*e**13*x**2 + 1200*d**3*e**14
*x**3 + 900*d**2*e**15*x**4 + 360*d*e**16*x**5 + 60*e**17*x**6) + 19278000*
d**6*e**4*x**2*log(d/e + x)/(60*d**6*e**11 + 360*d**5*e**12*x + 900*d**4*e*
*13*x**2 + 1200*d**3*e**14*x**3 + 900*d**2*e**15*x**4 + 360*d*e**16*x**5 +
60*e**17*x**6) + 40162500*d**6*e**4*x**2/(60*d**6*e**11 + 360*d**5*e**12*x
+ 900*d**4*e**13*x**2 + 1200*d**3*e**14*x**3 + 900*d**2*e**15*x**4 + 360*d*
e**16*x**5 + 60*e**17*x**6) + 246960*d**6*e**4*x*log(d/e + x)/(60*d**6*e**1
1 + 360*d**5*e**12*x + 900*d**4*e**13*x**2 + 1200*d**3*e**14*x**3 + 900*d**
2*e**15*x**4 + 360*d*e**16*x**5 + 60*e**17*x**6) + 563892*d**6*e**4*x/(60*d
**6*e**11 + 360*d**5*e**12*x + 900*d**4*e**13*x**2 + 1200*d**3*e**14*x**3 +
 900*d**2*e**15*x**4 + 360*d*e**16*x**5 + 60*e**17*x**6) + 59940*d**6*e**4*
log(d/e + x)/(60*d**6*e**11 + 360*d**5*e**12*x + 900*d**4*e**13*x**2 + 1200
*d**3*e**14*x**3 + 900*d**2*e**15*x**4 + 360*d*e**16*x**5 + 60*e**17*x**6)
+ 146853*d**6*e**4/(60*d**6*e**11 + 360*d**5*e**12*x + 900*d**4*e**13*x**2
+ 1200*d**3*e**14*x**3 + 900*d**2*e**15*x**4 + 360*d*e**16*x**5 + 60*e**17*
x**6) + 37800000*d**5*e**5*x**5*log(d/e + x)/(60*d**6*e**11 + 360*d**5*e**1
2*x + 900*d**4*e**13*x**2 + 1200*d**3*e**14*x**3 + 900*d**2*e**15*x**4 + 36
0*d*e**16*x**5 + 60*e**17*x**6) + 37800000*d**5*e**5*x**5/(60*d**6*e**11 +
360*d**5*e**12*x + 900*d**4*e**13*x**2 + 1200*d**3*e**14*x**3 + 900*d**2*e*
*15*x**4 + 360*d*e**16*x**5 + 60*e**17*x**6) + 1890000*d**5*e**5*x**4*log(d
/e + x)/(60*d**6*e**11 + 360*d**5*e**12*x + 900*d**4*e**13*x**2 + 1200*d**3
*e**14*x**3 + 900*d**2*e**15*x**4 + 360*d*e**16*x**5 + 60*e**17*x**6) + 283
5000*d**5*e**5*x**4/(60*d**6*e**11 + 360*d**5*e**12*x + 900*d**4*e**13*x**2
 + 1200*d**3*e**14*x**3 + 900*d**2*e**15*x**4 + 360*d*e**16*x**5 + 60*e**17
*x**6) + 25704000*d**5*e**5*x**3*log(d/e + x)/(60*d**6*e**11 + 360*d**5*e**
12*x + 900*d**4*e**13*x**2 + 1200*d**3*e**14*x**3 + 900*d**2*e**15*x**4 + 3
60*d*e**16*x**5 + 60*e**17*x**6) + 47124000*d**5*e**5*x**3/(60*d**6*e**11 +
 360*d**5*e**12*x + 900*d**4*e**13*x**2 + 1200*d**3*e**14*x**3 + 900*d**2*e
**15*x**4 + 360*d*e**16*x**5 + 60*e**17*x**6) + 617400*d**5*e**5*x**2*log(d
/e + x)/(60*d**6*e**11 + 360*d**5*e**12*x + 900*d**4*e**13*x**2 + 1200*d**3
*e**14*x**3 + 900*d**2*e**15*x**4 + 360*d*e**16*x**5 + 60*e**17*x**6) + 128
6250*d**5*e**5*x**2/(60*d**6*e**11 + 360*d**5*e**12*x + 900*d**4*e**13*x**2
 + 1200*d**3*e**14*x**3 + 900*d**2*e**15*x**4 + 360*d*e**16*x**5 + 60*e**17
*x**6) + 359640*d**5*e**5*x*log(d/e + x)/(60*d**6*e**11 + 360*d**5*e**12*x
+ 900*d**4*e**13*x**2 + 1200*d**3*e**14*x**3 + 900*d**2*e**15*x**4 + 360*d*
e**16*x**5 + 60*e**17*x**6) + 821178*d**5*e**5*x/(60*d**6*e**11 + 360*d**5*
e**12*x + 900*d**4*e**13*x**2 + 1200*d**3*e**14*x**3 + 900*d**2*e**15*x**4
+ 360*d*e**16*x**5 + 60*e**17*x**6) - 5100*d**5*e**5/(60*d**6*e**11 + 360*d
**5*e**12*x + 900*d**4*e**13*x**2 + 1200*d**3*e**14*x**3 + 900*d**2*e**15*x
```

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**4 + 360*d*e**16*x**5 + 60*e**17*x**6) + 6300000*d**4*e**6*x**6*log(d/e +
x)/(60*d**6*e**11 + 360*d**5*e**12*x + 900*d**4*e**13*x**2 + 1200*d**3*e**1
4*x**3 + 900*d**2*e**15*x**4 + 360*d*e**16*x**5 + 60*e**17*x**6) + 756000*d
**4*e**6*x**5*log(d/e + x)/(60*d**6*e**11 + 360*d**5*e**12*x + 900*d**4*e**
13*x**2 + 1200*d**3*e**14*x**3 + 900*d**2*e**15*x**4 + 360*d*e**16*x**5 + 6
0*e**17*x**6) + 756000*d**4*e**6*x**5/(60*d**6*e**11 + 360*d**5*e**12*x + 9
00*d**4*e**13*x**2 + 1200*d**3*e**14*x**3 + 900*d**2*e**15*x**4 + 360*d*e**
16*x**5 + 60*e**17*x**6) + 19278000*d**4*e**6*x**4*log(d/e + x)/(60*d**6*e*
*11 + 360*d**5*e**12*x + 900*d**4*e**13*x**2 + 1200*d**3*e**14*x**3 + 900*d
**2*e**15*x**4 + 360*d*e**16*x**5 + 60*e**17*x**6) + 28917000*d**4*e**6*x**
4/(60*d**6*e**11 + 360*d**5*e**12*x + 900*d**4*e**13*x**2 + 1200*d**3*e**14
*x**3 + 900*d**2*e**15*x**4 + 360*d*e**16*x**5 + 60*e**17*x**6) + 823200*d*
*4*e**6*x**3*log(d/e + x)/(60*d**6*e**11 + 360*d**5*e**12*x + 900*d**4*e**1
3*x**2 + 1200*d**3*e**14*x**3 + 900*d**2*e**15*x**4 + 360*d*e**16*x**5 + 60
*e**17*x**6) + 1509200*d**4*e**6*x**3/(60*d**6*e**11 + 360*d**5*e**12*x + 9
00*d**4*e**13*x**2 + 1200*d**3*e**14*x**3 + 900*d**2*e**15*x**4 + 360*d*e**
16*x**5 + 60*e**17*x**6) + 899100*d**4*e**6*x**2*log(d/e + x)/(60*d**6*e**1
1 + 360*d**5*e**12*x + 900*d**4*e**13*x**2 + 1200*d**3*e**14*x**3 + 900*d**
2*e**15*x**4 + 360*d*e**16*x**5 + 60*e**17*x**6) + 1873125*d**4*e**6*x**2/(
60*d**6*e**11 + 360*d**5*e**12*x + 900*d**4*e**13*x**2 + 1200*d**3*e**14*x*
*3 + 900*d**2*e**15*x**4 + 360*d*e**16*x**5 + 60*e**17*x**6) - 30600*d**4*e
**6*x/(60*d**6*e**11 + 360*d**5*e**12*x + 900*d**4*e**13*x**2 + 1200*d**3*e
**14*x**3 + 900*d**2*e**15*x**4 + 360*d*e**16*x**5 + 60*e**17*x**6) - 2218*
d**4*e**6/(60*d**6*e**11 + 360*d**5*e**12*x + 900*d**4*e**13*x**2 + 1200*d*
*3*e**14*x**3 + 900*d**2*e**15*x**4 + 360*d*e**16*x**5 + 60*e**17*x**6) - 9
00000*d**3*e**7*x**7/(60*d**6*e**11 + 360*d**5*e**12*x + 900*d**4*e**13*x**
2 + 1200*d**3*e**14*x**3 + 900*d**2*e**15*x**4 + 360*d*e**16*x**5 + 60*e**1
7*x**6) + 126000*d**3*e**7*x**6*log(d/e + x)/(60*d**6*e**11 + 360*d**5*e**1
2*x + 900*d**4*e**13*x**2 + 1200*d**3*e**14*x**3 + 900*d**2*e**15*x**4 + 36
0*d*e**16*x**5 + 60*e**17*x**6) + 7711200*d**3*e**7*x**5*log(d/e + x)/(60*d
**6*e**11 + 360*d**5*e**12*x + 900*d**4*e**13*x**2 + 1200*d**3*e**14*x**3 +
 900*d**2*e**15*x**4 + 360*d*e**16*x**5 + 60*e**17*x**6) + 7711200*d**3*e**
7*x**5/(60*d**6*e**11 + 360*d**5*e**12*x + 900*d**4*e**13*x**2 + 1200*d**3*
e**14*x**3 + 900*d**2*e**15*x**4 + 360*d*e**16*x**5 + 60*e**17*x**6) + 6174
00*d**3*e**7*x**4*log(d/e + x)/(60*d**6*e**11 + 360*d**5*e**12*x + 900*d**4
*e**13*x**2 + 1200*d**3*e**14*x**3 + 900*d**2*e**15*x**4 + 360*d*e**16*x**5
 + 60*e**17*x**6) + 926100*d**3*e**7*x**4/(60*d**6*e**11 + 360*d**5*e**12*x
 + 900*d**4*e**13*x**2 + 1200*d**3*e**14*x**3 + 900*d**2*e**15*x**4 + 360*d
*e**16*x**5 + 60*e**17*x**6) + 1198800*d**3*e**7*x**3*log(d/e + x)/(60*d**6
*e**11 + 360*d**5*e**12*x + 900*d**4*e**13*x**2 + 1200*d**3*e**14*x**3 + 90
0*d**2*e**15*x**4 + 360*d*e**16*x**5 + 60*e**17*x**6) + 2197800*d**3*e**7*x
**3/(60*d**6*e**11 + 360*d**5*e**12*x + 900*d**4*e**13*x**2 + 1200*d**3*e**
14*x**3 + 900*d**2*e**15*x**4 + 360*d*e**16*x**5 + 60*e**17*x**6) - 76500*d
**3*e**7*x**2/(60*d**6*e**11 + 360*d**5*e**12*x + 900*d**4*e**13*x**2 + 120
0*d**3*e**14*x**3 + 900*d**2*e**15*x**4 + 360*d*e**16*x**5 + 60*e**17*x**6)
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 - 13308*d**3*e**7*x/(60*d**6*e**11 + 360*d**5*e**12*x + 900*d**4*e**13*x**
2 + 1200*d**3*e**14*x**3 + 900*d**2*e**15*x**4 + 360*d*e**16*x**5 + 60*e**1
7*x**6) - 574*d**3*e**7/(60*d**6*e**11 + 360*d**5*e**12*x + 900*d**4*e**13*
x**2 + 1200*d**3*e**14*x**3 + 900*d**2*e**15*x**4 + 360*d*e**16*x**5 + 60*e
**17*x**6) + 112500*d**2*e**8*x**8/(60*d**6*e**11 + 360*d**5*e**12*x + 900*
d**4*e**13*x**2 + 1200*d**3*e**14*x**3 + 900*d**2*e**15*x**4 + 360*d*e**16*
x**5 + 60*e**17*x**6) - 18000*d**2*e**8*x**7/(60*d**6*e**11 + 360*d**5*e**1
2*x + 900*d**4*e**13*x**2 + 1200*d**3*e**14*x**3 + 900*d**2*e**15*x**4 + 36
0*d*e**16*x**5 + 60*e**17*x**6) + 1285200*d**2*e**8*x**6*log(d/e + x)/(60*d
**6*e**11 + 360*d**5*e**12*x + 900*d**4*e**13*x**2 + 1200*d**3*e**14*x**3 +
 900*d**2*e**15*x**4 + 360*d*e**16*x**5 + 60*e**17*x**6) + 246960*d**2*e**8
*x**5*log(d/e + x)/(60*d**6*e**11 + 360*d**5*e**12*x + 900*d**4*e**13*x**2
+ 1200*d**3*e**14*x**3 + 900*d**2*e**15*x**4 + 360*d*e**16*x**5 + 60*e**17*
x**6) + 246960*d**2*e**8*x**5/(60*d**6*e**11 + 360*d**5*e**12*x + 900*d**4*
e**13*x**2 + 1200*d**3*e**14*x**3 + 900*d**2*e**15*x**4 + 360*d*e**16*x**5
+ 60*e**17*x**6) + 899100*d**2*e**8*x**4*log(d/e + x)/(60*d**6*e**11 + 360*
d**5*e**12*x + 900*d**4*e**13*x**2 + 1200*d**3*e**14*x**3 + 900*d**2*e**15*
x**4 + 360*d*e**16*x**5 + 60*e**17*x**6) + 1348650*d**2*e**8*x**4/(60*d**6*
e**11 + 360*d**5*e**12*x + 900*d**4*e**13*x**2 + 1200*d**3*e**14*x**3 + 900
*d**2*e**15*x**4 + 360*d*e**16*x**5 + 60*e**17*x**6) - 102000*d**2*e**8*x**
3/(60*d**6*e**11 + 360*d**5*e**12*x + 900*d**4*e**13*x**2 + 1200*d**3*e**14
*x**3 + 900*d**2*e**15*x**4 + 360*d*e**16*x**5 + 60*e**17*x**6) - 33270*d**
2*e**8*x**2/(60*d**6*e**11 + 360*d**5*e**12*x + 900*d**4*e**13*x**2 + 1200*
d**3*e**14*x**3 + 900*d**2*e**15*x**4 + 360*d*e**16*x**5 + 60*e**17*x**6) -
 3444*d**2*e**8*x/(60*d**6*e**11 + 360*d**5*e**12*x + 900*d**4*e**13*x**2 +
 1200*d**3*e**14*x**3 + 900*d**2*e**15*x**4 + 360*d*e**16*x**5 + 60*e**17*x
**6) - 477*d**2*e**8/(60*d**6*e**11 + 360*d**5*e**12*x + 900*d**4*e**13*x**
2 + 1200*d**3*e**14*x**3 + 900*d**2*e**15*x**4 + 360*d*e**16*x**5 + 60*e**1
7*x**6) - 25000*d*e**9*x**9/(60*d**6*e**11 + 360*d**5*e**12*x + 900*d**4*e*
*13*x**2 + 1200*d**3*e**14*x**3 + 900*d**2*e**15*x**4 + 360*d*e**16*x**5 +
60*e**17*x**6) + 2250*d*e**9*x**8/(60*d**6*e**11 + 360*d**5*e**12*x + 900*d
**4*e**13*x**2 + 1200*d**3*e**14*x**3 + 900*d**2*e**15*x**4 + 360*d*e**16*x
**5 + 60*e**17*x**6) - 183600*d*e**9*x**7/(60*d**6*e**11 + 360*d**5*e**12*x
 + 900*d**4*e**13*x**2 + 1200*d**3*e**14*x**3 + 900*d**2*e**15*x**4 + 360*d
*e**16*x**5 + 60*e**17*x**6) + 41160*d*e**9*x**6*log(d/e + x)/(60*d**6*e**1
1 + 360*d**5*e**12*x + 900*d**4*e**13*x**2 + 1200*d**3*e**14*x**3 + 900*d**
2*e**15*x**4 + 360*d*e**16*x**5 + 60*e**17*x**6) + 359640*d*e**9*x**5*log(d
/e + x)/(60*d**6*e**11 + 360*d**5*e**12*x + 900*d**4*e**13*x**2 + 1200*d**3
*e**14*x**3 + 900*d**2*e**15*x**4 + 360*d*e**16*x**5 + 60*e**17*x**6) + 359
640*d*e**9*x**5/(60*d**6*e**11 + 360*d**5*e**12*x + 900*d**4*e**13*x**2 + 1
200*d**3*e**14*x**3 + 900*d**2*e**15*x**4 + 360*d*e**16*x**5 + 60*e**17*x**
6) - 76500*d*e**9*x**4/(60*d**6*e**11 + 360*d**5*e**12*x + 900*d**4*e**13*x
**2 + 1200*d**3*e**14*x**3 + 900*d**2*e**15*x**4 + 360*d*e**16*x**5 + 60*e*
*17*x**6) - 44360*d*e**9*x**3/(60*d**6*e**11 + 360*d**5*e**12*x + 900*d**4*
e**13*x**2 + 1200*d**3*e**14*x**3 + 900*d**2*e**15*x**4 + 360*d*e**16*x**5
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+ 60*e**17*x**6) - 8610*d*e**9*x**2/(60*d**6*e**11 + 360*d**5*e**12*x + 900
*d**4*e**13*x**2 + 1200*d**3*e**14*x**3 + 900*d**2*e**15*x**4 + 360*d*e**16
*x**5 + 60*e**17*x**6) - 2862*d*e**9*x/(60*d**6*e**11 + 360*d**5*e**12*x +
900*d**4*e**13*x**2 + 1200*d**3*e**14*x**3 + 900*d**2*e**15*x**4 + 360*d*e*
*16*x**5 + 60*e**17*x**6) - 270*d*e**9/(60*d**6*e**11 + 360*d**5*e**12*x +
900*d**4*e**13*x**2 + 1200*d**3*e**14*x**3 + 900*d**2*e**15*x**4 + 360*d*e*
*16*x**5 + 60*e**17*x**6) + 7500*e**10*x**10/(60*d**6*e**11 + 360*d**5*e**1
2*x + 900*d**4*e**13*x**2 + 1200*d**3*e**14*x**3 + 900*d**2*e**15*x**4 + 36
0*d*e**16*x**5 + 60*e**17*x**6) - 500*e**10*x**9/(60*d**6*e**11 + 360*d**5*
e**12*x + 900*d**4*e**13*x**2 + 1200*d**3*e**14*x**3 + 900*d**2*e**15*x**4
+ 360*d*e**16*x**5 + 60*e**17*x**6) + 22950*e**10*x**8/(60*d**6*e**11 + 360
*d**5*e**12*x + 900*d**4*e**13*x**2 + 1200*d**3*e**14*x**3 + 900*d**2*e**15
*x**4 + 360*d*e**16*x**5 + 60*e**17*x**6) - 5880*e**10*x**7/(60*d**6*e**11
+ 360*d**5*e**12*x + 900*d**4*e**13*x**2 + 1200*d**3*e**14*x**3 + 900*d**2*
e**15*x**4 + 360*d*e**16*x**5 + 60*e**17*x**6) + 59940*e**10*x**6*log(d/e +
 x)/(60*d**6*e**11 + 360*d**5*e**12*x + 900*d**4*e**13*x**2 + 1200*d**3*e**
14*x**3 + 900*d**2*e**15*x**4 + 360*d*e**16*x**5 + 60*e**17*x**6) - 30600*e
**10*x**5/(60*d**6*e**11 + 360*d**5*e**12*x + 900*d**4*e**13*x**2 + 1200*d*
*3*e**14*x**3 + 900*d**2*e**15*x**4 + 360*d*e**16*x**5 + 60*e**17*x**6) - 3
3270*e**10*x**4/(60*d**6*e**11 + 360*d**5*e**12*x + 900*d**4*e**13*x**2 + 1
200*d**3*e**14*x**3 + 900*d**2*e**15*x**4 + 360*d*e**16*x**5 + 60*e**17*x**
6) - 11480*e**10*x**3/(60*d**6*e**11 + 360*d**5*e**12*x + 900*d**4*e**13*x*
*2 + 1200*d**3*e**14*x**3 + 900*d**2*e**15*x**4 + 360*d*e**16*x**5 + 60*e**
17*x**6) - 7155*e**10*x**2/(60*d**6*e**11 + 360*d**5*e**12*x + 900*d**4*e**
13*x**2 + 1200*d**3*e**14*x**3 + 900*d**2*e**15*x**4 + 360*d*e**16*x**5 + 6
0*e**17*x**6) - 1620*e**10*x/(60*d**6*e**11 + 360*d**5*e**12*x + 900*d**4*e
**13*x**2 + 1200*d**3*e**14*x**3 + 900*d**2*e**15*x**4 + 360*d*e**16*x**5 +
 60*e**17*x**6) - 540*e**10/(60*d**6*e**11 + 360*d**5*e**12*x + 900*d**4*e*
*13*x**2 + 1200*d**3*e**14*x**3 + 900*d**2*e**15*x**4 + 360*d*e**16*x**5 +
60*e**17*x**6), Eq(m, -7)), (-2520000*d**10*log(d/e + x)/(20*d**5*e**11 + 1
00*d**4*e**12*x + 200*d**3*e**13*x**2 + 200*d**2*e**14*x**3 + 100*d*e**15*x
**4 + 20*e**16*x**5) - 5754000*d**10/(20*d**5*e**11 + 100*d**4*e**12*x + 20
0*d**3*e**13*x**2 + 200*d**2*e**14*x**3 + 100*d*e**15*x**4 + 20*e**16*x**5)
 - 12600000*d**9*e*x*log(d/e + x)/(20*d**5*e**11 + 100*d**4*e**12*x + 200*d
**3*e**13*x**2 + 200*d**2*e**14*x**3 + 100*d*e**15*x**4 + 20*e**16*x**5) -
26250000*d**9*e*x/(20*d**5*e**11 + 100*d**4*e**12*x + 200*d**3*e**13*x**2 +
 200*d**2*e**14*x**3 + 100*d*e**15*x**4 + 20*e**16*x**5) - 63000*d**9*e*log
(d/e + x)/(20*d**5*e**11 + 100*d**4*e**12*x + 200*d**3*e**13*x**2 + 200*d**
2*e**14*x**3 + 100*d*e**15*x**4 + 20*e**16*x**5) - 143850*d**9*e/(20*d**5*e
**11 + 100*d**4*e**12*x + 200*d**3*e**13*x**2 + 200*d**2*e**14*x**3 + 100*d
*e**15*x**4 + 20*e**16*x**5) - 25200000*d**8*e**2*x**2*log(d/e + x)/(20*d**
5*e**11 + 100*d**4*e**12*x + 200*d**3*e**13*x**2 + 200*d**2*e**14*x**3 + 10
0*d*e**15*x**4 + 20*e**16*x**5) - 46200000*d**8*e**2*x**2/(20*d**5*e**11 +
100*d**4*e**12*x + 200*d**3*e**13*x**2 + 200*d**2*e**14*x**3 + 100*d*e**15*
x**4 + 20*e**16*x**5) - 315000*d**8*e**2*x*log(d/e + x)/(20*d**5*e**11 + 10
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0*d**4*e**12*x + 200*d**3*e**13*x**2 + 200*d**2*e**14*x**3 + 100*d*e**15*x*
*4 + 20*e**16*x**5) - 656250*d**8*e**2*x/(20*d**5*e**11 + 100*d**4*e**12*x
+ 200*d**3*e**13*x**2 + 200*d**2*e**14*x**3 + 100*d*e**15*x**4 + 20*e**16*x
**5) - 856800*d**8*e**2*log(d/e + x)/(20*d**5*e**11 + 100*d**4*e**12*x + 20
0*d**3*e**13*x**2 + 200*d**2*e**14*x**3 + 100*d*e**15*x**4 + 20*e**16*x**5)
 - 1956360*d**8*e**2/(20*d**5*e**11 + 100*d**4*e**12*x + 200*d**3*e**13*x**
2 + 200*d**2*e**14*x**3 + 100*d*e**15*x**4 + 20*e**16*x**5) - 25200000*d**7
*e**3*x**3*log(d/e + x)/(20*d**5*e**11 + 100*d**4*e**12*x + 200*d**3*e**13*
x**2 + 200*d**2*e**14*x**3 + 100*d*e**15*x**4 + 20*e**16*x**5) - 37800000*d
**7*e**3*x**3/(20*d**5*e**11 + 100*d**4*e**12*x + 200*d**3*e**13*x**2 + 200
*d**2*e**14*x**3 + 100*d*e**15*x**4 + 20*e**16*x**5) - 630000*d**7*e**3*x**
2*log(d/e + x)/(20*d**5*e**11 + 100*d**4*e**12*x + 200*d**3*e**13*x**2 + 20
0*d**2*e**14*x**3 + 100*d*e**15*x**4 + 20*e**16*x**5) - 1155000*d**7*e**3*x
**2/(20*d**5*e**11 + 100*d**4*e**12*x + 200*d**3*e**13*x**2 + 200*d**2*e**1
4*x**3 + 100*d*e**15*x**4 + 20*e**16*x**5) - 4284000*d**7*e**3*x*log(d/e +
x)/(20*d**5*e**11 + 100*d**4*e**12*x + 200*d**3*e**13*x**2 + 200*d**2*e**14
*x**3 + 100*d*e**15*x**4 + 20*e**16*x**5) - 8925000*d**7*e**3*x/(20*d**5*e*
*11 + 100*d**4*e**12*x + 200*d**3*e**13*x**2 + 200*d**2*e**14*x**3 + 100*d*
e**15*x**4 + 20*e**16*x**5) - 41160*d**7*e**3*log(d/e + x)/(20*d**5*e**11 +
 100*d**4*e**12*x + 200*d**3*e**13*x**2 + 200*d**2*e**14*x**3 + 100*d*e**15
*x**4 + 20*e**16*x**5) - 93982*d**7*e**3/(20*d**5*e**11 + 100*d**4*e**12*x
+ 200*d**3*e**13*x**2 + 200*d**2*e**14*x**3 + 100*d*e**15*x**4 + 20*e**16*x
**5) - 12600000*d**6*e**4*x**4*log(d/e + x)/(20*d**5*e**11 + 100*d**4*e**12
*x + 200*d**3*e**13*x**2 + 200*d**2*e**14*x**3 + 100*d*e**15*x**4 + 20*e**1
6*x**5) - 12600000*d**6*e**4*x**4/(20*d**5*e**11 + 100*d**4*e**12*x + 200*d
**3*e**13*x**2 + 200*d**2*e**14*x**3 + 100*d*e**15*x**4 + 20*e**16*x**5) -
630000*d**6*e**4*x**3*log(d/e + x)/(20*d**5*e**11 + 100*d**4*e**12*x + 200*
d**3*e**13*x**2 + 200*d**2*e**14*x**3 + 100*d*e**15*x**4 + 20*e**16*x**5) -
 945000*d**6*e**4*x**3/(20*d**5*e**11 + 100*d**4*e**12*x + 200*d**3*e**13*x
**2 + 200*d**2*e**14*x**3 + 100*d*e**15*x**4 + 20*e**16*x**5) - 8568000*d**
6*e**4*x**2*log(d/e + x)/(20*d**5*e**11 + 100*d**4*e**12*x + 200*d**3*e**13
*x**2 + 200*d**2*e**14*x**3 + 100*d*e**15*x**4 + 20*e**16*x**5) - 15708000*
d**6*e**4*x**2/(20*d**5*e**11 + 100*d**4*e**12*x + 200*d**3*e**13*x**2 + 20
0*d**2*e**14*x**3 + 100*d*e**15*x**4 + 20*e**16*x**5) - 205800*d**6*e**4*x*
log(d/e + x)/(20*d**5*e**11 + 100*d**4*e**12*x + 200*d**3*e**13*x**2 + 200*
d**2*e**14*x**3 + 100*d*e**15*x**4 + 20*e**16*x**5) - 428750*d**6*e**4*x/(2
0*d**5*e**11 + 100*d**4*e**12*x + 200*d**3*e**13*x**2 + 200*d**2*e**14*x**3
 + 100*d*e**15*x**4 + 20*e**16*x**5) - 119880*d**6*e**4*log(d/e + x)/(20*d*
*5*e**11 + 100*d**4*e**12*x + 200*d**3*e**13*x**2 + 200*d**2*e**14*x**3 + 1
00*d*e**15*x**4 + 20*e**16*x**5) - 273726*d**6*e**4/(20*d**5*e**11 + 100*d*
*4*e**12*x + 200*d**3*e**13*x**2 + 200*d**2*e**14*x**3 + 100*d*e**15*x**4 +
 20*e**16*x**5) - 2520000*d**5*e**5*x**5*log(d/e + x)/(20*d**5*e**11 + 100*
d**4*e**12*x + 200*d**3*e**13*x**2 + 200*d**2*e**14*x**3 + 100*d*e**15*x**4
 + 20*e**16*x**5) - 315000*d**5*e**5*x**4*log(d/e + x)/(20*d**5*e**11 + 100
*d**4*e**12*x + 200*d**3*e**13*x**2 + 200*d**2*e**14*x**3 + 100*d*e**15*x**
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4 + 20*e**16*x**5) - 315000*d**5*e**5*x**4/(20*d**5*e**11 + 100*d**4*e**12*
x + 200*d**3*e**13*x**2 + 200*d**2*e**14*x**3 + 100*d*e**15*x**4 + 20*e**16
*x**5) - 8568000*d**5*e**5*x**3*log(d/e + x)/(20*d**5*e**11 + 100*d**4*e**1
2*x + 200*d**3*e**13*x**2 + 200*d**2*e**14*x**3 + 100*d*e**15*x**4 + 20*e**
16*x**5) - 12852000*d**5*e**5*x**3/(20*d**5*e**11 + 100*d**4*e**12*x + 200*
d**3*e**13*x**2 + 200*d**2*e**14*x**3 + 100*d*e**15*x**4 + 20*e**16*x**5) -
 411600*d**5*e**5*x**2*log(d/e + x)/(20*d**5*e**11 + 100*d**4*e**12*x + 200
*d**3*e**13*x**2 + 200*d**2*e**14*x**3 + 100*d*e**15*x**4 + 20*e**16*x**5)
- 754600*d**5*e**5*x**2/(20*d**5*e**11 + 100*d**4*e**12*x + 200*d**3*e**13*
x**2 + 200*d**2*e**14*x**3 + 100*d*e**15*x**4 + 20*e**16*x**5) - 599400*d**
5*e**5*x*log(d/e + x)/(20*d**5*e**11 + 100*d**4*e**12*x + 200*d**3*e**13*x*
*2 + 200*d**2*e**14*x**3 + 100*d*e**15*x**4 + 20*e**16*x**5) - 1248750*d**5
*e**5*x/(20*d**5*e**11 + 100*d**4*e**12*x + 200*d**3*e**13*x**2 + 200*d**2*
e**14*x**3 + 100*d*e**15*x**4 + 20*e**16*x**5) + 10200*d**5*e**5*log(d/e +
x)/(20*d**5*e**11 + 100*d**4*e**12*x + 200*d**3*e**13*x**2 + 200*d**2*e**14
*x**3 + 100*d*e**15*x**4 + 20*e**16*x**5) + 23290*d**5*e**5/(20*d**5*e**11
+ 100*d**4*e**12*x + 200*d**3*e**13*x**2 + 200*d**2*e**14*x**3 + 100*d*e**1
5*x**4 + 20*e**16*x**5) + 420000*d**4*e**6*x**6/(20*d**5*e**11 + 100*d**4*e
**12*x + 200*d**3*e**13*x**2 + 200*d**2*e**14*x**3 + 100*d*e**15*x**4 + 20*
e**16*x**5) - 63000*d**4*e**6*x**5*log(d/e + x)/(20*d**5*e**11 + 100*d**4*e
**12*x + 200*d**3*e**13*x**2 + 200*d**2*e**14*x**3 + 100*d*e**15*x**4 + 20*
e**16*x**5) - 4284000*d**4*e**6*x**4*log(d/e + x)/(20*d**5*e**11 + 100*d**4
*e**12*x + 200*d**3*e**13*x**2 + 200*d**2*e**14*x**3 + 100*d*e**15*x**4 + 2
0*e**16*x**5) - 4284000*d**4*e**6*x**4/(20*d**5*e**11 + 100*d**4*e**12*x +
200*d**3*e**13*x**2 + 200*d**2*e**14*x**3 + 100*d*e**15*x**4 + 20*e**16*x**
5) - 411600*d**4*e**6*x**3*log(d/e + x)/(20*d**5*e**11 + 100*d**4*e**12*x +
 200*d**3*e**13*x**2 + 200*d**2*e**14*x**3 + 100*d*e**15*x**4 + 20*e**16*x*
*5) - 617400*d**4*e**6*x**3/(20*d**5*e**11 + 100*d**4*e**12*x + 200*d**3*e*
*13*x**2 + 200*d**2*e**14*x**3 + 100*d*e**15*x**4 + 20*e**16*x**5) - 119880
0*d**4*e**6*x**2*log(d/e + x)/(20*d**5*e**11 + 100*d**4*e**12*x + 200*d**3*
e**13*x**2 + 200*d**2*e**14*x**3 + 100*d*e**15*x**4 + 20*e**16*x**5) - 2197
800*d**4*e**6*x**2/(20*d**5*e**11 + 100*d**4*e**12*x + 200*d**3*e**13*x**2
+ 200*d**2*e**14*x**3 + 100*d*e**15*x**4 + 20*e**16*x**5) + 51000*d**4*e**6
*x*log(d/e + x)/(20*d**5*e**11 + 100*d**4*e**12*x + 200*d**3*e**13*x**2 + 2
00*d**2*e**14*x**3 + 100*d*e**15*x**4 + 20*e**16*x**5) + 106250*d**4*e**6*x
/(20*d**5*e**11 + 100*d**4*e**12*x + 200*d**3*e**13*x**2 + 200*d**2*e**14*x
**3 + 100*d*e**15*x**4 + 20*e**16*x**5) - 4436*d**4*e**6/(20*d**5*e**11 + 1
00*d**4*e**12*x + 200*d**3*e**13*x**2 + 200*d**2*e**14*x**3 + 100*d*e**15*x
**4 + 20*e**16*x**5) - 60000*d**3*e**7*x**7/(20*d**5*e**11 + 100*d**4*e**12
*x + 200*d**3*e**13*x**2 + 200*d**2*e**14*x**3 + 100*d*e**15*x**4 + 20*e**1
6*x**5) + 10500*d**3*e**7*x**6/(20*d**5*e**11 + 100*d**4*e**12*x + 200*d**3
*e**13*x**2 + 200*d**2*e**14*x**3 + 100*d*e**15*x**4 + 20*e**16*x**5) - 856
800*d**3*e**7*x**5*log(d/e + x)/(20*d**5*e**11 + 100*d**4*e**12*x + 200*d**
3*e**13*x**2 + 200*d**2*e**14*x**3 + 100*d*e**15*x**4 + 20*e**16*x**5) - 20
5800*d**3*e**7*x**4*log(d/e + x)/(20*d**5*e**11 + 100*d**4*e**12*x + 200*d*
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*3*e**13*x**2 + 200*d**2*e**14*x**3 + 100*d*e**15*x**4 + 20*e**16*x**5) - 2
05800*d**3*e**7*x**4/(20*d**5*e**11 + 100*d**4*e**12*x + 200*d**3*e**13*x**
2 + 200*d**2*e**14*x**3 + 100*d*e**15*x**4 + 20*e**16*x**5) - 1198800*d**3*
e**7*x**3*log(d/e + x)/(20*d**5*e**11 + 100*d**4*e**12*x + 200*d**3*e**13*x
**2 + 200*d**2*e**14*x**3 + 100*d*e**15*x**4 + 20*e**16*x**5) - 1798200*d**
3*e**7*x**3/(20*d**5*e**11 + 100*d**4*e**12*x + 200*d**3*e**13*x**2 + 200*d
**2*e**14*x**3 + 100*d*e**15*x**4 + 20*e**16*x**5) + 102000*d**3*e**7*x**2*
log(d/e + x)/(20*d**5*e**11 + 100*d**4*e**12*x + 200*d**3*e**13*x**2 + 200*
d**2*e**14*x**3 + 100*d*e**15*x**4 + 20*e**16*x**5) + 187000*d**3*e**7*x**2
/(20*d**5*e**11 + 100*d**4*e**12*x + 200*d**3*e**13*x**2 + 200*d**2*e**14*x
**3 + 100*d*e**15*x**4 + 20*e**16*x**5) - 22180*d**3*e**7*x/(20*d**5*e**11
+ 100*d**4*e**12*x + 200*d**3*e**13*x**2 + 200*d**2*e**14*x**3 + 100*d*e**1
5*x**4 + 20*e**16*x**5) - 574*d**3*e**7/(20*d**5*e**11 + 100*d**4*e**12*x +
 200*d**3*e**13*x**2 + 200*d**2*e**14*x**3 + 100*d*e**15*x**4 + 20*e**16*x*
*5) + 15000*d**2*e**8*x**8/(20*d**5*e**11 + 100*d**4*e**12*x + 200*d**3*e**
13*x**2 + 200*d**2*e**14*x**3 + 100*d*e**15*x**4 + 20*e**16*x**5) - 1500*d*
*2*e**8*x**7/(20*d**5*e**11 + 100*d**4*e**12*x + 200*d**3*e**13*x**2 + 200*
d**2*e**14*x**3 + 100*d*e**15*x**4 + 20*e**16*x**5) + 142800*d**2*e**8*x**6
/(20*d**5*e**11 + 100*d**4*e**12*x + 200*d**3*e**13*x**2 + 200*d**2*e**14*x
**3 + 100*d*e**15*x**4 + 20*e**16*x**5) - 41160*d**2*e**8*x**5*log(d/e + x)
/(20*d**5*e**11 + 100*d**4*e**12*x + 200*d**3*e**13*x**2 + 200*d**2*e**14*x
**3 + 100*d*e**15*x**4 + 20*e**16*x**5) - 599400*d**2*e**8*x**4*log(d/e + x
)/(20*d**5*e**11 + 100*d**4*e**12*x + 200*d**3*e**13*x**2 + 200*d**2*e**14*
x**3 + 100*d*e**15*x**4 + 20*e**16*x**5) - 599400*d**2*e**8*x**4/(20*d**5*e
**11 + 100*d**4*e**12*x + 200*d**3*e**13*x**2 + 200*d**2*e**14*x**3 + 100*d
*e**15*x**4 + 20*e**16*x**5) + 102000*d**2*e**8*x**3*log(d/e + x)/(20*d**5*
e**11 + 100*d**4*e**12*x + 200*d**3*e**13*x**2 + 200*d**2*e**14*x**3 + 100*
d*e**15*x**4 + 20*e**16*x**5) + 153000*d**2*e**8*x**3/(20*d**5*e**11 + 100*
d**4*e**12*x + 200*d**3*e**13*x**2 + 200*d**2*e**14*x**3 + 100*d*e**15*x**4
 + 20*e**16*x**5) - 44360*d**2*e**8*x**2/(20*d**5*e**11 + 100*d**4*e**12*x
+ 200*d**3*e**13*x**2 + 200*d**2*e**14*x**3 + 100*d*e**15*x**4 + 20*e**16*x
**5) - 2870*d**2*e**8*x/(20*d**5*e**11 + 100*d**4*e**12*x + 200*d**3*e**13*
x**2 + 200*d**2*e**14*x**3 + 100*d*e**15*x**4 + 20*e**16*x**5) - 318*d**2*e
**8/(20*d**5*e**11 + 100*d**4*e**12*x + 200*d**3*e**13*x**2 + 200*d**2*e**1
4*x**3 + 100*d*e**15*x**4 + 20*e**16*x**5) - 5000*d*e**9*x**9/(20*d**5*e**1
1 + 100*d**4*e**12*x + 200*d**3*e**13*x**2 + 200*d**2*e**14*x**3 + 100*d*e*
*15*x**4 + 20*e**16*x**5) + 375*d*e**9*x**8/(20*d**5*e**11 + 100*d**4*e**12
*x + 200*d**3*e**13*x**2 + 200*d**2*e**14*x**3 + 100*d*e**15*x**4 + 20*e**1
6*x**5) - 20400*d*e**9*x**7/(20*d**5*e**11 + 100*d**4*e**12*x + 200*d**3*e*
*13*x**2 + 200*d**2*e**14*x**3 + 100*d*e**15*x**4 + 20*e**16*x**5) + 6860*d
*e**9*x**6/(20*d**5*e**11 + 100*d**4*e**12*x + 200*d**3*e**13*x**2 + 200*d*
*2*e**14*x**3 + 100*d*e**15*x**4 + 20*e**16*x**5) - 119880*d*e**9*x**5*log(
d/e + x)/(20*d**5*e**11 + 100*d**4*e**12*x + 200*d**3*e**13*x**2 + 200*d**2
*e**14*x**3 + 100*d*e**15*x**4 + 20*e**16*x**5) + 51000*d*e**9*x**4*log(d/e
 + x)/(20*d**5*e**11 + 100*d**4*e**12*x + 200*d**3*e**13*x**2 + 200*d**2*e*
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*14*x**3 + 100*d*e**15*x**4 + 20*e**16*x**5) + 51000*d*e**9*x**4/(20*d**5*e
**11 + 100*d**4*e**12*x + 200*d**3*e**13*x**2 + 200*d**2*e**14*x**3 + 100*d
*e**15*x**4 + 20*e**16*x**5) - 44360*d*e**9*x**3/(20*d**5*e**11 + 100*d**4*
e**12*x + 200*d**3*e**13*x**2 + 200*d**2*e**14*x**3 + 100*d*e**15*x**4 + 20
*e**16*x**5) - 5740*d*e**9*x**2/(20*d**5*e**11 + 100*d**4*e**12*x + 200*d**
3*e**13*x**2 + 200*d**2*e**14*x**3 + 100*d*e**15*x**4 + 20*e**16*x**5) - 15
90*d*e**9*x/(20*d**5*e**11 + 100*d**4*e**12*x + 200*d**3*e**13*x**2 + 200*d
**2*e**14*x**3 + 100*d*e**15*x**4 + 20*e**16*x**5) - 135*d*e**9/(20*d**5*e*
*11 + 100*d**4*e**12*x + 200*d**3*e**13*x**2 + 200*d**2*e**14*x**3 + 100*d*
e**15*x**4 + 20*e**16*x**5) + 2000*e**10*x**10/(20*d**5*e**11 + 100*d**4*e*
*12*x + 200*d**3*e**13*x**2 + 200*d**2*e**14*x**3 + 100*d*e**15*x**4 + 20*e
**16*x**5) - 125*e**10*x**9/(20*d**5*e**11 + 100*d**4*e**12*x + 200*d**3*e*
*13*x**2 + 200*d**2*e**14*x**3 + 100*d*e**15*x**4 + 20*e**16*x**5) + 5100*e
**10*x**8/(20*d**5*e**11 + 100*d**4*e**12*x + 200*d**3*e**13*x**2 + 200*d**
2*e**14*x**3 + 100*d*e**15*x**4 + 20*e**16*x**5) - 980*e**10*x**7/(20*d**5*
e**11 + 100*d**4*e**12*x + 200*d**3*e**13*x**2 + 200*d**2*e**14*x**3 + 100*
d*e**15*x**4 + 20*e**16*x**5) + 19980*e**10*x**6/(20*d**5*e**11 + 100*d**4*
e**12*x + 200*d**3*e**13*x**2 + 200*d**2*e**14*x**3 + 100*d*e**15*x**4 + 20
*e**16*x**5) + 10200*e**10*x**5*log(d/e + x)/(20*d**5*e**11 + 100*d**4*e**1
2*x + 200*d**3*e**13*x**2 + 200*d**2*e**14*x**3 + 100*d*e**15*x**4 + 20*e**
16*x**5) - 22180*e**10*x**4/(20*d**5*e**11 + 100*d**4*e**12*x + 200*d**3*e*
*13*x**2 + 200*d**2*e**14*x**3 + 100*d*e**15*x**4 + 20*e**16*x**5) - 5740*e
**10*x**3/(20*d**5*e**11 + 100*d**4*e**12*x + 200*d**3*e**13*x**2 + 200*d**
2*e**14*x**3 + 100*d*e**15*x**4 + 20*e**16*x**5) - 3180*e**10*x**2/(20*d**5
*e**11 + 100*d**4*e**12*x + 200*d**3*e**13*x**2 + 200*d**2*e**14*x**3 + 100
*d*e**15*x**4 + 20*e**16*x**5) - 675*e**10*x/(20*d**5*e**11 + 100*d**4*e**1
2*x + 200*d**3*e**13*x**2 + 200*d**2*e**14*x**3 + 100*d*e**15*x**4 + 20*e**
16*x**5) - 216*e**10/(20*d**5*e**11 + 100*d**4*e**12*x + 200*d**3*e**13*x**
2 + 200*d**2*e**14*x**3 + 100*d*e**15*x**4 + 20*e**16*x**5), Eq(m, -6)), (1
260000*d**10*log(d/e + x)/(12*d**4*e**11 + 48*d**3*e**12*x + 72*d**2*e**13*
x**2 + 48*d*e**14*x**3 + 12*e**15*x**4) + 2625000*d**10/(12*d**4*e**11 + 48
*d**3*e**12*x + 72*d**2*e**13*x**2 + 48*d*e**14*x**3 + 12*e**15*x**4) + 504
0000*d**9*e*x*log(d/e + x)/(12*d**4*e**11 + 48*d**3*e**12*x + 72*d**2*e**13
*x**2 + 48*d*e**14*x**3 + 12*e**15*x**4) + 9240000*d**9*e*x/(12*d**4*e**11
+ 48*d**3*e**12*x + 72*d**2*e**13*x**2 + 48*d*e**14*x**3 + 12*e**15*x**4) +
 37800*d**9*e*log(d/e + x)/(12*d**4*e**11 + 48*d**3*e**12*x + 72*d**2*e**13
*x**2 + 48*d*e**14*x**3 + 12*e**15*x**4) + 78750*d**9*e/(12*d**4*e**11 + 48
*d**3*e**12*x + 72*d**2*e**13*x**2 + 48*d*e**14*x**3 + 12*e**15*x**4) + 756
0000*d**8*e**2*x**2*log(d/e + x)/(12*d**4*e**11 + 48*d**3*e**12*x + 72*d**2
*e**13*x**2 + 48*d*e**14*x**3 + 12*e**15*x**4) + 11340000*d**8*e**2*x**2/(1
2*d**4*e**11 + 48*d**3*e**12*x + 72*d**2*e**13*x**2 + 48*d*e**14*x**3 + 12*
e**15*x**4) + 151200*d**8*e**2*x*log(d/e + x)/(12*d**4*e**11 + 48*d**3*e**1
2*x + 72*d**2*e**13*x**2 + 48*d*e**14*x**3 + 12*e**15*x**4) + 277200*d**8*e
**2*x/(12*d**4*e**11 + 48*d**3*e**12*x + 72*d**2*e**13*x**2 + 48*d*e**14*x*
*3 + 12*e**15*x**4) + 642600*d**8*e**2*log(d/e + x)/(12*d**4*e**11 + 48*d**
```

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3*e**12*x + 72*d**2*e**13*x**2 + 48*d*e**14*x**3 + 12*e**15*x**4) + 1338750
*d**8*e**2/(12*d**4*e**11 + 48*d**3*e**12*x + 72*d**2*e**13*x**2 + 48*d*e**
14*x**3 + 12*e**15*x**4) + 5040000*d**7*e**3*x**3*log(d/e + x)/(12*d**4*e**
11 + 48*d**3*e**12*x + 72*d**2*e**13*x**2 + 48*d*e**14*x**3 + 12*e**15*x**4
) + 5040000*d**7*e**3*x**3/(12*d**4*e**11 + 48*d**3*e**12*x + 72*d**2*e**13
*x**2 + 48*d*e**14*x**3 + 12*e**15*x**4) + 226800*d**7*e**3*x**2*log(d/e +
x)/(12*d**4*e**11 + 48*d**3*e**12*x + 72*d**2*e**13*x**2 + 48*d*e**14*x**3
+ 12*e**15*x**4) + 340200*d**7*e**3*x**2/(12*d**4*e**11 + 48*d**3*e**12*x +
 72*d**2*e**13*x**2 + 48*d*e**14*x**3 + 12*e**15*x**4) + 2570400*d**7*e**3*
x*log(d/e + x)/(12*d**4*e**11 + 48*d**3*e**12*x + 72*d**2*e**13*x**2 + 48*d
*e**14*x**3 + 12*e**15*x**4) + 4712400*d**7*e**3*x/(12*d**4*e**11 + 48*d**3
*e**12*x + 72*d**2*e**13*x**2 + 48*d*e**14*x**3 + 12*e**15*x**4) + 41160*d*
*7*e**3*log(d/e + x)/(12*d**4*e**11 + 48*d**3*e**12*x + 72*d**2*e**13*x**2
+ 48*d*e**14*x**3 + 12*e**15*x**4) + 85750*d**7*e**3/(12*d**4*e**11 + 48*d*
*3*e**12*x + 72*d**2*e**13*x**2 + 48*d*e**14*x**3 + 12*e**15*x**4) + 126000
0*d**6*e**4*x**4*log(d/e + x)/(12*d**4*e**11 + 48*d**3*e**12*x + 72*d**2*e*
*13*x**2 + 48*d*e**14*x**3 + 12*e**15*x**4) + 151200*d**6*e**4*x**3*log(d/e
 + x)/(12*d**4*e**11 + 48*d**3*e**12*x + 72*d**2*e**13*x**2 + 48*d*e**14*x*
*3 + 12*e**15*x**4) + 151200*d**6*e**4*x**3/(12*d**4*e**11 + 48*d**3*e**12*
x + 72*d**2*e**13*x**2 + 48*d*e**14*x**3 + 12*e**15*x**4) + 3855600*d**6*e*
*4*x**2*log(d/e + x)/(12*d**4*e**11 + 48*d**3*e**12*x + 72*d**2*e**13*x**2
+ 48*d*e**14*x**3 + 12*e**15*x**4) + 5783400*d**6*e**4*x**2/(12*d**4*e**11
+ 48*d**3*e**12*x + 72*d**2*e**13*x**2 + 48*d*e**14*x**3 + 12*e**15*x**4) +
 164640*d**6*e**4*x*log(d/e + x)/(12*d**4*e**11 + 48*d**3*e**12*x + 72*d**2
*e**13*x**2 + 48*d*e**14*x**3 + 12*e**15*x**4) + 301840*d**6*e**4*x/(12*d**
4*e**11 + 48*d**3*e**12*x + 72*d**2*e**13*x**2 + 48*d*e**14*x**3 + 12*e**15
*x**4) + 179820*d**6*e**4*log(d/e + x)/(12*d**4*e**11 + 48*d**3*e**12*x + 7
2*d**2*e**13*x**2 + 48*d*e**14*x**3 + 12*e**15*x**4) + 374625*d**6*e**4/(12
*d**4*e**11 + 48*d**3*e**12*x + 72*d**2*e**13*x**2 + 48*d*e**14*x**3 + 12*e
**15*x**4) - 252000*d**5*e**5*x**5/(12*d**4*e**11 + 48*d**3*e**12*x + 72*d*
*2*e**13*x**2 + 48*d*e**14*x**3 + 12*e**15*x**4) + 37800*d**5*e**5*x**4*log
(d/e + x)/(12*d**4*e**11 + 48*d**3*e**12*x + 72*d**2*e**13*x**2 + 48*d*e**1
4*x**3 + 12*e**15*x**4) + 2570400*d**5*e**5*x**3*log(d/e + x)/(12*d**4*e**1
1 + 48*d**3*e**12*x + 72*d**2*e**13*x**2 + 48*d*e**14*x**3 + 12*e**15*x**4)
 + 2570400*d**5*e**5*x**3/(12*d**4*e**11 + 48*d**3*e**12*x + 72*d**2*e**13*
x**2 + 48*d*e**14*x**3 + 12*e**15*x**4) + 246960*d**5*e**5*x**2*log(d/e + x
)/(12*d**4*e**11 + 48*d**3*e**12*x + 72*d**2*e**13*x**2 + 48*d*e**14*x**3 +
 12*e**15*x**4) + 370440*d**5*e**5*x**2/(12*d**4*e**11 + 48*d**3*e**12*x +
72*d**2*e**13*x**2 + 48*d*e**14*x**3 + 12*e**15*x**4) + 719280*d**5*e**5*x*
log(d/e + x)/(12*d**4*e**11 + 48*d**3*e**12*x + 72*d**2*e**13*x**2 + 48*d*e
**14*x**3 + 12*e**15*x**4) + 1318680*d**5*e**5*x/(12*d**4*e**11 + 48*d**3*e
**12*x + 72*d**2*e**13*x**2 + 48*d*e**14*x**3 + 12*e**15*x**4) - 30600*d**5
*e**5*log(d/e + x)/(12*d**4*e**11 + 48*d**3*e**12*x + 72*d**2*e**13*x**2 +
48*d*e**14*x**3 + 12*e**15*x**4) - 63750*d**5*e**5/(12*d**4*e**11 + 48*d**3
*e**12*x + 72*d**2*e**13*x**2 + 48*d*e**14*x**3 + 12*e**15*x**4) + 42000*d*
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*4*e**6*x**6/(12*d**4*e**11 + 48*d**3*e**12*x + 72*d**2*e**13*x**2 + 48*d*e
**14*x**3 + 12*e**15*x**4) - 7560*d**4*e**6*x**5/(12*d**4*e**11 + 48*d**3*e
**12*x + 72*d**2*e**13*x**2 + 48*d*e**14*x**3 + 12*e**15*x**4) + 642600*d**
4*e**6*x**4*log(d/e + x)/(12*d**4*e**11 + 48*d**3*e**12*x + 72*d**2*e**13*x
**2 + 48*d*e**14*x**3 + 12*e**15*x**4) + 164640*d**4*e**6*x**3*log(d/e + x)
/(12*d**4*e**11 + 48*d**3*e**12*x + 72*d**2*e**13*x**2 + 48*d*e**14*x**3 +
12*e**15*x**4) + 164640*d**4*e**6*x**3/(12*d**4*e**11 + 48*d**3*e**12*x + 7
2*d**2*e**13*x**2 + 48*d*e**14*x**3 + 12*e**15*x**4) + 1078920*d**4*e**6*x*
*2*log(d/e + x)/(12*d**4*e**11 + 48*d**3*e**12*x + 72*d**2*e**13*x**2 + 48*
d*e**14*x**3 + 12*e**15*x**4) + 1618380*d**4*e**6*x**2/(12*d**4*e**11 + 48*
d**3*e**12*x + 72*d**2*e**13*x**2 + 48*d*e**14*x**3 + 12*e**15*x**4) - 1224
00*d**4*e**6*x*log(d/e + x)/(12*d**4*e**11 + 48*d**3*e**12*x + 72*d**2*e**1
3*x**2 + 48*d*e**14*x**3 + 12*e**15*x**4) - 224400*d**4*e**6*x/(12*d**4*e**
11 + 48*d**3*e**12*x + 72*d**2*e**13*x**2 + 48*d*e**14*x**3 + 12*e**15*x**4
) + 13308*d**4*e**6*log(d/e + x)/(12*d**4*e**11 + 48*d**3*e**12*x + 72*d**2
*e**13*x**2 + 48*d*e**14*x**3 + 12*e**15*x**4) + 27725*d**4*e**6/(12*d**4*e
**11 + 48*d**3*e**12*x + 72*d**2*e**13*x**2 + 48*d*e**14*x**3 + 12*e**15*x*
*4) - 12000*d**3*e**7*x**7/(12*d**4*e**11 + 48*d**3*e**12*x + 72*d**2*e**13
*x**2 + 48*d*e**14*x**3 + 12*e**15*x**4) + 1260*d**3*e**7*x**6/(12*d**4*e**
11 + 48*d**3*e**12*x + 72*d**2*e**13*x**2 + 48*d*e**14*x**3 + 12*e**15*x**4
) - 128520*d**3*e**7*x**5/(12*d**4*e**11 + 48*d**3*e**12*x + 72*d**2*e**13*
x**2 + 48*d*e**14*x**3 + 12*e**15*x**4) + 41160*d**3*e**7*x**4*log(d/e + x)
/(12*d**4*e**11 + 48*d**3*e**12*x + 72*d**2*e**13*x**2 + 48*d*e**14*x**3 +
12*e**15*x**4) + 719280*d**3*e**7*x**3*log(d/e + x)/(12*d**4*e**11 + 48*d**
3*e**12*x + 72*d**2*e**13*x**2 + 48*d*e**14*x**3 + 12*e**15*x**4) + 719280*
d**3*e**7*x**3/(12*d**4*e**11 + 48*d**3*e**12*x + 72*d**2*e**13*x**2 + 48*d
*e**14*x**3 + 12*e**15*x**4) - 183600*d**3*e**7*x**2*log(d/e + x)/(12*d**4*
e**11 + 48*d**3*e**12*x + 72*d**2*e**13*x**2 + 48*d*e**14*x**3 + 12*e**15*x
**4) - 275400*d**3*e**7*x**2/(12*d**4*e**11 + 48*d**3*e**12*x + 72*d**2*e**
13*x**2 + 48*d*e**14*x**3 + 12*e**15*x**4) + 53232*d**3*e**7*x*log(d/e + x)
/(12*d**4*e**11 + 48*d**3*e**12*x + 72*d**2*e**13*x**2 + 48*d*e**14*x**3 +
12*e**15*x**4) + 97592*d**3*e**7*x/(12*d**4*e**11 + 48*d**3*e**12*x + 72*d*
*2*e**13*x**2 + 48*d*e**14*x**3 + 12*e**15*x**4) - 1722*d**3*e**7/(12*d**4*
e**11 + 48*d**3*e**12*x + 72*d**2*e**13*x**2 + 48*d*e**14*x**3 + 12*e**15*x
**4) + 4500*d**2*e**8*x**8/(12*d**4*e**11 + 48*d**3*e**12*x + 72*d**2*e**13
*x**2 + 48*d*e**14*x**3 + 12*e**15*x**4) - 360*d**2*e**8*x**7/(12*d**4*e**1
1 + 48*d**3*e**12*x + 72*d**2*e**13*x**2 + 48*d*e**14*x**3 + 12*e**15*x**4)
 + 21420*d**2*e**8*x**6/(12*d**4*e**11 + 48*d**3*e**12*x + 72*d**2*e**13*x*
*2 + 48*d*e**14*x**3 + 12*e**15*x**4) - 8232*d**2*e**8*x**5/(12*d**4*e**11
+ 48*d**3*e**12*x + 72*d**2*e**13*x**2 + 48*d*e**14*x**3 + 12*e**15*x**4) +
 179820*d**2*e**8*x**4*log(d/e + x)/(12*d**4*e**11 + 48*d**3*e**12*x + 72*d
**2*e**13*x**2 + 48*d*e**14*x**3 + 12*e**15*x**4) - 122400*d**2*e**8*x**3*l
og(d/e + x)/(12*d**4*e**11 + 48*d**3*e**12*x + 72*d**2*e**13*x**2 + 48*d*e*
*14*x**3 + 12*e**15*x**4) - 122400*d**2*e**8*x**3/(12*d**4*e**11 + 48*d**3*
e**12*x + 72*d**2*e**13*x**2 + 48*d*e**14*x**3 + 12*e**15*x**4) + 79848*d**
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2*e**8*x**2*log(d/e + x)/(12*d**4*e**11 + 48*d**3*e**12*x + 72*d**2*e**13*x
**2 + 48*d*e**14*x**3 + 12*e**15*x**4) + 119772*d**2*e**8*x**2/(12*d**4*e**
11 + 48*d**3*e**12*x + 72*d**2*e**13*x**2 + 48*d*e**14*x**3 + 12*e**15*x**4
) - 6888*d**2*e**8*x/(12*d**4*e**11 + 48*d**3*e**12*x + 72*d**2*e**13*x**2
+ 48*d*e**14*x**3 + 12*e**15*x**4) - 477*d**2*e**8/(12*d**4*e**11 + 48*d**3
*e**12*x + 72*d**2*e**13*x**2 + 48*d*e**14*x**3 + 12*e**15*x**4) - 2000*d*e
**9*x**9/(12*d**4*e**11 + 48*d**3*e**12*x + 72*d**2*e**13*x**2 + 48*d*e**14
*x**3 + 12*e**15*x**4) + 135*d*e**9*x**8/(12*d**4*e**11 + 48*d**3*e**12*x +
 72*d**2*e**13*x**2 + 48*d*e**14*x**3 + 12*e**15*x**4) - 6120*d*e**9*x**7/(
12*d**4*e**11 + 48*d**3*e**12*x + 72*d**2*e**13*x**2 + 48*d*e**14*x**3 + 12
*e**15*x**4) + 1372*d*e**9*x**6/(12*d**4*e**11 + 48*d**3*e**12*x + 72*d**2*
e**13*x**2 + 48*d*e**14*x**3 + 12*e**15*x**4) - 35964*d*e**9*x**5/(12*d**4*
e**11 + 48*d**3*e**12*x + 72*d**2*e**13*x**2 + 48*d*e**14*x**3 + 12*e**15*x
**4) - 30600*d*e**9*x**4*log(d/e + x)/(12*d**4*e**11 + 48*d**3*e**12*x + 72
*d**2*e**13*x**2 + 48*d*e**14*x**3 + 12*e**15*x**4) + 53232*d*e**9*x**3*log
(d/e + x)/(12*d**4*e**11 + 48*d**3*e**12*x + 72*d**2*e**13*x**2 + 48*d*e**1
4*x**3 + 12*e**15*x**4) + 53232*d*e**9*x**3/(12*d**4*e**11 + 48*d**3*e**12*
x + 72*d**2*e**13*x**2 + 48*d*e**14*x**3 + 12*e**15*x**4) - 10332*d*e**9*x*
*2/(12*d**4*e**11 + 48*d**3*e**12*x + 72*d**2*e**13*x**2 + 48*d*e**14*x**3
+ 12*e**15*x**4) - 1908*d*e**9*x/(12*d**4*e**11 + 48*d**3*e**12*x + 72*d**2
*e**13*x**2 + 48*d*e**14*x**3 + 12*e**15*x**4) - 135*d*e**9/(12*d**4*e**11
+ 48*d**3*e**12*x + 72*d**2*e**13*x**2 + 48*d*e**14*x**3 + 12*e**15*x**4) +
 1000*e**10*x**10/(12*d**4*e**11 + 48*d**3*e**12*x + 72*d**2*e**13*x**2 + 4
8*d*e**14*x**3 + 12*e**15*x**4) - 60*e**10*x**9/(12*d**4*e**11 + 48*d**3*e*
*12*x + 72*d**2*e**13*x**2 + 48*d*e**14*x**3 + 12*e**15*x**4) + 2295*e**10*
x**8/(12*d**4*e**11 + 48*d**3*e**12*x + 72*d**2*e**13*x**2 + 48*d*e**14*x**
3 + 12*e**15*x**4) - 392*e**10*x**7/(12*d**4*e**11 + 48*d**3*e**12*x + 72*d
**2*e**13*x**2 + 48*d*e**14*x**3 + 12*e**15*x**4) + 5994*e**10*x**6/(12*d**
4*e**11 + 48*d**3*e**12*x + 72*d**2*e**13*x**2 + 48*d*e**14*x**3 + 12*e**15
*x**4) + 6120*e**10*x**5/(12*d**4*e**11 + 48*d**3*e**12*x + 72*d**2*e**13*x
**2 + 48*d*e**14*x**3 + 12*e**15*x**4) + 13308*e**10*x**4*log(d/e + x)/(12*
d**4*e**11 + 48*d**3*e**12*x + 72*d**2*e**13*x**2 + 48*d*e**14*x**3 + 12*e*
*15*x**4) - 6888*e**10*x**3/(12*d**4*e**11 + 48*d**3*e**12*x + 72*d**2*e**1
3*x**2 + 48*d*e**14*x**3 + 12*e**15*x**4) - 2862*e**10*x**2/(12*d**4*e**11
+ 48*d**3*e**12*x + 72*d**2*e**13*x**2 + 48*d*e**14*x**3 + 12*e**15*x**4) -
 540*e**10*x/(12*d**4*e**11 + 48*d**3*e**12*x + 72*d**2*e**13*x**2 + 48*d*e
**14*x**3 + 12*e**15*x**4) - 162*e**10/(12*d**4*e**11 + 48*d**3*e**12*x + 7
2*d**2*e**13*x**2 + 48*d*e**14*x**3 + 12*e**15*x**4), Eq(m, -5)), (-2520000
*d**10*log(d/e + x)/(42*d**3*e**11 + 126*d**2*e**12*x + 126*d*e**13*x**2 +
42*e**14*x**3) - 4620000*d**10/(42*d**3*e**11 + 126*d**2*e**12*x + 126*d*e*
*13*x**2 + 42*e**14*x**3) - 7560000*d**9*e*x*log(d/e + x)/(42*d**3*e**11 +
126*d**2*e**12*x + 126*d*e**13*x**2 + 42*e**14*x**3) - 11340000*d**9*e*x/(4
2*d**3*e**11 + 126*d**2*e**12*x + 126*d*e**13*x**2 + 42*e**14*x**3) - 88200
*d**9*e*log(d/e + x)/(42*d**3*e**11 + 126*d**2*e**12*x + 126*d*e**13*x**2 +
 42*e**14*x**3) - 161700*d**9*e/(42*d**3*e**11 + 126*d**2*e**12*x + 126*d*e
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**13*x**2 + 42*e**14*x**3) - 7560000*d**8*e**2*x**2*log(d/e + x)/(42*d**3*e
**11 + 126*d**2*e**12*x + 126*d*e**13*x**2 + 42*e**14*x**3) - 7560000*d**8*
e**2*x**2/(42*d**3*e**11 + 126*d**2*e**12*x + 126*d*e**13*x**2 + 42*e**14*x
**3) - 264600*d**8*e**2*x*log(d/e + x)/(42*d**3*e**11 + 126*d**2*e**12*x +
126*d*e**13*x**2 + 42*e**14*x**3) - 396900*d**8*e**2*x/(42*d**3*e**11 + 126
*d**2*e**12*x + 126*d*e**13*x**2 + 42*e**14*x**3) - 1799280*d**8*e**2*log(d
/e + x)/(42*d**3*e**11 + 126*d**2*e**12*x + 126*d*e**13*x**2 + 42*e**14*x**
3) - 3298680*d**8*e**2/(42*d**3*e**11 + 126*d**2*e**12*x + 126*d*e**13*x**2
 + 42*e**14*x**3) - 2520000*d**7*e**3*x**3*log(d/e + x)/(42*d**3*e**11 + 12
6*d**2*e**12*x + 126*d*e**13*x**2 + 42*e**14*x**3) - 264600*d**7*e**3*x**2*
log(d/e + x)/(42*d**3*e**11 + 126*d**2*e**12*x + 126*d*e**13*x**2 + 42*e**1
4*x**3) - 264600*d**7*e**3*x**2/(42*d**3*e**11 + 126*d**2*e**12*x + 126*d*e
**13*x**2 + 42*e**14*x**3) - 5397840*d**7*e**3*x*log(d/e + x)/(42*d**3*e**1
1 + 126*d**2*e**12*x + 126*d*e**13*x**2 + 42*e**14*x**3) - 8096760*d**7*e**
3*x/(42*d**3*e**11 + 126*d**2*e**12*x + 126*d*e**13*x**2 + 42*e**14*x**3) -
 144060*d**7*e**3*log(d/e + x)/(42*d**3*e**11 + 126*d**2*e**12*x + 126*d*e*
*13*x**2 + 42*e**14*x**3) - 264110*d**7*e**3/(42*d**3*e**11 + 126*d**2*e**1
2*x + 126*d*e**13*x**2 + 42*e**14*x**3) + 630000*d**6*e**4*x**4/(42*d**3*e*
*11 + 126*d**2*e**12*x + 126*d*e**13*x**2 + 42*e**14*x**3) - 88200*d**6*e**
4*x**3*log(d/e + x)/(42*d**3*e**11 + 126*d**2*e**12*x + 126*d*e**13*x**2 +
42*e**14*x**3) - 5397840*d**6*e**4*x**2*log(d/e + x)/(42*d**3*e**11 + 126*d
**2*e**12*x + 126*d*e**13*x**2 + 42*e**14*x**3) - 5397840*d**6*e**4*x**2/(4
2*d**3*e**11 + 126*d**2*e**12*x + 126*d*e**13*x**2 + 42*e**14*x**3) - 43218
0*d**6*e**4*x*log(d/e + x)/(42*d**3*e**11 + 126*d**2*e**12*x + 126*d*e**13*
x**2 + 42*e**14*x**3) - 648270*d**6*e**4*x/(42*d**3*e**11 + 126*d**2*e**12*
x + 126*d*e**13*x**2 + 42*e**14*x**3) - 839160*d**6*e**4*log(d/e + x)/(42*d
**3*e**11 + 126*d**2*e**12*x + 126*d*e**13*x**2 + 42*e**14*x**3) - 1538460*
d**6*e**4/(42*d**3*e**11 + 126*d**2*e**12*x + 126*d*e**13*x**2 + 42*e**14*x
**3) - 126000*d**5*e**5*x**5/(42*d**3*e**11 + 126*d**2*e**12*x + 126*d*e**1
3*x**2 + 42*e**14*x**3) + 22050*d**5*e**5*x**4/(42*d**3*e**11 + 126*d**2*e*
*12*x + 126*d*e**13*x**2 + 42*e**14*x**3) - 1799280*d**5*e**5*x**3*log(d/e
+ x)/(42*d**3*e**11 + 126*d**2*e**12*x + 126*d*e**13*x**2 + 42*e**14*x**3)
- 432180*d**5*e**5*x**2*log(d/e + x)/(42*d**3*e**11 + 126*d**2*e**12*x + 12
6*d*e**13*x**2 + 42*e**14*x**3) - 432180*d**5*e**5*x**2/(42*d**3*e**11 + 12
6*d**2*e**12*x + 126*d*e**13*x**2 + 42*e**14*x**3) - 2517480*d**5*e**5*x*lo
g(d/e + x)/(42*d**3*e**11 + 126*d**2*e**12*x + 126*d*e**13*x**2 + 42*e**14*
x**3) - 3776220*d**5*e**5*x/(42*d**3*e**11 + 126*d**2*e**12*x + 126*d*e**13
*x**2 + 42*e**14*x**3) + 214200*d**5*e**5*log(d/e + x)/(42*d**3*e**11 + 126
*d**2*e**12*x + 126*d*e**13*x**2 + 42*e**14*x**3) + 392700*d**5*e**5/(42*d*
*3*e**11 + 126*d**2*e**12*x + 126*d*e**13*x**2 + 42*e**14*x**3) + 42000*d**
4*e**6*x**6/(42*d**3*e**11 + 126*d**2*e**12*x + 126*d*e**13*x**2 + 42*e**14
*x**3) - 4410*d**4*e**6*x**5/(42*d**3*e**11 + 126*d**2*e**12*x + 126*d*e**1
3*x**2 + 42*e**14*x**3) + 449820*d**4*e**6*x**4/(42*d**3*e**11 + 126*d**2*e
**12*x + 126*d*e**13*x**2 + 42*e**14*x**3) - 144060*d**4*e**6*x**3*log(d/e
+ x)/(42*d**3*e**11 + 126*d**2*e**12*x + 126*d*e**13*x**2 + 42*e**14*x**3)
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- 2517480*d**4*e**6*x**2*log(d/e + x)/(42*d**3*e**11 + 126*d**2*e**12*x + 1
26*d*e**13*x**2 + 42*e**14*x**3) - 2517480*d**4*e**6*x**2/(42*d**3*e**11 +
126*d**2*e**12*x + 126*d*e**13*x**2 + 42*e**14*x**3) + 642600*d**4*e**6*x*l
og(d/e + x)/(42*d**3*e**11 + 126*d**2*e**12*x + 126*d*e**13*x**2 + 42*e**14
*x**3) + 963900*d**4*e**6*x/(42*d**3*e**11 + 126*d**2*e**12*x + 126*d*e**13
*x**2 + 42*e**14*x**3) - 186312*d**4*e**6*log(d/e + x)/(42*d**3*e**11 + 126
*d**2*e**12*x + 126*d*e**13*x**2 + 42*e**14*x**3) - 341572*d**4*e**6/(42*d*
*3*e**11 + 126*d**2*e**12*x + 126*d*e**13*x**2 + 42*e**14*x**3) - 18000*d**
3*e**7*x**7/(42*d**3*e**11 + 126*d**2*e**12*x + 126*d*e**13*x**2 + 42*e**14
*x**3) + 1470*d**3*e**7*x**6/(42*d**3*e**11 + 126*d**2*e**12*x + 126*d*e**1
3*x**2 + 42*e**14*x**3) - 89964*d**3*e**7*x**5/(42*d**3*e**11 + 126*d**2*e*
*12*x + 126*d*e**13*x**2 + 42*e**14*x**3) + 36015*d**3*e**7*x**4/(42*d**3*e
**11 + 126*d**2*e**12*x + 126*d*e**13*x**2 + 42*e**14*x**3) - 839160*d**3*e
**7*x**3*log(d/e + x)/(42*d**3*e**11 + 126*d**2*e**12*x + 126*d*e**13*x**2
+ 42*e**14*x**3) + 642600*d**3*e**7*x**2*log(d/e + x)/(42*d**3*e**11 + 126*
d**2*e**12*x + 126*d*e**13*x**2 + 42*e**14*x**3) + 642600*d**3*e**7*x**2/(4
2*d**3*e**11 + 126*d**2*e**12*x + 126*d*e**13*x**2 + 42*e**14*x**3) - 55893
6*d**3*e**7*x*log(d/e + x)/(42*d**3*e**11 + 126*d**2*e**12*x + 126*d*e**13*
x**2 + 42*e**14*x**3) - 838404*d**3*e**7*x/(42*d**3*e**11 + 126*d**2*e**12*
x + 126*d*e**13*x**2 + 42*e**14*x**3) + 24108*d**3*e**7*log(d/e + x)/(42*d*
*3*e**11 + 126*d**2*e**12*x + 126*d*e**13*x**2 + 42*e**14*x**3) + 44198*d**
3*e**7/(42*d**3*e**11 + 126*d**2*e**12*x + 126*d*e**13*x**2 + 42*e**14*x**3
) + 9000*d**2*e**8*x**8/(42*d**3*e**11 + 126*d**2*e**12*x + 126*d*e**13*x**
2 + 42*e**14*x**3) - 630*d**2*e**8*x**7/(42*d**3*e**11 + 126*d**2*e**12*x +
 126*d*e**13*x**2 + 42*e**14*x**3) + 29988*d**2*e**8*x**6/(42*d**3*e**11 +
126*d**2*e**12*x + 126*d*e**13*x**2 + 42*e**14*x**3) - 7203*d**2*e**8*x**5/
(42*d**3*e**11 + 126*d**2*e**12*x + 126*d*e**13*x**2 + 42*e**14*x**3) + 209
790*d**2*e**8*x**4/(42*d**3*e**11 + 126*d**2*e**12*x + 126*d*e**13*x**2 + 4
2*e**14*x**3) + 214200*d**2*e**8*x**3*log(d/e + x)/(42*d**3*e**11 + 126*d**
2*e**12*x + 126*d*e**13*x**2 + 42*e**14*x**3) - 558936*d**2*e**8*x**2*log(d
/e + x)/(42*d**3*e**11 + 126*d**2*e**12*x + 126*d*e**13*x**2 + 42*e**14*x**
3) - 558936*d**2*e**8*x**2/(42*d**3*e**11 + 126*d**2*e**12*x + 126*d*e**13*
x**2 + 42*e**14*x**3) + 72324*d**2*e**8*x*log(d/e + x)/(42*d**3*e**11 + 126
*d**2*e**12*x + 126*d*e**13*x**2 + 42*e**14*x**3) + 108486*d**2*e**8*x/(42*
d**3*e**11 + 126*d**2*e**12*x + 126*d*e**13*x**2 + 42*e**14*x**3) - 6678*d*
*2*e**8/(42*d**3*e**11 + 126*d**2*e**12*x + 126*d*e**13*x**2 + 42*e**14*x**
3) - 5000*d*e**9*x**9/(42*d**3*e**11 + 126*d**2*e**12*x + 126*d*e**13*x**2
+ 42*e**14*x**3) + 315*d*e**9*x**8/(42*d**3*e**11 + 126*d**2*e**12*x + 126*
d*e**13*x**2 + 42*e**14*x**3) - 12852*d*e**9*x**7/(42*d**3*e**11 + 126*d**2
*e**12*x + 126*d*e**13*x**2 + 42*e**14*x**3) + 2401*d*e**9*x**6/(42*d**3*e*
*11 + 126*d**2*e**12*x + 126*d*e**13*x**2 + 42*e**14*x**3) - 41958*d*e**9*x
**5/(42*d**3*e**11 + 126*d**2*e**12*x + 126*d*e**13*x**2 + 42*e**14*x**3) -
 53550*d*e**9*x**4/(42*d**3*e**11 + 126*d**2*e**12*x + 126*d*e**13*x**2 + 4
2*e**14*x**3) - 186312*d*e**9*x**3*log(d/e + x)/(42*d**3*e**11 + 126*d**2*e
**12*x + 126*d*e**13*x**2 + 42*e**14*x**3) + 72324*d*e**9*x**2*log(d/e + x)
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/(42*d**3*e**11 + 126*d**2*e**12*x + 126*d*e**13*x**2 + 42*e**14*x**3) + 72
324*d*e**9*x**2/(42*d**3*e**11 + 126*d**2*e**12*x + 126*d*e**13*x**2 + 42*e
**14*x**3) - 20034*d*e**9*x/(42*d**3*e**11 + 126*d**2*e**12*x + 126*d*e**13
*x**2 + 42*e**14*x**3) - 945*d*e**9/(42*d**3*e**11 + 126*d**2*e**12*x + 126
*d*e**13*x**2 + 42*e**14*x**3) + 3000*e**10*x**10/(42*d**3*e**11 + 126*d**2
*e**12*x + 126*d*e**13*x**2 + 42*e**14*x**3) - 175*e**10*x**9/(42*d**3*e**1
1 + 126*d**2*e**12*x + 126*d*e**13*x**2 + 42*e**14*x**3) + 6426*e**10*x**8/
(42*d**3*e**11 + 126*d**2*e**12*x + 126*d*e**13*x**2 + 42*e**14*x**3) - 102
9*e**10*x**7/(42*d**3*e**11 + 126*d**2*e**12*x + 126*d*e**13*x**2 + 42*e**1
4*x**3) + 13986*e**10*x**6/(42*d**3*e**11 + 126*d**2*e**12*x + 126*d*e**13*
x**2 + 42*e**14*x**3) + 10710*e**10*x**5/(42*d**3*e**11 + 126*d**2*e**12*x
 + 126*d*e**13*x**2 + 42*e**14*x**3) + 46578*e**10*x**4/(42*d**3*e**11 + 126
*d**2*e**12*x + 126*d*e**13*x**2 + 42*e**14*x**3) + 24108*e**10*x**3*log(d/
e + x)/(42*d**3*e**11 + 126*d**2*e**12*x + 126*d*e**13*x**2 + 42*e**14*x**3
) - 20034*e**10*x**2/(42*d**3*e**11 + 126*d**2*e**12*x + 126*d*e**13*x**2 +
 42*e**14*x**3) - 2835*e**10*x/(42*d**3*e**11 + 126*d**2*e**12*x + 126*d*e*
*13*x**2 + 42*e**14*x**3) - 756*e**10/(42*d**3*e**11 + 126*d**2*e**12*x + 1
26*d*e**13*x**2 + 42*e**14*x**3), Eq(m, -4)), (3150000*d**10*log(d/e + x)/(
140*d**2*e**11 + 280*d*e**12*x + 140*e**13*x**2) + 4725000*d**10/(140*d**2*
e**11 + 280*d*e**12*x + 140*e**13*x**2) + 6300000*d**9*e*x*log(d/e + x)/(14
0*d**2*e**11 + 280*d*e**12*x + 140*e**13*x**2) + 6300000*d**9*e*x/(140*d**2
*e**11 + 280*d*e**12*x + 140*e**13*x**2) + 126000*d**9*e*log(d/e + x)/(140*
d**2*e**11 + 280*d*e**12*x + 140*e**13*x**2) + 189000*d**9*e/(140*d**2*e**1
1 + 280*d*e**12*x + 140*e**13*x**2) + 3150000*d**8*e**2*x**2*log(d/e + x)/(
140*d**2*e**11 + 280*d*e**12*x + 140*e**13*x**2) + 252000*d**8*e**2*x*log(d
/e + x)/(140*d**2*e**11 + 280*d*e**12*x + 140*e**13*x**2) + 252000*d**8*e**
2*x/(140*d**2*e**11 + 280*d*e**12*x + 140*e**13*x**2) + 2998800*d**8*e**2*l
og(d/e + x)/(140*d**2*e**11 + 280*d*e**12*x + 140*e**13*x**2) + 4498200*d**
8*e**2/(140*d**2*e**11 + 280*d*e**12*x + 140*e**13*x**2) - 1050000*d**7*e**
3*x**3/(140*d**2*e**11 + 280*d*e**12*x + 140*e**13*x**2) + 126000*d**7*e**3
*x**2*log(d/e + x)/(140*d**2*e**11 + 280*d*e**12*x + 140*e**13*x**2) + 5997
600*d**7*e**3*x*log(d/e + x)/(140*d**2*e**11 + 280*d*e**12*x + 140*e**13*x*
*2) + 5997600*d**7*e**3*x/(140*d**2*e**11 + 280*d*e**12*x + 140*e**13*x**2)
 + 288120*d**7*e**3*log(d/e + x)/(140*d**2*e**11 + 280*d*e**12*x + 140*e**1
3*x**2) + 432180*d**7*e**3/(140*d**2*e**11 + 280*d*e**12*x + 140*e**13*x**2
) + 262500*d**6*e**4*x**4/(140*d**2*e**11 + 280*d*e**12*x + 140*e**13*x**2)
 - 42000*d**6*e**4*x**3/(140*d**2*e**11 + 280*d*e**12*x + 140*e**13*x**2) +
 2998800*d**6*e**4*x**2*log(d/e + x)/(140*d**2*e**11 + 280*d*e**12*x + 140*
e**13*x**2) + 576240*d**6*e**4*x*log(d/e + x)/(140*d**2*e**11 + 280*d*e**12
*x + 140*e**13*x**2) + 576240*d**6*e**4*x/(140*d**2*e**11 + 280*d*e**12*x +
 140*e**13*x**2) + 2097900*d**6*e**4*log(d/e + x)/(140*d**2*e**11 + 280*d*e
**12*x + 140*e**13*x**2) + 3146850*d**6*e**4/(140*d**2*e**11 + 280*d*e**12*
x + 140*e**13*x**2) - 105000*d**5*e**5*x**5/(140*d**2*e**11 + 280*d*e**12*x
 + 140*e**13*x**2) + 10500*d**5*e**5*x**4/(140*d**2*e**11 + 280*d*e**12*x +
 140*e**13*x**2) - 999600*d**5*e**5*x**3/(140*d**2*e**11 + 280*d*e**12*x +
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140*e**13*x**2) + 288120*d**5*e**5*x**2*log(d/e + x)/(140*d**2*e**11 + 280*
d*e**12*x + 140*e**13*x**2) + 4195800*d**5*e**5*x*log(d/e + x)/(140*d**2*e*
*11 + 280*d*e**12*x + 140*e**13*x**2) + 4195800*d**5*e**5*x/(140*d**2*e**11
 + 280*d*e**12*x + 140*e**13*x**2) - 714000*d**5*e**5*log(d/e + x)/(140*d**
2*e**11 + 280*d*e**12*x + 140*e**13*x**2) - 1071000*d**5*e**5/(140*d**2*e**
11 + 280*d*e**12*x + 140*e**13*x**2) + 52500*d**4*e**6*x**6/(140*d**2*e**11
 + 280*d*e**12*x + 140*e**13*x**2) - 4200*d**4*e**6*x**5/(140*d**2*e**11 +
280*d*e**12*x + 140*e**13*x**2) + 249900*d**4*e**6*x**4/(140*d**2*e**11 + 2
80*d*e**12*x + 140*e**13*x**2) - 96040*d**4*e**6*x**3/(140*d**2*e**11 + 280
*d*e**12*x + 140*e**13*x**2) + 2097900*d**4*e**6*x**2*log(d/e + x)/(140*d**
2*e**11 + 280*d*e**12*x + 140*e**13*x**2) - 1428000*d**4*e**6*x*log(d/e + x
)/(140*d**2*e**11 + 280*d*e**12*x + 140*e**13*x**2) - 1428000*d**4*e**6*x/(
140*d**2*e**11 + 280*d*e**12*x + 140*e**13*x**2) + 931560*d**4*e**6*log(d/e
 + x)/(140*d**2*e**11 + 280*d*e**12*x + 140*e**13*x**2) + 1397340*d**4*e**6
/(140*d**2*e**11 + 280*d*e**12*x + 140*e**13*x**2) - 30000*d**3*e**7*x**7/(
140*d**2*e**11 + 280*d*e**12*x + 140*e**13*x**2) + 2100*d**3*e**7*x**6/(140
*d**2*e**11 + 280*d*e**12*x + 140*e**13*x**2) - 99960*d**3*e**7*x**5/(140*d
**2*e**11 + 280*d*e**12*x + 140*e**13*x**2) + 24010*d**3*e**7*x**4/(140*d**
2*e**11 + 280*d*e**12*x + 140*e**13*x**2) - 699300*d**3*e**7*x**3/(140*d**2
*e**11 + 280*d*e**12*x + 140*e**13*x**2) - 714000*d**3*e**7*x**2*log(d/e +
x)/(140*d**2*e**11 + 280*d*e**12*x + 140*e**13*x**2) + 1863120*d**3*e**7*x*
log(d/e + x)/(140*d**2*e**11 + 280*d*e**12*x + 140*e**13*x**2) + 1863120*d*
*3*e**7*x/(140*d**2*e**11 + 280*d*e**12*x + 140*e**13*x**2) - 241080*d**3*e
**7*log(d/e + x)/(140*d**2*e**11 + 280*d*e**12*x + 140*e**13*x**2) - 361620
*d**3*e**7/(140*d**2*e**11 + 280*d*e**12*x + 140*e**13*x**2) + 18750*d**2*e
**8*x**8/(140*d**2*e**11 + 280*d*e**12*x + 140*e**13*x**2) - 1200*d**2*e**8
*x**7/(140*d**2*e**11 + 280*d*e**12*x + 140*e**13*x**2) + 49980*d**2*e**8*x
**6/(140*d**2*e**11 + 280*d*e**12*x + 140*e**13*x**2) - 9604*d**2*e**8*x**5
/(140*d**2*e**11 + 280*d*e**12*x + 140*e**13*x**2) + 174825*d**2*e**8*x**4/
(140*d**2*e**11 + 280*d*e**12*x + 140*e**13*x**2) + 238000*d**2*e**8*x**3/(
140*d**2*e**11 + 280*d*e**12*x + 140*e**13*x**2) + 931560*d**2*e**8*x**2*lo
g(d/e + x)/(140*d**2*e**11 + 280*d*e**12*x + 140*e**13*x**2) - 482160*d**2*
e**8*x*log(d/e + x)/(140*d**2*e**11 + 280*d*e**12*x + 140*e**13*x**2) - 482
160*d**2*e**8*x/(140*d**2*e**11 + 280*d*e**12*x + 140*e**13*x**2) + 66780*d
**2*e**8*log(d/e + x)/(140*d**2*e**11 + 280*d*e**12*x + 140*e**13*x**2) + 1
00170*d**2*e**8/(140*d**2*e**11 + 280*d*e**12*x + 140*e**13*x**2) - 12500*d
*e**9*x**9/(140*d**2*e**11 + 280*d*e**12*x + 140*e**13*x**2) + 750*d*e**9*x
**8/(140*d**2*e**11 + 280*d*e**12*x + 140*e**13*x**2) - 28560*d*e**9*x**7/(
140*d**2*e**11 + 280*d*e**12*x + 140*e**13*x**2) + 4802*d*e**9*x**6/(140*d*
*2*e**11 + 280*d*e**12*x + 140*e**13*x**2) - 69930*d*e**9*x**5/(140*d**2*e*
*11 + 280*d*e**12*x + 140*e**13*x**2) - 59500*d*e**9*x**4/(140*d**2*e**11 +
 280*d*e**12*x + 140*e**13*x**2) - 310520*d*e**9*x**3/(140*d**2*e**11 + 280
*d*e**12*x + 140*e**13*x**2) - 241080*d*e**9*x**2*log(d/e + x)/(140*d**2*e*
*11 + 280*d*e**12*x + 140*e**13*x**2) + 133560*d*e**9*x*log(d/e + x)/(140*d
**2*e**11 + 280*d*e**12*x + 140*e**13*x**2) + 133560*d*e**9*x/(140*d**2*e**
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11 + 280*d*e**12*x + 140*e**13*x**2) - 9450*d*e**9/(140*d**2*e**11 + 280*d*
e**12*x + 140*e**13*x**2) + 8750*e**10*x**10/(140*d**2*e**11 + 280*d*e**12*
x + 140*e**13*x**2) - 500*e**10*x**9/(140*d**2*e**11 + 280*d*e**12*x + 140*
e**13*x**2) + 17850*e**10*x**8/(140*d**2*e**11 + 280*d*e**12*x + 140*e**13*
x**2) - 2744*e**10*x**7/(140*d**2*e**11 + 280*d*e**12*x + 140*e**13*x**2) +
 34965*e**10*x**6/(140*d**2*e**11 + 280*d*e**12*x + 140*e**13*x**2) + 23800
*e**10*x**5/(140*d**2*e**11 + 280*d*e**12*x + 140*e**13*x**2) + 77630*e**10
*x**4/(140*d**2*e**11 + 280*d*e**12*x + 140*e**13*x**2) + 80360*e**10*x**3/
(140*d**2*e**11 + 280*d*e**12*x + 140*e**13*x**2) + 66780*e**10*x**2*log(d/
e + x)/(140*d**2*e**11 + 280*d*e**12*x + 140*e**13*x**2) - 18900*e**10*x/(1
40*d**2*e**11 + 280*d*e**12*x + 140*e**13*x**2) - 3780*e**10/(140*d**2*e**1
1 + 280*d*e**12*x + 140*e**13*x**2), Eq(m, -3)), (-12600000*d**10*log(d/e +
 x)/(2520*d*e**11 + 2520*e**12*x) - 12600000*d**10/(2520*d*e**11 + 2520*e**
12*x) - 12600000*d**9*e*x*log(d/e + x)/(2520*d*e**11 + 2520*e**12*x) - 5670
00*d**9*e*log(d/e + x)/(2520*d*e**11 + 2520*e**12*x) - 567000*d**9*e/(2520*
d*e**11 + 2520*e**12*x) + 6300000*d**8*e**2*x**2/(2520*d*e**11 + 2520*e**12
*x) - 567000*d**8*e**2*x*log(d/e + x)/(2520*d*e**11 + 2520*e**12*x) - 15422
400*d**8*e**2*log(d/e + x)/(2520*d*e**11 + 2520*e**12*x) - 15422400*d**8*e*
*2/(2520*d*e**11 + 2520*e**12*x) - 2100000*d**7*e**3*x**3/(2520*d*e**11 + 2
520*e**12*x) + 283500*d**7*e**3*x**2/(2520*d*e**11 + 2520*e**12*x) - 154224
00*d**7*e**3*x*log(d/e + x)/(2520*d*e**11 + 2520*e**12*x) - 1728720*d**7*e*
*3*log(d/e + x)/(2520*d*e**11 + 2520*e**12*x) - 1728720*d**7*e**3/(2520*d*e
**11 + 2520*e**12*x) + 1050000*d**6*e**4*x**4/(2520*d*e**11 + 2520*e**12*x)
 - 94500*d**6*e**4*x**3/(2520*d*e**11 + 2520*e**12*x) + 7711200*d**6*e**4*x
**2/(2520*d*e**11 + 2520*e**12*x) - 1728720*d**6*e**4*x*log(d/e + x)/(2520*
d*e**11 + 2520*e**12*x) - 15104880*d**6*e**4*log(d/e + x)/(2520*d*e**11 + 2
520*e**12*x) - 15104880*d**6*e**4/(2520*d*e**11 + 2520*e**12*x) - 630000*d*
*5*e**5*x**5/(2520*d*e**11 + 2520*e**12*x) + 47250*d**5*e**5*x**4/(2520*d*e
**11 + 2520*e**12*x) - 2570400*d**5*e**5*x**3/(2520*d*e**11 + 2520*e**12*x)
 + 864360*d**5*e**5*x**2/(2520*d*e**11 + 2520*e**12*x) - 15104880*d**5*e**5
*x*log(d/e + x)/(2520*d*e**11 + 2520*e**12*x) + 6426000*d**5*e**5*log(d/e +
 x)/(2520*d*e**11 + 2520*e**12*x) + 6426000*d**5*e**5/(2520*d*e**11 + 2520*
e**12*x) + 420000*d**4*e**6*x**6/(2520*d*e**11 + 2520*e**12*x) - 28350*d**4
*e**6*x**5/(2520*d*e**11 + 2520*e**12*x) + 1285200*d**4*e**6*x**4/(2520*d*e
**11 + 2520*e**12*x) - 288120*d**4*e**6*x**3/(2520*d*e**11 + 2520*e**12*x)
+ 7552440*d**4*e**6*x**2/(2520*d*e**11 + 2520*e**12*x) + 6426000*d**4*e**6*
x*log(d/e + x)/(2520*d*e**11 + 2520*e**12*x) - 11178720*d**4*e**6*log(d/e +
 x)/(2520*d*e**11 + 2520*e**12*x) - 11178720*d**4*e**6/(2520*d*e**11 + 2520
*e**12*x) - 300000*d**3*e**7*x**7/(2520*d*e**11 + 2520*e**12*x) + 18900*d**
3*e**7*x**6/(2520*d*e**11 + 2520*e**12*x) - 771120*d**3*e**7*x**5/(2520*d*e
**11 + 2520*e**12*x) + 144060*d**3*e**7*x**4/(2520*d*e**11 + 2520*e**12*x)
- 2517480*d**3*e**7*x**3/(2520*d*e**11 + 2520*e**12*x) - 3213000*d**3*e**7*
x**2/(2520*d*e**11 + 2520*e**12*x) - 11178720*d**3*e**7*x*log(d/e + x)/(252
0*d*e**11 + 2520*e**12*x) + 4339440*d**3*e**7*log(d/e + x)/(2520*d*e**11 +
2520*e**12*x) + 4339440*d**3*e**7/(2520*d*e**11 + 2520*e**12*x) + 225000*d*
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*2*e**8*x**8/(2520*d*e**11 + 2520*e**12*x) - 13500*d**2*e**8*x**7/(2520*d*e
**11 + 2520*e**12*x) + 514080*d**2*e**8*x**6/(2520*d*e**11 + 2520*e**12*x)
- 86436*d**2*e**8*x**5/(2520*d*e**11 + 2520*e**12*x) + 1258740*d**2*e**8*x*
*4/(2520*d*e**11 + 2520*e**12*x) + 1071000*d**2*e**8*x**3/(2520*d*e**11 + 2
520*e**12*x) + 5589360*d**2*e**8*x**2/(2520*d*e**11 + 2520*e**12*x) + 43394
40*d**2*e**8*x*log(d/e + x)/(2520*d*e**11 + 2520*e**12*x) - 2404080*d**2*e*
*8*log(d/e + x)/(2520*d*e**11 + 2520*e**12*x) - 2404080*d**2*e**8/(2520*d*e
**11 + 2520*e**12*x) - 175000*d*e**9*x**9/(2520*d*e**11 + 2520*e**12*x) + 1
0125*d*e**9*x**8/(2520*d*e**11 + 2520*e**12*x) - 367200*d*e**9*x**7/(2520*d
*e**11 + 2520*e**12*x) + 57624*d*e**9*x**6/(2520*d*e**11 + 2520*e**12*x) -
755244*d*e**9*x**5/(2520*d*e**11 + 2520*e**12*x) - 535500*d*e**9*x**4/(2520
*d*e**11 + 2520*e**12*x) - 1863120*d*e**9*x**3/(2520*d*e**11 + 2520*e**12*x
) - 2169720*d*e**9*x**2/(2520*d*e**11 + 2520*e**12*x) - 2404080*d*e**9*x*lo
g(d/e + x)/(2520*d*e**11 + 2520*e**12*x) + 340200*d*e**9*log(d/e + x)/(2520
*d*e**11 + 2520*e**12*x) + 340200*d*e**9/(2520*d*e**11 + 2520*e**12*x) + 14
0000*e**10*x**10/(2520*d*e**11 + 2520*e**12*x) - 7875*e**10*x**9/(2520*d*e*
*11 + 2520*e**12*x) + 275400*e**10*x**8/(2520*d*e**11 + 2520*e**12*x) - 411
60*e**10*x**7/(2520*d*e**11 + 2520*e**12*x) + 503496*e**10*x**6/(2520*d*e**
11 + 2520*e**12*x) + 321300*e**10*x**5/(2520*d*e**11 + 2520*e**12*x) + 9315
60*e**10*x**4/(2520*d*e**11 + 2520*e**12*x) + 723240*e**10*x**3/(2520*d*e**
11 + 2520*e**12*x) + 1202040*e**10*x**2/(2520*d*e**11 + 2520*e**12*x) + 340
200*e**10*x*log(d/e + x)/(2520*d*e**11 + 2520*e**12*x) - 136080*e**10/(2520
*d*e**11 + 2520*e**12*x), Eq(m, -2)), (500*d**10*log(d/e + x)/e**11 - 500*d
**9*x/e**10 + 25*d**9*log(d/e + x)/e**10 + 250*d**8*x**2/e**9 - 25*d**8*x/e
**9 + 765*d**8*log(d/e + x)/e**9 - 500*d**7*x**3/(3*e**8) + 25*d**7*x**2/(2
*e**8) - 765*d**7*x/e**8 + 98*d**7*log(d/e + x)/e**8 + 125*d**6*x**4/e**7 -
 25*d**6*x**3/(3*e**7) + 765*d**6*x**2/(2*e**7) - 98*d**6*x/e**7 + 999*d**6
*log(d/e + x)/e**7 - 100*d**5*x**5/e**6 + 25*d**5*x**4/(4*e**6) - 255*d**5*
x**3/e**6 + 49*d**5*x**2/e**6 - 999*d**5*x/e**6 - 510*d**5*log(d/e + x)/e**
6 + 250*d**4*x**6/(3*e**5) - 5*d**4*x**5/e**5 + 765*d**4*x**4/(4*e**5) - 98
*d**4*x**3/(3*e**5) + 999*d**4*x**2/(2*e**5) + 510*d**4*x/e**5 + 1109*d**4*
log(d/e + x)/e**5 - 500*d**3*x**7/(7*e**4) + 25*d**3*x**6/(6*e**4) - 153*d*
*3*x**5/e**4 + 49*d**3*x**4/(2*e**4) - 333*d**3*x**3/e**4 - 255*d**3*x**2/e
**4 - 1109*d**3*x/e**4 - 574*d**3*log(d/e + x)/e**4 + 125*d**2*x**8/(2*e**3
) - 25*d**2*x**7/(7*e**3) + 255*d**2*x**6/(2*e**3) - 98*d**2*x**5/(5*e**3)
+ 999*d**2*x**4/(4*e**3) + 170*d**2*x**3/e**3 + 1109*d**2*x**2/(2*e**3) + 5
74*d**2*x/e**3 + 477*d**2*log(d/e + x)/e**3 - 500*d*x**9/(9*e**2) + 25*d*x*
*8/(8*e**2) - 765*d*x**7/(7*e**2) + 49*d*x**6/(3*e**2) - 999*d*x**5/(5*e**2
) - 255*d*x**4/(2*e**2) - 1109*d*x**3/(3*e**2) - 287*d*x**2/e**2 - 477*d*x/
e**2 - 135*d*log(d/e + x)/e**2 + 50*x**10/e - 25*x**9/(9*e) + 765*x**8/(8*e
) - 14*x**7/e + 333*x**6/(2*e) + 102*x**5/e + 1109*x**4/(4*e) + 574*x**3/(3
*e) + 477*x**2/(2*e) + 135*x/e + 54*log(d/e + x)/e, Eq(m, -1)), (1814400000
*d**11*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670
*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5
+ 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 12054
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3840*e**11*m + 39916800*e**11) - 1814400000*d**10*e*m*x*(d + e*x)**m/(e**11
*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11
*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 10
5258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e*
*11) + 9072000*d**10*e*m*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*
e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13
339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*
e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 99792000*d**10*e*(d + e*
x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 +
357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**
11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m +
 39916800*e**11) + 907200000*d**9*e**2*m**2*x**2*(d + e*x)**m/(e**11*m**11
+ 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 +
 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076
*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) -
9072000*d**9*e**2*m**2*x*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*
e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13
339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*
e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 30844800*d**9*e**2*m**2*
(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*
m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995
730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e*
*11*m + 39916800*e**11) + 907200000*d**9*e**2*m*x**2*(d + e*x)**m/(e**11*m*
*11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m*
*7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 10525
8076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11
) - 99792000*d**9*e**2*m*x*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 192
5*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 +
13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 15091797
6*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 647740800*d**9*e**2*m*
(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*
m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995
730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e*
*11*m + 39916800*e**11) + 3392928000*d**9*e**2*(d + e*x)**m/(e**11*m**11 +
66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2
637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e
**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) - 30
2400000*d**8*e**3*m**3*x**3*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 19
25*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 +
 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 1509179
76*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 4536000*d**8*e**3*m**
3*x**2*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670
*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5
+ 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 12054
3840*e**11*m + 39916800*e**11) - 30844800*d**8*e**3*m**3*x*(d + e*x)**m/(e*
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*11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e*
*11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 +
 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800
*e**11) + 493920*d**8*e**3*m**3*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10
+ 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m*
*6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150
917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) - 907200000*d**8*e*
*3*m**2*x**3*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 +
 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11
*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 +
 120543840*e**11*m + 39916800*e**11) + 54432000*d**8*e**3*m**2*x**2*(d + e*
x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 +
357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**
11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m +
 39916800*e**11) - 647740800*d**8*e**3*m**2*x*(d + e*x)**m/(e**11*m**11 + 6
6*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 26
37558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e*
*11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 148
17600*d**8*e**3*m**2*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**1
1*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 133395
35*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**1
1*m**2 + 120543840*e**11*m + 39916800*e**11) - 604800000*d**8*e**3*m*x**3*(
d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m
**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 459957
30*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**
11*m + 39916800*e**11) + 49896000*d**8*e**3*m*x**2*(d + e*x)**m/(e**11*m**1
1 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7
 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 1052580
76*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11)
- 3392928000*d**8*e**3*m*x*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 192
5*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 +
13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 15091797
6*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 147682080*d**8*e**3*m*
(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*
m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995
730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e*
*11*m + 39916800*e**11) + 488980800*d**8*e**3*(d + e*x)**m/(e**11*m**11 + 6
6*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 26
37558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e*
*11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 756
00000*d**7*e**4*m**4*x**4*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925
*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 1
3339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976
*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) - 1512000*d**7*e**4*m**4*
x**3*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e
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**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 +
45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 1205438
40*e**11*m + 39916800*e**11) + 15422400*d**7*e**4*m**4*x**2*(d + e*x)**m/(e
**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e
**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4
+ 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 3991680
0*e**11) - 493920*d**7*e**4*m**4*x*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**
10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11
*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 +
150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 719280*d**7*e*
*4*m**4*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 3267
0*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5
 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 1205
43840*e**11*m + 39916800*e**11) + 453600000*d**7*e**4*m**3*x**4*(d + e*x)**
m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 3574
23*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m
**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 399
16800*e**11) - 21168000*d**7*e**4*m**3*x**3*(d + e*x)**m/(e**11*m**11 + 66*
e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637
558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**1
1*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 33929
2800*d**7*e**4*m**3*x**2*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*
e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13
339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*
e**11*m**2 + 120543840*e**11*m + 39916800*e**11) - 14817600*d**7*e**4*m**3*
x*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**1
1*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 459
95730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*
e**11*m + 39916800*e**11) + 27332640*d**7*e**4*m**3*(d + e*x)**m/(e**11*m**
11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**
7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258
076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11)
 + 831600000*d**7*e**4*m**2*x**4*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10
 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m
**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 15
0917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) - 52920000*d**7*e*
*4*m**2*x**3*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 +
 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11
*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 +
 120543840*e**11*m + 39916800*e**11) + 2020334400*d**7*e**4*m**2*x**2*(d +
e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8
+ 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e
**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m
 + 39916800*e**11) - 147682080*d**7*e**4*m**2*x*(d + e*x)**m/(e**11*m**11 +
 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 +
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2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*
e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 3
87691920*d**7*e**4*m**2*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e
**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 133
39535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e
**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 453600000*d**7*e**4*m*x**
4*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**1
1*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 459
95730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*
e**11*m + 39916800*e**11) - 33264000*d**7*e**4*m*x**3*(d + e*x)**m/(e**11*m
**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m
**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 1052
58076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**1
1) + 1696464000*d**7*e**4*m*x**2*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10
 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m
**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 15
0917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) - 488980800*d**7*e
**4*m*x*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 3267
0*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5
 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 1205
43840*e**11*m + 39916800*e**11) + 2432604960*d**7*e**4*m*(d + e*x)**m/(e**1
1*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**1
1*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 1
05258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e
**11) + 5696697600*d**7*e**4*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1
925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6
+ 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917
976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) - 15120000*d**6*e**5*m
**5*x**5*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 326
70*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**
5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120
543840*e**11*m + 39916800*e**11) + 378000*d**6*e**5*m**5*x**4*(d + e*x)**m/
(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423
*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**
4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916
800*e**11) - 5140800*d**6*e**5*m**5*x**3*(d + e*x)**m/(e**11*m**11 + 66*e**
11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558
*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m
**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 246960*d
**6*e**5*m**5*x**2*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*
m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535
*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*
m**2 + 120543840*e**11*m + 39916800*e**11) - 719280*d**6*e**5*m**5*x*(d + e
*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 +
 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e*
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*11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m
+ 39916800*e**11) - 61200*d**6*e**5*m**5*(d + e*x)**m/(e**11*m**11 + 66*e**
11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558
*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m
**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) - 15120000
0*d**6*e**5*m**4*x**5*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**
11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339
535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**
11*m**2 + 120543840*e**11*m + 39916800*e**11) + 6426000*d**6*e**5*m**4*x**4
*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11
*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 4599
5730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e
**11*m + 39916800*e**11) - 123379200*d**6*e**5*m**4*x**3*(d + e*x)**m/(e**1
1*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**1
1*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 1
05258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e
**11) + 7655760*d**6*e**5*m**4*x**2*(d + e*x)**m/(e**11*m**11 + 66*e**11*m*
*10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**1
1*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 +
 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) - 27332640*d**6
*e**5*m**4*x*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 +
 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11
*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 +
 120543840*e**11*m + 39916800*e**11) - 2754000*d**6*e**5*m**4*(d + e*x)**m/
(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423
*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**
4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916
800*e**11) - 529200000*d**6*e**5*m**3*x**5*(d + e*x)**m/(e**11*m**11 + 66*e
**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 26375
58*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11
*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 291060
00*d**6*e**5*m**3*x**4*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e*
*11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 1333
9535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e*
*11*m**2 + 120543840*e**11*m + 39916800*e**11) - 899640000*d**6*e**5*m**3*x
**3*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e*
*11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 4
5995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 12054384
0*e**11*m + 39916800*e**11) + 81249840*d**6*e**5*m**3*x**2*(d + e*x)**m/(e*
*11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e*
*11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 +
 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800
*e**11) - 387691920*d**6*e**5*m**3*x*(d + e*x)**m/(e**11*m**11 + 66*e**11*m
**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**
11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3
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+ 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) - 49266000*d**
6*e**5*m**3*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 +
32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*
m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 +
120543840*e**11*m + 39916800*e**11) - 756000000*d**6*e**5*m**2*x**5*(d + e*
x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 +
357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**
11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m +
 39916800*e**11) + 48006000*d**6*e**5*m**2*x**4*(d + e*x)**m/(e**11*m**11 +
 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 +
2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*
e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) - 1
912377600*d**6*e**5*m**2*x**3*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 +
1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6
 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 15091
7976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 318331440*d**6*e**5
*m**2*x**2*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 3
2670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m
**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 1
20543840*e**11*m + 39916800*e**11) - 2432604960*d**6*e**5*m**2*x*(d + e*x)*
*m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357
423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*
m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39
916800*e**11) - 437886000*d**6*e**5*m**2*(d + e*x)**m/(e**11*m**11 + 66*e**
11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558
*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m
**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) - 36288000
0*d**6*e**5*m*x**5*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*
m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535
*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*
m**2 + 120543840*e**11*m + 39916800*e**11) + 24948000*d**6*e**5*m*x**4*(d +
 e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8
 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*
e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*
m + 39916800*e**11) - 1130976000*d**6*e**5*m*x**3*(d + e*x)**m/(e**11*m**11
 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7
+ 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 10525807
6*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) +
 244490400*d**6*e**5*m*x**2*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 19
25*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 +
 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 1509179
76*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) - 5696697600*d**6*e**5*
m*x*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e*
*11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 4
5995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 12054384
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0*e**11*m + 39916800*e**11) - 1933552800*d**6*e**5*m*(d + e*x)**m/(e**11*m*
*11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m*
*7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 10525
8076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11
) - 3392928000*d**6*e**5*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*
e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13
339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*
e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 2520000*d**5*e**6*m**6*x
**6*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e*
*11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 4
5995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 12054384
0*e**11*m + 39916800*e**11) - 75600*d**5*e**6*m**6*x**5*(d + e*x)**m/(e**11
*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11
*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 10
5258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e*
*11) + 1285200*d**5*e**6*m**6*x**4*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**
10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11
*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 +
150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) - 82320*d**5*e**
6*m**6*x**3*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 +
32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*
m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 +
120543840*e**11*m + 39916800*e**11) + 359640*d**5*e**6*m**6*x**2*(d + e*x)*
*m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357
423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*
m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39
916800*e**11) + 61200*d**5*e**6*m**6*x*(d + e*x)**m/(e**11*m**11 + 66*e**11
*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e
**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**
3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 26616*d**5
*e**6*m**6*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 3
2670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m
**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 1
20543840*e**11*m + 39916800*e**11) + 37800000*d**5*e**6*m**5*x**6*(d + e*x)
**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 35
7423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11
*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 3
9916800*e**11) - 1587600*d**5*e**6*m**5*x**5*(d + e*x)**m/(e**11*m**11 + 66
*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 263
7558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**
11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 3470
0400*d**5*e**6*m**5*x**4*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*
e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13
339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*
e**11*m**2 + 120543840*e**11*m + 39916800*e**11) - 2716560*d**5*e**6*m**5*x
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**3*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e*
*11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 4
5995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 12054384
0*e**11*m + 39916800*e**11) + 14025960*d**5*e**6*m**5*x**2*(d + e*x)**m/(e*
*11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e*
*11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 +
 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800
*e**11) + 2754000*d**5*e**6*m**5*x*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**
10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11
*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 +
150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 1357416*d**5*e
**6*m**5*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 326
70*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**
5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120
543840*e**11*m + 39916800*e**11) + 214200000*d**5*e**6*m**4*x**6*(d + e*x)*
*m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357
423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*
m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39
916800*e**11) - 10962000*d**5*e**6*m**4*x**5*(d + e*x)**m/(e**11*m**11 + 66
*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 263
7558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**
11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 3174
44400*d**5*e**6*m**4*x**4*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925
*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 1
3339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976
*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) - 32187120*d**5*e**6*m**4
*x**3*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*
e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 +
 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543
840*e**11*m + 39916800*e**11) + 207512280*d**5*e**6*m**4*x**2*(d + e*x)**m/
(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423
*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**
4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916
800*e**11) + 49266000*d**5*e**6*m**4*x*(d + e*x)**m/(e**11*m**11 + 66*e**11
*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e
**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**
3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 28612200*d
**5*e**6*m**4*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9
+ 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**1
1*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2
+ 120543840*e**11*m + 39916800*e**11) + 567000000*d**5*e**6*m**3*x**6*(d +
e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8
+ 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e
**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m
 + 39916800*e**11) - 32886000*d**5*e**6*m**3*x**5*(d + e*x)**m/(e**11*m**11
```

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 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7
+ 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 10525807
6*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) +
 1152824400*d**5*e**6*m**3*x**4*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10
+ 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m*
*6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150
917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) - 160277040*d**5*e*
*6*m**3*x**3*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 +
 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11
*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 +
 120543840*e**11*m + 39916800*e**11) + 1410148440*d**5*e**6*m**3*x**2*(d +
e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8
+ 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e
**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m
 + 39916800*e**11) + 437886000*d**5*e**6*m**3*x*(d + e*x)**m/(e**11*m**11 +
 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 +
2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*
e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 3
18992760*d**5*e**6*m**3*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e
**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 133
39535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e
**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 690480000*d**5*e**6*m**2*
x**6*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e
**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 +
45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 1205438
40*e**11*m + 39916800*e**11) - 43394400*d**5*e**6*m**2*x**5*(d + e*x)**m/(e
**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e
**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4
+ 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 3991680
0*e**11) + 1717027200*d**5*e**6*m**2*x**4*(d + e*x)**m/(e**11*m**11 + 66*e*
*11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 263755
8*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*
m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) - 2937177
60*d**5*e**6*m**2*x**3*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e*
*11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 1333
9535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e*
*11*m**2 + 120543840*e**11*m + 39916800*e**11) + 4064651280*d**5*e**6*m**2*
x**2*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e
**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 +
45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 1205438
40*e**11*m + 39916800*e**11) + 1933552800*d**5*e**6*m**2*x*(d + e*x)**m/(e*
*11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e*
*11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 +
 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800
*e**11) + 1983530784*d**5*e**6*m**2*(d + e*x)**m/(e**11*m**11 + 66*e**11*m*
```

```
*10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**1
1*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 +
 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 302400000*d**
5*e**6*m*x**6*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9
+ 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**1
1*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2
+ 120543840*e**11*m + 39916800*e**11) - 19958400*d**5*e**6*m*x**5*(d + e*x)
**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 35
7423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11
*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 3
9916800*e**11) + 848232000*d**5*e**6*m*x**4*(d + e*x)**m/(e**11*m**11 + 66*
e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637
558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**1
1*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) - 16299
3600*d**5*e**6*m*x**3*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**
11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339
535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**
11*m**2 + 120543840*e**11*m + 39916800*e**11) + 2848348800*d**5*e**6*m*x**2
*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11
*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 4599
5730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e
**11*m + 39916800*e**11) + 3392928000*d**5*e**6*m*x*(d + e*x)**m/(e**11*m**
11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**
7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258
076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11)
 + 6521026464*d**5*e**6*m*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925
*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 1
3339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976
*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 8853546240*d**5*e**6*(d
 + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m*
*8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 4599573
0*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**1
1*m + 39916800*e**11) - 360000*d**4*e**7*m**7*x**7*(d + e*x)**m/(e**11*m**1
1 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7
 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 1052580
76*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11)
+ 12600*d**4*e**7*m**7*x**6*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 19
25*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 +
 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 1509179
76*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) - 257040*d**4*e**7*m**7
*x**5*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*
e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 +
 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543
840*e**11*m + 39916800*e**11) + 20580*d**4*e**7*m**7*x**4*(d + e*x)**m/(e**
11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**
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11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 +
105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*
e**11) - 119880*d**4*e**7*m**7*x**3*(d + e*x)**m/(e**11*m**11 + 66*e**11*m*
*10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**1
1*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 +
 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) - 30600*d**4*e*
*7*m**7*x**2*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 +
 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11
*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 +
 120543840*e**11*m + 39916800*e**11) - 26616*d**4*e**7*m**7*x*(d + e*x)**m/
(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423
*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**
4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916
800*e**11) - 3444*d**4*e**7*m**7*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10
 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m
**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 15
0917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) - 7560000*d**4*e**
7*m**6*x**7*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 +
32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*
m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 +
120543840*e**11*m + 39916800*e**11) + 327600*d**4*e**7*m**6*x**6*(d + e*x)*
*m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357
423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*
m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39
916800*e**11) - 7968240*d**4*e**7*m**6*x**5*(d + e*x)**m/(e**11*m**11 + 66*
e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637
558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**1
1*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 74088
0*d**4*e**7*m**6*x**4*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**
11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339
535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**
11*m**2 + 120543840*e**11*m + 39916800*e**11) - 4915080*d**4*e**7*m**6*x**3
*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11
*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 4599
5730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e
**11*m + 39916800*e**11) - 1407600*d**4*e**7*m**6*x**2*(d + e*x)**m/(e**11*
m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*
m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105
258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**
11) - 1357416*d**4*e**7*m**6*x*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 +
 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**
6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 1509
17976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) - 192864*d**4*e**7*m
**6*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e*
*11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 4
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5995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 12054384
0*e**11*m + 39916800*e**11) - 63000000*d**4*e**7*m**5*x**7*(d + e*x)**m/(e*
*11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e*
*11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 +
 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800
*e**11) + 3150000*d**4*e**7*m**5*x**6*(d + e*x)**m/(e**11*m**11 + 66*e**11*
m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e*
*11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3
 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) - 91249200*d*
*4*e**7*m**5*x**5*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m
**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*
e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m
**2 + 120543840*e**11*m + 39916800*e**11) + 10084200*d**4*e**7*m**5*x**4*(d
 + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m*
*8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 4599573
0*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**1
1*m + 39916800*e**11) - 78521400*d**4*e**7*m**5*x**3*(d + e*x)**m/(e**11*m*
*11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m*
*7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 10525
8076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11
) - 26010000*d**4*e**7*m**5*x**2*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10
 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m
**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 15
0917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) - 28612200*d**4*e*
*7*m**5*x*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32
670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m*
*5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 12
0543840*e**11*m + 39916800*e**11) - 4580520*d**4*e**7*m**5*(d + e*x)**m/(e*
*11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e*
*11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 +
 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800
*e**11) - 264600000*d**4*e**7*m**4*x**7*(d + e*x)**m/(e**11*m**11 + 66*e**1
1*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*
e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m*
*3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 14616000*
d**4*e**7*m**4*x**6*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11
*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 1333953
5*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11
*m**2 + 120543840*e**11*m + 39916800*e**11) - 484520400*d**4*e**7*m**4*x**5
*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11
*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 4599
5730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e
**11*m + 39916800*e**11) + 64209600*d**4*e**7*m**4*x**4*(d + e*x)**m/(e**11
*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11
*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 10
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5258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e*
*11) - 608391000*d**4*e**7*m**4*x**3*(d + e*x)**m/(e**11*m**11 + 66*e**11*m
**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**
11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3
+ 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) - 243576000*d*
*4*e**7*m**4*x**2*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m
**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*
e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m
**2 + 120543840*e**11*m + 39916800*e**11) - 318992760*d**4*e**7*m**4*x*(d +
 e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8
 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*
e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*
m + 39916800*e**11) - 59787840*d**4*e**7*m**4*(d + e*x)**m/(e**11*m**11 + 6
6*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 26
37558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e*
*11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) - 584
640000*d**4*e**7*m**3*x**7*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 192
5*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 +
13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 15091797
6*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 34637400*d**4*e**7*m**
3*x**6*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670
*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5
+ 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 12054
3840*e**11*m + 39916800*e**11) - 1265664960*d**4*e**7*m**3*x**5*(d + e*x)**
m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 3574
23*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m
**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 399
16800*e**11) + 193637220*d**4*e**7*m**3*x**4*(d + e*x)**m/(e**11*m**11 + 66
*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 263
7558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**
11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) - 2294
982720*d**4*e**7*m**3*x**3*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 192
5*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 +
13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 15091797
6*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) - 1185719400*d**4*e**7*m
**3*x**2*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 326
70*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**
5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120
543840*e**11*m + 39916800*e**11) - 1983530784*d**4*e**7*m**3*x*(d + e*x)**m
/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 35742
3*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m*
*4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 3991
6800*e**11) - 463042356*d**4*e**7*m**3*(d + e*x)**m/(e**11*m**11 + 66*e**11
*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e
**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**
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3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) - 635040000*
d**4*e**7*m**2*x**7*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11
*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 1333953
5*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11
*m**2 + 120543840*e**11*m + 39916800*e**11) + 39488400*d**4*e**7*m**2*x**6*
(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*
m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995
730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e*
*11*m + 39916800*e**11) - 1543268160*d**4*e**7*m**2*x**5*(d + e*x)**m/(e**1
1*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**1
1*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 1
05258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e
**11) + 261036720*d**4*e**7*m**2*x**4*(d + e*x)**m/(e**11*m**11 + 66*e**11*
m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e*
*11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3
 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) - 3659217120*
d**4*e**7*m**2*x**3*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11
*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 1333953
5*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11
*m**2 + 120543840*e**11*m + 39916800*e**11) - 2663240400*d**4*e**7*m**2*x**
2*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**1
1*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 459
95730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*
e**11*m + 39916800*e**11) - 6521026464*d**4*e**7*m**2*x*(d + e*x)**m/(e**11
*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11
*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 10
5258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e*
*11) - 2127097056*d**4*e**7*m**2*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10
 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m
**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 15
0917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) - 259200000*d**4*e
**7*m*x**7*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 3
2670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m
**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 1
20543840*e**11*m + 39916800*e**11) + 16632000*d**4*e**7*m*x**6*(d + e*x)**m
/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 35742
3*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m*
*4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 3991
6800*e**11) - 678585600*d**4*e**7*m*x**5*(d + e*x)**m/(e**11*m**11 + 66*e**
11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558
*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m
**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 12224520
0*d**4*e**7*m*x**4*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*
m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535
*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*
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m**2 + 120543840*e**11*m + 39916800*e**11) - 1898899200*d**4*e**7*m*x**3*(d
 + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m*
*8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 4599573
0*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**1
1*m + 39916800*e**11) - 1696464000*d**4*e**7*m*x**2*(d + e*x)**m/(e**11*m**
11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**
7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258
076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11)
 - 8853546240*d**4*e**7*m*x*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 19
25*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 +
 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 1509179
76*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) - 5364581040*d**4*e**7*
m*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**1
1*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 459
95730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*
e**11*m + 39916800*e**11) - 5728060800*d**4*e**7*(d + e*x)**m/(e**11*m**11
+ 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 +
 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076
*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) +
45000*d**3*e**8*m**8*x**8*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925
*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 1
3339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976
*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) - 1800*d**3*e**8*m**8*x**
7*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**1
1*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 459
95730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*
e**11*m + 39916800*e**11) + 42840*d**3*e**8*m**8*x**6*(d + e*x)**m/(e**11*m
**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m
**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 1052
58076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**1
1) - 4116*d**3*e**8*m**8*x**5*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 +
1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6
 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 15091
7976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 29970*d**3*e**8*m**
8*x**4*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670
*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5
+ 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 12054
3840*e**11*m + 39916800*e**11) + 10200*d**3*e**8*m**8*x**3*(d + e*x)**m/(e*
*11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e*
*11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 +
 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800
*e**11) + 13308*d**3*e**8*m**8*x**2*(d + e*x)**m/(e**11*m**11 + 66*e**11*m*
*10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**1
1*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 +
 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 3444*d**3*e**
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8*m**8*x*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 326
70*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**
5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120
543840*e**11*m + 39916800*e**11) + 954*d**3*e**8*m**8*(d + e*x)**m/(e**11*m
**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m
**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 1052
58076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**1
1) + 1260000*d**3*e**8*m**7*x**8*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10
 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m
**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 15
0917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) - 57600*d**3*e**8*
m**7*x**7*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32
670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m*
*5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 12
0543840*e**11*m + 39916800*e**11) + 1542240*d**3*e**8*m**7*x**6*(d + e*x)**
m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 3574
23*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m
**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 399
16800*e**11) - 164640*d**3*e**8*m**7*x**5*(d + e*x)**m/(e**11*m**11 + 66*e*
*11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 263755
8*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*
m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 1318680
*d**3*e**8*m**7*x**4*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**1
1*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 133395
35*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**1
1*m**2 + 120543840*e**11*m + 39916800*e**11) + 489600*d**3*e**8*m**7*x**3*(
d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m
**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 459957
30*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**
11*m + 39916800*e**11) + 692016*d**3*e**8*m**7*x**2*(d + e*x)**m/(e**11*m**
11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**
7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258
076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11)
 + 192864*d**3*e**8*m**7*x*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 192
5*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 +
13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 15091797
6*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 57240*d**3*e**8*m**7*(
d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m
**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 459957
30*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**
11*m + 39916800*e**11) + 14490000*d**3*e**8*m**6*x**8*(d + e*x)**m/(e**11*m
**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m
**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 1052
58076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**1
1) - 730800*d**3*e**8*m**6*x**7*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10
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+ 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m*
*6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150
917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 21848400*d**3*e**
8*m**6*x**6*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 +
32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*
m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 +
120543840*e**11*m + 39916800*e**11) - 2609544*d**3*e**8*m**6*x**5*(d + e*x)
**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 35
7423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11
*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 3
9916800*e**11) + 23316660*d**3*e**8*m**6*x**4*(d + e*x)**m/(e**11*m**11 + 6
6*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 26
37558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e*
*11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 960
8400*d**3*e**8*m**6*x**3*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*
e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13
339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*
e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 14984808*d**3*e**8*m**6*
x**2*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e
**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 +
45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 1205438
40*e**11*m + 39916800*e**11) + 4580520*d**3*e**8*m**6*x*(d + e*x)**m/(e**11
*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11
*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 10
5258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e*
*11) + 1482516*d**3*e**8*m**6*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 +
1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6
 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 15091
7976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 88200000*d**3*e**8*
m**5*x**8*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32
670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m*
*5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 12
0543840*e**11*m + 39916800*e**11) - 4788000*d**3*e**8*m**5*x**7*(d + e*x)**
m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 3574
23*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m
**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 399
16800*e**11) + 156794400*d**3*e**8*m**5*x**6*(d + e*x)**m/(e**11*m**11 + 66
*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 263
7558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**
11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) - 2090
9280*d**3*e**8*m**5*x**5*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*
e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13
339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*
e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 210988800*d**3*e**8*m**5
*x**4*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*
```

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e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 +
 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543
840*e**11*m + 39916800*e**11) + 98532000*d**3*e**8*m**5*x**3*(d + e*x)**m/(
e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*
e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4
 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 399168
00*e**11) + 173802480*d**3*e**8*m**5*x**2*(d + e*x)**m/(e**11*m**11 + 66*e*
*11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 263755
8*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*
m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 5978784
0*d**3*e**8*m**5*x*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*
m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535
*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*
m**2 + 120543840*e**11*m + 39916800*e**11) + 21636720*d**3*e**8*m**5*(d + e
*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 +
 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e*
*11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m
+ 39916800*e**11) + 304605000*d**3*e**8*m**4*x**8*(d + e*x)**m/(e**11*m**11
 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7
+ 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 10525807
6*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) -
 17476200*d**3*e**8*m**4*x**7*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 +
1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6
 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 15091
7976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 614711160*d**3*e**8
*m**4*x**6*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 3
2670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m
**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 1
20543840*e**11*m + 39916800*e**11) - 90095124*d**3*e**8*m**4*x**5*(d + e*x)
**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 35
7423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11
*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 3
9916800*e**11) + 1030038930*d**3*e**8*m**4*x**4*(d + e*x)**m/(e**11*m**11 +
 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 +
2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*
e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 5
57623800*d**3*e**8*m**4*x**3*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1
925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6
+ 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917
976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 1151261772*d**3*e**8
*m**4*x**2*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 3
2670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m
**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 1
20543840*e**11*m + 39916800*e**11) + 463042356*d**3*e**8*m**4*x*(d + e*x)**
m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 3574
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23*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m
**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 399
16800*e**11) + 194510106*d**3*e**8*m**4*(d + e*x)**m/(e**11*m**11 + 66*e**1
1*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*
e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m*
*3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 590940000
*d**3*e**8*m**3*x**8*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**1
1*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 133395
35*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**1
1*m**2 + 120543840*e**11*m + 39916800*e**11) - 35330400*d**3*e**8*m**3*x**7
*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11
*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 4599
5730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e
**11*m + 39916800*e**11) + 1311932160*d**3*e**8*m**3*x**6*(d + e*x)**m/(e**
11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**
11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 +
105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*
e**11) - 207117120*d**3*e**8*m**3*x**5*(d + e*x)**m/(e**11*m**11 + 66*e**11
*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e
**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**
3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 2636041320
*d**3*e**8*m**3*x**4*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**1
1*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 133395
35*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**1
1*m**2 + 120543840*e**11*m + 39916800*e**11) + 1678226400*d**3*e**8*m**3*x*
*3*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**
11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45
995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840
*e**11*m + 39916800*e**11) + 4252278624*d**3*e**8*m**3*x**2*(d + e*x)**m/(e
**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e
**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4
+ 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 3991680
0*e**11) + 2127097056*d**3*e**8*m**3*x*(d + e*x)**m/(e**11*m**11 + 66*e**11
*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e
**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**
3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 1102270680
*d**3*e**8*m**3*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**
9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e*
*11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**
2 + 120543840*e**11*m + 39916800*e**11) + 588060000*d**3*e**8*m**2*x**8*(d
+ e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**
8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730
*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11
*m + 39916800*e**11) - 36223200*d**3*e**8*m**2*x**7*(d + e*x)**m/(e**11*m**
11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**
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7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258
076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11)
 + 1399154400*d**3*e**8*m**2*x**6*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**1
0 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*
m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 1
50917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) - 233278416*d**3*
e**8*m**2*x**5*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9
 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**
11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2
 + 120543840*e**11*m + 39916800*e**11) + 3219137640*d**3*e**8*m**2*x**4*(d
+ e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**
8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730
*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11
*m + 39916800*e**11) + 2340981600*d**3*e**8*m**2*x**3*(d + e*x)**m/(e**11*m
**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m
**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 1052
58076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**1
1) + 7687286352*d**3*e**8*m**2*x**2*(d + e*x)**m/(e**11*m**11 + 66*e**11*m*
*10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**1
1*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 +
 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 5364581040*d*
*3*e**8*m**2*x*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9
 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**
11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2
 + 120543840*e**11*m + 39916800*e**11) + 3842860824*d**3*e**8*m**2*(d + e*x
)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 3
57423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**1
1*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m +
39916800*e**11) + 226800000*d**3*e**8*m*x**8*(d + e*x)**m/(e**11*m**11 + 66
*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 263
7558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**
11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) - 1425
6000*d**3*e**8*m*x**7*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**
11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339
535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**
11*m**2 + 120543840*e**11*m + 39916800*e**11) + 565488000*d**3*e**8*m*x**6*
(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*
m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995
730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e*
*11*m + 39916800*e**11) - 97796160*d**3*e**8*m*x**5*(d + e*x)**m/(e**11*m**
11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**
7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258
076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11)
 + 1424174400*d**3*e**8*m*x**4*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 +
 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**
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6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 1509
17976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 1130976000*d**3*e*
*8*m*x**3*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32
670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m*
*5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 12
0543840*e**11*m + 39916800*e**11) + 4426773120*d**3*e**8*m*x**2*(d + e*x)**
m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 3574
23*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m
**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 399
16800*e**11) + 5728060800*d**3*e**8*m*x*(d + e*x)**m/(e**11*m**11 + 66*e**1
1*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*
e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m*
*3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 753072336
0*d**3*e**8*m*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9
+ 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**1
1*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2
+ 120543840*e**11*m + 39916800*e**11) + 6346771200*d**3*e**8*(d + e*x)**m/(
e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*
e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4
 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 399168
00*e**11) - 5000*d**2*e**9*m**9*x**9*(d + e*x)**m/(e**11*m**11 + 66*e**11*m
**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**
11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3
+ 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 225*d**2*e**
9*m**9*x**8*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 +
32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*
m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 +
120543840*e**11*m + 39916800*e**11) - 6120*d**2*e**9*m**9*x**7*(d + e*x)**m
/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 35742
3*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m*
*4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 3991
6800*e**11) + 686*d**2*e**9*m**9*x**6*(d + e*x)**m/(e**11*m**11 + 66*e**11*
m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e*
*11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3
 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) - 5994*d**2*e
**9*m**9*x**5*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9
+ 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**1
1*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2
+ 120543840*e**11*m + 39916800*e**11) - 2550*d**2*e**9*m**9*x**4*(d + e*x)*
*m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357
423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*
m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39
916800*e**11) - 4436*d**2*e**9*m**9*x**3*(d + e*x)**m/(e**11*m**11 + 66*e**
11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558
*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m
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**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) - 1722*d**
2*e**9*m**9*x**2*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m*
*9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e
**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m*
*2 + 120543840*e**11*m + 39916800*e**11) - 954*d**2*e**9*m**9*x*(d + e*x)**
m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 3574
23*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m
**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 399
16800*e**11) - 135*d**2*e**9*m**9*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**1
0 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*
m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 1
50917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) - 180000*d**2*e**
9*m**8*x**9*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 +
32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*
m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 +
120543840*e**11*m + 39916800*e**11) + 8775*d**2*e**9*m**8*x**8*(d + e*x)**m
/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 35742
3*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m*
*4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 3991
6800*e**11) - 257040*d**2*e**9*m**8*x**7*(d + e*x)**m/(e**11*m**11 + 66*e**
11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558
*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m
**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 30870*d*
*2*e**9*m**8*x**6*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m
**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*
e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m
**2 + 120543840*e**11*m + 39916800*e**11) - 287712*d**2*e**9*m**8*x**5*(d +
 e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8
 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*
e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*
m + 39916800*e**11) - 130050*d**2*e**9*m**8*x**4*(d + e*x)**m/(e**11*m**11
+ 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 +
 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076
*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) -
239544*d**2*e**9*m**8*x**3*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 192
5*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 +
13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 15091797
6*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) - 98154*d**2*e**9*m**8*x
**2*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e*
*11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 4
5995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 12054384
0*e**11*m + 39916800*e**11) - 57240*d**2*e**9*m**8*x*(d + e*x)**m/(e**11*m*
*11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m*
*7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 10525
8076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11
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) - 8505*d**2*e**9*m**8*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e
**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 133
39535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e
**11*m**2 + 120543840*e**11*m + 39916800*e**11) - 2730000*d**2*e**9*m**7*x*
*9*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**
11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45
995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840
*e**11*m + 39916800*e**11) + 141750*d**2*e**9*m**7*x**8*(d + e*x)**m/(e**11
*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11
*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 10
5258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e*
*11) - 4443120*d**2*e**9*m**7*x**7*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**
10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11
*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 +
150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 572124*d**2*e*
*9*m**7*x**6*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 +
 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11
*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 +
 120543840*e**11*m + 39916800*e**11) - 5718276*d**2*e**9*m**7*x**5*(d + e*x
)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 3
57423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**1
1*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m +
39916800*e**11) - 2769300*d**2*e**9*m**7*x**4*(d + e*x)**m/(e**11*m**11 + 6
6*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 26
37558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e*
*11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) - 545
6280*d**2*e**9*m**7*x**3*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*
e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13
339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*
e**11*m**2 + 120543840*e**11*m + 39916800*e**11) - 2386692*d**2*e**9*m**7*x
**2*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e*
*11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 4
5995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 12054384
0*e**11*m + 39916800*e**11) - 1482516*d**2*e**9*m**7*x*(d + e*x)**m/(e**11*
m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*
m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105
258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**
11) - 234090*d**2*e**9*m**7*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 19
25*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 +
 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 1509179
76*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) - 22680000*d**2*e**9*m*
*6*x**9*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 3267
0*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5
 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 1205
43840*e**11*m + 39916800*e**11) + 1237950*d**2*e**9*m**6*x**8*(d + e*x)**m/
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(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423
*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**
4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916
800*e**11) - 41126400*d**2*e**9*m**6*x**7*(d + e*x)**m/(e**11*m**11 + 66*e*
*11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 263755
8*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*
m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 5659500
*d**2*e**9*m**6*x**6*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**1
1*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 133395
35*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**1
1*m**2 + 120543840*e**11*m + 39916800*e**11) - 60851088*d**2*e**9*m**6*x**5
*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11
*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 4599
5730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e
**11*m + 39916800*e**11) - 31839300*d**2*e**9*m**6*x**4*(d + e*x)**m/(e**11
*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11
*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 10
5258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e*
*11) - 67924032*d**2*e**9*m**6*x**3*(d + e*x)**m/(e**11*m**11 + 66*e**11*m*
*10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**1
1*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 +
 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) - 32184180*d**2
*e**9*m**6*x**2*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**
9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e*
*11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**
2 + 120543840*e**11*m + 39916800*e**11) - 21636720*d**2*e**9*m**6*x*(d + e*
x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 +
357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**
11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m +
 39916800*e**11) - 3691170*d**2*e**9*m**6*(d + e*x)**m/(e**11*m**11 + 66*e*
*11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 263755
8*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*
m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) - 1122450
00*d**2*e**9*m**5*x**9*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e*
*11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 1333
9535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e*
*11*m**2 + 120543840*e**11*m + 39916800*e**11) + 6374025*d**2*e**9*m**5*x**
8*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**1
1*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 459
95730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*
e**11*m + 39916800*e**11) - 222211080*d**2*e**9*m**5*x**7*(d + e*x)**m/(e**
11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**
11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 +
105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*
e**11) + 32440254*d**2*e**9*m**5*x**6*(d + e*x)**m/(e**11*m**11 + 66*e**11*
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m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e*
*11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3
 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) - 374798826*d
**2*e**9*m**5*x**5*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*
m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535
*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*
m**2 + 120543840*e**11*m + 39916800*e**11) - 213304950*d**2*e**9*m**5*x**4*
(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*
m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995
730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e*
*11*m + 39916800*e**11) - 499622244*d**2*e**9*m**5*x**3*(d + e*x)**m/(e**11
*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11
*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 10
5258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e*
*11) - 261415098*d**2*e**9*m**5*x**2*(d + e*x)**m/(e**11*m**11 + 66*e**11*m
**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**
11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3
+ 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) - 194510106*d*
*2*e**9*m**5*x*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9
 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**
11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2
 + 120543840*e**11*m + 39916800*e**11) - 36710415*d**2*e**9*m**5*(d + e*x)*
*m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357
423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*
m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39
916800*e**11) - 336420000*d**2*e**9*m**4*x**9*(d + e*x)**m/(e**11*m**11 + 6
6*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 26
37558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e*
*11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 197
07975*d**2*e**9*m**4*x**8*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925
*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 1
3339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976
*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) - 714314160*d**2*e**9*m**
4*x**7*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670
*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5
+ 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 12054
3840*e**11*m + 39916800*e**11) + 109598790*d**2*e**9*m**4*x**6*(d + e*x)**m
/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 35742
3*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m*
*4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 3991
6800*e**11) - 1351239408*d**2*e**9*m**4*x**5*(d + e*x)**m/(e**11*m**11 + 66
*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 263
7558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**
11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) - 8377
74450*d**2*e**9*m**4*x**4*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925
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*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 1
3339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976
*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) - 2184934056*d**2*e**9*m*
*4*x**3*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 3267
0*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5
 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 1205
43840*e**11*m + 39916800*e**11) - 1295069706*d**2*e**9*m**4*x**2*(d + e*x)*
*m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357
423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*
m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39
916800*e**11) - 1102270680*d**2*e**9*m**4*x*(d + e*x)**m/(e**11*m**11 + 66*
e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637
558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**1
1*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) - 23855
6745*d**2*e**9*m**4*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11
*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 1333953
5*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11
*m**2 + 120543840*e**11*m + 39916800*e**11) - 590620000*d**2*e**9*m**3*x**9
*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11
*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 4599
5730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e
**11*m + 39916800*e**11) + 35442000*d**2*e**9*m**3*x**8*(d + e*x)**m/(e**11
*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11
*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 10
5258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e*
*11) - 1324392480*d**2*e**9*m**3*x**7*(d + e*x)**m/(e**11*m**11 + 66*e**11*
m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e*
*11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3
 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 211477336*d
**2*e**9*m**3*x**6*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*
m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535
*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*
m**2 + 120543840*e**11*m + 39916800*e**11) - 2752660584*d**2*e**9*m**3*x**5
*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11
*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 4599
5730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e
**11*m + 39916800*e**11) - 1843915200*d**2*e**9*m**3*x**4*(d + e*x)**m/(e**
11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**
11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 +
105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*
e**11) - 5397281200*d**2*e**9*m**3*x**3*(d + e*x)**m/(e**11*m**11 + 66*e**1
1*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*
e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m*
*3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) - 374583904
8*d**2*e**9*m**3*x**2*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**
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11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339
535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**
11*m**2 + 120543840*e**11*m + 39916800*e**11) - 3842860824*d**2*e**9*m**3*x
*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11
*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 4599
5730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e
**11*m + 39916800*e**11) - 1011746160*d**2*e**9*m**3*(d + e*x)**m/(e**11*m*
*11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m*
*7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 10525
8076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11
) - 547920000*d**2*e**9*m**2*x**9*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**1
0 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*
m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 1
50917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 33477300*d**2*e
**9*m**2*x**8*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9
 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**1
1*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2
 + 120543840*e**11*m + 39916800*e**11) - 1280059200*d**2*e**9*m**2*x**7*(d +
 e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8
 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*
e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*
m + 39916800*e**11) + 210698040*d**2*e**9*m**2*x**6*(d + e*x)**m/(e**11*m**
11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**
7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258
076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11)
 - 2860144992*d**2*e**9*m**2*x**5*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**1
0 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*
m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 1
50917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) - 2038480200*d**2
*e**9*m**2*x**4*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**
9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e*
*11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**
2 + 120543840*e**11*m + 39916800*e**11) - 6600448608*d**2*e**9*m**2*x**3*(d
 + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m*
*8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 4599573
0*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**1
1*m + 39916800*e**11) - 5546320920*d**2*e**9*m**2*x**2*(d + e*x)**m/(e**11*
m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*
m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105
258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**
11) - 7530723360*d**2*e**9*m**2*x*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**1
0 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*
m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 1
50917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) - 2697071580*d**2
*e**9*m**2*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 3
```

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2670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m
**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 1
20543840*e**11*m + 39916800*e**11) - 201600000*d**2*e**9*m*x**9*(d + e*x)**
m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 3574
23*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m
**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 399
16800*e**11) + 12474000*d**2*e**9*m*x**8*(d + e*x)**m/(e**11*m**11 + 66*e**
11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558
*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m
**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) - 48470400
0*d**2*e**9*m*x**7*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*
m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535
*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*
m**2 + 120543840*e**11*m + 39916800*e**11) + 81496800*d**2*e**9*m*x**6*(d +
 e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8
 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*
e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*
m + 39916800*e**11) - 1139339520*d**2*e**9*m*x**5*(d + e*x)**m/(e**11*m**11
 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7
+ 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 10525807
6*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) -
 848232000*d**2*e**9*m*x**4*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 19
25*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 +
 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 1509179
76*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) - 2951182080*d**2*e**9*
m*x**3*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670
*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5
+ 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 12054
3840*e**11*m + 39916800*e**11) - 2864030400*d**2*e**9*m*x**2*(d + e*x)**m/(
e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*
e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4
 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 399168
00*e**11) - 6346771200*d**2*e**9*m*x*(d + e*x)**m/(e**11*m**11 + 66*e**11*m
**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**
11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3
+ 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) - 4095133200*d
**2*e**9*m*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 3
2670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m
**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 1
20543840*e**11*m + 39916800*e**11) - 2694384000*d**2*e**9*(d + e*x)**m/(e**
11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**
11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 +
105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*
e**11) + 500*d*e**10*m**10*x**10*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10
 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m
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**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 15
0917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) - 25*d*e**10*m**10
*x**9*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*
e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 +
 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543
840*e**11*m + 39916800*e**11) + 765*d*e**10*m**10*x**8*(d + e*x)**m/(e**11*
m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*
m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105
258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**
11) - 98*d*e**10*m**10*x**7*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 19
25*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 +
 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 1509179
76*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 999*d*e**10*m**10*x**
6*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**1
1*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 459
95730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*
e**11*m + 39916800*e**11) + 510*d*e**10*m**10*x**5*(d + e*x)**m/(e**11*m**1
1 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7
 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 1052580
76*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11)
+ 1109*d*e**10*m**10*x**4*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925
*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 1
3339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976
*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 574*d*e**10*m**10*x**3*
(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*
m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995
730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e*
*11*m + 39916800*e**11) + 477*d*e**10*m**10*x**2*(d + e*x)**m/(e**11*m**11
+ 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 +
 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076
*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) +
135*d*e**10*m**10*x*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11
*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 1333953
5*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11
*m**2 + 120543840*e**11*m + 39916800*e**11) + 54*d*e**10*m**10*(d + e*x)**m
/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 35742
3*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m*
*4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 3991
6800*e**11) + 22500*d*e**10*m**9*x**10*(d + e*x)**m/(e**11*m**11 + 66*e**11
*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e
**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**
3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) - 1175*d*e**
10*m**9*x**9*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 +
 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11
*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 +
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 120543840*e**11*m + 39916800*e**11) + 37485*d*e**10*m**9*x**8*(d + e*x)**m
/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 35742
3*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m*
*4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 3991
6800*e**11) - 4998*d*e**10*m**9*x**7*(d + e*x)**m/(e**11*m**11 + 66*e**11*m
**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**
11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3
 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 52947*d*e**1
0*m**9*x**6*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 +
32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*
m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 +
120543840*e**11*m + 39916800*e**11) + 28050*d*e**10*m**9*x**5*(d + e*x)**m/
(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423
*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**
4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916
800*e**11) + 63213*d*e**10*m**9*x**4*(d + e*x)**m/(e**11*m**11 + 66*e**11*m
**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**
11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3
 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 33866*d*e**1
0*m**9*x**3*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 +
32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*
m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 +
120543840*e**11*m + 39916800*e**11) + 29097*d*e**10*m**9*x**2*(d + e*x)**m/
(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423
*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**
4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916
800*e**11) + 8505*d*e**10*m**9*x*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10
 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m
**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 15
0917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 3510*d*e**10*m**
9*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**1
1*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 459
95730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*
e**11*m + 39916800*e**11) + 435000*d*e**10*m**8*x**10*(d + e*x)**m/(e**11*m
**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m
**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 1052
58076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**1
1) - 23550*d*e**10*m**8*x**9*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1
925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6
 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917
976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 780300*d*e**10*m**8*
x**8*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e
**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 +
45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 1205438
40*e**11*m + 39916800*e**11) - 108192*d*e**10*m**8*x**7*(d + e*x)**m/(e**11
```

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*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11
*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 10
5258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e*
*11) + 1192806*d*e**10*m**8*x**6*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10
 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m
**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 15
0917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 657900*d*e**10*m
**8*x**5*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 326
70*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**
5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120
543840*e**11*m + 39916800*e**11) + 1543728*d*e**10*m**8*x**4*(d + e*x)**m/(
e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*
e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4
 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 399168
00*e**11) + 861000*d*e**10*m**8*x**3*(d + e*x)**m/(e**11*m**11 + 66*e**11*m
**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**
11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3
+ 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 769878*d*e**
10*m**8*x**2*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 +
 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11
*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 +
 120543840*e**11*m + 39916800*e**11) + 234090*d*e**10*m**8*x*(d + e*x)**m/(
e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*
e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4
 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 399168
00*e**11) + 100440*d*e**10*m**8*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10
+ 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m*
*6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150
917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 4725000*d*e**10*m
**7*x**10*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32
670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m*
*5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 12
0543840*e**11*m + 39916800*e**11) - 263550*d*e**10*m**7*x**9*(d + e*x)**m/(
e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*
e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4
 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 399168
00*e**11) + 9028530*d*e**10*m**7*x**8*(d + e*x)**m/(e**11*m**11 + 66*e**11*
m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e*
*11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3
 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) - 1298892*d*e
**10*m**7*x**7*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9
 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**
11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2
 + 120543840*e**11*m + 39916800*e**11) + 14907078*d*e**10*m**7*x**6*(d + e*
x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 +
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357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**
11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m +
 39916800*e**11) + 8583300*d*e**10*m**7*x**5*(d + e*x)**m/(e**11*m**11 + 66
*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 263
7558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**
11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 2107
3218*d*e**10*m**7*x**4*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e*
*11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 1333
9535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e*
*11*m**2 + 120543840*e**11*m + 39916800*e**11) + 12319188*d*e**10*m**7*x**3
*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11
*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 4599
5730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e
**11*m + 39916800*e**11) + 11559618*d*e**10*m**7*x**2*(d + e*x)**m/(e**11*m
**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m
**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 1052
58076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**1
1) + 3691170*d*e**10*m**7*x*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 19
25*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 +
 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 1509179
76*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 1663740*d*e**10*m**7*
(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*
m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995
730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e*
*11*m + 39916800*e**11) + 31636500*d*e**10*m**6*x**10*(d + e*x)**m/(e**11*m
**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m
**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 1052
58076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**1
1) - 1808625*d*e**10*m**6*x**9*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 +
 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**
6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 1509
17976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 63761985*d*e**10*m
**6*x**8*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 326
70*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**
5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120
543840*e**11*m + 39916800*e**11) - 9485322*d*e**10*m**6*x**7*(d + e*x)**m/(
e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*
e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4
 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 399168
00*e**11) + 113175711*d*e**10*m**6*x**6*(d + e*x)**m/(e**11*m**11 + 66*e**1
1*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*
e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m*
*3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 68132430*
d*e**10*m**6*x**5*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m
**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*
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e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m
**2 + 120543840*e**11*m + 39916800*e**11) + 175848585*d*e**10*m**6*x**4*(d
+ e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**
8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730
*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11
*m + 39916800*e**11) + 108594486*d*e**10*m**6*x**3*(d + e*x)**m/(e**11*m**1
1 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7
 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 1052580
76*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11)
+ 108073413*d*e**10*m**6*x**2*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 +
1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6
 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 15091
7976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 36710415*d*e**10*m*
*6*x*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e
**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 +
45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 1205438
40*e**11*m + 39916800*e**11) + 17637102*d*e**10*m**6*(d + e*x)**m/(e**11*m*
*11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m*
*7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 10525
8076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11
) + 134662500*d*e**10*m**5*x**10*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10
 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m
**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 15
0917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) - 7855575*d*e**10*
m**5*x**9*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32
670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m*
*5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 12
0543840*e**11*m + 39916800*e**11) + 283723965*d*e**10*m**5*x**8*(d + e*x)**
m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 3574
23*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m
**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 399
16800*e**11) - 43462902*d*e**10*m**5*x**7*(d + e*x)**m/(e**11*m**11 + 66*e*
*11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 263755
8*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*
m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 5375389
23*d*e**10*m**5*x**6*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**1
1*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 133395
35*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**1
1*m**2 + 120543840*e**11*m + 39916800*e**11) + 338198850*d*e**10*m**5*x**5*
(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*
m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995
730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e*
*11*m + 39916800*e**11) + 920950197*d*e**10*m**5*x**4*(d + e*x)**m/(e**11*m
**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m
**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 1052
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58076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**1
1) + 605966634*d*e**10*m**5*x**3*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10
 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m
**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 15
0917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 648390393*d*e**1
0*m**5*x**2*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 +
32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*
m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 +
120543840*e**11*m + 39916800*e**11) + 238556745*d*e**10*m**5*x*(d + e*x)**m
/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 35742
3*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m*
*4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 3991
6800*e**11) + 124791030*d*e**10*m**5*(d + e*x)**m/(e**11*m**11 + 66*e**11*m
**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**
11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3
+ 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 361840000*d*
e**10*m**4*x**10*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m*
*9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e
**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m*
*2 + 120543840*e**11*m + 39916800*e**11) - 21456200*d*e**10*m**4*x**9*(d +
e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8
+ 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e
**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m
 + 39916800*e**11) + 790573950*d*e**10*m**4*x**8*(d + e*x)**m/(e**11*m**11
+ 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 +
 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076
*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) -
124152868*d*e**10*m**4*x**7*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 19
25*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 +
 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 1509179
76*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 1584809604*d*e**10*m*
*4*x**6*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 3267
0*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5
 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 1205
43840*e**11*m + 39916800*e**11) + 1039002600*d*e**10*m**4*x**5*(d + e*x)**m
/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 35742
3*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m*
*4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 3991
6800*e**11) + 2988020842*d*e**10*m**4*x**4*(d + e*x)**m/(e**11*m**11 + 66*e
**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 26375
58*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11
*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 211199
2820*d*e**10*m**4*x**3*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e*
*11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 1333
9535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e*
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*11*m**2 + 120543840*e**11*m + 39916800*e**11) + 2472565752*d*e**10*m**4*x*
*2*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**
11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45
995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840
*e**11*m + 39916800*e**11) + 1011746160*d*e**10*m**4*x*(d + e*x)**m/(e**11*
m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*
m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105
258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**
11) + 595543860*d*e**10*m**4*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1
925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6
 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917
976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 586350000*d*e**10*m*
*3*x**10*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 326
70*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**
5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120
543840*e**11*m + 39916800*e**11) - 35223700*d*e**10*m**3*x**9*(d + e*x)**m/
(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423
*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**
4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916
800*e**11) + 1318850820*d*e**10*m**3*x**8*(d + e*x)**m/(e**11*m**11 + 66*e*
*11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 263755
8*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*
m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) - 2113660
08*d*e**10*m**3*x**7*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**1
1*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 133395
35*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**1
1*m**2 + 120543840*e**11*m + 39916800*e**11) + 2770574652*d*e**10*m**3*x**6
*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11
*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 4599
5730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e
**11*m + 39916800*e**11) + 1882828200*d*e**10*m**3*x**5*(d + e*x)**m/(e**11
*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11
*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 10
5258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e*
*11) + 5698073052*d*e**10*m**3*x**4*(d + e*x)**m/(e**11*m**11 + 66*e**11*m*
*10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**1
1*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 +
 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 4345999672*d*
e**10*m**3*x**3*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**
9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e*
*11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**
2 + 120543840*e**11*m + 39916800*e**11) + 5686792092*d*e**10*m**3*x**2*(d +
 e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8
 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*
e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*
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m + 39916800*e**11) + 2697071580*d*e**10*m**3*x*(d + e*x)**m/(e**11*m**11 +
 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 +
2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*
e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 1
888225560*d*e**10*m**3*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e*
*11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 1333
9535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e*
*11*m**2 + 120543840*e**11*m + 39916800*e**11) + 513288000*d*e**10*m**2*x**
10*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**
11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45
995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840
*e**11*m + 39916800*e**11) - 31143600*d*e**10*m**2*x**9*(d + e*x)**m/(e**11
*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11
*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 10
5258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e*
*11) + 1180639800*d*e**10*m**2*x**8*(d + e*x)**m/(e**11*m**11 + 66*e**11*m*
*10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**1
1*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 +
 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) - 192240720*d*e
**10*m**2*x**7*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9
 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**
11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2
 + 120543840*e**11*m + 39916800*e**11) + 2573344080*d*e**10*m**2*x**6*(d +
e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8
+ 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e
**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m
 + 39916800*e**11) + 1800430560*d*e**10*m**2*x**5*(d + e*x)**m/(e**11*m**11
 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7
+ 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 10525807
6*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) +
 5688131976*d*e**10*m**2*x**4*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 +
1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6
 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 15091
7976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 4652224080*d*e**10*
m**2*x**3*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32
670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m*
*5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 12
0543840*e**11*m + 39916800*e**11) + 6938747280*d*e**10*m**2*x**2*(d + e*x)*
*m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357
423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*
m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39
916800*e**11) + 4095133200*d*e**10*m**2*x*(d + e*x)**m/(e**11*m**11 + 66*e*
*11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 263755
8*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*
m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 3795710
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544*d*e**10*m**2*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m*
*9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e
**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m*
*2 + 120543840*e**11*m + 39916800*e**11) + 181440000*d*e**10*m*x**10*(d + e
*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 +
 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e*
*11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m
+ 39916800*e**11) - 11088000*d*e**10*m*x**9*(d + e*x)**m/(e**11*m**11 + 66*
e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637
558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**1
1*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 42411
6000*d*e**10*m*x**8*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11
*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 1333953
5*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11
*m**2 + 120543840*e**11*m + 39916800*e**11) - 69854400*d*e**10*m*x**7*(d +
e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8
+ 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e
**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m
 + 39916800*e**11) + 949449600*d*e**10*m*x**6*(d + e*x)**m/(e**11*m**11 + 6
6*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 26
37558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e*
*11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 678
585600*d*e**10*m*x**5*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**
11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339
535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**
11*m**2 + 120543840*e**11*m + 39916800*e**11) + 2213386560*d*e**10*m*x**4*(
d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m
**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 459957
30*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**
11*m + 39916800*e**11) + 1909353600*d*e**10*m*x**3*(d + e*x)**m/(e**11*m**1
1 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7
 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 1052580
76*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11)
+ 3173385600*d*e**10*m*x**2*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 19
25*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 +
 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 1509179
76*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 2694384000*d*e**10*m*
x*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**1
1*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 459
95730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*
e**11*m + 39916800*e**11) + 4353860160*d*e**10*m*(d + e*x)**m/(e**11*m**11
+ 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 +
 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076
*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) +
2155507200*d*e**10*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*
```

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m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535
*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*
m**2 + 120543840*e**11*m + 39916800*e**11) + 500*e**11*m**10*x**11*(d + e*x
)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 3
57423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**1
1*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m +
39916800*e**11) - 25*e**11*m**10*x**10*(d + e*x)**m/(e**11*m**11 + 66*e**11
*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e
**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**
3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 765*e**11*
m**10*x**9*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 3
2670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m
**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 1
20543840*e**11*m + 39916800*e**11) - 98*e**11*m**10*x**8*(d + e*x)**m/(e**1
1*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**1
1*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 1
05258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e
**11) + 999*e**11*m**10*x**7*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1
925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6
+ 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917
976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 510*e**11*m**10*x**6
*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11
*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 4599
5730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e
**11*m + 39916800*e**11) + 1109*e**11*m**10*x**5*(d + e*x)**m/(e**11*m**11
+ 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 +
 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076
*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) +
574*e**11*m**10*x**4*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**1
1*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 133395
35*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**1
1*m**2 + 120543840*e**11*m + 39916800*e**11) + 477*e**11*m**10*x**3*(d + e*
x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 +
357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**
11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m +
 39916800*e**11) + 135*e**11*m**10*x**2*(d + e*x)**m/(e**11*m**11 + 66*e**1
1*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*
e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m*
*3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 54*e**11*
m**10*x*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 3267
0*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5
 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 1205
43840*e**11*m + 39916800*e**11) + 27500*e**11*m**9*x**11*(d + e*x)**m/(e**1
1*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**1
1*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 1
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05258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e
**11) - 1400*e**11*m**9*x**10*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 +
1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6
 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 15091
7976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 43605*e**11*m**9*x*
*9*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**
11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45
995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840
*e**11*m + 39916800*e**11) - 5684*e**11*m**9*x**8*(d + e*x)**m/(e**11*m**11
 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7
+ 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 10525807
6*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) +
 58941*e**11*m**9*x**7*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e*
*11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 1333
9535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e*
*11*m**2 + 120543840*e**11*m + 39916800*e**11) + 30600*e**11*m**9*x**6*(d +
 e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8
 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*
e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*
m + 39916800*e**11) + 67649*e**11*m**9*x**5*(d + e*x)**m/(e**11*m**11 + 66*
e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637
558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**1
1*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 35588
*e**11*m**9*x**4*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m*
*9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e
**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m*
*2 + 120543840*e**11*m + 39916800*e**11) + 30051*e**11*m**9*x**3*(d + e*x)*
*m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357
423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*
m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39
916800*e**11) + 8640*e**11*m**9*x**2*(d + e*x)**m/(e**11*m**11 + 66*e**11*m
**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**
11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3
+ 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 3510*e**11*m
**9*x*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*
e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 +
 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543
840*e**11*m + 39916800*e**11) + 660000*e**11*m**8*x**11*(d + e*x)**m/(e**11
*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11
*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 10
5258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e*
*11) - 34125*e**11*m**8*x**10*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 +
1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6
 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 15091
7976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 1080180*e**11*m**8*
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x**9*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e
**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 +
45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 1205438
40*e**11*m + 39916800*e**11) - 143178*e**11*m**8*x**8*(d + e*x)**m/(e**11*m
**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m
**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 1052
58076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**1
1) + 1510488*e**11*m**8*x**7*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1
925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6
+ 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917
976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 798150*e**11*m**8*x*
*6*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**
11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45
995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840
*e**11*m + 39916800*e**11) + 1796580*e**11*m**8*x**5*(d + e*x)**m/(e**11*m*
*11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m*
*7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 10525
8076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11
) + 962598*e**11*m**8*x**4*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 192
5*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 +
13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 15091797
6*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 828072*e**11*m**8*x**3
*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11
*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 4599
5730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e
**11*m + 39916800*e**11) + 242595*e**11*m**8*x**2*(d + e*x)**m/(e**11*m**11
 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7
+ 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 10525807
6*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) +
 100440*e**11*m**8*x*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**1
1*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 133395
35*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**1
1*m**2 + 120543840*e**11*m + 39916800*e**11) + 9075000*e**11*m**7*x**11*(d
+ e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**
8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730
*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11
*m + 39916800*e**11) - 475500*e**11*m**7*x**10*(d + e*x)**m/(e**11*m**11 +
66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2
637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e
**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 15
270930*e**11*m**7*x**9*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e*
*11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 1333
9535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e*
*11*m**2 + 120543840*e**11*m + 39916800*e**11) - 2056236*e**11*m**7*x**8*(d
 + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m*
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*8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 4599573
0*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**1
1*m + 39916800*e**11) + 22063914*e**11*m**7*x**7*(d + e*x)**m/(e**11*m**11
+ 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 +
 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076
*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) +
11872800*e**11*m**7*x**6*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*
e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13
339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*
e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 27248130*e**11*m**7*x**5
*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11
*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 4599
5730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e
**11*m + 39916800*e**11) + 14902188*e**11*m**7*x**4*(d + e*x)**m/(e**11*m**
11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**
7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258
076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11)
 + 13099374*e**11*m**7*x**3*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 19
25*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 +
 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 1509179
76*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 3925260*e**11*m**7*x*
*2*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**
11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45
995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840
*e**11*m + 39916800*e**11) + 1663740*e**11*m**7*x*(d + e*x)**m/(e**11*m**11
 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7
+ 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 10525807
6*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) +
 78886500*e**11*m**6*x**11*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 192
5*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 +
13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 15091797
6*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) - 4180575*e**11*m**6*x**
10*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**
11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45
995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840
*e**11*m + 39916800*e**11) + 135990225*e**11*m**6*x**9*(d + e*x)**m/(e**11*
m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*
m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105
258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**
11) - 18577566*e**11*m**6*x**8*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 +
 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**
6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 1509
17976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 202618179*e**11*m*
*6*x**7*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 3267
0*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5
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 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 1205
43840*e**11*m + 39916800*e**11) + 111048930*e**11*m**6*x**6*(d + e*x)**m/(e
**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e
**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4
+ 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 3991680
0*e**11) + 260141457*e**11*m**6*x**5*(d + e*x)**m/(e**11*m**11 + 66*e**11*m
**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**
11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3
+ 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 145552050*e*
*11*m**6*x**4*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9
+ 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**1
1*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2
+ 120543840*e**11*m + 39916800*e**11) + 131192649*e**11*m**6*x**3*(d + e*x)
**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 35
7423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11
*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 3
9916800*e**11) + 40401585*e**11*m**6*x**2*(d + e*x)**m/(e**11*m**11 + 66*e*
*11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 263755
8*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*
m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 1763710
2*e**11*m**6*x*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9
 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**
11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2
 + 120543840*e**11*m + 39916800*e**11) + 451027500*e**11*m**5*x**11*(d + e*
x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 +
357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**
11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m +
 39916800*e**11) - 24133200*e**11*m**5*x**10*(d + e*x)**m/(e**11*m**11 + 66
*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 263
7558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**
11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 7938
19845*e**11*m**5*x**9*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**
11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339
535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**
11*m**2 + 120543840*e**11*m + 39916800*e**11) - 109860156*e**11*m**5*x**8*(
d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m
**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 459957
30*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**
11*m + 39916800*e**11) + 1216593189*e**11*m**5*x**7*(d + e*x)**m/(e**11*m**
11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**
7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258
076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11)
 + 678861000*e**11*m**5*x**6*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1
925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6
+ 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917
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976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 1624344537*e**11*m**
5*x**5*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670
*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5
+ 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 12054
3840*e**11*m + 39916800*e**11) + 931750092*e**11*m**5*x**4*(d + e*x)**m/(e*
*11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e*
*11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 +
 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800
*e**11) + 864537219*e**11*m**5*x**3*(d + e*x)**m/(e**11*m**11 + 66*e**11*m*
*10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**1
1*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 +
 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 275267160*e**
11*m**5*x**2*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 +
 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11
*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 +
 120543840*e**11*m + 39916800*e**11) + 124791030*e**11*m**5*x*(d + e*x)**m/
(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423
*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**
4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916
800*e**11) + 1708465000*e**11*m**4*x**11*(d + e*x)**m/(e**11*m**11 + 66*e**
11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558
*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m
**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) - 92156375
*e**11*m**4*x**10*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m
**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*
e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m
**2 + 120543840*e**11*m + 39916800*e**11) + 3060365670*e**11*m**4*x**9*(d +
 e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8
 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*
e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*
m + 39916800*e**11) - 428393182*e**11*m**4*x**8*(d + e*x)**m/(e**11*m**11 +
 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 +
2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*
e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 4
810043142*e**11*m**4*x**7*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925
*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 1
3339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976
*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 2729996850*e**11*m**4*x
**6*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e*
*11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 4
5995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 12054384
0*e**11*m + 39916800*e**11) + 6671821630*e**11*m**4*x**5*(d + e*x)**m/(e**1
1*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**1
1*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 1
05258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e
```

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**11) + 3929892722*e**11*m**4*x**4*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**
10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11
*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 +
150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 3769346538*e**
11*m**4*x**3*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 +
 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11
*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 +
 120543840*e**11*m + 39916800*e**11) + 1250302905*e**11*m**4*x**2*(d + e*x)
**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 35
7423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11
*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 3
9916800*e**11) + 595543860*e**11*m**4*x*(d + e*x)**m/(e**11*m**11 + 66*e**1
1*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*
e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m*
*3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 420475000
0*e**11*m**3*x**11*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*
m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535
*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*
m**2 + 120543840*e**11*m + 39916800*e**11) - 228329500*e**11*m**3*x**10*(d
+ e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**
8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730
*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11
*m + 39916800*e**11) + 7643442420*e**11*m**3*x**9*(d + e*x)**m/(e**11*m**11
 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7
+ 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 10525807
6*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) -
 1080436084*e**11*m**3*x**8*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 19
25*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 +
 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 1509179
76*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 12279432276*e**11*m**
3*x**7*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670
*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5
+ 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 12054
3840*e**11*m + 39916800*e**11) + 7077841200*e**11*m**3*x**6*(d + e*x)**m/(e
**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e
**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4
+ 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 3991680
0*e**11) + 17650156420*e**11*m**3*x**5*(d + e*x)**m/(e**11*m**11 + 66*e**11
*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e
**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**
3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 1068197813
2*e**11*m**3*x**4*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m
**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*
e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m
**2 + 120543840*e**11*m + 39916800*e**11) + 10631923596*e**11*m**3*x**3*(d
```

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+ e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**
8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730
*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11
*m + 39916800*e**11) + 3708817740*e**11*m**3*x**2*(d + e*x)**m/(e**11*m**11
 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7
+ 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 10525807
6*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) +
 1888225560*e**11*m**3*x*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*
e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13
339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*
e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 6376788000*e**11*m**2*x*
*11*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e*
*11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 4
5995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 12054384
0*e**11*m + 39916800*e**11) - 348156900*e**11*m**2*x**10*(d + e*x)**m/(e**1
1*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**1
1*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 1
05258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e
**11) + 11731446360*e**11*m**2*x**9*(d + e*x)**m/(e**11*m**11 + 66*e**11*m*
*10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**1
1*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 +
 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) - 1671802776*e*
*11*m**2*x**8*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9
+ 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**1
1*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2
+ 120543840*e**11*m + 39916800*e**11) + 19196791992*e**11*m**2*x**7*(d + e*
x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 +
357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**
11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m +
 39916800*e**11) + 11214571560*e**11*m**2*x**6*(d + e*x)**m/(e**11*m**11 +
66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2
637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e
**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 28
480424184*e**11*m**2*x**5*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925
*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 1
3339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976
*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 17690223096*e**11*m**2*
x**4*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e
**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 +
45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 1205438
40*e**11*m + 39916800*e**11) + 18312331464*e**11*m**2*x**3*(d + e*x)**m/(e*
*11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e*
*11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 +
 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800
*e**11) + 6792204780*e**11*m**2*x**2*(d + e*x)**m/(e**11*m**11 + 66*e**11*m
```

```
**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**
11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3
+ 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 3795710544*e
**11*m**2*x*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 +
32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*
m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 +
120543840*e**11*m + 39916800*e**11) + 5314320000*e**11*m*x**11*(d + e*x)**m
/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 35742
3*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m*
*4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 3991
6800*e**11) - 291380400*e**11*m*x**10*(d + e*x)**m/(e**11*m**11 + 66*e**11*
m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e*
*11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3
 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 9869234400*
e**11*m*x**9*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 +
 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11
*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 +
 120543840*e**11*m + 39916800*e**11) - 1415539440*e**11*m*x**8*(d + e*x)**m
/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 35742
3*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m*
*4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 3991
6800*e**11) + 16389514080*e**11*m*x**7*(d + e*x)**m/(e**11*m**11 + 66*e**11
*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e
**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**
3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 9680738400
*e**11*m*x**6*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9
+ 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**1
1*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2
+ 120543840*e**11*m + 39916800*e**11) + 24965914464*e**11*m*x**5*(d + e*x)*
*m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357
423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*
m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39
916800*e**11) + 15866025840*e**11*m*x**4*(d + e*x)**m/(e**11*m**11 + 66*e**
11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558
*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m
**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 17050880
160*e**11*m*x**3*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m*
*9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e
**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m*
*2 + 120543840*e**11*m + 39916800*e**11) + 6789517200*e**11*m*x**2*(d + e*x
)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 3
57423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**1
1*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m +
39916800*e**11) + 4353860160*e**11*m*x*(d + e*x)**m/(e**11*m**11 + 66*e**11
*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e
```

```
**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**
3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 1814400000
*e**11*x**11*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 +
 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11
*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 +
 120543840*e**11*m + 39916800*e**11) - 99792000*e**11*x**10*(d + e*x)**m/(e
**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e
**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4
+ 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 3991680
0*e**11) + 3392928000*e**11*x**9*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10
 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m
**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 15
0917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) - 488980800*e**11*
x**8*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e
**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 +
45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 1205438
40*e**11*m + 39916800*e**11) + 5696697600*e**11*x**7*(d + e*x)**m/(e**11*m*
*11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m*
*7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 10525
8076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11
) + 3392928000*e**11*x**6*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925
*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 1
3339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976
*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 8853546240*e**11*x**5*(
d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m
**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 459957
30*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**
11*m + 39916800*e**11) + 5728060800*e**11*x**4*(d + e*x)**m/(e**11*m**11 +
66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2
637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e
**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11) + 63
46771200*e**11*x**3*(d + e*x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11
*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e**11*m**6 + 1333953
5*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3 + 150917976*e**11
*m**2 + 120543840*e**11*m + 39916800*e**11) + 2694384000*e**11*x**2*(d + e*
x)**m/(e**11*m**11 + 66*e**11*m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 +
357423*e**11*m**7 + 2637558*e**11*m**6 + 13339535*e**11*m**5 + 45995730*e**
11*m**4 + 105258076*e**11*m**3 + 150917976*e**11*m**2 + 120543840*e**11*m +
 39916800*e**11) + 2155507200*e**11*x*(d + e*x)**m/(e**11*m**11 + 66*e**11*
m**10 + 1925*e**11*m**9 + 32670*e**11*m**8 + 357423*e**11*m**7 + 2637558*e*
*11*m**6 + 13339535*e**11*m**5 + 45995730*e**11*m**4 + 105258076*e**11*m**3
 + 150917976*e**11*m**2 + 120543840*e**11*m + 39916800*e**11), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2292 vs. $2(588) = 1176$.

Time = 0.27 (sec) , antiderivative size = 2292, normalized size of antiderivative = 3.90

$$\int (d+ex)^m \left(3+2x+5x^2\right)^3 \left(2+x+3x^2-5x^3+4x^4\right)\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)^m*(5*x^2+2*x+3)^3*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="m
axima")
```

```
[Out] 135*(e^2*(m + 1)*x^2 + d*e*m*x - d^2)*(e*x + d)^m/((m^2 + 3*m + 2)*e^2) + 5
4*(e*x + d)^(m + 1)/(e*(m + 1)) + 477*((m^2 + 3*m + 2)*e^3*x^3 + (m^2 + m)*
d*e^2*x^2 - 2*d^2*e*m*x + 2*d^3)*(e*x + d)^m/((m^3 + 6*m^2 + 11*m + 6)*e^3)
 + 574*((m^3 + 6*m^2 + 11*m + 6)*e^4*x^4 + (m^3 + 3*m^2 + 2*m)*d*e^3*x^3 -
3*(m^2 + m)*d^2*e^2*x^2 + 6*d^3*e*m*x - 6*d^4)*(e*x + d)^m/((m^4 + 10*m^3 +
 35*m^2 + 50*m + 24)*e^4) + 1109*((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*e^5*x
^5 + (m^4 + 6*m^3 + 11*m^2 + 6*m)*d*e^4*x^4 - 4*(m^3 + 3*m^2 + 2*m)*d^2*e^3
*x^3 + 12*(m^2 + m)*d^3*e^2*x^2 - 24*d^4*e*m*x + 24*d^5)*(e*x + d)^m/((m^5
+ 15*m^4 + 85*m^3 + 225*m^2 + 274*m + 120)*e^5) + 510*((m^5 + 15*m^4 + 85*m
^3 + 225*m^2 + 274*m + 120)*e^6*x^6 + (m^5 + 10*m^4 + 35*m^3 + 50*m^2 + 24*
m)*d*e^5*x^5 - 5*(m^4 + 6*m^3 + 11*m^2 + 6*m)*d^2*e^4*x^4 + 20*(m^3 + 3*m^2
 + 2*m)*d^3*e^3*x^3 - 60*(m^2 + m)*d^4*e^2*x^2 + 120*d^5*e*m*x - 120*d^6)*(
e*x + d)^m/((m^6 + 21*m^5 + 175*m^4 + 735*m^3 + 1624*m^2 + 1764*m + 720)*e^
6) + 999*((m^6 + 21*m^5 + 175*m^4 + 735*m^3 + 1624*m^2 + 1764*m + 720)*e^7*
x^7 + (m^6 + 15*m^5 + 85*m^4 + 225*m^3 + 274*m^2 + 120*m)*d*e^6*x^6 - 6*(m^
5 + 10*m^4 + 35*m^3 + 50*m^2 + 24*m)*d^2*e^5*x^5 + 30*(m^4 + 6*m^3 + 11*m^2
 + 6*m)*d^3*e^4*x^4 - 120*(m^3 + 3*m^2 + 2*m)*d^4*e^3*x^3 + 360*(m^2 + m)*d
^5*e^2*x^2 - 720*d^6*e*m*x + 720*d^7)*(e*x + d)^m/((m^7 + 28*m^6 + 322*m^5
+ 1960*m^4 + 6769*m^3 + 13132*m^2 + 13068*m + 5040)*e^7) - 98*((m^7 + 28*m^
6 + 322*m^5 + 1960*m^4 + 6769*m^3 + 13132*m^2 + 13068*m + 5040)*e^8*x^8 + (
m^7 + 21*m^6 + 175*m^5 + 735*m^4 + 1624*m^3 + 1764*m^2 + 720*m)*d*e^7*x^7 -
 7*(m^6 + 15*m^5 + 85*m^4 + 225*m^3 + 274*m^2 + 120*m)*d^2*e^6*x^6 + 42*(m^
5 + 10*m^4 + 35*m^3 + 50*m^2 + 24*m)*d^3*e^5*x^5 - 210*(m^4 + 6*m^3 + 11*m^
2 + 6*m)*d^4*e^4*x^4 + 840*(m^3 + 3*m^2 + 2*m)*d^5*e^3*x^3 - 2520*(m^2 + m)
*d^6*e^2*x^2 + 5040*d^7*e*m*x - 5040*d^8)*(e*x + d)^m/((m^8 + 36*m^7 + 546*
m^6 + 4536*m^5 + 22449*m^4 + 67284*m^3 + 118124*m^2 + 109584*m + 40320)*e^8
) + 765*((m^8 + 36*m^7 + 546*m^6 + 4536*m^5 + 22449*m^4 + 67284*m^3 + 11812
4*m^2 + 109584*m + 40320)*e^9*x^9 + (m^8 + 28*m^7 + 322*m^6 + 1960*m^5 + 67
69*m^4 + 13132*m^3 + 13068*m^2 + 5040*m)*d*e^8*x^8 - 8*(m^7 + 21*m^6 + 175*
m^5 + 735*m^4 + 1624*m^3 + 1764*m^2 + 720*m)*d^2*e^7*x^7 + 56*(m^6 + 15*m^5
 + 85*m^4 + 225*m^3 + 274*m^2 + 120*m)*d^3*e^6*x^6 - 336*(m^5 + 10*m^4 + 35
*m^3 + 50*m^2 + 24*m)*d^4*e^5*x^5 + 1680*(m^4 + 6*m^3 + 11*m^2 + 6*m)*d^5*e
^4*x^4 - 6720*(m^3 + 3*m^2 + 2*m)*d^6*e^3*x^3 + 20160*(m^2 + m)*d^7*e^2*x^2
 - 40320*d^8*e*m*x + 40320*d^9)*(e*x + d)^m/((m^9 + 45*m^8 + 870*m^7 + 9450
```

```
*m^6 + 63273*m^5 + 269325*m^4 + 723680*m^3 + 1172700*m^2 + 1026576*m + 3628
80)*e^9) - 25*((m^9 + 45*m^8 + 870*m^7 + 9450*m^6 + 63273*m^5 + 269325*m^4
+ 723680*m^3 + 1172700*m^2 + 1026576*m + 362880)*e^10*x^10 + (m^9 + 36*m^8
+ 546*m^7 + 4536*m^6 + 22449*m^5 + 67284*m^4 + 118124*m^3 + 109584*m^2 + 40
320*m)*d*e^9*x^9 - 9*(m^8 + 28*m^7 + 322*m^6 + 1960*m^5 + 6769*m^4 + 13132*
m^3 + 13068*m^2 + 5040*m)*d^2*e^8*x^8 + 72*(m^7 + 21*m^6 + 175*m^5 + 735*m^
4 + 1624*m^3 + 1764*m^2 + 720*m)*d^3*e^7*x^7 - 504*(m^6 + 15*m^5 + 85*m^4 +
 225*m^3 + 274*m^2 + 120*m)*d^4*e^6*x^6 + 3024*(m^5 + 10*m^4 + 35*m^3 + 50*
m^2 + 24*m)*d^5*e^5*x^5 - 15120*(m^4 + 6*m^3 + 11*m^2 + 6*m)*d^6*e^4*x^4 +
60480*(m^3 + 3*m^2 + 2*m)*d^7*e^3*x^3 - 181440*(m^2 + m)*d^8*e^2*x^2 + 3628
80*d^9*e*m*x - 362880*d^10)*(e*x + d)^m/((m^10 + 55*m^9 + 1320*m^8 + 18150*
m^7 + 157773*m^6 + 902055*m^5 + 3416930*m^4 + 8409500*m^3 + 12753576*m^2 +
10628640*m + 3628800)*e^10) + 500*((m^10 + 55*m^9 + 1320*m^8 + 18150*m^7 +
157773*m^6 + 902055*m^5 + 3416930*m^4 + 8409500*m^3 + 12753576*m^2 + 106286
40*m + 3628800)*e^11*x^11 + (m^10 + 45*m^9 + 870*m^8 + 9450*m^7 + 63273*m^6
 + 269325*m^5 + 723680*m^4 + 1172700*m^3 + 1026576*m^2 + 362880*m)*d*e^10*x
^10 - 10*(m^9 + 36*m^8 + 546*m^7 + 4536*m^6 + 22449*m^5 + 67284*m^4 + 11812
4*m^3 + 109584*m^2 + 40320*m)*d^2*e^9*x^9 + 90*(m^8 + 28*m^7 + 322*m^6 + 19
60*m^5 + 6769*m^4 + 13132*m^3 + 13068*m^2 + 5040*m)*d^3*e^8*x^8 - 720*(m^7
+ 21*m^6 + 175*m^5 + 735*m^4 + 1624*m^3 + 1764*m^2 + 720*m)*d^4*e^7*x^7 + 5
040*(m^6 + 15*m^5 + 85*m^4 + 225*m^3 + 274*m^2 + 120*m)*d^5*e^6*x^6 - 30240
*(m^5 + 10*m^4 + 35*m^3 + 50*m^2 + 24*m)*d^6*e^5*x^5 + 151200*(m^4 + 6*m^3
+ 11*m^2 + 6*m)*d^7*e^4*x^4 - 604800*(m^3 + 3*m^2 + 2*m)*d^8*e^3*x^3 + 1814
400*(m^2 + m)*d^9*e^2*x^2 - 3628800*d^10*e*m*x + 3628800*d^11)*(e*x + d)^m/
((m^11 + 66*m^10 + 1925*m^9 + 32670*m^8 + 357423*m^7 + 2637558*m^6 + 133395
35*m^5 + 45995730*m^4 + 105258076*m^3 + 150917976*m^2 + 120543840*m + 39916
800)*e^11)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10965 vs. 2(588) = 1176.

Time = 0.39 (sec) , antiderivative size = 10965, normalized size of antiderivative = 18.65

$$\int (d+ex)^m \left(3+2x+5x^2\right)^3 \left(2+x+3x^2-5x^3+4x^4\right)\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)^m*(5*x^2+2*x+3)^3*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="g
iac")
```

```
[Out] (500*(e*x + d)^m*e^11*m^10*x^11 + 500*(e*x + d)^m*d*e^10*m^10*x^10 - 25*(e*
x + d)^m*e^11*m^10*x^10 + 27500*(e*x + d)^m*e^11*m^9*x^11 - 25*(e*x + d)^m*
d*e^10*m^10*x^9 + 765*(e*x + d)^m*e^11*m^10*x^9 + 22500*(e*x + d)^m*d*e^10*
m^9*x^10 - 1400*(e*x + d)^m*e^11*m^9*x^10 + 660000*(e*x + d)^m*e^11*m^8*x^1
1 + 765*(e*x + d)^m*d*e^10*m^10*x^8 - 98*(e*x + d)^m*e^11*m^10*x^8 - 5000*(
e*x + d)^m*d^2*e^9*m^9*x^9 - 1175*(e*x + d)^m*d*e^10*m^9*x^9 + 43605*(e*x +
```

```
 d)^m*e^11*m^9*x^9 + 435000*(e*x + d)^m*d*e^10*m^8*x^10 - 34125*(e*x + d)^m
*e^11*m^8*x^10 + 9075000*(e*x + d)^m*e^11*m^7*x^11 - 98*(e*x + d)^m*d*e^10*
m^10*x^7 + 999*(e*x + d)^m*e^11*m^10*x^7 + 225*(e*x + d)^m*d^2*e^9*m^9*x^8
+ 37485*(e*x + d)^m*d*e^10*m^9*x^8 - 5684*(e*x + d)^m*e^11*m^9*x^8 - 180000
*(e*x + d)^m*d^2*e^9*m^8*x^9 - 23550*(e*x + d)^m*d*e^10*m^8*x^9 + 1080180*(
e*x + d)^m*e^11*m^8*x^9 + 4725000*(e*x + d)^m*d*e^10*m^7*x^10 - 475500*(e*x
 + d)^m*e^11*m^7*x^10 + 78886500*(e*x + d)^m*e^11*m^6*x^11 + 999*(e*x + d)^
m*d*e^10*m^10*x^6 + 510*(e*x + d)^m*e^11*m^10*x^6 - 6120*(e*x + d)^m*d^2*e^
9*m^9*x^7 - 4998*(e*x + d)^m*d*e^10*m^9*x^7 + 58941*(e*x + d)^m*e^11*m^9*x^
7 + 45000*(e*x + d)^m*d^3*e^8*m^8*x^8 + 8775*(e*x + d)^m*d^2*e^9*m^8*x^8 +
780300*(e*x + d)^m*d*e^10*m^8*x^8 - 143178*(e*x + d)^m*e^11*m^8*x^8 - 27300
00*(e*x + d)^m*d^2*e^9*m^7*x^9 - 263550*(e*x + d)^m*d*e^10*m^7*x^9 + 152709
30*(e*x + d)^m*e^11*m^7*x^9 + 31636500*(e*x + d)^m*d*e^10*m^6*x^10 - 418057
5*(e*x + d)^m*e^11*m^6*x^10 + 451027500*(e*x + d)^m*e^11*m^5*x^11 + 510*(e*
x + d)^m*d*e^10*m^10*x^5 + 1109*(e*x + d)^m*e^11*m^10*x^5 + 686*(e*x + d)^m
*d^2*e^9*m^9*x^6 + 52947*(e*x + d)^m*d*e^10*m^9*x^6 + 30600*(e*x + d)^m*e^1
1*m^9*x^6 - 1800*(e*x + d)^m*d^3*e^8*m^8*x^7 - 257040*(e*x + d)^m*d^2*e^9*m
^8*x^7 - 108192*(e*x + d)^m*d*e^10*m^8*x^7 + 1510488*(e*x + d)^m*e^11*m^8*x
^7 + 1260000*(e*x + d)^m*d^3*e^8*m^7*x^8 + 141750*(e*x + d)^m*d^2*e^9*m^7*x
^8 + 9028530*(e*x + d)^m*d*e^10*m^7*x^8 - 2056236*(e*x + d)^m*e^11*m^7*x^8
- 22680000*(e*x + d)^m*d^2*e^9*m^6*x^9 - 1808625*(e*x + d)^m*d*e^10*m^6*x^9
 + 135990225*(e*x + d)^m*e^11*m^6*x^9 + 134662500*(e*x + d)^m*d*e^10*m^5*x^
10 - 24133200*(e*x + d)^m*e^11*m^5*x^10 + 1708465000*(e*x + d)^m*e^11*m^4*x
^11 + 1109*(e*x + d)^m*d*e^10*m^10*x^4 + 574*(e*x + d)^m*e^11*m^10*x^4 - 59
94*(e*x + d)^m*d^2*e^9*m^9*x^5 + 28050*(e*x + d)^m*d*e^10*m^9*x^5 + 67649*(
e*x + d)^m*e^11*m^9*x^5 + 42840*(e*x + d)^m*d^3*e^8*m^8*x^6 + 30870*(e*x +
d)^m*d^2*e^9*m^8*x^6 + 1192806*(e*x + d)^m*d*e^10*m^8*x^6 + 798150*(e*x + d
)^m*e^11*m^8*x^6 - 360000*(e*x + d)^m*d^4*e^7*m^7*x^7 - 57600*(e*x + d)^m*d
^3*e^8*m^7*x^7 - 4443120*(e*x + d)^m*d^2*e^9*m^7*x^7 - 1298892*(e*x + d)^m*
d*e^10*m^7*x^7 + 22063914*(e*x + d)^m*e^11*m^7*x^7 + 14490000*(e*x + d)^m*d
^3*e^8*m^6*x^8 + 1237950*(e*x + d)^m*d^2*e^9*m^6*x^8 + 63761985*(e*x + d)^m
*d*e^10*m^6*x^8 - 18577566*(e*x + d)^m*e^11*m^6*x^8 - 112245000*(e*x + d)^m
*d^2*e^9*m^5*x^9 - 7855575*(e*x + d)^m*d*e^10*m^5*x^9 + 793819845*(e*x + d)
^m*e^11*m^5*x^9 + 361840000*(e*x + d)^m*d*e^10*m^4*x^10 - 92156375*(e*x + d
)^m*e^11*m^4*x^10 + 4204750000*(e*x + d)^m*e^11*m^3*x^11 + 574*(e*x + d)^m*
d*e^10*m^10*x^3 + 477*(e*x + d)^m*e^11*m^10*x^3 - 2550*(e*x + d)^m*d^2*e^9*
m^9*x^4 + 63213*(e*x + d)^m*d*e^10*m^9*x^4 + 35588*(e*x + d)^m*e^11*m^9*x^4
 - 4116*(e*x + d)^m*d^3*e^8*m^8*x^5 - 287712*(e*x + d)^m*d^2*e^9*m^8*x^5 +
657900*(e*x + d)^m*d*e^10*m^8*x^5 + 1796580*(e*x + d)^m*e^11*m^8*x^5 + 1260
0*(e*x + d)^m*d^4*e^7*m^7*x^6 + 1542240*(e*x + d)^m*d^3*e^8*m^7*x^6 + 57212
4*(e*x + d)^m*d^2*e^9*m^7*x^6 + 14907078*(e*x + d)^m*d*e^10*m^7*x^6 + 11872
800*(e*x + d)^m*e^11*m^7*x^6 - 7560000*(e*x + d)^m*d^4*e^7*m^6*x^7 - 730800
*(e*x + d)^m*d^3*e^8*m^6*x^7 - 41126400*(e*x + d)^m*d^2*e^9*m^6*x^7 - 94853
22*(e*x + d)^m*d*e^10*m^6*x^7 + 202618179*(e*x + d)^m*e^11*m^6*x^7 + 882000
00*(e*x + d)^m*d^3*e^8*m^5*x^8 + 6374025*(e*x + d)^m*d^2*e^9*m^5*x^8 + 2837
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23965*(e*x + d)^m*d*e^10*m^5*x^8 - 109860156*(e*x + d)^m*e^11*m^5*x^8 - 336
420000*(e*x + d)^m*d^2*e^9*m^4*x^9 - 21456200*(e*x + d)^m*d*e^10*m^4*x^9 +
3060365670*(e*x + d)^m*e^11*m^4*x^9 + 586350000*(e*x + d)^m*d*e^10*m^3*x^10
 - 228329500*(e*x + d)^m*e^11*m^3*x^10 + 6376788000*(e*x + d)^m*e^11*m^2*x^
11 + 477*(e*x + d)^m*d*e^10*m^10*x^2 + 135*(e*x + d)^m*e^11*m^10*x^2 - 4436
*(e*x + d)^m*d^2*e^9*m^9*x^3 + 33866*(e*x + d)^m*d*e^10*m^9*x^3 + 30051*(e*
x + d)^m*e^11*m^9*x^3 + 29970*(e*x + d)^m*d^3*e^8*m^8*x^4 - 130050*(e*x + d
)^m*d^2*e^9*m^8*x^4 + 1543728*(e*x + d)^m*d*e^10*m^8*x^4 + 962598*(e*x + d)
^m*e^11*m^8*x^4 - 257040*(e*x + d)^m*d^4*e^7*m^7*x^5 - 164640*(e*x + d)^m*d
^3*e^8*m^7*x^5 - 5718276*(e*x + d)^m*d^2*e^9*m^7*x^5 + 8583300*(e*x + d)^m*
d*e^10*m^7*x^5 + 27248130*(e*x + d)^m*e^11*m^7*x^5 + 2520000*(e*x + d)^m*d^
5*e^6*m^6*x^6 + 327600*(e*x + d)^m*d^4*e^7*m^6*x^6 + 21848400*(e*x + d)^m*d
^3*e^8*m^6*x^6 + 5659500*(e*x + d)^m*d^2*e^9*m^6*x^6 + 113175711*(e*x + d)^
m*d*e^10*m^6*x^6 + 111048930*(e*x + d)^m*e^11*m^6*x^6 - 63000000*(e*x + d)^
m*d^4*e^7*m^5*x^7 - 4788000*(e*x + d)^m*d^3*e^8*m^5*x^7 - 222211080*(e*x +
d)^m*d^2*e^9*m^5*x^7 - 43462902*(e*x + d)^m*d*e^10*m^5*x^7 + 1216593189*(e*
x + d)^m*e^11*m^5*x^7 + 304605000*(e*x + d)^m*d^3*e^8*m^4*x^8 + 19707975*(e
*x + d)^m*d^2*e^9*m^4*x^8 + 790573950*(e*x + d)^m*d*e^10*m^4*x^8 - 42839318
2*(e*x + d)^m*e^11*m^4*x^8 - 590620000*(e*x + d)^m*d^2*e^9*m^3*x^9 - 352237
00*(e*x + d)^m*d*e^10*m^3*x^9 + 7643442420*(e*x + d)^m*e^11*m^3*x^9 + 51328
8000*(e*x + d)^m*d*e^10*m^2*x^10 - 348156900*(e*x + d)^m*e^11*m^2*x^10 + 53
14320000*(e*x + d)^m*e^11*m*x^11 + 135*(e*x + d)^m*d*e^10*m^10*x + 54*(e*x
+ d)^m*e^11*m^10*x - 1722*(e*x + d)^m*d^2*e^9*m^9*x^2 + 29097*(e*x + d)^m*d
*e^10*m^9*x^2 + 8640*(e*x + d)^m*e^11*m^9*x^2 + 10200*(e*x + d)^m*d^3*e^8*m
^8*x^3 - 239544*(e*x + d)^m*d^2*e^9*m^8*x^3 + 861000*(e*x + d)^m*d*e^10*m^8
*x^3 + 828072*(e*x + d)^m*e^11*m^8*x^3 + 20580*(e*x + d)^m*d^4*e^7*m^7*x^4
+ 1318680*(e*x + d)^m*d^3*e^8*m^7*x^4 - 2769300*(e*x + d)^m*d^2*e^9*m^7*x^4
 + 21073218*(e*x + d)^m*d*e^10*m^7*x^4 + 14902188*(e*x + d)^m*e^11*m^7*x^4
- 75600*(e*x + d)^m*d^5*e^6*m^6*x^5 - 7968240*(e*x + d)^m*d^4*e^7*m^6*x^5 -
 2609544*(e*x + d)^m*d^3*e^8*m^6*x^5 - 60851088*(e*x + d)^m*d^2*e^9*m^6*x^5
 + 68132430*(e*x + d)^m*d*e^10*m^6*x^5 + 260141457*(e*x + d)^m*e^11*m^6*x^5
 + 37800000*(e*x + d)^m*d^5*e^6*m^5*x^6 + 3150000*(e*x + d)^m*d^4*e^7*m^5*x
^6 + 156794400*(e*x + d)^m*d^3*e^8*m^5*x^6 + 32440254*(e*x + d)^m*d^2*e^9*m
^5*x^6 + 537538923*(e*x + d)^m*d*e^10*m^5*x^6 + 678861000*(e*x + d)^m*e^11*
m^5*x^6 - 264600000*(e*x + d)^m*d^4*e^7*m^4*x^7 - 17476200*(e*x + d)^m*d^3*
e^8*m^4*x^7 - 714314160*(e*x + d)^m*d^2*e^9*m^4*x^7 - 124152868*(e*x + d)^m
*d*e^10*m^4*x^7 + 4810043142*(e*x + d)^m*e^11*m^4*x^7 + 590940000*(e*x + d)
^m*d^3*e^8*m^3*x^8 + 35442000*(e*x + d)^m*d^2*e^9*m^3*x^8 + 1318850820*(e*x
 + d)^m*d*e^10*m^3*x^8 - 1080436084*(e*x + d)^m*e^11*m^3*x^8 - 547920000*(e
*x + d)^m*d^2*e^9*m^2*x^9 - 31143600*(e*x + d)^m*d*e^10*m^2*x^9 + 117314463
60*(e*x + d)^m*e^11*m^2*x^9 + 181440000*(e*x + d)^m*d*e^10*m*x^10 - 2913804
00*(e*x + d)^m*e^11*m*x^10 + 1814400000*(e*x + d)^m*e^11*x^11 + 54*(e*x + d
)^m*d*e^10*m^10 - 954*(e*x + d)^m*d^2*e^9*m^9*x + 8505*(e*x + d)^m*d*e^10*m
^9*x + 3510*(e*x + d)^m*e^11*m^9*x + 13308*(e*x + d)^m*d^3*e^8*m^8*x^2 - 98
154*(e*x + d)^m*d^2*e^9*m^8*x^2 + 769878*(e*x + d)^m*d*e^10*m^8*x^2 + 24259
```

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5*(e*x + d)^m*e^11*m^8*x^2 - 119880*(e*x + d)^m*d^4*e^7*m^7*x^3 + 489600*(e
*x + d)^m*d^3*e^8*m^7*x^3 - 5456280*(e*x + d)^m*d^2*e^9*m^7*x^3 + 12319188*
(e*x + d)^m*d*e^10*m^7*x^3 + 13099374*(e*x + d)^m*e^11*m^7*x^3 + 1285200*(e
*x + d)^m*d^5*e^6*m^6*x^4 + 740880*(e*x + d)^m*d^4*e^7*m^6*x^4 + 23316660*(
e*x + d)^m*d^3*e^8*m^6*x^4 - 31839300*(e*x + d)^m*d^2*e^9*m^6*x^4 + 1758485
85*(e*x + d)^m*d*e^10*m^6*x^4 + 145552050*(e*x + d)^m*e^11*m^6*x^4 - 151200
00*(e*x + d)^m*d^6*e^5*m^5*x^5 - 1587600*(e*x + d)^m*d^5*e^6*m^5*x^5 - 9124
9200*(e*x + d)^m*d^4*e^7*m^5*x^5 - 20909280*(e*x + d)^m*d^3*e^8*m^5*x^5 - 3
74798826*(e*x + d)^m*d^2*e^9*m^5*x^5 + 338198850*(e*x + d)^m*d*e^10*m^5*x^5
 + 1624344537*(e*x + d)^m*e^11*m^5*x^5 + 214200000*(e*x + d)^m*d^5*e^6*m^4*
x^6 + 14616000*(e*x + d)^m*d^4*e^7*m^4*x^6 + 614711160*(e*x + d)^m*d^3*e^8*
m^4*x^6 + 109598790*(e*x + d)^m*d^2*e^9*m^4*x^6 + 1584809604*(e*x + d)^m*d*
e^10*m^4*x^6 + 2729996850*(e*x + d)^m*e^11*m^4*x^6 - 584640000*(e*x + d)^m*
d^4*e^7*m^3*x^7 - 35330400*(e*x + d)^m*d^3*e^8*m^3*x^7 - 1324392480*(e*x +
d)^m*d^2*e^9*m^3*x^7 - 211366008*(e*x + d)^m*d*e^10*m^3*x^7 + 12279432276*(
e*x + d)^m*e^11*m^3*x^7 + 588060000*(e*x + d)^m*d^3*e^8*m^2*x^8 + 33477300*
(e*x + d)^m*d^2*e^9*m^2*x^8 + 1180639800*(e*x + d)^m*d*e^10*m^2*x^8 - 16718
02776*(e*x + d)^m*e^11*m^2*x^8 - 201600000*(e*x + d)^m*d^2*e^9*m*x^9 - 1108
8000*(e*x + d)^m*d*e^10*m*x^9 + 9869234400*(e*x + d)^m*e^11*m*x^9 - 9979200
0*(e*x + d)^m*e^11*x^10 - 135*(e*x + d)^m*d^2*e^9*m^9 + 3510*(e*x + d)^m*d*
e^10*m^9 + 3444*(e*x + d)^m*d^3*e^8*m^8*x - 57240*(e*x + d)^m*d^2*e^9*m^8*x
 + 234090*(e*x + d)^m*d*e^10*m^8*x + 100440*(e*x + d)^m*e^11*m^8*x - 30600*
(e*x + d)^m*d^4*e^7*m^7*x^2 + 692016*(e*x + d)^m*d^3*e^8*m^7*x^2 - 2386692*
(e*x + d)^m*d^2*e^9*m^7*x^2 + 11559618*(e*x + d)^m*d*e^10*m^7*x^2 + 3925260
*(e*x + d)^m*e^11*m^7*x^2 - 82320*(e*x + d)^m*d^5*e^6*m^6*x^3 - 4915080*(e*
x + d)^m*d^4*e^7*m^6*x^3 + 9608400*(e*x + d)^m*d^3*e^8*m^6*x^3 - 67924032*(
e*x + d)^m*d^2*e^9*m^6*x^3 + 108594486*(e*x + d)^m*d*e^10*m^6*x^3 + 1311926
49*(e*x + d)^m*e^11*m^6*x^3 + 378000*(e*x + d)^m*d^6*e^5*m^5*x^4 + 34700400
*(e*x + d)^m*d^5*e^6*m^5*x^4 + 10084200*(e*x + d)^m*d^4*e^7*m^5*x^4 + 21098
8800*(e*x + d)^m*d^3*e^8*m^5*x^4 - 213304950*(e*x + d)^m*d^2*e^9*m^5*x^4 +
920950197*(e*x + d)^m*d*e^10*m^5*x^4 + 931750092*(e*x + d)^m*e^11*m^5*x^4 -
 151200000*(e*x + d)^m*d^6*e^5*m^4*x^5 - 10962000*(e*x + d)^m*d^5*e^6*m^4*x
^5 - 484520400*(e*x + d)^m*d^4*e^7*m^4*x^5 - 90095124*(e*x + d)^m*d^3*e^8*m
^4*x^5 - 1351239408*(e*x + d)^m*d^2*e^9*m^4*x^5 + 1039002600*(e*x + d)^m*d*
e^10*m^4*x^5 + 6671821630*(e*x + d)^m*e^11*m^4*x^5 + 567000000*(e*x + d)^m*
d^5*e^6*m^3*x^6 + 34637400*(e*x + d)^m*d^4*e^7*m^3*x^6 + 1311932160*(e*x +
d)^m*d^3*e^8*m^3*x^6 + 211477336*(e*x + d)^m*d^2*e^9*m^3*x^6 + 2770574652*(
e*x + d)^m*d*e^10*m^3*x^6 + 7077841200*(e*x + d)^m*e^11*m^3*x^6 - 635040000
*(e*x + d)^m*d^4*e^7*m^2*x^7 - 36223200*(e*x + d)^m*d^3*e^8*m^2*x^7 - 12800
59200*(e*x + d)^m*d^2*e^9*m^2*x^7 - 192240720*(e*x + d)^m*d*e^10*m^2*x^7 +
19196791992*(e*x + d)^m*e^11*m^2*x^7 + 226800000*(e*x + d)^m*d^3*e^8*m*x^8
+ 12474000*(e*x + d)^m*d^2*e^9*m*x^8 + 424116000*(e*x + d)^m*d*e^10*m*x^8 -
 1415539440*(e*x + d)^m*e^11*m*x^8 + 3392928000*(e*x + d)^m*e^11*x^9 + 954*
(e*x + d)^m*d^3*e^8*m^8 - 8505*(e*x + d)^m*d^2*e^9*m^8 + 100440*(e*x + d)^m
*d*e^10*m^8 - 26616*(e*x + d)^m*d^4*e^7*m^7*x + 192864*(e*x + d)^m*d^3*e^8*
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m^7*x - 1482516*(e*x + d)^m*d^2*e^9*m^7*x + 3691170*(e*x + d)^m*d*e^10*m^7*
x + 1663740*(e*x + d)^m*e^11*m^7*x + 359640*(e*x + d)^m*d^5*e^6*m^6*x^2 - 1
407600*(e*x + d)^m*d^4*e^7*m^6*x^2 + 14984808*(e*x + d)^m*d^3*e^8*m^6*x^2 -
 32184180*(e*x + d)^m*d^2*e^9*m^6*x^2 + 108073413*(e*x + d)^m*d*e^10*m^6*x^
2 + 40401585*(e*x + d)^m*e^11*m^6*x^2 - 5140800*(e*x + d)^m*d^6*e^5*m^5*x^3
 - 2716560*(e*x + d)^m*d^5*e^6*m^5*x^3 - 78521400*(e*x + d)^m*d^4*e^7*m^5*x
^3 + 98532000*(e*x + d)^m*d^3*e^8*m^5*x^3 - 499622244*(e*x + d)^m*d^2*e^9*m
^5*x^3 + 605966634*(e*x + d)^m*d*e^10*m^5*x^3 + 864537219*(e*x + d)^m*e^11*
m^5*x^3 + 75600000*(e*x + d)^m*d^7*e^4*m^4*x^4 + 6426000*(e*x + d)^m*d^6*e^
5*m^4*x^4 + 317444400*(e*x + d)^m*d^5*e^6*m^4*x^4 + 64209600*(e*x + d)^m*d^
4*e^7*m^4*x^4 + 1030038930*(e*x + d)^m*d^3*e^8*m^4*x^4 - 837774450*(e*x + d
)^m*d^2*e^9*m^4*x^4 + 2988020842*(e*x + d)^m*d*e^10*m^4*x^4 + 3929892722*(e
*x + d)^m*e^11*m^4*x^4 - 529200000*(e*x + d)^m*d^6*e^5*m^3*x^5 - 32886000*(
e*x + d)^m*d^5*e^6*m^3*x^5 - 1265664960*(e*x + d)^m*d^4*e^7*m^3*x^5 - 20711
7120*(e*x + d)^m*d^3*e^8*m^3*x^5 - 2752660584*(e*x + d)^m*d^2*e^9*m^3*x^5 +
 1882828200*(e*x + d)^m*d*e^10*m^3*x^5 + 17650156420*(e*x + d)^m*e^11*m^3*x
^5 + 690480000*(e*x + d)^m*d^5*e^6*m^2*x^6 + 39488400*(e*x + d)^m*d^4*e^7*m
^2*x^6 + 1399154400*(e*x + d)^m*d^3*e^8*m^2*x^6 + 210698040*(e*x + d)^m*d^2
*e^9*m^2*x^6 + 2573344080*(e*x + d)^m*d*e^10*m^2*x^6 + 11214571560*(e*x + d
)^m*e^11*m^2*x^6 - 259200000*(e*x + d)^m*d^4*e^7*m*x^7 - 14256000*(e*x + d)
^m*d^3*e^8*m*x^7 - 484704000*(e*x + d)^m*d^2*e^9*m*x^7 - 69854400*(e*x + d)
^m*d*e^10*m*x^7 + 16389514080*(e*x + d)^m*e^11*m*x^7 - 488980800*(e*x + d)^
m*e^11*x^8 - 3444*(e*x + d)^m*d^4*e^7*m^7 + 57240*(e*x + d)^m*d^3*e^8*m^7 -
 234090*(e*x + d)^m*d^2*e^9*m^7 + 1663740*(e*x + d)^m*d*e^10*m^7 + 61200*(e
*x + d)^m*d^5*e^6*m^6*x - 1357416*(e*x + d)^m*d^4*e^7*m^6*x + 4580520*(e*x
+ d)^m*d^3*e^8*m^6*x - 21636720*(e*x + d)^m*d^2*e^9*m^6*x + 36710415*(e*x +
 d)^m*d*e^10*m^6*x + 17637102*(e*x + d)^m*e^11*m^6*x + 246960*(e*x + d)^m*d
^6*e^5*m^5*x^2 + 14025960*(e*x + d)^m*d^5*e^6*m^5*x^2 - 26010000*(e*x + d)^
m*d^4*e^7*m^5*x^2 + 173802480*(e*x + d)^m*d^3*e^8*m^5*x^2 - 261415098*(e*x
+ d)^m*d^2*e^9*m^5*x^2 + 648390393*(e*x + d)^m*d*e^10*m^5*x^2 + 275267160*(
e*x + d)^m*e^11*m^5*x^2 - 1512000*(e*x + d)^m*d^7*e^4*m^4*x^3 - 123379200*(
e*x + d)^m*d^6*e^5*m^4*x^3 - 32187120*(e*x + d)^m*d^5*e^6*m^4*x^3 - 6083910
00*(e*x + d)^m*d^4*e^7*m^4*x^3 + 557623800*(e*x + d)^m*d^3*e^8*m^4*x^3 - 21
84934056*(e*x + d)^m*d^2*e^9*m^4*x^3 + 2111992820*(e*x + d)^m*d*e^10*m^4*x^
3 + 3769346538*(e*x + d)^m*e^11*m^4*x^3 + 453600000*(e*x + d)^m*d^7*e^4*m^3
*x^4 + 29106000*(e*x + d)^m*d^6*e^5*m^3*x^4 + 1152824400*(e*x + d)^m*d^5*e^
6*m^3*x^4 + 193637220*(e*x + d)^m*d^4*e^7*m^3*x^4 + 2636041320*(e*x + d)^m*
d^3*e^8*m^3*x^4 - 1843915200*(e*x + d)^m*d^2*e^9*m^3*x^4 + 5698073052*(e*x
+ d)^m*d*e^10*m^3*x^4 + 10681978132*(e*x + d)^m*e^11*m^3*x^4 - 756000000*(e
*x + d)^m*d^6*e^5*m^2*x^5 - 43394400*(e*x + d)^m*d^5*e^6*m^2*x^5 - 15432681
60*(e*x + d)^m*d^4*e^7*m^2*x^5 - 233278416*(e*x + d)^m*d^3*e^8*m^2*x^5 - 28
60144992*(e*x + d)^m*d^2*e^9*m^2*x^5 + 1800430560*(e*x + d)^m*d*e^10*m^2*x^
5 + 28480424184*(e*x + d)^m*e^11*m^2*x^5 + 302400000*(e*x + d)^m*d^5*e^6*m*
x^6 + 16632000*(e*x + d)^m*d^4*e^7*m*x^6 + 565488000*(e*x + d)^m*d^3*e^8*m*
x^6 + 81496800*(e*x + d)^m*d^2*e^9*m*x^6 + 949449600*(e*x + d)^m*d*e^10*m*x
```

```
^6 + 9680738400*(e*x + d)^m*e^11*m*x^6 + 5696697600*(e*x + d)^m*e^11*x^7 +
26616*(e*x + d)^m*d^5*e^6*m^6 - 192864*(e*x + d)^m*d^4*e^7*m^6 + 1482516*(e
*x + d)^m*d^3*e^8*m^6 - 3691170*(e*x + d)^m*d^2*e^9*m^6 + 17637102*(e*x + d
)^m*d*e^10*m^6 - 719280*(e*x + d)^m*d^6*e^5*m^5*x + 2754000*(e*x + d)^m*d^5
*e^6*m^5*x - 28612200*(e*x + d)^m*d^4*e^7*m^5*x + 59787840*(e*x + d)^m*d^3*
e^8*m^5*x - 194510106*(e*x + d)^m*d^2*e^9*m^5*x + 238556745*(e*x + d)^m*d*e
^10*m^5*x + 124791030*(e*x + d)^m*e^11*m^5*x + 15422400*(e*x + d)^m*d^7*e^4
*m^4*x^2 + 7655760*(e*x + d)^m*d^6*e^5*m^4*x^2 + 207512280*(e*x + d)^m*d^5*
e^6*m^4*x^2 - 243576000*(e*x + d)^m*d^4*e^7*m^4*x^2 + 1151261772*(e*x + d)^
m*d^3*e^8*m^4*x^2 - 1295069706*(e*x + d)^m*d^2*e^9*m^4*x^2 + 2472565752*(e*
x + d)^m*d*e^10*m^4*x^2 + 1250302905*(e*x + d)^m*e^11*m^4*x^2 - 302400000*(
e*x + d)^m*d^8*e^3*m^3*x^3 - 21168000*(e*x + d)^m*d^7*e^4*m^3*x^3 - 8996400
00*(e*x + d)^m*d^6*e^5*m^3*x^3 - 160277040*(e*x + d)^m*d^5*e^6*m^3*x^3 - 22
94982720*(e*x + d)^m*d^4*e^7*m^3*x^3 + 1678226400*(e*x + d)^m*d^3*e^8*m^3*x
^3 - 5397281200*(e*x + d)^m*d^2*e^9*m^3*x^3 + 4345999672*(e*x + d)^m*d*e^10
*m^3*x^3 + 10631923596*(e*x + d)^m*e^11*m^3*x^3 + 831600000*(e*x + d)^m*d^7
*e^4*m^2*x^4 + 48006000*(e*x + d)^m*d^6*e^5*m^2*x^4 + 1717027200*(e*x + d)^
m*d^5*e^6*m^2*x^4 + 261036720*(e*x + d)^m*d^4*e^7*m^2*x^4 + 3219137640*(e*x
 + d)^m*d^3*e^8*m^2*x^4 - 2038480200*(e*x + d)^m*d^2*e^9*m^2*x^4 + 56881319
76*(e*x + d)^m*d*e^10*m^2*x^4 + 17690223096*(e*x + d)^m*e^11*m^2*x^4 - 3628
80000*(e*x + d)^m*d^6*e^5*m*x^5 - 19958400*(e*x + d)^m*d^5*e^6*m*x^5 - 6785
85600*(e*x + d)^m*d^4*e^7*m*x^5 - 97796160*(e*x + d)^m*d^3*e^8*m*x^5 - 1139
339520*(e*x + d)^m*d^2*e^9*m*x^5 + 678585600*(e*x + d)^m*d*e^10*m*x^5 + 249
65914464*(e*x + d)^m*e^11*m*x^5 + 3392928000*(e*x + d)^m*e^11*x^6 - 61200*(
e*x + d)^m*d^6*e^5*m^5 + 1357416*(e*x + d)^m*d^5*e^6*m^5 - 4580520*(e*x + d
)^m*d^4*e^7*m^5 + 21636720*(e*x + d)^m*d^3*e^8*m^5 - 36710415*(e*x + d)^m*d
^2*e^9*m^5 + 124791030*(e*x + d)^m*d*e^10*m^5 - 493920*(e*x + d)^m*d^7*e^4*
m^4*x - 27332640*(e*x + d)^m*d^6*e^5*m^4*x + 49266000*(e*x + d)^m*d^5*e^6*m
^4*x - 318992760*(e*x + d)^m*d^4*e^7*m^4*x + 463042356*(e*x + d)^m*d^3*e^8*
m^4*x - 1102270680*(e*x + d)^m*d^2*e^9*m^4*x + 1011746160*(e*x + d)^m*d*e^1
0*m^4*x + 595543860*(e*x + d)^m*e^11*m^4*x + 4536000*(e*x + d)^m*d^8*e^3*m^
3*x^2 + 339292800*(e*x + d)^m*d^7*e^4*m^3*x^2 + 81249840*(e*x + d)^m*d^6*e^
5*m^3*x^2 + 1410148440*(e*x + d)^m*d^5*e^6*m^3*x^2 - 1185719400*(e*x + d)^m
*d^4*e^7*m^3*x^2 + 4252278624*(e*x + d)^m*d^3*e^8*m^3*x^2 - 3745839048*(e*x
 + d)^m*d^2*e^9*m^3*x^2 + 5686792092*(e*x + d)^m*d*e^10*m^3*x^2 + 370881774
0*(e*x + d)^m*e^11*m^3*x^2 - 907200000*(e*x + d)^m*d^8*e^3*m^2*x^3 - 529200
00*(e*x + d)^m*d^7*e^4*m^2*x^3 - 1912377600*(e*x + d)^m*d^6*e^5*m^2*x^3 - 2
93717760*(e*x + d)^m*d^5*e^6*m^2*x^3 - 3659217120*(e*x + d)^m*d^4*e^7*m^2*x
^3 + 2340981600*(e*x + d)^m*d^3*e^8*m^2*x^3 - 6600448608*(e*x + d)^m*d^2*e^
9*m^2*x^3 + 4652224080*(e*x + d)^m*d*e^10*m^2*x^3 + 18312331464*(e*x + d)^m
*e^11*m^2*x^3 + 453600000*(e*x + d)^m*d^7*e^4*m*x^4 + 24948000*(e*x + d)^m*
d^6*e^5*m*x^4 + 848232000*(e*x + d)^m*d^5*e^6*m*x^4 + 122245200*(e*x + d)^m
*d^4*e^7*m*x^4 + 1424174400*(e*x + d)^m*d^3*e^8*m*x^4 - 848232000*(e*x + d)
^m*d^2*e^9*m*x^4 + 2213386560*(e*x + d)^m*d*e^10*m*x^4 + 15866025840*(e*x +
 d)^m*e^11*m*x^4 + 8853546240*(e*x + d)^m*e^11*x^5 + 719280*(e*x + d)^m*d^7
```

```
*e^4*m^4 - 2754000*(e*x + d)^m*d^6*e^5*m^4 + 28612200*(e*x + d)^m*d^5*e^6*m
^4 - 59787840*(e*x + d)^m*d^4*e^7*m^4 + 194510106*(e*x + d)^m*d^3*e^8*m^4 -
 238556745*(e*x + d)^m*d^2*e^9*m^4 + 595543860*(e*x + d)^m*d*e^10*m^4 - 308
44800*(e*x + d)^m*d^8*e^3*m^3*x - 14817600*(e*x + d)^m*d^7*e^4*m^3*x - 3876
91920*(e*x + d)^m*d^6*e^5*m^3*x + 437886000*(e*x + d)^m*d^5*e^6*m^3*x - 198
3530784*(e*x + d)^m*d^4*e^7*m^3*x + 2127097056*(e*x + d)^m*d^3*e^8*m^3*x -
3842860824*(e*x + d)^m*d^2*e^9*m^3*x + 2697071580*(e*x + d)^m*d*e^10*m^3*x
+ 1888225560*(e*x + d)^m*e^11*m^3*x + 907200000*(e*x + d)^m*d^9*e^2*m^2*x^2
 + 54432000*(e*x + d)^m*d^8*e^3*m^2*x^2 + 2020334400*(e*x + d)^m*d^7*e^4*m^
2*x^2 + 318331440*(e*x + d)^m*d^6*e^5*m^2*x^2 + 4064651280*(e*x + d)^m*d^5*
e^6*m^2*x^2 - 2663240400*(e*x + d)^m*d^4*e^7*m^2*x^2 + 7687286352*(e*x + d)
^m*d^3*e^8*m^2*x^2 - 5546320920*(e*x + d)^m*d^2*e^9*m^2*x^2 + 6938747280*(e
*x + d)^m*d*e^10*m^2*x^2 + 6792204780*(e*x + d)^m*e^11*m^2*x^2 - 604800000*
(e*x + d)^m*d^8*e^3*m*x^3 - 33264000*(e*x + d)^m*d^7*e^4*m*x^3 - 1130976000
*(e*x + d)^m*d^6*e^5*m*x^3 - 162993600*(e*x + d)^m*d^5*e^6*m*x^3 - 18988992
00*(e*x + d)^m*d^4*e^7*m*x^3 + 1130976000*(e*x + d)^m*d^3*e^8*m*x^3 - 29511
82080*(e*x + d)^m*d^2*e^9*m*x^3 + 1909353600*(e*x + d)^m*d*e^10*m*x^3 + 170
50880160*(e*x + d)^m*e^11*m*x^3 + 5728060800*(e*x + d)^m*e^11*x^4 + 493920*
(e*x + d)^m*d^8*e^3*m^3 + 27332640*(e*x + d)^m*d^7*e^4*m^3 - 49266000*(e*x
+ d)^m*d^6*e^5*m^3 + 318992760*(e*x + d)^m*d^5*e^6*m^3 - 463042356*(e*x + d
)^m*d^4*e^7*m^3 + 1102270680*(e*x + d)^m*d^3*e^8*m^3 - 1011746160*(e*x + d)
^m*d^2*e^9*m^3 + 1888225560*(e*x + d)^m*d*e^10*m^3 - 9072000*(e*x + d)^m*d^
9*e^2*m^2*x - 647740800*(e*x + d)^m*d^8*e^3*m^2*x - 147682080*(e*x + d)^m*d
^7*e^4*m^2*x - 2432604960*(e*x + d)^m*d^6*e^5*m^2*x + 1933552800*(e*x + d)^
m*d^5*e^6*m^2*x - 6521026464*(e*x + d)^m*d^4*e^7*m^2*x + 5364581040*(e*x +
d)^m*d^3*e^8*m^2*x - 7530723360*(e*x + d)^m*d^2*e^9*m^2*x + 4095133200*(e*x
 + d)^m*d*e^10*m^2*x + 3795710544*(e*x + d)^m*e^11*m^2*x + 907200000*(e*x +
 d)^m*d^9*e^2*m*x^2 + 49896000*(e*x + d)^m*d^8*e^3*m*x^2 + 1696464000*(e*x
+ d)^m*d^7*e^4*m*x^2 + 244490400*(e*x + d)^m*d^6*e^5*m*x^2 + 2848348800*(e*
x + d)^m*d^5*e^6*m*x^2 - 1696464000*(e*x + d)^m*d^4*e^7*m*x^2 + 4426773120*
(e*x + d)^m*d^3*e^8*m*x^2 - 2864030400*(e*x + d)^m*d^2*e^9*m*x^2 + 31733856
00*(e*x + d)^m*d*e^10*m*x^2 + 6789517200*(e*x + d)^m*e^11*m*x^2 + 634677120
0*(e*x + d)^m*e^11*x^3 + 30844800*(e*x + d)^m*d^9*e^2*m^2 + 14817600*(e*x +
 d)^m*d^8*e^3*m^2 + 387691920*(e*x + d)^m*d^7*e^4*m^2 - 437886000*(e*x + d)
^m*d^6*e^5*m^2 + 1983530784*(e*x + d)^m*d^5*e^6*m^2 - 2127097056*(e*x + d)^
m*d^4*e^7*m^2 + 3842860824*(e*x + d)^m*d^3*e^8*m^2 - 2697071580*(e*x + d)^m
*d^2*e^9*m^2 + 3795710544*(e*x + d)^m*d*e^10*m^2 - 1814400000*(e*x + d)^m*d
^10*e*m*x - 99792000*(e*x + d)^m*d^9*e^2*m*x - 3392928000*(e*x + d)^m*d^8*e
^3*m*x - 488980800*(e*x + d)^m*d^7*e^4*m*x - 5696697600*(e*x + d)^m*d^6*e^5
*m*x + 3392928000*(e*x + d)^m*d^5*e^6*m*x - 8853546240*(e*x + d)^m*d^4*e^7*
m*x + 5728060800*(e*x + d)^m*d^3*e^8*m*x - 6346771200*(e*x + d)^m*d^2*e^9*m
*x + 2694384000*(e*x + d)^m*d*e^10*m*x + 4353860160*(e*x + d)^m*e^11*m*x +
2694384000*(e*x + d)^m*e^11*x^2 + 9072000*(e*x + d)^m*d^10*e*m + 647740800*
(e*x + d)^m*d^9*e^2*m + 147682080*(e*x + d)^m*d^8*e^3*m + 2432604960*(e*x +
 d)^m*d^7*e^4*m - 1933552800*(e*x + d)^m*d^6*e^5*m + 6521026464*(e*x + d)^m
```

```
*d^5*e^6*m - 5364581040*(e*x + d)^m*d^4*e^7*m + 7530723360*(e*x + d)^m*d^3*
e^8*m - 4095133200*(e*x + d)^m*d^2*e^9*m + 4353860160*(e*x + d)^m*d*e^10*m
+ 2155507200*(e*x + d)^m*e^11*x + 1814400000*(e*x + d)^m*d^11 + 99792000*(e
*x + d)^m*d^10*e + 3392928000*(e*x + d)^m*d^9*e^2 + 488980800*(e*x + d)^m*d
^8*e^3 + 5696697600*(e*x + d)^m*d^7*e^4 - 3392928000*(e*x + d)^m*d^6*e^5 +
8853546240*(e*x + d)^m*d^5*e^6 - 5728060800*(e*x + d)^m*d^4*e^7 + 634677120
0*(e*x + d)^m*d^3*e^8 - 2694384000*(e*x + d)^m*d^2*e^9 + 2155507200*(e*x +
d)^m*d*e^10)/(e^11*m^11 + 66*e^11*m^10 + 1925*e^11*m^9 + 32670*e^11*m^8 + 3
57423*e^11*m^7 + 2637558*e^11*m^6 + 13339535*e^11*m^5 + 45995730*e^11*m^4 +
 105258076*e^11*m^3 + 150917976*e^11*m^2 + 120543840*e^11*m + 39916800*e^11
)
```

Mupad [B] (verification not implemented)

Time = 18.10 (sec) , antiderivative size = 4341, normalized size of antiderivative = 7.38

$$\int (d+ex)^m \left(3+2x+5x^2\right)^3 \left(2+x+3x^2-5x^3+4x^4\right)\,dx = \text{Too large to display}$$

```
[In] int((d + e*x)^m*(2*x + 5*x^2 + 3)^3*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2),x)
```

```
[Out] (500*x^11*(d + e*x)^m*(10628640*m + 12753576*m^2 + 8409500*m^3 + 3416930*m^
4 + 902055*m^5 + 157773*m^6 + 18150*m^7 + 1320*m^8 + 55*m^9 + m^10 + 362880
0))/(120543840*m + 150917976*m^2 + 105258076*m^3 + 45995730*m^4 + 13339535*
m^5 + 2637558*m^6 + 357423*m^7 + 32670*m^8 + 1925*m^9 + 66*m^10 + m^11 + 39
916800) + ((d + e*x)^m*(2155507200*d*e^10 + 99792000*d^10*e + 1814400000*d^
11 - 2694384000*d^2*e^9 + 6346771200*d^3*e^8 - 5728060800*d^4*e^7 + 8853546
240*d^5*e^6 - 3392928000*d^6*e^5 + 5696697600*d^7*e^4 + 488980800*d^8*e^3 +
 3392928000*d^9*e^2 - 4095133200*d^2*e^9*m + 7530723360*d^3*e^8*m - 5364581
040*d^4*e^7*m + 6521026464*d^5*e^6*m - 1933552800*d^6*e^5*m + 2432604960*d^
7*e^4*m + 147682080*d^8*e^3*m + 647740800*d^9*e^2*m + 3795710544*d*e^10*m^2
 + 1888225560*d*e^10*m^3 + 595543860*d*e^10*m^4 + 124791030*d*e^10*m^5 + 17
637102*d*e^10*m^6 + 1663740*d*e^10*m^7 + 100440*d*e^10*m^8 + 3510*d*e^10*m^
9 + 54*d*e^10*m^10 - 2697071580*d^2*e^9*m^2 + 3842860824*d^3*e^8*m^2 - 2127
097056*d^4*e^7*m^2 + 1983530784*d^5*e^6*m^2 - 437886000*d^6*e^5*m^2 + 38769
1920*d^7*e^4*m^2 + 14817600*d^8*e^3*m^2 + 30844800*d^9*e^2*m^2 - 1011746160
*d^2*e^9*m^3 + 1102270680*d^3*e^8*m^3 - 463042356*d^4*e^7*m^3 + 318992760*d
^5*e^6*m^3 - 49266000*d^6*e^5*m^3 + 27332640*d^7*e^4*m^3 + 493920*d^8*e^3*m
^3 - 238556745*d^2*e^9*m^4 + 194510106*d^3*e^8*m^4 - 59787840*d^4*e^7*m^4 +
 28612200*d^5*e^6*m^4 - 2754000*d^6*e^5*m^4 + 719280*d^7*e^4*m^4 - 36710415
*d^2*e^9*m^5 + 21636720*d^3*e^8*m^5 - 4580520*d^4*e^7*m^5 + 1357416*d^5*e^6
*m^5 - 61200*d^6*e^5*m^5 - 3691170*d^2*e^9*m^6 + 1482516*d^3*e^8*m^6 - 1928
64*d^4*e^7*m^6 + 26616*d^5*e^6*m^6 - 234090*d^2*e^9*m^7 + 57240*d^3*e^8*m^7
 - 3444*d^4*e^7*m^7 - 8505*d^2*e^9*m^8 + 954*d^3*e^8*m^8 - 135*d^2*e^9*m^9
+ 4353860160*d*e^10*m + 9072000*d^10*e*m))/(e^11*(120543840*m + 150917976*m
```

```
^2 + 105258076*m^3 + 45995730*m^4 + 13339535*m^5 + 2637558*m^6 + 357423*m^7
 + 32670*m^8 + 1925*m^9 + 66*m^10 + m^11 + 39916800)) + (x*(d + e*x)^m*(435
3860160*e^11*m + 2155507200*e^11 + 3795710544*e^11*m^2 + 1888225560*e^11*m^
3 + 595543860*e^11*m^4 + 124791030*e^11*m^5 + 17637102*e^11*m^6 + 1663740*e
^11*m^7 + 100440*e^11*m^8 + 3510*e^11*m^9 + 54*e^11*m^10 - 6346771200*d^2*e
^9*m + 5728060800*d^3*e^8*m - 8853546240*d^4*e^7*m + 3392928000*d^5*e^6*m -
 5696697600*d^6*e^5*m - 488980800*d^7*e^4*m - 3392928000*d^8*e^3*m - 997920
00*d^9*e^2*m + 4095133200*d*e^10*m^2 + 2697071580*d*e^10*m^3 + 1011746160*d
*e^10*m^4 + 238556745*d*e^10*m^5 + 36710415*d*e^10*m^6 + 3691170*d*e^10*m^7
 + 234090*d*e^10*m^8 + 8505*d*e^10*m^9 + 135*d*e^10*m^10 - 7530723360*d^2*e
^9*m^2 + 5364581040*d^3*e^8*m^2 - 6521026464*d^4*e^7*m^2 + 1933552800*d^5*e
^6*m^2 - 2432604960*d^6*e^5*m^2 - 147682080*d^7*e^4*m^2 - 647740800*d^8*e^3
*m^2 - 9072000*d^9*e^2*m^2 - 3842860824*d^2*e^9*m^3 + 2127097056*d^3*e^8*m^
3 - 1983530784*d^4*e^7*m^3 + 437886000*d^5*e^6*m^3 - 387691920*d^6*e^5*m^3
- 14817600*d^7*e^4*m^3 - 30844800*d^8*e^3*m^3 - 1102270680*d^2*e^9*m^4 + 46
3042356*d^3*e^8*m^4 - 318992760*d^4*e^7*m^4 + 49266000*d^5*e^6*m^4 - 273326
40*d^6*e^5*m^4 - 493920*d^7*e^4*m^4 - 194510106*d^2*e^9*m^5 + 59787840*d^3*
e^8*m^5 - 28612200*d^4*e^7*m^5 + 2754000*d^5*e^6*m^5 - 719280*d^6*e^5*m^5 -
 21636720*d^2*e^9*m^6 + 4580520*d^3*e^8*m^6 - 1357416*d^4*e^7*m^6 + 61200*d
^5*e^6*m^6 - 1482516*d^2*e^9*m^7 + 192864*d^3*e^8*m^7 - 26616*d^4*e^7*m^7 -
 57240*d^2*e^9*m^8 + 3444*d^3*e^8*m^8 - 954*d^2*e^9*m^9 + 2694384000*d*e^10
*m - 1814400000*d^10*e*m))/(e^11*(120543840*m + 150917976*m^2 + 105258076*m
^3 + 45995730*m^4 + 13339535*m^5 + 2637558*m^6 + 357423*m^7 + 32670*m^8 + 1
925*m^9 + 66*m^10 + m^11 + 39916800)) + (x^8*(d + e*x)^m*(13068*m + 13132*m
^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040)*(45000*d^3*m - 29
302*e^3*m - 97020*e^3 - 2940*e^3*m^2 - 98*e^3*m^3 + 16065*d*e^2*m^2 + 225*d
^2*e*m^2 + 765*d*e^2*m^3 + 84150*d*e^2*m + 2475*d^2*e*m))/(e^3*(120543840*m
 + 150917976*m^2 + 105258076*m^3 + 45995730*m^4 + 13339535*m^5 + 2637558*m^
6 + 357423*m^7 + 32670*m^8 + 1925*m^9 + 66*m^10 + m^11 + 39916800)) + (3*x^
2*(m + 1)*(d + e*x)^m*(302400000*d^9*m + 1365044400*e^9*m + 898128000*e^9 +
 899023860*e^9*m^2 + 337248720*e^9*m^3 + 79518915*e^9*m^4 + 12236805*e^9*m^
5 + 1230390*e^9*m^6 + 78030*e^9*m^7 + 2835*e^9*m^8 + 45*e^9*m^9 - 954676800
*d^2*e^7*m + 1475591040*d^3*e^6*m - 565488000*d^4*e^5*m + 949449600*d^5*e^4
*m + 81496800*d^6*e^3*m + 565488000*d^7*e^2*m + 1255120560*d*e^8*m^2 + 1512
000*d^8*e*m^2 + 640476804*d*e^8*m^3 + 183711780*d*e^8*m^4 + 32418351*d*e^8*
m^5 + 3606120*d*e^8*m^6 + 247086*d*e^8*m^7 + 9540*d*e^8*m^8 + 159*d*e^8*m^9
 - 894096840*d^2*e^7*m^2 + 1086837744*d^3*e^6*m^2 - 322258800*d^4*e^5*m^2 +
 405434160*d^5*e^4*m^2 + 24613680*d^6*e^3*m^2 + 107956800*d^7*e^2*m^2 - 354
516176*d^2*e^7*m^3 + 330588464*d^3*e^6*m^3 - 72981000*d^4*e^5*m^3 + 6461532
0*d^5*e^4*m^3 + 2469600*d^6*e^3*m^3 + 5140800*d^7*e^2*m^3 - 77173726*d^2*e^
7*m^4 + 53165460*d^3*e^6*m^4 - 8211000*d^4*e^5*m^4 + 4555440*d^5*e^4*m^4 +
82320*d^6*e^3*m^4 - 9964640*d^2*e^7*m^5 + 4768700*d^3*e^6*m^5 - 459000*d^4*
e^5*m^5 + 119880*d^5*e^4*m^5 - 763420*d^2*e^7*m^6 + 226236*d^3*e^6*m^6 - 10
200*d^4*e^5*m^6 - 32144*d^2*e^7*m^7 + 4436*d^3*e^6*m^7 - 574*d^2*e^7*m^8 +
1057795200*d*e^8*m + 16632000*d^8*e*m))/(e^9*(120543840*m + 150917976*m^2 +
```

```
 105258076*m^3 + 45995730*m^4 + 13339535*m^5 + 2637558*m^6 + 357423*m^7 + 3
2670*m^8 + 1925*m^9 + 66*m^10 + m^11 + 39916800)) + (x^6*(d + e*x)^m*(274*m
 + 225*m^2 + 85*m^3 + 15*m^4 + m^5 + 120)*(2520000*d^5*m + 16112940*e^5*m +
 28274400*e^5 + 3649050*e^5*m^2 + 410550*e^5*m^3 + 22950*e^5*m^4 + 510*e^5*
m^5 + 679140*d^2*e^3*m + 4712400*d^3*e^2*m + 3378618*d*e^4*m^2 + 12600*d^4*
e*m^2 + 538461*d*e^4*m^3 + 37962*d*e^4*m^4 + 999*d*e^4*m^5 + 205114*d^2*e^3
*m^2 + 899640*d^3*e^2*m^2 + 20580*d^2*e^3*m^3 + 42840*d^3*e^2*m^3 + 686*d^2
*e^3*m^4 + 7912080*d*e^4*m + 138600*d^4*e*m))/(e^5*(120543840*m + 150917976
*m^2 + 105258076*m^3 + 45995730*m^4 + 13339535*m^5 + 2637558*m^6 + 357423*m
^7 + 32670*m^8 + 1925*m^9 + 66*m^10 + m^11 + 39916800)) + (x^3*(d + e*x)^m*
(3*m + m^2 + 2)*(3765361680*e^8*m - 302400000*d^8*m + 3173385600*e^8 + 1921
430412*e^8*m^2 + 551135340*e^8*m^3 + 97255053*e^8*m^4 + 10818360*e^8*m^5 +
741258*e^8*m^6 + 28620*e^8*m^7 + 477*e^8*m^8 - 1475591040*d^2*e^6*m + 56548
8000*d^3*e^5*m - 949449600*d^4*e^4*m - 81496800*d^5*e^3*m - 565488000*d^6*e
^2*m + 894096840*d*e^7*m^2 - 1512000*d^7*e*m^2 + 354516176*d*e^7*m^3 + 7717
3726*d*e^7*m^4 + 9964640*d*e^7*m^5 + 763420*d*e^7*m^6 + 32144*d*e^7*m^7 + 5
74*d*e^7*m^8 - 1086837744*d^2*e^6*m^2 + 322258800*d^3*e^5*m^2 - 405434160*d
^4*e^4*m^2 - 24613680*d^5*e^3*m^2 - 107956800*d^6*e^2*m^2 - 330588464*d^2*e
^6*m^3 + 72981000*d^3*e^5*m^3 - 64615320*d^4*e^4*m^3 - 2469600*d^5*e^3*m^3
- 5140800*d^6*e^2*m^3 - 53165460*d^2*e^6*m^4 + 8211000*d^3*e^5*m^4 - 455544
0*d^4*e^4*m^4 - 82320*d^5*e^3*m^4 - 4768700*d^2*e^6*m^5 + 459000*d^3*e^5*m^
5 - 119880*d^4*e^4*m^5 - 226236*d^2*e^6*m^6 + 10200*d^3*e^5*m^6 - 4436*d^2*
e^6*m^7 + 954676800*d*e^7*m - 16632000*d^7*e*m))/(e^8*(120543840*m + 150917
976*m^2 + 105258076*m^3 + 45995730*m^4 + 13339535*m^5 + 2637558*m^6 + 35742
3*m^7 + 32670*m^8 + 1925*m^9 + 66*m^10 + m^11 + 39916800)) + (x^4*(d + e*x)
^m*(11*m + 6*m^2 + m^3 + 6)*(75600000*d^7*m + 894096840*e^7*m + 954676800*e
^7 + 354516176*e^7*m^2 + 77173726*e^7*m^3 + 9964640*e^7*m^4 + 763420*e^7*m^
5 + 32144*e^7*m^6 + 574*e^7*m^7 - 141372000*d^2*e^5*m + 237362400*d^3*e^4*m
 + 20374200*d^4*e^3*m + 141372000*d^5*e^2*m + 271709436*d*e^6*m^2 + 378000*
d^6*e*m^2 + 82647116*d*e^6*m^3 + 13291365*d*e^6*m^4 + 1192175*d*e^6*m^5 + 5
6559*d*e^6*m^6 + 1109*d*e^6*m^7 - 80564700*d^2*e^5*m^2 + 101358540*d^3*e^4*
m^2 + 6153420*d^4*e^3*m^2 + 26989200*d^5*e^2*m^2 - 18245250*d^2*e^5*m^3 + 1
6153830*d^3*e^4*m^3 + 617400*d^4*e^3*m^3 + 1285200*d^5*e^2*m^3 - 2052750*d^
2*e^5*m^4 + 1138860*d^3*e^4*m^4 + 20580*d^4*e^3*m^4 - 114750*d^2*e^5*m^5 +
29970*d^3*e^4*m^5 - 2550*d^2*e^5*m^6 + 368897760*d*e^6*m + 4158000*d^6*e*m)
)/(e^7*(120543840*m + 150917976*m^2 + 105258076*m^3 + 45995730*m^4 + 133395
35*m^5 + 2637558*m^6 + 357423*m^7 + 32670*m^8 + 1925*m^9 + 66*m^10 + m^11 +
 39916800)) - (25*x^10*(d + e*x)^m*(11*e - 20*d*m + e*m)*(1026576*m + 11727
00*m^2 + 723680*m^3 + 269325*m^4 + 63273*m^5 + 9450*m^6 + 870*m^7 + 45*m^8
+ m^9 + 362880))/(e*(120543840*m + 150917976*m^2 + 105258076*m^3 + 45995730
*m^4 + 13339535*m^5 + 2637558*m^6 + 357423*m^7 + 32670*m^8 + 1925*m^9 + 66*
m^10 + m^11 + 39916800)) - (x^5*(d + e*x)^m*(50*m + 35*m^2 + 10*m^3 + m^4 +
 24)*(15120000*d^6*m - 271709436*e^6*m - 368897760*e^6 - 82647116*e^6*m^2 -
 13291365*e^6*m^3 - 1192175*e^6*m^4 - 56559*e^6*m^5 - 1109*e^6*m^6 + 474724
80*d^2*e^4*m + 4074840*d^3*e^3*m + 28274400*d^4*e^2*m - 16112940*d*e^5*m^2
```

```
+ 75600*d^5*e*m^2 - 3649050*d*e^5*m^3 - 410550*d*e^5*m^4 - 22950*d*e^5*m^5
- 510*d*e^5*m^6 + 20271708*d^2*e^4*m^2 + 1230684*d^3*e^3*m^2 + 5397840*d^4*
e^2*m^2 + 3230766*d^2*e^4*m^3 + 123480*d^3*e^3*m^3 + 257040*d^4*e^2*m^3 + 2
27772*d^2*e^4*m^4 + 4116*d^3*e^3*m^4 + 5994*d^2*e^4*m^5 - 28274400*d*e^5*m
+ 831600*d^5*e*m))/(e^6*(120543840*m + 150917976*m^2 + 105258076*m^3 + 4599
5730*m^4 + 13339535*m^5 + 2637558*m^6 + 357423*m^7 + 32670*m^8 + 1925*m^9 +
 66*m^10 + m^11 + 39916800)) - (x^7*(d + e*x)^m*(1764*m + 1624*m^2 + 735*m^
3 + 175*m^4 + 21*m^5 + m^6 + 720)*(360000*d^4*m - 3378618*e^4*m - 7912080*e
^4 - 538461*e^4*m^2 - 37962*e^4*m^3 - 999*e^4*m^4 + 673200*d^2*e^2*m + 2930
2*d*e^3*m^2 + 1800*d^3*e*m^2 + 2940*d*e^3*m^3 + 98*d*e^3*m^4 + 128520*d^2*e
^2*m^2 + 6120*d^2*e^2*m^3 + 97020*d*e^3*m + 19800*d^3*e*m))/(e^4*(120543840
*m + 150917976*m^2 + 105258076*m^3 + 45995730*m^4 + 13339535*m^5 + 2637558*
m^6 + 357423*m^7 + 32670*m^8 + 1925*m^9 + 66*m^10 + m^11 + 39916800)) - (5*
x^9*(d + e*x)^m*(1000*d^2*m - 3213*e^2*m - 16830*e^2 - 153*e^2*m^2 + 55*d*e
*m + 5*d*e*m^2)*(109584*m + 118124*m^2 + 67284*m^3 + 22449*m^4 + 4536*m^5 +
 546*m^6 + 36*m^7 + m^8 + 40320))/(e^2*(120543840*m + 150917976*m^2 + 10525
8076*m^3 + 45995730*m^4 + 13339535*m^5 + 2637558*m^6 + 357423*m^7 + 32670*m
^8 + 1925*m^9 + 66*m^10 + m^11 + 39916800))
```

3.368 $\int (d+ex)^m \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right.$

Optimal result

Integrand size = 38, antiderivative size = 432

$$
\begin{aligned}
&\int (d+ex)^m \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right) dx \\
&= \frac{\left(5d^2-2de+3e^2\right)^2 \left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right) (d+ex)^{1+m}}{e^9(1+m)} \\
&\quad - \frac{\left(5d^2-2de+3e^2\right) \left(160d^5+127d^4e+88d^3e^2-4d^2e^3+64de^4-11e^5\right) (d+ex)^{2+m}}{e^9(2+m)} \\
&\quad + \frac{\left(2800d^6+945d^5e+1665d^4e^2+370d^3e^3+888d^2e^4-195de^5+107e^6\right) (d+ex)^{3+m}}{e^9(3+m)} \\
&\quad - \frac{\left(5600d^5+1575d^4e+2220d^3e^2+370d^2e^3+592de^4-65e^5\right) (d+ex)^{4+m}}{e^9(4+m)} \\
&\quad + \frac{\left(7000d^4+1575d^3e+1665d^2e^2+185de^3+148e^4\right) (d+ex)^{5+m}}{e^9(5+m)} \\
&\quad - \frac{\left(5600d^3+945d^2e+666de^2+37e^3\right) (d+ex)^{6+m}}{e^9(6+m)} \\
&\quad + \frac{\left(2800d^2+315de+111e^2\right) (d+ex)^{7+m}}{e^9(7+m)} - \frac{5(160d+9e)(d+ex)^{8+m}}{e^9(8+m)} + \frac{100(d+ex)^{9+m}}{e^9(9+m)}
\end{aligned}
$$

```
[Out] (5*d^2-2*d*e+3*e^2)^2*(4*d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)*(e*x+d)^(1+m)/e
^9/(1+m)-(5*d^2-2*d*e+3*e^2)*(160*d^5+127*d^4*e+88*d^3*e^2-4*d^2*e^3+64*d*e
^4-11*e^5)*(e*x+d)^(2+m)/e^9/(2+m)+(2800*d^6+945*d^5*e+1665*d^4*e^2+370*d^3
*e^3+888*d^2*e^4-195*d*e^5+107*e^6)*(e*x+d)^(3+m)/e^9/(3+m)-(5600*d^5+1575*
d^4*e+2220*d^3*e^2+370*d^2*e^3+592*d*e^4-65*e^5)*(e*x+d)^(4+m)/e^9/(4+m)+(7
000*d^4+1575*d^3*e+1665*d^2*e^2+185*d*e^3+148*e^4)*(e*x+d)^(5+m)/e^9/(5+m)-
(5600*d^3+945*d^2*e+666*d*e^2+37*e^3)*(e*x+d)^(6+m)/e^9/(6+m)+(2800*d^2+315
*d*e+111*e^2)*(e*x+d)^(7+m)/e^9/(7+m)-5*(160*d+9*e)*(e*x+d)^(8+m)/e^9/(8+m)
+100*(e*x+d)^(9+m)/e^9/(9+m)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 432, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$, Rules used = {1642}

$$\begin{aligned}
&\int (d+ex)^m \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right) \, dx \\
&=\frac{\left(2800d^2+315de+111e^2\right)(d+ex)^{m+7}}{e^9(m+7)} \\
&\quad-\frac{\left(5600d^3+945d^2e+666de^2+37e^3\right)(d+ex)^{m+6}}{e^9(m+6)} \\
&\quad+\frac{\left(5d^2-2de+3e^2\right)^2\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)(d+ex)^{m+1}}{e^9(m+1)} \\
&\quad+\frac{\left(7000d^4+1575d^3e+1665d^2e^2+185de^3+148e^4\right)(d+ex)^{m+5}}{e^9(m+5)} \\
&\quad-\frac{\left(5d^2-2de+3e^2\right)\left(160d^5+127d^4e+88d^3e^2-4d^2e^3+64de^4-11e^5\right)(d+ex)^{m+2}}{e^9(m+2)} \\
&\quad-\frac{\left(5600d^5+1575d^4e+2220d^3e^2+370d^2e^3+592de^4-65e^5\right)(d+ex)^{m+4}}{e^9(m+4)} \\
&\quad+\frac{\left(2800d^6+945d^5e+1665d^4e^2+370d^3e^3+888d^2e^4-195de^5+107e^6\right)(d+ex)^{m+3}}{e^9(m+3)} \\
&\quad-\frac{5(160d+9e)(d+ex)^{m+8}}{e^9(m+8)}+\frac{100(d+ex)^{m+9}}{e^9(m+9)}
\end{aligned}$$

```
[In] Int[(d + e*x)^m*(3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

```
[Out] ((5*d^2 - 2*d*e + 3*e^2)^2*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*(d
 + e*x)^(1 + m))/(e^9*(1 + m)) - ((5*d^2 - 2*d*e + 3*e^2)*(160*d^5 + 127*d^
4*e + 88*d^3*e^2 - 4*d^2*e^3 + 64*d*e^4 - 11*e^5)*(d + e*x)^(2 + m))/(e^9*(
2 + m)) + ((2800*d^6 + 945*d^5*e + 1665*d^4*e^2 + 370*d^3*e^3 + 888*d^2*e^4
 - 195*d*e^5 + 107*e^6)*(d + e*x)^(3 + m))/(e^9*(3 + m)) - ((5600*d^5 + 157
5*d^4*e + 2220*d^3*e^2 + 370*d^2*e^3 + 592*d*e^4 - 65*e^5)*(d + e*x)^(4 + m
))/(e^9*(4 + m)) + ((7000*d^4 + 1575*d^3*e + 1665*d^2*e^2 + 185*d*e^3 + 148
*e^4)*(d + e*x)^(5 + m))/(e^9*(5 + m)) - ((5600*d^3 + 945*d^2*e + 666*d*e^2
 + 37*e^3)*(d + e*x)^(6 + m))/(e^9*(6 + m)) + ((2800*d^2 + 315*d*e + 111*e^
2)*(d + e*x)^(7 + m))/(e^9*(7 + m)) - (5*(160*d + 9*e)*(d + e*x)^(8 + m))/(
e^9*(8 + m)) + (100*(d + e*x)^(9 + m))/(e^9*(9 + m))
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{(5d^2-2de+3e^2)^2\,(4d^4+5d^3e+3d^2e^2-de^3+2e^4)\,(d+ex)^m}{e^8} \right. \\
&+ \frac{(-800d^7-315d^6e-666d^5e^2-185d^4e^3-592d^3e^4+195d^2e^5-214de^6+33e^7)\,(d+ex)^{1+m}}{e^8} \\
&+ \frac{(2800d^6+945d^5e+1665d^4e^2+370d^3e^3+888d^2e^4-195de^5+107e^6)\,(d+ex)^{2+m}}{e^8} \\
&+ \frac{(-5600d^5-1575d^4e-2220d^3e^2-370d^2e^3-592de^4+65e^5)\,(d+ex)^{3+m}}{e^8} \\
&+ \frac{(7000d^4+1575d^3e+1665d^2e^2+185de^3+148e^4)\,(d+ex)^{4+m}}{e^8} \\
&+ \frac{(-5600d^3-945d^2e-666de^2-37e^3)\,(d+ex)^{5+m}}{e^8} \\
&+ \frac{(2800d^2+315de+111e^2)\,(d+ex)^{6+m}}{e^8} - \frac{5(160d+9e)(d+ex)^{7+m}}{e^8} \\
&\left. + \frac{100(d+ex)^{8+m}}{e^8} \right) dx \\
&= \frac{(5d^2-2de+3e^2)^2\,(4d^4+5d^3e+3d^2e^2-de^3+2e^4)\,(d+ex)^{1+m}}{e^9(1+m)} \\
&- \frac{(5d^2-2de+3e^2)\,(160d^5+127d^4e+88d^3e^2-4d^2e^3+64de^4-11e^5)\,(d+ex)^{2+m}}{e^9(2+m)} \\
&+ \frac{(2800d^6+945d^5e+1665d^4e^2+370d^3e^3+888d^2e^4-195de^5+107e^6)\,(d+ex)^{3+m}}{e^9(3+m)} \\
&- \frac{(5600d^5+1575d^4e+2220d^3e^2+370d^2e^3+592de^4-65e^5)\,(d+ex)^{4+m}}{e^9(4+m)} \\
&+ \frac{(7000d^4+1575d^3e+1665d^2e^2+185de^3+148e^4)\,(d+ex)^{5+m}}{e^9(5+m)} \\
&- \frac{(5600d^3+945d^2e+666de^2+37e^3)\,(d+ex)^{6+m}}{e^9(6+m)} \\
&+ \frac{(2800d^2+315de+111e^2)\,(d+ex)^{7+m}}{e^9(7+m)} \\
&- \frac{5(160d+9e)(d+ex)^{8+m}}{e^9(8+m)} + \frac{100(d+ex)^{9+m}}{e^9(9+m)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 391, normalized size of antiderivative = 0.91

$$\int (d+ex)^m \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right)\,dx$$

$$= \frac{(d+ex)^{1+m}\left(\frac{(5d^2-2de+3e^2)^2(4d^4+5d^3e+3d^2e^2-de^3+2e^4)}{1+m} - \frac{(5d^2-2de+3e^2)(160d^5+127d^4e+88d^3e^2-4d^2e^3+64de^4-11e^5)(d+ex)}{2+m} + \right.}{}$$

```
[In] Integrate[(d + e*x)^m*(3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x
]
```

```
[Out] ((d + e*x)^(1 + m)*(((5*d^2 - 2*d*e + 3*e^2)^2*(4*d^4 + 5*d^3*e + 3*d^2*e^2
 - d*e^3 + 2*e^4))/(1 + m) - ((5*d^2 - 2*d*e + 3*e^2)*(160*d^5 + 127*d^4*e
+ 88*d^3*e^2 - 4*d^2*e^3 + 64*d*e^4 - 11*e^5)*(d + e*x))/(2 + m) + ((2800*d
^6 + 945*d^5*e + 1665*d^4*e^2 + 370*d^3*e^3 + 888*d^2*e^4 - 195*d*e^5 + 107
*e^6)*(d + e*x)^2)/(3 + m) - ((5600*d^5 + 1575*d^4*e + 2220*d^3*e^2 + 370*d
^2*e^3 + 592*d*e^4 - 65*e^5)*(d + e*x)^3)/(4 + m) + ((7000*d^4 + 1575*d^3*e
 + 1665*d^2*e^2 + 185*d*e^3 + 148*e^4)*(d + e*x)^4)/(5 + m) - ((5600*d^3 +
945*d^2*e + 666*d*e^2 + 37*e^3)*(d + e*x)^5)/(6 + m) + ((2800*d^2 + 315*d*e
 + 111*e^2)*(d + e*x)^6)/(7 + m) - (5*(160*d + 9*e)*(d + e*x)^7)/(8 + m) +
(100*(d + e*x)^8)/(9 + m)))/e^9
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3221 vs. 2(432) = 864.

Time = 0.39 (sec) , antiderivative size = 3222, normalized size of antiderivative = 7.46

method	result	size
gosper	Expression too large to display	3222
risch	Expression too large to display	3895
parallelrisch	Expression too large to display	6428

```
[In] int((e*x+d)^m*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x,method=_RETURNVERBO
SE)
```

```
[Out] 1/e^9*(e*x+d)^(1+m)/(m^9+45*m^8+870*m^7+9450*m^6+63273*m^5+269325*m^4+72368
0*m^3+1172700*m^2+1026576*m+362880)*(100*e^8*m^8*x^8-45*e^8*m^8*x^7+3600*e^
8*m^7*x^8-800*d*e^7*m^7*x^7+111*e^8*m^8*x^6-1665*e^8*m^7*x^7+54600*e^8*m^6*
x^8+315*d*e^7*m^7*x^6-22400*d*e^7*m^6*x^7-37*e^8*m^8*x^5+4218*e^8*m^7*x^6-2
5830*e^8*m^6*x^7+453600*e^8*m^5*x^8+5600*d^2*e^6*m^6*x^6-666*d*e^7*m^7*x^5+
9450*d*e^7*m^6*x^6-257600*d*e^7*m^5*x^7+148*e^8*m^8*x^4-1443*e^8*m^7*x^5+67
044*e^8*m^6*x^6-218610*e^8*m^5*x^7+2244900*e^8*m^4*x^8-1890*d^2*e^6*m^6*x^5
+117600*d^2*e^6*m^5*x^6+185*d*e^7*m^7*x^4-21312*d*e^7*m^6*x^5+114660*d*e^7*
```

```
m^5*x^6-1568000*d*e^7*m^4*x^7+65*e^8*m^8*x^3+5920*e^8*m^7*x^4-23532*e^8*m^6
*x^5+579642*e^8*m^5*x^6-1098405*e^8*m^4*x^7+6728400*e^8*m^3*x^8-33600*d^3*e
^5*m^5*x^5+3330*d^2*e^6*m^6*x^4-45360*d^2*e^6*m^5*x^5+980000*d^2*e^6*m^4*x^
6-592*d*e^7*m^7*x^3+6290*d*e^7*m^6*x^4-274392*d*e^7*m^5*x^5+727650*d*e^7*m^
4*x^6-5415200*d*e^7*m^3*x^7+107*e^8*m^8*x^2+2665*e^8*m^7*x^3+99160*e^8*m^6*
x^4-208458*e^8*m^5*x^5+2965809*e^8*m^4*x^6-3332385*e^8*m^3*x^7+11812400*e^8
*m^2*x^8+9450*d^3*e^5*m^5*x^4-504000*d^3*e^5*m^4*x^5-740*d^2*e^6*m^6*x^3+89
910*d^2*e^6*m^5*x^4-415800*d^2*e^6*m^4*x^5+4116000*d^2*e^6*m^3*x^6-195*d*e^
7*m^7*x^2-21312*d*e^7*m^6*x^3+86210*d*e^7*m^5*x^4-1831500*d*e^7*m^4*x^5+259
5285*d*e^7*m^3*x^6-10505600*d*e^7*m^2*x^7+33*e^8*m^8*x+4494*e^8*m^7*x^2+458
90*e^8*m^6*x^3+902800*e^8*m^5*x^4-1090353*e^8*m^4*x^5+9134412*e^8*m^3*x^6-5
906520*e^8*m^2*x^7+10958400*e^8*m*x^8+168000*d^4*e^4*m^4*x^4-13320*d^3*e^5*
m^5*x^3+179550*d^3*e^5*m^4*x^4-2856000*d^3*e^5*m^3*x^5+1776*d^2*e^6*m^6*x^2
-22200*d^2*e^6*m^5*x^3+922410*d^2*e^6*m^4*x^4-1871100*d^2*e^6*m^3*x^5+90944
00*d^2*e^6*m^2*x^6-214*d*e^7*m^7*x-7410*d*e^7*m^6*x^2-311392*d*e^7*m^5*x^3+
611240*d*e^7*m^4*x^4-6805854*d*e^7*m^3*x^5+5159700*d*e^7*m^2*x^6-10454400*d
*e^7*m*x^7+18*e^8*m^8+1419*e^8*m^7*x+79608*e^8*m^6*x^2+430690*e^8*m^5*x^3+4
850404*e^8*m^4*x^4-3422907*e^8*m^3*x^5+16387596*e^8*m^2*x^6-5519340*e^8*m*x
^7+4032000*e^8*x^8-37800*d^4*e^4*m^4*x^3+1680000*d^4*e^4*m^3*x^4+2220*d^3*e
^5*m^5*x^2-306360*d^3*e^5*m^4*x^3+1181250*d^3*e^5*m^3*x^4-7560000*d^3*e^5*m
^2*x^5+390*d^2*e^6*m^6*x+58608*d^2*e^6*m^5*x^2-256040*d^2*e^6*m^4*x^3+45454
50*d^2*e^6*m^3*x^4-4345110*d^2*e^6*m^2*x^5+9878400*d^2*e^6*m*x^6-33*d*e^7*m
^7-8560*d*e^7*m^6*x-115440*d*e^7*m^5*x^2-2365632*d*e^7*m^4*x^3+2395565*d*e^
7*m^3*x^4-13971348*d*e^7*m^2*x^5+5227740*d*e^7*m*x^6-4032000*d*e^7*x^7+792*
e^8*m^7+25872*e^8*m^6*x+772326*e^8*m^5*x^2+2389985*e^8*m^4*x^3+15608080*e^8
*m^3*x^4-6238718*e^8*m^2*x^5+15456528*e^8*m*x^6-2041200*e^8*x^7-672000*d^5*
e^3*m^3*x^3+39960*d^4*e^4*m^4*x^2-567000*d^4*e^4*m^3*x^3+5880000*d^4*e^4*m^
2*x^4-3552*d^3*e^5*m^5*x+59940*d^3*e^5*m^4*x^2-2464200*d^3*e^5*m^3*x^3+3449
250*d^3*e^5*m^2*x^4-9206400*d^3*e^5*m*x^5+214*d^2*e^6*m^6+14040*d^2*e^6*m^5
*x+758352*d^2*e^6*m^4*x^2-1420800*d^2*e^6*m^3*x^3+11302020*d^2*e^6*m^2*x^4-
4887540*d^2*e^6*m*x^5+4032000*d^2*e^6*x^6-1386*d*e^7*m^6-142096*d*e^7*m^5*x
-945750*d*e^7*m^4*x^2-9939088*d*e^7*m^3*x^3+5136710*d*e^7*m^2*x^4-14497488*
d*e^7*m*x^5+2041200*d*e^7*x^6+14868*e^8*m^6+260106*e^8*m^5*x+4453233*e^8*m^
4*x^2+7946185*e^8*m^3*x^3+29064240*e^8*m^2*x^4-5957592*e^8*m*x^5+5754240*e^
8*x^6+113400*d^5*e^3*m^3*x^2-4032000*d^5*e^3*m^2*x^3-4440*d^4*e^4*m^4*x+799
200*d^4*e^4*m^3*x^2-2457000*d^4*e^4*m^2*x^3+8400000*d^4*e^4*m*x^4-390*d^3*e
^5*m^5-110112*d^3*e^5*m^4*x+588300*d^3*e^5*m^3*x^2-8325000*d^3*e^5*m^2*x^3+
4479300*d^3*e^5*m*x^4-4032000*d^3*e^5*x^5+8346*d^2*e^6*m^5+202800*d^2*e^6*m
^4*x+4821840*d^2*e^6*m^3*x^2-3899060*d^2*e^6*m^2*x^3+13346640*d^2*e^6*m*x^4
-2041200*d^2*e^6*x^5-24486*d*e^7*m^5-1260460*d*e^7*m^4*x-4332705*d*e^7*m^3*
x^2-22675968*d*e^7*m^2*x^3+5510040*d*e^7*m*x^4-5754240*d*e^7*x^5+155232*e^8
*m^5+1567797*e^8*m^4*x+15458076*e^8*m^3*x^2+15254460*e^8*m^2*x^3+28238400*e
^8*m*x^4-2237760*e^8*x^5+2016000*d^6*e^2*m^2*x^2-79920*d^5*e^3*m^3*x+136080
0*d^5*e^3*m^2*x^2-7392000*d^5*e^3*m*x^3+3552*d^4*e^4*m^4-111000*d^4*e^4*m^3
*x+4995000*d^4*e^4*m^2*x^2-3969000*d^4*e^4*m*x^3+4032000*d^4*e^4*x^4-13650*
```

```
d^3*e^5*m^4-1296480*d^3*e^5*m^3*x+2497500*d^3*e^5*m^2*x^2-11908080*d^3*e^5*
m*x^3+2041200*d^3*e^5*x^4+133750*d^2*e^6*m^4+1485900*d^2*e^6*m^3*x+15351744
*d^2*e^6*m^2*x^2-4950600*d^2*e^6*m*x^3+5754240*d^2*e^6*x^4-235620*d*e^7*m^4
-6385546*d*e^7*m^3*x-10840440*d*e^7*m^2*x^2-25553088*d*e^7*m*x^3+2237760*d*
e^7*x^4+983682*e^8*m^4+5752131*e^8*m^3*x+31059532*e^8*m^2*x^2+15207660*e^8*
m*x^3+10741248*e^8*x^4-226800*d^6*e^2*m^2*x+6048000*d^6*e^2*m*x^2+4440*d^5*
e^3*m^3-1438560*d^5*e^3*m^2*x+3288600*d^5*e^3*m*x^2-4032000*d^5*e^3*x^3+106
560*d^4*e^4*m^3-954600*d^4*e^4*m^2*x+9990000*d^4*e^4*m*x^2-2041200*d^4*e^4*
x^3-189150*d^3*e^5*m^3-7050720*d^3*e^5*m^2*x+4204680*d^3*e^5*m*x^2-5754240*
d^3*e^5*x^3+1126710*d^2*e^6*m^3+5693610*d^2*e^6*m^2*x+21972672*d^2*e^6*m*x^
2-2237760*d^2*e^6*x^3-1332177*d*e^7*m^3-18145060*d*e^7*m^2*x-13242060*d*e^7
*m*x^2-10741248*d*e^7*x^3+3864168*e^8*m^3+12377178*e^8*m^2*x+32300304*e^8*m
*x^2+5896800*e^8*x^3-4032000*d^7*e*m*x+79920*d^6*e^2*m^2-2268000*d^6*e^2*m*
x+4032000*d^6*e^2*x^2+106560*d^5*e^3*m^2-7112880*d^5*e^3*m*x+2041200*d^5*e^
3*x^2+1189920*d^4*e^4*m^2-3085800*d^4*e^4*m*x+5754240*d^4*e^4*x^2-1296750*d
^3*e^5*m^2-16602048*d^3*e^5*m*x+2237760*d^3*e^5*x^2+5258836*d^2*e^6*m^2+102
93660*d^2*e^6*m*x+10741248*d^2*e^6*x^2-4419954*d*e^7*m^2-25828944*d*e^7*m*x
-5896800*d*e^7*x^2+9162072*e^8*m^2+13944744*e^8*m*x+12942720*e^8*x^2+226800
*d^7*e*m-4032000*d^7*e*x+1358640*d^6*e^2*m-2041200*d^6*e^2*x+848040*d^5*e^3
*m-5754240*d^5*e^3*x+5860800*d^4*e^4*m-2237760*d^4*e^4*x-4396860*d^3*e^5*m-
10741248*d^3*e^5*x+12886224*d^2*e^6*m+5896800*d^2*e^6*x-7957224*d*e^7*m-129
42720*d*e^7*x+11946528*e^8*m+5987520*e^8*x+4032000*d^8+2041200*d^7*e+575424
0*d^6*e^2+2237760*d^5*e^3+10741248*d^4*e^4-5896800*d^3*e^5+12942720*d^2*e^6
-5987520*d*e^7+6531840*e^8)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2796 vs. 2(432) = 864.

Time = 0.31 (sec) , antiderivative size = 2796, normalized size of antiderivative = 6.47

$$\int (d+ex)^m \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right) \, dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)^m*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="f
ricas")
```

```
[Out] (18*d*e^8*m^8 + 100*(e^9*m^8 + 36*e^9*m^7 + 546*e^9*m^6 + 4536*e^9*m^5 + 22
449*e^9*m^4 + 67284*e^9*m^3 + 118124*e^9*m^2 + 109584*e^9*m + 40320*e^9)*x^
9 + 4032000*d^9 + 2041200*d^8*e + 5754240*d^7*e^2 + 2237760*d^6*e^3 + 10741
248*d^5*e^4 - 5896800*d^4*e^5 + 12942720*d^3*e^6 - 5987520*d^2*e^7 + 653184
0*d*e^8 - 5*(408240*e^9 - (20*d*e^8 - 9*e^9)*m^8 - (560*d*e^8 - 333*e^9)*m^
7 - 14*(460*d*e^8 - 369*e^9)*m^6 - 14*(2800*d*e^8 - 3123*e^9)*m^5 - 7*(1934
0*d*e^8 - 31383*e^9)*m^4 - 7*(37520*d*e^8 - 95211*e^9)*m^3 - 216*(1210*d*e^
8 - 5469*e^9)*m^2 - 36*(2800*d*e^8 - 30663*e^9)*m)*x^8 - 33*(d^2*e^7 - 24*d
*e^8)*m^7 + (5754240*e^9 - 3*(15*d*e^8 - 37*e^9)*m^8 - 2*(400*d^2*e^7 + 675
```

```
*d*e^8 - 2109*e^9)*m^7 - 12*(1400*d^2*e^7 + 1365*d*e^8 - 5587*e^9)*m^6 - 14
*(10000*d^2*e^7 + 7425*d*e^8 - 41403*e^9)*m^5 - 21*(28000*d^2*e^7 + 17655*d
*e^8 - 141229*e^9)*m^4 - 28*(46400*d^2*e^7 + 26325*d*e^8 - 326229*e^9)*m^3
- 36*(39200*d^2*e^7 + 20745*d*e^8 - 455211*e^9)*m^2 - 144*(4000*d^2*e^7 + 2
025*d*e^8 - 107337*e^9)*m)*x^7 + 2*(107*d^3*e^6 - 693*d^2*e^7 + 7434*d*e^8)
*m^6 - (2237760*e^9 - 37*(3*d*e^8 - e^9)*m^8 - 3*(105*d^2*e^7 + 1184*d*e^8
- 481*e^9)*m^7 - 4*(1400*d^3*e^6 + 1890*d^2*e^7 + 11433*d*e^8 - 5883*e^9)*m
^6 - 6*(14000*d^3*e^6 + 11550*d^2*e^7 + 50875*d*e^8 - 34743*e^9)*m^5 - (476
000*d^3*e^6 + 311850*d^2*e^7 + 1134309*d*e^8 - 1090353*e^9)*m^4 - 3*(420000
*d^3*e^6 + 241395*d^2*e^7 + 776186*d*e^8 - 1140969*e^9)*m^3 - 2*(767200*d^3
*e^6 + 407295*d^2*e^7 + 1208124*d*e^8 - 3119359*e^9)*m^2 - 24*(28000*d^3*e^
6 + 14175*d^2*e^7 + 39960*d*e^8 - 248233*e^9)*m)*x^6 - 6*(65*d^4*e^5 - 1391
*d^3*e^6 + 4081*d^2*e^7 - 25872*d*e^8)*m^5 + (10741248*e^9 - 37*(d*e^8 - 4*
e^9)*m^8 - 74*(9*d^2*e^7 + 17*d*e^8 - 80*e^9)*m^7 - 2*(945*d^3*e^6 + 8991*d
^2*e^7 + 8621*d*e^8 - 49580*e^9)*m^6 - 2*(16800*d^4*e^5 + 17955*d^3*e^6 + 9
2241*d^2*e^7 + 61124*d*e^8 - 451400*e^9)*m^5 - (336000*d^4*e^5 + 236250*d^3
*e^6 + 909090*d^2*e^7 + 479113*d*e^8 - 4850404*e^9)*m^4 - 2*(588000*d^4*e^5
 + 344925*d^3*e^6 + 1130202*d^2*e^7 + 513671*d*e^8 - 7804040*e^9)*m^3 - 12*
(140000*d^4*e^5 + 74655*d^3*e^6 + 222444*d^2*e^7 + 91834*d*e^8 - 2422020*e^
9)*m^2 - 144*(5600*d^4*e^5 + 2835*d^3*e^6 + 7992*d^2*e^7 + 3108*d*e^8 - 196
100*e^9)*m)*x^5 + 2*(1776*d^5*e^4 - 6825*d^4*e^5 + 66875*d^3*e^6 - 117810*d
^2*e^7 + 491841*d*e^8)*m^4 + (5896800*e^9 + (148*d*e^8 + 65*e^9)*m^8 + (185
*d^2*e^7 + 5328*d*e^8 + 2665*e^9)*m^7 + 2*(1665*d^3*e^6 + 2775*d^2*e^7 + 38
924*d*e^8 + 22945*e^9)*m^6 + 2*(4725*d^4*e^5 + 38295*d^3*e^6 + 32005*d^2*e^
7 + 295704*d*e^8 + 215345*e^9)*m^5 + (168000*d^5*e^4 + 141750*d^4*e^5 + 616
050*d^3*e^6 + 355200*d^2*e^7 + 2484772*d*e^8 + 2389985*e^9)*m^4 + (1008000*
d^5*e^4 + 614250*d^4*e^5 + 2081250*d^3*e^6 + 974765*d^2*e^7 + 5668992*d*e^8
 + 7946185*e^9)*m^3 + 6*(308000*d^5*e^4 + 165375*d^4*e^5 + 496170*d^3*e^6 +
 206275*d^2*e^7 + 1064712*d*e^8 + 2542410*e^9)*m^2 + 36*(28000*d^5*e^4 + 14
175*d^4*e^5 + 39960*d^3*e^6 + 15540*d^2*e^7 + 74592*d*e^8 + 422435*e^9)*m)*
x^4 + 3*(1480*d^6*e^3 + 35520*d^5*e^4 - 63050*d^4*e^5 + 375570*d^3*e^6 - 44
4059*d^2*e^7 + 1288056*d*e^8)*m^3 + (12942720*e^9 + (65*d*e^8 + 107*e^9)*m^
8 - 2*(296*d^2*e^7 - 1235*d*e^8 - 2247*e^9)*m^7 - 4*(185*d^3*e^6 + 4884*d^2
*e^7 - 9620*d*e^8 - 19902*e^9)*m^6 - 2*(6660*d^4*e^5 + 9990*d^3*e^6 + 12639
2*d^2*e^7 - 157625*d*e^8 - 386163*e^9)*m^5 - (37800*d^5*e^4 + 266400*d^4*e^
5 + 196100*d^3*e^6 + 1607280*d^2*e^7 - 1444235*d*e^8 - 4453233*e^9)*m^4 - 4
*(168000*d^6*e^3 + 113400*d^5*e^4 + 416250*d^4*e^5 + 208125*d^3*e^6 + 12793
12*d^2*e^7 - 903370*d*e^8 - 3864519*e^9)*m^3 - 4*(504000*d^6*e^3 + 274050*d
^5*e^4 + 832500*d^4*e^5 + 350390*d^3*e^6 + 1831056*d^2*e^7 - 1103505*d*e^8
- 7764883*e^9)*m^2 - 48*(28000*d^6*e^3 + 14175*d^5*e^4 + 39960*d^4*e^5 + 15
540*d^3*e^6 + 74592*d^2*e^7 - 40950*d*e^8 - 672923*e^9)*m)*x^3 + 2*(39960*d
^7*e^2 + 53280*d^6*e^3 + 594960*d^5*e^4 - 648375*d^4*e^5 + 2629418*d^3*e^6
- 2209977*d^2*e^7 + 4581036*d*e^8)*m^2 + (5987520*e^9 + (107*d*e^8 + 33*e^9
)*m^8 - (195*d^2*e^7 - 4280*d*e^8 - 1419*e^9)*m^7 + 4*(444*d^3*e^6 - 1755*d
^2*e^7 + 17762*d*e^8 + 6468*e^9)*m^6 + 2*(1110*d^4*e^5 + 27528*d^3*e^6 - 50
```

```
700*d^2*e^7 + 315115*d*e^8 + 130053*e^9)*m^5 + (39960*d^5*e^4 + 55500*d^4*e
^5 + 648240*d^3*e^6 - 742950*d^2*e^7 + 3192773*d*e^8 + 1567797*e^9)*m^4 + (
113400*d^6*e^3 + 719280*d^5*e^4 + 477300*d^4*e^5 + 3525360*d^3*e^6 - 284680
5*d^2*e^7 + 9072530*d*e^8 + 5752131*e^9)*m^3 + 6*(336000*d^7*e^2 + 189000*d
^6*e^3 + 592740*d^5*e^4 + 257150*d^4*e^5 + 1383504*d^3*e^6 - 857805*d^2*e^7
 + 2152412*d*e^8 + 2062863*e^9)*m^2 + 72*(28000*d^7*e^2 + 14175*d^6*e^3 + 3
9960*d^5*e^4 + 15540*d^4*e^5 + 74592*d^3*e^6 - 40950*d^2*e^7 + 89880*d*e^8
+ 193677*e^9)*m)*x^2 + 12*(18900*d^8*e + 113220*d^7*e^2 + 70670*d^6*e^3 + 4
88400*d^5*e^4 - 366405*d^4*e^5 + 1073852*d^3*e^6 - 663102*d^2*e^7 + 995544*
d*e^8)*m + (6531840*e^9 + 3*(11*d*e^8 + 6*e^9)*m^8 - 2*(107*d^2*e^7 - 693*d
*e^8 - 396*e^9)*m^7 + 6*(65*d^3*e^6 - 1391*d^2*e^7 + 4081*d*e^8 + 2478*e^9)
*m^6 - 2*(1776*d^4*e^5 - 6825*d^3*e^6 + 66875*d^2*e^7 - 117810*d*e^8 - 7761
6*e^9)*m^5 - 3*(1480*d^5*e^4 + 35520*d^4*e^5 - 63050*d^3*e^6 + 375570*d^2*e
^7 - 444059*d*e^8 - 327894*e^9)*m^4 - 2*(39960*d^6*e^3 + 53280*d^5*e^4 + 59
4960*d^4*e^5 - 648375*d^3*e^6 + 2629418*d^2*e^7 - 2209977*d*e^8 - 1932084*e
^9)*m^3 - 12*(18900*d^7*e^2 + 113220*d^6*e^3 + 70670*d^5*e^4 + 488400*d^4*e
^5 - 366405*d^3*e^6 + 1073852*d^2*e^7 - 663102*d*e^8 - 763506*e^9)*m^2 - 14
4*(28000*d^8*e + 14175*d^7*e^2 + 39960*d^6*e^3 + 15540*d^5*e^4 + 74592*d^4*
e^5 - 40950*d^3*e^6 + 89880*d^2*e^7 - 41580*d*e^8 - 82962*e^9)*m)*x)*(e*x +
 d)^m/(e^9*m^9 + 45*e^9*m^8 + 870*e^9*m^7 + 9450*e^9*m^6 + 63273*e^9*m^5 +
269325*e^9*m^4 + 723680*e^9*m^3 + 1172700*e^9*m^2 + 1026576*e^9*m + 362880*
e^9)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65193 vs. 2(410) = 820.

Time = 13.90 (sec) , antiderivative size = 65193, normalized size of antiderivative = 150.91

$$\int (d+ex)^m \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right)\,dx=\text{Too large to display}$$

```
[In] integrate((e*x+d)**m*(5*x**2+2*x+3)**2*(4*x**4-5*x**3+3*x**2+x+2),x)
```

```
[Out] Piecewise((d**m*(100*x**9/9 - 45*x**8/8 + 111*x**7/7 - 37*x**6/6 + 148*x**5
/5 + 65*x**4/4 + 107*x**3/3 + 33*x**2/2 + 18*x), Eq(e, 0)), (84000*d**8*log
(d/e + x)/(840*d**8*e**9 + 6720*d**7*e**10*x + 23520*d**6*e**11*x**2 + 4704
0*d**5*e**12*x**3 + 58800*d**4*e**13*x**4 + 47040*d**3*e**14*x**5 + 23520*d
**2*e**15*x**6 + 6720*d*e**16*x**7 + 840*e**17*x**8) + 228300*d**8/(840*d**
8*e**9 + 6720*d**7*e**10*x + 23520*d**6*e**11*x**2 + 47040*d**5*e**12*x**3
+ 58800*d**4*e**13*x**4 + 47040*d**3*e**14*x**5 + 23520*d**2*e**15*x**6 + 6
720*d*e**16*x**7 + 840*e**17*x**8) + 672000*d**7*e*x*log(d/e + x)/(840*d**8
*e**9 + 6720*d**7*e**10*x + 23520*d**6*e**11*x**2 + 47040*d**5*e**12*x**3 +
 58800*d**4*e**13*x**4 + 47040*d**3*e**14*x**5 + 23520*d**2*e**15*x**6 + 67
20*d*e**16*x**7 + 840*e**17*x**8) + 1742400*d**7*e*x/(840*d**8*e**9 + 6720*
d**7*e**10*x + 23520*d**6*e**11*x**2 + 47040*d**5*e**12*x**3 + 58800*d**4*e
```

```
**13*x**4 + 47040*d**3*e**14*x**5 + 23520*d**2*e**15*x**6 + 6720*d*e**16*x*
*7 + 840*e**17*x**8) + 4725*d**7*e/(840*d**8*e**9 + 6720*d**7*e**10*x + 235
20*d**6*e**11*x**2 + 47040*d**5*e**12*x**3 + 58800*d**4*e**13*x**4 + 47040*
d**3*e**14*x**5 + 23520*d**2*e**15*x**6 + 6720*d*e**16*x**7 + 840*e**17*x**
8) + 2352000*d**6*e**2*x**2*log(d/e + x)/(840*d**8*e**9 + 6720*d**7*e**10*x
 + 23520*d**6*e**11*x**2 + 47040*d**5*e**12*x**3 + 58800*d**4*e**13*x**4 +
47040*d**3*e**14*x**5 + 23520*d**2*e**15*x**6 + 6720*d*e**16*x**7 + 840*e**
17*x**8) + 5762400*d**6*e**2*x**2/(840*d**8*e**9 + 6720*d**7*e**10*x + 2352
0*d**6*e**11*x**2 + 47040*d**5*e**12*x**3 + 58800*d**4*e**13*x**4 + 47040*d
**3*e**14*x**5 + 23520*d**2*e**15*x**6 + 6720*d*e**16*x**7 + 840*e**17*x**8
) + 37800*d**6*e**2*x/(840*d**8*e**9 + 6720*d**7*e**10*x + 23520*d**6*e**11
*x**2 + 47040*d**5*e**12*x**3 + 58800*d**4*e**13*x**4 + 47040*d**3*e**14*x*
*5 + 23520*d**2*e**15*x**6 + 6720*d*e**16*x**7 + 840*e**17*x**8) - 1665*d**
6*e**2/(840*d**8*e**9 + 6720*d**7*e**10*x + 23520*d**6*e**11*x**2 + 47040*d
**5*e**12*x**3 + 58800*d**4*e**13*x**4 + 47040*d**3*e**14*x**5 + 23520*d**2
*e**15*x**6 + 6720*d*e**16*x**7 + 840*e**17*x**8) + 4704000*d**5*e**3*x**3*
log(d/e + x)/(840*d**8*e**9 + 6720*d**7*e**10*x + 23520*d**6*e**11*x**2 + 4
7040*d**5*e**12*x**3 + 58800*d**4*e**13*x**4 + 47040*d**3*e**14*x**5 + 2352
0*d**2*e**15*x**6 + 6720*d*e**16*x**7 + 840*e**17*x**8) + 10740800*d**5*e**
3*x**3/(840*d**8*e**9 + 6720*d**7*e**10*x + 23520*d**6*e**11*x**2 + 47040*d
**5*e**12*x**3 + 58800*d**4*e**13*x**4 + 47040*d**3*e**14*x**5 + 23520*d**2
*e**15*x**6 + 6720*d*e**16*x**7 + 840*e**17*x**8) + 132300*d**5*e**3*x**2/(
840*d**8*e**9 + 6720*d**7*e**10*x + 23520*d**6*e**11*x**2 + 47040*d**5*e**1
2*x**3 + 58800*d**4*e**13*x**4 + 47040*d**3*e**14*x**5 + 23520*d**2*e**15*x
**6 + 6720*d*e**16*x**7 + 840*e**17*x**8) - 13320*d**5*e**3*x/(840*d**8*e**
9 + 6720*d**7*e**10*x + 23520*d**6*e**11*x**2 + 47040*d**5*e**12*x**3 + 588
00*d**4*e**13*x**4 + 47040*d**3*e**14*x**5 + 23520*d**2*e**15*x**6 + 6720*d
*e**16*x**7 + 840*e**17*x**8) + 185*d**5*e**3/(840*d**8*e**9 + 6720*d**7*e*
*10*x + 23520*d**6*e**11*x**2 + 47040*d**5*e**12*x**3 + 58800*d**4*e**13*x*
*4 + 47040*d**3*e**14*x**5 + 23520*d**2*e**15*x**6 + 6720*d*e**16*x**7 + 84
0*e**17*x**8) + 5880000*d**4*e**4*x**4*log(d/e + x)/(840*d**8*e**9 + 6720*d
**7*e**10*x + 23520*d**6*e**11*x**2 + 47040*d**5*e**12*x**3 + 58800*d**4*e*
*13*x**4 + 47040*d**3*e**14*x**5 + 23520*d**2*e**15*x**6 + 6720*d*e**16*x**
7 + 840*e**17*x**8) + 12250000*d**4*e**4*x**4/(840*d**8*e**9 + 6720*d**7*e*
*10*x + 23520*d**6*e**11*x**2 + 47040*d**5*e**12*x**3 + 58800*d**4*e**13*x*
*4 + 47040*d**3*e**14*x**5 + 23520*d**2*e**15*x**6 + 6720*d*e**16*x**7 + 84
0*e**17*x**8) + 264600*d**4*e**4*x**3/(840*d**8*e**9 + 6720*d**7*e**10*x +
23520*d**6*e**11*x**2 + 47040*d**5*e**12*x**3 + 58800*d**4*e**13*x**4 + 470
40*d**3*e**14*x**5 + 23520*d**2*e**15*x**6 + 6720*d*e**16*x**7 + 840*e**17*
x**8) - 46620*d**4*e**4*x**2/(840*d**8*e**9 + 6720*d**7*e**10*x + 23520*d**
6*e**11*x**2 + 47040*d**5*e**12*x**3 + 58800*d**4*e**13*x**4 + 47040*d**3*e
**14*x**5 + 23520*d**2*e**15*x**6 + 6720*d*e**16*x**7 + 840*e**17*x**8) + 1
480*d**4*e**4*x/(840*d**8*e**9 + 6720*d**7*e**10*x + 23520*d**6*e**11*x**2
+ 47040*d**5*e**12*x**3 + 58800*d**4*e**13*x**4 + 47040*d**3*e**14*x**5 + 2
3520*d**2*e**15*x**6 + 6720*d*e**16*x**7 + 840*e**17*x**8) - 444*d**4*e**4/
```

```
(840*d**8*e**9 + 6720*d**7*e**10*x + 23520*d**6*e**11*x**2 + 47040*d**5*e**
12*x**3 + 58800*d**4*e**13*x**4 + 47040*d**3*e**14*x**5 + 23520*d**2*e**15*
x**6 + 6720*d*e**16*x**7 + 840*e**17*x**8) + 4704000*d**3*e**5*x**5*log(d/e
 + x)/(840*d**8*e**9 + 6720*d**7*e**10*x + 23520*d**6*e**11*x**2 + 47040*d*
*5*e**12*x**3 + 58800*d**4*e**13*x**4 + 47040*d**3*e**14*x**5 + 23520*d**2*
e**15*x**6 + 6720*d*e**16*x**7 + 840*e**17*x**8) + 8624000*d**3*e**5*x**5/(
840*d**8*e**9 + 6720*d**7*e**10*x + 23520*d**6*e**11*x**2 + 47040*d**5*e**1
2*x**3 + 58800*d**4*e**13*x**4 + 47040*d**3*e**14*x**5 + 23520*d**2*e**15*x
**6 + 6720*d*e**16*x**7 + 840*e**17*x**8) + 330750*d**3*e**5*x**4/(840*d**8
*e**9 + 6720*d**7*e**10*x + 23520*d**6*e**11*x**2 + 47040*d**5*e**12*x**3 +
 58800*d**4*e**13*x**4 + 47040*d**3*e**14*x**5 + 23520*d**2*e**15*x**6 + 67
20*d*e**16*x**7 + 840*e**17*x**8) - 93240*d**3*e**5*x**3/(840*d**8*e**9 + 6
720*d**7*e**10*x + 23520*d**6*e**11*x**2 + 47040*d**5*e**12*x**3 + 58800*d*
*4*e**13*x**4 + 47040*d**3*e**14*x**5 + 23520*d**2*e**15*x**6 + 6720*d*e**1
6*x**7 + 840*e**17*x**8) + 5180*d**3*e**5*x**2/(840*d**8*e**9 + 6720*d**7*e
**10*x + 23520*d**6*e**11*x**2 + 47040*d**5*e**12*x**3 + 58800*d**4*e**13*x
**4 + 47040*d**3*e**14*x**5 + 23520*d**2*e**15*x**6 + 6720*d*e**16*x**7 + 8
40*e**17*x**8) - 3552*d**3*e**5*x/(840*d**8*e**9 + 6720*d**7*e**10*x + 2352
0*d**6*e**11*x**2 + 47040*d**5*e**12*x**3 + 58800*d**4*e**13*x**4 + 47040*d
**3*e**14*x**5 + 23520*d**2*e**15*x**6 + 6720*d*e**16*x**7 + 840*e**17*x**8
) - 195*d**3*e**5/(840*d**8*e**9 + 6720*d**7*e**10*x + 23520*d**6*e**11*x**
2 + 47040*d**5*e**12*x**3 + 58800*d**4*e**13*x**4 + 47040*d**3*e**14*x**5 +
 23520*d**2*e**15*x**6 + 6720*d*e**16*x**7 + 840*e**17*x**8) + 2352000*d**2
*e**6*x**6*log(d/e + x)/(840*d**8*e**9 + 6720*d**7*e**10*x + 23520*d**6*e**
11*x**2 + 47040*d**5*e**12*x**3 + 58800*d**4*e**13*x**4 + 47040*d**3*e**14*
x**5 + 23520*d**2*e**15*x**6 + 6720*d*e**16*x**7 + 840*e**17*x**8) + 352800
0*d**2*e**6*x**6/(840*d**8*e**9 + 6720*d**7*e**10*x + 23520*d**6*e**11*x**2
 + 47040*d**5*e**12*x**3 + 58800*d**4*e**13*x**4 + 47040*d**3*e**14*x**5 +
23520*d**2*e**15*x**6 + 6720*d*e**16*x**7 + 840*e**17*x**8) + 264600*d**2*e
**6*x**5/(840*d**8*e**9 + 6720*d**7*e**10*x + 23520*d**6*e**11*x**2 + 47040
*d**5*e**12*x**3 + 58800*d**4*e**13*x**4 + 47040*d**3*e**14*x**5 + 23520*d*
*2*e**15*x**6 + 6720*d*e**16*x**7 + 840*e**17*x**8) - 116550*d**2*e**6*x**4
/(840*d**8*e**9 + 6720*d**7*e**10*x + 23520*d**6*e**11*x**2 + 47040*d**5*e*
*12*x**3 + 58800*d**4*e**13*x**4 + 47040*d**3*e**14*x**5 + 23520*d**2*e**15
*x**6 + 6720*d*e**16*x**7 + 840*e**17*x**8) + 10360*d**2*e**6*x**3/(840*d**
8*e**9 + 6720*d**7*e**10*x + 23520*d**6*e**11*x**2 + 47040*d**5*e**12*x**3
 + 58800*d**4*e**13*x**4 + 47040*d**3*e**14*x**5 + 23520*d**2*e**15*x**6 + 6
720*d*e**16*x**7 + 840*e**17*x**8) - 12432*d**2*e**6*x**2/(840*d**8*e**9 +
6720*d**7*e**10*x + 23520*d**6*e**11*x**2 + 47040*d**5*e**12*x**3 + 58800*d
**4*e**13*x**4 + 47040*d**3*e**14*x**5 + 23520*d**2*e**15*x**6 + 6720*d*e**
16*x**7 + 840*e**17*x**8) - 1560*d**2*e**6*x/(840*d**8*e**9 + 6720*d**7*e**
10*x + 23520*d**6*e**11*x**2 + 47040*d**5*e**12*x**3 + 58800*d**4*e**13*x**
4 + 47040*d**3*e**14*x**5 + 23520*d**2*e**15*x**6 + 6720*d*e**16*x**7 + 840
*e**17*x**8) - 535*d**2*e**6/(840*d**8*e**9 + 6720*d**7*e**10*x + 23520*d**
6*e**11*x**2 + 47040*d**5*e**12*x**3 + 58800*d**4*e**13*x**4 + 47040*d**3*e
```

```
**14*x**5 + 23520*d**2*e**15*x**6 + 6720*d*e**16*x**7 + 840*e**17*x**8) + 6
72000*d*e**7*x**7*log(d/e + x)/(840*d**8*e**9 + 6720*d**7*e**10*x + 23520*d
**6*e**11*x**2 + 47040*d**5*e**12*x**3 + 58800*d**4*e**13*x**4 + 47040*d**3
*e**14*x**5 + 23520*d**2*e**15*x**6 + 6720*d*e**16*x**7 + 840*e**17*x**8) +
 672000*d*e**7*x**7/(840*d**8*e**9 + 6720*d**7*e**10*x + 23520*d**6*e**11*x
**2 + 47040*d**5*e**12*x**3 + 58800*d**4*e**13*x**4 + 47040*d**3*e**14*x**5
 + 23520*d**2*e**15*x**6 + 6720*d*e**16*x**7 + 840*e**17*x**8) + 132300*d*e
**7*x**6/(840*d**8*e**9 + 6720*d**7*e**10*x + 23520*d**6*e**11*x**2 + 47040
*d**5*e**12*x**3 + 58800*d**4*e**13*x**4 + 47040*d**3*e**14*x**5 + 23520*d*
*2*e**15*x**6 + 6720*d*e**16*x**7 + 840*e**17*x**8) - 93240*d*e**7*x**5/(84
0*d**8*e**9 + 6720*d**7*e**10*x + 23520*d**6*e**11*x**2 + 47040*d**5*e**12*
x**3 + 58800*d**4*e**13*x**4 + 47040*d**3*e**14*x**5 + 23520*d**2*e**15*x**
6 + 6720*d*e**16*x**7 + 840*e**17*x**8) + 12950*d*e**7*x**4/(840*d**8*e**9
+ 6720*d**7*e**10*x + 23520*d**6*e**11*x**2 + 47040*d**5*e**12*x**3 + 58800
*d**4*e**13*x**4 + 47040*d**3*e**14*x**5 + 23520*d**2*e**15*x**6 + 6720*d*e
**16*x**7 + 840*e**17*x**8) - 24864*d*e**7*x**3/(840*d**8*e**9 + 6720*d**7*
e**10*x + 23520*d**6*e**11*x**2 + 47040*d**5*e**12*x**3 + 58800*d**4*e**13*
x**4 + 47040*d**3*e**14*x**5 + 23520*d**2*e**15*x**6 + 6720*d*e**16*x**7 +
840*e**17*x**8) - 5460*d*e**7*x**2/(840*d**8*e**9 + 6720*d**7*e**10*x + 235
20*d**6*e**11*x**2 + 47040*d**5*e**12*x**3 + 58800*d**4*e**13*x**4 + 47040*
d**3*e**14*x**5 + 23520*d**2*e**15*x**6 + 6720*d*e**16*x**7 + 840*e**17*x**
8) - 4280*d*e**7*x/(840*d**8*e**9 + 6720*d**7*e**10*x + 23520*d**6*e**11*x*
*2 + 47040*d**5*e**12*x**3 + 58800*d**4*e**13*x**4 + 47040*d**3*e**14*x**5
+ 23520*d**2*e**15*x**6 + 6720*d*e**16*x**7 + 840*e**17*x**8) - 495*d*e**7/
(840*d**8*e**9 + 6720*d**7*e**10*x + 23520*d**6*e**11*x**2 + 47040*d**5*e**
12*x**3 + 58800*d**4*e**13*x**4 + 47040*d**3*e**14*x**5 + 23520*d**2*e**15*
x**6 + 6720*d*e**16*x**7 + 840*e**17*x**8) + 84000*e**8*x**8*log(d/e + x)/(
840*d**8*e**9 + 6720*d**7*e**10*x + 23520*d**6*e**11*x**2 + 47040*d**5*e**1
2*x**3 + 58800*d**4*e**13*x**4 + 47040*d**3*e**14*x**5 + 23520*d**2*e**15*x
**6 + 6720*d*e**16*x**7 + 840*e**17*x**8) + 37800*e**8*x**7/(840*d**8*e**9
+ 6720*d**7*e**10*x + 23520*d**6*e**11*x**2 + 47040*d**5*e**12*x**3 + 58800
*d**4*e**13*x**4 + 47040*d**3*e**14*x**5 + 23520*d**2*e**15*x**6 + 6720*d*e
**16*x**7 + 840*e**17*x**8) - 46620*e**8*x**6/(840*d**8*e**9 + 6720*d**7*e*
*10*x + 23520*d**6*e**11*x**2 + 47040*d**5*e**12*x**3 + 58800*d**4*e**13*x*
*4 + 47040*d**3*e**14*x**5 + 23520*d**2*e**15*x**6 + 6720*d*e**16*x**7 + 84
0*e**17*x**8) + 10360*e**8*x**5/(840*d**8*e**9 + 6720*d**7*e**10*x + 23520*
d**6*e**11*x**2 + 47040*d**5*e**12*x**3 + 58800*d**4*e**13*x**4 + 47040*d**
3*e**14*x**5 + 23520*d**2*e**15*x**6 + 6720*d*e**16*x**7 + 840*e**17*x**8)
- 31080*e**8*x**4/(840*d**8*e**9 + 6720*d**7*e**10*x + 23520*d**6*e**11*x**
2 + 47040*d**5*e**12*x**3 + 58800*d**4*e**13*x**4 + 47040*d**3*e**14*x**5 +
 23520*d**2*e**15*x**6 + 6720*d*e**16*x**7 + 840*e**17*x**8) - 10920*e**8*x
**3/(840*d**8*e**9 + 6720*d**7*e**10*x + 23520*d**6*e**11*x**2 + 47040*d**5
*e**12*x**3 + 58800*d**4*e**13*x**4 + 47040*d**3*e**14*x**5 + 23520*d**2*e*
*15*x**6 + 6720*d*e**16*x**7 + 840*e**17*x**8) - 14980*e**8*x**2/(840*d**8*
e**9 + 6720*d**7*e**10*x + 23520*d**6*e**11*x**2 + 47040*d**5*e**12*x**3 +
```

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58800*d**4*e**13*x**4 + 47040*d**3*e**14*x**5 + 23520*d**2*e**15*x**6 + 672
0*d*e**16*x**7 + 840*e**17*x**8) - 3960*e**8*x/(840*d**8*e**9 + 6720*d**7*e
**10*x + 23520*d**6*e**11*x**2 + 47040*d**5*e**12*x**3 + 58800*d**4*e**13*x
**4 + 47040*d**3*e**14*x**5 + 23520*d**2*e**15*x**6 + 6720*d*e**16*x**7 + 8
40*e**17*x**8) - 1890*e**8/(840*d**8*e**9 + 6720*d**7*e**10*x + 23520*d**6*
e**11*x**2 + 47040*d**5*e**12*x**3 + 58800*d**4*e**13*x**4 + 47040*d**3*e**
14*x**5 + 23520*d**2*e**15*x**6 + 6720*d*e**16*x**7 + 840*e**17*x**8), Eq(m
, -9)), (-336000*d**8*log(d/e + x)/(420*d**7*e**9 + 2940*d**6*e**10*x + 882
0*d**5*e**11*x**2 + 14700*d**4*e**12*x**3 + 14700*d**3*e**13*x**4 + 8820*d*
*2*e**14*x**5 + 2940*d*e**15*x**6 + 420*e**16*x**7) - 871200*d**8/(420*d**7
*e**9 + 2940*d**6*e**10*x + 8820*d**5*e**11*x**2 + 14700*d**4*e**12*x**3 +
14700*d**3*e**13*x**4 + 8820*d**2*e**14*x**5 + 2940*d*e**15*x**6 + 420*e**1
6*x**7) - 2352000*d**7*e*x*log(d/e + x)/(420*d**7*e**9 + 2940*d**6*e**10*x
+ 8820*d**5*e**11*x**2 + 14700*d**4*e**12*x**3 + 14700*d**3*e**13*x**4 + 88
20*d**2*e**14*x**5 + 2940*d*e**15*x**6 + 420*e**16*x**7) - 5762400*d**7*e*x
/(420*d**7*e**9 + 2940*d**6*e**10*x + 8820*d**5*e**11*x**2 + 14700*d**4*e**
12*x**3 + 14700*d**3*e**13*x**4 + 8820*d**2*e**14*x**5 + 2940*d*e**15*x**6
+ 420*e**16*x**7) - 18900*d**7*e*log(d/e + x)/(420*d**7*e**9 + 2940*d**6*e*
*10*x + 8820*d**5*e**11*x**2 + 14700*d**4*e**12*x**3 + 14700*d**3*e**13*x**
4 + 8820*d**2*e**14*x**5 + 2940*d*e**15*x**6 + 420*e**16*x**7) - 49005*d**7
*e/(420*d**7*e**9 + 2940*d**6*e**10*x + 8820*d**5*e**11*x**2 + 14700*d**4*e
**12*x**3 + 14700*d**3*e**13*x**4 + 8820*d**2*e**14*x**5 + 2940*d*e**15*x**
6 + 420*e**16*x**7) - 7056000*d**6*e**2*x**2*log(d/e + x)/(420*d**7*e**9 +
2940*d**6*e**10*x + 8820*d**5*e**11*x**2 + 14700*d**4*e**12*x**3 + 14700*d*
*3*e**13*x**4 + 8820*d**2*e**14*x**5 + 2940*d*e**15*x**6 + 420*e**16*x**7)
- 16111200*d**6*e**2*x**2/(420*d**7*e**9 + 2940*d**6*e**10*x + 8820*d**5*e*
*11*x**2 + 14700*d**4*e**12*x**3 + 14700*d**3*e**13*x**4 + 8820*d**2*e**14*
x**5 + 2940*d*e**15*x**6 + 420*e**16*x**7) - 132300*d**6*e**2*x*log(d/e + x
)/(420*d**7*e**9 + 2940*d**6*e**10*x + 8820*d**5*e**11*x**2 + 14700*d**4*e*
*12*x**3 + 14700*d**3*e**13*x**4 + 8820*d**2*e**14*x**5 + 2940*d*e**15*x**6
 + 420*e**16*x**7) - 324135*d**6*e**2*x/(420*d**7*e**9 + 2940*d**6*e**10*x
+ 8820*d**5*e**11*x**2 + 14700*d**4*e**12*x**3 + 14700*d**3*e**13*x**4 + 88
20*d**2*e**14*x**5 + 2940*d*e**15*x**6 + 420*e**16*x**7) - 6660*d**6*e**2/(
420*d**7*e**9 + 2940*d**6*e**10*x + 8820*d**5*e**11*x**2 + 14700*d**4*e**12
*x**3 + 14700*d**3*e**13*x**4 + 8820*d**2*e**14*x**5 + 2940*d*e**15*x**6 +
420*e**16*x**7) - 11760000*d**5*e**3*x**3*log(d/e + x)/(420*d**7*e**9 + 294
0*d**6*e**10*x + 8820*d**5*e**11*x**2 + 14700*d**4*e**12*x**3 + 14700*d**3*
e**13*x**4 + 8820*d**2*e**14*x**5 + 2940*d*e**15*x**6 + 420*e**16*x**7) - 2
4500000*d**5*e**3*x**3/(420*d**7*e**9 + 2940*d**6*e**10*x + 8820*d**5*e**11
*x**2 + 14700*d**4*e**12*x**3 + 14700*d**3*e**13*x**4 + 8820*d**2*e**14*x**
5 + 2940*d*e**15*x**6 + 420*e**16*x**7) - 396900*d**5*e**3*x**2*log(d/e + x
)/(420*d**7*e**9 + 2940*d**6*e**10*x + 8820*d**5*e**11*x**2 + 14700*d**4*e*
*12*x**3 + 14700*d**3*e**13*x**4 + 8820*d**2*e**14*x**5 + 2940*d*e**15*x**6
 + 420*e**16*x**7) - 906255*d**5*e**3*x**2/(420*d**7*e**9 + 2940*d**6*e**10
*x + 8820*d**5*e**11*x**2 + 14700*d**4*e**12*x**3 + 14700*d**3*e**13*x**4 +
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 8820*d**2*e**14*x**5 + 2940*d*e**15*x**6 + 420*e**16*x**7) - 46620*d**5*e*
*3*x/(420*d**7*e**9 + 2940*d**6*e**10*x + 8820*d**5*e**11*x**2 + 14700*d**4
*e**12*x**3 + 14700*d**3*e**13*x**4 + 8820*d**2*e**14*x**5 + 2940*d*e**15*x
**6 + 420*e**16*x**7) + 370*d**5*e**3/(420*d**7*e**9 + 2940*d**6*e**10*x +
8820*d**5*e**11*x**2 + 14700*d**4*e**12*x**3 + 14700*d**3*e**13*x**4 + 8820
*d**2*e**14*x**5 + 2940*d*e**15*x**6 + 420*e**16*x**7) - 11760000*d**4*e**4
*x**4*log(d/e + x)/(420*d**7*e**9 + 2940*d**6*e**10*x + 8820*d**5*e**11*x**
2 + 14700*d**4*e**12*x**3 + 14700*d**3*e**13*x**4 + 8820*d**2*e**14*x**5 +
2940*d*e**15*x**6 + 420*e**16*x**7) - 21560000*d**4*e**4*x**4/(420*d**7*e**
9 + 2940*d**6*e**10*x + 8820*d**5*e**11*x**2 + 14700*d**4*e**12*x**3 + 1470
0*d**3*e**13*x**4 + 8820*d**2*e**14*x**5 + 2940*d*e**15*x**6 + 420*e**16*x*
*7) - 661500*d**4*e**4*x**3*log(d/e + x)/(420*d**7*e**9 + 2940*d**6*e**10*x
 + 8820*d**5*e**11*x**2 + 14700*d**4*e**12*x**3 + 14700*d**3*e**13*x**4 + 8
820*d**2*e**14*x**5 + 2940*d*e**15*x**6 + 420*e**16*x**7) - 1378125*d**4*e*
*4*x**3/(420*d**7*e**9 + 2940*d**6*e**10*x + 8820*d**5*e**11*x**2 + 14700*d
**4*e**12*x**3 + 14700*d**3*e**13*x**4 + 8820*d**2*e**14*x**5 + 2940*d*e**1
5*x**6 + 420*e**16*x**7) - 139860*d**4*e**4*x**2/(420*d**7*e**9 + 2940*d**6
*e**10*x + 8820*d**5*e**11*x**2 + 14700*d**4*e**12*x**3 + 14700*d**3*e**13*
x**4 + 8820*d**2*e**14*x**5 + 2940*d*e**15*x**6 + 420*e**16*x**7) + 2590*d*
*4*e**4*x/(420*d**7*e**9 + 2940*d**6*e**10*x + 8820*d**5*e**11*x**2 + 14700
*d**4*e**12*x**3 + 14700*d**3*e**13*x**4 + 8820*d**2*e**14*x**5 + 2940*d*e*
*15*x**6 + 420*e**16*x**7) - 592*d**4*e**4/(420*d**7*e**9 + 2940*d**6*e**10
*x + 8820*d**5*e**11*x**2 + 14700*d**4*e**12*x**3 + 14700*d**3*e**13*x**4 +
 8820*d**2*e**14*x**5 + 2940*d*e**15*x**6 + 420*e**16*x**7) - 7056000*d**3*
e**5*x**5*log(d/e + x)/(420*d**7*e**9 + 2940*d**6*e**10*x + 8820*d**5*e**11
*x**2 + 14700*d**4*e**12*x**3 + 14700*d**3*e**13*x**4 + 8820*d**2*e**14*x**
5 + 2940*d*e**15*x**6 + 420*e**16*x**7) - 10584000*d**3*e**5*x**5/(420*d**7
*e**9 + 2940*d**6*e**10*x + 8820*d**5*e**11*x**2 + 14700*d**4*e**12*x**3 +
14700*d**3*e**13*x**4 + 8820*d**2*e**14*x**5 + 2940*d*e**15*x**6 + 420*e**1
6*x**7) - 661500*d**3*e**5*x**4*log(d/e + x)/(420*d**7*e**9 + 2940*d**6*e**
10*x + 8820*d**5*e**11*x**2 + 14700*d**4*e**12*x**3 + 14700*d**3*e**13*x**4
 + 8820*d**2*e**14*x**5 + 2940*d*e**15*x**6 + 420*e**16*x**7) - 1212750*d**
3*e**5*x**4/(420*d**7*e**9 + 2940*d**6*e**10*x + 8820*d**5*e**11*x**2 + 147
00*d**4*e**12*x**3 + 14700*d**3*e**13*x**4 + 8820*d**2*e**14*x**5 + 2940*d*
e**15*x**6 + 420*e**16*x**7) - 233100*d**3*e**5*x**3/(420*d**7*e**9 + 2940*
d**6*e**10*x + 8820*d**5*e**11*x**2 + 14700*d**4*e**12*x**3 + 14700*d**3*e*
*13*x**4 + 8820*d**2*e**14*x**5 + 2940*d*e**15*x**6 + 420*e**16*x**7) + 777
0*d**3*e**5*x**2/(420*d**7*e**9 + 2940*d**6*e**10*x + 8820*d**5*e**11*x**2
 + 14700*d**4*e**12*x**3 + 14700*d**3*e**13*x**4 + 8820*d**2*e**14*x**5 + 29
40*d*e**15*x**6 + 420*e**16*x**7) - 4144*d**3*e**5*x/(420*d**7*e**9 + 2940*
d**6*e**10*x + 8820*d**5*e**11*x**2 + 14700*d**4*e**12*x**3 + 14700*d**3*e*
*13*x**4 + 8820*d**2*e**14*x**5 + 2940*d*e**15*x**6 + 420*e**16*x**7) - 195
*d**3*e**5/(420*d**7*e**9 + 2940*d**6*e**10*x + 8820*d**5*e**11*x**2 + 1470
0*d**4*e**12*x**3 + 14700*d**3*e**13*x**4 + 8820*d**2*e**14*x**5 + 2940*d*e
**15*x**6 + 420*e**16*x**7) - 2352000*d**2*e**6*x**6*log(d/e + x)/(420*d**7
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*e**9 + 2940*d**6*e**10*x + 8820*d**5*e**11*x**2 + 14700*d**4*e**12*x**3 +
14700*d**3*e**13*x**4 + 8820*d**2*e**14*x**5 + 2940*d*e**15*x**6 + 420*e**1
6*x**7) - 2352000*d**2*e**6*x**6/(420*d**7*e**9 + 2940*d**6*e**10*x + 8820*
d**5*e**11*x**2 + 14700*d**4*e**12*x**3 + 14700*d**3*e**13*x**4 + 8820*d**2
*e**14*x**5 + 2940*d*e**15*x**6 + 420*e**16*x**7) - 396900*d**2*e**6*x**5*l
og(d/e + x)/(420*d**7*e**9 + 2940*d**6*e**10*x + 8820*d**5*e**11*x**2 + 147
00*d**4*e**12*x**3 + 14700*d**3*e**13*x**4 + 8820*d**2*e**14*x**5 + 2940*d*
e**15*x**6 + 420*e**16*x**7) - 595350*d**2*e**6*x**5/(420*d**7*e**9 + 2940*
d**6*e**10*x + 8820*d**5*e**11*x**2 + 14700*d**4*e**12*x**3 + 14700*d**3*e*
*13*x**4 + 8820*d**2*e**14*x**5 + 2940*d*e**15*x**6 + 420*e**16*x**7) - 233
100*d**2*e**6*x**4/(420*d**7*e**9 + 2940*d**6*e**10*x + 8820*d**5*e**11*x**
2 + 14700*d**4*e**12*x**3 + 14700*d**3*e**13*x**4 + 8820*d**2*e**14*x**5 +
2940*d*e**15*x**6 + 420*e**16*x**7) + 12950*d**2*e**6*x**3/(420*d**7*e**9 +
 2940*d**6*e**10*x + 8820*d**5*e**11*x**2 + 14700*d**4*e**12*x**3 + 14700*d
**3*e**13*x**4 + 8820*d**2*e**14*x**5 + 2940*d*e**15*x**6 + 420*e**16*x**7)
 - 12432*d**2*e**6*x**2/(420*d**7*e**9 + 2940*d**6*e**10*x + 8820*d**5*e**1
1*x**2 + 14700*d**4*e**12*x**3 + 14700*d**3*e**13*x**4 + 8820*d**2*e**14*x*
*5 + 2940*d*e**15*x**6 + 420*e**16*x**7) - 1365*d**2*e**6*x/(420*d**7*e**9
+ 2940*d**6*e**10*x + 8820*d**5*e**11*x**2 + 14700*d**4*e**12*x**3 + 14700*
d**3*e**13*x**4 + 8820*d**2*e**14*x**5 + 2940*d*e**15*x**6 + 420*e**16*x**7
) - 428*d**2*e**6/(420*d**7*e**9 + 2940*d**6*e**10*x + 8820*d**5*e**11*x**2
 + 14700*d**4*e**12*x**3 + 14700*d**3*e**13*x**4 + 8820*d**2*e**14*x**5 + 2
940*d*e**15*x**6 + 420*e**16*x**7) - 336000*d*e**7*x**7*log(d/e + x)/(420*d
**7*e**9 + 2940*d**6*e**10*x + 8820*d**5*e**11*x**2 + 14700*d**4*e**12*x**3
 + 14700*d**3*e**13*x**4 + 8820*d**2*e**14*x**5 + 2940*d*e**15*x**6 + 420*e
**16*x**7) - 132300*d*e**7*x**6*log(d/e + x)/(420*d**7*e**9 + 2940*d**6*e**
10*x + 8820*d**5*e**11*x**2 + 14700*d**4*e**12*x**3 + 14700*d**3*e**13*x**4
 + 8820*d**2*e**14*x**5 + 2940*d*e**15*x**6 + 420*e**16*x**7) - 132300*d*e*
*7*x**6/(420*d**7*e**9 + 2940*d**6*e**10*x + 8820*d**5*e**11*x**2 + 14700*d
**4*e**12*x**3 + 14700*d**3*e**13*x**4 + 8820*d**2*e**14*x**5 + 2940*d*e**1
5*x**6 + 420*e**16*x**7) - 139860*d*e**7*x**5/(420*d**7*e**9 + 2940*d**6*e*
*10*x + 8820*d**5*e**11*x**2 + 14700*d**4*e**12*x**3 + 14700*d**3*e**13*x**
4 + 8820*d**2*e**14*x**5 + 2940*d*e**15*x**6 + 420*e**16*x**7) + 12950*d*e*
*7*x**4/(420*d**7*e**9 + 2940*d**6*e**10*x + 8820*d**5*e**11*x**2 + 14700*d
**4*e**12*x**3 + 14700*d**3*e**13*x**4 + 8820*d**2*e**14*x**5 + 2940*d*e**1
5*x**6 + 420*e**16*x**7) - 20720*d*e**7*x**3/(420*d**7*e**9 + 2940*d**6*e**
10*x + 8820*d**5*e**11*x**2 + 14700*d**4*e**12*x**3 + 14700*d**3*e**13*x**4
 + 8820*d**2*e**14*x**5 + 2940*d*e**15*x**6 + 420*e**16*x**7) - 4095*d*e**7
*x**2/(420*d**7*e**9 + 2940*d**6*e**10*x + 8820*d**5*e**11*x**2 + 14700*d**
4*e**12*x**3 + 14700*d**3*e**13*x**4 + 8820*d**2*e**14*x**5 + 2940*d*e**15*
x**6 + 420*e**16*x**7) - 2996*d*e**7*x/(420*d**7*e**9 + 2940*d**6*e**10*x +
 8820*d**5*e**11*x**2 + 14700*d**4*e**12*x**3 + 14700*d**3*e**13*x**4 + 882
0*d**2*e**14*x**5 + 2940*d*e**15*x**6 + 420*e**16*x**7) - 330*d*e**7/(420*d
**7*e**9 + 2940*d**6*e**10*x + 8820*d**5*e**11*x**2 + 14700*d**4*e**12*x**3
 + 14700*d**3*e**13*x**4 + 8820*d**2*e**14*x**5 + 2940*d*e**15*x**6 + 420*e
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**16*x**7) + 42000*e**8*x**8/(420*d**7*e**9 + 2940*d**6*e**10*x + 8820*d**5
*e**11*x**2 + 14700*d**4*e**12*x**3 + 14700*d**3*e**13*x**4 + 8820*d**2*e**
14*x**5 + 2940*d*e**15*x**6 + 420*e**16*x**7) - 18900*e**8*x**7*log(d/e + x
)/(420*d**7*e**9 + 2940*d**6*e**10*x + 8820*d**5*e**11*x**2 + 14700*d**4*e*
*12*x**3 + 14700*d**3*e**13*x**4 + 8820*d**2*e**14*x**5 + 2940*d*e**15*x**6
 + 420*e**16*x**7) - 46620*e**8*x**6/(420*d**7*e**9 + 2940*d**6*e**10*x + 8
820*d**5*e**11*x**2 + 14700*d**4*e**12*x**3 + 14700*d**3*e**13*x**4 + 8820*
d**2*e**14*x**5 + 2940*d*e**15*x**6 + 420*e**16*x**7) + 7770*e**8*x**5/(420
*d**7*e**9 + 2940*d**6*e**10*x + 8820*d**5*e**11*x**2 + 14700*d**4*e**12*x*
*3 + 14700*d**3*e**13*x**4 + 8820*d**2*e**14*x**5 + 2940*d*e**15*x**6 + 420
*e**16*x**7) - 20720*e**8*x**4/(420*d**7*e**9 + 2940*d**6*e**10*x + 8820*d*
*5*e**11*x**2 + 14700*d**4*e**12*x**3 + 14700*d**3*e**13*x**4 + 8820*d**2*e
**14*x**5 + 2940*d*e**15*x**6 + 420*e**16*x**7) - 6825*e**8*x**3/(420*d**7*
e**9 + 2940*d**6*e**10*x + 8820*d**5*e**11*x**2 + 14700*d**4*e**12*x**3 + 1
4700*d**3*e**13*x**4 + 8820*d**2*e**14*x**5 + 2940*d*e**15*x**6 + 420*e**16
*x**7) - 8988*e**8*x**2/(420*d**7*e**9 + 2940*d**6*e**10*x + 8820*d**5*e**1
1*x**2 + 14700*d**4*e**12*x**3 + 14700*d**3*e**13*x**4 + 8820*d**2*e**14*x*
*5 + 2940*d*e**15*x**6 + 420*e**16*x**7) - 2310*e**8*x/(420*d**7*e**9 + 294
0*d**6*e**10*x + 8820*d**5*e**11*x**2 + 14700*d**4*e**12*x**3 + 14700*d**3*
e**13*x**4 + 8820*d**2*e**14*x**5 + 2940*d*e**15*x**6 + 420*e**16*x**7) - 1
080*e**8/(420*d**7*e**9 + 2940*d**6*e**10*x + 8820*d**5*e**11*x**2 + 14700*
d**4*e**12*x**3 + 14700*d**3*e**13*x**4 + 8820*d**2*e**14*x**5 + 2940*d*e**
15*x**6 + 420*e**16*x**7), Eq(m, -8)), (168000*d**8*log(d/e + x)/(60*d**6*e
**9 + 360*d**5*e**10*x + 900*d**4*e**11*x**2 + 1200*d**3*e**12*x**3 + 900*d
**2*e**13*x**4 + 360*d*e**14*x**5 + 60*e**15*x**6) + 411600*d**8/(60*d**6*e
**9 + 360*d**5*e**10*x + 900*d**4*e**11*x**2 + 1200*d**3*e**12*x**3 + 900*d
**2*e**13*x**4 + 360*d*e**14*x**5 + 60*e**15*x**6) + 1008000*d**7*e*x*log(d
/e + x)/(60*d**6*e**9 + 360*d**5*e**10*x + 900*d**4*e**11*x**2 + 1200*d**3*
e**12*x**3 + 900*d**2*e**13*x**4 + 360*d*e**14*x**5 + 60*e**15*x**6) + 2301
600*d**7*e*x/(60*d**6*e**9 + 360*d**5*e**10*x + 900*d**4*e**11*x**2 + 1200*
d**3*e**12*x**3 + 900*d**2*e**13*x**4 + 360*d*e**14*x**5 + 60*e**15*x**6) +
 18900*d**7*e*log(d/e + x)/(60*d**6*e**9 + 360*d**5*e**10*x + 900*d**4*e**1
1*x**2 + 1200*d**3*e**12*x**3 + 900*d**2*e**13*x**4 + 360*d*e**14*x**5 + 60
*e**15*x**6) + 46305*d**7*e/(60*d**6*e**9 + 360*d**5*e**10*x + 900*d**4*e**
11*x**2 + 1200*d**3*e**12*x**3 + 900*d**2*e**13*x**4 + 360*d*e**14*x**5 + 6
0*e**15*x**6) + 2520000*d**6*e**2*x**2*log(d/e + x)/(60*d**6*e**9 + 360*d**
5*e**10*x + 900*d**4*e**11*x**2 + 1200*d**3*e**12*x**3 + 900*d**2*e**13*x**
4 + 360*d*e**14*x**5 + 60*e**15*x**6) + 5250000*d**6*e**2*x**2/(60*d**6*e**
9 + 360*d**5*e**10*x + 900*d**4*e**11*x**2 + 1200*d**3*e**12*x**3 + 900*d**
2*e**13*x**4 + 360*d*e**14*x**5 + 60*e**15*x**6) + 113400*d**6*e**2*x*log(d
/e + x)/(60*d**6*e**9 + 360*d**5*e**10*x + 900*d**4*e**11*x**2 + 1200*d**3*
e**12*x**3 + 900*d**2*e**13*x**4 + 360*d*e**14*x**5 + 60*e**15*x**6) + 2589
30*d**6*e**2*x/(60*d**6*e**9 + 360*d**5*e**10*x + 900*d**4*e**11*x**2 + 120
0*d**3*e**12*x**3 + 900*d**2*e**13*x**4 + 360*d*e**14*x**5 + 60*e**15*x**6)
 + 6660*d**6*e**2*log(d/e + x)/(60*d**6*e**9 + 360*d**5*e**10*x + 900*d**4*
```

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e**11*x**2 + 1200*d**3*e**12*x**3 + 900*d**2*e**13*x**4 + 360*d*e**14*x**5
+ 60*e**15*x**6) + 16317*d**6*e**2/(60*d**6*e**9 + 360*d**5*e**10*x + 900*d
**4*e**11*x**2 + 1200*d**3*e**12*x**3 + 900*d**2*e**13*x**4 + 360*d*e**14*x
**5 + 60*e**15*x**6) + 3360000*d**5*e**3*x**3*log(d/e + x)/(60*d**6*e**9 +
360*d**5*e**10*x + 900*d**4*e**11*x**2 + 1200*d**3*e**12*x**3 + 900*d**2*e*
*13*x**4 + 360*d*e**14*x**5 + 60*e**15*x**6) + 6160000*d**5*e**3*x**3/(60*d
**6*e**9 + 360*d**5*e**10*x + 900*d**4*e**11*x**2 + 1200*d**3*e**12*x**3 +
900*d**2*e**13*x**4 + 360*d*e**14*x**5 + 60*e**15*x**6) + 283500*d**5*e**3*
x**2*log(d/e + x)/(60*d**6*e**9 + 360*d**5*e**10*x + 900*d**4*e**11*x**2 +
1200*d**3*e**12*x**3 + 900*d**2*e**13*x**4 + 360*d*e**14*x**5 + 60*e**15*x*
*6) + 590625*d**5*e**3*x**2/(60*d**6*e**9 + 360*d**5*e**10*x + 900*d**4*e**
11*x**2 + 1200*d**3*e**12*x**3 + 900*d**2*e**13*x**4 + 360*d*e**14*x**5 + 6
0*e**15*x**6) + 39960*d**5*e**3*x*log(d/e + x)/(60*d**6*e**9 + 360*d**5*e**
10*x + 900*d**4*e**11*x**2 + 1200*d**3*e**12*x**3 + 900*d**2*e**13*x**4 + 3
60*d*e**14*x**5 + 60*e**15*x**6) + 91242*d**5*e**3*x/(60*d**6*e**9 + 360*d*
*5*e**10*x + 900*d**4*e**11*x**2 + 1200*d**3*e**12*x**3 + 900*d**2*e**13*x*
*4 + 360*d*e**14*x**5 + 60*e**15*x**6) + 370*d**5*e**3/(60*d**6*e**9 + 360*
d**5*e**10*x + 900*d**4*e**11*x**2 + 1200*d**3*e**12*x**3 + 900*d**2*e**13*
x**4 + 360*d*e**14*x**5 + 60*e**15*x**6) + 2520000*d**4*e**4*x**4*log(d/e +
 x)/(60*d**6*e**9 + 360*d**5*e**10*x + 900*d**4*e**11*x**2 + 1200*d**3*e**1
2*x**3 + 900*d**2*e**13*x**4 + 360*d*e**14*x**5 + 60*e**15*x**6) + 3780000*
d**4*e**4*x**4/(60*d**6*e**9 + 360*d**5*e**10*x + 900*d**4*e**11*x**2 + 120
0*d**3*e**12*x**3 + 900*d**2*e**13*x**4 + 360*d*e**14*x**5 + 60*e**15*x**6)
 + 378000*d**4*e**4*x**3*log(d/e + x)/(60*d**6*e**9 + 360*d**5*e**10*x + 90
0*d**4*e**11*x**2 + 1200*d**3*e**12*x**3 + 900*d**2*e**13*x**4 + 360*d*e**1
4*x**5 + 60*e**15*x**6) + 693000*d**4*e**4*x**3/(60*d**6*e**9 + 360*d**5*e*
*10*x + 900*d**4*e**11*x**2 + 1200*d**3*e**12*x**3 + 900*d**2*e**13*x**4 +
360*d*e**14*x**5 + 60*e**15*x**6) + 99900*d**4*e**4*x**2*log(d/e + x)/(60*d
**6*e**9 + 360*d**5*e**10*x + 900*d**4*e**11*x**2 + 1200*d**3*e**12*x**3 +
900*d**2*e**13*x**4 + 360*d*e**14*x**5 + 60*e**15*x**6) + 208125*d**4*e**4*
x**2/(60*d**6*e**9 + 360*d**5*e**10*x + 900*d**4*e**11*x**2 + 1200*d**3*e**
12*x**3 + 900*d**2*e**13*x**4 + 360*d*e**14*x**5 + 60*e**15*x**6) + 2220*d*
*4*e**4*x/(60*d**6*e**9 + 360*d**5*e**10*x + 900*d**4*e**11*x**2 + 1200*d**
3*e**12*x**3 + 900*d**2*e**13*x**4 + 360*d*e**14*x**5 + 60*e**15*x**6) - 29
6*d**4*e**4/(60*d**6*e**9 + 360*d**5*e**10*x + 900*d**4*e**11*x**2 + 1200*d
**3*e**12*x**3 + 900*d**2*e**13*x**4 + 360*d*e**14*x**5 + 60*e**15*x**6) +
1008000*d**3*e**5*x**5*log(d/e + x)/(60*d**6*e**9 + 360*d**5*e**10*x + 900*
d**4*e**11*x**2 + 1200*d**3*e**12*x**3 + 900*d**2*e**13*x**4 + 360*d*e**14*
x**5 + 60*e**15*x**6) + 1008000*d**3*e**5*x**5/(60*d**6*e**9 + 360*d**5*e**
10*x + 900*d**4*e**11*x**2 + 1200*d**3*e**12*x**3 + 900*d**2*e**13*x**4 + 3
60*d*e**14*x**5 + 60*e**15*x**6) + 283500*d**3*e**5*x**4*log(d/e + x)/(60*d
**6*e**9 + 360*d**5*e**10*x + 900*d**4*e**11*x**2 + 1200*d**3*e**12*x**3 +
900*d**2*e**13*x**4 + 360*d*e**14*x**5 + 60*e**15*x**6) + 425250*d**3*e**5*
x**4/(60*d**6*e**9 + 360*d**5*e**10*x + 900*d**4*e**11*x**2 + 1200*d**3*e**
12*x**3 + 900*d**2*e**13*x**4 + 360*d*e**14*x**5 + 60*e**15*x**6) + 133200*
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d**3*e**5*x**3*log(d/e + x)/(60*d**6*e**9 + 360*d**5*e**10*x + 900*d**4*e**
11*x**2 + 1200*d**3*e**12*x**3 + 900*d**2*e**13*x**4 + 360*d*e**14*x**5 + 6
0*e**15*x**6) + 244200*d**3*e**5*x**3/(60*d**6*e**9 + 360*d**5*e**10*x + 90
0*d**4*e**11*x**2 + 1200*d**3*e**12*x**3 + 900*d**2*e**13*x**4 + 360*d*e**1
4*x**5 + 60*e**15*x**6) + 5550*d**3*e**5*x**2/(60*d**6*e**9 + 360*d**5*e**1
0*x + 900*d**4*e**11*x**2 + 1200*d**3*e**12*x**3 + 900*d**2*e**13*x**4 + 36
0*d*e**14*x**5 + 60*e**15*x**6) - 1776*d**3*e**5*x/(60*d**6*e**9 + 360*d**5
*e**10*x + 900*d**4*e**11*x**2 + 1200*d**3*e**12*x**3 + 900*d**2*e**13*x**4
 + 360*d*e**14*x**5 + 60*e**15*x**6) - 65*d**3*e**5/(60*d**6*e**9 + 360*d**
5*e**10*x + 900*d**4*e**11*x**2 + 1200*d**3*e**12*x**3 + 900*d**2*e**13*x**
4 + 360*d*e**14*x**5 + 60*e**15*x**6) + 168000*d**2*e**6*x**6*log(d/e + x)/
(60*d**6*e**9 + 360*d**5*e**10*x + 900*d**4*e**11*x**2 + 1200*d**3*e**12*x*
*3 + 900*d**2*e**13*x**4 + 360*d*e**14*x**5 + 60*e**15*x**6) + 113400*d**2*
e**6*x**5*log(d/e + x)/(60*d**6*e**9 + 360*d**5*e**10*x + 900*d**4*e**11*x*
*2 + 1200*d**3*e**12*x**3 + 900*d**2*e**13*x**4 + 360*d*e**14*x**5 + 60*e**
15*x**6) + 113400*d**2*e**6*x**5/(60*d**6*e**9 + 360*d**5*e**10*x + 900*d**
4*e**11*x**2 + 1200*d**3*e**12*x**3 + 900*d**2*e**13*x**4 + 360*d*e**14*x**
5 + 60*e**15*x**6) + 99900*d**2*e**6*x**4*log(d/e + x)/(60*d**6*e**9 + 360*
d**5*e**10*x + 900*d**4*e**11*x**2 + 1200*d**3*e**12*x**3 + 900*d**2*e**13*
x**4 + 360*d*e**14*x**5 + 60*e**15*x**6) + 149850*d**2*e**6*x**4/(60*d**6*e
**9 + 360*d**5*e**10*x + 900*d**4*e**11*x**2 + 1200*d**3*e**12*x**3 + 900*d
**2*e**13*x**4 + 360*d*e**14*x**5 + 60*e**15*x**6) + 7400*d**2*e**6*x**3/(6
0*d**6*e**9 + 360*d**5*e**10*x + 900*d**4*e**11*x**2 + 1200*d**3*e**12*x**3
 + 900*d**2*e**13*x**4 + 360*d*e**14*x**5 + 60*e**15*x**6) - 4440*d**2*e**6
*x**2/(60*d**6*e**9 + 360*d**5*e**10*x + 900*d**4*e**11*x**2 + 1200*d**3*e*
*12*x**3 + 900*d**2*e**13*x**4 + 360*d*e**14*x**5 + 60*e**15*x**6) - 390*d*
*2*e**6*x/(60*d**6*e**9 + 360*d**5*e**10*x + 900*d**4*e**11*x**2 + 1200*d**
3*e**12*x**3 + 900*d**2*e**13*x**4 + 360*d*e**14*x**5 + 60*e**15*x**6) - 10
7*d**2*e**6/(60*d**6*e**9 + 360*d**5*e**10*x + 900*d**4*e**11*x**2 + 1200*d
**3*e**12*x**3 + 900*d**2*e**13*x**4 + 360*d*e**14*x**5 + 60*e**15*x**6) -
24000*d*e**7*x**7/(60*d**6*e**9 + 360*d**5*e**10*x + 900*d**4*e**11*x**2 +
1200*d**3*e**12*x**3 + 900*d**2*e**13*x**4 + 360*d*e**14*x**5 + 60*e**15*x*
*6) + 18900*d*e**7*x**6*log(d/e + x)/(60*d**6*e**9 + 360*d**5*e**10*x + 900
*d**4*e**11*x**2 + 1200*d**3*e**12*x**3 + 900*d**2*e**13*x**4 + 360*d*e**14
*x**5 + 60*e**15*x**6) + 39960*d*e**7*x**5*log(d/e + x)/(60*d**6*e**9 + 360
*d**5*e**10*x + 900*d**4*e**11*x**2 + 1200*d**3*e**12*x**3 + 900*d**2*e**13
*x**4 + 360*d*e**14*x**5 + 60*e**15*x**6) + 39960*d*e**7*x**5/(60*d**6*e**9
 + 360*d**5*e**10*x + 900*d**4*e**11*x**2 + 1200*d**3*e**12*x**3 + 900*d**2
*e**13*x**4 + 360*d*e**14*x**5 + 60*e**15*x**6) + 5550*d*e**7*x**4/(60*d**6
*e**9 + 360*d**5*e**10*x + 900*d**4*e**11*x**2 + 1200*d**3*e**12*x**3 + 900
*d**2*e**13*x**4 + 360*d*e**14*x**5 + 60*e**15*x**6) - 5920*d*e**7*x**3/(60
*d**6*e**9 + 360*d**5*e**10*x + 900*d**4*e**11*x**2 + 1200*d**3*e**12*x**3
+ 900*d**2*e**13*x**4 + 360*d*e**14*x**5 + 60*e**15*x**6) - 975*d*e**7*x**2
/(60*d**6*e**9 + 360*d**5*e**10*x + 900*d**4*e**11*x**2 + 1200*d**3*e**12*x
**3 + 900*d**2*e**13*x**4 + 360*d*e**14*x**5 + 60*e**15*x**6) - 642*d*e**7*
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x/(60*d**6*e**9 + 360*d**5*e**10*x + 900*d**4*e**11*x**2 + 1200*d**3*e**12*
x**3 + 900*d**2*e**13*x**4 + 360*d*e**14*x**5 + 60*e**15*x**6) - 66*d*e**7/
(60*d**6*e**9 + 360*d**5*e**10*x + 900*d**4*e**11*x**2 + 1200*d**3*e**12*x*
*3 + 900*d**2*e**13*x**4 + 360*d*e**14*x**5 + 60*e**15*x**6) + 3000*e**8*x*
*8/(60*d**6*e**9 + 360*d**5*e**10*x + 900*d**4*e**11*x**2 + 1200*d**3*e**12
*x**3 + 900*d**2*e**13*x**4 + 360*d*e**14*x**5 + 60*e**15*x**6) - 2700*e**8
*x**7/(60*d**6*e**9 + 360*d**5*e**10*x + 900*d**4*e**11*x**2 + 1200*d**3*e*
*12*x**3 + 900*d**2*e**13*x**4 + 360*d*e**14*x**5 + 60*e**15*x**6) + 6660*e
**8*x**6*log(d/e + x)/(60*d**6*e**9 + 360*d**5*e**10*x + 900*d**4*e**11*x**
2 + 1200*d**3*e**12*x**3 + 900*d**2*e**13*x**4 + 360*d*e**14*x**5 + 60*e**1
5*x**6) + 2220*e**8*x**5/(60*d**6*e**9 + 360*d**5*e**10*x + 900*d**4*e**11*
x**2 + 1200*d**3*e**12*x**3 + 900*d**2*e**13*x**4 + 360*d*e**14*x**5 + 60*e
**15*x**6) - 4440*e**8*x**4/(60*d**6*e**9 + 360*d**5*e**10*x + 900*d**4*e**
11*x**2 + 1200*d**3*e**12*x**3 + 900*d**2*e**13*x**4 + 360*d*e**14*x**5 + 6
0*e**15*x**6) - 1300*e**8*x**3/(60*d**6*e**9 + 360*d**5*e**10*x + 900*d**4*
e**11*x**2 + 1200*d**3*e**12*x**3 + 900*d**2*e**13*x**4 + 360*d*e**14*x**5
 + 60*e**15*x**6) - 1605*e**8*x**2/(60*d**6*e**9 + 360*d**5*e**10*x + 900*d*
*4*e**11*x**2 + 1200*d**3*e**12*x**3 + 900*d**2*e**13*x**4 + 360*d*e**14*x*
*5 + 60*e**15*x**6) - 396*e**8*x/(60*d**6*e**9 + 360*d**5*e**10*x + 900*d**
4*e**11*x**2 + 1200*d**3*e**12*x**3 + 900*d**2*e**13*x**4 + 360*d*e**14*x**
5 + 60*e**15*x**6) - 180*e**8/(60*d**6*e**9 + 360*d**5*e**10*x + 900*d**4*e
**11*x**2 + 1200*d**3*e**12*x**3 + 900*d**2*e**13*x**4 + 360*d*e**14*x**5 +
 60*e**15*x**6), Eq(m, -7)), (-336000*d**8*log(d/e + x)/(60*d**5*e**9 + 300
*d**4*e**10*x + 600*d**3*e**11*x**2 + 600*d**2*e**12*x**3 + 300*d*e**13*x**
4 + 60*e**14*x**5) - 767200*d**8/(60*d**5*e**9 + 300*d**4*e**10*x + 600*d**
3*e**11*x**2 + 600*d**2*e**12*x**3 + 300*d*e**13*x**4 + 60*e**14*x**5) - 16
80000*d**7*e*x*log(d/e + x)/(60*d**5*e**9 + 300*d**4*e**10*x + 600*d**3*e**
11*x**2 + 600*d**2*e**12*x**3 + 300*d*e**13*x**4 + 60*e**14*x**5) - 3500000
*d**7*e*x/(60*d**5*e**9 + 300*d**4*e**10*x + 600*d**3*e**11*x**2 + 600*d**2
*e**12*x**3 + 300*d*e**13*x**4 + 60*e**14*x**5) - 56700*d**7*e*log(d/e + x)
/(60*d**5*e**9 + 300*d**4*e**10*x + 600*d**3*e**11*x**2 + 600*d**2*e**12*x*
*3 + 300*d*e**13*x**4 + 60*e**14*x**5) - 129465*d**7*e/(60*d**5*e**9 + 300*
d**4*e**10*x + 600*d**3*e**11*x**2 + 600*d**2*e**12*x**3 + 300*d*e**13*x**4
 + 60*e**14*x**5) - 3360000*d**6*e**2*x**2*log(d/e + x)/(60*d**5*e**9 + 300
*d**4*e**10*x + 600*d**3*e**11*x**2 + 600*d**2*e**12*x**3 + 300*d*e**13*x**
4 + 60*e**14*x**5) - 6160000*d**6*e**2*x**2/(60*d**5*e**9 + 300*d**4*e**10*
x + 600*d**3*e**11*x**2 + 600*d**2*e**12*x**3 + 300*d*e**13*x**4 + 60*e**14
*x**5) - 283500*d**6*e**2*x*log(d/e + x)/(60*d**5*e**9 + 300*d**4*e**10*x +
 600*d**3*e**11*x**2 + 600*d**2*e**12*x**3 + 300*d*e**13*x**4 + 60*e**14*x*
*5) - 590625*d**6*e**2*x/(60*d**5*e**9 + 300*d**4*e**10*x + 600*d**3*e**11*
x**2 + 600*d**2*e**12*x**3 + 300*d*e**13*x**4 + 60*e**14*x**5) - 39960*d**6
*e**2*log(d/e + x)/(60*d**5*e**9 + 300*d**4*e**10*x + 600*d**3*e**11*x**2 +
 600*d**2*e**12*x**3 + 300*d*e**13*x**4 + 60*e**14*x**5) - 91242*d**6*e**2/
(60*d**5*e**9 + 300*d**4*e**10*x + 600*d**3*e**11*x**2 + 600*d**2*e**12*x**
3 + 300*d*e**13*x**4 + 60*e**14*x**5) - 3360000*d**5*e**3*x**3*log(d/e + x)
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/(60*d**5*e**9 + 300*d**4*e**10*x + 600*d**3*e**11*x**2 + 600*d**2*e**12*x*
*3 + 300*d*e**13*x**4 + 60*e**14*x**5) - 5040000*d**5*e**3*x**3/(60*d**5*e*
*9 + 300*d**4*e**10*x + 600*d**3*e**11*x**2 + 600*d**2*e**12*x**3 + 300*d*e
**13*x**4 + 60*e**14*x**5) - 567000*d**5*e**3*x**2*log(d/e + x)/(60*d**5*e*
*9 + 300*d**4*e**10*x + 600*d**3*e**11*x**2 + 600*d**2*e**12*x**3 + 300*d*e
**13*x**4 + 60*e**14*x**5) - 1039500*d**5*e**3*x**2/(60*d**5*e**9 + 300*d**
4*e**10*x + 600*d**3*e**11*x**2 + 600*d**2*e**12*x**3 + 300*d*e**13*x**4 +
60*e**14*x**5) - 199800*d**5*e**3*x*log(d/e + x)/(60*d**5*e**9 + 300*d**4*e
**10*x + 600*d**3*e**11*x**2 + 600*d**2*e**12*x**3 + 300*d*e**13*x**4 + 60*
e**14*x**5) - 416250*d**5*e**3*x/(60*d**5*e**9 + 300*d**4*e**10*x + 600*d**
3*e**11*x**2 + 600*d**2*e**12*x**3 + 300*d*e**13*x**4 + 60*e**14*x**5) - 22
20*d**5*e**3*log(d/e + x)/(60*d**5*e**9 + 300*d**4*e**10*x + 600*d**3*e**11
*x**2 + 600*d**2*e**12*x**3 + 300*d*e**13*x**4 + 60*e**14*x**5) - 5069*d**5
*e**3/(60*d**5*e**9 + 300*d**4*e**10*x + 600*d**3*e**11*x**2 + 600*d**2*e**
12*x**3 + 300*d*e**13*x**4 + 60*e**14*x**5) - 1680000*d**4*e**4*x**4*log(d/
e + x)/(60*d**5*e**9 + 300*d**4*e**10*x + 600*d**3*e**11*x**2 + 600*d**2*e*
*12*x**3 + 300*d*e**13*x**4 + 60*e**14*x**5) - 1680000*d**4*e**4*x**4/(60*d
**5*e**9 + 300*d**4*e**10*x + 600*d**3*e**11*x**2 + 600*d**2*e**12*x**3 + 3
00*d*e**13*x**4 + 60*e**14*x**5) - 567000*d**4*e**4*x**3*log(d/e + x)/(60*d
**5*e**9 + 300*d**4*e**10*x + 600*d**3*e**11*x**2 + 600*d**2*e**12*x**3 + 3
00*d*e**13*x**4 + 60*e**14*x**5) - 850500*d**4*e**4*x**3/(60*d**5*e**9 + 30
0*d**4*e**10*x + 600*d**3*e**11*x**2 + 600*d**2*e**12*x**3 + 300*d*e**13*x*
*4 + 60*e**14*x**5) - 399600*d**4*e**4*x**2*log(d/e + x)/(60*d**5*e**9 + 30
0*d**4*e**10*x + 600*d**3*e**11*x**2 + 600*d**2*e**12*x**3 + 300*d*e**13*x*
*4 + 60*e**14*x**5) - 732600*d**4*e**4*x**2/(60*d**5*e**9 + 300*d**4*e**10*
x + 600*d**3*e**11*x**2 + 600*d**2*e**12*x**3 + 300*d*e**13*x**4 + 60*e**14
*x**5) - 11100*d**4*e**4*x*log(d/e + x)/(60*d**5*e**9 + 300*d**4*e**10*x +
600*d**3*e**11*x**2 + 600*d**2*e**12*x**3 + 300*d*e**13*x**4 + 60*e**14*x**
5) - 23125*d**4*e**4*x/(60*d**5*e**9 + 300*d**4*e**10*x + 600*d**3*e**11*x*
*2 + 600*d**2*e**12*x**3 + 300*d*e**13*x**4 + 60*e**14*x**5) - 1776*d**4*e*
*4/(60*d**5*e**9 + 300*d**4*e**10*x + 600*d**3*e**11*x**2 + 600*d**2*e**12*
x**3 + 300*d*e**13*x**4 + 60*e**14*x**5) - 336000*d**3*e**5*x**5*log(d/e +
x)/(60*d**5*e**9 + 300*d**4*e**10*x + 600*d**3*e**11*x**2 + 600*d**2*e**12*
x**3 + 300*d*e**13*x**4 + 60*e**14*x**5) - 283500*d**3*e**5*x**4*log(d/e +
x)/(60*d**5*e**9 + 300*d**4*e**10*x + 600*d**3*e**11*x**2 + 600*d**2*e**12*
x**3 + 300*d*e**13*x**4 + 60*e**14*x**5) - 283500*d**3*e**5*x**4/(60*d**5*e
**9 + 300*d**4*e**10*x + 600*d**3*e**11*x**2 + 600*d**2*e**12*x**3 + 300*d*
e**13*x**4 + 60*e**14*x**5) - 399600*d**3*e**5*x**3*log(d/e + x)/(60*d**5*e
**9 + 300*d**4*e**10*x + 600*d**3*e**11*x**2 + 600*d**2*e**12*x**3 + 300*d*
e**13*x**4 + 60*e**14*x**5) - 599400*d**3*e**5*x**3/(60*d**5*e**9 + 300*d**
4*e**10*x + 600*d**3*e**11*x**2 + 600*d**2*e**12*x**3 + 300*d*e**13*x**4 +
60*e**14*x**5) - 22200*d**3*e**5*x**2*log(d/e + x)/(60*d**5*e**9 + 300*d**4
*e**10*x + 600*d**3*e**11*x**2 + 600*d**2*e**12*x**3 + 300*d*e**13*x**4 + 6
0*e**14*x**5) - 40700*d**3*e**5*x**2/(60*d**5*e**9 + 300*d**4*e**10*x + 600
*d**3*e**11*x**2 + 600*d**2*e**12*x**3 + 300*d*e**13*x**4 + 60*e**14*x**5)
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- 8880*d**3*e**5*x/(60*d**5*e**9 + 300*d**4*e**10*x + 600*d**3*e**11*x**2 +
 600*d**2*e**12*x**3 + 300*d*e**13*x**4 + 60*e**14*x**5) - 195*d**3*e**5/(6
0*d**5*e**9 + 300*d**4*e**10*x + 600*d**3*e**11*x**2 + 600*d**2*e**12*x**3
+ 300*d*e**13*x**4 + 60*e**14*x**5) + 56000*d**2*e**6*x**6/(60*d**5*e**9 +
300*d**4*e**10*x + 600*d**3*e**11*x**2 + 600*d**2*e**12*x**3 + 300*d*e**13*
x**4 + 60*e**14*x**5) - 56700*d**2*e**6*x**5*log(d/e + x)/(60*d**5*e**9 + 3
00*d**4*e**10*x + 600*d**3*e**11*x**2 + 600*d**2*e**12*x**3 + 300*d*e**13*x
**4 + 60*e**14*x**5) - 199800*d**2*e**6*x**4*log(d/e + x)/(60*d**5*e**9 + 3
00*d**4*e**10*x + 600*d**3*e**11*x**2 + 600*d**2*e**12*x**3 + 300*d*e**13*x
**4 + 60*e**14*x**5) - 199800*d**2*e**6*x**4/(60*d**5*e**9 + 300*d**4*e**10
*x + 600*d**3*e**11*x**2 + 600*d**2*e**12*x**3 + 300*d*e**13*x**4 + 60*e**1
4*x**5) - 22200*d**2*e**6*x**3*log(d/e + x)/(60*d**5*e**9 + 300*d**4*e**10*
x + 600*d**3*e**11*x**2 + 600*d**2*e**12*x**3 + 300*d*e**13*x**4 + 60*e**14
*x**5) - 33300*d**2*e**6*x**3/(60*d**5*e**9 + 300*d**4*e**10*x + 600*d**3*e
**11*x**2 + 600*d**2*e**12*x**3 + 300*d*e**13*x**4 + 60*e**14*x**5) - 17760
*d**2*e**6*x**2/(60*d**5*e**9 + 300*d**4*e**10*x + 600*d**3*e**11*x**2 + 60
0*d**2*e**12*x**3 + 300*d*e**13*x**4 + 60*e**14*x**5) - 975*d**2*e**6*x/(60
*d**5*e**9 + 300*d**4*e**10*x + 600*d**3*e**11*x**2 + 600*d**2*e**12*x**3 +
 300*d*e**13*x**4 + 60*e**14*x**5) - 214*d**2*e**6/(60*d**5*e**9 + 300*d**4
*e**10*x + 600*d**3*e**11*x**2 + 600*d**2*e**12*x**3 + 300*d*e**13*x**4 + 6
0*e**14*x**5) - 8000*d*e**7*x**7/(60*d**5*e**9 + 300*d**4*e**10*x + 600*d**
3*e**11*x**2 + 600*d**2*e**12*x**3 + 300*d*e**13*x**4 + 60*e**14*x**5) + 94
50*d*e**7*x**6/(60*d**5*e**9 + 300*d**4*e**10*x + 600*d**3*e**11*x**2 + 600
*d**2*e**12*x**3 + 300*d*e**13*x**4 + 60*e**14*x**5) - 39960*d*e**7*x**5*lo
g(d/e + x)/(60*d**5*e**9 + 300*d**4*e**10*x + 600*d**3*e**11*x**2 + 600*d**
2*e**12*x**3 + 300*d*e**13*x**4 + 60*e**14*x**5) - 11100*d*e**7*x**4*log(d/
e + x)/(60*d**5*e**9 + 300*d**4*e**10*x + 600*d**3*e**11*x**2 + 600*d**2*e*
*12*x**3 + 300*d*e**13*x**4 + 60*e**14*x**5) - 11100*d*e**7*x**4/(60*d**5*e
**9 + 300*d**4*e**10*x + 600*d**3*e**11*x**2 + 600*d**2*e**12*x**3 + 300*d*
e**13*x**4 + 60*e**14*x**5) - 17760*d*e**7*x**3/(60*d**5*e**9 + 300*d**4*e*
*10*x + 600*d**3*e**11*x**2 + 600*d**2*e**12*x**3 + 300*d*e**13*x**4 + 60*e
**14*x**5) - 1950*d*e**7*x**2/(60*d**5*e**9 + 300*d**4*e**10*x + 600*d**3*e
**11*x**2 + 600*d**2*e**12*x**3 + 300*d*e**13*x**4 + 60*e**14*x**5) - 1070*
d*e**7*x/(60*d**5*e**9 + 300*d**4*e**10*x + 600*d**3*e**11*x**2 + 600*d**2*
e**12*x**3 + 300*d*e**13*x**4 + 60*e**14*x**5) - 99*d*e**7/(60*d**5*e**9 +
300*d**4*e**10*x + 600*d**3*e**11*x**2 + 600*d**2*e**12*x**3 + 300*d*e**13*
x**4 + 60*e**14*x**5) + 2000*e**8*x**8/(60*d**5*e**9 + 300*d**4*e**10*x + 6
00*d**3*e**11*x**2 + 600*d**2*e**12*x**3 + 300*d*e**13*x**4 + 60*e**14*x**5
) - 1350*e**8*x**7/(60*d**5*e**9 + 300*d**4*e**10*x + 600*d**3*e**11*x**2 +
 600*d**2*e**12*x**3 + 300*d*e**13*x**4 + 60*e**14*x**5) + 6660*e**8*x**6/(
60*d**5*e**9 + 300*d**4*e**10*x + 600*d**3*e**11*x**2 + 600*d**2*e**12*x**3
 + 300*d*e**13*x**4 + 60*e**14*x**5) - 2220*e**8*x**5*log(d/e + x)/(60*d**5
*e**9 + 300*d**4*e**10*x + 600*d**3*e**11*x**2 + 600*d**2*e**12*x**3 + 300*
d*e**13*x**4 + 60*e**14*x**5) - 8880*e**8*x**4/(60*d**5*e**9 + 300*d**4*e**
10*x + 600*d**3*e**11*x**2 + 600*d**2*e**12*x**3 + 300*d*e**13*x**4 + 60*e*
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*14*x**5) - 1950*e**8*x**3/(60*d**5*e**9 + 300*d**4*e**10*x + 600*d**3*e**1
1*x**2 + 600*d**2*e**12*x**3 + 300*d*e**13*x**4 + 60*e**14*x**5) - 2140*e**
8*x**2/(60*d**5*e**9 + 300*d**4*e**10*x + 600*d**3*e**11*x**2 + 600*d**2*e*
*12*x**3 + 300*d*e**13*x**4 + 60*e**14*x**5) - 495*e**8*x/(60*d**5*e**9 + 3
00*d**4*e**10*x + 600*d**3*e**11*x**2 + 600*d**2*e**12*x**3 + 300*d*e**13*x
**4 + 60*e**14*x**5) - 216*e**8/(60*d**5*e**9 + 300*d**4*e**10*x + 600*d**3
*e**11*x**2 + 600*d**2*e**12*x**3 + 300*d*e**13*x**4 + 60*e**14*x**5), Eq(m
, -6)), (84000*d**8*log(d/e + x)/(12*d**4*e**9 + 48*d**3*e**10*x + 72*d**2*
e**11*x**2 + 48*d*e**12*x**3 + 12*e**13*x**4) + 175000*d**8/(12*d**4*e**9 +
 48*d**3*e**10*x + 72*d**2*e**11*x**2 + 48*d*e**12*x**3 + 12*e**13*x**4) +
336000*d**7*e*x*log(d/e + x)/(12*d**4*e**9 + 48*d**3*e**10*x + 72*d**2*e**1
1*x**2 + 48*d*e**12*x**3 + 12*e**13*x**4) + 616000*d**7*e*x/(12*d**4*e**9 +
 48*d**3*e**10*x + 72*d**2*e**11*x**2 + 48*d*e**12*x**3 + 12*e**13*x**4) +
18900*d**7*e*log(d/e + x)/(12*d**4*e**9 + 48*d**3*e**10*x + 72*d**2*e**11*x
**2 + 48*d*e**12*x**3 + 12*e**13*x**4) + 39375*d**7*e/(12*d**4*e**9 + 48*d*
*3*e**10*x + 72*d**2*e**11*x**2 + 48*d*e**12*x**3 + 12*e**13*x**4) + 504000
*d**6*e**2*x**2*log(d/e + x)/(12*d**4*e**9 + 48*d**3*e**10*x + 72*d**2*e**1
1*x**2 + 48*d*e**12*x**3 + 12*e**13*x**4) + 756000*d**6*e**2*x**2/(12*d**4*
e**9 + 48*d**3*e**10*x + 72*d**2*e**11*x**2 + 48*d*e**12*x**3 + 12*e**13*x*
*4) + 75600*d**6*e**2*x*log(d/e + x)/(12*d**4*e**9 + 48*d**3*e**10*x + 72*d
**2*e**11*x**2 + 48*d*e**12*x**3 + 12*e**13*x**4) + 138600*d**6*e**2*x/(12*
d**4*e**9 + 48*d**3*e**10*x + 72*d**2*e**11*x**2 + 48*d*e**12*x**3 + 12*e**
13*x**4) + 19980*d**6*e**2*log(d/e + x)/(12*d**4*e**9 + 48*d**3*e**10*x + 7
2*d**2*e**11*x**2 + 48*d*e**12*x**3 + 12*e**13*x**4) + 41625*d**6*e**2/(12*
d**4*e**9 + 48*d**3*e**10*x + 72*d**2*e**11*x**2 + 48*d*e**12*x**3 + 12*e**
13*x**4) + 336000*d**5*e**3*x**3*log(d/e + x)/(12*d**4*e**9 + 48*d**3*e**10
*x + 72*d**2*e**11*x**2 + 48*d*e**12*x**3 + 12*e**13*x**4) + 336000*d**5*e*
*3*x**3/(12*d**4*e**9 + 48*d**3*e**10*x + 72*d**2*e**11*x**2 + 48*d*e**12*x
**3 + 12*e**13*x**4) + 113400*d**5*e**3*x**2*log(d/e + x)/(12*d**4*e**9 + 4
8*d**3*e**10*x + 72*d**2*e**11*x**2 + 48*d*e**12*x**3 + 12*e**13*x**4) + 17
0100*d**5*e**3*x**2/(12*d**4*e**9 + 48*d**3*e**10*x + 72*d**2*e**11*x**2 +
48*d*e**12*x**3 + 12*e**13*x**4) + 79920*d**5*e**3*x*log(d/e + x)/(12*d**4*
e**9 + 48*d**3*e**10*x + 72*d**2*e**11*x**2 + 48*d*e**12*x**3 + 12*e**13*x*
*4) + 146520*d**5*e**3*x/(12*d**4*e**9 + 48*d**3*e**10*x + 72*d**2*e**11*x*
*2 + 48*d*e**12*x**3 + 12*e**13*x**4) + 2220*d**5*e**3*log(d/e + x)/(12*d**
4*e**9 + 48*d**3*e**10*x + 72*d**2*e**11*x**2 + 48*d*e**12*x**3 + 12*e**13*
x**4) + 4625*d**5*e**3/(12*d**4*e**9 + 48*d**3*e**10*x + 72*d**2*e**11*x**2
 + 48*d*e**12*x**3 + 12*e**13*x**4) + 84000*d**4*e**4*x**4*log(d/e + x)/(12
*d**4*e**9 + 48*d**3*e**10*x + 72*d**2*e**11*x**2 + 48*d*e**12*x**3 + 12*e*
*13*x**4) + 75600*d**4*e**4*x**3*log(d/e + x)/(12*d**4*e**9 + 48*d**3*e**10
*x + 72*d**2*e**11*x**2 + 48*d*e**12*x**3 + 12*e**13*x**4) + 75600*d**4*e**
4*x**3/(12*d**4*e**9 + 48*d**3*e**10*x + 72*d**2*e**11*x**2 + 48*d*e**12*x*
*3 + 12*e**13*x**4) + 119880*d**4*e**4*x**2*log(d/e + x)/(12*d**4*e**9 + 48
*d**3*e**10*x + 72*d**2*e**11*x**2 + 48*d*e**12*x**3 + 12*e**13*x**4) + 179
820*d**4*e**4*x**2/(12*d**4*e**9 + 48*d**3*e**10*x + 72*d**2*e**11*x**2 + 4
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8*d*e**12*x**3 + 12*e**13*x**4) + 8880*d**4*e**4*x*log(d/e + x)/(12*d**4*e*
*9 + 48*d**3*e**10*x + 72*d**2*e**11*x**2 + 48*d*e**12*x**3 + 12*e**13*x**4
) + 16280*d**4*e**4*x/(12*d**4*e**9 + 48*d**3*e**10*x + 72*d**2*e**11*x**2
+ 48*d*e**12*x**3 + 12*e**13*x**4) + 1776*d**4*e**4*log(d/e + x)/(12*d**4*e
**9 + 48*d**3*e**10*x + 72*d**2*e**11*x**2 + 48*d*e**12*x**3 + 12*e**13*x**
4) + 3700*d**4*e**4/(12*d**4*e**9 + 48*d**3*e**10*x + 72*d**2*e**11*x**2 +
48*d*e**12*x**3 + 12*e**13*x**4) - 16800*d**3*e**5*x**5/(12*d**4*e**9 + 48*
d**3*e**10*x + 72*d**2*e**11*x**2 + 48*d*e**12*x**3 + 12*e**13*x**4) + 1890
0*d**3*e**5*x**4*log(d/e + x)/(12*d**4*e**9 + 48*d**3*e**10*x + 72*d**2*e**
11*x**2 + 48*d*e**12*x**3 + 12*e**13*x**4) + 79920*d**3*e**5*x**3*log(d/e +
 x)/(12*d**4*e**9 + 48*d**3*e**10*x + 72*d**2*e**11*x**2 + 48*d*e**12*x**3
+ 12*e**13*x**4) + 79920*d**3*e**5*x**3/(12*d**4*e**9 + 48*d**3*e**10*x + 7
2*d**2*e**11*x**2 + 48*d*e**12*x**3 + 12*e**13*x**4) + 13320*d**3*e**5*x**2
*log(d/e + x)/(12*d**4*e**9 + 48*d**3*e**10*x + 72*d**2*e**11*x**2 + 48*d*e
**12*x**3 + 12*e**13*x**4) + 19980*d**3*e**5*x**2/(12*d**4*e**9 + 48*d**3*e
**10*x + 72*d**2*e**11*x**2 + 48*d*e**12*x**3 + 12*e**13*x**4) + 7104*d**3*
e**5*x*log(d/e + x)/(12*d**4*e**9 + 48*d**3*e**10*x + 72*d**2*e**11*x**2 +
48*d*e**12*x**3 + 12*e**13*x**4) + 13024*d**3*e**5*x/(12*d**4*e**9 + 48*d**
3*e**10*x + 72*d**2*e**11*x**2 + 48*d*e**12*x**3 + 12*e**13*x**4) - 195*d**
3*e**5/(12*d**4*e**9 + 48*d**3*e**10*x + 72*d**2*e**11*x**2 + 48*d*e**12*x*
*3 + 12*e**13*x**4) + 2800*d**2*e**6*x**6/(12*d**4*e**9 + 48*d**3*e**10*x +
 72*d**2*e**11*x**2 + 48*d*e**12*x**3 + 12*e**13*x**4) - 3780*d**2*e**6*x**
5/(12*d**4*e**9 + 48*d**3*e**10*x + 72*d**2*e**11*x**2 + 48*d*e**12*x**3 +
12*e**13*x**4) + 19980*d**2*e**6*x**4*log(d/e + x)/(12*d**4*e**9 + 48*d**3*
e**10*x + 72*d**2*e**11*x**2 + 48*d*e**12*x**3 + 12*e**13*x**4) + 8880*d**2
*e**6*x**3*log(d/e + x)/(12*d**4*e**9 + 48*d**3*e**10*x + 72*d**2*e**11*x**
2 + 48*d*e**12*x**3 + 12*e**13*x**4) + 8880*d**2*e**6*x**3/(12*d**4*e**9 +
48*d**3*e**10*x + 72*d**2*e**11*x**2 + 48*d*e**12*x**3 + 12*e**13*x**4) + 1
0656*d**2*e**6*x**2*log(d/e + x)/(12*d**4*e**9 + 48*d**3*e**10*x + 72*d**2*
e**11*x**2 + 48*d*e**12*x**3 + 12*e**13*x**4) + 15984*d**2*e**6*x**2/(12*d*
*4*e**9 + 48*d**3*e**10*x + 72*d**2*e**11*x**2 + 48*d*e**12*x**3 + 12*e**13
*x**4) - 780*d**2*e**6*x/(12*d**4*e**9 + 48*d**3*e**10*x + 72*d**2*e**11*x*
*2 + 48*d*e**12*x**3 + 12*e**13*x**4) - 107*d**2*e**6/(12*d**4*e**9 + 48*d*
*3*e**10*x + 72*d**2*e**11*x**2 + 48*d*e**12*x**3 + 12*e**13*x**4) - 800*d*
e**7*x**7/(12*d**4*e**9 + 48*d**3*e**10*x + 72*d**2*e**11*x**2 + 48*d*e**12
*x**3 + 12*e**13*x**4) + 630*d*e**7*x**6/(12*d**4*e**9 + 48*d**3*e**10*x +
72*d**2*e**11*x**2 + 48*d*e**12*x**3 + 12*e**13*x**4) - 3996*d*e**7*x**5/(1
2*d**4*e**9 + 48*d**3*e**10*x + 72*d**2*e**11*x**2 + 48*d*e**12*x**3 + 12*e
**13*x**4) + 2220*d*e**7*x**4*log(d/e + x)/(12*d**4*e**9 + 48*d**3*e**10*x
+ 72*d**2*e**11*x**2 + 48*d*e**12*x**3 + 12*e**13*x**4) + 7104*d*e**7*x**3*
log(d/e + x)/(12*d**4*e**9 + 48*d**3*e**10*x + 72*d**2*e**11*x**2 + 48*d*e*
*12*x**3 + 12*e**13*x**4) + 7104*d*e**7*x**3/(12*d**4*e**9 + 48*d**3*e**10*
x + 72*d**2*e**11*x**2 + 48*d*e**12*x**3 + 12*e**13*x**4) - 1170*d*e**7*x**
2/(12*d**4*e**9 + 48*d**3*e**10*x + 72*d**2*e**11*x**2 + 48*d*e**12*x**3 +
12*e**13*x**4) - 428*d*e**7*x/(12*d**4*e**9 + 48*d**3*e**10*x + 72*d**2*e**
```

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11*x**2 + 48*d*e**12*x**3 + 12*e**13*x**4) - 33*d*e**7/(12*d**4*e**9 + 48*d
**3*e**10*x + 72*d**2*e**11*x**2 + 48*d*e**12*x**3 + 12*e**13*x**4) + 300*e
**8*x**8/(12*d**4*e**9 + 48*d**3*e**10*x + 72*d**2*e**11*x**2 + 48*d*e**12*
x**3 + 12*e**13*x**4) - 180*e**8*x**7/(12*d**4*e**9 + 48*d**3*e**10*x + 72*
d**2*e**11*x**2 + 48*d*e**12*x**3 + 12*e**13*x**4) + 666*e**8*x**6/(12*d**4
*e**9 + 48*d**3*e**10*x + 72*d**2*e**11*x**2 + 48*d*e**12*x**3 + 12*e**13*x
**4) - 444*e**8*x**5/(12*d**4*e**9 + 48*d**3*e**10*x + 72*d**2*e**11*x**2 +
 48*d*e**12*x**3 + 12*e**13*x**4) + 1776*e**8*x**4*log(d/e + x)/(12*d**4*e*
*9 + 48*d**3*e**10*x + 72*d**2*e**11*x**2 + 48*d*e**12*x**3 + 12*e**13*x**4
) - 780*e**8*x**3/(12*d**4*e**9 + 48*d**3*e**10*x + 72*d**2*e**11*x**2 + 48
*d*e**12*x**3 + 12*e**13*x**4) - 642*e**8*x**2/(12*d**4*e**9 + 48*d**3*e**1
0*x + 72*d**2*e**11*x**2 + 48*d*e**12*x**3 + 12*e**13*x**4) - 132*e**8*x/(1
2*d**4*e**9 + 48*d**3*e**10*x + 72*d**2*e**11*x**2 + 48*d*e**12*x**3 + 12*e
**13*x**4) - 54*e**8/(12*d**4*e**9 + 48*d**3*e**10*x + 72*d**2*e**11*x**2 +
 48*d*e**12*x**3 + 12*e**13*x**4), Eq(m, -5)), (-67200*d**8*log(d/e + x)/(1
2*d**3*e**9 + 36*d**2*e**10*x + 36*d*e**11*x**2 + 12*e**12*x**3) - 123200*d
**8/(12*d**3*e**9 + 36*d**2*e**10*x + 36*d*e**11*x**2 + 12*e**12*x**3) - 20
1600*d**7*e*x*log(d/e + x)/(12*d**3*e**9 + 36*d**2*e**10*x + 36*d*e**11*x**
2 + 12*e**12*x**3) - 302400*d**7*e*x/(12*d**3*e**9 + 36*d**2*e**10*x + 36*d
*e**11*x**2 + 12*e**12*x**3) - 18900*d**7*e*log(d/e + x)/(12*d**3*e**9 + 36
*d**2*e**10*x + 36*d*e**11*x**2 + 12*e**12*x**3) - 34650*d**7*e/(12*d**3*e*
*9 + 36*d**2*e**10*x + 36*d*e**11*x**2 + 12*e**12*x**3) - 201600*d**6*e**2*
x**2*log(d/e + x)/(12*d**3*e**9 + 36*d**2*e**10*x + 36*d*e**11*x**2 + 12*e*
*12*x**3) - 201600*d**6*e**2*x**2/(12*d**3*e**9 + 36*d**2*e**10*x + 36*d*e*
*11*x**2 + 12*e**12*x**3) - 56700*d**6*e**2*x*log(d/e + x)/(12*d**3*e**9 +
36*d**2*e**10*x + 36*d*e**11*x**2 + 12*e**12*x**3) - 85050*d**6*e**2*x/(12*
d**3*e**9 + 36*d**2*e**10*x + 36*d*e**11*x**2 + 12*e**12*x**3) - 26640*d**6
*e**2*log(d/e + x)/(12*d**3*e**9 + 36*d**2*e**10*x + 36*d*e**11*x**2 + 12*e
**12*x**3) - 48840*d**6*e**2/(12*d**3*e**9 + 36*d**2*e**10*x + 36*d*e**11*x
**2 + 12*e**12*x**3) - 67200*d**5*e**3*x**3*log(d/e + x)/(12*d**3*e**9 + 36
*d**2*e**10*x + 36*d*e**11*x**2 + 12*e**12*x**3) - 56700*d**5*e**3*x**2*log
(d/e + x)/(12*d**3*e**9 + 36*d**2*e**10*x + 36*d*e**11*x**2 + 12*e**12*x**3
) - 56700*d**5*e**3*x**2/(12*d**3*e**9 + 36*d**2*e**10*x + 36*d*e**11*x**2
+ 12*e**12*x**3) - 79920*d**5*e**3*x*log(d/e + x)/(12*d**3*e**9 + 36*d**2*e
**10*x + 36*d*e**11*x**2 + 12*e**12*x**3) - 119880*d**5*e**3*x/(12*d**3*e**
9 + 36*d**2*e**10*x + 36*d*e**11*x**2 + 12*e**12*x**3) - 4440*d**5*e**3*log
(d/e + x)/(12*d**3*e**9 + 36*d**2*e**10*x + 36*d*e**11*x**2 + 12*e**12*x**3
) - 8140*d**5*e**3/(12*d**3*e**9 + 36*d**2*e**10*x + 36*d*e**11*x**2 + 12*e
**12*x**3) + 16800*d**4*e**4*x**4/(12*d**3*e**9 + 36*d**2*e**10*x + 36*d*e*
*11*x**2 + 12*e**12*x**3) - 18900*d**4*e**4*x**3*log(d/e + x)/(12*d**3*e**9
 + 36*d**2*e**10*x + 36*d*e**11*x**2 + 12*e**12*x**3) - 79920*d**4*e**4*x**
2*log(d/e + x)/(12*d**3*e**9 + 36*d**2*e**10*x + 36*d*e**11*x**2 + 12*e**12
*x**3) - 79920*d**4*e**4*x**2/(12*d**3*e**9 + 36*d**2*e**10*x + 36*d*e**11*
x**2 + 12*e**12*x**3) - 13320*d**4*e**4*x*log(d/e + x)/(12*d**3*e**9 + 36*d
**2*e**10*x + 36*d*e**11*x**2 + 12*e**12*x**3) - 19980*d**4*e**4*x/(12*d**3
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*e**9 + 36*d**2*e**10*x + 36*d*e**11*x**2 + 12*e**12*x**3) - 7104*d**4*e**4
*log(d/e + x)/(12*d**3*e**9 + 36*d**2*e**10*x + 36*d*e**11*x**2 + 12*e**12*
x**3) - 13024*d**4*e**4/(12*d**3*e**9 + 36*d**2*e**10*x + 36*d*e**11*x**2 +
 12*e**12*x**3) - 3360*d**3*e**5*x**5/(12*d**3*e**9 + 36*d**2*e**10*x + 36*
d*e**11*x**2 + 12*e**12*x**3) + 4725*d**3*e**5*x**4/(12*d**3*e**9 + 36*d**2
*e**10*x + 36*d*e**11*x**2 + 12*e**12*x**3) - 26640*d**3*e**5*x**3*log(d/e
+ x)/(12*d**3*e**9 + 36*d**2*e**10*x + 36*d*e**11*x**2 + 12*e**12*x**3) - 1
3320*d**3*e**5*x**2*log(d/e + x)/(12*d**3*e**9 + 36*d**2*e**10*x + 36*d*e**
11*x**2 + 12*e**12*x**3) - 13320*d**3*e**5*x**2/(12*d**3*e**9 + 36*d**2*e**
10*x + 36*d*e**11*x**2 + 12*e**12*x**3) - 21312*d**3*e**5*x*log(d/e + x)/(1
2*d**3*e**9 + 36*d**2*e**10*x + 36*d*e**11*x**2 + 12*e**12*x**3) - 31968*d*
*3*e**5*x/(12*d**3*e**9 + 36*d**2*e**10*x + 36*d*e**11*x**2 + 12*e**12*x**3
) + 780*d**3*e**5*log(d/e + x)/(12*d**3*e**9 + 36*d**2*e**10*x + 36*d*e**11
*x**2 + 12*e**12*x**3) + 1430*d**3*e**5/(12*d**3*e**9 + 36*d**2*e**10*x + 3
6*d*e**11*x**2 + 12*e**12*x**3) + 1120*d**2*e**6*x**6/(12*d**3*e**9 + 36*d*
*2*e**10*x + 36*d*e**11*x**2 + 12*e**12*x**3) - 945*d**2*e**6*x**5/(12*d**3
*e**9 + 36*d**2*e**10*x + 36*d*e**11*x**2 + 12*e**12*x**3) + 6660*d**2*e**6
*x**4/(12*d**3*e**9 + 36*d**2*e**10*x + 36*d*e**11*x**2 + 12*e**12*x**3) -
4440*d**2*e**6*x**3*log(d/e + x)/(12*d**3*e**9 + 36*d**2*e**10*x + 36*d*e**
11*x**2 + 12*e**12*x**3) - 21312*d**2*e**6*x**2*log(d/e + x)/(12*d**3*e**9
+ 36*d**2*e**10*x + 36*d*e**11*x**2 + 12*e**12*x**3) - 21312*d**2*e**6*x**2
/(12*d**3*e**9 + 36*d**2*e**10*x + 36*d*e**11*x**2 + 12*e**12*x**3) + 2340*
d**2*e**6*x*log(d/e + x)/(12*d**3*e**9 + 36*d**2*e**10*x + 36*d*e**11*x**2
+ 12*e**12*x**3) + 3510*d**2*e**6*x/(12*d**3*e**9 + 36*d**2*e**10*x + 36*d*
e**11*x**2 + 12*e**12*x**3) - 428*d**2*e**6/(12*d**3*e**9 + 36*d**2*e**10*x
 + 36*d*e**11*x**2 + 12*e**12*x**3) - 480*d*e**7*x**7/(12*d**3*e**9 + 36*d*
*2*e**10*x + 36*d*e**11*x**2 + 12*e**12*x**3) + 315*d*e**7*x**6/(12*d**3*e*
*9 + 36*d**2*e**10*x + 36*d*e**11*x**2 + 12*e**12*x**3) - 1332*d*e**7*x**5/
(12*d**3*e**9 + 36*d**2*e**10*x + 36*d*e**11*x**2 + 12*e**12*x**3) + 1110*d
*e**7*x**4/(12*d**3*e**9 + 36*d**2*e**10*x + 36*d*e**11*x**2 + 12*e**12*x**
3) - 7104*d*e**7*x**3*log(d/e + x)/(12*d**3*e**9 + 36*d**2*e**10*x + 36*d*e
**11*x**2 + 12*e**12*x**3) + 2340*d*e**7*x**2*log(d/e + x)/(12*d**3*e**9 +
36*d**2*e**10*x + 36*d*e**11*x**2 + 12*e**12*x**3) + 2340*d*e**7*x**2/(12*d
**3*e**9 + 36*d**2*e**10*x + 36*d*e**11*x**2 + 12*e**12*x**3) - 1284*d*e**7
*x/(12*d**3*e**9 + 36*d**2*e**10*x + 36*d*e**11*x**2 + 12*e**12*x**3) - 66*
d*e**7/(12*d**3*e**9 + 36*d**2*e**10*x + 36*d*e**11*x**2 + 12*e**12*x**3) +
 240*e**8*x**8/(12*d**3*e**9 + 36*d**2*e**10*x + 36*d*e**11*x**2 + 12*e**12
*x**3) - 135*e**8*x**7/(12*d**3*e**9 + 36*d**2*e**10*x + 36*d*e**11*x**2 +
12*e**12*x**3) + 444*e**8*x**6/(12*d**3*e**9 + 36*d**2*e**10*x + 36*d*e**11
*x**2 + 12*e**12*x**3) - 222*e**8*x**5/(12*d**3*e**9 + 36*d**2*e**10*x + 36
*d*e**11*x**2 + 12*e**12*x**3) + 1776*e**8*x**4/(12*d**3*e**9 + 36*d**2*e**
10*x + 36*d*e**11*x**2 + 12*e**12*x**3) + 780*e**8*x**3*log(d/e + x)/(12*d*
*3*e**9 + 36*d**2*e**10*x + 36*d*e**11*x**2 + 12*e**12*x**3) - 1284*e**8*x*
*2/(12*d**3*e**9 + 36*d**2*e**10*x + 36*d*e**11*x**2 + 12*e**12*x**3) - 198
*e**8*x/(12*d**3*e**9 + 36*d**2*e**10*x + 36*d*e**11*x**2 + 12*e**12*x**3)
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- 72*e**8/(12*d**3*e**9 + 36*d**2*e**10*x + 36*d*e**11*x**2 + 12*e**12*x**3
), Eq(m, -4)), (33600*d**8*log(d/e + x)/(12*d**2*e**9 + 24*d*e**10*x + 12*e
**11*x**2) + 50400*d**8/(12*d**2*e**9 + 24*d*e**10*x + 12*e**11*x**2) + 672
00*d**7*e*x*log(d/e + x)/(12*d**2*e**9 + 24*d*e**10*x + 12*e**11*x**2) + 67
200*d**7*e*x/(12*d**2*e**9 + 24*d*e**10*x + 12*e**11*x**2) + 11340*d**7*e*l
og(d/e + x)/(12*d**2*e**9 + 24*d*e**10*x + 12*e**11*x**2) + 17010*d**7*e/(1
2*d**2*e**9 + 24*d*e**10*x + 12*e**11*x**2) + 33600*d**6*e**2*x**2*log(d/e
+ x)/(12*d**2*e**9 + 24*d*e**10*x + 12*e**11*x**2) + 22680*d**6*e**2*x*log(
d/e + x)/(12*d**2*e**9 + 24*d*e**10*x + 12*e**11*x**2) + 22680*d**6*e**2*x/
(12*d**2*e**9 + 24*d*e**10*x + 12*e**11*x**2) + 19980*d**6*e**2*log(d/e + x
)/(12*d**2*e**9 + 24*d*e**10*x + 12*e**11*x**2) + 29970*d**6*e**2/(12*d**2*
e**9 + 24*d*e**10*x + 12*e**11*x**2) - 11200*d**5*e**3*x**3/(12*d**2*e**9 +
 24*d*e**10*x + 12*e**11*x**2) + 11340*d**5*e**3*x**2*log(d/e + x)/(12*d**2
*e**9 + 24*d*e**10*x + 12*e**11*x**2) + 39960*d**5*e**3*x*log(d/e + x)/(12*
d**2*e**9 + 24*d*e**10*x + 12*e**11*x**2) + 39960*d**5*e**3*x/(12*d**2*e**9
 + 24*d*e**10*x + 12*e**11*x**2) + 4440*d**5*e**3*log(d/e + x)/(12*d**2*e**
9 + 24*d*e**10*x + 12*e**11*x**2) + 6660*d**5*e**3/(12*d**2*e**9 + 24*d*e**
10*x + 12*e**11*x**2) + 2800*d**4*e**4*x**4/(12*d**2*e**9 + 24*d*e**10*x +
12*e**11*x**2) - 3780*d**4*e**4*x**3/(12*d**2*e**9 + 24*d*e**10*x + 12*e**1
1*x**2) + 19980*d**4*e**4*x**2*log(d/e + x)/(12*d**2*e**9 + 24*d*e**10*x +
12*e**11*x**2) + 8880*d**4*e**4*x*log(d/e + x)/(12*d**2*e**9 + 24*d*e**10*x
 + 12*e**11*x**2) + 8880*d**4*e**4*x/(12*d**2*e**9 + 24*d*e**10*x + 12*e**1
1*x**2) + 10656*d**4*e**4*log(d/e + x)/(12*d**2*e**9 + 24*d*e**10*x + 12*e*
*11*x**2) + 15984*d**4*e**4/(12*d**2*e**9 + 24*d*e**10*x + 12*e**11*x**2) -
 1120*d**3*e**5*x**5/(12*d**2*e**9 + 24*d*e**10*x + 12*e**11*x**2) + 945*d*
*3*e**5*x**4/(12*d**2*e**9 + 24*d*e**10*x + 12*e**11*x**2) - 6660*d**3*e**5
*x**3/(12*d**2*e**9 + 24*d*e**10*x + 12*e**11*x**2) + 4440*d**3*e**5*x**2*l
og(d/e + x)/(12*d**2*e**9 + 24*d*e**10*x + 12*e**11*x**2) + 21312*d**3*e**5
*x*log(d/e + x)/(12*d**2*e**9 + 24*d*e**10*x + 12*e**11*x**2) + 21312*d**3*
e**5*x/(12*d**2*e**9 + 24*d*e**10*x + 12*e**11*x**2) - 2340*d**3*e**5*log(d
/e + x)/(12*d**2*e**9 + 24*d*e**10*x + 12*e**11*x**2) - 3510*d**3*e**5/(12*
d**2*e**9 + 24*d*e**10*x + 12*e**11*x**2) + 560*d**2*e**6*x**6/(12*d**2*e**
9 + 24*d*e**10*x + 12*e**11*x**2) - 378*d**2*e**6*x**5/(12*d**2*e**9 + 24*d
*e**10*x + 12*e**11*x**2) + 1665*d**2*e**6*x**4/(12*d**2*e**9 + 24*d*e**10*
x + 12*e**11*x**2) - 1480*d**2*e**6*x**3/(12*d**2*e**9 + 24*d*e**10*x + 12*
e**11*x**2) + 10656*d**2*e**6*x**2*log(d/e + x)/(12*d**2*e**9 + 24*d*e**10*
x + 12*e**11*x**2) - 4680*d**2*e**6*x*log(d/e + x)/(12*d**2*e**9 + 24*d*e**
10*x + 12*e**11*x**2) - 4680*d**2*e**6*x/(12*d**2*e**9 + 24*d*e**10*x + 12*
e**11*x**2) + 1284*d**2*e**6*log(d/e + x)/(12*d**2*e**9 + 24*d*e**10*x + 12
*e**11*x**2) + 1926*d**2*e**6/(12*d**2*e**9 + 24*d*e**10*x + 12*e**11*x**2)
 - 320*d*e**7*x**7/(12*d**2*e**9 + 24*d*e**10*x + 12*e**11*x**2) + 189*d*e*
*7*x**6/(12*d**2*e**9 + 24*d*e**10*x + 12*e**11*x**2) - 666*d*e**7*x**5/(12
*d**2*e**9 + 24*d*e**10*x + 12*e**11*x**2) + 370*d*e**7*x**4/(12*d**2*e**9
+ 24*d*e**10*x + 12*e**11*x**2) - 3552*d*e**7*x**3/(12*d**2*e**9 + 24*d*e**
10*x + 12*e**11*x**2) - 2340*d*e**7*x**2*log(d/e + x)/(12*d**2*e**9 + 24*d*
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e**10*x + 12*e**11*x**2) + 2568*d*e**7*x*log(d/e + x)/(12*d**2*e**9 + 24*d*
e**10*x + 12*e**11*x**2) + 2568*d*e**7*x/(12*d**2*e**9 + 24*d*e**10*x + 12*
e**11*x**2) - 198*d*e**7/(12*d**2*e**9 + 24*d*e**10*x + 12*e**11*x**2) + 20
0*e**8*x**8/(12*d**2*e**9 + 24*d*e**10*x + 12*e**11*x**2) - 108*e**8*x**7/(
12*d**2*e**9 + 24*d*e**10*x + 12*e**11*x**2) + 333*e**8*x**6/(12*d**2*e**9
 + 24*d*e**10*x + 12*e**11*x**2) - 148*e**8*x**5/(12*d**2*e**9 + 24*d*e**10*
x + 12*e**11*x**2) + 888*e**8*x**4/(12*d**2*e**9 + 24*d*e**10*x + 12*e**11*
x**2) + 780*e**8*x**3/(12*d**2*e**9 + 24*d*e**10*x + 12*e**11*x**2) + 1284*
e**8*x**2*log(d/e + x)/(12*d**2*e**9 + 24*d*e**10*x + 12*e**11*x**2) - 396*
e**8*x/(12*d**2*e**9 + 24*d*e**10*x + 12*e**11*x**2) - 108*e**8/(12*d**2*e*
*9 + 24*d*e**10*x + 12*e**11*x**2), Eq(m, -3)), (-336000*d**8*log(d/e + x)/
(420*d*e**9 + 420*e**10*x) - 336000*d**8/(420*d*e**9 + 420*e**10*x) - 33600
0*d**7*e*x*log(d/e + x)/(420*d*e**9 + 420*e**10*x) - 132300*d**7*e*log(d/e
+ x)/(420*d*e**9 + 420*e**10*x) - 132300*d**7*e/(420*d*e**9 + 420*e**10*x)
+ 168000*d**6*e**2*x**2/(420*d*e**9 + 420*e**10*x) - 132300*d**6*e**2*x*log
(d/e + x)/(420*d*e**9 + 420*e**10*x) - 279720*d**6*e**2*log(d/e + x)/(420*d
*e**9 + 420*e**10*x) - 279720*d**6*e**2/(420*d*e**9 + 420*e**10*x) - 56000*
d**5*e**3*x**3/(420*d*e**9 + 420*e**10*x) + 66150*d**5*e**3*x**2/(420*d*e**
9 + 420*e**10*x) - 279720*d**5*e**3*x*log(d/e + x)/(420*d*e**9 + 420*e**10*
x) - 77700*d**5*e**3*log(d/e + x)/(420*d*e**9 + 420*e**10*x) - 77700*d**5*e
**3/(420*d*e**9 + 420*e**10*x) + 28000*d**4*e**4*x**4/(420*d*e**9 + 420*e**
10*x) - 22050*d**4*e**4*x**3/(420*d*e**9 + 420*e**10*x) + 139860*d**4*e**4*
x**2/(420*d*e**9 + 420*e**10*x) - 77700*d**4*e**4*x*log(d/e + x)/(420*d*e**
9 + 420*e**10*x) - 248640*d**4*e**4*log(d/e + x)/(420*d*e**9 + 420*e**10*x)
 - 248640*d**4*e**4/(420*d*e**9 + 420*e**10*x) - 16800*d**3*e**5*x**5/(420*
d*e**9 + 420*e**10*x) + 11025*d**3*e**5*x**4/(420*d*e**9 + 420*e**10*x) - 4
6620*d**3*e**5*x**3/(420*d*e**9 + 420*e**10*x) + 38850*d**3*e**5*x**2/(420*
d*e**9 + 420*e**10*x) - 248640*d**3*e**5*x*log(d/e + x)/(420*d*e**9 + 420*e
**10*x) + 81900*d**3*e**5*log(d/e + x)/(420*d*e**9 + 420*e**10*x) + 81900*d
**3*e**5/(420*d*e**9 + 420*e**10*x) + 11200*d**2*e**6*x**6/(420*d*e**9 + 42
0*e**10*x) - 6615*d**2*e**6*x**5/(420*d*e**9 + 420*e**10*x) + 23310*d**2*e*
*6*x**4/(420*d*e**9 + 420*e**10*x) - 12950*d**2*e**6*x**3/(420*d*e**9 + 420
*e**10*x) + 124320*d**2*e**6*x**2/(420*d*e**9 + 420*e**10*x) + 81900*d**2*e
**6*x*log(d/e + x)/(420*d*e**9 + 420*e**10*x) - 89880*d**2*e**6*log(d/e + x
)/(420*d*e**9 + 420*e**10*x) - 89880*d**2*e**6/(420*d*e**9 + 420*e**10*x) -
 8000*d*e**7*x**7/(420*d*e**9 + 420*e**10*x) + 4410*d*e**7*x**6/(420*d*e**9
 + 420*e**10*x) - 13986*d*e**7*x**5/(420*d*e**9 + 420*e**10*x) + 6475*d*e**
7*x**4/(420*d*e**9 + 420*e**10*x) - 41440*d*e**7*x**3/(420*d*e**9 + 420*e**
10*x) - 40950*d*e**7*x**2/(420*d*e**9 + 420*e**10*x) - 89880*d*e**7*x*log(d
/e + x)/(420*d*e**9 + 420*e**10*x) + 13860*d*e**7*log(d/e + x)/(420*d*e**9
+ 420*e**10*x) + 13860*d*e**7/(420*d*e**9 + 420*e**10*x) + 6000*e**8*x**8/(
420*d*e**9 + 420*e**10*x) - 3150*e**8*x**7/(420*d*e**9 + 420*e**10*x) + 932
4*e**8*x**6/(420*d*e**9 + 420*e**10*x) - 3885*e**8*x**5/(420*d*e**9 + 420*e
**10*x) + 20720*e**8*x**4/(420*d*e**9 + 420*e**10*x) + 13650*e**8*x**3/(420
*d*e**9 + 420*e**10*x) + 44940*e**8*x**2/(420*d*e**9 + 420*e**10*x) + 13860
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*e**8*x*log(d/e + x)/(420*d*e**9 + 420*e**10*x) - 7560*e**8/(420*d*e**9 + 4
20*e**10*x), Eq(m, -2)), (100*d**8*log(d/e + x)/e**9 - 100*d**7*x/e**8 + 45
*d**7*log(d/e + x)/e**8 + 50*d**6*x**2/e**7 - 45*d**6*x/e**7 + 111*d**6*log
(d/e + x)/e**7 - 100*d**5*x**3/(3*e**6) + 45*d**5*x**2/(2*e**6) - 111*d**5*
x/e**6 + 37*d**5*log(d/e + x)/e**6 + 25*d**4*x**4/e**5 - 15*d**4*x**3/e**5
 + 111*d**4*x**2/(2*e**5) - 37*d**4*x/e**5 + 148*d**4*log(d/e + x)/e**5 - 20
*d**3*x**5/e**4 + 45*d**3*x**4/(4*e**4) - 37*d**3*x**3/e**4 + 37*d**3*x**2/
(2*e**4) - 148*d**3*x/e**4 - 65*d**3*log(d/e + x)/e**4 + 50*d**2*x**6/(3*e*
*3) - 9*d**2*x**5/e**3 + 111*d**2*x**4/(4*e**3) - 37*d**2*x**3/(3*e**3) + 7
4*d**2*x**2/e**3 + 65*d**2*x/e**3 + 107*d**2*log(d/e + x)/e**3 - 100*d*x**7
/(7*e**2) + 15*d*x**6/(2*e**2) - 111*d*x**5/(5*e**2) + 37*d*x**4/(4*e**2) -
 148*d*x**3/(3*e**2) - 65*d*x**2/(2*e**2) - 107*d*x/e**2 - 33*d*log(d/e + x
)/e**2 + 25*x**8/(2*e) - 45*x**7/(7*e) + 37*x**6/(2*e) - 37*x**5/(5*e) + 37
*x**4/e + 65*x**3/(3*e) + 107*x**2/(2*e) + 33*x/e + 18*log(d/e + x)/e, Eq(m
, -1)), (4032000*d**9*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**
7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3
 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) - 4032000*d**8*e*m*x*(d
 + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 632
73*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 10
26576*e**9*m + 362880*e**9) + 226800*d**8*e*m*(d + e*x)**m/(e**9*m**9 + 45*
e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*
m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9)
 + 2041200*d**8*e*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 +
9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 11
72700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) + 2016000*d**7*e**2*m**2*x*
*2*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6
 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2
 + 1026576*e**9*m + 362880*e**9) - 226800*d**7*e**2*m**2*x*(d + e*x)**m/(e*
*9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 +
 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m +
 362880*e**9) + 79920*d**7*e**2*m**2*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8
 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 72
3680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) + 201600
0*d**7*e**2*m*x**2*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 +
 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1
172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) - 2041200*d**7*e**2*m*x*(d
 + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 632
73*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 10
26576*e**9*m + 362880*e**9) + 1358640*d**7*e**2*m*(d + e*x)**m/(e**9*m**9 +
 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e
**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e
**9) + 5754240*d**7*e**2*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*
m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m*
*3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) - 672000*d**6*e**3*m
**3*x**3*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9
```

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*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**
9*m**2 + 1026576*e**9*m + 362880*e**9) + 113400*d**6*e**3*m**3*x**2*(d + e*
x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e*
*9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576
*e**9*m + 362880*e**9) - 79920*d**6*e**3*m**3*x*(d + e*x)**m/(e**9*m**9 + 4
5*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**
9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**
9) + 4440*d**6*e**3*m**3*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*
m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m*
*3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) - 2016000*d**6*e**3*
m**2*x**3*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**
9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e*
*9*m**2 + 1026576*e**9*m + 362880*e**9) + 1134000*d**6*e**3*m**2*x**2*(d +
e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*
e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 10265
76*e**9*m + 362880*e**9) - 1358640*d**6*e**3*m**2*x*(d + e*x)**m/(e**9*m**9
 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325
*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880
*e**9) + 106560*d**6*e**3*m**2*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870
*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e
**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) - 1344000*d**6
*e**3*m*x**3*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*
e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700
*e**9*m**2 + 1026576*e**9*m + 362880*e**9) + 1020600*d**6*e**3*m*x**2*(d +
e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*
e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 10265
76*e**9*m + 362880*e**9) - 5754240*d**6*e**3*m*x*(d + e*x)**m/(e**9*m**9 +
45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e*
*9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e*
*9) + 848040*d**6*e**3*m*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*
m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m*
*3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) + 2237760*d**6*e**3*
(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 6
3273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 +
1026576*e**9*m + 362880*e**9) + 168000*d**5*e**4*m**4*x**4*(d + e*x)**m/(e*
*9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 +
 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m +
 362880*e**9) - 37800*d**5*e**4*m**4*x**3*(d + e*x)**m/(e**9*m**9 + 45*e**9
*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4
 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) + 3
9960*d**5*e**4*m**4*x**2*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*
m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m*
*3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) - 4440*d**5*e**4*m**
4*x*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6
 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**
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2 + 1026576*e**9*m + 362880*e**9) + 3552*d**5*e**4*m**4*(d + e*x)**m/(e**9*
m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 26
9325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 36
2880*e**9) + 1008000*d**5*e**4*m**3*x**4*(d + e*x)**m/(e**9*m**9 + 45*e**9*
m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4
 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) - 45
3600*d**5*e**4*m**3*x**3*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*
m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m*
*3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) + 719280*d**5*e**4*m
**3*x**2*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9
*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**
9*m**2 + 1026576*e**9*m + 362880*e**9) - 106560*d**5*e**4*m**3*x*(d + e*x)*
*m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*
m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e*
*9*m + 362880*e**9) + 106560*d**5*e**4*m**3*(d + e*x)**m/(e**9*m**9 + 45*e*
*9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m*
*4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) +
 1848000*d**5*e**4*m**2*x**4*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e
**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**
9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) - 1096200*d**5*e
**4*m**2*x**3*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450
*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 117270
0*e**9*m**2 + 1026576*e**9*m + 362880*e**9) + 3556440*d**5*e**4*m**2*x**2*(
d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63
273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1
026576*e**9*m + 362880*e**9) - 848040*d**5*e**4*m**2*x*(d + e*x)**m/(e**9*m
**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269
325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362
880*e**9) + 1189920*d**5*e**4*m**2*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 +
 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 7236
80*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) + 1008000*
d**5*e**4*m*x**4*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9
450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 117
2700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) - 680400*d**5*e**4*m*x**3*(d
 + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 632
73*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 10
26576*e**9*m + 362880*e**9) + 2877120*d**5*e**4*m*x**2*(d + e*x)**m/(e**9*m
**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269
325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362
880*e**9) - 2237760*d**5*e**4*m*x*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 +
870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 72368
0*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) + 5860800*d
**5*e**4*m*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e*
*9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e
**9*m**2 + 1026576*e**9*m + 362880*e**9) + 10741248*d**5*e**4*(d + e*x)**m/
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(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**
5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*
m + 362880*e**9) - 33600*d**4*e**5*m**5*x**5*(d + e*x)**m/(e**9*m**9 + 45*e
**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m
**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9)
 + 9450*d**4*e**5*m**5*x**4*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**
9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*
m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) - 13320*d**4*e**5*
m**5*x**3*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**
9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e*
*9*m**2 + 1026576*e**9*m + 362880*e**9) + 2220*d**4*e**5*m**5*x**2*(d + e*x
)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**
9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*
e**9*m + 362880*e**9) - 3552*d**4*e**5*m**5*x*(d + e*x)**m/(e**9*m**9 + 45*
e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*
m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9)
 - 390*d**4*e**5*m**5*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**
7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3
+ 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) - 336000*d**4*e**5*m**4
*x**5*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m*
*6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m
**2 + 1026576*e**9*m + 362880*e**9) + 141750*d**4*e**5*m**4*x**4*(d + e*x)*
*m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*
m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e*
*9*m + 362880*e**9) - 266400*d**4*e**5*m**4*x**3*(d + e*x)**m/(e**9*m**9 +
45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e*
*9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e*
*9) + 55500*d**4*e**5*m**4*x**2*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 87
0*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*
e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) - 106560*d**4
*e**5*m**4*x*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*
e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700
*e**9*m**2 + 1026576*e**9*m + 362880*e**9) - 13650*d**4*e**5*m**4*(d + e*x)
**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9
*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e
**9*m + 362880*e**9) - 1176000*d**4*e**5*m**3*x**5*(d + e*x)**m/(e**9*m**9
+ 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*
e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*
e**9) + 614250*d**4*e**5*m**3*x**4*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 +
 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 7236
80*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) - 1665000*
d**4*e**5*m**3*x**3*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7
+ 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 +
1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) + 477300*d**4*e**5*m**3*x
**2*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6
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 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**
2 + 1026576*e**9*m + 362880*e**9) - 1189920*d**4*e**5*m**3*x*(d + e*x)**m/(
e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5
 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m
 + 362880*e**9) - 189150*d**4*e**5*m**3*(d + e*x)**m/(e**9*m**9 + 45*e**9*m
**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 +
 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) - 168
0000*d**4*e**5*m**2*x**5*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*
m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m*
*3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) + 992250*d**4*e**5*m
**2*x**4*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9
*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**
9*m**2 + 1026576*e**9*m + 362880*e**9) - 3330000*d**4*e**5*m**2*x**3*(d + e
*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e
**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 102657
6*e**9*m + 362880*e**9) + 1542900*d**4*e**5*m**2*x**2*(d + e*x)**m/(e**9*m*
*9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 2693
25*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 3628
80*e**9) - 5860800*d**4*e**5*m**2*x*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8
 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723
680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) - 1296750
*d**4*e**5*m**2*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 94
50*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172
700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) - 806400*d**4*e**5*m*x**5*(d
 + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 6327
3*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 102
6576*e**9*m + 362880*e**9) + 510300*d**4*e**5*m*x**4*(d + e*x)**m/(e**9*m**
9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 26932
5*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 36288
0*e**9) - 1918080*d**4*e**5*m*x**3*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 +
 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 7236
80*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) + 1118880*
d**4*e**5*m*x**2*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9
450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 117
2700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) - 10741248*d**4*e**5*m*x*(d
 + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 6327
3*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 102
6576*e**9*m + 362880*e**9) - 4396860*d**4*e**5*m*(d + e*x)**m/(e**9*m**9 +
45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e*
*9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e*
*9) - 5896800*d**4*e**5*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m
**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**
3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) + 5600*d**3*e**6*m**6
*x**6*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m*
*6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m
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**2 + 1026576*e**9*m + 362880*e**9) - 1890*d**3*e**6*m**6*x**5*(d + e*x)**m
/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m*
*5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9
*m + 362880*e**9) + 3330*d**3*e**6*m**6*x**4*(d + e*x)**m/(e**9*m**9 + 45*e
**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m
**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9)
- 740*d**3*e**6*m**6*x**3*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9
*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m
**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) + 1776*d**3*e**6*m*
*6*x**2*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*
m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9
*m**2 + 1026576*e**9*m + 362880*e**9) + 390*d**3*e**6*m**6*x*(d + e*x)**m/(
e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5
 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m
 + 362880*e**9) + 214*d**3*e**6*m**6*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8
 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 72
3680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) + 84000*
d**3*e**6*m**5*x**6*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7
+ 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 +
1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) - 35910*d**3*e**6*m**5*x*
*5*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6
+ 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2
 + 1026576*e**9*m + 362880*e**9) + 76590*d**3*e**6*m**5*x**4*(d + e*x)**m/(
e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5
 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m
 + 362880*e**9) - 19980*d**3*e**6*m**5*x**3*(d + e*x)**m/(e**9*m**9 + 45*e*
*9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m*
*4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) +
 55056*d**3*e**6*m**5*x**2*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**
9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*
m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) + 13650*d**3*e**6*
m**5*x*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m
**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*
m**2 + 1026576*e**9*m + 362880*e**9) + 8346*d**3*e**6*m**5*(d + e*x)**m/(e*
*9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 +
 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m +
 362880*e**9) + 476000*d**3*e**6*m**4*x**6*(d + e*x)**m/(e**9*m**9 + 45*e**
9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**
4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) -
236250*d**3*e**6*m**4*x**5*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**
9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*
m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) + 616050*d**3*e**6
*m**4*x**4*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e*
*9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e
**9*m**2 + 1026576*e**9*m + 362880*e**9) - 196100*d**3*e**6*m**4*x**3*(d +
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e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*
e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 10265
76*e**9*m + 362880*e**9) + 648240*d**3*e**6*m**4*x**2*(d + e*x)**m/(e**9*m*
*9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 2693
25*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 3628
80*e**9) + 189150*d**3*e**6*m**4*x*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 +
 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 7236
80*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) + 133750*d
**3*e**6*m**4*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450
*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 117270
0*e**9*m**2 + 1026576*e**9*m + 362880*e**9) + 1260000*d**3*e**6*m**3*x**6*(
d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63
273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1
026576*e**9*m + 362880*e**9) - 689850*d**3*e**6*m**3*x**5*(d + e*x)**m/(e**
9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 +
269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m +
362880*e**9) + 2081250*d**3*e**6*m**3*x**4*(d + e*x)**m/(e**9*m**9 + 45*e**
9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**
4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) -
832500*d**3*e**6*m**3*x**3*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**
9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*
m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) + 3525360*d**3*e**
6*m**3*x**2*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e
**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*
e**9*m**2 + 1026576*e**9*m + 362880*e**9) + 1296750*d**3*e**6*m**3*x*(d + e
*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e
**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 102657
6*e**9*m + 362880*e**9) + 1126710*d**3*e**6*m**3*(d + e*x)**m/(e**9*m**9 +
45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e*
*9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e*
*9) + 1534400*d**3*e**6*m**2*x**6*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 +
870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 72368
0*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) - 895860*d*
*3*e**6*m**2*x**5*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 +
9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 11
72700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) + 2977020*d**3*e**6*m**2*x*
*4*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6
+ 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2
 + 1026576*e**9*m + 362880*e**9) - 1401560*d**3*e**6*m**2*x**3*(d + e*x)**m
/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m*
*5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9
*m + 362880*e**9) + 8301024*d**3*e**6*m**2*x**2*(d + e*x)**m/(e**9*m**9 + 4
5*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**
9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**
9) + 4396860*d**3*e**6*m**2*x*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*
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e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e*
*9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) + 5258836*d**3*
e**6*m**2*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**
9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e*
*9*m**2 + 1026576*e**9*m + 362880*e**9) + 672000*d**3*e**6*m*x**6*(d + e*x)
**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9
*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e
**9*m + 362880*e**9) - 408240*d**3*e**6*m*x**5*(d + e*x)**m/(e**9*m**9 + 45
*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9
*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9
) + 1438560*d**3*e**6*m*x**4*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e
**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**
9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) - 745920*d**3*e*
*6*m*x**3*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**
9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e*
*9*m**2 + 1026576*e**9*m + 362880*e**9) + 5370624*d**3*e**6*m*x**2*(d + e*x
)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**
9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*
e**9*m + 362880*e**9) + 5896800*d**3*e**6*m*x*(d + e*x)**m/(e**9*m**9 + 45*
e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*
m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9)
 + 12886224*d**3*e**6*m*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m
**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**
3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) + 12942720*d**3*e**6*
(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 6
3273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 +
1026576*e**9*m + 362880*e**9) - 800*d**2*e**7*m**7*x**7*(d + e*x)**m/(e**9*
m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 26
9325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 36
2880*e**9) + 315*d**2*e**7*m**7*x**6*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8
 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 72
3680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) - 666*d*
*2*e**7*m**7*x**5*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 +
9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 11
72700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) + 185*d**2*e**7*m**7*x**4*(
d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63
273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1
026576*e**9*m + 362880*e**9) - 592*d**2*e**7*m**7*x**3*(d + e*x)**m/(e**9*m
**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269
325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362
880*e**9) - 195*d**2*e**7*m**7*x**2*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8
+ 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723
680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) - 214*d**
2*e**7*m**7*x*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450
*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 117270
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0*e**9*m**2 + 1026576*e**9*m + 362880*e**9) - 33*d**2*e**7*m**7*(d + e*x)**
m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m
**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**
9*m + 362880*e**9) - 16800*d**2*e**7*m**6*x**7*(d + e*x)**m/(e**9*m**9 + 45
*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9
*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9
) + 7560*d**2*e**7*m**6*x**6*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e
**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**
9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) - 17982*d**2*e**
7*m**6*x**5*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e
**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*
e**9*m**2 + 1026576*e**9*m + 362880*e**9) + 5550*d**2*e**7*m**6*x**4*(d + e
*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e
**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 102657
6*e**9*m + 362880*e**9) - 19536*d**2*e**7*m**6*x**3*(d + e*x)**m/(e**9*m**9
 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325
*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880
*e**9) - 7020*d**2*e**7*m**6*x**2*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 +
870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 72368
0*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) - 8346*d**2
*e**7*m**6*x*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*
e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700
*e**9*m**2 + 1026576*e**9*m + 362880*e**9) - 1386*d**2*e**7*m**6*(d + e*x)*
*m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*
m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e*
*9*m + 362880*e**9) - 140000*d**2*e**7*m**5*x**7*(d + e*x)**m/(e**9*m**9 +
45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e*
*9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e*
*9) + 69300*d**2*e**7*m**5*x**6*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 87
0*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*
e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) - 184482*d**2
*e**7*m**5*x**5*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 94
50*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172
700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) + 64010*d**2*e**7*m**5*x**4*(
d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63
273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1
026576*e**9*m + 362880*e**9) - 252784*d**2*e**7*m**5*x**3*(d + e*x)**m/(e**
9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 +
269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m +
362880*e**9) - 101400*d**2*e**7*m**5*x**2*(d + e*x)**m/(e**9*m**9 + 45*e**9
*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4
 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) - 1
33750*d**2*e**7*m**5*x*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m*
*7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3
 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) - 24486*d**2*e**7*m**5
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*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 +
63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 +
 1026576*e**9*m + 362880*e**9) - 588000*d**2*e**7*m**4*x**7*(d + e*x)**m/(e
**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5
+ 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m
+ 362880*e**9) + 311850*d**2*e**7*m**4*x**6*(d + e*x)**m/(e**9*m**9 + 45*e*
*9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m*
*4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) -
 909090*d**2*e**7*m**4*x**5*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e*
*9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9
*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) + 355200*d**2*e**
7*m**4*x**4*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e
**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*
e**9*m**2 + 1026576*e**9*m + 362880*e**9) - 1607280*d**2*e**7*m**4*x**3*(d
+ e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 6327
3*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 102
6576*e**9*m + 362880*e**9) - 742950*d**2*e**7*m**4*x**2*(d + e*x)**m/(e**9*
m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 26
9325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 36
2880*e**9) - 1126710*d**2*e**7*m**4*x*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**
8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 7
23680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) - 23562
0*d**2*e**7*m**4*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9
450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 117
2700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) - 1299200*d**2*e**7*m**3*x**
7*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 +
 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2
+ 1026576*e**9*m + 362880*e**9) + 724185*d**2*e**7*m**3*x**6*(d + e*x)**m/(
e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5
 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m
 + 362880*e**9) - 2260404*d**2*e**7*m**3*x**5*(d + e*x)**m/(e**9*m**9 + 45*
e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*
m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9)
 + 974765*d**2*e**7*m**3*x**4*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*
e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e*
*9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) - 5117248*d**2*
e**7*m**3*x**3*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 945
0*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 11727
00*e**9*m**2 + 1026576*e**9*m + 362880*e**9) - 2846805*d**2*e**7*m**3*x**2*
(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 6
3273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 +
1026576*e**9*m + 362880*e**9) - 5258836*d**2*e**7*m**3*x*(d + e*x)**m/(e**9
*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 2
69325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 3
62880*e**9) - 1332177*d**2*e**7*m**3*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8
```

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 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 72
3680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) - 141120
0*d**2*e**7*m**2*x**7*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**
7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3
+ 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) + 814590*d**2*e**7*m**2
*x**6*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m*
*6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m
**2 + 1026576*e**9*m + 362880*e**9) - 2669328*d**2*e**7*m**2*x**5*(d + e*x)
**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9
*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e
**9*m + 362880*e**9) + 1237650*d**2*e**7*m**2*x**4*(d + e*x)**m/(e**9*m**9
+ 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*
e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*
e**9) - 7324224*d**2*e**7*m**2*x**3*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8
+ 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723
680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) - 5146830
*d**2*e**7*m**2*x**2*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7
 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 +
 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) - 12886224*d**2*e**7*m**
2*x*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6
 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**
2 + 1026576*e**9*m + 362880*e**9) - 4419954*d**2*e**7*m**2*(d + e*x)**m/(e*
*9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 +
 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m +
 362880*e**9) - 576000*d**2*e**7*m*x**7*(d + e*x)**m/(e**9*m**9 + 45*e**9*m
**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 +
 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) + 340
200*d**2*e**7*m*x**6*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7
 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 +
 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) - 1150848*d**2*e**7*m*x*
*5*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6
+ 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2
 + 1026576*e**9*m + 362880*e**9) + 559440*d**2*e**7*m*x**4*(d + e*x)**m/(e*
*9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 +
 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m +
 362880*e**9) - 3580416*d**2*e**7*m*x**3*(d + e*x)**m/(e**9*m**9 + 45*e**9*
m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4
+ 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) - 29
48400*d**2*e**7*m*x**2*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m*
*7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3
 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) - 12942720*d**2*e**7*m
*x*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6
+ 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2
 + 1026576*e**9*m + 362880*e**9) - 7957224*d**2*e**7*m*(d + e*x)**m/(e**9*m
**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269
```

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325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362
880*e**9) - 5987520*d**2*e**7*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*
e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e*
*9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) + 100*d*e**8*m*
*8*x**8*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*
m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9
*m**2 + 1026576*e**9*m + 362880*e**9) - 45*d*e**8*m**8*x**7*(d + e*x)**m/(e
**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5
+ 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m
+ 362880*e**9) + 111*d*e**8*m**8*x**6*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**
8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 7
23680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) - 37*d*
e**8*m**8*x**5*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 945
0*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 11727
00*e**9*m**2 + 1026576*e**9*m + 362880*e**9) + 148*d*e**8*m**8*x**4*(d + e*
x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e*
*9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576
*e**9*m + 362880*e**9) + 65*d*e**8*m**8*x**3*(d + e*x)**m/(e**9*m**9 + 45*e
**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m
**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9)
+ 107*d*e**8*m**8*x**2*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m*
*7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3
 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) + 33*d*e**8*m**8*x*(d
+ e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 6327
3*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 102
6576*e**9*m + 362880*e**9) + 18*d*e**8*m**8*(d + e*x)**m/(e**9*m**9 + 45*e*
*9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m*
*4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) +
 2800*d*e**8*m**7*x**8*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m*
*7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3
 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) - 1350*d*e**8*m**7*x**
7*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 +
 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2
+ 1026576*e**9*m + 362880*e**9) + 3552*d*e**8*m**7*x**6*(d + e*x)**m/(e**9*
m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 26
9325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 36
2880*e**9) - 1258*d*e**8*m**7*x**5*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 +
 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 7236
80*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) + 5328*d*e
**8*m**7*x**4*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450
*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 117270
0*e**9*m**2 + 1026576*e**9*m + 362880*e**9) + 2470*d*e**8*m**7*x**3*(d + e*
x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e*
*9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576
*e**9*m + 362880*e**9) + 4280*d*e**8*m**7*x**2*(d + e*x)**m/(e**9*m**9 + 45
```

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*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9
*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9
) + 1386*d*e**8*m**7*x*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m*
*7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3
 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) + 792*d*e**8*m**7*(d +
 e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273
*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026
576*e**9*m + 362880*e**9) + 32200*d*e**8*m**6*x**8*(d + e*x)**m/(e**9*m**9
+ 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*
e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*
e**9) - 16380*d*e**8*m**6*x**7*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870
*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e
**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) + 45732*d*e**8
*m**6*x**6*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e*
*9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e
**9*m**2 + 1026576*e**9*m + 362880*e**9) - 17242*d*e**8*m**6*x**5*(d + e*x)
**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9
*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e
**9*m + 362880*e**9) + 77848*d*e**8*m**6*x**4*(d + e*x)**m/(e**9*m**9 + 45*
e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*
m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9)
 + 38480*d*e**8*m**6*x**3*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9
*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m
**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) + 71048*d*e**8*m**6
*x**2*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m*
*6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m
**2 + 1026576*e**9*m + 362880*e**9) + 24486*d*e**8*m**6*x*(d + e*x)**m/(e**
9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 +
269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m +
362880*e**9) + 14868*d*e**8*m**6*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 8
70*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680
*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) + 196000*d*e
**8*m**5*x**8*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450
*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 117270
0*e**9*m**2 + 1026576*e**9*m + 362880*e**9) - 103950*d*e**8*m**5*x**7*(d +
e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*
e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 10265
76*e**9*m + 362880*e**9) + 305250*d*e**8*m**5*x**6*(d + e*x)**m/(e**9*m**9
+ 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*
e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*
e**9) - 122248*d*e**8*m**5*x**5*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 87
0*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*
e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) + 591408*d*e*
*8*m**5*x**4*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*
e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700
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*e**9*m**2 + 1026576*e**9*m + 362880*e**9) + 315250*d*e**8*m**5*x**3*(d + e
*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e
**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 102657
6*e**9*m + 362880*e**9) + 630230*d*e**8*m**5*x**2*(d + e*x)**m/(e**9*m**9 +
 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e
**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e
**9) + 235620*d*e**8*m**5*x*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e*
*9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9
*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) + 155232*d*e**8*m
**5*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6
 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**
2 + 1026576*e**9*m + 362880*e**9) + 676900*d*e**8*m**4*x**8*(d + e*x)**m/(e
**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5
+ 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m
+ 362880*e**9) - 370755*d*e**8*m**4*x**7*(d + e*x)**m/(e**9*m**9 + 45*e**9*
m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4
+ 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) + 11
34309*d*e**8*m**4*x**6*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m*
*7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3
 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) - 479113*d*e**8*m**4*x
**5*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6
 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**
2 + 1026576*e**9*m + 362880*e**9) + 2484772*d*e**8*m**4*x**4*(d + e*x)**m/(
e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5
 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m
 + 362880*e**9) + 1444235*d*e**8*m**4*x**3*(d + e*x)**m/(e**9*m**9 + 45*e**
9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**
4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) +
3192773*d*e**8*m**4*x**2*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*
m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m*
*3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) + 1332177*d*e**8*m**
4*x*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6
 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**
2 + 1026576*e**9*m + 362880*e**9) + 983682*d*e**8*m**4*(d + e*x)**m/(e**9*m
**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269
325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362
880*e**9) + 1313200*d*e**8*m**3*x**8*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8
 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 72
3680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) - 737100
*d*e**8*m**3*x**7*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 +
9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 11
72700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) + 2328558*d*e**8*m**3*x**6*
(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 6
3273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 +
1026576*e**9*m + 362880*e**9) - 1027342*d*e**8*m**3*x**5*(d + e*x)**m/(e**9
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*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 2
69325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 3
62880*e**9) + 5668992*d*e**8*m**3*x**4*(d + e*x)**m/(e**9*m**9 + 45*e**9*m*
*8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 +
723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) + 3613
480*d*e**8*m**3*x**3*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7
 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 +
 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) + 9072530*d*e**8*m**3*x*
*2*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6
+ 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2
 + 1026576*e**9*m + 362880*e**9) + 4419954*d*e**8*m**3*x*(d + e*x)**m/(e**9
*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 2
69325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 3
62880*e**9) + 3864168*d*e**8*m**3*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 +
870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 72368
0*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) + 1306800*d
*e**8*m**2*x**8*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 94
50*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172
700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) - 746820*d*e**8*m**2*x**7*(d
+ e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 6327
3*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 102
6576*e**9*m + 362880*e**9) + 2416248*d*e**8*m**2*x**6*(d + e*x)**m/(e**9*m*
*9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 2693
25*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 3628
80*e**9) - 1102008*d*e**8*m**2*x**5*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8
+ 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723
680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) + 6388272
*d*e**8*m**2*x**4*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 +
9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 11
72700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) + 4414020*d*e**8*m**2*x**3*
(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 6
3273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 +
1026576*e**9*m + 362880*e**9) + 12914472*d*e**8*m**2*x**2*(d + e*x)**m/(e**
9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 +
269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m +
362880*e**9) + 7957224*d*e**8*m**2*x*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8
 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 72
3680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) + 916207
2*d*e**8*m**2*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450
*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 117270
0*e**9*m**2 + 1026576*e**9*m + 362880*e**9) + 504000*d*e**8*m*x**8*(d + e*x
)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**
9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*
e**9*m + 362880*e**9) - 291600*d*e**8*m*x**7*(d + e*x)**m/(e**9*m**9 + 45*e
**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m
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**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9)
+ 959040*d*e**8*m*x**6*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m*
*7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3
 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) - 447552*d*e**8*m*x**5
*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 +
63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 +
 1026576*e**9*m + 362880*e**9) + 2685312*d*e**8*m*x**4*(d + e*x)**m/(e**9*m
**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269
325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362
880*e**9) + 1965600*d*e**8*m*x**3*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 +
870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 72368
0*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) + 6471360*d
*e**8*m*x**2*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*
e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700
*e**9*m**2 + 1026576*e**9*m + 362880*e**9) + 5987520*d*e**8*m*x*(d + e*x)**
m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m
**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**
9*m + 362880*e**9) + 11946528*d*e**8*m*(d + e*x)**m/(e**9*m**9 + 45*e**9*m*
*8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 +
723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) + 6531
840*d*e**8*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e*
*9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e
**9*m**2 + 1026576*e**9*m + 362880*e**9) + 100*e**9*m**8*x**9*(d + e*x)**m/
(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**
5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*
m + 362880*e**9) - 45*e**9*m**8*x**8*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8
 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 72
3680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) + 111*e*
*9*m**8*x**7*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*
e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700
*e**9*m**2 + 1026576*e**9*m + 362880*e**9) - 37*e**9*m**8*x**6*(d + e*x)**m
/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m*
*5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9
*m + 362880*e**9) + 148*e**9*m**8*x**5*(d + e*x)**m/(e**9*m**9 + 45*e**9*m*
*8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 +
723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) + 65*e
**9*m**8*x**4*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450
*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 117270
0*e**9*m**2 + 1026576*e**9*m + 362880*e**9) + 107*e**9*m**8*x**3*(d + e*x)*
*m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*
m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e*
*9*m + 362880*e**9) + 33*e**9*m**8*x**2*(d + e*x)**m/(e**9*m**9 + 45*e**9*m
**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 +
 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) + 18*
e**9*m**8*x*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e
```

```
**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*
e**9*m**2 + 1026576*e**9*m + 362880*e**9) + 3600*e**9*m**7*x**9*(d + e*x)**
m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m
**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**
9*m + 362880*e**9) - 1665*e**9*m**7*x**8*(d + e*x)**m/(e**9*m**9 + 45*e**9*
m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4
+ 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) + 42
18*e**9*m**7*x**7*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 +
9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 11
72700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) - 1443*e**9*m**7*x**6*(d +
e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*
e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 10265
76*e**9*m + 362880*e**9) + 5920*e**9*m**7*x**5*(d + e*x)**m/(e**9*m**9 + 45
*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9
*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9
) + 2665*e**9*m**7*x**4*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m
**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**
3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) + 4494*e**9*m**7*x**3
*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 +
63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 +
 1026576*e**9*m + 362880*e**9) + 1419*e**9*m**7*x**2*(d + e*x)**m/(e**9*m**
9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 26932
5*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 36288
0*e**9) + 792*e**9*m**7*x*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9
*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m
**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) + 54600*e**9*m**6*x
**9*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6
 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**
2 + 1026576*e**9*m + 362880*e**9) - 25830*e**9*m**6*x**8*(d + e*x)**m/(e**9
*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 2
69325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 3
62880*e**9) + 67044*e**9*m**6*x**7*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 +
 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 7236
80*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) - 23532*e*
*9*m**6*x**6*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*
e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700
*e**9*m**2 + 1026576*e**9*m + 362880*e**9) + 99160*e**9*m**6*x**5*(d + e*x)
**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9
*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e
**9*m + 362880*e**9) + 45890*e**9*m**6*x**4*(d + e*x)**m/(e**9*m**9 + 45*e*
*9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m*
*4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) +
 79608*e**9*m**6*x**3*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**
7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3
+ 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) + 25872*e**9*m**6*x**2*
```

```
(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 6
3273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 +
1026576*e**9*m + 362880*e**9) + 14868*e**9*m**6*x*(d + e*x)**m/(e**9*m**9 +
 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e
**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e
**9) + 453600*e**9*m**5*x**9*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e
**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**
9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) - 218610*e**9*m*
*5*x**8*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*
m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9
*m**2 + 1026576*e**9*m + 362880*e**9) + 579642*e**9*m**5*x**7*(d + e*x)**m/
(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**
5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*
m + 362880*e**9) - 208458*e**9*m**5*x**6*(d + e*x)**m/(e**9*m**9 + 45*e**9*
m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4
+ 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) + 90
2800*e**9*m**5*x**5*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7
+ 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 +
1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) + 430690*e**9*m**5*x**4*(
d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63
273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1
026576*e**9*m + 362880*e**9) + 772326*e**9*m**5*x**3*(d + e*x)**m/(e**9*m**
9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 26932
5*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 36288
0*e**9) + 260106*e**9*m**5*x**2*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 87
0*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*
e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) + 155232*e**9
*m**5*x*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*
m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9
*m**2 + 1026576*e**9*m + 362880*e**9) + 2244900*e**9*m**4*x**9*(d + e*x)**m
/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m*
*5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9
*m + 362880*e**9) - 1098405*e**9*m**4*x**8*(d + e*x)**m/(e**9*m**9 + 45*e**
9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**
4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) +
2965809*e**9*m**4*x**7*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m*
*7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3
 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) - 1090353*e**9*m**4*x*
*6*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6
+ 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2
 + 1026576*e**9*m + 362880*e**9) + 4850404*e**9*m**4*x**5*(d + e*x)**m/(e**
9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 +
269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m +
362880*e**9) + 2389985*e**9*m**4*x**4*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**
8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 7
```

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23680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) + 44532
33*e**9*m**4*x**3*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 +
9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 11
72700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) + 1567797*e**9*m**4*x**2*(d
 + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 632
73*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 10
26576*e**9*m + 362880*e**9) + 983682*e**9*m**4*x*(d + e*x)**m/(e**9*m**9 +
45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e*
*9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e*
*9) + 6728400*e**9*m**3*x**9*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e
**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**
9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) - 3332385*e**9*m
**3*x**8*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9
*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**
9*m**2 + 1026576*e**9*m + 362880*e**9) + 9134412*e**9*m**3*x**7*(d + e*x)**
m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m
**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**
9*m + 362880*e**9) - 3422907*e**9*m**3*x**6*(d + e*x)**m/(e**9*m**9 + 45*e*
*9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m*
*4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) +
 15608080*e**9*m**3*x**5*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*
m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m*
*3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) + 7946185*e**9*m**3*
x**4*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**
6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m*
*2 + 1026576*e**9*m + 362880*e**9) + 15458076*e**9*m**3*x**3*(d + e*x)**m/(
e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5
 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m
 + 362880*e**9) + 5752131*e**9*m**3*x**2*(d + e*x)**m/(e**9*m**9 + 45*e**9*
m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4
+ 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) + 38
64168*e**9*m**3*x*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 +
9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 11
72700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) + 11812400*e**9*m**2*x**9*(
d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63
273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1
026576*e**9*m + 362880*e**9) - 5906520*e**9*m**2*x**8*(d + e*x)**m/(e**9*m*
*9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 2693
25*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 3628
80*e**9) + 16387596*e**9*m**2*x**7*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 +
 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 7236
80*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) - 6238718*
e**9*m**2*x**6*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 945
0*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 11727
00*e**9*m**2 + 1026576*e**9*m + 362880*e**9) + 29064240*e**9*m**2*x**5*(d +
```

```
 e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273
*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026
576*e**9*m + 362880*e**9) + 15254460*e**9*m**2*x**4*(d + e*x)**m/(e**9*m**9
 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325
*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880
*e**9) + 31059532*e**9*m**2*x**3*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 8
70*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680
*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) + 12377178*e
**9*m**2*x**2*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450
*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 117270
0*e**9*m**2 + 1026576*e**9*m + 362880*e**9) + 9162072*e**9*m**2*x*(d + e*x)
**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9
*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e
**9*m + 362880*e**9) + 10958400*e**9*m*x**9*(d + e*x)**m/(e**9*m**9 + 45*e*
*9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m*
*4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) -
 5519340*e**9*m*x**8*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7
 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 +
 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) + 15456528*e**9*m*x**7*(
d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63
273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1
026576*e**9*m + 362880*e**9) - 5957592*e**9*m*x**6*(d + e*x)**m/(e**9*m**9
+ 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*
e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*
e**9) + 28238400*e**9*m*x**5*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e
**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**
9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) + 15207660*e**9*
m*x**4*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m
**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*
m**2 + 1026576*e**9*m + 362880*e**9) + 32300304*e**9*m*x**3*(d + e*x)**m/(e
**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5
+ 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m
+ 362880*e**9) + 13944744*e**9*m*x**2*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**
8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 7
23680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) + 11946
528*e**9*m*x*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*
e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700
*e**9*m**2 + 1026576*e**9*m + 362880*e**9) + 4032000*e**9*x**9*(d + e*x)**m
/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m*
*5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9
*m + 362880*e**9) - 2041200*e**9*x**8*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**
8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 7
23680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) + 57542
40*e**9*x**7*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*
e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700
```

```
*e**9*m**2 + 1026576*e**9*m + 362880*e**9) - 2237760*e**9*x**6*(d + e*x)**m
/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m*
*5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9
*m + 362880*e**9) + 10741248*e**9*x**5*(d + e*x)**m/(e**9*m**9 + 45*e**9*m*
*8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 +
723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) + 5896
800*e**9*x**4*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450
*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 117270
0*e**9*m**2 + 1026576*e**9*m + 362880*e**9) + 12942720*e**9*x**3*(d + e*x)*
*m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*
m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e*
*9*m + 362880*e**9) + 5987520*e**9*x**2*(d + e*x)**m/(e**9*m**9 + 45*e**9*m
**8 + 870*e**9*m**7 + 9450*e**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 +
 723680*e**9*m**3 + 1172700*e**9*m**2 + 1026576*e**9*m + 362880*e**9) + 653
1840*e**9*x*(d + e*x)**m/(e**9*m**9 + 45*e**9*m**8 + 870*e**9*m**7 + 9450*e
**9*m**6 + 63273*e**9*m**5 + 269325*e**9*m**4 + 723680*e**9*m**3 + 1172700*
e**9*m**2 + 1026576*e**9*m + 362880*e**9), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1414 vs. 2(432) = 864.

Time = 0.24 (sec) , antiderivative size = 1414, normalized size of antiderivative = 3.27

$$\int (d+ex)^m \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right)\,dx=\text{Too large to display}$$

```
[In] integrate((e*x+d)^m*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="m
axima")
```

```
[Out] 33*(e^2*(m + 1)*x^2 + d*e*m*x - d^2)*(e*x + d)^m/((m^2 + 3*m + 2)*e^2) + 18
*(e*x + d)^(m + 1)/(e*(m + 1)) + 107*((m^2 + 3*m + 2)*e^3*x^3 + (m^2 + m)*d
*e^2*x^2 - 2*d^2*e*m*x + 2*d^3)*(e*x + d)^m/((m^3 + 6*m^2 + 11*m + 6)*e^3)
+ 65*((m^3 + 6*m^2 + 11*m + 6)*e^4*x^4 + (m^3 + 3*m^2 + 2*m)*d*e^3*x^3 - 3*
(m^2 + m)*d^2*e^2*x^2 + 6*d^3*e*m*x - 6*d^4)*(e*x + d)^m/((m^4 + 10*m^3 + 3
5*m^2 + 50*m + 24)*e^4) + 148*((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*e^5*x^5
+ (m^4 + 6*m^3 + 11*m^2 + 6*m)*d*e^4*x^4 - 4*(m^3 + 3*m^2 + 2*m)*d^2*e^3*x^
3 + 12*(m^2 + m)*d^3*e^2*x^2 - 24*d^4*e*m*x + 24*d^5)*(e*x + d)^m/((m^5 + 1
5*m^4 + 85*m^3 + 225*m^2 + 274*m + 120)*e^5) - 37*((m^5 + 15*m^4 + 85*m^3 +
 225*m^2 + 274*m + 120)*e^6*x^6 + (m^5 + 10*m^4 + 35*m^3 + 50*m^2 + 24*m)*d
*e^5*x^5 - 5*(m^4 + 6*m^3 + 11*m^2 + 6*m)*d^2*e^4*x^4 + 20*(m^3 + 3*m^2 + 2
*m)*d^3*e^3*x^3 - 60*(m^2 + m)*d^4*e^2*x^2 + 120*d^5*e*m*x - 120*d^6)*(e*x
+ d)^m/((m^6 + 21*m^5 + 175*m^4 + 735*m^3 + 1624*m^2 + 1764*m + 720)*e^6) +
 111*((m^6 + 21*m^5 + 175*m^4 + 735*m^3 + 1624*m^2 + 1764*m + 720)*e^7*x^7
+ (m^6 + 15*m^5 + 85*m^4 + 225*m^3 + 274*m^2 + 120*m)*d*e^6*x^6 - 6*(m^5 +
10*m^4 + 35*m^3 + 50*m^2 + 24*m)*d^2*e^5*x^5 + 30*(m^4 + 6*m^3 + 11*m^2 + 6
```

```
*m)*d^3*e^4*x^4 - 120*(m^3 + 3*m^2 + 2*m)*d^4*e^3*x^3 + 360*(m^2 + m)*d^5*e
^2*x^2 - 720*d^6*e*m*x + 720*d^7)*(e*x + d)^m/((m^7 + 28*m^6 + 322*m^5 + 19
60*m^4 + 6769*m^3 + 13132*m^2 + 13068*m + 5040)*e^7) - 45*((m^7 + 28*m^6 +
322*m^5 + 1960*m^4 + 6769*m^3 + 13132*m^2 + 13068*m + 5040)*e^8*x^8 + (m^7
+ 21*m^6 + 175*m^5 + 735*m^4 + 1624*m^3 + 1764*m^2 + 720*m)*d*e^7*x^7 - 7*(
m^6 + 15*m^5 + 85*m^4 + 225*m^3 + 274*m^2 + 120*m)*d^2*e^6*x^6 + 42*(m^5 +
10*m^4 + 35*m^3 + 50*m^2 + 24*m)*d^3*e^5*x^5 - 210*(m^4 + 6*m^3 + 11*m^2 +
6*m)*d^4*e^4*x^4 + 840*(m^3 + 3*m^2 + 2*m)*d^5*e^3*x^3 - 2520*(m^2 + m)*d^6
*e^2*x^2 + 5040*d^7*e*m*x - 5040*d^8)*(e*x + d)^m/((m^8 + 36*m^7 + 546*m^6
+ 4536*m^5 + 22449*m^4 + 67284*m^3 + 118124*m^2 + 109584*m + 40320)*e^8) +
100*((m^8 + 36*m^7 + 546*m^6 + 4536*m^5 + 22449*m^4 + 67284*m^3 + 118124*m^
2 + 109584*m + 40320)*e^9*x^9 + (m^8 + 28*m^7 + 322*m^6 + 1960*m^5 + 6769*m
^4 + 13132*m^3 + 13068*m^2 + 5040*m)*d*e^8*x^8 - 8*(m^7 + 21*m^6 + 175*m^5
+ 735*m^4 + 1624*m^3 + 1764*m^2 + 720*m)*d^2*e^7*x^7 + 56*(m^6 + 15*m^5 + 8
5*m^4 + 225*m^3 + 274*m^2 + 120*m)*d^3*e^6*x^6 - 336*(m^5 + 10*m^4 + 35*m^3
 + 50*m^2 + 24*m)*d^4*e^5*x^5 + 1680*(m^4 + 6*m^3 + 11*m^2 + 6*m)*d^5*e^4*x
^4 - 6720*(m^3 + 3*m^2 + 2*m)*d^6*e^3*x^3 + 20160*(m^2 + m)*d^7*e^2*x^2 - 4
0320*d^8*e*m*x + 40320*d^9)*(e*x + d)^m/((m^9 + 45*m^8 + 870*m^7 + 9450*m^6
 + 63273*m^5 + 269325*m^4 + 723680*m^3 + 1172700*m^2 + 1026576*m + 362880)*
e^9)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6226 vs. 2(432) = 864.

Time = 0.32 (sec) , antiderivative size = 6226, normalized size of antiderivative = 14.41

$$\int (d+ex)^m \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right)\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)^m*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="g
iac")
```

```
[Out] (100*(e*x + d)^m*e^9*m^8*x^9 + 100*(e*x + d)^m*d*e^8*m^8*x^8 - 45*(e*x + d)
^m*e^9*m^8*x^8 + 3600*(e*x + d)^m*e^9*m^7*x^9 - 45*(e*x + d)^m*d*e^8*m^8*x^
7 + 111*(e*x + d)^m*e^9*m^8*x^7 + 2800*(e*x + d)^m*d*e^8*m^7*x^8 - 1665*(e*
x + d)^m*e^9*m^7*x^8 + 54600*(e*x + d)^m*e^9*m^6*x^9 + 111*(e*x + d)^m*d*e^
8*m^8*x^6 - 37*(e*x + d)^m*e^9*m^8*x^6 - 800*(e*x + d)^m*d^2*e^7*m^7*x^7 -
1350*(e*x + d)^m*d*e^8*m^7*x^7 + 4218*(e*x + d)^m*e^9*m^7*x^7 + 32200*(e*x
+ d)^m*d*e^8*m^6*x^8 - 25830*(e*x + d)^m*e^9*m^6*x^8 + 453600*(e*x + d)^m*e
^9*m^5*x^9 - 37*(e*x + d)^m*d*e^8*m^8*x^5 + 148*(e*x + d)^m*e^9*m^8*x^5 + 3
15*(e*x + d)^m*d^2*e^7*m^7*x^6 + 3552*(e*x + d)^m*d*e^8*m^7*x^6 - 1443*(e*x
 + d)^m*e^9*m^7*x^6 - 16800*(e*x + d)^m*d^2*e^7*m^6*x^7 - 16380*(e*x + d)^m
*d*e^8*m^6*x^7 + 67044*(e*x + d)^m*e^9*m^6*x^7 + 196000*(e*x + d)^m*d*e^8*m
^5*x^8 - 218610*(e*x + d)^m*e^9*m^5*x^8 + 2244900*(e*x + d)^m*e^9*m^4*x^9 +
 148*(e*x + d)^m*d*e^8*m^8*x^4 + 65*(e*x + d)^m*e^9*m^8*x^4 - 666*(e*x + d)
```

```
^m*d^2*e^7*m^7*x^5 - 1258*(e*x + d)^m*d*e^8*m^7*x^5 + 5920*(e*x + d)^m*e^9*
m^7*x^5 + 5600*(e*x + d)^m*d^3*e^6*m^6*x^6 + 7560*(e*x + d)^m*d^2*e^7*m^6*x
^6 + 45732*(e*x + d)^m*d*e^8*m^6*x^6 - 23532*(e*x + d)^m*e^9*m^6*x^6 - 1400
00*(e*x + d)^m*d^2*e^7*m^5*x^7 - 103950*(e*x + d)^m*d*e^8*m^5*x^7 + 579642*
(e*x + d)^m*e^9*m^5*x^7 + 676900*(e*x + d)^m*d*e^8*m^4*x^8 - 1098405*(e*x +
 d)^m*e^9*m^4*x^8 + 6728400*(e*x + d)^m*e^9*m^3*x^9 + 65*(e*x + d)^m*d*e^8*
m^8*x^3 + 107*(e*x + d)^m*e^9*m^8*x^3 + 185*(e*x + d)^m*d^2*e^7*m^7*x^4 + 5
328*(e*x + d)^m*d*e^8*m^7*x^4 + 2665*(e*x + d)^m*e^9*m^7*x^4 - 1890*(e*x +
d)^m*d^3*e^6*m^6*x^5 - 17982*(e*x + d)^m*d^2*e^7*m^6*x^5 - 17242*(e*x + d)^
m*d*e^8*m^6*x^5 + 99160*(e*x + d)^m*e^9*m^6*x^5 + 84000*(e*x + d)^m*d^3*e^6
*m^5*x^6 + 69300*(e*x + d)^m*d^2*e^7*m^5*x^6 + 305250*(e*x + d)^m*d*e^8*m^5
*x^6 - 208458*(e*x + d)^m*e^9*m^5*x^6 - 588000*(e*x + d)^m*d^2*e^7*m^4*x^7
- 370755*(e*x + d)^m*d*e^8*m^4*x^7 + 2965809*(e*x + d)^m*e^9*m^4*x^7 + 1313
200*(e*x + d)^m*d*e^8*m^3*x^8 - 3332385*(e*x + d)^m*e^9*m^3*x^8 + 11812400*
(e*x + d)^m*e^9*m^2*x^9 + 107*(e*x + d)^m*d*e^8*m^8*x^2 + 33*(e*x + d)^m*e^
9*m^8*x^2 - 592*(e*x + d)^m*d^2*e^7*m^7*x^3 + 2470*(e*x + d)^m*d*e^8*m^7*x^
3 + 4494*(e*x + d)^m*e^9*m^7*x^3 + 3330*(e*x + d)^m*d^3*e^6*m^6*x^4 + 5550*
(e*x + d)^m*d^2*e^7*m^6*x^4 + 77848*(e*x + d)^m*d*e^8*m^6*x^4 + 45890*(e*x
+ d)^m*e^9*m^6*x^4 - 33600*(e*x + d)^m*d^4*e^5*m^5*x^5 - 35910*(e*x + d)^m*
d^3*e^6*m^5*x^5 - 184482*(e*x + d)^m*d^2*e^7*m^5*x^5 - 122248*(e*x + d)^m*d
*e^8*m^5*x^5 + 902800*(e*x + d)^m*e^9*m^5*x^5 + 476000*(e*x + d)^m*d^3*e^6*
m^4*x^6 + 311850*(e*x + d)^m*d^2*e^7*m^4*x^6 + 1134309*(e*x + d)^m*d*e^8*m^
4*x^6 - 1090353*(e*x + d)^m*e^9*m^4*x^6 - 1299200*(e*x + d)^m*d^2*e^7*m^3*x
^7 - 737100*(e*x + d)^m*d*e^8*m^3*x^7 + 9134412*(e*x + d)^m*e^9*m^3*x^7 + 1
306800*(e*x + d)^m*d*e^8*m^2*x^8 - 5906520*(e*x + d)^m*e^9*m^2*x^8 + 109584
00*(e*x + d)^m*e^9*m*x^9 + 33*(e*x + d)^m*d*e^8*m^8*x + 18*(e*x + d)^m*e^9*
m^8*x - 195*(e*x + d)^m*d^2*e^7*m^7*x^2 + 4280*(e*x + d)^m*d*e^8*m^7*x^2 +
1419*(e*x + d)^m*e^9*m^7*x^2 - 740*(e*x + d)^m*d^3*e^6*m^6*x^3 - 19536*(e*x
 + d)^m*d^2*e^7*m^6*x^3 + 38480*(e*x + d)^m*d*e^8*m^6*x^3 + 79608*(e*x + d)
^m*e^9*m^6*x^3 + 9450*(e*x + d)^m*d^4*e^5*m^5*x^4 + 76590*(e*x + d)^m*d^3*e
^6*m^5*x^4 + 64010*(e*x + d)^m*d^2*e^7*m^5*x^4 + 591408*(e*x + d)^m*d*e^8*m
^5*x^4 + 430690*(e*x + d)^m*e^9*m^5*x^4 - 336000*(e*x + d)^m*d^4*e^5*m^4*x^
5 - 236250*(e*x + d)^m*d^3*e^6*m^4*x^5 - 909090*(e*x + d)^m*d^2*e^7*m^4*x^5
 - 479113*(e*x + d)^m*d*e^8*m^4*x^5 + 4850404*(e*x + d)^m*e^9*m^4*x^5 + 126
0000*(e*x + d)^m*d^3*e^6*m^3*x^6 + 724185*(e*x + d)^m*d^2*e^7*m^3*x^6 + 232
8558*(e*x + d)^m*d*e^8*m^3*x^6 - 3422907*(e*x + d)^m*e^9*m^3*x^6 - 1411200*
(e*x + d)^m*d^2*e^7*m^2*x^7 - 746820*(e*x + d)^m*d*e^8*m^2*x^7 + 16387596*(
e*x + d)^m*e^9*m^2*x^7 + 504000*(e*x + d)^m*d*e^8*m*x^8 - 5519340*(e*x + d)
^m*e^9*m*x^8 + 4032000*(e*x + d)^m*e^9*x^9 + 18*(e*x + d)^m*d*e^8*m^8 - 214
*(e*x + d)^m*d^2*e^7*m^7*x + 1386*(e*x + d)^m*d*e^8*m^7*x + 792*(e*x + d)^m
*e^9*m^7*x + 1776*(e*x + d)^m*d^3*e^6*m^6*x^2 - 7020*(e*x + d)^m*d^2*e^7*m^
6*x^2 + 71048*(e*x + d)^m*d*e^8*m^6*x^2 + 25872*(e*x + d)^m*e^9*m^6*x^2 - 1
3320*(e*x + d)^m*d^4*e^5*m^5*x^3 - 19980*(e*x + d)^m*d^3*e^6*m^5*x^3 - 2527
84*(e*x + d)^m*d^2*e^7*m^5*x^3 + 315250*(e*x + d)^m*d*e^8*m^5*x^3 + 772326*
(e*x + d)^m*e^9*m^5*x^3 + 168000*(e*x + d)^m*d^5*e^4*m^4*x^4 + 141750*(e*x
```

```
+ d)^m*d^4*e^5*m^4*x^4 + 616050*(e*x + d)^m*d^3*e^6*m^4*x^4 + 355200*(e*x +
 d)^m*d^2*e^7*m^4*x^4 + 2484772*(e*x + d)^m*d*e^8*m^4*x^4 + 2389985*(e*x +
d)^m*e^9*m^4*x^4 - 1176000*(e*x + d)^m*d^4*e^5*m^3*x^5 - 689850*(e*x + d)^m
*d^3*e^6*m^3*x^5 - 2260404*(e*x + d)^m*d^2*e^7*m^3*x^5 - 1027342*(e*x + d)^
m*d*e^8*m^3*x^5 + 15608080*(e*x + d)^m*e^9*m^3*x^5 + 1534400*(e*x + d)^m*d^
3*e^6*m^2*x^6 + 814590*(e*x + d)^m*d^2*e^7*m^2*x^6 + 2416248*(e*x + d)^m*d*
e^8*m^2*x^6 - 6238718*(e*x + d)^m*e^9*m^2*x^6 - 576000*(e*x + d)^m*d^2*e^7*
m*x^7 - 291600*(e*x + d)^m*d*e^8*m*x^7 + 15456528*(e*x + d)^m*e^9*m*x^7 - 2
041200*(e*x + d)^m*e^9*x^8 - 33*(e*x + d)^m*d^2*e^7*m^7 + 792*(e*x + d)^m*d
*e^8*m^7 + 390*(e*x + d)^m*d^3*e^6*m^6*x - 8346*(e*x + d)^m*d^2*e^7*m^6*x +
 24486*(e*x + d)^m*d*e^8*m^6*x + 14868*(e*x + d)^m*e^9*m^6*x + 2220*(e*x +
d)^m*d^4*e^5*m^5*x^2 + 55056*(e*x + d)^m*d^3*e^6*m^5*x^2 - 101400*(e*x + d)
^m*d^2*e^7*m^5*x^2 + 630230*(e*x + d)^m*d*e^8*m^5*x^2 + 260106*(e*x + d)^m*
e^9*m^5*x^2 - 37800*(e*x + d)^m*d^5*e^4*m^4*x^3 - 266400*(e*x + d)^m*d^4*e^
5*m^4*x^3 - 196100*(e*x + d)^m*d^3*e^6*m^4*x^3 - 1607280*(e*x + d)^m*d^2*e^
7*m^4*x^3 + 1444235*(e*x + d)^m*d*e^8*m^4*x^3 + 4453233*(e*x + d)^m*e^9*m^4
*x^3 + 1008000*(e*x + d)^m*d^5*e^4*m^3*x^4 + 614250*(e*x + d)^m*d^4*e^5*m^3
*x^4 + 2081250*(e*x + d)^m*d^3*e^6*m^3*x^4 + 974765*(e*x + d)^m*d^2*e^7*m^3
*x^4 + 5668992*(e*x + d)^m*d*e^8*m^3*x^4 + 7946185*(e*x + d)^m*e^9*m^3*x^4
 - 1680000*(e*x + d)^m*d^4*e^5*m^2*x^5 - 895860*(e*x + d)^m*d^3*e^6*m^2*x^5
 - 2669328*(e*x + d)^m*d^2*e^7*m^2*x^5 - 1102008*(e*x + d)^m*d*e^8*m^2*x^5 +
 29064240*(e*x + d)^m*e^9*m^2*x^5 + 672000*(e*x + d)^m*d^3*e^6*m*x^6 + 3402
00*(e*x + d)^m*d^2*e^7*m*x^6 + 959040*(e*x + d)^m*d*e^8*m*x^6 - 5957592*(e*
x + d)^m*e^9*m*x^6 + 5754240*(e*x + d)^m*e^9*x^7 + 214*(e*x + d)^m*d^3*e^6*
m^6 - 1386*(e*x + d)^m*d^2*e^7*m^6 + 14868*(e*x + d)^m*d*e^8*m^6 - 3552*(e*
x + d)^m*d^4*e^5*m^5*x + 13650*(e*x + d)^m*d^3*e^6*m^5*x - 133750*(e*x + d)
^m*d^2*e^7*m^5*x + 235620*(e*x + d)^m*d*e^8*m^5*x + 155232*(e*x + d)^m*e^9*
m^5*x + 39960*(e*x + d)^m*d^5*e^4*m^4*x^2 + 55500*(e*x + d)^m*d^4*e^5*m^4*x
^2 + 648240*(e*x + d)^m*d^3*e^6*m^4*x^2 - 742950*(e*x + d)^m*d^2*e^7*m^4*x^
2 + 3192773*(e*x + d)^m*d*e^8*m^4*x^2 + 1567797*(e*x + d)^m*e^9*m^4*x^2 - 6
72000*(e*x + d)^m*d^6*e^3*m^3*x^3 - 453600*(e*x + d)^m*d^5*e^4*m^3*x^3 - 16
65000*(e*x + d)^m*d^4*e^5*m^3*x^3 - 832500*(e*x + d)^m*d^3*e^6*m^3*x^3 - 51
17248*(e*x + d)^m*d^2*e^7*m^3*x^3 + 3613480*(e*x + d)^m*d*e^8*m^3*x^3 + 154
58076*(e*x + d)^m*e^9*m^3*x^3 + 1848000*(e*x + d)^m*d^5*e^4*m^2*x^4 + 99225
0*(e*x + d)^m*d^4*e^5*m^2*x^4 + 2977020*(e*x + d)^m*d^3*e^6*m^2*x^4 + 12376
50*(e*x + d)^m*d^2*e^7*m^2*x^4 + 6388272*(e*x + d)^m*d*e^8*m^2*x^4 + 152544
60*(e*x + d)^m*e^9*m^2*x^4 - 806400*(e*x + d)^m*d^4*e^5*m*x^5 - 408240*(e*x
 + d)^m*d^3*e^6*m*x^5 - 1150848*(e*x + d)^m*d^2*e^7*m*x^5 - 447552*(e*x + d
)^m*d*e^8*m*x^5 + 28238400*(e*x + d)^m*e^9*m*x^5 - 2237760*(e*x + d)^m*e^9*
x^6 - 390*(e*x + d)^m*d^4*e^5*m^5 + 8346*(e*x + d)^m*d^3*e^6*m^5 - 24486*(e
*x + d)^m*d^2*e^7*m^5 + 155232*(e*x + d)^m*d*e^8*m^5 - 4440*(e*x + d)^m*d^5
*e^4*m^4*x - 106560*(e*x + d)^m*d^4*e^5*m^4*x + 189150*(e*x + d)^m*d^3*e^6*
m^4*x - 1126710*(e*x + d)^m*d^2*e^7*m^4*x + 1332177*(e*x + d)^m*d*e^8*m^4*x
 + 983682*(e*x + d)^m*e^9*m^4*x + 113400*(e*x + d)^m*d^6*e^3*m^3*x^2 + 7192
80*(e*x + d)^m*d^5*e^4*m^3*x^2 + 477300*(e*x + d)^m*d^4*e^5*m^3*x^2 + 35253
```

```
60*(e*x + d)^m*d^3*e^6*m^3*x^2 - 2846805*(e*x + d)^m*d^2*e^7*m^3*x^2 + 9072
530*(e*x + d)^m*d*e^8*m^3*x^2 + 5752131*(e*x + d)^m*e^9*m^3*x^2 - 2016000*(
e*x + d)^m*d^6*e^3*m^2*x^3 - 1096200*(e*x + d)^m*d^5*e^4*m^2*x^3 - 3330000*
(e*x + d)^m*d^4*e^5*m^2*x^3 - 1401560*(e*x + d)^m*d^3*e^6*m^2*x^3 - 7324224
*(e*x + d)^m*d^2*e^7*m^2*x^3 + 4414020*(e*x + d)^m*d*e^8*m^2*x^3 + 31059532
*(e*x + d)^m*e^9*m^2*x^3 + 1008000*(e*x + d)^m*d^5*e^4*m*x^4 + 510300*(e*x
+ d)^m*d^4*e^5*m*x^4 + 1438560*(e*x + d)^m*d^3*e^6*m*x^4 + 559440*(e*x + d)
^m*d^2*e^7*m*x^4 + 2685312*(e*x + d)^m*d*e^8*m*x^4 + 15207660*(e*x + d)^m*e
^9*m*x^4 + 10741248*(e*x + d)^m*e^9*x^5 + 3552*(e*x + d)^m*d^5*e^4*m^4 - 13
650*(e*x + d)^m*d^4*e^5*m^4 + 133750*(e*x + d)^m*d^3*e^6*m^4 - 235620*(e*x
+ d)^m*d^2*e^7*m^4 + 983682*(e*x + d)^m*d*e^8*m^4 - 79920*(e*x + d)^m*d^6*e
^3*m^3*x - 106560*(e*x + d)^m*d^5*e^4*m^3*x - 1189920*(e*x + d)^m*d^4*e^5*m
^3*x + 1296750*(e*x + d)^m*d^3*e^6*m^3*x - 5258836*(e*x + d)^m*d^2*e^7*m^3*
x + 4419954*(e*x + d)^m*d*e^8*m^3*x + 3864168*(e*x + d)^m*e^9*m^3*x + 20160
00*(e*x + d)^m*d^7*e^2*m^2*x^2 + 1134000*(e*x + d)^m*d^6*e^3*m^2*x^2 + 3556
440*(e*x + d)^m*d^5*e^4*m^2*x^2 + 1542900*(e*x + d)^m*d^4*e^5*m^2*x^2 + 830
1024*(e*x + d)^m*d^3*e^6*m^2*x^2 - 5146830*(e*x + d)^m*d^2*e^7*m^2*x^2 + 12
914472*(e*x + d)^m*d*e^8*m^2*x^2 + 12377178*(e*x + d)^m*e^9*m^2*x^2 - 13440
00*(e*x + d)^m*d^6*e^3*m*x^3 - 680400*(e*x + d)^m*d^5*e^4*m*x^3 - 1918080*(
e*x + d)^m*d^4*e^5*m*x^3 - 745920*(e*x + d)^m*d^3*e^6*m*x^3 - 3580416*(e*x
+ d)^m*d^2*e^7*m*x^3 + 1965600*(e*x + d)^m*d*e^8*m*x^3 + 32300304*(e*x + d)
^m*e^9*m*x^3 + 5896800*(e*x + d)^m*e^9*x^4 + 4440*(e*x + d)^m*d^6*e^3*m^3 +
 106560*(e*x + d)^m*d^5*e^4*m^3 - 189150*(e*x + d)^m*d^4*e^5*m^3 + 1126710*
(e*x + d)^m*d^3*e^6*m^3 - 1332177*(e*x + d)^m*d^2*e^7*m^3 + 3864168*(e*x +
d)^m*d*e^8*m^3 - 226800*(e*x + d)^m*d^7*e^2*m^2*x - 1358640*(e*x + d)^m*d^6
*e^3*m^2*x - 848040*(e*x + d)^m*d^5*e^4*m^2*x - 5860800*(e*x + d)^m*d^4*e^5
*m^2*x + 4396860*(e*x + d)^m*d^3*e^6*m^2*x - 12886224*(e*x + d)^m*d^2*e^7*m
^2*x + 7957224*(e*x + d)^m*d*e^8*m^2*x + 9162072*(e*x + d)^m*e^9*m^2*x + 20
16000*(e*x + d)^m*d^7*e^2*m*x^2 + 1020600*(e*x + d)^m*d^6*e^3*m*x^2 + 28771
20*(e*x + d)^m*d^5*e^4*m*x^2 + 1118880*(e*x + d)^m*d^4*e^5*m*x^2 + 5370624*
(e*x + d)^m*d^3*e^6*m*x^2 - 2948400*(e*x + d)^m*d^2*e^7*m*x^2 + 6471360*(e*
x + d)^m*d*e^8*m*x^2 + 13944744*(e*x + d)^m*e^9*m*x^2 + 12942720*(e*x + d)^
m*e^9*x^3 + 79920*(e*x + d)^m*d^7*e^2*m^2 + 106560*(e*x + d)^m*d^6*e^3*m^2
+ 1189920*(e*x + d)^m*d^5*e^4*m^2 - 1296750*(e*x + d)^m*d^4*e^5*m^2 + 52588
36*(e*x + d)^m*d^3*e^6*m^2 - 4419954*(e*x + d)^m*d^2*e^7*m^2 + 9162072*(e*x
 + d)^m*d*e^8*m^2 - 4032000*(e*x + d)^m*d^8*e*m*x - 2041200*(e*x + d)^m*d^7
*e^2*m*x - 5754240*(e*x + d)^m*d^6*e^3*m*x - 2237760*(e*x + d)^m*d^5*e^4*m*
x - 10741248*(e*x + d)^m*d^4*e^5*m*x + 5896800*(e*x + d)^m*d^3*e^6*m*x - 12
942720*(e*x + d)^m*d^2*e^7*m*x + 5987520*(e*x + d)^m*d*e^8*m*x + 11946528*(
e*x + d)^m*e^9*m*x + 5987520*(e*x + d)^m*e^9*x^2 + 226800*(e*x + d)^m*d^8*e
*m + 1358640*(e*x + d)^m*d^7*e^2*m + 848040*(e*x + d)^m*d^6*e^3*m + 5860800
*(e*x + d)^m*d^5*e^4*m - 4396860*(e*x + d)^m*d^4*e^5*m + 12886224*(e*x + d)
^m*d^3*e^6*m - 7957224*(e*x + d)^m*d^2*e^7*m + 11946528*(e*x + d)^m*d*e^8*m
 + 6531840*(e*x + d)^m*e^9*x + 4032000*(e*x + d)^m*d^9 + 2041200*(e*x + d)^
m*d^8*e + 5754240*(e*x + d)^m*d^7*e^2 + 2237760*(e*x + d)^m*d^6*e^3 + 10741
```

```
248*(e*x + d)^m*d^5*e^4 - 5896800*(e*x + d)^m*d^4*e^5 + 12942720*(e*x + d)^
m*d^3*e^6 - 5987520*(e*x + d)^m*d^2*e^7 + 6531840*(e*x + d)^m*d*e^8)/(e^9*m
^9 + 45*e^9*m^8 + 870*e^9*m^7 + 9450*e^9*m^6 + 63273*e^9*m^5 + 269325*e^9*m
^4 + 723680*e^9*m^3 + 1172700*e^9*m^2 + 1026576*e^9*m + 362880*e^9)
```

Mupad [B] (verification not implemented)

Time = 15.43 (sec) , antiderivative size = 2625, normalized size of antiderivative = 6.08

$$\int (d+ex)^m \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right) \, dx = \text{Too large to display}$$

```
[In] int((d + e*x)^m*(2*x + 5*x^2 + 3)^2*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2),x)
```

```
[Out] ((d + e*x)^m*(6531840*d*e^8 + 2041200*d^8*e + 4032000*d^9 - 5987520*d^2*e^7
 + 12942720*d^3*e^6 - 5896800*d^4*e^5 + 10741248*d^5*e^4 + 2237760*d^6*e^3
+ 5754240*d^7*e^2 - 7957224*d^2*e^7*m + 12886224*d^3*e^6*m - 4396860*d^4*e^
5*m + 5860800*d^5*e^4*m + 848040*d^6*e^3*m + 1358640*d^7*e^2*m + 9162072*d*
e^8*m^2 + 3864168*d*e^8*m^3 + 983682*d*e^8*m^4 + 155232*d*e^8*m^5 + 14868*d
*e^8*m^6 + 792*d*e^8*m^7 + 18*d*e^8*m^8 - 4419954*d^2*e^7*m^2 + 5258836*d^3
*e^6*m^2 - 1296750*d^4*e^5*m^2 + 1189920*d^5*e^4*m^2 + 106560*d^6*e^3*m^2 +
 79920*d^7*e^2*m^2 - 1332177*d^2*e^7*m^3 + 1126710*d^3*e^6*m^3 - 189150*d^4
*e^5*m^3 + 106560*d^5*e^4*m^3 + 4440*d^6*e^3*m^3 - 235620*d^2*e^7*m^4 + 133
750*d^3*e^6*m^4 - 13650*d^4*e^5*m^4 + 3552*d^5*e^4*m^4 - 24486*d^2*e^7*m^5
+ 8346*d^3*e^6*m^5 - 390*d^4*e^5*m^5 - 1386*d^2*e^7*m^6 + 214*d^3*e^6*m^6 -
 33*d^2*e^7*m^7 + 11946528*d*e^8*m + 226800*d^8*e*m))/(e^9*(1026576*m + 117
2700*m^2 + 723680*m^3 + 269325*m^4 + 63273*m^5 + 9450*m^6 + 870*m^7 + 45*m^
8 + m^9 + 362880)) + (100*x^9*(d + e*x)^m*(109584*m + 118124*m^2 + 67284*m^
3 + 22449*m^4 + 4536*m^5 + 546*m^6 + 36*m^7 + m^8 + 40320))/(1026576*m + 11
72700*m^2 + 723680*m^3 + 269325*m^4 + 63273*m^5 + 9450*m^6 + 870*m^7 + 45*m
^8 + m^9 + 362880) + (x*(d + e*x)^m*(11946528*e^9*m + 6531840*e^9 + 9162072
*e^9*m^2 + 3864168*e^9*m^3 + 983682*e^9*m^4 + 155232*e^9*m^5 + 14868*e^9*m^
6 + 792*e^9*m^7 + 18*e^9*m^8 - 12942720*d^2*e^7*m + 5896800*d^3*e^6*m - 107
41248*d^4*e^5*m - 2237760*d^5*e^4*m - 5754240*d^6*e^3*m - 2041200*d^7*e^2*m
 + 7957224*d*e^8*m^2 + 4419954*d*e^8*m^3 + 1332177*d*e^8*m^4 + 235620*d*e^8
*m^5 + 24486*d*e^8*m^6 + 1386*d*e^8*m^7 + 33*d*e^8*m^8 - 12886224*d^2*e^7*m
^2 + 4396860*d^3*e^6*m^2 - 5860800*d^4*e^5*m^2 - 848040*d^5*e^4*m^2 - 13586
40*d^6*e^3*m^2 - 226800*d^7*e^2*m^2 - 5258836*d^2*e^7*m^3 + 1296750*d^3*e^6
*m^3 - 1189920*d^4*e^5*m^3 - 106560*d^5*e^4*m^3 - 79920*d^6*e^3*m^3 - 11267
10*d^2*e^7*m^4 + 189150*d^3*e^6*m^4 - 106560*d^4*e^5*m^4 - 4440*d^5*e^4*m^4
 - 133750*d^2*e^7*m^5 + 13650*d^3*e^6*m^5 - 3552*d^4*e^5*m^5 - 8346*d^2*e^7
*m^6 + 390*d^3*e^6*m^6 - 214*d^2*e^7*m^7 + 5987520*d*e^8*m - 4032000*d^8*e*
m))/(e^9*(1026576*m + 1172700*m^2 + 723680*m^3 + 269325*m^4 + 63273*m^5 + 9
450*m^6 + 870*m^7 + 45*m^8 + m^9 + 362880)) - (x^5*(d + e*x)^m*(50*m + 35*m
^2 + 10*m^3 + m^4 + 24)*(33600*d^4*m - 244200*e^4*m - 447552*e^4 - 49580*e^
```

```
4*m^2 - 4440*e^4*m^3 - 148*e^4*m^4 + 47952*d^2*e^2*m + 7067*d*e^3*m^2 + 189
0*d^3*e*m^2 + 888*d*e^3*m^3 + 37*d*e^3*m^4 + 11322*d^2*e^2*m^2 + 666*d^2*e^
2*m^3 + 18648*d*e^3*m + 17010*d^3*e*m))/(e^4*(1026576*m + 1172700*m^2 + 723
680*m^3 + 269325*m^4 + 63273*m^5 + 9450*m^6 + 870*m^7 + 45*m^8 + m^9 + 3628
80)) - (x^7*(d + e*x)^m*(800*d^2*m - 1887*e^2*m - 7992*e^2 - 111*e^2*m^2 +
405*d*e*m + 45*d*e*m^2)*(1764*m + 1624*m^2 + 735*m^3 + 175*m^4 + 21*m^5 + m
^6 + 720))/(e^2*(1026576*m + 1172700*m^2 + 723680*m^3 + 269325*m^4 + 63273*
m^5 + 9450*m^6 + 870*m^7 + 45*m^8 + m^9 + 362880)) + (x^6*(d + e*x)^m*(274*
m + 225*m^2 + 85*m^3 + 15*m^4 + m^5 + 120)*(5600*d^3*m - 7067*e^3*m - 18648
*e^3 - 888*e^3*m^2 - 37*e^3*m^3 + 1887*d*e^2*m^2 + 315*d^2*e*m^2 + 111*d*e^
2*m^3 + 7992*d*e^2*m + 2835*d^2*e*m))/(e^3*(1026576*m + 1172700*m^2 + 72368
0*m^3 + 269325*m^4 + 63273*m^5 + 9450*m^6 + 870*m^7 + 45*m^8 + m^9 + 362880
)) + (x^4*(d + e*x)^m*(11*m + 6*m^2 + m^3 + 6)*(168000*d^5*m + 732810*e^5*m
 + 982800*e^5 + 216125*e^5*m^2 + 31525*e^5*m^3 + 2275*e^5*m^4 + 65*e^5*m^5
+ 93240*d^2*e^3*m + 239760*d^3*e^2*m + 244200*d*e^4*m^2 + 9450*d^4*e*m^2 +
49580*d*e^4*m^3 + 4440*d*e^4*m^4 + 148*d*e^4*m^5 + 35335*d^2*e^3*m^2 + 5661
0*d^3*e^2*m^2 + 4440*d^2*e^3*m^3 + 3330*d^3*e^2*m^3 + 185*d^2*e^3*m^4 + 447
552*d*e^4*m + 85050*d^4*e*m))/(e^5*(1026576*m + 1172700*m^2 + 723680*m^3 +
269325*m^4 + 63273*m^5 + 9450*m^6 + 870*m^7 + 45*m^8 + m^9 + 362880)) - (5*
x^8*(d + e*x)^m*(81*e - 20*d*m + 9*e*m)*(13068*m + 13132*m^2 + 6769*m^3 + 1
960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040))/(e*(1026576*m + 1172700*m^2 + 723
680*m^3 + 269325*m^4 + 63273*m^5 + 9450*m^6 + 870*m^7 + 45*m^8 + m^9 + 3628
80)) + (x^2*(m + 1)*(d + e*x)^m*(2016000*d^7*m + 7957224*e^7*m + 5987520*e^
7 + 4419954*e^7*m^2 + 1332177*e^7*m^3 + 235620*e^7*m^4 + 24486*e^7*m^5 + 13
86*e^7*m^6 + 33*e^7*m^7 - 2948400*d^2*e^5*m + 5370624*d^3*e^4*m + 1118880*d
^4*e^3*m + 2877120*d^5*e^2*m + 6443112*d*e^6*m^2 + 113400*d^6*e*m^2 + 26294
18*d*e^6*m^3 + 563355*d*e^6*m^4 + 66875*d*e^6*m^5 + 4173*d*e^6*m^6 + 107*d*
e^6*m^7 - 2198430*d^2*e^5*m^2 + 2930400*d^3*e^4*m^2 + 424020*d^4*e^3*m^2 +
679320*d^5*e^2*m^2 - 648375*d^2*e^5*m^3 + 594960*d^3*e^4*m^3 + 53280*d^4*e^
3*m^3 + 39960*d^5*e^2*m^3 - 94575*d^2*e^5*m^4 + 53280*d^3*e^4*m^4 + 2220*d^
4*e^3*m^4 - 6825*d^2*e^5*m^5 + 1776*d^3*e^4*m^5 - 195*d^2*e^5*m^6 + 6471360
*d*e^6*m + 1020600*d^6*e*m))/(e^7*(1026576*m + 1172700*m^2 + 723680*m^3 + 2
69325*m^4 + 63273*m^5 + 9450*m^6 + 870*m^7 + 45*m^8 + m^9 + 362880)) - (x^3
*(d + e*x)^m*(3*m + m^2 + 2)*(672000*d^6*m - 6443112*e^6*m - 6471360*e^6 -
2629418*e^6*m^2 - 563355*e^6*m^3 - 66875*e^6*m^4 - 4173*e^6*m^5 - 107*e^6*m
^6 + 1790208*d^2*e^4*m + 372960*d^3*e^3*m + 959040*d^4*e^2*m - 732810*d*e^5
*m^2 + 37800*d^5*e*m^2 - 216125*d*e^5*m^3 - 31525*d*e^5*m^4 - 2275*d*e^5*m^
5 - 65*d*e^5*m^6 + 976800*d^2*e^4*m^2 + 141340*d^3*e^3*m^2 + 226440*d^4*e^2
*m^2 + 198320*d^2*e^4*m^3 + 17760*d^3*e^3*m^3 + 13320*d^4*e^2*m^3 + 17760*d
^2*e^4*m^4 + 740*d^3*e^3*m^4 + 592*d^2*e^4*m^5 - 982800*d*e^5*m + 340200*d^
5*e*m))/(e^6*(1026576*m + 1172700*m^2 + 723680*m^3 + 269325*m^4 + 63273*m^5
 + 9450*m^6 + 870*m^7 + 45*m^8 + m^9 + 362880))
```

3.369 $\int (d+ex)^m \left(3+2x+5x^2\right) \left(2+x+3x^2-5x^3+4x^4\right)$

Optimal result

Integrand size = 36, antiderivative size = 292

$$
\begin{aligned}
&\int (d+ex)^m \left(3+2x+5x^2\right) \left(2+x+3x^2-5x^3+4x^4\right) \, dx \\
&= \frac{(5d^2-2de+3e^2)\,(4d^4+5d^3e+3d^2e^2-de^3+2e^4)\,(d+ex)^{1+m}}{e^7(1+m)} \\
&\quad - \frac{(120d^5+85d^4e+68d^3e^2+12d^2e^3+42de^4-7e^5)\,(d+ex)^{2+m}}{e^7(2+m)} \\
&\quad + \frac{(300d^4+170d^3e+102d^2e^2+12de^3+21e^4)\,(d+ex)^{3+m}}{e^7(3+m)} \\
&\quad - \frac{2(200d^3+85d^2e+34de^2+2e^3)\,(d+ex)^{4+m}}{e^7(4+m)} \\
&\quad + \frac{(300d^2+85de+17e^2)\,(d+ex)^{5+m}}{e^7(5+m)} - \frac{(120d+17e)(d+ex)^{6+m}}{e^7(6+m)} + \frac{20(d+ex)^{7+m}}{e^7(7+m)}
\end{aligned}
$$

```
[Out] (5*d^2-2*d*e+3*e^2)*(4*d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)*(e*x+d)^(1+m)/e^7
/(1+m)-(120*d^5+85*d^4*e+68*d^3*e^2+12*d^2*e^3+42*d*e^4-7*e^5)*(e*x+d)^(2+m
)/e^7/(2+m)+(300*d^4+170*d^3*e+102*d^2*e^2+12*d*e^3+21*e^4)*(e*x+d)^(3+m)/e
^7/(3+m)-2*(200*d^3+85*d^2*e+34*d*e^2+2*e^3)*(e*x+d)^(4+m)/e^7/(4+m)+(300*d
^2+85*d*e+17*e^2)*(e*x+d)^(5+m)/e^7/(5+m)-(120*d+17*e)*(e*x+d)^(6+m)/e^7/(6
+m)+20*(e*x+d)^(7+m)/e^7/(7+m)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {1642}

$$\begin{aligned}
&\int (d+ex)^m \left(3+2x+5x^2\right) \left(2+x+3x^2-5x^3+4x^4\right)\,dx \\
&= \frac{(300d^2+85de+17e^2)\,(d+ex)^{m+5}}{e^7(m+5)} - \frac{2(200d^3+85d^2e+34de^2+2e^3)\,(d+ex)^{m+4}}{e^7(m+4)} \\
&\quad + \frac{(5d^2-2de+3e^2)\,(4d^4+5d^3e+3d^2e^2-de^3+2e^4)\,(d+ex)^{m+1}}{e^7(m+1)} \\
&\quad + \frac{(300d^4+170d^3e+102d^2e^2+12de^3+21e^4)\,(d+ex)^{m+3}}{e^7(m+3)} \\
&\quad - \frac{(120d^5+85d^4e+68d^3e^2+12d^2e^3+42de^4-7e^5)\,(d+ex)^{m+2}}{e^7(m+2)} \\
&\quad - \frac{(120d+17e)(d+ex)^{m+6}}{e^7(m+6)} + \frac{20(d+ex)^{m+7}}{e^7(m+7)}
\end{aligned}$$

```
[In] Int[(d + e*x)^m*(3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

```
[Out] ((5*d^2 - 2*d*e + 3*e^2)*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*(d +
 e*x)^(1 + m))/(e^7*(1 + m)) - ((120*d^5 + 85*d^4*e + 68*d^3*e^2 + 12*d^2*e
^3 + 42*d*e^4 - 7*e^5)*(d + e*x)^(2 + m))/(e^7*(2 + m)) + ((300*d^4 + 170*d
^3*e + 102*d^2*e^2 + 12*d*e^3 + 21*e^4)*(d + e*x)^(3 + m))/(e^7*(3 + m)) -
(2*(200*d^3 + 85*d^2*e + 34*d*e^2 + 2*e^3)*(d + e*x)^(4 + m))/(e^7*(4 + m))
 + ((300*d^2 + 85*d*e + 17*e^2)*(d + e*x)^(5 + m))/(e^7*(5 + m)) - ((120*d
+ 17*e)*(d + e*x)^(6 + m))/(e^7*(6 + m)) + (20*(d + e*x)^(7 + m))/(e^7*(7 +
 m))
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\text{integral} = \int \left(\frac{(20d^6+17d^5e+17d^4e^2+4d^3e^3+21d^2e^4-7de^5+6e^6)(d+ex)^m}{e^6} + \frac{(-120d^5-85d^4e-68d^3e^2-12d^2e^3-42de^4+7e^5)(d+ex)^{1+m}}{e^6} + \frac{(300d^4+170d^3e+102d^2e^2+12de^3+21e^4)(d+ex)^{2+m}}{e^6} - \frac{2(200d^3+85d^2e+34de^2+2e^3)(d+ex)^{3+m}}{e^6} + \frac{(300d^2+85de+17e^2)(d+ex)^{4+m}}{e^6} + \frac{(-120d-17e)(d+ex)^{5+m}}{e^6} + \frac{20(d+ex)^{6+m}}{e^6}\right) dx$$

$$= \frac{(5d^2-2de+3e^2)(4d^4+5d^3e+3d^2e^2-de^3+2e^4)(d+ex)^{1+m}}{e^7(1+m)} - \frac{(120d^5+85d^4e+68d^3e^2+12d^2e^3+42de^4-7e^5)(d+ex)^{2+m}}{e^7(2+m)} + \frac{(300d^4+170d^3e+102d^2e^2+12de^3+21e^4)(d+ex)^{3+m}}{e^7(3+m)} - \frac{2(200d^3+85d^2e+34de^2+2e^3)(d+ex)^{4+m}}{e^7(4+m)} + \frac{(300d^2+85de+17e^2)(d+ex)^{5+m}}{e^7(5+m)} - \frac{(120d+17e)(d+ex)^{6+m}}{e^7(6+m)} + \frac{20(d+ex)^{7+m}}{e^7(7+m)}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.89

$$\int (d+ex)^m (3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)\, dx$$

$$= \frac{(d+ex)^{1+m}\left(\frac{(5d^2-2de+3e^2)(4d^4+5d^3e+3d^2e^2-de^3+2e^4)}{1+m} - \frac{(120d^5+85d^4e+68d^3e^2+12d^2e^3+42de^4-7e^5)(d+ex)}{2+m} + \frac{(300d^4+170d^3e+}{}\right.}{}$$

```
[In] Integrate[(d + e*x)^m*(3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

```
[Out] ((d + e*x)^(1 + m)*(((5*d^2 - 2*d*e + 3*e^2)*(4*d^4 + 5*d^3*e + 3*d^2*e^2 -
 d*e^3 + 2*e^4))/(1 + m) - ((120*d^5 + 85*d^4*e + 68*d^3*e^2 + 12*d^2*e^3 +
 42*d*e^4 - 7*e^5)*(d + e*x))/(2 + m) + ((300*d^4 + 170*d^3*e + 102*d^2*e^2
 + 12*d*e^3 + 21*e^4)*(d + e*x)^2)/(3 + m) - (2*(200*d^3 + 85*d^2*e + 34*d*
e^2 + 2*e^3)*(d + e*x)^3)/(4 + m) + ((300*d^2 + 85*d*e + 17*e^2)*(d + e*x)^
4)/(5 + m) - ((120*d + 17*e)*(d + e*x)^5)/(6 + m) + (20*(d + e*x)^6)/(7 + m
)))/e^7
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1219 vs. 2(292) = 584.

Time = 0.50 (sec) , antiderivative size = 1220, normalized size of antiderivative = 4.18

method	result	size
norman	Expression too large to display	1220
gosper	Expression too large to display	1504
risch	Expression too large to display	1908
parallelrisch	Expression too large to display	3215

```
[In] int((e*x+d)^m*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x,method=_RETURNVERBOSE
)
```

```
[Out] d*(6*e^6*m^6-7*d*e^5*m^5+162*e^6*m^5+42*d^2*e^4*m^4-175*d*e^5*m^4+1770*e^6*
m^4+24*d^3*e^3*m^3+924*d^2*e^4*m^3-1715*d*e^5*m^3+9990*e^6*m^3+408*d^4*e^2*
m^2+432*d^3*e^3*m^2+7518*d^2*e^4*m^2-8225*d*e^5*m^2+30624*e^6*m^2+2040*d^5*
e*m+5304*d^4*e^2*m+2568*d^3*e^3*m+26796*d^2*e^4*m-19278*d*e^5*m+48168*e^6*m
+14400*d^6+14280*d^5*e+17136*d^4*e^2+5040*d^3*e^3+35280*d^2*e^4-17640*d*e^5
+30240*e^6)/e^7/(m^7+28*m^6+322*m^5+1960*m^4+6769*m^3+13132*m^2+13068*m+504
0)*exp(m*ln(e*x+d))+(20*d*m-17*e*m-119*e)/e/(m^2+13*m+42)*x^6*exp(m*ln(e*x+
d))+(17*d*e^2*m^3-4*e^3*m^3+85*d^2*e*m^2+221*d*e^2*m^2-72*e^3*m^2+600*d^3*m
+595*d^2*e*m+714*d*e^2*m-428*e^3*m-840*e^3)/e^3/(m^4+22*m^3+179*m^2+638*m+8
40)*x^4*exp(m*ln(e*x+d))+(21*d*e^4*m^5+7*e^5*m^5+12*d^2*e^3*m^4+462*d*e^4*m
^4+175*e^5*m^4+204*d^3*e^2*m^3+216*d^2*e^3*m^3+3759*d*e^4*m^3+1715*e^5*m^3+
1020*d^4*e*m^2+2652*d^3*e^2*m^2+1284*d^2*e^3*m^2+13398*d*e^4*m^2+8225*e^5*m
^2+7200*d^5*m+7140*d^4*e*m+8568*d^3*e^2*m+2520*d^2*e^3*m+17640*d*e^4*m+1927
8*e^5*m+17640*e^5)/e^5/(m^6+27*m^5+295*m^4+1665*m^3+5104*m^2+8028*m+5040)*x
^2*exp(m*ln(e*x+d))+20/(7+m)*x^7*exp(m*ln(e*x+d))-(17*d*e*m^2-17*e^2*m^2+12
0*d^2*m+119*d*e*m-221*e^2*m-714*e^2)/e^2/(m^3+18*m^2+107*m+210)*x^5*exp(m*l
n(e*x+d))-(4*d*e^3*m^4-21*e^4*m^4+68*d^2*e^2*m^3+72*d*e^3*m^3-462*e^4*m^3+3
40*d^3*e*m^2+884*d^2*e^2*m^2+428*d*e^3*m^2-3759*e^4*m^2+2400*d^4*m+2380*d^3
*e*m+2856*d^2*e^2*m+840*d*e^3*m-13398*e^4*m-17640*e^4)/e^4/(m^5+25*m^4+245*
m^3+1175*m^2+2754*m+2520)*x^3*exp(m*ln(e*x+d))-(-7*d*e^5*m^6-6*e^6*m^6+42*d
^2*e^4*m^5-175*d*e^5*m^5-162*e^6*m^5+24*d^3*e^3*m^4+924*d^2*e^4*m^4-1715*d*
e^5*m^4-1770*e^6*m^4+408*d^4*e^2*m^3+432*d^3*e^3*m^3+7518*d^2*e^4*m^3-8225*
d*e^5*m^3-9990*e^6*m^3+2040*d^5*e*m^2+5304*d^4*e^2*m^2+2568*d^3*e^3*m^2+267
96*d^2*e^4*m^2-19278*d*e^5*m^2-30624*e^6*m^2+14400*d^6*m+14280*d^5*e*m+1713
6*d^4*e^2*m+5040*d^3*e^3*m+35280*d^2*e^4*m-17640*d*e^5*m-48168*e^6*m-30240*
e^6)/e^6/(m^7+28*m^6+322*m^5+1960*m^4+6769*m^3+13132*m^2+13068*m+5040)*x*ex
p(m*ln(e*x+d))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1448 vs. $2(292) = 584$.

Time = 0.29 (sec) , antiderivative size = 1448, normalized size of antiderivative = 4.96

$$\int (d+ex)^m \left(3+2x+5x^2\right) \left(2+x+3x^2-5x^3+4x^4\right) \, dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)^m*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="fri
cas")
```

```
[Out] (6*d*e^6*m^6 + 20*(e^7*m^6 + 21*e^7*m^5 + 175*e^7*m^4 + 735*e^7*m^3 + 1624*
e^7*m^2 + 1764*e^7*m + 720*e^7)*x^7 + 14400*d^7 + 14280*d^6*e + 17136*d^5*e
^2 + 5040*d^4*e^3 + 35280*d^3*e^4 - 17640*d^2*e^5 + 30240*d*e^6 - (14280*e^
7 - (20*d*e^6 - 17*e^7)*m^6 - 2*(150*d*e^6 - 187*e^7)*m^5 - 170*(10*d*e^6 -
 19*e^7)*m^4 - 20*(225*d*e^6 - 697*e^7)*m^3 - (5480*d*e^6 - 31433*e^7)*m^2
- 2*(1200*d*e^6 - 17323*e^7)*m)*x^6 - (7*d^2*e^5 - 162*d*e^6)*m^5 + (17136*
e^7 - 17*(d*e^6 - e^7)*m^6 - (120*d^2*e^5 + 289*d*e^6 - 391*e^7)*m^5 - 3*(4
00*d^2*e^5 + 595*d*e^6 - 1173*e^7)*m^4 - 5*(840*d^2*e^5 + 1003*d*e^6 - 3145
*e^7)*m^3 - 2*(3000*d^2*e^5 + 3179*d*e^6 - 18224*e^7)*m^2 - 12*(240*d^2*e^5
 + 238*d*e^6 - 3417*e^7)*m)*x^5 + (42*d^3*e^4 - 175*d^2*e^5 + 1770*d*e^6)*m
^4 - (5040*e^7 - (17*d*e^6 - 4*e^7)*m^6 - (85*d^2*e^5 + 323*d*e^6 - 96*e^7)
*m^5 - (600*d^3*e^4 + 1105*d^2*e^5 + 2227*d*e^6 - 904*e^7)*m^4 - (3600*d^3*
e^4 + 4505*d^2*e^5 + 6817*d*e^6 - 4224*e^7)*m^3 - 5*(1320*d^3*e^4 + 1411*d^
2*e^5 + 1836*d*e^6 - 2036*e^7)*m^2 - 6*(600*d^3*e^4 + 595*d^2*e^5 + 714*d*e
^6 - 1968*e^7)*m)*x^4 + (24*d^4*e^3 + 924*d^3*e^4 - 1715*d^2*e^5 + 9990*d*e
^6)*m^3 + (35280*e^7 - (4*d*e^6 - 21*e^7)*m^6 - (68*d^2*e^5 + 84*d*e^6 - 52
5*e^7)*m^5 - (340*d^3*e^4 + 1088*d^2*e^5 + 652*d*e^6 - 5187*e^7)*m^4 - (240
0*d^4*e^3 + 3400*d^3*e^4 + 5644*d^2*e^5 + 2268*d*e^6 - 25599*e^7)*m^3 - 4*(
1800*d^4*e^3 + 1955*d^3*e^4 + 2584*d^2*e^5 + 844*d*e^6 - 16338*e^7)*m^2 - 4
*(1200*d^4*e^3 + 1190*d^3*e^4 + 1428*d^2*e^5 + 420*d*e^6 - 19929*e^7)*m)*x^
3 + (408*d^5*e^2 + 432*d^4*e^3 + 7518*d^3*e^4 - 8225*d^2*e^5 + 30624*d*e^6)
*m^2 + (17640*e^7 + 7*(3*d*e^6 + e^7)*m^6 + (12*d^2*e^5 + 483*d*e^6 + 182*e
^7)*m^5 + 3*(68*d^3*e^4 + 76*d^2*e^5 + 1407*d*e^6 + 630*e^7)*m^4 + (1020*d^
4*e^3 + 2856*d^3*e^4 + 1500*d^2*e^5 + 17157*d*e^6 + 9940*e^7)*m^3 + (7200*d
^5*e^2 + 8160*d^4*e^3 + 11220*d^3*e^4 + 3804*d^2*e^5 + 31038*d*e^6 + 27503*
e^7)*m^2 + 6*(1200*d^5*e^2 + 1190*d^4*e^3 + 1428*d^3*e^4 + 420*d^2*e^5 + 29
40*d*e^6 + 6153*e^7)*m)*x^2 + 6*(340*d^6*e + 884*d^5*e^2 + 428*d^4*e^3 + 44
66*d^3*e^4 - 3213*d^2*e^5 + 8028*d*e^6)*m + (30240*e^7 + (7*d*e^6 + 6*e^7)*
m^6 - (42*d^2*e^5 - 175*d*e^6 - 162*e^7)*m^5 - (24*d^3*e^4 + 924*d^2*e^5 -
1715*d*e^6 - 1770*e^7)*m^4 - (408*d^4*e^3 + 432*d^3*e^4 + 7518*d^2*e^5 - 82
25*d*e^6 - 9990*e^7)*m^3 - 6*(340*d^5*e^2 + 884*d^4*e^3 + 428*d^3*e^4 + 446
6*d^2*e^5 - 3213*d*e^6 - 5104*e^7)*m^2 - 24*(600*d^6*e + 595*d^5*e^2 + 714*
d^4*e^3 + 210*d^3*e^4 + 1470*d^2*e^5 - 735*d*e^6 - 2007*e^7)*m)*x)*(e*x + d
)^m/(e^7*m^7 + 28*e^7*m^6 + 322*e^7*m^5 + 1960*e^7*m^4 + 6769*e^7*m^3 + 131
32*e^7*m^2 + 13068*e^7*m + 5040*e^7)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26165 vs. $2(272) = 544$.

Time = 4.85 (sec) , antiderivative size = 26165, normalized size of antiderivative = 89.61

$$\int (d+ex)^m \left(3+2x+5x^2\right) \left(2+x+3x^2-5x^3+4x^4\right) \, dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)**m*(5*x**2+2*x+3)*(4*x**4-5*x**3+3*x**2+x+2),x)

[Out] Piecewise((d**m*(20*x**7/7 - 17*x**6/6 + 17*x**5/5 - x**4 + 7*x**3 + 7*x**2
/2 + 6*x), Eq(e, 0)), (1200*d**6*log(d/e + x)/(60*d**6*e**7 + 360*d**5*e**8
*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*
d*e**12*x**5 + 60*e**13*x**6) + 2940*d**6/(60*d**6*e**7 + 360*d**5*e**8*x +
 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e*
*12*x**5 + 60*e**13*x**6) + 7200*d**5*e*x*log(d/e + x)/(60*d**6*e**7 + 360*
d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x*
*4 + 360*d*e**12*x**5 + 60*e**13*x**6) + 16440*d**5*e*x/(60*d**6*e**7 + 360
*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x
**4 + 360*d*e**12*x**5 + 60*e**13*x**6) + 170*d**5*e/(60*d**6*e**7 + 360*d*
*5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4
 + 360*d*e**12*x**5 + 60*e**13*x**6) + 18000*d**4*e**2*x**2*log(d/e + x)/(6
0*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 +
 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6) + 37500*d**4*e**2*
x**2/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10
*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6) + 1020*d**4
*e**2*x/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e*
*10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6) - 34*d**
4*e**2/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**
10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6) + 24000*d
**3*e**3*x**3*log(d/e + x)/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*
x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e
**13*x**6) + 44000*d**3*e**3*x**3/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**
4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5
 + 60*e**13*x**6) + 2550*d**3*e**3*x**2/(60*d**6*e**7 + 360*d**5*e**8*x + 9
00*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**1
2*x**5 + 60*e**13*x**6) - 204*d**3*e**3*x/(60*d**6*e**7 + 360*d**5*e**8*x +
 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e*
*12*x**5 + 60*e**13*x**6) + 4*d**3*e**3/(60*d**6*e**7 + 360*d**5*e**8*x + 9
00*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**1
2*x**5 + 60*e**13*x**6) + 18000*d**2*e**4*x**4*log(d/e + x)/(60*d**6*e**7 +
 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**
11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6) + 27000*d**2*e**4*x**4/(60*d**6
*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d
**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6) + 3400*d**2*e**4*x**3/(6
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0*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 +
 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6) - 510*d**2*e**4*x*
*2/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x
**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6) + 24*d**2*e**
4*x/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*
x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6) - 21*d**2*e*
*4/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x
**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6) + 7200*d*e**5
*x**5*log(d/e + x)/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1
200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**
6) + 7200*d*e**5*x**5/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2
 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*
x**6) + 2550*d*e**5*x**4/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x*
*2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**
13*x**6) - 680*d*e**5*x**3/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*
x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e
**13*x**6) + 60*d*e**5*x**2/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9
*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*
e**13*x**6) - 126*d*e**5*x/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*
x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e
**13*x**6) - 14*d*e**5/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2
 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13
*x**6) + 1200*e**6*x**6*log(d/e + x)/(60*d**6*e**7 + 360*d**5*e**8*x + 900*
d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x
**5 + 60*e**13*x**6) + 1020*e**6*x**5/(60*d**6*e**7 + 360*d**5*e**8*x + 900
*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*
x**5 + 60*e**13*x**6) - 510*e**6*x**4/(60*d**6*e**7 + 360*d**5*e**8*x + 900
*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*
x**5 + 60*e**13*x**6) + 80*e**6*x**3/(60*d**6*e**7 + 360*d**5*e**8*x + 900*
d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x
**5 + 60*e**13*x**6) - 315*e**6*x**2/(60*d**6*e**7 + 360*d**5*e**8*x + 900*
d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x
**5 + 60*e**13*x**6) - 84*e**6*x/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4
*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5
 + 60*e**13*x**6) - 60*e**6/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*
x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e
**13*x**6), Eq(m, -7)), (-7200*d**6*log(d/e + x)/(60*d**5*e**7 + 300*d**4*e
**8*x + 600*d**3*e**9*x**2 + 600*d**2*e**10*x**3 + 300*d*e**11*x**4 + 60*e*
*12*x**5) - 16440*d**6/(60*d**5*e**7 + 300*d**4*e**8*x + 600*d**3*e**9*x**2
 + 600*d**2*e**10*x**3 + 300*d*e**11*x**4 + 60*e**12*x**5) - 36000*d**5*e*x
*log(d/e + x)/(60*d**5*e**7 + 300*d**4*e**8*x + 600*d**3*e**9*x**2 + 600*d*
*2*e**10*x**3 + 300*d*e**11*x**4 + 60*e**12*x**5) - 75000*d**5*e*x/(60*d**5
*e**7 + 300*d**4*e**8*x + 600*d**3*e**9*x**2 + 600*d**2*e**10*x**3 + 300*d*
e**11*x**4 + 60*e**12*x**5) - 1020*d**5*e*log(d/e + x)/(60*d**5*e**7 + 300*
d**4*e**8*x + 600*d**3*e**9*x**2 + 600*d**2*e**10*x**3 + 300*d*e**11*x**4 +
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 60*e**12*x**5) - 2329*d**5*e/(60*d**5*e**7 + 300*d**4*e**8*x + 600*d**3*e*
*9*x**2 + 600*d**2*e**10*x**3 + 300*d*e**11*x**4 + 60*e**12*x**5) - 72000*d
**4*e**2*x**2*log(d/e + x)/(60*d**5*e**7 + 300*d**4*e**8*x + 600*d**3*e**9*
x**2 + 600*d**2*e**10*x**3 + 300*d*e**11*x**4 + 60*e**12*x**5) - 132000*d**
4*e**2*x**2/(60*d**5*e**7 + 300*d**4*e**8*x + 600*d**3*e**9*x**2 + 600*d**2
*e**10*x**3 + 300*d*e**11*x**4 + 60*e**12*x**5) - 5100*d**4*e**2*x*log(d/e
+ x)/(60*d**5*e**7 + 300*d**4*e**8*x + 600*d**3*e**9*x**2 + 600*d**2*e**10*
x**3 + 300*d*e**11*x**4 + 60*e**12*x**5) - 10625*d**4*e**2*x/(60*d**5*e**7
+ 300*d**4*e**8*x + 600*d**3*e**9*x**2 + 600*d**2*e**10*x**3 + 300*d*e**11*
x**4 + 60*e**12*x**5) - 204*d**4*e**2/(60*d**5*e**7 + 300*d**4*e**8*x + 600
*d**3*e**9*x**2 + 600*d**2*e**10*x**3 + 300*d*e**11*x**4 + 60*e**12*x**5) -
 72000*d**3*e**3*x**3*log(d/e + x)/(60*d**5*e**7 + 300*d**4*e**8*x + 600*d*
*3*e**9*x**2 + 600*d**2*e**10*x**3 + 300*d*e**11*x**4 + 60*e**12*x**5) - 10
8000*d**3*e**3*x**3/(60*d**5*e**7 + 300*d**4*e**8*x + 600*d**3*e**9*x**2 +
600*d**2*e**10*x**3 + 300*d*e**11*x**4 + 60*e**12*x**5) - 10200*d**3*e**3*x
**2*log(d/e + x)/(60*d**5*e**7 + 300*d**4*e**8*x + 600*d**3*e**9*x**2 + 600
*d**2*e**10*x**3 + 300*d*e**11*x**4 + 60*e**12*x**5) - 18700*d**3*e**3*x**2
/(60*d**5*e**7 + 300*d**4*e**8*x + 600*d**3*e**9*x**2 + 600*d**2*e**10*x**3
 + 300*d*e**11*x**4 + 60*e**12*x**5) - 1020*d**3*e**3*x/(60*d**5*e**7 + 300
*d**4*e**8*x + 600*d**3*e**9*x**2 + 600*d**2*e**10*x**3 + 300*d*e**11*x**4
+ 60*e**12*x**5) + 12*d**3*e**3/(60*d**5*e**7 + 300*d**4*e**8*x + 600*d**3*
e**9*x**2 + 600*d**2*e**10*x**3 + 300*d*e**11*x**4 + 60*e**12*x**5) - 36000
*d**2*e**4*x**4*log(d/e + x)/(60*d**5*e**7 + 300*d**4*e**8*x + 600*d**3*e**
9*x**2 + 600*d**2*e**10*x**3 + 300*d*e**11*x**4 + 60*e**12*x**5) - 36000*d*
*2*e**4*x**4/(60*d**5*e**7 + 300*d**4*e**8*x + 600*d**3*e**9*x**2 + 600*d**
2*e**10*x**3 + 300*d*e**11*x**4 + 60*e**12*x**5) - 10200*d**2*e**4*x**3*log
(d/e + x)/(60*d**5*e**7 + 300*d**4*e**8*x + 600*d**3*e**9*x**2 + 600*d**2*e
**10*x**3 + 300*d*e**11*x**4 + 60*e**12*x**5) - 15300*d**2*e**4*x**3/(60*d*
*5*e**7 + 300*d**4*e**8*x + 600*d**3*e**9*x**2 + 600*d**2*e**10*x**3 + 300*
d*e**11*x**4 + 60*e**12*x**5) - 2040*d**2*e**4*x**2/(60*d**5*e**7 + 300*d**
4*e**8*x + 600*d**3*e**9*x**2 + 600*d**2*e**10*x**3 + 300*d*e**11*x**4 + 60
*e**12*x**5) + 60*d**2*e**4*x/(60*d**5*e**7 + 300*d**4*e**8*x + 600*d**3*e*
*9*x**2 + 600*d**2*e**10*x**3 + 300*d*e**11*x**4 + 60*e**12*x**5) - 42*d**2
*e**4/(60*d**5*e**7 + 300*d**4*e**8*x + 600*d**3*e**9*x**2 + 600*d**2*e**10
*x**3 + 300*d*e**11*x**4 + 60*e**12*x**5) - 7200*d*e**5*x**5*log(d/e + x)/(
60*d**5*e**7 + 300*d**4*e**8*x + 600*d**3*e**9*x**2 + 600*d**2*e**10*x**3 +
 300*d*e**11*x**4 + 60*e**12*x**5) - 5100*d*e**5*x**4*log(d/e + x)/(60*d**5
*e**7 + 300*d**4*e**8*x + 600*d**3*e**9*x**2 + 600*d**2*e**10*x**3 + 300*d*
e**11*x**4 + 60*e**12*x**5) - 5100*d*e**5*x**4/(60*d**5*e**7 + 300*d**4*e**
8*x + 600*d**3*e**9*x**2 + 600*d**2*e**10*x**3 + 300*d*e**11*x**4 + 60*e**1
2*x**5) - 2040*d*e**5*x**3/(60*d**5*e**7 + 300*d**4*e**8*x + 600*d**3*e**9*
x**2 + 600*d**2*e**10*x**3 + 300*d*e**11*x**4 + 60*e**12*x**5) + 120*d*e**5
*x**2/(60*d**5*e**7 + 300*d**4*e**8*x + 600*d**3*e**9*x**2 + 600*d**2*e**10
*x**3 + 300*d*e**11*x**4 + 60*e**12*x**5) - 210*d*e**5*x/(60*d**5*e**7 + 30
0*d**4*e**8*x + 600*d**3*e**9*x**2 + 600*d**2*e**10*x**3 + 300*d*e**11*x**4
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 + 60*e**12*x**5) - 21*d*e**5/(60*d**5*e**7 + 300*d**4*e**8*x + 600*d**3*e*
*9*x**2 + 600*d**2*e**10*x**3 + 300*d*e**11*x**4 + 60*e**12*x**5) + 1200*e*
*6*x**6/(60*d**5*e**7 + 300*d**4*e**8*x + 600*d**3*e**9*x**2 + 600*d**2*e**
10*x**3 + 300*d*e**11*x**4 + 60*e**12*x**5) - 1020*e**6*x**5*log(d/e + x)/(
60*d**5*e**7 + 300*d**4*e**8*x + 600*d**3*e**9*x**2 + 600*d**2*e**10*x**3 +
 300*d*e**11*x**4 + 60*e**12*x**5) - 1020*e**6*x**4/(60*d**5*e**7 + 300*d**
4*e**8*x + 600*d**3*e**9*x**2 + 600*d**2*e**10*x**3 + 300*d*e**11*x**4 + 60
*e**12*x**5) + 120*e**6*x**3/(60*d**5*e**7 + 300*d**4*e**8*x + 600*d**3*e**
9*x**2 + 600*d**2*e**10*x**3 + 300*d*e**11*x**4 + 60*e**12*x**5) - 420*e**6
*x**2/(60*d**5*e**7 + 300*d**4*e**8*x + 600*d**3*e**9*x**2 + 600*d**2*e**10
*x**3 + 300*d*e**11*x**4 + 60*e**12*x**5) - 105*e**6*x/(60*d**5*e**7 + 300*
d**4*e**8*x + 600*d**3*e**9*x**2 + 600*d**2*e**10*x**3 + 300*d*e**11*x**4 +
 60*e**12*x**5) - 72*e**6/(60*d**5*e**7 + 300*d**4*e**8*x + 600*d**3*e**9*x
**2 + 600*d**2*e**10*x**3 + 300*d*e**11*x**4 + 60*e**12*x**5), Eq(m, -6)),
(3600*d**6*log(d/e + x)/(12*d**4*e**7 + 48*d**3*e**8*x + 72*d**2*e**9*x**2
 + 48*d*e**10*x**3 + 12*e**11*x**4) + 7500*d**6/(12*d**4*e**7 + 48*d**3*e**8
*x + 72*d**2*e**9*x**2 + 48*d*e**10*x**3 + 12*e**11*x**4) + 14400*d**5*e*x*
log(d/e + x)/(12*d**4*e**7 + 48*d**3*e**8*x + 72*d**2*e**9*x**2 + 48*d*e**1
0*x**3 + 12*e**11*x**4) + 26400*d**5*e*x/(12*d**4*e**7 + 48*d**3*e**8*x + 7
2*d**2*e**9*x**2 + 48*d*e**10*x**3 + 12*e**11*x**4) + 1020*d**5*e*log(d/e +
 x)/(12*d**4*e**7 + 48*d**3*e**8*x + 72*d**2*e**9*x**2 + 48*d*e**10*x**3 +
12*e**11*x**4) + 2125*d**5*e/(12*d**4*e**7 + 48*d**3*e**8*x + 72*d**2*e**9*
x**2 + 48*d*e**10*x**3 + 12*e**11*x**4) + 21600*d**4*e**2*x**2*log(d/e + x)
/(12*d**4*e**7 + 48*d**3*e**8*x + 72*d**2*e**9*x**2 + 48*d*e**10*x**3 + 12*
e**11*x**4) + 32400*d**4*e**2*x**2/(12*d**4*e**7 + 48*d**3*e**8*x + 72*d**2
*e**9*x**2 + 48*d*e**10*x**3 + 12*e**11*x**4) + 4080*d**4*e**2*x*log(d/e +
x)/(12*d**4*e**7 + 48*d**3*e**8*x + 72*d**2*e**9*x**2 + 48*d*e**10*x**3 + 1
2*e**11*x**4) + 7480*d**4*e**2*x/(12*d**4*e**7 + 48*d**3*e**8*x + 72*d**2*e
**9*x**2 + 48*d*e**10*x**3 + 12*e**11*x**4) + 204*d**4*e**2*log(d/e + x)/(1
2*d**4*e**7 + 48*d**3*e**8*x + 72*d**2*e**9*x**2 + 48*d*e**10*x**3 + 12*e**
11*x**4) + 425*d**4*e**2/(12*d**4*e**7 + 48*d**3*e**8*x + 72*d**2*e**9*x**2
 + 48*d*e**10*x**3 + 12*e**11*x**4) + 14400*d**3*e**3*x**3*log(d/e + x)/(12
*d**4*e**7 + 48*d**3*e**8*x + 72*d**2*e**9*x**2 + 48*d*e**10*x**3 + 12*e**1
1*x**4) + 14400*d**3*e**3*x**3/(12*d**4*e**7 + 48*d**3*e**8*x + 72*d**2*e**
9*x**2 + 48*d*e**10*x**3 + 12*e**11*x**4) + 6120*d**3*e**3*x**2*log(d/e + x
)/(12*d**4*e**7 + 48*d**3*e**8*x + 72*d**2*e**9*x**2 + 48*d*e**10*x**3 + 12
*e**11*x**4) + 9180*d**3*e**3*x**2/(12*d**4*e**7 + 48*d**3*e**8*x + 72*d**2
*e**9*x**2 + 48*d*e**10*x**3 + 12*e**11*x**4) + 816*d**3*e**3*x*log(d/e + x
)/(12*d**4*e**7 + 48*d**3*e**8*x + 72*d**2*e**9*x**2 + 48*d*e**10*x**3 + 12
*e**11*x**4) + 1496*d**3*e**3*x/(12*d**4*e**7 + 48*d**3*e**8*x + 72*d**2*e*
*9*x**2 + 48*d*e**10*x**3 + 12*e**11*x**4) + 12*d**3*e**3/(12*d**4*e**7 + 4
8*d**3*e**8*x + 72*d**2*e**9*x**2 + 48*d*e**10*x**3 + 12*e**11*x**4) + 3600
*d**2*e**4*x**4*log(d/e + x)/(12*d**4*e**7 + 48*d**3*e**8*x + 72*d**2*e**9*
x**2 + 48*d*e**10*x**3 + 12*e**11*x**4) + 4080*d**2*e**4*x**3*log(d/e + x)/
(12*d**4*e**7 + 48*d**3*e**8*x + 72*d**2*e**9*x**2 + 48*d*e**10*x**3 + 12*e
```

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**11*x**4) + 4080*d**2*e**4*x**3/(12*d**4*e**7 + 48*d**3*e**8*x + 72*d**2*e
**9*x**2 + 48*d*e**10*x**3 + 12*e**11*x**4) + 1224*d**2*e**4*x**2*log(d/e +
 x)/(12*d**4*e**7 + 48*d**3*e**8*x + 72*d**2*e**9*x**2 + 48*d*e**10*x**3 +
12*e**11*x**4) + 1836*d**2*e**4*x**2/(12*d**4*e**7 + 48*d**3*e**8*x + 72*d*
*2*e**9*x**2 + 48*d*e**10*x**3 + 12*e**11*x**4) + 48*d**2*e**4*x/(12*d**4*e
**7 + 48*d**3*e**8*x + 72*d**2*e**9*x**2 + 48*d*e**10*x**3 + 12*e**11*x**4)
 - 21*d**2*e**4/(12*d**4*e**7 + 48*d**3*e**8*x + 72*d**2*e**9*x**2 + 48*d*e
**10*x**3 + 12*e**11*x**4) - 720*d*e**5*x**5/(12*d**4*e**7 + 48*d**3*e**8*x
 + 72*d**2*e**9*x**2 + 48*d*e**10*x**3 + 12*e**11*x**4) + 1020*d*e**5*x**4*
log(d/e + x)/(12*d**4*e**7 + 48*d**3*e**8*x + 72*d**2*e**9*x**2 + 48*d*e**1
0*x**3 + 12*e**11*x**4) + 816*d*e**5*x**3*log(d/e + x)/(12*d**4*e**7 + 48*d
**3*e**8*x + 72*d**2*e**9*x**2 + 48*d*e**10*x**3 + 12*e**11*x**4) + 816*d*e
**5*x**3/(12*d**4*e**7 + 48*d**3*e**8*x + 72*d**2*e**9*x**2 + 48*d*e**10*x*
*3 + 12*e**11*x**4) + 72*d*e**5*x**2/(12*d**4*e**7 + 48*d**3*e**8*x + 72*d*
*2*e**9*x**2 + 48*d*e**10*x**3 + 12*e**11*x**4) - 84*d*e**5*x/(12*d**4*e**7
 + 48*d**3*e**8*x + 72*d**2*e**9*x**2 + 48*d*e**10*x**3 + 12*e**11*x**4) -
7*d*e**5/(12*d**4*e**7 + 48*d**3*e**8*x + 72*d**2*e**9*x**2 + 48*d*e**10*x*
*3 + 12*e**11*x**4) + 120*e**6*x**6/(12*d**4*e**7 + 48*d**3*e**8*x + 72*d**
2*e**9*x**2 + 48*d*e**10*x**3 + 12*e**11*x**4) - 204*e**6*x**5/(12*d**4*e**
7 + 48*d**3*e**8*x + 72*d**2*e**9*x**2 + 48*d*e**10*x**3 + 12*e**11*x**4) +
 204*e**6*x**4*log(d/e + x)/(12*d**4*e**7 + 48*d**3*e**8*x + 72*d**2*e**9*x
**2 + 48*d*e**10*x**3 + 12*e**11*x**4) + 48*e**6*x**3/(12*d**4*e**7 + 48*d*
*3*e**8*x + 72*d**2*e**9*x**2 + 48*d*e**10*x**3 + 12*e**11*x**4) - 126*e**6
*x**2/(12*d**4*e**7 + 48*d**3*e**8*x + 72*d**2*e**9*x**2 + 48*d*e**10*x**3
+ 12*e**11*x**4) - 28*e**6*x/(12*d**4*e**7 + 48*d**3*e**8*x + 72*d**2*e**9*
x**2 + 48*d*e**10*x**3 + 12*e**11*x**4) - 18*e**6/(12*d**4*e**7 + 48*d**3*e
**8*x + 72*d**2*e**9*x**2 + 48*d*e**10*x**3 + 12*e**11*x**4), Eq(m, -5)), (
-2400*d**6*log(d/e + x)/(6*d**3*e**7 + 18*d**2*e**8*x + 18*d*e**9*x**2 + 6*
e**10*x**3) - 4400*d**6/(6*d**3*e**7 + 18*d**2*e**8*x + 18*d*e**9*x**2 + 6*
e**10*x**3) - 7200*d**5*e*x*log(d/e + x)/(6*d**3*e**7 + 18*d**2*e**8*x + 18
*d*e**9*x**2 + 6*e**10*x**3) - 10800*d**5*e*x/(6*d**3*e**7 + 18*d**2*e**8*x
 + 18*d*e**9*x**2 + 6*e**10*x**3) - 1020*d**5*e*log(d/e + x)/(6*d**3*e**7 +
 18*d**2*e**8*x + 18*d*e**9*x**2 + 6*e**10*x**3) - 1870*d**5*e/(6*d**3*e**7
 + 18*d**2*e**8*x + 18*d*e**9*x**2 + 6*e**10*x**3) - 7200*d**4*e**2*x**2*lo
g(d/e + x)/(6*d**3*e**7 + 18*d**2*e**8*x + 18*d*e**9*x**2 + 6*e**10*x**3) -
 7200*d**4*e**2*x**2/(6*d**3*e**7 + 18*d**2*e**8*x + 18*d*e**9*x**2 + 6*e**
10*x**3) - 3060*d**4*e**2*x*log(d/e + x)/(6*d**3*e**7 + 18*d**2*e**8*x + 18
*d*e**9*x**2 + 6*e**10*x**3) - 4590*d**4*e**2*x/(6*d**3*e**7 + 18*d**2*e**8
*x + 18*d*e**9*x**2 + 6*e**10*x**3) - 408*d**4*e**2*log(d/e + x)/(6*d**3*e*
*7 + 18*d**2*e**8*x + 18*d*e**9*x**2 + 6*e**10*x**3) - 748*d**4*e**2/(6*d**
3*e**7 + 18*d**2*e**8*x + 18*d*e**9*x**2 + 6*e**10*x**3) - 2400*d**3*e**3*x
**3*log(d/e + x)/(6*d**3*e**7 + 18*d**2*e**8*x + 18*d*e**9*x**2 + 6*e**10*x
**3) - 3060*d**3*e**3*x**2*log(d/e + x)/(6*d**3*e**7 + 18*d**2*e**8*x + 18*
d*e**9*x**2 + 6*e**10*x**3) - 3060*d**3*e**3*x**2/(6*d**3*e**7 + 18*d**2*e*
*8*x + 18*d*e**9*x**2 + 6*e**10*x**3) - 1224*d**3*e**3*x*log(d/e + x)/(6*d*
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*3*e**7 + 18*d**2*e**8*x + 18*d*e**9*x**2 + 6*e**10*x**3) - 1836*d**3*e**3*
x/(6*d**3*e**7 + 18*d**2*e**8*x + 18*d*e**9*x**2 + 6*e**10*x**3) - 24*d**3*
e**3*log(d/e + x)/(6*d**3*e**7 + 18*d**2*e**8*x + 18*d*e**9*x**2 + 6*e**10*
x**3) - 44*d**3*e**3/(6*d**3*e**7 + 18*d**2*e**8*x + 18*d*e**9*x**2 + 6*e**
10*x**3) + 600*d**2*e**4*x**4/(6*d**3*e**7 + 18*d**2*e**8*x + 18*d*e**9*x**
2 + 6*e**10*x**3) - 1020*d**2*e**4*x**3*log(d/e + x)/(6*d**3*e**7 + 18*d**2
*e**8*x + 18*d*e**9*x**2 + 6*e**10*x**3) - 1224*d**2*e**4*x**2*log(d/e + x)
/(6*d**3*e**7 + 18*d**2*e**8*x + 18*d*e**9*x**2 + 6*e**10*x**3) - 1224*d**2
*e**4*x**2/(6*d**3*e**7 + 18*d**2*e**8*x + 18*d*e**9*x**2 + 6*e**10*x**3) -
 72*d**2*e**4*x*log(d/e + x)/(6*d**3*e**7 + 18*d**2*e**8*x + 18*d*e**9*x**2
 + 6*e**10*x**3) - 108*d**2*e**4*x/(6*d**3*e**7 + 18*d**2*e**8*x + 18*d*e**
9*x**2 + 6*e**10*x**3) - 42*d**2*e**4/(6*d**3*e**7 + 18*d**2*e**8*x + 18*d*
e**9*x**2 + 6*e**10*x**3) - 120*d*e**5*x**5/(6*d**3*e**7 + 18*d**2*e**8*x +
 18*d*e**9*x**2 + 6*e**10*x**3) + 255*d*e**5*x**4/(6*d**3*e**7 + 18*d**2*e*
*8*x + 18*d*e**9*x**2 + 6*e**10*x**3) - 408*d*e**5*x**3*log(d/e + x)/(6*d**
3*e**7 + 18*d**2*e**8*x + 18*d*e**9*x**2 + 6*e**10*x**3) - 72*d*e**5*x**2*l
og(d/e + x)/(6*d**3*e**7 + 18*d**2*e**8*x + 18*d*e**9*x**2 + 6*e**10*x**3)
- 72*d*e**5*x**2/(6*d**3*e**7 + 18*d**2*e**8*x + 18*d*e**9*x**2 + 6*e**10*x
**3) - 126*d*e**5*x/(6*d**3*e**7 + 18*d**2*e**8*x + 18*d*e**9*x**2 + 6*e**1
0*x**3) - 7*d*e**5/(6*d**3*e**7 + 18*d**2*e**8*x + 18*d*e**9*x**2 + 6*e**10
*x**3) + 40*e**6*x**6/(6*d**3*e**7 + 18*d**2*e**8*x + 18*d*e**9*x**2 + 6*e*
*10*x**3) - 51*e**6*x**5/(6*d**3*e**7 + 18*d**2*e**8*x + 18*d*e**9*x**2 + 6
*e**10*x**3) + 102*e**6*x**4/(6*d**3*e**7 + 18*d**2*e**8*x + 18*d*e**9*x**2
 + 6*e**10*x**3) - 24*e**6*x**3*log(d/e + x)/(6*d**3*e**7 + 18*d**2*e**8*x
+ 18*d*e**9*x**2 + 6*e**10*x**3) - 126*e**6*x**2/(6*d**3*e**7 + 18*d**2*e**
8*x + 18*d*e**9*x**2 + 6*e**10*x**3) - 21*e**6*x/(6*d**3*e**7 + 18*d**2*e**
8*x + 18*d*e**9*x**2 + 6*e**10*x**3) - 12*e**6/(6*d**3*e**7 + 18*d**2*e**8*
x + 18*d*e**9*x**2 + 6*e**10*x**3), Eq(m, -4)), (1800*d**6*log(d/e + x)/(6*
d**2*e**7 + 12*d*e**8*x + 6*e**9*x**2) + 2700*d**6/(6*d**2*e**7 + 12*d*e**8
*x + 6*e**9*x**2) + 3600*d**5*e*x*log(d/e + x)/(6*d**2*e**7 + 12*d*e**8*x +
 6*e**9*x**2) + 3600*d**5*e*x/(6*d**2*e**7 + 12*d*e**8*x + 6*e**9*x**2) + 1
020*d**5*e*log(d/e + x)/(6*d**2*e**7 + 12*d*e**8*x + 6*e**9*x**2) + 1530*d*
*5*e/(6*d**2*e**7 + 12*d*e**8*x + 6*e**9*x**2) + 1800*d**4*e**2*x**2*log(d/
e + x)/(6*d**2*e**7 + 12*d*e**8*x + 6*e**9*x**2) + 2040*d**4*e**2*x*log(d/e
 + x)/(6*d**2*e**7 + 12*d*e**8*x + 6*e**9*x**2) + 2040*d**4*e**2*x/(6*d**2*
e**7 + 12*d*e**8*x + 6*e**9*x**2) + 612*d**4*e**2*log(d/e + x)/(6*d**2*e**7
 + 12*d*e**8*x + 6*e**9*x**2) + 918*d**4*e**2/(6*d**2*e**7 + 12*d*e**8*x +
6*e**9*x**2) - 600*d**3*e**3*x**3/(6*d**2*e**7 + 12*d*e**8*x + 6*e**9*x**2)
 + 1020*d**3*e**3*x**2*log(d/e + x)/(6*d**2*e**7 + 12*d*e**8*x + 6*e**9*x**
2) + 1224*d**3*e**3*x*log(d/e + x)/(6*d**2*e**7 + 12*d*e**8*x + 6*e**9*x**2
) + 1224*d**3*e**3*x/(6*d**2*e**7 + 12*d*e**8*x + 6*e**9*x**2) + 72*d**3*e*
*3*log(d/e + x)/(6*d**2*e**7 + 12*d*e**8*x + 6*e**9*x**2) + 108*d**3*e**3/(
6*d**2*e**7 + 12*d*e**8*x + 6*e**9*x**2) + 150*d**2*e**4*x**4/(6*d**2*e**7
+ 12*d*e**8*x + 6*e**9*x**2) - 340*d**2*e**4*x**3/(6*d**2*e**7 + 12*d*e**8*
x + 6*e**9*x**2) + 612*d**2*e**4*x**2*log(d/e + x)/(6*d**2*e**7 + 12*d*e**8
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*x + 6*e**9*x**2) + 144*d**2*e**4*x*log(d/e + x)/(6*d**2*e**7 + 12*d*e**8*x
 + 6*e**9*x**2) + 144*d**2*e**4*x/(6*d**2*e**7 + 12*d*e**8*x + 6*e**9*x**2)
 + 126*d**2*e**4*log(d/e + x)/(6*d**2*e**7 + 12*d*e**8*x + 6*e**9*x**2) + 1
89*d**2*e**4/(6*d**2*e**7 + 12*d*e**8*x + 6*e**9*x**2) - 60*d*e**5*x**5/(6*
d**2*e**7 + 12*d*e**8*x + 6*e**9*x**2) + 85*d*e**5*x**4/(6*d**2*e**7 + 12*d
*e**8*x + 6*e**9*x**2) - 204*d*e**5*x**3/(6*d**2*e**7 + 12*d*e**8*x + 6*e**
9*x**2) + 72*d*e**5*x**2*log(d/e + x)/(6*d**2*e**7 + 12*d*e**8*x + 6*e**9*x
**2) + 252*d*e**5*x*log(d/e + x)/(6*d**2*e**7 + 12*d*e**8*x + 6*e**9*x**2)
+ 252*d*e**5*x/(6*d**2*e**7 + 12*d*e**8*x + 6*e**9*x**2) - 21*d*e**5/(6*d**
2*e**7 + 12*d*e**8*x + 6*e**9*x**2) + 30*e**6*x**6/(6*d**2*e**7 + 12*d*e**8
*x + 6*e**9*x**2) - 34*e**6*x**5/(6*d**2*e**7 + 12*d*e**8*x + 6*e**9*x**2)
+ 51*e**6*x**4/(6*d**2*e**7 + 12*d*e**8*x + 6*e**9*x**2) - 24*e**6*x**3/(6*
d**2*e**7 + 12*d*e**8*x + 6*e**9*x**2) + 126*e**6*x**2*log(d/e + x)/(6*d**2
*e**7 + 12*d*e**8*x + 6*e**9*x**2) - 42*e**6*x/(6*d**2*e**7 + 12*d*e**8*x +
 6*e**9*x**2) - 18*e**6/(6*d**2*e**7 + 12*d*e**8*x + 6*e**9*x**2), Eq(m, -3
)), (-1440*d**6*log(d/e + x)/(12*d*e**7 + 12*e**8*x) - 1440*d**6/(12*d*e**7
 + 12*e**8*x) - 1440*d**5*e*x*log(d/e + x)/(12*d*e**7 + 12*e**8*x) - 1020*d
**5*e*log(d/e + x)/(12*d*e**7 + 12*e**8*x) - 1020*d**5*e/(12*d*e**7 + 12*e*
*8*x) + 720*d**4*e**2*x**2/(12*d*e**7 + 12*e**8*x) - 1020*d**4*e**2*x*log(d
/e + x)/(12*d*e**7 + 12*e**8*x) - 816*d**4*e**2*log(d/e + x)/(12*d*e**7 + 1
2*e**8*x) - 816*d**4*e**2/(12*d*e**7 + 12*e**8*x) - 240*d**3*e**3*x**3/(12*
d*e**7 + 12*e**8*x) + 510*d**3*e**3*x**2/(12*d*e**7 + 12*e**8*x) - 816*d**3
*e**3*x*log(d/e + x)/(12*d*e**7 + 12*e**8*x) - 144*d**3*e**3*log(d/e + x)/(
12*d*e**7 + 12*e**8*x) - 144*d**3*e**3/(12*d*e**7 + 12*e**8*x) + 120*d**2*e
**4*x**4/(12*d*e**7 + 12*e**8*x) - 170*d**2*e**4*x**3/(12*d*e**7 + 12*e**8*
x) + 408*d**2*e**4*x**2/(12*d*e**7 + 12*e**8*x) - 144*d**2*e**4*x*log(d/e +
 x)/(12*d*e**7 + 12*e**8*x) - 504*d**2*e**4*log(d/e + x)/(12*d*e**7 + 12*e*
*8*x) - 504*d**2*e**4/(12*d*e**7 + 12*e**8*x) - 72*d*e**5*x**5/(12*d*e**7 +
 12*e**8*x) + 85*d*e**5*x**4/(12*d*e**7 + 12*e**8*x) - 136*d*e**5*x**3/(12*
d*e**7 + 12*e**8*x) + 72*d*e**5*x**2/(12*d*e**7 + 12*e**8*x) - 504*d*e**5*x
*log(d/e + x)/(12*d*e**7 + 12*e**8*x) + 84*d*e**5*log(d/e + x)/(12*d*e**7 +
 12*e**8*x) + 84*d*e**5/(12*d*e**7 + 12*e**8*x) + 48*e**6*x**6/(12*d*e**7 +
 12*e**8*x) - 51*e**6*x**5/(12*d*e**7 + 12*e**8*x) + 68*e**6*x**4/(12*d*e**
7 + 12*e**8*x) - 24*e**6*x**3/(12*d*e**7 + 12*e**8*x) + 252*e**6*x**2/(12*d
*e**7 + 12*e**8*x) + 84*e**6*x*log(d/e + x)/(12*d*e**7 + 12*e**8*x) - 72*e*
*6/(12*d*e**7 + 12*e**8*x), Eq(m, -2)), (20*d**6*log(d/e + x)/e**7 - 20*d**
5*x/e**6 + 17*d**5*log(d/e + x)/e**6 + 10*d**4*x**2/e**5 - 17*d**4*x/e**5 +
 17*d**4*log(d/e + x)/e**5 - 20*d**3*x**3/(3*e**4) + 17*d**3*x**2/(2*e**4)
- 17*d**3*x/e**4 + 4*d**3*log(d/e + x)/e**4 + 5*d**2*x**4/e**3 - 17*d**2*x*
*3/(3*e**3) + 17*d**2*x**2/(2*e**3) - 4*d**2*x/e**3 + 21*d**2*log(d/e + x)/
e**3 - 4*d*x**5/e**2 + 17*d*x**4/(4*e**2) - 17*d*x**3/(3*e**2) + 2*d*x**2/e
**2 - 21*d*x/e**2 - 7*d*log(d/e + x)/e**2 + 10*x**6/(3*e) - 17*x**5/(5*e) +
 17*x**4/(4*e) - 4*x**3/(3*e) + 21*x**2/(2*e) + 7*x/e + 6*log(d/e + x)/e, E
q(m, -1)), (14400*d**7*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m*
*5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 504
```

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0*e**7) - 14400*d**6*e*m*x*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**
7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m +
 5040*e**7) + 2040*d**6*e*m*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e*
*7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m
+ 5040*e**7) + 14280*d**6*e*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e*
*7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m
+ 5040*e**7) + 7200*d**5*e**2*m**2*x**2*(d + e*x)**m/(e**7*m**7 + 28*e**7*m
**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 1
3068*e**7*m + 5040*e**7) - 2040*d**5*e**2*m**2*x*(d + e*x)**m/(e**7*m**7 +
28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7
*m**2 + 13068*e**7*m + 5040*e**7) + 408*d**5*e**2*m**2*(d + e*x)**m/(e**7*m
**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 1313
2*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 7200*d**5*e**2*m*x**2*(d + e*x)**
m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m*
*3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 14280*d**5*e**2*m*x*(d +
 e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*
e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 5304*d**5*e**2*m*
(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6
769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 17136*d**5*e*
*2*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4
+ 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 2400*d**4*
e**3*m**3*x**3*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 196
0*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7)
+ 1020*d**4*e**3*m**3*x**2*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**
7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m +
 5040*e**7) - 408*d**4*e**3*m**3*x*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 +
 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*
e**7*m + 5040*e**7) + 24*d**4*e**3*m**3*(d + e*x)**m/(e**7*m**7 + 28*e**7*m
**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 1
3068*e**7*m + 5040*e**7) - 7200*d**4*e**3*m**2*x**3*(d + e*x)**m/(e**7*m**7
 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e
**7*m**2 + 13068*e**7*m + 5040*e**7) + 8160*d**4*e**3*m**2*x**2*(d + e*x)**
m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m*
*3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 5304*d**4*e**3*m**2*x*(d
 + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 676
9*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 432*d**4*e**3*m
**2*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4
 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 4800*d**4
*e**3*m*x**3*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*
e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) +
7140*d**4*e**3*m*x**2*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**
5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040
*e**7) - 17136*d**4*e**3*m*x*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e
**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m
 + 5040*e**7) + 2568*d**4*e**3*m*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 3
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22*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e*
*7*m + 5040*e**7) + 5040*d**4*e**3*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 +
 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*
e**7*m + 5040*e**7) + 600*d**3*e**4*m**4*x**4*(d + e*x)**m/(e**7*m**7 + 28*
e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m*
*2 + 13068*e**7*m + 5040*e**7) - 340*d**3*e**4*m**4*x**3*(d + e*x)**m/(e**7
*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13
132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 204*d**3*e**4*m**4*x**2*(d + e*
x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**
7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 24*d**3*e**4*m**4*x*
(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6
769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 42*d**3*e**4*
m**4*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**
4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 3600*d**
3*e**4*m**3*x**4*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1
960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7
) - 3400*d**3*e**4*m**3*x**3*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e
**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m
 + 5040*e**7) + 2856*d**3*e**4*m**3*x**2*(d + e*x)**m/(e**7*m**7 + 28*e**7*
m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 +
13068*e**7*m + 5040*e**7) - 432*d**3*e**4*m**3*x*(d + e*x)**m/(e**7*m**7 +
28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7
*m**2 + 13068*e**7*m + 5040*e**7) + 924*d**3*e**4*m**3*(d + e*x)**m/(e**7*m
**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 1313
2*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 6600*d**3*e**4*m**2*x**4*(d + e*x
)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7
*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 7820*d**3*e**4*m**2*x
**3*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4
 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 11220*d**
3*e**4*m**2*x**2*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1
960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7
) - 2568*d**3*e**4*m**2*x*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7
*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m +
5040*e**7) + 7518*d**3*e**4*m**2*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 3
22*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e*
*7*m + 5040*e**7) + 3600*d**3*e**4*m*x**4*(d + e*x)**m/(e**7*m**7 + 28*e**7
*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 +
 13068*e**7*m + 5040*e**7) - 4760*d**3*e**4*m*x**3*(d + e*x)**m/(e**7*m**7
+ 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e*
*7*m**2 + 13068*e**7*m + 5040*e**7) + 8568*d**3*e**4*m*x**2*(d + e*x)**m/(e
**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 +
 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 5040*d**3*e**4*m*x*(d + e*x)
**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*
m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 26796*d**3*e**4*m*(d +
 e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*
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e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 35280*d**3*e**4*(
d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 67
69*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 120*d**2*e**5*
m**5*x**5*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**
7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 85*
d**2*e**5*m**5*x**4*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5
+ 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e
**7) - 68*d**2*e**5*m**5*x**3*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*
e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*
m + 5040*e**7) + 12*d**2*e**5*m**5*x**2*(d + e*x)**m/(e**7*m**7 + 28*e**7*m
**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 1
3068*e**7*m + 5040*e**7) - 42*d**2*e**5*m**5*x*(d + e*x)**m/(e**7*m**7 + 28
*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m
**2 + 13068*e**7*m + 5040*e**7) - 7*d**2*e**5*m**5*(d + e*x)**m/(e**7*m**7
+ 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e*
*7*m**2 + 13068*e**7*m + 5040*e**7) - 1200*d**2*e**5*m**4*x**5*(d + e*x)**m
/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**
3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 1105*d**2*e**5*m**4*x**4*
(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6
769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 1088*d**2*e**
5*m**4*x**3*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e
**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 2
28*d**2*e**5*m**4*x**2*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m*
*5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 504
0*e**7) - 924*d**2*e**5*m**4*x*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322
*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7
*m + 5040*e**7) - 175*d**2*e**5*m**4*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6
 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 1306
8*e**7*m + 5040*e**7) - 4200*d**2*e**5*m**3*x**5*(d + e*x)**m/(e**7*m**7 +
28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7
*m**2 + 13068*e**7*m + 5040*e**7) + 4505*d**2*e**5*m**3*x**4*(d + e*x)**m/(
e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3
+ 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 5644*d**2*e**5*m**3*x**3*(d
 + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 676
9*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 1500*d**2*e**5*
m**3*x**2*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**
7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 751
8*d**2*e**5*m**3*x*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 +
 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e*
*7) - 1715*d**2*e**5*m**3*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7
*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m +
5040*e**7) - 6000*d**2*e**5*m**2*x**5*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**
6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 130
68*e**7*m + 5040*e**7) + 7055*d**2*e**5*m**2*x**4*(d + e*x)**m/(e**7*m**7 +
 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**
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7*m**2 + 13068*e**7*m + 5040*e**7) - 10336*d**2*e**5*m**2*x**3*(d + e*x)**m
/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**
3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 3804*d**2*e**5*m**2*x**2*
(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6
769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 26796*d**2*e*
*5*m**2*x*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**
7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 822
5*d**2*e**5*m**2*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1
960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7
) - 2880*d**2*e**5*m*x**5*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7
*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m +
5040*e**7) + 3570*d**2*e**5*m*x**4*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 +
 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*
e**7*m + 5040*e**7) - 5712*d**2*e**5*m*x**3*(d + e*x)**m/(e**7*m**7 + 28*e*
*7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2
 + 13068*e**7*m + 5040*e**7) + 2520*d**2*e**5*m*x**2*(d + e*x)**m/(e**7*m**
7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*
e**7*m**2 + 13068*e**7*m + 5040*e**7) - 35280*d**2*e**5*m*x*(d + e*x)**m/(e
**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 +
 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 19278*d**2*e**5*m*(d + e*x)*
*m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m
**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 17640*d**2*e**5*(d + e*
x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**
7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 20*d*e**6*m**6*x**6*
(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6
769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 17*d*e**6*m**
6*x**5*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m
**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 17*d*e
**6*m**6*x**4*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960
*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) -
 4*d*e**6*m**6*x**3*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5
+ 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e
**7) + 21*d*e**6*m**6*x**2*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**
7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m +
 5040*e**7) + 7*d*e**6*m**6*x*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*
e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*
m + 5040*e**7) + 6*d*e**6*m**6*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322
*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7
*m + 5040*e**7) + 300*d*e**6*m**5*x**6*(d + e*x)**m/(e**7*m**7 + 28*e**7*m*
*6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13
068*e**7*m + 5040*e**7) - 289*d*e**6*m**5*x**5*(d + e*x)**m/(e**7*m**7 + 28
*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m
**2 + 13068*e**7*m + 5040*e**7) + 323*d*e**6*m**5*x**4*(d + e*x)**m/(e**7*m
**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 1313
2*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 84*d*e**6*m**5*x**3*(d + e*x)**m/
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(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3
 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 483*d*e**6*m**5*x**2*(d +
e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e
**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 175*d*e**6*m**5*x*
(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6
769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 162*d*e**6*m*
*5*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4
+ 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 1700*d*e**
6*m**4*x**6*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e
**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 1
785*d*e**6*m**4*x**5*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5
 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*
e**7) + 2227*d*e**6*m**4*x**4*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*
e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*
m + 5040*e**7) - 652*d*e**6*m**4*x**3*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**
6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 130
68*e**7*m + 5040*e**7) + 4221*d*e**6*m**4*x**2*(d + e*x)**m/(e**7*m**7 + 28
*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m
**2 + 13068*e**7*m + 5040*e**7) + 1715*d*e**6*m**4*x*(d + e*x)**m/(e**7*m**
7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*
e**7*m**2 + 13068*e**7*m + 5040*e**7) + 1770*d*e**6*m**4*(d + e*x)**m/(e**7
*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13
132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 4500*d*e**6*m**3*x**6*(d + e*x)
**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*
m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 5015*d*e**6*m**3*x**5*
(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6
769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 6817*d*e**6*m
**3*x**4*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7
*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 2268
*d*e**6*m**3*x**3*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 +
1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**
7) + 17157*d*e**6*m**3*x**2*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e*
*7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m
+ 5040*e**7) + 8225*d*e**6*m**3*x*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 +
322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e
**7*m + 5040*e**7) + 9990*d*e**6*m**3*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**
6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 130
68*e**7*m + 5040*e**7) + 5480*d*e**6*m**2*x**6*(d + e*x)**m/(e**7*m**7 + 28
*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m
**2 + 13068*e**7*m + 5040*e**7) - 6358*d*e**6*m**2*x**5*(d + e*x)**m/(e**7*
m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 131
32*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 9180*d*e**6*m**2*x**4*(d + e*x)*
*m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m
**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 3376*d*e**6*m**2*x**3*(
d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 67
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69*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 31038*d*e**6*m
**2*x**2*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7
*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 1927
8*d*e**6*m**2*x*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 19
60*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7)
 + 30624*d*e**6*m**2*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5
 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*
e**7) + 2400*d*e**6*m*x**6*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**
7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m +
 5040*e**7) - 2856*d*e**6*m*x**5*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 3
22*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e*
*7*m + 5040*e**7) + 4284*d*e**6*m*x**4*(d + e*x)**m/(e**7*m**7 + 28*e**7*m*
*6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13
068*e**7*m + 5040*e**7) - 1680*d*e**6*m*x**3*(d + e*x)**m/(e**7*m**7 + 28*e
**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**
2 + 13068*e**7*m + 5040*e**7) + 17640*d*e**6*m*x**2*(d + e*x)**m/(e**7*m**7
 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e
**7*m**2 + 13068*e**7*m + 5040*e**7) + 17640*d*e**6*m*x*(d + e*x)**m/(e**7*
m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 131
32*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 48168*d*e**6*m*(d + e*x)**m/(e**
7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 1
3132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 30240*d*e**6*(d + e*x)**m/(e**
7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 1
3132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 20*e**7*m**6*x**7*(d + e*x)**m
/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**
3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 17*e**7*m**6*x**6*(d + e*
x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**
7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 17*e**7*m**6*x**5*(d
 + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 676
9*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 4*e**7*m**6*x**
4*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 +
 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 21*e**7*m**
6*x**3*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m
**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 7*e**7
*m**6*x**2*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e*
*7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 6*
e**7*m**6*x*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e
**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 4
20*e**7*m**5*x**7*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 +
1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**
7) - 374*e**7*m**5*x**6*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m
**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 50
40*e**7) + 391*e**7*m**5*x**5*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*
e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*
m + 5040*e**7) - 96*e**7*m**5*x**4*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 +
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 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*
e**7*m + 5040*e**7) + 525*e**7*m**5*x**3*(d + e*x)**m/(e**7*m**7 + 28*e**7*
m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 +
13068*e**7*m + 5040*e**7) + 182*e**7*m**5*x**2*(d + e*x)**m/(e**7*m**7 + 28
*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m
**2 + 13068*e**7*m + 5040*e**7) + 162*e**7*m**5*x*(d + e*x)**m/(e**7*m**7 +
 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**
7*m**2 + 13068*e**7*m + 5040*e**7) + 3500*e**7*m**4*x**7*(d + e*x)**m/(e**7
*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13
132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 3230*e**7*m**4*x**6*(d + e*x)**
m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m*
*3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 3519*e**7*m**4*x**5*(d +
 e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*
e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 904*e**7*m**4*x**
4*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 +
 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 5187*e**7*m
**4*x**3*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7
*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 1890
*e**7*m**4*x**2*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 19
60*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7)
 + 1770*e**7*m**4*x*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5
+ 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e
**7) + 14700*e**7*m**3*x**7*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e*
*7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m
+ 5040*e**7) - 13940*e**7*m**3*x**6*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6
+ 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068
*e**7*m + 5040*e**7) + 15725*e**7*m**3*x**5*(d + e*x)**m/(e**7*m**7 + 28*e*
*7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2
 + 13068*e**7*m + 5040*e**7) - 4224*e**7*m**3*x**4*(d + e*x)**m/(e**7*m**7
+ 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e*
*7*m**2 + 13068*e**7*m + 5040*e**7) + 25599*e**7*m**3*x**3*(d + e*x)**m/(e*
*7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 +
13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 9940*e**7*m**3*x**2*(d + e*x)
**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*
m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 9990*e**7*m**3*x*(d +
e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e
**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 32480*e**7*m**2*x*
*7*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4
+ 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 31433*e**7
*m**2*x**6*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e*
*7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 36
448*e**7*m**2*x**5*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 +
 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e*
*7) - 10180*e**7*m**2*x**4*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**
7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m +
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 5040*e**7) + 65352*e**7*m**2*x**3*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 +
 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*
e**7*m + 5040*e**7) + 27503*e**7*m**2*x**2*(d + e*x)**m/(e**7*m**7 + 28*e**
7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2
+ 13068*e**7*m + 5040*e**7) + 30624*e**7*m**2*x*(d + e*x)**m/(e**7*m**7 + 2
8*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*
m**2 + 13068*e**7*m + 5040*e**7) + 35280*e**7*m*x**7*(d + e*x)**m/(e**7*m**
7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*
e**7*m**2 + 13068*e**7*m + 5040*e**7) - 34646*e**7*m*x**6*(d + e*x)**m/(e**
7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 1
3132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 41004*e**7*m*x**5*(d + e*x)**m
/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**7*m**
3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 11808*e**7*m*x**4*(d + e*
x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 6769*e**
7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 79716*e**7*m*x**3*(d
 + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4 + 676
9*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 36918*e**7*m*x*
*2*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**4
+ 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 48168*e**7
*m*x*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*m**
4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 14400*e*
*7*x**7*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e**7*
m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) - 14280
*e**7*x**6*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960*e*
*7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) + 17
136*e**7*x**5*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 1960
*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7) -
 5040*e**7*x**4*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 + 19
60*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e**7)
 + 35280*e**7*x**3*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**5 +
 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040*e*
*7) + 17640*e**7*x**2*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**
5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040
*e**7) + 30240*e**7*x*(d + e*x)**m/(e**7*m**7 + 28*e**7*m**6 + 322*e**7*m**
5 + 1960*e**7*m**4 + 6769*e**7*m**3 + 13132*e**7*m**2 + 13068*e**7*m + 5040
*e**7), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 788 vs. $2(292) = 584$.

Time = 0.22 (sec) , antiderivative size = 788, normalized size of antiderivative = 2.70

$$\int (d+ex)^m \left(3+2x+5x^2\right) \left(2+x+3x^2-5x^3+4x^4\right)\,dx$$

$$= \frac{7\,(e^2(m+1)x^2+demx-d^2)(ex+d)^m}{(m^2+3\,m+2)e^2} + \frac{6\,(ex+d)^{m+1}}{e(m+1)}$$

$$+ \frac{21\,((m^2+3\,m+2)e^3x^3+(m^2+m)de^2x^2-2\,d^2emx+2\,d^3)(ex+d)^m}{(m^3+6\,m^2+11\,m+6)e^3}$$

$$- \frac{4\,((m^3+6\,m^2+11\,m+6)e^4x^4+(m^3+3\,m^2+2\,m)de^3x^3-3\,(m^2+m)d^2e^2x^2+6\,d^3emx-6\,d^4)(ex+}{(m^4+10\,m^3+35\,m^2+50\,m+24)e^4}$$

$$+ \frac{17\,((m^4+10\,m^3+35\,m^2+50\,m+24)e^5x^5+(m^4+6\,m^3+11\,m^2+6\,m)de^4x^4-4\,(m^3+3\,m^2+2\,m)d}{(m^5+15\,m^4+85\,m^3+225\,m^2+274\,m+120)}$$

$$- \frac{17\,((m^5+15\,m^4+85\,m^3+225\,m^2+274\,m+120)e^6x^6+(m^5+10\,m^4+35\,m^3+50\,m^2+24\,m)de^5x^5 -}{(m^6+21\,m^5+175\,m}$$

$$+ \frac{20\,((m^6+21\,m^5+175\,m^4+735\,m^3+1624\,m^2+1764\,m+720)e^7x^7+(m^6+15\,m^5+85\,m^4+225\,m^3 -}{}$$

```
[In] integrate((e*x+d)^m*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="max
ima")
```

```
[Out] 7*(e^2*(m + 1)*x^2 + d*e*m*x - d^2)*(e*x + d)^m/((m^2 + 3*m + 2)*e^2) + 6*(
e*x + d)^(m + 1)/(e*(m + 1)) + 21*((m^2 + 3*m + 2)*e^3*x^3 + (m^2 + m)*d*e^
2*x^2 - 2*d^2*e*m*x + 2*d^3)*(e*x + d)^m/((m^3 + 6*m^2 + 11*m + 6)*e^3) - 4
*((m^3 + 6*m^2 + 11*m + 6)*e^4*x^4 + (m^3 + 3*m^2 + 2*m)*d*e^3*x^3 - 3*(m^2
 + m)*d^2*e^2*x^2 + 6*d^3*e*m*x - 6*d^4)*(e*x + d)^m/((m^4 + 10*m^3 + 35*m^
2 + 50*m + 24)*e^4) + 17*((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*e^5*x^5 + (m^
4 + 6*m^3 + 11*m^2 + 6*m)*d*e^4*x^4 - 4*(m^3 + 3*m^2 + 2*m)*d^2*e^3*x^3 + 1
2*(m^2 + m)*d^3*e^2*x^2 - 24*d^4*e*m*x + 24*d^5)*(e*x + d)^m/((m^5 + 15*m^4
 + 85*m^3 + 225*m^2 + 274*m + 120)*e^5) - 17*((m^5 + 15*m^4 + 85*m^3 + 225*
m^2 + 274*m + 120)*e^6*x^6 + (m^5 + 10*m^4 + 35*m^3 + 50*m^2 + 24*m)*d*e^5*
x^5 - 5*(m^4 + 6*m^3 + 11*m^2 + 6*m)*d^2*e^4*x^4 + 20*(m^3 + 3*m^2 + 2*m)*d
^3*e^3*x^3 - 60*(m^2 + m)*d^4*e^2*x^2 + 120*d^5*e*m*x - 120*d^6)*(e*x + d)^
m/((m^6 + 21*m^5 + 175*m^4 + 735*m^3 + 1624*m^2 + 1764*m + 720)*e^6) + 20*(
(m^6 + 21*m^5 + 175*m^4 + 735*m^3 + 1624*m^2 + 1764*m + 720)*e^7*x^7 + (m^6
 + 15*m^5 + 85*m^4 + 225*m^3 + 274*m^2 + 120*m)*d*e^6*x^6 - 6*(m^5 + 10*m^4
 + 35*m^3 + 50*m^2 + 24*m)*d^2*e^5*x^5 + 30*(m^4 + 6*m^3 + 11*m^2 + 6*m)*d^
3*e^4*x^4 - 120*(m^3 + 3*m^2 + 2*m)*d^4*e^3*x^3 + 360*(m^2 + m)*d^5*e^2*x^2
 - 720*d^6*e*m*x + 720*d^7)*(e*x + d)^m/((m^7 + 28*m^6 + 322*m^5 + 1960*m^4
 + 6769*m^3 + 13132*m^2 + 13068*m + 5040)*e^7)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3099 vs. 2(292) = 584.

Time = 0.29 (sec) , antiderivative size = 3099, normalized size of antiderivative = 10.61

$$\int (d+ex)^m \left(3+2x+5x^2\right) \left(2+x+3x^2-5x^3+4x^4\right) \, dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)^m*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="gia
c")
```

```
[Out] (20*(e*x + d)^m*e^7*m^6*x^7 + 20*(e*x + d)^m*d*e^6*m^6*x^6 - 17*(e*x + d)^m
*e^7*m^6*x^6 + 420*(e*x + d)^m*e^7*m^5*x^7 - 17*(e*x + d)^m*d*e^6*m^6*x^5 +
 17*(e*x + d)^m*e^7*m^6*x^5 + 300*(e*x + d)^m*d*e^6*m^5*x^6 - 374*(e*x + d)
^m*e^7*m^5*x^6 + 3500*(e*x + d)^m*e^7*m^4*x^7 + 17*(e*x + d)^m*d*e^6*m^6*x^
4 - 4*(e*x + d)^m*e^7*m^6*x^4 - 120*(e*x + d)^m*d^2*e^5*m^5*x^5 - 289*(e*x
+ d)^m*d*e^6*m^5*x^5 + 391*(e*x + d)^m*e^7*m^5*x^5 + 1700*(e*x + d)^m*d*e^6
*m^4*x^6 - 3230*(e*x + d)^m*e^7*m^4*x^6 + 14700*(e*x + d)^m*e^7*m^3*x^7 - 4
*(e*x + d)^m*d*e^6*m^6*x^3 + 21*(e*x + d)^m*e^7*m^6*x^3 + 85*(e*x + d)^m*d^
2*e^5*m^5*x^4 + 323*(e*x + d)^m*d*e^6*m^5*x^4 - 96*(e*x + d)^m*e^7*m^5*x^4
- 1200*(e*x + d)^m*d^2*e^5*m^4*x^5 - 1785*(e*x + d)^m*d*e^6*m^4*x^5 + 3519*
(e*x + d)^m*e^7*m^4*x^5 + 4500*(e*x + d)^m*d*e^6*m^3*x^6 - 13940*(e*x + d)^
m*e^7*m^3*x^6 + 32480*(e*x + d)^m*e^7*m^2*x^7 + 21*(e*x + d)^m*d*e^6*m^6*x^
2 + 7*(e*x + d)^m*e^7*m^6*x^2 - 68*(e*x + d)^m*d^2*e^5*m^5*x^3 - 84*(e*x +
d)^m*d*e^6*m^5*x^3 + 525*(e*x + d)^m*e^7*m^5*x^3 + 600*(e*x + d)^m*d^3*e^4*
m^4*x^4 + 1105*(e*x + d)^m*d^2*e^5*m^4*x^4 + 2227*(e*x + d)^m*d*e^6*m^4*x^4
 - 904*(e*x + d)^m*e^7*m^4*x^4 - 4200*(e*x + d)^m*d^2*e^5*m^3*x^5 - 5015*(e
*x + d)^m*d*e^6*m^3*x^5 + 15725*(e*x + d)^m*e^7*m^3*x^5 + 5480*(e*x + d)^m*
d*e^6*m^2*x^6 - 31433*(e*x + d)^m*e^7*m^2*x^6 + 35280*(e*x + d)^m*e^7*m*x^7
 + 7*(e*x + d)^m*d*e^6*m^6*x + 6*(e*x + d)^m*e^7*m^6*x + 12*(e*x + d)^m*d^2
*e^5*m^5*x^2 + 483*(e*x + d)^m*d*e^6*m^5*x^2 + 182*(e*x + d)^m*e^7*m^5*x^2
- 340*(e*x + d)^m*d^3*e^4*m^4*x^3 - 1088*(e*x + d)^m*d^2*e^5*m^4*x^3 - 652*
(e*x + d)^m*d*e^6*m^4*x^3 + 5187*(e*x + d)^m*e^7*m^4*x^3 + 3600*(e*x + d)^m
*d^3*e^4*m^3*x^4 + 4505*(e*x + d)^m*d^2*e^5*m^3*x^4 + 6817*(e*x + d)^m*d*e^
6*m^3*x^4 - 4224*(e*x + d)^m*e^7*m^3*x^4 - 6000*(e*x + d)^m*d^2*e^5*m^2*x^5
 - 6358*(e*x + d)^m*d*e^6*m^2*x^5 + 36448*(e*x + d)^m*e^7*m^2*x^5 + 2400*(e
*x + d)^m*d*e^6*m*x^6 - 34646*(e*x + d)^m*e^7*m*x^6 + 14400*(e*x + d)^m*e^7
*x^7 + 6*(e*x + d)^m*d*e^6*m^6 - 42*(e*x + d)^m*d^2*e^5*m^5*x + 175*(e*x +
d)^m*d*e^6*m^5*x + 162*(e*x + d)^m*e^7*m^5*x + 204*(e*x + d)^m*d^3*e^4*m^4*
x^2 + 228*(e*x + d)^m*d^2*e^5*m^4*x^2 + 4221*(e*x + d)^m*d*e^6*m^4*x^2 + 18
90*(e*x + d)^m*e^7*m^4*x^2 - 2400*(e*x + d)^m*d^4*e^3*m^3*x^3 - 3400*(e*x +
 d)^m*d^3*e^4*m^3*x^3 - 5644*(e*x + d)^m*d^2*e^5*m^3*x^3 - 2268*(e*x + d)^m
*d*e^6*m^3*x^3 + 25599*(e*x + d)^m*e^7*m^3*x^3 + 6600*(e*x + d)^m*d^3*e^4*m
^2*x^4 + 7055*(e*x + d)^m*d^2*e^5*m^2*x^4 + 9180*(e*x + d)^m*d*e^6*m^2*x^4
- 10180*(e*x + d)^m*e^7*m^2*x^4 - 2880*(e*x + d)^m*d^2*e^5*m*x^5 - 2856*(e*
```

```
x + d)^m*d*e^6*m*x^5 + 41004*(e*x + d)^m*e^7*m*x^5 - 14280*(e*x + d)^m*e^7*
x^6 - 7*(e*x + d)^m*d^2*e^5*m^5 + 162*(e*x + d)^m*d*e^6*m^5 - 24*(e*x + d)^
m*d^3*e^4*m^4*x - 924*(e*x + d)^m*d^2*e^5*m^4*x + 1715*(e*x + d)^m*d*e^6*m^
4*x + 1770*(e*x + d)^m*e^7*m^4*x + 1020*(e*x + d)^m*d^4*e^3*m^3*x^2 + 2856*
(e*x + d)^m*d^3*e^4*m^3*x^2 + 1500*(e*x + d)^m*d^2*e^5*m^3*x^2 + 17157*(e*x
 + d)^m*d*e^6*m^3*x^2 + 9940*(e*x + d)^m*e^7*m^3*x^2 - 7200*(e*x + d)^m*d^4
*e^3*m^2*x^3 - 7820*(e*x + d)^m*d^3*e^4*m^2*x^3 - 10336*(e*x + d)^m*d^2*e^5
*m^2*x^3 - 3376*(e*x + d)^m*d*e^6*m^2*x^3 + 65352*(e*x + d)^m*e^7*m^2*x^3 +
 3600*(e*x + d)^m*d^3*e^4*m*x^4 + 3570*(e*x + d)^m*d^2*e^5*m*x^4 + 4284*(e*
x + d)^m*d*e^6*m*x^4 - 11808*(e*x + d)^m*e^7*m*x^4 + 17136*(e*x + d)^m*e^7*
x^5 + 42*(e*x + d)^m*d^3*e^4*m^4 - 175*(e*x + d)^m*d^2*e^5*m^4 + 1770*(e*x
+ d)^m*d*e^6*m^4 - 408*(e*x + d)^m*d^4*e^3*m^3*x - 432*(e*x + d)^m*d^3*e^4*
m^3*x - 7518*(e*x + d)^m*d^2*e^5*m^3*x + 8225*(e*x + d)^m*d*e^6*m^3*x + 999
0*(e*x + d)^m*e^7*m^3*x + 7200*(e*x + d)^m*d^5*e^2*m^2*x^2 + 8160*(e*x + d)
^m*d^4*e^3*m^2*x^2 + 11220*(e*x + d)^m*d^3*e^4*m^2*x^2 + 3804*(e*x + d)^m*d
^2*e^5*m^2*x^2 + 31038*(e*x + d)^m*d*e^6*m^2*x^2 + 27503*(e*x + d)^m*e^7*m^
2*x^2 - 4800*(e*x + d)^m*d^4*e^3*m*x^3 - 4760*(e*x + d)^m*d^3*e^4*m*x^3 - 5
712*(e*x + d)^m*d^2*e^5*m*x^3 - 1680*(e*x + d)^m*d*e^6*m*x^3 + 79716*(e*x +
 d)^m*e^7*m*x^3 - 5040*(e*x + d)^m*e^7*x^4 + 24*(e*x + d)^m*d^4*e^3*m^3 + 9
24*(e*x + d)^m*d^3*e^4*m^3 - 1715*(e*x + d)^m*d^2*e^5*m^3 + 9990*(e*x + d)^
m*d*e^6*m^3 - 2040*(e*x + d)^m*d^5*e^2*m^2*x - 5304*(e*x + d)^m*d^4*e^3*m^2
*x - 2568*(e*x + d)^m*d^3*e^4*m^2*x - 26796*(e*x + d)^m*d^2*e^5*m^2*x + 192
78*(e*x + d)^m*d*e^6*m^2*x + 30624*(e*x + d)^m*e^7*m^2*x + 7200*(e*x + d)^m
*d^5*e^2*m*x^2 + 7140*(e*x + d)^m*d^4*e^3*m*x^2 + 8568*(e*x + d)^m*d^3*e^4*
m*x^2 + 2520*(e*x + d)^m*d^2*e^5*m*x^2 + 17640*(e*x + d)^m*d*e^6*m*x^2 + 36
918*(e*x + d)^m*e^7*m*x^2 + 35280*(e*x + d)^m*e^7*x^3 + 408*(e*x + d)^m*d^5
*e^2*m^2 + 432*(e*x + d)^m*d^4*e^3*m^2 + 7518*(e*x + d)^m*d^3*e^4*m^2 - 822
5*(e*x + d)^m*d^2*e^5*m^2 + 30624*(e*x + d)^m*d*e^6*m^2 - 14400*(e*x + d)^m
*d^6*e*m*x - 14280*(e*x + d)^m*d^5*e^2*m*x - 17136*(e*x + d)^m*d^4*e^3*m*x
- 5040*(e*x + d)^m*d^3*e^4*m*x - 35280*(e*x + d)^m*d^2*e^5*m*x + 17640*(e*x
 + d)^m*d*e^6*m*x + 48168*(e*x + d)^m*e^7*m*x + 17640*(e*x + d)^m*e^7*x^2 +
 2040*(e*x + d)^m*d^6*e*m + 5304*(e*x + d)^m*d^5*e^2*m + 2568*(e*x + d)^m*d
^4*e^3*m + 26796*(e*x + d)^m*d^3*e^4*m - 19278*(e*x + d)^m*d^2*e^5*m + 4816
8*(e*x + d)^m*d*e^6*m + 30240*(e*x + d)^m*e^7*x + 14400*(e*x + d)^m*d^7 + 1
4280*(e*x + d)^m*d^6*e + 17136*(e*x + d)^m*d^5*e^2 + 5040*(e*x + d)^m*d^4*e
^3 + 35280*(e*x + d)^m*d^3*e^4 - 17640*(e*x + d)^m*d^2*e^5 + 30240*(e*x + d
)^m*d*e^6)/(e^7*m^7 + 28*e^7*m^6 + 322*e^7*m^5 + 1960*e^7*m^4 + 6769*e^7*m^
3 + 13132*e^7*m^2 + 13068*e^7*m + 5040*e^7)
```

Mupad [B] (verification not implemented)

Time = 14.29 (sec) , antiderivative size = 1425, normalized size of antiderivative = 4.88

$$\int (d+ex)^m \left(3+2x+5x^2\right) \left(2+x+3x^2-5x^3+4x^4\right) \, dx = \text{Too large to display}$$

```
[In] int((d + e*x)^m*(2*x + 5*x^2 + 3)*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2),x)
```

```
[Out] ((d + e*x)^m*(30240*d*e^6 + 14280*d^6*e + 14400*d^7 - 17640*d^2*e^5 + 35280
*d^3*e^4 + 5040*d^4*e^3 + 17136*d^5*e^2 - 19278*d^2*e^5*m + 26796*d^3*e^4*m
 + 2568*d^4*e^3*m + 5304*d^5*e^2*m + 30624*d*e^6*m^2 + 9990*d*e^6*m^3 + 177
0*d*e^6*m^4 + 162*d*e^6*m^5 + 6*d*e^6*m^6 - 8225*d^2*e^5*m^2 + 7518*d^3*e^4
*m^2 + 432*d^4*e^3*m^2 + 408*d^5*e^2*m^2 - 1715*d^2*e^5*m^3 + 924*d^3*e^4*m
^3 + 24*d^4*e^3*m^3 - 175*d^2*e^5*m^4 + 42*d^3*e^4*m^4 - 7*d^2*e^5*m^5 + 48
168*d*e^6*m + 2040*d^6*e*m))/(e^7*(13068*m + 13132*m^2 + 6769*m^3 + 1960*m^
4 + 322*m^5 + 28*m^6 + m^7 + 5040)) + (20*x^7*(d + e*x)^m*(1764*m + 1624*m^
2 + 735*m^3 + 175*m^4 + 21*m^5 + m^6 + 720))/(13068*m + 13132*m^2 + 6769*m^
3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040) - (x*(d + e*x)^m*(35280*d^2*e
^5*m - 30240*e^7 - 30624*e^7*m^2 - 9990*e^7*m^3 - 1770*e^7*m^4 - 162*e^7*m^
5 - 6*e^7*m^6 - 48168*e^7*m + 5040*d^3*e^4*m + 17136*d^4*e^3*m + 14280*d^5*
e^2*m - 19278*d*e^6*m^2 - 8225*d*e^6*m^3 - 1715*d*e^6*m^4 - 175*d*e^6*m^5 -
 7*d*e^6*m^6 + 26796*d^2*e^5*m^2 + 2568*d^3*e^4*m^2 + 5304*d^4*e^3*m^2 + 20
40*d^5*e^2*m^2 + 7518*d^2*e^5*m^3 + 432*d^3*e^4*m^3 + 408*d^4*e^3*m^3 + 924
*d^2*e^5*m^4 + 24*d^3*e^4*m^4 + 42*d^2*e^5*m^5 - 17640*d*e^6*m + 14400*d^6*
e*m))/(e^7*(13068*m + 13132*m^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 +
m^7 + 5040)) - (x^3*(d + e*x)^m*(3*m + m^2 + 2)*(2400*d^4*m - 13398*e^4*m -
 17640*e^4 - 3759*e^4*m^2 - 462*e^4*m^3 - 21*e^4*m^4 + 2856*d^2*e^2*m + 428
*d*e^3*m^2 + 340*d^3*e*m^2 + 72*d*e^3*m^3 + 4*d*e^3*m^4 + 884*d^2*e^2*m^2 +
 68*d^2*e^2*m^3 + 840*d*e^3*m + 2380*d^3*e*m))/(e^4*(13068*m + 13132*m^2 +
6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040)) - (x^5*(d + e*x)^m*(5
0*m + 35*m^2 + 10*m^3 + m^4 + 24)*(120*d^2*m - 221*e^2*m - 714*e^2 - 17*e^2
*m^2 + 119*d*e*m + 17*d*e*m^2))/(e^2*(13068*m + 13132*m^2 + 6769*m^3 + 1960
*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040)) + (x^4*(d + e*x)^m*(11*m + 6*m^2 + m
^3 + 6)*(600*d^3*m - 428*e^3*m - 840*e^3 - 72*e^3*m^2 - 4*e^3*m^3 + 221*d*e
^2*m^2 + 85*d^2*e*m^2 + 17*d*e^2*m^3 + 714*d*e^2*m + 595*d^2*e*m))/(e^3*(13
068*m + 13132*m^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040)) +
 (x^2*(m + 1)*(d + e*x)^m*(7200*d^5*m + 19278*e^5*m + 17640*e^5 + 8225*e^5*
m^2 + 1715*e^5*m^3 + 175*e^5*m^4 + 7*e^5*m^5 + 2520*d^2*e^3*m + 8568*d^3*e^
2*m + 13398*d*e^4*m^2 + 1020*d^4*e*m^2 + 3759*d*e^4*m^3 + 462*d*e^4*m^4 + 2
1*d*e^4*m^5 + 1284*d^2*e^3*m^2 + 2652*d^3*e^2*m^2 + 216*d^2*e^3*m^3 + 204*d
^3*e^2*m^3 + 12*d^2*e^3*m^4 + 17640*d*e^4*m + 7140*d^4*e*m))/(e^5*(13068*m
+ 13132*m^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040)) - (x^6*
(d + e*x)^m*(119*e - 20*d*m + 17*e*m)*(274*m + 225*m^2 + 85*m^3 + 15*m^4 +
m^5 + 120))/(e*(13068*m + 13132*m^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^
6 + m^7 + 5040))
```

3.370 $\int \frac{(d+ex)^m\left(2+x+3x^2-5x^3+4x^4\right)}{3+2x+5x^2}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 255

$$\begin{aligned}
&\int \frac{(d+ex)^m\left(2+x+3x^2-5x^3+4x^4\right)}{3+2x+5x^2}\,dx \\
&= \frac{\left(100d^2+165de+81e^2\right)(d+ex)^{1+m}}{125e^3(1+m)} - \frac{(40d+33e)(d+ex)^{2+m}}{25e^3(2+m)} + \frac{4(d+ex)^{3+m}}{5e^3(3+m)} \\
&\quad - \frac{\left(6412i-423\sqrt{14}\right)(d+ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{5(d+ex)}{5d-e+i\sqrt{14}e}\right)}{3500\left(5id-\left(i+\sqrt{14}\right)e\right)(1+m)} \\
&\quad - \frac{\left(6412i+423\sqrt{14}\right)(d+ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{5(d+ex)}{5d-\left(1+i\sqrt{14}\right)e}\right)}{3500\left(5id-\left(i-\sqrt{14}\right)e\right)(1+m)}
\end{aligned}$$

```
[Out] 1/125*(100*d^2+165*d*e+81*e^2)*(e*x+d)^(1+m)/e^3/(1+m)-1/25*(40*d+33*e)*(e*
x+d)^(2+m)/e^3/(2+m)+4/5*(e*x+d)^(3+m)/e^3/(3+m)-1/3500*(e*x+d)^(1+m)*hyper
geom([1, 1+m],[2+m],5*(e*x+d)/(5*d-e*(1+I*14^(1/2))))*(6412*I+423*14^(1/2))
/(1+m)/(5*I*d-e*(I-14^(1/2)))-1/3500*(e*x+d)^(1+m)*hypergeom([1, 1+m],[2+m]
,5*(e*x+d)/(5*d-e+I*14^(1/2)*e))*(6412*I-423*14^(1/2))/(1+m)/(5*I*d-e*(I+14
^(1/2)))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {1642, 70}

$$\begin{aligned}
&\int \frac{(d+ex)^m (2+x+3x^2-5x^3+4x^4)}{3+2x+5x^2} \, dx \\
&= \frac{(100d^2+165de+81e^2)(d+ex)^{m+1}}{125e^3(m+1)} - \frac{(40d+33e)(d+ex)^{m+2}}{25e^3(m+2)} + \frac{4(d+ex)^{m+3}}{5e^3(m+3)} \\
&\quad - \frac{\left(-423\sqrt{14}+6412i\right)(d+ex)^{m+1} \text{Hypergeometric2F1}\left(1, m+1, m+2, \frac{5(d+ex)}{5d+i\sqrt{14}e-e}\right)}{3500(m+1)\left(5id-\left(\sqrt{14}+i\right)e\right)} \\
&\quad - \frac{\left(423\sqrt{14}+6412i\right)(d+ex)^{m+1} \text{Hypergeometric2F1}\left(1, m+1, m+2, \frac{5(d+ex)}{5d-\left(1+i\sqrt{14}\right)e}\right)}{3500(m+1)\left(5id-\left(-\sqrt{14}+i\right)e\right)}
\end{aligned}$$

```
[In] Int[((d + e*x)^m*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2),x]
```

```
[Out] ((100*d^2 + 165*d*e + 81*e^2)*(d + e*x)^(1 + m))/(125*e^3*(1 + m)) - ((40*d
 + 33*e)*(d + e*x)^(2 + m))/(25*e^3*(2 + m)) + (4*(d + e*x)^(3 + m))/(5*e^3
*(3 + m)) - ((6412*I - 423*Sqrt[14])*(d + e*x)^(1 + m)*Hypergeometric2F1[1,
 1 + m, 2 + m, (5*(d + e*x))/(5*d - e + I*Sqrt[14]*e)])/(3500*((5*I)*d - (I
 + Sqrt[14])*e)*(1 + m)) - ((6412*I + 423*Sqrt[14])*(d + e*x)^(1 + m)*Hyper
geometric2F1[1, 1 + m, 2 + m, (5*(d + e*x))/(5*d - (1 + I*Sqrt[14])*e)])/(3
500*((5*I)*d - (I - Sqrt[14])*e)*(1 + m))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{(100d^2+165de+81e^2)(d+ex)^m}{125e^2} + \frac{\left(\frac{458}{125}+\frac{423i}{125\sqrt{14}}\right)(d+ex)^m}{2-2i\sqrt{14}+10x} \right. \\
&\quad \left. + \frac{\left(\frac{458}{125}-\frac{423i}{125\sqrt{14}}\right)(d+ex)^m}{2+2i\sqrt{14}+10x} + \frac{(-40d-33e)(d+ex)^{1+m}}{25e^2} + \frac{4(d+ex)^{2+m}}{5e^2} \right) dx \\
&= \frac{(100d^2+165de+81e^2)(d+ex)^{1+m}}{125e^3(1+m)} - \frac{(40d+33e)(d+ex)^{2+m}}{25e^3(2+m)} + \frac{4(d+ex)^{3+m}}{5e^3(3+m)} \\
&\quad + \frac{\left(6412-423i\sqrt{14}\right)\int \frac{(d+ex)^m}{2+2i\sqrt{14}+10x}\,dx}{1750} + \frac{\left(6412+423i\sqrt{14}\right)\int \frac{(d+ex)^m}{2-2i\sqrt{14}+10x}\,dx}{1750} \\
&= \frac{(100d^2+165de+81e^2)(d+ex)^{1+m}}{125e^3(1+m)} - \frac{(40d+33e)(d+ex)^{2+m}}{25e^3(2+m)} + \frac{4(d+ex)^{3+m}}{5e^3(3+m)} \\
&\quad - \frac{\left(6412i-423\sqrt{14}\right)(d+ex)^{1+m}\,{}_2F_1\left(1,1+m;2+m;\frac{5(d+ex)}{5d-e+i\sqrt{14}e}\right)}{3500\left(5id-\left(i+\sqrt{14}\right)e\right)(1+m)} \\
&\quad - \frac{\left(6412i+423\sqrt{14}\right)(d+ex)^{1+m}\,{}_2F_1\left(1,1+m;2+m;\frac{5(d+ex)}{5d-\left(1+i\sqrt{14}\right)e}\right)}{3500\left(5id-\left(i-\sqrt{14}\right)e\right)(1+m)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.87

$$\int \frac{(d+ex)^m(2+x+3x^2-5x^3+4x^4)}{3+2x+5x^2}\,dx$$
$$= \frac{(d+ex)^{1+m} \Bigg(\frac{28(100d^2+165de+81e^2)}{e^3(1+m)} - \frac{140(40d+33e)(d+ex)}{e^3(2+m)} + \frac{2800(d+ex)^2}{e^3(3+m)} - \frac{\left(6412i+423\sqrt{14}\right)\text{Hypergeometric2F1}\Big(1,1+m,2+m}{\left(5id+\left(-i+\sqrt{14}\right)e\right)(1+m)}}{3500}$$

```
[In] Integrate[((d + e*x)^m*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2),x
]
```

```
[Out] ((d + e*x)^(1 + m)*((28*(100*d^2 + 165*d*e + 81*e^2))/(e^3*(1 + m)) - (140*
(40*d + 33*e)*(d + e*x))/(e^3*(2 + m)) + (2800*(d + e*x)^2)/(e^3*(3 + m)) -
 ((6412*I + 423*Sqrt[14])*Hypergeometric2F1[1, 1 + m, 2 + m, (5*(d + e*x))/
(5*d + (-1 - I*Sqrt[14])*e)])/(((5*I)*d + (-I + Sqrt[14])*e)*(1 + m)) - ((-
6412*I + 423*Sqrt[14])*Hypergeometric2F1[1, 1 + m, 2 + m, (5*(d + e*x))/(5*
d + I*(I + Sqrt[14])*e)])/(((-5*I)*d + (I + Sqrt[14])*e)*(1 + m))))/3500
```

Maple [F]

$$\int \frac{(ex+d)^m(4x^4-5x^3+3x^2+x+2)}{5x^2+2x+3}dx$$

```
[In] int((e*x+d)^m*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x)
```

```
[Out] int((e*x+d)^m*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x)
```

Fricas [F]

$$\int \frac{(d+ex)^m(2+x+3x^2-5x^3+4x^4)}{3+2x+5x^2}\,dx = \int \frac{(4\,x^4-5\,x^3+3\,x^2+x+2)(ex+d)^m}{5\,x^2+2\,x+3}\,dx$$

```
[In] integrate((e*x+d)^m*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="fri
cas")
```

```
[Out] integral((4*x^4 - 5*x^3 + 3*x^2 + x + 2)*(e*x + d)^m/(5*x^2 + 2*x + 3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m(2+x+3x^2-5x^3+4x^4)}{3+2x+5x^2}\,dx = \text{Timed out}$$

```
[In] integrate((e*x+d)**m*(4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(d+ex)^m(2+x+3x^2-5x^3+4x^4)}{3+2x+5x^2}\,dx = \int \frac{(4\,x^4-5\,x^3+3\,x^2+x+2)(ex+d)^m}{5\,x^2+2\,x+3}\,dx$$

```
[In] integrate((e*x+d)^m*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="max
ima")
```

```
[Out] integrate((4*x^4 - 5*x^3 + 3*x^2 + x + 2)*(e*x + d)^m/(5*x^2 + 2*x + 3), x)
```

Giac [F]

$$\int \frac{(d+ex)^m (2+x+3x^2-5x^3+4x^4)}{3+2x+5x^2}\,dx = \int \frac{(4\,x^4-5\,x^3+3\,x^2+x+2)(ex+d)^m}{5\,x^2+2\,x+3}\,dx$$

```
[In] integrate((e*x+d)^m*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="gia
c")
```

```
[Out] integrate((4*x^4 - 5*x^3 + 3*x^2 + x + 2)*(e*x + d)^m/(5*x^2 + 2*x + 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m (2+x+3x^2-5x^3+4x^4)}{3+2x+5x^2}\,dx = \int \frac{(d+e\,x)^m\,(4\,x^4-5\,x^3+3\,x^2+x+2)}{5\,x^2+2\,x+3}\,dx$$

```
[In] int(((d + e*x)^m*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(2*x + 5*x^2 + 3),x)
```

```
[Out] int(((d + e*x)^m*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(2*x + 5*x^2 + 3), x)
```

3.371 $\int \frac{(d+ex)^m\left(2+x+3x^2-5x^3+4x^4\right)}{\left(3+2x+5x^2\right)^2}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 377

$$\int \frac{(d+ex)^m\left(2+x+3x^2-5x^3+4x^4\right)}{\left(3+2x+5x^2\right)^2}\,dx$$
$$= \frac{4(d+ex)^{1+m}}{25e(1+m)} - \frac{(1367d-293e+(423d-1367e)x)(d+ex)^{1+m}}{700\left(5d^2-2de+3e^2\right)\left(3+2x+5x^2\right)}$$
$$+ \frac{\left(80360d^2-32144de+48216e^2+i\sqrt{14}(6565d^2-2de(1313-3206m)+e^2(3939-98m))-5922dem -\right.}{19600\left(5d+i\left(i+\sqrt{14}\right)e\right)\left(5d^2-2de+\right.}$$
$$+ \frac{\left(80360d^2-32144de+48216e^2-i\sqrt{14}(6565d^2-2de(1313-3206m)+e^2(3939-98m))-5922dem -\right.}{19600\left(5d-\left(1+i\sqrt{14}\right)e\right)\left(5d^2-2de\right.}$$

```
[Out] 4/25*(e*x+d)^(1+m)/e/(1+m)-1/700*(1367*d-293*e+(423*d-1367*e)*x)*(e*x+d)^(1
+m)/(5*d^2-2*d*e+3*e^2)/(5*x^2+2*x+3)+1/19600*(e*x+d)^(1+m)*hypergeom([1, 1
+m],[2+m],5*(e*x+d)/(5*d-e*(1+I*14^(1/2))))*(80360*d^2-32144*d*e+48216*e^2-
5922*d*e*m+19138*e^2*m-I*(6565*d^2-2*d*e*(1313-3206*m)+e^2*(3939-98*m))*14^
(1/2))/(5*d^2-2*d*e+3*e^2)/(1+m)/(5*d-e*(1+I*14^(1/2)))+1/19600*(e*x+d)^(1+
m)*hypergeom([1, 1+m],[2+m],5*(e*x+d)/(5*d-e+I*14^(1/2)*e))*(80360*d^2-3214
4*d*e+48216*e^2-5922*d*e*m+19138*e^2*m+I*(6565*d^2-2*d*e*(1313-3206*m)+e^2*
(3939-98*m))*14^(1/2))/(5*d^2-2*d*e+3*e^2)/(1+m)/(5*d+I*e*(I+14^(1/2)))
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {1662, 1642, 70}

$$\int \frac{(d+ex)^m\,(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2}\,dx$$

$$= \frac{(i\sqrt{14}(6565d^2-2de(1313-3206m)+e^2(3939-98m))+80360d^2-5922dem-32144de+19138e^2m+}{19600(m+1)\left(5d+i\left(\sqrt{14}+i\right)e\right)(5d^2-2}$$

$$+\frac{(-i\sqrt{14}(6565d^2-2de(1313-3206m)+e^2(3939-98m))+80360d^2-5922dem-32144de+19138e^2}{19600(m+1)\left(5d-\left(1+i\sqrt{14}\right)e\right)(5d}$$

$$-\frac{(x(423d-1367e)+1367d-293e)(d+ex)^{m+1}}{700\,(5x^2+2x+3)\,(5d^2-2de+3e^2)}+\frac{4(d+ex)^{m+1}}{25e(m+1)}$$

```
[In] Int[((d + e*x)^m*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^2,x]
```

```
[Out] (4*(d + e*x)^(1 + m))/(25*e*(1 + m)) - ((1367*d - 293*e + (423*d - 1367*e)*
x)*(d + e*x)^(1 + m))/(700*(5*d^2 - 2*d*e + 3*e^2)*(3 + 2*x + 5*x^2)) + ((8
0360*d^2 - 32144*d*e + 48216*e^2 + I*Sqrt[14]*(6565*d^2 - 2*d*e*(1313 - 320
6*m) + e^2*(3939 - 98*m)) - 5922*d*e*m + 19138*e^2*m)*(d + e*x)^(1 + m)*Hyp
ergeometric2F1[1, 1 + m, 2 + m, (5*(d + e*x))/(5*d - e + I*Sqrt[14]*e)])/(1
9600*(5*d + I*(I + Sqrt[14])*e)*(5*d^2 - 2*d*e + 3*e^2)*(1 + m)) + ((80360*
d^2 - 32144*d*e + 48216*e^2 - I*Sqrt[14]*(6565*d^2 - 2*d*e*(1313 - 3206*m)
+ e^2*(3939 - 98*m)) - 5922*d*e*m + 19138*e^2*m)*(d + e*x)^(1 + m)*Hypergeo
metric2F1[1, 1 + m, 2 + m, (5*(d + e*x))/(5*d - (1 + I*Sqrt[14])*e)])/(1960
0*(5*d - (1 + I*Sqrt[14])*e)*(5*d^2 - 2*d*e + 3*e^2)*(1 + m))
```

<u>Rule 70</u>

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

<u>Rule 1642</u>

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

<u>Rule 1662</u>

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f =
```

```
Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[Polynom
ialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1)*((f*(b*c*d - b^2*e + 2*a*c*e) - a*g*(2*c*d - b*e) + c*(
f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e
 + a*e^2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), I
nt[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*
(c*d^2 - b*d*e + a*e^2)*Q + f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m + 2)
- 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m +
b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f*(2
*c*d - b*e))*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, m}, x] &
& PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && L
tQ[p, -1] &&  !(IGtQ[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || I
LtQ[p + 1/2, 0]))
```

Rubi steps

$$\text{integral} = -\frac{(1367d-293e+(423d-1367e)x)(d+ex)^{1+m}}{700\left(5d^2-2de+3e^2\right)\left(3+2x+5x^2\right)} + \frac{\int \frac{(d+ex)^m\left(\frac{2}{25}\left(1845d^2-de(738-1367m)+e^2(1107-293m)\right)-\frac{2}{25}\left(4620d^2-3de(616+141m)+e^2(2772+1367m)\right)x+\frac{224}{5}\left(5d^2-2de+3e^2\right)x^2\right)}{3+2x+5x^2}\,d}{56\left(5d^2-2de+3e^2\right)}$$

$$= -\frac{(1367d-293e+(423d-1367e)x)(d+ex)^{1+m}}{700\left(5d^2-2de+3e^2\right)\left(3+2x+5x^2\right)} + \frac{\int \left(\frac{224}{25}\left(5d^2-2de+3e^2\right)(d+ex)^m + \frac{\left(-\frac{2296d^2}{5}+\frac{4592de}{25}-\frac{6888e^2}{25}+\frac{846dem}{25}-\frac{2734e^2m}{25}-\frac{1}{25}i\sqrt{\frac{2}{7}}\left(6565d^2-2626de+39\right.\right.}{2-2i\sqrt{14}+10x}\right.}{}$$

$$= \frac{4(d+ex)^{1+m}}{25e(1+m)} - \frac{(1367d-293e+(423d-1367e)x)(d+ex)^{1+m}}{700\left(5d^2-2de+3e^2\right)\left(3+2x+5x^2\right)} - \frac{\left(80360d^2-32144de+48216e^2-i\sqrt{14}\left(6565d^2-2de(1313-3206m)+e^2(3939-98m)\right)-59\right.}{9800\left(5d^2-2de+3e^2\right)} - \frac{\left(80360d^2-32144de+48216e^2+i\sqrt{14}\left(6565d^2-2de(1313-3206m)+e^2(3939-98m)\right)-59\right.}{9800\left(5d^2-2de+3e^2\right)}$$

$$= \frac{4(d+ex)^{1+m}}{25e(1+m)} - \frac{(1367d-293e+(423d-1367e)x)(d+ex)^{1+m}}{700\left(5d^2-2de+3e^2\right)\left(3+2x+5x^2\right)} + \frac{\left(80360d^2-32144de+48216e^2+i\sqrt{14}\left(6565d^2-2de(1313-3206m)+e^2(3939-98m)\right)-59\right.}{19600\left(5d+i\left(i+\sqrt{14}\right)e\right)\left(5d^2-2de+\right.} + \frac{\left(80360d^2-32144de+48216e^2-i\sqrt{14}\left(6565d^2-2de(1313-3206m)+e^2(3939-98m)\right)-59\right.}{19600\left(5d-\left(1+i\sqrt{14}\right)e\right)\left(5d^2-2de-\right.}$$

Mathematica [A] (verified)

Time = 1.35 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.17

$$\int \frac{(d+ex)^m (2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2}\,dx$$

$$= \frac{(d+ex)^{1+m}\left(\frac{3136}{e+em}-\frac{28(d(1367+423x)-e(293+1367x))}{(5d^2-2de+3e^2)(3+2x+5x^2)}+\frac{56\left(287i+31\sqrt{14}\right)\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{5(d+ex)}{5d+\left(-1-i\sqrt{14}\right)e}\right)}{\left(5id+\left(-i+\sqrt{14}\right)e\right)(1+m)}+\right.}{}$$

```
[In] Integrate[((d + e*x)^m*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^2
,x]
```

```
[Out] ((d + e*x)^(1 + m)*(3136/(e + e*m) - (28*(d*(1367 + 423*x) - e*(293 + 1367*
x)))/((5*d^2 - 2*d*e + 3*e^2)*(3 + 2*x + 5*x^2)) + (56*(287*I + 31*Sqrt[14]
)*Hypergeometric2F1[1, 1 + m, 2 + m, (5*(d + e*x))/(5*d + (-1 - I*Sqrt[14])
*e)])/(((5*I)*d + (-I + Sqrt[14])*e)*(1 + m)) + (56*(-287*I + 31*Sqrt[14])*
Hypergeometric2F1[1, 1 + m, 2 + m, (5*(d + e*x))/(5*d + I*(I + Sqrt[14])*e)
])/(((-5*I)*d + (I + Sqrt[14])*e)*(1 + m)) - (Sqrt[14]*(((2115*d^2 + d*e*(-
846 + (-6412 + (423*I)*Sqrt[14])*m) + e^2*(1269 + (98 - (1367*I)*Sqrt[14])*
m))*Hypergeometric2F1[1, 1 + m, 2 + m, (5*(d + e*x))/(5*d + (-1 - I*Sqrt[14
])*e)])/((5*I)*d + (-I + Sqrt[14])*e) - ((2115*d^2 - d*e*(846 + (6412 + (42
3*I)*Sqrt[14])*m) + e^2*(1269 + (98 + (1367*I)*Sqrt[14])*m))*Hypergeometric
2F1[1, 1 + m, 2 + m, (5*(d + e*x))/(5*d + I*(I + Sqrt[14])*e)])/((5*I)*d -
(I + Sqrt[14])*e)))/((5*d^2 - 2*d*e + 3*e^2)*(1 + m))))/19600
```

Maple [F]

$$\int \frac{(ex+d)^m (4x^4-5x^3+3x^2+x+2)}{(5x^2+2x+3)^2}dx$$

```
[In] int((e*x+d)^m*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x)
```

```
[Out] int((e*x+d)^m*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x)
```

Fricas [F]

$$\int \frac{(d+ex)^m (2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2}\,dx = \int \frac{(4\,x^4-5\,x^3+3\,x^2+x+2)(ex+d)^m}{(5\,x^2+2\,x+3)^2}\,dx$$

```
[In] integrate((e*x+d)^m*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="f
ricas")
```

```
[Out] integral((4*x^4 - 5*x^3 + 3*x^2 + x + 2)*(e*x + d)^m/(25*x^4 + 20*x^3 + 34*
x^2 + 12*x + 9), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m (2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2}\,dx = \text{Timed out}$$

```
[In] integrate((e*x+d)**m*(4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3)**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(d+ex)^m (2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2}\,dx = \int \frac{(4\,x^4-5\,x^3+3\,x^2+x+2)(ex+d)^m}{(5\,x^2+2\,x+3)^2}\,dx$$

```
[In] integrate((e*x+d)^m*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="m
axima")
```

```
[Out] integrate((4*x^4 - 5*x^3 + 3*x^2 + x + 2)*(e*x + d)^m/(5*x^2 + 2*x + 3)^2,
x)
```

Giac [F]

$$\int \frac{(d+ex)^m (2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2}\,dx = \int \frac{(4\,x^4-5\,x^3+3\,x^2+x+2)(ex+d)^m}{(5\,x^2+2\,x+3)^2}\,dx$$

```
[In] integrate((e*x+d)^m*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="g
iac")
```

```
[Out] integrate((4*x^4 - 5*x^3 + 3*x^2 + x + 2)*(e*x + d)^m/(5*x^2 + 2*x + 3)^2,
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m (2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2} \, dx = \int \frac{(d+e\,x)^m \left(4\,x^4-5\,x^3+3\,x^2+x+2\right)}{\left(5\,x^2+2\,x+3\right)^2} \, dx$$

```
[In] int(((d + e*x)^m*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(2*x + 5*x^2 + 3)^2,x)
[Out] int(((d + e*x)^m*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(2*x + 5*x^2 + 3)^2, x)
```

3.372 $\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{\left(a+bx+cx^2\right)^3} \, dx$

Optimal result

Integrand size = 38, antiderivative size = 528

$$\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{\left(a+bx+cx^2\right)^3} \, dx =$$

$$-\frac{ab^3ch+bc^2(c^2d+acf-3a^2h)-ab^4i-ab^2c(cg-4ai)-2ac^2(c^2e-acg+a^2i)+(2c^5d-c^4(be+2af}{2c^4\,(b^2-4ac)\,(a+bx+cx^2}$$

$$+\frac{b^5ch+b^3c^2(cf-8ah)+2bc^3(3c^2d+acf+11a^2h)-b^6i-b^4c(cg-11ai)-16a^2c^3(cg-2ai)-b^2c^2(3}{2c^4\,(b^2}$$

$$-\frac{(12c^5d-c^4(6be-4af)+2c^3(b^2f-3abg+6a^2h)-b^5i+10ab^3ci-30a^2bc^2i)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\,(b^2-4ac)^{5/2}}$$

$$+\frac{i\log\left(a+bx+cx^2\right)}{2c^3}$$

```
[Out] 1/2*(-a*b^3*c*h-b*c^2*(-3*a^2*h+a*c*f+c^2*d)+a*b^4*i+a*b^2*c*(-4*a*i+c*g)+2
*a*c^2*(a^2*i-a*c*g+c^2*e)-(2*c^5*d-c^4*(2*a*f+b*e)+c^3*(2*a^2*h+3*a*b*g+b^
2*f)-b^5*i+b^3*c*(5*a*i+b*h)-b*c^2*(5*a^2*i+4*a*b*h+b^2*g))*x)/c^4/(-4*a*c+
b^2)/(c*x^2+b*x+a)^2+1/2*(b^5*c*h+b^3*c^2*(-8*a*h+c*f)+2*b*c^3*(11*a^2*h+a*
c*f+3*c^2*d)-b^6*i-b^4*c*(-11*a*i+c*g)-16*a^2*c^3*(-2*a*i+c*g)-b^2*c^2*(39*
a^2*i-5*a*c*g+3*c^2*e)+2*c*(6*c^5*d-c^4*(-2*a*f+3*b*e)+c^3*(-10*a^2*h-3*a*b
*g+b^2*f)+2*b^5*i-b^3*c*(15*a*i+b*h)+a*b*c^2*(25*a*i+8*b*h))*x)/c^4/(-4*a*c
+b^2)^2/(c*x^2+b*x+a)-(12*c^5*d-c^4*(-4*a*f+6*b*e)+2*c^3*(6*a^2*h-3*a*b*g+b
^2*f)-b^5*i+10*a*b^3*c*i-30*a^2*b*c^2*i)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/
2))/c^3/(-4*a*c+b^2)^(5/2)+1/2*i*ln(c*x^2+b*x+a)/c^3
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 528, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {1674, 648, 632, 212, 642}

$$\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{(a+bx+cx^2)^3}\,dx =$$

$$-\frac{\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(2c^3(6a^2h-3abg+b^2f)-30a^2bc^2i+10ab^3ci-c^4(6be-4af)+b^5(-i)+12c^5d\right)}{c^3\left(b^2-4ac\right)^{5/2}}$$

$$-\frac{x(c^3(2a^2h+3abg+b^2f)-bc^2(5a^2i+4abh+b^2g)+b^3c(5ai+bh)-c^4(2af+be)+b^5(-i)+2c^5d)+bc}{2c^4\left(b^2-4ac\right)\left(a+bx+cx\right.}$$

$$+\frac{-b^2c^2(39a^2i-5acg+3c^2e)+2cx(c^3(-10a^2h-3abg+b^2f)-b^3c(15ai+bh)-c^4(3be-2af)+abc^2(2}{2c^4\,(}$$

$$+\frac{i\log\left(a+bx+cx^2\right)}{2c^3}$$

```
[In] Int[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(a + b*x + c*x^2)^3,x]
```

```
[Out] -1/2*(a*b^3*c*h + b*c^2*(c^2*d + a*c*f - 3*a^2*h) - a*b^4*i - a*b^2*c*(c*g
- 4*a*i) - 2*a*c^2*(c^2*e - a*c*g + a^2*i) + (2*c^5*d - c^4*(b*e + 2*a*f) +
 c^3*(b^2*f + 3*a*b*g + 2*a^2*h) - b^5*i + b^3*c*(b*h + 5*a*i) - b*c^2*(b^2
*g + 4*a*b*h + 5*a^2*i))*x)/(c^4*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2) + (b^5*
c*h + b^3*c^2*(c*f - 8*a*h) + 2*b*c^3*(3*c^2*d + a*c*f + 11*a^2*h) - b^6*i
- b^4*c*(c*g - 11*a*i) - 16*a^2*c^3*(c*g - 2*a*i) - b^2*c^2*(3*c^2*e - 5*a*
c*g + 39*a^2*i) + 2*c*(6*c^5*d - c^4*(3*b*e - 2*a*f) + c^3*(b^2*f - 3*a*b*g
 - 10*a^2*h) + 2*b^5*i - b^3*c*(b*h + 15*a*i) + a*b*c^2*(8*b*h + 25*a*i))*x
)/(2*c^4*(b^2 - 4*a*c)^2*(a + b*x + c*x^2)) - ((12*c^5*d - c^4*(6*b*e - 4*a
*f) + 2*c^3*(b^2*f - 3*a*b*g + 6*a^2*h) - b^5*i + 10*a*b^3*c*i - 30*a^2*b*c
^2*i)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^3*(b^2 - 4*a*c)^(5/2)) + (
i*Log[a + b*x + c*x^2])/(2*c^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\text{integral} = -\frac{ab^3ch+bc^2(c^2d+acf-3a^2h)-ab^4i-ab^2c(cg-4ai)-2ac^2(c^2e-acg+a^2i)+(2c^5d-c^4(be+2af}{2c^4\left(b^2-4ac\right)\left(a+bx+cx^2\right.} - \frac{\int \frac{6cd-3be+2af+\frac{b^2f+abg-2a^2h}{c}-\frac{b^5i}{c^4}+\frac{b^3(bh+3ai)}{c^3}-\frac{b\left(b^2g+2abh-a^2i\right)}{c^2}-\frac{2\left(b^2-4ac\right)\left(c^2g+b^2i-c(bh+ai)\right)x}{c^3}-\frac{2\left(b^2-4ac\right)(ch-bi)x^2}{c^2}+2\left(4a-\frac{b^2}{c}\right)ix^3}{\left(a+bx+cx^2\right)^2}}{2\left(b^2-4ac\right)}$$

$$= -\frac{ab^3ch+bc^2(c^2d+acf-3a^2h)-ab^4i-ab^2c(cg-4ai)-2ac^2(c^2e-acg+a^2i)+(2c^5d-c^4(b}{2c^4\left(b^2-4ac\right)(a+b} + \frac{b^5ch+b^3c^2(cf-8ah)+2bc^3(3c^2d+acf+11a^2h)-b^6i-b^4c(cg-11ai)-16a^2c^3(cg-2ai)-}{} + \frac{\int \frac{2\left(6c^2d-3bce+b^2f+2acf-3abg+6a^2h+\frac{ab^3i}{c^2}-\frac{7a^2bi}{c}\right)+\frac{2\left(b^2-4ac\right)^2ix}{c^2}}{a+bx+cx^2}\,dx}{2\left(b^2-4ac\right)^2}$$

$$= -\frac{ab^3ch+bc^2(c^2d+acf-3a^2h)-ab^4i-ab^2c(cg-4ai)-2ac^2(c^2e-acg+a^2i)+(2c^5d-c^4(be}{2c^4\,(b^2-4ac)\,(a+bx}$$
$$+\frac{b^5ch+b^3c^2(cf-8ah)+2bc^3(3c^2d+acf+11a^2h)-b^6i-b^4c(cg-11ai)-16a^2c^3(cg-2ai)-}{}$$
$$+\frac{i\int\frac{b+2cx}{a+bx+cx^2}\,dx}{2c^3}$$
$$+\frac{(12c^5d-c^4(6be-4af)+2c^3(b^2f-3abg+6a^2h)-b^5i+10ab^3ci-30a^2bc^2i)\int\frac{1}{a+bx+cx^2}\,dx}{2c^3\,(b^2-4ac)^2}$$

$$= -\frac{ab^3ch+bc^2(c^2d+acf-3a^2h)-ab^4i-ab^2c(cg-4ai)-2ac^2(c^2e-acg+a^2i)+(2c^5d-c^4(be}{2c^4\,(b^2-4ac)\,(a+bx}$$
$$+\frac{b^5ch+b^3c^2(cf-8ah)+2bc^3(3c^2d+acf+11a^2h)-b^6i-b^4c(cg-11ai)-16a^2c^3(cg-2ai)-}{}$$
$$+\frac{i\log(a+bx+cx^2)}{2c^3}$$
$$-\frac{(12c^5d-c^4(6be-4af)+2c^3(b^2f-3abg+6a^2h)-b^5i+10ab^3ci-30a^2bc^2i)\,\mathrm{Subst}(\int\frac{1}{b^2-4ac-x^2}}{c^3\,(b^2-4ac)^2}$$

$$= -\frac{ab^3ch+bc^2(c^2d+acf-3a^2h)-ab^4i-ab^2c(cg-4ai)-2ac^2(c^2e-acg+a^2i)+(2c^5d-c^4(be}{2c^4\,(b^2-4ac)\,(a+bx}$$
$$+\frac{b^5ch+b^3c^2(cf-8ah)+2bc^3(3c^2d+acf+11a^2h)-b^6i-b^4c(cg-11ai)-16a^2c^3(cg-2ai)-}{}$$
$$-\frac{(12c^5d-c^4(6be-4af)+2c^3(b^2f-3abg+6a^2h)-b^5i+10ab^3ci-30a^2bc^2i)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\,(b^2-4ac)^{5/2}}$$
$$+\frac{i\log(a+bx+cx^2)}{2c^3}$$

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 488, normalized size of antiderivative = 0.92

$$\int\frac{d+ex+fx^2+gx^3+hx^4+ix^5}{(a+bx+cx^2)^3}\,dx$$
$$=\frac{\frac{b^5ix+b^4(ai-chx)+2c^2(a^3i-c^3dx+ac^2(e+fx)-a^2c(g+hx))+b^2c(-4a^2i-c^2fx+ac(g+4hx))+b^3c(cgx-a(h+5ix))+bc^2(c^2(-d+ex)-ac(f+3gx)+}{(b^2-4ac)(a+x(b+cx))^2}}{}$$

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(a + b*x + c*x^2)^3,x]
```

```
[Out] ((b^5*i*x + b^4*(a*i - c*h*x) + 2*c^2*(a^3*i - c^3*d*x + a*c^2*(e + f*x) -
a^2*c*(g + h*x)) + b^2*c*(-4*a^2*i - c^2*f*x + a*c*(g + 4*h*x)) + b^3*c*(c*
```

```
g*x - a*(h + 5*i*x)) + b*c^2*(c^2*(-d + e*x) - a*c*(f + 3*g*x) + a^2*(3*h +
 5*i*x)))/((b^2 - 4*a*c)*(a + x*(b + c*x))^2) + (-(b^6*i) + b^5*c*(h + 4*i*
x) + b^3*c^2*(c*f - 8*a*h - 30*a*i*x) - b^4*c*(-11*a*i + c*(g + 2*h*x)) + 4
*c^3*(8*a^3*i + 3*c^3*d*x + a*c^2*f*x - a^2*c*(4*g + 5*h*x)) + b^2*c^2*(-39
*a^2*i + c^2*(-3*e + 2*f*x) + a*c*(5*g + 16*h*x)) + 2*b*c^3*(3*c^2*(d - e*x
) + a*c*(f - 3*g*x) + a^2*(11*h + 25*i*x)))/((b^2 - 4*a*c)^2*(a + x*(b + c*
x))) + (2*c*(12*c^5*d + c^4*(-6*b*e + 4*a*f) + 2*c^3*(b^2*f - 3*a*b*g + 6*a
^2*h) - b^5*i + 10*a*b^3*c*i - 30*a^2*b*c^2*i)*ArcTan[(b + 2*c*x)/Sqrt[-b^2
 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2) + c*i*Log[a + x*(b + c*x)])/(2*c^4)
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 769, normalized size of antiderivative = 1.46

method	result
default	$\frac{\frac{\left(25a^2bc^2i-10a^2c^3h-15ab^3ci+8ab^2c^2h-3abc^3g+2ac^4f+2b^5i-b^4ch+b^2c^3f-3bc^4e+6c^5d\right)x^3}{c^2\left(16a^2c^2-8ab^2c+b^4\right)}+\frac{\left(32a^3c^3i+11a^2b^2c^2i+2a^2bc^3h-16a^2c^4g-19a\right.}{}}{}$
risch	Expression too large to display

```
[In] int((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^3,x,method=_RETURNVERBOSE
)
```

```
[Out] ((25*a^2*b*c^2*i-10*a^2*c^3*h-15*a*b^3*c*i+8*a*b^2*c^2*h-3*a*b*c^3*g+2*a*c^
4*f+2*b^5*i-b^4*c*h+b^2*c^3*f-3*b*c^4*e+6*c^5*d)/c^2/(16*a^2*c^2-8*a*b^2*c+
b^4)*x^3+1/2*(32*a^3*c^3*i+11*a^2*b^2*c^2*i+2*a^2*b*c^3*h-16*a^2*c^4*g-19*a
*b^4*c*i+8*a*b^3*c^2*h-a*b^2*c^3*g+6*a*b*c^4*f+3*b^6*i-b^5*c*h-b^4*c^2*g+3*
b^3*c^3*f-9*b^2*c^4*e+18*b*c^5*d)/(16*a^2*c^2-8*a*b^2*c+b^4)/c^3*x^2+(31*a^
3*b*c^2*i-6*a^3*c^3*h-22*a^2*b^3*c*i+10*a^2*b^2*c^2*h-5*a^2*b*c^3*g-2*a^2*c
^4*f+3*a*b^5*i-a*b^4*c*h-a*b^3*c^2*g+5*a*b^2*c^3*f-5*a*b*c^4*e+10*a*c^5*d-b
^3*c^3*e+2*b^2*c^4*d)/(16*a^2*c^2-8*a*b^2*c+b^4)/c^3*x+1/2/c^3*(24*a^4*c^2*
i-21*a^3*b^2*c*i+10*a^3*b*c^2*h-8*a^3*c^3*g+3*a^2*b^4*i-a^2*b^3*c*h-a^2*b^2
*c^2*g+6*a^2*b*c^3*f-8*a^2*c^4*e-a*b^2*c^3*e+10*a*b*c^4*d-b^3*c^3*d)/(16*a^
2*c^2-8*a*b^2*c+b^4))/(c*x^2+b*x+a)^2+1/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*(1/2
*(16*a^2*c^2*i-8*a*b^2*c*i+b^4*i)/c*ln(c*x^2+b*x+a)+2*(-7*a^2*b*i*c+6*a^2*c
^2*h+a*b^3*i-3*a*b*c^2*g+2*a*c^3*f+b^2*c^2*f-3*b*c^3*e+6*c^4*d-1/2*(16*a^2*
c^2*i-8*a*b^2*c*i+b^4*i)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2
)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1730 vs. $2(518) = 1036$.

Time = 0.37 (sec) , antiderivative size = 3480, normalized size of antiderivative = 6.59

$$\int \frac{d + ex + fx^2 + gx^3 + hx^4 + ix^5}{(a + bx + cx^2)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^3,x, algorithm="fri
cas")
```

```
[Out] [1/2*(2*(6*(b^2*c^6 - 4*a*c^7)*d - 3*(b^3*c^5 - 4*a*b*c^6)*e + (b^4*c^4 - 2
*a*b^2*c^5 - 8*a^2*c^6)*f - 3*(a*b^3*c^4 - 4*a^2*b*c^5)*g - (b^6*c^2 - 12*a
*b^4*c^3 + 42*a^2*b^2*c^4 - 40*a^3*c^5)*h + (2*b^7*c - 23*a*b^5*c^2 + 85*a^
2*b^3*c^3 - 100*a^3*b*c^4)*i)*x^3 + (18*(b^3*c^5 - 4*a*b*c^6)*d - 9*(b^4*c^
4 - 4*a*b^2*c^5)*e + 3*(b^5*c^3 - 2*a*b^3*c^4 - 8*a^2*b*c^5)*f - (b^6*c^2 -
 3*a*b^4*c^3 + 12*a^2*b^2*c^4 - 64*a^3*c^5)*g - (b^7*c - 12*a*b^5*c^2 + 30*
a^2*b^3*c^3 + 8*a^3*b*c^4)*h + (3*b^8 - 31*a*b^6*c + 87*a^2*b^4*c^2 - 12*a^
3*b^2*c^3 - 128*a^4*c^4)*i)*x^2 - (12*a^2*c^5*d - 6*a^2*b*c^4*e - 6*a^3*b*c
^3*g + 12*a^4*c^3*h + (12*c^7*d - 6*b*c^6*e - 6*a*b*c^5*g + 12*a^2*c^5*h +
2*(b^2*c^5 + 2*a*c^6)*f - (b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*i)*x^4 +
2*(12*b*c^6*d - 6*b^2*c^5*e - 6*a*b^2*c^4*g + 12*a^2*b*c^4*h + 2*(b^3*c^4 +
 2*a*b*c^5)*f - (b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*i)*x^3 + (12*(b^2*c
^5 + 2*a*c^6)*d - 6*(b^3*c^4 + 2*a*b*c^5)*e + 2*(b^4*c^3 + 4*a*b^2*c^4 + 4*
a^2*c^5)*f - 6*(a*b^3*c^3 + 2*a^2*b*c^4)*g + 12*(a^2*b^2*c^3 + 2*a^3*c^4)*h
 - (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*i)*x^2 + 2*(a^2*b^2*c^
3 + 2*a^3*c^4)*f - (a^2*b^5 - 10*a^3*b^3*c + 30*a^4*b*c^2)*i + 2*(12*a*b*c^
5*d - 6*a*b^2*c^4*e - 6*a^2*b^2*c^3*g + 12*a^3*b*c^3*h + 2*(a*b^3*c^3 + 2*a
^2*b*c^4)*f - (a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2)*i)*x)*sqrt(b^2 - 4*a*
c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/
(c*x^2 + b*x + a)) - (b^5*c^3 - 14*a*b^3*c^4 + 40*a^2*b*c^5)*d - (a*b^4*c^3
 + 4*a^2*b^2*c^4 - 32*a^3*c^5)*e + 6*(a^2*b^3*c^3 - 4*a^3*b*c^4)*f - (a^2*b
^4*c^2 + 4*a^3*b^2*c^3 - 32*a^4*c^4)*g - (a^2*b^5*c - 14*a^3*b^3*c^2 + 40*a
^4*b*c^3)*h + 3*(a^2*b^6 - 11*a^3*b^4*c + 36*a^4*b^2*c^2 - 32*a^5*c^3)*i +
2*(2*(b^4*c^4 + a*b^2*c^5 - 20*a^2*c^6)*d - (b^5*c^3 + a*b^3*c^4 - 20*a^2*b
*c^5)*e + (5*a*b^4*c^3 - 22*a^2*b^2*c^4 + 8*a^3*c^5)*f - (a*b^5*c^2 + a^2*b
^3*c^3 - 20*a^3*b*c^4)*g - (a*b^6*c - 14*a^2*b^4*c^2 + 46*a^3*b^2*c^3 - 24*
a^4*c^4)*h + (3*a*b^7 - 34*a^2*b^5*c + 119*a^3*b^3*c^2 - 124*a^4*b*c^3)*i)*
x + ((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*i*x^4 + 2*(b^7*
c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*i*x^3 + (b^8 - 10*a*b^6*c
 + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*i*x^2 + 2*(a*b^7 - 12*a^2
*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*i*x + (a^2*b^6 - 12*a^3*b^4*c + 48*
a^4*b^2*c^2 - 64*a^5*c^3)*i)*log(c*x^2 + b*x + a))/(a^2*b^6*c^3 - 12*a^3*b^
4*c^4 + 48*a^4*b^2*c^5 - 64*a^5*c^6 + (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*
c^7 - 64*a^3*c^8)*x^4 + 2*(b^7*c^4 - 12*a*b^5*c^5 + 48*a^2*b^3*c^6 - 64*a^3
```

```
*b*c^7)*x^3 + (b^8*c^3 - 10*a*b^6*c^4 + 24*a^2*b^4*c^5 + 32*a^3*b^2*c^6 - 1
28*a^4*c^7)*x^2 + 2*(a*b^7*c^3 - 12*a^2*b^5*c^4 + 48*a^3*b^3*c^5 - 64*a^4*b
*c^6)*x), 1/2*(2*(6*(b^2*c^6 - 4*a*c^7)*d - 3*(b^3*c^5 - 4*a*b*c^6)*e + (b^
4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*f - 3*(a*b^3*c^4 - 4*a^2*b*c^5)*g - (b^6*c
^2 - 12*a*b^4*c^3 + 42*a^2*b^2*c^4 - 40*a^3*c^5)*h + (2*b^7*c - 23*a*b^5*c^
2 + 85*a^2*b^3*c^3 - 100*a^3*b*c^4)*i)*x^3 + (18*(b^3*c^5 - 4*a*b*c^6)*d -
9*(b^4*c^4 - 4*a*b^2*c^5)*e + 3*(b^5*c^3 - 2*a*b^3*c^4 - 8*a^2*b*c^5)*f - (
b^6*c^2 - 3*a*b^4*c^3 + 12*a^2*b^2*c^4 - 64*a^3*c^5)*g - (b^7*c - 12*a*b^5*
c^2 + 30*a^2*b^3*c^3 + 8*a^3*b*c^4)*h + (3*b^8 - 31*a*b^6*c + 87*a^2*b^4*c^
2 - 12*a^3*b^2*c^3 - 128*a^4*c^4)*i)*x^2 - 2*(12*a^2*c^5*d - 6*a^2*b*c^4*e
- 6*a^3*b*c^3*g + 12*a^4*c^3*h + (12*c^7*d - 6*b*c^6*e - 6*a*b*c^5*g + 12*a
^2*c^5*h + 2*(b^2*c^5 + 2*a*c^6)*f - (b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4
)*i)*x^4 + 2*(12*b*c^6*d - 6*b^2*c^5*e - 6*a*b^2*c^4*g + 12*a^2*b*c^4*h + 2
*(b^3*c^4 + 2*a*b*c^5)*f - (b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*i)*x^3 +
 (12*(b^2*c^5 + 2*a*c^6)*d - 6*(b^3*c^4 + 2*a*b*c^5)*e + 2*(b^4*c^3 + 4*a*b
^2*c^4 + 4*a^2*c^5)*f - 6*(a*b^3*c^3 + 2*a^2*b*c^4)*g + 12*(a^2*b^2*c^3 + 2
*a^3*c^4)*h - (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*i)*x^2 + 2*
(a^2*b^2*c^3 + 2*a^3*c^4)*f - (a^2*b^5 - 10*a^3*b^3*c + 30*a^4*b*c^2)*i + 2
*(12*a*b*c^5*d - 6*a*b^2*c^4*e - 6*a^2*b^2*c^3*g + 12*a^3*b*c^3*h + 2*(a*b^
3*c^3 + 2*a^2*b*c^4)*f - (a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2)*i)*x)*sqrt
(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - (b^5
*c^3 - 14*a*b^3*c^4 + 40*a^2*b*c^5)*d - (a*b^4*c^3 + 4*a^2*b^2*c^4 - 32*a^3
*c^5)*e + 6*(a^2*b^3*c^3 - 4*a^3*b*c^4)*f - (a^2*b^4*c^2 + 4*a^3*b^2*c^3 -
32*a^4*c^4)*g - (a^2*b^5*c - 14*a^3*b^3*c^2 + 40*a^4*b*c^3)*h + 3*(a^2*b^6
- 11*a^3*b^4*c + 36*a^4*b^2*c^2 - 32*a^5*c^3)*i + 2*(2*(b^4*c^4 + a*b^2*c^5
 - 20*a^2*c^6)*d - (b^5*c^3 + a*b^3*c^4 - 20*a^2*b*c^5)*e + (5*a*b^4*c^3 -
22*a^2*b^2*c^4 + 8*a^3*c^5)*f - (a*b^5*c^2 + a^2*b^3*c^3 - 20*a^3*b*c^4)*g
- (a*b^6*c - 14*a^2*b^4*c^2 + 46*a^3*b^2*c^3 - 24*a^4*c^4)*h + (3*a*b^7 - 3
4*a^2*b^5*c + 119*a^3*b^3*c^2 - 124*a^4*b*c^3)*i)*x + ((b^6*c^2 - 12*a*b^4*
c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*i*x^4 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2
*b^3*c^3 - 64*a^3*b*c^4)*i*x^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^
3*b^2*c^3 - 128*a^4*c^4)*i*x^2 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 -
 64*a^4*b*c^3)*i*x + (a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)
*i)*log(c*x^2 + b*x + a))/(a^2*b^6*c^3 - 12*a^3*b^4*c^4 + 48*a^4*b^2*c^5 -
64*a^5*c^6 + (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*x^4 + 2
*(b^7*c^4 - 12*a*b^5*c^5 + 48*a^2*b^3*c^6 - 64*a^3*b*c^7)*x^3 + (b^8*c^3 -
10*a*b^6*c^4 + 24*a^2*b^4*c^5 + 32*a^3*b^2*c^6 - 128*a^4*c^7)*x^2 + 2*(a*b^
7*c^3 - 12*a^2*b^5*c^4 + 48*a^3*b^3*c^5 - 64*a^4*b*c^6)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{\left(a+bx+cx^2\right)^3}\,dx = \text{Timed out}$$

```
[In] integrate((i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(c*x**2+b*x+a)**3,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{\left(a+bx+cx^2\right)^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^3,x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 646, normalized size of antiderivative = 1.22

$$\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{\left(a+bx+cx^2\right)^3}\,dx = \frac{\left(12\,c^5d-6\,bc^4e+2\,b^2c^3f+4\,ac^4f-6\,abc^3g+12\,a^2c^3h-b^5i+10\,ab^3ci-30\,a^2bc^2i\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^4c^3-8\,ab^2c^4+16\,a^2c^5\right)\sqrt{-b^2+4\,ac}} + \frac{i\log\left(cx^2+bx+a\right)}{2\,c^3} - \frac{b^3c^3d-10\,abc^4d+ab^2c^3e+8\,a^2c^4e-6\,a^2bc^3f+a^2b^2c^2g+8\,a^3c^3g+a^2b^3ch-10\,a^3bc^2h-3\,a^2b^4i+21}{}$$

```
[In] integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^3,x, algorithm="gia
c")
```

```
[Out] (12*c^5*d - 6*b*c^4*e + 2*b^2*c^3*f + 4*a*c^4*f - 6*a*b*c^3*g + 12*a^2*c^3*
h - b^5*i + 10*a*b^3*c*i - 30*a^2*b*c^2*i)*arctan((2*c*x + b)/sqrt(-b^2 + 4
```

```
*a*c))/((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt(-b^2 + 4*a*c)) + 1/2*i*lo
g(c*x^2 + b*x + a)/c^3 - 1/2*(b^3*c^3*d - 10*a*b*c^4*d + a*b^2*c^3*e + 8*a^
2*c^4*e - 6*a^2*b*c^3*f + a^2*b^2*c^2*g + 8*a^3*c^3*g + a^2*b^3*c*h - 10*a^
3*b*c^2*h - 3*a^2*b^4*i + 21*a^3*b^2*c*i - 24*a^4*c^2*i - 2*(6*c^6*d - 3*b*
c^5*e + b^2*c^4*f + 2*a*c^5*f - 3*a*b*c^4*g - b^4*c^2*h + 8*a*b^2*c^3*h - 1
0*a^2*c^4*h + 2*b^5*c*i - 15*a*b^3*c^2*i + 25*a^2*b*c^3*i)*x^3 - (18*b*c^5*
d - 9*b^2*c^4*e + 3*b^3*c^3*f + 6*a*b*c^4*f - b^4*c^2*g - a*b^2*c^3*g - 16*
a^2*c^4*g - b^5*c*h + 8*a*b^3*c^2*h + 2*a^2*b*c^3*h + 3*b^6*i - 19*a*b^4*c*
i + 11*a^2*b^2*c^2*i + 32*a^3*c^3*i)*x^2 - 2*(2*b^2*c^4*d + 10*a*c^5*d - b^
3*c^3*e - 5*a*b*c^4*e + 5*a*b^2*c^3*f - 2*a^2*c^4*f - a*b^3*c^2*g - 5*a^2*b
*c^3*g - a*b^4*c*h + 10*a^2*b^2*c^2*h - 6*a^3*c^3*h + 3*a*b^5*i - 22*a^2*b^
3*c*i + 31*a^3*b*c^2*i)*x)/((c*x^2 + b*x + a)^2*(b^2 - 4*a*c)^2*c^3)
```

Mupad [B] (verification not implemented)

Time = 15.65 (sec) , antiderivative size = 1027, normalized size of antiderivative = 1.95

$$\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{\left(a+bx+cx^2\right)^3}\,dx$$

$$= \frac{\mathrm{atan}\left(\frac{x\,\left(32\,a^2\,c^5\,{\left(4\,a\,c-b^2\right)}^{5/2}+2\,b^4\,c^3\,{\left(4\,a\,c-b^2\right)}^{5/2}-16\,a\,b^2\,c^4\,{\left(4\,a\,c-b^2\right)}^{5/2}\right)}{c^2\,{\left(4\,a\,c-b^2\right)}^5}+\frac{\left(32\,a^2\,c^5\,{\left(4\,a\,c-b^2\right)}^{5/2}+2\,b^4\,c^3\,{\left(4\,a\,c-b^2\right)}^{5/2}-16\,a\,b^2\,c^4\,(}{2\,c^5\,{\left(4\,a\,c-b^2\right)}^5\,\left(16\,a^2\,c^2-8\right.}\right.}{}$$

$$-\frac{\ln\left(c\,x^2+b\,x+a\right)\,\left(-1024\,i\,a^5\,c^5+1280\,i\,a^4\,b^2\,c^4-640\,i\,a^3\,b^4\,c^3+160\,i\,a^2\,b^6\,c^2-20\,i\,a\,b^8\,c+i\,b^{10}\right)}{2\,\left(1024\,a^5\,c^8-1280\,a^4\,b^2\,c^7+640\,a^3\,b^4\,c^6-160\,a^2\,b^6\,c^5+20\,a\,b^8\,c^4-b^{10}\,c^3\right)}$$

$$-\frac{\frac{-24\,i\,a^4\,c^2+21\,i\,a^3\,b^2\,c-10\,h\,a^3\,b\,c^2+8\,g\,a^3\,c^3-3\,i\,a^2\,b^4+h\,a^2\,b^3\,c+g\,a^2\,b^2\,c^2-6\,f\,a^2\,b\,c^3+8\,e\,a^2\,c^4+e\,a\,b^2\,c^3-10\,d\,a\,b\,c^4+d\,b^3\,c^3}{2\,c^3\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}-\frac{x^2\,\left(32\,i\,a^3\right.}{}}{}$$

```
[In] int((d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(a + b*x + c*x^2)^3,x)
```

```
[Out] (atan((x*(32*a^2*c^5*(4*a*c - b^2)^(5/2) + 2*b^4*c^3*(4*a*c - b^2)^(5/2) -
16*a*b^2*c^4*(4*a*c - b^2)^(5/2)))/(c^2*(4*a*c - b^2)^5) + ((32*a^2*c^5*(4*
a*c - b^2)^(5/2) + 2*b^4*c^3*(4*a*c - b^2)^(5/2) - 16*a*b^2*c^4*(4*a*c - b^
2)^(5/2))*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4))/(2*c^5*(4*a*c - b^2)^5*(b
^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(12*c^5*d - b^5*i + 2*b^2*c^3*f + 12*a^2*c^3
*h + 4*a*c^4*f - 6*b*c^4*e - 6*a*b*c^3*g + 10*a*b^3*c*i - 30*a^2*b*c^2*i))/
(c^3*(4*a*c - b^2)^(5/2)) - (log(a + b*x + c*x^2)*(b^10*i - 1024*a^5*c^5*i
+ 160*a^2*b^6*c^2*i - 640*a^3*b^4*c^3*i + 1280*a^4*b^2*c^4*i - 20*a*b^8*c*i
))/(2*(1024*a^5*c^8 - b^10*c^3 + 20*a*b^8*c^4 - 160*a^2*b^6*c^5 + 640*a^3*b
^4*c^6 - 1280*a^4*b^2*c^7)) - ((8*a^2*c^4*e + b^3*c^3*d + 8*a^3*c^3*g - 3*a
^2*b^4*i - 24*a^4*c^2*i + a^2*b^2*c^2*g - 10*a*b*c^4*d + a*b^2*c^3*e - 6*a^
2*b*c^3*f + a^2*b^3*c*h - 10*a^3*b*c^2*h + 21*a^3*b^2*c*i)/(2*c^3*(b^4 + 16
*a^2*c^2 - 8*a*b^2*c)) - (x^2*(3*b^6*i - 9*b^2*c^4*e - 16*a^2*c^4*g + 3*b^3
*c^3*f - b^4*c^2*g + 32*a^3*c^3*i + 18*b*c^5*d - b^5*c*h + 11*a^2*b^2*c^2*i
```

```
 + 6*a*b*c^4*f - 19*a*b^4*c*i - a*b^2*c^3*g + 8*a*b^3*c^2*h + 2*a^2*b*c^3*h
))/(2*c^3*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x*(2*a^2*c^4*f - 2*b^2*c^4*d +
 b^3*c^3*e + 6*a^3*c^3*h - 10*a*c^5*d - 3*a*b^5*i - 10*a^2*b^2*c^2*h + 5*a*
b*c^4*e + a*b^4*c*h - 5*a*b^2*c^3*f + a*b^3*c^2*g + 5*a^2*b*c^3*g + 22*a^2*
b^3*c*i - 31*a^3*b*c^2*i))/(c^3*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (x^3*(6*c
^5*d + 2*b^5*i + b^2*c^3*f - 10*a^2*c^3*h + 2*a*c^4*f - 3*b*c^4*e - b^4*c*h
 - 3*a*b*c^3*g - 15*a*b^3*c*i + 8*a*b^2*c^2*h + 25*a^2*b*c^2*i))/(c^2*(b^4
+ 16*a^2*c^2 - 8*a*b^2*c)))/(x^2*(2*a*c + b^2) + a^2 + c^2*x^4 + 2*a*b*x +
2*b*c*x^3)
```

3.373 $\int \frac{d+ex+fx^2+gx^3+hx^4+jx^5+kx^6+lx^7+mx^8}{a+bx+cx^2}\,dx$

Optimal result

Integrand size = 53, antiderivative size = 765

$$\int \frac{d+ex+fx^2+gx^3+hx^4+jx^5+kx^6+lx^7+mx^8}{a+bx+cx^2}\,dx$$

$$= \frac{(c^6f-c^5(bg+ah)+c^4(b^2h+2abj+a^2k)+b^6m-b^4c(bl+5am)+b^2c^2(b^2k+4abl+6a^2m)-c^3(b^3j+}{c^7}$$

$$+\frac{(c^5g-c^4(bh+aj)+c^3(b^2j+2abk+a^2l)-b^5m+b^3c(bl+4am)-bc^2(b^2k+3abl+3a^2m))\,x^2}{2c^6}$$

$$+\frac{(c^4h-c^3(bj+ak)+b^4m-b^2c(bl+3am)+c^2(b^2k+2abl+a^2m))\,x^3}{3c^5}$$

$$+\frac{(c^3j-c^2(bk+al)-b^3m+bc(bl+2am))\,x^4}{4c^4}$$

$$+\frac{(c^2k+b^2m-c(bl+am))\,x^5}{5c^3}+\frac{(cl-bm)x^6}{6c^2}+\frac{mx^7}{7c}$$

$$-\frac{(2c^8d-c^7(be+2af)+c^6(b^2f+3abg+2a^2h)-c^5(b^3g+4ab^2h+5a^2bj+2a^3k)+b^8m-b^6c(bl+8an}{}$$

$$+\frac{(c^7e-c^6(bf+ag)+c^5(b^2g+2abh+a^2j)-c^4(b^3h+3ab^2j+3a^2bk+a^3l)-b^7m+b^5c(bl+6am)-b}{2c^8}$$

```
[Out] (c^6*f-c^5*(a*h+b*g)+c^4*(a^2*k+2*a*b*j+b^2*h)+b^6*m-b^4*c*(5*a*m+b*l)+b^2*
c^2*(6*a^2*m+4*a*b*l+b^2*k)-c^3*(a^3*m+3*a^2*b*l+3*a*b^2*k+b^3*j))*x/c^7+1/
2*(c^5*g-c^4*(a*j+b*h)+c^3*(a^2*l+2*a*b*k+b^2*j)-b^5*m+b^3*c*(4*a*m+b*l)-b*
c^2*(3*a^2*m+3*a*b*l+b^2*k))*x^2/c^6+1/3*(c^4*h-c^3*(a*k+b*j)+b^4*m-b^2*c*(
3*a*m+b*l)+c^2*(a^2*m+2*a*b*l+b^2*k))*x^3/c^5+1/4*(c^3*j-c^2*(a*l+b*k)-b^3*
m+b*c*(2*a*m+b*l))*x^4/c^4+1/5*(c^2*k+b^2*m-c*(a*m+b*l))*x^5/c^3+1/6*(-b*m+
c*l)*x^6/c^2+1/7*m*x^7/c+1/2*(c^7*e-c^6*(a*g+b*f)+c^5*(a^2*j+2*a*b*h+b^2*g)
-c^4*(a^3*l+3*a^2*b*k+3*a*b^2*j+b^3*h)-b^7*m+b^5*c*(6*a*m+b*l)-b^3*c^2*(10*
a^2*m+5*a*b*l+b^2*k)+b*c^3*(4*a^3*m+6*a^2*b*l+4*a*b^2*k+b^3*j))*ln(c*x^2+b*
```

```
x+a)/c^8-(2*c^8*d-c^7*(2*a*f+b*e)+c^6*(2*a^2*h+3*a*b*g+b^2*f)-c^5*(2*a^3*k+
5*a^2*b*j+4*a*b^2*h+b^3*g)+b^8*m-b^6*c*(8*a*m+b*l)+b^4*c^2*(20*a^2*m+7*a*b*
l+b^2*k)-b^2*c^3*(16*a^3*m+14*a^2*b*l+6*a*b^2*k+b^3*j)+c^4*(2*a^4*m+7*a^3*b
*l+9*a^2*b^2*k+5*a*b^3*j+b^4*h))*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c^8/
(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 4.74 (sec) , antiderivative size = 765, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {1671, 648, 632, 212, 642}

$$\int \frac{d+ex+fx^2+gx^3+hx^4+jx^5+kx^6+lx^7+mx^8}{a+bx+cx^2}\,dx$$
$$=\frac{x^3(c^2(a^2m+2abl+b^2k)-b^2c(3am+bl)-c^3(ak+bj)+b^4m+c^4h)}{3c^5}$$
$$+\frac{x^2(c^3(a^2l+2abk+b^2j)-bc^2(3a^2m+3abl+b^2k)+b^3c(4am+bl)-c^4(aj+bh)+b^5(-m)+c^5g)}{2c^6}$$
$$+\frac{\log\left(a+bx+cx^2\right)\left(c^5(a^2j+2abh+b^2g)-b^3c^2(10a^2m+5abl+b^2k)-c^4(a^3l+3a^2bk+3ab^2j+b^3h)+\right.}{2c^8}$$
$$+\frac{x(c^4(a^2k+2abj+b^2h)+b^2c^2(6a^2m+4abl+b^2k)-c^3(a^3m+3a^2bl+3ab^2k+b^3j)-b^4c(5am+bl)-}{c^7}$$
$$-\frac{\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(c^6(2a^2h+3abg+b^2f)+b^4c^2(20a^2m+7abl+b^2k)-c^5(2a^3k+5a^2bj+4ab^2h+b^3g\right.}{}$$
$$+\frac{x^4(-c^2(al+bk)+bc(2am+bl)+b^3(-m)+c^3j)}{4c^4}$$
$$+\frac{x^5(-c(am+bl)+b^2m+c^2k)}{5c^3}+\frac{x^6(cl-bm)}{6c^2}+\frac{mx^7}{7c}$$

```
[In] Int[(d + e*x + f*x^2 + g*x^3 + h*x^4 + j*x^5 + k*x^6 + l*x^7 + m*x^8)/(a +
b*x + c*x^2),x]
```

```
[Out] ((c^6*f - c^5*(b*g + a*h) + c^4*(b^2*h + 2*a*b*j + a^2*k) + b^6*m - b^4*c*(
b*l + 5*a*m) + b^2*c^2*(b^2*k + 4*a*b*l + 6*a^2*m) - c^3*(b^3*j + 3*a*b^2*k
 + 3*a^2*b*l + a^3*m))*x)/c^7 + ((c^5*g - c^4*(b*h + a*j) + c^3*(b^2*j + 2*
a*b*k + a^2*l) - b^5*m + b^3*c*(b*l + 4*a*m) - b*c^2*(b^2*k + 3*a*b*l + 3*a
^2*m))*x^2)/(2*c^6) + ((c^4*h - c^3*(b*j + a*k) + b^4*m - b^2*c*(b*l + 3*a*
m) + c^2*(b^2*k + 2*a*b*l + a^2*m))*x^3)/(3*c^5) + ((c^3*j - c^2*(b*k + a*l
) - b^3*m + b*c*(b*l + 2*a*m))*x^4)/(4*c^4) + ((c^2*k + b^2*m - c*(b*l + a*
m))*x^5)/(5*c^3) + ((c*l - b*m)*x^6)/(6*c^2) + (m*x^7)/(7*c) - ((2*c^8*d -
c^7*(b*e + 2*a*f) + c^6*(b^2*f + 3*a*b*g + 2*a^2*h) - c^5*(b^3*g + 4*a*b^2*
h + 5*a^2*b*j + 2*a^3*k) + b^8*m - b^6*c*(b*l + 8*a*m) + b^4*c^2*(b^2*k + 7
*a*b*l + 20*a^2*m) - b^2*c^3*(b^3*j + 6*a*b^2*k + 14*a^2*b*l + 16*a^3*m) +
c^4*(b^4*h + 5*a*b^3*j + 9*a^2*b^2*k + 7*a^3*b*l + 2*a^4*m))*ArcTanh[(b + 2
```

```
*c*x)/Sqrt[b^2 - 4*a*c]])/(c^8*Sqrt[b^2 - 4*a*c]) + ((c^7*e - c^6*(b*f + a*
g) + c^5*(b^2*g + 2*a*b*h + a^2*j) - c^4*(b^3*h + 3*a*b^2*j + 3*a^2*b*k + a
^3*l) - b^7*m + b^5*c*(b*l + 6*a*m) - b^3*c^2*(b^2*k + 5*a*b*l + 10*a^2*m)
+ b*c^3*(b^3*j + 4*a*b^2*k + 6*a^2*b*l + 4*a^3*m))*Log[a + b*x + c*x^2])/(2
*c^8)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

integral

$$=\int\Bigg(\frac{c^6f-c^5(bg+ah)+c^4(b^2h+2abj+a^2k)+b^6m-b^4c(bl+5am)+b^2c^2(b^2k+4abl+6a^2m)-c^3(b^3j}{c^7}$$

$$+\frac{(c^5g-c^4(bh+aj)+c^3(b^2j+2abk+a^2l)-b^5m+b^3c(bl+4am)-bc^2(b^2k+3abl+3a^2m))\,x}{c^6}$$

$$+\frac{(c^4h-c^3(bj+ak)+b^4m-b^2c(bl+3am)+c^2(b^2k+2abl+a^2m))\,x^2}{c^5}$$

$$+\frac{(c^3j-c^2(bk+al)-b^3m+bc(bl+2am))\,x^3}{c^4}+\frac{(c^2k+b^2m-c(bl+am))\,x^4}{c^3}$$

$$+\frac{(cl-bm)x^5}{c^2}+\frac{mx^6}{c}$$

$$+\frac{c^7d-ac^6f+ac^5(bg+ah)-ac^4(b^2h+2abj+a^2k)-ab^6m+ab^4c(bl+5am)-ab^2c^2(b^2k+4abl+6a^2}{}$$

$$=\frac{(c^6f-c^5(bg+ah)+c^4(b^2h+2abj+a^2k)+b^6m-b^4c(bl+5am)+b^2c^2(b^2k+4abl+6a^2m)-c^3}{c^7}$$

$$+\frac{(c^5g-c^4(bh+aj)+c^3(b^2j+2abk+a^2l)-b^5m+b^3c(bl+4am)-bc^2(b^2k+3abl+3a^2m))\,x^2}{2c^6}$$

$$+\frac{(c^4h-c^3(bj+ak)+b^4m-b^2c(bl+3am)+c^2(b^2k+2abl+a^2m))\,x^3}{3c^5}$$

$$+\frac{(c^3j-c^2(bk+al)-b^3m+bc(bl+2am))\,x^4}{4c^4}$$

$$+\frac{(c^2k+b^2m-c(bl+am))\,x^5}{5c^3}+\frac{(cl-bm)x^6}{6c^2}+\frac{mx^7}{7c}$$

$$+\int\frac{c^7d-ac^6f+ac^5(bg+ah)-ac^4(b^2h+2abj+a^2k)-ab^6m+ab^4c(bl+5am)-ab^2c^2(b^2k+4abl+6a^2m)+ac^3(b^3j+3ab^2k+3a^2bl+a^3m)+}{}$$

$$=\frac{(c^6f-c^5(bg+ah)+c^4(b^2h+2abj+a^2k)+b^6m-b^4c(bl+5am)+b^2c^2(b^2k+4abl+6a^2m)-c^3}{c^7}$$

$$+\frac{(c^5g-c^4(bh+aj)+c^3(b^2j+2abk+a^2l)-b^5m+b^3c(bl+4am)-bc^2(b^2k+3abl+3a^2m))\,x^2}{2c^6}$$

$$+\frac{(c^4h-c^3(bj+ak)+b^4m-b^2c(bl+3am)+c^2(b^2k+2abl+a^2m))\,x^3}{3c^5}$$

$$+\frac{(c^3j-c^2(bk+al)-b^3m+bc(bl+2am))\,x^4}{4c^4}$$

$$+\frac{(c^2k+b^2m-c(bl+am))\,x^5}{5c^3}+\frac{(cl-bm)x^6}{6c^2}+\frac{mx^7}{7c}$$

$$+\frac{(c^7e-c^6(bf+ag)+c^5(b^2g+2abh+a^2j)-c^4(b^3h+3ab^2j+3a^2bk+a^3l)-b^7m+b^5c(bl+6a}{2c^8}$$

$$+\frac{(2c^8d-c^7(be+2af)+c^6(b^2f+3abg+2a^2h)-c^5(b^3g+4ab^2h+5a^2bj+2a^3k)+b^8m-b^6c(bl}{}$$

$$
\begin{aligned}
&= \frac{(c^6f - c^5(bg+ah) + c^4(b^2h+2abj+a^2k) + b^6m - b^4c(bl+5am) + b^2c^2(b^2k+4abl+6a^2m) -}{c^7} \\
&\quad + \frac{(c^5g - c^4(bh+aj) + c^3(b^2j+2abk+a^2l) - b^5m + b^3c(bl+4am) - bc^2(b^2k+3abl+3a^2m))\,x}{2c^6} \\
&\quad + \frac{(c^4h - c^3(bj+ak) + b^4m - b^2c(bl+3am) + c^2(b^2k+2abl+a^2m))\,x^3}{3c^5} \\
&\quad + \frac{(c^3j - c^2(bk+al) - b^3m + bc(bl+2am))\,x^4}{4c^4} \\
&\quad + \frac{(c^2k + b^2m - c(bl+am))\,x^5}{5c^3} + \frac{(cl-bm)x^6}{6c^2} + \frac{mx^7}{7c} \\
&\quad + \frac{(c^7e - c^6(bf+ag) + c^5(b^2g+2abh+a^2j) - c^4(b^3h+3ab^2j+3a^2bk+a^3l) - b^7m + b^5c(bl+6}{2c^8} \\
&\quad - \frac{(2c^8d - c^7(be+2af) + c^6(b^2f+3abg+2a^2h) - c^5(b^3g+4ab^2h+5a^2bj+2a^3k) + b^8m - b^6c(}{}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{(c^6f - c^5(bg+ah) + c^4(b^2h+2abj+a^2k) + b^6m - b^4c(bl+5am) + b^2c^2(b^2k+4abl+6a^2m) -}{c^7} \\
&\quad + \frac{(c^5g - c^4(bh+aj) + c^3(b^2j+2abk+a^2l) - b^5m + b^3c(bl+4am) - bc^2(b^2k+3abl+3a^2m))\,x}{2c^6} \\
&\quad + \frac{(c^4h - c^3(bj+ak) + b^4m - b^2c(bl+3am) + c^2(b^2k+2abl+a^2m))\,x^3}{3c^5} \\
&\quad + \frac{(c^3j - c^2(bk+al) - b^3m + bc(bl+2am))\,x^4}{4c^4} \\
&\quad + \frac{(c^2k + b^2m - c(bl+am))\,x^5}{5c^3} + \frac{(cl-bm)x^6}{6c^2} + \frac{mx^7}{7c} \\
&\quad - \frac{(2c^8d - c^7(be+2af) + c^6(b^2f+3abg+2a^2h) - c^5(b^3g+4ab^2h+5a^2bj+2a^3k) + b^8m - b^6c(}{} \\
&\quad + \frac{(c^7e - c^6(bf+ag) + c^5(b^2g+2abh+a^2j) - c^4(b^3h+3ab^2j+3a^2bk+a^3l) - b^7m + b^5c(bl+6}{2c^8}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 754, normalized size of antiderivative = 0.99

$$\int \frac{d+ex+fx^2+gx^3+hx^4+jx^5+kx^6+lx^7+mx^8}{a+bx+cx^2}\,dx$$

$$= \frac{420c(c^6f - c^5(bg+ah) + c^4(b^2h+2abj+a^2k) + b^6m - b^4c(bl+5am) + b^2c^2(b^2k+4abl+6a^2m) - c^3(b}{}$$

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4 + j*x^5 + k*x^6 + l*x^7 + m*x^8)
/(a + b*x + c*x^2),x]
```

```
[Out] (420*c*(c^6*f - c^5*(b*g + a*h) + c^4*(b^2*h + 2*a*b*j + a^2*k) + b^6*m - b
^4*c*(b*l + 5*a*m) + b^2*c^2*(b^2*k + 4*a*b*l + 6*a^2*m) - c^3*(b^3*j + 3*a
*b^2*k + 3*a^2*b*l + a^3*m))*x + 210*c^2*(c^5*g - c^4*(b*h + a*j) + c^3*(b^
2*j + 2*a*b*k + a^2*l) - b^5*m + b^3*c*(b*l + 4*a*m) - b*c^2*(b^2*k + 3*a*b
*l + 3*a^2*m))*x^2 + 140*c^3*(c^4*h - c^3*(b*j + a*k) + b^4*m - b^2*c*(b*l
+ 3*a*m) + c^2*(b^2*k + 2*a*b*l + a^2*m))*x^3 + 105*c^4*(c^3*j - c^2*(b*k +
 a*l) - b^3*m + b*c*(b*l + 2*a*m))*x^4 + 84*c^5*(c^2*k + b^2*m - c*(b*l + a
*m))*x^5 + 70*c^6*(c*l - b*m)*x^6 + 60*c^7*m*x^7 + (420*(2*c^8*d - c^7*(b*e
 + 2*a*f) + c^6*(b^2*f + 3*a*b*g + 2*a^2*h) - c^5*(b^3*g + 4*a*b^2*h + 5*a^
2*b*j + 2*a^3*k) + b^8*m - b^6*c*(b*l + 8*a*m) + b^4*c^2*(b^2*k + 7*a*b*l +
 20*a^2*m) - b^2*c^3*(b^3*j + 6*a*b^2*k + 14*a^2*b*l + 16*a^3*m) + c^4*(b^4
*h + 5*a*b^3*j + 9*a^2*b^2*k + 7*a^3*b*l + 2*a^4*m))*ArcTan[(b + 2*c*x)/Sqr
t[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + 210*(c^7*e - c^6*(b*f + a*g) + c^5*(
b^2*g + 2*a*b*h + a^2*j) - c^4*(b^3*h + 3*a*b^2*j + 3*a^2*b*k + a^3*l) - b^
7*m + b^5*c*(b*l + 6*a*m) - b^3*c^2*(b^2*k + 5*a*b*l + 10*a^2*m) + b*c^3*(b
^3*j + 4*a*b^2*k + 6*a^2*b*l + 4*a^3*m))*Log[a + x*(b + c*x)])/(420*c^8)
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 1086, normalized size of antiderivative = 1.42

method	result	size
default	Expression too large to display	1086
risch	Expression too large to display	49911

```
[In] int((m*x^8+l*x^7+k*x^6+j*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a),x,metho
d=_RETURNVERBOSE)
```

```
[Out] -1/c^7*(a*b^2*c^3*m*x^3-1/2*c^6*g*x^2-b^6*m*x-c^6*f*x-1/5*c^6*k*x^5-1/4*c^6
*j*x^4-1/3*c^6*h*x^3-1/6*c^6*l*x^6-1/7*m*x^7*c^6+1/2*b*c^5*h*x^2+m*c^3*a^3*
x-a^2*c^4*k*x+h*c^5*a*x+b^5*c*l*x-b^4*c^2*k*x+b^3*c^3*j*x-b^2*c^4*h*x+b*c^5
*g*x-1/2*a^2*c^4*l*x^2+1/2*a*c^5*j*x^2+1/2*b^5*c*m*x^2-1/2*b^4*c^2*l*x^2+1/
2*b^3*c^3*k*x^2-1/2*b^2*c^4*j*x^2-1/3*a^2*c^4*m*x^3+1/3*a*c^5*k*x^3-1/3*b^4
*c^2*m*x^3+1/3*b^3*c^3*l*x^3-1/3*b^2*c^4*k*x^3+1/3*b*c^5*j*x^3+1/5*a*c^5*m*
x^5-1/5*b^2*c^4*m*x^5+1/5*b*c^5*l*x^5+1/4*a*c^5*l*x^4+1/4*b^3*c^3*m*x^4-1/4
*b^2*c^4*l*x^4+1/4*b*c^5*k*x^4+1/6*b*c^5*m*x^6-1/2*a*b*c^4*m*x^4-2/3*a*b*c^
4*l*x^3+3/2*a^2*b*c^3*m*x^2-2*a*b^3*c^2*m*x^2+3/2*a*b^2*c^3*l*x^2-a*b*c^4*k
*x^2-6*a^2*b^2*c^2*m*x+3*a^2*b*c^3*l*x+5*a*b^4*c*m*x-4*a*b^3*c^2*l*x+3*a*b^
2*c^3*k*x-2*a*b*c^4*j*x)+1/c^7*(1/2*(4*a^3*b*c^3*m-a^3*c^4*l-10*a^2*b^3*c^2
*m+6*a^2*b^2*c^3*l-3*a^2*b*c^4*k+a^2*c^5*j+6*a*b^5*c*m-5*a*b^4*c^2*l+4*a*b^
3*c^3*k-3*a*b^2*c^4*j+2*a*b*c^5*h-a*c^6*g-b^7*m+b^6*c*l-b^5*c^2*k+b^4*c^3*j
-b^3*c^4*h+b^2*c^5*g-b*c^6*f+c^7*e)/c*ln(c*x^2+b*x+a)+2*(a^4*c^3*m-6*a^3*b^
2*c^2*m+3*a^3*b*c^3*l-a^3*c^4*k+5*a^2*b^4*c*m-4*a^2*b^3*c^2*l+3*a^2*b^2*c^3
*k-2*a^2*b*c^4*j+a^2*c^5*h-a*b^6*m+a*b^5*c*l-a*b^4*c^2*k+a*b^3*c^3*j-a*b^2*
c^4*h+a*b*c^5*g-a*c^6*f+c^7*d-1/2*(4*a^3*b*c^3*m-a^3*c^4*l-10*a^2*b^3*c^2*m
```

```
+6*a^2*b^2*c^3*l-3*a^2*b*c^4*k+a^2*c^5*j+6*a*b^5*c*m-5*a*b^4*c^2*l+4*a*b^3*
c^3*k-3*a*b^2*c^4*j+2*a*b*c^5*h-a*c^6*g-b^7*m+b^6*c*l-b^5*c^2*k+b^4*c^3*j-b
^3*c^4*h+b^2*c^5*g-b*c^6*f+c^7*e)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(
4*a*c-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.62 (sec) , antiderivative size = 2643, normalized size of antiderivative = 3.45

$$\int \frac{d+ex+fx^2+gx^3+hx^4+jx^5+kx^6+lx^7+mx^8}{a+bx+cx^2}\,dx = \text{Too large to display}$$

```
[In] integrate((m*x^8+l*x^7+k*x^6+j*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a),x
, algorithm="fricas")
```

```
[Out] [1/420*(60*(b^2*c^7 - 4*a*c^8)*m*x^7 + 70*((b^2*c^7 - 4*a*c^8)*l - (b^3*c^6
 - 4*a*b*c^7)*m)*x^6 + 84*((b^2*c^7 - 4*a*c^8)*k - (b^3*c^6 - 4*a*b*c^7)*l
+ (b^4*c^5 - 5*a*b^2*c^6 + 4*a^2*c^7)*m)*x^5 + 105*((b^2*c^7 - 4*a*c^8)*j -
 (b^3*c^6 - 4*a*b*c^7)*k + (b^4*c^5 - 5*a*b^2*c^6 + 4*a^2*c^7)*l - (b^5*c^4
 - 6*a*b^3*c^5 + 8*a^2*b*c^6)*m)*x^4 + 140*((b^2*c^7 - 4*a*c^8)*h - (b^3*c^
6 - 4*a*b*c^7)*j + (b^4*c^5 - 5*a*b^2*c^6 + 4*a^2*c^7)*k - (b^5*c^4 - 6*a*b
^3*c^5 + 8*a^2*b*c^6)*l + (b^6*c^3 - 7*a*b^4*c^4 + 13*a^2*b^2*c^5 - 4*a^3*c
^6)*m)*x^3 + 210*((b^2*c^7 - 4*a*c^8)*g - (b^3*c^6 - 4*a*b*c^7)*h + (b^4*c^
5 - 5*a*b^2*c^6 + 4*a^2*c^7)*j - (b^5*c^4 - 6*a*b^3*c^5 + 8*a^2*b*c^6)*k +
(b^6*c^3 - 7*a*b^4*c^4 + 13*a^2*b^2*c^5 - 4*a^3*c^6)*l - (b^7*c^2 - 8*a*b^5
*c^3 + 19*a^2*b^3*c^4 - 12*a^3*b*c^5)*m)*x^2 + 210*(2*c^8*d - b*c^7*e + (b^
2*c^6 - 2*a*c^7)*f - (b^3*c^5 - 3*a*b*c^6)*g + (b^4*c^4 - 4*a*b^2*c^5 + 2*a
^2*c^6)*h - (b^5*c^3 - 5*a*b^3*c^4 + 5*a^2*b*c^5)*j + (b^6*c^2 - 6*a*b^4*c^
3 + 9*a^2*b^2*c^4 - 2*a^3*c^5)*k - (b^7*c - 7*a*b^5*c^2 + 14*a^2*b^3*c^3 -
7*a^3*b*c^4)*l + (b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 2*a^4
*c^4)*m)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^
2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 420*((b^2*c^7 - 4*a*c^8)*f - (
b^3*c^6 - 4*a*b*c^7)*g + (b^4*c^5 - 5*a*b^2*c^6 + 4*a^2*c^7)*h - (b^5*c^4 -
 6*a*b^3*c^5 + 8*a^2*b*c^6)*j + (b^6*c^3 - 7*a*b^4*c^4 + 13*a^2*b^2*c^5 - 4
*a^3*c^6)*k - (b^7*c^2 - 8*a*b^5*c^3 + 19*a^2*b^3*c^4 - 12*a^3*b*c^5)*l + (
b^8*c - 9*a*b^6*c^2 + 26*a^2*b^4*c^3 - 25*a^3*b^2*c^4 + 4*a^4*c^5)*m)*x + 2
10*((b^2*c^7 - 4*a*c^8)*e - (b^3*c^6 - 4*a*b*c^7)*f + (b^4*c^5 - 5*a*b^2*c^
6 + 4*a^2*c^7)*g - (b^5*c^4 - 6*a*b^3*c^5 + 8*a^2*b*c^6)*h + (b^6*c^3 - 7*a
*b^4*c^4 + 13*a^2*b^2*c^5 - 4*a^3*c^6)*j - (b^7*c^2 - 8*a*b^5*c^3 + 19*a^2*
b^3*c^4 - 12*a^3*b*c^5)*k + (b^8*c - 9*a*b^6*c^2 + 26*a^2*b^4*c^3 - 25*a^3*
b^2*c^4 + 4*a^4*c^5)*l - (b^9 - 10*a*b^7*c + 34*a^2*b^5*c^2 - 44*a^3*b^3*c^
3 + 16*a^4*b*c^4)*m)*log(c*x^2 + b*x + a))/(b^2*c^8 - 4*a*c^9), 1/420*(60*(
b^2*c^7 - 4*a*c^8)*m*x^7 + 70*((b^2*c^7 - 4*a*c^8)*l - (b^3*c^6 - 4*a*b*c^7
)*m)*x^6 + 84*((b^2*c^7 - 4*a*c^8)*k - (b^3*c^6 - 4*a*b*c^7)*l + (b^4*c^5 -
```

```
 5*a*b^2*c^6 + 4*a^2*c^7)*m)*x^5 + 105*((b^2*c^7 - 4*a*c^8)*j - (b^3*c^6 -
4*a*b*c^7)*k + (b^4*c^5 - 5*a*b^2*c^6 + 4*a^2*c^7)*l - (b^5*c^4 - 6*a*b^3*c
^5 + 8*a^2*b*c^6)*m)*x^4 + 140*((b^2*c^7 - 4*a*c^8)*h - (b^3*c^6 - 4*a*b*c^
7)*j + (b^4*c^5 - 5*a*b^2*c^6 + 4*a^2*c^7)*k - (b^5*c^4 - 6*a*b^3*c^5 + 8*a
^2*b*c^6)*l + (b^6*c^3 - 7*a*b^4*c^4 + 13*a^2*b^2*c^5 - 4*a^3*c^6)*m)*x^3 +
 210*((b^2*c^7 - 4*a*c^8)*g - (b^3*c^6 - 4*a*b*c^7)*h + (b^4*c^5 - 5*a*b^2*
c^6 + 4*a^2*c^7)*j - (b^5*c^4 - 6*a*b^3*c^5 + 8*a^2*b*c^6)*k + (b^6*c^3 - 7
*a*b^4*c^4 + 13*a^2*b^2*c^5 - 4*a^3*c^6)*l - (b^7*c^2 - 8*a*b^5*c^3 + 19*a^
2*b^3*c^4 - 12*a^3*b*c^5)*m)*x^2 - 420*(2*c^8*d - b*c^7*e + (b^2*c^6 - 2*a*
c^7)*f - (b^3*c^5 - 3*a*b*c^6)*g + (b^4*c^4 - 4*a*b^2*c^5 + 2*a^2*c^6)*h -
(b^5*c^3 - 5*a*b^3*c^4 + 5*a^2*b*c^5)*j + (b^6*c^2 - 6*a*b^4*c^3 + 9*a^2*b^
2*c^4 - 2*a^3*c^5)*k - (b^7*c - 7*a*b^5*c^2 + 14*a^2*b^3*c^3 - 7*a^3*b*c^4)
*l + (b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 2*a^4*c^4)*m)*sqr
t(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 420
*((b^2*c^7 - 4*a*c^8)*f - (b^3*c^6 - 4*a*b*c^7)*g + (b^4*c^5 - 5*a*b^2*c^6
+ 4*a^2*c^7)*h - (b^5*c^4 - 6*a*b^3*c^5 + 8*a^2*b*c^6)*j + (b^6*c^3 - 7*a*b
^4*c^4 + 13*a^2*b^2*c^5 - 4*a^3*c^6)*k - (b^7*c^2 - 8*a*b^5*c^3 + 19*a^2*b^
3*c^4 - 12*a^3*b*c^5)*l + (b^8*c - 9*a*b^6*c^2 + 26*a^2*b^4*c^3 - 25*a^3*b^
2*c^4 + 4*a^4*c^5)*m)*x + 210*((b^2*c^7 - 4*a*c^8)*e - (b^3*c^6 - 4*a*b*c^7
)*f + (b^4*c^5 - 5*a*b^2*c^6 + 4*a^2*c^7)*g - (b^5*c^4 - 6*a*b^3*c^5 + 8*a^
2*b*c^6)*h + (b^6*c^3 - 7*a*b^4*c^4 + 13*a^2*b^2*c^5 - 4*a^3*c^6)*j - (b^7*
c^2 - 8*a*b^5*c^3 + 19*a^2*b^3*c^4 - 12*a^3*b*c^5)*k + (b^8*c - 9*a*b^6*c^2
 + 26*a^2*b^4*c^3 - 25*a^3*b^2*c^4 + 4*a^4*c^5)*l - (b^9 - 10*a*b^7*c + 34*
a^2*b^5*c^2 - 44*a^3*b^3*c^3 + 16*a^4*b*c^4)*m)*log(c*x^2 + b*x + a))/(b^2*
c^8 - 4*a*c^9)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2+gx^3+hx^4+jx^5+kx^6+lx^7+mx^8}{a+bx+cx^2}\,dx=\text{Timed out}$$

```
[In] integrate((m*x**8+l*x**7+k*x**6+j*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(c*x**2+
b*x+a),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex+fx^2+gx^3+hx^4+jx^5+kx^6+lx^7+mx^8}{a+bx+cx^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((m*x^8+l*x^7+k*x^6+j*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a),x
, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 981, normalized size of antiderivative = 1.28

$$\begin{aligned}\int \frac{d+ex+fx^2+gx^3+hx^4+jx^5+kx^6+lx^7+mx^8}{a+bx+cx^2}\,dx \\ = \frac{60\,c^6mx^7+70\,c^6lx^6-70\,bc^5mx^6+84\,c^6kx^5-84\,bc^5lx^5+84\,b^2c^4mx^5-84\,ac^5mx^5+105\,c^6jx^4-105\,bc^5}{} \\ + \frac{(c^7e-bc^6f+b^2c^5g-ac^6g-b^3c^4h+2\,abc^5h+b^4c^3j-3\,ab^2c^4j+a^2c^5j-b^5c^2k+4\,ab^3c^3k-3\,a^2bc^4k}{2\,c^8} \\ + \frac{(2\,c^8d-bc^7e+b^2c^6f-2\,ac^7f-b^3c^5g+3\,abc^6g+b^4c^4h-4\,ab^2c^5h+2\,a^2c^6h-b^5c^3j+5\,ab^3c^4j-5\,a^2}{}\end{aligned}$$

```
[In] integrate((m*x^8+l*x^7+k*x^6+j*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a),x
, algorithm="giac")
```

```
[Out] 1/420*(60*c^6*m*x^7 + 70*c^6*l*x^6 - 70*b*c^5*m*x^6 + 84*c^6*k*x^5 - 84*b*c
^5*l*x^5 + 84*b^2*c^4*m*x^5 - 84*a*c^5*m*x^5 + 105*c^6*j*x^4 - 105*b*c^5*k*
x^4 + 105*b^2*c^4*l*x^4 - 105*a*c^5*l*x^4 - 105*b^3*c^3*m*x^4 + 210*a*b*c^4
*m*x^4 + 140*c^6*h*x^3 - 140*b*c^5*j*x^3 + 140*b^2*c^4*k*x^3 - 140*a*c^5*k*
x^3 - 140*b^3*c^3*l*x^3 + 280*a*b*c^4*l*x^3 + 140*b^4*c^2*m*x^3 - 420*a*b^2
*c^3*m*x^3 + 140*a^2*c^4*m*x^3 + 210*c^6*g*x^2 - 210*b*c^5*h*x^2 + 210*b^2*
c^4*j*x^2 - 210*a*c^5*j*x^2 - 210*b^3*c^3*k*x^2 + 420*a*b*c^4*k*x^2 + 210*b
^4*c^2*l*x^2 - 630*a*b^2*c^3*l*x^2 + 210*a^2*c^4*l*x^2 - 210*b^5*c*m*x^2 +
840*a*b^3*c^2*m*x^2 - 630*a^2*b*c^3*m*x^2 + 420*c^6*f*x - 420*b*c^5*g*x + 4
20*b^2*c^4*h*x - 420*a*c^5*h*x - 420*b^3*c^3*j*x + 840*a*b*c^4*j*x + 420*b^
4*c^2*k*x - 1260*a*b^2*c^3*k*x + 420*a^2*c^4*k*x - 420*b^5*c*l*x + 1680*a*b
^3*c^2*l*x - 1260*a^2*b*c^3*l*x + 420*b^6*m*x - 2100*a*b^4*c*m*x + 2520*a^2
```

```
*b^2*c^2*m*x - 420*a^3*c^3*m*x)/c^7 + 1/2*(c^7*e - b*c^6*f + b^2*c^5*g - a*
c^6*g - b^3*c^4*h + 2*a*b*c^5*h + b^4*c^3*j - 3*a*b^2*c^4*j + a^2*c^5*j - b
^5*c^2*k + 4*a*b^3*c^3*k - 3*a^2*b*c^4*k + b^6*c*l - 5*a*b^4*c^2*l + 6*a^2*
b^2*c^3*l - a^3*c^4*l - b^7*m + 6*a*b^5*c*m - 10*a^2*b^3*c^2*m + 4*a^3*b*c^
3*m)*log(c*x^2 + b*x + a)/c^8 + (2*c^8*d - b*c^7*e + b^2*c^6*f - 2*a*c^7*f
- b^3*c^5*g + 3*a*b*c^6*g + b^4*c^4*h - 4*a*b^2*c^5*h + 2*a^2*c^6*h - b^5*c
^3*j + 5*a*b^3*c^4*j - 5*a^2*b*c^5*j + b^6*c^2*k - 6*a*b^4*c^3*k + 9*a^2*b^
2*c^4*k - 2*a^3*c^5*k - b^7*c*l + 7*a*b^5*c^2*l - 14*a^2*b^3*c^3*l + 7*a^3*
b*c^4*l + b^8*m - 8*a*b^6*c*m + 20*a^2*b^4*c^2*m - 16*a^3*b^2*c^3*m + 2*a^4
*c^4*m)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^8)
```

Mupad [B] (verification not implemented)

Time = 17.97 (sec) , antiderivative size = 2779, normalized size of antiderivative = 3.63

$$\int \frac{d+ex+fx^2+gx^3+hx^4+jx^5+kx^6+lx^7+mx^8}{a+bx+cx^2}\,dx = \text{Too large to display}$$

```
[In] int((d + e*x + f*x^2 + g*x^3 + h*x^4 + j*x^5 + k*x^6 + l*x^7 + m*x^8)/(a +
b*x + c*x^2),x)
```

```
[Out] x^6*(l/(6*c) - (b*m)/(6*c^2)) + x*(f/c + (b*((a*(j/c - (a*(l/c - (b*m)/c^2)
)/c + (b*((b*(l/c - (b*m)/c^2))/c - k/c + (a*m)/c^2))/c))/c - g/c + (b*(h/c
 - (b*(j/c - (a*(l/c - (b*m)/c^2))/c + (b*((b*(l/c - (b*m)/c^2))/c - k/c +
(a*m)/c^2))/c))/c + (a*((b*(l/c - (b*m)/c^2))/c - k/c + (a*m)/c^2))/c))/c))
/c - (a*(h/c - (b*(j/c - (a*(l/c - (b*m)/c^2))/c + (b*((b*(l/c - (b*m)/c^2)
)/c - k/c + (a*m)/c^2))/c))/c + (a*((b*(l/c - (b*m)/c^2))/c - k/c + (a*m)/c
^2))/c))/c) + x^4*(j/(4*c) - (a*(l/c - (b*m)/c^2))/(4*c) + (b*((b*(l/c - (b
*m)/c^2))/c - k/c + (a*m)/c^2))/(4*c)) - x^2*((a*(j/c - (a*(l/c - (b*m)/c^2
))/c + (b*((b*(l/c - (b*m)/c^2))/c - k/c + (a*m)/c^2))/c))/(2*c) - g/(2*c)
+ (b*(h/c - (b*(j/c - (a*(l/c - (b*m)/c^2))/c + (b*((b*(l/c - (b*m)/c^2))/c
 - k/c + (a*m)/c^2))/c))/c + (a*((b*(l/c - (b*m)/c^2))/c - k/c + (a*m)/c^2)
)/c))/(2*c)) + x^3*(h/(3*c) - (b*(j/c - (a*(l/c - (b*m)/c^2))/c + (b*((b*(l
/c - (b*m)/c^2))/c - k/c + (a*m)/c^2))/c))/(3*c) + (a*((b*(l/c - (b*m)/c^2)
)/c - k/c + (a*m)/c^2))/(3*c)) - x^5*((b*(l/c - (b*m)/c^2))/(5*c) - k/(5*c)
 + (a*m)/(5*c^2)) + (log((2*c^9*x*(-(2*c^8*d + b^8*m + b^2*c^6*f + 2*a^2*c^
6*h - b^3*c^5*g + b^4*c^4*h - 2*a^3*c^5*k - b^5*c^3*j + b^6*c^2*k + 2*a^4*c
^4*m - 2*a*c^7*f - b*c^7*e - b^7*c*l + 9*a^2*b^2*c^4*k - 14*a^2*b^3*c^3*l +
 20*a^2*b^4*c^2*m - 16*a^3*b^2*c^3*m + 3*a*b*c^6*g - 8*a*b^6*c*m - 4*a*b^2*
c^5*h + 5*a*b^3*c^4*j - 5*a^2*b*c^5*j - 6*a*b^4*c^3*k + 7*a*b^5*c^2*l + 7*a
^3*b*c^4*l)^2/(c^16*(4*a*c - b^2)))^(1/2) - b^8*m - 2*c^8*d - b^2*c^6*f - 2
*a^2*c^6*h + b^3*c^5*g - b^4*c^4*h + 2*a^3*c^5*k + b^5*c^3*j - b^6*c^2*k -
2*a^4*c^4*m + 2*a*c^7*f + b*c^7*e + b^7*c*l + b*c^8*(-(2*c^8*d + b^8*m + b^
2*c^6*f + 2*a^2*c^6*h - b^3*c^5*g + b^4*c^4*h - 2*a^3*c^5*k - b^5*c^3*j + b
^6*c^2*k + 2*a^4*c^4*m - 2*a*c^7*f - b*c^7*e - b^7*c*l + 9*a^2*b^2*c^4*k -
```

```
14*a^2*b^3*c^3*l + 20*a^2*b^4*c^2*m - 16*a^3*b^2*c^3*m + 3*a*b*c^6*g - 8*a*
b^6*c*m - 4*a*b^2*c^5*h + 5*a*b^3*c^4*j - 5*a^2*b*c^5*j - 6*a*b^4*c^3*k + 7
*a*b^5*c^2*l + 7*a^3*b*c^4*l)^2/(c^16*(4*a*c - b^2)))^(1/2) - 9*a^2*b^2*c^4
*k + 14*a^2*b^3*c^3*l - 20*a^2*b^4*c^2*m + 16*a^3*b^2*c^3*m - 3*a*b*c^6*g +
 8*a*b^6*c*m + 4*a*b^2*c^5*h - 5*a*b^3*c^4*j + 5*a^2*b*c^5*j + 6*a*b^4*c^3*
k - 7*a*b^5*c^2*l - 7*a^3*b*c^4*l)*(2*c^8*d + b^8*m + 2*c^9*x*(-(2*c^8*d +
b^8*m + b^2*c^6*f + 2*a^2*c^6*h - b^3*c^5*g + b^4*c^4*h - 2*a^3*c^5*k - b^5
*c^3*j + b^6*c^2*k + 2*a^4*c^4*m - 2*a*c^7*f - b*c^7*e - b^7*c*l + 9*a^2*b^
2*c^4*k - 14*a^2*b^3*c^3*l + 20*a^2*b^4*c^2*m - 16*a^3*b^2*c^3*m + 3*a*b*c^
6*g - 8*a*b^6*c*m - 4*a*b^2*c^5*h + 5*a*b^3*c^4*j - 5*a^2*b*c^5*j - 6*a*b^4
*c^3*k + 7*a*b^5*c^2*l + 7*a^3*b*c^4*l)^2/(c^16*(4*a*c - b^2)))^(1/2) + b^2
*c^6*f + 2*a^2*c^6*h - b^3*c^5*g + b^4*c^4*h - 2*a^3*c^5*k - b^5*c^3*j + b^
6*c^2*k + 2*a^4*c^4*m - 2*a*c^7*f - b*c^7*e - b^7*c*l + b*c^8*(-(2*c^8*d +
b^8*m + b^2*c^6*f + 2*a^2*c^6*h - b^3*c^5*g + b^4*c^4*h - 2*a^3*c^5*k - b^5
*c^3*j + b^6*c^2*k + 2*a^4*c^4*m - 2*a*c^7*f - b*c^7*e - b^7*c*l + 9*a^2*b^
2*c^4*k - 14*a^2*b^3*c^3*l + 20*a^2*b^4*c^2*m - 16*a^3*b^2*c^3*m + 3*a*b*c^
6*g - 8*a*b^6*c*m - 4*a*b^2*c^5*h + 5*a*b^3*c^4*j - 5*a^2*b*c^5*j - 6*a*b^4
*c^3*k + 7*a*b^5*c^2*l + 7*a^3*b*c^4*l)^2/(c^16*(4*a*c - b^2)))^(1/2) + 9*a
^2*b^2*c^4*k - 14*a^2*b^3*c^3*l + 20*a^2*b^4*c^2*m - 16*a^3*b^2*c^3*m + 3*a
*b*c^6*g - 8*a*b^6*c*m - 4*a*b^2*c^5*h + 5*a*b^3*c^4*j - 5*a^2*b*c^5*j - 6*
a*b^4*c^3*k + 7*a*b^5*c^2*l + 7*a^3*b*c^4*l))*(b^9*m - b^2*c^7*e - 4*a^2*c^
7*g + b^3*c^6*f - b^4*c^5*g + b^5*c^4*h + 4*a^3*c^6*j - b^6*c^3*j - 4*a^4*c
^5*l + b^7*c^2*k + 4*a*c^8*e - b^8*c*l - 13*a^2*b^2*c^5*j + 19*a^2*b^3*c^4*
k - 26*a^2*b^4*c^3*l + 25*a^3*b^2*c^4*l + 34*a^2*b^5*c^2*m - 44*a^3*b^3*c^3
*m - 4*a*b*c^7*f - 10*a*b^7*c*m + 5*a*b^2*c^6*g - 6*a*b^3*c^5*h + 8*a^2*b*c
^6*h + 7*a*b^4*c^4*j - 8*a*b^5*c^3*k - 12*a^3*b*c^5*k + 9*a*b^6*c^2*l + 16*
a^4*b*c^4*m))/(2*(4*a*c^9 - b^2*c^8)) + (m*x^7)/(7*c) + (atan(b/(4*a*c - b^
2)^(1/2) + (2*c*x)/(4*a*c - b^2)^(1/2))*(2*c^8*d + b^8*m + b^2*c^6*f + 2*a^
2*c^6*h - b^3*c^5*g + b^4*c^4*h - 2*a^3*c^5*k - b^5*c^3*j + b^6*c^2*k + 2*a
^4*c^4*m - 2*a*c^7*f - b*c^7*e - b^7*c*l + 9*a^2*b^2*c^4*k - 14*a^2*b^3*c^3
*l + 20*a^2*b^4*c^2*m - 16*a^3*b^2*c^3*m + 3*a*b*c^6*g - 8*a*b^6*c*m - 4*a*
b^2*c^5*h + 5*a*b^3*c^4*j - 5*a^2*b*c^5*j - 6*a*b^4*c^3*k + 7*a*b^5*c^2*l +
 7*a^3*b*c^4*l))/(c^8*(4*a*c - b^2)^(1/2))
```

3.374 $\int \left(1+4x-7x^2\right)^3 \left(2+5x+x^2\right) \sqrt{3+2x+5x^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 208

$$
\begin{aligned}
&\int \left(1+4x-7x^2\right)^3 \left(2+5x+x^2\right) \sqrt{3+2x+5x^2}\,dx \\
&= -\frac{77159983(1+5x)\sqrt{3+2x+5x^2}}{31250000} - \frac{1968340667\left(3+2x+5x^2\right)^{3/2}}{131250000} \\
&\quad + \frac{1045360143x\left(3+2x+5x^2\right)^{3/2}}{43750000} + \frac{98060877x^2\left(3+2x+5x^2\right)^{3/2}}{4375000} \\
&\quad - \frac{90960857x^3\left(3+2x+5x^2\right)^{3/2}}{1575000} - \frac{888751x^4\left(3+2x+5x^2\right)^{3/2}}{105000} \\
&\quad + \frac{190939x^5\left(3+2x+5x^2\right)^{3/2}}{3000} - \frac{50519x^6\left(3+2x+5x^2\right)^{3/2}}{2250} \\
&\quad - \frac{343}{50}x^7\left(3+2x+5x^2\right)^{3/2} - \frac{540119881\operatorname{arcsinh}\left(\frac{1+5x}{\sqrt{14}}\right)}{15625000\sqrt{5}}
\end{aligned}
$$

```
[Out] -1968340667/131250000*(5*x^2+2*x+3)^(3/2)+1045360143/43750000*x*(5*x^2+2*x+
3)^(3/2)+98060877/4375000*x^2*(5*x^2+2*x+3)^(3/2)-90960857/1575000*x^3*(5*x
^2+2*x+3)^(3/2)-888751/105000*x^4*(5*x^2+2*x+3)^(3/2)+190939/3000*x^5*(5*x^
2+2*x+3)^(3/2)-50519/2250*x^6*(5*x^2+2*x+3)^(3/2)-343/50*x^7*(5*x^2+2*x+3)^
(3/2)-540119881/78125000*arcsinh(1/14*(1+5*x)*14^(1/2))*5^(1/2)-77159983/31
250000*(1+5*x)*(5*x^2+2*x+3)^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1675, 654, 626, 633, 221}

$$\int \left(1+4x-7x^2\right)^3 \left(2+5x+x^2\right) \sqrt{3+2x+5x^2}\,dx = -\frac{540119881\operatorname{arcsinh}\left(\frac{5x+1}{\sqrt{14}}\right)}{15625000\sqrt{5}} + \frac{98060877(5x^2+2x+3)^{3/2}\,x^2}{4375000} + \frac{1045360143(5x^2+2x+3)^{3/2}\,x}{43750000} - \frac{1968340667(5x^2+2x+3)^{3/2}}{131250000} - \frac{77159983(5x+1)\sqrt{5x^2+2x+3}}{31250000} - \frac{343}{50}\left(5x^2+2x+3\right)^{3/2}x^7 - \frac{50519(5x^2+2x+3)^{3/2}\,x^6}{2250} + \frac{190939(5x^2+2x+3)^{3/2}\,x^5}{3000} - \frac{888751(5x^2+2x+}{105000}$$

```
[In] Int[(1 + 4*x - 7*x^2)^3*(2 + 5*x + x^2)*Sqrt[3 + 2*x + 5*x^2],x]
```

```
[Out] (-77159983*(1 + 5*x)*Sqrt[3 + 2*x + 5*x^2])/31250000 - (1968340667*(3 + 2*x
 + 5*x^2)^(3/2))/131250000 + (1045360143*x*(3 + 2*x + 5*x^2)^(3/2))/4375000
0 + (98060877*x^2*(3 + 2*x + 5*x^2)^(3/2))/4375000 - (90960857*x^3*(3 + 2*x
 + 5*x^2)^(3/2))/1575000 - (888751*x^4*(3 + 2*x + 5*x^2)^(3/2))/105000 + (1
90939*x^5*(3 + 2*x + 5*x^2)^(3/2))/3000 - (50519*x^6*(3 + 2*x + 5*x^2)^(3/2
))/2250 - (343*x^7*(3 + 2*x + 5*x^2)^(3/2))/50 - (540119881*ArcSinh[(1 + 5*
x)/Sqrt[14]])/(15625000*Sqrt[5])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
```

```
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\text{integral} = -\frac{343}{50}x^7\left(3+2x+5x^2\right)^{3/2}+\frac{1}{50}\int\sqrt{3+2x+5x^2}\left(100+1450x+5750x^2-3050x^3 -43550x^4+6350x^5+110453x^6-50519x^7\right)\,dx$$

$$= -\frac{50519x^6(3+2x+5x^2)^{3/2}}{2250}-\frac{343}{50}x^7\left(3+2x+5x^2\right)^{3/2} +\frac{\int\sqrt{3+2x+5x^2}(4500+65250x+258750x^2-137250x^3-1959750x^4+1195092x^5+5728170x}{2250}$$

$$= \frac{190939x^5(3+2x+5x^2)^{3/2}}{3000}-\frac{50519x^6(3+2x+5x^2)^{3/2}}{2250}-\frac{343}{50}x^7\left(3+2x+5x^2\right)^{3/2} +\frac{\int\sqrt{3+2x+5x^2}(180000+2610000x+10350000x^2-5490000x^3-164312550x^4-26662530x^5}{90000}$$

$$= -\frac{888751x^4(3+2x+5x^2)^{3/2}}{105000}+\frac{190939x^5(3+2x+5x^2)^{3/2}}{3000}-\frac{50519x^6(3+2x+5x^2)^{3/2}}{2250} -\frac{343}{50}x^7\left(3+2x+5x^2\right)^{3/2}+\frac{\int\sqrt{3+2x+5x^2}(6300000+91350000x+362250000x^2+127800360x^3}{3150000}$$

$$= -\frac{90960857x^3(3+2x+5x^2)^{3/2}}{1575000}-\frac{888751x^4(3+2x+5x^2)^{3/2}}{105000} +\frac{190939x^5(3+2x+5x^2)^{3/2}}{3000}-\frac{50519x^6(3+2x+5x^2)^{3/2}}{2250} -\frac{343}{50}x^7\left(3+2x+5x^2\right)^{3/2}+\frac{\int\sqrt{3+2x+5x^2}(189000000+2740500000x+5998636278 0x^2+52952}{94500000}$$

$$\begin{aligned}
&= \frac{98060877x^2(3+2x+5x^2)^{3/2}}{4375000} - \frac{90960857x^3(3+2x+5x^2)^{3/2}}{1575000} \\
&\quad - \frac{888751x^4(3+2x+5x^2)^{3/2}}{105000} \\
&\quad + \frac{190939x^5(3+2x+5x^2)^{3/2}}{3000} - \frac{50519x^6(3+2x+5x^2)^{3/2}}{2250} \\
&\quad - \frac{343}{50}x^7\left(3+2x+5x^2\right)^{3/2} + \frac{\int \sqrt{3+2x+5x^2}(4725000000 - 249204741480x + 1128988954440x^2)}{2362500000} \\
&= \frac{1045360143x(3+2x+5x^2)^{3/2}}{43750000} + \frac{98060877x^2(3+2x+5x^2)^{3/2}}{4375000} \\
&\quad - \frac{90960857x^3(3+2x+5x^2)^{3/2}}{1575000} - \frac{888751x^4(3+2x+5x^2)^{3/2}}{105000} \\
&\quad + \frac{190939x^5(3+2x+5x^2)^{3/2}}{3000} - \frac{50519x^6(3+2x+5x^2)^{3/2}}{2250} \\
&\quad - \frac{343}{50}x^7\left(3+2x+5x^2\right)^{3/2} + \frac{\int (-3292466863320 - 10629039601800x)\sqrt{3+2x+5x^2}\,dx}{47250000000} \\
&= -\frac{1968340667(3+2x+5x^2)^{3/2}}{131250000} + \frac{1045360143x(3+2x+5x^2)^{3/2}}{43750000} \\
&\quad + \frac{98060877x^2(3+2x+5x^2)^{3/2}}{4375000} \\
&\quad - \frac{90960857x^3(3+2x+5x^2)^{3/2}}{1575000} - \frac{888751x^4(3+2x+5x^2)^{3/2}}{105000} \\
&\quad + \frac{190939x^5(3+2x+5x^2)^{3/2}}{3000} - \frac{50519x^6(3+2x+5x^2)^{3/2}}{2250} \\
&\quad - \frac{343}{50}x^7\left(3+2x+5x^2\right)^{3/2} - \frac{77159983\int \sqrt{3+2x+5x^2}\,dx}{3125000} \\
&= -\frac{77159983(1+5x)\sqrt{3+2x+5x^2}}{31250000} - \frac{1968340667(3+2x+5x^2)^{3/2}}{131250000} \\
&\quad + \frac{1045360143x(3+2x+5x^2)^{3/2}}{43750000} + \frac{98060877x^2(3+2x+5x^2)^{3/2}}{4375000} \\
&\quad - \frac{90960857x^3(3+2x+5x^2)^{3/2}}{1575000} - \frac{888751x^4(3+2x+5x^2)^{3/2}}{105000} \\
&\quad + \frac{190939x^5(3+2x+5x^2)^{3/2}}{3000} - \frac{50519x^6(3+2x+5x^2)^{3/2}}{2250} \\
&\quad - \frac{343}{50}x^7\left(3+2x+5x^2\right)^{3/2} - \frac{540119881\int \frac{1}{\sqrt{3+2x+5x^2}}\,dx}{15625000}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{77159983(1+5x)\sqrt{3+2x+5x^2}}{31250000} - \frac{1968340667(3+2x+5x^2)^{3/2}}{131250000} \\
&\quad + \frac{1045360143x(3+2x+5x^2)^{3/2}}{43750000} + \frac{98060877x^2(3+2x+5x^2)^{3/2}}{4375000} \\
&\quad - \frac{90960857x^3(3+2x+5x^2)^{3/2}}{1575000} - \frac{888751x^4(3+2x+5x^2)^{3/2}}{105000} \\
&\quad + \frac{190939x^5(3+2x+5x^2)^{3/2}}{3000} - \frac{50519x^6(3+2x+5x^2)^{3/2}}{2250} \\
&\quad - \frac{343}{50}x^7\left(3+2x+5x^2\right)^{3/2} - \frac{\left(77159983\sqrt{\frac{7}{10}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1+\frac{x^2}{56}}}\,dx,x,2+10x\right)}{31250000} \\
&= -\frac{77159983(1+5x)\sqrt{3+2x+5x^2}}{31250000} - \frac{1968340667(3+2x+5x^2)^{3/2}}{131250000} \\
&\quad + \frac{1045360143x(3+2x+5x^2)^{3/2}}{43750000} + \frac{98060877x^2(3+2x+5x^2)^{3/2}}{4375000} \\
&\quad - \frac{90960857x^3(3+2x+5x^2)^{3/2}}{1575000} - \frac{888751x^4(3+2x+5x^2)^{3/2}}{105000} \\
&\quad + \frac{190939x^5(3+2x+5x^2)^{3/2}}{3000} - \frac{50519x^6(3+2x+5x^2)^{3/2}}{2250} \\
&\quad - \frac{343}{50}x^7\left(3+2x+5x^2\right)^{3/2} - \frac{540119881\sinh^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{15625000\sqrt{5}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.48

$$
\begin{aligned}
&\int\left(1+4x-7x^2\right)^3\left(2+5x+x^2\right)\sqrt{3+2x+5x^2}\,dx \\
&= \frac{\sqrt{3+2x+5x^2}(-93436408944+57768004650x+78839046795x^2-17642392275x^3-56757413000x^4-2}{1968750000} \\
&\quad + \frac{540119881\log\left(-1-5x+\sqrt{5}\sqrt{3+2x+5x^2}\right)}{15625000\sqrt{5}}
\end{aligned}
$$

```
[In] Integrate[(1 + 4*x - 7*x^2)^3*(2 + 5*x + x^2)*Sqrt[3 + 2*x + 5*x^2],x]
```

```
[Out] (Sqrt[3 + 2*x + 5*x^2]*(-93436408944 + 57768004650*x + 78839046795*x^2 - 17
642392275*x^3 - 56757413000*x^4 - 225922362500*x^5 + 34674656250*x^6 + 4975
93468750*x^7 - 248031875000*x^8 - 67528125000*x^9))/1968750000 + (540119881
*Log[-1 - 5*x + Sqrt[5]*Sqrt[3 + 2*x + 5*x^2]])/(15625000*Sqrt[5])
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.36

method	result
risch	$-\frac{\left(67528125000 x^{9}+248031875000 x^{8}-497593468750 x^{7}-34674656250 x^{6}+225922362500 x^{5}+56757413000 x^{4}+17642392275 x^{3}-7883\right.}{1968750000}$
trager	$\left(-\frac{343}{10} x^{9}-\frac{56693}{450} x^{8}+\frac{2274713}{9000} x^{7}+\frac{369863}{21000} x^{6}-\frac{18073789}{157500} x^{5}-\frac{56757413}{1968750} x^{4}-\frac{235231897}{26250000} x^{3}+\frac{5255936453}{131250000} x^{2}+\right.$
default	$-\frac{77159983(10x+2)\sqrt{5x^2+2x+3}}{62500000}-\frac{540119881\sqrt{5}\ \operatorname{arcsinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{78125000}-\frac{1968340667\left(5x^2+2x+3\right)^{\frac{3}{2}}}{131250000}+\frac{98060877x^2\left(5x^2+2\right.}{4375000}$

```
[In] int((-7*x^2+4*x+1)^3*(x^2+5*x+2)*(5*x^2+2*x+3)^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -1/1968750000*(67528125000*x^9+248031875000*x^8-497593468750*x^7-3467465625
0*x^6+225922362500*x^5+56757413000*x^4+17642392275*x^3-78839046795*x^2-5776
8004650*x+93436408944)*(5*x^2+2*x+3)^(1/2)-540119881/78125000*5^(1/2)*arcsi
nh(5/14*14^(1/2)*(x+1/5))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.47

$$\int \left(1+4x-7x^2\right)^3\left(2+5x+x^2\right)\sqrt{3+2x+5x^2}\,dx = -\frac{1}{1968750000}\left(67528125000\,x^9+248031875000\,x^8-497593468750\,x^7-34674656250\,x^6+2259223625\right. \\ +\frac{540119881}{156250000}\sqrt{5}\log\left(\sqrt{5}\sqrt{5\,x^2+2\,x+3}(5\,x+1)-25\,x^2-10\,x-8\right)$$

```
[In] integrate((-7*x^2+4*x+1)^3*(x^2+5*x+2)*(5*x^2+2*x+3)^(1/2),x, algorithm="fr
icas")
```

```
[Out] -1/1968750000*(67528125000*x^9 + 248031875000*x^8 - 497593468750*x^7 - 3467
4656250*x^6 + 225922362500*x^5 + 56757413000*x^4 + 17642392275*x^3 - 788390
46795*x^2 - 57768004650*x + 93436408944)*sqrt(5*x^2 + 2*x + 3) + 540119881/
156250000*sqrt(5)*log(sqrt(5)*sqrt(5*x^2 + 2*x + 3)*(5*x + 1) - 25*x^2 - 10
*x - 8)
```

Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.48

$$\begin{aligned}&\int \left(1+4x-7x^2\right)^3 \left(2+5x+x^2\right) \sqrt{3+2x+5x^2}\,dx\\&=\sqrt{5x^2+2x+3}\bigg(-\frac{343x^9}{10}-\frac{56693x^8}{450}+\frac{2274713x^7}{9000}+\frac{369863x^6}{21000}-\frac{18073789x^5}{157500}\\&\quad-\frac{56757413x^4}{1968750}-\frac{235231897x^3}{26250000}+\frac{5255936453x^2}{131250000}+\frac{385120031x}{13125000}-\frac{648863951}{13671875}\bigg)\\&\quad-\frac{540119881\sqrt{5}\operatorname{asinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{78125000}\end{aligned}$$

```
[In] integrate((-7*x**2+4*x+1)**3*(x**2+5*x+2)*(5*x**2+2*x+3)**(1/2),x)
```

```
[Out] sqrt(5*x**2 + 2*x + 3)*(-343*x**9/10 - 56693*x**8/450 + 2274713*x**7/9000 +
 369863*x**6/21000 - 18073789*x**5/157500 - 56757413*x**4/1968750 - 2352318
97*x**3/26250000 + 5255936453*x**2/131250000 + 385120031*x/13125000 - 64886
3951/13671875) - 540119881*sqrt(5)*asinh(5*sqrt(14)*(x + 1/5)/14)/78125000
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.85

$$\begin{aligned}&\int \left(1+4x-7x^2\right)^3 \left(2+5x+x^2\right) \sqrt{3+2x+5x^2}\,dx\\&=-\frac{343}{50}\left(5x^2+2x+3\right)^{\frac{3}{2}}x^7-\frac{50519}{2250}\left(5x^2+2x+3\right)^{\frac{3}{2}}x^6+\frac{190939}{3000}\left(5x^2+2x+3\right)^{\frac{3}{2}}x^5\\&\quad-\frac{888751}{105000}\left(5x^2+2x+3\right)^{\frac{3}{2}}x^4-\frac{90960857}{1575000}\left(5x^2+2x+3\right)^{\frac{3}{2}}x^3\\&\quad+\frac{98060877}{4375000}\left(5x^2+2x+3\right)^{\frac{3}{2}}x^2+\frac{1045360143}{43750000}\left(5x^2+2x+3\right)^{\frac{3}{2}}x\\&\quad-\frac{1968340667}{131250000}\left(5x^2+2x+3\right)^{\frac{3}{2}}-\frac{77159983}{6250000}\sqrt{5x^2+2x+3}x\\&\quad-\frac{540119881}{78125000}\sqrt{5}\operatorname{arsinh}\left(\frac{1}{14}\sqrt{14}(5x+1)\right)-\frac{77159983}{31250000}\sqrt{5x^2+2x+3}\end{aligned}$$

```
[In] integrate((-7*x^2+4*x+1)^3*(x^2+5*x+2)*(5*x^2+2*x+3)^(1/2),x, algorithm="ma
xima")
```

```
[Out] -343/50*(5*x^2 + 2*x + 3)^(3/2)*x^7 - 50519/2250*(5*x^2 + 2*x + 3)^(3/2)*x^
6 + 190939/3000*(5*x^2 + 2*x + 3)^(3/2)*x^5 - 888751/105000*(5*x^2 + 2*x +
3)^(3/2)*x^4 - 90960857/1575000*(5*x^2 + 2*x + 3)^(3/2)*x^3 + 98060877/4375
```

```
000*(5*x^2 + 2*x + 3)^(3/2)*x^2 + 1045360143/43750000*(5*x^2 + 2*x + 3)^(3/
2)*x - 1968340667/131250000*(5*x^2 + 2*x + 3)^(3/2) - 77159983/6250000*sqrt
(5*x^2 + 2*x + 3)*x - 540119881/78125000*sqrt(5)*arcsinh(1/14*sqrt(14)*(5*x
 + 1)) - 77159983/31250000*sqrt(5*x^2 + 2*x + 3)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.44

$$\int \left(1+4x-7x^2\right)^3 \left(2+5x+x^2\right) \sqrt{3+2x+5x^2}\,dx =$$

$$-\frac{1}{1968750000}\left(5\left(\left(5\left(10\left(25\left(5\left(49\left(140\left(315\,x+1157\right)x-324959\right)x-1109589\right)x+36147578\right)x+227029\right.\right.\right.\right.\right.\right.$$

$$+\frac{540119881}{78125000}\sqrt{5}\log\left(-\sqrt{5}\left(\sqrt{5}x-\sqrt{5\,x^2+2\,x+3}\right)-1\right)$$

```
[In] integrate((-7*x^2+4*x+1)^3*(x^2+5*x+2)*(5*x^2+2*x+3)^(1/2),x, algorithm="gi
ac")
```

```
[Out] -1/1968750000*(5*((5*(10*(25*(5*(49*(140*(315*x + 1157)*x - 324959)*x - 110
9589)*x + 36147578)*x + 227029652)*x + 705695691)*x - 15767809359)*x - 1155
3600930)*x + 93436408944)*sqrt(5*x^2 + 2*x + 3) + 540119881/78125000*sqrt(5
)*log(-sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3)) - 1)
```

Mupad [B] (verification not implemented)

Time = 15.62 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.06

$$\int \left(1+4x-7x^2\right)^3 \left(2+5x+x^2\right) \sqrt{3+2x+5x^2}\,dx$$

$$\begin{aligned}
&= \frac{98060877\,x^2\,{\left(5\,x^2+2\,x+3\right)}^{3/2}}{4375000}-\frac{90960857\,x^3\,{\left(5\,x^2+2\,x+3\right)}^{3/2}}{1575000}\\
&-\frac{888751\,x^4\,{\left(5\,x^2+2\,x+3\right)}^{3/2}}{105000}+\frac{190939\,x^5\,{\left(5\,x^2+2\,x+3\right)}^{3/2}}{3000}\\
&-\frac{50519\,x^6\,{\left(5\,x^2+2\,x+3\right)}^{3/2}}{2250}-\frac{343\,x^7\,{\left(5\,x^2+2\,x+3\right)}^{3/2}}{50}\\
&-\frac{3048580429\,\sqrt{5}\,\ln\left(\sqrt{5\,x^2+2\,x+3}+\frac{\sqrt{5}\,\left(5\,x+1\right)}{5}\right)}{156250000}\\
&-\frac{3048580429\,\left(\frac{x}{2}+\frac{1}{10}\right)\,\sqrt{5\,x^2+2\,x+3}}{43750000}\\
&-\frac{1968340667\,\sqrt{5\,x^2+2\,x+3}\,\left(200\,x^2+20\,x+108\right)}{5250000000}+\frac{1045360143\,x\,{\left(5\,x^2+2\,x+3\right)}^{3/2}}{43750000}\\
&+\frac{1968340667\,\sqrt{5}\,\ln\left(2\,\sqrt{5\,x^2+2\,x+3}+\frac{\sqrt{5}\,\left(10\,x+2\right)}{5}\right)}{156250000}
\end{aligned}$$

```
[In] int((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(1/2)*(4*x - 7*x^2 + 1)^3,x)
```

```
[Out] (98060877*x^2*(2*x + 5*x^2 + 3)^(3/2))/4375000 - (90960857*x^3*(2*x + 5*x^2
 + 3)^(3/2))/1575000 - (888751*x^4*(2*x + 5*x^2 + 3)^(3/2))/105000 + (19093
9*x^5*(2*x + 5*x^2 + 3)^(3/2))/3000 - (50519*x^6*(2*x + 5*x^2 + 3)^(3/2))/2
250 - (343*x^7*(2*x + 5*x^2 + 3)^(3/2))/50 - (3048580429*5^(1/2)*log((2*x +
 5*x^2 + 3)^(1/2) + (5^(1/2)*(5*x + 1))/5))/156250000 - (3048580429*(x/2 +
1/10)*(2*x + 5*x^2 + 3)^(1/2))/43750000 - (1968340667*(2*x + 5*x^2 + 3)^(1/
2)*(20*x + 200*x^2 + 108))/5250000000 + (1045360143*x*(2*x + 5*x^2 + 3)^(3/
2))/43750000 + (1968340667*5^(1/2)*log(2*(2*x + 5*x^2 + 3)^(1/2) + (5^(1/2)
*(10*x + 2))/5))/156250000
```

3.375 $\int \left(1+4x-7x^2\right)^2 \left(2+5x+x^2\right) \sqrt{3+2x+5x^2}\, dx$

Optimal result

Integrand size = 35, antiderivative size = 166

$$\begin{aligned}\int \left(1+4x-7x^2\right)^2 \left(2+5x+x^2\right) \sqrt{3+2x+5x^2}\, dx &= -\frac{2521723(1+5x)\sqrt{3+2x+5x^2}}{1250000}+\frac{198439\left(3+2x+5x^2\right)^{3/2}}{750000}\\ &\quad+\frac{1781669x\left(3+2x+5x^2\right)^{3/2}}{250000}-\frac{77509x^2\left(3+2x+5x^2\right)^{3/2}}{25000}-\frac{25277x^3\left(3+2x+5x^2\right)^{3/2}}{3000}\\ &\quad+\frac{989}{200}x^4\left(3+2x+5x^2\right)^{3/2}+\frac{49}{40}x^5\left(3+2x+5x^2\right)^{3/2}-\frac{17652061\operatorname{arcsinh}\left(\frac{1+5x}{\sqrt{14}}\right)}{625000\sqrt{5}}\end{aligned}$$

```
[Out] 198439/750000*(5*x^2+2*x+3)^(3/2)+1781669/250000*x*(5*x^2+2*x+3)^(3/2)-7750
9/25000*x^2*(5*x^2+2*x+3)^(3/2)-25277/3000*x^3*(5*x^2+2*x+3)^(3/2)+989/200*
x^4*(5*x^2+2*x+3)^(3/2)+49/40*x^5*(5*x^2+2*x+3)^(3/2)-17652061/3125000*arcs
inh(1/14*(1+5*x)*14^(1/2))*5^(1/2)-2521723/1250000*(1+5*x)*(5*x^2+2*x+3)^(1
/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used

$= \{1675, 654, 626, 633, 221\}$

$$\int \left(1+4x-7x^2\right)^2 \left(2+5x+x^2\right) \sqrt{3+2x+5x^2}\,dx$$

$$= -\frac{17652061 \operatorname{arcsinh}\left(\frac{5x+1}{\sqrt{14}}\right)}{625000\sqrt{5}} - \frac{77509(5x^2+2x+3)^{3/2} x^2}{25000} + \frac{1781669(5x^2+2x+3)^{3/2} x}{250000} + \frac{198439(5x^2+2x+3)^{3/2}}{750000} - \frac{2521723(5x+1)\sqrt{5x^2+2x+3}}{1250000} + \frac{49}{40}(5x^2+2x+3)^{3/2} x^5 + \frac{989}{200}(5x^2+2x+3)^{3/2} x^4 - \frac{25277(5x^2+2x+3)^{3/2} x^3}{3000}$$

```
[In] Int[(1 + 4*x - 7*x^2)^2*(2 + 5*x + x^2)*Sqrt[3 + 2*x + 5*x^2],x]
```

```
[Out] (-2521723*(1 + 5*x)*Sqrt[3 + 2*x + 5*x^2])/1250000 + (198439*(3 + 2*x + 5*x
^2)^(3/2))/750000 + (1781669*x*(3 + 2*x + 5*x^2)^(3/2))/250000 - (77509*x^2
*(3 + 2*x + 5*x^2)^(3/2))/25000 - (25277*x^3*(3 + 2*x + 5*x^2)^(3/2))/3000
+ (989*x^4*(3 + 2*x + 5*x^2)^(3/2))/200 + (49*x^5*(3 + 2*x + 5*x^2)^(3/2))/
40 - (17652061*ArcSinh[(1 + 5*x)/Sqrt[14]])/(625000*Sqrt[5])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{49}{40}x^5\left(3+2x+5x^2\right)^{3/2} \\
&\quad+\frac{1}{40}\int\sqrt{3+2x+5x^2}\left(80+840x+1800x^2-3760x^3-7935x^4+6923x^5\right)\,dx \\
&= \frac{989}{200}x^4\left(3+2x+5x^2\right)^{3/2}+\frac{49}{40}x^5\left(3+2x+5x^2\right)^{3/2} \\
&\quad+\frac{\int\sqrt{3+2x+5x^2}(2800+29400x+63000x^2-214676x^3-353878x^4)\,dx}{1400} \\
&= -\frac{25277x^3(3+2x+5x^2)^{3/2}}{3000}+\frac{989}{200}x^4\left(3+2x+5x^2\right)^{3/2}+\frac{49}{40}x^5\left(3+2x+5x^2\right)^{3/2} \\
&\quad+\frac{\int\sqrt{3+2x+5x^2}(84000+882000x+5074902x^2-3255378x^3)\,dx}{42000} \\
&= -\frac{77509x^2(3+2x+5x^2)^{3/2}}{25000}-\frac{25277x^3(3+2x+5x^2)^{3/2}}{3000}+\frac{989}{200}x^4\left(3+2x+5x^2\right)^{3/2} \\
&\quad+\frac{49}{40}x^5\left(3+2x+5x^2\right)^{3/2}+\frac{\int\sqrt{3+2x+5x^2}(2100000+41582268x+149660196x^2)\,dx}{1050000} \\
&= \frac{1781669x(3+2x+5x^2)^{3/2}}{250000}-\frac{77509x^2(3+2x+5x^2)^{3/2}}{25000}-\frac{25277x^3(3+2x+5x^2)^{3/2}}{3000} \\
&\quad+\frac{989}{200}x^4\left(3+2x+5x^2\right)^{3/2}+\frac{49}{40}x^5\left(3+2x+5x^2\right)^{3/2}+\frac{\int(-406980588+83344380x)\sqrt{3+2x+5x^2}\,d}{21000000} \\
&= \frac{198439(3+2x+5x^2)^{3/2}}{750000}+\frac{1781669x(3+2x+5x^2)^{3/2}}{250000} \\
&\quad-\frac{77509x^2(3+2x+5x^2)^{3/2}}{25000}-\frac{25277x^3(3+2x+5x^2)^{3/2}}{3000} \\
&\quad+\frac{989}{200}x^4\left(3+2x+5x^2\right)^{3/2}+\frac{49}{40}x^5\left(3+2x+5x^2\right)^{3/2}-\frac{2521723\int\sqrt{3+2x+5x^2}\,dx}{125000}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{2521723(1+5x)\sqrt{3+2x+5x^2}}{1250000} \\
&\quad + \frac{198439(3+2x+5x^2)^{3/2}}{750000} + \frac{1781669x(3+2x+5x^2)^{3/2}}{250000} \\
&\quad - \frac{77509x^2(3+2x+5x^2)^{3/2}}{25000} - \frac{25277x^3(3+2x+5x^2)^{3/2}}{3000} \\
&\quad + \frac{989}{200}x^4\left(3+2x+5x^2\right)^{3/2} + \frac{49}{40}x^5\left(3+2x+5x^2\right)^{3/2} - \frac{17652061\int\frac{1}{\sqrt{3+2x+5x^2}}\,dx}{625000} \\
&= -\frac{2521723(1+5x)\sqrt{3+2x+5x^2}}{1250000} \\
&\quad + \frac{198439(3+2x+5x^2)^{3/2}}{750000} + \frac{1781669x(3+2x+5x^2)^{3/2}}{250000} \\
&\quad - \frac{77509x^2(3+2x+5x^2)^{3/2}}{25000} - \frac{25277x^3(3+2x+5x^2)^{3/2}}{3000} \\
&\quad + \frac{989}{200}x^4\left(3+2x+5x^2\right)^{3/2} + \frac{49}{40}x^5\left(3+2x+5x^2\right)^{3/2} - \frac{\left(2521723\sqrt{\frac{7}{10}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1+\frac{x^2}{56}}}\,dx, x, 2+10x\right)}{1250000} \\
&= -\frac{2521723(1+5x)\sqrt{3+2x+5x^2}}{1250000} \\
&\quad + \frac{198439(3+2x+5x^2)^{3/2}}{750000} + \frac{1781669x(3+2x+5x^2)^{3/2}}{250000} \\
&\quad - \frac{77509x^2(3+2x+5x^2)^{3/2}}{25000} - \frac{25277x^3(3+2x+5x^2)^{3/2}}{3000} \\
&\quad + \frac{989}{200}x^4\left(3+2x+5x^2\right)^{3/2} + \frac{49}{40}x^5\left(3+2x+5x^2\right)^{3/2} - \frac{17652061\sinh^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{625000\sqrt{5}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.54

$$
\begin{aligned}
&\int\left(1+4x-7x^2\right)^2\left(2+5x+x^2\right)\sqrt{3+2x+5x^2}\,dx \\
&= \frac{\sqrt{3+2x+5x^2}(-4588584+44333650x+23531995x^2+15583725x^3-65693000x^4-107112500x^5+1019}{3750000} \\
&\quad + \frac{17652061\log\left(-1-5x+\sqrt{5}\sqrt{3+2x+5x^2}\right)}{625000\sqrt{5}}
\end{aligned}
$$

```
[In] Integrate[(1 + 4*x - 7*x^2)^2*(2 + 5*x + x^2)*Sqrt[3 + 2*x + 5*x^2],x]
```

```
[Out] (Sqrt[3 + 2*x + 5*x^2]*(-4588584 + 44333650*x + 23531995*x^2 + 15583725*x^3
 - 65693000*x^4 - 107112500*x^5 + 101906250*x^6 + 22968750*x^7))/3750000 +
(17652061*Log[-1 - 5*x + Sqrt[5]*Sqrt[3 + 2*x + 5*x^2]])/(625000*Sqrt[5])
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.39

method	result
risch	$\frac{\left(22968750 x^{7}+101906250 x^{6}-107112500 x^{5}-65693000 x^{4}+15583725 x^{3}+23531995 x^{2}+44333650 x-4588584\right) \sqrt{5 x^{2}+2 x+3}}{3750000}-\frac{17652}{}$
trager	$\left(\frac{49}{8} x^{7}+\frac{1087}{40} x^{6}-\frac{8569}{300} x^{5}-\frac{65693}{3750} x^{4}+\frac{207783}{50000} x^{3}+\frac{4706399}{750000} x^{2}+\frac{886673}{75000} x-\frac{191191}{156250}\right) \sqrt{5 x^{2}+2 x+3}+\frac{17}{}$
default	$-\frac{2521723(10 x+2) \sqrt{5 x^{2}+2 x+3}}{2500000}-\frac{17652061 \sqrt{5}\, \operatorname{arcsinh}\left(\frac{5 \sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{3125000}+\frac{198439\left(5 x^{2}+2 x+3\right)^{\frac{3}{2}}}{750000}+\frac{49 x^{5}\left(5 x^{2}+2 x+3\right)^{\frac{3}{2}}}{40}+\frac{98}{}$

```
[In] int((-7*x^2+4*x+1)^2*(x^2+5*x+2)*(5*x^2+2*x+3)^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/3750000*(22968750*x^7+101906250*x^6-107112500*x^5-65693000*x^4+15583725*x
^3+23531995*x^2+44333650*x-4588584)*(5*x^2+2*x+3)^(1/2)-17652061/3125000*5^
(1/2)*arcsinh(5/14*14^(1/2)*(x+1/5))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.52

$$
\begin{aligned}
&\int \left(1+4x-7x^{2}\right)^{2}\left(2+5x+x^{2}\right)\sqrt{3+2x+5x^{2}}\,dx \\
&= \frac{1}{3750000}\left(22968750\,x^{7}+101906250\,x^{6}-107112500\,x^{5}-65693000\,x^{4}+15583725\,x^{3}+23531995\,x^{2}+44\right. \\
&\quad +\frac{17652061}{6250000}\sqrt{5}\log\left(\sqrt{5}\sqrt{5\,x^{2}+2\,x+3}(5\,x+1)-25\,x^{2}-10\,x-8\right)
\end{aligned}
$$

```
[In] integrate((-7*x^2+4*x+1)^2*(x^2+5*x+2)*(5*x^2+2*x+3)^(1/2),x, algorithm="fr
icas")
```

```
[Out] 1/3750000*(22968750*x^7 + 101906250*x^6 - 107112500*x^5 - 65693000*x^4 + 15
583725*x^3 + 23531995*x^2 + 44333650*x - 4588584)*sqrt(5*x^2 + 2*x + 3) + 1
7652061/6250000*sqrt(5)*log(sqrt(5)*sqrt(5*x^2 + 2*x + 3)*(5*x + 1) - 25*x^
2 - 10*x - 8)
```

Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.51

$$\int \left(1+4x-7x^2\right)^2 \left(2+5x+x^2\right) \sqrt{3+2x+5x^2}\, dx = \sqrt{5x^2+2x+3} \cdot \left(\frac{49x^7}{8}+\frac{1087x^6}{40}-\frac{8569x^5}{300}-\frac{65693x^4}{3750}+\frac{207783x^3}{50000}+\frac{4706399x^2}{750000}+\frac{886673x}{75000}-\frac{191191}{156250}\right) - \frac{17652061\sqrt{5}\operatorname{asinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{3125000}$$

```
[In] integrate((-7*x**2+4*x+1)**2*(x**2+5*x+2)*(5*x**2+2*x+3)**(1/2),x)
```

```
[Out] sqrt(5*x**2 + 2*x + 3)*(49*x**7/8 + 1087*x**6/40 - 8569*x**5/300 - 65693*x*
*4/3750 + 207783*x**3/50000 + 4706399*x**2/750000 + 886673*x/75000 - 191191
/156250) - 17652061*sqrt(5)*asinh(5*sqrt(14)*(x + 1/5)/14)/3125000
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.86

$$\int \left(1+4x-7x^2\right)^2 \left(2+5x+x^2\right) \sqrt{3+2x+5x^2}\, dx = \frac{49}{40}\left(5x^2+2x+3\right)^{\frac{3}{2}}x^5 + \frac{989}{200}\left(5x^2+2x+3\right)^{\frac{3}{2}}x^4 - \frac{25277}{3000}\left(5x^2+2x+3\right)^{\frac{3}{2}}x^3 - \frac{77509}{25000}\left(5x^2+2x+3\right)^{\frac{3}{2}}x^2 + \frac{1781669}{250000}\left(5x^2+2x+3\right)^{\frac{3}{2}}x + \frac{198439}{750000}\left(5x^2+2x+3\right)^{\frac{3}{2}} - \frac{2521723}{250000}\sqrt{5x^2+2x+3}x - \frac{17652061}{3125000}\sqrt{5}\operatorname{arsinh}\left(\frac{1}{14}\sqrt{14}(5x+1)\right) - \frac{2521723}{1250000}\sqrt{5x^2+2x+3}$$

```
[In] integrate((-7*x^2+4*x+1)^2*(x^2+5*x+2)*(5*x^2+2*x+3)^(1/2),x, algorithm="ma
xima")
```

```
[Out] 49/40*(5*x^2 + 2*x + 3)^(3/2)*x^5 + 989/200*(5*x^2 + 2*x + 3)^(3/2)*x^4 - 2
5277/3000*(5*x^2 + 2*x + 3)^(3/2)*x^3 - 77509/25000*(5*x^2 + 2*x + 3)^(3/2)
*x^2 + 1781669/250000*(5*x^2 + 2*x + 3)^(3/2)*x + 198439/750000*(5*x^2 + 2*
x + 3)^(3/2) - 2521723/250000*sqrt(5*x^2 + 2*x + 3)*x - 17652061/3125000*sq
rt(5)*arcsinh(1/14*sqrt(14)*(5*x + 1)) - 2521723/1250000*sqrt(5*x^2 + 2*x +
 3)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.49

$$\int \left(1+4x-7x^2\right)^2 \left(2+5x+x^2\right) \sqrt{3+2x+5x^2}\,dx$$
$$= \frac{1}{3750000}\left(5\left(\left(5\left(10\left(25\left(15\left(245\,x+1087\right)x-17138\right)x-262772\right)x+623349\right)x+4706399\right)x+8866730\right)x\right.$$
$$+\frac{17652061}{3125000}\sqrt{5}\log\left(-\sqrt{5}\left(\sqrt{5}x-\sqrt{5\,x^2+2\,x+3}\right)-1\right)$$

```
[In] integrate((-7*x^2+4*x+1)^2*(x^2+5*x+2)*(5*x^2+2*x+3)^(1/2),x, algorithm="gi
ac")
```

```
[Out] 1/3750000*(5*((5*(10*(25*(15*(245*x + 1087)*x - 17138)*x - 262772)*x + 6233
49)*x + 4706399)*x + 8866730)*x - 4588584)*sqrt(5*x^2 + 2*x + 3) + 17652061
/3125000*sqrt(5)*log(-sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3)) - 1)
```

Mupad [B] (verification not implemented)

Time = 15.04 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.13

$$\int \left(1+4x-7x^2\right)^2 \left(2+5x+x^2\right) \sqrt{3+2x+5x^2}\,dx$$
$$= \frac{989\,x^4\,\left(5\,x^2+2\,x+3\right)^{3/2}}{200}-\frac{25277\,x^3\,\left(5\,x^2+2\,x+3\right)^{3/2}}{3000}-\frac{77509\,x^2\,\left(5\,x^2+2\,x+3\right)^{3/2}}{25000}$$
$$+\frac{49\,x^5\,\left(5\,x^2+2\,x+3\right)^{3/2}}{40}-\frac{33915049\,\sqrt{5}\,\ln\left(\sqrt{5\,x^2+2\,x+3}+\frac{\sqrt{5}\,\left(5\,x+1\right)}{5}\right)}{6250000}$$
$$-\frac{4845007\,\left(\frac{x}{2}+\frac{1}{10}\right)\,\sqrt{5\,x^2+2\,x+3}}{250000}+\frac{198439\,\sqrt{5\,x^2+2\,x+3}\,\left(200\,x^2+20\,x+108\right)}{30000000}$$
$$+\frac{1781669\,x\,\left(5\,x^2+2\,x+3\right)^{3/2}}{250000}-\frac{1389073\,\sqrt{5}\,\ln\left(2\,\sqrt{5\,x^2+2\,x+3}+\frac{\sqrt{5}\,\left(10\,x+2\right)}{5}\right)}{6250000}$$

```
[In] int((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(1/2)*(4*x - 7*x^2 + 1)^2,x)
```

```
[Out] (989*x^4*(2*x + 5*x^2 + 3)^(3/2))/200 - (25277*x^3*(2*x + 5*x^2 + 3)^(3/2))
/3000 - (77509*x^2*(2*x + 5*x^2 + 3)^(3/2))/25000 + (49*x^5*(2*x + 5*x^2 +
3)^(3/2))/40 - (33915049*5^(1/2)*log((2*x + 5*x^2 + 3)^(1/2) + (5^(1/2)*(5*
x + 1))/5))/6250000 - (4845007*(x/2 + 1/10)*(2*x + 5*x^2 + 3)^(1/2))/250000
 + (198439*(2*x + 5*x^2 + 3)^(1/2)*(20*x + 200*x^2 + 108))/30000000 + (1781
669*x*(2*x + 5*x^2 + 3)^(3/2))/250000 - (1389073*5^(1/2)*log(2*(2*x + 5*x^2
 + 3)^(1/2) + (5^(1/2)*(10*x + 2))/5))/6250000
```

3.376 $\int \left(1+4 x-7 x^2\right) \left(2+5 x+x^2\right) \sqrt{3+2 x+5 x^2} \, dx$

Optimal result

Integrand size = 33, antiderivative size = 124

$$\begin{aligned}\int \left(1+4 x-7 x^2\right) \left(2+5 x+x^2\right) \sqrt{3+2 x+5 x^2} \, dx &= -\frac{4633(1+5x)\sqrt{3+2x+5x^2}}{12500}+\frac{7819(3+2x+5x^2)^{3/2}}{7500}+\frac{2149x(3+2x+5x^2)^{3/2}}{2500}\\ &\quad -\frac{289}{250}x^2\left(3+2x+5x^2\right)^{3/2}-\frac{7}{30}x^3\left(3+2x+5x^2\right)^{3/2}-\frac{32431\text{arcsinh}\left(\frac{1+5x}{\sqrt{14}}\right)}{6250\sqrt{5}}\end{aligned}$$

```
[Out] 7819/7500*(5*x^2+2*x+3)^(3/2)+2149/2500*x*(5*x^2+2*x+3)^(3/2)-289/250*x^2*(
5*x^2+2*x+3)^(3/2)-7/30*x^3*(5*x^2+2*x+3)^(3/2)-32431/31250*arcsinh(1/14*(1
+5*x)*14^(1/2))*5^(1/2)-4633/12500*(1+5*x)*(5*x^2+2*x+3)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {1675, 654, 626, 633, 221}

$$\begin{aligned}\int \left(1+4 x-7 x^2\right) \left(2+5 x+x^2\right) \sqrt{3+2 x+5 x^2} \, dx &= -\frac{32431\text{arcsinh}\left(\frac{5x+1}{\sqrt{14}}\right)}{6250\sqrt{5}}-\frac{289}{250}\left(5x^2+2x+3\right)^{3/2}x^2+\frac{2149\left(5x^2+2x+3\right)^{3/2}x}{2500}\\ &\quad +\frac{7819\left(5x^2+2x+3\right)^{3/2}}{7500}-\frac{4633(5x+1)\sqrt{5x^2+2x+3}}{12500}-\frac{7}{30}\left(5x^2+2x+3\right)^{3/2}x^3\end{aligned}$$

```
[In] Int[(1 + 4*x - 7*x^2)*(2 + 5*x + x^2)*Sqrt[3 + 2*x + 5*x^2],x]
```

```
[Out] (-4633*(1 + 5*x)*Sqrt[3 + 2*x + 5*x^2])/12500 + (7819*(3 + 2*x + 5*x^2)^(3/
2))/7500 + (2149*x*(3 + 2*x + 5*x^2)^(3/2))/2500 - (289*x^2*(3 + 2*x + 5*x^
2)^(3/2))/250 - (7*x^3*(3 + 2*x + 5*x^2)^(3/2))/30 - (32431*ArcSinh[(1 + 5*
x)/Sqrt[14]])/(6250*Sqrt[5])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{7}{30}x^3\left(3+2x+5x^2\right)^{3/2}+\frac{1}{30}\int\sqrt{3+2x+5x^2}\left(60+390x+273x^2-867x^3\right)\,dx\\ &= -\frac{289}{250}x^2\left(3+2x+5x^2\right)^{3/2}\\ &\quad-\frac{7}{30}x^3\left(3+2x+5x^2\right)^{3/2}+\frac{1}{750}\int\sqrt{3+2x+5x^2}\left(1500+14952x+12894x^2\right)\,dx\end{aligned}$$

$$
\begin{aligned}
&= \frac{2149x(3+2x+5x^2)^{3/2}}{2500} - \frac{289}{250}x^2\left(3+2x+5x^2\right)^{3/2} \\
&\quad - \frac{7}{30}x^3\left(3+2x+5x^2\right)^{3/2} + \frac{\int(-8682+234570x)\sqrt{3+2x+5x^2}\,dx}{15000} \\
&= \frac{7819(3+2x+5x^2)^{3/2}}{7500} + \frac{2149x(3+2x+5x^2)^{3/2}}{2500} \\
&\quad - \frac{289}{250}x^2\left(3+2x+5x^2\right)^{3/2} - \frac{7}{30}x^3\left(3+2x+5x^2\right)^{3/2} - \frac{4633\int\sqrt{3+2x+5x^2}\,dx}{1250} \\
&= -\frac{4633(1+5x)\sqrt{3+2x+5x^2}}{12500} + \frac{7819(3+2x+5x^2)^{3/2}}{7500} + \frac{2149x(3+2x+5x^2)^{3/2}}{2500} \\
&\quad - \frac{289}{250}x^2\left(3+2x+5x^2\right)^{3/2} - \frac{7}{30}x^3\left(3+2x+5x^2\right)^{3/2} - \frac{32431\int\frac{1}{\sqrt{3+2x+5x^2}}\,dx}{6250} \\
&= -\frac{4633(1+5x)\sqrt{3+2x+5x^2}}{12500} + \frac{7819(3+2x+5x^2)^{3/2}}{7500} \\
&\quad + \frac{2149x(3+2x+5x^2)^{3/2}}{2500} - \frac{289}{250}x^2\left(3+2x+5x^2\right)^{3/2} \\
&\quad - \frac{7}{30}x^3\left(3+2x+5x^2\right)^{3/2} - \frac{\left(4633\sqrt{\frac{7}{10}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1+\frac{x^2}{56}}}\,dx,x,2+10x\right)}{12500} \\
&= -\frac{4633(1+5x)\sqrt{3+2x+5x^2}}{12500} + \frac{7819(3+2x+5x^2)^{3/2}}{7500} + \frac{2149x(3+2x+5x^2)^{3/2}}{2500} \\
&\quad - \frac{289}{250}x^2\left(3+2x+5x^2\right)^{3/2} - \frac{7}{30}x^3\left(3+2x+5x^2\right)^{3/2} - \frac{32431\sinh^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{6250\sqrt{5}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.64

$$
\begin{aligned}
&\int\left(1+4x-7x^2\right)\left(2+5x+x^2\right)\sqrt{3+2x+5x^2}\,dx \\
&= \frac{\sqrt{3+2x+5x^2}(103386+105400x+129895x^2+48225x^3-234250x^4-43750x^5)}{37500} \\
&\quad + \frac{32431\log\left(-1-5x+\sqrt{5}\sqrt{3+2x+5x^2}\right)}{6250\sqrt{5}}
\end{aligned}
$$

```
[In] Integrate[(1 + 4*x - 7*x^2)*(2 + 5*x + x^2)*Sqrt[3 + 2*x + 5*x^2],x]
```

```
[Out] (Sqrt[3 + 2*x + 5*x^2]*(103386 + 105400*x + 129895*x^2 + 48225*x^3 - 234250
*x^4 - 43750*x^5))/37500 + (32431*Log[-1 - 5*x + Sqrt[5]*Sqrt[3 + 2*x + 5*x
^2]])/(6250*Sqrt[5])
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.44

method	result
risch	$-\frac{(43750x^5+234250x^4-48225x^3-129895x^2-105400x-103386)\sqrt{5x^2+2x+3}}{37500}-\frac{32431\sqrt{5}\ \operatorname{arcsinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{31250}$
trager	$\left(-\frac{7}{6}x^5-\frac{937}{150}x^4+\frac{643}{500}x^3+\frac{25979}{7500}x^2+\frac{1054}{375}x+\frac{17231}{6250}\right)\sqrt{5x^2+2x+3}-$ 32431 RootOf$\left(_Z^2-5\right)$ ln$\big($5 RootO
default	$-\frac{4633(10x+2)\sqrt{5x^2+2x+3}}{25000}-\frac{32431\sqrt{5}\ \operatorname{arcsinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{31250}+\frac{7819(5x^2+2x+3)^{\frac{3}{2}}}{7500}-\frac{7x^3(5x^2+2x+3)^{\frac{3}{2}}}{30}-$ $289x^2(5x^2+$ / 250

```
[In] int((-7*x^2+4*x+1)*(x^2+5*x+2)*(5*x^2+2*x+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/37500*(43750*x^5+234250*x^4-48225*x^3-129895*x^2-105400*x-103386)*(5*x^2
+2*x+3)^(1/2)-32431/31250*5^(1/2)*arcsinh(5/14*14^(1/2)*(x+1/5))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.62

$$\int \left(1+4x-7x^2\right)\left(2+5x+x^2\right)\sqrt{3+2x+5x^2}\,dx =$$
$$-\frac{1}{37500}\left(43750\,x^5+234250\,x^4-48225\,x^3-129895\,x^2-105400\,x-103386\right)\sqrt{5\,x^2+2\,x+3}$$
$$+\frac{32431}{62500}\sqrt{5}\log\left(\sqrt{5}\sqrt{5\,x^2+2\,x+3}(5\,x+1)-25\,x^2-10\,x-8\right)$$

```
[In] integrate((-7*x^2+4*x+1)*(x^2+5*x+2)*(5*x^2+2*x+3)^(1/2),x, algorithm="fric
as")
```

```
[Out] -1/37500*(43750*x^5 + 234250*x^4 - 48225*x^3 - 129895*x^2 - 105400*x - 1033
86)*sqrt(5*x^2 + 2*x + 3) + 32431/62500*sqrt(5)*log(sqrt(5)*sqrt(5*x^2 + 2*
x + 3)*(5*x + 1) - 25*x^2 - 10*x - 8)
```

Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.57

$$\begin{aligned}&\int (1+4x-7x^2)\left(2+5x+x^2\right)\sqrt{3+2x+5x^2}\,dx\\&=\sqrt{5x^2+2x+3}\left(-\frac{7x^5}{6}-\frac{937x^4}{150}+\frac{643x^3}{500}+\frac{25979x^2}{7500}+\frac{1054x}{375}+\frac{17231}{6250}\right)\\&\quad-\frac{32431\sqrt{5}\operatorname{asinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{31250}\end{aligned}$$

```
[In] integrate((-7*x**2+4*x+1)*(x**2+5*x+2)*(5*x**2+2*x+3)**(1/2),x)
```

```
[Out] sqrt(5*x**2 + 2*x + 3)*(-7*x**5/6 - 937*x**4/150 + 643*x**3/500 + 25979*x**
2/7500 + 1054*x/375 + 17231/6250) - 32431*sqrt(5)*asinh(5*sqrt(14)*(x + 1/5
)/14)/31250
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.88

$$\begin{aligned}&\int (1+4x-7x^2)\left(2+5x+x^2\right)\sqrt{3+2x+5x^2}\,dx\\&=-\frac{7}{30}\left(5x^2+2x+3\right)^{\frac{3}{2}}x^3-\frac{289}{250}\left(5x^2+2x+3\right)^{\frac{3}{2}}x^2\\&\quad+\frac{2149}{2500}\left(5x^2+2x+3\right)^{\frac{3}{2}}x+\frac{7819}{7500}\left(5x^2+2x+3\right)^{\frac{3}{2}}-\frac{4633}{2500}\sqrt{5x^2+2x+3}x\\&\quad-\frac{32431}{31250}\sqrt{5}\operatorname{arsinh}\left(\frac{1}{14}\sqrt{14}(5x+1)\right)-\frac{4633}{12500}\sqrt{5x^2+2x+3}\end{aligned}$$

```
[In] integrate((-7*x^2+4*x+1)*(x^2+5*x+2)*(5*x^2+2*x+3)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] -7/30*(5*x^2 + 2*x + 3)^(3/2)*x^3 - 289/250*(5*x^2 + 2*x + 3)^(3/2)*x^2 + 2
149/2500*(5*x^2 + 2*x + 3)^(3/2)*x + 7819/7500*(5*x^2 + 2*x + 3)^(3/2) - 46
33/2500*sqrt(5*x^2 + 2*x + 3)*x - 32431/31250*sqrt(5)*arcsinh(1/14*sqrt(14)
*(5*x + 1)) - 4633/12500*sqrt(5*x^2 + 2*x + 3)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.58

$$\int \left(1+4x-7x^2\right)\left(2+5x+x^2\right)\sqrt{3+2x+5x^2}\,dx = -\frac{1}{37500}\left(5\left(\left(5\left(10\left(175x+937\right)x-1929\right)x-25979\right)x-21080\right)x-103386\right)\sqrt{5x^2+2x+3} + \frac{32431}{31250}\sqrt{5}\log\left(-\sqrt{5}\left(\sqrt{5}x-\sqrt{5x^2+2x+3}\right)-1\right)$$

```
[In] integrate((-7*x^2+4*x+1)*(x^2+5*x+2)*(5*x^2+2*x+3)^(1/2),x, algorithm="giac
")
```

```
[Out] -1/37500*(5*((5*(10*(175*x + 937)*x - 1929)*x - 25979)*x - 21080)*x - 10338
6)*sqrt(5*x^2 + 2*x + 3) + 32431/31250*sqrt(5)*log(-sqrt(5)*(sqrt(5)*x - sq
rt(5*x^2 + 2*x + 3)) - 1)
```

Mupad [B] (verification not implemented)

Time = 14.39 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.23

$$\int \left(1+4x-7x^2\right)\left(2+5x+x^2\right)\sqrt{3+2x+5x^2}\,dx = \frac{7819\sqrt{5x^2+2x+3}\,\left(200x^2+20x+108\right)}{300000} - \frac{7x^3\left(5x^2+2x+3\right)^{3/2}}{30} - \frac{10129\sqrt{5}\,\ln\left(\sqrt{5x^2+2x+3}+\frac{\sqrt{5}\,(5x+1)}{5}\right)}{62500} - \frac{1447\left(\frac{x}{2}+\frac{1}{10}\right)\sqrt{5x^2+2x+3}}{2500} - \frac{289x^2\left(5x^2+2x+3\right)^{3/2}}{250} + \frac{2149x\left(5x^2+2x+3\right)^{3/2}}{2500} - \frac{54733\sqrt{5}\,\ln\left(2\sqrt{5x^2+2x+3}+\frac{\sqrt{5}\,(10x+2)}{5}\right)}{62500}$$

```
[In] int((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(1/2)*(4*x - 7*x^2 + 1),x)
```

```
[Out] (7819*(2*x + 5*x^2 + 3)^(1/2)*(20*x + 200*x^2 + 108))/300000 - (7*x^3*(2*x
+ 5*x^2 + 3)^(3/2))/30 - (10129*5^(1/2)*log((2*x + 5*x^2 + 3)^(1/2) + (5^(1
/2)*(5*x + 1))/5))/62500 - (1447*(x/2 + 1/10)*(2*x + 5*x^2 + 3)^(1/2))/2500
 - (289*x^2*(2*x + 5*x^2 + 3)^(3/2))/250 + (2149*x*(2*x + 5*x^2 + 3)^(3/2))
/2500 - (54733*5^(1/2)*log(2*(2*x + 5*x^2 + 3)^(1/2) + (5^(1/2)*(10*x + 2))
/5))/62500
```

3.377 $\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{1+4x-7x^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 187

$$\begin{aligned}
&\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{1+4x-7x^2}\,dx \\
&= -\frac{1}{490}(397+35x)\sqrt{3+2x+5x^2}-\frac{8233\text{arcsinh}\left(\frac{1+5x}{\sqrt{14}}\right)}{1715\sqrt{5}} \\
&\quad -\frac{3}{343}\sqrt{\frac{1}{11}\left(497041-146555\sqrt{11}\right)}\text{arctanh}\left(\frac{23-\sqrt{11}+\left(17-5\sqrt{11}\right)x}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right) \\
&\quad +\frac{3}{343}\sqrt{\frac{1}{11}\left(497041+146555\sqrt{11}\right)}\text{arctanh}\left(\frac{23+\sqrt{11}+\left(17+5\sqrt{11}\right)x}{\sqrt{2\left(125+17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)
\end{aligned}$$

```
[Out] -8233/8575*arcsinh(1/14*(1+5*x)*14^(1/2))*5^(1/2)-1/490*(397+35*x)*(5*x^2+2
*x+3)^(1/2)-3/3773*arctanh((23+x*(17-5*11^(1/2))-11^(1/2))/(5*x^2+2*x+3)^(1
/2)/(250-34*11^(1/2))^(1/2))*(5467451-1612105*11^(1/2))^(1/2)+3/3773*arctan
h((23+11^(1/2)+x*(17+5*11^(1/2)))/(5*x^2+2*x+3)^(1/2)/(250+34*11^(1/2))^(1/
2))*(5467451+1612105*11^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1080, 1090, 633, 221, 1046, 738, 212}

$$\begin{aligned}
&\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{1+4x-7x^2}\,dx \\
&\quad = -\frac{8233\text{arcsinh}\left(\frac{5x+1}{\sqrt{14}}\right)}{1715\sqrt{5}} \\
&\qquad - \frac{3}{343}\sqrt{\frac{1}{11}\left(497041-146555\sqrt{11}\right)}\text{arctanh}\left(\frac{\left(17-5\sqrt{11}\right)x-\sqrt{11}+23}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right) \\
&\qquad + \frac{3}{343}\sqrt{\frac{1}{11}\left(497041+146555\sqrt{11}\right)}\text{arctanh}\left(\frac{\left(17+5\sqrt{11}\right)x+\sqrt{11}+23}{\sqrt{2\left(125+17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right) \\
&\qquad - \frac{1}{490}\sqrt{5x^2+2x+3}(35x+397)
\end{aligned}$$

```
[In] Int[((2 + 5*x + x^2)*Sqrt[3 + 2*x + 5*x^2])/(1 + 4*x - 7*x^2),x]
```

```
[Out] -1/490*((397 + 35*x)*Sqrt[3 + 2*x + 5*x^2]) - (8233*ArcSinh[(1 + 5*x)/Sqrt[
14]])/(1715*Sqrt[5]) - (3*Sqrt[(497041 - 146555*Sqrt[11])/11]*ArcTanh[(23 -
 Sqrt[11] + (17 - 5*Sqrt[11])*x)/(Sqrt[2*(125 - 17*Sqrt[11])]*Sqrt[3 + 2*x
+ 5*x^2])])/343 + (3*Sqrt[(497041 + 146555*Sqrt[11])/11]*ArcTanh[(23 + Sqrt
[11] + (17 + 5*Sqrt[11])*x)/(Sqrt[2*(125 + 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x
^2])])/343
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1046

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1080

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(B*c*f*(2*p
 + 2*q + 3) + C*(b*f*p - c*e*(2*p + q + 2)) + 2*c*C*f*(p + q + 1)*x)*(a + b
*x + c*x^2)^p*((d + e*x + f*x^2)^(q + 1)/(2*c*f^2*(p + q + 1)*(2*p + 2*q +
3))), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + b*x + c*x
^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[p*(b*d - a*e)*(C*(c*e - b*f)*(q + 1) -
 c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(b^2*C*d*f*p + a*c*(C*(2*d*f
- e^2*(2*p + q + 2)) + f*(B*e - 2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)
*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e
*f*p*(b^2 - 4*a*c) - b*c*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e +
2*A*f)*(2*p + 2*q + 3))))*x + (p*(c*e - b*f)*(C*(c*e - b*f)*(q + 1) - c*(C*
e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(e^
2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x^2,
x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*
q + 3, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1090

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\text{integral} = -\frac{1}{490}(397+35x)\sqrt{3+2x+5x^2} - \frac{1}{490}\int \frac{-3442-13408x-16466x^2}{(1+4x-7x^2)\sqrt{3+2x+5x^2}}\,dx$$

$$= -\frac{1}{490}(397+35x)\sqrt{3+2x+5x^2} + \frac{\int \frac{40560+159720x}{(1+4x-7x^2)\sqrt{3+2x+5x^2}}\,dx}{3430} - \frac{8233\int \frac{1}{\sqrt{3+2x+5x^2}}\,dx}{1715}$$

$$= -\frac{1}{490}(397+35x)\sqrt{3+2x+5x^2} - \frac{8233\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{56}}}\,dx, x, 2+10x\right)}{3430\sqrt{70}} + \frac{\left(12\left(14641-5028\sqrt{11}\right)\right)\int \frac{1}{\left(4-2\sqrt{11}-14x\right)\sqrt{3+2x+5x^2}}\,dx}{3773} + \frac{\left(12\left(14641+5028\sqrt{11}\right)\right)\int \frac{1}{\left(4+2\sqrt{11}-14x\right)\sqrt{3+2x+5x^2}}\,dx}{3773}$$

$$= -\frac{1}{490}(397+35x)\sqrt{3+2x+5x^2} - \frac{8233\sinh^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{1715\sqrt{5}} - \frac{\left(24\left(14641-5028\sqrt{11}\right)\right)\text{Subst}\biggl(\int \frac{1}{2352+112\left(4-2\sqrt{11}\right)+20\left(4-2\sqrt{11}\right)^2-x^2}\,dx, x, \frac{-84-2\left(4-2\sqrt{11}\right)-\bigl(28+10\bigl(4}{\sqrt{3+2x+5x^2}}}{3773} - \frac{\left(24\left(14641+5028\sqrt{11}\right)\right)\text{Subst}\biggl(\int \frac{1}{2352+112\left(4+2\sqrt{11}\right)+20\left(4+2\sqrt{11}\right)^2-x^2}\,dx, x, \frac{-84-2\left(4+2\sqrt{11}\right)-\bigl(28+10\bigl(4}{\sqrt{3+2x+5x^2}}}{3773}$$

$$= -\frac{1}{490}(397+35x)\sqrt{3+2x+5x^2} - \frac{8233\sinh^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{1715\sqrt{5}} - \frac{3\sqrt{5467451-1612105\sqrt{11}}\tanh^{-1}\left(\frac{23-\sqrt{11}+\left(17-5\sqrt{11}\right)x}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)}{3773} + \frac{3\sqrt{5467451+1612105\sqrt{11}}\tanh^{-1}\left(\frac{23+\sqrt{11}+\left(17+5\sqrt{11}\right)x}{\sqrt{2\left(125+17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)}{3773}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.35 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.25

$$\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{1+4x-7x^2}\,dx$$
$$=\frac{1}{490}(-397-35x)\sqrt{3+2x+5x^2}+\frac{8233\log\left(-1-5x+\sqrt{5}\sqrt{3+2x+5x^2}\right)}{1715\sqrt{5}}$$
$$+\frac{6}{343}\text{RootSum}\Big[83-16\sqrt{5}\#1-70\#1^2+8\sqrt{5}\#1^3$$
$$+7\#1^4\&,\frac{3317\log\left(-\sqrt{5}x+\sqrt{3+2x+5x^2}-\#1\right)+676\sqrt{5}\log\left(-\sqrt{5}x+\sqrt{3+2x+5x^2}-\#1\right)\#1-13}{-4\sqrt{5}-35\#1+6\sqrt{5}\#1^2+7\#1^3}$$

```
[In] Integrate[((2 + 5*x + x^2)*Sqrt[3 + 2*x + 5*x^2])/(1 + 4*x - 7*x^2),x]
```

```
[Out] ((-397 - 35*x)*Sqrt[3 + 2*x + 5*x^2])/490 + (8233*Log[-1 - 5*x + Sqrt[5]*Sq
rt[3 + 2*x + 5*x^2]])/(1715*Sqrt[5]) + (6*RootSum[83 - 16*Sqrt[5]*#1 - 70*#
1^2 + 8*Sqrt[5]*#1^3 + 7*#1^4 & , (3317*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5
*x^2] - #1] + 676*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1
 - 1331*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1^2)/(-4*Sqrt[5] -
35*#1 + 6*Sqrt[5]*#1^2 + 7*#1^3) & ])/343
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.19

method	result
risch	$-\frac{(397+35x)\sqrt{5x^2+2x+3}}{490}-\frac{8233\sqrt{5}\,\text{arcsinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{8575}+\frac{6\left(-5028+1331\sqrt{11}\right)\sqrt{11}\,\text{arctanh}\left(\frac{250-34\sqrt{11}+}{\sqrt{250-34\sqrt{11}}\sqrt{245\left(x-\frac{2}{7}+\frac{\sqrt{11}}{7}\right.}}\right.}{3773\sqrt{250-34\sqrt{11}}}$
default	$-\frac{(10x+2)\sqrt{5x^2+2x+3}}{140}-\frac{\sqrt{5}\,\text{arcsinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{25}-\frac{3\left(-61+13\sqrt{11}\right)\sqrt{11}\left(\frac{\sqrt{245\left(x-\frac{2}{7}+\frac{\sqrt{11}}{7}\right)^2+49\left(\frac{34}{7}-\frac{10\sqrt{11}}{7}\right)\left(x-\frac{2}{7}+\frac{\sqrt{11}}{7}\right)}}{49}\right.}{}$
trager	$\left(-\frac{397}{490}-\frac{x}{14}\right)\sqrt{5x^2+2x+3}+\frac{8233\,\text{RootOf}\left(\_Z^2-5\right)\ln\left(-5\,\text{RootOf}\left(\_Z^2-5\right)x+5\sqrt{5x^2+2x+3}-\text{RootOf}\left(\_Z^2-5\right)\right)}{8575}$

```
[In] int((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/490*(397+35*x)*(5*x^2+2*x+3)^(1/2)-8233/8575*5^(1/2)*arcsinh(5/14*14^(1/
2)*(x+1/5))+6/3773*(-5028+1331*11^(1/2))*11^(1/2)/(250-34*11^(1/2))^(1/2)*a
rctanh(49/2*(500/49-68/49*11^(1/2)+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2)
))/(250-34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/
2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2))+6/3773*(5028+1331*11^(1/2)
)*11^(1/2)/(250+34*11^(1/2))^(1/2)*arctanh(49/2*(500/49+68/49*11^(1/2)+(34/
7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2)))/(250+34*11^(1/2))^(1/2)/(245*(x-2/7-
1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2
))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 304 vs. $2(133) = 266$.

Time = 0.27 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.63

$$\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{1+4x-7x^2}\,dx$$

$$= \frac{3}{7546}\sqrt{11}\sqrt{146555\sqrt{11}+497041}\log\left(\frac{6\left(\sqrt{5x^2+2x+3}\sqrt{146555\sqrt{11}+497041}(87\sqrt{11}-265)+651\right.}{x}\right.$$

$$-\frac{3}{7546}\sqrt{11}\sqrt{146555\sqrt{11}+497041}\log\left(-\frac{6\left(\sqrt{5x^2+2x+3}\sqrt{146555\sqrt{11}+497041}(87\sqrt{11}-265)-\right.}{x}\right.$$

$$-\frac{1}{15092}\sqrt{11}\sqrt{-5275980\sqrt{11}+17893476}\log\left(-\frac{\sqrt{5x^2+2x+3}(87\sqrt{11}+265)\sqrt{-5275980\sqrt{11}+17}}{x}\right.$$

$$+\frac{1}{15092}\sqrt{11}\sqrt{-5275980\sqrt{11}+17893476}\log\left(\frac{\sqrt{5x^2+2x+3}(87\sqrt{11}+265)\sqrt{-5275980\sqrt{11}+1789}}{x}\right.$$

$$-\frac{1}{490}\sqrt{5x^2+2x+3}(35x+397)$$

$$+\frac{8233}{17150}\sqrt{5}\log\left(\sqrt{5}\sqrt{5x^2+2x+3}(5x+1)-25x^2-10x-8\right)$$

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1),x, algorithm="fric
as")
```

```
[Out] 3/7546*sqrt(11)*sqrt(146555*sqrt(11) + 497041)*log(6*(sqrt(5*x^2 + 2*x + 3)
*sqrt(146555*sqrt(11) + 497041)*(87*sqrt(11) - 265) + 6517*sqrt(11)*(x + 3)
 + 19551*x - 32585)/x) - 3/7546*sqrt(11)*sqrt(146555*sqrt(11) + 497041)*log
(-6*(sqrt(5*x^2 + 2*x + 3)*sqrt(146555*sqrt(11) + 497041)*(87*sqrt(11) - 26
5) - 6517*sqrt(11)*(x + 3) - 19551*x + 32585)/x) - 1/15092*sqrt(11)*sqrt(-5
275980*sqrt(11) + 17893476)*log(-(sqrt(5*x^2 + 2*x + 3)*(87*sqrt(11) + 265)
*sqrt(-5275980*sqrt(11) + 17893476) + 39102*sqrt(11)*(x + 3) - 117306*x + 1
95510)/x) + 1/15092*sqrt(11)*sqrt(-5275980*sqrt(11) + 17893476)*log((sqrt(5
```

```
*x^2 + 2*x + 3)*(87*sqrt(11) + 265)*sqrt(-5275980*sqrt(11) + 17893476) - 39
102*sqrt(11)*(x + 3) + 117306*x - 195510)/x) - 1/490*sqrt(5*x^2 + 2*x + 3)*
(35*x + 397) + 8233/17150*sqrt(5)*log(sqrt(5)*sqrt(5*x^2 + 2*x + 3)*(5*x +
1) - 25*x^2 - 10*x - 8)
```

Sympy [F]

$$\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{1+4x-7x^2}\,dx = -\int \frac{2\sqrt{5x^2+2x+3}}{7x^2-4x-1}\,dx - \int \frac{5x\sqrt{5x^2+2x+3}}{7x^2-4x-1}\,dx - \int \frac{x^2\sqrt{5x^2+2x+3}}{7x^2-4x-1}\,dx$$

```
[In] integrate((x**2+5*x+2)*(5*x**2+2*x+3)**(1/2)/(-7*x**2+4*x+1),x)
```

```
[Out] -Integral(2*sqrt(5*x**2 + 2*x + 3)/(7*x**2 - 4*x - 1), x) - Integral(5*x*sq
rt(5*x**2 + 2*x + 3)/(7*x**2 - 4*x - 1), x) - Integral(x**2*sqrt(5*x**2 + 2
*x + 3)/(7*x**2 - 4*x - 1), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 500 vs. $2(133) = 266$.

Time $= 0.33$ (sec) , antiderivative size $= 500$, normalized size of antiderivative $= 2.67$

$$\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{1+4x-7x^2}\,dx$$

$$= \frac{1}{188650}\sqrt{11}\Bigg(975\sqrt{11}\sqrt{2}\sqrt{17\sqrt{11}+125}\,\operatorname{arsinh}\Bigg(\frac{5\sqrt{11}\sqrt{7}\sqrt{2}x}{7\,|14\,x-2\sqrt{11}-4|}+\frac{17\sqrt{7}\sqrt{2}x}{7\,|14\,x-2\sqrt{11}-4|}+$$

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1),x, algorithm="maxi
ma")
```

```
[Out] 1/188650*sqrt(11)*(975*sqrt(11)*sqrt(2)*sqrt(17*sqrt(11) + 125)*arcsinh(5/7
*sqrt(11)*sqrt(7)*sqrt(2)*x/abs(14*x - 2*sqrt(11) - 4) + 17/7*sqrt(7)*sqrt(
2)*x/abs(14*x - 2*sqrt(11) - 4) + 1/7*sqrt(11)*sqrt(7)*sqrt(2)/abs(14*x - 2
*sqrt(11) - 4) + 23/7*sqrt(7)*sqrt(2)/abs(14*x - 2*sqrt(11) - 4)) - 1225*sq
rt(11)*sqrt(5*x^2 + 2*x + 3)*x - 16466*sqrt(11)*sqrt(5)*arcsinh(5/14*sqrt(7
)*sqrt(2)*x + 1/14*sqrt(7)*sqrt(2)) - 6825*sqrt(11)*sqrt(-34/49*sqrt(11) +
250/49)*arcsinh(5/7*sqrt(11)*sqrt(7)*sqrt(2)*x/abs(14*x + 2*sqrt(11) - 4) -
 17/7*sqrt(7)*sqrt(2)*x/abs(14*x + 2*sqrt(11) - 4) + 1/7*sqrt(11)*sqrt(7)*s
qrt(2)/abs(14*x + 2*sqrt(11) - 4) - 23/7*sqrt(7)*sqrt(2)/abs(14*x + 2*sqrt(
11) - 4)) + 4575*sqrt(2)*sqrt(17*sqrt(11) + 125)*arcsinh(5/7*sqrt(11)*sqrt(
7)*sqrt(2)*x/abs(14*x - 2*sqrt(11) - 4) + 17/7*sqrt(7)*sqrt(2)*x/abs(14*x -
```

```
2*sqrt(11) - 4) + 1/7*sqrt(11)*sqrt(7)*sqrt(2)/abs(14*x - 2*sqrt(11) - 4)
+ 23/7*sqrt(7)*sqrt(2)/abs(14*x - 2*sqrt(11) - 4)) + 32025*sqrt(-34/49*sqrt
(11) + 250/49)*arcsinh(5/7*sqrt(11)*sqrt(7)*sqrt(2)*x/abs(14*x + 2*sqrt(11)
 - 4) - 17/7*sqrt(7)*sqrt(2)*x/abs(14*x + 2*sqrt(11) - 4) + 1/7*sqrt(11)*sq
rt(7)*sqrt(2)/abs(14*x + 2*sqrt(11) - 4) - 23/7*sqrt(7)*sqrt(2)/abs(14*x +
2*sqrt(11) - 4)) - 13895*sqrt(11)*sqrt(5*x^2 + 2*x + 3))
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.77

$$\begin{aligned}\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{1+4x-7x^2}\,dx = & -\frac{1}{490}\sqrt{5\,x^2+2\,x+3}(35\,x+397) \\ & +\frac{8233}{8575}\sqrt{5}\log\left(-5\sqrt{5}x-\sqrt{5}+5\sqrt{5\,x^2+2\,x+3}\right) \\ & +2.61475869687464\log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}\right. \\ & \left.+4.41924736459000\right) \\ & -0.276245077121866\log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}\right. \\ & \left.+1.25295163054000\right) \\ & -2.61475869687464\log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}\right. \\ & \left.-1.02258038113000\right) \\ & +0.276245077121866\log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}\right. \\ & \left.-2.09411235400000\right)\end{aligned}$$

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1),x, algorithm="giac
")
```

```
[Out] -1/490*sqrt(5*x^2 + 2*x + 3)*(35*x + 397) + 8233/8575*sqrt(5)*log(-5*sqrt(5
)*x - sqrt(5) + 5*sqrt(5*x^2 + 2*x + 3)) + 2.61475869687464*log(-sqrt(5)*x
+ sqrt(5*x^2 + 2*x + 3) + 4.41924736459000) - 0.276245077121866*log(-sqrt(5
)*x + sqrt(5*x^2 + 2*x + 3) + 1.25295163054000) - 2.61475869687464*log(-sqr
t(5)*x + sqrt(5*x^2 + 2*x + 3) - 1.02258038113000) + 0.276245077121866*log(
-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) - 2.09411235400000)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{1+4x-7x^2}\,dx=\int \frac{\left(x^2+5\,x+2\right)\,\sqrt{5\,x^2+2\,x+3}}{-7\,x^2+4\,x+1}\,dx$$

```
[In] int(((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(1/2))/(4*x - 7*x^2 + 1),x)
[Out] int(((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(1/2))/(4*x - 7*x^2 + 1), x)
```

3.378 $\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{(1+4x-7x^2)^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 199

$$\begin{aligned}\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{(1+4x-7x^2)^2}\,dx &= \frac{3(3+61x)\sqrt{3+2x+5x^2}}{154\,(1+4x-7x^2)} + \frac{1}{49}\sqrt{5}\text{arcsinh}\left(\frac{1+5x}{\sqrt{14}}\right) \\ &\quad - \frac{\sqrt{\frac{325022311+39132731\sqrt{11}}{1397}}\text{arctanh}\left(\frac{23-\sqrt{11}+\left(17-5\sqrt{11}\right)x}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)}{2156} \\ &\quad + \frac{\sqrt{\frac{325022311-39132731\sqrt{11}}{1397}}\text{arctanh}\left(\frac{23+\sqrt{11}+\left(17+5\sqrt{11}\right)x}{\sqrt{2\left(125+17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)}{2156}\end{aligned}$$

```
[Out] 1/49*arcsinh(1/14*(1+5*x)*14^(1/2))*5^(1/2)+3/154*(3+61*x)*(5*x^2+2*x+3)^(1
/2)/(-7*x^2+4*x+1)+1/3011932*arctanh((23+11^(1/2)+x*(17+5*11^(1/2)))/(5*x^2
+2*x+3)^(1/2)/(250+34*11^(1/2))^(1/2))*(454056168467-54668425207*11^(1/2))^
(1/2)-1/3011932*arctanh((23+x*(17-5*11^(1/2))-11^(1/2))/(5*x^2+2*x+3)^(1/2)
/(250-34*11^(1/2))^(1/2))*(454056168467+54668425207*11^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1068, 1090, 633, 221, 1046, 738, 212}

$$\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{(1+4x-7x^2)^2}\,dx$$

$$= \frac{1}{49}\sqrt{5}\text{arcsinh}\left(\frac{5x+1}{\sqrt{14}}\right) - \frac{\sqrt{\frac{325022311+39132731\sqrt{11}}{1397}}\text{arctanh}\left(\frac{\left(17-5\sqrt{11}\right)x-\sqrt{11}+23}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right)}{2156}$$

$$+ \frac{\sqrt{\frac{325022311-39132731\sqrt{11}}{1397}}\text{arctanh}\left(\frac{\left(17+5\sqrt{11}\right)x+\sqrt{11}+23}{\sqrt{2\left(125+17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right)}{2156} + \frac{3\sqrt{5x^2+2x+3}(61x+3)}{154\,(-7x^2+4x+1)}$$

```
[In] Int[((2 + 5*x + x^2)*Sqrt[3 + 2*x + 5*x^2])/(1 + 4*x - 7*x^2)^2,x]
```

```
[Out] (3*(3 + 61*x)*Sqrt[3 + 2*x + 5*x^2])/(154*(1 + 4*x - 7*x^2)) + (Sqrt[5]*Arc
Sinh[(1 + 5*x)/Sqrt[14]])/49 - (Sqrt[(325022311 + 39132731*Sqrt[11])/1397]*
ArcTanh[(23 - Sqrt[11] + (17 - 5*Sqrt[11])*x)/(Sqrt[2*(125 - 17*Sqrt[11])]*
Sqrt[3 + 2*x + 5*x^2])])/2156 + (Sqrt[(325022311 - 39132731*Sqrt[11])/1397]
*ArcTanh[(23 + Sqrt[11] + (17 + 5*Sqrt[11])*x)/(Sqrt[2*(125 + 17*Sqrt[11])]
*Sqrt[3 + 2*x + 5*x^2])])/2156
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1046

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1068

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(A*b*c - 2*
a*B*c + a*b*C - (c*(b*B - 2*A*c) - C*(b^2 - 2*a*c))*x)*(a + b*x + c*x^2)^(p
 + 1)*((d + e*x + f*x^2)^q/(c*(b^2 - 4*a*c)*(p + 1))), x] - Dist[1/(c*(b^2
 - 4*a*c)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*
Simp[e*q*(A*b*c - 2*a*B*c + a*b*C) - d*(c*(b*B - 2*A*c)*(2*p + 3) + C*(2*a*
c - b^2*(p + 2))) + (2*f*q*(A*b*c - 2*a*B*c + a*b*C) - e*(c*(b*B - 2*A*c)*(
2*p + q + 3) + C*(2*a*c*(q + 1) - b^2*(p + q + 2))))*x - f*(c*(b*B - 2*A*c)
*(2*p + 2*q + 3) + C*(2*a*c*(2*q + 1) - b^2*(p + 2*q + 2)))*x^2, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2
 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] &&  !IGtQ[q, 0]
```

Rule 1090

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{3(3+61x)\sqrt{3+2x+5x^2}}{154\,(1+4x-7x^2)}-\frac{1}{308}\int\frac{-948-188x+220x^2}{(1+4x-7x^2)\sqrt{3+2x+5x^2}}\,dx\\ &= \frac{3(3+61x)\sqrt{3+2x+5x^2}}{154\,(1+4x-7x^2)}+\frac{\int\frac{6416+436x}{(1+4x-7x^2)\sqrt{3+2x+5x^2}}\,dx}{2156}+\frac{5}{49}\int\frac{1}{\sqrt{3+2x+5x^2}}\,dx\end{aligned}$$

$$= \frac{3(3+61x)\sqrt{3+2x+5x^2}}{154\left(1+4x-7x^2\right)} + \frac{1}{98}\sqrt{\frac{5}{14}}\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{56}}}\,dx, x, 2+10x\right)$$

$$+ \frac{\left(1199-11446\sqrt{11}\right)\int \frac{1}{\left(4-2\sqrt{11}-14x\right)\sqrt{3+2x+5x^2}}\,dx}{5929}$$

$$+ \frac{\left(1199+11446\sqrt{11}\right)\int \frac{1}{\left(4+2\sqrt{11}-14x\right)\sqrt{3+2x+5x^2}}\,dx}{5929}$$

$$= \frac{3(3+61x)\sqrt{3+2x+5x^2}}{154\left(1+4x-7x^2\right)} + \frac{1}{49}\sqrt{5}\sinh^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)$$

$$- \frac{\left(2\left(1199-11446\sqrt{11}\right)\right)\text{Subst}\left(\int \frac{1}{2352+112\left(4-2\sqrt{11}\right)+20\left(4-2\sqrt{11}\right)^2-x^2}\,dx, x, \frac{-84-2\left(4-2\sqrt{11}\right)-\left(28+10\left(4-2\right.\right.}{\sqrt{3+2x+5x^2}}\right.}{5929}$$

$$- \frac{\left(2\left(1199+11446\sqrt{11}\right)\right)\text{Subst}\left(\int \frac{1}{2352+112\left(4+2\sqrt{11}\right)+20\left(4+2\sqrt{11}\right)^2-x^2}\,dx, x, \frac{-84-2\left(4+2\sqrt{11}\right)-\left(28+10\left(4+2\right.\right.}{\sqrt{3+2x+5x^2}}\right.}{5929}$$

$$= \frac{3(3+61x)\sqrt{3+2x+5x^2}}{154\left(1+4x-7x^2\right)} + \frac{1}{49}\sqrt{5}\sinh^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)$$

$$- \frac{\sqrt{\frac{325022311+39132731\sqrt{11}}{1397}}\tanh^{-1}\left(\frac{23-\sqrt{11}+\left(17-5\sqrt{11}\right)x}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)}{2156}$$

$$+ \frac{\sqrt{\frac{325022311-39132731\sqrt{11}}{1397}}\tanh^{-1}\left(\frac{23+\sqrt{11}+\left(17+5\sqrt{11}\right)x}{\sqrt{2\left(125+17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)}{2156}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.53 (sec) , antiderivative size = 427, normalized size of antiderivative = 2.15

$$\int \frac{\left(2+5x+x^2\right)\sqrt{3+2x+5x^2}}{\left(1+4x-7x^2\right)^2}\,dx$$

$$= \frac{-\frac{5145(3+61x)\sqrt{3+2x+5x^2}}{-1-4x+7x^2} - 5390\sqrt{5}\log\left(-1-5x+\sqrt{5}\sqrt{3+2x+5x^2}\right) - 55\text{RootSum}\left[83-16\sqrt{5}\#1-70\#1\right.}{}$$

```
[In] Integrate[((2 + 5*x + x^2)*Sqrt[3 + 2*x + 5*x^2])/(1 + 4*x - 7*x^2)^2,x]
```

```
[Out] ((-5145*(3 + 61*x)*Sqrt[3 + 2*x + 5*x^2])/(-1 - 4*x + 7*x^2) - 5390*Sqrt[5]
*Log[-1 - 5*x + Sqrt[5]*Sqrt[3 + 2*x + 5*x^2]] - 55*RootSum[83 - 16*Sqrt[5]
```

```
*#1 - 70*#1^2 + 8*Sqrt[5]*#1^3 + 7*#1^4 & , (-314239*Log[-(Sqrt[5]*x) + Sqr
t[3 + 2*x + 5*x^2] - #1] + 28462*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x +
5*x^2] - #1]*#1 - 11221*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1^2
)/(-4*Sqrt[5] - 35*#1 + 6*Sqrt[5]*#1^2 + 7*#1^3) & ] - 6*Sqrt[5]*RootSum[83
 - 16*Sqrt[5]*#1 - 70*#1^2 + 8*Sqrt[5]*#1^3 + 7*#1^4 & , (599633*Sqrt[5]*Lo
g[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1] - 391895*Log[-(Sqrt[5]*x) + Sq
rt[3 + 2*x + 5*x^2] - #1]*#1 + 21462*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*
x + 5*x^2] - #1]*#1^2)/(-4*Sqrt[5] - 35*#1 + 6*Sqrt[5]*#1^2 + 7*#1^3) & ])/
264110
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.18

method	result
risch	$-\frac{3(3+61x)\sqrt{5x^2+2x+3}}{154(7x^2-4x-1)}+\frac{\sqrt{5}\,\operatorname{arcsinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{49}+\frac{\left(-11446+109\sqrt{11}\right)\sqrt{11}\,\operatorname{arctanh}\left(\frac{250-34\sqrt{11}+\frac{49\left(\frac{34}{7}\right.}{}}{\sqrt{250-34\sqrt{11}}\sqrt{245\left(x-\frac{2}{7}+\frac{\sqrt{11}}{7}\right)^2+4}}\right.}{11858\sqrt{250-34\sqrt{11}}}$
trager	Expression too large to display
default	Expression too large to display

```
[In] int((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1)^2,x,method=_RETURNVERBOS
E)
```

```
[Out] -3/154*(3+61*x)/(7*x^2-4*x-1)*(5*x^2+2*x+3)^(1/2)+1/49*5^(1/2)*arcsinh(5/14
*14^(1/2)*(x+1/5))+1/11858*(-11446+109*11^(1/2))*11^(1/2)/(250-34*11^(1/2))
^(1/2)*arctanh(49/2*(500/49-68/49*11^(1/2)+(34/7-10/7*11^(1/2))*(x-2/7+1/7*
11^(1/2)))/(250-34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/
7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2))+1/11858*11^(1/2)*(
11446+109*11^(1/2))/(250+34*11^(1/2))^(1/2)*arctanh(49/2*(500/49+68/49*11^(
1/2)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2)))/(250+34*11^(1/2))^(1/2)/(24
5*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+3
4*11^(1/2))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 378 vs. $2(145) = 290$.

Time = 0.26 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.90

$$\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{(1+4x-7x^2)^2}\,dx =$$

$$-\frac{\sqrt{1397}(7\,x^2-4\,x-1)\sqrt{39132731\,\sqrt{11}+325022311}\log\left(-\frac{\sqrt{1397}\sqrt{5\,x^2+2\,x+3}\sqrt{39132731\,\sqrt{11}+325022311}\left(16943\right.}{}\right.}{}$$

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1)^2,x, algorithm="fr
icas")
```

```
[Out] -1/6023864*(sqrt(1397)*(7*x^2 - 4*x - 1)*sqrt(39132731*sqrt(11) + 325022311
)*log(-(sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*sqrt(39132731*sqrt(11) + 325022311
)*(16943*sqrt(11) + 235367) + 26119953475*sqrt(11)*(x + 3) - 78359860425*x
+ 130599767375)/x) - sqrt(1397)*(7*x^2 - 4*x - 1)*sqrt(39132731*sqrt(11) +
325022311)*log((sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*sqrt(39132731*sqrt(11) + 3
25022311)*(16943*sqrt(11) + 235367) - 26119953475*sqrt(11)*(x + 3) + 783598
60425*x - 130599767375)/x) + sqrt(1397)*(7*x^2 - 4*x - 1)*sqrt(-39132731*sq
rt(11) + 325022311)*log((sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*(16943*sqrt(11) -
 235367)*sqrt(-39132731*sqrt(11) + 325022311) + 26119953475*sqrt(11)*(x + 3
) + 78359860425*x - 130599767375)/x) - sqrt(1397)*(7*x^2 - 4*x - 1)*sqrt(-3
9132731*sqrt(11) + 325022311)*log(-(sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*(16943
*sqrt(11) - 235367)*sqrt(-39132731*sqrt(11) + 325022311) - 26119953475*sqrt
(11)*(x + 3) - 78359860425*x + 130599767375)/x) - 61468*sqrt(5)*(7*x^2 - 4*
x - 1)*log(-sqrt(5)*sqrt(5*x^2 + 2*x + 3)*(5*x + 1) - 25*x^2 - 10*x - 8) +
117348*sqrt(5*x^2 + 2*x + 3)*(61*x + 3))/(7*x^2 - 4*x - 1)
```

Sympy [F]

$$\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{(1+4x-7x^2)^2}\,dx = \int \frac{(x^2+5x+2)\sqrt{5x^2+2x+3}}{(7x^2-4x-1)^2}\,dx$$

```
[In] integrate((x**2+5*x+2)*(5*x**2+2*x+3)**(1/2)/(-7*x**2+4*x+1)**2,x)
```

```
[Out] Integral((x**2 + 5*x + 2)*sqrt(5*x**2 + 2*x + 3)/(7*x**2 - 4*x - 1)**2, x)
```

Maxima [F]

$$\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{(1+4x-7x^2)^2}\,dx = \int \frac{\sqrt{5\,x^2+2\,x+3}(x^2+5\,x+2)}{(7\,x^2-4\,x-1)^2}\,dx$$

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1)^2,x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(5*x^2 + 2*x + 3)*(x^2 + 5*x + 2)/(7*x^2 - 4*x - 1)^2, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{(1+4x-7x^2)^2}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1)^2,x, algorithm="gi
ac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{184473632,[8]%%%}+%%%{%%{[421654016,0]:[1,0,-5]%%},[7]%%%}+%%
%{-248
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{(1+4x-7x^2)^2}\,dx=\int \frac{\left(x^2+5\,x+2\right)\,\sqrt{5\,x^2+2\,x+3}}{{\left(-7\,x^2+4\,x+1\right)}^2}\,dx$$

```
[In] int(((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(1/2))/(4*x - 7*x^2 + 1)^2,x)
```

```
[Out] int(((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(1/2))/(4*x - 7*x^2 + 1)^2, x)
```

3.379 $\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{(1+4x-7x^2)^3}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 213

$$\begin{aligned}
&\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{(1+4x-7x^2)^3}\,dx \\
&= \frac{3(3+61x)\sqrt{3+2x+5x^2}}{308\,(1+4x-7x^2)^2} - \frac{(272941-813113x)\sqrt{3+2x+5x^2}}{1721104\,(1+4x-7x^2)} \\
&\quad - \frac{\sqrt{\frac{6492253020949-11879169071\sqrt{11}}{1397}}\operatorname{arctanh}\left(\frac{23-\sqrt{11}+\left(17-5\sqrt{11}\right)x}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)}{491744} \\
&\quad + \frac{\sqrt{\frac{6492253020949+11879169071\sqrt{11}}{1397}}\operatorname{arctanh}\left(\frac{23+\sqrt{11}+\left(17+5\sqrt{11}\right)x}{\sqrt{2\left(125+17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)}{491744}
\end{aligned}$$

```
[Out] 3/308*(3+61*x)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1)^2-1/1721104*(272941-81311
3*x)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1)-1/686966368*arctanh((23+x*(17-5*11^
(1/2))-11^(1/2))/(5*x^2+2*x+3)^(1/2)/(250-34*11^(1/2))^(1/2))*(906967747026
5753-16595199192187*11^(1/2))^(1/2)+1/686966368*arctanh((23+11^(1/2)+x*(17+
5*11^(1/2)))/(5*x^2+2*x+3)^(1/2)/(250+34*11^(1/2))^(1/2))*(9069677470265753
+16595199192187*11^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1068, 1074, 1046, 738, 212}

$$\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{(1+4x-7x^2)^3}\,dx$$
$$= -\frac{\sqrt{\frac{6492253020949-11879169071\sqrt{11}}{1397}}\operatorname{arctanh}\left(\frac{\left(17-5\sqrt{11}\right)x-\sqrt{11}+23}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right)}{491744}$$
$$+\frac{\sqrt{\frac{6492253020949+11879169071\sqrt{11}}{1397}}\operatorname{arctanh}\left(\frac{\left(17+5\sqrt{11}\right)x+\sqrt{11}+23}{\sqrt{2\left(125+17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right)}{491744}$$
$$-\frac{\sqrt{5x^2+2x+3}(272941-813113x)}{1721104\,(-7x^2+4x+1)}+\frac{3(61x+3)\sqrt{5x^2+2x+3}}{308\,(-7x^2+4x+1)^2}$$

```
[In] Int[((2 + 5*x + x^2)*Sqrt[3 + 2*x + 5*x^2])/(1 + 4*x - 7*x^2)^3,x]
```

```
[Out] (3*(3 + 61*x)*Sqrt[3 + 2*x + 5*x^2])/(308*(1 + 4*x - 7*x^2)^2) - ((272941 -
 813113*x)*Sqrt[3 + 2*x + 5*x^2])/(1721104*(1 + 4*x - 7*x^2)) - (Sqrt[(6492
253020949 - 11879169071*Sqrt[11])/1397]*ArcTanh[(23 - Sqrt[11] + (17 - 5*Sq
rt[11])*x)/(Sqrt[2*(125 - 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x^2])])/491744 + (
Sqrt[(6492253020949 + 11879169071*Sqrt[11])/1397]*ArcTanh[(23 + Sqrt[11] +
(17 + 5*Sqrt[11])*x)/(Sqrt[2*(125 + 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x^2])])/
491744
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1046

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
```

```
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1068

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(A*b*c - 2*
a*B*c + a*b*C - (c*(b*B - 2*A*c) - C*(b^2 - 2*a*c))*x)*(a + b*x + c*x^2)^(p
 + 1)*((d + e*x + f*x^2)^q/(c*(b^2 - 4*a*c)*(p + 1))), x] - Dist[1/(c*(b^2
- 4*a*c)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*
Simp[e*q*(A*b*c - 2*a*B*c + a*b*C) - d*(c*(b*B - 2*A*c)*(2*p + 3) + C*(2*a*
c - b^2*(p + 2))) + (2*f*q*(A*b*c - 2*a*B*c + a*b*C) - e*(c*(b*B - 2*A*c)*(
2*p + q + 3) + C*(2*a*c*(q + 1) - b^2*(p + q + 2))))*x - f*(c*(b*B - 2*A*c)
*(2*p + 2*q + 3) + C*(2*a*c*(2*q + 1) - b^2*(p + 2*q + 2)))*x^2, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2
 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] &&  !IGtQ[q, 0]
```

Rule 1074

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x +
c*x^2)^(p + 1)*((d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f))*(p + 1)))*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) +
(A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c
*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d
 - a*f)))*x), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rubi steps

$$\text{integral} = \frac{3(3+61x)\sqrt{3+2x+5x^2}}{308\left(1+4x-7x^2\right)^2} - \frac{1}{616}\int \frac{-3012-1564x-3220x^2}{\left(1+4x-7x^2\right)^2\sqrt{3+2x+5x^2}}\,dx$$

$$= \frac{3(3+61x)\sqrt{3+2x+5x^2}}{308\left(1+4x-7x^2\right)^2} - \frac{(272941-813113x)\sqrt{3+2x+5x^2}}{1721104\left(1+4x-7x^2\right)} + \frac{\int \frac{47581408+28345408x}{\left(1+4x-7x^2\right)\sqrt{3+2x+5x^2}}\,dx}{27537664}$$

$$= \frac{3(3+61x)\sqrt{3+2x+5x^2}}{308\left(1+4x-7x^2\right)^2} - \frac{(272941-813113x)\sqrt{3+2x+5x^2}}{1721104\left(1+4x-7x^2\right)} + \frac{\left(1391962-1740003\sqrt{11}\right)\int \frac{1}{\left(4-2\sqrt{11}-14x\right)\sqrt{3+2x+5x^2}}\,dx}{1352296} + \frac{\left(1391962+1740003\sqrt{11}\right)\int \frac{1}{\left(4+2\sqrt{11}-14x\right)\sqrt{3+2x+5x^2}}\,dx}{1352296}$$

$$= \frac{3(3+61x)\sqrt{3+2x+5x^2}}{308\left(1+4x-7x^2\right)^2} - \frac{(272941-813113x)\sqrt{3+2x+5x^2}}{1721104\left(1+4x-7x^2\right)} + \frac{\left(-1391962+1740003\sqrt{11}\right)\text{Subst}\left(\int \frac{1}{2352+112\left(4-2\sqrt{11}\right)+20\left(4-2\sqrt{11}\right)^2-x^2}\,dx, x, \frac{-84-2\left(4-2\sqrt{11}\right)-\left(28+\right.}{\sqrt{3+2x+5x}}\right.}{676148} - \frac{\left(1391962+1740003\sqrt{11}\right)\text{Subst}\left(\int \frac{1}{2352+112\left(4+2\sqrt{11}\right)+20\left(4+2\sqrt{11}\right)^2-x^2}\,dx, x, \frac{-84-2\left(4+2\sqrt{11}\right)-\left(28+10\right.}{\sqrt{3+2x+5x^2}}\right.}{676148}$$

$$= \frac{3(3+61x)\sqrt{3+2x+5x^2}}{308\left(1+4x-7x^2\right)^2} - \frac{(272941-813113x)\sqrt{3+2x+5x^2}}{1721104\left(1+4x-7x^2\right)} - \frac{\sqrt{\frac{6492253020949-11879169071\sqrt{11}}{1397}}\tanh^{-1}\left(\frac{23-\sqrt{11}+\left(17-5\sqrt{11}\right)x}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)}{491744} + \frac{\sqrt{\frac{6492253020949+11879169071\sqrt{11}}{1397}}\tanh^{-1}\left(\frac{23+\sqrt{11}+\left(17+5\sqrt{11}\right)x}{\sqrt{2\left(125+17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)}{491744}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.72 (sec) , antiderivative size = 602, normalized size of antiderivative = 2.83

$$\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{(1+4x-7x^2)^3}\,dx$$
$$=\frac{\frac{5764801\sqrt{3+2x+5x^2}(-31807+106279x+737577x^2-813113x^3)}{(1+4x-7x^2)^2}-60545521580434\mathrm{RootSum}\left[83-16\sqrt{5}\#1-70\#1^2+8\sqrt{}\right.}{}$$

```
[In] Integrate[((2 + 5*x + x^2)*Sqrt[3 + 2*x + 5*x^2])/(1 + 4*x - 7*x^2)^3,x]
```

```
[Out] ((5764801*Sqrt[3 + 2*x + 5*x^2]*(-31807 + 106279*x + 737577*x^2 - 813113*x^
3))/(1 + 4*x - 7*x^2)^2 - 60545521580434*RootSum[83 - 16*Sqrt[5]*#1 - 70*#1
^2 + 8*Sqrt[5]*#1^3 + 7*#1^4 & , Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] -
 #1]/(-4*Sqrt[5] - 35*#1 + 6*Sqrt[5]*#1^2 + 7*#1^3) & ] + 20661853520*RootS
um[83 - 16*Sqrt[5]*#1 - 70*#1^2 + 8*Sqrt[5]*#1^3 + 7*#1^4 & , (-465*Log[-(S
qrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1] + 7*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt
[3 + 2*x + 5*x^2] - #1]*#1)/(-4*Sqrt[5] - 35*#1 + 6*Sqrt[5]*#1^2 + 7*#1^3)
& ] + 22*RootSum[83 - 16*Sqrt[5]*#1 - 70*#1^2 + 8*Sqrt[5]*#1^3 + 7*#1^4 & ,
 (3751778663030*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1 +
 2597308755559*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1^2)/(-4*Sqr
t[5] - 35*#1 + 6*Sqrt[5]*#1^2 + 7*#1^3) & ] - 6*RootSum[83 - 16*Sqrt[5]*#1
- 70*#1^2 + 8*Sqrt[5]*#1^3 + 7*#1^4 & , (-11648778057271*Log[-(Sqrt[5]*x) +
 Sqrt[3 + 2*x + 5*x^2] - #1] + 13372446682211*Sqrt[5]*Log[-(Sqrt[5]*x) + Sq
rt[3 + 2*x + 5*x^2] - #1]*#1 + 9645047011740*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*
x + 5*x^2] - #1]*#1^2)/(-4*Sqrt[5] - 35*#1 + 6*Sqrt[5]*#1^2 + 7*#1^3) & ])/
1417403151472
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.08

method	result
risch	$-\frac{(813113x^3-737577x^2-106279x+31807)\sqrt{5x^2+2x+3}}{245872(7x^2-4x-1)^2}+\frac{\left(1740003+126542\sqrt{11}\right)\sqrt{11}\,\operatorname{arctanh}\left(\frac{250+34\sqrt{11}+\frac{49(}{}}{\sqrt{250+34\sqrt{11}}\sqrt{245\left(x-\frac{2}{7}-\frac{\sqrt{11}}{7}\right)^2+}}\right.}{2704592\sqrt{250+34\sqrt{11}}}$
trager	Expression too large to display
default	Expression too large to display

```
[In] int((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1)^3,x,method=_RETURNVERBOS
E)
```

```
[Out] -1/245872*(813113*x^3-737577*x^2-106279*x+31807)/(7*x^2-4*x-1)^2*(5*x^2+2*x
+3)^(1/2)+1/2704592*(1740003+126542*11^(1/2))*11^(1/2)/(250+34*11^(1/2))^(1
/2)*arctanh(49/2*(500/49+68/49*11^(1/2)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^
(1/2)))/(250+34*11^(1/2))^(1/2)/(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*1
1^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2))+1/2704592*(-1740003+1
26542*11^(1/2))*11^(1/2)/(250-34*11^(1/2))^(1/2)*arctanh(49/2*(500/49-68/49
*11^(1/2)+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2)))/(250-34*11^(1/2))^(1/2
)/(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+
250-34*11^(1/2))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 390 vs. $2(160) = 320$.

Time = 0.27 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.83

$$\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{(1+4x-7x^2)^3}\,dx =$$

$$-\frac{\sqrt{1397}(49\,x^4-56\,x^3+2\,x^2+8\,x+1)\sqrt{11879169071\,\sqrt{11}+6492253020949}\,\log\left(\frac{\sqrt{1397}\sqrt{5\,x^2+2\,x+3}\sqrt{1187}}{}\right.}{}$$

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1)^3,x, algorithm="fr
icas")
```

```
[Out] -1/1373932736*(sqrt(1397)*(49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)*sqrt(11879169
071*sqrt(11) + 6492253020949)*log((sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*sqrt(11
879169071*sqrt(11) + 6492253020949)*(4822219*sqrt(11) - 37335441) + 5690716
98870455*sqrt(11)*(x + 3) + 1707215096611365*x - 2845358494352275)/x) - sqr
t(1397)*(49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)*sqrt(11879169071*sqrt(11) + 649
2253020949)*log(-(sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*sqrt(11879169071*sqrt(11
) + 6492253020949)*(4822219*sqrt(11) - 37335441) - 569071698870455*sqrt(11)
*(x + 3) - 1707215096611365*x + 2845358494352275)/x) + sqrt(1397)*(49*x^4 -
 56*x^3 + 2*x^2 + 8*x + 1)*sqrt(-11879169071*sqrt(11) + 6492253020949)*log(
-(sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*(4822219*sqrt(11) + 37335441)*sqrt(-1187
9169071*sqrt(11) + 6492253020949) + 569071698870455*sqrt(11)*(x + 3) - 1707
215096611365*x + 2845358494352275)/x) - sqrt(1397)*(49*x^4 - 56*x^3 + 2*x^2
 + 8*x + 1)*sqrt(-11879169071*sqrt(11) + 6492253020949)*log((sqrt(1397)*sqr
t(5*x^2 + 2*x + 3)*(4822219*sqrt(11) + 37335441)*sqrt(-11879169071*sqrt(11)
 + 6492253020949) - 569071698870455*sqrt(11)*(x + 3) + 1707215096611365*x -
 2845358494352275)/x) + 5588*(813113*x^3 - 737577*x^2 - 106279*x + 31807)*s
qrt(5*x^2 + 2*x + 3))/(49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)
```

Sympy [F]

$$\begin{aligned}\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{(1+4x-7x^2)^3}\,dx &= -\int \frac{2\sqrt{5x^2+2x+3}}{343x^6-588x^5+189x^4+104x^3-27x^2-12x-1}\,dx \\ &\quad -\int \frac{5x\sqrt{5x^2+2x+3}}{343x^6-588x^5+189x^4+104x^3-27x^2-12x-1}\,dx \\ &\quad -\int \frac{x^2\sqrt{5x^2+2x+3}}{343x^6-588x^5+189x^4+104x^3-27x^2-12x-1}\,dx\end{aligned}$$

```
[In] integrate((x**2+5*x+2)*(5*x**2+2*x+3)**(1/2)/(-7*x**2+4*x+1)**3,x)
```

```
[Out] -Integral(2*sqrt(5*x**2 + 2*x + 3)/(343*x**6 - 588*x**5 + 189*x**4 + 104*x*
*3 - 27*x**2 - 12*x - 1), x) - Integral(5*x*sqrt(5*x**2 + 2*x + 3)/(343*x**
6 - 588*x**5 + 189*x**4 + 104*x**3 - 27*x**2 - 12*x - 1), x) - Integral(x**
2*sqrt(5*x**2 + 2*x + 3)/(343*x**6 - 588*x**5 + 189*x**4 + 104*x**3 - 27*x*
*2 - 12*x - 1), x)
```

Maxima [F]

$$\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{(1+4x-7x^2)^3}\,dx = \int -\frac{\sqrt{5\,x^2+2\,x+3}(x^2+5\,x+2)}{(7\,x^2-4\,x-1)^3}\,dx$$

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1)^3,x, algorithm="ma
xima")
```

```
[Out] -integrate(sqrt(5*x^2 + 2*x + 3)*(x^2 + 5*x + 2)/(7*x^2 - 4*x - 1)^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 378 vs. $2(160) = 320$.

Time = 0.30 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.77

$$\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{(1+4x-7x^2)^3}\,dx$$

$$= \frac{6200558\left(\sqrt{5}x-\sqrt{5\,x^2+2\,x+3}\right)^7-835775\,\sqrt{5}\left(\sqrt{5}x-\sqrt{5\,x^2+2\,x+3}\right)^6-190947036\left(\sqrt{5}x-\sqrt{5\,x^2}\right.}{430276\left(7\left(\sqrt{5}x-\sqrt{5\,x}\right.\right.}$$

$$+0.139051039089329\log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}+4.41924736459000\right)$$
$$-0.138209741946100\log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}+1.25295163054000\right)$$
$$-0.139051039089329\log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}-1.02258038113000\right)$$
$$+0.138209741946100\log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}-2.09411235400000\right)$$

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1)^3,x, algorithm="gi
ac")
```

```
[Out] 1/430276*(6200558*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^7 - 835775*sqrt(5)*(s
qrt(5)*x - sqrt(5*x^2 + 2*x + 3))^6 - 190947036*(sqrt(5)*x - sqrt(5*x^2 + 2
*x + 3))^5 - 92732607*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^4 + 81632
1374*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^3 + 419437335*sqrt(5)*(sqrt(5)*x -
 sqrt(5*x^2 + 2*x + 3))^2 - 765111048*sqrt(5)*x - 376983161*sqrt(5) + 76511
1048*sqrt(5*x^2 + 2*x + 3))/(7*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^4 - 8*sq
rt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^3 - 70*(sqrt(5)*x - sqrt(5*x^2 +
2*x + 3))^2 + 16*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3)) + 83)^2 + 0.13
9051039089329*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) + 4.41924736459000) -
0.138209741946100*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) + 1.25295163054000
) - 0.139051039089329*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) - 1.0225803811
3000) + 0.138209741946100*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) - 2.094112
35400000)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{(1+4x-7x^2)^3}\,dx = \int \frac{\left(x^2+5\,x+2\right)\sqrt{5\,x^2+2\,x+3}}{\left(-7\,x^2+4\,x+1\right)^3}\,dx$$

```
[In] int(((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(1/2))/(4*x - 7*x^2 + 1)^3,x)
```

```
[Out] int(((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(1/2))/(4*x - 7*x^2 + 1)^3, x)
```

3.380 $\int \left(1+4x-7x^2\right)^3 \left(2+5x+x^2\right) \left(3+2x+5x^2\right)^{3/2} \, dx$

Optimal result

Integrand size = 35, antiderivative size = 231

$$\begin{aligned}
&\int \left(1+4x-7x^2\right)^3 \left(2+5x+x^2\right) \left(3+2x+5x^2\right)^{3/2} \, dx = \\
&-\frac{479652579(1+5x)\sqrt{3+2x+5x^2}}{312500000}-\frac{22840599(1+5x)\left(3+2x+5x^2\right)^{3/2}}{62500000} \\
&-\frac{6133820867\left(3+2x+5x^2\right)^{5/2}}{1203125000}+\frac{837379699x\left(3+2x+5x^2\right)^{5/2}}{72187500} \\
&+\frac{2173004363x^2\left(3+2x+5x^2\right)^{5/2}}{173250000}-\frac{190236913x^3\left(3+2x+5x^2\right)^{5/2}}{4950000} \\
&-\frac{796559x^4\left(3+2x+5x^2\right)^{5/2}}{123750}+\frac{1031177x^5\left(3+2x+5x^2\right)^{5/2}}{20625} \\
&-\frac{61103x^6\left(3+2x+5x^2\right)^{5/2}}{3300}-\frac{343}{60}x^7\left(3+2x+5x^2\right)^{5/2}-\frac{3357568053\operatorname{arcsinh}\left(\frac{1+5x}{\sqrt{14}}\right)}{156250000\sqrt{5}}
\end{aligned}$$

```
[Out] -22840599/62500000*(1+5*x)*(5*x^2+2*x+3)^(3/2)-6133820867/1203125000*(5*x^2
+2*x+3)^(5/2)+837379699/72187500*x*(5*x^2+2*x+3)^(5/2)+2173004363/173250000
*x^2*(5*x^2+2*x+3)^(5/2)-190236913/4950000*x^3*(5*x^2+2*x+3)^(5/2)-796559/1
23750*x^4*(5*x^2+2*x+3)^(5/2)+1031177/20625*x^5*(5*x^2+2*x+3)^(5/2)-61103/3
300*x^6*(5*x^2+2*x+3)^(5/2)-343/60*x^7*(5*x^2+2*x+3)^(5/2)-3357568053/78125
0000*arcsinh(1/14*(1+5*x)*14^(1/2))*5^(1/2)-479652579/312500000*(1+5*x)*(5*
x^2+2*x+3)^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1675, 654, 626, 633, 221}

$$\int \left(1+4x-7x^2\right)^3 \left(2+5x+x^2\right) \left(3+2x+5x^2\right)^{3/2}\,dx =$$

$$-\frac{3357568053\operatorname{arcsinh}\left(\frac{5x+1}{\sqrt{14}}\right)}{156250000\sqrt{5}}+\frac{2173004363(5x^2+2x+3)^{5/2}\,x^2}{173250000}$$
$$+\frac{837379699(5x^2+2x+3)^{5/2}\,x}{72187500}-\frac{6133820867(5x^2+2x+3)^{5/2}}{1203125000}$$
$$-\frac{22840599(5x+1)\,(5x^2+2x+3)^{3/2}}{62500000}-\frac{479652579(5x+1)\sqrt{5x^2+2x+3}}{312500000}$$
$$-\frac{343}{60}(5x^2{+}2x{+}3)^{5/2}\,x^7-\frac{61103(5x^2+2x+3)^{5/2}\,x^6}{3300}+\frac{1031177(5x^2+2x+3)^{5/2}\,x^5}{20625}-\frac{796559(5x^2+2x+}{123750}$$

```
[In] Int[(1 + 4*x - 7*x^2)^3*(2 + 5*x + x^2)*(3 + 2*x + 5*x^2)^(3/2),x]
```

```
[Out] (-479652579*(1 + 5*x)*Sqrt[3 + 2*x + 5*x^2])/312500000 - (22840599*(1 + 5*x
)*(3 + 2*x + 5*x^2)^(3/2))/62500000 - (6133820867*(3 + 2*x + 5*x^2)^(5/2))/
1203125000 + (837379699*x*(3 + 2*x + 5*x^2)^(5/2))/72187500 + (2173004363*x
^2*(3 + 2*x + 5*x^2)^(5/2))/173250000 - (190236913*x^3*(3 + 2*x + 5*x^2)^(5
/2))/4950000 - (796559*x^4*(3 + 2*x + 5*x^2)^(5/2))/123750 + (1031177*x^5*(
3 + 2*x + 5*x^2)^(5/2))/20625 - (61103*x^6*(3 + 2*x + 5*x^2)^(5/2))/3300 -
(343*x^7*(3 + 2*x + 5*x^2)^(5/2))/60 - (3357568053*ArcSinh[(1 + 5*x)/Sqrt[1
4]])/(156250000*Sqrt[5])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

integral

$$= -\frac{343}{60}x^7\left(3+2x+5x^2\right)^{5/2}+\frac{1}{60}\int \left(3+2x+5x^2\right)^{3/2}\left(120+1740x+6900x^2-3660x^3-52260x^4+7620x^5+131103x^6-61103x^7\right)\,dx$$

$$= -\frac{61103x^6\left(3+2x+5x^2\right)^{5/2}}{3300}-\frac{343}{60}x^7\left(3+2x+5x^2\right)^{5/2}+\frac{\int \left(3+2x+5x^2\right)^{3/2}\left(6600+95700x+379500x^2-201300x^3-2874300x^4+1518954x^5+8249416x^6\right)\,d}{3300}$$

$$= \frac{1031177x^5\left(3+2x+5x^2\right)^{5/2}}{20625}-\frac{61103x^6\left(3+2x+5x^2\right)^{5/2}}{3300}-\frac{343}{60}x^7\left(3+2x+5x^2\right)^{5/2}+\frac{\int \left(3+2x+5x^2\right)^{3/2}\left(330000+4785000x+18975000x^2-10065000x^3-267456240x^4-47793540x^5\right)\,dx}{165000}$$

$$= -\frac{796559x^4\left(3+2x+5x^2\right)^{5/2}}{123750}+\frac{1031177x^5\left(3+2x+5x^2\right)^{5/2}}{20625}-\frac{61103x^6\left(3+2x+5x^2\right)^{5/2}}{3300}-\frac{343}{60}x^7\left(3+2x+5x^2\right)^{5/2}+\frac{\int \left(3+2x+5x^2\right)^{3/2}\left(14850000+215325000x+853875000x^2+120597480x^3-\right.}{7425000}$$

$$= -\frac{190236913x^3\left(3+2x+5x^2\right)^{5/2}}{4950000}-\frac{796559x^4\left(3+2x+5x^2\right)^{5/2}}{123750}+\frac{1031177x^5\left(3+2x+5x^2\right)^{5/2}}{20625}-\frac{61103x^6\left(3+2x+5x^2\right)^{5/2}}{3300}-\frac{343}{60}x^7\left(3+2x+5x^2\right)^{5/2}+\frac{\int \left(3+2x+5x^2\right)^{3/2}\left(594000000+8613000000x+136882933020x^2+13038026\right.}{297000000}$$

$$
\begin{aligned}
&= \frac{2173004363x^2(3+2x+5x^2)^{5/2}}{173250000} - \frac{190236913x^3(3+2x+5x^2)^{5/2}}{4950000} \\
&\quad - \frac{796559x^4(3+2x+5x^2)^{5/2}}{123750} + \frac{1031177x^5(3+2x+5x^2)^{5/2}}{20625} - \frac{61103x^6(3+2x+5x^2)^{5/2}}{3300} \\
&\quad - \frac{343}{60}x^7\left(3+2x+5x^2\right)^{5/2} + \frac{\int \left(3+2x+5x^2\right)^{3/2}\left(20790000000 - 480826570680x + 3617480299680x^2\right)\,dx}{10395000000}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{837379699x(3+2x+5x^2)^{5/2}}{72187500} + \frac{2173004363x^2(3+2x+5x^2)^{5/2}}{173250000} \\
&\quad - \frac{190236913x^3(3+2x+5x^2)^{5/2}}{4950000} - \frac{796559x^4(3+2x+5x^2)^{5/2}}{123750} \\
&\quad + \frac{1031177x^5(3+2x+5x^2)^{5/2}}{20625} - \frac{61103x^6(3+2x+5x^2)^{5/2}}{3300} \\
&\quad - \frac{343}{60}x^7\left(3+2x+5x^2\right)^{5/2} + \frac{\int (-10228740899040 - 39747159218160x)\left(3+2x+5x^2\right)^{3/2}\,dx}{311850000000}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{6133820867(3+2x+5x^2)^{5/2}}{1203125000} + \frac{837379699x(3+2x+5x^2)^{5/2}}{72187500} \\
&\quad + \frac{2173004363x^2(3+2x+5x^2)^{5/2}}{173250000} - \frac{190236913x^3(3+2x+5x^2)^{5/2}}{4950000} \\
&\quad - \frac{796559x^4(3+2x+5x^2)^{5/2}}{123750} + \frac{1031177x^5(3+2x+5x^2)^{5/2}}{20625} - \frac{61103x^6(3+2x+5x^2)^{5/2}}{3300} \\
&\quad - \frac{343}{60}x^7\left(3+2x+5x^2\right)^{5/2} - \frac{22840599\int \left(3+2x+5x^2\right)^{3/2}\,dx}{3125000}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{22840599(1+5x)\left(3+2x+5x^2\right)^{3/2}}{62500000} - \frac{6133820867(3+2x+5x^2)^{5/2}}{1203125000} \\
&\quad + \frac{837379699x(3+2x+5x^2)^{5/2}}{72187500} + \frac{2173004363x^2(3+2x+5x^2)^{5/2}}{173250000} \\
&\quad - \frac{190236913x^3(3+2x+5x^2)^{5/2}}{4950000} - \frac{796559x^4(3+2x+5x^2)^{5/2}}{123750} \\
&\quad + \frac{1031177x^5(3+2x+5x^2)^{5/2}}{20625} - \frac{61103x^6(3+2x+5x^2)^{5/2}}{3300} \\
&\quad - \frac{343}{60}x^7\left(3+2x+5x^2\right)^{5/2} - \frac{479652579\int \sqrt{3+2x+5x^2}\,dx}{31250000}
\end{aligned}
$$

$$\begin{aligned}
&= -\frac{479652579(1+5x)\sqrt{3+2x+5x^2}}{312500000} - \frac{22840599(1+5x)\left(3+2x+5x^2\right)^{3/2}}{62500000} \\
&\quad - \frac{6133820867\left(3+2x+5x^2\right)^{5/2}}{1203125000} + \frac{837379699x\left(3+2x+5x^2\right)^{5/2}}{72187500} \\
&\quad + \frac{2173004363x^2\left(3+2x+5x^2\right)^{5/2}}{173250000} - \frac{190236913x^3\left(3+2x+5x^2\right)^{5/2}}{4950000} \\
&\quad - \frac{796559x^4\left(3+2x+5x^2\right)^{5/2}}{123750} + \frac{1031177x^5\left(3+2x+5x^2\right)^{5/2}}{20625} \\
&\quad - \frac{61103x^6\left(3+2x+5x^2\right)^{5/2}}{3300} - \frac{343}{60}x^7\left(3+2x+5x^2\right)^{5/2} - \frac{3357568053\int \frac{1}{\sqrt{3+2x+5x^2}}\,dx}{156250000}
\end{aligned}$$

$$\begin{aligned}
&= -\frac{479652579(1+5x)\sqrt{3+2x+5x^2}}{312500000} - \frac{22840599(1+5x)\left(3+2x+5x^2\right)^{3/2}}{62500000} \\
&\quad - \frac{6133820867\left(3+2x+5x^2\right)^{5/2}}{1203125000} + \frac{837379699x\left(3+2x+5x^2\right)^{5/2}}{72187500} \\
&\quad + \frac{2173004363x^2\left(3+2x+5x^2\right)^{5/2}}{173250000} - \frac{190236913x^3\left(3+2x+5x^2\right)^{5/2}}{4950000} \\
&\quad - \frac{796559x^4\left(3+2x+5x^2\right)^{5/2}}{123750} + \frac{1031177x^5\left(3+2x+5x^2\right)^{5/2}}{20625} - \frac{61103x^6\left(3+2x+5x^2\right)^{5/2}}{3300} \\
&\quad - \frac{343}{60}x^7\left(3+2x+5x^2\right)^{5/2} - \frac{\left(479652579\sqrt{\frac{7}{10}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{56}}}\,dx, x, 2+10x\right)}{312500000}
\end{aligned}$$

$$\begin{aligned}
&= -\frac{479652579(1+5x)\sqrt{3+2x+5x^2}}{312500000} - \frac{22840599(1+5x)\left(3+2x+5x^2\right)^{3/2}}{62500000} \\
&\quad - \frac{6133820867\left(3+2x+5x^2\right)^{5/2}}{1203125000} + \frac{837379699x\left(3+2x+5x^2\right)^{5/2}}{72187500} \\
&\quad + \frac{2173004363x^2\left(3+2x+5x^2\right)^{5/2}}{173250000} - \frac{190236913x^3\left(3+2x+5x^2\right)^{5/2}}{4950000} \\
&\quad - \frac{796559x^4\left(3+2x+5x^2\right)^{5/2}}{123750} + \frac{1031177x^5\left(3+2x+5x^2\right)^{5/2}}{20625} \\
&\quad - \frac{61103x^6\left(3+2x+5x^2\right)^{5/2}}{3300} - \frac{343}{60}x^7\left(3+2x+5x^2\right)^{5/2} - \frac{3357568053\sinh^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{156250000\sqrt{5}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.47

$$\int \left(1+4x-7x^2\right)^3 \left(2+5x+x^2\right) \left(3+2x+5x^2\right)^{3/2} \, dx = \frac{\sqrt{3+2x+5x^2}(-10506617068392+6352777129950x+15865844408685x^2+190416882396}{} + \frac{3357568053 \log \left(-1-5x+\sqrt{5}\sqrt{3+2x+5x^2}\right)}{156250000\sqrt{5}}$$

```
[In] Integrate[(1 + 4*x - 7*x^2)^3*(2 + 5*x + x^2)*(3 + 2*x + 5*x^2)^(3/2),x]
```

```
[Out] (Sqrt[3 + 2*x + 5*x^2]*(-10506617068392 + 6352777129950*x + 15865844408685*
x^2 + 19041688239675*x^3 + 2573089891000*x^4 - 85130334087500*x^5 - 5210683
0406250*x^6 + 72918247281250*x^7 + 30505457500000*x^8 + 148393743750000*x^9
 - 125007421875000*x^10 - 30950390625000*x^11))/216562500000 + (3357568053*
Log[-1 - 5*x + Sqrt[5]*Sqrt[3 + 2*x + 5*x^2]])/(156250000*Sqrt[5])
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.37

method	result
risch	$-\frac{(30950390625000x^{11}+125007421875000x^{10}-148393743750000x^{9}-30505457500000x^{8}-72918247281250x^{7}+52106830406250x^{6}}{21656}$
trager	$\left(-\frac{1715}{12}x^{11}-\frac{76195}{132}x^{10}+\frac{376873}{550}x^{9}+\frac{1743169}{12375}x^{8}+\frac{333340559}{990000}x^{7}-\frac{555806191}{2310000}x^{6}-\frac{6810426727}{17325000}x^{5}+\frac{2573089891}{216562500}\right.$
default	$-\frac{22840599(10x+2)\left(5x^2+2x+3\right)^{\frac{3}{2}}}{125000000}-\frac{479652579(10x+2)\sqrt{5x^2+2x+3}}{625000000}-\frac{3357568053\sqrt{5}\ \operatorname{arcsinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{781250000}-\frac{6133820867}{120}$

```
[In] int((-7*x^2+4*x+1)^3*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -1/216562500000*(30950390625000*x^11+125007421875000*x^10-148393743750000*x
^9-30505457500000*x^8-72918247281250*x^7+52106830406250*x^6+85130334087500*
x^5-2573089891000*x^4-19041688239675*x^3-15865844408685*x^2-6352777129950*x
+10506617068392)*(5*x^2+2*x+3)^(1/2)-3357568053/781250000*5^(1/2)*arcsinh(5
/14*14^(1/2)*(x+1/5))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.46

$$\int \left(1+4x-7x^2\right)^3 \left(2+5x+x^2\right) \left(3+2x+5x^2\right)^{3/2}\,dx =$$

$$-\frac{1}{216562500000}\left(30950390625000\,x^{11}+125007421875000\,x^{10}-148393743750000\,x^{9}-30505457500000\,x^{8}-\right.$$

$$+\frac{3357568053}{1562500000}\sqrt{5}\log\left(\sqrt{5}\sqrt{5\,x^2+2\,x+3}(5\,x+1)-25\,x^2-10\,x-8\right)$$

```
[In] integrate((-7*x^2+4*x+1)^3*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2),x, algorithm="fr
icas")
```

```
[Out] -1/216562500000*(30950390625000*x^11 + 125007421875000*x^10 - 1483937437500
00*x^9 - 30505457500000*x^8 - 72918247281250*x^7 + 52106830406250*x^6 + 851
30334087500*x^5 - 2573089891000*x^4 - 19041688239675*x^3 - 15865844408685*x
^2 - 6352777129950*x + 10506617068392)*sqrt(5*x^2 + 2*x + 3) + 3357568053/1
562500000*sqrt(5)*log(sqrt(5)*sqrt(5*x^2 + 2*x + 3)*(5*x + 1) - 25*x^2 - 10
*x - 8)
```

Sympy [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.48

$$\begin{aligned}&\int \left(1+4x-7x^2\right)^3 \left(2+5x+x^2\right) \left(3+2x\right.\\&\left.+5x^2\right)^{3/2}\,dx = \sqrt{5x^2+2x+3}\left(-\frac{1715x^{11}}{12}-\frac{76195x^{10}}{132}+\frac{376873x^{9}}{550}\right.\\&+\frac{1743169x^{8}}{12375}+\frac{333340559x^{7}}{990000}-\frac{555806191x^{6}}{2310000}-\frac{6810426727x^{5}}{17325000}+\frac{2573089891x^{4}}{216562500}\\&\left.+\frac{253889176529x^{3}}{2887500000}+\frac{352574320193x^{2}}{4812500000}+\frac{14117282511x}{481250000}-\frac{145925237061}{3007812500}\right)\\&-\frac{3357568053\sqrt{5}\operatorname{asinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{781250000}\end{aligned}$$

```
[In] integrate((-7*x**2+4*x+1)**3*(x**2+5*x+2)*(5*x**2+2*x+3)**(3/2),x)
```

```
[Out] sqrt(5*x**2 + 2*x + 3)*(-1715*x**11/12 - 76195*x**10/132 + 376873*x**9/550
+ 1743169*x**8/12375 + 333340559*x**7/990000 - 555806191*x**6/2310000 - 681
0426727*x**5/17325000 + 2573089891*x**4/216562500 + 253889176529*x**3/28875
00000 + 352574320193*x**2/4812500000 + 14117282511*x/481250000 - 1459252370
61/3007812500) - 3357568053*sqrt(5)*asinh(5*sqrt(14)*(x + 1/5)/14)/78125000
0
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.89

$$\begin{aligned}\int \left(1+4x-7x^2\right)^3\left(2+5x+x^2\right)\left(3+2x+5x^2\right)^{3/2}\,dx &= -\frac{343}{60}\left(5\,x^2+2\,x+3\right)^{\frac{5}{2}}x^7\\ &-\frac{61103}{3300}\left(5\,x^2+2\,x+3\right)^{\frac{5}{2}}x^6+\frac{1031177}{20625}\left(5\,x^2+2\,x+3\right)^{\frac{5}{2}}x^5\\ &-\frac{796559}{123750}\left(5\,x^2+2\,x+3\right)^{\frac{5}{2}}x^4-\frac{190236913}{4950000}\left(5\,x^2+2\,x+3\right)^{\frac{5}{2}}x^3\\ &+\frac{2173004363}{173250000}\left(5\,x^2+2\,x+3\right)^{\frac{5}{2}}x^2+\frac{837379699}{72187500}\left(5\,x^2+2\,x+3\right)^{\frac{5}{2}}x\\ &-\frac{6133820867}{1203125000}\left(5\,x^2+2\,x+3\right)^{\frac{5}{2}}-\frac{22840599}{12500000}\left(5\,x^2+2\,x+3\right)^{\frac{3}{2}}x\\ &-\frac{22840599}{62500000}\left(5\,x^2+2\,x+3\right)^{\frac{3}{2}}-\frac{479652579}{62500000}\sqrt{5\,x^2+2\,x+3}x\\ &-\frac{3357568053}{781250000}\sqrt{5}\,\mathrm{arsinh}\left(\frac{1}{14}\sqrt{14}(5\,x+1)\right)-\frac{479652579}{312500000}\sqrt{5\,x^2+2\,x+3}\end{aligned}$$

```
[In] integrate((-7*x^2+4*x+1)^3*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2),x, algorithm="ma
xima")
```

```
[Out] -343/60*(5*x^2 + 2*x + 3)^(5/2)*x^7 - 61103/3300*(5*x^2 + 2*x + 3)^(5/2)*x^
6 + 1031177/20625*(5*x^2 + 2*x + 3)^(5/2)*x^5 - 796559/123750*(5*x^2 + 2*x
+ 3)^(5/2)*x^4 - 190236913/4950000*(5*x^2 + 2*x + 3)^(5/2)*x^3 + 2173004363
/173250000*(5*x^2 + 2*x + 3)^(5/2)*x^2 + 837379699/72187500*(5*x^2 + 2*x +
3)^(5/2)*x - 6133820867/1203125000*(5*x^2 + 2*x + 3)^(5/2) - 22840599/12500
000*(5*x^2 + 2*x + 3)^(3/2)*x - 22840599/62500000*(5*x^2 + 2*x + 3)^(3/2) -
 479652579/62500000*sqrt(5*x^2 + 2*x + 3)*x - 3357568053/781250000*sqrt(5)*
arcsinh(1/14*sqrt(14)*(5*x + 1)) - 479652579/312500000*sqrt(5*x^2 + 2*x + 3
)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.44

$$\begin{aligned}&\int \left(1+4x-7x^2\right)^3\left(2+5x+x^2\right)\left(3+2x+5x^2\right)^{3/2}\,dx =\\ &-\frac{1}{216562500000}\left(5\left(\left(5\left(10\left(25\left(5\left(7\left(20\left(105\left(875\left(77\,x+311\right)x-323034\right)x-6972676\right)x-333340559\right)x+1\right.\right.\right.\right.\right.\right.\right.\right.\\ &+\frac{3357568053}{781250000}\sqrt{5}\log\left(-\sqrt{5}\left(\sqrt{5}x-\sqrt{5\,x^2+2\,x+3}\right)-1\right)\end{aligned}$$

```
[In] integrate((-7*x^2+4*x+1)^3*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2),x, algorithm="gi
ac")
```

```
[Out] -1/216562500000*(5*((5*(10*(25*(5*(7*(20*(105*(875*(77*x + 311)*x - 323034)
*x - 6972676)*x - 333340559)*x + 1667418573)*x + 13620853454)*x - 102923595
64)*x - 761667529587)*x - 3173168881737)*x - 1270555425990)*x + 10506617068
392)*sqrt(5*x^2 + 2*x + 3) + 3357568053/781250000*sqrt(5)*log(-sqrt(5)*(sqr
t(5)*x - sqrt(5*x^2 + 2*x + 3)) - 1)
```

Mupad [F(-1)]

Timed out.

$$\int \left(1+4x-7x^2\right)^3\left(2+5x+x^2\right)\left(3+2x+5x^2\right)^{3/2}\,dx=\int \left(x^2+5\,x+2\right)\,\left(5\,x^2+2\,x+3\right)^{3/2}\,\left(-7\,x^2+4\,x+1\right)^3\,dx$$

```
[In] int((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(3/2)*(4*x - 7*x^2 + 1)^3,x)
```

```
[Out] int((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(3/2)*(4*x - 7*x^2 + 1)^3, x)
```

3.381 $\int \left(1+4x-7x^2\right)^2 \left(2+5x+x^2\right) \left(3+2x+5x^2\right)^{3/2} \, dx$

Optimal result

Integrand size = 35, antiderivative size = 189

$$\begin{aligned}
\int \left(1+4x-7x^2\right)^2 \left(2+5x+x^2\right) \left(3+2x+5x^2\right)^{3/2} \, dx = & -\frac{14501781(1+5x)\sqrt{3+2x+5x^2}}{6250000}-\frac{690561(1+5x)\left(3+2x+5x^2\right)^{3/2}}{1250000} \\
& +\frac{505667(3+2x+5x^2)^{5/2}}{2187500}+\frac{86721x(3+2x+5x^2)^{5/2}}{21875} \\
& -\frac{219271x^2(3+2x+5x^2)^{5/2}}{105000}-\frac{18379x^3(3+2x+5x^2)^{5/2}}{3000} \\
& +\frac{581}{150}x^4\left(3+2x+5x^2\right)^{5/2}+\frac{49}{50}x^5\left(3+2x+5x^2\right)^{5/2}-\frac{101512467\operatorname{arcsinh}\left(\frac{1+5x}{\sqrt{14}}\right)}{3125000\sqrt{5}}
\end{aligned}$$

```
[Out] -690561/1250000*(1+5*x)*(5*x^2+2*x+3)^(3/2)+505667/2187500*(5*x^2+2*x+3)^(5
/2)+86721/21875*x*(5*x^2+2*x+3)^(5/2)-219271/105000*x^2*(5*x^2+2*x+3)^(5/2)
-18379/3000*x^3*(5*x^2+2*x+3)^(5/2)+581/150*x^4*(5*x^2+2*x+3)^(5/2)+49/50*x
^5*(5*x^2+2*x+3)^(5/2)-101512467/15625000*arcsinh(1/14*(1+5*x)*14^(1/2))*5^
(1/2)-14501781/6250000*(1+5*x)*(5*x^2+2*x+3)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1675, 654, 626, 633, 221}

$$\int \left(1+4x-7x^2\right)^2 \left(2+5x+x^2\right) \left(3+2x+5x^2\right)^{3/2} \, dx = -\frac{101512467\text{arcsinh}\left(\frac{5x+1}{\sqrt{14}}\right)}{3125000\sqrt{5}} - \frac{219271(5x^2+2x+3)^{5/2} x^2}{105000} + \frac{86721(5x^2+2x+3)^{5/2} x}{21875} + \frac{505667(5x^2+2x+3)^{5/2}}{2187500} - \frac{690561(5x+1)\left(5x^2+2x+3\right)^{3/2}}{1250000} - \frac{14501781(5x+1)\sqrt{5x^2+2x+3}}{6250000} + \frac{49}{50}\left(5x^2+2x+3\right)^{5/2} x^5 + \frac{581}{150}\left(5x^2+2x+3\right)^{5/2} x^4 - \frac{18379(5x^2+2x+3)^{5/2} x^3}{3000}$$

```
[In] Int[(1 + 4*x - 7*x^2)^2*(2 + 5*x + x^2)*(3 + 2*x + 5*x^2)^(3/2),x]
```

```
[Out] (-14501781*(1 + 5*x)*Sqrt[3 + 2*x + 5*x^2])/6250000 - (690561*(1 + 5*x)*(3
 + 2*x + 5*x^2)^(3/2))/1250000 + (505667*(3 + 2*x + 5*x^2)^(5/2))/2187500 +
(86721*x*(3 + 2*x + 5*x^2)^(5/2))/21875 - (219271*x^2*(3 + 2*x + 5*x^2)^(5/
2))/105000 - (18379*x^3*(3 + 2*x + 5*x^2)^(5/2))/3000 + (581*x^4*(3 + 2*x +
 5*x^2)^(5/2))/150 + (49*x^5*(3 + 2*x + 5*x^2)^(5/2))/50 - (101512467*ArcSi
nh[(1 + 5*x)/Sqrt[14]])/(3125000*Sqrt[5])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
```

```
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{49}{50}x^5\left(3+2x+5x^2\right)^{5/2} \\
&\quad+\frac{1}{50}\int\left(3+2x+5x^2\right)^{3/2}\left(100+1050x+2250x^2-4700x^3-9735x^4+8715x^5\right)\,dx \\
&= \frac{581}{150}x^4\left(3+2x+5x^2\right)^{5/2}+\frac{49}{50}x^5\left(3+2x+5x^2\right)^{5/2} \\
&\quad+\frac{\int\left(3+2x+5x^2\right)^{3/2}\left(4500+47250x+101250x^2-316080x^3-551370x^4\right)\,dx}{2250} \\
&= -\frac{18379x^3\left(3+2x+5x^2\right)^{5/2}}{3000}+\frac{581}{150}x^4\left(3+2x+5x^2\right)^{5/2}+\frac{49}{50}x^5\left(3+2x+5x^2\right)^{5/2} \\
&\quad+\frac{\int\left(3+2x+5x^2\right)^{3/2}\left(180000+1890000x+9012330x^2-6578130x^3\right)\,dx}{90000} \\
&= -\frac{219271x^2\left(3+2x+5x^2\right)^{5/2}}{105000}-\frac{18379x^3\left(3+2x+5x^2\right)^{5/2}}{3000}+\frac{581}{150}x^4\left(3+2x+5x^2\right)^{5/2} \\
&\quad+\frac{49}{50}x^5\left(3+2x+5x^2\right)^{5/2}+\frac{\int\left(3+2x+5x^2\right)^{3/2}\left(6300000+105618780x+374634720x^2\right)\,dx}{3150000} \\
&= \frac{86721x\left(3+2x+5x^2\right)^{5/2}}{21875}-\frac{219271x^2\left(3+2x+5x^2\right)^{5/2}}{105000}-\frac{18379x^3\left(3+2x+5x^2\right)^{5/2}}{3000} \\
&\quad+\frac{581}{150}x^4\left(3+2x+5x^2\right)^{5/2}+\frac{49}{50}x^5\left(3+2x+5x^2\right)^{5/2}+\frac{\int(-934904160+546120360x)\left(3+2x+5x^2\right)^{3}}{94500000} \\
&= \frac{505667\left(3+2x+5x^2\right)^{5/2}}{2187500}+\frac{86721x\left(3+2x+5x^2\right)^{5/2}}{21875} \\
&\quad-\frac{219271x^2\left(3+2x+5x^2\right)^{5/2}}{105000}-\frac{18379x^3\left(3+2x+5x^2\right)^{5/2}}{3000} \\
&\quad+\frac{581}{150}x^4\left(3+2x+5x^2\right)^{5/2}+\frac{49}{50}x^5\left(3+2x+5x^2\right)^{5/2}-\frac{690561\int\left(3+2x+5x^2\right)^{3/2}\,dx}{62500}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{690561(1+5x)\left(3+2x+5x^2\right)^{3/2}}{1250000} \\
&\quad + \frac{505667\left(3+2x+5x^2\right)^{5/2}}{2187500} + \frac{86721x\left(3+2x+5x^2\right)^{5/2}}{21875} \\
&\quad - \frac{219271x^2\left(3+2x+5x^2\right)^{5/2}}{105000} - \frac{18379x^3\left(3+2x+5x^2\right)^{5/2}}{3000} \\
&\quad + \frac{581}{150}x^4\left(3+2x+5x^2\right)^{5/2} + \frac{49}{50}x^5\left(3+2x+5x^2\right)^{5/2} - \frac{14501781\int\sqrt{3+2x+5x^2}\,dx}{625000}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{14501781(1+5x)\sqrt{3+2x+5x^2}}{6250000} - \frac{690561(1+5x)\left(3+2x+5x^2\right)^{3/2}}{1250000} \\
&\quad + \frac{505667\left(3+2x+5x^2\right)^{5/2}}{2187500} + \frac{86721x\left(3+2x+5x^2\right)^{5/2}}{21875} \\
&\quad - \frac{219271x^2\left(3+2x+5x^2\right)^{5/2}}{105000} - \frac{18379x^3\left(3+2x+5x^2\right)^{5/2}}{3000} \\
&\quad + \frac{581}{150}x^4\left(3+2x+5x^2\right)^{5/2} + \frac{49}{50}x^5\left(3+2x+5x^2\right)^{5/2} - \frac{101512467\int\frac{1}{\sqrt{3+2x+5x^2}}\,dx}{3125000}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{14501781(1+5x)\sqrt{3+2x+5x^2}}{6250000} - \frac{690561(1+5x)\left(3+2x+5x^2\right)^{3/2}}{1250000} \\
&\quad + \frac{505667\left(3+2x+5x^2\right)^{5/2}}{2187500} + \frac{86721x\left(3+2x+5x^2\right)^{5/2}}{21875} \\
&\quad - \frac{219271x^2\left(3+2x+5x^2\right)^{5/2}}{105000} - \frac{18379x^3\left(3+2x+5x^2\right)^{5/2}}{3000} \\
&\quad + \frac{581}{150}x^4\left(3+2x+5x^2\right)^{5/2} + \frac{49}{50}x^5\left(3+2x+5x^2\right)^{5/2} - \frac{\left(14501781\sqrt{\frac{7}{10}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1+\frac{x^2}{56}}}\,dx, x, 2+10x\right)}{6250000}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{14501781(1+5x)\sqrt{3+2x+5x^2}}{6250000} - \frac{690561(1+5x)\left(3+2x+5x^2\right)^{3/2}}{1250000} \\
&\quad + \frac{505667\left(3+2x+5x^2\right)^{5/2}}{2187500} + \frac{86721x\left(3+2x+5x^2\right)^{5/2}}{21875} \\
&\quad - \frac{219271x^2\left(3+2x+5x^2\right)^{5/2}}{105000} - \frac{18379x^3\left(3+2x+5x^2\right)^{5/2}}{3000} \\
&\quad + \frac{581}{150}x^4\left(3+2x+5x^2\right)^{5/2} + \frac{49}{50}x^5\left(3+2x+5x^2\right)^{5/2} - \frac{101512467\sinh^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{3125000\sqrt{5}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.52

$$\int \left(1+4x-7x^2\right)^2 \left(2+5x+x^2\right) \left(3+2x+5x^2\right)^{3/2}\,dx = \frac{\sqrt{3+2x+5x^2}(-249003936+2291675850x+3721040355x^2+5959365525x^3-322759700}{13125} + \frac{101512467\log\left(-1-5x+\sqrt{5}\sqrt{3+2x+5x^2}\right)}{3125000\sqrt{5}}$$

```
[In] Integrate[(1 + 4*x - 7*x^2)^2*(2 + 5*x + x^2)*(3 + 2*x + 5*x^2)^(3/2),x]
```

```
[Out] (Sqrt[3 + 2*x + 5*x^2]*(-249003936 + 2291675850*x + 3721040355*x^2 + 595936
5525*x^3 - 3227597000*x^4 - 12554262500*x^5 - 4105593750*x^6 - 5561281250*x
^7 + 15281875000*x^8 + 3215625000*x^9))/131250000 + (101512467*Log[-1 - 5*x
 + Sqrt[5]*Sqrt[3 + 2*x + 5*x^2]])/(3125000*Sqrt[5])
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.40

method	result
risch	$\frac{(3215625000x^9+15281875000x^8-5561281250x^7-4105593750x^6-12554262500x^5-3227597000x^4+5959365525x^3+3721040355x^2+}{131250000}$
trager	$\left(\frac{49}{2}x^9+\frac{3493}{30}x^8-\frac{25423}{600}x^7-\frac{43793}{1400}x^6-\frac{1004341}{10500}x^5-\frac{3227597}{131250}x^4+\frac{79458207}{1750000}x^3+\frac{248069357}{8750000}x^2+\frac{15277839}{875000}x\right.$
default	$-\frac{690561(10x+2)(5x^2+2x+3)^{\frac{3}{2}}}{2500000}-\frac{14501781(10x+2)\sqrt{5x^2+2x+3}}{12500000}-\frac{101512467\sqrt{5}\ \operatorname{arcsinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{15625000}+\frac{505667(5x^2+2x}{2187500}$

```
[In] int((-7*x^2+4*x+1)^2*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/131250000*(3215625000*x^9+15281875000*x^8-5561281250*x^7-4105593750*x^6-1
2554262500*x^5-3227597000*x^4+5959365525*x^3+3721040355*x^2+2291675850*x-24
9003936)*(5*x^2+2*x+3)^(1/2)-101512467/15625000*5^(1/2)*arcsinh(5/14*14^(1/
2)*(x+1/5))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.51

$$\int \left(1+4x-7x^2\right)^2 \left(2+5x+x^2\right) \left(3+2x+5x^2\right)^{3/2} \, dx = \frac{1}{131250000} \left(3215625000\,x^9 + 15281875000\,x^8 - 5561281250\,x^7 - 4105593750\,x^6 - 125542625 \right. + \frac{101512467}{31250000}\sqrt{5}\log\left(\sqrt{5}\sqrt{5\,x^2+2\,x+3}(5\,x+1) - 25\,x^2 - 10\,x - 8\right)$$

```
[In] integrate((-7*x^2+4*x+1)^2*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2),x, algorithm="fr
icas")
```

```
[Out] 1/131250000*(3215625000*x^9 + 15281875000*x^8 - 5561281250*x^7 - 4105593750
*x^6 - 12554262500*x^5 - 3227597000*x^4 + 5959365525*x^3 + 3721040355*x^2 +
 2291675850*x - 249003936)*sqrt(5*x^2 + 2*x + 3) + 101512467/31250000*sqrt(
5)*log(sqrt(5)*sqrt(5*x^2 + 2*x + 3)*(5*x + 1) - 25*x^2 - 10*x - 8)
```

Sympy [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.52

$$\int \left(1+4x-7x^2\right)^2 \left(2+5x+x^2\right) \left(3+2x+5x^2\right)^{3/2} \, dx = \sqrt{5x^2+2x+3} \cdot \left(\frac{49x^9}{2} + \frac{3493x^8}{30} - \frac{25423x^7}{600} - \frac{43793x^6}{1400} - \frac{1004341x^5}{10500} - \frac{3227597x^4}{131250} + \frac{79458207x^3}{1750000} + \frac{248069357x^2}{8750000} + \frac{15277839x}{875000} - \frac{5187582}{2734375}\right) - \frac{101512467\sqrt{5}\operatorname{asinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{15625000}$$

```
[In] integrate((-7*x**2+4*x+1)**2*(x**2+5*x+2)*(5*x**2+2*x+3)**(3/2),x)
```

```
[Out] sqrt(5*x**2 + 2*x + 3)*(49*x**9/2 + 3493*x**8/30 - 25423*x**7/600 - 43793*x
**6/1400 - 1004341*x**5/10500 - 3227597*x**4/131250 + 79458207*x**3/1750000
 + 248069357*x**2/8750000 + 15277839*x/875000 - 5187582/2734375) - 10151246
7*sqrt(5)*asinh(5*sqrt(14)*(x + 1/5)/14)/15625000
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.91

$$\int \left(1+4x-7x^2\right)^2\left(2+5x+x^2\right)\left(3+2x+5x^2\right)^{3/2}\,dx=\frac{49}{50}\left(5\,x^2+2\,x+3\right)^{\frac{5}{2}}x^5 + \frac{581}{150}\left(5\,x^2+2\,x+3\right)^{\frac{5}{2}}x^4 - \frac{18379}{3000}\left(5\,x^2+2\,x+3\right)^{\frac{5}{2}}x^3 - \frac{219271}{105000}\left(5\,x^2+2\,x+3\right)^{\frac{5}{2}}x^2 + \frac{86721}{21875}\left(5\,x^2+2\,x+3\right)^{\frac{5}{2}}x + \frac{505667}{2187500}\left(5\,x^2+2\,x+3\right)^{\frac{5}{2}} - \frac{690561}{250000}\left(5\,x^2+2\,x+3\right)^{\frac{3}{2}}x - \frac{690561}{1250000}\left(5\,x^2+2\,x+3\right)^{\frac{3}{2}} - \frac{14501781}{1250000}\sqrt{5\,x^2+2\,x+3}x - \frac{101512467}{15625000}\sqrt{5}\,\mathrm{arsinh}\left(\frac{1}{14}\sqrt{14}(5\,x+1)\right) - \frac{14501781}{6250000}\sqrt{5\,x^2+2\,x+3}$$

```
[In] integrate((-7*x^2+4*x+1)^2*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2),x, algorithm="ma
xima")
```

```
[Out] 49/50*(5*x^2 + 2*x + 3)^(5/2)*x^5 + 581/150*(5*x^2 + 2*x + 3)^(5/2)*x^4 - 1
8379/3000*(5*x^2 + 2*x + 3)^(5/2)*x^3 - 219271/105000*(5*x^2 + 2*x + 3)^(5/
2)*x^2 + 86721/21875*(5*x^2 + 2*x + 3)^(5/2)*x + 505667/2187500*(5*x^2 + 2*
x + 3)^(5/2) - 690561/250000*(5*x^2 + 2*x + 3)^(3/2)*x - 690561/1250000*(5*
x^2 + 2*x + 3)^(3/2) - 14501781/1250000*sqrt(5*x^2 + 2*x + 3)*x - 101512467
/15625000*sqrt(5)*arcsinh(1/14*sqrt(14)*(5*x + 1)) - 14501781/6250000*sqrt(
5*x^2 + 2*x + 3)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.49

$$\int \left(1+4x-7x^2\right)^2\left(2+5x+x^2\right)\left(3+2x+5x^2\right)^{3/2}\,dx=\frac{1}{131250000}(5\,((5\,(10\,(25\,(5\,(7\,(140\,(105\,x+499)x-25423)x-131379)x-2008682)x-129 + \frac{101512467}{15625000}\sqrt{5}\log\left(-\sqrt{5}\left(\sqrt{5}x-\sqrt{5\,x^2+2\,x+3}\right)-1\right)$$

```
[In] integrate((-7*x^2+4*x+1)^2*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2),x, algorithm="gi
ac")
```

```
[Out] 1/131250000*(5*((5*(10*(25*(5*(7*(140*(105*x + 499)*x - 25423)*x - 131379)*
x - 2008682)*x - 12910388)*x + 238374621)*x + 744208071)*x + 458335170)*x -
 249003936)*sqrt(5*x^2 + 2*x + 3) + 101512467/15625000*sqrt(5)*log(-sqrt(5)
*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3)) - 1)
```

Mupad [F(-1)]

Timed out.

$$\int \left(1+4x-7x^2\right)^2 \left(2+5x+x^2\right) \left(3+2x+5x^2\right)^{3/2}\,dx=\int \left(x^2+5\,x+2\right)\,\left(5\,x^2+2\,x+3\right)^{3/2}\,\left(-7\,x^2+4\,x+1\right)^2\,dx$$

```
[In] int((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(3/2)*(4*x - 7*x^2 + 1)^2,x)
[Out] int((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(3/2)*(4*x - 7*x^2 + 1)^2, x)
```

3.382 $\int \left(1+4x-7x^2\right)\left(2+5x+x^2\right)\left(3+2x+5x^2\right)^{3/2}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 147

$$\begin{aligned}
&\int \left(1+4x-7x^2\right)\left(2+5x+x^2\right)\left(3+2x+5x^2\right)^{3/2}\,dx= \\
&-\frac{128779(1+5x)\sqrt{3+2x+5x^2}}{250000}-\frac{18397(1+5x)\left(3+2x+5x^2\right)^{3/2}}{150000} \\
&+\frac{149509(3+2x+5x^2)^{5/2}}{262500}+\frac{2809x(3+2x+5x^2)^{5/2}}{5250} \\
&-\frac{1163x^2(3+2x+5x^2)^{5/2}}{1400}-\frac{7}{40}x^3\left(3+2x+5x^2\right)^{5/2}-\frac{901453\operatorname{arcsinh}\left(\frac{1+5x}{\sqrt{14}}\right)}{125000\sqrt{5}}
\end{aligned}$$

```
[Out] -18397/150000*(1+5*x)*(5*x^2+2*x+3)^(3/2)+149509/262500*(5*x^2+2*x+3)^(5/2)
+2809/5250*x*(5*x^2+2*x+3)^(5/2)-1163/1400*x^2*(5*x^2+2*x+3)^(5/2)-7/40*x^3
*(5*x^2+2*x+3)^(5/2)-901453/625000*arcsinh(1/14*(1+5*x)*14^(1/2))*5^(1/2)-1
28779/250000*(1+5*x)*(5*x^2+2*x+3)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used

$= \{1675, 654, 626, 633, 221\}$

$$\int \left(1+4x-7x^2\right)\left(2+5x+x^2\right)\left(3+2x+5x^2\right)^{3/2}\,dx = -\frac{901453\operatorname{arcsinh}\left(\frac{5x+1}{\sqrt{14}}\right)}{125000\sqrt{5}}-\frac{1163\left(5x^2+2x+3\right)^{5/2}x^2}{1400}+\frac{2809\left(5x^2+2x+3\right)^{5/2}x}{5250}+\frac{149509\left(5x^2+2x+3\right)^{5/2}}{262500}-\frac{18397(5x+1)\left(5x^2+2x+3\right)^{3/2}}{150000}-\frac{128779(5x+1)\sqrt{5x^2+2x+3}}{250000}-\frac{7}{40}\left(5x^2+2x+3\right)^{5/2}x^3$$

```
[In] Int[(1 + 4*x - 7*x^2)*(2 + 5*x + x^2)*(3 + 2*x + 5*x^2)^(3/2),x]
```

```
[Out] (-128779*(1 + 5*x)*Sqrt[3 + 2*x + 5*x^2])/250000 - (18397*(1 + 5*x)*(3 + 2*
x + 5*x^2)^(3/2))/150000 + (149509*(3 + 2*x + 5*x^2)^(5/2))/262500 + (2809*
x*(3 + 2*x + 5*x^2)^(5/2))/5250 - (1163*x^2*(3 + 2*x + 5*x^2)^(5/2))/1400 -
 (7*x^3*(3 + 2*x + 5*x^2)^(5/2))/40 - (901453*ArcSinh[(1 + 5*x)/Sqrt[14]])/
(125000*Sqrt[5])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{7}{40}x^3\left(3+2x+5x^2\right)^{5/2}+\frac{1}{40}\int\left(3+2x+5x^2\right)^{3/2}\left(80+520x+343x^2-1163x^3\right)\,dx\\
&= -\frac{1163x^2\left(3+2x+5x^2\right)^{5/2}}{1400}\\
&\quad-\frac{7}{40}x^3\left(3+2x+5x^2\right)^{5/2}+\frac{\int\left(3+2x+5x^2\right)^{3/2}\left(2800+25178x+22472x^2\right)dx}{1400}\\
&= \frac{2809x\left(3+2x+5x^2\right)^{5/2}}{5250}-\frac{1163x^2\left(3+2x+5x^2\right)^{5/2}}{1400}\\
&\quad-\frac{7}{40}x^3\left(3+2x+5x^2\right)^{5/2}+\frac{\int(16584+598036x)\left(3+2x+5x^2\right)^{3/2}\,dx}{42000}\\
&= \frac{149509\left(3+2x+5x^2\right)^{5/2}}{262500}+\frac{2809x\left(3+2x+5x^2\right)^{5/2}}{5250}-\frac{1163x^2\left(3+2x+5x^2\right)^{5/2}}{1400}\\
&\quad-\frac{7}{40}x^3\left(3+2x+5x^2\right)^{5/2}-\frac{18397\int\left(3+2x+5x^2\right)^{3/2}\,dx}{7500}\\
&= -\frac{18397(1+5x)\left(3+2x+5x^2\right)^{3/2}}{150000}+\frac{149509\left(3+2x+5x^2\right)^{5/2}}{262500}+\frac{2809x\left(3+2x+5x^2\right)^{5/2}}{5250}\\
&\quad-\frac{1163x^2\left(3+2x+5x^2\right)^{5/2}}{1400}-\frac{7}{40}x^3\left(3+2x+5x^2\right)^{5/2}-\frac{128779\int\sqrt{3+2x+5x^2}\,dx}{25000}\\
&= -\frac{128779(1+5x)\sqrt{3+2x+5x^2}}{250000}-\frac{18397(1+5x)\left(3+2x+5x^2\right)^{3/2}}{150000}\\
&\quad+\frac{149509\left(3+2x+5x^2\right)^{5/2}}{262500}+\frac{2809x\left(3+2x+5x^2\right)^{5/2}}{5250}\\
&\quad-\frac{1163x^2\left(3+2x+5x^2\right)^{5/2}}{1400}-\frac{7}{40}x^3\left(3+2x+5x^2\right)^{5/2}-\frac{901453\int\frac{1}{\sqrt{3+2x+5x^2}}\,dx}{125000}\\
&= -\frac{128779(1+5x)\sqrt{3+2x+5x^2}}{250000}-\frac{18397(1+5x)\left(3+2x+5x^2\right)^{3/2}}{150000}\\
&\quad+\frac{149509\left(3+2x+5x^2\right)^{5/2}}{262500}+\frac{2809x\left(3+2x+5x^2\right)^{5/2}}{5250}-\frac{1163x^2\left(3+2x+5x^2\right)^{5/2}}{1400}\\
&\quad-\frac{7}{40}x^3\left(3+2x+5x^2\right)^{5/2}-\frac{\left(128779\sqrt{\frac{7}{10}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1+\frac{x^2}{56}}}\,dx,x,2+10x\right)}{250000}
\end{aligned}$$

$$= -\frac{128779(1+5x)\sqrt{3+2x+5x^2}}{250000} - \frac{18397(1+5x)\left(3+2x+5x^2\right)^{3/2}}{150000} + \frac{149509(3+2x+5x^2)^{5/2}}{262500} + \frac{2809x(3+2x+5x^2)^{5/2}}{5250} - \frac{1163x^2(3+2x+5x^2)^{5/2}}{1400} - \frac{7}{40}x^3\left(3+2x+5x^2\right)^{5/2} - \frac{901453\sinh^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{125000\sqrt{5}}$$

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.61

$$\int \left(1+4x-7x^2\right)\left(2+5x+x^2\right)\left(3+2x+5x^2\right)^{3/2}\,dx = \frac{\sqrt{3+2x+5x^2}(22275576+36695150x+86464445x^2+78608475x^3-28373000x^4-4823750}{5250000} + \frac{901453\log\left(-1-5x+\sqrt{5}\sqrt{3+2x+5x^2}\right)}{125000\sqrt{5}}$$

```
[In] Integrate[(1 + 4*x - 7*x^2)*(2 + 5*x + x^2)*(3 + 2*x + 5*x^2)^(3/2),x]
```

```
[Out] (Sqrt[3 + 2*x + 5*x^2]*(22275576 + 36695150*x + 86464445*x^2 + 78608475*x^3
 - 28373000*x^4 - 48237500*x^5 - 127406250*x^6 - 22968750*x^7))/5250000 + (
901453*Log[-1 - 5*x + Sqrt[5]*Sqrt[3 + 2*x + 5*x^2]])/(125000*Sqrt[5])
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.44

method	result
risch	$-\frac{(22968750x^7+127406250x^6+48237500x^5+28373000x^4-78608475x^3-86464445x^2-36695150x-22275576)\sqrt{5x^2+2x+3}}{5250000} - \frac{90145}{}$
trager	$\left(-\frac{35}{8}x^7-\frac{1359}{56}x^6-\frac{3859}{420}x^5-\frac{28373}{5250}x^4+\frac{1048113}{70000}x^3+\frac{17292889}{1050000}x^2+\frac{733903}{105000}x+\frac{928149}{218750}\right)\sqrt{5x^2+2x+3} -$
default	$-\frac{18397(10x+2)\left(5x^2+2x+3\right)^{\frac{3}{2}}}{300000} - \frac{128779(10x+2)\sqrt{5x^2+2x+3}}{500000} - \frac{901453\sqrt{5}\ \operatorname{arcsinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{625000} + \frac{149509\left(5x^2+2x+3\right)^{\frac{5}{2}}}{262500} -$

```
[In] int((-7*x^2+4*x+1)*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5250000*(22968750*x^7+127406250*x^6+48237500*x^5+28373000*x^4-78608475*x
^3-86464445*x^2-36695150*x-22275576)*(5*x^2+2*x+3)^(1/2)-901453/625000*5^(1
/2)*arcsinh(5/14*14^(1/2)*(x+1/5))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.59

$$\int \left(1+4x-7x^2\right)\left(2+5x+x^2\right)\left(3+2x+5x^2\right)^{3/2}\,dx =$$
$$-\frac{1}{5250000}\left(22968750\,x^7+127406250\,x^6+48237500\,x^5+28373000\,x^4-78608475\,x^3-86464445\,x^2-366\right.$$
$$+\frac{901453}{1250000}\sqrt{5}\log\left(\sqrt{5}\sqrt{5\,x^2+2\,x+3}(5\,x+1)-25\,x^2-10\,x-8\right)$$

```
[In] integrate((-7*x^2+4*x+1)*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2),x, algorithm="fric
as")
```

```
[Out] -1/5250000*(22968750*x^7 + 127406250*x^6 + 48237500*x^5 + 28373000*x^4 - 78
608475*x^3 - 86464445*x^2 - 36695150*x - 22275576)*sqrt(5*x^2 + 2*x + 3) +
901453/1250000*sqrt(5)*log(sqrt(5)*sqrt(5*x^2 + 2*x + 3)*(5*x + 1) - 25*x^2
 - 10*x - 8)
```

Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.58

$$\int \left(1+4x-7x^2\right)\left(2+5x+x^2\right)\left(3+2x\right.$$
$$\left.+5x^2\right)^{3/2}\,dx = \sqrt{5x^2+2x+3}\left(-\frac{35x^7}{8}-\frac{1359x^6}{56}-\frac{3859x^5}{420}-\frac{28373x^4}{5250}\right.$$
$$\left.+\frac{1048113x^3}{70000}+\frac{17292889x^2}{1050000}+\frac{733903x}{105000}+\frac{928149}{218750}\right)-\frac{901453\sqrt{5}\operatorname{asinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{625000}$$

```
[In] integrate((-7*x**2+4*x+1)*(x**2+5*x+2)*(5*x**2+2*x+3)**(3/2),x)
```

```
[Out] sqrt(5*x**2 + 2*x + 3)*(-35*x**7/8 - 1359*x**6/56 - 3859*x**5/420 - 28373*x
**4/5250 + 1048113*x**3/70000 + 17292889*x**2/1050000 + 733903*x/105000 + 9
28149/218750) - 901453*sqrt(5)*asinh(5*sqrt(14)*(x + 1/5)/14)/625000
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.94

$$\int \left(1+4x-7x^2\right)\left(2+5x+x^2\right)\left(3+2x+5x^2\right)^{3/2}\,dx = -\frac{7}{40}\left(5\,x^2+2\,x+3\right)^{\frac{5}{2}}x^3 - \frac{1163}{1400}\left(5\,x^2+2\,x+3\right)^{\frac{5}{2}}x^2 + \frac{2809}{5250}\left(5\,x^2+2\,x+3\right)^{\frac{5}{2}}x + \frac{149509}{262500}\left(5\,x^2+2\,x+3\right)^{\frac{5}{2}} - \frac{18397}{30000}\left(5\,x^2+2\,x+3\right)^{\frac{3}{2}}x - \frac{18397}{150000}\left(5\,x^2+2\,x+3\right)^{\frac{3}{2}} - \frac{128779}{50000}\sqrt{5\,x^2+2\,x+3}x - \frac{901453}{625000}\sqrt{5}\,\mathrm{arsinh}\left(\frac{1}{14}\sqrt{14}(5\,x+1)\right) - \frac{128779}{250000}\sqrt{5\,x^2+2\,x+3}$$

```
[In] integrate((-7*x^2+4*x+1)*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2),x, algorithm="maxi
ma")
```

```
[Out] -7/40*(5*x^2 + 2*x + 3)^(5/2)*x^3 - 1163/1400*(5*x^2 + 2*x + 3)^(5/2)*x^2 +
 2809/5250*(5*x^2 + 2*x + 3)^(5/2)*x + 149509/262500*(5*x^2 + 2*x + 3)^(5/2
) - 18397/30000*(5*x^2 + 2*x + 3)^(3/2)*x - 18397/150000*(5*x^2 + 2*x + 3)^
(3/2) - 128779/50000*sqrt(5*x^2 + 2*x + 3)*x - 901453/625000*sqrt(5)*arcsin
h(1/14*sqrt(14)*(5*x + 1)) - 128779/250000*sqrt(5*x^2 + 2*x + 3)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.56

$$\int \left(1+4x-7x^2\right)\left(2+5x+x^2\right)\left(3+2x+5x^2\right)^{3/2}\,dx = -\frac{1}{5250000}\left(5\left(\left(5\left(10\left(25\left(15\left(245\,x+1359\right)x+7718\right)x+113492\right)x-3144339\right)x-17292889\right)x-7339030\right)x\right. + \frac{901453}{625000}\sqrt{5}\log\left(-\sqrt{5}\left(\sqrt{5}x-\sqrt{5\,x^2+2\,x+3}\right)-1\right)$$

```
[In] integrate((-7*x^2+4*x+1)*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2),x, algorithm="giac
")
```

```
[Out] -1/5250000*(5*((5*(10*(25*(15*(245*x + 1359)*x + 7718)*x + 113492)*x - 3144
339)*x - 17292889)*x - 7339030)*x - 22275576)*sqrt(5*x^2 + 2*x + 3) + 90145
3/625000*sqrt(5)*log(-sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3)) - 1)
```

Mupad [F(-1)]

Timed out.

$$\int \left(1+4x-7x^2\right) \left(2+5x+x^2\right) \left(3+2x+5x^2\right)^{3/2} \, dx = \int \left(x^2+5\,x+2\right)\,\left(5\,x^2+2\,x+3\right)^{3/2}\,\left(-7\,x^2+4\,x+1\right) \,d x$$

```
[In] int((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(3/2)*(4*x - 7*x^2 + 1),x)
[Out] int((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(3/2)*(4*x - 7*x^2 + 1), x)
```

3.383 $\int \frac{(2+5x+x^2)(3+2x+5x^2)^{3/2}}{1+4x-7x^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 210

$$\begin{aligned}
\int \frac{(2+5x+x^2)(3+2x+5x^2)^{3/2}}{1+4x-7x^2}\,dx &= -\frac{3(571621+196105x)\sqrt{3+2x+5x^2}}{240100} \\
&- \frac{1}{980}(267+35x)\left(3+2x+5x^2\right)^{3/2} - \frac{34425687\operatorname{arcsinh}\left(\frac{1+5x}{\sqrt{14}}\right)}{840350\sqrt{5}} \\
&- \frac{6\sqrt{\frac{2}{11}\left(8098902607-2434122235\sqrt{11}\right)}\operatorname{arctanh}\left(\frac{23-\sqrt{11}+\left(17-5\sqrt{11}\right)x}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)}{16807} \\
&+ \frac{6\sqrt{\frac{2}{11}\left(8098902607+2434122235\sqrt{11}\right)}\operatorname{arctanh}\left(\frac{23+\sqrt{11}+\left(17+5\sqrt{11}\right)x}{\sqrt{2\left(125+17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)}{16807}
\end{aligned}$$

```
[Out] -1/980*(267+35*x)*(5*x^2+2*x+3)^(3/2)-34425687/4201750*arcsinh(1/14*(1+5*x)
*14^(1/2))*5^(1/2)-3/240100*(571621+196105*x)*(5*x^2+2*x+3)^(1/2)-6/184877*
arctanh((23+x*(17-5*11^(1/2))-11^(1/2))/(5*x^2+2*x+3)^(1/2)/(250-34*11^(1/2
))^(1/2))*(178175857354-53550689170*11^(1/2))^(1/2)+6/184877*arctanh((23+11
^(1/2)+x*(17+5*11^(1/2)))/(5*x^2+2*x+3)^(1/2)/(250+34*11^(1/2))^(1/2))*(178
175857354+53550689170*11^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1080, 1090, 633, 221, 1046, 738, 212}

$$\begin{aligned}\int \frac{(2+5x+x^2)\left(3+2x+5x^2\right)^{3/2}}{1+4x-7x^2}\,dx = &-\frac{34425687\operatorname{arcsinh}\left(\frac{5x+1}{\sqrt{14}}\right)}{840350\sqrt{5}}\\ &-\frac{6\sqrt{\frac{2}{11}\left(8098902607-2434122235\sqrt{11}\right)}\operatorname{arctanh}\left(\frac{\left(17-5\sqrt{11}\right)x-\sqrt{11}+23}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right)}{16807}\\ &+\frac{6\sqrt{\frac{2}{11}\left(8098902607+2434122235\sqrt{11}\right)}\operatorname{arctanh}\left(\frac{\left(17+5\sqrt{11}\right)x+\sqrt{11}+23}{\sqrt{2\left(125+17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right)}{16807}\\ &-\frac{1}{980}(35x+267)\left(5x^2+2x+3\right)^{3/2}-\frac{3(196105x+571621)\sqrt{5x^2+2x+3}}{240100}\end{aligned}$$

```
[In] Int[((2 + 5*x + x^2)*(3 + 2*x + 5*x^2)^(3/2))/(1 + 4*x - 7*x^2),x]
```

```
[Out] (-3*(571621 + 196105*x)*Sqrt[3 + 2*x + 5*x^2])/240100 - ((267 + 35*x)*(3 +
2*x + 5*x^2)^(3/2))/980 - (34425687*ArcSinh[(1 + 5*x)/Sqrt[14]])/(840350*Sq
rt[5]) - (6*Sqrt[(2*(8098902607 - 2434122235*Sqrt[11]))/11]*ArcTanh[(23 - S
qrt[11] + (17 - 5*Sqrt[11])*x)/(Sqrt[2*(125 - 17*Sqrt[11])]*Sqrt[3 + 2*x +
5*x^2])])/16807 + (6*Sqrt[(2*(8098902607 + 2434122235*Sqrt[11]))/11]*ArcTan
h[(23 + Sqrt[11] + (17 + 5*Sqrt[11])*x)/(Sqrt[2*(125 + 17*Sqrt[11])]*Sqrt[3
 + 2*x + 5*x^2])])/16807
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1046

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1080

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(B*c*f*(2*p
 + 2*q + 3) + C*(b*f*p - c*e*(2*p + q + 2)) + 2*c*C*f*(p + q + 1)*x)*(a + b
*x + c*x^2)^p*((d + e*x + f*x^2)^(q + 1)/(2*c*f^2*(p + q + 1)*(2*p + 2*q +
3))), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + b*x + c*x
^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[p*(b*d - a*e)*(C*(c*e - b*f)*(q + 1) -
 c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(b^2*C*d*f*p + a*c*(C*(2*d*f
- e^2*(2*p + q + 2)) + f*(B*e - 2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)
*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e
*f*p*(b^2 - 4*a*c) - b*c*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e +
2*A*f)*(2*p + 2*q + 3))))*x + (p*(c*e - b*f)*(C*(c*e - b*f)*(q + 1) - c*(C*
e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(e^
2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x^2,
x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*
q + 3, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1090

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\text{integral} = -\frac{1}{980}(267+35x)\left(3+2x+5x^2\right)^{3/2} - \frac{\int \frac{\left(-20358-79272x-100854x^2\right)\sqrt{3+2x+5x^2}}{1+4x-7x^2}\,dx}{2940}$$

$$
\begin{aligned}
&= -\frac{3(571621+196105x)\sqrt{3+2x+5x^2}}{240100} \\
&\quad -\frac{1}{980}(267+35x)\left(3+2x+5x^2\right)^{3/2}+\frac{\int \frac{50805108+282031632x+413108244x^2}{(1+4x-7x^2)\sqrt{3+2x+5x^2}}\,dx}{1440600} \\
&= -\frac{3(571621+196105x)\sqrt{3+2x+5x^2}}{240100}-\frac{1}{980}(267+35x)\left(3+2x+5x^2\right)^{3/2} \\
&\quad -\frac{\int \frac{-768744000-3626654400x}{(1+4x-7x^2)\sqrt{3+2x+5x^2}}\,dx}{10084200}-\frac{34425687\int \frac{1}{\sqrt{3+2x+5x^2}}\,dx}{840350} \\
&= -\frac{3(571621+196105x)\sqrt{3+2x+5x^2}}{240100}-\frac{1}{980}(267+35x)\left(3+2x+5x^2\right)^{3/2} \\
&\quad -\frac{34425687\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{56}}}\,dx,x,2+10x\right)}{1680700\sqrt{70}} \\
&\quad +\frac{\left(24\left(2770361-877397\sqrt{11}\right)\right)\int \frac{1}{\left(4-2\sqrt{11}-14x\right)\sqrt{3+2x+5x^2}}\,dx}{184877} \\
&\quad +\frac{\left(24\left(2770361+877397\sqrt{11}\right)\right)\int \frac{1}{\left(4+2\sqrt{11}-14x\right)\sqrt{3+2x+5x^2}}\,dx}{184877} \\
&= -\frac{3(571621+196105x)\sqrt{3+2x+5x^2}}{240100} \\
&\quad -\frac{1}{980}(267+35x)\left(3+2x+5x^2\right)^{3/2}-\frac{34425687\sinh^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{840350\sqrt{5}} \\
&\quad -\frac{\left(48\left(2770361-877397\sqrt{11}\right)\right)\text{Subst}\Bigg(\int \frac{1}{2352+112\left(4-2\sqrt{11}\right)+20\left(4-2\sqrt{11}\right)^2-x^2}\,dx,x,\frac{-84-2\left(4-2\sqrt{11}\right)-\big(28}{\sqrt{3+2x+}}}{184877} \\
&\quad -\frac{\left(48\left(2770361+877397\sqrt{11}\right)\right)\text{Subst}\Bigg(\int \frac{1}{2352+112\left(4+2\sqrt{11}\right)+20\left(4+2\sqrt{11}\right)^2-x^2}\,dx,x,\frac{-84-2\left(4+2\sqrt{11}\right)-\big(28}{\sqrt{3+2x+}}}{184877}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{3(571621+196105x)\sqrt{3+2x+5x^2}}{240100} \\
&\quad -\frac{1}{980}(267+35x)\left(3+2x+5x^2\right)^{3/2}-\frac{34425687\sinh^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{840350\sqrt{5}} \\
&\quad -\frac{6\sqrt{178175857354-53550689170\sqrt{11}}\tanh^{-1}\left(\frac{23-\sqrt{11}+\left(17-5\sqrt{11}\right)x}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)}{184877} \\
&\quad +\frac{6\sqrt{\frac{2}{11}\left(8098902607+2434122235\sqrt{11}\right)}\tanh^{-1}\left(\frac{23+\sqrt{11}+\left(17+5\sqrt{11}\right)x}{\sqrt{2\left(125+17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)}{16807}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.56 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.21

$$
\begin{aligned}
&\int \frac{(2+5x+x^2)\left(3+2x+5x^2\right)^{3/2}}{1+4x-7x^2}\,dx = \frac{\sqrt{3+2x+5x^2}(-1911108-744870x-344225x^2-42875x^3)}{240100} \\
&+\frac{34425687\log\left(-1-5x+\sqrt{5}\sqrt{3+2x+5x^2}\right)}{840350\sqrt{5}} \\
&-\frac{12\text{RootSum}\left[83-16\sqrt{5}\#1-70\#1^2+8\sqrt{5}\#1^3+7\#1^4\&,\frac{-648783\sqrt{5}\log\left(-\sqrt{5}x+\sqrt{3+2x+5x^2}-\#1\right)-533850\log\left(-\sqrt{5}x\right.}{-4\sqrt{5}-35}\right.}{16807\sqrt{5}}
\end{aligned}
$$

```
[In] Integrate[((2 + 5*x + x^2)*(3 + 2*x + 5*x^2)^(3/2))/(1 + 4*x - 7*x^2),x]
```

```
[Out] (Sqrt[3 + 2*x + 5*x^2]*(-1911108 - 744870*x - 344225*x^2 - 42875*x^3))/2401
00 + (34425687*Log[-1 - 5*x + Sqrt[5]*Sqrt[3 + 2*x + 5*x^2]])/(840350*Sqrt[
5]) - (12*RootSum[83 - 16*Sqrt[5]*#1 - 70*#1^2 + 8*Sqrt[5]*#1^3 + 7*#1^4 &
, (-648783*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1] - 533850*
Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1 + 251851*Sqrt[5]*Log[-(Sq
rt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1^2)/(-4*Sqrt[5] - 35*#1 + 6*Sqrt[5
]*#1^2 + 7*#1^3) & ])/(16807*Sqrt[5])
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.11

method	result
risch	$-\frac{\left(42875x^{3}+344225x^{2}+744870x+1911108\right)\sqrt{5x^{2}+2x+3}}{240100}-\frac{34425687\sqrt{5}\,\operatorname{arcsinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{4201750}+\frac{12\left(-877397+251851\sqrt{11}\right)}{}$
trager	Expression too large to display
default	Expression too large to display

```
[In] int((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/240100*(42875*x^3+344225*x^2+744870*x+1911108)*(5*x^2+2*x+3)^(1/2)-34425
687/4201750*5^(1/2)*arcsinh(5/14*14^(1/2)*(x+1/5))+12/184877*(-877397+25185
1*11^(1/2))*11^(1/2)/(250-34*11^(1/2))^(1/2)*arctanh(49/2*(500/49-68/49*11^
(1/2)+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2)))/(250-34*11^(1/2))^(1/2)/(2
45*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-
34*11^(1/2))^(1/2))+12/184877*11^(1/2)*(877397+251851*11^(1/2))/(250+34*11^
(1/2))^(1/2)*arctanh(49/2*(500/49+68/49*11^(1/2)+(34/7+10/7*11^(1/2))*(x-2/
7-1/7*11^(1/2)))/(250+34*11^(1/2))^(1/2)/(245*(x-2/7-1/7*11^(1/2))^2+49*(34
/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 326 vs. $2(152) = 304$.

Time = 0.29 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.55

$$\int \frac{\left(2+5x+x^{2}\right)\left(3+2x+5x^{2}\right)^{3/2}}{1+4x-7x^{2}}\,dx=\frac{3}{184877}\sqrt{11}\sqrt{2}\sqrt{2434122235\sqrt{11}+8098902607}\log\left(\frac{12\left(\sqrt{2}\sqrt{5}x\right.}{}\right.$$

$$-\frac{3}{184877}\sqrt{11}\sqrt{2}\sqrt{2434122235\sqrt{11}+8098902607}\log\left(-\frac{12\left(\sqrt{2}\sqrt{5x^{2}+2x+3}\sqrt{2434122235\sqrt{11}+809}\right.}{}\right.$$

$$-\frac{1}{739508}\sqrt{11}\sqrt{-701027203680\sqrt{11}+2332483950816}\log\left(-\frac{\sqrt{5x^{2}+2x+3}\left(7690\sqrt{11}+24697\right)\sqrt{-701}}{}\right.$$

$$+\frac{1}{739508}\sqrt{11}\sqrt{-701027203680\sqrt{11}+2332483950816}\log\left(\frac{\sqrt{5x^{2}+2x+3}\left(7690\sqrt{11}+24697\right)\sqrt{-70102}}{}\right.$$

$$-\frac{1}{240100}\left(42875x^{3}+344225x^{2}+744870x+1911108\right)\sqrt{5x^{2}+2x+3}$$

$$+\frac{34425687}{8403500}\sqrt{5}\log\left(\sqrt{5}\sqrt{5x^{2}+2x+3}(5x+1)-25x^{2}-10x-8\right)$$

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1),x, algorithm="fric
as")
```

```
[Out] 3/184877*sqrt(11)*sqrt(2)*sqrt(2434122235*sqrt(11) + 8098902607)*log(12*(sq
rt(2)*sqrt(5*x^2 + 2*x + 3)*sqrt(2434122235*sqrt(11) + 8098902607)*(7690*sq
rt(11) - 24697) + 40555291*sqrt(11)*(x + 3) + 121665873*x - 202776455)/x) -
 3/184877*sqrt(11)*sqrt(2)*sqrt(2434122235*sqrt(11) + 8098902607)*log(-12*(
sqrt(2)*sqrt(5*x^2 + 2*x + 3)*sqrt(2434122235*sqrt(11) + 8098902607)*(7690*
sqrt(11) - 24697) - 40555291*sqrt(11)*(x + 3) - 121665873*x + 202776455)/x)
 - 1/739508*sqrt(11)*sqrt(-701027203680*sqrt(11) + 2332483950816)*log(-(sqr
t(5*x^2 + 2*x + 3)*(7690*sqrt(11) + 24697)*sqrt(-701027203680*sqrt(11) + 23
32483950816) + 486663492*sqrt(11)*(x + 3) - 1459990476*x + 2433317460)/x) +
 1/739508*sqrt(11)*sqrt(-701027203680*sqrt(11) + 2332483950816)*log((sqrt(5
*x^2 + 2*x + 3)*(7690*sqrt(11) + 24697)*sqrt(-701027203680*sqrt(11) + 23324
83950816) - 486663492*sqrt(11)*(x + 3) + 1459990476*x - 2433317460)/x) - 1/
240100*(42875*x^3 + 344225*x^2 + 744870*x + 1911108)*sqrt(5*x^2 + 2*x + 3)
+ 34425687/8403500*sqrt(5)*log(sqrt(5)*sqrt(5*x^2 + 2*x + 3)*(5*x + 1) - 25
*x^2 - 10*x - 8)
```

Sympy [F]

$$
\begin{aligned}
\int \frac{(2+5x+x^2)\,(3+2x+5x^2)^{3/2}}{1+4x-7x^2}\,dx &= -\int \frac{6\sqrt{5x^2+2x+3}}{7x^2-4x-1}\,dx \\
&\quad - \int \frac{19x\sqrt{5x^2+2x+3}}{7x^2-4x-1}\,dx - \int \frac{23x^2\sqrt{5x^2+2x+3}}{7x^2-4x-1}\,dx \\
&\quad - \int \frac{27x^3\sqrt{5x^2+2x+3}}{7x^2-4x-1}\,dx - \int \frac{5x^4\sqrt{5x^2+2x+3}}{7x^2-4x-1}\,dx
\end{aligned}
$$

```
[In] integrate((x**2+5*x+2)*(5*x**2+2*x+3)**(3/2)/(-7*x**2+4*x+1),x)
```

```
[Out] -Integral(6*sqrt(5*x**2 + 2*x + 3)/(7*x**2 - 4*x - 1), x) - Integral(19*x*s
qrt(5*x**2 + 2*x + 3)/(7*x**2 - 4*x - 1), x) - Integral(23*x**2*sqrt(5*x**2
 + 2*x + 3)/(7*x**2 - 4*x - 1), x) - Integral(27*x**3*sqrt(5*x**2 + 2*x + 3
)/(7*x**2 - 4*x - 1), x) - Integral(5*x**4*sqrt(5*x**2 + 2*x + 3)/(7*x**2 -
 4*x - 1), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 535 vs. $2(152) = 304$.

Time = 0.39 (sec) , antiderivative size = 535, normalized size of antiderivative = 2.55

$$\int \frac{(2+5x+x^2)\left(3+2x+5x^2\right)^{3/2}}{1+4x-7x^2}\,dx = \frac{1}{92438500}\sqrt{11}\left(19500\sqrt{11}\sqrt{2}\left(17\sqrt{11}+125\right)^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{5\sqrt{11}}{7\left|14x-\right.}\right.\right.$$

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1),x, algorithm="maxi
ma")
```

```
[Out] 1/92438500*sqrt(11)*(19500*sqrt(11)*sqrt(2)*(17*sqrt(11) + 125)^(3/2)*arcsi
nh(5/7*sqrt(11)*sqrt(7)*sqrt(2)*x/abs(14*x - 2*sqrt(11) - 4) + 17/7*sqrt(7)
*sqrt(2)*x/abs(14*x - 2*sqrt(11) - 4) + 1/7*sqrt(11)*sqrt(7)*sqrt(2)/abs(14
*x - 2*sqrt(11) - 4) + 23/7*sqrt(7)*sqrt(2)/abs(14*x - 2*sqrt(11) - 4)) - 3
00125*sqrt(11)*(5*x^2 + 2*x + 3)^(3/2)*x - 3344250*sqrt(11)*(-34/49*sqrt(11
) + 250/49)^(3/2)*arcsinh(5/7*sqrt(11)*sqrt(7)*sqrt(2)*x/abs(14*x + 2*sqrt(
11) - 4) - 17/7*sqrt(7)*sqrt(2)*x/abs(14*x + 2*sqrt(11) - 4) + 1/7*sqrt(11)
*sqrt(7)*sqrt(2)/abs(14*x + 2*sqrt(11) - 4) - 23/7*sqrt(7)*sqrt(2)/abs(14*x
 + 2*sqrt(11) - 4)) + 91500*sqrt(2)*(17*sqrt(11) + 125)^(3/2)*arcsinh(5/7*s
qrt(11)*sqrt(7)*sqrt(2)*x/abs(14*x - 2*sqrt(11) - 4) + 17/7*sqrt(7)*sqrt(2)
*x/abs(14*x - 2*sqrt(11) - 4) + 1/7*sqrt(11)*sqrt(7)*sqrt(2)/abs(14*x - 2*s
qrt(11) - 4) + 23/7*sqrt(7)*sqrt(2)/abs(14*x - 2*sqrt(11) - 4)) + 15692250*
(-34/49*sqrt(11) + 250/49)^(3/2)*arcsinh(5/7*sqrt(11)*sqrt(7)*sqrt(2)*x/abs
(14*x + 2*sqrt(11) - 4) - 17/7*sqrt(7)*sqrt(2)*x/abs(14*x + 2*sqrt(11) - 4)
 + 1/7*sqrt(11)*sqrt(7)*sqrt(2)/abs(14*x + 2*sqrt(11) - 4) - 23/7*sqrt(7)*s
qrt(2)/abs(14*x + 2*sqrt(11) - 4)) - 2289525*sqrt(11)*(5*x^2 + 2*x + 3)^(3/
2) - 20591025*sqrt(11)*sqrt(5*x^2 + 2*x + 3)*x - 68851374*sqrt(11)*sqrt(5)*
arcsinh(5/14*sqrt(7)*sqrt(2)*x + 1/14*sqrt(7)*sqrt(2)) - 60020205*sqrt(11)*
sqrt(5*x^2 + 2*x + 3))
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.73

$$\int \frac{(2+5x+x^2)\left(3+2x+5x^2\right)^{3/2}}{1+4x-7x^2}\,dx = -\frac{1}{240100}\,(35\,(35\,(35\,x+281)x+21282)x+1911108)\sqrt{5\,x^2+2\,x+3} + \frac{34425687}{4201750}\,\sqrt{5}\log\left(-5\,\sqrt{5}x-\sqrt{5}+5\,\sqrt{5\,x^2+2\,x+3}\right) + 19.3580321168561\,\log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}+4.41924736459000\right) - 0.773682164624264\,\log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}+1.25295163054000\right) - 19.3580321168561\,\log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}-1.02258038113000\right) + 0.773682164625454\,\log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}-2.09411235400000\right)$$

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1),x, algorithm="giac
")
```

```
[Out] -1/240100*(35*(35*(35*x + 281)*x + 21282)*x + 1911108)*sqrt(5*x^2 + 2*x + 3
) + 34425687/4201750*sqrt(5)*log(-5*sqrt(5)*x - sqrt(5) + 5*sqrt(5*x^2 + 2*
x + 3)) + 19.3580321168561*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) + 4.41924
736459000) - 0.773682164624264*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) + 1.2
5295163054000) - 19.3580321168561*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) -
1.02258038113000) + 0.773682164625454*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3
) - 2.09411235400000)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2+5x+x^2)\left(3+2x+5x^2\right)^{3/2}}{1+4x-7x^2}\,dx = \int \frac{\left(x^2+5\,x+2\right)\,\left(5\,x^2+2\,x+3\right)^{3/2}}{-7\,x^2+4\,x+1}\,dx$$

```
[In] int(((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(3/2))/(4*x - 7*x^2 + 1),x)
```

```
[Out] int(((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(3/2))/(4*x - 7*x^2 + 1), x)
```

3.384 $\int \frac{(2+5x+x^2)(3+2x+5x^2)^{3/2}}{(1+4x-7x^2)^2} \, dx$

Optimal result

Integrand size = 35, antiderivative size = 222

$$\begin{aligned}\int \frac{(2+5x+x^2)(3+2x+5x^2)^{3/2}}{(1+4x-7x^2)^2} \, dx &= \frac{(5826+3395x)\sqrt{3+2x+5x^2}}{3773} \\ &+ \frac{3(3+61x)(3+2x+5x^2)^{3/2}}{154(1+4x-7x^2)} + \frac{16691\operatorname{arcsinh}\left(\frac{1+5x}{\sqrt{14}}\right)}{2401\sqrt{5}} \\ &- \frac{\sqrt{\frac{1}{22}\left(52175400311-13155376531\sqrt{11}\right)}\operatorname{arctanh}\left(\frac{23-\sqrt{11}+\left(17-5\sqrt{11}\right)x}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)}{26411} \\ &- \frac{\sqrt{\frac{1}{22}\left(52175400311+13155376531\sqrt{11}\right)}\operatorname{arctanh}\left(\frac{23+\sqrt{11}+\left(17+5\sqrt{11}\right)x}{\sqrt{2\left(125+17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)}{26411}\end{aligned}$$

```
[Out] 3/154*(3+61*x)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1)+16691/12005*arcsinh(1/14*
(1+5*x)*14^(1/2))*5^(1/2)+1/3773*(5826+3395*x)*(5*x^2+2*x+3)^(1/2)-1/581042
*arctanh((23+x*(17-5*11^(1/2))-11^(1/2))/(5*x^2+2*x+3)^(1/2)/(250-34*11^(1/
2))^(1/2))*(1147858806842-289418283682*11^(1/2))^(1/2)-1/581042*arctanh((23
+11^(1/2)+x*(17+5*11^(1/2)))/(5*x^2+2*x+3)^(1/2)/(250+34*11^(1/2))^(1/2))*(
1147858806842+289418283682*11^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {1068, 1080, 1090, 633, 221, 1046, 738, 212}

$$\int \frac{(2+5x+x^2)(3+2x+5x^2)^{3/2}}{(1+4x-7x^2)^2}\,dx = \frac{16691\text{arcsinh}\left(\frac{5x+1}{\sqrt{14}}\right)}{2401\sqrt{5}} - \frac{\sqrt{\frac{1}{22}\left(52175400311-13155376531\sqrt{11}\right)}\text{arctanh}\left(\frac{\left(17-5\sqrt{11}\right)x-\sqrt{11}+23}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right)}{26411} - \frac{\sqrt{\frac{1}{22}\left(52175400311+13155376531\sqrt{11}\right)}\text{arctanh}\left(\frac{\left(17+5\sqrt{11}\right)x+\sqrt{11}+23}{\sqrt{2\left(125+17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right)}{26411} + \frac{3(61x+3)\left(5x^2+2x+3\right)^{3/2}}{154\left(-7x^2+4x+1\right)} + \frac{(3395x+5826)\sqrt{5x^2+2x+3}}{3773}$$

```
[In] Int[((2 + 5*x + x^2)*(3 + 2*x + 5*x^2)^(3/2))/(1 + 4*x - 7*x^2)^2,x]
```

```
[Out] ((5826 + 3395*x)*Sqrt[3 + 2*x + 5*x^2])/3773 + (3*(3 + 61*x)*(3 + 2*x + 5*x
^2)^(3/2))/(154*(1 + 4*x - 7*x^2)) + (16691*ArcSinh[(1 + 5*x)/Sqrt[14]])/(2
401*Sqrt[5]) - (Sqrt[(52175400311 - 13155376531*Sqrt[11])/22]*ArcTanh[(23 -
 Sqrt[11] + (17 - 5*Sqrt[11])*x)/(Sqrt[2*(125 - 17*Sqrt[11])]*Sqrt[3 + 2*x
+ 5*x^2])])/26411 - (Sqrt[(52175400311 + 13155376531*Sqrt[11])/22]*ArcTanh[
(23 + Sqrt[11] + (17 + 5*Sqrt[11])*x)/(Sqrt[2*(125 + 17*Sqrt[11])]*Sqrt[3 +
 2*x + 5*x^2])])/26411
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1046

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1068

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(A*b*c - 2*
a*B*c + a*b*C - (c*(b*B - 2*A*c) - C*(b^2 - 2*a*c))*x)*(a + b*x + c*x^2)^(p
 + 1)*((d + e*x + f*x^2)^q/(c*(b^2 - 4*a*c)*(p + 1))), x] - Dist[1/(c*(b^2
- 4*a*c)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*
Simp[e*q*(A*b*c - 2*a*B*c + a*b*C) - d*(c*(b*B - 2*A*c)*(2*p + 3) + C*(2*a*
c - b^2*(p + 2))) + (2*f*q*(A*b*c - 2*a*B*c + a*b*C) - e*(c*(b*B - 2*A*c)*(
2*p + q + 3) + C*(2*a*c*(q + 1) - b^2*(p + q + 2))))*x - f*(c*(b*B - 2*A*c)
*(2*p + 2*q + 3) + C*(2*a*c*(2*q + 1) - b^2*(p + 2*q + 2)))*x^2, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2
 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] &&  !IGtQ[q, 0]
```

Rule 1080

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(B*c*f*(2*p
 + 2*q + 3) + C*(b*f*p - c*e*(2*p + q + 2)) + 2*c*C*f*(p + q + 1)*x)*(a + b
*x + c*x^2)^p*((d + e*x + f*x^2)^(q + 1)/(2*c*f^2*(p + q + 1)*(2*p + 2*q +
3))), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + b*x + c*x
^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[p*(b*d - a*e)*(C*(c*e - b*f)*(q + 1) -
 c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(b^2*C*d*f*p + a*c*(C*(2*d*f
- e^2*(2*p + q + 2)) + f*(B*e - 2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)
*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e
*f*p*(b^2 - 4*a*c) - b*c*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e +
2*A*f)*(2*p + 2*q + 3))))*x + (p*(c*e - b*f)*(C*(c*e - b*f)*(q + 1) - c*(C*
e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(e^
2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x^2,
x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] && NeQ[b^2 - 4*a*c,
```

```
0] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*
q + 3, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1090

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\text{integral} = \frac{3(3+61x)\left(3+2x+5x^2\right)^{3/2}}{154\left(1+4x-7x^2\right)} - \frac{1}{308}\int \frac{\sqrt{3+2x+5x^2}\left(-912+724x+3880x^2\right)}{1+4x-7x^2}\,dx$$

$$= \frac{(5826+3395x)\sqrt{3+2x+5x^2}}{3773} + \frac{3(3+61x)\left(3+2x+5x^2\right)^{3/2}}{154\left(1+4x-7x^2\right)} + \frac{\int \frac{700200-4304880x-7344040x^2}{\left(1+4x-7x^2\right)\sqrt{3+2x+5x^2}}\,dx}{150920}$$

$$= \frac{(5826+3395x)\sqrt{3+2x+5x^2}}{3773} + \frac{3(3+61x)\left(3+2x+5x^2\right)^{3/2}}{154\left(1+4x-7x^2\right)} - \frac{\int \frac{2442640+59510320x}{\left(1+4x-7x^2\right)\sqrt{3+2x+5x^2}}\,dx}{1056440} + \frac{16691\int \frac{1}{\sqrt{3+2x+5x^2}}\,dx}{2401}$$

$$= \frac{(5826+3395x)\sqrt{3+2x+5x^2}}{3773} + \frac{3(3+61x)\left(3+2x+5x^2\right)^{3/2}}{154\left(1+4x-7x^2\right)} + \frac{16691\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{56}}}\,dx, x, 2+10x\right)}{4802\sqrt{70}} - \frac{\left(2\left(8182669-1701489\sqrt{11}\right)\right)\int \frac{1}{\left(4-2\sqrt{11}-14x\right)\sqrt{3+2x+5x^2}}\,dx}{290521} - \frac{\left(2\left(8182669+1701489\sqrt{11}\right)\right)\int \frac{1}{\left(4+2\sqrt{11}-14x\right)\sqrt{3+2x+5x^2}}\,dx}{290521}$$

$$= \frac{(5826+3395x)\sqrt{3+2x+5x^2}}{3773} + \frac{3(3+61x)\left(3+2x+5x^2\right)^{3/2}}{154\left(1+4x-7x^2\right)} + \frac{16691\sinh^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{2401\sqrt{5}} + \frac{\left(4\left(8182669-1701489\sqrt{11}\right)\right)\text{Subst}\Bigg(\int \frac{1}{2352+112\left(4-2\sqrt{11}\right)+20\left(4-2\sqrt{11}\right)^2-x^2}\,dx, x, \frac{-84-2\left(4-2\sqrt{11}\right)-\big(28+}{\sqrt{3+2x+5x}}}{290521} + \frac{\left(4\left(8182669+1701489\sqrt{11}\right)\right)\text{Subst}\Bigg(\int \frac{1}{2352+112\left(4+2\sqrt{11}\right)+20\left(4+2\sqrt{11}\right)^2-x^2}\,dx, x, \frac{-84-2\left(4+2\sqrt{11}\right)-\big(28+}{\sqrt{3+2x+5x}}}{290521}$$

$$= \frac{(5826+3395x)\sqrt{3+2x+5x^2}}{3773} + \frac{3(3+61x)\left(3+2x+5x^2\right)^{3/2}}{154\left(1+4x-7x^2\right)} + \frac{16691\sinh^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{2401\sqrt{5}}$$

$$- \frac{\sqrt{\frac{1}{22}\left(52175400311-13155376531\sqrt{11}\right)}\tanh^{-1}\left(\frac{23-\sqrt{11}+\left(17-5\sqrt{11}\right)x}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)}{26411}$$

$$- \frac{\sqrt{\frac{1}{22}\left(52175400311+13155376531\sqrt{11}\right)}\tanh^{-1}\left(\frac{23+\sqrt{11}+\left(17+5\sqrt{11}\right)x}{\sqrt{2\left(125+17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)}{26411}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.70 (sec) , antiderivative size = 447, normalized size of antiderivative = 2.01

$$\int \frac{\left(2+5x+x^2\right)\left(3+2x+5x^2\right)^{3/2}}{\left(1+4x-7x^2\right)^2}\,dx = \frac{\frac{1715\sqrt{3+2x+5x^2}\left(-12975-81181x+34265x^2+2695x^3\right)}{-1-4x+7x^2} - 17992898\sqrt{5}\log(-1 -}{}$$

```
[In] Integrate[((2 + 5*x + x^2)*(3 + 2*x + 5*x^2)^(3/2))/(1 + 4*x - 7*x^2)^2,x]
```

```
[Out] ((1715*Sqrt[3 + 2*x + 5*x^2]*(-12975 - 81181*x + 34265*x^2 + 2695*x^3))/(-1
 - 4*x + 7*x^2) - 17992898*Sqrt[5]*Log[-1 - 5*x + Sqrt[5]*Sqrt[3 + 2*x + 5*
x^2]] + 44*Sqrt[5]*RootSum[83 - 16*Sqrt[5]*#1 - 70*#1^2 + 8*Sqrt[5]*#1^3 +
7*#1^4 & , (25954129*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]
 - 19416530*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1 + 2717099*Sqr
t[5]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1^2)/(-4*Sqrt[5] - 35*
#1 + 6*Sqrt[5]*#1^2 + 7*#1^3) & ] - 6*Sqrt[5]*RootSum[83 - 16*Sqrt[5]*#1 -
70*#1^2 + 8*Sqrt[5]*#1^3 + 7*#1^4 & , (225782939*Sqrt[5]*Log[-(Sqrt[5]*x) +
 Sqrt[3 + 2*x + 5*x^2] - #1] - 137400830*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x +
5*x^2] - #1]*#1 + 7775369*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2]
- #1]*#1^2)/(-4*Sqrt[5] - 35*#1 + 6*Sqrt[5]*#1^2 + 7*#1^3) & ])/12941390
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.10

method	result
risch	$\frac{\left(2695x^3+34265x^2-81181x-12975\right)\sqrt{5x^2+2x+3}}{52822x^2-30184x-7546}+\frac{16691\sqrt{5}\ \operatorname{arcsinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{12005}-\frac{\left(1701489+743879\sqrt{11}\right)\sqrt{11}\ \operatorname{arctanh}\left(\right.}{}$
trager	Expression too large to display
default	Expression too large to display

```
[In] int((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1)^2,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/7546*(2695*x^3+34265*x^2-81181*x-12975)/(7*x^2-4*x-1)*(5*x^2+2*x+3)^(1/2)
+16691/12005*5^(1/2)*arcsinh(5/14*14^(1/2)*(x+1/5))-1/290521*(1701489+74387
9*11^(1/2))*11^(1/2)/(250+34*11^(1/2))^(1/2)*arctanh(49/2*(500/49+68/49*11^
(1/2)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2)))/(250+34*11^(1/2))^(1/2)/(2
45*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+
34*11^(1/2))^(1/2))-1/290521*(-1701489+743879*11^(1/2))*11^(1/2)/(250-34*11
^(1/2))^(1/2)*arctanh(49/2*(500/49-68/49*11^(1/2)+(34/7-10/7*11^(1/2))*(x-2
/7+1/7*11^(1/2)))/(250-34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(1/2))^2+49*(3
4/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 378 vs. 2(164) = 328.

Time = 0.28 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.70

$$\int \frac{(2+5x+x^2)\left(3+2x+5x^2\right)^{3/2}}{\left(1+4x-7x^2\right)^2}\,dx=\frac{5\,\sqrt{11}(7\,x^2-4\,x-1)\sqrt{26310753062\,\sqrt{11}+104350800622}\log\left(\right.}{}$$

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1)^2,x, algorithm="fr
icas")
```

```
[Out] 1/5810420*(5*sqrt(11)*(7*x^2 - 4*x - 1)*sqrt(26310753062*sqrt(11) + 1043508
00622)*log((sqrt(5*x^2 + 2*x + 3)*sqrt(26310753062*sqrt(11) + 104350800622)
*(16206*sqrt(11) - 68441) + 1795191685*sqrt(11)*(x + 3) + 5385575055*x - 89
75958425)/x) - 5*sqrt(11)*(7*x^2 - 4*x - 1)*sqrt(26310753062*sqrt(11) + 104
350800622)*log(-(sqrt(5*x^2 + 2*x + 3)*sqrt(26310753062*sqrt(11) + 10435080
0622)*(16206*sqrt(11) - 68441) - 1795191685*sqrt(11)*(x + 3) - 5385575055*x
 + 8975958425)/x) - 5*sqrt(11)*(7*x^2 - 4*x - 1)*sqrt(-26310753062*sqrt(11)
 + 104350800622)*log(-(sqrt(5*x^2 + 2*x + 3)*(16206*sqrt(11) + 68441)*sqrt(
-26310753062*sqrt(11) + 104350800622) + 1795191685*sqrt(11)*(x + 3) - 53855
75055*x + 8975958425)/x) + 5*sqrt(11)*(7*x^2 - 4*x - 1)*sqrt(-26310753062*s
qrt(11) + 104350800622)*log((sqrt(5*x^2 + 2*x + 3)*(16206*sqrt(11) + 68441)
```

```
*sqrt(-26310753062*sqrt(11) + 104350800622) - 1795191685*sqrt(11)*(x + 3) +
 5385575055*x - 8975958425)/x) + 4039222*sqrt(5)*(7*x^2 - 4*x - 1)*log(-sqr
t(5)*sqrt(5*x^2 + 2*x + 3)*(5*x + 1) - 25*x^2 - 10*x - 8) + 770*(2695*x^3 +
 34265*x^2 - 81181*x - 12975)*sqrt(5*x^2 + 2*x + 3))/(7*x^2 - 4*x - 1)
```

Sympy [F]

$$\int \frac{(2+5x+x^2)(3+2x+5x^2)^{3/2}}{(1+4x-7x^2)^2}\,dx = \int \frac{(x^2+5x+2)(5x^2+2x+3)^{\frac{3}{2}}}{(7x^2-4x-1)^2}\,dx$$

```
[In] integrate((x**2+5*x+2)*(5*x**2+2*x+3)**(3/2)/(-7*x**2+4*x+1)**2,x)
[Out] Integral((x**2 + 5*x + 2)*(5*x**2 + 2*x + 3)**(3/2)/(7*x**2 - 4*x - 1)**2,
x)
```

Maxima [F]

$$\int \frac{(2+5x+x^2)(3+2x+5x^2)^{3/2}}{(1+4x-7x^2)^2}\,dx = \int \frac{(5\,x^2+2\,x+3)^{\frac{3}{2}}(x^2+5\,x+2)}{(7\,x^2-4\,x-1)^2}\,dx$$

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1)^2,x, algorithm="ma
xima")
[Out] integrate((5*x^2 + 2*x + 3)^(3/2)*(x^2 + 5*x + 2)/(7*x^2 - 4*x - 1)^2, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(2+5x+x^2)(3+2x+5x^2)^{3/2}}{(1+4x-7x^2)^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1)^2,x, algorithm="gi
ac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{63274455776,[8]%%%}+%%%{%%{[144627327488,0]:[1,0,-5]%%},[7]%%
%}+%%%
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2+5x+x^2)\,(3+2x+5x^2)^{3/2}}{(1+4x-7x^2)^2}\,dx=\int \frac{(x^2+5\,x+2)\,\left(5\,x^2+2\,x+3\right)^{3/2}}{\left(-7\,x^2+4\,x+1\right)^2}\,dx$$

```
[In] int(((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(3/2))/(4*x - 7*x^2 + 1)^2,x)
[Out] int(((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(3/2))/(4*x - 7*x^2 + 1)^2, x)
```

3.385 $\int \frac{(2+5x+x^2)(3+2x+5x^2)^{3/2}}{(1+4x-7x^2)^3} \, dx$

Optimal result

Integrand size = 35, antiderivative size = 234

$$\begin{aligned}\int \frac{(2+5x+x^2)(3+2x+5x^2)^{3/2}}{(1+4x-7x^2)^3} \, dx &= -\frac{(9495-37088x)\sqrt{3+2x+5x^2}}{23716(1+4x-7x^2)} \\ &+ \frac{3(3+61x)(3+2x+5x^2)^{3/2}}{308(1+4x-7x^2)^2} - \frac{5}{343}\sqrt{5}\operatorname{arcsinh}\left(\frac{1+5x}{\sqrt{14}}\right) \\ &- \frac{\sqrt{\frac{62294197250171-2085440742055\sqrt{11}}{2794}}\operatorname{arctanh}\left(\frac{23-\sqrt{11}+\left(17-5\sqrt{11}\right)x}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)}{332024} \\ &+ \frac{\sqrt{\frac{62294197250171+2085440742055\sqrt{11}}{2794}}\operatorname{arctanh}\left(\frac{23+\sqrt{11}+\left(17+5\sqrt{11}\right)x}{\sqrt{2\left(125+17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)}{332024}\end{aligned}$$

```
[Out] 3/308*(3+61*x)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1)^2-5/343*arcsinh(1/14*(1+5
*x)*14^(1/2))*5^(1/2)-1/23716*(9495-37088*x)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*
x+1)-1/927675056*arctanh((23+x*(17-5*11^(1/2))-11^(1/2))/(5*x^2+2*x+3)^(1/2
)/(250-34*11^(1/2))^(1/2))*(17404998711697774-5826721433301670*11^(1/2))^(
1/2)+1/927675056*arctanh((23+11^(1/2)+x*(17+5*11^(1/2)))/(5*x^2+2*x+3)^(1/2
)/(250+34*11^(1/2))^(1/2))*(17404998711697774+5826721433301670*11^(1/2))^(
1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1068, 1090, 633, 221, 1046, 738, 212}

$$\int \frac{(2+5x+x^2)\left(3+2x+5x^2\right)^{3/2}}{\left(1+4x-7x^2\right)^3}\,dx = -\frac{5}{343}\sqrt{5}\text{arcsinh}\left(\frac{5x+1}{\sqrt{14}}\right) - \frac{\sqrt{\frac{62294197250171-2085440742055\sqrt{11}}{2794}}\text{arctanh}\left(\frac{\left(17-5\sqrt{11}\right)x-\sqrt{11}+23}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right)}{332024} + \frac{\sqrt{\frac{62294197250171+2085440742055\sqrt{11}}{2794}}\text{arctanh}\left(\frac{\left(17+5\sqrt{11}\right)x+\sqrt{11}+23}{\sqrt{2\left(125+17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right)}{332024} + \frac{3(61x+3)\left(5x^2+2x+3\right)^{3/2}}{308\left(-7x^2+4x+1\right)^2} - \frac{(9495-37088x)\sqrt{5x^2+2x+3}}{23716\left(-7x^2+4x+1\right)}$$

```
[In] Int[((2 + 5*x + x^2)*(3 + 2*x + 5*x^2)^(3/2))/(1 + 4*x - 7*x^2)^3,x]
```

```
[Out] -1/23716*((9495 - 37088*x)*Sqrt[3 + 2*x + 5*x^2])/(1 + 4*x - 7*x^2) + (3*(3
 + 61*x)*(3 + 2*x + 5*x^2)^(3/2))/(308*(1 + 4*x - 7*x^2)^2) - (5*Sqrt[5]*Ar
cSinh[(1 + 5*x)/Sqrt[14]])/343 - (Sqrt[(62294197250171 - 2085440742055*Sqrt
[11])/2794]*ArcTanh[(23 - Sqrt[11] + (17 - 5*Sqrt[11])*x)/(Sqrt[2*(125 - 17
*Sqrt[11])]*Sqrt[3 + 2*x + 5*x^2])])/332024 + (Sqrt[(62294197250171 + 20854
40742055*Sqrt[11])/2794]*ArcTanh[(23 + Sqrt[11] + (17 + 5*Sqrt[11])*x)/(Sqr
t[2*(125 + 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x^2])])/332024
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1046

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1068

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(A*b*c - 2*
a*B*c + a*b*C - (c*(b*B - 2*A*c) - C*(b^2 - 2*a*c))*x)*(a + b*x + c*x^2)^(p
 + 1)*((d + e*x + f*x^2)^q/(c*(b^2 - 4*a*c)*(p + 1))), x] - Dist[1/(c*(b^2
- 4*a*c)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*
Simp[e*q*(A*b*c - 2*a*B*c + a*b*C) - d*(c*(b*B - 2*A*c)*(2*p + 3) + C*(2*a*
c - b^2*(p + 2))) + (2*f*q*(A*b*c - 2*a*B*c + a*b*C) - e*(c*(b*B - 2*A*c)*(
2*p + q + 3) + C*(2*a*c*(q + 1) - b^2*(p + q + 2))))*x - f*(c*(b*B - 2*A*c)
*(2*p + 2*q + 3) + C*(2*a*c*(2*q + 1) - b^2*(p + 2*q + 2)))*x^2, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2
 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] &&  !IGtQ[q, 0]
```

Rule 1090

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\text{integral} = \frac{3(3+61x)\left(3+2x+5x^2\right)^{3/2}}{308\left(1+4x-7x^2\right)^2} - \frac{1}{616}\int \frac{\sqrt{3+2x+5x^2}\left(-2976-652x+440x^2\right)}{\left(1+4x-7x^2\right)^2}\,dx$$

$$= -\frac{(9495-37088x)\sqrt{3+2x+5x^2}}{23716\left(1+4x-7x^2\right)} + \frac{3(3+61x)\left(3+2x+5x^2\right)^{3/2}}{308\left(1+4x-7x^2\right)^2} + \frac{\int \frac{1024152+715224x+96800x^2}{(1+4x-7x^2)\sqrt{3+2x+5x^2}}\,dx}{189728}$$

$$= -\frac{(9495-37088x)\sqrt{3+2x+5x^2}}{23716\left(1+4x-7x^2\right)} + \frac{3(3+61x)\left(3+2x+5x^2\right)^{3/2}}{308\left(1+4x-7x^2\right)^2} - \frac{\int \frac{-7265864-5393768x}{(1+4x-7x^2)\sqrt{3+2x+5x^2}}\,dx}{1328096} - \frac{25}{343}\int \frac{1}{\sqrt{3+2x+5x^2}}\,dx$$

$$= -\frac{(9495-37088x)\sqrt{3+2x+5x^2}}{23716\left(1+4x-7x^2\right)} + \frac{3(3+61x)\left(3+2x+5x^2\right)^{3/2}}{308\left(1+4x-7x^2\right)^2} - \frac{1}{686}\left(5\sqrt{\frac{5}{14}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{56}}}\,dx, x, 2+10x\right) - \frac{\left(-7416431+7706073\sqrt{11}\right)\int \frac{1}{\left(4-2\sqrt{11}-14x\right)\sqrt{3+2x+5x^2}}\,dx}{1826132} + \frac{\left(7416431+7706073\sqrt{11}\right)\int \frac{1}{\left(4+2\sqrt{11}-14x\right)\sqrt{3+2x+5x^2}}\,dx}{1826132}$$

$$= -\frac{(9495-37088x)\sqrt{3+2x+5x^2}}{23716\left(1+4x-7x^2\right)} + \frac{3(3+61x)\left(3+2x+5x^2\right)^{3/2}}{308\left(1+4x-7x^2\right)^2} - \frac{5}{343}\sqrt{5}\sinh^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)$$

$$- \frac{\left(7416431-7706073\sqrt{11}\right)\text{Subst}\Bigg(\int \frac{1}{2352+112\left(4-2\sqrt{11}\right)+20\left(4-2\sqrt{11}\right)^2-x^2}\,dx, x, \frac{-84-2\left(4-2\sqrt{11}\right)-\Big(28+10\Big(4}{\sqrt{3+2x+5x^2}}}{913066}$$

$$- \frac{\left(7416431+7706073\sqrt{11}\right)\text{Subst}\Bigg(\int \frac{1}{2352+112\left(4+2\sqrt{11}\right)+20\left(4+2\sqrt{11}\right)^2-x^2}\,dx, x, \frac{-84-2\left(4+2\sqrt{11}\right)-\Big(28+10\Big(4}{\sqrt{3+2x+5x^2}}}{913066}$$

$$
\begin{aligned}
&= -\frac{(9495-37088x)\sqrt{3+2x+5x^2}}{23716\,(1+4x-7x^2)} \\
&\quad + \frac{3(3+61x)\,(3+2x+5x^2)^{3/2}}{308\,(1+4x-7x^2)^2} - \frac{5}{343}\sqrt{5}\sinh^{-1}\left(\frac{1+5x}{\sqrt{14}}\right) \\
&\quad - \frac{\left(7706073-674221\sqrt{11}\right)\tanh^{-1}\left(\frac{23-\sqrt{11}+\left(17-5\sqrt{11}\right)x}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)}{332024\sqrt{22\left(125-17\sqrt{11}\right)}} \\
&\quad + \frac{\left(7706073+674221\sqrt{11}\right)\tanh^{-1}\left(\frac{23+\sqrt{11}+\left(17+5\sqrt{11}\right)x}{\sqrt{2\left(125+17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)}{332024\sqrt{22\left(125+17\sqrt{11}\right)}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.81 (sec) , antiderivative size = 636, normalized size of antiderivative = 2.72

$$
\begin{aligned}
&\int \frac{(2+5x+x^2)\,(3+2x+5x^2)^{3/2}}{(1+4x-7x^2)^3}\,dx = \frac{\sqrt{3+2x+5x^2}(-7416+42767x+246464x^2-189161x^3)}{23716\,(-1-4x+7x^2)^2} \\
&+ \frac{5}{343}\sqrt{5}\log\left(-1-5x+\sqrt{5}\sqrt{3+2x+5x^2}\right) \\
&- \frac{\text{RootSum}\Big[83-16\sqrt{5}\#1-70\#1^2+8\sqrt{5}\#1^3+7\#1^4\&, \frac{4506829\sqrt{5}\log\left(-\sqrt{5}x+\sqrt{3+2x+5x^2}-\#1\right)-1320270\log\left(-\sqrt{5}x\right.}{-4\sqrt{5}-35\#}}{33614\sqrt{5}} \\
&+ \frac{\text{RootSum}\Big[83-16\sqrt{5}\#1-70\#1^2+8\sqrt{5}\#1^3+7\#1^4\&, \frac{-16323208013227\sqrt{5}\log\left(-\sqrt{5}x+\sqrt{3+2x+5x^2}-\#1\right)+151120773}{}}{71748713246\sqrt{5}} \\
&- \frac{3\text{RootSum}\Big[83-16\sqrt{5}\#1-70\#1^2+8\sqrt{5}\#1^3+7\#1^4\&, \frac{-4192656948824863\sqrt{5}\log\left(-\sqrt{5}x+\sqrt{3+2x+5x^2}-\#1\right)+24518}{}}{34726377211064\sqrt{}}
\end{aligned}
$$

```
[In] Integrate[((2 + 5*x + x^2)*(3 + 2*x + 5*x^2)^(3/2))/(1 + 4*x - 7*x^2)^3,x]
```

```
[Out] (Sqrt[3 + 2*x + 5*x^2]*(-7416 + 42767*x + 246464*x^2 - 189161*x^3))/(23716*
(-1 - 4*x + 7*x^2)^2) + (5*Sqrt[5]*Log[-1 - 5*x + Sqrt[5]*Sqrt[3 + 2*x + 5*
x^2]])/343 - RootSum[83 - 16*Sqrt[5]*#1 - 70*#1^2 + 8*Sqrt[5]*#1^3 + 7*#1^4
 & , (4506829*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1] - 1320
270*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1 + 64435*Sqrt[5]*Log[-
(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1^2)/(-4*Sqrt[5] - 35*#1 + 6*Sqr
t[5]*#1^2 + 7*#1^3) & ]/(33614*Sqrt[5]) + RootSum[83 - 16*Sqrt[5]*#1 - 70*#
```

```
1^2 + 8*Sqrt[5]*#1^3 + 7*#1^4 & , (-16323208013227*Sqrt[5]*Log[-(Sqrt[5]*x)
 + Sqrt[3 + 2*x + 5*x^2] - #1] + 151120773150070*Log[-(Sqrt[5]*x) + Sqrt[3
+ 2*x + 5*x^2] - #1]*#1 + 21832390993791*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[3
+ 2*x + 5*x^2] - #1]*#1^2)/(-4*Sqrt[5] - 35*#1 + 6*Sqrt[5]*#1^2 + 7*#1^3) &
 ]/(71748713246*Sqrt[5]) - (3*RootSum[83 - 16*Sqrt[5]*#1 - 70*#1^2 + 8*Sqrt
[5]*#1^3 + 7*#1^4 & , (-4192656948824863*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[3
+ 2*x + 5*x^2] - #1] + 24518831643829090*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x +
5*x^2] - #1]*#1 + 3523608887504055*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x
+ 5*x^2] - #1]*#1^2)/(-4*Sqrt[5] - 35*#1 + 6*Sqrt[5]*#1^2 + 7*#1^3) & ])/(3
4726377211064*Sqrt[5])
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.05

method	result
risch	$-\frac{\left(189161 x^{3}-246464 x^{2}-42767 x +7416\right) \sqrt{5 x^{2}+2 x +3}}{23716 \left(7 x^{2}-4 x -1\right)^{2}}-\frac{5 \sqrt{5}\, \operatorname{arcsinh}\left(\frac{5 \sqrt{14}\, \left(x +\frac{1}{5}\right)}{14}\right)}{343}+\frac{\left(7706073+674221 \sqrt{11}\right) \sqrt{11}\, \operatorname{arctanh}\left(\right.}{}$
trager	Expression too large to display
default	Expression too large to display

```
[In] int((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1)^3,x,method=_RETURNVERBOS
E)
```

```
[Out] -1/23716*(189161*x^3-246464*x^2-42767*x+7416)/(7*x^2-4*x-1)^2*(5*x^2+2*x+3)
^(1/2)-5/343*5^(1/2)*arcsinh(5/14*14^(1/2)*(x+1/5))+1/3652264*(7706073+6742
21*11^(1/2))*11^(1/2)/(250+34*11^(1/2))^(1/2)*arctanh(49/2*(500/49+68/49*11
^(1/2)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2)))/(250+34*11^(1/2))^(1/2)/(
245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250
+34*11^(1/2))^(1/2))+1/3652264*(-7706073+674221*11^(1/2))*11^(1/2)/(250-34*
11^(1/2))^(1/2)*arctanh(49/2*(500/49-68/49*11^(1/2)+(34/7-10/7*11^(1/2))*(x
-2/7+1/7*11^(1/2)))/(250-34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(1/2))^2+49*
(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 447 vs. $2(176) = 352$.

Time = 0.29 (sec) , antiderivative size = 447, normalized size of antiderivative = 1.91

$$\int \frac{(2+5x+x^2)(3+2x+5x^2)^{3/2}}{(1+4x-7x^2)^3}\,dx =$$

$$-\frac{\sqrt{2794}(49\,x^4-56\,x^3+2\,x^2+8\,x+1)\sqrt{2085440742055\,\sqrt{11}+62294197250171}\log\left(\frac{\sqrt{2794}\sqrt{5\,x^2+2\,x+3}\sqrt{208}}{}\right.}{}$$

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1)^3,x, algorithm="fr
icas")
```

```
[Out] -1/1855350112*(sqrt(2794)*(49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)*sqrt(20854407
42055*sqrt(11) + 62294197250171)*log((sqrt(2794)*sqrt(5*x^2 + 2*x + 3)*sqrt
(2085440742055*sqrt(11) + 62294197250171)*(11840590*sqrt(11) - 83479737) +
542667120256069*sqrt(11)*(x + 3) + 16280013607680207*x - 27133356012800345
)/x) - sqrt(2794)*(49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)*sqrt(2085440742055*sq
rt(11) + 62294197250171)*log(-(sqrt(2794)*sqrt(5*x^2 + 2*x + 3)*sqrt(208544
0742055*sqrt(11) + 62294197250171)*(11840590*sqrt(11) - 83479737) - 5426671
202560069*sqrt(11)*(x + 3) - 16280013607680207*x + 27133356012800345)/x) +
sqrt(2794)*(49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)*sqrt(-2085440742055*sqrt(11)
 + 62294197250171)*log(-(sqrt(2794)*sqrt(5*x^2 + 2*x + 3)*(11840590*sqrt(11
) + 83479737)*sqrt(-2085440742055*sqrt(11) + 62294197250171) + 542667120256
0069*sqrt(11)*(x + 3) - 16280013607680207*x + 27133356012800345)/x) - sqrt(
2794)*(49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)*sqrt(-2085440742055*sqrt(11) + 62
294197250171)*log((sqrt(2794)*sqrt(5*x^2 + 2*x + 3)*(11840590*sqrt(11) + 83
479737)*sqrt(-2085440742055*sqrt(11) + 62294197250171) - 5426671202560069*s
qrt(11)*(x + 3) + 16280013607680207*x - 27133356012800345)/x) - 13522960*sq
rt(5)*(49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)*log(sqrt(5)*sqrt(5*x^2 + 2*x + 3)
*(5*x + 1) - 25*x^2 - 10*x - 8) + 78232*(189161*x^3 - 246464*x^2 - 42767*x
+ 7416)*sqrt(5*x^2 + 2*x + 3))/(49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)
```

Sympy [F]

$$\int \frac{(2+5x+x^2)\,(3+2x+5x^2)^{3/2}}{(1+4x-7x^2)^3}\,dx = -\int \frac{6\sqrt{5x^2+2x+3}}{343x^6-588x^5+189x^4+104x^3-27x^2-12x-1}\,dx - \int \frac{19x\sqrt{5x^2+2x+3}}{343x^6-588x^5+189x^4+104x^3-27x^2-12x-1}\,dx - \int \frac{23x^2\sqrt{5x^2+2x+3}}{343x^6-588x^5+189x^4+104x^3-27x^2-12x-1}\,dx - \int \frac{27x^3\sqrt{5x^2+2x+3}}{343x^6-588x^5+189x^4+104x^3-27x^2-12x-1}\,dx - \int \frac{5x^4\sqrt{5x^2+2x+3}}{343x^6-588x^5+189x^4+104x^3-27x^2-12x-1}\,dx$$

```
[In] integrate((x**2+5*x+2)*(5*x**2+2*x+3)**(3/2)/(-7*x**2+4*x+1)**3,x)
```

```
[Out] -Integral(6*sqrt(5*x**2 + 2*x + 3)/(343*x**6 - 588*x**5 + 189*x**4 + 104*x*
*3 - 27*x**2 - 12*x - 1), x) - Integral(19*x*sqrt(5*x**2 + 2*x + 3)/(343*x*
*6 - 588*x**5 + 189*x**4 + 104*x**3 - 27*x**2 - 12*x - 1), x) - Integral(23
*x**2*sqrt(5*x**2 + 2*x + 3)/(343*x**6 - 588*x**5 + 189*x**4 + 104*x**3 - 2
7*x**2 - 12*x - 1), x) - Integral(27*x**3*sqrt(5*x**2 + 2*x + 3)/(343*x**6
- 588*x**5 + 189*x**4 + 104*x**3 - 27*x**2 - 12*x - 1), x) - Integral(5*x**
4*sqrt(5*x**2 + 2*x + 3)/(343*x**6 - 588*x**5 + 189*x**4 + 104*x**3 - 27*x*
*2 - 12*x - 1), x)
```

Maxima [F]

$$\int \frac{(2+5x+x^2)\,(3+2x+5x^2)^{3/2}}{(1+4x-7x^2)^3}\,dx = \int -\frac{(5\,x^2+2\,x+3)^{\frac{3}{2}}(x^2+5\,x+2)}{(7\,x^2-4\,x-1)^3}\,dx$$

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1)^3,x, algorithm="ma
xima")
```

```
[Out] -integrate((5*x^2 + 2*x + 3)^(3/2)*(x^2 + 5*x + 2)/(7*x^2 - 4*x - 1)^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 411 vs. $2(176) = 352$.

Time = 0.33 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.76

$$\int \frac{(2+5x+x^2)\,(3+2x+5x^2)^{3/2}}{(1+4x-7x^2)^3}\,dx = \frac{5}{343}\sqrt{5}\log\left(-\sqrt{5}\left(\sqrt{5}x-\sqrt{5\,x^2+2\,x+3}\right)-1\right)$$

$$+\frac{264327\,\left(\sqrt{5}x-\sqrt{5\,x^2+2\,x+3}\right)^7-3224225\,\sqrt{5}\left(\sqrt{5}x-\sqrt{5\,x^2+2\,x+3}\right)^6-87069759\,\left(\sqrt{5}x-\sqrt{5\,x^2+2}\right.}{83006\left(7\,\left(\sqrt{5}x-\sqrt{5\,x^2}\right.\right.}$$

$$+\,0.474028359166807\,\log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}+4.41924736459000\right)$$

$$-\,0.424017987131739\,\log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}+1.25295163054000\right)$$

$$-\,0.474028359166807\,\log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}-1.02258038113000\right)$$

$$+\,0.424017987131739\,\log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}-2.09411235400000\right)$$

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1)^3,x, algorithm="gi
ac")
```

```
[Out] 5/343*sqrt(5)*log(-sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3)) - 1) + 1/830
06*(264327*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^7 - 3224225*sqrt(5)*(sqrt(5)
*x - sqrt(5*x^2 + 2*x + 3))^6 - 87069759*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3)
)^5 - 36535763*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^4 + 416818149*(s
qrt(5)*x - sqrt(5*x^2 + 2*x + 3))^3 + 204858869*sqrt(5)*(sqrt(5)*x - sqrt(5
*x^2 + 2*x + 3))^2 - 411908789*sqrt(5)*x - 187277977*sqrt(5) + 411908789*sq
rt(5*x^2 + 2*x + 3))/(7*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^4 - 8*sqrt(5)*(
sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^3 - 70*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3
))^2 + 16*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3)) + 83)^2 + 0.474028359
166807*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) + 4.41924736459000) - 0.42401
7987131739*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) + 1.25295163054000) - 0.4
74028359166807*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) - 1.02258038113000) +
 0.424017987131739*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) - 2.0941123540000
0)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2+5x+x^2)\,(3+2x+5x^2)^{3/2}}{(1+4x-7x^2)^3}\,dx=\int \frac{(x^2+5\,x+2)\,\left(5\,x^2+2\,x+3\right)^{3/2}}{\left(-7\,x^2+4\,x+1\right)^3}\,dx$$

```
[In] int(((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(3/2))/(4*x - 7*x^2 + 1)^3,x)
[Out] int(((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(3/2))/(4*x - 7*x^2 + 1)^3, x)
```

3.386 $\int \frac{(1+4x-7x^2)^3(2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 185

$$\int \frac{(1+4x-7x^2)^3(2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx = -\frac{16515809\sqrt{3+2x+5x^2}}{156250}+\frac{5793077x\sqrt{3+2x+5x^2}}{75000} +\frac{40722851x^2\sqrt{3+2x+5x^2}}{750000} -\frac{5160533x^3\sqrt{3+2x+5x^2}}{50000} -\frac{47807x^4\sqrt{3+2x+5x^2}}{3750} +\frac{26159}{300}x^5\sqrt{3+2x+5x^2}-\frac{1141}{40}x^6\sqrt{3+2x+5x^2} -\frac{343}{40}x^7\sqrt{3+2x+5x^2}-\frac{77513689\operatorname{arcsinh}\left(\frac{1+5x}{\sqrt{14}}\right)}{625000\sqrt{5}}$$

```
[Out] -77513689/3125000*arcsinh(1/14*(1+5*x)*14^(1/2))*5^(1/2)-16515809/156250*(5
*x^2+2*x+3)^(1/2)+5793077/75000*x*(5*x^2+2*x+3)^(1/2)+40722851/750000*x^2*(
5*x^2+2*x+3)^(1/2)-5160533/50000*x^3*(5*x^2+2*x+3)^(1/2)-47807/3750*x^4*(5*
x^2+2*x+3)^(1/2)+26159/300*x^5*(5*x^2+2*x+3)^(1/2)-1141/40*x^6*(5*x^2+2*x+3
)^(1/2)-343/40*x^7*(5*x^2+2*x+3)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {1675, 654, 633, 221}

$$\begin{aligned}\int \frac{(1+4x-7x^2)^3(2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx = &-\frac{77513689\operatorname{arcsinh}\left(\frac{5x+1}{\sqrt{14}}\right)}{625000\sqrt{5}}\\ &+\frac{40722851\sqrt{5x^2+2x+3}x^2}{750000}\\ &+\frac{5793077\sqrt{5x^2+2x+3}x}{75000}\\ &-\frac{16515809\sqrt{5x^2+2x+3}}{156250}\\ &-\frac{343}{40}\sqrt{5x^2+2x+3}x^7-\frac{1141}{40}\sqrt{5x^2+2x+3}x^6\\ &+\frac{26159}{300}\sqrt{5x^2+2x+3}x^5-\frac{47807\sqrt{5x^2+2x+3}x^4}{3750}\\ &-\frac{5160533\sqrt{5x^2+2x+3}x^3}{50000}\end{aligned}$$

```
[In] Int[((1 + 4*x - 7*x^2)^3*(2 + 5*x + x^2))/Sqrt[3 + 2*x + 5*x^2],x]
```

```
[Out] (-16515809*Sqrt[3 + 2*x + 5*x^2])/156250 + (5793077*x*Sqrt[3 + 2*x + 5*x^2]
)/75000 + (40722851*x^2*Sqrt[3 + 2*x + 5*x^2])/750000 - (5160533*x^3*Sqrt[3
 + 2*x + 5*x^2])/50000 - (47807*x^4*Sqrt[3 + 2*x + 5*x^2])/3750 + (26159*x^
5*Sqrt[3 + 2*x + 5*x^2])/300 - (1141*x^6*Sqrt[3 + 2*x + 5*x^2])/40 - (343*x
^7*Sqrt[3 + 2*x + 5*x^2])/40 - (77513689*ArcSinh[(1 + 5*x)/Sqrt[14]])/(6250
00*Sqrt[5])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
```

```
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
&\text{integral}\\
&=-\frac{343}{40}x^7\sqrt{3+2x+5x^2}\\
&\quad+\frac{1}{40}\int\frac{80+1160x+4600x^2-2440x^3-34840x^4+5080x^5+89803x^6-39935x^7}{\sqrt{3+2x+5x^2}}\,dx\\
&=-\frac{1141}{40}x^6\sqrt{3+2x+5x^2}-\frac{343}{40}x^7\sqrt{3+2x+5x^2}\\
&\quad+\frac{\int\frac{2800+40600x+161000x^2-85400x^3-1219400x^4+896630x^5+3662260x^6}{\sqrt{3+2x+5x^2}}\,dx}{1400}\\
&=\frac{26159}{300}x^5\sqrt{3+2x+5x^2}-\frac{1141}{40}x^6\sqrt{3+2x+5x^2}-\frac{343}{40}x^7\sqrt{3+2x+5x^2}\\
&\quad+\frac{\int\frac{84000+1218000x+4830000x^2-2562000x^3-91515900x^4-13385960x^5}{\sqrt{3+2x+5x^2}}\,dx}{42000}\\
&=-\frac{47807x^4\sqrt{3+2x+5x^2}}{3750}+\frac{26159}{300}x^5\sqrt{3+2x+5x^2}-\frac{1141}{40}x^6\sqrt{3+2x+5x^2}\\
&\quad-\frac{343}{40}x^7\sqrt{3+2x+5x^2}+\frac{\int\frac{2100000+30450000x+120750000x^2+96581520x^3-2167423860x^4}{\sqrt{3+2x+5x^2}}\,dx}{1050000}\\
&=-\frac{5160533x^3\sqrt{3+2x+5x^2}}{50000}-\frac{47807x^4\sqrt{3+2x+5x^2}}{3750}\\
&\quad+\frac{26159}{300}x^5\sqrt{3+2x+5x^2}-\frac{1141}{40}x^6\sqrt{3+2x+5x^2}\\
&\quad-\frac{343}{40}x^7\sqrt{3+2x+5x^2}+\frac{\int\frac{42000000+609000000x+21921814740x^2+17103597420x^3}{\sqrt{3+2x+5x^2}}\,dx}{21000000}\\
&=\frac{40722851x^2\sqrt{3+2x+5x^2}}{750000}-\frac{5160533x^3\sqrt{3+2x+5x^2}}{50000}\\
&\quad-\frac{47807x^4\sqrt{3+2x+5x^2}}{3750}+\frac{26159}{300}x^5\sqrt{3+2x+5x^2}-\frac{1141}{40}x^6\sqrt{3+2x+5x^2}\\
&\quad-\frac{343}{40}x^7\sqrt{3+2x+5x^2}+\frac{\int\frac{630000000-93486584520x+243309234000x^2}{\sqrt{3+2x+5x^2}}\,dx}{315000000}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{5793077x\sqrt{3+2x+5x^2}}{75000} + \frac{40722851x^2\sqrt{3+2x+5x^2}}{750000} - \frac{5160533x^3\sqrt{3+2x+5x^2}}{50000} \\
&\quad - \frac{47807x^4\sqrt{3+2x+5x^2}}{3750} + \frac{26159}{300}x^5\sqrt{3+2x+5x^2} - \frac{1141}{40}x^6\sqrt{3+2x+5x^2} \\
&\quad - \frac{343}{40}x^7\sqrt{3+2x+5x^2} + \frac{\int \frac{-723627702000-1664793547200x}{\sqrt{3+2x+5x^2}}\,dx}{3150000000} \\
&= -\frac{16515809\sqrt{3+2x+5x^2}}{156250} + \frac{5793077x\sqrt{3+2x+5x^2}}{75000} + \frac{40722851x^2\sqrt{3+2x+5x^2}}{750000} \\
&\quad - \frac{5160533x^3\sqrt{3+2x+5x^2}}{50000} - \frac{47807x^4\sqrt{3+2x+5x^2}}{3750} + \frac{26159}{300}x^5\sqrt{3+2x+5x^2} \\
&\quad - \frac{1141}{40}x^6\sqrt{3+2x+5x^2} - \frac{343}{40}x^7\sqrt{3+2x+5x^2} - \frac{77513689\int \frac{1}{\sqrt{3+2x+5x^2}}\,dx}{625000} \\
&= -\frac{16515809\sqrt{3+2x+5x^2}}{156250} + \frac{5793077x\sqrt{3+2x+5x^2}}{75000} \\
&\quad + \frac{40722851x^2\sqrt{3+2x+5x^2}}{750000} - \frac{5160533x^3\sqrt{3+2x+5x^2}}{50000} \\
&\quad - \frac{47807x^4\sqrt{3+2x+5x^2}}{3750} + \frac{26159}{300}x^5\sqrt{3+2x+5x^2} - \frac{1141}{40}x^6\sqrt{3+2x+5x^2} \\
&\quad - \frac{343}{40}x^7\sqrt{3+2x+5x^2} - \frac{77513689\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{56}}}\,dx, x, 2+10x\right)}{1250000\sqrt{70}} \\
&= -\frac{16515809\sqrt{3+2x+5x^2}}{156250} + \frac{5793077x\sqrt{3+2x+5x^2}}{75000} + \frac{40722851x^2\sqrt{3+2x+5x^2}}{750000} \\
&\quad - \frac{5160533x^3\sqrt{3+2x+5x^2}}{50000} - \frac{47807x^4\sqrt{3+2x+5x^2}}{3750} + \frac{26159}{300}x^5\sqrt{3+2x+5x^2} \\
&\quad - \frac{1141}{40}x^6\sqrt{3+2x+5x^2} - \frac{343}{40}x^7\sqrt{3+2x+5x^2} - \frac{77513689\sinh^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{625000\sqrt{5}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.48

$$
\begin{aligned}
&\int \frac{(1+4x-7x^2)^3\,(2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx \\
&= \frac{\sqrt{3+2x+5x^2}(-396379416+289653850x+203614255x^2-387039975x^3-47807000x^4+326987500x^5}{3750000} \\
&\quad + \frac{77513689\log\left(-1-5x+\sqrt{5}\sqrt{3+2x+5x^2}\right)}{625000\sqrt{5}}
\end{aligned}
$$

```
[In] Integrate[((1 + 4*x - 7*x^2)^3*(2 + 5*x + x^2))/Sqrt[3 + 2*x + 5*x^2],x]
```

```
[Out] (Sqrt[3 + 2*x + 5*x^2]*(-396379416 + 289653850*x + 203614255*x^2 - 38703997
5*x^3 - 47807000*x^4 + 326987500*x^5 - 106968750*x^6 - 32156250*x^7))/37500
00 + (77513689*Log[-1 - 5*x + Sqrt[5]*Sqrt[3 + 2*x + 5*x^2]])/(625000*Sqrt[
5])
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.35

method	result
risch	$-\frac{\left(32156250x^{7}+106968750x^{6}-326987500x^{5}+47807000x^{4}+387039975x^{3}-203614255x^{2}-289653850x+396379416\right)\sqrt{5x^{2}+2x+3}}{3750000} -$
trager	$\left(-\frac{343}{40}x^{7}-\frac{1141}{40}x^{6}+\frac{26159}{300}x^{5}-\frac{47807}{3750}x^{4}-\frac{5160533}{50000}x^{3}+\frac{40722851}{750000}x^{2}+\frac{5793077}{75000}x-\frac{16515809}{156250}\right)\sqrt{5x^{2}+2x}$
default	$-\frac{77513689\sqrt{5}\ \operatorname{arcsinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{3125000}-\frac{16515809\sqrt{5x^{2}+2x+3}}{156250}+\frac{40722851x^{2}\sqrt{5x^{2}+2x+3}}{750000}+\frac{5793077x\sqrt{5x^{2}+2x+3}}{75000}-$

```
[In] int((-7*x^2+4*x+1)^3*(x^2+5*x+2)/(5*x^2+2*x+3)^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -1/3750000*(32156250*x^7+106968750*x^6-326987500*x^5+47807000*x^4+387039975
*x^3-203614255*x^2-289653850*x+396379416)*(5*x^2+2*x+3)^(1/2)-77513689/3125
000*5^(1/2)*arcsinh(5/14*14^(1/2)*(x+1/5))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.47

$$\int \frac{\left(1+4x-7x^2\right)^3\left(2+5x+x^2\right)}{\sqrt{3+2x+5x^2}}\,dx = -\frac{1}{3750000}\left(32156250\,x^7+106968750\,x^6-326987500\,x^5+47807000\,x^4+387039975\,x^3-203614255\,x^2\right. + \frac{77513689}{6250000}\sqrt{5}\log\left(\sqrt{5}\sqrt{5\,x^2+2\,x+3}(5\,x+1)-25\,x^2-10\,x-8\right)$$

```
[In] integrate((-7*x^2+4*x+1)^3*(x^2+5*x+2)/(5*x^2+2*x+3)^(1/2),x, algorithm="fr
icas")
```

```
[Out] -1/3750000*(32156250*x^7 + 106968750*x^6 - 326987500*x^5 + 47807000*x^4 + 3
87039975*x^3 - 203614255*x^2 - 289653850*x + 396379416)*sqrt(5*x^2 + 2*x +
3) + 77513689/6250000*sqrt(5)*log(sqrt(5)*sqrt(5*x^2 + 2*x + 3)*(5*x + 1) -
 25*x^2 - 10*x - 8)
```

Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.46

$$\int \frac{(1+4x-7x^2)^3(2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx = \sqrt{5x^2+2x+3}\left(-\frac{343x^7}{40}-\frac{1141x^6}{40}+\frac{26159x^5}{300}-\frac{47807x^4}{3750}-\frac{5160533x^3}{50000}+\frac{40722851x^2}{750000}+\frac{5793077x}{75000}-\frac{16515809}{156250}\right)-\frac{77513689\sqrt{5}\operatorname{asinh}\left(\frac{5\sqrt{14}(x+\frac{1}{5})}{14}\right)}{3125000}$$

```
[In] integrate((-7*x**2+4*x+1)**3*(x**2+5*x+2)/(5*x**2+2*x+3)**(1/2),x)
```

```
[Out] sqrt(5*x**2 + 2*x + 3)*(-343*x**7/40 - 1141*x**6/40 + 26159*x**5/300 - 4780
7*x**4/3750 - 5160533*x**3/50000 + 40722851*x**2/750000 + 5793077*x/75000 -
 16515809/156250) - 77513689*sqrt(5)*asinh(5*sqrt(14)*(x + 1/5)/14)/3125000
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.80

$$\int \frac{(1+4x-7x^2)^3(2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx = -\frac{343}{40}\sqrt{5x^2+2x+3}x^7-\frac{1141}{40}\sqrt{5x^2+2x+3}x^6+\frac{26159}{300}\sqrt{5x^2+2x+3}x^5-\frac{47807}{3750}\sqrt{5x^2+2x+3}x^4-\frac{5160533}{50000}\sqrt{5x^2+2x+3}x^3+\frac{40722851}{750000}\sqrt{5x^2+2x+3}x^2+\frac{5793077}{75000}\sqrt{5x^2+2x+3}x-\frac{77513689}{3125000}\sqrt{5}\operatorname{arsinh}\left(\frac{1}{14}\sqrt{14}(5x+1)\right)-\frac{16515809}{156250}\sqrt{5x^2+2x+3}$$

```
[In] integrate((-7*x^2+4*x+1)^3*(x^2+5*x+2)/(5*x^2+2*x+3)^(1/2),x, algorithm="ma
xima")
```

```
[Out] -343/40*sqrt(5*x^2 + 2*x + 3)*x^7 - 1141/40*sqrt(5*x^2 + 2*x + 3)*x^6 + 261
59/300*sqrt(5*x^2 + 2*x + 3)*x^5 - 47807/3750*sqrt(5*x^2 + 2*x + 3)*x^4 - 5
160533/50000*sqrt(5*x^2 + 2*x + 3)*x^3 + 40722851/750000*sqrt(5*x^2 + 2*x +
 3)*x^2 + 5793077/75000*sqrt(5*x^2 + 2*x + 3)*x - 77513689/3125000*sqrt(5)*
arcsinh(1/14*sqrt(14)*(5*x + 1)) - 16515809/156250*sqrt(5*x^2 + 2*x + 3)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.44

$$\int \frac{(1+4x-7x^2)^3(2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx = -\frac{1}{3750000}(5((5(10(175(15(49x+163)x-7474)x+191228)x+15481599)x-40722851)x-579307 + \frac{77513689}{3125000}\sqrt{5}\log\left(-\sqrt{5}\left(\sqrt{5}x-\sqrt{5x^2+2x+3}\right)-1\right)$$

```
[In] integrate((-7*x^2+4*x+1)^3*(x^2+5*x+2)/(5*x^2+2*x+3)^(1/2),x, algorithm="gi
ac")
```

```
[Out] -1/3750000*(5*((5*(10*(175*(15*(49*x + 163)*x - 7474)*x + 191228)*x + 15481
599)*x - 40722851)*x - 57930770)*x + 396379416)*sqrt(5*x^2 + 2*x + 3) + 775
13689/3125000*sqrt(5)*log(-sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3)) - 1)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+4x-7x^2)^3(2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx = \int \frac{(x^2+5x+2)\,(-7x^2+4x+1)^3}{\sqrt{5x^2+2x+3}}\,dx$$

```
[In] int(((5*x + x^2 + 2)*(4*x - 7*x^2 + 1)^3)/(2*x + 5*x^2 + 3)^(1/2),x)
```

```
[Out] int(((5*x + x^2 + 2)*(4*x - 7*x^2 + 1)^3)/(2*x + 5*x^2 + 3)^(1/2), x)
```

3.387 $\int \frac{(1+4x-7x^2)^2(2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 143

$$\int \frac{(1+4x-7x^2)^2(2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx = -\frac{22053\sqrt{3+2x+5x^2}}{31250}+\frac{36073x\sqrt{3+2x+5x^2}}{1875} - \frac{207427x^2\sqrt{3+2x+5x^2}}{37500} - \frac{33259x^3\sqrt{3+2x+5x^2}}{2500}+\frac{5131}{750}x^4\sqrt{3+2x+5x^2} + \frac{49}{30}x^5\sqrt{3+2x+5x^2}-\frac{1719097\operatorname{arcsinh}\left(\frac{1+5x}{\sqrt{14}}\right)}{31250\sqrt{5}}$$

```
[Out] -1719097/156250*arcsinh(1/14*(1+5*x)*14^(1/2))*5^(1/2)-22053/31250*(5*x^2+2
*x+3)^(1/2)+36073/1875*x*(5*x^2+2*x+3)^(1/2)-207427/37500*x^2*(5*x^2+2*x+3)
^(1/2)-33259/2500*x^3*(5*x^2+2*x+3)^(1/2)+5131/750*x^4*(5*x^2+2*x+3)^(1/2)+
49/30*x^5*(5*x^2+2*x+3)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used

$= \{1675, 654, 633, 221\}$

$$\int \frac{\left(1+4x-7x^2\right)^2\left(2+5x+x^2\right)}{\sqrt{3+2x+5x^2}}\,dx = -\frac{1719097\operatorname{arcsinh}\left(\frac{5x+1}{\sqrt{14}}\right)}{31250\sqrt{5}} - \frac{207427\sqrt{5x^2+2x+3}x^2}{37500} + \frac{36073\sqrt{5x^2+2x+3}x}{1875} - \frac{22053\sqrt{5x^2+2x+3}}{31250} + \frac{49}{30}\sqrt{5x^2+2x+3}x^5 + \frac{5131}{750}\sqrt{5x^2+2x+3}x^4 - \frac{33259\sqrt{5x^2+2x+3}x^3}{2500}$$

```
[In] Int[((1 + 4*x - 7*x^2)^2*(2 + 5*x + x^2))/Sqrt[3 + 2*x + 5*x^2],x]
```

```
[Out] (-22053*Sqrt[3 + 2*x + 5*x^2])/31250 + (36073*x*Sqrt[3 + 2*x + 5*x^2])/1875
 - (207427*x^2*Sqrt[3 + 2*x + 5*x^2])/37500 - (33259*x^3*Sqrt[3 + 2*x + 5*x
^2])/2500 + (5131*x^4*Sqrt[3 + 2*x + 5*x^2])/750 + (49*x^5*Sqrt[3 + 2*x + 5
*x^2])/30 - (1719097*ArcSinh[(1 + 5*x)/Sqrt[14]])/(31250*Sqrt[5])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{49}{30}x^5\sqrt{3+2x+5x^2}+\frac{1}{30}\int \frac{60+630x+1350x^2-2820x^3-6135x^4+5131x^5}{\sqrt{3+2x+5x^2}}\,dx\\
&= \frac{5131}{750}x^4\sqrt{3+2x+5x^2}+\frac{49}{30}x^5\sqrt{3+2x+5x^2}\\
&\quad+\frac{1}{750}\int \frac{1500+15750x+33750x^2-132072x^3-199554x^4}{\sqrt{3+2x+5x^2}}\,dx\\
&= -\frac{33259x^3\sqrt{3+2x+5x^2}}{2500}+\frac{5131}{750}x^4\sqrt{3+2x+5x^2}\\
&\quad+\frac{49}{30}x^5\sqrt{3+2x+5x^2}+\frac{\int \frac{30000+315000x+2470986x^2-1244562x^3}{\sqrt{3+2x+5x^2}}\,dx}{15000}\\
&= -\frac{207427x^2\sqrt{3+2x+5x^2}}{37500}-\frac{33259x^3\sqrt{3+2x+5x^2}}{2500}+\frac{5131}{750}x^4\sqrt{3+2x+5x^2}\\
&\quad+\frac{49}{30}x^5\sqrt{3+2x+5x^2}+\frac{\int \frac{450000+12192372x+43287600x^2}{\sqrt{3+2x+5x^2}}\,dx}{225000}\\
&= \frac{36073x\sqrt{3+2x+5x^2}}{1875}-\frac{207427x^2\sqrt{3+2x+5x^2}}{37500}-\frac{33259x^3\sqrt{3+2x+5x^2}}{2500}\\
&\quad+\frac{5131}{750}x^4\sqrt{3+2x+5x^2}+\frac{49}{30}x^5\sqrt{3+2x+5x^2}+\frac{\int \frac{-125362800-7939080x}{\sqrt{3+2x+5x^2}}\,dx}{2250000}\\
&= -\frac{22053\sqrt{3+2x+5x^2}}{31250}+\frac{36073x\sqrt{3+2x+5x^2}}{1875}-\frac{207427x^2\sqrt{3+2x+5x^2}}{37500}\\
&\quad-\frac{33259x^3\sqrt{3+2x+5x^2}}{2500}+\frac{5131}{750}x^4\sqrt{3+2x+5x^2}\\
&\quad+\frac{49}{30}x^5\sqrt{3+2x+5x^2}-\frac{1719097\int \frac{1}{\sqrt{3+2x+5x^2}}\,dx}{31250}\\
&= -\frac{22053\sqrt{3+2x+5x^2}}{31250}+\frac{36073x\sqrt{3+2x+5x^2}}{1875}-\frac{207427x^2\sqrt{3+2x+5x^2}}{37500}\\
&\quad-\frac{33259x^3\sqrt{3+2x+5x^2}}{2500}+\frac{5131}{750}x^4\sqrt{3+2x+5x^2}\\
&\quad+\frac{49}{30}x^5\sqrt{3+2x+5x^2}-\frac{1719097\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{56}}}\,dx,x,2+10x\right)}{62500\sqrt{70}}\\
&= -\frac{22053\sqrt{3+2x+5x^2}}{31250}+\frac{36073x\sqrt{3+2x+5x^2}}{1875}-\frac{207427x^2\sqrt{3+2x+5x^2}}{37500}\\
&\quad-\frac{33259x^3\sqrt{3+2x+5x^2}}{2500}+\frac{5131}{750}x^4\sqrt{3+2x+5x^2}\\
&\quad+\frac{49}{30}x^5\sqrt{3+2x+5x^2}-\frac{1719097\sinh^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{31250\sqrt{5}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.55

$$\int \frac{\left(1+4x-7x^2\right)^2\left(2+5x+x^2\right)}{\sqrt{3+2x+5x^2}}\,dx$$
$$=\frac{\sqrt{3+2x+5x^2}(-132318+3607300x-1037135x^2-2494425x^3+1282750x^4+306250x^5)}{187500}$$
$$+\frac{1719097\log\left(-1-5x+\sqrt{5}\sqrt{3+2x+5x^2}\right)}{31250\sqrt{5}}$$

```
[In] Integrate[((1 + 4*x - 7*x^2)^2*(2 + 5*x + x^2))/Sqrt[3 + 2*x + 5*x^2],x]
```

```
[Out] (Sqrt[3 + 2*x + 5*x^2]*(-132318 + 3607300*x - 1037135*x^2 - 2494425*x^3 + 1
282750*x^4 + 306250*x^5))/187500 + (1719097*Log[-1 - 5*x + Sqrt[5]*Sqrt[3 +
 2*x + 5*x^2]])/(31250*Sqrt[5])
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.38

method	result
risch	$\frac{(306250x^5+1282750x^4-2494425x^3-1037135x^2+3607300x-132318)\sqrt{5x^2+2x+3}}{187500}-\frac{1719097\sqrt{5}\,\operatorname{arcsinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{156250}$
trager	$\left(\frac{49}{30}x^5+\frac{5131}{750}x^4-\frac{33259}{2500}x^3-\frac{207427}{37500}x^2+\frac{36073}{1875}x-\frac{22053}{31250}\right)\sqrt{5x^2+2x+3}+\frac{1719097\operatorname{RootOf}\left(\_Z^2-5\right)\ln\left(-\right.}{}$
default	$-\frac{1719097\sqrt{5}\,\operatorname{arcsinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{156250}-\frac{22053\sqrt{5x^2+2x+3}}{31250}+\frac{49x^5\sqrt{5x^2+2x+3}}{30}+\frac{5131x^4\sqrt{5x^2+2x+3}}{750}-\frac{33259x^3\sqrt{5x^2+2x}}{2500}$

```
[In] int((-7*x^2+4*x+1)^2*(x^2+5*x+2)/(5*x^2+2*x+3)^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/187500*(306250*x^5+1282750*x^4-2494425*x^3-1037135*x^2+3607300*x-132318)*
(5*x^2+2*x+3)^(1/2)-1719097/156250*5^(1/2)*arcsinh(5/14*14^(1/2)*(x+1/5))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.54

$$\int \frac{(1+4x-7x^2)^2\,(2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx$$
$$= \frac{1}{187500}\left(306250\,x^5 + 1282750\,x^4 - 2494425\,x^3 - 1037135\,x^2 + 3607300\,x - 132318\right)\sqrt{5\,x^2+2\,x+3}$$
$$+ \frac{1719097}{312500}\,\sqrt{5}\log\left(\sqrt{5}\sqrt{5\,x^2+2\,x+3}(5\,x+1) - 25\,x^2 - 10\,x - 8\right)$$

```
[In] integrate((-7*x^2+4*x+1)^2*(x^2+5*x+2)/(5*x^2+2*x+3)^(1/2),x, algorithm="fr
icas")
```

```
[Out] 1/187500*(306250*x^5 + 1282750*x^4 - 2494425*x^3 - 1037135*x^2 + 3607300*x
- 132318)*sqrt(5*x^2 + 2*x + 3) + 1719097/312500*sqrt(5)*log(sqrt(5)*sqrt(5
*x^2 + 2*x + 3)*(5*x + 1) - 25*x^2 - 10*x - 8)
```

Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.50

$$\int \frac{(1+4x-7x^2)^2\,(2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx = \sqrt{5x^2+2x+3}\cdot\left(\frac{49x^5}{30} + \frac{5131x^4}{750} - \frac{33259x^3}{2500} - \frac{207427x^2}{37500} + \frac{36073x}{1875} - \frac{22053}{31250}\right) - \frac{1719097\sqrt{5}\operatorname{asinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{156250}$$

```
[In] integrate((-7*x**2+4*x+1)**2*(x**2+5*x+2)/(5*x**2+2*x+3)**(1/2),x)
```

```
[Out] sqrt(5*x**2 + 2*x + 3)*(49*x**5/30 + 5131*x**4/750 - 33259*x**3/2500 - 2074
27*x**2/37500 + 36073*x/1875 - 22053/31250) - 1719097*sqrt(5)*asinh(5*sqrt(
14)*(x + 1/5)/14)/156250
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.80

$$\int \frac{(1+4x-7x^2)^2(2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx = \frac{49}{30}\sqrt{5\,x^2+2\,x+3}x^5 + \frac{5131}{750}\sqrt{5\,x^2+2\,x+3}x^4 - \frac{33259}{2500}\sqrt{5\,x^2+2\,x+3}x^3 - \frac{207427}{37500}\sqrt{5\,x^2+2\,x+3}x^2 + \frac{36073}{1875}\sqrt{5\,x^2+2\,x+3}x - \frac{1719097}{156250}\sqrt{5}\,\operatorname{arsinh}\left(\frac{1}{14}\sqrt{14}(5\,x+1)\right) - \frac{22053}{31250}\sqrt{5\,x^2+2\,x+3}$$

```
[In] integrate((-7*x^2+4*x+1)^2*(x^2+5*x+2)/(5*x^2+2*x+3)^(1/2),x, algorithm="ma
xima")
```

```
[Out] 49/30*sqrt(5*x^2 + 2*x + 3)*x^5 + 5131/750*sqrt(5*x^2 + 2*x + 3)*x^4 - 3325
9/2500*sqrt(5*x^2 + 2*x + 3)*x^3 - 207427/37500*sqrt(5*x^2 + 2*x + 3)*x^2 +
 36073/1875*sqrt(5*x^2 + 2*x + 3)*x - 1719097/156250*sqrt(5)*arcsinh(1/14*s
qrt(14)*(5*x + 1)) - 22053/31250*sqrt(5*x^2 + 2*x + 3)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.50

$$\int \frac{(1+4x-7x^2)^2(2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx = \frac{1}{187500}\left(5\left(\left(5\left(70\left(175\,x+733\right)x-99777\right)x-207427\right)x+721460\right)x-132318\right)\sqrt{5\,x^2+2\,x+3} + \frac{1719097}{156250}\sqrt{5}\log\left(-\sqrt{5}\left(\sqrt{5}x-\sqrt{5\,x^2+2\,x+3}\right)-1\right)$$

```
[In] integrate((-7*x^2+4*x+1)^2*(x^2+5*x+2)/(5*x^2+2*x+3)^(1/2),x, algorithm="gi
ac")
```

```
[Out] 1/187500*(5*((5*(70*(175*x + 733)*x - 99777)*x - 207427)*x + 721460)*x - 13
2318)*sqrt(5*x^2 + 2*x + 3) + 1719097/156250*sqrt(5)*log(-sqrt(5)*(sqrt(5)*
x - sqrt(5*x^2 + 2*x + 3)) - 1)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+4x-7x^2)^2\,(2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx = \int \frac{\left(x^2+5\,x+2\right)\,\left(-7\,x^2+4\,x+1\right)^2}{\sqrt{5\,x^2+2\,x+3}}\,dx$$

```
[In] int(((5*x + x^2 + 2)*(4*x - 7*x^2 + 1)^2)/(2*x + 5*x^2 + 3)^(1/2),x)
[Out] int(((5*x + x^2 + 2)*(4*x - 7*x^2 + 1)^2)/(2*x + 5*x^2 + 3)^(1/2), x)
```

3.388 $\int \frac{(1+4x-7x^2)(2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 101

$$\begin{aligned}\int \frac{(1+4x-7x^2)(2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx &= \frac{463}{125}\sqrt{3+2x+5x^2}+\frac{59}{30}x\sqrt{3+2x+5x^2}\\ &\quad-\frac{571}{300}x^2\sqrt{3+2x+5x^2}\\ &\quad-\frac{7}{20}x^3\sqrt{3+2x+5x^2}-\frac{1901\operatorname{arcsinh}\left(\frac{1+5x}{\sqrt{14}}\right)}{250\sqrt{5}}\end{aligned}$$

```
[Out] -1901/1250*arcsinh(1/14*(1+5*x)*14^(1/2))*5^(1/2)+463/125*(5*x^2+2*x+3)^(1/
2)+59/30*x*(5*x^2+2*x+3)^(1/2)-571/300*x^2*(5*x^2+2*x+3)^(1/2)-7/20*x^3*(5*
x^2+2*x+3)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {1675, 654, 633, 221}

$$\begin{aligned}\int \frac{(1+4x-7x^2)(2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx &= -\frac{1901\operatorname{arcsinh}\left(\frac{5x+1}{\sqrt{14}}\right)}{250\sqrt{5}}-\frac{571}{300}\sqrt{5x^2+2x+3}x^2\\ &\quad+\frac{59}{30}\sqrt{5x^2+2x+3}x+\frac{463}{125}\sqrt{5x^2+2x+3}\\ &\quad-\frac{7}{20}\sqrt{5x^2+2x+3}x^3\end{aligned}$$

```
[In] Int[((1 + 4*x - 7*x^2)*(2 + 5*x + x^2))/Sqrt[3 + 2*x + 5*x^2],x]
```

```
[Out] (463*Sqrt[3 + 2*x + 5*x^2])/125 + (59*x*Sqrt[3 + 2*x + 5*x^2])/30 - (571*x^
2*Sqrt[3 + 2*x + 5*x^2])/300 - (7*x^3*Sqrt[3 + 2*x + 5*x^2])/20 - (1901*Arc
Sinh[(1 + 5*x)/Sqrt[14]])/(250*Sqrt[5])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{7}{20}x^3\sqrt{3+2x+5x^2}+\frac{1}{20}\int\frac{40+260x+203x^2-571x^3}{\sqrt{3+2x+5x^2}}\,dx\\
&= -\frac{571}{300}x^2\sqrt{3+2x+5x^2}-\frac{7}{20}x^3\sqrt{3+2x+5x^2}+\frac{1}{300}\int\frac{600+7326x+5900x^2}{\sqrt{3+2x+5x^2}}\,dx\\
&= \frac{59}{30}x\sqrt{3+2x+5x^2}-\frac{571}{300}x^2\sqrt{3+2x+5x^2}-\frac{7}{20}x^3\sqrt{3+2x+5x^2}+\frac{\int\frac{-11700+55560x}{\sqrt{3+2x+5x^2}}\,dx}{3000}\\
&= \frac{463}{125}\sqrt{3+2x+5x^2}+\frac{59}{30}x\sqrt{3+2x+5x^2}-\frac{571}{300}x^2\sqrt{3+2x+5x^2}\\
&\quad-\frac{7}{20}x^3\sqrt{3+2x+5x^2}-\frac{1901}{250}\int\frac{1}{\sqrt{3+2x+5x^2}}\,dx
\end{aligned}$$

$$
\begin{aligned}
&= \frac{463}{125}\sqrt{3+2x+5x^2} + \frac{59}{30}x\sqrt{3+2x+5x^2} - \frac{571}{300}x^2\sqrt{3+2x+5x^2} \\
&\quad - \frac{7}{20}x^3\sqrt{3+2x+5x^2} - \frac{1901\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{56}}}\,dx, x, 2+10x\right)}{500\sqrt{70}} \\
&= \frac{463}{125}\sqrt{3+2x+5x^2} + \frac{59}{30}x\sqrt{3+2x+5x^2} - \frac{571}{300}x^2\sqrt{3+2x+5x^2} \\
&\quad - \frac{7}{20}x^3\sqrt{3+2x+5x^2} - \frac{1901\sinh^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{250\sqrt{5}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.68

$$
\begin{aligned}
\int \frac{(1+4x-7x^2)(2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx &= \frac{\sqrt{3+2x+5x^2}(5556+2950x-2855x^2-525x^3)}{1500} \\
&\quad + \frac{1901\log\left(-1-5x+\sqrt{5}\sqrt{3+2x+5x^2}\right)}{250\sqrt{5}}
\end{aligned}
$$

```
[In] Integrate[((1 + 4*x - 7*x^2)*(2 + 5*x + x^2))/Sqrt[3 + 2*x + 5*x^2],x]
[Out] (Sqrt[3 + 2*x + 5*x^2]*(5556 + 2950*x - 2855*x^2 - 525*x^3))/1500 + (1901*L
og[-1 - 5*x + Sqrt[5]*Sqrt[3 + 2*x + 5*x^2]])/(250*Sqrt[5])
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.45

method	result
risch	$-\frac{(525x^3+2855x^2-2950x-5556)\sqrt{5x^2+2x+3}}{1500}-\frac{1901\sqrt{5}\,\operatorname{arcsinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{1250}$
trager	$\left(-\frac{7}{20}x^3-\frac{571}{300}x^2+\frac{59}{30}x+\frac{463}{125}\right)\sqrt{5x^2+2x+3}-\frac{1901\operatorname{RootOf}(\_Z^2-5)\ln(5\operatorname{RootOf}(\_Z^2-5)x+\operatorname{RootOf}(\_Z^2}{1250}$
default	$-\frac{1901\sqrt{5}\,\operatorname{arcsinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{1250}+\frac{463\sqrt{5x^2+2x+3}}{125}-\frac{7x^3\sqrt{5x^2+2x+3}}{20}-\frac{571x^2\sqrt{5x^2+2x+3}}{300}+\frac{59x\sqrt{5x^2+2x+3}}{30}$

```
[In] int((-7*x^2+4*x+1)*(x^2+5*x+2)/(5*x^2+2*x+3)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/1500*(525*x^3+2855*x^2-2950*x-5556)*(5*x^2+2*x+3)^(1/2)-1901/1250*5^(1/2
)*arcsinh(5/14*14^(1/2)*(x+1/5))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.66

$$\begin{aligned}\int \frac{(1+4x-7x^2)(2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx &= -\frac{1}{1500}\left(525\,x^3+2855\,x^2-2950\,x-5556\right)\sqrt{5\,x^2+2\,x+3}\\ &\quad+\frac{1901}{2500}\sqrt{5}\log\left(\sqrt{5}\sqrt{5\,x^2+2\,x+3}(5\,x+1)-25\,x^2-10\,x-8\right)\end{aligned}$$

```
[In] integrate((-7*x^2+4*x+1)*(x^2+5*x+2)/(5*x^2+2*x+3)^(1/2),x, algorithm="fric
as")
```

```
[Out] -1/1500*(525*x^3 + 2855*x^2 - 2950*x - 5556)*sqrt(5*x^2 + 2*x + 3) + 1901/2
500*sqrt(5)*log(sqrt(5)*sqrt(5*x^2 + 2*x + 3)*(5*x + 1) - 25*x^2 - 10*x - 8
)
```

Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.57

$$\begin{aligned}\int \frac{(1+4x-7x^2)(2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx &= \sqrt{5x^2+2x+3}\left(-\frac{7x^3}{20}-\frac{571x^2}{300}+\frac{59x}{30}+\frac{463}{125}\right)\\ &\quad-\frac{1901\sqrt{5}\operatorname{asinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{1250}\end{aligned}$$

```
[In] integrate((-7*x**2+4*x+1)*(x**2+5*x+2)/(5*x**2+2*x+3)**(1/2),x)
```

```
[Out] sqrt(5*x**2 + 2*x + 3)*(-7*x**3/20 - 571*x**2/300 + 59*x/30 + 463/125) - 19
01*sqrt(5)*asinh(5*sqrt(14)*(x + 1/5)/14)/1250
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.79

$$\begin{aligned}\int \frac{(1+4x-7x^2)(2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx &= -\frac{7}{20}\sqrt{5\,x^2+2\,x+3}x^3\\ &\quad-\frac{571}{300}\sqrt{5\,x^2+2\,x+3}x^2+\frac{59}{30}\sqrt{5\,x^2+2\,x+3}x\\ &\quad-\frac{1901}{1250}\sqrt{5}\operatorname{arsinh}\left(\frac{1}{14}\sqrt{14}(5\,x+1)\right)\\ &\quad+\frac{463}{125}\sqrt{5\,x^2+2\,x+3}\end{aligned}$$

```
[In] integrate((-7*x^2+4*x+1)*(x^2+5*x+2)/(5*x^2+2*x+3)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] -7/20*sqrt(5*x^2 + 2*x + 3)*x^3 - 571/300*sqrt(5*x^2 + 2*x + 3)*x^2 + 59/30
*sqrt(5*x^2 + 2*x + 3)*x - 1901/1250*sqrt(5)*arcsinh(1/14*sqrt(14)*(5*x + 1
)) + 463/125*sqrt(5*x^2 + 2*x + 3)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.61

$$\begin{aligned}\int \frac{(1+4x-7x^2)(2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx &= -\frac{1}{1500}\,(5\,((105\,x+571)x-590)x-5556)\sqrt{5\,x^2+2\,x+3} \\ &\quad+\frac{1901}{1250}\,\sqrt{5}\log\left(-\sqrt{5}\left(\sqrt{5}x-\sqrt{5\,x^2+2\,x+3}\right)-1\right)\end{aligned}$$

```
[In] integrate((-7*x^2+4*x+1)*(x^2+5*x+2)/(5*x^2+2*x+3)^(1/2),x, algorithm="giac
")
```

```
[Out] -1/1500*(5*((105*x + 571)*x - 590)*x - 5556)*sqrt(5*x^2 + 2*x + 3) + 1901/1
250*sqrt(5)*log(-sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3)) - 1)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+4x-7x^2)(2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx = \int \frac{(x^2+5\,x+2)\,(-7\,x^2+4\,x+1)}{\sqrt{5\,x^2+2\,x+3}}\,dx$$

```
[In] int(((5*x + x^2 + 2)*(4*x - 7*x^2 + 1))/(2*x + 5*x^2 + 3)^(1/2),x)
```

```
[Out] int(((5*x + x^2 + 2)*(4*x - 7*x^2 + 1))/(2*x + 5*x^2 + 3)^(1/2), x)
```

3.389 $\int \frac{2+5x+x^2}{(1+4x-7x^2)\sqrt{3+2x+5x^2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 164

$$
\begin{aligned}
&\int \frac{2+5x+x^2}{(1+4x-7x^2)\sqrt{3+2x+5x^2}}\,dx \\
&= -\frac{\operatorname{arcsinh}\left(\frac{1+5x}{\sqrt{14}}\right)}{7\sqrt{5}} \\
&\quad - \frac{3}{14}\sqrt{\frac{4091-1055\sqrt{11}}{2794}}\operatorname{arctanh}\left(\frac{23-\sqrt{11}+\left(17-5\sqrt{11}\right)x}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right) \\
&\quad + \frac{3}{14}\sqrt{\frac{4091+1055\sqrt{11}}{2794}}\operatorname{arctanh}\left(\frac{23+\sqrt{11}+\left(17+5\sqrt{11}\right)x}{\sqrt{2\left(125+17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)
\end{aligned}
$$

```
[Out] -1/35*arcsinh(1/14*(1+5*x)*14^(1/2))*5^(1/2)-3/39116*arctanh((23+x*(17-5*11
^(1/2))-11^(1/2))/(5*x^2+2*x+3)^(1/2)/(250-34*11^(1/2))^(1/2))*(11430254-29
47670*11^(1/2))^(1/2)+3/39116*arctanh((23+11^(1/2)+x*(17+5*11^(1/2)))/(5*x^
2+2*x+3)^(1/2)/(250+34*11^(1/2))^(1/2))*(11430254+2947670*11^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {1090, 633, 221, 1046, 738, 212}

$$\begin{aligned}&\int \frac{2+5x+x^2}{(1+4x-7x^2)\sqrt{3+2x+5x^2}}\,dx\\&=-\frac{\operatorname{arcsinh}\left(\frac{5x+1}{\sqrt{14}}\right)}{7\sqrt{5}}\\&\quad-\frac{3}{14}\sqrt{\frac{4091-1055\sqrt{11}}{2794}}\operatorname{arctanh}\left(\frac{\left(17-5\sqrt{11}\right)x-\sqrt{11}+23}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right)\\&\quad+\frac{3}{14}\sqrt{\frac{4091+1055\sqrt{11}}{2794}}\operatorname{arctanh}\left(\frac{\left(17+5\sqrt{11}\right)x+\sqrt{11}+23}{\sqrt{2\left(125+17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right)\end{aligned}$$

```
[In] Int[(2 + 5*x + x^2)/((1 + 4*x - 7*x^2)*Sqrt[3 + 2*x + 5*x^2]),x]
```

```
[Out] -1/7*ArcSinh[(1 + 5*x)/Sqrt[14]]/Sqrt[5] - (3*Sqrt[(4091 - 1055*Sqrt[11])/2
794]*ArcTanh[(23 - Sqrt[11] + (17 - 5*Sqrt[11])*x)/(Sqrt[2*(125 - 17*Sqrt[1
1])]*Sqrt[3 + 2*x + 5*x^2])])/14 + (3*Sqrt[(4091 + 1055*Sqrt[11])/2794]*Arc
Tanh[(23 + Sqrt[11] + (17 + 5*Sqrt[11])*x)/(Sqrt[2*(125 + 17*Sqrt[11])]*Sqr
t[3 + 2*x + 5*x^2])])/14
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1046

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1090

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\text{integral} = -\left(\frac{1}{7}\int \frac{1}{\sqrt{3+2x+5x^2}}\,dx\right) - \frac{1}{7}\int \frac{-15-39x}{\left(1+4x-7x^2\right)\sqrt{3+2x+5x^2}}\,dx$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{56}}}\,dx, x, 2+10x\right)}{14\sqrt{70}} + \frac{1}{77}\left(3\left(143-61\sqrt{11}\right)\right)\int \frac{1}{\left(4-2\sqrt{11}-14x\right)\sqrt{3+2x+5x^2}}\,dx + \frac{1}{77}\left(3\left(143+61\sqrt{11}\right)\right)\int \frac{1}{\left(4+2\sqrt{11}-14x\right)\sqrt{3+2x+5x^2}}\,dx$$

$$= -\frac{\sinh^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{7\sqrt{5}} - \frac{1}{77}\left(6\left(143 -61\sqrt{11}\right)\right)\text{Subst}\left(\int \frac{1}{2352+112\left(4-2\sqrt{11}\right)+20\left(4-2\sqrt{11}\right)^2-x^2}\,dx, x, \frac{-84-2\left(4-2\sqrt{11}\right)}{\sqrt{3+}}\right.$$

$$- \frac{1}{77}\left(6\left(143 +61\sqrt{11}\right)\right)\text{Subst}\left(\int \frac{1}{2352+112\left(4+2\sqrt{11}\right)+20\left(4+2\sqrt{11}\right)^2-x^2}\,dx, x, \frac{-84-2\left(4+2\sqrt{11}\right)}{\sqrt{3+}}\right.$$

$$= -\frac{\sinh^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{7\sqrt{5}} - \frac{3}{14}\sqrt{\frac{4091-1055\sqrt{11}}{2794}}\tanh^{-1}\left(\frac{23-\sqrt{11}+\left(17-5\sqrt{11}\right)x}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)$$
$$+\frac{3}{14}\sqrt{\frac{4091+1055\sqrt{11}}{2794}}\tanh^{-1}\left(\frac{23+\sqrt{11}+\left(17+5\sqrt{11}\right)x}{\sqrt{2\left(125+17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.29 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.29

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)\sqrt{3+2x+5x^2}}\,dx$$
$$= \frac{\log\left(-1-5x+\sqrt{5}\sqrt{3+2x+5x^2}\right)}{7\sqrt{5}} + \frac{3}{14}\mathrm{RootSum}\Big[83-16\sqrt{5}\#1-70\#1^2+8\sqrt{5}\#1^3$$
$$+7\#1^4\&, \frac{29\log\left(-\sqrt{5}x+\sqrt{3+2x+5x^2}-\#1\right)+10\sqrt{5}\log\left(-\sqrt{5}x+\sqrt{3+2x+5x^2}-\#1\right)\#1-13\,\mathrm{lo}}{-4\sqrt{5}-35\#1+6\sqrt{5}\#1^2+7\#1^3}$$

```
[In] Integrate[(2 + 5*x + x^2)/((1 + 4*x - 7*x^2)*Sqrt[3 + 2*x + 5*x^2]),x]
```

```
[Out] Log[-1 - 5*x + Sqrt[5]*Sqrt[3 + 2*x + 5*x^2]]/(7*Sqrt[5]) + (3*RootSum[83 -
 16*Sqrt[5]*#1 - 70*#1^2 + 8*Sqrt[5]*#1^3 + 7*#1^4 & , (29*Log[-(Sqrt[5]*x)
 + Sqrt[3 + 2*x + 5*x^2] - #1] + 10*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x
 + 5*x^2] - #1]*#1 - 13*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1^2
)/(-4*Sqrt[5] - 35*#1 + 6*Sqrt[5]*#1^2 + 7*#1^3) & ])/14
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.24

method	result
default	$-\frac{\sqrt{5}\,\operatorname{arcsinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{35}+\frac{3\left(61+13\sqrt{11}\right)\sqrt{11}\,\operatorname{arctanh}\left(\frac{250+34\sqrt{11}+\frac{49\left(\frac{34}{7}+\frac{10\sqrt{11}}{7}\right)\left(x-\frac{2}{7}-\frac{\sqrt{11}}{7}\right)}{2}}{\sqrt{250+34\sqrt{11}}\sqrt{245\left(x-\frac{2}{7}-\frac{\sqrt{11}}{7}\right)^2+49\left(\frac{34}{7}+\frac{10\sqrt{11}}{7}\right)\left(x-\frac{2}{7}-\frac{\sqrt{11}}{7}\right)+250}}\right.}{154\sqrt{250+34\sqrt{11}}}$
trager	$-\frac{\mathrm{RootOf}\left(\_Z^2+382515364\,\mathrm{RootOf}\left(24095456\_Z^4-3240072\_Z^2+29241\right)^2-51436143\right)\ln\left(-\frac{9459586880128\,\mathrm{RootOf}\left(\_Z^2+382\right.}{}\right.}{}$

```
[In] int((x^2+5*x+2)/(-7*x^2+4*x+1)/(5*x^2+2*x+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/35*5^(1/2)*arcsinh(5/14*14^(1/2)*(x+1/5))+3/154*(61+13*11^(1/2))*11^(1/2
)/(250+34*11^(1/2))^(1/2)*arctanh(49/2*(500/49+68/49*11^(1/2)+(34/7+10/7*11
^(1/2))*(x-2/7-1/7*11^(1/2)))/(250+34*11^(1/2))^(1/2)/(245*(x-2/7-1/7*11^(1
/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2))
+3/154*(-61+13*11^(1/2))*11^(1/2)/(250-34*11^(1/2))^(1/2)*arctanh(49/2*(500
/49-68/49*11^(1/2)+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2)))/(250-34*11^(1
/2))^(1/2)/(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*1
1^(1/2))+250-34*11^(1/2))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 297 vs. $2(114) = 228$.

Time = 0.27 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.81

$$
\begin{aligned}
&\int \frac{2+5x+x^2}{(1+4x-7x^2)\sqrt{3+2x+5x^2}}\,dx = \\
&-\frac{3}{78232}\sqrt{2794}\sqrt{1055\sqrt{11}+4091}\log\left(\frac{3\left(\sqrt{2794}\sqrt{5x^2+2x+3}\sqrt{1055\sqrt{11}+4091}(172\sqrt{11}-715)+\right.}{x}\right. \\
&+\frac{3}{78232}\sqrt{2794}\sqrt{1055\sqrt{11}+4091}\log\left(-\frac{3\left(\sqrt{2794}\sqrt{5x^2+2x+3}\sqrt{1055\sqrt{11}+4091}(172\sqrt{11}-715)\right.}{x}\right. \\
&-\frac{1}{78232}\sqrt{2794}\sqrt{-9495\sqrt{11}+36819}\log\left(-\frac{\sqrt{2794}\sqrt{5x^2+2x+3}(172\sqrt{11}+715)\sqrt{-9495\sqrt{11}+3681}}{x}\right. \\
&+\frac{1}{78232}\sqrt{2794}\sqrt{-9495\sqrt{11}+36819}\log\left(\frac{\sqrt{2794}\sqrt{5x^2+2x+3}(172\sqrt{11}+715)\sqrt{-9495\sqrt{11}+36819}}{x}\right. \\
&+\frac{1}{70}\sqrt{5}\log\left(\sqrt{5}\sqrt{5x^2+2x+3}(5x+1)-25x^2-10x-8\right)
\end{aligned}
$$

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)/(5*x^2+2*x+3)^(1/2),x, algorithm="fric
as")
```

```
[Out] -3/78232*sqrt(2794)*sqrt(1055*sqrt(11) + 4091)*log(3*(sqrt(2794)*sqrt(5*x^2
 + 2*x + 3)*sqrt(1055*sqrt(11) + 4091)*(172*sqrt(11) - 715) + 185801*sqrt(1
1)*(x + 3) + 557403*x - 929005)/x) + 3/78232*sqrt(2794)*sqrt(1055*sqrt(11)
+ 4091)*log(-3*(sqrt(2794)*sqrt(5*x^2 + 2*x + 3)*sqrt(1055*sqrt(11) + 4091)
*(172*sqrt(11) - 715) - 185801*sqrt(11)*(x + 3) - 557403*x + 929005)/x) - 1
/78232*sqrt(2794)*sqrt(-9495*sqrt(11) + 36819)*log(-(sqrt(2794)*sqrt(5*x^2
+ 2*x + 3)*(172*sqrt(11) + 715)*sqrt(-9495*sqrt(11) + 36819) + 557403*sqrt(
11)*(x + 3) - 1672209*x + 2787015)/x) + 1/78232*sqrt(2794)*sqrt(-9495*sqrt(
11) + 36819)*log((sqrt(2794)*sqrt(5*x^2 + 2*x + 3)*(172*sqrt(11) + 715)*sqr
```

```
t(-9495*sqrt(11) + 36819) - 557403*sqrt(11)*(x + 3) + 1672209*x - 2787015)/
x) + 1/70*sqrt(5)*log(sqrt(5)*sqrt(5*x^2 + 2*x + 3)*(5*x + 1) - 25*x^2 - 10
*x - 8)
```

Sympy [F]

$$\begin{aligned}&\int \frac{2+5x+x^2}{(1+4x-7x^2)\sqrt{3+2x+5x^2}}\,dx\\&=-\int \frac{5x}{7x^2\sqrt{5x^2+2x+3}-4x\sqrt{5x^2+2x+3}-\sqrt{5x^2+2x+3}}\,dx\\&\quad-\int \frac{x^2}{7x^2\sqrt{5x^2+2x+3}-4x\sqrt{5x^2+2x+3}-\sqrt{5x^2+2x+3}}\,dx\\&\quad-\int \frac{2}{7x^2\sqrt{5x^2+2x+3}-4x\sqrt{5x^2+2x+3}-\sqrt{5x^2+2x+3}}\,dx\end{aligned}$$

```
[In] integrate((x**2+5*x+2)/(-7*x**2+4*x+1)/(5*x**2+2*x+3)**(1/2),x)
```

```
[Out] -Integral(5*x/(7*x**2*sqrt(5*x**2 + 2*x + 3) - 4*x*sqrt(5*x**2 + 2*x + 3) -
 sqrt(5*x**2 + 2*x + 3)), x) - Integral(x**2/(7*x**2*sqrt(5*x**2 + 2*x + 3)
 - 4*x*sqrt(5*x**2 + 2*x + 3) - sqrt(5*x**2 + 2*x + 3)), x) - Integral(2/(7
*x**2*sqrt(5*x**2 + 2*x + 3) - 4*x*sqrt(5*x**2 + 2*x + 3) - sqrt(5*x**2 + 2
*x + 3)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 465 vs. 2(114) = 228.

Time = 0.31 (sec) , antiderivative size = 465, normalized size of antiderivative = 2.84

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)\sqrt{3+2x+5x^2}}\,dx = -\frac{1}{10780}\sqrt{11}\left(28\sqrt{11}\sqrt{5}\operatorname{arsinh}\left(\frac{5}{14}\sqrt{7}\sqrt{2}x+\frac{1}{14}\sqrt{7}\sqrt{2}\right)-\frac{1365\sqrt{11}\sqrt{2}\operatorname{arsinh}\left(\frac{5\sqrt{11}\sqrt{7}\sqrt{2}x}{7\left|14x-2\sqrt{11}-4\right|}+\frac{1}{7|14}\right.}{\sqrt{17\sqrt{}}}\right.$$

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)/(5*x^2+2*x+3)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] -1/10780*sqrt(11)*(28*sqrt(11)*sqrt(5)*arcsinh(5/14*sqrt(7)*sqrt(2)*x + 1/1
4*sqrt(7)*sqrt(2)) - 1365*sqrt(11)*sqrt(2)*arcsinh(5/7*sqrt(11)*sqrt(7)*sqr
t(2)*x/abs(14*x - 2*sqrt(11) - 4) + 17/7*sqrt(7)*sqrt(2)*x/abs(14*x - 2*sqr
t(11) - 4) + 1/7*sqrt(11)*sqrt(7)*sqrt(2)/abs(14*x - 2*sqrt(11) - 4) + 23/7
```

```
*sqrt(7)*sqrt(2)/abs(14*x - 2*sqrt(11) - 4))/sqrt(17*sqrt(11) + 125) + 390*
sqrt(11)*arcsinh(5/7*sqrt(11)*sqrt(7)*sqrt(2)*x/abs(14*x + 2*sqrt(11) - 4)
- 17/7*sqrt(7)*sqrt(2)*x/abs(14*x + 2*sqrt(11) - 4) + 1/7*sqrt(11)*sqrt(7)*
sqrt(2)/abs(14*x + 2*sqrt(11) - 4) - 23/7*sqrt(7)*sqrt(2)/abs(14*x + 2*sqrt
(11) - 4))/sqrt(-34/49*sqrt(11) + 250/49) - 6405*sqrt(2)*arcsinh(5/7*sqrt(1
1)*sqrt(7)*sqrt(2)*x/abs(14*x - 2*sqrt(11) - 4) + 17/7*sqrt(7)*sqrt(2)*x/ab
s(14*x - 2*sqrt(11) - 4) + 1/7*sqrt(11)*sqrt(7)*sqrt(2)/abs(14*x - 2*sqrt(1
1) - 4) + 23/7*sqrt(7)*sqrt(2)/abs(14*x - 2*sqrt(11) - 4))/sqrt(17*sqrt(11)
 + 125) - 1830*arcsinh(5/7*sqrt(11)*sqrt(7)*sqrt(2)*x/abs(14*x + 2*sqrt(11)
 - 4) - 17/7*sqrt(7)*sqrt(2)*x/abs(14*x + 2*sqrt(11) - 4) + 1/7*sqrt(11)*sq
rt(7)*sqrt(2)/abs(14*x + 2*sqrt(11) - 4) - 23/7*sqrt(7)*sqrt(2)/abs(14*x +
2*sqrt(11) - 4))/sqrt(-34/49*sqrt(11) + 250/49))
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.76

$$\begin{aligned}\int \frac{2+5x+x^2}{(1+4x-7x^2)\sqrt{3+2x+5x^2}}\,dx = {} & \frac{1}{35}\sqrt{5}\log\left(-5\sqrt{5}x-\sqrt{5}+5\sqrt{5x^2+2x+3}\right) \\ & + 0.353184817631429\log\Big(-\sqrt{5}x+\sqrt{5x^2+2x+3} \\ & + 4.41924736459000\Big) \\ & - 0.0986339689905714\log\Big(-\sqrt{5}x \\ & + \sqrt{5x^2+2x+3}+1.25295163054000\Big) \\ & - 0.353184817631429\log\Big(-\sqrt{5}x+\sqrt{5x^2+2x+3} \\ & - 1.02258038113000\Big) \\ & + 0.0986339689905714\log\Big(-\sqrt{5}x \\ & + \sqrt{5x^2+2x+3}-2.09411235400000\Big)\end{aligned}$$

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)/(5*x^2+2*x+3)^(1/2),x, algorithm="giac
")
```

```
[Out] 1/35*sqrt(5)*log(-5*sqrt(5)*x - sqrt(5) + 5*sqrt(5*x^2 + 2*x + 3)) + 0.3531
84817631429*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) + 4.41924736459000) - 0.
0986339689905714*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) + 1.25295163054000)
 - 0.353184817631429*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) - 1.02258038113
000) + 0.0986339689905714*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) - 2.094112
35400000)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)\sqrt{3+2x+5x^2}}\,dx = \int \frac{x^2+5\,x+2}{\sqrt{5\,x^2+2\,x+3}\,\left(-7\,x^2+4\,x+1\right)}\,dx$$

```
[In] int((5*x + x^2 + 2)/((2*x + 5*x^2 + 3)^(1/2)*(4*x - 7*x^2 + 1)),x)
[Out] int((5*x + x^2 + 2)/((2*x + 5*x^2 + 3)^(1/2)*(4*x - 7*x^2 + 1)), x)
```

3.390 $\int \frac{2+5x+x^2}{\left(1+4x-7x^2\right)^2\sqrt{3+2x+5x^2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 178

$$\begin{aligned}\int \frac{2+5x+x^2}{\left(1+4x-7x^2\right)^2\sqrt{3+2x+5x^2}}\,dx &= -\frac{3(40-371x)\sqrt{3+2x+5x^2}}{5588\left(1+4x-7x^2\right)} \\ &\quad -\frac{\sqrt{\frac{3027900955+14035681\sqrt{11}}{2794}}\operatorname{arctanh}\left(\frac{23-\sqrt{11}+\left(17-5\sqrt{11}\right)x}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)}{11176} \\ &\quad +\frac{\sqrt{\frac{3027900955-14035681\sqrt{11}}{2794}}\operatorname{arctanh}\left(\frac{23+\sqrt{11}+\left(17+5\sqrt{11}\right)x}{\sqrt{2\left(125+17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)}{11176}\end{aligned}$$

```
[Out] -3/5588*(40-371*x)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1)+1/31225744*arctanh((2
3+11^(1/2)+x*(17+5*11^(1/2)))/(5*x^2+2*x+3)^(1/2)/(250+34*11^(1/2))^(1/2))*
(8459955268270-39215692714*11^(1/2))^(1/2)-1/31225744*arctanh((23+x*(17-5*1
1^(1/2))-11^(1/2))/(5*x^2+2*x+3)^(1/2)/(250-34*11^(1/2))^(1/2))*(8459955268
270+39215692714*11^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {1074, 1046, 738, 212}

$$
\begin{aligned}
&\int \frac{2+5x+x^2}{(1+4x-7x^2)^2\sqrt{3+2x+5x^2}}\,dx \\
&= -\frac{\sqrt{\frac{3027900955+14035681\sqrt{11}}{2794}}\operatorname{arctanh}\left(\frac{\left(17-5\sqrt{11}\right)x-\sqrt{11}+23}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right)}{11176} \\
&\quad+\frac{\sqrt{\frac{3027900955-14035681\sqrt{11}}{2794}}\operatorname{arctanh}\left(\frac{\left(17+5\sqrt{11}\right)x+\sqrt{11}+23}{\sqrt{2\left(125+17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right)}{11176} \\
&\quad-\frac{3\sqrt{5x^2+2x+3}(40-371x)}{5588\left(-7x^2+4x+1\right)}
\end{aligned}
$$

```
[In] Int[(2 + 5*x + x^2)/((1 + 4*x - 7*x^2)^2*Sqrt[3 + 2*x + 5*x^2]),x]
```

```
[Out] (-3*(40 - 371*x)*Sqrt[3 + 2*x + 5*x^2])/(5588*(1 + 4*x - 7*x^2)) - (Sqrt[(3
027900955 + 14035681*Sqrt[11])/2794]*ArcTanh[(23 - Sqrt[11] + (17 - 5*Sqrt[
11])*x)/(Sqrt[2*(125 - 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x^2])])/11176 + (Sqrt
[(3027900955 - 14035681*Sqrt[11])/2794]*ArcTanh[(23 + Sqrt[11] + (17 + 5*Sq
rt[11])*x)/(Sqrt[2*(125 + 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x^2])])/11176
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1046

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
```

```
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1074

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x +
c*x^2)^(p + 1)*((d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f))*(p + 1)))*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) +
(A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c
*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d
 - a*f)))*x), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{3(40-371x)\sqrt{3+2x+5x^2}}{5588\,(1+4x-7x^2)} - \frac{\int \frac{-52136-29544x}{(1+4x-7x^2)\sqrt{3+2x+5x^2}}\,dx}{44704} \\
&= -\frac{3(40-371x)\sqrt{3+2x+5x^2}}{5588\,(1+4x-7x^2)} - \frac{\left(-40623+53005\sqrt{11}\right)\int \frac{1}{\left(4-2\sqrt{11}-14x\right)\sqrt{3+2x+5x^2}}\,dx}{61468} \\
&\quad + \frac{\left(40623+53005\sqrt{11}\right)\int \frac{1}{\left(4+2\sqrt{11}-14x\right)\sqrt{3+2x+5x^2}}\,dx}{61468}
\end{aligned}$$

$$= -\frac{3(40-371x)\sqrt{3+2x+5x^2}}{5588\,(1+4x-7x^2)}$$

$$-\frac{\left(40623-53005\sqrt{11}\right)\text{Subst}\left(\int\frac{1}{2352+112\left(4-2\sqrt{11}\right)+20\left(4-2\sqrt{11}\right)^2-x^2}\,dx,x,\frac{-84-2\left(4-2\sqrt{11}\right)-\left(28+10\left(4-2\right.\right.}{\sqrt{3+2x+5x^2}}\right.}{30734}$$

$$-\frac{\left(40623+53005\sqrt{11}\right)\text{Subst}\left(\int\frac{1}{2352+112\left(4+2\sqrt{11}\right)+20\left(4+2\sqrt{11}\right)^2-x^2}\,dx,x,\frac{-84-2\left(4+2\sqrt{11}\right)-\left(28+10\left(4+2\right.\right.}{\sqrt{3+2x+5x^2}}\right.}{30734}$$

$$= -\frac{3(40-371x)\sqrt{3+2x+5x^2}}{5588\,(1+4x-7x^2)}$$

$$-\frac{\sqrt{\frac{3027900955+14035681\sqrt{11}}{2794}}\tanh^{-1}\left(\frac{23-\sqrt{11}+\left(17-5\sqrt{11}\right)x}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)}{11176}$$

$$+\frac{\sqrt{\frac{3027900955-14035681\sqrt{11}}{2794}}\tanh^{-1}\left(\frac{23+\sqrt{11}+\left(17+5\sqrt{11}\right)x}{\sqrt{2\left(125+17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)}{11176}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.44 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.98

$$\int\frac{2+5x+x^2}{\left(1+4x-7x^2\right)^2\sqrt{3+2x+5x^2}}\,dx$$

$$= -\frac{3(-40+371x)\sqrt{3+2x+5x^2}}{5588\,(-1-4x+7x^2)}-\frac{1}{49}\text{RootSum}\Big[83-16\sqrt{5}\#1-70\#1^2+8\sqrt{5}\#1^3$$

$$+7\#1^4\&,\frac{-397\log\left(-\sqrt{5}x+\sqrt{3+2x+5x^2}-\#1\right)+7\sqrt{5}\log\left(-\sqrt{5}x+\sqrt{3+2x+5x^2}-\#1\right)\#1}{-4\sqrt{5}-35\#1+6\sqrt{5}\#1^2+7\#1^3}\&\Big]$$

$$+\frac{3\text{RootSum}\Big[83-16\sqrt{5}\#1-70\#1^2+8\sqrt{5}\#1^3+7\#1^4\&,\frac{-1510889\log\left(-\sqrt{5}x+\sqrt{3+2x+5x^2}-\#1\right)+238966\sqrt{5}\log\left(-\right.}{-4\sqrt{5}-3}}{547624}$$

```
[In] Integrate[(2 + 5*x + x^2)/((1 + 4*x - 7*x^2)^2*Sqrt[3 + 2*x + 5*x^2]),x]
```

```
[Out] (-3*(-40 + 371*x)*Sqrt[3 + 2*x + 5*x^2])/(5588*(-1 - 4*x + 7*x^2)) - RootSu
m[83 - 16*Sqrt[5]*#1 - 70*#1^2 + 8*Sqrt[5]*#1^3 + 7*#1^4 & , (-397*Log[-(Sq
rt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1] + 7*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[
3 + 2*x + 5*x^2] - #1]*#1)/(-4*Sqrt[5] - 35*#1 + 6*Sqrt[5]*#1^2 + 7*#1^3) &
 ]/49 + (3*RootSum[83 - 16*Sqrt[5]*#1 - 70*#1^2 + 8*Sqrt[5]*#1^3 + 7*#1^4 &
```

```
 , (-1510889*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1] + 238966*Sqrt[5
]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1 - 60319*Log[-(Sqrt[5]*x
) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1^2)/(-4*Sqrt[5] - 35*#1 + 6*Sqrt[5]*#1^2
+ 7*#1^3) & ])/547624
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.24

method	result
risch	$-\frac{3(-40+371x)\sqrt{5x^2+2x+3}}{5588(7x^2-4x-1)}+\frac{\left(-53005+3693\sqrt{11}\right)\sqrt{11}\ \operatorname{arctanh}\left(\frac{250-34\sqrt{11}+\frac{49\left(\frac{34}{7}-\frac{10\sqrt{11}}{7}\right)\left(x-\frac{2}{7}+\frac{\sqrt{11}}{7}\right)}{2}}{\sqrt{250-34\sqrt{11}}\sqrt{245\left(x-\frac{2}{7}+\frac{\sqrt{11}}{7}\right)^2+49\left(\frac{34}{7}-\frac{10\sqrt{11}}{7}\right)\left(x-\frac{2}{7}+\frac{\sqrt{11}}{7}\right)+}}\right.}{122936\sqrt{250-34\sqrt{11}}}$
trager	Expression too large to display
default	$\frac{161\sqrt{11}\ \operatorname{arctanh}\left(\frac{250+34\sqrt{11}+\frac{49\left(\frac{34}{7}+\frac{10\sqrt{11}}{7}\right)\left(x-\frac{2}{7}-\frac{\sqrt{11}}{7}\right)}{2}}{\sqrt{250+34\sqrt{11}}\sqrt{245\left(x-\frac{2}{7}-\frac{\sqrt{11}}{7}\right)^2+49\left(\frac{34}{7}+\frac{10\sqrt{11}}{7}\right)\left(x-\frac{2}{7}-\frac{\sqrt{11}}{7}\right)+250+34\sqrt{11}}}\right)}{484\sqrt{250+34\sqrt{11}}}-\frac{161\sqrt{11}\ \operatorname{arctanh}\left(\frac{}{\sqrt{250-34\sqrt{11}}\sqrt{2}}\right.}{}$

```
[In] int((x^2+5*x+2)/(-7*x^2+4*x+1)^2/(5*x^2+2*x+3)^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -3/5588*(-40+371*x)/(7*x^2-4*x-1)*(5*x^2+2*x+3)^(1/2)+1/122936*(-53005+3693
*11^(1/2))*11^(1/2)/(250-34*11^(1/2))^(1/2)*arctanh(49/2*(500/49-68/49*11^(
1/2)+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2)))/(250-34*11^(1/2))^(1/2)/(24
5*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-3
4*11^(1/2))^(1/2))+1/122936*(53005+3693*11^(1/2))*11^(1/2)/(250+34*11^(1/2)
)^(1/2)*arctanh(49/2*(500/49+68/49*11^(1/2)+(34/7+10/7*11^(1/2))*(x-2/7-1/7
*11^(1/2)))/(250+34*11^(1/2))^(1/2)/(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10
/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 330 vs. 2(129) = 258.

Time = 0.25 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.85

$$\int \frac{2+5x+x^2}{\left(1+4x-7x^2\right)^2\sqrt{3+2x+5x^2}}\,dx =$$

$$-\frac{\sqrt{2794}(7\,x^2-4\,x-1)\sqrt{14035681\,\sqrt{11}+3027900955}\,\log\left(-\frac{\sqrt{2794}\sqrt{5\,x^2+2\,x+3}\sqrt{14035681\,\sqrt{11}+3027900955}\,\left(71796\right.}{}\right.}{}$$

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)^2/(5*x^2+2*x+3)^(1/2),x, algorithm="fr
icas")
```

```
[Out] -1/62451488*(sqrt(2794)*(7*x^2 - 4*x - 1)*sqrt(14035681*sqrt(11) + 30279009
55)*log(-(sqrt(2794)*sqrt(5*x^2 + 2*x + 3)*sqrt(14035681*sqrt(11) + 3027900
955)*(71796*sqrt(11) + 567523) + 265381033753*sqrt(11)*(x + 3) - 7961431012
59*x + 1326905168765)/x) - sqrt(2794)*(7*x^2 - 4*x - 1)*sqrt(14035681*sqrt(
11) + 3027900955)*log((sqrt(2794)*sqrt(5*x^2 + 2*x + 3)*sqrt(14035681*sqrt(
11) + 3027900955)*(71796*sqrt(11) + 567523) - 265381033753*sqrt(11)*(x + 3)
 + 796143101259*x - 1326905168765)/x) + sqrt(2794)*(7*x^2 - 4*x - 1)*sqrt(-
14035681*sqrt(11) + 3027900955)*log((sqrt(2794)*sqrt(5*x^2 + 2*x + 3)*(7179
6*sqrt(11) - 567523)*sqrt(-14035681*sqrt(11) + 3027900955) + 265381033753*s
qrt(11)*(x + 3) + 796143101259*x - 1326905168765)/x) - sqrt(2794)*(7*x^2 -
4*x - 1)*sqrt(-14035681*sqrt(11) + 3027900955)*log(-(sqrt(2794)*sqrt(5*x^2
+ 2*x + 3)*(71796*sqrt(11) - 567523)*sqrt(-14035681*sqrt(11) + 3027900955)
- 265381033753*sqrt(11)*(x + 3) - 796143101259*x + 1326905168765)/x) + 3352
8*sqrt(5*x^2 + 2*x + 3)*(371*x - 40))/(7*x^2 - 4*x - 1)
```

Sympy [F]

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)^2\sqrt{3+2x+5x^2}}\,dx = \int \frac{x^2+5x+2}{\sqrt{5x^2+2x+3}\,(7x^2-4x-1)^2}\,dx$$

```
[In] integrate((x**2+5*x+2)/(-7*x**2+4*x+1)**2/(5*x**2+2*x+3)**(1/2),x)
```

```
[Out] Integral((x**2 + 5*x + 2)/(sqrt(5*x**2 + 2*x + 3)*(7*x**2 - 4*x - 1)**2), x
)
```

Maxima [F]

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)^2\sqrt{3+2x+5x^2}}\,dx = \int \frac{x^2+5\,x+2}{(7\,x^2-4\,x-1)^2\sqrt{5\,x^2+2\,x+3}}\,dx$$

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)^2/(5*x^2+2*x+3)^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate((x^2 + 5*x + 2)/((7*x^2 - 4*x - 1)^2*sqrt(5*x^2 + 2*x + 3)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 276 vs. $2(129) = 258$.

Time = 0.31 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.55

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)^2\sqrt{3+2x+5x^2}}\,dx$$

$$= \frac{3\Big(1231\,(\sqrt{5}x-\sqrt{5\,x^2+2\,x+3})^3+1735\,\sqrt{5}(\sqrt{5}x-\sqrt{5\,x^2+2\,x+3})^2-3913\,\sqrt{5}x-3989\,\sqrt{5}}{2794\Big(7\,(\sqrt{5}x-\sqrt{5\,x^2+2\,x+3})^4-8\,\sqrt{5}(\sqrt{5}x-\sqrt{5\,x^2+2\,x+3})^3-70\,(\sqrt{5}x-\sqrt{5\,x^2+2\,x+3})^2+16}$$

$$+ 0.0924287071106453\log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}+4.41924736459000\right)$$

$$- 0.0938608034604765\log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}+1.25295163054000\right)$$

$$- 0.0924287071106453\log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}-1.02258038113000\right)$$

$$+ 0.0938608034604765\log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}-2.09411235400000\right)$$

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)^2/(5*x^2+2*x+3)^(1/2),x, algorithm="gi
ac")
```

```
[Out] 3/2794*(1231*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^3 + 1735*sqrt(5)*(sqrt(5)*
x - sqrt(5*x^2 + 2*x + 3))^2 - 3913*sqrt(5)*x - 3989*sqrt(5) + 3913*sqrt(5*
x^2 + 2*x + 3))/(7*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^4 - 8*sqrt(5)*(sqrt(
5)*x - sqrt(5*x^2 + 2*x + 3))^3 - 70*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^2
+ 16*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3)) + 83) + 0.0924287071106453
*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) + 4.41924736459000) - 0.09386080346
04765*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) + 1.25295163054000) - 0.092428
7071106453*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) - 1.02258038113000) + 0.0
938608034604765*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) - 2.09411235400000)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)^2\sqrt{3+2x+5x^2}}\,dx = \int \frac{x^2+5\,x+2}{\sqrt{5\,x^2+2\,x+3}\,(-7\,x^2+4\,x+1)^2}\,dx$$

```
[In] int((5*x + x^2 + 2)/((2*x + 5*x^2 + 3)^(1/2)*(4*x - 7*x^2 + 1)^2),x)
```

```
[Out] int((5*x + x^2 + 2)/((2*x + 5*x^2 + 3)^(1/2)*(4*x - 7*x^2 + 1)^2), x)
```

3.391 $\int \frac{2+5x+x^2}{\left(1+4x-7x^2\right)^3\sqrt{3+2x+5x^2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 227

$$\begin{aligned}
&\int \frac{2+5x+x^2}{\left(1+4x-7x^2\right)^3\sqrt{3+2x+5x^2}}\,dx \\
&= -\frac{3(40-371x)\sqrt{3+2x+5x^2}}{11176\left(1+4x-7x^2\right)^2}-\frac{7(409769-1189370x)\sqrt{3+2x+5x^2}}{62451488\left(1+4x-7x^2\right)} \\
&\quad -\frac{7\left(39370231-2538725\sqrt{11}\right)\operatorname{arctanh}\left(\frac{23-\sqrt{11}+\left(17-5\sqrt{11}\right)x}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)}{124902976\sqrt{22\left(125-17\sqrt{11}\right)}} \\
&\quad +\frac{7\left(39370231+2538725\sqrt{11}\right)\operatorname{arctanh}\left(\frac{23+\sqrt{11}+\left(17+5\sqrt{11}\right)x}{\sqrt{2\left(125+17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)}{124902976\sqrt{22\left(125+17\sqrt{11}\right)}}
\end{aligned}$$

```
[Out] -3/11176*(40-371*x)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1)^2-7/62451488*(409769
-1189370*x)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1)-7/124902976*arctanh((23+x*(1
7-5*11^(1/2))-11^(1/2))/(5*x^2+2*x+3)^(1/2)/(250-34*11^(1/2))^(1/2))*(39370
231-2538725*11^(1/2))/(2750-374*11^(1/2))^(1/2)+7/124902976*arctanh((23+11^
(1/2)+x*(17+5*11^(1/2)))/(5*x^2+2*x+3)^(1/2)/(250+34*11^(1/2))^(1/2))*(3937
0231+2538725*11^(1/2))/(2750+374*11^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {1074, 1046, 738, 212}

$$\begin{aligned}
&\int \frac{2+5x+x^2}{(1+4x-7x^2)^3\sqrt{3+2x+5x^2}}\,dx \\
&= -\frac{7\left(39370231-2538725\sqrt{11}\right)\operatorname{arctanh}\left(\frac{\left(17-5\sqrt{11}\right)x-\sqrt{11}+23}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right)}{124902976\sqrt{22\left(125-17\sqrt{11}\right)}} \\
&\quad+\frac{7\left(39370231+2538725\sqrt{11}\right)\operatorname{arctanh}\left(\frac{\left(17+5\sqrt{11}\right)x+\sqrt{11}+23}{\sqrt{2\left(125+17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right)}{124902976\sqrt{22\left(125+17\sqrt{11}\right)}} \\
&\quad-\frac{7\sqrt{5x^2+2x+3}(409769-1189370x)}{62451488\left(-7x^2+4x+1\right)}-\frac{3(40-371x)\sqrt{5x^2+2x+3}}{11176\left(-7x^2+4x+1\right)^2}
\end{aligned}$$

```
[In] Int[(2 + 5*x + x^2)/((1 + 4*x - 7*x^2)^3*Sqrt[3 + 2*x + 5*x^2]),x]
```

```
[Out] (-3*(40 - 371*x)*Sqrt[3 + 2*x + 5*x^2])/(11176*(1 + 4*x - 7*x^2)^2) - (7*(4
09769 - 1189370*x)*Sqrt[3 + 2*x + 5*x^2])/(62451488*(1 + 4*x - 7*x^2)) - (7
*(39370231 - 2538725*Sqrt[11])*ArcTanh[(23 - Sqrt[11] + (17 - 5*Sqrt[11])*x
)/(Sqrt[2*(125 - 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x^2])])/(124902976*Sqrt[22*
(125 - 17*Sqrt[11])]) + (7*(39370231 + 2538725*Sqrt[11])*ArcTanh[(23 + Sqrt
[11] + (17 + 5*Sqrt[11])*x)/(Sqrt[2*(125 + 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x
^2])])/(124902976*Sqrt[22*(125 + 17*Sqrt[11])])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1046

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1074

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x +
c*x^2)^(p + 1)*((d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f))*(p + 1)))*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) +
(A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c
*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d
 - a*f)))*x), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] && !( !IntegerQ[p] && ILtQ[q, -1]) && !
IGtQ[q, 0]
```

Rubi steps

$$\text{integral} = -\frac{3(40-371x)\sqrt{3+2x+5x^2}}{11176\left(1+4x-7x^2\right)^2} - \frac{\int \frac{-130024-81000x-89040x^2}{\left(1+4x-7x^2\right)^2\sqrt{3+2x+5x^2}}\,dx}{89408}$$

$$= -\frac{3(40-371x)\sqrt{3+2x+5x^2}}{11176\left(1+4x-7x^2\right)^2} - \frac{7(409769-1189370x)\sqrt{3+2x+5x^2}}{62451488\left(1+4x-7x^2\right)} + \frac{\int \frac{2194737984+1137348800x}{\left(1+4x-7x^2\right)\sqrt{3+2x+5x^2}}\,dx}{3996895232}$$

$$= -\frac{3(40-371x)\sqrt{3+2x+5x^2}}{11176\left(1+4x-7x^2\right)^2} - \frac{7(409769-1189370x)\sqrt{3+2x+5x^2}}{62451488\left(1+4x-7x^2\right)} + \frac{\left(7\left(27925975-39370231\sqrt{11}\right)\right)\int \frac{1}{\left(4-2\sqrt{11}-14x\right)\sqrt{3+2x+5x^2}}\,dx}{686966368} + \frac{\left(7\left(27925975+39370231\sqrt{11}\right)\right)\int \frac{1}{\left(4+2\sqrt{11}-14x\right)\sqrt{3+2x+5x^2}}\,dx}{686966368}$$

$$= -\frac{3(40-371x)\sqrt{3+2x+5x^2}}{11176\left(1+4x-7x^2\right)^2} - \frac{7(409769-1189370x)\sqrt{3+2x+5x^2}}{62451488\left(1+4x-7x^2\right)} - \frac{\left(7\left(27925975-39370231\sqrt{11}\right)\right)\text{Subst}\left(\int \frac{1}{2352+112\left(4-2\sqrt{11}\right)+20\left(4-2\sqrt{11}\right)^2-x^2}\,dx, x, \frac{-84-2\left(4-2\sqrt{11}\right)-(2}{\sqrt{3+2x-}}\right.}{343483184} - \frac{\left(7\left(27925975+39370231\sqrt{11}\right)\right)\text{Subst}\left(\int \frac{1}{2352+112\left(4+2\sqrt{11}\right)+20\left(4+2\sqrt{11}\right)^2-x^2}\,dx, x, \frac{-84-2\left(4+2\sqrt{11}\right)-(2}{\sqrt{3+2x-}}\right.}{343483184}$$

$$= -\frac{3(40-371x)\sqrt{3+2x+5x^2}}{11176\left(1+4x-7x^2\right)^2} - \frac{7(409769-1189370x)\sqrt{3+2x+5x^2}}{62451488\left(1+4x-7x^2\right)} - \frac{7\left(39370231-2538725\sqrt{11}\right)\tanh^{-1}\left(\frac{23-\sqrt{11}+\left(17-5\sqrt{11}\right)x}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)}{124902976\sqrt{22\left(125-17\sqrt{11}\right)}} + \frac{7\left(39370231+2538725\sqrt{11}\right)\tanh^{-1}\left(\frac{23+\sqrt{11}+\left(17+5\sqrt{11}\right)x}{\sqrt{2\left(125+17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)}{124902976\sqrt{22\left(125+17\sqrt{11}\right)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.70 (sec) , antiderivative size = 433, normalized size of antiderivative = 1.91

$$\int \frac{2+5x+x^2}{\left(1+4x-7x^2\right)^3\sqrt{3+2x+5x^2}}\,dx$$

$$= \frac{\frac{235298\sqrt{3+2x+5x^2}\left(-3538943+3071502x+53381041x^2-58279130x^3\right)}{\left(1+4x-7x^2\right)^2} - 1796775175713\text{RootSum}\left[83-16\sqrt{5}\#1-70\#1^2+\right.}{}$$

```
[In] Integrate[(2 + 5*x + x^2)/((1 + 4*x - 7*x^2)^3*Sqrt[3 + 2*x + 5*x^2]),x]
```

```
[Out] ((235298*Sqrt[3 + 2*x + 5*x^2]*(-3538943 + 3071502*x + 53381041*x^2 - 58279
130*x^3))/(1 + 4*x - 7*x^2)^2 - 1796775175713*RootSum[83 - 16*Sqrt[5]*#1 -
70*#1^2 + 8*Sqrt[5]*#1^3 + 7*#1^4 & , Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x
^2] - #1]/(-4*Sqrt[5] - 35*#1 + 6*Sqrt[5]*#1^2 + 7*#1^3) & ] + 11176*RootSu
m[83 - 16*Sqrt[5]*#1 - 70*#1^2 + 8*Sqrt[5]*#1^3 + 7*#1^4 & , (10486671792*S
qrt[5]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1 + 6928653865*Log[-
(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1^2)/(-4*Sqrt[5] - 35*#1 + 6*Sqr
t[5]*#1^2 + 7*#1^3) & ] - 3*RootSum[83 - 16*Sqrt[5]*#1 - 70*#1^2 + 8*Sqrt[5
]*#1^3 + 7*#1^4 & , (36376673721218*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x
 + 5*x^2] - #1]*#1 + 26508461599305*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2
] - #1]*#1^2)/(-4*Sqrt[5] - 35*#1 + 6*Sqrt[5]*#1^2 + 7*#1^3) & ])/146947102
23424
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.02

method	result
risch	$-\frac{\left(58279130 x^{3}-53381041 x^{2}-3071502 x+3538943\right) \sqrt{5 x^{2}+2 x+3}}{62451488 \left(7 x^{2}-4 x-1\right)^{2}}+\frac{7\left(-39370231+2538725 \sqrt{11}\right) \sqrt{11}\, \operatorname{arctanh}\left(\frac{2}{\sqrt{250-34 \sqrt{11}}\, \sqrt{245}}\right.}{1373932736 \sqrt{2}}$
trager	Expression too large to display
default	Expression too large to display

```
[In] int((x^2+5*x+2)/(-7*x^2+4*x+1)^3/(5*x^2+2*x+3)^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -1/62451488*(58279130*x^3-53381041*x^2-3071502*x+3538943)/(7*x^2-4*x-1)^2*(
5*x^2+2*x+3)^(1/2)+7/1373932736*(-39370231+2538725*11^(1/2))*11^(1/2)/(250-
34*11^(1/2))^(1/2)*arctanh(49/2*(500/49-68/49*11^(1/2)+(34/7-10/7*11^(1/2))
*(x-2/7+1/7*11^(1/2)))/(250-34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(1/2))^2+
49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2))+7/1373
932736*(39370231+2538725*11^(1/2))*11^(1/2)/(250+34*11^(1/2))^(1/2)*arctanh
(49/2*(500/49+68/49*11^(1/2)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2)))/(25
0+34*11^(1/2))^(1/2)/(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x
-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 390 vs. 2(174) = 348.

Time = 0.29 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.72

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)^3\sqrt{3+2x+5x^2}}\,dx =$$

$$-\frac{\sqrt{2794}(49\,x^4-56\,x^3+2\,x^2+8\,x+1)\sqrt{1283973697005131\,\sqrt{11}+82616280769148425}\log\left(-\frac{\sqrt{2794}\sqrt{5\,x^2}}{}\right.}{}$$

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)^3/(5*x^2+2*x+3)^(1/2),x, algorithm="fr
icas")
```

```
[Out] -1/697957829888*(sqrt(2794)*(49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)*sqrt(128397
3697005131*sqrt(11) + 82616280769148425)*log(-(sqrt(2794)*sqrt(5*x^2 + 2*x
+ 3)*sqrt(1283973697005131*sqrt(11) + 82616280769148425)*(358684877*sqrt(11
) + 2940638404) + 723215097220611079 7*sqrt(11)*(x + 3) - 216964529166183323
91*x + 3616075486103055398 5)/x) - sqrt(2794)*(49*x^4 - 56*x^3 + 2*x^2 + 8*x
 + 1)*sqrt(1283973697005131*sqrt(11) + 82616280769148425)*log((sqrt(2794)*s
qrt(5*x^2 + 2*x + 3)*sqrt(1283973697005131*sqrt(11) + 82616280769148425)*(3
58684877*sqrt(11) + 2940638404) - 7232150972206110797*sqrt(11)*(x + 3) + 21
696452916618332391*x - 3616075486103055398 5)/x) + sqrt(2794)*(49*x^4 - 56*x
^3 + 2*x^2 + 8*x + 1)*sqrt(-1283973697005131*sqrt(11) + 82616280769148425)*
log((sqrt(2794)*sqrt(5*x^2 + 2*x + 3)*(358684877*sqrt(11) - 2940638404)*sqr
t(-1283973697005131*sqrt(11) + 82616280769148425) + 7232150972206110797*sqr
t(11)*(x + 3) + 21696452916618332391*x - 36160754861030553985)/x) - sqrt(27
94)*(49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)*sqrt(-1283973697005131*sqrt(11) + 8
2616280769148425)*log(-(sqrt(2794)*sqrt(5*x^2 + 2*x + 3)*(358684877*sqrt(11
) - 2940638404)*sqrt(-1283973697005131*sqrt(11) + 82616280769148425) - 7232
150972206110797*sqrt(11)*(x + 3) - 21696452916618332391*x + 361607548610305
53985)/x) + 11176*(58279130*x^3 - 53381041*x^2 - 3071502*x + 3538943)*sqrt(
5*x^2 + 2*x + 3))/(49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)
```

Sympy [F]

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)^3\sqrt{3+2x+5x^2}}\,dx =$$

$$-\int \frac{5x}{343x^6\sqrt{5x^2+2x+3}-588x^5\sqrt{5x^2+2x+3}+189x^4\sqrt{5x^2+2x+3}+104x^3\sqrt{5x^2+2x+3}-27x^2\sqrt{}}$$

$$-\int \frac{x^2}{343x^6\sqrt{5x^2+2x+3}-588x^5\sqrt{5x^2+2x+3}+189x^4\sqrt{5x^2+2x+3}+104x^3\sqrt{5x^2+2x+3}-27x^2\sqrt{}}$$

$$-\int \frac{2}{343x^6\sqrt{5x^2+2x+3}-588x^5\sqrt{5x^2+2x+3}+189x^4\sqrt{5x^2+2x+3}+104x^3\sqrt{5x^2+2x+3}-27x^2\sqrt{}}$$

```
[In] integrate((x**2+5*x+2)/(-7*x**2+4*x+1)**3/(5*x**2+2*x+3)**(1/2),x)
```

```
[Out] -Integral(5*x/(343*x**6*sqrt(5*x**2 + 2*x + 3) - 588*x**5*sqrt(5*x**2 + 2*x
 + 3) + 189*x**4*sqrt(5*x**2 + 2*x + 3) + 104*x**3*sqrt(5*x**2 + 2*x + 3) -
 27*x**2*sqrt(5*x**2 + 2*x + 3) - 12*x*sqrt(5*x**2 + 2*x + 3) - sqrt(5*x**2
 + 2*x + 3)), x) - Integral(x**2/(343*x**6*sqrt(5*x**2 + 2*x + 3) - 588*x**
5*sqrt(5*x**2 + 2*x + 3) + 189*x**4*sqrt(5*x**2 + 2*x + 3) + 104*x**3*sqrt(
5*x**2 + 2*x + 3) - 27*x**2*sqrt(5*x**2 + 2*x + 3) - 12*x*sqrt(5*x**2 + 2*x
 + 3) - sqrt(5*x**2 + 2*x + 3)), x) - Integral(2/(343*x**6*sqrt(5*x**2 + 2*
x + 3) - 588*x**5*sqrt(5*x**2 + 2*x + 3) + 189*x**4*sqrt(5*x**2 + 2*x + 3)
+ 104*x**3*sqrt(5*x**2 + 2*x + 3) - 27*x**2*sqrt(5*x**2 + 2*x + 3) - 12*x*s
qrt(5*x**2 + 2*x + 3) - sqrt(5*x**2 + 2*x + 3)), x)
```

Maxima [F]

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)^3\sqrt{3+2x+5x^2}}\,dx = \int -\frac{x^2+5\,x+2}{(7\,x^2-4\,x-1)^3\sqrt{5\,x^2+2\,x+3}}\,dx$$

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)^3/(5*x^2+2*x+3)^(1/2),x, algorithm="ma
xima")
```

```
[Out] -integrate((x^2 + 5*x + 2)/((7*x^2 - 4*x - 1)^3*sqrt(5*x^2 + 2*x + 3)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 378 vs. 2(174) = 348.

Time = 0.30 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.67

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)^3\sqrt{3+2x+5x^2}}\,dx$$

$$= \frac{124397525\,(\sqrt{5}x-\sqrt{5\,x^2+2\,x+3})^7+26796567\,\sqrt{5}(\sqrt{5}x-\sqrt{5\,x^2+2\,x+3})^6-3595807617\,(\sqrt{5}x-}{31225744\,(7\,(}$$

$$+\,0.0423989586659649\,\log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}+4.41924736459000\right)$$

$$-\,0.0446437606656958\,\log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}+1.25295163054000\right)$$

$$-\,0.0423989586659649\,\log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}-1.02258038113000\right)$$

$$+\,0.0446437606656958\,\log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}-2.09411235400000\right)$$

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)^3/(5*x^2+2*x+3)^(1/2),x, algorithm="gi
ac")
```

```
[Out] 1/31225744*(124397525*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^7 + 26796567*sqrt
(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^6 - 3595807617*(sqrt(5)*x - sqrt(5*
x^2 + 2*x + 3))^5 - 1719888775*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^
4 + 17096132999*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^3 + 8328401413*sqrt(5)*
(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^2 - 16383202915*sqrt(5)*x - 7800623485*
sqrt(5) + 16383202915*sqrt(5*x^2 + 2*x + 3))/(7*(sqrt(5)*x - sqrt(5*x^2 + 2
*x + 3))^4 - 8*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^3 - 70*(sqrt(5)*
x - sqrt(5*x^2 + 2*x + 3))^2 + 16*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3
)) + 83)^2 + 0.0423989586659649*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) + 4.
41924736459000) - 0.0446437606656958*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3)
 + 1.25295163054000) - 0.0423989586659649*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x
 + 3) - 1.02258038113000) + 0.0446437606656958*log(-sqrt(5)*x + sqrt(5*x^2
+ 2*x + 3) - 2.09411235400000)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2+5x+x^2}{\left(1+4x-7x^2\right)^3\sqrt{3+2x+5x^2}}\,dx=\int \frac{x^2+5\,x+2}{\sqrt{5\,x^2+2\,x+3}\,{\left(-7\,x^2+4\,x+1\right)}^3}\,dx$$

```
[In] int((5*x + x^2 + 2)/((2*x + 5*x^2 + 3)^(1/2)*(4*x - 7*x^2 + 1)^3),x)
[Out] int((5*x + x^2 + 2)/((2*x + 5*x^2 + 3)^(1/2)*(4*x - 7*x^2 + 1)^3), x)
```

3.392 $\int \frac{(1+4x-7x^2)^3(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 166

$$\begin{aligned}\int \frac{(1+4x-7x^2)^3(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx &= \frac{16(6122807-5338217x)}{546875\sqrt{3+2x+5x^2}} \\ &+ \frac{15715799\sqrt{3+2x+5x^2}}{156250} - \frac{3192602x\sqrt{3+2x+5x^2}}{46875} \\ &- \frac{2583293x^2\sqrt{3+2x+5x^2}}{187500} + \frac{393659x^3\sqrt{3+2x+5x^2}}{12500} \\ &- \frac{25921x^4\sqrt{3+2x+5x^2}}{3750} - \frac{343}{150}x^5\sqrt{3+2x+5x^2} + \frac{50047657\operatorname{arcsinh}\left(\frac{1+5x}{\sqrt{14}}\right)}{156250\sqrt{5}}\end{aligned}$$

```
[Out] 50047657/781250*arcsinh(1/14*(1+5*x)*14^(1/2))*5^(1/2)+16/546875*(6122807-5
338217*x)/(5*x^2+2*x+3)^(1/2)+15715799/156250*(5*x^2+2*x+3)^(1/2)-3192602/4
6875*x*(5*x^2+2*x+3)^(1/2)-2583293/187500*x^2*(5*x^2+2*x+3)^(1/2)+393659/12
500*x^3*(5*x^2+2*x+3)^(1/2)-25921/3750*x^4*(5*x^2+2*x+3)^(1/2)-343/150*x^5*
(5*x^2+2*x+3)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used

= {1674, 1675, 654, 633, 221}

$$\int \frac{(1+4x-7x^2)^3\,(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx = \frac{50047657\operatorname{arcsinh}\left(\frac{5x+1}{\sqrt{14}}\right)}{156250\sqrt{5}} - \frac{2583293\sqrt{5x^2+2x+3}x^2}{187500} - \frac{3192602\sqrt{5x^2+2x+3}x}{46875} + \frac{15715799\sqrt{5x^2+2x+3}}{156250} + \frac{16(6122807-5338217x)}{546875\sqrt{5x^2+2x+3}} - \frac{343}{150}\sqrt{5x^2+2x+3}x^5 - \frac{25921\sqrt{5x^2+2x+3}x^4}{3750} + \frac{393659\sqrt{5x^2+2x+3}x^3}{12500}$$

```
[In] Int[((1 + 4*x - 7*x^2)^3*(2 + 5*x + x^2))/(3 + 2*x + 5*x^2)^(3/2),x]
```

```
[Out] (16*(6122807 - 5338217*x))/(546875*Sqrt[3 + 2*x + 5*x^2]) + (15715799*Sqrt[
3 + 2*x + 5*x^2])/156250 - (3192602*x*Sqrt[3 + 2*x + 5*x^2])/46875 - (25832
93*x^2*Sqrt[3 + 2*x + 5*x^2])/187500 + (393659*x^3*Sqrt[3 + 2*x + 5*x^2])/1
2500 - (25921*x^4*Sqrt[3 + 2*x + 5*x^2])/3750 - (343*x^5*Sqrt[3 + 2*x + 5*x
^2])/150 + (50047657*ArcSinh[(1 + 5*x)/Sqrt[14]])/(156250*Sqrt[5])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
```

```
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

integral

$$= \frac{16(6122807-5338217x)}{546875\sqrt{3+2x+5x^2}} + \frac{1}{28}\int \frac{\frac{473724104}{78125}+\frac{94462228x}{15625}-\frac{40822404x^2}{3125}-\frac{1210328x^3}{625}+\frac{1866704x^4}{125}-\frac{138572x^5}{25}-\frac{9604x^6}{5}}{\sqrt{3+2x+5x^2}}\,dx$$

$$= \frac{16(6122807-5338217x)}{546875\sqrt{3+2x+5x^2}} - \frac{343}{150}x^5\sqrt{3+2x+5x^2} + \frac{1}{840}\int \frac{\frac{2842344624}{15625}+\frac{566773368x}{3125}-\frac{244934424x^2}{625}-\frac{7261968x^3}{125}+\frac{11920524x^4}{25}-\frac{725788x^5}{5}}{\sqrt{3+2x+5x^2}}\,dx$$

$$= \frac{16(6122807-5338217x)}{546875\sqrt{3+2x+5x^2}} - \frac{25921x^4\sqrt{3+2x+5x^2}}{3750} - \frac{343}{150}x^5\sqrt{3+2x+5x^2} + \frac{\int \frac{\frac{2842344624}{625}+\frac{566773368x}{125}-\frac{244934424x^2}{25}+\frac{1447488x^3}{5}+\frac{66134712x^4}{5}}{\sqrt{3+2x+5x^2}}\,dx}{21000}$$

$$= \frac{16(6122807-5338217x)}{546875\sqrt{3+2x+5x^2}} + \frac{393659x^3\sqrt{3+2x+5x^2}}{12500} - \frac{25921x^4\sqrt{3+2x+5x^2}}{3750} - \frac{343}{150}x^5\sqrt{3+2x+5x^2} + \frac{\int \frac{\frac{11369378496}{125}+\frac{2267093472x}{25}-\frac{1574950104x^2}{5}-\frac{433993224x^3}{5}}{\sqrt{3+2x+5x^2}}\,dx}{420000}$$

$$= \frac{16(6122807-5338217x)}{546875\sqrt{3+2x+5x^2}} - \frac{2583293x^2\sqrt{3+2x+5x^2}}{187500} + \frac{393659x^3\sqrt{3+2x+5x^2}}{12500} - \frac{25921x^4\sqrt{3+2x+5x^2}}{3750} - \frac{343}{150}x^5\sqrt{3+2x+5x^2} + \frac{\int \frac{\frac{34108135488}{25}+1881047952x-4290857088x^2}{\sqrt{3+2x+5x^2}}\,dx}{6300000}$$

$$= \frac{16(6122807-5338217x)}{546875\sqrt{3+2x+5x^2}} - \frac{3192602x\sqrt{3+2x+5x^2}}{46875} - \frac{2583293x^2\sqrt{3+2x+5x^2}}{187500} + \frac{393659x^3\sqrt{3+2x+5x^2}}{12500} - \frac{25921x^4\sqrt{3+2x+5x^2}}{3750} - \frac{343}{150}x^5\sqrt{3+2x+5x^2} + \frac{\int \frac{\frac{132579127296}{5}+31683050784x}{\sqrt{3+2x+5x^2}}\,dx}{63000000}$$

$$= \frac{16(6122807-5338217x)}{546875\sqrt{3+2x+5x^2}} + \frac{15715799\sqrt{3+2x+5x^2}}{156250} - \frac{3192602x\sqrt{3+2x+5x^2}}{46875} - \frac{2583293x^2\sqrt{3+2x+5x^2}}{187500} + \frac{393659x^3\sqrt{3+2x+5x^2}}{12500} - \frac{25921x^4\sqrt{3+2x+5x^2}}{3750} - \frac{343}{150}x^5\sqrt{3+2x+5x^2} + \frac{50047657\int \frac{1}{\sqrt{3+2x+5x^2}}\,dx}{156250}$$

$$= \frac{16(6122807-5338217x)}{546875\sqrt{3+2x+5x^2}} + \frac{15715799\sqrt{3+2x+5x^2}}{156250} - \frac{3192602x\sqrt{3+2x+5x^2}}{46875} - \frac{2583293x^2\sqrt{3+2x+5x^2}}{187500} + \frac{393659x^3\sqrt{3+2x+5x^2}}{12500} - \frac{25921x^4\sqrt{3+2x+5x^2}}{3750} - \frac{343}{150}x^5\sqrt{3+2x+5x^2} + \frac{50047657\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{56}}}\,dx, x, 2+10x\right)}{312500\sqrt{70}}$$

$$= \frac{16(6122807-5338217x)}{546875\sqrt{3+2x+5x^2}} + \frac{15715799\sqrt{3+2x+5x^2}}{156250} - \frac{3192602x\sqrt{3+2x+5x^2}}{46875} - \frac{2583293x^2\sqrt{3+2x+5x^2}}{187500} + \frac{393659x^3\sqrt{3+2x+5x^2}}{12500} - \frac{25921x^4\sqrt{3+2x+5x^2}}{3750} - \frac{343}{150}x^5\sqrt{3+2x+5x^2} + \frac{50047657\sinh^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{156250\sqrt{5}}$$

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.54

$$\int \frac{(1+4x-7x^2)^3\,(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx = \frac{3155769618 - 1045703388x + 2135143465x^2 - 1795638985x^3 - 1748}{6562500\sqrt{3+2x+}} - \frac{50047657\log\left(-1-5x+\sqrt{5}\sqrt{3+2x+5x^2}\right)}{156250\sqrt{5}}$$

```
[In] Integrate[((1 + 4*x - 7*x^2)^3*(2 + 5*x + x^2))/(3 + 2*x + 5*x^2)^(3/2),x]
```

```
[Out] (3155769618 - 1045703388*x + 2135143465*x^2 - 1795638985*x^3 - 174819575*x^
4 + 897612625*x^5 - 256821250*x^6 - 75031250*x^7)/(6562500*Sqrt[3 + 2*x + 5
*x^2]) - (50047657*Log[-1 - 5*x + Sqrt[5]*Sqrt[3 + 2*x + 5*x^2]])/(156250*S
qrt[5])
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.39

method	result
risch	$-\frac{75031250x^7+256821250x^6-897612625x^5+174819575x^4+1795638985x^3-2135143465x^2+1045703388x-3155769618}{6562500\sqrt{5x^2+2x+3}}+\frac{500476}{}$
trager	$-\frac{75031250x^7+256821250x^6-897612625x^5+174819575x^4+1795638985x^3-2135143465x^2+1045703388x-3155769618}{6562500\sqrt{5x^2+2x+3}}+\frac{500476}{}$
default	$\frac{\frac{176049701x}{1093750}+\frac{176049701}{5468750}}{\sqrt{5x^2+2x+3}}+\frac{175268451}{390625\sqrt{5x^2+2x+3}}+\frac{61004099x^2}{187500\sqrt{5x^2+2x+3}}-\frac{50047657x}{156250\sqrt{5x^2+2x+3}}+\frac{50047657\sqrt{5}\,\operatorname{arcsinh}\left(\frac{5\sqrt{14}\,(x}{14}\right.}{781250}$

```
[In] int((-7*x^2+4*x+1)^3*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -1/6562500*(75031250*x^7+256821250*x^6-897612625*x^5+174819575*x^4+17956389
85*x^3-2135143465*x^2+1045703388*x-3155769618)/(5*x^2+2*x+3)^(1/2)+50047657
/781250*5^(1/2)*arcsinh(5/14*14^(1/2)*(x+1/5))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.67

$$\int \frac{(1+4x-7x^2)^3\,(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx = \frac{1051000797\,\sqrt{5}(5\,x^2+2\,x+3)\log\left(-\sqrt{5}\sqrt{5\,x^2+2\,x+3}(5\,x+1)-\right.}{}$$

```
[In] integrate((-7*x^2+4*x+1)^3*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2),x, algorithm="fr
icas")
```

```
[Out] 1/32812500*(1051000797*sqrt(5)*(5*x^2 + 2*x + 3)*log(-sqrt(5)*sqrt(5*x^2 +
2*x + 3)*(5*x + 1) - 25*x^2 - 10*x - 8) - 5*(75031250*x^7 + 256821250*x^6 -
 897612625*x^5 + 174819575*x^4 + 1795638985*x^3 - 2135143465*x^2 + 10457033
88*x - 3155769618)*sqrt(5*x^2 + 2*x + 3))/(5*x^2 + 2*x + 3)
```

Sympy [F]

$$
\begin{aligned}
&\int \frac{(1+4x-7x^2)^3\,(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx = \\
&-\int\left(-\frac{29x}{5x^2\sqrt{5x^2+2x+3}+2x\sqrt{5x^2+2x+3}+3\sqrt{5x^2+2x+3}}\right)dx \\
&-\int\left(-\frac{115x^2}{5x^2\sqrt{5x^2+2x+3}+2x\sqrt{5x^2+2x+3}+3\sqrt{5x^2+2x+3}}\right)dx \\
&-\int\frac{61x^3}{5x^2\sqrt{5x^2+2x+3}+2x\sqrt{5x^2+2x+3}+3\sqrt{5x^2+2x+3}}\,dx \\
&-\int\frac{871x^4}{5x^2\sqrt{5x^2+2x+3}+2x\sqrt{5x^2+2x+3}+3\sqrt{5x^2+2x+3}}\,dx \\
&-\int\left(-\frac{127x^5}{5x^2\sqrt{5x^2+2x+3}+2x\sqrt{5x^2+2x+3}+3\sqrt{5x^2+2x+3}}\right)dx \\
&-\int\left(-\frac{2065x^6}{5x^2\sqrt{5x^2+2x+3}+2x\sqrt{5x^2+2x+3}+3\sqrt{5x^2+2x+3}}\right)dx \\
&-\int\frac{1127x^7}{5x^2\sqrt{5x^2+2x+3}+2x\sqrt{5x^2+2x+3}+3\sqrt{5x^2+2x+3}}\,dx \\
&-\int\frac{343x^8}{5x^2\sqrt{5x^2+2x+3}+2x\sqrt{5x^2+2x+3}+3\sqrt{5x^2+2x+3}}\,dx \\
&-\int\left(-\frac{2}{5x^2\sqrt{5x^2+2x+3}+2x\sqrt{5x^2+2x+3}+3\sqrt{5x^2+2x+3}}\right)dx
\end{aligned}
$$

```
[In] integrate((-7*x**2+4*x+1)**3*(x**2+5*x+2)/(5*x**2+2*x+3)**(3/2),x)
```

```
[Out] -Integral(-29*x/(5*x**2*sqrt(5*x**2 + 2*x + 3) + 2*x*sqrt(5*x**2 + 2*x + 3)
 + 3*sqrt(5*x**2 + 2*x + 3)), x) - Integral(-115*x**2/(5*x**2*sqrt(5*x**2 +
 2*x + 3) + 2*x*sqrt(5*x**2 + 2*x + 3) + 3*sqrt(5*x**2 + 2*x + 3)), x) - In
tegral(61*x**3/(5*x**2*sqrt(5*x**2 + 2*x + 3) + 2*x*sqrt(5*x**2 + 2*x + 3)
+ 3*sqrt(5*x**2 + 2*x + 3)), x) - Integral(871*x**4/(5*x**2*sqrt(5*x**2 + 2
*x + 3) + 2*x*sqrt(5*x**2 + 2*x + 3) + 3*sqrt(5*x**2 + 2*x + 3)), x) - Inte
gral(-127*x**5/(5*x**2*sqrt(5*x**2 + 2*x + 3) + 2*x*sqrt(5*x**2 + 2*x + 3)
+ 3*sqrt(5*x**2 + 2*x + 3)), x) - Integral(-2065*x**6/(5*x**2*sqrt(5*x**2 +
 2*x + 3) + 2*x*sqrt(5*x**2 + 2*x + 3) + 3*sqrt(5*x**2 + 2*x + 3)), x) - In
tegral(1127*x**7/(5*x**2*sqrt(5*x**2 + 2*x + 3) + 2*x*sqrt(5*x**2 + 2*x + 3
) + 3*sqrt(5*x**2 + 2*x + 3)), x) - Integral(343*x**8/(5*x**2*sqrt(5*x**2 +
 2*x + 3) + 2*x*sqrt(5*x**2 + 2*x + 3) + 3*sqrt(5*x**2 + 2*x + 3)), x) - In
tegral(-2/(5*x**2*sqrt(5*x**2 + 2*x + 3) + 2*x*sqrt(5*x**2 + 2*x + 3) + 3*s
qrt(5*x**2 + 2*x + 3)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.89

$$\int \frac{(1+4x-7x^2)^3\,(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx = -\frac{343\,x^7}{30\,\sqrt{5\,x^2+2\,x+3}} - \frac{29351\,x^6}{750\,\sqrt{5\,x^2+2\,x+3}} + \frac{1025843\,x^5}{7500\,\sqrt{5\,x^2+2\,x+3}} - \frac{998969\,x^4}{37500\,\sqrt{5\,x^2+2\,x+3}} - \frac{51303971\,x^3}{187500\,\sqrt{5\,x^2+2\,x+3}} + \frac{61004099\,x^2}{187500\,\sqrt{5\,x^2+2\,x+3}} + \frac{50047657}{781250}\,\sqrt{5}\,\mathrm{arsinh}\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right) - \frac{87141949\,x}{546875\,\sqrt{5\,x^2+2\,x+3}} + \frac{525961603}{1093750\,\sqrt{5\,x^2+2\,x+3}}$$

```
[In] integrate((-7*x^2+4*x+1)^3*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2),x, algorithm="ma
xima")
```

```
[Out] -343/30*x^7/sqrt(5*x^2 + 2*x + 3) - 29351/750*x^6/sqrt(5*x^2 + 2*x + 3) + 1
025843/7500*x^5/sqrt(5*x^2 + 2*x + 3) - 998969/37500*x^4/sqrt(5*x^2 + 2*x +
 3) - 51303971/187500*x^3/sqrt(5*x^2 + 2*x + 3) + 61004099/187500*x^2/sqrt(
5*x^2 + 2*x + 3) + 50047657/781250*sqrt(5)*arcsinh(1/14*sqrt(14)*(5*x + 1))
 - 87141949/546875*x/sqrt(5*x^2 + 2*x + 3) + 525961603/1093750/sqrt(5*x^2 +
 2*x + 3)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.49

$$\int \frac{(1+4x-7x^2)^3\,(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx = -\frac{50047657}{781250}\,\sqrt{5}\log\left(-\sqrt{5}\left(\sqrt{5}x-\sqrt{5\,x^2+2\,x+3}\right)-1\right) - \frac{(35\,((5\,(35\,(70\,(175\,x+599)x-146549)x+998969)x+51303971)x-61004099)x+1045703388)x-31}{6562500\,\sqrt{5\,x^2+2\,x+3}}$$

```
[In] integrate((-7*x^2+4*x+1)^3*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2),x, algorithm="gi
ac")
```

```
[Out] -50047657/781250*sqrt(5)*log(-sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3)) -
 1) - 1/6562500*((35*((5*(35*(70*(175*x + 599)*x - 146549)*x + 998969)*x +
51303971)*x - 61004099)*x + 1045703388)*x - 3155769618)/sqrt(5*x^2 + 2*x +
3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+4x-7x^2)^3\,(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx = \int \frac{(x^2+5\,x+2)\,\left(-7\,x^2+4\,x+1\right)^3}{\left(5\,x^2+2\,x+3\right)^{3/2}}\,dx$$

```
[In] int(((5*x + x^2 + 2)*(4*x - 7*x^2 + 1)^3)/(2*x + 5*x^2 + 3)^(3/2),x)
[Out] int(((5*x + x^2 + 2)*(4*x - 7*x^2 + 1)^3)/(2*x + 5*x^2 + 3)^(3/2), x)
```

3.393 $\int \frac{(1+4x-7x^2)^2(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 124

$$\begin{aligned}\int \frac{(1+4x-7x^2)^2(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx &= -\frac{8(12983+136602x)}{21875\sqrt{3+2x+5x^2}}\\ &\quad-\frac{5086\sqrt{3+2x+5x^2}}{3125}-\frac{8749x\sqrt{3+2x+5x^2}}{1250}+\frac{203}{100}x^2\sqrt{3+2x+5x^2}\\ &\quad+\frac{49}{100}x^3\sqrt{3+2x+5x^2}+\frac{89583\operatorname{arcsinh}\left(\frac{1+5x}{\sqrt{14}}\right)}{1250\sqrt{5}}\end{aligned}$$

```
[Out] 89583/6250*arcsinh(1/14*(1+5*x)*14^(1/2))*5^(1/2)-8/21875*(12983+136602*x)/
(5*x^2+2*x+3)^(1/2)-5086/3125*(5*x^2+2*x+3)^(1/2)-8749/1250*x*(5*x^2+2*x+3)
^(1/2)+203/100*x^2*(5*x^2+2*x+3)^(1/2)+49/100*x^3*(5*x^2+2*x+3)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1674, 1675, 654, 633, 221}

$$\begin{aligned}\int \frac{(1+4x-7x^2)^2(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx &= \frac{89583\operatorname{arcsinh}\left(\frac{5x+1}{\sqrt{14}}\right)}{1250\sqrt{5}}\\ &\quad+\frac{203}{100}\sqrt{5x^2+2x+3}x^2-\frac{8749\sqrt{5x^2+2x+3}x}{1250}-\frac{5086\sqrt{5x^2+2x+3}}{3125}\\ &\quad-\frac{8(136602x+12983)}{21875\sqrt{5x^2+2x+3}}+\frac{49}{100}\sqrt{5x^2+2x+3}x^3\end{aligned}$$

```
[In] Int[((1 + 4*x - 7*x^2)^2*(2 + 5*x + x^2))/(3 + 2*x + 5*x^2)^(3/2),x]
```

```
[Out] (-8*(12983 + 136602*x))/(21875*Sqrt[3 + 2*x + 5*x^2]) - (5086*Sqrt[3 + 2*x
+ 5*x^2])/3125 - (8749*x*Sqrt[3 + 2*x + 5*x^2])/1250 + (203*x^2*Sqrt[3 + 2*
x + 5*x^2])/100 + (49*x^3*Sqrt[3 + 2*x + 5*x^2])/100 + (89583*ArcSinh[(1 +
5*x)/Sqrt[14]])/(1250*Sqrt[5])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{8(12983+136602x)}{21875\sqrt{3+2x+5x^2}}+\frac{1}{28}\int\frac{\frac{4291112}{3125}-\frac{296716x}{625}-\frac{194012x^2}{125}+\frac{23716x^3}{25}+\frac{1372x^4}{5}}{\sqrt{3+2x+5x^2}}\,dx\\
&= -\frac{8(12983+136602x)}{21875\sqrt{3+2x+5x^2}}+\frac{49}{100}x^3\sqrt{3+2x+5x^2}\\
&\quad+\frac{1}{560}\int\frac{\frac{17164448}{625}-\frac{1186864x}{125}-\frac{837788x^2}{25}+17052x^3}{\sqrt{3+2x+5x^2}}\,dx\\
&= -\frac{8(12983+136602x)}{21875\sqrt{3+2x+5x^2}}+\frac{203}{100}x^2\sqrt{3+2x+5x^2}\\
&\quad+\frac{49}{100}x^3\sqrt{3+2x+5x^2}+\frac{\int\frac{\frac{51493344}{125}-\frac{6118392x}{25}-\frac{2939664x^2}{5}}{\sqrt{3+2x+5x^2}}\,dx}{8400}\\
&= -\frac{8(12983+136602x)}{21875\sqrt{3+2x+5x^2}}-\frac{8749x\sqrt{3+2x+5x^2}}{1250}+\frac{203}{100}x^2\sqrt{3+2x+5x^2}\\
&\quad+\frac{49}{100}x^3\sqrt{3+2x+5x^2}+\frac{\int\frac{\frac{147081648}{25}-\frac{3417792x}{5}}{\sqrt{3+2x+5x^2}}\,dx}{84000}\\
&= -\frac{8(12983+136602x)}{21875\sqrt{3+2x+5x^2}}-\frac{5086\sqrt{3+2x+5x^2}}{3125}-\frac{8749x\sqrt{3+2x+5x^2}}{1250}\\
&\quad+\frac{203}{100}x^2\sqrt{3+2x+5x^2}+\frac{49}{100}x^3\sqrt{3+2x+5x^2}+\frac{89583\int\frac{1}{\sqrt{3+2x+5x^2}}\,dx}{1250}\\
&= -\frac{8(12983+136602x)}{21875\sqrt{3+2x+5x^2}}-\frac{5086\sqrt{3+2x+5x^2}}{3125}\\
&\quad-\frac{8749x\sqrt{3+2x+5x^2}}{1250}+\frac{203}{100}x^2\sqrt{3+2x+5x^2}\\
&\quad+\frac{49}{100}x^3\sqrt{3+2x+5x^2}+\frac{89583\text{Subst}\left(\int\frac{1}{\sqrt{1+\frac{x^2}{56}}}\,dx,x,2+10x\right)}{2500\sqrt{70}}\\
&= -\frac{8(12983+136602x)}{21875\sqrt{3+2x+5x^2}}-\frac{5086\sqrt{3+2x+5x^2}}{3125}-\frac{8749x\sqrt{3+2x+5x^2}}{1250}\\
&\quad+\frac{203}{100}x^2\sqrt{3+2x+5x^2}+\frac{49}{100}x^3\sqrt{3+2x+5x^2}+\frac{89583\sinh^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{1250\sqrt{5}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.64

$$\int \frac{(1+4x-7x^2)^2\,(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx = \frac{-168536-1298674x-280805x^2-515655x^3+194775x^4+42875x^5}{17500\sqrt{3+2x+5x^2}} - \frac{89583\log\left(-1-5x+\sqrt{5}\sqrt{3+2x+5x^2}\right)}{1250\sqrt{5}}$$

```
[In] Integrate[((1 + 4*x - 7*x^2)^2*(2 + 5*x + x^2))/(3 + 2*x + 5*x^2)^(3/2),x]
```

```
[Out] (-168536 - 1298674*x - 280805*x^2 - 515655*x^3 + 194775*x^4 + 42875*x^5)/(1
7500*Sqrt[3 + 2*x + 5*x^2]) - (89583*Log[-1 - 5*x + Sqrt[5]*Sqrt[3 + 2*x +
5*x^2]])/(1250*Sqrt[5])
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.44

method	result
risch	$\frac{42875x^5+194775x^4-515655x^3-280805x^2-1298674x-168536}{17500\sqrt{5x^2+2x+3}}+\frac{89583\sqrt{5}\ \operatorname{arcsinh}\left(\frac{5\sqrt{14}\,\left(x+\frac{1}{5}\right)}{14}\right)}{6250}$
trager	$\frac{42875x^5+194775x^4-515655x^3-280805x^2-1298674x-168536}{17500\sqrt{5x^2+2x+3}}+\frac{89583\operatorname{RootOf}(\_Z^2-5)\ln(5\operatorname{RootOf}(\_Z^2-5)x+\operatorname{RootOf}(\_Z}{6250}$
default	$-\frac{5564(10x+2)}{21875\sqrt{5x^2+2x+3}}-\frac{28506}{3125\sqrt{5x^2+2x+3}}+\frac{49x^5}{20\sqrt{5x^2+2x+3}}+\frac{1113x^4}{100\sqrt{5x^2+2x+3}}-\frac{14733x^3}{500\sqrt{5x^2+2x+3}}-\frac{8023x^2}{500\sqrt{5x^2+2x+3}}-$

```
[In] int((-7*x^2+4*x+1)^2*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/17500*(42875*x^5+194775*x^4-515655*x^3-280805*x^2-1298674*x-168536)/(5*x^
2+2*x+3)^(1/2)+89583/6250*5^(1/2)*arcsinh(5/14*14^(1/2)*(x+1/5))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.82

$$\int \frac{(1+4x-7x^2)^2\,(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx = 627081\,\sqrt{5}(5\,x^2+2\,x+3)\log\left(-\sqrt{5}\sqrt{5\,x^2+2\,x+3}(5\,x+1)-25\,x^2\right.$$

```
[In] integrate((-7*x^2+4*x+1)^2*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2),x, algorithm="fr
icas")
```

```
[Out] 1/87500*(627081*sqrt(5)*(5*x^2 + 2*x + 3)*log(-sqrt(5)*sqrt(5*x^2 + 2*x + 3
)*(5*x + 1) - 25*x^2 - 10*x - 8) + 5*(42875*x^5 + 194775*x^4 - 515655*x^3 -
 280805*x^2 - 1298674*x - 168536)*sqrt(5*x^2 + 2*x + 3))/(5*x^2 + 2*x + 3)
```

Sympy [F]

$$\int \frac{(1+4x-7x^2)^2\,(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx = \int \frac{(x^2+5x+2)\,(7x^2-4x-1)^2}{(5x^2+2x+3)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((-7*x**2+4*x+1)**2*(x**2+5*x+2)/(5*x**2+2*x+3)**(3/2),x)
```

```
[Out] Integral((x**2 + 5*x + 2)*(7*x**2 - 4*x - 1)**2/(5*x**2 + 2*x + 3)**(3/2),
x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.92

$$\begin{aligned}\int \frac{(1+4x-7x^2)^2\,(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx &= \frac{49\,x^5}{20\,\sqrt{5\,x^2+2\,x+3}} \\ &+ \frac{1113\,x^4}{100\,\sqrt{5\,x^2+2\,x+3}} - \frac{14733\,x^3}{500\,\sqrt{5\,x^2+2\,x+3}} - \frac{8023\,x^2}{500\,\sqrt{5\,x^2+2\,x+3}} \\ &+ \frac{89583}{6250}\,\sqrt{5}\,\mathrm{arsinh}\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right) - \frac{649337\,x}{8750\,\sqrt{5\,x^2+2\,x+3}} - \frac{42134}{4375\,\sqrt{5\,x^2+2\,x+3}}\end{aligned}$$

```
[In] integrate((-7*x^2+4*x+1)^2*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2),x, algorithm="ma
xima")
```

```
[Out] 49/20*x^5/sqrt(5*x^2 + 2*x + 3) + 1113/100*x^4/sqrt(5*x^2 + 2*x + 3) - 1473
3/500*x^3/sqrt(5*x^2 + 2*x + 3) - 8023/500*x^2/sqrt(5*x^2 + 2*x + 3) + 8958
3/6250*sqrt(5)*arcsinh(1/14*sqrt(14)*(5*x + 1)) - 649337/8750*x/sqrt(5*x^2
+ 2*x + 3) - 42134/4375/sqrt(5*x^2 + 2*x + 3)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.57

$$\int \frac{(1+4x-7x^2)^2(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx = -\frac{89583}{6250}\sqrt{5}\log\left(-\sqrt{5}\left(\sqrt{5}x-\sqrt{5\,x^2+2\,x+3}\right)-1\right) + \frac{(35\,((35\,(35\,x+159)x-14733)x-8023)x-1298674)x-168536}{17500\,\sqrt{5\,x^2+2\,x+3}}$$

```
[In] integrate((-7*x^2+4*x+1)^2*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2),x, algorithm="gi
ac")
```

```
[Out] -89583/6250*sqrt(5)*log(-sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3)) - 1) +
 1/17500*((35*((35*(35*x + 159)*x - 14733)*x - 8023)*x - 1298674)*x - 16853
6)/sqrt(5*x^2 + 2*x + 3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+4x-7x^2)^2(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx = \int \frac{(x^2+5\,x+2)\,(-7\,x^2+4\,x+1)^2}{(5\,x^2+2\,x+3)^{3/2}}\,dx$$

```
[In] int(((5*x + x^2 + 2)*(4*x - 7*x^2 + 1)^2)/(2*x + 5*x^2 + 3)^(3/2),x)
```

```
[Out] int(((5*x + x^2 + 2)*(4*x - 7*x^2 + 1)^2)/(2*x + 5*x^2 + 3)^(3/2), x)
```

3.394 $\int \frac{(1+4x-7x^2)(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 82

$$\int \frac{(1+4x-7x^2)(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx = -\frac{2(2321+2449x)}{875\sqrt{3+2x+5x^2}} - \frac{261}{250}\sqrt{3+2x+5x^2} - \frac{7}{50}x\sqrt{3+2x+5x^2} + \frac{149\text{arcsinh}\left(\frac{1+5x}{\sqrt{14}}\right)}{25\sqrt{5}}$$

```
[Out] 149/125*arcsinh(1/14*(1+5*x)*14^(1/2))*5^(1/2)-2/875*(2321+2449*x)/(5*x^2+2
*x+3)^(1/2)-261/250*(5*x^2+2*x+3)^(1/2)-7/50*x*(5*x^2+2*x+3)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {1674, 1675, 654, 633, 221}

$$\int \frac{(1+4x-7x^2)(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx = \frac{149\text{arcsinh}\left(\frac{5x+1}{\sqrt{14}}\right)}{25\sqrt{5}} - \frac{7}{50}\sqrt{5x^2+2x+3}x - \frac{261}{250}\sqrt{5x^2+2x+3} - \frac{2(2449x+2321)}{875\sqrt{5x^2+2x+3}}$$

```
[In] Int[((1 + 4*x - 7*x^2)*(2 + 5*x + x^2))/(3 + 2*x + 5*x^2)^(3/2),x]
```

```
[Out] (-2*(2321 + 2449*x))/(875*Sqrt[3 + 2*x + 5*x^2]) - (261*Sqrt[3 + 2*x + 5*x^
2])/250 - (7*x*Sqrt[3 + 2*x + 5*x^2])/50 + (149*ArcSinh[(1 + 5*x)/Sqrt[14]]
)/(25*Sqrt[5])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2(2321+2449x)}{875\sqrt{3+2x+5x^2}}+\frac{1}{28}\int\frac{\frac{15736}{125}-\frac{3948x}{25}-\frac{196x^2}{5}}{\sqrt{3+2x+5x^2}}\,dx\\ &= -\frac{2(2321+2449x)}{875\sqrt{3+2x+5x^2}}-\frac{7}{50}x\sqrt{3+2x+5x^2}+\frac{1}{280}\int\frac{\frac{34412}{25}-\frac{7308x}{5}}{\sqrt{3+2x+5x^2}}\,dx\end{aligned}$$

$$= -\frac{2(2321+2449x)}{875\sqrt{3+2x+5x^2}} - \frac{261}{250}\sqrt{3+2x+5x^2} - \frac{7}{50}x\sqrt{3+2x+5x^2} + \frac{149}{25}\int \frac{1}{\sqrt{3+2x+5x^2}}\,dx$$

$$= -\frac{2(2321+2449x)}{875\sqrt{3+2x+5x^2}} - \frac{261}{250}\sqrt{3+2x+5x^2} - \frac{7}{50}x\sqrt{3+2x+5x^2} + \frac{149\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{56}}}\,dx, x, 2+10x\right)}{50\sqrt{70}}$$

$$= -\frac{2(2321+2449x)}{875\sqrt{3+2x+5x^2}} - \frac{261}{250}\sqrt{3+2x+5x^2} - \frac{7}{50}x\sqrt{3+2x+5x^2} + \frac{149\sinh^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{25\sqrt{5}}$$

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.84

$$\int \frac{(1+4x-7x^2)(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx = \frac{-2953-2837x-1925x^2-245x^3}{350\sqrt{3+2x+5x^2}} - \frac{149\log\left(-1-5x+\sqrt{5}\sqrt{3+2x+5x^2}\right)}{25\sqrt{5}}$$

```
[In] Integrate[((1 + 4*x - 7*x^2)*(2 + 5*x + x^2))/(3 + 2*x + 5*x^2)^(3/2),x]
```

```
[Out] (-2953 - 2837*x - 1925*x^2 - 245*x^3)/(350*Sqrt[3 + 2*x + 5*x^2]) - (149*Lo
g[-1 - 5*x + Sqrt[5]*Sqrt[3 + 2*x + 5*x^2]])/(25*Sqrt[5])
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.55

method	result
risch	$-\frac{245x^3+1925x^2+2837x+2953}{350\sqrt{5x^2+2x+3}} + \frac{149\sqrt{5}\operatorname{arcsinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{125}$
trager	$-\frac{245x^3+1925x^2+2837x+2953}{350\sqrt{5x^2+2x+3}} - \frac{149\operatorname{RootOf}\left(\_Z^2-5\right)\ln\left(-5\operatorname{RootOf}\left(\_Z^2-5\right)x+5\sqrt{5x^2+2x+3}-\operatorname{RootOf}\left(\_Z^2-5\right)\right)}{125}$
default	$-\frac{751(10x+2)}{3500\sqrt{5x^2+2x+3}} - \frac{1001}{125\sqrt{5x^2+2x+3}} - \frac{7x^3}{10\sqrt{5x^2+2x+3}} - \frac{11x^2}{2\sqrt{5x^2+2x+3}} - \frac{149x}{25\sqrt{5x^2+2x+3}} + \frac{149\sqrt{5}\operatorname{arcsinh}\left(\frac{5\sqrt{14}\left(x+\right.}{14}\right.}{125}$

```
[In] int((-7*x^2+4*x+1)*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/350*(245*x^3+1925*x^2+2837*x+2953)/(5*x^2+2*x+3)^(1/2)+149/125*5^(1/2)*a
rcsinh(5/14*14^(1/2)*(x+1/5))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.12

$$\int \frac{(1+4x-7x^2)(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx = \frac{1043\sqrt{5}(5x^2+2x+3)\log\left(-\sqrt{5}\sqrt{5x^2+2x+3}(5x+1)-25x^2-1\right.}{1750\,(5x^2+2}$$

```
[In] integrate((-7*x^2+4*x+1)*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2),x, algorithm="fric
as")
```

```
[Out] 1/1750*(1043*sqrt(5)*(5*x^2 + 2*x + 3)*log(-sqrt(5)*sqrt(5*x^2 + 2*x + 3)*(
5*x + 1) - 25*x^2 - 10*x - 8) - 5*(245*x^3 + 1925*x^2 + 2837*x + 2953)*sqrt
(5*x^2 + 2*x + 3))/(5*x^2 + 2*x + 3)
```

Sympy [F]

$$\begin{aligned}\int \frac{(1+4x-7x^2)(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx =\\ &-\int \left(-\frac{13x}{5x^2\sqrt{5x^2+2x+3}+2x\sqrt{5x^2+2x+3}+3\sqrt{5x^2+2x+3}}\right)dx\\ &-\int \left(-\frac{7x^2}{5x^2\sqrt{5x^2+2x+3}+2x\sqrt{5x^2+2x+3}+3\sqrt{5x^2+2x+3}}\right)dx\\ &-\int \frac{31x^3}{5x^2\sqrt{5x^2+2x+3}+2x\sqrt{5x^2+2x+3}+3\sqrt{5x^2+2x+3}}\,dx\\ &-\int \frac{7x^4}{5x^2\sqrt{5x^2+2x+3}+2x\sqrt{5x^2+2x+3}+3\sqrt{5x^2+2x+3}}\,dx\\ &-\int \left(-\frac{2}{5x^2\sqrt{5x^2+2x+3}+2x\sqrt{5x^2+2x+3}+3\sqrt{5x^2+2x+3}}\right)dx\end{aligned}$$

```
[In] integrate((-7*x**2+4*x+1)*(x**2+5*x+2)/(5*x**2+2*x+3)**(3/2),x)
```

```
[Out] -Integral(-13*x/(5*x**2*sqrt(5*x**2 + 2*x + 3) + 2*x*sqrt(5*x**2 + 2*x + 3)
 + 3*sqrt(5*x**2 + 2*x + 3)), x) - Integral(-7*x**2/(5*x**2*sqrt(5*x**2 + 2
*x + 3) + 2*x*sqrt(5*x**2 + 2*x + 3) + 3*sqrt(5*x**2 + 2*x + 3)), x) - Inte
gral(31*x**3/(5*x**2*sqrt(5*x**2 + 2*x + 3) + 2*x*sqrt(5*x**2 + 2*x + 3) +
3*sqrt(5*x**2 + 2*x + 3)), x) - Integral(7*x**4/(5*x**2*sqrt(5*x**2 + 2*x +
 3) + 2*x*sqrt(5*x**2 + 2*x + 3) + 3*sqrt(5*x**2 + 2*x + 3)), x) - Integral
(-2/(5*x**2*sqrt(5*x**2 + 2*x + 3) + 2*x*sqrt(5*x**2 + 2*x + 3) + 3*sqrt(5*
x**2 + 2*x + 3)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.98

$$\int \frac{(1+4x-7x^2)(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx = -\frac{7\,x^3}{10\,\sqrt{5\,x^2+2\,x+3}} - \frac{11\,x^2}{2\,\sqrt{5\,x^2+2\,x+3}} + \frac{149}{125}\,\sqrt{5}\operatorname{arsinh}\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right) - \frac{2837\,x}{350\,\sqrt{5\,x^2+2\,x+3}} - \frac{2953}{350\,\sqrt{5\,x^2+2\,x+3}}$$

```
[In] integrate((-7*x^2+4*x+1)*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2),x, algorithm="maxi
ma")
```

```
[Out] -7/10*x^3/sqrt(5*x^2 + 2*x + 3) - 11/2*x^2/sqrt(5*x^2 + 2*x + 3) + 149/125*
sqrt(5)*arcsinh(1/14*sqrt(14)*(5*x + 1)) - 2837/350*x/sqrt(5*x^2 + 2*x + 3)
 - 2953/350/sqrt(5*x^2 + 2*x + 3)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.76

$$\int \frac{(1+4x-7x^2)(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx = -\frac{149}{125}\,\sqrt{5}\log\left(-\sqrt{5}\left(\sqrt{5}x - \sqrt{5\,x^2+2\,x+3}\right) - 1\right) - \frac{(35\,(7\,x+55)x+2837)x+2953}{350\,\sqrt{5\,x^2+2\,x+3}}$$

```
[In] integrate((-7*x^2+4*x+1)*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2),x, algorithm="giac
")
```

```
[Out] -149/125*sqrt(5)*log(-sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3)) - 1) - 1/
350*((35*(7*x + 55)*x + 2837)*x + 2953)/sqrt(5*x^2 + 2*x + 3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+4x-7x^2)(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx = \int \frac{(x^2+5\,x+2)\,(-7\,x^2+4\,x+1)}{(5\,x^2+2\,x+3)^{3/2}}\,dx$$

```
[In] int(((5*x + x^2 + 2)*(4*x - 7*x^2 + 1))/(2*x + 5*x^2 + 3)^(3/2),x)
```

```
[Out] int(((5*x + x^2 + 2)*(4*x - 7*x^2 + 1))/(2*x + 5*x^2 + 3)^(3/2), x)
```

3.395 $\int \frac{2+5x+x^2}{(1+4x-7x^2)(3+2x+5x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 166

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)(3+2x+5x^2)^{3/2}}\,dx = -\frac{131-605x}{3556\sqrt{3+2x+5x^2}} - \frac{3\sqrt{\frac{281693-25015\sqrt{11}}{1397}}\,\mathrm{arctanh}\left(\frac{23-\sqrt{11}+\left(17-5\sqrt{11}\right)x}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)}{1016} + \frac{3\sqrt{\frac{281693+25015\sqrt{11}}{1397}}\,\mathrm{arctanh}\left(\frac{23+\sqrt{11}+\left(17+5\sqrt{11}\right)x}{\sqrt{2\left(125+17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)}{1016}$$

```
[Out] 1/3556*(-131+605*x)/(5*x^2+2*x+3)^(1/2)-3/1419352*arctanh((23+x*(17-5*11^(1
/2))-11^(1/2))/(5*x^2+2*x+3)^(1/2)/(250-34*11^(1/2))^(1/2))*(393525121-3494
5955*11^(1/2))^(1/2)+3/1419352*arctanh((23+11^(1/2)+x*(17+5*11^(1/2)))/(5*x
^2+2*x+3)^(1/2)/(250+34*11^(1/2))^(1/2))*(393525121+34945955*11^(1/2))^(1/2
)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used

$= \{1074, 1046, 738, 212\}$

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)\left(3+2x+5x^2\right)^{3/2}}\,dx = -\frac{3\sqrt{\frac{281693-25015\sqrt{11}}{1397}}\operatorname{arctanh}\left(\frac{\left(17-5\sqrt{11}\right)x-\sqrt{11}+23}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right)}{1016} + \frac{3\sqrt{\frac{281693+25015\sqrt{11}}{1397}}\operatorname{arctanh}\left(\frac{\left(17+5\sqrt{11}\right)x+\sqrt{11}+23}{\sqrt{2\left(125+17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right)}{1016} - \frac{131-605x}{3556\sqrt{5x^2+2x+3}}$$

```
[In] Int[(2 + 5*x + x^2)/((1 + 4*x - 7*x^2)*(3 + 2*x + 5*x^2)^(3/2)),x]
```

```
[Out] -1/3556*(131 - 605*x)/Sqrt[3 + 2*x + 5*x^2] - (3*Sqrt[(281693 - 25015*Sqrt[
11])/1397]*ArcTanh[(23 - Sqrt[11] + (17 - 5*Sqrt[11])*x)/(Sqrt[2*(125 - 17*
Sqrt[11])]*Sqrt[3 + 2*x + 5*x^2])])/1016 + (3*Sqrt[(281693 + 25015*Sqrt[11]
)/1397]*ArcTanh[(23 + Sqrt[11] + (17 + 5*Sqrt[11])*x)/(Sqrt[2*(125 + 17*Sqr
t[11])]*Sqrt[3 + 2*x + 5*x^2])])/1016
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1046

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1074

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x +
```

```
c*x^2)^(p + 1)*((d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f))*(p + 1)))*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) +
(A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c
*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d
 - a*f)))*x), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rubi steps

$$\text{integral} = -\frac{131-605x}{3556\sqrt{3+2x+5x^2}} + \frac{\int \frac{13776+14112x}{(1+4x-7x^2)\sqrt{3+2x+5x^2}}\,dx}{28448}$$

$$= -\frac{131-605x}{3556\sqrt{3+2x+5x^2}} + \frac{\left(21\left(66-53\sqrt{11}\right)\right)\int \frac{1}{\left(4-2\sqrt{11}-14x\right)\sqrt{3+2x+5x^2}}\,dx}{2794} + \frac{\left(21\left(66+53\sqrt{11}\right)\right)\int \frac{1}{\left(4+2\sqrt{11}-14x\right)\sqrt{3+2x+5x^2}}\,dx}{2794}$$

$$= -\frac{131-605x}{3556\sqrt{3+2x+5x^2}} - \frac{\left(21\left(66-53\sqrt{11}\right)\right)\text{Subst}\Bigg(\int \frac{1}{2352+112\left(4-2\sqrt{11}\right)+20\left(4-2\sqrt{11}\right)^2-x^2}\,dx, x, \frac{-84-2\left(4-2\sqrt{11}\right)-\left(28+10\left(4-2\sqrt{11}\right)\right)}{\sqrt{3+2x+5x^2}}}{1397} - \frac{\left(21\left(66+53\sqrt{11}\right)\right)\text{Subst}\Bigg(\int \frac{1}{2352+112\left(4+2\sqrt{11}\right)+20\left(4+2\sqrt{11}\right)^2-x^2}\,dx, x, \frac{-84-2\left(4+2\sqrt{11}\right)-\left(28+10\left(4+2\sqrt{11}\right)\right)}{\sqrt{3+2x+5x^2}}}{1397}$$

$$= -\frac{131-605x}{3556\sqrt{3+2x+5x^2}} - \frac{3\sqrt{\frac{281693-25015\sqrt{11}}{1397}}\tanh^{-1}\left(\frac{23-\sqrt{11}+\left(17-5\sqrt{11}\right)x}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)}{1016}$$
$$+ \frac{3\sqrt{\frac{281693+25015\sqrt{11}}{1397}}\tanh^{-1}\left(\frac{23+\sqrt{11}+\left(17+5\sqrt{11}\right)x}{\sqrt{2\left(125+17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)}{1016}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.40 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.20

$$\int \frac{2+5x+x^2}{\left(1+4x-7x^2\right)\left(3+2x+5x^2\right)^{3/2}}\,dx = \frac{-131+605x}{3556\sqrt{3+2x+5x^2}}$$
$$+ \frac{3}{254}\text{RootSum}\Bigg[83-16\sqrt{5}\#1-70\#1^2+8\sqrt{5}\#1^3$$
$$+7\#1^4\&, \frac{22\log\left(-\sqrt{5}x+\sqrt{3+2x+5x^2}-\#1\right)+41\sqrt{5}\log\left(-\sqrt{5}x+\sqrt{3+2x+5x^2}-\#1\right)\#1-21\log(}{-4\sqrt{5}-35\#1+6\sqrt{5}\#1^2+7\#1^3}$$

```
[In] Integrate[(2 + 5*x + x^2)/((1 + 4*x - 7*x^2)*(3 + 2*x + 5*x^2)^(3/2)),x]
```

```
[Out] (-131 + 605*x)/(3556*Sqrt[3 + 2*x + 5*x^2]) + (3*RootSum[83 - 16*Sqrt[5]*#1
 - 70*#1^2 + 8*Sqrt[5]*#1^3 + 7*#1^4 & , (22*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*
x + 5*x^2] - #1] + 41*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1
]*#1 - 21*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1^2)/(-4*Sqrt[5]
- 35*#1 + 6*Sqrt[5]*#1^2 + 7*#1^3) & ])/254
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.26

method	result
risch	$\frac{-131+605x}{3556\sqrt{5x^2+2x+3}}+\frac{21\left(53+6\sqrt{11}\right)\sqrt{11}\,\mathrm{arctanh}\left(\frac{250+34\sqrt{11}+\frac{49\left(\frac{34}{7}+\frac{10\sqrt{11}}{7}\right)\left(x-\frac{2}{7}-\frac{\sqrt{11}}{7}\right)}{2}}{\sqrt{250+34\sqrt{11}}\sqrt{245\left(x-\frac{2}{7}-\frac{\sqrt{11}}{7}\right)^2+49\left(\frac{34}{7}+\frac{10\sqrt{11}}{7}\right)\left(x-\frac{2}{7}-\frac{\sqrt{11}}{7}\right)+250+34\sqrt{11}}}\right)}{5588\sqrt{250+34\sqrt{11}}}+$
trager	$\frac{-131+605x}{3556\sqrt{5x^2+2x+3}}+\frac{27\,\mathrm{RootOf}\left(12905329536\_Z^4-4015815408\_Z^2+285305881\right)\ln\left(\frac{8804847847027176806 4x\,\mathrm{RootOf}\left(12905329536\_\right.}{}\right.}{}$
default	$-\frac{10x+2}{196\sqrt{5x^2+2x+3}}-\frac{3\left(61+13\sqrt{11}\right)\sqrt{11}\left(\frac{1}{7\left(\frac{250}{49}+\frac{34\sqrt{11}}{49}\right)\sqrt{5\left(x-\frac{2}{7}-\frac{\sqrt{11}}{7}\right)^2+\left(\frac{34}{7}+\frac{10\sqrt{11}}{7}\right)\left(x-\frac{2}{7}-\frac{\sqrt{11}}{7}\right)+\frac{250}{49}+\frac{34\sqrt{11}}{49}}}-\frac{}{7\left(\frac{250}{49}+\frac{34\sqrt{}}{49}\right.}\right.}{}$

```
[In] int((x^2+5*x+2)/(-7*x^2+4*x+1)/(5*x^2+2*x+3)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3556*(-131+605*x)/(5*x^2+2*x+3)^(1/2)+21/5588*(53+6*11^(1/2))*11^(1/2)/(2
50+34*11^(1/2))^(1/2)*arctanh(49/2*(500/49+68/49*11^(1/2)+(34/7+10/7*11^(1/
2))*(x-2/7-1/7*11^(1/2)))/(250+34*11^(1/2))^(1/2)/(245*(x-2/7-1/7*11^(1/2))
^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2))+21/
5588*(-53+6*11^(1/2))*11^(1/2)/(250-34*11^(1/2))^(1/2)*arctanh(49/2*(500/49
-68/49*11^(1/2)+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2)))/(250-34*11^(1/2)
)^(1/2)/(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(
1/2))+250-34*11^(1/2))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 333 vs. 2(117) = 234.

Time = 0.27 (sec) , antiderivative size = 333, normalized size of antiderivative = 2.01

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)\left(3+2x+5x^2\right)^{3/2}}\,dx =$$

$$-\frac{21\sqrt{1397}(5\,x^2+2\,x+3)\sqrt{25015\,\sqrt{11}+281693}\log\left(\frac{3\left(\sqrt{1397}\sqrt{5\,x^2+2\,x+3}\sqrt{25015\,\sqrt{11}+281693}\left(1335\,\sqrt{11}-8173\right)+235967\right.}{x}\right.}{}$$

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)/(5*x^2+2*x+3)^(3/2),x, algorithm="fric
as")
```

```
[Out] -1/19870928*(21*sqrt(1397)*(5*x^2 + 2*x + 3)*sqrt(25015*sqrt(11) + 281693)*
log(3*(sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*sqrt(25015*sqrt(11) + 281693)*(1335
*sqrt(11) - 8173) + 23596727*sqrt(11)*(x + 3) + 70790181*x - 117983635)/x)
- 21*sqrt(1397)*(5*x^2 + 2*x + 3)*sqrt(25015*sqrt(11) + 281693)*log(-3*(sqr
t(1397)*sqrt(5*x^2 + 2*x + 3)*sqrt(25015*sqrt(11) + 281693)*(1335*sqrt(11)
- 8173) - 23596727*sqrt(11)*(x + 3) - 70790181*x + 117983635)/x) + 7*sqrt(1
397)*(5*x^2 + 2*x + 3)*sqrt(-225135*sqrt(11) + 2535237)*log(-(sqrt(1397)*sq
rt(5*x^2 + 2*x + 3)*(1335*sqrt(11) + 8173)*sqrt(-225135*sqrt(11) + 2535237)
 + 70790181*sqrt(11)*(x + 3) - 212370543*x + 353950905)/x) - 7*sqrt(1397)*(
5*x^2 + 2*x + 3)*sqrt(-225135*sqrt(11) + 2535237)*log((sqrt(1397)*sqrt(5*x^
2 + 2*x + 3)*(1335*sqrt(11) + 8173)*sqrt(-225135*sqrt(11) + 2535237) - 7079
0181*sqrt(11)*(x + 3) + 212370543*x - 353950905)/x) - 5588*sqrt(5*x^2 + 2*x
 + 3)*(605*x - 131))/(5*x^2 + 2*x + 3)
```

Sympy [F]

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)\left(3+2x+5x^2\right)^{3/2}}\,dx =$$
$$-\int \frac{5x}{35x^4\sqrt{5x^2+2x+3}-6x^3\sqrt{5x^2+2x+3}+8x^2\sqrt{5x^2+2x+3}-14x\sqrt{5x^2+2x+3}-3\sqrt{5x^2+2x+}}$$
$$-\int \frac{x^2}{35x^4\sqrt{5x^2+2x+3}-6x^3\sqrt{5x^2+2x+3}+8x^2\sqrt{5x^2+2x+3}-14x\sqrt{5x^2+2x+3}-3\sqrt{5x^2+2x+}}$$
$$-\int \frac{2}{35x^4\sqrt{5x^2+2x+3}-6x^3\sqrt{5x^2+2x+3}+8x^2\sqrt{5x^2+2x+3}-14x\sqrt{5x^2+2x+3}-3\sqrt{5x^2+2x+}}$$

```
[In] integrate((x**2+5*x+2)/(-7*x**2+4*x+1)/(5*x**2+2*x+3)**(3/2),x)
```

```
[Out] -Integral(5*x/(35*x**4*sqrt(5*x**2 + 2*x + 3) - 6*x**3*sqrt(5*x**2 + 2*x +
3) + 8*x**2*sqrt(5*x**2 + 2*x + 3) - 14*x*sqrt(5*x**2 + 2*x + 3) - 3*sqrt(5
*x**2 + 2*x + 3)), x) - Integral(x**2/(35*x**4*sqrt(5*x**2 + 2*x + 3) - 6*x
**3*sqrt(5*x**2 + 2*x + 3) + 8*x**2*sqrt(5*x**2 + 2*x + 3) - 14*x*sqrt(5*x*
*2 + 2*x + 3) - 3*sqrt(5*x**2 + 2*x + 3)), x) - Integral(2/(35*x**4*sqrt(5*
x**2 + 2*x + 3) - 6*x**3*sqrt(5*x**2 + 2*x + 3) + 8*x**2*sqrt(5*x**2 + 2*x
+ 3) - 14*x*sqrt(5*x**2 + 2*x + 3) - 3*sqrt(5*x**2 + 2*x + 3)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 777 vs. $2(117) = 234$.

Time = 0.33 (sec) , antiderivative size = 777, normalized size of antiderivative = 4.68

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)\left(3+2x+5x^2\right)^{3/2}}\,dx =$$

$$-\frac{1}{4312}\sqrt{11}\left(\frac{20\sqrt{11}x}{\sqrt{5x^2+2x+3}} - \frac{7890\sqrt{11}x}{17\sqrt{11}\sqrt{5x^2+2x+3}+125\sqrt{5x^2+2x+3}} + \frac{7890\sqrt{11}}{17\sqrt{11}\sqrt{5x^2+2x+3}-1}\right.$$

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)/(5*x^2+2*x+3)^(3/2),x, algorithm="maxi
ma")
```

```
[Out] -1/4312*sqrt(11)*(20*sqrt(11)*x/sqrt(5*x^2 + 2*x + 3) - 7890*sqrt(11)*x/(17
*sqrt(11)*sqrt(5*x^2 + 2*x + 3) + 125*sqrt(5*x^2 + 2*x + 3)) + 7890*sqrt(11
)*x/(17*sqrt(11)*sqrt(5*x^2 + 2*x + 3) - 125*sqrt(5*x^2 + 2*x + 3)) - 13377
*sqrt(11)*sqrt(2)*arcsinh(5/7*sqrt(11)*sqrt(7)*sqrt(2)*x/abs(14*x - 2*sqrt(
11) - 4) + 17/7*sqrt(7)*sqrt(2)*x/abs(14*x - 2*sqrt(11) - 4) + 1/7*sqrt(11)
*sqrt(7)*sqrt(2)/abs(14*x - 2*sqrt(11) - 4) + 23/7*sqrt(7)*sqrt(2)/abs(14*x
 - 2*sqrt(11) - 4))/(17*sqrt(11) + 125)^(3/2) + 4*sqrt(11)/sqrt(5*x^2 + 2*x
 + 3) - 26280*x/(17*sqrt(11)*sqrt(5*x^2 + 2*x + 3) + 125*sqrt(5*x^2 + 2*x +
 3)) - 26280*x/(17*sqrt(11)*sqrt(5*x^2 + 2*x + 3) - 125*sqrt(5*x^2 + 2*x +
3)) + 156*sqrt(11)*arcsinh(5/7*sqrt(11)*sqrt(7)*sqrt(2)*x/abs(14*x + 2*sqrt
(11) - 4) - 17/7*sqrt(7)*sqrt(2)*x/abs(14*x + 2*sqrt(11) - 4) + 1/7*sqrt(11
)*sqrt(7)*sqrt(2)/abs(14*x + 2*sqrt(11) - 4) - 23/7*sqrt(7)*sqrt(2)/abs(14*
x + 2*sqrt(11) - 4))/(-34/49*sqrt(11) + 250/49)^(3/2) - 62769*sqrt(2)*arcsi
nh(5/7*sqrt(11)*sqrt(7)*sqrt(2)*x/abs(14*x - 2*sqrt(11) - 4) + 17/7*sqrt(7)
*sqrt(2)*x/abs(14*x - 2*sqrt(11) - 4) + 1/7*sqrt(11)*sqrt(7)*sqrt(2)/abs(14
*x - 2*sqrt(11) - 4) + 23/7*sqrt(7)*sqrt(2)/abs(14*x - 2*sqrt(11) - 4))/(17
*sqrt(11) + 125)^(3/2) + 2244*sqrt(11)/(17*sqrt(11)*sqrt(5*x^2 + 2*x + 3) +
 125*sqrt(5*x^2 + 2*x + 3)) - 2244*sqrt(11)/(17*sqrt(11)*sqrt(5*x^2 + 2*x +
 3) - 125*sqrt(5*x^2 + 2*x + 3)) - 732*arcsinh(5/7*sqrt(11)*sqrt(7)*sqrt(2)
*x/abs(14*x + 2*sqrt(11) - 4) - 17/7*sqrt(7)*sqrt(2)*x/abs(14*x + 2*sqrt(11
) - 4) + 1/7*sqrt(11)*sqrt(7)*sqrt(2)/abs(14*x + 2*sqrt(11) - 4) - 23/7*sqr
t(7)*sqrt(2)/abs(14*x + 2*sqrt(11) - 4))/(-34/49*sqrt(11) + 250/49)^(3/2) +
 12678/(17*sqrt(11)*sqrt(5*x^2 + 2*x + 3) + 125*sqrt(5*x^2 + 2*x + 3)) + 12
678/(17*sqrt(11)*sqrt(5*x^2 + 2*x + 3) - 125*sqrt(5*x^2 + 2*x + 3)))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.67

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)(3+2x+5x^2)^{3/2}}\,dx = \frac{605\,x-131}{3556\,\sqrt{5\,x^2+2\,x+3}} + 0.0477059376663667\,\log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}+4.41924736459000\right) - 0.0352174957838020\,\log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}+1.25295163054000\right) - 0.0477059376663667\,\log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}-1.02258038113000\right) + 0.0352174957838020\,\log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}-2.09411235400000\right)$$

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)/(5*x^2+2*x+3)^(3/2),x, algorithm="giac
")
```

```
[Out] 1/3556*(605*x - 131)/sqrt(5*x^2 + 2*x + 3) + 0.0477059376663667*log(-sqrt(5
)*x + sqrt(5*x^2 + 2*x + 3) + 4.41924736459000) - 0.0352174957838020*log(-s
qrt(5)*x + sqrt(5*x^2 + 2*x + 3) + 1.25295163054000) - 0.0477059376663667*l
og(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) - 1.02258038113000) + 0.0352174957838
020*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) - 2.09411235400000)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)(3+2x+5x^2)^{3/2}}\,dx = \int \frac{x^2+5\,x+2}{(5\,x^2+2\,x+3)^{3/2}\,(-7\,x^2+4\,x+1)}\,dx$$

```
[In] int((5*x + x^2 + 2)/((2*x + 5*x^2 + 3)^(3/2)*(4*x - 7*x^2 + 1)),x)
```

```
[Out] int((5*x + x^2 + 2)/((2*x + 5*x^2 + 3)^(3/2)*(4*x - 7*x^2 + 1)), x)
```

3.396 $\int \frac{2+5x+x^2}{\left(1+4x-7x^2\right)^2\left(3+2x+5x^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 215

$$
\begin{aligned}
&\int \frac{2+5x+x^2}{\left(1+4x-7x^2\right)^2\left(3+2x+5x^2\right)^{3/2}}\,dx = \\
&-\frac{76567+22755x}{19870928\sqrt{3+2x+5x^2}}-\frac{3(40-371x)}{5588\left(1+4x-7x^2\right)\sqrt{3+2x+5x^2}} \\
&-\frac{7\left(541543-5144\sqrt{11}\right)\operatorname{arctanh}\left(\frac{23-\sqrt{11}+\left(17-5\sqrt{11}\right)x}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)}{2838704\sqrt{22\left(125-17\sqrt{11}\right)}} \\
&+\frac{7\left(541543+5144\sqrt{11}\right)\operatorname{arctanh}\left(\frac{23+\sqrt{11}+\left(17+5\sqrt{11}\right)x}{\sqrt{2\left(125+17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)}{2838704\sqrt{22\left(125+17\sqrt{11}\right)}}
\end{aligned}
$$

```
[Out] 1/19870928*(-76567-22755*x)/(5*x^2+2*x+3)^(1/2)-3/5588*(40-371*x)/(-7*x^2+4
*x+1)/(5*x^2+2*x+3)^(1/2)-7/2838704*arctanh((23+x*(17-5*11^(1/2))-11^(1/2))
/(5*x^2+2*x+3)^(1/2)/(250-34*11^(1/2))^(1/2))*(541543-5144*11^(1/2))/(2750-
374*11^(1/2))^(1/2)+7/2838704*arctanh((23+11^(1/2)+x*(17+5*11^(1/2)))/(5*x^
2+2*x+3)^(1/2)/(250+34*11^(1/2))^(1/2))*(541543+5144*11^(1/2))/(2750+374*11
^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {1074, 1046, 738, 212}

$$
\begin{aligned}
&\int \frac{2+5x+x^2}{\left(1+4x-7x^2\right)^2\left(3+2x+5x^2\right)^{3/2}}\,dx = \\
&-\frac{7\left(541543-5144\sqrt{11}\right)\operatorname{arctanh}\left(\frac{\left(17-5\sqrt{11}\right)x-\sqrt{11}+23}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right)}{2838704\sqrt{22\left(125-17\sqrt{11}\right)}} \\
&+\frac{7\left(541543+5144\sqrt{11}\right)\operatorname{arctanh}\left(\frac{\left(17+5\sqrt{11}\right)x+\sqrt{11}+23}{\sqrt{2\left(125+17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right)}{2838704\sqrt{22\left(125+17\sqrt{11}\right)}} \\
&-\frac{3(40-371x)}{5588\left(-7x^2+4x+1\right)\sqrt{5x^2+2x+3}}-\frac{22755x+76567}{19870928\sqrt{5x^2+2x+3}}
\end{aligned}
$$

```
[In] Int[(2 + 5*x + x^2)/((1 + 4*x - 7*x^2)^2*(3 + 2*x + 5*x^2)^(3/2)),x]
```

```
[Out] -1/19870928*(76567 + 22755*x)/Sqrt[3 + 2*x + 5*x^2] - (3*(40 - 371*x))/(558
8*(1 + 4*x - 7*x^2)*Sqrt[3 + 2*x + 5*x^2]) - (7*(541543 - 5144*Sqrt[11])*Ar
cTanh[(23 - Sqrt[11] + (17 - 5*Sqrt[11])*x)/(Sqrt[2*(125 - 17*Sqrt[11])]*Sq
rt[3 + 2*x + 5*x^2])])/(2838704*Sqrt[22*(125 - 17*Sqrt[11])]) + (7*(541543
+ 5144*Sqrt[11])*ArcTanh[(23 + Sqrt[11] + (17 + 5*Sqrt[11])*x)/(Sqrt[2*(125
 + 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x^2])])/(2838704*Sqrt[22*(125 + 17*Sqrt[1
1])])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1046

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1074

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x +
c*x^2)^(p + 1)*((d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f))*(p + 1)))*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) +
(A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c
*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d
 - a*f)))*x), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] && !( !IntegerQ[p] && ILtQ[q, -1]) && !
IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{3(40-371x)}{5588\left(1+4x-7x^2\right)\sqrt{3+2x+5x^2}}-\frac{\int \frac{-50216-37752x-89040x^2}{\left(1+4x-7x^2\right)\left(3+2x+5x^2\right)^{3/2}}\,dx}{44704}\\ &= -\frac{76567+22755x}{19870928\sqrt{3+2x+5x^2}}-\frac{3(40-371x)}{5588\left(1+4x-7x^2\right)\sqrt{3+2x+5x^2}}-\frac{\int \frac{-476004480-32263168x}{\left(1+4x-7x^2\right)\sqrt{3+2x+5x^2}}\,dx}{1271739392}\end{aligned}$$

$$= -\frac{76567+22755x}{19870928\sqrt{3+2x+5x^2}} - \frac{3(40-371x)}{5588\left(1+4x-7x^2\right)\sqrt{3+2x+5x^2}} + \frac{\left(7\left(56584-541543\sqrt{11}\right)\right)\int \frac{1}{\left(4-2\sqrt{11}-14x\right)\sqrt{3+2x+5x^2}}\,dx}{15612872} + \frac{\left(7\left(56584+541543\sqrt{11}\right)\right)\int \frac{1}{\left(4+2\sqrt{11}-14x\right)\sqrt{3+2x+5x^2}}\,dx}{15612872}$$

$$= -\frac{76567+22755x}{19870928\sqrt{3+2x+5x^2}} - \frac{3(40-371x)}{5588\left(1+4x-7x^2\right)\sqrt{3+2x+5x^2}} - \frac{\left(7\left(56584-541543\sqrt{11}\right)\right)\text{Subst}\left(\int \frac{1}{2352+112\left(4-2\sqrt{11}\right)+20\left(4-2\sqrt{11}\right)^2-x^2}\,dx, x, \frac{-84-2\left(4-2\sqrt{11}\right)-\left(28+10\right.}{\sqrt{3+2x+5x^2}}\right.}{7806436} - \frac{\left(7\left(56584+541543\sqrt{11}\right)\right)\text{Subst}\left(\int \frac{1}{2352+112\left(4+2\sqrt{11}\right)+20\left(4+2\sqrt{11}\right)^2-x^2}\,dx, x, \frac{-84-2\left(4+2\sqrt{11}\right)-\left(28+10\right.}{\sqrt{3+2x+5x^2}}\right.}{7806436}$$

$$= -\frac{76567+22755x}{19870928\sqrt{3+2x+5x^2}} - \frac{3(40-371x)}{5588\left(1+4x-7x^2\right)\sqrt{3+2x+5x^2}} - \frac{7\left(541543-5144\sqrt{11}\right)\tanh^{-1}\left(\frac{23-\sqrt{11}+\left(17-5\sqrt{11}\right)x}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)}{2838704\sqrt{22\left(125-17\sqrt{11}\right)}} + \frac{7\left(541543+5144\sqrt{11}\right)\tanh^{-1}\left(\frac{23+\sqrt{11}+\left(17+5\sqrt{11}\right)x}{\sqrt{2\left(125+17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)}{2838704\sqrt{22\left(125+17\sqrt{11}\right)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.66 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.93

$$\int \frac{2+5x+x^2}{\left(1+4x-7x^2\right)^2\left(3+2x+5x^2\right)^{3/2}}\,dx = \frac{503287-3628805x-444949x^2-159285x^3}{19870928\sqrt{3+2x+5x^2}\left(-1-4x+7x^2\right)} + \frac{\text{RootSum}\left[83-16\sqrt{5}\#1-70\#1^2+8\sqrt{5}\#1^3+7\#1^4\&, \frac{116685\sqrt{5}\log\left(-\sqrt{5}x+\sqrt{3+2x+5x^2}-\#1\right)+205710\log\left(-\sqrt{5}x+\right.}{-4\sqrt{5}-35\#1}\right.}{258064\sqrt{5}} - \frac{3\text{RootSum}\left[83-16\sqrt{5}\#1-70\#1^2+8\sqrt{5}\#1^3+7\#1^4\&, \frac{746007\sqrt{5}\log\left(-\sqrt{5}x+\sqrt{3+2x+5x^2}-\#1\right)-1016580\log\left(-\sqrt{5}x\right.}{-4\sqrt{5}-35}\right.}{2838704\sqrt{5}}$$

```
[In] Integrate[(2 + 5*x + x^2)/((1 + 4*x - 7*x^2)^2*(3 + 2*x + 5*x^2)^(3/2)),x]
```

```
[Out] (503287 - 3628805*x - 444949*x^2 - 159285*x^3)/(19870928*Sqrt[3 + 2*x + 5*x
^2]*(-1 - 4*x + 7*x^2)) + RootSum[83 - 16*Sqrt[5]*#1 - 70*#1^2 + 8*Sqrt[5]*
#1^3 + 7*#1^4 & , (116685*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2]
- #1] + 205710*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1 + 8351*Sqr
t[5]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1^2)/(-4*Sqrt[5] - 35*
#1 + 6*Sqrt[5]*#1^2 + 7*#1^3) & ]/(258064*Sqrt[5]) - (3*RootSum[83 - 16*Sqr
t[5]*#1 - 70*#1^2 + 8*Sqrt[5]*#1^3 + 7*#1^4 & , (746007*Sqrt[5]*Log[-(Sqrt[
5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1] - 1016580*Log[-(Sqrt[5]*x) + Sqrt[3 + 2
*x + 5*x^2] - #1]*#1 + 42623*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^
2] - #1]*#1^2)/(-4*Sqrt[5] - 35*#1 + 6*Sqrt[5]*#1^2 + 7*#1^3) & ])/(2838704
*Sqrt[5])
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.07

method	result
risch	$-\frac{159285x^3+444949x^2+3628805x-503287}{19870928(7x^2-4x-1)\sqrt{5x^2+2x+3}}+\frac{7\left(-541543+5144\sqrt{11}\right)\sqrt{11}\,\operatorname{arctanh}\Bigg(\frac{250-34\sqrt{11}+\frac{49\left(\frac{34}{7}-\frac{10\sqrt{11}}{7}\right)\big(x}{2}}{\sqrt{250-34\sqrt{11}}\sqrt{245\left(x-\frac{2}{7}+\frac{\sqrt{11}}{7}\right)^2+49\big(\frac{34}{7}-\frac{10\sqrt{1}}{7}}}}{31225744\sqrt{250-34\sqrt{11}}}$
trager	Expression too large to display
default	Expression too large to display

```
[In] int((x^2+5*x+2)/(-7*x^2+4*x+1)^2/(5*x^2+2*x+3)^(3/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -1/19870928*(159285*x^3+444949*x^2+3628805*x-503287)/(7*x^2-4*x-1)/(5*x^2+2
*x+3)^(1/2)+7/31225744*(-541543+5144*11^(1/2))*11^(1/2)/(250-34*11^(1/2))^(
1/2)*arctanh(49/2*(500/49-68/49*11^(1/2)+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11
^(1/2)))/(250-34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*
11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2))+7/31225744*(541543+5
144*11^(1/2))*11^(1/2)/(250+34*11^(1/2))^(1/2)*arctanh(49/2*(500/49+68/49*1
1^(1/2)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2)))/(250+34*11^(1/2))^(1/2)/
(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+25
0+34*11^(1/2))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 392 vs. $2(162) = 324$.

Time = 0.27 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.82

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)^2\,(3+2x+5x^2)^{3/2}}\,dx =$$

$$-\frac{7\sqrt{1397}(35\,x^4-6\,x^3+8\,x^2-14\,x-3)\sqrt{4294093814065\,\sqrt{11}+35653135368317}\log\left(-\frac{\sqrt{1397}\sqrt{5\,x^2+2\,x+3}}{}\right.}{}$$

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)^2/(5*x^2+2*x+3)^(3/2),x, algorithm="fr
icas")
```

```
[Out] -1/111038745664*(7*sqrt(1397)*(35*x^4 - 6*x^3 + 8*x^2 - 14*x - 3)*sqrt(4294
093814065*sqrt(11) + 35653135368317)*log(-(sqrt(1397)*sqrt(5*x^2 + 2*x + 3)
*sqrt(4294093814065*sqrt(11) + 35653135368317)*(5609479*sqrt(11) + 77949905
) + 2865029444171587*sqrt(11)*(x + 3) - 8595088332514761*x + 14325147220857
935)/x) - 7*sqrt(1397)*(35*x^4 - 6*x^3 + 8*x^2 - 14*x - 3)*sqrt(42940938140
65*sqrt(11) + 35653135368317)*log((sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*sqrt(42
94093814065*sqrt(11) + 35653135368317)*(5609479*sqrt(11) + 77949905) - 2865
029444171587*sqrt(11)*(x + 3) + 8595088332514761*x - 14325147220857935)/x)
+ 7*sqrt(1397)*(35*x^4 - 6*x^3 + 8*x^2 - 14*x - 3)*sqrt(-4294093814065*sqrt
(11) + 35653135368317)*log((sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*(5609479*sqrt(
11) - 77949905)*sqrt(-4294093814065*sqrt(11) + 35653135368317) + 2865029444
171587*sqrt(11)*(x + 3) + 8595088332514761*x - 14325147220857935)/x) - 7*sq
rt(1397)*(35*x^4 - 6*x^3 + 8*x^2 - 14*x - 3)*sqrt(-4294093814065*sqrt(11) +
 35653135368317)*log(-(sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*(5609479*sqrt(11) -
 77949905)*sqrt(-4294093814065*sqrt(11) + 35653135368317) - 286502944417158
7*sqrt(11)*(x + 3) - 8595088332514761*x + 14325147220857935)/x) + 5588*(159
285*x^3 + 444949*x^2 + 3628805*x - 503287)*sqrt(5*x^2 + 2*x + 3))/(35*x^4 -
 6*x^3 + 8*x^2 - 14*x - 3)
```

Sympy [F]

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)^2\,(3+2x+5x^2)^{3/2}}\,dx = \int \frac{x^2+5x+2}{(5x^2+2x+3)^{\frac{3}{2}}\,(7x^2-4x-1)^2}\,dx$$

```
[In] integrate((x**2+5*x+2)/(-7*x**2+4*x+1)**2/(5*x**2+2*x+3)**(3/2),x)
```

```
[Out] Integral((x**2 + 5*x + 2)/((5*x**2 + 2*x + 3)**(3/2)*(7*x**2 - 4*x - 1)**2)
, x)
```

Maxima [F]

$$\int \frac{2+5x+x^2}{\left(1+4x-7x^2\right)^2\left(3+2x+5x^2\right)^{3/2}}\,dx = \int \frac{x^2+5\,x+2}{\left(7\,x^2-4\,x-1\right)^2\left(5\,x^2+2\,x+3\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)^2/(5*x^2+2*x+3)^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate((x^2 + 5*x + 2)/((7*x^2 - 4*x - 1)^2*(5*x^2 + 2*x + 3)^(3/2)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.37

$$\int \frac{2+5x+x^2}{\left(1+4x-7x^2\right)^2\left(3+2x+5x^2\right)^{3/2}}\,dx = \frac{25230\,x+13397}{903224\,\sqrt{5\,x^2+2\,x+3}}$$
$$+\frac{3\left(42623\left(\sqrt{5}x-\sqrt{5\,x^2+2\,x+3}\right)^3+77302\,\sqrt{5}\left(\sqrt{5}x-\sqrt{5\,x^2+2\,x+3}\right)^2-275511\,\sqrt{5}x-219860\right.}{709676\left(7\left(\sqrt{5}x-\sqrt{5\,x^2+2\,x+3}\right)^4-8\,\sqrt{5}\left(\sqrt{5}x-\sqrt{5\,x^2+2\,x+3}\right)^3-70\left(\sqrt{5}x-\sqrt{5\,x^2+2\,x+3}\right)^2+1\right.}$$
$$+0.0218058276254033\,\log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}+4.41924736459000\right)$$
$$-0.0332874364433911\,\log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}+1.25295163054000\right)$$
$$-0.0218058276254033\,\log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}-1.02258038113000\right)$$
$$+0.0332874364433911\,\log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}-2.09411235400000\right)$$

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)^2/(5*x^2+2*x+3)^(3/2),x, algorithm="gi
ac")
```

```
[Out] 1/903224*(25230*x + 13397)/sqrt(5*x^2 + 2*x + 3) + 3/709676*(42623*(sqrt(5)
*x - sqrt(5*x^2 + 2*x + 3))^3 + 77302*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x
 + 3))^2 - 275511*sqrt(5)*x - 219860*sqrt(5) + 275511*sqrt(5*x^2 + 2*x + 3)
)/(7*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^4 - 8*sqrt(5)*(sqrt(5)*x - sqrt(5*
x^2 + 2*x + 3))^3 - 70*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^2 + 16*sqrt(5)*(
sqrt(5)*x - sqrt(5*x^2 + 2*x + 3)) + 83) + 0.0218058276254033*log(-sqrt(5)*
x + sqrt(5*x^2 + 2*x + 3) + 4.41924736459000) - 0.0332874364433911*log(-sqr
t(5)*x + sqrt(5*x^2 + 2*x + 3) + 1.25295163054000) - 0.0218058276254033*log
(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) - 1.02258038113000) + 0.033287436443391
1*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) - 2.09411235400000)
```

Mupad [**F(-1)**]

Timed out.

$$\int \frac{2+5x+x^2}{\left(1+4x-7x^2\right)^2\left(3+2x+5x^2\right)^{3/2}}\,dx=\int \frac{x^2+5\,x+2}{\left(5\,x^2+2\,x+3\right)^{3/2}\,\left(-7\,x^2+4\,x+1\right)^2}\,dx$$

```
[In] int((5*x + x^2 + 2)/((2*x + 5*x^2 + 3)^(3/2)*(4*x - 7*x^2 + 1)^2),x)
[Out] int((5*x + x^2 + 2)/((2*x + 5*x^2 + 3)^(3/2)*(4*x - 7*x^2 + 1)^2), x)
```

3.397 $\int \frac{2+5x+x^2}{(1+4x-7x^2)^3 (3+2x+5x^2)^{3/2}} \, dx$

Optimal result

Integrand size = 35, antiderivative size = 250

$$\begin{aligned}\int \frac{2+5x+x^2}{(1+4x-7x^2)^3 (3+2x+5x^2)^{3/2}} \, dx = & -\frac{5(461370781+1118731375x)}{222077491328\sqrt{3+2x+5x^2}} \\ & -\frac{3(40-371x)}{11176\,(1+4x-7x^2)^2\,\sqrt{3+2x+5x^2}}-\frac{2701733-9148874x}{62451488\,(1+4x-7x^2)\,\sqrt{3+2x+5x^2}} \\ & -\frac{7\left(2792860024-84865895\sqrt{11}\right)\operatorname{arctanh}\left(\frac{23-\sqrt{11}+\left(17-5\sqrt{11}\right)x}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)}{31725355904\sqrt{22\left(125-17\sqrt{11}\right)}} \\ & +\frac{7\left(2792860024+84865895\sqrt{11}\right)\operatorname{arctanh}\left(\frac{23+\sqrt{11}+\left(17+5\sqrt{11}\right)x}{\sqrt{2\left(125+17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)}{31725355904\sqrt{22\left(125+17\sqrt{11}\right)}}\end{aligned}$$

```
[Out] -5/222077491328*(461370781+1118731375*x)/(5*x^2+2*x+3)^(1/2)-3/11176*(40-37
1*x)/(-7*x^2+4*x+1)^2/(5*x^2+2*x+3)^(1/2)+1/62451488*(-2701733+9148874*x)/(
-7*x^2+4*x+1)/(5*x^2+2*x+3)^(1/2)-7/31725355904*arctanh((23+x*(17-5*11^(1/2
))-11^(1/2))/(5*x^2+2*x+3)^(1/2)/(250-34*11^(1/2))^(1/2))*(2792860024-84865
895*11^(1/2))/(2750-374*11^(1/2))^(1/2)+7/31725355904*arctanh((23+11^(1/2)+
x*(17+5*11^(1/2)))/(5*x^2+2*x+3)^(1/2)/(250+34*11^(1/2))^(1/2))*(2792860024
+84865895*11^(1/2))/(2750+374*11^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {1074, 1046, 738, 212}

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)^3 (3+2x+5x^2)^{3/2}} \, dx =$$

$$-\frac{7\left(2792860024-84865895\sqrt{11}\right)\operatorname{arctanh}\left(\frac{\left(17-5\sqrt{11}\right)x-\sqrt{11}+23}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right)}{31725355904\sqrt{22\left(125-17\sqrt{11}\right)}}$$
$$+\frac{7\left(2792860024+84865895\sqrt{11}\right)\operatorname{arctanh}\left(\frac{\left(17+5\sqrt{11}\right)x+\sqrt{11}+23}{\sqrt{2\left(125+17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right)}{31725355904\sqrt{22\left(125+17\sqrt{11}\right)}}$$
$$-\frac{2701733-9148874x}{62451488\left(-7x^2+4x+1\right)\sqrt{5x^2+2x+3}}$$
$$-\frac{5(1118731375x+461370781)}{222077491328\sqrt{5x^2+2x+3}}-\frac{3(40-371x)}{11176\left(-7x^2+4x+1\right)^2\sqrt{5x^2+2x+3}}$$

```
[In] Int[(2 + 5*x + x^2)/((1 + 4*x - 7*x^2)^3*(3 + 2*x + 5*x^2)^(3/2)),x]
```

```
[Out] (-5*(461370781 + 1118731375*x))/(222077491328*Sqrt[3 + 2*x + 5*x^2]) - (3*(
40 - 371*x))/(11176*(1 + 4*x - 7*x^2)^2*Sqrt[3 + 2*x + 5*x^2]) - (2701733 -
 9148874*x)/(62451488*(1 + 4*x - 7*x^2)*Sqrt[3 + 2*x + 5*x^2]) - (7*(279286
0024 - 84865895*Sqrt[11])*ArcTanh[(23 - Sqrt[11] + (17 - 5*Sqrt[11])*x)/(Sq
rt[2*(125 - 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x^2])])/(31725355904*Sqrt[22*(12
5 - 17*Sqrt[11])]) + (7*(2792860024 + 84865895*Sqrt[11])*ArcTanh[(23 + Sqrt
[11] + (17 + 5*Sqrt[11])*x)/(Sqrt[2*(125 + 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x
^2])])/(31725355904*Sqrt[22*(125 + 17*Sqrt[11])])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1046

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1074

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x +
c*x^2)^(p + 1)*((d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f))*(p + 1)))*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) +
(A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c
*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d
 - a*f)))*x), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{3(40-371x)}{11176\left(1+4x-7x^2\right)^2\sqrt{3+2x+5x^2}}-\frac{\int \frac{-128104-89208x-178080x^2}{\left(1+4x-7x^2\right)^2\left(3+2x+5x^2\right)^{3/2}}\,dx}{89408}\\
&= -\frac{3(40-371x)}{11176\left(1+4x-7x^2\right)^2\sqrt{3+2x+5x^2}}\\
&\quad -\frac{2701733-9148874x}{62451488\left(1+4x-7x^2\right)\sqrt{3+2x+5x^2}}+\frac{\int \frac{1722335552+835857088x+5855279360x^2}{\left(1+4x-7x^2\right)\left(3+2x+5x^2\right)^{3/2}}\,dx}{3996895232}
\end{aligned}
$$

$$= -\frac{5(461370781+1118731375x)}{222077491328\sqrt{3+2x+5x^2}} - \frac{3(40-371x)}{11176\left(1+4x-7x^2\right)^2\sqrt{3+2x+5x^2}} - \frac{2701733-9148874x}{62451488\left(1+4x-7x^2\right)\sqrt{3+2x+5x^2}} + \frac{\int \frac{18802583181312+4258231147520x}{(1+4x-7x^2)\sqrt{3+2x+5x^2}}\,dx}{113703675559936}$$

$$= -\frac{5(461370781+1118731375x)}{222077491328\sqrt{3+2x+5x^2}} - \frac{3(40-371x)}{11176\left(1+4x-7x^2\right)^2\sqrt{3+2x+5x^2}} - \frac{2701733-9148874x}{62451488\left(1+4x-7x^2\right)\sqrt{3+2x+5x^2}} + \frac{\left(7\left(933524845-2792860024\sqrt{11}\right)\right)\int \frac{1}{\left(4-2\sqrt{11}-14x\right)\sqrt{3+2x+5x^2}}\,dx}{174489457472} + \frac{\left(7\left(933524845+2792860024\sqrt{11}\right)\right)\int \frac{1}{\left(4+2\sqrt{11}-14x\right)\sqrt{3+2x+5x^2}}\,dx}{174489457472}$$

$$= -\frac{5(461370781+1118731375x)}{222077491328\sqrt{3+2x+5x^2}} - \frac{3(40-371x)}{11176\left(1+4x-7x^2\right)^2\sqrt{3+2x+5x^2}} - \frac{2701733-9148874x}{62451488\left(1+4x-7x^2\right)\sqrt{3+2x+5x^2}} - \frac{\left(7\left(933524845-2792860024\sqrt{11}\right)\right)\text{Subst}\left(\int \frac{1}{2352+112\left(4-2\sqrt{11}\right)+20\left(4-2\sqrt{11}\right)^2-x^2}\,dx, x, \frac{-84-2\left(4-2\sqrt{1}\right.}{\surd}\right.}{87244728736} - \frac{\left(7\left(933524845+2792860024\sqrt{11}\right)\right)\text{Subst}\left(\int \frac{1}{2352+112\left(4+2\sqrt{11}\right)+20\left(4+2\sqrt{11}\right)^2-x^2}\,dx, x, \frac{-84-2\left(4+2\sqrt{1}\right.}{\surd}\right.}{87244728736}$$

$$= -\frac{5(461370781+1118731375x)}{222077491328\sqrt{3+2x+5x^2}} - \frac{3(40-371x)}{11176\left(1+4x-7x^2\right)^2\sqrt{3+2x+5x^2}} - \frac{2701733-9148874x}{62451488\left(1+4x-7x^2\right)\sqrt{3+2x+5x^2}} - \frac{7\left(2792860024-84865895\sqrt{11}\right)\tanh^{-1}\left(\frac{23-\sqrt{11}+\left(17-5\sqrt{11}\right)x}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)}{31725355904\sqrt{22\left(125-17\sqrt{11}\right)}} + \frac{7\left(2792860024+84865895\sqrt{11}\right)\tanh^{-1}\left(\frac{23+\sqrt{11}+\left(17+5\sqrt{11}\right)x}{\sqrt{2\left(125+17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)}{31725355904\sqrt{22\left(125+17\sqrt{11}\right)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.92 (sec) , antiderivative size = 607, normalized size of antiderivative = 2.43

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)^3\,(3+2x+5x^2)^{3/2}}\,dx = \frac{-\frac{1715(14298727813+7828199499x-148022158802x^2+109737266678x^3-200208943655x}{(1+4x-7x^2)^2\sqrt{3+2x+5x^2}}}{}$$

```
[In] Integrate[(2 + 5*x + x^2)/((1 + 4*x - 7*x^2)^3*(3 + 2*x + 5*x^2)^(3/2)),x]
```

```
[Out] ((-1715*(14298727813 + 7828199499*x - 148022158802*x^2 + 109737266678*x^3 -
 200208943655*x^4 + 274089186875*x^5))/((1 + 4*x - 7*x^2)^2*Sqrt[3 + 2*x +
5*x^2]) + 2324168*Sqrt[5]*RootSum[83 - 16*Sqrt[5]*#1 - 70*#1^2 + 8*Sqrt[5]*
#1^3 + 7*#1^4 & , (-4989740*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2
] - #1] + 3790865*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1 + 40044
9*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1^2)/(-4*Sqrt[5]
- 35*#1 + 6*Sqrt[5]*#1^2 + 7*#1^3) & ] + 22*Sqrt[5]*RootSum[83 - 16*Sqrt[5]
*#1 - 70*#1^2 + 8*Sqrt[5]*#1^3 + 7*#1^4 & , (-3200991286865*Sqrt[5]*Log[-(S
qrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1] + 18470877323690*Log[-(Sqrt[5]*x) +
 Sqrt[3 + 2*x + 5*x^2] - #1]*#1 + 2296522946389*Sqrt[5]*Log[-(Sqrt[5]*x) +
Sqrt[3 + 2*x + 5*x^2] - #1]*#1^2)/(-4*Sqrt[5] - 35*#1 + 6*Sqrt[5]*#1^2 + 7*
#1^3) & ] - 9*Sqrt[5]*RootSum[83 - 16*Sqrt[5]*#1 - 70*#1^2 + 8*Sqrt[5]*#1^3
 + 7*#1^4 & , (-8189062651053*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x
^2] - #1] + 39132066594240*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#
1 + 5875617407695*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1
^2)/(-4*Sqrt[5] - 35*#1 + 6*Sqrt[5]*#1^2 + 7*#1^3) & ])/380862897627520
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.96

method	result
risch	$-\frac{274089186875x^5-200208943655x^4+109737266678x^3-148022158802x^2+7828199499x+14298727813}{222077491328(7x^2-4x-1)^2\sqrt{5x^2+2x+3}}+\frac{7\left(2792860024+84865895\right.}{}$
trager	Expression too large to display
default	Expression too large to display

```
[In] int((x^2+5*x+2)/(-7*x^2+4*x+1)^3/(5*x^2+2*x+3)^(3/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -1/222077491328*(274089186875*x^5-200208943655*x^4+109737266678*x^3-1480221
58802*x^2+7828199499*x+14298727813)/(7*x^2-4*x-1)^2/(5*x^2+2*x+3)^(1/2)+7/3
```

```
48978914944*(2792860024+84865895*11^(1/2))*11^(1/2)/(250+34*11^(1/2))^(1/2)
*arctanh(49/2*(500/49+68/49*11^(1/2)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/
2)))/(250+34*11^(1/2))^(1/2)/(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(
1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2))+7/348978914944*(-2792860
024+84865895*11^(1/2))*11^(1/2)/(250-34*11^(1/2))^(1/2)*arctanh(49/2*(500/4
9-68/49*11^(1/2)+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2)))/(250-34*11^(1/2
))^(1/2)/(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^
(1/2))+250-34*11^(1/2))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 452 vs. 2(193) = 386.

Time = 0.37 (sec) , antiderivative size = 452, normalized size of antiderivative = 1.81

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)^3\,(3+2x+5x^2)^{3/2}}\,dx =$$

$$-\frac{7\sqrt{1397}(245\,x^6-182\,x^5+45\,x^4-124\,x^3+27\,x^2+26\,x+3)\sqrt{746933147106396414 67\sqrt{11}+89626649}}{}$$

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)^3/(5*x^2+2*x+3)^(3/2),x, algorithm="fr
icas")
```

```
[Out] -1/1240969021540864*(7*sqrt(1397)*(245*x^6 - 182*x^5 + 45*x^4 - 124*x^3 + 2
7*x^2 + 26*x + 3)*sqrt(746933147106396414 67*sqrt(11) + 896266498377233657855
5)*log(-(sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*sqrt(746933147106396414 67*sqrt(11
) + 89626649837723365785 5)*(37271563201*sqrt(11) + 407780707037) + 75502120
686844055144479*sqrt(11)*(x + 3) - 226506362060532165433437*x + 37751060343
4220275722395)/x) - 7*sqrt(1397)*(245*x^6 - 182*x^5 + 45*x^4 - 124*x^3 + 27
*x^2 + 26*x + 3)*sqrt(746933147106396414 67*sqrt(11) + 89626649837723365785 5
)*log((sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*sqrt(746933147106396414 67*sqrt(11)
+ 89626649837723365785 5)*(37271563201*sqrt(11) + 407780707037) - 7550212068
6844055144479*sqrt(11)*(x + 3) + 226506362060532165433437*x - 3775106034342
20275722395)/x) + 7*sqrt(1397)*(245*x^6 - 182*x^5 + 45*x^4 - 124*x^3 + 27*x
^2 + 26*x + 3)*sqrt(-746933147106396414 67*sqrt(11) + 89626649837723365785 5)
*log((sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*(37271563201*sqrt(11) - 407780707037
)*sqrt(-746933147106396414 67*sqrt(11) + 89626649837723365785 5) + 7550212068
6844055144479*sqrt(11)*(x + 3) + 226506362060532165433437*x - 3775106034342
20275722395)/x) - 7*sqrt(1397)*(245*x^6 - 182*x^5 + 45*x^4 - 124*x^3 + 27*x
^2 + 26*x + 3)*sqrt(-746933147106396414 67*sqrt(11) + 89626649837723365785 5)
*log(-(sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*(37271563201*sqrt(11) - 40778070703
7)*sqrt(-746933147106396414 67*sqrt(11) + 89626649837723365785 5) - 755021206
86844055144479*sqrt(11)*(x + 3) - 226506362060532165433437*x + 377510603434
220275722395)/x) + 5588*(274089186875*x^5 - 200208943655*x^4 + 109737266678
```

```
*x^3 - 148022158802*x^2 + 7828199499*x + 14298727813)*sqrt(5*x^2 + 2*x + 3)
)/(245*x^6 - 182*x^5 + 45*x^4 - 124*x^3 + 27*x^2 + 26*x + 3)
```

Sympy [F]

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)^3\,(3+2x+5x^2)^{3/2}}\,dx =$$

$$-\int \frac{5}{1715x^8\sqrt{5x^2+2x+3}-2254x^7\sqrt{5x^2+2x+3}+798x^6\sqrt{5x^2+2x+3}-866x^5\sqrt{5x^2+2x+3}+640x^4}$$

$$-\int \frac{x^2}{1715x^8\sqrt{5x^2+2x+3}-2254x^7\sqrt{5x^2+2x+3}+798x^6\sqrt{5x^2+2x+3}-866x^5\sqrt{5x^2+2x+3}+640x^4}$$

$$-\int \frac{2}{1715x^8\sqrt{5x^2+2x+3}-2254x^7\sqrt{5x^2+2x+3}+798x^6\sqrt{5x^2+2x+3}-866x^5\sqrt{5x^2+2x+3}+640x^4}$$

```
[In] integrate((x**2+5*x+2)/(-7*x**2+4*x+1)**3/(5*x**2+2*x+3)**(3/2),x)
```

```
[Out] -Integral(5*x/(1715*x**8*sqrt(5*x**2 + 2*x + 3) - 2254*x**7*sqrt(5*x**2 + 2
*x + 3) + 798*x**6*sqrt(5*x**2 + 2*x + 3) - 866*x**5*sqrt(5*x**2 + 2*x + 3)
 + 640*x**4*sqrt(5*x**2 + 2*x + 3) + 198*x**3*sqrt(5*x**2 + 2*x + 3) - 110*
x**2*sqrt(5*x**2 + 2*x + 3) - 38*x*sqrt(5*x**2 + 2*x + 3) - 3*sqrt(5*x**2 +
 2*x + 3)), x) - Integral(x**2/(1715*x**8*sqrt(5*x**2 + 2*x + 3) - 2254*x**
7*sqrt(5*x**2 + 2*x + 3) + 798*x**6*sqrt(5*x**2 + 2*x + 3) - 866*x**5*sqrt(
5*x**2 + 2*x + 3) + 640*x**4*sqrt(5*x**2 + 2*x + 3) + 198*x**3*sqrt(5*x**2
+ 2*x + 3) - 110*x**2*sqrt(5*x**2 + 2*x + 3) - 38*x*sqrt(5*x**2 + 2*x + 3)
- 3*sqrt(5*x**2 + 2*x + 3)), x) - Integral(2/(1715*x**8*sqrt(5*x**2 + 2*x +
 3) - 2254*x**7*sqrt(5*x**2 + 2*x + 3) + 798*x**6*sqrt(5*x**2 + 2*x + 3) -
866*x**5*sqrt(5*x**2 + 2*x + 3) + 640*x**4*sqrt(5*x**2 + 2*x + 3) + 198*x**
3*sqrt(5*x**2 + 2*x + 3) - 110*x**2*sqrt(5*x**2 + 2*x + 3) - 38*x*sqrt(5*x*
*2 + 2*x + 3) - 3*sqrt(5*x**2 + 2*x + 3)), x)
```

Maxima [F]

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)^3\,(3+2x+5x^2)^{3/2}}\,dx = \int -\frac{x^2+5\,x+2}{(7\,x^2-4\,x-1)^3(5\,x^2+2\,x+3)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)^3/(5*x^2+2*x+3)^(3/2),x, algorithm="ma
xima")
```

```
[Out] -integrate((x^2 + 5*x + 2)/((7*x^2 - 4*x - 1)^3*(5*x^2 + 2*x + 3)^(3/2)), x
)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 397 vs. $2(193) = 386$.

Time = 0.30 (sec) , antiderivative size = 397, normalized size of antiderivative = 1.59

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)^3\,(3+2x+5x^2)^{3/2}}\,dx = \frac{501205\,x + 1702037}{458837792\,\sqrt{5\,x^2+2\,x+3}}$$

$$+\frac{6871871279\,\left(\sqrt{5}x-\sqrt{5\,x^2+2\,x+3}\right)^7 + 4012856750\,\sqrt{5}\left(\sqrt{5}x-\sqrt{5\,x^2+2\,x+3}\right)^6 - 223088535693\,\left(\sqrt{5}\right.}{7931338}$$

$$+\,0.0107382277384513\,\log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}+4.41924736459000\right)$$

$$-\,0.0142619066316905\,\log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}+1.25295163054000\right)$$

$$-\,0.0107382277384513\,\log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}-1.02258038113000\right)$$

$$+\,0.0142619066316905\,\log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}-2.09411235400000\right)$$

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)^3/(5*x^2+2*x+3)^(3/2),x, algorithm="gi
ac")
```

```
[Out] 1/458837792*(501205*x + 1702037)/sqrt(5*x^2 + 2*x + 3) + 1/7931338976*(6871
871279*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^7 + 4012856750*sqrt(5)*(sqrt(5)*
x - sqrt(5*x^2 + 2*x + 3))^6 - 223088535693*(sqrt(5)*x - sqrt(5*x^2 + 2*x +
 3))^5 - 100577598176*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^4 + 12550
97956673*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^3 + 566810398070*sqrt(5)*(sqrt
(5)*x - sqrt(5*x^2 + 2*x + 3))^2 - 1246245909011*sqrt(5)*x - 561299654796*s
qrt(5) + 1246245909011*sqrt(5*x^2 + 2*x + 3))/(7*(sqrt(5)*x - sqrt(5*x^2 +
2*x + 3))^4 - 8*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^3 - 70*(sqrt(5)
*x - sqrt(5*x^2 + 2*x + 3))^2 + 16*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x +
3)) + 83)^2 + 0.0107382277384513*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) + 4
.41924736459000) - 0.0142619066316905*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3
) + 1.25295163054000) - 0.0107382277384513*log(-sqrt(5)*x + sqrt(5*x^2 + 2*
x + 3) - 1.02258038113000) + 0.0142619066316905*log(-sqrt(5)*x + sqrt(5*x^2
 + 2*x + 3) - 2.09411235400000)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2+5x+x^2}{\left(1+4x-7x^2\right)^3 \left(3+2x+5x^2\right)^{3/2}}\,dx=\int \frac{x^2+5\,x+2}{{\left(5\,x^2+2\,x+3\right)}^{3/2}\,{\left(-7\,x^2+4\,x+1\right)}^3}\,dx$$

```
[In] int((5*x + x^2 + 2)/((2*x + 5*x^2 + 3)^(3/2)*(4*x - 7*x^2 + 1)^3),x)
[Out] int((5*x + x^2 + 2)/((2*x + 5*x^2 + 3)^(3/2)*(4*x - 7*x^2 + 1)^3), x)
```

3.398 $\int (a+cx^2)^p (A+Cx^2) (d+fx^2)^q \, dx$

Optimal result

Integrand size = 26, antiderivative size = 166

$$\int (a+cx^2)^p (A+Cx^2) (d+fx^2)^q \, dx = Ax(a+cx^2)^p \left(1+\frac{cx^2}{a}\right)^{-p} (d+fx^2)^q \left(1+\frac{fx^2}{d}\right)^{-q} \text{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{cx^2}{a},-\frac{fx^2}{d}\right) + \frac{1}{3}Cx^3(a+cx^2)^p \left(1+\frac{cx^2}{a}\right)^{-p} (d+fx^2)^q \left(1+\frac{fx^2}{d}\right)^{-q} \text{AppellF1}\left(\frac{3}{2},-p,-q,\frac{5}{2},-\frac{cx^2}{a},-\frac{fx^2}{d}\right)$$

```
[Out] A*x*(c*x^2+a)^p*(f*x^2+d)^q*AppellF1(1/2,-p,-q,3/2,-c*x^2/a,-f*x^2/d)/((1+c
*x^2/a)^p)/((1+f*x^2/d)^q)+1/3*C*x^3*(c*x^2+a)^p*(f*x^2+d)^q*AppellF1(3/2,-
p,-q,5/2,-c*x^2/a,-f*x^2/d)/((1+c*x^2/a)^p)/((1+f*x^2/d)^q)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used

$= \{545, 441, 440, 525, 524\}$

$$\int \left(a+cx^2\right)^p \left(A+Cx^2\right) \left(d+fx^2\right)^q \, dx = Ax\left(a+cx^2\right)^p \left(\frac{cx^2}{a}+1\right)^{-p} \left(d+fx^2\right)^q \left(\frac{fx^2}{d}+1\right)^{-q} \text{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{cx^2}{a},-\frac{fx^2}{d}\right) + \frac{1}{3}Cx^3\left(a+cx^2\right)^p \left(\frac{cx^2}{a}+1\right)^{-p} \left(d+fx^2\right)^q \left(\frac{fx^2}{d}+1\right)^{-q} \text{AppellF1}\left(\frac{3}{2},-p,-q,\frac{5}{2},-\frac{cx^2}{a},-\frac{fx^2}{d}\right)$$

```
[In] Int[(a + c*x^2)^p*(A + C*x^2)*(d + f*x^2)^q,x]
```

```
[Out] (A*x*(a + c*x^2)^p*(d + f*x^2)^q*AppellF1[1/2, -p, -q, 3/2, -((c*x^2)/a), -
((f*x^2)/d)])/((1 + (c*x^2)/a)^p*(1 + (f*x^2)/d)^q) + (C*x^3*(a + c*x^2)^p*
(d + f*x^2)^q*AppellF1[3/2, -p, -q, 5/2, -((c*x^2)/a), -((f*x^2)/d)])/(3*(1
 + (c*x^2)/a)^p*(1 + (f*x^2)/d)^q)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
```

```
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= A\int \left(a+cx^2\right)^p \left(d+fx^2\right)^q \, dx + C\int x^2\left(a+cx^2\right)^p \left(d+fx^2\right)^q \, dx \\
&= \left(A\left(a+cx^2\right)^p \left(1+\frac{cx^2}{a}\right)^{-p}\right)\int \left(1+\frac{cx^2}{a}\right)^p \left(d+fx^2\right)^q \, dx \\
&\quad + \left(C\left(a+cx^2\right)^p \left(1+\frac{cx^2}{a}\right)^{-p}\right)\int x^2\left(1+\frac{cx^2}{a}\right)^p \left(d+fx^2\right)^q \, dx \\
&= \left(A\left(a+cx^2\right)^p \left(1+\frac{cx^2}{a}\right)^{-p} \left(d+fx^2\right)^q \left(1+\frac{fx^2}{d}\right)^{-q}\right)\int \left(1+\frac{cx^2}{a}\right)^p \left(1+\frac{fx^2}{d}\right)^q \, dx \\
&\quad + \left(C\left(a+cx^2\right)^p \left(1+\frac{cx^2}{a}\right)^{-p} \left(d+fx^2\right)^q \left(1+\frac{fx^2}{d}\right)^{-q}\right)\int x^2\left(1+\frac{cx^2}{a}\right)^p \left(1+\frac{fx^2}{d}\right)^q \, dx \\
&= Ax\left(a+cx^2\right)^p \left(1+\frac{cx^2}{a}\right)^{-p} \left(d+fx^2\right)^q \left(1+\frac{fx^2}{d}\right)^{-q} F_1\left(\frac{1}{2};-p,-q;\frac{3}{2};-\frac{cx^2}{a},-\frac{fx^2}{d}\right) \\
&\quad + \frac{1}{3}Cx^3\left(a+cx^2\right)^p \left(1+\frac{cx^2}{a}\right)^{-p} \left(d+fx^2\right)^q \left(1+\frac{fx^2}{d}\right)^{-q} F_1\left(\frac{3}{2};-p,-q;\frac{5}{2};-\frac{cx^2}{a},\right. \\
&\qquad \left. -\frac{fx^2}{d}\right)
\end{aligned}
$$

Mathematica [A] (warning: unable to verify)

Time = 0.32 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.46

$$
\begin{aligned}
&\int \left(a+cx^2\right)^p \left(A+Cx^2\right) \left(d+fx^2\right)^q \, dx = \frac{1}{3}x\left(a+cx^2\right)^p \left(d\right. \\
&\left. +fx^2\right)^q \left(\frac{9aAd\,\text{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{cx^2}{a},-\frac{fx^2}{d}\right)}{3ad\,\text{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{cx^2}{a},-\frac{fx^2}{d}\right)+2x^2\left(cdp\,\text{AppellF1}\left(\frac{3}{2},1-p,-q,\frac{5}{2},-\frac{cx^2}{a},-\frac{fx^2}{d}\right)+\right.}\right. \\
&\qquad \left. +Cx^2\left(1+\frac{cx^2}{a}\right)^{-p} \left(1+\frac{fx^2}{d}\right)^{-q} \text{AppellF1}\left(\frac{3}{2},-p,-q,\frac{5}{2},-\frac{cx^2}{a},-\frac{fx^2}{d}\right)\right)
\end{aligned}
$$

```
[In] Integrate[(a + c*x^2)^p*(A + C*x^2)*(d + f*x^2)^q,x]
```

```
[Out] (x*(a + c*x^2)^p*(d + f*x^2)^q*((9*a*A*d*AppellF1[1/2, -p, -q, 3/2, -((c*x^
2)/a), -((f*x^2)/d)])/(3*a*d*AppellF1[1/2, -p, -q, 3/2, -((c*x^2)/a), -((f*
x^2)/d)] + 2*x^2*(c*d*p*AppellF1[3/2, 1 - p, -q, 5/2, -((c*x^2)/a), -((f*x^
2)/d)] + a*f*q*AppellF1[3/2, -p, 1 - q, 5/2, -((c*x^2)/a), -((f*x^2)/d)]))
+ (C*x^2*AppellF1[3/2, -p, -q, 5/2, -((c*x^2)/a), -((f*x^2)/d)])/((1 + (c*x
^2)/a)^p*(1 + (f*x^2)/d)^q)))/3
```

Maple [F]

$$\int \left(c\,x^{2}+a\right)^{p} \left(C\,x^{2}+A\right) \left(f\,x^{2}+d\right)^{q}d x$$

```
[In] int((c*x^2+a)^p*(C*x^2+A)*(f*x^2+d)^q,x)
```

```
[Out] int((c*x^2+a)^p*(C*x^2+A)*(f*x^2+d)^q,x)
```

Fricas [F]

$$\int \left(a+cx^2\right)^p \left(A+Cx^2\right) \left(d+fx^2\right)^q \, dx=\int { {\left(C x^{2} + A\right)} {\left(c x^{2} + a\right)}^{p} {\left(f x^{2} + d\right)}^{q} \,d x }$$

```
[In] integrate((c*x^2+a)^p*(C*x^2+A)*(f*x^2+d)^q,x, algorithm="fricas")
```

```
[Out] integral((C*x^2 + A)*(c*x^2 + a)^p*(f*x^2 + d)^q, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+cx^2\right)^p \left(A+Cx^2\right) \left(d+fx^2\right)^q \, dx=\text{Timed out}$$

```
[In] integrate((c*x**2+a)**p*(C*x**2+A)*(f*x**2+d)**q,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \left(a+cx^2\right)^p \left(A+Cx^2\right) \left(d+fx^2\right)^q \, dx = \int \left(Cx^2+A\right)\left(cx^2+a\right)^p \left(fx^2+d\right)^q \, dx$$

```
[In] integrate((c*x^2+a)^p*(C*x^2+A)*(f*x^2+d)^q,x, algorithm="maxima")
[Out] integrate((C*x^2 + A)*(c*x^2 + a)^p*(f*x^2 + d)^q, x)
```

Giac [F]

$$\int \left(a+cx^2\right)^p \left(A+Cx^2\right) \left(d+fx^2\right)^q \, dx = \int \left(Cx^2+A\right)\left(cx^2+a\right)^p \left(fx^2+d\right)^q \, dx$$

```
[In] integrate((c*x^2+a)^p*(C*x^2+A)*(f*x^2+d)^q,x, algorithm="giac")
[Out] integrate((C*x^2 + A)*(c*x^2 + a)^p*(f*x^2 + d)^q, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+cx^2\right)^p \left(A+Cx^2\right) \left(d+fx^2\right)^q \, dx = \int \left(C\,x^2+A\right)\,\left(c\,x^2+a\right)^p\,\left(f\,x^2+d\right)^q \, dx$$

```
[In] int((A + C*x^2)*(a + c*x^2)^p*(d + f*x^2)^q,x)
[Out] int((A + C*x^2)*(a + c*x^2)^p*(d + f*x^2)^q, x)
```

3.399 $\int (A+Bx)\left(a+cx^2\right)^p\left(d+fx^2\right)^q\,dx$

Optimal result

Integrand size = 24, antiderivative size = 167

$$\int (A+Bx)\left(a+cx^2\right)^p\left(d+fx^2\right)^q\,dx = Ax\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\left(d+fx^2\right)^q\left(1+\frac{fx^2}{d}\right)^{-q}\text{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{cx^2}{a},-\frac{fx^2}{d}\right)+\frac{B\left(a+cx^2\right)^{1+p}\left(d+fx^2\right)^q\left(\frac{c(d+fx^2)}{cd-af}\right)^{-q}\text{Hypergeometric2F1}\left(1+p,-q,2+p,-\frac{f(a+cx^2)}{cd-af}\right)}{2c(1+p)}$$

```
[Out] A*x*(c*x^2+a)^p*(f*x^2+d)^q*AppellF1(1/2,-p,-q,3/2,-c*x^2/a,-f*x^2/d)/((1+c
*x^2/a)^p)/((1+f*x^2/d)^q)+1/2*B*(c*x^2+a)^(p+1)*(f*x^2+d)^q*hypergeom([-q,
 p+1],[2+p],-f*(c*x^2+a)/(-a*f+c*d))/c/(p+1)/((c*(f*x^2+d)/(-a*f+c*d))^q)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1024, 441, 440, 455, 72, 71}

$$\int (A+Bx)\left(a+cx^2\right)^p\left(d+fx^2\right)^q\,dx = Ax\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}\left(d+fx^2\right)^q\left(\frac{fx^2}{d}+1\right)^{-q}\text{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{cx^2}{a},-\frac{fx^2}{d}\right)+\frac{B\left(a+cx^2\right)^{p+1}\left(d+fx^2\right)^q\left(\frac{c(d+fx^2)}{cd-af}\right)^{-q}\text{Hypergeometric2F1}\left(p+1,-q,p+2,-\frac{f(cx^2+a)}{cd-af}\right)}{2c(p+1)}$$

```
[In] Int[(A + B*x)*(a + c*x^2)^p*(d + f*x^2)^q,x]
```

```
[Out] (A*x*(a + c*x^2)^p*(d + f*x^2)^q*AppellF1[1/2, -p, -q, 3/2, -((c*x^2)/a), -
((f*x^2)/d)])/((1 + (c*x^2)/a)^p*(1 + (f*x^2)/d)^q) + (B*(a + c*x^2)^(1 + p
)*(d + f*x^2)^q*Hypergeometric2F1[1 + p, -q, 2 + p, -((f*(a + c*x^2))/(c*d
- a*f))])/(2*c*(1 + p)*((c*(d + f*x^2))/(c*d - a*f))^q)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1024

```
Int[((g_) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q
_), x_Symbol] :> Dist[g, Int[(a + c*x^2)^p*(d + f*x^2)^q, x], x] + Dist[h,
Int[x*(a + c*x^2)^p*(d + f*x^2)^q, x], x] /; FreeQ[{a, c, d, f, g, h, p, q}
```

```
, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= A\int \left(a+cx^2\right)^p \left(d+fx^2\right)^q \,dx + B\int x\left(a+cx^2\right)^p \left(d+fx^2\right)^q \,dx \\
&= \frac{1}{2}B\text{Subst}\left(\int (a+cx)^p(d+fx)^q \,dx, x, x^2\right) \\
&\quad + \left(A\left(a+cx^2\right)^p \left(1+\frac{cx^2}{a}\right)^{-p}\right)\int \left(1+\frac{cx^2}{a}\right)^p \left(d+fx^2\right)^q \,dx \\
&= \frac{1}{2}\left(B\left(d+fx^2\right)^q \left(\frac{c\left(d+fx^2\right)}{cd-af}\right)^{-q}\right)\text{Subst}\left(\int (a+cx)^p \left(\frac{cd}{cd-af}+\frac{cfx}{cd-af}\right)^q \,dx, x, x^2\right) \\
&\quad + \left(A\left(a+cx^2\right)^p \left(1+\frac{cx^2}{a}\right)^{-p} \left(d+fx^2\right)^q \left(1+\frac{fx^2}{d}\right)^{-q}\right)\int \left(1+\frac{cx^2}{a}\right)^p \left(1+\frac{fx^2}{d}\right)^q \,dx \\
&= Ax\left(a+cx^2\right)^p \left(1+\frac{cx^2}{a}\right)^{-p} \left(d+fx^2\right)^q \left(1+\frac{fx^2}{d}\right)^{-q} F_1\left(\frac{1}{2}; -p, -q; \frac{3}{2}; -\frac{cx^2}{a}, -\frac{fx^2}{d}\right) \\
&\quad + \frac{B\left(a+cx^2\right)^{1+p}\left(d+fx^2\right)^q \left(\frac{c\left(d+fx^2\right)}{cd-af}\right)^{-q} {}_2F_1\left(1+p, -q; 2+p; -\frac{f\left(a+cx^2\right)}{cd-af}\right)}{2c(1+p)}
\end{aligned}
$$

Mathematica [A] (warning: unable to verify)

Time = 0.27 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.41

$$
\int (A+Bx)\left(a+cx^2\right)^p \left(d+fx^2\right)^q \,dx = \frac{1}{2}x\left(a+cx^2\right)^p \left(d+fx^2\right)^q \left(Bx\left(1+\frac{cx^2}{a}\right)^{-p}\left(1+\frac{fx^2}{d}\right)^{-q}\text{AppellF1}\left(1, -p, -q, 2, -\frac{cx^2}{a}, -\frac{fx^2}{d}\right) + \frac{6aAd\,\text{AppellF1}\left(\frac{1}{2}, -p, -q, \frac{3}{2}, -\frac{cx^2}{a}, -\frac{fx^2}{d}\right)}{3ad\,\text{AppellF1}\left(\frac{1}{2}, -p, -q, \frac{3}{2}, -\frac{cx^2}{a}, -\frac{fx^2}{d}\right) + 2x^2\left(cdp\,\text{AppellF1}\left(\frac{3}{2}, 1-p, -q, \frac{5}{2}, -\frac{cx^2}{a}, -\frac{fx^2}{d}\right) + afq\,\text{App}\right.}\right.
$$

```
[In] Integrate[(A + B*x)*(a + c*x^2)^p*(d + f*x^2)^q,x]
```

```
[Out] (x*(a + c*x^2)^p*(d + f*x^2)^q*((B*x*AppellF1[1, -p, -q, 2, -((c*x^2)/a), -
((f*x^2)/d)])/((1 + (c*x^2)/a)^p*(1 + (f*x^2)/d)^q) + (6*a*A*d*AppellF1[1/2
, -p, -q, 3/2, -((c*x^2)/a), -((f*x^2)/d)])/(3*a*d*AppellF1[1/2, -p, -q, 3/
2, -((c*x^2)/a), -((f*x^2)/d)] + 2*x^2*(c*d*p*AppellF1[3/2, 1 - p, -q, 5/2,
 -((c*x^2)/a), -((f*x^2)/d)] + a*f*q*AppellF1[3/2, -p, 1 - q, 5/2, -((c*x^2
)/a), -((f*x^2)/d)]))))/2
```

Maple [F]

$$\int (Bx+A)\left(c\,x^{2}+a\right)^{p}\left(f\,x^{2}+d\right)^{q}dx$$

```
[In] int((B*x+A)*(c*x^2+a)^p*(f*x^2+d)^q,x)
[Out] int((B*x+A)*(c*x^2+a)^p*(f*x^2+d)^q,x)
```

Fricas [F]

$$\int (A+Bx)\left(a+cx^2\right)^p\left(d+fx^2\right)^q\,dx=\int (Bx+A)(cx^2+a)^p(fx^2+d)^q\,dx$$

```
[In] integrate((B*x+A)*(c*x^2+a)^p*(f*x^2+d)^q,x, algorithm="fricas")
[Out] integral((B*x + A)*(c*x^2 + a)^p*(f*x^2 + d)^q, x)
```

Sympy [F(-1)]

Timed out.

$$\int (A+Bx)\left(a+cx^2\right)^p\left(d+fx^2\right)^q\,dx=\text{Timed out}$$

```
[In] integrate((B*x+A)*(c*x**2+a)**p*(f*x**2+d)**q,x)
[Out] Timed out
```

Maxima [F]

$$\int (A+Bx)\left(a+cx^2\right)^p\left(d+fx^2\right)^q\,dx=\int (Bx+A)(cx^2+a)^p(fx^2+d)^q\,dx$$

```
[In] integrate((B*x+A)*(c*x^2+a)^p*(f*x^2+d)^q,x, algorithm="maxima")
[Out] integrate((B*x + A)*(c*x^2 + a)^p*(f*x^2 + d)^q, x)
```

Giac [F]

$$\int (A+Bx)\left(a+cx^2\right)^p\left(d+fx^2\right)^q\,dx=\int (Bx+A)\left(cx^2+a\right)^p\left(fx^2+d\right)^q\,dx$$

```
[In] integrate((B*x+A)*(c*x^2+a)^p*(f*x^2+d)^q,x, algorithm="giac")
[Out] integrate((B*x + A)*(c*x^2 + a)^p*(f*x^2 + d)^q, x)
```

Mupad [F(-1)]

Timed out.

$$\int (A+Bx)\left(a+cx^2\right)^p\left(d+fx^2\right)^q\,dx=\int \left(c\,x^2+a\right)^p\,\left(f\,x^2+d\right)^q\,\left(A+B\,x\right)\,dx$$

```
[In] int((a + c*x^2)^p*(d + f*x^2)^q*(A + B*x),x)
[Out] int((a + c*x^2)^p*(d + f*x^2)^q*(A + B*x), x)
```

3.400 $\int (a+cx^2)^p (A+Bx+Cx^2)(d+fx^2)^q\,dx$

Optimal result

Integrand size = 29, antiderivative size = 252

$$\int (a+cx^2)^p \left(A+Bx+Cx^2\right)\left(d+fx^2\right)^q\,dx = Ax(a+cx^2)^p\left(1+\frac{cx^2}{a}\right)^{-p}(d+fx^2)^q\left(1+\frac{fx^2}{d}\right)^{-q}\text{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{cx^2}{a},-\frac{fx^2}{d}\right)+\frac{1}{3}Cx^3(a+cx^2)^p\left(1+\frac{cx^2}{a}\right)^{-p}(d+fx^2)^q\left(1+\frac{fx^2}{d}\right)^{-q}\text{AppellF1}\left(\frac{3}{2},-p,-q,\frac{5}{2},-\frac{cx^2}{a},-\frac{fx^2}{d}\right)+\frac{B(a+cx^2)^{1+p}(d+fx^2)^q\left(\frac{c(d+fx^2)}{cd-af}\right)^{-q}\text{Hypergeometric2F1}\left(1+p,-q,2+p,-\frac{f(a+cx^2)}{cd-af}\right)}{2c(1+p)}$$

```
[Out] A*x*(c*x^2+a)^p*(f*x^2+d)^q*AppellF1(1/2,-p,-q,3/2,-c*x^2/a,-f*x^2/d)/((1+c
*x^2/a)^p)/((1+f*x^2/d)^q)+1/3*C*x^3*(c*x^2+a)^p*(f*x^2+d)^q*AppellF1(3/2,-
p,-q,5/2,-c*x^2/a,-f*x^2/d)/((1+c*x^2/a)^p)/((1+f*x^2/d)^q)+1/2*B*(c*x^2+a)
^(p+1)*(f*x^2+d)^q*hypergeom([-q, p+1],[2+p],-f*(c*x^2+a)/(-a*f+c*d))/c/(p+
1)/((c*(f*x^2+d)/(-a*f+c*d))^q)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used

$= \{6874, 441, 440, 455, 72, 71, 525, 524\}$

$$\int \left(a+cx^2\right)^p \left(A+Bx+Cx^2\right) \left(d+fx^2\right)^q \, dx = Ax\left(a+cx^2\right)^p \left(\frac{cx^2}{a}+1\right)^{-p} \left(d+fx^2\right)^q \left(\frac{fx^2}{d}+1\right)^{-q} \text{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{cx^2}{a},-\frac{fx^2}{d}\right) + \frac{1}{3}Cx^3\left(a+cx^2\right)^p \left(\frac{cx^2}{a}+1\right)^{-p} \left(d+fx^2\right)^q \left(\frac{fx^2}{d}+1\right)^{-q} \text{AppellF1}\left(\frac{3}{2},-p,-q,\frac{5}{2},-\frac{cx^2}{a},-\frac{fx^2}{d}\right) + \frac{B\left(a+cx^2\right)^{p+1} \left(d+fx^2\right)^q \left(\frac{c(d+fx^2)}{cd-af}\right)^{-q} \text{Hypergeometric2F1}\left(p+1,-q,p+2,-\frac{f(cx^2+a)}{cd-af}\right)}{2c(p+1)}$$

```
[In] Int[(a + c*x^2)^p*(A + B*x + C*x^2)*(d + f*x^2)^q,x]
```

```
[Out] (A*x*(a + c*x^2)^p*(d + f*x^2)^q*AppellF1[1/2, -p, -q, 3/2, -((c*x^2)/a), -
((f*x^2)/d)])/((1 + (c*x^2)/a)^p*(1 + (f*x^2)/d)^q) + (C*x^3*(a + c*x^2)^p*
(d + f*x^2)^q*AppellF1[3/2, -p, -q, 5/2, -((c*x^2)/a), -((f*x^2)/d)])/(3*(1
 + (c*x^2)/a)^p*(1 + (f*x^2)/d)^q) + (B*(a + c*x^2)^(1 + p)*(d + f*x^2)^q*H
ypergeometric2F1[1 + p, -q, 2 + p, -((f*(a + c*x^2))/(c*d - a*f))])/(2*c*(1
 + p)*((c*(d + f*x^2))/(c*d - a*f))^q)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = \int \left(A\left(a+cx^2\right)^p \left(d+fx^2\right)^q + Bx\left(a+cx^2\right)^p \left(d+fx^2\right)^q + Cx^2\left(a+cx^2\right)^p \left(d+fx^2\right)^q\right) dx$$

$$= A\int \left(a+cx^2\right)^p \left(d+fx^2\right)^q dx + B\int x\left(a+cx^2\right)^p \left(d+fx^2\right)^q dx + C\int x^2\left(a+cx^2\right)^p \left(d+fx^2\right)^q dx$$

$$
\begin{aligned}
&= \frac{1}{2}B\text{Subst}\left(\int (a+cx)^p(d+fx)^q\,dx, x, x^2\right) \\
&\quad + \left(A\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\right)\int\left(1+\frac{cx^2}{a}\right)^p\left(d+fx^2\right)^q\,dx \\
&\quad + \left(C\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\right)\int x^2\left(1+\frac{cx^2}{a}\right)^p\left(d+fx^2\right)^q\,dx
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{1}{2}\left(B\left(d+fx^2\right)^q\left(\frac{c\left(d+fx^2\right)}{cd-af}\right)^{-q}\right)\text{Subst}\left(\int (a+cx)^p\left(\frac{cd}{cd-af}+\frac{cfx}{cd-af}\right)^q\,dx, x, x^2\right) \\
&\quad + \left(A\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\left(d+fx^2\right)^q\left(1+\frac{fx^2}{d}\right)^{-q}\right)\int\left(1+\frac{cx^2}{a}\right)^p\left(1+\frac{fx^2}{d}\right)^q\,dx \\
&\quad + \left(C\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\left(d+fx^2\right)^q\left(1+\frac{fx^2}{d}\right)^{-q}\right)\int x^2\left(1+\frac{cx^2}{a}\right)^p\left(1+\frac{fx^2}{d}\right)^q\,dx
\end{aligned}
$$

$$
\begin{aligned}
&= Ax\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\left(d+fx^2\right)^q\left(1+\frac{fx^2}{d}\right)^{-q}F_1\left(\frac{1}{2};-p,-q;\frac{3}{2};-\frac{cx^2}{a},-\frac{fx^2}{d}\right) \\
&\quad + \frac{1}{3}Cx^3\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\left(d+fx^2\right)^q\left(1+\frac{fx^2}{d}\right)^{-q}F_1\left(\frac{3}{2};-p,-q;\frac{5}{2};-\frac{cx^2}{a},-\frac{fx^2}{d}\right) \\
&\quad + \frac{B\left(a+cx^2\right)^{1+p}\left(d+fx^2\right)^q\left(\frac{c\left(d+fx^2\right)}{cd-af}\right)^{-q}{}_2F_1\left(1+p,-q;2+p;-\frac{f\left(a+cx^2\right)}{cd-af}\right)}{2c(1+p)}
\end{aligned}
$$

Mathematica [A] (warning: unable to verify)

Time = 0.40 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.20

$$
\begin{aligned}
&\int\left(a+cx^2\right)^p\left(A+Bx+Cx^2\right)\left(d+fx^2\right)^q\,dx = \frac{1}{6}x\left(a+cx^2\right)^p\left(d+fx^2\right)^q\Biggl(3Bx\left(1+\frac{cx^2}{a}\right)^{-p}\left(1+\frac{fx^2}{d}\right)^{-q}\text{AppellF1}\left(1,-p,-q,2,-\frac{cx^2}{a},-\frac{fx^2}{d}\right) \\
&\quad + \frac{18aAd\,\text{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{cx^2}{a},-\frac{fx^2}{d}\right)}{3ad\,\text{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{cx^2}{a},-\frac{fx^2}{d}\right)+2x^2\Bigl(cdp\,\text{AppellF1}\left(\frac{3}{2},1-p,-q,\frac{5}{2},-\frac{cx^2}{a},-\frac{fx^2}{d}\right)+afq\,\text{Appe}} \\
&\quad + 2Cx^2\left(1+\frac{cx^2}{a}\right)^{-p}\left(1+\frac{fx^2}{d}\right)^{-q}\text{AppellF1}\left(\frac{3}{2},-p,-q,\frac{5}{2},-\frac{cx^2}{a},-\frac{fx^2}{d}\right)\Biggr)
\end{aligned}
$$

```
[In] Integrate[(a + c*x^2)^p*(A + B*x + C*x^2)*(d + f*x^2)^q,x]
```

```
[Out] (x*(a + c*x^2)^p*(d + f*x^2)^q*((3*B*x*AppellF1[1, -p, -q, 2, -((c*x^2)/a),
 -((f*x^2)/d)])/((1 + (c*x^2)/a)^p*(1 + (f*x^2)/d)^q) + (18*a*A*d*AppellF1[
1/2, -p, -q, 3/2, -((c*x^2)/a), -((f*x^2)/d)])/(3*a*d*AppellF1[1/2, -p, -q,
 3/2, -((c*x^2)/a), -((f*x^2)/d)] + 2*x^2*(c*d*p*AppellF1[3/2, 1 - p, -q, 5
/2, -((c*x^2)/a), -((f*x^2)/d)] + a*f*q*AppellF1[3/2, -p, 1 - q, 5/2, -((c*
x^2)/a), -((f*x^2)/d)])) + (2*C*x^2*AppellF1[3/2, -p, -q, 5/2, -((c*x^2)/a)
, -((f*x^2)/d)])/((1 + (c*x^2)/a)^p*(1 + (f*x^2)/d)^q)))/6
```

Maple [F]

$$\int \left(c\,x^{2}+a\right)^{p} \left(C\,x^{2}+Bx+A\right) \left(f\,x^{2}+d\right)^{q}dx$$

```
[In] int((c*x^2+a)^p*(C*x^2+B*x+A)*(f*x^2+d)^q,x)
```

```
[Out] int((c*x^2+a)^p*(C*x^2+B*x+A)*(f*x^2+d)^q,x)
```

Fricas [F]

$$\int \left(a+cx^2\right)^p \left(A+Bx+Cx^2\right) \left(d+fx^2\right)^q \,dx=\int (Cx^{2}+Bx+A)(cx^{2}+a)^{p}(fx^{2}+d)^{q}\,dx$$

```
[In] integrate((c*x^2+a)^p*(C*x^2+B*x+A)*(f*x^2+d)^q,x, algorithm="fricas")
```

```
[Out] integral((C*x^2 + B*x + A)*(c*x^2 + a)^p*(f*x^2 + d)^q, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+cx^2\right)^p \left(A+Bx+Cx^2\right) \left(d+fx^2\right)^q \,dx=\text{Timed out}$$

```
[In] integrate((c*x**2+a)**p*(C*x**2+B*x+A)*(f*x**2+d)**q,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \left(a+cx^2\right)^p \left(A+Bx+Cx^2\right) \left(d+fx^2\right)^q \, dx = \int \left(Cx^2+Bx+A\right)\left(cx^2+a\right)^p\left(fx^2+d\right)^q \, dx$$

```
[In] integrate((c*x^2+a)^p*(C*x^2+B*x+A)*(f*x^2+d)^q,x, algorithm="maxima")
[Out] integrate((C*x^2 + B*x + A)*(c*x^2 + a)^p*(f*x^2 + d)^q, x)
```

Giac [F]

$$\int \left(a+cx^2\right)^p \left(A+Bx+Cx^2\right) \left(d+fx^2\right)^q \, dx = \int \left(Cx^2+Bx+A\right)\left(cx^2+a\right)^p\left(fx^2+d\right)^q \, dx$$

```
[In] integrate((c*x^2+a)^p*(C*x^2+B*x+A)*(f*x^2+d)^q,x, algorithm="giac")
[Out] integrate((C*x^2 + B*x + A)*(c*x^2 + a)^p*(f*x^2 + d)^q, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+cx^2\right)^p \left(A+Bx+Cx^2\right) \left(d+fx^2\right)^q \, dx = \int \left(c\,x^2+a\right)^p \left(f\,x^2+d\right)^q \left(C\,x^2+B\,x+A\right) \, dx$$

```
[In] int((a + c*x^2)^p*(d + f*x^2)^q*(A + B*x + C*x^2),x)
[Out] int((a + c*x^2)^p*(d + f*x^2)^q*(A + B*x + C*x^2), x)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count i
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
```

```
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well")
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_count
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1]))  #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != −1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2∗leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larg
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2∗leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#               Albert Rich to use with Sagemath. This is used to
#               grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#               'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#               issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

    #debug=False
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-t
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
```

```
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):



    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)



    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_c
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_result


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```